Noisy-Signalling Models of Organizational Decision Making

by

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Abstract

This thesis consists of three separate papers concerning the use of communication channels and intermediaries in organizations. A noisy-signalling model of strategic communication is introduced in the first chapter, and expanded upon in the remainder of the thesis.

In the second part of the first chapter, I use the core noisy-signalling model to study organizational design of a single channel of communication. The results of the analysis provide a rational for the variation in communication processes observed across organizations, as well as costly political lobbying and advertising campaigns.

In the second chapter, I extend the core model to allow the informed party to choose among multiple communication channels when conversing with the decision maker. The model suggests that polarization across communication channels may be an efficient response to "bandwidth" concerns facing decision-makers of large corporations or unqualified management. Conversely, coexistence of partisan and non-partisan channels within an organization or community (e.g. tabloids and professional news sources in the journalism industry) may also be socially efficient for other environments.

In the third chapter, I consider a different extension of the core model by allowing the two parties to communicate via a strategic intermediary. I use the model to provide a possible explanation for the variety of roles communication intermediaries play in different organizations, the correlation between control-rights and communication hierarchies in organizations, as well usage of third-party, conflict-resolution arrangements.

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Chapter 1

Designing Communication Channels for Efficient Information Transmission

Abstract

Organizations use a variety of communication channels to manage information transmission (e.g. E-mail, face-to-face meetings, video chats, etc.). This paper studies a noisy-signalling model in which informed parties can use communication channels to either transmit useful information to decision makers, or engage in influence activities to "jam" otherwise informative signals. I find that the simultaneous presence of cost and noise in communication generates a potential two-sided inefficiency, resulting either too much unproductive signal jamming when biases are large or too little productive information transmission when informed parties are unmotivated. If communication channels affect the cost and effect of influence on observed signals, organizations may wish to tailor communication channels to manage this two-sided inefficiency. I formally set up this "channel design problem" and characterize optimally-designed communication channels for a range of organizational environments. Finally, I apply the results to provide a rational for the variation in communication practices observed across organizations, as well as costly political lobbying and advertising campaigns.

1.1. Introduction

Organizations use a variety of communication channels to manage information transmission. These channels range from written and/or non-interactive (e.g. office memos, E-mails, text messages), to visual and/or interactive (e.g. face-to-face meetings, video chats, seminars). Channels can also be distinguished by formality requirements (e.g. formal board-room presentations vs. impromptu discussions in an "open-door" office), capacity for visuals (e.g. use room with projector vs. room without), and travel requirements (e.g. phone call vs. drive to corporate head quarters to meet in person).

The sheer variety of communication channels observed in organizations suggests that information transmission may not as simple as merely sending a message and having it received. Instead, communication may be a complicated process requiring both *effort* on the part of experts to explain information clearly and accurately, as well as *expertise* on the part of decision makers to correctly process complicated information, and react appropriately. In other words, communication is often both *costly* and *noisy*. The table below shows four commonplace communication channels and how they may vary along these two dimensions:

Cost of Communication \ Noise in Communication	Low	High
Low	Open-door offices	E-mails/memos
High	Formal boardroom meetings/seminars	Restricted floors

Table 1: Four Example Channels

This paper presents a model of strategic communication in a setting where communication between an informed party and a decision maker is costly and noisy. Formally, an informed agent must exert effort to influence a noisy signal observed by a principal (decision maker). Communication between the two parties occurs via a communication channel, which I model as determining the cost and marginal effect of the agent's communication effort on the principal's signal. The paper focuses on attacking two questions: 1) How do aspects of the communication channel and organizational environment (i.e. parameters of the noisy-signalling game) affect how information is transmitted?; and 2) How should an organization optimally design a (single) communication channel given the environment and objectives its members face?

The model highlights several predictions regarding how communication channels affect information transmission in organizations. Specifically, the noisy-signalling nature of communication in this model generates a dual tension: The expert's communication incentives are directly affected by the principal's anticipated response to noisy signals, while the principal's incentives to respond to noisy signals are directly affected by the agent's expected communication effort. In other words, communication effort for each agent type and responsiveness to influenced signals are strategic complements.

The dual tension described above generates a potential two-sided inefficiency. When the agent lacks motivation to communicate useful information, the principal will lack incentive to respond to observed signals, which in turn further diminishes the agent's incentives to exert effort in transmitting information. On the other hand, if the agent has strong incentives to "jam" the principal's signal to avoid generation of certain informative signal realizations (e.g. by misrepresentation of facts, embellishment of irrelevant details, distraction, etc.), the principal may anticipate this and adjust her decision making accordingly, potentially amplifying the incentives for the agent to engage in additional unproductive signal jamming.

This potential two-sided inefficiency necessitates tailoring communication channels to specific organizational environments and objectives. I find that for any implemented communication channel, the ex-ante expected payoff for both parties can be expressed as the sum of two effects: the expertise effect and the influence effect. The former effect captures the principal's ability to process information without the agent's added influence, and is always increasing in communication effort and responsiveness.

On the other hand, the influence effect captures the added welfare effect of the agent's ability to influence the principal's signal *after* observing private information. This effect is increasing in the principal's equilibrium responsiveness when the agent has a large stake in the decision, but on average has similar preferences to the principal regarding the extensive margin of the decision. However, when the agent has a large ex-ante bias for a particular extensive margin, the influence effect may be negative as communication will primarily be used for unproductive signal jamming (as in Gibbons, 2005, and Powell, 2015). Consequently, a low cost/high effect communication channel is a *double-edged sword*. It may be beneficial for some organizations to adopt, and detrimental for others, depending on experts' stake in decision making as well as existing expected biases.

The results of the model provide a rational for various organizational observations regarding internal communication practices. The model speaks to the vast variation observed in communication practices observed across organizations. While some organizations implement communication facilitating practices such as open-door policies or company-wide social events and mixers, others adopt much stricter practices, e.g. strict communication protocol in the military. The results of the model indeed suggests that different organizations may wish to implement vastly different communication processes based on internal characteristics. The model also provides a rational for permitting costly partisan lobbying or advertising, as failed attempts at persuasion in such environments can also provide decision makers with useful information via the expertise effect.

The current paper relates to several strands of literature. The broad field of information economics has introduced several models of strategic communication including cheap talk (e.g. Crawford and Sobel, 1982), signalling (e.g. Spence 1973), and verifiable information (e.g. Grossman, 1981; Milgrom, 1981). This paper uses a noisy-signalling model, first introduced in Matthews and Mirman (1983), to model strategic communication. While noisy-signalling models are not as commonly used as other models of strategic communication, adoption of this model is crucial for capturing the desired environment of costly and noisy communication, and has important implications regarding usage and design of communication channels in organizations.

Classic papers in the strategic communication literature tend to focus on analyzing equilibrium behavior during communication, rather than the design of communication processes themselves. However, the newer field of information design (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2016) studies the design of information transmission rules under sender commitment. In contrast, this paper focuses on a setting where neither the information sender nor the receiver can pre-commit to a particular strategy. It is therefore more closely related to Bernheim (1991), who studies how corporate tax policies affect firms' incentives to signal quality (noiselessly) via issuance of dividends.

The paper also contributes to various strands of literature in organizational economics. Closely related is the literature on influence activities in organizations. Key papers in this literature include Milgrom and Roberts (1988), Gibbons (2005), and Powell (2015). These papers focus on models of uninformed influence and, in the latter two, influence activities are purely a deadweight loss for the organization. In the setting I study, influence activities may be used by some agent-types for productive information transmission, rather than unproductive signal jamming. Thus, I find that facilitating such influence activities may be desirable for certain types of organizations.

Also related in the organizational economics literature is literature on authority and communication in organizations. Alonso, Dessein and Matouschek (2008); Rantakari (2008), and Friebel and Raith (2010) study settings in which delegation of control rights imply an endogenous communication structure within the organization. In the current paper, control rights are always held fixed with the decision maker. Instead I focus on the design of the noisy-signalling game played between the two parties.[1]

The remainder of the paper is structured as follows: Section 2 sets up the formal model. Section 3 describes equilibrium behavior for a fixed channel of communication and comparative static effects. Section 4 discusses welfare for a fixed channel of communication and studies the channel design problem. Section 5 provides a review of further related literature. Section 6 concludes.

1.2. Players and Environment

In this section, I set up the formal environment I will study in the remainder of the paper and discuss various assumptions of the model.

1.2.1 Model Set Up

The core model consists of a noisy-signalling game between two parties: a principal (P) and an agent (A). The principal is a decision maker who is responsible for making a decision $d \in \mathbb{R}$. The agent is an expert who privately observes an underlying state $\theta \in \Theta$, which determines the marginal payoff of the decision d for both parties. The parties payoffs are given by

$$\begin{aligned} u_P(\theta, d) &= \theta d - \frac{1}{2} d^2 \\ u_A(\theta, e, d) &= \alpha(\theta) d - \frac{\kappa}{2} e^2 \end{aligned}$$

where I assume $E_\theta\{\theta\} = 0$, $\alpha(.)$ is strictly increasing, and $E_\theta\{\alpha(\theta)\} = E_\alpha > 0$. The assumption that $\alpha(.)$ is increasing restricts us to an interesting setting where A can play a positive welfare role. However, note that because $E_\theta\{\theta\} = 0$ while $E_\theta\{\alpha(\theta)\} > 0$, A has an ex-ante bias for positive decisions ($d > 0$) relative to P.

The second term in the agent's payoff function ($-\frac{\kappa}{2}e^2$) will be discussed shortly. For now, note that P's preferred state-contingent decision is to set $d = \theta$, while A is a "directional empire builder" who always prefers either $d = -\infty$ or $d = \infty$, depending on the sign of $\alpha(\theta)$. As discussed above, the agent may also disagree with the principal on the preferred sign of d as well (i.e. disagree on the extensive margin as well as intensive margin). The size of the parameter E_α captures the degree of this extensive margin disagreement and will be an important parameter when studying the channel design problem. Another important parameter will be $\sigma_{\alpha,\theta} = cov\{\alpha(\theta), \theta\}$, which captures the agent's stake in decision making. Specifically, when the slope of $\alpha(\theta)$ is large, the agent has strong incentives to influence decision making by transmitting useful information. Thus, the agent has strong incentives to influence decision making when either E_α or $\sigma_{\alpha,\theta}$ is large. However, influence activities driven by the former will primarily be socially unproductive (signal jamming) while influence activities driven by the latter will be socially productive (information transmission).

Influence activities in this model take the form of *communication effort* for the agent. Recall that only A observes θ. We allow A to communicate with P via a *communication*

[1] Communication is assumed to be cheap talk in the above three papers.

channel. The communication channel is characterized by $c = (\kappa, \gamma)$, and determines a noisy-signalling game played between P and A. Later, I will study the channel design problem of picking a communication channel that maximizes organizational welfare. For now I will hold the channel $c = (\kappa, \gamma)$ fixed.

In the noisy-signalling game generated by channel c, A can exert communication effort e at cost $\frac{\kappa}{2}e^2$ to influence the distribution of a noisy signal observed by P. We will denote this signal as $s \in \{L, R\}$ and assume that

$$\Pr\{s = R|\theta, e\} = \gamma e + g(\theta)$$

Thus, the communication channel determines the marginal cost of communication effort (κ), as well as its marginal effect on $P's$ noisy signal (γ). In fact, due to the linear-quadratic nature of this game, it is without loss of generality to replace the communication channel $c = (\kappa, \gamma)$ with an alternative channel given by $c' = (k, 1)$ where $k = \frac{\kappa}{\gamma^2}$. We will adopt this normalization for the remainder of the paper, and assume the communication channel is characterized by only the parameter k. For expositional purposes, I will refer to k as the "marginal cost of influence". However, it should also be implicitly understood that this parameter should be interpreted as additionally encoding information about the marginal effect of influence when using communication channel c. I provide a further discussion about interpreting the model's signalling technology at the end of this section. For now, additionally note that the function $g(.)$ captures P's exogenous expertise. Specifically, $g(.)$ encodes P's ability to infer the underlying state without communication effort from A. We will assume that $g(.)$ is strictly increasing. This assumption is without loss of generality if $g(.)$ is also restricted to be monotonic. Note that P is more of an exogenous expert herself when $cov_\theta\{g(\theta), \theta\} = \sigma_{g,\theta}$ is large.

To summarize, the timing of the game is as follows[2]:

1. Nature chooses $\theta \in \Theta \subseteq \mathbb{R}$, observed by only A
2. A chooses communication effort $e \in \mathbb{R}$ at cost $\frac{k}{2}e^2$
3. P observes $s \in \{L, R\}$ where
$$\Pr\{s = R|\theta, e\} = e + g(\theta)$$
4. P makes decision $d \in \mathbb{R}$
5. Payoffs realized

Throughout, it is convenient to assume that Θ is discrete and finite and that the following symmetry condition holds:

$$E_\theta\{g(\theta)\} = \frac{1}{2}$$

Suppose $\theta > 0$. Given the above symmetry assumption, it is intuitive to say that the type-θ agent is "transmitting information" if he chooses $e > 0$ and "signal jamming" if he chooses $e < 0$. Similarly, if $\theta < 0$, we will say that say that the type-θ agent is transmitting information if he chooses $e < 0$ and signal jamming if he chooses $e > 0$. We

[2]Note that the timing is written using the normalized channel characterized by $c' = (k, 1)$ where $\frac{\kappa}{\gamma^2}$.

will also denote $f(\theta_i)$ to be the prior probability that $\theta = \theta_i$ is realized, which is assumed to be common knowledge to both parties.

Finally, let $\underline{\theta} = \min_{\theta \in \Theta}\{\theta\}$ and $\overline{\theta} = \max_{\theta \in \Theta}\{\theta\}$. We will assume that k is large enough so that

$$\frac{\alpha(\theta)}{k}(\overline{\theta} - \underline{\theta}) + g(\theta) \in [0,1] \tag{1}$$
and
$$\frac{\alpha(\theta)}{k}(\underline{\theta} - \overline{\theta}) + g(\theta) \in [0,1]$$

For all $\theta \in \Theta$. Condition (1) is a no-shifting-support condition that implies it is never a rationalizable strategy for any agent type to choose a level of communication effort that degenerates the principal's signal.[3] Thus, this model is intended to capture situations where it is too costly to completely eliminate noise in communication.

1.2.2. Discussion of Formal Setting

Several features of the model merit further discussion. Recall that a key assumption of this model is that communication is both costly and noisy. While this is non-standard in much of the strategic communication literature, I argue that noisy-signalling is a fundamental part of various communication interactions in organizations. An academic seminar is an extreme example of such an interaction, as attendees do not observe the speaker's preparation for the seminar (which may be considerable), but only the seminar itself. Analogous situations arise in firms and other organizations: Time spent preparing for meetings, writing memos, making slide shows, etc. are typically not observed directly, but are often expected and relied upon by decision makers. These can all be considered examples of communication that is both costly and noisy (i.e. noisy signalling).

It is also worth discussing how we might map real-world communication channels (e.g. E-mail, F2F meetings, video chats) to the formal channels described in the model. To reiterate, this model focuses on capturing two incentive-relevant characteristics of communication channels: the cost of influencing perceptions of decision makers and the effect costly efforts have on the distribution of these perceptions. Naturally, there is an equivalence between these two characteristics as communication effort does not have a clear scale of measurement. Thus, a communication channel in the model, characterized by a particular k, can actually map to several different kinds of communication channels in reality. These channels may seem physically different, but the model suggests that they may in fact equivalent in terms of implied equilibrium behavior and welfare. For example, in Table 1 in the introduction, the upward diagonal can be thought of as two different channels, each with a moderate level k. Thus, the model suggests that, even though these two channels may look physically different to an observer, from a design perspective, they both achieve similar outcomes. With that said, in Palida 2020a, I show that observable differences between channels (that may otherwise have the same effective marginal cost of influence) can be leveraged to attain additional information transmission (see section 5).

The preference structure used in this model is also somewhat non-standard with respect to the strategic communication literature. While empire-building preferences are common in the internal capital markets literature (see section 5), applied models of

[3] i.e. choose e so that $\Pr\{s = R|\theta, e\} \in \{0,1\}$

strategic communication often use the "quadratic-bias" preference specification, where both the sender and receiver wish to match decision making to the underlying state, plus a player-contingent bias. This model adopts empire-building preferences partially for tractability and partially to capture realistic organizational settings in which experts often only care about the extensive margin of a decision. For example, a researcher may prefer a particular project to work on, but will always prefer as much funding as possible for his desired project. Similar incentives can be observed with branch managers of large firms, local politicians, leaders of specific task forces, etc. It is worth mentioning that both the current model's specification as well as the quadratic-bias specification are both special cases of a more general specification where both parties preferences are quadratic in the decision.[4]

Finally, the model assumes a binary-signal specification for the principal's signal. While this may seem like a restrictive assumption, it can actually be shown that all results are qualitatively identical in a model with a richer signal-space.[5]

1.3. Equilibrium for a Fixed Communication Channel

In this section I present positive results regarding behavior in equilibrium, for a fixed channel of communication with marginal cost of influence given by k. In this model, I will focus on pure strategies for both parties. This is without loss of generality as all perfect Bayesian equilibria (PBE) of the game must be in pure-strategies. In fact, as we will see, the model admits a unique (pure-strategy) PBE as long as k is large enough to satisfy (1). A pure strategy for P is a signal-contingent investment rule $d(s)$ for $s \in \{L, R\}$. A pure strategy for A is a communication-effort profile $e(\theta)$, defined on $\theta \in \Theta$.

Suppose P believes A is playing communication-effort profile $e^*(\theta)$. Her best response requires her to choose $d^*(s)$ to solve

$$\max_d E_\theta\{\theta|s; e^*(.)\}d - \frac{1}{2}d^2 \tag{2}$$

which implies that we must have

$$d^*(s) = E_\theta\{\theta|s; e^*(.)\} \tag{3}$$

in any PBE.

[4] The current model can be extended to study this more general environment. In the next section we will see that all strategies and payoffs can be expressed in terms of a single equilibrium quantity representing the difference in decision making across the two signal realizations. If we allow the agent to have a quadratic cost for the decision as well, equilibrium will be characterized by two quantities: the equilibrium decision spread, plus a quantity that captures the total cost of decision making.

[5] In particular, one can study a model in which $s \in \{s_1, s_2..., s_K\}$ and

$$\Pr\{s = s_k|\theta, e) = \gamma(s_k)e + g(\theta)$$

where the function $\gamma(.)$ enters exogenously as an "influence-distortion function" that satisfies $\sum_{k-1}^{K} .\gamma(s_k) = 0$. This alternative model yields identical expressions to the ones derived in this paper, with a slight reinterpretation of the key equilibrium parameter (see section 3).

Turning to the agent, if A believes that P is playing according to the strategy $d^*(s)$, he must choose $e^*(\theta)$ to solve

$$\max_e \alpha(\theta)[d^*(L) + (e + g(\theta))\,\Delta^*] - \frac{k}{2}e^2 \tag{4}$$

where

$$\Delta^* = d^*(R) - d^*(L) \tag{5}$$

is the equilibrium decision spread. The first-order condition of (4), combined with the no-shifting-support condition (1), implies that the following is a necessary condition in any PBE:

$$e^*(\theta) = \frac{\alpha(\theta)}{k}\Delta^* \tag{6}$$

In other words, all agent-types play a level of communication effort that is the ratio $\frac{\alpha(\theta)}{k}$, multiplied by the equilibrium decision spread Δ^*.

Given (6) we can now compute expressions for $d^*(L)$ and $d^*(R)$ in terms of Δ^* and exogenous model parameters. Then we can characterize all PBE by finding solutions to the single-variable equation (5). In particular, we have that

$$E_\theta\{\theta|s; e^*(.)\} = \sum_{\theta\in\Theta} \theta f^*(\theta; s)$$

where

$$f^*(\theta; s) = \frac{f(\theta)\Pr\{s|\theta; e^*(.)\}}{\sum_{\theta'\in\Theta} f(\theta')\Pr\{s|\theta'; e^*(.)\}}$$

This gives us that[6]

$$d^*(L) = \frac{-(\sigma_{g,\theta} + \frac{\sigma_{\alpha,\theta}}{k})\Delta^*}{\frac{1}{2} - \frac{E_\alpha}{k}\Delta^*} \tag{7}$$

and

$$d^*(R) = \frac{\sigma_{g,\theta} + \frac{\sigma_{\alpha,\theta}}{k}\Delta^*}{\frac{1}{2} + \frac{E_\alpha}{k}\Delta^*} \tag{8}$$

Using the two expressions above, our key equilibrium equation is then given by

$$\Delta^* = \frac{\sigma_{g,\theta} + \frac{\sigma_{\alpha,\theta}}{k}\Delta^*}{\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*\right)^2} \tag{9}$$

Thus, finding the PBE of this model reduces to finding the roots of the cubic polynomial implied by (9).

It is clear that in any PBE we must have that $d^*(R) - d^*(L) = \Delta^*$ solve (9). However, there will typically be three solutions to this equation. The proposition below establishes that only one of them corresponds to a legitimate PBE:

Proposition 1 *Under condition (1) there is a unique PBE in which*

1. *$e^*(\theta)$ is given by (6)*
2. *$d^*(L)$ and $d^*(R)$ are given by (7) and (8) respectively*

[6] Recall $E\alpha = E_\theta\{\alpha(\theta)\}, \sigma_{\alpha,\theta} = cov_\theta\{\alpha(\theta), \theta\}$, and $\sigma_{g,\theta} = cov_\theta\{g(\theta), \theta\}$.

3. Δ^* is given by the median of the solutions to (9)

Proof. See appendix. ■

In general, there will be three solutions to (9). One will imply that $\Delta^* < 0$ while the other two will imply that $\Delta^* > 0$. The proposition argues that only the smallest positive solution corresponds to a legitimate PBE. The other two solutions correspond to settings in which probability bounds are being violated, so that the formula implied by Bayes' rule becomes invalid.

One result of this model is that it admits a unique PBE. This is typically not the case for standard signalling models, which usually exhibit equilibrium-selection problems. Our equilibrium uniqueness is driven by modeling of communication as both costly and noisy (i.e. noisy signalling as opposed to standard Spencian signalling). If P directly observed the quantity $e + g(\theta)$, we would have a variety of equilibria in this model, differing qualitatively as well as quantitatively, depending on how off-equilibrium path beliefs are specified.

Two general points should be made about noisy versus noiseless signalling: 1) many of the outcomes that would be predicted in a standard signalling model, are in fact not realistic if signals are typically observed with noise (e.g. translation of a complicated piece of information); and 2) In situations where noise in communication is expected, clever designing of communication channels can allow organization leaders a significant degree of control over how information is transmitted within the organization. The second point provides motivation for studying the channel design problem in the next section of this paper.[7]

Before moving to welfare and channel design, I will state the following corollary to Proposition 1:

Corollary 1 *In the unique PBE of the game, we have that $\Delta^* > 0$. Furthermore, we have that Δ^* is strictly decreasing in k, and strictly increasing in $E_\alpha, \sigma_{\alpha,\theta}$, and $\sigma_{g,\theta}$.*

Proof. Implied by the fact that the RHS of (9) is increasing in Δ^*. ■

The equilibrium decision spread Δ^* is a sufficient statistic for determining all parties strategies, and captures both the A's average marginal incentives for a unit of communication effort as well as $P's$ responsiveness to observed signals in equilibrium. The corollary states that Δ^* (and thus average communication effort and responsiveness) is decreasing in the cost of communication effort k, and increasing in the agent's ex-ante bias E_α, the covariance between P and A's marginal decision valuations $\sigma_{\alpha,\theta}$, and P's exogenous expertise $\sigma_{g,\theta}$.

1.3.1. Discussion: Dual Tension

As noted above, the equilibrium decision spread Δ^* captures P's responsiveness to signals in equilibrium, as well as determines A's incentives to influence decision making. Thus, this model exhibits a dual tension, in which the expert's communication effort incentives are directly affected by the principal's anticipated response to noisy signals, while the principal's incentives to respond to noisy signals are directly affected by the agent's expected communication effort. As we will see in the next section, this dual tension can

[7] In regards to the first point regarding equilibrium uniqueness, I direct the reader to Carlsson and Dasgupta (1995), who study noise-proof equilibria in signalling games.

create a potential two-sided inefficiency: welfare suboptimal behavior on the part of the agent will induce welfare suboptimal behavior on the part of the principal and vice versa.

To see these forces more clearly, suppose that the two parties could pre-commit to a decision spread Δ^{ctr}, where the superscript "ctr" stands for "contractual". Specifically, in this hypothetical situation, P is restricted to choose $d(R) - d(L) = \Delta^{ctr}$. If Δ^{ctr} is chosen to maximize the sum of the two parties' expected payoffs, we can consider this outcome as a constrained first best.

Returning to the non-contractual case, if P has incentives to increase $\Delta = d(R) - d(L)$ relative to Δ^{ctr}, A will anticipate this and all agent types will increase communication effort in absolute value. Anticipating this increase P will increase Δ by even more, and so forth, leading to an equilibrium with $\Delta^* >> \Delta^{ctr}$. A similar spiral will occur when P has incentives to decrease Δ relative to Δ^{ctr}, which will lead to an equilibrium with $\Delta^* << \Delta^{ctr}$.

The next section will focus on why communication effort and responsiveness may either be too high or too low, and how an organization may wish to design a communication channel (in this model, choose k) to manage potential two-sided inefficiencies.

1.4. Welfare and Channel Design

In this section, we will study the welfare implications of the communication game described in the prior two sections. We will first study how welfare of the two parties depends on characteristics of the communication channel, as well as exogenous parameters of the model. I will then introduce and motivate the channel design problem, and provide an analysis of this problem.

1.4.1. Welfare for a Fixed Communication Channel

Recall from the prior section that any communication channel (characterized by its marginal cost of influence k) admits a unique PBE under (1). This implies that, in situations where communication is inherently noisy, information transmission and welfare can be highly sensitive to the communication channel selected by the organization. Here we will examine how welfare of the two parties depends on the cost of influence along a particular channel, and how these effects also depend on exogenous characteristics of the organization.

In this model, all equilibrium strategies are characterized by the equilibrium quantity Δ^* solving (9). Thus, we can derive expressions for both parties welfare in terms of only Δ^* and exogenous parameters. In particular, the ex-ante expected payoffs for the two parties are given by

$$W_P^* = E_{\theta,s}\{\theta d^*(s) - \frac{1}{2}d^*(s)^2 | e^*(.), d^*(.)\} \quad (10)$$

$$W_A^* = E_{\theta,s}\{\alpha(\theta) d^*(s) - \frac{1}{2}e^*(\theta)^2 | e^*(.), d^*(.)\} \quad (11)$$

The proposition below provides analytic expressions for the two quantities above:

Proposition 2 *In the unique PBE, we have that P's ex-ante expected payoff is given by*

$$W_P^* = \underbrace{\frac{(\sigma_{g,\theta})^2}{2\left[\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*\right)^2\right]}}_{Expertise\ effect} + \left(\frac{1}{2k}\right)\underbrace{\left[\frac{\sigma_{g,\theta}\sigma_{\alpha,\theta}}{\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*\right)^2} + \sigma_{\alpha,\theta}\Delta^*\right]}_{Influence\ effect}\Delta^* \quad (12)$$

and A's ex-ante expected payoff is given by

$$W_A^* = \underbrace{\frac{\sigma_{g,\alpha}\sigma_{g,\theta}}{\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*\right)^2}}_{Expertise\ effect} + \left(\frac{1}{2k}\right)\underbrace{\left[\frac{2\sigma_{g,\alpha}\sigma_{\alpha,\theta}\Delta^*}{\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*\right)^2} + \left(\sigma_\alpha^2 - (E_\alpha)^2\right)\Delta^*\right]}_{Influence\ effect} \quad (13)$$

Proof. Implied by the strategies described in Proposition 1. ■

Importantly, note that both parties' welfare expressions consist of the sum of two terms. We will refer to the first term in (12) and (13) as the "expertise" effect and the second in both expressions as the "influence effect".

It can be shown that the expertise effect is essentially the welfare the two parties would attain if no influence occurred, but P instead observed a signal $s' \in \{L, R\}$ where

$$\Pr\{s' = R\} = g(\theta) + \frac{E_\alpha}{k}\Delta^*$$

and made her decision d solely off of the realization of s'. The expertise effect is essentially saying that knowledge of the agent's ex-ante incentives to signal jam, exogenously makes P's signal more informative (even if A never observes θ). Importantly, the expertise effect is always increasing in Δ^*, i.e. more communication effort/responsiveness always increases the expertise effect. The reason for this is as follows: If P knows that, on average, A will be trying to influence realization of $s = R$ (recall $E_\alpha > 0$ by assumption) then signal realizations of $s = L$ become extremely informative, incentivizing P to choose $d^*(L) << 0$ precisely in situations where it would be beneficial for both parties (recall $\sigma_{a,\theta} > 0$). Thus, while parties lose when $s = R$ is realized, they both gain relatively more when $s = L$ is realized.[8]

The second effect, the influence effect, captures both the additional welfare gain associated with the agent influencing s conditional on being informed of the underlying state θ, as well as the total expected cost of communication effort. For the principal, this effect is always positive. Thus, in this model, the principal always benefits from the agent's influence.

