FEEDBACK COMPLEXITY AND MARKET ADJUSTMENT: AN EXPERIMENTAL APPROACH

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ABSTRACT

Recent research on human behavior in dynamic settings has revealed a set of common "misperceptions of feedback" that cause performance in dynamics' systems to deteriorate dramatically in the presence of lags, stock accumulations, side-effects, and non-linearities. Yet these findings have not been tested in a market environment. At the same time, studies in experimental economics have demonstrated that the precise form of the price-setting institution has significant effects on market behavior, even where such institutions are essentially equivalent from a neoclassical perspective.

This thesis explores the relationship between feedback structure and market behavior by observing human subjects in six experimental markets, involving two alternative feedback structures: a complex condition with a three-period lag in production and a multiplier effect on industry demand and a simple condition without these elements, and three alternative price regimes: fixed prices, where market imbalances accommodated by inventories, posted prices, where each firm sets its own price, and clearing prices, where the market-clearing price vector is found by the computer. While neoclassical economic theory would predict virtually no differences in the performance and behavior accross the six experimental conditions, a behavioral perspective predicts large and systematic differences.

The results conform to the behavioral predictions: Performance, relative to optimal, is substantially lower in the complex markets, and the effect is largest under fixed prices and smallest under clearing prices. Market institutions can therefore substantially affect and ameliorate the problems observed in non-market experiments, but their ability to do so depends critically on the efficiency of the price system. Conversely, feedback structure can have substantial effects on market behavior beyond what neoclassical theory predicts.

While markets in the simple conditions show no systematic dynamic behavior, markets in the complex conditions all show characteristic patterns of oscillation (in the fixed- and posted conditions) and an initial overshoot

followed by gradual settlement to equilibrium (in the clearing condition). Subjects' decisions were fitted to simple decision rules, which were then embedded in a simulation model of each market. The simulation models successfully reproduced the essential features of the observed behavior, thus providing an explicit link between a micro-level description of individual agents' behavior and the macro-level market behavior.

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NOTE ON SOFTWARE AND DOCUMENTATION

The experimental markets in this thesis were created on a local-area network of Apple Macintosh computers, using Hypercard version 1.2.2 and Hypercard stacks created by the author. The stacks used a set of CODE resources, also written by the author, to handle networking communications and data plotting.

The data compilation and compression was performed using Microsoft Excel, version 2.2 and, later, version 3.0. The majority of tasks were carried out using macro files.

The statistical analysis was done using SYSTAT version 5.0 and, later, version 5.1. Regressions of individual subject data was done using script files. The SYSTAT output was saved in text files which were then parsed using Microsoft Excel scripts to transform them into numerical tables. All simulations were done in Excel as well.

All data files, scripts, and software is avialable upon request for documentation purposes.

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1. Introduction

Homo economicus, the hyper-rational workhorse of economic theory, is in trouble these days. Psychologists, anthropologists, philosophers, and other social scientists, even a sizable number of heretic economists, continue to accumulate evidence on the systematic and substantial fallacies and shortcomings of human decision making.

The evidence is impressive and convincing, but does it matter? This question has been the subject of a long-standing debate between the economists on one side and just about everyone else on the other. Economists argue that, for the purposes of economic theory, one can safely assume that markets behave *as if* all its agents were rational. Even though individuals may not act optimally, market forces will compensate for the misbeliefs and miscalculations of the simple-minded. In the short run, prices that are out of line will be moved towards their appropriate values by "informed" speculators and arbitrageurs, as the term goes. In the longer run, financial incentives will stimulate agents to learn to improve their performance or hire expert help, or the Darwinian process of competitive selection will weed out the inefficient.

The skeptics respond that the mechanisms summoned in support of orthodoxy can easily go awry. Numerous examples, both theoretical and empirical, have demonstrated that misperceptions and biases in judgment and choice can persist in market settings. Delays, side-effects, and external disturbing influences makes it difficult to learn from experience in real markets. And cumulative learning can be lost as new inexperienced actors enter the market. Evolution through competitive selection may be too slow to keep up with the rate of change in the environment, and organizations may lack the genetic mechanisms that can pass on a successful strategy to later generations.

Thus, the question whether it "matters" cannot be answered in the abstract or the general. Instead, the agenda should be one of charting the borders of *terra neoclassica*. What are the proper domains of neoclassical equilibrium theory where it elegantly and economically explains the facts? What are the key attributes that determine this domain? And, most importantly, where the neoclassical approach must strain or where it fails outright to explain the facts, what theoretical framework can take its place?

Such endeavors require explicit attention to the dynamic processes of market adjustment, with theories based on, and corroborated by, actual observations of human decision makers. Furthermore, they require frameworks, or paradigms, that can serve to organize and unify the research.

The author believes that a fertile paradigm is found in the concepts and theories of engineering feedback control theory, particularly when this theory is combined with laboratory experiments where human subjects play the rule of actors in the system.

In recent years, a number of experiments with human behavior in dynamic feedback systems have revealed that the structure of these systems strongly condition their behavior: Humans exhibit a set of characteristic responses which have become known as "misperceptions of feedback." It appears that the mental models carried in the human mind are incomplete: they do not sufficiently take into account such elements as lags, self-reinforcing feedback and side effects, and non-linear constraints. The result is that when people act in such systems they create behaviors that are very far from optimal.

The motivating question of this thesis is the relationship between feedback complexity and market behavior. In light of previous experimental results, might not the feedback structure of a market strongly condition its behavior? Conversely can one doubt that the presence of market incentives and market forces would modify the performance of systems without a market institution?

In addressing these questions, the thesis also tries to forge a link between the psychologist's detailed studies of individual decision making and the economist's studies of aggregate phenomena. Without a link between these spheres of interest, there is no hope of integrating them into a deeper and broader understanding of the real economy.

The approach of the study is one of using laboratory experiments as the empirical testing ground and a combination of statistical analysis and computer simulation as the tools of theory-building. Chapter 2 discusses the background, motivation as well as the basic approach.

The basic question, the relationship between feedback structure and market behavior, is expressed in an experimental design that varies both the feedback complexity and the market institution.

Complexity is introduced in the form of a production lag and a multiplier effect on demand. In previous experiments such elements interact with decision making to reduce the stability of the system.

An efficient price system might well relieve these problems by stabilizing the system, but prior experimental studies in economics economics have shown that the details of the market institution matter a great deal. To explore the significance of alternative price institutions, the design examines a range of price institutions, from fixed prices, where supply and demand must be coordinated through changes in buffer stocks, to the ideal Walrasian market, where prices automatically move to equate supply and demand and the need for buffer stocks is eliminated. Inserted in the middle is the more realistic case where firms set their own prices while maintaining buffer stocks to accommodate demand.

Together, crossing the experimental treatments of feedback structure (simple and complex) with price regime (fixed, clearing, and posted prices) defines six experimental conditions. It can be shown that in neoclassical stochastic equilibrium the performance and behavior of markets in the six experimental conditions should vary little and that, if firms used available data to estimate the structural parameters in the system, all markets should converge rapidly to this equilibrium. In contrast, the resultss of non-market experimental studies would predict large and systematic variations accross the conditions. Chapter 3 describes the experimental hypotheses, design, and method.

The analysis of the experimental results falls in two parts. The first part focuses on comparing the experimental hypotheses to actual outcomes. Are there small differences accross conditions, as neoclassical theory would predict, or, if there are, do these differences accord with the behavioral hypotheses motivating the design? These questions are the subject of Chapter 4.

The second part of the analysis seeks to describe decision making of individual firms and use simulation to explore how this micro-level behavior could lead to the observed aggregate outcomes. Chapters 5, 6, and 7 describe the results for the fixed-, clearing-, and posted-price conditions, respectively.

Finally, Chapter 8 summarizes the results and discusses their validity and implications for further research. The results clearly show that both sides of the basic research question are relevant: Market institutions can significantly improve the performance of complex dynamic systems. At the same time, the feedback structure of a market has strong implications for its behavior. The benefits of a market mechanism are therefore not automatic; they depend on the feedback properties created by the particular system and market institution. In particular, dynamic complexity may, despite market institutions, lead to outcomes that cannot be explained by neoclassical theory.

Yet these outcomes can be regenerated surprisingly well in simple behavioral simulation models that reflect the bounded rationality of the human mind. The closing note, therefore, is one of optimism: Although *homo economicus* may have withdrawn from a large part of our theoretical landscape, there are others who stand ready to take the field.

2. Motivation and Background

Is human decision making rational, or do people suffer from systematic errors and biases? Does it make any difference to the aggregate behavior of markets? These questions have been the subject of intense debate for many years in psychology and economics. A vast body of psychological evidence suggests individual behavior is flawed. But economists have challenged the relevance of this evidence with the (increasingly rare) contention that agents are more rational in real market settings, or the more frequent argument that the market compensates for individual departures from rationality through dynamic processes of adaption and arbitrage, competitive selection, and organizational learning. Resolving the debate will require explicit attention to these dynamics, in both theoretical and empirical analysis. Human beings must be observed in dynamic market settings.

Studies of decision making in dynamic settings, where actions influence future options as well as present outcomes, are sparse. But already, the evidence indicates that people have a limited ability to anticipate and discern the dynamic behavior implied by the stocks, flows, and mutual interdependence of the elements of the systems in which they operate, and as a result fail to adjust their behavior appropriately in alternative dynamic environments. So far, however, there have been no attempts to test whether these findings persist in the presence of market mechanisms and financial incentives.

At the same time, experimental economists have studied a wide variety of institutional arrangements and shown how the details of the market institution systematically affect market adjustment and efficiency, even when such institutions are essentially equivalent from a neoclassical perspective. Yet, with notable exceptions, there has been little explicit attention to dynamics; while the markets studied often have a great deal of "static" or "combinatorial" complexity, they lack such real-world features as lags, stock-and-flow structure, endogenous supply and demand functions, and non-linearities, and the main emphasis has been on settlement to equilibrium states rather than the dynamics of the market adjustment process.

To be relevant for the study of market dynamics, empirical studies must involve markets, and they must involve dynamics. The motivation for this thesis is to develop this area of inquiry by joining the experimental studies of dynamic decision making and market institutions.

The first part of this chapter reviews the contrast between conceptions of choice in psychology and economics and highlights the need for an explicit treatment of dynamics. The section following reviews the relevant experimental results from studies of dynamic decision making and from experimental studies of markets. The review reveals a need to extend experimental economics to incorporate explicit studies of dynamics and the structural features of the system that might shape these dynamics. The last section discusses the approach taken in the study, emphasizing the need for tools that link behavior at the individual or "micro" level with outcomes at the aggregate or "macro" level of concern to economists. It is argued that such a link can be forged by taking a feedback control view of the market and, within this framework, to combine the procedures from experimental economics with computer simulation, statistical analysis of individual decisions, and process-tracing data. The next chapter will discuss the specific experimental method, treatment variables, and hypotheses of the study.

2.1 Individual and market rationality: The need for a dynamic perspective

This section reviews the debate surrounding the validity of the common joint assumptions of equilibrium and rationality in neoclassical economic theory. A consideration of each of the standard arguments in defense of neoclassical assumptions, combined with the inherent methodological limits of the neoclassical approach, reveals a need for explicit treatment of disequilibrium dynamics, supported by empirical studies of how people actually behave in economic settings. Although recent years have seen an increasing interest in dynamic, disequilibrium economic models, the assumptions employed in these models have not yet been subjected to direct empirical testing.

The debate between psychology and economics

A schism has long existed between mainstream economics and psychology concerning proper axioms to characterize human action (Hogarth and Reder 1987). Regardless of the conclusions of that debate, it is clear that the inconsistency of views represents a challenge to both disciplines for achieving a reconciliation.[1]

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1 The use of such general terms as "economics/economists" and "psychology/psychologists" in this discussion does little justice to the wide range of research paradigms and opinions in both disciplines. Even more specific terms, such as "mainstream, neoclassical economics" or "behavioral decision theory," skirt many nuances and exceptions. The terms should therefore be understood as shorthand for two opposite ends of the spectrum of opinions in the debate.

Chapter 2

Economists have traditionally relied on the joint assumptions of perfect rationality and equilibrium, often known as the "neoclassical" approach. People are assumed to be rational in the sense that they maximize their expected utility or profits, given constraints of resources and information, and markets are assumed to be in equilibrium in the sense that the expectations of individual agents are mutually consistent. Moreover, since economics has traditionally been a non-experimental science, economists rely on statistical analysis of aggregate historical data for empirical tests of their theories. The unique strength of this approach is the fact that only the substantive aspects of a decision matter; the process through which decisions are made is irrelevant (Simon 1978). On this basis, the field has developed an impressive deductive system and body of knowledge.

Psychologists, on the other hand, have emphasized description of the process of decision making and the extent to which individuals conform to the tenets of rational choice. This area of psychology has become known as "behavioral decision theory," as distinct from "decision theory," which concerns the normative aspects of decision making. A large body of empirical evidence, both experimental and historical, makes it abundantly clear that many, if not all, of the tenets of rational choice are frequently violated: It appears that human judgment and choice follow certain simplifying heuristics or "mental procedures" that work well under some conditions, but also lead to systematic and substantial deviations from utility-maximizing behavior in other situations. The range of phenomena is too extensive to recount here; surveys can be found in Hogarth (1987), Kahneman, et al. (1982), Slovic, et al. (1977), and Thaler (1987).

However, psychologists have paid relatively little attention to the aggregate implications of individual behavior. Indeed, it is mainly in the link between the behavior of individuals and the aggregate economic outcomes that economists take issue with psychologists: In defending the assumption of rationality, economists have appealed to the efficiency of the market mechanism. Even though individuals may appear "irrational," the argument goes, their irrationality is an experimental artifact, or their interaction in the marketplace will result in overall outcomes that are for all practical purposes indistinguishable from outcomes based on rational behavior. A number of arguments are forwarded to support this contention, each of which has been challenged by skeptics of the rational-equilibrium approach.

Argument 1: Experiments lack realism and incentives, limiting their external validity.

Economists argue that financial incentives will induce participants both to put more effort into making accurate judgments and correct decisions, and to learn and improve their performance, particularly when the stakes are high. However, both theoretical arguments and empirical evidence make it clear that economic incentives do not necessarily eliminate systematic biases and errors in a variety of static tasks. Parallel experiments run with or without monetary incentives have, with some exceptions, produced two basic conclusions: 1) monetary incentives induce subjects to pay more attention to the task, thus reducing unsystematic errors (inconsistency) in behavior, and 2) the systematic violations of rationality tend to persist, or even become somewhat stronger, in the presence of incentives.[2]

Others argue that behavioral decision research on the effect of incentives has been conducted in laboratory settings with relatively low monetary rewards, and therefore the results cannot be transferred to real markets with large financial stakes. Yet some phenomena have been replicated in real-world settings. For instance, Lichtenstein and Slovic (1973) found strong evidence of preference reversals among professional gamblers in a Las Vegas casino. Furthermore, as Thaler (1987) argues, there is no *a priori* reason why the intuitions that guide individual decision making in experimental settings should not also apply in real-life situations.

Alternatively one might argue that in real life decision makers have a variety of tools at their disposal, such as decision support systems or consulting services. Yet real life is also much more complex than laboratory markets, creating a bigger challenge to the decision maker.

Moreover, while analytical methods certainly help to promote consistency and understanding, any model must be based on initial

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2 Examples can be found in Grether 1980; Grether and Plott 1979; Knetsch and Sinden 1984; Lichtenstein, et al. 1977; Lichtenstein and Slovic 1971, 1973; Pommerehne, et al. 1982; Reilly 1982; Tversky and Kahneman 1981. The failure of incentives to improve performance is not universal, however. The binary prediction task, in which subjects are asked repeatedly to predict a two-outcome lottery draw, is a case in point. Without monetary incentives, people usually fail to use the optimal rule, which is always to predict the most likely outcome; instead, their guesses tend to match the frequency of the two outcomes, in a phenomenon known as "probability matching." When monetary incentives are added, however, subjects' responses do move closer (though not all the way) to the optimal strategy (see Edwards, 1956; Siegel, 1961; Tversky and Edwards, 1966). For further discussion of the effect of incentives on performance, see Hogarth, et al. (1989).

assumptions that can only come from intuition. An econometric model, for example, involves assumptions both about the structure of its equations and about future values of exogenous variables. With such a wide range of possible assumptions, a subtle process of selection is bound to take place: If the output from the model does not "seem plausible," i.e. does not accord with intuition, then the model and its exogenous inputs are changed to make a better fit--sometimes referred in the profession as "add factoring".

These "mental models" probably exert much greater influence over decisions than is generally believed (Sterman 1987b, 1988a, b). For instance, Sterman found that professional forecasts of inflation (1987b) and energy demand forecasts (1988a) could both be described quite well by a simple trend extrapolation and, in the case of energy demand forecasts, that there is a systematic "political" bias in the forecasts of particular organizations (1988a).

Furthermore, there is considerable evidence that complex econometric forecasting models are outperformed by simple trend extrapolation or naive forecasts (e.g. the M-competition of Makridakis, et. al 1982; Ahlers and Lakonishok 1983), and that professional forecasters and businessmen who presumably use such models are outperformed by ordinary consumers (Gramlish 1983; Bryan and Gavin 1988; Thies 1986).

In short, there is little empirical ground for believing that the use of "expert tools" would necessarily improve performance or eliminate the phenomena observed in experiments.

Argument 2: Competitive selection will favor rational agents

It is often argued that in the long run, competition will weed out inefficient actors (Alchian 1950; Friedman 1953; Winter 1964). But when one considers exactly how competitive selection would work in reality, one finds several ways in which the process may be effectively prevented from improving performance. First, evolutionary processes may work too slowly and with too little force relative to the rate of change in a complex world characterized by large external shocks and technological innovation. In such a world, the survival of a firm may be more a function of its luck and financial robustness than the "optimality" of its decisions. Second, competitive selection is only relative--the best-performing firms may not be even close to optimal. In fact, there is considerable evidence that biological evolution has produced frequent "dead ends" (Gould 1989). Third, while irrational agents may indeed fare poorly in the fight for economic survival, the market may continue to attract new inexperienced agents to take their place (Camerer 1987). Biological evolution relies on a genetic mechanism to replicate and pass down adaptive genes to future generations. Social and economic evolution, on the other hand, must rely on the imperfect process of individual and organizational learning. Thus, the benefits of evolution are far from automatic, and any satisfactory approach must thus show explicitly how the process of competitive selection would work.

There has been a growing interest among economists in the process of market evolution, e.g. through innovation and competitive selection. The interest has fostered a new field, "evolutionary economics," which addresses these questions directly, using a variety of analytical techniques (see e.g.

Axelrod 1987; Culbertson 1982; Day 1989; Goodwin 1989; Iwai 1984b; Kauffman 1988; Miller 1989; Nelson and Winter 1982). In the typical model, agents learn over time by selecting decision rules through trial and error or hill-climbing, or agents with fixed decision rules are subjected to competitive selection. As a result of this learning and/or selection, the system as a whole evolves over time, often in surprising and unpredictable ways. In particular, there are a number of plausible situations where it is not the "best and the brightest" who survive. For instance, technological "lock-in" and network externalities, scale economies, or "free riding" in technological innovation can all put the system on a sub-optimal path (see e.g. Arthur 1988; Nelson and Winter 1982; Nelson and Winter 1974; Iwai 1984a, b; Day 1989; Goodwin 1989). Or irrational traders who underestimate the riskiness of assets may over time come to dominate the market, as they on average receive higher returns than risk-averse rational investors (De Long, et. al. 1991).

These studies all show that plausible assumptions about learning and competitive selection can yield suboptimal behavior. But whether and how fast evolution actually occurs is still an empirical question requiring experimental testing and field research, very little of which has been done.

Argument 3: Arbitrageurs and speculators will eliminate inefficiencies

Some argue that in a well-functioning market not everyone needs to be perfectly rational: A few clever agents will, through arbitrage and speculation, move prices so that they reflect the true trade-offs in the system. In a way, non-rational agents learn "implicitly" from rational agents so that, in long term equilibrium, the system will behave as if all agents were rational (Camerer 1987). In turn, the main argument in defense of equilibrium is that, whenever the system is not in equilibrium, it will present profitable opportunities which, when acted on, will drive it back toward equilibrium. Therefore, persistent disequilibrium that could be corrected through arbitrage or speculation is highly unlikely.

This is not to say that suboptimal resource allocation in general cannot persist. For instance, institutional or informational constraints, such as wage rigidity or limited information, may prevent markets from clearing. But this kind of disequilibrium is created by structural constraints on information availability and allowable actions, and retains the notion that agents behave optimally, given such constraints--there are no exploitable opportunities for excess profits.

Yet there are also theoretical arguments why rational agents would not necessarily move the market to rational equilibrium. A series of vignettes have demonstrated how non-rational agents can dominate a market that also has rational agents. For instance, Arrow (1982) argues that rational investors can only realize gains from wrongly priced securities when the market corrects itself; if prices continue to be dominated by "irrational" investors, better-informed investors cannot profit from their knowledge, at least in the short run. Haltiwanger and Waldman (1985) show how "synergy effects" (where agent j benefits if other agents do the same thing that j does) allow naive agents to affect prices disproportionately. Akerlof and Yellen (1985a, b) show how deviations from optimal behavior which have only second-order effects for the individual agent can have first-order effects on the market. Russell and Thaler (1985) provide examples of competitive markets where agents cannot earn arbitrage profits from mistakes of less sophisticated agents, so that those mistakes may persist.

Argument 4: Agents will learn to perform better

Another argument in support of rationality posits that, over time, people will learn to perform better, or they will hire expert assistance. The market should therefore over time be dominated by rational "experts." Ultimately, however, this is an empirical question, and much work is still needed to study learning in real-world environments.

One question to be asked is whether "experts" are indeed more rational (i.e. less prone to common psychological biases) than novices. As already mentioned, the study of professional forecasters leaves room for doubt (see above). In addition, numerous studies in psychology have compared expert and novice performance, and the general conclusion is that experts seem just as prone to biases and errors. For instance, McNeil, et al. (1982) found that doctors are very susceptible to the "framing effects" observed by Tversky and Kahneman (1981). Lichtenstein, et. al (1977) surveyed numerous studies of overconfidence in judgment that hold for experts as well as novices. Experts frequently distinguish themselves by being more confident in the accuracy of their judgment rather than being more accurate (Oskamp 1965, p. 287). One notable exception is weather forecasters, who, as a result of very frequent and tangible feedback, are very aware of the limitations of their predictions (Murphy and Winkler 1977). Kahneman and Tversky (1973, Tversky and Kahneman 1971) find that the intuitive judgments of individuals who have had extensive training in statistics, professors of psychology, for instance, continue to rely on "the law of small numbers." Moreover, even if experts were indeed clearly better than novices, the sad fact remains that real-world experts die or retire, and markets may continue to attract naive, inexperienced agents, making it less obvious that wisdom will predominate.

An equally relevant question is whether individuals can learn from market experience. While experimental studies specifically concerned with learning in markets are sparse, the theoretical aspects of learning and convergence to rational expectations equilibrium has received considerable attention in economics in recent years (see e.g. Anderson, et al. 1988; Blume and Bray 1982; Bray and Savin 1986; Frydman 1982; Frydman and Phelps 1983; Marcet and Sargent 1989a, b).[3] Although these studies represent a large step forward, they typically assume that agents know the structure of the market, except for a finite set of parameters, and that they optimally estimate these parameters (using Bayes' rule). Yet there is little evidence that humans are Bayesian learners. On the contrary, numerous studies have demonstrated that people tend to ignore or underrate base rates in one-time judgments under uncertainty (Kahneman, et. al 1982, parts II to IV) while they tend to

3 Camerer (1987) explicitly considered learning in an experimental market. Testing for evidence of the "representation bias," he found that experience and market forces do partially alleviate biases for some subjects, but not all.

underrate new information when updating judgments in multi-stage evaluations (Edwards 1968).

Recently, a number of researchers have tried to make less restrictive assumptions about learning, primarily by using computer simulation instead of analytical techniques. An ongoing project by Rust, et al. (1989) seeks to reproduce the convergence of double oral auctions using computer-simulation tournaments, in the tradition of Axelrod (1987). Elliott (1990) simulates learning in a repeated prisoner's dilemma game. Marimon, et al. (1989) use the same framework to examine how a value-less good (money) can evolve to become the medium of exchange in a simple exchange economy. Both of these works adopt the "classifier" model of induction, where agents start more or less from a position of complete ignorance about the system and learn by a "Darwinian" evolution of their decision rules (see Holland, et al. 1986). On the other hand, there have been no attempts yet to combine these efforts with experimental testing.

However, the "classifier" models depend on the "bucket-brigade" heuristic which assigns credit (or attributes causality) to different conditions in the environment. Yet humans judgment of cause and effect are subject to numerous fallacies (e.g., Einhorn 1982; Einhorn and Hogarth 1978; Bar-Hillel 1980). In particular, one must ask whether or not a market offers a good environment for learning, and whether favorable conditions for learning are the rule or the exception in real economic markets. As mentioned before, the issue is largely unexplored, but some researchers find grounds for doubts:

> The necessary feedback is often lacking for the decisions faced by managers, entrepreneurs, and politicians because (i) outcomes are commonly delayed and not easily attributable to a particular action; (ii) variability in the environment degrades the reliability

> of the feedback, especially where outcomes of low probability are involved; (iii) there is often no information about what the outcome would have been if another decision had been taken; and (iv) most important decisions are unique and therefore provide little opportunity for learning (See Einhorn and Hogarth 1978). The conditions for organizational learning are hardly better. Learning surely occurs, for both individuals and organizations, but any claim that a particular error will be eliminated by experience must be supported by demonstrating that the conditions for effective learning are satisfied. (Tversky and Kahneman 1987, pp. 90-91.)

Empirical evidence of irrational behavior in markets

Experimental evidence suggests that market arbitrage has limited ability to correct individual biases. For instance, it appears that the well-known "representativeness bias" in Bayesian inference (Kahneman and Tversky 1982) is reduced but does not disappear in experimental markets, even with experienced subjects (Camerer 1987). In another experiment, Camerer, et al. (1987) report similar results for the "hindsight bias" (Fischoff 1980). In the "bubble" studies of Smith, et al. (1987), experimental stock markets frequently show evidence of speculative bubbles, even with experienced subjects, although the fundamental value of the stock is known to the participants. Berg, et al. (1985) find that the presence of experimenter arbitrage diminishes but does not eliminate preference reversals. (It should be noted, though, that in the study of Knez, et al. (1985), anomalies in subjects' preferences for assets disappear under repeated trading for money).

A further cause for skepticism toward orthodox theory are the numerous examples of seemingly irrational behavior in real markets. Although it is inherently more difficult to assess rationality in a complex world (Zeckhauser 1987), there are empirical "anomalies" in real markets, i.e. phenomena that are difficult or impossible to explain using rational-agent models.[4] For instance, studies of people's assessment of low-probability, high-loss events and their decisions to purchase insurance have demonstrated seemingly irrational attitudes toward risk (Kunreuther, et al. 1978). Empirical evidence from stock prices seems to suggest that investors overreact to new information (see e.g. Arrow 1982; De Bondt and Thaler 1985; Dreman 1982; Shiller 1981). Another example is closed-end mutual funds, which are frequently traded at an initial premium and later at a substantial discount from the market value of their portfolios (Lee, et al. 1990a).

The evidence is all the more interesting because such phenomena can often be explained in a fairly simple way from typical biases and errors found in psychology. For instance, Shefrin and Statman (1984) argue that many firms' dividend-paying policies are inconsistent with orthodox financial theory but are consistent with the "prospect theory" of Kahneman and Tversky (1979), or with neo-Freudian theories of "self-control" (Shefrin and Thaler 1981; Schelling 1984a, b). Thaler (1980 1985) argues that certain anomalous consumer habits and firm pricing strategies may be derived from the system of "mental accounting" that people appear to use in making decisions. Akerlof and Dickens (1982) show how labor market anomalies might result from workers resolving cognitive dissonance by altering their beliefs. Lee, et al. (1990b) found evidence that the fluctuations in closed-end mutual fund discounts are driven by investor sentiments.

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4 Indeed, such "anomalies" are sufficiently common that a regular column in the Journal of Economic Perspectives is devoted exclusively to this topic.

Chapter 2

The limits of the neoclassical approach

In addition to the debate surrounding the empirical validity of rationality assumptions, a number of economists have pointed out that the neoclassical approach has inherent methodological limitations that are too severe to be ignored.

First, the simplicity and beauty of rational equilibrium is sometimes strained: Kenneth Arrow questions the usefulness of modern dynamic "rational-expectations" theories in macro-economics since they attribute enormous sophistication to decision makers in the economy. "Each individual agent is in effect using as much information as would be required for a central planner," thus undermining the much-praised informational efficiency of the price system (Arrow 1987).

Second, there has long been a concern among economists that the preoccupation with equilibrium and rationality has excluded insights into a range of important economic phenomena (Kaldor 1972; Leijonhufvud 1968; Phelps-Brown 1972). The criticism has at times been quite harsh:

> ... in my view, the prevailing theory of value--what I call, in a shorthand way, 'equilibrium economics'--is barren and irrelevant as an apparatus of thought to deal with the manner of operation of economic forces, or as an instrument for non-trivial predictions concerning the effects of economic changes, whether induced by political action or by other causes. I should go further and say that the powerful attraction of the habits of thought engendered by 'equilibrium economics' has become a major obstacle to the development of economics as a *science*--meaning by the term 'science' a body of theorems based on assumptions that are *empirically* derived (from observations) and which embody hypotheses that are capable of verification both in regard to the assumptions and the predictions.
> (Kaldor 1972, p. 1237; emphasis in original)

Regardless of the epistemological issues raised by Kaldor and others, it is fair to say that many economic issues, such as innovation, learning, the role of entrepreneurs, the evolution of markets, increasing returns, and competitor behavior, are inherently dynamic and complex and thus difficult to treat meaningfully in an equilibrium context. Indeed, recent developments in non-linear modeling point to the possibility that economic systems may exhibit "chaotic" behavior and may thus be inherently unpredictable (Arthur 1988; Barnett and Chen 1988; Boldrin 1988; Day 1989). Sterman (1989a) showed how chaos can arise in an experimental economic system.

Third, while rationality is certainly a strong assumption about agent behavior, it is often neither necessary nor sufficient to explain aggregate phenomena (Simon 1979). Neoclassical theory building and testing frequently require additional "bolstering" assumptions, such as perfect competition, complete markets, separable utility functions, constant or decreasing returns to scale, etc. (Arrow 1982, 1987; Simon 1987). Further, theories are often supplemented with ad hoc assumptions that limit rationality, such as in Lucas' famous model (1972) in which firms cannot distinguish real price shocks from inflation caused by monetary changes. Conversely, there is often a simple explanation for an empirical phenomenon based on assumptions that behavior is reasonable, perhaps, but not rational or optimal, whereas a rationalization of that phenomenon may take enormous contortions on the part of the theorists (Simon 1979, 1987).

Fourth, when testing theory, economists have relied almost exclusively on aggregate data because the assumption of rationality makes study of individual decision making irrelevant--yet the record of using such data to settle theoretical economic debates has not been impressive. As Leamer (1983) points out, due to the large number of possible variables and equations that could be used to express the same theory, econometric inference is both "whimsical" in its assumptions and "fragile" in its conclusions. Leontieff laments that

> In no other field of empirical inquiry has so massive and sophisticated a statistical machinery been used with such indifferent results. ... The same well-known set of figures are used again and again in all possible combinations to pit different theoretical models against each other in formal statistical combat. ... True advance can be achieved only through an iterative process in which improved theoretical formulation raises new empirical questions and the answers to these questions, in their turn, lead to new theoretical insights. (1971, pp. 3, 5.)

While Leamer advocates a more systematic attention to the limits of econometric inference, Leontieff argues that to truly advance knowledge economists need to use other data sources as well, such as direct observation of how business decisions are actually made (1971, 1982; Simon 1984).

The need for a dynamic approach

There is a striking feature that pervades the debate about individual versus market rationality: Except for the question of whether experimental results are externally valid, all substantive defenses of the neoclassical approach rely on arguments that are inherently dynamic. Arbitrage, competitive selection, and learning are all processes that take place over time and out of equilibrium.

Many believe that economic theory could be enriched by explicit models of the dynamic, disequilibrium adjustment process of markets, of individual behavior in such dynamics settings, and of the process of individual learning. The inquiry would be guided by two aims: First, to establish the limits of neoclassical theory, and second to provide alternative theories of the behavior of markets that fall outside those limits. Thus, rather than debating whether markets are rational or not, one should ask such questions as:

- When does it matter that individuals are not rational, i.e., under what circumstances will market mechanisms correct or compensate for individual mistakes and false beliefs? Can one generalize such circumstances?
- When dynamics and non-rationality are of essential importance, what are the "laws of motion" that govern the economic system? Can such laws be generalized to other systems?
- Under what circumstances will agents need to learn in order to improve their behavior? Alternatively, when will simple adaptive heuristics (with no explicit learning) be sufficient to create an efficient aggregate market?
- When learning is called for, when will it be possible for agents to learn from experience?
- What are the heuristics or rules that govern individual actions, and how might such heuristics change over time as a result of learning?
- What are the processes the govern the evolution of the market, i.e. how do agents enter and exit the market and change their relative importance?
- How does the dynamic behavior of the market arise from individual adaptive actions and learning, and from evolutionary forces?

Although many steps have been taken toward dynamic disequilibrium theories, the integration of psychology and economics will require that both the assumptions and the predictions of such theories be tested empirically.

One arena of testing is the area of laboratory experiments, which is the subject reviewed in the next section.

2.2 Dynamic decision making and experimental economics: The need for a synthesis

To be relevant to the study of market dynamics, experiments must involve both dynamics and markets. Unfortunately, the union of these two research areas is practically empty. Although research in the area of human decision making in dynamic settings is sparse compared with the extensive work on static decision tasks, the evidence clearly suggests that structural elements common in economic systems, such as non-linearities, lags, stocks and flows, feedback loops, and side effects, strongly and systematically affect the behavior and performance of the decision maker. But these effects have not been studied in the context of market forces. Conversely, experimental market studies have put little emphasis on dynamics, choosing instead to focus on the effects of alternative institutional arrangements on market equilibria and, occasionally, the speed of market convergence. In particular, the markets studied use simple, exogenous reward schedules for buying and selling and the market is usually "reset" before each trading period.

The feedback paradigm

Most work in behavioral decision research has viewed decision making as an isolated, discrete choice between alternatives. There is, however, another way of viewing decision making, frequently referred to as the "dynamic decision-making" or "feedback-system" paradigm. The feedback view of decision making views human behavior as a stream of individual decisions, where the actions taken influence the context of future actions. At the most general level, all human decisions take place in the context of a feedback system:

> An information-feedback system exists whenever the environment leads to a decision that results in action which affects the environment and thereby influences future decisions. This is a definition that encompasses every conscious and subconscious decision made by people. (Forrester 1961, p. 14)

If one considers all states of nature, including the knowledge of the decision maker, Forrester's statement is obviously true but perhaps too general to guide research in decision making. Yet, as Hogarth (1981) and Klayman and Ha (1987) have observed, many real-life decisions are in fact dynamic and partially repetitive, where individuals interact continuously with a surrounding system.

Hogarth (1981) points out that the systematic errors and biases observed in static decision-making tasks may not be very relevant for real-world situations if people have the opportunity to correct their mistakes. One would expect humans to perform well in systems where there is access to immediate and undistorted outcome feedback (i.e. where all consequences of actions are immediately and fully known), and where there is little commitment involved in previous decisions (i.e. where past actions do not put severe limits on future options). A relevant example of such a system is the typical experimental market where supply and demand schedules are exogenous and the system is "reset" every period.

On the other hand, humans may have significant difficulties in managing "complex" systems with lags, stocks, flows, and feedback loops. Here feedback can be distorted or delayed and actions can have consequences that are distant in space and time. In the field of system dynamics, experience with numerous simulation models of such systems has revealed a set of common characteristics. In a word, complex feedback systems tend to be "counterintuitive," i.e. they defy human intuition because of our limited capacity to anticipate the dynamic behavior arising from complex, non-linear feedback relationships:

> First, social systems are inherently insensitive to most policy changes that people select in an effort to alter the behavior of the system. In fact, a social system tends to draw our attention to the very points at which an attempt to intervene will fail. Our experience, which has been developed from contact with simple systems, leads us to look near the symptoms of trouble for a cause. When we look, we discover that the social system presents us with an apparent cause that is plausible according to what we have learned from simple systems. But this apparent cause is usually a coincident occurrence that, like the trouble symptom itself, is being produced by the feedback-loop dynamics of a larger system...A second characteristic of social systems is that all of them seem to have a few sensitive influence points through which the behavior of the system can be changed. These influence points are not in the locations most people expect. (Forrester 1970, p. 216.)

Most work on the effects of feedback on performance has centered on "outcome" feedback, where subjects receive information about the accuracy of their judgments or the outcomes of their decisions in a series of choices, but the decisions do not feed back to alter the environment. The world is "reset" after each decision to exogenously specified conditions (Edwards 1990). Particularly common is the "multiple-cue probability learning" (MCPL) paradigm, in which subjects are asked to estimate the value of some variable based on a set of cues (independent variables) (see Brehmer 1980a; Brehmer and Kuylenstierna 1980). Even here, where outcome feedback is immediate though perhaps blurred by random noise, the general conclusion from such studies is that human learning capability is limited: While people can learn simple linear relations quite rapidly and efficiently, their performance deteriorates dramatically in the face of non-linearities, noise, or correlated (redundant) cues (Brehmer 1980a, b).

It is important, however, to distinguish the "outcome" feedback used in MCPL from "action" feedback, in which the environment and the choice set are functions of past choices. Action feedback systems warrant study both because of the potentially "counterintuitive" nature of such systems and because, in most real-life situations, actions have consequences. One must "learn by doing" without knowing how things would have turned out if one had done otherwise. As Einhorn (1982) has pointed out, this makes learning inherently more difficult and introduces the possibility of "false" or illusory learning.

Early studies in dynamic decision making

In the 1960's and early 1970's, there was some interest among psychologists in dynamic decision making (i.e. decision making with action feedback). The starting point of these efforts were two pioneering papers by Edwards (1962) and Toda (1962).

Edwards, partly inspired by the optimism from early experiments on "man as intuitive statistician" (e.g. Peterson and Beach 1967), proposed a methodology of comparing observed behavior to an "ideal" (optimal) decision maker in dynamic information sampling and processing tasks. In an experiment involving updating prior beliefs based on a stream of new evidence, he found that people are basically "good statisticians," i.e. they revise their opinion in the right direction, but that they do so too slowly or insufficiently. He named this phenomenon "conservatism."[5]

Toda based his work on the idea that human behavior may be much more adaptable and appropriate than static discrete choice experiments suggest:

> Man and rat are both incredibly stupid in an experimental room. On the other hand, psychology has paid little attention to the things they do in their normal habitats; man drives a car, plays complicated games, and organizes society, and rat is troublesomely cunning in the kitchen. (Toda 1962, p. 165.)

Thus, he argued, subjects should be observed in their interaction with the relevant environment rather than in isolated situations. He advocated using artificially created, dynamic "microcosms" in order to "study how people plan, or how people, individually or collectively organize their behavior over time."

Later, Rapoport and Wallsten (1972) and Rapoport (1975) suggested a general research paradigm for such studies that emphasized comparisons to optimal behavior. It was believed that, through the use of constrained optimization techniques, the "logic underlying the planned behavior ... in a well-defined environmental context" could be uncovered and form the basis of a deductive science of dynamic decision behavior (Toda and Miyamae 1967; Rapoport 1975).

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5 Yet the subsequent work of Kahneman, Tversky, and others has since provided overwhelming evidence that people are frequently not even remotely Bayesian in their approach to uncertainty. See e.g. Kahneman, et al., eds., 1982, parts II, IV and VI.

This is not to say that people would necessarily be assumed to be optimal in the objective sense implied by the environmental features. Limitations imposed by human cognitive abilities could be included as additional constraints in the mathematical optimization problem:

> Dynamic decision theory then primarily compares real with optimal behavior, leading to modifiable 'normative-descriptive' models ... For example, if an observed discrepancy is reasonably and meaningfully interpreted as due to a limitation in [the decision maker's] information processing capability, such as his finite memory or his limited capability to project the effects of his decision into the future, this limitation may be incorporated into the model as an additional constraint. The optimal policy may then be derived under this constraint and compared to actual behavior. One may successively collect data, compare actual to optimal behavior, and improve the model so that, finally, decisions can be interpreted as optimal under given perceptual, intellectual, and cognitive constraints. Dynamic decision theory may prove useful only if discrepancies between optimal and actual decisions are small, systematic, and the constraints are psychologically interpretable.
> (Rapoport and Wallsten 1972, p. 167.)

The approach was applied to a variety of comparatively simple dynamic settings, such as inventory control tasks (Rapoport 1966), multi-stage betting (Rapoport 1970; Rapoport and Jones 1970; Rapoport, et al. 1970; Funk, et al. 1979), signal detection and Bayesian revision (Edwards 1962; Shuford 1964), and multi-stage search and optimal stopping rules (Rapoport and Tversky 1970).

However, the original ambitions of a general deductive theory of dynamic decision making were not satisfied, for several reasons. First, practical constraints in computer hardware and software at the time limited the experimental setups. Second, the emphasis on optimal solutions limited the study to systems that were so simple as to border on the trivial or to more realistic systems that were impossible to solve without so many additional assumptions that the interpretation of experimental results would be ambiguous (Slovic, et al. 1977). A third problem, sometimes referred to as the "curse of insensitivity" is that dynamic programming problems often have "flat optima" so that moderate or even substantial deviations from optimal decision behavior produce only small changes in payoffs to the decision maker. This fact made it difficult to discriminate between criteria that subjects may apply in their choices.

As a result, dynamic decision making did not receive much attention in mainstream behavioral decision research for a decade (Slovic, et al. 1977, p.14), although studies continued on a more ad hoc basis in related fields, e.g. in ergonomic studies of man-machine interactions (e.g. Allen 1986; Rasmussen 1974; Sheridan and Johanssen 1974; Tzelgov, et al. 1985).

The revival of dynamics

In the past decade, however, there has been a renewed interest in the field, in large part motivated by the recognition that dynamics are important:

> The failure to study and evaluate judgment and choice as continuous processes has had two important, negative consequences. First, insufficient attention has been paid to the effects of feedback between organism and environment. Second, although judgmental performance has been evaluated according to principles of optimal behavior implied by decision theory and the probability calculus, few researchers have questioned whether the assumptions of such models apply to continuous processes. (Hogarth 1981, p. 198.)

The recent work has been more exploratory in nature and modest in its ambitions, but already, it seems clear from the results that certain feedback systems are difficult for humans to manage, due to what Sterman (1989b) denotes systematic "misperceptions of feedback."

Berndt Brehmer's work (1987, 1988) exemplifies an eclectic approach in which subjects are observed operating in computerized "microcosms" similar to Toda's (1962) idea, except that Brehmer compared actual behavior to a baseline strategy rather than optimized behavior, since the optimal strategy in such complex and realistic systems is ill defined, even unknowable. He conducted a series of experiments in which subjects managed a simulated forest fire department. People underestimated the positive feedback mechanism causing the exponential growth of a forest fire and the delays involved in deploying fire teams, they maintained too much central control, and they reacted to circumstances after the fact rather than anticipating them, as would have been more appropriate in a system with such long response lags.

Similar difficulties in dealing with positive-feedback systems and exponential processes have been documented in multi-stage betting tasks (Rapoport and Jones 1970; Funk, et al. 1979): Subjects' behavior is unduly affected by past wins or losses and by the size of the current capital stock.[6] In forecasting tasks, Wagenaar and colleagues (Timmers and Wagenaar 1977; Wagenaar and Sagaria 1975; Wagenaar and Timmers 1978, 1979) have documented a tendency for subjects to underestimate substantially the rate of change implied by exponential growth.

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6 The probability of winning in each bet, p, is known to participants. Assuming either constant absolute or constant relative risk aversion, the optimal strategy is to invest a constant proportion of current wealth, where this proportion depends on p and the degree of risk aversion.

Hogarth and Makridakis (1981b) compared the performance of simple computer-created strategies in competition with human players in a dynamic business game which was rich with details. Interestingly enough, although the computer strategies were selected more or less arbitrarily, with no attempt to optimize their performance, they actually outperformed a significant fraction of the human players, even though the latter spent many hours discussing each decision: The computerized player outperformed 41% of the human players when using a deterministic set of rules, and 19% when some random noise was added to the rule. It appears that consistency in decision making is valuable in itself, even if the rules of thumb used are crude and sub-optimal.

Kleinmuntz and Thomas (1987) observed students and doctors in a simulated medical diagnosis-and-treatment task and found a tendency for both groups to order too many diagnostic tests and wait too long before selecting treatments, thus achieving a survival rate of their "patients" substantially below reasonable "benchmark" decision rules. Although much of the excessive diagnostic testing may be due to the participants' preconceptions of "what doctors do," it also suggests that subjects had an inherently static conception of the task as a discrete one-time decision of what treatment to use rather than as a dynamic process of trial and error.

Broadbent and Aston (1978) investigated human control of an econometric simulation model of the British economy. Subjects were ask to make decisions on government expenditure, tax rates, and money supply targets in order to minimize unemployment, inflation, and trade deficits. The authors did not consider absolute measures of performance such as comparisons to benchmark strategies, but instead measured subjects' knowledge of the structure and parameters of the system (through pre- and post-task questionnaires). Interestingly, although subjects seem to improve their ability to achieve their policy objectives during the course of the game, the experience had little effect on subjects' qualitative understanding of the system. Moreover, subjects' accuracy in forecasting the impact of specific policy changes improved significantly in the course of the game when this impact was immediate (e.g. the effect of fiscal policy on unemployment), but failed to get better when effects were lagged (e.g. fiscal policy's impact on inflation).

Dörner (1980, 1989a, b) observed students and professionals managing a variety of simulated systems, ranging from a small township (1980) to an ecological system (1989b). The majority of subjects failed to achieve even close to optimal or "benchmark" results. Dörner attributed these failures to a variety of cognitive factors, including frequent changes of strategy and inconsistency in maintaining a particular policy ("thematic vagabonding"), a tendency for "ballistic" actions which ignore compensating feedback relations in the system, and, generally, a tendency to focus on short term, immediate effects rather than more long-term and fundamental processes.

It is interesting to note that such behavior is entirely appropriate in the typical environment from which humans evolved. The hunter-gatherer's environment was dominated by constant, large "external shocks" (sabre-tooth tigers, etc.), requiring instant attention and response, and the link between action and result was immediate. But in the complex interrelated world of modern human life, a focus on only short-term, local outcomes can lead to destabilizing, even catastrophic results. And, equally important, a short attention span (in time and space) prevents one from learning from experience.

These issues were highlighted in two experiments by John Sterman, investigating human behavior in a multistage production-distribution system (1989c), and a investmet multiplier-accelerator economy (1989b), respectively. In both cases, the majority of subjects performed very poorly. Subjects had difficulty adjusting their decisions to reflect the production or shipping delays in the system, and they were often misled by disequilibria caused by non-linearities and self-reinforcing mechanisms in the system. As a result, decision rules estimated for each subject, when simulated on a computer, often resulted in unstable, even chaotic behavior (Sterman 1989a). Sterman attributed the observed problems to decision makers' "misperceptions of feedback," including

- a failure to account for time lags and stock-and-flow structure,
- a failure to account for feedback from actions to the environment,
- a failure to recognize tradeoffs between short- and long-run effects of actions, and
- a tendency to view dynamics as externally imposed rather than endogenously generated.

The performance in these experiments seemingly contrasts with the results of similar studies involving dynamically simple systems. For instance, MacKinnon and Wearing (1980, 1985) found in a study of human performance managing a simple (first-order) system with no delays, that people performed very close to optimal. Yet, from a feedback control perspective, this should not be surprising since such systems suffer from the "curse of insensitivity" (or "blessing," depending on how you look at it):

Almost any rule that provides corrective action in the right direction will perform close to optimal. Thus, these results are further evidence that feedback structure is an important determinant of behavior.

Experimental economics

Although economics is often considered a non-experimental science, controlled experiments using "artificial" laboratory markets with human participants has evolved to become a recognized separate field (Plott 1982, 1987; Smith 1982; Roth 1987). The field shows much promise in linking psychology and economics, but, as it is currently practiced, it has inherited much of the axiomatic, deductive nature of neoclassical economics. Although experimenters recognize a role for more exploratory work (see e.g. Roth 1987, Chapter 1), the tendency is still to focus on tests of neoclassical theory by comparing equilibrium states. Accordingly, researchers have been concerned primarily with steady-state outcomes and market averages and less with the process through which such outcomes emerged. In particular, there have been few attempts to provide psychological explanations of the results.

There are, of course, exceptions to this tendency. For instance, dynamics played an important role in Carlson's early experiments on the "cobweb theorem" (1967), experimental studies of speculative bubbles (Camerer and Weigelt 1986; Smith, et al. 1987), and in Daniels and Plott's studies of inflationary expectations (1987).

Conversely, many psychological phenomena have been investigated in the context of economic transactions, e.g. in the work on willingness-to-accept vs. willingness-to-pay (Knetsch and Sinden, 1984; Coursey, et al., 1984), Camerer, et al.'s work on probability biases (Camerer, 1987, Camerer, et al., 1987), and Berg, et al.'s work on preference reversals in market settings (1985). Easley and Ledyard (1986) have considered how the results of experimental auctions can arise from explicit psychological assumptions about individual behavior.

Most of the systems investigated in experimental economics have extremely simple feedback structures, even though they may nontheless be very complex to solve in a game theory framework. With some exceptions (e.g. Smith, et. al. 1987; Daniels and Plott 1987), market supply and demand are "reset" each trading period, thus making each period essentially independent and identical. Profits or "utility" are typically simple functions of the number of goods sold or bought, thus making it easy for the decision maker to relate rewards to the decision to trade a given quantity at a given price.

This can at least partly explain the remarkable convergence of these markets: Time and again, competitive equilibrium can arise out of a seemingly chaotic auction process (Plott 1982, 1987; Smith 1982, 1986). (However, only the double auction invariably produces convergence--under other price institutions, convergence is often slower, and equilibria differ from neoclassical predictions.) When the link between action and reward is made less direct, performance deteriorates. For instance, in "winner's curse" experiments, where subjects bid competitively for an item but receive a noisy signal of its associated reward, the price of the item is inevitably bid up too high (Thaler 1988).

Chapter 2

Feedback structure and market dynamics

As mentioned before, research on decision making in dynamically complex systems strongly suggests that humans have a limited capacity to anticipate the dynamics of such systems and, consequently, do not adapt their behavior appropriately to the structure of the system. Moreover, effective learning from experience is likely only when there is accurate and immediate feedback about the outcome of alternative actions and the opportunity for repeated actions, "holding everything else constant." As a result, both performance and learning are significantly degraded when feedback is delayed or distorted and when cause and effect are distant in time and space.

When viewed from this perspective, most studies in experimental economics, with their simple cost and payoff functions, are likely to overestimate the effectiveness of the market mechanism. As Richard Thaler observes, "some markets offer opportunities for learning, but few, if any offer the instantaneous feedback used in most market experiments" (1987, p. 124). On the other hand, the existing studies of dynamic decision making, with their lack of market mechanisms, may overestimate the impact of feedback complexity.

It seems important, therefore, to link the study of dynamic decision making in complex feedback systems with the experimental study of markets. Real-life markets have many of the elements that cause difficulties in dynamic decision-making studies:

- lags in information acquisition and in physical processes such as production, shipments, and capital investment;

- positive feedback loops, for instance resulting from multiplier effects on demand, economies of scale, learning curves, network externalities, technological standards, or extrapolative expectations;
- non-linearities due to capacity and budget constraints, physical constraints, and technological discontinuities, and institutional processes;
- stock accumulations, such as money balances, inventories and backlogs, durable goods, or plant and equipment.

Of course, no economist would take issue with the fact that constraints on information, production adjustment, etc. affect the performance of the market. Indeed, informational constraints and gestation lags in investments have played key roles in rational-expectations models of the business cycle (which has always been difficult to explain in a neoclassical framework). The most famous example of the importance of informational structure is Robert Lucas' (1972) explanation of how monetary policy can affect output, based on the structural constraint that firms cannot distinguish "local" from "global" price changes.

What is suggested here, however, is that feedback structure can affect markets far beyond what neoclassical theory would predict, due to the difficulty people have in anticipating market dynamics, and due to the fact that feedback complexity affects people's ability to learn and adapt to the market environment. Thus, the hypothesis underlying this work is that feedback structure influences the behavior of the agents in systematic ways which are not necessarily optimal or rational. Under some circumstances, it is possible that people make systematic, persistent, and substantial errors of judgment and action, leading to aggregate market behavior different from that predicted by neoclassical theory. Put differently, neoclassical theory is likely to underestimate the significance of feedback structure; conversely, experiments in dynamic decision making are likely to overestimate the effects of errors because they lack market processes that may partially correct or compensate for them. That is, there is a critical need to examine empirically how the micro-structure of decision making interacts with physical and informational structure of economic institutions to produce dynamic macro-behavior.

The next section discusses how the integration of micro-level decisions and macro-level outcomes can be achieved by combining the protocols of experimental economics with the psychologist's detailed study of individual behavior, and how the critical link between the two levels is established through computer simulation.

2.3 Approach

The approach taken in the thesis builds on the traditions of three fields: Experimental economics, psychology, and system dynamics. Experimental economists study markets in the laboratory, using performance-based rewards to recreate real-world financial incentives. Psychologists, in turn, have developed a number of methods for studying decision making, including statistical analysis of judgments and choices, analysis of verbal protocols, and interpretation of patterns of information access and use. The field of system dynamics employs a feedback control perspective to decision making, emphasizing the interaction of the individual with their surrounding system, and it relies extensively on computer simulation as a means of analyzing such systems. It is argued here that these diverse methods can be combined through the use of computer simulation, and that this combined approach may provide a fruitful avenue for integrating micro-level decisions and macro-level outcomes because the different elements of the approach mutually reinforce each other.

Methodological barriers between economics and psychology

Although economics and psychology could benefit from closer interaction, the integration of the two fields has been slow, in large part because they have focused on different aspects of human behavior, and because they employ different methods and criteria for testing theories.

Psychologists have concentrated on discrete, individual decisions, isolated from an aggregate-level context, with a detailed emphasis on the process through which decisions are made. Underlying this approach is an implicit belief that one needs to understand the parts in detail before one can describe how the system as a whole works.

Economists, on the other hand, are concerned with aggregate market-level outcomes, and individual behavior is important only insofar as it affects these outcomes. This emphasis finds its most extreme form in Milton Friedman's empirical positivism (1953), which argues that micro-level assumptions about behavior are not merely irrelevant but not even testable in any direct way: Only comparison of aggregate outcomes matters in validation of theory. However, as Herbert Simon has pointed out, this view is unduly restrictive and logically flawed:

> [Friedman's argument] would be true, of course, if we had no microscopes, so that the micro-level behavior was not directly observable. But we do have microscopes. There are many techniques for observing decision-making behavior, even at second-by-second intervals if that is wanted. In testing our economic theories, we do not have to depend on the rough

> aggregate time-series that are the main grist for the econometric mill, or even upon company financial statements. (1979, p. 495).

Most economists, it is fair to say, do not fully agree with Friedman's position, and there is fairly widespread awareness of the limits of aggregate statistical procedures for verifying theories (Leamer 1983). Yet mainstream economic research continues to rely on these methods, in large part because there is no obvious elegant way in which to incorporate less restrictive micro-level assumptions in the theory. To overcome this problem, one thus needs analytical tools for linking individual actions and aggregate consequences (Coleman 1987). In the following section, it is argued that computer simulation of feedback systems, when combined with experimental and statistical techniques, may provide one such tool.

Combining experiments and simulation

Simulation, experiments, and statistical analysis each has strengths and weaknesses; when used in concert, they can mutually reinforce each other. Although there may be many variations, the method typically involves the following steps (Sterman 1987a):

- First, a computer simulation model is built of the system in question, including assumptions about how actors in that system make their decisions. Such simulations help generate hypotheses about the structural features of the system and of individual behaviors that shape aggregate outcomes.
- Second, the original simulation model is transformed into a game, in which the simulated agents in the model are replaced with human decision makers. The players in the game can be rewarded in accordance with their performance. The experiment is run under different treatment conditions including different institutional and structural features.
- Third, the decisions made by individual participants in the game are analyzed in an attempt to find heuristics or rules of thumb that

guide these decisions. Such rules typically take the form of equations specifying an output (the decision) as a function of a limited number of inputs or "cues" (the states of the system). The individual-level analysis can make use of a variety of techniques, such as multiple regression, time-series analysis, factor analysis, and protocol analysis.

- Fourth, the estimated decision rules are incorporated into a new simulation model and the simulated aggregate output of this model is compared to the observed aggregate results of the experiments. The aim of the analysis is to identify rules or heuristics that seem to guide the decisions of individuals in the market, and that, when implemented in a computer simulation model, reproduce the observed aggregate behavior of the market in whatever way appropriate.

A good example of the use of computer simulation in this fashion is found in Sterman's work on the "economic long wave game" (1987a, 1989a, b). Sterman's experimental study was motivated by a simulation model that showed how a simple one-sector capital-producing economy could produce long large-amplitude cycles in capital investment, using plausible, boundedly rational decision rules on the part of capital producers. The simulation model was then transformed into a game in which a human player made the capital-investment decisions that were otherwise produced by these decision rules in the original model. Sterman found that the vast majority of subjects produced outcomes very similar to those of the original model. Moreover, their individual decisions generally fit the decision rules proposed in the model very well, and the between-subject differences in performance in the game could be explained to a large extent by differences in the estimated parameters of those rules.

Computer simulation makes very explicit both the dynamic process of market adjustment and the link between individual-level assumptions and system-level outcomes. In principle, analytical methods are equally explicit, but the demands of mathematical tractability often make it necessary to simplify the system under study or to assume that the system is in equilibrium. The main advantage of using simulation is the flexibility that comes from not having to rely on analytical solutions: There is, in principle, no limit to the complexity of a computer simulation model. This then leaves one free to incorporate realistic assumptions about the physical and informational structure of markets and about the behavioral rules of its agents. This flexibility is particularly important because many interesting dynamic phenomena, such as "chaos," "counter-intuitive behavior," and "shifts in feedback loop dominance" are inherently non-linear (Anderson, et al. 1988; Forrester 1970).

There is a corresponding cost of using simulation, since one loses the mathematical elegance and generality of the "grand theorems" of neoclassical theory. Simulation models run the risk of being arbitrary, fragmented, or, as some would say, "ad hoc."[7] On the other hand, this cost seems appropriate in the exploratory stage. Hopefully, as the evidence accumulates, one would be able to extract more general principles based on commonalities in the findings of individual projects. For instance, after years of experience with applying feedback models to a wide range of problems, researchers have identified a number of "generic" or "archetypal" feedback structures which recur in many different guises and in many diverse contexts (Senge 1990, Chapter 6).

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7 Interestingly enough, the Latin phrase "ad hoc" actually means "for the purpose at hand," the very same phrase used by Milton Friedman (1953): "... whether a theory is realistic 'enough' can be settled only by seeing whether it yields predictions that are good enough for the purpose at hand ..." (p. 41).

The analysis of feedback systems through simulation can also play an important role in the experimental design by providing a clear purpose and direction to the questions to be investigated by the experiments. Variations in individual behavior may or may not be important for aggregate outcomes, and the identification of "leverage points" through simulation modeling can help define the important aspects of behavior that need to be recorded and analyzed.

Evaluating individual performance

One direction in which to enhance experimental economics would be to supplement aggregate-level comparisons with attempts to measure directly whether individual actions conform to the optimality assumptions of economic theory. In some cases, it is possible to derive an optimal decision rule, which relates decisions to the current and expected future state of the system. To test for optimality, one then needs a measure of expectations. The standard approach in modern economic theory is to assume that expectations are rational, in which case actual outcomes should differ from their expected value only by a random error. The assumption of rational expectations often has very strong implications for the relations between variables, relations which can be tested statistically. But this approach can only test the joint assumption of rational decision making and rational expectations. In using aggregate historical data, economists often have no other choice. Further, the hypothesis is usually rejected in direct empirical tests (see e.g. Lovell 1976)

In laboratory experiments, however, it is possible to get at least a partial impression of such expectations directly by asking subjects for forecasts of future prices and sales and giving them a "bonus" reward for good forecasting performance. An example of this method is found in Williams (1987). Using the forecasts one can separate deviations from optimality into expectational errors and failure to act consistently with one's expectations.[8] In evaluating the forecasts one can use powerful tools from economics and statistics, such as variance bounds (Shiller 1989, part II) and orthogonality of forecast errors (e.g. Sargent 1979, Chapter X). And the forecasts can then be used as inputs to the optimal decision rule which can then be compared to actual decisions, using standard statistical techniques.

But comparing behavior to optimal rules has its limitations. Interesting problems are usually too hard to solve analytically, and it is difficult to get much insight into "what happens" when subjects to not follow an optimal rule. In that case, one much search for alternative descriptions of behavior and develop other criteria for evaluating performance.

This is one way in which simulation models can offer help. In cases where analytical optimal rules cannot be found, the use of simulation still

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8 Three caveats are in order. First, complete separation of expectation and action is, in principle, impossible: It is always possible--though not necessarily good science (Leamer 1983)--to rationalize any behavior by evoking "hidden" or "excluded" factors not considered in the analysis. Perhaps the most relevant such factor may be the "Peso problem," where a small probability of a large "shock" may systematically affect decisions even though the shock may never be observed during the experiment. On the other hand, verbal protocol data could reveal such factors. Second, there are cases where the optimal rule requires expectations about "many" (more than one or two), or even infinitely many variables (e.g. all future prices), in which case a full measure of relevant expectations is impractical. On the other hand, even partial information about expectations is valuable and may be enough in practice, since the optimal policy is unlikely to be sensitive to variables very far out into the future. Third, if subjects suspect that their original forecast was wrong, they may depart from an otherwise optimal policy in order to manipulate the system to reduce the forecasting error. It is important, therefore, that the forecasting reward be small relative to the profit-based reward, though still large enough to encourage accurate prediction. Note, though, that the conflict of interest is not relevant when subjects make simultaneous decisions about their forecasts and the actions which can influence the variable to be forecasted.

permits an objective evaluation of both individual and aggregate performance. Observed behavior can be compared with bench-mark strategies found to give "reasonable" results, and deviations in performance can be related to deviations from the bench-mark rules. The experimental evidence so far suggests that simple bench-mark strategies can often substantially out-perform subjects (see e.g. Kleinmuntz and Thomas 1987; Hogarth and Makridakis 1981; Sterman 1989b, c).

Identifying and validating decision rules

A central task in developing and testing behavioral theories is the search for the heuristics that individuals appear to use. It may at first hand seem implausible that human decisions in complex environment should fit simple decision rules. There are, however, some a priori reasons for being optimistic.

First, apparently complex behavior may in fact be a function of the complexity of the environment rather than of the decision maker. While the social systems around us are indeed exceedingly complex, it does not follow that the behavior of individuals in these systems is similarly inscrutable. As alluded to in the previous section, the experience in system dynamics simulation models of complex feedback systems has shown that very simple decision rules can, in their interaction with the system, create highly complex, counter-intuitive behavior over time (Forrester 1970). Herbert Simon has argued forcefully for adopting the research hypothesis that individual behavior is guided by relatively simple rules:

> A man, viewed as a behaving system, is quite simple. The apparent complexity of his behavior over time is largely a

> reflection of the environment in which he finds himself... (1981, p. 126)
>
> ... a great deal can be learned about rational decision making by taking into account, at the outset, the limitations upon the capacities and complexity of the organism, and by taking account of the fact that the environment to which it must adapt possesses properties that permit further simplification of its choice mechanisms. It may be useful, therefore, to ask: How simple a set of choice mechanisms can we postulate and still obtain the gross features of observed adaptive choice behavior? (1956, pp. 129-30)

A set of simple rules may in fact be enough to allow individuals to function under most circumstances, even though, in their interaction with the environment, the decision makers appear to be making very complicated choices. For instance, a simple adaptive rule that adjusts prices in reaction to low inventories and/or high costs may be sufficient to allow a firm to survive in a competitive environment and even operate at close to maximum profitability (Cyert and March 1963; Day and Groves 1975; Crain, et al. 1984). In a sense, this approach to decision making turns the "curse of insensitivity" of optimal rules into a blessing: Given the nature of its environment, an organism may use a wide variety of heuristics and still survive, as long as the heuristics create a reasonable feedback relationship between organism and environment.

Second, the feedback systems perspective allows one to focus on the feedback properties of a decision heuristic, i.e. how it responds to alternative states of the system, without knowing in detail how the decisions were made. Some aspects of the process are likely to be more important than others. Experience from engineering feedback control suggests that the precise mathematical or numerical form of a decision rule is often less important than its structural characteristics, such as what information cues it relies on, the relative weight it attaches to each cue, and whether it has proportional, integral, or differential elements. For instance, Sterman's work on the stock-adjustment task shows that a key determinant of performance is the extent to which the supply line of already ordered but not yet completed production is factored into future orders (1989b, c).

Third, while the space of possible decision rules may seem very large, it is reduced considerably if one makes the reasonable assumption that any conjectured decision rule should exhibit some intended rationality: The rule should work toward accomplishing some goal, such as adjusting a stock to a desired value. Moreover, the characteristics of "bounded rationality" may be of much help in providing candidates for decision rules (Morecroft 1983). For instance, decision makers are likely to give greatest weight to readily available (salient) and certain information, to rely on rules of thumb, and to factor large, multi-objective decisions into smaller problems and consider each problem in isolation, typically in a sequential fashion.

A final reason for optimism comes from studies of static judgment tasks (Kleinmuntz 1990). One of the remarkable and consistent findings of behavioral decision theory is that human judgment and choice can often be captured very well by simple linear functions of the input cues (Dawes 1979). Part of the success of linear rules can be explained by limitations in people's ability to combine different inputs to the judgment process. It appears that people are fairly good at recognizing both which cues are important, the first-order direction of their influence, and, to some extent, the fact that cues may be correlated (Dawes 1979; Camerer 1981; Einhorn, et al. 1979), but they find it almost impossible to learn higher-order, non-linear interactions among cues (Brehmer 1980a, b).

Much of the reason for the success of statistically fitted linear models is the fact that, by construction, they will almost invariably capture a large part of the relationships underlying the data. Therefore, linear rules can reproduce decisions based on many different cognitive processes, including strategies that are non-linear or that use information not used in the linear rule, and, conversely, very different cognitive processes could lead to nearly identical behavior (Kleinmuntz 1990; see also Dawes 1979; Dawes and Corrigan 1974; Einhorn and Hogarth 1975). It seems plausible that, in dynamic tasks, the decisions made as a function of the input variables (cues), will similarly be largely linear, as evidenced in the success of Sterman's simple linear stock-adjustment rule (1989b, c).[9]

The robustness of linear rules is a definite strength, but it is also a limitation (Payne, et al. 1978). As Kleinmuntz (1990) has pointed out, linear rules can efficiently identify "misperceptions of feedback" and show their implications for behavior when embedded in a simulation model, but they cannot offer full insights into the cognitive causes of these misperceptions, precisely because so many different cognitive processes can lead to the same outcome. If one is concerned primarily with the implications of people's behavior for performance and aggregate behavior over time, this may not be of concern. As argued above, this is indeed one of the strengths of combining experiments with simulation models. But to make further progress into understanding why these misperceptions occur and how one might eliminate

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9 Sterman's rule was not fully linear but incorporated an institutional constraint (non-negative orders).

them through training or other means, one needs to go beyond statistical analysis of the outcomes of the rules.

For instance, the low weight on supply line information in Sterman's production-distribution experiments (1989c) has at least two different interpretations. It could be that people simply do not understand the importance of the supply line, but it is equally possible that the decision makers appreciate the importance of that variable, but have difficulty remembering or reconstructing what its value is. (In the production-distribution game, supply lines were not explicitly presented to the subjects (1989c). On the other hand, in Sterman's capital investment game (1989b), in which subjects showed similar low weights on the supply line, that information was explicitly provided. This would seem to lend weight to the interpretation that people fail to appreciate the importance of that variable.)

Process-tracing data on the actual thought processes of the individual can therefore be valuable as an adjunct to the statistical analysis of decisions. Such data can be obtained through the use of verbal protocols, which provide extremely rich information on the cognitive processes of the decision maker. The extent to which verbal protocols reflect actual thought processes is still a controversial issue (see e.g. the introduction in Payne, et al. 1978). The most accurate method seems to be "concurrent verbalizations," achieved by asking subjects to "think aloud" while deliberating a decision. In contrast, asking people for introspective after-the-fact accounts about how or why they make a particular choice is fraught with distortions of the reasoning process and "hindsight biases" (Nisbett and Wilson 1977).

The "think-aloud" procedure is surprisingly nonintrusive once subjects get used to it, and, as a general rule, has little effect on performance, except for slowing the problem-solving process (Ericsson and Simon 1984). Recent evidence suggests, however, that concurrent verbal protocols can sometimes lower or raise performance in problem-solving tasks, although there is no theory of why and how particular tasks are affected by verbalization (Russo, et. al 1989). Thus, the effects of verbalization should preferably be tested empirically case by case, i.e. the experimental design should include a control group without verbalization.[10] Although this need for testing adds to the cost of conducting the experiment, the point still stands that verbalizations provide a valuable additional source of information and insight.

Apart from these reasons for optimism, one also needs to consider what are appropriate criteria for validation of a particular proposed heuristic. In a statistical analysis of the results, the criteria of fit are changed substantially as one moves from a static to a dynamic perspective. The focus is shifted from fitting individual decisions as accurately as possible to given input cues to replicating dynamic patterns of behavior. In particular, there are several levels at which to measure whether a model of behavior reproduces observed outcomes.

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10 In the present study, it was decided to bypass the issue by excluding concurrent verbalization altogether in the main data set. Some pilot experiments did include verbalization, thus allowing for some insight into its possible effects, but the investigation of this issue was considered outside the scope of the dissertation.

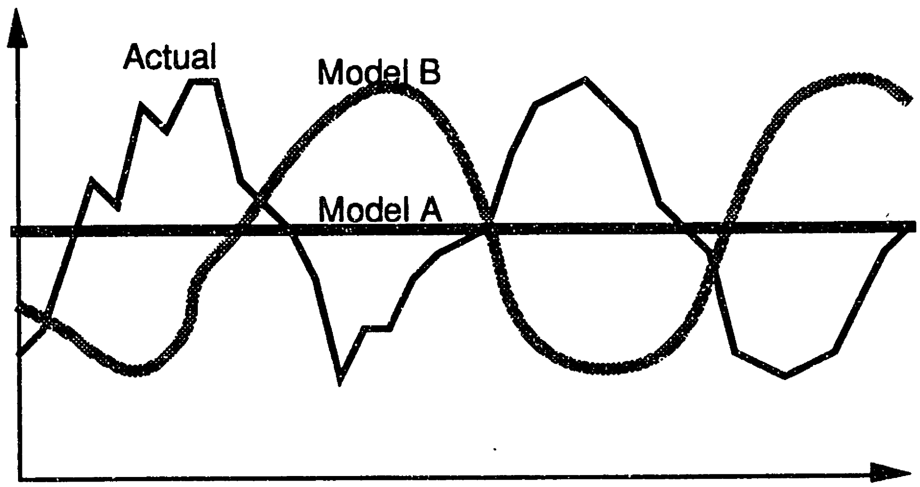

Figure 2.1: Illustration of appropriate measures of fit

Although Model A has a better fit when measured by the mean squared error, Model B is better at capturing the pattern of behavior.

Traditional measures of fit, such as R^2, which emphasize point-by-point correspondence between model and observation, are only one of several sets of measures. Least-squares-type measures are appropriate when fitting observed individual decisions to an equation that specifies these decision as a function of input cues, where these cues are taken as independent variables. But when these estimated rules are incorporated in a complete endogenous simulation of the market, other measures would be more appropriate.

The situation is illustrated in Figure 2.1, which shows a hypothetical observed outcome and two alternative hypothetical simulations. Although Model A clearly has a smaller mean squared error, yet Model B captures the pattern of behavior better. Therefore, measures that summarize such patterns are more appropriate. Examples would be the serial covariance or the spectral density of the observed dynamics, covariances and phase relationships between variables in the system, overall performance, average speed of convergence, etc.

2.4 Summary

The question of whether neoclassical assumptions are adequate for economic theory or whether human cognitive limitations may affect aggregate market behavior has been the subject of much debate but little testing. A review of the arguments in support of rationality and equilibrium reveals that the mechanisms through which the market is supposed to correct or compensate for the fallibility of individual agents are all dynamic in nature. To further the debate thus requires explicit attention to dynamic processes, and to how humans behave in dynamic settings.

Yet it is only recently that human behavior in dynamic settings has received much attention. Despite the paucity of research, it seems clear that people have a limited ability to anticipate the dynamic implications of the system structure in which they operate: They generally fail to appreciate the importance of stock and flow structures, lags, non-linearities, side effects, and short- vs. long-term tradeoffs, tending instead to assume that dynamics are externally imposed on the system rather than generated internally by their interaction with it.

But these important results have not yet been tested in a market environment. Studies in experimental economics have focused on markets with relatively simple dynamic structures, choosing instead to emphasize the importance of alternative exchange institutions. The evidence from these studies is that the precise form of the price-setting institution has significant effects on market behavior, even where such institutions are essentially equivalent from a neoclassical perspective.

Developing explicit behavioral theories of market adjustment requires a way to both observe individual behavior and to draw the implications of this behavior for the aggregate behavior of the market. The approach advocated here is to combine the protocols of experimental economics with the detailed observation of individual decisions used in psychology, and to use feedback control theory and computer simulation as a framework for the whole experiment. Dynamic simulation models can help both in the initial design of experiments and in the subsequent data analysis, by identifying the aspects of behavior most important to the dynamics of the market, and by testing whether conjectured rules governing individual decisions also lead to the observed market-level outcome.

3. Hypotheses and experimental method

The previous chapter highlighted the need for a dynamic perspective in assessing whether limits to individual rationality affect aggregate market outcomes. In particular, the poor performance of humans in experimental studies of dynamically complex systems leads to two related research questions: 1) To what extent can market mechanisms and financial incentives alleviate the problems observed in non-market dynamic decision making experiments? 2) What is the effect of feedback complexity on market behavior and performance?

In addition, the extensive evidence from experimental economics of the importance of price-setting institutions for market behavior makes it important to consider the role of alternative institutions in modifying the effects of feedback complexity on market behavior. The exact way in which prices are determined and imbalances between demand and supply manifest themselves are themselves important structural elements of a market and may have large consequences for the behavior of that market.

Thus, the experimental treatments chosen for this study have two dimensions. One dimension is "feedback complexity," introduced in the form of production lags and a multiplier effect on demand. The other dimension is the price-setting institution (or "price regime"), which ranges from the extreme of completely fixed prices to the other extreme of automatic price clearing, with posted seller prices as a more realistic intermediate case. The next section of this chapter describes and motivates the main treatment variables in more depth, ending with a detailed mathematical description of the market system used for the experiments.

Underlying the selected treatments are a set of hypotheses (or expectations) regarding their effect. Most basically, it is expected under a behavioral paradigm that markets with feedback complexity will be less stable and less efficient than the corresponding simple markets. However, this effect is expected to be large when prices are fixed and small when prices automatically clear the market. In the posted-price regime, the effect of complexity is likely to be larger than in the clearing-price regime, but the size of the effect relative to the fixed-price regime is ambiguous: If subjects are able to use prices effectively to control inventories, thus bypassing the long production lag, then the effect of complexity would be smaller than in the fixed-price regime. On the other hana, if the variance in prices is large and unpredictable, the additional variance in sales could cause performance to deteriorate more than in the fixed-price regime.

In contrast, all the treatment effects on performance relative to optimal are either zero or virtually zero when agents have rational expectations and actions. Section 2 discusses the expected treatment effects, under various assumptions about individual behavior, illustrated with simulation examples.

The final section describes the experimental method. A detailed mathematical description of the experimental market can be found in Appendix A. The simulation examples are documented in Appendix B.

Chapter 3

3.1. Experimental treatment design

3.1.1. Feedback complexity

Most studies in experimental economics have involved markets which are relatively simple from a feedback perspective. In particular, markets are usually "reset" each period so that past decisions do not influence current or future options. Yet the evidence from studies in non-market settings indicates that, while human performance is generally very good in dynamically simple settings, it degrades significantly in the presence of delays, non-linearities, and self-reinforcing feedback. In the few experimental studies of markets without "resets," such as the work on speculative bubbles (e.g. Smith, et al. 1987), markets often show substantial and systematic fluctuation which are difficult to explain from a rational-equilibrium perspective. One dimension in the experimental design is therefore the contrast between markets with simple and complex feedback structures.

Sterman (1989b) identifies the following elements of feedback complexity which seem to cause problems: Stock-and-flow structure (especially cumulative delays), self-reinforcing mechanisms, and non-linearities. These elements abound in real markets but have generally not been considered in experimental market studies. This study will focus on the first two elements: Delays and self-reinforcing feedback.

In the typical experimental market, supply and demand are expressed as simple tables that define payoffs to subjects as an instantaneous function of units bought or sold. By manipulating these payoff functions, the experimenter can create demand and supply incentives analogous to the textbook static demand and supply curves. In real world markets, however, production is subject to physical constraints, such as capacity limits, manufacturing cycle times, and availability of raw materials, and there are numerous delays in capacity investment, production, communication, data collection and reporting. Management must often set production schedules and acquire capacity well in advance of the time the goods are actually sold.

Such lags may be significant for the behavior of the market. The classical example is the cobweb model, in which a one-period production lag can result in cyclical prices and quantities (Nerlove 1958). In experimental economics, the studies of Mestelman and colleagues (1987), and Mestelman and Welland (1987) have demonstrated that markets do behave differently when production must be determined in advance of sales.[1]

From a behavioral perspective, delays may be important since much of the low performance observed in dynamic decision-making experiments can be traced to subjects' misperceptions of the consequences of lags in the system, particularly when such lags take the form of accumulations, such as pipeline delays, rather than simple time lags in information or outcomes. Humans tend to underestimate the length and importance of such pipeline delays, and confuse short-term effects with longer-term consequences (see Chapter 2). It

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1 Mestelman and Welland (1987) find that advance production in oral double auction markets does not affect the distribution of prices but does lower both production and efficiency, relative to the "normal" case of "production to demand." In a similar study of posted offer markets, Mestelman *et al* (1987) find that prices in markets with advance production converge faster and closer to the competitive equilibrium than do posted-offer markets with production to demand. It is important to note, however, that inventory carryover between periods was not allowed: as in most experimental economics studies, markets were "reset" each period, with the likely effect of stabilizing the system relative to real markets where inventories accumulate.

is conceivable, therefore, that production lags could introduce a tendency for instability analogous to the classical cob-web theorem. On the other hand, the cob-web model, in which prices are assumed to equate output and demand each time period, is likely to underestimate the importance of lags since imbalances do not accumulate in inventories and backlogs. If one relaxes the price-clearing assumption, the possibility of accumulation could be much more destabilizing.

Another typical feature in experimental markets is that demand (in the form of buyer redemption schedules) is determined exogenously. In the regional, national, or global economies of the real world, however, demand is determined endogenously in a system where producers and consumers in many markets are intimately linked. To go beyond micro-level studies of single markets, one must therefore move from the static seller and buyer schedules to dynamic, endogenously determined versions.

A particularly important endogenous process is the self-reinforcing feedback known as the "multiplier" effect from production on consumption: An increase in production tends to raise demand, both because outputs from the economy are used as inputs in production itself, and because increased activity and associated increases in employment improve the incomes of consumers. Increased demand in turn induces producers to raise output.

In long-run equilibrium, a multiplier effect simply changes the equilibrium of the system. In a dynamic setting, however, a multiplier can lead to substantial fluctuations, unless agents in the system anticipate its effects. The classic example is the famous multiplier-accelerator model of Samuelson 1939 and Hicks 1950, but other examples abound (e.g. Arthur

1988; Frisch 1933; Goodwin 1951; Low 1980; Mass 1975; Metzler 1941; Sterman 1985).

In dynamic decision-making experiments, such positive feedback mechanisms are known to create difficulties for humans. For instance, in Sterman's investment experiment, the capital-investment multiplier was chiefly responsible for the wild fluctuations in capacity and production subjects generate (Sterman 1989b). And subjects in Brehmer's fire-fighting task consistently underestimated the self-reinforcing growth of forest fires (Brehmer 1987).

Accordingly, the design of this study involves two versions of the market system:

- A simple condition, where (1) production initiated at the beginning of each period becomes available for storage or delivery during that same period, and (2) where industry demand is unaffected by the level of activity in the market;
- a complex condition, where (1) there is a lag of three periods between the time production is started and the time it becomes available for storage or delivery, and (2) where industry demand is influenced by the level of production in the market.

Thus, the complex condition presents subjects with an environment containing (1) stock and flow structures which accumulate imbalances between demand and supply and create long lags, and (2) positive feedback processes which arise from linkages between firms and consumers.

Ideally, it would be desirable to examine the separate effects of lags and multiplier effects by introducing these elements separately as well as in combination, but resource constraints on time, subjects and money precluded a complete investigation. However, Diehl (1991) investigates the separate effects of these elements in a non-market context (a cost-minimizing inventory control problem with quadratic adjustment and inventory costs). The combination of multiplier effects and lags increases the objective difficulty of the task (i.e. the expected minimum cost using the optimal control rule) by a factor of 100 or more![2] He finds that subjects' average costs are about four times higher than optimal across all conditions, indicating that the objective difficulty of the task is the strongest determinant of performance. A significant source of the suboptimal performance is a general tendency for subjects to "under-control," i.e. to react too sluggishly to imbalances in the system. While subjects generally adjust their actions in the right direction to reflect changes in the feedback structure of the task, this adjustment is insufficient. Moreover, there seems to be a threshhold of complexity beyond which subjects tend to "give up" adapting their behavior and revert to simplified rules that ignore the added feedback elements altogether.

The present design seeks to establish a range of possible outcomes by comparing two points on a scale between simple and complex structures. "Filling in the blanks" between these two extremes is an obvious suggestion for further research. (This is not to say that the design tests the extremes of that scale. The system still leaves out many feedback mechanisms and stock-and-flow structures which could further destabilize and/or complicate the

2 However, the main reason for this dramatic increase in difficulty is the quadratic adjustment costs in production. In the present study where there are no adjustment costs and inventory costs are proportional, the objective difficulty varies very little across experimental conditions.

system, such as money and credit, speculation, endogenous factor prices, and technological change.)

3.1.2. Price-setting institution

An important question motivating this research is the extent to which market mechanisms can alleviate the problems observed in non-market dynamic decision-making experiments. These mechanisms involve three unique features: Financial incentives, competitive selection, and a price system. As discussed in the previous chapter, there are several ways in which market mechanisms may improve behavior. One frequently discussed mechanism is financial incentives, which might induce participants to "try harder." Another suggestion is the possibility that, over time, the best performing agents will dominate the market through competitive selection. The merits and shortcomings of these two arguments have already been discussed in Chapter 2, and the focus here will be on the third mechanism, the existence of a price system to coordinate supply and demand.

The textbook explanation of how the price system coordinates and stabilizes economic activity, restated in feedback control terms, argues that the price system creates set of powerful negative (balancing) feedback loops. If, for instance, there is an excess of demand over supply, prices will rise so as to increase supply and decrease demand, thus bringing the two into balance. As is well known in engineering control theory, introducing such negative feedback loops tends to stabilize the system, *provided the feedback has no significant delays and that its strength ("gain") is not too high.*[3]

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3 See, e.g., Ogata (1990).

This explanation leaves many questions unanswered, however. For instance, what happens exactly while prices are out of line with their equilibrium values? How are prices actually set in the market, and by whom? And how do pressures for changing prices manifest themselves to individual agents? While economic theory tends to assume that prices always move to equate supply and demand, prices in real markets are set by agents in the market, working in some institutional setting and established process, such as sealed bid auctions, negotiated contracts, or, the most common in goods markets, posted prices. To what extent do these institutional processes reproduce the ideal price-clearing mechanism?

Addressing these questions is a major concern of experimental economics. It is a common finding in that field that the details of the price-setting institutions matter; markets that are theoretically equivalent may behave quite differently in the laboratory (see reviews in Plott 1986b, 1987). Indeed, it appears that the only mechanism which consistently produces a rapid convergence of prices to equate supply and demand is the double auction. All other mechanisms, such as posted prices, sealed bids, and English and Dutch auctions take longer to converge and frequently introduce systematic departures from neoclassical equilibrium (Smith 1986). Thus, one would suspect that, in the dynamically more complex systems considered here, the nature of the price-setting system would have strong implications for the market adjustment process.

If prices do not clear the market, some other form of coordination must determine goods bought and sold.

> When an economy is in equilibrium, the role of markets, financial institutions and money tends to disappear. The

> institutions such as organized markets, firms, and banks are the carriers of process and a major part of the information and communication flow of an economy. In disequilibrium they appear clearly ... Many different institutions may have the same static efficiency properties, but it is possible that they manifest considerably different dynamic properties. The questions concerning the selection of optimal ... institutions in a fully dynamic context have hardly been asked in a precise form, let alone answered. (Shubik 1979, p. 354)

One possibility, employed in most experimental markets (and in economic theory in general), is that production occurs only in connection with an actual transaction, and thus only when both buyer and seller explicitly agree to it. For instance, if there is an excess of demand over supply, some buyers are unable to complete desired purchases while sellers produce and sell all the goods they wish to. Conversely, an excess supply means that buyers are satisfied while some sellers do not produce all that they would have desired, given prices. (This mechanism includes the usual form of posted-price markets, where sellers specify a maximum amount they're willing to sell.)

An alternative which frequently occurs in real markets is that short term demand/supply imbalances are met through changes in buffer stocks, notably inventories and order backlogs. Buyers complete most, if not all, of their desired purchases, while producers determine output independently of buyer decisions. Excess production accumulates in inventories while excess demand fills order books and lengthens delivery lags.

While buffer stocks create some flexibility in the system, they also make the dynamics of the economy more complex and potentially less stable because they accumulate imbalances. For instance, if there are lags in adjusting production, an excess of demand will continue to draw down inventories (or swell backlogs) in a cumulative fashion until production can be increased. As a result, the production rate must overshoot demand in order to restore inventories and backlogs to their desired level, thus amplifying unanticipated demand changes--a phenomenon well known as the inventory accelerator (Metzler 1941). Depending on the time-series properties of incoming demand, the presence of an inventory can actually lead to larger variance in production than in demand, thus counteracting the very purpose of the buffer stock. The fact that inventory investment is the dominant feature of business cycles[4] suggests that it must play an important part in macroeconomic dynamics (Abramovitz 1950; Blinder and Maccini 1991; Holt, et al. 1960; Metzler 1941; Zarnowitz 1985).

Similar fluctuations are at the heart of many of the observed dysfunctional behaviors in dynamic decision making studies, particularly when there is a lag between the initiation and completion of production. For instance, in both of Sterman's experiments (Sterman 1989b; Sterman 1989c), the inventory accelerator created very large fluctuations in the system in response to a small step increase in exogenous demand. In light of such evidence, one would expect that buffer stocks can potentially create instability in the market, particularly when production lags are large. Further, to the extent the price system helps equate supply and demand in each period, the system should be more stable.[5]

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4 Inventory investments account for 87% of the drop in GNP during the average post-war U.S. recession (Blinder and Maccini 1991).

5 Another important factor in Sterman's experiments was the variable, endogenously determined time lags in the systems he observed. Capacity limits and non-negativity constraints on inventories meant that delivery delays could lengthen substantially when capacity or inventories were insufficient. The experiments in this thesis do not involve non-linear constraints and as such leave out an important potential source of instability.

At one extreme, a perfectly functioning price system would obviate the need for buffer stocks altogether (ignoring the speculative role of such buffers), as agents in the market receive and act on price information alone and fully adjust prices to clear the market at all times. Indeed, early experiments by Carlson (1967) , showed that "classical" cobweb markets with price clearing and a one-period production lag were remarkably stable, even in conditions where the cobweb theory predicted substantial instability. Carlson attributed the stability to subjects' use of a moving-average or cumulative average expectation formation process, whose stabilizing effects were first discussed by Nerlove (1958) and Auster (1971). In this condition, firm's are involved in a pure "quantity game" where all attention can be focused on finding the output level that, given the actions of other firms, maximizes profits. There is no need to control inventories.

The price clearing, and the consequent absence of accumulation of imbalances, in the cobweb model makes it more stable but also less realistic. Perfect price clearing is a useful abstraction, but an ideal rarely if ever realized in actual markets. The situations which come closest to this ideal are the highly organized and liquid markets for stocks, bonds, commodities, and certain local agricultural markets. Yet even these markets show large departures from market clearing in extreme conditions, such as the October 1987 crash. On the other hand, for experimental purposes, perfect price clearing represents a useful benchmark against which to evaluate the results.

At the other extreme of this scale is a system in which prices are completely fixed. In this situation, the full burden of adjustment lies on buffer stocks, and agents would receive and act on quantity information alone. Again, exact real-world equivalents may not exist, although some centrally planned economies may come close to this situation. In the absence of a price system, the system becomes fully susceptible to the dynamics of stock accumulation, and one would expect experimental outcomes to be similar to the results of previous studies in dynamic decision making, such as Sterman's (1989b, c) experiments, unless the larger financial incentives provided have a (presumably beneficial) effect.

Most real-world markets fall somewhere in between these two extremes. A very common form of price setting is posted offers, where producers and sellers announce the price of their product and maintain inventories or order books to accommodate buyer demand. This institution applies approximately to most consumer non-durable manufactured goods as well as many consumer durables. Although manufactured goods are usually not sold directly by the producers, it seems a fair approximation for the purpose of this study to consolidate manufacturers, wholesalers and retailers into one producing unit.[6]

It is not obvious how this more realistic setting might compare to the two previous extremes. On the one hand, the fact that producers can use prices to control demand may be strongly stabilizing. On the other hand, the environment becomes more complex and the opportunities for mistakes more numerous when prices are fluctuating. One could thus imagine that

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6 If anything, the fact that real production systems involve a chain of inventories and deliveries exacerbates the possible inventory fluctuations in the system. Empirical studies of business cycles in different sectors of the economy have shown that fluctuations increase as one moves from retail to wholesale to manufacturing output (Zarnowitz 1985). This empirical fact was strongly reproduced in Sterman's 1989c experiment.

performance may actually worsen compared to the fixed-price case. In the section below, these issues are discussed in more detail.

To explore the effects of alternative price regimes, the experimental design thus involves the following three price-setting institutions:

- **Fixed prices**: All prices are completely fixed and equal. Fluctuations in demand are accommodated entirely by changes in inventories. (All firms receive an equal share of total market demand.)
- **Posted seller prices**: Each firm sets its own price and production rate, and demand is fully accommodated by changes in inventories.
- **Clearing prices**: Prices move to equate demand to the given supply each period. In this condition, the need for inventories is eliminated. The market-clearing price vector, given this period's output and demand function, is found by the computer, which thus functions as a perfect Walrasian auctioneer.

One might of course imagine many other institutions akin to the schemes employed in experimental economics or used in real markets. Yet the three cases chosen here represent a fairly simple progression on a continuum, going from the absence of a price system, to the "best of all possible worlds." Extending the study to incorporate other forms of price setting is an obvious task in further research.

3.1.3. Treatment design

To summarize, the experimental treatment involves two dimensions, system complexity (simple vs. complex) and price-setting institution or "regime" (fixed prices, posted seller prices, and clearing prices). The combination of these treatments defines six experimental conditions, as illustrated in Table 3.1.1

The design is between subjects rather than within. A within-subjects design would perhaps eliminate some between-subject variance and thus increase statistical power, but it would also confound the treatment effects with an experience effect when more than one game is run per subject.[7]

3.1.4. Market structure

The market used for the experiment is perhaps best considered to be a regional industry where the level of activity and employment in that industry may have a significant effect on aggregate demand in the region. The products of the industry have some limited degree of differentiation (reflected in large but finite elasticities of substitution) but the market is otherwise close to the perfect-competition ideal.

Figure 3.1.1 shows a schematic representation of an individual firm and its interaction with the market. The market consists of K firms and a "consumer" sector. The conception of the market is similar in many ways to the classic monopolistic-competition model of Chamberlin (1956). The market is symmetric in the sense that all firms have the same structure and parameters. However, each firm produces a distinct product and consumers differentiate between products from different firms. Thus, each firm holds some monopoly power.[8]

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7 However, to begin to explore the experience issue, some subjects played two successive games, one with and one without complexity, in the same price regime. Occasional reference is made to this supplementary information, but it is not included in analysis except where explicitly stated.

8 This assumption implies non-zero profits in competitive equilibrium even though production exhibits constant returns to scale. Moreover, it allows a consistent way of distributing demand among firms when individual prices differ.

Firms are operated by the subjects in the experiment while the demand side is represented by a computer. Modeling the demand side of the market by computer was done to highlight the experimental hypotheses that relate to firms' ability to anticipate implications of feedback complexity, as described above. If consumers were also represented by human actors, additional phenomena, such as buyer expectations and inconsistencies in buyer decisions, would greatly complicate the interpretation of the results. Previous experiments have shown that buyer "counterspeculation" frequently prevents sellers from extracting monopoly rents, even when there is a single seller facing many buyers (Plott 1982, p. 1493). One would expect, therefore, that a version of the experiment with both human buyers and producers would exhibit fewer cases of seller collusion, and would generally show slower adjustment to equilibrium, due to the additional noise introduced by inconsistency in buyer decisions. However, the effects of the experimental treatments on market behavior would likely remain the same.

Time is divided into discrete periods. At the beginning of each period, t, each firm, i, must decide how much production, $y_{i,t}$, to initiate and, in the posted-price condition, what price, $p_{i,t}$, to charge for its product this period. The firm must make these decisions *ex ante*, i.e. without knowing the demand it faces this period.

Each firm maintains a goods inventory, $n_{i,t}$, to accommodate fluctuations in demand. Inventories can be negative, corresponding to an order backlog. The inventory is decreased by sales, $x_{i,t}$, and increased by production. There may be a lag of δ periods between the initiation of production, and the time it arrives in inventory. Thus, we have

(3.1.1) $$n_{i,t+1} = n_{i,t} + y_{i,t-\delta} - x_{i,t}\,.$$

Profits, $v_{i,t}$, each period are the difference between revenue and costs. Costs consist of production cost and inventory/backlog holding costs. Production costs are proportional to output and, to simplify accounting, are incurred at the point of sale (ordering and/or delivery) rather than during production. Inventory/backlog holding costs are proportional to the absolute value of the inventory at the beginning of the period. (In standard control models, inventory costs are often quadratic, since it simplifies the derivation of optimal policies. The absolute-value formulation was chosen to simplify the task and computation on the part of subjects, although it does make the computation of optimal policies more difficult.) Given unit production costs ω and unit inventory/backlog holding costs γ, firm i's profit in period t is

(3.1.2) $$v_{i,t} = p_{i,t}x_{i,t} - \omega x_{i,t} - \gamma \mid n_{i,t} \mid .$$

It is assumed that buyer utility is a Constant-Elasticity-of-Substitution (CES) function of goods bought from individual firms. If the elasticity of substitution is ε, purchases of individual goods, $x_{i,t}$, can be combined into an aggregate good, X_t, according to

(3.1.3) $$\tilde{X}_t = \left(\frac{1}{K}(x_{1,t}{}^{(\varepsilon-1)/\varepsilon} + \ldots + x_{K,t}{}^{(\varepsilon-1)/\varepsilon})\right)^{\varepsilon/(\varepsilon-1)} .$$

Moreover, it is possible to define an aggregate price index, $\tilde{P}_t$, so that the total expenditures are equal to $\tilde{P}_t\tilde{X}_t$, i.e. so that $\tilde{P}_t$ can be interpreted as the price of the aggregate good, $\tilde{X}_t$. The index $\tilde{P}_t$ is a function of individual prices, $p_{i,t}$, according to

(3.1.4) $$\tilde{P}_t = \left(\frac{1}{K}(p_{1,t}{}^{1-\varepsilon} + \ldots + p_{K,t}{}^{1-\varepsilon})\right)^{1/(1-\varepsilon)}.$$

If consumers maximize their utility given total expenditures, the demand for firm i's product, $x_{i,t}$, can be derived to be

(3.1.5) $$x_{i,t} = \tilde{X}_t \, (p_{i,t}/\tilde{P}_t)^{-\varepsilon}.$$[9]

Note that demand does not depend on inventory. It is assumed that all costs to the consumer from delivery delays or low inventory coverage are embodied in the inventory/backlog holding costs. In other words, firms are assumed to compensate their customers fully for any such costs. Note also that there is no inertia or brand loyalty in purchases: Any firm's market share is strictly a function of the firm's price this period, relative to the market-aggregate price.

The aggregate demand in period t, $\tilde{X}_t$, in turn depends on the aggregate price, $\tilde{P}_t$. The elasticity of aggregate demand with respect to the aggregate price is assumed to be a constant, μ, around the competitive-equilibrium price. Moreover, this "industry price elasticity" is assumed to be substantially lower than the "individual price elasticity," ε, reflecting the idea that the goods offered by firms are fairly close substitutes while the overall elasticity of demand in the industry is low. As the aggregate price moves further away from the competitive-equilibrium value, however, aggregate demand

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9 In some versions of the game (not included in the primary data set), demand was furthermore subject to a random multiplicative error, $u_{i,t}$, i.e.

$$x_{i,t} = \tilde{X}_t \, (p_{i,t}/\tilde{P}_t)^{-\varepsilon}(1 + u_{i,t}).$$

The errors $u_{i,t}$ were normally distributed with mean zero and standard deviation 0.1. The errors were uncorrelated across time but correlated across firms with a correlation coefficient ρ=0.5.

Although the "noisy" version of the game is not part of the primary data set, these extra data are sometimes used in the analysis, in which case it will be explicitly stated.

becomes a linear function of price (i.e. elasticity increases for rising prices and decreases for lower prices).[10] Specifically, aggregate demand is formulated around a "reference" point (indicated by a star *), i.e. as a "reference" aggregate demand, X_t^*, multiplied by an "effect of price on demand," according to

(3.1.7) $\tilde{X}_t = X_t^* f(\tilde{P}_t/p^*)$;

(3.1.8) $p^* = \omega\varepsilon/(\varepsilon-1)$;

(3.1.9) $f(1) = 1; f'(.) < 0; f'(1) = -\mu$.

Thus, when $\tilde{P}_t$ is equal to the constant reference price, p^*, defined by (3.1.8), aggregate demand is equal to reference demand. Figure 3.1.2 shows a plot of the function f(.). One can show that, if the number of firms is very large, or if firms do not consider the effect of their own actions on aggregate quantities, then the competitive-equilibrium price is equal to the constant p^* (independent of both X_t and $\tilde{P}_t$). This follows from the fact that the elasticity of individual-firm demand is constant (see Appendix A.)

The "reference" aggregate demand, X_t^*, consists of a constant "exogenous demand component," G, and of a variable "multiplier" component proportional to the market average production. The average production is the sum of current average production starts, Y_t and the average "supply line," S_t. Thus,

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10 If aggregate price-elasticity of demand was constant and less than unity for all prices, colluding firms could earn arbitrarily large profits by charging an arbitrarily large price. Conversely, demand would go to infinity as prices approach zero.

(3.1.10) $X_t^* = (1-\alpha)G_t + \alpha\frac{1}{\delta+1}(Y_t+S_t);\ 0 \leq \alpha \leq 1;$[11]

(3.1.11) $Y_t = (y_{1,t} + \ldots + y_{K,t})/K;$

(3.1.12) $S_t = (Y_{t-\delta} + \ldots + Y_{t-1}).$

The fact that reference demand, X_t^*, depends on the total level of production activity introduces a multiplier effect into the system. This effect can be interpreted either as a Keynesian consumption multiplier, where wage income is related to the level of activity, or as an input-output multiplier, where firms use one another's products as inputs to the production process. Equations (3.1.10-3.1.12) imply that the steady-state competitive-equilibrium production of the system is G, independent of α.

Table 3.1.2 summarizes the parameter values chosen for the experiments. The demand multiplier, α, and the production lag, δ, are both experimental treatment variables, as discussed above. In the "simple" case, they are both zero (i.e. there is neither a multiplier effect on demand nor a production lag. In the "complex" case, α has a value of 0.5 and the production lag is 3 periods.

A marginal propensity to consume of 0.5 is lower than macroeconomic estimates for this parameter which typically range between 0.9 and 0.95. The lower value was chosen primarily because simulation experiments have shown that the system becomes prone to unrealistically large fluctuations for high values of α. Employing a lower value of α is thus an *a fortiori*

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11 Note that the total units in production, Y_t+S_t is divided by $\delta+1$, the "length" of the production process. This normalization implies that the gain of the multiplier feedback loop is constant for varying values of the production lag δ.

assumption: If the multiplier has strong effects with this value, these are likely to be even larger for values closer to the macro-economic estimate. In addition, the market system is not intended to represent a complete macroeconomy: A regional economy will have a lower multiplier because some part of demand is satisfied by firms outside the region.

The value of the unit inventory cost, γ, relative to the production cost, ω, was chosen to balance the need for making it reasonably easy for experimental subjects to earn positive profits while at the same time motivating them to control inventories. The particular value was based on simulations and pilot experiments. Out of 97 subjects, only 5 (3 in the fixed-price complex condition and 2 in the posted-price complex condition) suffered a cumulative loss, in all cases due to large inventory fluctuations.

Of course, the value of γ has a direct influence on absolute performance, as measured by profits; in the extreme, a value of zero would make inventory fluctuations completely inconsequential for profits. Although it might appear that evaluations of performance would therefore depend on γ and thus lack generality, the problem is circumvented by comparing performance to benchmark strategies, such as rational expectations, or "reasonable" behavioral decision rules, rather than comparing absolute performance across conditions. Furthermore, when such benchmark strategies are used, inventory costs are very small (see the section below). In particular, inventory costs are zero in equilibrium in the absence of external noise inputs. Another measure of performance is the conformance of subjects' decisions to optimal or "reasonable" rules. Interestingly, the optimal decision rule for production and pricing does not depend on γ/ω, except in the posted-price complex condition (see Appendix A).

A fairly high value of 2.5 was chosen for the own-price elasticity of individual firm demand, ε, to reflect the idea that firms' products are close substitutes. The industry elasticity of demand, μ, is substantially lower (.75) expressing the assumption that there are no close substitutes for the industry's product as a whole. The fact that industry demand is inelastic ($\mu < 1$) accords well with empirical findings. In fact, 0.75 may well be a high estimate:

> [E]conometric studies tend to indicate very low price elasticities of demand whenever anyone attempts to estimate the entire structure of demand elasticities. In the Hauthakker and Taylor 1970 study , only 17 out of 83 products have price elasticities of demand greater than one ... Fifty-three products have zero elasticities ... Another eight have elasticities of less than 0.5 (Thurow 1983, p. 11).

The other parameters in the aggregate industry demand curve were chosen to limit the maximum profits possible (if all firms colluded) to about twice the competitive profit level.

The unit production cost, ω, and the equilibrium output level, G, determine only the "scale" of the actual numeric values of variables. They can be set arbitrarily without changing the more fundamental parameters that determine relationships between variables.[12] By changing these two parameters from game to game, the comparisons between games are made

12 The choice of ω and G is not completely without consequence in the actual games, since prices are rounded to nearest 1/100 and quantities are rounded to nearest whole units. On the other hand, for the values chosen, the rounding error was typically less than 0.1%.

more difficult. (Although the protocol was already secure from "crosstalk" between current and prospective subjects, the changing of parameters further minimized the chance of communication.) The actual values chosen for each session are provided in Appendix D.

3.2. Experimental hypotheses.

This section outlines in detail the expected effects of the main experimental treatments, under alternative hypotheses about the behavior of individuals in the market. These hypotheses and the treatment effects that they imply provide benchmarks against which to evaluate the experimental outcomes described in subsequent chapters.

One extreme benchmark is provided by the hypothesis that people act rationally, in the sense that they both fully understand the structure of the system in which they are operating and the dynamic consequences of that structure. Since participants are given all relevant information about the structure of the system, except for specific functional forms and associated parameters, such as the elasticity of industry demand, a rational agent would use past data to estimate these unknown parameters, assuming some reasonable functional form of the unknown equations in the system. For instance, people may assume that unknown functions are linear around the operating (equilibrium) point of the market. This is in fact an excellent approximation even though these functions are non-linear, as long as the deviations from equilibrium are small. As will be evident in the simulations below, it is in fact not so important what precise functional forms people use, as long as departures from the current operating point of the system are small.

Such a high degree of rationality is presumed to characterize the behavior of agents in much of the economic literature, including the rational-expectations school (e.g. Begg 1982) and the optimal learning literature (e.g Blume and Bray 1982; Bray and Savin 1986; Fourgeaud, 1986; Frydman 1982; Frydman and Phelps 1983; Marcet and Sargent 1989 a, b). Although many researchers may not believe that every individual does in fact behave with such sophistication, it is also clear that a large part of the literature nonetheless considers it a valid assumption.

Apart from the prominence of the assumption of perfect rationality in economics, there is another important reason for including that hypothesis as a benchmark measure. In effect, optimal performance is a measure of the "intrinsic" difficulty of the task. When comparing performance across experimental conditions, it is performance relative to the (optimal) benchmark that is of relevance, and not the absolute performance. As it turns out, the optimal performance does not differ much across the six conditions in this study, but with more challenging parameters (such as a stronger multiplier effect) or with exogenous shocks, there could be a difference of several orders of magnitude between optimal performance in the easy and the difficult tasks (see e.g. Diehl 1991).

Another, less restrictive set of benchmarks derive from the hypothesis that people are "reasonable," in the sense that their actions reflect the current feedback environment and their understanding of its consequences. While people do not necessarily act optimally in the strictest sense (i.e. they do not optimally estimate unknown structural parameters and they do not choose the optimal decision, given these estimates and their expectations of the future), they nonetheless modify their actions appropriately to reflect the possible short term distortions of prices and demand, caused by the structural complexities of the system.

Finally, a set of benchmark results arise from the hypothesis that decisions are "rational" or appropriate in the context of the decision makers' mental model of the system, but that, due to bounds on rationality, this mental model is <u>incomplete</u> and omits important feedback relationships in the system. The hypothesized decision rules express an <u>intended</u> rationality that will indeed work well in a simple feedback environment. But in more complex environments, the added feedback relationships render the rules inadequate, leading to poor performance. A fully rational agent would have to adjust the rules to reflect the added complexity. The hypothesis states that actual decision makers make only incomplete adjustments to their behavior and that, because they have difficulty attributing the dysfunctional behavior to their own actions, will learn only slowly, or not at all.

The comparison of the performance of appropriate and less appropriate behavioral heuristics with the optimal policy also gives a sense of the "lenience" or "forgiveness" of the task at hand--another measure of the difficulty of the task. In Chapter 2, it was mentioned that many dynamic optimization problems exhibit "flat" optima, i.e. performance is not reduced much by moderate departures from the optimal policy. Thus, one would expect that appropriate behavioral heuristics can do almost as well. On the other hand, if a behavioral heuristic does not reflect the feedback structure of the environment, performance may be degraded significantly, as seen, for instance, in Sterman's experiments (1989b, c).

As will be seen in the analysis below (and in Appendices A and B), the simple experimental conditions and the price-clearing complex condition all provide fairly lenient environments for the decision makers in the system: A wide range of heuristics will lead to performance close to the optimal. On the other hand, those complex-feedback conditions where imbalances are allowed to accumulate in inventories or backlogs are inherently less forgiving: While appropriate heuristics perform almost as well as the optimal rule set, a heuristic which fails to take certain features of the structure of the system into account will perform very poorly.

In the following, each of these three hypotheses are described in more detail for each of the six experimental treatment conditions, and their implications for the expected dynamics of the market and for individual performance (profits) are examined through computer simulation. The last part of the section summarizes the results as a set of conjectures about the separate and combined effect of the two main dimensions in the design, system complexity and price regime. To clarify the argument, the exposition starts with the fixed price regime (conditions 1 and 2 in Table 3.1.1), followed by the clearing-price regime (conditions 5 and 6 in Table 3.1.1), and ending with the posted-price regime (conditions 3 and 4 in Table 3.1.1).

3.2.1. Fixed prices, simple structure

The decision task in this condition is a simple inventory control problem. Subjects must choose production so as to minimize inventory/backlog costs, subject to a constant incoming stream of orders. The obvious "solution" to this problem is to produce the amount that one expects to sell in any particular period, and adjust production to reflect a possible inventory or backlog at the beginning of the period. Thus, the optimal decision rule (denoted by a star (*) superscript) takes the form

(3.2.1) $y_{i,t}^* = \text{Max}\,\{\,0\,,\, E_{i,t}\{x_{i,t}\} - (n^* - n_{i,t})/\tau^* + u_{i,t}\,\}$;[13]

(3.2.2) $E_{i,t}\{x_{i,t}\} = G$,

(3.2.3) $n^* = 0$,

(3.2.4) $\tau^* = 1$, where

$y_{i,t}$ is production by firm i during period t,
$E_{i,t}$ is the mathematical expectation of future variables, given the information available to firm i at the beginning of period t,
$x_{i,t}$ is firm i's sales (orders) received during period t,
$n_{i,t}$ is the inventory (or backlog, if negative) of firm i at the beginning of period t,
n^* is the desired inventory (equal to zero),
τ^* is the "time to adjust inventory" (equal to 1),
$u_{i,t}$ is a random, zero-mean error, and
G is the constant demand.

Note that production is constrained to be non-negative--if indeed agents are acting optimally, this constraint will virtually never be binding, as the system quickly adjusts to (stochastic) equilibrium and remains there. Thus, ignoring the constraints will be an excellent approximation to the true optimal rule. The random error $u_{i,t}$ is included to distinguish the strict rational hypothesis that agents always choose exactly the optimal decision (Var{u} = 0) from the

13 The fact that inventory/backlog costs are proportional to the absolute value of $n_{i,t}$ means that the optimal rules, both here and in the following conditions, use the median rather than the expected value of future sales. However, if the distribution of demand is approximately symmetric, substituting the expected value is a good approximation (for further details, see Appendix A.) As mentioned previously, the absolute-value formulation was chosen to simplify the task and computation for subjects during the experiment.

weaker hypothesis that people's choices and expectations do not differ from the optimal in any systematic way (Var{u} > 0).

