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The Effect of Unbalanced Voltages on the Operation of a Three Phase Synchronous Motor.

By

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Approved W.V.L.

Contents

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Introduction.

Synchronous Motors are not used for ordinary power work such as, in mills, hoisting, and railway trains, because they have very poor starting torques unless they are supplied with compensating windings. Consequently they are used in cases where it is not necessary to start any appreciable load. Their use may therefore be applied in substations, where motor generator sets are required. Also in connection with frequency changers, and as synchronous condensers.

The operation of synchronous motors under balanced conditions is well understood, and more or less thoroughly worked out. Since it is usually assumed in practice that the voltages are balanced, or are very nearly balanced, little attention has been paid to the operation of these types of motors under unbalanced conditions.

However, in practice the voltages of three phase systems are almost always unbalanced to a greater or less degree. Since this is true, the author of this paper has made a study of a three phase synchronous motor in order to determine whether or not it is possible to predict with some precission, the charactéristics of such a motor when the voltages are badly unbalanced.

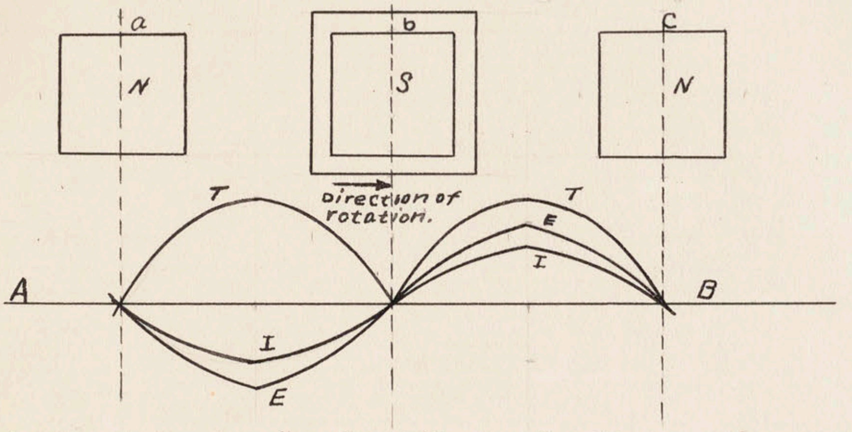
The greater part of the problem of trying to predict the operation of synchronous motors under unbalanced conditions consists in determining the values of the impedances to the uniphase and reverse phase components of the impressed voltages and currents which are unbalanced. With all the constants of a motor known, including the uniphase and reverse phase impedances, it is then only necessary to consider the effect of each component of the impressed voltage separately, and finally to combine these effects to obtain the operating characteristics of the machine.

Since the reverse phase impedance is a very important factor in the prediction of the operating characteristics of a motor, a great deal of time has been spent in trying to arrive at some logically correct value to assign to this impedance, which will take account, as nearly as possible, of all the variations due to the unbalancing of the line voltages.

Due to the lack of time all corrections of instrument readings have been neglected. But care has been taken to see that in every case the instrument needles were set at the exact zero point of the scale. Also all decimal readings have been estimated as accurately as the eye would permit, and as small a scale instrument as practicable was used in every case in order that the readings taken might be as accurate as possible.

Theory of the Operation of a Synchronous Motor.

In order to explain the theory of operation of a synchronous motor, a single phase motor having a concentrated winding will be assumed. The explanation given here is the same as that given by Professor Lawrence



in his text-book, "Principles of Alternating Current Machinery."

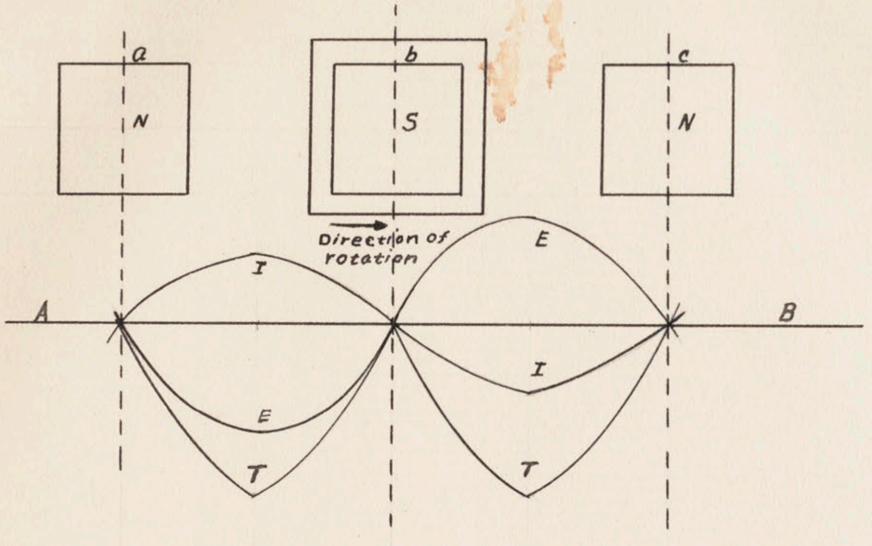
The rectangles marked N, V, and S, are the ends of the pole faces, and the rectangle "d" represents the single phase armature winding. The electromotive force induced in the armature winding will be zero for the position shown. The direction of rotation is such that the armature moves from left to right relative to the poles. The electromotive force will be called positive when it acts in a clockwise direction.

Let the armature be driven at a uniform speed. The electromotive force generated in the coil while it moves from left to right is plotted on the reference line A.B. in Fig. 1. Let the armature circuit be closed through a load of such constants that the current is in phase with the generated voltage. This current is shown by curve I.

While the coil moves from a to b, the face of the coil towards the poles will be south. There is, therefore, a force of attraction between it and the pole "a", and a force of repulsion between it and the pole "b". That is, during the movement from a to b there is a torque which opposes the motion of the coil. The product of the instantaneous values of the current and voltage at any instance gives the power developed. The torque is proportional to this product also, since the speed is constant. While the coil travels from b to c the current and voltage both reverse, but since their product is still positive the sign of the torque does not change. The torque is intermittant but it is always positive and since it opposes the motion of the coil, it corresponds to generator action.

If the load on the generator is such that the current is not in phase with the voltage, the torque curve will have both positive and negative loops, and the average torque will be proportional to the difference between the areas inclosed by these loops. It will be positive for any angle of lag or lead less than 9.0 degrees. A study of fig. 1 will show this.

If in some way, while the generator is running with the current and voltage in phase, the current be reversed, the values of torque, voltage, and current will be as shown in figure 2. In this case the current and voltage are exactly 180 degrees out of phase, and their product, which is still proportional to the torque, is negative,



and corresponds to motor action.

The current in the coil while it passes from a to b is in a clockwise direction, and causes the face of the coil towards the poles to be a north pole. There is, therefore, a force of repulsion between the coil and the pole "a" and a force of attraction between the coil and the pole "b". The resultant of these two forces assist the motions of the coil and produces motor action. If the current leads or lags the voltage, the tongue curve will have both positive and negative loops as in the case for generator action, and the average tongue will be the resultant of the areas of the loops. It will always be negative for values of phase angle less than 30 degrees. A leading current demagnetizes and a lagging current magnetizes the field in a motor, and produces just the opposite effect in a generator.

The foregoing explanation applies to a single-phase machine. In a poly-phase machine the torque is the algebraic sum of the torques developed by all the phases and is constant if the currents and voltages are sine waves and the impressed voltages and currents are balanced.

Name Plate Data

A - C Machine

Alternating Current Generator.

No. 1776422; Type A. N. 1 4

15 H.P. 1800 R.P.M. 12 K.W. 39.5 Amps.

60 cycles; 220 volts. 3 ϕ Y con.

G. E. Schenectady, New York, U. S. A.

D - C Machine

Direct Current Motor.

No. 955771. Shunt: Type R.F.

Form A. 1. 7. H.P. 15

Volts 230 Amps 59.

Temperature Rise 50° 2 hours.

550 / 1650 R.P.M.

G. E. Schenectady, New York, U.S.A.

Method of Solving Unbalanced Circuits.

The method used in handling a three phase unbalanced system of voltages or currents depends upon the fact that any unbalanced three phase system may be replaced by three component systems of vectors. The components of the first of these are identical. They are zero when the vector sum of the original three vectors is zero. They are called by several names, zero sequence, residual, or uniphase components. The components of the second system of vectors are balance, and form a three phase system of the same phase order as the original vectors. This system is called the direct phase, or direct sequence system. They are displaced in phase by 120 degrees just the same as any balanced three phase system of voltages or currents. The third set of components comprise what is called the reverse phase or reverse sequence system. This system consists of three vectors equal in magnitude and displaced in phase by 120 degrees, but their phase order is exactly opposite to that of the original vectors. They, therefore, form a balanced three phase system of opposite or reverse phase order; hence, the name reverse phase system.

The magnitude of the direct and reverse phase components of an unbalance system of voltages or currents may be found very easily by a graphical method. Let us consider the three line voltages of a three phase system. The vector sum of these voltages must be zero whether the system is balanced or not. Since the sum of the line voltages is zero there can be no uniphase components present. Since the vector sum of these voltages is zero they can be represented by a triangle. In the figure on page 8a let $\bar{V}^{1}_{12}$, $\bar{V}^{1}_{23}$, and $\bar{V}^{1}_{31}$ represent the voltages. Construct an equilateral triangle 2-4-3 on $\bar{V}^{1}_{23}$ as a base. The side 2-4 of this triangle is $\bar{V}^{1}_{23}$ rotated through 60 degrees in a positive direction. The diagonal 1-4 = $\bar{V}^{1}_{14}$ divided by the square root of three is equal to the migintude of the direct phase component of phase 1-2. Draw the isosceles triangle 1-5-4 on 1-4 as a base, with 30 degree angles at 1 and 4 as shown in fig. page 8a. Then

$$1\text{-}5 = \bar{V}'_{15} = \frac{\bar{V}'_{14}}{\sqrt{3}} \quad 30^{\circ} = \frac{1}{\sqrt{3}} \left(\bar{V}'_{12} \quad \bar{V}'_{23} \quad 60^{\circ} \right) \quad 30^{\circ}$$

= Vd-12 = direct phase component for phase 1-2.

The reverse phase component of the impressed voltages can be determined by a similar construction as shown by the red line in the figure. Using $\bar{V}'_{23}$ as a base construct the equilateral triangle 2-3-4' as shown. Then draw 1-4, on 1-4' as a base construct the isosceles triangle, 1-4'-5 with 30 degree angles at 1 and 4: then $\frac{\bar{V}'_{14}}{\sqrt{3}} \; -30^{\circ}$ = the reverse phase component of phase 1-4.

If, however, uniphase components are present the vectors do not form a closed triangle, consequently the uniphase components must be determined by a mathematical solution. In this case the angles between the vectors must be

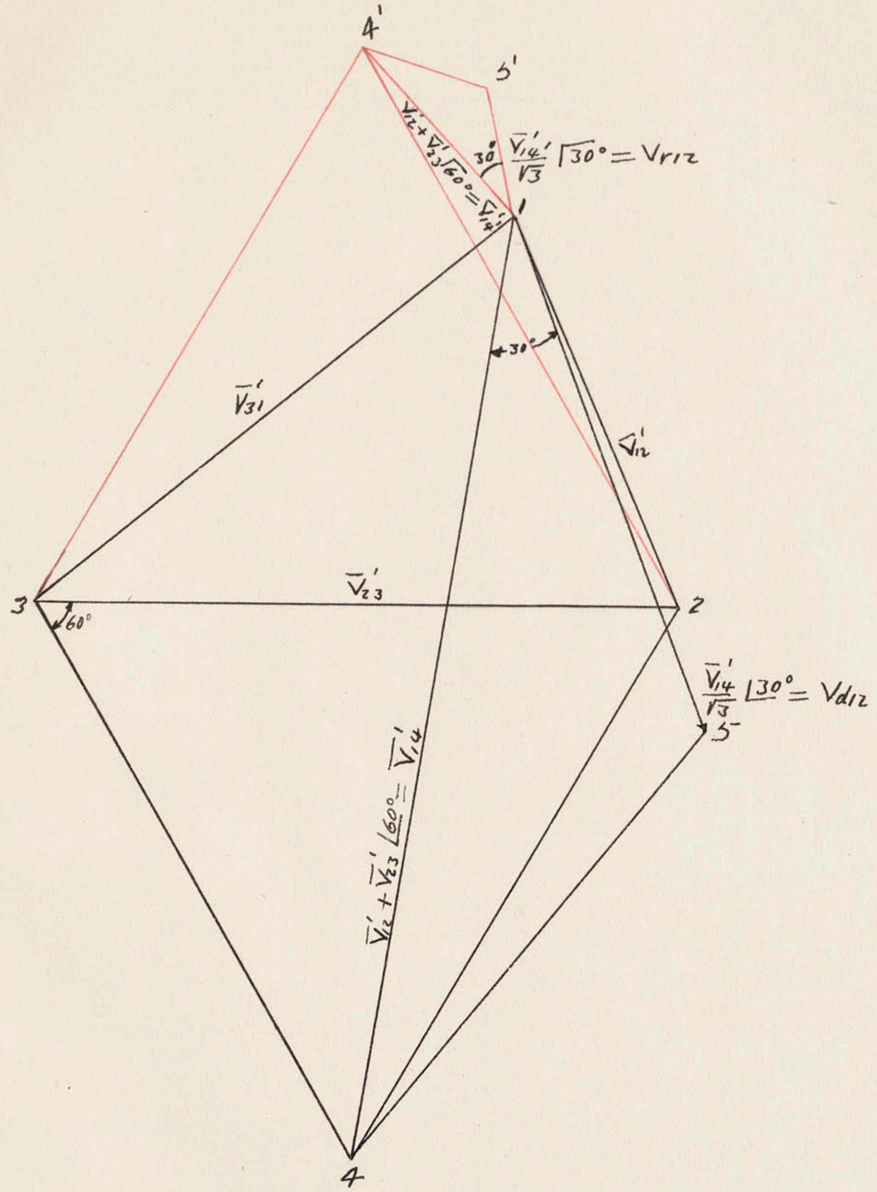
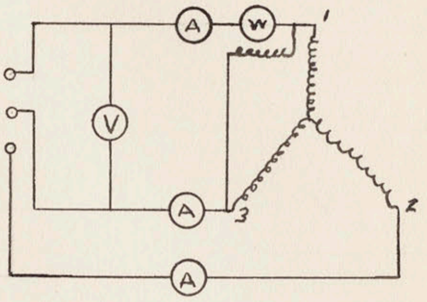


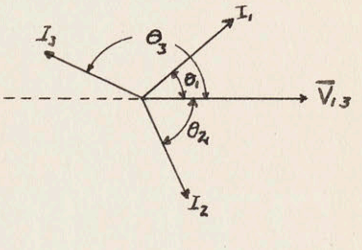
Diagram for graphical Solution of unbalanced Threephase line Voltages.

known.

