Modular Gas-Cooled Reactor Economics: The Application of Contingent-Claims Analysis

by

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Abstract

This dissertation concerns an economic evaluation of a direct Brayton-cycle modular gas-cooled reactor, the MGR. This particular concept is an offshoot of a larger R&D program concerning a steam-cycle version known as the MHTGR. Presently, the MGR is in the early stages of research. It is hoped that its development will follow a successful demonstration of an MHTGR prototype. It is envisioned that it will be among the next generation of advanced nuclear reactors to be deployed in the next century. Among the attractive characteristics of this technology are that it can be configured such that a catastrophic nuclear accident becomes an impossibility; that its simplicity of design and operation give it the potential to reach a wider market than will other forms of nuclear power; and that it promises to provide an economic, clean and reliable source of electricity relative to other modes of power generation.

The motivation for this dissertation is that there appears to be a preoccupation with comparing the MGR to the competing technologies on the basis of levelized busbar cost. While this is the standard, and universally used, measure of power generation economics, we contend that it is inappropriate in an environment characterized by uncertainty. In its place, we propose the method of contingent-claims analysis from the school of modern finance theory. We explain and demonstrate how it provides a far superior measure of true economic value.

The thesis consists of four chapters. The first provides a description of the specific version of the MGR under study, the MGR-GTS, a 200 MWth direct Brayton-cycle design. The second provides a cost estimate of a 6-module power plant along with a comparison to competing technologies on the basis of levelized busbar cost. The third demonstrates the application of contingent-claims analysis to investment program and capital asset valuation. We consider the general problem of power plant investment programs along with specific application to the MGR. The final chapter explains the implications of the analysis to investment policy at the level of the firm as well as its significance to the nation's R&D policies and long-term energy strategy.

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May God love them all!

Contents

Chapter 1

Overview of the Modular Gas-Cooled Reactor Technology

1.1 Introduction

While nuclear power is currently out of favor in the United States as well as in a number of other countries, there is, nonetheless, a significant research effort ongoing in this country and abroad concerning the development of the next generation of nuclear power technology. The major concepts under development are the advanced light water reactor (ALWR), the liquid metal reactor (LMR) and the modular high temperature gas-cooled reactor (MHTGR). The common thread joining all current nuclear energy research is the objective of creating a design which will be cost-effective to build and operate, highly reliable and absolutely safe from any sort of catastrophic failure. In these respects the MHTGR has great potential as well as some clear advantages relative to the competing technologies. We will address these attributes in detail in Section 1.3.

Presently, the R&D program on the MHTGR concept is being sponsored by the U. S. Department of Energy (DOE) in collaboration with a private-sector consortium, Gas-Cooled Reactor Associates (GCRA). The design now under study builds on over 30 years of general gas-cooled reactor experience both in the U. S. and in Western Europe, and on several specific HTGR programs and projects, the most notable of which were carried out in Germany.[1]

More than any other yet conceived form of nuclear power, the MHTGR incorporates the principle of passive, or inherent, safety. The design philosophy guiding the research program is the development of a reactor which not only meets the most stringent requirements of the Nuclear Regulatory Commission (NRC) and of industry, but is also non-interfering and non-threatening toward the public under all imaginable worst-case accident scenarios[2]. The MHTGR achieves this level of inherent safety by abandoning the reliance on highly engineered and complex-to-manage systems of defense in depth such as are employed in other nuclear technologies. The MHTGR is designed such that the fission process is self-controlling. Equally important, the residual heat, i.e., heat generated when reactor is shut down, can be removed by completely passive phenomena. This means that the core will remain below the failure point for any possible combination of conditions. In other words, a "meltdown" is rendered impossible by reliance on basic physical principles and not on engineered devices. Because of this, a runaway chain reaction is an impossibility independent of any and all operator actions. Nonetheless, well-conceived safety systems are incorporated into the MHTGR design in order to protect the equipment, assure the safety of the operating personnel, and reassure the public in the event of an accident of force majeure.

A second facet of the design philosophy is the concept of modularization. The idea here is that an MHTGR power plant will consist of a number of independent power generating modules which will be operated from a single control center and will share all of the other peripheral facilities and services needed to operate a power plant. The envisaged advantages of this concept are:

- **Design Standardization**: Once a standardized design is tested, proved and

[1] For an excellent overview of the history of operating experience and R&D programs on the gas-cooled reactor, the reader is referred to chapter 1 of Staudt[40]. For an overview of the status of a number of R&D issues concerning the MHTGR, the reader is referred to the *Proceedings of the 1988 Intersociety Energy Conversion Engineering Conference* [1] as well as to an issue of *Energy* [5] which was entirely dedicated to the MHTGR. For a more detailed and technical look at the AVR and THTR programs in Germany, the reader is referred to a series of articles which were published in *Nuclear Engineering and Science* [6] and the *Journal of Nuclear Materials* [7].

[2] See Silady & Millunzi [39] for an overview of the safety aspects of the MHTGR

approved, a number of benefits accrue. To begin with, the regulatory process for a new plant and for each individual module addition can be significantly accelerated and made less costly. In addition, future engineering expenses are sharply reduced, and the overall construction lead-time is greatly shortened. The result is not only a cost savings but a reduction in the perceived riskiness of the investment.

- **Flexible Capacity Expansion**: The ability to add small increments of capacity quickly greatly enhances the flexibility of long-term power system planning, minimizes the risk of overcapacity, and reduces the costs associated with financing capital which is not yet productive[3]. Of course, this is the rationale driving the growth of gas combustion turbine-based generating capacity; however, the MHTGR would enjoy the additional advantage of not being exposed to the fuel delivery risk which characterizes gas.

- **Serial Production**: Since the design will be standardized and the modules will be produced in volume, it becomes possible to complete a greater proportion of the work in a factory. The factory environment not only allows for much more effective quality control but also creates the conditions which promote the learning process. We may also anticipate that the uniform design will lead to productivity gains from learning at the construction site. The net result is that we may expect costs to be driven down significantly over time as production volume grows.

1.2 Description of the MGR-GTS

While the current DOE-GCRA research effort is focused on the development of an MHTGR employing a steam cycle for energy conversion[4], the specific design concept

[3]In the electric power industry these carrying costs are known as "Allowance for Funds Used During Construction" (AFUDC).

[4]Current designs descend from an MHTGR study which was performed by a previous consortium. A detailed description of the initial design proposed by this group is given in [4].

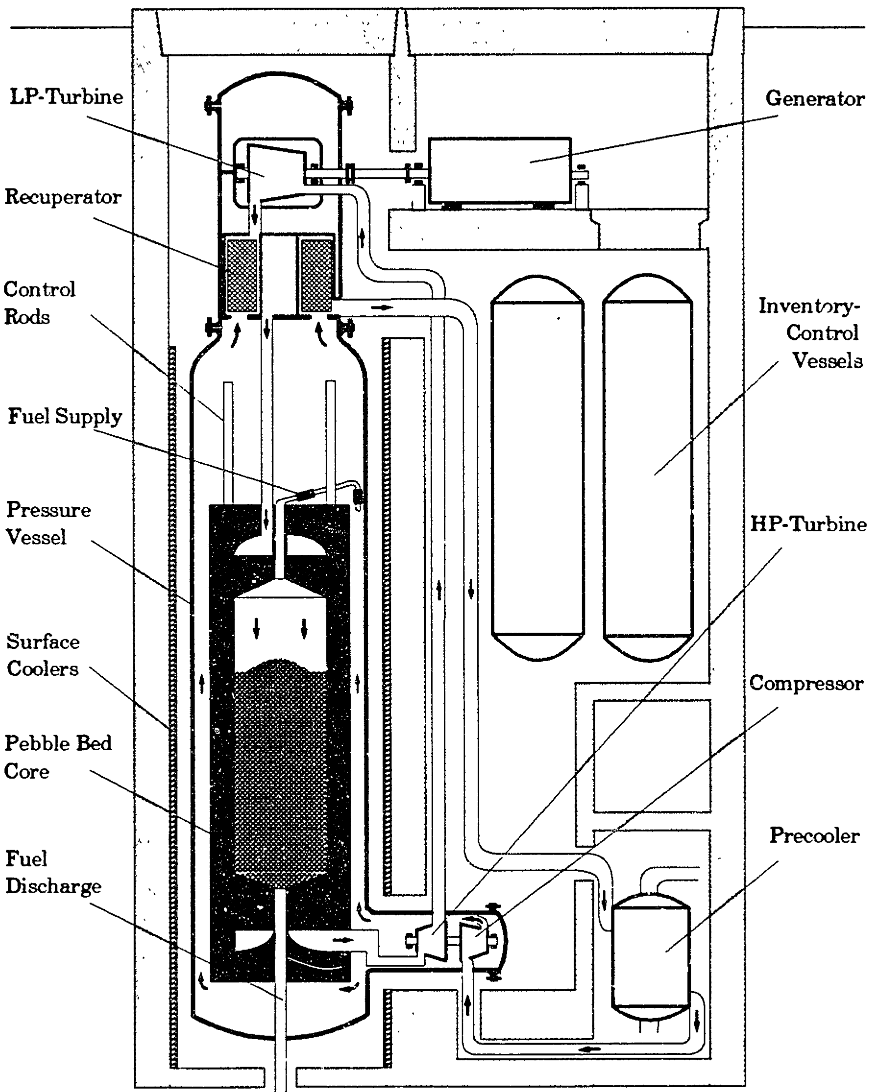
which we will consider in this study is a direct Brayton-cycle version, developed by Yan, Kunitomi & Lidsky [44], termed the MGR-GTS[5]. The basic module design is illustrated in Figure 1-1

It consists of a 200 MWt pebble bed reactor which serves to heat the working fluid, helium, to a maximum temperature of 850° C (1,562° F) at the outlet. The helium reaches a maximum pressure of 8.2 MPa (1,189 psi) within the closed cycle. It serves to drive a split-shaft turbomachinery power conversion unit consisting of a high-pressure turbine coupled to a compressor, and a low-pressure turbine. The latter drives a synchronous, 60 Hz generator. The net output of the module is 90 MWe and the overall cycle efficiency is 45%.

