Essays on the Risk Premium in the U.S. Financial Market

by

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B.A., Kyoto University (1983)

Submitted to the Department of Economics
in partial fulfillment of the requirements
for the Degree of

Doctor of Philosophy in Economics

at the

Massachusetts Institute of Technology
June, 1992

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ABSTRACT

This thesis theoretically and empirically studies the asset pricing behavior with trading among heterogeneous agents. The basic idea is that heterogeneous agents bear different degrees of aggregate shocks. In other words, aggregate shocks are disproportionately distributed among heterogeneous agents through financial markets. For implementing this idea, all of the asset pricing models discussed in this thesis introduce two types of agents who differ with respect to investment opportunities, degrees of relative risk aversion (RRA), or state—dependent utility functions. In these models, one type of consumer is a risk—taker, while the other type is a risk—hedger. The asset pricing behavior, consequently, reflects the trading between these two types. By applying this simple framework to stock/bond markets, I can consistently explain the dynamics of risk premia observed in the U.S. financial market. In addition, this framework resolves some important empirical puzzles which have arisen in the context of representative agent models (RA models).

Chapter 1 analyzes a continuous—time asset pricing model in which a significant portion of consumers are excluded from the risky asset market (stock market) due to participation costs. This model can be considered as an extreme case of the basic framework, because stockholders are pure risk—takers in the sense that they bear the whole aggregate risk, while non—stockholders are pure risk—hedgers who are completely free from any aggregate shocks for an instantaneous period. The implications of this model are significantly different from those of RA models in the following respect. First, the model can yield much larger risk premia than those predicted by RA models, because stockholders require high compensation for bearing the entire aggregate shock. Second, the model is able to endogenize leverage ratios (bond/equity ratios) which represent how much risk is borne by stockholders. Time—varying leverage ratios produce rich dynamics of risk premia, even if the underlying stochastic process is formulated in a simple way. These sets of predictions help to resolve the equity premium puzzle (Mehra and Prescott [1985]) and too weak correlation between aggregate consumption growth and asset returns (Hall [1988]). For analytical purposes, all endogenous variables are characterized in

terms of a closed—form stationary distribution. The model is empirically supported by the U.S. time—series data.

Chapter 2 investigates the qualitative implications of the equity premium dynamics common to the basic framework. In addition, these implications are empirically nested against those of RA models. For this purpose, three examples are constructed within the basic framework: (i) heterogeneous degrees of RRA (Dumas [1989]), (ii) wealth—dependent RRA (Marcus [1989]), (iii) heterogeneous investment opportunities (the model developed in Chapter 1), while the implications of RA models are derived from the asset pricing model due to Cox, Ingersoll and Ross [1985a]. In the above three examples, the risk premium dynamics are driven by time—varying attitudes of risk—takers; risk—takers require higher premia for holding risky assets after recessions, while they are satisfied with lower compensations after booms. In RA models, on the other hand, the risk premium dynamics mainly reflect time—varying riskiness of the underlying technology. These hypotheses can be differentiated by examining the response of risk premia to realized aggregate shocks. The estimation results show that the conditional mean of risk premia in the U.S. stock market significantly reflects the trading between risk—takers and risk—hedgers, while its conditional volatility is not levered mainly by the trading. These results are consistent with the calibration results in Chapter 1.

Chapter 3 empirically examines liquidity effects on yield spreads in bond markets; then, it discusses how a monetary sector can be introduced into the basic framework. By estimating the VAR system which consists of production growth, inflation, and money supply, both aggregate shocks (technological shocks) and monetary shocks are identified. The empirical investigation shows that liquidity has an immediately strong effect on nominal/real short—run interest, while it persistently affects nominal long—run interest; consequently, an expansionary monetary policy in recessions widens yield spreads initially, and narrows them later. Combined with the effect through the trading between risk—takers and risk—hedgers, liquidity effects can successfully explain one stylized fact in the U.S. bond market: yield curves are inverted late in business expansions, while they become steeper in the latter stages of recessions. As final remarks on this thesis, Chapter 3 suggests that liquidity effects may work in the basic framework by assuming that injected money by an open market operation is disproportionately distributed between risk—takers and risk—hedgers. The disproportionate distribution of injected money may also interact with that of aggregate shocks (already built in the basic framework) through the response of a monetary policy to aggregate shocks.

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ACKNOWLEDGEMENTS

I would like to acknowledge my indebtedness to my teachers and my fellow students who helped me during various stages of my graduate study. John Heaton's advices have played a decisive role at many points; I reached the basic idea of Chapter 1 through intensive discussions with him. Olivier Blanchard's comments have been valuable and encouraging; his remarks on this project have been full of economic insights. Both John and Olivier have encouraged and disciplined me up to the last minute of completing this thesis. Roland Benabou has been a rigorous and careful reader of earlier versions of this thesis; his suggestions have substantially improved Chapters 1 and 2. Deborah Lucas, Danny Quah, and Jean—Luc Villa have provided many helpful and detailed comments on Chapter 1, while Andrew Bernard and James Poterba on Chapter 2, Joseph Beaulieu on Chapter 3. Joe and Stephen Jen caught a number of infelicities of style and content in earlier versions of all chapters. I have received constructive comments from the participants of the MIT money workshop, the MIT money lunch seminar, and recruiting seminars at several institutions. I thank Richard Eckaus for presiding at the Graduate Committee. I am also grateful to the Sumitomo Trust and Banking Company for financial support.

CONTENTS

Chapter 1

On the Stationary Distribution of Risk Premia in an Economy with Stockholders and Non-stockholders: A Simple Explanation for the Mehra and Prescott Puzzle

1. Introduction

Since Mehra and Prescott [1985] claimed that the unconditional mean of the risk premia observed in the U.S. stock market is much larger than that predicted by representative agent models (hereafter, RA models), serious efforts have been made to resolve this puzzle (hereafter, the MP puzzle). Within the context of RA models, the setting imposed by Mehra and Prescott has been generalized in three directions. First, an attempt is made to disentangle risk aversion attitudes from intertemporal substitution. Examples of this approach include the habit formation hypothesis (Constantinides [1990]) and a special class of non–expected utility theory (Epstein and Zin [1989], Weil [1990] and others). Second, exogenous stochastic processes (technology or endowment) are characterized in a more sophisticated way. For example, Rietz [1988] extends the state space of endowment processes, while Cecchetti, Lam and Mark [1990] explicitly deal with a change in regimes by adopting Markov switching models[1]. Third,

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[1] Both models assume that one catastrophic state is realized with a small probability. In their set–up, consumers demand much risk–free asset in order to avoid such a catastrophic state; consequently, risk–free returns are lowered, while risk premia become large.

Benninga and Protopapadakis [1990] and Cecchetti et al. [1990] relate large risk premia to leverage effects, although leverage ratios (bond/equity ratios) are treated as exogenous variables.

Another strand of research attributes market incompleteness as the source of large risk premia[2]. Among these models, large risk premia are caused by a particular stochastic shock which is more volatile than aggregate shocks. Mankiw [1986] and Weil [1989, 1990] assume that idiosyncratic shocks are not insured among individual consumers due to incomplete insurance. A more radical deviation from complete markets is to introduce noise created by irrational traders. DeLong, Shleifer, Summers and Waldmann [1990] study how risk premia are affected by the interaction between rational and irrational traders.

In this chapter, we try to view the MP puzzle in its different perspective. Unlike the studies mentioned above, we still assume a simple time–additive utility as well as a simple technological process, and we exclude any stochastic shocks except a unique aggregate shock. The only significant departure of the proposed model from the Mehra and Prescott setting is to suppose that investment opportunities differ among consumers; a subset of consumers in this model are completely excluded from the risky asset market due to participation costs.

In this economy, risk–free assets are issued to participation–constrained consumers (non–stockholders) by unconstrained consumers (stockholders), while financial claims to risky physical assets are traded only among unconstrained consumers. Its main difference from RA models is that aggregate shocks are concentrated on stockholders. Because stockholders

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[2] Mehra and Prescott [1985] themselves suggest that large risk premia might be a consequence of market incompleteness.

require high compensation for bearing the entire aggregate shock, this model can yield risk premia much larger than those predicted by RA models. In addition, while RA models treat leverage effects as exogenous, our model is able to endogenize leverage ratios which represent how much risk is borne by stockholders. Time—varying leverage effects produce rich dynamics of risk premia as well as of risk—free returns, even if the underlying stochastic process is characterized in a simple way.

Our approach is also different from those with incomplete insurance in that the heterogeneous consumption growth among consumers is regarded as a consequence of different investment opportunities, not as a product of undiversified shocks. Our interpretation is motivated by the following facts. First, Mankiw and Zeldes [1990] report that many consumers do not hold any equities and find that the consumption process of stockholders is more volatile and more correlated with risky asset returns than that of non—stockholders. Second, Lucas [1990] claims that incomplete insurance can be made partially complete (almost perfectly complete in her example) by trading through financial markets; therefore, the incompleteness of insurance markets alone cannot yield large risk premia as long as consumers are allowed to access financial markets. Recent studies on models with incomplete insurance put more emphasis on the interaction between the incompleteness of insurance markets and other market frictions such as liquidity constraints, transaction costs and so on. (Aiyagari and Gertler [1990], Marcet and Singleton [1991], Heaton and Lucas [1991])

Moreover, the economy described by our model contrasts sharply with that represented by models with liquidity constraints; incomplete capital markets, in the latter, make it difficult for the poor to borrow money from the rich, while heterogeneous investment opportunities, in the former, enable

stockholders (likely to be rich) to borrow capital from non–stockholders (maybe poor) through the risk–free asset market. Such a keen difference between them can potentially change how we view the empirical failure of Euler equations in the consumption literature. We reinterpret this failure as reflecting heterogeneous investment opportunities, not due to liquidity constraints. To see whether this view is appropriate, our model is applied to the empirical puzzle raised by Hall [1988]: why is the U.S. aggregate consumption growth not so correlated with asset returns ?

The model is treated in continuous–time. Our analytical goal is to establish the existence of a stationary distribution which can characterize all endogenous variables, especially risk premia; without its existence, the long–run equilibrium might degenerate to a point distribution which is identical to the equilibrium of RA models. Our empirical goal is to examine a set of predictions about risk premia in the following context: their long–run average in the U.S. stock market and their dynamic behavior in the post–war U.S. stock market.

This chapter is organized as follows. Section 2 sets up the model. In Section 3, the properties of the model are compared with those of a simple version of RA models. Section 4 reports how well the model is supported by the U.S. time–series data. In addition, it evaluates how successfully the model resolves the MP puzzle through a "calibration" type discussion. Section 5 concludes. The proofs of the propositions in the text are presented in an appendix.

2. Model Set–Up

In this section, we discuss the set–up of our model and derive the

equilibrium solution. Our final goal is to characterize the dynamics of all endogenous variables in terms of a closed–form stationary distribution.

(physical assets and financial assets)

It is assumed that one linear risky technology is available in terms of physical assets. Its productivity is exogenously given, or completely independent of other economic activities. We further assume that it is driven only by a unique aggregate shock. Any idiosyncratic shocks are abstracted from this model. Returns on physical assets are characterized by a geometric Brownian motion.

$$(1) \qquad \frac{dS(t)}{S(t)} = \alpha \, dt + \sigma \, dB(t),$$

where dB(t) is a standard Brownian motion.

With respect to financial assets, claims to physical assets are traded in the risky asset market. In the risk–free asset market, any consumer is allowed to issue risk–free assets directly to other consumers (home leverage) *or* indirectly to them through firms (firm leverage); in either case, risk–free returns are endogenized by the market–clearing condition.

(optimization problems for stockholders and non–stockholders)

In this economy, two types of consumers trade with each other in financial markets. The first type of consumers (hereafter, stockholders) can participate in the risky asset market as well as in the risk–free asset market, while the second type (hereafter, non–stockholders) can trade *only* in

the risk—free asset market. Although a type of each consumer is *exogenously* given in our setting, I will later justify such an exogenous differentiation by introducing fixed costs for participating in the risky asset market.

Infinitely—lived stockholders maximize the expected value of discounted utility by investing their wealth over risk—free assets and risky assets. They are allowed to borrow physical goods from the risk—free asset market. They also earn an exogenous income in addition to financial asset returns. The optimization problem is constructed as follows with the transversality condition (boundedness of value functions). A subscript i denotes an index of an individual.

$$(2) \qquad V^s(W_i(0)) = \max_{\{c_i(t),\ x_i(t)\}} E_0 \int_0^\infty e^{-\rho t} u(c_i(t))\ dt,$$

s.t.

$$(3) \quad dW_i(t) = \{[(1-x_i(t))r(t)+x_i(t)\alpha]W_i(t)+I_i(t)-c_i(t)\}dt + x_i(t)W_i(t)\sigma dB(t),$$

given $W_i(0)$,

where $W_i(t)$: total own capital,

$I_i(t)$: exogenous income other than from financial assets,

ρ: discount rate,

$r(t)$: risk—free return,

$c_i(t)$: consumption,

$x_i(t)$: ratio of risky asset to total asset.

On the other hand, infinitely—lived non—stockholders optimize the expected value of discounted utility by holding risk—free assets only They also get an exogenous income other than from financial assets. Unless they have such an exogenous income against which they can borrow, they always behave as lenders in the risk—free asset market. They solve the following problem with the transversality condition.

$$(4) \qquad V^{ns}(W_i(0)) = \max_{\{c_i(t)\}} E_0 \int_0^{\infty} e^{-\rho t} u(c_i(t))\, dt,$$

s.t.

$$(5) \qquad dW_i(t) = \{r(t)\cdot W_i(t) + I_i(t) - c_i(t)\}\, dt,$$

given $W_i(0)$.

(participation costs)

We interpret the above type—differentiation among consumers as being motivated by *initial fixed costs* which are incurred upon participating in the risky asset market. For non—stockholders, a decrease in benefits due to participation costs is assumed to exceed an increase in benefits due to expanding investment opportunities. In other words, non—stockholders are constrained by equation (6) over time.

$$(6) \quad \begin{cases} \text{not participate if } V^{s}(W_i(t) - F_i(t)) < V^{ns}(W_i(t)), \\ \text{participate, otherwise,} \end{cases} \quad \forall\, t,$$

where $F_i(t)$: initial fixed cost.

The initial fixed costs defined as $F_i(t)$ may cover from *physical costs* such as costs of collecting information, transaction fees, transportation costs, to *mental costs* such as difficulty with understanding schemes, stress accompanied by transactions, and so on. Any cost is supposed to be measured in units of physical goods. After deriving the equilibrium solution, we will calculate the critical cost/wealth ratios at which consumers are indifferent between holding risk–free assets only and participating in the risky asset market.

Under this setting, heterogeneous portfolio behavior arises from the two sources; the first source is from differences in financial wealth, while the second source is from heterogeneous participation costs. The first type of heterogeneity is obviously important. According to the 1983 Survey of Consumer Finances (Avery, Elliehausen and Kennickell [1988]), the upper 0.5th percentile households (sorted by total assets including physical assets net of liabilities) invest more than half of their financial assets in public stocks, while the lower 90th percentile households deposit about 80% of their financial wealth in checking/saving accounts.

Preceding empirical findings indicate that the second source of heterogeneity is important as well, or that non–wealth factors are also significant in determining individual portfolios. Mankiw and Zeldes [1990] report that a half of consumers with more than $10,000 of financial assets (1984 valuation) do not invest in any equities. Several micro–based studies of household portfolios, including Blume, Crockett and Friend [1974], King and Leape [1984], and Mankiw and Zeldes [1990], also show that holding equities is associated with household characteristics such as occupations,

living locations, education levels and so on. The association with higher education could be the most important evidence among them; how easily they can obtain knowledge required for equity investments might depend on how educated they are.

Some skepticism may be cast on a significance of participation costs, provided that mutual funds or pension funds are widely available. These types of funds are supposed to make equity investments more accessible for a wide range of consumers; however, it is not so obvious in the U.S. economy as it seems. Concerning mutual funds, most existing funds invest their capital mainly in money markets such as commercial papers, certificates of deposits, treasury bills, not in equity markets. According to the Investment Company Institute [1990], only one quarter of capital in mutual funds is invested in equity markets in 1989. In addition, selecting one specific mutual fund does not seem an easy job for consumers; there are 1224 mutual funds which hold equity in 1989, whose prospectuses differ significantly in terms of investment policy, minimum initial (subsequent) capital, transaction fees.

With respect to pension funds, on the other hand, workers joining pension funds cannot be necessarily considered as virtual stockholders. Under the defined—benefit plans (a dominant type of pension funds in the U.S.), the residual claimants are actually the stock—owners who invest in the company sponsoring this type of pension funds. If the pension fund yields asset returns higher than the previously—defined—benefits for the workers, the excess earnings (residuals) are eventually attributed to the stock—owners through an increase in the stock price of the sponsoring

company[3].

(representative agent model)

Before solving the equilibrium with participation constraints, we briefly discuss a simple version of RA model under this set—up. The equilibrium of this version of RA model will serve as a reference point in studying the properties of our model. For deriving its equilibrium solution, we need additional assumptions: (i) the participation costs ($F_i(t)$'s) are equal to zero; therefore, all consumers participate in the risky asset market, (ii) there is no exogenous income ($I_i(t) = 0$), (iii) $u(\cdot)$ is the utility function with a constant degree of relative risk aversion (CRRA).

The following statements are easily proved (see Merton [1971]): (i) given constant risk—free returns, the optimal portfolio rule for (2) is independent of individual wealth levels, (ii) since all consumers are homogeneous except for initial wealth, their optimal portfolio is identical among consumers, (iii) given (ii), no one can be either of borrowers or lenders in the equilibrium of the risk—free asset market, in other words, $x_i(t)$ in (3) is equal to one over time and across consumers, (iv) conversely, risk—free returns are constant if $x_i(t)$ is equal to one, (v) therefore, risk—free returns are indeed constant in the market equilibrium. Figure 1—1 depicts the flow of funds in the market equilibrium of the RA model.

The main endogenous variables in the market equilibrium are summarized as follows (Merton [1971]).

(risk—free returns)

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3 Mankiw and Zeldes [1990] report several statistics concerning this issue.

$$(7) \qquad r(t) = \alpha - \gamma \cdot \sigma^2$$

γ: a degree of relative risk aversion

(risky asset returns)

$$(8) \qquad \frac{dP(t)}{P(t)} = \alpha \, dt + \sigma \, dB(t)$$

(ex—ante risk premia)

$$(9) \qquad \Pi(t) = E_t \frac{dP(t)}{P(t)} / dt - r(t)$$

$$= \gamma \cdot \sigma^2$$

(individual consumption/wealth growth)

$$(10) \qquad \frac{dc_i(t)}{c_i(t)} = \frac{dW_i(t)}{W_i(t)} = (\alpha - a) \, dt + \sigma \, dB(t),$$

where $a = [\rho + \alpha \cdot (\gamma - 1)]/\gamma + (1 - \gamma) \cdot \sigma^2/2$

(dynamic budget constraint of aggregate economy)

$$(11) \qquad dW(t) = (\alpha - a) \cdot W(t) dt + \sigma \cdot W(t) \cdot dB(t),$$

where $W(t) = \Sigma_i \, W_i(t)$.

If the marginal propensity to consume, a ($= [\rho + \alpha \cdot (\gamma - 1)]/\gamma + (1 - \gamma) \cdot \sigma^2/2$) is greater than zero, the transversality condition is satisfied (see Merton [1971]). Because of the characteristics of the CRRA utility function

($u'(0) = \infty$), the above solution is free from bankruptcy ($W_i(t) > 0 \quad \forall t$).

Equation (9) represents the essence of the MP puzzle; if the conditional variance of the production process (the endowment process in the context of Mehra and Prescott [1986]) is observed to be very low, an unreasonably high degree of RRA is required to obtain large risk premia. In addition, (9) shows that the risk premia of the RA model abstract any dynamics from their behavior[4]. As shown later, the model yields larger risk premia as well as richer dynamics, once participation constraints are taken into consideration.

(optimal policy)

Returning to the model with participation constraints, several issues will be discussed in the following sequence: the optimal rules, the market equilibrium and the dynamics.

Generally speaking, it is impossible to obtain closed—forms of optimal portfolio/consumption rules for (2) and (4), because (i) risk—free returns ($r(t)$) are not constant over time, and (ii) optimal rules depend crucially on the future process of an exogenous income $I_i(t)$. One possible remedy to avoid these difficulties is to assume that

(A.1) the utility function is logarithmic,

(A.2) there is no exogenous income ($I_i(t) = 0$),

(A.3) the discount rate is positive ($\rho > 0$).

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[4] The absence of dynamics is specific to this setting; it is not universal among a wider class of RA models. If the underlying stochastic process is characterized in a more complicated way, RA models can potentially produce richer dynamics of risk premia.

(A.1) makes the optimal rules independent of the future evolution of risk–free returns because price effects are exactly canceled out by income effects in the logarithmic utility. Although this assumption is pretty useful for analytical operation, it imposes a strong restriction on preferences. In Section 4, we will relax (A.1) by using the CRRA utility at the expense of rigor. As discussed in Appendix 2, an exogenous income will be introduced in a simple way by replacing (A.2). (A.3) will be used in proving the boundedness of value functions.

In addition to the above, we assume that

(A.4) consumers do not trade outside financial markets.

This assumption rules out insurance contracts among consumers, even if there exists some room to insure risks between the two types of consumers. (A.4) may be motivated by moral hazard or adverse selection in insurance markets. Given (A.4), our model is similar to models with incomplete insurance in that both models assume that insurance markets are incomplete. They still differ, however, in that heterogeneous consumption growth in our model is driven not by idiosyncratic shocks, but by heterogeneous investment opportunities.

Together with (A.1) — (A.3), Hakansson [1971] and Merton [1971] show that the following rules are *candidates* for the optimal consumption–portfolio rules of stockholders.

$$(12) \qquad c(t) = \rho\ W(t)$$

$$(13) \qquad x(t) = \frac{\alpha - r(t)}{\sigma^2}$$

(12) is a candidate for the optimal consumption rule of non–stockholders as well. There is no portfolio problem for non–stockholders. Propositions 2 to 3 in Appendix 1 prove the boundedness of value functions and the non–bankruptcy condition ($W_i^{ns}(t)$, $W_j^s(t) > 0 \quad \forall\ t$). The former condition guarantees that the above candidates are actually optimal, while the latter keeps consumers from exiting financial markets.

(equilibrium of risk–free asset market)

Our model virtually has two markets, the physical goods market and the risk–free asset market, because the risky asset market can be identified with the physical goods market. First of all, we pay attention to the equilibrium of the risk–free asset market; then, we check whether the physical goods market is in equilibrium given the former equilibrium (verification of Walras' law).

We denote the financial wealth of non–stockholders as $W_i^{ns}(t)$, $i = 1, 2, \ldots, I$. Similarly, $W_j^s(t)$, $j = 1, 2, \ldots, J$ for stockholders. The total wealth $W(t)$ is the sum of the two.

$$(14) \qquad W(t) = W^{ns}(t) + W^s(t),$$

$$\text{where} \qquad W^{ns}(t) = \Sigma_{i=1}^{I} W_i^{ns}(t), \qquad W^s(t) = \Sigma_{j=1}^{J} W_j^s(t).$$

The total wealth of non–stockholders, $W^{ns}(t)$, automatically appears on the demand side of the risk–free asset market, because non–stockholders

do not have any alternative investment opportunities. On the other hand, stockholders can finance risky asset investments by issuing risk—free assets. The total supply of risk—free assets is, then, equal to the amount of risky assets in excess of their own capital, or to $(x(t) - 1)\cdot W^s(t)$. Hence, the following equality should hold in the equilibrium of the risk—free asset market.

$$(15) \qquad W^{ns}(t) = (x(t) - 1)\cdot W^s(t)$$

$$\text{or} \qquad x(t) = 1 + y(t), \quad \text{where } y(t) = \frac{W^{ns}(t)}{W^s(t)}.$$

$y(t)$ represents the financial wealth distribution between both types of consumers. As long as the equilibrium path is free from the bankruptcy ($W_i^c(t)$, $W_j^{uc}(t) > 0 \quad \forall\ t$), $y(t)$ is always non—negative.

Once $y(t)$ is determined, asset returns can be derived from the individual optimal rules and their budget constrains. Given $y(t)$, the portfolio rule of stockholders, (13), determines risk—free returns.

$$(16) \qquad r(t) = \alpha - (1 + y(t))\cdot\sigma^2$$

The above risk—free returns are lower than that of the RA model ((7) with γ=1) as a consequence that positive demand for risk—free assets from non—stockholders creates a downward pressure on risk—free returns.

How to define risky asset returns depends on whether stockholders issue risk—free assets directly to non—stockholders (unleveraged or home—leveraged), *or* indirectly to them through firms (leveraged or firm—leveraged).

Different risky asset returns are observed between both ways of financing, although both are equivalent to each other in terms of resource allocations (Modigliani and Miller theorem). For simplicity, we omit a mixed case between the two types of financing. Figure 1—2 depicts the flow of funds for both types of financing. Under firm leverage, y(t) can be interpreted as leverage ratios (debt/equity ratios).

In the case of home leverage, firms are 100 % equity—financed; accordingly, risky asset returns (liability side) are identical to returns on physical assets (asset side). From equation (1), unleveraged risky asset returns are

$$(17) \qquad \frac{dP(t)}{P(t)} = \alpha\ dt + \sigma\ dB(t).$$

On the other hand, risky assets turn out to be residual claims under firm leverage. Accordingly, leveraged risky asset returns are equal to the total returns of physical assets *net of* interest payments of risk—free assets. That is,

$$(18) \qquad \frac{dP_l(t)}{P_l(t)} = [(1+y(t))\cdot\alpha - r(t)\cdot y(t)]\ dt + (1+y(t))\cdot\sigma\cdot dB(t)$$
$$=[\alpha + y(t)\cdot(1 + y(t))\cdot\sigma^2]dt + (1 + y(t))\cdot\sigma\cdot dB(t).$$

Ex—ante risky asset returns for both types of financing are derived as follows.

$$(19) \qquad E_t\frac{dP(t)}{P(t)} = \alpha\ dt$$

$$(20) \qquad E_t \frac{dP_l(t)}{P_l(t)} = [\alpha + y(t)\cdot(1 + y(t))\cdot\sigma^2]\ dt$$

Under firm leverage, the pay–off from the physical production is split between safe/lower returns (risk–free returns) and risky/higher returns (leveraged risky asset returns). Ex–ante leveraged returns are, consequently, higher than ex–ante unleveraged returns, while ex–post leveraged returns are more volatile than ex–post unleveraged returns. The effect due to firm leverage on conditional moments of risky returns is called the *leverage effect.*

Ex–ante risk premia are also defined in two different ways, corresponding to unleveraged returns (Π) or to leveraged returns (Π_l).

$$(21) \qquad \Pi(t) = (1 + y(t))\sigma^2$$

$$(22) \qquad \Pi_l(t) = (1 + y(t))^2\sigma^2$$

Compared with the risk premia of the RA model ((9) with $\gamma = 1$), our model always produces larger risk premia than those predicted by the RA model, unless y(t) degenerates to zero. One difference between (9) and (21) reflects lower risk–free returns due to positive demand from non–stockholders, while the other difference between (21) and (22) is a consequence of the leverage effect. Restating these findings in terms of the MP puzzle, higher equity returns are driven by the leverage effect, while lower risk–free returns are caused by positive demand from non–stockholders.

(equilibrium of physical goods market)

The following development confirms that the physical goods market is indeed in equilibrium given the equilibrium of the risk–free asset market. The individual wealth growth processes for both types of consumers are derived as below.

$$(23) \qquad \frac{dW^{ns}{}_i(t)}{W^{ns}{}_i(t)} = [\alpha - (1 + y(t))\cdot\sigma^2 - \rho]\ dt$$

$$(24) \qquad \frac{dW^{s}{}_j(t)}{W^{s}{}_j(t)} = [\alpha+(y(t)+y(t)^2)\sigma^2-\rho]\ dt + (1+y(t))\cdot\sigma\cdot dB(t)$$

As (23) and (24) show, the difference in the conditional mean of the wealth growth between the two types of consumers exactly reflects leveraged risk premia $(= (1 + y(t))^2\sigma^2)$.

Aggregating (23) and (24) over all consumers, we obtain the dynamic budget constraint of the aggregate economy. The aggregate dynamic budget constraint is equivalent to the market equilibrium of the physical goods at each date.

$$(25) \qquad dW(t) = [\alpha\cdot W(t) - c^{ns}(t) - c^{s}(t)]dt + \sigma\cdot W(t)dB(t)$$

$$= (\alpha - \rho)\cdot W(t)dt + \sigma\cdot W(t)dB(t),$$

$$\text{or} \qquad \frac{dW(t)}{W(t)} = (\alpha - \rho)\ dt + \sigma\ dB(t),$$

where $c^{ns}(t) = \Sigma_{i=1}^{I}\ c^{ns}{}_i(t)$, $c^{s}(t) = \Sigma_{j=1}^{J}\ c^{s}{}_j(t)$.

As (25) shows, the aggregate wealth growth of our model is the same process as that of the RA model ((11) with $\gamma = 1$). In other words, the same aggregate process can be interpreted by either of two models (RA models and our model) which have different implications about asset pricing. As discussed in Section 3, this aspect of our model may provide a clue for resolving the inconsistency between aggregate consumption growth and asset returns which has been pointed out in the context of RA models.

Because marginal propensities to consume out of wealth (MPC_w) are constant (equal to ρ), consumption growth is identical to wealth growth at both levels, individual and aggregate.

$$(26) \qquad \frac{dc^{ns}_i(t)}{c^{ns}_i(t)} = \frac{dW^{ns}_i(t)}{W^{ns}_i(t)}$$

$$(27) \qquad \frac{dc^{s}_j(t)}{c^{s}_j(t)} = \frac{dW^{s}_j(t)}{W^{s}_j(t)}$$

$$(28) \qquad \frac{dc(t)}{c(t)} = \frac{dW(t)}{W(t)}$$

As the above equations indicate, individual consumption growth differs between non–stockholders and stockholders; the consumption (wealth) growth of stockholders is faster and more volatile than that of non–stockholders.

(stationary distribution)

Our next task is to characterize the dynamics of y(t) which

represent those of non–stockholders/stockholders wealth ratios, or leverage ratios (bond/equity ratios) under firm leverage. How rich dynamics our model produces depends on how y(t) behaves over time. If y(t) degenerates to zero, the prediction of the model is identical to that of the RA model. On the other hand, if y(t) follows a certain type of stationary distribution over time, the model can yield cyclical movements of asset returns, providing testable predictions specific to our model. The existence of such a stationary distribution also guarantees that both types of consumers coexist in financial markets (non–bankruptcy condition).