Let us consider the three line currents in the following circuit. If it is desired to find the angle between the current vectors $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ proceed as follows: With I_1, I_2, I_3 at some known values; read the wattmeter with its current coil in line 1 as shown; then change it to line 2 and line 3 respectively, leaving the potential coil in its same position, and take the readings of the wattmeter. Call these readings W_1, W_2, and W_3, and call the voltage between the lines 1 and 3 V+J 0. Take this as the axis of reference. Then $\cos \theta_1 = \frac{W1}{I_1 V}$; $\cos \theta_2 = \frac{W_2}{I_2 V}$, and $\cos \theta_3 = \frac{W_3}{I_3 V}$. Then since the angular dis-



placement of the three currents is known with respect to a single axis of reference, the angles between the currents can be found. With these angles known the vector expressions for the currents can be written.



The vector solution for the direct, reverse, and uniphase components is as follows: Let $\bar{I}_a$ = the uniphase component, $\bar{I}_d$ = the direct phase component, and $\bar{I}_r$ = the reverse phase components, all currents expressed as vectors.

The subscripts 1, 2, 3, refer to phases 1, 2, and 3

(1) $\bar{I}_u = \frac{\bar{I}_1 \quad \bar{I}_2 \quad \bar{I}_3}{3}$ assume clockwise rotation for the direct phase components.

(2) $I_{d1} + I_{d2} + I_{d3} = 0$; (3) $\bar{I}_{r1} + \bar{I}_{r2} + \bar{I}_{r3} = 0$; Subtract $\bar{I}_u$ from each of the three phase currents, call these values $\bar{I}_1'$, $\bar{I}_2'$etc. Then;

(4) $\bar{I}_1' = \bar{I}_1 - \bar{I}_u = \bar{I}_{d1} + \bar{I}_{r1} = \bar{I}_{d1} \overline{|0} + \bar{I}_{r1} \underline{|0}$

(5) $\bar{I}_2' = \bar{I}_2 - \bar{I}_u = \bar{I}_{d2} + \bar{I}_{r2} = \bar{I}_{d2} \overline{|120^\circ} + I_{r2} \underline{|120^\circ}$

(6) $\bar{I}_3' = \bar{I}_3 - \bar{I}_u = \bar{I}_{d3} + \bar{I}_{r3} = \bar{I}_{d3} \overline{|240} + \bar{I}_{r3} \underline{|240}$

Rotate each of the vectors in equation 5 through 120° in a clockwise direction by applying the operator 120°, then:

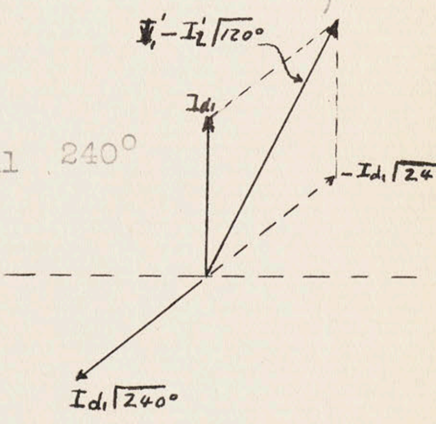
$\bar{I}_2' \overline{|120^\circ} = \bar{I}_{d1} \overline{|240^\circ} + \bar{I}_{r1} \underline{|0^\circ}$ (7)

Subtracting this equation from (4);

$\left(\bar{I}_1' - \bar{I}_2' \; 120^\circ \right) = \left(\bar{I}_{d1} \underline{|0} - \bar{I}_{d1} \overline{|240} \right) + \left(\bar{I}_{r1} \underline{|0} - \bar{I}_{r1} \underline{|0} \right)$

$= \bar{I}_{d1} \underline{|0} - \bar{I}_{d1} \overline{|240}$

The currents; $\bar{I}_1' - \bar{I}_2' \underline{|120^\circ}$, $\bar{I}_{d1}0$, and $\bar{I}_{d1}$ 240° are shown in the accompanying diagram.



from Fig. 2 it can be seen that;

$\left(\bar{I}_1' - \bar{I}_2' \overline{|120^\circ} \right) = \left(\bar{I}_1' + \bar{I}_2' \underline{|60^\circ} \right)$

$= \sqrt{3}\ \bar{I}_{d1} \overline{|30^\circ}$ (8) 1'' $\bar{I}_{d1} = \frac{1}{\sqrt{3}} \left\{ \bar{I}_1' + \bar{I}_2' \underline{|60^\circ} \right\} \underline{|30^\circ}$

$$= \frac{1}{\sqrt{3}} \left\{ \bar{I}_1' \underline{|30^\circ} + \bar{I}_2' \underline{|90^\circ} \right\} \quad (9)$$

Since $\bar{I}_1' = \bar{I}_{d1} + \bar{I}_{r1}$ equation (4)

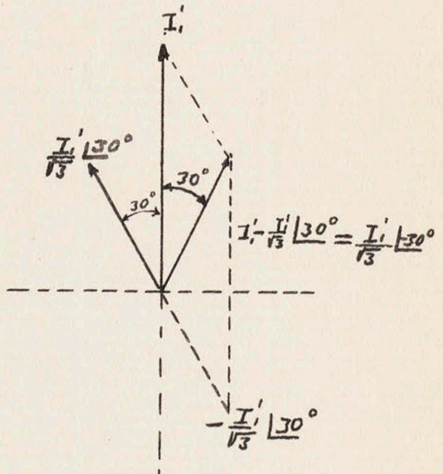
$$\bar{I}_{r1} = \bar{I}_1' - \bar{I}_{d1} = \bar{I}_1' - \frac{I_1'}{\sqrt{3}} \underline{|30^\circ} - \frac{\bar{I}_2'}{\sqrt{3}} \underline{|90^\circ} \quad (10)$$

From Fig. 3 it will be seen that equation 10 reduces to,

$$\bar{I}_{r1} = \frac{1}{\sqrt{3}} \left\{ \bar{I}_1' \overline{|30^\circ} - \bar{I}_2' \overline{|90^\circ} \right\} = \frac{1}{\sqrt{3}} \left\{ \bar{I}_1' - \bar{I}_2' \underline{|120^\circ} \right\} \overline{|30^\circ}$$

$$= \frac{1}{3} \left\{ \bar{I}_1' + \bar{I}_2' \overline{|60^\circ} \right\} \overline{|30^\circ} = \frac{1}{\sqrt{3}} \left\{ \bar{I}_1' \overline{|30^\circ} + \bar{I}_2' \overline{|90^\circ} \right\} \quad (11)$$

Since the direct phase and reverse phase components of phase 1 have been determined, it is now only necessary to apply the proper operators to these components to obtain $\bar{I}_{d2}$, $\bar{I}_{r2}$, $\bar{I}_{d3}$ and $\bar{I}_{r3}$. Thus the phase values are as follows:



$\bar{I}_{d1} = \bar{I}_{d1} \underline{|0^\circ}$ $\qquad$ $\bar{I}_{r1} = \bar{I}_{r1} \underline{|0^\circ}$

$\bar{I}_{d2} = \bar{I}_{d1} \overline{|120^\circ}$ $\qquad$ $\bar{I}_{r2} = \bar{I}_{r1} \underline{|120^\circ}$

$I_{d3} = \bar{I}_{d1} \overline{|240^\circ}$ $\qquad$ $I_{r3} = \bar{I}_{r1} \underline{|240^\circ}$

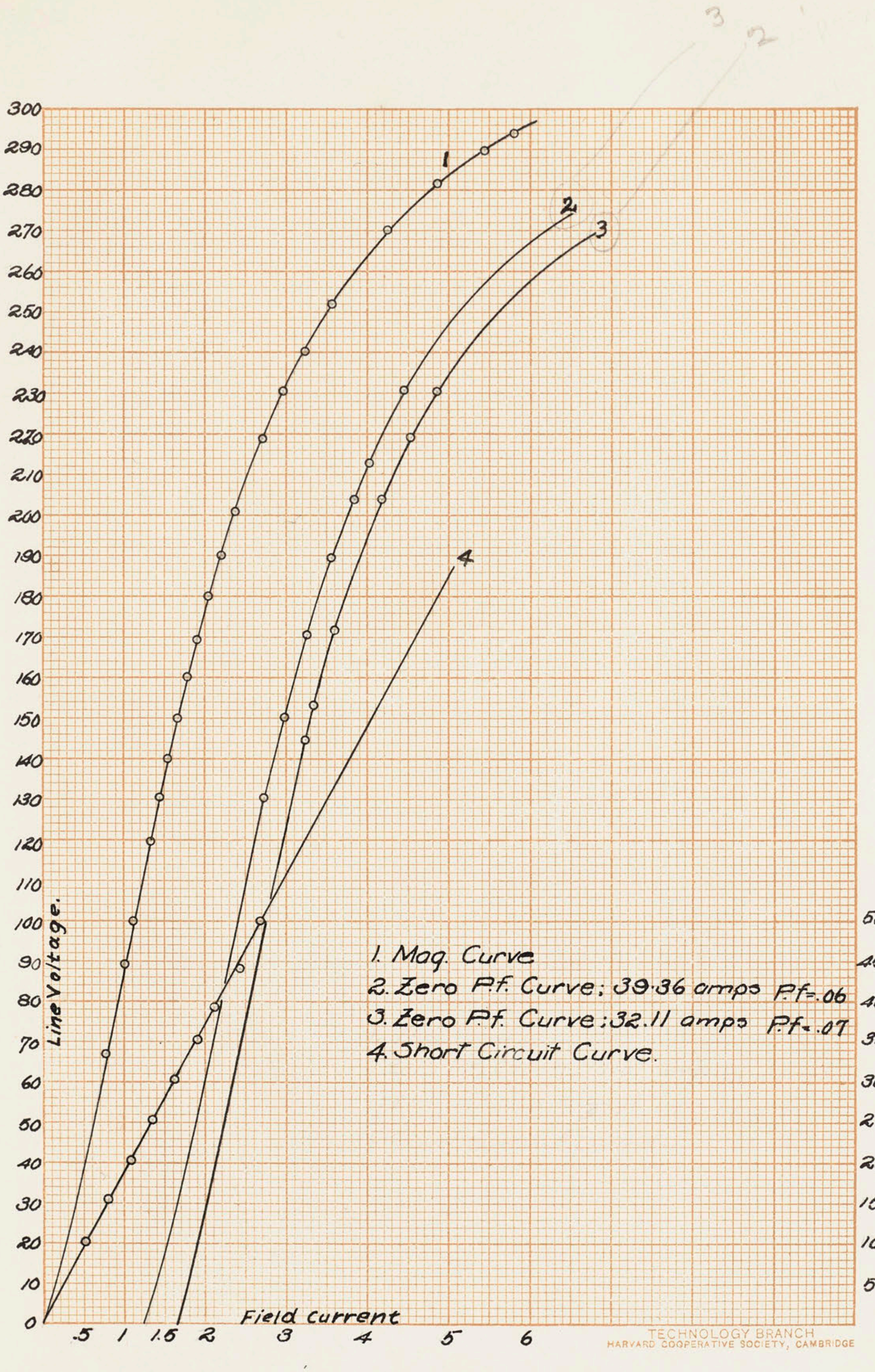
Resistance of the Armature.

The ohmic resistance of the armature is small and is easily found by the drop of potential method as will be explained later. It is of little practical value because it cannot be used in alternating current work, if accurate results are to be obtained. It is the resistance offered by a conductor to the passage of a direct current through the conductor. The potential drop from one end of the conductor to the other is then given by the product of this resistance and the current flowing.

The effective resistance is the resistance which is always used in connection with alternating current work. It is the resistance offered by a conductor to the passage of an alternative current through the conductor, and is almost always greater than the ohmic resistance. In either the A-c or the d-c case the power absorbed by a circuit in which there is no rotating machinery is given by $P = I^2$ or $R = \frac{P}{I^2}$. However in the a-c case the power is greater, and consequently the value of R must increase, assuming the current to remain constant. The increase in power in the a-c case is due to two reasons; first, local losses are produced in the conductor itself and in the surrounding material, due to a changing flux caused by the alternating current. Secondly, the current in the conductor is not uniformly distributed over its cross section. The now uniform distribution of the current being due to the difference in the reactance of elements of the conductor taken parallel to its axis.

Direct Phase Impedance (Synchronous Impedance).

The synchronous impedance was measured in two ways; the short circuit method, and the zero power factor method. From the curve sheet on page 13a the value of Zs may be obtained by either the zero power factor method or the short circuit method. To obtain Zs by the short circuit method proceed as follows. Take any value of field current, and from the short circuit curve obtain the value of the armature current corresponding. Call this value of current I. For the same value of field current obtain from the magnetization curve the corresponding value of voltage V. Then $Zs = \frac{V}{\sqrt{3}I}$. This value of Zs is greater than the normal value of Zs under operative conditions. To obtain the value of Zs by the zero power factor method proceed as follows: Take any value of field current and obtain the corresponding value of Voltage V_1 from the zero power factor curve. With the same value of field current obtain the corresponding value of voltage 2 from the magnetization curve. Then the value of Zs_1 is given by $Z'_s = \frac{V_2 - V_1}{\sqrt{3}\ I}$ where I is the value of armature current at which the zero power factor curve was taken. These two values of synchronous impedance will be different. The value of Zs obtained by the short circuit method is greater on account of low saturation. The value of Z'_s is



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120
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70
60
50
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30
20
10
0
Line Voltage.
Field current
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1.5
2
3
4
5
6
50
45
40
35
30
25
20
15
10
5
1
2
3
4
1. Mag. Curve
2. Zero P.f. Curve: 39.36 amps P.f=.06
3. Zero P.f. Curve: 32.11 amps P.f=.07
4. Short Circuit Curve.
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not exactly correct due to low power factor, but it does not contain the greater error due to low saturation. Call the ratio of these two impedances, that is $\frac{Z_s'}{Z_s} = C$. This factor will be used later in connection with the reverse phase impedance.

