Nonlinear Analysis of Topology-Optimized Scissor-Like Elements During Deployment and Structural Performance Analysis

by

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B.S. Civil Engineering
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Submitted to the Department of Civil and Environmental Engineering on August 17, 2020 in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Civil and Environmental Engineering

Abstract
Deployable structures, known for their flexibility and adaptability, have gained popularity in applications such as emergency shelters, aerospace structures, and sports facility roofs. One of many commonly used deployable mechanisms is Scissor-Like Elements, which is generally made from two straight rods connected by a scissor hinge. To improve the quality of the flexibility, deployable scissor structures must be lightweight. This thesis will use topology optimization, a free-form design technology, to design low-weight rods within chains of deployable scissor elements. This will be done with the aim of finding a trade-off relationship between the structural performance and the required energy for the deployment of scissor chains. From the trade-off relationship, the balance between the structural stiffness, material saving, and power saving is investigated.

The first segment of the study focuses on reducing the self-weight of a deployable scissor chain while ensuring its structural performance through topology optimization. The scissor chain, intended as a retractable roof component, is formed by three pairs of scissors made of two identical bars with homogeneous steel material. Topology optimization tasks with varying volume constraints (50-90%) are performed in Abaqus on the static scissor chain in its open state. To avoid support condition influence, only the scissor bars in the center pair are optimized. The resulting topology-optimized bar geometries are found to have resemblance to hollow core sections.

The second part of the thesis establishes the nonlinear force-displacement relationship of the deployable scissor chain and the structural performance of the deployed structure. The topology-optimized center scissor pair is extracted and postprocessed to form a chain with three identical optimized scissor pairs in closed position. Using nonlinear analysis, this structure is computationally deployed to its fully open state. The resulting nonlinear external force required for deployment is recorded. A similar nonlinear force-displacement pattern is discovered in all cases. Furthermore, the total energy required to fully deploy the structure is calculated. Interestingly, both the maximum force and the total energy are found to increase nearly linearly with increasing self-weight. The structural performance of each case is studied by determining the center deflection under a service load. The weight reduction is found to affect the stiffness, with the 50% design showing a drastic reduction compared to the unoptimized case. The stiffness sacrifice decreases with weight reduction such that the 80% and 90% cases experience only mild decreases compared to the unoptimized chain. By comparison of the weight-deflection and the weight-energy to deploy relationships, the study illustrates a trade-off relationship. An ideal point for the case study in this work seems to appear between 60% and 80% volume limits, where the required external work for deployment is reduced by more than 36.8% while the deflection is only increased by less than 22.4% compared to the reference ($V=100\%V_0$).

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Title: Assistant Professor of Civil and Environmental Engineering
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At last, I would like to acknowledge Professor Chuck Hoberman, whose class “Transformable Design Methods” inspired the topic of this thesis.
Chapter 1

Introduction

1.1 Deployable Scissor Structures

Known for their flexibility and adaptability, deployable structures have gained popularity in applications such as emergency shelters, aerospace structures, and sports facility roofs (Gantes et al. 1994). One of many deployable mechanisms commonly used is the Scissor-Like Elements (SLEs). Scissor-Like elements in two-dimensional forms, which are shown in Figure 1.1-1, can be made from two straight rods connected by scissor hinges.

With two equal-length straight bars, the linkage translates in a simple straight line (Figure 1.1-1(a)). Curvature is introduced when the two bars that form a pair of scissors are uneven in length (Figure 1.1-1(b)).

![Figure 1.1-1](image)

Figure 1.1-1. (a) Straight scissor linkage (b) Curved scissor linkage (Mira et al. 2004)
The assembly can be deployed by either pulling or pushing from the ends. The chain of linkages will elongate in length \( (L') \) when the height is reduced and vice versa (Figure 1.1-2).

![Figure 1.1-2. Scissor chain deployment mechanism (Escrig 1985)](image)

*Structural Applications of Scissor-Like-Elements (SLEs)*

Deployable scissor structures have wide applications in structures as they are lightweight and can be folded compactly for easy transportation. If designed properly, the structure can have a high volume-expansion ratio (Mira et al. 2004). Additionally, the translation mechanism allows for rapid deployment, where the structure expands and contracts swiftly in great distance.

The first traceable account of a flat, deployable scissor structure dates to the 16th century when Leonardo Da Vinci introduced a lifting mechanism (Figure 1.1-3) consisting of “X-structures” in the “Madrid I” notebook (Valcarcel and Escrig 1994).
In 1961, Emilio Perez Piñero's moveable theater design introduced the world to the versatile functionality of these scissor elements. He designed a theater using SLEs that could be folded or unfolded in every direction and transported as needed with no permanent site (Peña Fernández-Serrano 2015). When fully expanded, the theater is capable of housing 500 people. Figure 1.1-4 is a photograph of Emilio Perez Piñero holding a 1/10 scale prototype of the retractable roof of the moveable theatre.

Figure 1.1-4. Emilio Perez Piñero’s 1/10 scale moveable theatre prototype (Peña Fernández-Serrano 1961)
Some of the main contributors to the development of scissor-like deployable structures in the past decade or so are Felix Escrig, Chuck Hoberman and Sergio Pellegrino. Hoberman invented and patented the angulated scissor elements which were used in the famous Hoberman sphere toy design and the retractable Iris Dome at MoMA (Figure 1.1-5) (Hoberman 1990). Hoberman’s simple angulated elements were furthered by You and Pellegrino who introduced *generalized angulated elements* which afford greater freedom in the configurations of foldable building blocks (You and Pellegrino 1997).