We may know by intuition, however, that y(t) might degenerate to zero under the assumptions imposed so far. By construction, a change in y(t) reflects a difference in wealth growth between the two types of consumers. As mentioned before, the differential of the expected wealth growth between the two types is identical to leveraged risk premia. Since risk premia are always ex–ante positive, the wealth growth of stockholders is faster than that of non–stockholders on average (hereafter, we call this effect on a change in y(t) the *risk premium effect*). Without any offsetting factors for the risk premium effect, the wealth growth of stockholders would eventually dominate that of non–stockholders, or y(t) might degenerate to zero in the long–run.

As one candidate for canceling out the risk premium effect, we can list up a difference in discount rates between the two types of consumers. If the discount rates of stockholders are higher than that of non–stockholders, the former consume more than the latter; consequently, the wealth growth of non–stockholders is faster than that of stockholders (hereafter, we call this effect on a change in y(t) the *differential discount effect*). It would be, however, difficult to literally take the differential

discount effect; we usually think that stockholders identified with rich consumers are likely to be more patient[5]. For dealing with such an interpretation difficulty, we construct the discussion as follows. First of all, we examine how large differential discount effect is required to cancel out the risk premium effect and to yield some stationary distribution of y(t). Second, in Appendix 2, we inquire how the differential discount effect can be interpreted in alternative contexts.

The use of Ito's lemma leads to the following Ito process of y(t) under different discount rates between stockholders and non–stockholders.

$$(29) \qquad dy(t) = (\rho^{s} - \rho^{ns})\cdot y(t)\cdot dt - (1 + y(t))\cdot y(t)\cdot \sigma\cdot dB(t)$$

where ρ^{s}: stockholder's discount rate,

ρ^{ns}: non–stockholder's discount rate.

We can present two propositions with respect to the dynamics of the above Ito process.

Proposition 1

(1) If $\mu \leq 0$ where $\mu = \dfrac{2(\rho^{s} - \rho^{ns})}{\sigma^{2}}$, then the distribution of y(t) asymptotically degenerates to a point distribution y(t) = 0 for a finite interval $0 < y < \eta < \infty$.

(2) If $0 < \mu \leq 1$, then the distribution of y(t) asymptotically degenerates to a point distribution y(t) = 0 in the neighborhood of the 0 —

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[5] The same criticism can be applied to the utility function proposed in Uzawa [1968]. He claims that for an equilibrium to be stable a discount rate is an increasing function of a consumption level.

equilibrium.

proof: see Appendix 1.

Proposition 2

If $\mu > 1$ where $\mu = \dfrac{2(\rho^{s} - \rho^{ns})}{\sigma^2}$, then y(t) follows a stationary distribution whose density function is $\pi(y) = \dfrac{m}{\sigma^2} \cdot \dfrac{y^{\mu-2}}{(1+y)^{2+\mu}} \cdot \exp(\frac{\mu}{1+y})$. m is picked such that $\int_0^{\infty} \pi(y)dy = 1$.

proof: see Appendix 1.

A parameter μ plays a crucial role in both propositions. Its numerator represents the differential discount effect, while its denominator is related to the risk premium effect (notice that leveraged risk premia are proportional to σ^2, given y(t)). Proposition 1 shows that if the differential discount effect is small relative to the risk premium effect (μ is equal to or smaller than 1), y(t) degenerates to zero in the long—run.

On the other hand, Proposition 2 demonstrates that if the differential discount effect rate is beyond some critical point relative to the risk premium effect (μ is greater than 1), y(t) neither degenerates to zero, nor explodes to infinity; rather, it has a stationary distribution. This stationary distribution can be derived in a closed—form. Figure 2 depicts the density functions of y(t). If μ is larger, the mode of y(t) shifts rightward and the average of y (t) is raised. Table 1 reports the averages and the standard errors of y(t) for several sets of parameters, from which we can observe that the larger μ is, the higher expectation and the more volatility emerge in the process of y(t).

The source of the above stationarity is motivated as follows. The differential discount effect constantly works to raise y(t), while how much the risk premium effect works to lower y(t) depends on y(t) itself. The higher y(t) is, the more the risk premium effect is accelerated through the leverage effect. When y(t) becomes higher, the accelerated risk premium effect dominates the differential discount effect, thereby lowering y(t). Conversely, when y(t) becomes smaller, y(t) tends to be raised as a result that the differential discount effect dominates the weakened risk premium effect.

In addition, the properties of the stationary distribution are interpreted as follows. Since a larger μ promotes the risk premium effect relative to the differential discount effect, it shift the stationary distribution of y(t) rightward (see Figure 2); consequently, a larger μ yields a higher expectation of y(t). A larger μ also raises the variance of y(t), because the wealth growth of stockholders fluctuate more due to the leverage effect through higher y(t).

The above rivalry between the risk premium effect and the differential discount effect also makes y(t) move inertially over time, yielding the positive serial correlation of y(t). To see this more precisely, we approximate the serial correlation of y(t) for a certain interval of time. Since the diffusion part of the Ito process of dy(t)/y(t) is still a function of y(t), it is impossible to derive the transition probability in a closed–form. For an approximation purpose, we replace y(t) in the diffusion part by the unconditional expectation of y(t), or $\bar{y}$, and obtain the following transition probability (See Arnold [1974]).

$$(30) \qquad P(y(s),\ y(t),\ Y) = P(\ y(t) \leq Y \mid y(s)\ ) \text{ where } s < t,$$

$$= \frac{1}{\sqrt{2\pi}\ \sigma} \int_{\infty}^{Z} \exp\left(-\frac{u^2\sigma^2}{2}\right) du,$$

$$\text{where } Z = \log\left(\frac{Y}{x}\right) - \left[\int_s^t (n^u - n^{uc}) - \frac{1}{2}(1+\bar{y})\right]\cdot(t-s).$$

Then, the unconditional serial covariance is defined as

$$(31) \qquad Cov(s\ t) = \int_0^1\int_0^{\infty}(y(s)-\bar{y})(y(t)-\bar{y})P(y(s),y(t),y)\cdot dy(t)\cdot g(z)\cdot dz.$$

The serial correlation is equal to the serial covariance divided by the unconditional variance of y(t). Table 2 reports the serial correlation for a set of parameters, from which we observe that the larger μ is, the lower correlation is. The reason for this relationship is that larger volatilities due to higher μ through the leverage effect tend to dominate the serially correlated movements of y(t). As equations (16), (21) and (22) show, the serial correlation of y(t) creates the serial correlation of risk–free returns and of the two types of risk premia.

(practical interpretation of time–varying y(t))

We can interpret the serial correlation of y(t) as follows. Since realized aggregate shocks are concentrated primarily on stockholders, changes in y(t) are caused mainly by changes in the wealth of stockholders (the denominator of y(t)). Because it takes some time to recover changes in the wealth of stockholders to its previous level, realized aggregate shocks have persistent effects on y(t). Such persistent effects of realized shocks cause the serial correlation in y(t).

From a different point of view, serially correlated risk premia can be interpreted as reflecting how stockholders change their attitude toward future risk. If they bear a negative aggregate shock (increasing y(t)), stockholders are more reluctant to take future risk, or demand higher risk premia for a given level of risk until their wealth position is recovered. On the other hand, if they bear a positive aggregate shock (decreasing y(t)), they are more willing to take future risk while their wealth position is relatively high. As a result, simple technological shocks can create the serial correlation in risk premia.

With respect to the serial correlation of ex—ante asset returns, our model contrasts sharply with RA models. RA models attribute time—varying investment opportunities as the source of the serial correlation; realized aggregate shocks affect the serial correlation *only through* changing investment opportunities (e.g. autoregressive conditional heteroskedasticity models). The serial correlation in our model is, on the other hand, motivated *directly* by realized aggregate shocks. Such a contrasting difference between them provides us with a set of testable implications. Chapter 2 empirically tests the implications by nesting this model against RA models with time—varying investment opportunities.

3. Model Properties with Numerical Examples

This section addresses three sets of issues with numerical examples. First, the model properties are further discussed with respect to asset pricing as well as consumption growth. Second, an attempt is made to resolve the empirical puzzle raised by Hall [1988]: why is the U.S. aggregate consumption growth not so correlated with asset returns ? The last task in

this section is to calculate the critical fixed cost/wealth ratios at which consumers are indifferent between holding risk–free assets only and participating in the risky asset market. Based on this result, we inquire into how heavy participation costs bind non–stockholders.

When we refer to the RA model throughout this section, it implies the solution from (7) to (12) with $\gamma = 1$, or the solution of our model with $y(t) = 0 \quad \forall\ t$. For numerical examples, underlying parameters are set as follows: $\alpha = 0.04$, $\rho^s = 0.02$, $\rho^s - \rho^{ns} = 0.005$, 0.01, 0.015, 0.02, 0.025, 0.03, $\sigma = 0.01$, 0.02, 0.03. If we pick some annual statistics corresponding to the U.S. economy (discussed in detail later), σ ranges from 0.01 to 0.04[6], the average of y(t) is around 0.6, and the average risk premia are from 6 % to 7 %, depending on the sample period.

(asset pricing and consumption growth)

In Table 3, the averages of risk–free returns and the two types of risk premia are evaluated in terms of the stationary distribution. Compared with the RA model, our model yields lower risk–free returns (4th column) and higher risk premia (5th and 6th columns). We further make two remarks on leveraged risk premia. First, the averages of leveraged risk premia are independent of conditional variances of productivity shocks, σ^2. The reason for this is that an increase in σ raises risk premia directly, while it lowers risk premia indirectly through a decrease in μ; consequently, both effects exactly cancel. Second, in order to make leveraged risk premia close to the U.S. counterpart (6% to 7%), the model requires the averages

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[6] Among underlying parameters, σ is the most easily available, because σ is the same as the variance of the aggregate consumption growth. See (25) and (28).

of y(t) (interpretable as debt/equity ratios under firm leverage) to be much higher than the U.S. reality (around 0.6). It implies that this simple model cannot provide us with a perfect resolution for the MP puzzle, although its prediction is in the right direction. In Section 4, we try to close this gap between our model and the U.S. reality by relaxing the assumption of the logarithmic utility function.

In Table 4, the averages of drifts and of diffusions for aggregate and individual consumption growth are evaluated in terms of the stationary distribution. With respect to conditional volatilities, (i) the consumption growth of non–stockholders is completely smooth due to the absence of diffusion parts, (ii) the consumption growth of stockholders is as volatile as leveraged risky asset returns (7th column), (iii) there is no difference in the aggregate consumption volatilities between our model and the RA model (4th column).

Table 4 also confirms that the equilibrium solution of our model is actually stationary. To see this, we need to show that consumption growth rates are equivalent between the two types of consumers in terms of long–run averages. If a long–run average growth is defined as (log c(T) – log c(0))/T, it can be approximated by (average drift) – 0.5×(average diffusion) (see Arnold [1974]). Based on this approximation, we can easily check the equality of long–run consumption growth rates between the two types.

(Hall's puzzle)

As discussed in the previous section and shown in the above numerical examples, asset pricing behavior significantly differs between our model and the RA model, while aggregate consumption behavior looks alike in both of them. This contrast between asset pricing and consumption

growth can provide us with a clue to empirical puzzles which have been raised within the context of RA models: why does the U.S. consumption grow "excessively" smoothly ? or why are the U.S. risky asset returns "excessively" volatile ?

Taking the specification due to Hall [1988] for example, we show how seriously an empirical test based on the RA model creates a specification error. Suppose empirical researchers estimate the following equation á la Hall[7].

$$\log c(t+1) = \theta \cdot R(t+1) + \log c(t) + \text{error term}$$

where c(t) is per capita consumption, while R(t+1) corresponds to either among P(t+1)/P(t), $P_1(t+1)/P_1(t)$ or r(t) in our context. In the context of the RA model, θ can be interpreted as the elasticity of intertemporal substitution (EIS) or the inverse of RRA. The least square estimator of θ is calculated from Cov{log[c(t+1)]—log[c(t)], R(t+1)}/Var(R(t+1)). Its continuous—time counterpart is equal to Cov[dc(t)/c(t), R(t)]/Var[R(t)]. Table 5 reports the estimated θ's for several sets of parameters.

If they regress aggregate consumption growth on risky asset returns with the data generated by the RA model, the estimated θ is exactly equal to one (5th column). It indeed corresponds to the EIS or the inverse of RRA for the logarithmic utility function. Even with the aggregate data generated by our model, they get the same result as above from regressing aggregate consumption growth on unleveraged risky asset returns (5th

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[7] Using several sets of data, Hall [1988] reports that estimated θ's on real Treasury bill rates are 0.346 (0.337), –0.40 (0.20), –0.03 (0.38), 0.10 (0.23), while those on real stock returns are 0.066 (0.050), 0.03 (0.10), where numbers in parentheses are standard errors.

column).

However, the estimated θ becomes significantly lower than one, once regressors are switched from unleveraged to leveraged risky asset returns (7th column). The reason for this is that volatilities of risky asset returns are raised through the leverage effect, while the conditional variances of aggregate consumption growth are independent of the leverage effect; accordingly, volatile regressors bring about smaller estimated coefficients on themselves.

If risk–free returns are used as regressors, the above specification is quite misleading even in the context of the RA model. Since no consumers hold risk–free assets along the equilibrium of the RA model, the time–series of risk–free returns cannot provide any relevant information for individual intertemporal allocation; risk–free returns are actually constant in our version of RA model.

If risk–free returns are drawn from our model, estimation results are more embarrassing. Since non–stockholders are lenders in the risk–free asset market, their financial wealth increases in response to an increase in risk–free returns, while the wealth of stockholders respond in the other direction because they are borrowers; accordingly, such opposite movements between the two types of consumers are exactly canceled out at aggregate levels. Again, the movement of risk–free returns cannot provide any effective information about intertemporal allocations at aggregate levels. What is worse, the estimated θ's may pick a seeming relationship which is completely independent of intertemporal substitution. When our model follows the stationary distribution, an increase in $W^{ns}(t)$ lowers risk–free returns by creating more demand for risk–free assets, while it raises aggregate consumption growth because consumers with lower discount rates

(non–stockholders) hold a larger portion of total wealth ($\rho^{ns} < \rho^{s}$ for stationarity). The estimated θ's turn out to be negative, if they pick such a negative correlation between risk–free returns and aggregate consumption growth (3rd column).

Under the RA model specification, very small θ's can be interpreted as very low EIS's, extremely high RRA's, or too smooth consumption growth, while negative θ's may be considered as an evidence of violating the second order condition. Some researchers may use them as an evidence for liquidity constraints. Our model, however, provides another way to interpret these estimators. According to our model, small θ's on risky asset returns reflect the leverage effect, while negative θ's on risk–free [illegible] are caused by the differential discount effect.

(critical cost/wealth ratio)

As for the third issue, we show how heavy fixed costs bind non–stockholders along the equilibrium path. To see this, we calculate the critical cost/wealth ratios at which consumers are indifferent between holding risk–free assets only and participating in the risky asset market. In other words, we derive $F_i(0)/W_i(0)$ such that

$$(32) \qquad V^{ns}(W_i(0)) = V^{s}(W_i(0) - F_i(0)),$$

$$\text{or } E_0 \int_0^{\infty} e^{-\rho t}\cdot\log(\rho W^{ns}{}_i(t))\, dt = E_0 \int_0^{\infty} e^{-\rho t}\cdot\log(\rho W^{s}{}_i(t))\, dt,$$

$$\text{where } W^{ns}{}_i(0) = W_i(0),\ W^{s}{}_i(0) = W_i(0) - F_i(0),\ \rho = \rho^{ns}.$$

The dynamic paths of $W^{ns}_i(t)$ and $W^{s}_i(t)$ can be approximated as follows (See Arnold [1974]).

$$(33) \quad W^{ns}_i(t) = W_i(0)\cdot\exp[\ (\alpha - (1 + \bar{y})\sigma^2 - \rho)\cdot t\ ],$$

$$(34) \quad W^{s}_i(t) = [W_i(0)-F_i(0)]\exp[(\alpha+\tfrac{1}{2}(\bar{y}^2-1)\sigma^2-\rho)\cdot t+(1+\bar{y})\sigma\textstyle\int_0^t dB(t)],$$

where $\bar{y} = E\ y(t)$.

Plugging (33) and (34) in (32), we obtain

$$(35) \quad \text{critical } \frac{F_i(0)}{W_i(0)} = [\exp(\frac{1}{\rho}\cdot\frac{(1+\bar{y})^2\sigma^2}{2}) - 1]/\exp(\frac{1}{\rho}\cdot\frac{(1+\bar{y})^2\sigma^2}{2}).$$

From (22), $(1 + \bar{y})^2\cdot\sigma^2 < E\ \Pi_1$ by Jensen's inequality. By using this inequality, (35) is rearranged as a function of leveraged risk premia.

$$(36) \quad \text{critical } \frac{F_i(0)}{W_i(0)} < [\exp(\frac{1}{2\rho}\cdot E\ \Pi_1) - 1]/\exp(\frac{1}{2\rho}\cdot E\ \Pi_1)$$

We use the right hand side of (36) as critical ratios such that our approximation via (33) and (34) may not favor lower critical ratios too much.

Based on the RHS of (36), Table 6 reports the critical cost/wealth ratios for several sets of leveraged risk premia and discount rates. It also adds the results from finite horizon cases[8]. Since discount rates are set on

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[8] In calculating the finite horizon cases, we still assume that y(t) follows

an annual basis, a unit of time can be interpreted as annual. If the U.S. risk premia (about 6 % to 7 %) are assumed to correspond to leveraged risk premia in our model, the 6th or the 7th row for each value of ρ in Table 6 may be closer to the U.S. reality.

We have two remarks on this table. First, the critical cost/wealth ratios are very sensitive to time horizon, especially up to 50 years; the more the time horizon is extended, the more the critical ratios increase. Second, there is some trade—off in the effects of discount rates on the critical ratios. On the one hand, an increase in discount rates narrows differences in far—future utility between the two types of consumers, because it discounts them significantly. On the other hand, an increase in discount rates widens near—future differences, because it promotes near—future consumption rather than far—future one. Hence, an increase in discount rates raises the critical ratios for the short time horizon, while it lowers them for the long time horizon.

Though a firm judgment is difficult, we may not justify too high critical ratios; it may be impossible to identify what such heavy costs would actually imply in the real world. If the time horizon is finite, but still longer rather than myopic, relatively small values of critical ratios can sustain the equilibrium path. For example, critical ratios ranging from 13 % to 20 % for 25 year time horizon can lead to 6 % risk premia in our model.

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the stationary distribution defined in Proposition 2. This procedure is not so misleading as it looks. First, the portfolio rule (13) still holds even with finite horizon cases. Second, the optimal consumption rules for two types of consumers change exactly in the same direction as time approaches the final date. That is, MPC_w's at time t for both consumers are equal to $\rho/(1-\exp(-\rho(T-t)))$, where T is a time horizon. Thanks to these reasons, a relative difference in wealth growth between them, or y(t), may still behave in almost the same way as infinite horizon cases.

4. Empirical Tests

Section 4 examines whether the U.S. data actually support the advantageous properties of our model over a simple version of RA model: (i) larger risk premia and (ii) richer dynamics of asset returns. These two issues are tested separately. With respect to the second issue, we test whether y(t) can predict actual asset returns consistently. For addressing the first issue, we develop a simple calibration which relaxes the logarithmic utility function, because the previous section shows that our simple setting cannot yield risk premia large enough to be consistent with the actual data.

(how to measure y(t))

Throughout this section, we assume that y(t)'s in our model correspond to leverage ratios or to debt/equity ratios (hereafter D/E ratios) constructed from the liability of aggregate private firms (non–financial sectors). For justifying so, we make the following assumptions. First, we classify asset classes into risk–free assets (bonds) and risky assets (equities), according to whether asset–holders are residual claimants. Second, we assume that all risk–free assets are held by non–stockholders only. This assumption ignores the fact that even stockholders hold risk–free assets primarily because of transaction convenience. Third, perfect firm leverage is assumed to be prevailing; therefore, risky asset returns are leveraged. Although our model cannot determine optimal debt/equity mixes in corporate capital structure, this simple assumption may be justified by tax reasons; deducting interest payments from corporate revenues favors debt–financing to equity–financing. Fourth, we exclude government bonds from risk–free assets, because government bonds differ from risk–free assets our model assumes in that they are backed by tax liabilities, not by production

activities. With bonds supplied by a government, equation (15) (the equilibrium condition of the risk–free asset market) needs to be revised as

$$(37) \qquad W^{ns}(t) = (x(t) - 1)\cdot W^{s}(t) + G(t),$$

G(t): government bonds.

From this condition, the portfolio choice relevant for stockholders, x(t), is not equal to $1 + W^{ns}(t)/W^{s}(t)$, but equal to $1 + [W^{ns}(t) - G(t)]/W^{s}(t)$. Since D/E ratios of private firms correspond to $[W^{ns}(t) - G(t)]/W^{s}(t)$ under firm leverage, using D/E ratios of private firms as y(t) is equivalent to excluding government bonds from risk–free assets. Finally, we treat the whole economy as if it were closed as our model assumes. Because capital markets have been integrated globally, this assumption may be one possible source of misspecifications.

(U.S. time series data)

The U.S. balance sheets (Board of Governors [1990]) provide us with reliable and convenient D/E ratios for our empirical test; (i) the series of annual sheets cover the entire post—war era, (ii) the sheets are reported every year, and (iii) each item of liabilities is evaluated at market price basis. Although the U.K., Canada and Japan adopt methods similar to the U.S. government's, their balance sheets are still less suitable to our test in terms of covered periods, reporting intervals, and valuation methods.

Besides the balance sheets, we use the following U.S. data for our estimation; (i) stock returns are constructed from the Standard and Poor's series of composite indexes and dividend/price ratios, (ii) the returns of

6—month commercial paper are used as proxies for risk—free asset returns, and (iii) the consumption price index (total items) is adopted in calculating ex—post inflation rates. The sample period is covered from 1945 to 1989. All data are available from Council of Economic Advisors [1991].

Empirical implications are derived from equations (16) and (22). Discrete—time counterparts to these ex—post relationships are specified as follows.

$$(38) \qquad \text{ex—post } r(t) = r^n(t) - \frac{CPI(t+1) - CPI(t)}{CPI(t)}$$

$$= \alpha - (1 + y(t))\cdot\sigma^2 + \xi(t+1),$$

where $r^n(t)$ is a nominal interest rate, CPI(t) is a price index, $\xi(t+1)$ is a prediction error, $E_t\xi(t+1) = 0$.

$$(39) \qquad \text{ex—post } \Pi(t) = \frac{P_1(t+1) - P_1(t)}{P_1(t)} - r(t)$$

$$= (1 + y(t))^2\cdot\sigma^2 + (1 + y(t))\cdot\sigma\cdot\eta(t+1),$$

where $\eta(t+1)$ is a prediction error, $E_t\eta(t+1) = 0$, $Var_t\eta(t+1) = 1$.

Although prediction errors in our model signify unexpected productivity shocks, those in the empirical specification, $\xi(t+1)$ and $\sigma\cdot\eta(t+1)$, may represent any exogenous shocks.

We regress ex–post returns (risk–free returns and risk premia) on one–period lagged D/E ratios rather than estimate (38) and (39) directly, because we are interested in whether y(t) consistently predicts asset returns, *not* whether the whole structure of our model is consistent with the U.S. data. According to the above equations, the signs of coefficients on lagged D/E ratios should be negative for ex–post risk–free returns, while they should be positive for ex–post risk premia.

Before reporting the estimation results, we look over the post–war data. For this purpose, we attach Figures 3 to 5. Figure 3 depicts the time–series of D/E ratios (average = 59.4 %, standard error = 16.3 %, autocorrelation = 0.78) where the vertical lines imply the troughs of the U.S. business cycle. Figure 4 draws the series of ex–post real risk–free returns (average = 0.29 %, standard error = 3.61 %), while Figure 5 traces the forward moving averages of ex–post risk premia (average = 7.54 %, standard error = 13.54 %) to remove volatile exogenous shocks for visual purpose (for example, the 1980 data point is the value averaging from 1980 to 1984). The series of D/E ratios show a long–run wave. Reflecting the U.S. business cycle, D/E ratios tend to be high in recessions and low in booms. For instance, D/E ratios remain at lower levels in 1960's when the U.S. economy performed well, while they reach higher levels from mid–1970's to early 1980's when the U.S. economy suffered from the consecutive realization of negative shocks.

If the U.S. data are consistent with our model, higher risk–free returns and smaller risk premia are tend to be observed when positive aggregate shocks are realized (lower D/E ratios), while lower risk–free returns and larger risk premia are more likely to be recorded when negative aggregate shocks are realized (higher D/E ratios). Such a relationship holds

for ex–post risk premia over the entire post–war era as well as for risk–free returns up to mid–1970's. In late 1970's to 1980's, however, risk–free returns are observed to be very high even under high D/E ratios.

Table 7 reports the estimated parameters for the pre–oil shock periods (1945 — 1973) and for the entire post–war era. In estimating equations, we include a surprised change in a nominal oil price[9] as an additional regressor, such that prediction errors about oil prices may be sorted out from other kinds of exogenous shocks. These surprised changes are calculated from the AR(1) process of the growth rates of nominal oil prices[10]. All standard errors are corrected by Newey–West estimators.

With respect to risk–free returns, the signs of coefficients on lagged D/E ratios are negative as well as significant (5 % size) for the sub–sample; however, they are insignificant for the full–sample. As Durbin–Watson statistics show, considering surprises about oil prices can help to remove serially–correlated residuals for the sub–sample, while it cannot eliminate them for the full–sample at all. These results indicate that our model lacks something important in explaining the post–oil shock behavior of risk–free returns.

On the other hand, the estimation results about risk premia show that the signs of coefficients on lagged D/E ratios are positive as well as significant for both samples. The Durbin–Watson statistics take reasonable values if surprises about oil prices are included as regressors. As far as

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9 We use the series of the crude oil price index (1982 =100) in DeGolye & MacNaughton [1990].

10 The estimated process is $y(t) = \underset{(0.026)}{0.030} - \underset{(0.122)}{0.021} \cdot y(t-1)$, D.W. = 1.93, where $y(\cdot)$ is a nominal oil growth, and ( )'s are standard errors. Since the coefficient on $y(t-1)$ is almost zero, the estimated surprises are approximately equal to the deviations from the average growth rate.

this simple specification is concerned, our model can fit the time–series of risk premia almost perfectly.

We make two additional remarks on the estimation results. First, according to our empirical investigation, the bull states of the U.S. stock market in mid–1980's originate in consecutive negative shocks up to early 1980's; positive shocks in mid–1980's tend to be magnified under very high D/E ratios. Since stock prices fell drastically due to the first and to the second oil shocks (October, 1973 and December, 1978) in most countries, the same source may be responsible for bull stock markets among these countries in mid–1980's[11].

Second, the inability of our model to explain too high risk–free returns in late 1970's to 1980's may be attributed to: (i) the intensive issue of government bonds, (ii) the tight monetary policy up to mid–1980's, and (iii) the globally integrated capital markets. Although we consider the first factor to some extent by excluding government bonds from risk–free assets (if government bonds were included in risk–free assets, higher y(t) in 1980's would cause more serious inconsistency with our model), the other two factors are completely outside our model. We may need to introduce a government sector, a monetary sector, or a foreign sector so as to reconcile our model with the post–oil shock behavior of the U.S. money market (see Section 4 in Chapter 3).

(simple calibration)

We finally try to answer how well our model can resolve the MP puzzle. By relaxing the logarithmic utility function, we calculate how large

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11 Similarily in Japan, high stock yields in mid–1980's follow high D/E ratios in mid–1970's to early 1980's.

RRA our model requires to match the observed moments of the U.S. risk premia. For this purpose, an equilibrium with the CRRA utility is approximated in a heroic way. We adopt the following approximated portfolio rule (see Section 2 in Chapter 2).

$$(40) \qquad x = \frac{\alpha - r}{\gamma\sigma^2}$$

γ: RRA

In order to be compatible with this portfolio rule, risk–free returns should be

$$(41) \qquad r = \alpha - (1 + y)\cdot\gamma\cdot\sigma^2.$$

Since the first line of equation (18) still holds in this context, leveraged risky asset returns are equal to

$$(42) \qquad \frac{dP_1(t)}{P_1(t)} = [\alpha + y\cdot(1 + y)\cdot\gamma\cdot\sigma^2]\cdot dt + (1 + y)\cdot\sigma\cdot dB(t).$$

Risk premia corresponding to the above leveraged risky returns are defined as follows.

$$(43) \qquad \Pi_1 = (1 + y)^2\cdot\gamma\cdot\sigma^2$$

Equation (43) combines the effect of RRA (see (9)) with the leverage effect (see (22)). As a result, a larger γ as well as a higher y can help to

generate larger risk premia.

Based on (43), we can calculate how large RRA is required to yield the same order of risk premia as those observed in the U.S. economy. To see this, we borrow several statistics from Mehra and Prescott [1985] where the sample period is from 1889 to 1978; the standard error of per capita aggregate consumption growth (σ) = 3.57 %, the average of risk premia (E $\Pi_1(t)$) = 6.18 %, the standard error of risky asset returns (SE($\frac{dP_1(t)}{P_1(t)}$)) = 16.54 %. We set y as follows: y = 0.0 for the RA model, while y = 0.6 for our model where y is borrowed from the U.S. post—war average of D/E ratios. We assume that the MPC_w at aggregate level is constant over time; under this assumption, the aggregate wealth growth is equal to the aggregate consumption growth. Table 8 reports how large RRA is required to match the above risk premia. In addition, the standard errors of risky asset returns (volatilities) are derived from equation (42).

The calculation result seems double—edged. On the one hand, our model can reduce the required RRA drastically from 48.5 to 18.9 and increase the volatility from 3.6 to 5.7. Such a drastic reduction in the required RRA is consistent with the results reported in Mankiw and Zeldes [1990] where they estimate RRA to be 100.4 from the RA specification, while they reduce the estimated RRA to 35.2 by using the consumption data consisting of stockholders only.