Uniphase Impedance.

The uniphase impedance is the impedance to the zero sequence component of the current. It was measured by impressing a low single phase voltage across the neutral and the interconnection of the three phase terminals of the motor. This impedance is very small as compared to the direct phase impedance. It decreases a little as the value of the voltage impressed increases. The variation of this impedance is small; the greatest deviation from the average value being about 6%.

The uniphase impedance was measured with the rotor of the motor stationary, and with it revolving at synchronous speed. In the latter case the average value of the impedance was found to be 3.7% less than in the former. This difference was due to the fact that when the rotor of the motor is at stand still it is not exactly symmetrical with respect to the armature winding. This conclusion was arrived at by taking readings of the voltage and current for different positions of the rotor with respect to the armature, and getting the average value of the impedance. The average in this case was the same as that obtained by driving the rotor at synchronous speed. Therefore the net effect of having the rotor of the motor rotate at synchronous speed while taking measurements for the zero sequence impedance, is to average up the effects of the disymmetry of the rotor with respect to the stationary winding, and thus get a more accurate value of the impedance.

Reverse Phase Impedance.

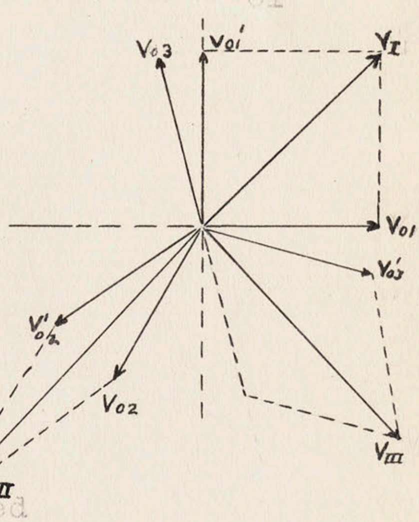
The impedance to the reverse sequence component of the unbalanced impressed current, is less than the direct phase synchronous impedance as measured by the zero power factor method at the same degree of satuation. From actual measurements with the field circuit open the value of Zr is greater than Zd. But Zr in this case must be measured at a very much lower satuation than Zd. Consequently a direct comparison cannot be made unless Zr is corrected for satuation. A correction was made based on the assumption that the ratio of Zd as measured by the short-circuit method, to Zd as measured by the zero power factor method, is the same as the ratio of Zr at the low satuation at which it was measured, to Zr if it could be measured at normal satuation. This assumption is justifiable because a point on the no lood satuation curve higher than one for short-circuit conditions, with the same value of current cannot be reached, when Zr is determined by the method used. Therefore, it is probably better to use this corrected value of Zr for more accurate calculations.

Values of current and voltage were taken under several conditions in order to try to get a value Zr which would be as nearly as possible the same as the actual value of Zr under normal conditions of operation. It was found that when the field was short-circuited through an ammeter the voltage required to cause the same value of current to flow was very much less than in the case when the field was open. This should be expected however, because the field winding is equivalent to the secondary winding of a transformer, with respect to the armature winding. The field winding is a single phase winding, and can therefore have only a single phase current flowing in it when it is short-circuited.

Before analyzing the significance of the readings of V and I with the field short circuited it will be necessary to mention how the data was obtained for the values of Zr. A low voltage was impressed on the armature of such phase order that the machine would run in the correct direction if allowed to run as a motor. Then with the impressed voltage remaining the same as to phase order, the motor was driven in the reverse direction, by a shunt motor, at synchronous speed. Under this condition a current will be induced in the field which is not of fundamental frequency. The current induced in the field is a second harmonic as shown by the oscillograph picture on page 18a. This second harmonic current causes a voltage to be induced in the armature of tripple frequency.

Since there are tripple frequency components of voltage present; the ratio of the voltmeter readings to the ammeter readings cannot give a correct value of impedance. This is true because the values of reactance are not the same for tripple frequency as for fundamental frequency. The value of a reactancefor the third harmonic is three times that for fundamental frequency. Consequently if either V or I is made up of 1st, third, etc. harmonic components, the ratio of V to I is meaningless.

If a direct current voltage be impressed on the field the value of the reverse phase impedance is again not equal to the ratio of the voltage impressed on the armature to the current flowing. In this case there is also a second harmonic current induced in the field winding. The direct current induced a voltage in the armature which is of opposite phase sequence to the impressed voltage. Therefore the voltages as measured by a voltmeter across the terminals of the three phases are unbalanced. This may be illustrated by the vector diagram shown below. Let V_{o1} V_{o2} V_{o3}, the impressed voltages, and V'_{o1}, V'_{o2}, V'_{o3} be the voltages induced by the direct current in the field. Then the vector sum of these voltages will be the live voltage as read by the voltmeter. From the diagram it can be seen that the live voltages V_I, V_{II}, and V_{III} will be unbalanced. The observed data agreed with this fact, and the actual values of voltages as measured by voltmeter between the three armature terminals

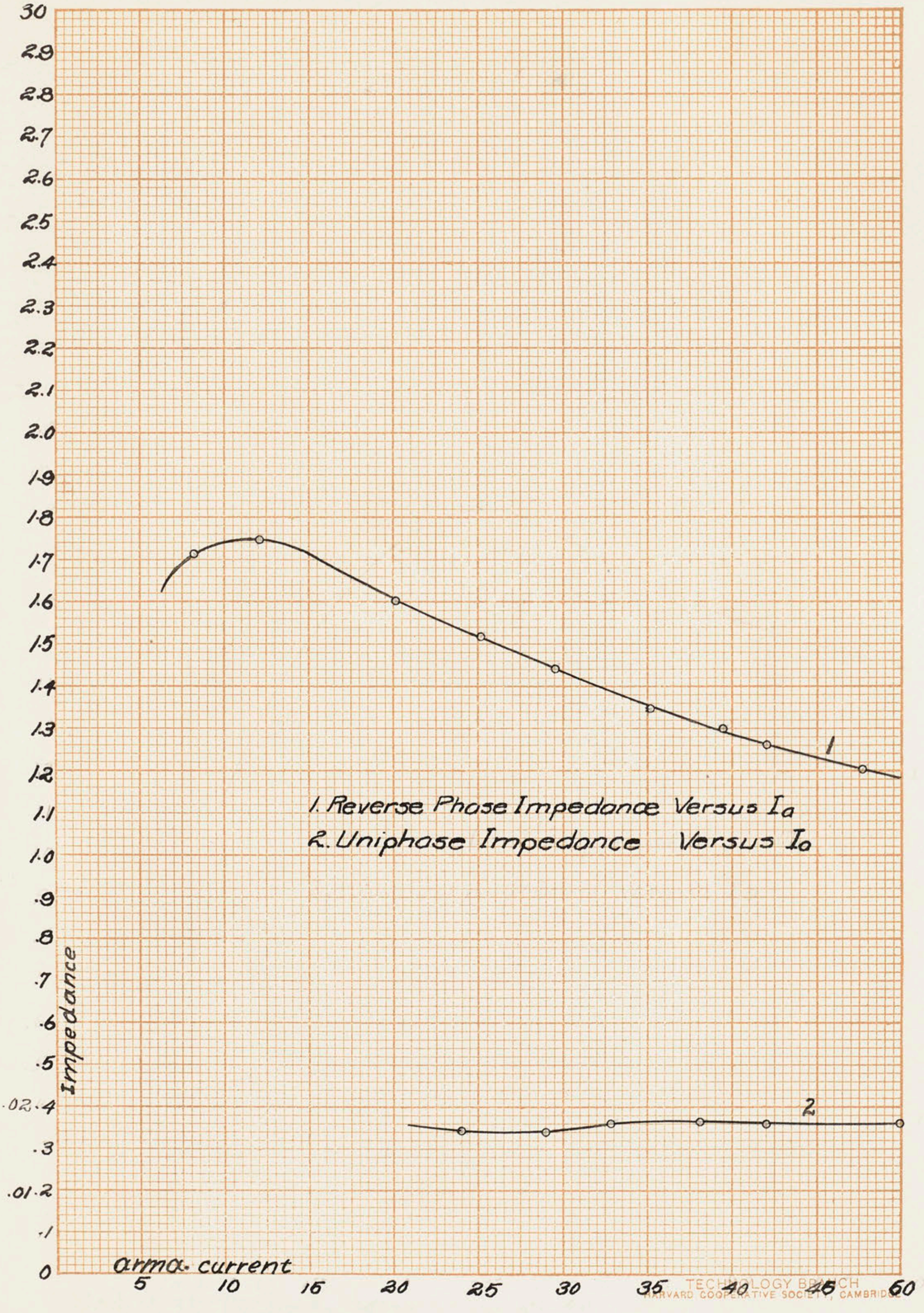


were not equal.

If a large value of field current is impressed there is a synchronizing action between the motor voltage and the main line voltage. In this case, however, no steady readings could be obtained on the meters, except that the motor was driven above or below synchronous speed.

The condition which corresponded to operating conditions was the case when a direct current was caused to flow in the field. Consequently it seems that the reverse phase impedance should have been determined under this condition. As stated above, data were taken with a direct current in the field, but no method was found by which the value of the reverse phase impedance could be obtained from these data. It is evident however, that a direct ratio of V to I is not a value to be used as an impedance due to the presence of the harmonics as pointed out above. Therefore the value of the reverse phase impedance used was determined from values of V and I with the field open, and then corrected as stated above.

The curve I on page 25a shows the variation of the reverse phase impedance with the armature current. For low values of current the impedance seems to be a little low, but it is increasing and reaches a clearly defined maximum and then decreases for further increases in current. No explanation is offered for the low value of the impedance for small values of current. The decrease after the maximum can


30
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2.1
2.0
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1.3
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.3
.01 .2
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Impedance
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30
35
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45
50
1
2
1. Reverse Phase Impedance Versus I_a
2. Uniphase Impedance Versus I_a
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be explained from the standpoint of saturation. That is, one would expect for the impedance to be less the higher the saturation just as the direct phase impedance decreases with increasing saturation. Of the many runs made for the determination of the reverse phase impedance, all observed data indicate that the value of this impedance starts at a lower value rises to a maximum and then decreases again.

Final Discussion

The following effects may be said to be due to the unbalanceing of the line voltages of a synchronous motor. With the neutral connected to the line (1) A large component of the fundamental line current flows through the neutral. (2) Even harmonic currents are introduced in the field winding. (3) The armature current of the motor contains three components of currents; uniphase, reverse phase, and direct phase currents.

The effect of the neutral current is to increase the copper loss in the motor and therefore lower its efficiency. The neutral current is composed of the uniphase components of the armature current, and is equal to three times the value of any one.

The even harmonics in the field increase the core loss, and cause a third harmonic voltage to be induced in the armature winding, which increases the copper loss.

The direct phase component of the armature current produces a torque which is available at the pulley of the motor. The reverse phase component of the armature current produces a torque which is opposite to the direct phase torque. Consequently there is a decreased torque when the voltages are unbalanced.

If the neutral is not connected, there can be no uniphase components of current. There will however, be reverse, and direct components of the armature current.

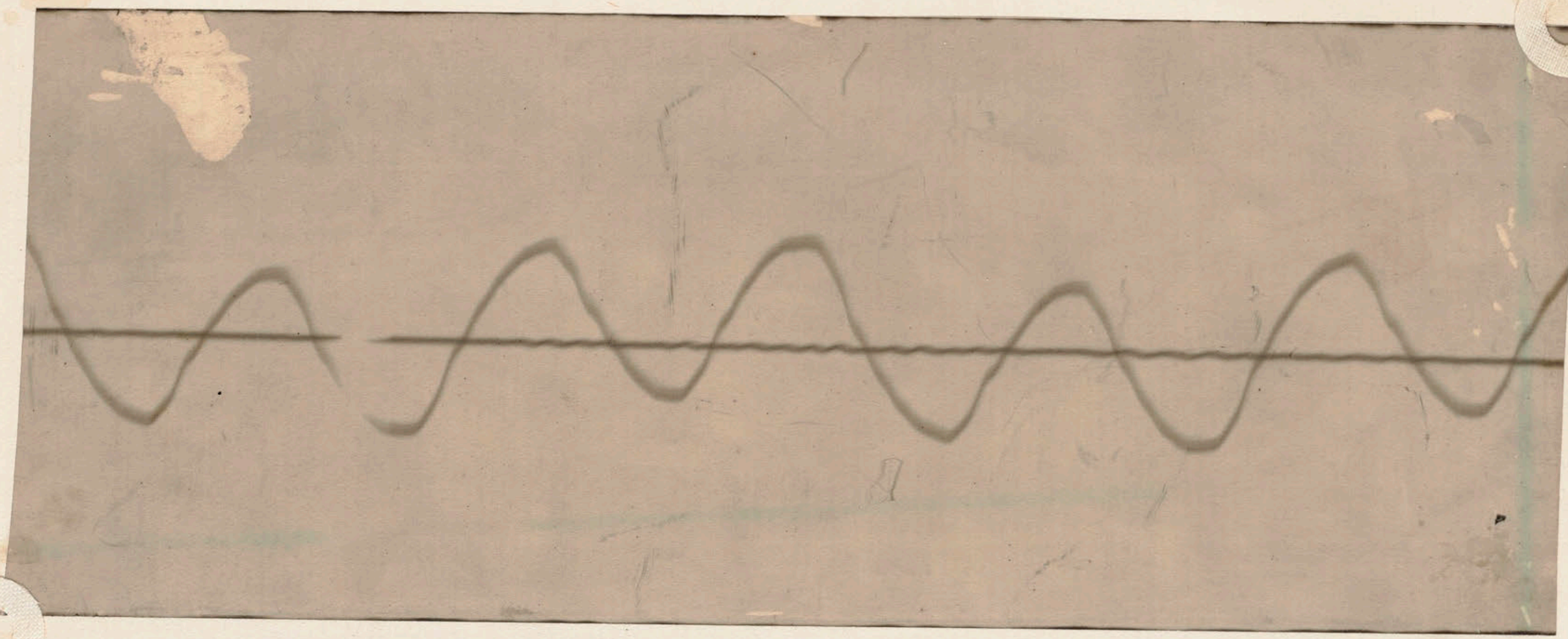
Therefore the same diminution of torque will be present whether the neutral is connected or not.