![Iris dome at the Museum of Modern Art, New York](image)

*Figure 1.1-5. Iris dome at the Museum of Modern Art, New York (Hoberman Associates 2020)*

In recent years, some of the applications of SLEs in structural designs include disaster relief emergency shelters (Mira et al. 2004), emergency bridges (Figure 1.1-6) and sports facilities (Figure 1.1-7). With their versatility and adaptability, deployable scissor structures have great potential in engineering design.

To further improve the quality of flexibility, it is important for deployable scissor structures to be lightweight. Heavy members not only undermine the architectural appeal of the structure, but also induce significant strains in the members themselves and the connecting moveable joints. The unintended deformations due to large self-weight compromises the integrity of
architecture and may even pose considerable risk to the safety of the occupants. Thus, if such problems exist, measures must be taken to reduce the weight of the deployable structure while ensuring the structural performance.

Figure 1.1-6. Emergency mobile bridge (Hiroshima University 2020)

Figure 1.1-7. The Salt Lake City Olympic Arch (Salt Lake Tribune 2020)

1.2 Structural Topology Optimization

One way of designing lightweight structure is to use topology optimization. Topology optimization has long been used by engineers as a tool, for instance, to minimize the amount
of material in a structure, while maintaining its structural strength. Topology optimization may be applied to structures for the benefit of weight saving. It is widely used in lightweight design of structures in automotive and aerospace industry, as well as in civil engineering, material science, and biomechanics (Zhang et al. 2016, Bujny et al. 2016, Zhang and Zhu 2018). In structural engineering, some topology optimization applications are emerging over the past decade. The Shenzhen CITIC Financial Center Towers in Shenzhen, China (Figure 1.2-1), for instance, is an example of the structural optimization application on a large scale. The optimized exoskeleton truss layout promoted material efficiency while ensured overall stiffness of the towers (SOM 2020).

![Shenzhen CITIC Financial Center Towers (SOM 2020)](image)

Figure 1.2-1. Shenzhen CITIC Financial Center Towers (SOM 2020)

The purpose of topology optimization is to find an optimized layout of a structure within a specified region (Bendsøe et al. 2003). It is one of the branches of structural optimization methods differing from sizing and shape optimization. In a sizing optimization problem, the objective is to minimize a physical quantity of a structure such as peak stress or deflection by varying, for instance, the material thickness distribution of structural member. In a shape
optimization problem, on the other hand, the objective is to find the optimized shape of a design domain such as the shape of the holes in a perforated beam (Figure 1.2-2(b)). It is important to note that in both sizing and shape optimizations, there is no change in the number of members. Opposite, in topology optimization some part or member of the structure can be added or deleted. Thus, a new layout will be generated based on the chosen criteria of the optimizer. Figure 1.2-2 illustrates and compares the three different types of structural optimization.

Applying topology optimization to deployable structures has additional advantages other than the manufacturing cost savings and environmental benefit due to material saving. Disaster relief shelter, for example, can become more portable with reduced self-weight, which leads to more ease for transportation to any remote areas that lacks accessibility. On a larger scale, a lighter weight topology-optimized retractable stadium roof, for instance, means more ease during deployment that is usually driven by hydraulic power systems, resulting in energy saving.

1.3 Related Works and Research Motivation
In the field of deployable structures, optimization has been used to solve a variety of problems. Deployable structures are often optimized for desired properties such as stiffness, self-weight etc. Shape optimization of global structure and sizing optimization of individual elements are common approaches to such problem setup. (Mira et al. 2015, Dai et al. 2014). Other researches have focused on topology optimization of local components such as self-deployable joints (Ferraro and Pellegrino 2019) and layout of the discrete beams that comprise the Universal Scissor Component (USC) (Mira et al. 2015).

In their study, Dai et al. 2014 applied shape optimization on the double-ring deployable truss (DRDT) structure itself. By varying the geometry of the repeating deployable units and their height, the stiffness and the performances of the entire antenna system were optimized. The optimized weight was obtained with the stiffness constraint via a mix of genetic algorithm (GA) as a stochastic search optimization algorithm and gradient-based optimizer, being subject to system stiffness and constraints of member sections (Dai et al. 2014).

Mira et al. 2015 proposed an optimized design of a universal scissor component (USC). It is a generalized element that can adapt to all traditional scissor types (Figure 1.3-2) by combining the respective characteristics in one component. First, a global shape study was performed to determine the dimensions for the USC. The sizing optimization was set to
maximize the structural performance subjecting to the self-weight constraint. Then, for a more
detailed design, a topology optimization was implemented to determine the positions of the
pinholes in the mast and strut for minimum weight. The beam layout within a USC depended
on the pinhole positions, for a potential beam member exists between two pinholes. Figure 1.3-
4 summarizes the three optimized topologies (Mira et al. 2015).

Figure 1.3-2. T-shape universal scissor component: (a) as a translational unit; (b) as a polar unit; and
(c) as an angulated unit (Mira et al. 2015)

Figure 1.3-3. Different positions of the pinholes in the mast (vertical bar) and strut (horizontal bar)
(Mira et al. 2015)
Ferraro and Pellegrino explored the design of a self-deployable thin-shell joint with cutouts at its tip. The objective of the research was to strategically position cutouts on the joints so that they can fold without failing, while maximizing the bending stiffness. The optimized shape and position of the cutouts were determined by topology optimization with a novel level-set method. The method simplified the 3-D problem into a 2-D one. (Ferraro and Pellegrino 2019)

The existing related work has proven structural optimization effective in deployable structure applications. The studies mentioned above have different focuses. Dai et al. 2014 applied shape and size optimization on the entire truss. Because the bars were treated as truss members, the optimized results vary solely in cross-section areas and lengths. No material is removed from the full bars. Mira et al. 2015 also used full bars as main structural component.
Topology optimization is performed on an adapting component for universal application. Likewise did Ferraro and Pellegrino 2019 who conducted topology optimization on a 3-D joint member that undergoes large displacement.