The same cannot be said for the agent, however. For the agent, the influence effect may either be increasing or decreasing in Δ^*. The key determinant of the sign for this derivative is the size of the agents ex-ante bias, E_α, relative to the correlation between the two parties' preferences, captured by the parameter $\sigma_{\alpha,\theta}$ (i.e. the expected slope of $\alpha(\theta)$). When the former dominates the latter, the agent has strong incentives to exert communication effort, but not to transmit useful information. Instead the agent primarily exerts communication effort to distort P's signal so that $s = R$ is realized more frequently than without influence. This implies that, when $s = R$ is indeed realized, P anticipates that this is largely due to uninformative signal-jamming and dampens her responsiveness to this signal realization. Thus, communication effort largely becomes a deadweight loss (see Gibbons, 2005, and Powell, 2015). On the other hand, when $\sigma_{\alpha,\theta}$ is large relative to

[8] The net welfare increase is because the function $f(x) = x(1-x)$ is maximized at $x = \frac{1}{2}$.

E_α, communication effort is primarily used for information transmission as opposed to unproductive signal jamming. So in this case, the agent's utility is strictly increasing in Δ^* as well.

1.4.2. The Channel Design Problem

The fact that incentivizing communication effort can have either positive or negative effects on the parties' welfare, depending on characteristics of the organization, suggests an active role for designing communication channels tailored specifically to the environment the organization operates in. Furthermore, the fact that only a limited number of equilibria exist when communication is noisy (in this model, exactly one equilibrium exists for any channel) suggests that organization designers can impose a significant degree of control over how communication occurs within the organization, by implementing customized communication channels. Here, I use the noisy-signalling model developed thus far to model this design process formally.

Recall that, in this model, a communication channel is characterized by the marginal cost of influence the agent must pay to influence the principal's signal, k. Designing a communication channel, in this setting, therefore reduces picking a value of k that optimizes a welfare criterion. Here, we will take the sum of the two parties ex-ante expected payoffs as the welfare criterion. A common foundation for this criterion is that the principal offers a take-it-or-leave-it wage contract to the agent, prior to the start of the game.

Let us denote

$$\begin{aligned} W_P^*(k) &= E_{\theta,s}\{\theta d^*(s) - \frac{1}{2}d^*(s)^2 | e^*(.), d^*(.), k\} \quad (14) \\ W_A^*(k) &= E_{\theta,s}\{\alpha(\theta) d^*(s) - \frac{1}{2}e^*(\theta)^2 | e^*(.), d^*(.), k\} \quad (15) \end{aligned}$$

Using Proposition 2, we can express the sum of the two parties' expected utilities as[9]

$$W^*(k) = \left(\frac{\sigma_{g,\theta}}{2} + \sigma_{g,\alpha}\right)\Delta^*(k) + \frac{(\sigma_{\alpha,\theta} + \sigma_\alpha^2 - (E_\alpha)^2)}{2k}(\Delta^*(k))^2 \quad (16)$$

where $\Delta^*(k)$ solves

$$\Delta^*(k) = \frac{\sigma_{g,\theta} + \frac{\sigma_{\alpha,\theta}}{k}\Delta^*(k)}{\frac{1}{4} - \left(\frac{E_\alpha}{k}\Delta^*(k)\right)^2} \quad (17)$$

We will assume that the organization is able to choose among a collection of communication channels characterized by the interval $[\underline{k}, \overline{k}] \subseteq \mathbb{R}$, where $\underline{k}$ is assumed to be large enough so that (1) is satisfied. The channel design problem can now be stated as:

$$\max_{k \in [\underline{k}, \overline{k}]} W^*(k) \quad (18)$$

We will denote k^o to be a solution to the above problem.

By looking at the second term in the expression for $W^*(k)$ in (16), we can see that $k^o = \underline{k}$ when

$$\sigma_{\alpha,\theta} \geq (E_\alpha)^2 - \sigma_\alpha^2 \quad (19)$$

[9]For studying the channel design problem, it is useful to combine the expertise effect of the two parties, with the first term of the influence effect, which gives expression (16).

Specifically, when the agent's preferences are strongly aligned with the principal's preferences ($\sigma_{\alpha,\theta}$) and his ex-ante bias ($(E_\alpha)^2$) is small, it is optimal for the organization to implement the lowest-cost communication channel and maximize both the agent's incentives to exert communication effort, as well as the principal's responsiveness to observed signals. In this environment, the agent is primarily using communication effort to transmit mutually beneficial information rather than signal jam. Thus, it is optimal to facilitate this process by lowering the cost of influence by as much as possible.

When condition (19) is not satisfied, the expertise effect and the influence effect move in opposite directions with respect to k. In this environment, the agent is suffering a large signal-jamming deadweight loss from having the ability to influence the principal's decision making. It may still be desirable to increase communication effort (via reducing k) however, as the expertise effect is still positive for both parties. Nevertheless, we can derive a condition under which it is optimal for the organization to minimize the amount of equilibrium communication effort and responsiveness.

In particular, we can compute the derivative of $W^*(k)$ as

$$\begin{aligned}\frac{dW^*(k)}{dk} &= \left[\frac{\sigma_{g,\theta}}{2} + \sigma_{g,\alpha} + \frac{(\sigma_{\alpha,\theta} + \sigma_\alpha^2 - (E_\alpha)^2)}{k_A}\Delta^*(k)\right]\frac{d\Delta^*(k)}{dk} \\ &\quad - \frac{(\sigma_{\alpha,\theta} + \sigma_\alpha^2 - (E_\alpha)^2)}{2k_A^2}\left(\Delta^*(k)\right)^2\end{aligned}$$

When the overall influence effect is negative (condition (19) fails) we can see that the second term in the expression above will always be positive. Thus, it will be optimal to decrease communication effort and responsiveness by as much as possible ($k^o = \overline{k}$) if

$$\frac{\sigma_{g,\theta}}{2} + \sigma_{g,\alpha} < \frac{((E_\alpha)^2 - \sigma_\alpha^2 - \sigma_{\alpha,\theta})}{k_A}\Delta^*(\overline{k}) \tag{20}$$

where $\Delta^*(\overline{k})$ solves

$$\Delta^*(\overline{k}) = \frac{\sigma_{g,\theta} + \frac{\sigma_{\alpha,\theta}}{\overline{k}}\Delta^*(\overline{k})}{\frac{1}{4} - \left(\frac{E_\alpha}{\overline{k}}\Delta^*(\overline{k})\right)^2} \tag{21}$$

Condition (20) can be interpreted as follows: While the aggregate influence effect is decreasing in Δ^* (and thus increasing in k), the expertise effect is always positive for both parties. However, the expertise effect goes to zero as the principal becomes less and less of an expert herself ($g(\theta)$ weakly correlated with $\theta, \alpha(\theta)$).[10] Thus the welfare gain the parties attain from the expertise effect does not cover the signal-jamming deadweight loss suffered by the agent. Consequently, combination of misaligned incentives and poor information-processing abilities provides a sufficient condition for reducing communication effort and responsiveness by as much as possible.

Whenever parameter values satisfy neither (19) or (20), it can be shown that a variety of arrangements may be optimal depending on the parameter values. These arrangements include ones in which $k^o \in (\underline{k}, \overline{k})$. The figure below depicts such a scenario in a numerical

[10]Note that because $\alpha(\theta)$ was assumed to be increasing in θ, $\sigma_{g,\alpha}$ will typically be small when $\sigma_{g,\theta}$ is small.

example where all functions of θ are linear:

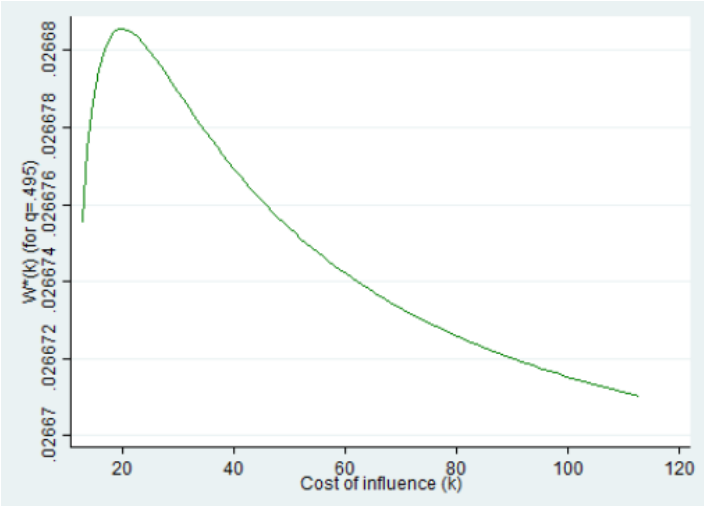

Figure 1: Optimal Communication Channel

To conclude this section, I state the proposition below, summarizing the key insights attained from analysis of the channel design problem.

Proposition 3 *The following statements are true regarding the channel design problem described in (18)*

1. *If*
$$\sigma_{\alpha,\theta} \geq (E_\alpha)^2 - \sigma_\alpha^2 \tag{22}$$
then it is optimal to choose $k^o = \underline{k}$.

2. *If*
$$\frac{\sigma_{g,\theta}}{2} + \sigma_{g,\alpha} < \frac{((E_\alpha)^2 - \sigma_\alpha^2 - \sigma_{\alpha,\theta})}{k} \Delta^*(\overline{k}) \tag{23}$$
where $\Delta^*(\overline{k})$ *solves (21), then it is optimal to choose* $k^o = \overline{k}$.

3. *If neither of the conditions in 1) and 2) are satisfied, then the optimal communication may maximize communication effort/responsiveness* ($k^o = \underline{k}$), *minimize communication effort/responsiveness* ($k^o = \overline{k}$), *or neither* ($k^o \in (\underline{k}, \overline{k})$).

1.4.3. Discussion: Implications for Organizations

Formal analysis of the channel design problem gives predictions regarding what kinds of communication channels are likely to be used by different organizations. Here I discuss two key findings of the model, and what they predict regarding communication processes in real-world organizations.

First, the model establishes that different organizations will adopt different communication channels, depending on the incentives of informed parties and decision makers.[11]

[11] The result regarding adapting the communication channel to the organizational environment is also observed in Milgrom and Roberts (1988) and Powell (2015). The current model can be thought of as combining these two models into one model where the two (costly) tasks are "signal-jamming" and "information-transmission", and some agent types will engage entirely in the former while other's will engage entirely in the latter.

Indeed, we see vastly different communication channels in different organizations. For example, the software-development company Valve has installed wheels on the desks of all employees, so any employee meeting with another can do so with the assistance of any personal visuals or useful supplementary materials. Similarly, open-door offices, which can drastically reduce search costs associated with making contact with decision makers, has become a common practice in many modern firms. In the context of the current models, these practices are most effective in improving organizational welfare in situations where experts have preferences that are well aligned (ex-ante) with decision makers, and decision makers have sufficient expertise on the topic at hand to process transmission of complicated information. Such a situation is especially representative of Valve, whose employees all have equal decision authority and perform tasks that simultaneously improve their own future career prospectives and Valve's overall profitability.[12]

The facilitating communication practices described above are in stark contrast to practices adopted by other organizations. Restricted floors and executive-only amenities are also commonplace in some large organizations. In the military, soldiers are often reprimanded, even fired, for attempting to converse with officers ranking higher than their supervising officer.[13][14] The model suggests that such restrictive practices are necessary in situations where excessive signal-jamming is expected, i.e. when experts have large ex-ante biases relative to decision makers. An example of this would be an organization in which an informed party, may be assigned to an undesirable task if decision makers discover his information (e.g. lower-level employees or low-ranking soldiers).

The second main prediction of the model is that incentivizing communication effort may be desirable from a societal standpoint, even if communicators are primarily using this effort to signal-jam. While this may seem like a strange finding, there are many situations in the real world where large communication costs are borne, even when they seem to not be transmitting much (if any) useful information. Product advertising and political lobbying are often associated with large campaigning costs which are often viewed as a deadweight losses.[15] While the model indeed suggests that a deadweight loss is born by information senders in these situations, a high-stakes environment makes signal realizations that are undesirable for senders (on average), very information for the decision makers. For example, in political campaigns, voters often respond much more strongly to bad news about a particular candidate than good news.[16] The model suggests that such arrangements may actually be desirable from an ex-ante perspective, despite the incurred signal-jamming deadweight loss for information senders.

[12] See Valve handbook: http://media.steampowered.com/apps/valve/Valve_Handbook_LowRes.pdf

[13] For example, Captain Brett Crozier was recently relieved of command for requesting assistance from individuals outside his chain of command. Source: https://www.defenseone.com/threats/2020/04/aircraft-carrier-captain-fired-poor-judgement-over-coronavirus-letter/164336/

[14] Palida 2020b extends the current model to allow for communication intermediaries. One of the effects intermediation has communication is equivalent to increasing the marginal cost of influence for the agent.

[15] See, for example, Pettinger (2016): https://www.economicshelp.org/blog/150/economics/economics-of-advertising/

[16] See, for example, Ellickson, Lovett, and Shachar (2019).

1.5. Further Related Literature

In this section I discuss several other strands of literature related to the setting and results from the current paper: I also discuss some in-progress research regarding communication channels as well as potential future research.

A crucial assumption of this model is that communication is modeled as being both costly and noisy. In other words, communication is noisy signalling as opposed to either cheap talk or Spencian signalling. While noisy-signaling models are not as commonly studied as other information transmission models, several papers do adopt this framework. To my knowledge, the model was first formally introduced in Matthews and Mirman (1983), in the context of product-market competition between firms. Carlsson and Dasgupta (1995) expand on the theory presented in Matthews and Mirman (1982) and derive additional general properties for noisy-signaling games. Choe and Park (2012) characterize conditions under which useful information can be transmitted in a noisy-signaling game. This paper instead focuses on the welfare consequences of noisy-signalling games and applies the analysis to study usage and design of communication channels in organizations.

In two separate papers (Palida 2020a,b) I apply the noisy-signalling framework developed here to study usage of multiple channels in organizations and communication intermediation. Palida 2020a shows that the dual-tension generated by costly and noisy communication provides important implications for what kinds of information should be sent through which channels. In particular, the model microfounds encouragement of "healthy discussion" channels which are used explicitly by those with important information, irrespective of partisan allegiances. The model also suggests that polarization across communication channels may be an efficient response to "bandwidth" concerns facing decision-makers of large corporations or unqualified management.

In Palida 2020b, I show that communication intermediaries have two opposing effects on communication effort and responsiveness. The first effect is equivalent to raising the marginal cost of communication for the agent, while the second effect is equivalent to increasing the exogenous expertise of the principal. Thus, whether or not an intermediary improves organizational welfare depends on both the qualities of the intermediary as well as the underlying tension facing the principal and agent. In particular, the model formalizes: 1) use of communication intermediaries to mitigate unproductive influence activities (e.g. government bureaucracy); 2) communication via middle management in large organizations; and 3) third-party conflict resolution arrangements.

Another related paper is Dewatripont and Tirole (2005) who use a moral-hazard-in-teams model study *modes* of communication in organizations. As defined in the psychology literature (see Petty and Cacioppo 1981,1986), two kinds of communication modes are frequently discussed: 1) "issue-relevant messages" that convey information specifically about the issue at hand; and 2) "cue messages" which are not directly related to the issue, but instead signal credibility of the speaker.[17] While the authors characterize properties of equilibria for various parameter values, they also do not focus organizational design, which is the focus of the current paper.

The model presented in this paper also has very natural applications to internal capital markets (ICM). Gertner and Scharfstein (2013) provide an overview of the theoretical literature on ICM, but key papers include Harris and Raviv (1996), Scharfstein and Stein

[17] An example of the former would be submitting results of an experiment to a supervisor, while an example of the latter would be a degree from a prestigious college.

(2000), Wulf (2009), and Friebel and Raith (2010). A common assumption in ICM models is that the decision maker can commit to a decision rule ex-ante.[18] Thus, many of these models correspond to mechanism design problems. In contrast, in my model I assume that all of the decision maker's signals are non-contractible. This is a very reasonable assumption, especially in situations where signals are intangible or privately observed, e.g. general impressions or "hunches".

1.6. Conclusion

This paper studies how organizations use and design communication channels for efficient information transmission. Modeling communication channels as noisy-signalling games with varying parameters, I find that simultaneous cost and noise in communication generates a potential two-sided inefficiency: welfare suboptimal behavior on the part of the information senders induce welfare suboptimal behavior on the part of organizational decision makers (and vice versa). Specifically, costly and noisy communication between the two parties is prone to either too much uninformative signal jamming or too little productive information transmission.

To manage this two-sided inefficiency, organizations may have strong incentives to tailor communication channels specifically to the environment and objectives its members face. I show that for any communication channel, welfare can be decomposed into two effects: the expertise effect and the influence effect. Furthermore, these effects may or may not move in the same direction with respect to communication effort and organizational responsiveness. In particular, when information senders have large ex-ante biases for particular decisions, communication effort will primarily be unproductive signal jamming, and implementing high-cost or noisy communication channels can help dampen the resulting deadweight loss. On the other hand, when the sender's preferences covary strongly with decision makers, but ex-ante biases are still small, it will be optimal for organizations to facilitate communication as much as possible by selecting the communication channel with the lowest possible effect cost of influence. For moderate values of these parameters, I find that a variety of design structures may be optimal, including channels that neither maximize or minimize communication.

The results of the model provide a rationale for various observed organizational practices. The model speaks to the vast variety of communication channels we observe used across organizations. The model also suggests that encouraging costly lobbying or advertising endeavors may in fact be desirable, as failed attempts at persuasion in such environments can also provide decision makers with useful information.

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Chapter 2

Partisan and Non-Partisan Communication Channels

Abstract

Organizational decision makers are often accessible via a variety of communication channels (e.g. E-mail, face-to-face meetings, phone calls, etc.), and the channel selected by an informed party can signal information about the sender's preferences. This paper uses a noisy-signalling model to study how and why partisan and non-partisan communication channels arise in organizations. Polarized organizations use only partisan channels and information transmission occurs primarily via cheap talk (channel selection). On the other hand, healthy-discussion equilibria include partisan channels for moderates, while holders of important information instead use non-partisan or discussion channels. The model suggests that polarization across communication channels may be an efficient response to "bandwidth" concerns facing decision-makers of large corporations or unqualified management. Conversely, coexistence of partisan and non-partisan channels within an organization or community (e.g. tabloids and professional news sources in the journalism industry) may also be socially efficient for other environments. Finally, I use a special case of the model to study the usage of endorsements in organizations and communities.

2.1. Introduction

Organization members can surface issues with decision makers via a variety of communication channels. An employee can schedule a face-to-face meeting with a supervisor to discuss an issue, or choose to converse via E-mail instead. Beyond firms, news stories can be reported on a variety of media sources, e.g. professional news channels and tabloid publications, and academics can present ideas using a variety of mediums, e.g. scholarly papers and blog posts.

Often times, the channel used by an informed party to communicate can signal additional information regarding the sender's preferences. This is perhaps most clearly seen "non-firm" communities such as the journalism industry, where we observe usage of *partisan channels*, which directly signal how the sender would like the audience to interpret presented facts (e.g. conservative and liberal publications), as well as *non-partisan* channels, where direct endorsements by reporters would be unexpected or unacceptable (e.g. professional news networks). Channel selection as a signal is conceivable within firms as well. One may naturally expect that information received by a manager via gossip or a rumor will bear a different reaction than information that was presented in a pre-scheduled, formal meeting. Furthermore, many firms now attempt to establish "healthy-discussions" between employees, which are accomplished by non-partisan channels whose userbases consist of organization members with varying opinions on important issues. How does the reputation of a communication channel (i.e. partisan vs non-partisan) effect the incentives of users and decision makers, and why do communication channels gain these reputations in the first place?

The current paper attacks the two questions above using a model costly and noisy communication. Formally, the model consists of a noisy-signalling game between two parties: 1) A principal, or decision maker, who must choose among two projects to invest in, as well as the scale of investment; and 2) an agent who is privately informed on the marginal returns of both projects. The agent will first observe the state of the world and then choose: 1) an observable communication channel, from a pre-specified set of feasible alternatives; and 2) unobservable communication effort, the marginal cost of which is determined by the channel selected. In addition to observing which communication channel was used, the principal observes a noisy signal correlated with the underlying state of the world and the agent's communication effort. Finally, the principal chooses which project to invest in and the size of the investment.

I find that two forms of equilibria may lie on the Pareto frontier for different environments. *Polarized* equilibria use only partisan channels, whose userbases consist entirely of supporters for only one of the two projects. Polarized equilibria optimal when the principal's signal is sufficiently uninformative without the agent's effort. This is because costless extensive-margin signalling via channel selection (i.e. cheap talk) becomes a more efficient mode of information transmission in this environment. Thus, the model suggests that polarization across communication channels may be an efficient response to "bandwidth" concerns facing executives of large firms, or unqualified management.

The second form of equilibrium does make use of non-partisan or *discussion channels*. In particular, *Healthy-discussion* equilibria include partisan channels for moderate supporters of one of the two projects, as well as a discussion channel used by agent types with important information, or information that shifts both parties preferences sufficiently far from prior biases. Healthy-discussion equilibria optimal when: 1) the principal has some exogenous expertise on her own, i.e. her signal is sufficiently informative without communication effort; and 2) irrelevant information, i.e. information that *does not* shift preferences far away from prior beliefs, is prevalent.

Formally, when the principal is relatively well informed without communication effort, she is willing to tie her investment more closely to observed signals (larger intensive margin of investment), boosting incentives for productive communication effort along the discussion channel. Furthermore, removal of moderate partisans from the discussion channel also mitigates a signal-jamming deadweight loss they would suffer in a full-pooling equilibrium. One interpretation of this is that many organizations that typically make use of non-partisan channels, e.g. professional news networks in journalism and seminars/refereed publications in academia, may benefit from also allowing outlets designed specifically for partisan declarations. Under this view, the longtime coexistence of tabloid journalism with professional reporting, as well as the use of blog posts, informal research discussions, and non-refereed publications in academia, are not inconsistent with characteristics of optimally designed communication arrangements predicted by the model. The paper also discusses reasons to restrict usage of inferior communication channels (e.g. E-mail relative to face-to-face), and studies the usage of explicit endorsements in organizations and communities.

The current paper relates to the broad field of information economics, which has introduced several models of strategic communication. Classic models in this literature include "signaling" models (e.g. Spence 1978), "cheap-talk" models (e.g. Crawford and Sobel 1983), and "verifiable-information" models (e.g. Grossman 1981, Milgrom 1981). As discussed in more detail in Palida (2020a), none of these models capture the dual notion that communication both takes effort to be successful, and can result in miscommunication even when effort is exerted. In other words, communication is both *costly* and *noisy*.

To best capture this dual notion, this paper adopts a noisy-signaling framework, first studied in Matthews and Mirman (1983).[19] While this may seem like a pure technical difference, it is actually quite significant. In a noisy-signaling game, the sender's action cannot be determined by the receiver with certainty (in equilibrium). Consequently, I find non-partisan channels may be desirable, as miscommunication risk simultaneously filters out moderates with irrelevant information and induces communication effort from those with important information.

Two other closely related papers are Palida (2020a) and Palida (2020b). Palida 2020a studies usage and design of a single communication channel, and presents the core framework used in this paper. Palida (2020b) extends the framework from Palida (2020a) to study usage of communication intermediaries in organizations.

The remainder of the paper is structured as follows: Section 2 reviews further related literature; Section 3 sets up the formal model; Section 4 solves for the continuation equilibrium given a channel-selection profile for the agent; Section 5 solves the agent's channel-selection problem and introduces the notion of partisan and non-partisan channels, in the context of the model; Section 6 studies optimal PBE or communication arrangements; Section 7 applies the model to study usage of endorsements; and Section 8 concludes and discusses future research.

2.2. Related Literature

The current paper relates to several other strands of literature. In general, the paper concerns designing communication processes between informed parties and decision makers. This broad design environment encapsulates both the settings studied in the mechanism design (e.g. Groves, 1973; Myerson and Satterthwaite (1983)) and information design (e.g. Kamenica and Gentzkow, 2011; Bergemann and Morris (2016)) literatures. Both these literatures focus on settings where either the sender or receiver of information can commit to an action or decision rule and focus on designing this rule. In the former literature the receiver can commit to a cheap talk-contingent decision rule, while in the latter (upon applying a revelation principle) the sender can commit to a type-contingent, decision-recommendation rule. In contrast, in this model, neither the sender nor receiver can pre-commit to a strategy, and instead we examine the welfare consequences of different equilibrium channel-usage arrangements.

The authority and delegation literature in organizational economics (Dessein, 2002; Alonso, Dessein and Matouschek, 2008; Rantakari, 2008) can also be thought of as falling under the broad umbrella of designing communication processes. In particular, these models relax the strategy-commitment assumption, and instead focus on allocating decision rights within organizations. Consequently, these models can be thought of as designing who within an organization will talk to whom.

The current paper is similar to the organizational design literature in that no strategy pre-commitment is possible. However, while decision rights are always fixed with the

[19] While I believe that noisy signaling best captures the communication processes observed in motivating examples, the framework provides another technical benefit. In particular, noisy signaling models largely avoid equilibrium uniqueness issues typically found in signaling and cheap-talk models. For the current paper, this greatly assists in deriving analytic expressions for key equilibrium quanitites.

principal in my setting, the agent potentially has access to a richer set of communication channels to choose from. Such flexibility is not possible in above-mentioned papers, as communication is restricted to be cheap talk in those models.

A growing body of literature has also emerged which focuses on studying environments where information senders have multiple ways of communicating with decision makers. Austen-Smith and Banks (2000) study a model in which a privately informed sender can communicate with a decision maker via both cheap talk and money burning (Spencian signalling). Two other related papers are Kolotilin and Li (2018), who study a repeated cheap-talk model with relational transfers, and Venables (2013), who studies the role of mission statements in organizations by examining a model combining cheap-talk with money burning at a later date. All three models find that constrained-optimal outcomes may use both forms of information transmission.

While I also find that there may be significant benefits to allowing multiple forms of information transmission, the current paper combines cheap-talk (channel selection) with noisy-signaling (communication effort). As discussed above, the fact that communication is noisy in my model allows the potential optimality of discussion channels in equilibrium.

Two other very related works are Mathias Dewatripont and Jean Tirole's 1999 and 2005 papers (DT99 and DT05) on endogenous advocacy and modes of communication respectively. In DT99, the authors present a communication model that provides a rational for creating (or tolerating) advocates in organizations. The advocates studied in this model play a similar role to users of discussion channels in the current model in that both are granted high-powered incentives to engage in socially productive communication. However, discussion channels arises endogenously as an equilibrium phenomenon in the current model, while in DT99 advocates are created artificially by the organization's compensation scheme.

In DT05, the authors construct a moral-hazard-in-teams model to study the use of communication "modes" in organizations. As defined in the psychology literature (see Petty and Cacioppo 1981,1986), two kinds of communication modes are frequently discussed: 1) "issue-relevant messages" that convey information specifically about the issue at hand; and 2) "cue messages" which are not directly related to the issue, but instead signal credibility of the speaker.[20] While DT05 characterize properties of equilibria for various parameter values, they does not discuss organizational design, which is the focus of the current paper.

Finally, the model presented in this paper also has very natural applications to internal capital markets (ICM). Gertner and Scharfstein (2013) provide an overview of the theoretical literature on ICM, but key papers include Harris and Raviv (1996), Scharfstein and Stein (2000), Wulf (2009), and Friebel and Raith (2010). A common assumption in ICM models is that the decision maker can commit to a decision rule ex-ante.[21] In contrast, in my model I assume that all of the decision maker's signals are non-contractible. This is a very reasonable assumption, especially in situations where signals are intangible or privately observed, e.g. a general impressions or "hunches".

[20] An example of the former would be submitting results of an experiment to a supervisor, while an example of the latter would be a degree from a prestigious college.

[21] One exception to this is Friebel and Raith (2010), who study a model of cheap-talk communication.

2.3. Players and Environment

In this section, I set up the formal environment I will study in the remainder of the paper, and discuss various assumptions of the model.

2.3.1. Model Set Up

The model consists of a noisy-signaling game between two players: a principal (P), endowed with formal decision rights, and a privately-informed agent (A). Throughout the paper, the principal's decision will be interpreted as investment in one of two projects. Specifically, I assume that P is responsible for choosing $d \in \mathbb{R}$, where $d < 0$ corresponds to investment in what will be called the "left" (L) project and $d > 0$ corresponds to investment in the "right" (R) project. Note that P can also choose $d = 0$ which corresponds to the status quo of investment in neither project.[22] More generally, the environment captures any setting in which a decision consists of both an extensive and an intensive margin (see discussion at end of section).