The optimal rule requires full knowledge of the parameters in the system. However, a simple behavioral rule that appropriately addresses both expected demand and inventory/backlog adjustment will, for most demand patterns, perform close to optimal. Table 3.2.1 compares the outcomes, both in terms of profits and in terms of market variance in production, of the optimal rule (*) with the performance of the following simple adaptive rule

(3.2.5) $y_{i,t} = \text{Max}\{\, 0\,,\, x_{i,t}^{e} + (n^{*} - n_{i,t})/\tau_n + u_{i,t}\,\}$;

(3.2.6) $x_{i,t}^{e} = x_{i,t-1}^{e} + (x_{i,t-1} - x_{i,t-1}^{e})/\tau_x + w_{i,t}$, where

$w_{i,t}$ is a random error.[14]

The rule expresses the idea that people use an adaptive forecast $x_{i,t}^{e}$ (here simple exponential smoothing) of future sales as the anchor for their production decision, adjusting this anchor by an inventory correction term. As is evident from the table, the performance of the two sets of rules is very close indeed.

The rule (3.2.5-3.2.6) is not the only one to perform well: A wide range of possible rules will also closely approximate the optimal policy, provided they have the required feedback from sales and inventory , as illustrated in Figure 3.2.1. As long as the rule reacts to excess (insufficient) inventory by decreasing (increasing) production, inventory will not deviate "much" from

14 The random errors u and w are both assumed to be normally distributed with a mean of zero, and to be both serially and cross-sectionally uncorrelated. (This assumption is not crucial for the results, however).

its desired level, thanks to the negative (regulating) feedback loop created by the rule. If, furthermore, the rule takes expected sales (and possible systematic patterns in sales) into account, inventory imbalances can be kept at a minimum for a wide range of demand patterns.

One may rightly argue that this condition in the design has little to do with an economic market. In particular, since there is neither a multiplier effect nor any variance in prices, the performance of individual subjects is independent of the actions of other subjects (except for the time spent waiting for others to make their decisions). This condition is nonetheless included because it serves as a useful benchmark for assessing the outcomes in the other cells of the design. Moreover, the condition is of some intrinsic interest since it resembles the simple dynamic decision making tasks where humans have generally done very well, such as the tasks in MacKinnon and Wearing's (1985) study. The expectation here is likewise that subjects will perform well, i.e. close to optimal in this condition.[15]

3.2.2. Fixed prices, complex structure

When the complexity of time lags and the multiplier feedback is introduced, however, performance relative to optimal is likely to suffer systematically, compared to the simple condition, unless subjects correctly perceive the nature of the feedback environment. The fixed-price complex condition is analogous to the systems studied by Sterman (1989b, c), and one

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15 It is possible that subjects become so bored with the task that they will introduce significant fluctuation in their decisions, simply to "see what happens." Such "flukes" are likely to be unsystematic and to be well represented by the random error term in the equation.

would thus expect to see similar results--poor performance relative to optimal and systematic self-organized fluctuations.

The task is still to set production so as to minimize expected inventory costs, given expected future demand and the current state of one's inventory. Now, however, there is the added complication that one must consider the "supply line" of production already initiated but not yet completed. Unless proper account is taken of this supply line, the system can become very unstable. In addition, one must consider the fact that past and future patterns of demand are influenced by the level of production through the multiplier effect.

The greater complexity of this system is reflected in the more complicated optimal rule[16]

(3.2.7) $$y_{i,t}^* = \text{Max}\{\, 0 \,,\, E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^* + u_{i,t}\},$$

(3.2.8) $$E_t\{x_{i,t+\delta}\} = G + \frac{1}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t + (N^* - N_t)/\tau^* + \beta^*(S^* - S_t)/\tau^*),$$

(3.2.9) $$s_{i,t}^* = E_t\{x_{i,t} + \dots + x_{i,t+\delta-1}\}$$

$$= S^* + Z_t + \frac{\delta}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t + (N^* - N_t)/\tau^* + \beta^*(S^* - S_t)/\tau^*),$$ where

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16 The rule assumes that the number of firms is very large or, alternatively, that firms do not consider the effect of their <u>own</u> actions on the system as a whole (although they do take into account that all other firms will act the same way). The derivations also assume that the distribution of random errors are symmetric so that the expected value can be substituted for the median. When the number of firms is less than infinite, the optimal rule becomes too complicated (at least for the author) to derive analytically. On the other hand, if deviations from equilibrium are random and approximately normally distributed, an optimal rule which does take into account the effects of each firm's own actions on the market can be found numerically, using linear-quadratic optimal control methods. In any case, these rules do not differ substantially in their performance or time behavior from the rule described here. See Appendix A for further details.

(3.2.10) $Z_t \equiv \frac{\alpha}{\delta+1}\left[Y_{t-\delta}\text{-}G + 2(Y_{t-\delta+1}\text{-}G) + \ldots + \delta(Y_{t-1}\text{-}G)\right]$, and

(3.2.11) $S^* = \delta G$; $N^* = n^* = 0$; $\tau^* = \beta^* = 1$, where
- α is the marginal propensity to consume,
- δ is the production lag,
- G is the long-run steady-state output level,
- $s_{i,t}$ is the "supply line" if previously initiated but not yet completed production, equal to $y_{i,t-1} + \ldots + y_{i,t-\delta}$, and
- $s_{i,t}^*$ is the "desired" supply line,
- S^* is the aggregate desired steady-state supply line
- N^* is the aggregate desired inventory
- Z_t is an auxiliary variable, and
- β^* is the weight on supply line adjustments relative to inventory adjustments (i.e. the "time to adjust supply line" is τ^*/β).

The equations (3.2.7-3.2.11) show that, in addition to the expected-demand and inventory-adjustment components, the production decision now also contains a supply-line adjustment term: The decision maker must take into account the current state of the supply line and how it differs from the "desired" supply line, the latter being equal to the total expected sales during the next δ time periods. Moreover, because of the presence of the multiplier effect, the expected demand is now a function of the current "disequilibrium." (If α is set to zero, the equations reduce to a form similar to the simple case, except for the supply-line adjustment term, where the desired supply line is now simply equal to the constant steady-state supply line, δG).

One consequence of the rule (3.2.7-3.2.11) would be that all imbalances disappear after exactly δ periods (the length of the production lag), provided agents know the structure of the system completely. Under the "strict" rational hypothesis, the system should thus jump to its equilibrium state three periods after firms have had enough time to estimate the parameters in the system.[17]

In contrast, almost any behavioral rule will have some residual consequences beyond that time. However, if the rules are "reasonable," the differences from optimal behavior will be small. One possible rule, which mimics the optimal rule but substitutes simpler expectations, is

(3.2.12) $y_{i,t} = \text{Max}\ \{\ 0\ , x_{i,t}^{e} + (n^{*} - n_{i,t})/\tau_{n} + \beta(s_{i,t}^{d} - s_{i,t})/\tau_{n} + u_{i,t}\ \}$,

(3.2.13) $x_{i,t}^{e} = x_{i,t-1}^{e} + (x_{i,t-1} - x_{i,t-1}^{e})/\tau_{x} + v_{i,t}$,

(3.2.14) $s_{i,t}^{d} = \delta\theta x_{i,t}^{e} + (1-\theta)s_{i}^{c};\ 0 \le \theta \le 1$, where
$x_{i,t}^{e}$ is the expected sales,
$s_{i,t}^{d}$ is the desired supply line,
$v_{i,t}$ is a random error, and
$\beta, \tau_{x}, \tau_{n}, \theta, s_{i}^{c}$ are parameters.

As in the previous rule (3.2.6), subjects are assumed to use a simple adaptive forecast of future sales as the anchor for their decision. They then modify this anchor with a correction for inventory and a correction for the supply line of production not yet completed. Thus, the rule reflects three components that must enter in any appropriate stock-adjustment task: 1) One must produce to meet expected demand (outflow), 2) One must modify production to adjust inventory toward its desired level, and 3) One must take into account already ordered but not yet completed production.

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17 Assuming agents know or correctly assume that the relations in the system are linear and, significantly, that the exogenous demand component, G, is constant, they must estimate two parameters, G and α in the equation $X_t = G(1-\alpha) + (\alpha/(\delta+1))(Y_t+S_t) + \text{Error}_t$. This can be done with only two observations, although a third observation is necessary to infer that there is no external noise in the system. Thus, agents could use a reasonable behavioral rule (or a random rule) during the first three rounds and then jump to the rational strategy.

In (3.2.12), the parameter β reflects the degree to which attention is paid to the third part, the supply line correction. A value of 1 implies that subjects fully account for production already placed but not yet received (as in the optimal rule), while a value of 0 indicates that they completely ignore it. The parameter θ indicates the degree to which agents adjust their desired supply line to reflect long-term average demand. If $\theta=1$, agents use their adaptive forecast of demand as the basis for the desired supply line; if $\theta=0$, agents have a constant desired supply line, equal to s_i^d. From a control-theory perspective, the target supply line must be a variable which over time will adjust to reflect the long-term average throughput; otherwise, the rule would exhibit a steady-state error if the equilibrium throughput, G, changes. At the same time, a variable desired supply line introduces a destabilizing element of amplification into the system, analogous to the inventory accelerator: If demand is expected to change permanently by an amount dX, the supply line must increase by δdX. From a cognitive perspective, however, one must also consider whether subjects are in fact able to fully realize the required adjustment of the supply line, especially if they do not pay much attention to the supply line correction in the first place (i.e. if $\beta<<1$). Thus, if an analysis of the actual decisions, using a constant desired supply line, yields evidence of a low supply attention (β), it seems unwarranted to consider the possibility of a variable desired supply line.[18]

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18 Sterman (1989c) found $\theta=0$ (constant desired supply line) produced excellent results in estimating subjects' decision rules. Further, there was strong evidence that s_i^c was far too small, even for the initial equilibrium throughput, strengthening the notion that subjects do not understand the need for a supply-line correction, much less a variable supply-line target.

The rule (3.2.12-3.2.14) is virtually identical to the rule employed by Sterman (1989b, c), and, as in Sterman's experiments, the value of the parameter β is of critical importance to the stability of the system. Table 3.2.2 compares the performance of the optimal rule with those of the behavioral rule (3.2.12-3.2.14), for three alternate values of β. It is quite clear that, as long as subjects are putting sufficient weight on the supply line, performance closely matches the optimal result. On the other hand, performance deteriorates dramatically with less attention to supply lines (lower β). Thus, the value of the parameter β is both very important for performance and at the same time captures many of the adjustments in behavior that are necessary when moving from the simple system to the more complex.

The effect of the supply line parameter on behavior is further illustrated in Figure 3.2.2, which compares the time pattern of behavior of the rational response with the three versions of the behavioral rule employed in Table 3.2.2. When insufficient attention is paid to the supply line, large oscillations occur, which contrast sharply with the adjustment pattern under rational expectations shown in Figure 3.2.2. On the other hand, an appropriate behavioral rule ($\beta = 1$) is not dramatically different from the rational response.

It is important to emphasize that the rule (3.2.12-3.2.14) employs only the minimum elements necessary in any stock-adjustment task. In particular, there is no account in the rule of the multiplier feedback: Expectations are simply smoothed (or possibly extrapolated) historical data. In contrast, the optimal rule (3.2.7-3.2.11) takes explicit account of the multiplier effect, as the parameter α enters expectations in complex ways. In spite of its lack of consideration for the multiplier effect, the behavioral rule performs quite well for parameters which place adequate weight on the supply-line correction. On the other hand, consideration of the multiplier effect would be critical for higher values of the marginal propensity to consume, α (e.g. in a complete macroeconomic system).[19]

Another point which bears directly on the motivation for this study is that, although (3.2.12-3.2.14) looks very similar to the optimal rule in many respects, the fact that agents may use it with appropriate parameters and thus perform well does not mean that they possess the information-processing skills assumed in rational-expectations theory: Instead, the rule is justified on behavioral, boundedly-rational grounds as a simple anchoring-and-adjustment heuristic. *That the rule leads to good convergence is a function of the structure of the system, not of the individuals in that system.* Furthermore, to the extent that subjects ignore the supply-line adjustment ($\beta << 1$), the market converges slowly or not at all.

The relationship between the decision rule and the resulting aggregate behavior can be seen in Figure 3.2.3 which shows the added feedback relationships caused by the production lag and the multiplier. Unlike the simple case in Figure 3.2.1, there are now many ways in which the system may fail to adjust to equilibrium in a smooth and rapid manner. First, the addition of the pipeline delay creates a delay in the negative feedback loop regulating inventories. It is well known from control theory (e.g. Graham 1977) that adding delays in a negative feedback loop can cause oscillation if the reaction of production to inventory discrepancies (the "gain") is too large and

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19 For instance, if α were .90, the system using the same decision rule as in Figure 3.2.2 becomes highly unstable, leading to a limit cycle.

if decisions are not modified by the supply line correction. In the Figure 3.2.3, the key to avoiding this source of oscillation is the negative feedback loop regulating the supply line; if this loop is weak or absent (i.e. if the supply line weight β is low), the system will oscillate. In addition, the multiplier link creates two self-reinforcing (positive) feedback loops which can further destabilize the system. If agents rapidly adjust their expectations in response to changes in demand (i.e. if τ_x is small), the positive loop on the right in the diagram can be highly destabilizing, resulting in larger and longer-lasting cycles.[20]

To summarize, the effect of introducing production lags and multiplier feedback in the fixed-price regime is strongly dependent on the degree to which subjects take appropriate account of their supply lines. The performance of optimal policy does not differ substantially from the simple case, and, with the chosen set of parameters, an appropriate behavioral rule will do almost as well as optimal. If, on the other hand, there is insufficient supply-line correction, the result can be large, sustained oscillations, leading to significantly lower performance. Since previous experiments (Sterman 1989b, c) strongly suggest that few subjects pay full attention to the supply line, one would expect oscillations to be common.

3.2.3. Clearing prices, simple structure

The decision task facing the individual firm is changed dramatically with the introduction of a price-clearing market system. In this regime the

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20 If agents have extrapolative expectations, i.e. they project recent changes in demand into the future, the result is larger cycles but with a higher frequency.

need for inventories is eliminated altogether, as a built-in "auctioneer" finds the set of prices which will clear the market each period. Each firm experiences demand just sufficient to absorb the firm's output in that period. The task is now to determine the output level which will maximize profit (revenue less production cost)--since inventories are automatically kept at their desired level (zero), there are no inventory costs to consider. The firm must consider how its own price will change as a function of the output it produces, and how its price will be affected by the decisions of other firms. Moreover, it may consider how its own actions may affect the decisions of other firms in the future. In effect, the firm is involved in a dynamic, multi-period version of the textbook "Cournot" or "quantity" game.[21]

Because of the strategic aspect of the task, it is not trivial to define an "optimal" or "rational" policy. One can, however, define two archetypal strategies: Complete collusion (or cooperation) and competition (or defection). In the former case, firms are assumed to seek the level of production which would maximize the profits of all firms, resulting in a "collusive" equilibrium, even though any individual firm would benefit from deviating from this level. In the latter case, each firm is assumed to seek its own profit-maximizing output level, given the expected output level of other firms, leading to a "competitive" (non-cooperative, or Nash) equilibrium, with higher output and lower prices than in the collusive equilibrium.

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21 In the Cournot game, firms consider their own and their competitors' output (quantities) to be the choice variable and prices as dependent variables. In the Bertrand game, firms instead consider prices to be the choice variables and output to be the dependent variables. The Nash equilibria of the two games are sometimes different.

In Appendix A, it is shown that the optimal non-cooperative strategy is to produce the output which results in an expected price equal to the competitive-equilibrium price, p^*, i.e.

(3.2.15) $$E_{i,t}\{p_{i,t}\} = E_{i,t}\{\tilde{P}_t(x_{i,t}/\tilde{X}_t)^{-1/\varepsilon}\} = p^*,$$

which yields the optimal policy

(3.2.16) $$y_{i,t}^* = \left(E_{i,t}\{\tilde{X}_t^{1/\varepsilon}\,\tilde{P}_t\}/p^*\right)^{\varepsilon} \approx E_{i,t}\{\tilde{X}_t\}\; E_{i,t}\{\tilde{P}_t/p^*\}^{\varepsilon}.$$[22]

If all firms follow this strategy, every firm should thus produce the competitive-equilibrium output, G, every period, i.e.

(3.2.17) $y_{i,t}^* = G + u_{i,t}$, where
$u_{i,t}$ is a zero-mean random variable.

As before, the random variable u has been included to distinguish the strict rational-expectations hypothesis ($Var\{u\} = 0$) from the weak hypothesis ($Var\{u\}>0$). In a corresponding fashion, cooperating firms should produce the constant output, G_M, which maximizes joint profits for all firms.

Being simply a constant, the strategy in rational-expectations equilibrium is not very interesting. Of more interest is the extent to which firms can learn to find the individual and/or joint optimizing output level as

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22 The CES aggregate sales $\tilde{X}$ is approximated with the arithmetic average, X, and the CES aggregate price, P, is approximated with the trade-weighted average P, since only those measures are given to subjects in the experiment. Since the CES aggregate $\tilde{X}$ is a convex function of its component sales, the arithmetic average, X, is always greater than or equal to $\tilde{X}$. Moreover, since $\tilde{X}\tilde{P} = XP$, where P is the trade-weighted average, P is always less than or equal to $\tilde{P}$. In practice, however, because the elasticity of substitution is quite high (2.5), both measures are closely correlated and, except for large variances in the individual components, almost identical. (For details, see Appendix A).

the game progresses, and the extent to which they can signal and achieve collusion.

In some respects, the simple price-clearing condition is close to the standard double-auction scheme employed in experimental economics. The double auction is known to produce rapid convergence to competitive equilibrium even with few sellers and buyers. Convergence is rapid even when production must be determined in advance of transactions, although market efficiency during the convergence process is somewhat lower in this latter condition (Mestelman and Welland 1987).

From a behavioral and feedback-control perspective, it is significant that agents receive immediate and accurate feedback from their actions each period and that this feedback is undistorted by the multiplier effect. Because price clearing eliminates inventory accumulations, the market is effectively "reset" each period, as in the typical experimental economics protocol (Smith 1982; Plott 1982). Although there are also many differences between the double auction and the computer-mediated price clearing employed here, it seems reasonable to expect that the price-clearing simple market will converge quite rapidly to a range close to the competitive equilibrium.

How might boundedly-rational agents behave? If agents lack understanding of the task or the cognitive capacity to solve the game for the optimal strategy, they might instead use a "hill-climbing" heuristic for production. Such heuristics would gradually adjust production over time in an attempt to move closer to the profit-maximizing output level, based on an assessment of the local gradient of profits with respect to production. One such rule is the following:

(3.2.18) $y_{i,t} = Y_{t-1} \exp\{ a_1 \Delta v_{i,t-1} + a_2 \Delta V_{t-1} + u_{i,t} \}$;

(3.2.19) $\Delta v_{i,t} = (v_{i,t} - V_t)/(y_{i,t} - Y_t)$;

(3.2.20) $\Delta V_t = (V_t - V_{t-1})/(Y_t - Y_{t-1})$, where
$v_{i,t}$ is the profit of firm i during time t,
V_t is the average profit of all firms during period t,
Y_t is the average output per firm during period t, and
$u_{i,t}$ is a zero-mean random variable.

The rule expresses the idea that each firm anchors its output decision on the market average output level the previous period and then adjusts its anchor according to how profitable this output level was compared to "other" decisions. The adjustment has two components. One compares profitability to the market average, and one tracks market profitability over time. Both components express the simple idea that, "if profits are higher (lower) by producing more (less), continue to do so, otherwise, produce less (more)." If the first component dominates, firms act competitively: They consider only how well they do relative to others; if the second component dominates, firms act cooperatively: They consider market profitability as a whole.

The cooperative component is analogous to a "hill-climbing" algorithm. The competitive component is not, since performance is compared to the market average and decisions are anchored on the average output. A true hill-climbing algorithm would use individual decisions and performance during the two previous periods. Therefore, the algorithm will not reach the optimum performance for any player in isolation (holding all other players' actions constant). (The outcomes of the hill-climbing algorithm for a variety of parameter values are investigated in Appendix B.) The present algorithm was chosen because it considers the fact that performance depends on what other agents do in a simple way. A simple competitive hill-climbing mechanism can be unstable because it ignores competitor behavior.

Although the heuristic (3.2.18-3.2.20) is simple compared to the mathematical derivations required to find the optimal collusive or competitive price, it is not in any sense trivial from a cognitive perspective: It requires computing four differences from eight variables and, from these differences compute two ratios, which in turn function as cues for the decision rule.[23] Yet these calculations should not be interpreted literally. The psychological interpretation is that subjects can compare profits and outputs and, in a rough way, determine whether "they're moving in the right direction." Moreover, the ratios express the assumption that subjects have some ability to correct for the fact that a larger change in action (production) leads to larger change in results (profits). Finally, it is unlikely that subjects will pay attention to both the cooperative and the competitive portion of the rule each period. More likely, they will attend to one or the other in any given period. The coefficients thus represent a combination of average "attention" and "intention," and period-to-period variation in attention is subsumed in the random error term. This is an error where alternative sources of data, such as verbal protocols, eye-movements, or information-access, would be helpful.

Figure 3.2.4 shows an example of a simulation where all firms are assumed to act competitively ($a_1 > 0$, $a_2 = 0$). The market converges in a fairly

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23 The log-linear form of (3.2.18) may also look complicated, but the form was chosen for technical rather than substantive reasons, to force production to be positive when performing the simulations. Multiplicative, additive, or dummy-variable forms of the same rule perform identically in most respects.

orderly fashion to the competitive-equilibrium range. Conversely, if firms acted cooperatively, the market converges to the collusive equilibrium, as shown in Figure 3.2.5.

The hypothesized outcomes are illustrated in Table 3.2.3, which compares the results of the optimal rule with the outcome of rule (3.2.18-3.2.20), for both the competitive and the cooperative case .

It is noteworthy that even a simple myopic rule like (3.2.18-3.2.20) leads to orderly convergence in the simple price-clearing market. One would therefore expect that, in the absence of disturbances generated, for example, by attempts to signal collusion or "punish" other agents for defecting, the market would converge reasonably fast, especially when one considers agents' ability to look beyond just the previous two periods in their search for the "best" output level. (In the experimental setup (described in Section 3.3) it is relatively easy for subjects to compare both their own profits and output and the market average profit and output for all past periods.)

The ability of the simple clearing-price market to adjust toward equilibrium with hill-climbing rules like (3.2.18-3.2.20) is illustrated in Figure 3.2.6. The hill-climbing creates a negative feedback loop which drives the system toward the top of the hill. If firms primarily use individual profits as the criterion, the left loop in the diagram dominates, and the system is moved toward the competitive equilibrium. Conversely, if aggregate market profit is the dominant criterion, the other negative loop guides the system toward the collusive equilibrium. The positive feedback loop in the diagram allows the system to drift over time in search for equilibrium: If, for instance average output is below the competitive level, competitive agents will tend to raise their output relative to this average (since they perceive a positive gradient in profits). The next period, the average will thus go up and, assuming output is still below competitive equilibrium, agents will now anchor on this higher value and raise output still further. The process will continue until there are no incentives for agents to deviate from the market average output.

One feature of the system is that the optimum is quite flat, i.e. profits do not change dramatically for variations around the optimal output level. Although the steepness of the optimum is a function of the elasticity of demand, it remains quite flat for a wide range of this parameter. For the elasticity chosen for the experiments, an deviation of plus or minus 50% reduces profits by 10% or less. Figure 3.2.7 shows individual-firm profits (relative to optimal) as a function of output (relative to optimal) for high and low values of ε, the elasticity of individual-firm demand.

On the other hand, there is considerable leeway for improving profits through collusion, as indicated in Figure 3.2.8. In the simple condition ($\alpha=0$), firms could roughly double their profits by colluding. (In the complex condition, the multiplier mechanism reduces the collusive profits because demand is depressed by the low collusive output level.)

Thus, while <u>production</u> may show large variation, <u>profits</u> are likely to be fairly close to the equilibrium (optimal) profits for any <u>individual</u> firm. The largest deciding factor in profits is therefore likely to be the extent to which firms can cooperate. Moreover, one would expect it to be easier for firms to collude in the simple than in the complex condition. In the latter, the long delay between output decisions and their direct effect on prices, combined with the distorting effect of the multiplier, both makes it more difficult for firms to signal collusion and detect cooperation or defection, and it is more difficult to find the collusive steady-state output level.

3.2.4. Clearing prices, complex structure

The agent's task is more difficult in the complex-environment, price-clearing condition. First, there is a three-period lag between the time the output level is set and the time that production is sold, and therefore between the time decisions are made to the time the outcome (profits) of those decisions are realized. Second, demands, and therefore the market-clearing prices, are influenced by the multiplier feedback.

It is important to point out, however, that objectively, the task is still essentially equal to the simple case, since all market history (in the form of previous production in the pipe-line) is cleared by the time the decision comes to fruition.[24] Because of the price-clearing mechanism, the market is effectively "reset" every period. This situation is very different from the case where imbalances are allowed to accumulate in inventories or backlogs.

Thus, the optimal policy remains the same as in the simple condition, i.e. competitive firms should produce the output which is expected to lead to the constant price p*, and the rational-expectations equilibrium output

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24 There is, however, a difference in the objective learning task: Agents must now estimate the multiplier parameter α in addition to the aggregate and individual demand elasticities and the constant G. This requires three time periods of data.

remains the constant G. Likewise, the monopolistic rational-expectations policy is to produce the constant output G_M.[25]

From a behavioral perspective, however, the task is considerably more difficult. The lag between action and result, and the multiplier's distortion of prices in the short term degrades the quality of feedback received by individual decision makers. As such, one would expect them to learn more slowly (if at all) to find their best output level. Moreover, if subjects misperceive short-term changes in prices caused by the multiplier feedback for exogenous trends in demand, the market could fluctuate more before settling to equilibrium, as illustrated below. Finally, as mentioned above, signalling is much more difficult when there is a long lag before one can change one's price and one would expect to see less successful collusion than in the simple case.

The multiplier effect will result in a trend in prices which could lead the market to overshoot its equilibrium point as it tries to adjust. For instance, if firms initially expand output starting from a position of above-equilibrium price (below-equilibrium output), prices will start to rise due to the multiplier effect. This may induce firms to expand production further. Only three periods later will the "truth" be revealed, as prices suddenly drop substantially. In the meantime, firms who fail to understand the effect of the multiplier feedback have probably expanded output beyond the equilibrium level, so that prices now drop below their equilibrium value. In response, firms may cut back production, thus further aggravating the drop in prices in

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25 The value of G_M differs from the simple case, because the multiplier effect changes the steady-state price (see Figure 3.2.8 and Appendix A).

the short run. The cycle could continue for some time, with successive over- and under-shooting.

(3.2.21) $y_{i,t} = X_{t-1} \exp\{ a_1 \Delta v_{i,t-1} + a_2 \Delta V_{t-1} + u_{i,t} \};$

(3.2.22) $\Delta v_{i,t} = (v_{i,t} - V_t)/(x_{i,t} - X_t);$

(3.2.23) $\Delta V_t = (V_t - V_{t-1})/(X_t - X_{t-1})$, where
$v_{i,t}$ is the profit of firm i during time t,
V_t is the average profit of all firms during period t,
X_t is the average output per firm during period t, and
$u_{i,t}$ is a zero-mean random variable.

The mechanism is illustrated in Figure 3.2.9, which shows the outcome of the rule (3.2.21-3.2.23). The rule is the same as the one used in the simple clearing-price case in Figure 3.2.4, except that sales, x, rather than production, y, figures on the right-hand side of the equations. One sees a substantial initial overshoot in production before the market gradually settles toward equilibrium. The intuition for this behavior is as follows: Initially, prices are substantially above unit production costs and firms perceive an incentive to raise output. As output increases, the multiplier effect raises prices still further in the short term, leading to yet further increases in output because firms attribute the endogenously generated change in prices to an exogenous "boom". Eventually, however, the increased output arrives on the market, causing a sharp drop in prices. The self-reinforcing process is now reversed. Firms lower their output, causing prices to fall still further in the short term. However, firms also have information at this point about their profitability relative to the market average. Hence, the perceived profit gradients are more accurate, and the movements in the market more gentle. The result is a more gradual settlement toward the competitive equilibrium. Many other candidate rules share this general pattern of adjustment to varying degrees. Thus, one would expect the general pattern to be similar to Figure 3.2.9.

Table 3.2.4 compares the results of the optimal rule with the outcome of rule (3.2.21-3.2.23), for both the competitive and the cooperative case. The table, like the tables in the other experimental conditions, excludes the first 10 rounds of the simulation, to allow for some initial settling. Since the overshoot observed in the behavioral simulation in Figure 3.2.9 takes place mostly during those intial 10 time periods, Table 3.2.4 understates the difference between the optimal and the behavioral adjustment process.

Figure 3.2.10 shows a feedback loop diagram which illustrates the cause of the initial overshoot and possible oscillation. Indeed, the mechanism is much the same as in the fixed-price complex case, though it operates through price signals rather than quantity signals. On the other hand, there is likely to be much less instability in this system because there are no physical cumulative processes in the system, due to the automatic price clearing.

The objective difficulty of the simple and complex conditions is virtually the same (except for initial learning of parameters--see previous footnote), but if firms instead use behavioral rules similar to (3.2.21-3.2.23), one would expect the market to take longer to settle to equilibrium in the complex condition, with larger variance in production along the way. And, as discussed above, one would expect it to be difficult for firms to achieve cooperation.

An additional factor, mentioned above, is the fact that monopoly profits are somewhat lower than in the simple case. Due to the multiplier feedback, demand is depressed when output is lowered toward the collusive level so that prices, and thus profits, will be lower than if this effect was absent. Thus, if all firms cooperate, profits are about 50% higher than in competitive equilibrium, as opposed to about twice as high in the simple case (see Figure 3.2.8 and Appendix A). Note, however, that the competitive profits are the same for the two cases.

In terms of performance, average profits would therefore be lower in this case, to the extent that firms do indeed cooperate in the simple case. Again, however, since profits are a fairly flat function of output, the deviations are likely to be small.

If, on the other hand, firms are "rational" in the sense that they correctly interpret the effects of the multiplier feedback and have no difficulty managing the lag between decision and outcome, one would not expect there to be any significant change in variability or profits from the simple case (except for the effect of the lower collusive profit level).

3.2.5. *Posted prices, simple structure*

In the posted-price, simple condition, firms have control over both production and prices, and they must maximize profits by both minimizing inventory costs and maximizing revenue less cost of goods sold.

In Appendix A, it is shown that the decision task of the firm can be effectively decomposed into two separate tasks, corresponding to the tasks in the simple price-clearing and the simple fixed-price condition, respectively. This is so because the firm can control its inventory directly through production. Firms are thus free to search for the "best" price while separately setting inventory policy, except in the rare case where its inventory is larger than its expected sales at this price. (Should the latter be the case, the non-negativity constraint on production becomes binding, and the firm should then lower its price to help clear out its inventory. Since, by lowering its price, firms are likely to clear such excess during a single period, large excess inventories are likely to be short-lived.[26]) In Appendix A, it is shown that the optimal policy of a non-cooperative firm, assuming the non-negativity constraint on production is not binding, is

(3.2.24) $y_{i,t}{}^* = \mathrm{Max}\ \{\ 0\ ,\ E_{i,t}\{x_{i,t}\} - (n^* - n_{i,t})/\tau^* + u_{i,t}\ \}$;[27]

$n^* = 0;\ \tau^* = 1;$

(3.2.25) $p_{i,t}{}^* = p^* \exp\{w_{i,t}\}$, where

n^* is the desired inventory level,
τ^* is the "inventory adjustment time",
w is the unit production cost,
p^* is the steady-state competitive-equilibrium price, and
$u_{i,t}$ and $w_{i,t}$ are zero-mean random errors.[28]

Note that to the extent that firms realize the separability of production and prices and are able to avoid extreme fluctuations in inventories, the market is effectively reset every period. Structurally, the posted-price simple condition is therefore similar to the posted seller-price auction schemes investigated in experimental economics, except that buyers are "fully revealing" their preferences here, whereas the other experiments have employed human buyers which may exhibit varying forms of strategic

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26 However, Diehl (1992) finds that, even in such simple systems, subjects do not always achieve rapid inventory adjustment.

27 As in equation 3.2.1, it is assumed that distribution of x is symmetric around the mean.

28 The log-linear equation for prices was chosen to avoid a negative price in the simulations. For small error variances, an additive error could also be used.

behavior.[29] One would therefore expect the dynamics of the market to be dominated by the search for the profit-maximizing price level and from attempts at collusion, i.e. the pattern of prices may look much like the simple price-clearing case.

But there are some important differences between the simple posted- and clearing-price conditions. First, if the "random" variation in subjects' decisions is approximately the same (as a percent of the mean) for all decisions, then the unsystematic errors in both sales and prices in the posted-price condition will be about ε times larger than in the clearing-price condition. Second, the inventory accelerator is now active: Since inventories are now allowed to fluctuate, errors in expectations and decisions will lead to inventory costs which detract from profits. Moreover, the need to adjust inventories adds another source of variation in production. Thus, one would expect profits to be somewhat lower and the variance in production to be somewhat higher than in the price-clearing condition. Third, since the elasticity of individual-firm demand is constant, the profit-maximizing price is independent of the actions of other firms, thus making it easier for firms to find the optimum.[30] In contrast, the optimal output in the price-clearing condition depends on the output decisions of other firms. Thus, one would expect learning to be faster and decisions to be guided more by attempts at collusion than by searches for the best, non-cooperative price-output level.

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29 To the extent that inconsistencies in buyer behavior creates additional fluctuations and ambiguity, one can regard the absence of human buyers as an *a fortiori* condition in the experiment: If the market fails to converge in this simpler environment, it is unlikely to converge in the more complicated setting with human buyers.

30 This is of course an artifact of the particular formulation of the market structure used. On the other hand, to the extent that price elasticities in real markets are approximately constant, this would continue to hold approximately.

Unless there are systematic differences in the dynamics of collusion between the two conditions, one would therefore expect variance in prices to be lower in the simple posted-price case than in the simple clearing-price case.

Figure 3.2.11 shows the behavior of a simple decision rule which combines the heuristics presented above in the simple fixed-price and clearing-price conditions, respectively, except that prices rather than output are now used in the search for the best price-output point. The rule is

(3.2.26) $y_{i,t} = \mathrm{Max}\{\,0\,,\, x_{i,t}{}^{e} + (n^{*} - n_{i,t})/\tau_n + u_{i,t}\,\};\ n^{*} = 0;$

(3.2.27) $x_{i,t}{}^{e} = x_{i,t-1}{}^{e} + (x_{i,t-1} - x_{i,t-1}{}^{e})/\tau_x + w_{i,t},$

(3.2.28) $p_{i,t} = P_{t-1}\exp\{\,a_1 \Delta v^{g}_{i,t-1} + a_2 \Delta V^{g}_{i,t-1} + m_{i,t}\,\};$

(3.2.29) $\Delta v^{g}_{i,t} = (v^{g}_{i,t} - V^{g}_{i,t}/(p_{i,t} - P_t);$

(3.2.30) $\Delta V^{g}_{t} = (V^{g}_{t} - V^{g}_{t-1})/(P_t - P_{t-1}),$

(3.2.31) $v^{g}_{i,t} = (p_{i,t} - \omega)x_{i,t},$

(3.2.32) $V^{g}_{t} = (P_t - \omega)X_t,$ where

- $v^{g}_{i,t}$ is the "gross" profit of firm i (revenue less cost of goods sold) during period t,
- ω is the unit production cost,
- n^{*} is the desired inventory level,
- V^{g}_{t} is the average "gross" profit of all firms during period t,
- Y_t is the average output per firm during period t, and
- $u_{i,t}$, $w_{i,t}$ and $m_{i,t}$ are a zero-mean random errors.[31]

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31 As in the clearing-price condition, the log-linear form of (3.2.28) was chosen for technical rather than substantive reasons, to force prices to be positive when performing the simulations. Multiplicative, additive, or dummy-variable forms of the same rule perform identically in most respects.

The cognitive interpretation of the various components of the rule remains the same as when those components were introduced above. However, the task is definitely more complicated now, since subjects must attend to both the production and the pricing problem simultaneously. In addition, the rule presumes that subjects realize that the inventory costs are irrelevant when considering the pricing decision: Only the "operating" profits (revenue less cost of goods sold) matters. If subjects include inventory costs when "calculating" the gradient of profits with respect to prices, they could be led far astray.

In the figure, firms are assumed not to cooperate ($a_1>0$, $a_2=0$), and, as in the price-clearing simple condition (Figure 3.2.4), the market settles smoothly to a range close to the competitive equilibrium. If firms used the market-average profits as their criterion ($a_1=0$, $a_2>0$), the market would settle around the collusive equilibrium (cf. Appendix B).

Table 3.2.5 compares the average results of the rule (3.2.26-3.2.32) with the optimal rule with respect to profits and variance in prices and production. The results in the table should not be seen as critical values against which to test the conjectured behavioral rule. Rather, they should be seen as demonstrations that plausible decision rules can lead to convergence of the market, with only moderate variance in production and prices. The test of any specific behavioral decision rule must be conducted at the level of the individual by fitting observed decisions to the rule. It is interesting to note in Table 3.2.5, however, that profits and variability of output are not very sensitive to the variation in the random error terms: Most of the variation in output is caused by the search process.

Figure 3.2.12 shows a feedback-loop diagram of the simple posted-price market. It is evident from the figure that, although it is more complicated than the corresponding fixed-price and clearing-price conditions (Figures 3.2.1 and 3.2.6, respectively), the possibility of separating the inventory control task from the pricing task creates two systems which are both dominated by stable, negative feedback loops. As in the price-clearing condition, the positive feedback loop in the pricing system represents the "market learning" process through which the anchor for agents' decisions (the market average price) is gradually shifted until there are no incentives for individual agents to deviate from the anchor. In feedback control terms, the mechanism amounts to an "integral control" which assures that the market will be consistent in the steady state with agents' intentions (if such a steady state is reached at all!).

3.2.6. Posted prices, complex structure

The task becomes much more complicated when one moves from the simple to the complex posted-price condition. Firms must still maximize profits by minimizing inventory costs and maximizing revenue less production cost. But unlike in the simple posted-price condition, these two elements of the task are now inter-linked because production control of inventories can only be achieved with a lag. Moreover, firms must anticipate the effect of the multiplier feedback. The multiplier can make the search for the best price level much more difficult, since the multiplier will affect "gross" profits (revenue less cost of goods sold).

It can be shown that the optimal policy for small and moderate deviations from equilibrium is to use prices to control inventories and produce the competitive-equilibrium output, ignoring inventory and supply-line imbalances.[32] Thus, the optimal policy mimics the price-clearing, complex condition. Note that, because firms no longer control inventories immediately through production, the role of prices and production adjustments are <u>reversed</u> from the simple posted-price condition.[33]

Thus, for small or moderate random variations, firms should strive to set their price so that their expected sales during the period, $x_{i,t}$, will be close to production coming on line that period, $y_{i,t-\delta}$, plus the net inventory at the beginning of the period, $n_{i,t}$. Given that inventory is thus controlled through prices, the output decision is similar to the complex clearing-price condition, i.e. output should be chosen so as to maximize operating profits (revenue less cost of goods sold), ignoring inventory adjustments. In mathematical terms, this approximate optimal policy is

(3.2.33) $$y_{i,t}^* \approx E_{i,t}\{X_{t+\delta}\} \left(E_{i,t}\{P_{t+\delta}\}/p^*\right)^{\varepsilon} + u_{i,t},$$

(3.2.34) $$p_{i,t}^* \approx E_{i,t}\{P_{t+\delta}\} \left(E_{i,t}\{X_{t+\delta}\}/(n_{i,t}+y_{i,t-\delta})\right)^{1/\varepsilon} \exp\{w_{i,t}\},$$ where

X is the average sales,
P is the (trade-weighted) average price,
p^* is the steady-state competitive-equilibrium price, and
$u_{i,t}$ and $w_{i,t}$ are zero-mean random errors.[34]

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32 The optimal, competitive, rational-expectations policy is derived in Appendix A. The derivation is exact for the case with no random errors in decisions and with infinitely many firms. For the stochastic case, and for a finite number of firms, the system can be approximated by a linear-quadratic control problem, as demonstrated in Appendix A.

33 Following the terminology in Leijonhufvud (1968), the simple posted-price market adjusts in a Walrasian manner (price is the decision variable while quantity follows from prices) while the complex posted-price market adjusts in a Mashallian manner (quantity is the decision variable while prices move to clear supply and demand.) As has been shown both by Leijonhufvud (1968) and others (e.g. Tobin 1975), the two alternative forms of adjustment may have significant implications for market dynamics and stability.

34 As in the clearing-price rule (3.2.15-3.2.16), the CES aggregate sales $\tilde{X}$ is approximated with the arithmetic average, X, and the CES aggregate price, P, is approximated with

Although the rule (3.2.33-3.2.34) approximates the optimal policy <u>if firms know the structural parameters</u> in the market, in practice, firms must estimate these parameters from past data. Moreover, the aggregate demand function is sufficiently complicated in form to prevent perfect estimation of all the parameters: Instead, firms must rely on simplified structural equations for their estimation. It turns out that the learning process which employs the rule (3.2.33-3.2.34) is not very stable (cf. Appendix B). An alternative rule is to keep prices constant at their estimated target level and to make all adjustments in production, following the optimal fixed-price rule (3.2.7-3.2.11). This latter procedure shows better convergence (cf. Appendix B), although firms should switch to the former rule once the parameters were fully learned.

What, then, is the expected behavior and performance of this condition, compared to those of the other conditions? The expected results cannot be stated unambiguously, because the outcomes depend on the relative strength of many competing factors. Indeed the range of possible behaviors, using equally plausible decision rules, is very wide. The complicated nature of the problem is illustrated in the causal loop (or influence) diagram in Figure 3.2.13.

To get a sense of the range of possible behavior, a set of simulations were performed with the following decision-rule set

(3.2.35) $y_{i,t} = \mathrm{Max}\,\{\, 0\,,\, x_{i,t}^{e} + (n^{*} - n_{i,t})/\tau_n + \beta(s_{i,t}^{d} - s_{i,t})/\tau_n + u_{i,t} \,\},$

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the trade-weighted average P. The log-linear form for price was chosen to avoid negative prices in the simulations.

(3.2.36) $x_{i,t}^{e} = x_{i,t-1}^{e} + (x_{i,t-1} - x_{i,t-1}^{e})/\tau_x + v_{i,t}$,

(3.2.37) $s_{i,t}^{d} = \delta\theta x_{i,t}^{e} + (1-\theta)s_i^c;\ 0 \leq \theta \leq 1$,

(3.2.38) $p_{i,t} = P_{t-1} \exp\{ a_1 \Delta v_{i,t-1}^{g} + a_2 \Delta V_{i,t-1}^{g} + a_3(n^{*}-n_{i,t+1}^{e}) + m_{i,t} \}$,

(3.2.39) $\Delta v_{i,t}^{g} = (v_{i,t}^{g} - V_{i,t}^{g})/(p_{i,t} - P_t)$,

(3.2.40) $\Delta V_t^{g} = (V_t^{g} - V_{t-1}^{g})/(P_t - P_{t-1})$,

(3.2.41) $v_{i,t}^{g} = (p_{i,t}-\omega)x_{i,t}$,

(3.2.42) $V_t^{g} = (P_t - \omega)X_t$,

(3.2.43) $n_{i,t+1}^{e} = n_{i,t} + y_{i,t-\delta} - x_{i,t}^{e}$, where
$x_{i,t}^{e}$ is the expected sales,
$s_{i,t}^{d}$ is the desired supply line,
$v_{i,t}^{g}$ is the "gross" profit of firm i (revenue less cost of goods sold) during period t,
V_t^{g} is the average "gross" profit of all firms during period t,
$u_{i,t}$, $w_{i,t}$ and $m_{i,t}$ are a zero-mean random errors, and
$a_1, a_2, a_3, \beta, \tau_x, \tau_n, \theta, s_i^c$ are parameters.[35]

The rule set is essentially a combination of the decision rules for the fixed-price complex case and the posted-price simple case, except for an added term in the price decision which attempts to regulate (expected) inventory imbalances by adjusting the price. Thus, firms continue to use a production rule which anchors on expected demand and, depending on the parameters τ_n and β, also takes inventory and supply-line imbalances into account. In their pricing decision, firms anchor on the previous period's market-average price and then adjust it according to both the perceived direction of greater

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35 The log-linear form of (3.2.38) was chosen to force production to be positive when performing the simulations.

individual and/or joint profit margins (reflected in a_1 and a_2, respectively), and whether an excess inventory or backlog is expected (reflected in a_3).

Note that, although any individual part of the rule set is no more complex than the previously suggested rules, there are now many factors that each subject must attend to simultaneously. The information-processing load on the decision maker is thus increased considerably, and one would expect it to be more difficult for subjects to follow such a rule set consistently. This is important to keep in mind when evaluating the simulated performance of the rule: Actual decisions may not achieve the level of performance shown by appropriate versions of the simulated rule.

One possible outcome is that the posted-price complex condition is more stable than the corresponding fixed-price condition since firms can use prices to control their inventories more directly. This case is illustrated in Figure 3.2.14. The figure shows the adjustment pattern of production and prices if firms do not pay attention to the supply line (β=0), but do use prices control inventories (i.e. a_3>0). At the same time, firms are assumed to seek their individual profit-maximizing price level (i.e. a_1>0). (The other parameters are identical to those used in Figure 3.2.2 and 3.2.11, respectively). It is evident that the system can both be stabilized and reach the competitive equilibrium.

On the other hand, there are many ways that the system could fail to show such convergence. One trivial source of fluctuation is inconsistencies in decisions, expressed as random errors in the simulations. In Figure 3.2.14, the random errors u and w have both been assumed to have a standard deviation of 5%, yet the standard deviation in both prices and production is 5 to 10 times greater than that.[36] More serious, however, are systematic sources of variance, caused by inappropriate weights in the decision rules. For instance, if firms adjust their decisions aggressively in response to a perceived gradient in profitability, the result can be highly unstable behavior, as evidenced in Figure 3.2.15. The figure shows the result of raising the coefficient of profitability, a_1, from 0.2 to 0.5, with all other parameters kept the same as in Figure 3.2.14. Although a value of 0.5 is stable in the simple posted-price condition (cfr. Figure 3.2.11), the additional feedback effects on demand in the complex posted-price condition render the system unstable.

The example suggests that the range of parameters (or, more generally, behavioral rules) that assure stable and rapid convergence in the complex posted-price condition are quite narrow. The possibilities are too numerous to recount here (indeed they have probably only just been explored). Appendix B lists a number of simulations, and it is evident from those simulations that stable and rapid convergence in the complex posted-price condition is the exception rather than the rule. In contrast, the other experimental conditions (excepting the complex fixed-price case) show convergence for a much wider range of parameters.

The simulations in Appendix B further demonstrate that convergence in the complex posted-price condition is much more difficult, if firms attempt to collude: It is extremely difficult to find the appropriate price level, since profits are distorted by the multiplier effect. Thus, one would expect it to be

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36 Part of the greater variance is inherent in the structure of the task. In particular, a variation in prices of about 5% will cause sales to vary about 25% (since the elasticity of demand is 2.5).

very difficult for firms to (consciously) charge monopoly prices, unless they are all very moderate and conservative in modifying their prices.

To summarize, the decision task in the complex posted-price condition is exceedingly complicated, and the range of possible behaviors is very wide. One would thus expect there to be quite a large variance between markets in this condition: Some markets may happen to concentrate on policies which are stabilizing and lead to competitive or collusive equilibrium. But most will probably fall outside this relatively narrow range of policies, resulting in large variances in both prices and output. Rational agents (with consistent strategies regarding collusion vs. non-cooperation) would do well in the complex posted-price setting; profits and market variance would not differ much from the other experimental conditions. But it is unlikely that real, boundedly rational agents will perform that well, even though some behavioral rules can be found which lead to convergence.

3.2.7. Expected effects of the main treatment variables

The choice of treatment variables was motivated by a set of conjectures about how the important features of real industries would influence market behavior. The design involves six experimental conditions which are reproduced here:

- Condition 1: Fixed prices, simple environment.
- Condition 2: Fixed prices, complex environment.
- Condition 3: Posted prices, simple environment.
- Condition 4: Posted prices, complex environment.
- Condition 5: Clearing prices, simple environment.
- Condition 6: Clearing prices, complex environment.

The simulations and mathematical derivations above and in Appendices A and B demonstrate that, if firms act according to the standard neoclassical assumptions of non-cooperation and rationality, the differences between these six conditions would be very small: In all cases, the markets should settle smoothly and rapidly (after an initial learning period) to the non-cooperative equilibrium.

If firms are assumed to be engaging in strategic behavior, the question of market convergence becomes more complicated. If all firms were committed to achieving full collusion from the outset and never defected from the coalition, one would again expect rational agents to move the market rapidly to (collusive) equilibrium. Such a situation is unlikely, though, and it is more plausible that continuous attempts at achieving or defecting from cooperation would occur. There is no *a priori* reason to expect such attempts to follow a systematic pattern, and one would thus expect them to be essentially random, except perhaps for an "average" level of cooperation. Thus, it is reasonable to hope that strategic behavior can be captured by a systematic (constant) effect and a random-error term. If firms make unsystematic (random) departures from the optimal non-cooperative policy, the market should converge quickly and remain within a range defined by a stochastic stationary state.

One measure of the intrinsic difficulty of each condition is the degree to which such unsystematic errors are either amplified or damped out by the market system (assuming firms act optimally on average). The analysis has revealed that the consequences of these errors are small in all conditions if the systematic component of firms' decisions remains the optimal rule. Thus, although all performance and behavior should be compared to the optimal (or near-optimal) benchmarks in each condition, it is also true here that these benchmarks are very close across conditions.

If one abandons this latter assumption and instead assumes that firms follow some conjectured behavioral decision rule, the differences between the conditions become significant. These "behavioral hypotheses" can be summarized as follows.

- Complexity will decrease stability, relative to optimal, in all three price regimes

  The combined effects of the multiplier and production lags are likely to cause significant fluctuations in production and/or prices.

- Complexity will decrease profits, relative to optimal, in posted- and fixed-price regimes

  In the fixed-price and posted-price regimes, the main source of decreased profits will be inventory costs, resulting from the excess variance in production and sales. In the clearing-price regime, profits will vary less, due to the lack of inventory costs and a relatively flat profit function.

- The effects of complexity will be weakest in the clearing-price regime.

  Market-clearing prices eliminate the process of inventory accumulation, thus making the dynamics of the system relatively simpler. In particular, all history is "erased" from the market system by the time the decisions made each period take effect; for the purposes of decision making, the market is thus reset each period, both in the simple and the complex condition.

  In the complex condition, production lags and the multiplier effect may create some excess fluctuation compared to the simple condition. As the experimental markets are initialized with output below the competitive-equilibrium level, the fluctuation is likely to take the form of an initial "overshoot" of production above its equilibrium level before the market settles toward equilibrium. Yet the effect is likely to be much smaller than under fixed prices and posted seller prices.

- <u>The effects of complexity will be strong in the fixed-price regime</u>.

  Under fixed prices, the presence of lags and multiplier effects will result in large, long-term fluctuations, as people mistake temporary self-created increases in demand for long term changes and/or fail to properly take their supply lines into account. In contrast, markets in the simple condition under fixed prices will adjust fairly quickly to the steady state, since the control of the firm constitutes a simple inventory adjustment task with stable demand and immediate output control.

- <u>The effects of complexity in the posted-price regime will be much stronger than in the clearing-price regime, but may be smaller or larger than in the fixed-price regime.</u>

  The response in the posted-price complex condition could be more stable than in the corresponding fixed-price condition, since firms can control inventories directly through their prices. Equally, if not more likely, however is that fluctuations in inventories quickly translate into price fluctuations, which makes it difficult for firms both to anticipate the average market price and thus to control their own inventories, and difficult to interpret prices as signals of collusion. The result would be large excess variance in prices and production.

  In contrast, the simple condition under posted seller prices does not imply any need for inventory control via prices, as inventories are adjusted immediately by changes in production. Thus, firms can concentrate completely on strategic considerations in their pricing. Accordingly, the adjustment toward equilibrium is likely to be rapid and signals of cooperation are likely to take place.

- <u>Complexity will slow learning in all three price regimes</u>

  In general, increased complexity and large price and/or quantity variations in the market will slow the rate of learning significantly, to a point where a full approach to rational equilibrium effectively never takes place. Moreover, in making inferences from their experience, subjects may develop erroneous beliefs about the workings of the market, thus perpetuating deviations from rational equilibrium.

- <u>Collusion will be most evident in the simple (posted and clearing price) conditions and least evident in the complex posted-price condition</u>.

  Signalling collusion and detecting such signals is difficult in the complex posted-price condition, where prices are also likely to be used to control inventories. It is also difficult in the clearing-price complex condition because decisions only manifest with a lag. In contrast, firms are free to concentrate on collusive behavior in the corresponding simple conditions.

3.3. Experimental Method

3.3.1. Setting and procedure

Each market involved between four and seven subjects (firms). It was desirable that there be as many players in the market as possible. First, many firms would best mimic the conditions for perfect competition. Second, a large number of players would also minimize the effect of any extreme behavior of particular individuals and bring the sample distribution of subject behavior closer to the population distribution. In practice, the experience from experimental economics is that markets with only a handful of sellers behave competitively. A commonly used rule of thumb in the field is that five agents are enough to assure competitive conditions (Plott 1982, 1987). (Complete details of each experimental market, including the number of subjects in each market, is provided in Appendix D).

The market was implemented on a local-area-network of Apple Macintosh computers that automatically administered and recorded all decisions and other events in the course of the experimental session. Up to three separate markets could be run simultaneously on the network. The recording ability has provided a wealth of data on what information subjects accessed, and the timing of their actions.[37]

Subjects sat at separate terminals in the network, each person managing one firm in the market. The terminals were all set up in the same room, but with the distance between terminals large enough to prevent subjects from looking at the terminals of their competitors.[38]

Before the beginning of the actual session, a set of written instructions was distributed to the subjects. The instructions described the objectives of the research and the market structure in general terms, what decisions subjects would make in the game, and the basis for rewards. Each person also received a short questionnaire requesting basic demographic data about their age, educational background, and experience in economics, system dynamics, and statistics or econometrics. All instructions and questionnaires are reproduced in Appendix C.

After reading the instructions and filling out the questionnaire, subjects had an opportunity to ask questions and become familiar with the mechanics of the game in a practice session of either 3 periods (in games with no production lags) or 6 periods (in games with lags).[39] After this period,

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37 The software used for the experiment was programmed by the author, using Apple Hypercard™, with the addition of certain external routines, written in Assembly or C, to handle network communication and data plotting.

38 In selected sessions, not included in the primary data set, one randomly selected participant from each market was asked to sit in a separate room and generate "think aloud" protocols (Ericsson and Simon 1984) while making his or her decisions. These concurrent protocols were taped for later analysis.

39 In the simple condition, subjects thus had three time periods in which their decisions would not affect their reward (except for the inventory remaining at the end of the third period). Similarly, in the complex condition, the first three decision periods were of

cumulative profits and forecasting scores were reset to zero, while other variables were unchanged. Thus, the practice period also provided an opportunity for subjects to explore prices and quantities and learn the structure and parameters of the system. In each case, the system was initialized with a level of production of 67% of the competitive-equlibrium level. The initial price (in "period 0") was set at the level which would clear the market with that level of output. In the fixed-price condition, prices were then lowered to the competitive-equilibrium level and held constant there.

At the beginning of each period, subjects made their production decision and, if applicable, set their price. After all decisions had been collected, the computers calculated demand or prices for each firm, updated the information, and advanced time to the beginning of the next period.

In addition to the actual decisions, the computers solicited forecasts of aggregate market sales and/or prices from the subjects. These forecasts provide valuable information about players' expectations to be used in the analysis. To provide an incentive for accurate forecasts, the computers kept a forecasting "score" (the root mean squared error) that provided the basis for a bonus at the end of the game. The bonus was small (several dollars) relative to the profit-related reward in order to avoid attempts to manipulate the market in to reduce prediction errors.

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little consequence for the reward (again except for the remaining inventory) while, due to the production lag, the production decisions made after the third period would affect profits after the end of the practice round.

Subjects were free to take as long as they wished to make their decisions. The game was stopped after three hours or after subjects had played 50 time periods, whichever came first.[40] While subjects were not informed of the 50-period maximum, they were told that the game would be stopped within a fixed (real) time. Thus, as was pointed out to the subjects, taking longer to deliberate decisions would decrease the number of time periods they could play, reducing their profits (assuming positive average profits per period). The end of the game was not announced before subjects had made their decisions for the last round.

After completing the experiment, subjects received a money award (mailed by check) in proportion to their accumulated profits in the game, plus a "bonus" award for forecasting performance. The profit-based award was based on a fixed conversion factor between "paper" dollars earned in the game and "real" dollars, while the forecasting bonus depended on the rank of the forecast score (relative to the scores of other subjects in the same game). When subjects submitted both price and demand forecasts, each forecast was ranked separately. The forecast "bonus" ranged linearly from $0 for the worst to $10 for the best forecast (average $5).

All participants were guaranteed a minimum payment of $10 for participating in the game, even if their performance would indicate a negative reward. Moreover, subjects were free to leave at any time during the experiment and still collect their $10. (Only one person elected to do this, and that particular market was discarded from the data set.) The average payout

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40 The average length of each game was 44 time periods. The minimum was 35 time periods. In 7 out of the 24 markets, the maximum of 50 periods was reached.

per subject was $34.80; 4 out of 97 subjects received only the minimum $10 payment while the maximum payment was $63. Since each experimental session lasted slightly over three hours, this payout accords with the typical payout to experimental economics subjects of about $10 per hour (Plott 1986a).

3.3.2. Subjects

Subjects were primarily graduate and undergraduate students in economics and management at M.I.T. and Harvard University. The median age was 23 years (the average was 24 years). Table 3.3.1 summarizes the background of the experimental subjects in the main data set. It is also evident from the table that most subjects had received some formal education in economics and quantitative fields such as statistics or operations research. A fair number had taken advanced courses in these areas.

Subjects were recruited by announcements in classes and posted or distributed flyers. To the extent possible, subjects in each session were balanced with respect to the educational background and affiliation of participants. Volunteers were asked to indicate which of several alternative days they were available; this provided some leeway in assigning people to experimental sessions. The experimental condition applied in any given session was randomly determined. However, to facilitate initial instruction and minimize speculation about the experimental treatment variables and hypotheses, markets run simultaneously were assigned the same experimental condition.

3.3.3. *Information availability and display*

An important question is how much information subjects should be given, and in what form. Information availability is itself an important element of structure and will change the nature of the system both because it changes the objective estimation and decision-making problem for each firm, and because it interacts with human cognitive limitations to influence behavior. Further, the manner in which information is displayed and the degree of interactivity and control subjects have over the information display can have a large impact on cognitive processing and performance (see e.g. Johnson, et. al 1988; Kleinmuntz and Schkade 1990).

The access to information is both particularly relevant and complicated with regard to the process of learning. The initial qualitative and quantitative knowledge of the structure of the system in fact defines the learning task for the participants, both objectively and psychologically. If, for instance, rational agents knew everything about the system, including parameter values and its current state, the market should instantaneously settle to a rational-expectations equilibrium. If the structure was known, except for parameter values, the market may or may not settle to equilibrium, depending on the a priori beliefs of agents and the particular structure in question (Blume and Bray 1982; Bray and Savin 1986; Frydman and Phelps 1983). If key qualitative features of the system structure are not known, it is not clear what "rational" learning would mean.

One could address the issue of information access by making it an experimental variable, but the issue *per se* was not the central concern here. In any case, the issue has not received much formal attention in experimental economics; in the typical market experiment, subjects are usually provided with relatively little structural information and no decision aids. (Indeed, since auction processes are quite fast-paced and chaotic, it is not clear that such aids would be of much use.) The attempt here was to strike a reasonable balance between realism and simplicity of the experimental setup.

During the game, firms had "full aggregate information" in the sense that they could observe past values of all their "own" variables, such as price, inventory, and production, and past values of the market average values of these variables.[41, 42] In real-world markets, such data are generally available. They did not, however, have full information about the individual variables of other firms. In addition to the market average price (weighted by sales), firms also received some information about the distribution of individual prices, in the form of the highest and lowest price in the market.

In the introductory description of the market (see Appendix C) subjects were told all the relevant structural relationships in the system, such as the factors affecting industry and individual-firm demand, but they were not given precise numerical or mathematical information about the relationships, other than an indication of the relative size of elasticities. Subjects also they knew that all firms were identical in costs and structure--

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41 In the versions of the game that included random disturbances, these random shocks could not be observed directly by the firms. However, it is theoretically possible (though unlikely to happen in reality) for firms to infer these from observing the other variables, provided that they have estimated (learned) the structural parameters in the system from observing past data.

42 The averages provided were simple arithmetic means (in the case of production, sales, inventories, etc.), except the average price, which was a trade-weighted average. The averages for sales and prices thus differ slightly from the non-linear averages (3.1.3) and (3.1.4), respectively (cfr footnotes in previous sections).

only the products differed. In real-world settings, such general features are the most that is known, at least initially, and functional forms and parameters must be estimated intuitively or statistically.

Of particular interest is the fact that subjects were told that certain "external factors" could influence demand:

> Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.
>
> The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.
>
> (Appendix C, emphasis in original).

In fact, there were no such exogenous disturbances in the system (except in the auxiliary data set, where a there was random white noise input). It seems fair to say that in real life, external shocks are both unobservable and the nature of the process generating them is unknown (although this issue is at the heart of much of the debate over rational-expectations models).

From a psychological perspective, the fact that subjects did not know the nature of external shocks (i.e. that they were absent) is significant: Studies of attribution of failure or success have revealed a tendency to attribute success to one's skill and failures to chance or factors outside one's control (e.g. Langer 1975; Miller 1976). In the context of dynamic systems, there is an equivalent tendency to attribute observed behavior to factors outside "the system," rather than see it as internally generated. In Sterman's production-distribution experiment, for instance, subjects invariably attributed the oscillations they experienced in the game to exogenous fluctuations in demand, even though the actual demand was a one-time step function (1989c). Since learning is contingent upon the ability to relate cause and effect, the tendency to "externalize" behavior and to may be a significant obstacle to effective market adjustment and learning.

Figure 3.3.1 shows an example of the "main" computer screen faced by the participants. The relations between the stock of inventory and the flows of production and sales were shown in diagrammatic form in the top half of the screen. On the bottom left part, a simple "summary report" showed last-period revenues, prices, costs, and profits, and accumulated profits. On the right, the players entered their decisions and forecasts.

The bottom right section contained "pop-up menus" for accessing historical data, both in the form of tables and graphs, and an "OK" button when the player was ready to submit his or her decision(s). Subjects had access to five tables, four of which were "pre-defined" with variables, as listed in Table 3.3.3. But any time they wished, players could modify their tables, using "pop-up menus" and "buttons" to choose from a very large list of variables, reproduced in Table 3.3.2. In addition, subjects had access to three time plots and three plots of one variable against another, as shown in Table 3.3.4.[43] Figures 3.3.2 and 3.3.3 show an examples of a table and a graph, respectively.

43 Although the software allowed for redefining graphs in a similar manner as tables, the feature was disabled to save time.

In order to minimize the specific effects of information displays and decision aids on behavior, the same information display was maintained across all experimental conditions, with only the smallest modifications necessary to accommodate the different conditions. For instance, in the price-clearing condition there is no need for inventories, but they were nonetheless included in the display (with a constant value of 0). Likewise, the price information was included in the fixed-Sales condition even though it is of little interest.

3.3.4. Comparison with typical experimental-economics procedure

The setting for these experiments deviates in some ways from the usual procedure in experimental economics, but key elements of the standard procedure are retained:

- Time is divided into discrete periods.
- Subjects are paid real money in accordance with their performance (accumulated profits).
- Performance information is private.
- There is no "cover story," and products are abstract.

In contrast to the typical procedure, however, this work employs

- an extensive and tightly controlled "decision support system," in the form of computerized displays and access to historical data;
- market institutions, such as inventories, fixed prices or price clearing, etc., which are different from the typical auction or posted bid/offer institutions used in experimental economics; in particular, the market is not "reset" between periods;[44]

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44 Even the posted-seller price regime differs slightly from the typical posted-price experimental market. In the latter, firms announce a price and a maximum amount they

- in this particular work, an emphasis on the supply side of the market, with the demand side modeled by a computer;
- extensive data collection, including verbal protocols, information-access data, and solicited forecasts.

It is worth pointing out that the amount of information and the degree of sophistication and flexibility with which it is presented is much greater here than in the normal experimental-economics procedure. By simple "mouse-clicks" and key-strokes, subjects could gain access to a very large amount of information, and they were given considerable flexibility in the display of that information.

Moreover, all key-strokes and mouse-clicks were registered by the software. Thus, whenever subjects entered decisions, asked for information, redefined tables, etc., these actions were recorded by the computer, creating a detailed audit trail of the information subjects attended to and the time they spent accessing and contemplating information.

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are willing to sell. Here, firms do not have control over how much they will sell each period but must fully accommodate "customers" by backlogging orders or running down inventories.

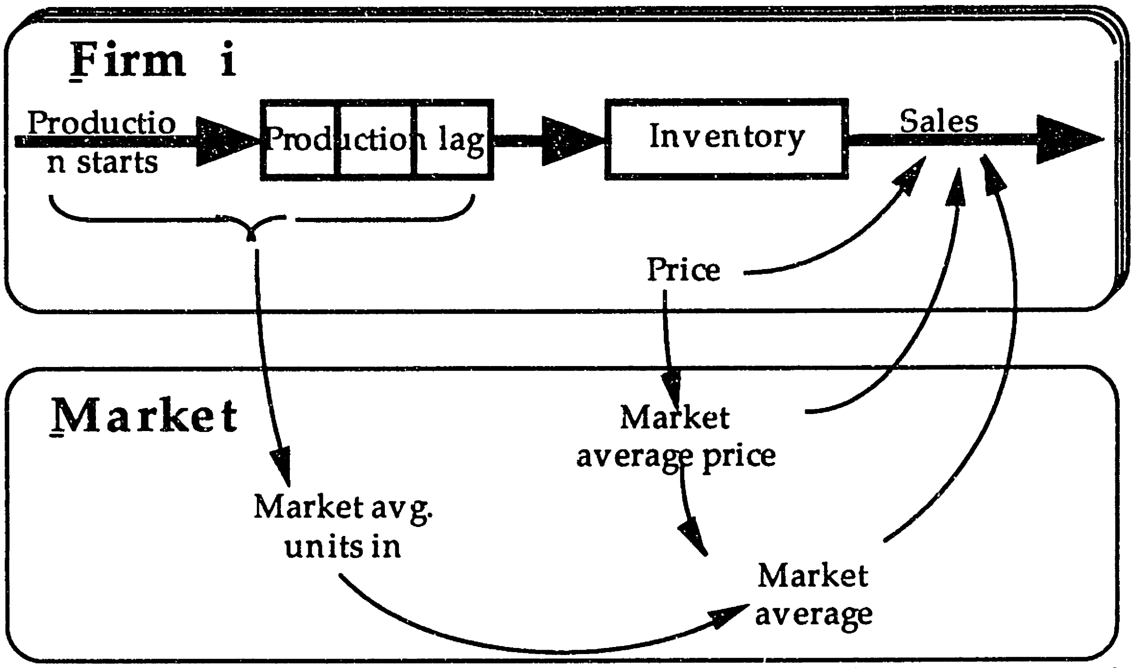

Figure 3.1.1: Schematic representation of the structure of the experimental market

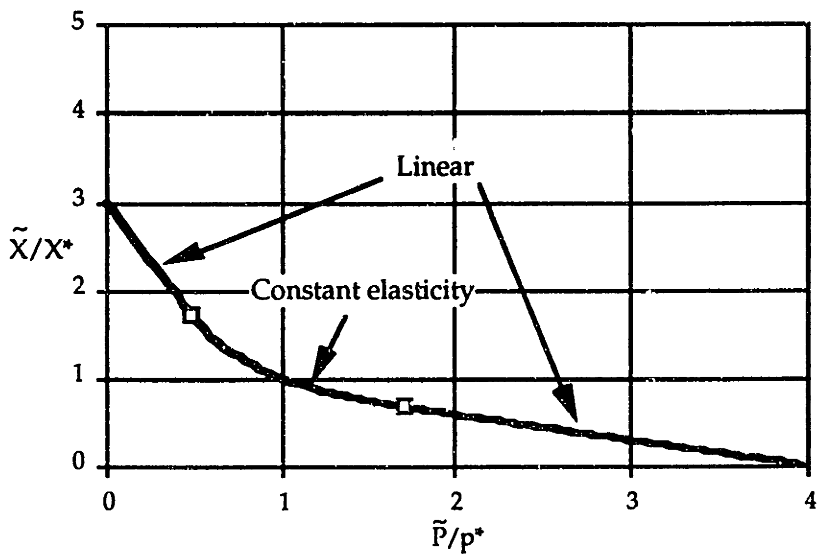

Figure 3.1.2: Plot of aggregate demand, X_t, relative to "reference" aggregate demand, X_t^*, as a function of aggregate price, P_t, relative to "reference" price, p^*

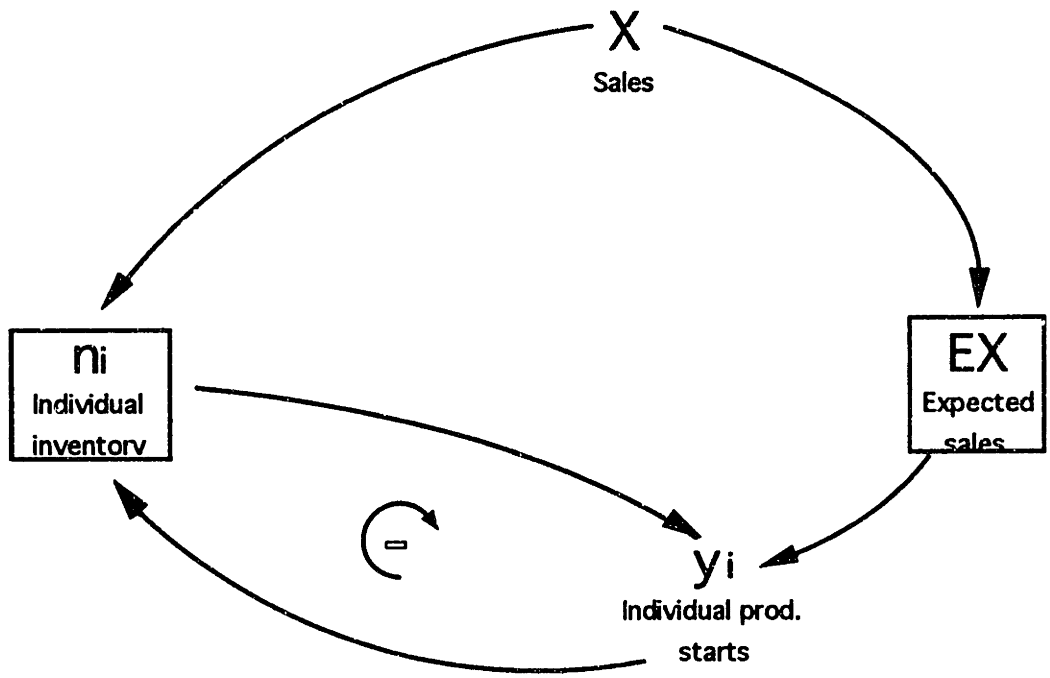

Figure 3.2.1: Causal loop diagram of the simple fixed-price condition

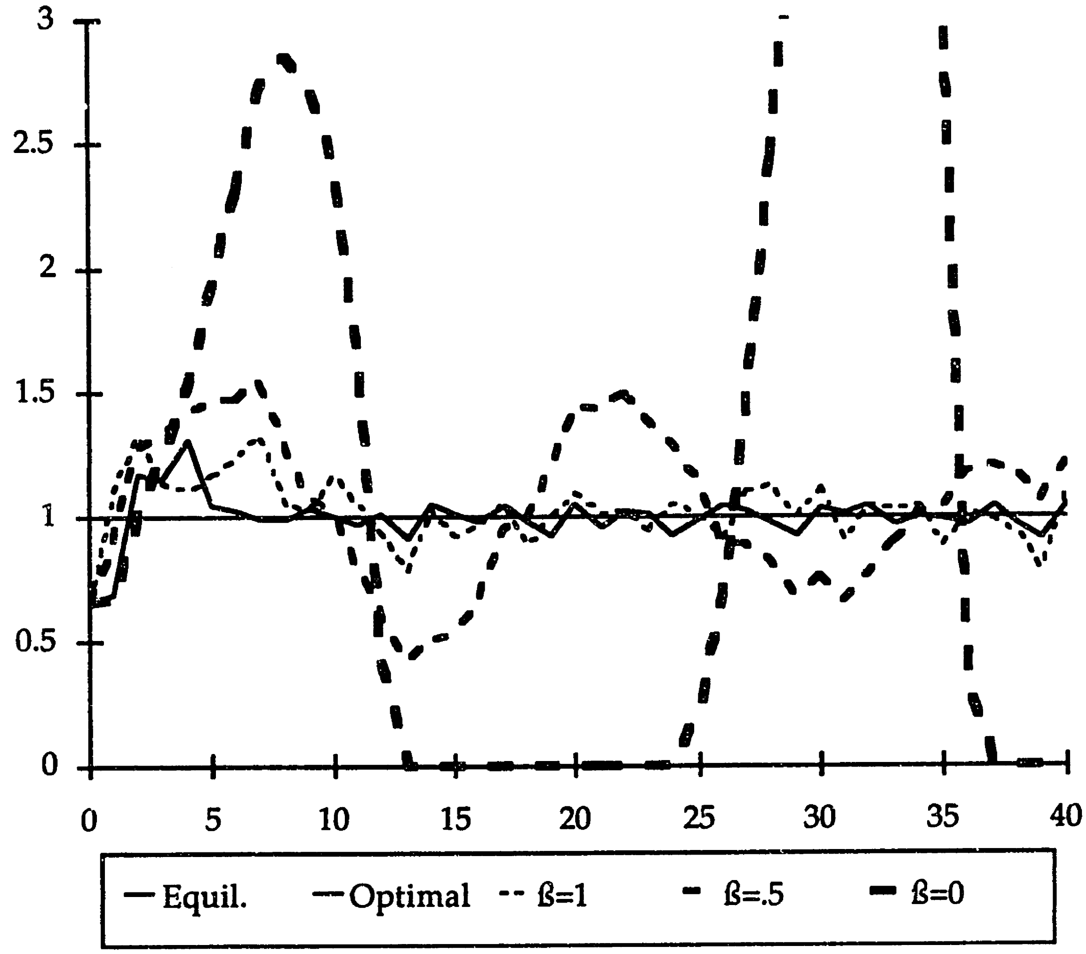

Figure 3.2.2: Comparison of the behavior of optimal and behavioral rules in the complex fixed-price condition

The simulation compares the optimal response (using the minimum-variance criterion) with the behavioral rule (3.2.12-3.2.14), using three alternative value for the parameter β. The other parameters in the rule are $n^*=0$, $\tau_n=1$, $\tau_x=2$, $\theta=0.5$, $s_i^c=\delta G$, Std(u)=0.05, Std(v)=0.05.

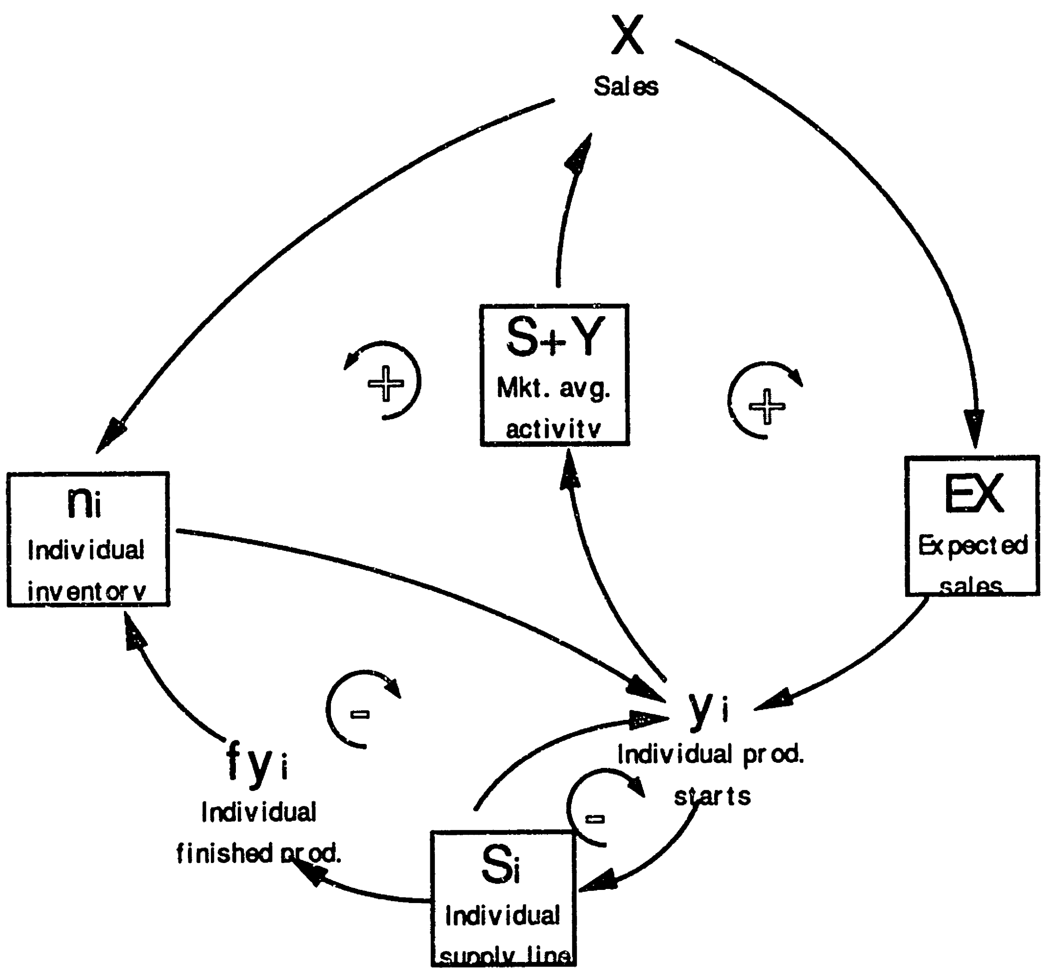

Figure 3.2.3: Causal loop diagram of the complex fixed-price condition

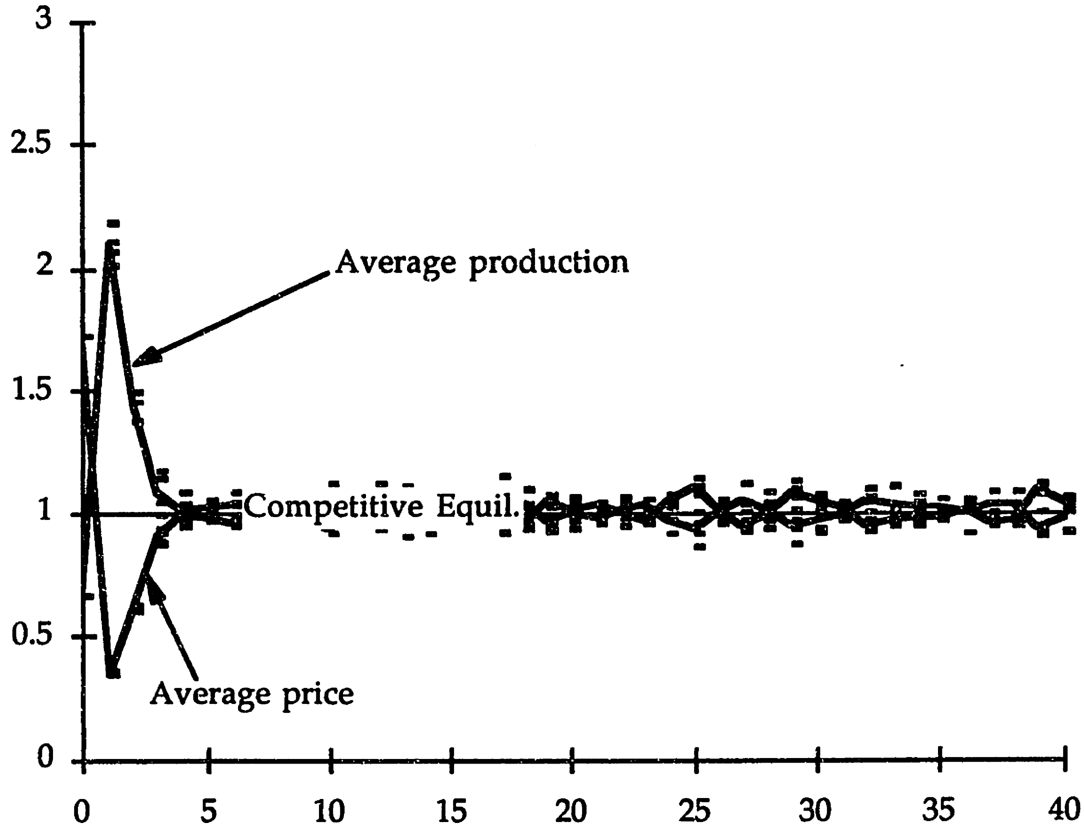

Figure 3.2.4: Example of competitive behavioral adjustment, simple clearing-price condition

The figure shows the outcome of simulating the rule 3.2.18-3.2.20 with the following parameters $a_1=1$, $a_2=0$, Std{u}=0.05. Moreover, during the first round, where firms have no information about the marginal profit since all their output is the same, they use the "gross margin," i.e. price less unit production cost. The "-" marks show output and price for the four individual individual firms in the market.

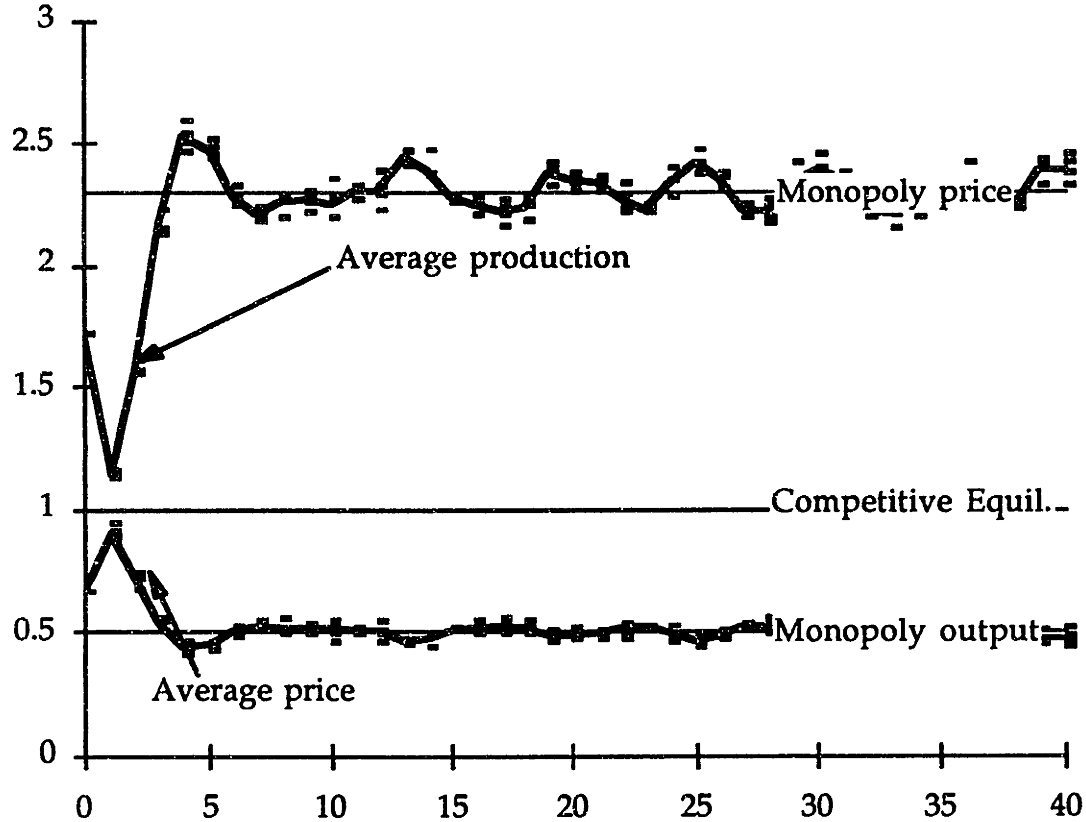

Figure 3.2.5: Example of cooperative behavioral adjustment in the simple clearing-price condition

The figure shows the outcome of simulating the rule 3.2.18-3.2.20 with the following parameters a_1=0, a_2=0.25, Std{u}=0.05. Moreover, during the first round, where firms have no information about the marginal profit since all their output is the same, they use the "gross margin," i.e. price less unit production cost. The "-" marks show output and price for each of the four individual firms in the market. Note the tendency to oscillate around the equilibrium. The oscillation is a result of the relatively high gain represented by the parameter a_2.

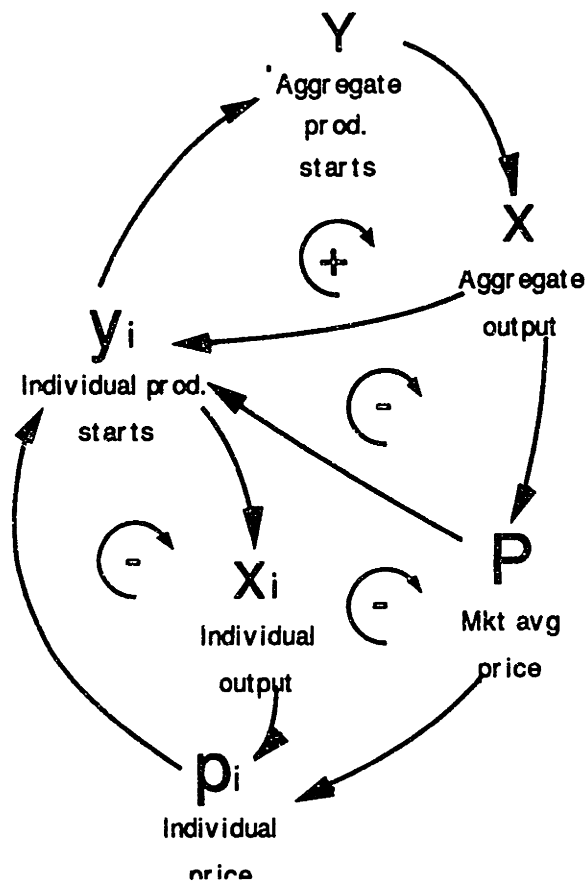

Figure 3.2.6: Causal loop diagram of the simp'e clearing-price condition.

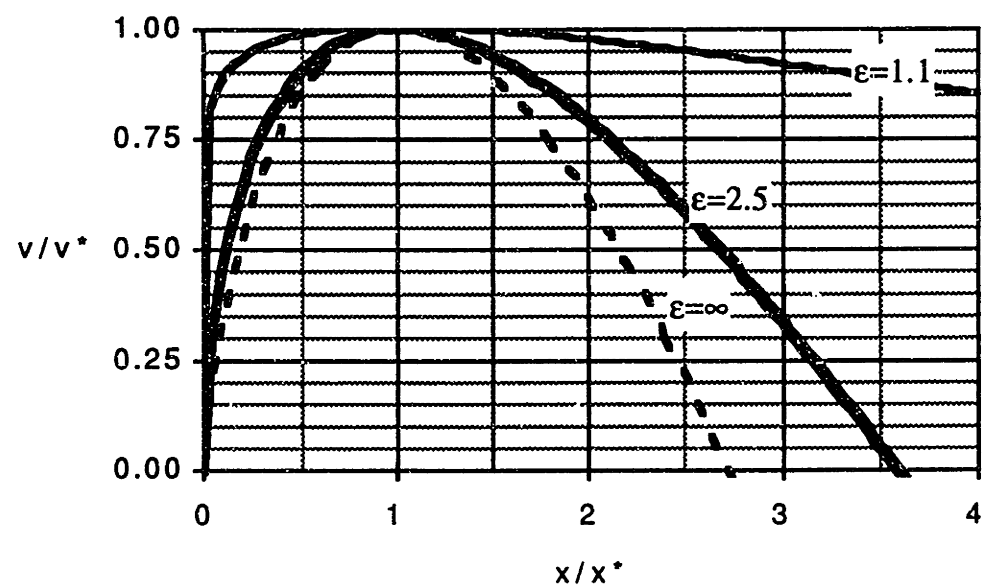

Figure 3.2.7: Individual-firm profit function, simple clearing condition

The figure shows profits (v) relative to optimal (v*), as a function of output (x) relative to optimal (x*) in the price-clearing condition, assuming the number of firms, K is very large.

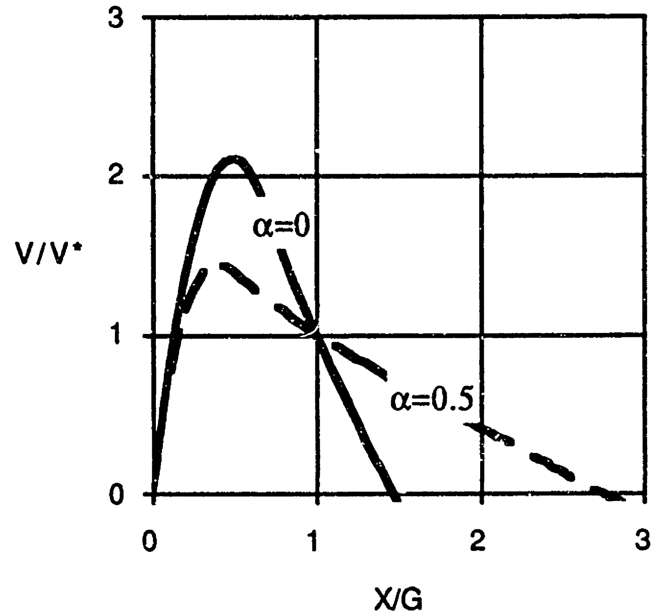

Figure 3.2.8: Steady state profit function, clearing-price condition.
The figure shows aggregate profits relative to competitive profits (V/V*), as a function of aggregate output relative to competitive output (X/G) in the price-clearing condition.

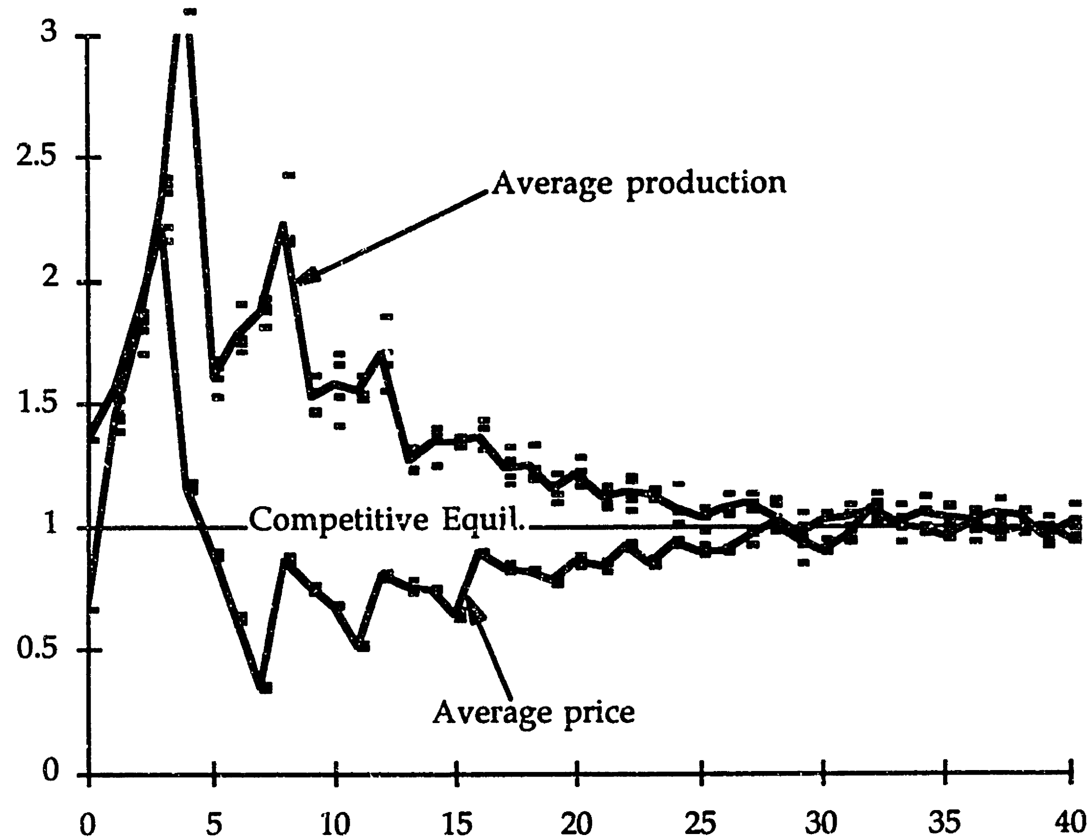

Figure 3.2.9: Example of competitive behavioral adjustment in the complex clearing-price condition.

The figure shows the outcome of simulating the rule 3.2.21-3.2.23 with the following parameters $a_1=1$, $a_2=0$, Std{u}=0.05. Moreover, during the first three rounds, where firms have no information about the marginal profit since all their output is the same, they use the "gross margin," i.e. price less unit production cost. The "-" marks show output and price for the four individual firms. Note the tendency to oscillate. It is caused by a combination of a relatively high gain represented by the parameter a_2 and the three-period production lag.

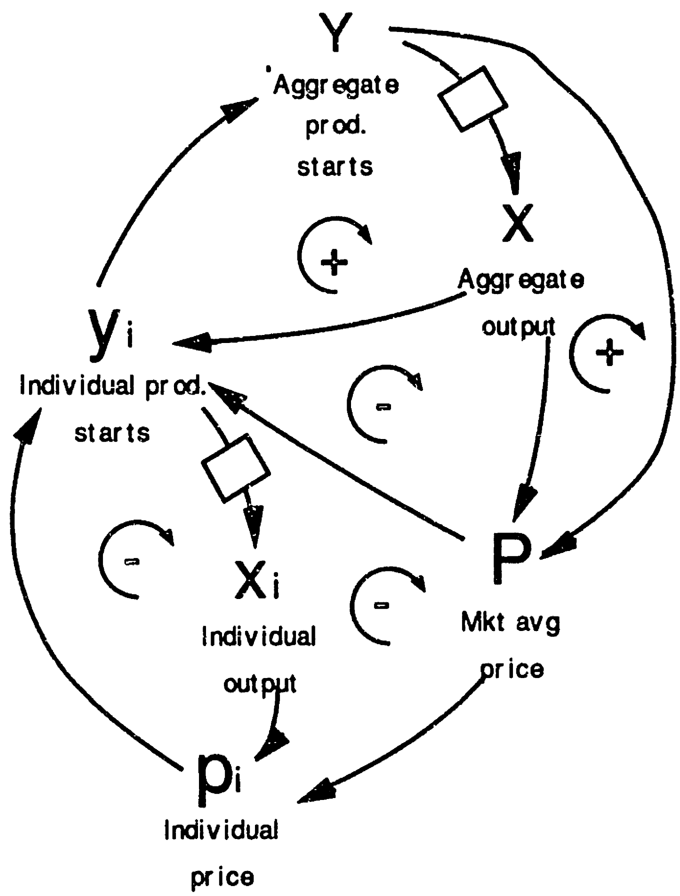

Figure 3.2.10: Causal loop diagram of the complex price-clearing condition

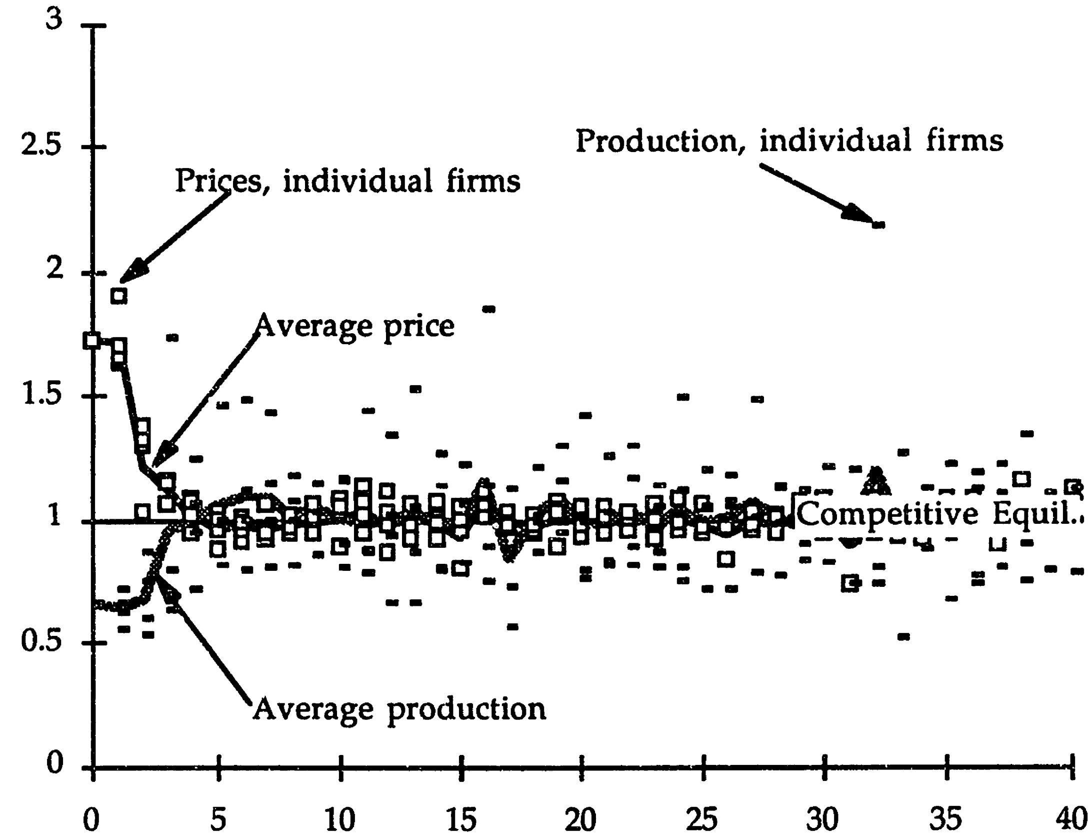

Figure 3.2.11: Example of competitive behavioral adjustment in the simple posted-price condition

The figure shows the outcome of simulating the rule 3.2.24-3.2.30 with the following parameters a_1=0.5, a_2=0, Std{u}=Std{m}=0.05, Std{w}=0, τ_x=2, τ_n=1. Moreover, during the first round, where firms have no information about the marginal profit since all their prices are the same, the marginal profit is set to zero.

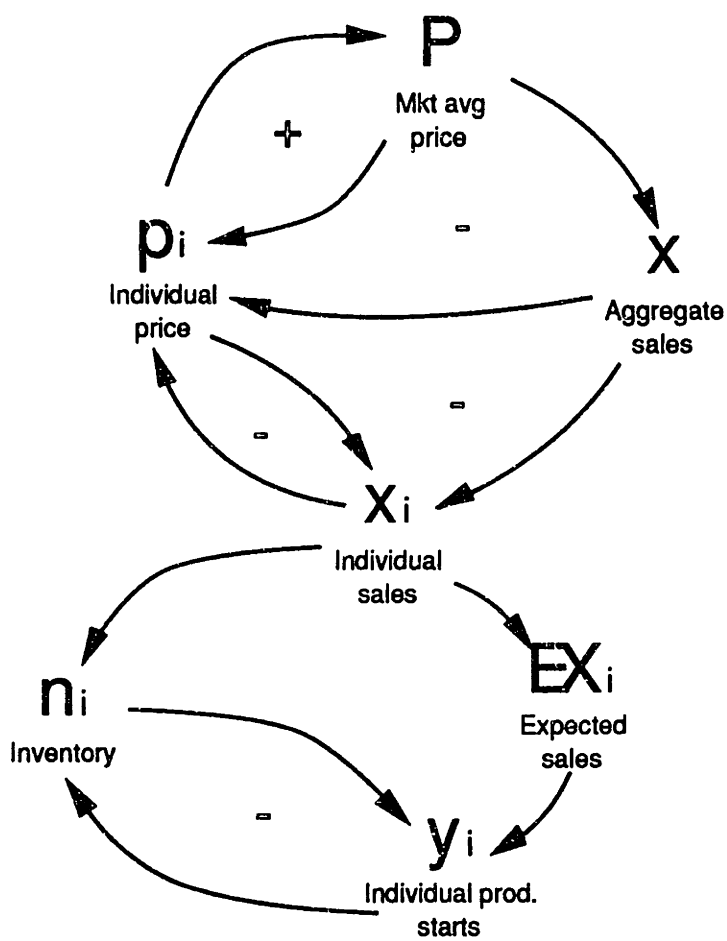

Figure 3.2.12: Causal loop diagram of the simple posted-price condition

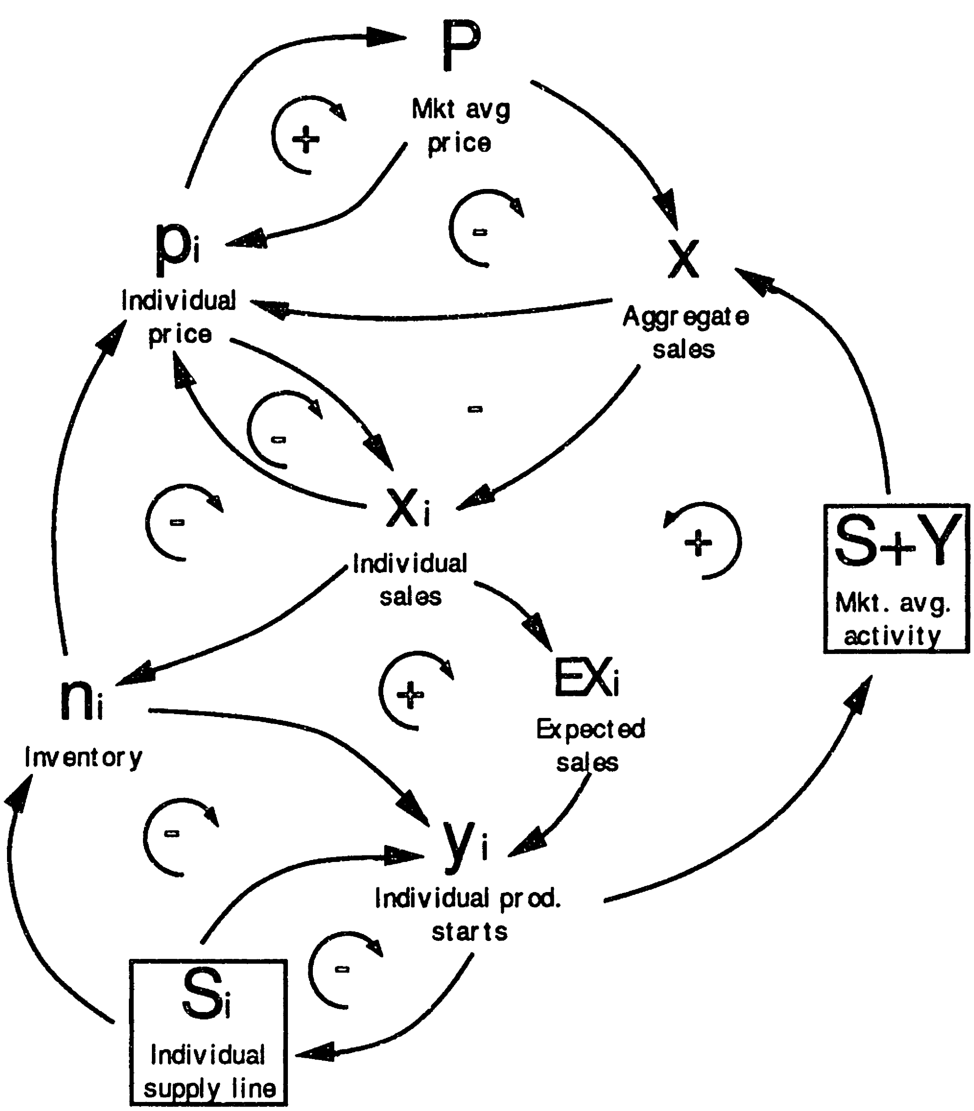

Figure 3.2.13: Causal loop diagram of the complex posted-price condition

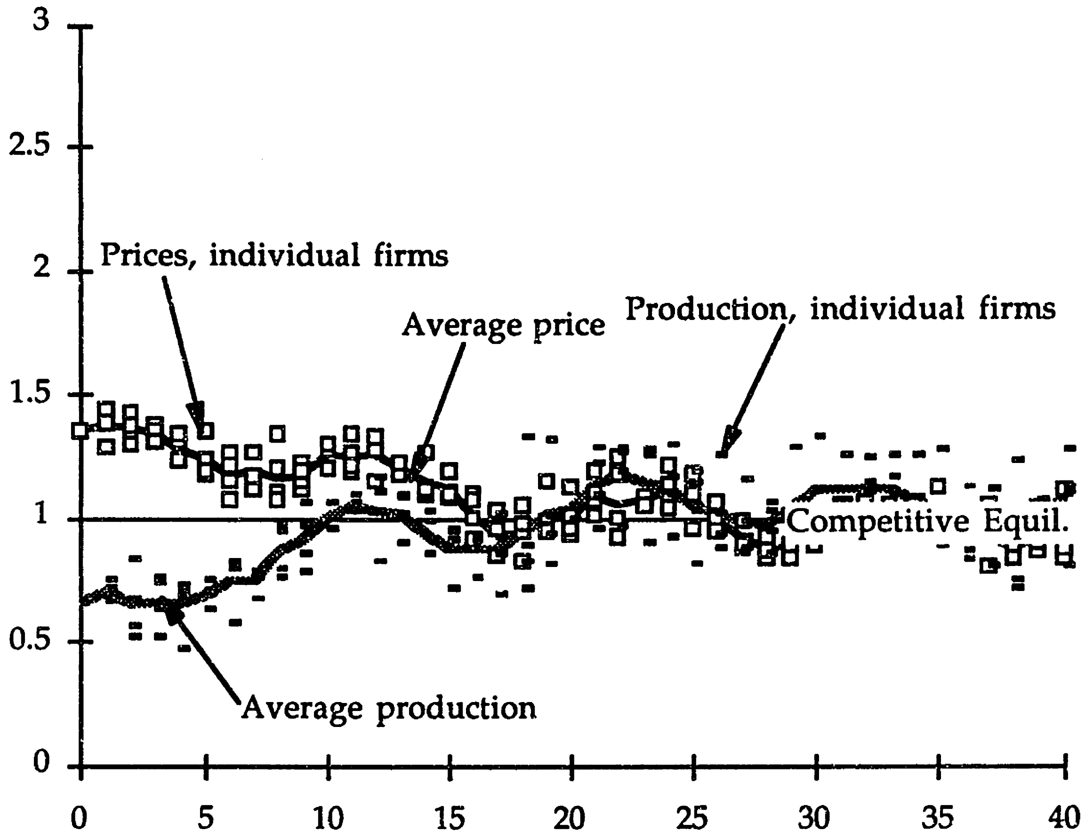

Figure 3.2.14: Example of behavior, complex posted-price market

The figure shows the outcome of simulating the rule 3.2.33-3.2.41 where firms pay no attention to the supply line (β=0) but use prices to adjust inventories (a_3>0). Specifically, the simulation involved the following parameters a_1=0.2, a_2=0, a_3=0.2, τ_n=2, β=0, $s^c = \delta G$, θ=0, Std{u} = Std{m} = 0.05, Std{w}=0, τ_x=2. Moreover, during the first round, where firms have no information about the marginal profit since all their prices are the same, the marginal profit is set to zero.

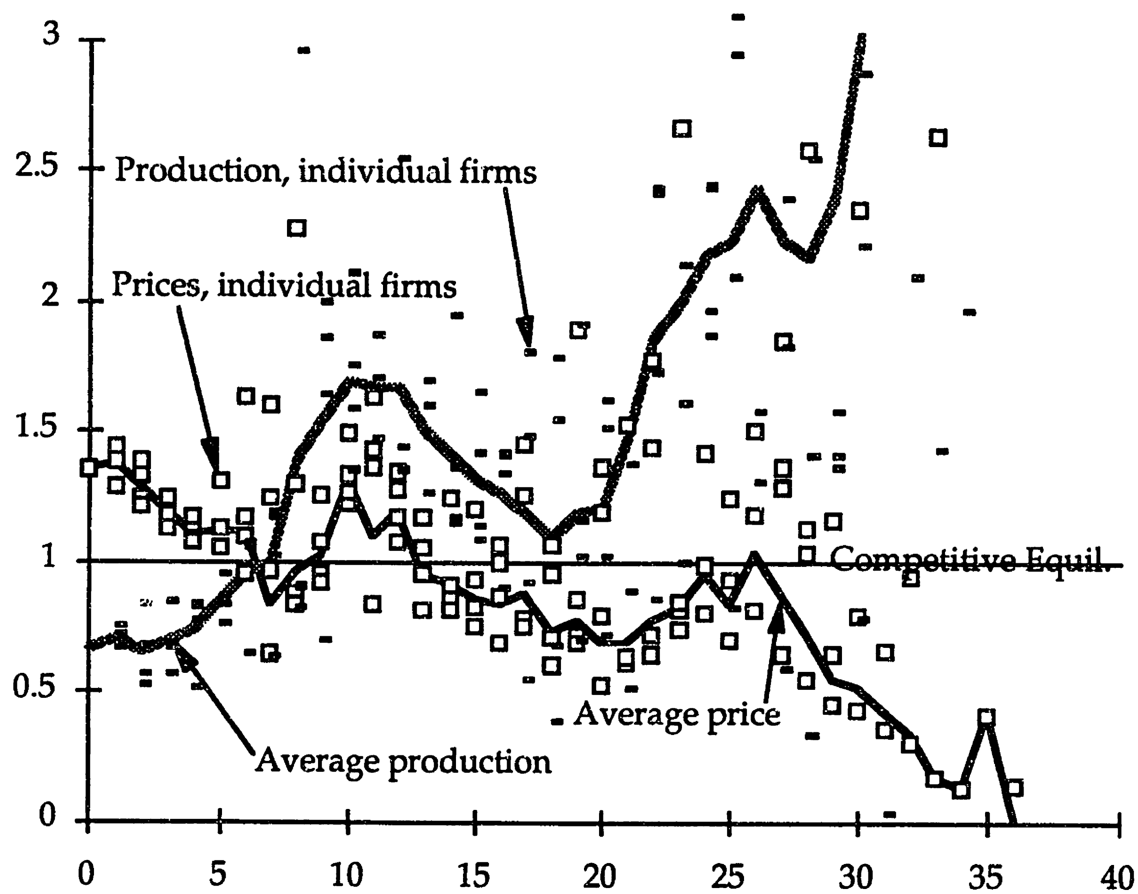

Figure 3.2.15: Example of unstable behavior, complex posted-price condition

The figure shows the outcome of simulating the rule 3.2.33-3.2.41 with the same parameters as in Figure 3.2.14, except that a_1=0.5. When the aggressiveness of price responses to marginal profits (a_1) is increased, the system becomes unstable.