This thesis aims at studying the required energy and structural response of topology-optimized scissor-like structures during the transformation from a compact state to an expanded one. As such, this thesis is most related to Ferraro and Pellegrino 2019’s study. However, in this work, the focus will be on conducting topology optimization on the bars of a deployable unit, leaving the joints untouched. The scissor bars, which are the main structural components forming the scissor chain, are treated as 3-D beam members that allow both axial and flexural deformations. Through topology optimization, the deployable scissor chain structure has great potential for material savings while maintaining structural performance. Like Ferraro and Pellegrino’s “self-deployable joint”, the scissor chain also undergoes large deformation leading to geometric nonlinearities during deployment. In this work, the large deformations are not included in the topology optimized design, as the design will be conducted in the deployed state. Instead, nonlinear finite element analyses will be performed on the topology-optimized design to simulate deployment and estimate the work needed to fully open the new designs.

The rest of the thesis will be structured as follows. In Chapter 2, the methodology used will be detailed including the procedures for topology optimization, nonlinear finite element analyses and structural performance analyses. Chapter 3 will show the results of the three main segments with a discussion on the trade-off relationship between the work saving and structural performance. Chapter 4 will be a conclusion that summarizes all the procedures and findings with some future work recommendations.
Chapter 2

Methodology

2.1 Topology Optimization: Problem Formulation, SIMP Method and Filtering

Topology Optimization Problem Formulation

An efficient and robust method of approaching a topology optimization problem is to formulate it as a formal optimization problem and solve it using a mathematical optimizer. Typically, the following problem formulation is implemented:

\[
\begin{align*}
\min_{\varphi} & \quad f = F^T d & \quad \text{Objective} \\
\text{subject to} & \quad K(\varphi)d = F & \quad \text{Linear Elastic Equilibrium Constraint} \\
& \quad \sum_{e \in \Omega} \rho_e (\varphi)v_e \leq V & \quad \text{Volume Constraint} \\
& \quad \varphi_{min} \leq \varphi_i \leq \varphi_{max} & \quad \forall \ i \in \Omega
\end{align*}
\]

\(K\): global stiffness matrix of the design domain

\(F\): external force vector
The design variables, \( \varphi \), controls the material distribution, \( \rho \), that will affect the value of the chosen objective function (the relation between \( \varphi \) and \( \rho \) will be detailed shortly). The objective function, \( f \), represents the quantity that is being minimized for best performance. The most common objective is to minimize the compliance which, in turn, leads to maximizing the elastic stiffness of a linear elastic structure. The constraints are the characteristics that the solution must satisfy. Common examples of the constraints are the equilibrium condition of the structure and the maximum amount of material to be distributed (also known as the volume constraint). Note that in (2) a linear elastic equilibrium constraint is applied. Problem formulations that include nonlinear equilibrium constraints (as in reality needed if modeling deployment) have been suggested e.g. by Bruns and Tsorterelli 2001 and Pedersen et al. 2001.

The traditional approach to topology optimization is the discretization of a domain into a mesh of finite elements (Bendsøe et al. 2003). Each grid is either filled with solid when the material is required at the location or removed of material where it is not needed. Thus, the density distribution, \( \rho \), of material within a design domain, \( \Omega \), is discrete, and each element is assigned a binary value (0 or 1). The design variables, \( \varphi \), control the densities through the filtering operation (as will be described later). For example, Figure 2.1-1 shows an optimized binary material layout of a loaded beam. The solid elements with density 1 are black, whereas
the void elements with density 0 are white.

![Figure 2.1-1. Optimized binary distribution of density simply supported beam example (Liu et al. 2018)](image)

The binary density distribution of the design domain, however, has its drawbacks when it comes to solving the problem. It is crucial to note that the number of design variables increases with the increasing number of meshes assigned to the domain. To obtain an accurate result using finite element method, the solver often requires a fine mesh. As solving the binary problem requires the use of non-gradient based (stochastic) optimizers, it is computationally expensive for a high number of design variables. (Sigmund 2011).

**SIMP Method**

The most commonly used approach used to solve the discrete (pixelated) problem is the Solid Isotropic Material with Penalization (SIMP) method (Bendsøe 1989). SIMP replaces the integer or discrete variables with continuous variables and then introduce some form of penalty that steers the solution to discrete 0-1 values. The introduction of a continuous relative density distribution function relaxes the binary constraints in the problem and thus, with the continuous variable formulation, the optimizer can apply a gradient-based method to solve the problem.
For each element, the assigned relative density can vary between 0 and 1, which allows the assignment of intermediate densities for elements ($\rho_{\text{min}} \leq \rho_e \leq 1$). A small stabilizer $\rho_{\text{min}} > 0$ (minimum relative element density) is needed for all elements to avoid singularity of global stiffness matrix.

The Young’s modulus, $E_e(\rho_e)$, at each element varies continuously with the material relative density $\rho_e$ at a penalty $p$:

$$E_e(\rho_e) = (\rho_e^p + \rho_{\text{min}}) \cdot E_{e0}$$  \hspace{1cm} (4)

$E_{e0}$ is the original Young’s modulus of the isotropic material.