The agent is an expert who privately observes an underlying state $\theta \in \Theta$, which determines the marginal payoff of the decision d for both parties. The parties payoffs are given by

$$u_P(\theta, d) = \theta d - \frac{1}{2}d^2 \tag{24}$$

and

$$u_A(\theta, e, d) = \alpha(\theta)d - \frac{k(c)}{2}e^2 \tag{25}$$

where I assume $E_\theta\{\theta\} = 0$, $\alpha(.)$ is strictly increasing, and $E_\theta\{\alpha(\theta)\} = E_\alpha > 0$. The assumption that $\alpha(.)$ is increasing restricts us to an interesting setting where A can play a positive welfare role. However, note that because $E_\theta\{\theta\} = 0$ while $E_\theta\{\alpha(\theta)\} > 0$, A has a slight ex-ante bias for positive decisions ($d > 0$) relative to P.

The second term in the agent's payoff function ($-\frac{k(c)}{2}e^2$) will be discussed shortly. For now, note that P's preferred state-contingent decision is to set $d = \theta$, while A is a "directional empire builder" who always prefers either $d = -\infty$ or $d = \infty$, depending on the sign of $\alpha(\theta)$. As discussed above, the agent may also disagree with the principal on the preferred sign of d as well (i.e. disagree on the extensive margin as well as intensive margin). The size of the parameter E_α captures the degree of this extensive margin disagreement and will be an important parameter when studying optimal communication arrangements. Another important parameter will be $\sigma_{\alpha,\theta} = cov\{\alpha(\theta), \theta\}$, which captures the agents stake in decision making. Specifically, when the slope of $\alpha(\theta)$ is large, the agent has strong incentives to influence decision making by transmitting useful information. Thus, the agent has strong incentives to influence decision making when either E_α or $\sigma_{\alpha,\theta}$ is large. However, influence activities driven by the former will primarily be socially unproductive (signal jamming) while influence activities driven by the latter will be socially productive (information transmission).

Because A has personal interests in the decision d, we allow A to exert *communication effort* $e \in \mathbb{R}$ to influence the distribution of a noisy signal observed by the principal, s. To engage in these influence activities, A first chooses a communication channel c from a set of available communication channels C. The communication channel selected by the

[22] In Palida (2019), I show that this set-up can be microfounded using a specification where P is allowed to invest in both projects, but always optimally chooses to invest only in one.

agent determines his marginal cost of influence activities ($k(c)$) as shown in the agents payoff function given in (25).

We will assume that the noisy signal s is binary, $s \in \{L, R\}$, with distribution characterized by

$$\Pr\{s = R|\theta, e\} = e + g(\theta) \tag{26}$$

The function $g(.)$ is P's *expertise* function, which encodes her ability to infer the underlying state without communication effort from the agent. We will assume that $g(.)$ is increasing in θ, so a realization of $s = R$ can be interpreted as a "hunch" that the R project is the profitable one (for P) and $s = L$ is a "hunch" that the L project is the profitable one. The increasing assumption is without loss of generality if $g(.)$ is restricted to be monotonic. Furthermore, we will assume that for all $c \in C$, $k(c)$ is large enough so that it is never a rationalizable strategy for A to choose communication effort large enough to degenerate s. This condition will be referred to the no-shifting-support condition or NSS. The NSS condition implies that we are restricting attention to cases where it is simply too costly to completely eliminate noise in communication (see discussion at end of section).

We can now state the general timing of this interaction as follows:

1. Nature chooses $\theta \in \Theta$, observed by A
2. A chooses $c \in C$ and $e \in \mathbb{R}$
3. P observes $c \in C$ and $s \in \{L, R\}$, and chooses $d \in \mathbb{R}$
4. Payoffs realized as in expressions (24) and (25)

We will focus on pure-strategy perfect Bayesian equilibria (PBE).[23] P's strategy consists of a channel and signal-contingent decision rule $d(c, s)$, while $A's$ strategy consists of type-contingent channel and communication-effort profile $c(\theta)$ and $e(\theta)$.

2.3.2. Discussion of Formal Setting

Several features of the model merit further discussion. First, the project-investment interpretation of the decision d is largely for expositional convenience. As mentioned, the principal's decision in this model can capture any scenario in which the decision maker must make a decision that consists of both an extensive and intensive margin. For example, d can also be interpreted as a recommendation for promotion into a particular position. In this case the extensive margin would be the position P recommends A for, and the intensive margin would be the time P spends lobbying on A's behalf.

A second crucial component of this model is that communication is both costly and noisy. While this is non-standard in much of the strategic communication literature, I argue that noisy-signalling is a fundamental part of various communication interactions in organizations. An academic seminar is an extreme example of such an interaction, as attendees do not observe the speaker's preparation for the seminar (which may be considerable), but only the seminar itself. Analogous situations arise in firms and other organizations: Time spent preparing for meetings, writing memos, making slide shows, etc. are typically not observed directly, but are often expected and relied upon by decision

[23]Mixed-strategy PBE may exist in this setting. For simplicity, the current paper does not consider these equilibria.

makers. These can all be considered examples of communication that is both costly and noisy (i.e. noisy signalling).

It is worth mentioning that the parameters $k(c)$ for $c \in C$ can actually be interpreted as capturing either the cost of influencing perceptions of decision makers or the effect costly efforts have on the distribution of these perceptions (see Palida 2020a). Naturally, there is an equivalence between these two characteristics as communication effort does not have a clear scale of measurement. Thus, a communication channel in the model, characterized by a particular $k(c)$, can actually map to several different kinds of communication channels in reality. These channels may seem physically different, but the model suggests that they may in fact equivalent in terms of implied equilibrium behavior and welfare. For example, in the table below, the upward diagonal can be thought of as two different channels, each with moderate levels of $k(c)$. With that said, we will see that observable differences between channels (that may otherwise have the same effective marginal cost of influence) can be leveraged to attain additional information transmission.

		Noise in Communication	
		Low	High
Cost of Communication	Low	Open-door offices	E-mails/memos
	High	Formal boardroom meetings/seminars	Restricted floors

Table 1: Four Example Channels

The preference structure used in this model is also somewhat non-standard with respect to the strategic communication literature. While empire-building preferences are common in the internal-capital markets literature (see Gertner and Scharfstein, 2013 for a review), applied models of strategic communication often use the "quadratic-bias" preference specification, where both the sender and receiver wish to match decision making to the underlying state, plus a player-contingent bias. This model adopts empire-building preferences partially for tractability and partially to capture realistic organizational settings in which experts often only care about the extensive margin of a decision. For example, a researcher may prefer a particular project to work on, but will always prefer as much funding as possible for his desired project. Similar incentives can be observed with branch managers of large firms, local politicians, leaders of specific task forces, etc. It is worth mentioning that both the current model's specification as well as the quadratic-bias specification are both special cases of a more general specification where both parties preferences are quadratic in the decision.[24]

[24] The current model can be extended to study this more general environment. In the next section we will see that all strategies and payoffs can be expressed in terms of a single equilibrium quantity representing the difference in decision making across the two signal realizations. If we allow the agent to have a quadratic cost for the decision as well, equilibrium will be characterized by two quantities: the equilibrium decision spread, plus a quantity that captures the total cost of decision making.

Finally, the model assumes a binary-signal specification for the principal's signal. While this may seem like a restrictive assumption, it can actually be shown that all results are qualitatively identical in a model with a richer signal-space.[25]

2.4. Equilibrium for a Fixed Communication Channel

Our first goal is to solve for all possible equilibria of the game described in the previous section. To do this, let us suppose that the agent is playing a channel profile given by $c^*(\theta)$ in a PBE. We will first solve for the equilibrium effort profile and decision rule, conditional on

$$\theta \in \{\theta \in \Theta | c^*(\theta) = c\} = \Theta^*(c) \tag{27}$$

We will denote these quantities as $e^*(\theta)$ and $d^*(c, s)$ respectively.

For P to be optimizing given $e^*(\theta)$, she must choose $d^*(c, s)$ to solve

$$\max_d E_\theta\{\theta | c, s; c^*(.), e^*(.)\} d - \frac{1}{2} d^2 \tag{28}$$

The first-order condition to the above is necessary in any PBE and implies that

$$d^*(c, s) = E_\theta\{\theta | c, s; c^*(.), e^*(.)\} \tag{29}$$

Similarly, since we are assuming $\theta \in \Theta^*(c)$, for A to be optimizing given $d^*(c, s)$, he must choose $e^*(\theta)$ to solve

$$\max_e \alpha(\theta)[d^*(c, L) + (e + g(\theta))\Delta^*(c)] - \frac{k(c)}{2} e^2 \tag{30}$$

where

$$\Delta^*(c) = d^*(c, R) - d^*(c, L) \tag{31}$$

Under the NSS condition, the first-order condition to (30) is necessary in any PBE, and implies that

$$e^*(\theta) = \frac{\alpha(\theta)}{k(c)} \Delta^*(c) \tag{32}$$

Given (29) and (32), we can derive an implicit analytic expression for $\Delta^*(c)$ in terms of exogenous model parameters. In particular, denote the quantity

$$f^*(\theta; c, s) = \frac{f(\theta) \Pr\{s | \theta, c; e^*(.)\}}{\sum_{\theta' \in \Theta} f(\theta') \Pr\{s | \theta', c; e^*(.)\}} \tag{33}$$

to be $P's$ posterior belief regarding the probability that the state of the world is θ, conditional on A's communication-effort profile given in (32) and P observing s. We have that

$$d^*(c, s) = \sum_{\theta \in \Theta} \theta f^*(\theta; c, s) \tag{34}$$

which will imply that

$$\Delta^*(c) = \frac{\sigma^*_{g,\theta}(c) + \frac{\sigma^*_{\alpha,\theta}(c)}{k(c)} \Delta^*(c)}{(1 - E^*_g(c) - \frac{E^*_\alpha(c)}{k(c)} \Delta^*(c))(E^*_g(c) + \frac{E^*_\alpha(c)}{k(c)} \Delta^*(c))} \tag{35}$$

[25] The proof of this is available upon request.

where

$$\begin{aligned}
E_g^*(c) &= E_\theta\{g(\theta)|\theta \in \Theta^*(c)\} \\
E_\alpha^*(c) &= E_\theta\{\alpha(\theta)|\theta \in \Theta^*(c)\} \\
\sigma_{g,\theta}^*(c) &= cov_\theta\{g(\theta), \theta|\theta \in \Theta^*(c)\} \\
\sigma_{\alpha,\theta}^*(c) &= cov_\theta\{\alpha(\theta), \theta|\theta \in \Theta^*(c)\}
\end{aligned}$$

Given a PBE channel profile $c^*(\theta)$, one can identify all possible "continuation" PBE of the game by solving equation (35) for all $c \in range\{c^*(\theta)\}$ and checking that probability bounds on $P's$ signal s are never violated at the candidate equilibrium.[26] In Palida (2020a) I show that (35) can in fact admit only one solution that does not imply a violation of probability bounds on s. Thus, any PBE channel profile in this model, admits a unique continuation effort profile and decision rule. This is summarized in the proposition below:

Proposition 4 *For any PBE channel profile $c^*(\theta)$, there exists a unique corresponding PBE effort profile $e^*(\theta)$ and decision rule $d^*(c, s)$ characterized by (29), (32), and the median of the solutions to (35) for each $c \in range\{c^*(\theta)\}$.*

Proof. See Palida (2020a), Proposition 1. ■

In general, there will be three solutions to (35). One will imply that $\Delta^*(c) < 0$ while the other two will imply that $\Delta^*(c) > 0$. The proposition argues that only the smallest positive solution corresponds to a legitimate PBE. The other two solutions correspond to settings in which probability bounds are violated, so that the formula implied by Bayes' rule becomes invalid.

The following corollary establishes some useful comparative static results, which will be important for analysis of welfare across different candidate channel profiles.

Corollary 2 *For all $c \in range\{c^*(\theta)\}$, we have that $\Delta^*(c) > 0$. Furthermore, we have that $\Delta^*(c)$ is strictly decreasing in $k(c)$, and strictly increasing in $E_\alpha^*(c), \sigma_{\alpha,\theta}^*(c)$, and $\sigma_{g,\theta}^*(c)$.*

Proof. Implied by the fact that the RHS of (35) is increasing in $\Delta^*(c)$. ■

The next section will focus on the agent's channel-selection problem which will give us insight into what kinds of channel profiles can be sustained in a PBE. For now, however, consider a PBE characterized by channel profile $c^*(\theta)$ and let $W^*(c)$ be the sum of P and $A's$ ex-ante expected payoffs conditional on $\theta \in \Theta^*(c)$. We can show that

$$\begin{aligned}
W^*(c) &= \underbrace{(\frac{E_\theta^*(c)}{2} + E_\alpha^*(c))E_\theta^*(c)}_{\text{Sorting}} + \qquad (36) \\
&\underbrace{\left(\frac{\sigma_{g,\theta}^*(c)}{2} + \sigma_{g,\alpha}^*(c)\right)\Delta^*(c)}_{\text{Expertise}} + \underbrace{\frac{(\sigma_{\alpha,\theta}^*(c) + \sigma_{\alpha,\alpha}^* - (E_\alpha^*(c))^2)}{2k(c)}(\Delta^*(c))^2}_{\text{Influence}}
\end{aligned}$$

The second two terms in the expression above are introduced in Palida (2020a). the expertise effect can (roughly) be interpreted as the welfare the two parties would attain if no influence occurred, but P instead observed a signal $s' \in \{L, R\}$ where

$$\Pr\{s' = R\} = g(\theta) + \frac{E_\alpha^*(c)}{k(c)}\Delta^*(c) \qquad (37)$$

[26] It is not exactly a continuation equilibrium, as c and e are chosen at the same time.

and made her decision d solely off of the realization of s'. The influence effect, captures both the additional welfare gain associated with the agent influencing s conditional on being informed of the underlying state θ, as well as the total expected cost of communication effort.

The first term in the expression for $W^*(c)$ is new to this paper. This "sorting effect" can be interpreted as the joint expected payoff the parties would attain on channel c, if P ignored her signal s and simply based her decision off of her observation of channel c being used.

Comparison of the sorting effect with the other two effects will be crucial in determining optimal PBE channel profiles, or communication arrangements in section 6. Specifically, selection of a particular channel, *by itself*, provides the principal with information regarding the underlying state θ. Thus, channel selection has a *cheap-talk* role in this model, as well as determining the noisy-signalling game played by the two parties. This cheap-talk role of channel usage will be important when we discuss partisan and non-partisan channels in the the next section.

2.4.1. Discussion: Dual Tension

As noted above, the equilibrium decision spread for channel c, $\Delta^*(c)$, captures P's responsiveness to equilibrium signal realizations for channel c, as well as A's incentives to influence decision making when using channel c. Thus, this model exhibits a dual tension, in which the agent's communication effort incentives are directly affected by the principal's anticipated response to noisy signals, while the principal's incentives to respond to noisy signals are directly affected by the agent's expected communication effort. This dual tension can create a potential two-sided inefficiency: welfare suboptimal behavior on the part of the agent will induce welfare suboptimal behavior on the part of the principal and vice versa.

To see these forces more clearly, let us suppose that there is only one channel, as in Palida (2020a). Suppose that the two parties could pre-commit to a decision spread Δ^{ctr}, where the superscript "ctr" stands for "contractual". Specifically, in this hypothetical situation, P is restricted to choose $d(R) - d(L) = \Delta^{ctr}$. If Δ^{ctr} is chosen to maximize the sum of the two parties expected payoffs, we can consider this outcome as a constrained first best.

Returning to the non-contractual case, if P has incentives to increase $\Delta = d(R) - d(L)$ relative to Δ^{ctr}, A will anticipate this and all agent types will increase communication effort in absolute value. Anticipating this increase P will increase Δ^{ctr} by even more, and so forth, leading to an equilibrium with $\Delta^* >> \Delta^{ctr}$. A similar spiral will occur when P has incentives to decrease Δ relative to Δ^{ctr}, which will lead to an equilibrium with $\Delta^* << \Delta^{ctr}$.

When we study optimal communication arrangements in Section 6, we will see how equilibrium sorting into different channels can help manage this potential two-sided inefficiency.

2.5. Partisan and Non-Partisan Communication Channels

In the previous section, we solved for the equilibrium-effort profile and decision rule, given a channel profile. In this section we identify which channel profiles are sustainable in a PBE. In other words, we are interested in understanding what kind of informal *communication arrangements* are feasible.[27] To address this question, let us examine the agent's channel-selection problem.

Suppose we have a PBE with $c^*(\theta) = c$ for some $\theta \in \Theta$. Recall from the previous section that he will choose

$$e^*(\theta) = \frac{\alpha(\theta)}{k(c)}\Delta^*(c) \tag{38}$$

and will therefore face an expected utility of

$$\alpha(\theta)\left[d^*(c, L) + \left(\frac{\alpha(\theta)}{2k(c)}\Delta^*(c) + g(\theta)\right)\Delta^*(c)\right] \tag{39}$$

Alternatively, if this agent type were to deviate to channel c', the best he can do would be to choose

$$e'(\theta) = \frac{\alpha(\theta)}{k(c')}\Delta^*(c') \tag{40}$$

and his expected utility will be

$$\alpha(\theta)\left[d^*(c', L) + \left(\frac{\alpha(\theta)}{2k(c')}\Delta^*(c') + g(\theta)\right)\Delta^*(c')\right] \tag{41}$$

Let us denote $\Omega = \Theta \times C \times \mathbb{R} \times \mathbb{R}_+$ & define the function $v : \Omega \to \mathbb{R}$ as

$$v(\theta; c, \Delta, d) = d + \left(\frac{\alpha(\theta)}{2k(c)}\Delta + g(\theta)\right)\Delta \tag{42}$$

Given the empire-building nature of the agent's preferences, (39) and (41) imply that if $c^*(\theta) = c$ in a PBE, we must have that

$$v(\theta; c, \Delta^*(c), d^*(c, L)) \geq \max_{c' \in C} v(\theta, c', \Delta^*(c'), d^*(c', L)) \tag{43}$$

if $\alpha(\theta) > 0$, and

$$v(\theta; c, \Delta^*(c), d^*(c, L)) \leq \max_{c' \in C} v(\theta, c', \Delta^*(c'), d^*(c', L)) \tag{44}$$

if $\alpha(\theta) < 0$. The quantity $v(\theta; c, \Delta^*(c), d^*(c, L))$ captures the expected value of P's equilibrium decision net of communication costs, conditional on a θ-type agent selecting channel c and optimally choosing communication effort. The figure below depicts how agents of various types would select among three channels, $c, c', c'' \in C$, in a hypothetical

[27] In what follows, I will use the term PBE and communication arrangement interchangebly.

PBE.

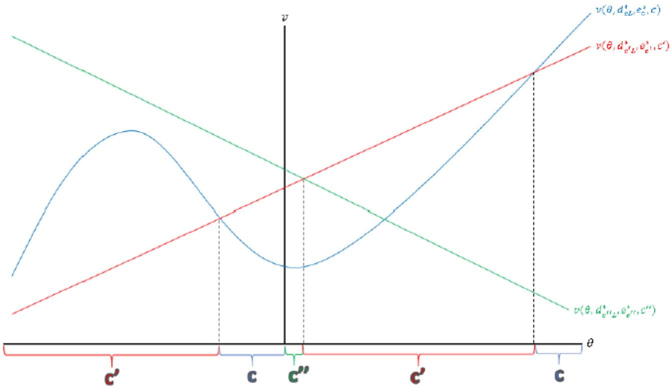

Figure 2: Example Pooling Partition

To place some more structure on the problem, I now impose a single-crossing condition on the function $v(.)$ which will greatly reduce the number of possible PBE pooling partitions.

Definition 1 *The function $v(\theta, c, \Delta, d)$ satisfies the "single-crossing condition" (SCC) if for every two vectors $(c, \Delta, d), (c', \Delta', d') \in C \times \mathbb{R} \times \mathbb{R}_+$, exactly one of the two following statements hold:*

1. $v(\theta, c, \Delta, d) = v(\theta, c', \Delta', d')$ *for all* $\theta \in \Theta$
2. *There exists a* $\theta^{crit} \in \Theta$ *such that* $v(\theta, c, \Delta, d) > v(\theta, c', \Delta', d')$ *for all* $\theta < \theta^{crit}$ *and* $v(\theta, c, \Delta, d) < v(\theta, c', \Delta', d')$ *for all* $\theta > \theta^{crit}$

SCC requires that any two functions $v(\theta, c, \Delta, d)$ and $v(\theta, c', \Delta', d')$ must cross at most once in θ, v-space, or coincide entirely. One case that satisfies SCC is when $\alpha(\theta)$ and $g(\theta)$ are both linear in θ. In this case, $v(.)$ is also linear in θ, so that SCC is automatically satisfied.

Also note that the above definition of single crossing does not exclude situations in which $v(\theta, c, \Delta, d) > v(\theta, c', \Delta', d')$ for all $\theta \in \Theta$, for some $(c, \Delta, d), (c', \Delta', d') \in C \times \mathbb{R} \times \mathbb{R}_+$. If this were to be the case in a PBE, we would have that all agent types with $\alpha(\theta) > 0$ prefer channel c, and all agent types with $\alpha(\theta) < 0$ prefer the channel c'. This scenario is depicted in panel a) of the figure below:

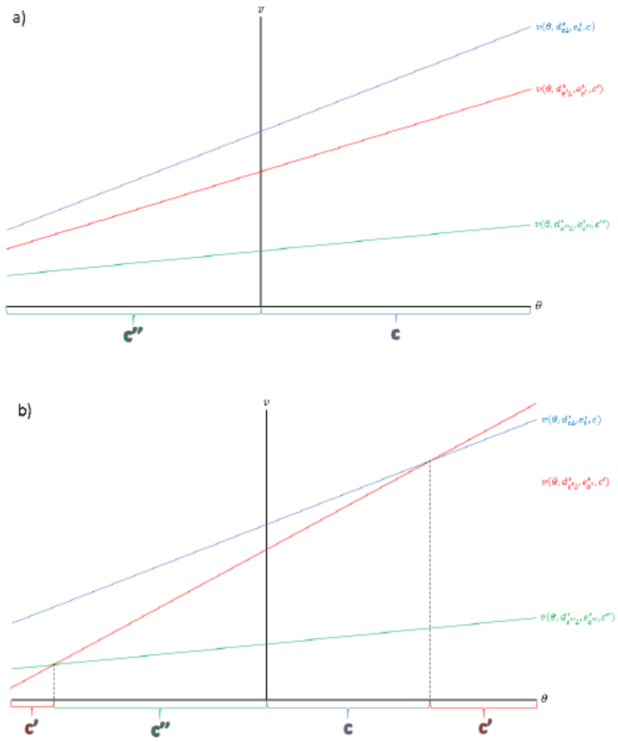

Figure 3: Pooling Under SCC

Panel b) of Figure 2 highlights a non-monotone form of pooling. In this partition, a group of moderate partisans voluntarily separate out of the channel used by the more extreme agent types. More generally, the two forms of pooling observed in Figure 2 are important for the main result of this section. I first provide a formal definition of partisan and non-partisan channels relating to the two forms of pooling observed above. Then I present the proposition and discuss its implications for equilibrium outcomes.

Definition 2 *A group $\Theta^{prt} \subseteq \Theta$ is called a "partisan group" if it is a convex set and $sign\{\alpha(\theta)\} = sign\{\alpha(\theta')\}$ for all $\theta, \theta' \in \Theta^{prt}$. A channel is "partisan" in a PBE if its user base consists of a single partisan group. A channel is non-partisan if its user base consists of two opposing partisan groups.*[28]

A partisan group is simply a group of agent types within a range of the type space that all prefer investment in the same project. In a PBE, a channel will be referred to as a partisan channel if it is used by a single partisan group. Importantly, partisan channels identify the extensive margin preferences of the agent. A non-partisan channel, includes two partisan groups, but the two groups will each prefer different projects. In particular, selection of a non-partisan channel will not directly signal extensive margin preferences to the principal. I now state the main proposition of the section which establishes that, under SCC, there are only three possible forms of PBE pooling:

Proposition 5 *Suppose that $v(.)$ satisfies SCC and the PBE channel profile $c^*(\theta)$ is such that for any $c, c' \in range\{c^*(.)\}$ there exists some $\theta \in \Theta$ in which $v(\theta; c, \Delta^*(c), d^*(c, L)) \neq v(\theta; c', \Delta^*(c'), d^*(c', L))$. The following statements are true:*

[28] "Opposing": members of two partisan groups prefer different projects.

1. *If $c \in range\{c^*(.)\}$, then c must be either a partisan or non-partisan channel.*

2. *There can be at most one partisan channel per project*

3. *There can be at most one non-partisan channel in any PBE, and it must contain the most extreme agent types $\underline{\theta} = \min_\Theta\{\theta\}$ and $\overline{\theta} = \max_\Theta\{\theta\}$.*

The proposition above shows that, under SCC, any PBE pooling group will be of the partisan or non-partisan form, unless there is mass-indifference across two or more of the channels. Furthermore, there can be at most one partisan channel for each of the projects, and at most one non-partisan channel which contains the extremes of the type space. The figure below depicts the three possible forms of PBE pooling under SCC:

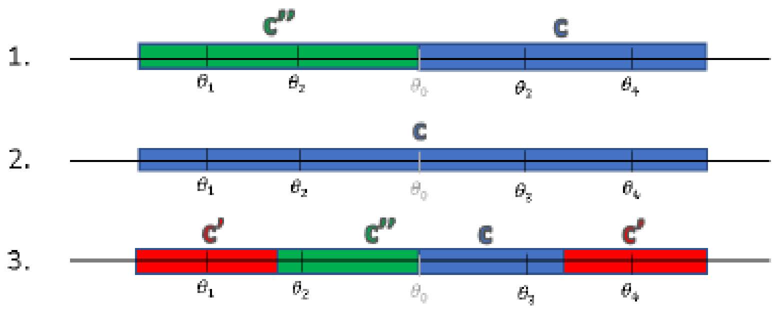

Figure 4: Three Forms of PBE Pooling[29]

I denote equilibria using the first from of pooling shown above as *polarized* equilibria. A polarized equilibrium uses exactly two partisan channels, each consisting of all supporters of the corresponding project. Polarized equilibria provide the principal full disclosure regarding the agent's extensive-margin preferences, i.e. which of the two projects he prefers. However, channel selection alone tells the principal very little about how she should select the intensive margin of the decision, i.e. the size of the investment that should be made. A potential negative effect of this is that the principal will generally be conservative with her investment regardless of the signals she observes. As discussed in the previous section, the two-sided inefficiency present in this environment of costly and noisy communication may therefore diminish the agent's incentives to further distinguish himself via costly communication effort. Essentially, polarized equilibria exemplify the classic "boy-who-cried-wolf" tension. Reliance on cheap talk for communication crowds out potentially more informative forms of communication (i.e. communication effort in this case) and kills responsiveness to observed signals.

The second two equilibria depicted in the figure both make use of non-partisan channels. The first form is a full-pooling equilibrium in which only one non-partisan channel is used, while the second form has a non-partisan channel as well as partisan channel(s) consisting of moderate supporters for one of the two projects.

In both equilibria with non-partisan channels, selection of the non-partisan channel does not directly signal the agent's extensive margin preferences. Thus, there are strong incentives for users of this channel to achieve this signalling via costly communication effort instead. These channels therefore resemble the notion of fostering a "healthy discussion" often stated as a goal of various observed organizational channels. For this

[29] θ_0 is such that $\alpha(\theta_0) = 0$. Note that $\theta_0 \notin \Theta$.

reason, I will sometimes refer to non-partisan channels as *discussion channels*, and the third equilibrium form depicted in Figure 4 will be referred to as a *healthy-discussion* equilibria.[30][31]

It is worth discussing why moderate partisans are willing to filter out of the discussion channel in a healthy-discussion equilibrium. Consider a version of the game where the agent has no private information, but still has the ability to influence the principal's signal via communication effort. A version of this game is studied in Gibbons (2005) and Powell (2015) and is often called a *signal-jamming* game. A major result in the signal-jamming literature is that the uniformed agent will typically be better off if he could pre-commit to avoiding signal-jamming. This is because his influence is simply anticipated by the principal in expectation and subtracted out of her decision. A signal-jamming deadweight loss therefore becomes a self-fulfilling prophecy in these models.

In the current model, the agent does have decision-relevant private information, but some agent types have information that is more important than others. In particular, because $\alpha(\theta)$ is increasing, agent types who have observed extreme values of θ will have information that shifts *both* parties preferences far away from their ex-ante biases. These agent types will have incentives to engage in levels of communication effort that are far more extreme than the expected level in a non-partisan channel. On the other hand, those who have not discovered important information (i.e. θ is very close to E_θ) would optimally choose a level of influence that is very close to what an uninformed agent-type would choose if they were to select the non-partisan channel. Therefore, if an alternative channel is available, these agent types would strictly prefer to take that option, as selecting the discussion channel would yield them the same form of deadweight loss as an uninformed agent would suffer in a pure signal-jamming interaction (i.e. no private information).