On the other hand, our specification still needs relatively high RRA and runs short of the observed volatility. Since the U.S. aggregate consumption growth is smoother in the post—war era (σ = 1.00 % in 1949–1958, 1.00 % in 1959–1968, 1.40 % in 1969–1978) than in the pre—war era (σ = 4.90 % in 1889–1898, 5.31 % in 1899–1908, 3.07 % in 1909–1918),

this negative aspects become more serious in using the post—war consumption data only. As far as matching moments is concerned, our approach may need to exploit several components invented for solving the MP puzzle within the RA models. In reward for such a marriage, we may prevent one model from depending too much on a specific parameter. For example, a weak complementarity of consumption might successfully generate large risk premia with the help of the leverage effect[12].

5. Concluding Remarks

In this chapter, we show that the model with participation constraints can produce larger risk premia as well as richer dynamics of asset returns than those predicted by a simple version of RA model. The source of large risk premia is due to the high concentration of aggregate shocks on stockholders, where a degree of concentration is measured by leverage ratios. This implication can further be tested by exploiting different degrees of concentration among countries. Saito [1991] examines the above implication by the data of OECD countries. He finds that risk premia are larger and risk-free returns are lower in countries with higher leverage ratios (e.g. Japan and North European countries) than in those with lower leverage ratios (e.g. the U.S. and Canada).

With respect to asset pricing dynamics, our model can successfully establish the relationship between business cycles and asset pricing behavior. This model has a clear prediction about what happens to asset returns in recessions and in booms. In this respect, our model contrasts sharply with

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[12] One objection to the habit formation hypothesis is that it requires a very strong complementarity in order to match the observation.

RA models or models with incomplete insurance whose predictions about asset pricing dynamics depend deeply on how underlying processes (endowment, productivity, idiosyncratic shocks) are characterized. Based on our simple model, we can still discuss asset pricing behavior in the macro–economic context without heavily parameterizing models or without any intensive reference to micro–data.

APPENDIX

1. Proofs of Propositions

Proof of Proposition 1

(first part): When $f(0) = 0$ and $\sigma(0) = 0$ for an Ito process $dy(t) = f(y(t))\cdot dt + \sigma(y(t))\cdot dB(t)$, the 0 — equilibrium is stable from the right for an interval $0 < y < \eta$, if and only if $\int_0^{\eta} \exp\{ \int_u^{\eta} \frac{2\cdot f(x)}{\sigma(x)^2} dx \} du < \infty$ (Gihnman and Skorohod [1972]). Applying this theorem to the Ito process (29), we have to show

$$\int_0^{\eta} \exp\{ \int_u^{\eta} \frac{2(\rho^s - \rho^{ns})x}{(1+x)^2 x^2 \sigma^2} dx \} du < \infty$$

in order to prove the stability around $y(t) = 0$. After rearranging, we have

$$\int_0^{\eta} \exp\{ \int_u^{\eta} \frac{2(\rho^s - \rho^{ns})x}{(1+x)^2 x^2 \sigma^2} dx \} du = \int_0^{\eta} \{(\frac{1+u}{u})^{\mu} \cdot \exp(\frac{-\mu}{1+u})\} du.$$

Since $\lim_{u\to\infty}\{(\frac{1+u}{u})\cdot\exp(\frac{-1}{1+u})\} = 1$ and $d\{(\frac{1+u}{u})\cdot\exp(\frac{-1}{1+u})\}/du < 0$, we obtain

$$\{(\frac{1+u}{u})\cdot\exp(\frac{-1}{1+u})\} \geq 1 \text{ for } 0 \leq u \leq \infty.$$

Hence, for $\mu \leq 0$,

$$\int_0^{\eta}\{ (\frac{1 + u}{u})^{\mu}\cdot\exp(\frac{-\mu}{1 + u}) \}du = \int_0^{\eta}\{ (\frac{1 + u}{u})\cdot\exp(\frac{-1}{1 + u}) \}^{\mu} du$$

$$\leq \int_0^{\eta}\{ (\frac{1 + u}{u})\cdot\exp(\frac{-1}{1 + u}) \}^{0} du$$

$$= \eta$$

Therefore, if $\mu \leq 0$ and $0 < \eta < \infty$, then $\int_0^{\eta}\exp\{ \int_u^{\eta}\frac{2(\rho^s - \rho^{ns})x}{(1+x)^2x^2\sigma^2} dx \}du < \infty$. (Q.E.D.)

(second part): Similarly, we have to show

$$\lim_{\epsilon\to 0} \int_0^{\epsilon}\exp\{ \int_u^{\epsilon}\frac{2(\rho^s - \rho^{ns})x}{(1+x)^2x^2\sigma^2} dx \} du < \infty$$

in order to prove the stability at $y(t) = 0$.

$$\lim_{\epsilon\to 0}\int_0^{\epsilon}\exp\{\int_u^{\epsilon}\frac{2(\rho^s - \rho^{ns})x}{(1+x)^2x^2\sigma^2}dx\}du = \lim_{\epsilon\to 0} [(1+\epsilon)^{\mu}\cdot\epsilon^{-\mu}\cdot\exp(\frac{-\mu}{1 + \epsilon})]\cdot\epsilon$$

$$= \lim_{\epsilon\to 0} [(1 + \epsilon)^{\mu}\cdot\epsilon^{1-\mu}\cdot\exp(\frac{-\mu}{1 + \epsilon})]$$

If $\mu \leq 1$, then all three products are finite in the last right hand side. Hence, $\lim_{\epsilon\to 0}\int_0^{\epsilon}\exp\{ \int_u^{\epsilon}\frac{2(\rho^s - \rho^{ns})x}{(1+x)^2x^2\sigma^2} dx \} du$ is actually finite. (Q.E.D.)

Proof of Proposition 2

If and only if the stationary distribution of y(t) exits for an Ito process $dy(t) = f(y(t))\cdot dt + \sigma(y(t))\cdot dB(t)$, then it is given by $\pi(y) = \frac{m}{\sigma(y)^2}\cdot\exp\{\int^y \frac{2f(x)}{\sigma^2(x)}\,dx\}$, where m is picked such that $\int_0^\infty \pi(x)\,dx = 1$ [13]. We apply this formula to the Ito process (29). For the moment, we assume the existence of $\pi(y)$ for (29); then, we determine which ranges of parameters actually guarantee the existence of the derived stationary distribution. $\pi(y)$ is defined as follows.

$$\pi(y) = \frac{m}{(1+y)^2 y^2 \sigma^2}\cdot\exp\{\int^y \frac{2(\rho^s - \rho^{ns})x}{(1+x)^2 x^2 \sigma^2}\,dx\}$$

After rearranging, we have

$$\pi(y) = \frac{m}{\sigma^2}\cdot\frac{y^{\mu-2}}{(1+y)^{2+\mu}}\cdot\exp(\frac{\mu}{1+y}).$$

By a change of variable $z = 1/(1+y)$, $0 < z < 1$, the density function of z, g(z), is

$$g(z) = \pi(y)\left|\frac{dy}{dz}\right|$$

$$= k\cdot(1-z)^{(\mu-1)-1} z^{3-i}\exp(\mu z).$$

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[13] Bhattacharya and Majumdar [1980] and Malliaris [1982] survey this class of stationary distribution. Bourguignon [1974] and Merton [1975] apply this type of stationary distribution to stochastic growth models.

If $\mu - 1 > 0$, $\int_0^\infty g(x)dx$ is a linear transformation of the moment generating function of the beta function. It is known that the moment generating function exists for the beta function (see Johnson and Kotz [1970]). Hence, if $\mu > 1$, $\int_0^\infty g(x)dx$ actually exists and k can be picked such that $\int_0^\infty g(x)dx = 1$. (Q.E.D.)

Proof of Proposition 3

Proposition 3

The value functions are bounded for non–stockholders and stockholders.

proof:

(non–stockholders): We have to show $V^c(W_i(0)) < B < \infty$. Since $r(t) \leq \alpha - \sigma^2$,

$$V^c(W_i(0)) = \int_0^\infty e^{-\rho t}(\log \rho + \log W(0))dt + E_0\int_0^\infty e^{-\rho t}(r(t)+n^c-\rho)\cdot t\cdot dt$$

$$\leq \int_0^\infty e^{-\rho t}(\log \rho + \log W(0))\cdot dt + \int_0^\infty e^{-\rho t}(\alpha - \sigma^2 + n^c - \rho)\cdot t\cdot dt$$

$$= \frac{1}{\rho}(\log \rho + \log W(0)) + \frac{1}{\rho^2}\cdot(\alpha - \sigma^2 + n^c - \rho)$$

As far as ρ is positive, the last right hand side is finite. Consequently, $V^c(W_i(0))$ is bounded. (Q.E.D.)

(stockholders): Given that $\rho > 0$, $r(t) = \alpha - (1 + y(t))\cdot\sigma^2$, and $y(t)$

follows the stationary distribution defined in Proposition 2, the Bellman equation for the optimization problem (2) is expressed with the optimal rules (12) and (13) as follows.

$$\rho^s \cdot V(W(t)|y(t)) = \log(\rho^s W(t))$$

$$+ [\alpha(1+y(t)) - (\alpha - (1+y(t))\cdot\sigma^2)\cdot y(t) - \rho^s]\cdot W(t)\cdot V_w(W(t)|y(t))$$

$$+ \frac{1}{2}(1+y(t))^2\sigma^2 W(t)^2 V_{ww}(W(t)|y(t)) + (\rho^s - \rho^{ns})y(t)V_y(W(t)|y(t))$$

$$+ (1+y(t))^2 y(t)^2 V_{yy}(W(t)|y(t)) + (1+y(t))^2 y(t)\sigma^2 V_{wy}(W(t)|y(t))$$

If the guess solution $V(W(t)|y(t)) = \frac{1}{\rho}\cdot\log W(t) + v(y(t))$ is plugged in the above Bellman equation, we obtain the following linear differential equation of $v(y(t))$.

$$(1+y(t))^2\cdot y(t)^2\cdot v''(y(t)) + (\rho^s - \rho^{ns})\cdot y(t)\cdot v'(y(t)) - \rho\cdot v(y(t))$$

$$= -\log\rho - \frac{1}{2\rho}(1+y(t))^2\sigma^2 - \frac{1}{\rho}[\alpha - \rho - (1+y(t))\sigma^2]$$

By Picard's Theorem, as far as $0 < y(t) < \infty$, $v(y(t))$ can always have an arbitrary finite value. Hence, given finite positive $W(t)$ and $y(t)$, the value function $V(W(t)|y(t))$ is bounded. (Q.E.D.)

2. Alternative Interpretation for Differential Discount Effect

We propose several alternative factors which play the same role as the differential discount effect as follows: (a) exogenous incomes flow more to the financial assets of non–stockholders than to those of stockholders, (b) the assets of stockholders are handed down generation by generation *with some friction* such as a weak altruistic motivation, inheritance taxes, and so on, (c) a government redistributes assets from stockholders to non–stockholders, (d) new cohorts with financial assets flow to non–stockholders more than to stockholders.

If (A.2) is replaced by the following assumption (A.2'), the first factor (a) can be introduced into our model.

> (A.2') an exogenous income/wealth ratio, $n = I_i(t)/W_i(t)$ is constant over time, but the ratio for non–stockholders is higher than that of stockholders ($n^{ns} > n^s$).

Under assumption (A.2'), an exogenous income can be treated as constant excess returns on financial assets. As a result, the previous set of solutions still holds except for the individual/aggregate wealth growth processes.

$$\frac{dW^{ns}_i(t)}{W^{ns}_i(t)} = [\alpha - (1 + y(t))\cdot\sigma^2 + n^{ns} - \rho]\ dt$$

$$\frac{dW^{s}_j(t)}{W^{s}_j(t)} = [\alpha+(y(t)+y(t)^2)\sigma^2+n^s-\rho]dt + (1 + y(t))\sigma dB(t)$$

$$dW(t) = [\alpha\cdot W(t)+I^{ns}(t)+I^s(t)-c^{ns}(t)-c^s(t)]dt + \sigma\cdot W(t)dB(t)$$

$$= (\alpha-\rho+\frac{y(t)}{1+y(t)}n^{ns}+\frac{1}{1+y(t)}n^{s})W(t)dt \ +\sigma W(t)dB(t),$$

where $I^{ns}(t) = n^{ns}\cdot\Sigma_i^I \ W_i(t)$, $I^s(t) = n^s\cdot\Sigma_j^J \ W_j(t)$.

The Ito process of y(t) in this case is equivalent to equation (29), if $(\rho^s - \rho^{ns})$ is replaced by $(n^{ns} - n^s)$.

$$dy(t) = (n^{ns} - n^s)\cdot y(t)\cdot dt - (1 + y(t))\cdot y(t)\cdot\sigma\cdot dB(t)$$

The last factor (d) may be captured by adding the following assumption (A.5) instead of replacing (A.2).

(A.5) non–stockholders (stockholders) with financial assets $n^{ns}\cdot W^{ns}(t)$ $(n^s\cdot W^s(t))$ constantly flow to the economy. n^{ns} is higher than n^s.

This assumption makes the same Ito process of y(t) as under (A.2') available again, while it does not change the individual optimization problem unlike under (A.2').

One example of the second factor (b) combined with the third factor (c) is an inheritance tax. If we assume that a probability of death per unit time is equal to π, and that inheritance taxes with 100τ % are imposed on financial assets upon death of stockholders (wealthy consumers), the *total* wealth growth of stockholders is changed from (24) to

$$\frac{dW^s(t)}{W^s(t)} = [\alpha+(y(t)+y(t)^2)\sigma^2-\rho-\pi\tau]dt \ + \ (1+y(t))\sigma dB(t).$$

As a consequence, the Ito process of y(t) is slightly modified as follows.

$$dy(t) = \pi\tau \cdot y(t) \cdot dt - (1 + y(t)) \cdot y(t) \cdot \sigma \cdot dB(t)$$

As long as the variance of the technological process is not large, even a low probability of death is likely to satisfy the condition for generating the stationary distribution (e.g. $\pi = 0.03$, $\tau = 0.3$, $\sigma = 0.1$).

Table 1:
Averages and Standard Errors of y(t)

σ	$\rho^s - \rho^{ns}$	ν	y(t) average	s.e.
0.01	0.005	100.0	7.132	5.818
0.01	0.010	200.0	10.435	8.306
0.01	0.015	300.0	12.883	10.268
0.01	0.020	400.0	15.188	11.939
0.01	0.025	500.0	17.402	13.292
0.01	0.030	600.0	19.306	14.386
0.02	0.005	25.0	3.149	2.791
0.02	0.010	50.0	4.799	4.047
0.02	0.015	75.0	6.066	5.007
0.02	0.020	100.0	7.132	5.818
0.02	0.025	125.0	8.078	6.531
0.02	0.030	150.0	8.939	7.172
0.03	0.005	11.1	1.823	1.772
0.03	0.010	22.2	2.921	2.617
0.03	0.015	33.3	3.765	3.260
0.03	0.020	44.4	4.476	3.802
0.03	0.025	55.6	5.104	4.278
0.03	0.030	66.7	5.671	4.708

Table 2:
Serial Correlation of y(t)

σ	$\rho^{s}-\rho^{ns}$	μ	time–interval	y(t) average	y(t) serial–correlation
0.01	0.01	200.0	0.5	10.438	0.164
0.01	0.01	200.0	1.0	10.438	0.160
0.01	0.02	400.0	0.5	15.111	0.146
0.01	0.02	400.0	1.0	15.111	0.144
0.01	0.03	600.0	0.5	18.697	0.141
0.01	0.03	600.0	1.0	18.697	0.138
0.03	0.01	22.2	0.5	2.921	0.527
0.03	0.01	22.2	1.0	2.921	0.519
0.03	0.02	44.4	0.5	4.476	0.379
0.03	0.02	44.4	1.0	4.476	0.372
0.03	0.03	66.7	0.5	5.671	0.296
0.03	0.03	66.7	1.0	5.671	0.287
0.05	0.01	8.0	0.5	1.423	0.704
0.05	0.01	8.0	1.0	1.423	0.700
0.05	0.02	16.0	0.5	2.353	0.582
0.05	0.02	16.0	1.0	2.353	0.578
0.05	0.03	24.0	0.5	3.068	0.500
0.05	0.03	24.0	1.0	3.068	0.494

Table 3:
Averages of Risk–free Returns and Risk Premia
Evaluated in Terms of Stationary Distribution

σ	$\rho^s - \rho^{ns}$	y(t) average	risk–free returns	risk premia home leverage	risk premia firm leverage
RA Model					
0.01		0.0	0.0399	0.0001	
Constrained Model					
0.01	0.005	7.134	0.0392	0.0008	0.0100
0.01	0.010	10.438	0.0389	0.0011	0.0200
0.01	0.015	12.973	0.0386	0.0014	0.0300
0.01	0.020	15.111	0.0384	0.0016	0.0400
0.01	0.025	16.994	0.0382	0.0018	0.0500
0.01	0.030	18.697	0.0380	0.0020	0.0600
RA Model					
0.02		0.0	0.0396	0.0004	
Constrained Model					
0.02	0.005	3.149	0.0383	0.0017	0.0100
0.02	0.010	4.799	0.0377	0.0023	0.0200
0.02	0.015	6.066	0.0372	0.0028	0.0300
0.02	0.020	7.134	0.0367	0.0033	0.0400
0.02	0.025	8.075	0.0364	0.0036	0.0500
0.02	0.030	8.926	0.0360	0.0040	0.0600
RA Model					
0.03		0.0	0.0391	0.0009	
Constrained Model					
0.03	0.005	1.823	0.0375	0.0025	0.0100
0.03	0.010	2.921	0.0365	0.0035	0.0200
0.03	0.015	3.765	0.0357	0.0043	0.0300
0.03	0.020	4.476	0.0351	0.0049	0.0400
0.03	0.025	5.104	0.0345	0.0055	0.0500
0.03	0.030	5.671	0.0340	0.0060	0.0600

Table 4:
Drifts and Diffusions for Consumption Growth Processes Evaluated in Terms of Stationary Distribution

σ	$\rho^s - \rho^{ns}$	aggregate drift	aggregate diffusion	non-stockholders drift	stockholders drift	stockholders diffusion
RA Model						
0.01		0.0	0.0001			
Constrained Model						
0.01	0.005	0.0042	0.0001	0.0042	0.0092	0.0100
0.01	0.010	0.0089	0.0001	0.0089	0.0189	0.0200
0.01	0.015	0.0137	0.0001	0.0136	0.0286	0.0300
0.01	0.020	0.0184	0.0001	0.0184	0.0384	0.0400
0.01	0.025	0.0233	0.0001	0.0232	0.0482	0.0500
0.01	0.030	0.0281	0.0001	0.0280	0.0580	0.0600
RA Model						
0.02		0.0	0.0004			
Constrained Model						
0.02	0.005	0.0035	0.0004	0.0033	0.0083	0.0100
0.02	0.010	0.0079	0.0004	0.0077	0.0177	0.0200
0.02	0.015	0.0124	0.0004	0.0122	0.0272	0.0300
0.02	0.020	0.0169	0.0004	0.0167	0.0367	0.0400
0.02	0.025	0.0216	0.0004	0.0214	0.0464	0.0500
0.02	0.030	0.0262	0.0004	0.0260	0.0560	0.0600
RA Model						
0.03		0.0	0.0009			
Constrained Model						
0.03	0.005	0.0029	0.0009	0.0025	0.0075	0.0100
0.03	0.010	0.0069	0.0009	0.0065	0.0165	0.0200
0.03	0.015	0.0112	0.0009	0.0107	0.0257	0.0300
0.03	0.020	0.0155	0.0009	0.0151	0.0351	0.0400
0.03	0.025	0.0200	0.0009	0.0195	0.0445	0.0500
0.03	0.030	0.0244	0.0009	0.0240	0.0540	0.0600

Table 5:
Covariance between Aggregate Consumption Growth and Asset Returns

(unit = dt, except for the third column where unit = $(dt)^2$)

		risk–free return	risky asset return			
			home leverage		firm leverage	
σ	$\rho^s - \rho^{ns}$	covar	covar	θ	covar	θ
RA Model						
0.01		0.0	0.0001	1.0000		
Constrained Model						
0.01	0.005	–.0012	0.0001	1.0000	0.0008	0.0813
0.01	0.010	–.0025	0.0001	1.0000	0.0011	0.0572
0.01	0.015	–.0039	0.0001	1.0000	0.0014	0.0466
0.01	0.020	–.0057	0.0001	1.0000	0.0016	0.0403
0.01	0.025	–.0069	0.0001	1.0000	0.0018	0.0360
0.01	0.030	–.0073	0.0001	1.0000	0.0020	0.0328
RA Model						
0.02		0.0	0.0004	1.0000		
Constrained Model						
0.02	0.005	–.0011	0.0004	1.0000	0.0017	0.1660
0.02	0.010	–.0023	0.0004	1.0000	0.0023	0.1160
0.02	0.015	–.0036	0.0004	1.0000	0.0028	0.0942
0.02	0.020	–.0048	0.0004	1.0000	0.0033	0.0813
0.02	0.025	–.0062	0.0004	1.0000	0.0036	0.0726
0.02	0.030	–.0074	0.0004	1.0000	0.0040	0.0662
RA Model						
0.03		0.0	0.0009	1.0000		
Constrained Model						
0.03	0.005	–.0009	0.0009	1.0000	0.0025	0.2541
0.03	0.010	–.0021	0.0009	1.0000	0.0035	0.1765
0.03	0.015	–.0033	0.0009	1.0000	0.0043	0.1429
0.03	0.020	–.0045	0.0009	1.0000	0.0049	0.1232
0.03	0.025	–.0058	0.0009	1.0000	0.0055	0.1099
0.03	0.030	–.0070	0.0009	1.0000	0.0060	0.1001

Table 6:
Critical Cost/Wealth Ratios

(unit = %)

risk premia under firm leverage	time horizon (unit = year) 10	25	50	100	200	300	∞
ρ = 2 %							
1 %	0.4	2.2	6.4	13.8	20.3	21.8	22.1
2 %	0.9	4.4	12.4	25.7	36.5	38.8	39.3
3 %	1.3	6.5	18.0	36.0	49.4	52.1	52.7
4 %	1.7	8.6	23.2	44.8	59.7	62.6	63.2
5 %	2.2	10.7	28.1	52.4	67.9	70.7	71.4
6 %	2.6	12.7	32.8	59.0	74.5	77.2	77.7
7 %	3.0	14.6	37.1	64.7	79.6	82.1	82.6
8 %	3.4	16.5	41.0	69.5	83.7	86.0	86.4
ρ = 4 %							
1 %	0.8	3.3	7.2	10.7	11.7	11.8	11.8
2 %	1.5	6.4	13.8	20.3	22.1	22.1	22.1
3 %	2.3	9.4	20.0	28.9	31.2	31.2	31.2
4 %	3.0	12.4	25.7	36.5	39.3	39.3	39.3
5 %	3.8	15.2	31.0	43.3	46.4	46.5	46.5
6 %	4.5	18.0	36.0	49.5	52.7	52.8	52.8
7 %	5.3	20.7	40.6	54.9	58.2	58.3	58.4
8 %	6.0	23.2	44.8	59.6	63.1	63.2	63.2
ρ = 5 %							
1 %	1.0	3.6	6.5	7.9	8.0	8.0	8.0
2 %	2.0	7.1	12.5	15.1	15.4	15.4	15.4
3 %	3.0	10.5	18.1	21.8	22.1	22.1	22.1
4 %	4.0	13.7	23.4	27.9	28.3	28.3	28.3
5 %	5.0	16.8	28.4	33.6	34.1	34.1	34.1
6 %	5.9	19.9	33.0	38.9	39.4	39.4	39.4
7 %	6.9	22.8	37.4	43.7	44.2	44.2	44.2
8 %	7.8	25.5	41.3	48.0	48.6	48.6	48.6

Table 7:
Regression Results

(standard errors in parentheses)

dependent variable	ex–post real commercial paper rate			
sample period	1945–73	1945–73	1945–89	1945–89
lagged D/E ratio	–0.1463 (0.064)	–0.0754 (0.039)	–0.0260 (0.033)	–0.0057 (0.031)
oil price surprise		–0.2937 (0.049)		–0.0864 (0.027)
constant	0.0700 (0.033)	0.0401 (0.020)	0.0192 (0.021)	0.0093 (0.019)
# of samples	27	27	44	44
R–square	0.1678	0.6835	0.0138	0.2174
SSR	0.0329	0.0105	0.0562	0.0444
Durbin–Watson	0.9369	1.6361	0.5935	0.7879

dependent variable	ex–post risk premium			
sample period	1945–73	1945–73	1945–89	1945–89
lagged D/E ratio	0.6396 (0.197)	0.6638 (0.180)	0.1908 (0.123)	0.2461 (0.119)
oil price surprise		–0.5492 (0.222)		–0.2565 (0.100)
constant	–0.2326 (0.101)	–0.2399 (0.092)	–0.0437 (0.076)	–0.0669 (0.072)
# of samples	27	27	44	44
R–square	0.2957	0.4385	0.0539	0.1936
SSR	0.3015	0.2404	0.7419	0.6281
Durbin–Watson	1.6659	2.1093	1.7121	2.0605

Table 8:
Required RRA, RA Model v.s. Our Model

	RA Model	Our Model
average y(t)	0.0	0.6
RRA	48.5	18.9
volatility	3.6	5.7

Figure 1 - 1 Flow of Funds in RA Model

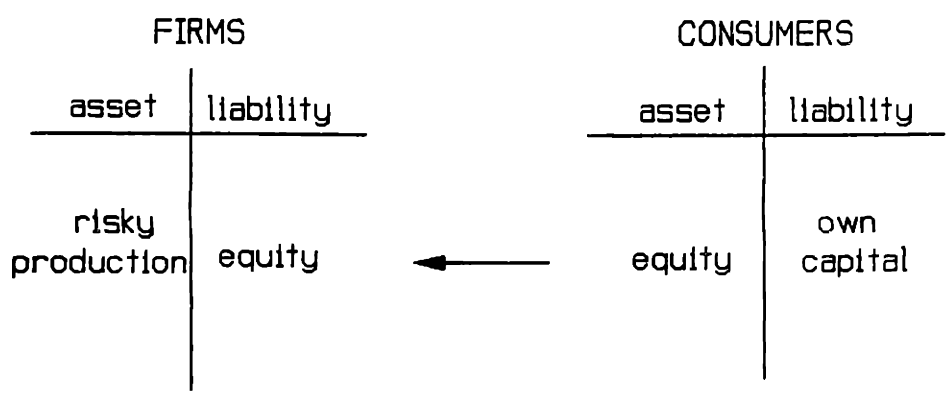
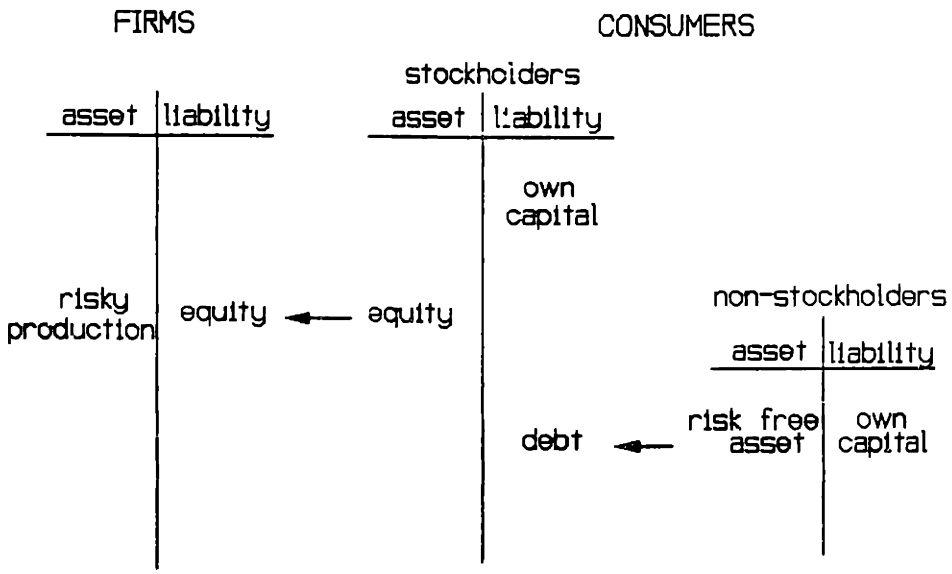