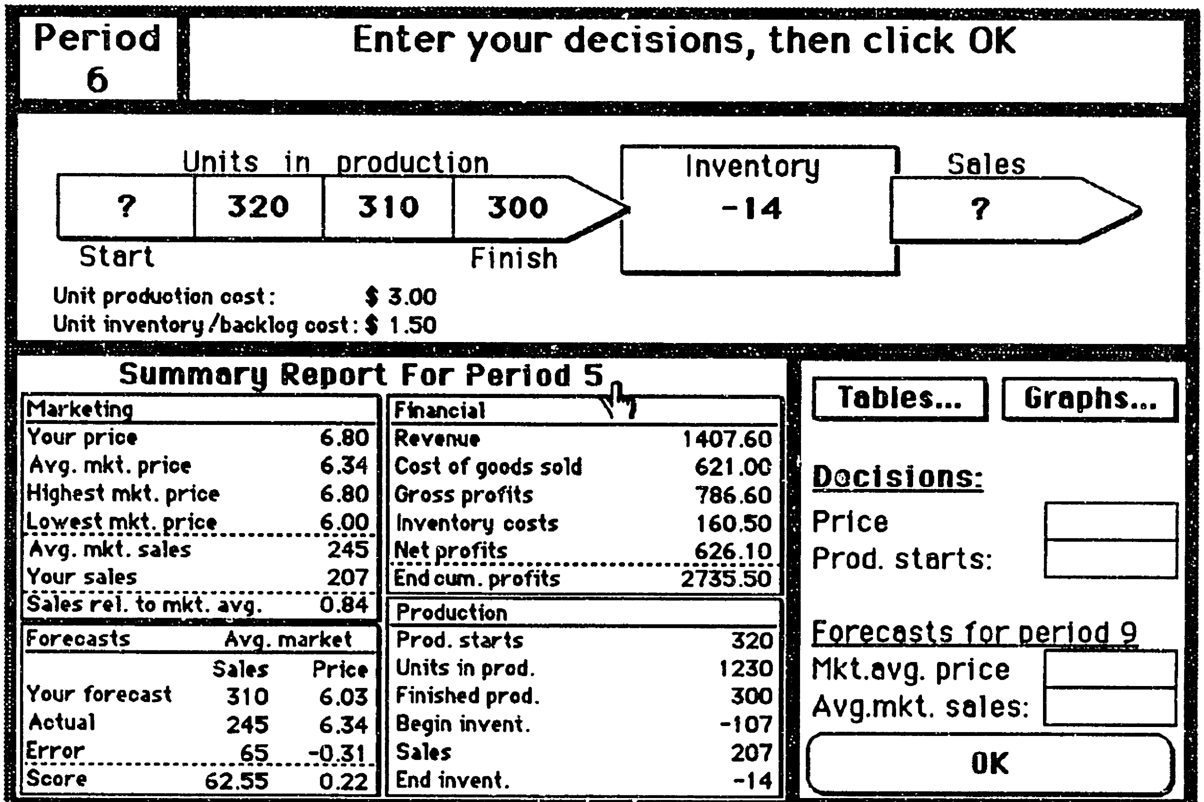

Figure 3.3.1: Example of screen display : (posted-price, complex condition)

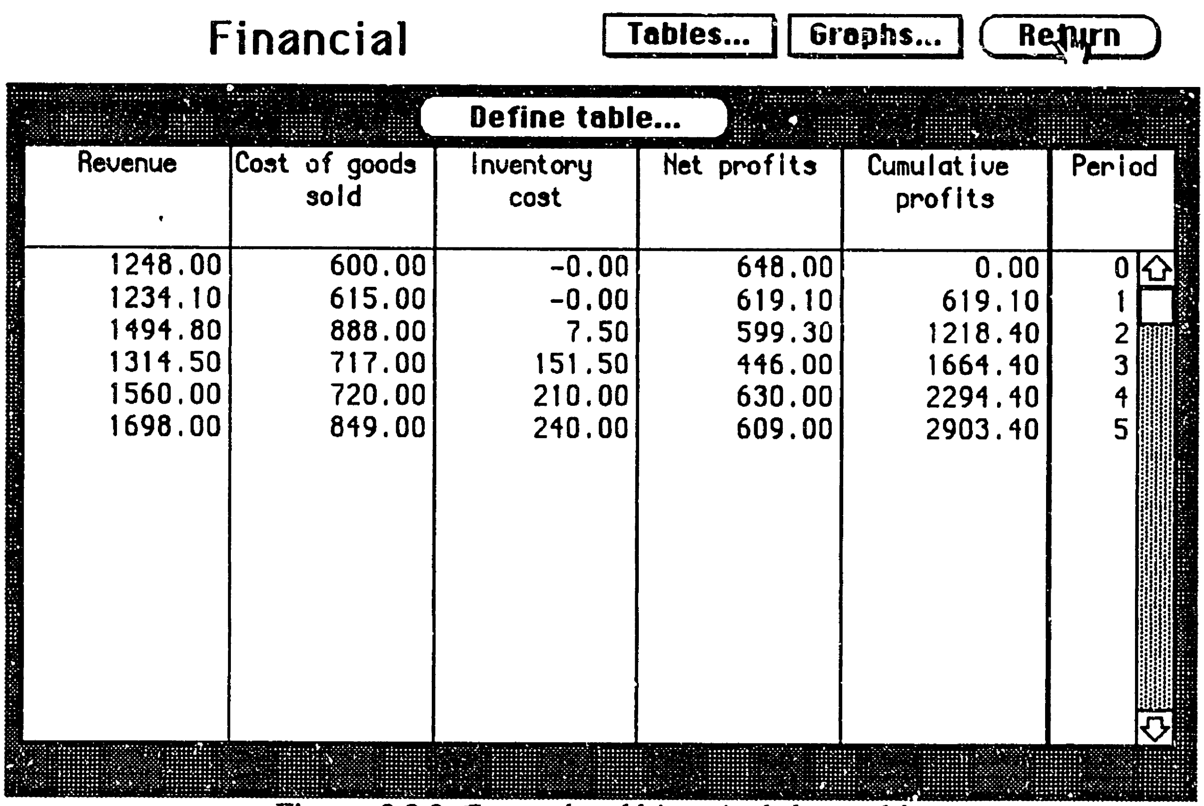

Figure 3.3.2: Example of historical data table

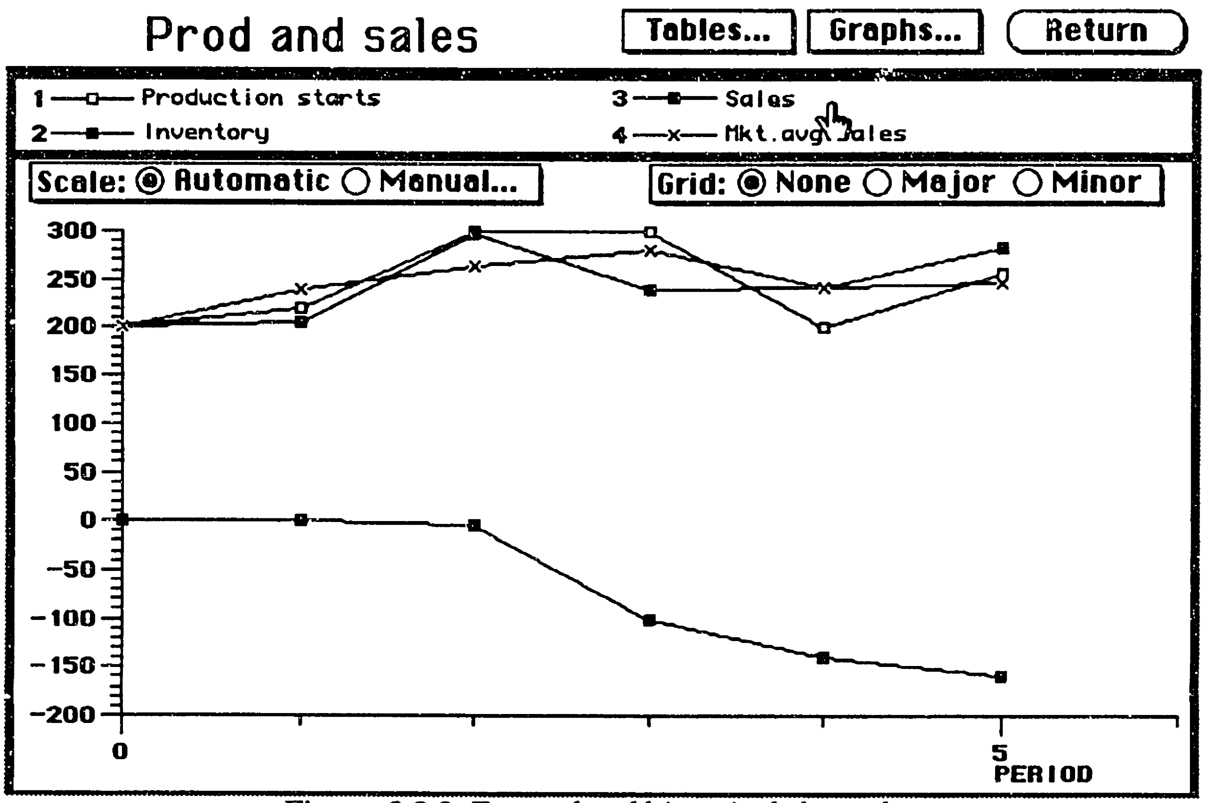

Figure 3.3.3: Example of historical data plot

Chapter 3 Figures

System Structure	Price regime		
	Fixed • Prices are constant • Demand met via inventory/backlog fluctuations	**Posted** • Sellers set prices • Demand met via inventory/backlog fluctuations	**Clearing** • Market-clearing prices found by computer • No inventories or backlogs needed
Simple • No production lags • No multiplier effect on demand	Condition 1	Condition 3	Condition 5
Complex • 3-period production lag • Multiplier effect on demand	Condition 2	Condition 4	Condition 6

Table 3.1.1: Experimental treatment design

Symbol	Text	Value
α	Marginal propensity to consume	0 or 0.5
δ	Production lag	0 or 3
γ/ω	Unit inventory cost, relative to unit production cost	0.5
ε	Price elasticity of individual-firm demand	2.5
μ	Price elasticity of industry demand around competitive equilibrium	0.75
π_0	Ratio of price to "reference" price at which aggregate demand is zero	4
χ_0	Ratio of demand to "reference" demand when aggregate price is zero	3
ω	Unit production cost	Arbitrary
G	Equilibrium output level	Arbitrary
σ	Standard deviation of random error in demand (selected cases only)	.10
ρ	Cross-firm correlation of random error in demand (selected cases only)	.50

Table 3.1.2: System parameter values.
The parameters for random errors apply only in those cases where such errors were introduced (not in the primary data set).

	Optimal rule	Behavioral rule $\tau_n = 1, \tau_x = 2$
$S\{u_{i,t}\}$	**Average profits**	
0	0.400	0.400
.1	0.376 (± 0.001)	0.376 (± 0.001)
$S\{u_{i,t}\}$	**Standard deviation of avg. production**	
0	0	0.000
.1	0.073 (± 0.008)	0.072 (± 0.014)

Table 3.2.1: Comparison of outcomes, simple fixed-price condition

Comparison of performance (profits) and standard deviation in average production resulting from the optimal rule (3.2.1-3.2.4) vs. the simple behavioral rule (3.2.5-3.26), respectively. $S\{u\} \equiv \sqrt{Var\{u\}}$. The "standard deviation in average production" is defined as the sample standard deviation of the average output per firm each time period, i.e.

std. dev. = $\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(Y_t - \bar{Y})^2}$; $Y_t = \frac{1}{K}\sum_{i=1}^{K} y_{i,t}$; $\bar{Y} = \frac{1}{T}\sum_{t=1}^{T} Y_t$. Measures are computed from period 11 through period 40. Standard deviations of measures is the sample standard deviation of the measure based on 20 simulations.

Variation: Prod-uction	Variation: Expec-ted sales	Optimal rule	Behavioral rule ($\tau_n = 2, \tau_x = 1, \theta = 0.5$) $\beta=0$	$\beta=.5$	$\beta=1$
$S\{u_{i,t}\}$	$S\{v_{i,t}\}$	**Average profits**			
0	0	0.400	-0.485	0.299	0.381
0	.1		-0.459 (± 0.034)	0.268 (± 0.025)	0.326 (± 0.007)
.1	0	0.377 (± 0.002)	-0.450 (± 0.034)	0.291 (± 0.009)	0.368 (± 0.002)
.1	.1		-0.459 (± 0.044)	0.263 (± 0.024)	0.324 (± 0.007)
$S\{u_{i,t}\}$	$S\{v_{i,t}\}$	**Standard deviation of average production**			
0	0	0	1.838	0.166	0.027
0	.1		1.588 (± 0.176)	0.204 (± 0.047)	0.141 (± 0.020)
.1	0	0.070 (± 0.010)	1.705 (± 0.071)	0.181 (± 0.017)	0.075 (± 0.008)
.1	.1		1.563 (± 0.166)	0.205 (± 0.044)	0.160 (± 0.023)

Table 3.2.2: Comparison of outcomes, complex fixed-price condition

The table compares performance (profits) and standard deviation in average production resulting from optimal behavior vs. three versions of a simple behavioral rule (3.2.12-3.2.14), respectively. In the optimal case, firms use the first three periods to estimate the structural parameters and then adopt the optimal, rational-expectations policy (3.2.7-3.2.11), based on these estimates (while continuing to update their parameter estimates, using data from the last 5 time periods, as documented in Appendix B). Standard deviations are given in parentheses underneath each entry. Measures are computed from period 11 through period 40. Standard deviations of measures is the sample standard deviation of the measure based on 20 simulations.

Variation Production	Optimal rule competitive	Optimal rule cooperative	Behavioral rule competitive $a_1 = 0.5$, $a_2 = 0$	Behavioral rule cooperative $a_1 = 0$, $a_2 = 0.25$
$S\{u_{i,t}\}$	Average profits			
0.01	0.401 (± 0.001)	0.844 (± 0.000)	0.400 (± 0.002)	0.844 (± 0.000)
.1	0.401 (± 0.009)	0.834 (± 0.002)	0.377 (± 0.014)	0.840 (± 0.002)
$S\{u_{i,t}\}$	Standard deviation of average production			
0.01	0.006 (± 0.001)	0.005 (± 0.001)	0.006 (± 0.001)	0.003 (± 0.001)
.1	0.050 (± 0.006)	0.048 (± 0.007)	0.061 (± 0.013)	0.031 (± 0.007)

Table 3.2.3: Comparison of outcomes, simple price-clearing condition

The table compares performance (profits) and standard deviation in average production resulting from the optimal non-cooperative rule (3.2.17), the optimal cooperative rule ($y = G_M$), and two versions of the simple behavioral rule (3.2.18-3.2.20), respectively. In the optimal cases, firms use the first three time periods to estimate the structural parameters in the system; henceforth, they adopt the optimal rational-expectations policy while continuing to update their parameter estimates, using the data from the last 10 time periods, (see Appendix B). Measures are computed from period 11 through period 40. Standard deviations of measures is the sample standard deviation of the measure based on 20 simulations.

Variation Prod-uction	Optimal rule competitive	Optimal rule cooperative	Behavioral rule competitive $a_1 = 0.5$, $a_2 = 0$	Behavioral rule cooperative $a_1 = 0$, $a_2 = 0.25$
$S\{u_{i,t}\}$	Average profits			
0.01	0.408 (± 0.001)	0.561 (± 0.001)	0.325 (± 0.007)	-0.997 (± 5.370)
0.1	0.407* (± 0.005)	0.553† (± 0.005)	0.321 (± 0.018)	0.221 (± 0.339)
$S\{u_{i,t}\}$	Standard deviation of average production			
0.01	0.017 (± 0.001)	0.041 (± 0.001)	0.161 (±0.012)	7.692 (± 29.995)
0.1	0.051* (± 0.006)	0.063† (± 0.010)	0.192 (± 0.036)	1.038 (± 2.126)

*) 3 out of the 20 simulations failed to converge properly. Since firms base their structural parameter estimates on a simplified (misspecified) aggregate demand equation, these estimates can at times be "thrown off" and lead to a crash of the simulation (see Appendix B).

†) 2 out of the 20 simulations failed to converge properly. Since firms base their structural parameter estimates on a simplified (misspecified) aggregate demand equation, these estimates can at times be "thrown off" and lead to a crash of the simulation (see Appendix B).

Table 3.2.4: Comparison of outcomes in the complex, clearing-price condition

The table compares performance (profits) and standard deviation in average production resulting from the optimal non-cooperative rule, the optimal cooperative rule, and two versions of the simple behavioral rule (3.2.21-3.2.23), respectively. In the optimal cases, firms use the first three time periods to estimate the structural parameters in the system; henceforth, they adopt the optimal rational-expectations policy while continuing to update their parameter estimates, using the data from the last 10 time periods, (see Appendix B). Measures are computed from period 11 through period 40. Standard deviations of measures is the sample standard deviation of the measure based on 20 simulations.

Variation: Production	Variation: Prices	Optimal rule: competitive	Optimal rule: cooperative	Behavioral rule (τ_n=1, τ_x=2, S{v}=0): competitive $a_1 = 0.5, a_2 = 0$	Behavioral rule (τ_n=1, τ_x=2, S{v}=0): cooperative $a_1 = 0, a_2 = 0.2$
Var{$u_{i,t}$}	Var{$m_{i,t}$}	**Average profits**			
0	0.01	0.394 (± 0.001)	0.841 (± 0.000)	0.128 (± 0.036)	0.746 (± 0.016)
0	0.1	0.341 (± 0.010)	0.810 (± 0.002)	0.118 (± 0.054)	0.720 (± 0.019)
0.1	0.01	0.375 (± 0.002)	0.819 (± 0.001)	0.139 (± 0.052)	0.741 (± 0.016)
0.1	0.1	0.337 (± 0.008)	0.800 (± 0.002)	0.116 (± 0.071)	0.739 (± 0.005)
Var{$u_{i,t}$}	Var{$m_{i,t}$}	**Standard deviation of average production**			
0	0.01	0.004 (± 0.000)	0.003 (± 0.001)	0.224 (± 0.010)	0.107 (± 0.009)
0	0.1	0.040 (± 0.005)	0.033 (± 0.003)	0.263 (± 0.026)	0.116 (± 0.011)
0.1	0.01	0.072 (± 0.011)	0.073 (± 0.010)	0.231 (± 0.020)	0.125 (± 0.013)
0.1	0.1	0.080 (± 0.011)	0.078 (± 0.015)	0.280 (± 0.038)	0.131 (± 0.012)

Table 3.2.5: Comparison of outcomes in the simple, posted-price condition
The table compares performance (profits) and standard deviation in average production resulting from the optimal non-cooperative rule, the optimal cooperative rule, and two versions of the simple behavioral rule (3.2.26-3.2.32), respectively. In the optimal cases, firms use the first three time periods to estimate the structural parameters in the system; henceforth, they adopt the optimal rational-expectations policy (3.2.24-3.2.25) while continuing to update their parameter estimates, using the data from the last 10 time periods, (see Appendix B). Measures are computed from period 11 through period 40. Standard deviations of measures is the sample standard deviation of the measure based on 20 simulations.

Chapter 3 Tables

Educational status				
Undergraduate	Master's	Ph.D.	Other	Total
43	45	7	2	97
Affiliation				
MIT Sloan	Other MIT	Harvard	Other	Total
43	20	29	5	97
Background in...	None	Elementary	Intermediate	Advanced
Economics	19	41	26	11
Stat./OR	28	29	30	10
System dynamics	72	21	4	

Table 3.3.1: Summary of subject backgrounds.

Financial		**Production and sales**		**Prices, etc.**	**Forecasts**
Revenue	Mkt. avg. revenue	Production starts	Mkt. avg. prod. starts	Price	Sales forecast
Cost of goods sold	Mkt. avg. cost of goods sold	Units in production	Mkt. avg. units in prod.	Market avg. Price	Sales forecast error
Gross profits	Mkt. avg. gross profits	Finished production	Mkt. avg. fin. prod.	Market highest Price	Sales forecast score
Inventory costs	Mkt. avg. invent. costs	Inventory	Mkt. avg. inventory	Market lowest Price	Sales forecast
Net profits	Mkt. avg. net profits	Sales	Mkt. avg. sales	Price/mkt. avg. price	Sales forecast error
Cumulative profits	Mkt. avg. cum. profits			Sales/mkt. avg. sales	Sales forecast score

Table 3.3.2: Historical data available to subjects during experiment.

Table number				
1	2	3	4	5
Variables				
Revenue	Production starts	Price	Sales	<empty>
Cost of goods sold	Units in production	Mkt. avg. Price	Mkt. avg. Sales	<empty>
Inventory costs	Finished production	Price forecast	Sales forecast	<empty>
Net profits	Inventory	Price forecast error	Sales forecast error	<empty>
Cumulative profits	Sales	Price forecast score	Sales forecast score	<empty>

Table 3.3.3.: Initial definition of historical data tables available to subjects

Graph no. 1	2	3	4	5	6
Horizontal axis					
Time	Time	Time	Price/mkt. avg. price	Mkt. avg. sales	Mkt.avg. units in prod.
Vertical axis					
Gross profits	Production starts	Price	Sales/mkt. avg. sales	Mkt. avg. price	Mkt. avg. sales
Inventory costs	Inventory	Mkt. avg. price			
Net profits	Sales	Mkt. lowest price			
Mkt. avg. net profits	Mkt. avg. sales	Mkt. highest price			

Table 3.3.4: Historical data plots available to subjects.

4. Market performance, dynamics, and rationality.

4.1 Introduction

Part of the purpose of this study is to explore whether "neoclassical theory" adequately describes or explains the observed market behavior across all the experimental conditions. First, however, one should point out that the work does not test of neoclassical theory in the widest sense. For instance, in calculating the "neoclassical" benchmarks, it was assumed that firms had fixed and consistent expectations about each other's willingness to collude and that prices and quantities were not used to convey information about such intentions. It is possible that, when one considers such signalling explicitly, the benchmarks are changed. Nonetheless, as a shorthand, the benchmarks will be referred to as "neoclassical."

If the markets conform well to neoclassical theory, there is little need to probe further into the behavior of individual subjects; established neoclassical theory will suffice. If, on the other hand, the market behavior diverges systematically and substantially from neoclassical predictions, then further investigation is necessary.

This chapter takes the first step in the analysis of the experimental results by comparing the aggregate market behavior with that predicted by neoclassical theory. As seen in Chapter 3, the theory makes predictions about both market performance and dynamics. In the next section, the average profits earned by firms in each of the six experimental conditions are compared to these prediction. In Section 4.3, the dynamic behavior of the markets is presented and analyzed, both with respect to the amount of variation in prices and quantities and with respect to the pattern of this variation.

It is useful to distinguish between "neoclassical theory" and "rational expectations." The neoclassical theory presumes that agents act optimally, given their expectations of future states of the world. Rational-expectations theories further add the assumption that agents correctly use all the available information in forming their expectations and that they therefore do not make systematic forecasting errors. In most empirical work, researchers are forced to test the joint hypothesis. Indeed, the comparisions in the first two sections of this chapter are in that tradition. However, it is also possible here to separate the two elements. One can test for rational expectations directly by evaluating the quality of the forecasts solicited from subjects during the experiment.[1] Likewise, one can to some extent test whether subjects' decisions are optimal, given their (stated) expectations, by fitting the observed decisions to the optimal decision rule. Section 4.4 evaluates the rationality of subjects' forecasts and looks for evidence of learning.

Although economists normally assume that agents possess perfect substantive rationality (Simon 1978), most would probably concede that humans are limited in their information-processing capacity. Given such limitations, decision makers are faced with a tradeoff between decision-making time or effort and the accuracy of the decision (Johnson and Payne 1985). A rational economic agent with limited information-processing capacity would make an appropriate tradeoff between the cost (effort, time)

1 An interesting variation on the experiment would involve giving subjects the "correct" forecasts each period, based on the rational-expectations theory.

and the benefit (accuracy) of decision making. In the present experiment, the complexity of the decision task and the relative importance of making accurate decisions differs across the experimental conditions, and one can use these objective properties of the task to generate a set of hypotheses about how decision-making effort should vary across the conditions. In Section 4.5, the average deliberation time of subjects in the experiment is used as a proxy measure for decision effort and compared to these hypotheses.

Most of this chapter is devoted to the testing of rationality, but the experimental design was motivated by an alternative set of behavioral hypotheses about the effects of complexity and price institution on market behavior. The final section concludes by reviewing the extent to which these hypotheses are supported or rejected by the data. In the following three chapters, the behavior of individual firms and of the market as a whole will be analyzed in more detail. Therefore, most attempts at explaining the observed behavior will be relegated to these chapters.

4.2 Profits

One compact measure of market behavior is the average profit earned by the participant firms. This measure is also the most relevant measure of subject performance, since rewards were explicitly tied to the profits accumulated in the game.

The analysis in the previous chapter showed that, if agents are rational in the sense that they have both accurate expectations and act consistently with those expectations, performance across the six experimental conditions should vary very little, except for the effects of collusion. Thus, the simulations in Chapter 3 and Appendix B show that agents who either know the parameters of the system or correctly estimate them from past data as the game progresses will realize profits very close to the theoretical maximum, given the degree of collusion, even with fairly substantial random variation in output and price decisions.

These results are reproduced in Figures 4.2.1 and 4.2.2, which show the expected simulated profits (excluding the first 10 periods) of rational competitive and rational collusive agents, respectively. The first 10 periods have been excluded to minimize variations caused by initial learning and experimentation, and profits are broken into two components: "Gross profits" (profits before inventory costs), and net profits (after inventory costs).

The "gross" profits are primarily related to the price-output operating point of the market, i.e. a measure of the degree of successful collusion, whereas inventory costs are a function both of firm's production policy and the overall variation in prices and output, i.e., a measure of the degree of successful control. The relative importance of these two components is inherently different in the three pairs of price regimes. In the fixed-price regimes, there is no possibility of collusion and inventory costs are therefore the primary determinant of performance. Conversely, in the clearing-price regimes, inventory costs are eliminated and performance depends instead on reaching the best price-output point. The posted-price regimes involve elements of both.

It is evident from the figure that competitive rational agents should realize virtually the same gross profits in all six conditions. Fully cooperating agents, on the other hand, would realize somewhat lower profits in the complex conditions than in the simple ones. Profits are lowered because the multiplier effect on demand makes the steady-state aggregate demand curve flatter (more elastic) in the complex condition (cf. Appendix A). Thus, to measure differences in the degree of collusion, one must correct for this effect by normalizing the profit measure.

From a neoclassical perspective, there is no a priori reason to believe that agents would collude to different extents across the six experimental conditions, except for the obvious fact that collusion is not possible in the fixed-price regime.[2] Thus, if one does find such systematic differences in the degree of collusion, one must rely on other theories to explain it.

In contrast, the behavioral hypotheses reviewed in the previous chapter predict large deviations across the conditions. First, the complex markets are likely to show lower profits than the simple ones, mainly because of higher inventory costs but also possibly because of lower degrees of collusion. Second, the drop in profits is likely to be large in the fixed-price condition and small in the clearing price condition. The posted-price condition is more ambiguous: Profits could be relatively unaffected by the introduction of complexity, or they could drop substantially. Given the wide range of possible behaviors, one would expect the experimental outcomes to show fairly large variance in the the posted-price complex condition.

Figure 4.2.3 shows the actual average profits earned in each of the six conditions (excluding the first 10 periods). It appears that, unlike the

2 One qualification is in order: It is possible that the experimental conditions could be different from a game-theoretic perspective. However, such an analysis is considered beyond the scope of this thesis. In any cases, the differences are likely to be subtle and quite marginal compared to the strong observed experimental treatment effects.

simulations in Figures 4.2.1 and 4.2.2, there are substantial differences in the actual profits across the six conditions in Figure 4.2.3. This visual impression is confirmed in the analysis of variance of (normalized) gross profits in Table 4.2.1 and (the logarithm of) inventory costs in Table 4.2.2.[3]

In Table 4.2.1, gross profits have been normalized by the collusive and competitive profits to an index which is 0 at the competitive profit level and 1 at the collusive profit level. In a sense, the index measures the average degree of collusion in the market. The table shows a two-way analysis of variance of this index using price regime (P) and complexity (C) as factors. While the price regime has no significant effect, the effect of complexity is significant (p=3.1%); on average, gross profits in the complex conditions are 10-15% lower than in the corresponding simple conditions, sometimes even falling below the competitive equilibrium level.[4]

Likewise, the hypothesis of constant inventory costs in the non-clearing conditions in Table 4.2.2 is rejected, this time strongly ($p<0.1\%$).[5] Much of the significance comes from the very large effect of complexity on inventory costs--on average, observed inventory costs are about 13 times larger in the complex conditions! But there is also a strong interaction effect,

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3 The analysis treats each market as one observation. One would expect the correlation between firm profits in one market to be very high, since all firms are related through the aggregate variables of the market. Indeed, firm profits are significantly correlated within markets in all cases except the fixed-price simple condition. Thus, one cannot treat each firm as an independent observation of the condition.

4 A similar analysis to the ANOVA in Table 4.2.1 was performed on the logarithms of the unnormalized gross profits. The results are very similar: there is a significant effect of complexity (4.8%) and no significant price-regime effect.

5 Inventory costs were transformed with logarithms in order to minimize differences in within-cell variance. The logged measures passed the Bartlett test for homogeneity of group variances at the 10% level.

indicating that the effect of complexity on inventory costs is much lower in the posted-price than in the fixed-price regime.

Thus, while the data do not support the hypothesis of little or no treatment effects on performance, they do confirm many of the predictions of the behavioral hypotheses: Profits, relative to optimal, are lowered by the introduction of complexity in all three price regimes, sometimes dramatically. Most of the lower profits stem from higher inventory costs (except of course in the price-clearing conditions), but profits before inventory costs are lower as well. As a result, the effect of complexity on net profit is very large in the fixed-price and posted-price regimes, and smaller in the clearing-price regime. Finally, it is evident from Figure 4.2.3 that there is much greater variance in profits in the complex posted-price and fixed-price cases than in the other four conditions.[6] This latter result accords with the simulation results in the previous chapter and in Appendix B which showed these two conditions to be the most sensitive to systematic and unsystematic errors.

4.3 Market dynamics and convergence

Although average profits are a compact and convenient measure of market outcomes, it says relatively little about the dynamic process of market adjustment. This section investigates the dynamic behavior of the markets. As in the previous section, much of the analysis is a comparison of observed behavior to the hypothesis of stochastic, rational-expectations equilibrium (or rather least-squares learning convergence to such an equilibrium).

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6 The Bartlett test for homogeneity of group variances on net profits shows significant differences ($p<0.1\%$).

Chapter 4

The simulation analysis in Chapter 3 and Appendix B showed that if firms act rationally in the sense that they correctly estimate the parameters of the system as historical data accumulate, and if they act to maximize their profits given their expectations of future price, and, finally, if their expectations are consistent, then markets should converge quite rapidly in all the six experimental conditions. This is illustrated in Figures 4.3.1 and 4.3.2, which show examples of simulations of the convergence to competitive equilibrium under optimal learning, assuming that firms' decision are subject to uncorrelated random errors with moderate variance. For both prices and quantities, the adjustment is fairly rapid in all six conditions--an apparent stationary state is established after about 10 time periods. The variation in market averages differs across conditions, but in all cases it is lower than the variance of the random error (see Appendices A and B).

The series of figures 4.3.3 through 4.3.8 show the actual behavior of production and prices in each market for each of the six experimental conditions. A quick glance at these figures reveals that there are significant differences in the pattern of behavior across the conditions. Roughly speaking, the complex markets generally show much larger and longer term variation in prices and quantities, and less tendency to converge to or remain in equilibrium, than do the corresponding simple markets. In several of the complex markets with inventory accumulation, there appears to be persistent cyclical movements.

In the simple condition with fixed prices (Figure 4.3.3), production settles fairly quickly in the expected range. Apart from a few occasional departures from the equilibrium level, production is constant and equal to its steady-state value. The task facing the decision maker in this condition is a simple inventory control problem, with a constant exogenous outflow. Previous experiments have shown that, under such simple circumstances, humans perform quite well (Diehl 1982; MacKinnon and Wearing 1985). Much of the good performance is due to the inherent simplicity of the task. As shown in the previous chapter, almost any decision rule which takes into account both expected future outflow and the current value of the stock will achieve rapid and stable convergence.

By comparison, the variation in production is dramatically larger in the fixed-price complex condition (Figure 4.3.4). All markets show substantial cycles of "boom and bust." Initially, the increase in demand caused by the initial lowering of prices to the competitive-equilibrium level leads to a depletion of inventories before additional output has gone through the supply line. In the face of rising demand and falling inventories, firms raise their production. This increase in production leads to yet a further increase in demand, which in turn causes firms to raise production further. Because of the production delay and the continuous accumulation of imbalances in the inventories, firms have great difficulty catching up with demand. The upward spiral continues until the increase in production leads to accumulation of inventories, at which point all firms cut their production, leading to a decrease in demand. The cycle is now turned around, and the result is a subsequent "recession", where production falls below its equilibrium level. In some markets, this cycle is exceedingly large; in Market 25, output peaks at around four times the equilibrium value. And none of the markets show any sign of being in equilibrium at the end of the game. In contrast, the "optimal" agents in Figure 4.3.1 realize the strength of the multiplier effect and make an immediate initial correction in production, taking into account the fact that this correction will not take place until after a delay. Henceforth, firms produce the equilibrium output, and the steady state is achieved three time periods later.

The markets with clearing prices also show marked differences between the simple and the complex condition, although the effects are different than in the fixed-price condition. The markets in the simple clearing-price condition show no systematic pattern of behavior (Figure 4.3.5). Some appear to settle in a range close to, or slightly above, the competitive price equilibrium, but with a fair amount of short-term fluctuation. Others show some longer-term fluctuation.

In contrast, the complex clearing-price markets all display a distinct "boom and bust" pattern of initial dramatic overshoot in production, followed by a gradual downward adjustment in output, closely resembling the behavioral simulation example shown in Figure 3.2.9 of Chapter 3. The behavior of Mkt 19 in Figure 4.3.6 is typical and can serve as an example: At the outset of the game, prices are substantially above production costs, and most firms raise their output. However, due to the production lag, the increase in output does not show up on the market right away. Instead, the increase in production increases demand through the multiplier effect, causing prices to rise. The increase in prices apparently induces firms to increase output further, raising prices even higher. Thus, for the first three time periods, prices climb above twice the competitive-equilibrium level. When the increased output arrives in period four, prices drop sharply and firms start lowering their production. During the next three time periods, prices continue to drop, both because the finished output is still increasing and because the lower new production starts depressing prices through the multiplier effect. Finally, around period 9, prices reach a low point of about half the competitive equilibrium value. From then on, prices climb slowly and steadily toward and beyond the competitive equilibrium level as firms very gradually lower their output.

Although the clearing-price complex condition shows a substantial initial boom and bust, the cycle is not sustained as it is in the corresponding fixed-price condition. A key structural difference between the fixed and clearing price regimes is the lack of cumulative effects of market imbalances in the latter. Thus, the market-clearing system effectively "forgets" past imbalances after they have gone through the pipeline delay, making it more forgiving of errors.

Finally, the posted-price markets also show some effects of complexity on behavior, although the effect seems much less uniform than in the other two price regimes. In all the markets of the simple condition (Figure 4.3.7), the behavior of prices is relatively calm, and in three out of the four markets, prices seem to be driven down toward the competitive level. In the fourth market ("Mkt 32"), firms change their prices very little throughout most of the game. In all of the posted-price simple markets, prices are closely correlated, both in time and across firms within a given period. In contrast, output shows much more short-term fluctuation, again with the exception of Market 32. In all markets, inventories are kept closely in check and never depart substantially from the desired level.

The posted-price complex condition generally shows larger variance in prices and production compared to the simple case, although the variance differs from market to market (Figure 4.3.8). One market ("Mkt 16"), the oscillations in prices and output are very dramatic; indeed, the fluctuations appear to grow larger with time. In two other markets, (Mkt 18 and Mkt 38), both prices and inventories and, in Mkt 18 also production, show a moderate but quite regular cycle. (In Mkt 38, production shows very little variation; most of the fluctuation is in prices and inventories.) The last market (Mkt 17) shows relatively little variance in output or prices, except for a one-time peak in prices. In all, the responses in the four markets are quite varied, though it is also clear that the majority of the markets have large cyclical variance in the variables.

The effects of the experimental treatments can be seen more compactly in Figure 4.3.9, which compares the observed variation in output and prices, respectively, to the variation one would expect in rational-expectations equilibrium. The difficulty involved in such comparisons is of course that the variability of the rational-expectations depends directly on the variance of the random noise in the decisions. However, it seems reasonable to assume that the random errors have approximately the same variance in each of the six rational-expectations cases--at least there is no reason to expect it would be otherwise. Under this assumption, the benchmarks may provide a sense for whether the relative observed variance across conditions is in line with theory.

The first thing to note in Figure 4.3.9 is that the observed variance is about 5 to 10 times larger than the benchmarks, except for the fixed-price, simple condition where it is about 10 times smaller! Thus, it is very unlikely that differences in variance across the six conditions can be explained simply by different amounts of noise. (The magnitude of the variation is in itself a sign of departures from rational expectations equilibrium: When the random errors get large enough it becomes difficult to call the behavior rational.) Second, the variance in output is generally larger in the complex conditions than in the corresponding simple ones. This is particularly dramatic in the fixed-price condition. (Using the logarithm of the variance as the dependent measure, a two-way analysis of variance using price regime and complexity as factors yields highly significant main effects and interactions ($p<0.1\%$). However, much of the result is due to the dramatic contrast in the fixed-price regime. Conducting the same analysis on the price-varying regimes only yields no significant effects.)

The analysis of the overall variability of the markets provides a rough measure of the degree of stability of these markets, but if fails to capture the differences in the pattern of behavior over time. As has been pointed out in the discussion above, the patterns of behavior of the markets do not all resemble the rational-expectations simulations in Figures 4.3.1 and 4.3.2. In order to capture the time behavior of the variation, the data were subjected to a spectral analysis and compared with rational-expectations stochastic equilibria.[7] Figures 4.3.10 and 4.3.11 show the observed output and price spectra, respectively compared with the benchmark spectra one would expect to see in rational-expectations equilibrium.[8] Note that the figures show the

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7 Spectral analysis breaks the variance of a time series into components at different frequencies. A rapidly varying, short-term fluctuation will have much of its variance concentrated at high frequencies while long-term movements will appear as high variance at low frequencies. A completely random (white noise) process will have equal variance at all frequencies.

8 The benchmark spectra are derived in Appendix A. The derivations assumed that that linearization around the operating point was valid. Spectra based on simulated rational-expectations markets showed good conformance with the derived ones.

spectral density for each market, i.e. the spectrum normalized by the variance of that market.

The rational spectra were derived on the assumption that all random errors were uncorrelated white noise. The spectrum of such noise is a horizontal line. However, the structure of the system modifies this spectrum in some cases, mostly shifting the variance toward the high-frequency region. The intuition for this shift lies in the process of inventory correction: If, for instance, a firm has produced too much output in one period, the firm's inventory (or, if there is a delay, its pipeline) will be too high. In the subsequent period, the firm therefore corrects for this by lowering its output. A similar process occurs when firms use prices to control inventories (as in the posted complex condition). Therefore, positive departures from equilibrium will generally be followed by negative departures and vice versa, yielding a negatively correlated, high-frequency variation.

The fixed-price simple condition does not show substantial departures from the optimal spectrum. (The measures in this condition are somewhat unreliable because the variance in output is minimal.) In the fixed-price complex condition, the optimal spectrum is identical to the simple one, but the observed variance is almost entirely concentrated in the low end of the spectrum. Thus, three out of four markets have a sharp peak in the spectrum around a frequency of .05 to .1, corresponding to cycles with a period of 10 to 20 rounds. The exception to this pattern is Mkt 25, in which the variance is more diffuse. A glance back at Figure 4.3.4 reveals that this market exhibits an overshoot and collapse and does not recover before the end of the game.

In the clearing-price conditions, the production behavior of firms in rational-expectations equilibrium is a white noise process, since the optimal policy is to produce a constant output. The spectrum for output should therefore be flat. The spectrum for prices should either also be flat (in the simple condition) or somewhat shifted toward the high end (in the complex condition).[9]

The observed behavior in the clearing simple condition is generally not significantly different from the optimal. Although two markets do show some tendency for low-frequency behavior, the deviation is not dramatic. In the complex clearing case, however, three out of four markets show quite strong concentrations in the low frequency range, both for prices and for output. A glance back at Figure 4.3.6 shows that the markets are not stationary after the first 10 periods: Most of them are experiencing a persistent slow drift in prices and output.

In the posted-price simple condition, the optimal spectrum of production is shifted toward the high-frequency end of the spectrum. This is so because the rational-expectations policy involves charging a constant price (plus a random error) and correcting inventory imbalances by adjusting production, hence leading to negative serial correlation as explained above. The price spectrum should, in equilibrium, be flat. The observed production spectra are not dramatically different, although none of them show any tendency of concentration in the high frequencies. The spectrum of one market (Mkt 23) is concentrated more in the low frequencies, but since this

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9 The moderate negative correlation in prices in the complex clearing condition has no intuitive interpretation. it is the result of the influence of the multiplier effect on prices.

market shows very low variation in average output (cf. Figure 4.3.7), the interpretation of the spectrum is unreliable. The price spectra show more tendency toward the low frequencies. Much of this tendency is due to slow drifts in prices--there seems to be no tendency to cyclical behavior.

In the posted-price complex condition, the rational policy is to mimic the clearing-price condition, i.e. to produce a constant output (except for the random error) and to correct current imbalances in inventories and finished production by adjusting prices. Hence, the spectrum for output should be flat while the spectrum for prices should be slightly shifted toward the high frequency end (cfr. the discussion above). In contrast, the observed spectra in both prices and, to some extent, output are shifted toward the low end of the spectrum. Three out of the four markets show a peak in the price spectrum corresponding to a cycle of period 10 to 20. The last market (Mkt 16) shows additional concentration at still lower frequencies. Indeed, a glance back at Figure 4.3.8 confirms that two of the markets (Mkt 18 and Mkt 38) show a distinct cycle in prices while one market (Mkt 16) shows something akin to an exploding oscillation. The last market (Mkt 17) shows no distinct pattern.

In summary, in all of the complex conditions and most of the simple ones, the variance is too big and concentrated too much in the low frequencies to be interpreted as unsystematic noise; the results are an indication that, as a rule, the markets are in continuous disequilibrium, exhibiting persistent cycles, drift, or more irregular variation. Moreover, it is clear that the differences in market behavior across the experimental conditions are much larger than would be expected in rational-expectations stationary states.

4.4 Forecasting performance

As part of the experimental protocol, subjects were asked to provide a forecast of aggregate market demand and (except in the fixed-price conditions) the aggregate market price. In the simple conditions, the forecast was for the current time period; in the complex conditions, the forecast was for three time periods later. Subjects were given an incentive for accurate forecasting through a small bonus, based on the rank of the "root mean squared error" of their forecast (see Chapter 3).

Assuming that the solicited forecasts directly measure agents' true expectations, one can test rational-expectations hypothesis directly by comparing forecasts to actual outcomes.[10] The hypothesis states that agents' expectations of a certain variable are based on a model that equals the true model of the system, i.e., that they use that model to form their expectations, given all the relevant information that is avialable to them. Thus, the actual outcome should differ from the forecast only by a random error which is uncorrelated with any of the information available to the agent at the time the expectation was held (Muth 1961).

The classic test of that hypothesis amounts to regressing the actual value on the forecast (or a stock price or some other variable that should equal the expected actual value), i.e.

(4.4.1) Actual = a + b*Forecast + error,

[10] It should be noted that the use of such "survey data" is still considered controversial among some economists, particularly adherents to the rational-expectations paradigm (e.g. Prescott (1973)) who claim that survey data is either redundant or subject to various idiosyncratic limitations of the data collection.

and then testing the joint hypothesis a=0, b=1. It may seem more natural to regress the forecast on the actual outcome, but this procedure will systematically underestimate the coeffient b because the "explanatory" variable is correlated with the error term. The assumption underlying the classic test is that only the actual outcome, not the forecast, is subject to random variation.[11] This equation is a relatively weak test of rationality: It tests only the absence of any simple biases (in either slope or intercept), yet many forecasts that would pass this test are nonetheless inefficient because they do not take all information fully into account. In particular, a forecast with a large errors of random or non-random nature might very well reduce the significance level of the test, since this variation is interpreted as variance in the outcome, not the forecast. Therefore, if forecasts fail this test, it is quite a strong rejection of the rational-expectations hypothesis.

An alternative test is the variance-bounds analysis. This test derives from the fact that if actual outcomes differ from forecasts only by a random uncorrelated error, the variance of the forecasts must be less than the variance of the outcome (Shiller 1981). Other tests of rational expectations involve regressing the forecast error on various variables in the information set available to the forecaster, most notably the past forecast errors and past values of the actual outcomes (e.g. Lovell 1986). Under the hypothesis, all coefficients in such a regression should be zero.

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11 If both the forecast and the outcome include random errors, neither method will lead to consistent estimates. In this case, one might nonetheless get a consistent estimate of the constant term,a by regressing Actual - Mean[Actual] = a + b*(Forecast - Mean[Actual]). Moreover, the asymptotic covariance matrix of the estimates equals that estimated in the standard procedure, so the standard t-test on the coefficient a is valid.

As it turns out, the hypothesis of rational expectations in the experiment is decisively rejected in the classic test, thus making further tests unnecessary. Tables 4.4.1 and 4.4.2 show a summary of the results of the classic test, in both linear and logarithmic form, for demand and price forecasts, respectively. In the vast majority of cases, the hypothesis is rejected at the 1% level or lower. It should be noted, however, that the Durbin-Watson statistic indicates substantial autocorrelation in the residuals, thus inflating the significance of the test (Pindyck and Rubinfeld 1981). However, the presence of serial correlation in the errors is itself evidence of inefficient forecasting, in violation of the rational-expectations hypothesis.

One might ask whether subjects over time learn to become better forecasters. This is an important question because the rational-expectations hypothesis implicitly assumes that agents have gone through a learning process (Lucas 1987). As discussed in Chapter 2, the measurement of learning is clouded by the fact that the observed market behavior is a combination of dynamics arising from an unchanging set of decision rules, and changes in the decision rules themselves. Thus, an unchanging set of rules may lead to a stable adjustment toward equilibrium which might look like a learning process where there is none. Likewise, it is a lot easier to make accurate predictions when the system changes very little over time. In the classic test, the problem shows up as multicollinearity, reducing the significance of the test. Thus, fewer failures to pass the test do not necessarily imply learning whereas continued failures is evidence of the lack of learning.

Figures 4.4.1 and 4.4.2 show the result of breaking the sample into the first and second part of the game and applying the rationality test for either half. At first glance, it would appear that some learning is indeed taking place, but the degree of learning differs across the experimental conditions. In the simple conditions, the fraction of subjects who do not fail the classic test increases in the second half of the game for both price and demand forecasts. Yet a large fraction (typically about half) still fail the test at the 5% level during the second half of the game. Moreover, as just noted, the constructed test can only reject the hypothesis of learning, not confirm it. Thus, while it is possible that some moderate learning is taking place in the simple conditions, agents still do not reach the rational-expectations ideal.

In the complex conditions, there is evidence that little or no learning is taking place: Generally, the fraction of subjects passing the classic test is about the same or lower in the second half of the game. And that fraction is very small or zero in all cases. The only exception to this pattern is the price forecasts in the clearing complex condition, where the fraction passing the test goes from zero to 25%. However, as seen in section 4.3, the variation in prices is considerably lower in the second half of the game, as the market slowly recovers from its initial sharp overshoot. The lower variation makes the forecasting task easier and reduces the statistical power of the classic test.

It is fair to conclude that the rational-expectations hypothesis is quite firmly rejected in the forecasting data and that the learning that does take place in expectation formation is modest and concentrated in the simple experimental conditions; the complex conditions show no convincing evidence of learning.

4.5 Time spent on decisions

Given the limitations of human information processing, particularly under time pressure, humans cannot always act optimally in sufficiently difficult situations. But it is possible that they consciously or unconsciously recognize the tradeoff between mental effort and accuracy and thus adjust their mental effort appropriately to fit the properties of the task at hand (Johnson and Payne 1985). Two important determinants of this tradeoff is the inherent complexity of the task, i.e. the amount of information processing needed to improve decisions, and the "forgiveness" or "leniency" of the task, i.e. how much room there is for departures from the "best" choice before performance suffers noticeably.

In this experiment, the time elapsed between the beginning of a time period and the execution of a new set of decisions was recorded by computers. These data may be a coarse indicator of mental effort involved in each decision (Johnson, et al. 1991). By comparing observed deliberation times to those one could expect from the objective properties of the decision task, one can get an impression about whether subjects appropriately traded off effort for accuracy.

The experimental protocol did not involve overt time pressure. Subjects were free to take as much time as they wanted to make their decisions. However, they were also told that their rewards depended proportionally on the amount of profits accumulated in the game and that they had a fixed time period in which to play as many rounds as they could. Each group of subjects in a market thus faced a tradeoff between making better versus making more decisions, although the incentive for any individual to make faster decisions was diluted by the fact that subjects had to wait for the last person to finish before advancing to the next round.

Since both the information processing requirements and the leniency of the task differ across the six experimental conditions, one would expect deliberation times to vary as well. First, the mental effort required varies because the number of decisions was not the same for each of the six experimental conditions. One would therefore expect the deliberation time to be higher in those conditions that have more decisions and forecasts, all other things equal. Under fixed prices, subjects had to make one production decision and one forecast of future sales. Under clearing prices, subjects made these same two choices plus a forecast of future price. Finally, under posted prices, subjects decided on both their price and their production and gave a forecast of both future sales and price.

Second, the "arithmetic" of inventory control may be simpler than the task of finding the optimal price-output pair. The inventory control task involves three steps: Form an expectation of future demand and anchor production on this figure. Then, look at current inventory and adjust production to account for a current excess or insufficient inventory. Finally, (in the complex condition only,) look at the pipeline of unfinished production and adjust production if there is "too much" or "too little" in the pipeline. All these steps involve addition or at least simple linear operations. In contrast, the task of finding the optimal price-output position involves calculating marginal profits as a function of price, relative to market average price. None of these figures are readily available in the information display but must be calculated from tables or graphs.[12] The posted-price condition is

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12 Of course, one might imagine that subjects use a search procedure rather than trying to calculate marginal profits. However, such a search is difficult because profits depend not

the most difficult because it involves both inventory control and price-output optimization.

Third, both the clearing-price and the posted-price regimes allow for possible collusion among firms. Here, firms must also consider whether to cooperate or defect and how to get other firms to cooperate. All of these task characteristics speak for the following ranking of deliberation times in both the simple and complex conditions: Fixed < clearing < posted.

On the other hand, the leniency of the tasks does not follow the same ordering. In the clearing-price regime, profits are not very sensitive to deviations in the output decisions around the optimal price-output point (cfr. Figure 3.2.7 in Chapter 3). In the two other regimes, the cost of excess or insufficient inventory makes it more important to set production at the right level. Thus, even though the clearing-price regime involves one more forecast to make than the fixed-price regime, the overall deliberation time may be smaller, all other things equal, since the output decision is less crucial for performance. This would, all other things equal, call for the following ranking of deliberation times: Clearing < fixed < posted.

Finally, one would expect there to be a strong effect of complexity on decision times. In all three price regimes, the introduction of complexity makes the task more difficult, though the added difficulty varies in the three price regimes. One would therefore expect there to be some interaction effects on deliberation time between price regime and complexity.

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only on the decision parameter (price or output) but on other variables as well (other prices, the multiplier effect, etc.).

In the fixed-price regimes, the difference between the simple and complex condition is probably the largest. The simple fixed-price condition amounts to only a trivial inventory control task in the face of constant demand. In the corresponding complex condition, the task is complicated by the time lag and the supply line correction, and by the possible large variations in demand caused by the multiplier effect.

In the clearing-price regime, the introduction of complexity is somewhat less important, since inventory accumulations are absent in this condition. Thus, by the time the started production is ready to sell, the system will have "forgotten" all its history. In contrast, inventory accumulations in the other two price regimes perpetuate past errors in the form of inventory imbalances. The correct procedure, not including the search for the target price level, in the clearing-price regime involves 4 steps: 1) Form expectations of the future average output level; 2) Form expectations of the resulting future average price level; 3) Anchor output decision on the expected average output; 4) Adjust output up or down, depending on whether the expected average price is above or below the target level. The main complicating effect of complexity is that the multiplier effect on demand influences the price level in the complex condition whereas the average price in the simple condition only depends on average output. However, since profits are not particularly sensitive to deviations from the optimal output choice, the agent can get by quite well as long as the forecasted average price and output are not too far off. Thus, one would expect the clearing-price regime to show a smaller effect of complexity on deliberation time than the two other regimes.

The posted-price regime involves both the inventory-control element of the fixed-price regime and the price-output search and strategic considerations of the clearing-price regimes. Since each of these elements is compounded by the complexity treatment, one might expect the effect of complexity on deliberation time to be largest in the posted-price regime. On the other hand the treatment effects are unlikely to be additive in this fashion. The simple posted-price condition is "at least an order of magnitude" more complicated than the simple fixed-price condition since it involves both strategic behavior aspects, a variable demand, and a search for the best output-price position. Thus, whether the effect of complexity is larger in the fixed-price or the posted-price regime is ambiguous.

To summarize, the expected differences in deliberation times are based on the assumption that subjects on average will use more mental effort (measured by deliberation time) in the more difficult tasks (from an information-processing point of view), but that the effect will also depend on the leniency of the system, i.e. how important it is to be close to optimal. The considerations above leads to the following hypotheses about average deliberation times in the six experimental conditions:

H1	Fixed simple	<	Clearing simple	<	Posted simple,
H2.1	Fixed complex	<	Clearing complex	<	Posted complex,
H2.2	Clearing complex	<	Fixed complex	<	Posted complex,
H3	0	<	Clearing complex - Clearing simple	<	Fixed complex - Fixed simple,
H4	0	<	Clearing complex - Clearing simple	<	Posted complex - Posted simple.

The hypotheses H2.1 and H2.2 are alternatives, depending on whether the task complexity dominates (H2.1) or the leniency effect dominates (H2.2). (The fixed-price simple condition is so trivial that the leniency effect is unlikely to dominate the task complexity effect in the simple conditions. Hence, H2.2 only applies to the complex conditions.)

Figure 4.5.1 shows the average time taken to deliberate decisions during the experiment for each subject, market, and experimental condition. The first 10 rounds have been excluded to remove the variance caused by subjects' learning the mechanics of the game, setting up tables and graphs, etc.[13]

A glance at Figure 4.5.1 reveals that there do appear to be significant differences in deliberation times across the six experimental conditions. The expected ranking based on task complexity does indeed occur for the simple condition (H1). In the complex condition, the clearing-price measure is lower than the corresponding fixed-price measure, concurring with the ranking where the leniency effect (H2.2) dominates the task complexity effect (H2.1).

However, the effect of introducing complexity shows some surprising violations of the hypotheses: While the effect is strong in the fixed-price condition, it absent in the clearing-price condition and appears to be negative in the posted-price condition.

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13 The data analysis was conducted for a much wider variety of dependent measures, excluding or including the first 10 rounds, using either averages or medians, and taking logarithms to assure more uniform within-cell variances. The substantive results remained virtually the same in all measures.

These results are expressed more formally in the two-way analysis of variance in Table 4.5.1.[14] Both the main effects (price regime and complexity) and their interaction are highly significant. Moreover, although much of this significance might come from the fact that the simple fixed-price condition is so much lower than the others, Test 1 in Table 4.5.1 of the hypothesis that the other five cell means are equal is still thoroughly rejected. The last part of the table shows the two-tailed test of a non-zero effect of complexity in the posted-price regime, yielding a significance level of 6%.[15] (If one includes the supplementary data set in the analysis, the results remains the same as in Table 4.5.1, except that the negative effect of complexity in the posted-price regime is larger, raising the significance to p=0.015.) Thus, although there is some statistical evidence of a negative effect of complexity on deliberation time in the posted-price regime, that effect is certainly not positive.

This "rebound" effect is consistent with the findings of Diehl (1992), where complexities of the same type as here--delays and positive feedback--caused subjects to become cautious and "under-control."

What might account the reduced effort? There is no question that the decision task in the complex posted-price condition is extremely complicated,

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14 The analysis was done on market averages (i.e. treating each market as an observation) rather than individual subject data because of significant within-market, within-cell correlation in the latter. Thus, individual subject data cannot be considered independent observations of the same experimental condition. One can think of several explanations for the correlation, but most likely, it is caused by a combination of "implicit peer pressure" and the fact that the particular dynamics of any market strongly affect the difficulty of the decision task. The "implicit peer pressure" would lead students who are often the last to decide to feel guilty and the fastest ones to feel they can take a little longer than they otherwise would.

15 The same test done with the other dependent measures mentioned in note 13 did not show any significance levels below 10%.

if one considers all aspects and consequences of the decisions. Hence, it is possible that the subjects in this condition simply give up trying to "figure out the system" in detail. Instead of worrying about optimizing, they may resort to "damage control." Subjects' pricing policy may revert to a simple heuristic along the following lines, ignoring any attempts at searching for the best price-output point or signalling collusion: 1) form expectation about average price, and anchor on this average, and 2) adjust the anchor up or down, depending on whether inventory is negative or positive.

A slightly different explanation may be offered for the absence of any complexity effect in the clearing-price regime. The lack of any effect is certainly surprising, since both communication and figuring out the best price-output level is much harder in the clearing-price regime. On the other hand, the fact that both the simple and the complex price-clearing conditions are quite forgiving of errors may induce subjects over time, as they discover this leniency, to worry less about making "the right" decisions and instead try to make more decisions.

In conclusion, the results do not indicate that the observed mental effort uniformly follows the objective properties of the task; although there is evidence for some of the effects one would expect based on this perspective, there does seem to be a threshhold level of complexity beyond which mental effort is reduced, or at least not increased. Instead, subjects appear to resort to simplified heuristics.

4.6 Conclusion

The analysis in this chapter was focused primarily on the degree to which the observed market behavior conformed to the predictions of a neoclassical, optimal-learning framework. Briefly stated, neoclassical theory would predict small or no effects of the experimental treatments on market dynamics and performance and a rapid settlement of the markets toward equilibrium in all cases.

The actual behavior, however, shows large and systematic treatment effects in all aspects of the analysis. First, the introduction of complexity results in substantially lower performance, relative to optimal, in all three price regimes, most dramatically in the fixed-price regime. Much of the decrease in profits is the result of excessive inventory fluctuations, but there is also evidence that the introduction of complexity made it more difficult for firms to find the price-output level that would maximize profits before inventory costs.

Second, the introduction of complexity results in a significant increase in market volatility far above the effects to be expected from the changes in the objective difficulty of the task. In the fixed-price condition, the complexity leads to large, sustained cycles in output. In the posted-price complex condition, there is likewise a tendency for cycles. In the clearing-price complex condition, the variation takes the form of a large single overshoot, followed by a gradual drift down to equilibrium. Such behavior is in stark contrast to the rational market adjustment process and to the nature of variation in stochastic equilibrium, where the variables in the market should be either serially uncorrelated or negatively correlated.

Third, a direct test of expectation formation shows massive evidence of systematic forecast biases. Moreover, there is little evidence of learning, even though the forecasting task sometimes becomes easier when the variability in the market is reduced over time (e.g. in the clearing-price complex conditions).

The experimental results clearly diverge from the neoclassical benchmarks. Does this imply that neoclassical theory is wrong? As was mentioned at the beginning of the chapter, the result is not necessarily a refutation of neoclassical theory in general. First, it is remotely possible that the behavior in the price-varying conditions could be rationalized by considering the strategic behavior of firms, for instance their use of prices and quantities to signal intentions to collude. Yet such explanations do not apply in the fixed-price regimes where there is no strategic interactions. Second, the tests in this chapter were either tests of the joint hypothesis of rational expectations and optimal behavior or a test of expectations only. It is still possible that subjects were behaving optimally, given their expectations but that their expectations were out of joint. However, as will be seen in the following chapters, firms' decision rules, given their stated expectations, do not fit the optimal rules either. Hence, the results give grounds to abandon a neoclassical explanation, if not beyond all doubt, at least beyond all reasonable doubt.

From the point of view of the behavioral hypotheses described in the previous chapter, however, the data are more promising. The hypotheses are based on the notion of "misperceptions of feedback" (Sterman, 1989b, c). Three important aspects of such misperceptions are

- little understanding of time delays, i.e. a lack of appreciation of the length of time delays and insufficient correction for cumulative delayed effects,
- lack of attention to feedback effects, i.e. a tendency to ignore side-effects and compensating processes in the system, and
- an "open-loop" perspective, i.e. a tendency to attribute problems or success to external factors unrelated to one's own actions.

The lack of attention and understanding of time delays can explain the fact that the introduction of production lags in the complex conditions produces large cyclical behavior, particularly in the fixed-price condition. In the clearing-price complex condition, the effect is much smaller. This is so because the process of market clearing literally "clears" the system by preventing market imbalances from accumulating in inventories.

The lack of attention to feedback effects is also evident in the large fluctuations in the complex condition. The feedback effect in question is the multiplier effect on demand. In the clearing-price complex markets, the large initial overshoot is clear evidence that subjects do not take account of the fact that prices will initially rise when they raise their production. Instead, they appear to take the rising prices as evidence of a "business cycle upturn" and raise their output still further. A similar effect can be seen in some of the fixed-price markets, particularly Mkt 25. In the face of rising demand, firms raise their output which raises demand still further. If firms now attribute the rising demand to an externally imposed surge in demand, and if they extrapolate this short-term trend further, the result can be a huge overshoot like the one seen in Mkt 25, as output and demand rise ever higher in a self-reinforcing process.

The overshoot-and-collapse behavior is thus evidence of both a lack of attention to feedback effects and of an "open-loop" perspective. The tendency to attribute causes to external factors also has important implications for learning. If causes are wrongly attributed, true learning may never occur. The lack of improvement in forecasting performance is an indication that learning is indeed limited. It is quite possible, therefore, that market convergence, when it is observed, it simply a function of an unchanging set of decision rules, with little or no learning taking place on the part of firms.

The observed behavior thus shows strong evidence of misperceptions of feedback, although the consequences of these misperception are quite different in the three price regimes. In the fixed-price regime, the result is the observed sustained cycles. In the clearing-price regime, the automatic market clearing suppresses the accumulation of imbalances and thus makes the system much more forgiving of poor attention to delays. In the posted-price regime, the possibility of using prices to control inventories makes the system potentially easier to handle. However, in three out of the four markets in that condition, inventories and prices continue to fluctuate in a cyclical manner throughout the task. Thus, the cycle involving output and inventories in the fixed-price condition is replaced with one involving prices and inventories in the posted-price condition.

In the following three chapters, all of these notions are investigated in more detail by analyzing the individual decisions of subjects and, through the use of simulation, investigating the consequences of individual decision making for aggregate market behavior.

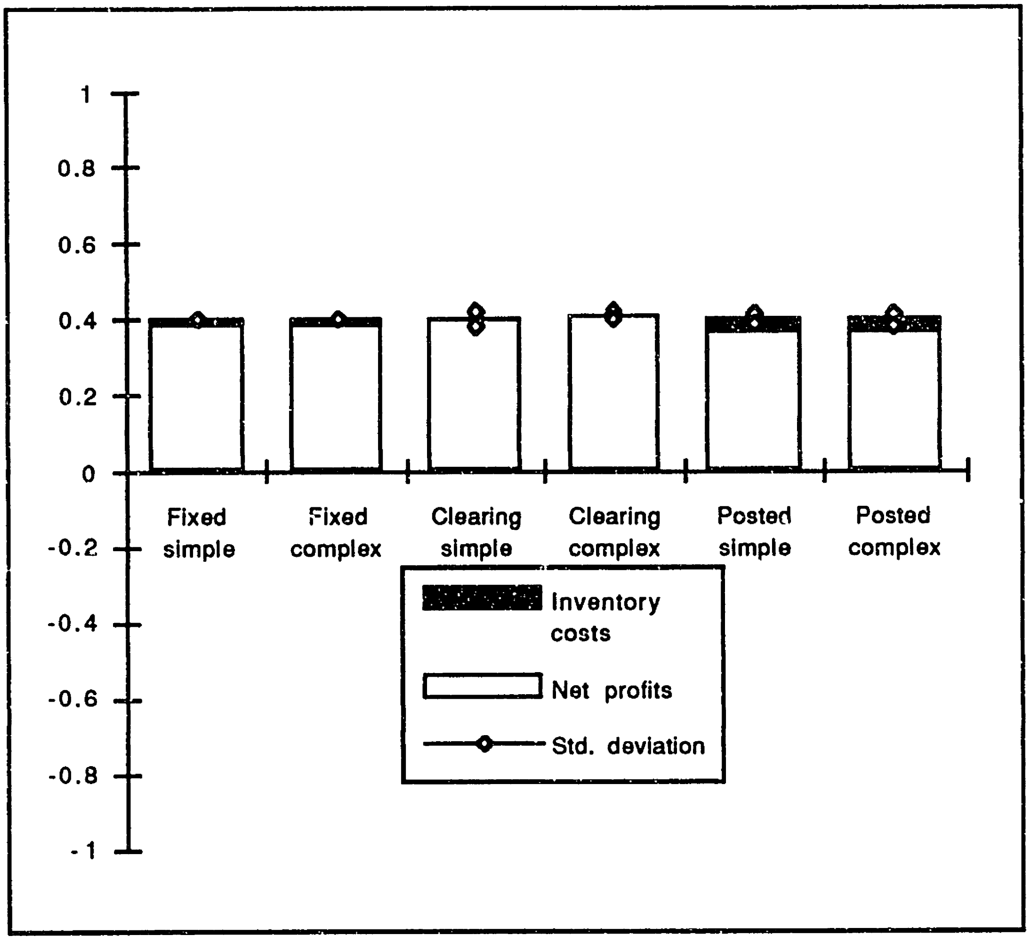


Figure 4.2.1: Simulated performance of rational competitive agents

The figure shows the simulated gross profits, inventory costs, and net profits for each of the six experimental conditions, under the assumption that firms correctly estimate the structural parameters of the system and follow a rational-expectations, competitive policy, except for a moderate amount of random error. The standard deviation of these errors 5% for both output and price decisions. All values shown are relative to the competitive equilibrium revenue (which is the same for all six conditions). The results shown are averages of 20 simulations for each condition. The first 10 periods have been excluded to allow for initial learning and adjustment. Also shown is the sample standard deviation of net profits.

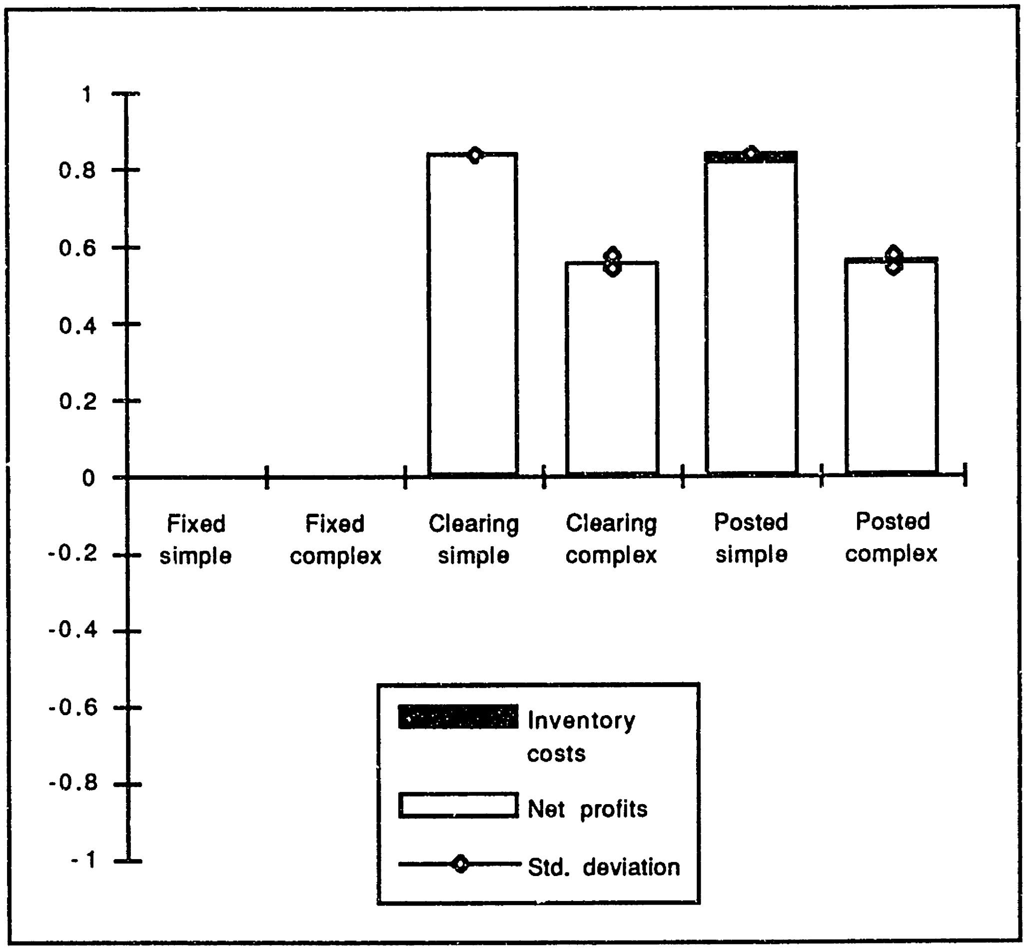


Figure 4.2.2: Simulated performance of rational colluding agents

The figure shows the simulated gross profits, inventory costs, and net profits for each of the six experimental conditions, under the assumption that firms correctly estimate the structural parameters of the system and follow a rational-expectations, fully cooperative policy, except for a moderate amount of random error. The standard deviation of these errors 5% for both output and price decisions. All values shown are relative to the competitive equilibrium revenue (which is the same for all six conditions). The results shown are averages of 20 simulations for each condition. The first 10 periods have been excluded to allow for initial learning and adjustment. Also shown is the sample standard deviation of net profits.

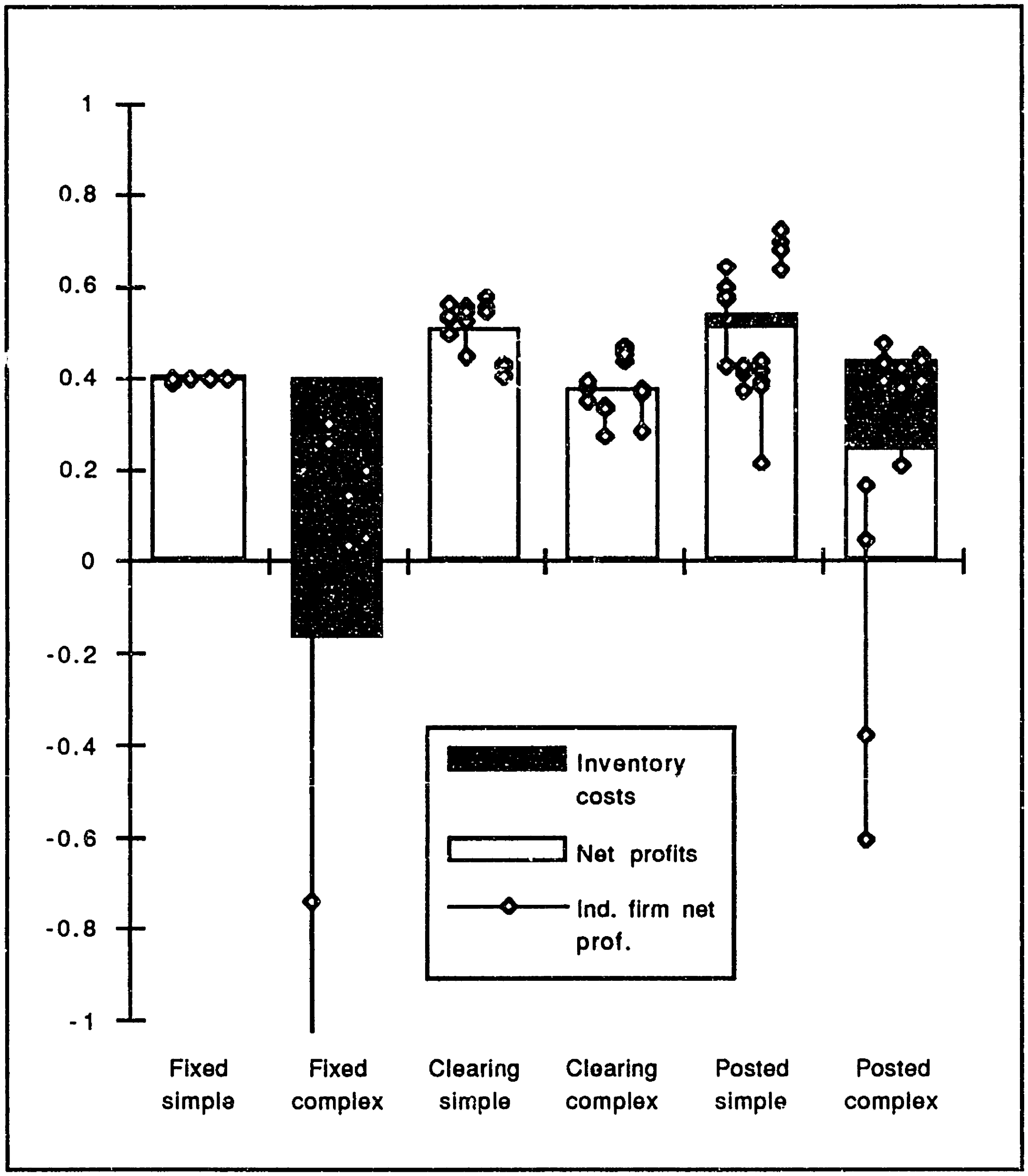


Figure 4.2.3: Actual profits and inventory costs

The figure shows the actual average gross profits, inventory costs and net profits for each of the six experimental conditions, excluding the first 10 time periods. The columns indicate averages for that experimental condition while the diamonds shows average profits for individual firms, where firms in the same market are connected with a vertical line. All values shown are relative to the competitive equilibrium revenue (which is the same for all six conditions).

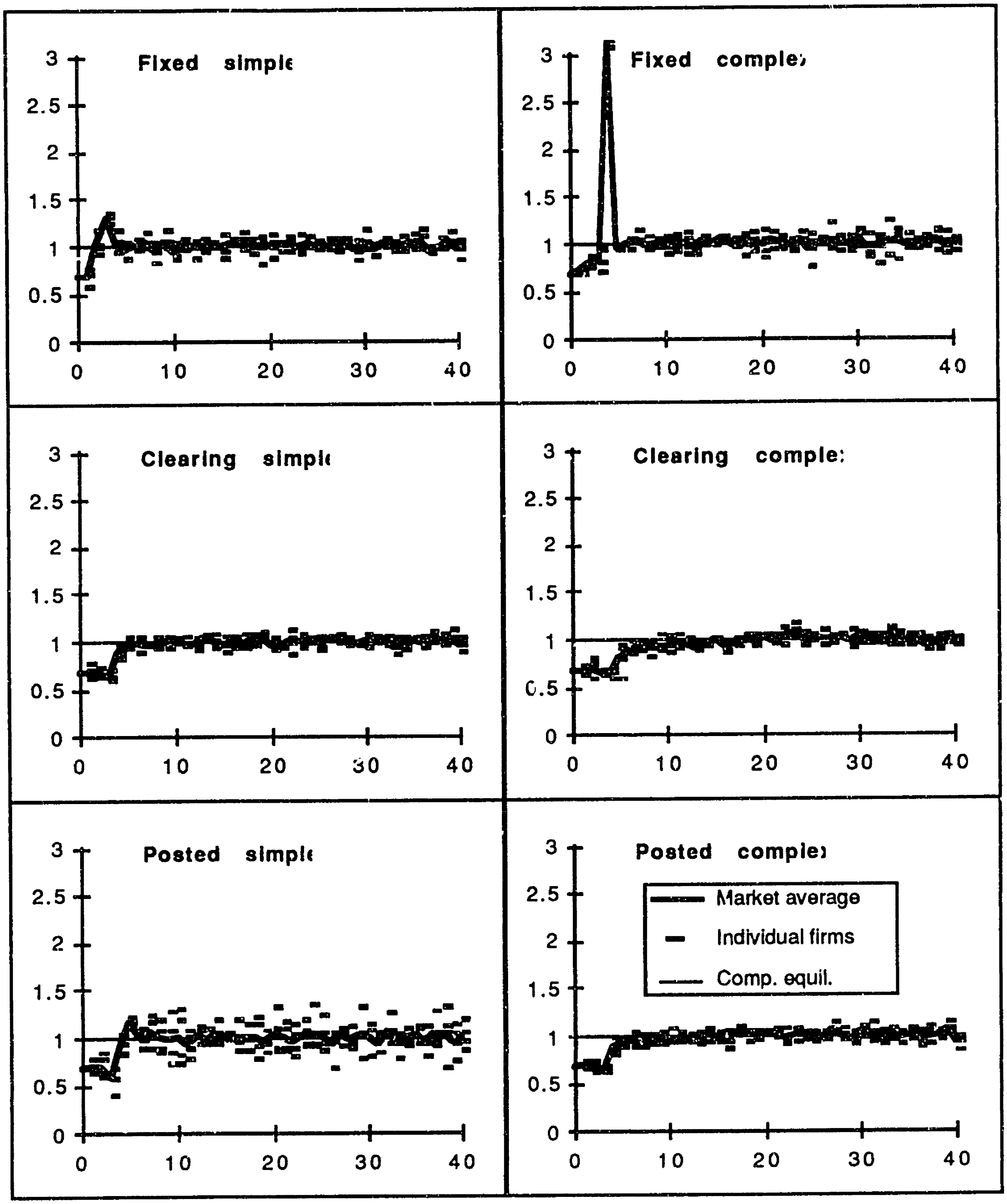


Figure 4.3.1: Examples of production behavior in simulated optimal markets

The figure shows examples of market production behavior under the assumption that firms correctly estimate the structural parameters in the system and act to maximize their expected profits, given their expectation of non-cooperative rational-expectations equilibrium. A moderate amount of random errors was introduced in the decision rules. The standard deviation of this error was .05 for both price and output.

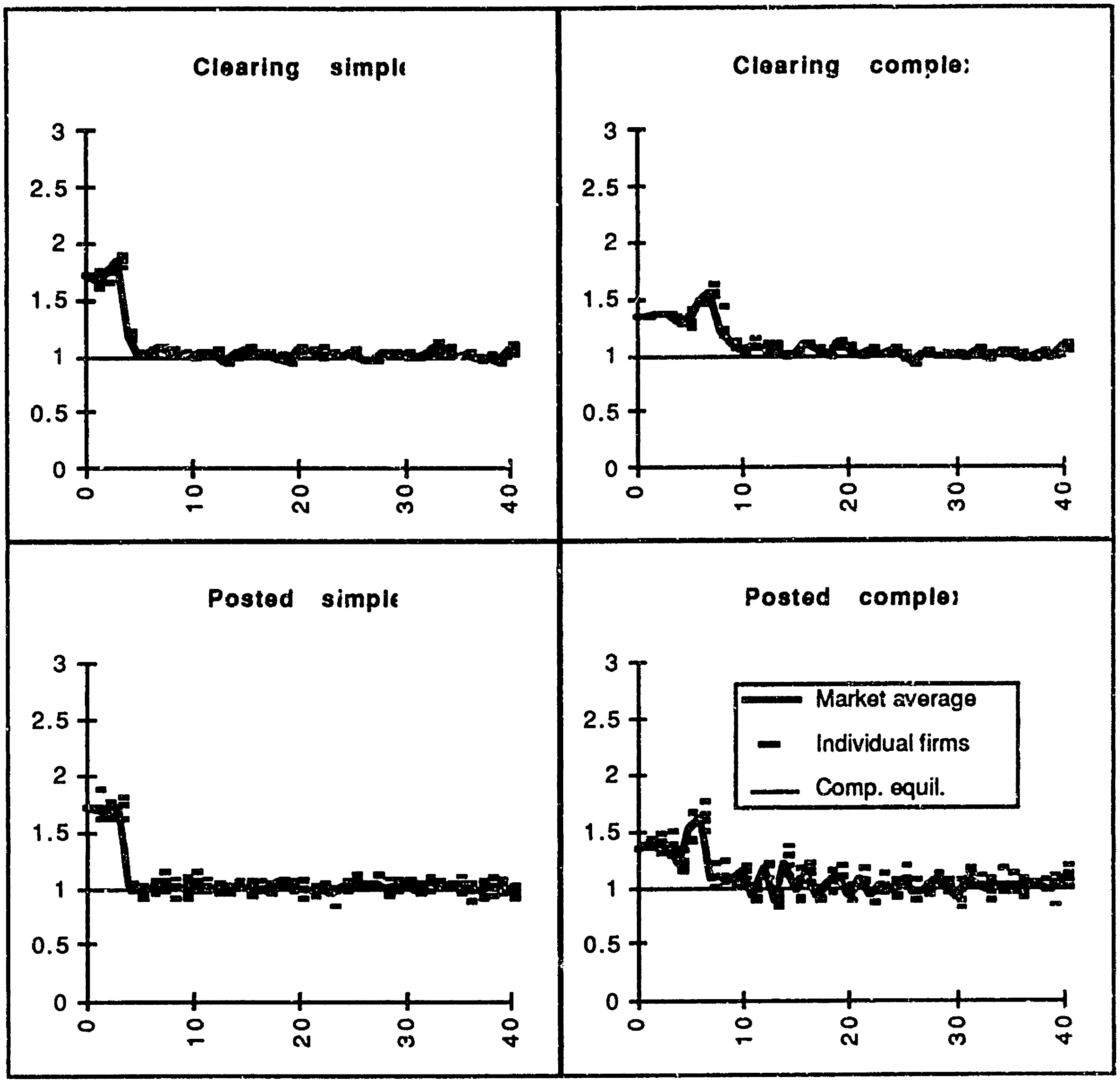
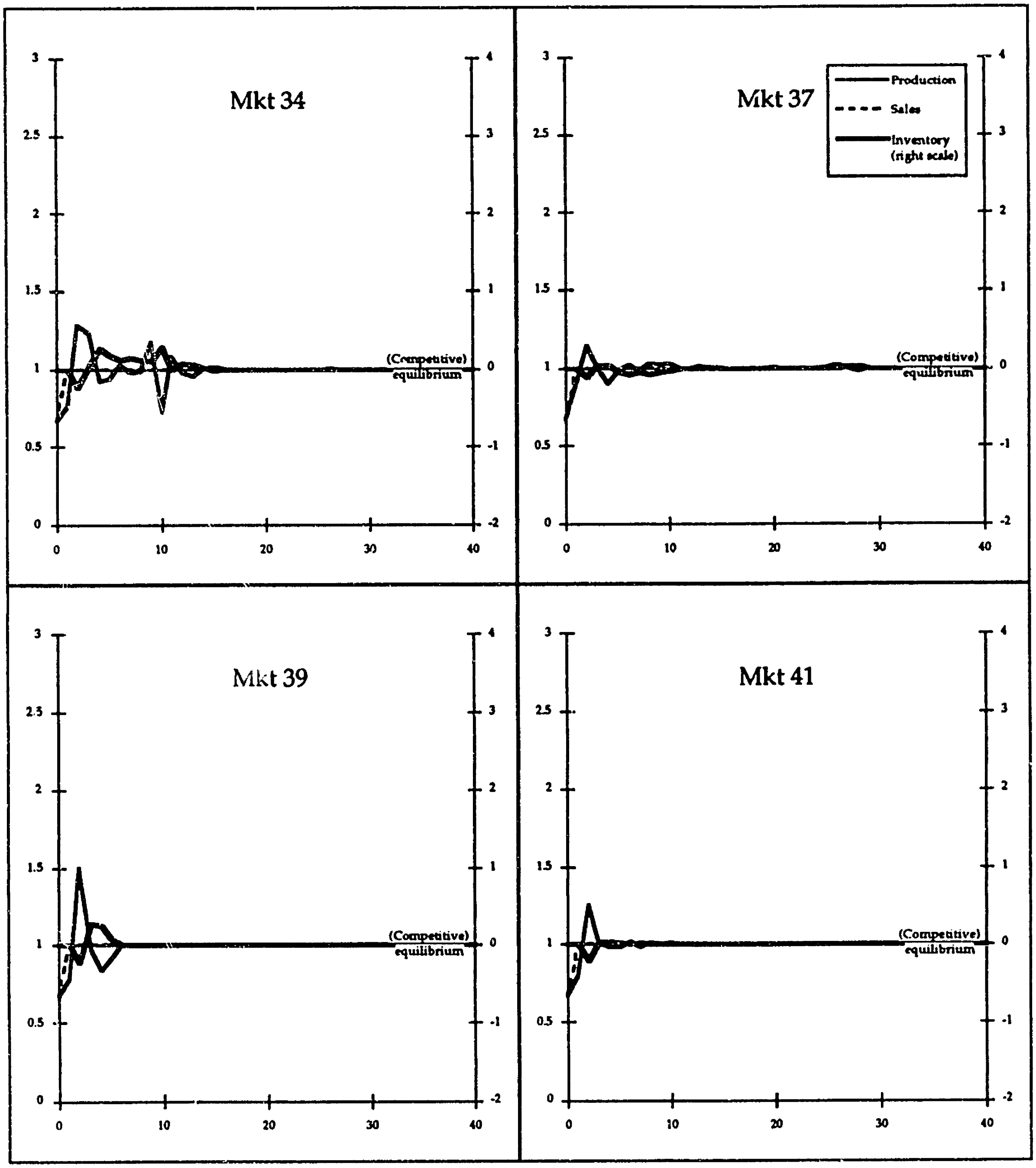


Figure 4.3.2: Examples of price behavior in simulated optimal markets

The figure shows examples of market price behavior under the assumption that firms correctly estimate the structural parameters in the system and act to maximize their expected profits, given their expectation of non-cooperative rational-expectations equilibrium. A moderate amount of random errors was introduced in the decision rules. The standard deviation of this error was .05 for both price and output.



<u>Figure 4.3.3: Observed behavior of market averages in the fixed-price simple condition</u>

The figure shows the market-average production, sales, and inventory for each of the four markets in the condition, relative to the equilibrium output level. Inventory is shown on the right-hand scale.

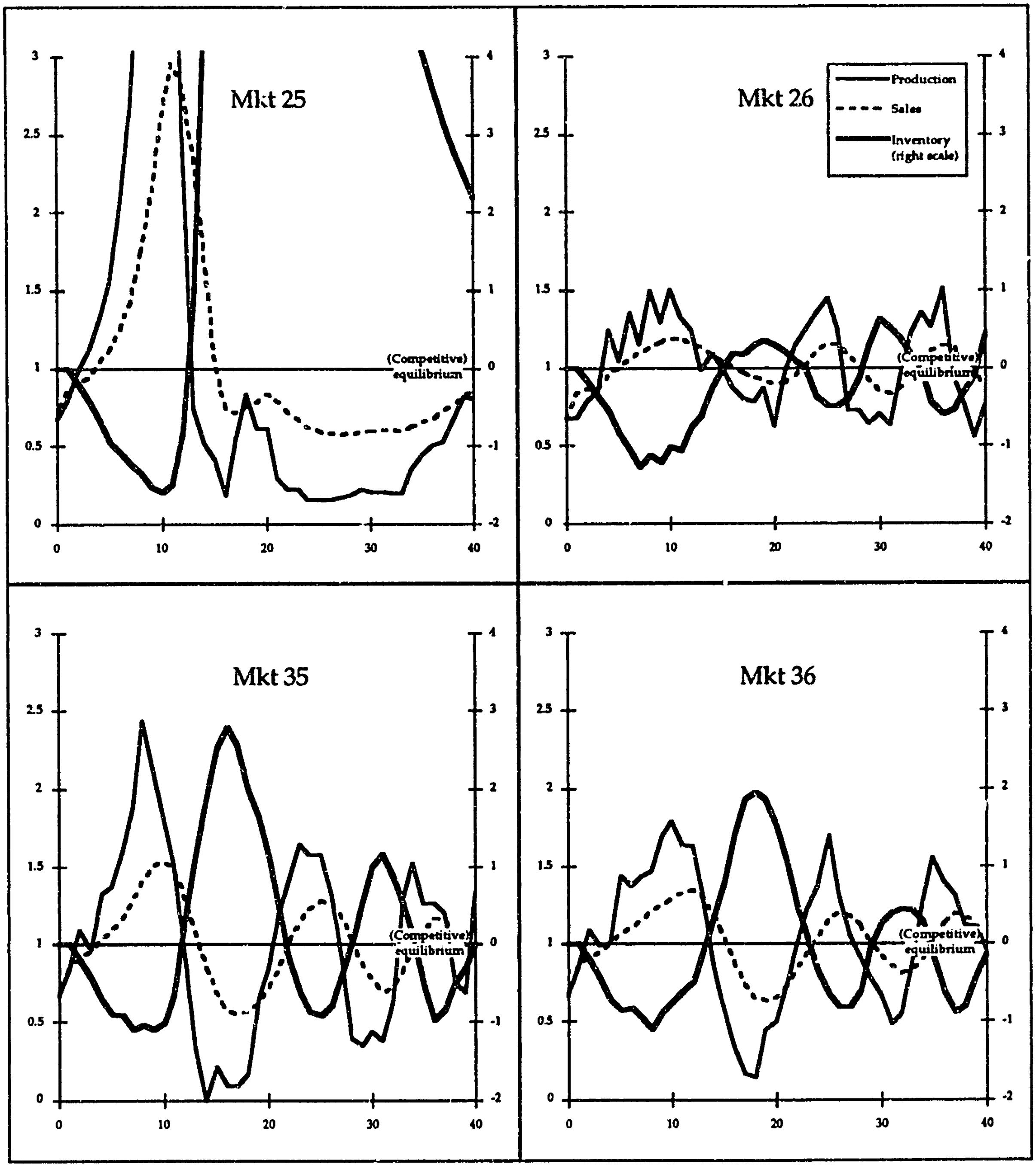


Figure 4.3.4: Observed behavior of market averages in the fixed-price complex condition

The figure shows the market-average production, sales, and inventory for each of the four markets in the condition, relative to the equilibrium output level. Inventory is shown on the right-hand scale.

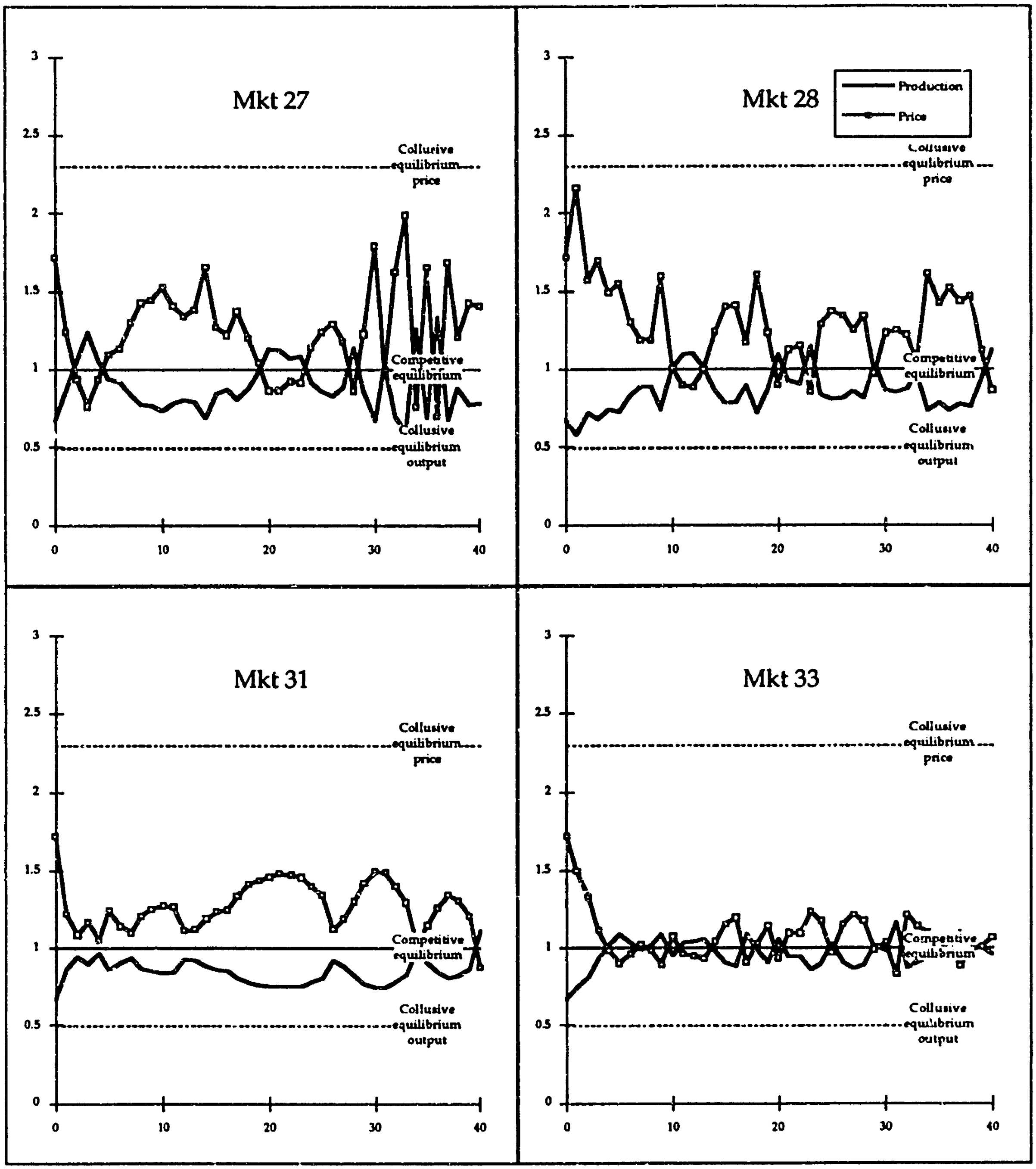
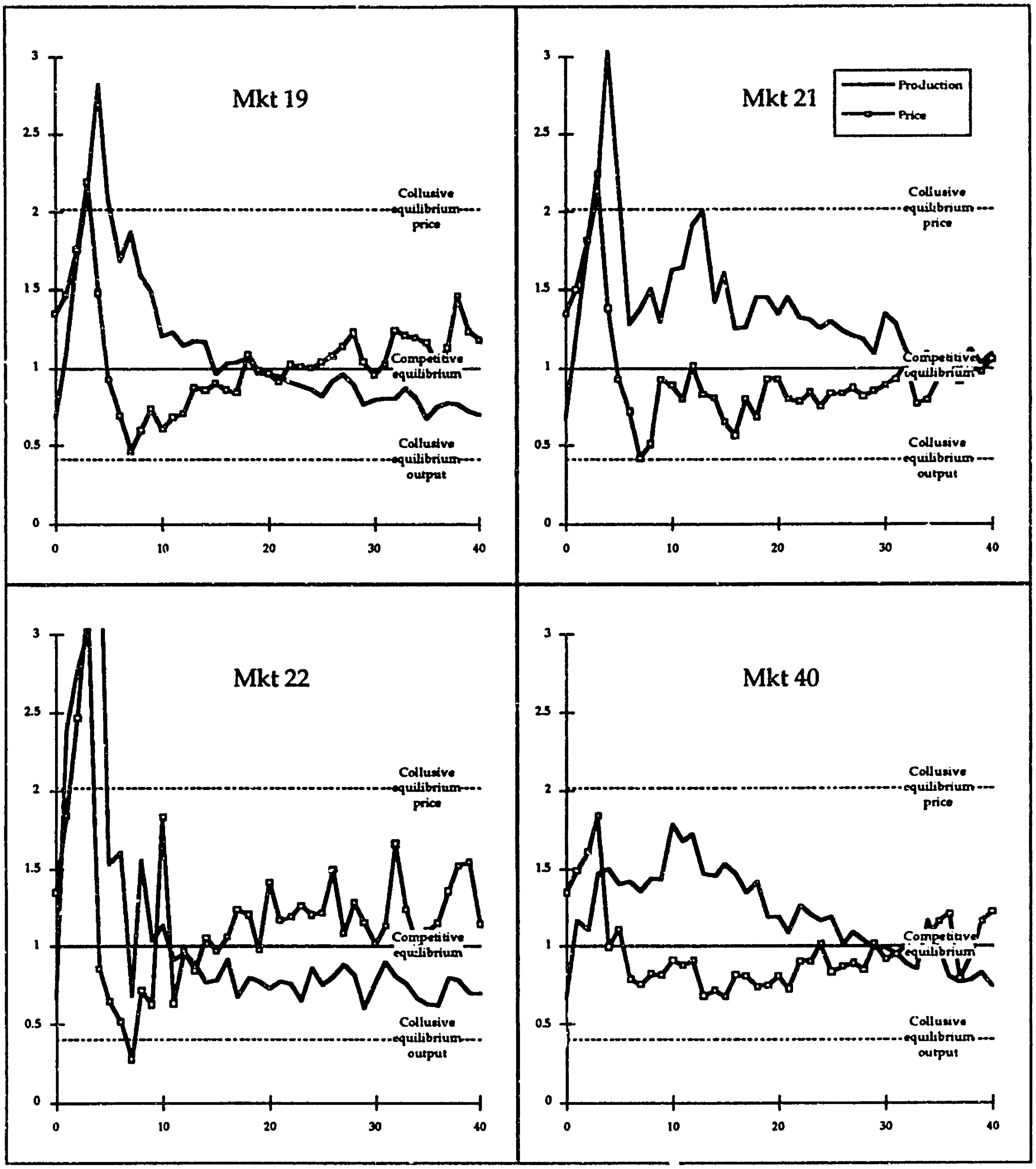


Figure 4.3.5: Observed behavior of market averages in the clearing-price simple condition

The figure shows the market-average production and price, relative to their respective competitive-equilibrium values, for each of the four markets in the condition. Also shown is the collusive-equilibrium price and output.



<u>Figure 4.3.6: Observed behavior of market averages, clearing-price complex condition</u>

The figure shows the market-average production and price, relative to their respective competitive-equilibrium values, for each of the four markets in the condition. Also shown is the collusive-equilibrium price and output.

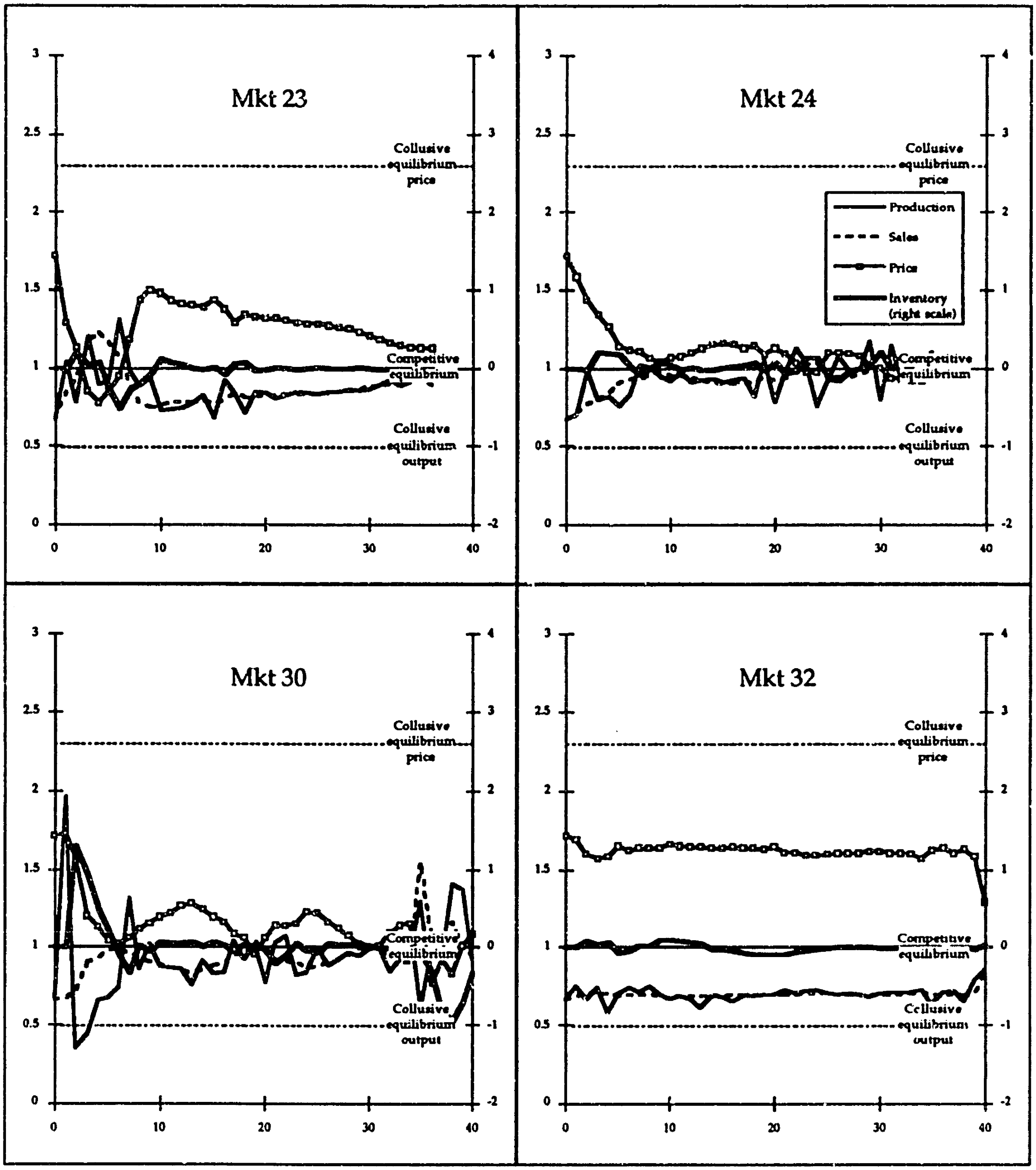


Figure 4.3.7: Observed behavior of market averages in the posted-price simple condition

The figure shows the market average production, sales, and inventory relative to the competitive-equilibrium output, and the market-average price relative to the competitive-equilibrium price. Also shown is the collusive-equilibrium price and output. Inventory is shown on the right-hand scale.

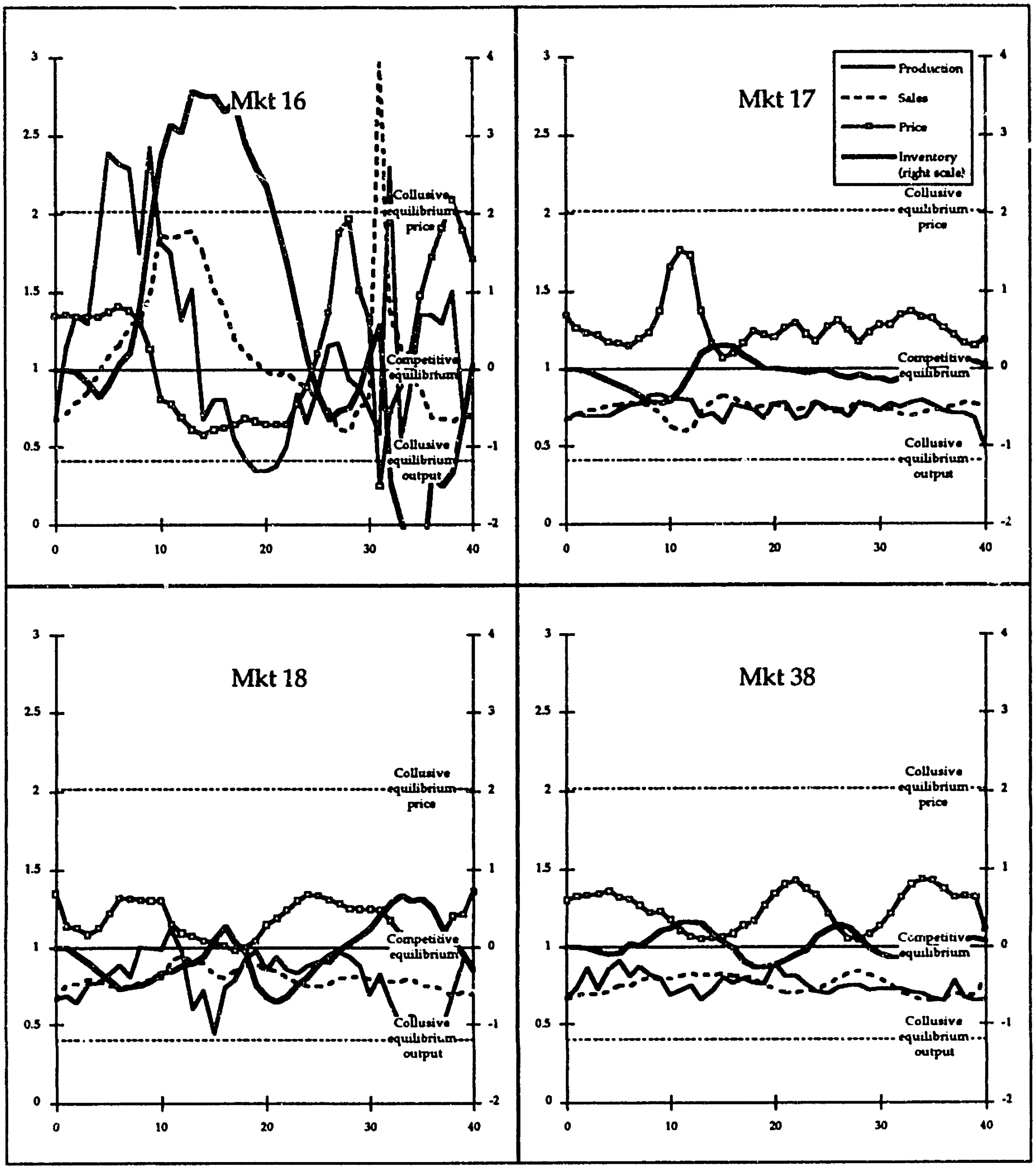


Figure 4.3.8: Observed behavior of market averages in the posted-price complex condition

The figure shows the market average production, sales, and inventory relative to the competitive-equilibrium output, and the market-average price relative to the competitive-equilibrium price. Also shown is the collusive-equilibrium price and output. Inventory is shown on the right-hand scale.

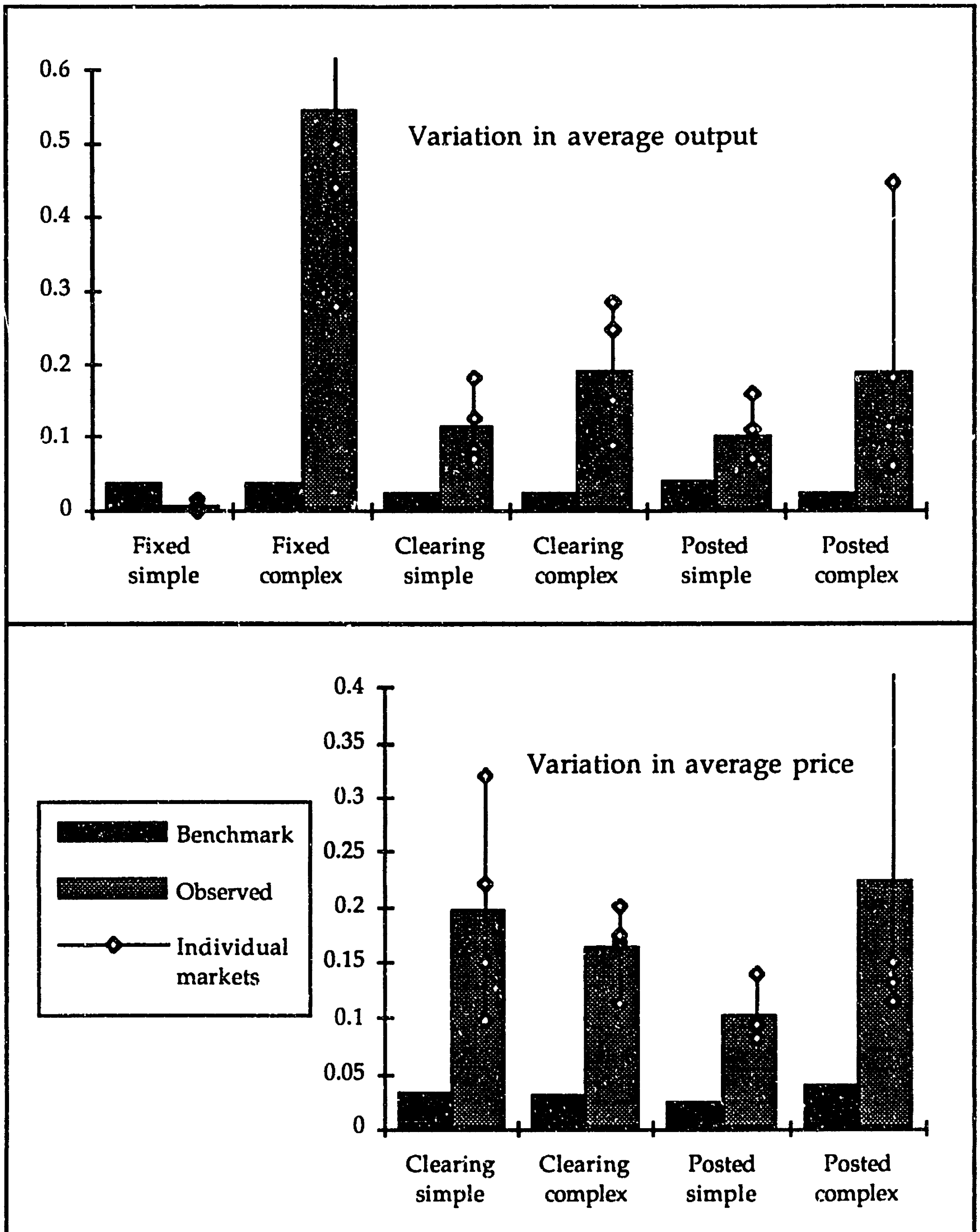


Figure 4.3.9: Comparison of "benchmark" and actual observed variation in average output and price (first 10 periods excluded).

The figure compares the standard deviations in average production and average price, respectively, to a benchmark value. The benchmark was found by deriving the variance in rational-expectations competitive equilibrium, assuming that firms' decision are subject to a small to moderate amount of random error (see Appendix A). In the figure, a standard deviation of 5% and was used for both price and output decisions. The number of firms was assumed to be four. Note that the benchmark variance of the aggregate is lower than 5% in all cases.