Conveniently, the healthy-discussion equilibrium removes the very individuals who would diminish the value of the discussion channel the most. In particular, because the principal knows that agent types whose preferences are primarily being determined by ex-ante biases are being removed from the discussion channel, she has incentives to tie her decision more closely to observed signals. This in turn provides high-powered incentives for communication effort precisely from agent types with important information to communicate. In other words, healthy-discussion can improve both sides of the two-sided inefficiency discussed in the previous section: 1) It provides an escape valve for moderate partisans who would suffer a signal-jamming deadweight loss if they were restricted to using a bipartisan channel; and 2) it amplifies the incentives for communication effort on the part of those with important information to communicate. In fact, in the next section I will analyze a special case of this model in which the healthy-discussion equilibrium *always* Pareto dominates the full-pooling equilibrium.

[30]For clarity, we will continue to refer to the second equilibrium form simply as the full-pooling equilibrium. This equilibrium form will not play a large role when we study optimal communication arrangements.

[31]It is not necessary for a healthy-discussion equilibrium to have partisan channels for both projects, as is depicted in Figure 3. The special case analyzed in the next section will have healthy-discussion equilibria with only one partisan channel.

2.6. Optimal Communication Arrangements

The previous two sections derive primarily positive results on equilibrium behavior in this model. I now focus on developing normative statements regarding optimal organizational design. The first part of this subsection introduces a parameterized version of the model presented in sections 3 and 4. Under this parameterization I am able to identify all PBE and derive near closed-form expressions for welfare in each of them. Given the parametric nature of the model, I can also connect the optimal communication arrangements predicted by the model to several environments faced by a variety of real organizations.

As will be shown in this setting, the solution to this design problem in this parametric setting will always include usage of exactly two communication channels.[32] However, the assumption that parties can coordinate on an optimal PBE is a potentially a strong one. The final part of this section, therefore, relaxes the Pareto coordination assumption and asks if channel-menu restrictions can be used to manage communication in this environment.

2.6.1. Parametric Restrictions

To facilitate welfare analysis, in this section I focus on a special case of the model. As mentioned, the simplified setting is general enough to be applicable to a wide variety of organizational scenarios. Simultaneously, it provides enough parameterization to allow full characterization of all PBE, and direct comparisons of equilibrium quantities (e.g. welfare) across different PBE.

To begin description of the special case, I first assume that the underlying state θ can take one of three types: $\theta \in \{-1, 0, 1\}$. The agent's preferences will be characterized by

$$\alpha(\theta) = \theta + \beta \tag{45}$$

where $\beta \in (0, 1)$. Recall that the probability of a particular state θ being realized is given by $f(\theta)$. To reduce the number of parameters, I assume that the distribution of θ is symmetric in the sense that $f(-1) = f(1) = \frac{f}{2}$ and $f(0) = 1 - f$ where $f \in [0, 1]$.

From an ex-ante perspective, A has a strict bias for investment in the R project, captured by the parameter β. On the other hand, P prefers the status quo of no investment in either project ($d = 0$), ex-ante. When $\theta = -1$ or $\theta = 1$ is realized, both parties gain from investment in the corresponding project (the L project or R project, respectively) as $\beta < 1$. The probability of this preference alignment is given by the parameter f. On the other hand, when $\theta = 0$, P and A disagree on their preferred investment decision. Specifically, P prefers the status quo of no investment, while A strictly favors investment in the R project. This occurs with probability $1 - f$.

The next assumption I make is an assumption on the functional form of $g(\theta)$. In particular, I assume that

$$g(\theta) = \frac{1}{2} + \left(g - \frac{1}{2}\right)\theta \tag{46}$$

with $g \in \left(\frac{1}{2}, 1\right)$. The parameter g captures the accuracy of P's exogenous information. When $g = \frac{1}{2}$, the signal s is completely uninformative if not supplemented by communication effort. Furthermore, $P's$ expectation of θ conditional on s, becomes larger in

[32] In other words, full-pooling equilibria will never be optimal.

magnitude as $g \to 1$ (if no influence occurs). Note that, absent influence, the distribution of s conditional on θ is symmetric and has $\Pr\{s = R|\theta = 0, e = 0\} = \frac{1}{2}$.

With the addition of the above assumption on $g(\theta)$, all conditions for Proposition 2 are satisfied, and we can conclude that any channel used in a PBE of this setting must be either a partisan or discussion channel, or have all three agent types indifferent between the two channels. We can see that only the former is possible, as any other PBE would have the $\theta = 1$ agent separating[33], which would imply a profitable deviation for the $\theta = 0$ agent.

To conclude the description of this setting, I restrict the channel menu to be $C = \{cl, op\}$ ("closed" and "open") with $k(cl) = \infty$ and $k(op) = k > \underline{k}$ where

$$\underline{k} = \frac{2(1+\beta)}{1-g} \tag{47}$$

The expression for $\underline{k}$ is attained by evaluating the NSS condition in the context of the above setting, at $\theta = 1$. The restriction to two communication channels is without loss of generality since there are only three types and full separation is impossible.[34]

Additionally, as the focus of this section will be on examining welfare rankings across different PBE, little is lost by assuming that influence is infinitely costly for one of the channels. This is because in any strictly semi-separating PBE with only three possible types, one agent type will always fully reveal himself to the principal. Clearly, this agent-type will never exert any communication effort. Welfare for the PBE is then determined by which channel is chosen by the other two types. If we had $k(cl) < \infty$, determining which channel the pooling group should use in a Pareto optimal PBE essentially reduces to determining the optimal communication effort cost for a single-channel model, in which the type space consists only of types in the pooling group. This involves studying the trade-off between the signal-jamming deadweight loss suffered by some channel users, against the benefits of information transmission incentivized in others. As this problem is discussed in detail in Palida (2020a), I direct the reader there for more information on that particular design question.

I will call this setting the symmetric, positive-bias, 3-type (SPB3) setting. To summarize, the SPB3 setting consists of the set-up described in section 3, with the additional three assumptions:

SPB3 Assumptions

1. $\theta \in \{-1, 0, 1\}$ where $\Pr\{\theta = -1\} = \Pr\{\theta = 1\} = \frac{f}{2}$
2. $\alpha(\theta) = \theta + \beta$ and $g(\theta) = \frac{1}{2} + \left(g - \frac{1}{2}\right)\theta$ where $\beta \in (0,1)$ and $g \in \left(\frac{1}{2}, 1\right)$
3. $C = \{cl, op\}$ with $k(cl) = \infty$ and $k(op) = k$, where $k > \underline{k} = \frac{2(1+b)}{1-\lambda}$

2.6.2 Optimal Equilibria

In the SPB3 setting, there are six potential PBE to consider. Each is characterized by two properties: 1) the PBE pooling partition $p \in \{plr, fp, hd\}$ ("polarized", "full-pooling", and "healthy discussion"); and 2) the channel used by the non-singleton pooling group, $c \in \{cl, op\}$. Thus, all equilibrium quantities will be denoted with a superscript p, c.

[33] Recall we have restricted attention to pure strategies.

[34] The $\theta = 0$ agent would always deviate to the channel used by the $\theta = 1$ agent.

Because communication effort will be strictly positive in at most one of the two channels in any PBE, it should be understood that the equilibrium decision spread $\Delta^{p,c}$ refers to the value this quantity takes in the channel where the user base consists of more than a single agent-type.

We can use expression (36) to derive expressions for the joint ex-ante expected payoffs in each of six PBE. Furthermore, these expressions will depend only on: 1) the equilibrium quantity $\Delta^{p,c}$; and 2) moments of the distribution $\theta|\Theta^p$, where Θ^p consists of the agent types in the non-singleton pooling group in PBE p, c. The lemma below establishes that only three of the six PBE are candidates for optimality:

Lemma 1 *The following relationships hold:*

1. $W^{fp,cl} < W^{hd,cl}$
2. $W^{fp,op} < W^{hd,op}$
3. $W^{hd,cl} < W^{hd,op}$.

Proof. See appendix. ■

The first two items of the lemma establish that full-pooling on a non-partisan channel is essentially a suboptimally designed discussion channel. Removal of the moderate partisan whose preferences are based entirely on his ex-ante bias (the $\theta = 0$ agent) always helps. In the first case, where communication effort is completely restricted, filtering allows the agent types with important information (the $\theta = -1$ and $\theta = 1$ agents) to signal the importance of their information via channel selection (cheap talk). In the second case, filtering out of the middle type generates a positive reinforcing cycle of responsive decision making and large efforts to communicate information related to the extensive margin of the decision. Because, healthy-discussion equilibria incentivize communication effort precisely from the agent types who have useful information to communicate, item 3) of the lemma establishes that the lower cost channel should always be used as the discussion channel.

The table below lists expressions for the three welfare effects from (36) in each of the three optimal-PBE candidates[35]:

	Sorting	Expertise	Influence
plr, cl	$\frac{3f(1+x)}{4}$	$\frac{3f(1-x)(g-\frac{1}{2})}{4}\Delta^{plr,cl}$	0
plr, op	$\frac{3f(1+x)}{4}$	$\frac{3f(1-x)(g-\frac{1}{2})}{4}\Delta^{plr,op}$	$\frac{f[2-x(3+\frac{(1-f)\beta}{f})]^2}{4k}\left(\Delta^{plr,op}\right)^2$
hd, op	0	$\frac{3f\left(g-\frac{1}{2}\right)}{2}\Delta^{hd,op}$	$\frac{f\left(2-\beta^2\right)}{2k}\left(\Delta^{fd,op}\right)^2$

Table 2: Three Welfare Effects in SPB3 Setting[36]

The table highlights that the two polarized equilibria provide strong sorting effects, as the sign of $\alpha(\theta)$ is always conveyed to P via channel selection. The main difference

[35] The sorting effect in the table below includes the joint expected payoff both parties attain when the singleton pooling-group agent-type is realized, weighted by the probability of this type.

[36] Note $x = \frac{f/2}{1-f/2} = \Pr\{\theta = R|\theta \in \{C, R\}\}$

between the plr, cl- and plr, op-equilibrium is that further communication via communication effort only occurs in the latter equilibria. The plr, cl-equilibrium is therefore a way to eliminate costly influence while still maintaining some information transmission via cheap talk (channel selection).

While the hd, op-equilibrium forfeits the sorting effect, it has other welfare benefits. First, it can be shown that in the fd, op-equilibrium, the $\theta = 0$ type agent always obtains a negative ex-ante expected payoff. This is because he chooses a level of communication effort that is exactly equal to the average level of influence across all three types. Thus, in the SPB3 setting, the $\theta = 0$ agent plays the role of the moderate partisan who suffers a signal-jamming deadweight loss when pooled with agent types who bear more important information. The hd, op-equilibrium corrects for this welfare loss by allowing the middle type a channel in which he is not held to the same standards as users of the discussion channel. In particular, upon observing the channel used by the $\theta = 0$ agent in the hd, op-equilibrium, P will choose an investment of 0 following realization of either signal. Thus, there is no need for the $\theta = 0$ agent to exert any communication effort.

As suggested in the previous section, separation of the middle type has a second effect. Specifically, it induces the extreme agent types to communicate *more* with the principal. This is because, without the middle type, the principal is willing to make larger investments in each project following realization of either signal. This then increases the marginal gain from exerting communication effort for the $\theta = -1$ and $\theta = 1$-type agents. Thus, the expertise effect and influence effect in the hd, op-equilibrium will tend to be larger than these effects in the other two equilibria. Furthermore, because $\beta < 1$, the influence effect in this equilibrium will always be positive, which will not necessarily be the case in the plr, op-equilibrium for values of β close to 1.

I now state two propositions establishing sufficient conditions under which polarized and healthy-discussion equilibria are optimal. It is convenient to begin with examining the latter communication arrangement.

Proposition 6 *There exists a* $g^{crit} \in (\frac{1}{2}, 1)$ *and a* $f^{crit} \in (0, 1)$ *such that, if* $g > g^{crit}$ *and* $f < f^{crit}$*, the optimal equilibrium is the* hd, op*-equilibrium. If* $g < g^{crit}$ *then the* hd, op*-equilibrium cannot be the optimal equilibrium.*

Proof. See appendix. ■

When f is very small, the sorting effects in the two polarized equilibria are small as well, and communication effort becomes important for informed decision making. However, when $g < g^{crit}$, the fd, op-equilibrium will be strictly dominated by the $plr, cl-$equilibrium. In this case, incentives to communicate are so low that any information-transmission benefit provided by the hd, op-equilibrium cannot surpass the smallest possible sorting effect in the plr, cl-equilibrium. Because the plr, cl-equilibrium is always weakly larger than its sorting effect alone, the hd, op-equilibrium cannot perform better than the plr, cl-equilibrium when the agent does not have strong enough incentives to exert communication effort. These incentives will be diminished when P lacks confidence in her own signal, which occurs when g takes a value too close to $\frac{1}{2}$. Thus, optimality of the hd, op-equilibrium requires both frequent disagreements (f small), as well as a decision maker with some degree of personal expertise (g large).

When $g < g^{crit}$ the optimal equilibrium must either be the plr, cl-equilibrium or the plr, op-equilibrium. In this region, even though the hd, op-equilibrium cannot perform better than the plr, cl-equilibrium, the plr, op-equilibrium may still be optimal because it also allows a positive sorting effect, in addition to incentivizing communication effort.

The proposition below establishes a sufficient condition for the plr, op-equilibrium to be optimal in this region.

Proposition 7 *If*

$$x \in \left(\frac{1-\beta-\sqrt{1-2\beta-2\beta^2}}{3}, \frac{1-\beta+\sqrt{1-2\beta-2\beta^2}}{3} \right) \tag{48}$$

then $W^{plr,op} > W^{plr,cl}$*. If* $g < g^{crit}$ *holds as well, then the optimal equilibrium is the* plr, op*-equilibrium.*

Proof. See appendix. ■

Recall that $x = \frac{f}{2-f} = \Pr\{\theta = R | \theta \in \{0, 1\}\}$. The above proposition establishes that allowing the agent to influence decision making makes partisan pooling more attractive when the likelihood of the $\theta = 1$ type agent is moderate. The reason for this is once again the trade-off between the information transmission benefits of informative communication, and the standard signal-jamming deadweight loss of anticipated influence activities. When f is close to 0, influence is primarily used by the $\theta = 0$ type to jam $P's$ signal. Anticipating this, P simply adjusts her decision rule to account for the misaligned preferences, making communication effort largely a deadweight loss. Similarly, when f is close to 1, the $\theta = 1$ agent communicates excessively to prevent realization of $s = L$. Once again, since the principal anticipates this, communication effort becomes wasteful. On the other hand, when both agent types in the partisan pooling group are sufficiently likely, the levels of communication effort exerted by the two types are different enough for communication effort to play a useful information-transmission role.

When f falls in the interval specified in the proposition and $g < g^{crit}$, the plr, op-equilibrium will dominate both of the other two equilibria. Consider the case where there is no bias between the two party's marginal valuation for the decision, i.e. $\beta = 0$. In this case, the plr, op-equilibrium dominates the plr, cl-equilibrium whenever $f \in \left(0, \frac{2}{3}\right)$ which does not depend on any model parameters, particularly g. Therefore, in this case there always exists some g small enough so that plr, op-equilibrium is optimal.

Numerically, parameter vectors exist in which the optimal equilibrium is given by the plr, cl-equilibrium. In particular this occurs when $g < g^{crit}$ and β is large. However, the analytic expression describing this optimal region is not particularly informative. The figure below numerically computes the optimal PBE in f, g-space for small, medium, and large values for the agent's bias β. We can see that the optimal PBE regions in the numerical example follow closely the implications of the above two propositions:

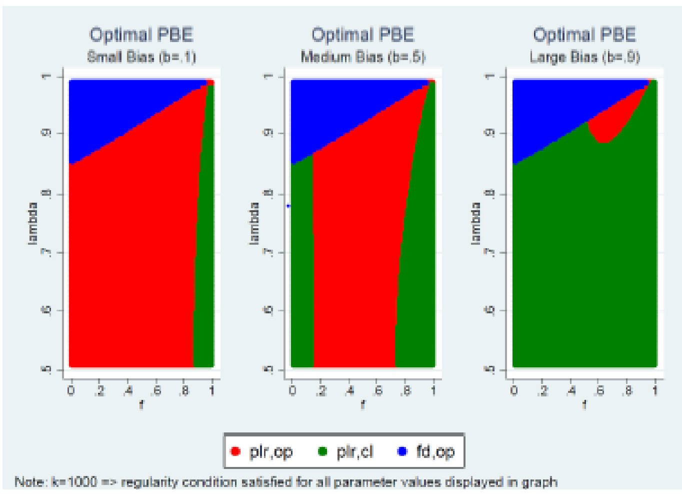

Figure 5: Optimal PBE in the SPB3 Setting

In all three panels of the figure above, the plr, op-equilibrium is optimal in the region shaded red, the plr, cl-equilibrium is optimal in the region shaded green, and the hd, op-equilibrium is optimal in the region shaded blue. It is easy to see that for small, medium, and large values of the agent's bias β, the hd, op equilibrium is optimal when the probability of an extreme type f is small and the accuracy of the principal's exogenous information, g, is large. This is consistent with the sufficient condition derived in proposition 3.

While the region in which the plr, op-equilibrium is optimal is largely invariant with respect to the bias term β,[37] comparing welfare across the other two equilibria requires examination of this parameter. When the bias is small (first panel), the plr, op-equilibrium dominates the plr, cl-equilibrium as long as f is not too large. As the bias term β gets larger (second and third panels), the region in which the plr, op-equilibrium dominates the plr, cl-equilibrium shrinks. Furthermore, this shrinkage occurs from both sides as both low and high values of f deteriorate the communication benefits provided by the plr, op-equilibrium. Again both these findings are consistent with the sufficient conditions derived in Proposition 4 above.

2.6.3. Restricting Channels

The organizational economics literature has previously noted that efficient organizational design may involve restricting communication rather than facilitating it. For example, Milgrom and Roberts (1988) suggest "closing communication channels for some decisions" to mitigate less productive forms of influence such as "buttering up the boss" for promotions or perks. Similarly, in Palida (2020a), I find that the optimal design of a single communication channel may involve deliberately making influence activities prohibitively costly or noisy.

In the context of the current model, one way to restrict communication is to shut down all but one communication channel, effectively forcing a full-pooling equilibrium. While the optimal equilibrium in the SPB3 setting always uses two channels, the assumption that parties can Pareto coordinate is a strong one. I now ask if there may be a reason to cut access to a particular communication channel to prevent realization of a "bad" equilibrium in which both channels are used.

Proposition 8 *As $f \to 1$, we have that $W^{fp,op} > W^{hd,cl}$*

Proof. As $f \to 1$ we have that $W^{fp,op} \to W^{hd,op} > W^{hd,cl}$ ■

The proposition above establishes that cutting the high-cost communication channel can prevent a potentially inferior equilibrium from forming. In particular, the fp, op-equilibrium is the unique equilibrium that would arise if the agent could not use the closed communication channel. When the probability of the $\theta = 0$ type agent is sufficiently low, there is little welfare loss in requiring this agent type pool with the two extreme types. However, allowing usage of the closed channel facilitates a Pareto dominated equilibrium in which the open communication channel is misused by the $\theta = 0$-type agent. Because the influenceable communication channel is flooded by the type who is not communicating

[37] In the figure, it appears as though the region in which the hd, op-equilibrium is optimal is invariant with respect to the bias parameter β. However, this has not yet been proven analytically.

useful information, P does not rely on her signal when she observes this channel being used. Consequently, those with useful information have no incentive to use that channel, and therefore keep their information to themselves.

The current model would suggest the restrictive communication practices mentioned above can serve a positive welfare role in certain situations. In particular, they can halt formation of inferior equilibria characterized by moderate partisans flooding superior channels without transmitting useful information.

2.6.4. Discussion: Applications

This section provides three main predictions regarding communication arrangements in real organizations. First, the model predicts that polarization across channels may be an optimal response to "bandwidth" concerns facing decision makers of large organizations, or unqualified management. As discussed in the next section, one example of this would be usage of primarily informal channels of communication within an organization, in which costless partisan endorsements (i.e. cheap talk) are used as the primary means of communication.

Relatedly, a Google employee was recently terminated for circulating an internal memo arguing that "the low number of women in technical positions was a result of biological differences instead of discrimination."[38] While there are likely many factors that contributed to the termination of this employee[39], the model suggests that firmly defining content expectations for a particular channel of communication may be a necessary measure to maintain some degree of communication when decision makers are not experts on a particular issue at hand.

A second prediction of the model is that many organizations that do make use of discussion channels, may also benefit from allowing outlets with less stringent restrictions on voicing personal opinions. Professional news channels and academic seminars resemble discussion channels for the journalistic and academic communities respectively. Nevertheless, both communities simultaneously make use of additional channels of communication as well. As mentioned previously, tabloid journalism has long coexisted alongside professional news networks. Relatedly, while scholarly seminars and articles are perhaps the channels that are most commonly associated with the academic community, researchers do have additional options for relaying their findings to their peers. Blog posts and non-refereed publications are common in academia, as well as informal conversations in passing or during conferences or mixers.

Given that the model predicts simultaneous use of partisan and non-partisan channels within an organization when decision makers have some degree of expertise themselves and irrelevant information is prevalent, the communication arrangements observed in the academic and journalistic communities are not inconsistent with what the model would predict to be characteristics of optimally designed organizations. The next section studies a special case of the model to examine the practice of explicit endorsements in organizations and communities.

Finally, the model predicts that it may be desirable for organization leaders to cut access to inferior communication channels (i.e. higher-cost communication channels). It actually is not uncommon for organizations to discourage usage of channels that may be considered inferior. E-mail communication is typically considered to be excessively

[38] Source: https://www.nytimes.com/2017/08/07/business/google-women-engineer-fired-memo.html
[39] e.g. PR concerns

impersonal and time consuming, leading many to suggest banning usage of this channel altogether. Indeed, organizations such as French IT company Atos, have actually implemented such bans, with Atos' CEO prohibiting internal E-mail for the company's 80,000 employees.[40] Additionally, it is common practice in professional organizations to require that certain topics be discussed using a specific protocol. For example, company policy may require that complaints about other coworkers be lodged in private face-to-face meetings, with a member of the legal team or human resources present.

The current model would suggest the restrictive communication practices mentioned above can serve a positive welfare role in certain situations. In particular, they can halt formation of inferior equilibria characterized by moderate partisans flooding superior channels without transmitting useful information.

2.7. Endorsements

An endorsement is an act of giving one's public approval of someone or something. Endorsements are observed in many organizational settings. Celebrities often endorse politicians, platforms, or products, publicly on various media sources. In doing so, they can provide both information about the underlying topic, as well as a direct statement regarding how they would like the audience to interpret the presented information. Endorsements are observed within firms as well. An company executive trying to push for a particular initiative will likely need to present evidence (e.g. statistics, news stories, anecdotes, etc.) to argue that the initiative is in the best interest of the company. While much of this evidence may to be subject to multiple interpretations, she may be able to guide these interpretations by saying up-front that she strongly believes the presented evidence supports the initiative.

However, direct endorsements would be unexpected, or even unacceptable, in other organizational settings. For example, a researcher may write academic paper to present a new methodology for solving an existing problem or provide a novel explanation for an observed behavior. Nevertheless, the language used in scholarly writing typically establishes the paper's results as being a contribution, extension, or alternative to existing results in related literature, rather than a better version or replacement. Similarly, in the case of professional journalism, news reports and articles may strategically choose words and expressions to target a degree of political slant (e.g. Gentzkow and Shapiro, 2006), but stories are typically communicated without reporters directly endorsing any particular side of an issue.[41]

What role do endorsements play in the transmission of information? Which organizations are likely to use the practice of endorsements and which are likely to abstain from such behaviors? Fortunately, we can use a special case of the model developed thus far to answer these two questions.

Let us assume that there are three communication channels $C = \{L, X, R\}$ where $k(c) = k$ for all $c \in C$. One interpretation of this setting is a situation in which there are three observably different channels which all have the same marginal cost of influence. For example, two of the channels could be given by the upward diagonal in Table 1.

Here, we will interpret this setting slightly differently. We will think of this setting as capturing an environment in which there is only one communication channel, but the

[40] Source: https://www.bbc.com/worklife/article/20150324-the-companies-that-banned-email

[41] See Ward (2019), among others, for more details on journalistic objectivity.

agent is also allowed to endorse either the L or R project costlessly. Selection of $c = X$ corresponds to a case where no endorsement is made by the agent. Selection of $c = L$ or $c = R$ corresponds to the agent endorsing the L project or R project, respectively. Note that, regardless of endorsement, the agent can still exert communication effort to influence the noisy signal s. Communication effort in this scenario can be interpreted as actions such as translating esoteric facts supporting the agent's preferred decision or discretely misrepresenting or hiding facts that support the the principal's investment in the agent's less preferred project.

The results from the previous sections highlight several qualitative predictions regarding the effects of endorsements on equilibrium behavior. Allowing direct endorsements on a particular channel can help mitigate bad misunderstandings in which complicated information can be easily misinterpreted. For example, political platforms are often multifaceted with various subplatforms for different issues. A direct endorsement of a politician allows some information transmission without requiring effort in explaining each of the subplatforms.

On the other hand, excessive usage of endorsements can crowd out more potentially more productive forms of information transmission. When a communication channel is flooded by individuals who only support a particular cause because of their ex-ante biases, it becomes harder for decision makers to determine the information content of signals generated on that channel. Thus, arguments that are supplemented with direct endorsements are more likely to be taken "with a grain of salt". Anticipating this lack of responsiveness, those with important information then have little incentive to exert effort in transmitting additional information (e.g. translating complicated statistical findings) to supplement their endorsement.

Analysis of optimal communication arrangements in the previous section implies direct predictions regarding how endorsements are likely to be used in different organizations. Organizations that must frequently make large-scale decision that must be closely adapted to a volatile environment may rely exclusively on endorsements (i.e. optimal polarization). An example of this could be a start up with limited financial reserves or an interest group dealing with a sensitive societal issue. Such organizations may resemble "rubber-stamping" arrangements where decision makers trust their experts enough to effectively make decisions themselves. Police radio codes can also be thought of as a form of an endorsement-driven communication arrangement. In this case, the code stated by the officer or dispatcher costlessly signals exactly what kind of service is required, eliminating the need to describe in detail the necessitating environment.

The results of the previous section additionally suggest that organization members may make direct endorsements in some cases, but not in others. Indeed, reporters may remain objective about a political candidate in stories written on behalf of the news network. Nevertheless, the reporter may express more partisan views on the candidate using other sources, e.g. social media, personal interviews, etc. Within firms, complaints against a coworker can be lodged in the form of an angry tirade or a formal summary of the events that took place.

One final point regarding endorsements should be made. While the SPB3 setting highlights that healthy-discussion equilibria may be optimal for a wide array of environments, the possibility of costless endorsements suggest that achieving it within an organization may not be a simple task. This is particularly true in cases where the principal and agent would both benefit ex-post if users of the discussion channel actually did reveal their partisan subgroup.[42] To ensure that moderate partisans are kept out of the discussion channel, however, the principal must make clear ex-ante that any form of endorsement will be ignored, even if there are circumstances in which they could be used more efficiently ex-post.

In the current model, discussion-channel users are essentially playing a babbling equilibrium in terms of their endorsement. Specifically, the principal assumes any endorsement is uncorrelated with the agent's type and therefore serves no purpose. Given this, no agent type has any incentive to try to inform the principal via an endorsement in the first place. Another form of endorsement prohibition, which is perhaps more relevant to the organizational applications described thus far, is that a violation of an established cultural tradition may lead to negative impressions regarding the violator's beliefs, preferences, or unobserved practices. For example, an academic who expresses extreme partisan views in a scholarly paper or during a formal seminar may be perceived as unprofessional or socially insensitive. The model currently does not allow for this interpretation, but can be easily extended with the inclusion of a "socially inept" agent type, that is realized with some probability.

2.8. Conclusion and Future Research

In this paper, I study how communication channels are chosen and used within organizations. In particular, I examine a noisy-signaling game between a principal (decision maker) and a privately-informed agent. The key novelty of my environment is that the agent can observably choose a communication channel, from a preselected menu, which determines the cost of his unobservable communication effort. The choice of communication channel therefore serves as an observable signal that supplements a separate noisy signal, in information transmission.

I find that channel selection itself serves as an informative signal of the agent's type. As a result, the agent's incentives to communicate using any particular channel depends not only on the cost of communication effort along that channel, but also on the equilibrium distribution of other agent types using the channel. Under relatively general assumptions, two forms of equilibria may arise on the Pareto frontier: *polarized* equilibria and *healthy-discussion* equilibria..

Polarized equilibria use only *partisan* channels consisting of like-minded supporters of a particular cause. In such equilibria, information transmission occurs primarily via observation of the selected communication channel, but responsiveness to observed signals may be dampened due to the decision maker's expectations of the user's partisan biases.