The foregoing effects are more or less evident, and are always present when the voltages across the terminals of a synchronous motor are unbalanced. The main purpose of this thesis was to determine whether or not the components of the current caused by the unbalancing could be determined, when the constants of the machine, including the uniphase and reverse impedances, were known.

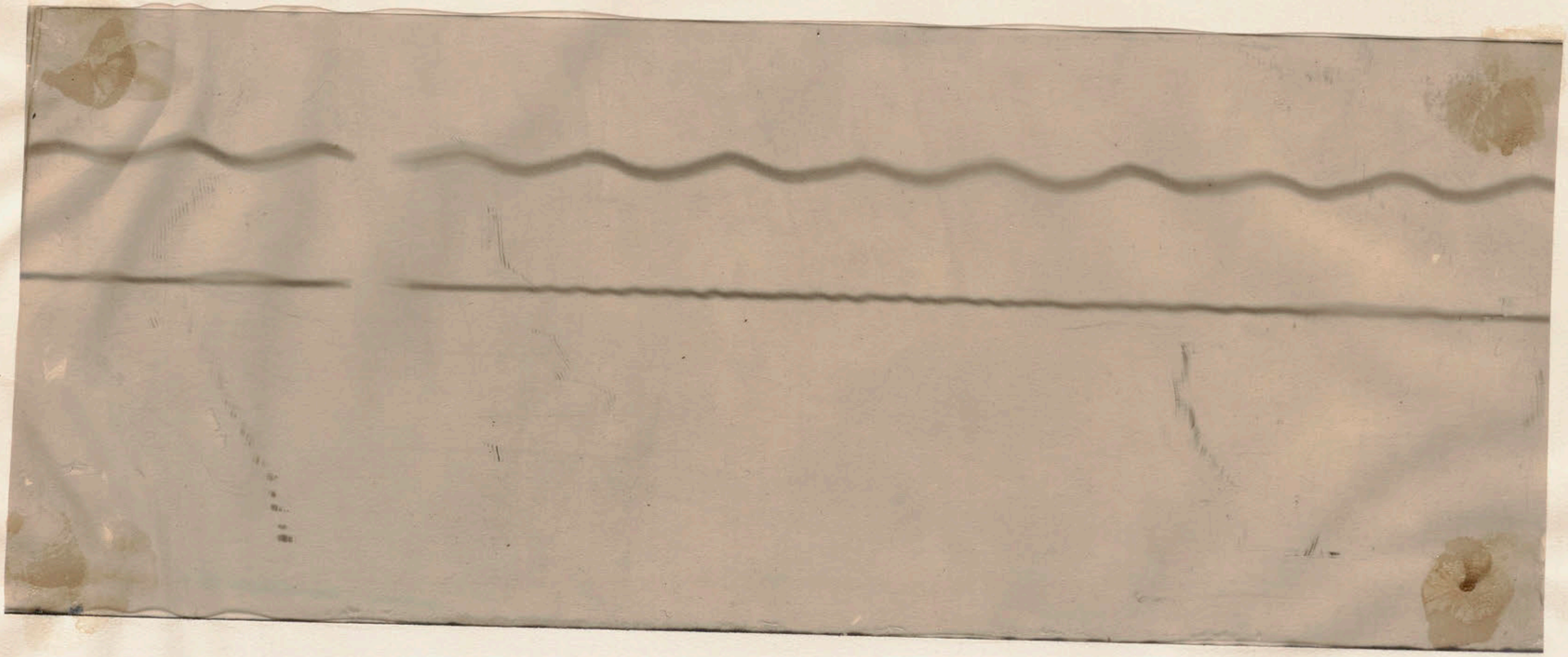
The calculations are carried out on page 34 The values of the components of the armature current did not check even approximately with the experimental values. The solution of the equations is very difficult, and involve many transformations from polar to rectangular coordinates and vice versa. Since the time was very limitted the solution was only worked through once and checked once. Therefore there may be unseen errors in the solution.

Conclusions.

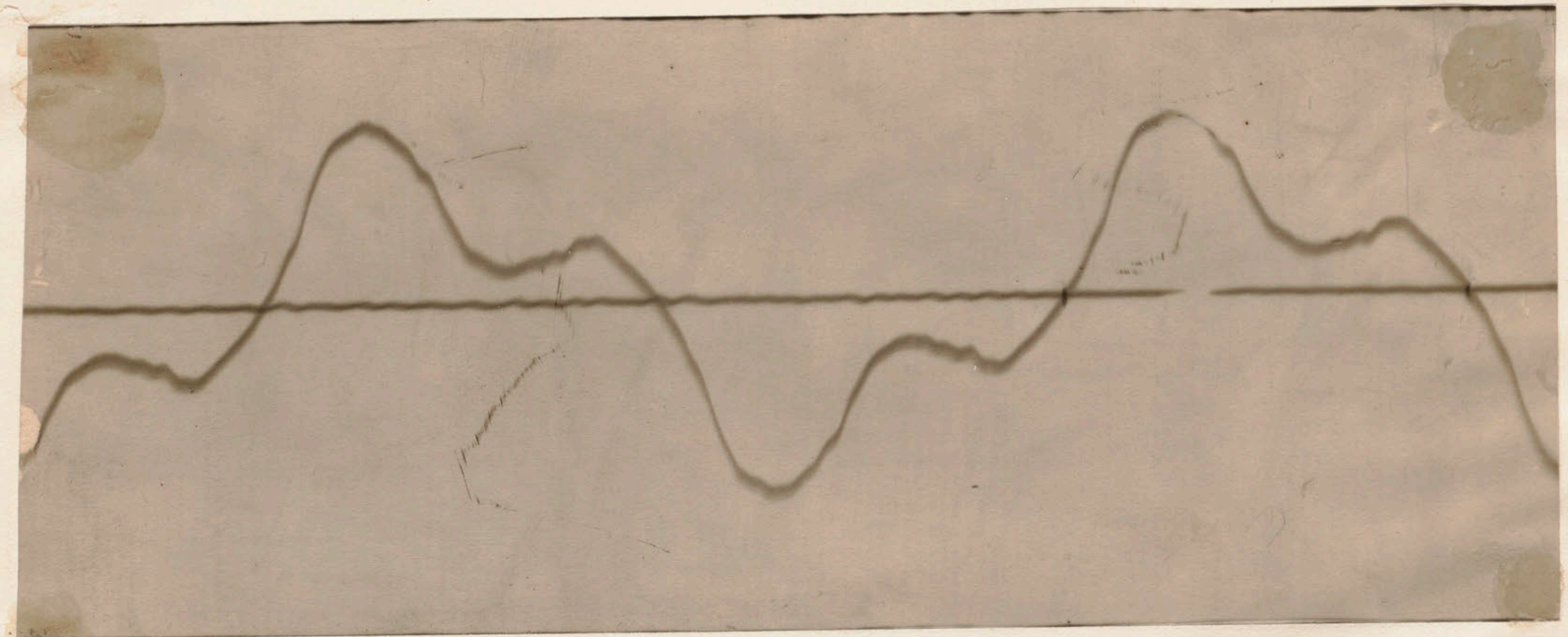
The final results indicate that the components of the armature current can not be calculated from the constants of the machine. These results are not conclusive, because time did not permit enough calculations to be made, and enough data to be observed. The method is a theoretical one, and has not been verified by experiment so far as the writer was able to ascertain. It seems probable, however, that the calculated results and the experimental values would come out closer, if the effect of the harmonics could be lessened, and the calculations made very slowly and checked, so that all the complex quantities would be represented exactly by the equations given them.



Neutral Current When Voltages are Balanced.

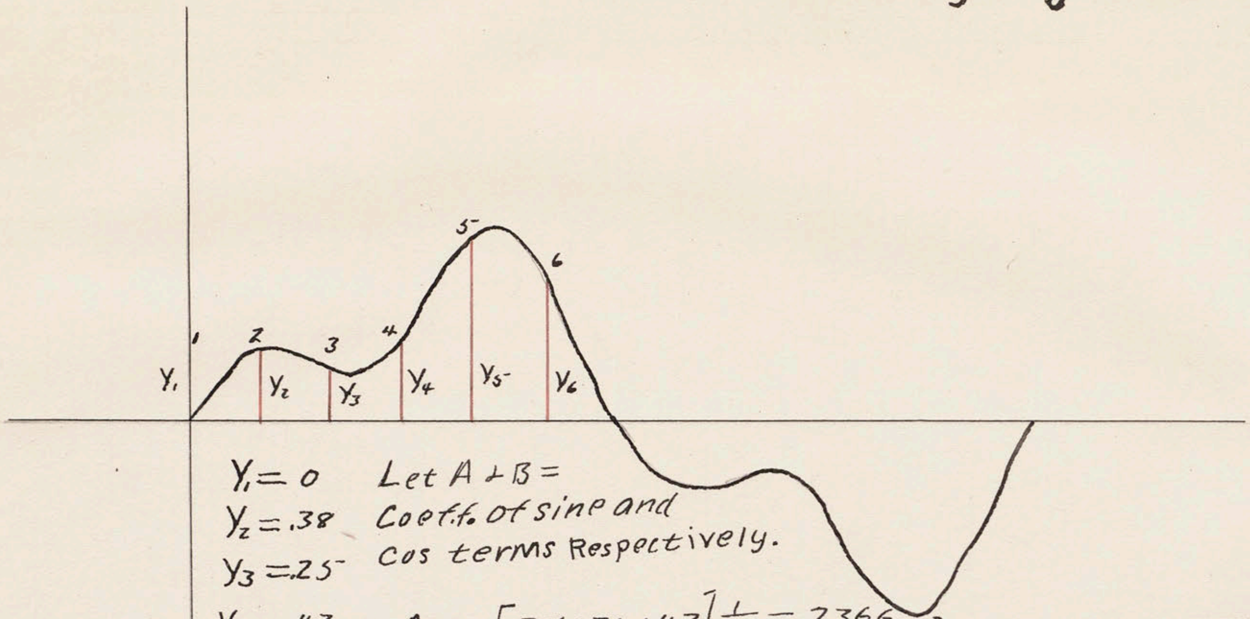


Field Current When Voltages are Unbalanced.



Neutral Current When Voltages are Unbalanced.

Fischer-Hinnen Method of Analysing a Wave.



$Y_1 = 0$

$Y_2 = .38$

$Y_3 = .25$

$Y_4 = .43$

$Y_5 = .95$

$Y_6 = .76$

Let A + B = Coeff. of sine and cos terms Respectively.

$A_3 = [.38 + .76 + .43]\frac{1}{3} = .2366$

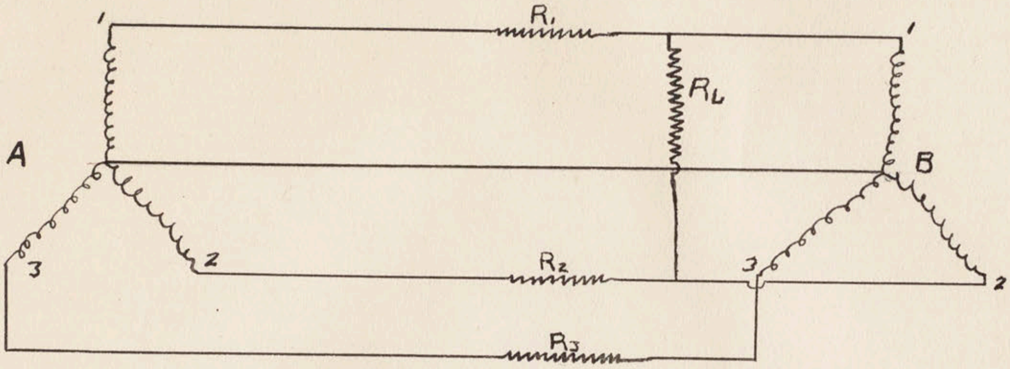
$B_3 = \frac{1}{3}[0 - .25 + .95] = .2333$

$C_3 = \sqrt{(.2366)^2 + (.2333)^2} = .3339$

$A_1 = \frac{1}{3}[(.38 + .76).5 + (.25 + .95).866 + (.43 \times 1)] = (2.039)\frac{1}{3} = .6796$

$B_1 = \frac{1}{3}[(.25 - .95).5 + (.38 - .76).866] = -.2263 \quad C_1 = \sqrt{(.6796)^2 + (-.2263)^2} = .715$

% of third Harmonic $= \frac{.3339 \times 100}{.715} = 46.65\%$.



At A the voltages are balanced;

Let; $\underset{+}{V}$ = direct phase component, $\underset{-}{V}$ = reverse phase component, and V_0 = uniphase component; and; $\underset{+}{I}$, $\underset{-}{I}$, $\underset{0}{I}$, = the respective components of current.

$$R_0 = \frac{R_1 + R_2 + R_3}{3} = .28 \text{ ohms};$$

$$\underset{+}{R} = \frac{R_1 + R_2\overline{|120^\circ} + R_3\underline{|120^\circ}}{3} = .146 - J.15 = .209\overline{|45.8^\circ}$$

$$\underset{-}{R} = \frac{R_1 + R\underline{|120^\circ} + R_3\overline{|120^\circ}}{3} = .146 + J.15 = .209\underline{|45.8^\circ}$$

For Motor: $Z_r = .208 + J.369 = .423\underline{|60.6^\circ} = \underset{-}{Z}^M$

$$Z_d = 1.96 + J5.5 = 5.84\underline{|70.3^\circ} = \underset{+}{Z}^M$$

$$Z_0 = .0181$$

R_L = 4.34 ohms.

