Financial Fragility with SAM?

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ABSTRACT

Shared Appreciation Mortgages feature mortgage payments that adjust with house prices. They are designed to stave off borrower default by providing payment relief when house prices fall. Some argue that SAMs may help prevent the next foreclosure crisis. However, home owners' gains from payment relief are mortgage lenders' losses. A general equilibrium model in which financial intermediaries channel savings from saver to borrower households shows that indexation of mortgage payments to aggregate house prices increases financial fragility, reduces risk-sharing, and leads to expensive financial sector bailouts. In contrast, indexation to local house prices reduces financial fragility and improves risk-sharing.

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The $10 trillion market in U.S. mortgage debt is the world's largest consumer debt market and second largest fixed income market. Mortgages are not only the largest liability for U.S. households, but also the largest asset of the U.S. financial sector.[1] Given the heavy exposure of the financial sector to mortgages, large house price declines and the default waves that accompany them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the financial crisis of 2008 to 2011, as U.S. house prices fell by 30% nationwide, and by much more in some regions, pushing roughly 25% of U.S. home owners underwater by 2010 and leading to seven million foreclosures. Large losses on real estate loans caused several U.S. banks to collapse during the crisis, while the stress to surviving banks' balance sheets led them to dramatically tighten mortgage lending standards, precluding many home owners from refinancing into lower interest rates.[2] Homeowners' reduced ability to tap into their housing wealth short-circuited the stimulative consumption response from lower mortgage rates that policy makers had hoped for.

This experience led economists and policy makers to ask whether a different mortgage finance system would result in a better risk-sharing arrangement between borrowers and lenders.[3] While contracts offering alternative allocations of interest rate risk are already widely available — most notably, the adjustable rate mortgage (ARM), which offers nearly perfect pass-through of interest rates — contracts offering alternative divisions of *house price* risk are still rare. Recently, however, some fintech lenders have begun to offer such contracts,

[1]Banks and credit unions hold $3 trillion in mortgage loans directly on their balance sheets in the form of whole loans, and an additional $2.2 trillion in the form of MBSs. Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITs in the definition of the financial sector adds another $1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and government-sponsored enterprise (GSE) portfolios adds a further $2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

[2]Charge-off rates of residential real estate loans at U.S. banks went from 0.1% in mid-2006 to 2.8% in mid-2009, returning to their initial value only in mid-2016.

[3]The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015.

most prominently the shared appreciation mortgage (SAM), which indexes mortgage payments to house price changes.[4]

A SAM contract ensures that a borrower receives payment relief in bad states of the world, potentially reducing mortgage defaults and the associated deadweight losses to society. However, SAMs impose losses on mortgage lenders in these adverse aggregate states, which may increase financial fragility at inopportune times. Our paper is the first to study how SAM contracts affect the allocation of house price risk between mortgage borrowers, financial intermediaries, and savers in a general equilibrium framework, and to propose a shift in the mortgage design literature from a focus on *household risk management* to one on *system-wide risk management*. The main goal of this paper is to quantitatively assess whether SAMs present a better arrangement to the overall economy than standard fixed-rate mortgages (FRMs).

We begin with a rich baseline model in which mortgage borrowers obtain long-term, defaultable, prepayable, nominal mortgages from financial intermediaries. These intermediaries are financed with short-term deposits raised from savers and equity raised from their shareholders, subject to realistic capital requirements, and are bailed out by the government in the event of insolvency. Borrowers face idiosyncratic house valuation shocks while banks face idiosyncratic profit shocks, which influence their respective optimal default decisions. We solve the model using a state-of-the-art global nonlinear solution technique that allows for occasionally binding constraints.

To evaluate the mortgage system's resilience to adverse scenarios, our model economy transits between a normal state and a housing-crash state featuring high house price uncertainty and a decrease in aggregate home values, in addition to aggregate business-cycle income risk. With FRMs, the arrival of a housing-crash state leads to higher rates of bor-

[4]Examples of startups in this space are Unison Home Ownership Investors, Point Digital Finance, Own Home Finance, and Patch Homes. In addition, similar contracts have been offered to faculty at Stanford University for leasehold purchases over the past 15 years (Landvoigt, Piazzesi, and Schneider, 2014).

rower defaults, bank losses, and failures, along with large decreases in borrower consumption as the financial sector contracts.[5]

To study the impact of alternative mortgage contracts, we consider SAM economies in which mortgage payments are indexed to either aggregate house prices or local house prices. We contrast the effects of alternative schemes on the model's key externalities: the deadweight losses and risk-sharing consequences of borrower and bank default. Our main result is that indexation to aggregate (national) house prices reduces borrower welfare even though it slightly reduces mortgage defaults, due to a severe increase in financial fragility. These contracts lead mortgage lenders to absorb aggregate house price declines, which causes a wave of bank failures and triggers bailouts ultimately funded by taxpayers, including the borrowers. Equilibrium house prices are lower and decrease more in crises due to higher mortgage spreads as credit supply contracts. Ironically, while overall welfare declines, intermediary welfare increases as banks enjoy large gains from increased mortgage payments in housing expansions and can charge higher mortgage spreads in a riskier financial system.

In sharp contrast, indexation of mortgage payments to the local component of house price risk only can eliminate up to half of mortgage defaults while reducing systemic risk. Banks' geographically diversified portfolios of SAMs allow them to offset the cost of debt forgiveness in areas where house prices fall by collecting higher mortgage payments in areas where house prices rise. Lower mortgage defaults substantially reduce bank failures and dampen fluctuations in intermediary net worth, stabilizing the financial system and reducing deadweight losses. Banking becomes safer, but also less profitable, due to a fall in mortgage spreads and in the value of the bailout option. As a result, the welfare of borrowers and savers rises, at the expense of bank owners. Overall welfare increases. The empirically relevant case, which we label regional indexation, combines aggregate and local indexation and generates modest welfare benefits for the economy.

[5]Throughout the paper, we use "default" to refer to permanent nonpayment or foreclosure, rather than late payment or delinquency.

Last, we examine the consequences of several alternative SAM implementations. Indexing interest payments only—which are fixed only until the next borrower mortgage transaction—has much weaker effects than indexing principal. Asymmetric indexation, which allows payments to fall but never to rise, dramatically decreases mortgage default rates, but does so by shrinking average household leverage rather than by improving risk-sharing, while also leading to major financial fragility. In robustness analysis, we show that our results continue to hold when savers cannot hold any mortgage debt directly, when we vary the risk absorption capacity of the intermediaries, when indexation is partial, and when bank bailouts are financed with government debt rather than instantaneous taxation. Our results imply that macro-financial considerations should play an important role in the design of mortgage contracts indexed to house prices.

Literature Review. This paper contributes to the literature that studies innovative mortgage contracts. Shiller and Weiss (1999) are the first to discuss the idea of home equity insurance policies. SAMs were first considered in detail by Caplin, Chan, Freeman, and Tracy (1997); Caplin, Carr, Pollock, and Tong (2007); Caplin, Cunningham, Engler, and Pollock (2008). They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but also a useful tool to prevent a mortgage crisis in the first place. More recently, Mian and Sufi (2014) propose Shared Responsibility Mortgages (SRMs), mortgages whose payments fall when the local house price index goes down, and return to the initial payment upon recovery, with lenders receiving a share of home value appreciation upon sale. Mian and Sufi (2014) argue that foreclosure avoidance raises house prices, shares wealth losses more equitably between borrowers and lenders, and boosts borrower spending and aggregate consumption after house price declines. Posner and Zingales (2009) propose allowing mortgagors to swap debt for equity following the 2008 housing crash, which would have created SAMs as a form of ex-post modification. We build on this literature through our analysis of intermediary and financial risk, which interacts with the borrower balance sheet risk discussed in these studies.

Kung (2015) studies the effect of the disappearance of nonagency mortgages for house prices, mortgage rates, and default rates in an industrial organization model of the Los Angeles housing market. While not the emphasis of his work, he also evaluates the hypothetical introduction of SAMs in the 2003 to 2007 period and finds that SAMs would have enjoyed substantial uptake, partially supplanting nonagency loans, but would have further exacerbated the boom and would not have mitigated the bust. Piskorski and Tchistyi (2018) also study mortgage design in a risk-neutral environment. They emphasize asymmetric information about home values between borrowers and lenders and derive the optimal mortgage contract. The latter takes the form of a Home Equity Insurance Mortgage that eliminates the strategic default option and insures borrowers' home equity. Relative to these papers, we provide a quantitative equilibrium model of the entire U.S. housing market, with risk-averse lenders and with endogenously determined risk-free rate and mortgage risk premium. Our emphasis on imperfect risk-sharing and financial fragility complements their approach.

Our paper is distinct from prior quantitative literature on mortgage design in that we study the endogenous interaction of contract design and intermediary risk-bearing capacity in general equilibrium. Guren, Krishnamurthy, and McQuade (2018) and Campbell, Clara, and Cocco (2018) investigate the interaction of ARM and FRM contracts with monetary policy. These authors focus on interest rate risk, contrasting, for example, ARMs and FRMs, as well as novel contracts with various forms of optionality.[6] Both papers model a rich borrower risk profile that includes a life cycle and uninsurable idiosyncratic income risk. Perhaps because interest rate risk is easier for banks to hedge than house price risk, these

[6]Related work on contract schemes other than house price indexation include Piskorski and Tchistyi (2011), who study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract; Kalotay (2015), who considers automatically refinancing mortgages or ratchet mortgages (whose interest rate only adjusts down); and Eberly and Krishnamurthy (2014), who propose a mortgage contract that automatically refinances from a FRM into an ARM, even when the loan is underwater.

papers abstract from implications for financial fragility and use exogenous lender SDFs to price loans. In contrast, our framework studies the impact on financial fragility of changing banks' contractual exposure to house price risk that is difficult to hedge. As a result, our model emphasizes a rich intermediation sector with capital requirements, bank failures, and bailouts while featuring a much simpler borrower sector. We see these approaches as highly complementary.

More generally, our paper connects to the quantitative macro-housing literature, providing a novel and tractable general equilibrium setting for analyzing the interaction between the housing and financial sectors.[7] Our paper also contributes to the literature on the amplification of business cycle shocks provided by credit frictions, focusing specifically on key features of the mortgage market.[8] Finally, we provide a general equilibrium counterpart to recent empirical work that finds strong responses of consumption and default rates to changes in mortgage interest rates and house prices.[9]

[7] Elenev, Landvoigt, and Van Nieuwerburgh (2016) study the role the role of default insurance provided by the GSEs. Gete and Zecchetto (2018) study the redistributive role of the Federal Housing Agency. Greenwald (2018) studies the interaction between payment-to-income and loan-to-value constraints in a model of monetary shock transmission through the mortgage market, but without default. Favilukis, Ludvigson, and Van Nieuwerburgh (2017) study the role of relaxed down payment constraints in explaining the house price boom. Corbae and Quintin (2015) investigate the effect of risky mortgage innovation in a general equilibrium model with default. Guren and McQuade (2017) study the interaction of foreclosures and house prices in a model with search.

[8] See, for example, Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Kiyotaki and Moore (1997), and Gertler and Karadi (2011). A second generation of models has added nonlinear dynamics and a richer financial sector. See, for example, Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2013, 2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2017), and Elenev, Landvoigt, and Van Nieuwerburgh (2018).

[9] See, for example, Mian and Sufi (2009); Mian, Rao, and Sufi (2013), Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), Fuster and Willen (2015).

Overview. The rest of the paper proceeds as follows. Section I presents empirical facts motivating our analysis. Section II presents the theoretical model, while Section III discusses its calibration. Section IV presents a baseline set of results for the FRM economy without indexation, while Section V contains our main results on mortgage indexation. Extensions and robustness checks are presented in Section VI. Section VII concludes. The Appendix derives the model's equilibrium conditions, while additional information on the computational solution, model extensions, and empirics can be found in the Internet Appendix.[10]

I. Motivating Empirical Evidence

This section presents motivating empirical evidence for the model that follows. We combine house price data from the Federal Housing Finance Agency with loan performance data from Freddie Mac to create a quarterly panel at the ZIP-3 level. We present three main facts, illustrated by the plots in Figure 1.

First, house price growth is a key determinant of mortgage defaults and loan losses. Figure 1 Panel A displays coefficient estimates from a binned regression of the loan loss rate (the ratio of total loan losses to original principal balance) on each loan's three-digit ZIP-level house price growth over the five years following origination. The estimates show that loan losses are near zero in areas that experience positive house price growth, but are steeply increasing as house prices decline, reaching 15% for areas that experience house price declines of 50% or more. This link is important to establish in light of findings by Ganong and Noel (2019b) and others demonstrating that liquidity shocks are a key determinant of mortgage default.[11]

[10] The Internet Appendix is available in the online version of the article on the Journal of Finance website.

[11] An extension of our model that allows for double-trigger default (Internet Appendix Section III) is able to reconcile these findings because, while negative liquidity events may be necessary for default, they are not sufficient. Since borrowers with positive home equity can choose to sell the property rather than enter foreclosure, while underwater borrowers cannot, we still observe a strong link between household leverage

[Figure 1 about here.]

Second, house price growth at the local level is also a key determinant of delinquency and lender losses. This is not obvious, since the total house price growth used in Figure 1 Panel A may be correlated with national economic conditions. For example, many of the largest losses occurred during the housing bust period, in the wake of a major financial crisis and recession. To control for this possibility, Figure 1 Panel B shows a similar binned regression using the *relative* house price growth after removing the national average, and controlling for time effects, to absorb the influence of the national environment on losses. The estimates based on local variation are nearly identical to our original estimates, providing evidence that it is indeed house prices, not confounding national conditions, that drive our findings.

Third, we verify that local house price growth explains much of the variation in outcomes during the recent housing bust, an episode of particular significance to advocates of mortgage indexation. To do so, we restrict our sample to the 2007 vintage of loans — the worst performing vintage in our data. By construction, these loans experienced close to identical national conditions. Figure 1 Panel C shows that the link between local house price variation and losses is similar during the crisis period, increasing with the size of the loss, and approaching 15% for the worst-performing areas. Last, Figure 1 Panel D compares the house price growth histograms for losses and repaid balances from the 2007 vintage and shows that mortgage loan losses are heavily concentrated in areas that experience average house price declines of 35% or more. Indeed, 77% of losses for this vintage occurred in areas that experienced house price growth below the national average.

Figures IA.3 and IA.4 in Internet Appendix Section IV show that nearly identical patterns hold for delinquency and foreclosure rates. In sum, these results highlight that both national and local house price dynamics are key drivers of mortgage borrower and lender outcomes.

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and default.

II. Model

A. Overview

The model is designed to study how mortgage risk is shared in society. We set up a model of incomplete risk-sharing between three types of agents: mortgage borrowers (denoted B), savers (denoted S), and intermediaries (denoted I). Figure 2 depicts the model structure.

Savers are relatively patient — hence the saver label — and can invest in both safe assets and risky mortgage debt. Impatient borrowers want to take on long-term mortgage debt. A key friction in our model is that savers have a comparative disadvantage in holding mortgage assets. This creates a role for an intermediation sector with expertise in evaluating mortgage credit and prepayment risk: to transform risky long-term mortgages into safe short-term debt. Intermediaries use their equity capital to buffer mortgage losses. However, the intermediation sector has limited capacity to absorb losses. It relies on the government as ultimate guarantor of the short-term debt it issues. Thus, mortgage intermediation between borrowers and savers is subject to frictions stemming from the default options of both borrowers and banks. Mortgage default results in foreclosure, which comes with a resource loss to society. Similarly, bank default leads to costly liquidations and the loss of resources.

[Figure 2 about here.]

With traditional FRMs, borrowers bear the majority of house price risk. Large declines in aggregate house prices lead to an increase in mortgage foreclosures and loss rates, consistent with the empirical evidence from Section I. The indexation contracts we study implement a different allocation of risk between borrowers, intermediaries, and savers. Indexation of mortgage debt to house prices explicitly shifts house price risk to banks, reducing borrower foreclosures, while potentially making banks more fragile. We study how the welfare of each agent as well as overall welfare are affected by indexation.

The economy is hit with two persistent sources of aggregate risk, described in detail in the calibration section. The first exogenous state fluctuates between a "normal" state and a "housing-crash" state. The housing-crash state is associated with a higher mortgage default rate (engineered through the variable $\sigma_{\omega,t}$ defined below) and lower aggregate house prices (engineered through the variable ξ_t below). The second exogenous state is aggregate labor income, which fluctuates with the business cycle ($\varepsilon_{y,t}$). We define a "financial recession" as a transition from the normal state to the housing-crash state combined with a low realization of the aggregate income shock. House prices, safe interest rates, mortgage interest rates, and mortgage default and prepayment rates are all determined in equilibrium. Equilibrium objects depend on the exogenous state of the economy, just described, and on the endogenous wealth distribution. The wealth distribution consists of five continuous state variables: borrower wealth, intermediary wealth, saver wealth, mortgage principal outstanding, and promised mortgage interest payments. We denote the state vector by $\mathcal{S}_t$.

We first characterize the equilibrium with FRMs and calibrate the model to U.S. data. The next part of the analysis studies how indexation of mortgage payments to house prices changes the equilibrium. The key question is whether mortgage indexation can improve risk-sharing and overall welfare, which is driven in large part by the performance of the economy during financial recessions.

While borrowers face idiosyncratic house quality shocks and banks idiosyncratic profit shocks, all incompleteness in our model stems from imperfect risk-sharing *across* the three household types. We assume perfect risk-sharing *among* the agents of a given type. This structure allows for a fraction of borrowers and intermediaries to default in equilibrium, while also allowing for aggregation to a representative household within each type. The upshot is that the wealth distribution, which is a state variable, remains manageable. Others, such as Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and Guren, Krishnamurthy, and McQuade (2018), allow for imperfect risk-sharing among borrowers but do not have an intermediary sector. When computing equilibria, they approximate the wealth distribution with a similar number of state variables as in our model. Given our focus on how mortgage indexation affects financial fragility and welfare, we use our computational degrees of freedom to provide a richer model of the intermediary sector.

In one special case of the model, discussed in Section VI.E, savers are not allowed to invest directly in mortgage loans. Rather, they participate only indirectly in the sharing of mortgage credit and prepayment risk by paying for bank bailouts through taxes and through general equilibrium effects on safe interest rates. Banks adjust their capital structure to manage the increased risk they bear. More generally, the welfare effects of indexation naturally depend on the risk absorption capacity of the intermediary sector.

