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# Nonlinear negotiation approaches for complex-network optimization: a study inspired by Wi-Fi channel assignment

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**Abstract** At the present time, Wi-Fi networks are everywhere. They operate in unlicensed radio-frequency spectrum bands (divided in channels), which are highly congested. The purpose of this paper is to tackle the problem of channel assignment in Wi-Fi networks. To this end, we have modeled the networks as multilayer graphs, in a way that frequency channel assignment becomes a graph coloring problem. For a high number and variety of scenarios, we have solved the problem with two different automated negotiation techniques: a hill-climbing mediated negotiation and a simulated annealing mediated negotiation. As an upper bound reference for the performance of these two techniques, we have also solved the problem using a particle swarm optimizer. Results show that the annealer negotiator behaves as the best choice because it is able to obtain even better results than the particle swarm optimizer in the most complex scenarios under study, with running times one order of magnitude below. Moreover, we study how different properties of the network layout affect to the performance gain that the annealer is able to obtain with respect to the particle swarm optimizer. Finally, we show how the different strategic behavior of the participants affects the results.

## 1 Introduction

In our current society everything is interconnected, with the Internet network as the prime example. Internet ubiquity and popularity have grown impressively in the last decades. Because of this, many of the current research problems can be modeled as interconnected nodes, i.e. as networks. We can find real-world

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problems that can be modeled by networks in key strategic fields like transportation (Ghavidelshyooki et al 2017), energy (Valori et al 2016), industrial processes (Bernini et al 2016), medical disciplines like neurology (Fornito 2016) and communication networks. This last domain is the one this paper focuses on.

More specifically, we are focused in communication networks where nodes have an important feature: they are self-interested. We will compare two different families of methods to address this kind of problems: optimization techniques and automated negotiation. The first family, optimization techniques, are well suited to large-scale problems, as networked systems use to be. However, these techniques fail when there are self-interested nodes that ignore the optimal solution taking a decision that improves their performance or utility, but severely decreases the total network performance. Thus, there is a number of works that are focused on detecting self-interested nodes (Kumar et al 2016; Banchs et al 2016). The second family, automated negotiation techniques, can reach solutions in a timely manner and consider selfish nodes in their intrinsic behavior (Ren et al 2009) so nodes will be less predisposed to deviate from the solutions given (in fact, they are usually called *agreements*). Although automated negotiation techniques could be worth to solve the type of problems we deal with in this paper, they have been barely used to solve complex networked problems (De Jonge and Sierra 2015), as their applications have been focused on designing tools for collaboration, e-commerce or decision-making support (Fujita et al 2017). In this paper we show that automated negotiation also behaves as a very interesting tool to solve complex network optimization problems where there are conflicts of interest.

The specific problem we tackle in this paper is channel assignment in Wireless Local Area Networks (WLAN) operating in infrastructure mode, i.e. consisting in access points (APs) and clients attached to those APs. This is a complex network problem that includes selfish behavior as nodes are only interested in their performance, not in the global performance of the whole network. Thus, the problem consists of assigning the frequency channel to each AP of the network that minimizes interferences and, therefore, maximizes the performance of their clients. As there are interferences between channels and the number of APs to be assigned with a channel is usually much higher than the total number of available channels, this becomes a very complex problem.

This problem fits within other generic well-known research problems like FAP (Frequency Assignment Problem) (Aardal et al 2007; FAP 2017) and the more generic graph coloring problem (Jensen and Toft 2011; Tuza et al 2003), as frequencies can be considered as colors. Regarding the graph coloring problem we can emphasize the work (Malaguti and Toth 2010) where there is a survey of the generic vertex coloring problem (VCP), whose objective is to assign a color to each vertex using different colors on adjacent vertices and minimizing the total number of colors required (Orden et al 2018). In addition to this problem, in (Malaguti and Toth 2010) it is also included a survey on other generalizations of the VCP, like the *Bandwidth Coloring Problem* (BCP), where distance between colors is taken into account forbidding those colorings where distance between two connected vertices is below a certain value. Other works that consider distances between colors are (Griggs et al 2009; Sharp 2007; Bodlaender et al 2000). All these works considering distances between colors, i.e. including hard restrictions in the graph coloring process, are different from our problem because they are focused on minimizing the largest color assigned to the vertices, while in our prob-

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lem we have a predefined number of colors (the available spectrum band where the technology is able to operate in) and we want to minimize interferences. The main works in FAP can be found in the survey (Aardal et al 2007). Although this survey is mainly devoted to channel assignment in cellular networks, military applications or satellite communications, it also includes a brief mapping of the channel assignment problem to WLAN networks. IEEE 802.11 networks (commercially known as Wi-Fi networks) are the most widespread (and, from a practical point of view, unique) WLAN networks. Although these networks can operate in different unlicensed frequency bands, the most widely used is the 2.4 GHz band, that contains 11 possible and partially overlapped channels (Gimenez-Guzman et al in press). As they are partially overlapped, only three frequencies (the first, the sixth and the eleventh channels) do not collide among them. For that reason it is often considered a three-colors problem (Aardal et al 2007), although in this work we consider the whole set of possible channels.

More closely related to our specific research are the works directly addressing channel assignment in Wi-Fi networks. It is interesting to highlight that the number of works in this field is quite limited if we consider Wi-Fi impact in our daily lives, as we are surrounded by a large and increasing number of Wi-Fi networks. This scarcity is probably due to the high complexity of the problem, being NP-hard, as stated in (Chieochan et al 2010). It is precisely in (Chieochan et al 2010) where we can find a survey of channel assignment in Wi-Fi networks. It is interesting to emphasize the works (Mishra et al 2005, 2006) as they are probably the most similar works to ours, not only in terms of their scope (channel assignment for Wi-Fi networks) but also in terms of the problem modelling (the Wi-Fi network is modeled as a graph), although we focus on the use of nonlinear negotiation techniques to solve this type of problems. In (McDiarmid and Reed 2000; Narayanan 2002) authors also use graphs for channel assignment, but not specifically in Wi-Fi. Finally, in (Abusubaih et al 2007) authors propose a coordination protocol for dynamic channel assignment in Wi-Fi networks.

In spite of all the above-mentioned works, the objective and contributions of our proposal are different from them. Our first contribution is to model the Wi-Fi infrastructure network as a multilayer graph composed by three layers. Our second contribution, and probably the most prominent, is to show that nonlinear negotiation techniques are powerful tools to assign channels to APs in comparison to well-known centralized optimization techniques like a particle swarm optimizer. Our third contribution consists of analyzing how graph properties, or network layout in terms of topology, contribute to the performance gains that nonlinear negotiation approaches offer. Our fourth contribution is the study of how the different strategic behavior of the participants affects the results. A preliminary and shorter version of this work was presented in (de La Hoz et al 2017), but in this paper we make not only a more in-depth study of the problem, including a more accurate multilayer model, but also a much more detailed experimental evaluation and analysis section.

The rest of this paper is organized as follows. In Section 2 we describe the graph-based problem model. Section 3 describes the negotiation scenario and approach. Section 4 includes the description of the performed experiments and a discussion of the obtained results. Finally, the last section concludes the paper summarizing our main contributions.

## 2 System modelling

### 2.1 Wi-Fi networks

Wi-Fi technology is the most widespread technology to deploy wireless local area networks. It is based on the family of IEEE 802.11 standards and it operates in unlicensed frequency bands, with the 2.4 GHz frequency band as the most popular. This band is divided into 11 partially overlapped channels (Ng and Szymanski 2012), although this number is dependent on the world region where the network operates. Our focus is to choose the channel where each access point (AP) will operate, and therefore, the channel where each wireless device (WD) will work, as the channel used by a WD is the same than the one used by its associated AP. This type of Wi-Fi architecture, the most widely deployed, is called infrastructure mode, where we have two different types of nodes: APs and WDs/clients, being examples in this last category devices like laptops, smartphones, TVs... From a user's point of view, the access point is usually a wireless router. Note that WDs are able to communicate to each other only through their associated APs. The decision of the channel used must aim to minimize interferences, and, therefore, optimize network throughput.