The entire reactor system is housed in an underground, steel-reinforced concrete silo which stands 38 m in height and is 20 m diameter. The silo extends from -35 m to +3 m relative to ground-level. The reactor vessel is 25 m in height with an outer diameter of 6.2 m. The wall thickness averages 15 cm. It will be made of a high-temperature steel alloy, namely, $2\frac{1}{4}$ Cr-1 Mo, and will weigh approximately 695 metric tons (765 tons). The metallic internal members of the reactor will be made of an even more highly temperature-resistant alloy, such as Incoloy 800-HT. Within the reactor vessel the pebble bed core is housed in a mass of graphite which performs the functions of neutron reflection and heat absorption The graphite assembly stands 14 m high and is 5.5 m in diameter. Its total weight is 590 metric tons (650 tons). The inner cavity which forms the core is 9.6 m high and 3 m in diameter. It holds some 365,000 graphite covered pebbles, 6 cm in diameter, each containing 7 g of low-enriched UO_2 fuel in the form of 10,000 0.5 mm kernels. Each fuel kernel is encapsulated in one coating each of porous carbon, pyrolytic carbon and silicon carbide. The mean power density of the core is 3 W/cm^3.

[5]In general terms, the R&D strategy is that the steam-cycle version will be the first-generation plant, to be followed by future generations of increasingly sophisticated Brayton-cycle designs.

Figure 1-1: The MGR-GTS Module



Reactor shutdown is assured by means of 6 movable control rods and 18 KLAK assemblies which are located along the perimeter of the core within the surrounding graphite reflectors. Each KLAK consists of a number of small absorber spheres, containing boron which is the active neutron-absorbing material. The spheres are held in a chamber immediately above the core and are readily dropped upon command into the channel below which stands alongside the perimeter of the core. In this way they serve as back-up to the control rods for the purpose of reducing the rate of fission.

Under normal operation, helium enters the reactor core from above at 585° C and 8 MPa; it sweeps downward through the core and attains a maximum temperature of 850° C at the outlet. It passes through the HP-turbine where it gives up some energy used to drive the accompanying compressor (10,000 rpm). It leaves the HP-turbine at 741° C and 5.9 MPa, passes through the $2\frac{1}{4}$ Cr-1 Mo primary duct (1 m diameter) and then drives the LP-turbine and power generator (3,600 rpm). It leaves the LP-turbine at 610° C and 4.1 MPa and enters the low-pressure side of the recuperator where it gives up energy to the helium stream discharged from the compressor. It exits the recuperator at 163° C and 4 MPa and passes through a length of primary duct to the precooler where waste heat is rejected to the water-based cooling system. It leaves the precooler at 30° C and 4 MPa and enters the compressor from which it exits at 137° C and 8.2 MPa. It then enters the reactor vessel where it rises through the annulus formed by the vessel wall and the outer perimeter of the graphite material surrounding the core. This serves the dual function of preheating the helium and cooling the vessel. The stream then enters the high-pressure side of the recuperator, gains additional energy, and exits at 585° C and 8 MPa, whereupon it re-enters the reactor core.

Some of the more important technical specifications of the module and its major components are listed in Table 1.1. A more extensive technical description and analysis of the reactor system and the balance of plant is beyond the scope of this study.[6]

[6]The interested reader is referred to Yan, Kunitomi & Lidsky [44] for a detailed technical analysis of the reactor and all associated components, in particular, the turbomachinery and heat exchangers. This report also includes a brief analysis of the plant performance under transient load. For a description and analysis of the plant control system and a detailed analysis of the plant performance

Table 1.1:

Turbomachinery	Compressor	HP-Turbine	LP-Turbine
Power Rating (MW)	82.1	82.6	97.7
Rotational Speed (RPM)	10,000	10,000	3,600
Pressure Ratio	2.04	1.32	1.45
Efficiency (%)	93.1	93.1	93.1
Inlet Pressure (MPa)	4.01	7.8	5.88
Inlet Temperature (°C)	30	850	741
Outlet Temperature (°C)	137	741	610
Stages	15	3	6
Maximum Tip Diameter (m)	0.733	0.864	1.709
Last Stage Hub-to-Tip Ratio	0.866	0.799	0.90

Generator	
Electric Power Output (MW)	93.2
Rotational Speed (RPM)	3,600
Efficiency (%)	95.0
Rotor Length/Diameter	3.5/1.24

Heat Exchangers	Recuperator	Precooler
Type	Counterflow	Crossflow
Surface	SF-PF 1/9-24.12	S 1.5-1.0
Effectiveness (%)	95	93
Total Pressure Drop (%)	1.20	0.10
Total Volume (m^3)	15.84	5.30
Total Heat Transfer Area (m^2)	17,933	1,168
Total Frontal Area (m^2)	13.00	9.36

Plant Performance	
Plant Power Generation (MWe)	93.2
Station Loads (MWe)	3.0
Net Electric Output (MWe)	90.2
Net Electric Efficiency (MWe)	45.1

We point out that this design incorporates state-of-the-art technology in a number of areas, most notable of which are the coated fuel kernels; the high-temperature alloys; the high-efficiency turbomachinery with active magnetic bearings; and the high-efficiency, compact heat exchangers. However, the design is in no way reliant on any yet to be achieved technological breakthrough. So, while we recognize that an immense amount of work lies between moving from a design on paper to a prototype, we have, nonetheless, an MGR concept which is capable of being built and demonstrated in the near term.

We also note that, while the issues of social acceptance, regulatory compliance, radioactive waste disposal, etc. are of extreme importance, and their resolution is critical to any future that nuclear power may have, they are beyond the scope of this study. Since we are concerned with the specific question of power plant investment valuation and the larger issue of R&D strategy, suffice it to say:

- The role of R&D is to demonstrate a technology to the point where it is ready for deployment, or, alternatively, to demonstrate that it has no future. If and when the time arrives that the social, economic and environmental advantages of some form of nuclear power prove to be sufficiently compelling, the technology must be ready or the nation as a whole will suffer a loss.

- There is a growing market for nuclear power technology beyond the U. S., and this market will widen with the advent of the safer advanced technologies. Presently, U. S. firms in the industry are well positioned vis-a-vis their foreign competitors, but do not dominate as they once did. An intense level of R&D is needed to maintain competitiveness. However, the current sentiment of the public and the regulatory barriers facing the industry are a tremendous disincentive toward continued private-sector R&D for obvious reasons. Hence, the industry is in need of government support of R&D now more than ever in order to keep pace. We argue that it would be suboptimal to allow current political sentiment to dictate long-term R&D strategy and thereby run the risk of lim-

under transient load and under accident conditions, see Yan [43] and Yan & Lidsky [45].

iting the options available to future generations. Furthermore, it may be that the next generation of nuclear technology is deployed elsewhere before being accepted in the United States.

- While some would question the continued spending of public funds on projects of dubious long-term cost-effectiveness, we argue that while the risks are high so are the potential rewards. One need only consider capturing a very small fraction of the future additions of new and replacement power generating capacity in the relevant markets to realize just how significant the rewards could be[7] Moreover, the benefits will accrue largely to society as a whole in the form of whatever advantages the MHTGR eventually proves to offer. We will present the theoretical framework which provides the rationale and motivation for such high-risk R&D.

1.3 Attributes

We now consider those attributes of the MGR-GTS which differentiate it from other technologies and make it an attractive and potentially valuable source of energy for the future.

- **Inherent Safety of the Reactor**: By this we refer to the fact that under no conditions whatsoever can the reactor damage the integrity of the coated fuel kernels in such a way as to release radioactive material. The low power density and negative temperature coefficient of the core are such that when the power output of the reactor is increased, the temperature initially rises; however, this brings about a reduction in the fission rate due to the specific characteristics of the neutron cross-section.[8] The result is a subsequent decrease in the power output and a stabilization on the temperature. In short, the fission process is

[7]As a rough gauge of the size of the market in question, we note that, worldwide, the current annual rate of capacity additions and replacements is in the order of 100 Gigawatts [36].