Figure 1 - 2 Flow of Funds in Model with Participation Constraints

< HOME LEVERAGE >



< FIRM LEVERAGE >

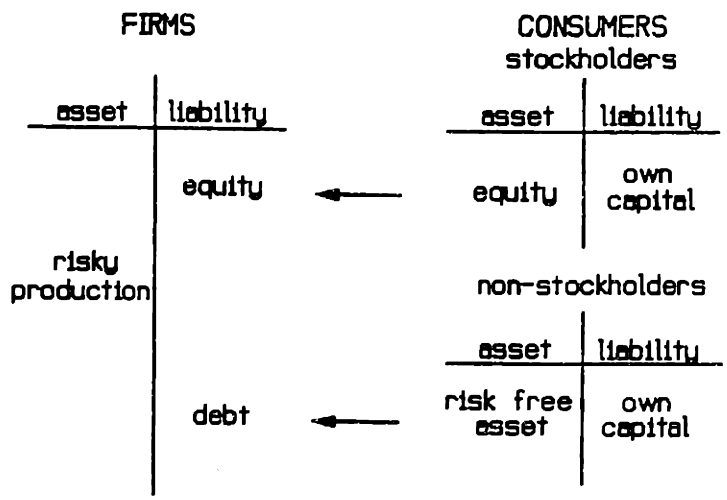


Figure 2: Density Function of Y

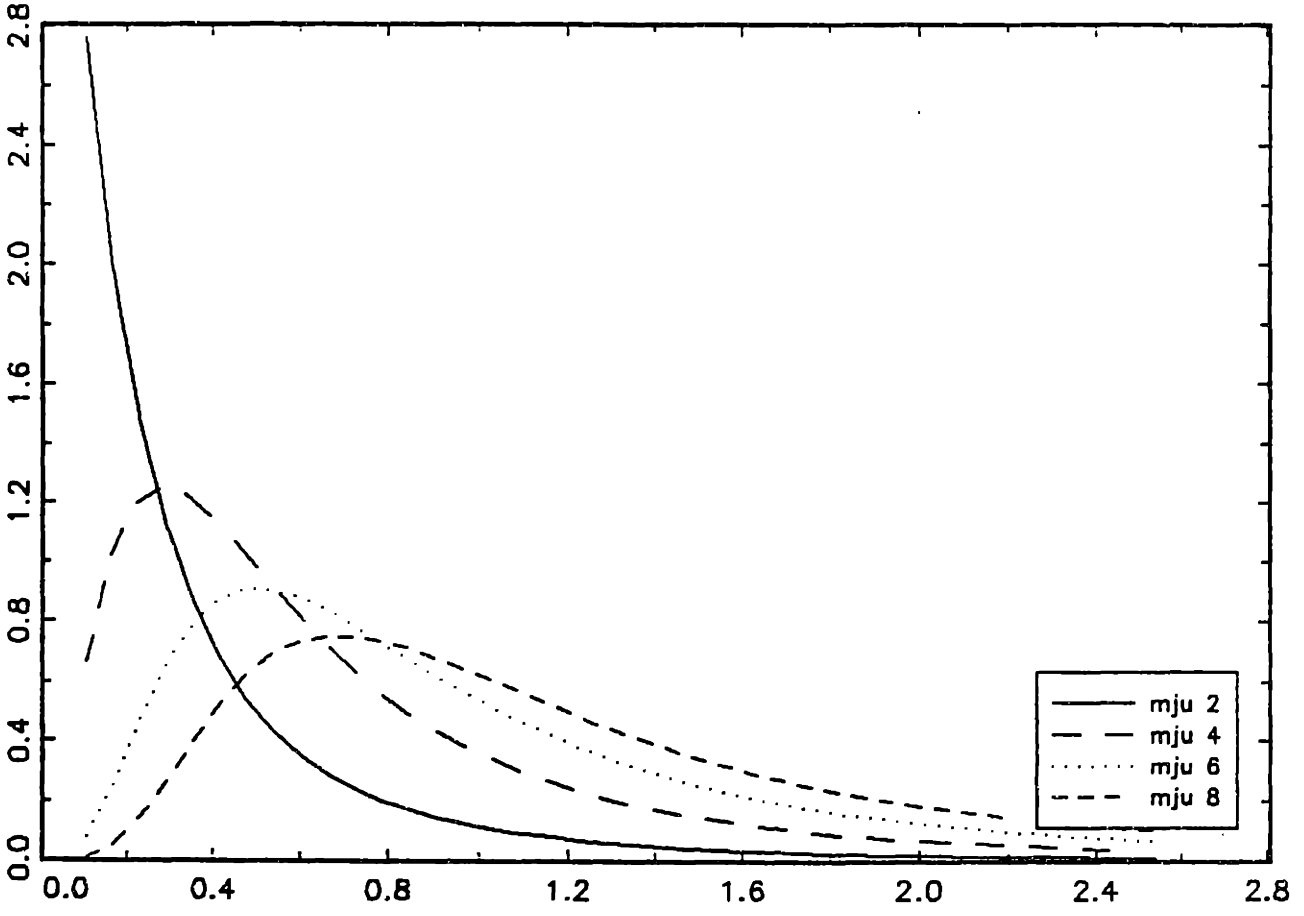
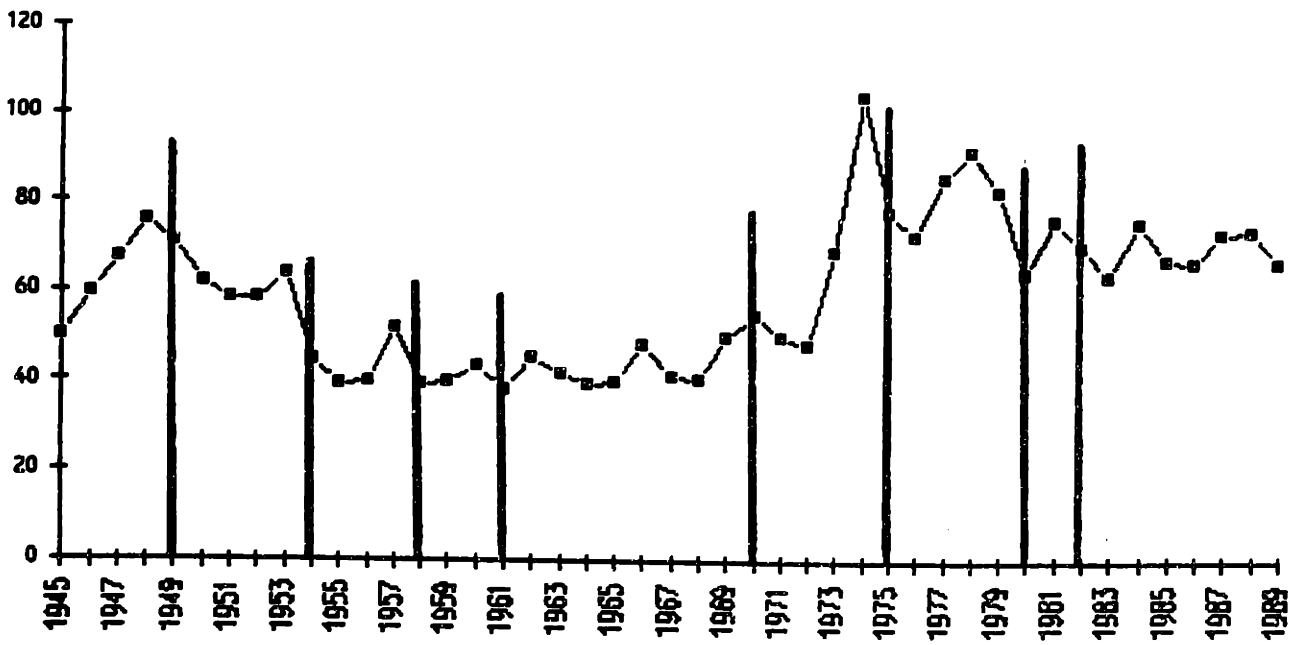


Figure 3: U.S. Debt/Equity Ratios



Vertical line : Troughs of Business Cycles

Figure 4: U.S. Real Risk-Free Returns vs Debt/Equity Ratios

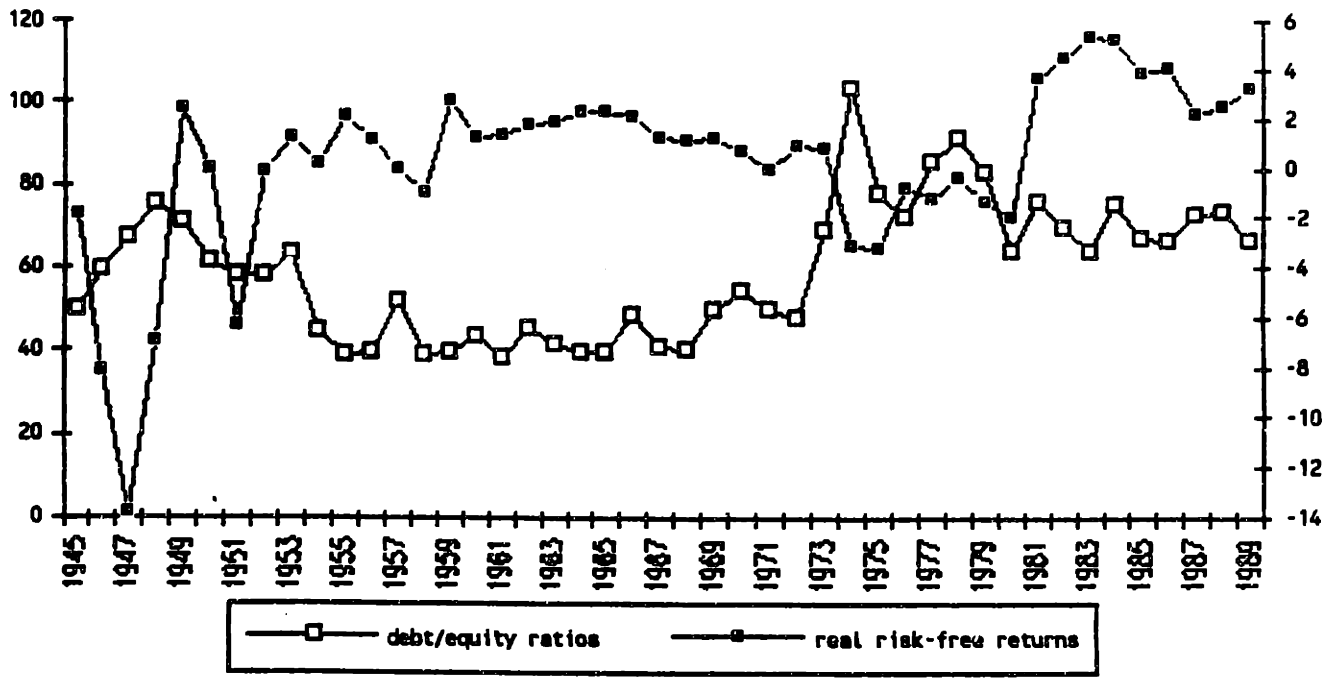
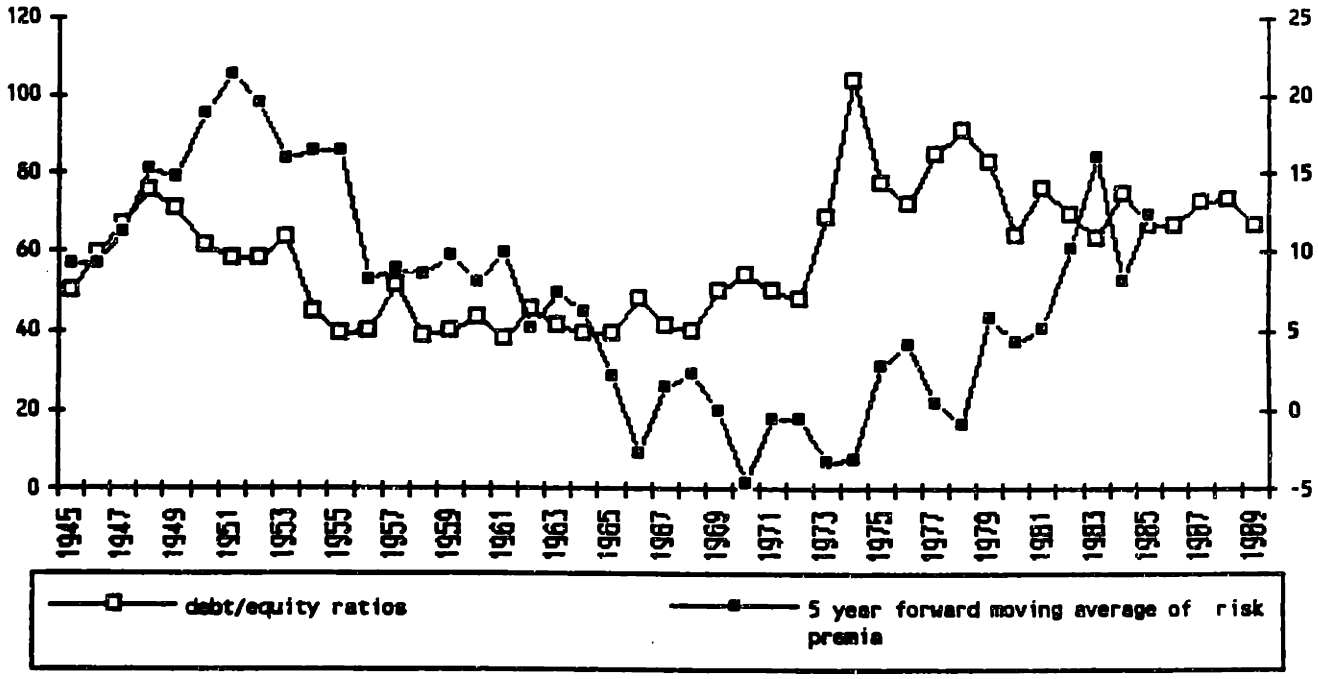


Figure 5: U.S. Risk Premia vs Debt/Equity Ratios



Chapter 2

On the Response of Risk Premia to Aggregate Shocks: Time–Varying Risk vs. Trading among Heterogeneous Consumers

1. Introduction

The lack of empirical support for representative agent models (RA models) does not necessarily imply that the asset pricing behavior is independent of aggregate economic activities. Analysts in the business world have recognized that a set of macroeconomic variables may have predictive power for future asset returns, while academic scholars have also documented some empirical relationships between asset pricing and aggregate economy (e.g. Schwert [1989]). If RA models fail in reconciling these facts[1], alternative models should relate the asset pricing behavior to macroeconomic activities or to business cycles.

What is missing in RA models[2], if anything ? Although RA models capture the interaction between the underlying technology and the behavior of individual agents, they abstract any explicit effects of trading among heterogeneous agents on asset pricing. The latter aspect of RA

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[1] Although predictability of asset returns is not necessarily inconsistent with the implications of RA models, RA models may fail in consistently explaining the observed relationships between asset prices and macro economic variables.

[2] What is meant by RA models in this context is a class of models where a representative agent utility is independent of the initial wealth distribution among agents (aggregation preperty, see Huang and Litzenberger [1988]).

models is contradictory with what the market participants think about the actual financial markets; they believe that asset returns are determined in the financial markets such that the demand from some types of agents are equal to the supply by other types of agents. Based on such a belief, they often adopt the supply–and–demand–for–credit method in forecasting interest rates (e.g. Kaufman [1986]).

From the above point of view, this chapter investigates the forces that drive the dynamics of the risk premia observed in the post–war U.S. stock market. For this purpose, we propose the following hypotheses, (i) time–varying investment opportunities and (ii) time–varying attitudes of risk–takers. The first hypothesis is developed within RA models; the risk premium dynamics reflect time–varying riskiness of the underlying technology. If time–varying riskiness is specified as a function of aggregate variables, RA models can predict the relationship between risk premia and macroeconomic activities.

On the other hand, the second hypothesis is obtained from models with heterogeneous agents (HA models) in which asset pricing is determined by the interaction between risk–takers and risk–hedgers. In this chapter, we construct three examples for the second hypothesis: (i) heterogeneous degrees of relative risk aversion (RRA), (ii) wealth–dependent risk aversion (RRA is a decreasing function of the level of individual wealth), and (iii) heterogeneous investment opportunities (a subset of consumers are excluded from the stock market). The roles of risk–takers are played by consumers with lower RRA, richer consumers, and stockholders respectively.

These three examples have in common the following properties. In equilibrium, risk–takers hold more risky physical assets by issuing risk–free bonds to risk–hedgers, while risk–hedgers demand risk–free bonds in order to

hedge the wealth fluctuations arising from aggregate shocks. Since risk–takers bear more aggregate risk than risk–hedgers, the wealth of the former is more volatile than that of the latter. Because their willingness to bear risks is proportional to the level of their own wealth (a characteristic of the utility function with a constant degree of RRA), risk–takers demand higher risk premia for holding risky assets when their wealth decreases in response to adverse shocks (larger risk premia following recessions), while they demand lower risk premia when they experience favorite shocks (smaller risk premia after booms). If risk–takers issue risk–free assets indirectly to risk–hedgers through firms (firm leverage), time–varying attitudes of risk–takers are reflected not only in the conditional mean of risk premia, but also in its conditional volatility.

The above two hypotheses, RA models and HA models, can be empirically differentiated in terms of how risk premia respond to realized aggregate shocks. Realized aggregate shocks, in RA models, can affect the equity premium dynamics only through their changing investment opportunities, while the risk premium dynamics in HA models are driven by the wealth fluctuations of risk–takers due to realized shocks. These two hypotheses, consequently, can have different empirical implications in relation to macroeconomic activities.

This chapter is organized as follows. In Section 2, a simple version of RA models with time–varying risk is built on Cox, Ingersoll and Ross [1985a], while three examples for HA models are constructed on the basis of Dumas [1989], Marcus [1989], and the model developed in Chapter 1. Section 3 derives some empirical implications from these theoretical models and constructs tests for comparing the above competing hypotheses. Section 4 concludes. An appendix discusses an alternative specification of RA

models with time–varying risk based on the Markov switching model (Hamilton [1989], Cecchetti, Lam and Mark [1990a, b]).

2. Theoretical Predictions under Two Hypotheses

In this section, we present a simple framework by which RA models with time–varying investment opportunities can be differentiated empirically from HA models where two types of agents trade with each other. For doing so, we construct continuous–time asset pricing models based on the two hypotheses and derive the testable predictions about the risk premium dynamics. Throughout this section, we assume that: (i) only one risky technology is available in terms of physical assets, (ii) returns on physical assets are driven only by aggregate shocks, (iii) risk–free returns are endogenized by the market–clearing condition, and (iv) consumers trade with one another only through financial markets.

2–1. RA Models with Time–varying Investment Opportunities (basic set–up)

Based on Cox, Ingersoll and Ross [1985a], we derive risk premia from RA models with time–varying investment opportunities. In this economy, all consumers are identical except for the levels of their wealth. We assume that each consumer faces a linear risky technology defined as below, where the conditional expectation of productivity ($\alpha(\cdot)$) and its conditional variance ($\sigma(\cdot)$) are changing over time. One state variable, b(t), is assumed to drive the dynamics of these conditional moments, while b(t) itself follows the Ito process.

$$(1) \qquad \frac{dS}{S} = \alpha(b(t))dt + \sigma(b(t))dB_1(t),$$

$$(2) \qquad db(t) = g(b(t))dt + w(b(t))dB_2(t),$$

where $dB_1(t)$ and $dB_2(t)$ are standard Brownian motions.

Exogenous shocks, $dB_1(t)$ and $dB_2(t)$, are interpreted as representing aggregate shocks in this economy.

Given the above investment opportunity, each infinitely–lived consumer solves the following optimization problem in which his discounted utility is maximized by investing his wealth over risky and risk–free assets.

$$(3) \qquad V(W_i(t),b(t)) = E_t\int_t^{\infty} e^{-\rho(\tau-t)}u(c_i(\tau))d\tau,$$

s.t.

$$dW_i(t)=[\{(1-x_i(t))r(t)+x_i(t)\alpha(b(t))\}W_i(t)-c_i(t)]dt+x_i(t)W_i(t)\cdot\sigma(b(t))dB_1(t),$$

where $u(\cdot)$: utility function,

ρ: discount rate,

$x_i(t)$: risky asset/total asset ratio of consumer i,

$W_i(t)$: wealth level of consumer i,

$c_i(t)$: consumption level of consumer i,

$r(t)$: risk–free returns.

The assumption of identical consumers implies that no one holds

risk–free assets in market equilibrium ($x_i(t) = 1 \ \forall$ t and i). Given this equilibrium condition, and assuming that u(c) is a utility function with a constant degree of RRA (γ) ($u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$), Cox, Ingersoll and Ross [1985a] show that ex–post risk premia, $\pi(t) = \frac{dS}{S} - r(t)dt$, is derived as

$$(4) \quad \pi(t) = \{\gamma\sigma(b(t))^2 - \frac{V_{wb}}{V_w} w(b(t))\sigma(b(t))\sigma_{12}\}dt + \sigma(b(t))dB_1(t),$$

where σ_{12} is a covariance between $dB_1(t)$ and $dB_2(t)$.

The first term on the right hand side of equation (4) implies that ex–ante risk premia are proportional to the conditional volatility of risky technology. On the other hand, the third term indicates that the conditional volatility of risk premia is identical to that of the underlying technology. We, however, do not know much about the properties of the second term, since it is difficult to derive a closed–form solution for that term. What Cox, Ingersoll and Ross [1985a] show is that (i) $\frac{V_{wb}}{V_w}$ is independent of $W_i(t) \ \forall$ t and i, and (ii) if γ is closer to 1 (closer to a log–utility specification), $\frac{V_{wb}}{V_w}$ approaches zero.

(empirical implications)

In terms of empirical implications, we focus on the following: (i) ex–ante risk premia are positively correlated with time–varying risk, and (ii) the conditional volatility of risk premia is affected only by technological

riskiness. If we assume that the implication from the first term in equation (4) is not weakened by the behavior of its second term at all, equation (4) implies that ex–ante risk premia ($E_t\pi(t)$) are linear functions of $\sigma(b(t))^2$ with a constant multiplier of γ (RRA).

2–2. Models with Trading among Heterogeneous Consumers: Three Examples (basic set–up)

As an alternative hypothesis to the above RA model, we construct HA models in which asset pricing is determined by the interaction between risk–takers and risk–hedgers. In this sub–section, three examples are constructed for this class of models: (i) heterogeneous degrees of RRA, (ii) wealth–dependent RRA, and (iii) heterogeneous investment opportunities.

The basic set–up for these models is as follows. Each agent faces a constant investment opportunity whose conditional expectation of productivity is α, while its conditional volatility is σ^2.

$$(5) \qquad \frac{dS}{S} = \alpha\ dt + \sigma\ dB(t),$$

where dB(t) is a standard Brownian motion.

Again, an exogenous shock, dB(t), is interpreted as representing an aggregate shock. In this set–up, the technological side is not responsible for the risk premium dynamics unlike in the previous model.

Consumers are heterogeneous with respect to RRA, γ_i. Given the above technological opportunity, each infinitely–lived consumer solves the following optimization problem in which his discounted utility is maximized

by investing his wealth over risky and risk–free assets.

$$(6) \qquad V(W_i(t),r(t)) = E_t\int_t^{\infty} e^{-\rho(\tau-t)}u(c_i(\tau))d\tau,$$

s.t.

$$dW_i(t) = [\{(1-x_i(t))r(t)+x_i(t)\alpha W_i(t) - c_i(t)]dt + x_i(t)W_i(t)\sigma dB_1(t),$$

$$\text{where } u(c) = \frac{c^{1-\gamma_i} - 1}{1 - \gamma_i}.$$

If $x_i(t)$ (risky asset share) is greater than one, he is a borrower in the risk–free asset market.

Since risk–free returns change over time depending on the market clearing condition, a simple optimal portfolio rule is not available for the above optimization problem. An optimal portfolio rule is characterized only in terms of the value function, $V(W_i(t), r(t))$.

$$(7) \qquad x_i(t) = \frac{\alpha - r(t)}{-\dfrac{V_{ww}W}{V_w}\sigma^2} - \frac{V_{wr}}{V_{ww}W}\kappa(r(t)),$$

where κ is a function of $r(t)$.

As a degree of RRA (γ_i) approaches 1 (closer to a log–utility function) in the right hand side of equation (7), $-\dfrac{V_{ww}W}{V_w}$ (the denominator of the

first term) approaches γ_i, while the second term vanishes. For simplicity, we approximate the above portfolio rule as follows.

$$(7') \qquad x_i(t) = \frac{\alpha - r(t)}{\gamma_i \sigma^2}$$

Equation (7') implies that risky asset shares are inversely proportional to a degree of RRA.

Throughout the three examples, the first type of consumers are risk–takers who hold more risky assets than their own capital by financing additional resources from the risk–free asset market, while the second type of consumers are risk–hedgers who demand risk–free assets in order to hedge against the wealth fluctuations. In these models, the following equilibrium condition must hold in the risk–free asset market.

$$(8) \qquad (x_1(t) - 1)W_1(t) = (1 - x_2(t))W_2(t),$$

where $W_1(t)$: the total wealth of the first type, risk–takers,
$W_2(t)$: the total wealth of the second type, risk–hedgers.

The left hand side of equation (8) corresponds to the supply by risk–takers, while the right hand side is equal to the demand from risk–hedgers. Risk–free returns, therefore, are determined by the trading between the two types of consumers.

Because, up to the last example, we assume that risk–takers issue risk–free assets *directly* to risk–hedgers (home leverage), risky asset returns are the same as returns on physical assets (equation (5)). In other words,

they are completely independent of the trading between the two types of agents. Accordingly, we can define ex–post risk premia ($\pi(t)$) as $\frac{dS}{S} - r(t)dt \quad (= \alpha\ dt + \sigma\ dB(t) - r(t)dt)$.

For notational convenience, we define the ratio of the wealth of risk–takers to aggregate wealth as

$$(9) \qquad \omega_1(t) = \frac{W_1(t)}{W_1(t) + W_2(t)}.$$

Since risk premia in all examples depend on the wealth distribution among heterogeneous consumers, the time–series behavior of $\omega_1(t)$ can determine the time path of risk premia.

(example 1: heterogeneous degrees of risk aversion)

The first example is borrowed from Dumas [1989], where the RRA of the first type of consumers is lower than that of the second type of consumers, $\gamma_1 < \gamma_2$. In this example, consumers with lower RRA are risk–takers, because they are more robust to the wealth fluctuations due to aggregate shocks than those with higher RRA. The market equilibrium condition is

$$(10) \qquad \left(\frac{\alpha - r(t)}{\gamma_1 \sigma^2} - 1\right)W_1(t) = \left(1 - \frac{\alpha - r(t)}{\gamma_2 \sigma^2}\right)W_2(t).$$

Based on the equilibrium risk–free returns from equation (10), ex–post risk premia are derived as

$$(11) \qquad \pi(t) = \frac{\gamma_1 \gamma_2 \sigma^2}{\omega_1(t)(\gamma_2 - \gamma_1) + \gamma_1} dt + \sigma \, dB(t).$$

If the wealth of risk–takers occupies the aggregate wealth ($\omega_1(t) = 1$), ex–ante risk premia completely reflect the risk aversion of risk–takers, or $E_t\pi(t) = \gamma_1\sigma^2 dt$, while if their wealth is negligible relative to the aggregate wealth ($\omega_1(t) = 0$), ex–ante risk premia represent the risk aversion of risk–hedgers, or $E_t\pi(t) = \gamma_2\sigma^2 dt$. Depending on the time–series of $\omega_1(t)$, therefore, ex–ante risk premia fluctuate between $\gamma_1\sigma^2 dt$ and $\gamma_2\sigma^2 dt$.

(example 2: wealth–dependent RRA)

The second example is an extended case of Marcus [1989], where RRA is a decreasing function of the level of individual wealth. Assuming that there exist a wealthy consumer and a poor consumer, we specify each wealth–dependent RRA as follows.

$$(12) \qquad \gamma_1(t) = \left(\frac{W_1(t)}{W_1(t) - \overline{W}}\right)^m \cdot \gamma,$$

$$\gamma_2(t) = \left(\frac{W_2(t)}{W_2(t) - \overline{W}}\right)^m \cdot \gamma,$$

where $m > 1$ and $\overline{W} > 0$.

If the first type is wealthier than the second type at all times, or $W_1(t) > W_2(t)$ $\forall$ t, the RRA of the first type is always lower than that of the second type; therefore, a wealthier consumer is a risk–taker. The market

equilibrium condition is

$$(13) \qquad \left(\frac{\alpha - r(t)}{\gamma_1(t)\sigma^2} - 1\right)W_1(t) = \left(1 - \frac{\alpha - r(t)}{\gamma_2(t)\sigma^2}\right)W_2(t).$$

From equation (13), ex–post risk premia are derived as

$$(14) \quad \pi(t) = \frac{1}{\omega_1^{1-m}(\omega_1-\bar{\omega})^m+(1-\omega_1)^{1-m}(1-\omega_1-\bar{\omega})^m}\gamma\sigma^2 dt + \sigma dB(t)$$

Since Marcus [1989] treats only the special case where m = 1, risk premia in his model are completely independent of the wealth distribution between the rich and the poor ($E_t\pi(t) = \frac{1}{1 - 2\bar{\omega}(t)} \gamma\sigma^2 dt$) [3]. In our case with m > 1, the time path of $\omega_1(t)$ still affects ex–ante risk premia as long as $\bar{\omega}(t)$ is positive (if $\bar{\omega}(t) = 0$, $E_t\pi(t) = \gamma\sigma^2 dt$).

(example 3: heterogeneous investment opportunities and leverage effect)

The third case is proposed by Chapter 1, where a subset of consumers are completely excluded from the stock market (risky asset market). We can consider this model as an extreme case of the above two examples, because stockholders are pure risk–takers in the sense that they bear the entire aggregate risk, while non–stockholders are pure risk–hedgers who are completely free from any aggregate shocks for an instantaneous period. The market equilibrium condition is

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[3] Implicitly due to this reason, Marcus [1989] considers his model as belonging to a class of RA models.

$$(15) \qquad \left(\frac{\alpha - r(t)}{\gamma\sigma^2} - 1\right)W_1(t) = W_2(t).$$

The left hand side of equation (15) corresponds to the capital financed through the risk–free asset market by stockholders. On the other hand, the total wealth of non–stockholders appears on the demand side, because they do not have any alternative investment opportunities. From equation (15), ex–post risk premia are derived as

$$(16) \qquad \pi(t) = \frac{\gamma\sigma^2}{\omega_1(t)}\, dt + \sigma\, dB(t).$$

Similar to the previous examples, the time–series of ex–ante risk premia depend on the evolution of the wealth distribution between stockholders and non–stockholders, $\omega_1(t)$.

Unlike the others, however, this example can explicitly treat the leverage effect. If stockholders issue risk–free assets *indirectly* to non–stockholders through firms (firm leverage), risky asset returns are also affected by the trading between the two types of consumers. In the case of firm leverage, risky asset returns are equal to the total returns of physical assets *net of* the interest payments of risk–free assets divided by the wealth of stockholders. (See Section 2 in Chapter 1.)

$$(17) \qquad \frac{dP(t)}{P(t)} = \frac{1}{W_1(t)}\{(\alpha+\sigma dB(t))(W_1(t)+W_2(t))-r(t)\cdot W_2(t)\}$$

Combined equation (17) with equation (15), ex–post risk premia can be

defined in terms of leveraged risky asset returns.

$$(18) \qquad \pi_1(t) = \frac{\gamma\sigma^2}{\omega_1(t)^2}\, dt + \frac{\sigma}{\omega_1(t)}\, dB(t)$$

As equation (18) implies, ex–ante risk premia are more sensitive to changes in $\omega_1(t)$ than in equation (16) (notice that $0 \leq \omega_1(t) \leq 1$), while the conditional volatility of risk premia is influenced not only by the technological side (σ), but also by the trading between the two types of consumers ($\omega_1(t)$).

(empirical implications)

Given equations (11), (14), (16) and (18), the risk premium dynamics depend crucially on how the wealth distribution, $\omega_1(t)$, evolves over time. Although $\omega_1(t)$ is difficult to observe directly, we can point out the following relationship between the short–run properties of $\omega_1(t)$ and the realized sequence of aggregate shocks[4]:

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[4] Each example has a different long–run property.
(1) By intensive numerical studies, Dumas [1989] shows that $\omega_1(t)$ is positively and uniquely related to the aggregate wealth level; therefore, risk premia approach $\gamma_1\sigma^2$ if the aggregate wealth grows over time, while risk premia come close to $\gamma_2\sigma^2$ if the aggregate wealth declines over time. Dumas [1989] establishes the conditions for the aggregate wealth to grow (decline) forever.
(2) In the second example, $\bar{\omega}(t)$ degenerates to 0 if the aggregate wealth grows over time. Once $\bar{\omega}(t) = 0$, risk premia are completely independent of the evolution of the wealth distribution, $\omega_1(t)$.
(3) Section 2 in Chapter 1 proposes several conditions in which the wealth of stockholders does not dominate that of non–stockholders, or $\omega_1(t)$ does not degenerate to one.