In this paper we have chosen centrally managed Wi-Fi networks, i.e. we consider that there is a wireless controller that is responsible of selecting the frequency channel of the APs it has in charge, and also to collect information related to the utility they are achieving. This type of networks, whose number is increasing (Baid and Raychaudhuri 2015), represents not only the typical enterprise Wi-Fi environment, but also the increasing number of centrally managed home Wi-Fi networks, where the Internet Service Provider (ISP) manages the APs remotely. In this setting, the role of the mediator is assigned to an external device where the wireless controllers are attached, typically using a wired network. Regarding the implementation of centrally managed schemes, in (Cisco 2007) Cisco presents a network architecture that is able to perform it. Another prominent trend in networks is their virtualization. In this sense, Software-Defined Networking (SDN) is a flexible alternative for the management of networks, including Wi-Fi networks. Using SDN, the implementation of centrally managed techniques, once the channel assignment is known, is rather straightforward, as the SDN controller applies this assignment through its Open Flow southbound API (Seyedbrahimi et al 2016). In summary, all these facts will contribute to make centralized techniques even more applicable in a near future.

### 2.2 Multilayer graph model

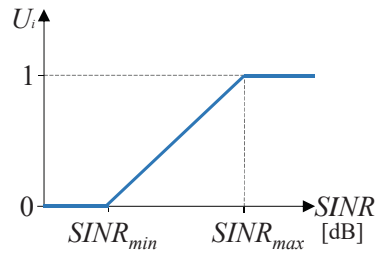
As it is stated in Section 1, FAP is a specific instance of the more generic graph coloring problem, a widely studied field, some of which main results are summarized in (Jensen and Toft 2011; Tuza et al 2003). For that reason, it is natural to model the problem of assigning channels as a graph. A frequency assignment graph is composed by a set of vertices, that represents the devices, and by edges connecting them. This way, channel assignment is reduced to a vertex coloring problem (Orden et al in press). With this model and the typical vertex coloring problem (VCP), as described in Section 1, we are not able to capture all the pe-

cularities of our Wi-Fi channel assignment problem due to the following reasons. First, in Wi-Fi networks we have two different types of vertices, APs and WDs. Therefore, we do not have to color all the vertices, only the ones that represent APs. Note that WDs will take the same color of their AP. We could be tempted to not include WDs in the graph, but this would neglect their effect in terms of interferences. Second, the meaning of the edges in the graph would not be clear, as it could mean interferences or wireless connections or the provider to which an access point belongs to. Third, we do not intend to only avoid monochromatic edges, because, as has been shown, the distance between colors (or channels) is crucial in Wi-Fi networks in terms of interferences. In conclusion, a plain graph along with the VCP is not accurate enough to capture all the peculiarities of wireless networks (E. Z. Tragos et al 2013).

Once defined the lack of information that a plain graph leads to, we next show the model we propose to more accurately capture the features of Wi-Fi deployments. We use a multilayer network graph (Kivelä et al 2014) in which each layer models a given relation between network elements. More specifically, the graph is composed by three layers. For each layer we differentiate two different types of vertices: APs and WDs. The information that each layer captures is as follows:

- *Layer a* shows the relation between WDs and APs, i.e. in *layer a* edges represent the attachment between each WD and the AP it is associated to. Although a different assumption could be done, we have linked each WD with its closest AP. Remember that when we assign a color to an AP, all the WDs it has associated will also receive the same color, i.e. all the nodes linked in *layer a* will receive the same color.
- *Layer b* models the interference between neighboring nodes, so two nodes are linked provided the distance between them is lower than a value  $R$ , that is obtained from the sensitivity of the reception antennas. Note that two WDs do not collide if they are associated to the same AP, as these communications are coordinated by the AP. The same applies for an AP and all the WDs it has associated. The interference model used is discussed in Section 2.4.
- Finally, *layer c* links the access points that are controlled by a network provider or ISP. This is a common situation, as it is usual to have a small number of network providers coexisting in the same place. It is very interesting to include this layer, as each provider is able to control their own APs. Of course, this is a general situation, as we could consider that there is only a network provider for the case of the management of the wireless network in a corporation, for example. Moreover, the inclusion of this layer is essential for the negotiation point of view, because an ISP could sacrifice the performance of a node for the sake of improving the performance of others, what enables trade-offs in negotiations.

Note that this model does not restrict the communication between WDs associated to different APs, as in the most cases APs are connected to the rest of the network (including the rest of APs and to the Internet in general) through wired links. As these wired links do not affect at all the wireless communications, they do not appear in the multilayer graph.



**Fig. 1** Relation between utility and  $SINR$ .

### 2.3 Utility of a channel assignment

The performance of a certain channel assignment (network coloring) will be shown in terms of its utility. We define by  $U_i$  the utility of node  $i$  (either an AP or a WD) as a measure ranging from 0 to 1 that depends on the throughput perceived by the user. When the throughput is equal to its maximum value we will have  $U_i = 1$  and when the throughput decreases the utility will also monotonically decrease. When the node cannot keep connected to the network, and therefore its throughput equals 0, then  $U_i = 0$ . In other words, the utility of a node can be considered as a normalized throughput. A parallel reasoning can be done in terms of the signal to noise ratio of terminal  $i$ , defined by  $SINR_i$ . As it is shown in (A. Bazzi 2011), when  $SINR$  is above a certain value  $SINR_{max}$ , the network throughput reaches its maximum value for a node, so higher  $SINR$  values will not lead to better throughputs. Additionally, when  $SINR$  is below a certain value  $SINR_{min}$  the wireless node cannot keep connected, so its throughput (and utility) equals to 0. Note that the values for  $SINR_{max}$  and  $SINR_{min}$  thresholds have been taken from a realistic point of view (Geier 2017). A graphical representation that shows the relation between  $SINR$  and  $U_i$  is shown in Fig. 1.

For the moment, we have only defined the utility for a certain node. However, as we are interested in the whole utility of a particular channel assignment, we will sum the utility for all the nodes in the graph, i.e.

$$U = \sum_{\forall i} U_i.$$

Moreover, and for the right operation of the nonlinear negotiation techniques, we have also to define the utility for a provider  $p_i$ , defined by  $U_{p_i}$ . This value  $U_{p_i}$  is defined as the sum of the utilities for all its APs and the WDs attached to those APs

$$U_{p_i} = \sum_{\forall i \in p_i} U_i.$$

### 2.4 Interference model

As seen in Section 2.3, the utility of a particular channel assignment depends on the  $SINR$  value of each node ( $SINR_i$ ). As usual,  $SINR_i$  is computed as the ratio

between the received power of the desired signal ( $P_i$ ) and the sum of the received unsought interferences ( $I_{j \rightarrow i}$ ):

$$SINR_i = \frac{P_i}{\sum_{\forall j \in J} I_{j \rightarrow i}},$$

being  $J$  the set of wireless nodes (either WDs or APs) that produce interferences to node  $i$ . Note that, as APs will have so many  $SINR$  values as the number of WDs they have associated, we will take as  $SINR$  for each AP the worst case, i.e. the minimum  $SINR$  value.

Finally, to be able to compute the signal strengths required for computing  $SINR$  we have to take into account interfering signals. In fact, the weight of the edges of the graph *layer*  $b$ , as described in Section 2.2, represent these interferences. Three are the main elements that affect the signal strength: distance ( $d$ ), co-channel interference ( $W_{xy}$ ) and activity index ( $\psi$ ). In the following, we describe the impact of these three elements.