From Figure 2.1-2, it is observed that when the penalty $p > 1$, the increase in stiffness of the element is small compared to the volume assigned. This means, the intermediate densities are unfavorable as the gain in stiffness cannot justify the volume expenditure when the volume constraint is active. As such penalty in stiffness of intermediate densities increase with $p$ value, $p \geq 3$ is usually required to obtain a nearly binary design (Bendsøe and Sigmund 1999).
Figure 2.1-3. Design domain: a cantilevered beam subject to downward point load at free end

Figure 2.1-4. Topology optimized results with mesh size: 16x10 (produced by author)

Figure 2.1-3 is an illustration of a continuum topology optimization problem setup: A simple cantilever beam is subject to a downward point load at the free end. The cantilever beam is topology optimized both with and without SIMP implementation. Figure 2.1-4 shows the difference in results whether SIMP is implemented, illustrating how SIMP method regularizes the result of a continuum topology optimization problem.

Figure 2.1-4(a) gives a result beam element with varying material thickness when no SIMP is applied. Such design is sometimes difficult to realize as it can be limited by 3D printing technique and printer resolution (Zhu et al. 2017). With the implementation of SIMP (Figure 2.1-4(b)), the structural member can be made of homogenous material with mostly macroscopic changes in the geometry.
**Filtering**

By applying SIMP, a binary solid-void structure is achieved. Further inspections suggest that the method alone still cannot lead to satisfactory designs. Commonly, solutions will have disconnectivities in the diagonal members (see e.g. Figure 2.1-4(b)). Moreover, when the mesh is further refined, micro-perforations begin to appear in the materials (Figure 2.1-5). That is, the solution does not converge to a macroscopic continuous geometry. The differences between Figure 2.1-4(b) and Figure 2.1-5 indicate that with finer meshes, smaller micro-voids occur in the material, resulting in a checkerboard pattern. This problem is commonly referred to as *mesh dependency* (Bourdin 2001).

![Checkerboards](image)

Figure 2.1-5. Unfiltered optimized results with checkerboard patterns 80x50 and 160x100 mesh (Carstensen 2019)

Thus, to ensure connectivity and avoid mesh dependency, a smoothing filter is often applied after the optimizer implements a penalty method such as SIMP. There are a number of existing filtering approaches and they vary in ways how the function, $f$, is modified, which connects the design variables, $\varphi$, and the element densities, $\rho$.,
Density filtering is a commonly used filtering technique nowadays. For given values of the design variables, the filtering function, $f$, gives weighted density distributions of the surrounding elements (Bruns and Tortorelli 2001). That is, when applying the density filter, density distribution becomes a function of the design variables, i.e. $\rho = f(\varphi)$.

![Figure 2.1-6 Optimized cantilever beam with density filter 160x100 mesh (Carstensen 2019)](image)

Introduced by Sigmund in 1997, sensitivity filter applies a smoothing filter to the derivatives of the objective function (Sigmund 1997). In the sensitivity filtering approach, no filtering is directly conducted on the density, $\rho$, itself. Instead, the sensitivities (derivatives) of the objective function are filtered.

![Figure 2.1-7 Optimized cantilever beam with sensitivity filter 160x100 mesh (Carstensen 2019)](image)
For both filters, the weighting criteria can often base on the distance between the centroid locations of the element, \( e \), and its neighboring elements within a circle with radius, \( r_{\text{min}} \). The weight factors can be taken, for instance, linear decreasing outward for the elements from its neighborhood center.

![Figure 2.1-8 Neighboring elements in density and sensitivity filters (Nana et al. 2016)](image)

Other existing filters include the Heaviside filter: the design variables, \( \rho \), are first filtered using the linear density filter, and the result is then mapped to a physical density by a Heaviside approximation (Guest et al. 2004).

### 2.2 Modeling and Optimization Setups in Abaqus

The first half of the study involves the 3-D modeling and topology optimization of the deployable scissor chain in the load-resisting state (extended or open position). Note that though a deployable structure, topology optimization in this work is not applied during deployment but rather on the fully deployed static scissor chain structure. The fully extended scissor chain is optimized since it is subjected to the highest loads (gravity and climate loads). Additionally, the optimization tasks are carried out solely on the center pair to avoid the
influence of support reactions. One considerable advantage of this method is the significant reduction in design variables (only one out of three pairs is optimized) and computational power demand compared to the optimization of the structure while it is deploying (a nonlinear topology optimization problem). The following workflow summarizes the main steps taken.

**Workflow:**

*Step 1:* Model three pairs of scissors in their *open* positions and define material properties in Abaqus.

*Step 2:* Assign *hinge* connectors to the scissor pairs.

*Step 3.* Define *boundary conditions* and apply *body force* to the structure.

*Step 4.* Create topology optimization tasks of the center scissor pair with various *volume constraints* (50% to 90% of initial volume with 10% increments) in Tosca Structure.

*Step 5.* Run the optimization tasks and extract results in *.STL* format.

The scissor pair consists of two identical solid bars connected in the center. The top and bottom bar are oriented at a 120-degree angle with respect to each other.

![Perspective view](image1.png) ![Top view](image2.png)

(a) Perspective view  
(b) Top view

Figure 2.2-1. Scissor pair unit geometry in *open* position

The material property of the scissor bars is defined under the assumption of linear elastic
deformation. Steel is selected as the structural material with a Young’s modulus of 29000 ksi and a Poisson’s ratio of 0.3. Three circular holes (diameter = 1in) are cut through the rectangular bars indicating the locations of “hinge” connectors.