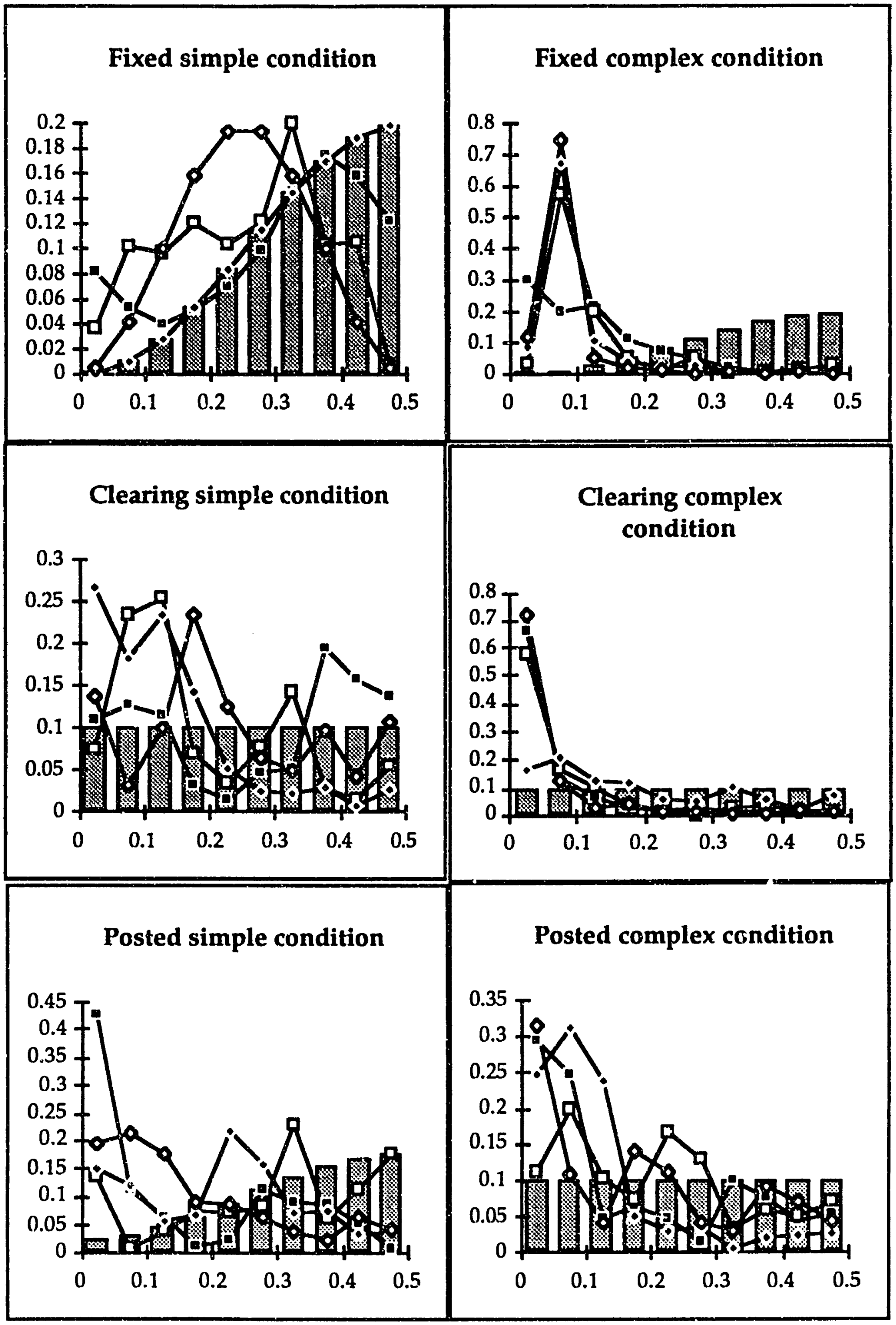


Figure 4.3.10: Comparison of observed spectral density in the variation in average output with rational-expectations benchmarks. (See caption for Figure 4.3.11)

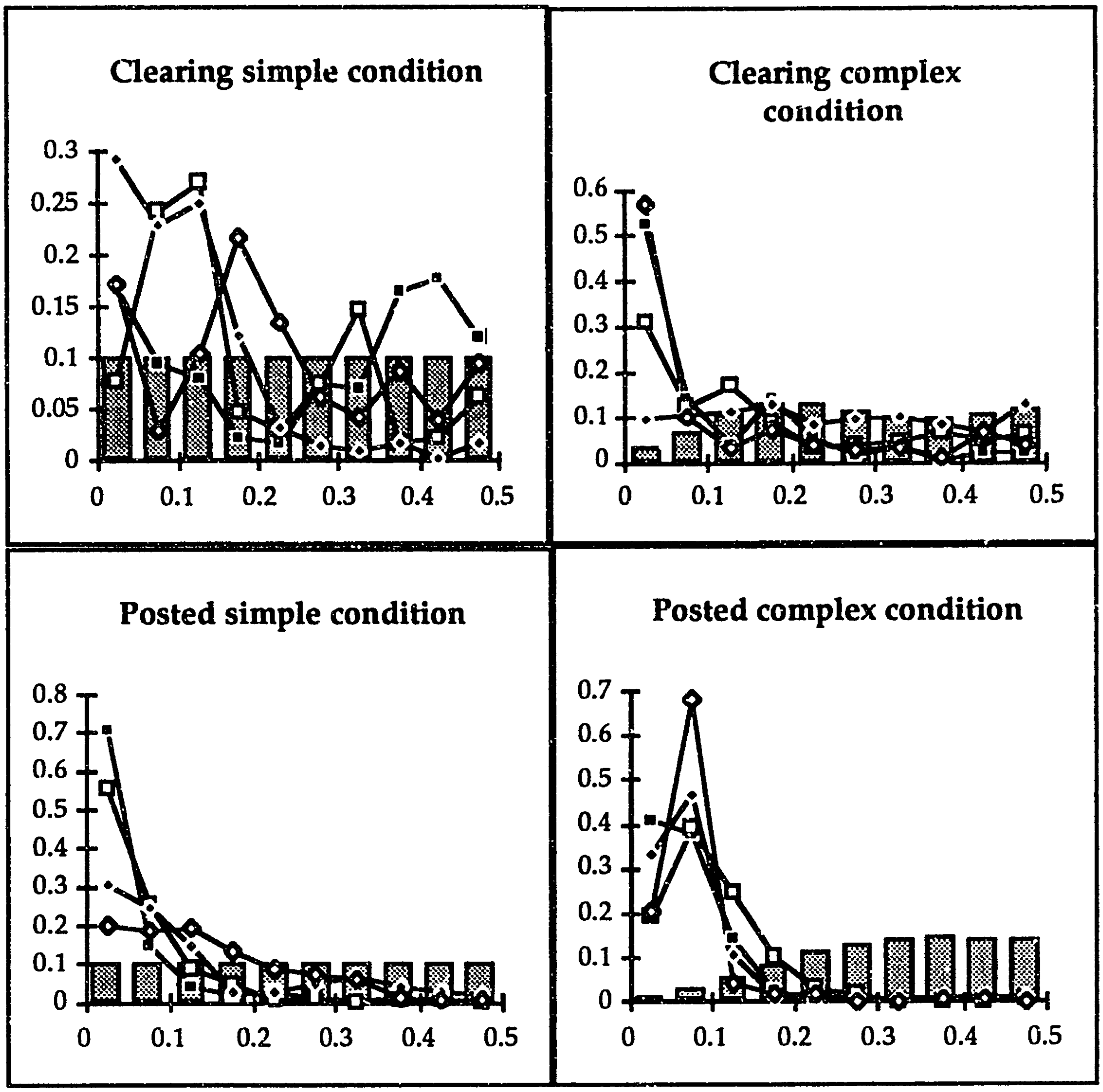


Figure 4.3.11: Comparison of observed spectral density in the variation in average price with rational-expectations benchmarks.

The figures show the spectral density for each market in each condition and compares it to the theoretical spectrum that would prevail in rational-expectations stochastic equilibrium (see derivation in Appendix A). For the posted-price condition, the benchmark presumes that the error variance is equal in both prices and output. (This assumption is not critical--the spectra do not change substantially with different error variances in the two decisions.) The first 10 periods have been excluded from the observations to allow for some initial learning and adjustment. The horizontal scale represents frequency intervals while the vertical scale shows the fraction of the total variance in each market that falls in that frequency range.

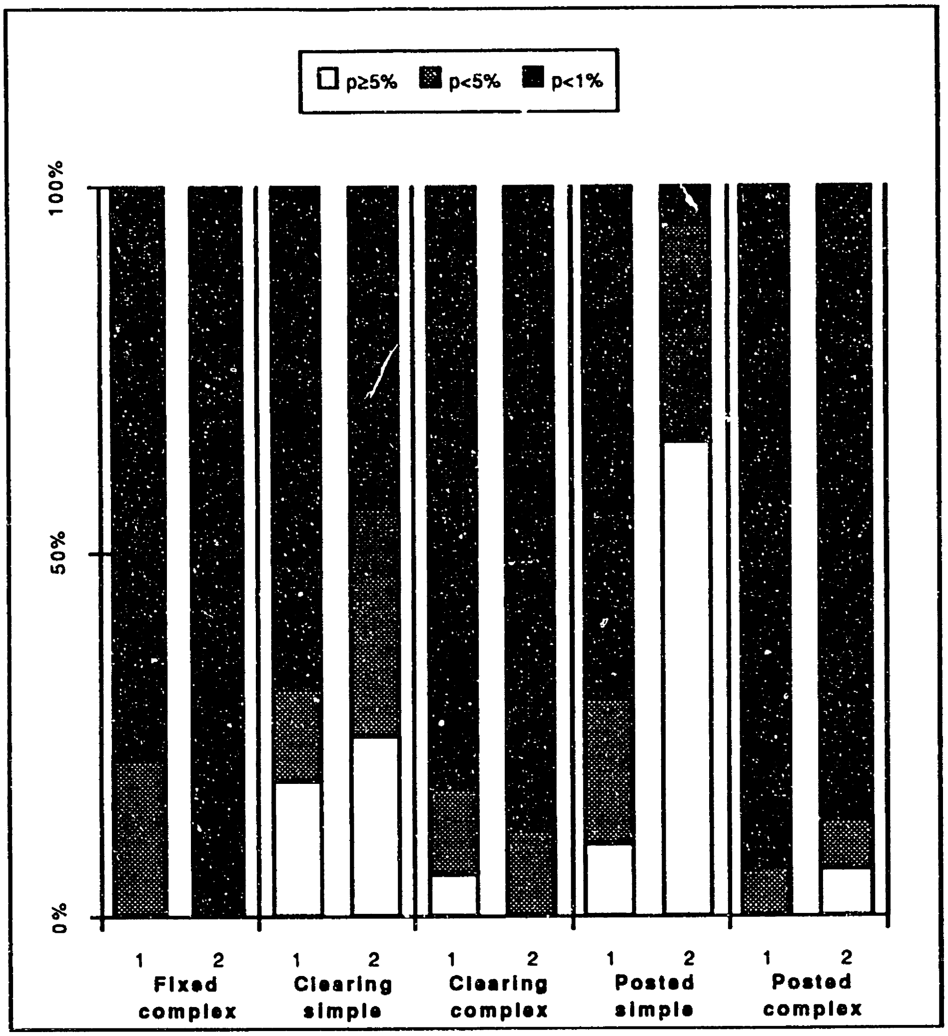


Figure 4.4.1: Test of learning in demand forecasts

The figure compares the outcome of the same test for rationality as in Table 4.4.1 (in linear rather than log-linear form) by splitting the sample in two parts: time periods 1 through 18 ("1"), and time periods 19 through the end of the game ("2"). The bars indicate the fraction of subjects for which the test failed at the 1% and the 5% level or was accepted at the 5% level, respectively. The forecasts in the simple fixed-price condition had too little variance to allow statistical analysis: The task was a trivial one of forecasting a constant demand.

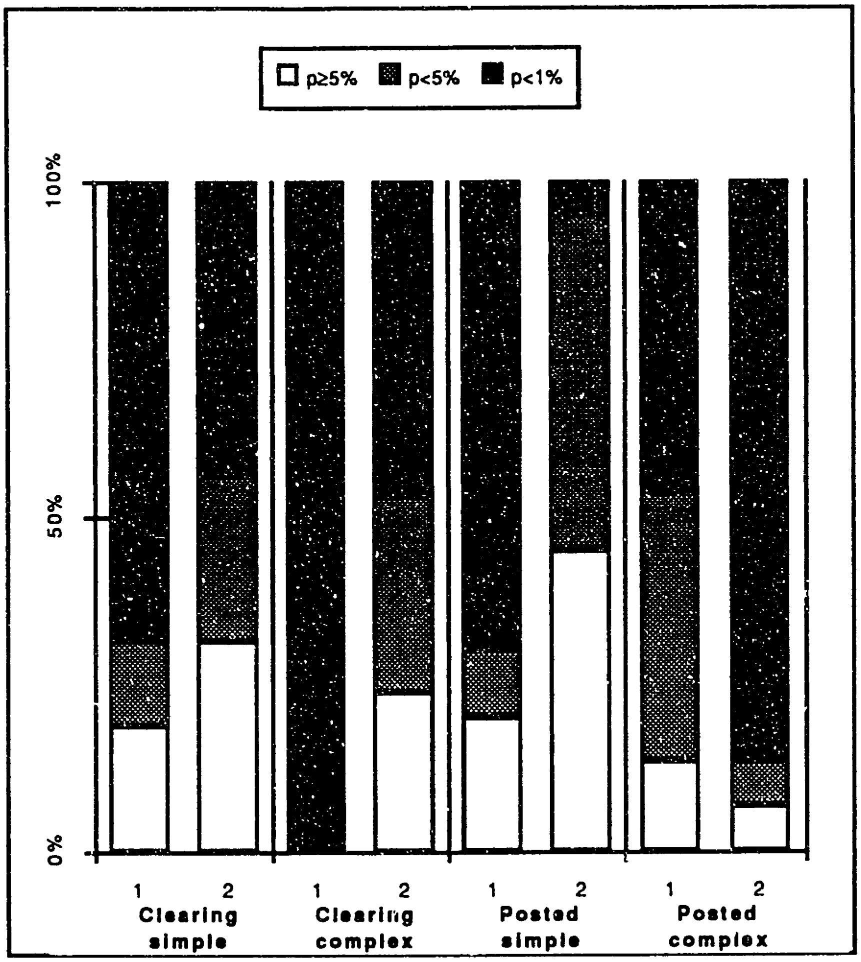


Figure 4.4.2: Test of learning in price forecasts

The figure compares the outcome of the same test for rationality as in Table 4.4.1 (in linear rather than log-linear form) by splitting the sample in two parts: time periods 1 through 18 ("1"), and time periods 19 through the end of the game ("2"). The bars indicate the fraction of subjects for which the test failed at the 1% and the 5% level or was accepted at the 5% level, respectively.

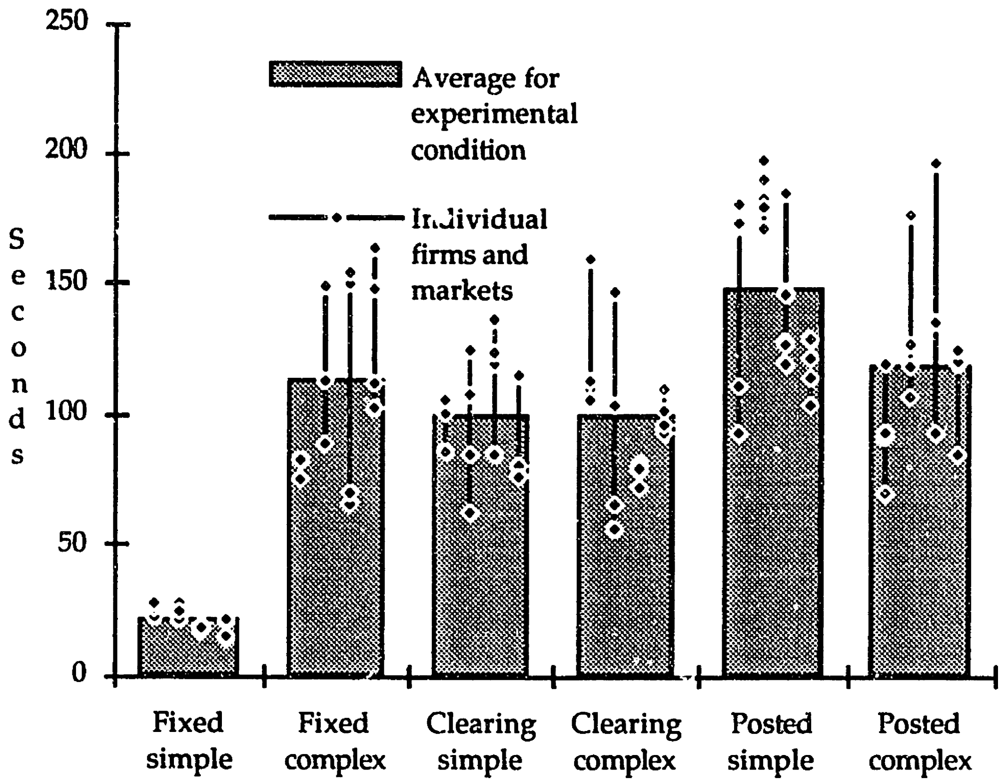


Figure 4.5.1: Average time to deliberate decisions, by experimental condition (First 10 periods excluded).

The table shows the average time taken from the beginning of a round until the subject was ready to execute his or her decisions. Subjects had to enter all decisions and forecasts before before submitting them to execution, and the figure shows the total time used deliberating, not the time per decision.

DEP VAR: IGP10 N: 16 MULTIPLE R: .633 SQUARED MULTIPLE R: .401

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
C	0.074	1	0.074	5.957	0.031
P	0.021	1	0.021	1.700	0.217
C*P	0.005	1	0.005	0.373	0.553
ERROR	0.149	12	0.012		

Table 4.2.1

Analysis of variance of (the log of) gross profits

The table shows the two-way analysis of variance of normalized average profits before inventory costs in each market, excluding the first 10 time periods, using price condition (P) and complexity (C) as factors. The normalization was

$$\text{Index} = \frac{\text{Profits before inventory costs - Competitive equilibrium profits}}{\text{Collusive equilibrium profits}}$$

The fixed-price conditions were excluded since profits before inventory costs are not changeable in the long run in this regime.

DEP VAR: LNC10 N: 16 MULTIPLE R: .893 SQUARED MULTIPLE R: .798

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
C	81.308	1	81.308	30.144	0.000
P	13.070	1	13.070	4.846	0.048
C*P	33.654	1	33.654	12.477	0.004
ERROR	32.368	12	2.697		

Table 4.2.2: Analysis of variance of (the log of) inventory costs

The table shows the two-way analysis of variance of the logarithm of the average inventory costs in each market, excluding the first 10 time periods, using price condition (P) and complexity (C) as factors. The clearing-price conditions have been excluded since inventories are identically zero in this condition.

Experimental condition	Actual = a + b*Forecast, Test (a=0,b=1)				Log(Act.) = a+b*Log(Fcst.), Test (a=0,b=1)			
	p <1%	p <5%	p ≥5%	DW <1.25	p <1%	p <5%	p ≥5%	DW <1.25
Fixed simple	***	***	***	***	***	***	***	***
Fixed complex	14	0	0	14	14	0	0	14
Clearing simple	15	1	0	6	15	1	0	5
Clearing complex	11	2	4	11*	11	2	4	11*
Posted simple	14	6	0	13	14	6	0	14
Posted complex	14	1	0	15	14	1	0	15

* The DW statistic was missing in 6 instances, due to a software error

*** The forecasts in this condition had too little variance to allow statistical analysis. The task was a trivial one of forecasting a constant demand.

Table 4.4.1: Test of rationality in demand forecasts.

The table shows the number of subjects where the test was rejected at the 1% level, rejected at the 5% level, or accepted at the 5% level, respectively. Moreover, it shows the number of cases where the Durbin-Watson statistic was below 1.25, which would indicate significant serial correlation in the errors ($p < 5\%$).

Experimental condition	Actual = a + b*Forecast, Test (a=0,b=1)				Log(Act.) = a+b*Log(Fcst.), Test (a=0,b=1)			
	p <1%	p <5%	p ≥5%	DW <1.25	p <1%	p <5%	p ≥5%	DW <1.25
Clearing simple	15	0	1	5	15	0	1	6
Clearing complex	17	0	0	13	16	1	0	16
Posted simple	16	1	3	19	13	4	3	16
Posted complex	14	1	0	15	14	1	0	15

Table 4.4.2: Test of rationality in price forecasts.

The table shows the number of subjects where the test was rejected at the 1% level, rejected at the 5% level, or accepted at the 5% level, respectively. Moreover, it shows the number of cases where the Durbin-Watson statistic was below 1.25, which would indicate significant serial correlation in the errors ($p < 5\%$).

DEP VAR: ATD10 N: 24 MULTIPLE R: .918 SQUARED MULTIPLE R: .842

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
P	18591.361	2	9295.681	25.030	0.000
C	2424.575	1	2424.575	6.528	0.020
P*C	14593.798	2	7296.899	19.648	0.000
ERROR	6684.949	18	371.386		

TEST 1 "Only fixed simple different?"

SOURCE	SS	DF	MS	F	P
HYPOTHESIS	19943.903	3	6647.968	17.900	0.000
ERROR	6684.949	18	371.386		

TEST 2 "Posted simple = posted complex"

SOURCE	SS	DF	MS	F	P
HYPOTHESIS	1489.889	1	1489.889	4.012	0.060
ERROR	6684.949	18	371.386		

Table 4.5.1: Analysis of variance of deliberation times

Two-way analysis of variance with the average deliberation time in each market, excluding the first 10 periods (ATD10), as a function of the two factors price regime (P) and complexity (C). Test 1 is a contrast test of whether all five cells in the design excepting the fixed-price simple condition are equal. Test 2 is a contrast test of the hypothesis that the posted simple and the posted complex conditions are equal.

5 Behavior under fixed prices

5.1 Introduction

The previous chapter concentrated on a comparison of the observed market outcomes with those predicted by neoclassical theories. It was evident from the analysis that experimental markets showed substantial departures from these theories, and that these departures varied systematically with the experimental treatments. In the concluding section of the chapter, the alternative behavioral hypotheses were discussed, and it was argued that much of the observed behavior was consistent with the notion of "misperceptions of feedback." This and the following three chapters probe further into this idea, with one chapter devoted to each of the three price regimes in the experimental design.

The method of the chapter follows and extends the lines of Sterman's (1981b) approach and represents the main attempt at a methodological contribution of the thesis: To identify well-fitting decision rules at the individual level and then use simulation to link these micro-level rules to the macro-level outcomes. In investigating these matters, one can define a hierarchy of analysis where more and more variables are made to vary endogenously. At the lowest level is the fitting of decisions to the given output cues, which are taken to be exogenous, as is typically done in much multiple-cue probability-learning (MCPL) research in psychology (Hammond 1955; Camerer 1981). At the next level, some of these input cues, such as inventories, or lagged values of the dependent variable, are made endogenous and allowed to vary while the market environment is taken to be exogenous to the firm. Finally, at the highest level, all variables are allowed to vary endogenously. At this point, it would be surprising if one could find a good point-by-point fit of the simulated and the actual behavior, due to the inconsistencies in the actual behavior of the subjects. However, the modes of behavior (oscillation, convergence, etc.) should be the same, and other measures or criteria of fit must be used, such as average variability and performance.

The analysis in this and the following three chapters thus starts with an analysis of the individual decisions and forecasts made by the subject, trying through regression techniques to identify heuristics that guide these decisions. The investigation follows the MCPL tradition by estimating a single equation relating decisions (outputs) to the relevant input cues. The chapter starts with an analysis of the output decisions of firms. A simple decision rule is posited which builds on Sterman's generic stock management rule (Sterman 1989). The rule is estimated and the degree of fit discussed. Also, the degree of learning, manifested as a change in the parameters in the rule, is investigated.

The following section conducts a similar analysis of the forecasts solicited from the subjects. As was evident from the tests for rational expectations in the previous chapter, forecasts are not unbiased predictors of future variables. The purpose of the analysis here is to uncover what rules subjects might use in making their forecasts.

Then, in Section 5.4, the estimated decision and forecasting rules are combined in a simulation of the entire market, and these simulated results are compared to the actual outcomes. The analysis in this section represents a challenging test of the predictive ability of the estimated decision rules: Not only must they fit the observed decisions well in traditional statistical terms, they must also reproduce the observed patterns of behavior when all variables are allowed to vary endogenously.

While Section 5.4 emphasizes the full quality and pattern of behavior by presenting the full simulations in graphical form, Section 5.5 employs a more compact and formal comparison by computing two key summary measures of market outcomes: Average performance, measured by profits earned, and average variability, measured by the variance in production.

Section 5.6 attempts to explain in a compact manner the observed variation in both individual firm and market outcomes by identifying key determinants of performance: What are the important features of decision making and forecasting that are most crucial for performance? How much of the observed variation can be explained by a few key features?

Section 5.7 concludes the chapter by relating the findings to the notions of misperceptions of feedback and other general behavioral hypotheses.

The simple fixed-price condition has been completely excluded from the analysis here, in large part because the variance in decisions and forecasts is simply too small to allow any form of statistical analysis. As was shown in the previous chapter, all of the firms in the simple fixed-price condition achieved rapid approach to equilibrium, though perhaps through slightly different methods. However, once the equilibrium prevails, it is impossible to distinguish the alternative routes through which it was reached. In any case, as was argued in Chapter 3, almost any reasonable decision rule will assure rapid convergence in that condition, so that the details of individual decision rules matter little for performance in any case.

5.2 Analysis of output decisions

In the fixed-price games, all subjects must make one decision each period, namely how much production to initiate. In addition, they are asked to provide a forecast of future average demand. The analysis of the optimal decision rule in Chapter 3 and Appendix A showed that the optimal decision rule can be interpreted as having three components: Expected future demand, a correction for inventory, and a correction for supply line. Indeed, any reasonable rule would take these three factors into account (Sterman 1989). The procedure is thus:

1) Forecast expected demand at the time the initiated production will be finished.

2) Look at current inventory and compare it to the desired level; adjust production upward if there is a deficit, downward if there is a surplus.

3) Look at the current amount in the supply line of production and compare this to the desired level (given expectations of future demand); adjust production upward if there is too little in the pipeline, downward if there is too much.

4) If the resulting desired production turns out to be negative, the actual production is constrained at zero.

In formal terms, as presented in Chapter 3, the decision rule can be written

(5.2.1) $y_t = \text{Max}\ \{\ 0\ ,\ x_{i,t}^{e} + \alpha_n(n^{*} - n_t) + \alpha_s(s_t^{d} - s_t) + u_t\ \},$

x_t^e is the expected sales,
s_t^d is the desired supply line,
u_t is a random error, and
α_n, α_s, n^* are parameters.

If one furthermore makes the simplification that the desired supply line, $s_{i,t}^d$, is constant, and that the desired inventory, n^*, is zero, the equation becomes

$$(5.2.2) \qquad y_t = \text{Max}\{\, 0, x_t^e - \alpha_n n_t + \alpha_s s^d - \alpha_s s_t + u_t\}.$$

The assumption that the desired supply line is constant can be justified by the fact that, as will be seen below, most subjects put relatively little weight on the supply line correction in the first place. Thus, it seems implausible that they should further adjust their long-term supply-line goal when making the supply line correction.[1] In any case, the matter has little importance if the weight on the supply line correction is already small.

Table 5.2.1 shows the result of estimating equation (5.2.2), using the solicited forecasts for the expectation x_t^e, and using non-linear least-squares estimation.[2] It is evident from the table that the fit is very good indeed; the R^2 vary between .86 and .99. (It should be noted, though, that the measure of fit, R^2, differs from the conventional measure. The conventional R^2 measures variations around the sample mean. This is appropriate in a regression with a constant term, where the residuals must sum to zero. Here, the equation contains no constant term, and the R^2 is instead based on variations around zero, i.e.

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1 Sterman (1989c) found excellents fits assuming a constant supply line.

2 The non-linear least squares estimate is the parameter values which minimize the sum of squared residuals. Although the equation 5.2.2 is in fact a Tobit model which could be estimated using a well-known maximum-likelihood method (Amemeyia 1973), the small-sample properties of this method are not well-known. Simulations performed by the author comparing the two methods indicated that the non-linear least-squares method gives better estimates in the case at hand.

(5.2.3) $$R^2 = 1 - \frac{\sum_t e_t^2}{\sum_t y_t^2}.$$

This measure thus gives the regression credit for the ability to predict the absolute production level, not just variations around the mean.

It is also evident that most subjects appear to pay some attention to their inventory levels: The coefficient α_n is significantly greater than zero in all but two cases, though it is also generally less than unity. In contrast, the attention to supply line is much lower or absent: The coefficient α_s is significantly greater than zero in only 4 out of 14 cases (in one case it is significantly negative). Moreover, it is smaller than the inventory coefficient in all of the 14 cases!

In conclusion, one must say therefore that the simple production equation (5.2.2) does a good job of predicting the observed decisions and that subjects generally pay little attention to the supply line. These results are fully in accord with Sterman's (1989b, c) findings.

5.3 Analysis of forecasts

In a fully endogenous simulation, it would be necessary to also model expectations. Therefore, the solicited forecasts were fitted to a simple equation, relating the forecast to past values of the forecasted variables. The following equation was used

(5.3.1) $$F_t = F_{t-1} + a_1(X_{t-1} - F_{t-1}) + a_2(X_{t-1} - X_{t-2}) + e_t.$$

The equation is a combination of an adaptive-expectations model and an extrapolative-expectations model. (Several additional versions of equation (5.3.1) were estimated, including the use of logarithms and ratios. However, the substantive results remain unchanged.) The equation expresses the idea that subjects anchor on last period's forecast and then modify it with last period's forecast error and, possibly, with the observed recent trend in the forecasted variable.

Table 5.3.1 shows the results of estimating the equation (5.3.1). In the regression, the independent variable is the change in the forecast, F_t-F_{t-1}. The measure of fit used was

$$(5.3.2) \qquad R^2 = 1 - \frac{\sum_t e_t^2}{\sum_t (F_t - F_{t-1})^2}.$$

The measure varies between a low of 0.05 and a high of 0.75, with an average of 0.44--a more modest result than the fit in production decisions, though still not bad. However, unlike the latter, the measure does not give credit to the equation for predicting the absolute level of the forecast.

As one would expect, the adaptive parameter, a_1, are all significant and no greater than unity. This indicates some conservatism in judgment, where subjects only adjust their forecast gradually toward recent history. The extrapolative parameter, a_2, is significant and positive in all cases, though the values vary a great deal. In fact, in over half the cases, the parameter is greater than unity, indicating that changes in demand are expected to accelerate. Extrapolative forecasts are particularly prevalent in the first market (Mkt 25). Looking back at the observed behavior of that market (the observed outcomes are reproduced in Figure 5.4.1), it seems clear that subjects are extrapolating extensively during the first part of the game, thus creating the very large overshoot. It seems likely, therefore, that a lot of extrapolation will create more instability.

5.4 Simulated versus actual behavior

The most interesting question of the analysis in this chapter is whether the decision- and forecast rules can also function to reproduce the observed patterns of behavior. Thus, in the following, the estimated parameters in these rules were used in a simulation model of the entire market.

Figure 5.4.1 shows the behavior of these simulation models, with no external noise anywhere in the models. The original outcomes are reproduced in Figure 5.4.2. Three things become readily apparent when comparing the two sets of figures: First, the relative variability of the four markets is the same as the observed actual behavior: Mkt 25 shows a large overshoot and collapse, Mkt 26 shows some tendency to oscillate, while Mkt 35 and Mkt 36 show more sustained cycles. Second, the periodicity and phase relationships between production, sales, and inventory are approximately the same in the simulated and actual outcomes. Third, the variation is smaller in all of the simulated markets except possibly Mkt 25, where simulated inventories are more stable but production varies a bit more than the observed. The regularity of this pattern is quite striking, especially considering the fact that the outcome is completely endogenously generated!

The slightly greater stability of the simulated markets without noise is an example of the well-known bootstrap effect (Dawes 1979). The phenomenon can arise in a wide variety of judgment tasks, and most likely has its origins in the fact that random variance in any decision or judgment can be shown analytically to decrease performance in a wide variety of situations (Camerer 1981). The effect has also been shown to be present in complicated, dynamic decision tasks, such as realistic simulations of competitive markets (Hogarth and Makridakis 1981b).

In order to "remove" the bootstrap effect, the markets were simulated again, this time introducing a normally distributed random error in both forecasts and decisions, with a variance equal to the mean squared error of the regression. Figure 5.4.3 shows examples of such simulated behavior for each of the four markets (only inventories are shown).

Although there is still some residual bootstrapping effect, the simulated stochastic behavior of the market appears to be very close to the observed outcome. Thus, one must say that the reproduction is excellent! It is interesting to note that in three of the four markets, the simulated market does slightly better (i.e. is slightly more stable) in the first part of the run but slightly worse or no better than the observed market in the second part of the run. Thus, it appears that there is a limited amount of learning happening in the real system. Since the simulation assumes the parameters are constant throughout the course of the game, it produces an "average" performance throughout.

5.5 Simulated versus actual performance

In order to compare the simulated and observed outcomes in a more compact and formal way, two key performance measures were computed and compared. The first of these is the average profits earned. The observed profits were compared both to the deterministic simulations and to the average for a large number of the stochastic simulations. The results are shown in Figure 5.5.1.

As can be seen, the correlation between the simulated stochastic and observed average performance is very close. The correlation coefficient between observed and simulated deterministic and stochastic profits is .997 and .999, respectively. While the deterministic simulations clearly show a boot-strapping effect (significant at p=.009), the hypothesis that the observed and stochastic performance are equal cannot be rejected at levels below p=.25.

The close correspondence between simulated and observed performance is also present for individual firms, as can be seen in Figure 5.5.2. The correlation coefficients for the deterministic and stochastic profits with observed profits are .962 and .982, respectively. There is a significant boot-strapping effect in the deterministic case (p<.0005) but not in the stochastic one, although the hypothesis that observed and simulated stochastic profits are "equal" is rejected at p=.04.

In short, one must conclude that the simulated stochastic performance is an excellent predictor of the observed performance, both for the individual firms and for the market as a whole.

The second key indicator is the average variability in the market, measured by the average standard deviation of production. Figure 5.5.3 compares the observed and simulated variability for each of the four markets. As before, the correspondence seems quite close. The correlation coefficient between observed and simulated deterministic and stochastic variability (the logarithm of the variance) is .936 and .957, respectively. The figure also confirms the impression from Figure 5.5.3 that the variability of the simulated markets are slightly lower, except for Mkt 25. (In this market stability and variance are not correlated due to the non-negativity constraint on production.)

It is thus interesting to note that, although the regressions of individual decisions and forecasts may not always fit very well, though they generally do, the overall performance of the market is very close to the observed. In other words, the fit seems to become better in the aggregation, at least along the key dimensions of variability and profits.

5.6 Key determinants of performance

Although there is clearly a bootstrapping effect when one replaces subjects by linear approximations, the effect is dwarfed by the difference between the markets themselves. It is thus of great interest to be able to determine what features of the market seem to be the most crucial determinants of performance.

The simulations in Chapter 3 demonstrated that the attention to the supply line was a key determinant of performance. Thus, one would expect there to be a correlation between observed performance and the estimated weight, α_s on the supply correction in the production equation 5.2.2. The lower this weight, the more oscillation one would expect, and the lower profits.

Another factor which may exacerbate the problem is the degree to which subjects extrapolate changes in demand. Such extrapolation is evidence of lack of appreciation of the endogenous influences on demand. If firms strongly extrapolate recent changes in demand, the strength of the self-reinforcing feedback loop involving the multiplier effect is greatly increased. Thus, one would expect the degree of extrapolation (measured by the parameter a_2 in equation 5.3.1) to worsen performance.

Finally, the effects of insufficient attention to the supply line and failure to take the multiplier into effect when assessing future demand are made much worse if subjects react very aggressively to inventory imbalances: If firms are quite calm and conservative in their reaction to an inventory imbalance, the changes in production, and thus in demand, will be much slower and smaller. Although the adjustment will be more costly than the prompt aggressive and <u>correct</u> response in the optimal policy, the slow conservative policy may work well because it avoids setting off the above-mentioned destabilizing mechanisms. Thus, one would expect that, given an insufficient attention to supply lines and given a tendency to extrapolate demand, a greater aggressiveness of inventory adjustment (measured by the parameter α_n in equation 5.2.2) will worsen performance.

In Figure 5.6.1, the correlation of profits with each of these three parameters is shown, both for individual firms and for market averages. Visual inspection of the figure suggests that the correlation, both for market averages and for individual firms, has the right sign in each of the three cases. Thus, profits are negatively correlated with extrapolation (a_2) and inventory adjustment aggressiveness (α_n), and positively correlated with supply line attention (α_s). The correlations are investigated more formally in Table 5.6.1, which shows a regression of profits on the three parameters. It can be seen that both the decision-rule parameters are highly significant with the correct sign while the extrapolation parameter is not significant. Thus, one must conclude that there is strong evidence that the two decision rule parameters are key determinants of performance--the R^2 of the regression is .95. One cannot reject the hypothesis that extrapolation has no effect on performance: Its effect is dwarfed by the supply-line and inventory adjustment parameters.

5.7 Summary and conclusions

This chapter analyzed the observed behavior in the fixed-price complex condition by positing a simple stock-adjustment rule and a simple adaptive-extrapolative rule for expectation formation. The analysis in Section 5.2 showed that the simple stock-management rule fits the observed decisions excellently: A very high proportion of the observed variance was explained by the rule. Moreover, the analysis revealed that none of the subjects paid full attention to the necessary supply-line correction: The weights on the supply line were all lower than the corresponding weights on inventories. Such low weights can be interpreted as evidence that subjects are not fully perceiving the importance of the long delays involved in production and the need to correct for actions taken earlier but not yet manifested. In the answers to the post-game questionnaires, most subjects express some frustration at the difficulty of "keeping up" or "correcting mistakes," and a few mention directly that they were surprised at the significance of the production lag.

The forecasting rule showed a more modest fit, though still quite high by most standards in studies of judgment (see e.g. Dawes and Corrigan 1974). The parameter estimates also showed a marked tendency to extrapolate changes in demand. The tendency to extrapolate demand is evidence that subjects do not fully understand the importance of the endogenous forces in the system: It shows a tendency to view changes as externally-imposed factors over which one has relatively little control. Indeed, an informal content analysis of the post-game questionnaires revealed that 12 out of the 14 subjects emphasized forecasting "the business cycle" or "trends" or "shifts" in demand by extending recent trends. Only two subjects made any reference to the multiplier effect. The following quote from a participant in Mkt 35 is typical:

> For the first 15 to 20 periods, I was just trying to keep up and my profits swung from 2500 to -1500. Then, once I had the general pattern of a complete business cycle, I was able to make estimates of the average increase or decrease per period. That way, I could numerically estimate how much the sales would move to a reasonable degree and thus how much to set production targets. It became clear after a while that given the instability of sales and the constant prices, profit maximization became simply inventory minimization.
>
> The major problem was determining the timing of the peaks and troughs of the business cycle, and my guess is that it's mostly due to external factors and thus hard to pinpoint exactly.

Moreover, when asked to sketch out their guess of the pattern of external factors affecting demand (there were none!), all but one subject sketched a pattern resembling the observed average sales in that market.

The analysis in Section 5.4 showed that it is possible to explain most, if not all, features of the observed behavior in terms of the suggested simple decision and forecast rules. The fit between observed and simulated behavior must be characterized as excellent, although the endogenous simulation will naturally not achieve a point-by-point fit to history. The simulations showed a classic bootstrapping effect: When the estimated rules were simulated with no random variation, the simulated markets showed greater stability and better performance than the observed. However, when the "noise was put back in," the simulated markets showed patterns very similar to the observed.

Thus, the complete characterization of the behavior must include the "inconsistencies" in human decision making which interact with the systematic portions to produce the outcome.

One might argue that an explanation involving random noise is no explanation at all. However, it is a well-known result from engineering control theory that systems behave differently when subjected to deterministic and stochastic inputs: A system which may be highly damped in its response to a step input may exhibit sustained oscillations when "driven" by noise. The noise provides a continuous source of energy which fuels the oscillation. The explanatory power lies in the ability to predict how different systems, in this case different markets, will react to noise inputs, given the systematic components of the decision rules of that market.

In Section 5.5, the observed average profits and average variability were correlated with the corresponding simulated measures. The correlation was very high, indicating that the simulated outcomes are good predictors of the actual. Thus, by testing individual subjects in isolation to determine the parameters of their decision rule, one might very well be able to predict quite closely how these subjects, when brought together in a market, will make that market behave and perform.

Finally, Section 5.6 identified two key components of performance: The degree of attention to the supply line, and the aggressiveness of responses to inventory imbalances. Both effects had the expected direction and were highly significant: A lower weight on the supply line correction drastically lowers performance; a high (i.e. aggressive) weight on inventory correction, given the low observed attention to the supply line, likewise lowers performance. The last component expected to affect performance, the degree of extrapolation, did not turn out to be significant. The failure to show significance does not mean the effect isn't there, however. The coefficient still has the right sign, and with such a small sample size, compared to the number of parameters, the significance test has very low power.

In conclusion, the analysis has clearly demonstrated that it is possible to both identify heuristics at the individual level and to integrate this micro-level observation with macro-level outcomes, through the use of simulation. Moreover, the results show strong evidence of "misperceptions of feedback": Subjects showed a marked tendency to extrapolate demand and attribute changes to external causes, insufficient attention to the supply line correction, and general underestimation of the importance of the production lag. Moreover, the analysis showed that these misperceptions had direct, predictable, and substantial consequences for market behavior and performance.

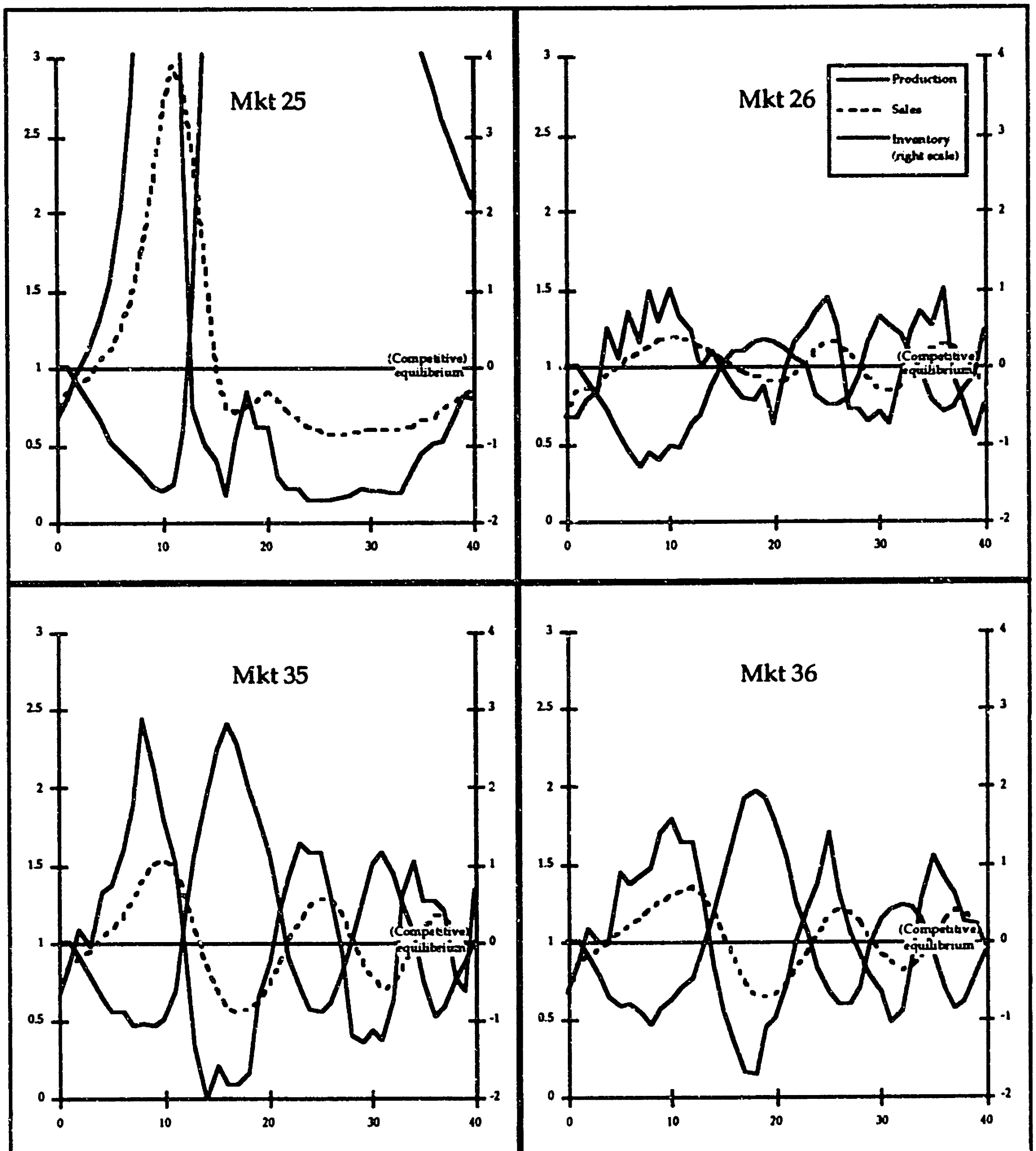


Figure 5.4.1: Observed behavior of market averages in the fixed complex condition

All measures are relative to the competitive-equilibrium output.

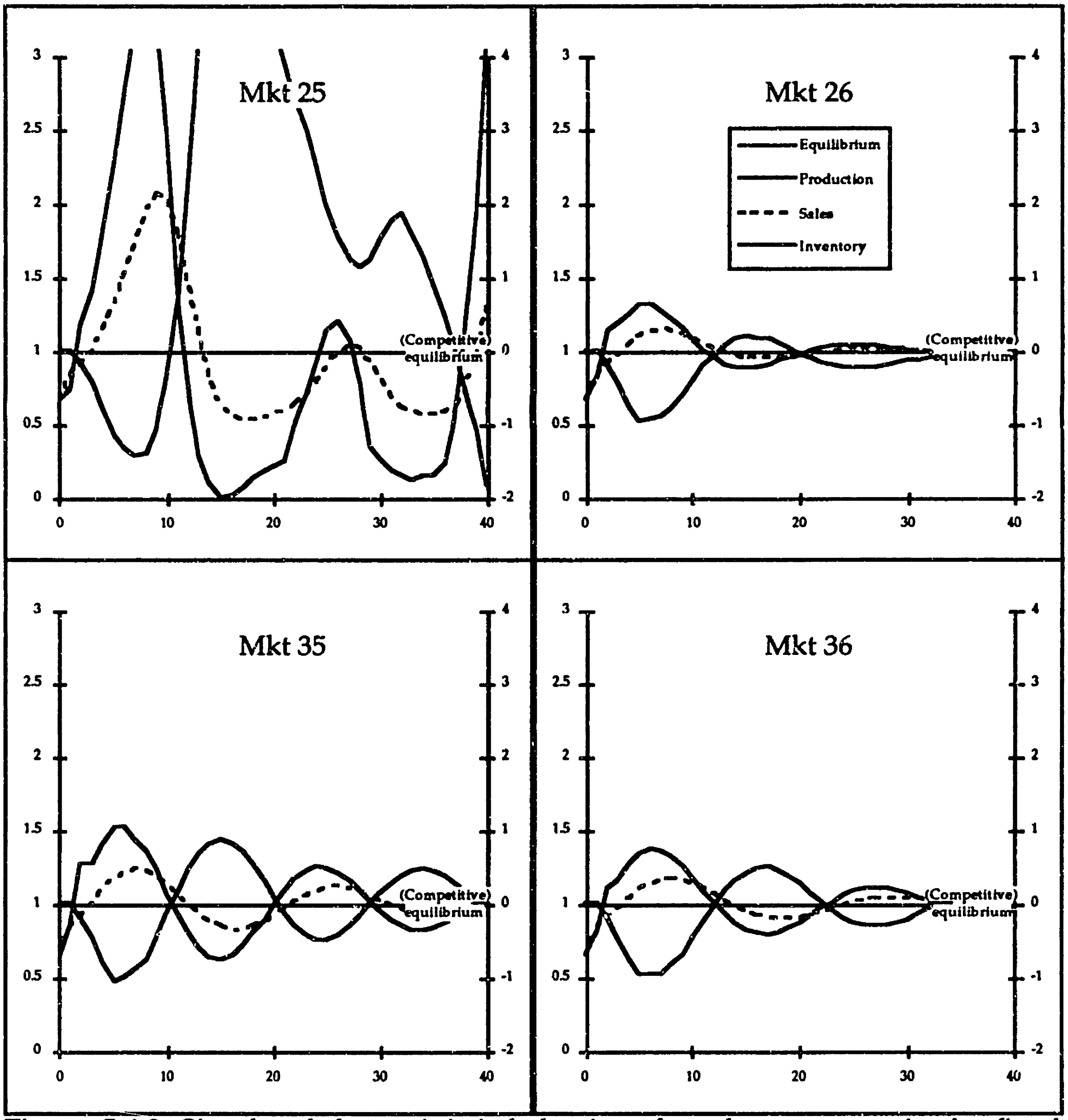


Figure 5.4.2: Simulated deterministic behavior of market averages in the fixed complex condition

The figure shows the output of simulation models in which firms were assumed to follow the decision- and forecast rules proposed in the analysis in Section 5.3. The estimated parameters for each subject were entered in the model in a "deterministic" simulation with no random variation in the decision rule. All measures are relative to the competitive-equilibrium output.

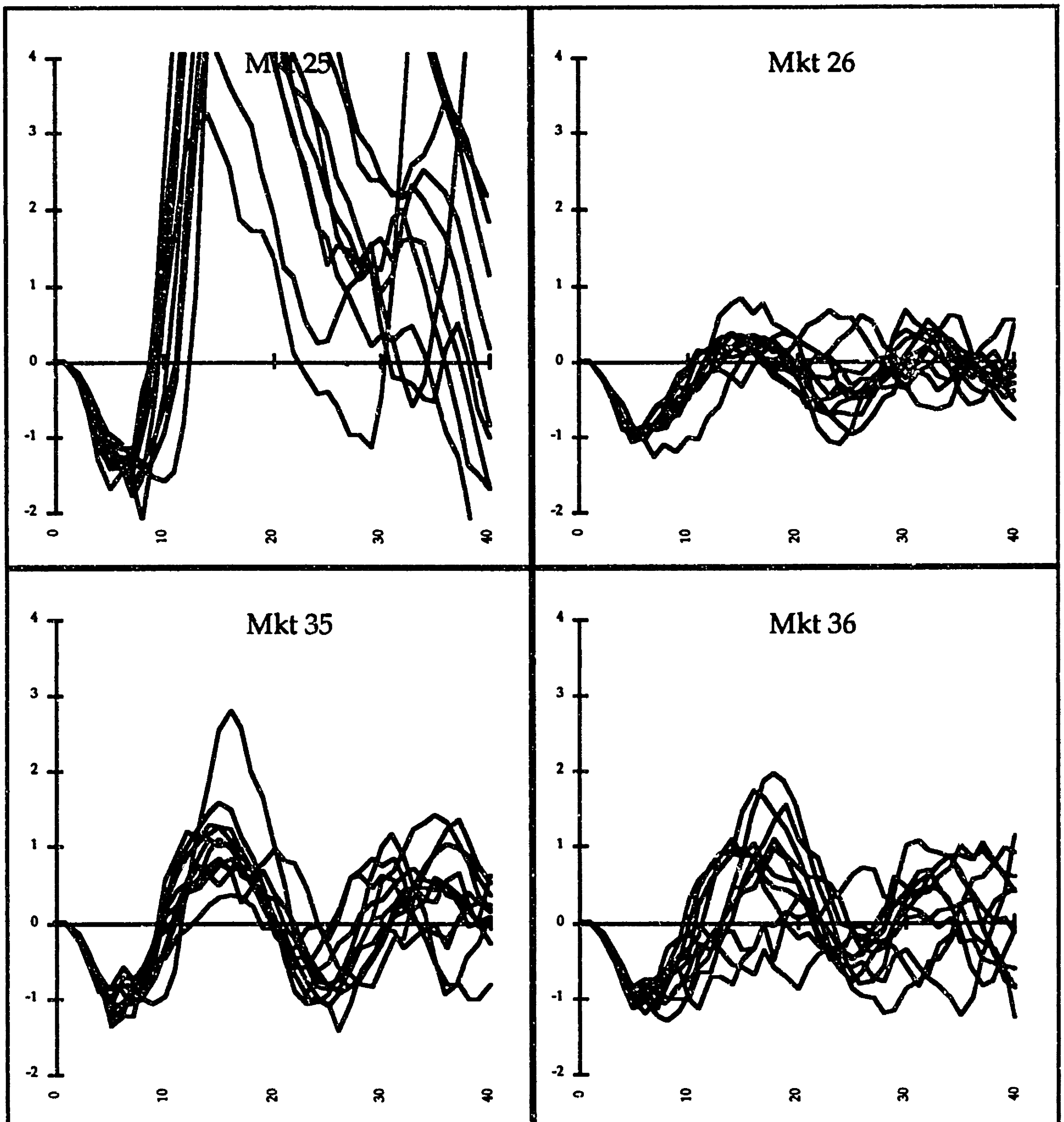


Figure 5.4.3: Simulated stochastic behavior of inventories in the fixed complex condition

The figure shows the results of simulating the same models as in Figure 4.3.2, but this time introducing a random (normally-distributed) error with a variance equal to the mean squared error of the regressions of the decision rules. The figure compares the observed actual outcome (fat line) with 10 simulations (thin lines). For clarity, only the average inventory level is included. Measures are relative to the competitive-equilibrium output.

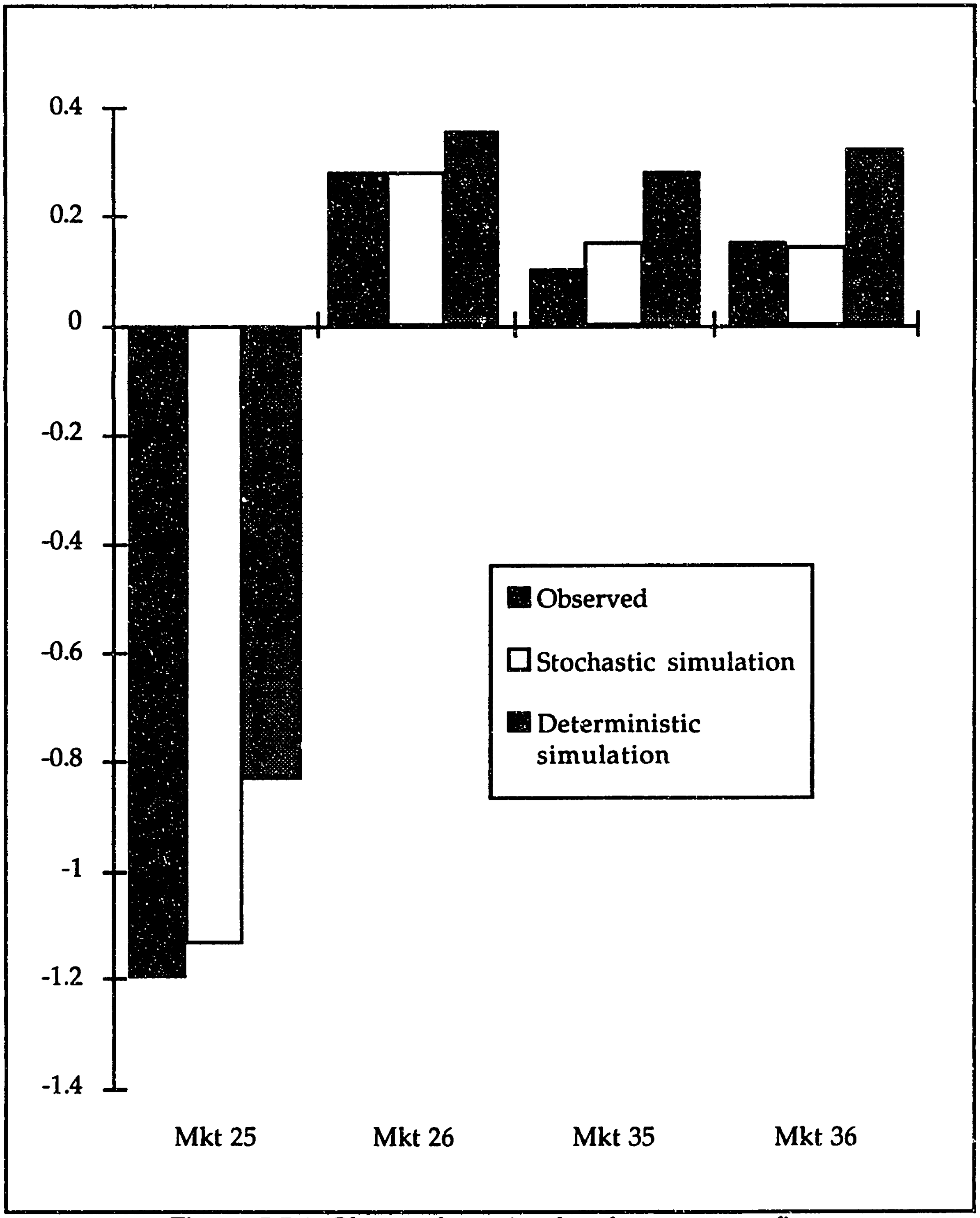


Figure 5.5.1: Observed vs. simulated average profits

The figure compares the observed market average profits to the corresponding measure for the deterministic simulation in Figure 5.4.2 and the average of 20 stochastic simulations like those in Figure 5.4.3. (The deterministic result is shown last to highlight the boot-strapping effect, i.e. the fact that the simulated profits, though strongly correlated with the observed, are also consistently somewhat higher.) All measures are relative to the competitive-equilibrium output times the competitive-equilibrium price. Thus normalized, the competitive-equilibrium profit is 0.4.

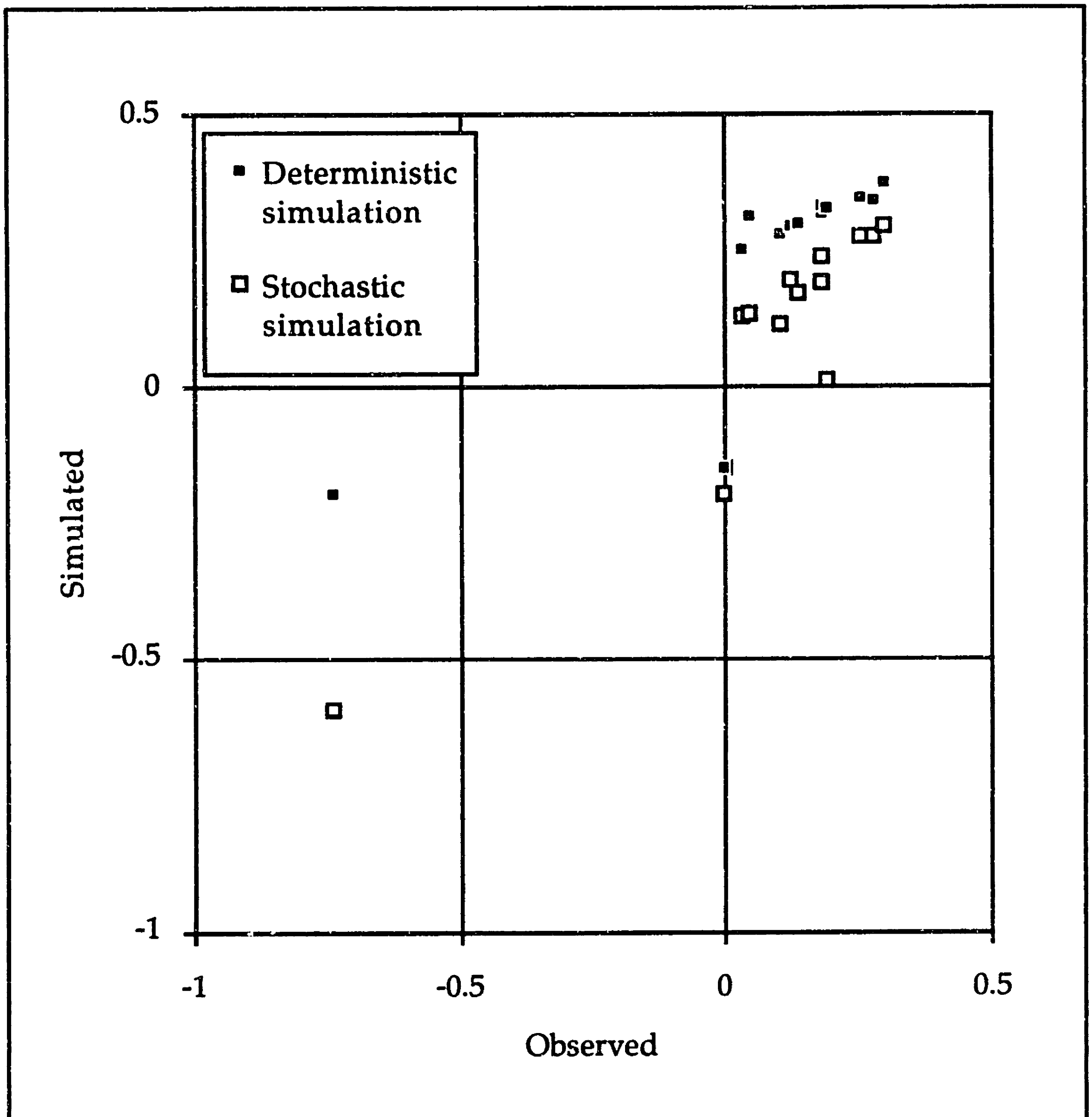


Figure 5.5.2: Correlation between observed and simulated individual-firm profits

The simulated and observed profits of one firm were so low that they fall outside the chosen scale of the chart. The observed profits were -2.6 while the simulated deterministic and stochastic profits were -2.1 and -2.9, respectively. All measures are relative to the competitive-equilibrium output times the competitive-equilibrium price. Thus normalized, the competitive-equilibrium profit is 0.4.

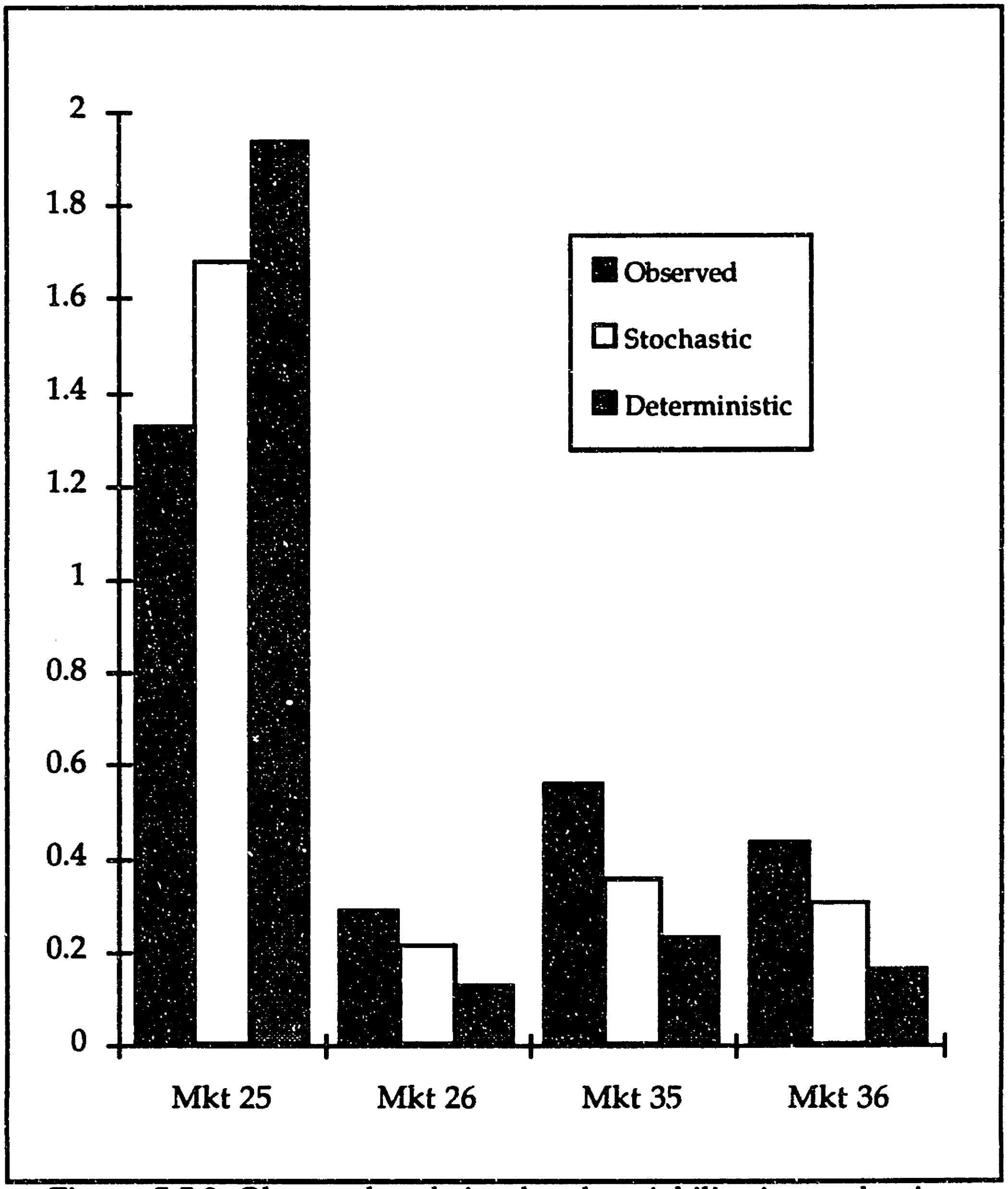
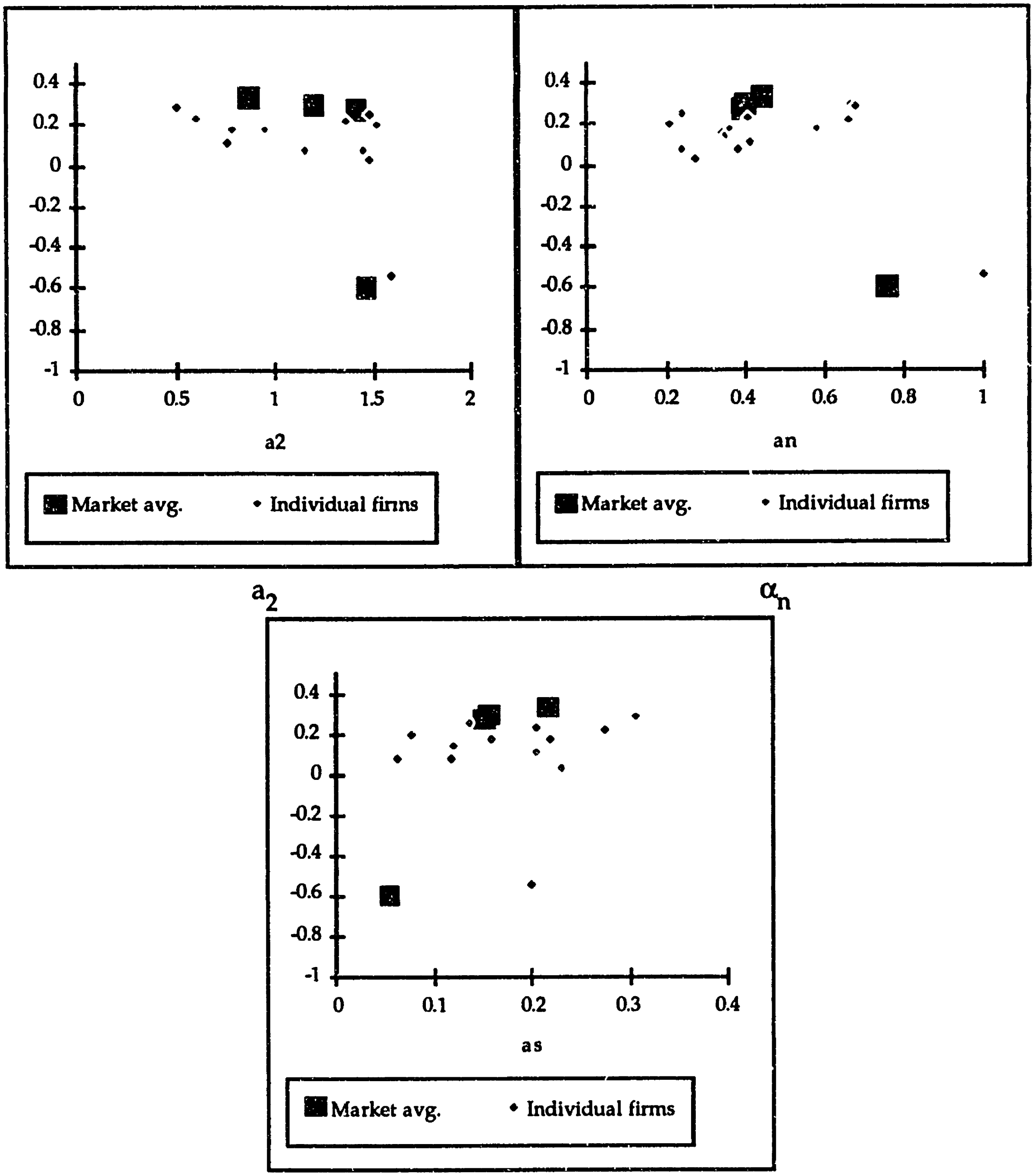


Figure 5.5.3: Observed and simulated variability in production

The figure compares the observed standard deviation in average production with the corresponding simulated measure, using the same simulation models as in the previous section, i.e. a deterministic and a stochastic model, respectively. All measures are relative to the competitive-equilibrium output.



α_s

Figure 5.6.1: Correlation of firm- and market-average profits with decision rule parameters

The figure shows the correlation between the observed average profits earned and three key parameters in the decision rules: the degree of extrapolation in the forecasting equation (a_2), the aggressiveness of the inventory correction (α_n), and the degree of supply line correction (α_s). The graphs plots both the values for individual firms and the average for each of the four markets. The measure shown is profits relative to equilibrium output times equilibrium price. Thus normalized, the equilibrium profits are 0.4.

Market	Firm	α_n	Std. error*		α_s	Std. error*		s^d	Std. error*		R^2
Mkt 25	1	.28	(0.21)		.23	(0.2)		1.44	(1.00)		.91
Mkt 25	2	1.00	†		.20	(0.03)	a	3.04	(0.77)	a	.86
Mkt 25	3	1.00	†		-.27	(0.03)	a	1.55	(0.88)	c	.94
Mkt 26	1	.24	(0.24)		.14	(0.19)		2.64	(0.96)	a	.97
Mkt 26	2	.68	(0.36)	c	.31	(0.21)		3.00	(0.37)	a	.97
Mkt 26	3	.41	(0.22)	c	.21	(0.17)		2.73	(0.6)	a	.94
Mkt 35	1	.24	(0.12)	b	.06	(0.09)		3.58	(1.84)	c	.95
Mkt 35	2	.41	(0.12)	a	.20	(0.09)	b	2.59	(0.56)	a	.88
Mkt 35	3	.35	(0.18)	b	.12	(0.1)		3.58	(0.95)	a	.97
Mkt 35	4	.58	(0.17)	a	.22	(0.1)	b	3.52	(0.5)	a	.95
Mkt 36	1	.36	(0.17)	b	.16	(0.13)		2.53	(0.79)	a	.90
Mkt 36	2	.39	(0.12)	a	.12	(0.08)		3.34	(0.98)	a	.85
Mkt 36	3	.20	(0.19)		.07	(0.14)		2.83	(1.55)	c	.99
Mkt 36	4	.66	(0.24)	a	.27	(0.14)	b	3.33	(0.42)	a	.97

† The estimated parameter was greater than 1. The equation was reestimated with the parameter constraint to the interval [0,1].
* The value shown is the computed asymptotic standard error of the estimate, based on the estimated Hessian (2nd derivative) matrix of the loss function (the sum of squared residuals). The small-sample properties of this estimate are not known, and the results should therefore be interpreted with caution. Assuming the estimate is normally distributed, the column also shows the result of testing for a non-zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.

Table 5.2.1: Parameter estimates and R^2 of the decision rules 5.3.2

Market	Firm	a_1	Std. error*		a_2	Std. error*		R^2
Mkt 25	1	.45	(.14)	a	1.47	(.42)	a	.23
Mkt 25	2	.29	(.07)	a	1.58	(.15)	a	.75
Mkt 25	3	.09	(.06)	a	1.31	(.15)	a	.72
Mkt 26	1	.71	(.16)	a	1.47	(.33)	a	.43
Mkt 26	2	.47	(.16)	a	0.50	(.25)	a	.30
Mkt 26	3	.32	(.13)	a	0.59	(.21)	a	.35
Mkt 35	1	.62	(.16)	a	1.45	(.27)	a	.41
Mkt 35	2	.28	(.12)	a	0.75	(.24)	a	.25
Mkt 35	3	.93	(.16)	a	2.51	(.35)	a	.56
Mkt 35	4	.37	(.13)	a	0.94	(.2)	a	.41
Mkt 36	1	.22	(.11)	a	0.77	(.18)	a	.57
Mkt 36	2	.36	(.1)	a	1.15	(.17)	a	.60
Mkt 36	3	.58	(.15)	a	1.52	(.23)	a	.54
Mkt 36	4	.21	(.15)	a	0.07	(.15)	a	.05

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.

Table 5.3.1: Parameter estimates and R^2 in the forecasting rule 5.3.1

DEP VAR: NPOB N: 14 MULTIPLE R: .972 SQUARED MULTIPLE R: .946
ADJUSTED SQUARED MULTIPLE R: .929 STANDARD ERROR OF ESTIMATE: 0.176

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	0.061	0.200	0.000	.	0.304	0.768
A21	0.022	0.099	0.017	0.893	0.218	0.832
ALFAN1	-1.307	0.192	-0.522	0.919	-6.791	0.000
ALFAS1	3.385	0.376	0.715	0.859	8.992	0.000

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	5.366	3	1.789	57.989	0.000
RESIDUAL	0.308	10	0.031		

Table 5.6.1: Regression of average firm profits as a function of decision-rule parameters.

The regression treats each firm as an observation because the market-average sample size of four is too small to allow analysis. However, when each of the three parameters are regressed in isolation on the market averages, the significance levels are approximately the same or lower as those computed in the regression.

6 Behavior under clearing prices

6.1 Introduction

In the clearing-price condition, the price vector that will clear the market each period is found automatically by the computer so that firms sell all of their finished output each period and inventory or backlog accumulation is eliminated. The condition corresponds in some respects to most experimental-economics settings where there is typically no distinction between production and sales or transactions. In many ways, one might therefore consider the clearing-price complex condition an extension of the typical experimental market through the introduction of lags and complexity, but with many of the characteristics of the institutional setting otherwise preserved.

Since firms do not determine prices directly, they have only a single decision to make each period, namely their output level. Because prices are now varying, they are also asked each period for both a demand- and a price forecast. Unlike the fixed-price conditions in the previous chapter, there is now a possibility for collusive actions, signalling, and other strategic interactions between firms. Moreover, since there is no inventory accumulation, subjects' attention can be fully focused on this aspect.

The process of searching for the best price-output point, signalling collusion to other firms, punishing defectors, etc., is likely to be more difficult to model with simple regression equations than is the stock-adjustment task in the fixed-price condition: Firms are likely to react to cues and events which may be quite specific to particular recent patterns in prices and output.

Nonetheless, the fact that all four of the markets in the complex clearing-price condition show roughly the same pattern indicates that subjects respond systematically to certain cues in prices.

Indeed, the analysis below shows that for the simple clearing-price, the observed pattern of price changes is replaced in the simulation by a smooth adjustment toward an equilibrium which is approximately equal to the average of the observed time series. Any other pattern, such as short- or long-term fluctuation, is lost. In the complex case, however, the main feature--the initial overshoot followed by the gradual settlement--can be reproduced by a simple decision rule that uses parameters estimated from the observed decisions.

Since expectations of future price and sales are clearly important elements of decision making, Section 6.2 investigates the solicited forecasts from the experiment, fitting them to a simple adaptive-extrapolative rule similar to the rule used in the fixed-price condition. Moreover, the price- and demand forecasts are analyzed together for internal consistency. A higher price forecast should, all else equal, lead to a lower demand forecast, and vice versa. Since all subjects receive the same input stream of historical aggregate price and demand, the residuals of the simple forecasting rules are also compared across subjects to detect whether subjects deviate from the rule in a systematic way.

Section 6.3 analyzes the output decisions in the simple clearing-price condition, by fitting them to a simple adaptive rule that is based on perceived profit gradients (similar to the rule proposed in Chapter 3). The simple rule is further tested by embedding it in an endogenous simulation of the entire market, using the estimated parameters in decision- and forecasting rules. In similar fashion, Section 6.4 compares the observed decisions in the complex clearing-price condition to the optimal rule and to an adaptive decision rule. Finally, section 6.5 summarizes the results and discusses the insights from the analysis.

6.2 Behavior of forecasts

The analysis in Chapter 4 showed that the forecasts of firms fail the test of "rational expectations", i.e. they are not unbiased or efficient predictors of actual prices and quantities. It is always possible to rationalize the observed forecasts by saying that subjects made particular assumptions about the behavior of other firms which turned out not to be fulfilled, but one would at least expect the forecasts of rational firms to be internally consistent: a prediction of higher prices should, ceteris paribus, also imply a prediction of lower demand.

In the simple condition, there is a one-to-one relationship between aggregate demand and aggregate price, defined by the aggregate demand curve. Since the game software allowed subjects to plot the two variables against each other and there is no noise in demand, one would expect them to quickly discern the relationship and, accordingly, adjust their forecasts to lie on or close to the demand curve.

In the complex condition, on the other hand, demand depends on both aggregate price and, through the multiplier effect, on the aggregate supply line. Consequently, one cannot measure whether a particular pair of price- and demand forecasts is internally consistent, since the third variable, the forecast of the future aggregate supply line, is not explicitly stated.

Nonetheless, some impression of consistency can be gained by making assumptions about this variable. In light of the results in the previous chapter, where subjects paid very little attention to the supply line and the multiplier, it is unlikely that subjects in the clearing condition should make very complicated or sophisticated forecasts of this variable--most likely, they ignore it.

In the analysis below, the consistency of price and demand forecasts is tested by comparing the demand forecast to the demand implied by the price forecast. In the simple condition, the relationship is determined by the aggregate demand curve. In the complex condition, two alternative derived demand measures were used: a "steady state" demand, equal to the long term steady-state demand implied by the given price forecast if prices remained constant forever at that level, and an "instantaneous" demand, equal to the demand implied by the price if the supply line remained equal to its current value. The two measures represent opposite ends of a range of assumptions, from a complete steady-state adjustment to no adjustment at all of the supply line.

Figure 6.2.1 plots the demand forecasts and the derived demand based on the price forecasts for the simple clearing condition. If the two forecasts were internally consistent, the points should all lie on a 45° line. Indeed, for most firms, the points fall quite close to that line, indicating that subjects are aware of the demand-curve relationship.

In similar fashion, Figure 6.2.2 plots the observed and derived demand forecasts in the complex clearing condition. Both the "instantaneous" and the "steady-state" demand measures are shown. Again, there is clearly a strong correlation between the measures, though the consistency appears substantially lower than in the simple condition. Moreover, the forecasts appear to conform more to the "instantaneous" demand curve than to the long-run steady state.

These observations are confirmed in Table 6.2.1 which shows an analysis of the "average percentage discrepancy" between the observed demand forecast and the derived instantaneous demand measure. The error has further been normalized by the standard deviation of the derived measure to allow for the fact that the variance in the forecasts themselves differs between markets.[1] Table 6.2.1 shows a 2-by-2 factorial analysis of variance of this measure, averaged over each market, using price regime (P = clearing or posted) and complexity (C = simple or complex) as factors.[2] It is clear that there is a significant effect of complexity on consistency but no effects of price regime. On average, the normalized standard error in the two complex conditions was almost twice that of the simple conditions (cf. Table 6.2.1).

Nevertheless, forecasts in both the simple and complex clearing-price condition clearly conform partly to the demand-curve relationship. One

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1 Specifically, the measure is

$$\frac{\sqrt{\frac{1}{n}\Sigma(F - D)^2}}{\sqrt{\frac{1}{n}\Sigma D^2 - (\frac{1}{n}\Sigma D)^2}}, \text{ or } \frac{\sqrt{\frac{1}{n}\Sigma(F - D)^2}}{\sqrt{\text{Var}\{D\}}}.$$

where F is the forecast, D is the derived demand measure, and n is the number of observations.

2 The analysis thus treats each market as one observation. This is necessary since the consistency measures are correlated between firms within a particular market.

might ask whether the observed relationship between the two forecasts might be the result of a conscious consideration on the part of the subjects, or whether it is simply an artifact of the fact that the market tends to move along such a demand curve. For instance, if firms simply gave recent aggregate price and sales as their forecast, these variables would obviously be absolutely consistent with the structural relationships in the system

To address this question, the forecasts were first regressed on the same adaptive-extrapolative rule used for the demand forecasts in Chapter 5. For the demand forecast, X_t^e, the equation is

(6.2.1) $$X_t^e = X_{t-1}^e + a_1(X_{t-1} - X_{t-1}^e) + a_2(X_{t-1} - X_{t-2}).$$

A similar equation was used for the price forecast.

The demand forecasts in the complex clearing-price condition present a special case, however. Since demand (or sales) is always equal to completed production, forecasting demand three periods into the future means forecasting the current period's aggregate production starts. Therefore, the demand forecasts in this condition were assumed to depend on a mixture of sales, X, and production starts, Y, i.e.,

(6.2.2) $$X_t^e = X_{t-1}^e + a_1(X_{t-1} - X_{t-1}^e) + a_2(Y_{t-1} - X_{t-1}^e) + a_3(X_{t-1} - X_{t-2}) + a_4(Y_{t-1} - Y_{t-2}).$$

One would expect that, since sales, X, is simply a delayed measure of output it does not carry as much information about future output as does the more recent measure, Y. Moreover, both numbers were readily avialable to subjects, both in the main display and in the tables and graphs. Therefore, subjects ought to put more weight on the aggregate production starts than on sales.

Tables 6.2.2 and 6.2.3 show the regression results for the clearing simple and the clearing complex condition, respectively. Generally, the rules provide a reasonably good explanation of the observed changes in the forecasts, though the fit is marginally better in the simple condition. The average correlation coefficient, R^2, for both rules in the simple condition is .58, and the coefficients are all in the expected range.[3] In the complex condition, the demand forecasting rule has an average R^2 of .49 while the price forecasting rule does not perform quite as well--the average R^2 is .31. The coefficients in both rules are all within (or at least not significantly outside of) the expected range.[4]

There is some evidence that firms in the clearing complex conditions do indeed use recent production rather than sales in forecasting future sales. The average adjustment weight on production, a_2, is .35 while the corresponding weight on sales, a_1, is .20. Table 6.2.3 also shows a test for whether firms over time "learn" to put more weight on production than on sales when making their sales forecast. The test was performed by using separate coefficients for the first and second half of the sample and then testing whether the two coefficient sets were equal. There is little evidence of any change, except in the last market (Mkt 40), where three out of five firms

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3 The "adjustment" coefficient, a_1 in equation 6.2.1 should be between 0 and 1. The extrapolative coefficient could have a wider range of values: a positive value would indicate a trend extrapolation while a negative one would indicate regressive expectations, i.e. the belief that demand will regress towards the mean.

4 The two "adjustment" coefficients, a_1 and a_2 in equation 6.2.2 should both be non-zero ansd should sum to less than or equal to 1.

do show evidence of having changed their decision rule. Indeed, those very same firms also show evidence of putting a high weight on Y as opposed to X.

Now, to detect whether firms consciously adjust their two forecasts to make them mutually consistent, the residuals of the two rules were correlated with each other. If there were no conscious adjustment, one would not expect correlations in the residuals. If, on the other hand, there were full adjustment, the residuals should be negatively correlated, with a coefficient approximately equal to the slope of the demand curve.[5]

Table 6.2.4 shows the results. In the simple condition, it is very clear that the residuals are indeed correlated, and the coefficients are all close to though slightly lower than the observed average slope of the demand curve (about -0.75). In the complex condition, about half the firms also show significantly correlated residuals, and all but one of the coefficients are negative. In short, there is evidence that firms do indeed consciously consider the demand-curve relationship when making their forecasts, especially in the simple condition.

Since all firms in a market receive the same cue inputs to their forecasts, i.e., they see the same pattern of aggregate price and demand, it is conceivable that the residuals are also correlated across firms within markets. Accordingly, Table 6.2.4 also shows the result of testing for this correlation.[6] There is clearly a significant correlation of residuals in the simple condition,

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5 However, correlated residuals could reflect common variation in some other variable not captured in the equations. Hence, a negative correlation is perhaps an indication but not a proof that subjects accound for the demand curve.

6 The test statistic is $\chi^2 = -[\ N-1-(2p+5)/6)\ ]\ \ln|\mathbf{R}|$, where N is the sample size, p is the number of variables, and $|\mathbf{R}|$ is the determinant of the Pearson correlation matrix.

both for price- and for demand forecasts. The complex condition shows no significant correlation.

The correlation in the residuals across firms in the simple condition indicates that the simple adaptive-extrapolative rule does not capture all of the systematic forecasting behavior: all firms are "doing something" beyond what is expressed in the rule.[7] For instance, it is possible that the "true" rule contains higher-order non-linear terms reflecting the belief that the market variables stay within a certain range. Another possibility is that the forecasts are tied to the strategic interactions between firms. However, a full investigation would require a much more detailed look at both the period-to-period decisions of the firms and the protocol data. For the purposes of the present study, the simple rule does quite well in that it captures at least half of the observed variation in the movement of forecasts.

6.3 Behavior in the simple condition

The optimal, non-cooperative decision rules are identical in the simple and the complex clearing-price condition. The structure of the market implies that a profit-maximizing, non-cooperative firm should strive to bring its own price as close as possible to the competitive-equilibrium price. If firms followed this rule (subject only to a random error) and if they further had rational expectations, prices should vary randomly around the competitive-equilibrium price. Clearly, neither the simple nor the complex markets pass

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7 Significant cross-correlation in the residuals indicates that some estimation efficiency could be gained by using e.g. Zellner's method for "seemingly unrelated regressions" (see e.g. Johnston (1972), p. 238), Unfortunately, software errors in the statistics package used for the analysis prevented the author from carrying out this analysis.

this stringent requirement, and one must therefore search for alternative explanations of the observations.[8]

As already mentioned in the introduction, there is evidence in the post-game questionnaire data that most firms were highly aware of the strategic interactions between them and their competitors. 10 of the 16 firms in the simple clearing condition explicitly mention attempts at collusion, signalling, defection, or other strategic considerations, in their answers. Moreover, all but one firm discuss their strategy in terms relative to the market average. Here are some examples of responses:

> My production strategy was basically dictated by the effect it would have upon prices and on the results this would have upon my opponents. When I saw a trend->high production, I went for low myself, thereby maximizing my own price without too much influence on theirs. The average mkt. price and sales were undoubtedly the most important numbers...

> I was trying to keep prices high by keeping down production. After the first practice round, it was obvious that smaller production led to higher prices. Since people kept producing more, I tried to send a signal by producing 1 unit. I tried to decide as quickly as possible in order to have more periods (avg. time ~ 20 seconds). I tried to find my optimum varying the amount greatly in early rounds. The most important factors to me were last period profits, avg. production, my production. I alternated from trying to send messages to trying to make more money.

> The first few periods I tried to determine where to produce. It seemed that being a little above the average was best but the

8 The approximate optimal, non-cooperative production rule, based on expected aggregate output, X^e and price P^e (by the time the output is ready for delivery), is $\ln(y) = \text{const} + \ln(X^e) - (1/\varepsilon)\ln(P^e)$. The observed output was regressed on the observed forecasts according to this equation, and the coefficients in this regressions were then tested for having the correct values. In both the simple and complex clearing condition, every firm failed the test.

> average would rise and profits would shrink. My production depended largely on the production from the turn before, since most of the time the spread didn't make it worthwhile to be below the average. Eventually, if we had gotten to 500 and stayed there we would have been well off. But no!
>
> Early in the game, I had been setting my production at about 100 units above average. This seemed to work well since in the financial table, I was continually about average. I never really varied from this plan, though at times, I would set production levels at different amounts (from 50-200 units) above my forecast.
>
> I was trying to lead the other players into a quantity which gave us all high profits (e.g. the quantity the computer showed for the 1st round, 500). As long as I was substantially under the market average for sales, I maintained my production level. When people began raising production for a few rounds, I drastically raised my production to hurt profits and send a message. This worked beautifully, for several rounds after my 'punishment'. Then, for some reason, someone raised their production several times in a row. I repeated the strategy several times--each time success for a while. Then someone raised again, cutting profits.

The last of the responses illustrates a behavior that is difficult to capture with a simple regression equation: the player is apparently alternating between attempts at signalling collusion by keeping production low and periodic "punishments", in which he or she raises output to depress prices. Figure 6.3.1 reproduces the observed aggregate market behavior of the four markets in the clearing-price simple condition. The player in question was in "Mkt 31", and as can be seen from the figure, the result of his or her behavior is a fairly regular fluctuation in price and quantity.

Most responses also indicate, explicitly or implicitly, that firms monitor their own profits relative to the market average and, to a lesser extent, monitor trends in the market-average profits as well. This accords well with the hypothesized rule proposed in Chapter 3 which would anchor on the previous market-average output and then adjust this anchor up or down, depending on the perceived profit gradient (see equations 3.2.18-3.2.20 in Chapter 3). A modified, slightly more general rule which instead anchors on a weighted (geometric) average of the forecast of average output and the firm's own output last period is

(6.3.1) $\ln (y_t) = (1-a_1) \ln(y_{t-1}) + a_1 \ln(X_t^e) + a_2\Delta v_{t-1} + a_3\Delta V_{t-1};$

(6.3.2) $\Delta v_t = (v_t - V_t)/(y_t - Y_t);$

(6.3.3) $\Delta V_t = (V_t - V_{t-1})/(Y_t - Y_{t-1}),$

where v_t is the firm's own profit V_t is the market-average profit.

The results of estimating this rule are shown in Table 6.3.1. The average R^2 is .59 which is quite good, considering the considerable observed variation in the markets. Moreover, the estimates of the coefficient a_1 is in the expected range (between 0 and 1), most of them quite close to unity, indicating that firms anchor mainly on their forecast of aggregate demand and less on their own output last period. The two profit-adjustment coefficients, a_2 and a_3, are generally not very significant, although they tend to have the right sign (positive). One reason for the low significance may be multicollinearity and/or low variance in the explanatory variables: once firms anchor on a particular output level relative to average, the individual profit gradient, v, stays more or less constant and is highly correlated with the aggregate gradient, V.

To test the ability of the rule to reproduce the observed market behavior, it was embedded in a simulation model. The resulting endogenous simulations are shown in Figure 6.3.2. When comparing the simulated behavior to the observed behavior in Figure 6.3.1, it is quite clear that the rule fails to capture most of the observed dynamics. Instead, it leads to a steady and in most cases rapid adjustment to equilibrium. Interestingly, it appears that the resulting equilibrium price and output approximately equal the average price- and output level in the observed markets.

It is not very surprising that the simulated decision rule leads to an orderly convergence: The coefficients in the decision rule implicitly define an equilibrium or steady state, both for an individual firm (for a given aggregate output level) and for the market as a whole. Whenever the market moves away from this equilibrium, the profit gradients change, and as long as the coefficients in the rule are in the right range (positive, but not too large), the result will be a pressure for output to move back toward the equilibrium. In control-engineering terms, the system is thus essentially a simple first-order system with a negative (balancing) feedback mechanism. Such systems always produce orderly convergence to equilibrium.[9]

It is encouraging to observe that each equilibrium implied by the estimated coefficients in a market is close to the average observed position of that market. Although this claim would have to be supported with more theoretical and statistical analysis, it appears that the rule captures the most important determinants of the average operating point of the market. In other words, one can determine the aggregate outcome by investigating how

9 Strictly speaking, only continuous first-order negative-feedback systems are guaranteed to converge. In discrete systems, high gains can lead to rapid oscillations because the discrete time period acts as a lag in the negative feedback loops. Here, however, the estimated gains are well within the stable region.

much weight individual firms put on individual and aggregate profitability, respectively.

6.4 Behavior in the complex condition

As previously mentioned , the optimal, non-cooperative policy in both the simple and complex clearing-price conditions is to bring price as close to the competitive-equilibrium level as possible. The result would be prices fluctuating randomly around this level.

However, a glance at the observed market behavior, reproduced in Figure 6.4.1, makes it clear that the movements in prices and quantities are not random. Unlike the simple condition, the markets in the complex clearing condition all show a characteristic pattern. Initially, there is a surge in both production and prices, followed by a sharp drop in prices as the increase in supply hits the market, followed in turn by a gradual decline in output and increase in prices.

As in the simple condition, many subjects undoubtedly also try to engage in strategic behavior. Out of 17 subjects, 7 explicitly mention trying to "outwit" or "signal" or otherwise manipulate the market. Interestingly, however, the proportion 7/17 is lower than the corresponding ratio 10/16 in the simple condition. (The one-tailed Fisher-Irwin exact test for equal proportions shows a significant difference at p=5.7%.[10] Yet the result should be taken with a grain of salt, given that it is based on the authors own

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10 For a description of the Fisher-Irwin test, see e.g. Marasculio and Serlin 1988, p. 200.

subjective assessment of the responses.) Here are some examples of responses:

> At first I thought the best strategy would be to produce slightly more than the market average to capture extra profits than average and not drive others into overproducing also as they produce at the market average. This resulted in brutal competition so we cut back production significantly and thus were able to make a profit. [...] I tried signalling several times my willingness to lower production which worked to some extent but may have cost me more than I gained.

> I was slightly above the market average profit. My aim was not to be way above the market average profit but to have this average be very high [...]

> Initially, I overproduced and lost about $1500 more than the market. [...] I was not surprised by the results because competition was very fierce after the 10th period and there was never an opportunity to make any major gains over the market average for net profits. My strategy throughout the game was to be conservative and to try to meet anticipated market price and average sales volume. Unfortunately, I was severely hurt by the initial errors in judgment.

> At first, I was shooting in the dark. The initial plunge hurt, but I stayed positive, due to lower production--aha! So during the course of the game, my production decreased, using my profit level vs. the average profit level as a measure of success.[11]

> ... My output was too high at first, so I tried to bring it down gradually in order to bring my price up. I looked at the average market profit [...] to compare how I was doing with the rest of the market.

> I compared my period's profits with the average and tried to pick quantities that gave me the greatest spread from the average as well as bring the highest overall [profits].

> My production decisions were made on a profit maximization basis. I followed long term trends and tried to keep my

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11 This subject's profit-gradient parameter (a_2 in equation 6.3.1) was significantly greater than zero ($p<.05$).

> production below the average market production rate. I basically used trial and error to find the best level. Initially, I tried to flood the market and made a huge loss, so I figured it does not pay to try and outwit competitive firms. It took me about 10 periods to settle on my strategy. The only result that surprised me was when I tried to flood the market, when prices were high, I made a big loss. I guess this is a somewhat unrealistic aspect of the model (markets cannot really react this quickly, can they). I was fairly consistent with my strategy after period 10, although I raised production near the end when prices rose.

In the discussion in Chapter 3, it was hypothesized that strategic interactions between firms would be more difficult in the complex condition because of the time lag in production and one should therefore see fewer instances of collusion, signalling, etc. Both the fact that profits tend to be lower in the complex condition (see Chapter 4) and that relatively fewer subjects mention attempts at collusion, support this hypothesis.

The similarity of the behavior pattern in the four markets in Figure 6.4.1 indicates that the heuristics used by the participants share certain common characteristics. The simulations in Chapter 3 already indicated the likelihood of a "boom and bust", if firms initially mistake the internally generated price rise as an external increase in demand. Indeed, only a single subject mentions the multiplier effect, and this only in explaining that he or she was "too conservative" in their output:

> I should have produced more units and also expand the economic multiplier effect. My sales were always below average sales.

The rest of the subjects attribute the initial peak and sharp price decline to factors such as "a boom and bust", or "fierce competition". Recall that subjects were asked after the trial to sketch the pattern of the external influences on demand. Ten out of sixteen subjects plotted a "boom" during the very first periods of the game, or a subsequent "bust" during the first half or third of the game, or both. Figure 6.4.2 shows some examples. Only one subject came close to the truth--that there were no external factors--by drawing a horizontal line through zero with the comment "I really had no idea about any external factors." The responses thus reveal a marked tendency to attribute the observed boom and bust to factors unrelated to the feedback structure of the system.

The likely explanation for the characteristic boom-and-bust pattern of the initial stages of the game is straightforward: Initially, firms note a large unit profit margin (price minus unit production costs) and thus believe, correctly, that they can increase profits by expanding output. The supply line therefore starts to grow, in turn increasing aggregate demand through the multiplier effect. The finished output remains constant for the first three periods, due to the production lag, and consequently, prices must rise to clear the market. Firms observe the increase in prices and attribute it to an impending boom. The expectation of a boom induces firms to further increase their output, in the process creating a self-reinforcing feedback loop. After a few periods, however, the increased production arrives on the market, creating a sudden, sharp drop in prices. Now, the self-reinforcing feedback loop is reversed, as firms cut back production, depressing demand, causing further drops in prices. However, unlike the very beginning of the game, where all firms sell the same amount, they are now able to observe whether their profits are above or below the market average. Moreover, at this stage firms have also gained a rough sense of the aggregate profit function. As a result, and since no inventories accumulate, the remaining part of the game is dominated by a gradual adjustment process, as firms respond to both the individual and the market-average profit pressures.

The protocols also provide clues to the appropriate decision rule to use in the experiments: Most subjects describe their output decisions in terms relative to either the market average or their own previous output levels. Furthermore, the perceived profit gradient (with respect to deviations from the market average) appears to be an important cue: About 12 out of the 17 subjects speak explicitly of monitoring their own versus the aggregate level. Moreover, several mention a desire to bring aggregate output down and prices up, while at the same time staying at or slightly above the market average output level.

It appears, in other words, that subjects are responding to the same cues as those expressed in the adaptive rule (6.3.1-3) in the simple condition. However, the perceived average profit gradient, ΔV, is an unreliable measure in the complex condition, due to the disturbing effect of the multiplier. In consequence, the rule does not perform well when embedded in an endogenous simulation: Instead of replicating the observed characteristic behavior pattern, it tends to generate steady upward drifts in output with no similarity to the observed markets.

Therefore, a decision rule was used which substitutes the original ΔV with a simpler proxy measure of aggregate profitability, in the form of a log-linear function of the expected aggregate price level. Specifically, the following rule was estimated:

(6.4.1) $$\ln(y_t) = a_o + (1-a_1)\ln(y_{t-1}) + a_1\ln(X_t^e) + a_2\ln(P_t^e) + a_3\Delta v_{t-1},$$

where X_t^e and P_t^e are the observed forecasts of aggregate demand and price, respectively. The rule suggests that subjects anchor on a weighted average of their own previous output and their forecast of aggregate demand (with weight a_1) and then adjust this anchor, both with respect to their own perceived profit gradient, Δv, and with respect to a function of the observed forecast of the aggregate price ($a_o + a_1 \ln(P_t^e)$). One would expect a_1 to be between zero and one and a_2 and a_3 to be greater than zero.

The estimation results are shown in Table 6.4.1. The estimated coefficients largely fall within the expected range, although there are some exceptions. The weighting parameter a_1 is significant in all but one case and not significantly outside the expected interval between zero and one. Further, 12 out of 17 firms respond significantly to either the price-forecast cue (a_2) or the marginal profit cue (a_3). Yet, several of the significant coefficients are negative. Negative coefficients could typically arise from attempts at signalling collusion or being "contrarian."[12]

The estimated decision- and forecasting rules were embedded in an endogenous simulation model. The results of the deterministic simulations are shown in Figure 6.4.3. Comparing the simulations to the observed outcomes in Figure 6.4.1, one can discern several similarities. First, the model reproduces the characteristic pattern of boom and bust, followed by a slow and gradual adjustment. Second, the shape and relative size of the initial spike in output is preserved. For instance, both the simulated and the observed "Mkt 22" show a sharp but very short surge in output whereas both

12 The protocol data might shed further light on these matters, but time constaints prevented the author from further exploring this avenue.

actual and simulated in "Mkt 19" show a more extended period of high production; and both the simulated and the actual "Mkt 40" show a much smaller overshoot. Third, the long-term adjustment pattern is reproduced: The models settle at approximately the same level as the observed markets, and with approximately the same speed.

The most striking difference is of course the lack of short-term fluctuations in the simulation, and a smaller initial overshoot. As was the case in the fixed-price condition in Chapter 5, a significant "bootstrap" effect arises from replacing actual decisions with an estimated rule. The fact that the initial overshoot is smaller should not be surprising: the regression estimates capture the average of the entire game, thus in effect bringing the benefit of later experience to bear on the early stages of the simulations. For instance, it is likely that subjects may initially greatly extrapolate the observed increase in prices and only later learn not to do so. In the simulation, however, the average extrapolation coefficient is used throughout.

6.5 Summary and Conclusions

The analysis of the observed price- and demand forecasts shows that subjects have a good general understanding of the aggregate demand-curve relationship: Although a large part of the observed forecasts can be explained by a simple adaptive-extrapolative rule, firms also consciously (though partially) adjust their forecasts to be consistent with the aggregate demand relationship. In the simple condition, where the demand curve is stable and readily observable, the consistency between the two forecasts is quite high. But in the complex condition, consistency is significantly lower, and only slightly under half the firms appear to consciously adjust their two forecasts to.

An across-firm, within-markets analysis of the residuals from the simple adaptive-extrapolative forecasting rule reveals that firms in the simple condition are all responding to their common input cues in a manner that is not captured entirely by the simple rule. An obvious next step would be to use Zellner estimation to take advantage of the correlation. However, it would be more interesting to try to uncover the source of the correlation in order to refine the model of expectation formation.

In the simple condition, attempts at capturing the observed decisions by a simple adaptive rule has mixed success: From the questionnaire responses, it is clear that most firms engage in strategic interactions that are difficult to capture in a simple cue-weight model. Simulations of the simple clearing-price markets with a proposed adaptive decision rule that responds to perceived profit gradients all show a gradual and orderly adjustment to a long-term equilibrium which, interestingly, falls close to the approximate average operating point of each market.

The orderly adjustment is an inherent property of the simple feedback structure of the markets in this condition. To model the specific dynamics--if this is even possible--would require a more event-oriented, pattern-specific approach. One promising avenue might be the use of classifier systems or similar methods (Axelrod, 1987; Holland, et. al, 1986; Elliott, 1990), though the best one could reasonably hope for here is qualititative matching of the patterns of signalling, defection, and punishment, rather than point-by-point fits.

In the clearing-price complex condition, several firms are likewise engaging in strategic behavior, though apparently with less frequency than the firms in the simple condition. Yet the feedback structure and the way in which the subjects interact with this structure produces a characteristic time pattern for these markets. A simple decision rule was proposed which anchors on the previous output and forecasts of future output and responds to both the perceived profit gradient and the forecasted aggregate price (as a proxy for the aggregate long-term profit gradient). When the markets are simulated with the estimated parameters in this rule, all important features of the dynamic behavior patterns are reproduced quite successfully.

That the behavior of the complex markets can be reproduced so well is an indirect confirmation of the underlying hypothesis motivating this study: that the feedback structure of markets can strongly influence their behavior. In contrast, behavior in the simple condition, where the feedback structure is stable and favors convergence, short-term, event-oriented fluctuations dominate.

Moreover, the informal analysis of the protocol data lends further support to the hypothesis that people tend to attribute the observed fluctuations of aggregate prices to external influences rather than to internal, self-generated dynamics: None of the subjects attributed the initial boom and bust to the production lag and the multiplier effects. Instead, most attributed it either to external "business cycles" or to "the competition." This finding is consistent with Sterman's (1989c) findings.

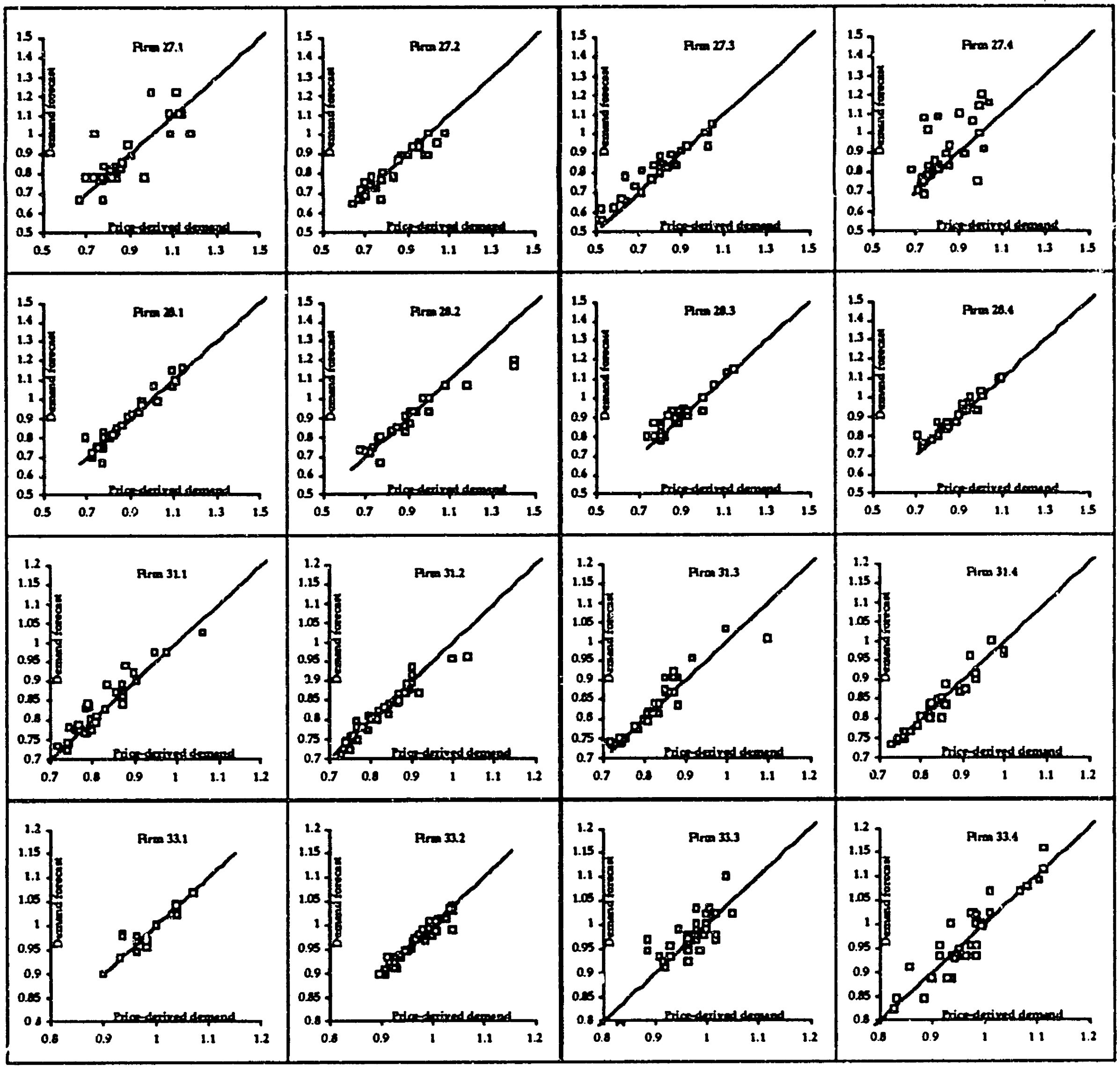

Figure 6.2.1: Consistency of observed demand- and price forecasts in the simple clearing-price condition

The figure shows plots for each firm of the observed demand forecasts (on the vertical axis) against the demand (on the horizontal axis) that would obtain at the observed price forecasts.

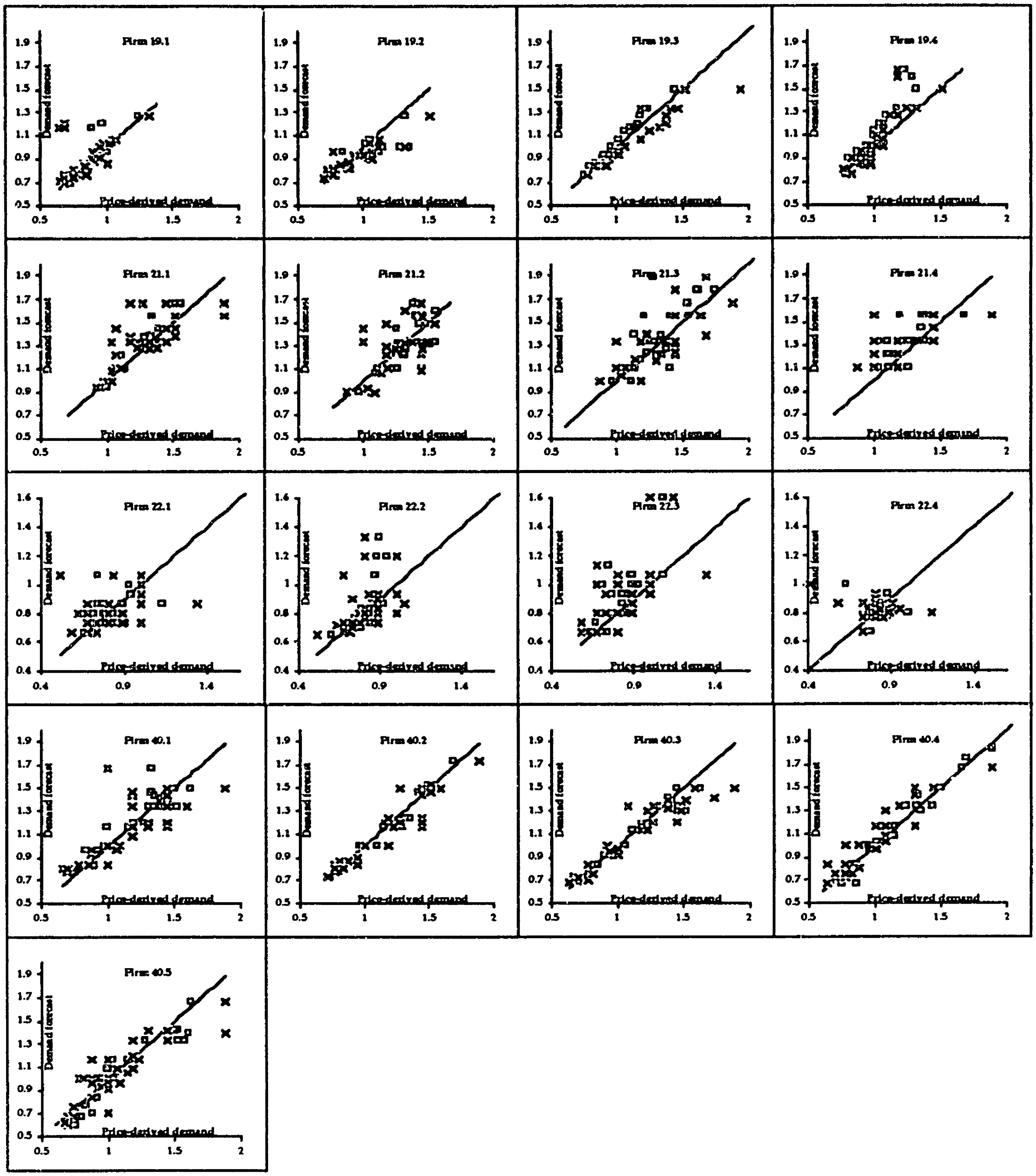

Figure 6.2.2: Consistency of demand- and price forecasts, complex clearing-price condition

The figure shows plots for each firm of the observed demand forecasts (on the vertical axis) against two alternative demand measures (on the horizontal axis) derived from the observed price forecasts. The first measure (cross symbols) is steady-state demand that would obtain at the given forecasted price. The second measure (square symbols) is the "instantaneous" demand that would obtain at the given forecasted price assuming that the aggregate supply line remains constant.

