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# Mixed QCD-electroweak corrections to Higgs production via gluon fusion in the small mass approximation

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**ABSTRACT:** We compute the mixed QCD-electroweak corrections to the cross section for the production of a Higgs boson via gluon fusion, in the limit of a small mass of the electroweak gauge bosons. This limit is regular and we calculate it by setting the  $W, Z$  masses to zero in the Feynman rules for their propagators. Our analytic results provide an independent check, in a non-trivial limit, of a recent exact computation for the three-loop mixed QCD and electroweak virtual corrections [1] and the corresponding contribution to the cross section in the soft-virtual approximation [2]. From our calculation in the small mass approximation, we can infer the second term in the expansion of the cross section around the threshold limit with its exact dependence on the masses of the  $W, Z$  bosons. Furthermore we find that in the small mass approximation the non-factorizable contributions from the real radiation, so far unknown for full gauge boson mass dependence, are modest in comparison to the known factorizable and virtual contributions to the full  $\mathcal{O}(\alpha_s^3 \alpha^2)$  mixed QCD and electroweak cross-section. This furnishes a new phenomenological test of estimates [3] for the mixed QCD and electroweak corrections, which were based on the hypothesis of factorization of QCD and electroweak corrections.

**KEYWORDS:** Higgs Physics, Effective Field Theories, Perturbative QCD

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**1 Introduction**

The discovery of the Higgs boson [4, 5] at the Large Hadron Collider (LHC) at CERN has been the ultimate success of the Standard Model (SM) of particle physics, and a terrific start for the LHC physics program. With that, the LHC community has entered a phase of precision measurements with emphasis on the properties, couplings and quantum numbers of the Higgs boson. In parallel, the theory community has made a decades long effort in the computation of the Higgs production cross section via the dominant gluon-fusion production mechanism, at ever increasing accuracy. The leading-order production cross section was computed in the 70's [6], and the next-to-leading-order (NLO) QCD corrections were computed in the 90's [7, 8]. The NLO corrections are large ( $\sim 80 - 100\%$ ) casting doubt whether a perturbative expansion in the strong coupling constant  $\alpha_s$  would yield a reliable theoretical estimate of the cross section.

The next-to-next-to-leading-order (NNLO) QCD corrections [9–11] were computed in the Higgs Effective Field Theory (HEFT), i.e. in the limit of a top quark much heavier than the Higgs boson, while all other quarks are taken as massless, and turned out to be significant ( $\sim 10 - 20\%$ ), but smaller than the NLO corrections, indicating that the  $\alpha_s$  expansion might be stabilising. Recently, the next-to-next-to-next-to-leading-order ( $N^3\text{LO}$ ) QCD corrections in HEFT have been computed [12, 13].<sup>1</sup> This result led to a theory uncertainty which turned out to be small [37], putting on solid ground QCD predictions in perturbation theory for the gluon fusion cross section. The  $N^3\text{LO}$  cross section shows a

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<sup>1</sup>This result requires the knowledge of the three-loop virtual corrections [14, 15], the  $N^3\text{LO}$  Wilson coefficient in the heavy-top effective theory [16–18], the renormalization and mass-factorization terms at lower orders [19–23], the two-loop matrix elements for Higgs plus jet production [24] and the corresponding two-loop soft-gluon current [25, 26], the real-virtual-virtual contributions [27, 28], the one-loop corrections to Higgs production is association with a parton [29, 30], the real-real-virtual and the triple-real corrections at [31] and beyond [13, 32] threshold. The combination of these elements allowed the calculation of the cross section at  $N^3\text{LO}$  in perturbative QCD for the production of a Higgs boson at threshold [33–35] and of the next-to-soft corrections to it [36].

remarkable stability with respect to the choice of renormalisation and factorisation scale, with a typical scale variation of less than  $\pm 2\%$ . At this level of precision other effects are important, which are not captured by the QCD perturbative expansion in the combined heavy top-quark/massless light-quarks limit of HEFT (see for example ref. [37] for a comprehensive study).

Finite quark-mass effects for all flavours are known exactly through NLO [8, 38–41]. These amount to a  $\sim -7\%$  change [41] to the gluon-fusion cross section. At NNLO, contributions due to the top quark have been evaluated as a systematic expansion around the infinite top-quark mass limit [42, 43] finding corrections of less than 1%. Light quark flavour effects, of which the interference of diagrams with top and bottom quark loops is the most relevant, are not yet known at NNLO. Assuming a typical NNLO K-factor as for the top-quark contributions in the infinite top-quark limit, one can expect a contribution of the order  $\sim 7\%_{\text{NLO}} \times 20\%_{K_{\text{NNLO/NLO}}} \sim 1.5\%$ . This is similar to the  $\pm 2\%$  scale-variation of the N<sup>3</sup>LO corrections discussed above.

A different class of contributions arise at two loops due to the quark coupling to electroweak (EW) vector bosons  $V = W, Z$  through a quark loop, followed by the gauge coupling of the EW bosons to the Higgs boson. The two-loop EW contributions due to light-quark flavours are dominant and were computed analytically in ref. [44]. The two-loop amplitude was completed with the addition of heavy-quark flavour contributions in ref. [45]. These two-loop EW contributions are equal to a  $\sim +5.2\%$  increase of the leading order gluon-fusion cross section. Electroweak production of a Higgs boson involving initial state quarks at the same power in all coupling constant was considered in ref. [46] and was shown to be numerically small.