Our findings are driven largely by the difference in fragility between the financing arrangements of intermediaries and households. Households borrow in long-term mortgages whose credit constraints must be satisfied only at the origination of the loan, and due to infrequent refinancing, typically have sizable equity buffers. As a result, when hit by an adverse shock, relatively few households default on their loans, while the rest are not forced to delever, limiting further propagation. In contrast, intermediaries borrow in the form of short-term deposits, hold small equity buffers, and must satisfy their capital requirements each period. When intermediaries sustain losses, some banks fail, while those that do not must rapidly delever, restricting credit supply. A contraction of mortgage credit increases mortgage rates and the user cost of housing, which depresses house prices and worsens bank losses further, creating a feedback loop. This strong asymmetry between the consequences of losses for households and intermediaries underlies our core finding that shifting risk between these groups is not neutral, but instead can have large effects on financial fragility.

B. Setup: Preferences and Endowments

There is a continuum of agents of each type with population shares χ_j; $\chi_B + \chi_S + \chi_I = 1$. To allow for nontrivial risk premia, an agent of type $j \in \{B, S, I\}$ has preferences following Epstein and Zin (1989), so that lifetime utility is given by

$$U_t^j = \left\{ (1-\beta_j) \left(u_t^j\right)^{1-1/\psi} + \beta_j \left(\mathbb{E}_t \left[\left(U_{t+1}^j\right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{1}$$

$$u_t^j = (C_t^j)^{1-\xi_t}(H_t^j)^{\xi_t}, \tag{2}$$

where C_t^j is nondurable consumption and H_t^j is housing services. Borrowers, intermediary households, and savers have different degrees of patience β_j, but all households have the same risk aversion γ and intertemporal elasticity ψ. Naturally, borrowers (and intermediaries) are less patient than savers. The preference parameter ξ_t governs the demand for housing services and varies with the exogenous state of the economy, taking on a low value during the housing-crash state. We denote by Λ^j the intertemporal marginal rate of substitution or stochastic discount factor of agent j, spelled out in the Appendix.

All agents are endowed with nonhousing and housing goods. The nonhousing endowment Y_t consists of a stationary stochastic component $\tilde{Y}_t$ and a deterministic component that grows at a constant rate g, $Y_t = e^{gt}\tilde{Y}_t$, where $\mathbb{E}(\tilde{Y}_t) = 1$ and

$$\log \tilde{Y}_t = \rho_y \log \tilde{Y}_{t-1} + \sigma_y \varepsilon_{y,t}, \qquad \varepsilon_{y,t} \sim N(0,1). \tag{3}$$

The transitory shocks to the aggregate endowment $\varepsilon_{y,t}$ are the second source of aggregate risk. Each agent type j receives a fixed share s_j of the overall endowment Y_t; this can be interpreted as labor income.

Agents are also endowed with housing. The stock of housing is fixed at $\bar{K}$, and produces housing services that grow at the same rate g as the nondurable endowment. Housing requires a maintenance cost that is equal to fraction ν^K of the value of the housing stock. This cost is rebated lump-sum to households.[12] To ensure that the borrowers are the marginal pricers

[12]In our endowment economy, housing maintenance stands in for residential investment, which strongly comoves with house prices in the data. The assumption that maintenance is a fraction of current house prices creates this correlation in the model. However, the assumption also means that different versions of the model, for example, with mortgage indexation, which have different steady-state house price levels, feature different maintenance expenditure levels. Rebating maintenance expenditure avoids the undesirable effect that higher (lower) house price levels lead to lower (higher) consumption.

of housing, we fix intermediary and saver demand for housing to be $H_t^I = \bar{K}^I$ and $H_t^S = \bar{K}^S$.

C. Mortgages

Mortgage Contracts. Like in the U.S., mortgages are long-term, nominal, defaultable, prepayable contracts. For tractability, mortgages are modeled as perpetuities with outstanding loan balance and interest payments that decline geometrically. One unit of debt yields payments of $1, \delta, \delta^2, \ldots$ until either prepayment or default; the fraction $(1-\delta)$ captures the scheduled amortization of principal. Mortgage interest payments can be deducted from taxes. New mortgages have fixed mortgage rate r_t^* and principal M_t^*, and are subject to a loan-to-value constraint, shown below in (20), that is applied at origination only.

Refinancing. After the default decision has taken place (explained below), nondefaulting borrowers can choose to refinance. Refinancers prepay the principal balance on the existing loan before they obtain a new mortgage loan. They re-optimize their housing position. Since borrowers in the model tend to borrow up to their credit limit when taking out new loans, as is typical in reality, adjustments in borrower leverage generally occur at times of refinancing. Refinancing has an important effect on financial fragility because borrower leverage is a key determinant of default.

We assume a transaction cost for obtaining a new mortgage that is proportional to the new loan balance, $\kappa_{i,t} M_t^*$, where $\kappa_{i,t}$ is drawn i.i.d. across borrowers and over time from a distribution with CDF Γ_κ. Since these costs largely stand in for nonmonetary frictions such as inertia, they are rebated to borrowers and do not impose an aggregate resource cost. Following Greenwald (2018), we assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on $\kappa_{i,t}$ and all current values and expectations of aggregate variables, but cannot depend on the borrower's individual loan characteristics.[13] We guess and verify that the optimal plan for the borrower is to

[13]This assumption keeps the problem tractable by removing the distribution of loans as a state variable

refinance whenever $\kappa_{i,t} \leq \bar{\kappa}_t$, where $\bar{\kappa}_t(\mathcal{S}_t)$ is a threshold cost that makes the borrower indifferent between refinancing and not refinancing and that depends on the entire state of the economy $\mathcal{S}_t$. The fraction of nondefaulting borrowers who choose to refinance is therefore

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t). \tag{4}$$

Once the threshold cost (or refinancing rate) is known, the total transaction cost per unit of debt is defined by

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa \, d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa \, d\Gamma_\kappa. \tag{5}$$

As shown in the Appendix, borrowers refinance both to lower their mortgage rate (standard rate refi incentive) and to extract home equity (cash-out refi incentive).

House Quality Shocks. Before deciding whether to refinance a loan, borrowers can choose to default on the loan. Upon default, the housing collateral backing the loan is seized by the intermediary. To obtain an aggregated model in which there is fractional default and the default rate responds endogenously to macroeconomic conditions, we introduce stochastic processes $\omega_{i,t}$ for each borrower i that influence the quality of borrowers' houses. We decompose house quality into two components, $\omega_{i,t} = \omega^L_{i,t}\omega^U_{i,t}$, where $\omega^L_{i,t}$ is a local component that shifts prices in an area relative to the national average while $\omega^U_{i,t}$ is an uninsurable component that shifts an individual house price relative to its local area. The key idea is that payments on shared appreciation mortgages can potentially be indexed to local house prices, most likely at the Metropolitan Statistical Area (MSA) or ZIP-code level. For moral hazard reasons, lenders would be reluctant to index payments to the individual house price component, hence the label uninsurable. Both components are drawn i.i.d. from independent

while maintaining the realistic feature that an endogenous fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

log-normal distributions:

$$\log \omega_{i,t}^{L} \sim N\left(-\frac{1}{2}\alpha\sigma_{\omega,t}^{2},\ \ \alpha\sigma_{\omega,t}^{2}\right), \tag{6}$$

$$\log \omega_{i,t}^{U} \sim N\left(-\frac{1}{2}(1-\alpha)\sigma_{\omega,t}^{2},\ \ (1-\alpha)\sigma_{\omega,t}^{2}\right), \tag{7}$$

ensuring that each process has unit mean unity and that the local and uninsurable components account for α and $1-\alpha$ of the cross-sectional variance of $\omega_{i,t}$, respectively. The cross-sectional dispersion $\sigma_{\omega,t}$ takes a low value in normal times and a high value in housing-crash times, fluctuating with the aggregate state of the economy. While the assumption that local and individual house values are drawn i.i.d. is not realistic, we show in Section II.B of the Internet Appendix that our functional form can be micro-founded based on more realistic AR(1) processes.[14]

Mortgage Indexation. In addition to the standard mortgage contracts defined above, we introduce SAMs, whose payments are indexed to house prices. We allow SAM contracts to insure households in two ways. First, mortgage payments can be indexed to the aggregate (national) house price p_t. In this case, the principal balance and the scheduled principal and interest payments on each existing mortgage loan are multiplied by

$$\zeta_{p,t} = \left(\frac{p_t}{p_{t-1}}\right)^{\iota_p}. \tag{8}$$

The special cases $\iota_p = 0$ and $\iota_p = 1$ correspond to the cases of no indexation and complete insurance against aggregate house price risk.

Second, mortgage contracts can also be indexed against shocks to the local component

[14] The intuition is that due to perfect insurance within the borrower family and the symmetric form of indexation, swapping the identities of two individual borrower agents is irrelevant. Drawing i.i.d. can therefore be thought of as drawing from a more persistent process and then randomly reshuffling the identities of the individual borrowers.

$\omega_{i,t}^L$ of house values. The principal balance and payment on the loan backed by a house that experiences local house quality growth $\omega_{i,t}^L$ are multiplied by

$$\zeta_\omega(\omega_{i,t}^L) = \left(\omega_{i,t}^L\right)^{\iota_\omega}. \tag{9}$$

The special cases $\iota_\omega = 0$ and $\iota_\omega = 1$ correspond to no insurance and complete insurance against cross-sectional local house price risk, respectively.

Indexation of mortgage principal and payments to the aggregate and local components of house prices can be combined by setting $\iota_p = 1$ and $\iota_\omega = 1$. This is the main case of interest, which we refer to as *regional* indexation. Indeed, regional (MSA- or ZIP-level) house prices are the product of national house prices and the local component of house prices that is orthogonal to the national index.

Mortgage Default. As with refinancing, borrowers must commit to a default plan that can depend in an unrestricted way on $\omega_{i,t}^L, \omega_{i,t}^U$, and the aggregate states, but not on a borrower's individual loan conditions. We guess and verify that the optimal plan for the borrower is to default whenever $\omega_{i,t}^U \leq \bar{\omega}_t^U$, where $\bar{\omega}_t^U$ is the threshold value of uninsurable (individual-level) house quality that makes a borrower indifferent between defaulting and not defaulting. The level of the default threshold depends on the aggregate state $\mathcal{S}_t$, the local component $\omega_{i,t}^L$, and the level of mortgage payment indexation. Given $\bar{\omega}_t^U$, the fraction of nondefaulting borrowers $Z_{N,t}$ is

$$Z_{N,t} = \int \left(1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U)\right) \, d\Gamma_{\omega,t}^L, \tag{10}$$

where $\Gamma_{\omega,t}^U$ and $\Gamma_{\omega,t}^L$ are the CDFs of $\omega_{i,t}^U$ and $\omega_{i,t}^L$, respectively. The integral is needed because $\bar{\omega}_t^U$ depends on $\omega_{i,t}^L$. The share of housing held by nondefaulting borrower households is

$$Z_{K,t} = \int \left(\int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U \, d\Gamma_{\omega,t}^U \right) \omega_{i,t}^L \, d\Gamma_{\omega,t}^L, \tag{11}$$

where the inner-most integral contains a selection effect –borrowers only keep their housing when their idiosyncratic quality shock is sufficiently good– while the outer integral again accounts for dependence of $\bar{\omega}_t^U$ on local house quality.

The fractions of principal and interest payments retained and not defaulted on are denoted by $Z_{M,t}$ and $Z_{A,t}$, respectively, and are given by

$$Z_{M,t} = Z_{A,t} = \int\int \underbrace{\left(1 - \Gamma_{\omega,t}^U\left(\bar{\omega}_t^U\right)\right)}_{\text{remove defaulters}} \underbrace{\left(\omega_{i,t}^L\right)^{\iota_\omega}}_{\text{indexation}} \, d\Gamma_{\omega,t}^L. \tag{12}$$

The first term in the integral above removes the fraction of debt that is defaulted on, while the second term adjusts for indexation of debt to local prices.[15]

Equations (13) to (15) describe the evolution of the aggregate outstanding mortgage principal balance, interest payments, and housing stock:

$$M_{t+1}^B = \bar{\pi}^{-1}\zeta_{p,t+1}\Big[Z_{R,t}Z_{N,t}M_t^* + \delta(1 - Z_{R,t})Z_{M,t}M_t^B\Big] \tag{13}$$

$$A_{t+1}^B = \bar{\pi}^{-1}\zeta_{p,t+1}\Big[Z_{R,t}Z_{N,t}r_t^*M_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^B\Big] \tag{14}$$

$$K_{t+1}^B = Z_{R,t}Z_{N,t}K_t^* + (1 - Z_{R,t})Z_{K,t}K_t^B. \tag{15}$$

Since mortgages are nominal contracts, dividing by the gross inflation rate $\bar{\pi}$ expresses balance and payments in real terms. Aggregate indexation influences the laws of motion by directly scaling principal and interest payments to aggregate house price growth, through the term $\zeta_{p,t+1}$. Under full aggregate indexation ($\iota_p = 1$), a 20% national house price decline reduces mortgage principal balances and interest payments by 20%. Local indexation, whose direct effects wash out in aggregate, instead influences the default decision through the threshold $\bar{\omega}_t^U$, thereby affecting $Z_{N,t}$, $Z_{M,t}$, $Z_{A,t}$, and $Z_{K,t}$.

Under a standard mortgage contract without indexation ($\iota_p = \iota_\omega = 0$), households default

[15] While $Z_{A,t}$ and $Z_{M,t}$ are identical in the baseline indexation case, it is convenient to define them separately since they will diverge under separate indexation of interest and principal in Section VI.A.

when the market value of debt, which includes the option value of waiting to default, exceeds the market value of the housing collateral. [16] Default happens when households suffer a large house price drop, the origin of which could be at the national, local, or individual level. Indexation to national (local) house prices adjusts the mortgage balance and payments so as to stabilize current leverage, thereby reducing default due to national (local) house price declines. It is straightforward to show that for the limiting case in which all cross-sectional house price risk is insurable ($\alpha = 1$) and this risk is fully indexed ($\iota_\omega = 1$, $\iota_p = 1$), we obtain $Z_{N,t} = Z_{M,t} = Z_{A,t} = Z_{K,t} = 1$. Full indexation prevents all mortgage defaults.

Recovery Rate on Foreclosed Mortgages. As discussed below, mortgages can be held by intermediaries ("banks") and by savers. The housing collateral backing a defaulted mortgage is seized by the holder of that mortgage. After paying maintenance on this so-called real estate owned (REO) housing for one period at the higher depreciation rate $\nu^{REO} > \nu^K$, the mortgage holder sells the REO housing to a specialized intermediary, a REO firm, at a price p_t^{REO} determined in equilibrium. The recovery rate X_t on foreclosed mortgages (per unit of principal outstanding) is

$$X_t = \frac{(1 - Z_{K,t})K_t^B(p_t^{REO} - \nu^{REO}p_t)}{M_t^B}. \tag{16}$$

Note that X_t is taken as given by each individual mortgage holder. An individual bank does not internalize the effect of its mortgage debt issuance on the overall recovery rate.

PO and IO Strips. After being originated by banks, mortgages can be traded on secondary markets by banks ($j = I$) and savers ($j = S$). Although each mortgage vintage has a different fixed interest rate r_t^* and hence a different secondary market price, we show in the

[16] While all defaults in the baseline model are therefore strategic, Section III of the Internet Appendix presents a model extension in which borrower default is driven by liquidity shocks and borrowers face a penalty for strategically defaulting.

Appendix that any portfolio of loans (vintages) can be replicated using two instruments: an interest-only (IO) strip and a principal-only (PO) strip. Let q_t^A and q_t^M be the market prices of IO and PO strips, respectively. The cash flow CF^j from a generic portfolio of M_t^j units of POs and A_t^j units of IOs equals

$$CF_t^j = X_t M_t^j + Z_{M,t}\Big(1 - \delta + \delta Z_{R,t}\Big) M_t^j + Z_{A,t} A_t^j, \tag{17}$$

for $j = I, S$. The first term reflects principal recovery from defaulted mortgages. On the nondefaulted mortgage principal, $Z_{M,t} M_t^j$, the investor receives scheduled principal amortization $(1 - \delta)$ as well as unscheduled principal prepayments of the outstanding mortgage balance $\delta Z_{R,t}$. On the IOs, the investor receives $Z_{A,t} A_t^j$ since Z_A is the nondefaulted fraction and each unit of IOs pays 1 in interest in the current period (recall from (14) that the interest rate is folded into the definition of A). The ex-dividend value of this portfolio of POs and IOs after current-period cash flows have been made, EDV, is

$$EDV_t^j = \delta(1 - Z_{R,t}) \left(q_t^A Z_{A,t} A_t^j + q_t^M Z_{M,t} M_t^j \right). \tag{18}$$

A fraction $\delta(1 - Z_{R,t}) Z_{A,t}$ of IOs remain outstanding after default and refinancing decisions and after principal amortization. Each unit is worth q_t^A. Similarly, a fraction $\delta(1 - Z_{R,t}) Z_{M,t}$ of POs remain outstanding; each unit is worth q_t^M.

D. *Borrowers*

Given this setup, individual borrowers' problems aggregate to that of a representative borrower. The endogenous state variables to the borrower are the promised payment A_t^B on the stock of all mortgage debt, the outstanding principal balance on all mortgage debt M_t^B, and the stock of borrower-owned housing K_t^B. The representative borrower's choice variables are nondurable consumption C_t^B, housing services consumption H_t^B, the amount of housing K_t^* and new loans M_t^* taken on by refinancing borrowers, the refinancing fraction

$Z_{R,t}$, and the default policy $\bar{\omega}_t^U$, which implicitly determines $(Z_{N,t}, Z_{M,t}, Z_{A,t}, Z_{K,t})$.

The borrower maximizes expected utility in (1) subject to the budget constraint

$$\begin{aligned} C_t^B = \underbrace{(1-\tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t}\left(Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B\right)}_{\text{net new borrowing}} - \underbrace{(1-\delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1-\tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\ - \underbrace{p_t Z_{R,t}\left[Z_{N,t}K_t^* - Z_{K,t}K_t^B\right]}_{\text{owned housing}} - \underbrace{\nu^K p_t Z_K K_t^B}_{\text{maintenance}} - \underbrace{\rho_t\left(H_t^B - K_t^B\right)}_{\text{rental housing}} \\ - \underbrace{\left(\Psi(Z_{R,t}) - \bar{\Psi}_t\right)Z_{N,t}M_t^*}_{\text{net refinancing costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}} + \underbrace{R_t^B}_{\text{maintenance rebate}}, \end{aligned} \tag{19}$$

the loan-to-value (LTV) constraint

$$M_t^* \leq \phi^K p_t K_t^*, \tag{20}$$

and the laws of motion (13) to (15). Borrower consumption equals after-tax labor income, where τ is the income tax rate, plus net new mortgage borrowing (mortgage principal on new loans minus outstanding principal balance on refinanced loans), minus scheduled principal amortization on outstanding mortgages, minus interest payment on mortgages taking into account the tax shield, minus net new housing purchased by refinancing borrowers, minus housing maintenance expenses to offset depreciation, minus rental expenses (the rental rate ρ_t times the difference between housing services consumed and owned), minus net refinancing costs associated (zero in equilibrium), minus taxes raised on borrowers to pay for intermediary bailouts (defined below in (39)), plus a rebate of maintenance costs ($R_t^B = \nu^K p_t Z_K K_t^B$ in equilibrium). Equation (20) caps new mortgage debt at a maximum LTV ratio of ϕ^K. We discuss the borrower's optimality conditions in the Appendix.