On the other hand, equilibria may take a healthy-discussion form, where moderate partisans filter into separate channels providing users of the discussion, or *non-partisan*, channel high-powered incentives to exert effort in communicating valuable information. Such equilibria can increase communication effort and responsiveness, precisely in the states where it is efficient to do so.

The model suggests that key determinants of the optimal communication arrangement are: 1) expertise of the decision maker; and 2) value in distinguishing important information from irrelevant information. Specifically, I find that polarization across communication channels may be an efficient response to "bandwidth" concerns facing executives of large organizations or unqualified leadership. Furthermore, I find that organizations

[42] The hd, op-equilibrium in the SPB3 setting is an example of such a situation.

that typically make use of non-partisan channels, e.g. research seminars in the academic community, may also benefit from allowing separate channels for moderate partisans to express their personal opinions, e.g. blog posts and non-refereed publications. The model also suggests that cutting inferior communication channels can assist in preventing realization of inferior equilibria, and provides a rational for the use of direct endorsements in organizations.

The issues discussed in this paper raise further questions. In particular, certain organizational choices often involve an individual deciding *who* to talk to, rather than *how* to talk to them. While the current paper focuses on the latter, one can ask how the model relates to one in which informed agents choose the audience of their communication efforts, rather than a communication channel.

Palida (2020b) shows that usage of a communication intermediary is strategically and welfare equivalent to playing a direct communication, noisy-signalling game in which influence is more costly but the principal is exogenously more of an expert. In other words, using a communication intermediary is like using a different communication channel. Thus, many of the results derived in this paper can be also be used to analyze situations in which organization members choose a path on which to send information along. Examples of this in organizations include, usage of skip-level or town-hall meetings in hierarchical organizations or back channel communication, such as gossip or rumors.

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Chapter 3

Communication Intermediaries

Abstract

Intermediaries are often used to manage information flow from experts to decision makers. This paper studies a model of strategic communication where communication is both costly and noisy (i.e. noisy signalling). In this setting, I find that communication intermediaries can improve organizational welfare in three scenarios: 1) When the preferences of the expert are badly misaligned with the decision maker, intermediaries can mitigate incentives for unproductive signal-jamming; 2) When the expert does not have a large stake in the decision, the intermediary can accept the burden of communication instead; and 3) When the intermediary has especially strong incentives to influence decision making, she can induce an equilibrium with significantly more information transmission and responsive decision making. The model provides a possible explanation for the variety of roles communication intermediaries play in different organizations, the correlation between control-rights and communication hierarchies in organizations, as well usage of third-party conflict resolution arrangements.

3.1. Introduction

Information flow is a crucial process in various organizations, and intermediaries are frequently observed managing interactions between experts and decision makers. Some communication hierarchies are formally or informally enforced as in the military or government. In other cases, these hierarchies are not directly enforced, but nevertheless closely follow the formal control-rights structure of the organization, e.g. communication via middle management in large firms. Still, other organizations use third-party intermediaries who are not even directly part of the organization's control hierarchy, e.g. human-resource representatives and professional conflict-resolution specialists.

Furthermore, casual observation suggests that communication intermediaries are used for different reasons in different organizations. In some organizations, intermediaries serve a restrictive role, mitigating unproductive lobbying or influence activities. For example, the time an employee spends "buttering up the boss" can detract from other more productive organizational activities, making it necessary to disconnect these individuals from those with formal authority. On the other hand, intermediation may be necessary to guide decision making if experts are unmotivated or unable to communicate ideas effectively. In fact, some intermediaries seem to be specifically hand-picked for their ability to facilitate conversation. Councillors, therapists, and conflict mediators are often hired specifically to break down barriers of communication between parties with conflicting interests.

The widespread usage of communication intermediaries in organizations, as well as the variance observed in the roles they play across organizations, introduces two questions regarding organizational communication: 1) What effect does intermediation have on how private information is transmitted?; and 2) when can the organization benefit from using a communication hierarchy as opposed to direct communication? Additionally, one would expect that the answers questions 1) and 2) should depend both on the environment the organization faces, as well as the qualities of available intermediaries, e.g. speaking skills, incentives, prior expertise on issue etc..

This paper uses a noisy-signalling model of strategic communication to examine the positive and normative effects of communication intermediation in organizations. Specifically, an expert, or agent, discovers a complicated piece of private information relevant to an decision that must be made by a principal. A crucial ingredient of the model is that communication in this setting is costly and noisy. In particular, attempts by the agent to communicate information require exertion of costly communication effort, e.g. preparing a slide show, translating esoteric facts, rehearsing, etc. Furthermore, this communication effort is observed by the receiver with noise, e.g. the audience of a seminar does not see time spent preparing for the seminar, but only the seminar itself.

We will focus on analysis of two design structures, or *communication structures*. In direct communication, the agent observes his private information and then plays a noisy-signalling game with the principal. In addition to direct communication, we will also assume that the organization has a third member, called an intermediary, who also has some stake in the decision (i.e. is a strategic player). In hierarchical communication, the agent plays a noisy-signalling game with the intermediary, who then plays a noisy-signalling game with the principal after observing an influenced signal from the agent. The paper focuses on comparing equilibrium and welfare properties across these two communication structures.

Formal modeling of cost and noise in communication reveals several novel results regarding communication intermediation. The noisy-signalling nature of communication generates a potential two-sided inefficiency in direct communication: The agent is incentivized to exert communication effort when he believes the principal will be responsive to her signals, while the principal will be responsive to her signals when she believes the agent has exerted sufficient communication effort. In other words, communication effort and responsiveness are strategic complements. Thus, if the agent has incentives to signal-jam in an unproductive way, e.g. exert effort in hiding or misrepresenting facts, the principal will respond excessively to unanticipated signals, further incentivizing unproductive signal jamming. On the other hand, if the agent lacks motivation to exert effort in communicating information that would be useful for the principal, she will be hesitant to respond to observed signals, further reducing incentives for costly but productive information transmission.

I find that intermediation can have large effects on equilibrium behavior and payoffs. In particular, the model highlights that intermediation brings two opposing effects on communication effort and responsiveness. The *hierarchy-of-no effect* captures the idea that noisy communication implies information is lost as it passes through hierarchical links, and this exerts downward pressure on communication effort incentives and responsive decision making. On the other hand, usage of an intermediary provides an additional informative signal, which from the perspective of the agent, is equivalent to exogenously decreasing the amount of noise the principal observes in her influenced signal. This *collaboration effect* works against the hierarchy-of-no effect and exerts upward pressure on communication effort incentives and responsive decision making.

Given that intermediation bears two opposing effects on equilibrium strategies, I find that usage of the intermediary can generate one of three optimal hierarchical forms, depending on the qualities of the intermediary. A *restrictive hierarchy* is generated by an intermediary with a low stake in decision making, and will reduce both communication effort on the part of the agent as well as the principal's responsiveness to signals. In a restrictive hierarchy, the hierarchy-of-no effect dominates the collaboration effect, but this may be desirable for the organization from an ex-ante perspective if the agent will bear a large signal-jamming deadweight loss under direct communication.

A *necessary hierarchy* is generated by an intermediary with moderate incentives to influence decision making. These hierarchies will stifle incentives for communication effort on the part of the agent, but will lead to more responsive decision making on the part of the principal. What occurs in this environment is that the hierarchy-of-no effect dominates the collaboration effect in terms of the agent's incentives, but the collaboration effect dominates the hierarchy-of-no effect from the perspective of the principal's incentives. I find that necessary hierarchies may be crucial for generating some degree of organizational adaptation, when experts are unmotivated or unable to communicate effectively or lack stake in decision making.

Finally, *facilitating hierarchies* are generated by intermediaries with strong incentives to influence decision making, and increase both communication effort and responsiveness relative to direct communication. This can either be because the intermediary has a very large stake in decision making himself, or because he is skilled in persuading decision makers at low a cost (e.g. may be a particularly eloquent speaker, or speak same "language" as principal). Facilitating hierarchies can spur significantly more productive information transmission when organizational incentives are well aligned from an ex-ante perspective (i.e. before the agent's information is revealed). However, they can exacerbate the signal-jamming deadweight loss when incentives between the principal and agent are badly misaligned.

The model provides a possible explanation for a variety communication practices observed in organizations. It speaks to variety of hierarchical forms observed in communication relationships across organizations, and establishes usage of different forms as welfare-optimal responses to various organizational environments. Furthermore, the model provides a possible rational for the connection between control hierarchies and communication hierarchies often observed in organizations, as well as usage of specialized third-party mediators as communication facilitating devices.

The remainder of the paper is structured as follows: Section 2 provides a summary of related literature, section 3 introduces the formal model, section 4 solves for the equilibrium in both direct and hierarchical communication. Section 5 formally describes the three hierarchical forms discussed above in the context of the model, and examines the welfare consequences of intermediation for a range of environments.

3.2. Related Literature

The current paper relates to several strands of literature. The broad field of information economics has introduced several models of strategic communication including cheap talk (e.g. Crawford and Sobel, 1982), signalling (e.g. Spence 1973), and verifiable information (e.g. Grossman, 1981; Milgrom, 1981). This paper uses a noisy-signalling model, first introduced in Matthews and Mirman (1983), to model strategic communication. While noisy-signalling models are not as commonly used as other models of strategic communication, adoption of this model is crucial for capturing the desired environment of costly and noisy communication, and has important implications regarding the usage of communication intermediaries.

Two relatively more recent papers in information economics, Ivanov (2010) and Ambrus, Azevedo, and Kamada (2013), both study models of hierarchical cheap talk. These papers also find that communication intermediaries may generate additional information transmission between two communicating parties, but only in mixed-strategy equilibria. In contrast, the current model does not admit any mixed-strategy equilibria and the mechanism through which intermediation improves communication is via providing incentives for costly information transmission, rather than garbling information.

The paper also contributes to various strands of literature in organizational economics. Closely related is the literature on influence activities in organizations. Key papers in this literature include Milgrom and Roberts (1988), Gibbons (2005), and Powell (2015). These papers focus on models of uninformed influence and, in the latter two, influence activities are purely a deadweight loss for the organization. In the setting I study, influence activities may be used by some agent-types for productive information transmission, rather than unproductive signal jamming.

Also related in the organizational economics literature is literature on authority and communication in organizations. Alonso, Dessein and Matouschek (2008); Rantakari (2008), and Friebel and Raith (2010) study settings in which delegation of control rights imply an endogenous communication structure within the organization. In the current paper, control rights are always held fixed with the decision maker. Instead I focus on the design of the noisy-signalling game played between the interested parties.

Finally, in two closely-related companion papers (Palida, 2020a, 2020b) I apply the core noisy-signalling framework used in this paper to study usage and design of communication channels in organizations (e.g. E-mail vs. face-to-face communication). Palida 2020a studies design of a single channel of communication, while Palida 2020b studies an environment where experts have multiple communication channels, i.e. multiple noisy-signalling games, to use at their discretion.

3.3. Economic Environment

In this section, I introduce the formal model and discuss its interpretations and assumptions. The core model is based on the one studied in Palida 2020a and 2020b, with the addition of a communication intermediary.

3.3.1. Model Set Up

The model has three players: a principal (P), an agent (A), and an intermediary (M). The principal is the decision maker who is responsible for making a decision $d \in \mathbb{R}$. Payoffs for P,A, and M depend on the decision d that is made by P, as well as an underlying state $\theta \in \Theta \subseteq \mathbb{R}$. P does not observe θ directly, but can potentially observe *one* of two informative signals, s_{AP} and s_{MP}.

As discussed shortly, the main design structure of this model will be determining which one of the two signals P sees. We will assume that Θ is discrete and finite, and $\Pr\{\theta = \theta_i\} = f(\theta_i)$ which is common knowledge to all players. Furthermore, we will assume that $E_\theta\{\theta\} = 0$, which will help simplify expressions.

The agent is a privately-informed expert who does observe the underlying state θ. Because A has personal interests in the decision d, we allow A to exert *communication effort* e_{AP} to influence the distribution of s_{AP} at cost $\frac{k_{AP}}{2}e_{AP}^2$. The subscript "AP" on these quantities denotes that they pertain to direct communication between the principal and agent.

Additionally, we will allow A to exert communication effort e_{AM} to influence the distribution of a separate signal that is observed only by M, s_{AM}. The cost of this effort is given by $\frac{k_{AM}}{2}e_{AM}^2$. We will assume that k_{AP} and k_{AM} are large enough so that it is never a rationalizable strategy for A to choose a level of communication effort that is large enough so that the distribution of either s_{AP} or s_{AM} becomes degenerate (see (49) and (50) below). This condition will be referred to the no-shifting-support condition for the agent, or NSS-A.

The intermediary (M) does not have private information of his own, but can attain some insight into the underlying state θ by observing s_{AM}. Because M also has interests in the decision d, we allow him to exert communication effort e_{MP} to influence the signal s_{MP}. Similar to the agent, we will assume that e_{MP} costs M an amount of $\frac{k_{MP}}{2}e_{MP}^2$, and that k_{MP} is large enough so that it is never a rationalizable strategy for M to fully degenerate s_{MP} (see (49) below). This condition will be referred to the no-shifting-support condition for the manager, or NSS-M. Note that NSS-A and NSS-M together imply that we are restricting attention to cases where it is simply too costly to completely eliminate noise in communication (see discussion at end of section).

We will focus on comparing two organizational design choices, or *communication structures*. In the first communication structure, $c = dc$ ("direct communication",) P only observes the signal s_{AP} and does not observe the signal s_{MP}. In the second, $c = hc$ ("hierarchical communication") P only observes the signal s_{MP} and does not observe the signal s_{AP}. This design environment can be interpreted as a situation in which bandwidth concerns prevent P from communicating directly with too many people (see discussion at end of section). We will assume that both of P's signals are binary with $s_{MP}, s_{AP} \in \{L, R\}$ and

$$\Pr\{s_{nP} = R|\theta, e_{nP}\} = e_{nP} + g_P(\theta) \tag{49}$$

for $n \in \{A, M\}$. We will assume a similar structure for the signal s_{AM} observed by M. In particular, we assume that s_{AM} also takes the value of either L or R, where

$$\Pr\{s_{AM} = R|\theta, e_{AM}\} = e_{AM} + g_M(\theta) \tag{50}$$

The functions $g_P(.)$ and $g_M(.)$ are P and M's *expertise* functions, which encode their ability to infer the underlying state without communication effort from the agent. We will assume that $g_P(.)$ and $g_M(.)$ are both increasing functions and that $E_\theta\{g_P(\theta)\} = E_\theta\{g_M(\theta)\} = \frac{1}{2}$. The increasing assumption is without loss of generality if $g_P(.)$ and $g_M(.)$ are monotonic. The symmetry assumption is to simplify expressions, and does not have a qualitative effect on results.

We can now state the general timing of this interaction as follows:

1. Communication structure $c \in \{dc, hc\}$ chosen

2. Nature chooses $\theta \in \Theta$, observed by A

3. A observes θ and chooses e_{AM} and e_{AP}

4. M observes s_{AM} and chooses e_{MP}

5. P observes $1_{\{c=dc\}} \cdot s_{AP}$ and $1_{\{c=hc\}} \cdot s_{MP}$, and chooses $d \in \mathbb{R}$

6. Payoffs realized

The notation $1_{\{c=dc\}} \cdot s_{AP}$ means that P sees only the signal s_{AP} if $c = dc$ is chosen in stage 1) of the game. Similarly, $1_{\{c=hc\}} \cdot s_{MP}$ means that P sees only the signal s_{MP} if $c = hc$ is chosen in stage 1 of the game. In section 5, we will study how the organization selects c in stage 1 of the game, based on the environment it faces as well as the incentives of the three players. But first, we will study both of these communication structures in isolation in section 4.

Payoffs in this game are given by the following:

$$u_P(\theta, d) = \theta d - \frac{1}{2}d^2 \tag{51}$$

$$u_A(\theta, d; c) = \alpha(\theta)d - \left(\frac{k_{AP}}{2}e_{AP}^2 + \frac{k_{AM}}{2}e_{AM}^2\right) \tag{52}$$

$$u_M(\theta, d; c) = \mu(\theta)d - \frac{k_{MP}}{2}e_{MP}^2 \tag{53}$$

In particular, note that P's preferred state-contingent decision is to set $d = \theta$, while A and M are "directional empire builders" who always prefer either $d = -\infty$ or $d = \infty$, depending on the sign of $\alpha(\theta)$ and $\mu(\theta)$. We will assume that both $\alpha(.)$ and $\mu(.)$ are increasing, so that there is partial preference, and that $E_\theta\{\alpha(\theta)\} = E_\alpha > 0$ and $E_\theta\{\mu(\theta)\} = 0$. The size of the parameter E_α captures the degree to which A disagrees with P regarding the extensive margin of the decision. This will be an important parameter in determining when an how intermediation should be used. Another important parameter will be $\sigma_{\alpha,\theta} = cov\{\alpha(\theta), \theta\}$, which captures the agent's stake in decision making. Specifically, when the slope of $\alpha(\theta)$ is large, the agent has strong incentives to influence decision making by transmitting useful information. Thus, the agent has strong incentives to influence decision making when either E_α or $\sigma_{\alpha,\theta}$ is large. As in Palida 2020a and 2020b, influence activities driven by the former will primarily be socially unproductive (signal jamming) while influence activities driven by the latter will be socially productive (information transmission).

For simplicity we assume that M does not have an extensive-margin bias relative to P ($E_\theta\{\mu(\theta)\} = 0$). The implications of the intermediary having an extensive-margin bias are very similar to that of the agent having the bias, and would not contribute very much to the analysis of this setting. However, the parameter $\sigma_{\mu,\theta} = cov\{\mu(\theta), \theta\}$ will be important, as this quantity captures the manager's stake in decision making, and thus his incentives to exert effort in transmitting useful information to P.

3.3.2. Discussion of Formal Setting

Several features of the model merit further discussion. First, a crucial component of the model is potential communication via an intermediary. In terms of interpreting this intermediary, the player M can capture several different kinds of organizational individuals. In the case of gossip or back-channel communication, M can be a coworker, peer, or acquaintance of A. In the case of upward communication, M can be a middle manager, supervising officer, or local government official. M can also be a third-party arbitrator or conflict resolution specialist. As we will see, the model allows for M to play any one of these roles, depending on his own characteristics and the environment the organization faces.

A second crucial component of this model is that communication is both costly and noisy. While this is non-standard in much of the strategic communication literature, I argue that noisy-signalling is a fundamental part of various communication interactions in organizations. An academic seminar is an extreme example of such an interaction, as attendees do not observe the speaker's preparation for the seminar (which may be considerable), but only the seminar itself. Analogous situations arise in firms and other organizations: Time spent preparing for meetings, writing memos, making slide shows, etc. are typically not observed directly, but are often expected and relied upon by decision makers. These can all be considered examples of communication that is both costly and noisy (i.e. noisy signalling).

In this model, the parameters k_{AP}, k_{AM}, and k_{MP} encode both the marginal effect and marginal cost of communication effort on observed signals. Thus, these parameters have various interpretations. For example, we may have $k_{MP} < k_{AP}$ if: 1) M has stronger communication skills than A; 2) M has access to less noisy communication channel than A (e.g. face-to-face meeting as opposed to E-mail, see Palida 2020a); or 3) P exogenously trusts M more due to pre-existing social capital. The results of this model do not require us to take a stance on what these parameters are actually capturing, and therefore the model can be applied to a variety of communication environments that resemble noisy-signalling.

Third, the design structures considered in this paper restrict attention to a case where the principal can observe only one of the two signals s_{AP} and s_{MP}. This is to capture the idea that decision makers often face bandwidth concerns that prevent them from having a direct line of communication with too many separate organization members. There are of course other interesting settings that this assumption rules out. For example, P may see separate signals from A and M if they are in different divisions, and each acquire their own information regarding a topic. Furthermore, A may have the option of sending information both directly *and* hierarchically (e.g. talk to middle manager *and* participate in town hall meeting). While these alternative settings are indeed realistic, I save their analysis for future research (see section 6).

The preference structure used in this model is also somewhat non-standard with respect to the strategic communication literature. While empire-building preferences are common in the internal capital markets literature (see section 5), applied models of strategic communication often use the "quadratic-bias" preference specification, where both the sender and receiver wish to match decision making to the underlying state, plus a player-contingent bias. This model adopts empire-building preferences partially for tractability and partially to capture realistic organizational settings in which experts and middle management often only care about the extensive margin of a decision. For example, a researcher may prefer a particular project to work on, but will always prefer as much funding as possible for his desired project. Similar incentives can be observed with branch managers of large firms, local politicians, leaders of specific task forces, etc. It is worth mentioning that both the current model's specification as well as the quadratic-bias specification are both special cases of a more general specification where both parties preferences are quadratic in the decision.

Finally, the model assumes a binary-signal specification for the principal's signal. While this may seem like a restrictive assumption, it can actually be shown that all results are qualitatively identical in a model with a richer signal-space.

3.4. Equilibrium and Welfare for Direct and Hierarchical Communication

In this section, we will characterize equilibrium strategies and derive expressions for the parties' payoffs in the two communication structures $c = dc$ and $c = hc$. Recall that in direct communication ($c = dc$), the principal does not communicate with M (does not observe s_{MP}). In hierarchical communication ($c = hc$) the principal does not communicate with the agent (does not observe s_{AP}).

3.4.1. Direct Communication

The effective timing of the interaction under direct communication can be written as follows[43]:

1. Nature chooses $\theta \in \Theta$
2. A observes θ and chooses e_{AP}
3. P observes s_{AP} and chooses $d \in \mathbb{R}$
4. Payoffs realized

Note that, because P does not observe s_{MP}, M has no reason to choose $e_{MP} \neq 0$. Given this, there is no incentive for A to choose $e_{AM} \neq 0$. Thus, we will simply ignore these quantities in analysis of direct communication. In particular, the PBE for $c = dc$ is characterized by a type-contingent effort profile $e^{dc}_{AP}(\theta)$ for the agent, and a signal contingent decision rule $d^{dc}(s_{AP})$ for the principal.

Suppose P believes A is playing communication-effort profile $e^{dc}_{AP}(\theta)$. Her best response requires her to choose $d^{dc}(s_{AP})$ to solve

$$\max_{d} E_{\theta}\{\theta | s_{AP}; e^{dc}_{AP}(.)\}d - \frac{1}{2}d^2 \tag{54}$$

which implies that we must have

$$d^{dc}(s_{AP}) = E_{\theta}\{\theta | s_{AP}; e^{dc}_{AP}(.)\} \tag{55}$$

in any PBE.

Similarly, if A believes P is playing decision rule $d^{dc}(s_{AP})$, he will must choose $e^{dc}_{AP}(\theta)$ to solve

$$\max_{e_{AP}} \alpha(\theta)[(1 - e_{AP} - g_P(\theta))d^{dc}(L) + (e_{AP} + g_P(\theta))d^{dc}(R)] - \frac{k_{AP}}{2}e^2_{AP} \tag{56}$$

which gives a first-order condition of

$$e^{dc}_{AP}(\theta) = \frac{\alpha(\theta)}{k_{AP}}\Delta^{dc} \tag{57}$$

where $\Delta^{dc} = d^{dc}(R) - d^{dc}(L)$.

[43] Please see Palida 2020a for a fuller treatment of this case.

The conditions (55) and (57) are necessary for a PBE under NSS-A. Combining these conditions, in any PBE we must have that

$$d^{dc}(L) = \frac{-\left(\sigma_{g_P,\theta} + \frac{\sigma_{\alpha,\theta}}{k_{AP}}\Delta^{dc}\right)}{\frac{1}{2} - \frac{E_\alpha}{k_{AP}}\Delta^{dc}} \tag{58}$$

$$d^{dc}(R) = \frac{\sigma_{g_P,\theta} + \frac{\sigma_{\alpha,\theta}}{k_{AP}}\Delta^{dc}}{\frac{1}{2} + \frac{E_\alpha}{k_{AP}}\Delta^{dc}} \tag{59}$$

which then implies that PBE are characterized by all Δ^{dc} that satisfy

$$\Delta^{dc} = \frac{\sigma_{g_P,\theta} + \frac{\sigma_{\alpha,\theta}}{k_{AP}}\Delta^{dc}}{\frac{1}{4} - \left(\frac{E_\alpha}{k_{AP}}\Delta^{dc}\right)^2} \tag{60}$$

but do not lead to strategies that violate probability bounds on s_{AP}.

Proposition 9 *There exists a unique Δ^{dc} that satisfies (60) and does not violate probability bounds on s_{AP}.*

Proof. See Palida 2020a, Proposition 1. ■

In direct communication there is a unique PBE characterized by the equilibrium decision spread Δ^{dc}. It can also be shown that Δ^{dc} will be decreasing in k_{AP} and increasing in $\sigma_{g_P,\theta}, \sigma_{\alpha,\theta}$, and E_α.

Uniqueness of equilibria in direct communication implies a unique vector of equilibrium payoffs for all three parties. We can compute these expected payoffs solely as functions of Δ^{dc} and exogenous model parameters. In particular, using the conditions (55), (57), and the expressions for $d^{dc}(L)$ and $d^{dc}(R)$, we have that payoffs for the three parties' are given by:

$$W_P^{dc} = \frac{\sigma_{g_P,\theta}}{2}\Delta^{dc} + \frac{\sigma_{\alpha,\theta}}{k_{AP}}\left(\Delta^{dc}\right)^2 \tag{61}$$

$$W_A^{dc} = \sigma_{g_P,\alpha}\Delta^{dc} + \frac{\sigma_\alpha^2 - (E_\alpha)^2}{2k_{AP}}\left(\Delta^{dc}\right)^2 \tag{62}$$

$$W_M^{dc} = \sigma_{g_P,\mu}\Delta^{dc} + \frac{\sigma_{\alpha,\mu}}{k_{AP}}\left(\Delta^{dc}\right)^2 \tag{63}$$

We will typically focus on the sum of the parties expected payoffs as the welfare criterion of interest.[44] Using the expressions above, this quantity can be expressed as

$$W^{dc} = \underbrace{\frac{\sigma_{g_P,\theta}\left(\frac{\sigma_{g_P,\theta}}{2} + \sigma_{g_P,\alpha} + \sigma_{g_P,\mu}\right)}{\frac{1}{4} - \left(\frac{E_\alpha}{k_{AP}}\Delta^{dc}\right)^2}}_{\text{Expertise effect}} + \underbrace{\left[\frac{\frac{\sigma_{\alpha,\theta}}{k_{AP}}\left(\frac{\sigma_{g_P,\theta}}{2} + \sigma_{g_P,\alpha} + \sigma_{g_P,\mu}\right)}{\frac{1}{4} - \left(\frac{E_\alpha}{k_{AP}}\Delta^{dc}\right)^2} + \left(\frac{\sigma_{\alpha,\theta} + \sigma_\alpha^2 + 2\sigma_{\alpha,\mu} - (E_\alpha)^2}{2k_{AP}}\right)\Delta^{dc}\right]\Delta^{dc}}_{\text{Influence effect}} \tag{64}$$

[44] Note that in direct communication, the intermediary does not actually participate in the interaction. For completeness, we will still include his payoff in discussions of welfare.

As discussed in Palida 2020a, the expertise effect can be interpreted as the welfare the three parties would attain if no influence occurred, but P instead observed a signal $s' \in \{L, R\}$ where

$$\Pr\{s' = R\} = g_P(\theta) + \frac{E_\alpha}{k_{AP}} \Delta^{dc} \tag{65}$$

and made her decision d solely off of the realization of s'. The expertise effect is essentially saying that knowledge of the agent's ex-ante incentives to signal jam, exogenously makes P's signal more informative (even if A never observes θ). Importantly, the expertise effect is always increasing in Δ^{dc}, i.e. more communication effort/responsiveness always increases the expertise effect. The reason for this is as follows: If P knows that, on average, A will be trying to influence realization of $s_{AP} = R$ (recall $E_\alpha > 0$ by assumption) then signal realizations of $s_{AP} = L$ become extremely informative, incentivizing P to choose $d^{dc}(L) << 0$ precisely in situations where it would be beneficial for both parties (recall $\sigma_{a,\theta} > 0$). Thus, while parties loose when $s_{AP} = R$ is realized, they all gain relatively more when $s_{AP} = L$ is realized.[45]

The second effect, the influence effect, captures both the additional welfare gain associated with the agent influencing s_{AP} conditional on being informed of the underlying state θ, as well as the total expected cost of communication effort. For the principal, this effect is always positive. Thus, in this model, the principal always benefits from the agent's influence.