At A; $\underset{+}{V} = V + J0$ $\qquad I_L = \frac{V_{12}}{R_L}$ in line 1.

$\underset{-}{V} = 0$ $\qquad I_L = -\frac{V_{12}}{R_L}$ in line 2.

$\underset{0}{V} = 0$

$$I_{0L} = 0;\ \underset{+}{I}_{1L} = \frac{1}{3}\frac{V_{12}}{R_L} + \left[-\frac{V_{12}}{R_L}\overline{|120^\circ}\right] + 0 = \frac{1}{\sqrt{3}}\frac{V_{12}}{R_L}\underline{|30^\circ}$$

$$\underset{-}{I}_{1L} = \frac{1}{3}\left[\frac{V}{R_L} + \left(-\frac{V_{12}}{R_L}\right)\underline{|120^\circ} + 0\right] = \frac{1}{\sqrt{3}}\frac{V_{12}}{R_L}\overline{|30^\circ}$$

$$\underset{+}{I}_L = \frac{1}{R_L}\left[\underset{+}{I}_1\underset{+}{Z}^M + \underset{-}{I}_1\underset{-}{Z}^M\underline{|60^\circ}\right]$$

$$\underset{-}{I}_L = \frac{1}{R_L}\left[\underset{+}{I}_1\underset{+}{Z}^M\overline{|60^\circ} + \underset{-}{I}_1\underset{-}{Z}^M\right]$$

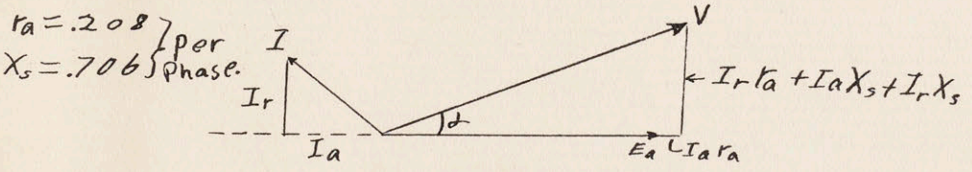
(1) $$\text{Line } I_1 = \left[\underset{+}{I}_1 + \frac{1}{R_L}\left(\underset{+}{I}_1\underset{+}{Z}^M + \underset{-}{I}_1\underset{-}{Z}^M\underline{|60^\circ}\right)\right] + \left[\underset{-}{I}_1 + \frac{1}{R_L}\left(\underset{+}{I}_1\underset{+}{Z}^M\overline{|60^\circ} + \underset{-}{I}_1\underset{-}{Z}^M\right) + I_0\right]$$

$$= \left[\underset{+}{I}_1\underbrace{\left(1 + \frac{\underset{+}{Z}^M}{R_L}\right)}_{=a} + \underset{-}{I}_1\underbrace{\frac{\underset{-}{Z}^M}{R_L}\underline{|60}}_{=b}\right] + \left[\underset{-}{I}_1\underbrace{\left(1 + \frac{\underset{-}{Z}^M}{R_L}\right)}_{=c} + \left(\underset{+}{I}_1\underbrace{\frac{\underset{+}{Z}^M}{R_L}\overline{|60^\circ}}_{=d}\right)\right] + I_0$$

Solution of the unbalanced circuit:

On page 34 is shown the equivalent circuit of the unbalanced system. Let A be a large generator, (large as compared to the motor so that the unbalanced current taken by the motor will not have any effect on the voltages of the generator) generating balanced voltages. The line impedances are as shown. A load R is connected across lines 1 and 3. The load R causes the voltages to be-come unbalanced across the terminals of the motor.

In order to solve this circuit the motor must be replaced by an impedance which is equivalent, so far as voltages and currents are concerned, to the motor. This impedance can not be found exactly, but it can be found very closely by making an approximation as shown below.



P = power per phase = 1020 watts. $I_a = \frac{1020\sqrt{3}}{227} = 7.75$

$E_a = 227$, taken from mag. curve, for known value of I_f.
A value of V must be assumed: From the known value of V at A page 34. V can be taken at about 229 Volts.
Then: From the vector diagram for a Syn. motor,

$$\left[\frac{229}{\sqrt{3}}(\cos\alpha + j\sin\alpha)\right]^2 = \left[\frac{227}{\sqrt{3}}\right]^2 + \left[-.208 I_r\right]^2 + \left[.706 I_r\right]^2 + \left[.775 \times .706\right]^2 + \left[.208 \times 7.75\right]^2$$

$I_r = 21.8$ amps. $\quad \tan\alpha = \frac{7.75 \times .208}{.208 \times 23.1 + 7.75 \times .706 + 23.1 \times .706} = .067$

$I = \sqrt{(7.75)^2 + (21.8)^2} = 23.1$ $\qquad \alpha = 3.5^\circ$

$I = 23.1\angle{-71.5^\circ}$

$$Z = \frac{229\angle 3.5^\circ}{\sqrt{3}} \times \frac{1}{23.1\angle{-71.5^\circ}} = 5.74\angle 75^\circ$$

This value Z is very close, because the value of V has been approximated by several trials.

In equation 1; Let the Coefficients of the currents be as follows;

I_{+}, Coef. = a

and→ $I_{=}$, " = b

$I_{\neq}$, " = c

$I_{\mp}$ " = d

Then:

$$(2)\quad V_{+} = [a R_0 + Z^M_{+} + d Z_{=}] I_{+} + [b R_0 + c R_{=}] I_{=} + I_0 R_{+}$$

$$(3)\quad V_{=} = 0 = [d R_0 + a R_{+}] I_{+} + [c R_0 + b R_{=} + Z^M_{=}] I_{=} + R_{=} I_0$$

$$(4)\quad V_0 = 0 = [a R_{=} + d R_{+}] I_{+} + [b R_{=} + c R_{+}] I_{=} + [R_0 + Z^M_0] I_0$$

Introduce the following Coeff.

In (2) Coef. of $I_{+} = K_1$ In (3) Coef. of $I_{+} = K_4$

" " " " $I_{=} = K_2$ " " " $I_{=} = K_5$

" " " " $I_0 = K_3$ " " " $I_0 = K_6$

In (4) Coef. of $I_{+} = K_7$

" " " " $I_{=} = K_8$

" " " " $I_0 = K_9$

Then: $V_{+} = K_1 I_{+} + K_2 I_{=} + K_3 I_0 \quad (5)$

$$0 = K_4 I_{+} + K_5 I_{=} + K_6 I_0 \quad (6)$$

$$0 = K_7 I_{+} + K_8 I_{=} + K_9 I_0 \quad (7)$$

$$I_0 = \frac{132.5 - K_1 I_{+} - K_2 I_{=}}{K_3} \quad \text{From (5)}$$

Sub: in 6 & 7

$$0 = K_4 I_{+} + K_5 I_{=} + K_6 \left[\frac{132.5 - K_1 I_{+} - K_2 I_{=}}{K_3}\right] \quad (8)$$

$$0 = K_7 I_{+} + K_8 I_{=} + K_9 \left[\frac{132.5 - K_1 I_{+} - K_2 I_{=}}{K_3}\right] \quad (9)$$

$$I_{+} = \frac{K_2 K_6 I_{=} - K_6\, 132.5 - K_5 K_3 I_{=}}{K_3 K_4 - K_1 K_6} \quad (10)$$

Values of the Constants.

$a = 1.342 + J1.28 = 1.85\angle 43.6°$ $c = 1.048 + J.085 = 1.056\angle 4.7°$

$b = -.0495 + J.0837 = .0975\angle 120.6°$ $d = 1.280 + J.342 = 1.321\angle 15°$

$K_1 = 1.995 + J6.44 = 6.73\angle 72.7°$ $K_2 = .156 + J.164 = .227\angle 46.4°$

$K_3 = .146 - J.15 = .209\overline{\angle 45.8°}$ $K_4 = .744 + J.081 = .75\angle 6.2°$

$K_5 = .508 + J.402 = .646\angle 38.3°$ $K_6 = .146 + J.15 = .209\angle 45.8°$

$K_7 = .238 + J.246 = .342\angle 46°$ $K_8 = .146 - J.14 = .202\overline{\angle 43.8°}$

$K_9 = .28 - J.0181 = .284\overline{\angle 3.7°}$

sub. the constants in eq. 7, 8, 9, page 35.

Then: $I_d = 17.6\overline{\angle 73°}$

$I_r = 17.25\angle 210°$

$I_u = 15.3\overline{\angle 6°}$

Comparison of calculated and measured values of currents.

I_u

Measured;......... $11.2 \angle 222^\circ$

Calculated;....... $15.3 \angle 6^\circ$

I_d

Measured;......... $25.3 \angle 83^\circ$

Calculated;....... $17.6 \angle 73^\circ$

I_m

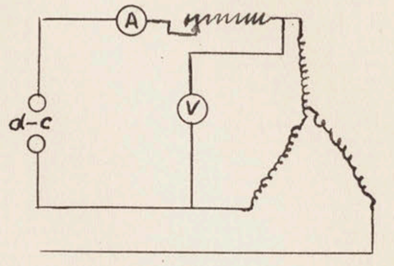
Measured;........ $19.7 \angle .9^\circ$

Calculated;....... $17.3 \angle 210$

Appendix A.

Ohmic Resistance.

The ohmic resistance was measured by the drop of potential method. The diagram of connections is shown in the accompaning figure. A resistance was placed in series with the armature in order to reduce the impressed voltage. Several values of V and I were measured between each pair of terminals, also for different positions of the rotor with respect to the armature. The value of the ohmic resistance is then given by: $V_{oh} = \frac{V}{2I}$ ohms per phase.



Effective Resistance.

The effective resistance was measured by the short-circuit method. As follows: Let P_1 equal the power input to the shunt motor when it is driving the synchronous motor with its armature circuit, and field circuit open. Let P_2 equal the power input to the shunt motor when the synchronous motor is short-circuited, and P_{sp} = the stray power of the shunt motor. Then synchronous motor friction and windage = $P_1 - (I_{de})^2 V_{de} - P_{sp.} = P_w$ where I_{de} and V_{de} = the shunt motor armature current and armature resistance respectively. Let P_c = care loss of the synchronous motor corresponding to short circuit field current. Then the effective resistance per phase is given by:

$$r_e = \frac{P_2 - P_w - (I_{d-c})^2 r_{d-c} - P_c}{3 I^2}$$

where I^2 is the short-circuit phase current of the synchronous motor.

Magnetization Curve.

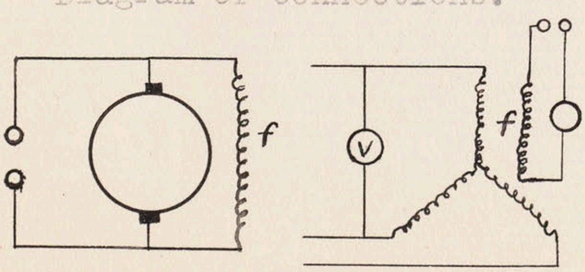
The magnetization curve of the motor was obtained by driving it as a synchronous generator with a shunt motor. The speed of the shunt motor was adjusted until the frequency of the generator was at its rated value of 60 cycles per second. The field current of the synchronous generator was varied from a value which gave about 65 volts to a value which gave about 300 volts.

The form of the magnetization curve shows that the machine operates somewhat below the knee of the curve when it is operated on 230 volts. This causes the machine to have a higher synchronous impedance than it would have if it operated at the knee of the curve under normal voltage conditions.

Several sets of data were taken, and in each case the curves were all coincident with each other for the same value of field current; thus showing that the data was accurate.

The curve is plotted with live voltage as ordinate and field current as abscissal. Therefore in making calculations from the curve the voltage must be divided by the square root of three in order to get phase voltages.

Diagram of connections.



Shunt motor was directly connected to synchronous motor.

Zero Power Factor Curve.

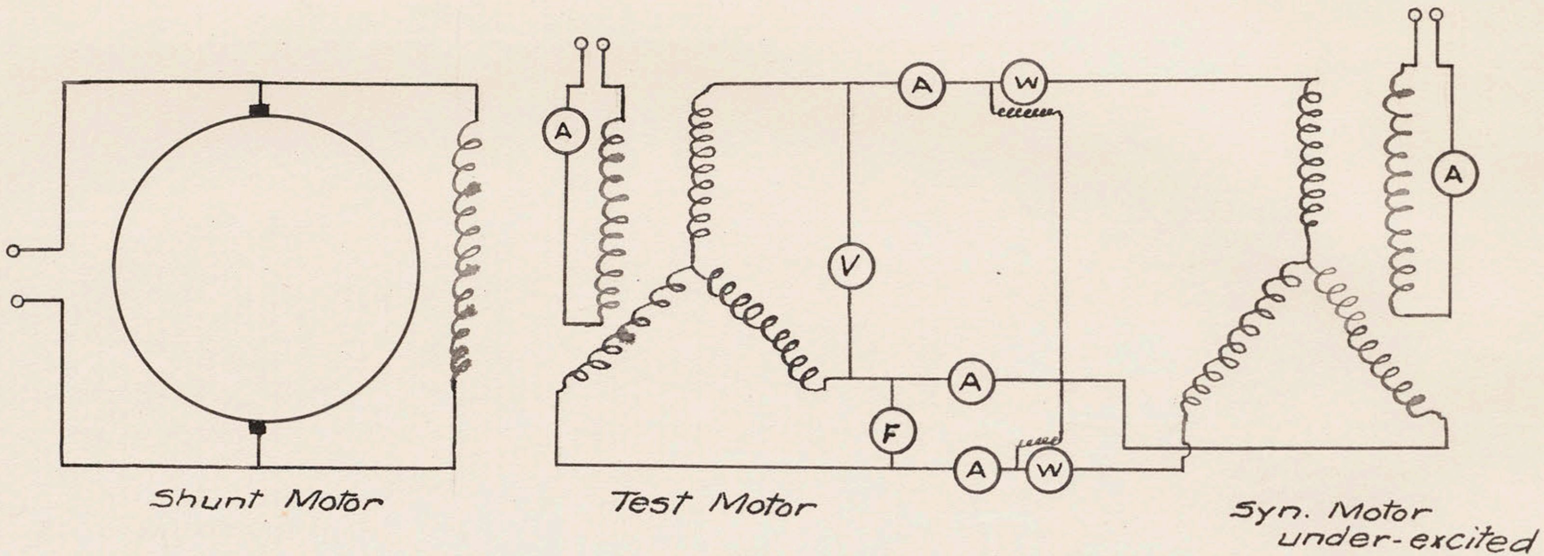
A zero power factor curve is a load saturation curve taken when an alterator is supplying a reactive load at a very low power factor. The curve has the same form as the no load saturation curve.

