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Citation: Jia-Richards, Oliver and Paulo Lozano. "Analytical Framework for Staging of Space Propulsion Systems." *Journal of Propulsion and Power* 36, 4 (July 2020): dx.doi.org/10.2514/1.b37722. © 2020 by the American Institute of Aeronautics and Astronautics

As Published: <http://dx.doi.org/10.2514/1.b37722>

Publisher: American Institute of Aeronautics and Astronautics (AIAA)

Persistent URL: <https://hdl.handle.net/1721.1/130048>

Version: Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

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An Analytical Framework for Staging of Space Propulsion Systems

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Staging of space propulsion systems would allow lifetime limitations inherent to small propulsion systems to be bypassed in order to enable high- ΔV capabilities for small spacecraft, in particular mass and volume constrained CubeSats. In addition, staging can be used to provide redundancy in the propulsion system, counteract thruster degradation, or open up new avenues of mission optimization. Analytical approximations are developed in order to provide a computationally-simple approach to the design, analysis, definition of propulsion technology requirements, and online autonomous decision making for systems that make use of staging of propulsion elements. In addition, the analytical approximations provide insight into the dependencies of performance metrics on system parameters. Equations are developed for any mission, defined by its ΔV , and then specialized for escape trajectories.

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Nomenclature

| | | |
|------------|---|--|
| a_p | = | propulsive acceleration |
| c | = | exhaust velocity |
| F | = | propulsion system thrust |
| g | = | standard acceleration due to gravity |
| I_{sp} | = | specific impulse |
| L | = | stage lifetime |
| \bar{m} | = | time-averaged spacecraft mass |
| m_0 | = | initial spacecraft wet mass |
| m_f | = | fuel mass |
| m_s | = | stage dry mass |
| N | = | number of stages |
| r_0 | = | initial orbital radius |
| S | = | escape time scalar factor |
| t_{esc} | = | escape time |
| T_s | = | staged trajectory firing time |
| T_u | = | un-staged trajectory firing time |
| v_0 | = | initial orbital velocity |
| γ | = | ratio of propulsive to gravitational accelerations |
| μ | = | gravitational parameter |
| σ_L | = | stage lifetime distribution |
| τ | = | firing time ratio |
| ζ | = | thrust reduction factor |

I. Introduction

The use of staging has long been implemented on launch vehicles as a strategy to maximize payload delivered to orbit given the high ΔV requirements of a launch trajectory. Once in orbit, the paradigm for spacecraft has been to use a single “stage” due in part to the lower ΔV requirements

and in part to the ability to use low-thrust but high-efficiency electric propulsion systems. However, for small spacecraft such as CubeSats, the complexity of miniaturizing propulsion devices in general has so far prevented the development of high- ΔV propulsion systems compatible with the small form factor [1].

The lack of high- ΔV propulsion systems has limited the types of missions that can be performed with CubeSats [2]. In particular, any deep-space mission where such a small spacecraft independently propels itself from Earth orbit and into deep-space is not possible with current propulsion technology [3]. To date, only two CubeSats have left Earth orbit as part of the Mars Cube One (MarCO) mission. Both spacecraft had to rideshare with the InSight lander as their propulsion systems were only capable of ~ 40 m/s of ΔV and were intended for attitude control and trajectory correction maneuvers [4]. In order to open up deep-space and leverage the affordability of small spacecraft, high- ΔV propulsion capabilities need to be developed.

Electrospray propulsion is a promising technology due to its high efficiency and compactness and several devices using microfabrication techniques have been proposed [5–9]. In this work, the ion Electrospray Propulsion System (iEPS) is used [9]. These thrusters use near-zero vapor pressure ionic liquids as the propellant [10–12] which allows the propellant to be exposed to the vacuum of space without boiling, thereby eliminating the need for pressurized tanks and complex propellant management systems. In addition, bipolar operation of pairs of electrospray thrusters can eliminate the need for a neutralization cathode [13]. Figure 1 shows the ΔV versus wet mass for iEPS, with parameters given by Table 1, as well as CubeSat propulsion systems based in cold-gas, monopropellant, and electric propulsion technology [14] when used on a 4 kg, 3U CubeSat. In order to enable independent deep-space missions with CubeSats, it is assumed that the largest starting orbital radius of the spacecraft is geostationary orbit. In this case, the propulsion system needs to be capable of producing at least 2.7 km/s of ΔV to achieve escape when low-thrust gravity losses have been accounted for. All available propulsion systems fall short of the 2.7 km/s ΔV cutoff by at least an order of magnitude. However, compared to the other options, iEPS has the best performance-to-mass ratio.