A pertinent question is then how large are the NLO QCD corrections to the two-loop EW contributions due to light-quark flavours, given the experience from pure QCD corrections which are large at NLO and NNLO. That would require combining the three-loop virtual corrections, recently computed [1, 47], with the real emission contributions from two-loop four-point functions, which depend on four mass scales,  $s, t, m_H^2, m_V^2$ , and are as yet unknown.

The NLO QCD corrections have been computed in the unphysical limit when the Higgs mass is much smaller than the EW-boson mass  $m_H \ll m_V$  [3], by means of effective theory methods. In this limit, all mixed QCD-EW amplitudes factorise into a product of a Wilson coefficient and the same QCD amplitude which emerges in pure QCD corrections in HEFT. Thus, in the  $m_H \ll m_V$  limit, mixed QCD-EW corrections follow the same perturbative pattern as in the pure QCD corrections and amount to  $\sim +5\%$  of the all-orders QCD cross section.

There has been no progress towards a complete computation of the mixed QCD-EW corrections for a long time until a recent breakthrough in refs. [1, 47], which computes the three-loop virtual corrections to that process for arbitrary values of the gauge and Higgs boson masses. Using these results, the same authors computed with a full mass dependence the NLO QCD corrections in the soft gluon approximation [2], for which the real emission contributions are given by an almost universal factor. Ref [2] could challenge earlier estimates of the mixed QCD-EW corrections since the three-loop amplitude contains

a hard contribution which differs from the amplitude computed in the heavy mass effective theory. The phenomenological outcome of the work in ref. [2] was that the difference due to the three-loop virtual contribution is small with respect to the prescription of ref. [3].

However, the factorisation of mixed QCD-EW corrections can also break down due to hard real radiation for the physical values of the Higgs boson and the EW boson masses,  $m_H > m_V$ . While one should reasonably expect that this “non-factorising” contribution which is not captured by the soft approximation is not large, the amount of the violation of the factorisation ansatz cannot be predicted by extrapolating from the un-physical region  $m_H \ll m_V$ , due to the presence of a threshold at  $m_H = 2m_V$ . It is therefore still necessary that the mixed QCD-EW corrections be computed with a full mass dependence.

A complete computation is challenging. So far, only the planar master integrals for the two-loop light-fermion EW corrections to Higgs plus jet production, with arbitrary values of the gauge and Higgs boson masses, have been computed [48]. In this paper, we consider the approximation of small EW boson masses,  $m_V^2 \ll m_H^2, s, |t|$ . This approximation shares more of the complications of the full computation than the heavy mass  $m_V \rightarrow \infty$  limit. For example, the number of loops is the same and a non-factorisable part of the cross section from hard real radiation is also present. We compute this small mass approximation of the cross section analytically using the methods described in ref. [28]. Then, we identify the parts of the cross section which are clearly factorizable and compare it to the remainder. In this respect, we point out that also the second term in the threshold expansion (next-to-soft) also factorises and we can therefore determine fully its dependence on the mass of the electroweak gauge bosons.

The scope of the calculation presented in this paper is triadic. First, we provide a significant check of the calculation of refs. [1, 47] for the three-loop amplitude. Second, our analytic results offer a check and/or a boundary condition for a future computation of the cross section with arbitrary masses. Third, we obtain for the first time a phenomenological estimate of the relative importance of the non-factorisable hard real radiation contributions.

## 2 Mixed QCD-EW corrections to the $gg \rightarrow H$ cross section

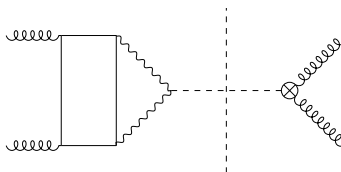
The hadronic cross section for Higgs boson production in the gluon-gluon channel is

$$\begin{aligned} \sigma &= \int_0^1 dx_1 \int_0^1 dx_2 f_{g/h_1}(x_1, \mu_F^2) f_{g/h_2}(x_2, \mu_F^2) \hat{\sigma}_{gg}(S, m_H^2, \mu_R^2, \mu_F^2) \\ &= \tau \int_\tau^1 \frac{dz}{z} \int_{\tau/z}^1 \frac{dx_1}{x_1} f_{g/h_1}(x_1, \mu_F^2) f_{g/h_2}\left(\frac{\tau}{x_1 z}, \mu_F^2\right) \hat{\sigma}_{gg}(z, \mu_R^2, \mu_F^2), \end{aligned} \quad (2.1)$$