It is impossible to get absolutely zero power factor experimentally, but it can be approximated by using a synchronous motor as a load, rùning at no load with its field over excited.

The connections for this run are shown on the following page. The synchronous motor was driven by a shunt motor. Another synchronous motor was used as a load, and was over excited as stated above. The armature current was held constant and the field current of the motor acting as a synchronous generator was varied. Values of field current and terminal voltage were observed. The frequency was held constant at 60 cycles. The power out-put was also observed.

From this data the zero power factor curves could be plotted. Two curves were plotted, one for 32.11 amperes and one for 39.67 amperes armature current. The power factors were .0746 and .0605 respectively.

Diagram of Connections
for
Zero Power Factor Run



Short Circuit Curve.

A short circuit curve is a curve showing the relation between the field current of a synchronous machine and the armature current when the armature is short circuited.

The diagram of connections is shown on the following page.

The synchronous motor was driven as a generator by a shunt motor. The armature was short circuited through three ammeters, and the field current was varied in steps until about 1.20 full rated current was reached. The values of field current and armature current were observed. The short circuit curve was plotted from this data. The speed was held constant at 1800 R.P.M. (Synchronous Speed). The short circuit curve is a straight line over the range of saturation through which it is possible to carry it.

Core Loss, and F+W Run.

The core loss and friction and windage of the synchronous motor was found by driving it as a generator by the shunt motor. The diagram of connections is shown below.

First, drive the motor with its field and armature circuits open. The power input to the shunt motor minus the shunt motor stray power and armature copper loss give the friction and windage loss of the synchronous motor.

Secondly, drive the synchronous motor with the shunt motor at synchronous speed, and vary the field current of the synchronous machine, taking readings of shunt motor current and voltage, and synchronous motor field current. Then for any particular value of synchronous motor field current the core loss is equal to the input to the shunt motor minus the stray power of the shunt motor, the armature loss of the shunt motor, and the friction and windage of the synchronous motor.

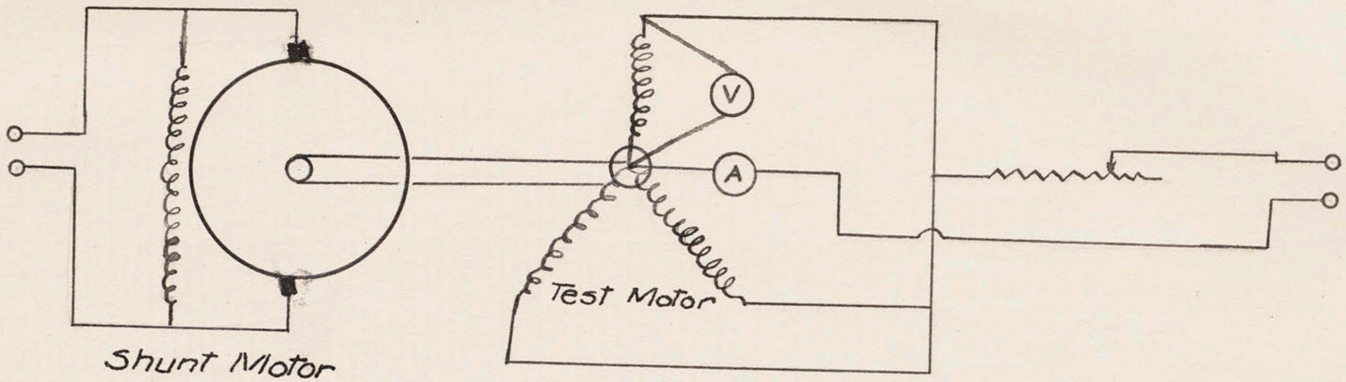
Method of Determining the Zero Sequence Impedance.

The zero sequence impedance was measured with the field of the motor rotating, and with it stationary. Average values were taken in each case from a number of observed readings.

The diagram of connections is shown on page 29a. The three armature terminals were connected together. A low single phase voltage was impressed across the neutral and the inter connection of the three phases. An ammeter was put in the line as shown and a voltmeter connected between the neutral and one line. The value of the current and voltage was varied in steps until the current reached about 56 amperes. From the readings of the voltmeter and ammeter the zero sequence impedance was obtained. $Z_o = \frac{V}{3I}$ where Zo is the zero sequence impedance per phase and V and I, the magnitude of the alternating current voltage and current respectively.

The voltmeter was connected alternately between first one line and the other and the neutral, in order to see if the zero sequence impedance was the same for all the phases. The rotor of the machine was also rotated by hand to see if there was any effect due to the position of the field. The values of Zo obtained in this way were averaged,

Diagram of Connections
for
Uniphase Impedance Run

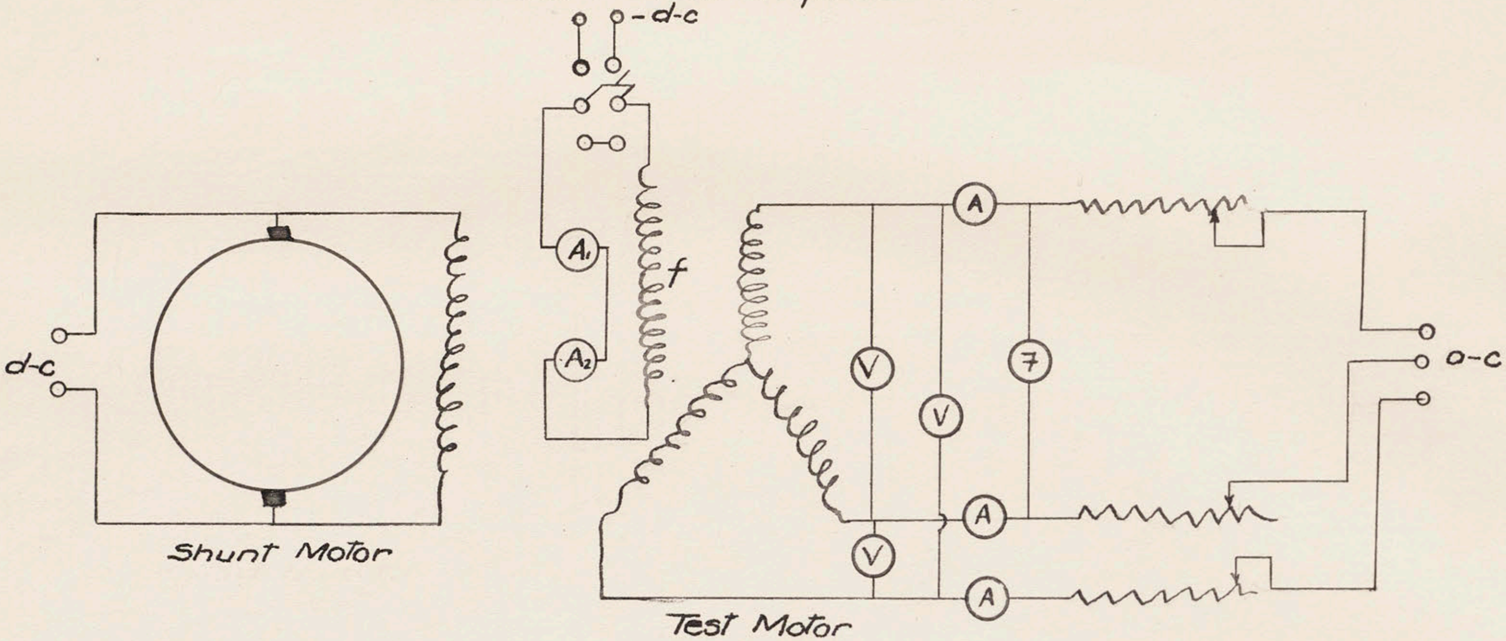


and this average compared with the average value of
Eo obtained when the motor was driven at synchronous
speed by a shunt motor.

Reverse Phase Impedance Run.

The connections for the reverse phase impedance run are shown on the following page. Three equal resistances were placed in each line in order to reduce the impressed voltage. The connections to the motor are such that it would run in its correct direction of rotation if allowed to operate as a motor. The shunt motor, which was directly connected to the synchronous motor, was connected so as to drive the synchronous motor backwards with respect to the impressed voltage across the three terminals. The speed was adjusted so that the frequency was at its rated value. A series of readings were taken for three separate cases. First; with the field circuit of the synchronous motor open, second, with the field circuit short circuited, and third, with a direct current in the field. An alternating current and a direct current ammeter was placed in the field circuit during the last run. It was found that the alternative current meter read a little more than the direct current meter, due to the presence of a second harmonic current induced in the field. As stated in the main part of this thesis no values of the reverse phase impedance were obtained except in the first case, that is with the field circuit open. The reverse phase impedance in this case is given by; $Zd = \frac{V}{3I}$ where V is the line voltage impressed, and I is the current flowing as measured by the three ammeters.

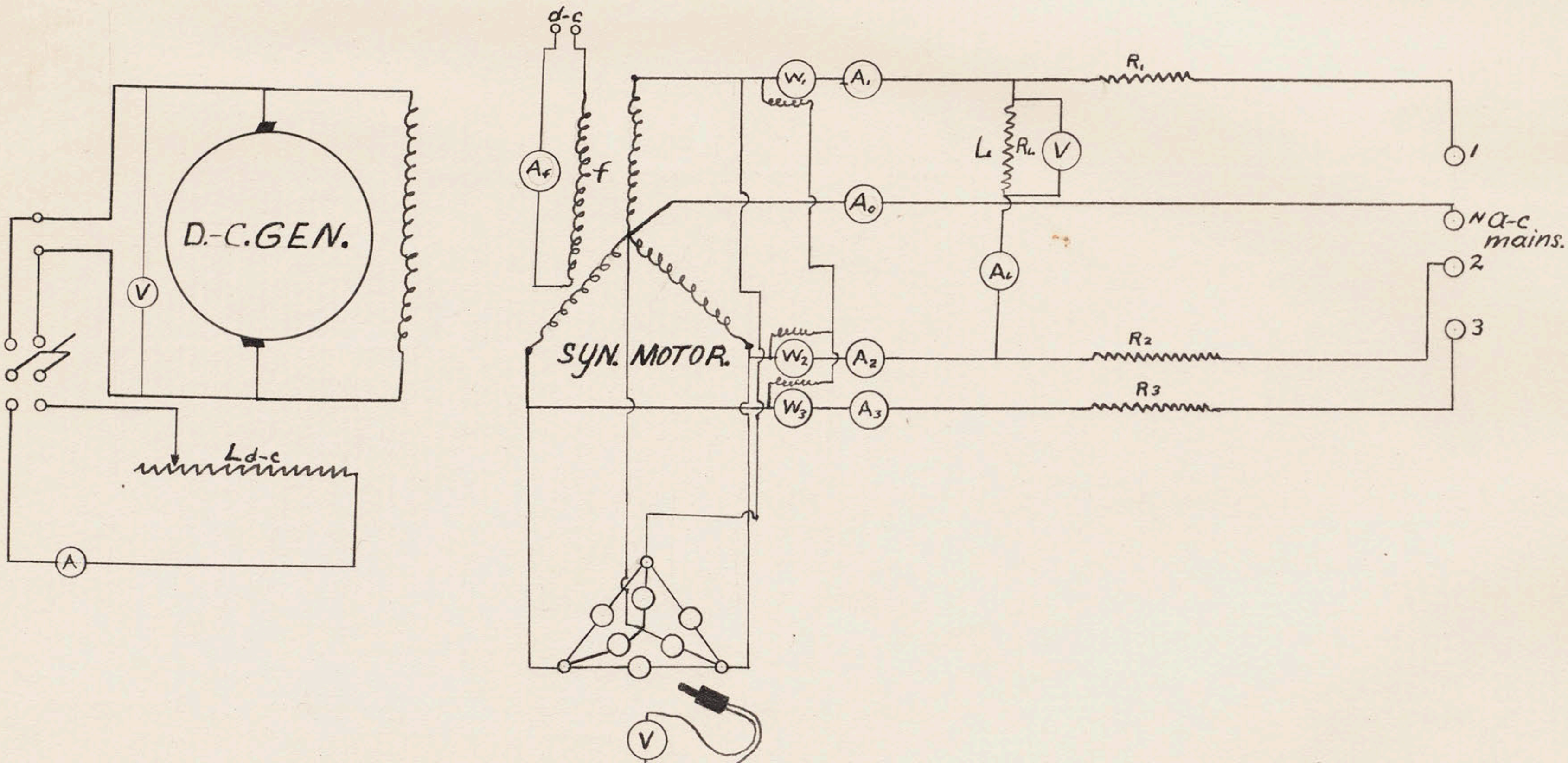
Diagram of Connections
for
Reverse Phase Impedance Run.



Unbalanced Run.

A carbon rheostat was placed in each of the three lines leading from the main panel to the auxilliary switchboard and a single phase load was put across one line. The complete diagram of connections for this run is shown on page . The voltage was reduced to about 200 volts between the lines 1 and 2. The direct current generator was used as a load. The generator was loaded by a resistance which could be varied. The load "L" across the two lines of the synchronous motor was a resistance known constants; R + Jo. The shunt generator was loaded in steps until one of the ammeters in the lines leading to the synchronous motor read full rated current. Readings of all the instruments were taken as near as possible at the same time. Also the angular displacement between the currents was found for each value of armature current as stated elsewhere in this thesis. The value of the three line resistances R_1, R_2, and R_3 were also measured.