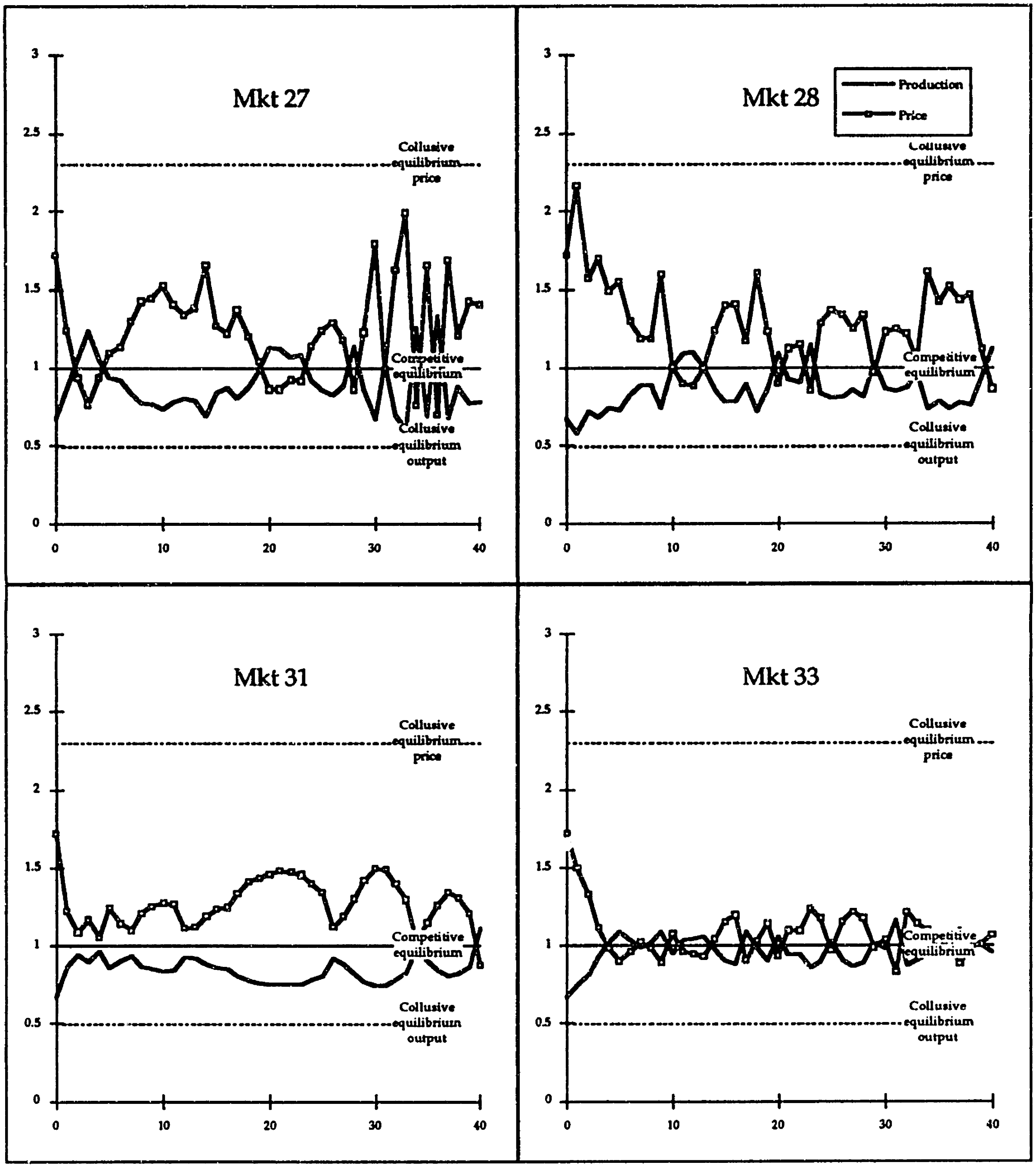

Figure 6.3.1: Observed behavior of markets in the simple clearing-price condition

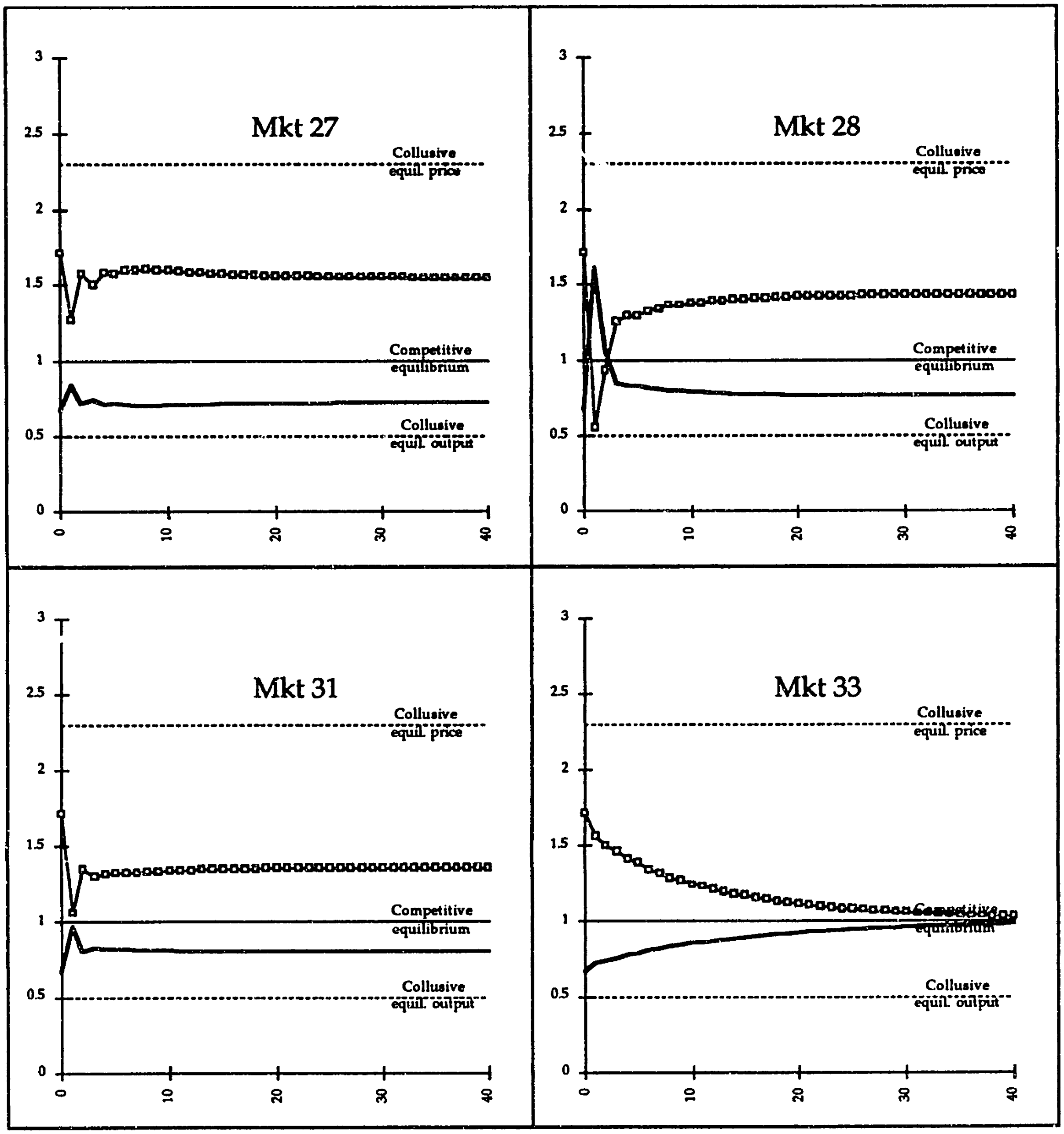

Figure 6.3.2: Simulated endogenous behavior of markets in the simple clearing-price condition

The figure shows the result of simulating the simple decision rule (6.3.1-3) and the adaptive-extrapolative forecasting rule (6.2.1), using the statistically estimated parameters for each firm.

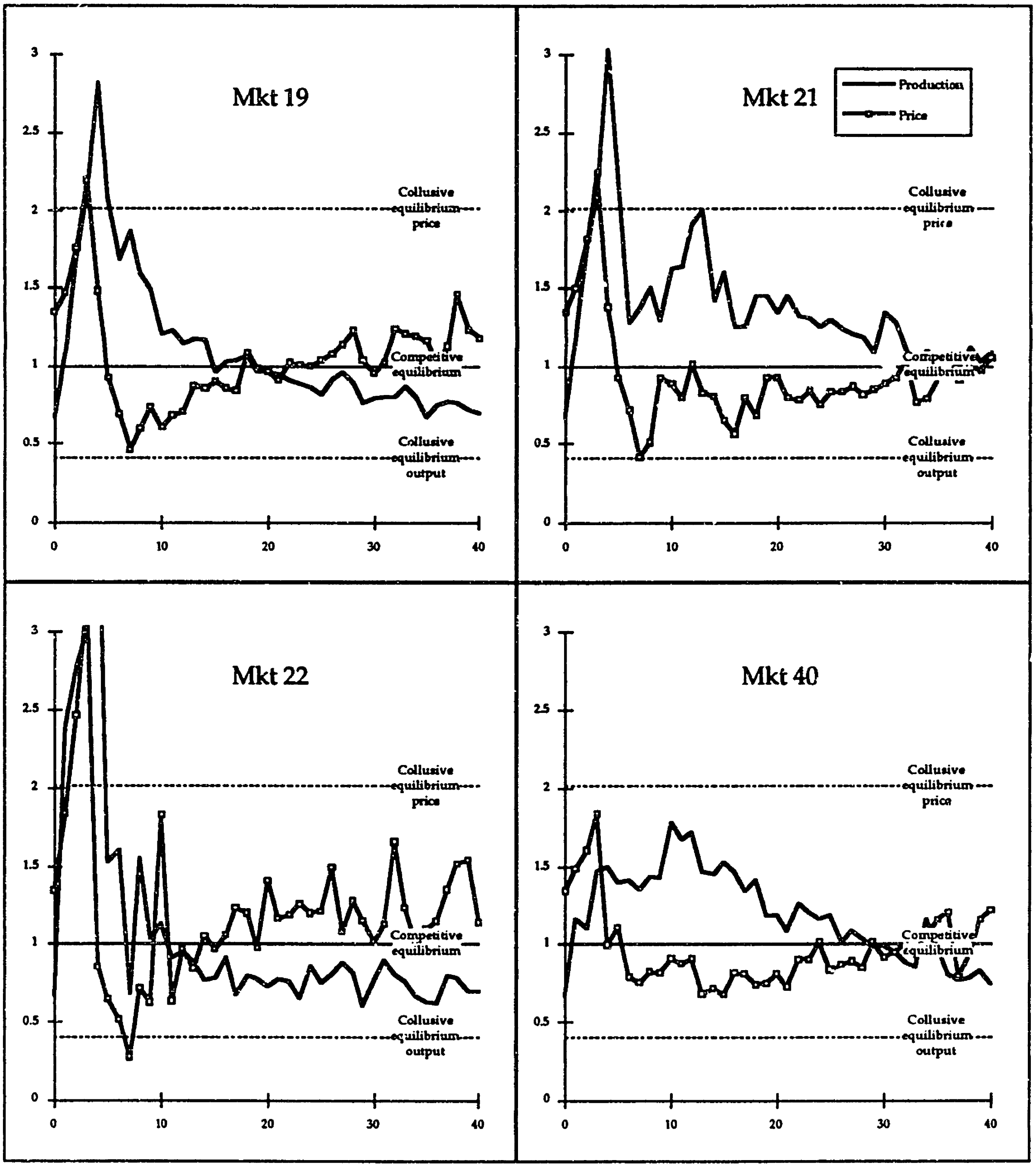

Figure 6.4.1: Observed behavior of markets in the complex clearing-price condition

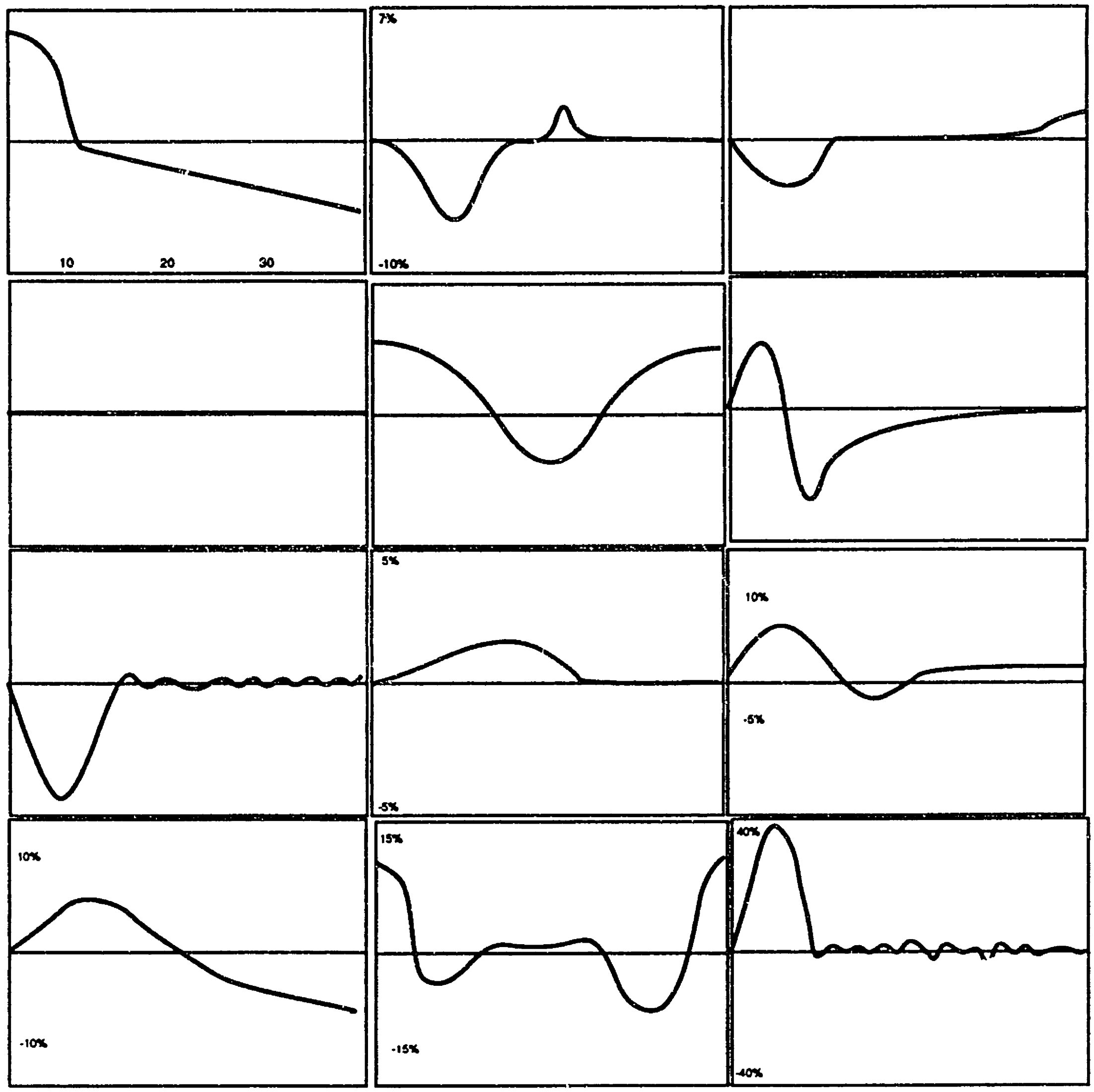

Figure 6.4.2: Examples of subjects' sketches of external influences: in the complex clearing-price condition

In the post-game questionnaire, subjects were asked to sketch their guess at the pattern of "external factors" which might have influenced aggregate demand. The figure shows examples of these sketches.

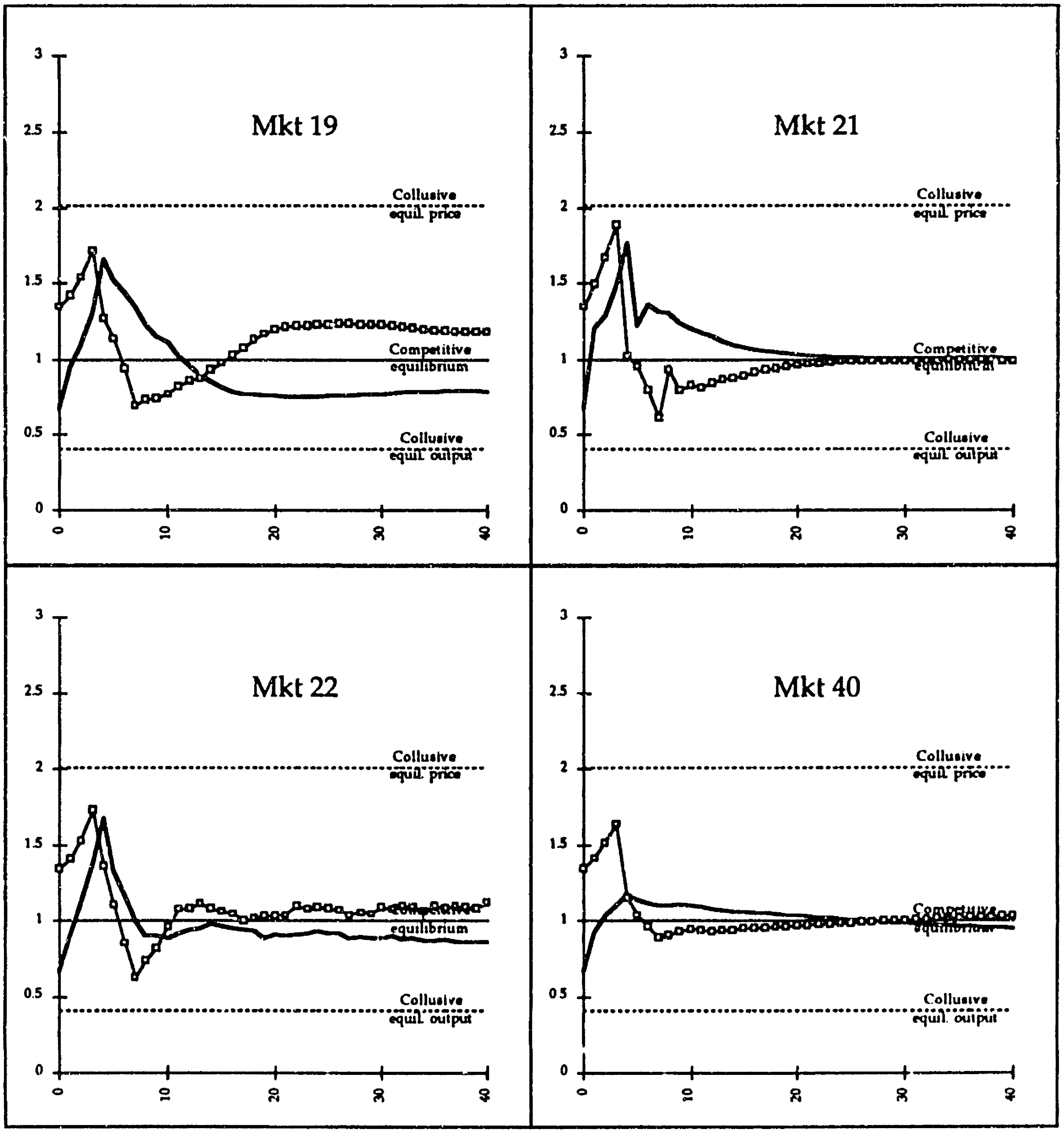

Figure 6.4.3: Simulated endogenous behavior of markets in the complex clearing-price condition

The figure shows the simulated market behavior using the decision rule 6.4.1 and the adaptive-extrapolative forecasting rules (6.2.1), all with the regression parameter estimates for each firm. To initialize the model, the first forecast variables are equal to the observed historical forecasts for each firm in the first period.

Condition	Mean
Clearing simple	0.438
Clearing complex	0.731
Posted simple	0.505
Posted complex	0.877

DEP VAR:RELEI N:16 MULTIPLE R: 0.610 SQUARED MULTIPLE R: 0.372

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
P	0.043	1	0.043	0.591	0.457
C	0.477	1	0.477	6.504	0.025
P*C	0.002	1	0.002	0.029	0.868
ERROR	0.879	12	0.073		

Table 6.2.1: Analysis of variance of the consistency of forecasts in the posted- and clearing-price conditions.

The table shows an analysis of the "average percentage discrepancy" between the observed demand forecast and the derived instantaneous demand measure. The error has further been normalized by the standard deviation of the derived measure to allow for the fact that the variance in the forecasts themselves differs between markets. Specifically, the measure is

$$\frac{\sqrt{\frac{1}{n}\Sigma(F-D)^2}}{\sqrt{\frac{1}{n}\Sigma D^2-(\frac{1}{n}\Sigma D)^2}}, \text{ or } \frac{\sqrt{\frac{1}{n}\Sigma(F-D)^2}}{\sqrt{Var\{D\}}},$$

where F is the forecast, D is the derived demand measure, and n is the number of observations.

Market & Firm	Demand forecasts $F_t=F_{t-1}+a_1(X_{t-1}-F_{t-1})+a_2(X_{t-1}-X_{t-2})$					Price forecasts $F_t=F_{t-1}+b_1(P_{t-1}-F_{t-1})+b_2(P_{t-1}-P_{t-2})$				
	a_1	Std. error*	a_2	Std. error*	R^2	b_1	Std. error*	b_2	Std. error*	R^2
Mkt 27 1	1.01	(.16) a	-.41	(.13) a	.56	.97	(.20) a	-.31	(.14) b	.48
2	.96	(.17) a	-.27	(.16) c	.52	.37	(.12) a	.03	(.10)	.41
3	.21	(.10) b	.31	(.08) a	.78	.12	(.09)	.69	(.09) a	.91
4	.88	(.16) a	-.38	(.10) a	.46	.54	(.14) a	-.12	(.09)	.37
Mkt 28 1	1.04	(.18) a	.15	(.20)	.60	.93	(.20) a	-.05	(.16)	.63
2	1.21	(.19) a	-.20	(.16)	.69	.86	(.16) a	-.05	(.17)	.59
3	.24	(.14)	.31	(.12) b	.57	.41	(.17) b	.07	(.13)	.37
4	.45	(.11) a	.24	(.10) b	.90	.47	(.11) a	.08	(.10)	.71
Mkt 31 1	.65	(.14) a	.55	(.14) a	.89	1.02	(.13) a	.26	(.14) c	.93
2	.43	(.13) a	-.02	(.13)	.22	.58	(.16) a	.16	(.20)	.30
3	.83	(.19) a	-.29	(.16) c	.32	.63	(.18) a	-.14	(.17)	.26
4	.51	(.15) a	.52	(.13) a	.73	.83	(.19) a	.23	(.16)	.76
Mkt 33 1	.48	(.11) a	.10	(.10)	.65	.48	(.11) a	.16	(.11)	.70
2	.57	(.10) a	.00	(.08)	.68	.69	(.10) a	-.11	(.11)	.74
3	.45	(.14) a	.07	(.12)	.34	.68	(.18) a	-.01	(.15)	.39
4	1.02	(.20) a	-.12	(.14)	.37	1.38	(.22) a	-.23	(.14)	.58

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.

For all regressions, the Durbin-Watson statistic indicated acceptance of no serial correlation at either the 5% or the 1% level, i.e., the statistics were inside both the significant correlation region and the "indeterminate" region.

Table 6.2.2: Parameter estimates in the proposed adaptive-extrapolative demand- and price forecasting rule, clearing simple condition

The first three time periods have been excluded to allow for initial learning.

Market & Firm	Demand forecasts $F_t=F_{t-1}+a_1(X_{t-1}-F_{t-1})+a_2(Y_{t-1}-F_{t-1}) + a_3(X_{t-1}-X_{t-2}) + a_4(Y_{t-1}-Y_{t-2})$						Price forecasts $F_t=F_{t-1}+b_1(P_{t-1}-F_{t-1}) +b_2(P_{t-1}-P_{t-2})$				
	a_1*	a_2*	a_3*	a_4*	R^2	Chg ?	b_1	Std. error*	b_2	Std. error*	R^2
Mkt 1	-0.01	0.44 a	0.10	0.54 a	.86	.99	.28	(.13) b	.30	(.17) c	.27
19 2	0.04	0.68 a	0.05	0.28	.76	.95	.28	(.14) c	-.17	(.17)	.10
3	-0.21	1.19 a	-0.19	0.14	.37	.81	.03	(.08)	.27	(.08) a	.30
4	0.63 a	0.18 b	-0.30 c	0.08	.45	.84	.22	(.06) a	.04	(.08)	.39--
Mkt 1	0.08	0.28 a	0.56 a	-0.53 a	.67	.79	.08	(.09)	.50	(.10) a	.58
21 2	0.47 a	0.13	0.36 b	-0.10	.60	.95	.27	(.10) b	-.07	(.12)	.16
3	0.42 b	-0.08	0.77 a	0.01	.72	.97	.75	(.14) a	.55	(.15) a	.82
4	0.32 c	0.19 b	0.03	-0.21 c	.42	.99	.10	(.09)	.21	(.10) b	.22-
Mkt 1	0.26 a	0.26 a	-0.24 a	-0.40 a	.68	.51	.13	(.08)	.36	(.07) a	.75
22 2	0.03	0.03	0.15 a	-0.05	.49-	.71	.17	(.08) b	.11	(.07)	.30
3	0.15 b	0.14 a	0.18 a	0.09	.74	.67	.23	(.07) a	.10	(.07)	.47
4	0.24 a	1.39 a	-0.06	-0.66 a	.89++	.77	.34	(.11) a	.26	(.10) b	.71
Mkt 1	0.43 a	0.23 b	-0.11	-0.03	.42	.01	.53	(.12) a	.09	(.11)	.70
40 2	-0.06	0.45 a	0.32 a	-0.19	.44	.05	.27	(.12) b	.42	(.11) a	.68
3	0.31 a	-0.05	0.06	0.16	.38	.14	.27	(.10) a	.30	(.11) a	.57
4	0.38 a	0.01	0.46 a	0.18	.54	.99	.24	(.10) b	.42	(.13) a	.53
5	-0.14	0.40 b	0.73 a	-0.09	.41	.03	.41	(.11) a	.08	(.20)	.40

* Significance of t-test for zero parameter: a: p<.01, b: p<.05, c: p<.10.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
-- Durbin-Watson test indicated significant negative autocorrelation at the 1% level.

Table 6.2.3: Parameter estimates in the proposed adaptive-extrapolative demand- and price forecasting rule, clearing complex condition

The first three time periods have been excluded to allow for initial learning. Since aggregate sales, X, is simply a lagged value of aggregate production, Y, the forecast of X is assumed to depend on both X and Y. The column "Chg?" shows a test for whether the forecasting rule remain the same over time by splitting the sample in two halves and testing for stability of the coefficients. The value shown is the probability of the observed difference under the null hypothesis of unchanging coefficients.

Clearing simple condition							Clearing complex condition						
Mkt & Firm		Coefficient*	Std. error**	R^2	Bartlett test*** X	P	Mkt & Firm		Coefficient*	Std. error**	R^2	Bartlett test*** X	P
Mkt 27	1	-.37	(.09) a	.39	.02	.09	Mkt 19	1	-.14	(.06) b	.14	.16	.69
	2	-.64	(.13) a	.48				2	-.45	(.22) b	.14		
	3	-.35	(.07) a	.44				3	-.44	(.47)	.03		
	4	-.38	(.16) b	.18				4	-.32	(.31)	.04		
Mkt 28	1	-.71	(.09) a	.68	.04	.40	Mkt 21	1	-.53	(.31)	.09	.41	.39
	2	-.36	(.05) a	.67				2	.12	(.29)	.01		
	3	-.33	(.05) a	.61				3	-1.22	(.50) b	.16		
	4	-.24	(.04) a	.51				4	-.24	(.19)	.05		
Mkt 31	1	-.32	(.10) a	.25	.00	.00	Mkt 22	1	-.18	(.15)	.04	.15	.54
	2	-.52	(.06) a	.74				2	-.39	(.08) a	.38		
	3	-.52	(.03) a	.89				3	-.21	(.11) c	.09		
	4	-.45	(.05) a	.74				4	-.10	(.13)	.02		
Mkt 33	1	-1.04	(.37) a	.17	.00	.00	Mkt 40	1	-.16	(.24)	.01	.62	.83
	2	-.69	(.45)	.06				2	-.85	(.15) a	.50		
	3	-.99	(.22) a	.36				3	-.56	(.11) a	.46		
	4	-1.02	(.20) a	.41				4	-.67	(.09) a	.61		
								5	-.56	(.19) a	.20		

* Coefficient "a" in the regression EX = constant + a EP, where EX and EP are the residuals in the regression of demand and price forecasts on the simple adaptive-extrapolative rule (shown in Tables 7.2.1 and 7.2.2), respectively.

** Significance of t-test for zero parameter: a: p<.01, b: p<.05, c: p<.10.

*** Bartlett Chi-square test for no correlation in residuals accross subjects in a given market. The value shown is the probability of the observed correlation, under the null hypothesis of no correlation, for the demand forecasts (X) and the price forecasts (P), respectively.

Table 6.2.4: Relation between price- and demand forecast residuals within subjects, and test for correlations of residuals accross subjects

Market & Firm		Production equation $\ln(y_t/y_{t-1}) = a_1\ln(X^e_t/y_{t-1}) + a_2\Delta v_{t-1} + a_3\Delta V_{t-1}$						
		a_1	Std. error*	a_2	Std. error*	a_3	Std. error*	R^2
Mkt 27	1	1.02	(.15) a	.16	(.18)	.09	(.07)	.62
	2	.95	(.17) a	.14	(.24)	.26	(.11) b	.49
	3	.19	(.09) b	.04	(.05)	.01	(.01)	.15
	4	1.15	(.12) a	.43	(.17) b	-.09	(.06)	.84
Mkt 28	1	.95	(.28) a	1.62	(1.50)	.78	(.27) a	.60
	2	1.31	(.11) a	.27	(.10) b	.08	(.03) a	.81
	3	.38	(.17) b	.14	(.08) c	-.06	(.04)	.28
	4	.65	(.09) a	.03	(.03)	-.01	(.01)	.70
Mkt 31	1	.77	(.13) a	.04	(.10)	-.06	(.02) b	.48
	2	.89	(.12) a	.16	(.21)	.05	(.04)	.61
	3	.94	(.12) a	.90	(.29) a	.24	(.05) a	.72
	4	.54	(.10) a	-.16	(.11)	-.03	(.02)	.48++
Mkt 33	1	.93	(.10) a	.09	(.05) c	.00	(.01)	.68
	2	.75	(.06) a	.00	(.01)	.03	(.00) a	.80+
	3	.70	(.12) a	.36	(.29)	-.15	(.03) a	.51
	4	1.15	(.18) a	-.30	(.42)	.27	(.05) a	.60

* Significance of t-test for zero parameter: a: p<.01, b: p<.05, c: p<.10.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
\+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.

Table 6.3.1: Parameter estimates in the proposed output decision rule, clearing simple condition

Output, y(t), is assumed to anchor on a weighted average of the firm's own output the previous period, y(t-1) and the expected aggregate output this period, $X^e(t)$. The anchor is modified by the perceived own profit gradient, $\Delta v = (v(t)-v(t-1))/(y(t)-y(t-1))$ and the perceived aggregate profit gradient, $\Delta V = (V(t)-V(t-1))/(Y(t)-Y(t-1))$, where v and V is the individual and aggregate profit, respectively.

Market & Firm		Production equation $\ln(y_t/y_{t-1}) = a_0 + a_1\ln(X^e_t/y_{t-1}) + a_2\ln(P^e_t) + a_3\Delta v_{t-1}$								
		a_0	Std. error*	a_1	Std. error*	a_2	Std. error*	a_3	Std. error*	R^2
Mkt 19	1	-.13	(.08) c	.63	(.16) a	-.04	(.43)	.53	(.28) c	.34
	2	-.05	(.02) a	.33	(.11) a	.23	(.09) b	.26	(.06) a	.63+
	3	-.03	(.01) b	.77	(.04) a	-.07	(.08)	.27	(.06) a	.94
	4	-.01	(.01)	.19	(.06) a	.10	(.12)	.03	(.07)	.28
Mkt 21	1	-.09	(.03) a	.72	(.13) a	.48	(.23) b	-.08	(.12)	.45
	2	.03	(.09)	.76	(.12) a	.51	(.40)	.74	(.28) b	.53
	3	.04	(.02) c	.15	(.04) a	.14	(.09)	.04	(.11)	.39
	4	-.14	(.07) c	1.06	(.13) a	.78	(.57)	.35	(.29)	.63
Mkt 22	1	-.08	(.04) c	.73	(.12) a	.53	(.24) b	-.06	(.15)	.46
	2	.15	(.10)	.55	(.15) a	-1.36	(.54) b	1.16	(.28) a	.41++
	3	-.01	(.05)	.01	(.06)	-.06	(.11)	.09	(.06)	.06
	4	-.05	(.06)	1.21	(.09) a	1.48	(.25) a	-.38	(.16) b	.82
Mkt 40	1	-.14	(.04) a	1.12	(.14) a	.55	(.27) b	-.11	(.20)	.61
	2	-.06	(.04)	.79	(.14) a	.06	(.31)	.08	(.21)	.46
	3	-.14	(.04) a	.83	(.15) a	.45	(.20) b	.19	(.11)	.44
	4	-.07	(.02) a	.65	(.10) a	.05	(.10)	.11	(.05) b	.61
	5	.13	(.04) a	.37	(.10) a	-.35	(.11) a	.35	(.12) a	.46

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
\+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.

Table 6.4.1: Parameter estimates in the proposed production rule, clearing complex condition

Chapter 6 Tables

7 Behavior under posted prices

7.1 Introduction

The final experimental condition involves posted seller prices, where firms set their own price and where demand is fully accommodated through changes in inventories or backlogs. Thus, each firm must make two decisions each period: how much production to initiate and what price to charge for their product. The decision task involves elements of both strategic considerations (finding the best price-output operating point) and of controlling inventory fluctuations. The previous two price regimes involved only one or the other of these elements.

Given the greater scope in decision making, one would expect that the posted-price conditions would both be somewhat more difficult to analyze and that the variation between markets would be greater. Indeed, the variability is quite large, especially in the complex posted-price condition. Nonetheless, the structural characteristics of the complex posted-price condition are strong enough to create certain patterns of behavior which, as will be demonstrated below, can be reproduced in a simulation model using behavioral rules for price and output. In the corresponding simple condition, however, the behavior is governed less by the feedback characteristics of the environment and more by the feedbacks amoung the firms themselves. Consequently, a simple regression-based simulation model has difficulty capturing the dynamics of the simple markets.

The chapter is organized as follows. Section 7.2 duplicates the analysis in Chapter 6 of the price and demand forecasts by comparing them to the simple adaptive-extrapolative rule; the two forecasts are checked for internal consistency, and the residuals of the simple adaptive-extrapolative rule are analyzed to detect whether subjects respond systematically to other cues than those assumed in the rule.

The next section describes the analysis of output and price decisions in the simple posted-price condition, building partly on an informal review of the questionnaire data collected in the experiment, and partly on regressions of simple rules. As mentioned before, efforts at reproducing the particular time pattern of behavior in this condition were not successful, and consequently this section does not contain any endogenous simulations.

Section 7.4 analyzes the behavior in the complex posted-price condition, where most markets show a distinct pattern of fluctuation in prices and inventories. A simple pricing rule is proposed which can produce inventory-price cycles of the observed periodicity and phase relationships. Finally, section 7.5 summarizes the results and concludes.

7.2 Behavior of forecasts

The analysis in Chapter 4 clearly showed that the hypothesis of rational expectations does not hold in the case of the experimental data: Subjects' forecasts are clearly not unbiased or efficient predictors of the actual value. However, one might expect that the price- and the demand forecast would at least have some consistency, i.e., that a prediction of higher prices would, ceteris paribus, also mean a prediction of lower demand.

In the simple condition, there is a constant relationship between the aggregate price and aggregate demand. It is therefore very likely that subjects would discover the demand curve and take it into account when forming their respective forecasts, particularly since the data display provided subjects witha scatter plot of industry demand versus industry price.

In the posted complex condition, there is no unique correspondence between price and demand because demand also depends on the amount of production in the pipeline (the multiplier effect). Still, a higher price would in most cases mean a lower demand, and one might expect subjects to apply such a relationship when forming their forecasts.

Figure 7.2.1 plots the demand- and price forecast pairs (omitting the first 10 periods) for each firm in the simple posted-price condition, compared to the true aggregate demand curve. The forecasts fall quite close to this curve, indicating a high internal consistency in forecasts of price and demand. Again, since the true demand curve was directly observable by the subjects this is not surprising.

In the complex condition, no such fixed relationship exists. It is therefore difficult to determine whether subjects are consistent in their forecasting. Figure 7.2.2 shows the demand forecast plotted against two alternative functions of the price forecast: the long-run steady-state demand implied by the forecasted price, and the instantaneous demand, assuming that the pipeline remains unchanged. It is evident that the subjects' forecasts fall closer to the instantaneous than to the long-run steady-state demand. Thus, it appears the subjects do gauge the short-term elasticity of aggregate demand and use this in forming their forecasts, but that subjects do not assume or try to calculate steady state conditions.

In parallel to the analysis in the previous chapter, the consistency of the forecasts can be assessed by comparing the observed demand forecasts to the derived demand measures based on the observed price forecast. In the simple condition, there is only one such measure, based on the fixed aggregate demand curve. In the complex condition, the "instantaneous" demand measure is used because it has the highest correlation with the observed forecasts and thus appears to reflect subjects' assumptions more closely than does the "steady-state" measure.[1] The results of this analysis are provided in Table 6.2.1 in the previous chapter. The table shows an analysis of the "average percentage discrepancy" between the observed demand forecast and the derived instantaneous demand measure. The error has further been normalized by the standard deviation of the derived measure to allow for the fact that the variance in the forecasts themselves differs between markets. Specifically, the measure is

$$\frac{\sqrt{\frac{1}{n}\Sigma(F - D)^2}}{\sqrt{\frac{1}{n}\Sigma D^2 - (\frac{1}{n}\Sigma D)^2}} \text{ or } \frac{\sqrt{\frac{1}{n}\Sigma(F - D)^2}}{\sqrt{Var\{D\}}},$$

where F is the forecast, D is the derived demand measure, and n is the number of observations.

It is evident from the table that there is a significant effect of complexity on consistency. The average index in the posted-price complex condition is

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1 One must keep in mind that the analysis of forecasting consistency in the complex condition is not a strict test of "rationality" because the observed forecast pairs could be "rationalized" with the appropriate value of the unobserved supply line forecast.

about 75% larger than in the corresponding simple condition, a significant effect (p=2.5%).

One might argue that the observed relationships between price- and demand forecasts might be a side product of simple extrapolation of history. If, for instance, prices are rising, demand is likely to be falling, and a simple adaptive-extrapolative forecast would produce results that would lie close to the demand curve.

To address this question, subjects' forecasts were first regressed on a combined adaptive-extrapolative rule similar to the one in Chapter 5. For the demand forecast, X_t^e, the equation is

(7.2.1) $$X_t^e = X_{t-1}^e + a_1(X_{t-1} - X_{t-1}^e) + a_2(X_{t-1} - X_{t-2}).$$

A similar equation was used for the price forecast.

It was very clear from the informal feedback given by the participants that their previous-period forecast was the anchor on which they based their next forecast: Several subjects complained about the fact that, in the complex condition, they were not able to see their own previous forecast because the software tabulated forecasts in the time period they pertained to (i.e., three periods hence), not the period they were made. Many subjects therefore manually kept a written log of their forecasts.

Tables 7.2.1 and 7.2.2 show the results of these estimations for both the demand- and price forecast in the simple and complex condition, respectively. Generally, the rule explains the observed forecasts quite well. In the simple condition, the average correlation coefficient, R^2, is .67; in the complex condition, the average is .45.

Following this analysis, the correlation of the residuals of the demand- and the price forecasts was measured to see if departures from the rule were consistent with the demand-curve relationship. A significant negative correlation between the residual in the demand- and the price forecast would indicate that subjects not only use simple smoothing or extrapolation of the individual forecasts but also try to adjust their two forecasts to each other.

Table 7.2.3 shows the results. It is very clear that, for both the simple and the complex condition, there is substantial negative correlation between the price- and demand forecast residuals, indicating that, when subjects depart from the simple rule, they do so in some systematic fashion, taking into account the demand-curve relationships. Although the correlation coefficient should depend on the particular time pattern of prices and sales, one would generally expect it to be close to the elasticity of the (instantaneous) aggregate demand curve, i.e. around -0.75. Indeed, except for three cases which show positive correlation, most coefficients in Table 7.2.3 are between -0.25 and -1.

The fact that subjects appear to adjust their forecasts fairly systematically begs the question whether there might also be a correlation accross subjects in a given market. To that end, Bartlett's Chi-squared test for significant correlation was applied to the correlation accross subjects, for both the demand and the price forecasts.[2] The results of the test are also shown in Table 7.2.3. From the table, it is clear that in most markets, there is indeed a strong correlation between the residuals, indicating that subjects' departures from the conjectured simple rule are systematic and, to a large extent, "the same" for all subjects.[3]

One possible explanation for this correlation is that subjects' forecasts are non-linear for large variations in prices and demand. Although the linear forecasting rule captures most of the variation well, it does not perform correctly in extreme cases. For instance, if a very low price was suddenly observed one period, many subjects might consider it a "fluke" and therefore assign less weight to it than normal. Such effects might be captured by including higher-order non-linear terms in the forecasting rule, but given the limited sample size, it is doubtful that one could obtain reliable parameter estimates. Moreover, for the purposes of capturing "most" of the observed behavior, the simple linear rule does quite well.

7.3 Behavior in the simple condition

Because inventories can be regulated directly through production in the simple posted-price condition, the optimal strategy is to work from the price decision to the output decision, i.e. first to determine what price to

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2 The test statistic is $\chi^2 = -\{N-1-(2p+5)/6\}\ln|\mathbf{R}|$, where N is the sample size, p is the number of variables, and $|\mathbf{R}|$ is the determinant of the correlation matrix.

3 Significant cross-correlation in the residuals indicates that some estimation efficiency could be gained by using e.g. Zellner's method for "seemingly unrelated regressions" (see e.g. Johnston (1972), p. 238), Unfortunately, software errors in the statistics package used for the analysis prevented the author from carrying out this analysis.

charge and then, based on this price and the expected aggregate price and demand, to set production to minimize expected inventory costs.[4]

Interestingly, subjects' responses in the post-game questionnaires reflect revealed that subjects indeed followed such a strategy. Subjects in the posted simple condition clearly considered pricing to be the most important task and discuss at length how they attempt to maximize profits through various pricing strategies, such as price leading, signalling, free riding, etc. Many subjects mention the difficult balance between trying to keep prices high while at the same time trying to price below the competitors to increase sales. If output decisions were mentioned at all, they were described as "trying to minimize inventory costs". Moreover, when prompted directly in Question 4 of the questionnaire all but one subject said pricing was more important than output. Here are a few examples of responses to the question:

> Price. Production (individual) can be more easily adjusted if you pay attention. Everyone suffers when the price moves down.
>
> Price is the most important. It gives you the demand and you can adjust production afterwards. But the two have to be coordinated.
>
> Prices most important. Once you picked a rough price range, production followed ...
>
> Price. Price decides the position in the market. Production is the next decision.
>
> Price. Competitor was playing havoc with this.
>
> Price, easily. I set my production to match price.
>
> Both are interrelated. I worked via price to production.

---

4 The formal treatment of the optimal policy (under rational expectations) is contained in Appendix A.

Since much of subjects' concerns center around collusion, signalling, and other strategic considerations, one might expect that their decision making might be quite event oriented and responsive primarily to recent particular patterns of pricing rather than to the current value of such variables. To the extent that this is the case, a regression equation might not do very well. In contrast, the output decision is fairly mechanical and should therefore lend itself well to simple regression equations. Indeed, this turns out to be the case.

For the pricing decision, a number of regressions were performed, based on the suggested rules from the simulations in Chapter 3. Under these rules, the price is governed by the perceived marginal profit: If the perceived profit gradient is positive, people raise their price, if it is negative, they lower it. Moreover, the rule includes both the individual-firm profit gradient and the market average profit gradient. Firms who cooperate will put more weight on the market profit gradient whereas "egotistical" firms will use only their individual profit gradient.

Unfortunately, unless individual prices vary a great deal over time, the profit gradients are fairly constant and highly collinear, which makes statistical estimation very difficult in the actual data. If prices do not move very much, both profit gradients are more or less constant, and it is impossible to identify the coefficients. Moreover, since the profit-gradient rule in the simple condition is a first-order negative feedback loop, it will always produce a uniform convergence toward equilibrium. Thus, any non-monotone behavior will not be captured by the rule.

Thus, attempts at identifying a simple adaptive pricing equation in the simple condition were not successful: The endogenously simulated markets failed to reproduce the particular pattern of behavior observed. They showed prices settling rapidly (in a few time periods) to a constant value whereas the observed prices show more varied patterns.

From the questionnaire data, it appears that some subjects were passive "market followers" while others deliberately tried to manipulate the market. It is possible that the markets showing the most flucuation in prices were dominated by the latter category of players. By alternatively attempting to signal collusion by keeping prices high and defecting or punishing other defectors by lowering their prices, such players could easily create continous fluctuations in the price level between the competitive and the collusive level.

The output decisions, on the other hand, fit regressions quite well. A number of alternative decision rules were employed, all of which showed quite good fit and explanatory power. As an example, Table 7.3.1 shows the estimation results of the following equation:

$$(7.3.1) \qquad y(t) = x^e(t) + a_2\, n(t);$$

$$(7.3.2) \qquad x^e(t) = X^e(t) + a_o + a_1\, X^e(t)\, \frac{p(t)-P^e(t)}{P^e(t)}$$

The equation expresses the idea that, after forming their forecasts, or expectations of market average sales and price, $X^e(t)$ and $P^e(t)$, and after deciding on their own price, p(t), firms form an expectation of what their own sales will be, $x^e(t)$. The expectation anchors on the forecast of market average sales, and adjusts this average by a factor reflecting any discrepancy between the firms' own price and the expected market average price.[5] If subjects make this adjustment correctly, the coefficient a_1 should be approximately equal to the firm's own-price elasticity of demand. Finally, output is adjusted to eliminate inventory imbalances. If subjects attempt to eliminate inventories fully in one period, the coefficient a_2 should be -1. The coefficient a_0 shouid be close to zero.

The table shows that, with a few exceptions, subjects' production decisions conform well to this simple rule. The correlation coefficients are quite high, and the coefficients are in the expected range, though generally numerically somewhat smaller than implied by full adjustment.[6] This accords well with the general result in behavioral decision making that, in the anchoring and adjustment process, humans tend to correct their initial anchored decision insufficiently.

In summary, the markets in the simple posted-price condition are driven to a high degree by subjects' pricing strategies: Whenever prices are above the competitive level, each firm has an incentive to undercut the others, thus stealing market share. The dynamic interplay of these considerations, and the signalling and "punishment" actions that go along with such strategic behavior is not well modeled by the simple regression techniques otherwise used in this study.

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5 Subjects can obtain the price-on-sales effect quite accurately by looking at one of the graphs displayed by the software. Indeed, several subjects stated in their questionnaire responses that they explicitly did so.

6 Two of the inventory adjustment parameters are numerically significantly above 1. One explanation for this is that subjects use last period's sales as an important factor in forming their expectation of this period's sales. Since current inventory is likely to be negatively correlated with last period's sales, omitting the latter from the regression could inflate the estimate of the inventory correction coefficient.

The production decisions, on the other hand, are more programmatic and thus lend themselves better to regression analysis. However, since the market dynamics are dominated by pricing behavior, the endogenous simulations of the estimated decision rules failed to produce similar patterns in market adjustment.

7.4 Behavior in the complex condition

In the complex posted-price condition, inventories can no longer be controlled immediately through production. Consequently, an appropriate strategy would make extensive use of prices to control inventories while output would be held fairly constant, based on long-run expected sales (see Appendix A).

The inventory-control component in price, combined with the long lags from initiation to completion of production makes it much more difficult for firms to signal collusion or punish defections. Variability in prices is costly, due to the resulting inventory fluctuations.

Moreover, the presence of the multiplier effect makes it more difficult for firms to discern whether they are operating in the right price-output range or not: Sales, and thus profits, are affected by the current amount of production in the pipeline, which may vary substantially over time.

Given these task characteristics, one would expect firms to devote less attention to strategic-interaction issues and more attention to controlling inventories and judging trends. Indeed, these notions are reflected in the questionnaire responses: Although many subjects watch competitors' pricing behavior closely, there are few references to strategic interactions, compared to the corresponding simple condition. Analysis of the questionnaire responses revealed that in the simple posted-price condition, 16 out of 20 firms referred explicitly to terms such as "signal", "collusion", "cooperate", "free rider", "support prices", "price leader" etc. In the complex condition, only 3 out of 15 firms referred explicitly to any attempts at strategic interaction.[7,8]

Moreover, the questionnaire responses reflect the importance of using prices to control inventories in the complex condition: Whereas only 1 out of 20 firms in the simple condition mentions using price to control inventory, 9 out of 15 firms do so in the complex condition.[9] Here are some examples:

> The price decision generally attempted to clear out the planned production and inventory.
>
> I attempted to use my price-setting to manipulate what my sales in that period would be. If I wanted to dampen demand, I overcharged, if I wanted to boost sales, I undercharged.
>
> I set price above or below market price depending on how I needed to manipulate my inventory ... Price was my primary decision maker. Production stayed relatively constant.
>
> Optimal price seemed to be about 6.3, and demand could support 375 units at this price. I tried to hold things there, so I matched production to my anticipated sales. I occasionally used price to throttle demand to stabilize inventory but more commonly, I regulated production based on anticipated sales and inventories.
>
> I played it safe--mostly keeping my prices close the the average market price, except when I was trying to unload inventory.

---

7 The questionnaire responses were read and coded by the author. Thus, the results should only be taken as an indication, not as a formal analysis of the protocols.

8 The Fisher-Irwin exact test of the hypothesis of equal proportions is strongly rejected ($p<.0005$).

9 The one-tailed Fisher-Irwin exact test rejects the hypothesis of equal proportions at $p=.001$. (For a description of this test, see e.g. Marascuilo and Serlin, 1988, p. 200.)

> I tried to make my prices follow the market to minimize my inventory. If I had positive inventory, I had to sell at lower prices to get rid of them.

In both the simple and the complex condition, the questionnaire responses also showed a strong tendency to anchor price decisions on the market average price or the projected future average price. 15 out of the 20 firms in the simple condition, and 11 out of the 15 firms in the complex condition, described their pricing position in terms relative to the market average rather than in absolute levels.

To the extent that firms anchor on the market average (or their forecast of this variable), a disequilibrium, either in profitability or in inventories, can have a cumulative effect on prices: If the firms' combined adjustments around the anchor result in a new average that is lower (higher) than the current average, then prices will continue to drift lower until the conditions affecting the individual firms' adjustments change enough to reverse the trend.

In control-engineering terms, pricing in effect become an integrative control. Integral control eliminates steady-state error (because corrections continue to grow larger until the discrepancy is eliminated.) However, such cumulative controls can be destabilizing. In the case at hand, an integrative aggregate pricing rule can create a price-inventory cycle in the posted complex condition: If firms respond to inventory imbalances by adjusting their prices downward, and if they also anchor their decisions on last period's average price, then prices will continue to drift lower as long as there is an excess inventory. Eventually, the lower prices increase sales enough that inventories fall back toward their desired level, but by the time inventory equilibrium has been reached, average price may now be close to its minimum, due to the cumulative drift, and sales may be substantially below production. The result would be that inventories continue to fall below their desired level, causing the process to reverse.

If production also responds to inventory imbalances and if firms do not account sufficiently for the amount of production in the pipeline, the cycle could be further amplified, as it was observed in the fixed-price complex condition. Finally, extrapolative expectations could create further amplification in the system.

A review of the observed behavior of the posted-price complex markets, reproduced in Figure 7.4.1, suggests that in three out of the four markets some form of cycle is indeed present. The strength of the cycle varied greatly, from very large (Mkt 16) to moderate (Mkt 18 and 38) to non-existent (Mkt 17). The nature of the cycle can be more clearly seen in Figure 7.4.2, which plots the average price against the average inventory, for each of the four markets.[10]

A "pure" inventory-price cycle, i.e. one where prices rise as long as inventories are negative and fall as long as they're positive would, with the appropriate scaling of the variables, result in a circular motion in the phase diagram in Figure 7.4.1, with prices leading one quarter cycle, or 90°, ahead of inventories. If, on the other hand, prices are anchored on a constant value and then adjusted in proportion to the inventory discrepancy, the result should be a straight line with negative slope in the phase diagram, as prices

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10 I am grateful to Erik Mosekilde for suggesting this analysis.

and inventories are exactly 180° out of phase. The actual data appear to approximate ellipses with their major axis along a line with negative slope, indicating a phase shift between prices and inventories somewhere between 90° and 180°, indicating some combination of a proportional and an integral control.[11]

To test the above qualitative theory, the following set of decision rules were estimated:

(7.4.1) $p_t = p_o + a_1(p^e_t - p_o) + a_2 n_t;$

(7.4.2) $y_t = y_{t-1} + b_1(x_{t-1} - y_{t-1}) + b_2 n_t.$

The pricing rule (7.4.1) expresses the idea that subjects anchor their decision on a weighted average of a long-term target price level, p_o and their expectations of the aggregate price level, p^e_t. For many subjects, the weight, a_1, on the forecasted average is unity, suggesting that they exclusively follow trends. The anchor is then adjusted to regulate inventory imbalances. A high inventory suggests prices should be lowered to increase sales, and vice versa, indicating that the coefficient a_2 should be negative.

The rule (7.4.1) does not include profit-gradient terms like the rules used in the clearing-price conditions. Instead, the attempt of firms to position themselves in the market is captured by the target price parameter, p_o. (Pricing rules using the profit gradients explicitly did not produce as convincing results when simulated endogenously.)

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11 A similar phase plot of the simple posted-price condition shows no tendency for circular or elliptical motion: The trajectories more closely resemble scattered clouds.

The production rule (7.4.2) suggests that subjects anchor their decision on last period's output. (Last period's output is clearly shown and is quite salient in the information display, and regressions not involving lagged values of the decision frequently had high serial correlation of the residuals, suggesting that the previous period's output should be included.) The anchor is then gradually adjusted toward recent sales figures, expressed in the second term on the right-hand side. Finally, output may also be responsive to inventory imbalances, expressed in the third term in the equation.

The equation (7.4.2) does not include a supply-line correction term. In light of the results of the fixed-price analysis in Chapter 5 and given the low weight assigned to the inventory correction (see below), it would seem highly unlikely that such a term would be significant. In fact, when such a term was included in the equation, only two cases showed significant coefficients, and one of the two had the wrong sign. (A negative coefficient could arise if firms do not pay any attention to the supply line *per se* but anchor on their production in previous periods.) Since the equations including this term did not produce convincing simulation results, the supply line term was dropped and the equations reestimated.

The estimation results of the two equations are shown in Table 7.4.1 and Table 7.4.2, respectively. The pricing equation fares better than the output equation: The average R^2 in the pricing equation is .66, which is quite high for a regression with a constant term, and it is above .5 in all but two cases. Moreover, the inventory coefficient is negative and, in all but one case, significant. Likewise, the coefficient for the price forecast is significant and close to unity for all but two cases. Only the constant term is often not significant, indicating that a large number of subjects anchor exclusively on their forecast of the average price. It is fair to conclude, therefore, that pricing is guided in large part by a combination of following the trend and the desire to regulate inventories through price.

The production equation (7.4.2), on the other hand, has less explanatory power (the average R^2 is only .26). But at least most subjects show a significant adjustment to past incoming demand (parameter b_1), with the coefficient in the right range between 0 and 1 in all cases. The inventory coefficient, b_2, on the other hand, is only significant in 2-3 cases. Moreover, in one of those cases, the coefficient is positive! Thus, it appears that subjects adjust their production gradually to reflect long-term expected sales and that they only rarely use production to regulate inventories. But together these two factors explain only about 25% of the variability in the rate of change of production.[12]

The next step in the analysis is to incorporate the estimated equation into a simulation to test if the observed tendency to price-inventory cycles recurs. Figure 7.4.3 shows the simulation results, with no random noise

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12 It is important to note, however, that the correlation coefficient pertains to the rate of change of output, not the absolute level. Thus, the measure shown is

$R^2 = 1 - \Sigma e_t^2 / \Sigma(y_t - y_{t-1})^2$.

If, instead, one used the variance around zero in the denominator,

$R^2 = 1 - \Sigma e_t^2 / \Sigma y_t^2$,

the correlations would of course be much higher.

As mentioned in a previous note, a number of other regressions were performed. Some of these did fare better in terms of the correlation coefficient, but such direct comparisons of R^2 are generally not valid (Pindyck and Rubinfeld, 1981). Moreover, in all of the regressions, the inventory coefficient were significant in only few cases.

inputs to the decision rules.[13] The first three time periods incorporate the actual historical decisions, for two reasons: First, the first three periods constituted a practice round and, since one would therefore expect there to be a fair amount of experimentation, were excluded from the regression analysis. Second, the initial decisions provide a "jolt" to the system, moving it out of equilibrium, so that the subsequent adjustment process is more clearly revealed than if one incorporated adaptive rules from the beginning of the simulation.

Comparing the simulations in Figure 7.4.3 to the observed outcomes in Figure 7.4.1, it is evident that the proposed simple decisions rules do indeed capture a great deal of the pattern of behavior in each market. The relative variability in each market is the same. Moreover, the phase relationships and relative variability of the key variables is similar to the observed outcomes, and the periodicity of the simulated fluctuation is in the same range as the observed.

As was the case in the fixed-price condition, there is clearly a strong boot-strapping effect when the estimated rules are simulated without random noise, so that the simulated markets appear to be much more stable than the observed behavior. However, as is well-known in control theory, a damped system, when driven by random noise, can show sustained cycles.

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13 In the actual simulation, the parameter a_1 in the pricing equation (7.4.1) was restricted to the interval between 0 and 1. (If a_1 is 1, the value of p_0 is irrelevant since it cancels out in the equation.) The constraint on a_1 was binding for 5 out of the 17 subjects, but in none of these instances were the estimates significantly larger than 1.

Indeed, the cyclical tendency increases when the observed unexplained variance in the decision is "put back in" in the form of serially and cross-sectionally uncorrelated noise with a standard deviation equal to the standard error of estimates in the regressions. Figure 7.4.4 compares the observed and deterministic simulated outcome with an example of a stochastic simulation, where the decisions are subject to a random error (with a variance equal to the mean squared regression residual).

Thus, the complex posted-price markets do show some tendency for a price-inventory cycle, though the prominence and regularity of such a cycle varies greatly from market to market. Yet, as was demonstrated by the endogenous simulations, the mechanism of the cycle, and therefore its qualitative characteristics, remains the same: Because firms anchor their pricing decisions on recent market averages, imbalances create a cumulative drift in prices which, when combined with the inventory accumulation, creates a tendency to oscillate.

7.5 Summary and conclusions

In the first part of this chapter, the price and demand forecasts were investigated, both to test their internal consistency and to see whether they could be reproduced by a simple adaptive-extrapolative rule. Indeed, subjects did exhibit good internal consistency between price- and demand forecasts, particularly in the simple condition, but the simple rule nonetheless captures a large portion of the observed variation in the forecasts. It appears, therefore, that subjects use some combination of smoothing and extrapolation of both demand and price forecasting, and adjust the two forecast to reflect the downward-sloping aggregate demand curve.

Then, the simple posted-price condition was analyzed with the aim of finding decision rules that would produce the observed market outcomes. For the production decision, which in the simple condition is quite straightforward and mechanical, a simple regression rule which would take into account both the expected aggregated demand and price, the firm's own price, and its inventory, performed well.

However, a simple pricing rule which responds to the individual and aggregate profit gradients (hill-climbing), was unable to reproduce the observed market dynamics. Indeed, a reading of the post-game questionnaires revealed that subjects' pricing strategies were concerned with strategic interaction and signalling, and as such were quite event-oriented and dependent on specific recent pricing patterns.

From a feedback perspective, it is not surprising that a simple profit-maximizing rule will not reproduce particular fluctuations in prices: A hill-climbing routine constitutes a first-order negative feedback loop driving prices toward the level that maximizes individual and/or average profits with little chance of overshooting or oscillation.

Thus, to achieve more insight into the "fine structure" of the observed pricing behavior, one would have to investigate the interaction of individual firms in more detail, probably including specific models of cooperation and defection, such as those presented in Axelrod (1984, 1987).

In contrast, the complex posted-price condition shows more regularity in the nature of the pricing behavior, though the behavior varies greatly from market to market. Although subjects may also have been concerned about competitive dynamics and strategies in this condition, most of their attention was focused on the need to control inventories and anticipate trends in prices and demand.

Because inventories cannot be controlled directly through production in the complex condition, they must be regulated by prices. Although this direct means of control makes it much easier to keep inventories in check than in the corresponding fixed-price condition, the observed markets nonetheless showed a tendency for cycles, primarily in prices and inventories.

The observed cycles were explained by the observation that firms to a great extent anchor their pricing on recent market averages, creating a cumulative effect of disequilibrium on prices. Indeed, a simple pricing rule in which price depends on the forecasted average market price and on current inventory when estimated and embedded in an endogenous simulation model of the market, was able to reproduce the cycle with the same relative variability, periodicity, and phase relationships between variables as those observed in the experiment.

The fact that it was possible to reproduce much of the behavior of the complex posted-price markets is encouraging because it indicates that the feedback structure of the system has a powerful and predictable influence on the adjustment process of the market. It is precisely in the presence of such complicating elements as delays, self-reinforcing feedback and non-linearities that the approach in this thesis shows the most promise.

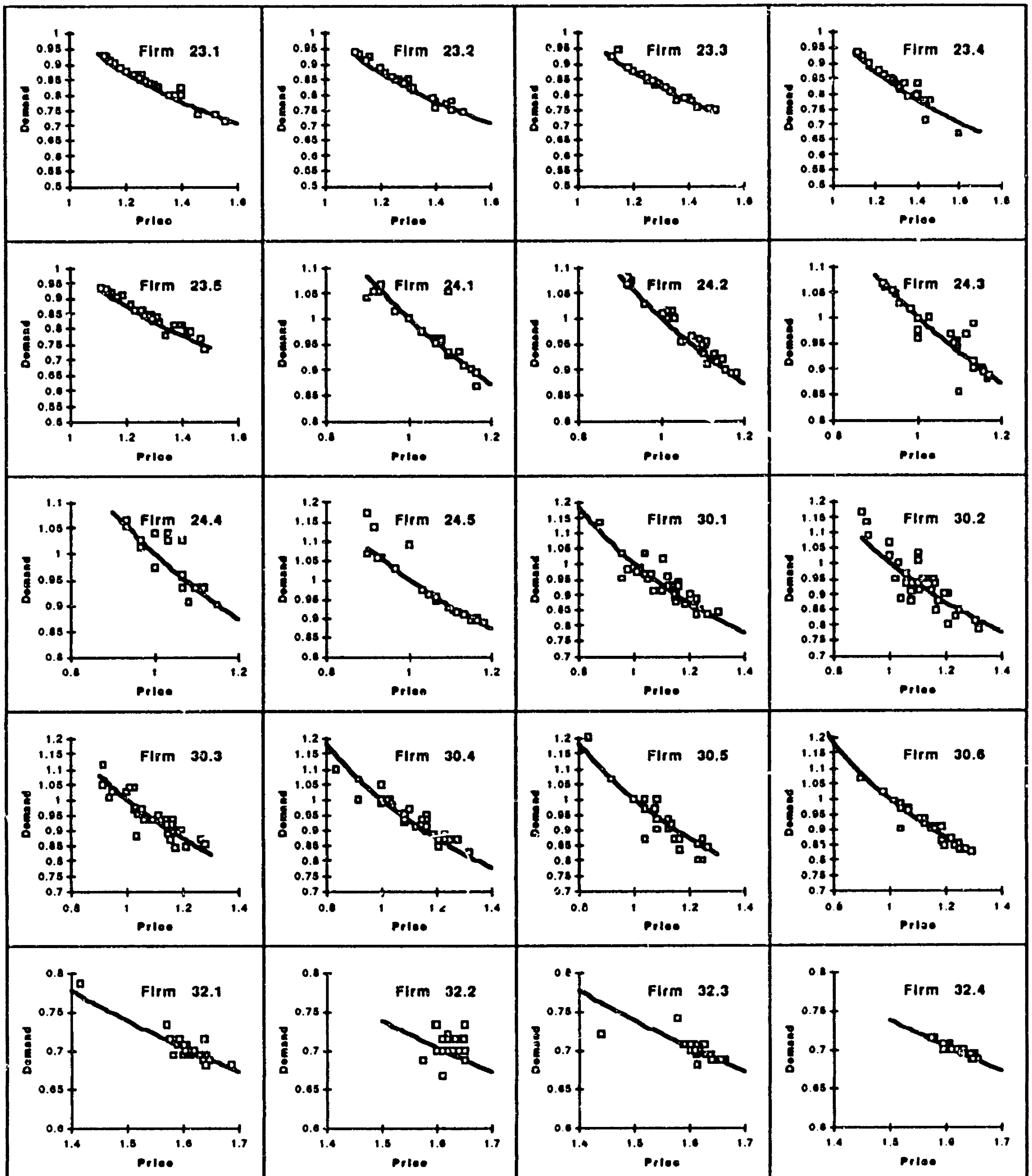

Figure 7.2.1: Demand versus price forecasts in the simple posted-price condition

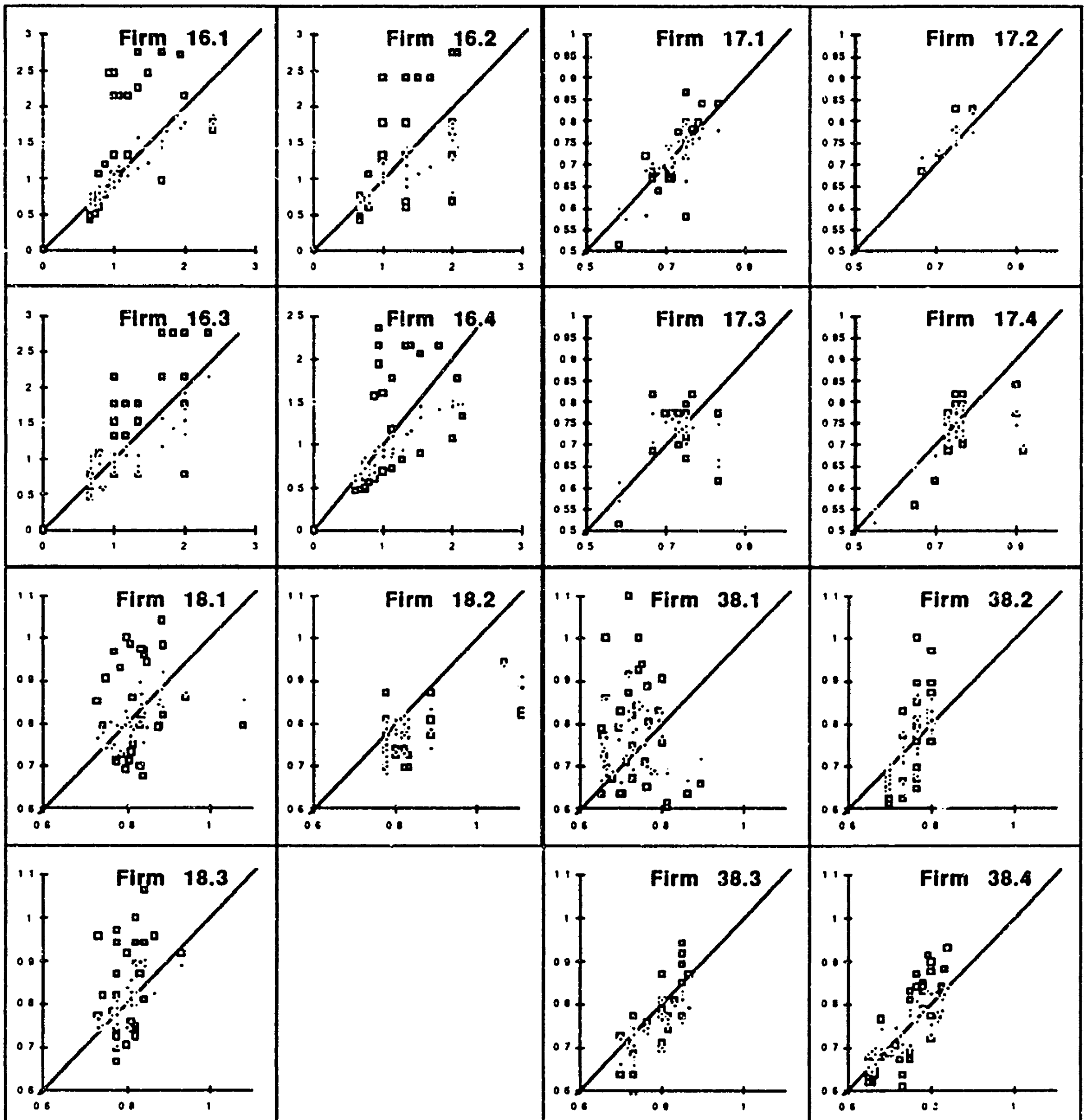

Figure 7.2.2: Demand versus price forecasts in the complex posted-price condition

The figure compares the observed demand forecasts to two alternative derived demands, based on the price forecast. The observed demand forecast is plotted on the horizontal axis; the derived measures are plotted on the vertical axis. The first measure (diamond symbols) is the "instantaneous" demand, which presumes that the production pipeline remains unchanged from its current value. The second measure (square symbolds) presumes that the system will have reached a steady state with the given price level.

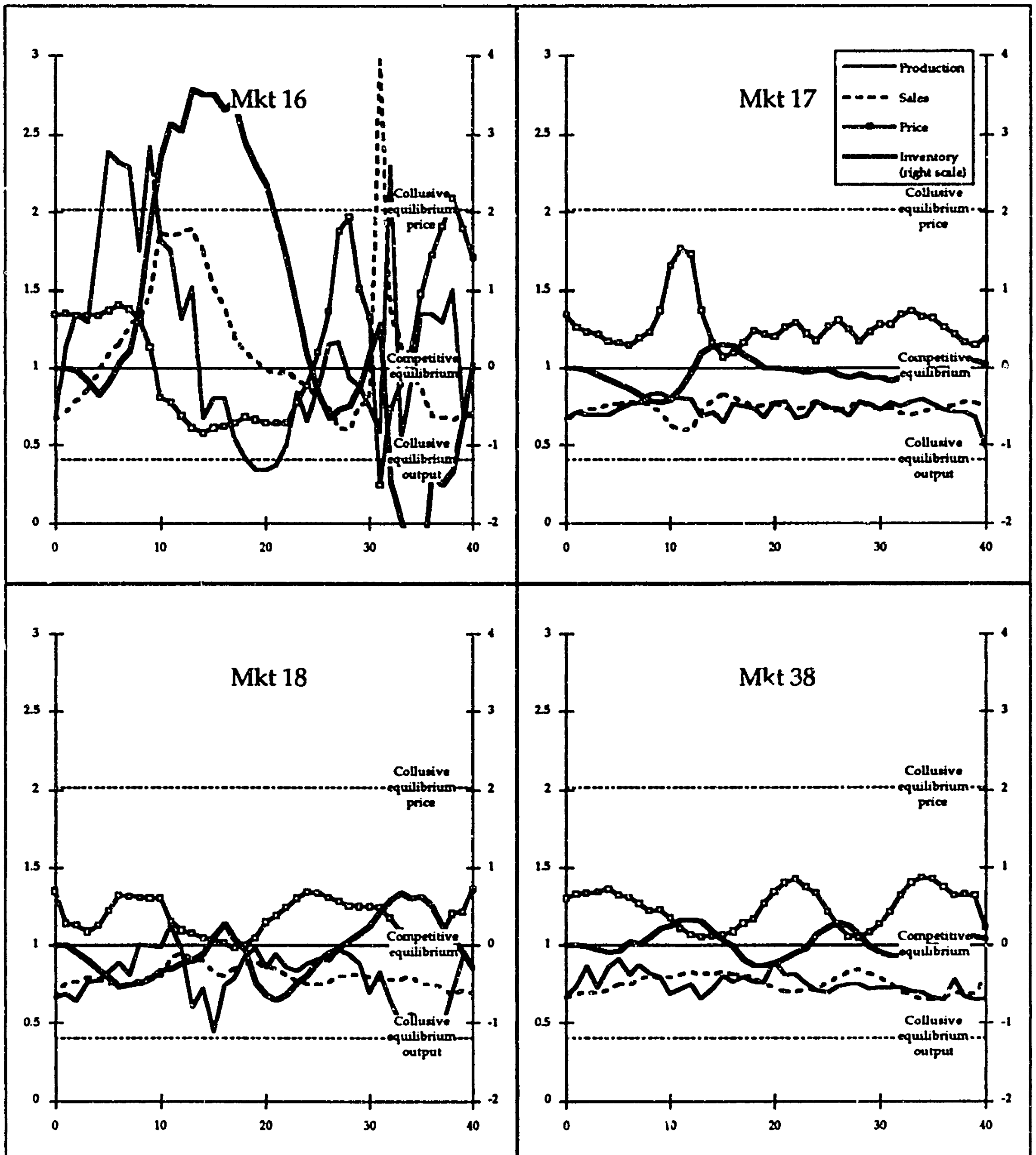

Figure 7.4.1: Observed behavior of markets in the posted-price complex condition

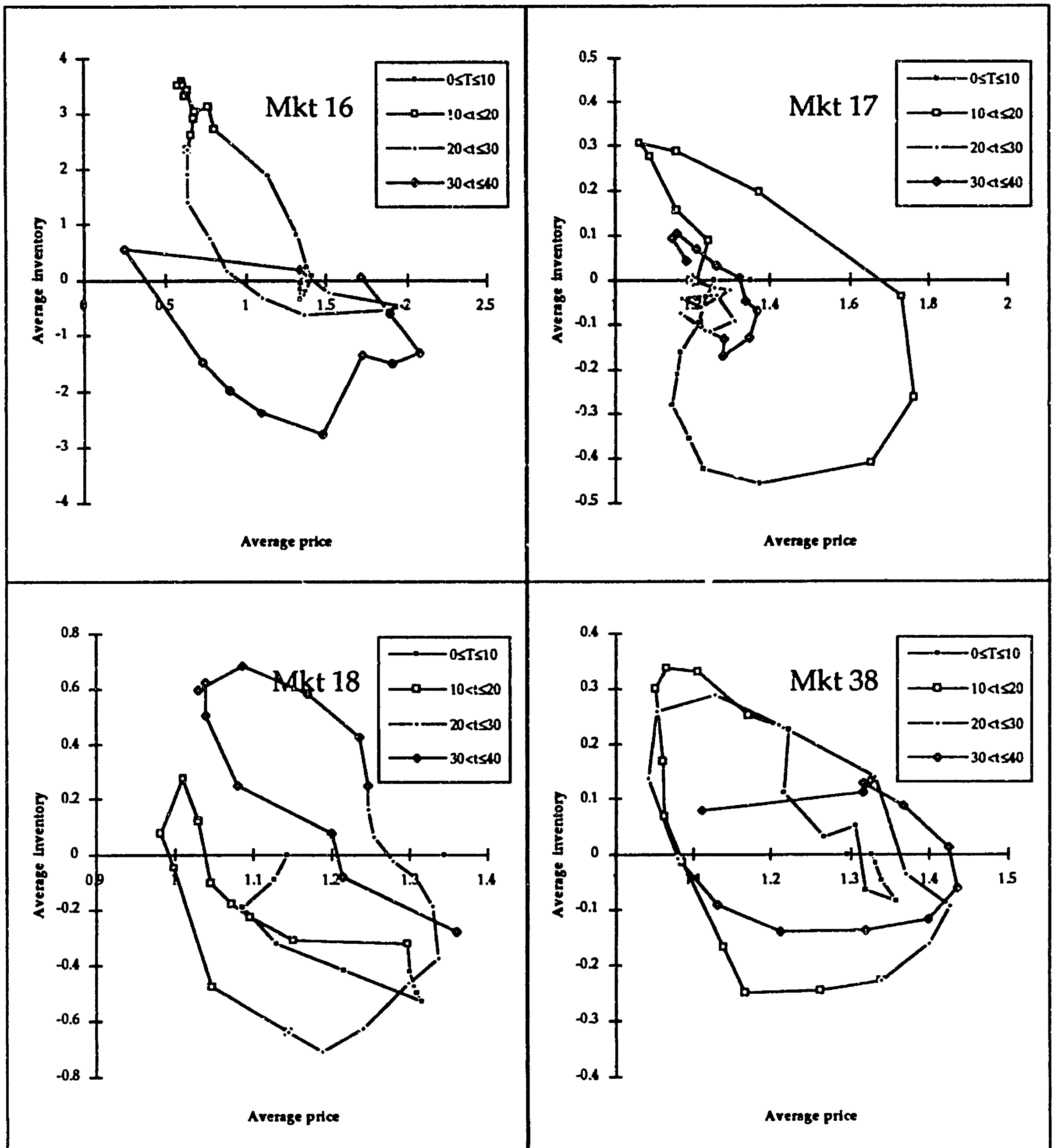

Figure 7.4.2: Observed behavior of average price versus average inventory in the posted-price complex condition

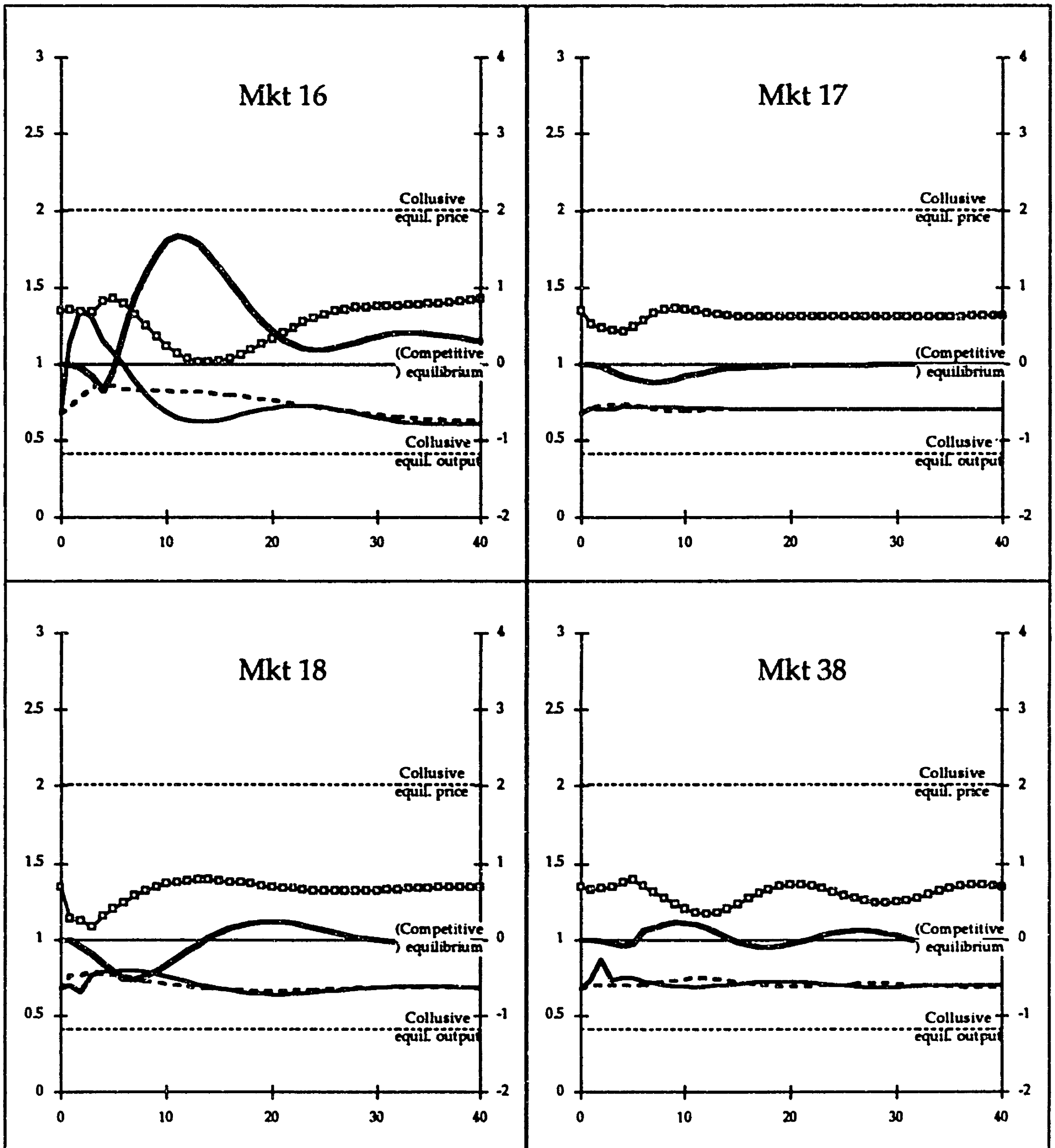

Figure 7.4.3: Simulated deterministic behavior of markets in the posted-price complex condition

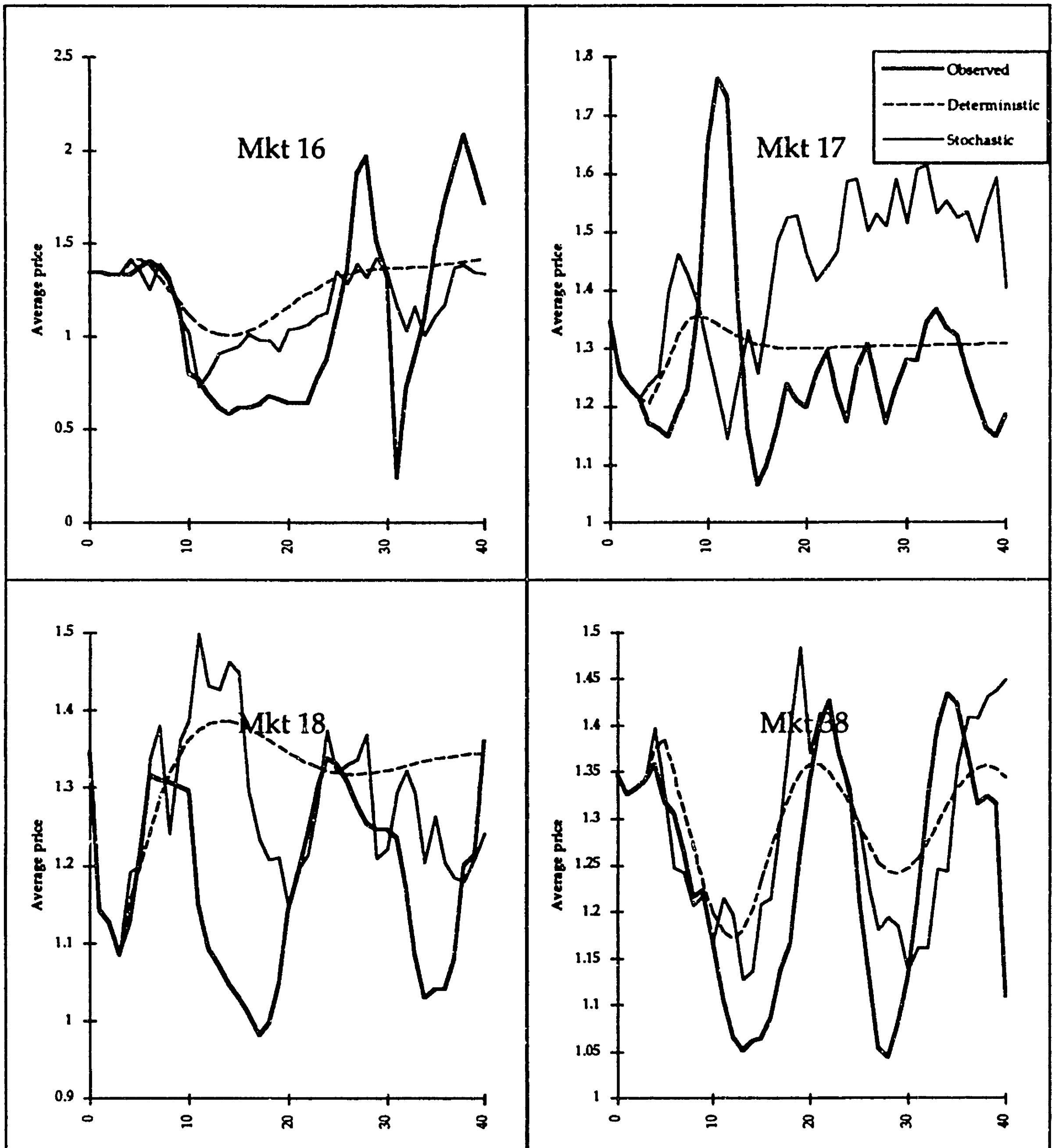

Figure 7.4.4: Comparison of observed and simulated deterministic and stochastic behavior of average price in the posted-price complex condition

The figure shows the observed average price pattern, the simulated deterministic price pattern, and an example of a simulated stochastic price pattern, where decisions are subject to random noise with a variance equal to the mean squared regression residual for each rule. (The standard error was limited to a maximum of .1)

Market & Firm	Demand forecasts $F_t=F_{t-1}+a_1(X_{t-1}-F_{t-1})+a_2(X_{t-1}-X_{t-2})$					Price forecasts $F_t=F_{t-1}+b_1(P_{t-1}-F_{t-1})+b_2(P_{t-1}-P_{t-2})$				
	a_1	Std. error*	a_2	Std. error*	R^2	b_1	Std. error*	b_2	Std. error*	R^2
Mkt 23 1	1.03	(.15) a	.47	(.09) a	.92	.85	(.13) a	.49	(.09) a	.93
2	1.30	(.23) a	.10	(.15)	.89	1.25	(.08) a	.08	(.05)	.92
3	1.42	(.19) a	.14	(.36)	.63	1.56	(.17) a	-.29	(.14) b	.79
4	.85	(.15) a	.67	(.16) a	.77	.60	(.15) a	.65	(.13) a	.83
5	.52	(.10) a	.83	(.11) a	.92	.59	(.08) a	.59	(.06) a	.95
Mkt 24 1	1.03	(.20) a	.34	(.17) c	.65	.90	(.17) a	.53	(.16) a	.76
2	1.33	(.15) a	-1.28	(.37) a	.73 ++	.93	(.12) a	.33	(.10) a	.77
3	.86	(.15) a	.59	(.20) a	.72	.73	(.18) a	.41	(.17) b	.74
4	.58	(.16) a	.27	(.26)	.36	.28	(.12) b	.54	(.11) a	.55
5	.95	(.19) a	.28	(.26)	.50	.67	(.17) a	.37	(.19) c	.64
Mkt 30 1	.30	(.08) a	-.13	(.07) c	.29	.62	(.15) a	-.05	(.14)	.49
2	.85	(.17) a	-.27	(.14) c	.44	.84	(.20) a	-.03	(.15)	.39
3	.28	(.08) a	.17	(.07) b	.70	.47	(.10) a	.14	(.10)	.79
4	.49	(.10) a	-.37	(.11) a	.41	.80	(.12) a	-.34	(.13) b	.58
5	.46	(.13) a	-.10	(.14)	.29	.45	(.15) a	.06	(.14)	.34
6	.40	(.20) c	-.25	(.21)	.20	.72	(.24) a	-.18	(.26)	.66
Mkt 32 1	.83	(.05) a	-.71	(.14) a	.87	.71	(.17) a	-.58	(.14) a	.38
2	.77	(.16) a	.72	(.22) a	.71	.22	(.12) c	.57	(.14) a	.78+
3	.99	(.20) a	.91	(.23) a	.86	1.20	(.13) a	.10	(.11)	.83
4	1.20	(.12) a	-.35	(.15) b	.83	.82	(.18) a	.05	(.19)	.95

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.

Table 7.2.1: Parameter estimates in the proposed adaptive-extrapolative demand and price forecasting rule, posted simple condition

Market & Firm	Demand forecasts $F_t=F_{t-1}+a_1(X_{t-1}-F_{t-1})$ $+a_2(X_{t-1}-X_{t-2})$					Price forecasts $F_t=F_{t-1}+b_1(P_{t-1}-F_{t-1})$ $+b_2(P_{t-1}-P_{t-2})$				
	a_1*	a_2*	a_3*	a_4*	R^2	b_1	Std. error*	b_2	Std. error*	R^2
Mkt 16 1	.23	(.09) b	.11	(.08)	.43	.44	(.13) a	.37	(.13) a	.77
2	.21	(.09) b	-.10	(.16)	.12	.61	(.13) a	-.05	(.17)	.44
3	.19	(.09) b	.27	(.09) a	.49	.33	(.12) b	.20	(.19)	.22
4	.21	(.09) b	-.05	(.08)	.15	.51	(.12) a	-.12	(.14)	.36
Mkt 17 1	.82	(.18) a	-.11	(.26)	.57	.69	(.17) a	.49	(.15) a	.81
2	.10	(.06)	.18	(.10) c	.21	.04	(.03)	.08	(.05)	.16
3	1.00	(.16) a	.70	(.25) a	.62	.56	(.15) a	.48	(.21) b	.49
4	.45	(.15) a	.66	(.19) a	.40	.67	(.18) a	.51	(.18) a	.45
Mkt 18 1	.99	(.17) a	-.17	(.37)	.51	.54	(.14) a	.26	(.16)	.51
2	.14	(.10)	.39	(.33)	.07	.16	(.10)	.38	(.18) b	.22--
3	.53	(.13) a	.00	(.18)	.41	.65	(.18) a	.35	(.15) b	.66
Mkt 38 1	.46	(.12) a	-.28	(.31)	.25	.72	(.14) a	.55	(.14) a	.74
2	.21	(.04) a	-.03	(.15)	.45	.43	(.08) a	.40	(.10) a	.77
3	.45	(.12) a	.02	(.24)	.30	.50	(.12) a	.03	(.17)	.43
4	.88	(.11) a	.47	(.19) b	.67 ++	.82	(.17) a	.32	(.16) b	.79

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
-- Durbin-Watson test indicated significant negative autocorrelation at the 1% level.

Table 7.2.2: Parameter estimates in the proposed adaptive-extrapolative demand- and price forecasting rule, posted complex condition

Posted simple condition						Posted complex condition					
Mkt & Firm	Coefficient*	Std. error**	R^2	Bartlett test*** X	P	Mkt & Firm	Coefficient*	Std. error**	R^2	Bartlett test*** X	P
Mkt 23 1	-.98	(.15) a	.64	.02	.01	Mkt 16 1	-.48	(.21) b	.14	.11	.00
2	-.58	(.08) a	.68			2	-.07	(.14)	.01		
3	-.42	(.26)	.10			3	-.23	(.09) b	.16		
4	.08	(.13)	.02			4	-.38	(.12) a	.22		
5	-.06	(.24)	.00								
Mkt 24 1	-.18	(.13)	.07	.52	.00	Mkt 17 1	-.26	(.10) b	.19	.01	.48
2	.78	(.40) c	.14			2	-.67	(.05) a	.87		
3	-.35	(.19) c	.13			3	-.14	(.06) b	.14		
4	-.95	(.22) a	.45			4	-.18	(.06) a	.24		
5	-.76	(.15) a	.54								
Mkt 30 1	-.42	(.10) a	.40	.00	.00	Mkt 18 1	.06	(.14)	.01	.75	.68
2	-.96	(.13) a	.64			2	-.77	(.12) a	.62		
3	-.32	(.16) c	.13			3	-.17	(.13)	.06		
4	-.47	(.09) a	.49								
5	-.65	(.12) a	.53								
Mkt 32 1	-.37	(.04) a	.79	.52	.00	Mkt 38 1	.17	(.34)	.01	.00	.97
2	-.12	(.18)	.01			2	-.06	(.37)	.00		
3	-.18	(.08) b	.15			3	-.39	(.18) b	.12		
4	-3.54	(.78) a	.42			4	-.64	(.14) a	.39		

* Coefficient "a" in the regression EX = constant + a EP, where EX and EP are the residuals in the regression of demand and price forecasts on the simple adaptive-extrapolative rule (shown in Tables 7.2.1 and 7.2.2), respectively.

** Significance of t-test for zero parameter: a: p<.01, b: p<.05, c: p<.10.

*** Bartlett Chi-square test for no correlation in residuals accross subjects in a given market. The value shown is the probability of the observed correlation, under the null hypothesis of no correlation, for the demand forecasts (X) and the price forecasts (P), respectively.

Table 7.2.3: Relation between price- and demand forecast residuals within subjects, and test for correlations of residuals accross subjects

Market & Firm		Production equation $y_t = X_t^e + a_o + a_1 X_t^e (p_t - P_t^e)/P_t^e + a_2 n_t$						
		a_o	Std. error*	a_1	Std. error*	a_2	Std. error*	R^2
Mkt 23	1	.01	(.01)	-1.51	(.15) a	-1.17	(.05) a	.96+
	2	.07	(.02) a	-.76	(.06) a	-1.12	(.17) a	.92
	3	-.33	(.01) a	-.42	(.02) a	-1.02	(.04) a	.97++
	4	.03	(.03)	-2.13	(.43) a	-.88	(.08) a	.82
	5	.11	(.05) b	-3.26	(.42) a	-1.17	(.11) a	.79++
Mkt 24	1	.09	(.03) a	-2.13	(.41) a	-.56	(.10) a	.75
	2	.04	(.02) b	-2.85	(.44) a	-.55	(.12) a	.70
	3	.06	(.01) a	-2.24	(.17) a	-1.09	(.13) a	.89
	4	.01	(.03)	-2.66	(.48) a	-.60	(.17) a	.64
	5	-.04	(.03)	-.98	(.09) a	-.99	(.21) a	.83++
Mkt 30	1	-.01	(.01)	-.84	(.30) a	-.84	(.08) a	.81
	2	-.02	(.07)	-1.26	(.39) a	-.62	(.10) a	.57
	3	.07	(.04) c	-1.96	(.50) a	-.80	(.13) a	.56++
	4	.04	(.10)	-.47	(.48)	-.27	(.08) a	.28
	5	-.02	(.05)	-1.00	(.17) a	-1.01	(.13) a	.74
	6	.07	(.03) a	-2.94	(.45) a	-.72	(.15) a	.75+
Mkt 32	1	.01	(.01)	-.79	(.47)	-.32	(.08) a	.36+
	2	-.05	(.01) a	-.82	(.32) b	-.07	(.05)	.23++
	3	-.01	(.00) b	-1.91	(.07) a	-1.44	(.05) a	.99
	4	.05	(.01) a	-1.54	(.17) a	-.98	(.07) a	.89++

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.

Table 7.3.1: Parameter estimates in the proposed production rule, posted simple condition

Market & Firm		Production equation $y_t=y_{t-1} + b_1(x_{t-1}-y_{t-1}) + b_2n_t$				
		b_1	Std. error*	b_2	Std. error*	R^2
Mkt 16	1	.24	(.10)b	-.04	(.06)	.13
	2	.30	(.10)a	-.04	(.04)	.24
	3	.33	(.15)b	-.19	(.18)	.13
	4	.73	(.03)a	.01	(.02)	.93++
Mkt 17	1	.33	(.08)a	-.01	(.04)	.35
	2	.28	(.07)a	-.03	(.03)	.31
	3	.03	(.04)	-.05	(.05)	.05
	4	.04	(.05)	-.02	(.04)	.04
Mkt 18	1	.44	(.13)a	-.08	(.05)	.25
	2	.34	(.10)a	.34	(.12)a	.58+
	3	.08	(.05)c	.00	(.03)	.09
Mkt 38	1	.66	(.11)a	-.17	(.09)c	.46--
	2	.07	(.05)	-.05	(.02)b	.12
	3	.43	(.14)a	-.05	(.13)	.19--
	4	.06	(.10)	-.12	(.09)	.05

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.
-- Durbin-Watson test indicated significant negative autocorrelation at the 1% level.

Table 7.4.1: Parameter estimates in the proprosed production rule posted complex condition

Market & Firm		Pricing equation $p_t=a_o+a_1(P^e_t-a_o)+a_2n_t$						
		a_o	Std. error*	a_1	Std. error*	a_2	Std. error*	R^2
Mkt 16	1	.19	(.19)	1.04	(.15)a	-.11	(.04)b	.70++
	2	.68	(.13)a	.50	(.10)a	-.07	(.01)a	.77+
	3	1.10	(.32)a	.57	(.28)b	-.60	(.15)a	.55
	4	.20	(.14)	.83	(.12)a	-.05	(.01)a	.69+
Mkt 17	1	.02	(.10)	1.05	(.07)a	-.11	(.05)c	.87
	2	.35	(1.13)	.75	(.92)	-.16	(.09)c	.08+
	3	.49	(.11)a	.80	(.09)a	-1.16	(.16)a	.82
	4	.47	(.07)a	.56	(.06)a	-.28	(.05)a	.93
Mkt 18	1	-.05	(.14)	1.05	(.12)a	-.03	(.01)c	.74+
	2	1.10	(.55)c	.15	(.45)	-.22	(.10)b	.12
	3	-.04	(.13)	1.05	(.11)a	-.06	(.06)	.83
Mkt 38	1	.23	(.08)a	.78	(.07)a	-.26	(.04)a	.80
	2	-.08	(.09)	1.08	(.07)a	-.23	(.03)a	.88
	3	.34	(.22)	.80	(.17)a	-.24	(.08)a	.53
	4	.22	(.12)c	.85	(.09)a	-.32	(.08)a	.74++

* Significance of t-test for zero parameter: a: $p<.01$, b: $p<.05$, c: $p<.10$.
++ Durbin-Watson test indicated significant positive autocorrelation at the 1% level.
\+ Durbin-Watson test indicated significant positive autocorrelation at the 5% level.

Table 7.4.2: Parameter estimates in the proposed pricing rule, posted complex condition

8 Conclusion

8.1 Recapitulation

Motivation

Are the neoclassical assumptions of perfect rationality and market equilibrium adequate for economic theory, or is it necessary to take human cognitive limitations and market disequilibrium into account? This subject has been the subject of intense debate, primarily between economists and psychologists.

The review in Chapter 2 of the mechanisms summoned in support of rationality and equilibrium revealed that they are all dynamic in nature. Processes such as arbitrage, speculation, learning, and competitive selection are supposed to compensate for the fallibility of individual agents and drive the market toward equilibrium, yet they are all disequilibrium phenomena that take place over time. Therefore, the rationality debate cannot advance without explicit attention to these dynamic processes, and, equally important, without studying how humans behave in dynamic market settings.

Recent research on human behavior in dynamic settings makes it clear that people's mental models of dynamic systems are incomplete when it comes to representing "feedback complexity," in the form of stock- and flow structure, lags, non-linearities, side effects, and tradeoffs between the short and the long term. Humans instead tend to view problems as externally imposed rather than generated internally by their interaction with the system. Consequently, human performance in dynamic systems deteriorates dramatically with increasing "feedback complexity."

Yet these findings have not been tested in or applied to a market environment. Studies in experimental economics have instead focused on the behavior of alternative exchange institutions, using simple demand- and supply payoff schedules. The evidence from these studies makes it clear that the precise form of the price-setting institution has significant effects on market behavior, even where such institutions are essentially equivalent from a neoclassical perspective.

The thesis was thus motivated by two related fundamental research questions:

1) To what extent will market institutions alleviate or alter the findings from dynamic decision-making studies?

2) What are the effects of feedback complexity on market behavior?

Design and hypotheses

To address these questions, six experiments were conducted, involving two alternative feedback structures and three price regimes. The feedback structures were

- **a simple condition**, where (1) production initiated at the beginning of each period becomes available for storage or delivery during that same period, and (2) where industry demand is unaffected by the average level of activity in the market, and
- **a complex condition**, where (1) there is a three-period lag between the time period production is initiated and the time it becomes available for storage or delivery, and (2) where industry demand is influenced by the average level of production in the market.

The price regimes were

- **fixed prices**, where prices are constant and fluctuations in demand are accommodated entirely by changes in inventories;

- **posted seller prices**, where each firm sets its own price and production rate, and demand is fully accommodated by changes in inventories; and

- **clearing prices**, where the market-clearing price vector is found by the computer, which thus functions as a perfect Walrasian auctioneer.

It was shown that in neoclassical stochastic equilibrium the performance and behavior of markets in the six experimental conditions should vary little and that, if firms used available data to estimate the structural parameters in the system, all markets should converge rapidly to stochastic equilibrium.

In contrast, the experience from dynamic decision-making studies lead to the following predictions about the main experimental treatment effects:

1) **Complexity should decrease stability in all three price regimes.** The combined effects of the multiplier and production lags should cause significant fluctuations in production and/or prices.

2) **Complexity should decrease profits, relative to optimal, in posted- and fixed-price regimes.** In the fixed-price and posted-price regimes, the main source of decreased profits should be inventory costs, resulting from the excess variance in production and sales. In the clearing-price regime, profits should vary less, due to the lack of inventory costs and a relatively flat profit function.

3) **The effects of complexity should be smallest in the clearing-price regime,** because market-clearing prices eliminate the process of inventory accumulation, thus "erasing" past imbalances.

4) **The effects of complexity should be large in the fixed-price regime.** The presence of lags and multiplier effects will result in large, long-term fluctuations. Without these elements, markets should adjust quickly to equilibrium.

5) **The effects of complexity in the posted-price regime should be larger than in the clearing-price regime, but may be smaller or larger than in the fixed-price regime.** If firms in the posted-price complex condition use prices effectively to control inventories, the markets should be more stable than under fixed prices. But high variation

in prices also makes it more difficult to control inventories and to maximize profit margins.

6) **Collusion should be most evident in the simple (posted and clearing price) conditions and least evident in the complex posted-price condition.** Signalling collusion and detecting such signals is difficult in the complex posted-price condition, where prices are also likely to be used to control inventories. It is also difficult in the clearing-price complex condition because the consequences of decisions only manifest with a lag. In contrast, firms are free to concentrate on collusive behavior in the corresponding simple conditions.

The analysis of the experimental results fell in two parts. The first part focused on whether the observed market behavior conformed to the neoclassical or the behavioral predictions. The second part sought to describe decision making of individual firms and using simulation to explore how this micro-level behavior could lead to the observed aggregate outcomes.

Aggregate outcomes

As mentioned, neoclassical theory predicts only negligible effects of the experimental treatments on market dynamics and performance relative to optimal and a rapid settlement the markets towards equilibrium in all cases. The actual behavior, however, showed large and systematic treatment effects.

First, the introduction of complexity lowered performance in all three price regimes. The effect was largest in the fixed-price regime and smallest in the clearing-price regime. Much of the decrease in profits was the result of excessive inventory fluctuations, but firms in the complex conditions also had lower profits before inventory costs. These results accord well with the behavioral predictions 2, 3, 4, 5, and 6 above.

Second, the introduction of complexity resulted in a significant increase in market volatility, in concurrence with prediction 1 above. In the fixed-price condition, the complexity led to large, sustained cycles in output. In the posted-price complex condition, there was likewise a tendency for cycles, involving prices and inventories. In the clearing-price complex condition, the variation took the form of a large single overshoot, followed by a gradual shift down to equilibrium. In contrast, the rational market adjustment process is rapid and orderly, and in the subsequent stochastic equilibrium, the variables in the market should be either serially uncorrelated or negatively correlated.

Third, a direct test of expectation formation showed clear evidence of systematic forecast biases. And there was no convincing evidence of learning even in the cases where the forecasting task became easier because the variability in the market declined over time.

In short, the experimental results clearly support the "behavioral" hypotheses motivating the experimental design.

It is important to emphasize that this result is not just simply the triviality that "if one gives people a harder task, they don't do as well." The key finding is that people's performance relative to the benchmark performance is lower in the complex condition. In the present study, it so happens that the (neoclassical) benchmarks do not vary much between the conditions, but this is not generally the case. In most cases, making the task "harder" means that the optimal performance would also decrease. The interesting result is that complexity degrades the ability of people to achieve the potential performance in the task.

Moreover, the study reveals not only that "people don't do as well" in the "harder" tasks but also that behavior departs from optimal in systematic, predictable ways, related to misperceptions of feedback. The second part of the analysis sought to describe the behavior of the markets explicitly by estimating decisions rules for the individual subjects and then simulating these rules to see if they could replicate the aggregate behavior of the markets.

Behavior under fixed prices

In the fixed-price complex condition, the observed decisions were regressed on a simple stock-adjustment rule and a simple adaptive-extrapolative rule for expectation formation. The simple stock-management rule fit the decisions excellently. Moreover, the analysis revealed that none of the subjects paid full attention to the supply-line correction that is crucial for stability: the weights on the supply line were all lower than the corresponding weights on inventories. Such low weights can be interpreted as evidence that subjects are not fully perceiving the importance of the long delays involved in production and the need to correct for actions taken earlier but not yet manifested.

The parameter estimates in the forecasting rule revealed a marked tendency to extrapolate movements in sales. The extrapolation of the past was indirect evidence that subjects did not fully understand the importance of the endogenous forces in the system, primarily the multiplier effect on demand, and instead viewed changes as externally imposed factors over which they had no control. This interpretation was further confirmed in the questionnaire responses of the subjects.

By inserting the estimated decision rules in a simulation model of each market, it was possible to explain the essential characteristics of the observed market-level outcomes. Both the pattern of fluctuation and the variations from market to market in the length and strength of the oscillations were reproduced closely by the models. Thus, differences between the markets were explained almost completely by differences in the parameters of the decision rules of individual firms.

The simulations also identified two key determinants of performance: the degree of attention to the supply line, and the aggressiveness of responses to inventory imbalances. Both effects had the expected direction and were highly significant: a lower weight on the supply line correction drastically lowered performance; a high (i.e. aggressive) weight on inventory correction, given the low observed attention to the supply line, likewise lowered performance.

Behavior under clearing prices

In the simple clearing-price condition, the markets showed no consistent characteristic behavior patterns, and attempts at capturing the observed decisions by a simple adaptive rule had mixed success: From the questionnaire responses, it was clear that most firms engaged in strategic interactions that are difficult to capture in a simple cue-weight model. Simulations of the simple clearing-price markets with a proposed "hill-climbing" decision rule all showed a gradual and orderly adjustment to a long-term equilibrium which, interestingly, fell very near the average operating point of each market.

The orderly adjustment is an inherent property of the feedback structure of the markets in the simple condition: A hill-climbing procedure constitutes a first-order negative feedback loop driving prices towards the level that maximizes individual and/or average profits with little chance of overshooting or oscillation.

In the corresponding complex condition, on the other hand, the feedback structure and the way in which the subjects interact with this structure produced a characteristic pattern of a large overshoot in production, followed by a gradual adjustment toward equilibrium. A simple decision rule was proposed which anchored on the previous output and forecasts of future output and responded to both the perceived profit gradient and the forecasted aggregate price (as a proxy for the aggregate long-term profit gradient). When the markets were simulated with the estimated parameters in this rule, all important features of the dynamic behavior patterns were reproduced quite successfully.

As in the fixed-price condition, the informal analysis of the protocol data indicated that people tended to attribute the observed history of aggregate demand to external influences rather than to internal, self-generated processes: None of the subjects attributed the initial boom and bust to the production lag and the multiplier effects. Instead, most attributed it either to external "business cycles" or to "the competition."

Behavior under posted prices

As in the simple clearing-price condition, the behavior of prices in the simple posted-price condition was dominated by strategic interactions between firms which could not be captured by a simple adaptive decision rule.

The production decisions, which in this condition should be quite straight-forward and mechanical, were reproduced quite well by a simple rule which took into account both the expected aggregated demand and price, the firm's own price, and its inventory.

But markets in this condition are primarily driven by prices. Adaptive "hill-climbing" pricing rules were unable to reproduce the observed market dynamics. Instead, markets would quickly converge to an equilibrium approximately equal to the average operating point of the observed markets. As in the clearing-price condition, the convergence of these rules is inherent in the feedback structure of the markets in this condition.

In contrast, the complex posted-price condition showed more regularity in the pricing behavior in any given market, though the behavior varied greatly from market to market. Although subjects were also concerned about competitive dynamics and strategies in this condition, much of their attention was focused on the need to control inventories and anticipate trends in prices and demand.

Inventories cannot be controlled directly through production in the complex condition, but they can be regulated by prices. Although this direct means of control makes it much easier to keep inventories in check than in the corresponding fixed-price condition, the observed markets nonetheless showed a tendency for cycles, primarily in prices and inventories.

The observed cycles were explained by the observation that firms to a great extent anchored their pricing on recent market averages, creating a cumulative effect of disequilibrium on prices. A simple pricing rule, in which price depended on the forecasted average market price and on the current inventory, was estimated and embedded in an endogenous simulation model of the market. The simulations reproduced cycles with the same relative variability, periodicity, and phase relationships between variables as those observed in the experiment.

8.2 Discussion

Feedback complexity and market adjustment

The basic research question motivating this research concerned the relationship between feedback structure and market adjustment: How can market institutions alleviate or alter the problems observed in non-market experiments? And, conversely, how does feedback structure affect market behavior?