(a) Since risk–takers bear more aggregate shocks than risk–hedgers, the wealth of risk–takers fluctuates more. Accordingly, the relative share of their wealth, $\omega_1(t)$, responds positively to realized aggregate shocks.

(b) Realized aggregate shocks have persistent effects on changes in $\omega_1(t)$; consequently, $\omega_1(t)$ are positively serially correlated. Although this implication can not be derived directly from the above simple analyses, Dumas [1989] shows this property by an intensive numerical calculation, while Section 2 in Chapter 1 proves it by approximating the serial correlation function of $\omega_1(t)$. This property is the result of the fact that it takes some time for the wealth of risk–takers to recover its change due to aggregate shocks.

Since $\dfrac{\partial\ E_t\,\pi(t)}{\partial\ \omega_1(t)} < 0$, and given the above properties (a) and (b), ex–ante risk premia respond negatively to the lagged series of aggregate shocks. Intuitively, this prediction can be understood as follows. With negative shocks realized, risk–takers suffer from severer capital losses because they hold more risky assets (a decrease in $\omega_1(t)$); consequently, in the subsequent periods, they become more reluctant to take risks from the other consumers as their wealth decreases. In other words, they demand higher compensations (premia) for holding risky assets. In the case of favorite shocks, the above mechanism works in the opposite way.

If firm leverage is adopted as in equation (18), the same prediction as above can be applied to the conditional volatility of risk premia, because $\dfrac{\partial\ \mathrm{Var}_t\,\pi_1(t)}{\partial\ \omega_1(t)} < 0$. Since the leverage ratio (debt/equity ratio) corresponds to the non–stockholder/stockholder wealth ratio in the third example with firm leverage, it is inversely proportional to $\omega_1(t)$. When

negative shocks are realized (a decrease in $\omega_1(t)$), higher leverage ratios raise the volatility of risky asset returns, while lower ratios reduce its volatility after positive shocks are realized (an increase in $\omega_1(t)$).

2–3. Summary

To summarize, if the underlying technology is represented by equation (1), the dynamics of risk premia can be characterized as

$$(19) \qquad \pi(t) = f(\sigma(t),\omega_1(t))dt + v(\sigma(t),\omega_1(t))dB_1(t),$$

where f( ) is the conditional mean of risk premia, while v( ) is the conditional volatility.

In RA models, the conditional volatility of aggregate risk ($\sigma(t)$) is the driving force that determines the time path of risk premia, and both $\frac{\partial f}{\partial \sigma(t)}$ and $\frac{\partial v}{\partial \sigma(t)}$ are positive. In HA models, on the other hand, the evolution of the wealth distribution ($\omega_1(t)$) motivates the risk premium dynamics, and both $\frac{\partial f}{\partial \omega_1(t)}$ and $\frac{\partial v}{\partial \omega_1(t)}$ are negative. As discussed above, $\omega_1(t)$ may be replaced by $\lambda(L)\epsilon(t)$ where $\epsilon(t)$'s are the lagged sequence of aggregate shocks with $\lambda(\tau) > 0$ for some duration; accordingly, f( ) and v( ) are negatively correlated with lagged shocks, $\epsilon(\tau)$'s, for some interval.

3. Empirical Implementation and Estimation Results

3–1. Empirical Implementation

In this section, we specify the underlying technological process (equation (1)) and the risk premium process (equation (19)) such that the two hypotheses, RA models and HA models, can be empirically nested. Returns on physical assets, y(t), are assumed to follow the AR process defined by equation (20), where a building block, $\epsilon(t)$, represents a unique aggregate shock.

$$(20) \qquad y(t+1) = \text{constant} + \theta(L)\cdot y(t) + \epsilon(t+1),$$

where $\epsilon(t+1)$ distributes with the conditional mean 0 and the conditional variance $\sigma_\epsilon(t+1)^2$. $\sigma_\epsilon(t+1)^2$'s are supposed to be a measurable function of the information set available at time t.

For modeling $\sigma_\epsilon(t+1)^2$ and specifying equation (19), we adopt three different types of specifications: (i) squared residual specification, (ii) absolute residual specification, and (iii) maximum likelihood estimator (MLE). In addition, the appendix discusses another type of specification of the conditional variance based on the Markov switching model (Hamilton [1989]).

(squared residual specification)

The first type of specification exploits the fact that the conditional variance, $\sigma_\epsilon(t+1)^2$, can be approximated by the conditional mean of squared residuals, $E_t\epsilon(t+1)^2$ (Amemiya [1977]). It is further assumed that $E_t\epsilon(t+1)^2$ is a linear function of the past sequence of squared residuals. Under these assumptions, $\sigma_\epsilon(t+1)^2$ is specified as

$$(21) \qquad \sigma_\epsilon(t+1)^2 \simeq E_t\epsilon(t+1)^2,$$

where $\epsilon(t+1)^2 = \text{constant} + \alpha(L)\cdot\epsilon(t)^2 + \nu(t+1)$ with $E_t\nu(t+1) = 0$.

Given the above approximated conditional variance, the risk premium process (equation (19)) is assumed to be formulated as

$$(22) \qquad \pi(t+1) = \text{constant} + \gamma\, E_t\epsilon(t+1)^2 + \lambda(L)\ \epsilon(t) + \xi(t+1),$$

where $E_t\xi(t+1) = 0$.

In RA models, γ is positive and it can be interpreted as RRA. In HA models, on the other hand, $\lambda(\tau)$'s are expected to be negative for some duration.

Following the squared residual specification, the conditional volatility of risk premia, $\sigma_\xi(t+1)^2$, is also specified as

$$(23) \qquad \sigma_\xi(t+1)^2 \simeq E_t\xi(t+1)^2,$$

where $\xi(t+1)^2 = \text{constant} + \eta\, E_t\epsilon(t+1)^2 + \delta(L)\ \epsilon(t) + \zeta(t+1)$ with $E_t\zeta(t+1) = 0$.

If the observation is completely consistent with the RA specification, η is expected to be one, while the other parameters including a constant term are equal to zero. If the leverage effect works in HA models, $\delta(\tau)$'s are expected to be negative for some interval.

In terms of estimation procedures, we estimate equations (20) to (23) *sequentially* by the ordinary least squares estimator (OLS estimators).

As Pagan [1984] shows, however, the estimated standard errors by OLS are inconsistent under the multi–stage estimation, although the estimated parameters themselves are consistent. In this study, the inconsistencies of standard errors are corrected by the two step method proposed by Newey [1984].

(absolute residual specification)

The second type of specification exploits the fact that the conditional standard error, $\sigma_\epsilon(t+1)$, is linearly proportional to the conditional mean of absolute values of residuals, $E_t|\epsilon(t+1)|$ (Glejser [1969]). It is further assumed that $E_t|\epsilon(t+1)|$ is a linear function of the past sequence of absolute values of residuals. Under these assumptions, $\sigma_\epsilon(t+1)$ is specified as

$$(24) \qquad \sigma_\epsilon(t+1) = \psi\ E_t|\epsilon(t+1)|,$$

where $|\epsilon(t+1)| = \text{constant} + \alpha(L)\cdot|\epsilon(t)| + \nu(t+1)$ with $E_t\nu(t+1) = 0$, ψ is a positive constant.

Given the above conditional standard error, the risk premium process (equation (19)) is assumed to be specified as

$$(25) \qquad \pi(t+1) = \text{constant} + \gamma\ E_t|\epsilon(t+1)| + \lambda(L)\ \epsilon(t) + \xi(t+1),$$

where $E_t\xi(t+1) = 0$.

In RA models, γ is positive, although it cannot be interpreted as RRA as

in equation (22). In HA models, $\lambda(\tau)$'s are expected to be negative for some duration.

Following the absolute residual specification, the conditional standard error of risk premia, $\sigma_\xi(t+1)$, is also specified as

$$(26) \qquad \sigma_\xi(t+1) \simeq \varphi \; E_t|\xi(t+1)|,$$

where $|\xi(t+1)| = \text{constant} + \eta \; E_t|\epsilon(t+1)| + \delta(L) \; \epsilon(t) + \zeta(t+1)$ with $E_t\zeta(t+1) = 0$, φ is a positive constant.

The interpretation of parameters is exactly the same as in the first specification.

We follow the same estimation strategy as in the first specification. In comparing the second specification with the first, the efficiency of the first specification deteriorates once the distribution of residuals ($\epsilon(t)$'s) deviates from the normal distribution, while the efficiency of the second is robust to such a deviation (see Dividian and Carroll [1987]). Due to this reason, Schwert [1989] and Kandel and Stambaugh [1990] adopt the second specification in modeling the conditional volatility of stock returns.

(maximum likelihood estimator)

In the third estimation, we impose more stringent specifications on the structure of $\sigma_\epsilon(t+1)^2$ and $\sigma_\xi(t+1)^2$, while equations (20) and (22) are still adopted. Following Engle [1982], the ARCH (autoregressive conditional heteroskedasticity) structure is given for characterizing $\sigma_\epsilon(t+1)^2$.

$$(27) \qquad \epsilon(t+1) = z_1(t+1)\cdot\sigma_\epsilon(t+1),$$

where $\sigma_\epsilon(t+1)^2 = \alpha(0) + \alpha(L)\cdot\epsilon(t)^2$ with positive $\alpha(\cdot)$'s, $z_1(t+1)$ follows the standard normal distribution.

On the other hand, $\sigma_\xi(t+1)^2$ is specified by the exponential ARCH proposed by Nelson [1991] in order to avoid negative variances due to the leverage effect.

$$(28) \qquad \xi(t+1) = z_2(t+1)\cdot\sigma_\xi(t+1),$$

$$\text{where } \sigma_\xi(t+1)^2 = \exp(\delta(0) + \delta(L)\cdot\epsilon(t) + a_1|\xi(t)| + a_2\xi(t))$$

Since it is assumed that $z_1(t+1)$ and $z_2(t+1)$ follow the standard bivariate normal distribution with a correlation coefficient ρ, the above specification may allow non–aggregate shocks to drive the risk premium dynamics[5]. Negative $\delta(\cdot)$'s can capture the leverage effect due to aggregate shocks, while negative a_2 can represent the leverage effect by non–aggregate shocks. If an estimated ρ is close to one, the conditional volatility of risk premia is driven mainly by the aggregate technological process, otherwise, it suggests the significance of other shocks in the risk premium dynamics.

By MLE, equations (20), (22), (27), and (28) can be estimated *simultaneously*.

3–2. Estimation Results

(data)

If we try to estimate aggregate shocks from the data rigorously

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5 Cutler, Poterba, and Summers [1990] emphasize the importance of non–fundamental shocks as driving forces for equity pricing.

corresponding to the underlying models, we need to obtain the time–series of aggregate physical assets. It would be, however, very difficult to do so, especially for higher frequencies. In this study, we use the industrial production index as a proxy for the aggregate physical asset by assuming that the former is proportional to the latter. We define the shocks to the growth rates of the industrial production indexes as the aggregate shocks corresponding to the underlying models. The total industrial production indexes (seasonally–adjusted) are released by Board of Governors of the Federal Reserve System. Its sample period covers the entire post—war era. For our research purpose, we construct quarterly growth rates.

Ex—post risk premia are defined as ex–post risky asset returns net of risk–free returns. In this study, risky asset returns are derived from the Standard & Poor's composite stock price indexes together with its dividend yields, while the three month treasury bill rates are used as risk–free returns. Corresponding to the production index growth, ex–post risk premia are calculated in a quarterly frequency for the post—war era.

(estimation results: multi–stage OLS estimation)

Table 1—A to 1—D report the results of the first and the second specifications by the multi–stage OLS estimation. The AR process performs quite well in estimating the production process (equation (20)) (Table 1—A). This simple AR approximation can successfully remove the positive serial correlation of estimated residuals (very high significance level of Q statistics). In estimating time–varying aggregate risk, the squared residual specification (equation (21)) yields almost the same results about signs of parameters as the absolute residual specification (equation (25)) (Table 1—B). The former, however, can more successfully remove the positive serial

correlation of estimated residuals than the latter.

When the risk premium processes (equations (22) and (25)) are estimated *without* considering the effects due to HA models, the specifications work very poorly (the first and the third columns in Table 1—C). First, estimated residuals are highly serially correlated in both types of specifications (extremely low significance levels of Q statistics). It indicates that hidden variables other than time–varying aggregate risk may have predictive power for future risk premia. Second, $\hat{\gamma}$ in equation (22) is estimated to be very large; large RRA is required for explaining a large variation of the observed risk premia by a small variation of time–varying risk. These two observations may be interpreted as another phase of the Mehra and Prescott Puzzle (Mehra and Prescott [1985]) which claims that the large risk premia observed in the U.S. stock market cannot be successfully explained by RA models.

If the lagged sequence of aggregate shocks are included in the above risk premium process, the estimation results are substantially improved (the second and the fourth columns in Table 1—C). First, the inclusion of the lagged aggregate shocks helps to remove the serial correlation of estimated residual in both types of specifications (higher significance level of Q statistics). Second, the signs of $\hat{\lambda}$'s are estimated to be consistent with the prediction of HA models. The observed risk premia respond negatively to lagged aggregate shocks up to 6 quarters. Some of estimated λ's are significantly negative; $\hat{\lambda}(1)$ and $\hat{\lambda}(4)$ are significant at a 2.5 % level in both types of specifications, while $\hat{\lambda}(3)$ and $\hat{\lambda}(5)$ are significant at a 10 % level in the squared residual specification.

The advantage of HA models over RA models may be confirmed by a visual comparison as well. Figures 1 and 2 draw the fitted values of

risk premia for the two cases (without the HA specification and with it) together with the 1 year forward moving averages of the observed ex–post risk premia. As Figure 1 shows, the fitted values from RA models cannot trace the actual behavior of risk premia very well. In Figure 2, on the other hand, the fitted values from HA models can trace the actual series relatively well.

The conditional volatility of risk premia is estimated via equations (23) and (26) (Table 1–D). As insignificant or negative $\hat{\eta}$'s show, the conditional volatility of risk premia is not driven very much by time–varying risk of the underlying technology. In addition, the leverage effect due to aggregate shocks is detected only weakly, because $\hat{\delta}$'s are estimated to be insignificantly negative.

(estimation results: MLE)

Table 2 reports the estimation results of the MLE specification. Although their significance levels are a little worse than those of the multi–stage estimation, the overall results are quite similar to the other two types of specifications in that (i) the observed risk premia respond negatively to realized aggregate shocks up to from 6 quarters to 8 quarters, and (ii) the leverage effect due to aggregate shocks is detected only weakly.

The interesting differences from the previous results should be pointed out. First, the lagged shocks are included in the list of variables (shifting from specification 1 to specification 4), RRA ($\hat{\gamma}$)'s are estimated to be smaller. In other words, the effects due to HA models help to predict the conditional mean of risk premia under smaller RRA's. Second, very low estimated $\hat{\rho}$ (around 0.1) implies that aggregate shocks are not strongly correlated with the shocks which motivate the unpredictable part of risk

premium dynamics. Third, unlike estimated $\hat{\delta}$'s, $\hat{a}_2$ in equation (28) is estimated to be significantly negative. It suggests that the conditional volatility of risk premia is levered mainly by non–aggregate shocks. These three observations are consistent with the calibration results of Section 4 in Chapter 1; the model with heterogeneous investment opportunities can match the observed conditional mean of risk premia under relatively small RRA's, while the predicted volatility is far from the observed volatility.

4. Conclusion

This chapter shows that the conditional mean of risk premia in the U.S. stock market reflects the trading among heterogeneous agents or the interaction between risk–takers and risk–hedgers; the effects due to HA models have high predictive power for future levels of equity premia. The estimation results, however, suggest that the conditional volatility of risk premia is levered mainly by non–aggregate shocks, although the leverage effect due to aggregate shocks is weakly detected. In other words, as far as the first moments are concerned, the relationship between equity pricing and the U.S. economy may be established through considering the trading among heterogeneous consumers. Further theoretical models, however, need to be developed for explaining the second moment of the observed risk premia.

The empirical investigation in this chapter may provide us with some theoretical background for the practice of the market participants. Why do they forecast asset returns based on a set of macroeconomic variables ? Why do they pay serious attention to the asset demand and the asset supply from different types of agents ? HA models may

rationalize forecasting techniques and investment strategies which have been used frequently in the business world, but have not yet been justified theoretically.

In response to the empirical results in this study, it may be productive to extend the predictions of HA models in several directions and to apply them to other phenomena observed in the U.S. financial markets. As Chapter 1 shows, the model with heterogeneous investment opportunities can well explain the Mehra and Prescott Puzzle (Mehra and Prescott [1985]) as well as the time–series behavior of short–run interest rates. Since it is well–known that yield spreads have predictive power for other economic variables and respond to business cycles in the U.S. economy, HA models may impose some theoretical restrictions on the relationship between yield spreads and macroeconomic variables. Chapter 3 pursues this line of research.

APPENDIX: Alternative Specification of Time–varying Risk

This appendix discusses another type of specification of the conditional volatility of the underlying technology based on the Markov switching model (Hamilton [1989]). If the conditional aggregate risk is negatively correlated with a large shift of economic activities (high risk in a recession and low risk in a boom), we may not differentiate the two hypotheses (RA models and HA models) from each other. For both hypotheses, ex–ante risk premia respond negatively to realized aggregate shocks which are large enough to change an economic state from one to the other[6].

In fact, Cecchetti, Lam and Mark [1990a, b] propose such a possibility by adopting the Markov switching model, although their interest is mainly in the mean reversion of asset returns in lower frequencies, not in the risk premium dynamics in higher frequencies. Setting a long sample period (1869 to 1985 in [1990a], 1890 to 1987 in [1990b]), they identify the first regime with 2.5 % annual growth economy and the second with –15.4 % growth. According to our calculation based on their estimation (Table A–1)[7], the conditional aggregate risk (conditional standard error) in a bad economy is two times as high as in a good economy. Reflecting this difference, ex–ante risk premia in a bad economy are more than two times as large as in a good economy (a degree of RRA is set to be from 1.4 to 3.0 for this calculation). In their model, ex–ante risk premia are indeed negatively correlated with shocks which are large enough to change regimes.

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[6] This kind of technological process may be explained by operating leverage; when firms have large costs, net profits fall faster than revenues if demand falls (see Schwert [1989]). It is not a purely technological phenomenon, but a combined effect of technological sides with demand sides.

[7] The calculation is based on their [1990b] version.

Their result, however, is seriously sensitive to the choice of sample periods. To see this, we use the results due to Hamilton [1989] where he sets a sample period from 1952 to 1984. Unlike Cecchetti et al. [1990a, b], his estimation identifies the second regime with a moderate recession (—1.4 % annual growth rate). Based on this estimation (Table 1—A), the conditional aggregate risk (conditional standard errors) for both regimes turns out to be almost identical; consequently, ex—ante risk premia in both states are nearly the same as each other[8] [9]. The above simple calculation shows that Markov switching models are not so powerful for explaining the post—war dynamics of risk premia.

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[8] The reason why an ex—ante risk premium in a good state is a little larger than in a bad state regardless of lower conditional standard error in a good state is that consumption smoothing incentives dominate risk averse incentives. In a good (bad) state, consumers save (dissave) more money in a risk—free asset; as a result, a risk—free return gets lower (higher), or a risk premium becomes larger (smaller).

[9] Rietz [1988] claims that the predictions of RA models are very sensitive to introducing a catastrophic state even with an extremely small probability.

Table 1: Multi-Stage Estimation

Table 1-A: Underlying Production Process (1948:III to 1990:IV)

$$y(t+1) = \text{constant} + \theta(L)\, y(t) + \epsilon(t+1)$$

	Models with Squared Residuals		Models with Absolute Values of Residuals	
	RA Model	adding HA Models	RA Model	adding HA Model
constant	0.00901 (0.00255)			
$\theta(1)$	0.34103 (0.12251)			
$\theta(2)$	-0.03905 (0.08116)			
$\theta(3)$	0.06879 (0.06973)	same as left	same as left	same as left
$\theta(4)$	-0.20347 (0.08039)			
$\theta(5)$	-0.15242 (0.08098)			
R^2	0.21544			
significance level of Q value	0.95108			

(i) The numbers in parentheses are the standard errors corrected by the two-step method.

Table 1-B: Time-varying Volatility of Underlying Production (1951:II to 1990:IV)

$$\epsilon(t+1)^2 = \text{constant} + \alpha(L)\ \epsilon(t)^2 + \nu(t+1)$$
$$\text{or } |\epsilon(t+1)| = \text{constant} + \alpha(L)\ |\epsilon(t)| + \nu(t+1)$$

	Models with Squared Residuals		Models with Absolute Values of Residuals	
	RA Model	adding HA Models	RA Model	adding HA Model
constant	0.00030		0.00787	
	(0.00011)		(0.00276)	
$\alpha(1)$	0.28384		0.29054	
	(0.16623)		(0.11893)	
$\alpha(2)$	-0.12433		-0.11972	
	(0.07741)		(0.07980)	
$\alpha(3)$	0.03589		0.10397	
	(0.03771)		(0.06160)	
$\alpha(4)$	-0.04140	same	-0.07063	same
	(0.03569)	as left	(0.06228)	as left
$\alpha(5)$	0.06584		0.13817	
	(0.05556)		(0.07259)	
$\alpha(6)$	-0.00213		-0.02863	
	(0.05639)		(0.07645)	
$\alpha(7)$	0.02940		0.02137	
	(0.08733)		(0.09912)	
$\alpha(8)$	0.09936		0.12941	
	(0.07372)		(0.07263)	
R^2	0.09375		0.11750	
significance level of Q value	0.93912		0.70619	

(i) The numbers in parentheses are the standard errors corrected by the two-step method.

Table 1-C: Regressing Ex-post Risk Premium (1951:II to 1990:IV)

$$\Pi(t+1) = \text{constant} + \gamma\, E_t \epsilon(t+1)^2 + \lambda(L)\ \epsilon(t) + \xi(t+1)$$

$$\text{or } \Pi(t+1) = \text{constant} + \gamma\, E_t |\epsilon(t+1)| + \lambda(L)\ \epsilon(t) + \xi(t+1)$$

	Models with Squared Residuals		Models with Absolute Values of Residuals	
	RA Model	adding HA Models	RA Model	adding HA Model
constant	-.00150	-0.00039	-0.02260	-0.01567
	(0.01110)	(0.00932)	(0.01877)	(0.01622)
γ	38.94637	37.65150	2.63401	2.19781
	(21.8408)	(15.5062)	(1.21643)	(0.96374)
$\lambda(1)$		-0.53695		-0.54587
		(0.24119)		(0.25735)
$\lambda(2)$		-0.15383		-0.19199
		(0.24021)		(0.24078)
$\lambda(3)$		-0.34084		-0.29512
		(0.25305)		(0.25791)
$\lambda(4)$		-0.48261		-0.48108
		(0.23446)		(0.24055)
$\lambda(5)$		-0.32322		-0.29566
		(0.24727)		(0.24380)
$\lambda(6)$		-0.23625		-0.19233
		(0.21971)		(0.21307)
$\lambda(7)$		0.23061		0.22968
		(0.25126)		(0.24809)
$\lambda(8)$		0.31351		0.27467
		(0.18307)		(0.18519)
R^2	0.03233	0.11033	0.03653	0.10715
significance level of Q value	0.03045	0.47509	0.04342	0.48044

(i) The numbers in parentheses are the standard errors corrected by the two-step method.

Table 1-D: Time-varying Volatility of Risk Premium (1951:II to 1990:IV)

$$\xi(t+1)^2 = \text{constant} + \eta\ E_t\epsilon(t+1)^2 + \delta(L)\ \epsilon(t) + \zeta(t+1)$$

$$\text{or } |\xi(t+1)| = \text{constant} + \eta\ E_t|\epsilon(t+1)| + \delta(L)\ \epsilon(t) + \zeta(t+1)$$

	Models with Squared Residuals		Models with Absolute Values of Residuals	
	RA Model	adding HA Models	RA Model	adding HA Model
constant	0.00444	0.00476	0.05239	0.05276
	(0.00064)	(0.00733)	(0.01084)	(0.00386)
η	1.59508	-1.67949	0.18570	-0.95539
	(1.51801)	(0.57123)	(0.69043)	(0.32160)
$\delta(1)$		-0.02080		-0.07940
		(0.02140)		(0.13836)
$\delta(2)$		-0.02940		-0.24195
		(0.02356)		(0.16576)
$\delta(3)$		-0.02085		0.01763
		(0.01994)		(0.13645)
$\delta(4)$		0.01410		-0.04905
		(0.02455)		(0.15122)
R^2	0.00314	0.00997	0.00046	0.01479
significance level of Q value	0.99495	0.99824	0.96772	0.92295

(i) The numbers in parentheses are the standard errors corrected by the two-step method.

Table 2:
Estimating Complete System by Maximum Likelihood Estimator
(1951:II to 1990:IV)

$$y(t+1) = \text{constant} + \theta(L)\ y(t) + \epsilon(t+1),$$

$$\text{where } \sigma_{\epsilon}(t+1)^2 = \alpha(0) + \alpha(L)\ \epsilon(t)^2$$

	Spec 1	Spec 2	Spec 3	Spec 4
constant	0.00885	0.00861	0.00853	0.00899
	(0.00201)	(0.00196)	(0.00201)	(0.00199)
$\theta(1)$	0.37419	0.38662	0.39385	0.40727
	(0.09463)	(0.10247)	(0.10430)	(0.09908)
$\theta(2)$	-0.01619	-0.02546	-0.02237	-0.03128
	(0.08951)	(0.10196)	(0.10434)	(0.10231)
$\theta(3)$	0.06054	0.05959	0.07214	0.06413
	(0.07325)	(0.07834)	(0.08247)	(0.08037)
$\theta(4)$	-0.07597	-0.09372	-0.08114	-0.12730
	(0.08531)	(0.08524)	(0.08928)	(0.08342)
$\theta(5)$	-0.10779	-0.10409	-0.12400	-0.12271
	(0.08323)	(0.08158)	(0.08086)	(0.07767)
$\alpha(0)$	0.00022	0.00021	0.00021	0.00022
	(0.00004)	(0.00004)	(0.00004)	(0.00004)
$\alpha(1)$	0.25058	0.27609	0.26432	0.29288
	(0.08685)	(0.10653)	(0.10932)	(0.12090)
$\alpha(5)$	0.16737	0.14357	0.17106	0.05661
	(0.09332)	(0.10960)	(0.12834)	(0.07573)
$\alpha(7)$	0.10523	0.11991	0.12486	0.18174
	(0.05742)	(0.06502)	(0.06991)	(0.08504)
$\alpha(8)$	0.00414	0.00402	0.00424	0.00457
	(0.00126)	(0.00134)	(0.00140)	(0.00145)

(i) The numbers in parentheses are the standard errors.
(ii) $\alpha(2)$, $\alpha(3)$, $\alpha(4)$, and $\alpha(6)$ are set to be zero, since these estimated coefficients are negative.

Table 2: [Continued]

$$\Pi(t+1) = \text{constant} + \gamma\, E_t\sigma(t+1)^2 + \lambda(L)\ \epsilon(t) + \xi(t+1)$$

	Spec 1	Spec 2	Spec 3	Spec 4
constant	-.00490	-0.00291	-0.00149	0.01936
	(0.01142)	(0.01165)	(0.01212)	(0.01038)
γ	47.53056	38.13363	33.77765	6.39011
	(21.3110)	(23.5576)	(22.8539)	(17.0659)
$\lambda(1)$		-0.48992	-0.40709	-0.29112
		(0.34244)	(0.36607)	(0.30042)
$\lambda(2)$		-0.10174	-0.08353	-0.25794
		(0.42718)	(0.37525)	(0.32640)
$\lambda(3)$		-0.29061	-0.44290	-0.46778
		(0.34745)	(0.31649)	(0.31983)
$\lambda(4)$		-0.53859	-0.43826	-0.41980
		(0.36454)	(0.34051)	(0.27882)
$\lambda(5)$		-0.39238	-0.52459	-0.29636
		(0.32809)	(0.35006)	(0.26969)
$\lambda(6)$		-0.30745	-0.42057	-0.72401
		(0.35914)	(0.37643)	(0.26505)
$\lambda(7)$		0.16239	-0.05696	-0.07659
		(0.31214)	(0.32840)	(0.29094)
$\lambda(8)$		0.17211	0.08810	-0.01275
		(0.43600)	(0.38987)	(0.29933)

(i) The numbers in parentheses are the standard errors.

Table 2: [Continued]

$$\sigma_{\xi}(t+1)^2 = \exp(\ \delta(0) + \delta(L)\epsilon(t) + a_1|\xi(t)| + a_2\xi(t)\ )$$

	Spec 1	Spec 2	Spec 3	Spec 4
$\delta(0)$	-5.28918	-5.35910	-5.42043	-4.96418
	(0.09775)	(0.10459)	(0.11551)	(0.19044)
$\delta(1)$			-10.75627	-12.37772
			(8.18695)	(8.68696)
$\delta(2)$			-8.66130	-9.89279
			(7.14139)	(8.31361)
$\delta(3)$			-5.54742	-6.27931
			(7.26653)	(8.54520)
$\delta(4)$			7.61473	8.00799
			(6.66577)	(7.65425)
a_1				-10.43960
				(3.54316)
a_2				-7.49223
				(2.43390)
ρ b/w	0.09773	0.09605	0.10138	0.12024
z_1 & z_2	(0.10807)	(0.11534)	(0.11815)	(0.13005)
log-likelihood	888.634	894.543	897.068	901.375

(i) The numbers in parentheses are the standard errors.