[8]The interested reader is referred to chapter 7 of Lamarsh [19] for a good introductory explanation of temperature coefficient and reactor behavior.

self-controlling[9]. Under the worst-case scenario of reactor shutdown, total loss of coolant and no lowering of control rods, the center of the core will reach a maximum temperature of 1550° C before stabilizing[10]; well within the design limits of the system[11]. We note, moreover, that the natural stability of the core reactivity is independent of any and all operator actions.

- **Passive Heat Dissipation:** By this we refer to the fact that the walls of the reactor vessel are not insulated and there is a more than adequate natural rate of heat dissipation from the system. Under normal operation the core of the reactor is at a maximum temperature of 980° C at its center while the vessel is at only 142° C due to the rapid dissipation of heat into the reactor cavity cooling system[12] (RCCS) as well as into the earth via the concrete silo. Under the worst-case accident the RCCS will continue to dissipate heat by means of the evaporation of boiling water until its reservoir is depleted. At that point the rejection of heat solely into the earth will still be adequate from the standpoint of safety; however, the concrete walls of the silo will suffer damage. Under the worst conditions the maximum temperature experienced by the reactor vessel is 380° C. In no case is its integrity compromised.

- **Reduced Diffusion of Radionuclides:** By this we refer to the coated fuel kernel technology. The coatings, previously described, are designed to isolate the uranium oxide fuel and to retain the gaseous fission products. The technology has already been successfully demonstrated in Germany over an extended period of reactor operation. What is more, it continues to be improved through

[9]An actual real-time test of this behavior was demonstrated on the AVR reactor in Germany over a 24-hour shutdown period. Refer to [18] for a detailed description of the test and its results.

[10]This estimate is based on Izenson [17]

[11]Significant fission product release through the silicon carbide coating on the fuel kernels has been demonstrated to occur only at temperatures greater than 1600° C.

[12]The RCCS consists of cooling panels mounted in the cavity in which the reactor vessel stands and along the exterior of the vessel wall Heat is absorbed by means of water circulated through the panels. The heat is rejected at the surface via a heat exchanger. The system includes a reservoir which hold a back-up supply of water such that, in the event that the heat exchanger or pumps are disabled, water will continue to be supplied by gravity feed and heat will be dissipated by boiling. The RCCS will continue to function for at least one week before operator intervention is required.

the development of better performing materials as well as through the exercise of more effective quality control over the manufacturing processes. We may expect, then, that the MGR will experience a very low rate of radionuclide diffusion, relative to other nuclear technologies. As a result, the operating personnel will be exposed to much lower levels of radioactivity, as was the case in Germany, and that there will be significantly less contamination of equipment and material.

- **Underground Placement:** The underground silo concept is appealing from the standpoint of aesthetics. More importantly, though, it makes the structure much more resistant to seismic events. In addition, the design is such that the graphite core cannot be attacked by a current of air. This eliminates the possibility of a graphite fire, regardless of the accident scenario.

- **Easier Siting:** The fact that the MGR is designed to have no impact on the public will make the siting issue much less problematic. In addition, the costs associated with providing for emergency evacuation of the surrounding population are eliminated. Another favorable characteristic is that, by virtue of its modularity and underground placement, the MGR can be sited at locations that would prove to be inadequate for other technologies. Lastly, the MGR lends itself ideally to the floating power plant concept. By this we refer to the placement of plants on barges in protected waters close to population centers. The dual advantage of this concept is that the need to purchase high-cost land is avoided and the expenses associated with power transmission are reduced.

- **Safe Disposal of Spent Fuel:** The graphite-coated pebbles provide the ideal medium for long-term storage of spent fuel. There is no need at all to tamper with them since they are designed to retain the radionuclides in isolation. They may be easily tested upon removal from the reactor to verify their integrity, then they may be stored indefinitely. We point out that the political and technical issues surrounding the long-term disposal of spent fuel are beyond the scope of this study.

- **Reduced Risk of Diversion of Material:** From a purely practical point of view, fuel pebbles would be one of the most inconvenient and ineffective sources of material for illicit weapons production. First, the uranium is only low-enriched in quality; second, tracking the pebble count makes inventory management more efficient; third, the coated fuel kernels can be reprocessed only with highly sophisticated equipment and with much difficulty.

- **High Reliability:** One of the underlying principles driving the MGR concept is that the simplicity of the technology's design and operation will lead to higher reliability and ease of maintenance. In addition, the expectation is that the staffing requirements will be much lower than they are for other advanced technologies. This along with a reduced maintenance requirement will result in lower operating costs. While the reliability remains to be demonstrated[13], the principle of simplicity is sound.

- **On-Line Refueling:** The MGR is equipped with an automated fuel handling system which removes pebbles individually from the bottom of the reactor vessel, inspects them, rejects those that are spent, returns those that are not to the pebble bed core through the top of the reactor vessel, and adds fresh pebbles as needed. This system serves to maintain a uniform power density in the core. Furthermore, it eliminates the need to shut down the reactor for periodic refueling, thereby allowing a higher capacity factor to be achieved.

- **Brayton-Cycle Efficiency:** The MGR-GTS achieves a relatively high overall thermal efficiency of 45%, yet it only begins to tap the potential of the Brayton cycle. As the technology progresses and the operating temperatures are driven higher, future designs will have the ability to achieve efficiencies in the neighborhood of 60%.

[13]We note that the German AVR system demonstrated a high level of reliability. The reactor was in operation over a 21-year period, from 1967 to 1978. The overall utilization rate was 66.4%. However, it improved as experienced was gained over the life of the project and attained a high of 92% in 1976 [46].

- **Multi-Use Facility**: In the future, the MGR will be more than simply a power generating plant. Subsequent designs will offer the capability of simultaneously producing high quality steam which could be employed in applications such as chemical processing, enhanced oil recovery, synthetic fuel production, water desalination, district heating, etc.[14] This multi-use capability will not only increase the value of the plant but will give the MGR a competitive advantage in certain markets for power.

1.4 Major Uncertainties

As we have explained, the MGR-GTS design is meant for near-term application and, consequently, employs existing technology which has been proven in other contexts. Nonetheless, much uncertainty remains to be resolved through a significant R&D program. Furthermore, future generations of the MGR will continue to push the limits of the various technologies. Briefly, among the more salient issues are the following:

- **Materials Performance**: The most important challenge relative to materials concerns the steel alloys to be used in the reactor vessel and internals. The alloys mentioned earlier are capable of withstanding the design temperatures, but their long-term performance in operating environment must be demonstrated. In addition, as the design temperatures of future MGR's rises, new alloys will have to be developed. A second issue concerns the graphite internals. In order to take full advantage of the on-line refueling capability, it is necessary to minimize the need to shut down the reactor for other reasons, namely, graphite reflector replacement. Consequently, there is a need to develop a graphite which will be longer-lasting and more resistant to oxidation than is presently available.

- **Coated-Fuel Performance**: While the silicon carbide coated-fuel technology has been demonstrated under the MGR operating conditions, the ability to

[14]The interested reader is referred to McDonald [23] [24] who describes these future possibilities in some detail.

manufacture the kernels and pebbles on a large scale and with a coating failure rate in the order of 10^{-5} remains as a challenge. It will also be necessary to demonstrate that the long-term rate of fission product diffusion through the system can be kept to levels that are acceptably low.

- **Component Performance**: The major components, i.e., heat exchangers, turbomachinery, control systems, etc., are designed within the limits of existing technology, but remain to be built and demonstrated. Many questions await to be resolved. For example, the preliminary designs of the heat exchangers are based on technology developed for the aerospace industry. While the technology will certainly transfer, it remains to adapt the equipment to perform under the operating conditions of the MGR, and with the very high long-term level of reliability which will be required.

- **System Performance**: Once the components are built and tested on an individual basis, there will remain the challenge of demonstrating the reactor system as a whole. There are critical questions to be answered relative to the dynamic response of the reactor to temperature and pressure transients, to sudden loss of load or loss of coolant as well as to other serious accident conditions. Undoubtably the most important issue to be addressed concerns proving that the nuclear fission process is inherently self-controlling. In the event that all of these near-term technical issues are resolved favorably, there will still remain the question of demonstrating that, over the long term, the system is capable of operating at a high level of reliability.

- **Manufacturing Issues**: As mentioned earlier, the MGR's production economics rely heavily on the expectation of driving costs down by virtue of a standardized design, a higher percentage of the work performed in the factory and the implementation of modularized construction techniques. This raises the question as to how significant will be the long-term gains from learning and modularization. Our sense is that the productivity gains will have to be important in order for the MGR to become a commercial success. Unfortunately, the uncertainties surrounding this issue will not be resolved through R&D; they will require a commercial undertaking. However, such a venture becomes all the more problematic in the face of this level of risk.