While electrospray thrusters have a high performance-to-mass ratio, the ΔV that can be pro-

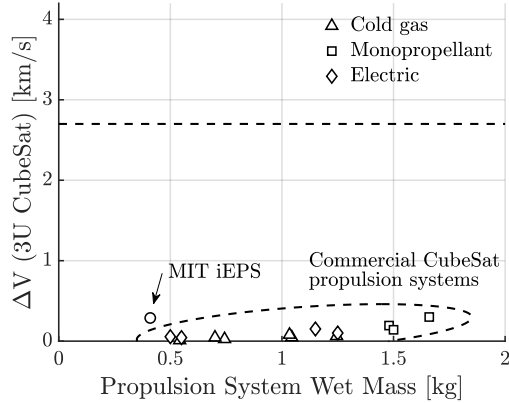


Fig. 1 Comparison of ΔV versus wet mass for 3U CubeSat Propulsion systems

Table 1 ion Electro spray Propulsion System performance metrics

| Metric | Value |
|------------------|---------|
| Thrust | 0.64 mN |
| Specific impulse | 1000 s |
| Lifetime | 500 hr |
| Stage dry mass | 0.1 kg |

duced by an electro spray thruster system is limited by the operational lifetime of the thrusters themselves [15, 16]. For iEPS thrusters, the two main life-limiting mechanisms are propellant accumulation on the extractor grid as well as arcing between isolated tips on the emitter array and the extractor grid. The beam of ions which is extracted from an emitter tip leaves in a conical shape with observed half angles of up to 60 degrees [9]. The beam can therefore impact the extractor grid and allow propellant to accumulate on the extractor grid or backspray onto the emitter array. If enough propellant accumulates, an ionic liquid connection can form between the emitter array and extractor grid causing an electrical short on the thruster and rendering it inoperable [8]. In addition, not all tips on the emitter array will be identical due to difficulties with repeatable manufacturing and inherent material non-uniformity [17, 18]. The non-uniformity between emitter tips can lead to non-uniformity in the emitted current over a particular array [19]. While most emitter tips will operate as intended, some emitter tips might have unstable menisci [20] which can lead to erratic liquid emission and occasional electrical discharges between the emitter tip and extractor grid [21]

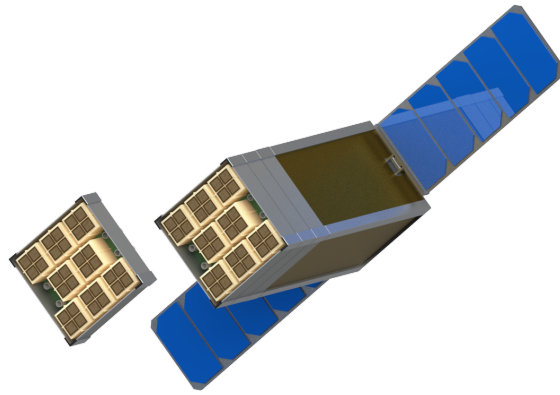


Fig. 2 Conceptual image of staging on a 3U CubeSat

degrading the thruster over time.

In order to develop a high- ΔV propulsion system based around electro spray thrusters, all of these lifetime limitations need to be overcome. Two strategies could be taken: improve the lifetime of individual thrusters through a better understanding and mitigation of these mechanisms or bypass the lifetime limitations of individual thrusters through the use of staging. While the former strategy is continuously being explored and likely will bring lifetime improvements in the future, the latter presents itself as a strategy that could enable high- ΔV capabilities with existing electro spray technology. Furthermore, the use of staging systems in interplanetary missions would provide additional redundancy and reliability, even for thrusters with improved lifetime, as sets of fresh units could replace functioning, albeit degraded ones.

Staging of electro spray thrusters was originally analyzed in [22] to explore reductions in the transfer time of a lunar mission. Since the spacecraft drops off structural mass at each staging event, the acceleration of each stage is increased relative to a single-stage system. Staging with electro spray thrusters was further analyzed in [3] for enabling deep-space exploration with CubeSats. A laboratory demonstration of an electro spray-thruster-based staging system was conducted in [23] in order to demonstrate the mechanical and electrical feasibility of such a configuration. Figure 2 shows a conceptual image of an electro spray thruster staging system on a 3U CubeSat. Each stage is composed of an array of electro spray thrusters. As they reach their lifetime limits, they are ejected from the spacecraft exposing a new array of thrusters to continue the mission.

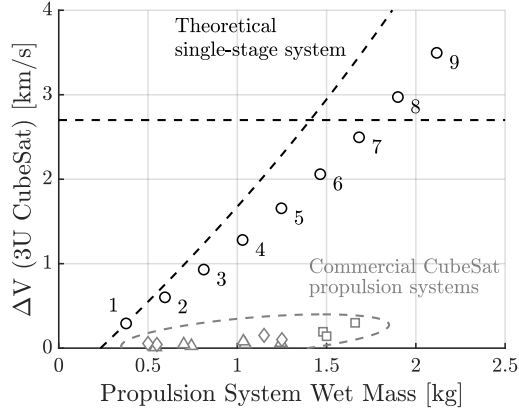


Fig. 3 ΔV versus wet mass for a 3U CubeSat compatible stage-based system

In terms of ΔV production, Fig. 3 shows an example of ΔV versus wet mass for systems with one to nine stages and performance metrics given by Table 1 as well as a theoretical single-stage system with the same performance metrics. An eight-stage system can provide enough ΔV to reach deep space and does not significantly increase the mass of the propulsion system compared to the theoretical single-stage system. However, while the single-stage system is not possible with current propulsion technology, the stage-based system is. The concept and mechanical feasibility of a staging approach with electrospray thrusters has been vetted by previous work [3, 22, 23]. This paper focuses on the analysis of these propulsion systems.