The experiments clearly showed that market institutions can have a substantial effect on behavior and performance: The fixed-price complex condition showed large fluctuations of the same nature as those observed in previous non-market experiments. When posted prices were introduced, a substantial part of this fluctuation was eliminated, although markets still continued to show cyclical tendencies. When perfect market-clearing prices were introduced, the fluctuations were transformed into a single overshoot followed by a gradual settlement towards steady state.

It is important to note that the improvements in behavior gained through the introduction of markets were primarily due to the benefits from the extra feedback structure added by the price system, not by an increase in "rationality." On the contrary, both the market outcomes and the questionnaire responses clearly demonstrated that misperceptions of feedback persist in market settings. Whether these misperceptions manifest as dysfunctional behavior is therefore a question of the dynamic properties of the system, of its forgiveness of error.

The benefits from market institutions are not automatic. Only in the clearing-price condition were the oscillations completely eliminated (and then only replaced by a one-time boom and bust, not a rapid adjustment to equilibrium.) In the more realistic case where sellers set their own prices, the complex markets still showed considerable fluctuations, though generally smaller than the fixed-price markets. The beneficial effects are thus dependent on the efficiency of the price system, and it is an open question whether real-world markets can approach the efficiency of the clearing-price conditions. Posted seller prices is one of the most prevalent price institutions in developed economies.

Another qualification is that the observed stability of the clearing-price markets may not be robust to changes in the structural parameters of the system. Variability might well rise substantially if the strength of the multiplier were increased, or if the aggregate elasticity of demand were reduced. In the experiments, the "marginal propensity to consume" had a value of .5, which is much lower than the empirical aggregate estimate of .9 to .95. Moreover, a price-elasticity of aggregate demand of .75 is probably in the high range, given that many industries have much lower empirical elasticities (Hauthakker and Taylor, 1970). (If the system in question is the entire economy, the long-run nominal price elasticity of demand would be closer to unity to the extent that nominal equilibrium expenditures are constant. Yet equilibrium elasticities would be largely irrelevant if the system was dominated by disequilibirum dynamics with fluctuating nominal expenditures.)

Qualifications and caveats

To put the results in the proper perspective, it is necessary to consider the factors that might limit their validity and generality. The most obvious such issue is a general one pertaining to all experiments: do laboratory experiments reveal anything about real-world markets? This question of external validity was discussed in Chapter 2, and the arguments will only be summarized here.

In the experimental-economics field, the traditional argument in support of external validity is that since subjects play for real money the markets are no less real than any other market. Yet this argument rests on the axioms of neoclassical economics and cannot be used to support experiments designed to question these axioms.

A more fruitful approach is to turn the question around: Why should one expect decision makers to act differently in a laboratory setting than in the real world? It is certainly possible that the financial rewards in the experiment were too small to induce subjects to do their utmost. On the other hand, parallel experiments run with or without monetary incentives have shown that it is mostly the unsystematic (random) errors that are reduced while the systematic ones persist, or even become somewhat stronger, in the presence of incentives. Several well-known biases persist even with large monetary awards (see citations in Chapter 2).

Real-life decision makers may have a variety decision support systems or consulting services at their disposal, yet real life is also orders of magnitude more complex than laboratory markets. Whether one factor outweighs the other is not clear. Analytical methods certainly help promote consistency and understanding, but any tool must be based on initial assumptions that are subject to the strengths and weaknesses of intuition.

Ultimately, the question of external validity can only be settled empirically. Some of the empirically testable implications are discussed in the next section.

Another issue is the internal validity of the experiments. A large part of the analysis was based on comparisons with rational-expectations stochastic equilibrium, assuming firms were not cooperating. Yet one might ask whether such a benchmark is a fair comparison; is it really a test of neoclassical theory?

Rational firms ought to learn from all the available information at their disposal and to incorporate this information optimally in their decisions. In practice, this leads to the assumption in the optimal-learning literature that firms know everything about the system except for certain parameters, which must be estimated from past data. In the experiments, it was theoretically possible for firms to find these parameters after three or four time periods, but it required that they already knew what equations to estimate. Is it really reasonable to expect them to know that? Is it not too stringent a requirement of rationality?

Yes, one would be inclined to say, it is unreasonable. Yet if one wishes to maintain the notion of perfect substantive rationality, one cannot escape a dilemma: In the service of realism, one may wish to assume that agents know very little about the system at the outset and must gain that knowledge with experience. Yet the less this initial knowledge, the more Herculean becomes the task of gaining this knowledge and, all the while, acting optimally on any new evidence.

Another criticism one might levy is that the non-cooperative benchmarks assume that firms do not use prices to convey information (signal) to each other. Recently, economists have created a large body of work that deals with information asymmetries and the role of prices as carriers of hidden information. The markets in the experiments, having relatively few players and ample opportunity and incentives for collusion, would be an obvious area to apply such theories. It is conceivable, for instance, that one could explain the excess variation in prices and inventories in the complex posted condition as the result of firms' rational attempts to collude: firms would balance the additional inventory costs with the potential benefits from collusion.

This argument illustrates the problem that has impeded progress in the debate between psychology and economics for so long: Unless one is able and willing to use data gathered from observations of the actual making of economic decisions, one is always free to propose ad hoc auxillary hypotheses to save the axioms of rationality. Such "ex-post-ulation" is, in this author's view, of very little value.

One must return, then, to the data from the experiments, including the verbal protocols, and ask whether they can really be explained by such theories. The protocols certainly did not reveal any evidence of the sophistication and precision that one would expect to find in dynamic game theory with asymmetric information. Moreover, the fixed-price condition involved no element of strategic interaction--how could game theory explain the large oscillation observed in the complex condition here?

This is not to say that the game-theoretic, strategic aspects of the markets are uninteresting or unimportant. On the contrary, the failure of simple adaptive rules to reproduce the observed variation in the simple posted- and clearing-price condition indicates the need for an explicit model of the strategic interactions between firms. The claim is that the observed effects of complexity can be explained and reproduced by a feedback model that accords well with both the experience from previous studies of dynamic decision-making, the observed individual decisions, and the observed verbal responses.

Implications and suggestions for further research

The analytical tradition in psychology holds that to understand the whole one must first understand the parts that make up that whole. Viewed from this perspective, the incredible complexity of the human mind can quickly lead one to pessimism: If the parts are so difficult to understand, how can one even begin to understand the whole? The message of this thesis is an optimistic one: Human decisions may appear inscrutable when viewed individually and out of context. But in the context of a dynamic task environment the aspects of human decisions that are of real importance to the performance of the system reveal themselves. Hence, complexity does not mean mystery. On the contrary, the dynamic complexity of a system can bring more regularity to behavior and, hence, more fertile grounds for theory-building.

The results in this work and the other recent studies of dynamic decision making suggest that one way to bridge the gap between economics and psychology is the feedback systems approach. It provides a way to link the individual, micro-level description of human behavior to the aggregate, macro-level outcomes of interest to economists, sociologists, social psychologists, and policy-makers.

The possibilities are vast and varied, but in the interest of concreteness, one should start closest to the work at hand with some immediate extensions to the thesis. The experiments have generated a vast amount of data, only a fraction of which has been analyzed so far. For instance, the cross-firm correlation in the forecasting residuals should be investigated further, and a similar analysis of residuals should be used for the other decision rules. Although one could use statistical methods, such as Zellner estimation, to take advantage of the correlation, a more interesting endeavor would be to try to uncover the source of the correlation: What is it that all subjects respond to that is not reflected in the simple rule?

Another extension is to look closely at the questionnaire data and try to correlate the responses with the observed behavior of each subject. Do the stated strategies appear to correlate with the actions? Might the responses provide clues to a model of strategic interactions? In addition to the questionnaire responses, a number of concurrent verbal "think-aloud" protocols were collected. These protocols could provide extremely detailed information about the decision processes of the individuals which might give further ideas for modeling. One could also correlate the behavior and performance of individual subjects with the data on their educational background. Do subjects with extensive background in economics do better than others? Do they do better in the simple conditions which mirror "textbook" markets and worse in the complex conditions?

Finally, it would be interesting to see whether the effects observed in the laboratory also occur in the real world. An interesting project would be to try to classify real-world industries along scales of "feedback complexity" and market institution and then see the extent to which these scales explain differences in the behavior of these markets. The difficulty, naturally, lies in defining the independent variables correctly. But it should be possible to find objective measures, such as the time lags between product development, production, and delivery, and the degree to which previous actions can be altered along the way. Moreover, it should be possible to develop a measure of how organized the markets are. For instance, agricultural commodities are traded in highly organized and centralized exchanges which even include futures contracts. Conversely, automobiles are traded in dispersed dealer networks where local dealers have much more leverage over what price to charge for their product.

Based on the results in this thesis one would predict that industries with longer time delays and stronger commitments to production levels would show more cyclical tendencies than would markets where supply can quickly be adjusted to match demand. And to the extent that the centralized exchanges in the real-world can mimic the price-clearing condition in the experiments, industries should be relatively less prone to instability. (The presence of irrational speculators in such exchanges could, on the other hand, make matters worse by destabilizing prices.) Finally, industries with effects equivalent to the multiplier in the experiments--strong word-of-mouth effects or technological externalities, for instance--should be less stable.

Beyond these immediate extensions, there lies an agenda of investigating other economic systems. Of particular interest to this author is the simulation of macroeconomic systems with endogenous money supplies, capital stocks, etc. A particularly interesting step would be to try to make the connection to the rest of the experimental-economics literature by trying to provide explicit behavioral dynamic models of the processes, such as auctions, sealed bids, etc., that are the subject of this field. Might it be possible to explain the observed, frequently puzzling, regularities with such models? Certainly, there is no lack of challenges or questions ahead.

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Appendix A:
Mathematical description and derivations

The following appendix contains the mathematical description of the experimental market and the derivations used in the analysis in the thesis. The appendix is fairly voluminous, in large part because it also serves as a record of the derivations. The first section describes the market structure and derives the competitive and collusive equilibria. Following this are three sections, one for each price regime, in which the optimal policies and the rational-expectations stochastic equilibria are derived. In some cases one cannot find exact analytic expression but must instead rely on linear- or quadratric approximations and numerical analysis. Section 5 shows the variance and spectral density of output and prices in rational-expectations equilibrium, for each of the six regimes. The last three sections contain subsidiary notes on optimal control of linear systems with first-order terms, comparison of the simple averages X, P to the non-linear aggregates $\tilde{X}, \tilde{P}$, and on the expectation $E\{|n|\}$, respectively.

Notation

To facilitate reading of equations, the following conventions are observed in the following:

- Upper-case symbols (X,Y,N, etc.) denote market averages or aggregates while lower-case symbols (x, y, n, etc.) denote the corresponding individual firm variables. Moreover, for non-linear aggregations, a superscript squiggle (~) will be used. Otherwise, the aggregates are all arithmetic averages.

- Boldface symbols (**z**, **Z**, **u**, etc.) denote vectors while matrices are denoted by boldface with a double underline ($\underline{\underline{\mathbf{P}}}$, $\underline{\underline{\mathbf{Q}}}$, $\underline{\underline{\mathbf{R}}}$, etc.).

- Parameters are denoted by greek letters while variables and derived constants are denoted by roman letters.

- At its **first introduction**, each symbol is printed in **boldface**.

- Likewise, **assumptions** will be highlighted whenever made.

Appendix A

1. Market system

Firms

The market consists of **K firms**, indexed by **i = 1, ..., K. Time** is divided into discrete periods, indexed by **t = 0, 1, ...** .

At the beginning of each period, t, each firm, i, must decide how much **production to initiate, $y_{i,t}$,** to initiate and what **price, $p_{i,t}$,** to charge for its product this period. Production starts are constrained to be non-negative. The firm must make these decisions *ex ante,* i.e. without knowing the demand it faces this period.

Each firm maintains a goods **inventory, $n_{i,t}$,** to accommodate fluctuations in demand. Inventories can be negative, corresponding to an order backlog. The inventory is decreased by **sales,** $x_{i,t}$, and increased by production. There may be a **lag of δ periods** between the initiation of production, and the time it arrives in inventory. Thus, the movement of inventories/backlogs is described by

(1.1) $$n_{i,t+1} = n_{i,t} + y_{i,t-\delta} \text{ for all i,t.}$$

Profits, $v_{i,t}$, each period is the difference between revenue and costs. Costs consist of production cost, proportional to sales, and inventory holding costs, proportional to the absolute value of the inventory at the beginning of the period. Production costs and revenues are assumed to be incurred at the time the goods are ordered by buyers, regardless of when the good is actually produced and/or delivered. Given **unit production costs** ω and **unit inventory/backlog holding costs** γ, firm i's profit in period t is

(1.2) $$v_{i,t} = p_{i,t}x_{i,t} - \omega x_{i,t} - \gamma | n_{i,t} | \text{ for all i,t.}$$

The inventory backlog cost is conceived as a combination of holding and processing costs and, in the case of a backlog, a "discount" offered to consumers to compensate for lost utility due to late delivery.

Firms have "full aggregate information" in the sense that they can observe past values of all their "own" variables, such as price, inventory, and production, and they can observe past values of the market average values of these variables.

Demand side

The demand side of the market is assumed to consist of an arbitrary but large number of buyers who purchase (or order) goods from individual firms so as to maximize their utility. It is assumed that buyer utility is a Constant-Elasticity-of-Substitution (CES) function of goods bought from individual firms, with an **elasticity of substitution** $\varepsilon > 1$, and that buyer utility is "obtained" at the time of purchase or ordering, regardless of when the good is in fact delivered. (The loss of utility from late delivery is assumed to be fully compensated by firms, in the form of the backlog cost, γ.) Thus, buyer utility is an instantaneous function of the "**aggregate good**", $\tilde{X}_t$, determined by

$$(1.3) \qquad \tilde{X}_t = \left(\frac{1}{K}\left(x_{1,t}^{(\varepsilon-1)/\varepsilon} + \dots + x_{K,t}^{(\varepsilon-1)/\varepsilon}\right)\right)^{\varepsilon/(\varepsilon-1)}, \text{ for all t.}$$

Given the **average amount of consumer spending per firm, I** on all the goods in the market during a particular period, consumers thus solve the problem of maximizing $\tilde{X}$ over the individual goods x_i, subject to the budget constraint

$$(1.4) \qquad p_{1,t}x_{1,t} + \dots + p_{K,t}x_{K,t} = K\, I, \text{ for all t.}$$

The first-order conditions for this problem yield

$$(1.5) \qquad \frac{x_{i,t}^{(\varepsilon-1)/\varepsilon}}{x_{j,t}^{(\varepsilon-1)/\varepsilon}} = \frac{p_{i,t}^{1-\varepsilon}}{p_{j,t}^{1-\varepsilon}}, \text{ for all i, j, t.}$$

Summing this expression over all j and inserting into (1.3) yields

$$(1.6) \qquad x_{i,t} = \tilde{X}_t(p_{i,t}/\tilde{P}_t)^{-\varepsilon}\ \forall\ i, t,$$

where the "**aggregate" price**, $\tilde{P}_t$, is defined by

$$(1.7) \qquad \tilde{P}_t = \left(\frac{1}{K}(p_{1,t}^{1-\varepsilon} + \dots + p_{K,t}^{1-\varepsilon})\right)^{1/(1-\varepsilon)}, \forall\ t.$$

Multiplying (1.7) by $p_{i,t}$, summing over i, and inserting in the budget constrant (1.4) further yields

$$(1.8) \qquad I_t = \tilde{X}_t\tilde{P}_t, \forall\ t.$$

Thus, $\tilde{P}_t$ can be interpreted as an aggregate price index, or the "price" of the aggregate good $\tilde{X}_t$.

Buyers are further assumed to embed their total purchases from this market in the context of a larger optimization problem involving other markets, savings, leasure, etc., so that the aggregate market demand (or demand for the aggregate good) in period t, $\tilde{X}_t$, is a function of the the aggregate price, $\tilde{P}_t$.

Moreover, the aggregate market demand is assumed to depend on the overall level of activity in that market, i.e. there is a "multiplier" effect from production to demand. Economically, the effect can be interpreted either as a Keynesian consumption multiplier, where wage income is related to the level of activity, or as an input-output mutliplier, where firms use part of each others products as inputs to the production process.

Specifically, $\tilde{X}_t$ is determined by

(1.9) $\tilde{X}_t = X_t^* f(\tilde{P}_t);$

(1.10) $X_t^* = (1-\alpha)G_t + \alpha\frac{1}{\delta+1}(Y_t+S_t); 0\leq \alpha \leq 1;$

(1.11) $Y_t = (y_{1,t} + ... + y_{K,t})/K;$

(1.12) $S_t = (Y_{t-\delta} + ... + Y_{t-1}), \forall$ t.

X_t^* is the **"reference demand,"** which is multiplied by the function f(.) of the aggregate price to get actual demand. The "reference" demand consists of **autonomous demand, G_t,** and a proportion attributed to the multiplier effect. The parameter α is the indicates the **strength of the multiplier effect**, i.e. the increase in reference demand for each unit increase in the average production level, i.e., the average production over all firms and all δ production stages. In equations (1.10) to (1.12) a distinction is made for notational purposes between the **average production starts, Y_t** and the **average "supply line", S_t,** of previously started but not yet completed production.

The **price-dependence of industry demand, f(.)**, is assumed to have a **constant elasticity, μ,** around the **perfect competition equilibrium price, p^*.**

(p^* is a function only of the unit production cost, ω, and the elasticity of substitution, ε, as shown below but can be considered to be a derived parameter of the system.) The elasticity μ is assumed to be less than unity, reflecting the idea that, while the goods offered by different firms in the market are fairly close substitutes, the overall industry demand is inelastic. As the aggregate price moves further away from the competitive-equilibrium value, however, aggregate demand becomes a linear function of price. (If aggregate price-elasticity of demand was constant and less than unity for all prices, colluding firms could earn arbitrarily large profits by charging an arbitrarily large price.) Figure 1.1 shows a plot of the function f(.).

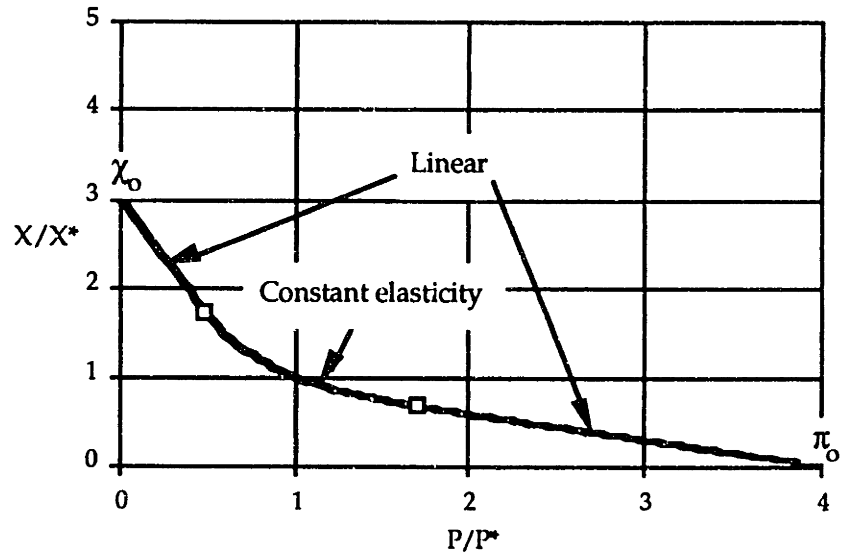

Figure 1.1
Plot of aggregate demand relative to "reference" aggregate demand, as a function of aggregate price relative to "reference" price.

Specifically, the function f(.) is formulated in terms of the ratio $\tilde{P}_t/p^*$, according to

$$(1.16) \qquad f(\tilde{P}_t) = \begin{cases} \chi_0 - b_1\tilde{P}_t/p^*, & 0 \le \tilde{P}_t/p^* < c_1; \\ (\tilde{P}_t/p^*)^{-\mu}, & c_1 \le \tilde{P}_t/p^* \le c_2; \\ a_2 - b_2\tilde{P}_t/p^*, & c_2 < \tilde{P}_t/p^* \le \pi_0; \\ 0, & \pi_0 < \tilde{P}_t/p^*, \forall\, t. \end{cases}$$

The parameter χ_0 is the ratio of demand to reference demand at zero price, and π_0 is the ratio of price to competitive equilibrium price at which

demand is zero. Given these cut-off points and the requirement that f(.) be differentiable, the **derived parameters b_1, c_1, a_2, b_2, and c_2** fulfill

$$(1.17) \qquad b_1 = \frac{\mu\chi_o}{c_1(1+\mu)};\ c_1 = \left(\frac{\chi_o}{1+\mu}\right)^{-1/\mu};\ a_2 = b_2\pi_o;\ b_2 = \mu c_2^{-1-\mu};\ c_2 = \frac{\mu\pi_o}{1+\mu}.$$

Steady-state competitive equilibrium

If the market is in steady-state equilibrium with a constant autonomous demand component, G_t=G, inventories will be zero. If **furthermore all firms are assumed to act competitively,** they face the profit maximization problem

$$(1.18) \qquad \text{Max } v_i(p_i) = x_i(p_i-\omega),$$

where x_i is determined by the structural equations given above. The **steady state,** requires that (assuming firms are producing approximately the same outout)

$$X^* = G(1-\alpha) + \alpha\,\tilde{X} = G(1-\alpha) + \alpha\,X^*\,f(\tilde{P}) \Rightarrow$$

$$(1.19) \qquad \tilde{X}_{ss} = G\frac{(1-\alpha)\,f(\tilde{P})}{1-\alpha\,f(\tilde{P})},$$ where "ss" stands for steady state.

The first-order condition, assuming other firms hold their prices (not their quantities) constant, yields

$$\frac{dv_i}{dp_i} = 0, \Rightarrow x_i + (p_i-\omega)\frac{dx_i}{dp_i} = 0, \Rightarrow$$

$$x_i + (p_i-\omega)\left(-\varepsilon x_i/p_i + (\varepsilon x_i/\tilde{P} + \frac{x_i}{\tilde{X}_{ss}}\frac{d\tilde{X}_{ss}}{d\tilde{P}})\frac{d\tilde{P}}{dp_i}\right) = 0, \Rightarrow$$

$$x_i + x_i(p_i-\omega)\left(-\varepsilon/p_i + (\varepsilon/\tilde{P} + \frac{f'(\tilde{P})}{f(\tilde{P})[1-\alpha f(\tilde{P})]})\frac{1}{K}(p_i/\tilde{P})^{-\varepsilon}\right) = 0, \Rightarrow$$

$$(1.20)\qquad p_i + (p_i-\omega)\left(-\varepsilon + \left(\varepsilon + \frac{\tilde{P}\,f'(\tilde{P})}{f(\tilde{P})}\frac{1}{1-\alpha f(\tilde{P})}\right)\frac{1}{K}(p_i/\tilde{P})^{1-\varepsilon}\right) = 0.$$

It is possible to show (numerically, at least) that (1.20) has only one positive solution for p_i for any given aggregate price level, $\tilde{P}$. This in turn implies that the symmetric equilibrium where all firms charge the same price and produce the same output is the only possible Nash equilibrium. If, moreover, the function f(.) is in the constant-elasticity region, the **competitive-equilibirum price for K firms, $p_K{}^*$,** is given by

$$(1.21)\qquad p_K{}^* = \frac{\omega\,\varepsilon_K}{\varepsilon_K - 1}, \text{ where } \varepsilon_K = \frac{K-1}{K}\varepsilon + \frac{1}{K}\mu\,[1 - \alpha\, f(p_K{}^*)]^{-1}.$$

The equation (1.21) cannot be solved analytically, but are easy to solve numerically. Table 1.1 shows the values of $p_K{}^*$ and teh **competitive-equilibrium output for K firms, $x_K{}^*$**,respectively, for various values of K. The assumption underlying these derivation was that firms optimize assuming other firms hold their <u>prices</u> constant (the so-called Bertrand game equilibrium). If, instead, firms assume that other firms hold their <u>sales</u> constant, one can go through a similar set of derivations, to find the **Cournot-equilibrium price, $p_K{}^{**}$, and output, $x_K{}^{**}$**, respectively. These are also listed in Table 1.1. If the number of firms goes to infinity, the solution converges to the **"perfect"-competitive-equilibrium price,**

$$(1.22)\qquad p^* = \omega\varepsilon/(\varepsilon - 1)$$

and the **"perfect"-competitive-equilibrium output, G.**

K	Bertrand equilibrium				Cournot equilibrium			
	p_K^*/p^*		x_K^*/G		p_K^{**}/p^*		x_K^{**}/G	
	α=0	α=.5	α=0	α=.5	α=0	α=.5	α=0	α=.5
2	1.56	1.27	0.56	0.72		1.66		0.52
3	1.25	1.13	0.73	0.84	2.08	1.26	0.58	0.72
4	1.16	1.09	0.80	0.88	1.64	1.16	0.69	0.81
5	1.12	1.07	0.85	0.91	1.45	1.12	0.76	0.85
6	1.10	1.05	0.87	0.93	1.35	1.09	0.80	0.88
7	1.08	1.04	0.89	0.94	1.29	1.08	0.83	0.90
8	1.07	1.04	0.91	0.95	1.24	1.07	0.85	0.91
9	1.06	1.03	0.92	0.95	1.21	1.06	0.87	0.92
10	1.05	1.03	0.93	0.96	1.18	1.05	0.88	0.93
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 1.1
Non-cooperative steady-state price and output level as a function of the number of firms.

Collusive equilibrium

If, instead, firms collude perfectly, they must solve the joint profit-maximization problem

(1.23) $\max V = (\tilde{P} - \omega)\tilde{X}$, where $\tilde{X}$ must satisfy (1.19).

It is clear that, since the elasticity of aggregate demand is $\mu<1$ around the constant-elasticity region of the function f(.), the collusive equilibrium must lie in the high-price, linear region.[1] By differentiating (1.23) and

1 If $\alpha > (1-1/\varepsilon)^{\mu}$ (which it isn't with the parameters chosen for the experiment), colluding firms could theoretically earn arbitrarily large profits by charging a price which would render the denominator in (1.19) zero: in that case the system has no equilibrium point, and demand can grow forever, fueled by the multiplier process.

inserting (1.19), one can find the profit-maximizing collusive price and output. The process is straightforward but tedious, and only the final results are shown here. The **collusive steady-state output, x^M**, is

$$(1.24) \qquad x^M = \begin{cases} qG/2, & \alpha = 0; \\ (1-\alpha)G\dfrac{1-q}{\alpha q}, & 0<\alpha<1, \end{cases}$$

and the **steady-state collusive price, p^M**, is

$$(1.25) \qquad p^M = \begin{cases} (\omega + \pi_0)/2, & \alpha = 0 \\ p^*(\alpha a_2 - 1 + q)/(\alpha b_2), & 0<\alpha<1 \end{cases}, \text{ where}$$

$$q = \sqrt{1 - \alpha(a_2 - b_2(1 - 1/\varepsilon))}.$$

Figure 1.2 shows steady-state profits and output as a function of price, all normalized by the competitive-equilibrium values. Thus, colluding firms could earn about 50% more in the complex condition (α=0.5) and about double in the simple condition (α=0).

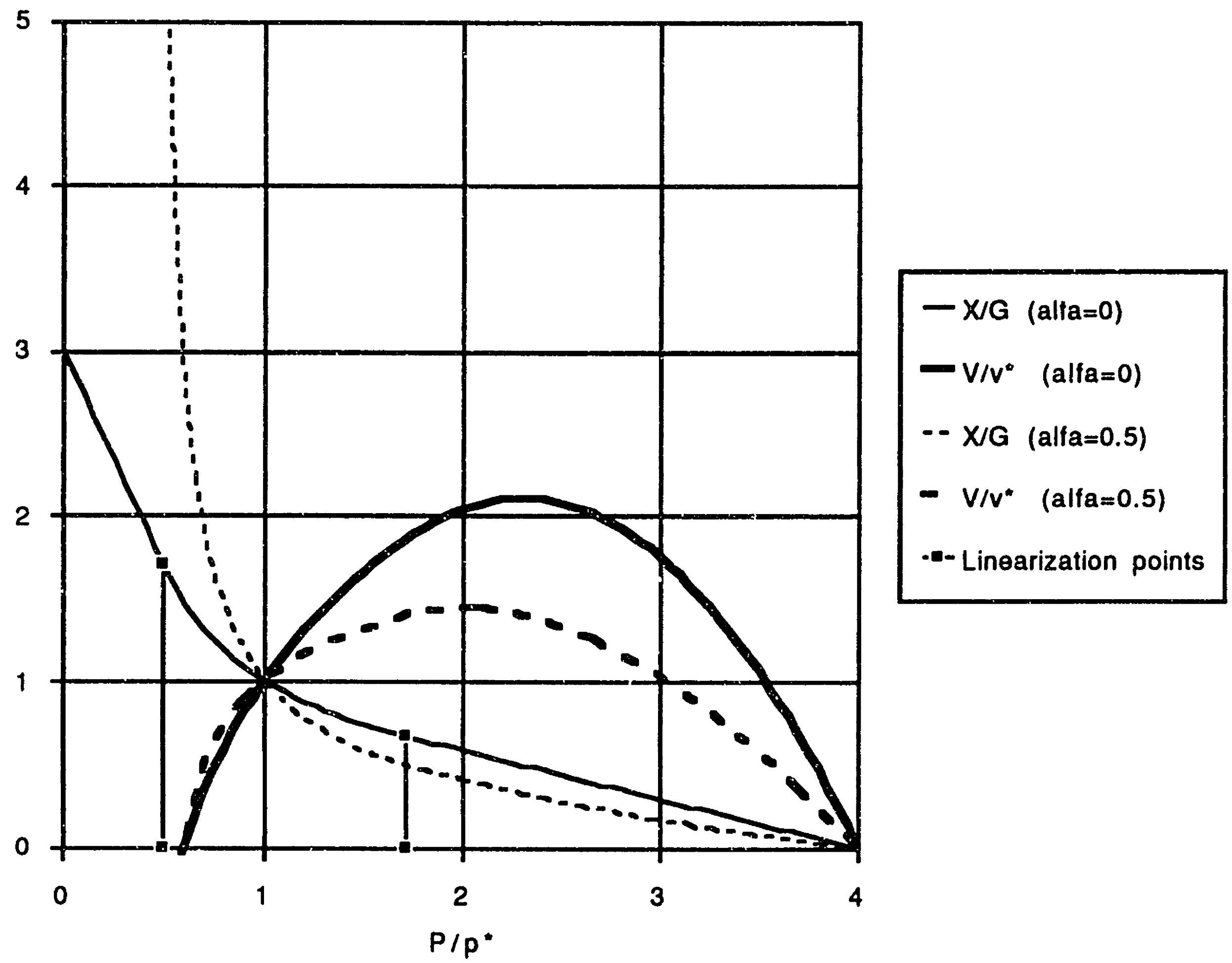

Figure 1.2
Steady-state output and profits as a function of price.
The parameters in f(.), etc., are those used in the experiment. Note that, for α>0, the system does not have a steady-state solution below a certain price level.

2. Fixed prices

Optimal policy

In the following, firms are assumed to know the complete structure of the system and its parameters. Moreover, in solving for rational expectations, firms are assumed to follow the optimal policy subject only to a random, serially uncorrelated error. Thus, this section does not consider optimal (Bayesian) learning.

Appendix A

Since prices are held fixed at the perfectly-competitive level, p^*, the task of each firm is to meet incoming demand and minimize inventory costs. The demand for each firm is the same, equal to the reference demand, X^*, i.e.,

$$x_{i,t} = (1-\alpha)G + \alpha\frac{1}{\delta+1}(Y_t+S_t). \tag{2.1}$$

Assuming first that the number of firms, K, is large or that firms believe the effect of their own actions on the aggregate to be negligible, and further assuming that the non-negativity constraint on production is not binding, the dynamic cost minimization problem can be decomposed into separate problems for each time period. One period's production decision affects only the inventory level $\delta+1$ periods hence; demand is assumed to be unaffected by the decision, and the inventory in later periods can be fully regulated by subsequent production decisions. Then, since costs are proportional to the absolute value of inventory, the problem becomes to choose output, $y_{i,t}$ to minimize the expected inventory costs, i.e., to minimize the expected absolute value of inventory. Using the results in Section 8, one finds the first-order condition

$$\begin{aligned} 0 &= \frac{\partial}{\partial y_{i,t}} E_t\{|n_{i,t+\delta+1}|\} \\ &= [1 - 2F(n_{i,t+\delta+1})]\frac{\partial n_{i,t+\delta+1}}{\partial y_{i,t}} \\ &= [1 - 2F(n_{i,t+\delta+1})], \end{aligned} \tag{2.2}$$

where E_t is the expectation operator and F(.) is the cumulative distribution function of $n_{i,t+\delta+1}$, conditional on $y_{i,t}$. The first-order condition thus requires the <u>median</u> of $n_{i,t+\delta+1}$ to be zero. Further, since, by successive substitution in equation (1.1),

$$n_{i,t+\delta+1} = n_{i,t} + y_{i,t-\delta} + \ldots + y_{i,t-1} + y_{i,t} - x_{i,t} - \ldots - x_{i,t+\delta}, \tag{2.3}$$

the optimal policy, $y_{i,t}^*$, is

$$y_{i,t}^* = M_t\{x_{i,t} + \ldots + x_{i,t+\delta}\} - (y_{i,t-\delta} + \ldots + y_{i,t-1} + n_{i,t}), \tag{2.4}$$

where M_t is the median operator. If the distribution of $x_{i,t} + \ldots + x_{i,t+\delta}$ is approximately symmetric, as will be assumed in the following, the median and the mean approximately coincide. One can now write the optimal production policy as

(2.5) $y_{i,t}^* = E_t\{x_{i,t} + ... + x_{i,t+\delta}\} - (y_{i,t-\delta} + ... + y_{i,t-1} + n_{i,t})$.

Defining the **individual "supply line", $s_{i,t}$,**

(2.6) $s_{i,t} = y_{i,t-\delta} + ... + y_{i,t-1}, \forall\ i,t,$

and the **desired "supply line", $s_{i,t}^*$,** as

(2.7) $s_{i,t}^* = E_t\{x_{i,t} + ... + x_{i,t+\delta-1}\}, \forall\ i,t,$

one can rewrite (2.5) as

(2.8) $y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*$, where

$\tau^* = \beta^* = 1, n^* = 0.$

Expressed this way, the optimal rule thus has the same general form as the suggested behavioral stock adjustment rule.

Rational expectations, perfect competition

In the following, firms are assumed to be following the optimal policy (2.8) up to a random, serially uncorrelated error, $u_{i,t}$ and to have rational expectations. The number of firms is still assumed to be very large. Thus,

(2.9) $y_{i,t} = y_{i,t}^* + u_{i,t}, \forall\ i, t.$

Since all firms receive identical demand and have the same aggregate information, the desired supply line and expected demand is the same for each and (2.9) is readily aggregated into

(2.10) $$Y_t = Y_t^* + U_t,$$
$$= E_t\{X_{t+\delta} - N_{t+\delta}\} + U_t,$$
$$= E_t\{X_t + ... + X_{t+\delta}\} - (Y_{t-\delta} + ... + Y_{t-1} + N_t) + U_t,$$
$$= E_t\{X_t + ... + X_{t+\delta}\} - (S_t + N_t) + U_t, \forall t,$$[2] where

(2.11) $U_t = \frac{1}{K}(u_{1,t} + ... + u_{K,t})$ is the aggregate random decision error.

Now consider

(2.12) $$E_t\{Y_{t+s}\} = E_t\{ E_{t+s}\{X_{t+\delta+s} - N_{t+\delta+s}\} + U_{t+s} \},$$
$$= E_t\{X_{t+\delta+s}\} - E_t\{N_{t+\delta+s}\} + E_t\{U_{t+s}\},$$
$$= E_t\{X_{t+\delta+s}\}, \forall t \forall s>0,$$

since the expected error is zero and expected future inventories are zero. Further inserting the equation for demand (2.1) yields

$$E_t\{Y_{t+s}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(E_t\{Y_{t+s}\} + ... + E_t\{Y_{t+\delta+s}\}), =>$$

(2.13) $$E_t\{Y_{t+s}\} - G = \frac{\alpha}{1-\alpha+\delta}\ \frac{1}{\delta+1}\ (E_t\{Y_{t+1+s}\}-G + ... + E_t\{Y_{t+\delta+s}\}-G), \forall t \forall s>0.$$

Since

$$\frac{\alpha}{1-\alpha+\delta} < 1,$$

the process (2.13) is stable (Y converges to G) going <u>backwards</u> in time. Conversely, the process is unstable going <u>forwards</u> in time. Thus, the only non-exploding solution to (2.13) is

(2.14) $$E_t\{Y_{t+s}\} = G; E_t\{X_{t+\delta+s}\} = G, \forall t \forall s>0.$$

---

2 Since all firm demands are equal, the non-linear aggregate demand, $\tilde{X}$ is identical to the simple arithmetic average, X, which is used in the following.

In effect, the rational-expectations solution requires that all observed imbalances are eliminated "immediately" by adjusting production, so that future production is on average in equilibrium. In the absense of future errors, production would exhibit a one-time spike or dip and then jump to equilibrium.

To find Y_t, first consider demand in the current and future δ periods.

(2.15) $$E_t\{X_t\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-\delta} + \dots + Y_{t-1} + E_t\{Y_t\}),$$

$$E_t\{X_{t+1}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-\delta+1} + \dots + Y_{t-1} + E_t\{Y_t\} + E_t\{Y_{t+1}\}),$$

...,

$$E_t\{X_{t+\delta}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} + \dots + E_t\{Y_{t+\delta}\}).$$

Summing the δ+1 equations (2.15), and using (2.14), one gets

(2.16) $$E_t\{X_t + \dots + X_{t+\delta}\} = (\delta+1)(1-\alpha)G +$$

$$\frac{\alpha}{\delta+1}\left[Y_{t-\delta} + 2Y_{t-\delta+1} + \dots + \delta Y_{t-1} + (\delta+1)E_t\{Y_t\} + G\delta(\delta+1)/2\right],$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t, \forall\, t,$$

where the **auxillary variable, Z_t,** is defined by

(2.17) $$Z_t = \frac{\alpha}{\delta+1}\left[Y_{t-\delta}-G + 2(Y_{t-\delta+1}-G) + \dots + \delta(Y_{t-1}- G)\right], \forall\, t.$$

Inserting this in (2.10) allows one to solve for $E_t\{Y_t\}$.

$$E_t\{Y_t\} = E_t\{X_t + \dots + X_{t+\delta}\} - (S_t + N_t)$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t - (S_t + N_t), \forall\, t, \Rightarrow$$

(2.18) $$E_t\{Y_t\} = G + \frac{1}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\, t.$$

One can now find $E_t\{X_{t+\delta}\}$ by inserting (2.17) and (2.18) in (2.15), to get

(2.19) $$E_t\{X_{t+\delta}\} = (1-\alpha)G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} + \delta G)$$

$$= G + \frac{\alpha}{\delta+1}(E_t\{Y_t\} - G)$$

$$= G + \frac{1}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\ t.$$

In similar fashion, one finds S_t^* to be

$$S_t^* = E_t\{X_t + \ldots + X_{t+\delta}\} - E_t\{X_{t+\delta}\} = \tag{2.20}$$

$$= (\delta+1)G + \alpha(E_t\{Y_t\} - G) + Z_t - G - \frac{\alpha}{\delta+1}(E_t\{Y_t\} - G)$$

$$= \delta G + \frac{\delta}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)) + Z_t, \forall\ t.$$

To summarize, under rational expectations with many firms, and in the absence of binding non-negativity constraints on production, the production policy is

$$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*, \tag{2.21}$$

$$E_t\{x_{i,t+\delta}\} = G + \frac{1}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)),$$

$$s_{i,t}^* = \delta G + Z_t + \frac{\delta}{\delta+1}\frac{\alpha}{1-\alpha}(Z_t - (S_t - \delta G + N_t)), \forall\ t, \text{ where}$$

$$Z_t = \frac{\alpha}{\delta+1}\left[Y_{t-\delta} - G + 2(Y_{t-\delta+1} - G) + \ldots + \delta(Y_{t-1} - G)\right], \forall\ t, \text{ and}$$

$$n^* = 0;\ \tau^* = \beta^* = 1.$$

Note that if $\delta = 0$, $Z_t = s_{i,t}^* = s_{i,t} = 0$, and (2.21) reduces to

$$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^*, \tag{2.22}$$

$$E_t\{x_{i,t+\delta}\} = G - \frac{\alpha}{1-\alpha}N_t, \forall\ t.$$

Likewise, if $\alpha = 0$, (2.21) reduces to

$$y_{i,t}^* = E_t\{x_{i,t+\delta}\} + (n^* - n_{i,t})/\tau^* + \beta^*(s_{i,t}^* - s_{i,t})/\tau^*, \tag{2.23}$$

$$E_t\{x_{i,t+\delta}\} = G,$$

$$s_{i,t}^* = \delta G, \forall t.$$

Performance in stochastic equilibrium

The the aggregate production can be written as

$$\text{(2.24)} \qquad Y_t = E_t\{Y_t\} + U_t$$

$$= G + \frac{1}{1-\alpha}(Z_t - (S_t - \delta G + N_t)) + U_t.$$

Taking first differences on both sides yield

$$\text{(2.25)} \qquad (1-L)Y_t = \frac{1}{1-\alpha}((1-L)Z_t - (1-L)(S_t + N_t)) + (1-L)U_t,$$

where L is the lag operator $L\, x_t = x_{t-1}$; $L\, E_t x_t = E_{t-1}\, x_{t-1}$. Taking first differences on both sides of (2.17) yields

$$\text{(2.26)} \qquad (1-L)Z_t = \frac{\alpha}{\delta+1}\Big[Y_{t-\delta} + 2Y_{t-\delta+1} + \dots + \delta Y_{t-1} - Y_{t-\delta-1} - 2Y_{t-\delta} - \dots - \delta Y_{t-1}\Big]$$

$$= \frac{\alpha}{\delta+1}\Big[\delta Y_{t-1} - Y_{t-\delta-1} - \dots - Y_{t-2}\Big]$$

$$= \frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}).$$

Likewise, taking first differences on both sides of (1.12) yields

$$\text{(2.27)} \qquad (1-L)S_t = Y_{t-\delta} + \dots + Y_{t-1} - Y_{t-\delta-1} - \dots - Y_{t-2}$$

$$= Y_{t-1} - Y_{t-\delta-1}.$$

Finally, aggregating the equation (1.1) for inventories yields

$$\text{(2.28)} \qquad (1-L)N_t = Y_{t-\delta-1} - X_{t-1}.$$

By inserting this results and the equation for demand (2.1) into (2.24) one gets

$$(1-L)Y_t = \frac{1}{1-\alpha}\Big(\frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}) - (Y_{t-1} - X_{t-1})\Big) + (1-L)U_t,$$

$$= \frac{1}{1-\alpha}\Big(\frac{\alpha}{\delta+1}(\delta Y_{t-1} - S_{t-1}) - Y_{t-1} + (1-\alpha)G + \frac{\alpha}{\delta+1}(Y_{t-1} + S_{t-1})\Big) + (1-L)U_t,$$

$$= G - Y_{t-1} + (1-L)U_t, \Rightarrow$$

(2.29) $$Y_t - G = U_t - U_{t-1}.$$

Further, inserting (2.29) in (2.1) yields

(2.30) $$X_t - G = \frac{\alpha}{\delta+1}(Y_t - G + ... + Y_{t-\delta} - G)$$

$$= \frac{\alpha}{\delta+1}(U_t - U_{t-1} + ... + U_{t-\delta} - U_{t-\delta-1})$$

$$= \frac{\alpha}{\delta+1}(U_t - U_{t-\delta-1}).$$

Finally, inserting in the equation for inventory yields

$$(1-L)N_t = L^{\delta+1}Y_t - LX_t$$

$$= L^{\delta+1}(1-L)U_t - \frac{\alpha}{\delta+1}L(1 - L^{\delta+1})U_t, \Rightarrow$$

(2.31) $$N_t = \left(L^{\delta+1} - \frac{\alpha}{\delta+1}L\frac{1 - L^{\delta+1}}{1-L}\right)U_t$$

$$= -\frac{\alpha}{\delta+1}(U_{t-1} + ... + U_{t-\delta}) + (1 - \frac{\alpha}{\delta+1})U_{t-\delta-1}.$$

Turning now to the production and inventory of individual firms, one first observes from (2.9) and (2.21) that

(2.32) $$y_{i,t} - G = Y_t - G - U_t + S_t + N_t - s_{i,t} - n_{i,t} + u_{i,t}.$$

Taking first differences on both sides, one gets

$$(1\text{-}L)y_{i,t} = (1\text{-}L)Y_t - (1\text{-}L)U_t + (1\text{-}L)(S_t + N_t) - (1\text{-}L)(s_{i,t} + n_{i,t}) + (1\text{-}L)u_{i,t}$$

$$= (1\text{-}L)Y_t - (Y_t - G) + (Y_{t-1} - X_{t-1}) - (y_{i,t-1} - x_{i,t-1}) + (1\text{-}L)u_{i,t}$$

$$= G - y_{i,t-1} - X_{t-1} + x_{i,t-1} + (1\text{-}L)u_{i,t}, \Rightarrow$$

(2.33) $$y_{i,t} - G = (1\text{-}L)u_{i,t} = u_{i,t} - u_{i,t-1}.$$

In similar fashion, one finds for inventories that

$$(1\text{-}L)n_{i,t} = L^{\delta+1}y_{i,t} - Lx_{i,t}$$

$$= L^{\delta+1}(1\text{-}L)u_{i,t} - \frac{\alpha}{\delta+1}L(1 - L^{\delta+1})U_t, \Rightarrow$$

(2.34) $$n_{i,t} = L^{\delta+1}u_{i,t} - \frac{\alpha}{\delta+1}L\frac{1 - L^{\delta+1}}{1\text{-}L}U_t = u_{i,t-\delta-1} - \frac{\alpha}{\delta+1}(U_{t-1} + \dots + U_{t-\delta-1}).$$

It is now possible to calculate the expected invenotry costs, if, for instance, one further **makes the assumption that the errors, u, are normally distributed.** Since n must then also be normally distributed, one finds from Section 8 one finds the expected absolute value of inventory is

(2.35) $$E_t\{|n_{i,t}|\} = \sqrt{\frac{2}{\pi}}\sigma_n,$$

where σ_n^2 **is the variance of** $n_{i,t}$. Now, further **assume that individual errors are serially uncorrelated but correlated accross firms in any given period with correlation coefficent** $E\{u_i u_j\} = \rho$. Then, by squaring (2.34) and taking expectations on both sides, one finds that the **variance,** $V\{\,.\,\}$ of $n_{i,t}$ is

(2.36) $$V\{n_{i,t}\} = \sigma_n^2 = V\{u_{i,t-\delta-1}\}$$

$$+ \left(\frac{\alpha}{\delta+1}\right)^2 (V\{U_{t-1}\} + \dots + V\{U_{t-\delta-1}\}) - 2\frac{\alpha}{\delta+1}E\{U_{t-\delta-1}u_{i,t-\delta-1}\}$$

$$= \sigma_u^2 + \frac{\alpha^2}{\delta+1}V\{U\} - 2\frac{\alpha}{\delta+1}E\{u_i U\},$$

where σ_u **is the variance of** $u_{i,t}$. Further, using (2.11) and taking expectations readily yields

(2.37) $$V\{U\} = (\tfrac{1}{K})^2 E\{u_1^2 + \ldots + u_K^2 + 2u_1u_2 + \ldots + 2u_1u_K + 2u_2u_3 + \ldots + 2u_{K-1}u_K\}$$

$$= \frac{1}{K}\sigma_u^2 + \frac{K-1}{K}\rho\,\sigma_u^2 = (\frac{1}{K} + \frac{K-1}{K}\rho)\,\sigma_u^2, \text{ and}$$

(2.38) $$E\{u_iU\} = (\frac{1}{K} + \frac{K-1}{K}\rho)\,\sigma_u^2.$$

Inserting this in (2.36) one further finds

(2.39) $$V\{n_{i,t}\} = \sigma_u^2\left(1 - \frac{1-(1-\alpha)^2}{\delta+1}(\rho(1-1/K) + 1/K)\right)$$

It can be seen from (2.39) that the variance of inventories is less than or equal to the variance in production, due to the fact that production affects demand. If, for instance, production is too high, demand will also increase somewhat, thus partly offsetting the full impact of the error on inventories. As expected, the effect is greater for higher correlation of errors accross firms, fewer firms, a higher multiplier effect, or a shorter time lag. Under the assumptions used, the average inventory cost will be

(2.40) $$\gamma E\{|n_{i,t}|\} = \gamma\,\sigma_u\sqrt{\frac{2}{\pi}\left(1 - \frac{1-(1-\alpha)^2}{\delta+1}(\rho(1-1/K) + 1/K)\right)}$$

Minimum-variance criterion in stochastic equilibrium

The rational-expectations solution above was derived under the assumption that the number of firms was essentially infinite. When the number of firms is finite, the optimal policy cannot be derived analytically, but one can instead use optimal control theory to obtain a numerical solution. In this section, the optimal policy is found, using these methods, and both the policy and its performance is compared to the infinite-firm policy at the end of the section.

If the distribution of inventories are approximately normal with a zero mean, the average inventory costs are proportional to the standard deviation of inventories (cf. Section 8.) Thus, minimizing the variance in inventories will also minimize average costs. This means that standard methods for optimal control in linear systems can be used (as long as the non-negativity constraint on production is not binding.)

In such linear systems, the optimal policy is a linear feedback rule, i.e. the optimal production is a linear function of the system states. From the point of view of an individual firm the states of the system are its own inventory and supply line, and the aggregate inventory and supply line. Thus, the optimal policy of the task of an individual firm has the form

(2.41) $$y_{i,t}^* = -(g_{o1}n_{i,t}+g_{11}y_{i,t-1} +\ldots+ g_{\delta 1}y_{i,t-\delta})$$

$$-(g_{o2}N_t+g_{12}Y_{t-1}+\ldots+g_{\delta 2}Y_{t-\delta})+u_{i,t}, \forall\ i,t,$$

or, in vector notation,

(2.42) $$y_{i,t}^* = -\mathbf{g}\begin{bmatrix}\mathbf{z}_{i,t}\\ \mathbf{Z}_t\end{bmatrix} \equiv -\mathbf{g}\,\tilde{\mathbf{z}}_{i,t},\ \text{where}$$

$$\mathbf{g} = [g_{o1}\, g_{11} \cdots g_{\delta 1}\ \ g_{o2}\, g_{12} \cdots g_{\delta 2}],$$

$$\mathbf{z}_{i,t} = [n_{i,t}\, y_{i,t-1} \cdots y_{i,t-3}]'\text{, and}$$

$$\mathbf{Z}_t = [N_t\, Y_{t-1} \cdots Y_{t-3})]'.$$

Moreover, knowing that all other firms follow the same policy, one can aggregate (2.42) to get the equations of motions for the entire system. The manipulations are straight-forward but laborious. The result, for δ=3, is

(2.43) $$\tilde{\mathbf{z}}_{i,t+1} = \begin{bmatrix}\mathbf{z}_{i,t+1}\\ \mathbf{Z}_{t+1}\end{bmatrix} = \underline{\underline{\mathbf{P}}}\,\tilde{\mathbf{z}}_{i,t} + \mathbf{q}\, y_{i,t} + \underline{\underline{\mathbf{R}}}\,\tilde{\mathbf{u}}_{i,t},\ \text{where}$$

$$\underline{\underline{\mathbf{P}}} = \begin{bmatrix}\underline{\underline{\mathbf{P}}}_{11} \underline{\underline{\mathbf{P}}}_{12}\\ \underline{\underline{\mathbf{P}}}_{21} \underline{\underline{\mathbf{P}}}_{22}\end{bmatrix},\ \tilde{\mathbf{u}}_{i,t} = \begin{bmatrix}u_{i,t}\\ U_t\end{bmatrix},$$

$$\underline{\underline{\mathbf{P}}}_{11} = \begin{bmatrix} 1-\frac{\alpha\gamma_{o1}}{K(\delta+1)} & -\frac{\alpha\gamma_{11}}{K(\delta+1)} & -\frac{\alpha\gamma_{21}}{K(\delta+1)} & 1-\frac{\alpha\gamma_{31}}{K(\delta+1)} \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

Appendix A

$$\underline{\underline{P}}_{12} = \frac{\alpha}{(\delta+1)}\begin{bmatrix} \gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K} & \gamma_{11}-1+\frac{(K-1)\gamma_{12}}{K} & \gamma_{21}-1+\frac{(K-1)\gamma_{22}}{K} & \gamma_{31}-1+\frac{(K-1)\gamma_{32}}{K} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\underline{\underline{P}}_{21} = \frac{1}{K}\begin{bmatrix} -\frac{\alpha\gamma_{o1}}{\delta+1} & -\frac{\alpha\gamma_{11}}{\delta+1} & -\frac{\alpha\gamma_{21}}{\delta+1} & -\frac{\alpha\gamma_{31}}{\delta+1} \\ \gamma_{o1} & \gamma_{11} & \gamma_{21} & \gamma_{31} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\underline{\underline{P}}_{22} =$$

$$\begin{bmatrix} 1+\frac{\alpha}{\delta+1}(\gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K}), & \frac{\alpha}{\delta+1}(\gamma_{11}+\frac{(K-1)\gamma_{12}}{K}-1), & \frac{\alpha}{\delta+1}(\gamma_{21}+\frac{(K-1)\gamma_{22}}{K}-1), & 1+\frac{\alpha}{\delta+1}(\gamma_{31}+\frac{(K-1)\gamma_{32}}{K}-1) \\ -(\gamma_{o1}+\frac{(K-1)\gamma_{o2}}{K}) & -(\gamma_{11}+\frac{(K-1)\gamma_{12}}{K}) & -(\gamma_{21}+\frac{(K-1)\gamma_{22}}{K}) & -(\gamma_{31}+\frac{(K-1)\gamma_{32}}{K}) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$q = \begin{bmatrix} -\frac{\alpha}{K(\delta+1)} \\ 1 \\ 0 \\ 0 \\ -\frac{\alpha}{K(\delta+1)} \\ 1/K \\ 0 \\ 0 \end{bmatrix}, \underline{\underline{R}} = \begin{bmatrix} 0 & -\frac{\alpha}{\delta+1} \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{\alpha}{\delta+1} \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

That individual errors, $u_{i,t}$, do not seem to make any difference in the system is due to the fact that the aggregate effects of this error are subsumed in the aggregate error, U_t. Thus, rather than distinguish between $u_{i,t}$ and all other $u_{j,t}$, we have chosen to consider U_t as the relevant variable. (Note that U_t and $u_{i,t}$ are correlated, even if $u_{i,t}$ is uncorrelated with $u_{j,t}$).

The individual firms seeks to minimize the variance of its inventory, i.e. it faces the optimization problem

(2.44) $\min J_i = E\{ \tilde{z}_{i,t} \underline{\underline{F}} \tilde{z}_{i,t}' \}$ for all t, where,

(2.45) $$\underline{\underline{F}} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

and subject to the equations of motion (2.43). The optimal policy, i.e. the optimal "gain vector," **g**, is

(2.46) $$\mathbf{g} = (q'\underline{\underline{H}}\, q)^{-1} q'\underline{\underline{H}}\,\underline{\underline{P}},$$

where $\underline{\underline{H}}$ is the solution to the matrix Riccatti equation

(2.47) $$\underline{\underline{H}} = \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{P}} - \underline{\underline{P}}'\underline{\underline{H}}\, q(q'\underline{\underline{H}}\, q)^{-1} q'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}.$$ [3]

The equations (2.43), (2.45), (2.46), and (2.47) together identify the rational-expectations solution. They cannot be solved analytically but must instead be determined numerically. The Riccatti equation, taken as an iterative method for finding $\underline{\underline{H}}$, will always converge, as long as the system is controllable and observable and the covariance matrix of the errors is positive definite (see e.g. F.C. Schweppe, F.C. *Uncertain dynamic systems.* Englewood Cliffs, NJ: Prentice-Hall, 1973). The complication here, however, is that the feedback gain, **g**, also enters into the system matrix, $\underline{\underline{P}}$. In practice, the solutions were found by first iterating $\underline{\underline{H}}$ for a given **g**, then iterating **g** for a given $\underline{\underline{H}}$, etc.

Table 2.1 shows the optimal feedback gain for various values of K, the number of firms. As K goes to infinity, the optimal control law converges to the solution found analytically in the previous section.

---

3 This result can be found in any standard textbook on optimal control.

	Gain in individual feedback rules, $y_{i,t} = -g^* \tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$.								Aggregate gain $Y_t = -G^* \tilde{Z}_t$, $\tilde{Z}_t = (N_t, \ldots, Y_{t-3})$.			
K	$n_{i,t}$	$y_{i,t-1}$	$y_{i,t-2}$	$y_{i,t-3}$	N_t	Y_{t-1}	Y_{t-2}	Y_{t-3}	N_t	Y_{t-1}	Y_{t-2}	Y_{t-3}
1									1.143	1.000	1.000	1.000
2	0.800	0.933	0.867	0.800	0.800	0.200	0.400	0.600	1.600	1.133	1.267	1.400
3	0.870	0.957	0.913	0.870	0.870	0.217	0.435	0.652	1.739	1.174	1.348	1.522
4	0.903	0.968	0.935	0.903	0.903	0.226	0.452	0.677	1.806	1.194	1.387	1.581
5	0.923	0.974	0.949	0.923	0.923	0.231	0.462	0.692	1.846	1.205	1.410	1.615
10	0.962	0.987	0.975	0.962	0.962	0.241	0.481	0.722	1.924	1.228	1.456	1.684
∞	1.000	1.000	1.000	1.000	1.000	0.250	0.500	0.750	2.000	1.250	1.500	1.750

Table 2.1

Optimal decision rules in fixed-price complex condition, using minimum variance criterion, as a function of the number of firms, K.

We note that, for four or more firms, the optimal policy is quite close to the infinite-firm policy: the coefficients are nearly the same. Thus, it is probably not too bad an approximation to use the infinite-firm policy with four or more firms.

To find the average inventory costs, first find the covariance matrix of the noise, i.e.,

$$(2.48) \qquad E\{\tilde{u}_{i,t}\, \tilde{u}_{i,t}'\} = \underline{\underline{W}} = \begin{bmatrix} E\{u_{i,t}^2\} & E\{u_{i,t}U_t\} \\ E\{u_{i,t}U_t\} & E\{U_t^2\} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{K} + \rho\frac{K-1}{K} \\ \frac{1}{K} + \rho\frac{K-1}{K} & \frac{1}{K} + \rho\frac{K-1}{K} \end{bmatrix} \sigma^2.$$

Then, the steady-state covariance matrix of the system states, $E\{\, \tilde{z}_{i,t}\, \tilde{z}_{i,t}'\,\} = \underline{\underline{\Gamma}}$ satisfies the matrix equation

$$(2.49) \qquad \underline{\underline{\Gamma}} = (\underline{\underline{P}} - q\,g)\,\underline{\underline{\Gamma}}\,(\underline{\underline{P}} - q\,g)' + \underline{\underline{R}}\,\underline{\underline{W}}\,\underline{\underline{R}}'.$$

The equation is easy to solve numerically. The computed variances in output and inventories are shown in Table 2.2. Also shown is the performance resulting from following the infinite-firm optimal policy derived previously. The coefficients in Table 2.1 indicated that the finite-firm and infinite-firm policies are not very different. The 2.2 shows that the performances of the two rules are very close, i.e., that the infinite-firm policy does almost as well as the optimal, even with only a single firm! For instance, in the case of a single firm, the optimal policy results in an average reduction in inventory costs of only 3% compared to the infinite-firm policy. Moreover, if a random-error standard deviation of about 10% of the average production level, inventory costs in the infinite-firm policy amount only to a few percent of average profits. So the difference in performance is neglible.

K	ρ	$V\{n_{i,t}\}/\sigma^2$		$V\{y_{i,t}\}/\sigma^2$		$V\{N_t\}/\sigma^2$		$V\{Y_t\}/\sigma^2$	
1		0.766	*0.812*	1.750	*2.000*	0.766	*0.812*	1.750	*2.000*
2	0	0.898	*0.906*	1.875	*2.000*	0.391	*0.406*	0.937	*1.000*
	0.5	0.840	*0.859*	1.875	*2.000*	0.586	*0.609*	1.406	*1.500*
	1	0.781	*0.812*	1.875	*2.000*	0.781	*0.812*	1.875	*2.000*
3	0	0.934	*0.937*	1.917	*2.000*	0.264	*0.270*	0.639	*0.666*
	0.5	0.863	*0.875*	1.917	*2.000*	0.527	*0.541*	1.278	*1.333*
	1	0.791	*0.812*	1.917	*2.000*	0.791	*0.812*	1.917	*2.000*
4	0	0.951	*0.953*	1.937	*2.000*	0.199	*0.203*	0.484	*0.500*
	0.5	0.874	*0.882*	1.937	*2.000*	0.497	*0.507*	1.211	*1.250*
	1	0.796	*0.812*	1.937	*2.000*	0.796	*0.812*	1.937	*2.000*
5	0	0.961	*0.962*	1.950	*2.000*	0.160	*0.162*	0.390	*0.400*
	0.5	0.880	*0.887*	1.950	*2.000*	0.479	*0.487*	1.170	*1.200*
	1	0.799	*0.812*	1.950	*2.000*	0.799	*0.812*	1.950	*2.000*
10	0	0.981	*0.981*	1.975	*2.000*	0.081	*0.081*	0.197	*0.200*
	0.5	0.893	*0.896*	1.975	*2.000*	0.443	*0.446*	1.086	*1.100*
	1	0.806	*0.812*	1.975	*2.000*	0.806	*0.812*	1.975	*2.000*
∞	0	1.000	*1.000*	2.000	*2.000*	0.000	*0.000*	0.000	*0.000*
	0.5	0.906	*0.906*	2.000	*2.000*	0.406	*0.406*	1.000	*1.000*
	1	0.812	*0.812*	2.000	*2.000*	0.812	*0.812*	2.000	*2.000*

Table 2.2
Variance of production and inventories in stochastic equilibrium.
The figures shown are relative to the random-error variance, using the optimal minimum-variance decision rule (plain text) and the infinite-firm rule (*italic numbers*), respectively. The numbers are shown for three alternative values of the cross-firm correlation, ρ, of the random errors.

3. Clearing prices

In this condition, the computer determines the set of prices which will equate output (i.e. finished production) and demand each period. Thus, inventories are constantly equal to zero. **Assuming firms act competitively,** the optimization problem of the individual firm is

$$(3.1) \qquad \text{Max } E_{i,t}\{p_{i,t}x_{i,t} - \omega x_{i,t}\}, \text{ where } p_{i,t} = \tilde{P}_t(x_{i,t}/\tilde{X}_t)^{-1/\varepsilon}.$$

If the number of firms, K, is very large, the aggregate quantities are independent of the individual firm's actions, and the first-order conditions become

$$(3.2) \qquad E_{i,t}\{\tilde{P}_t\tilde{X}_t^{1/\varepsilon}\}\, x_{i,t}^{-1/\varepsilon}\, (\varepsilon-1)/\varepsilon - \omega = 0, \Rightarrow$$

$$x_{i,t}^* = E_{i,t}\{\tilde{P}_t\tilde{X}_t^{1/\varepsilon}\}^{-1/\varepsilon}(p^*)^{1/\varepsilon}.$$

Note that this means that

$$(3.3) \qquad E_{i,t}\{p_{i,t}\} = p^*.$$

If, futher, variances are small, one can use the approximation $E\{f(x)\} \approx f(E\{x\})$, so that the approximate optimal competitive policy for many firms becomes

$$(3.4) \qquad x_{i,t}^* = E_{i,t}\{\tilde{X}_t\}\, (E_{i,t}\{\tilde{P}_t\}/p^*)^{-1/\varepsilon}.$$

If the number of firms, K, is small, and one can no longer ignore the effect of the firm's actions on aggregate variables, one must turn to the derivations in Section 1. If the variances are small, one may use the equations in that section directly, substituting aggregates with their expected values, conditional on the choice of $x_{i,t}$. It would seem most natural to use the Cournot equilibrium (where other firms' output is assumed to be constant) instead of the Bertrand equilibrium (where other firms' prices are assumed to be constant.) As mentioned in Section 1, the equation (1.21), or, for that matter, the corresponding Cournot equation, has no analytical solution, except when all firms have equal output and there is no multiplier effect. **Under rational expectations with small random deviations,** however, the solution would approximately equal the constant output. Hence, the **optimal policy in rational-expectations competitive equilibrium** is

$$(3.5) \qquad x_{i,t}^* = x_K^* \text{ (Bertrand) or}$$

$$(3.6) \qquad x_{i,t}^* = x_K^{**} \text{ (Cournot).}$$

Note that this is so for both the simple and complex condition since, in the latter, the pipeline is completely cleared for past outputs by the time the current output decision becomes available for sale.

4. Posted prices

Simple condition

In the simple posted-price condition, the firm seeks to maximize its profits before inventory costs while minimizing the latter. It is obvious that, unless the non-negativity constraint on production is binding, the two problems can be separated: since production is costless, the optimal policy is always to choose the output that will minimize expected inventory costs. Conversely, in choosing its price, the firm can ignore inventory considerations. Furthermore, for **small error variances**, one can make the approximation E{ f(x) } ≈ f(E{x}), and the derivations for the steady-state equilibria in Section 1 apply to the price decision. **Assuming also that firms act competitively**, the approximate optimal pricing policy is the price that solves

(4.1) $$p_i + (p_i-\omega)\left(-\varepsilon + (\varepsilon + \frac{E\{\tilde{P}\}\, f'(E\{\tilde{P}\})}{f(E\{\tilde{P}\})}\frac{1}{1-\alpha f(E\{\tilde{P}\})})\frac{1}{K}(p_i/E\{\tilde{P}\})^{1-\varepsilon}\right) = 0,$$

where the expectation E{ } is conditional upon p_i (cf. Section 1), and the time subscripts have been dropped to simplify notation. Further, it is shown in Section 8 that the optimal production policy is the one that sets the median of next period's inventory to zero. Moreover, **if the distribution of demand is approximately symmetric**, the median and the expected value coincide. Finally, **assume that $\tilde{P}$ is close to the competitive-equilibrium value, p_K^* or that K is very large**. Then, the optimal policy is

(4.2) $$p_{i,t}^* = p_K^*;$$

(4.3) $$y_{i,t}^* = E_{i,t}\{x_{i,t}\} + (n^*-n_{i,t})/\tau^*;\ n^*= 0;\ \tau^* = 1.$$

Complex condition

The market system in the posted complex condition is too complicated to solve analytically for the optimal policy, except in the special case where there are no random errors. However, the system can be approximated around the equilibrium point with a Taylor expansion, yielding a linear-quadratic optimal control problem which can be analyzed using standard techniques. In the following, the simple posted condition, the deterministic posted complex condition, and the linear-quadratic approximate posted complex condition are treated in turn.

Deterministic case:

As in the simple condition, since there is no direct cost of initiating production, y_t, firms should choose y_t to minimize expected inventory cost. Moreover, **assuming that the number of firms, K, is very large**, firms can ignore the effect of their output on demand, and the optimal policy is therefore to choose output so that $n_{t+\delta+1} = 0$. Note that the firm cannot influence earlier inventories since it is assumed that the multiplier effect of production on demand is negligible. Thus, the optimal deterministic output policy (for $K=\infty$) is

(4.3) $$y_{i,t}^* = x_{i,t} + ... + x_{i,t+\delta} - n_{i,t} - y_{i,t-1} - ... - y_{i,t-\delta}.$$

In effect, the production policy "clears the system memory" so that, beyond time $t+\delta$, the system remains in equilibrium, with output and prices at their competitive-equilibrium values G and p^*, respectively. Moreover, since inventory in time $t+\delta+1$ is "taken care of" by the output decision, $p_{t+\delta}$ is equal to the unconstrainted optimal price, p^*. It now remains to find the choice of prices that maximize the expected profits in the intervening periods. The firm thus faces the problem of choosing prices $p_{i,t}, ..., p_{i,t+\delta-1}$ to

(4.4) $$\text{Max } V = v_{i,t} + ... + v_{i,t+\delta}$$

$$= x_{i,t}(p_{i,t}-\omega) + ... + x_{i,t+\delta-1}(p_{i,t+\delta-1}-\omega) - \gamma|n_{i,t+1}| - ... - \gamma|n_{i,t+\delta}|.$$

To solve this problem, it is easiest to use "dynamic programming", i.e., to work backwards from the period $t+\delta-1$. First, consider the **inventory-clearing price, p^c**, i.e., the price that will bring next period's inventory to zero. The existence of this price presumes that the current inventory plus the production due to arrive in inventory together are non-negative; otherwise, the p^c is best thought of as "infinite." In the following, the firm index, i, has been dropped for notational convenience. One has

(4.5) $$p^c_{t+\delta-1} = \tilde{P}_{t+\delta-1}\left(\frac{n_{t+\delta-1}+y_{t-1}}{\tilde{X}_{t+\delta-1}}\right)^{-1/\varepsilon},\ n_{t+\delta-1} + y_{t-1} > 0.$$

Note that in the deterministic case, V is not differentiable around this point. Instead of a stationarity condition, consider the effect separately of small changes in price above or below the clearing value. One has

(4.6) $$\frac{dV}{dp_{t+\delta-1}} = x_{t+\delta-1} + \frac{dx_{t+\delta-1}}{dp_{t+\delta-1}}(p_{t+\delta-1} - \omega + \gamma)$$

$$= x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1}) \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$\frac{dV}{dp_{t+\delta-1}} = x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1}) \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1}.$$

Consider now the conditions for the clearing price to be optimal. That would imply that the profit-function has a maximum at that point, i.e., that

$$\frac{dV}{dp_{t+\delta-1}} < 0 \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$\frac{dV}{dp_{t+\delta-1}} > 0 \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1}, \Rightarrow$$

$$1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1} < 0 \text{ for } p_{t+\delta-1} > p^c_{t+\delta-1}$$

$$1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1} > 0 \text{ for } p_{t+\delta-1} < p^c_{t+\delta-1}, \Rightarrow$$

(4.7) $$p^* - \gamma(1-1/\varepsilon) < p^c_{t+\delta-1} < p^* + \gamma(1-1/\varepsilon).$$

Thus, as long as the inventory-clearing price is within the range in (4.7), it is optimal to charge that price. If the clearing price falls outside this range, V will be differentiable and one can instead use the stationarity condition

(4.8) $$\frac{dV}{dp_{t+\delta-1}} = 0, \Rightarrow$$

$$x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega + \gamma)/p_{t+\delta-1}) = 0 \text{ for } p^c_{t+\delta-1} < p^*(1 - \frac{\gamma}{\omega});$$

$$x_{t+\delta-1}(1 - \varepsilon(p_{t+\delta-1} - \omega - \gamma)/p_{t+\delta-1}) = 0 \text{ for } p^c_{t+\delta-1} > p^*(1 + \frac{\gamma}{\omega}), \Rightarrow$$

$$p_{t+\delta-1} = p^* - \gamma(1-1/\varepsilon) \text{ for } p^c_{t+\delta-1} < p^* - \gamma(1-1/\varepsilon);$$

$$p_{t+\delta-1} = p^* + \gamma(1-1/\varepsilon) \text{ for } p^c_{t+\delta-1} > p^* + \gamma(1-1/\varepsilon).$$

Thus, the optimal rule, $p_{i,t+\delta-1}{}^*$ is to charge the clearing price if possible but to limit it to remain within the range (4.7). It is straightforward to show that V is differentiable with respect to the inventory $n_{t+\delta-1}$ and

(4.9) $$\frac{dV}{dn_{t+\delta-1}} = (1-1/\varepsilon)p_{t+\delta-1}{}^* - \omega \equiv g_{t+\delta-1} \in [-\gamma, \gamma].$$

Now consider the previous period's price, $p_{t+\delta-2}$. Again, the point of departure is the price that will clear inventories, but this time letting the next period's price vary endogenously. Hence, the differentiation of V is done "forwards in time" in the sense that all variables in the current or later periods are allowed to vary endogenously. One gets

(4.10) $$\frac{dV}{dp_{t+\delta-2}} = x_{t+\delta-2} + \frac{dx_{t+\delta-2}}{dp_{t+\delta-2}}(p_{t+\delta-2} - \omega + \gamma - \frac{dV}{dn_{i,t+\delta-1}})$$

$$= x_{t+\delta-2}(1 - \varepsilon(p_{t+\delta-2} - \omega + \gamma - g_{t+\delta-1})/p_{t+\delta-2}) \text{ for } p_{t+\delta-2} > p^c_{t+\delta-2}$$

$$\frac{dV}{dp_{t+\delta-2}} = x_{t+\delta-2}(1 - \varepsilon(p_{t+\delta-2} - \omega - \gamma - g_{t+\delta-1})/p_{t+\delta-2}) \text{ for } p_{t+\delta-2} < p^c_{t+\delta-2}.$$

In completely analogous fashion as before, one finds that the optimal policy is to charge the inventory-clearing price $p^c_{t+\delta-2}$, truncated in the interval

(4.11) $$p_{t+\delta-1}{}^* - \gamma(1-1/\varepsilon) < p^c_{t+\delta-2} < p_{t+\delta-1}{}^* + \gamma(1-1/\varepsilon).$$

Of course, $p_{t+\delta-1}{}^*$ is a function of $p_{t+\delta-1}$. Nonetheless, one can first compute the inventory-clearing price, then compute the optimal next-period price and then check whether (4.11) holds. If so, the optimal policy is the clearing price. If not, the fact that $p_{t+\delta-1}{}^*$ as a function of $p_{t+\delta-2}$ has a negative derivative means that one can always find a price $p_{t+\delta-2}^*$ for which the lower or upper limit in (4.11)--whichever is applicable--holds with equality.

One may continue analogously backwards in time, so that the complete optimal pricing policy is characterized by the conditions

$$
\begin{aligned}
(4.12)\qquad p_t^* &= p_{t+1}^* - \gamma(1-1/\varepsilon) && \text{for } p_t^c \le p_{t+1}^* - \gamma(1-1/\varepsilon) \\
&= p_t^c && \text{for } p_{t+1}^* - \gamma(1-1/\varepsilon) \le p_t^c \le p_{t+1}^* + \gamma(1-1/\varepsilon) \\
&= p_{t+1}^* + \gamma(1-1/\varepsilon) && \text{for } p_t^c > p_{t+1}^* + \gamma(1-1/\varepsilon); \\
\ldots \\
p_{t+\delta-1}^* &= p_{t+\delta}^* - \gamma(1-1/\varepsilon) && \text{for } p_{t+\delta-1}^c \le p_{t+\delta}^* - \gamma(1-1/\varepsilon) \\
&= p_{t+\delta-1}^c && \text{for } p_{t+\delta}^* - \gamma(1-1/\varepsilon) \le p_{t+\delta-1}^c \le p_{t+\delta}^* + \gamma(1-1/\varepsilon) \\
&= p_{t+\delta}^* + \gamma(1-1/\varepsilon) && \text{for } p_{t+\delta-1}^c > p_{t+\delta}^* + \gamma(1-1/\varepsilon); \\
p_{t+\delta}^* &= p^*.
\end{aligned}
$$

To actually calculate (4.12) requires a limited amount of trial and error but is doable in a finite number of steps. In practice, a wide variety of conditions lead to the inventory-clearing prices being optimal, including the initial conditions used in the experiment.

<u>Stochastic case</u>

An approximation to the optimal policy with less than infite firms and stochastic errors can be obtained by linearizing the equations of motion around the steady-state equilibrium and using a quadratic approximation to the profit function. In Section 6 it is shown that the first-order terms in the profit function can be eliminated by redefining the problem in terms of deviations from the steady state so that standard methods for linear-quadratic optimal control can be used.

The derivations are straightforward in principle but quite tedious. Therefore, only the final results will be presented here.[4] However, one part deserves special mention. The expected inventory cost are a function of the expected absolute inventory level. **If errors are normally distributed**, which will be assumed throughout, then it is shown in Section 8 that

$$(4.13)\qquad E\{|n|\} = \frac{2}{\sqrt{2\pi}}\sqrt{Var\{n\}}.$$

This is thus a deviation from all the other terms in the objective function which are proportional to the variance-covariance matrix E{z z'}. However, one can still achieve a quadratic approximation by considering that the variance in inventories consists of two components, one, call it s^2, that is related to the policy followed, and one, call it e^2, that comes from the random errors each period and is independent of the policy. (The two terms are analogous to the two terms of the right-hand side of equation (2.49) in the fixed-price case above.) Hence,

$$(4.14)\qquad Var\{n\} = s^2 + e^2, \Rightarrow$$

$$(4.15)\qquad E\{|n|\} = \frac{2}{\sqrt{2\pi}}\sqrt{s^2 + e^2}.$$

Now performing a first-order Taylor expansion of (4.15) with respect to s^2, one gets

$$(4.16)\qquad E\{|n|\} \approx \frac{2}{\sqrt{2\pi}}\left(\sqrt{e^2} + \frac{s^2}{2\sqrt{e^2+s^2}}\right) = \frac{2}{\sqrt{2\pi}}\left(\sqrt{e^2} + \frac{Var\{n\} - e^2}{2\sqrt{e^2+s^2}}\right)$$

Moreover, if $e^2 >> s^2$, a good approximation to the expected inventory cost is

$$(4.17)\qquad E\{\gamma|n|\} \approx \frac{\gamma e}{\sqrt{2\pi}} + \frac{\gamma}{e\sqrt{2\pi}} Var\{n\}.$$

Figure 8.1 shows a plot of the approximation compared with the true function. It is evident that even for s^2 approaching e^2, the approximation is excellent. Hence, as long as the systematic part of the variance is small relative to the unsystematic one, the profit function is well approximated by for purposes of the optimal-control derivations by

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[4] The derivations were done in *Mathematica* and are thus readily reproducible. The file is available upon request.

$$(4.18) \qquad v_{i,t} \approx x_{i,t}(p_{i,t}-\omega) + \frac{\gamma e}{\sqrt{2\pi}} - \frac{\gamma}{e\sqrt{2\pi}} n_{i,t}^{2}.$$

The results are shown in Tables 4.1 through 4.3. The gain vector, g, for the optimal production rule is shown in Table 4.1, for various values of both the number of firms and the standard deviations of the random errors in production and price, respectively. (These errors matter because the unsystematic error variance e^2 is part of the coefficients in the profit function, cf. equation 4.18). Table 4.2 shows the same results for the optimal price rule. It is quite clear that the rules mimic the inventory-clearing rule found in the deterministic case above: The coefficients for production are all quite low while the coefficients for prices are close to the linearized values from the clearing-price rule.

The performance results are shown in Table 4.3. The values shown are the reduction (i.e., a positive number) in profits as a percent of the steady-state profit. The reduction is partitioned into reduced profits before inventory costs, and inventory costs. It is quite clear that the reduction is very slight: by far the greatest portion comes from the unsystematic variance in inventories. This is further underlined by the last column of the table which shows the ratio of the systmatic and unsystematic error in inventory (s/e, cf. equation 4.14). That column also shows that s<<e, so that the quadratic approximation (4.17) is highly accurate (cf. also Figure 8.1).

K	Sy	Sp	n(t)	y(t-1)	y(t-2)	y(t-3)	N(t)	Y(t-1)	Y(t-2)	Y(t-3)
			Gain vector, g, in feedback rule, $y_{i,t} = -g\,\tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$							
4	.01	.01	-.03	-.03	-.07	-.03	.02	.09	-.03	.02
4	.05	.01	-.03	-.02	-.07	-.03	.02	.09	-.03	.02
4	.01	.05	-.04	.06	-.05	-.04	.05	.27	.07	.05
4	.05	.05	-.04	.06	-.05	-.04	.05	.27	.07	.05
4	.00	.10	-.04	.14	-.02	-.04	.12	.35	.17	.11
4	.10	.00	-.02	-.04	-.07	-.02	.02	.02	-.05	.02
4	.10	.10	-.04	.14	-.02	-.04	.12	.35	.17	.11
∞	.01	.01	.00	.02	.00	.00	.00	.16	.01	.00
∞	.05	.01	.00	.02	.00	.00	.00	.16	.01	.00
∞	.01	.05	.00	.10	.01	.00	.04	.40	.11	.04
∞	.05	.05	.00	.10	.01	.00	.04	.40	.11	.04
∞	.00	.10	.01	.17	.03	.01	.13	.49	.22	.11
∞	.10	.00	.00	.00	.00	.00	.00	.00	.00	.00
∞	.10	.10	.01	.17	.03	.01	.13	.49	.22	.11

Table 4.1
Coefficients in optimal linear-quadratic production rule.

K	Sy	Sp	n(t)	y(t-1)	y(t-2)	y(t-3)	N(t)	Y(t-1)	Y(t-2)	Y(t-3)
			Gain vector, g, in feedback rule, $p_{i,t} = -g\,\tilde{z}_{i,t}$, $\tilde{z}_{i,t} = (n_{i,t}, y_{i,t-1}, \ldots, Y_{t-3})$							
4	.01	.01	.48	.00	.01	.48	1.05	-.18	-.12	.87
4	.05	.01	.48	.00	.01	.48	1.05	-.17	-.12	.87
4	.01	.05	.44	.01	.05	.44	.83	-.08	.06	.68
4	.05	.05	.44	.01	.05	.44	.83	-.08	.06	.68
4	.00	.10	.40	.01	.07	.40	.70	-.01	.14	.57
4	.10	.00	.49	.00	.00	.49	1.11	-.20	-.18	.93
4	.10	.10	.40	.01	.07	.40	.70	-.01	.14	.57
∞	.01	.01	.39	.00	.01	.39	.85	-.13	-.08	.69
∞	.05	.01	.39	.00	.01	.39	.85	-.13	-.08	.69
∞	.01	.05	.36	.00	.04	.36	.66	-.04	.07	.53
∞	.05	.05	.36	.00	.04	.36	.66	-.04	.07	.53
∞	.00	.10	.33	.01	.06	.33	.55	.02	.13	.44
∞	.10	.00	.40	.00	.00	.40	.93	-.17	-.17	.77
∞	.10	.10	.33	.01	.06	.33	.55	.02	.13	.44

Table 4.2
Coefficients in optimal linear-quadratic pricing rule.

K	Sy	Sp	Reduction in gross profits % of steady-state profits	Inventory cost % of steady-state profits	Total reduction % of steady-state profits	Systematic relative to unsystematic inventory variance s/e
4	.01	.01	0.0	1.0	1.0	.04
4	.05	.01	0.2	1.0	1.2	.15
4	.01	.05	0.3	5.0	5.3	.12
4	.05	.05	0.4	5.0	5.4	.16
4	.00	.10	0.9	10.1	11.0	.20
4	.10	.00	0.8	0.3	1.2	.32
4	.10	.10	1.2	10.2	11.4	.25
∞	.01	.01	0.0	1.5	1.5	.03
∞	.05	.01	0.1	1.5	1.6	.07
∞	.01	.05	0.4	7.5	7.9	.10
∞	.05	.05	0.4	7.5	8.0	.12
∞	.00	.10	1.3	15.2	16.5	.17
∞	.10	.00	0.3	0.0	0.3	.14
∞	.10	.10	1.5	15.2	16.7	.20

Table 4.3
Performance of optimal linear-quadratic rules.

5. Variation in rational-expectations stochastic equilibrium.

In the following, it is assumed that the market is in rational-expectations stochastic equilibrium, i.e., that all firms have rational expectations and that they follow the optimal infinite-firm policies (K=∞), except for an independent and identically distributed random eror. Errors are assumed to be both serially and cross-sectionally uncorrelated. Hence,

(5.1) $$y_{i,t} = y_{i,t}^* + u_{i,t};$$

$$p_{i,t} = p_{i,t}^* + w_{i,t};$$

$$E\{u_{i,t}\} = E\{u_{i,t}u_{j,t}\} = E\{u_{i,t}u_{i,s}\} = 0;\ E\{u_{i,t}^2\} = \sigma_u^2,\ \forall\ i,\ j\neq i,\ s\neq t.$$

$$E\{w_{i,t}\} = E\{w_{i,t}w_{j,t}\} = E\{w_{i,t}w_{i,s}\} = 0;\ E\{w_{i,t}^2\} = \sigma_w^2,\ \forall\ i,\ j\neq i,\ s,\ t.$$

$$E\{u_{i,s}w_{j,t}\} = 0,\ \forall\ i,\ j,\ s,\ t.$$

It is also helpful to define the aggregate average errors

(5.2) $U_t = \frac{1}{K}(u_{1,t} + ... + u_{K,t});$

$$W_t = \frac{1}{K}(w_{1,t} + ... + w_{K,t}).$$

The derivations below are based on a linear first-order approximation around the equilibrium point. The approximation is derived by a Taylor expansion around the equilibrium with respect to the random noise variables. Hence, the results below hold only for small deviations from equilibrium. The results are summarized in Table 5.1, which shows the individual and aggregate production and prices as a function of the individual and random errors.

Condition	Y_t-G	y_t-G	P_t-p^*	p_t-p^*
Fixed simple	U_t-U_{t-1}	u_t-u_{t-1}		
Fixed complex	U_t-U_{t-1}	u_t-u_{t-1}		
Clearing simple	U_t	u_t	$-U_t/\mu$	$(U_t-u_t)/\varepsilon - U_t/\mu$
Clearing complex	U_t	u_t	$\frac{\alpha}{4\mu}(U_t+U_{t-1}+U_{t-2})+ \frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3}$	$\frac{\alpha}{4\mu}(U_t+U_{t-1}+U_{t-2})+ \frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3} + (U_{t-3} - u_{t-3})/\varepsilon$
Posted simple	$U_t-U_{t-1}-\mu V_{t-1}$	$u_t-u_{t-1} -\varepsilon v_{t-1} +(\varepsilon-\mu)W_{t-1}$	W_t	w_t
Posted complex	U_t	u_t	$W_t-W_{t-1}+ \frac{\alpha}{4\mu}(2U_{t-1}+U_{t-2}) + \frac{1}{\mu}(\frac{\alpha}{4}-1)U_{t-3}$	$w_t - w_{t-1} - u_{t-3}/\varepsilon + (\frac{1}{\varepsilon} - \frac{1}{\mu})U_{t-3}+ \frac{\alpha}{4\mu}(2U_{t-1}+U_{t-2}+U_{t-3})$

Table 5.1

Random deviations in stochastic, rational-expectations equilibrium.

The table shows the deviations of average and individual prices, as functions of the average and individual random errors. The expressions have been linearized around the mean.

From the analytical expressions in Table 5.1, it is straightforward to calculate the variance in average production (Table 5.2) and average price (Table 5.3).

Condition	Var{Y_t}	Value (K=4)	Var{$y_{i,t}$}	Value (K=4)
Fixed simple	$\frac{1}{K}2\sigma_u^2$	$0.50\,\sigma_u^2$	$2\sigma_u^2$	$2.00\,\sigma_u^2$
Fixed complex	$\frac{1}{K}2\sigma_u^2$	$0.50\,\sigma_u^2$	$2\sigma_u^2$	$2.00\,\sigma_u^2$
Clearing simple	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$
Clearing complex	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$
Posted simple	$\frac{1}{K}2\sigma_u^2+\frac{1}{K}\mu^2\sigma_w^2$	$0.50\,\sigma_u^2+$ $0.14\,\sigma_w^2$	$2\sigma_u^2+$ $[\frac{1}{K}\mu^2+\frac{K-1}{K}\varepsilon^2]\sigma_w^2$	$2.00\,\sigma_u^2+$ $4.83\,\sigma_w^2$
Posted complex	$\frac{1}{K}\sigma_u^2$	$0.25\,\sigma_u^2$	σ_u^2	$1.00\,\sigma_u^2$

Table 5.2
Variance of production in stochastic rational-expectations equilibrium

Condition	Var{P_t}	Value (K=4)	Var{$p_{i,t}$}	Value (K=4)
Clearing simple	$\frac{1}{K}\mu^{-2}\sigma_u^2$	$0.44\,\sigma_u^2$	$[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^{-2}]\sigma_u^2$	$0.56\,\sigma_u^2$
Clearing complex	$\frac{1}{K}\mu^{-2}(1+\alpha^2/4-\alpha/2)\sigma_u^2$	$0.36\,\sigma_u^2$	$[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^{-2}(1+\frac{1}{4}\alpha^2-\frac{1}{2}\alpha)]\sigma_u^2$	$0.48\,\sigma_u^2$
Posted simple	$\frac{1}{K}\sigma_w^2$	$0.25\,\sigma_w^2$	σ_w^2	$1.00\,\sigma_w^2$
Posted complex	$\frac{1}{K}2\sigma_w^2+\frac{1}{K}\mu^{-2}(1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)\sigma_u^2$	$0.50\,\sigma_w^2+$ $0.38\,\sigma_u^2$	$2\sigma_w^2+[\frac{K-1}{K}\varepsilon^{-2}+\frac{1}{K}\mu^{-2}(1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)]\sigma_u^2$	$2.00\,\sigma_w^2+$ $0.50\,\sigma_u^2$

Table 5.3
Variance of prices in stochastic rational-expectations equilibrium

In order to calculate the spectral density of the the variance, recall that, for a q'th-order moving-average process

(5.3) $$z_t = \theta_o e_t + \theta_1 e_{t-1} + \dots + \theta_q e_{t-q} = \theta(L)e_t;$$
$$E\{e_t\} = 0;\ E\{e_t e_{t-i}\} = 0 \text{ for } i\neq 0 \text{ and } \sigma^2 \text{ for } i=0,$$

the spectrum, i.e., the distribution of the variance accross frequencies, ω, is

(5.4) $$f(\omega) = \frac{\sigma^2}{2\pi}\theta(e^{i\omega})\theta(e^{-i\omega}),\ -\pi\leq\omega\leq\pi.$$

For a third-order process, this amounts to

$$(5.5) \qquad f(\omega)\frac{2\pi}{\sigma^2} = \frac{\sigma^2}{2\pi}\Big(\theta_o{}^2+\theta_1{}^2+\theta_2{}^2+\theta_3{}^2 + [\theta_o\theta_1+\theta_1\theta_2+\theta_2\theta_3]\cos(\omega) + [\theta_o\theta_2+\theta_1\theta_3]\cos(2\omega) + \theta_o\theta_3\cos(3\omega)\Big), \pi\leq\omega\leq\pi.$$

Note also that, since the Fourier transformation is linear, the spectrum of a sum of two such processes is simply the sum of the spectra.

The spectral density functions, f(ω) have been calculated in Table 5.4. The functions are plotted in Figure 5.1 under the assumption that, in the posted-price conditions, the error variances in prices (σ_w) and output (σ_u) are equal. Figures 5.2 through 5.5 plots f(ω) in the posted-price conditions for various assumptions about the relative sizes of σ_w and σ_u.

Condition	$f[\omega]\ Y_t\ -\pi\le\omega\le\pi$	$f[\omega]\ P_t\ -\pi\le\omega\le\pi$
Fixed simple	$\frac{1}{K}\frac{\sigma_u^2}{\pi}(1-\cos(\omega))$	
Fixed complex	$\frac{1}{K}\frac{\sigma_u^2}{\pi}(1-\cos(\omega))$	
Clearing simple	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{\sigma_u^2}{2\pi\mu^2}$
Clearing complex	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{\sigma_u^2}{2\pi\mu^2}\left[(1+\frac{\alpha^2}{4}-\frac{\alpha}{2}) + (\frac{3\alpha^2}{8}-\frac{\alpha}{2})\cos(\omega) + (\frac{\alpha^2}{4}-\frac{\alpha}{2})\cos(3\omega) + (\frac{\alpha^2}{8}-\frac{\alpha}{2})\cos(3\omega)\right]$
Posted simple	$\frac{1}{K}\frac{1}{2\pi}(2\sigma_u^2+\mu^2\sigma_w^2 - 2\cos(\omega))$	$\frac{1}{K}\frac{\sigma_w^2}{2\pi}$
Posted complex	$\frac{1}{K}\frac{\sigma_u^2}{2\pi}$	$\frac{1}{K}\frac{1}{\pi}\sigma_w^2+\frac{1}{K}\frac{1}{2\pi}(\frac{\sigma_u}{\mu})^2\left[1+\frac{3}{8}\alpha^2-\frac{1}{2}\alpha + (\frac{3}{8}\alpha^2-\frac{1}{2}\alpha)\cos(\omega) + (\frac{1}{4}\alpha^2-\frac{1}{2}\alpha)\cos(2\omega)\right]$

Table 5.4

Spectral densities of average price and production in stochastic, rational-expectations equilibrium.

The table shows the spectral density as a function of the frequency, ω. The expressions are based on the linearized expressions in Table 5.1.

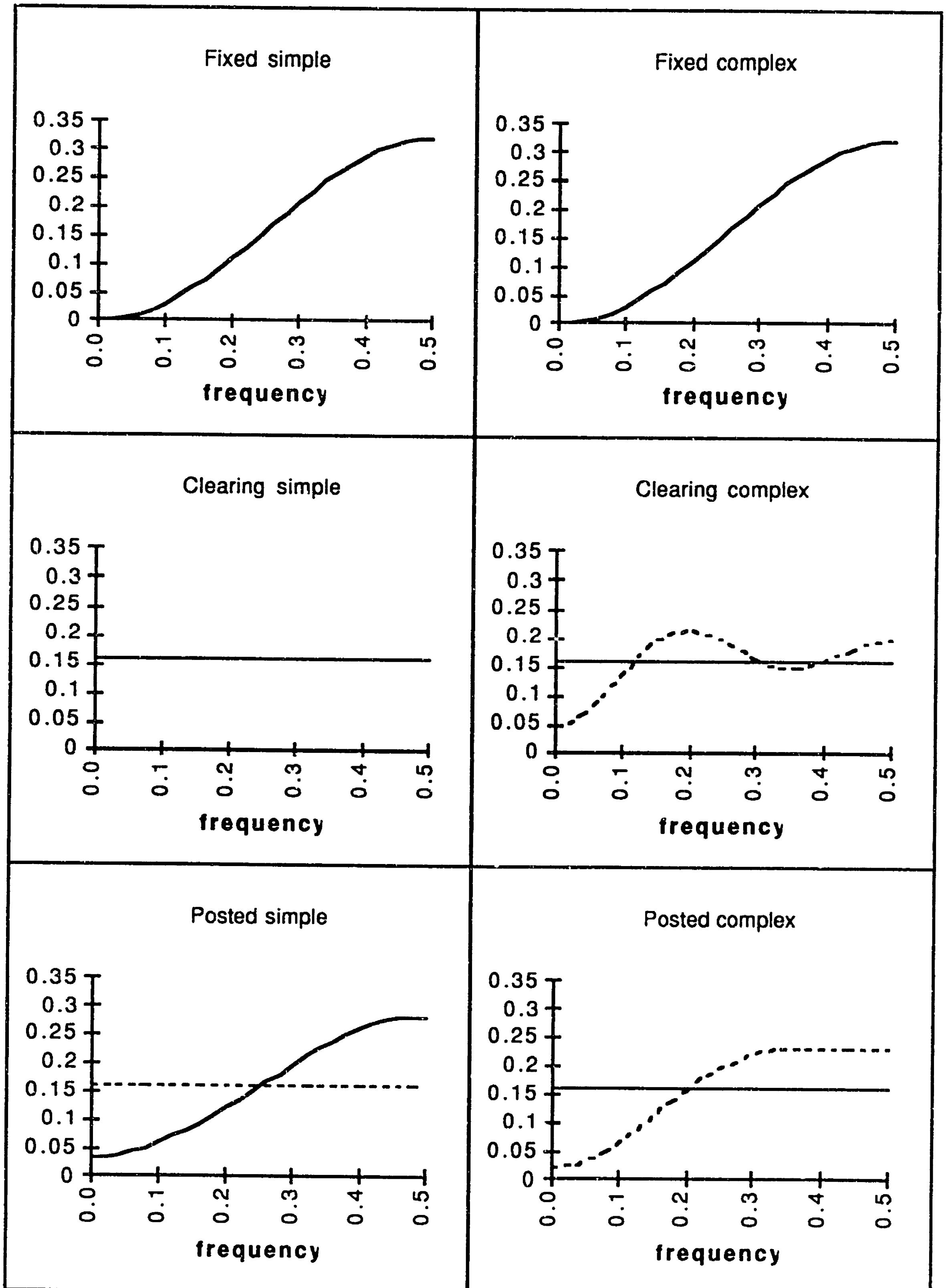

Figure 5.1

Spectral densities for $\sigma_u = \sigma_w$. Dashed line is prices, solid line is output.

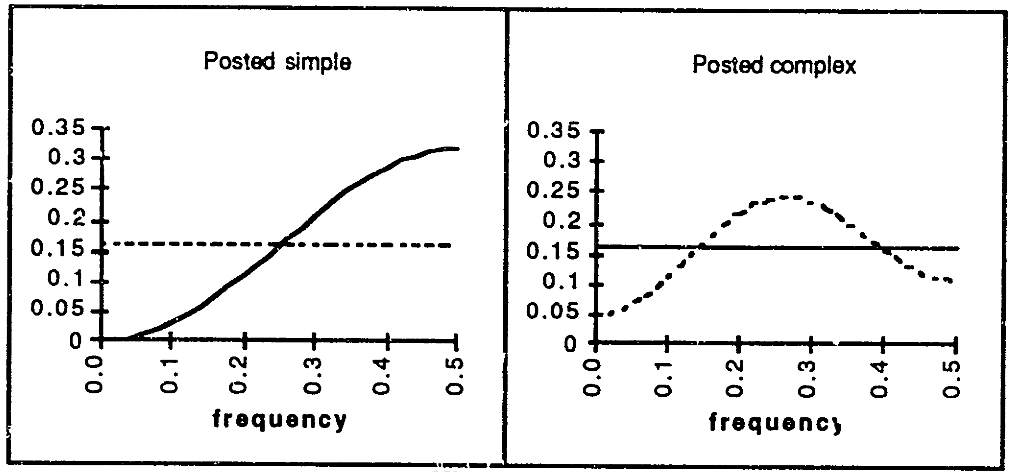

Figure 5.2

Spectral densities for posted prices, for $\sigma_u >> \sigma_w$

Dashed line is prices, solid line is output.

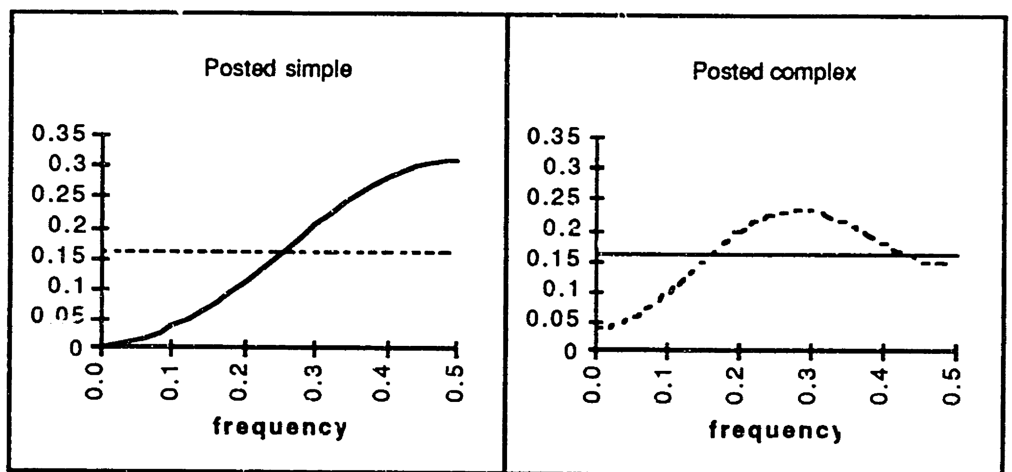

Figure 5.3

Spectral densities for posted prices, for $\sigma_u = 2.5\,\sigma_w$

Dashed line is prices, solid line is output.

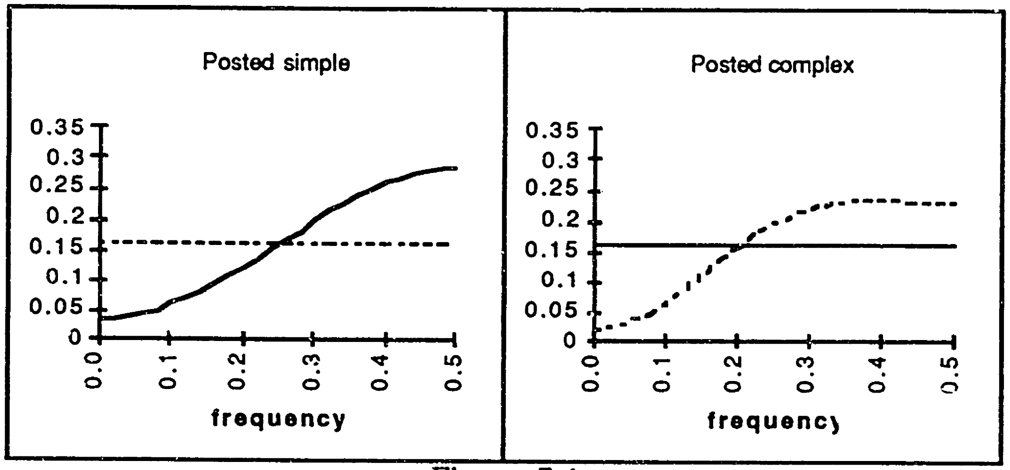

Figure 5.4

Spectral densities for posted prices, for $\sigma_u = \sigma_w$

Dashed line is prices, solid line is output.

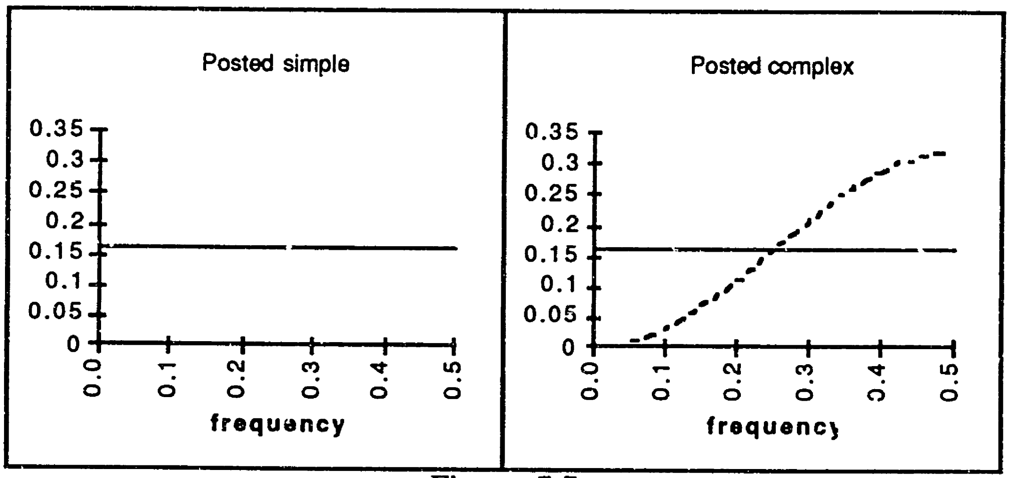

Figure 5.5

Spectral densities for posted prices, for $\sigma_u << \sigma_w$

Dashed line is prices, solid line is output.

6. Note regarding optimal control of linear (time invariant) systems with first-order terms

Normally, linear-quadratic optimal control problems do not involve first-order terms in the objective function. However, as is shown below, it is straightforward to transform a system with first-order terms into one with only quadratic terms in the objective function. Consider the control problem

(6.1) $\min J =$

$$\lim\{T\to\infty\}\ \frac{1}{T}\sum_{t=0}^{T-1}\left(\mathbf{x}_t'\,\tfrac{1}{2}\underline{\underline{\mathbf{F}}}_{xx}\,\mathbf{x}_t + \mathbf{u}_t'\,\tfrac{1}{2}\underline{\underline{\mathbf{F}}}_{uu}\,\mathbf{u}_t + \mathbf{x}_t'\,\underline{\underline{\mathbf{F}}}_{xu}\mathbf{u}_t + \mathbf{h}_x'\mathbf{x}_t + \mathbf{h}_u\mathbf{u}_t\right);$$

s.t.

(6.2) $$\mathbf{x}_{t+1} = \underline{\underline{\mathbf{P}}}\,\mathbf{x}_t + \underline{\underline{\mathbf{Q}}}\,\mathbf{u}_t + \mathbf{a}, \quad t\geq 0$$

(6.3) $$\mathbf{x}_o = \mathbf{x}^o \text{ (initial conditions).}$$

$\mathbf{x}$ is a n-dimensional vector of state variables, $\mathbf{u}$ is a p-dimensional vector of control variables, $\underline{\underline{\mathbf{F}}}_{xx}, \underline{\underline{\mathbf{F}}}_{uu}, \underline{\underline{\mathbf{F}}}_{xu}$ are positive semidefinite matrices of dimensions n x n, p x p, and n x p, respectively, and $\mathbf{h}_x$ and $\mathbf{h}_u$ are n- and p-dimensional vectors.

The first-order (necessary) conditions are (in addition to (6.2) and (6.3))

(6.4) $$\frac{\partial L}{\partial \mathbf{x}_t} = \underline{\underline{\mathbf{F}}}_{xx}\mathbf{x}_t + \underline{\underline{\mathbf{F}}}_{xu}\mathbf{u}_t + \mathbf{h}_x - \psi_t + \underline{\underline{\mathbf{P}}}'\psi_{t+1} = 0, \quad t\geq 0$$

(6.5) $$\frac{\partial L}{\partial \mathbf{u}_t} = \underline{\underline{\mathbf{F}}}_{xu}'\mathbf{x}_t + \underline{\underline{\mathbf{F}}}_{uu}\mathbf{u}_t + \mathbf{h}_u + \underline{\underline{\mathbf{Q}}}'\psi_{t+1} = 0, \quad t\geq 0,$$

where ψ is the n-dimensional co-state vector (i.e. the Lagrangian multiplier).

Now assume that there is some optimal operating point, $\mathbf{x}^*$, such that, if the system is at that point, it is optimal to let it remain there. More formally, assume that the linear system of equations

(6.6) $$\mathbf{x}^* = \underline{\underline{\mathbf{P}}}\,\mathbf{x}^* + \underline{\underline{\mathbf{Q}}}\,\mathbf{u}^* + \mathbf{a} \text{ (steady state);}$$

(6.7) $$\underline{\underline{F}}_{xx}x^* + \underline{\underline{F}}_{xu}u^* + h_x - \psi^* + \underline{\underline{P}}'\psi^* = 0;$$

(6.8) $$\underline{\underline{F}}_{xu}'x^* + \underline{\underline{F}}_{uu}u^* + h_u + \underline{\underline{Q}}'\psi^* = 0$$

has a solution (x^*,u^*,ψ^*), and define a new system in terms of the deviations from this operating point, as

(6.9) $$z_t = x_t - x^*, \qquad t \geq 0;$$

(6.10) $$v_t = u_t - u^*, \qquad t \geq 0;$$

(6.11) $$\phi_t = \psi_t - \psi^*, \qquad t \geq 0.$$

Now the first-order conditions can be rewritten as

(6.12) $$\underline{\underline{F}}_{xx}z_t + \underline{\underline{F}}_{xu}v_t - \phi_t + \underline{\underline{P}}'\phi_{t+1} = 0, \qquad t \geq 0;$$

(6.13) $$\underline{\underline{F}}_{uu}v_t + \underline{\underline{F}}_{xu}'z_t + \underline{\underline{Q}}'\phi_{t+1} = 0, \qquad t \geq 0,$$

(6.14) $$z_{t+1} = \underline{\underline{P}}\, z_t + \underline{\underline{Q}}\, v_t, \qquad t \geq 0.$$

The system is now on a form which lends itself to the standard linear-quadratic derivation. Thus, the optimal policy is

(6.15) $$u_t = u^* - g'(x_t - x^*), \text{ where}$$

(6.16) $$(\underline{\underline{P}} - \underline{\underline{I}})\; x^* + \underline{\underline{Q}}\; u^* = -a,$$

(6.17) $$\underline{\underline{F}}_{xx}\; x^* + \underline{\underline{F}}_{xu}\; u^* + (\underline{\underline{P}}' - \underline{\underline{I}})\; \psi^* = -h_x;$$

(6.18) $$\underline{\underline{F}}_{xu}'\; x^* + \underline{\underline{F}}_{uu}\; u^* + \underline{\underline{Q}}'\; \psi^* = -h_u,$$

and the gain vector, g, is defined by

(6.19) $$g = (\underline{\underline{F}}_{uu} + \underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{Q}})^{-1}(\underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}_{xu}'),$$

where $\underline{\underline{H}}$ is the solution to the Matrix-Riccatti equation

(6.20) $$\underline{\underline{H}} = \underline{\underline{F}}_{xx} + \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{P}} - (\underline{\underline{F}}_{xu} + \underline{\underline{P}}'\underline{\underline{H}}\,\underline{\underline{Q}})(\underline{\underline{F}}_{u} + \underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{Q}})^{-1}(\underline{\underline{Q}}'\underline{\underline{H}}\,\underline{\underline{P}} + \underline{\underline{F}}_{xu}').$$

7. Note regarding non-linear aggregates versus arithmetic averages

In the experimental software, firms observed not the non-linear aggregates $\tilde{X}$ and $\tilde{P}$ but the simple arithmetic average sales, X, and the trade-weighted average price, P, defined by

(7.1) $$X = \frac{1}{K}[x_1 + \ldots + x_K];$$

(7.2) $$P = \frac{x_1 p_1 + \ldots + x_K p_K}{x_1 + \ldots + x_K}.$$

From (1.4) and (1.8), it follows that

(7.3) $$X P = \tilde{X} \tilde{P}.$$

Moreover, since $\tilde{X}$ is a convex, homogenous function of its individual components, it must always be less than or equal to the average X. There is therefore a systematic bias between the two variables. X will generally overestimate $\tilde{X}$. Conversely, it follows from (7.3) that P will underestimate $\tilde{P}$. It is important, therefore, to determine just how important this bias may be. Accordingly, a set of Monte Carlo simulations were performed, with normally or log-normally independently distributed individual prices (with mean 1), and the aggregate and average price were compared. A similar analysis was done for sales. The results are shown in Figure 7.1. As can be seen from the figure, the correlation between the two measures is very high and the bias very small except for very large variances (σ=.5).