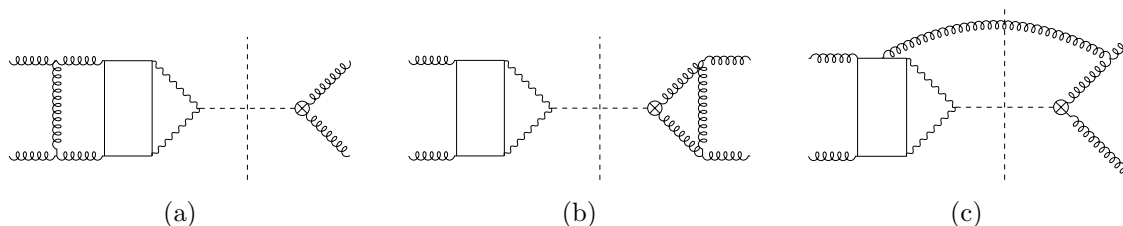
where  $m_H$  is the mass of the Higgs boson,

$$\tau = \frac{m_H^2}{S}, \quad z = \frac{m_H^2}{s} = \frac{m_H^2}{x_1 x_2 S} = \frac{\tau}{x_1 x_2}, \quad (2.2)$$

$S$  is the squared hadron centre-of-mass energy,  $f_i(x)$  are the parton distribution functions and  $\hat{\sigma}_{gg}$  the renormalised partonic cross section.



**Figure 1.** Representative Feynman diagram contributing to  $\mathcal{G}_{\text{HT}}^{(0)*} \mathcal{G}_{lf}^{(0)}$  of eq. (2.4). The vertical dashed line represents the interference between the two-loop light quark form factor on the left, and the top-loop form factor in HEFT on the right.



**Figure 2.** Examples of Feynman diagrams contributing to the  $\mathcal{O}(\alpha_s)$  corrections (2.5) to the interference between the two-loop light-quark form factor and the top-loop form factor in HEFT. Figure 2a: contribution to  $\mathcal{G}_{\text{HT}}^{(0)*} \mathcal{G}_{lf}^{(1)}$ . Figure 2b: contribution to  $\mathcal{G}_{\text{HT}}^{(1)*} \mathcal{G}_{lf}^{(0)}$ . Figure 2c: contribution to  $\mathcal{G}_{\text{HT}-g}^{(0)*} \mathcal{G}_{lf-g}^{(0)}$ .

The NLO contributions to  $\hat{\sigma}_{gg}$  due to mixed QCD-EW corrections are given in the heavy top mass limit, up to  $\mathcal{O}(\alpha_s^3 \alpha^2)$ , by

$$\hat{\sigma}_{gg}^{NLO,EW} = \sigma_0 \left[ \sigma_{\text{HT-EW}}^{(0)} + \frac{\alpha_s}{\pi} \sigma_{\text{HT-EW}}^{(1)} \right], \quad (2.3)$$

with

$$\sigma_{\text{HT-EW}}^{(0)} = 2\text{Re} \left[ \mathcal{G}_{\text{HT}}^{(0)*} \mathcal{G}_{lf}^{(0)} \right], \quad (2.4)$$

$$\sigma_{\text{HT-EW}}^{(1)} = 2\text{Re} \left[ \mathcal{G}_{\text{HT}}^{(0)*} \mathcal{G}_{lf}^{(1)} + \mathcal{G}_{\text{HT}}^{(1)*} \mathcal{G}_{lf}^{(0)} + \mathcal{G}_{\text{HT}-g}^{(0)*} \mathcal{G}_{lf-g}^{(0)} \right]. \quad (2.5)$$

Here  $\mathcal{G}_{lf}^{(0)}$  denotes the two-loop light-quark form factor and  $\mathcal{G}_{\text{HT}}^{(0)}$  is the top-loop form factor in the HEFT. Their interference is shown in figure 1.  $\sigma_{\text{HT-EW}}^{(1)}$  are the NLO corrections to it, as displayed in figure 2. The overall normalization of the cross section (2.3)

$$\sigma_0 = \frac{\alpha_s^2}{576\pi v^2} \quad (2.6)$$

is chosen such that the leading-order QCD contribution in HEFT reads

$$\hat{\sigma}_{gg}^{LO,QCD} = \sigma_0 |\mathcal{G}_{\text{HT}}^{(0)}|^2 = \sigma_0 \delta(1-z). \quad (2.7)$$

In order to study the different limits considered in this paper it is convenient to write the NLO mixed QCD-EW corrections by imposing the factorisation with respect to the leading-order contribution,

$$\hat{\sigma}_{gg}^{NLO,EW} = \sigma_0 \sigma_{\text{HT-EW}}^{(0)} \left[ \delta(1-z) + \frac{\alpha_s}{\pi} \left( \eta_{gg}^{\text{fact}} + \delta(1-z) \eta_{gg}^{\text{hard,V}} + N_c \eta_{gg}^{\text{hard,R}} \right) \right], \quad (2.8)$$

The leading order  $\sigma_{\text{HT-EW}}^{(0)}$  and the relative virtual corrections  $\eta_{gg}^{\text{hard,V}}$  have been computed in [44] and [1] respectively, retaining the full dependence on the EW bosons masses  $m_W$ ,  $m_Z$ , while the universal factorizable (see next section) contribution  $\eta_{gg}^{\text{fact}}$  is given by,

$$\eta_{gg}^{\text{fact}} = 2N_c \left\{ \zeta_2 \delta(1-z) + 2 \left[ \frac{\log(1-z)}{(1-z)} \right]_+ + \left[ \frac{1}{1-z} \right]_+ \log \left( \frac{m_H^2}{\mu^2} \right) + \left( z(1-z) + \frac{(1-z)}{z} - 1 \right) \log \left( \frac{m_H^2(1-z)^2}{\mu^2} \right) + 1 \right\} + \frac{11}{4} \delta(1-z). \quad (2.9)$$

The only contributions which are not known with full EW bosons mass dependence are the real emission corrections  $\eta_{gg}^{\text{hard,R}}$ .