DIAGRAM of CONNECTIONS
for
UNBALANCED RUN ON MOTOR.



An arrangement for measuring six Voltages With 1 Voltmeter.

Appendix B.

Ohmic Resistance Data

V_{12}	I_1	V_{23}	I_2	V_{31}	I_3
4.36	18.25	4.4	18.25	4.38	18.25
4.40	18.25	4.4	18.22	4.36	18.25
4.40	18.23	4.4	18.30	4.36	18.20
Ave: V_{12}	Ave. I_1	Ave. V_{23}	Ave. I_2	Ave. V_{31}	Ave. I_3
4.38	18.24	4.4	18.25	4.36	18.23

1.= .1200   2= .1204   3= .1195

Avepage value of V_{oh} per phase = .120 ohmis.

Short Circuit Run: Data for Effective Resistance.

Id-e	Vd-e	I	I'	I_2	I_3
6.40	225	.35	9.2	9.2	9.3
6.80	225	.47	12.1	12.1	12.1
7.20	226	.78	19.2	19.2	19.2
8.00	226	1.22	29.3	29.6	29.7
9.20	226	1.49	35.8	35.9	35.9
10.40	226	1.78	42.8	43.0	43.0

Speed; constant at 1800 V.P.M.

Values of Effective Resistance. Per Phase

.285

.276

.266

.142

.142

.139

Average value of Effective Resistance per phase.

Ve = .208 ohms per phase.

$$\text{Ratio of } \frac{V_e}{Voh} = 1.816$$

Data for Mag. Curve.

Field I	Line Volt; 1-2	Line volts 2-3	Line volts 3-1	Freq.
.75	67.5	67.5	67.5	60
.99	89.8	89.8	89.8	60
1.10	100.0	100.0	100.0	60
1.30	120.0	120.0	120.0	60
1.44	130.5	130.5	130.5	60
1.52	140.0	140.0	140.0	60
1.65	150.0	150.0	150.0	60
1.79	160.0	160.0	160.0	60
1.90	170.0	170.0	170.0	60
2.06	180.0	180.0	180.0	60
2.20	190.0	190.0	190.0	60
2.37	201.0	201.0	201.0	60
2.70	219.0	219.0	219.0	60
2.94	231.0	231.0	231.0	60
3.21	240.0	240.0	240.0	60
3.56	253.0	253.0	253.0	60
3.84	260.0	260.0	260.0	60
4.22	270.0	270.0	270.0	60
4.86	282.0	282.0	282.0	60
5.45	290.0	290.0	290.0	60
5.77	295.0	295.0	295.0	60

Short Circuit Curve, Data:

Line I.	Line I_2	Line I_3	Field I	Speed
18.2	18.5	18.3	.75	1800
20.8	21.2	20.8	.85	1800
26.6	27.0	26.6	1.11	1800
30.0	30.5	30.0	---	1800
36.2	36.8	36.0	1.15	1800
40.5	41.5	40.7	1.70	1800
45.5	46.7	45.8	1.90	1800
49.5	50.5	49.2	2.05	1800

Data for Zero Power Factor Curve. P.f. = .0605

Line V.	Field I.	I_1	I_2	I_3	Freg.
231.0	4.82	39.1	38.6	39.6	60
204.0	4.16	39.5	39.4	40.20	60
189.0		40.0	39.5	40.0	60
172.0	3.60	39.6	39.1	40.0	60
153.0	3.34	39.6	39.4	40.0	60
144.0	3.29	40.0	39.6	40.5	60

In put to Syn. Motor = 960 watts.

Average line current = 39.67 amperes.

Average power factor = $\frac{960}{3 \times 230 \times 39.67}$ = .0605

Data for Zero Power Factor Curve. I ave.= 32.11 P.f.=.0746

Line V.	Field I	I_1	I_2	I_3	Freg.
231	4.45	31.8	30.8	31.6	60
213	4.05	32.2	31.8	33.0	60
204	3.85	32.4	32.0	33.0	60
189	3.55	32.0	31.6	32.5	60
170	3.25	32.2	31.9	32.8	60
150	3.00	31.8	31.6	32.4	60
130	2.72	32.0	32.6	31.8	60
124	2.68	32.3	32.2	32.6	60

Input to Syn. Motor = 960 watts.

Average Line Current = 32.11

Average Power Factor = $\frac{960}{3 \times 32.11 \times 230}$ = .0746

Direct Phase Impedance

Short Circuit Method.

$Z_s = 2.31$

$Z_s = 2.81$

$Z_s = 2.85$

$Z_s = 2.74$

$Z_s = 2.58$

$Z_s = 2.54$

Average value of $Z_s = 2.64$ ohms per phase.

Zero power factor Method.

For 39.67 amperes at power factor of .0605

$Z_s = .736$ ohms per phase.

For 32.11 amperes at power factor of .0746

$Z_s = .793$ ohms per phase.

Ratio of Synchronous Impedance from Short Circuit Method to Sunchronous impedance from Zero power factor Method $= \dfrac{2.64}{.736} = 3.58$

Data for Zero Sequence Impedance.

I_1	V_1	I_2	V_2
33.8	1.86	24.3	1.26
37.6	2.22	29.0	1.55
43.2	2.40	33.4	1.85
48.0	2.66	38.2	2.10
52.4	2.90	42.6	2.33
56.8	3.15	47.2	2.55
		52.0	2.80
		56.2	3.05

(1) This data was taken with the armature of the synchronous motor at standstill:

(2) This data was taken with the armature of the synchronous motor rotating at synchronous speed.

Calculated Values of Zero Sequence Impedance.

1	2
.0183	.0172
.0193	.0172
.0185	.0184
.0185	.0183
.0185	.0182
.0185	.0180
	.0179
	.0181

Average values of Z_o

(1) Z_o = .0186 ohms per phase

(2) Z_o = .0179 " " "

Reverse Phase Run: Field Circuit Open:

I_1	I_2	I_3	V_{12}	V_{23}	V_3	Freq.
8.4	8.2	8.3	24.2	24.7	24.4	59
11.5	12.0	12.8	36.7	38.5	36.0	59
19.5	20.9	20.6	56.0	57.5	55.5	59
24.8	25.9	25.7	66.5	67.5	66.0	59
29.5	29.1	29.4	73.2	75.0	75.0	59
34.7	35.2	35.9	81.0	82.5	82.5	59
39.6	39.9	38.7	88.0	89.4	89.5	59
42.2	42.4	41.0	91.5	92.0	93.0	59
47.9	47.9	46.2	98.2	99.0	99.5	59

Reverse Phase Run, with Voltmeter Across Field

V	I_1	I_2	I_3	V	Freq.
14.0	5.0	5.0	5	13.5	58.9
27.5	10.1	10.1	10.0	57.5	58.6
40.0	14.0	14.1	14.2	123.0	58.7
51.5	18.9	19.0	18.9	300.0	58.9
63.2	23.2	23.6	23.5	444.0	58.9
73.0	26.6	27.0	27.0	495.0	58.9
82.8	30.4	30.8	30.6	580.0	58.9

Data For Reverse Phase Impedance: Field Short Circuited.

V_{12}	V_{23}	V_{31}	I_1	I_2	I_3	I	Freq.
8.3	8.8	8.4	8.2	8.2	8.1	.195	59
12.3	14.7	12.75	11.5	12.0	12.8	.290	59
19.7	21.8	20.3	19.5	20.9	25.7	.470	59
24.3	26.0	25.0	24.8	25.9	25.7	.590	59
27.5	28.6	29.5	29.6	30.3	29.0	.694	59
32.2	33.0	34.0	34.7	35.2	35.9	.815	59
36.5	37.0	38.3	39.6	39.9	38.7	.934	59
38.0	38.1	40.6	42.2	42.4	41.0	.995	59
44.0	44.0	45.2	47.9	47.9	40.2	1.140	59

Data For Reverse Phase Impedance: Field Current On.

V_{12}	V_{23}	V_{31}	I_1	I_2	I_3	I	I_{a-e}	Freq.
23.0	23.2	23.5	5.8	5.6	5.5	.25	.286	59.5
24.5	24.5	25.5	9.2	10.4	10.6	.25	.344	59.5
30.0	31.5	30.5	19.8	21.0	21.0	.25	.500	59.5
32.5	35.0	33.0	25.0	26.8	26.6	.25	.642	59.5
36.5	38.5	36.0	29.2	29.8	29.4	.25	.750	59.5
43.0	45.0	43.0	36.8	37.8	37.8	.25	.850	59.5
49.0	49.0	48.0	45.0	45.4	45.0	.25	1.050	59.5

Data For Reverse Phase Impedance With A Larger Field Current.

V_{12}	V_{23}	V_{31}	I_1	I_2	I_3	I	Speed.
220	219	220	8.0	7.7	7.8	2.96	1620
205	206	205	15.1	14.7	14.7	2.91	1620
202	202	204	22.0	21.9	22.0	2.9	1620
194	195	196	29.2	28.6	28.6	2.88	1620
192	196	193	36.4	36.0	35.6	2.90	1620
190	192	191	40.0	40.0	39.2	2.88	1620

Values of Zr; ohms per phase.

1.715	1.650	1.420	1.385
1.750	1.737	1.502	1.571
1.600	1.625	1.537	1.638
1.520	1.510	1.590	1.570
1.450	1.535	1.576	1.590
1.345	1.365	1.575	1.570
1.302	1.270		1.564
1.266			1.570
1.209			1.523
			1.523
			1.500
Averages			
1.492	1.527	1.533	1.546

Corrected Averages; for Saturation

$\frac{1.492}{3.58} = .404 \quad \frac{1.527}{3.58} = .426 \quad \frac{1.533}{3.58} = .428 \quad \frac{1.546}{3.58} = .432$

Average of Corrected Averages:

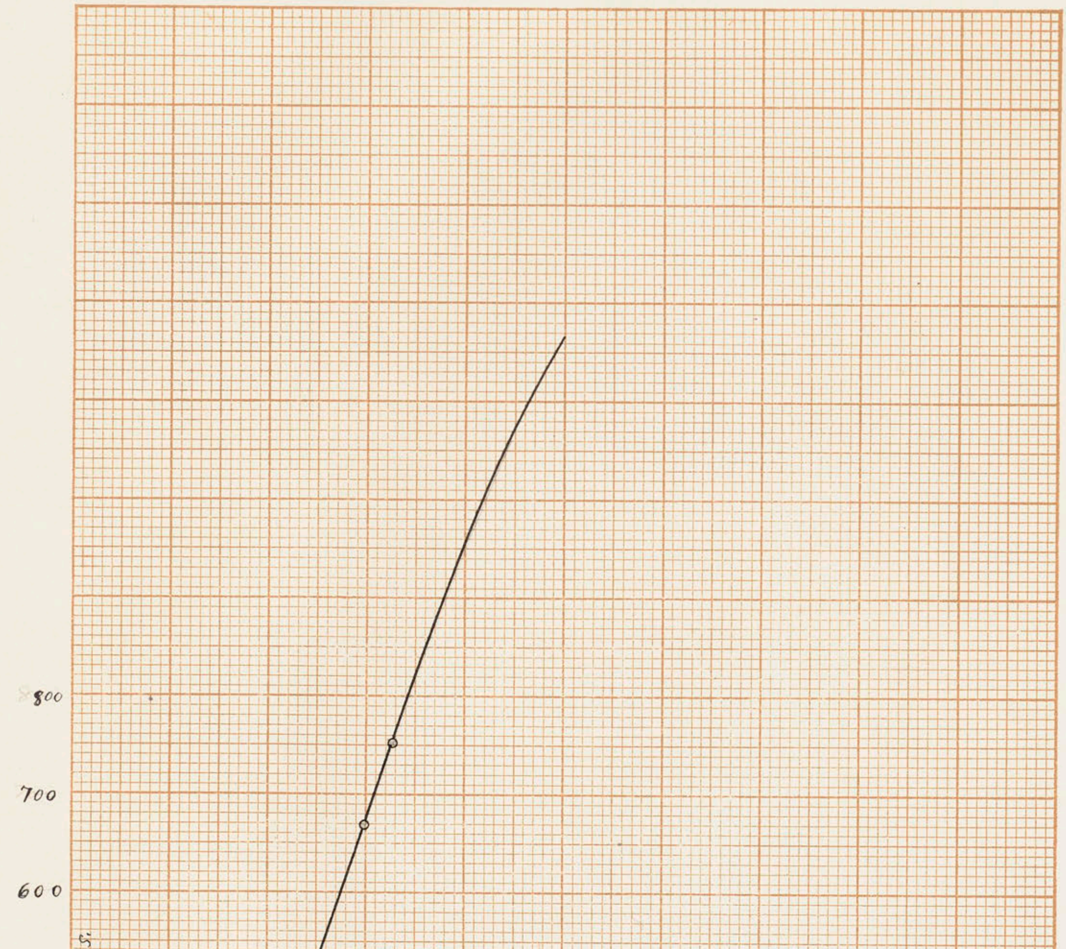
$$\frac{.404 \quad .426 \quad .428 \quad .432}{4} = .4225 \text{ ohms.}$$

Data for Care loss curve:

Line V.	Field I	D-c Motor V.	D-c Motor I	Freq.
0.0	0.00	224	5.6	60
108.0	1.15	225	6.0	60
149.0	1.59	225	6.7	60
180.0	2.00	225	7.4	60
215.0	2.54	224	8.0	60
230.0	2.80	225	8.6	60
260.0	3.27	225	9.0	60

Calculated care loss.

Pc watts	I
94	1.15
250	1.59
402	2.00
527	2.54
672	2.80
758	3.27


800
700
600
500
400
300
200
100
0
Core Loss in watts.
Core Loss curve for Syn. motor.
Field current
5
1
2
3
4

Data for unbalanced Run.