To analyze the impact of using this propulsion system, an analytical approach is taken. The analytical approach provides equations that explicitly show the dependencies of the system’s performance on the parameters of the individual stages which would be impractical to generate with numerical approaches. In addition, they provide the framework to move towards online autonomous decision making on spacecraft by providing a computationally simple method, through estimating the results of optimizations and Monte Carlo analysis with an algebraic equation, for determining propulsion system performance. Both of these attributes are important as they can provide insight into what propulsion system parameters to focus developments on, and allow practical implementation of autonomous decision making on small spacecraft where computational power and memory are limited.

The goal of the analytical approach is to develop equations that can answer key questions

about the performance of the propulsion system and provide insight about dependencies on system parameters. Specifically: how many stages are required, what are the dependencies on propulsion system parameters, how do staged and unstaged propulsion systems compare, and what is the probability of mission success? The answers to these questions stem from an approximation of the ΔV of the propulsion system. An approximation of the ΔV is developed, both in cases where fuel mass depletion is neglected and where fuel mass depletion is accounted for, that can easily be manipulated in order to answer the key questions above. In addition, the approximation of ΔV will be conservative, meaning that results derived from it will bound the true result. These analytical approximations can then be used in preliminary propulsion system and mission design, focusing of propulsion system technology developments, and online autonomous decision making.

II. ΔV Approximation

Neglecting fuel mass depletion, the ΔV for a stage-based propulsion system is given by

$$\Delta V = \sum_{i=1}^N \frac{FL}{m_0 - (i-1)m_s} \quad (1)$$

where F is the propulsion system thrust, L is the lifetime of each stage, m_0 is the initial spacecraft wet mass, m_s is the dry mass of each stage, and N is the total number of stages. This sum does not have an analytical solution for general values of the parameters due to the singularity if $m_0 = (i-1)m_s$. However, its value can be approximated by assuming that the impulse of each stage is applied to the time-averaged mass of the spacecraft, \bar{m} , which is given by

$$\bar{m} = m_0 - \frac{1}{2}(N-1)m_s \quad (2)$$

This approximation eliminates the summation index and allows for a simple approximation of the ΔV for the staging system

$$\Delta V \approx \frac{NFL}{m_0 - \frac{1}{2}(N-1)m_s} \quad (3)$$

The mass-averaging technique can be extended to account for the effects of fuel mass depletion. Assuming that each stage is identical in terms of thrust, specific impulse, and lifetime, then the total fuel mass used by the system, m_f , is

$$m_f = N \frac{FL}{c} \quad (4)$$

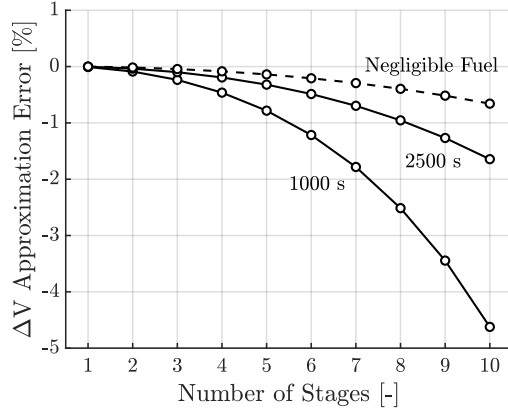


Fig. 4 Percent error in ΔV approximation versus number of stages

where c is the ideal exhaust velocity ($I_{sp}g$). Including the fuel mass in the time-averaged mass simply introduces a third term

$$\bar{m} = m_0 - \frac{1}{2}(N-1)m_s - \frac{1}{2}N\frac{FL}{c} \quad (5)$$

and allows for an approximation of the propulsion system ΔV when accounting for fuel mass depletion

$$\Delta V \approx \frac{NFL}{m_0 - \frac{1}{2}(N-1)m_s - \frac{1}{2}N\frac{FL}{c}} \quad (6)$$

Figure 4 shows the percent error in the ΔV approximation relative to the true ΔV of the system as defined by the rocket equation versus number of stages when accounting for fuel mass depletion. Results are plotted using propulsion system performance metrics from Table 1 as well as for a specific impulse of 2500 s and when fuel mass is negligible. As the specific impulse increases, the error in the ΔV approximation decreases. However, even for the lowest specific impulse considered (1000 s) the error in the ΔV approximation is small ($< 5\%$), including cases with a large number of stages. In the limit, as fuel mass becomes negligible, the error for a 10 stage system will approach $\sim 0.7\%$. In all cases, the approximation underestimates the ΔV of the system. Therefore, results derived from the approximation, such as the number of stages required to complete a mission, will be conservative and provide a tight bound of the true value.

It is worth noting that evaluating the ΔV of a staged propulsion system is not a difficult calculation nor is the approximation above the most accurate approximation that can be developed.

The value of this approximation is that it can easily be manipulated to approximate parameters of interest where algebraic solutions do not currently exist such as the number of stages, required stage impulse, or total firing time for a particular mission. In addition, the mass-averaging technique can be used to estimate the probability of mission success when the lifetime of each stage is a random variable. The remainder of this paper is focused on the manipulation of the ΔV approximation in order to approximate parameters of interest.