Distance is probably the most evident factor that affects signal strength. We have made use of the propagation model defined in (Green and Obaidat 2002), so we compute the power loss ( $P_{loss}$ ), expressed in dB, for a 2.4 GHz signal as

$$P_{loss} = 7.6 + 40 \log_{10} d - 20 \log_{10}(h_t h_r) + L_{obs},$$

being  $h_t$  and  $h_r$  the heights where transmission and reception antennas are located and  $L_{obs}$  the power losses due to obstacles in the propagation as walls or windows (expressed in dB). As a result of considering the distance between network nodes, the weight assigned to an edge  $ij$  will depend on the distance  $d$  between nodes  $i$  and  $j$ , so our graph is geometrical and no longer abstract, as it is usually the case for the VCP.

Probably, the most peculiar feature of Wi-Fi networks is the partial overlap between channels. This behavior is captured by adding the co-channel interference index to the signal propagation. In this way, a transmission in channel  $i$  will affect another transmission in channel  $j$  with a fraction of its nominal power depending on the “distance” between channels in the spectrum, represented with matrix  $W_{xy}$ . The values for this co-channel interference have been taken from the empirical study conducted in (Ng and Szymanski 2012). Note again that the VCP does not consider the distance between colors, so our model is more general.

Finally, we have considered that data flows do not occupy the frequency channels permanently, but they use the spectrum a certain ratio of time. It is obvious that when a node emits with a higher ratio, the interferences it causes to other nodes are more harmful. This fact is represented by the *activity index*, ( $\psi$ ). From all the above, the interference signal from node  $i$  to  $j$  ( $I_{j \rightarrow i}$ ) that operate in channels  $x$  and  $y$  respectively can be computed as:

$$I_{j \rightarrow i} = W_{xy} + \psi + P_t + G_t + G_r - P_{loss},$$

where every value is expressed in logarithmic scale and  $P_t$  represents the transmission power and  $G_t$  (resp.  $G_r$ ) represents the transmission (resp. reception) antenna gain.



### 3 Wi-Fi frequency assignment as a negotiation process

As stated above, the problem we want to tackle in this paper is the coordination between APs to select the most appropriate frequencies (channels) in order to reduce interference. We want to address this problem by means of automated negotiation (Fatima et al 2014). In this section, we describe the problem as a negotiation process, using the key elements traditionally used to characterize this kind of processes (Fatima et al 2001): the negotiation domain, the interaction protocol and the decision mechanisms.

#### 3.1 Negotiation domain

The negotiation domain defines the scope of the negotiation (basically what is negotiated and among whom). In this paper, we propose to see the channel assignment problem as a multiattribute negotiation, where an agreement would be to collectively assign a value to the channel assigned to each of the access points, being these channels the attributes (or *issues*) under negotiation. That is we will consider solutions or contracts  $S$  of the form  $S = \{s_i | i \in 1, \dots, n_{AP}\}$ , where  $n_{AP}$  is the number of access points. Here,  $s_i \in \{1, \dots, 11\}$  stands for the channel which has been assigned to the corresponding access point  $i$ .

As stated in the previous sections, we will assume a bilateral negotiation scenario, with two negotiating agents  $p_1, p_2$ , corresponding to two network providers (commonly ISPs), each of which has jurisdiction over a subset of the APs. This has the advantage that there are more works in the literature to compare with than for the three or more agents instance. Each agent will have a utility model based on the interference model described in Section 2. With these assumptions, the resulting utility spaces will be non-monotonic and highly rugged, with many local optima (Hattori et al 2007).

#### 3.2 Interaction protocol

The interaction protocol defines the rules of the negotiation process. There is a wide variety of protocol proposals in the literature for bilateral and multilateral negotiations (Rubinstein 1982; Hattori et al 2007). Since we expect the utility spaces to be highly rugged, and in a similar way as we did in (Marsa-Maestre et al 2009), we will use here a simple text mediation protocol (Klein et al 2003). In the following we briefly describe the protocol:

1. The mediator generates a random candidate contract ( $S_0^c$ ), thus effectively selecting a random channel for each AP as the initial solution.
2. At time  $t$  (starting in  $t = 0$ ), the mediator sends contract  $S_t^c$  as a proposal for the negotiating agents (i.e.,  $p_1, p_2$ ).
3. The negotiating agents then vote on the contract  $S_t^c$ , either accepting or rejecting it.
4. For  $t = t + 1$ , the mediator builds a new contract  $S_{t+1}^c$  taking into account the received feedback and goes back to step 2.
5. After a fixed number of iterations, the process ends and a final agreement is declared.

This protocol definition has to be augmented with the appropriate decision mechanisms (or strategies) for the negotiating agents and the mediator. In the following, we define these decision mechanisms.

### 3.3 Decision mechanisms

The decision mechanisms or strategies define how agents behave when facing the different situations that may occur during the negotiation. In our case, we have to define decision mechanisms for the negotiating agents and for the mediator. Negotiating agents have to decide which votes to cast when confronted with a proposal  $S^c$  from the mediator. We have considered two different strategies:

- *Hill-climber (HC)*: This is a greedy utility maximization approach, where agents accept a proposal when it yields at least the same utility that the previous commonly accepted proposal.
- *Annealer (SA)*: *Simulated annealing* (Klein et al 2003) is a widely used method to avoid getting stuck in local optima during optimization processes. Contrary to greedy utility maximization, there will be a finite probability  $P_a$  for the acceptance of a proposal even when it makes the agent to lose utility.  $P_a$  is defined as  $P_a = e^{-\frac{\Delta u}{\tau}}$ , where  $\Delta u$  is the utility loss for the new proposal, and  $\tau$  is an annealing temperature parameter, which linearly decreases to zero during the course of the negotiation according to  $\tau = \tau_0(1 - \frac{t}{T})$ , being  $\tau_0 = 1$  the initial temperature and  $T = 3000$  the maximum number of iterations of the algorithm. In this way, agents are more flexible at the beginning of the negotiation and become more greedy as the deadline approaches.

The mediator, on the other hand, has to decide which new contract to propose to the negotiating agents at each iteration. We have considered here a single-text mediation mechanism (Klein et al 2003) :

1. The new proposed contract  $S_{t+1}^c$  is built from a base contract  $S^b$ , which is the last contract upon which all negotiating agents have voted *accept*.
2.  $S_{t+1}^c$  is obtained by random, single-issue mutation from the base contract  $S^b$ . That is, the mediator randomly varies the assigned frequency for a randomly chosen AP.
3. When the deadline expires, the last contract upon which all negotiating agents have voted *accept* is assumed to be the final agreement.

This protocol has a number of desirable properties for our problem. First, it does not require revelation of agent preference information (agents just vote in favor or against proposed contracts), which makes it adequate for competitive settings. In addition, its computation and time cost is adjustable via the number of iterations, which makes it adequate for scenarios where performance is important (as it is the case of wireless networks applications). Finally, the protocol has been proven successful in academic nonlinear negotiation scenarios (Klein et al 2003; Lang and Fink 2015).

## 4 Experimental settings

### 4.1 Scenarios under study

For the scope of this work, we consider a typical, realistic Wi-Fi configuration, similar to the one used in (de la Hoz et al 2015). Readers are referred to this paper for the specifics of the Wi-Fi configuration parameters. From an intuitive point of view, these scenarios could represent a realistic situation in residential or commercial managed buildings, where there are a number of different households or small businesses which get their internet access from an AP belonging to one of the available ISPs, and where there is an entity in charge of the building which is interested in social welfare (i.e. that all households/businesses are satisfied with their experience in the building). In this setting, the different ISPs will play the role of negotiating agents, while the managing entity would play the role of mediator.

Furthermore, we assume that APs and WDs do not move during the experiments. Regarding WDs, we have placed them randomly throughout the scenario, while for the APs we have considered two different types of scenarios. In the first case, APs are distributed randomly, as in (Marsa-Maestre et al 2016). In the second one, APs are located in the junctions of a square grid. According to these positions of the deployed APs, we will call the first type of scenarios *random*, and the second one *square*.