E. *Intermediaries*

The intermediation sector consists of intermediary households ("bank owners"), mortgage lenders ("banks" for short), and REO firms. The intermediary households are equity holders of both the banks and the REO firms.

Bank Owners. Each period, bank owners receive labor income Y_t^I and dividends D_t^I and D_t^{REO} from all banks and REO firms, defined in equations (30) and (32) below. Bank owners choose consumption C_t^I to maximize (1) subject to the budget constraint

$$C_t^I \leq (1-\tau)Y_t^I + D_t^I + D_t^{REO} - T_t^I - \nu^K p_t H_t^I + R_t^I, \tag{21}$$

where T_t^I are taxes raised on intermediary households to pay for bank bailouts, defined in (39) below, and R_t^I is the lump-sum rebate of maintenance expenditure. Bank owners consume their fixed endowment of housing services each period, $H_t^I = \bar{K}^I$.

Banks' Portfolio Choice. There is a continuum of competitive banks. Banks maximize shareholder value, defined as the present discounted value of dividends valued using the shareholder SDF Λ^I, by optimally choosing new mortgage originations, short-term deposits, and IO and PO positions in the secondary market for mortgage debt.

Let A_t^I and M_t^I denote the bank's holdings of IO and PO strips, respectively, at the start of the period. After all shocks are realized and borrowers have made default decisions, banks originate new mortgages L_t^* to refinancers at interest rate r_t^*. They then re-optimize their portfolio of mortgages on the secondary market. That is, banks *supply* PO and IO strips,

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{M,t}M_t^I \tag{22}$$

$$\hat{A}_t^I = r_t^* L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I., \tag{23}$$

and *demand* a new portfolio of PO and IO strips $\tilde{M}_t^I$ and $\tilde{A}_t^I$, respectively. The market value of the bank's portfolio after portfolio rebalancing is

$$J_t^I = \underbrace{q_t^A \tilde{A}_t^I}_{\text{IO strips}} + \underbrace{q_t^M \tilde{M}_t^I}_{\text{PO strips}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}}. \tag{24}$$

Next period's beginning-of-period IO and PO strip holdings adjust current end-of-period holdings for inflation and indexation,

$$M^I_{t+1} = \bar{\pi}^{-1} \zeta_{p,t+1} \tilde{M}^I_t, \tag{25}$$

$$A^I_{t+1} = \bar{\pi}^{-1} \zeta_{p,t+1} \tilde{A}^I_t. \tag{26}$$

Banks' net worth at the beginning of period $t+1$ equals the cash flows on its IO and PO portfolio (A^I_{t+1}, M^I_{t+1}) plus the value of that portfolio minus deposit redemptions,

$$W^I_{t+1} = CF^I_{t+1} + EDV^I_{t+1} - \bar{\pi}^{-1} B^I_{t+1}, \tag{27}$$

using equations (17), (18), (25), and (26).

Banks' Problem. At the beginning of each period, aggregate shocks are realized and each bank receives an idiosyncratic profit/loss shock $\epsilon^I_t \sim F^I_\epsilon$, with $\mathrm{E}(\epsilon^I_t) = 0$. A high draw for ϵ^I_t represents a large idiosyncratic loss. The idiosyncratic profit shock captures unmodeled heterogeneity in banks' balance sheets. Banks then make their optimal default decision. The government seizes the defaulted banks, wipes out the equity holders, and makes whole the depositors. Bank owners then start new banks to replace the liquidated banks. Finally, all banks make optimal portfolio choice decisions.

In the Appendix we show that surviving and newly started banks face an identical portfolio choice problem. This property allows for aggregation across all banks. The problem solved by the representative bank, after default decisions have been made, is

$$V^I(W^I_t, \mathcal{S}_t) = \max_{L^*_t, \tilde{M}^I_t, \tilde{A}^I_t, B^I_{t+1}} W^I_t - J^I_t + \mathrm{E}_t \left[\Lambda^I_{t+1} \, F^I_{\epsilon,t+1} \left(V^I(W^I_{t+1}, \mathcal{S}_{t+1}) - \epsilon^{I,-}_{t+1} \right) \right], \tag{28}$$

given the definitions of J^I_t and W^I_t in (24) and (27), and the laws of motion (25) and (26). The function $F^I_{\epsilon,t+1} \equiv F^I_\epsilon(V^I(W^I_{t+1}, \mathcal{S}_{t+1}))$ is the probability of continuation, and the expectation of the loss realization ϵ^I_{t+1} conditional on continuation is $\epsilon^{I,-}_{t+1} = \mathrm{E}\left[\epsilon^I_{t+1} \,|\, \epsilon^I_{t+1} < V^I(W^I_{t+1}, \mathcal{S}_{t+1})\right]$. By the law of large numbers, the fraction of defaulting banks in the current period is $1 - F^I_{\epsilon,t}$.

The bank's portfolio choice is subject to a leverage constraint that limits the amount of deposit finance to a fraction ϕ^I of assets,

$$B^I_{t+1} \leq \phi^I \left(q^A_t \tilde{A}^I_t + q^M_t \tilde{M}^I_t \right). \tag{29}$$

Since banks enjoy limited liability and issue insured deposits, they have incentives to take on excessive risk in the form of high leverage. To curb this incentive, the Basel-style regulatory equity capital requirement limits bank leverage to $0 < \phi^I < 1$.

The aggregate dividend paid by banks to their shareholders is

$$D^I_t = F^I_{\epsilon,t} \left(W^I_t - J^I_t - \epsilon^{I,-}_t \right) - \left(1 - F^I_{\epsilon,t} \right) J^I_t. \tag{30}$$

The first term reflects dividends paid out from nondefaulting banks. Bank shareholders bear the burden of replacing liquidated banks by an equal measure of new banks and seeding them with new capital equal to that of continuing banks (J^I_t) — the second term.

Government Bailout. The government bails out defaulted banks at cost

$$\text{bailout}_t = \left(1 - F^I_{\epsilon,t} \right) \left[\epsilon^{I,+}_t - W^I_t + \eta EDV^I_t \right], \tag{31}$$

where $\epsilon^{I,+}_t = \mathrm{E}\left[\epsilon^I_t \,|\, \epsilon^I_t > V^I(W^I_t, \mathcal{S}^I_t)\right]$ is the expectation of the idiosyncratic loss ϵ^I_t conditional on default. The government absorbs the negative net worth of the defaulting banks, $\epsilon^{I,+}_t - W^I_t$. The last term captures deadweight losses from bank bankruptcies, which are a fraction η of the mortgage assets seized from the bankrupt banks. Government bailout always makes depositors whole. This deposit insurance is what makes deposits risk-free.

REO Firm's Problem. There is a continuum of competitive REO firms that are owned and operated by intermediary households. REO firms maximize the present discounted value of dividends by choosing how many foreclosed properties to buy, I_t^{REO},

$$D_t^{REO} = \underbrace{\Big[\rho_t + (S^{REO} - \nu^{REO})p_t\Big] K_t^{REO}}_{\text{REO net income}} - \underbrace{p_t^{REO} I_t^{REO}}_{\text{REO investment}}. \tag{32}$$

REO firms earn revenue from renting foreclosed homes to borrowers and gradually selling them back to borrowers at an exogenous rate S^{REO}. REO firms must pay for maintenance $\nu^{REO} p_t K_t^{REO}$, which unlike regular housing maintenance is not rebated and thus constitutes an aggregate resource cost. This cost is the deadweight loss from mortgage foreclosures. The law of motion of the REO housing stock is

$$K_{t+1}^{REO} = (1 - S^{REO}) K_t^{REO} + I_t^{REO}. \tag{33}$$

F. *Saver's Problem*

Savers can invest in risk-free debt and risky mortgage debt. To capture the comparative disadvantage of savers in holding mortgages relative to banks, savers face a cost of holding mortgages. When mortgages default, savers sell the collateral backing the defaulted mortgages to REO firms after one period of maintenance, just like banks do.

Savers enter the period with net worth W_t^S. They sell their holdings of mortgages into the secondary market; call this supply $(\hat{A}_t^S, \hat{M}_t^S)$. Savers then form an optimal portfolio of safe assets and mortgages $(\tilde{A}_t^S, \tilde{M}_t^S)$. For simplicity, we assume that savers only buy and sell mortgages in fixed combinations of IO and PO strips. Denote the post-trade value of their portfolio by

$$J_t^S = q_t^A \tilde{A}_t^S + q_t^M \tilde{M}_t^S + q_t^f B_{t+1}^S. \tag{34}$$

The laws of motion (25) and (26) equally apply to saver holdings. Net worth at the beginning of period $t+1$ equals the cash flows on their IO and PO portfolio (A^S_{t+1}, M^S_{t+1}) plus the ex-dividend value of that portfolio minus deposit redemptions,

$$W^S_{t+1} = CF^S_{t+1} + EDV^S_{t+1} + \bar{\pi}^{-1} B^S_{t+1}, \tag{35}$$

using equations (25), (26), (17), and (18).

Savers' problem can also be aggregated, so that the representative saver chooses non-durable consumption C^S_t, holdings of safe assets B^S_t, and mortgages $\tilde{M}^S_t$ to maximize expected lifetime utility (1) subject to the budget constraint

$$C^S_t \leq (1-\tau)Y^S_t + W^S_t - J^S_t - \frac{\varphi_0}{\varphi_1}\left(\tilde{M}^S_t\right)^{\varphi_1} - T^S_t - \nu^K p_t H^S_t + R^S_t \tag{36}$$

and restrictions that safe debt and mortgage holdings must be positive: $B^S_t \geq 0$ and $\tilde{M}^S_t \geq 0$. Savers consume their fixed endowment of housing services each period, $H^S_t = \bar{K}^S$, on which they pay maintenance expenses that are rebated lump-sum. Savers incur a cost for holding mortgages ($\varphi_0 > 0$) that is increasing in the amount of mortgage debt they own ($\varphi_1 > 0$); this cost is rebated lump-sum as part of R^S_t so that it does not represent a resource loss to society. This holding cost represents the comparative disadvantage of savers relative to banks for holding (screening and monitoring) mortgage debt.

G. Government

Discretionary government spending equals income taxes net of the mortgage interest deduction,

$$G_t = \tau(Y_t - Z_{A,t} A^B_t). \tag{37}$$

To finance bank bailout expenses in (31), the government issues risk-free short-term debt that trades at the same price as deposits. To service this debt, the government levies lump-sum taxes T^j_t on households of type j in period t. Total tax revenue from lump-sum taxation is $T_t = T_t^B + T_t^I + T_t^S$. Therefore, if B_t^G is the amount of government bonds outstanding at the beginning of t, the government budget constraint satisfies

$$\bar{\pi}^{-1} B_t^G + \text{bailout}_t = q_t^f B_{t+1}^G + T_t, . \tag{38}$$

Lump-sum taxes are levied in proportion to population shares χ_j and at a rate τ_L,

$$T_t^j = \chi_j \tau_L \left(\bar{\pi}^{-1} B_t^G + \text{bailout}_t \right), \ \forall j \in \{B, I, S\}. \tag{39}$$

When $\tau_L < 1$, this formulation implies gradual repayment of government debt following a bailout. When $\tau_L = 1$, the bailout is financed entirely with current taxes.[17]

H. Equilibrium

Given a sequence of endowment and housing-crash state realizations $[\varepsilon_{y,t}, (\sigma_{\omega,t}, \xi_t)]$, a competitive equilibrium is a sequence of saver allocations $(C_t^S, B_{t+1}^S, \tilde{M}_t^S, \tilde{A}_t^S, \hat{M}_t^S, \hat{A}_t^S)$, borrower allocations $(C_t^B, H_t^B, M_t^B, A_t^B, K_t^B, K_t^*, M_t^*, Z_{R,t}, \bar{\omega}_t^U)$, intermediary allocations $(C_t^I, M_t^I, A_t^I, K_t^{REO}, W_t^I, L_t^*, I_t^{REO}, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I)$, and prices $(r_t^*, q_t^M, q_t^A, q_t^f, p_t, p_t^{REO}, \rho_t)$ such that borrowers, intermediaries, and savers optimize and markets clear:

$$\begin{aligned}
\text{New mortgages:} \quad & Z_{R,t} Z_{N,t} M_t^* = L_t^* \\
\text{PO strips:} \quad & \tilde{M}_t^I + \tilde{M}_t^S = \hat{M}_t^I + \hat{M}_t^S \\
\text{IO strips:} \quad & \tilde{A}_t^I + \tilde{A}_t^S = \hat{A}_t^I + \hat{A}_t^S \\
\text{Deposits and Gov. Debt:} \quad & B_{t+1}^I + B_{t+1}^G = B_{t+1}^S \\
\text{Housing Purchases:} \quad & Z_{R,t} Z_{N,t} K_t^* = S^{REO} K_t^{REO} + Z_{R,t} Z_{K,t} K_t^B
\end{aligned}$$

[17] Equations (38) and (39) together imply that new bonds issued in t are $B_{t+1}^G = (1 - \tau_L)(q_t^f)^{-1} \left(\bar{\pi}^{-1} B_t^G + \text{bailout}_t \right)$. The case $\tau_L = 1$ implies $B_t^G = 0$, $\forall t$. To ensure stationarity of the government debt balance, τ_L needs to be large enough relative to the average risk-free rate. We verify that this is the case in our quantitative exercises. Results for $\tau_L < 1$ are discussed in Section VI.F.

$$\text{REO Purchases:} \quad I_t^{REO} = (1 - Z_{K,t})K_t^B$$

$$\text{Housing Services:} \quad H_t^B = K_t^B + K_t^{REO} = \bar{K}^B$$

$$\text{Resources:} \quad Y_t = C_t^B + C_t^I + C_t^S + G_t + DWL_t^b + MAINT_t,$$

where

$$DWL_t^b = \left(1 - F_{\epsilon,t}^I\right) \eta\delta(1 - Z_{R,t}) \left(Z_{A,t} q_t^A A_t^I + Z_{M,t} q_t^M M_t^I\right) \tag{40}$$

$$MAINT_t = \nu^{REO} p_t \Big[K_t^{REO} + (1 - Z_{K,t})K_t^B\Big]. \tag{41}$$

The resource constraint states that the endowment Y_t is spent on nondurable consumption, government consumption, deadweight losses from bank failures, and housing maintenance. Maintenance consists of payments for houses owned by REO firms, K_t^{REO}, or newly bought by REO firms from foreclosed borrowers $(1 - Z_{K,t})K_t^B$. Recall that regular maintenance by households is rebated and thus does not affect the resource constraint.

Section I of the Internet Appendix describes the system of equations that characterizes equilibrium and the numerical solution method. The model is solved using global projection methods. Since the integrals (11) and (12) lack a closed form, we evaluate them using Gauss-Hermite quadrature with 11 nodes in each dimension.

III. Calibration

This section describes the calibration procedure for the key variables used in the analysis, summarizing the full set of parameter values in Table I. The model is calibrated at a quarterly frequency. All data are for the period 1991.Q1 to 2016.Q1, the longest period of mortgage foreclosure data. Data sources are detailed in Section IV.B of the Internet Appendix.

Exogenous Shock Processes. Aggregate endowment shocks in (3) have quarterly persistence $\rho_y = 0.977$ and innovation volatility $\sigma_y = 0.81\%$. These are the observed persistence and innovation volatility of log real per-capita labor income. This autoregressive (AR) process is discretized as a five-state Markov Chain, following the Rouwenhorst (1995) method. We abstract from long-run endowment growth ($g = 0$). The average level of aggregate income (GDP) is normalized to one. The income tax rate is $\tau = 0.147$, the observed ratio of personal income tax revenue to personal income.

The housing-crash state follows a two-state Markov Chain, with state 0 indicating normal times, and state 1 indicating a housing crash. The probability of staying in the normal state in the next quarter is $\Pi_{00} = 97.5\%$ and the probability of staying in the housing-crash state in the next quarter is $\Pi_{11} = 92.5\%$. Under these parameters, the economy is in the normal state three-quarters of the time and in the housing-crash state one-quarter of the time. This matches the fraction of time between 1991.Q1 and 2016.Q4 that the U.S. economy was in the foreclosure crisis, and implies an average duration of the normal state of 10 years and an average duration of the housing-crash state of 3.33 years.[18] These transition probabilities are independent of the aggregate endowment state. The normal state has $\bar{\sigma}_{\omega,0} = 0.200$ and the housing-crash state has $\bar{\sigma}_{\omega,1} = 0.250$. These numbers allow the model to match an average mortgage default rate of 0.5% per quarter in expansions and 2.15% per quarter in financial recessions, which are periods defined by low endowment growth and high uncertainty. The unconditional mortgage default rate in the model is 0.97%. In the data, the average mortgage delinquency rate is 1.05% per quarter: 0.7% in normal times and 2.3% during the foreclosure crisis.

Local House Price Process. We calibrate the cross-sectional dispersion of the local housing quality process using MSA-level house prices indices from Federal Housing Finance Agency.

[18] Using a longer time series for the U.S. (1870 to 2011), Jordà, Schularick, and Taylor (2017) find that the U.S. was in a financial crisis in 20% of the sample years. For a larger sample of 17 developed nations, they find that one-quarter of recessions are financial crises. The same is true in our model. A financial recession, which is the combination of a decline in aggregate labor income and a housing crash, occurs in 7.5% of our model periods.

Specifically, we run the annual panel regression

$$\log HPI_{i,t} = \phi_t + \psi_i + \rho_\omega^{ann} \log HPI_{i,t-1} + \varepsilon_{i,t}, \tag{42}$$

where i indexes the MSA, and t indexes the year, and ϕ_t and ψ_i are year and MSA fixed effects.[19] Quarterly persistence is computed as $\rho_\omega = (\rho_\omega^{ann})^{1/4}$, which we estimate to be 0.977. Since this persistence parameter only matters for the indexation of local house price risk, it is appropriate to calibrate this parameter only to local house price data. To calibrate α, the share of house price variance at the local/regional level, we use (42) to compute the implied unconditional variance $\mathrm{Var}(\omega_{i,t}^L) = \mathrm{Var}(\varepsilon_{i,t})/(1 - (\rho_\omega^{ann})^2)$, which delivers an unconditional standard deviation at the MSA level of 11.5%. We set $\alpha = 0.25$, which generates an unconditional volatility of local house prices of 10.6% close to the data. Given our calibration for $\sigma_{\omega,t}$, it implies that the standard deviation of house prices is 10% in the model in normal times and 12.5% in financial recessions.