The same cannot be said for the agent, however. For the agent, the influence effect may either be increasing or decreasing in Δ^{dc}. The key determinant of the sign for this derivative is the size of the agents ex-ante bias, E_α, relative to the correlation between the two parties' preferences, captured by the parameter $\sigma_{\alpha,\theta}$ (i.e. the expected slope of $\alpha(\theta)$). When the former dominates the latter, the agent has strong incentives to communicate, but not to transmit useful information. Instead the agent primarily exerts communication effort to distort P's signal so that $s_{AP} = R$ is realized more frequently than without influence. This implies that, when $s_{AP} = R$ is indeed realized, P anticipates that this is largely due to uninformative signal-jamming and dampens her responsiveness to this signal realization. Thus, communication effort largely becomes a deadweight loss (see Gibbons, 2005, and Powell, 2015). On the other hand, when $\sigma_{\alpha,\theta}$ is large relative to E_α, communication effort is primarily used for information transmission as opposed to unproductive signal jamming. So in this case, the agent's utility is strictly increasing in Δ^{dc} as well.

In sum, communication effort and responsiveness under direct communication can be either too high or too low. In the former case, the agent's ex-ante bias primarily determines his state-contingent preferences (E_α large relative to $\sigma_{\alpha,\theta} = \sigma_{\alpha,\theta}$), so that communication effort is typically unproductive signal jamming that bears a deadweight loss for the agent. On the other hand, when the agent does not have a large ex-ante bias (E_α small) but lacks stake in the decision ($\sigma_{\alpha,\theta}$ small as well), communication effort would be used for information transmission, but the agent lacks motivation to exert it. A key focus of the next section is studying how communication intermediation can be used to manage this two-sided inefficiency.

[45] The net welfare increase is because the function $f(x) = x(1-x)$ is maximized at $x = \frac{1}{2}$.

Before concluding, we will rewrite (64) in a more compact form by combining the expertise effect with the first term of the influence effect:

$$W^{dc} = \left(\frac{\sigma_{g_P,\theta}}{2} + \sigma_{g_P,\alpha} + \sigma_{g_P,\mu}\right)\Delta^{dc} + \left(\frac{\sigma_{\alpha,\theta} + \sigma_\alpha^2 + 2\sigma_{\alpha,\mu} - (E_\alpha)^2}{2k_{AP}}\right)\left(\Delta^{dc}\right)^2 \tag{66}$$

This expression will be useful in comparing welfare outcomes between direct communication and hierarchical communication. With perhaps some abuse of notation, when welfare is written in this form, I will also refer to the first term as the expertise effect and the second as the influence effect.

3.4.2. Hierarchical Communication

We now study the second design structure, hierarchical communication ($c = hc$). The effective timing of the interaction can be expressed as follows:

1. Nature chooses $\theta \in \Theta$, observed by A
2. A observes θ and chooses e_{AM}
3. M observes s_{AM} and chooses e_{MP}
4. P observes s_{MP} and chooses $d \in \mathbb{R}$
5. Payoffs realized

Recall that because P does not observe s_{AP}, there is no incentive for A to choose $e_{AP} \neq 0$ under hierarchical communication. This, PBE in this communication structure are characterized by a type-contingent effort profile for the agent $e^{hc}_{AM}(\theta)$, a signal-contingent effort rule for the intermediary $e^{hc}_{MP}(s_{AP})$, and a signal-contingent decision rule for the principal $d^{hc}(s_{MP})$.

Suppose P observes s_{MP} and believes A and M are playing according to $e^{hc}_{AM}(.)$ and $e^{hc}_{MP}(.)$ in a PBE. The we must have that $d^{hc}(s_{MP})$ solves

$$\max_d E_\theta\{\theta|s_{MP}; e^{hc}_{AM}(.), e^{hc}_{MP}(.)\}d - \frac{1}{2}d^2 \tag{67}$$

or

$$d^{hc}(s_{MP}) = E_\theta\{\theta|s_{MP}; e^{hc}_{AM}(.), e^{hc}_{MP}(.)\} \tag{68}$$

Moving backwards, suppose M observed s_{AM}. He will choose $e^{hc}_{MP}(s_{AM})$ to solve

$$\max_{e_{MP}} E_\theta\{\mu(\theta)[d^{hc}(L) + (g_P(\theta) + e_{MP})\Delta^{hc}]|s_{MP}; e^{hc}_{AM}(.)\} - \frac{k_{MP}}{2}e^2_{MP} \tag{69}$$

where $\Delta^{hc} = d^{hc}(R) - d^{hc}(L)$. The first-order condition of the above gives us that $e^{hc}_M(s_M)$ must satisfy

$$e^{hc}_{MP}(s_M) = \frac{E_\theta\{\theta|s_{AM}; e^{hc}_{AM}(.)\}}{k_{MP}}\Delta^{hc} \tag{70}$$

Under NSS-M, condition (70) is necessary in any PBE. Denote the equilibrium quantity $\delta^{hc} = e^{hc}_M(R) - e^{hc}_M(L)$ to be M's equilibrium effort spread.

We can now turn to A's optimization. A will choose $e^{hc}_{AM}(\theta)$ to solve

$$\max_{e_{AM}} \alpha(\theta) \quad \{d^{hc}(L) + [(1 - g_M(\theta) - e_A)(e^{hc}_{MP}(L) + g_P(\theta)) + (e_{AM} + g_M(\theta))(e^{hc}_{MP}(R) + g_P(\theta))]\Delta^{hc}\} - \frac{k_{AM}}{2} e^2_{AM} \tag{71}$$

which has a first-order condition of

$$e^{hc}_{AM} = \frac{\alpha(\theta)}{k_{AM}} \delta^{hc} \Delta^{hc} \tag{72}$$

Under NSS-A we have that (72) is necessary in a PBE.

In the case of hierarchical communication, all equilibrium strategies are characterized by the two equilibrium sufficient statistics δ^{hc} and Δ^{hc}. Using (72) and Bayes' rule, we can show that

$$E_\theta\{\theta|L; e^{hc}_{AM}(.)\} = \frac{-\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_{AM}}\delta^{hc}\Delta^{hc}\right)}{\frac{1}{2} - \frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta^{hc}} \tag{73}$$

and

$$E_\theta\{\theta|R; e^{hc}_{AM}(.)\} = \frac{\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_{AM}}\delta^{hc}\Delta^{hc}}{\frac{1}{2} + \frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta^{hc}} \tag{74}$$

This then implies that

$$\delta^{hc} = \frac{\Delta^{hc}}{k_{MP}} \left[\frac{\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_{AM}}\delta^{hc}\Delta^{hc}}{\frac{1}{4} - \left(\frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta^{hc}\right)^2} \right] \tag{75}$$

Similarly, we can show that

$$d^{hc}(L) = -2\left[\sigma_{g_P,\theta} + \left(\sigma_{g_M,\theta} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\delta^{hc}\Delta^{hc}\right)\delta^{hc}\right] \tag{76}$$

and

$$d^{hc}(R) = 2\left[\sigma_{g_P,\theta} + \left(\sigma_{g_M,\theta} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\delta^{hc}\Delta^{hc}\right)\delta^{hc}\right] \tag{77}$$

Using the above expressions, we can show that

$$\Delta^{hc} = \frac{\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc}}{\frac{1}{4} - \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2} \tag{78}$$

Denote the RHS of the above as $\Delta(\delta^{hc})$. We can then characterize all equilibria of the model by solving the single-variable equation

$$\delta^{hc} = \frac{\Delta(\delta^{hc})}{k_{MP}} \left[\frac{\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_{AM}}\delta^{hc}\Delta(\delta^{hc})}{\frac{1}{4} - \left(\frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta(\delta^{hc})\right)^2} \right] \tag{79}$$

and checking that

$$\frac{\alpha(\theta)}{k_{AM}}\delta^{hc}\Delta(\delta^{hc}) + g_M(\theta) \in [0, 1] \tag{80}$$

for all $\theta \in \Theta$, and

$$\frac{E_\theta\{\theta|s_{AM}; e^{hc}_{AM}(.)\}}{k_{MP}}\Delta^{hc} + g_P(\theta) \in [0,1] \tag{81}$$

for $s_{AM} \in \{L, R\}$ and all $\theta \in \Theta$.

It is strongly conjectured that the RHS of (79) is a contraction mapping on the compact subset of $\mathbb{R}$ implied by (80), and (81). However, a formal proof of this has not been yet established. Nevertheless, we can characterize properties of a particular, intuitive PBE. This PBE will be referred to as the "intervention PBE" and is given by the limiting outcome of the following strategy-updating process:

1. P and A start by playing the strategies specified in the direct-communication equilibrium (characterized by Δ^{dc})
2. M best responds to P and A's strategies
3. P updates strategy to best respond to $A's$ strategy from 1 and M's strategy from 2
4. A updates strategy to best respond to M's communication effort from 2 and P's decision rule from 3
5. Steps 2-4 are repeated

Proposition 10 *The intervention PBE always exists and is unique. In other words the process described in steps 1-5 above, converge to a unique strategy profile characterized by the equilibrium quantities* δ^{hc} *and* Δ^{hc}

Proof. See appendix. ■

The intervention PBE can be thought of as the outcome that parties would arrive at if they started with direct communication, added an intermediary, and updated their strategies in a repeating sequential order of M, P, A. As mentioned above, it is strongly conjectured that the intervention PBE is the unique PBE of the game. However, showing that the RHS of (79) is a contraction mapping is algebraically tedious. Because the intervention PBE has a natural interpretation of a gradual adjustment process to an organizational design change, I will restrict attention to this PBE when discussing hierarchical communication. Any PBE of the hierarchical communication game discussed henceforth should be interpreted to be the intervention PBE. The corollary below describes comparative-static effects in this PBE:

Corollary 3 *The quantities* δ^{hc} *and* Δ^{hc} *are both decreasing in* k_{MP} *and increasing in* $\sigma_{g_M,\theta}$ *and* $\sigma_{\mu,\theta}$.

Proof. See appendix. ■

In hierarchical communication, welfare for the three parties will be denoted

$$\begin{aligned}
W^{hc}_P &= E_{\theta,s_{AM},s_{MP}}\{\theta d^{hc}(s_{MP}) - \frac{1}{2}\left(d^{hc}(s_{MP})\right)^2 | e^{hc}_{AM}(.), e^{hc}_{MP}(.).d^{hc}(.)\} \\
W^{hc}_A &= E_{\theta,s_{AM},s_{MP}}\{\alpha(\theta) d^{hc}(s_{MP}) - \frac{k_{AM}}{2}\left(e^{hc}_{AM}(\theta)\right)^2 | e^{hc}_{AM}(.), e^{hc}_{MP}(.).d^{hc}(.)\} \\
W^{hc}_M &= E_{\theta,s_{AM},s_{MP}}\{\mu(\theta) d^{hc}(s_{MP}) - \frac{k_{MP}}{2}\left(e^{hc}_{MP}(s_{AM})\right)^2 | e^{hc}_{AM}(.), e^{hc}_{MP}(.).d^{hc}(.)\}
\end{aligned}$$

We now prove a useful accounting result which links welfare properties under hierarchical-communication with a corresponding direct-communication game. Suppose the PBE of the hierarchical communication game is characterized by δ^{hc} and Δ^{hc}. Let us define an alternative cost parameter for the direct communicating setting as

$$\widehat{k_{AP}} = \frac{k_{AM}}{(\delta^{hc})^2} \tag{82}$$

Similarly, let us define an alternative expertise function for the principal:

$$\widehat{g_P}(\theta) = g_P(\theta) + e^{hc}_{MP}(L) + g_M(\theta)\delta^{hc} \tag{83}$$

The quantity $\widehat{g_P}(\theta)$ captures the probability $s_{MP} = R$ in the intervention PBE, given that the agent (unprofitably) deviates to exerting no communication effort. Thus, we have that $\widehat{g_P}(\theta) \in (0,1)$ for all $\theta \in \Theta$.

Consider a direct-communication game in which A must pay an influence cost of $\widehat{k_{AP}}$ and the distribution of s_{AP} is characterized by

$$\Pr\{s_{AP} = R|\theta, e_{AP}\} = e_{AP} + \widehat{g_P}(\theta)$$

In this game, equilibrium strategies and payoffs will be characterized by the equilibrium quantity $\widehat{\Delta^{dc}}$ which solves:

$$\widehat{\Delta^{dc}} = \frac{\sigma_{\widehat{g_P},\theta} + \frac{\sigma_{\alpha,\theta}}{\widehat{k_{AP}}}\Delta^{dc}}{\left(E\widehat{g_P} + \frac{E_\alpha}{\widehat{k_{AP}}}\Delta^{dc}\right)\left(1 - E\widehat{g_P} - \frac{E_\alpha}{\widehat{k_{AP}}}\Delta^{dc}\right)} \tag{84}$$

As shown in Palida 2020b[46], the above expression also implies a unique PBE which corresponds to the median of the three solutions to (84). We will denote payoffs to the three players in this unique PBE as $\widehat{W^{dc}_P}, \widehat{W^{dc}_A}$, and $\widehat{W^{dc}_M}$.

Proposition 11 *We have that*

$$\Delta^{hc} = \widehat{\Delta^{dc}}$$

and

$$\begin{aligned} W^{hc}_P &= \widehat{W^{dc}_P} \\ W^{hc}_A &= \widehat{W^{dc}_A} \\ W^{hc}_M &= \widehat{W^{dc}_M} - K^{hc} \end{aligned}$$

where

$$K^{hc} = \frac{k_{MP}}{2} E_{\theta,s_{AM}}\{\left(e^{hc}_{MP}(s_{AM})\right)^2 | e^{hc}_{AM}(.)\}$$

Proof. See appendix. ■

The implication of the above proposition is as follows: We can think of the hierarchical-communication setting as the parties' paying a cost of intermediation K^{hc}, and then

[46] Palida 2020a introduces the proof for a symmetric environment ($E_\theta\{\theta\} = 0$, and $E_\theta\{g(\theta)\} = \frac{1}{2}$), while Palida 2020b generalizes the proof to non-symmetric envirionments. Non-symmetric environments are necessary to consider when studying usage of communication intermediaries or mulitple communication channels.

switching to a different direct communication game where A bears a larger cost of influence ($\delta^{hc} \in (0,1)$ by NSS-M), but P is exogenously more of an expert ($\sigma_{\widehat{g_P},\theta} > \sigma_{g_P,\theta}$). In what follows, we will refer to the former effect as the *hierarchy-of-no effect* and the second as the *collaboration effect*.[47] These two effects will be very important in determining how and when intermediation should be used between two communication parties. They are discussed in more detail in the discussion section at the end of the section.

For now, let us note that we can easily derive analytic expressions for the three parties' payoffs as

$$W_P^{hc} = \frac{\sigma_{\widehat{g_P},\theta}}{2}\Delta^{hc} + \frac{\sigma_{\alpha,\theta}}{\widehat{k_{AP}}}\left(\Delta^{hc}\right)^2 \tag{85}$$

$$W_A^{hc} = \sigma_{\widehat{g_P},\alpha}\Delta^{hc} + \frac{\sigma_\alpha^2 - (E_\alpha)^2}{2\widehat{k_{AP}}}\left(\Delta^{hc}\right)^2 \tag{86}$$

$$W_M^{hc} = \sigma_{\widehat{g_P},\mu}\Delta^{hc} + \frac{\sigma_{\alpha,\mu}}{\widehat{k_{AP}}}\left(\Delta^{hc}\right)^2 - K^{hc} \tag{87}$$

where in this case we can show that

$$K^{hc} = \frac{1}{2}\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_{AM}}\delta^{hc}\Delta^{hc}\right)\delta^{hc}\Delta^{hc} \tag{88}$$

As before, we will typically focus on joint expected payoffs as the welfare criterion. Therefore, it is useful to write the expression for this quantity as well:

$$\begin{aligned} W^{hc} = & \left[\frac{\sigma_{g_P,\theta}}{2} + \sigma_{g_P,\alpha} + \sigma_{g_P,\mu} + \left(\frac{\sigma_{g_M}}{2} + \sigma_{g_M,\alpha} + \frac{\sigma_{g_M,\theta}}{2}\right)\delta^{hc}\right]\Delta^{hc} \\ & + \left(\frac{\sigma_{\alpha,\theta} + \sigma_\alpha^2 + \sigma_{\alpha,\mu} - (E_\alpha)^2}{2k_{AM}}\right)\left(\delta^{hc}\Delta^{hc}\right)^2 \end{aligned} \tag{89}$$

One can observe both the hierarchy-of-no and collaboration effects in the above expression, by comparing in to (66). The addition of the term $\left(\frac{\sigma_{g_M,\theta}}{2} + \sigma_{g_M,\alpha} + \frac{\sigma_{g_M,\mu}}{2}\right)\delta^{hc}$ in the first term of (89) arises from the collaboration effect, while a discount factor of $\left(\delta^{hc}\right)^2$ is added onto the second term due to the hierarchy-of-no effect.[48][49]

3.4.3. Discussion: Opposing Effects

A key result of this section is that usage of a communication intermediary has two opposing effects on the parties' strategies and payoffs: the hierarchy-of-no effect and the collaboration effect.

The hierarchy-of-no effect is quite intuitive. As additional layers of hierarchy are added between the source of information and decision makers, the noise in communication causes information to be lost every time it is transferred. This results in weaker equilibrium incentives for responsive decision making, which further diminishes incentives for experts to exert communication effort.

Indeed such concerns are observed in real organizations. Burkus (2012) argues that "as an idea moves through the different levels, the likelihood of rejection increases, since those

[47] Burkus (2012) attributes first usage of the term "Hierarchy-of-no" to Vanderbilt professor Dave Owens.

[48] Note that also $\Delta^{hc} \neq \Delta^{dc}$ in general.

[49] Note that, the above expression can also be written in terms of the expertise and influence effects, by additiviely separating the multiplier Δ^{hc} in the first term.

managers are further from the domain the idea applies to and less likely to understand its true value in that domain". Burkus (2012) additionally argues that such communication hierarchies are often derived from control-rights hierarchies. For example, employees may naturally surface issues with their immediate supervisors rather than upper-level executives. In the next section, I apply the framework developed thus far to examine why this natural linkage between control rights and "communication rights" may arise. I also study why such restrictive communication practices may actually be desirable for some organizations.

The second effect, the collaboration effect, is perhaps less intuitive at first glance. While the hierarchy-of-no effect exerts downward pressure on equilibrium communication effort and responsiveness, the collaboration effect boosts incentives for these actions. The effect can be interpreted as follows. The intermediary's processing of the agent's communication effort provides an additional opinion, or informative signal. When the principal communicates with the intermediary, she anticipates that the intermediary is influencing her signal based on informative signals the intermediary has observed. Thus, she will infer that her signal is also more informative than it would have been without the intermediary. From the point of view of the agent, this is equivalent to exogenously boosting the exogenous expertise of the decision maker. Thus, the collaboration effect exerts upward pressure on incentives to exert communication effort, as well as overall responsiveness to signals. Note that from (89) we can see that the collaboration effect of intermediation goes to zero, as the intermediary's expertise ($\sigma_{g_M,\theta}$) goes to zero.

Various organizational practices seem to leverage the collaboration effect of intermediation. Large organizations often make use of third-party mediators for conflict resolution. human-resource representatives and Ombudspersons are often used to resolve internal disputes. Additionally, therapists, councillors, and social workers are also used to facilitate communication between two disagreeing parties in families, schools, and governments. The purpose of these communication intermediaries is precisely to improve information transmission and responsive decision making. The next section will study when such arrangements are desirable and/or feasible, and how qualities of the intermediary affect how they operate in this collaborative role.

3.5. Comparison of Structures

In this section, we focus on comparing the two communication structures analyzed in the previous section: direct communication ($c = dc$) and hierarchical communication ($c = hc$). We will compare both positive implications about equilibrium strategies, as well as normative implications regarding optimal communication design structures.

For expositional convenience, we will work with a slightly more parameterized version of the model. First, we will assume that all functions of θ are linear. In particular, we will assume that the agent's and intermediary's preference functions are given as

$$\alpha(\theta) = \alpha\theta + \beta \tag{90}$$
$$\mu(\theta) = \mu\theta \tag{91}$$

The parameters α and μ determine the agent's and intermediary's stake in the decision d. We will assume that $\alpha, \mu > 0$, to maintain the partial preference alignment assumption. The parameter β captures A's ex-ante bias. As before, we will assume (without loss of generality) that $\beta > 0$, so that, relative to P and M, A has an ex-ante bias for positive decisions ($d > 0$).

Additionally, we will assume that P and M's exogenous-expertise functions are given by $g_P(\theta) = g_M(\theta) = g(\theta)$ where

$$g(\theta) = \frac{1}{2} + \left(g - \frac{1}{2}\right)\theta$$

with $g \in (0, \frac{1}{2})$. Also, we will assume that $\Theta = [-1, 1]$. Thus, we have $g(1) = g, g(-1) = 1 - g$, and $g(0) = \frac{1}{2}$. The assumption that $g_P(\theta) = g_M(\theta)$ is to reduce parameters.[50] Finally, also to reduce parameters and simplify notation, we will assume that $k_{AP} = k_{AM} = k_A$ and, for consistency, write $k_{MP} = k_M$.

Recall that equilibrium strategies and payoffs under direct communication structure are characterized by the equilibrium decision spread Δ^{dc}. On the other hand, strategies and payoffs in the hierarchical communication case are characterized by the equilibrium effort spread for the intermediary, δ^{hc}, and the equilibrium decision spread Δ^{hc}. Given the parametric assumptions above, these key quantities are given implicitly, in terms of exogenous parameters, by a solution to the system below:

$$\begin{aligned}
\Delta^{dc} &= \frac{\left(g + \frac{\alpha}{k_A}\Delta^{dc} - \frac{1}{2}\right)\sigma_\theta^2}{\frac{1}{4} - \left(\frac{\beta}{k_A}\Delta^{dc}\right)^2} \\
\delta^{hc} &= \frac{\mu}{k_M}\left[\frac{\left(g + \frac{\alpha}{k_A}\delta^{hc}\Delta^{hc} - \frac{1}{2}\right)\sigma_\theta^2}{\frac{1}{4} - \left(\frac{\beta}{k_A}\delta^{hc}\Delta^{hc}\right)^2}\right]\Delta^{hc} \\
\Delta^{hc} &= \frac{\left(g - \frac{1}{2}\right)\left(1 + \delta^{hc}\right)}{\frac{1}{4\sigma_\theta^2} - \frac{\alpha}{k_A}\left(\delta^{hc}\right)^2}
\end{aligned} \tag{92}$$

From the above, we can see that there are several parameters of interest. The ratio's $\frac{\alpha}{k_A}$ and $\frac{\beta}{k_A}$ capture the agent's incentives to exert communication effort for the purposes of productive information transmission and signal-jamming, respectively. Similarly, the ratio $\frac{\mu}{k_M}$ captures the intermediary's incentives to transmit information received from his signal s_{AM}. As we have assumed $E_\theta\{\mu(\theta)\} = 0$, M only exerts communication effort for information transmission.[51] Our primary results comparing regarding equilibrium strategies and welfare, across communication structures, will depend on how these parameters relate to each other.

[50] One can consider a case where $g_M(\theta) \neq g_M(\theta)$ but both are still linear. In this case we will have

$$\begin{aligned}
g_P(\theta) &= \frac{1}{2} + \left(g - \frac{1}{2}\right)\theta \\
g_M(\theta) &= \frac{1}{2} + \left(g + x - \frac{1}{2}\right)\theta
\end{aligned}$$

In terms of comparing communication structures, the effect of the parameter "x" will be very similar to the effect of the parameter μ.

[51] Formally, we say that a $\theta-$type agent is signal jamming if $\theta > 0$ (< 0) but A chooses $e_{An}(\theta) < 0$ (> 0) for $n = P$ if $c = dc$ and $n = M$ for $c = hc$.

Two other parameters that enter into expressions are g and σ_θ^2. The parameter g can be interpreted as how esoteric A's private information is. When $g \approx \frac{1}{2}$, P and M must rely heavily on communication effort from the agent to achieve any information transmission. When $g \approx 1$, P and M's signals are informative even if A does not exert any communication effort. The parameter σ_θ^2 is the variance of the underlying state θ. When σ_θ^2 is large, all parties have a larger stake in decision making (from an ex-ante perspective). These two parameters will not be crucial for the results of this section, but play important roles in the design of a single communication channel (Palida 2020a) and studying organizations in which members have access to multiple communication channels to use at their discretion (Palida 2020b).

3.5.1. Types of Hierarchies

As discussed in the introduction, communication hierarchies in organizations can take many different forms. Communication hierarchies can serve to restrict communication opportunities for those who may have incentives to interfere with decision making. For example, middle management in large firms and local political officials often prevent lower-level organization members from directly lobbying those with the most formal authority to make decisions. Often these arrangements are enforced via cultural norms, e.g. stigma from "going over a bosses head", while others are more strictly enforced via formal instruments.[52]

Communication hierarchies are also observed that seem to have the intention of improving communication between experts and decision makers. Middle management can play an important role in the transmission of information when experts are unmotivated or unable to communicate findings effectively. For example, senior researchers of R&D firms may provide a crucial link between novice experts with cutting-edge information and upper-level management.[53] Furthermore, intermediaries not associated with control hierarchies are often used to facilitate communication. For example, Ombudspersons and HR representatives may step in to facilitate resolution of a conflict between two employees.

I will now show, in the context of the current model, how these different forms of intermediated relationships arise based on the qualities of the organization and the intermediary. In particular, the hierarchical form the intermediary generates will depend on what intermediation does to incentives for the agent to exert communication effort and incentives for the principal to respond to her noisy signals.

In the equilibrium under direct-communication, these incentives are both jointly captured by the equilibrium decision spread Δ^{dc}. However, in the equilibrium under hierarchical communication, the agent's incentives are characterized by the equilibrium quantity $\delta^{hc}\Delta^{hc}$ (via expression (72)) while the principal's are characterized by the equilibrium decision spread Δ^{hc}. Thus, our analysis will consist of comparing the relative values of Δ^{dc}, Δ^{hc}, and $\delta^{hc}\Delta^{hc}$. The definition below describes three qualitative forms of hierarchies, based on what these values take in equilibrium. The following proposition establishes parameter spaces in which each of these forms arise.

[52] For example, Captain Brett Crozier was recently relieved of command for requesting assistance from individuals outside his chain of command. Source: https://www.defenseone.com/threats/2020/04/aircraft-carrier-captain-fired-poor-judgement-over-coronavirus-letter/164336/

[53] See, for example, the biotech firm Millenium, discussed in Wulf and Waggoner (2010).

Definition 3 *Consider the case of hierarchical communication, and suppose equilibrium is characterized by the quantities* δ^{hc} *and* Δ^{hc}*.*

1. *The hierarchy is a "restrictive hierarchy" if* $\delta^{hc}\Delta^{hc} < \Delta^{dc}$ *and* $\Delta^{hc} < \Delta^{dc}$*.*

2. *The hierarchy is a "necessary hierarchy" if* $\delta^{hc}\Delta^{hc} < \Delta^{dc}$ *but* $\Delta^{hc} > \Delta^{dc}$*.*

3. *The hierarchy is a "facilitating hierarchy" if* $\delta^{hc}\Delta^{hc} > \Delta^{dc}$ *but* $\Delta^{hc} > \Delta^{dc}$*.*

Proposition 12 *There exists a* $\underline{\mu}, \overline{\mu} \in \mathbb{R}$ *such that* $0 < \underline{\mu} < \overline{\mu} < \infty$ *and hierarchical communication takes the form of a restrictive hierarchy if* $\frac{\mu}{k_M} < \underline{\mu}$*, a necessary hierarchy if* $\frac{\mu}{k_M} \in (\underline{\mu}, \overline{\mu})$*, and a facilitating hierarchy if* $\frac{\mu}{k_M} > \overline{\mu}$*.*

Proof. Immediate from the fact that δ^{hc} and Δ^{hc} are decreasing in k_{MP} and increasing in $\sigma_{\mu,\theta}$ (see Corollary 3). ■

In the current model, Intermediation generates one of three hierarchical forms. The first of these, restrictive hierarchies, are generated by intermediaries with low-powered incentives for communication effort. This can occur either because the intermediary's stake in decision making is small (μ small) or his cost of influencing decision making is large (k_M large). In a restrictive hierarchy, the hierarchy-of-no effect dominates the collaboration effect, causing intermediation to result in less communication effort from all agent types, and less responsive decision making. Formally, in a restrictive hierarchy, we will have that $|e^{dc}_{AP}(\theta)| > |e^{hc}_{AM}(\theta)|)$ for all $\theta \in \Theta$, in addition to $\Delta^{dc} > \Delta^{hc}$.

Necessary hierarchies are similar to restrictive hierarchies in that they diminish communication-effort incentives for the agent, relative to direct communication ($\delta^{hc}\Delta^{hc} < \Delta^{dc}$). However, for a sufficiently motivated intermediary (μ sufficiently large), or an intermediary who can easily influence the principal's impressions (k_M sufficiently small), intermediation can still facilitate more responsive decision making ($\Delta^{hc} > \Delta^{dc}$). What occurs here is that, relative to the agents incentives, the hierarchy-of-no effect of intermediation dominates the collaboration effect. On the other hand, relative to the principal's incentives, the collaboration effect dominates the hierarchy-of-no effect. In other words, because the intermediary has stronger incentives to exert communication effort than the agent, the intermediary accepts the burden of communicating whatever information he can siphon from the agent. As the agent would not have transmitted very much information under direct communication anyway, the signal s_{MP} influenced by M in hierarchical communication after observing s_{AM}, is more informative that the signal s_{AP} influenced only by A in direct communication.