Before assigning “hinge” connectors to the predefined locations, some coupling constraints and wire features must be made. Figure 2.2-3 illustrates that a “structural distributing” coupling constraint is applied between the inner surface and the center point (control point: RP-1) of the hole in the top bar. The structural coupling method couples the movements (translational and rotational) of the reference node to those of the coupling nodes (Abaqus User Manual 2020).
The same coupling constraint is applied to the bottom bar. Then, the two control points are connected by a “wire” feature to which allows a “hinge” connector to attach. The “hinge” connector enables the desired relative rotational movement of the scissor pair. According to Abaqus’s definition, “hinge” section restricts all degrees of freedom but the rotational movement about its local X-axis (Orientation 1). Thus, a local coordinate system (Datum CSYS) must be created for each “hinge” assignment. In Figure 2.2-4, the highlighted local coordinate system marks the direction of the X-axis along the hinge shaft. It is important to note that in the study, no friction or rotational stiffness is defined. That is, the bars are assumed to rotate freely without resistance from the joints.

![Figure 2.2-4. “Hinge” connector type diagram and local coordinate system](image)

The same procedure is repeated on all center and edge joints of scissor bars. All three pairs of scissors eventually form a chain that resembles a component of a deployable roof (Figure 2.2-5).
To ensure the scissor chain can freely expand in the X-direction (U1) but not move in rigid body motion, the following boundary conditions are imposed (Figure 2.2-6). “0” means the DOF is constrained and “-” means unconstrained. Note that all rotational degrees of freedom are unconstrained.

Moreover, the retractable scissor chain will be optimized considering solely self-weight. Additional load such as roof snow load will be accounted during structural performance analysis. A body force [force/volume] value of $2.8e-4$ (k/in³) in the global negative-Y direction is assigned to all steel elements.

Five topology optimization tasks are implemented on the center scissor pair (Figure 2.2-7). Each task has the same objective to minimize strain energy (2.1(1)) but varying volume constraints (2.1(3)).
Figure 2.2-7. Five optimization tasks summary ($V < 50\% V_0$, $60\% V_0$, ..., $90\% V_0$)

Figure 2.2-8. Optimization task advanced setting

Under the “Advanced” tab of are some parameters defined for the optimization tasks, which have been discussed in Section 2.1. SIMP method is selected as the material interpolation technique.

**Mesh Study**

A mesh size study was performed on the example topology optimization task ($V = 90\% V_0$) to determine the suitable mesh size and geometric order for all optimization jobs. The
linear/quadratic element refers to a linear/quadratic shape function that results in a first/second order formation.

Table 2.2-1 Mesh Study Results

<table>
<thead>
<tr>
<th>Strain energy -Objective</th>
<th>Mesh Size (inch)</th>
<th>Time elapsed (seconds)</th>
<th>Mesh Size (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Linear</td>
<td>0.452</td>
<td>0.440</td>
<td>0.443</td>
</tr>
<tr>
<td>Quadratic</td>
<td>0.475</td>
<td>0.447</td>
<td>0.446</td>
</tr>
</tbody>
</table>

As shown in Table 2.2-1, a mesh size of 2 with linear elements yields the best results. It leads to a smallest value of strain energy of all eight trials without significant sacrifice in computation time. All subsequent optimization tasks will follow the same mesh settings.

![Figure 2.2-9. Strain energy objective and volume constraint (V<90%V₀) plot on corresponding design cycles 1-15](image)

As an example, Figure 2.2-9 shows that the $V<90\%V₀$ optimization task converges at cycle 15. Note that the strain energy is minimized and converged to approximately 0.45, and the volume constraint is satisfied at $V$ slightly under 0.9$V₀$. The optimized scissor pair is extracted from result and then imported to a new model for further analysis. Same steps are taken for all five optimization tasks.
2.3 Linear and Nonlinear Finite Element Analyses of Scissor Chains

The second half of the study aims at obtaining the external force required for deploying the scissor chain and investigating the scissor chain’s structural performance under different volume constraint. To achieve such a goal, the study performs (i) a nonlinear analysis of the scissor chain during deployment, and (ii) a linear analysis of the structure in the extended configuration under service load.

*Workflow:*

*Step 1:* Export optimized center scissors in .STL file to Rhino 6 to tidy up geometry and convert mesh to closed solid polysurface in .IGES format.

*Step 2:* Import .IGES file back into Abaqus. Orient the optimized scissor pair in its closed position.

*Step 3:* Create a linear array of the optimized structure in positive X-direction and form a chain of three pairs of scissors. Repeat section 2.2 Step 2 and Step 3.

*Step 4:* Create a linear step in the undeployed state for the applied self-weight.

*Step 5:* Apply displacement to the scissor chain to allow it to deploy and then create a
nonlinear step for the large displacement applied.

**Step 6:** Mesh the entire model and run the analysis.

**Step 7:** Record the total reaction forces in X-direction during deployment.

**Step 8:** Model three pairs of scissors in their *open* positions for each optimized result with $V < 50\%, 60\%, ..., 90\%V_0$ volume constraints. Repeat Section 2.2 Step 2 and Step 3 for each model.

**Step 9:** Apply snow load to determine mid-span deflection.

When the optimized result is first exported from Tosca, it cannot be used directly for analysis as it is in the form of *orphan meshes*. Four steps were implemented in Rhino 6 to tidy up geometry and convert to compatible solid polysurface for analytical models: (i) Apply Mesh > Mesh Repair > Fill Holes to all mesh components (ii) Apply Mesh > Mesh Repair > Unify Normals to all mesh components (iii) Join all component meshes with Mesh > Mesh Boolean > Union (iv) Enter "MeshtoNURB" at the command line to convert mesh to solid and then delete mesh. At last, to check that the result is a closed solid polysurface, type “What” at the command (Figure 2.3-1).