III. Required Number of Stages

Perhaps the main figure of merit for a staging system is how many stages are required to complete a mission. More stages implies a larger and more massive propulsion system and can greatly increase its mechanical complexity. Leveraging the ΔV approximation, the number of stages required to complete a mission, defined by its ΔV , can be approximated given the parameters of the propulsion system.

Rearranging Eq. 6 to solve for N , the number of stages, gives

$$N \approx \frac{m_0 + \frac{1}{2}m_s}{\frac{1}{2}m_s + \frac{FL}{\Delta V} + \frac{1}{2}\frac{FL}{c}} \quad (7)$$

The analytical formulation gives the dependencies of the number of stages on propulsion system parameters. There are some dependencies that are expected, such as increasing the thrust and/or lifetime of each stage will reduce the number of stages required to complete the mission. Perhaps counterintuitively, the dependency on exhaust velocity is such that greater exhaust velocities (typically associated with better propulsion systems) increases the required number of stages. This is an effect that can be seen from the mass averaging technique - a lower exhaust velocity means more fuel mass resulting in a lower average spacecraft mass.

The approximation for number of stages does permit non-integer values. In this case, the last stage in the staging system is only used to some fraction of its lifetime limit. When determining the required number of stages for the propulsion system, the output of Eq. 7 needs to be rounded up to the next highest integer value but the raw, non-integer, value can provide further insight into the safety margin of the propulsion system's ΔV .

Figure 5 shows a comparison of analytical and numerical calculations of the required number of

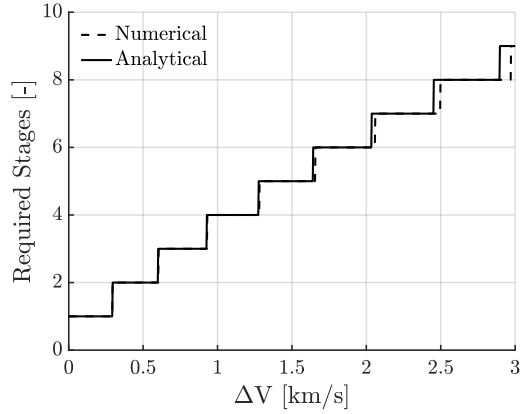


Fig. 5 Comparison of analytical and numerical calculations of required number of stages

stages for a 3U CubeSat with a propulsion system with performance metrics given by Table 1 for various ΔV requirements. For the analytical solution, the output of Eq. 7 has been rounded up to the next highest integer value for this comparison. The numerical solution is calculated for each ΔV by incrementally increasing the number of stages until the ΔV of the system is above the desired ΔV . The first integer value of number of stages for which this happens gives the required number of stages the system needs in order to produce a given ΔV . The analytical approximation predicts the true required number of stages quite well with differences occurring for high- ΔV requirements. As fuel mass becomes more negligible, the approximation of the required number of stages becomes practically exact with only a 9 m/s difference (0.3% error) in the prediction of the transition ΔV between an 8 and 9 stage system.

In all cases, the analytical approximation acts as a tight bound on the required number of stages. As the specific impulse of the propulsion system is increased and fuel mass depletion becomes less significant, the bound on the required number of stages from the analytical approximation will become tighter.

A. Parameter Tradeoff

While the purpose of this propulsion system is to improve propulsion capabilities without waiting for developments in propulsion technology, this analysis can be used to guide where to focus efforts in improvements for the underlying propulsion technology. Based on Eq. 7, the thrust and lifetime

of the propulsion system are the main drivers of the required number of stages and there is a weak dependence on the exhaust velocity.

Rearranging Eq. 7 to solve for the impulse of each stage, FL , gives

$$FL = \frac{\Delta V}{N} \left(\frac{m_0 - \frac{1}{2}(N-1)m_s}{1 + \frac{1}{2}\frac{\Delta V}{c}} \right) \quad (8)$$

which explicitly says that for a given mission and fixed number of stages, the impulse of each stage must be constant. This is not a surprising result, since for a fixed ΔV and number of stages, which sets the time-average mass of the spacecraft, the total impulse must be a constant. However, the value of this equation is to show the dependency of the required impulse of each stage versus mission and system parameters and to provide a computationally simple way for analyzing the tradeoff between thrust and lifetime.

Figure 6 shows the required number of stages to achieve 2.5 km/s ΔV with a propulsion system with baseline performance metrics given by Table 1 on a 3U CubeSat. Both the thrust and stage lifetime are varied from their baseline value to 4x and 2x their baseline value respectively and normalized by their respective baseline values. The regions between lines correspond to thrust and lifetime values that lead to a given number of required stages. The analytical and numerical solutions agree with no visually discernible difference between the two solutions. In fact, discernible differences between the solutions only start to occur at ΔV requirements of 7.5 km/s and greater. Therefore, the analytical solution provides a tight approximation of the tradeoff between thrust and lifetime for a large range of mission scenarios.