W_1	W_2	W_3	I_1	I_2	I_3	I_o	I_{d-c}	V_{d-c}	I_f
-11.4	1.8	8.8	23.5	28.0	17.0	37.5	0	231	2.9
3.0	2.8	7.7	17.6	40.0	32.0	37.5	21	202	2.9
2.2	3.1	9.2	21.6	48.0	40.0	39.0	30	181	2.9

V_1	V_2	V_3	V_{01}	V_{02}	V_{03}	I_L	V_L	W_1'	W_2'
225	228	208	132.5	123	125	51	221	17.2	-1.4
221	223	203	132	120	123	66.7	203	4.0	-265
221	221	196	131	188	120	80.0	196	2.2	-2.9

W_3'	V	I_1'	I_2'	I_3'	F
-.7	210	23.5	24	17	59
1.0	203	17.6	40	32	59
1.5	197	21.6	48	40	59

Resistances R_1, R_2, R_3;

V	I	R_1	V	I	R_2	V	I	R_3	V_L
20	35	.572	8.15	37	.22	1.7	38.1	.0458	221

I_L	R_L
51.	4.34

Calculated data for unbalanced Run.

Angle between currents and components of currents

Data No. I.

$$\cos \theta_1 = \frac{3440}{23.5 \times 210} = .597; \ \theta_1 = 45.8^\circ$$

$$\cos \theta_2 = \frac{-1400}{28 \times 210} = -.238; \ \theta_2 = 76.2^\circ$$

$$\cos \theta_3 = \frac{140}{17 \times 210} = .0392 \quad \theta_3 = 87.8^\circ$$

$I_1 = 16.35 + J16.8$

$I_2 = 6.67 - J25.5$

$I_3 = .89 + J17$

$I_o = 22.13 - J25.5 = 33.6 \ \overline{|49^\circ}$

$I_u = 1/3 \ (22.13 - J25.5) = 7.34 - J8.4$

$I_{r1} = 19.73 + J.3$

$I_{d1} = 3.15 + J25.1$

Data II.

$$\cos \theta_1 = \frac{1600}{17.6 \times 203} = .447; \ \theta_1 = 63.4^\circ$$

$$\cos \theta_2 = \frac{-2650}{40 \times 203} = -.327 \ \theta_2 = -70.9^\circ$$

$$\cos \theta_3 = \frac{400}{32 \times 203} = .0616 \ \#\theta_3 = 86.45^\circ$$

$I_1 = 7.9 + J15.7$

$I_2 = 13.1 - J37.8$

$I_3 = 2.01 + J31.8^\circ$

$I_o = 19 + J9.7 = 21.4 \ \underline{|27.1^\circ}$

$I_u = 6.33 + 3.23$

Calculated data for Unbalanced Run (con.)

Data II

$$\cos \theta_1 = \frac{880}{21.6 \times 196} = .2075; \quad \theta_1 = 78^\circ$$

$$\cos \theta_2 = \frac{-7900}{48 \times 196} = .3085; \quad \theta_2 = 72^\circ$$

$$\cos \theta_3 = \frac{-600}{40 \times 196} = .0765 \quad \theta_3 = 85.6^\circ$$

$I_1 = 4.48 + J21$

$I_2 = -15.2 + J46.6$

$I_3 = -3.14 - J39.9$

$I_o = -13.26 + 27.5$

$= 31 \angle 64^\circ$

$I_u = -4.42 + J9.16$

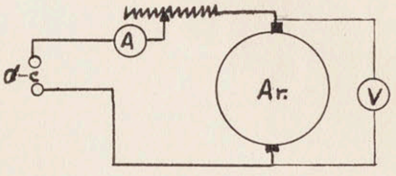
$I_{r1} = 48.81 + J15.61$

$I_{d1} = -21.9 + J3.95$

Appendix C.

Armature Resistance

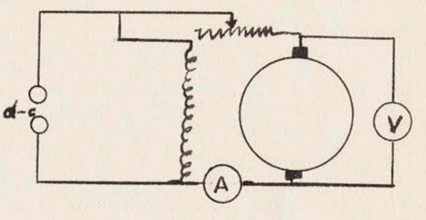
The armature resistance was measured by the drop of potential method. The diagram of connections is shown in the accompaning figure. A resistance was put in series with the armature to reduce the voltage impressed. A voltmeter and ammeter were placed as shown in the diagram.



Several readings were taken on each instrument, for different positions of the armature. The resistance is equal to the ratio of V to I.

Stray Power

The stray power of a direct current dynamo depends only on the speed and flux at which the dynamo operates. The diagram of connections for determining the stray power is shown in the accompaning figure. The resistance is used to vary the voltage on the armature, thereby varying the speed.



The input to the motor, at any speed is VI, and the stray power is $P_{sp} = VI_a - (I_a)^2 V_a$. where I_a is the armature current, V is the voltage across the armature measured as shown, and va is the resistance of the armature.

Core Loss

The core loss was obtained by driving the shunt motor by the synchronous motor, and measuring the input to the synchronous motor for different values of d-c motor voltage. With the direct current motor field open, the power input to the synchronous motor is equal to the friction and windage losses of the two machines, and I^2R_a loss and core loss of the synchronous motor. Then for any particular value of d-c voltage, the core loss of the d-c machine is equal to the input to the synchronous motor minus the sum of the power input to the synchronous motor and its armature copper loss.

The stray power curve and core loss curve for the d-c machine are plotted on page 49a.

Data for Shunt Motor

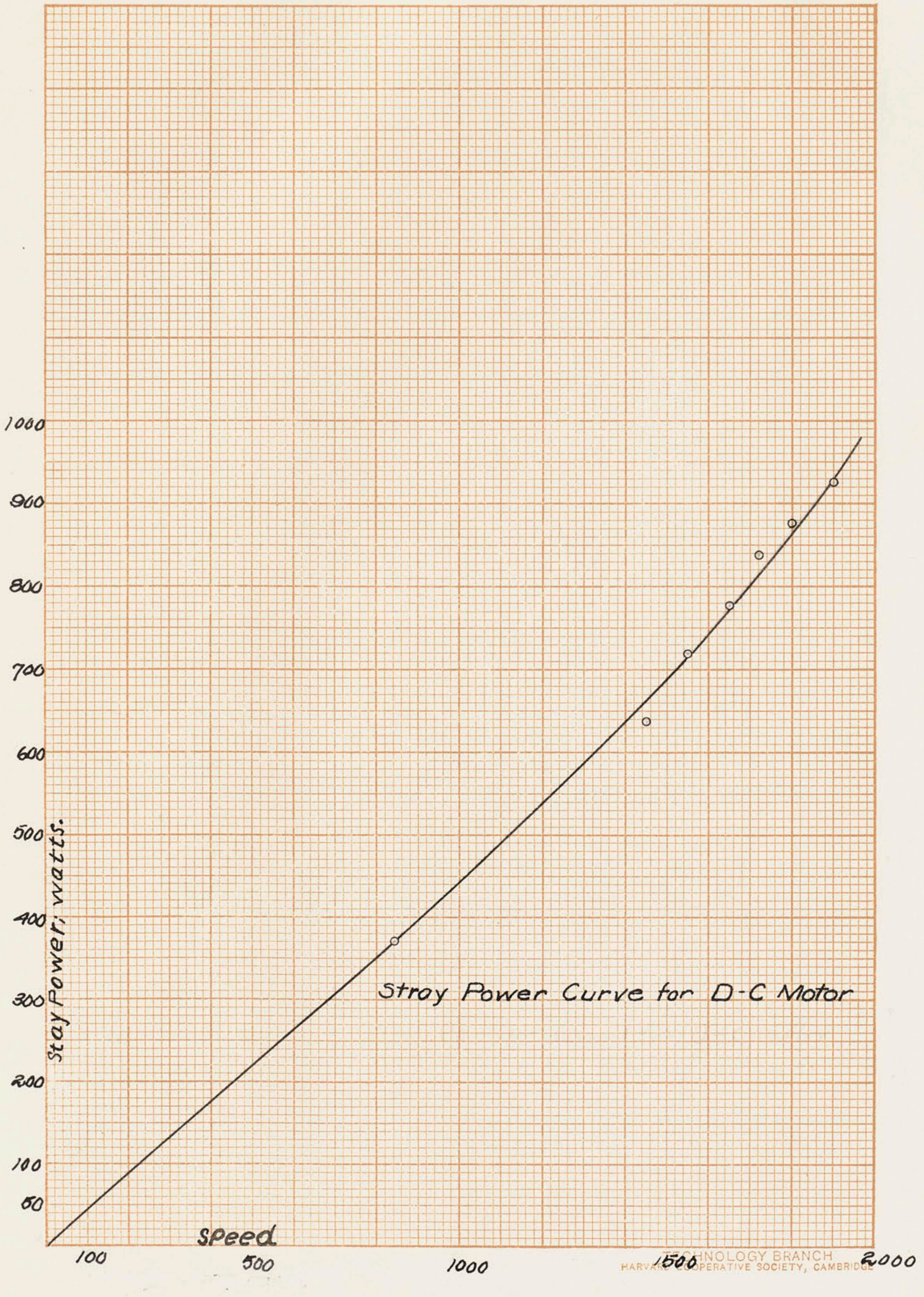
Armature Current	Resistance Voltage	Stray Power Data Current	Voltage	Speed
35.8	11.6	4.10	227	1900
35.9	11.6	4.10	220	1850
35.8	11.7	4.10	214	1800
35.8	11.7	4.10	206	1710
		4.00	195	1650
		4.00	182	1550
		4.10	157	1450
		3.50	107	870

Calculated values of Resistance and Stray power

Resistance	Stray power
.324	925
.323	896
.327	873
.327	839
	775
Average	723
	639
.326	370

Core Loss Data

V	W_1	W_2	W_3	Calculated values of core loss.
0	560	220	580	0
168	620	300	620	180
175	660	300	600	260
215	700	380	600	320
222	760	400	620	420
244	800	400	620	580


Stray Power Curve for D-C Motor
Stay Power; watts.
1000
900
800
700
600
500
400
300
200
100
50
Speed
100
500
1000
1500
2000
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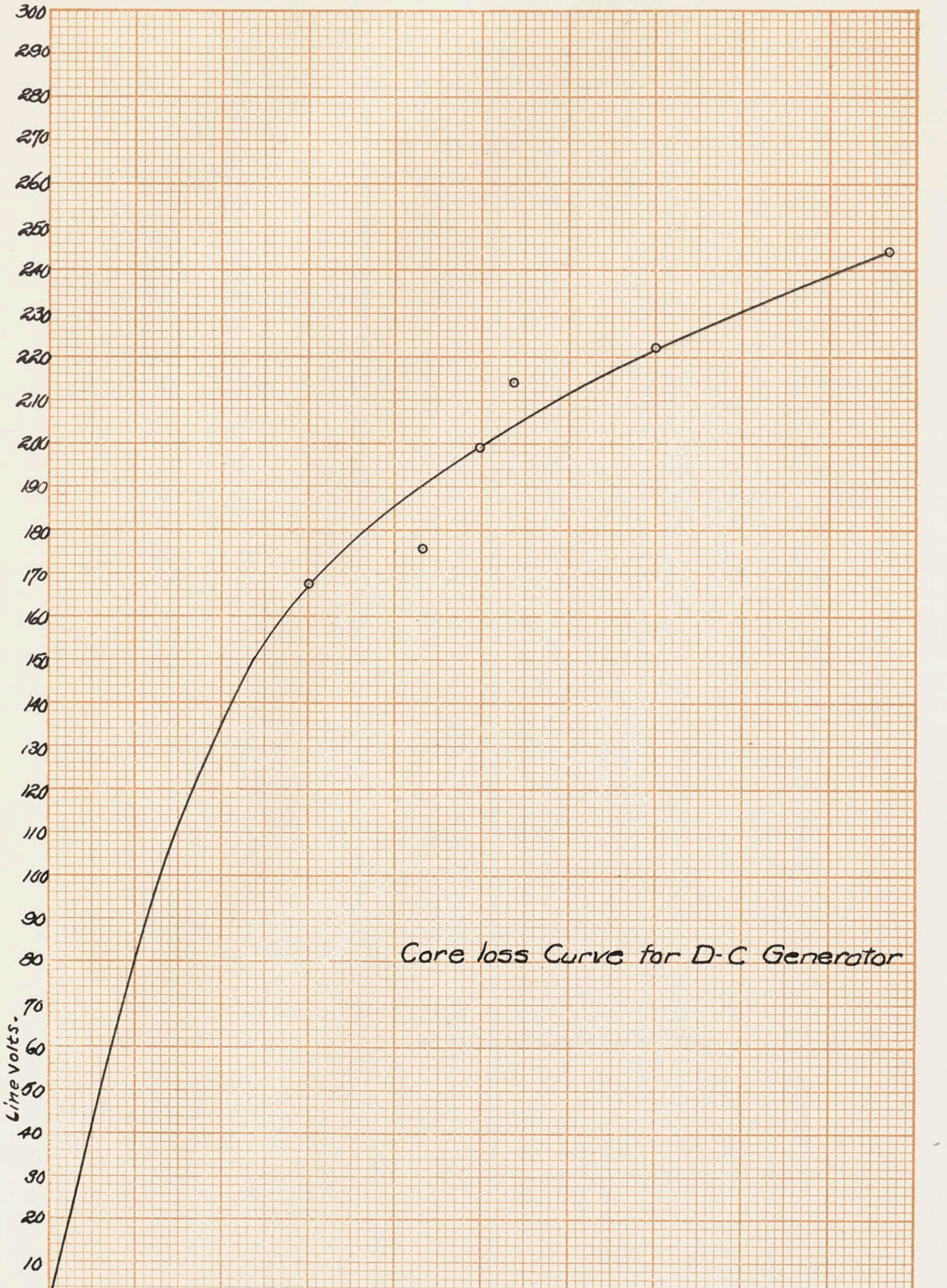
Core loss Curve for D-C Generator
Line volts.
Core Loss; in watts.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
270
280
290
300
30
60
80
120
150
180
210
240
270
300
330
360
390
420
450
480
510
540
570
600

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