The mixed QCD-EW corrections have been also estimated in the literature in ref. [3] in the heavy EW boson-mass limit, and they correspond to

$$\begin{aligned} \lim_{m_W, m_Z \rightarrow \infty} \eta_{gg}^{\text{hard,V}} &= \frac{7}{6}, \\ \lim_{m_W, m_Z \rightarrow \infty} \eta_{gg}^{\text{hard,R}} &= 2 \frac{(z^2 - z + 1)^2 \log(z)}{(z-1)z} - \frac{11}{6} \frac{(1-z)^3}{z} - 2. \end{aligned} \quad (2.10)$$

### 3 The small-mass approximation

In this section, we consider the first term of a small EW-boson mass expansion,  $m_V^2 \ll m_H^2, s, |t|$ . That is equivalent to take the massless limit for the propagators of the gauge bosons  $W$  and  $Z$  running in the loop, while retaining their mass in the couplings to the Higgs boson. More precisely, in the case of the  $W$  boson, we account for the first two generations of quarks, while for the  $Z$  boson we include the bottom quark as well. We neglect diagrams involving the top quark [45]. This is motivated by the fact that they contribute to the two-loop electroweak correction by just a few percent of the light-quark electroweak effects [49] and we expect a similar pattern at one higher order in QCD perturbation theory. All the light quarks are considered to be massless. In addition, we consider only the contributions from the gluon-gluon initial state, which is dominant. Specifically, we consider the gauge invariant subset of diagrams with only up to one final state parton. Contributions with two final state partons are separately gauge invariant and numerically sub-leading [50].

We generate all contributing Feynman diagrams using QGRAF [51] and perform the colour-, spinor- and tensor-algebra with standard methods. Within the framework of reverse unitarity [10, 52–54], all resulting expressions can be directly simplified by using integration by parts (IBP) identities [55, 56] and the reduction algorithm of Laporta [57]. The reduction is performed using KIRA [58] and FIRE5 [59]. The master integrals have been computed in refs. [15, 55, 60–64] and in ref. [28] as part of the computation of the inclusive Higgs cross-section at N<sup>3</sup>LO in perturbative QCD.

The primary analytic result of this paper is the computation of the mixed QCD-EW corrections in the approximation of a small EW-boson mass (light boson), as defined above.

We obtain,<sup>2</sup>

$$\hat{\sigma}_{gg,lb}^{NLO,EW} = \sigma_0 f_{HTEW}^{(0)} (3\zeta_3 - 2) \delta(1 - z) + \sigma_0 f_{HTEW}^{(0)} \frac{\alpha_s}{\pi} \left[ (3\zeta_3 - 2) \eta_{gg}^{\text{fact}} + \delta(1 - z) V_{gg,lb}^{\text{hard}} + N_c R_{gg,lb}^{\text{hard}} \right], \quad (3.1)$$

where,

$$f_{HTEW}^{(0)} = -\frac{3\alpha^2 v^2}{2 \sin^4 \theta_W m_H^2} \left[ \frac{2}{\cos^4 \theta_W} \left( \frac{5}{4} - \frac{7}{3} \sin^2 \theta_W + \frac{22}{9} \sin^4 \theta_W \right) + 4 \right], \quad (3.2)$$

and the overall factor  $(3\zeta_3 - 2)$  is the small EW-boson mass approximation, for both the  $W$  and  $Z$  bosons, of the two-loop light-quark form factor, while the non-universal part of the virtual cross section is

$$V_{gg,lb}^{\text{hard}} = N_c \left( \frac{31\zeta(3)}{4} - \frac{\pi^2\zeta(3)}{24} - \frac{15\zeta(5)}{2} - \frac{17}{12} + \frac{\pi^2}{3} \right) + \frac{1}{N_c} \left( -\frac{19\zeta(3)}{8} - \frac{\pi^2\zeta(3)}{24} + 5\zeta(5) - \frac{5}{3} + \frac{\pi^2}{48} - \frac{\pi^4}{1440} \right). \quad (3.3)$$

The real contributions,  $R_{gg,lb}^{\text{hard}}$ , can be expressed in terms of harmonic polylogarithms [65], defined iteratively as,

$$H(a_1, \dots, a_n, x) = \int_0^x dt f(a_1, t) H(a_2, \dots, a_n, t), \quad (3.4)$$

with  $a_i \in \{0, 1, -1\}$  and

$$f(0, x) = \frac{1}{x}, \quad f(1, x) = \frac{1}{1-x}, \quad f(-1, x) = \frac{1}{1+x}. \quad (3.5)$$