[Table 1 about here.]

Demographics, Income, and Housing Shares. We split the population into mortgage borrowers, savers, and bank owners as follows. We use the 1998 Survey of Consumer Finances (SCF) to calculate a loan-to-value ratio for every household. This ratio is zero for renters and for households who own their house free and clear. We define mortgage borrowers to be those households with a LTV ratio of at least 30%.[20] Those households make up 34.3% of households ($\chi_B = 0.343$) and earn 46.9% of labor income ($s_B = 0.469$). For parsimony, we set all housing shares equal to the corresponding income share. Since the aggregate housing stock $\bar{K}$ is normalized to one, $\bar{K}^B = 0.469$.

[19] Using quarterly house price data instead results in very similar estimates of the cross-sectional dispersion.

[20] Those households account for 88.2% of all mortgage debt and 81.6% of all mortgage payments.

To split the remaining households into savers and intermediary households, we again turn to the 1998 SCF and define a household's risky share as the ratio of direct and indirect equity holdings plus net business wealth to financial assets. We define intermediary households, the "shareholders" in the model, as those households with a risky share above a given cutoff. We choose the cutoff such that bank owners' population share is 5%, implying a risky share cutoff of 68.2%. The share of labor income for this group in the SCF is equal to $s_I = 6.2\%$. In Section VI.E, we check the sensitivity of our results to the relative size of intermediary households, which influences banks' risk absorption capacity. Savers make up the remaining $\chi_S = 60.7\%$ of the population, and receive the remaining $s_S = 46.9\%$ of labor income and of the housing stock.

Prepayment Costs. For the prepayment cost distribution, we assume a mixture distribution, such that the borrower draws an infinite prepayment cost with probability 3/4, while the borrower draws from a logistic distribution with probability 1/4, yielding

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t) = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(\frac{\bar{\kappa}_t - \mu_\kappa}{\sigma_\kappa}\right)}.$$

The calibration of the parameters follows Greenwald (2018).[21] The parameter σ_κ determining the sensitivity of prepayment to equity extraction and interest rate incentives is set to the estimate in Greenwald (2018) (0.152), while the parameter μ_κ is set to match the average quarterly prepayment rate of 3.76% found in Greenwald (2018).

[21]The parameters are fit to minimize the forecast error $LTV_t = Z_{R,t} LTV_t^* + (1 - Z_{R,t}) \delta HPA_t^{-1} LTV_{t-1}$, where LTV_t is the ratio of total mortgage debt to housing wealth, LTV_t^* is LTV at origination, and HPA_t is growth in house values. See Greenwald (2018), Section 4.2.

Mortgages. We set $\delta = 0.99565$ to match the fraction of principal that U.S. households amortize on mortgages.[22] The maximum LTV ratio at mortgage origination is $\phi^B = 0.85$, consistent with average mortgage underwriting norms.[23] Inflation $\bar{\pi}$ is set to the observed 0.57% per quarter (2.29% per year) over our sample period.

Banks. We set the maximum leverage that banks may take on to $\phi^I = 0.930$, following Elenev, Landvoigt, and Van Nieuwerburgh (2018), to capture the historical average leverage ratio of the leveraged financial sector. The idiosyncratic profit shock that hits banks has a standard deviation of $\sigma_\epsilon = 6.50\%$ per quarter. This delivers a bank failure rate of 0.30% per quarter, consistent with historical bank failure rate data from the Federal Deposit Insurance Corporation (FDIC). We assume a deadweight loss from bank bankruptcies of $\eta = 5.00\%$ of bank assets. Based on a study of bank failures from 1986 to 2007 (Bennett and Unal, 2015), the FDIC estimates that direct expenses of resolution for failed banks that are liquidated are 4.88% of assets.

Housing Maintenance and REOs. We set the regular housing maintenance cost equal to $\nu^K = 0.616\%$ per quarter or 2.46% per year. This is the average of the ratio of current-cost depreciation of privately owned residential fixed assets to the current-cost net stock of privately owned residential fixed assets at the end of the previous year (Bureau of Economic

[22]The average duration of a 30-year FRM is about seven years. This low duration is mostly the result of early prepayments. The parameter δ captures amortization absent refinancing. Put differently, households pay off much less than one-seventh of their mortgage principal each year in the absence of prepayment. A quarterly value of $\delta = 0.99565$ implies that 1.73% of principal is paid off in the first year of the mortgage, matching the first-year principal reduction on a 30-year FRM with a rate of 4.25%.

[23]The average LTV of purchase mortgages originated by Fannie and Freddie was in the 80% to 85% range during our sample period. However, that does not include second mortgages and home equity lines of credit. Our limit is a combined loan-to-value limit (CLTV). It also does not capture the lower down payments on nonconforming loans that became increasingly prevalent after 2000. Keys, Piskorski, Seru, and Vig (2012) document that CLTVs on nonconforming loans that rose from 85% to 95% between 2000 and 2007.

Analysis (BEA) Fixed Asset Tables 5.1 and 5.4). We calibrate the maintenance cost in the REO state to $\nu^{REO} = 2.20\%$ per quarter. This cost delivers REO housing prices that are 23.1% below regular housing prices on average. This is close to the observed fire-sale discounts (losses-given-default) reported by Fannie Mae and Freddie Mac during the foreclosure crisis.

We assume that $S^{REO} = 0.167$, so that one-sixth of the REO stock is sold back to the borrower households each quarter. It takes eight quarters for 75% of the REO stock to roll off. This generates REO crises that take some time to resolve, as they did in the data.

Savers. Savers' holding cost of mortgage securities has two parameters, the cost shifter φ_0 and the elasticity parameter φ_1. We set $\varphi_0 = 0.200$ to target an average saver share of mortgage holdings of 15%, and we set $\varphi_1 = 5.000$ to target a volatility of this share of 3%. We arrive at these targets by calculating the fraction of mortgage debt held outside the levered financial sector using the Financial Accounts of the United States, as detailed in Section IV.B of the Internet Appendix. The model produces an average share of 14.5% with a volatility of 2.55%.

Preferences. All agents have the same risk-aversion coefficient of $\gamma_j = 2.000$ and intertemporal elasticity of substitution $\psi = 1$. These are standard values in the literature. We set the value of the housing preference parameter in normal times $\bar{\xi}_0 = 0.210$ to match a ratio of housing expenditure to income for borrowers of 19%, a common estimate in the housing literature.[24] The model produces an expenditure ratio of 19.5%. To induce an additional house price drop, we set $\bar{\xi}_1 = 0.16$ in the housing-crash states. This additional variation yields a volatility of quarterly log national house price growth of 1.64%, matching the 1.66% in the data (Case-Shiller national home price index, deflated by Personal Consumption Expenditure

[24] Piazzesi, Schneider, and Tuzel (2007) obtain estimates between 18% and 20% based on national income account data (NIPA) and consumption micro data (CEX). Davis and Ortalo-Magné (2011) obtain a ratio of 18% after netting out 6% for utilities from the median value of 24% across MSAs using data on rents.

(PCE), 1991.Q1 to 2016.Q4).

For the time discount factors, we set $\beta^B = \beta^I = 0.950$ to target average borrower mortgage debt to housing wealth (LTV) of 64.3%, close to the corresponding value of 61.6% for the borrower population in the 1998 SCF. We set the discount rate of savers $\beta^D = 0.998$ to exactly match the observed nominal short rate of 3.1% per year or 0.76% per quarter.

With these parameters, the model generates a ratio of housing wealth to quarterly income for borrowers of 8.27, close to the 8.67 ratio for borrowers in the 1998 SCF. Total housing wealth, represents about 212.6% of annual GDP in the model and 153% in the data, that is, total housing wealth is overstated in the model. This discrepancy is an artifact of the model giving all agents the same housing-to-income ratio, while the "borrower" type holds relatively more housing in the data than the other groups. In equilibrium, only borrower holdings of housing are relevant, so the quantitative effect of exaggerating total housing wealth is minimal.

Government. We set the income tax rate τ in the model to match the average effective personal tax rate of 14.7% as reported by the BEA. We further set the fraction of bailout expenses funded through lump-sum taxation in the same period, τ_L, to 100%. This assumption guarantees that the outstanding balance of government debt B_t^G is always zero, which avoids government debt as a state variable. In Section VI.F, we test the sensitivity of our quantitative conclusions to a different taxation regime with a positive amount of government debt. We find that the assumption of instantaneous taxation does not significantly affect our quantitative conclusions about the different indexation schemes.

IV. Fixed-Rate Mortgage Benchmark

To establish a benchmark for the indexation results in the next section, we start by solving a model without indexation (No Index model). Mortgages are of the standard fixed-rate variety. Of particular importance is how the model behaves in a financial recession.

Unconditional Moments. We conduct a long simulation of the model and display the resulting averages of key prices and quantities in the first column of Table II. As discussed in the calibration section, the model generates an unconditional average mortgage debt to annual income ratio, LTV ratio among mortgage borrowers, and mortgage default, loss-given-default, and refinancing rates that match the data. The maximum LTV constraint, which only applies at origination and caps the LTV at 85%, always binds in simulation, consistent with the overwhelming majority of borrowers taking out new loans up to the limit.

On the intermediary side, the model matches the leverage ratio of the levered financial sector, which is 92.98% in the model. Banks' regulatory capital constraints bind in 100.00% of the periods in the baseline model. Bank equity capital represents 4.4% of annual GDP (17.6% of quarterly GDP) and 7.04% of bank assets in the model. Bank deposits (which go towards financing mortgage debt) represent just over 50.1% of annual GDP (200.3%/4). Bank dividends are 0.9% of GDP. The model generates a substantial amount of financial fragility. The bank default rate is 0.30% per quarter or 1.2% per year. Deadweight losses from bank bankruptcies represent 0.03% of GDP in an average year.

REO firms represent the other part of the intermediary sector. They spend 0.31% of GDP on housing maintenance, and pay 0.5% of GDP in dividends to their owners. REO firms earn high returns from investing in foreclosed properties and selling them back to borrowers: their return on equity is 5.4% per quarter.[25]

The mortgage rate, which was not directly targeted in the calibration, exceeds the short rate by 80 bps per quarter. This is close to the average spread between the 30-year FRM rate and the three-month T-bill rate of 89 bps per quarter over the 1991 to 2016 period. The mortgage spread compensates for time value of money, expected credit losses, and interest

[25]This return on equity in the model mimics the high returns earned by private equity firms. For example, the private equity fund Blackstone bought nearly 100,000 single-family homes in foreclosure during the financial crisis and recently exited that investment through an IPO of Invitation Homes. Internal rate of return targets of 20% per year are not uncommon for opportunistic real estate private equity funds.

rate, prepayment, and default risks. The expected excess return (risk premium) earned by banks on mortgages is 40 bps per quarter.

Financial Crises. To understand risk-sharing patterns in the benchmark FRM model, it is instructive to study how the economy behaves in a financial versus a nonfinancial recession. We define a nonfinancial recession event as a one-standard-deviation drop in aggregate income while the economy remains in the normal (nonhousing-crash) state. In a financial recession, the economy experiences the same decrease in aggregate income, but also transitions from the normal to the housing-crash state, leading to an increase in house value uncertainty ($\bar{\sigma}_{\omega,0} \rightarrow \bar{\sigma}_{\omega,1}$) and a decrease in housing utility ($\bar{\xi}_0 \rightarrow \bar{\xi}_1$). We simulate many such recessions to average over the endogenous state variables (namely, the wealth distribution). Figures 3 and 4 plot the impulse-response functions (IRFs), with financial recessions indicated by red circles and nonfinancial recessions in blue.[26] By construction, the blue and red lines coincide in the top left panel of Figure 3.

[Figure 3 about here.]

Figure 4 shows that a financial recession results in a significant increase in mortgage defaults. The risk on existing mortgages goes up but the fixed interest rates do not, leading the value of bank assets to fall. Faced with reduced equity, some banks fail, while the remaining ones are forced to delever in the wake of the losses they suffer, substantially shrinking both mortgage assets and deposit liabilities. As banks shed mortgage assets, savers expand their share of outstanding debt by over 50% relative to their pre-crisis position. To induce saver households to reduce demand deposits, the real interest rate falls (Figure 3). Savers' drop in deposit holdings is less than fully offset by their increase in mortgage holdings, and as a result saver consumption rises. Intermediary consumption drops heavily, as the owners of the intermediary sector absorb losses from their banks. Borrower consumption

[26]The simulations underlying these generalized IRF plots are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the non-housing-crash state $(\bar{\sigma}_{\omega,0}, \bar{\xi}_0)$.

also falls. Faced with higher mortgage rates, borrowers cut back on new borrowing, and they help pay for the bank bailouts through higher taxes. After the shock, the economy slowly recovers as high excess returns on mortgages eventually replenish bank equity.

[Figure 4 about here.]

V. Main Results on Mortgage Indexation

Our main exercise introduces indexation of mortgage principal and interest payments to house prices, and compares the resulting equilibrium to that in the No-Index economy. While the empirically relevant case — the Regional model — combines indexation to both aggregate and local house price shocks ($\iota_p = 1$ and $\iota_\omega = 1$), it turns out to be conceptually useful to seperate this case into an Aggregate model, with indexation only to national house prices ($\iota_p = 1$ and $\iota_\omega = 0$), and a Local model, with indexation only to the component of regional house prices that is orthogonal to the national index ($\iota_p = 0$ and $\iota_\omega = 1$). The two forms of indexation yield sharply different economic implications. Table II presents unconditional moments for the Aggregate, Local, and Regional models in columns (2), (3), and (4), respectively. Results under an extension of the model allowing for liquidity-driven "double-trigger" defaults can be found in Section III of the Internet Appendix and largely duplicate our benchmark findings.

[Table 2 about here.]

A. Aggregate Indexation

The conjecture in the literature is that indexing mortgage payments to aggregate house prices should reduce mortgage defaults and improves borrower's ability to smooth consumption. Perhaps surprisingly, we find that this conjecture does not hold up in general equilibrium. To the contrary, by adding to financial fragility (bank default rates nearly quadruple), aggregate indexation destabilizes borrower consumption (its volatility increases nearly 240%) while leaving mortgage default rates unchanged.

To understand this result, Figure 5 compares financial recessions in the No-Index (black line) and Aggregate (red line) models. Under aggregate indexation, banks find themselves exposed to increased risk through their loan portfolio, whose cash flows now fluctuate directly with aggregate house price movements. Although banks optimally choose to hold slightly more capital, the extra buffer is insufficient to protect their equity from the much greater risks they face. The rise in default risk increases the value of the bankruptcy option. Left with a trade-off between preserving franchise value and exploiting limited liability, banks optimally lean more toward their option to declare bankruptcy and saddle the government with the losses.

The combination of increased risk and the absence of precautionary capital means that the share of bank defaults upon entering a financial recession is vastly larger in the Aggregate economy, with nearly 40% of banks failing. This spike in bank failures necessitates a wave of government bailouts of bank deposits, placing a large tax burden of 0.6% of quarterly GDP on the population. This tax obligation depresses borrower consumption and housing demand, contributing to a larger drop in house prices relative to the benchmark. The breakdown in intermediation and risk-sharing is reflected in the upward spike in saver consumption while at the same time borrower and intermediary households have to cut consumption sharply.[27] Savers' holdings of mortgage debt provide some relief to offset the instability of intermediaries but only moderate the crisis.

[Figure 5 about here.]

Aggregate indexation provides a modest reduction in mortgage defaults in the financial recession. Although Aggregate indexation protects borrowers from the large decrease in na-

[27] In Section VI we allow the government to fund the bailout expenditure in part by issuing government debt. We confirm that these dynamics do not depend on the assumption of immediate taxation, but rather are a result of the breakdown in mortgage credit.

tional house prices, it is unable to stave off the increase in defaults due to higher idiosyncratic dispersion $\sigma_{\omega,t}$ that accompanies the financial recession. Importantly, Aggregate indexation provides equal relief to the hardest-hit and relatively unaffected regions/households alike. This indiscriminately targeted aid limits the policy's effect on the number of foreclosures.

The bottom half of Table II compares welfare and consumption outcomes across the different indexation regimes.[28] Aggregate indexation is bad for aggregate welfare. We present two schemes to aggregate the value functions of the three types of agents. The first simply adds up the value functions, which are expressed in consumption units and already reflect the population mass of each type of agent. This measure shows a 0.16% welfare loss from Aggregate indexation. The second approach computes the one-time payments each type of household would be willing to make to transition permanently from the No Index to the alternative indexation economy. Different agents have different valuations for a dollar of consumption since their SDFs differ. We weigh these consumption-equivalent values (CEVs) by the population shares and add up across the types. A positive CEV indicates that the indexation is a Pareto improvement after transfers. Aggregate indexation results in a negative 9.3% CEV, implying that agents would need to receive a one-time payment of 9.3% of aggregate consumption in the No-index economy to be willing to switch to Aggregate indexation. Thus, both measures suggest a welfare loss to society.

Underlying the aggregate welfare result are interesting distributional differences. Borrowers are made worse off (row 20). Their consumption is lower (row 23) and becomes much more volatile (row 26). The increased financial fragility from Aggregate indexation results in incredibly volatile intermediary wealth (W^I growth volatility goes up 1268.6%). Intermediary consumption growth volatility goes up 284.8%, and intermediary consumption falls sharply in a financial recession. These results point to a deterioration in risk-sharing between

[28]Since steady state comparisons may not reflect the welfare consequences of transiting between steady states, Table IA.III in the Internet Appendix presents welfare the complete welfare consequences of switching policies along the transition path. This procedure delivers highly similar implications for policy.

borrowers and intermediaries, further evidenced by an increase of 134.2% in the volatility of the log marginal utility ratio between these types (row 31). Despite the increase in consumption volatility, intermediaries are made better off (row 22). Aggregate indexation raises the average credit spread, mortgage risk premium, and REO returns, and hence the profitability of intermediation. Also, Aggregate indexation increases the value of banks' default option, allowing for very high consumption in good times but limited downside in bad times. Banks thrive when financial turmoil is large. Finally, savers' welfare decreases modestly (row 21) due to lower consumption in financial recessions, which are high marginal utility times. The latter is due in part to higher taxes that need to be raised to cover losses from bank bailouts. Since savers are more patient, they have a larger shadow value of consumption; their welfare loss weighs heavily in the CEV welfare measure. All told, insuring borrower exposure to aggregate house price risk paradoxically hurts the borrowers it was meant to help as well as savers but benefits financial intermediaries.