The final hierarchical form predicted by the model are facilitating hierarchies. These hierarchies increase incentives for *both* communication effort and responsiveness. Formally, the manager either has such a large stake in decision making (μ very large) or can very easily influence perceptions of the decision maker (k_M very small), that the collaboration effect of intermediation dominates the hierarchy-of-no effect for both P and A's incentives ($\Delta^{hc} > \Delta^{dc}$ *and* $\delta^{hc}\Delta^{hc} > \Delta^{dc}$). In other words, usage of a facilitating hierarchy spurs both dialog and action.

3.5.2. Optimal Intermediation

We now turn to examining when intermediation is desirable for the organization, from an ex-ante perspective. Recall that intermediation generates one of three hierarchical forms, depending on the incentives of the intermediary. We will study each of the hierarchical forms in isolation by comparing welfare across direct and hierarchical communication in the parameter region where the hierarchical form of focus constitutes the unique intervention PBE.

As mentioned earlier, we will focus on the parties' total ex-ante, expected payoff as the welfare criterion. It the parametric environment of this section, we can express the difference in this welfare criterion between the two communication structures as:

$$\begin{aligned} W^{hc}-W^{dc} &= \sigma_\theta^2\left(g-\frac{1}{2}\right)\left(\frac{1}{2}+\alpha+\mu\right)\left(\Delta^{hc}-\Delta^{dc}\right) \\ &+\sigma_\theta^2\left(g-\frac{1}{2}\right)\left(\frac{1}{2}+\alpha+\frac{\mu}{2}\right)\delta^{hc}\Delta^{hc} \\ &+\frac{\alpha\sigma_\theta^2\left(1+\alpha+2\mu\right)-\beta^2}{2k_A}\left[\left(\delta^{hc}\Delta^{hc}\right)^2-\left(\Delta^{dc}\right)^2\right] \end{aligned} \tag{93}$$

In the expression above we can see three terms. Recall that joint welfare in this model can be expressed as the sum of two effects: the expertise effect and the influence effect. The first term in the expression above (roughly) captures the difference between the expertise effects in the two communication structures while the third term (roughly) captures the difference in influence effects across the two structures.[54] The middle effect is the added benefit of the intermediary's informed influence which is only present under hierarchical communication.

Optimal Restrictive Hierarchies

Perhaps expectedly, the main welfare role restrictive hierarchies will play will be to curb incentives for signal jamming. Thus, to study when this hierarchical form may be desirable relative to direct communication, we will focus on an environment where incentives to signal jam in an uninformative manner are at its worst.

Specifically, let us assume her that $\alpha = 0$. Here, the agent's preferences are based entirely off of his ex-ante expected bias, and does not depend on the underlying state of the world. In this environment, the difference in welfare across the two structures becomes

$$\begin{aligned} W^{hc}-W^{dc} &= \sigma_\theta^2\left(g-\frac{1}{2}\right)\left(\frac{1}{2}+\mu\right)\left(\Delta^{hc}-\Delta^{dc}\right) \\ &+\sigma_\theta^2\left(g-\frac{1}{2}\right)\left(\frac{1}{2}+\frac{\mu}{2}\right)\delta^{hc}\Delta^{hc} \\ &-\frac{\beta^2}{2k_A}\left[\left(\delta^{hc}\Delta^{hc}\right)^2-\left(\Delta^{dc}\right)^2\right] \end{aligned} \tag{94}$$

Note that in a restrictive hierarchy we have that $\delta^{hc}\Delta^{hc} < \Delta^{dc}$ and $\Delta^{hc} < \Delta^{dc}$. Thus, the first term in the expression above will be negative, while the second two will be positive. In other words, restrictive hierarchies will yield a larger payoff than direct communication when signal-jamming deadweight losses are large. In this case, the organization can leverage the strong hierarchy-of-no effect as a tool to dissuade influence activities.

[54]The term "roughly" refers to the fact that the first term of the influence effect (which is always increasing in communication effort and responsiveness) has been combined with the expertise effect, in this expression, for expositional convenience.

We can derive a slightly more formal sufficient condition. Note that $W^{hc} \to 0$ as $\frac{\mu}{k_M} \to 0$. Thus, if $W^{dc} < 0$ then an intermediary who is sufficiently unmotivated can improve organizational welfare. Specifically we have the following sufficient condition for optimality of a restrictive hierarchy:

Proposition 13 *There exists a* $\underline{\mu}$ *such that if* $\frac{\mu}{k_M} < \underline{\mu}$ *and the following condition holds:*

$$\frac{\beta^2}{k_A} > \frac{2\sigma_\theta^2 \left(g - \frac{1}{2}\right)\left(\frac{1}{2} + \mu\right)}{\Delta^{dc}} \tag{95}$$

then we have that $W^{hc} - W^{dc} > 0$.

Proof. When (95) holds, we have that $W^{dc} < 0$, and $W^{hc} \to 0$ as $\frac{\mu}{k_M} \to 0$. So by continuity we must have that such a $\underline{\mu}$ exists. ■

Note that the RHS of (95) in decreasing in β because Δ^{dc} is increasing β. Thus the condition above says that restrictive hierarchies are welfare superior to direct communication when the agent's ex-ante bias is large.

Restrictive hierarchies are commonplace in many organizations. Various organizational communication practices demonstrate deliberate attempts to close communication channels between certain organization members and decision makers. Executive offices are often placed on restricted floors, decreasing the likelihood of chance encounters with lower-level employees. As discussed previously, military personnel are often reprimanded or even punished for attempting to communication with official outside of their chain of command. Politicians typically do not make personal contact information public and instead require interested parties to transmit information to them via local representatives. As discussed in Burkus (2012), these practices can stifle information flow and responsiveness to new data. Nevertheless, the model argues that these diminished communication incentives provided by restrictive hierarchies, may actually be desirable to mitigate unproductive signal jamming..

When (95) does not hold, the optimal communication structure may be either direct or hierarchical communication. If the influence effect in direct communication is strong enough, it may still be optimal to allow direct communication even if the agent is suffering a large signal jamming deadweight loss. Scenarios such as this are discussed in more detail in Palida (2020a) and can be likened to situations in which failures to persuade in an expected direction provides large amounts of information to decision makers.[55]

Optimal Necessary Hierarchies

We now turn to the second hierarchical form, necessary hierarchies. Recall that a necessary hierarchy is characterized by an intermediary with moderate incentives to influence decision making. To characterize environments where this hierarchical form is optimal, we will focus on a situation where there the agent has no ex-ante bias. Therefore, the agent always exerts communication effort for productive information transmission, as opposed to unproductive signal jamming. However, the agent may not have strong enough incentives to exert this effort, so the intermediary becomes a crucial link for information transmission.

[55] E.g. polls responding more strongly to negative news about a candididate than positive news. See, for example, Ellickson, Lovett, and Arison (2019).

Specifically, here we will assume that the parameter $\beta = 0$. This implies that expression (93) becomes:

$$W^{hc} - W^{dc} = \sigma_\theta^2 \left(g - \frac{1}{2}\right)\left(\frac{1}{2} + \alpha + \mu\right)\left(\Delta^{hc} - \Delta^{dc}\right) + \sigma_\theta^2 \left(g - \frac{1}{2}\right)\left(\frac{1}{2} + \alpha + \frac{\mu}{2}\right)\delta^{hc}\Delta^{hc} + \frac{\alpha\sigma_\theta^2 (1 + \alpha + 2\mu)}{2k_A}\left[\left(\delta^{hc}\Delta^{hc}\right)^2 - \left(\Delta^{dc}\right)^2\right] \quad (96)$$

In a restrictive hierarchy we have that $\Delta^{hc} > \Delta^{dc}$ but $\delta^{hc}\Delta^{hc} < \Delta^{dc}$. Thus the first two terms above will be positive, while the final term will be negative. Note that we will have

$$\lim_{\alpha \to 0} \Delta^{hc} > \lim_{\alpha \to 0} \Delta^{dc} \quad (97)$$

because under hierarchical communication, P's signal is influenced in an informative manner based on the signal observed by M.[56] Thus, we have the following proposition establishing a sufficient condition for optimality of a necessary hierarchy:

Proposition 14 *There exists an* $\underline{\alpha}$ *such that, if* $\alpha < \underline{\alpha}$*, then* $W^{hc} > W^{dc}$*.*

Proof. As $\alpha \to 0$ the final term in (96) goes to zero, while the first two remain positive. This proves the proposition. ■

Necessary hierarchies are optimal when the agent lacks motivation to communicate useful information. In such situations, the intermediary bears responsibility for influencing decision making based on whatever influenced signal he receives.

It is commonly observed that information flow within organizations follows control-rights hierarchies. For example, lower-level level employees in large firms will typically send information to decision makers by conversing through immediate supervisors. Various strands of economic literature suggest decision authority should be granted to individuals with the strongest incentives to behave in ways that are best for the organization as a whole. For example, the canonical property rights-theory model, introduced in Grossman and Hart (1886), predicts that control should be given to those who marginal investments are more important for organizational surplus.

One interpretation of a necessary hierarchy is an organization in which those delegated decision authority (e.g. middle management) also become the main communication intermediaries. This is because their natural incentives to behave in accordance with the goals of the organization also provides them with strong incentives to communicate effectively any useful information that they have learned from their subordinates.

On the other hand, when α is large, the agent becomes the efficient transmitter of information. Thus, using an intermediary in such situations is counterproductive to all parties. Such situations can be representative of small firms in which solvency of the organization depends heavily on adaptive decision making (e.g. start ups) or organizations whose members share a common objective (e.g. task forces or interest groups).

Finally, note that it may also be optimal to use an intermediary that generates a necessary hierarchy when α is small and β is large (i.e. incentives for unproductive signal

[56] Recall that s_{AM} is informative for M, via the function $g_M(\theta)$, even when $e_{AM}(\theta) = 0$ for all $\theta \in \Theta$.

jamming are large). In this case, the intermediary plays the same role as that in a restrictive hierarchy. Namely, the organization tries to leverage the hierarchy-of-no effect to mitigate unproductive influence activities by the agent. However, in such situations, it may actually be better for the organization to use a less motivated intermediary that generates a restrictive hierarchy instead.

Optimal Facilitating Hierarchies

The final hierarchical form predicted by the model are facilitating hierarchies. A facilitating hierarchy is characterized by an intermediary with strong incentives to influence decision making. In studying this hierarchical form we will allow there to be both an ex ante bias ($\beta > 0$) as well as partial preference alignment between the principal and agent ($\alpha > 0$). As we will see, facilitating hierarchies are a double-edged sword that can either improve welfare or result in excessive incentives for unproductive signal jamming.

In particular, recall the difference in joint welfare across structures is given by

$$\begin{aligned} W^{hc} - W^{dc} &= \sigma_\theta^2 \left(g - \frac{1}{2}\right)\left(\frac{1}{2} + \alpha + \mu\right)\left(\Delta^{hc} - \Delta^{dc}\right) \\ &+\sigma_\theta^2 \left(g - \frac{1}{2}\right)\left(\frac{1}{2} + \alpha + \frac{\mu}{2}\right)\delta^{hc}\Delta^{hc} \\ &+\frac{\alpha\sigma_\theta^2\left(1+\alpha+2\mu\right) - \beta^2}{2k_A}\left[\left(\delta^{hc}\Delta^{hc}\right)^2 - \left(\Delta^{dc}\right)^2\right] \end{aligned} \tag{98}$$

In a facilitating hierarchy, we have that both $\Delta^{hc} > \Delta^{dc}$ and $\delta^{hc}\Delta^{hc} > \Delta^{dc}$. Thus, the following proposition falls immediately out of expression (98):

Proposition 15 *Suppose that*

$$\beta \leq \sqrt{\alpha\sigma_\theta^2\left(1+\alpha+2\mu\right)} \tag{99}$$

the we must have that $W^{hc} > W^{dc}$.

Proof. Immediate from expression (98) and the fact that $\Delta^{hc} > \Delta^{dc}$ and $\delta^{hc}\Delta^{hc} > \Delta^{dc}$ in a facilitating hierarchy. ■

When the agent's ex-ante bias (β) is small relative to his preference-alignment parameter (α), facilitating hierarchies increase both information transmission on the part of the agent and responsiveness on the part of the principal. In this case, the organization leverages the strong collaboration effect of intermediation to generate what is the equivalent of a lower cost channel of communication between the principal and agent (see Palida (2020a)). Doing so is desirable because the communicating parties have incentives that are sufficiently aligned.

Facilitating hierarchies bear a strong resemblance to third-party, conflict-resolution practices. Staff (2020) describes three forms of third-party conflict resolution: Mediation, arbitration, and litigation. The form of conflict resolution that bears the most resemblance to the current model is mediation. In such arrangements, the intermediary helps parties "vent their feelings and fully explore their grievances". Unlike in arbitration and litigation, in mediation arrangements, the intermediary does not become the final decision maker. Examples of such arrangements in firms include conflict resolution via human-resource representatives or Ombudspersons, while examples outside of firms include usage of councillors or therapists in schools or families. The model suggests such arrangements are difficult to sustain because the intermediary must both have incentives to exert effort in conveying information and also be skilled at persuasion. Thus, only intermediaries with specialized human capital conducive to encouraging dialog are effective in generating facilitating hierarchies.[57]

When (99) does not hold, it is important to note that intermediation can actually amplify existing incentives for the agent to unproductively signal jam. Thus, a facilitating hierarchy is a double-edged sword. The model suggests it will only be used in scenarios where the main friction is a lack of productive information transmission, and not excessive signal jamming.

3.6. Conclusion and Future Research

This paper uses a noisy-signalling model to study the positive and normative effects of communication hierarchies in organizations. Simultaneous presence of cost and noise in communication generates a potential two-sided inefficiency in direct communication relationships, as communication effort and responsiveness are strategic complements. Therefore, communication effort can be inefficiently high if the agent has excessive incentives to signal jam in an uninformative manner, and inefficiently low if the agent lacks motivation to exert effort in productive information transmission.

I find that communication intermediaries can play a crucial role in managing this two-sided inefficiency. In particular, the model shows that intermediation can have two opposing effects on communication effort and responsiveness, relative to direct communication. The hierarchy-of-no effect exerts downward pressure on both these equilibrium quantities, while the collaboration effect exerts upward pressure on communication effort and responsiveness. Thus, intermediaries can be used to either increase or decrease these quantities, depending on qualities of the intermediary.

Intermediaries with weak incentives to influence decision making generate restrictive hierarchies in which the hierarchy-of-no effect dominates the collaboration effect. Necessary hierarchies are generated by intermediaries with moderate incentives to influence decision making, and in this hierarchical form, the hierarchy-of-no effect dominates the collaboration effect from the perspective of the agent's incentives, while the later dominates the former in regards to the principal's incentives. Finally, restrictive hierarchies are generated by intermediaries with strong incentives to influence decision making, and increase both communication effort and responsiveness relative to direct communication (i.e. the collaboration effect dominates hierarchy-of-no effect in both parties incentives).

I characterize sufficient conditions under which each of these three hierarchical forms would be preferred for an organization over direct communication between the principal and agent. The analysis provides possible rationales for the wide array of intermediated communication relationships observed across organizations, the frequent observed correlation between control-rights hierarchies and communication hierarchies, and the usage

[57] It is often stated that intermediaries in mediation relationships are intended to be "impartial" or "unbiased" in that they do not explicitly take the side of either party. It is important to understand that this is not the same concept as the intermediary not having a stake in the decision. All the intermediaries described in the examples above do indeed have stake in the decisions they manage as future usage of their services likely depends on how satisfied both parties are with the outcome realized in the current interaction. One can interpret the preferences of the intermediary in this model as a reduced form of these kinds of incentives.

of third-party conflict resolution arrangements.

Many questions remain to be answered. The current model focuses on a setting where bandwidth concerns prevent the decision maker from observing multiple noisy signals. In many situations decision makers can receive information from multiple sources. For example, the intermediary may generate some information on his own, and we can then ask whether or not the principal should see both signals or only one. In this environment, we could also examine communication-representative arrangements in which the intermediary and expert are allowed communicate revealed information to each other first, and then send a delegate to converse with the principal.

Also, many organizational environments allow information holders to send information via different intermediated paths at their discretion. For example, skip-level meetings are common practices in many organizations, in which lower-level employees have the opportunity to surface issues with the boss of their immediate supervisor. What information will be sent via the skip-level meeting and what information will be sent hierarchically via middle management? Palida 2020b, studies this question in the context of communication channels in organizations and additionally discusses how the results of that paper can be applied to settings in which informed parties can choose among multiple "intermediation paths".

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Appendix: Proofs

Proof of Proposition 1 (Unique PBE for Single Channel)

This proof is done in a more general setting where no symmetry assumptions on θ or $g(.)$ are made. For notational convenience we will denote moments of distributions in the following manner: Let $\Theta' \subseteq \Theta$ be a subset of agent types, and let $x(\theta)$ and $y(\theta)$ be arbitrary functions of θ. I will denote

$$\begin{aligned}
\underline{x}_{\Theta'} &= \min_{\theta}\{x(\theta)|\theta \in \Theta'\} \\
\overline{x}_{\Theta'} &= \max_{\theta}\{x(\theta)|\theta \in \Theta'\} \\
x_{\Theta'} &= E_{\theta}\{x(\theta)|\theta \in \Theta'\} \\
\widetilde{xy}_{\Theta'} &= E_{\theta}\{x(\theta)y(\theta)|\theta \in \Theta'\} \\
\widehat{xy}_{\Theta'} &= cov_{\theta}\{x(\theta), y(\theta)|\theta \in \Theta'\}
\end{aligned}$$

The proof will be done assuming $\Theta' = \Theta$ (i.e. all agent types) but note that the result holds for any subset of agent types as well.

Suppose P is playing strategy d_s^* in a PBE. The agent's first order conditions imply that we must have

$$\begin{aligned}
e^*(\theta) &= \alpha(\theta)\frac{\Delta^*}{k} && (100)\\
&= \alpha(\theta)e^* && (101)
\end{aligned}$$

Similarly, P's first-order conditions imply that

$$d_s^* = E\{\theta|s; e^*(.)\} \tag{102}$$

and, combined with fact that $e^*(\theta) = \alpha(\theta)e^*$ in any PBE, we can use Bayes rule to deduce that

$$d_L^* = \frac{\theta_\Theta - \widetilde{\alpha\theta}_\Theta e^* - \widetilde{g\theta}_\Theta}{1 - \alpha_\Theta e^* - g_\Theta} \tag{103}$$

and

$$d_R^* = \frac{\widetilde{\alpha\theta}_\Theta e^* + \widetilde{g\theta}_\Theta}{\alpha_\Theta e^* + g_\Theta} \tag{104}$$

Note that PBE strategies are determined entirely by the quantity e^*. Therefore an additional necessary condition for a PBE is that e^* must satisfy

$$ke^* = \frac{\widetilde{\alpha\theta}_\Theta e^* + \widetilde{g\theta}_\Theta}{\alpha_\Theta e^* + g_\Theta} - \frac{\theta_\Theta - \widetilde{\alpha\theta}_\Theta e^* - \widetilde{g\theta}_\Theta}{1 - \alpha_\Theta e^* - g_\Theta} \tag{105}$$

Rearranging this expression gives

$$l(e^*) = 0 \tag{106}$$

where the cubic function $l(e)$ is given by

$$l(e) = \alpha_\Theta^2 e^3 - \alpha_\Theta(1-2g_\Theta)e^2 - [g_\Theta(1-g_\Theta) - \frac{\widehat{\alpha\theta}_\Theta}{k}]e + \frac{\widehat{g\theta}_\Theta}{k} \tag{107}$$

Thus far we have established that condition (106) along with conditions (100), (103), and (104) are necessary conditions for a PBE strategy profile. However, these conditions are not sufficient by themselves, as a solution to (106) may result in a violation of item 3 in the definition of a PBE. On the otherhand, we will have found a PBE if the solution of (106) does not result in a violation of this requirement. Thus, a necessary and sufficient condition for a PBE is that: 1) (106) is satisfied; 2) strategies follow (100), (103), and (104); and 3) item 3 in the definition of a PBE is not violated.

Since $l(e)$ is a cubic polynomial, the equation in (106) can have one to three solutions. In fact, we will see that it will always have exactly three solutions. Furthemore, we will show that the median solution to (106) *never* leads to a violation of NSS, and the other two solutions *always* violate this condition. Together, this implies that the single-channel model admits a *unique* PBE.

To start, let us define the largest and smallest possible values e^* can take in a PBE. In particular, if $\alpha(\theta) > 0$, then item 3 in the PBE definition is satisfied if $e^* \in [-\frac{g(\theta)}{\alpha(\theta)}, \frac{1-g(\theta)}{\alpha(\theta)}]$. Similarly, If $\alpha(\theta) < 0$ then we must have that $e^* \in [\frac{1-g(\theta)}{\alpha(\theta)}, -\frac{g(\theta)}{\alpha(\theta)}]$. Since item 3 needs to hold for all $\theta \in \Theta$ in a PBE, we must have that $e^* \in [\underline{e}, \overline{e}]$ where

$$\underline{e} = \max\left\{\max_{\theta:\alpha(\theta)<0} \frac{1-g(\theta)}{\alpha(\theta)}, \max_{\theta:\alpha(\theta)>0} -\frac{g(\theta)}{\alpha(\theta)}\right\} \tag{108}$$

$$\overline{e} = \min\left\{\min_{\theta:\alpha(\theta)<0} -\frac{g(\theta)}{\alpha(\theta)}, \min_{\theta:\alpha(\theta)<0} \frac{1-g(\theta)}{\alpha(\theta)}\right\} \tag{109}$$

Ultimately, what we will establish is that $l(\underline{e}) > 0$ and $l(\overline{e}) < 0$. This will imply that $l(e)$ has three real roots, with exactly one between $[\underline{e}, \overline{e}]$ corresponding to a unique PBE.

A second implication of P's first-order condition is that we must have $\Delta^* \in [\underline{\theta}-\overline{\theta}, \overline{\theta}-\underline{\theta}]$. Given (103), and (104) we have that

$$\Delta^* = \frac{\widehat{\alpha\theta}_\Theta e^* + \widehat{g\theta}_\Theta}{g_\Theta(1-g_\Theta) + \alpha_\Theta(1-2g_\Theta)e^* - \alpha_\Theta^2(e^*)^2} \tag{110}$$

Combining the expression in (110) with the requirement that $\Delta^* \in [\underline{\theta}-\overline{\theta}, \overline{\theta}-\underline{\theta}]$ implies that

$$\widehat{\alpha\theta}_\Theta e^* + \widehat{g\theta}_\Theta \geq (\underline{\theta}-\overline{\theta})[g_\Theta(1-g_\Theta) + \alpha_\Theta(1-2g_\Theta)e^* - \alpha_\Theta^2(e^*)^2] \tag{111}$$

and

$$\widehat{\alpha\theta}_\Theta e^* + \widehat{g\theta}_\Theta \leq (\overline{\theta}-\underline{\theta})[g_\Theta(1-g_\Theta) + \alpha_\Theta(1-2g_\Theta)e^* - \alpha_\Theta^2(e^*)^2] \tag{112}$$

Now suppose that $\underline{e} = \frac{1-g(\theta)}{\alpha(\theta)}$ for some θ such that $\alpha(\theta) < 0$. (**??**) implies that

$$k > \frac{\alpha(\theta)(\underline{\theta}_\Theta - \overline{\theta}_\Theta)}{1-g(\theta)} \tag{113}$$

Thus we have that

$$l(\underline{e}) = \frac{1}{k}\left\{-k\underline{e}\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\underline{e}-\alpha_\Theta^2\underline{e}^2\right]+\widehat{\alpha\theta}_\Theta\underline{e}+\widehat{g\theta}_\Theta\right\} \tag{114}$$

$$> \frac{1}{k}\left\{-\left(\frac{\alpha(\theta)(\underline{\theta}_\Theta-\overline{\theta}_\Theta)}{1-g(\theta)}\right)\underline{e}\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\underline{e}-\alpha_\Theta^2\underline{e}^2\right]+\widehat{\alpha\theta}_\Theta\underline{e}+\widehat{g\theta}_\Theta\right\} \tag{115}$$

$$= \frac{1}{k}\left\{-(\underline{\theta}_\Theta-\overline{\theta}_\Theta)\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\underline{e}-\alpha_\Theta^2\underline{e}^2\right]+\widehat{\alpha\theta}_\Theta\underline{e}+\widehat{g\theta}_\Theta\right\} \tag{116}$$

$$\geq 0 \tag{117}$$

An similar argument will show that If $\underline{e} = -\frac{g(\theta)}{\alpha(\theta)}$ for some θ such that $\alpha(\theta) > 0$, we will still have that $l(\underline{e}) > 0$.

Now suppose that $\overline{e} = -\frac{g(\theta)}{\alpha(\theta)}$ for some θ such that $\alpha(\theta) < 0$. (**??**) implies that

$$k > \frac{\alpha(\theta)(\underline{\theta}_\Theta-\overline{\theta}_\Theta)}{-g(\theta)} \tag{118}$$

Thus we have that

$$l(\overline{e}) = \frac{1}{k}\left\{-k\overline{e}\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\overline{e}-\alpha_\Theta^2\overline{e}^2\right]+\widehat{\alpha\theta}_\Theta\overline{e}+\widehat{g\theta}_\Theta\right\} \tag{119}$$

$$> \frac{1}{k}\left\{-\left(\frac{\alpha(\theta)(\underline{\theta}_\Theta-\overline{\theta}_\Theta)}{-g(\theta)}\right)\overline{e}\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\overline{e}-\alpha_\Theta^2\overline{e}^2\right]+\widehat{\alpha\theta}_\Theta\overline{e}+\widehat{g\theta}_\Theta\right\} \tag{120}$$

$$= \frac{1}{k}\left\{-(\underline{\theta}_\Theta-\overline{\theta}_\Theta)\left[g_\Theta(1-g_\Theta)+\alpha_\Theta(1-2g_\Theta)\underline{e}-\alpha_\Theta^2\underline{e}^2\right]+\widehat{\alpha\theta}_\Theta\underline{e}+\widehat{g\theta}_\Theta\right\} \tag{121}$$

$$\leq 0 \tag{122}$$

An identical argument will show that If $\overline{e} = \frac{1-g(\theta)}{\alpha(\theta)}$ for some θ such that $\alpha(\theta) > 0$, we will still have that $l(\overline{e}) > 0$.

We have now established that $l(\underline{e}) > 0$ and $l(\overline{e}) > 0$. Intermediate value theorem then guarentees $l(e)$ has at least one $e^* \in (\underline{e}, \overline{e})$ such that $l(e^*) = 0$, corresponding to at least one PBE of the game. However, because the coefficient on the cubic term in $l(e)$ is positive, we know that $\lim_{e\to-\infty} l(e) = -\infty$ and $\lim_{e\to\infty} l(e) = \infty$. This implies that $l(e)$ must have two zeros outside of the interval $[\underline{e}, \overline{e}]$. Given that the cubic polynomials can have at most three zeros, we can conclude that the PBE characterized by the unique $e^* \in (\underline{e}, \overline{e})$ with $l(e^*) = 0$ is the unique PBE. Finally, since $l(0) = \frac{\widehat{g\theta}_\Theta}{k} \geq 0$, it must be the case that $e^* \geq 0$. Note that this also implies that $\Delta^* = d_R^* - d_L^* \geq 0$.

Proofs for Chapter 2

Proof of Proposition 5

Here I prove Proposition 2 from Section 6. I first repeat some key definitions, then I restate and prove the proposition.

Recall that the function $v(\theta; d, e, m)$ is given by

$$v(\theta; d, e, m) = d + \left(\frac{\alpha(\theta)}{2}e + \lambda(\theta)\right) k_m e \tag{123}$$

and, in a PBE, the quantity $v(\theta; d^*_{mL}, e^*_m, m)$ captures the expected value of P's decision net of communication costs, if a θ-type agent chooses channel m and selects communication effort optimally given the PBE decision rule.