We have considered scenarios with different number of APs and WDs per AP. More specifically, we have considered scenarios with 15, 50 and 100 APs and for each of these values, we have considered two different settings, with 1 and 5 WDs per AP. For simplicity, we use the nomenclature  $(i, j)$ , being  $i$  the number of APs and  $j$  the number of WDs, having the following combinations: (15, 15), (15, 75), (50, 50), (50, 250), (100, 100) and (100, 500). For each one, we consider 50 distinct settings, resulting in 600 scenarios (300 for random scenarios and 300 for square scenarios). These scenarios, which are an extended version of the ones in (Marsa-Maestre et al 2016), represent very different types of scenarios in terms not only of the problem size (we consider scenarios from a few tens to several hundreds of devices) but also in terms of variability, due to the random generation that produces the graphs. Moreover, the variability is higher if we consider that we have taken away from the scenarios those wirelessly unconnected nodes. As we consider a bilateral negotiation, we have evenly divided APs into two random groups, one for each agent. An example of one of the scenarios under study is the one shown in Figure 2, that corresponds to a square scenario of the category (100, 500).

### 4.2 Reference techniques

We have compared our results with the next approaches:

- *Random Reference*: the simplest possible technique, where each AP is assigned a channel in a random, uniform manner.
- *Augmented Lagrangian Particle Swarm Optimization (ALPSO)*: this is a parallel particle swarm optimizer, which solves nonlinear non-smooth constrained problems using an augmented Lagrange multiplier approach to handle con-

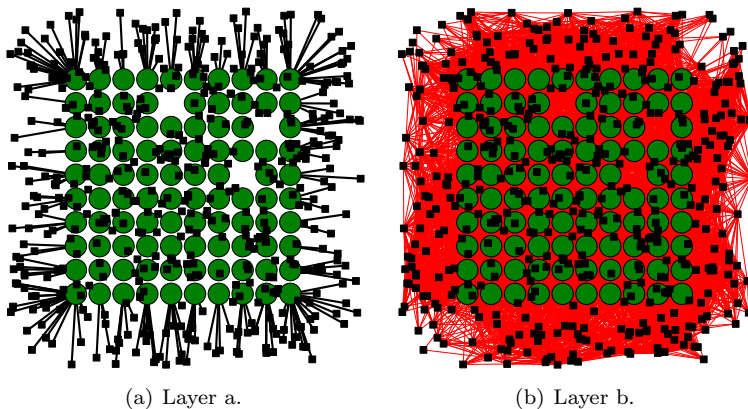


Fig. 2 (100, 500) square scenario example.

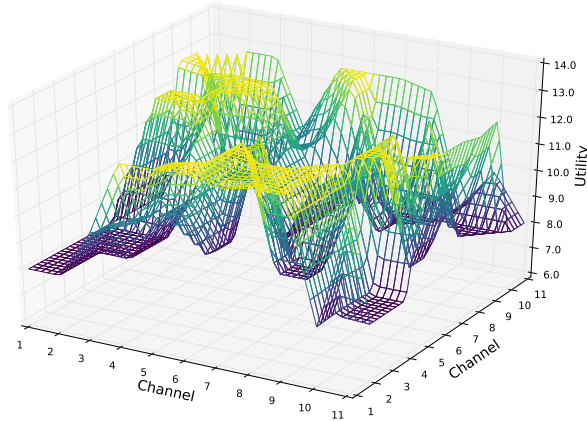
straints (Jansen and Perez 2011). This provides the perspective of centralized, complete-information optimization.

#### 4.3 Graph metrics under consideration

An interesting question we intend to give answer in this research is what is the effect of the network-related features of a problem on the adequacy of the different techniques to tackle the problem. With this long-term goal in mind, we have analyzed our set of scenarios according to a set of metrics taken from the graph theory literature. Some of these metrics are directly or indirectly related to graph size, such as the *order* (number of vertices), or the *diameter* (higher number of hops between any two vertices) (Newman 2010). The other metrics give us an idea of the structural properties of the graph regardless of its size. The *Wiener index* (Wiener 1947), for instance, assesses complexity considering distances between nodes, yielding a metric  $W(G) = \frac{1}{2} \sum_{i=0}^{|N|} \sum_{j=0}^{|N|} d(n_i, n_j)$ , with  $d(n_i, n_j)$  being the lowest number of hops of the path linking  $n_i$  and  $n_j$ . *Graph density* is the global relative connectedness of the graph (i.e. number of edges) when compared to a graph that is fully-connected. *Clustering coefficient* is a similar metric but with a local meaning, computing the density for each node's local cluster (i.e. its neighbors and itself) and then averaged. Finally, from the diverse centrality metrics used to rank the importance of vertices in a graph, we have chosen the betweenness centrality, which gives more importance to those nodes which are part of a bigger proportion of the shortest paths in the graph (Koschützki et al 2005).

### 5 Performance evaluation

Now we show the main results of the evaluation. The first results (Tables 1 and 2) show the performance obtained for each of the studied techniques for each of the scenario categories. Note that a scenario category is given in terms of the number



**Fig. 3** Two-dimensional projection of the utility function for a (15,15) scenario.

of APs and WDs. Also note that performance is given as the sum of utilities for both providers, i.e.  $U_{p_1} + U_{p_2}$ . Moreover, each category comprehends 50 different graphs and for each graph we have performed 10 different repetitions. Results show that, for the simplest scenarios all the techniques, with the exception of *random*, perform fairly similar. However, as the scenarios get more complex the annealer *SA* becomes the best choice, followed by the hill climber *HC* and the particle swarm optimizer *ALPSO*. Furthermore, *random* becomes worse as the complexity of the scenarios increase. The difference in performance between *HC* and *SA* means that the scenarios under study are highly nonlinear (Klein et al 2003; Marsa-Maestre et al 2009), since one of the strengths of the annealer is its ability to escape from local optima. As an illustration of this high ruggedness of the utility function, in Figure 3 we show a *two-dimensional projection* of the utility function for one of the simplest scenarios, with 15 access points and 15 wireless devices, over two random dimensions (remember there is one issue per AP, so the utility function is, in this case, 15-dimensional).

If we compare the utility of the random scenarios and their counterpart square scenarios, we perceive that the utility in random scenarios is lower. This is due to the fact that in the square scenarios, as the APs are evenly distributed, the number of APs that have no nearby clients, and therefore are removed from the graph, is lower than in the random setting. Finally, it is important to note that the results of the annealer *SA* are achieved 8 to 10 times faster than with the particle swarm *ALPSO*. The results in terms of the running time needed to obtain the previous performance results are summarized in Tables 3 and 4. The experiments have been run in an Intel<sup>®</sup> Core<sup>™</sup> i7-2600 machine with 8 CPUs@3.40GHz and 8GB RAM, running Ubuntu 14.04.4 LTS.