If  $a_i = 0$  for all  $i \in 1, \dots, n$  then,

$$H(0_n, x) = \frac{\log(x)^n}{n!}. \quad (3.6)$$

By using the following compact notation for the harmonic polylogarithms [65] of argument  $1 - z$ ,

$$H_{\dots, \pm n, \dots} \equiv H(\dots, \underbrace{0, \dots, 0}_{(n-1)\text{-times}}, \pm 1, \dots, 1 - z), \quad (3.7)$$

we get, in the physical domain  $z \in [0, 1]$ , the following real-valued expression,

$$R_{gg,lb}^{\text{hard}} = -3(z-1)H_{2,3} + \frac{1}{2}(z-1)H_{3,2} + (z-1)H_{4,0} + (z-1)H_{4,1} + (4-3z)H_{1,1,3} + \frac{1}{2}(-7z-6)H_{1,2,2} + (5z+1)H_{1,3,0}$$

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<sup>2</sup>We note that the general formula (2.8) enforces the factorisation with respect to born, and it is useful when comparing the small mass approximation and the heavy mass approximation for the EW bosons as done in section 4. On the other hand eq. (3.1) represents the small mass approximation computed in this paper, where in fact the NLO corrections do not factorise with respect to born.



$$\begin{aligned}
& + (5z + 1)H_{1,3,1} + H_{2,1,2} - 2(z - 1)H_{2,2,0} + (1 - 2z)H_{2,2,1} \\
& + \frac{1}{2}(z - 1)H_{3,0,0} + (-4z - 1)H_{1,1,1,2} + \frac{1}{2}(4z + 11)H_{1,1,2,0} \\
& + \frac{1}{2}(4z + 9)H_{1,1,2,1} - \frac{7}{2}(z + 1)H_{1,2,0,0} + \frac{1}{2}H_{1,2,1,1} \\
& - 3(z - 1)H_{2,0,0,0} - \frac{3}{2}(2z - 3)H_{1,1,0,0,0} + \frac{1}{2}(-8z - 3)H_{1,1,1,0,0} \\
& + \frac{1}{2} \left( -z^2 - 7z + \frac{2}{z} + 7 \right) H_{2,2} + \frac{1}{2} \left( z^2 + 6z - \frac{2}{z} - 3 \right) H_{3,0} \\
& + \frac{1}{2} \left( z^2 + 6z - \frac{2}{z} - 3 \right) H_{3,1} + \frac{1}{6} (-3z^2 - 21z + 4) H_{1,1,2} \\
& + \frac{1}{4} (2z^2 + 14z - 27) H_{1,2,0} + \frac{1}{12} (6z^2 + 54z - 23) H_{1,2,1} \\
& + \frac{1}{2} \left( -z^2 - 8z + \frac{2}{z} + 8 \right) H_{2,0,0} + \frac{1}{4} (-2z^2 - 16z + 7) H_{1,1,0,0} \\
& + \frac{1}{2} (2z - 3)H_{1,3} + \frac{1}{2} (z - 1)H_{2,1,1} - \frac{21}{4} H_{1,0,0,0} - zH_{1,1,1,0} \\
& + \left( \frac{1}{24} (9z^2 - 118z - 68) + (7z + 8)\zeta_2 \right) H_{1,2} + \left( \frac{1}{12} (3z^2 - 131z \right. \\
& \left. + \frac{36}{z} + 116) + 9(z - 1)\zeta_2 \right) H_{2,0} + \left( \frac{1}{24} (15z^2 - 146z - 22) \right. \\
& \left. - 2(2z + 1)\zeta_2 \right) H_{1,0,0} + \left( \frac{1}{2} (z^2 - 22z) + (5z - 16)\zeta_2 \right) H_{1,1,0} \\
& + \left( \frac{1}{8} (z - 1) \left( z + \frac{16}{z} - 9 \right) - (z - 1)\zeta_2 \right) H_3 + \frac{1}{12} \left( -55z + \frac{12}{z} \right. \\
& \left. + 67 \right) H_{2,1} + \left( \left( z^2 + 9z - \frac{2}{z} - 9 \right) \zeta_2 + \frac{1}{24} (z - 1) \left( 20z^2 - 3z \right. \right. \\
& \left. \left. - \frac{24}{z} - 194 \right) + 2(z + 2)\zeta_3 \right) H_2 + \left( \frac{1}{2} (-z^2 - 4z + 44) \zeta_2 \right. \\
& \left. + \frac{20z^4 - 43z^3 - 135z^2 - 58z + 192}{24(z - 1)} + 6(2z + 1)\zeta_3 \right) H_{1,0} - \frac{1}{48} z \log^4(z) \\
& + \left( \frac{1}{24} (-z^2 + 39z + 8) + \frac{1}{6} (-8z - 3)\zeta_2 \right) \log^3(z) + \frac{1}{16} (z - 19) \\
& \times (z - 1) \log^3(1 - z) + \left( \frac{1}{48} (z - 1) \left( 20z^2 - 3z - \frac{24}{z} - 32 \right) \right. \\
& \left. + \frac{1}{4} \left( -z - \frac{2}{z} - 7 \right) (z - 1)\zeta_2 \right) \log^2(1 - z) + \left( \frac{1}{4} (2z^2 + 20z - 7) \zeta_2 \right. \\
& \left. + \frac{20z^4 - 43z^3 - 275z^2 + 222z + 52}{48(z - 1)} + (7z + 4)\zeta_3 \right) \log^2(z) \\
& + \left( \frac{3(z^3 + 5z^2 - 11z + \frac{2}{z} + 3) \zeta_3}{2(z - 1)} + \frac{-14z^3 - 115z^2 + 212z - \frac{48}{z} - 35}{12(z - 1)} \right. \\
& \left. - \frac{3}{2} (z - 1) \left( z - \frac{2}{z} - 22 \right) \zeta_2 \right) \log(1 - z) + \left( \frac{1}{4} (9z^2 - 104z + 10) \zeta_2 \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{3(3z^3 - 11z^2 + 31z - 19)\zeta_3}{2(z-1)} + \frac{-17z^3 + 87z^2 - 135z + 47}{6(z-1)} \\
& - 3(2z+1)\zeta_4 \log(z) + \frac{3}{4} \left( z^2 + 6z - \frac{2}{z} - 5 \right) \zeta_4 + \left( \frac{1}{4}(z-1) \right. \\
& \times \left. \left( 23z - \frac{36}{z} - 147 \right) - 6 \right) \zeta_3 + \frac{1}{8}(z-1) \left( -20z^2 + 3z + \frac{24}{z} - 49 \right) \zeta_2 \\
& + \frac{1}{12}(213 - 44z)(z-1) + 4. \tag{3.8}
\end{aligned}$$