Next, recall the current paper's notion of single crossing:

Definition: *Let the set* $\Omega = [\underline{\theta}_\Theta, \overline{\theta}_\Theta] \times \mathbb{R}_+ \times M$. *The function* $v(\theta, d, e, m)$ *satisfies the "single-crossing condition" (SCC) if for every two vectors* $(d, e, m), (d', e', m') \in \Psi$, *exactly one of the two following statements hold:*

1. $v(\theta, d, e, m) = v(\theta, d', e', m')$ *for all* $\theta \in \Theta$
2. *There exists a* $\theta^{crit} \in \Theta$ *such that* $v(\theta, d, e, m) > v(\theta, d', e', m')$ *for all* $\theta < \theta^{crit}$ *and* $v(\theta, d, e, m) < v(\theta, d', e', m')$ *for all* $\theta > \theta^{crit}$

Finally, recall the formal definitions of partisan and advocates pooling groups:

Definition: *A group* $g \subseteq \Theta$ *is called a "partisan group" if it is a convex set*[58] *and* $sign\{\alpha(\theta)\} = sign\{\alpha(\theta')\}$ *for all* $\theta, \theta' \in g$.[59]

Definition: *Let* g_- *and* g_+ *be two partisan groups with* $\alpha(\theta) < 0$ *for all* $\theta \in g_-$ *and* $\alpha(\theta) > 0$ *for all* $\theta \in g_+$.[60] *The group* $g = g_- \cup g_+$ *is called an "advocates group".*

I now restate and prove the proposition:

Proposition 5: *Suppose that* $\alpha(\theta)$ *is strictly monotonic and* $v(\theta)$ *satisfies SCC. If channel profile* $m^*(\theta)$ *is played in a strictly semi-separating PBE and there does not exist* $m, m' \in M$ *such that all agent types are indifferent between* m *and* m', *the following statements are true:*

1. *Let* $g^*_m = \{\theta \in \Theta : m^*(\theta) = m\}$. *We must have that* g^*_m *is either a partisan group or an advocates group.*
2. *There can be at most one partisan group* g^*_{m-} *with* $\alpha(\theta) < 0$ *for all* $\theta \in g^*_{m-}$ *and at most one partisan group* g^*_{m+} *with* $\alpha(\theta) > 0$ *for all* $\theta \in g^*_{m+}$.
3. *There can be at most one advocates group* g^*_m *in any PBE. Furthermore, if* $\theta \in g^*_m$, *then* $\theta' \in g^*_m$ *for all* $\theta' \in \Theta$ *s.t.* $sign\{\alpha(\theta)\} = sign\{\alpha(\theta')\}$ *and* $|\theta| \leq |\theta'|$.

Proof. Suppose we have a strictly semi-separating PBE characterized by channel profile $m^*(\theta)$, and let $g^*_m = \{\theta : m^*(\theta) = m\}$ be one of the PBE pooling groups. In what follows it will also be useful to denote

$$g^*_{m+} = \{\theta \in g^*_m : \alpha(\theta) > 0\} \quad (124)$$
$$g^*_{m-} = \{\theta \in g^*_m : \alpha(\theta) < 0\} \quad (125)$$

It is easiest to start with showing item 2. Suppose g^*_m is a partisan group for the R project and there exists another partisan group, $g^*_{m'}$ using channel m', that also supports the R project. Because partisan groups must be convex, we must have that either $g^*_m >_{SSO} g^*_{m'}$ or vice versa.[61] WLG let us assume that $g^*_m >_{SSO} g$. If $\alpha(\theta)$ is strictly

[58] g is "convex" if for every $\theta < \theta'$ with $\theta, \theta' \in \Theta$ we have that $\theta'' \in [\theta, \theta'] \cap \Theta \Rightarrow \theta'' \in g$.

[59] "=" here is a weak equality, so that $sign\{0\} = sign\{\alpha(\theta)\}$ for all $\theta \in \Theta$.

[60] Note that these two sets must be disjoint as it was assumed that $\nexists \theta \in \Theta$ such that $\alpha(\theta) = 0$.

[61] "$>_{SSO}$" denotes the strong set order.

increasing (recall $\alpha(\theta)$ was assumed to be monotonic), then we will have that $d^*_{ms} > d^*_{m's}$ for both $s = L$ and $s = R$. Similarly, if $\alpha(\theta)$ is strictly decreasing, we will have that $d^*_{ms} < d^*_{m's}$ for both $s = L$ and $s = R$. In either case, we will have that all agent types in one of the two groups will wish to deviate to the other group. An identical argument can be used for partisan groups supporting the L project. Thus, item 2 is proven.

To establish item 1, we must show that, for any PBE pooling group g^*_m, the subsets g^*_{m+} and g^*_{m-} are convex. Suppose for contradiction that we had three agent types $\theta_1 < \theta_2 < \theta_3$ such that $\theta_1, \theta_3 \in g^*_m$ and $\theta_2 \in g^*_{m'}$ for $m \neq m'$. Without loss of generality, let us assume that $\theta_1, \theta_3 \in g^*_{m+}$. Since this is a PBE, we must have that

$$\begin{aligned} v(\theta_1; d^*_{mL}, e^*_m, m) &\geq v(\theta_1; d^*_{m'L}, e_{m'}, m') && (126)\\ v(\theta_2; d^*_{mL}, e^*_m, m) &\leq v(\theta_2; d^*_{m'L}, e_{m'}, m') && (127)\\ v(\theta_3; d^*_{mL}, e^*_m, m) &\geq v(\theta_3; d^*_{m'L}, e_{m'}, m') && (128) \end{aligned}$$

But since function $v(\theta, d, e, m)$ is continuous in all of its arguments, intermediate value theorem combined with the above three inequalities implies that the functions $v(\theta; d^*_{mL}, e^*_m, m)$ and $v(\theta; d^*_{m'L}, e_{m'}, m')$ must cross at least twice in θ, v-space. This is a violation of SCC and thus establishes the desired contradiction. Item 1 is now proven.

Item 3 of the proposition also comes from the single-crossing condition. Again consider θ_1, θ_2,and θ_3 such that $\alpha(\theta) > 0$ for $\theta \in \{\theta_1, \theta_2, \theta_3\}$ and $\theta_1 < \theta_2 < \theta_3$. Furthermore suppose that θ_1 and θ_3 were both part of advocate groups in a PBE characterized by $m^*(\theta)$. I will show that we must have $m^*(\theta_1) = m^*(\theta_2) = m^*(\theta_3)$.

Suppose $m^*(\theta_1) = m_1 \neq m_3 = m^*(\theta_3)$. It must be the case that

$$v(\theta_1; d^*_{m_1L}, e^*_{m_1}, m_1) \geq v(\theta_1; d^*_{m_3L}, e_{m_3}, m_3) \quad (129)$$

and

$$v(\theta_3; d^*_{m_1L}, e^*_{m_1}, m_1) \leq v(\theta_3; d^*_{m_3L}, e_{m_3}, m_2) \quad (130)$$

SCC combined with no mass-indifference implies that there must exist a $\theta_0 \in [\theta_1, \theta_3]$ such that $v(\theta; d^*_{m_1L}, e^*_{m_1}, m_1) > v(\theta; d^*_{m_3L}, e_{m_3}, m_3)$ for all $\theta < \theta_0$. But then this implies that all $\theta \in \Theta$ such that $\alpha(\theta) < 0$ strictly prefer using channel m_3 to m_1. Thus, θ_1 cannot be part of an advocates group, which contradicts the initial claim.

Now suppose $m^*(\theta_2) = m_2 \neq m_3$. From the previous part, this would imply that $m_2 \neq m_1 = m^*(\theta_1)$. Using a similar argument as before, SCC combined no mass indifference implies that all $\theta \in \Theta$ such that $\alpha(\theta) < 0$ strictly prefer using channel m_2 to m_1. But that implies that θ_1 cannot be part of an advocates group which contradicts the intitial claim.

An identical argument provides the same equality when $\alpha(\theta) < 0$ for $\theta \in \{\theta_1, \theta_2, \theta_3\}$. Together this implies that any PBE can have a maximum of one advocates group and the partisan subgroups of this advocates group must lie on the two extremes of the type space. This concludes the proof of item 3 as well as the full proposition. ■

SPB3 Setting Proofs

Here I provide proofs for Lemma 1 and propositions 6 and 7. Let us first remind ourselves of the parametric assumptions characterizing the SPB3 model:

SPB3 Assumptions

1. $\theta \in \{L, C, R\} = \{-1, 0, 1\}$ where $\Pr\{\theta = L\} = \Pr\{\theta = H\} = \frac{f}{2}$

2. $\alpha(\theta) = \theta + b$ and $\lambda(\theta) = \frac{1}{2} + \left(\lambda - \frac{1}{2}\right)\theta$ where $b \in (0,1)$ and $\lambda \in \left(\frac{1}{2}, 1\right)$

3. $M = \{c, o\}$ with $k_c = \infty$ and $k_o = k$, where $k > \underline{k} = \frac{2(1+b)}{1-\lambda}$

Recall from Section 3 that an expression for the ex-ante expected payoff in any PBE is given by

$$TW^* = \sum_{m \in M} \Pr\{\theta \in g^*_m\} W^*_m \tag{131}$$

where W^*_m was given in (??) to be:

$$W^*_m = \left(\frac{\theta_{g^*_m}}{2} + \alpha_{g^*_m}\right)\theta_{g^*_m} + \left[\left(\frac{\widehat{\lambda\theta}_{g^*_m}}{2} + \widehat{\lambda\alpha}_{g^*_m}\right) + \left(\frac{\widehat{\alpha\theta}_{g^*_m} + \widehat{\alpha\alpha}_{g^*_m} - \alpha^2_{g^*_m}}{2}\right) e^*_m\right] \Delta^*_m \tag{132}$$

In the SPB3 setting, it is easy to see that for any group $g \subseteq \Theta$ we must have

$$\alpha_g = \theta_g + b \tag{133}$$

$$\widehat{\lambda\alpha}_g = \widehat{\lambda\theta}_g = \left(\lambda - \frac{1}{2}\right)\widehat{\theta\theta}_g \tag{134}$$

$$\widehat{\alpha\theta}_g = \widehat{\alpha\alpha}_g = \widehat{\theta\theta}_g \tag{135}$$

Thus we can rewrite the expression for W^*_m as

$$W^*_m = \left(\frac{3\theta_{g^*_m}}{2} + b\right)\theta_{g^*_m} + \left[\frac{3\left(\lambda - \frac{1}{2}\right)\widehat{\theta\theta}_g}{2} + \left(\frac{2\widehat{\theta\theta}_g - (\theta_g + v)^2}{2}\right) e^*_m\right] \Delta^*_m \tag{136}$$

Furthermore, given the parametric Suppose in a PBE we have that there is a channel m such that $g^*_m = \{\theta\}$ for some $\theta \in \{L, C, R\}$. It is clear in this case that $W^*_m = \left(\frac{3\theta_{g^*_m}}{2} + b\right)\theta_{g^*_m}$ as we must have $\Delta^*_m = 0$ in the PBE. if g^*_m is not a singleton, there are three possible forms it can take: 1) $g = \{L, C, R\} = full$; 2) $g = \{C, R\} = part$; and 3) $g = \{L, R\} = adv$. Now we use (136) to compute $TW^{g,m}$ for each of the 6 PBE:

$$\begin{aligned}
TW^{full,o} &= \left[\frac{3f(\lambda - \frac{1}{2})}{2} + \left(\frac{2f - b^2}{2}\right) e^{full,o}\right] \Delta^{full,o} \\
TW^{full,c} &= \left[\frac{3f(\lambda - \frac{1}{2})}{2}\right] \Delta^{full,c} \\
TW^{part,o} &= \frac{f}{2}\left\{\frac{3(1-x)}{2} + \left[\frac{3(1-x)(\lambda - \frac{1}{2})}{2} + \left(\frac{2(1-x) - \frac{(x+b)^2}{x}}{2}\right) e^{part,o}\right] \Delta^{part,o}\right\} \\
TW^{part,c} &= \frac{f}{2}\left[\frac{3(1-x)}{2} + \frac{3(1-x)}{2}\Delta^{part,c}\right] \\
TW^{adv,o} &= f\left[\frac{3(\lambda - \frac{1}{2})}{2} + \left(\frac{2 - b^2}{2}\right) e^{adv,o}\right] \Delta^{adv,o} \\
TW^{adv,c} &= f\left[\frac{3(\lambda - \frac{1}{2})}{2}\right] \Delta^{adv,c}
\end{aligned}$$

Lemma 1: *The following relationships hold: 1)* $TW^{full,c} < TW^{adv,c}$*; 2)* $TW^{full,o} < TW^{adv,o}$*; 3)* $TW^{adv,c} < TW^{adv,o}$

Proof. Bayes rule implies that we must have $\Delta^{full,c} < \Delta^{adv,c}$. Item 1 is thus immediately established.

Next, note that, $\theta_{adv} = \theta_{full} = 0$, but $\widehat{\theta\theta}_{adv} = 1 > f = \widehat{\theta\theta}_{full}$. Therefore we must have that $e^{adv,o} > e^{full,o}$, and equivalently, $\Delta^{adv,o} > \Delta^{full,o}$. This then implies item 2.

Bayes rule implies that we must have $\Delta^{adv,o} > \Delta^{adv,c}$. Furthermore, since $b < 1$ by assumption, and $e^{adv,o} > 0$, we must have that $\left(\frac{2-b^2}{2}\right) e^{adv,o} > 0$. This then implies that $TW^{adv,c} < TW^{adv,o}$, which establishes the final item. ■

Proposition 6: *There exists a* $\widehat{\lambda} \in (\frac{1}{2}, 1)$ *and a* $\widehat{f} \in (0,1)$ *such that, if* $\lambda > \widehat{\lambda}$ *and* $f < \widehat{f}$, *the optimal equilibrium is the* adv, o*-equilibrium. If* $\lambda < \widehat{\lambda}$ *then the* adv, o*-equilibrium cannot be the optimal equilibrium.*

Proof. We have

$$\begin{aligned}
\lim_{f \to 0} \frac{TW^{part,c}}{TW^{adv,o}} &= \lim_{f \to 0} \frac{TW^{part,o}}{TW^{adv,o}} \\
&= \frac{3/4}{\left[\frac{3(\lambda - \frac{1}{2})}{2} + \left(\frac{2-b^2}{2}\right) e^{adv,o}\right] \Delta^{adv,o}} \\
&= \frac{3/4}{\mu^{adv,o}}
\end{aligned}$$

Note that the quantities $e^{adv,o}$ and $\Delta^{adv,o}$ do not depend on f. Therefore, if $\mu^{adv,o} > \frac{3}{4}$, we will have that there exists some f small enough so that the adv, o-equilibrium would be optimal.

Now note that

$$\begin{aligned}
\mu^{adv,o} &= \left[\frac{3(\lambda - \frac{1}{2})}{2} + \left(\frac{2-b^2}{2}\right) e^{adv,o}\right] \Delta^{adv,o} \\
&> \frac{3(\lambda - \frac{1}{2})}{2} \Delta^{adv,o} \\
&> \frac{3(\lambda - \frac{1}{2})}{2} \Delta^{adv,c} \\
&= \frac{3(\lambda - \frac{1}{2})}{2} \frac{(\lambda - \frac{1}{2})}{1/4}
\end{aligned}$$

Thus as $\lambda \to 1$, $\mu^{adv,o} \to \frac{3}{2} > \frac{3}{4}$. Therefore, we must have that there exists a $\widehat{\lambda} \in (\frac{1}{2}, 1)$ and a $\widehat{f} \in (0,1)$ such that if $\lambda > \widehat{\lambda}$ and $f < \widehat{f}$, the optimal equilibrium is the adv, o-equilibrium.[62] ■

Proposition 7: *If*

$$x \in \left(\frac{1 - b - \sqrt{1 - 2b - 2b^2}}{3}, \frac{1 - b + \sqrt{1 - 2b - 2b^2}}{3}\right) \tag{137}$$

then $TW^{part,o} > TW^{part,c}$. *If* $\lambda < \widehat{\lambda}$ *holds as well, then the optimal equilibrium is the* $part, o$*-equilibrium.*

[62] Note that the regularity condition on the cost of communicative effort implies that λ can never exceed $1 - \frac{2(1+b)}{k}$. However, we can simply assume k is large enough and take the upper limit of λ to be arbitrarily close to 1. In other words, there do exist parameter vectors in which this sufficient condition can be applied to conclude that the adv, o-equilibrium is optimal.

Proof. The proof is entirely algebra. A sufficient condition to have $TW^{part,o} > TW^{part,c}$ is for

$$2(1-x) - \frac{(x+b)^2}{x} > 0 \tag{138}$$

Quadratic formula then gives the specified range for x. ■

Proofs for Chapter 3

Intervention PBE

In this section we establish existence of the intervention PBE for the hierarchical communication setting, and comparative static effects of parameters. For convenience, the formal proposition and corollary are restated below:

Proposition: *The intervention PBE always exists and is unique. In other words, the following strategy-updating process,*

1. *P and A start by playing the strategies specified in the direct-communication equilibrium (characterized by Δ^{dc})*
2. *M best responds to P and A's strategies*
3. *P updates strategy to best respond to A's strategy from 1 and M's strategy from 2*
4. *A updates strategy to best respond to M's communication effort from 2 and P's decision rule from*
5. *Steps 2-4 are repeated*

converges to a unique strategy profile characterized by the equilibrium quantities $\delta^{hc} > 0$ and $\Delta^{hc} > 0$

Corollary: *In the intervention PBE, we have that δ^{hc} and Δ^{hc} are both decreasing in k_{MP} and increasing in $\sigma_{g_M,\theta}$ and $\sigma_{\mu,\theta}$.*

The process described in steps 1-5 above, generates a sequence $E_0, E_1, E_2, ...$ where $E_k = (\delta_k, \Delta_k)$. if this sequence converges in $\mathbb{R}^2$, then its limit point must be a PBE (and will obviously be unique). Thus, to prove the proposition, we will show that the sequence $\{E_k\}_{k=0}^{\infty}$ converges.

Let us start at stage 1, where P and A are playing direct communication strategies characterized by the equilibrium quantity Δ^{dc}. Now, instead of P seeing s_{AP} that was influenced by $e^{dc}_{AP}(\theta)$, assume instead that M observes s'_{AM} where

$$\Pr\{s'_{AM} = R|\theta, e^{dc}_{AP}(.)\} = e^{dc}_{AP}(\theta) + g_M(\theta) \tag{139}$$

Using Bayes' rule, we can compute M's best response given $e^{dc}_{AP}(.)$, $d^{dc}(L)$, and $d^{dc}(R)$ as

$$e'_M(L) = \frac{-\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_A}\Delta^{dc}\right)\Delta^{dc}}{k_M\left(\frac{1}{2} - \frac{\beta}{k_A}\Delta^{dc}\right)} \tag{140}$$

$$e'_M(R) = \frac{\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_A}\Delta^{dc}\right)\Delta^{dc}}{k_M\left(\frac{1}{2} - \frac{\beta}{k_A}\Delta^{dc}\right)} \tag{141}$$

Let us define $\delta' = e_M^{'}(R) - e_M^{'}(L)$.

Now looking at stage 3 of the steps above, We can compute P's best response to $e_{AP}^{dc}(.)$ and δ' as

$$\begin{aligned} d^{'}(L) &= -2\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta' + \frac{\sigma_{\alpha,\theta}}{k_A}\Delta^{dc}\delta'\right) \\ d^{'}(R) &= 2\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta' + \frac{\sigma_{\alpha,\theta}}{k_A}\Delta^{dc}\delta'\right) \end{aligned}$$

We will denote $\Delta' = d'(L) - d'(R)$.

Before proceeding we will define some useful functions. Suppose we are in hierarchical communication and A believes M and P are playing according to $\delta = e_{MP}(R) - e_{MP}(L)$ and $\Delta = d(R) - d(L)$. A's optimization problem implies that he will choose

$$e_A^*(\theta; \delta, \Delta) = \frac{\alpha(\theta)}{k_A}\delta\Delta \tag{142}$$

Similarly suppose M believes P and A are playing according to Δ and $e_A^*(\theta; \delta, \Delta)$. Let

$$\delta^*(\delta, \Delta) = e_M^*(R; \delta, \Delta) - e_M^*(L; \delta, \Delta) \tag{143}$$

be characterize M's best response, where

$$e_M^*(L) = \frac{-\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_A}\delta\Delta\right)\Delta}{k_M\left(\frac{1}{2} - \frac{\beta}{k_A}\delta\Delta\right)} \tag{144}$$

$$e_M^{*}(R) = \frac{\left(\sigma_{g_M,\mu} + \frac{\sigma_{\alpha,\mu}}{k_A}\delta\Delta\right)\Delta}{k_M\left(\frac{1}{2} - \frac{\beta}{k_A}\delta\Delta\right)} \tag{145}$$

Finally, suppose P believes that A and M are playing according to $e_A^*(\theta; \delta, \Delta)$ and $e_M^*(s_M; \delta, \Delta)$. Let

$$\Delta^*(\delta, \Delta) = d^*(R; \delta, \Delta) - d^*(L; \delta, \Delta) \tag{146}$$

characterize P's best response where

$$d^*(L; \delta, \Delta) = -2\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^*(L; \delta, \Delta) + \frac{\sigma_{\alpha,\theta}}{k_A}\delta\Delta\delta^*(L; \delta, \Delta)\right) \tag{147}$$

$$d^*(R; \delta, \Delta) = 2\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^*(L; \delta, \Delta) + \frac{\sigma_{\alpha,\theta}}{k_A}\delta\Delta\delta^*(L; \delta, \Delta)\right) \tag{148}$$

Let us define the function

$$E(\delta, \Delta) = \begin{bmatrix} \delta^*(\delta, \Delta) \\ \Delta^*(\delta, \Delta) \end{bmatrix}$$

We can generate a sequence $\{E_k\}_{k=0}^{\infty}$ by letting $E_0 = E(\delta', \Delta')$ and

$$E_k = E(E_{k-1}) \tag{149}$$

for all $k > 0$. Note that $\{E_k\}_{k=0}^{\infty}$ is bounded above an below by the no-shifting-support conditions. If $\Delta_0 > \Delta'$, then $\{E_n\}_{n=0}^{\infty}$ is a strictly increasing sequence and therefore much converge. Suppose $\Delta_0 < \Delta'$, then $\{E_n\}_{n=0}^{\infty}$ is a strictly decreasing sequence and therefore must converge.[63]

Moving to the corollary, suppose that $k_M > \widehat{k}_M$ and define $\{E_k\}_{k=0}^{\infty}$ and $\{\widehat{E}_k\}_{k=0}^{\infty}$ be the corresponding strategy-updating sequences. We must have that $\widehat{E}_k \geq E_k$ for all k which implies that the limit point of $\{\widehat{E}_k\}_{k=0}^{\infty}$ must be weakly larger than the one for $\{E_k\}_{k=0}^{\infty}$. A similar argument establishes the other comparative static results.

Welfare in Hierarchical Communication

In this section we prove the accounting result that allows us to interpret the outcome of the hierarchical-communication game with one from a corresponding direct-communication game: Let δ^{hc} and Δ^{hc} characterize an equilibrium of the hierarchical-communication game. Recall that we defined the quantities

$$\widehat{k_{AP}} = \frac{k_{AM}}{(\delta^{hc})^2} \tag{150}$$

and

$$\widehat{g_P}(\theta) = g_P(\theta) + e_{MP}^{hc}(L) + g_M(\theta)\delta^{hc} \tag{151}$$

and applied these quantities to study an alternative direct communication game where the agent faces a cost of influence of $\widehat{k_{AP}}$ and the principal's exogenous expertise function is characterized by $\widehat{g_P}(\theta)$. We will prove the following proposition:

Proposition: *We have that*

$$\Delta^{hc} = \widehat{\Delta^{dc}}$$

and

$$\begin{aligned} W_P^{hc} &= \widehat{W_P^{dc}} \\ W_A^{hc} &= \widehat{W_A^{dc}} \\ W_M^{hc} &= \widehat{W_M^{dc}} - K^{hc} \end{aligned}$$

where

$$K^{hc} = \frac{k_{MP}}{2} E_{\theta, s_{AM}}\{\left(e_{MP}^{hc}(s_{AM})\right)^2 | e_{AM}^{hc}(.)\}$$

To begin the proof, first note that we can write

$$\Delta^{hc} = \frac{\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2 \Delta^{hc}}{\frac{1}{2}} - \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2 \Delta^{hc}\right)}{\frac{1}{2}}$$

In the dc' game, we have that

$$\begin{aligned} d^{dc'}(L) &= \frac{-\left(\sigma_{g'_P.\theta} + \frac{\sigma_{\alpha,\theta}}{k'_{AP}}\Delta^{dc}\right)}{\frac{1}{2} - \frac{E_\alpha}{k'_{AP}}\Delta^{dc'}} \\ &= \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2 \Delta^{dc'}\right)}{\frac{1}{2} - e_M^{hc}(L) - \frac{\delta^{hc}}{2} - \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2 \Delta^{dc'}} \end{aligned}$$

[63] If $\Delta_0 = \Delta'$, then we have found a PBE.

Note that we can write

$$e_M^{hc}(L) = -\left(\frac{1}{2} + \frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta^{hc}\right)\delta^{hc}$$

which implies that

$$\begin{aligned} d^{dc'}(L) &= \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}\right)}{\frac{1}{2} + \left(\frac{1}{2} + \frac{E_\alpha}{k_{AM}}\delta^{hc}\Delta^{hc}\right)\delta^{hc} - \frac{\delta^{hc}}{2} - \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}} \\ &= \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}\right)}{\frac{1}{2} - \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2\left(\Delta^{dc'} - \Delta^{hc}\right)} \end{aligned}$$

Similarly, we can show that

$$d^{dc'}(R) = \frac{\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}}{\frac{1}{2} + \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2\left(\Delta^{dc'} - \Delta^{hc}\right)}$$

Thus, we have that $\Delta^{dc'}$ must solve

$$\Delta^{dc'} = \frac{\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}}{\frac{1}{2} + \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2\left(\Delta^{dc'} - \Delta^{hc}\right)} - \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{dc'}\right)}{\frac{1}{2} - \frac{E_\alpha}{k_{AM}}\left(\delta^{hc}\right)^2\left(\Delta^{dc'} - \Delta^{hc}\right)}$$

and if we let $\Delta^{dc'} = \Delta^{hc}$ then the RHS above gives

$$\frac{\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{hc}}{\frac{1}{2}} - \frac{-\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\right)^2\Delta^{hc}\right)}{\frac{1}{2}}$$

which must be equal to Δ^{hc}. Furthermore, because we know that this model admits a unique equilibrium, we have that the unique equilibrium in the dc' game is characterized by $\Delta^{dc'} = \Delta^{hc}$.

Now we can examine the parties' welfare in the dc' game. We have

$$W_P^{dc'} = \frac{1}{2}\left(\sigma_{g'_P,\theta}\Delta^{dc'} + \frac{\sigma_{\alpha,\theta}}{k'_{AP}}\left(\Delta^{dc'}\right)^2\right) \tag{152}$$

$$W_A^{dc'} = \sigma_{g'_P,\alpha}\Delta^{dc'} + \frac{\sigma_\alpha^2 - (E_\alpha)^2}{2k'_{AP}}\left(\Delta^{dc'}\right)^2 \tag{153}$$

$$W_M^{dc'} = \sigma_{g'_P,\mu}\Delta^{dc'} + \frac{\sigma_{\alpha,\mu}}{k'_{AP}}\left(\Delta^{dc'}\right)^2 \tag{154}$$

which reduces to

$$W_P^{dc'} = \frac{1}{2}\left[\left(\sigma_{g_P,\theta} + \sigma_{g_M,\theta}\delta^{hc}\right)\Delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\Delta^{hc}\right)^2\right] \tag{155}$$

$$W_A^{dc'} = \left(\sigma_{g_P,\alpha} + \sigma_{g_M,\alpha}\delta^{hc}\right)\Delta^{hc} + \frac{\sigma_\alpha^2 - (E_\alpha)^2}{2k_{AM}}\left(\delta^{hc}\Delta^{hc}\right)^2 \tag{156}$$

$$W_M^{dc'} = \left(\sigma_{g_P,\mu} + \sigma_{g_M,\mu}\delta^{hc}\right)\Delta^{hc} + \frac{\sigma_{\alpha,\theta}}{k_{AM}}\left(\delta^{hc}\Delta^{hc}\right)^2 \tag{157}$$

where

$$\frac{k_{MP}}{2} E_{\theta, s_{AM}} \{ \big(e^{hc}_{MP}(s_{AM}) \big)^2 | e^{hc}_{AM}(.) \} \tag{158}$$

Using the necessary conditions for a PBE in the hierarchical-communication game (conditions (72), (76), and (77)) one can directly compute expressions for W^{hc}_P, W^{hc}_A, and W^{hc}_M. Noting that $\Delta^{dc'} = \Delta^{hc}$, one can verify that these expressions satisfy

$$\begin{aligned} W^{hc}_P &= W^{dc'}_P \\ W^{dc'}_A &= W^{dc'}_A \\ W^{hc}_M &= W^{dc'}_M - K^{hc} \end{aligned}$$

where

$$K^{hc} = \frac{k_{MP}}{2} E_{\theta, s_{AM}} \{ \big(e^{hc}_{MP}(s_{AM}) \big)^2 | e^{hc}_{AM}(.) \} \tag{159}$$