Table A-1: Markov Switching Model

	Cecchetti et al. [1990a, b]		Hamilton [1989]	
GNP				
sample period	1869-1985, annual		1952:II-1984:IV, quarterly	
	high state	low state	high state	low state
average annual growth	0.0246	-0.1514	0.0474	-0.0142
conditional standard error	0.0423	0.0980	0.0356	0.0404
RISK PREMIA				
sample period	1890-1987, annual		1952:II-1984:IV, quarterly	
	high state	low state	high state	low state
RRA = 1.4	0.0026	0.0063	0.0014	0.0014
RRA = 2.0	0.0038	0.0089	0.0020	0.0020
RRA = 2.4	0.0046	0.0107	0.0025	0.0023
RRA = 3.0	0.0058	0.0134	0.0031	0.0029

Figure 1: Predicted Risk Premia by Time-varying Volatilities Only

fitted values vs. 1 year forward moving averages

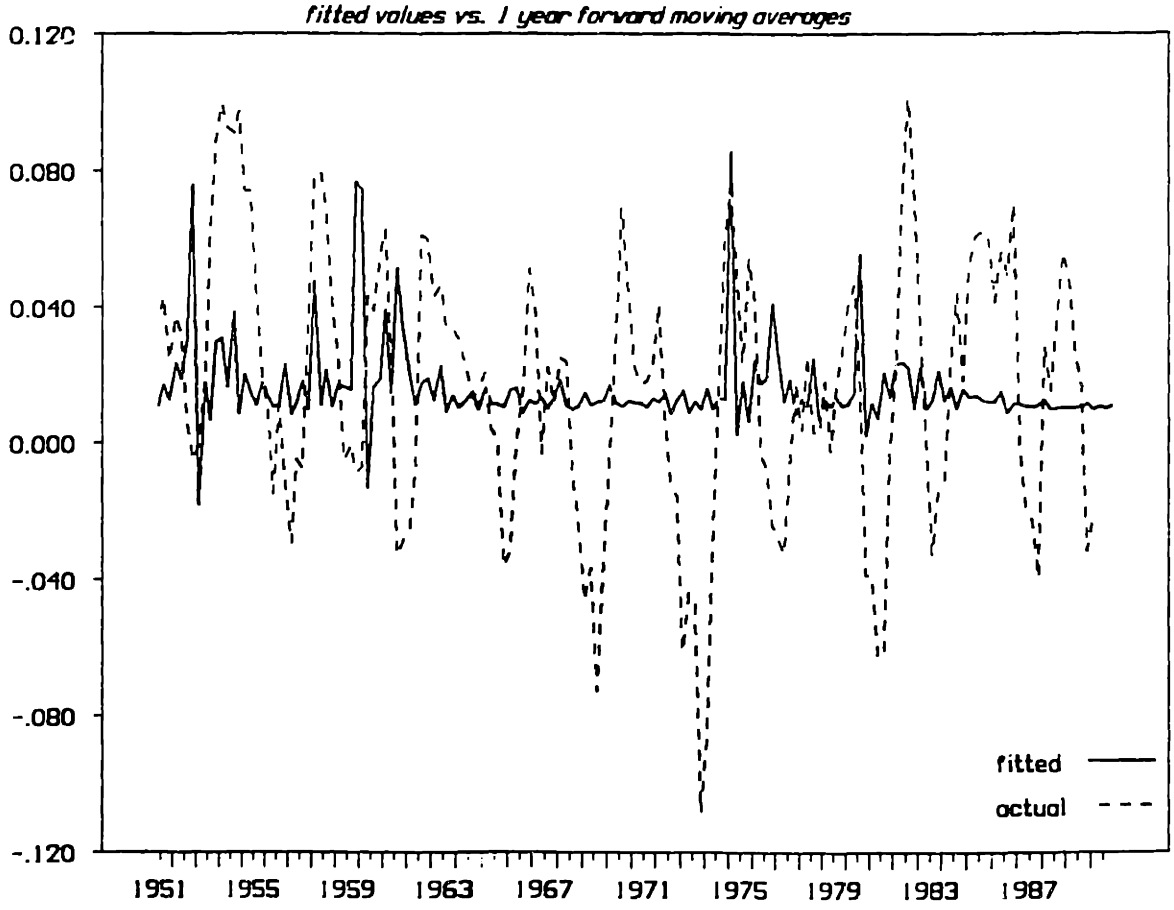
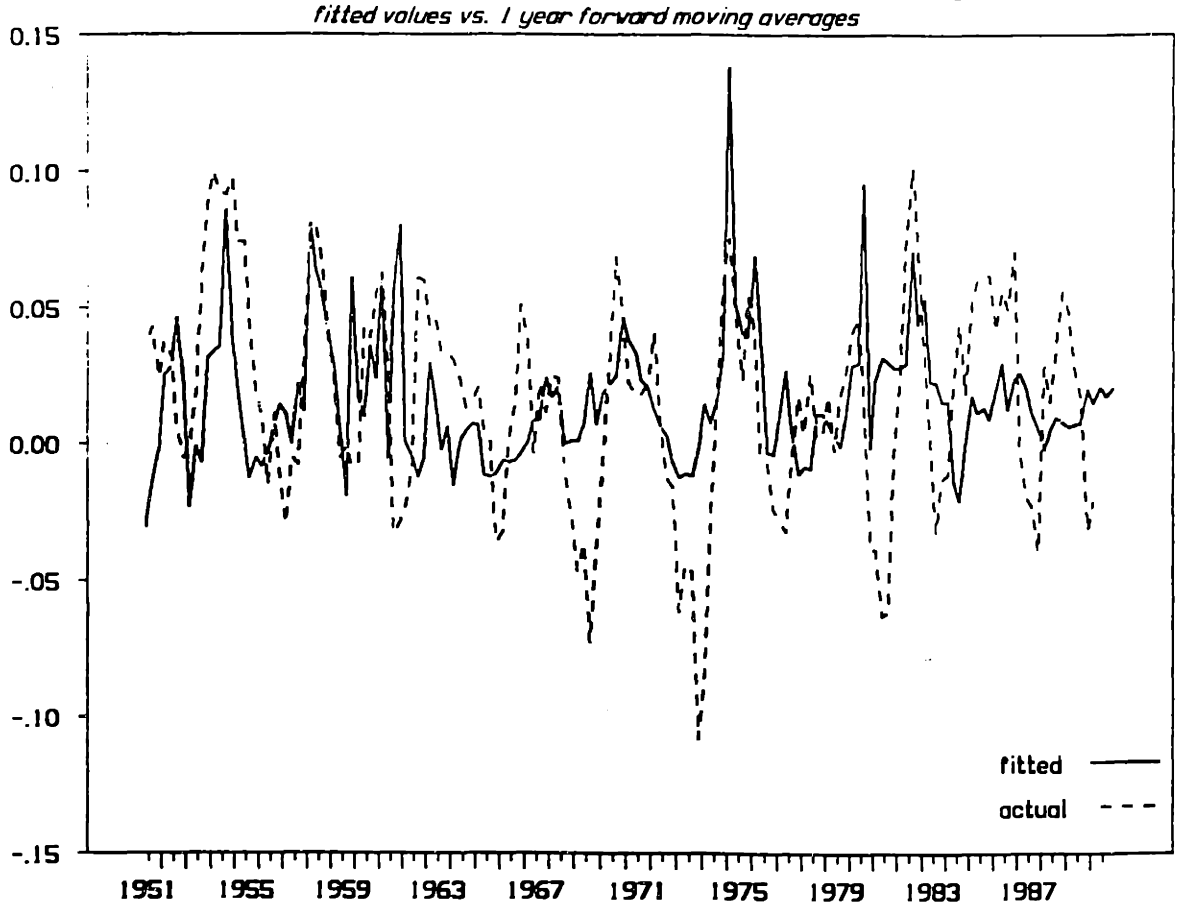


Figure 2: Predicted Risk Premia with Heterogeneous Agents

fitted values vs. 1 year forward moving averages



Chapter 3

Aggregate Shocks, Liquidity, and Bond Pricing: An Assessment of Conventional Views

1. Competing Hypotheses

In this chapter, I empirically examine several issues which are not covered in Chapters 1 and 2, however, have been conventionally considered as crucial factors in asset pricing models; then, I discuss how some important factors can be introduced into my framework. This chapter raises the following empirical questions: is the expected inflation reflected fully in nominal interest rates ? Do liquidity effects matter for determining short-run interest rates ? If so, are liquidity effects persistent ? How do aggregate shocks affect the demand/supply conditions of bond markets ? Is the representative agent (RA) framework[1] a good approximation for characterizing bond pricing ? For assessing these issues, I test which hypothesis can successfully explain the yield spread dynamics observed in the U.S. bond market. This chapter is organized and summarized as below.

In the U.S. bond market, yield curves are steeper late in recessions, while flatter or inverted in the latter stage of booms. Several hypotheses potentially explain this phenomenon. The first hypothesis emphasizes the effect of the expected inflation on nominal interest rates (originated from

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[1] As in Chapter 2, what is meant by RA models is defined in a narrow sense. See the first footnote in Chapter 2.

Fisher [1930], the Fisher hypothesis). This hypothesis claims that yield spreads just reflect the difference between the expected long–run and the expected short–run inflation. If the expected short–run inflation is pro–cyclical while the expected long–run inflation is stable (for example, a case where an economy moves along the short–run Phillips curve), the Fisher hypothesis predicts that yield spreads are counter–cyclical.

The second hypothesis considers liquidity effects on short–run interest rates. A monetary policy is expected to immediately affect short–run interest rates, while it is supposed to take some time to have an effect on long–run interest rates. Based on this explanation, an expansionary monetary policy in recessions initially lowers short–run interest relative to long–run interest (steep yield curves), while a contractionary monetary policy in booms initially raises short–run interest relative to long–run interest (inverted yield curves).

The third hypothesis focuses on real factors rather than on monetary ones within the RA framework (Hansen and Singleton [1983], Cox, Ingersoll and Ross [1981, 1985b], Breeden [1986], Campbell [1986] and others). In RA models, the term structure dynamics are driven by time–varying conditional moments of production growth (usually up to the second moment). A representative agent is more willing to value short–run bonds (a decrease in interest) if the conditional volatility (one–period ahead prediction) is larger, while he is more willing to discount short–run bonds (an increase in interest) if the expected growth (one–period ahead prediction) is higher. Accordingly, if an economic contraction makes the conditional volatility higher and the expected growth lower, yield curves become steeper in recessions, while yield curves are flatter in booms if an economic expansion changes these two conditional moments in the other way. See

the detail of RA models in an appendix.

Fourth, the models developed in Chapter 1 and Chapter 2 may also predict how yield spreads behave over business cycles. This class of models (heterogeneous agent models, HA models) emphasizes the effect of trading between risk–takers and risk–hedgers; risk–takers supply short–term bonds and hold more risky assets (long–lived assets) in their portfolio, while risk–hedgers demand short–term bonds for hedging the wealth fluctuations due to aggregate shocks. Since long–run interest is stable as far as the wealth distribution between the two types of consumers follows a stationary distribution, changes in yield spreads mainly reflect time–varying short–run interest rates. Because their willingness to bear risks is proportional to the level of their wealth, risk–takers are reluctant to supply short–term bonds (a decrease in short–run interest or large yield spreads) as their wealth decreases in response to adverse shocks. In the case of favorable shocks, they are willing to supply short–run bonds (an increase in short–run interest or small yield spreads). Since aggregate shocks have persistent effects on the willingness of risk–takers, yield spreads respond negatively to realized aggregate shocks for some duration (larger yield spreads after negative shocks and smaller yield spreads after positive shocks).

In Section 2, the above hypotheses are differentiated by examining how yield spreads respond to the driving forces of each hypothesis: the expected inflation, unexpected shocks to money supply, time–varying conditional moments of production growth, and aggregate shocks (technological shocks). For this purpose, each type of shocks is identified from the vector autoregressive processes (VAR) which contain production growth, inflation, and money supply. Based on the recent literature (Bernanke and Blinder [1990], Strongin [1991], Christiano and Eichenbaum

[1992], and others), the proxy for money supply is chosen carefully.

The properties of the estimated VAR system are consistent with the traditional macroeconomic view: (i) higher growth induces higher inflation contemporaneously and temporally (the Phillips curve), (ii) money supply Granger–causes production growth, (iii) unexpected shocks to money supply raise production growth quickly, but inflation with delay, (iv) a monetary policy becomes expansionary in response to negative shocks to either production growth or inflation.

Given these estimated processes, Section 3 reports that the empirical investigation favors the second and the fourth hypotheses, while either the first or the third hypothesis cannot explain the observation very well. The results are summarized as:

(a) unexpected shocks to money supply immediately have strongly negative effects on nominal/real short–run interest, while they affect long–run interest persistently. An expansionary monetary policy, consequently, widens yield spreads initially (for about half a year), and narrows them later.

(b) aggregate shocks have persistent and negative effects on yield spreads up to around two years. This effect makes yield curves very steep in the latter stage of recessions and inverted in the late stage of booms.

(c) the expected inflation is not reflected in yield spreads very much. The Fisher effect is weakly detected in the specification without monetary or aggregate shocks, while it vanishes with them.

(d) time–varying conditional moments of production growth cannot explain the dynamics of yield spreads at all.

Finally, I interpret some episodes in the U.S. bond market within the empirical framework of this chapter. In addition, I discuss the

possibility for HA models to introduce liquidity effects. Sharing ideas with Grossman and Weiss [1983], Rotemberg [1984], Lucas [1990], Fuerst [1990], and Christiano [1990], the following effect is stressed: injected money through an open market operation is disproportionately distributed between borrowers (risk–takers) and lenders (risk–hedgers) for some duration.

2. Specifications

2–1. Identification of Shocks

In order to identify exogenous shocks, I construct the VAR system which consists of production growth (y(t)), inflation (p(t)), and money supply (m(t)).

$$
\begin{aligned}
y(t+1) &= a_{10}+a_{11}(L)y(t)+a_{12}(L)p(t)+a_{13}(L)m(t)+\eta_1(t+1) \\
p(t+1) &= a_{20}+a_{21}(L)y(t)+a_{22}(L)p(t)+a_{23}(L)m(t)+\eta_2(t+1) \qquad (1) \\
m(t+1) &= a_{30}+a_{31}(L)y(t)+a_{32}(L)p(t)+a_{33}(L)m(t)+\eta_3(t+1)
\end{aligned}
$$

Following the Cholesky decomposition, η's are specified by orthogonal shocks, ϵ's.

$$
\begin{aligned}
\eta_i(t+1) &= \epsilon_i(t+1), \\
\eta_j(t+1) &= \varphi_1\epsilon_i(t+1) + \epsilon_j(t+1), \qquad (2) \\
\eta_k(t+1) &= \varphi_2\epsilon_i(t+1) + \varphi_3\epsilon_j(t+1) + \epsilon_k(t+1).
\end{aligned}
$$

How exogenous shocks are orthogonalized depends on the order of η's. The expected production growth and the expected inflation are defined as $E_t y(t+1)$ and $E_t p(t+1)$ respectively. The conditional volatility of production growth ($\sigma(t+1)^2$) is approximated by the conditional mean of $\epsilon_a(t+1)^2$ (see Section 3 in Chapter 2). It is further assumed that $E_t \epsilon_a(t+1)^2$ is a linear function of the past squared residuals of each shock. Under these assumptions, $\sigma(t+1)^2$ is specified as

$$(3) \qquad \sigma(t+1)^2 \simeq E_t \epsilon_a(t+1)^2,$$

where $\epsilon_a(t+1)^2 = \text{const.} + b_1(L)\epsilon_1(t)^2 + b_2(L)\epsilon_2(t)^2 + b_3(L)\epsilon_3(t)^2 + \nu(t+1)$ with $E_t \nu(t+1) = 0$.

I judge whether the above VAR system is reasonable by checking the following respects: does money supply Granger–cause production growth ? Is the contemporaneous relationship among η's plausible ? How does a monetary policy respond to exogenous shocks ? Is the Phillips curve observed ?

2–2. specifications of asset pricing equations

For nesting each hypothesis, nominal short–run interest (i_s), nominal long–run interest (i_l), ex–post real short–run interest (r_s), yield spreads (s) are regressed on the expected inflation ($E_t p(t+1)$), the past sequence of aggregate shocks (unexpected shocks to production growth, ϵ_a), and the lagged sequence of unexpected shocks to money supply (ϵ_m).

$$(4) \quad i_s(t+1) = \text{const.} + \phi_{is} E_t p(t+1) + \alpha_{is}(L)\epsilon_a(t) + \beta_{is}(L)\epsilon_m(t) + \xi_{is}(t+1)$$

$$(5)\quad i_l(t+1) = \text{const.}+\phi_{il}E_t p(t+1)+\alpha_{il}(L)\epsilon_a(t)+\beta_{il}(L)\epsilon_m(t)+\xi_{il}(t+1)$$

$$(6)\quad r_s(t+1) = \text{const.}+\phi_r E_t p(t+1)+\alpha_r(L)\epsilon_a(t)+\beta_r(L)\epsilon_m(t)+\zeta_r(t+1)$$

$$(7)\quad s(t+1) = \text{const.}+\phi_s E_t p(t+1)+\alpha_s(L)\epsilon_a(t)+\beta_s(L)\epsilon_m(t)+\xi_s(t+1)$$

For testing RA models, the asset pricing equations are specified as below.

$$(8)\quad r_s(t+1) = \text{const.}+\gamma_r E_t[\sigma(t+1)^2]+\delta_r E_t[y(t+1)]+\zeta_{RAr}(t+1)$$

$$(9)\quad s(t+1) = \text{const.}+\gamma_s E_t[\sigma(t+1)^2]+\delta_s E_t[y(t+1)]+\xi_{RAs}(t+1)$$

$i_s(t+1)$, $i_l(t+1)$, and $s(t+1)$ are measured at the beginning of time t+1, while $r_s(t+1)$ from the beginning to the end of time t+1. ξ's can be interpreted as measurement errors, while ζ's as prediction errors.

As far as the expected long–run inflation is stable, the Fisher hypothesis predicts that $\phi_{is}= 1$, $\phi_{il}= 0$, $\phi_r= 0$, and $\phi_s= -1$. If β_{is}'s, β_{il}'s, or β_r's are negative for some duration, liquidity effects are detected. In HA models, α_r's are expected to be positive for some periods. If the effect of the expected inflation on nominal interest is controlled by the term of $E_t p(t+1)$ in each equation, the signs of α_{is}'s, α_{il}'s, and α_s's can be considered as the trading effects on real interest rates (HA models); therefore, the theory predicts that α_{is}'s are positive, α_{il}'s are zero, and α_s's are negative for some duration. The combination of a negative γ_r and a positive δ_r is consistent with the implications of RA models, and so is that of a positive γ_s and a negative δ_s (see the appendix).

In terms of estimation method, equations (1) to (9) are estimated

sequentially by the ordinary least squares estimator (OLS estimator). The standard errors reported in this chapter are corrected by the two step method (see Section 2 in Chapter 3).

3. Estimation Results

3–1 Choices of Variables

(data sources)

Which variable is chosen as the proxy for money supply crucially determines the properties of the VAR system. As the recent studies point out (Bernanke and Blinder [1990], Strongin [1991], Christiano and Eichenbaum [1992], and others), the Federal Reserve changes broad money aggregates such as M1, M2, or total reserves when it accommodates money demand from private sectors, while it mainly changes nonborrowed reserves when it controls money supply; accordingly, unexpected shocks to broad money aggregates represent disturbances to money demand rather than to money supply. Following their findings, this study identifies as shocks to money supply the disturbances to the ratio of nonborrowed to total reserves. The higher this ratio is, the more expansionary monetary policy is.

With respect to other variables, I define production growth as the growth rate of the production index released by Board of Governors of the Federal Reserve System (seasonally–adjusted total industrial indexes), and calculate inflation rates from the consumption price indexes. The U.S. Treasury bill rates are used as nominal short–run interest rates (three month maturities), while the U.S. Government bond rates (twenty year maturities) are applied to nominal long–run interest rates. Yield spreads are defined as the differences between these two interest rates. The above

financial data are available from Ibbotson Associates [1991]. The nonborrowed/total reserve ratios are constructed in a monthly frequency from January 1959 to December 1990, while the sample periods of other data can cover the entire post–war era.

(stationarity)

Before estimating equations, I check whether there are unit roots in the above listed variables. Table 1 reports the results of the augmented Dickey–Fuller tests. All variables except nominal short–run/long–run interest rates do not contain any unit roots in the series. Detrending (assuming linear time trends) removes a unit root of nominal short–run interest; however, it cannot remove a unit root of nominal long–run interest. When I estimate equations (4) and (5), the results are reported for the case with time trends as well as without them. Since the detrended nominal long–run interest still has a unit root, the results concerning equation (5) should be interpreted carefully.

3–2. VAR System

(Granger causality)

In this subsection, the properties of the estimated VAR system are investigated in several dimensions. Table 2–1 reports the Granger causalities among the three variables. Both inflation and money supply Granger–cause production growth, while other causalities are not detected. In this system, the past sequence of money supply has a predictive power for the present state of production growth.

(contemporaneous correlation)

Table 2–2 reports the contemporaneous correlation among the estimated disturbances, η's. A positive correlation between η_1 and η_2 implies that the Phillips curve holds in a contemporaneous sense, although it is very weak. Monetary shocks become contemporaneously expansionary in response to negative shocks to either production or inflation.

Since the correlation among them are very weak, the order of η's does not significantly change the final estimation results (impulse responses and asset pricing equations). For convenience, the order is fixed from production growth, inflation, to money supply. Table 2–3 reports the estimated ϕ's in equation (2), given the above order.

(impulse responses)

Figures 1–1 to 1–3 draw the impulse responses of the three variables to each type of shocks. All units are standardized by the standard error of each type of orthogonal shocks. A monetary policy becomes expansionary in response to positive shocks to either production growth or inflation for about one year. Positive shocks to production induce higher inflation (the Phillips curve in a temporal sense), while positive shocks to inflation have an negative impact on production growth. Unexpected monetary shocks raise production growth quickly, but inflation with delay.

To summarize, the properties described by the above VAR system are consistent with the traditional macroeconomic view such as in Dornbusch and Fischer [1990].

3–3. Asset Pricing Equations

(Fisher hypothesis)

The estimated ϕ's (coefficients on the expected inflation) in Table 3 show that the expected inflation is reflected only partially in nominal interest rates; in other words, the Fisher hypothesis does not hold perfectly. An increase in the expectation of one period ahead inflation raises nominal short–run interest only by its half; consequently, it lowers real short–run interest. In addition, nominal long–run interest responds very sensitively to the expected near future inflation, although the expected long–run inflation is stable (p(t) has no unit root)[2]. Since both short–run and long–run interest reflect the expected inflation by almost the same magnitude, the effects due to the expected inflation are canceled in deriving yield spreads. If yield spreads are regressed on the expected inflation without aggregate or monetary shocks, the differences in the expected inflation is reflected weakly in yield spreads (the fourth column of Table 4).

(liquidity effects)

Figures 2–1 to 2–6 draws the series of α's (coefficients on aggregate shocks) and β's (coefficients on monetary shocks) with one–standard error upper/lower bands. As shown in these figures, liquidity significantly affect both nominal short–run and nominal long–run interest rates. Liquidity effects have a strongly negative impact on nominal short–run interest for initial 6 to 9 months, while they have a persistent influence on nominal long–run interest up to one and half a year although their effects are not so strong as those on short–run interest. Reflecting the difference of monetary shocks on these two interest rates, unexpected shocks to money

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[2] One reason for this is that long–term bonds traded in the real world are not pure discount–bonds, but mixed portfolios among short–run and long–run pure discount bonds.

supply widen yield spreads initially, and narrow them later. In addition, liquidity effects work on real short–run interest for initial half a year.

(RA models)

Table 4 reports the results of RA model specifications. The results show that time–varying conditional moments of production growth cannot explain the bond pricing behavior at all. With respect to real interest, the signs of the estimated parameters are consistent, but insignificant. In the case of yield spreads, the signs are inconsistent for both cases with the expected inflation and without it.

(HA models)

As Figures 2–1 to 2–4 show, aggregate shocks have a persistent and positive effect on nominal short–run interest (positive α_{is}'s), while they have no impact on nominal long–run interest (zero α_{il}'s). Accordingly, unexpected shocks to production growth narrow yield spreads for one and half a year (Figure 2–6, negative α_s's). In this respect, the results are consistent with the implications of HA models (if the term of $E_t p(t+1)$ in each equation controls the Fisher effect, the signs of α_{is}'s, α_{il}'s, and α_s's can be considered as the trading effects on real interest rates). The results, however, are mixed in that α_r's are not significantly positive.

(summary)

To summarize, the effects due to both liquidity and aggregate shocks well explain the dynamics of yield spreads as well as those of nominal interest rates. As shown by the R–squares (Table 3) or Figures 3–1 to 3–6, these lagged sequences can trace significant portions of the

actual movements in the U.S. bond market.

3–4. Discussions

Based on the empirical advantage of liquidity effects and HA models, I can interpret why the Federal Reserve always finds difficulty in narrowing yield spreads during recessions, even if it triggers an expansionary monetary policy. The lagged sequence of negative aggregate shocks itself makes yield spreads large, while an expansionary monetary policy initially widens yield spreads due to its big impact on nominal short–run interest. These combined effects put the Federal Reserve in trouble. Figure 4 visually explain these combined effects; it decomposes the time–series of yield spreads into the components due to monetary shocks and those due to aggregate shocks. In most periods except mid–1980's, both components have worked in the same direction. Given the above interpretation, the "Operation Twist" in early 1960's succeeded in narrowing yield spreads up to mid–1960's because the U.S. economy well performed in 1960's[3], while the Federal Reserve have had a hard time for fixing large yield spreads in early 1990's because the U.S. business cycle have already peaked out in July 1990 according to the NBER Business Cycle Dating Committee.

4. Remarks on Future Researches

Throughout my thesis, I have investigated how financial assets are priced with the trading between risk–takers (borrowers in bond markets) and risk–hedgers (lenders). The basic driving force in these models is that

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[3] Modigliani and Sutch [1966] conclude that the monetary policy was irrelevant for shaping yield curves in the first half of 1960's.

aggregate shocks are disproportionately borne by heterogeneous consumers; risk–takers bear more risks than risk–hedgers. This idea successfully explains several properties of the observed risk premia (Chapters 1 and 2) and the observed yield spreads (this chapter). As discussed in Chapter 1 (Section 4) and Chapter 2, however, my framework cannot explain large volatilities of risk premia very well. In addition, as shown in this chapter, liquidity effects on interest rates are too obvious to ignore.

The effect on asset pricing through a monetary policy is one of possibilities for fixing the above problems, since a monetary sector introduces another type of uncertainty (monetary shocks) and creates another source of asset pricing dynamics. By surveying the previous literature on liquidity effects with heterogeneous agents, I discuss how my framework can be extended for allowing liquidity effects.

In monetary versions of RA models, holding money is often motivated by cash–in–advance constraints. Cash–in–advance constraints alone, however, cannot yield liquidity effects. As far as agents can adjust money holdings immediately, injected money instantaneously raises a price level. Consequently, nominal short–run interest reflects the expected short–run inflation induced by the expected monetary growth. This property in cash–in–advance models produces predictions completely opposite to what I find in this chapter. Short–run interest increases in response to an increase in the expected short–run inflation caused by positive shocks to money supply, while long–run interest is stable as long as inflation follows an ergodic process (Backus, Gregory and Zin [1989]); consequently, positive shocks to money supply immediately narrow yield spreads. This prediction can carry over to more general settings than a standard version of cash–in–advance models (den Haan [1991]).

Grossman and Weiss [1983] and Rotemberg [1984] fix the above undesirable property of cash–in–advance models and create persistent liquidity effects on nominal/real interest in the following environment: (i) money is held for transaction reasons (cash–in–advance constraints), (ii) consumers infrequently withdraw money from banks due to transaction costs, (iii) consumers do not go to banks simultaneously. In this set–up, injected money through an open market operation is distributed only to consumers who happen to visit banks on that day. Both nominal and real interest rates, consequently, need to decrease sufficiently enough for visitors to absorb the entire injected money. What is more, it takes longer time for prices to fully reflect an increase in money supply, because consumers gradually spend injected money due to infrequent money withdrawals. In other words, injected money does not raise the expected inflation so much as in a standard version of cash–in–advance models, thereby keeping the expected inflation from dominating liquidity effects for some time. In addition, Rotemberg [1984] produces another source of persistence. Since those who do not visit banks (non–visitors) hold more money than visitors (who have no money just before withdrawal), inflation induced by money creation depresses the consumption of non–visitors. This effect shifts resources from consumption to capital, and makes liquidity effects more persistent.

With a similar spirit, but a different setting from the above models, Lucas [1990], Fuerst [1990] and Christiano [1990] develop cash–in–advance models where household sectors adjust money holdings more slowly than firm sectors. In this class of models, injected money is distributed solely to firms. For firms to absorb the entire injected money, nominal interest needs to be lower than in the case where injected money is proportionately

distributed among agents. One significant disadvantage of this simple setting is that the disproportionate distribution of monetary shocks between households and firms vanishes in the end of each period, because firms are eventually owned by households. In order to create persistence, Christiano [1991] proposes to introduce the utility with habit formation, while Christiano and Eichenbaum [1992] suggest the case where it takes longer than one period for households to adjust money holding due to adjustment costs.

The above basic mechanism is summarized as that injected money through an open market operation is disproportionately distributed among heterogeneous agents. My framework, especially the model developed in Chapter 1, can introduce this idea by assuming that injected money is absorbed solely by stockholders (borrowers in bond markets) through firms. The expected reward for this is that liquidity effects may be persistent, because firms are owned only by a subset of consumers, and injected money is still disproportionately distributed in firm sectors (stockholders) for some duration. In addition, the disproportionate distribution of monetary shocks may interact with that of aggregate shocks (already built in the basic framework) through the response of a monetary sector to aggregate shocks. Some predictions from my framework with money injection are expected to consistently explain the findings in this chapter.

APPENDIX: Implications of RA Models

In this appendix, we derive some implications on yield spreads from a simple version of RA models based on Hansen and Singleton [1983], Breeden [1986], and Campbell [1986]. Main assumptions for these models are: (i) an economy is an exchange economy, and (ii) a representative agent has a time–additive utility with a constant discount rate $(1 + \rho)$ and a constant RRA (γ).

Under these assumptions, an i–period discount bond (a pure discount bond) which certainly guarantees one unit of consumption at time t+i is valued as

$$\text{(A.1)} \qquad P_{it} = E_t[(1 + \rho)^{-i}(\frac{c_{t+i}}{c_t})^{-\gamma}],$$

at time t, where c_t is aggregate consumption. An interest rate on an i–period bond (r_{it}) is

$$\text{(A.2)} \qquad \log(1 + r_{it}) = -\frac{\log(P_{it})}{i}.$$

Due to assumption (i), aggregate consumption (c_t) in equation (A.1) can be replaced by aggregate production (Y_t). If we further assume that the growth rates of aggregate production $(\log Y_{t+1} - \log Y_t)$ are approximately normally distributed, equation (A.1) can be rearranged as

$$\text{(A.3)} \qquad \log(1+r_{it}) = \log(1+\rho) + \gamma E_t[\frac{\log Y_{t+i} - \log Y_t}{i}]$$

$$- \frac{\gamma^2}{2} \mathrm{Var}_t[\frac{\log Y_{t+i} - \log Y_t}{i^{1/2}}],$$

where E_t and Var_t denote the conditional expectation and the conditional volatility respectively. Equation (A.3) is easy to interpret intuitively. If the conditional expectation of production growth is higher up to time t + i, representative agents discount an i–period bond more (an increase in an interest rate). Representative agents, on the other hand, value an i–period bond more (a decrease in an interest rate), if the conditional volatility of production growth is larger up to time t + i.