**Table 1** Utility for different techniques in random scenarios (Marsa-Maestre et al 2016).

(APs,WDs)	Random		HC		SA		ALPSO	
	avg	std	avg	std	avg	std	avg	std
(15, 15)	12.45	1.90	15.88	0.02	15.86	0.04	15.86	0.03
(15, 75)	30.57	5.18	52.53	1.35	53.85	0.50	52.95	0.93
(50, 50)	29.17	4.15	50.40	0.89	51.08	0.52	50.06	0.98
(50, 250)	60.28	9.44	125.24	4.71	134.96	2.34	125.51	3.80
(100, 100)	45.37	5.48	84.90	2.39	88.33	1.52	83.53	2.25
(100, 500)	86.21	11.68	188.13	7.93	208.23	4.33	191.43	6.25

**Table 2** Utility for different techniques in square scenarios.

(APs,WDs)	Random		HC		SA		ALPSO	
	avg	std	avg	std	avg	std	avg	std
(15, 15)	16.57	2.67	24.18	0.05	24.15	0.06	24.15	0.06
(15, 75)	42.11	7.01	74.59	1.91	76.91	0.69	74.77	1.68
(50, 50)	32.96	4.43	60.26	1.34	61.80	0.82	59.48	1.45
(50, 250)	68.86	9.69	145.28	5.21	157.19	2.63	144.12	4.32
(100, 100)	49.36	4.55	91.05	2.40	94.64	1.63	89.11	2.38
(100, 500)	102.00	10.44	205.90	7.10	221.68	4.20	201.91	5.12

**Table 3** Run time (in seconds) for different techniques in random scenarios (Marsa-Maestre et al 2016).

(APs,WDs)	HC		SA		ALPSO	
	avg	std	avg	std	avg	std
(15, 15)	0.53	0.21	0.64	0.22	0.25	0.19
(15, 75)	5.79	1.22	5.96	1.23	5.86	2.00
(50, 50)	5.22	1.16	5.40	1.17	11.91	5.02
(50, 250)	69.39	6.44	69.32	6.36	285.89	74.37
(100, 100)	22.01	2.96	22.15	2.99	108.14	31.39
(100, 500)	330.38	17.23	326.90	16.61	3225.63	817.93

**Table 4** Run time (in seconds) for different techniques in square scenarios.

(APs,WDs)	HC		SA		ALPSO	
	avg	std	avg	std	avg	std
(15, 15)	1.35	0.12	1.55	0.13	0.65	0.10
(15, 75)	17.30	1.41	13.64	0.99	13.25	2.81
(50, 50)	13.05	1.07	13.64	1.23	23.53	4.42
(50, 250)	115.12	4.74	122.93	4.54	362.52	65.05
(100, 100)	47.97	3.23	47.02	3.22	187.34	33.32
(100, 500)	433.16	12.02	463.59	14.75	3985.70	797.10

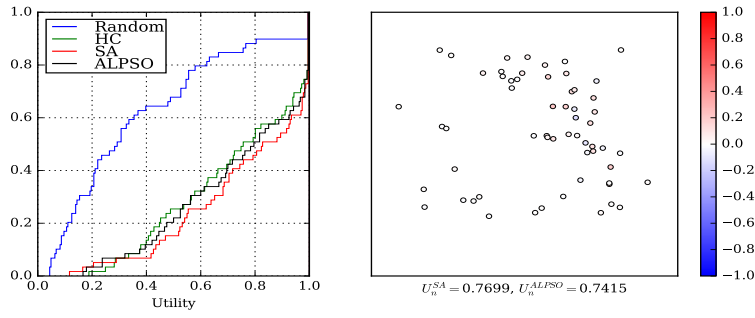
Tables 1 and 2 offer us a first and clear comparison between the different techniques under study. However, Figs. 4 and 5 show us a more profound insight for some specific selected scenarios (for the sake of space), although the same conclusions can be given for the rest. Note that, also for the sake of brevity, we have focused on the larger scenarios, omitting the categories (15, 15) and (15, 75).

Figures on the left show the cumulative distribution function (cdf) of the utility experienced by the different nodes in the network, so a point  $(x, y)$  represents that a fraction of  $y$  nodes have a utility below  $x$ . For that reason, lower curves are better. As an example, if we focus on Fig. 4a, and for  $Utility = 0.8$ , we can observe that for *SA*, around 45% of the nodes have utilities below 0.8. However, this percentage grows up to 50%, 52% and 90% for *ALPSO*, *HC* and *Random*, respectively. Obviously, as this percentage is lower, it means that there are less nodes with utility under 0.8, i.e. more nodes are above this utility. In all cases, we can conclude that the worst technique is clearly *Random*, while the best one is *SA*. Moreover, the performance of *HC* and *ALPSO* is fairly similar. Finally, to compare the performance of *SA* and *ALPSO*, in the figures on the right we show heat maps of the difference in the utility achieved by each node for *SA* and *ALPSO*, i.e.  $U_i^{SA} - U_i^{ALPSO}$ ,  $\forall i$ , being  $i$  the  $i$ -th node. For that reason, red colorings represent situations where *SA* outperforms *ALPSO*, while blue colorings represent the opposite situation. Note also that each heat map in the right corresponds to the same scenario whose results are shown in the cdf on the left. From those heat maps, as the color of most of the dots is very similar to white, we can conclude that the utilities given by *SA* and *ALPSO* are not very differently distributed, and that the differences between both techniques are distributed among a high number of nodes, i.e. many of the nodes are light red colored. Note that in some few cases, we have noted the situation given in Figure 4(c), where there are three nodes in dark red, so *SA* greatly improves *ALPSO* for them. It is also important to point out that we have not found any point in dark blue for any of the 600 scenarios under study. Finally, heat maps also show in their lower part the mean utility achieved by *SA* ( $U_n^{SA}$ ) and *ALPSO* ( $U_n^{ALPSO}$ ).

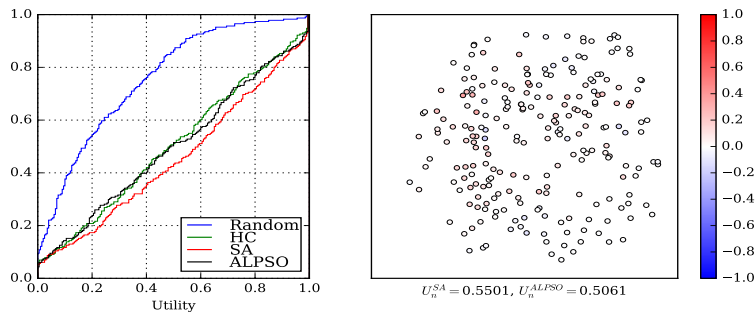
Next, we study how the different graph metrics described in Section 4.3 influence the gain that the annealer *SA* is able to obtain with respect to the particle swarm optimizer *ALPSO*. In Figure 6 it is shown the quotient between the mean utility reached by *SA* in the 10 runs for each graph and the same value reached by *ALPSO* for the same graph (the dashed line with value 1 represents the values where the performance of *SA* and *ALPSO* coincide). Note also that blue squares correspond to random scenarios while red crosses correspond to square ones.

Attending to the graph order (Figure 6a) we note an almost linear increasing trend with this metric, reaching performance gains up to 10% for the largest graphs. This behavior was also glimpsed in Tables 1 and 2. A similar behavior is shown for the Wiener index in Figure 6f. Figure 6b shows the opposite behavior when dealing with the average betweenness centrality. The best performance for *SA* is obtained for low values of centrality and this behavior can be explained by the fact that the negotiations of the annealer are expected to be easier as there are less interfering nodes, and therefore, more interfering vertices. A similar argument can be given for graph density where *SA* performs better for low density graphs, i.e. with less interferences, as shown in Figure 6c. In Figures 6d and 6e we show that there are optimal values for the diameter and cluster coefficient that make that *SA* performs better than *ALPSO*. In this last figure we have obtained an expected and interesting result, as the improvement achieved by *SA* in almost complete graphs (i.e. with high clustering coefficient) cannot be very high, as the room for improvement is more limited.