It is interesting to study the limiting behaviour of the real contributions to the cross section,  $R_{gg}^{\text{hard}}$ , around  $z = 0$  and  $z = 1$ . More precisely, in the proximity of  $z = 1$  we find

$$\begin{aligned}
R_{gg,lb}^{\text{hard}} = (1-z) \left\{ 4\zeta(3) + \frac{\pi^4}{60} - \frac{17}{4} + \left( 3\zeta(3) + 5 - \frac{3\pi^2}{2} \right) \log(1-z) \right. \\
\left. - \left( \frac{\pi^2}{12} - \frac{1}{2} \right) \log^2(1-z) + \frac{1}{4} \log^3(1-z) \right\} + \mathcal{O}((1-z)^2), \tag{3.9}
\end{aligned}$$

which, together with the limit  $z \rightarrow 1$  of eq. (2.9), yields the next-to-next-to-leading-power corrections to the threshold limit,  $z \rightarrow 1$ , while in the proximity of  $z = 0$ ,

$$R_{gg,lb}^{\text{hard}} = \frac{1}{z} \left( 2\zeta(3) - \frac{\pi^2}{3} \right) + \mathcal{O}(1), \tag{3.10}$$

which, together with the limit  $z \rightarrow 0$  of eq. (2.9), provides the high energy limit of the NLO corrections to the interference (2.4).

Finally we note that our small mass approximation is recovered from the general formula eq. (2.8) by using the results of refs. [1, 47], where we take the limits,

$$\begin{aligned}
\lim_{m_W, m_Z \rightarrow 0} \sigma_{\text{HT-EW}}^{(0)} &= f_{\text{HTEW}}^{(0)} (3\zeta_3 - 2), \\
\lim_{m_W, m_Z \rightarrow 0} \eta_{gg}^{\text{hard,V}} &= V_{gg,lb}^{\text{hard}} / (3\zeta_3 - 2), \tag{3.11}
\end{aligned}$$

and by identifying,

$$\lim_{m_W, m_Z \rightarrow 0} \eta_{gg}^{\text{hard,R}} = R_{gg,lb}^{\text{hard}} / (3\zeta_3 - 2). \tag{3.12}$$

Eq. (3.11) agrees with the results of ref. [2] when the latter are computed in the small mass approximation. This is a significant check of the computations in refs. [1, 47]. The behaviour of the factorizable soft and next-to-soft contributions of eq. (2.9) are in agreement with the expectations from the next-to-leading-power corrections [66–69] to colour-singlet production from gluon-gluon fusion at NLO [70].

## 4 Phenomenological analysis

Currently, the mixed QCD-EW contributions in the Higgs inclusive cross section are estimated by computing their relative size with respect to the leading order EW contributions, by taking the first term of a large EW-boson mass expansion,  $m_V^2 \gg m_H^2, s, |t|$ , that we term the limit of  $m_V \rightarrow \infty$ . These mass values are unphysical. However, it is reasonable to

expect that the relative size of the corrections is not very sensitive to the masses of the EW bosons and that they can be estimated by choosing a convenient value, albeit unphysical. In the  $m_V \rightarrow \infty$  limit, the mixed QCD-EW corrections factorize in terms of the square of a Wilson coefficient times a partonic cross section in an effective theory where the Higgs boson couples at tree level to gluons. The same type of factorization (with a different Wilson coefficient) holds for the pure QCD cross section in the HEFT, i.e. in the limit of an infinite top-quark mass. Therefore, in these mass limits, one finds very similar QCD K-factors for the top-quark and the electroweak contributions to the Higgs cross section.