Based on equation (A.3), we can derive yield spreads between long–run interest rates and risk–free returns (one–period interest rates). As a maturity (i) becomes longer and longer in equation (A.3), the second (third) term may be approximated by the unconditional expectation (volatility) of production growth (see Backus, Gregory and Zin [1989]). Accordingly, the above–defined yield spread (s_t) is characterized as follows.

$$\text{(A.4)} \quad s_t = \gamma\{AG - E_t[\log y_{t+1} - \log y_t]\} + \frac{\gamma^2}{2}\{\mathrm{Var}_t[\log y_{t+1} - \log y_t] - AV\},$$

where AG implies the average production growth, while AV corresponds to its unconditional volatility. Yield curves are steeper if the near–future conditional volatility is larger relative to the unconditional volatility, while they are flatter if the near–future growth is expected to be higher relative to the average growth.

Table 1: Augmented Dickey-Fuller Test
t-statistics

production growth	-7.280
inflation	-3.225
nonborrowed reserve ratio	-3.913
nominal t-bill rate (without trend)	-2.026
(with trend)	-3.259
nominal long-run rate (without trend)	-1.529
(with trend)	-2.367
ex-post real t-bill rate	-3.410
yield spread	-3.912

(1) sample periods: Feb. 1960 to Dec. 1990
for nonborrowed reserve ratio,
Jan. 1950 to Dec. 1990 for other variables,
(2) number of lags: 12 months
(3) critical values:
(without trend) -3.13(2.5%), -2.87(5%), -2.57(10%)
(with trend) -3.68(2.5%), -3.42(5%), -3.13(10%)

Table 2-1: Granger Causality and Serial Correlation p-values of F-tests and Q-statistics

	(dependent variable) production growth	inflation	nonborrowed reserve ratio
(explanatory variables, F-tests)			
production growth	0.002	0.141	0.316
inflation	0.021	0.000	0.208
nonborrowed reserve ratio	0.019	0.665	0.000
(Q-statistics)	0.961	0.677	0.940

(1) sample period: Jan. 1960 to Dec. 1990,
(2) number of lags: 24 months for the first two variables, 12 months for the third variable,

Table 2-2: Contemporaneous Correlation of Disturbances in VAR

	η_1	η_2	η_3
η_1	1.000		
η_2	0.076	1.000	
η_3	-0.134	-0.122	1.000

Table 2-3: Cholesky Decomposition

corrected standard errors are in parentheses.

$$\eta_1 = \epsilon_1$$
$$\eta_2 = \phi_1 \epsilon_1 + \epsilon_2$$
$$\eta_3 = \phi_2 \epsilon_1 + \phi_3 \epsilon_2 + \epsilon_3$$

ϕ_1	0.0231
	(0.0168)
ϕ_2	-0.2191
	(0.0675)
ϕ_3	-0.6026
	(0.2153)

Table 3: Regressing Bond Returns on Realized Shocks

corrected standard errors are in parentheses.

	R-square	Durbin-Watson	Coefficient on Expected Inflation
nominal t-bill rate (without trend)	0.5301	0.3178	0.4966 (0.0429)
(with trend)	0.7179	0.4084	0.3014 (0.0327)
nominal long-run rate (without trend)	0.4003	0.1490	0.5129 (0.0457)
(with trend)	0.7541	0.1457	0.2485 (0.0303)
ex-post real t-bill rate	0.3777	1.2321	-0.4903 (0.0540)
yield spread	0.4227	0.5620	0.0164 (0.0192)

(1) sample period: Jan. 1963 to Dec. 1990,
(2) number of lags: 36 months,

Table 4: Specifications of RA models

corrected standard errors are in parentheses.

	(dependent variable) ex-post real t-bill rate	yield spread	yield spread
constant	0.0013 (0.0004)	0.0012 (0.0002)	0.0018 (0.0002)
expected inflation			-0.1349 (0.0293)
expected growth	0.0018 (0.0314)	0.0333 (0.0125)	0.0130 (0.0135)
conditional volatility	-4.9605 (7.7522)	-2.0917 (2.9862)	-1.6908 (2.8647)
R-square	0.0014	0.0222	0.0950
Durbin-Watson	0.9454	0.4792	0.4997

(1) sample period: Jan. 1961 to Dec. 1990,

Figure 1-1: Responses to Aggregate Shocks

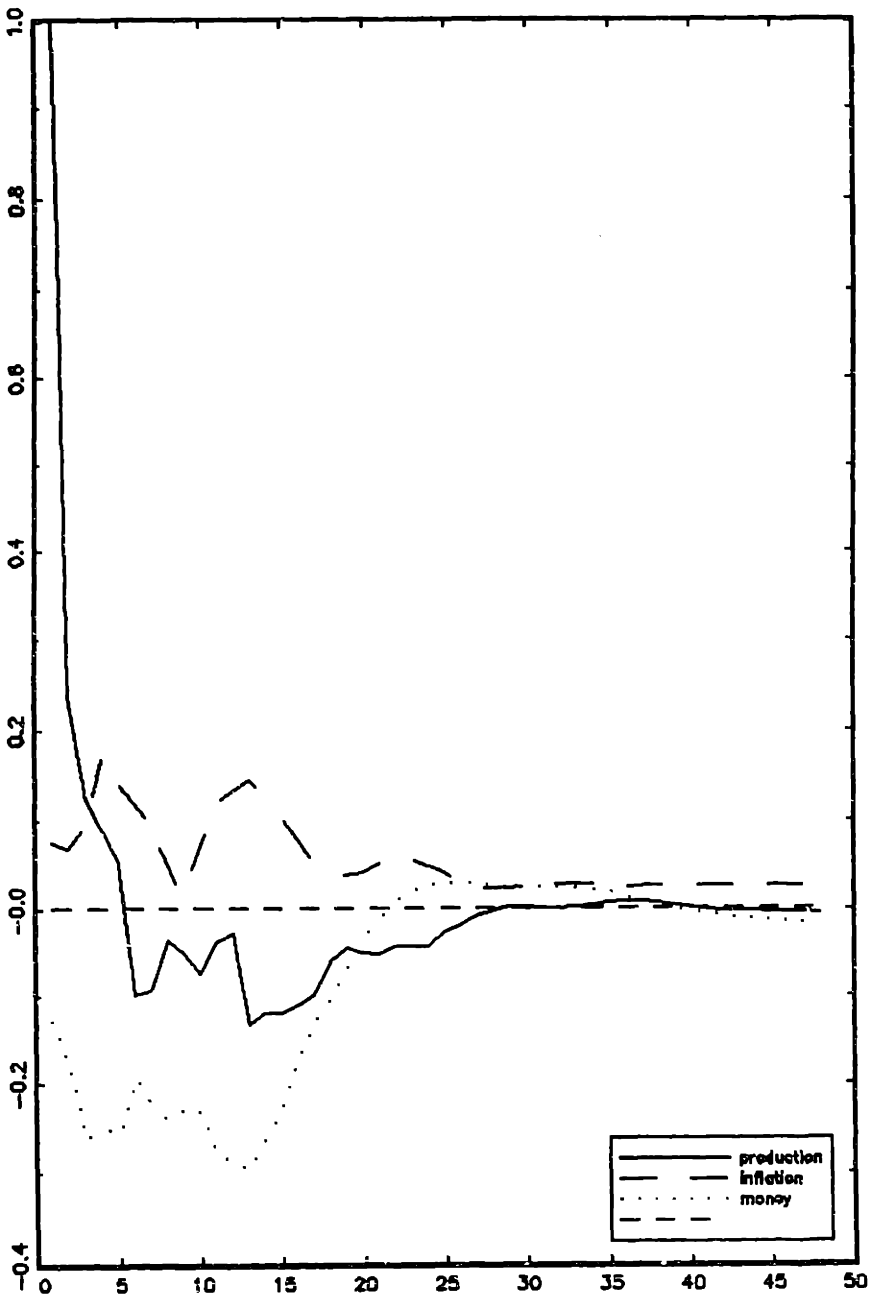


Figure 1-2: Responses to Price Shocks

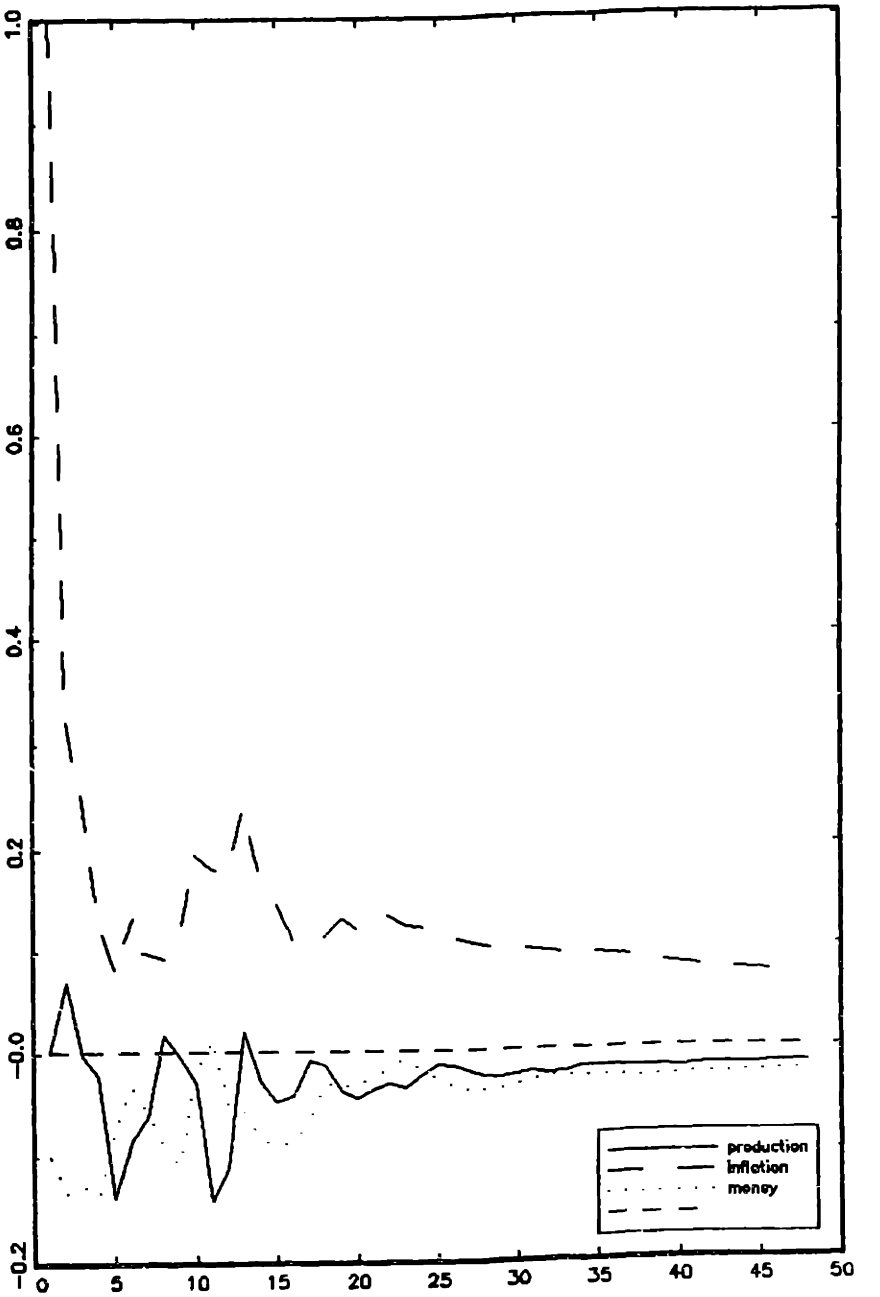


Figure 1-3: Responses to Monetary Shocks

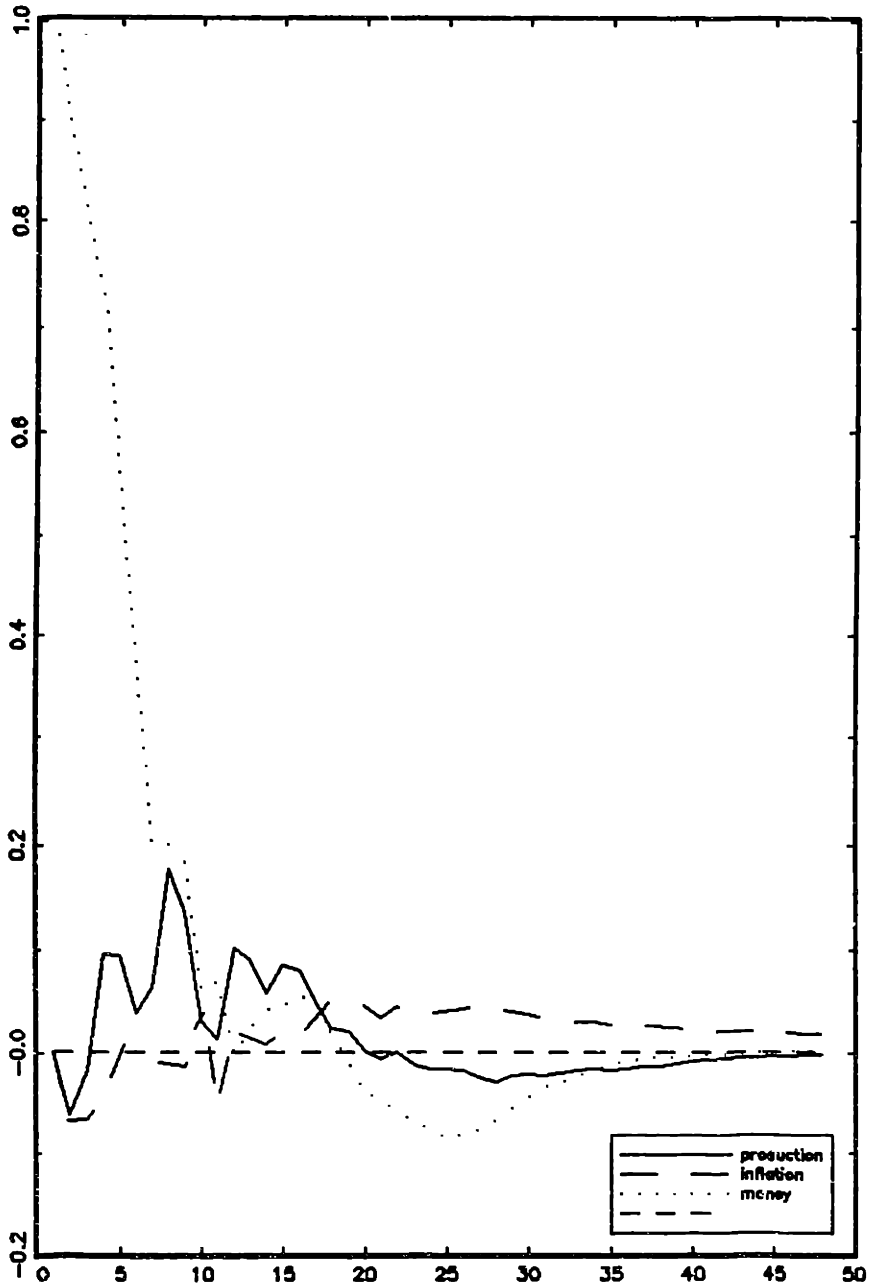


Figure 2-1: Effects on Nominal Short-Run Interest Rates (without trend)

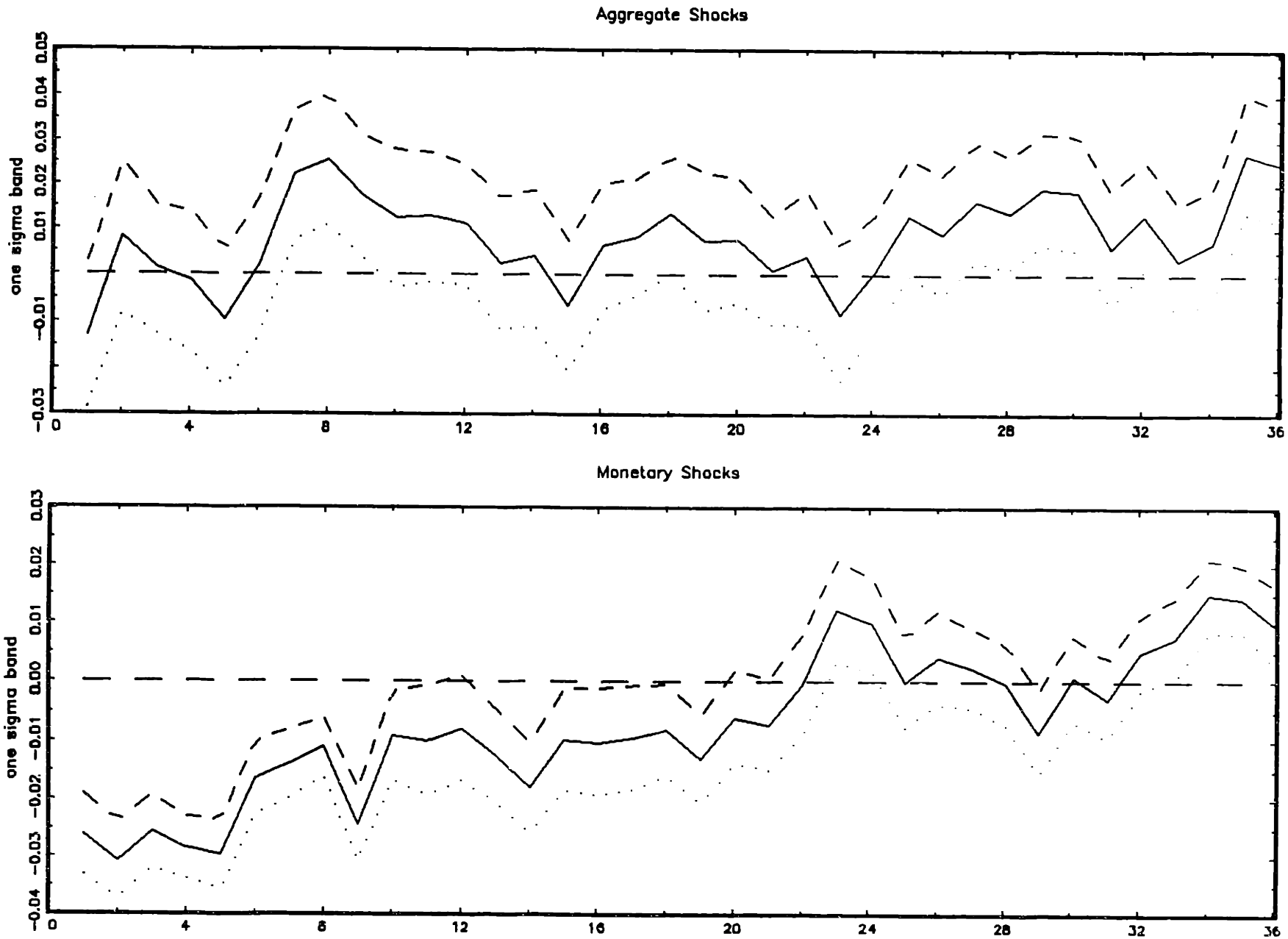


Figure 2-2: Effects on Nominal Short-Run Interest Rates (with trend)

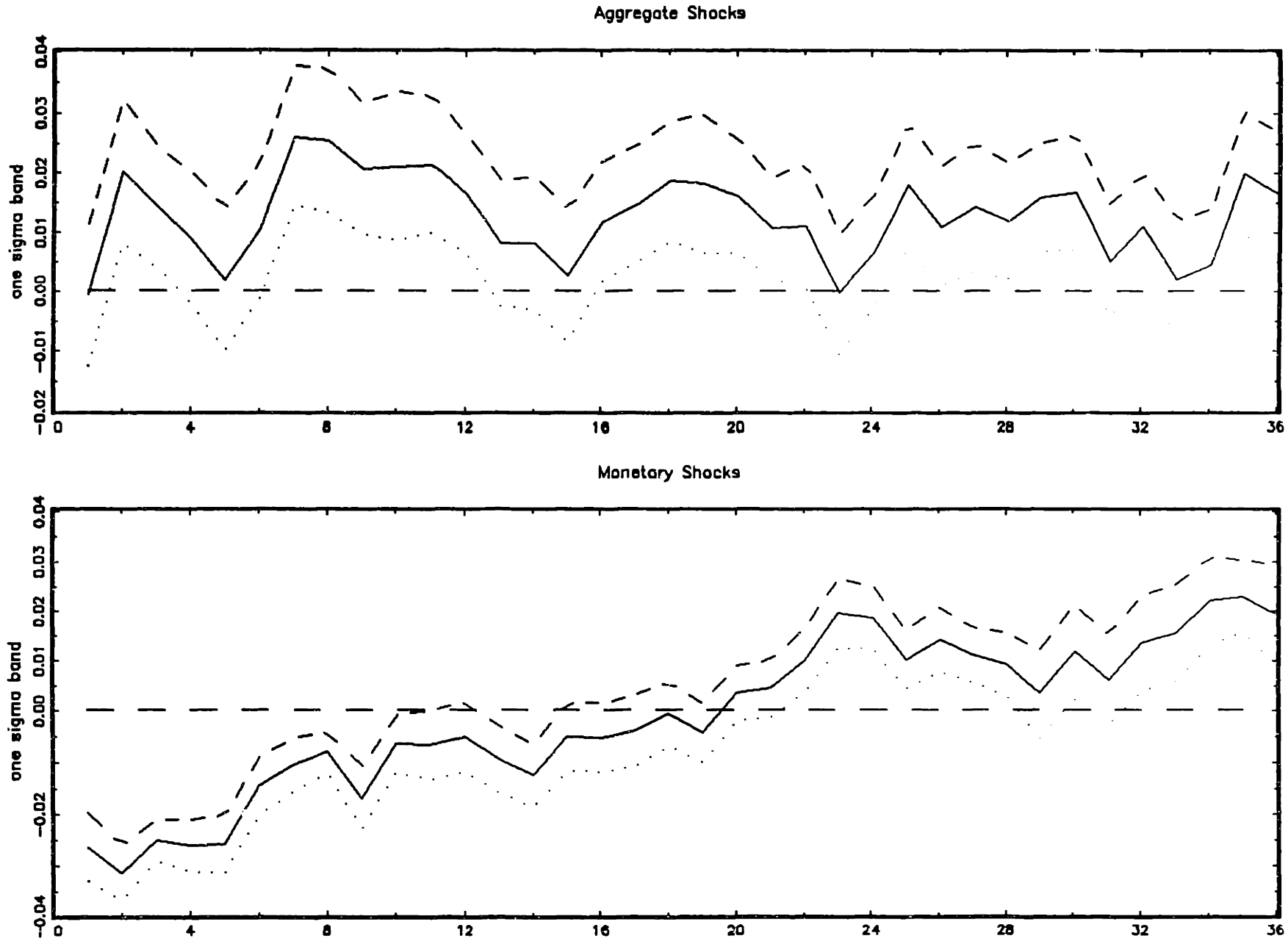


Figure 2–3: Effects on Nominal Long–Run Interest Rates (without trend)

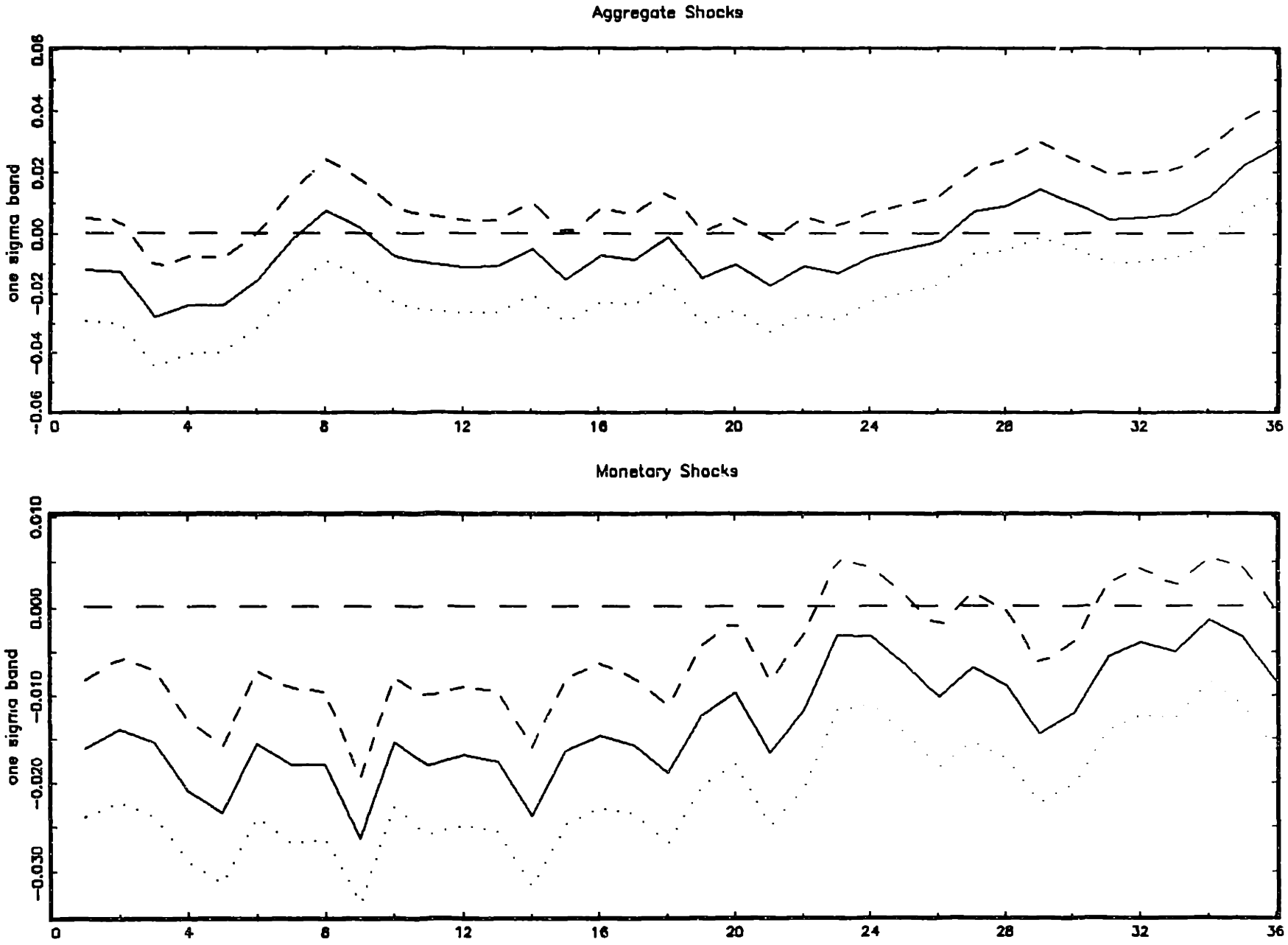


Figure 2-4: Effects on Nominal Long-Run Interest Rates (with trend)