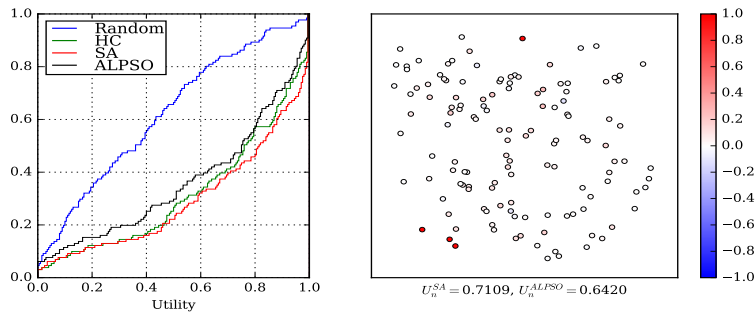
Finally, we now briefly discuss the strategic implications of the Wi-Fi negotiation setting. So far, we have just considered both agents playing the same strategy



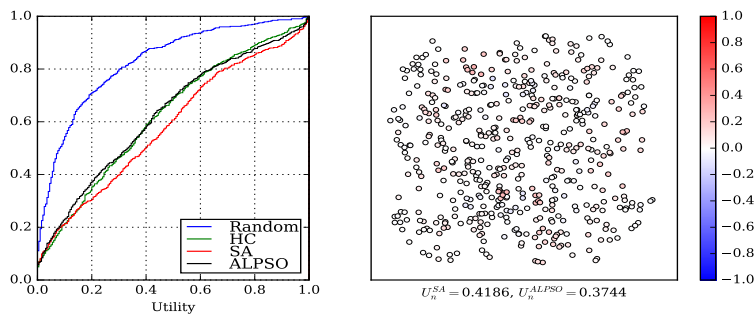
(a) (50, 50).



(b) (50, 250).



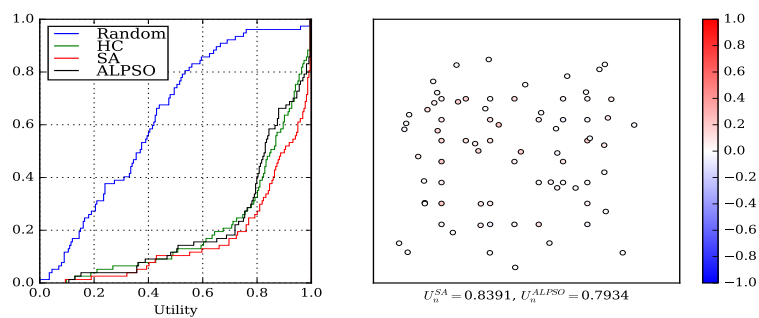
(c) (100, 100).



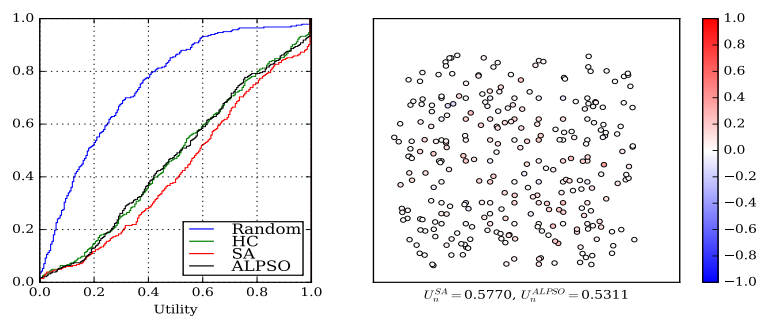
(d) (100, 500).

**Fig. 4** cdf of the utility and comparison of the utility achieved by SA and ALPSO in random scenarios.

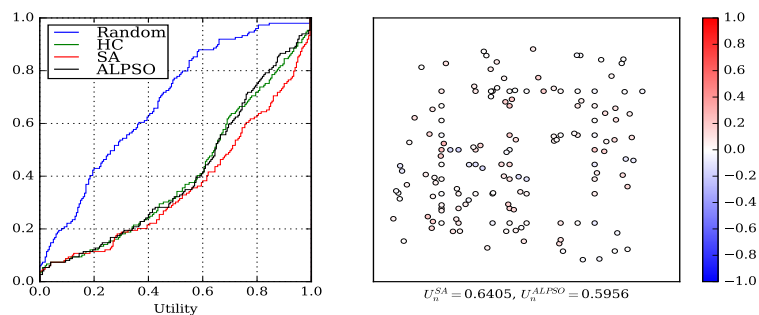




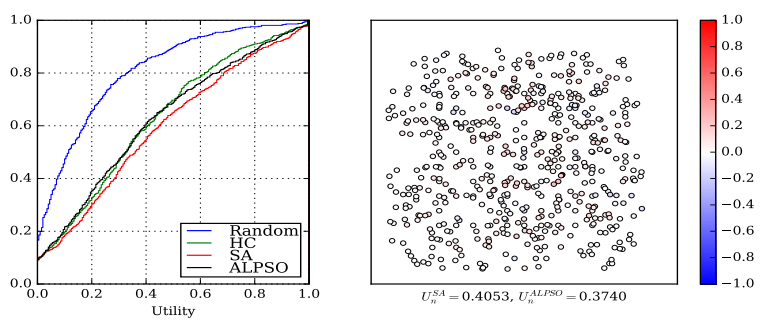
(a) (50, 50).



(b) (50, 250).

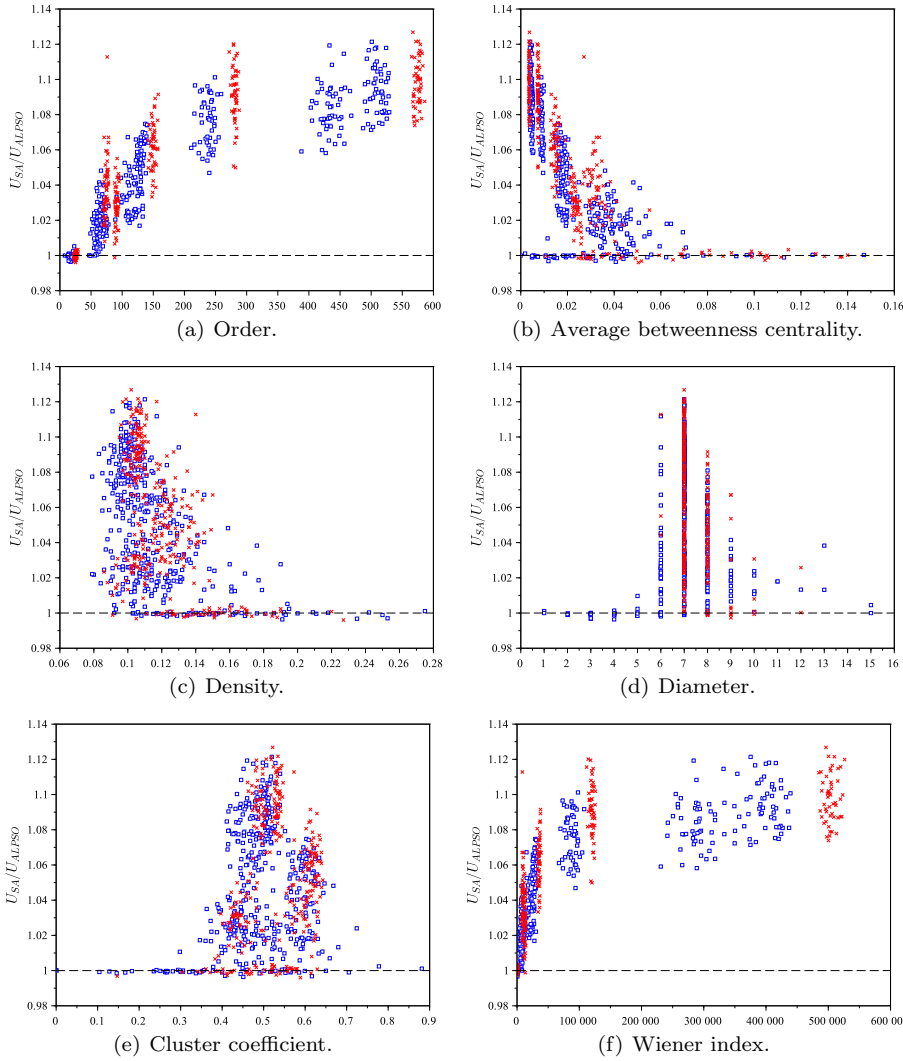


(c) (100, 100).



(d) (100, 500).