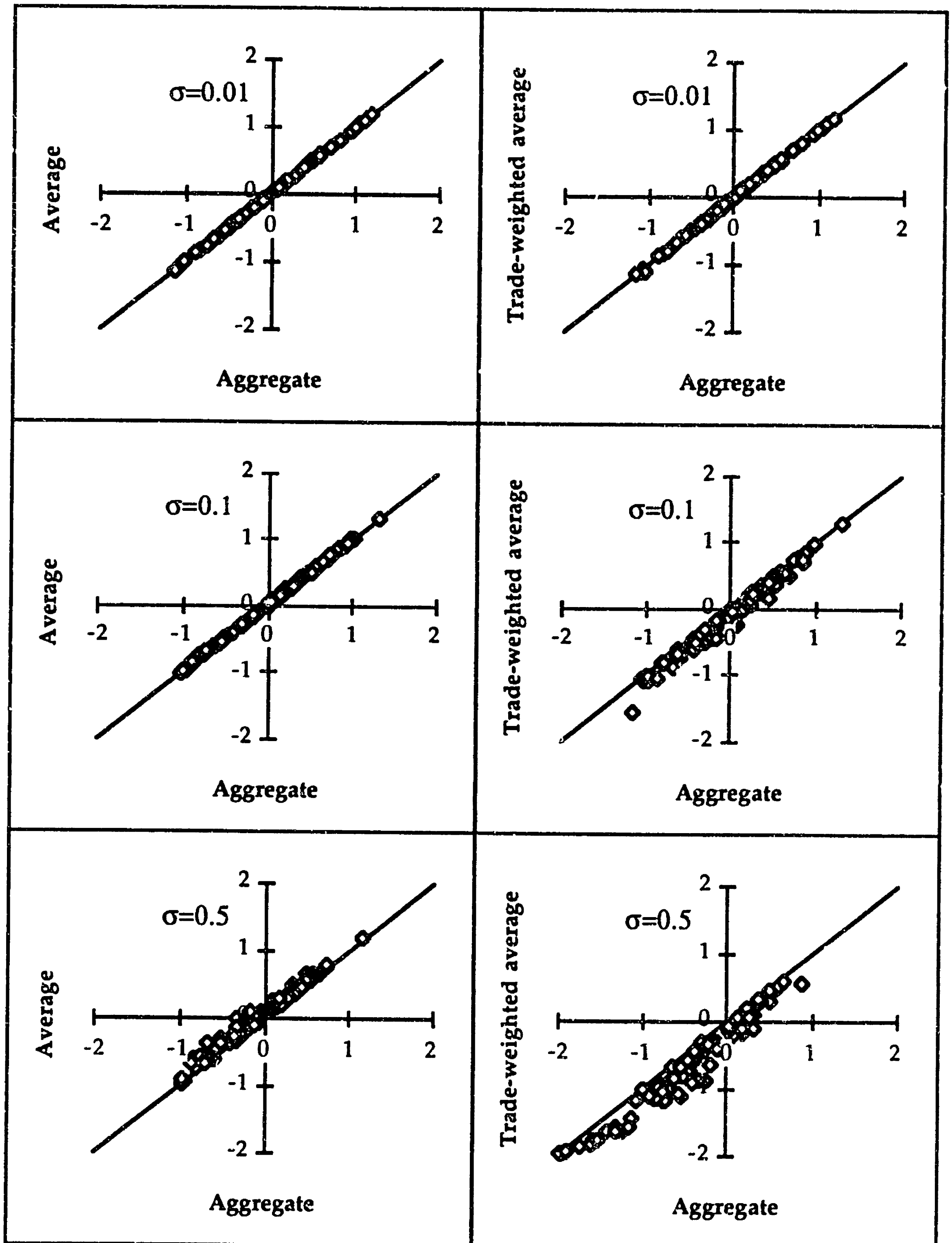

Figure 7.1
Plot of aggregate versus average price and demand

The figure shows the aggregates X and P plotted against the corresponding averages X and P, assuming that the individual x's and p's are normally distributed with mean 1 and variance σ^2. The results have been normalized to "Z-scores", i.e., $(X - 1)/\sigma$, etc.

8. Note regarding E{ | n | }.

If a **random variable, n,** with a **cumulative distribution function, $\tilde{F}(n)$** and an **expected value of** μ, one can write the expected absolute value of n as

(8.1) $$E\{ |n| \} = \int_{-\infty}^{0} -n \, d\tilde{F}(n) + \int_{0}^{\infty} n \, d\tilde{F}(n).$$

Suppose further that n can be written as

(8.2) $$n = \mu + t,$$

where the **distribution of t, F(t), is independent of** μ. Then,

(8.3) $$E\{ |n| \} = \int_{-\infty}^{-\mu} -(\mu+t)\, dF(t) + \int_{-\mu}^{\infty} (\mu+t)\, dF(t),$$

$$= -\mu \int_{-\infty}^{-\mu} dF(t) + \mu \int_{-\mu}^{\infty} dF(t) - \int_{-\infty}^{-\mu} t\, dF(t) + \int_{-\mu}^{\infty} t\, dF(t)$$

$$= \mu(1 - 2F(-\mu)) + \int_{-\mu}^{\infty} t\, dF(t) - \int_{-\infty}^{-\mu} t\, dF(t).$$

Moreover, by differentiating with respect to μ, one finds

(8.4) $$\frac{dE\{ |n| \}}{d\mu} = 1 - 2F(-\mu) + 2\mu\, dF(-\mu) - \mu\, dF(-\mu) - \mu\, dF(-\mu),$$

$$= 1 - 2F(-\mu).$$

The expected absolute value of n finds its minimum when

(8.5) $$\frac{dE\{ |n| \}}{d\mu} = 0, \Rightarrow$$

$$F(-\mu) = 1/2,$$

i.e. when μ is equal to minus the median of the distribution of the random error, t. Note that if the distribution of t is symmetric, the mean should also be zero.

If, in particular, n, is normally distributed with mean μ **and variance** σ^2, one gets

(8.6) $$dF(t) = \phi(t/\sigma)/\sigma = \frac{1}{\sigma\sqrt{2\pi}} e^{-(t/\sigma)^2/2}, \text{ and}$$

(8.5) $$F(t) = \Phi(t/\sigma) = \int_{-\infty}^{t/\sigma} \frac{1}{\sqrt{2\pi}} e^{-s^2/2} ds,$$

where Φ and ϕ are the cumulative and frequency distribution, respectively, of the standard normal distribution.

Inserting this in (8.3) yields

(8.6) $$E\{\, |n| \,\} = \mu(1 - 2\Phi(-\mu/\sigma)) + 2\sigma\phi(-\mu/\sigma).$$

If, in particular, μ is zero, (8.6) becomes

(8.7) $$E\{\, |n| \,\} = 2\sigma\phi(0) = \sqrt{\frac{2}{\pi}}\sigma.$$

One can also approximate the expression (8.6) with a second-order Taylor expansion around the point μ=0. One gets

(8.8) $$E\{|n|\} \approx \sqrt{\frac{2}{\pi}}\sigma[1 + \frac{1}{2}(\mu/\sigma)^2].$$

Figure 8.1 shows a plot of $E\{|n|\}/\sigma$ and the approximation (8.8) as a function of μ/σ. It is evident that, within one standard deviation, the approximation is very good.

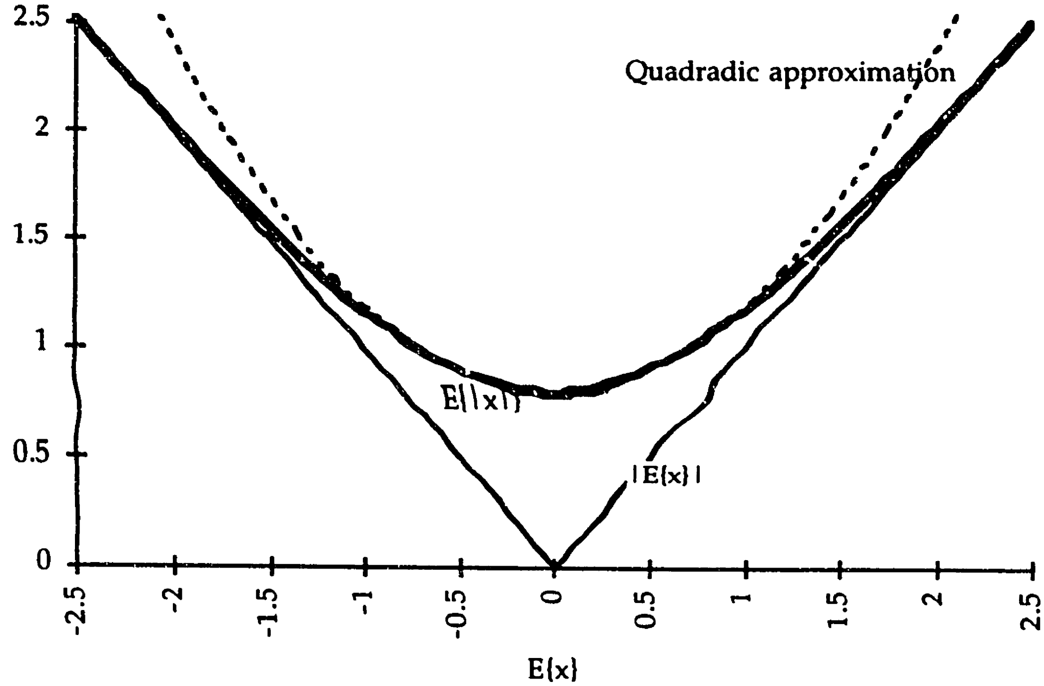

Figure 8.1
Plot of the expected absolute value of a normally distributed variable, x, as a function of its mean. Also shown is the quadratic Taylor approximation around the point x=0. All measures have been normalized by the standard deviation of x.

Appendix B: Simulations

In connection with the formation of the experimental hypotheses, a large number of simulations were performed to chart the range of possible behaviors one could expect. For each condition, two kinds of simulations were performed: A set of "optimal" simulations, based on the assumption that firms correctly estimate the structural parameters of the system and, once these parameters have become known, switch to the optimal decision rule, and a set of "behavioral" simulations, based on a simple set of decision rules with unchanging parameters. This appendix documents the simulation models in detail and tabulates the results for each of the six conditions. The first part lists the complete equations for each simulation model. The second part is a series of tables of the simulation outcomes.

Simulation models

Fixed-price simple condition

Optimal model

For this condition, the optimal model is functionally equivalent to the behavioral model with the parameters

$\tau = 1$; $a_1 = 1$; $a_2 = 0$; $\sigma_x = 0$ (see below).

Behavioral model

$y_{i,t} = \text{Max}\{0, x_{i,t}^e - n_{i,t}/\tau + u_{i,t}\}$, $t>0$;

$u_{i,t} \sim N(0, \sigma_y^2)$;

$y_{i,0} = x_0$;

$x_{i,t}{}^{e} = x_{i,t-1}{}^{e} + a_1(x_{i,t-1} - x_{i,t-1}{}^{e}) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t}$, t>0;

$x_{i,o}{}^{e} = x_o + w_{i,t}$;

$w_{i,t} \sim N(0, \sigma_x{}^2)$;

$x_o = \frac{2}{3}G$; G = 1; $p_o = p_{i,t} = p^* = 1$; K = 4.

Fixed-price complex condition

Optimal model

$y_{i,t} = \mathrm{Max}\{\, 0, \hat{G}_t + \hat{\alpha}_t(Z_t - 3\hat{G}_t - N_t - S_t)/(1-\hat{\alpha}_t) + Z_t + 3\hat{G}_t - n_{i,t} - s_{i,t} + u_{i,t} \,\}$, t>0;

$u_{i,t} \sim N(0, \sigma_y{}^2)$;

$\hat{G}_t = G$, $\hat{\alpha} = \alpha$ for **Est** false;

$\hat{G}_t = x_o$, $t \leq 3$, $\hat{G}_t = G$, $t > 3$, for **Est** true;

$\hat{\alpha}_t = 0$, $t \leq 3$, $\hat{\alpha}_t = \alpha$, t>3, for **Est** true;

$Z_t = \hat{\alpha}_t * (\frac{3}{4}Y_{t-1} + \frac{1}{2}Y_{t-2} + \frac{1}{4}Y_{t-3} - \frac{3}{2}\hat{G}_t)$, $t \geq 0$;

$x_o = \frac{2}{3}G$; G = 1; $p_o = p_{i,t} = p^* = 1$; K = 4.

Behavioral model

$y_{i,t} = \mathrm{Max}\{\, 0, x_{i,t}{}^{e} + (n^* - n_{i,t})/\tau + \beta\,(s_{i,t}{}^{d} - s_{i,t})/\tau \,\}$, t>0;

$u_{i,t} \sim N(0, \sigma_y{}^2)$;

$s_{i,t}{}^{d} = 3\theta x_{i,t}{}^{e} + (1-\theta)\, s_c$, t>0;

$y_{i,o} = x_o$;

$x_{i,t}^e = x_{i,t-1}^e + a_1(x_{i,t-1} - x_{i,t-1}^e) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t}$, $t>0$;

$x_{i,o}^e = x_o + w_{i,t}$;

$w_{i,t} \sim N(0, \sigma_x^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $p_o = p_{i,t} = p^* = 1$; $K = 4$.

Clearing-price simple condition

<u>Optimal model</u>

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Parameters known (**Est** false), no collusion (**M** false):

$y_{i,t} = G + u_{i,t}$, $t>0$.

Parameters known (**Est** false), with collusion (**M** true):

$y_{i,t} = x_K^M + u_{i,t}$, $t>0$.

Parameters unknown (**Est** true):

$y_{i,t} = x_o + u_{i,t}$, $0<t\leq3$;

$y_{i,t} = \hat{a}_o + \hat{a}_1\, p_{i,t}^d + u_{i,t}$, $t>3$;

$\hat{a}_o$ and $\hat{a}_1$ are estimated from the equation $X = a_o + a_1 P$, using data from the previous $\mathbf{N}_s$ periods.

No collusion (**M** false):

$p_{i,t}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>3$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

With collusion (**M** true):

$$p_{i,t}^{d} = \frac{1}{2}(\omega - \hat{a}_o/\hat{a}_1).$$

<u>Behavioral model</u>

$$\ln(y_{i,t}) = (1-a_o)\ln(y_{i,t-1}) + a_o\ln(Y_{t-1}) + a_1\,\Delta v_{i,t-1} + a_2\,\Delta V_{t-1} + a_3\,dv_{i,t-1} + u_{i,t};$$

$$y_{i,o} = x_o;$$

$$u_{i,t} \sim N(0, \sigma_y^2);$$

$$\Delta v_{i,t} = (v_{i,t}-V_t)/(x_{i,t}-X_t) \text{ if } |x_{i,t}-X_t| \geq \sigma_y/10, \text{ else } \Delta v_{i,t-1};$$

$$\Delta V_t = (V_t-V_{t-1})/(X_t-X_{t-1}) \text{ if } |X_t-X_{t-1}| \geq \sigma_y/10, \text{ else } \Delta V_{t-1};$$

$$dv_{i,t} = (v_{i,t}-v_{i,t-1} + b(V_t-V_{t-1}))/(x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1}))$$

$$\text{if } |x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1})| \geq \sigma_y, \text{ else } dv_{i,t-1}.$$

$$\Delta v_{i,o} = \Delta V_o = dv_{i,o} = p_o - \omega.$$

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\text{sgn}(\Delta v)$.

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Clearing-price complex condition

<u>Optimal model</u>

$$y_{i,o} = x_o.$$

$$u_{i,t} \sim N(0, \sigma_y^2);$$

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Parameters known (**Est** false), no collusion (**M** false):

$y_{i,t} = G + u_{i,t}$, $t>0$.

Parameters known (**Est** false), with collusion (**M** true):

$y_{i,t} = x_K^M + u_{i,t}$, $t>0$.

Parameters unknown (**Est** true):

No collusion (**M** false):

$y_{i,t} = x_o + u_{i,t}$, $0<t\leq 4$;

$y_{i,t} = \hat{G}_{i,t} + u_{i,t}$, $t>4$;

$\hat{G}_{i,t} = (\omega\, \hat{p}_{i,t}^* - \hat{a}_o)/(\hat{a}_1 + \hat{a}_2)$;

$\hat{p}_{i,t}^* = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>4$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

$\hat{a}_o$, $\hat{a}_1$, and $\hat{a}_2$ are estimated from the equation $P = a_o + a_1(S + Y)/4 + a_2X$, using data from the previous N_s periods.

With collusion (**M** true):

$y_{i,t} = \hat{x}_t^M + u_{i,t}$;

$\hat{x}_t^M = \frac{1}{2}(\omega-\hat{a}_o)/(\hat{a}_1+\hat{a}_2)$;

<u>Behavioral model</u>

$\ln(y_{i,t}) = (1-a_o)\ln(y_{i,t-1}) + a_o\ln(Y_{t-1}) + a_1\, \Delta v_{i,t-1} + a_2\, \Delta V_{t-1} + a_3\, dv_{i,t-1} + u_{i,t}$, $t>0$;

$y_{i,o} = x_o$;

$u_{i,t} \sim N(0, \sigma_y^2)$;

$\Delta v_{i,t} = (v_{i,t}-V_t)/(x_{i,t}-X_t)$ if $|x_{i,t}-X_t| \geq \sigma_y/10$, else $\Delta v_{i,t-1}$;

$\Delta V_t = (V_t-V_{t-1})/(X_t-X_{t-1})$ if $|X_t-X_{t-1}| \geq \sigma_y/10$, else ΔV_{t-1};

$dv_{i,t} = (v_{i,t}-v_{i,t-1} + b(V_t-V_{t-1}))/(x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1}))$

if $|x_{i,t}-x_{i,t-1} + b(X_t-X_{t-1})| \geq \sigma_y$, else $dv_{i,t-1}$.

$\Delta v_{i,t} = \Delta V_t = P_t - \omega$, $t \leq 3$;

$dv_{i,o} = p_o - \omega$;

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\text{sgn}(\Delta v)$.

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

Posted-price simple condition

Optimal model

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

$p_{i,t} = p_{i,t}^d$, $t>0$.

Parameters known (**Est** false), no collusion (**M** false):

$y_{i,t} = \text{Max}\{0, G - n_{i,t} + u_{i,t}\}$, $t>0$.

Parameters known (**Est** false), with collusion (**M** true):

$y_{i,t} = \text{Max}\{0, x_K^M - n_{i,t} + u_{i,t}\}$, $t>0$.

Parameters unknown (**Est** true):

$y_{i,t} = \mathrm{Max}\{0, x_o - n_{i,t} + u_{i,t}\}$, $0<t\leq 3$;

$y_{i,t} = \mathrm{Max}\{0, \hat{a}_o + \hat{a}_1\, p_{i,t}{}^d + u_{i,t}\}$, $t>3$;

$\hat{a}_o$ and $\hat{a}_1$ are estimated from the equation $X = a_o + a_1 P$, using data from the previous N_s periods.

No collusion (**M** false):

$p_{i,t}{}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>3$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

With collusion (**M** true):

$p_{i,t}{}^d = \frac{1}{2}(\omega - \hat{a}_o/\hat{a}_1)$.

<u>Behavioral model</u>

$y_{i,t} = \mathrm{Max}\{0, x_{i,t}{}^e - n_{i,t}/\tau + u_{i,t}\}$, $t>0$;

$x_{i,t}{}^e = x_{i,t-1}{}^e + \mathbf{a}_1(X_{t-1} - x_{i,t-1}{}^e) + \mathbf{a}_2(X_{t-1}-X_{t-2}) + w_{i,t}$, $t>0$;

$x_{i,o} = x_o + w_{i,o}$;

$w_{i,t} \sim N(0,\sigma_x{}^2)$;

$p_{i,t} = [\mathbf{b}_o \ln(P_{t-1}) + (1-\mathbf{b}_o)\ln(p_{i,t-1})]\,\mathrm{Exp}(\mathbf{b}_1 \Delta v_{i,t-1} + \mathbf{b}_2 \Delta V_{t-1} + r_{i,t})$, $t>0$;

$r_{i,t} \sim N(0,\sigma_p{}^2)$;

$p_{i,o} = p_o$;

$\Delta v_{i,t} = (v_{i,t}-V_t)/(p_{i,t}-P_t)$ if $|p_{i,t}-P_t| \geq \mathbf{d}$, else $\Delta v_{i,t-1}$;

$\Delta V_t = (V_t-V_{t-1})/(P_t-P_{t-1})$ if $|P_t-P_{t-1}| \geq \mathbf{d}$, else ΔV_{t-1};

$\Delta v_{i,o} = 0$; $\Delta V_o = 0$;

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\mathrm{sgn}(\Delta v)$.

Posted-price complex condition

Optimal model

$y_{i,o} = x_o$.

$u_{i,t} \sim N(0, \sigma_y^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $K = 4$; $p^* = 1$; p_o = steady-state price, given x_o.

$y_{i,t} = \mathrm{Max}\{0, x_{i,t}^d + (1\text{-}a)(3 - 1.5\hat{\alpha}_t)x_{i,t}^d/(1\text{-}\hat{\alpha}_t) - Z_t - n_{i,t} - s_{i,t}) + u_{i,t}\}$, $t>0$;

known (**Est** false), no collusion (**M** false):

$x_{i,t}^d = G$, $\hat{\alpha}_t = \alpha$.

Parameters known (**Est** false), with collusion (**M** true):

$x_{i,t}^d = x_M$, $\hat{\alpha}_t = \alpha$.

Parameters unknown (**Est** true):

$x_{i,t}^d = X_{t-1}$, $0<t\leq 3$;

$x_{i,t}^d = (\hat{a}_o + \hat{a}_1 p_{i,t}^d)/(1\text{-}\hat{a}_2)$, $t>3$;

$\hat{a}_o$, $\hat{a}_1$ and $\hat{a}_2$ are estimated from the equation $X = a_o + a_1 P + a_2 (S+Y)/4$, using data from the previous N_s periods.

No collusion (**M** false):

$p_{i,t}^d = \omega\, \hat{\varepsilon}_{i,t} / (1 - \hat{\varepsilon}_{i,t})$, $t>3$;

$\hat{\varepsilon}_{i,t}$ is estimated from the equation $\ln(x_i/X) = \varepsilon \ln(p_i/P)$, using data from the previous N_s periods.

With collusion (**M** true):

$p_{i,t}^{d} = \frac{1}{2}(\omega - \hat{a}_o/\hat{a}_1)$.

$p_{i,t} = a\, p_{i,t}^{c}\, [(N_t + Y_{t-\delta})/(n_{i,t}+y_{i,t-\delta})]^{1/\varepsilon} + (1\text{-}a)p_{i,t}^{d} + w_{i,t}$;

$w_{i,t} \sim N(0, \sigma_p^2)$;

Parameters known (Est false):

$p_{i,t}^{c} = P_t^{c}$ = true aggregate clearing price.

Parameters estimated (Est true):

$p_{i,t}^{c} = p_{i,t}^{d}$ for $0 \le t \le 3$;

$p_{i,t}^{c} = (N_t + Y_{t-\delta} - \hat{a}_o - \hat{a}_2(S_t + x_{i,t}^{d})/4)/\hat{a}_1$;

Z_t is defined in the optimal rule in Appendix A, but replacing α with $\hat{\alpha}$ and G with X_{t-1}.

<u>Behavioral model</u>

$y_{i,t} = \mathrm{Max}\{\, 0, x_{i,t}^{e} + (n^* - n_{i,t})/\tau + \beta\,(s_{i,t}^{d} - s_{i,t})/\tau \,\}$, $t>0$;

$u_{i,t} \sim N(0, \sigma_y^2)$;

$s_{i,t}^{d} = 3\theta x_{i,t}^{e} + (1\text{-}\theta)\, s_c$, $t>0$;

$y_{i,o} = x_o$;

$x_{i,t}^{e} = x_{i,t-1}^{e} + a_1(x_{i,t-1} - x_{i,t-1}^{e}) + a_2(x_{i,t-1} - x_{i,t-2}) + w_{i,t}$, $t>0$;

$x_{i,o}^{e} = x_o + w_{i,t}$;

$w_{i,t} \sim N(0, \sigma_x^2)$;

$x_o = \frac{2}{3}G$; $G = 1$; $n^* = 0$; $p_o = p_{i,t} = p^* = 1$; $K = 4$.

Appendix B

$p_{i,t} = P_{t-1}\ \mathrm{Exp}(\mathbf{b}_1 \Delta v_{i,t-1} + \mathbf{b}_2 \Delta V_{t-1} + \mathbf{b}_3(n_{i,t} + y_{i,t-\delta} - x_{i,t}^{e}) + r_{i,t}),\ t>0;$

$r_{i,t} \sim N(0, \sigma_p^2);$

$p_{i,o} = p_o;$

$\Delta v_{i,t} = (v_{i,t} - V_t)/(p_{i,t} - P_t)$ if $|p_{i,t} - P_t| \geq \mathbf{d}$, else $\Delta v_{i,t-1};$

$\Delta V_t = (V_t - V_{t-1})/(P_t - P_{t-1})$ if $|P_t - P_{t-1}| \geq \mathbf{d}$, else $\Delta V_{t-1};$

$\Delta v_{i,o} = 0;\ \ \Delta V_o = 0;$

If "Discrete" is true, then all the profit gradients, Δv, are replaced by $\mathrm{sgn}(\Delta v)$.

Simulation results

The results are based on 20 simulations of 40 time periods each for each combination of parameters. The symbol ### means that the values exceed 1000, indicating that the system is unstable. The column "Crash" indicates how many out of the 20 simulations that led to infinite or undefined quantities (as a results of instability.) Otherwise, the following results are calculated:

S{Y}, t>10	standard deviation in Y_t, t>10;
S{P}, t>10	standard deviation in P_t, t>10;
Rev., t>10	average revenue, t>10;
P.C., t>10	average production cost, t>10;
G.P., t>10	average gross profits, t>10;
In.C., t>10	average inventory costs, t>10;
Prof., t>10	average net profits, t>10;
S{Y}, t≤20	standard deviation in Y_t, t≤20;
P, t≤20	average P_t, t≤ 20;
S{P}, t≤20	standard deviation in P_t, t≤ 20;
S{Y}, t>20	standard deviation in Y_t, t> 20;
P, t>20	average P_t, t>20;
S{P}, t>20	standard deviation in P_t, t>20.

Fixed-price simple condition, behavioral and optimal model

a_1	a_2	τ	σ_r	σ_v	Crashes	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0.5	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
0.5	0	1	0	0.1	0	.060	.000	1.000	.600	.400	.028	.372	.108	1.000	.000	.060	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.009	±.000	±.000
0.5	0	1	0.1	0	0	.070	.000	1.000	.600	.400	.024	.376	.110	1.000	.000	.069	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.016	±.000	±.000
0.5	0	1	0.1	0.1	0	.096	.000	1.000	.600	.400	.037	.363	.128	1.000	.000	.096	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.003	±.003	±.017	±.000	±.000	±.015	±.000	±.000
0.5	0	2	0	0	0	.001	.000	1.000	.600	.400	.000	.400	.098	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
0.5	0	2	0	0.1	0	.054	.000	1.000	.600	.400	.041	.359	.112	1.000	.000	.054	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.005	±.005	±.016	±.000	±.000	±.008	±.000	±.000
0.5	0	2	0.1	0	0	.059	.000	1.000	.600	.400	.027	.373	.116	1.000	.000	.058	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.003	±.003	±.013	±.000	±.000	±.008	±.000	±.000
0.5	0	2	0.1	0.1	0	.078	.000	1.000	.600	.400	.048	.352	.123	1.000	.000	.080	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.005	±.005	±.020	±.000	±.000	±.012	±.000	±.000
1	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	1	0	0.1	0	.070	.000	1.000	.600	.400	.024	.376	.128	1.000	.000	.073	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.011	±.000	±.000	±.014	±.000	±.000
1	0	1	0.1	0	0	.073	.000	1.000	.600	.400	.025	.375	.132	1.000	.000	.074	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.001	±.001	±.012	±.000	±.000	±.010	±.000	±.000
1	0	1	0.1	0.1	0	.101	.000	1.000	.600	.400	.034	.366	.146	1.000	.000	.103	1.000	.000
						±.017	±.000	±.000	±.000	±.000	±.003	±.003	±.016	±.000	±.000	±.024	±.000	±.000
1	0	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	2	0	0.1	0	.058	.000	1.000	.600	.400	.027	.373	.124	1.000	.000	.057	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.010	±.000	±.000	±.012	±.000	±.000
1	0	2	0.1	0	0	.060	.000	1.000	.600	.400	.028	.372	.123	1.000	.000	.059	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.011	±.000	±.000	±.011	±.000	±.000
1	0	2	0.1	0.1	0	.079	.000	1.000	.600	.400	.040	.360	.138	1.000	.000	.078	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.005	±.005	±.019	±.000	±.000	±.011	±.000	±.000
1	0.5	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.094	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	1	0	0.1	0	.069	.000	1.000	.600	.400	.023	.377	.118	1.000	.000	.068	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.001	±.001	±.011	±.000	±.000	±.014	±.000	±.000
1	0.5	1	0.1	0	0	.070	.000	1.000	.600	.400	.024	.376	.115	1.000	.000	.072	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.002	±.002	±.014	±.000	±.000	±.014	±.000	±.000
1	0.5	1	0.1	0.1	0	.095	.000	1.000	.600	.400	.033	.367	.135	1.000	.000	.091	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.020	±.000	±.000	±.013	±.000	±.000
1	0.5	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	2	0	0.1	0	.059	.000	1.000	.600	.400	.027	.373	.106	1.000	.000	.060	1.000	.000
						±.007	±.000	±.000	±.000	±.000	±.002	±.002	±.012	±.000	±.000	±.010	±.000	±.000
1	0.5	2	0.1	0	0	.060	.000	1.000	.600	.400	.028	.372	.101	1.000	.000	.059	1.000	.000
						±.009	±.000	±.000	±.000	±.000	±.002	±.002	±.012	±.000	±.000	±.010	±.000	±.000
1	0.5	2	0.1	0.1	0	.082	.000	1.000	.600	.400	.039	.361	.123	1.000	.000	.083	1.000	.000
						±.011	±.000	±.000	±.000	±.000	±.004	±.004	±.017	±.000	±.000	±.014	±.000	±.000
1	1	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	1	0	0.1	0	.068	.000	1.000	.600	.400	.024	.376	.132	1.000	.000	.068	1.000	.000
						±.012	±.000	±.000	±.000	±.000	±.002	±.002	±.016	±.000	±.000	±.011	±.000	±.000
1	1	1	0.1	0	0	.071	.000	1.000	.600	.400	.023	.377	.133	1.000	.000	.069	1.000	.000
						±.013	±.000	±.000	±.000	±.000	±.002	±.002	±.017	±.000	±.000	±.016	±.000	±.000
1	1	1	0.1	0.1	0	.101	.000	1.000	.600	.400	.035	.365	.150	1.000	.000	.105	1.000	.000
						±.015	±.000	±.000	±.000	±.000	±.003	±.003	±.021	±.000	±.000	±.020	±.000	±.000
1	1	2	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.088	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	2	0	0.1	0	.055	.000	1.000	.600	.400	.028	.372	.104	1.000	.000	.053	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.013	±.000	±.000	±.012	±.000	±.000
1	1	2	0.1	0	0	.059	.000	1.000	.600	.400	.027	.373	.101	1.000	.000	.060	1.000	.000
						±.008	±.000	±.000	±.000	±.000	±.002	±.002	±.015	±.000	±.000	±.010	±.000	±.000
1	1	2	0.1	0.1	0	.081	.000	1.000	.600	.400	.040	.360	.121	1.000	.000	.081	1.000	.000
						±.010	±.000	±.000	±.000	±.000	±.003	±.003	±.018	±.000	±.000	±.011	±.000	±.000
1	0	1	0	0	0	.000	.000	1.000	.600	.400	.000	.400	.108	1.000	.000	.000	1.000	.000
						±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0	1	0.05	0	0	.037	.000	1.000	.600	.400	.012	.388	.116	1.000	.000	.038	1.000	.000
						±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000
1	0	1	0.1	0	0	.077	.000	1.000	.600	.400	.024	.376	.136	1.000	.000	.077	1.000	.000
						±.011	±.000	±.000	±.000	±.000	±.001	±.001	±.014	±.000	±.000	±.013	±.000	±.000
1	0	1	0.01	0	0	.007	.000	1.000	.600	.400	.002	.398	.109	1.000	.000	.008	1.000	.000
						±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000

Fixed-price complex condition optimal model

σ_y	Est	Crashes	S{Y} t>10	S{P} t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S{Y} t≤20	P t≤20	S{P} t≤20	S{Y} t>20	P t>20	S{P} t>20
0.01	###	0	.007	.000	1.000	.600	.400	.002	.398	.335	1.000	.000	.007	1.000	.000
			±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000
0.05	###	0	.035	.000	1.000	.600	.400	.012	.388	.337	1.000	.000	.034	1.000	.000
			±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.007	±.000	±.000
0.1	###	0	.071	.000	1.000	.600	.400	.023	.377	.344	1.000	.000	.075	1.000	.000
			±.008	±.000	±.001	±.000	±.000	±.002	±.002	±.013	±.000	±.000	±.012	±.000	±.000
0.01	###	0	.007	.000	1.000	.600	.400	.002	.398	.495	1.000	.000	.006	1.000	.000
			±.001	±.000	±.000	±.000	±.000	±.000	±.000	±.001	±.000	±.000	±.001	±.000	±.000
0.05	###	0	.035	.000	1.000	.600	.400	.012	.388	.497	1.000	.000	.035	1.000	.000
			±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.009	±.000	±.000	±.006	±.000	±.000
0.1	###	0	.071	.000	1.000	.600	.400	.024	.376	.499	1.000	.000	.072	1.000	.000
			±.010	±.000	±.000	±.000	±.000	±.002	±.002	±.021	±.000	±.000	±.012	±.000	±.000

Appendix B

Fixed-price condition, behavioral model

τ	β	θ	S_c	σ_y	a_1	a_2	σ_x	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
1	0	0.5	3	0	0.5	0	0	0	1.838	.000	1.194	.717	.478	.962	-.485	1.051	1.000	.000	2.053	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	0.5	0.5	3	0.05	0.5	0	0	0	.169	.000	.980	.588	.392	.095	.297	.284	1.000	.000	.141	1.000	.000
									±.006	±.000	±.001	±.000	±.000	±.003	±.003	±.005	±.000	±.000	±.007	±.000	±.000
1	1	0.5	3	0.1	0.5	0	0	0	.073	.000	1.002	.601	.401	.033	.368	.130	1.000	.000	.071	1.000	.000
									±.011	±.000	±.001	±.000	±.000	±.002	±.003	±.017	±.000	±.000	±.014	±.000	±.000
1	1	1	3	0	0.5	0	0	0	.101	.000	1.008	.605	.403	.069	.334	.202	1.000	.000	.073	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	1	3	0.05	0.5	0	0	0	.107	.000	1.008	.605	.403	.070	.333	.205	1.000	.000	.081	1.000	.000
									±.004	±.000	±.000	±.000	±.000	±.002	±.002	±.005	±.000	±.000	±.005	±.000	±.000
1	1	1	3	0.1	0.5	0	0	0	.117	.000	1.008	.605	.403	.073	.330	.213	1.000	.000	.097	1.000	.000
									±.007	±.000	±.001	±.001	±.000	±.003	±.003	±.006	±.000	±.000	±.008	±.000	±.000
1	1	1	3	0	0.5	0	0.05	0	.157	.000	1.009	.605	.404	.090	.314	.222	1.000	.000	.142	1.000	.000
									±.022	±.000	±.004	±.003	±.002	±.012	±.012	±.025	±.000	±.000	±.023	±.000	±.000
1	1	1	3	0	0.5	0	0.1	0	.257	.000	1.010	.606	.404	.133	.271	.295	1.000	.000	.256	1.000	.000
									±.034	±.000	±.009	±.006	±.004	±.019	±.017	±.043	±.000	±.000	±.039	±.000	±.000
1	1	1	3	0.1	0.4	0	0.1	0	.247	.000	1.012	.607	.405	.144	.261	.289	1.000	.000	.226	1.000	.000
									±.033	±.000	±.007	±.004	±.003	±.020	±.020	±.040	±.000	±.000	±.041	±.000	±.000
1	1	0	3	0	0.5	0	0	0	.006	.000	1.002	.601	.401	.004	.396	.153	1.000	.000	.001	1.000	.000
									±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000	±.000
1	1	0	3	0.05	0.5	0	0	0	.036	.000	1.002	.601	.401	.014	.387	.158	1.000	.000	.033	1.000	.000
									±.005	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.006	±.000	±.000
1	1	0	3	0.1	0.5	0	0	0	.069	.000	1.002	.601	.401	.025	.375	.163	1.000	.000	.067	1.000	.000
									±.014	±.000	±.001	±.000	±.000	±.001	±.001	±.013	±.000	±.000	±.016	±.000	±.000
1	1	0	3	0.05	1	0	0	0	.034	.000	1.001	.600	.400	.013	.388	.160	1.000	.000	.034	1.000	.000
									±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.008	±.000	±.000
1	1	0	3	0.05	1	1	0	0	.034	.000	1.000	.600	.400	.012	.388	.161	1.000	.000	.034	1.000	.000
									±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000

Clearing-price simple condition, optimal model

σ_y	N_s	M	Est	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
0	10	No	No	0	.000	.000	1.000	.600	.400	.000	.400	.005	1.000	.007	.005	1.000	.006
					±.000	±.000	±.000	±.000	±.001	±.000	±.001	±.001	±.002	±.001	±.001	±.002	±.001
0	10	No	Yes	0	.006	.008	1.000	.599	.401	.000	.401	.119	1.130	.256	.005	1.000	006
					±.001	±.001	±.000	±.001	±.001	±.000	±.001	±.001	±.003	±.004	±.001	±.001	±.001
0	10	Yes	No	0	.005	.017	1.142	.298	.844	.000	.844	.005	2.300	.018	.005	2.301	.017
					±.001	±.002	±.000	±.000	±.000	±.000	±.000	±.001	±.004	±.003	±.001	±.003	±.002
0	10	Yes	Yes	0	.005	.017	1.142	.298	.844	.000	.844	.062	2.212	.214	.005	2.300	.016
					±.001	±.002	±.000	±.000	±.000	±.000	±.000	±.001	±.004	±.004	±.001	±.003	±.003
0.1	10	No	No	0	.024	.033	1.001	.599	.401	.000	.401	.025	1.002	.034	.024	1.002	.032
					±.003	±.005	±.002	±.003	±.004	±.000	±.004	±.004	±.007	±.006	±.004	±.006	±.006
0.1	10	No	Yes	0	.025	.034	1.001	.599	.402	.000	.402	.121	1.127	.258	.026	1.002	.035
					±.002	±.003	±.002	±.003	±.004	±.000	±.004	±.007	±.014	±.023	±.003	±.007	±.005
0.1	10	Yes	No	0	.025	.085	1.139	.298	.841	.000	.841	.025	2.299	.085	.024	2.296	.083
					±.003	±.011	±.003	±.002	±.001	±.000	±.001	±.004	±.019	±.015	±.004	±.018	±.015
0.1	10	Yes	Yes	0	.025	.084	1.139	.298	.841	.000	.841	.065	2.211	.223	.025	2.298	.085
					±.003	±.009	±.002	±.002	±.000	±.000	±.000	±.005	±.023	±.018	±.003	±.014	±.012
0.1	10	No	No	0	.052	.071	1.000	.602	.397	.000	.397	.049	1.000	.066	.052	1.000	.070
					±.008	±.010	±.004	±.007	±.011	±.000	±.011	±.013	±.017	±.019	±.009	±.018	±.012
0.1	10	No	Yes	0	.047	.065	1.002	.599	.402	.000	.402	.124	1.134	.262	.047	1.004	.064
					±.007	±.010	±.003	±.005	±.007	±.000	±.007	±.010	±.028	±.030	±.008	±.011	±.012
0.1	10	Yes	No	0	.052	.175	1.131	.298	.832	000	.832	.053	2.306	.178	.051	2.290	.171
					±.007	±.022	±.007	±.006	±.002	±.000	±.002	±.007	±.039	±.022	±.009	±.041	±.029
0.1	10	Yes	Yes	0	.051	.170	1.133	.300	.833	.000	.833	.080	2.195	.268	.050	2.283	.168
					±.005	±.017	±.005	±.005	±.002	±.000	±.002	±.011	±.035	±.035	±.006	±.045	±.020
1	1	0	0	0	.034	.000	1.001	.600	.400	.013	.388	.160	1.000	.000	.034	1.000	.000
					±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.006	±.000	±.000	±.008	±.000	±.000
1	1	1	0	0	.034	.000	1.000	.600	.400	.012	.388	.161	1.000	.000	.034	1.000	.000
					±.006	±.000	±.000	±.000	±.000	±.001	±.001	±.005	±.000	±.000	±.007	±.000	±.000

Clearing-price simple condition, behavioral model

a1	a2	a3	a0	Discrete	b	σ_y	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
0.25	0	0	1	No	1	0.01	0	.418 ±.004	.009 ±.001	1.000 ±.001	.600 ±.002	.400 ±.002	.000 ±.000	.400 ±.002	.032 ±.004	1.030 ±.013	.049 ±.006	.006 ±.001	.999 ±.004	.008 ±.002
0.25	0	0	1	No	1	0.05	0	.417 ±.017	.047 ±.007	.999 ±.007	.602 ±.013	.398 ±.020	.000 ±.000	.398 ±.020	.047 ±.012	1.031 ±.032	.067 ±.018	.030 ±.007	.999 ±.035	.040 ±.009
0.25	0	0	1	No	1	0.1	0	.396 ±.043	.093 ±.022	.994 ±.017	.616 ±.032	.378 ±.049	.000 ±.000	.378 ±.049	.077 ±.020	1.002 ±.071	.103 ±.022	.070 ±.018	.983 ±.072	.091 ±.025
0.5	0	0	1	No	1	0.01	0	.397 ±.005	.008 ±.001	1.000 ±.001	.601 ±.001	.399 ±.002	.000 ±.000	.399 ±.002	.050 ±.017	.970 ±.011	.056 ±.015	.006 ±.001	.999 ±.004	.008 ±.002
0.5	0	0	1	No	1	0.05	0	.390 ±.017	.040 ±.007	.999 ±.004	.602 ±.006	.397 ±.010	.000 ±.000	.397 ±.010	.077 ±.068	.963 ±.034	.078 ±.039	.030 ±.006	.996 ±.018	.039 ±.008
0.5	0	0	1	No	1	0.1	0	.380 ±.018	.083 ±.015	.998 ±.007	.606 ±.013	.392 ±.020	.000 ±.000	.392 ±.020	.094 ±.031	.941 ±.038	.107 ±.033	.061 ±.016	.995 ±.029	.079 ±.018
1	0	0	1	No	1	0.01	0	.365 ±.024	.007 ±.001	1.000 ±.000	.600 ±.001	.400 ±.001	.000 ±.000	.400 ±.001	.291 ±.113	.930 ±.025	.177 ±.038	.005 ±.001	1.000 ±.002	.007 ±.001
1	0	0	1	No	1	0.05	0	.366 ±.018	.035 ±.005	.999 ±.002	.601 ±.004	.398 ±.006	.000 ±.000	.398 ±.006	.281 ±.082	.931 ±.023	.178 ±.033	.027 ±.004	.998 ±.011	.036 ±.006
1	0	0	1	No	1	0.1	0	.357 ±.021	.074 ±.011	.997 ±.005	.608 ±.009	.388 ±.014	.000 ±.000	.388 ±.014	.299 ±.079	.932 ±.025	.194 ±.033	.058 ±.011	.981 ±.023	.074 ±.013
2	0	0	1	No	1	0.01	0	.273 ±.073	.013 ±.005	1.000 ±.000	.600 ±.000	.400 ±.001	.000 ±.000	.400 ±.001	1.292 ±.522	.917 ±.024	.268 ±.030	.008 ±.003	1.000 ±.002	.010 ±.003
2	0	0	1	No	1	0.05	0	.270 ±.066	.053 ±.019	1.000 ±.001	.601 ±.003	.398 ±.004	.000 ±.000	.398 ±.004	1.317 ±.496	.918 ±.021	.277 ±.028	.036 ±.009	1.000 ±.006	.047 ±.012
2	0	0	1	No	1	0.1	0	.277 ±.029	.128 ±.047	.996 ±.010	.615 ±.027	.381 ±.037	.000 ±.000	.381 ±.037	1.147 ±.055	.923 ±.014	.286 ±.023	.137 ±.194	.992 ±.026	.125 ±.057
0.25	0	0	1	Yes	1	0.01	0	.396 ±.021	.189 ±.011	.997 ±.008	.615 ±.015	.382 ±.023	.000 ±.000	.382 ±.023	.149 ±.011	1.012 ±.040	.197 ±.013	.146 ±.006	1.001 ±.033	.189 ±.010
0.25	0	0	1	Yes	1	0.05	0	.398 ±.008	.186 ±.011	.997 ±.004	.615 ±.008	.382 ±.012	.000 ±.000	.382 ±.012	.140 ±.011	1.015 ±.031	.186 ±.011	.142 ±.015	1.000 ±.023	.186 ±.016
0.25	0	0	1	Yes	1	0.1	0	.401 ±.014	.196 ±.016	1.000 ±.005	.611 ±.008	.388 ±.013	.000 ±.000	.388 ±.013	.145 ±.013	1.014 ±.032	.194 ±.015	.146 ±.014	1.013 ±.036	.196 ±.024
0.5	0	0	1	Yes	1	0.01	0	.360 ±.031	.375 ±.025	.992 ±.014	.654 ±.028	.338 ±.042	.000 ±.000	.338 ±.042	.302 ±.021	1.038 ±.069	.388 ±.035	.305 ±.019	1.017 ±.068	.376 ±.036
0.5	0	0	1	Yes	1	0.05	0	.361 ±.032	.370 ±.034	.993 ±.013	.651 ±.024	.342 ±.037	.000 ±.000	.342 ±.037	.287 ±.026	1.043 ±.079	.376 ±.030	.302 ±.022	1.006 ±.073	.370 ±.046
0.5	0	0	1	Yes	1	0.1	0	.360 ±.039	.378 ±.033	.992 ±.015	.653 ±.030	.340 ±.044	.000 ±.000	.340 ±.044	.300 ±.030	1.031 ±.065	.396 ±.025	.293 ±.026	1.018 ±.063	.376 ±.032
1	0	0	1	Yes	1	0.01	0	.118 ±.077	.707 ±.070	.894 ±.035	.822 ±.061	.072 ±.093	.000 ±.000	.072 ±.093	.721 ±.097	1.059 ±.111	.742 ±.064	.754 ±.081	1.011 ±.140	.714 ±.072
1	0	0	1	Yes	1	0.05	0	.161 ±.106	.732 ±.068	.919 ±.051	.783 ±.081	.135 ±.129	.000 ±.000	.135 ±.129	.716 ±.082	1.103 ±.140	.761 ±.067	.700 ±.095	1.054 ±.145	.720 ±.086
1	0	0	1	Yes	1	0.1	0	.164 ±.062	.733 ±.082	.924 ±.033	.773 ±.054	.150 ±.080	.000 ±.000	.150 ±.080	.678 ±.061	1.047 ±.096	.708 ±.049	.710 ±.110	1.079 ±.151	.742 ±.113
2	0	0	1	Yes	1	0.01	0	-.526 ±.249	1.092 ±.347	.488 ±.269	.973 ±.586	-.486 ±.355	.000 ±.000	-.486 ±.355	2.245 ±.435	1.481 ±.602	1.275 ±.209	1.745 ±1.236	2.127 ±1.293	.865 ±.591
2	0	0	1	Yes	1	0.05	0	-.618 ±.348	1.042 ±.516	.430 ±.254	1.041 ±.682	-.611 ±.452	.000 ±.000	-.611 ±.452	2.401 ±.456	1.403 ±.659	1.253 ±.238	1.785 ±1.349	2.261 ±1.294	.910 ±.588
2	0	0	1	Yes	1	0.1	0	-.679 ±.212	1.169 ±.146	.619 ±.128	1.280 ±.352	-.661 ±.284	.000 ±.000	-.661 ±.284	2.288 ±.375	1.095 ±.137	1.113 ±.095	2.035 ±.761	1.442 ±.850	1.084 ±.263
0	0.1	0	1	No	1	0.01	0	.833 ±.000	.012 ±.003	1.142 ±.001	.298 ±.001	.844 ±.000	.000 ±.000	.844 ±.000	.067 ±.002	2.199 ±.005	.224 ±.005	.003 ±.001	2.300 ±.007	.011 ±.003
0	0.1	0	1	No	1	0.05	0	.832 ±.001	.059 ±.012	1.140 ±.004	.298 ±.004	.843 ±.001	.000 ±.000	.843 ±.001	.070 ±.007	2.200 ±.028	.233 ±.023	.016 ±.003	2.304 ±.027	.055 ±.011
0	0.1	0	1	No	1	0.1	0	.829 ±.003	.114 ±.026	1.143 ±.007	.304 ±.008	.839 ±.002	.000 ±.000	.839 ±.002	.070 ±.011	2.172 ±.063	.233 ±.034	.032 ±.008	2.267 ±.055	.109 ±.029
0	0.25	0	1	No	1	0.01	0	.830 ±.000	.011 ±.002	1.142 ±.000	.298 ±.000	.844 ±.000	.000 ±.000	.844 ±.000	.096 ±.001	2.224 ±.003	.289 ±.003	.003 ±.001	2.301 ±.002	.010 ±.002
0	0.25	0	1	No	1	0.05	0	.830 ±.001	.052 ±.010	1.141 ±.002	.298 ±.002	.843 ±.000	.000 ±.000	.843 ±.000	.096 ±.007	2.221 ±.013	.290 ±.018	.015 ±.003	2.301 ±.014	.051 ±.011
0	0.25	0	1	No	1	0.1	0	.826 ±.003	.105 ±.014	1.140 ±.004	.299 ±.004	.840 ±.001	.000 ±.000	.840 ±.001	.102 ±.013	2.229 ±.031	.310 ±.033	.031 ±.005	2.287 ±.026	.106 ±.017
0	0.5	0	1	No	1	0.01	0	.815 ±.001	.105 ±.006	1.138 ±.001	.297 ±.001	.841 ±.000	.000 ±.000	.841 ±.000	.171 ±.002	2.213 ±.005	.440 ±.005	.019 ±.002	2.303 ±.004	.065 ±.006
0	0.5	0	1	No	1	0.05	0	.813 ±.003	.132 ±.032	1.136 ±.003	.297 ±.002	.839 ±.002	.000 ±.000	.839 ±.002	.174 ±.008	2.211 ±.006	.446 ±.019	.030 ±.010	2.303 ±.013	.102 ±.035
0	0.5	0	1	No	1	0.1	0	.809 ±.004	.183 ±.035	1.132 ±.005	.298 ±.002	.834 ±.004	.000 ±.000	.834 ±.004	.178 ±.015	2.206 ±.024	.459 ±.031	.048 ±.013	2.303 ±.016	.164 ±.044
0	1	0	1	No	1	0.01	0	.547 ±.001	1.006 ±.002	.924 ±.001	.358 ±.001	.566 ±.001	.000 ±.000	.566 ±.001	.478 ±.002	2.070 ±.001	1.048 ±.001	.388 ±.002	2.093 ±.002	1.021 ±.002
0	1	0	1	No	1	0.05	0	.548 ±.003	1.005 ±.007	.925 ±.003	.359 ±.002	.567 ±.004	.000 ±.000	.567 ±.004	.478 ±.010	2.067 ±.007	1.047 ±.008	.385 ±.009	2.092 ±.008	1.017 ±.009
0	1	0	1	No	1	0.1	0	.549 ±.007	.996 ±.015	.927 ±.010	.357 ±.010	.570 ±.007	.000 ±.000	.570 ±.007	.490 ±.018	2.073 ±.063	1.042 ±.019	.388 ±.016	2.106 ±.041	1.008 ±.017

0	0.1	0	1	Yes	1	0.01	0	.703 ±.018	.111 ±.004	1.124 ±.010	.423 ±.012	.701 ±.022	.000 ±.000	.701 ±.022	.037 ±.001	1.604 ±.036	.112 ±.003	.037 ±.001	1.602 ±.068	.112 ±.005
0	0.1	0	1	Yes	1	0.05	0	.696 ±.057	.142 ±.028	1.118 ±.030	.428 ±.042	.690 ±.071	.000 ±.000	.690 ±.071	.047 ±.008	1.670 ±.126	.139 ±.022	.048 ±.011	1.589 ±.226	.134 ±.029
0	0.1	0	1	Yes	1	0.1	0	.645 ±.087	.225 ±.109	1.076 ±.088	.449 ±.094	.627 ±.114	.000 ±.000	.627 ±.114	.068 ±.019	1.575 ±.182	.185 ±.043	.071 ±.018	1.546 ±.548	.169 ±.054
0	0.25	0	1	Yes	1	0.01	0	.646 ±.009	.250 ±.005	1.100 ±.005	.455 ±.007	.644 ±.012	.000 ±.000	.644 ±.012	.098 ±.001	1.469 ±.020	.249 ±.004	.097 ±.001	1.485 ±.034	.253 ±.006
0	0.25	0	1	Yes	1	0.05	0	.613 ±.074	.251 ±.032	1.082 ±.036	.479 ±.058	.603 ±.094	.000 ±.000	.603 ±.094	.108 ±.009	1.426 ±.114	.257 ±.019	.108 ±.016	1.416 ±.229	.247 ±.036
0	0.25	0	1	Yes	1	0.1	0	.558 ±.181	.280 ±.065	1.054 ±.081	.528 ±.151	.527 ±.232	.000 ±.000	.527 ±.232	.130 ±.028	1.457 ±.278	.294 ±.032	.148 ±.059	1.326 ±.479	.256 ±.076
0	0.5	0	1	Yes	1	0.01	0	.531 ±.015	.424 ±.012	1.056 ±.007	.531 ±.012	.526 ±.019	.000 ±.000	.526 ±.019	.223 ±.003	1.292 ±.023	.427 ±.008	.222 ±.005	1.300 ±.040	.429 ±.013
0	0.5	0	1	Yes	1	0.05	0	.533 ±.049	.438 ±.038	1.056 ±.021	.528 ±.038	.529 ±.059	.000 ±.000	.529 ±.059	.228 ±.012	1.306 ±.096	.438 ±.032	.225 ±.017	1.318 ±.138	.437 ±.039
0	0.5	0	1	Yes	1	0.1	0	.488 ±.161	.464 ±.142	1.006 ±.150	.536 ±.165	.470 ±.202	.000 ±.000	.470 ±.202	.241 ±.043	1.318 ±.260	.444 ±.071	.243 ±.101	1.436 ±.689	.413 ±.102
0	1	0	1	Yes	1	0.01	0	.234 ±.017	.632 ±.014	.973 ±.007	.756 ±.013	.218 ±.020	.000 ±.000	.218 ±.020	.592 ±.007	1.072 ±.019	.643 ±.011	.599 ±.011	1.057 ±.028	.636 ±.015
0	1	0	1	Yes	1	0.05	0	.245 ±.090	.657 ±.072	.973 ±.036	.739 ±.075	.233 ±.110	.000 ±.000	.233 ±.110	.591 ±.035	1.090 ±.095	.658 ±.053	.586 ±.068	1.101 ±.166	.663 ±.083
0	1	0	1	Yes	1	0.1	0	.205 ±.207	.669 ±.128	.933 ±.093	.756 ±.172	.177 ±.260	.000 ±.000	.177 ±.260	.601 ±.080	1.105 ±.200	.677 ±.098	.618 ±.156	1.101 ±.347	.671 ±.135
0	0	0.25	0	No	1	0.01	0	-1.904 ±10.201	.091 ±.162	.968 ±.107	3.742 ±13.844	-2.774 ±13.942	.000 ±.000	-2.774 ±13.942	.038 ±.013	1.041 ±.031	.058 ±.024	7.687 ±33.457	.943 ±.188	.087 ±.152
0	0	0.25	0	No	1	0.1	0	.442 ±.043	.215 ±.121	1.027 ±.026	.588 ±.032	.439 ±.051	.000 ±.000	.439 ±.051	.097 ±.040	1.025 ±.077	.132 ±.053	.132 ±.073	1.124 ±.117	.222 ±.148
0	0	0.25	0	Yes	1	0.01	0	.747 ±.024	.357 ±.067	1.099 ±.016	.314 ±.025	.785 ±.016	.000 ±.000	.785 ±.016	.228 ±.023	1.971 ±.163	.595 ±.038	.109 ±.022	2.188 ±.174	.325 ±.055
0	0	0.25	0	Yes	1	0.1	0	.516 ±.072	.340 ±.108	1.066 ±.035	.528 ±.055	.539 ±.088	.000 ±.000	.539 ±.088	.150 ±.034	1.120 ±.143	.242 ±.098	.179 ±.038	1.365 ±.227	.347 ±.106
0	0	0.5	0	No	1	0.01	0	###	.478 ±.241	.895 ±.175	###	###	.000 ±.000	###	4.267 ±13.800	.991 ±.143	.388 ±.279	###	1.045 ±.253	.455 ±.247
0	0	0.5	0	No	1	0.1	0	-2.236 ±6.851	.395 ±.164	.980 ±.132	4.177 ±9.240	-3.197 ±9.355	.000 ±.000	-3.197 ±9.355	4.075 ±17.199	.984 ±.070	.237 ±.137	14.745 ±35.893	1.118 ±.334	.399 ±.167
0	0	0.5	0	Yes	1	0.01	0	.696 ±.036	.497 ±.056	1.008 ±.035	.287 ±.038	.722 ±.034	.000 ±.000	.722 ±.034	.289 ±.069	2.110 ±.219	.689 ±.090	.175 ±.022	2.292 ±.216	.500 ±.067
0	0	0.5	0	Yes	1	0.1	0	.594 ±.034	.527 ±.081	1.042 ±.027	.434 ±.031	.607 ±.038	.000 ±.000	.607 ±.038	.301 ±.039	1.620 ±.200	.617 ±.114	.301 ±.037	1.597 ±.144	.478 ±.089
0	0	1	0	No	1	0.01	0	###	.429 ±.249	.760 ±.330	###	###	.000 ±.000	###	###	1.077 ±.261	.701 ±.164	###	.879 ±.426	.344 ±.273
0	0	1	0	No	1	0.1	0	###	.510 ±.169	.724 ±.292	###	###	.000 ±.000	###	###	1.001 ±.307	.627 ±.145	###	.846 ±.425	.372 ±.249
0	0	1	0	Yes	1	0.01	0	.484 ±.037	.823 ±.046	.939 ±.049	.439 ±.080	.499 ±.039	.000 ±.000	.499 ±.039	.580 ±.057	1.774 ±.207	.880 ±.057	.517 ±.087	1.863 ±.214	.832 ±.046
0	0	1	0	Yes	1	0.1	0	.444 ±.060	.703 ±.112	.883 ±.042	.440 ±.092	.443 ±.072	.000 ±.000	.443 ±.072	.553 ±.091	1.854 ±.290	.824 ±.104	.579 ±.123	1.611 ±.288	.651 ±.122
0	0	0.1	0	No	0	0.01	0	###	.540 ±.272	.692 ±.396	###	###	.000 ±.000	###	###	1.554 ±.507	.639 ±.291	###	.873 ±.758	.387 ±.316
0	0	0.1	0	No	0	0.1	0	.406 ±.621	.389 ±.178	1.027 ±.168	.706 ±.683	.321 ±.844	.000 ±.000	.321 ±.844	.184 ±.224	1.484 ±.375	.315 ±.118	.451 ±.946	1.305 ±.485	.349 ±.130
0	0	0.1	0	Yes	0	0.01	0	.829 ±.001	.130 ±.009	1.140 ±.003	.302 ±.004	.838 ±.001	.000 ±.000	.838 ±.001	.073 ±.002	2.190 ±.020	.247 ±.006	.038 ±.003	2.280 ±.027	.131 ±.011
0	0	0.1	0	Yes	0	0.1	0	.692 ±.071	.268 ±.082	1.119 ±.031	.427 ±.054	.692 ±.085	.000 ±.000	.692 ±.085	.088 ±.024	1.608 ±.247	.239 ±.061	.088 ±.035	1.658 ±.263	.239 ±.086
0	0	0.2	0	No	0	0.01	0	###	.459 ±.343	.520 ±.433	###	###	.000 ±.000	###	###	1.280 ±.467	.861 ±.188	###	.631 ±.699	.305 ±.320
0	0	0.2	0	No	0	0.1	0	-5.089 ±17.524	.597 ±.207	.872 ±.257	7.589 ±23.826	-6.717 ±23.963	.000 ±.000	-6.717 ±23.963	3.906 ±9.562	1.454 ±.550	.626 ±.211	14.361 ±41.494	1.106 ±.590	.478 ±.215
0	0	0.2	0	Yes	0	0.01	0	.814 ±.003	.248 ±.011	1.127 ±.006	.303 ±.008	.824 ±.003	.000 ±.000	.824 ±.003	.104 ±.004	2.176 ±.043	.338 ±.014	.073 ±.002	2.270 ±.048	.248 ±.008
0	0	0.2	0	Yes	0	0.1	0	.786 ±.012	.318 ±.030	1.136 ±.006	.346 ±.013	.790 ±.011	.000 ±.000	.790 ±.011	.111 ±.017	2.005 ±.091	.351 ±.047	.097 ±.015	2.035 ±.079	.306 ±.042
0	0	0.5	0	No	0	0.01	0	###	.672 ±.247	.480 ±.308	###	###	.000 ±.000	###	###	1.525 ±.300	.962 ±.211	###	.509 ±.469	.406 ±.349
0	0	0.5	0	No	0	0.1	0	###	.622 ±.323	.497 ±.328	###	###	.000 ±.000	###	###	1.268 ±.405	.933 ±.125	###	.635 ±.628	.366 ±.348
0	0	0.5	0	Yes	0	0.01	0	.738 ±.001	.523 ±.010	1.003 ±.006	.255 ±.005	.749 ±.001	.000 ±.000	.749 ±.001	.215 ±.001	2.437 ±.017	.646 ±.003	.156 ±.003	2.520 ±.031	.533 ±.011
0	0	0.5	0	Yes	0	0.1	0	.717 ±.017	.583 ±.028	1.056 ±.014	.333 ±.028	.723 ±.023	.000 ±.000	.723 ±.023	.234 ±.016	2.110 ±.105	.655 ±.025	.209 ±.033	2.110 ±.138	.578 ±.036
0	0	0.25	0	No	1	0.05	0	.420 ±.044	.106 ±.098	1.006 ±.020	.603 ±.040	.403 ±.058	.000 ±.000	.403 ±.058	.057 ±.033	1.045 ±.057	.079 ±.032	.119 ±.252	1.025 ±.063	.111 ±.108

0	0	0.25	0	Yes	1	0.05	0	.580 ±.080	.424 ±.096	1.079 ±.033	.479 ±.059	.601 ±.090	.000 ±.000	.601 ±.090	.176 ±.045	1.365 ±.327	.373 ±.169	.184 ±.036	1.495 ±.253	.401 ±.096
0	0	0.5	0	No	1	0.05	0	###	.362 ±.219	.949 ±.159	###	###	.000 ±.000	###	1.493 ±6.067	.979 ±.069	.187 ±.181	###	1.070 ±.315	.376 ±.223
0	0	0.5	0	Yes	1	0.05	0	.644 ±.057	.517 ±.090	1.029 ±.028	.375 ±.061	.654 ±.067	.000 ±.000	.654 ±.067	.288 ±.042	1.874 ±.222	.659 ±.061	.246 ±.057	1.818 ±.290	.474 ±.103
0	0	1	0	No	1	0.05	0	###	.523 ±.140	.828 ±.208	###	###	.000 ±.000	###	###	1.075 ±.267	.636 ±.170	###	1.005 ±.394	.380 ±.216
0	0	1	0	Yes	1	0.05	0	.481 ±.044	.722 ±.126	.887 ±.072	.397 ±.093	.490 ±.052	.000 ±.000	.490 ±.052	.555 ±.073	1.831 ±.212	.855 ±.072	.461 ±.122	1.825 ±.345	.690 ±.157
0	0	0.1	0	No	0	0.05	0	-31.915 ±101.552	.465 ±.243	.748 ±.385	44.239 ±138.826	-43.491 ±138.873	.000 ±.000	-43.491 ±138.873	14.352 ±35.830	1.393 ±.611	.537 ±.223	93.335 ±380.293	.890 ±.748	.302 ±.224
0	0	0.1	0	Yes	0	0.05	0	.813 ±.008	.169 ±.028	1.155 ±.004	.334 ±.010	.821 ±.008	.000 ±.000	.821 ±.008	.070 ±.010	2.049 ±.077	.232 ±.034	.049 ±.008	2.089 ±.065	.163 ±.027
0	0	0.2	0	No	0	0.05	0	###	.594 ±.277	.836 ±.307	###	###	.000 ±.000	###	###	1.725 ±.444	.634 ±.253	###	1.137 ±.658	.499 ±.293
0	0	0.2	0	Yes	0	0.05	0	.806 ±.006	.274 ±.024	1.136 ±.005	.322 ±.011	.813 ±.008	.000 ±.000	.813 ±.008	.104 ±.006	2.135 ±.061	.337 ±.018	.082 ±.008	2.139 ±.080	.270 ±.027
0	0	0.5	0	No	0	0.05	0	###	.715 ±.268	.516 ±.298	###	###	.000 ±.000	###	###	1.375 ±.496	.958 ±.196	###	.741 ±.622	.563 ±.346
0	0	0.5	0	Yes	0	0.05	0	.727 ±.009	.581 ±.031	1.042 ±.022	.307 ±.031	.735 ±.012	.000 ±.000	.735 ±.012	.224 ±.010	2.270 ±.149	.650 ±.020	.197 ±.023	2.231 ±.155	.587 ±.033

Clearing-price complex condition, optimal model

Sy	Coll ude	Est	Ns	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	No	No	10	0	.005	.006	1.000	.600	.400	.000	.400	.005	1.079	.197	.005	1.000	.006
					±.001	±.001	±.000	±.001	±.000	±.000	±.000	±.001	±.000	±.001	±.001	±.001	±.001
0	No	Yes	10	0	.017	.031	.998	.590	.408	.000	.408	.121	1.17[illegible]	.174	.005	1.001	.006
					±.002	±.003	±.000	±.001	±.001	±.000	±.001	±.001	±.002	±.003	±.001	±.001	±.001
0	Yes	No	10	0	.005	.022	.823	.245	.577	.000	.577	.005	1.892	.296	.005	2.011	.023
					±.001	±.003	±.001	±.001	±.000	±.000	±.000	±.001	±.003	±.002	±.001	±.005	±.004
0	Yes	Yes	10	1	.041	.154	.840	.278	.562	.000	.562	.067	1.511	.153	.014	1.921	.074
					±.002	±.004	±.001	±.002	±.001	±.000	±.001	±.001	±.006	±.005	±.002	±.007	±.005
0.1	No	No	10	0	.026	.031	1.000	.599	.401	.000	.401	.024	1.081	.196	.025	1.002	.030
					±.003	±.005	±.002	±.003	±.002	±.000	±.002	±.004	±.002	±.003	±.004	±.006	±.006
0.1	No	Yes	10	0	.030	.048	.998	.590	.408	.000	.406	.125	1.178	.180	.025	1.002	.032
					±.005	±.007	±.002	±.004	±.002	±.000	±.002	±.007	±.009	±.016	±.005	±.004	±.006
0.1	Yes	No	10	0	.025	.114	.820	.245	.575	.000	.575	.023	1.894	.312	.026	2.010	.119
					±.004	±.020	±.003	±.003	±.001	±.000	±.001	±.004	±.017	±.009	±.005	±.021	±.024
0.1	Yes	Yes	10	0	.051	.191	.842	.285	.557	.000	.557	.071	1.508	.177	.034	1.882	.148
					±.008	±.017	±.005	±.008	±.003	±.000	±.003	±.006	±.021	±.025	±.007	±.051	±.027
0.1	No	No	10	0	.049	.062	1.000	.599	.401	.000	.401	.052	1.083	.206	.049	1.003	.060
					±.006	±.008	±.003	±.005	±.003	±.000	±.003	±.007	±.006	±.008	±.006	±.007	±.008
0.1	No	Yes	10	2	.052	.073	1.001	.595	.406	.000	.406	.141	1.174	.207	.050	1.003	.062
					±.005	±.010	±.003	±.006	±.004	±.000	±.004	±.016	±.020	±.029	±.007	±.006	±.009
0.1	Yes	Yes	10	2	.073	.263	.846	.297	.548	.000	.548	.098	1.507	.237	.062	1.808	.245
					±.025	±.045	±.015	±.025	±.011	±.000	±.011	±.025	±.068	±.052	±.025	±.144	±.067
0.1	Yes	No	10	0	.048	.215	.813	.245	.568	.000	.568	.048	1.897	.355	.048	2.009	.221
					±.005	±.029	±.005	±.004	±.003	±.000	±.003	±.009	±.031	±.025	±.006	±.032	±.032

Clearing-price complex condition, behavioral model

a1	a2	a3	a0	Discrete	b	Sy	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	0	0.1	0	No	0	0.01	2	###	1.068 ±.439	21.248 ±52.226	###	###	.000 ±.000	###	###	1.639 ±.309	.775 ±.430	###	1.790 ±.529	1.096 ±.451
0	0	0.1	0	No	0	0.05	0	###	.655 ±.375	1.207 ±.656	###	###	.000 ±.000	###	1.899 ±6.133	1.401 ±.114	.394 ±.247	###	1.560 ±.331	.699 ±.415
0	0	0.1	0	No	0	0.1	0	.173 ±.121	.340 ±.214	.922 ±.031	.443 ±.047	.480 ±.030	.000 ±.000	.480 ±.030	.090 ±.043	1.313 ±.055	.187 ±.110	.190 ±.147	1.360 ±.149	.374 ±.251
0	0	0.1	0	Yes	0	0.01	0	.129 ±.015	.417 ±.033	.851 ±.005	.335 ±.012	.516 ±.007	.000 ±.000	.516 ±.007	.082 ±.006	1.340 ±.017	.203 ±.020	.106 ±.020	1.733 ±.050	.433 ±.055
0	0	0.1	0	Yes	0	0.05	0	.097 ±.025	.290 ±.101	.881 ±.025	.370 ±.034	.511 ±.012	.000 ±.000	.511 ±.012	.068 ±.015	1.330 ±.029	.160 ±.045	.091 ±.025	1.541 ±.153	.310 ±.110
0	0	0.1	0	Yes	0	0.1	0	.094 ±.017	.211 ±.071	.925 ±.029	.440 ±.050	.485 ±.022	.000 ±.000	.485 ±.022	.070 ±.013	1.287 ±.042	.142 ±.041	.098 ±.023	1.312 ±.146	.224 ±.082
0	0	0.2	0	No	0	0.01	4	###	1.300 ±.415	###	###	###	.000 ±.000	###	###	1.911 ±.334	1.150 ±.361	###	2.020 ±.587	1.252 ±.471
0	0	0.2	0	No	0	0.05	1	###	.817 ±.447	3.795 ±6.224	###	###	.000 ±.000	###	###	1.480 ±.217	.631 ±.330	###	1.530 ±.475	.810 ±.487
0	0	0.2	0	No	0	0.1	0	25.265 ±77.721	.678 ±.320	1.282 ±.651	3.730 ±10.092	-2.448 ±9.838	.000 ±.000	-2.448 ±9.838	1.082 ±3.401	1.330 ±.128	.375 ±.218	30.108 ±93.481	1.411 ±.318	.736 ±.368
0	0	0.2	0	Yes	0	0.01	0	.199 ±.032	.688 ±.072	.814 ±.007	.330 ±.022	.484 ±.017	.000 ±.000	.484 ±.017	.150 ±.013	1.355 ±.028	.384 ±.033	.197 ±.043	1.844 ±.074	.741 ±.094
0	0	0.2	0	Yes	0	0.05	0	.181 ±.030	.618 ±.119	.822 ±.035	.326 ±.034	.496 ±.020	.000 ±.000	.496 ±.020	.152 ±.020	1.359 ±.036	.377 ±.050	.168 ±.039	1.850 ±.149	.642 ±.145
0	0	0.2	0	Yes	0	0.1	0	.168 ±.032	.445 ±.214	.899 ±.079	.416 ±.087	.483 ±.022	.000 ±.000	.483 ±.022	.124 ±.027	1.305 ±.076	.274 ±.097	.172 ±.040	1.496 ±.302	.475 ±.243
0	0	0.25	0	No	1	0.01	0	.033 ±.007	.039 ±.012	.966 ±.003	.532 ±.005	.434 ±.002	.000 ±.000	.434 ±.002	.029 ±.005	1.179 ±.005	.109 ±.004	.027 ±.012	1.077 ±.007	.035 ±.020
0	0	0.25	0	No	1	0.05	0	.059 ±.020	.085 ±.033	.970 ±.007	.535 ±.013	.435 ±.007	.000 ±.000	.435 ±.007	.047 ±.012	1.181 ±.013	.123 ±.013	.059 ±.026	1.081 ±.023	.086 ±.042
0	0	0.25	0	No	1	0.1	0	.108 ±.037	.155 ±.064	.979 ±.014	.544 ±.024	.435 ±.011	.000 ±.000	.435 ±.011	.073 ±.025	1.180 ±.023	.144 ±.022	.111 ±.049	1.090 ±.048	.167 ±.084
0	0	0.25	0	Yes	1	0.01	0	.220 ±.052	.287 ±.106	1.005 ±.021	.592 ±.031	.413 ±.014	.000 ±.000	.413 ±.014	.166 ±.018	1.115 ±.030	.237 ±.018	.223 ±.065	1.098 ±.081	.301 ±.139
0	0	0.25	0	Yes	1	0.05	0	.226 ±.049	.267 ±.084	1.035 ±.039	.630 ±.049	.406 ±.015	.000 ±.000	.406 ±.015	.154 ±.024	1.108 ±.021	.225 ±.018	.238 ±.058	1.049 ±.074	.293 ±.107
0	0	0.25	0	Yes	1	0.1	0	.228 ±.071	.276 ±.111	1.035 ±.042	.625 ±.050	.410 ±.015	.000 ±.000	.410 ±.015	.168 ±.040	1.108 ±.026	.235 ±.034	.239 ±.081	1.069 ±.104	.298 ±.133
0	0	0.5	0	No	0	0.01	18	###	1.234 ±.372	###	###	###	.000 ±.000	###	###	2.122 ±.178	1.374 ±.111	###	1.576 ±1.238	1.054 ±.701
0	0	0.5	0	No	0	0.05	7	###	1.041 ±.281	841.887 ±1613.026	###	###	.000 ±.000	"?#	###	1.624 ±.222	.973 ±.274	###	1.334 ±.504	.844 ±.415
0	0	0.5	0	No	0	0.1	4	###	1.003 ±.361	290.210 ±1035.321	###	###	.000 ±.000	###	###	1.543 ±.248	.950 ±.268	###	1.298 ±.466	.855 ±.498
0	0	0.5	0	Yes	1	0.01	0	.577 ±.257	.500 ±.097	1.160 ±.145	.780 ±.220	.380 ±.086	.000 ±.000	.380 ±.086	.424 ±.167	1.201 ±.073	.407 ±.067	.598 ±.297	1.060 ±.196	.499 ±.139
0	0	0.5	0	Yes	1	0.05	0	.664 ±.439	.540 ±.165	1.173 ±.242	.762 ±.290	.411 ±.079	.000 ±.000	.411 ±.079	.367 ±.131	1.219 ±.061	.427 ±.058	.680 ±.426	1.136 ±.186	.545 ±.227
0	0	0.5	0	Yes	1	0.1	0	.444 ±.090	.496 ±.126	1.107 ±.075	.691 ±.092	.416 ±.042	.000 ±.000	.416 ±.042	.352 ±.095	1.181 ±.096	.416 ±.084	.439 ±.103	1.114 ±.144	.480 ±.165
0	0	1	0	No	1	0.01	0	###	.310 ±.314	1.167 ±.402	###	###	.000 ±.000	###	.198 ±.091	1.010 ±.008	.342 ±.013	###	1.118 ±.220	.351 ±.371
0	0	1	0	No	1	0.05	0	###	.322 ±.407	###	###	###	.000 ±.000	###	2.958 ±12.493	1.034 ±.087	.379 ±.174	###	1.112 ±.354	.344 ±.418
0	0	1	0	No	1	0.1	0	51.279 ±166.867	.490 ±.425	3.016 ±6.377	11.087 ±36.440	-8.071 ±30.074	.000 ±.000	-8.071 ±30.074	40.644 ±180.953	1.031 ±.075	.379 ±.132	39.951 ±109.445	1.200 ±.354	.561 ±.481
0	0	1	0	Yes	1	0.01	0	2.733 ±2.902	.911 ±.206	1.547 ±.764	1.491 ±1.245	.056 ±.535	.000 ±.000	.056 ±.535	1.357 ±1.068	1.291 ±.101	.758 ±.113	2.986 ±3.425	1.309 ±.362	.922 ±.275
0	0	1	0	Yes	1	0.05	0	5.969 ±18.243	.914 ±.140	2.509 ±5.546	2.728 ±7.380	-.220 ±1.845	.000 ±.000	-.220 ±1.845	1.001 ±.572	1.263 ±.063	.741 ±.083	6.429 ±19.278	1.344 ±.246	.945 ±.190

0	0	1	0	Yes	1	0.1	0	1.999 ±1.992	.900 ±.158	1.467 ±.936	1.278 ±1.234	.190 ±.331	.000 ±.000	.190 ±.331	1.160 ±.965	1.283 ±.061	.772 ±.084	2.087 ±2.197	1.332 ±.266	.899 ±.217
0	0.1	0	1	No	1	0.01	0	###	1.114 ±.497	12.452 ±43.906	###	###	.000 ±.000	###	###	1.828 ±.423	.972 ±.489	###	2.111 ±.468	1.077 ±.506
0	0.1	0	1	No	1	0.05	0	.084 ±.011	.306 ±.025	.826 ±.007	.280 ±.011	.547 ±.005	.000 ±.000	.547 ±.005	.095 ±.008	1.415 ±.031	.176 ±.031	.045 ±.014	1.995 ±.065	.211 ±.049
0	0.1	0	1	No	1	0.1	0	.084 ±.013	.306 ±.046	.828 ±.012	.283 ±.014	.545 ±.006	.000 ±.000	.545 ±.006	.095 ±.010	1.416 ±.032	.189 ±.037	.047 ±.014	1.969 ±.079	.226 ±.060
0	0.1	0	1	Yes	1	0.01	0	.117 ±.014	.371 ±.027	.840 ±.010	.311 ±.019	.528 ±.010	.000 ±.000	.528 ±.010	.094 ±.009	1.357 ±.020	.203 ±.026	.077 ±.014	1.847 ±.098	.337 ±.050
0	0.1	0	1	Yes	1	0.05	0	.101 ±.013	.345 ±.037	.828 ±.008	.289 ±.015	.539 ±.008	.000 ±.000	.539 ±.008	.092 ±.007	1.390 ±.025	.177 ±.030	.064 ±.016	1.953 ±.079	.286 ±.066
0	0.1	0	1	Yes	1	0.1	0	.104 ±.016	.355 ±.040	.827 ±.011	.291 ±.019	.537 ±.009	.000 ±.000	.537 ±.009	.092 ±.010	1.389 ±.046	.176 ±.039	.072 ±.017	1.944 ±.094	.309 ±.067
0	0.25	0	1	No	1	0.01	0	###	1.147 ±.460	868.599 ±2573.545	###	###	.000 ±.000	###	###	2.166 ±.256	1.247 ±.309	###	2.496 ±.302	1.054 ±.503
0	0.25	0	1	No	1	0.05	0	###	.783 ±.441	351.248 ±1565.734	###	###	.000 ±.000	###	###	1.658 ±.310	.759 ±.360	###	2.177 ±.257	.675 ±.484
0	0.25	0	1	No	1	0.1	0	.088 ±.035	.398 ±.096	.776 ±.016	.236 ±.015	.540 ±.014	.000 ±.000	.540 ±.014	.188 ±.030	1.521 ±.090	.412 ±.107	.047 ±.021	2.250 ±.047	.254 ±.118
0	0.25	0	1	Yes	1	0.01	0	.210 ±.067	.688 ±.116	.779 ±.024	.296 ±.051	.483 ±.033	.000 ±.000	.483 ±.033	.233 ±.034	1.398 ±.071	.497 ±.055	.154 ±.062	2.061 ±.189	.658 ±.164
0	0.25	0	1	Yes	1	0.05	0	.147 ±.034	.577 ±.082	.769 ±.014	.258 ±.021	.511 ±.019	.000 ±.000	.511 ±.019	.220 ±.015	1.435 ±.051	.459 ±.030	.100 ±.024	2.202 ±.089	.500 ±.098
0	0.25	0	1	Yes	1	0.1	0	.132 ±.051	.531 ±.110	.769 ±.017	.251 ±.023	.518 ±.021	.000 ±.000	.518 ±.021	.214 ±.022	1.471 ±.055	.435 ±.064	.104 ±.021	2.208 ±.092	.475 ±.105
0	0.5	0	1	No	1	0.01	0	.114 ±.049	.531 ±.140	.715 ±.029	.205 ±.008	.509 ±.030	.000 ±.000	.509 ±.030	.308 ±.007	1.833 ±.037	.753 ±.038	.111 ±.061	2.290 ±.053	.515 ±.188
0	0.5	0	1	No	1	0.05	0	###	.814 ±.473	###	###	###	.000 ±.000	###	###	1.879 ±.192	.940 ±.298	###	2.258 ±.216	.791 ±.493
0	0.5	0	1	No	1	0.1	0	.497 ±1.264	.741 ±.421	.703 ±.057	.381 ±.520	.322 ±.563	.000 ±.000	.322 ±.563	.928 ±2.059	1.716 ±.171	.832 ±.266	.253 ±.483	2.211 ±.115	.691 ±.425
0	0.5	0	1	Yes	1	0.01	0	.431 ±.150	1.079 ±.154	.708 ±.059	.364 ±.098	.344 ±.066	.000 ±.000	.344 ±.066	.509 ±.100	1.464 ±.075	.828 ±.074	.375 ±.157	2.074 ±.283	1.091 ±.207
0	0.5	0	1	Yes	1	0.05	0	.318 ±.123	.933 ±.122	.711 ±.042	.302 ±.064	.409 ±.049	.000 ±.000	.409 ±.049	.451 ±.111	1.518 ±.095	.782 ±.080	.277 ±.079	2.144 ±.210	.924 ±.144
0	0.5	0	1	Yes	1	0.1	0	.264 ±.172	.840 ±.203	.698 ±.038	.267 ±.088	.431 ±.081	.000 ±.000	.431 ±.081	.423 ±.134	1.557 ±.110	.730 ±.103	.233 ±.083	2.274 ±.147	.838 ±.208
0	1	0	1	No	1	0.01	0	###	1.623 ±.232	###	###	###	.000 ±.000	###	###	1.843 ±.135	1.340 ±.205	###	2.225 ±.472	1.668 ±.212
0	1	0	1	No	1	0.05	0	###	1.245 ±.244	51.799 ±228.871	832.119 ±3719.592	-780.320 ±3490.721	.000 ±.000	-780.320 ±3490.721	###	1.728 ±.089	1.106 ±.161	708.723 ±3167.411	2.124 ±.163	1.279 ±.257
0	1	0	1	No	1	0.1	0	2.661 ±9.793	1.090 ±.196	.754 ±.366	.901 ±2.630	-.148 ±2.269	.000 ±.000	-.148 ±2.269	.867 ±.092	1.687 ±.072	1.002 ±.092	2.950 ±11.426	2.059 ±.126	1.114 ±.270
0	1	0	1	Yes	1	0.01	0	1.013 ±.699	1.338 ±.188	.634 ±.104	.543 ±.296	.091 ±.227	.000 ±.000	.091 ±.227	1.425 ±.609	1.713 ±.169	1.183 ±.101	.814 ±.483	2.098 ±.335	1.356 ±.201
0	1	0	1	Yes	1	0.05	0	.924 ±.497	1.345 ±.163	.632 ±.106	.546 ±.283	.085 ±.234	.000 ±.000	.085 ±.234	1.312 ±.203	1.707 ±.152	1.189 ±.083	.934 ±.582	2.093 ±.349	1.349 ±.190
0	1	0	1	Yes	1	0.1	0	1.197 ±1.094	1.343 ±.180	.680 ±.124	.647 ±.475	.033 ±.368	.000 ±.000	.033 ±.368	1.363 ±.634	1.633 ±.123	1.145 ±.105	1.117 ±1.142	2.054 ±.312	1.379 ±.182
0.25	0	0	1	No	1	0.01	0	.025 ±.002	.027 ±.003	.969 ±.002	.538 ±.004	.431 ±.002	.000 ±.000	.431 ±.002	.028 ±.003	1.171 ±.005	.113 ±.004	.015 ±.003	1.069 ±.004	.018 ±.004
0.25	0	0	1	No	1	0.05	0	.045 ±.010	.059 ±.013	.972 ±.005	.542 ±.010	.430 ±.004	.000 ±.000	.430 ±.004	.040 ±.008	1.173 ±.010	.119 ±.008	.041 ±.010	1.065 ±.015	.057 ±.015
0.25	0	0	1	No	1	0.1	0	.077 ±.021	.111 ±.027	.976 ±.015	.546 ±.025	.429 ±.011	.000 ±.000	.429 ±.011	.067 ±.018	1.172 ±.022	.145 ±.020	.076 ±.021	1.070 ±.036	.113 ±.029
0.25	0	0	1	Yes	1	0.01	0	.131 ±.009	.164 ±.008	.998 ±.007	.586 ±.011	.412 ±.005	.000 ±.000	.412 ±.005	.124 ±.005	1.101 ±.004	.211 ±.002	.131 ±.010	1.052 ±.017	.165 ±.010
0.25	0	0	1	Yes	1	0.05	0	.150 ±.015	.168 ±.011	1.007 ±.011	.608 ±.021	.399 ±.013	.000 ±.000	.399 ±.013	.139 ±.020	1.104 ±.013	.214 ±.010	.150 ±.019	1.014 ±.030	.167 ±.015
0.25	0	0	1	Yes	1	0.1	0	.156 ±.019	.162 ±.014	1.021 ±.016	.631 ±.026	.389 ±.013	.000 ±.000	.389 ±.013	.147 ±.022	1.103 ±.022	.216 ±.011	.155 ±.021	.983 ±.025	.159 ±.016
0.5	0	0	1	No	1	0.01	0	.015 ±.005	.022 ±.005	1.005 ±.002	.611 ±.005	.394 ±.003	.000 ±.000	.394 ±.003	.037 ±.010	1.065 ±.006	.207 ±.004	.008 ±.003	.991 ±.004	.011 ±.003

Appendix B

0.5	0	0	1	No	1	0.05	0	.035 ±.006	.044 ±.008	1.006 ±.006	.612 ±.011	.394 ±.005	.000 ±.000	.394 ±.005	.044 ±.008	1.064 ±.008	.210 ±.006	.033 ±.006	.992 ±.013	.041 ±.008
0.5	0	0	1	No	1	0.1	0	.061 ±.013	.073 ±.016	1.016 ±.014	.627 ±.024	.389 ±.011	.000 ±.000	.389 ±.011	.067 ±.017	1.063 ±.018	.217 ±.017	.062 ±.015	.978 ±.026	.073 ±.019
0.5	0	0	1	Yes	1	0.01	0	.355 ±.062	.335 ±.025	1.058 ±.048	.655 ±.075	.402 ±.031	.000 ±.000	.402 ±.031	.266 ±.070	1.192 ±.060	.364 ±.014	.361 ±.060	1.029 ±.080	.312 ±.039
0.5	0	0	1	Yes	1	0.05	0	.343 ±.049	.360 ±.046	1.055 ±.041	.651 ±.061	.404 ±.024	.000 ±.000	.404 ±.024	.267 ±.060	1.182 ±.049	.361 ±.021	.342 ±.056	1.042 ±.087	.349 ±.065
0.5	0	0	1	Yes	1	0.1	0	.336 ±.046	.344 ±.044	1.061 ±.036	.678 ±.062	.383 ±.031	.000 ±.000	.383 ±.031	.307 ±.050	1.119 ±.065	.360 ±.027	.343 ±.051	1.032 ±.072	.340 ±.053
1	0	0	1	No	1	0.01	0	.175 ±.034	.116 ±.009	1.051 ±.013	.734 ±.030	.318 ±.017	.000 ±.000	.318 ±.017	.529 ±.056	.928 ±.015	.450 ±.009	.040 ±.012	.949 ±.010	.041 ±.006
1	0	0	1	No	1	0.05	0	.180 ±.036	.119 ±.012	1.055 ±.014	.741 ±.033	.314 ±.019	.000 ±.000	.314 ±.019	.532 ±.063	.927 ±.016	.450 ±.013	.054 ±.015	.944 ±.015	.054 ±.008
1	0	0	1	No	1	0.1	0	.194 ±.045	.126 ±.014	1.065 ±.025	.755 ±.052	.309 ±.027	.000 ±.000	.309 ±.027	.524 ±.079	.927 ±.023	.452 ±.020	.077 ±.015	.934 ±.027	.074 ±.013
1	0	0	1	Yes	1	0.01	0	.920 ±.259	.637 ±.045	1.202 ±.099	.887 ±.145	.315 ±.067	.000 ±.000	.315 ±.067	.698 ±.214	1.239 ±.055	.654 ±.031	.978 ±.299	1.073 ±.078	.623 ±.068
1	0	0	1	Yes	1	0.05	0	.906 ±.225	.642 ±.047	1.197 ±.093	.890 ±.156	.307 ±.074	.000 ±.000	.307 ±.074	.716 ±.210	1.228 ±.064	.657 ±.027	.924 ±.270	1.079 ±.095	.632 ±.059
1	0	0	1	Yes	1	0.1	0	.901 ±.252	.646 ±.038	1.201 ±.065	.894 ±.108	.307 ±.058	.000 ±.000	.307 ±.058	.708 ±.242	1.219 ±.070	.662 ±.031	.909 ±.265	1.081 ±.067	.636 ±.046
2	0	0	1	No	1	0.01	0	12.61 8 ±4.55 9	.370 ±.005	2.155 ±.561	10.19 3 ±3.10 5	- 8.038 ±2.55 2	.000 ±.000	- 8.038 ±2.55 2	103.6 96 ±16.2 14	.984 ±.014	1.420 ±.015	.830 ±.283	.724 ±.036	.277 ±.010
2	0	0	1	No	1	0.05	0	14.21 8 ±7.20 1	.367 ±.010	2.378 ±1.00 4	11.23 7 ±4.68 3	- 8.860 ±3.70 0	.000 ±.000	- 8.860 ±3.70 0	108.5 23 ±23.6 54	.987 ±.021	1.423 ±.018	.948 ±.464	.710 ±.052	.280 ±.015
2	0	0	1	No	1	0.1	0	###	.437 ±.319	###	###	###	.000 ±.000	###	###	1.079 ±.383	1.435 ±.062	###	.728 ±.131	.350 ±.277
2	0	0	1	Yes	1	0.01	0	10.66 8 ±3.36 2	1.069 ±.065	3.178 ±.844	4.054 ±1.33 3	-.876 ±.846	.000 ±.000	-.876 ±.846	4.461 ±4.31 0	1.457 ±.060	1.176 ±.045	11.10 2 ±4.34 9	1.177 ±.132	1.024 ±.066
2	0	0	1	Yes	1	0.05	0	14.20 9 ±13.6 66	1.026 ±.119	4.321 ±3.23 7	5.545 ±4.75 3	- 1.225 ±1.63 2	.000 ±.000	- 1.225 ±1.63 2	11.84 6 ±17.8 05	1.427 ±.078	1.145 ±.074	10.28 0 ±6.18 6	1.183 ±.139	.989 ±.126
2	0	0	1	Yes	1	0.1	0	55.98 7 ±136. 480	.996 ±.132	11.84 6 ±24.9 26	17.46 5 ±43.1 84	- 5.619 ±18.8 13	.000 ±.000	- 5.619 ±18.8 13	14.65 4 ±19.3 56	1.406 ±.082	1.117 ±.080	62.23 3 ±162. 430	1.142 ±.207	.953 ±.171

Posted-price simple condition, optimal model

σy	σp	M	Est	Ns	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
0	0	No	Yes	10	0	.004 ±.001	.005 ±.001	1.000 ±.000	.600 ±.001	.400 ±.001	.005 ±.000	.395 ±.001	.129 ±.001	1.107 ±.001	.262 ±.002	.004 ±.001	1.000 ±.001	.005 ±.001
0	0	Yes	Yes	10	0	.003 ±.001	.012 ±.002	1.142 ±.000	.298 ±.000	.844 ±.000	.003 ±.000	.841 ±.000	.062 ±.000	2.212 ±.002	.214 ±.001	.003 ±.001	2.299 ±.003	.012 ±.002
0	0.1	No	Yes	10	0	.038 ±.005	.050 ±.007	1.001 ±.003	.605 ±.005	.396 ±.007	.054 ±.003	.342 ±.008	.135 ±.008	1.097 ±.012	.260 ±.016	.038 ±.007	.993 ±.013	.049 ±.009
0	0.1	Yes	Yes	10	0	.035 ±.004	.118 ±.015	1.142 ±.005	.305 ±.005	.837 ±.001	.027 ±.002	.810 ±.002	.067 ±.007	2.170 ±.026	.233 ±.025	.035 ±.005	2.249 ±.035	.114 ±.016
0	0	No	No	10	0	.008 ±.001	.005 ±.001	1.000 ±.000	.600 ±.001	.400 ±.001	.006 ±.000	.394 ±.001	.008 ±.001	1.000 ±.001	.005 ±.001	.008 ±.002	1.000 ±.001	.005 ±.001
0	0	No	Yes	10	0	.008 ±.001	.005 ±.001	1.000 ±.000	.600 ±.000	.400 ±.001	.006 ±.000	.394 ±.001	.129 ±.001	1.108 ±.001	.263 ±.002	.008 ±.002	1.000 ±.001	.005 ±.001
0	0	Yes	No	10	0	.008 ±.001	.012 ±.001	1.142 ±.000	.298 ±.000	.844 ±.000	.004 ±.000	.840 ±.000	.007 ±.002	2.299 ±.003	.011 ±.002	.008 ±.001	2.300 ±.004	.012 ±.001
0	0	Yes	Yes	10	0	.008 ±.001	.011 ±.001	1.142 ±.000	.298 ±.000	.844 ±.000	.004 ±.000	.840 ±.000	.063 ±.001	2.213 ±.002	.214 ±.002	.008 ±.001	2.299 ±.003	.011 ±.002
0	0.1	No	No	10	0	.020 ±.002	.025 ±.003	.999 ±.001	.603 ±.002	.397 ±.003	.027 ±.002	.370 ±.004	.019 ±.003	.996 ±.006	.024 ±.004	.021 ±.003	.996 ±.005	.026 ±.004
0	0.1	No	Yes	10	0	.020 ±.002	.024 ±.002	1.001 ±.001	.600 ±.002	.400 ±.003	.027 ±.002	.374 ±.004	.130 ±.003	1.106 ±.006	.262 ±.010	.019 ±.003	1.000 ±.004	.024 ±.003
0	0.1	Yes	No	10	0	.018 ±.002	.057 ±.008	1.141 ±.002	.299 ±.002	.842 ±.000	.014 ±.001	.829 ±.001	.017 ±.003	2.295 ±.010	.055 ±.010	.017 ±.002	2.295 ±.015	.057 ±.008
0	0.1	Yes	Yes	10	0	.018 ±.002	.060 ±.007	1.143 ±.002	.300 ±.002	.842 ±.000	.014 ±.001	.828 ±.001	.063 ±.002	2.194 ±.014	.213 ±.009	.019 ±.003	2.287 ±.011	.062 ±.008
0.1	0	No	No	10	0	.033 ±.005	.005 ±.001	1.000 ±.000	.600 ±.000	.400 ±.001	.013 ±.001	.387 ±.001	.034 ±.008	1.000 ±.001	.005 ±.001	.033 ±.006	1.000 ±.001	.005 ±.001
0.1	0	No	Yes	10	0	.035 ±.006	.005 ±.001	1.000 ±.000	.600 ±.000	.400 ±.000	.013 ±.001	.387 ±.001	.134 ±.003	1.108 ±.002	.263 ±.002	.035 ±.007	1.000 ±.001	.005 ±.001
0.1	0	Yes	No	10	0	.037 ±.007	.011 ±.002	1.142 ±.000	.298 ±.000	.844 ±.000	.012 ±.001	.832 ±.001	.039 ±.008	2.299 ±.003	.011 ±.002	.035 ±.007	2.299 ±.003	.011 ±.002
0.1	0	Yes	Yes	10	0	.036 ±.005	.012 ±.001	1.142 ±.000	.298 ±.000	.844 ±.000	.012 ±.001	.832 ±.001	.072 ±.004	2.211 ±.003	.213 ±.002	.037 ±.005	2.299 ±.002	.012 ±.002
0.1	0.1	No	No	10	0	.040 ±.005	.025 ±.003	1.000 ±.001	.602 ±.002	.398 ±.003	.029 ±.002	.369 ±.005	.039 ±.007	.997 ±.006	.026 ±.004	.041 ±.008	.997 ±.006	.025 ±.004
0.1	0.1	No	Yes	10	0	.038 ±.004	.024 ±.002	1.001 ±.001	.600 ±.002	.401 ±.002	.030 ±.002	.370 ±.003	.134 ±.004	1.109 ±.008	.264 ±.012	.039 ±.008	1.000 ±.006	.023 ±.003
0.1	0.1	Yes	No	10	0	.040 ±.005	.058 ±.006	1.142 ±.002	.300 ±.002	.842 ±.000	.018 ±.001	.824 ±.002	.038 ±.007	2.289 ±.013	.058 ±.012	.041 ±.006	2.289 ±.011	.058 ±.007
0.1	0.1	Yes	Yes	10	0	.039 ±.007	.058 ±.007	1.142 ±.002	.300 ±.002	.842 ±.000	.018 ±.001	.825 ±.002	.072 ±.006	2.202 ±.013	.216 ±.011	.038 ±.008	2.286 ±.014	.058 ±.009
0.1	0	No	No	10	0	.068 ±.008	.005 ±.001	1.000 ±.000	.600 ±.000	.400 ±.001	.025 ±.002	.375 ±.002	.067 ±.016	1.000 ±.001	.005 ±.001	.071 ±.010	1.000 ±.001	.005 ±.001
0.1	0	No	Yes	10	0	.069 ±.013	.005 ±.000	1.000 ±.000	.600 ±.000	.400 ±.001	.025 ±.002	.375 ±.002	.144 ±.007	1.107 ±.001	.262 ±.002	.072 ±.017	1.000 ±.001	.005 ±.001
0.1	0	Yes	Yes	10	0	.075 ±.011	.012 ±.001	1.142 ±.000	.298 ±.000	.844 ±.000	.024 ±.002	.819 ±.002	.098 ±.010	2.212 ±.003	.214 ±.001	.074 ±.012	2.299 ±.003	.012 ±.002
0.1	0.1	No	No	10	0	.075 ±.010	.024 ±.003	.999 ±.001	.603 ±.002	.397 ±.002	.036 ±.003	.361 ±.003	.073 ±.015	.999 ±.004	.024 ±.004	.076 ±.012	.994 ±.006	.024 ±.003
0.1	0.1	Yes	Yes	10	0	.074 ±.011	.058 ±.007	1.142 ±.002	.300 ±.002	.842 ±.000	.028 ±.003	.814 ±.003	.095 ±.012	2.204 ±.010	.214 ±.008	.075 ±.012	2.287 ±.012	.058 ±.009
0.1	0.1	No	Yes	10	0	.083 ±.012	.053 ±.006	1.001 ±.002	.606 ±.004	.395 ±.006	.059 ±.006	.336 ±.009	.150 ±.013	1.098 ±.014	.257 ±.022	.080 ±.016	.994 ±.011	.054 ±.008
0.1	0.1	Yes	Yes	10	0	.081 ±.014	.113 ±.017	1.141 ±.004	.303 ±.004	.837 ±.001	.036 ±.002	.802 ±.003	.098 ±.013	2.171 ±.022	.237 ±.023	.081 ±.015	2.268 ±.027	.110 ±.021
0.1	0	Yes	No	10	0	.070 ±.009	.011 ±.001	1.142 ±.000	.298 ±.001	.844 ±.000	.024 ±.001	.820 ±.001	.075 ±.014	2.300 ±.003	.011 ±.001	.067 ±.012	2.300 ±.003	.012 ±.002
0.1	0.1	No	Yes	10	0	.069 ±.012	.027 ±.004	1.000 ±.002	.602 ±.003	.398 ±.004	.036 ±.003	.362 ±.004	.148 ±.010	1.102 ±.008	.263 ±.013	.070 ±.015	.999 ±.005	.027 ±.005
0.1	0.1	Yes	No	10	0	.067 ±.012	.058 ±.010	1.141 ±.002	.299 ±.002	.842 ±.000	.027 ±.002	.815 ±.002	.070 ±.014	2.291 ±.011	.054 ±.011	.066 ±.013	2.292 ±.014	.060 ±.012
0.1	0.1	No	No	10	0	.079 ±.013	.051 ±.008	.999 ±.002	.608 ±.004	.391 ±.005	.057 ±.004	.334 ±.007	.079 ±.014	.994 ±.011	.049 ±.010	.078 ±.016	.985 ±.008	.052 ±.010
0.1	0.1	Yes	No	10	0	.076 ±.010	.116 ±.014	1.139 ±.004	.302 ±.004	.837 ±.001	.035 ±.002	.802 ±.003	.075 ±.010	2.266 ±.026	.111 ±.013	.078 ±.014	2.265 ±.029	.116 ±.017

Appendix B

Posted-price simple condition, behavioral model

τ	σy	bo	b1	b2	σp	Discrete	a1	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
1	0	1	0.25	0	0.01	No	0.5	0	.007 ±.002	.013 ±.004	1.000 ±.001	.600 ±.001	.400 ±.002	.006 ±.000	.394 ±.002	.123 ±.003	-.105 ±.011	.146 ±.015	.006 ±.002	.001 ±.005	.011 ±.003
1	0	1	0.25	0	0.05	No	0.5	0	.034 ±.009	.057 ±.021	.998 ±.002	.603 ±.004	.394 ±.006	.027 ±.003	.367 ±.007	.129 ±.010	-.094 ±.019	.159 ±.027	.032 ±.007	.017 ±.022	.052 ±.013
1	0	1	0.25	0	0.1	No	0.5	0	.073 ±.012	.131 ±.027	.993 ±.007	.619 ±.012	.374 ±.019	.056 ±.004	.319 ±.021	.151 ±.017	-.038 ±.054	.206 ±.044	.070 ±.016	.061 ±.046	.123 ±.043
1	0	1	0.25	0	0.01	Yes	0.5	0	.253 ±.052	.682 ±.425	.979 ±.014	.665 ±.030	.314 ±.044	.103 ±.006	.211 ±.048	.278 ±.049	.220 ±.186	.652 ±.333	.241 ±.060	.279 ±.278	.598 ±.443
1	0	1	0.25	0	0.05	Yes	0.5	0	.305 ±.121	.908 ±.555	.943 ±.140	.723 ±.180	.220 ±.319	.120 ±.010	.100 ±.317	.292 ±.093	.245 ±.233	.688 ±.392	.274 ±.091	.438 ±.382	.892 ±.736
1	0	1	0.25	0	0.1	Yes	0.5	0	.273 ±.139	.673 ±.329	.955 ±.132	.706 ±.170	.248 ±.302	.137 ±.010	.111 ±.307	.283 ±.069	.236 ±.203	.646 ±.448	.232 ±.052	.312 ±.227	.624 ±.381
1	0	1	0.5	0	0.01	Yes	0.5	0	.544 ±.291	3.149 ±3.314	.584 ±.417	1.187 ±.447	-.604 ±.864	.153 ±.084	-.756 ±.781	.712 ±.157	1.268 ±1.505	3.115 ±2.850	.390 ±.332	.965 ±1.828	2.721 ±3.495
1	0	1	0.5	0	0.05	Yes	0.5	0	.656 ±.360	4.836 ±6.765	.580 ±.397	1.269 ±.473	-.689 ±.868	.293 ±.108	-.982 ±.909	.844 ±.174	3.495 ±6.326	5.344 ±5.868	.536 ±.418	5.486 ±14.680	2.851 ±4.220
1	0	1	0.5	0	0.1	Yes	0.5	0	.335 ±.340	1.753 ±2.893	.191 ±.340	1.821 ±.458	-1.630 ±.792	.526 ±.238	-2.156 ±.948	1.000 ±.182	4.692 ±6.615	4.469 ±4.850	.155 ±.224	9.136 ±11.835	.299 ±.780
1	0	1	1	0	0.01	No	0.5	4	1.995 ±1.166	19.791 ±30.974	.361 ±.342	3.073 ±1.350	-2.712 ±1.688	1.999 ±1.177	-4.711 ±2.860	1.934 ±1.468	3.753 ±7.136	17.478 ±26.947	.968 ±1.304	-.052 ±8.342	8.119 ±29.516
1	0	1	1	0	0.05	No	0.5	1	.989 ±1.075	5.630 ±9.873	.583 ±.450	2.030 ±1.540	-1.448 ±1.974	1.196 ±1.257	-2.643 ±3.226	1.248 ±1.363	1.457 ±3.676	6.515 ±11.656	.543 ±1.045	-.051 ±2.124	1.837 ±5.317
1	0	1	1	0	0.1	No	0.5	0	.322 ±.528	.882 ±1.879	.837 ±.363	1.142 ±1.149	-.306 ±1.502	.507 ±.917	-.813 ±2.410	.595 ±.965	.241 ±.331	.643 ±.764	.260 ±.548	.376 ±.768	.942 ±2.316
1	0	1	1	0	0.01	Yes	0.5	0	.108 ±.211	.000 ±.000	.008 ±.025	2.079 ±.490	-2.071 ±.497	.460 ±.695	-2.531 ±1.189	1.247 ±.385	20.268 ±62.744	25.631 ±57.120	.003 ±.005	33.152 ±116.945	.000 ±.000
1	0	1	1	0	0.05	Yes	0.5	1	.103 ±.172	1.257 ±5.478	.003 ±.011	2.489 ±.444	-2.486 ±.444	1.063 ±.626	-3.549 ±1.070	1.455 ±.272	20.939 ±26.574	18.357 ±17.766	.025 ±.020	29.538 ±40.420	.000 ±.000
1	0	1	1	0	0.1	Yes	0.5	0	.111 ±.076	.000 ±.000	.000 ±.000	2.860 ±.615	-2.860 ±.615	1.571 ±.836	-4.432 ±1.451	1.547 ±.397	13.030 ±17.268	7.750 ±9.866	.102 ±.076	17.984 ±23.393	.000 ±.000
1	0	1	0	0.1	0.01	No	0.5	0	.014 ±.004	.013 ±.002	1.148 ±.002	.306 ±.002	.843 ±.001	.004 ±.000	.838 ±.001	.055 ±.003	-.417 ±.017	.056 ±.050	.006 ±.002	-.422 ±.004	.012 ±.002
1	0	1	0	0.1	0.05	No	0.5	0	.036 ±.008	.064 ±.024	1.152 ±.009	.317 ±.012	.836 ±.004	.015 ±.001	.820 ±.005	.055 ±.010	-.407 ±.029	.077 ±.038	.032 ±.007	-.431 ±.023	.057 ±.010
1	0	1	0	0.1	0.1	No	0.5	0	.062 ±.016	.125 ±.022	1.154 ±.007	.347 ±.022	.807 ±.020	.032 ±.004	.775 ±.024	.057 ±.024	-.415 ±.032	.120 ±.028	.056 ±.016	-.410 ±.032	.129 ±.027
1	0	1	0	0.1	0.01	Yes	0.5	0	.067 ±.004	.023 ±.003	1.132 ±.007	.297 ±.007	.835 ±.001	.017 ±.001	.818 ±.002	.093 ±.009	-.406 ±.020	.050 ±.047	.067 ±.004	-.416 ±.007	.022 ±.004
1	0	1	0	0.1	0.05	Yes	0.5	0	.076 ±.005	.062 ±.008	1.132 ±.005	.300 ±.004	.833 ±.002	.023 ±.001	.810 ±.003	.097 ±.009	-.423 ±.026	.069 ±.021	.076 ±.006	-.419 ±.012	.061 ±.007
1	0	1	0	0.1	0.1	Yes	0.5	0	.088 ±.009	.123 ±.016	1.134 ±.006	.307 ±.007	.827 ±.003	.034 ±.002	.793 ±.004	.103 ±.009	-.410 ±.036	.131 ±.032	.087 ±.011	-.407 ±.023	.124 ±.024
1	0	1	0	0.2	0.01	No	0.5	0	.006 ±.001	.012 ±.003	1.143 ±.002	.299 ±.002	.844 ±.000	.003 ±.000	.840 ±.000	.063 ±.006	-.420 ±.011	.034 ±.037	.006 ±.002	-.422 ±.003	.012 ±.003
1	0	1	0	0.2	0.05	No	0.5	0	.032 ±.006	.061 ±.010	1.144 ±.008	.304 ±.009	.840 ±.001	.014 ±.001	.826 ±.002	.069 ±.010	-.418 ±.018	.082 ±.038	.030 ±.007	-.431 ±.023	.059 ±.009
1	0	1	0	0.2	0.1	No	0.5	0	.058 ±.014	.121 ±.021	1.151 ±.011	.327 ±.018	.824 ±.010	.031 ±.003	.793 ±.012	.077 ±.012	-.418 ±.042	.126 ±.025	.058 ±.016	-.414 ±.033	.119 ±.028
1	0	1	0	0.2	0.01	Yes	0.5	0	.136 ±.015	.039 ±.012	1.114 ±.024	.309 ±.017	.805 ±.007	.033 ±.003	.772 ±.010	.135 ±.019	-.401 ±.021	.055 ±.037	.136 ±.016	-.407 ±.013	.038 ±.014
1	0	1	0	0.2	0.05	Yes	0.5	0	.146 ±.009	.074 ±.010	1.105 ±.007	.299 ±.007	.805 ±.003	.039 ±.002	.767 ±.005	.154 ±.012	-.399 ±.019	.078 ±.019	.146 ±.012	-.406 ±.020	.074 ±.015
1	0	1	0	0.2	0.1	Yes	0.5	0	.155 ±.012	.123 ±.021	1.101 ±.010	.302 ±.007	.799 ±.005	.046 ±.003	.753 ±.007	.153 ±.013	-.408 ±.037	.126 ±.031	.155 ±.015	-.395 ±.038	.123 ±.030
1	0	1	0	0.5	0.01	No	0.5	0	.006 ±.001	.012 ±.002	1.142 ±.001	.298 ±.001	.844 ±.000	.003 ±.000	.841 ±.000	.068 ±.006	-.412 ±.014	.039 ±.042	.006 ±.001	-.421 ±.003	.012 ±.002
1	0	1	0	0.5	0.05	No	0.5	0	.030 ±.006	.063 ±.011	1.142 ±.003	.300 ±.003	.841 ±.001	.015 ±.001	.827 ±.002	.072 ±.008	-.413 ±.021	.077 ±.037	.030 ±.007	-.424 ±.025	.064 ±.016

1	0	1	0	0.5	0.1	No	0.5	0	.061 ±.012	.119 ±.019	1.141 ±.006	.307 ±.007	.834 ±.003	.030 ±.002	.804 ±.004	.090 ±.014	-.409 ±.031	.124 ±.026	.060 ±.014	-.411 ±.030	.118 ±.024
1	0	1	0	0.5	0.01	Yes	0.5	20	###	###	###	###	###	###	###	###	###	###	###	###	###
1	0	1	0	0.5	0.05	Yes	0.5	20	###	###	###	###	###	###	###	###	###	###	###	###	###
1	0	1	0	0.5	0.1	Yes	0.5	20	###	###	###	###	###	###	###	.319 ±.011	-.359 ±.018	.163 ±.011	###	###	###
1	0	0	0.25	0	0.01	No	0.5	0	.015 ±.021	.040 ±.070	.997 ±.013	.625 ±.017	.372 ±.026	.105 ±.048	.267 ±.051	.141 ±.012	-.097 ±.033	.159 ±.073	.013 ±.022	-.003 ±.010	.026 ±.030
1	0	0	0.25	0	0.05	No	0.5	0	.071 ±.039	.189 ±.193	1.024 ±.025	.634 ±.040	.390 ±.044	.207 ±.093	.183 ±.110	.162 ±.040	-.089 ±.050	.249 ±.186	.066 ±.035	.003 ±.057	.158 ±.152
1	0	0	0.25	0	0.1	No	0.5	1	.146 ±.060	.321 ±.255	1.052 ±.021	.640 ±.028	.412 ±.020	.281 ±.083	.131 ±.090	.178 ±.030	-.083 ±.108	.247 ±.151	.154 ±.080	.015 ±.082	.356 ±.313
1	0	0	0.25	0	0.01	Yes	0.5	0	.049 ±.012	.312 ±.273	1.052 ±.021	.647 ±.036	.405 ±.036	.297 ±.091	.107 ±.108	.162 ±.017	.004 ±.068	.344 ±.200	.043 ±.010	.027 ±.119	.306 ±.286
1	0	0	0.25	0	0.05	Yes	0.5	0	.147 ±[illegible]	.229 ±.275	1.062 ±.015	.698 ±.064	.365 ±.052	.371 ±.100	-.006 ±.149	.196 ±.027	.042 ±.071	.269 ±.155	.132 ±.046	.038 ±.060	.220 ±.317
1	0	0	0.25	0	0.1	Yes	0.5	0	.379 ±.548	.384 ±.471	1.042 ±.050	.809 ±.294	.233 ±.336	.438 ±.234	-.205 ±.560	.219 ±.057	.078 ±.087	.396 ±.388	.395 ±.589	.074 ±.082	.320 ±.480
1	0	0	0.5	0	0.01	No	0.5	4	.258 ±.801	.086 ±.203	.939 ±.261	.917 ±1.002	.022 ±1.262	.384 ±.802	-.362 ±2.060	.369 ±.888	-.017 ±.102	.288 ±.393	.235 ±.913	.026 ±.088	.084 ±.240
1	0	0	0.5	0	0.05	No	0.5	2	.274 ±.823	.259 ±.461	.946 ±.266	.919 ±.983	.027 ±1.248	.434 ±.793	-.407 ±2.038	.331 ±.733	-.015 ±.139	.396 ±.703	.261 ±.861	.046 ±.131	.258 ±.540
1	0	0	0.5	0	0.1	No	0.5	1	1.066 ±1.428	.403 ±.370	.847 ±.326	1.370 ±1.209	-.524 ±1.533	.793 ±.895	-1.317 ±2.427	.554 ±.897	-.099 ±.258	.517 ±.447	.651 ±1.171	-.196 ±.873	.271 ±.292
1	0	0	0.5	0	0.01	Yes	0.5	0	.131 ±.090	.440 ±.417	1.029 ±.037	.684 ±.080	.345 ±.096	.268 ±.106	.078 ±.166	.249 ±.077	.075 ±.044	.562 ±.546	.112 ±.086	-.004 ±.269	.438 ±.419
1	0	0	0.5	0	0.05	Yes	0.5	0	1.053 ±1.196	3.256 ±4.890	.918 ±.251	1.404 ±.978	-.486 ±1.229	.911 ±.700	-1.397 ±1.927	.410 ±.150	.708 ±1.947	1.890 ±3.788	.750 ±.967	.639 ±2.186	3.421 ±5.204
1	0	0	0.5	0	0.1	Yes	0.5	0	1.816 ±1.158	1.201 ±2.243	.503 ±.374	2.798 ±1.198	-2.295 ±1.558	1.890 ±.909	-4.185 ±2.459	1.229 ±1.068	.918 ±1.737	2.793 ±4.554	1.248 ±1.240	-.557 ±1.919	.300 ±.895
1	0	0	1	0	0.01	No	0.5	7	.062 ±.172	.021 ±.016	.788 ±.449	1.494 ±1.734	-.706 ±2.183	.873 ±1.427	-1.579 ±3.610	.868 ±1.395	-.235 ±.927	.325 ±.351	.004 ±.002	-.262 ±.960	.015 ±.008
1	0	0	1	0	0.05	No	0.5	5	.889 ±1.312	.103 ±.166	.284 ±.424	3.277 ±1.555	-2.993 ±1.962	2.243 ±1.317	-5.236 ±3.265	1.979 ±1.558	-.043 ±.766	1.145 ±1.269	.771 ±1.506	-.336 ±1.141	.106 ±.196
1	0	0	1	0	0.1	No	0.5	1	1.322 ±1.206	.263 ±.521	.101 ±.167	4.184 ±.640	-4.083 ±.807	2.994 ±.557	-7.077 ±1.362	2.964 ±1.179	-.249 ±.884	.783 ±.607	.340 ±.925	-.759 ±1.532	.068 ±.226
1	0	0	1	0	0.01	Yes	0.5	0	.854 ±1.034	5.909 ±24.624	.586 ±.467	2.175 ±1.627	-1.589 ±2.065	1.417 ±1.243	-3.006 ±3.276	1.166 ±1.195	3.207 ±8.415	12.001 ±27.906	.512 ±.817	1.841 ±9.864	6.996 ±29.893
1	0	0	1	0	0.05	Yes	0.5	0	.460 ±.717	.025 ±.059	.024 ±.052	4.199 ±.667	-4.175 ±.671	2.975 ±.782	-7.150 ±1.451	2.739 ±.675	1.217 ±5.093	4.584 ±13.004	.031 ±.063	-.474 ±.897	.000 ±.000
1	0	0	1	0	0.1	Yes	0.5	0	.541 ±.866	.056 ±.208	.038 ±.126	3.638 ±.818	-3.600 ±.859	2.383 ±.905	-5.983 ±1.749	2.091 ±.859	-.181 ±.538	.262 ±.231	.317 ±.674	-.255 ±.564	.049 ±.220
1	0	1	0.5	0	0.01	No	0.5	0	.016 ±.039	.021 ±.039	1.000 ±.001	.601 ±.004	.399 ±.005	.008 ±.010	.391 ±.015	.136 ±.069	-.041 ±.039	.155 ±.142	.008 ±.005	.002 ±.005	.013 ±.009
1	0	1	0.5	0	0.05	No	0.5	0	.033 ±.007	.059 ±.012	1.000 ±.002	.603 ±.005	.397 ±.007	.029 ±.003	.368 ±.008	.128 ±.008	-.034 ±.020	.133 ±.036	.033 ±.008	.010 ±.019	.059 ±.015
1	0	1	0.5	0	0.1	No	0.5	0	.068 ±.018	.114 ±.037	1.000 ±.005	.607 ±.010	.393 ±.015	.060 ±.006	.333 ±.018	.140 ±.015	-.015 ±.029	.168 ±.042	.070 ±.021	.035 ±.039	.114 ±.047
1	0.05	1	0.5	0	0.01	No	0.5	0	.036 ±.007	.011 ±.003	1.000 ±.000	.600 ±.000	.400 ±.001	.014 ±.001	.386 ±.001	.126 ±.005	-.052 ±.014	.129 ±.030	.035 ±.009	.000 ±.003	.011 ±.003
1	0.05	1	0.5	0	0.05	No	0.5	0	.047 ±.007	.059 ±.012	.999 ±.002	.604 ±.004	.395 ±.006	.032 ±.002	.363 ±.007	.130 ±.007	-.039 ±.017	.143 ±.026	.047 ±.008	.009 ±.016	.054 ±.013
1	0.05	1	0.5	0	0.1	No	0.5	0	.076 ±.013	.127 ±.032	.998 ±.003	.612 ±.006	.385 ±.009	.061 ±.004	.324 ±.010	.149 ±.015	-.004 ±.037	.189 ±.057	.074 ±.013	.029 ±.034	.128 ±.032
1	0.1	1	0.5	0	0.01	No	0.5	0	.076 ±.011	.012 ±.003	1.000 ±.000	.600 ±.001	.400 ±.001	.025 ±.002	.376 ±.002	.139 ±.008	-.052 ±.020	.134 ±.034	.079 ±.014	.000 ±.003	.012 ±.003
1	0.1	1	0.5	0	0.05	No	0.5	0	.081 ±.009	.058 ±.008	1.000 ±.002	.603 ±.004	.397 ±.006	.039 ±.003	.358 ±.006	.150 ±.007	-.045 ±.027	.151 ±.038	.081 ±.011	.009 ±.017	.056 ±.011

1	0.1	1	0.5	0	0.1	No	0.5	0	.101	.128	.997	.613	.384	.066	.318	.158	-.008	.183	.102	.051	.126
									±.011	±.036	±.003	±.006	±.010	±.007	±.013	±.019	±.038	±.041	±.017	±.045	±.051
2	0	1	0.5	0	0.01	No	0.5	0	.008	.011	1.000	.601	.399	.007	.392	.134	-.055	.142	.006	.002	.010
									±.004	±.003	±.000	±.001	±.001	±.001	±.002	±.004	±.017	±.036	±.002	±.004	±.002
2	0	1	0.5	0	0.05	No	0.5	0	.030	.058	.999	.604	.396	.032	.364	.139	-.044	.146	.030	.014	.057
									±.006	±.012	±.002	±.003	±.005	±.002	±.004	±.007	±.019	±.034	±.007	±.015	±.015
2	0	1	0.5	0	0.1	No	0.5	0	.066	.129	.999	.609	.390	.065	.325	.153	-.007	.174	.066	.039	.132
									±.014	±.031	±.004	±.008	±.012	±.005	±.015	±.022	±.036	±.043	±.015	±.030	±.042
2	0.05	1	0.5	0	0.01	No	0.5	0	.030	.012	1.000	.600	.400	.015	.384	.136	-.051	.126	.029	.001	.011
									±.003	±.003	±.000	±.001	±.001	±.001	±.001	±.004	±.016	±.033	±.005	±.003	±.003
2	0.05	1	0.5	0	0.05	No	0.5	0	.042	.060	1.000	.603	.397	.035	.362	.144	-.036	.141	.039	.010	.059
									±.007	±.014	±.002	±.004	±.006	±.003	±.006	±.008	±.016	±.035	±.006	±.012	±.011
2	0.05	1	0.5	0	0.1	No	0.5	0	.064	.119	.999	.610	.389	.065	.324	.152	-.026	.188	.063	.042	.125
									±.015	±.028	±.005	±.008	±.013	±.007	±.014	±.018	±.038	±.050	±.016	±.037	±.034
2	0.1	1	0.5	0	0.01	No	0.5	0	.059	.013	1.000	.600	.400	.029	.371	.142	-.048	.129	.061	.000	.014
									±.010	±.004	±.000	±.001	±.001	±.003	±.004	±.008	±.014	±.027	±.013	±.003	±.005
2	0.1	1	0.5	0	0.05	No	0.5	0	.068	.058	.999	.603	.396	.043	.353	.151	-.030	.148	.069	.007	.055
									±.010	±.016	±.002	±.004	±.005	±.004	±.007	±.009	±.026	±.040	±.010	±.014	±.014
2	0.1	1	0.5	0	0.1	No	0.5	0	.080	.126	.997	.614	.382	.073	.310	.167	-.011	.185	.081	.053	.124
									±.012	±.032	±.003	±.006	±.008	±.008	±.012	±.019	±.045	±.066	±.013	±.033	±.026
1	0.1	1	0	0.2	0.01	No	0.5	0	.069	.013	1.143	.299	.844	.023	.820	.093	-.419	.033	.068	-.421	.012
									±.012	±.002	±.001	±.001	±.000	±.001	±.001	±.014	±.016	±.042	±.014	±.004	±.003
1	0.1	1	0	0.2	0.1	No	0.5	0	.100	.305	1.146	.320	.827	.039	.788	.107	-.427	.289	.095	-.419	.277
									±.014	±.226	±.010	±.014	±.007	±.003	±.008	±.022	±.087	±.231	±.016	±.064	±.217

Note: a_2 and σ_x are zero and $d=\sigma_p/2$ in all simulations.