![Figure 2.3-1. Geometry validity check result](image)
With the compatible geometry in .IGES format, the scissor bars are imported into Abaqus as individual parts. They are then re-oriented in their *closed* position with a 14-degree angle between the top and bottom bars (Figure 2.3-2). The angle is determined as the smallest possible value that allows for maximum compactness and also prohibits interference with neighboring components.

To create a chain of *retracted* scissors, the single pair is repeated by using a linear array in positive X-direction with appropriate distance between the units. Then, with same maneuvers as depicted in Section 2.2, *hinge* connectors are assigned at each center and edge joint. Similarly, identical *boundary conditions* and *body force* are applied to corresponding locations and components.
Now, the basic models are established and ready to deploy in simulation. A displacement of 200.922 inch is required to open the chain from its retracted position (14°) to extended position (120°). The large displacement, however, leads to geometric nonlinearity issues.

**Linear and nonlinear FEA analysis**

A linear static analysis is an analysis where the load-displacement relationship is linear. A linear static analysis is suitable for static structures where the deformation is deemed to not affect how the structure behaves. That is, the deformed shape of the structure after the load
being applied is assumed the same as the undeformed shape (small deformation). Thus, the structure’s stiffness matrix remains constant during load application and so requires less computation time compared to a nonlinear analysis on the same structure (Borst et al. 2012).

A nonlinear static analysis is an analysis where the load-displacement relationship is nonlinear. There are three sources of nonlinearity in the context of structural mechanics simulation (Abaqus User Manual 2020): (i) Material nonlinearity, (ii) boundary nonlinearity, and (iii) geometric nonlinearity. These effects result in a stiffness matrix which is not constant during the load application. The large translational/rotational displacement due to deployment is categorized as geometric nonlinearity. Therefore, to analyze a deployable structure during deployment where large deformation is prescribed, a nonlinear solving strategy is needed. The large deformations mean that the structure in the initial configuration (undeployed) and the final configuration (fully deployed) cannot assumed to be the same. Therefore, the stiffness matrix must be updated on each iteration as the structure deforms. To account for the geometric nonlinearity in Abaqus, the corresponding step (Step-2) must have its “NLgeom” switch “On”. This option includes the nonlinear effects of large displacement without large deformation (“large displacement, small strain”) while still allowing linear formulations. It can be considered an upgrading modification to the “Static, General” procedure. Note that Step-1 is associated with body force assignment. Since there is no significant strain occurring under self-weight while the chain remains undeployed, it is suitable to keep the “NLgeom” switch “Off”.

![Step Manager](image)
Displacement Control Procedure for External Force Determination

An essential objective of the study is to determine the variation in external force required to deploy the scissor chain composed of optimized components with varying volume constraints. The goal can be realized by prescribing load (load control) or by prescribing displacement (displacement control). The displacement control procedure causes a stress development within the specimen, which in turn results in nodal forces at the nodes where the displacements are prescribed. Summation of these forces gives the total reaction force, which, except for a minus sign, equals the equivalent external load that would be caused by the prescribed displacements (Borst et al. 2012).

In this study, displacement control is an intuitive selection as it mimics how most deployments occur. To obtain the corresponding reaction force at each support, a “History Outputs” called “Nodal_force” is created. It records the total nodal force at the tip where the displacement is applied. According to Borst et al. 2012, the total external load required to open the scissor chain is equivalent to the opposite value of “Nodal_force”. Figure 2.3-7 shows the “Nodal_force” plot of the optimized scissor chain with 90% volume constraint.
To analyze the structural performance of the scissor chain, which is potentially a component of retractable roof, the study wants to determine its deflections under service load with varying volume constraints. Cambridge, Massachusetts is selected as the area of study where the ground snow load is reported as 40 psf (ASCE/SEI 7-16 2017). The extended scissor chain covers an area of approximately 73 sqft (234 in × 45 in), which leads to 2925 lbs (2.925 kips) in snow load. The total snow load is evenly distributed to the center of each of the three scissor pairs (Figure 2.3-8). The deflection at the mid-span of the chain under unfactored snow load and self-weight is then recorded (Figure 2.3-9).
Figure 2.3-8. Equivalent concentrated snow load assignment

Figure 2.3-9. Mid-span deflection history output set-up
Chapter 3

Results and Discussions

3.1 Topology-Optimized Results

As recounted in Section 2.2, six solid rectangular beam elements compose the three identical scissor pairs and all components are subject to a body load [force/volume] value of $2.8 \times 10^{-4}$ (k/in$^3$) in the global negative-Y direction. A detailed description of applied boundary conditions can be found in Figure 2.2-6.

The topology optimization tasks were conducted when the scissor chain is in its open position. This is because the structure experiences the most stress and center deflection due to bending when the total span elongates as in the extended state. To avoid the influence of support reactions, a series of five topology optimization tasks were carried out solely on the center pair. The topology-optimized results of a deployable scissor chain are summarized in Figure 3.1-1.