The analytical solution is plotted with Eq. 8 for integer values of number of stages. The numerical solution is found by posing the problem as a multi-objective optimization and solving with a genetic algorithm. For a fixed number of stages, spacecraft initial wet mass, and stage dry mass the objective of the optimization is to minimize the required thrust and stage lifetime in order to achieve the given ΔV . The solution to the optimization provides a Pareto front, for a given number of stages, that describes the tradeoff between stage thrust and lifetime. The optimization is first solved assuming that only a single stage is used. Then, the number of stages is incremented and the optimization problem is solved again. The number of stages continues to be incremented until the minimum values of thrust and lifetime are inside the resulting Pareto front. Use of the

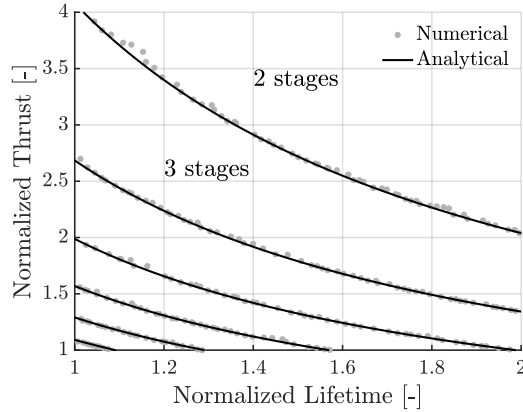


Fig. 6 Tradeoff in required number of stages between thrust and stage lifetime

analytical solution allows the repeated optimization to be avoided. This is particularly useful in cases where a large number of stages is required or when different values of ΔV are surveyed. In both cases, the optimization needs to be repeatedly solved whereas the analytical solution allows the trade-space to be quickly computed.

Knowledge of the tradeoff in thrust and lifetime can help direct propulsion technology development. For some systems, it might be easier to increase the thrust versus the lifetime while for others the opposite might be true. In either case, proportional increases in either parameter produce identical results in terms of required number of stages and the required fuel mass. However, there will be differences in system and mission architectures, for example mission times and power requirements.

IV. Comparison of Mission Time

The ΔV approximation can also be used to compare the amount of firing time required to complete a mission. While staging is used to enable missions which cannot be done with current propulsion technology, it also influences the resulting trajectory and this influence needs to be considered when designing missions.

The firing time for an un-staged trajectory is

$$T_u = \frac{m_0 c}{F} \left(1 - e^{-\Delta V/c} \right) \quad (9)$$

The firing time for the staged trajectory can be estimated with the required number of stages from

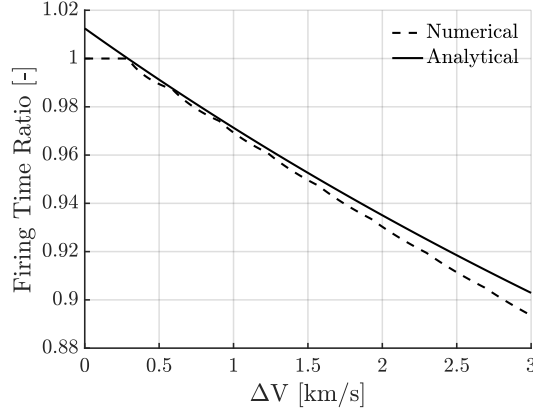


Fig. 7 Ratio of firing times for staged to un-staged propulsion systems

Eq. 7 as

$$T_s = NL = \frac{m_0 + \frac{1}{2}m_s}{\frac{1}{2}m_s + \frac{FL}{\Delta V} + \frac{1}{2}\frac{FL}{c}}L \quad (10)$$

Calculating the firing time ratio, τ , gives

$$\tau = \frac{T_s}{T_u} = \frac{1}{1 - e^{-\Delta V/c}} \left(\frac{1 + \frac{1}{2}\frac{m_s}{m_0}}{\frac{1}{2} + \frac{1}{2}\frac{m_s c}{FL} + \frac{c}{\Delta V}} \right) \quad (11)$$

Figure 7 shows analytical and numerical calculations of the ratio of firing times for staged to un-staged propulsion systems. The analytical solution is from Eq. 11 and the numerical solution is calculated from propagating the acceleration from the propulsion system through time, accounting for fuel mass depletion and staging events, until a desired ΔV is reached. The spacecraft is assumed to be a 3U CubeSat carrying a propulsion system with performance metrics from Table 1. The analytical solution predicts the true numerical ratio quite well with a $\sim 1\%$ error at 3 km/s of ΔV which is in line with the predicted ΔV error in Fig. 4. An additional benefit of staging, which can be seen in this figure, is that the firing time required to reach deep space is reduced by about 10% over a conventional system. This is due to the increased spacecraft acceleration as excess structural mass decreases when stages are ejected. The predicted 10% reduction from the analytical solution is also consistent with the observed 10% reduction in mission time seen in [22].

At low ΔV requirements (< 300 m/s), the analytical solution predicts that the stage propulsion system actually requires more firing time than the un-staged system. This is an artifact from the mass averaging technique when estimating the ΔV of the staged propulsion system in Eq. 5. The

mass averaging technique treats the stage-based system as an un-staged system that acts on the average mass of the spacecraft. In the situation that $N < 1$, which occurs when the mission can be completed in only a fraction of the lifetime of a single stage, the mass average is actually greater than the initial mass of the spacecraft, m_0 . This causes the subsequent approximations of firing time to be artificially inflated when, in reality, they should exactly equal the un-staged system's firing times.