Posted-price complex condition, optimal model

σ_y	σ_p	a	M	Est	Ns	Crash	S(Y) t>10	S(P) t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S(Y) t≤20	P t≤20	S(P) t≤20	S(Y) t>20	P t>20	S(P) t>20
0	0	1	No	No	10	0	.005 ±.001	.010 ±.001	1.000 ±.000	.600 ±.001	.400 ±.000	.005 ±.000	.395 ±.000	.005 ±.001	1.080 ±.001	.197 ±.001	.005 ±.001	1.000 ±.001	.010 ±.001
0	0	1	No	Yes	10	0	.009 ±.001	.037 ±.005	.999 ±.001	.600 ±.002	.400 ±.001	.006 ±.000	.394 ±.002	.122 ±.003	1.154 ±.002	.199 ±.005	.005 ±.001	1.000 ±.002	.010 ±.002
0	0	1	Yes	No	10	0	.005 ±.001	.023 ±.004	.823 ±.001	.246 ±.001	.577 ±.000	.001 ±.000	.576 ±.000	.005 ±.001	1.891 ±.004	.295 ±.002	.005 ±.001	2.011 ±.003	.023 ±.005
0	0	1	Yes	Yes	10	0	.040 ±.001	.147 ±.003	.840 ±.001	.276 ±.001	.563 ±.000	.001 ±.000	.562 ±.000	.064 ±.002	1.527 ±.003	.156 ±.004	.011 ±.001	1.929 ±.008	.068 ±.007
0	0.1	1	No	No	10	0	.005 ±.001	.040 ±.006	1.002 ±.001	.600 ±.001	.402 ±.001	.026 ±.002	.375 ±.001	.005 ±.001	1.082 ±.002	.199 ±.005	.005 ±.001	1.003 ±.002	.041 ±.009
0	0.1	1	No	Yes	10	3	.016 ±.008	.060 ±.013	.999 ±.002	.594 ±.007	.405 ±.005	.026 ±.003	.379 ±.007	.120 ±.012	1.166 ±.026	.201 ±.028	.009 ±.005	1.005 ±.003	.039 ±.007
0	0.1	1	Yes	No	10	0	.005 ±.001	.045 ±.006	.823 ±.000	.246 ±.000	.577 ±.000	.005 ±.000	.572 ±.000	.005 ±.001	1.890 ±.004	.297 ±.006	.005 ±.001	2.011 ±.004	.045 ±.010
0	0.1	1	Yes	Yes	10	5	.044 ±.005	.160 ±.014	.842 ±.004	.281 ±.008	.561 ±.004	.007 ±.001	.554 ±.005	.072 ±.025	1.518 ±.032	.171 ±.027	.015 ±.005	1.908 ±.039	.085 ±.026
0.1	0	1	No	No	10	0	.025 ±.003	.032 ±.005	1.000 ±.002	.600 ±.004	.400 ±.002	.005 ±.000	.395 ±.002	.025 ±.004	1.081 ±.003	.199 ±.004	.024 ±.003	1.000 ±.004	.032 ±.005
0.1	0	1	No	Yes	10	3	.036 ±.026	.068 ±.048	1.003 ±.007	.606 ±.013	.397 ±.007	.008 ±.004	.390 ±.010	.139 ±.037	1.155 ±.006	.224 ±.050	.035 ±.029	.998 ±.007	.046 ±.045
0.1	0	1	Yes	No	10	0	.025 ±.002	.116 ±.009	.820 ±.003	.245 ±.003	.575 ±.001	.001 ±.000	.574 ±.001	.025 ±.003	1.898 ±.017	.314 ±.011	.026 ±.004	2.009 ±.020	.119 ±.016
0.1	0	1	Yes	Yes	10	1	.058 ±.027	.202 ±.071	.842 ±.005	.284 ±.012	.559 ±.006	.002 ±.002	.556 ±.009	.071 ±.023	1.520 ±.017	.174 ±.018	.040 ±.038	1.899 ±.039	.161 ±.100
0.1	0.1	1	No	No	10	0	.026 ±.004	.047 ±.006	1.002 ±.002	.600 ±.002	.402 ±.002	.025 ±.002	.376 ±.002	.025 ±.004	1.083 ±.003	.202 ±.007	.025 ±.004	1.003 ±.003	.046 ±.008
0.1	0.1	1	No	Yes	10	3	.046 ±.042	.079 ±.041	1.006 ±.016	.608 ±.030	.399 ±.014	.028 ±.005	.370 ±.019	.139 ±.046	1.156 ±.008	.222 ±.057	.044 ±.043	1.001 ±.017	.068 ±.036
0.1	0.1	1	Yes	No	10	0	.025 ±.003	.117 ±.014	.821 ±.003	.246 ±.003	.575 ±.001	.006 ±.000	.569 ±.001	.024 ±.003	1.893 ±.018	.314 ±.013	.024 ±.004	2.007 ±.020	.116 ±.019
0.1	0.1	1	Yes	Yes	10	4	.050 ±.008	.184 ±.024	.843 ±.006	.282 ±.009	.560 ±.004	.007 ±.001	.553 ±.005	.113 ±.180	1.540 ±.021	.195 ±.030	.032 ±.010	1.886 ±.055	.149 ±.030
0.1	0	1	No	No	10	0	.050 ±.007	.063 ±.008	1.001 ±.003	.600 ±.005	.401 ±.003	.006 ±.000	.396 ±.003	.048 ±.010	1.083 ±.007	.202 ±.009	.051 ±.007	1.005 ±.008	.065 ±.011
0.1	0	1	No	Yes	10	8	.064 ±.021	.103 ±.040	1.006 ±.009	.608 ±.020	.398 ±.011	.008 ±.003	.390 ±.014	.163 ±.050	1.161 ±.015	.243 ±.060	.058 ±.012	1.000 ±.012	.077 ±.016
0.1	0	1	Yes	No	10	0	.051 ±.006	.231 ±.027	.812 ±.008	.246 ±.006	.566 ±.003	.002 ±.000	.564 ±.003	.049 ±.007	1.895 ±.027	.359 ±.022	.052 ±.009	2.010 ±.043	.239 ±.034
0.1	0	1	Yes	Yes	10	0	.068 ±.013	.252 ±.047	.844 ±.014	.290 ±.017	.554 ±.005	.004 ±.002	.550 ±.006	.079 ±.010	1.530 ±.034	.217 ±.045	.060 ±.019	1.854 ±.101	.236 ±.061
0.1	0.1	1	No	No	10	0	.049 ±.006	.074 ±.010	1.003 ±.003	.600 ±.006	.403 ±.004	.026 ±.002	.376 ±.004	.051 ±.007	1.086 ±.007	.205 ±.010	.049 ±.008	1.006 ±.010	.072 ±.013
0.1	0.1	1	No	Yes	10	3	.055 ±.012	.092 ±.017	1.004 ±.007	.602 ±.015	.402 ±.009	.027 ±.002	.376 ±.010	.141 ±.041	1.161 ±.021	.231 ±.048	.052 ±.011	1.007 ±.015	.080 ±.016
0.1	0.1	1	Yes	No	10	1	.048 ±.007	.218 ±.030	.811 ±.009	.245 ±.007	.566 ±.003	.006 ±.001	.561 ±.003	.048 ±.011	1.875 ±.034	.354 ±.033	.048 ±.008	2.027 ±.051	.215 ±.034
0.1	0.1	1	Yes	Yes	10	2	.063 ±.011	.241 ±.034	.838 ±.005	.281 ±.006	.557 ±.003	.008 ±.001	.549 ±.003	.078 ±.010	1.544 ±.028	.225 ±.035	.054 ±.010	1.890 ±.033	.229 ±.045

Posted-price complex condition, behavioral model

τ	σy	θ	b1	b2	b3	σp	a1	σx	Discrete	Crash	S[Y] t>10	S[P] t>10	Rev. t>10	P.C. t>10	G.P. t>10	In.C. t>10	Prof. t>10	S[Y] t≤20	P t≤20	S[P] t≤20	S[Y] t>20	P t>20	S[P] t>20
2	0	0	0.5	0	0	0.01	0.5	0	No	0	.023 ±.002	.006 ±.002	1.008 ±.001	.605 ±.001	.403 ±.001	.024 ±.002	.379 ±.002	.062 ±.003	1.032 ±.004	.090 ±.005	.005 ±.002	.999 ±.002	.006 ±.001
2	0	0	0.5	0	0	0.05	0.5	0	No	0	.040 ±.008	.034 ±.005	1.012 ±.004	.610 ±.006	.402 ±.003	.065 ±.010	.336 ±.011	.071 ±.010	1.029 ±.011	.099 ±.010	.029 ±.008	.996 ±.009	
2	0	0	0.5	0	0	0.1	0.5	0	No	0	.069 ±.016	.063 ±.008	1.027 ±.008	.630 ±.016	.398 ±.009	.134 ±.013	.264 ±.017	.086 ±.018	1.005 ±.019	.107 ±.016	.060 ±.018	.986 ±.020	.000 ±.000
2	0.1	0	0.5	0	0	0.01	0.5	0	No	0	.060 ±.009	.006 ±.001	1.007 ±.002	.605 ±.002	.403 ±.001	.038 ±.004	.365 ±.004	.082 ±.009	1.034 ±.006	.093 ±.007	.056 ±.011	1.000 ±.002	
2	0.1	0	0.5	0	0	0.05	0.5	0	No	0	.066 ±.010	.032 ±.004	1.011 ±.003	.610 ±.005	.401 ±.003	.074 ±.008	.327 ±.008	.092 ±.011	1.028 ±.011	.097 ±.010	.060 ±.013	.994 ±.008	
2	0.1	0	0.5	0	0	0.1	0.5	0	No	0	.093 ±.015	.068 ±.010	1.025 ±.011	.626 ±.013	.399 ±.006	.128 ±.022	.271 ±.023	.111 ±.025	1.025 ±.022	.124 ±.022	.084 ±.019	.986 ±.016	
2	0	0	1	0	0	0.01	0.5	0	No	9	###	.230 ±.198	.394 ±.381	###	###	###	###	139.696 ±431.294	.832 ±.233	.283 ±.191	###	.248 ±.419	
2	0	0	1	0	0	0.05	0.5	0	No	3	###	.225 ±.173	.663 ±.674	###	###	###	###	50.834 ±112.364	.795 ±.216	.298 ±.175	###	.320 ±.415	
2	0	0	1	0	0	0.1	0.5	0	No	0	###	.236 ±.161	.700 ±.551	###	###	###	###	49.671 ±168.952	.846 ±.211	.262 ±.166	###	.458 ±.441	
2	0.1	0	1	0	0	0.01	0.5	0	No	6	###	.178 ±.199	.488 ±.436	###	###	###	###	271.221 ±789.005	.798 ±.274	.291 ±.196	###	.382 ±.465	
2	0.1	0	1	0	0	0.05	0.5	0	No	1	###	.239 ±.188	.581 ±.709	###	###	###	###	50.524 ±125.934	.768 ±.218	.337 ±.194	###	.274 ±.409	
2	0.1	0	1	0	0	0.1	0.5	0	No	0	###	.227 ±.155	.543 ±.408	###	###	###	###	32.350 ±94.695	.798 ±.228	.283 ±.168	###	.350 ±.440	
2	0	0	0	0.1	0	0.01	0.5	0	No	1	.067 ±.034	.174 ±.053	.811 ±.051	.247 ±.048	.564 ±.017	.537 ±.074	.027 ±.077	.246 ±.062	1.765 ±.385	.280 ±.109	.031 ±.017	2.116 ±.187	
2	0	0	0	0.1	0	0.05	0.5	0	No	0	.054 ±.025	.167 ±.049	.836 ±.024	.265 ±.026	.571 ±.005	.508 ±.041	.063 ±.039	.199 ±.020	1.615 ±.150	.176 ±.035	.031 ±.011	2.018 ±.123	
2	0	0	0	0.1	0	0.1	0.5	0	No	0	.083 ±.042	.205 ±.076	.861 ±.027	.303 ±.039	.558 ±.014	.450 ±.060	.108 ±.048	.180 ±.040	1.459 ±.144	.164 ±.060	.049 ±.019	1.858 ±.183	
2	0.1	0	0	0.1	0	0.01	0.5	0	No	2	.100 ±.025	.227 ±.062	.831 ±.040	.269 ±.040	.562 ±.009	.504 ±.064	.058 ±.064	.233 ±.057	1.549 ±.173	.261 ±.137	.068 ±.015	2.033 ±.208	
2	0.1	0	0	0.1	0	0.05	0.5	0	No	0	.084 ±.017	.188 ±.050	.832 ±.023	.262 ±.025	.570 ±.005	.512 ±.039	.058 ±.037	.202 ±.023	1.632 ±.177	.161 ±.046	.069 ±.010	2.038 ±.099	
2	0.1	0	0	0.1	0	0.1	0.5	0	No	0	.097 ±.025	.207 ±.077	.851 ±.029	.291 ±.038	.560 ±.012	.468 ±.060	.092 ±.051	.199 ±.027	1.551 ±.186	.165 ±.042	.082 ±.020	1.888 ±.205	
2	0	0	0	0.2	0	0.01	0.5	0	No	6	.083 ±.053	.249 ±.120	.782 ±.036	.226 ±.032	.557 ±.015	.573 ±.052	-.016 ±.054	.280 ±.030	1.917 ±.425	.404 ±.101	.048 ±.036	2.197 ±.129	
2	0	0	0	0.2	0	0.05	0.5	0	No	0	.046 ±.026	.154 ±.067	.784 ±.031	.219 ±.023	.564 ±.010	.582 ±.037	-.018 ±.045	.246 ±.034	1.859 ±.225	.291 ±.094	.027 ±.012	2.235 ±.085	
2	0	0	0	0.2	0	0.1	0.5	0	No	0	.067 ±.025	.218 ±.059	.802 ±.026	.238 ±.022	.565 ±.007	.554 ±.036	.011 ±.041	.225 ±.018	1.761 ±.180	.249 ±.064	.052 ±.020	2.143 ±.124	
2	0.1	0	0	0.2	0	0.01	0.5	0	No	8	.130 ±.057	.340 ±.141	.768 ±.071	.227 ±.047	.540 ±.031	.570 ±.076	-.030 ±.098	.321 ±.106	1.881 ±.457	.480 ±.194	.093 ±.037	2.202 ±.215	
2	0.1	0	0	0.2	0	0.05	0.5	0	No	0	.084 ±.014	.189 ±.049	.772 ±.031	.212 ±.023	.560 ±.010	.592 ±.037	-.033 ±.045	.260 ±.034	1.899 ±.211	.341 ±.100	.069 ±.011	2.262 ±.133	
2	0.1	0	0	0.2	0	0.1	0.5	0	No	0	.085 ±.017	.200 ±.057	.796 ±.031	.232 ±.023	.564 ±.009	.563 ±.037	.001 ±.045	.248 ±.024	1.819 ±.189	.285 ±.051	.075 ±.015	2.145 ±.163	
2	0	0	0	0.5	0	0.01	0.5	0	No	19	##	##	##	##	##	##	##	.339 ±.023	2.232 ±.016	.690 ±.070	##	##	
2	0	0	0	0.5	0	0.05	0.5	0	No	4	.051 ±.025	.187 ±.065	.753 ±.020	.201 ±.011	.552 ±.011	.613 ±.019	-.061 ±.029	.289 ±.033	2.052 ±.156	.445 ±.088	.040 ±.013	2.281 ±.058	
2	0	0	0	0.5	0	0.1	0.5	0	No	0	.054 ±.026	.195 ±.053	.767 ±.022	.210 ±.013	.557 ±.009	.595 ±.022	-.039 ±.030	.270 ±.023	2.028 ±.152	.380 ±.091	.048 ±.016	2.223 ±.082	
2	0.1	0	0	0.5	0	0.01	0.5	0	No	16	.285 ±.236	.522 ±.200	.774 ±.034	.260 ±.057	.514 ±.039	.596 ±.053	-.082 ±.078	1.049 ±1.587	1.637 ±.395	.713 ±.167	.119 ±.102	2.173 ±.221	
2	0.1	0	0	0.5	0	0.05	0.5	0	No	4	.089 ±.019	.241 ±.069	.736 ±.024	.193 ±.012	.543 ±.013	.623 ±.023	-.080 ±.034	.287 ±.030	2.098 ±.184	.442 ±.100	.084 ±.018	2.341 ±.085	
2	0.1	0	0	0.5	0	0.1	0.5	0	No	0	.086 ±.017	.219 ±.042	.761 ±.021	.208 ±.012	.554 ±.010	.598 ±.023	-.045 ±.032	.269 ±.027	2.064 ±.191	.378 ±.072	.079 ±.014	2.249 ±.070	

2	0	0	0.5	0	0.1	0.01	0.5	0	No	0	.009 ±.002	.008 ±.002	1.004 ±.000	.601 ±.001	.403 ±.001	.014 ±.001	.389 ±.001	.056 ±.005	1.049 ±.006	.091 ±.012	.005 ±.002	1.000 ±.001
2	0	0	0.5	0	0.1	0.05	0.5	0	No	0	.030 ±.007	.037 ±.008	1.007 ±.003	.607 ±.005	.401 ±.003	.048 ±.006	.352 ±.006	.066 ±.009	1.043 ±.008	.099 ±.009	.027 ±.007	.995 ±.008
2	0	0	0.5	0	0.1	0.1	0.5	0	No	0	.061 ±.015	.071 ±.014	1.018 ±.005	.617 ±.010	.401 ±.006	.095 ±.010	.306 ±.013	.082 ±.014	1.034 ±.014	.112 ±.021	.059 ±.019	.991 ±.014
2	0.1	0	0.5	0	0.1	0.01	0.5	0	No	0	.087 ±.155	.023 ±.057	1.022 ±.082	.621 ±.090	.401 ±.008	.063 ±.168	.338 ±.176	.091 ±.054	1.044 ±.016	.095 ±.028	.092 ±.190	.995 ±.022
2	0.1	0	0.5	0	0.1	0.05	0.5	0	No	0	.064 ±.012	.039 ±.009	1.006 ±.004	.603 ±.005	.403 ±.002	.050 ±.006	.353 ±.007	.085 ±.015	1.045 ±.010	.094 ±.011	.062 ±.012	.999 ±.006
2	0.1	0	0.5	0	0.1	0.1	0.5	0	No	0	.086 ±.015	.073 ±.018	1.021 ±.008	.623 ±.013	.398 ±.008	.102 ±.014	.296 ±.018	.100 ±.020	1.025 ±.015	.103 ±.019	.086 ±.018	.983 ±.020
2	0	0	0.5	0	0.2	0.01	0.5	0	No	0	.008 ±.002	.010 ±.002	1.002 ±.001	.600 ±.000	.402 ±.001	.010 ±.001	.392 ±.001	.055 ±.004	1.054 ±.003	.091 ±.007	.006 ±.002	1.000 ±.001
2	0	0	0.5	0	0.2	0.05	0.5	0	No	0	.031 ±.007	.040 ±.009	1.006 ±.003	.605 ±.004	.401 ±.002	.039 ±.004	.361 ±.005	.061 ±.008	1.052 ±.009	.097 ±.018	.030 ±.008	.995 ±.006
2	0	0	0.5	0	0.2	0.1	0.5	0	No	0	.070 ±.013	.087 ±.014	1.017 ±.007	.616 ±.007	.402 ±.004	.082 ±.009	.319 ±.010	.081 ±.016	1.044 ±.013	.120 ±.022	.068 ±.014	.995 ±.011
2	0.1	0	0.5	0	0.2	0.01	0.5	0	No	1	.059 ±.015	.022 ±.024	1.004 ±.009	.603 ±.011	.402 ±.002	.026 ±.024	.376 ±.025	.078 ±.010	1.053 ±.005	.092 ±.009	.060 ±.019	.998 ±.008
2	0.1	0	0.5	0	0.2	0.05	0.5	0	No	0	.063 ±.006	.039 ±.008	1.006 ±.002	.605 ±.003	.401 ±.003	.042 ±.005	.359 ±.005	.082 ±.014	1.050 ±.004	.098 ±.010	.062 ±.010	.997 ±.006
2	0.1	0	0.5	0	0.2	0.1	0.5	0	No	0	.088 ±.016	.083 ±.021	1.019 ±.007	.619 ±.010	.400 ±.005	.088 ±.009	.313 ±.012	.100 ±.019	1.045 ±.009	.125 ±.021	.089 ±.021	.990 ±.011
5	0	0	0.5	0	0.2	0.01	0.5	0	No	0	.008 ±.001	.021 ±.004	1.002 ±.001	.591 ±.001	.411 ±.001	.027 ±.003	.384 ±.003	.059 ±.003	1.088 ±.003	.081 ±.006	.004 ±.001	1.007 ±.001
5	0	0	0.5	0	0.2	0.05	0.5	0	No	0	.025 ±.005	.046 ±.009	1.005 ±.003	.594 ±.004	.411 ±.002	.048 ±.006	.364 ±.007	.062 ±.005	1.087 ±.006	[illegible] ±.008	.023 ±.007	1.006 ±.005
5	0	0	0.5	0	0.2	0.1	0.5	0	No	0	.050 ±.013	.084 ±.015	1.018 ±.006	.608 ±.008	.410 ±.005	.089 ±.012	.321 ±.014	.077 ±.012	1.076 ±.010	.106 ±.021	.051 ±.016	1.001 ±.010
5	0.1	0	0.5	0	0.2	0.01	0.5	0	No	1	.054 ±.007	.026 ±.006	1.001 ±.004	.590 ±.006	.412 ±.003	.035 ±.005	.377 ±.004	.077 ±.009	1.086 ±.011	.087 ±.021	.053 ±.008	1.009 ±.008
5	0.1	0	0.5	0	0.2	0.05	0.5	0	No	0	.053 ±.008	.043 ±.007	1.006 ±.003	.594 ±.005	.412 ±.003	.050 ±.005	.362 ±.006	.078 ±.008	1.085 ±.009	.088 ±.011	.052 ±.009	1.007 ±.007
5	0.1	0	0.5	0	0.2	0.1	0.5	0	No	0	.069 ±.011	.080 ±.020	1.018 ±.008	[illegible] ±.008	.411 ±.006	.088 ±.011	.323 ±.011	.098 ±.015	1.083 ±.012	.116 ±.027	.066 ±.014	.997 ±.008
2	0	1	0.5	0	0.1	0.01	0.5	0	No	0	.021 ±.004	.025 ±.002	1.013 ±.001	.599 ±.002	.414 ±.001	.046 ±.003	.368 ±.002	.138 ±.005	1.087 ±.006	.080 ±.005	.012 ±.004	1.001 ±.002
2	0	1	0.5	0	0.1	0.05	0.5	0	No	0	.055 ±.012	.045 ±.008	1.018 ±.006	[illegible] ±.009	.412 ±.004	.073 ±.010	.339 ±.010	.149 ±.013	1.082 ±.012	.091 ±.012	.050 ±.013	.996 ±.011
2	0	1	0.5	0	0.1	0.1	0.5	0	No	0	.108 ±.028	.076 ±.016	1.042 ±.012	.637 ±.017	.405 ±.008	.116 ±.018	.288 ±.019	.192 ±.040	1.062 ±.027	.115 ±.017	.093 ±.024	.970 ±.017
2	0.1	1	0.5	0	0.1	0.01	0.5	0	No	0	.061 ±.007	.026 ±.003	1.012 ±.002	.599 ±.003	.414 ±.001	.052 ±.004	.361 ±.003	.148 ±.008	1.087 ±.008	.080 ±.005	.058 ±.009	1.001 ±.003
2	0.1	1	0.5	0	0.1	0.05	0.5	0	No	0	.077 ±.011	.047 ±.006	1.019 ±.005	.606 ±.008	.413 ±.003	.075 ±.006	.338 ±.008	.161 ±.011	1.056 ±.010	.090 ±.010	.075 ±.014	.994 ±.010
2	0.1	1	0.5	0	0.1	0.1	0.5	0	No	0	.112 ±.020	.075 ±.013	1.035 ±.013	.628 ±.016	.407 ±.007	.118 ±.016	.289 ±.018	.186 ±.024	1.071 ±.017	.108 ±.022	.112 ±.023	.975 ±.018
2	0	1	0.5	0	0.1	0.01	0.2	0	No	0	.009 ±.002	.030 ±.002	1.010 ±.001	.586 ±.002	.424 ±.001	.088 ±.003	.336 ±.002	.124 ±.004	1.101 ±.006	.073 ±.006	.006 ±.002	1.017 ±.002
2	0	1	0.5	0	0.1	0.05	0.2	0	No	0	.031 ±.008	.047 ±.006	1.013 ±.004	.589 ±.006	.424 ±.003	.098 ±.008	.325 ±.008	.128 ±.009	1.098 ±.012	.085 ±.013	.028 ±.009	1.015 ±.007
2	0	1	0.5	0	0.1	0.1	0.2	0	No	0	.061 ±.016	.079 ±.017	1.032 ±.010	.613 ±.011	.419 ±.004	.134 ±.013	.285 ±.013	.145 ±.017	1.081 ±.016	.100 ±.018	.061 ±.020	.995 ±.011
2	0.1	1	0.5	0	0.1	0.01	0.2	0	No	0	.055 ±.010	.032 ±.003	1.010 ±.002	.585 ±.003	.424 ±.002	.090 ±.005	.334 ±.004	.135 ±.006	1.101 ±.008	.073 ±.006	.056 ±.011	1.017 ±.003
2	0.1	1	0.5	0	0.1	0.05	0.2	0	No	0	.064 ±.008	.048 ±.008	1.016 ±.004	.593 ±.006	.422 ±.003	.104 ±.005	.318 ±.006	.138 ±.008	1.097 ±.011	.077 ±.006	.064 ±.009	1.009 ±.007
2	0.1	1	0.5	0	0.1	0.1	0.2	0	No	0	.082 ±.013	.077 ±.012	1.031 ±.013	.612 ±.017	.419 ±.005	.134 ±.015	.285 [illegible]	.158 ±.019	1.090 ±.021	.100 ±.023	.080 ±.015	.992 ±.018
2	0	1	0.5	0	0.2	0.01	0.2	0	No	0	.031 ±.002	.037 ±.002	.987 ±.001	.554 ±.002	.433 ±.001	.079 ±.002	.354 ±.002	.089 ±.003	1.146 ±.005	.067 ±.005	.015 ±.002	1.048 ±.002
2	0	1	0.5	0	0.2	0.05	0.2	0	No	0	.045 ±.009	.057 ±.009	.993 ±.003	.562 ±.006	.431 ±.003	.086 ±.004	.345 ±.006	.094 ±.009	1.141 ±.009	.083 ±.014	.032 ±.010	1.039 ±.008
2	0	1	0.5	0	0.2	0.1	0.2	0	No	0	.067 ±.015	.093 ±.021	1.010 ±.009	.583 ±.013	.427 ±.006	.112 ±.010	.315 ±.011	.118 ±.018	1.123 ±.018	.113 ±.034	.059 ±.014	1.022 ±.016
2	0.1	1	0.5	0	0.2	0.01	0.2	0	No	1	.083 ±.090	.049 ±.034	.991 ±.011	.561 ±.024	.430 ±.012	.085 ±.026	.345 ±.038	.105 ±.006	1.144 ±.008	.068 ±.005	.082 ±.109	1.042 [illegible]
2	0.1	1	0.5	0	0.2	0.05	0.2	0	No	0	.070 ±.010	.057 ±.008	.992 ±.005	.560 ±.007	.431 ±.004	.088 ±.004	.343 ±.006	.107 ±.008	1.143 ±.009	.080 ±.016	.065 ±.012	1.041 ±.011
2	0.1	1	0.5	0	0.2	0.1	0.2	0	No	0	.089 ±.017	.089 ±.013	1.012 ±.017	.585 ±.021	.427 ±.006	.116 ±.018	.312 ±.021	.133 ±.031	1.123 ±.018	.108 ±.022	.078 ±.014	1.023 ±.020
2	0	0	0.5	0	0	0.01	0.5	0	Yes	0	208.073 ±347.794	.295 ±.191	.766 ±.572	100.946 ±168.390	-100.180 ±168.827	119.580 ±200.744	-219.759 ±369.571	4.599 ±7.295	.684 ±.245	.394 ±.057	225.367 ±376.331	.382 ±.388

Appendix B

2	0	0	0.5	0	0	0.05	0.5	0	Yes	0	376.904 ±658.821	.296 ±.185	.829 ±.634	173.579 ±298.447	- 172.750 ±298.890	198.703 ±339.478	- 371.453 ±636.290	5.522 ±8.585	.676 ±.253	.396 ±.060	415.401 ±729.557	.325 ±.353
2	0	0	0.5	0	0	0.1	0.5	0	Yes	0	###	.227 ±.191	.584 ±.527	650.233 ±1139.949	- 649.649 ±1140.181	684.700 ±1163.787	###	9.324 ±12.385	.584 ±.220	.409 ±.060	###	.166 ±.269
2	0.1	0	0.5	0	0	0.01	0.5	0	Yes	0	135.077 ±286.412	.332 ±.156	.857 ±.530	65.596 ±138.649	- 64.739 ±139.017	77.434 ±165.271	- 142.173 ±304.288	3.057 ±5.989	.742 ±.213	.390 ±.054	146.347 ±309.945	.400 ±.366
2	0.1	0	0.5	0	0	0.05	0.5	0	Yes	0	144.147 ±295.763	.313 ±.136	.985 ±.521	68.609 ±141.497	- 67.624 ±141.917	79.947 ±167.485	- 147.571 ±309.384	2.875 ±5.813	.759 ±.208	.366 ±.056	157.400 ±321.350	.480 ±.394
2	0.1	0	0.5	0	0	0.1	0.5	0	Yes	0	###	.178 ±.179	.527 ±.540	###	###	###	###	15.267 ±20.676	.530 ±.237	.404 ±.062	###	.082 ±.190
2	0	0	1	0	0	0.01	0.5	0	Yes	2	###	.242 ±.231	.224 ±.279	###	###	###	###	23.250 ±39.092	.534 ±.265	.569 ±.105	###	.003 ±.007
2	0	0	1	0	0	0.05	0.5	0	Yes	0	###	.021 ±.076	.025 ±.068	###	###	###	###	460.469 ±906.022	.246 ±.150	.458 ±.162	###	.000 ±.000
2	0	0	1	0	0	0.1	0.5	0	Yes	0	###	.000 ±.000	.003 ±.004	###	###	###	###	###	.117 ±.041	.319 ±.059	###	.000 ±.000
2	0.1	0	1	0	0	0.01	0.5	0	Yes	2	###	.236 ±.245	.269 ±.326	###	###	###	###	22.372 ±32.640	.522 ±.271	.550 ±.144	###	.018 ±.050
2	0.1	0	1	0	0	0.05	0.5	0	Yes	0	###	.001 ±.002	.005 ±.007	###	###	###	###	635.301 ±1035.409	.196 ±.097	.409 ±.098	###	.000 ±.000
2	0.1	0	1	0	0	0.1	0.5	0	Yes	0	###	.000 ±.000	.002 ±.002	###	###	###	###	###	.124 ±.042	.331 ±.050	###	.000 ±.000
2	0	0	0	0.1	0	0.01	0.5	0	Yes	0	.033 ±.004	.186 ±.013	.748 ±.008	.196 ±.004	.552 ±.004	.620 ±.006	-.068 ±.010	.266 ±.011	1.997 ±.073	.358 ±.043	.033 ±.005	2.343 ±.038
2	0	0	0	0.1	0	0.05	0.5	0	Yes	0	.040 ±.008	.194 ±.018	.750 ±.012	.198 ±.006	.553 ±.006	.617 ±.011	-.065 ±.017	.276 ±.015	1.969 ±.091	.391 ±.049	.038 ±.007	2.318 ±.039
2	0	0	0	0.1	0	0.1	0.5	0	Yes	0	.068 ±.030	.250 ±.044	.760 ±.025	.207 ±.015	.553 ±.010	.601 ±.026	-.049 ±.035	.268 ±.016	1.874 ±.180	.396 ±.042	.052 ±.012	2.284 ±.061
2	0.1	0	0	0.1	0	0.01	0.5	0	Yes	0	.074 ±.010	.203 ±.019	.753 ±.010	.199 ±.005	.553 ±.005	.614 ±.011	-.060 ±.015	.273 ±.015	1.983 ±.096	.370 ±.050	.074 ±.014	2.309 ±.040
2	0.1	0	0	0.1	0	0.05	0.5	0	Yes	0	.073 ±.006	.214 ±.020	.750 ±.012	.198 ±.007	.551 ±.006	.615 ±.012	-.063 ±.018	.275 ±.020	1.994 ±.094	.369 ±.061	.071 ±.009	2.315 ±.048
2	0.1	0	0	0.1	0	0.1	0.5	0	Yes	0	.086 ±.020	.246 ±.033	.762 ±.016	.209 ±.010	.554 ±.007	.600 ±.016	-.046 ±.022	.268 ±.022	1.923 ±.111	.373 ±.071	.082 ±.017	2.257 ±.044
2	0	0	0	0.2	0	0.01	0.5	0	Yes	0	.057 ±.005	.358 ±.015	.690 ±.018	.175 ±.009	.515 ±.009	.651 ±.010	-.136 ±.019	.291 ±.015	2.213 ±.112	.486 ±.037	.058 ±.007	2.438 ±.065
2	0	0	0	0.2	0	0.05	0.5	0	Yes	0	.074 ±.011	.375 ±.025	.721 ±.016	.194 ±.008	.527 ±.008	.625 ±.014	-.098 ±.022	.294 ±.027	2.141 ±.097	.488 ±.065	.070 ±.008	2.312 ±.057
2	0	0	0	0.2	0	0.1	0.5	0	Yes	0	.077 ±.009	.386 ±.025	.723 ±.012	.196 ±.006	.526 ±.007	.622 ±.010	-.095 ±.016	.296 ±.025	2.096 ±.098	.500 ±.059	.074 ±.009	2.281 ±.047
2	0.1	0	0	0.2	0	0.01	0.5	0	Yes	0	.085 ±.012	.370 ±.020	.699 ±.011	.181 ±.006	.519 ±.006	.644 ±.011	-.125 ±.016	.293 ±.019	2.144 ±.107	.484 ±.053	.086 ±.012	2.423 ±.039
2	0.1	0	0	0.2	0	0.05	0.5	0	Yes	0	.091 ±.011	.363 ±.018	.723 ±.011	.194 ±.006	.529 ±.006	.626 ±.010	-.097 ±.015	.297 ±.026	2.106 ±.074	.478 ±.043	.094 ±.013	2.319 ±.051
2	0.1	0	0	0.2	0	0.1	0.5	0	Yes	0	.095 ±.010	.388 ±.028	.727 ±.017	.199 ±.010	.527 ±.008	.615 ±.016	-.088 ±.023	.282 ±.025	2.135 ±.079	.449 ±.064	.095 ±.014	2.280 ±.071
2	0	0	0	0.5	0	0.01	0.5	0	Yes	0	.138 ±.002	.838 ±.014	.619 ±.015	.193 ±.007	.426 ±.009	.626 ±.011	-.200 ±.018	.336 ±.030	2.184 ±.106	.883 ±.014	.139 ±.001	2.343 ±.046
2	0	0	0	0.5	0	0.05	0.5	0	Yes	7	.145 ±.006	.794 ±.051	.672 ±.050	.223 ±.026	.449 ±.025	.579 ±.033	-.130 ±.057	.321 ±.030	2.194 ±.151	.863 ±.045	.140 ±.003	2.129 ±.149
2	0	0	0	0.5	0	0.1	0.5	0	Yes	18	.154 ±.024	##	.700 ±.031	.241 ±.012	.459 ±.019	.537 ±.033	-.078 ±.052	.303 ±.043	2.018 ±.120	.802 ±.088	.154 ±.017	##
2	0.1	0	0	0.5	0	0.01	0.5	0	Yes	0	.144 ±.011	.835 ±.021	.622 ±.022	.195 ±.010	.428 ±.012	.623 ±.015	-.195 ±.026	.328 ±.034	2.200 ±.118	.871 ±.022	.147 ±.011	2.337 ±.059
2	0.1	0	0	0.5	0	0.05	0.5	0	Yes	12	.153 ±.013	.758 ±.044	.702 ±.032	.236 ±.018	.465 ±.015	.561 ±.031	-.096 ±.044	.317 ±.032	2.115 ±.118	.832 ±.037	.143 ±.017	2.041 ±.063

2	0.1	0	0	0.5	0	0.1	0.5	0	Yes	20	##	##	##	##	##	##	##	.295 ±.012	2.057 ±.125	.795 ±.045	##	##
2	0	0	0.5	0	0.1	0.01	0.5	0	Yes	0	.280 ±.062	.321 ±.034	1.129 ±.037	.758 ±.049	.372 ±.017	.311 ±.062	.060 ±.073	.258 ±.084	.981 ±.036	.320 ±.044	.275 ±.054	.972 ±.034
2	0	0	0.5	0	C.1	0.05	0.5	0	Yes	0	.336 ±.084	.306 ±.043	1.251 ±.044	.888 ±.056	.363 ±.018	.531 ±.067	-.167 ±.080	.357 ±.106	.917 ±.047	.297 ±.038	.334 ±.103	.916 ±.043
2	0	0	0.5	0	0.1	0.1	0.5	0	Yes	0	.449 ±.123	.315 ±.050	1.313 ±.044	.981 ±.062	.332 ±.032	.676 ±.082	-.345 ±.102	.550 ±.251	.896 ±.046	.335 ±.069	.423 ±.142	.862 ±.041
2	0.1	0	0.5	0	0.1	0.01	0.5	0	Yes	0	.250 ±.049	.305 ±.031	1.138 ±.049	.763 ±.056	.375 ±.015	.327 ±.080	.049 ±.088	.267 ±.078	.965 ±.042	.302 ±.040	.250 ±.067	.962 ±.034
2	0.1	0	0.5	0	0.1	0.05	0.5	C	Yes	0	.357 ±.070	.312 ±.043	1.254 ±.039	.901 ±.058	.353 ±.029	.535 ±.070	-.183 ±.090	.339 ±.061	.916 ±.035	.300 ±.037	.365 ±.097	.908 ±.041
2	0.1	0	0.5	0	0.1	0.1	0.5	0	Yes	0	.519 ±.142	.319 ±.043	1.318 ±.044	1.004 ±.066	.314 ±.063	.699 ±.083	-.384 ±.118	.415 ±.166	.912 ±.031	.305 ±.048	.530 ±.165	.857 ±.055
2	0	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.284 ±.050	.349 ±.056	1.128 ±.051	.736 ±.065	.392 ±.028	.272 ±.088	.121 ±.101	.260 ±.065	1.036 ±.045	.367 ±.054	.264 ±.061	.989 ±.054
2	0	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.351 ±.076	.339 ±.064	1.226 ±.041	.844 ±.045	.382 ±.025	.437 ±.068	-.055 ±.074	.320 ±.061	.964 ±.037	.311 ±.057	.360 ±.086	.959 ±.055
2	0	0	0.5	0	0.2	0.1	0.5	0	Yes	1	.427 ±.161	.380 ±.144	1.250 ±.061	.883 ±.040	.367 ±.056	.522 ±.066	-.155 ±.103	.352 ±.067	.963 ±.061	.335 ±.083	.450 ±.197	.958 ±.063
2	0.1	0	0.5	0	0.2	0.01	0.5	0	Yes	C	.290 ±.056	.343 ±.042	1.153 ±.054	.764 ±.058	.390 ±.011	.317 ±.092	.073 ±.095	.267 ±.053	1.016 ±.052	.350 ±.056	.285 ±.074	.979 ±.030
2	0.1	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.344 ±.079	.326 ±.077	1.219 ±.040	.833 ±.041	.386 ±.022	.434 ±.065	-.048 ±.066	.298 ±.057	.986 ±.035	.320 ±.052	.348 ±.103	.945 ±.055
2	0.1	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.361 ±.065	.334 ±.048	1.248 ±.039	.861 ±.038	.387 ±.016	.491 ±.059	-.104 ±.058	.359 ±.077	.957 ±.040	.328 ±.066	.353 ±.087	.953 ±.037
5	0	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.217 ±.041	.370 ±.072	1.107 ±.037	.711 ±.037	.396 ±.018	.274 ±.052	.122 ±.056	.210 ±.054	1.079 ±.054	.407 ±.089	.205 ±.054	1.013 ±.057
5	0	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.223 ±.051	.327 ±.055	1.182 ±.030	.783 ±.035	.399 ±.020	.399 ±.058	.001 ±.062	.250 ±.045	1.023 ±.040	.331 ±.066	.220 ±.056	.958 ±.028
5	0	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.273 ±.049	.345 ±.039	1.228 ±.022	.830 ±.033	[illegible] ±.025	.492 ±.047	-.094 ±.057	.276 ±.053	1.001 ±.031	.328 ±.045	.276 ±.057	.961 ±.030
5	0.1	0	0.5	0	0.2	0.01	0.5	0	Yes	0	.196 ±.032	.331 ±.043	1.125 ±.034	.730 ±.034	.394 ±.013	.286 ±.058	.108 ±.060	.202 ±.049	1.058 ±.046	.352 ±.037	.200 ±.035	.992 ±.034
5	0.1	0	0.5	0	0.2	0.05	0.5	0	Yes	0	.226 ±.032	.310 ±.031	1.188 ±.034	.800 ±.041	.388 ±.017	.410 ±.058	-.022 ±.066	.232 ±.040	1.010 ±.030	.313 ±.033	.231 ±.033	.948 ±.033
5	0.1	0	0.5	0	0.2	0.1	0.5	0	Yes	0	.292 ±.068	.348 ±.062	1.227 ±.030	.843 ±.026	.384 ±.022	.496 ±.041	-.112 ±.043	.295 ±.050	1.004 ±.039	.350 ±.062	.298 ±.081	.951 ±.040
2	0	1	0.5	0	0.1	0.01	0.5	0	Yes	0	.588 ±.119	.303 ±.040	1.259 ±.074	.914 ±.090	.345 ±.027	.370 ±.070	-.025 ±.085	.547 ±.088	.977 ±.049	.333 ±.035	.574 ±.138	.871 ±.054
2	0	1	0.5	0	0.1	0.05	0.5	0	Yes	0	.952 ±.427	.309 ±.092	1.434 ±.149	1.140 ±.219	.293 ±.109	.633 ±.214	-.340 ±.289	.848 ±.412	.907 ±.045	.327 ±.104	.935 ±.447	.813 ±.048
2	0	1	0.5	0	0.1	0.1	0.5	0	Yes	1	1.409 ±.927	.313 ±.058	1.687 ±.267	1.496 ±.425	.191 ±.177	.849 ±.276	-.658 ±.444	.956 ±.260	.871 ±.043	.321 ±.069	1.538 ±1.073	.745 ±.064
2	0.1	1	0.5	0	0.1	0.01	0.5	0	Yes	0	.577 ±.100	.301 ±.027	1.254 ±.083	.897 ±.089	.357 ±.021	.365 ±.091	-.008 ±.095	.560 ±.094	.961 ±.044	.323 ±.036	.586 ±.134	.884 ±.042
2	0.1	1	0.5	0	0.1	0.05	0.5	0	Yes	0	.817 ±.213	.289 ±.050	1.443 ±.119	1.134 ±.146	.309 ±.043	.600 ±.117	-.291 ±.143	.762 ±.218	.918 ±.050	.321 ±.056	.779 ±.226	.800 ±.037
2	0.1	1	0.5	0	0.1	0.1	0.5	0	Yes	0	1.445 ±.489	.363 ±.099	1.640 ±.173	1.447 ±.255	.194 ±.108	.859 ±.183	-.665 ±.278	.965 ±.230	.882 ±.056	.328 ±.058	1.582 ±.613	.746 ±.034
2	0	1	0.5	0	0.1	0.01	0.2	0	Yes	0	.311 ±.082	.309 ±.042	1.252 ±.099	.890 ±.106	.362 ±.046	.377 ±.098	-.015 ±.114	.363 ±.073	.987 ±.051	.315 ±.054	.322 ±.104	.893 ±.054
2	0	1	0.5	0	0.1	0.05	0.2	0	Yes	0	.450 ±.230	.291 ±.058	1.391 ±.077	1.044 ±.102	.347 ±.041	.576 ±.125	-.229 ±.155	.560 ±.274	.937 ±.033	.326 ±.067	.411 ±.190	.845 ±.039
2	0	1	0.5	0	0.1	0.1	0.2	0	Yes	0	.641 ±.255	.301 ±.051	1.595 ±.208	1.277 ±.227	.318 ±.036	.782 ±.168	-.464 ±.187	.657 ±.214	.906 ±.056	.322 ±.077	.645 ±.297	.784 ±.029
2	0.1	1	0.5	0	0.1	0.01	0.2	0	Yes	0	.282 ±.060	.298 ±.022	1.220 ±.060	.846 ±.071	.374 ±.023	.340 ±.062	.033 ±.072	.345 ±.098	.991 ±.046	.319 ±.042	.286 ±.057	.911 ±.050
2	0.1	1	0.5	0	0.1	0.05	0.2	0	Yes	0	.485 ±.212	.289 ±.058	1.431 ±.104	1.095 ±.127	.336 ±.034	.619 ±.114	-.283 ±.138	.501 ±.101	.923 ±.044	.308 ±.062	.509 ±.227	.837 ±.044
2	0.1	1	0.5	0	0.1	0.1	0.2	0	Yes	0	.572 ±.201	.290 ±.051	1.568 ±.146	1.250 ±.172	.318 ±.034	.744 ±.132	-.426 ±.158	.611 ±.129	.892 ±.045	.302 ±.062	.626 ±.216	.808 ±.047
2	0	1	0.5	0	0.2	0.01	0.2	0	Yes	0	.323 ±.080	.341 ±.046	1.226 ±.110	.839 ±.116	.387 ±.029	.325 ±.105	.062 ±.111	.313 ±.095	1.062 ±.045	.382 ±.040	.318 ±.083	.919 ±.063
2	0	1	0.5	0	0.2	0.05	0.2	0	Yes	0	.456 ±.157	.326 ±.053	1.390 ±.102	1.028 ±.127	.362 ±.032	.485 ±.090	-.123 ±.115	.395 ±.077	.994 ±.033	.312 ±.047	.462 ±.189	.858 ±.040
2	0	1	0.5	0	0.2	0.1	0.2	0	Yes	0	.601 ±.233	.351 ±.059	1.465 ±.093	1.118 ±.117	.348 ±.050	.576 ±.100	-.228 ±.132	.459 ±.075	.977 ±.043	.340 ±.055	.613 ±.264	.849 ±.047
2	0.1	1	0.5	0	0.2	0.01	0.2	0	Yes	0	.326 ±.108	.309 ±.047	1.224 ±.078	.838 ±.091	.386 ±.022	.338 ±.091	.047 ±.105	.304 ±.072	1.062 ±.042	.361 ±.052	.304 ±.108	.907 ±.054
2	0.1	1	0.5	0	0.2	0.05	0.2	0	Yes	0	.423 ±.115	.315 ±.043	1.369 ±.099	1.007 ±.107	.363 ±.022	.464 ±.076	-.102 ±.086	.394 ±.095	.989 ±.038	.319 ±.051	.428 ±.144	.867 ±.046
2	0.1	1	0.5	0	0.2	0.1	0.2	0	Yes	0	.511 ±.190	.340 ±.092	1.453 ±.067	1.101 ±.072	.351 ±.031	.550 ±.072	-.199 ±.088	.467 ±.080	.964 ±.031	.338 ±.046	.517 ±.258	.847 ±.044

Note: In all simulations, σ_x and a_2 are zero; β is one; s_c is 3; $d=\sigma_p$.

Appendix C: Questionnaires and instructions

Pre-game questionnaire

1. Date ___________

2. Firm ___________ (name indicated on your computer)

3. Your age ___________ (years)

4. What is your current educational status (check one)?

_____ Faculty member
_____ Doctoral student, ABD (all but dissertation)
_____ Doctoral student taking courses
_____ Master's student
_____ Sloan Fellow
_____ Undergraduate
_____ Other (please specify) ______________________________

5. With what department at M.I.T. are you mostly affiliated (check one)?

_____ Management (15), Behavioral and Policy Sciences
_____ Management (15), Economics and Finance
_____ Managmenet (15), Management Science
_____ Management (15), MBA/MOT/Sloan Fellow
_____ Economics (14)
_____ Politicial Science (17)
_____ Other (please specify) ______________________________

In the following, a "graduate course equivalent" is either one graduate-level course or two undergraduate courses.

6. What is your educational background in economics (don't count statistics or econometrics courses)?

_____ Advanced (more than 3 graduate course equivalents)
_____ Intermediate (2-3 graduate course equivalents)
_____ Introductory (1/2 - 1 graduate course equivalent)
_____ None

7. What is your educational background in relevant quantitative disciplines such as statistics, operations research, management science, engineering control theory (check one)?

______ Advanced (more than 3 graduate course equivalents)
______ Intermediate (2-3 graduate course equivalents)
______ Introductory (1/2 - 1 graduate course equivalent)
______ None

8. What is your educational background in "system dynamics" (check one)?

______ Several courses (e.g. 15.872/15.874, 15.873, individual study)
______ One course (e.g. 15.872/15.874)
______ None

9. Has anyone who has played this game before ever told you any datails about the game? (Answer yes or no) ________

10. Have you ever played any of the following simulation games (check any that apply)?

______ The market game during IAP, January 1990
______ The "People Express" Management Flight Simulator
______ The "Beer Distribution Game"
______ Bent Bakken's Real Estate Investment Game
______ Bent Bakken's Oil Tanker Game
______ Ernst Diehl's Control Game
______ The "Long Wave" Capital Investment Game.

Game instructions, fixed-price, simple condition

Goal

As in life, the purpose of the game is for you to maximize your profits. For the experimenters, the purpose is to investigate how markets with different structures and price regimes behave.

Your dollar reward is based on your cumulative net profits in the game. Your cumulative profits are converted to real money using a fixed exchange rate. On average, you should earn about $30 in three hours of playing.

In addition, you will receive a bonus for your forecasting performance (see below). On average, the bonus will amount to a couple of dollars.

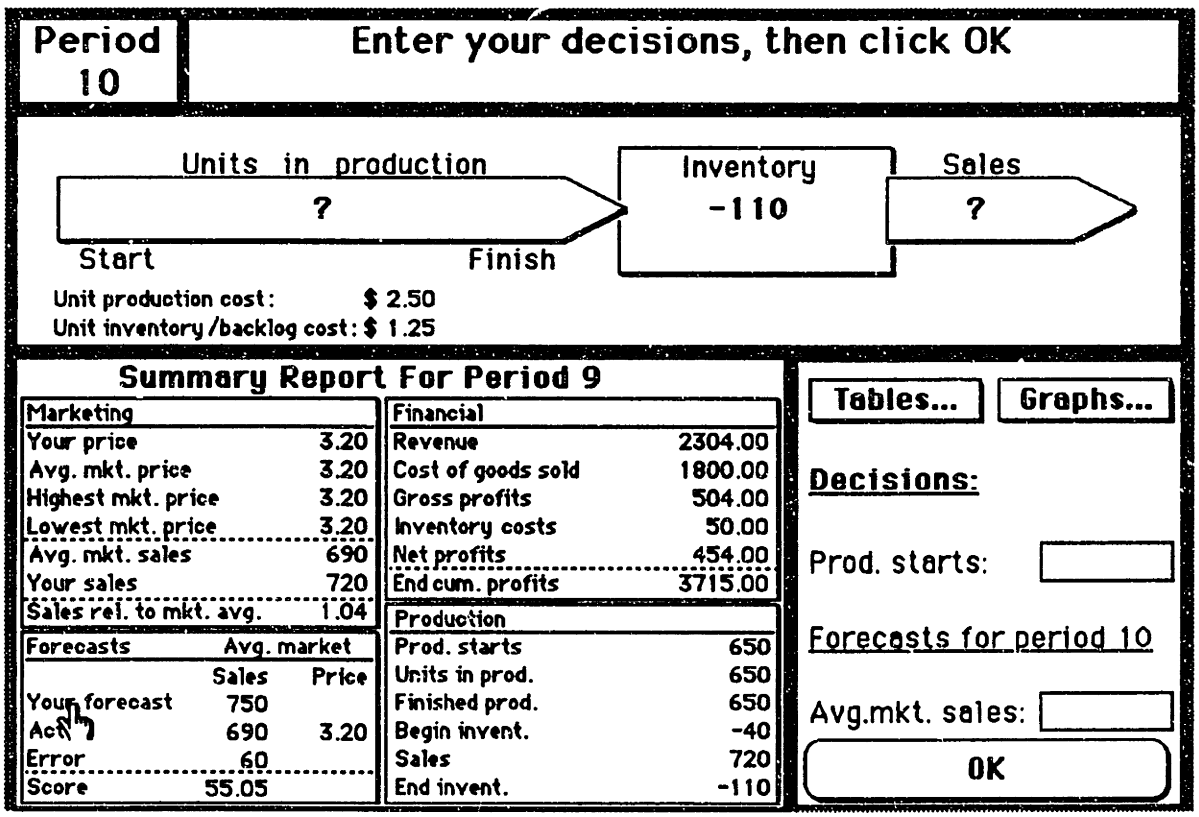

Figure 1

Your firm and the market

Figure 1 shows the screen that you will be seeing in the game. (The numbers in the figure are for illustration only--the numbers in your game will be different.)

Your firm competes with other firms in the market. All firms are identical with respect to production cost, inventory cost, etc. Each firm sells only one product, which for experimental reasons has been kept abstract. The products of different firms are quite alike but not identical. You can think of the product as a tire, a softdrink, or a men's shirt.

Production, inventory, and backlog

Each period, you have only one decision to make, namely how much to produce.

If you sell less than the finished production, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

Theoretically, there is no limit on how much you produce, but you cannot cancel initiated production or destroy your inventory, i.e. you cannot produce negative amounts.

Profits and costs

Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

Inventory/backlog costs are proportional to your inventory or backlog that you have at the beginning of each period. You might also say that your inventory/backlog cost is proportional to the absolute value of your inventory.

In the case where you have positive inventory, you can interpret the cost as a storage and financing charge. When you have a backlog (negative inventory), you can think of the cost as a rebate that you have to offer your customers to compensate for delayed delivery, or a penalty imposed on you for late delivery by a central planning ministry.

Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs.

Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

In this version of the game, prices are fixed by government decree.

Sales

Your sales are influenced by only one factor (in other versions of the market where prices are variable, sales would also be affected by price):

- External factors Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.

  The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.

Forecasts

In addition to your production decision, we ask you to make a forecast of the average sales per firm in the market (i.e. not the total market sales, and not your own sales). We ask you to forecast both of these figures for the current period.

On top of your profit-based reward, you will get a "bonus" of a couple of dollars, depending how you rank in your forecasting performance. Your performance is measured by a "score" which is the average standard deviation of your forecast (i.e. the root mean squared error). Thus, the lower your score, the better your performance. The fact that errors are squared before they're summed means that it is better to have many small errors than a few large ones.

Length of game

We cannot tell you exactly when the game will end. There is no time limit on your decisions, but, since rewards are based on cumulative profits, the more periods you play, the more money you will accumulate in the game, so there's an incentive to not take too long to make your decision. Moreover, since all the other players in the market must wait for the last player to make up his or her mind, we suggest that you do not keep your co-players waiting for too long.

Practice rounds

In order to familiarize you with the game and the mechanics of using the machine, you will have a practice session of 3 time periods .

At the end of period 4, your cumulative profits are then "reset" so that they are equal to the profits you made during that period. Your forecasting score is reset in the same manner. However, your inventory is not reset!!!

Making your decisions

When you are ready to make your decisions, click the mouse on the text fields in the lower right-hand corner of your screen to bring the cursor into the appropriate field. (Hitting the <tab> or <enter> or <return> key does the same thing.) Type in your decision and, when you're finished, click on the "OK" button to execute them. You will then be asked to confirm each decision.

After all participants have entered their decisions, the computers calculate the results for that period and advance to the next period.

Previous-period report

On the lower left part of the screen, you will see a summary report for the previous period. The report lists both prices for last period, your forecasting performance, your production, inventory and sales, and your profits, costs and cumulative profits.

Graphs and Tables

Just above your decision area in the lower left part of the screen, you will find two "pop-up menus" which allow you to look at data for previous periods in both graphical and tabular form. To use the menus, move the mouse to the appropriate menu and depress and hold down the mouse button, and the menu will "pop" up. Move the mouse (still holding down the button) to the desired item in the menu (items will turn black as you move over them) and then release the mouse button to select that graph or table.

Some graphs show variables over time while others plot one variables as a function of the other.

Tables list historical data in numerical form. Unlike the graphs, which cannot be changed, you can re-define the tables. Click on the "Define table..." button. Then use the "pop-up menus" that now appear above each column in the table to select the variable you want to list in that column. (The column-definition menus are hierarchical, i.e. each item in the main menu opens a sub-menu from which you choose your variable. It takes a bit of practice to use these.)

We encourage you to use the practice round to gain familiarity with the mechanics of the game software such as using the mouse, looking at graphs and tables, defining tables, etc.

Speaking with others

You are not allowed to communicate directly with the other players. Moreover, we request that you do not tell other people about the game until the end of the semester. We appreciate your cooperation in this matter.

Game instructions, fixed-price, complex condition

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As in life, the purpose of the game is for you to maximize your profits. For the experimenters, the purpose is to investigate how markets with different structures and price regimes behave.

Your dollar reward is based on your cumulative net profits in the game. Your cumulative profits are converted to real money using a fixed exchange rate. On average, you should earn about $30 in three hours of playing.

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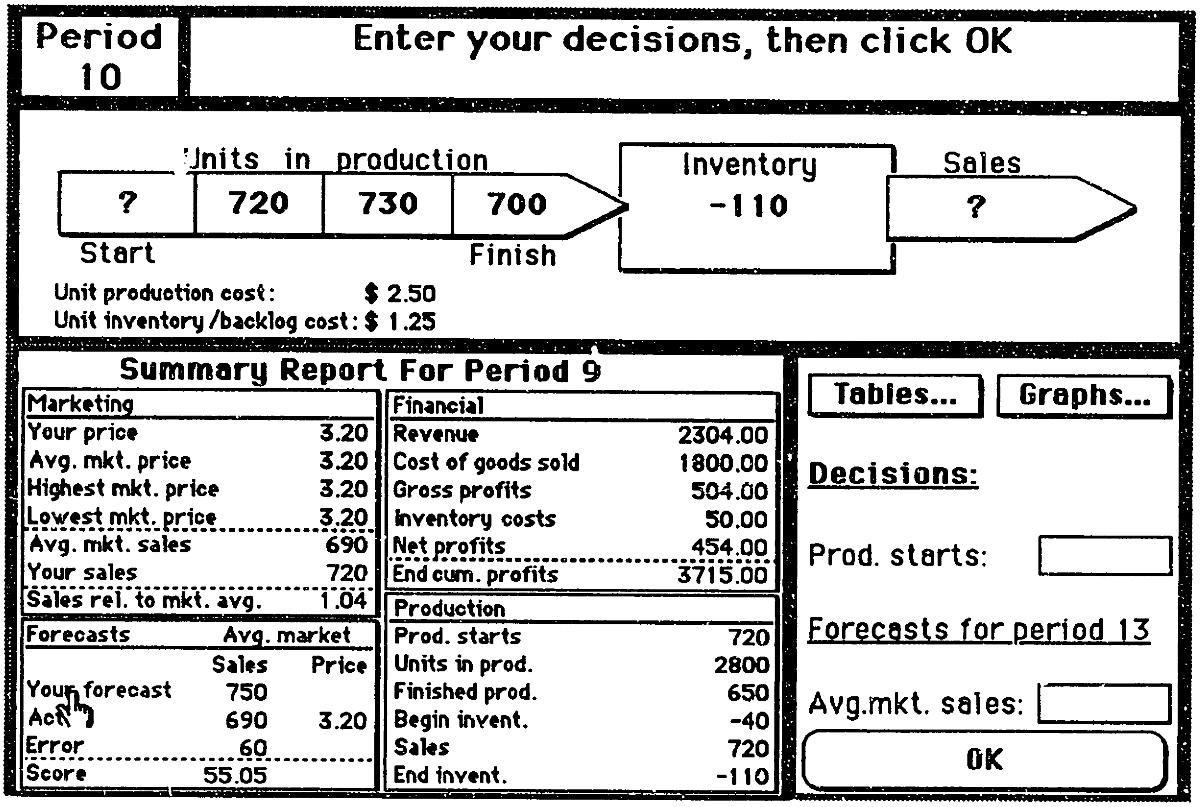

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Production, inventory, and backlog

Each period, you have only one decision to make, namely how much production to initiate. There is a three-period production lag between the time production is initiated and the time it becomes "finished", available for sale (or storage in inventory).

If you sell less than the finished production, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

Theoretically, there is no limit on how much you produce, but you cannot cancel initiated production or destroy your inventory, i.e. you cannot produce negative amounts.

Profits and costs

Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

Inventory/backlog costs are proportional to your inventory or backlog that you have at the beginning of each period. You might also say that your inventory/backlog cost is proportional to the absolute value of your inventory.

In the case where you have positive inventory, you can interpret the cost as a storage and financing charge. When you have a backlog (negative inventory), you can think of the cost as a rebate that you have to offer your customers to compensate for delayed delivery, or a penalty imposed on you for late delivery by a central planning ministry.

Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs.

Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

In this version of the game, prices are fixed by government decree.

Sales

Your sales are influenced by two factors (in other versions of the market where prices are variable, sales would also be affected by price):

- Multiplier effect You can think of the market you are in as part of a regional economy where, if every firm has a high level of production, more people will be employed, which in turn will influence incomes and thus demand for all goods in the region, including your product. Conversely, if all firms produce less, people are laid off, and demand for your product will fall somewhat.

  Thus, the demand for your product (and for the product of other firms) depends partly on the total amount of units in production. (Units in production includes both production started this period plus the amount in the "pipeline" started during the previous three periods).

- External factors Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.

  The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.

Forecasts

In addition to your production decision, we ask you to make a forecast of the average sales per firm in the market (i.e. not the total market sales, and not your own sales).

We ask you to forecast this figure 3 periods into the future. For instance, at the beginning of period 5, we ask you to forecast what you think the average sales per firm in the market will be in period 8.

On top of your profit-based reward, you will get a "bonus" of a couple of dollars, depending how you rank in your forecasting performance. Your performance is measured by a "score" which is the average standard deviation of your forecast (i.e. the root mean squared error). Thus, the lower your score, the better your performance. The fact that errors are squared before they're summed means that it is better to have many small errors than a few large ones.

Length of game

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Practice rounds

In order to familiarize you with the game and the mechanics of using the machine, you will have a practice session of 6 time periods .

At the end of period 7, your cumulative profits are then "reset" so that they are equal to the profits you made during that period. Your forecasting score is reset in the same manner.

However, your inventory and your production pipeline are not reset!!! Note that, due to the lag, your decisions will start to be important beginning in period 4.

Making your decisions

When you are ready to make your decisions, click the mouse on the text fields in the lower right-hand corner of your screen to bring the cursor into the appropriate field. (Hitting the <tab> or <enter> or <return> key does the same thing.) Type in your decision and, when you're finished, click on the "OK" button to execute them. You will then be asked to confirm each decision.

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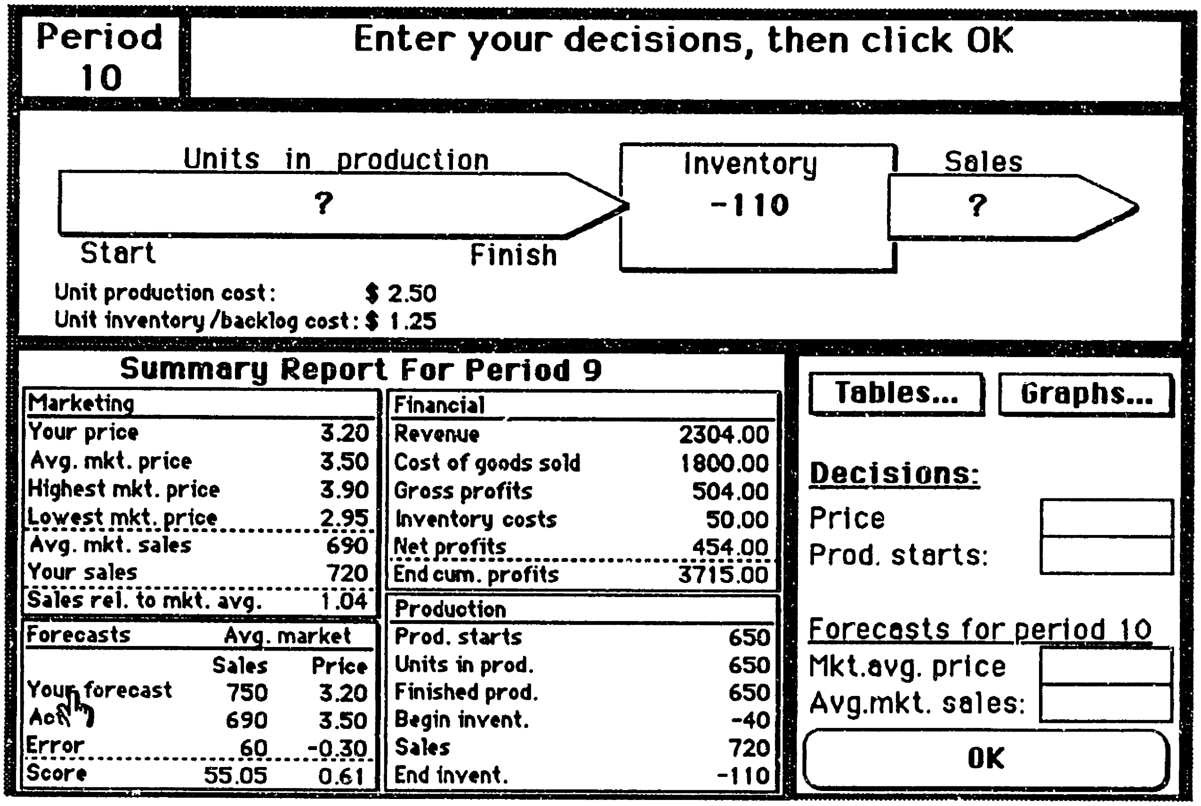

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Each period, you have two decisions to make: what price to charge for your product, and how much to produce.

Production, inventory, and backlog

If you sell less than what you produce, the rest piles up in your inventory. On the other hand, if you sell more than you have available, the excess sales accumulate in an order backlog (i.e. a negative inventory).

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Sales

Your sales are influenced by three factors:

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- The average market price charged by all firms in the market will affect how much the average firm sells, but this effect is weaker than the effect of how much you charge, relative to the other firms. Thus, if all firms increase their price, they will all sell somewhat less, but if only one firm raises its price, that firm will sell a lot less.

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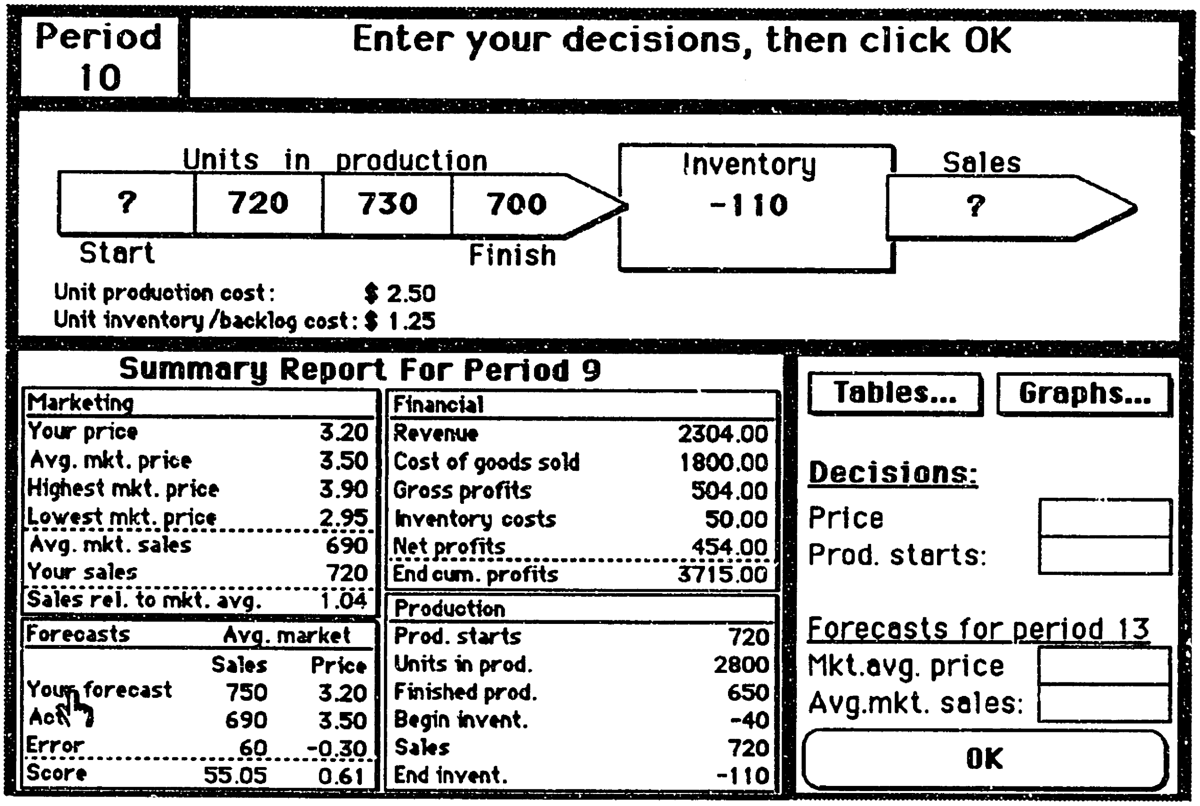

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There is a three-period production lag between the time production is initiated and the time it becomes "finished", available for sale (or storage in inventory).

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Note that both the unit production cost and the unit inventory/backlog cost are constant (and will not change at any time during the game).

Price

You are free to choose any price you want for your product. The only limitation is that price must be greater than zero. If you charge a higher price, you will make more profits per unit but also sell fewer units, as described below.

Sales

Your sales are influenced by four factors:

- Your price, relative to the average market price that period will have a strong effect on how much you sell. Since the products of your competitors are fairly similar to your product, customers are quite sensitive to price. Thus, if you charge a higher price than the market average, you will probably sell quite a bit less than the average firm does. Conversely, if you charge a lower price, you will sell more than the average.

- The average market price charged by all firms in the market will affect how much the average firm sells, but this effect is weaker than the effect of how much you charge, relative to the other firms. Thus, if all firms increase their price, they will all sell somewhat less, but if only one firm raises its price, that firm will sell a lot less.

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Game instructions, clearing-price, simple condition

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As in life, the purpose of the game is for you to maximize your profits. For the experimenters, the purpose is to investigate how markets with different structures and price regimes behave.

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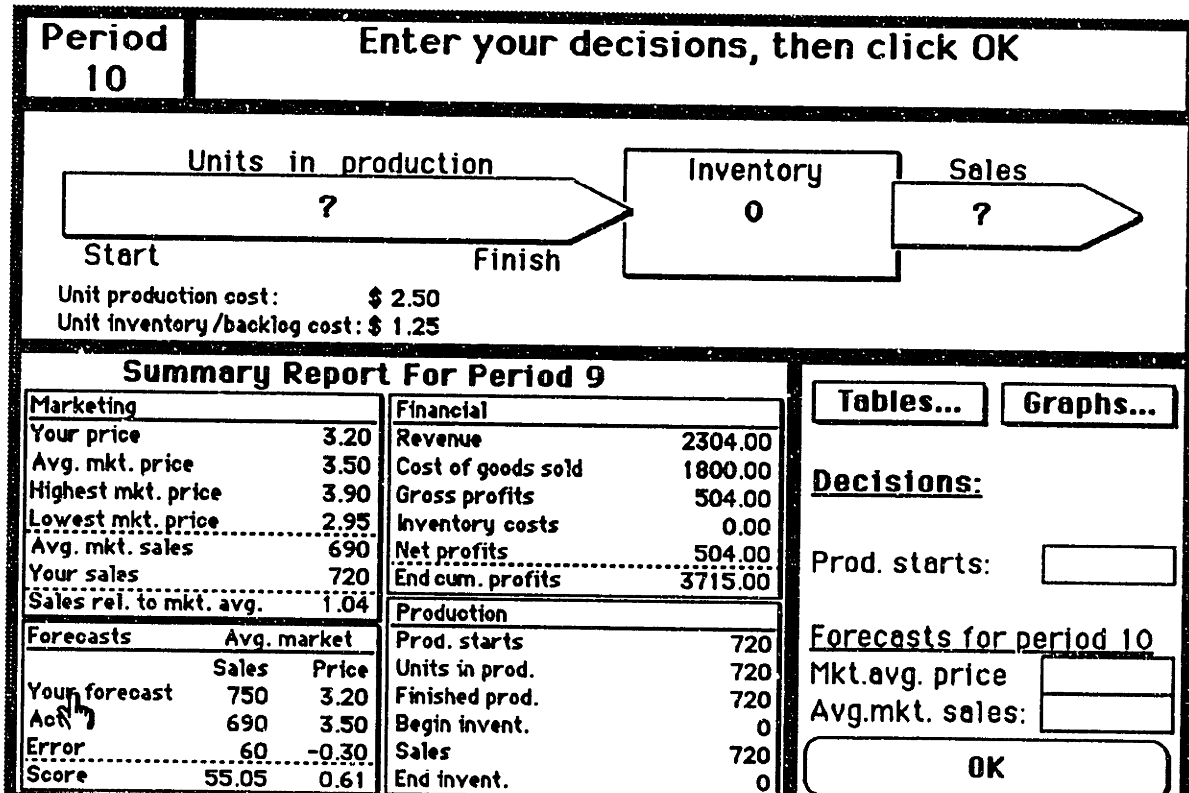

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Production, inventory, and backlog

Each period, you must decide how much production to initiate.

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Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs, but, since inventory is always zero in this version of the game, gross and net profits are the same.

Note that the unit production cost is constant (and will not change at any time during the game).

Price

The price you receive for your product depends on three factors:

- How much you are trying to sell, relative to the amounts supplied by other firms in the market. Since the products of your competitors are fairly similar to your product, this effect is relatively small. Thus, if you supply more than the market average, you will probably get a somewhat lower price, but not a dramatically lower price. Conversely, if you supply less than the market average, your price will be higher, but probably not a lot higher.

- The average amount supplied by all firms in the market will have a stronger effect on the average price received. Thus, if all firms increase their output, they will all receive a significantly lower price. Conversely, if everyone lowers their output, the average price will increase.

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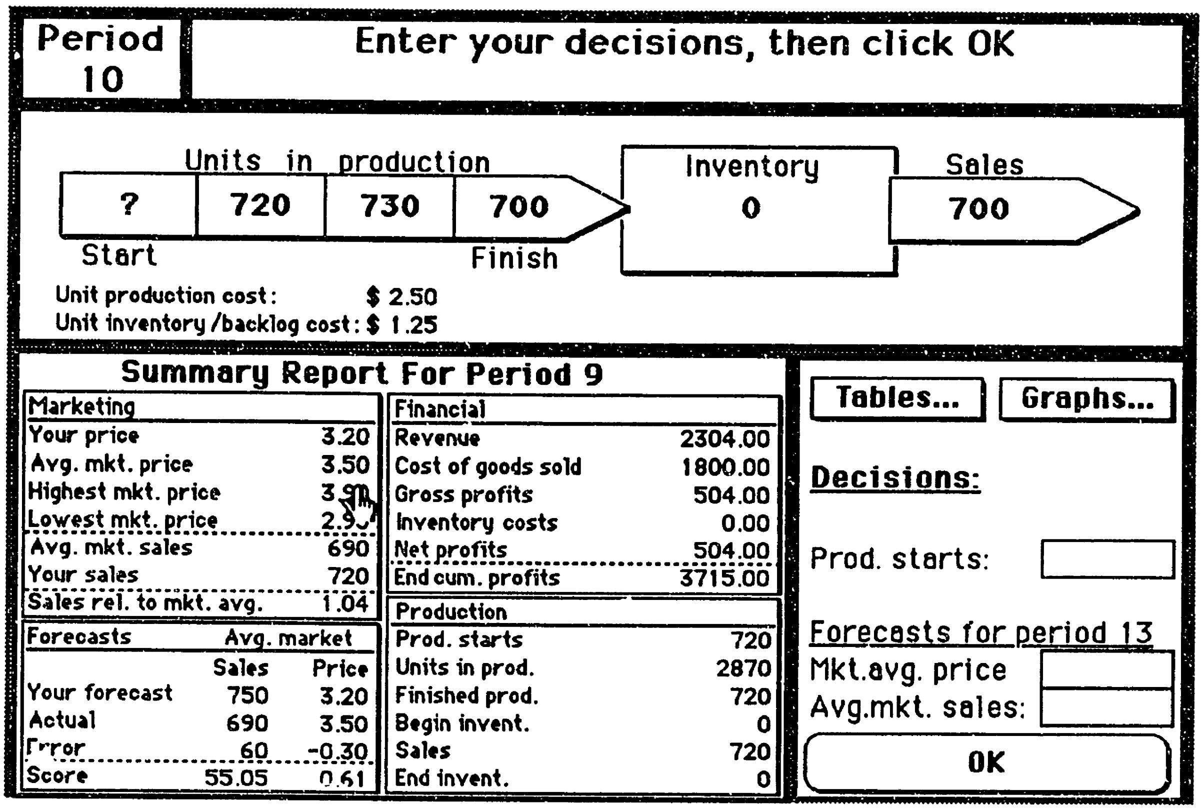

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Your profits each period is your revenue (sales x price) less your costs. Costs consist of production cost, incurred at the time of sale (called "cost of goods sold"), and inventory/backlog costs.

The "cost of goods sold" is simply proportional to sales, i.e. the unit production cost is constant.

Your gross profits are your revenues less cost of goods sold. Your net profits are gross profits less inventory/backlog costs, but, since inventory is always zero in this version of the game, gross and net profits are the same.

Note that the unit production cost is constant (and will not change at any time during the game).

Price

The price you receive for your product depends on four factors:

- How much you are trying to sell, relative to the amounts supplied by other firms in the market. Since the products of your competitors are fairly similar to your product, this effect is relatively small. Thus, if you supply more than the market average, you will probably get a somewhat lower price, but not a dramatically lower price. Conversely, if you supply less than the market average, your price will be higher, but probably not a lot higher.

- The average amount supplied by all firms in the market will have a stronger effect on the average price received. Thus, if all firms increase their output, they will all receive a significantly lower price. Conversely, if everyone lowers their output, the average price will increase.

Note that only current prices and supplies matter. Customers have no loyalty to your brand, and your market share and price in previous periods will not have any effect on your market share and price this period.

- Multiplier effect You can think of the market you are in as part of a regional economy where, if every firm has a high level of production, more people will be employed, which in turn will influence incomes and thus demand for all goods in the region, including your product. Conversely, if all firms produce less, people are laid off, and demand for your product will fall somewhat.

  Thus, the demand (i.e. the price) for your product (and for the product of other firms) depends partly on the total amount of units in production. (Units in production includes both production started this period plus the amount in the "pipeline" started during the previous three periods).

- External factors Demand for your product, as well as demand for the products of your competitors, can be influenced by exogenous factors which are independent of anything you or your competitors do in the game.

  The pattern of these "factors" will be known only to the experimenter, but you can be sure that there will not be a consistent trend of growth and decline. Moreover, these "factors" will remain in a "reasonable" range.

Forecasts

In addition to your production decision, we ask you to make both a forecast of the average sales per firm in the market (i.e. not the total market sales, and not your own sales) and a forecast of the average market price. The average market price is an average of all firms' prices, weighted by their sales.

We ask you to forecast both of these figures 3 periods into the future. For instance, at the beginning of period 5, we ask you to forecast what you think the average sales per firm and the average market price will be in period 8.

On top of your profit-based reward, you will get a "bonus" of a couple of dollars, depending how you rank in your forecasting performance. (The two forecasts will be ranked separately.) Your performance is measured by a "score" which is the average standard deviation of your forecast (i.e. the root mean squared error). Thus, the lower your score, the better your performance. The fact that errors are squared before they're summed means that it is better to have many small errors than a few large ones.

Length of game

We cannot tell you exactly when the game will end. There is no time limit on your decisions, but, since rewards are based on cumulative profits, the more periods you play, the more money you will accumulate in the game, so there's an incentive to not take too long to make your decision. Moreover, since all the other players in the market must wait for the last player to make up his or her mind, we suggest that you do not keep your co-players waiting for too long.

Practice rounds

In order to familiarize you with the game and the mechanics of using the machine, you will have a practice session of 6 time periods .

At the end of period 7, your cumulative profits are then "reset" so that they are equal to the profits you made during that period. Your forecasting score is reset in the same manner.

However, your your production pipeline is not reset!!! Note that, due to the lag, your decisions will start to be important beginning in period 4.

Making your decisions

When you are ready to make your decisions, click the mouse on the text fields in the lower right-hand corner of your screen to bring the cursor into the appropriate field. (Hitting the <tab> or <enter> or <return> key does the same thing.) Type in your decision and, when you're finished, click on the "OK" button to execute them. You will then be asked to confirm each decision.

After all participants have entered their decisions, the computers calculate the results for that period and advance to the next period.

Previous-period report

On the lower left part of the screen, you will see a summary report for the previous period. The report lists both prices for last period, your forecasting performance, your production, inventory and sales, and your profits, costs and cumulative profits.

Graphs and Tables

Just above your decision area in the lower left part of the screen, you will find two "pop-up menus" which allow you to look at data for previous periods in both graphical and tabular form. To use the menus, move the mouse to the appropriate menu and depress and hold down the mouse button, and the menu will "pop" up. Move the mouse (still holding down the button) to the desired item in the menu (items will turn black as you move over them) and then release the mouse button to select that graph or table.

Some graphs show variables over time while others plot one variables as a function of the other.

Tables list historical data in numerical form. Unlike the graphs, which cannot be changed, you can re-define the tables. Click on the "Define table..." button. Then use the "pop-up menus" that now appear above each column in the table to select the variable you want to list in that column. (The column-definition menus are hierarchical, i.e. each item in the main menu opens a sub-menu from which you choose your variable. It takes a bit of practice to use these.)

We encourage you to use the practice round to gain familiarity with the mechanics of the game software such as using the mouse, looking at graphs and tables, defining tables, etc.

Speaking with others

You are not allowed to communicate directly with the other players. Moreover, we request that you do not tell other people about the game until the end of the semester. We appreciate your cooperation in this matter.

Appendix C

Post-game questionnaire, simple conditions

Question 1

It was mentioned in the introduction to the game that overall market "demand" was influenced by two factors:

- Prices (both your own prices and the aggregate average price)
- "External factors" such as weather, macro-economic trends, etc.

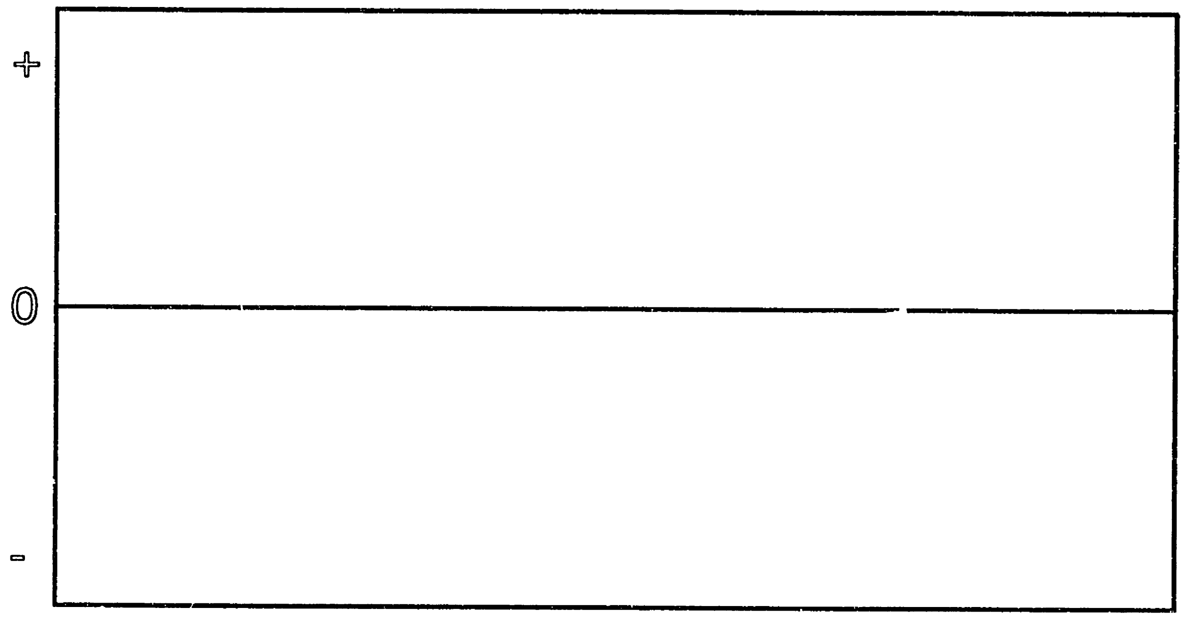
If prices were constant, or if you set your own price, a higher/lower "demand" was expressed as more/less sales. If you did not set prices but prices were found by the computer, a higher/lower "demand" was expressed in higher/lower prices.

Although you were not told what the pattern of the "external factors" was, we would like you to sketch, on the graph on the next page, a curve over time of your best guess at what you think the pattern of "outside factors" was. Remember that the factors work to raise or lower demand, <u>all other things equal</u>. Remember that demand is also influenced by prices.

The value 0 (zero), indicates a neutral influence or, if you will, the "average" influence of the external factors. Points where the curve goes above 0 would indicate times that the factors worked to raise demand, <u>all other things equal</u>, above what it would otherwise have been. Conversely, points below 0 indicate that demand, again <u>all other things equal</u> would fall below "normal".

Don't worry too much about the exact numerical values or timing--just give a rough picture of the <u>pattern</u> of the factors. If you can, try to indicate an approximate magnitude of the factors by scaling the graph. The scale is expressed as a percentage of normal. For instance, in case prices were fixed or set by you, the factors might have worked to raise sales approximately 10% above normal. In the price-clearing case (where prices were found by the computer), a value of 10% would indicate that prices were 10% above where they would have been if no such factors were present.

___ Approximate maximum (in %)



___ Approximate minimum (in %) Time period

Question 2

Tell us, in your own words, how you made your production decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to production?
- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 3

Ignore this question if you did not set your own price

Now tell us how you made your price decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to price?

- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 4

Ignore this question if you did not set your own price

Which do you feel was the "most important" decision--price or production? Explain.

Question 5

Although you do not know specifically how well other participants did in the game, you do know whether your profits were above or below the market average. What do you think you did "right" or "wrong" which led to your relative performance?

Post-game questionnaire, complex conditions

Question 1

It was mentioned in the introduction to the game that overall market "demand" was influenced by three factors:

- Prices (both your own prices and the aggregate average price)
- Overall level of production ("units in production") in the market (the multiplier effect).]
- "External factors" such as weather, macro-economic trends, etc.

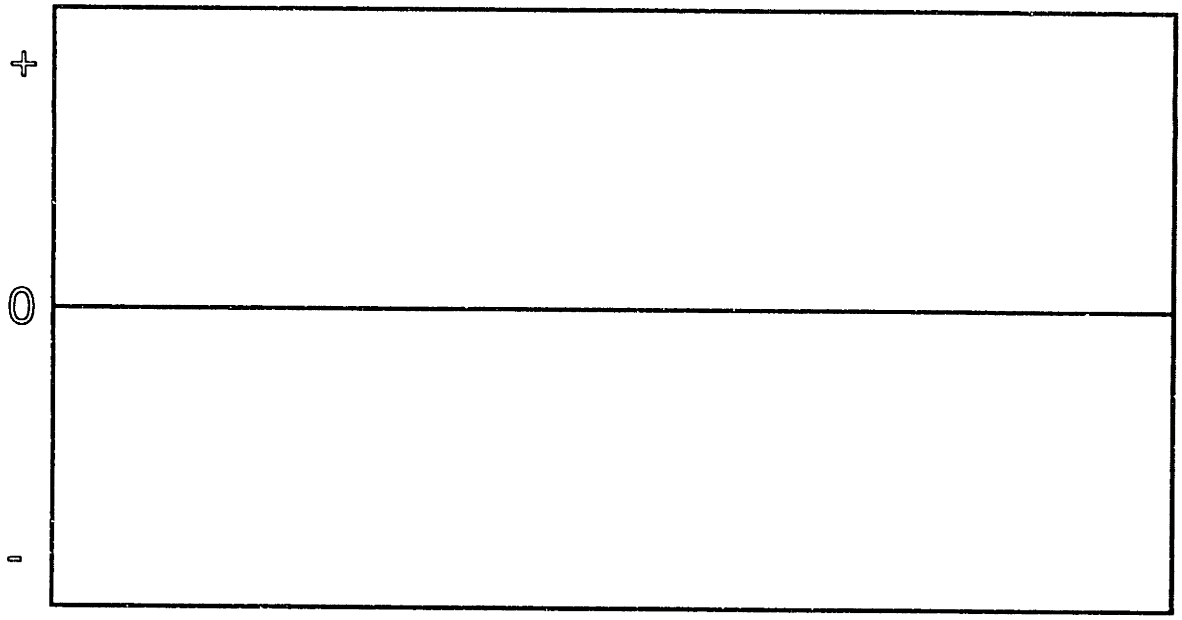
If prices were constant, or if you set your own price, a higher/lower "demand" was expressed as more/less sales. If you did not set prices but prices were found by the computer, a higher/lower "demand" was expressed in higher/lower prices.

Although you were not told what the pattern of the "external factors" was, we would like you to sketch, on the graph on the next page, a curve over time of your best guess at what you think the pattern of "outside factors" was. Remember that the factors work to raise or lower demand, all other things equal. Remember that demand is also influenced by prices and by the multiplier effect.

The value 0 (zero), indicates a neutral influence or, if you will, the "average" influence of the external factors. Points where the curve goes above 0 would indicate times that the factors worked to raise demand, all other things equal, above what it would otherwise have been. Conversely, points below 0 indicate that demand, again all other things equal would fall below "normal".

Don't worry too much about the exact numerical values or timing--just give a rough picture of the pattern of the factors. If you can, try to indicate an approximate magnitude of the factors by scaling the graph. The scale is expressed as a percentage of normal. For instance, in case prices were fixed or set by you, the factors might have worked to raise sales approximately 10% above normal. In the price-clearing case (where prices were found by the computer), a value of 10% would indicate that prices were 10% above where they would have been if no such factors were present.

___ Approximate maximum (in %)



___ Approximate minimum (in %) Time period

Question 2

Tell us, in your own words, how you made your production decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to production?
- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 3

Ignore this question if you did not set your own price

Now tell us how you made your price decision. Explain what happened in the course of the game. You might try to address the following questions in your explanation:

- What were you "trying to do"--i.e. what was your major strategy with regard to price?

- Did you have a specific procedure for arriving at a number? What were the factors of variables you primarily looked at or thought were the most relevant?
- How long did it take you to "settle" on your strategy?
- Were you surprised by the results? How?
- Were your consistent in your strategy, or did it change in the course of the game. How? When?

Question 4

Ignore this question if you did not set your own price

Which do you feel was the "most important" decision--price or production? Explain.

Question 5

Although you do not know specifically how well other participants did in the game, you do know whether your profits were above or below the market average. What do you think you did "right" or "wrong" which led to your relative performance?

Appendix D: Parameters of the experimental markets

Name	Date & time	Team	Firms	Price regime	Structure	ω	G	Periods played
Mkt 16	9/29/90 10 am	A	4	Posted	Complex	5.00	150	45
Mkt 17	9/29/90 10 am	B	4	Posted	Complex	2.50	600	40
Mkt 18	9/29/90 10 am	C	3	Posted	Complex	3.00	450	40
Mkt 19	9/29/90 1 pm	A	4	Clearing	Complex	5.00	150	40
Mkt 21	9/30/90 10 am	A	4	Clearing	Complex	3.00	450	42
Mkt 22	9/30/90 10 am	B	4	Clearing	Complex	1.80	750	50
Mkt 23	9/30/90 1 pm	A	5	Posted	Simple	3.00	450	36
Mkt 24	9/30/90 1 pm	B	5	Posted	Simple	1.80	750	35
Mkt 25	10/5/90 10 am	A	3	Fixed	Complex	3.00	450	50
Mkt 26	10/5/90 10 am	B	3	Fixed	Complex	1.80	750	40
Mkt 27	10/9/90 1 pm	A	4	Clearing	Simple	3.00	450	40
Mkt 28	10/9/90 1 pm	B	4	Clearing	Simple	1.80	750	40
Mkt 30	10/10/90 6 pm	A	6	Posted	Simple	7.20	150	40
Mkt 31	10/11/90 6 pm	A	4	Clearing	Simple	1.50	750	42
Mkt 32	10/11/90 6 pm	B	4	Posted	Simple	3.00	150	41
Mkt 33	10/11/90 6 pm	C	4	Clearing	Simple	1.20	450	50
Mkt 34	3/15/91 10 am	A	4	Fixed	Simple	3.00	300	50
Mkt 35	3/16/91 10 am	A	4	Fixed	Complex	3.00	300	45
Mkt 36	3/16/91 10 am	B	4	Fixed	Complex	1.20	450	40
Mkt 37	3/19/91 1 pm	A	4	Fixed	Simple	1.20	450	50
Mkt 38	4/26/91 1 pm	A	4	Posted	Complex	3.00	300	46
Mkt 39	5/5/91 1 pm	A	3	Fixed	Simple	3.00	300	50
Mkt 40	5/22/91 1 pm	A	5	Clearing	Complex	3.00	300	45
Mkt 41	5/24/91 1 pm	A	4	Fixed	Simple	3.00	300	50

Table D.1
Parameters of experimental markets in the primary data set

Name	Date & time	Team	Firms	Price regime	Structure	Game no.	ω	G	Periods played	Note
Mkt 01	1/24/90 10 am	A	5	Posted	Simple	1	1.00	130	23	1
Mkt 02	1/24/90 10 am	B	6	Posted	Complex	1	1.00	130	24	1
Mkt 03.1	4/23/90 10 am	A	5	Fixed	Complex	1	3.00	450	40	2
Mkt 03.2	4/23/90 10 am	A	5	Fixed	Simple	2	1.20	150	11	
Mkt 04.1	4/23/90 10 am	B	4	Fixed	Complex	1	1.20	150	40	3
Mkt 04.2	4/23/90 10 am	B	4	Fixed	Simple	2	3.00	450	15	
Mkt 05.1	4/24/90 10 am	A	4	Clearing	Simple	1	3.00	150	13	
Mkt 05.2	4/24/90 10 am	A	4	Clearing	Complex	2	1.20	450	20	
Mkt 06.1	4/24/90 10 am	B	5	Clearing	Simple	1	1.20	450	13	
Mkt 06.2	4/24/90 10 am	B	5	Clearing	Complex	2	3.00	150	20	
Mkt 07.1	4/25/90 10 am	A	4	Clearing	Complex	1	1.20	150	20	4,5
Mkt 07.2	4/25/90 10 am	A	4	Clearing	Simple	2	3.00	450	14	4
Mkt 08.1	4/25/90 10 am	B	4	Clearing	Complex	1	3.00	450	19	
Mkt 08.2	4/25/90 10 am	B	4	Clearing	Simple	2	1.20	150	15	
Mkt 09	4/26/90 10 am	A	7	Posted	Complex	1	1.20	150	26	
Mkt 10.1	4/27/90 10 am	A	4	Fixed	Simple	1	3.00	450	11	
Mkt 10.2	4/27/90 10 am	A	4	Fixed	Complex	2	1.20	150	21	
Mkt 11.1	4/27/90 10 am	B	4	Fixed	Simple	1	1.20	150	11	
Mkt 11.2	4/27/90 10 am	B	4	Fixed	Complex	2	3.00	450	17	
Mkt 12.1	4/28/90 10 am	A	4	Posted	Simple	1	1.20	150	11	
Mkt 12.2	4/28/90 10 am	A	4	Posted	Complex	2	3.00	450	13	
Mkt 13.1	4/28/90 10 am	B	4	Posted	Simple	1	3.00	450	13	6
Mkt 13.2	4/28/90 10 am	B	4	Posted	Complex	2	1.20	150	16	
Mkt 14	6/12/90 10 am	A	5	Posted	Complex	1	3.00	450	29	7
Mkt 15	6/12/90 10 am	B	4	Posted	Simple	1	1.20	150	22	7
Mkt 20	9/29/90 1 pm	B	5	Clearing	Complex	1	2.50	450	40	8
Mkt 29.1	10/9/90 6 pm	A	5	Posted	Complex	1	3.00	450	18	9
Mkt 29.2	10/9/90 6 pm	A	5	Posted	Complex	2	3.00	450	18	10

Notes

1. Pilot experiment. Subjects were not paid. Prices were constrained by the software to be in the range between 0.10 and 5.00. There was no trial period. Demand curve had constant elasticity (.75) for all prices. Intial output was 100 and initial price was 2.40. The standard deviation of random errors in demand, σ, was .05.
2. By mistake, the multiplier α was set to 0 during the first few rounds.
3. The multiplier α may have been set to 0 during the first few rounds by mistake.
4. The standard deviation of random errors in demand, σ, was set to 0.05 during the first few rounds by mistake.
5. Firm A3's game log missing until period 5.
6. almost all of firm B1's and B4's game logs missing.
7. Summer session participants. Subjects were not paid.
8. Firm B3 left early (Period 15). Firm B5's software crashed (Period 30).
9. Software crash in Period 18.
10. Game restarted with same conditions but no trial period.

Table D.2: Parameters of experimental markets in the secondary data set

Unlike the markets in the main data set, the demand of individual firms were subjected to a random, normally distributed percentage error with a standard deviation, σ, equal to 0.10. Errors were serially uncorrelated but correlated accross firms with a correlation coefficent, ρ, of .5. Furthermore, except for a few cases, all groups of participants played two games in succession with the same price regime but different feedback structures. The order of the games are indicated in the column "game number."