It is very important to check the validity of the phenomenological predictions for the mixed QCD-EW corrections, which are made by means of the factorization hypothesis. Recently, the authors of ref. [1] computed the three-loop QCD-EW corrections and calculated numerically their contribution to the Higgs cross section in the soft-virtual approximation [2]. This calculation replaces with an exact/physical result the contribution to the cross section which was approximated earlier by the NLO term of the Wilson coefficient. Ref. [2] found that the approximation based on the Wilson coefficient is phenomenologically good. While this observation strengthens the credibility of the existing phenomenological predictions, it is still an open question whether the pattern of perturbative corrections can be significantly altered in different ways. Modifications on the structure of the perturbative corrections are theoretically anticipated when the cross section is evaluated away from the  $m_V \rightarrow \infty$  limit. Specifically, in the  $\eta_{gg}^{\text{hard,R}}$  and  $\eta_{gg}^{\text{hard,V}}$  parts of the cross section. In this section, we compare the numerical impact of the  $m_V \rightarrow \infty$  limit with the  $m_V \rightarrow 0$  limit. This is the diametric reverse limit of the one used in previous estimates and it can reveal a potential breakdown of the phenomenological assumptions of ref. [3].

We use the NNLO PDF4LHC15 set [71], and take  $m_H = 125$  GeV,  $m_W = 80.398$  GeV,  $m_Z = 91.88$  GeV,  $\sin^2(\theta_W) = 0.2233$ ,  $\alpha = 1/128$ ,  $G_F = 1.16639 \times 10^{-5}/\text{GeV}^2$  and a center of mass energy of 13 TeV. We evolve the strong coupling constant  $\alpha_s$  to NNLO. In order to obtain a fast and reliable numerical implementation of the real corrections  $R_{gg,lb}^{\text{hard}}$ , we perform two power series expansions around the points  $z = 0$  and  $z = 1$ . We evaluate the series expansion around  $z = 1$  in the interval  $z \in [\frac{1}{2}, 1]$  and truncate the series at  $\mathcal{O}((1-z)^{50})$ . Similarly, we evaluate the expansion around  $z = 0$  in the interval  $z \in [0, \frac{1}{2})$  and truncate the series at  $\mathcal{O}(z^{50})$ . In this way we achieve for  $R_{gg,lb}^{\text{hard}}$  a numerical precision of at least  $10^{-10}$  within the full physical interval  $z \in [0, 1]$ .

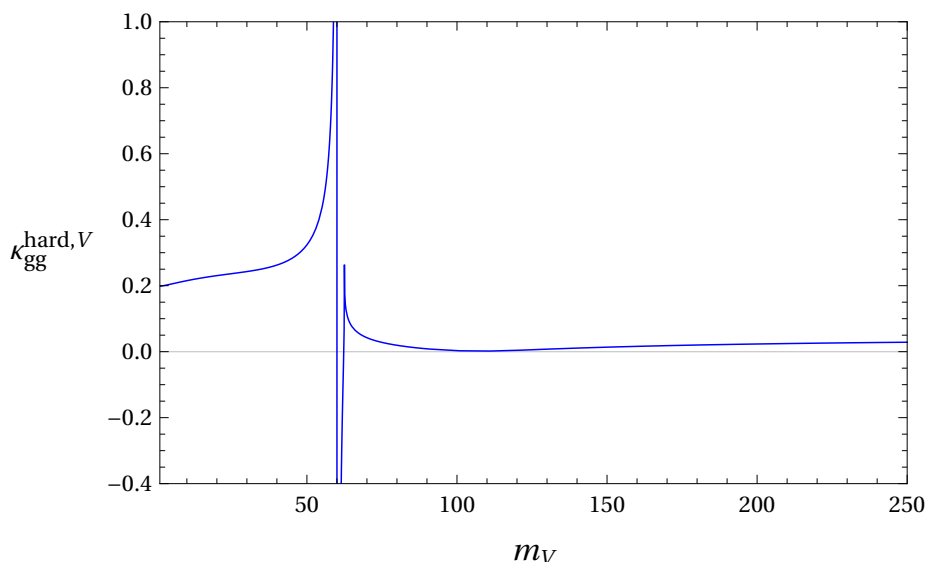
We then decompose the NLO QCD-EW corrections as,

$$\delta\sigma_{gg}^{NLO,EW} = \sigma_{gg}^{\text{fact}} + \sigma_{gg}^{\text{hard,V}} + \sigma_{gg}^{\text{hard,R}}, \tag{4.1}$$

where the three terms on the right-hand side correspond to the hadronic cross section contributions from the NLO terms in eq. (2.8). In table 1, we show the ratios of these terms to the leading order electroweak corrections in various approximations as a function of a common factorization and renormalization scale  $\mu$ . We notice that the universal factorised contributions are dominant in both the approximations. The hard virtual contributions are a factor of 5 larger in the small mass approximation with respect to the heavy boson-mass approximation, while the physical values are about a factor of 2.5 smaller than the heavy mass approximation. The hard real contributions, are about 50% larger in the small-mass