B. *Local Indexation*

Next, we turn to the Local economy ($\iota_p = 0, \iota_\omega = 1$), which indexes only to the local component of house values. In practice, such a contract would be implemented by subtracting an aggregate house price index from regional indexes and then indexing the debt of local borrowers to the local residual. For example, during the Great Recession house prices fell substantially more in Las Vegas than in Boston. Local indexation would have implied a reduction in mortgage debt for Las Vegas borrowers but an increase in debt for Boston borrowers. While such indexation is unlikely to ever see the light of day, it is an important building block for Regional indexation.

In sharp contrast to Aggregate indexation, indexing mortgage debt to relative local house prices stabilizes the financial sector while substantially reducing the frequency of borrower defaults. Figure 6 compares financial recessions in the No-Index and Local models. Although borrowers must absorb a similar decrease in aggregate house prices as in the baseline, Local indexation is still largely successful at reducing foreclosures. It sends targeted debt relief to households in areas where house prices fall the most.[29] Unlike in the Aggregate indexation case, the reduction in defaults under local indexation is not accompanied by large financial sector losses, since the diversifiable local shocks wash out. As a result, the rate of bank failures in a financial recession is markedly *lower* under Local indexation, a sign of improved financial stability. Savers hold a smaller share of mortgage debt directly when intermediaries are more stable.

Turning to unconditional moments in the third column of Table II, we observe that the average mortgage default rate falls precipitously, with a reduction of nearly half relative to the benchmark. While Aggregate and Local indexation are roughly equally effective at reducing default in a housing crash, when default is largely driven by aggregate house prices, local indexation is much more effective than Aggregate indexation in normal times, when default is primarily driven by local and idiosyncratic shocks. Facing less default risk, banks reduce mortgage interest rates. This pushes up house values and supports increased household borrowing. The higher average stock of mortgage debt is financed with a larger deposit base. While banks react to this reduced risk by holding as little capital as allowed, the required minimum is sufficient to ensure a large decrease in the rate of bank failures. The risk-free interest rate rises slightly as the supply of deposits expands to meet the demands of a larger intermediation sector. Overall, the banking system is both safer and larger in the Local economy, but it receives less compensation for risk on a per-loan basis.

[Figure 6 about here.]

The welfare effects of Local indexation are the reverse of those of Aggregate indexation.

[29]For intuition, recall that the average borrower in the model, similar to the data, has leverage around 65%. Thus, the typical borrower could absorb a very large decrease in aggregate house prices (on the order of the 2008 housing crash) and still remain above water. Instead, the typical defaulting borrower must *also* receive an adverse local or idiosyncratic shock. Effectively indexing against these shocks is therefore a potent force against default, even during an aggregate house price decline.

Local indexation is good for aggregate welfare according to both measures. The population-weighted welfare function increases by 0.183%, and agents would be willing to pay 44.9% of aggregate consumption to transition to Local indexation according to the CEV criterion. Borrowers and savers gain while intermediaries lose. Risk-sharing in the economy improves dramatically, as the volatility of marginal utility ratios between groups falls, especially between borrowers and intermediaries. Savers and intermediaries also see large reductions in consumption growth volatility, while borrowers experience increased volatility — albeit from a low level — due to larger housing and mortgage positions.[30]

In sum, indexation to local house price shocks is highly effective at reducing the risk of foreclosures and financial fragility. More intermediation ensues, which makes both borrowers and savers richer. However, the increased safety makes banking less profitable.

C. *Regional Indexation*

The fourth column of Table II reports results for the Regional model, which indexes mortgage principal and interest payments to both aggregate and local house price variation. Unsurprisingly, the simulation means in this column tend to lie between the Aggregate and Local cases in columns (2) and (3). Pairing Local and Aggregate indexation decreases the bank default rate in the Regional model relative to the Aggregate model. But the destabilizing effect of Aggregate indexation is still enough to increase bank defaults relative to the No-Index baseline. The high consumption and wealth growth volatilities of the intermediary are further signs of financial instability. The high degree of indexation in this economy strongly reduces the incentives to default, leading to the lowest borrower default rates across the four models. Aggregate welfare is 0.09% higher in the Regional model than in the No-

[30]The smaller changes in intermediary and saver consumption during crises (top row of Figure 6) emphasize this point. Savers earn higher interest rates under this system, while borrowers pay lower rates on their mortgages, helping to boost the consumption of each group. In contrast, intermediary households' mean consumption falls by 2.5% as dividends from REO firms and banks decline.

Index model according to the population-weighted measure, and the willingness to pay is 37.1% of according to the CEV measure.

VI. Extensions

We consider several extensions, with details relegated to the Internet Appendix.

A. Interest versus Principal Indexation

So far, our indexation applies to both interest payments and principal. However, a number of the contract proposals in the literature consider indexing to interest payments only. These proposals are motivated by empirical work by Fuster and Willen (2015) and Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), who suggest that households respond strongly to interest payment adjustments, and Ganong and Noel (2019a), who show that households respond little to principal adjustments, at least when the latter leave them underwater. We run experiments in which either interest or principal payments, but not both, are indexed to house prices. The corresponding default thresholds are derived in Section II.A of the Internet Appendix.

The first four columns of Table III contrast the No-Index and Regional models with Reg-IO and Reg-PO specifications that index interest-only and principal-only payments to regional house prices, respectively. The main result is that indexing interest only greatly dilutes the effects of indexation, reducing its ability to mitigate borrower defaults, while indexing principal only delivers results very similar to full indexation. Quantitatively, the Reg-IO model delivers a borrower default rate of 0.82%, which is much higher than the Regional model's 0.47% but is close to the 0.97% of the No Index model. The Reg-PO model's 0.51% default rate is nearly as low as that of the Regional model.

This result is perhaps surprising given that our baseline model mortgage payments are on average 75% interest and 25% principal, closely matching reality. The key to this result is that our model mortgages are prepayable, and our model borrowers (realistically) choose to refinance or renew them every six to seven years. But while a lower principal balance provides equity extraction opportunities at this time, the interest rate is reset upon receiving a new loan, wiping out further gains from interest indexation. As a result, the temporary gains from interest forgiveness under IO indexation are valued less than the permanent gains from principal forgiveness under PO indexation, leading to a smaller overall impact. By the same logic, forgiving interest payments is less costly to intermediaries than forgiving principal, mitigating their losses during housing declines, and avoiding an increase in financial fragility and bank defaults.

[Table 3 about here.]

B. Asymmetric Indexation

Some real-world SAM proposals consider reducing mortgage payments when house prices fall but not increasing payments when prices rise. We now study such asymmetric contracts. We assume indexation to both aggregate and local house price components (Regional model), but cap the maximum upward indexation in both dimensions. With asymmetric indexation, our assumption of i.i.d. house quality shocks $\omega_{i,t}$ is no longer equivalent to more realistic persistent $\omega_{i,t}$ processes. To address this, we model the $\omega^L_{i,t}$ and $\omega^U_{i,t}$ shocks as AR(1) processes. Section II.B of the Internet Appendix provides details on this extension and the corresponding optimality conditions.

Column (5) of Table III presents the results for the Reg-Asym case. We find that asymmetric indexation substantially alters the mortgage landscape. Banks now expect to take losses on average from indexation, since the debt relief they offer on the downside is no longer compensated by higher debt payments when house prices increase. As a result, banks set much higher mortgage rates ex-ante, 3.24% higher per year than without indexation, to compensate for the asymmetric transfers to households. At the aggregate, this has an effect similar to shortening the mortgage amortization schedule (lower δ), since borrowers make higher coupon payments in exchange for a much larger effective principal reduction each period, albeit one occurring largely through indexation rather than explicit principal payments. House prices are lower, reflecting the lower collateral value of housing under this more front-loaded contract. Lower house prices imply lower mortgage balances, lower deposits, and a smaller financial sector overall. Savers intermediate a larger share of mortgage debt.

Although borrowers partially compensate for the higher mortgage rates by increasing the refinancing rate, the faster effective amortization of these loans dominates, reducing household leverage. Lower leverage in turn virtually eliminates foreclosures, since it now takes much larger shocks to push borrowers underwater. Nonetheless, financial fragility is massively increased under Reg-Asym. When indexation is symmetric, the large losses that the financial sector suffers when house prices decline are partially offset by the expected gains from indexation as house prices rise. Asymmetric indexation removes this mitigating force, leading to an extremely high bank failure rate of 0.83%, more than twice as high as in the symmetric Regional model.

Turning to total welfare, the gain of +0.41% is the highest among all contracts we consider. These gains are driven by a decrease in deadweight losses from foreclosure, increasing aggregate consumption, and overpowering the deterioration in risk-sharing observed from this model's high volatilities of borrower consumption and intermediary wealth. However, we note that since these foreclosure reductions occur largely through lower household leverage, other measures to reduce household leverage (for instance, lowering maximum LTVs) might attain the same benefits without increasing financial fragility. Although borrowers must finance more bailouts under asymmetric indexation, they are more than compensated by house price gains in good times, which they retain under asymmetric indexation.

Asymmetric IO. Column (6) of Table III presents the asymmetric indexation of interest payments only (Asym-IO), leaving the principal balance and payments unindexed. Similar to the findings in Section VI.A, indexing interest only dilutes the positive welfare effects of the Reg-Asym contract. The Asym-IO model has a higher foreclosure rate (0.54% versus 0.13%) and a lower bank failure rate (0.29% versus 0.83%). Household leverage again falls, in part for the same reasons as in the Reg-Asym case, and in part due to a different and novel force. Because interest is reduced over time through indexation but principal is not, the effective interest rates on existing loans tend to be lower than the interest rates on new loans. Borrowers respond by refinancing their loans less often, causing longer periods between equity extractions and reducing average leverage. Overall, the Asym-IO contract is much less disruptive than the full Reg-Asym contract, delivering a substantial reduction in foreclosures while slightly reducing bank failures and improving measures of risk-sharing (rows 26 to 31).

Tail Indexation. The final contract type we consider is tail indexation (Reg-Tail), in which the borrower is responsible for the first 10% of regional price declines and the lender fully indexes any decline beyond that threshold. This scheme is similar to the Reg-Asym scheme, except that indexation kicks in at a positive level of losses instead of at zero losses. The resulting economy features a foreclosure rate of 0.34% and a bank failure rate of 0.38%, both of which are improvements over the Regional model. This superior performance is due to the more efficient intervention of the Reg-Tail model, which provides only enough relief to prevent households from becoming underwater, in contrast to the Reg-Asym model, which seeks to insure *all* house price declines. Avoiding excessive indexation allows for effective reduction in the default rate without overburdening the financial sector, limiting the increase in financial fragility.

C. Partial Indexation

So far we have considered full Aggregate indexation ($\iota_p = 1$), full Local indexation ($\iota_\omega = 1$), and both. But might intermediate levels of indexation be optimal? Panel A of

Figure 7 gradually adds Aggregate indexation to an economy that already has full Local indexation. Panel B gradually adds Local indexation to an economy with full Aggregate indexation. Panel C gradually adds both types of indexation in lock-step. The effect on the value function on each type of agent is indicated by bars and measured against the right axis, while the population-weighted aggregate welfare measure is indicated by a solid line plotted against the left axis. Each set of bars increases indexation by 25%. The end point in each panel is the same Regional economy but the starting point and hence the welfare changes are different.

[Figure 7 about here.]

Adding even a small amount of Aggregate indexation to an economy that already has full local indexation is not good for welfare. The gains to the borrowers and the intermediaries do not outweigh the losses to the savers. The Local indexation provides a good amount of financial sector stability. Adding 25% or 50% Aggregate indexation reduces borrower and bank default rates, decreases mortgage rates, and increases mortgage debt and house prices. When Aggregate indexation becomes greater than 50%, financial fragility increases and borrowers begin to lose relative to the world with only Local indexation.

Panel B shows that adding Local indexation to an economy that has Aggregate indexation monotonically increases welfare. Borrowers and savers gradually gain by more while intermediaries gradually lose. The same result holds in Panel C for an economy that gradually implements Regional indexation, starting from No Indexation.

D. *Tighter Bank Leverage Constraints*

A possible response to the destabilizing effects resulting from Aggregate indexation could be to tighten bank capital requirements. Solving the model with a minimum bank equity capital ratio of 10% rather than 7%, we find that tighter leverage does indeed reduce bank failure rates substantially from 1.08% to 0.22%, but it does not lower the welfare losses from Aggregate indexation. The reason is that tighter macro-prudential policy shrinks the banking sector. Aggregate indexation shrinks deposits by 6% when banks' minimum capital is 7% but by more than twice that (13%) when bank capital requirements are 10%. Reduced intermediation capacity results in higher credit spreads and larger welfare gains for banks. But savers' and borrowers' welfare losses are substantially larger, due to the larger decrease in deposits and the higher mortgage rates, respectively. Tighter bank capital requirements cannot rescue Aggregate indexation.

E. Risk Absorption Capacity

We perform two exercises to analyze the sensitivity of the results to the intermediary's risk absorption capacity.

Intermediary Population Share. In a first exercise, we change the population share of intermediaries from 5% to 3%, 4%, 6%, or 10%.[31] Regional indexation delivers an aggregate population-weighted welfare gain of 0.09% at the benchmark 5% population share. The welfare gain is increasing in the intermediaries' population share, from -0.77% at a 3% share to +0.18% at a 10% share (Internet Appendix Table IA.II). With lower intermediary risk absorption capacity, there is more financial fragility. Mortgage defaults are slightly higher but bank failure rates are substantially higher. Bank default rates are 7.5 times higher for the 3% than for the 10% economy. The 3% economy has much higher credit spreads and mortgage risk premia than the 10% economy, resulting in lower mortgage debt, lower house prices, and lower borrower welfare. In sum, we are getting the intuitive result that the welfare

[31]To identify the corresponding income shares for the intermediaries in these four model variants, we first find a new risky asset share cutoff in the SCF data that delivers the desired population share. The intermediary income share is then the observed share of income in the SCF for the resulting group of households whose risky asset share is above the cutoff. The saver income share is the income share of the complement group of households. The population share of savers changes in the opposite direction and by the same absolute value as that of the intermediary households.

effects of Regional indexation depend on the risk absorption capacity of the intermediary sector. Regional indexation achieves positive welfare effects only when intermediaries have sufficiently large risk absorption capacity.

Saver Holdings of Mortgages. In a second exercise, we switch off the ability of savers to directly hold mortgage debt by increasing the cost parameter φ_0 to a very high value. Intermediaries are responsible for all mortgage market intermediation. They choose to hold more equity capital in this economy, not only in dollar terms but also as a ratio of bank assets. They face less financial fragility as a result; baseline bank default rates are only 0.10% versus 0.30% in the model with saver holdings of mortgage debt. Aggregate indexation is better for overall welfare and Local indexation is slightly worse in the model without saver holdings. Regional indexation, which combines both, results in the same quantitative welfare gain in the models with and without saver holdings.

The key difference between both models is that risk-free rates fall to a greater extent during a financial recession in the economy without direct saver holdings. This benefits banks because it aids their subsequent recapitalization. The intuition for this effect is as follows. If savers cannot directly hold mortgages, then their only store of wealth are bank deposits. During recessions, banks significantly shrink their deposit issuance, which is an inward shift of the demand curve in the deposit market. As a result, the deposit interest rate drops sharply. When savers can also directly hold mortgages, their supply of deposits to banks effectively becomes more elastic. Therefore, the interest rate falls by less in housing recessions when savers can directly invest in mortgages. Since deposit interest rates fall by less, banks earn lower returns during the transition out of a housing recession, leading to a more sluggish recovery. In sum, our main indexation results on the merits of Aggregate and Local indexation are slightly *amplified* when savers hold mortgage debt directly.

F. Government Debt

Our baseline model assumes that the government raises lump-sum taxes to fully pay for bank bailouts within each period. When a large fraction of banks fail, the taxes required to fund the bailout reduce consumption, most notably in the aggregate indexation model (Figure 5). This immediate tax burden might be smaller if the government financed bailouts with debt, potentially reducing the severity of financial recessions. To test the sensitivity of housing-crash dynamics to different taxation regimes, we solve the Aggregate model with some tax smoothing. Each period, the government uses taxes to pay 80% of its outstanding liabilities (past debt plus expenses for current bailouts), with the remainder funded by new debt.

Internet Appendix Figure IA.2 compares housing-crash dynamics in the Aggregate indexation model with government debt ($\tau_L = 0.8$) to the Aggregate model with immediate taxation ($\tau_L = 1$). Borrower consumption falls by slightly less on impact, as the tax burden is postponed further into the future. However, saver consumption is substantially reduced, as savers must purchase the government debt that funds the bailout. To induce the savers to absorb this debt, the real risk-free interest rate, which is both the deposit rate and the yield on government debt, needs to increase compared to the immediate-taxation model. At this higher real rate, banks issue fewer deposits as government safe asset provision crowds out private safe asset production (Azzimonti and Yared, 2018). The higher real rate increases banks' funding cost and compresses mortgage spreads, depressing intermediary consumption. Banks respond to the lower supply of deposits by cutting their lending more sharply, which reduces the availability of mortgage credit to borrowers and this leads to a sharper drop in house prices. At the same time, higher funding costs reduce bank net worth, increasing the rate of bank failures. Perhaps surprisingly, financial fragility increases in the economy with government debt. The comparison demonstrates that the severe housing-crash dynamics with Aggregate indexation are not an artifact of the assumption of instantaneous taxation. Instead, the primary driver of the steep drop in house prices is the sharp contraction in the size of the financial sector. This contraction is only amplified when bailouts are funded through government debt since the higher cost of deposit funding leads to a larger decline in lending.

VII. Conclusion

Redesigning the mortgage market through product innovation may allow an economy to avoid a severe foreclosure crisis like the one that hit the U.S. economy in 2008. To this end, we study the implications of indexing mortgage payments to house prices in a general equilibrium model with incomplete risk-sharing, costly default, and a rich intermediation sector. A key finding is that indexing mortgage debt to aggregate house prices may increase financial fragility. Inflicting large losses on highly levered lenders in bad states of the world can lead to systemic risk (high bank failure rates), costly taxpayer-financed bailouts, larger house price declines, and higher risk premia on mortgages, all of which ultimately hurt the borrowers the indexation was intended to help. Moreover, aggregate indexation redistributes wealth from borrowers and savers towards bank owners, since a more fragile banking business also is a more profitable banking business. In sharp contrast, indexation of cross-sectional local house price risk is highly effective at reducing mortgage defaults and financial fragility. It increases welfare for borrowers and savers, while reducing it for intermediaries, as mortgage banking becomes safer but less profitable.