In summary, we have explained the potential which the MGR holds for the future and have briefly discussed some of the technological and institutional risks presently confronting us. At this point, we pose the question: How do we justify the immediate commitment to a costly R&D effort based on the hope of an economic gain whose magnitude is unknown, which may or may not materialize and, in any case, is far off in the future?

Chapter 2

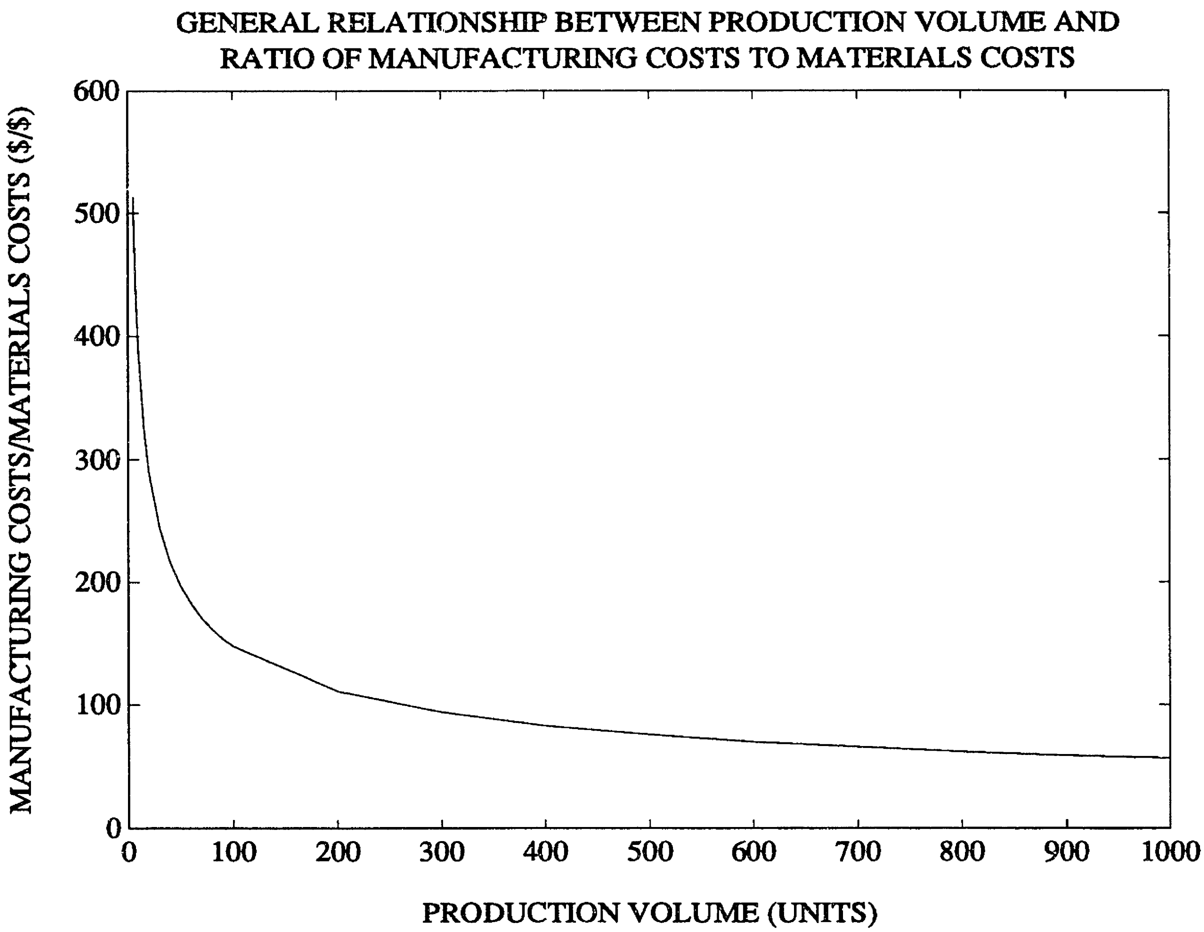
Cost Estimate of a 6-Module Plant

2.1 Cost Uncertainty

While a cost estimate is central to any economic evaluation or capital investment decision, it is not the purpose of this cost estimate to actually demonstrate that the MGR is, or is not, competitive relative to the alternative technologies. Given that the concept is some ten or more years from realization, it is much too early in the R&D cycle to make that determination. Rather, the objective is to estimate a most likely cost of the reference technology based on current information and to characterize the nature and importance of the uncertainties.

In Chapter 1, we considered the more important elements of uncertainty surrounding the MGR. With regard to the ultimate commercial success of the technology, however, we find that we may think of the greater portion of this uncertainty in terms of a single parameter: production volume. We believe that the most significant unknown relative to the ultimate costs is the rate at which units are ordered over the long term. This will ultimately influence the magnitude of the gains to be derived from learning and from modular construction practices. It will also determine how fully and how cost-effectively the manufacturing capacity dedicated to MGR production is utilized. These factors weigh heavily in their contribution to the ultimate production costs.

Figure 2-1:



Referring to Figure 2-1, we see a general relationship between the ratio of manufacturing costs to materials costs versus the production volume [35]. This graph is meant to illustrate how unit costs typically vary with production volume. Of course, the degree of convexity of the curve and the ultimate cost reduction will vary according to the component in question. For example, the cost of a reactor vessel would tend to stabilize at a lower production volume than would be the case for a component such as a control rod or a recuperator. Moreover, we would expect the ultimate cost reduction of the reactor vessel to be proportionately less significant than it would be for a control rod or a recuperator. This is because the reactor vessel has a relatively much higher proportion of its total cost embodied in the raw material.

The important point at our level of analysis is that the cost of a component may vary by an order of magnitude depending on the ultimate production volume. Furthermore, we find that the uncertainty related to production volume is much more significant in its impact on the ultimate costs than would be the other levels of uncertainty described in Chapter 1. Before proceeding toward a meaningful cost estimate, we must first assume that the MGR will either achieve a threshold volume or that it will not survive. So, we make the assumption that this volume will be realized, and we position our estimates down the curve.

2.2 Construction Schedule and Overnight Capital Cost

The development of the construction schedule and the cost estimate draws from previous research performed at MIT by Coxe [12] and Staudt [40]; from the industry study, previously mentioned, which consists of a detailed evaluation of the steam-cycle version of the MGR [4]; and on current information obtained from power plant construction firms and equipment manufacturers[1]. The following assumptions are made:

- A fully developed MGR industry exists. The design has been standardized; the modules to be built, including all components, are $N^{th} - of - a - kind$. Component delivery conforms to the schedule.

- The base case assumption is that regulatory requirements are such that those structures which are considered "safety-related" must be designed for safety with respect to seismic events but are not required to conform to the standards of nuclear-grade design which prevail in LWR construction. We do, however, run a sensitivity case which illustrates, to some extent, the incremental costs that such a requirement would impose.

[1]The major cost assumptions are listed in Appendix A.

- Most of the construction operations are performed on the basis of a 70-hour workweek employing a "rolling 4-10's" schedule (each crew works 4 successive 10-hour shifts followed by 4 days off). However, certain critical operations, i.e., construction of the reactor silo and installation of the equipment in the silo, are carried out on an around-the-clock basis.

- Field work is minimized to the extent possible by assembling components and sub-assemblies in the factory.