V. Mission Success Probability

Throughout the previous analysis, it was assumed that the lifetime of each stage was known exactly. However, this is not true to reality - the lifetime of a propulsion system, and therefore a stage, can have a significant degree of uncertainty. Uncertainty in stage lifetime will lead to uncertainty in the ΔV output of the propulsion system. Therefore, it is valuable to estimate the probability density of ΔV based on distributions of the stage lifetimes.

The ΔV of a propulsion system with variable stage lifetime is

$$\Delta V = \frac{F}{\bar{m}} \sum_{i=1}^N t_i \quad (12)$$

Where the mass average in this case is the average time-averaged mass. While this approximation will be close to the true time-averaged mass of the spacecraft, care will have to be taken when the distribution of the stage lifetimes is very wide and the firing times between stages varies significantly.

Assuming that the lifetime of each stage is normally distributed as

$$L_s \sim \mathcal{N}(L, \sigma_L^2) \quad (13)$$

then the distribution of the propulsion system ΔV will also be normally distributed with statistics

$$\mu = \frac{F}{\bar{m}} NL \quad (14)$$

$$\sigma = \frac{F}{\bar{m}} \sqrt{N} \sigma_L \quad (15)$$

Figure 8 shows a comparison of the analytical calculation of the ΔV probability density for a five stage propulsion system with performance metrics given by Table 1 on a 3U CubeSat versus a Monte Carlo analysis of the true density. The statistics of the stage lifetime were assumed to have

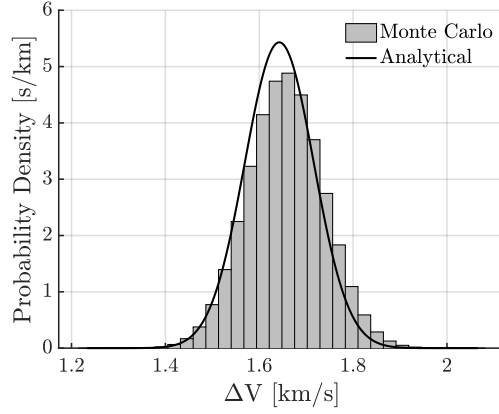


Fig. 8 ΔV probability density for a stage-based propulsion system

a mean given by Table 1 and a standard deviation of 50 hours. The analytical distribution is a fairly good fit for the Monte Carlo analysis but has a slight negative shift of the mean of 0.8% and a slight reduction in the standard deviation of 9%. As fuel mass depletion becomes less significant, the analytical solution will better fit the Monte Carlo analysis.

With the analytical approximation of the ΔV distribution, the probability of mission success can be approximated. Assuming that a 99.9% probability of success is desired, the cutoff ΔV can be calculated as three standard deviations away from the mean

$$\Delta V^* = \frac{F}{\dot{m}} \left(NL - 3\sqrt{N}\sigma_L \right) \quad (16)$$

For any missions with $\Delta V \leq \Delta V^*$ the mission will have at least a 99.9% probability of success. If a different probability of success is desired, then ΔV^* can easily be calculated based on the number of standard deviations away from the mean.

A. Deciding when to Stage

The approximation of ΔV distributions can also be used to determine when to eject a partially degraded stage. It is possible during the course of a mission, particularly with an array of electro-spray thrusters, that instead of an entire stage failing, its thrust is simply reduced by some factor. For a distributed array of electro-spray thrusters, this situation would be representative of a few thrusters having lower lifetimes than the rest of the stage. Alternatively, it might represent a stage that starts with an abnormally low thrust or natural thruster degradation.

In all representations, during the course of the mission the thrust of a particular stage, n , suddenly drops from its nominal thrust, F , to a degraded thrust, ζF , where $\zeta \in (0, 1)$, at some time t_n^- into its firing time. From a ΔV perspective, it is never advantageous to eject a partially degraded stage - the stage still has the capability to provide ΔV to the mission. However, from a time perspective a partially degraded stage should be ejected immediately as lower thrust corresponds to longer firing times for a given mission. To resolve this dispute, the problem can be viewed instead from the perspective of mission success probability: how long does a partially degraded stage need to be kept in order to guarantee a particular probability of mission success? Such a solution balances the need to provide enough ΔV to complete the mission with the desire not to operate with reduced thrust.

Assuming that the thrust reduction occurs on stage n then the ΔV produced by stages 1 to $n - 1$, $\Delta V_{1:n-1}$, is

$$\Delta V_{1:n-1} = F \sum_{i=1}^{n-1} \frac{t_i}{m_0 - (i-1)m_s} \quad (17)$$

which is a known value and does not require approximation. Given that the thrust reduction occurs some time, t_n^- , into the firing time of stage n then the ΔV produced by stage n prior to failure, ΔV_n^- , is

$$\Delta V_n^- = \frac{F t_n^-}{m_0 - (n-1)m_s} \quad (18)$$

and is known. The ΔV for the remaining stages in the system, $\Delta V_{n+1:N}$, is unknown and is a random variable. However, the cutoff ΔV^* from Eq. 16 can be used to determine what the minimum value of $\Delta V_{n+1:N}$ will be in 99.9% of scenarios. Therefore, in the “worst-case” scenario, $\Delta V_{n+1:N}$ is given by

$$\Delta V_{n+1:N} = \frac{F}{\bar{m}_{n+1:N}} \left[(N-n)L - 3\sqrt{N-n}\sigma_L \right] \quad (19)$$

where the time averaged spacecraft mass over the remaining stages is

$$\bar{m}_{n+1:N} = m_0 - \frac{1}{2}(N+n-1)m_s - \frac{1}{2}(N-n)\frac{FL}{c} \quad (20)$$