Our results show that mortgage indexation in a world in which intermediaries have limited liability and risk absorption capacity has important general equilibrium effects. Although potential benefits exist, indexation schemes must be carefully designed to achieve them. We conclude that less invasive approaches such as our tail indexation model that concentrate on limiting the severe losses that cause defaults, leaving mortgages unindexed when they appear far from default, could provide substantial benefits with minimal disruption to financial stability.

The framework proposed in this paper could be extended in several directions to allow for other costs and benefits of mortgage indexation. Considering imperfectly insurable idiosyncratic labor income risk and its interaction with mortgage indexation would be a fruitful extension. We conjecture that adding uninsurable individual income risk to our setup would further strengthen the benefits of local indexation, since indexation to local house price shocks would provide insurance against local labor market risk. A second promising extension could consider an economy in which indexed and non-indexed contracts co-exist, with the share of indexed contracts varying endogenously with the state of the economy. To the extent that these shares covary with the health of the financial sector, this might have important implications for the costs of indexation during a crisis.

Table I
Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Target/Source
		Technology	
Agg. income persistence	ρ_{TFP}	0.977	Real per capita labor income BEA
Agg. income st. dev.	σ_{TFP}	0.008	Real per capita labor income BEA
Profit shock st. dev.	σ_{ϵ}	0.065	FDIC bank failure rate
Transition: Normal $\rightarrow$ Normal	Π_{00}	0.975	Avg. length = 10Y
Transition: Crisis $\rightarrow$ Crisis	Π_{11}	0.925	25% of time in housing-crash state
		Demographics and Income	
Fraction of borrowers	χ_B	0.343	SCF 1998 population share LTV>0.30
Fraction of intermediaries	χ_I	0.050	Stock holders in SCF 1998
Borr. inc. and housing share	s_B	0.470	SCF 1998 income share LTV>0.30
Intermediary inc. and housing share	s_I	0.062	Income stock holders in SCF 1998
		Housing and Mortgages	
Housing stock	$\bar{K}$	1	Normalization
Housing XS persistence	ρ_{ω}	0.977	FHFA MSA-level regression
Housing XS dispersion (Normal)	$\bar{\sigma}_{\omega,0}$	0.200	Mortg. delinq. rate U.S. banks, no housing-crash state
Housing XS dispersion (Crisis)	$\bar{\sigma}_{\omega,1}$	0.250	Mortg. delinq. rate U.S. banks, housing-crash state
Local share of XS dispersion	α	0.25	FHFA MSA-level regression
Inflation rate	$\bar{\pi}$	1.006	2.29% CPI inflation
Mortgage duration	δ	0.996	Principal amortization on 30-yr FRM
Prepayment cost mean	μ_{κ}	0.370	Greenwald (2018)
Prepayment cost scale	s_{κ}	0.152	Greenwald (2018)
LTV limit	ϕ^K	0.850	LTV at origination
Maintenance cost (owner)	ν^K	0.616%	BEA Fixed Asset Tables
		Intermediaries	
Bank regulatory capital limit	ϕ^I	0.930	Financial sector leverage limit
Deadweight cost of bank failures	η	0.050	Bank receivership expense rate
Maintenance cost (REO)	ν^{REO}	0.022	REO discount: $p_{ss}^{REO}/p_{ss} = 0.725$
REO sale rate	S^{REO}	0.167	Length of foreclosure crisis
		Savers	
Mortgage holding cost, coeff.	φ_0	0.200	Avg. HH sector's share m. debt, FoF
Mortgage holding cost, expon.	φ_1	5.000	Vol. of HH sector's share m. debt, FoF
		Preferences	
Borrower discount factor	β_B	0.950	Borrower LTV, SCF
Intermediary discount factor	β_I	0.950	Equal to β_B
Saver discount factor	β_S	0.998	3% nominal short rate (annual)
Risk aversion	γ	2.000	Standard value
EIS	ψ	1.000	Standard value
Housing preference (Normal)	$\bar{\xi}_0$	0.210	Borrower hous. expend./income
Housing preference (Crisis)	$\bar{\xi}_1$	0.160	HP growth volatility
		Government	
Income tax rate	τ	0.147	Personal tax rate BEA
Bailout taxation rate	τ_L	1.0	Tractability, relaxed in Section VI.F

Table II
Results, Main Indexation Experiments

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with indexation to relative local prices (third column), and a model with both aggregate and local indexation (fourth column). Rows 18 to 31 of columns (2) to (4) calculate percentage differences relative to the benchmark model. All flow variables are quarterly. Welfare results including the transition between steady states can be found in Internet Appendix Table IA.III.

	No Index (1)	Aggregate (2)	Local (3)	Regional (4)
	Borrower			
1. Housing capital	0.456	0.456	0.462	0.463
2. Refi rate	3.82%	3.77%	3.76%	3.73%
3. Default rate	0.97%	0.98%	0.51%	0.47%
4. Household leverage	64.31%	64.29%	65.71%	65.61%
5. Mortgage debt to income	250.06%	239.42%	267.96%	261.95%
6. Loss-given-default rate	37.31%	35.52%	36.76%	35.75%
7. Loss rate	0.40%	0.40%	0.21%	0.19%
	Intermediary			
8. Bank equity ratio	7.04%	7.14%	7.13%	7.22%
9. Bank default rate	0.30%	1.08%	0.16%	0.40%
10. DWL of bank defaults	0.03%	0.10%	0.02%	0.04%
11. Deposits	2.003	1.882	2.174	2.098
12. Saver mortgage share	14.43%	15.81%	13.30%	14.17%
	Prices			
13. House price	8.533	8.161	8.832	8.616
14. Risk-free rate	0.76%	0.75%	0.77%	0.77%
15. Mortgage rate	1.56%	1.67%	1.37%	1.43%
16. Credit spread	0.80%	0.92%	0.60%	0.66%
17. Mortgage risk prem.	0.40%	0.52%	0.39%	0.46%
	Welfare			
18. Aggregate welfare	0.872	-0.16%	+0.18%	+0.09%
19. CEV welfare	+0.00%	-9.35%	+44.93%	+37.08%
20. Value function, B	0.398	-0.63%	+0.51%	+0.19%
21. Value function, S	0.408	-0.04%	+0.23%	+0.20%
22. Value function, I	0.066	+1.93%	-2.06%	-1.18%
	Consumption and Risk-sharing			
23. Consumption, B	0.382	-0.8%	+0.6%	+0.3%
24. Consumption, S	0.404	+0.0%	+0.1%	+0.2%
25. Consumption, I	0.067	+3.2%	-2.5%	-1.1%
26. Consumption gr vol, B	0.55%	+238.6%	+12.5%	+26.2%
27. Consumption gr vol, S	1.14%	-1.0%	-25.3%	-18.1%
28. Consumption gr vol, I	5.66%	+284.8%	-49.7%	+134.2%
29. Wealth gr vol, I	0.045	+1268.6%	-37.4%	+375.9%
30. log (MU B / MU S) vol	0.026	-0.3%	-9.2%	-35.1%
31. log (MU B / MU I) vol	0.068	+134.2%	-35.1%	+66.1%

Table III
Results, Alternative Indexation Schemes

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with regional indexation (second column), a model with regional interest indexation only (third column), a model with regional principal indexation only (fourth column), a model with regional asymmetric indexation (fifth column), and a model with regional asymmetric interest indexation only (sixth column). Rows 18 to 31 of columns (2) to (7) calculate percentage differences relative to the benchmark model. All flow variables are quarterly. Welfare results including the transition between steady states can be found in Internet Appendix Table IA.IV.

	No Index (1)	Regional (2)	Reg-IO (3)	Reg-PO (4)	Reg-Asym (5)	Asym-IO (6)	Reg-Tail (7)
	Borrower						
1. Housing capital	0.456	0.463	0.458	0.462	0.468	0.461	0.465
2. Refi rate	3.82%	3.73%	3.72%	3.76%	4.40%	3.54%	4.28%
3. Default rate	0.97%	0.47%	0.82%	0.51%	0.13%	0.54%	0.34%
4. Household leverage	64.31%	65.61%	65.19%	65.52%	58.47%	62.65%	60.03%
5. Mortgage debt to income	250.06%	261.95%	260.03%	265.09%	230.12%	257.11%	239.16%
6. Loss-given-default rate	37.31%	35.75%	38.88%	38.36%	33.71%	29.23%	36.63%
7. Loss rate	0.40%	0.19%	0.27%	0.22%	0.91%	0.35%	0.90%
	Intermediary						
8. Bank equity ratio	7.04%	7.22%	7.04%	7.13%	6.95%	6.81%	6.98%
9. Bank default rate	0.30%	0.40%	0.24%	0.32%	0.83%	0.29%	0.38%
10. DWL of bank defaults	0.03%	0.04%	0.02%	0.03%	0.07%	0.03%	0.03%
11. Deposits	2.003	2.098	2.090	2.135	1.803	1.991	1.898
12. Saver mortgage share	14.43%	14.17%	13.87%	13.82%	16.43%	14.09%	15.24%
	Prices						
13. House price	8.533	8.616	8.686	8.742	8.409	8.571	8.566
14. Risk-free rate	0.76%	0.77%	0.76%	0.75%	0.76%	0.76%	0.77%
15. Mortgage rate	1.56%	1.43%	1.43%	1.40%	2.37%	1.61%	2.10%
16. Credit spread	0.80%	0.66%	0.68%	0.65%	1.61%	0.84%	1.33%
17. Mortgage risk prem.	0.40%	0.46%	0.39%	0.43%	0.50%	0.41%	0.40%
	Welfare						
18. Aggregate welfare	0.872	+0.09%	+0.03%	+0.10%	+0.41%	+0.08%	+0.36%
19. CEV welfare	+0.00%	+37.08%	+7.74%	+14.77%	-13.52%	+3.01%	-4.94%
20. Value function, B	0.398	+0.19%	+0.16%	+0.32%	+1.75%	+0.56%	+1.54%
21. Value function, S	0.408	+0.20%	+0.04%	+0.08%	-0.10%	+0.01%	-0.05%
22. Value function, I	0.066	-1.18%	-0.78%	-1.14%	-4.49%	-2.38%	-4.25%
	Consumption and Risk-sharing						
23. Consumption, B	0.382	+0.3%	+0.2%	+0.5%	+2.1%	+0.7%	+1.9%
24. Consumption, S	0.404	+0.2%	+0.0%	+0.1%	-0.0%	+0.0%	-0.1%
25. Consumption, I	0.067	-1.1%	-0.7%	-1.4%	-5.2%	-2.9%	-5.4%
26. Consumption gr vol, B	0.55%	+26.2%	-13.3%	-24.2%	+57.3%	-1.0%	+15.6%
27. Consumption gr vol, S	1.14%	-18.1%	-9.9%	-14.8%	-20.9%	-19.7%	-8.8%
28. Consumption gr vol, I	5.66%	+134.2%	-6.4%	+96.9%	+128.6%	-25.7%	-26.6%
29. Wealth gr vol, I	0.045	+375.9%	-12.6%	+272.4%	+687.5%	-8.1%	+21.4%
30. log (MU B / MU S) vol	0.026	-35.1%	-9.6%	-37.5%	-13.2%	-12.6%	+8.6%
31. log (MU B / MU I) vol	0.068	+66.1%	-6.9%	+49.0%	+42.8%	-24.3%	-44.7%

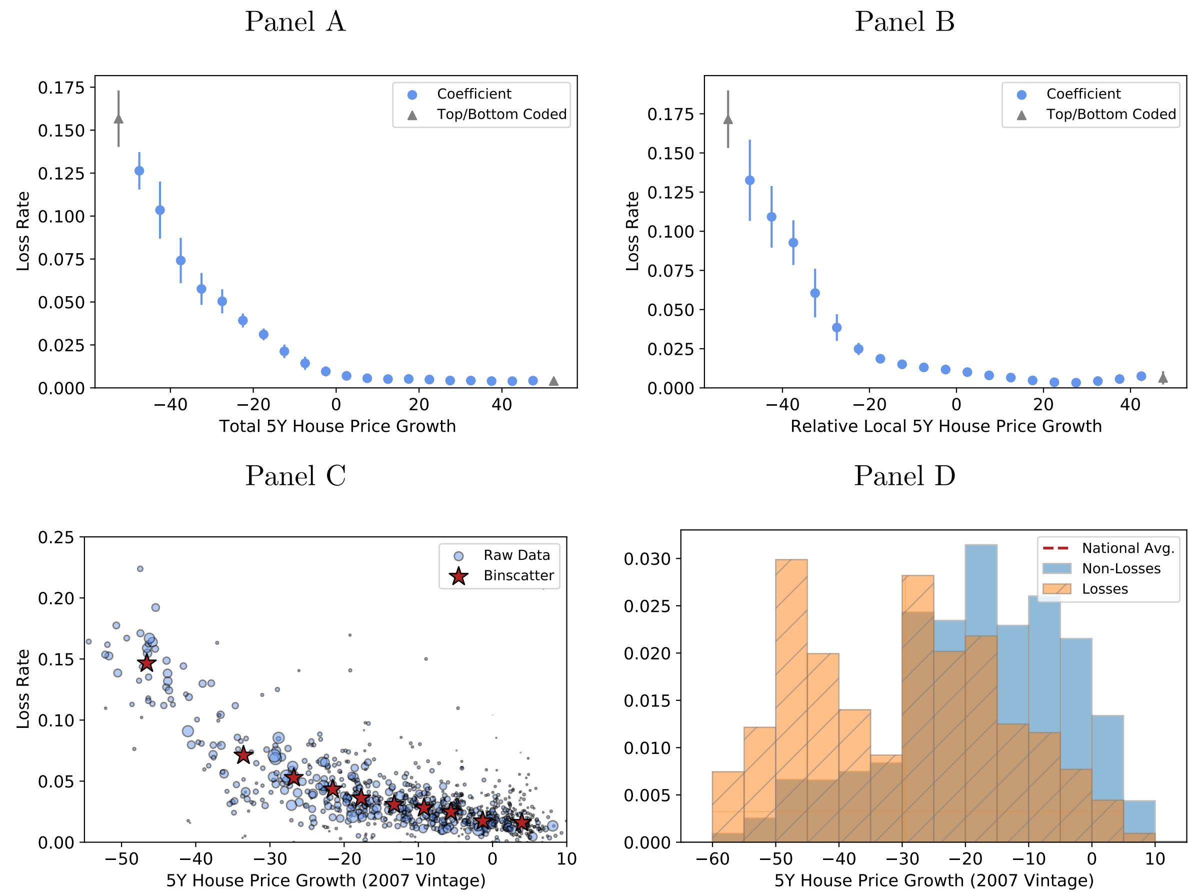

Figure 1. Loan losses versus house prices, Freddie Mac loan performance data. Panel A shows loss rate by total house price growth. Panel B shows loss rate by local house price growth. Panel C shows loss rate by house price growth for the 2007 vintage. Panel D shows losses versus repaid balances. Source is Freddie Mac Single Family Loan-Level Dataset. The "loss rate" is the ratio of total losses on the loan (losses to Freddie Mac plus recoveries from private mortgage insurers) to the original principal balance on the loan. "Total 5Y House Price Growth" is the percent growth in the FHFA all-transactions house price index in each ZIP-3 area. "Relative 5Y House Price Growth" is the same series, removing the mean of the series for each quarter. "5Y House Price Growth (2007 Vintage)" displays the "Total 5Y House Price Growth" series for loans originated in 2007. In Panel (D), "National Avg." displays the average national house price growth over the following five-year period, averaged over each quarter in 2007, "Nonlosses" display the share of repaid original principal balances in each house price growth bin, and "Losses" display the share of total losses in each house price growth bin. See Section IV.A of the Internet Appendix for further details.

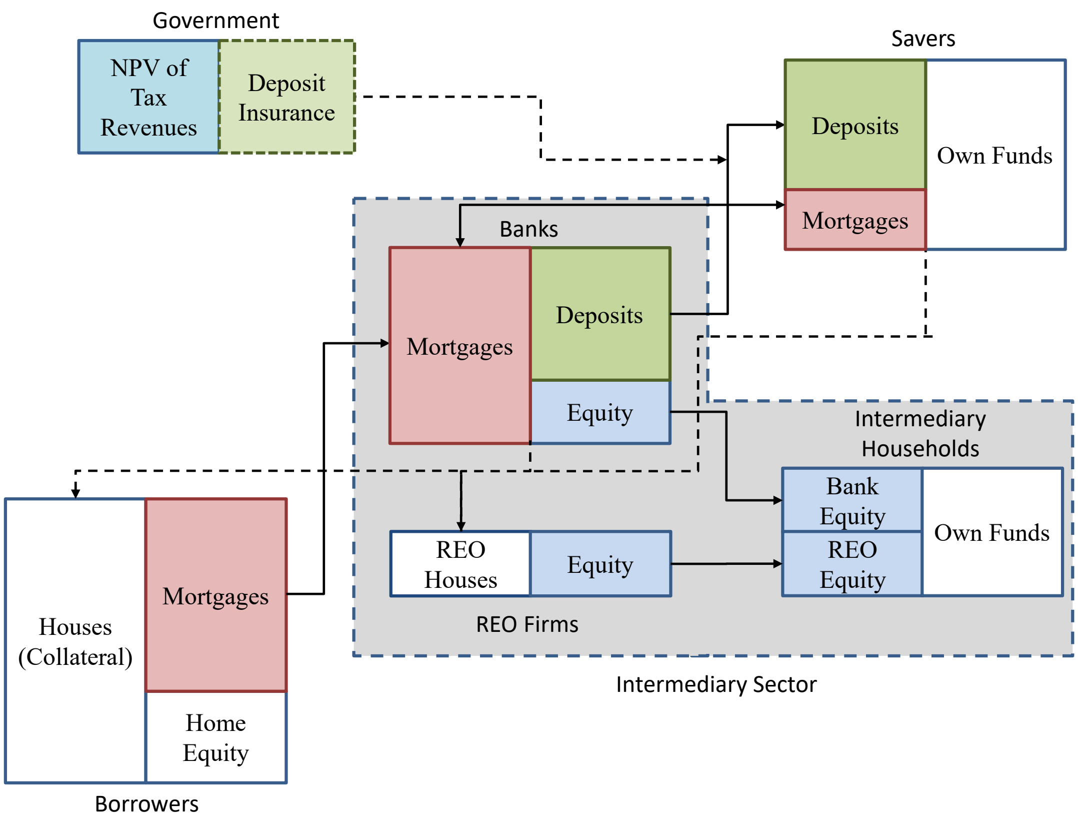

Figure 2. Model Structure.