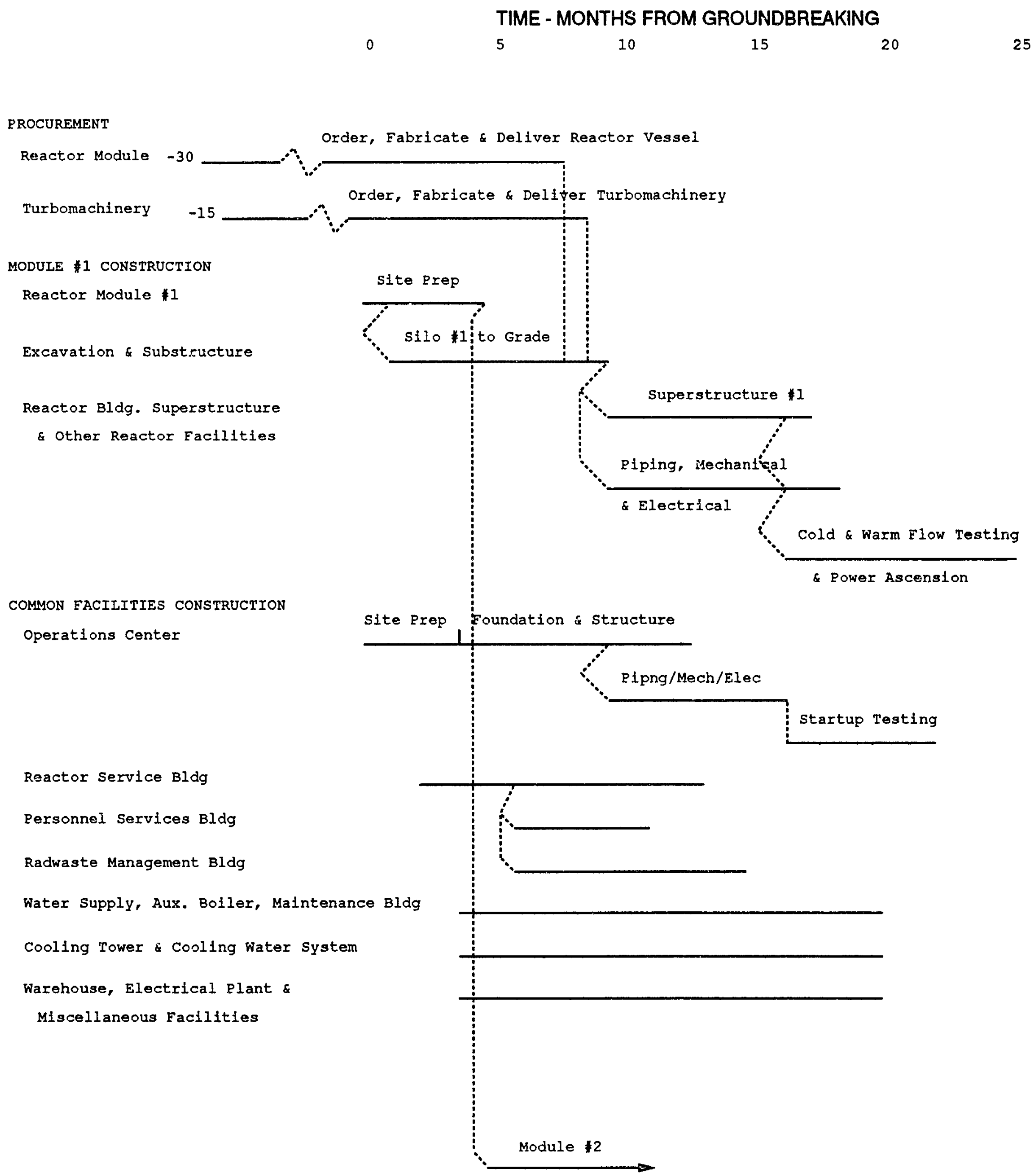
- Reactor metallic internals and core are preassembled and checked in the factory, disassembled for shipping and then reassembled upon delivery to the site. Once the metallic internals are installed in the reactor vessel, the core will be lifted and set in place as a unit.

The sequence of construction operations, as illustrated by the Construction Scheduling Chart, Figure 2-2, proceeds as follows:

1. **Site Preparation.** This includes the clearing and grading of the site, the building of access roads as well as the securing of the site. This work extends over a four month period and requires 100K man-hours.

2. **Silo Construction.** The excavation for the reactor silo emplacement begins one month after groundbreaking and requires two months and 60K man-hours to complete. The assumption is that augering is employed in conjunction with freezewall groundwater barriers. The excavation extends to a depth of 115 ft. (35 m) below grade and is about 82 ft. (25 m) in diameter. The silo is constructed using the slipform technique. The mat is 72 ft. (22 m) in diameter and 6.5 ft. (2 m) in thickness. The silo stands 125 ft. (38 m) in height from the top of the mat, to 10 ft. above grade, and is 66 ft. (20 m) in diameter. The cap is 5 ft. (1.5 m) thick. The silo requires approximately 11,000 yd^3 of concrete and 1,650 tons, or 300 lb/yd^3, of reinforcing steel. Construction of the silo is completed in two months. (Slip-form work advances at 1/4 ft/hr

Figure 2-2: Construction Scheduling Chart



around-the-clock). The estimated labor requirement of the construction phase is 190K man-hours.

3. **Reactor Installation.** The installation of the nuclear reactor system and the energy conversion equipment inside of the silo requires 185K man-hours. In effect, a crew averaging 60 craftsmen will work around-the-clock over a 4-month period to complete the following tasks:

   - Install water pump and instrumentation in bottom of reactor vessel cavity.
   - Install shutdown cooling circulator and heat exchanger in bottom of reactor vessel.
   - Install reactor cavity cooling system panels and ductwork in cavity to surround reactor vessel.
   - Lift and install reactor vessel into cavity.
   - Weld appendage to lower portion of reactor vessel to house the HP-turbine/compressor.
   - Install HP-turbine/compressor.
   - Install metallic internals in bottom of reactor vessel. Lift and install into the vessel the preassembled reactor graphite core and reflectors together with the metallic internals.
   - Install piping leading from HP-turbine/compressor and weld cover over the open end of the appendage.
   - Install fuel handling system and lines.
   - Install control rods and small absorber sphere shutdown system (KLAK).
   - Lift and install upper section of reactor vessel with recuperator and LP-turbine preassembled inside.
   - Install precooler.
   - Install piping leading from recuperator, LP-turbine and precooler.

- Install inventory control vessels (4)[2].
- Install control system, instrumentation and wiring.
- Install generator.
- Install upper reactor vessel head over LP-turbine.
- Install concrete cap over top of silo.

4. **Reactor Complex Superstructure.** Construction of these buildings, which are dedicated to a single module, follows the completion of equipment installation into the silo. These operations require a crew averaging 100 laborers and craftsmen to work over a 9-month period.

   - The reactor building is made of reinforced concrete and is "safety-related". It extends from 20 ft. below grade to 70 ft. above grade and is 90 ft. long by 60 ft. wide. Its construction will require 40 man-hours and $800 of materials per sq. ft.
   - The reactor auxiliary building is made of reinforced concrete and is "safety-related". It is a one-storey structure, 80 ft. long by 50 ft. wide, which contains the uninterruptible AC and DC power supply systems as well as the helium purification system. Its construction will require 16 man-hours and $400 of materials per sq. ft.

5. **Nuclear Island Common Facilities.** Simultaneous to the construction of the first module, the facilities designed to serve all planned modules are installed. Since these facilities are not "safety-related", we assume that modularization and factory fabrication techniques will be employed to the extent possible in order to reduce costs.

   - The reactor services building is a multi-level structure, made of reinforced concrete, which extends from 40 ft. below grade to 20 ft. above grade and

[2]Each of the four inventory vessels is 15 m in height, 3.5 m in diameter and weighs 73 metric tons (80 tons).

is 150 ft. long by 100 ft. wide. It contains all of the facilities related to reactor maintenance, new and spent fuel handling, storage and shipping. It houses the bridge crane which services all of the reactors and also holds the helium regeneration system. Its construction requires 12 man-hours and $200 of materials per sq. ft.

- The radioactive waste management building is a steel-framed, one-storey structure, 100 ft. by 100 ft., which contains all of the facilities for the handling and storage of gaseous, liquid and solid radioactive wastes. Its construction requires 2 man-hours and $80 of materials per sq. ft.
- The helium services building is a steel-framed. It is one-storey, 50 ft. by 50 ft. It houses the helium storage and transfer system. Its construction requires 2 man-hours and $80 of materials per sq. ft.

6. **Balance of Plant Facilities.** Simultaneous to the above-mentioned operations, the balance of plant, or non-nuclear common facilities, are constructed. As above, we assume that modularization and factory fabrication techniques will be employed to the extent possible in order to reduce costs. The following list includes all of the major buildings but is not comprehensive.

   - The operations center is a two-storey structure, made of reinforced concrete, and is 150 ft.long by 100 ft. wide. It houses the control center from which the operation of all of the reactor modules are managed. The security systems are also tied into the center. Its construction requires 4 man-hours and $140 of materials per sq. ft.
   - The personnel services building is a steel-framed, one-storey structure, 80 ft. by 80 ft., which houses the facilities dedicated to controlling the radiation exposure of the plant personnel. Its construction requires 1 man-hour and $70 of materials per sq. ft.
   - The maintenance building is a steel-framed, two-storey structure, 60 ft. by 120 ft. It houses the facilities and equipmemt dedicated to the repair and

maintenance of the plant's physical systems. Its construction requires 2 man-hours and $140 of materials per sq. ft.

- The auxiliary power building is a steel-framed, one-storey structure, 60 ft. by 60 ft., which houses the stand-by generators and related equipment. Its construction requires 1 man-hour and $70 of materials per sq. ft.
- The chilled water & auxiliary boiler building is a steel-framed, one-storey structure, 60 ft. by 120 ft. It houses the equipmemt related to treating and chilling water as well the stand-by boiler. Its construction requires 1 man-hour and $70 of materials per sq. ft.
- The warehouse is a steel-framed, one-storey building, 100 ft. by 200 ft. It houses the inventory of spare parts and miscellaneous. equipment. Its construction requires 1 man-hour and $70 of materials per sq. ft.

7. **Other major systems.** In addition to all that is described above, the plant includes the following systems and equipment which are worthy of separate mention:

    - The plant control system manages the overall operation of the plant's facilities. It is distinct from the individual reactor control systems.
    - The plant electrical system performs two major functions. First, it transfers the power generated at the plant to the grid via the high-voltage switchyard and two independent transmission systems. Second, it supplies and distributes both AC and DC power to the various facilities in the plant.
    - The transportation and lift equipment consists of trucks, forklifts, etc.
    - The air and water systems include the HVAC, drinking water distribution, wastewater handling facilities, etc.
    - The communications and security systems includes telephones, emergency lighting and power equipment, fire protection equipment, etc.