Therefore, the ΔV that the reduced thrust stage needs to produce, ΔV_n^+ , in order to guarantee

99.9% probability of mission success is

$$\Delta V_n^+ = \Delta V_d - \Delta V_{1:n-1} - \Delta V_n^- - \Delta V_{n+1:N} \quad (21)$$

where ΔV_d is the desired total ΔV for the mission. The required firing time can then be calculated as

$$t_n^+ = \frac{m_0 - (n-1)m_s}{\zeta F} \Delta V_n^+ \quad (22)$$

There are three regimes of value that t_n^+ can take. If $t_n^+ \leq 0$ then stage n can safely be ejected. If $0 < t_n^+ \leq t^*$ where t^* is a reasonably low firing time to expect the reduced thrust stage to be capable of firing for, then the stage can be kept and a mission success probability of 99.9% can be expected. However, if $t_n^+ > t^*$, then it's unlikely that the reduced thrust stage will be able to produce ΔV_n^+ and the mission success probability will be lower than 99.9%. The exact calculation of t^* will be highly dependent on the system as it will have to take into account coupling between the failure already observed on the stage and potential future failures, as well as the potential that further discrete drops in thrust may occur on the same stage. However, this analysis provides the framework for potential autonomous decision making regarding when to eject partially failed stages on spacecraft equipped with this kind of propulsion system.

For an example scenario, a five stage propulsion system with performance metrics given by Table 1 on a 3U CubeSat is desired to produce 1,250 m/s of ΔV . It is assumed that the lifetime of each stage is distributed with the mean given by Table 1 and standard deviation of 50 hours. The first stage in the system is allowed to fire for 100 hours at nominal thrust levels before experiencing a partial failure which causes a thrust reduction of 25%. Two situations are analyzed through Monte Carlo analysis with 10^6 samples each: the first stage is ejected immediately after experiencing the partial failure (immediate eject) and the first stage is fired at its reduced thrust level for a firing time based on Eq. 22 before ejection (hold and eject).

Figure 9 shows the resulting propulsion system ΔV probability density for both situations. In the immediate eject situation, the mission does not have a 99.9% probability of success and a substantial portion of the density is below ΔV_d . In fact, the numerical estimate of success probability is only 72.8%. However, in the hold and eject situation, the density is shifted such that the vast majority

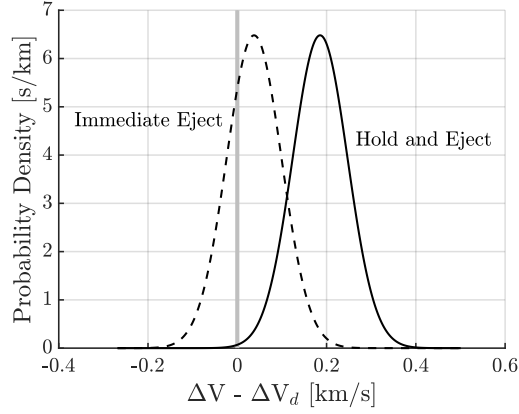


Fig. 9 Effect of holding partially failed stage before ejecting on ΔV probability density of the entire propulsion system

of the density is greater than ΔV_d . Numerically, the probability of success is estimated at 99.9%. In this scenario, to achieve 99.9% probability of success, the first stage was required to fire for 342 hours at reduced thrust level. This brings the total firing time of the first stage to 442 hours.

VI. Application to Escape

Specializing the above equations to escape missions requires approximating the ΔV required for escape. Wiesel [24] analyzes circle-circle coplanar orbit transfers and escape trajectories with a constant low-thrust propulsive acceleration from an energy perspective and provides an approximation of the escape time as

$$t_{\text{esc}} = \frac{v_0}{a_p} \left[1 - (2\gamma)^{1/4} \right] \quad (23)$$

where v_0 is the initial orbital velocity and γ is the ratio of the propulsive acceleration, a_p , to the gravitational acceleration at the start of the trajectory. While the approximation for the escape time is quite good, it can be improved based on numerical results. Solving for the scalar factor, “ $2^{1/4}$,” and assigning it the variable S ,

$$S = \frac{1 - \frac{a_p}{v_0} t_{\text{esc}}}{\gamma^{1/4}} \quad (24)$$

Table 2 shows the scalar factor for various acceleration ratios based on numerical propagation of the escape trajectories for both angular pointing and velocity pointing control laws. In the angular