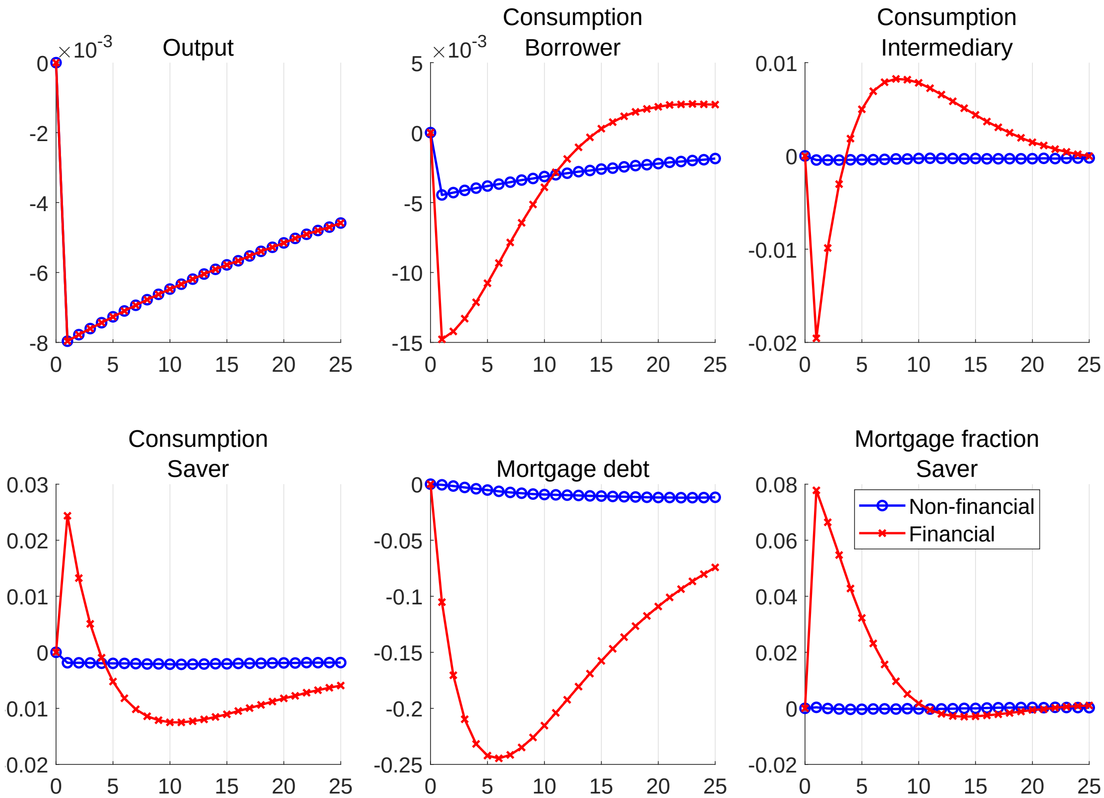

Figure 3. Financial versus nonfinancial recessions, benchmark model (part 1). Blue circles: non-financial recession. Red crosses: financial recession. Plots show deviations in levels from the ergodic steady state. Impulse responses are computed by simulating the response of the model after an initial shock with no additional realized shocks afterward. For the "nonfinancial recession" series the initial shock is a one standard-deviation decrease in output. For the "financial recession" series the initial shock is a one standard-deviation decrease in output combined with a transition from the normal state to the housing-crash state. "Output" is Y_t, "Consumption B" is C_t^B, "Consumption I" is C_t^I, "Consumption S" is C_t^S, "Mortgage Debt' 'is M_t^B, and "Mortg. fraction S" is the ratio $\hat{M}_t^S/M_t^B$.

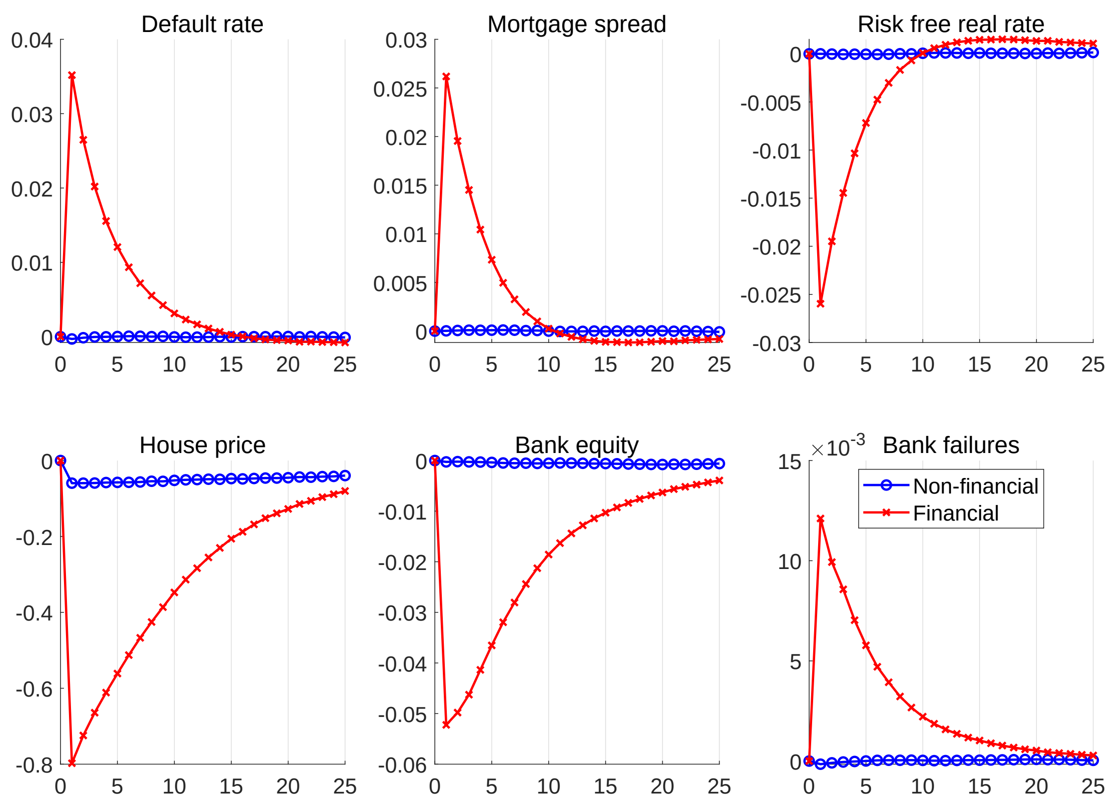

Figure 4. Financial versus nonfinancial recessions, benchmark model (part 2). Blue circles: non-financial recession. Red crosses: financial recession. Plots report deviations in levels from the ergodic steady state. Impulse responses are computed by simulating the response of the model after an initial shock with no additional realized shocks afterward. For the "nonfinancial recession" series, the initial shock is a one standard-deviation. decrease in output. For the "financial recession" series the initial shock is a standard-deviation decrease in output combined with a transition from the normal state to the housing-crash state. "Def. rate" is the default rate $Z_{D,t}$, "Mortgage spread" is rate on new mortgages r_t^* minus risk free real rate, "Risk free real rate" is $1/q_t^f(1+\bar{\pi})$, "House price" is p_t, "Bank equity" is W_t^I, and "Bank failures" is $(1-F_{\varepsilon,t}^I)$.

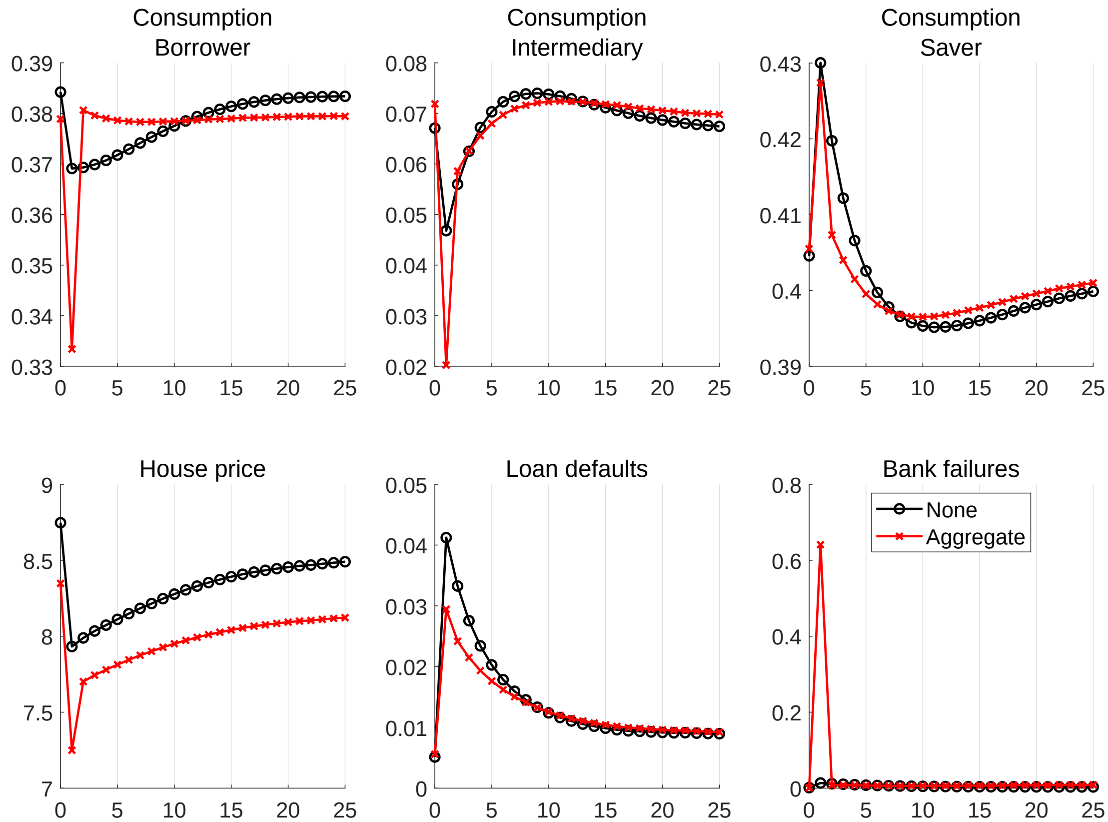

Figure 5. Financial recessions, benchmark versus aggregate model. Black circles: benchmark financial recession. Red crosses: aggregate indexation financial recession. Responses are plotted in levels. Impulse responses are computed by simulating the response of the model after an initial shock with no additional realized shocks afterward. For both series the initial shock is a one standard-deviation decrease in output combined with a transition from the normal state to the housing-crash state. The "No Index" model corresponds to $\iota_p = \iota_\omega = 0$, while the "Aggregate" model corresponds to $\iota_p = 1, \iota_\omega = 0$. "Consumption B" is C_t^B, "Consumption I" is C_t^I, "Consumption S" is C_t^S, "House price" is p_t, "Loan defaults" is $Z_{D,t}$, and "Bank failures" is $(1 - F_{\varepsilon,t}^I)$.

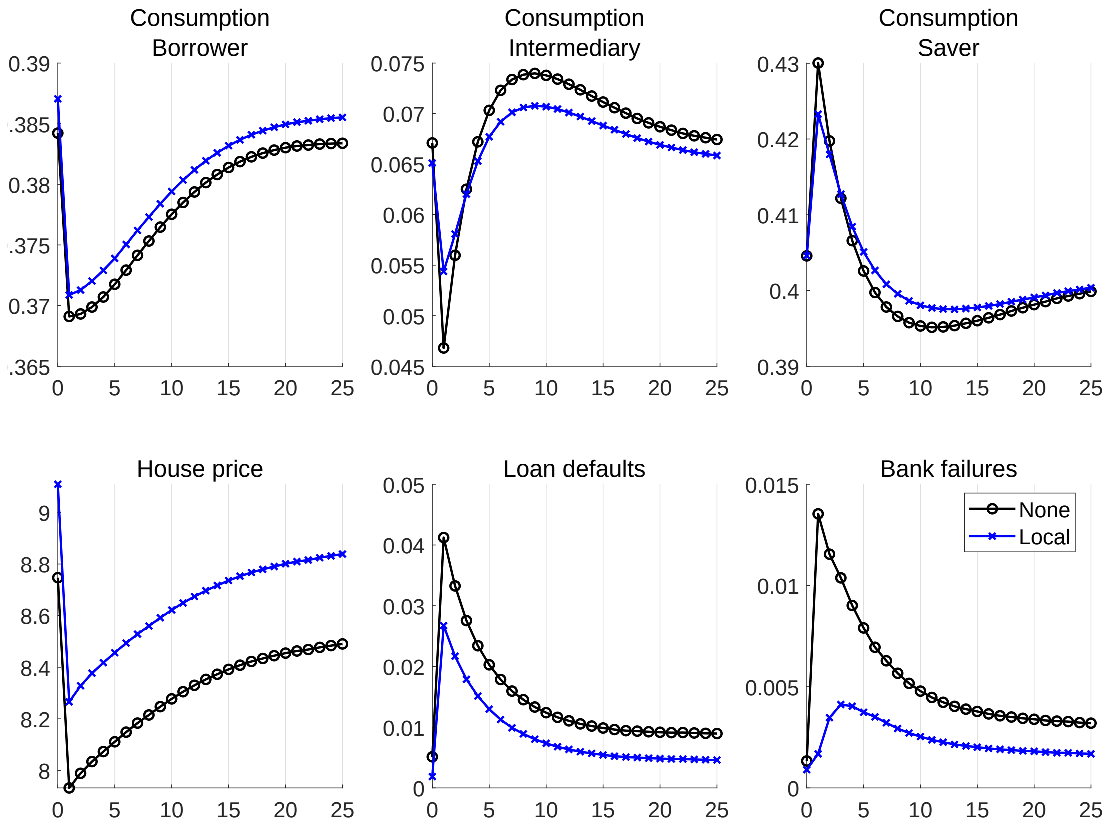

Figure 6. Financial recessions, benchmark versus local model. Black circles: benchmark financial recession. Blue crosses: local indexation financial recession. Responses are plotted in levels. Impulse responses are computed by simulating the response of the model after an initial shock with no additional realized shocks afterward. For both series, the initial shock is a one standard-deviation decrease in output combined with a transition from the normal state to the housing-crash state. The "No Index" model corresponds to $\iota_p = \iota_\omega = 0$, while the "Local" model corresponds to $\iota_p = 0, \iota_\omega = 1$. "Consumption B" is C_t^B, "Consumption I" is C_t^I, "Consumption S" is C_t^S, "House price" is p_t, "Loan defaults" is $Z_{D,t}$, and "Bank failures" is $(1 - F_{\varepsilon,t}^I)$.

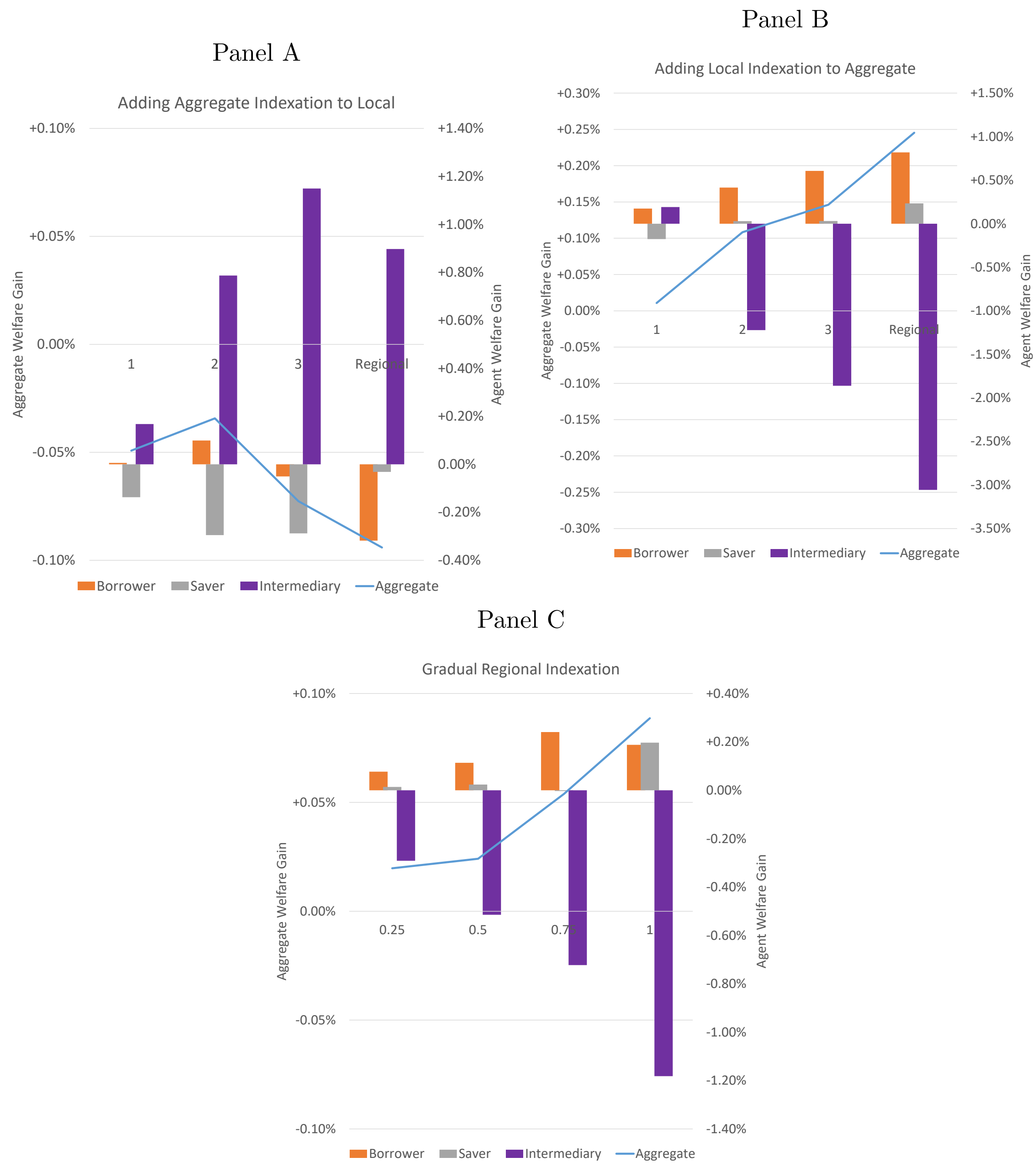

Figure 7. Partial indexation. Panel A shows the change in aggregate population-weighted welfare (left vertical axis), and individual agent welfare (right vertical axis), by gradually adding aggregate indexation to local indexation. Change is relative to an economy with only local indexation. Points 1, 2, and 3 on the horizontal axis indicate 25%, 50%, and 75% aggregate indexation, respectively. “Regional” is the regional indexation case with 100% aggregate and local indexation. Panel B performs the same exercise, but instead adds local indexation gradually to an economy with only aggregate indexation. Panel C goes gradually from the no-indexation benchmark to full regional indexation.