- The cooling tower and service water system consists principally of the 2-unit evaporative cooling tower along with its inlet and outlet piping systems.

8. **Test system, load fuel and conduct power ascent.** Within approximately 20 months from groundbreaking, the construction of the first module and the common facilities will be essentially complete. At that point the final phase of the project may begin. It employs a crew of 60 workers over a 5-month period and proceeds as follows:

   - Perform cold flow testing over the entire system. (1 month)
   - Load some fuel samples, test fuel handling and adjust fuel handling system as needed. (2 weeks)
   - Partially load reactor with fuel and perform warm flow testing. (1 month)
   - Complete fueling operation and perform hot flow testing. (6 weeks)
   - Connect plant to grid and execute power ascent. (1 month)

According to the above schedule, the first module becomes operational 25 months after ground-breaking. Since the plan is to begin the construction of the subsequent modules at 4-month intervals, we may anticipate to complete a 6-module plant within 4 years, barring unforeseen, site-specific problems. The detailed overnight capital cost estimates of a single module and the operations center are given in Figures 2-3 and 2-4, respectively. Note that we also consider some of the incremental costs that would result in the event of more stringent regulatory requirements.

2.3 Operating & Maintenance Costs

One of the underlying assumptions driving the MGR R&D program is that the technology, once proven and perfected, will demonstrate high reliability and be inexpensive to operate. Its simplicity of design and operation will allow for a reduced number of personnel and will be less costly to regulate and insure relative to other nuclear technologies. In addition, with continuous on-line refueling the MGR will be able to achieve a high annual capacity factor. While there is great uncertainty today as to whether or not these goals will be realized, we make the assumption that either they will be demonstrated during the R&D phase, or the technology will not be viable. With this in mind, our estimate of annual operating costs is based on the following:

- One-third of the fuel pebbles is replaced annually.
- A fee of 1 mill/kWh is assessed for the purpose of spent fuel disposal and radioactive waste management.
- A fee of 0.5 mill/kWh is assessed for the purpose of site decommissioning.
- On the average, 5% of the control rods and 10% of the reflector material are replaced annually.
- The complete staffing requirement is 15 personnel per module and 30 personnel assigned to the operations center. This assumes a certain degree of regulatory accommodation.

We note, furthermore, that we are estimating the operating life of the plant to be 40 years, based on the expected longevity of the reactor vessel. We estimate the decommissioning sinking-fund charge based on the assumption that at the end of the operating life of the plant each silo will be entombed in concrete. The operating expenses are listed in Figure 2-5. Lastly, we also consider a sensitivity case which includes the major incremental costs which would be incurred in the event that regulations require "nuclear safety grade" construction and more operating personnel.

2.4 Levelized Busbar Cost

Making use of the foregoing cost estimate, we now proceed to determine the levelized busbar cost for the reference 6-module MGR. At the same time, we will evaluate two competing technologies on the basis of their respective busbar costs. In the case of the MGR, we must take into account the fact that each module begins production as soon as it is completed. Therefore, we will consider the busbar cost on the basis of a single module and we will approximate its share of the cost of the common facilities on the basis of simple proration.

To calculate the levelized busbar cost, e, we will ignore the effect of income taxes and will make the further simplification of considering project lead-time to begin at the time of ground-breaking. We employ the formula

$$e = \frac{1000}{8766} \cdot \frac{\phi}{C_f} \cdot (\frac{I_0}{K})[1 + \frac{x+y}{2}]^{C_t} + \{\frac{1000}{8766} \cdot \frac{1}{C_f} \cdot (\frac{C_o}{K}) + C_v\}[1 + \frac{x \cdot (T-1)}{2}] \quad (2.1)$$

where

$$\begin{aligned}
\phi &= \frac{x \cdot (1+x)^T}{(1+x)^T - 1} = \text{fixed charge rate} \\
C_f &= \text{capacity factor} \\
\frac{I_0}{K} &= \text{overnight capital cost per KW (1991 \$)} \\
x &= \text{cost of capital (\%)} \\
y &= \text{annual inflation rate (\%)} \\
C_t &= \text{construction lead-time (yr)} \\
\frac{C_o}{K} &= \text{annual fixed operating cost per KW (1991 \$)} \\
C_v &= \text{variable operating cost, mill/kWh (1991 \$)} \\
T &= \text{operating lifetime (yr)}
\end{aligned}$$

Using cost information developed by the Electric Power Research Institute [16], we compare the MGR to a 400 MW Non-Integrated Coal Gasification/Combined Cycle plant[3] (CGCC) and a 210 MW Advanced Gas Combustion Turbine/Combined Cycle plant[4] (GTCC). The relevant data are:

[3]EPRI TAG Technology Number 17.2
[4]EPRI TAG Technology Number 46.2

	BASE CASE		
ITEM	MGR	CGCC	GTCC
Plant Capacity (MW)	90	400	210
Overnight Capital Cost ($/KW)	2,362	1,703	687
Fixed Operating Cost ($/KW-yr)	68.9	44.9	4.3
Variable Operating Cost (mills/kWh)	3.9	2.9	10.1
Plant Operating Life (Yr)	40	30	30
Capacity Factor	.90	.85	.90
Construction Lead-Time (Yr)	2	3	2
Inflation Rate (%/yr)	5.0	5.0	5.0
Cost of Capital (%)	11.5	11.5	11.5
Fixed Charge Rate	.1165	.1196	.1196
Levelized Busbar Cost (mills/kWh)	65.9	50.1	30.6

	SENSITIVITY CASE
Overnight Capital Cost ($/KW)	2,473
Fixed Operating Cost ($/KW-yr)	79.4
Variable Operating Cost (mills/kWh)	4.1
Levelized Busbar Cost (mills/kWh)	70.8

Clearly, on the basis of these busbar cost estimates one would be averse to investing in an MGR. We note, however, that, in addition to the high degree of cost uncertainty already discussed, there are some differences in methodology between this study and EPRI. Nonetheless, let us assume that these estimates represent the best available information. The important question for our purposes is to ask what decisions should be made on the basis of this information relative to the MGR R&D strategy and to long-term energy policy in general.

Figure 2-3:

OVERNIGHT CAPITAL COST ESTIMATE - SINGLE MODULE
(1991 $1000, 1000 Man-hours)

ITEM	LABOR TIME	LABOR COST	MATERIALS COST	EQUIPMENT COST	TOTAL COST
REACTOR COMPLEX	-------	-------	-------	-------	-------

Reactor Silo	250	9,400	2,500		11,900
Reactor Superstructure	216	6,480	4,320	3,000	13,800
Reactor Auxiliary Bldg	16	480	400	1,000	1,880
Shutdown Cooling System	10	400		2,000	2,400
Reactor Cavity Cooling System	5	200		500	700
Reactor Vessel	15	600		18,000	18,600
Reactor Core & Internals	15	600		25,000	25,600
Control Rods & KLAK	4	160		5,000	5,160
HP-Turbine	3	120		6,000	6,120
Compressor	3	120		5,000	5,120
LP-Turbine	3	120		5,000	5,120
Fuel Handling System	6	240		8,000	8,240
Precooler	3	120		1,000	1,120
Recuperator	3	120		3,000	3,120
Inventory Control Vessels	12	480		7,000	7,480
Generator	3	120		10,000	10,120
Control System & Instrumentation	20	800	1,000	5,000	6,800
Reactor Service & Miscellaneous Systems	25	1,000	5,000	5,000	11,000
Piping & Conduits	25	1,000	4,000	500	5,500
Electrical Wiring & Equipment	25	1,000	2,000	500	3,500
Test, Load Fuel & Power Ascent	90	3,600		3,000	6,600
Temporary Construction Facilities		3,000	1,000		4,000
Home Office Engineering		7,200			7,200
Home Office Project Management		2,880			2,880
Field Office Expenses & Supervision		5,760	1,000		6,760
Permits, Insurance & Taxes				10,000	10,000
Owner's Management Expenses		4,320			4,320
	-------	-------	-------	-------	-------
TOTAL	752	50,320	21,220	123,500	195,040

INCREMENTAL COSTS FOR NUCLEAR GRADE REQUIREMENT

Reactor Silo	10	400	500		900
Reactor Superstructure	54	1,620	1,080	3,000	5,700
Reactor Auxiliary Bldg	4	120	100	1,000	1,220
Other Added Costs	26	780	600	800	2,180
	-------	-------	-------	-------	-------
TOTAL	94	2,920	2,280	4,800	10,000

Figure 2-4:

OVERNIGHT CAPITAL COST ESTIMATE - COMMON FACILITIES
(1991 $1000, 1000 Man-hours)