Design Domain:
### Topology-Optimized Results

\( f: \text{compliance (strain energy)} \)
\( V: \text{volume} \)

<table>
<thead>
<tr>
<th>Perspective view</th>
<th>Top view</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &lt; 0.9V_0 )</td>
<td>![Image]</td>
</tr>
</tbody>
</table>

Objective: \( f = 0.44 \)
Constraint: \( V = 0.90V_0 \)

<table>
<thead>
<tr>
<th>Perspective view</th>
<th>Top view</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &lt; 0.8V_0 )</td>
<td>![Image]</td>
</tr>
</tbody>
</table>

Objective: \( f = 0.46 \)
Constraint: \( V = 0.80V_0 \)

<table>
<thead>
<tr>
<th>Perspective view</th>
<th>Top view</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V &lt; 0.7V_0 )</td>
<td>![Image]</td>
</tr>
</tbody>
</table>
The optimizer removes material from the two ends and the interior of the bars. With decreasing volume constraints, more material is taken from the core, making the scissor bars increasingly hollow. The optimization results approach the geometry of hollow rectangular
elements. Furthermore, as the volume percentage reaches 50%, the interior hollow space reaches the exterior. A few holes appear in the remaining outer shell, resulting in discontinuities in the exterior.

### 3.2 Nonlinear Finite Element Analysis Results

The second part of the thesis studied the nonlinear relationship between the applied displacement from a fully closed to fully open position of the chain of three scissors-links in the direction of deployment (global X-direction) and the external force in the same direction. The nonlinear analysis was conducted on the unoptimized scissor chain and all five topology-optimized cases. Large displacements were considered, however, yielding of the material was ignored. It is crucial to note that in this stage of analysis, the structure is subject to self-weight only as in a real-life deployment scenario - refer to Figure 2.3-4 to see a detailed illustration of the displacement applied and boundary conditions.

![Displacement](image)

(a) closed position
The nonlinear analysis result of the relationship between force and displacement of the unoptimized case \((V=100\%V_0)\) is shown in Figure 3.2-2. Within the analysis, the displacement used to open the scissor-chain was applied in constant displacement increments. In the first displacement increment, the force experiences a jump in value when the scissor chain begins to open from its resting undeployed state. Subsequently, the load increases with a decreasing rate and eventually plateaus at approximately 70.0\% of the entire prescribed displacement. When the structure approaches the completion of the deployment, the external force in the global X-direction gradually decreases.
Figure 3.2-3 is a collection of all load-displacement curves. All five topology-optimized and unoptimized deployable scissor chains show similar load-displacement relationship as described in the previous paragraph.

Table 3.2-1 summarizes the Degree of Deployment at peak load for all five optimized cases.
The Degree of Deployment (DoD) refers to the ratio between the displacement at peak load to the displacement at completion. From Table 3.2-1, it is observed that the DOD where the peak load occurs varies with different volume constraints. With decreasing volume percentage constraint, the DoD at maximum load also decreases. Based on the numerical results, external loads reach their maximum sooner for optimized scissor chain subjecting to less gravity load (smaller volume cap). The unoptimized case is the exception to the trend of peak loads.

Table 3.2-1. Degree of Deployment (DoD) at maximum load with different volume constraints

<table>
<thead>
<tr>
<th>Volume constraints</th>
<th>$V&lt;50%V_0$</th>
<th>$V&lt;60%V_0$</th>
<th>$V&lt;70%V_0$</th>
<th>$V&lt;80%V_0$</th>
<th>$V&lt;90%V_0$</th>
<th>$V=100%V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOD at max load</td>
<td>62.5%</td>
<td>70.0%</td>
<td>72.5%</td>
<td>75.1%</td>
<td>77.5%</td>
<td>70.0%</td>
</tr>
</tbody>
</table>

The maximum external load values corresponding to all unoptimized and optimized cases are plotted in Figure 3.2-4. Interestingly, a near linear relationship is observed between the peak external force and the percentage of volume constraints.

Figure 3.2-4. Peak external loads with different volume constraints
The total work, measured in [kip-in], is the integral of the external force over the entire displacement (from closed to open position). The total worked required for fullying deploying the scissor chains also experiences a near linear increase with greater self-weight (higher volume limit).

![Total Work Curve](image)

Figure 3.2-5. Total required work for the deployment of scissor chains with different volume constraints

3.3 Structural Performance Analysis Results

A structural performance analysis is also conducted on the scissor chains by determining their deflections under service load when fully deployed. As potentially a component of a retractable roof, the scissor structure is assume to be subject to a uniform snow load of 40 psf (ASCE/SEI 7-16 2017). See details of the loading conditions in Figure 2.3-8. The von Mises stress contours of all six cases are summarized in Figure 3.3-1. Stress gradients near the end supports are seen in all cases. In the $V<50\%V_0$ case, some local stress concentrations are observed, particularly near the exterior porosities in the scissor bars and the hinge connections. With increasing
volume constraints, the stress in the scissor pairs becomes more uniform. Though local stress concentrations still exist around the hinge connections. The concentrated stress distributions near the supports and hinges are expected. The interactions in these areas are applied through reference points which control the movement of the whole structure. Singularities will therefore inevitably occur around the joints, causing local stress concentrations.

Figure 3.3-1. The von Mises stress contours

Figure 3.3-2 shows the deflection contours in global Y-direction of all cases. The deflection gradients mostly occur in the scissor pairs of two ends. The deflections (absolute value) gradually increases from the end supports and remains relatively even in the center pair.
The center deflections corresponding to the respective volume constraints are recorded in Figure 3.3-3. The numbers in the vertical axis refer to the absolute values of deflection. The deflection experiences a significant drop from 50% to 60% volume constraints. From 60% to 80% volume constraints, the deflection undoes a moderate decrease. Later on, the deflection sees a mild decrease from 80% to 100%. All cases, including the 50%\(V_0\) one which is deemed to have a drastic increase in center deflection compared to the rest, satisfy the deflection requirement in industry practice (1/240 of total span) (The Massachusetts state building code 2016). For research analysis purpose, only snow load is considered in the analysis. In real-world design, however, additional loadings such as roof live load must be accounted and so the deflection curve might differ for varying loading conditions.
3.4 Discussions: Trade-Off Relationship Between Work Saving and Structural Performance