Table 2 Escape time scalar factor for various acceleration ratios

| γ | Angular Pointing | Velocity Pointing |
|-----------|------------------|-------------------|
| 10^{-2} | 0.7551 | 0.8053 |
| 10^{-3} | 0.7554 | 0.8081 |
| 10^{-4} | 0.7555 | 0.8082 |
| 10^{-5} | 0.7555 | 0.8082 |

pointing control law, the thrust is aligned with the component of the spacecraft's velocity that is perpendicular to the position vector. In both cases, the scalar factor is fairly constant and converges to a value as γ is decreased. For reference, a propulsion system with performance metrics given by Table 1 on a 3U CubeSat starting in geostationary orbit has a γ of 7×10^{-4} . Based on the numerical results, a better approximation for the escape time is

$$t_{\text{esc}} = \frac{v_0}{a_p} \left(1 - S\gamma^{1/4}\right) \quad (25)$$

where $S = 0.7555$ for an angular pointing control law and $S = 0.8082$ for a velocity pointing control law. The required ΔV for escape can then be calculated as

$$\Delta V_{\text{esc}} = a_p t_{\text{esc}} = v_0 \left(1 - S\gamma^{1/4}\right) \quad (26)$$

With this estimate for ΔV_{esc} , the analytical approximations developed previously can be specialized to escape missions. The required number of stages is

$$N = \frac{m_0 + \frac{1}{2}m_s}{\frac{1}{2}m_s + \frac{FL}{v_0(1-S\gamma^{1/4})} + \frac{1}{2}\frac{FL}{c}} \quad (27)$$

the required impulse that each stage needs to produce is

$$FL = \frac{1}{N} \left(\frac{m_0 - \frac{1}{2}(N-1)m_s}{\frac{1}{v_0(1-S\gamma^{1/4})} + \frac{1}{2}\frac{1}{c}} \right) \quad (28)$$

and the ratio of firing times for staged versus un-staged trajectories is

$$\tau = \frac{1}{1 - e^{-\frac{v_0}{c}(1-S\gamma^{1/4})}} \left(\frac{1 + \frac{1}{2}\frac{m_s}{m_0}}{\frac{1}{2} + \frac{1}{2}\frac{m_s c}{FL} + \frac{c}{v_0(1-S\gamma^{1/4})}} \right) \quad (29)$$

The probability of mission success can also be analytically estimated. Figure 10 shows the probability of achieving escape versus the standard deviation of stage lifetimes given a propulsion

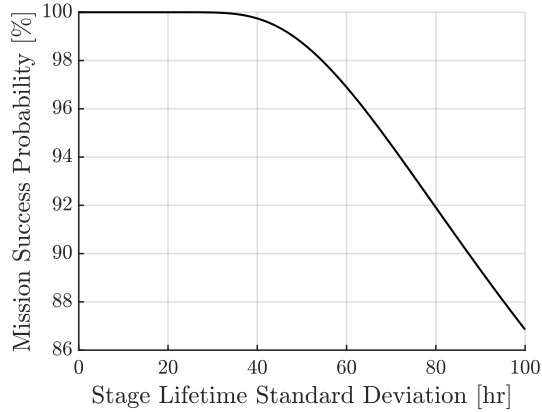


Fig. 10 Probability of mission success versus standard deviation of stage lifetime

system with mean performance metrics from Table 1 on a 3U CubeSat. For stage lifetime standard deviations below 40 hours the probability of mission success is quite high ($> 99.7\%$) and then decreases linearly for larger stage lifetime standard deviations.

VII. Conclusion

This work contributes towards the development of stage-based propulsion systems by providing tight analytical approximations of: ΔV output, required number of stages, tradeoff in propulsion system parameters, comparison of firing time for un-staged versus staged propulsion systems, and probability of mission success.

Approximations of ΔV output are conservative with errors on the order of 1% for a stage-based electro-spray propulsion system capable of propelling a 3U CubeSat to escape from geostationary orbit. The ΔV approximation allows for the required number of stages to be predicted with errors on the order of 0.1% in the prediction of the transition ΔV between steps in the required number of stages. In addition, the tradeoff in propulsion system thrust and lifetime can be analyzed to see how the required impulse of each stage depends on the mission and spacecraft parameters.

Analysis of the firing time for un-staged versus staged propulsion systems allows the impact of stage-based propulsion system on mission time and fuel mass consumption to be accounted for during preliminary mission design. Errors in the analytical prediction of the ratio of staged to un-staged firing times, and therefore fuel mass consumption, are on the order of 1% for a 3 km/s ΔV

mission.

Lastly, the distribution of ΔV due to uncertainty in stage lifetime can also be predicted with errors on the order of 1% for the mean ΔV output and below 10% for the standard deviation. The approximation for the ΔV distribution can then be extended to determine when to stage a partially failed stage in order to guarantee a specified mission success probability. The prediction was numerically verified to guarantee 99.9% probability of mission success where immediately ejecting the partially failed stage would result in only 72.8% probability of mission success.

All of the approximations developed in this work can be combined with analytical approximations of the ΔV required to achieve escape to specify the approximations for escape trajectories. The probability of mission success can be calculated for varying stage lifetime uncertainties in order to determine the required stage-lifetime distribution in order to guarantee specified levels of mission success probability thereby eliminating the need for repeated Monte Carlo analysis.

Funding Sources

Funding for this work was provided by the NASA Space Technology Mission Directorate through the Small Spacecraft Technology Program under grant 80NSSC18M0045 and through a NASA Space Technology Research Fellowship under grant 80NSSC18K1186. In addition, P. Lozano would like to thank the Miguel Aleman-Velasco foundation for its support.

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