ITEM	LABOR TIME	LABOR COST	MATERIALS COST	EQUIPMENT COST	TOTAL COST
GENERAL					
Site Preparation	100	3,000	2,000		5,000
NUCLEAR ISLAND					
Reactor Service Bldg	180	5,400	3,000	3,000	11,400
Radioactive Waste Management Bldg	20	600	800	1,000	2,400
Helium Services Bldg	5	150	200	1,000	1,350
BALANCE OF PLANT					
Operations Center	80	2,400	2,800	500	5,700
Personnel Services Bldg	6	192	448	200	840
Maintenance Bldg	14	432	1,008	200	1,640
Auxiliary Power Bldg	4	108	252	1,000	1,360
Chilled Water & Auxiliary Boiler Bldg	7	216	504	1,000	1,720
Warehouse	20	600	1,400	200	2,200
Plant Control System	10	300	1,000	9,000	10,300
Plant Electrical System	20	600		20,000	20,600
Transportation & Lift Equipment	5	150		3,000	3,150
Air & Water Systems Equipment	20	600		3,000	3,600
Communications & Security Systems	5	150		2,000	2,150
Cooling Tower & Service Water System	200	6,000		16,000	22,000
Temporary Construction Facilities		1,000	500		1,500
Home Office Engineering		1,440			1,440
Home Office Project Management		720			720
Field Office Expenses & Supervision		1,440	500		1,940
Permits, Insurance & Taxes				2,000	2,000
Owner's Management Expenses		2,160			2,160
TOTAL	697	27,658	14,412	63,100	105,170

Figure 2-5:

ANNUAL OPERATING & MAINTENANCE COST ESTIMATE
(1991 $1000)

ITEM	PERSONNEL FULL-TIME	PERSONNEL COST	MATERIALS COST	SERVICES COST	TOTAL COST
	-------	-------	-------	-------	-------
SINGLE MODULE - FUEL CYCLE					
Fuel Replacement			2,500	100	2,600
Waste Management				710	710
Decommissioning Charge				355	355
			-------	-------	-------
TOTAL			2,500	1,165	3,665
SINGLE MODULE - OPERATIONS					
Salaries, Benefits & Taxes	15	1,500			1,500
Insurance, Taxes & Fees				800	800
Control Rod & Reflector Replacement			1,500	100	1,600
Other Fixed O&M Expenses			250	250	500
Variable O&M Expenses			100	100	200
	-------	-------	-------	-------	-------
TOTAL	15	1,500	1,850	1,250	4,600
OPERATIONS CENTER					
Salaries, Benefits & Taxes	30	3,000			3,000
Insurance, Taxes & Fees				1,000	1,000
Other Fixed O&M Expenses			200	200	400
	-------	-------	-------	-------	-------
TOTAL	30	3,000	200	1,200	4,400

INCREMENTAL COSTS FOR REGULATORY COMPLIANCE

Additional Personnel per Module	5	500			500
Additional Decommissioning Charge				355	355
Other Fixed Costs per Module				100	100
Other Variable O&M Expenses per Module			50	50	100
	-------	-------	-------	-------	-------
TOTAL	5	500	50	505	1,055

Chapter 3

The Application of Contingent-Claims Analysis to the Investment Problem

3.1 The Impact of Real Options on Valuation

As was mentioned earlier, one of the stated, and vitally important, objectives of the MGR R&D program is to demonstrate that the technology will ultimately be capable of delivering electricity at a cost which is attractive relative to the competing technologies, namely, coal, gas and advanced nuclear reactors. By "attractive" what is meant is that it is generally held that the MGR will have to produce less costly electricity, by a factor of as much as 20%, in order to overcome certain barriers to entry. The yardstick which is being used to measure the relative cost performances is the traditional lifetime-levelized busbar cost. The objective of this chapter is not to determine that the MGR will or will not be cost competitive, but rather, to demonstrate that the yardstick being used is incorrect.

A methodology for arriving at a truer measure of economic value will be developed and will be demonstrated by means of a series of stylized examples. We will compare three different power plant investment programs, each one representing one unit of capacity and all three having the same levelized busbar cost. We will show that, although the projects would be equally valued according to the traditional measure, there are significant differences in their true economic value when we account for the impact of market uncertainty on the relative differences in capital intensity and construction lead-time.

To begin with, as the preceding cost analysis in Chapter 2 illustrates, busbar cost estimates are invariably based on deterministic "best guess" projections of capital outlay, time to build, operating costs, availability, demand, prices, interest rates, etc. The reality, however, is that there is uncertainty underlying all of the factors in question and that quantifying how uncertainty affects value is crucial to understanding the true economic worth of a long-lived capital investment. Unfortunately, the classical methods of valuation, e.g., discounted cash flow (DCF) analysis, are not capable of explaining how the various levels of uncertainty and their resolution over time really affect the value of a project. The use of such methods may result in a faulty evaluation of competing investment opportunities when it happens that the effects of uncertainty are asymmetric from one project to another. For example, in our case we would be interested in weighing the economic advantages of the MGR, i.e., its shorter lead-time and sequential deployment characteristics, against its higher capital costs in an environment of uncertainty in the cost of and demand for electricity. DCF analysis does not address this type of question.

To be sure, over the past ten years or so, there has been a growing awareness that smaller, shorter lead-time power plants are less risky, i.e., better investments. In fact, such plants represent virtually all that are being built at this time. Implicit in this fact is the realization by the investors that the logic of economies of scale is not the only one upon which to base power plant investment decisions. To the contrary, investor behavior demonstrates that there is incremental value value in the smaller, shorter lead-time projects which compensates for what may appear to be lost economies of scale, or higher levelized busbar costs. Yet, a review of the literature reveals a general lack of rigor in quantifying the value of such project characteristics. That is, except for the growing body of work employing the technique of contingent-claims analysis (CCA).

To my knowledge, CCA is the only method of analysis which permits a rigorous valuation of capital investment programs, of the type in question, based on rational investor behavior. The strength of this technique stems from the fact that it relies on prices and price movements observable in the market, or deducible from observed events, to determine the value of an investment in terms of the market price of risk. What is more, its results are independent of price forecasts and individual risk preferences.

Presently, we are interested in valuing power plant investment programs having the following characteristics:

- The construction of the plant is not instantaneous. It takes time to build, and at each point in time the investor holds the option to proceed, delay or abandon.
- The investment is lumpy in the sense that the increments of capacity expansion are large. The investor has an option to invest in more or less plant capacity at one time.
- The investment is irreversible. Once capital is sunk it may not be costlessly recovered.
- The construction of a plant on a site may confer to the investor the option to expand capacity at a future date.
- The asset is long-lived. At each point in time the operator holds the option to produce, shut down temporarily or abandon permanently.

While this list is by no means exhaustive, it captures the salient features of the problem at hand. In the parlance of CCA the above are referred to as real options. According to CCA, an investment program, or capital asset, is viewed as consisting of so many such options which are embedded in some sequence over its lifetime. As CCA originated from the discipline of finance theory, the valuation of real options employs the logic and mathematics which were originally devised for the valuation of options on financial assets. [1] The fundamental link between the two is the fact that, at any point in time, the market value of a financial asset represents investors' collective beliefs as to the economic value of that security's claim on the underlying cash flows. By the same token, the market value of a real asset represents investors' beliefs as to the future cash flow which it will generate. So, in short, CCA makes use of the information available in the market to deduce the value of an investment program in terms of what the rational investor would be willing to pay for that project, or asset.

The application of CCA to be employed here is based on the work of Merton [29] [31] [33], McDonald & Siegel [25] and Majd & Pindyck [20] as well as on Meehan [27]. In the context of the power plant investment decision, we are principally interested in two levels of real options. At the outset of and during the of construction of a plant there is uncertainty as to the ultimate value of the project. At any time during this phase one has the option to defer or abandon continued investment in response to changing market conditions. Subsequently, during the operating life of a plant there is uncertainty as to the prices of the inputs and outputs. At any point in time, if the revenue to be derived from continued production will not cover the variable operating costs, one has the option to temporarily shut down rather than to operate at a loss. These options materially affect the value of a project and are not captured in a rigorous manner in the DCF methods of analysis. Moreover, the value of these embedded options may vary significantly across technologies such that the failure to account for them may result in suboptimal investment decisions. So, our task is to construct a model which will value these options and then apply it to the power plant investment problem. Our ultimate objective is to gain additional insight as to the value of the MGR relative to the competing technologies.

[1] The pioneering works on the theory of option pricing were authored by Black & Scholes [8] and Merton [28].

3.2 Assumptions

Applications of CCA may become quite complex as added levels of uncertainty are built into the models. So, in order to keep the analysis manageable a number of simplifying assumptions are necessary. Nonetheless, we intend to preserve the essential character of the investment problem. To begin, we have a firm facing a capital investment decision with the following structure:

- The construction phase requires a certain amount of time during which cash outlays occur continuously.
- There is a maximum rate at which capital may be invested. In other words, there is a minimum time to build.
- At any time during construction, the project may be temporarily deferred and then subsequently recommenced at no additional cost. [2]
- Investment is irreversible and the capital in place has no alternative use nor salvage value. [3]
- The investment opportunity may be indefinitely deferred without expiring.
- The size of the project is fixed and the plant is not productive until it is complete. (Later, this assumption will be modified to analyze modularity.)
- The market value of the completed project is determined by the discounted value of the expected future cash flows over the operating lifetime. While the dynamics of the riskiness, or uncertainty, of these cash flows are indeed complex, we will model the process in terms of a single state variable, namely, the uncertain price of the output.