Finally, the trade-off relationship between the total work required to deploy the scissor chains and their deflection in the extended state is shown in Figure 3.4-1. That is, the deployable scissor chain with smaller gravity load requires less energy to deploy, but sacrifices stiffness when fully deployed. Depending on whether the greater stiffness or less power for deploying is prioritized in designs, engineers can learn from trade-off relationship and decide on an optimal design of judgment. An ideal point is shown to appear between 60% and 80% volume limits, where the required external energy is saved by more than 36.8% while the deflection is only increased by less than 22.4% compared to the reference \( V=100\%V_0 \).
Figure 3.4-1. Trade-off relationship between required work and deflection (stiffness)

Table 3.4-1. Center deflection and required work data

<table>
<thead>
<tr>
<th>Volume Fraction (%)</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Deflection, in</td>
<td>1.47 E-02</td>
<td>1.49 E-02</td>
<td>1.53 E-02</td>
<td>1.65 E-02</td>
<td>1.80 E-02</td>
<td>2.42 E-02</td>
</tr>
<tr>
<td>Work, kip-in</td>
<td>3.93 E-01</td>
<td>2.99 E-01</td>
<td>2.49 E-01</td>
<td>1.91 E-01</td>
<td>1.54 E-01</td>
<td>1.22 E-01</td>
</tr>
<tr>
<td>Deflection increase (%)</td>
<td>0.0 %</td>
<td>1.4 %</td>
<td>3.8 %</td>
<td>12.2 %</td>
<td>22.4 %</td>
<td>64.2 %</td>
</tr>
<tr>
<td>Work saving (%)</td>
<td>0.0 %</td>
<td>23.8 %</td>
<td>36.8 %</td>
<td>51.4 %</td>
<td>61.7 %</td>
<td>69.0 %</td>
</tr>
</tbody>
</table>
Chapter 4

Conclusions and Future Work

4.1 Conclusions

This study has presented a method of studying the trade-off relationship between the structural performance and the required work for deployment of topology-optimized deployable scissor chains. The approach allows engineers to find an optimal design of judgment balancing the structural stiffness, material savings and power savings based on design limits and priorities.

For deployable structures, full parametric topology optimization is inherently difficult due to geometric nonlinearities resulting from large displacement during deployment. To save computational power and time, the topology optimization tasks are conducted on the fully deployed state of the scissor chain with varying volume constraints ($V<90\% V_0, V<80\% V_0 ..., V<50\% V_0$). To further reduce the complexity of the problem and avoid support condition influence, only the center scissor pair is optimized. The geometry of topology-optimized scissor bar resembles that of hollow rectangular element, with a solid continuous outers shell and a hollowed interior. In the $V<50\% V_0$ case, some cavities are observed on the exterior of the scissor bars.
The second part of the thesis has determined the nonlinear force-displacement relationship of the deployable scissor chain. The optimized shapes from the first section are extracted for postprocessing. After a series of reorientation and linear array, a chain of three pairs of optimized scissor pairs in their closed position is formed. To transform the scissor chain from its currently closed to its fully open position, a displacement is applied at one end of the structure. The resulting nodal force where the displacement is applied is recorded as the external force required to fully open the scissor chain. The force experiences an initial jump at the first displacement increment, and gradually increases until a maximum force is reached. The external force slowly decreases towards the end phase of deployment after the plateau. Similar pattern of the force-displacement relationship is observed in every optimized and unoptimized case. Interestingly, a near linear relationship is discovered between the maximum external force and the self-weight (volume) of the scissor chains. By integrating the force over the displacement, the total energy required to fully deploy the structure is also calculated for each case. Curiously, a near linear relationship is also discovered between the total energy for deployment and the self-weight (volume) of the scissor chains.

The structural performance of each case is studied by determining the respective center deflection due to a uniform snow load applied on the extended scissor chain with the corresponding volume constraint. Based on the volume limit - deflection relationship, the scissor chain with 50% volume constraint has a drastic reduction in stiffness in comparison to the unoptimized case. The 60% and 70% cases have moderate reductions in stiffness. The 80% and 90% cases experience only mild decreases in stiffness, with little increase in deflection compared to the unoptimized chain. By combining the volume fraction - deflection curve with
the volume fraction - total energy curve from previous step, the study has finally obtained the trade-off relationship, realizing the research objective. An ideal point for the studied case seems to appear between 60% and 80% volume limits, where the required external work for deployment is reduced by more than 36.8% while the deflection is only increased by less than 22.4% compared to the reference \( (V=100\%V_0) \).

### 4.2 Future Work

This thesis has studied the deployable scissor chain entirely in FEM simulation models. To test the validity of the results, it is necessary to perform experiments to interpret the similarities and discrepancies between the numerical and experimental data. Some potential challenges of the experiment may include the manufacturing of the optimized scissor bars, installations of near frictionless hinge connectors, boundary support settings, etc. Furthermore, future studies can increase the number of scissor pairs to investigate how the span length affect the results. For instance, the sweet spot between the stiffness and the power saving likely to differ as the span length changes. That is, volume limits between 60% and 80% will likely no longer be the ideal point for a scissor chain with different span length and/or different numbers of scissor pairs.

Lastly, as the structure studied is intended to be a deployable roof component, cover plates that fill the gap between the scissor links are to be designed. The cover plates should be capable of folding and unfolding as the main structural component, the scissor chain, contracts and extends. Future work should consider overlapping rigid plates, flexible membranes, origami structures, etc. as potential roof coverings.
References