**Fig. 5** cdf of the utility and comparison of the utility achieved by SA and ALPSO in square scenarios.



**Fig. 6** Utility of SA relative to ALPSO for different graph metrics.

(i.e., hill-climbing or annealing) in the negotiation. Now we will see what happens if the agents may choose one or the other strategy freely. Table 5 shows the payoff matrices for both agents in all strategy combinations for the different *random* scenario categories. Note that, since the mechanisms are randomized, these are expected payoffs obtained as the average of all negotiations conducted. Payoffs are normalized to the maximum expected utility, to allow for easier interpretation. Each tuple in the table cells contains, therefore, the normalized expected utility for both agents, as  $(\bar{U}_n^{Provider1}, \bar{U}_n^{Provider2})$ . We can see that, for the smaller scenarios (Tables 5 (a) and (b)), SA is a best-response strategy in all cases, that is, regardless of the other agent's strategy, the best "move" for an agent is to use simulated annealing. Thus, we have an equilibrium in the (SA, SA) strategy profile,

		Provider 2				Provider 2	
		SA	HC			SA	HC
Provider 1	SA	(0.98, 0.98)	(1.00, 0.85)	Provider 1	SA	(1.00, 1.00)	(0.98, 0.92)
	HC	(0.85, 1.00)	(0.95, 0.95)		HC	(0.92, 0.98)	(0.94, 0.94)

		Provider 2				Provider 2	
		SA	HC			SA	HC
Provider 1	SA	(0.95, 0.95)	(0.95, 1.00)	Provider 1	SA	(0.80, 0.80)	(0.48, 1.00)
	HC	(1.00, 0.95)	(0.91, 0.91)		HC	(1.00, 0.48)	(0.78, 0.78)

		Provider 2				Provider 2	
		SA	HC			SA	HC
Provider 1	SA	(0.86, 0.86)	(0.69, 1.00)	Provider 1	SA	(0.84, 0.84)	(0.57, 1.00)
	HC	(1.00, 0.69)	(0.84, 0.84)		HC	(1.00, 0.57)	(0.72, 0.72)

**Table 5** Normalized payoff matrices for the different random scenario categories.

which is also the situation with the highest expected social welfare. This can be interpreted as having incentives for cooperation in these scenarios, since the SA strategy is more cooperative than the HC one (i.e., an annealer agents can accept a contract even when there is no immediate gain). However, for more complex scenarios (Tables 5 (c) to (f)), the dominant strategy is HC, which leads to an equilibrium in the (HC, HC) strategy profile. This is also the strategy profile with the lowest expected social welfare. The problem is especially relevant in the more complex scenarios (which are also the more realistic ones), where the social welfare loss, also known as Price of Anarchy (Koutsoupias and Papadimitriou 1999), is above 14%. This can be interpreted as these scenarios having strong incentives for competition which hamper the social welfare.

## 6 Discussion and conclusions

In this work we study the problem of coordinating frequency assignment for Wi-Fi access points. We consider an approach inspired in the well-known graph coloring problem. In contrast with the traditional viewpoint from discrete optimization, we provide a negotiation approach based on a simple-text mediation protocol and agent strategies based on simulated annealing. After experimental evaluation, our results are significantly better than the reference approaches. This is especially significant, because 1) it is the first time (to our knowledge) that nonlinear negotiation is used for complex network optimization in a realistic setting, and 2) the approach effectively addresses complex negotiation scalability, which has been a challenge to apply this kind of techniques to real settings. The advantage of using our negotiation approach is especially relevant in the more complex scenarios. In addition, there is a clear influence of the graph structural properties on the performance of the approach, which could be used to derive insights to guide the design of wireless coverage maps in settings where there are coexisting, competing networks.

There are a number of issues that are left outside the scope of this work. Probably the most important is the fact that we consider a static environment,

where wireless clients do not appear or disappear (or move) during the course of the negotiation. This imposes a performance constraint on the approaches considered, since the response time must be short enough to allow to provide a solution before the network has changed too much for the solution to be useful. Also, since we have used uniform distributions for provider AP placements, client placements, activity indexes and the like, the generated scenarios result in “symmetric” negotiations, in the sense that there are no fundamental “inequalities” that make one or the other provider to have more “negotiating power” in the negotiation. Scenarios more “biased”, where a provider has more APs or more clients than the other, or where clients have non-uniformly distributed activity indexes, will likely result in asymmetric negotiation settings where fairness would be an issue and should be studied.

There are a different future lines of research which emerge from this work. We are studying the creation of negotiation-based hyper-heuristics, to dynamically adapt the approaches to the metrics that characterize the scenarios. We are also working on distributed belief propagation as a way to conduct the negotiation, which raises some challenges in itself. Furthermore, we are enlarging the family of negotiation approaches under evaluation, and we are working on generalizing the approach to other network-structured real-world problems. Finally, we want to address the outlined strategic implications of the negotiation setting and introduce mechanisms to mitigate the detected incentives and their effects, applying a generalization of the work we did in this line for other nonlinear negotiation domains (Lopez-Carmona et al 2012).

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## References

- A Bazzi (2011) On Uncoordinated Multi User Multi RAT Combining. In: Vehicular Technology Conference (VTC Fall), 2011 IEEE, pp 1–6, DOI 10.1109/VETEFCF.2011.6093056
- Aardal KI, Van Hoesel SP, Koster AM, Mannino C, Sassano A (2007) Models and solution techniques for frequency assignment problems. *Annals of Operations Research* 153(1):79–129
- Abusubaih M, Gross J, Wolisz A (2007) An inter-access point coordination protocol for dynamic channel selection in IEEE802.11 wireless LANs. In: 1st IEEE Workshop on Automatic Communications and Network Management 2007 (ACNM 2007)
- Baid A, Raychaudhuri D (2015) Understanding channel selection dynamics in dense wi-fi networks. *IEEE Communications Magazine* 53(1):110–117
- Banchs A, Ortin J, Garcia-Saavedra A, Leith DJ, Serrano P (2016) Thwarting selfish behavior in 802.11 WLANs. *IEEE/ACM Transactions on Networking* 24(1):492–505
- Bernini R, Bondavalli A, Lollini P, Montecchi L (2016) Combining san and p-graphs for the analysis and optimization of industrial processes. In: Dependable Computing Conference (EDCC), 2016 12th European, IEEE, pp 197–207
- Bodlaender HL, Kloks T, Tan RB, van Leeuwen J (2000)  $\lambda$ -coloring of graphs. In: Annual Symposium on Theoretical Aspects of Computer Science, Springer, pp 395–406
- Chiochan S, Hossain E, Diamond J (2010) Channel assignment schemes for infrastructure-based 802.11 WLANs: A survey. *IEEE Communications Surveys & Tutorials* 12(1):124–136