		$\frac{\mu}{m_H} = 2$	$\frac{\mu}{m_H} = 1$	$\frac{\mu}{m_H} = \frac{1}{2}$	$\frac{\mu}{m_H} = \frac{1}{4}$	$\frac{\mu}{m_H} = \frac{1}{8}$
$K_{gg}^{\text{hard,R}}$	$m_V \rightarrow 0$	0.14	0.15	0.18	0.21	0.25
	$m_V \rightarrow \infty$	0.08	0.10	0.12	0.15	0.21
$K_{gg}^{\text{hard,V}}$	$m_V \rightarrow 0$	0.18	0.20	0.22	0.25	0.28
	$m_V \rightarrow \infty$	0.04	0.04	0.05	0.05	0.06
	phys	0.014	0.015	0.017	0.019	0.022
$K_{gg}^{\text{fact}}$		1.24	1.11	0.97	0.86	0.79
$K_{gg}^{\text{NLO,EW}}$	$m_V \rightarrow 0$	1.56	1.46	1.37	1.31	1.33
	$m_V \rightarrow \infty$	1.36	1.25	1.14	1.06	1.06

**Table 1.**  $K$ -factors of the various components of the NLO QCD-EW corrections within different approximations.



**Figure 3.** Plot of  $K_{gg}^{\text{hard,V}}$  for  $m_V = m_W = m_Z$  and  $\mu = m_H$

approximation when compared to the large-mass approximation. It is also interesting to study the contribution of the virtual corrections for intermediate values of the boson mass. In figure 3 we present the plot of  $K_{gg}^{\text{hard,V}}$  as a function of the boson mass  $m_V$  for  $\mu = m_H$ . On the left side of the plot (small mass approximation) the values approach  $\frac{\alpha_s}{\pi} \frac{V_{gg,lb}^{\text{hard}}}{3\zeta_3 - 2} = 0.20$ , while on the right side (heavy boson-mass approximation) the values approach  $\frac{\alpha_s}{\pi} \frac{7}{6} = 0.04$ , which correspond to the values reported in table 1.

Our results can be summarised by providing the values of the hadronic cross section in the different limits we have analysed. We set the reference scale to  $\mu = m_H/2$ . The pure QCD cross section in the HEFT is

$$\sigma_{gg}^{\text{NLO,QCD}} = 33.24 \text{ pb.} \tag{4.2}$$

By including the mixed QCD-EW corrections eq. (2.8) and taking  $\eta_{gg}^{\text{hard,V/R}}$  in the heavy (hb) and light (lb) EW-boson limits as defined in eqs. (2.10), (3.11), (3.12) we have, respectively

$$\sigma_{gg,hb}^{NLO,QCD-EW} = 35.01 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.32\%), \quad (4.3)$$

$$\sigma_{gg,lb}^{NLO,QCD-EW} = 35.20 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.90\%). \quad (4.4)$$

On the other hand, by taking the hard real contributions in the heavy and light EW bosons limits and keeping all the other contributions exact we have, respectively

$$\sigma_{gg,R(hb)}^{NLO,QCD-EW} = 34.98 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.23\%), \quad (4.5)$$

$$\sigma_{gg,R(lb)}^{NLO,QCD-EW} = 35.03 \text{ pb}, (= \sigma_{gg}^{NLO,QCD} + 5.39\%). \quad (4.6)$$

These results are in very good agreement with the (improved) soft-gluon approximation of ref. [2], where, by using our setup for PDFs and values of the parameters, the NLO QCD-EW corrections increase the QCD cross section by 5.33% .

With the recent calculation of the hard virtual contributions in refs. [1, 2] with full  $m_W, m_Z$  dependence, the only contribution which is known in an unphysical limit is the hard real. This appears in our calculation to be small. Moreover the full cross section appears to be relatively insensitive to different configurations of the boson masses for this contribution. We have therefore found no significant deviations which invalidate the phenomenological assumptions made in the estimates of mixed QCD-EW corrections in ref. [3]. Nevertheless, we would like to note that this is not a “bullet-proof” exclusion of the possibility of larger hard real corrections for physical EW-boson masses. A calculation for physical masses is therefore still motivated.

## 5 Conclusions

We have presented the calculation of the mixed QCD-EW corrections to the Higgs production cross section in the gluon-gluon channel. We neglect contributions from matrix elements with two final-state quarks. These particular contributions are separately gauge invariant and are numerically sub-leading as shown in ref. [50]. We work in the limit of massless propagators for the electroweak gauge bosons  $W$  and  $Z$ . Besides providing a non-trivial check of the recent exact calculation of the three-loop virtual corrections [1], we could calculate, in the small-mass approximation, the ratio of non-factorisable to factorisable contributions. A large ratio would challenge the phenomenological assumptions which enter the theoretical predictions for the inclusive Higgs boson production cross section at the LHC. We have reassuringly found a small ratio.

Besides its phenomenological significance to test the validity of prior theoretical predictions, our calculation will be a useful stepping stone for a future full determination of the mixed QCD-EW corrections for physical EW-boson masses.

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