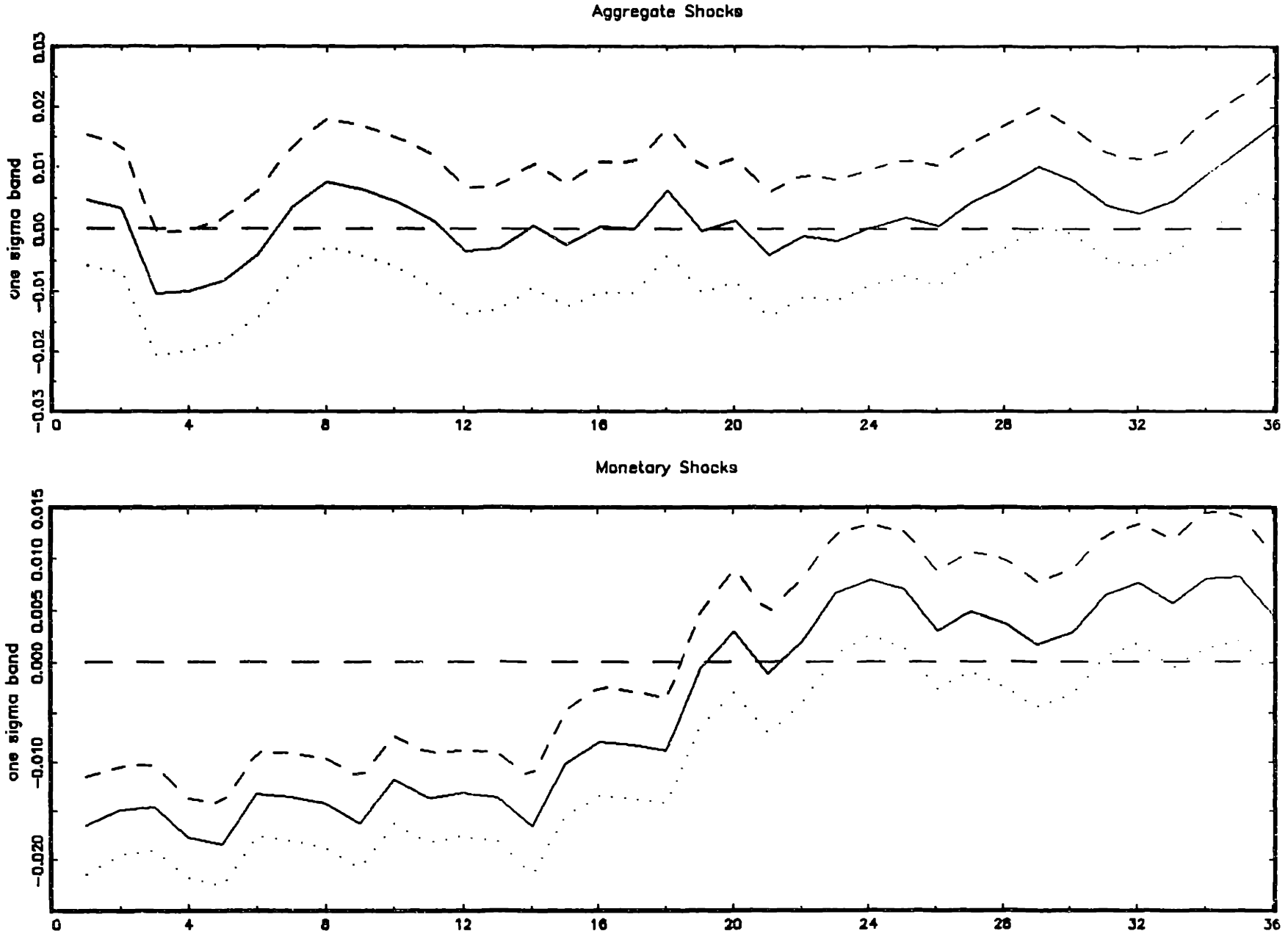


Figure 2-5: Effects on Real Short-Run Interest Rates

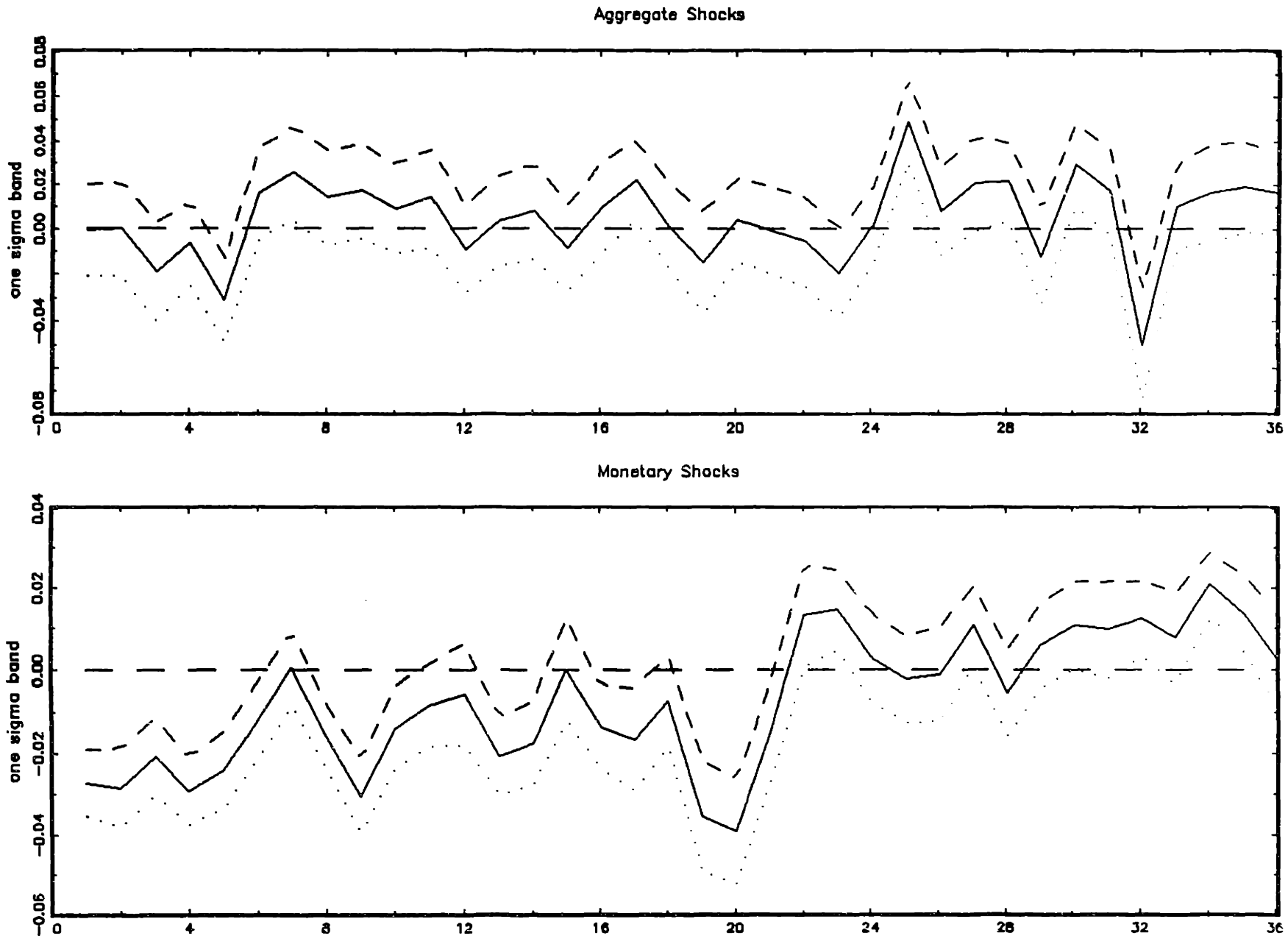
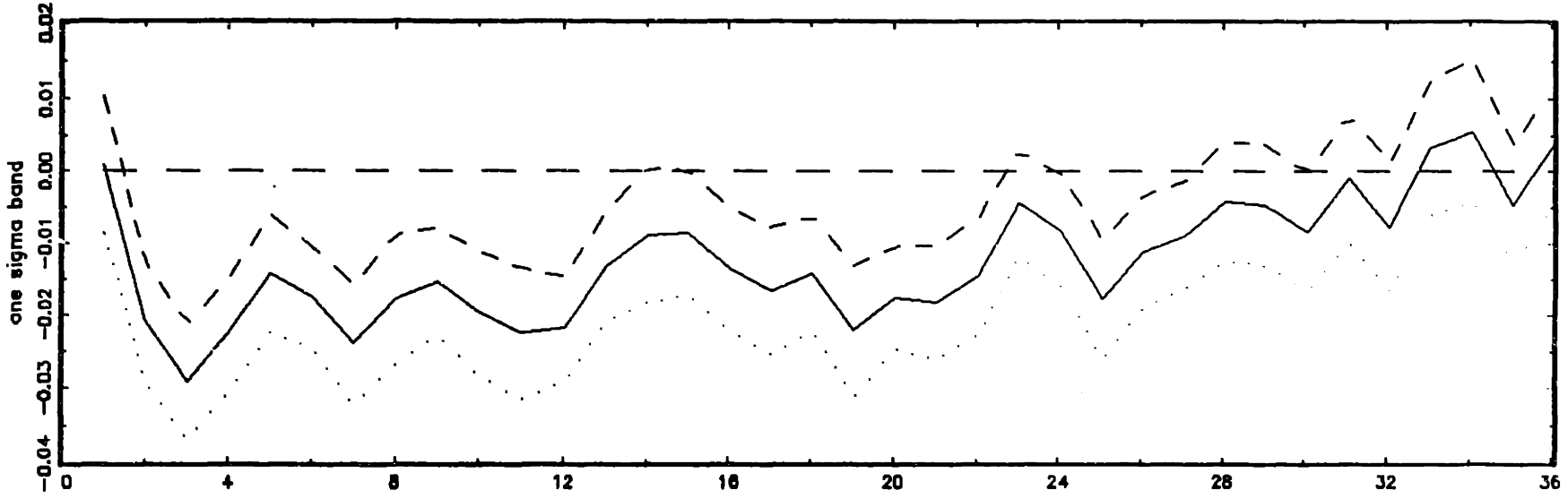


Figure 2-6: Effects on Yield Spreads

Aggregate Shocks



Monetary Shocks

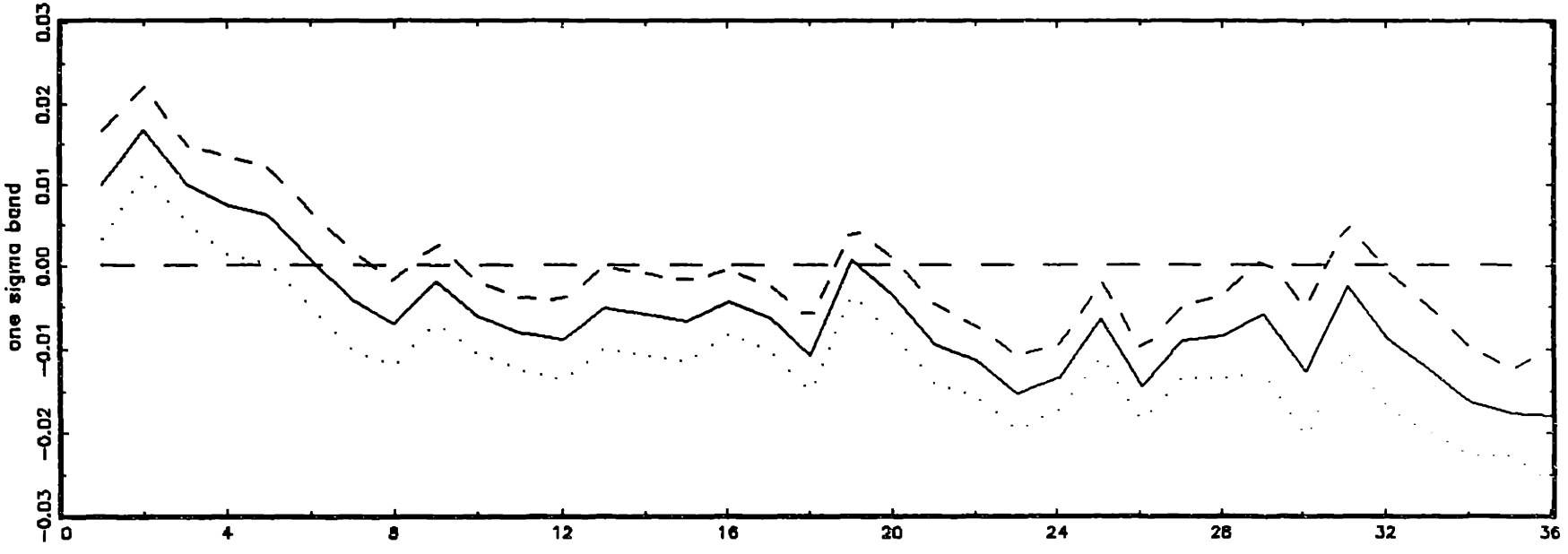


Figure 3–1: Nominal Short–Run Interest Rates (without trend)

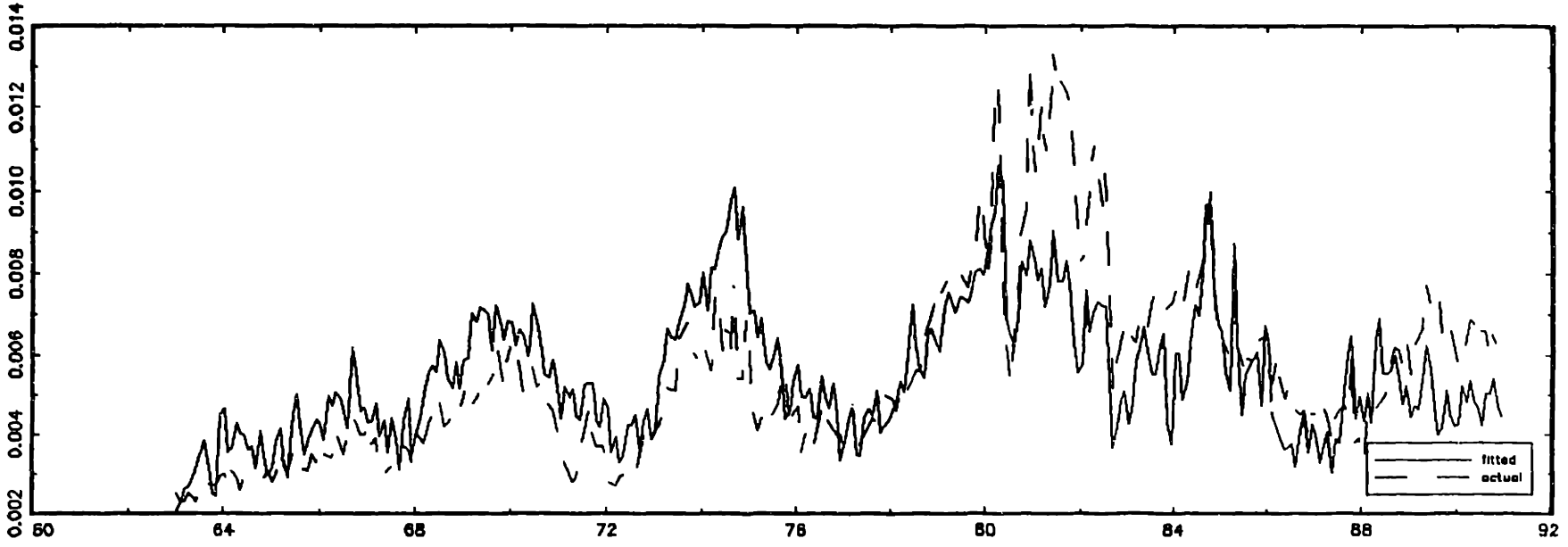


Figure 3–2: Nominal Short–Run Interest Rates (with trend)

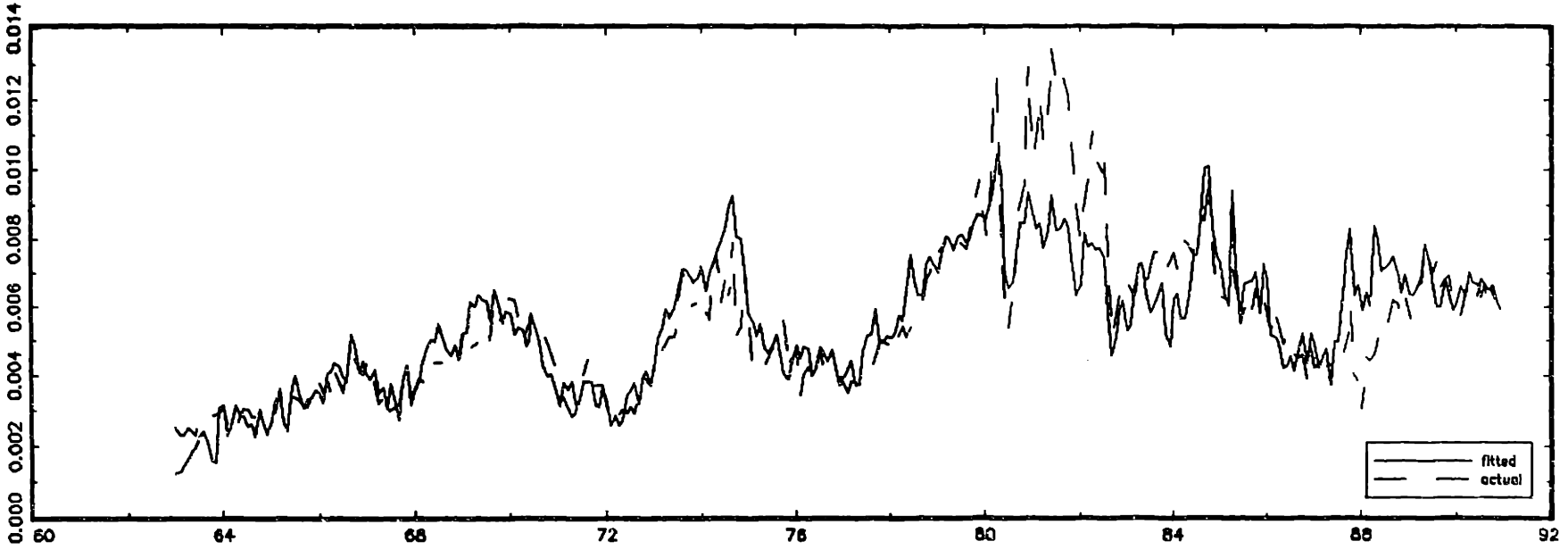


Figure 3-3: Nominal Long-Run Interest Rates (without trend)

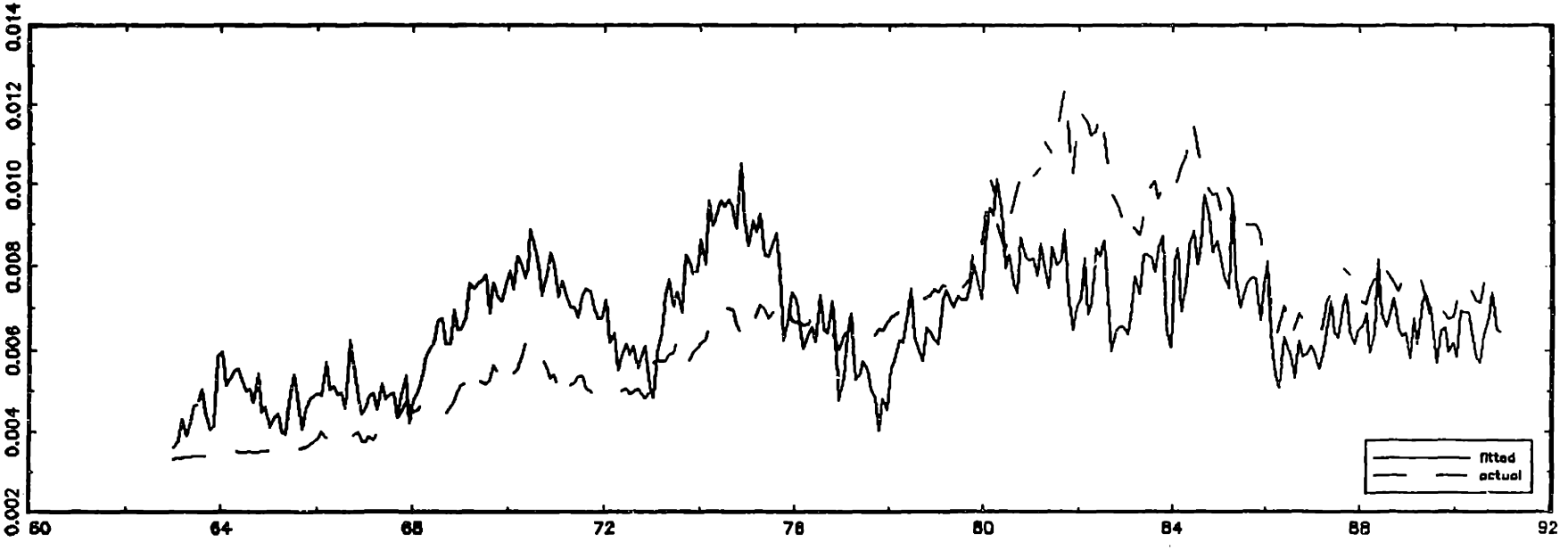


Figure 3-4: Nominal Long-Run Interest Rates (with trend)

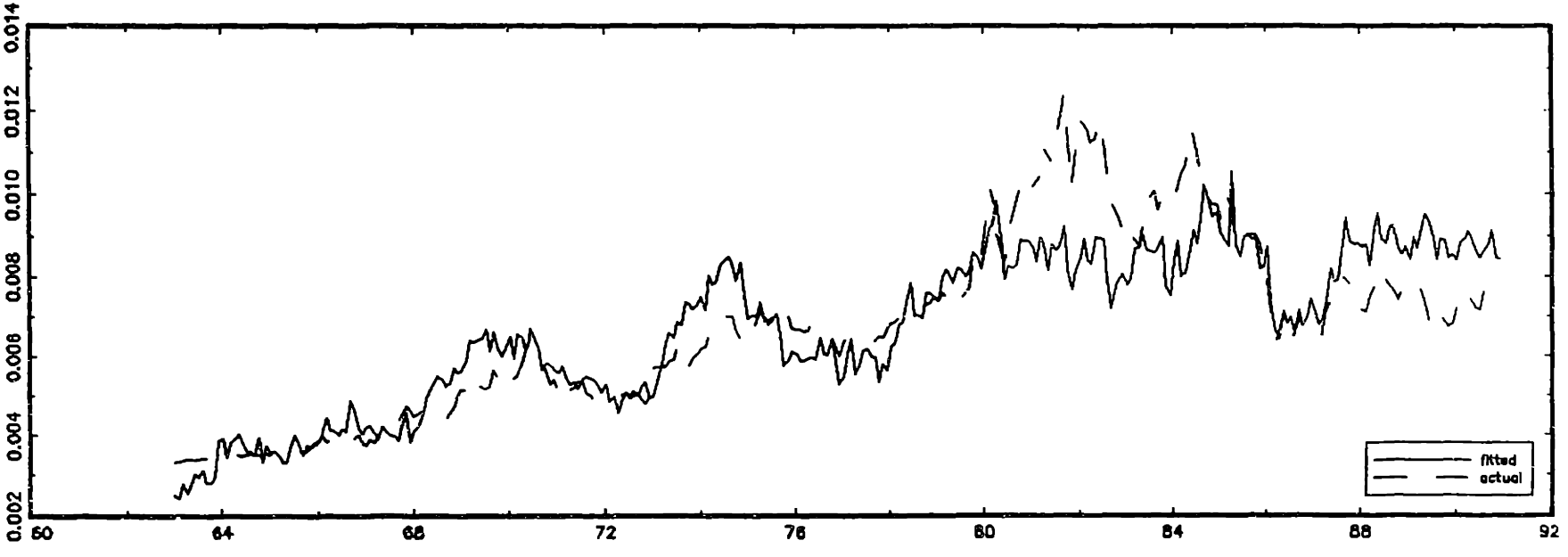


Figure 3–5: Real Interest Rates

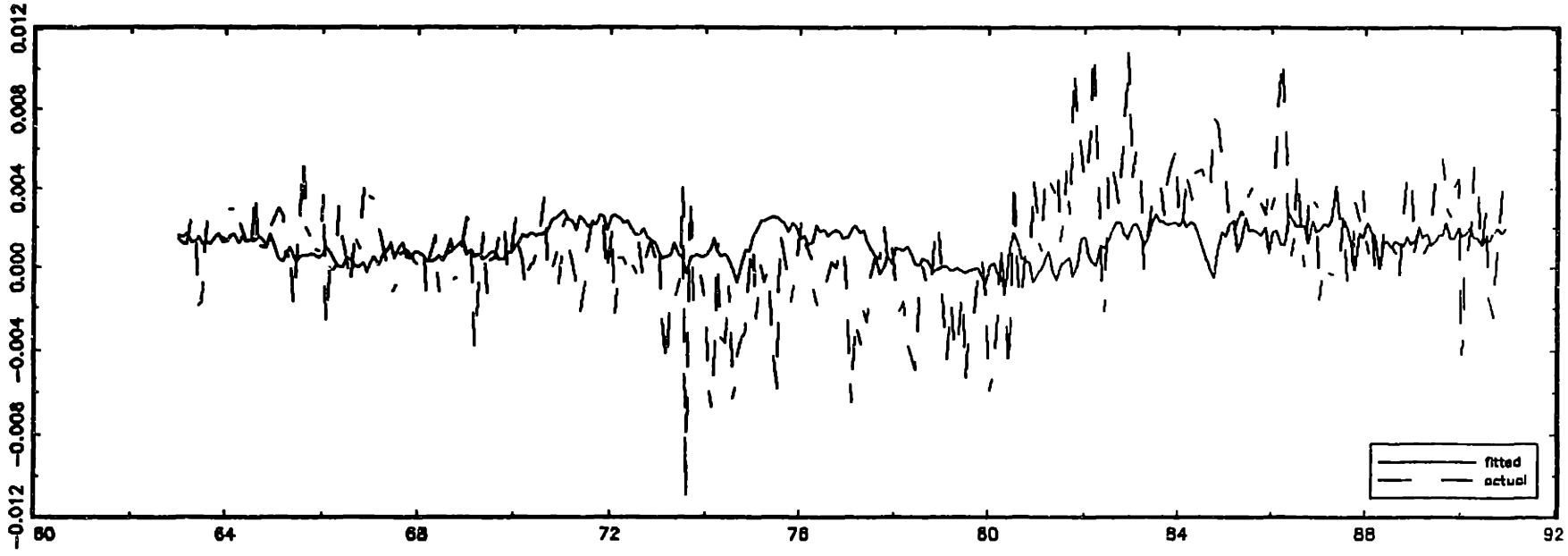


Figure 3–6: Yield Spreads

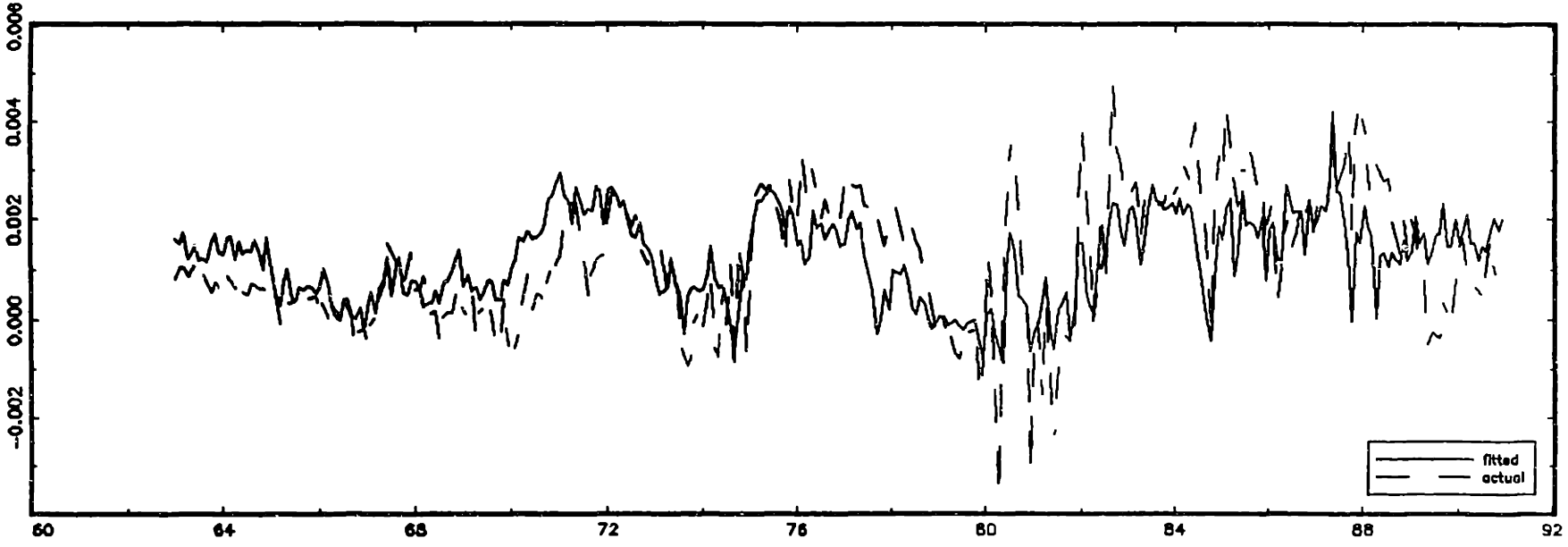
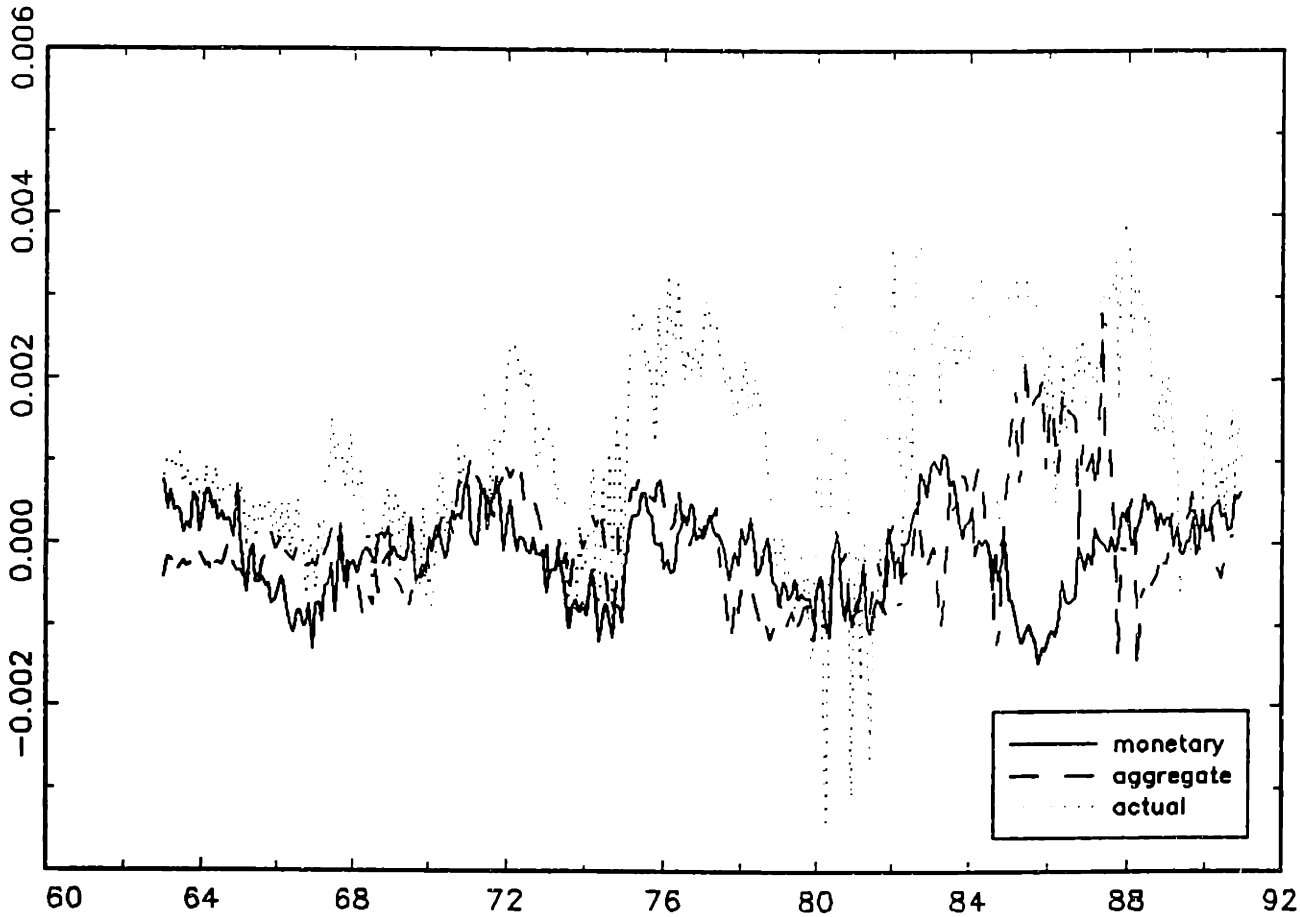


Figure 4: Yield Spreads: Monetary Shocks vs. Aggregate Shocks



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