- Cisco (2007) Radio resource management under unified wireless networks, cisco system technical note
- De Jonge D, Sierra C (2015) NB<sup>3</sup>: a multilateral negotiation algorithm for large, nonlinear agreement spaces with limited time. *Autonomous Agents and Multi-Agent Systems* 29(5):896–942
- de la Hoz E, Gimenez-Guzman JM, Marsa-Maestre I, Orden D (2015) Automated negotiation for resource assignment in wireless surveillance sensor networks. *Sensors* 15(11):29,547–29,568
- E Z Tragos, S Zeadally, A G Fragkiadakis, V A Siris (2013) Spectrum Assignment in Cognitive Radio Networks: A Comprehensive Survey. *IEEE Communications Surveys & Tutorials* 15(3):1108–1135, DOI 10.1109/SURV.2012.121112.00047
- FAP (2017) Fap web - a website about frequency assignment problems. <http://fap.zib.de/>
- Fatima S, Kraus S, Wooldridge M (2014) *Principles of Automated Negotiation*. Cambridge University Press, Cambridge
- Fatima SS, Wooldridge M, Jennings NR (2001) Optimal negotiation strategies for agents with incomplete information. In: *International Workshop on Agent Theories, Architectures, and Languages*, Springer, pp 377–392
- Fornito A (2016) Graph theoretic analysis of human brain networks. *fMRI Techniques and Protocols* pp 283–314
- Fujita K, Bai Q, Ito T, Zhang M, Ren F, Aydoğan R, Hadfi R (2017) Modern approaches to agent-based complex automated negotiation. *Studies in Computational Intelligence* 674
- Geier J (2017) How to: Define Minimum SNR Values for Signal Coverage. [http://www.wireless-nets.com/resources/tutorials/define\\_SNR\\_values.html](http://www.wireless-nets.com/resources/tutorials/define_SNR_values.html)
- Ghavidelisyooki M, Awasthi A, Allouche M, Berger J, Mitrovic Minic S (2017) Partitioning of transportation networks under disruption. *International Journal of Modelling and Simulation* pp 1–9
- Gimenez-Guzman JM, Marsa-Maestre I, Orden D, de la Hoz E, Ito T (in press) On the goodness of using orthogonal channels in wlan ieee 802.11 in realistic scenarios. *Wireless Communications and Mobile Computing*
- Green DB, Obaidat AS (2002) An accurate line of sight propagation performance model for ad-hoc 802.11 wireless LAN (WLAN) devices. In: *Communications, 2002. ICC 2002. IEEE International Conference on*, vol 5, pp 3424–3428 vol.5, DOI 10.1109/ICC.2002.997466
- Griggs JR, et al (2009) Graph labellings with variable weights, a survey. *Discrete Applied Mathematics* 157(12):2646–2658
- Hattori H, Klein M, Ito T (2007) Using Iterative Narrowing to Enable Multi-party Negotiations with Multiple Interdependent Issues. In: *Proceedings of the 6th International Joint Conference on Autonomous Agents and Multiagent Systems*, ACM, New York, NY, USA, AAMAS '07, pp 247:1–247:3, DOI 10.1145/1329125.1329424
- Jansen P, Perez R (2011) Constrained structural design optimization via a parallel augmented Lagrangian particle swarm optimization approach. *Computers & Structures* 89(13–14):1352–1366, DOI 10.1016/j.compstruc.2011.03.011
- Jensen TR, Toft B (2011) *Graph coloring problems*, vol 39. John Wiley & Sons
- Kivelä M, Arenas A, Barthelemy M, Gleeson JP, Moreno Y, Porter MA (2014) Multilayer networks. *Journal of Complex Networks* DOI 10.1093/comnet/cnu016
- Klein M, Faratin P, Sayama H, Bar-Yam Y (2003) Negotiating Complex Contracts. *Group Decision and Negotiation* 12(2):111–125, DOI 10.1023/A:1023068821218
- Koschützki D, Lehmann KA, Peeters L, Richter S, Tenfelde-Podehl D, Zlotowski O (2005) Centrality Indices. In: Brandes U, Erlebach T (eds) *Network Analysis: Methodological Foundations*, Springer Berlin Heidelberg, Berlin, Heidelberg, pp 16–61
- Koutsoupias E, Papadimitriou C (1999) Worst-case equilibria. In: *Stacs*, Springer, vol 99, pp 404–413
- Kumar S, Dutta K, Sharma G (2016) A detailed survey on selfish node detection techniques for mobile ad hoc networks. In: *Parallel, Distributed and Grid Computing (PDGC), 2016 Fourth International Conference on*, IEEE, pp 122–127
- de La Hoz E, Marsa-Maestre I, Gimenez-Guzman JM, Orden D, Klein M (2017) Multi-agent nonlinear negotiation for wi-fi channel assignment. In: *Proceedings of the 16th Conference on Autonomous Agents and MultiAgent Systems*, International Foundation for Autonomous Agents and Multiagent Systems, pp 1035–1043
- Lang F, Fink A (2015) Learning from the Metaheuristics: Protocols for Automated Negotiations. *Group Decision and Negotiation* 24(2):299–332, DOI 10.1007/s10726-014-9390-x

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- Lopez-Carmona MA, Marsa-Maestre I, Klein M, Ito T (2012) Addressing stability issues in mediated complex contract negotiations for constraint-based, non-monotonic utility spaces. *Autonomous Agents and Multi-Agent Systems* 24(3):485–535
- Malaguti E, Toth P (2010) A survey on vertex coloring problems. *International Transactions in Operational Research* 17(1):1–34
- Marsa-Maestre I, Lopez-Carmona MA, Velasco JR, Ito T, Klein M, Fujita K (2009) Balancing Utility and Deal Probability for Auction-based Negotiations in Highly Nonlinear Utility Spaces. In: *Proceedings of the 21st International Joint Conference on Artificial Intelligence*, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, IJCAI'09, pp 214–219
- Marsa-Maestre I, de la Hoz E, Gimenez-Guzman JM, Orden D, Klein M (2016) Nonlinear negotiation approaches for complex-network optimization: A study inspired by wi-fi channel assignment. In: *International Workshop on Conflict Resolution in Decision Making*, Springer, pp 51–65
- McDiarmid C, Reed B (2000) Channel assignment and weighted coloring. *Networks* 36(2):114–117
- Mishra A, Banerjee S, Arbaugh W (2005) Weighted coloring based channel assignment for WLANs. *ACM SIGMOBILE Mobile Computing and Communications Review* 9(3):19–31
- Mishra A, Brik V, Banerjee S, Srinivasan A, Arbaugh WA (2006) A client-driven approach for channel management in wireless LANs. In: *INFOCOM*
- Narayanan L (2002) Channel assignment and graph multicoloring. *Handbook of wireless networks and mobile computing* 8:71–94
- Newman M (2010) *Networks: An Introduction*. Oxford University Press, Oxford
- Ng SWK, Szymanski TH (2012) Interference measurements in an 802.11n Wireless Mesh Network testbed. In: *Electrical Computer Engineering (CCECE)*, 2012 25th IEEE Canadian Conference on, pp 1–6, DOI 10.1109/CCECE.2012.6334846
- Orden D, Gimenez-Guzman JM, Marsa-Maestre I, de la Hoz E (2018) Spectrum graph coloring and applications to Wi-Fi channel assignment. *Symmetry* 10(3):65
- Orden D, Marsa-Maestre I, Gimenez-Guzman JM, de la Hoz E, Alvarez-Suarez A (in press) Spectrum graph coloring to improve Wi-Fi channel assignment in a real-world scenario via edge contraction. *Discrete Applied Mathematics*
- Ren F, Zhang M, Sim KM (2009) Adaptive conceding strategies for automated trading agents in dynamic, open markets. *Wireless in the Healthcare* 46(3):704–716, DOI 10.1016/j.dss.2008.11.005
- Rubinstein A (1982) Perfect Equilibrium in a Bargaining Model. *Econometrica* 50(1):97–109, DOI 10.2307/1912531
- Seyedbrahimi M, Bouhafs F, Raschellà A, Mackay M, Shi Q (2016) Sdn-based channel assignment algorithm for interference management in dense wi-fi networks. In: *Networks and Communications (EuCNC)*, 2016 European Conference on, IEEE, pp 128–132
- Sharp A (2007) Distance coloring. In: *European Symposium on Algorithms*, Springer, pp 510–521
- Tuza Z, Gutin G, Plurnmer M, Tucker A, Burke E, Werra D, Kingston J (2003) Colorings and Related Topics. In: *Handbook of Graph Theory, Discrete Mathematics and Its Applications*, CRC Press, pp 340–483
- Valori L, Giannuzzi GL, Facchini A, Squartini T, Garlaschelli D, Basosi R (2016) A generation-attraction model for renewable energy flows in italy: A complex network approach. *The European Physical Journal Special Topics* 225(10):1913–1927
- Wiener H (1947) Structural Determination of Paraffin Boiling Points. *Journal of the American Chemical Society* 69(1):17–20, DOI 10.1021/ja01193a005