Appendix: Model Derivations

Stochastic Discount Factors

In our incomplete markets economy, we can construct a separate SDF for each representative household, $j = B, I, S$. Denote the certainty equivalent of future utility of type j by

$$CE_t^j = \mathbb{E}_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \tag{1}$$

where utility U_t^j is defined in equation (2). The SDF of agent j is then

$$\Lambda_{t+1}^j = \beta_j \left(\frac{U_{t+1}^j}{CE_t^j} \right)^{1/\psi - \gamma} \left(\frac{u_{t+1}^j}{u_t^j} \right)^{-1/\psi} \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-1}, \tag{2}$$

where $u_t^j = (C_t^j)^{1-\xi_t} (H_t^j)^{\xi_t}$, the standard definition with Epstein-Zin preferences.

Aggregation Across Vintages

This appendix shows that a portfolio of long-term FRMs issued in different periods (vintages) at vintage-specific mortgage rates can be completely summarized by two state variables: the portfolio's outstanding principal balance and the portfolio's promised interest payments.

Consider the complete distribution over $m_t(r)$, the start-of-period balance of a loan with interest rate r, as a state variable. Banks can freely choose their end-of-period holdings of these loans $\tilde{m}_t(r)$ by trading in the secondary market at price $q^m(r)$. In this case, the bank's problem is to choose new debt issuance L_t^*, new deposits B_{t+1}^I, and end-of-period loan holdings $\tilde{m}_t(r)$ to maximize shareholder value

$$V^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{m}_t(r), B_{t+1}^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I \, F_{\epsilon,t+1}^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right], \tag{3}$$

subject to the net worth identity

$$
\begin{aligned}
W_t^I = & \underbrace{\int \Big[X_t + Z_{A,t} r + Z_{M,t}\Big((1-\delta) + \delta Z_{R,t}\Big)\Big] m_t(r)\, dr}_{\text{payments on existing debt}} \\
& + \underbrace{\int q_t^m(r)\delta(1 - Z_{R,t}) Z_{M,t} m_t(r)\, dr}_{\text{secondary market sales}} - \underbrace{\bar{\pi}^{-1} B_t^I}_{\text{old deposits}},
\end{aligned}
\tag{4}
$$

The asset portfolio

$$
J_t^I = \underbrace{(1 - q_t^m(r_t^*))L_t^*}_{\text{net new debt}} + \underbrace{\int q_t^m(r)\tilde{m}_t(r)\, dr}_{\text{secondary market purchases}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}},
\tag{5}
$$

and the leverage constraint

$$
q_t^f B_{t+1}^I \leq \phi^I \int q_t^m(r)\tilde{m}_t(r)\, dr,
\tag{6}
$$

with law of motion (by vintage r)

$$
m_{t+1}(r) = \bar{\pi}^{-1}\zeta_{p,t+1}\tilde{m}_t(r),
\tag{7}
$$

where the recovery rate X_t is defined as in the main text. To obtain aggregation, we can split $q_t(r)$ into an IO strip with value q_t^M and a PO strip with value q_t^A, so that

$$
q_t^m(r) = r q_t^A + q_t^M.
\tag{8}
$$

Substituting this definition into equations (4) to (7) and applying the identities

$$
M_t^I = \int m_t(r)\, dr \tag{9}
$$

$$
A_t^I = \int r m_t(r)\, dr \tag{10}
$$

yields the aggregated intermediary problem of Section II.E. The same logic applies to the mortgage debt holdings of savers. Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant $Z_{R,t}$ across the r distribution), the prices q_t^A and q_t^M are independent of r. Furthermore, the effects of indexation are also independent of the vintage rate r.

Bank Aggregation and First-Order Conditions

Aggregation. The value of banks that do not default can be expressed recursively as

$$V_{ND}^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I - \epsilon_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I \max \left\{V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}), 0\right\}\right], \quad (11)$$

subject to the bank leverage constraint (29), the definitions of J_t^I and W_t^I in (24) and (27), respectively, and the transition laws for the aggregate supply of IO and PO strips in (22) to (26). The value of defaulting banks to shareholders is zero.

The value of the newly started bank that replaces a bank liquidated by the government after defaulting is given by

$$V_R^I(\mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I \max \left\{V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}), 0\right\}\right], \quad (12)$$

subject to the same set of constraints as the nondefaulting bank.

Beginning-of-period net worth W_t^I and the idiosyncratic profit shock ϵ_t^I are irrelevant for the portfolio choice of newly started banks. Inspecting equation (11), one can see that the optimization problem of nondefaulting banks is also independent of W_t^I and ϵ_t^I, since the value function is linear in those variables and they are determined before the portfolio decision. Taken together, this implies that all banks will choose identical portfolios at the end of the period. This property gives rise to aggregation, as we show next.

Starting from the value function in (11), we can define a value function net of the idiosyncratic profit shock

$$V^I(W_t^I, \mathcal{S}_t) = V_{ND}^I(W_t^I, \mathcal{S}_t) + \epsilon_t^I, \tag{13}$$

such that we can equivalently write the optimization problem of the nondefaulting bank after the default decision as

$$V^I(W_t^I, \mathcal{S}_t) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I, 0 \right\} \right], \tag{14}$$

subject to the same set of constraints as the original problem.

We can now take the expectation with respect to ϵ_t^I of the term in the expectation operator:

$$\begin{aligned}
&\mathbb{E}_\epsilon \left[\max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I, 0 \right\} \right] \\
&\quad = \mathrm{Prob}_\epsilon \left(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \mathbb{E}_\epsilon \left[V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^I \,|\, \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right] \\
&\quad = F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right),
\end{aligned} \tag{15}$$

where $\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[\epsilon_{t+1}^I \,|\, \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right]$ as in the main text. Inserting (15) into (14) gives the value function in (28) in the main text.

The value of the newly started bank with zero net worth is simply the value in (28) evaluated at $W_t^I = 0$: $V_R^I(\mathcal{S}_t) = V^I(0, \mathcal{S}_t)$.

First-Order Conditions. To derive the first-order conditions for the bank's problem, we formulate the Lagrangian

$$\begin{aligned}
\mathcal{L}^I(W_t^I, \mathcal{S}_t) = &\max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} \min_{\lambda_t^I} W_t^I - J_t^I + \mathrm{E}_t \left[\Lambda_{t+1}^I F_\epsilon^I \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) \right) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right] \\
&+ \lambda_t^I \left(\phi^I \left(q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - B_{t+1}^I \right) \right),
\end{aligned} \tag{16}$$

and further conjecture that

$$V^I(W_t^I, \mathcal{S}_t) = W_t^I + \mathcal{C}(\mathcal{S}_t), \tag{17}$$

where $\mathcal{C}(\mathcal{S}_t)$ is a function of the aggregate state variables but not individual bank net worth.

Before differentiating (16) to obtain first-order conditions (FOCs), note that the derivative of the term in the expectation operator with respect to future wealth, after substituting in this guess, is

$$\begin{aligned}
&\frac{\partial}{\partial W_{t+1}^I} F_\epsilon^I\left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1})\right)\Big(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-}\Big) \\
&\qquad = \frac{\partial}{\partial W_{t+1}^I}\left[F_\epsilon^I\left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1})\right)\left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1})\right) - \int_{-\infty}^{W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1})} \epsilon f_\epsilon^I(\epsilon)\, d\epsilon\right] \\
&\qquad = F_\epsilon^I\left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1})\right) \equiv F_{\epsilon,t+1}^I.
\end{aligned} \tag{18}$$

Using this result, and differentiating with respect to $L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I$, and λ_t^I, gives the FOCs:

$$1 = q_t^M + r_t^* q_t^A, \tag{19}$$

$$q_t^M = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1}\zeta_{p,t+1}\left[X_{t+1} + Z_{M,t+1}\Big((1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1})q_{t+1}^M\Big)\right]\right\}}{(1-\phi^I\lambda_t^I)}, \tag{20}$$

$$q_t^A = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1}\zeta_{p,t+1}\left[Z_{A,t+1}\Big(1 + \delta(1-Z_{R,t+1})q_{A,t+1}^A\Big)\right]\right\}}{(1-\phi^I\lambda_t^I)}, \tag{21}$$

$$q_t^f = \mathbb{E}_t\Big[\Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1}\Big] + \lambda_t^I, \tag{22}$$

and the usual complementary slackness condition for λ_t^I.

Recalling the definition of J_t^I,

$$J_t^I = (1 - r_t^* q_t^A - q_t^M)L_t^* + q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - q_t^f B_{t+1}^I, \tag{23}$$

we note that the term in front of L_t^* is zero due to FOC (19). We can substitute out prices q_t^M, q_t^A, and q_t^f from FOCs (20) to (22), both in J_t^I and in the constraint term in (16). Further inserting our guess from (17) on the left-hand side of (16), and canceling and collecting terms, we get

$$\mathcal{C}(\mathcal{S}_t) = \mathrm{E}_t \left[\Lambda_{t+1}^I \, F_\epsilon^I \left(W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}) \right) \left(\mathcal{C}(\mathcal{S}_{t+1}) - \epsilon_{t+1}^{I,-} \right) \right], \tag{24}$$

which confirms the conjecture, where $\mathcal{C}(\mathcal{S}_t)$ is the recursively defined value of the bankruptcy option to the bank. Note that without the option to default,

$$\epsilon_{t+1}^{I,-} = \mathrm{E}_\epsilon \left[\epsilon_{t+1}^I \right] = 0. \tag{25}$$

The equation in (24) then implies that $\mathcal{C}(\mathcal{S}_t) = 0$ and thus $V^I(W_t^I, \mathcal{S}_t) = W_t^I$. However, if the bank has the option to default, its value generally exceeds its financial wealth W_t^I by the bankruptcy option value $\mathcal{C}(\mathcal{S}_t)$. Deposit insurance creates bank franchise value.

Borrower Optimality

The optimality condition for new mortgage debt,

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV}, \tag{26}$$

equalizes the benefit of taking on additional debt — \$1 today — and the cost of carrying more debt in the future, in terms of both carrying more principal ($\Omega_{M,t}$) and higher interest payments ($\Omega_{A,t}$), plus the shadow cost of tightening the LTV constraint. The marginal continuation costs are defined recursively as

$$\Omega_{M,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \bar{\pi}^{-1} \zeta_{p,t+1} Z_{M,t+1} \Big[(1-\delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) \Omega_{M,t+1} \Big] \right\}, \tag{27}$$

$$\Omega_{A,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \bar{\pi}^{-1} \zeta_{p,t+1} Z_{A,t+1} \Big[(1-\tau) + \delta(1 - Z_{R,t+1}) \Omega_{A,t+1} \Big] \right\}, \tag{28}$$

where an extra unit of principal requires a regular principal amortization payment of $(1-\delta)$ in the case of nondefault, plus payment of the face value of prepaid debt, plus the continuation cost of nonprepaid debt. An extra promised payment requires a tax-deductible payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

The optimality condition for housing services consumption sets the rental rate equal to the marginal rate of substitution between housing services and nondurables,

$$\rho_t = \frac{u_{H,t}}{u_{C,t}} = \left(\frac{\xi_t}{1-\xi_t}\right)\left(\frac{C_t^B}{H_t^B}\right). \tag{29}$$

The borrower's optimality condition for new housing capital is

$$p_t = \frac{\mathbb{E}_t\left\{\Lambda_{t+1}^B\left[\rho_{t+1} + Z_{K,t+1}p_{t+1}\left(1-\nu^K-(1-Z_{R,t+1})\lambda_{t+1}^{LTV}\phi^K\right)\right]\right\}}{1-\lambda_t^{LTV}\phi^K}. \tag{30}$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by $(1-Z_{R,t+1})\lambda_{t+1}^{LTV}\phi^K$ because if the borrower does not choose to refinance, which occurs with probability $1-Z_{R,t+1}$, then she does not use the unit of housing to collateralize a new loan and therefore does not receive the collateral benefit.

The optimal refinancing rate is

$$\begin{aligned} Z_{R,t} = \Gamma_\kappa\Bigg\{ & \underbrace{(1-\Omega_{M,t}-\bar{r}_t\Omega_{A,t})\left(1-\frac{\delta Z_{M,t}M_t}{Z_{N,t}M_t^*}\right)}_{\text{equity extraction incentive}} + \underbrace{\Omega_{A,t}\left(\bar{r}_t-r_t^*\right)}_{\text{interest rate incentive}} \\ & - \underbrace{p_t\lambda_t^{LTV}\phi^K\left(\frac{Z_{N,t}K_t^*-Z_{K,t}K_t^B}{Z_{N,t}M_t^*}\right)}_{\text{collateral expense}}\Bigg\}, \end{aligned} \tag{31}$$

where $\bar{r}_t = A_t^B/M_t^B$ is the average interest rate on existing debt. The "equity extraction incentive" term represents the net gain from obtaining additional debt at the *existing* interest rate, while the "interest rate incentive" term represents the gain from moving from the existing to new interest rate. The stronger these incentives, the higher the refinancing rate. The "collateral expense" term arises because housing trades at a premium relative to the present value of its housing service flow due to its collateral value. If the borrower intends to obtain new debt by buying more housing collateral, the cost of paying this premium must be taken into account.

The optimality condition for the default rate pins down the default threshold $\bar{\omega}_t^U$ as a function of the aggregate state, as well as the value of the local component $(\omega_{i,t}^L)$,

$$\bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{\iota_\omega} \left(Q_{A,t} A_t^B + Q_{M,t} M_t^B \right)}{\omega_{i,t}^L Q_{K,t} K_t^B}, \tag{32}$$

where $Q_{A,t}$ and $Q_{M,t}$ are the marginal benefits of discharging interest payments and principal, respectively, and $Q_{K,t}$ is the marginal continuation value of housing, defined by

$$Q_{A,t} = \underbrace{(1-\tau)}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{A,t}}_{\text{continuation cost}} \tag{33}$$

$$Q_{M,t} = \underbrace{(\delta Z_{R,t} + (1-\delta))}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{M,t}}_{\text{continuation cost}} \tag{34}$$

$$Q_{K,t} = \Big[\underbrace{Z_{R,t}}_{\text{refi case}} + \underbrace{(1 - Z_{R,t})\left(1 - \lambda_t^{LTV} \phi^K\right)}_{\text{no refi case}} - \underbrace{\nu^K}_{\text{maint.}} \Big] p_t. \tag{35}$$

The marginal value of housing $Q_{K,t}$ is equal to the full market price p_t net of maintenance if used to collateralize a new loan (namely, if the borrower refinances), but is worth less if the borrower does not refinance next period due to the loss of collateral services. Equation (32) relates the benefit of defaulting on debt, which eliminates both the current payment and the continuation cost, potentially indexed by $\omega_{i,t}^L$, against the cost of losing a marginal unit of housing, which is scaled by both $\omega_{i,t}^L$ and $\omega_{i,t}^U$. Default occurs when the market value of the debt exceeds the market value of the collateral, meaning that the mark-to-market LTV exceeds one. The market value of debt reflects the option value of default and prepayment.

Because the option to delay default is valuable to the borrower, the market value of the debt tends to be below the book value of the debt. In other words, it can be optimal to continue servicing the debt when the book LTV (which contains the book value of debt in the numerator and ignores the value of delay) exceeds one. In the case of local indexation ($\iota_\omega = 1$), the market LTV is immunized from shocks to local house prices.

Intermediary Optimality

Bank owner. Given their preferences (2), the bank owner's budget constraint in (21) always holds with equality and the household's only choice, consumption C_t^I, is determined from the budget constraint. Bank owners trade equity shares of banks and REO firms in competitive markets. One could derive the market value of these firms from the intermediary household's first-order conditions. In equilibrium, the representative bank owner holds 100% of the outstanding shares, and thus these optimality conditions are not needed to solve for the model's dynamics. Nonetheless, the bank owner's optimization problem gives rise to the SDF Λ_{t+1}^I.

Banks. Optimality conditions for banks are discussed in Section VII of the Appendix.

REO Firms. The optimality condition for REO housing is

$$p_t^{REO} = \mathbb{E}_t \left\{ \Lambda_{t+1}^I \Big[\rho_{t+1} - \nu^{REO} p_{t+1} + S^{REO} p_{t+1} + (1 - S^{REO}) p_{t+1}^{REO} \Big] \right\}. \tag{36}$$

The right-hand side is the present discounted value of holding a unit of REO housing next period. This term is made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers and the portion kept in the REO state.

Saver Optimality

Savers' optimality condition for deposits, which are nominal contracts, is

$$q_t^f = \mathbb{E}_t\Big[\Lambda_{t+1}^S \bar{\pi}^{-1}\Big]. \tag{37}$$

Savers also trade IO and PO strips in the secondary market. Their choice variable is the amount of PO strips $\tilde{M}_t^S$. To write the FOC for this choice variable, it is useful to define

$$\hat{r}_t = \frac{\hat{A}_t^I + \hat{A}_t^S}{\hat{M}_t^I + \hat{M}_t^S}, \tag{38}$$

which is the effective interest rate paid on all debt supplied in secondary markets (including new debt).

Since savers always hold IO and PO strips in the same proportion as the market supply, their choice of $\tilde{M}_t^S$ implies a choice of IO strips of $\tilde{A}_t^S = \hat{r}_t \tilde{M}_t^S$. The FOC for $\tilde{M}_t^S$ is therefore

$$\begin{aligned} q_t^M + \hat{r}_t q_t^A + \varphi_0(\tilde{M}_t^S)^{\varphi_1 - 1} \\ &= \mathbb{E}_t\left\{\Lambda_{t+1}^S \bar{\pi}^{-1} \zeta_{p,t+1}\left[X_{t+1} + Z_{M,t+1}\Big((1-\delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^M\Big)\right]\right\} \\ &\quad + \hat{r}_t \mathbb{E}_t\left\{\Lambda_{t+1}^S \bar{\pi}^{-1} \zeta_{p,t+1}\left[Z_{A,t+1}\Big(1 + \delta(1 - Z_{R,t+1}) q_{A,t+1}^A\Big)\right]\right\} + \lambda_t^S, \end{aligned} \tag{39}$$

where λ_t^S is the Lagrange multiplier on the no-shorting constraint

$$\tilde{M}_t^S \geq 0. \tag{40}$$

The marginal cost of buying the combined portfolio of IO and PO strips on the left-hand side consists of the security prices and the marginal portfolio holding cost $\varphi_0(\tilde{M}_t^S)^{\varphi_1 - 1}$. Savers have a comparative disadvantage (relative to banks) at holding mortgage securities governed by the magnitude of the cost.

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