Under our scenario, which requires time to build, the process of investing in the construction of a plant is represented as a compound option. In each period the firm,

[2]This assumption may be relaxed by incorporating a switching cost into the problem formulation.
[3]This assumption may be relaxed. In fact, Myers & Majd [34] have studied the problem of valuing the option to abandon when the salvage value is uncertain.

as investor, holds the option to continue investment or to defer. Exercising one such option buys the right to proceed to the next stage of the program. But there is a cost incurred in exercising the option which is over and above the capital outlay in question. This cost derives from the fact that an option is worth more alive than dead. It must be accounted for in the decision-making process. So, at each point in time the firm must weigh the value of the option acquired against the cost of the option exercised. The optimal decision is a function of the current market value of a completed plant. That is, there is a critical value above which it is optimal to invest and below which it is optimal to defer. However, this value is, in turn, a function of the expected cash flow to be generated over the life of the plant once completed. So, we model the operation of the plant, the characteristics of the firm and the dynamics of the market in which it competes as follows:

- A single good is produced.
- The plant has a defined maximum output per period and the firm seeks to maximize profit in each period. However, we consider the plant in terms of a single unit of capacity; so, in each period that unit will either operate at full capacity or not at all.
- The capital structure of the firm is 100% equity.[4]
- The firm is a price-taker in a perfectly competitive market. The price of the good, i.e., electricity, is observable and evolves stochastically over time according to a diffusion process which we describe as

$$dP = \alpha_P P\, dt + \sigma_P P\, dz_P \tag{3.1}$$

where

$$\alpha_P \quad = \quad \mu_s - \delta_P = \text{expected growth rate of output price}$$

[4]See Mason & Merton [22] for an illustration of CCA applied to a project whose financial structure consists of equity, senior debt and junior debt along with a government loan guarantee.

σ_P = instantaneous standard deviation of output price

μ_s = equilibrium return on a financial asset having the same market covariance as the output price

δ_P = convenience yield, or opportunity cost, which accrues to the holder of the physical output[5]

dz_P = $\epsilon(t)(dt)^{1/2}$ = the increment of a Wiener process

$\epsilon_P(t)$ = a serially uncorrelated and normally distributed random variable

- The variable unit production cost, C_v, is constant and is known with certainty at t=0. (Later, this assumption will be relaxed by introducing a stochastic variable cost.)
- The firm may temporarily and costlessly[6] reduce production or shut down without affecting the future market prices of inputs or outputs. Consequently, with respect to the single unit of capacity under consideration, when $P(t_i) > C_v$, the unit will operate in period t_i; otherwise, it will shut down.
- The plant has a known operating life, T. We assume that capital does not depreciate until t=T, at which time capital disintegrates having no further value.

Basically, what we are describing is known in the electric power industry as a unit commitment model. It applies to those plants which are dispatched on demand. While it is presently true that in the electricity industry the price is generally fixed

[5]Note that the convenience yield represents a rate of return shortfall. That is, it is the difference between the market-required rate of return, given the riskiness, and the expected return. The fact that this difference exists implies that there is a benefit, or convenience, which accrues to the holder of the physical commodity; otherwise, no one would hold it. McDonald & Siegel [25] point out that, while the δ of a commodity which is physically storable must be greater than zero, there is no such restriction on the δ of a commodity, such as electricity, which cannot be stored. However, in the context of the present problem. δ_P is tied, in part, to the price dynamics of the fuel consumed at the margin in the electric power system in question. Consequently, it is not unreasonable to assume that $\delta_P > 0$ is a possibility.

[6]This assumption may be relaxed by introducing a switching cost to shut down and restart.

and it is demand which fluctuates, we are interested in measuring true economic value, consequently, the price of interest is the real-time marginal price[7]. This price, also called the avoided cost, best represents the real-time marginal value of electricity. In general, it fluctuates on an hourly basis. Its dynamics are driven, essentially, by two factors: the level of demand in the system and the cost of energy at the margin. The demand component may be further analyzed in terms of its effect on the marginal costs of generation, i.e., heat rate, of system transmission losses, of system reliability and of capacity. [8] The energy component, in turn, is tied to the price of the fuel which is used at the margin in the power system in question.

In reality, then, the firm faces uncertainty in both demand and price. In addition to the question of either operating or shutting down in each period, there will is also the issue of the variation in the level of the output of the plant corresponding to the demand. A complete analysis would incorporate a model of stochastic demand, along with that of price, but is beyond the scope of this study. We make the problems of stochastic demand and the "lumpiness" of capacity additions exogenous to the analysis by considering only a single KW of generating capacity. We assume, implicitly, that price adjusts stochastically to clear the market. Furthermore, we state, without proof, that this simplified analysis will yield results which are directionally similar to what would obtain in the case of a more comprehensive study.

We argue, moreover, that price is the more interesting of the two variables. In view of the evolution of the industry in the direction of the deregulation and decentralization of power generation, we may expect the increasing use of the real-time marginal cost, or spot price, as the basis for negotiating transactions between buyers and sellers of power. In time, as the market grows in breadth and depth, it will become efficient and the law of one price will obtain. Then, even plants operating under long-term, fixed-priced contracts will be valued off of the spot price by means

[7]An example of such a price is the *System – Lamda* of the New England Regional Power Pool (NEPOOL).

[8]Due to the physical limitations of the power generation and transmission system, the marginal price will vary not only in time, but also in space. See Schweppe, Caramanis, Tabors & Bohn [38] for a thorough treatment of the theory of real-time pricing of electricity.

of CCA.

So, while it would be ideal to develop a more elegant model of electricity consumption along with both the price and demand functions, for our purposes it will suffice to assume that Equation 3.1 provides an adequate description of the market dynamics. As long as the assumed stochastic process is a reasonable representation of the true price evolution, then the analysis will be valid.[9]

Finally, we make these further the assumptions concerning the economy:

- The market is complete, i.e., it is spanned by the portfolio of existing assets.
- The market is frictionless, i.e., there are no taxes, no transactions costs, no restrictions on short sales and no difference between the costs of lending and borrowing. Trading takes place continuously.
- The riskless interest rate, r, is constant and is known with certainty at t=0.[10]
- Inflation is ignored.

Just as we did in the case of the construction phase, we model the operating life of the plant as a compound option. In each period the operator holds the option to continue production, shut down if operating or restart if not producing. To the rational investor, the value of the plant consists of the discounted sum of the optimal, i.e., profit maximizing, operating decisions over time. Clearly, the option to avoid operating at a loss is a significant determinant of value.

3.3 The Option to Operate or Shut Down

The objective is to develop a model employing the technique of Majd & Pindyck which allows an investor to determine the value of a project to invest in the construction

[9]It is equally possible to perform the analysis using alternative stochastic processes. For example, the mean reverting process, $dP = \alpha_P(P^* - P)\,dt + \sigma_P\,dz$, may prove to be more plausible in certain cases, particularly for electricity. The use of this process, however, makes the mathematics significantly more complex, yet the results will generally prove to be directionally similar to those obtained under the assumed diffusion process.

[10]This assumption may be relaxed by incorporating a model of the term structure of interest rates into the analysis.

of an industrial plant. However, whereas the authors assume that the value of a completed plant is the exogenous stochastic variable priced in the market, we will incorporate the model of McDonald & Siegel which determines the value of a plant as a function of the stochastic price of the output.[11]

Consider a unit of capital capable of producing a unit of output of price, P, having a unit variable production cost, C_v[12], during each period of time, t. If P(t)< C_v, no production will occur. Hence, the cash flow at time t is $\Pi(t) = \max[0, P(t) - C_v]$.

We seek to determine V(0), the value at t=0 of all future expected cash flows, $\Pi(t)$. As seen from t=0, for each future time period the operator holds an option to produce or to shut down depending on the price, P(t). To value this contingent claim we employ the dynamic portfolio hedging strategy developed by Merton [29]. This technique assumes that investors are able to continuously and costlessly trade securities in order to adjust their portfolios as desired. While this is not an exact representation of the market, the technique has demonstrated itself to be a sufficiently robust method of accurately describing how securities are actually priced in the market.

In the context of the financial markets, Merton postulates the existence of a firm whose market value evolves stochastically, and applies dynamic portfolio hedging to derive the value of a financial asset, i.e., a contingent claim on the firm, as a function of the value of the firm and of time. By way of extension, we postulate the existence of a commodity whose market value evolves stochastically, and employ the technique to derive the value of a plant, modelled as a contingent claim, as a function of the value of the commodity and of time. Our logic holds under the assumption that investors price real assets in the same manner by which they price financial assets. In addition, the assumption of a complete market implies that the securities do exist which will permit investors to synthesize the derivative securities which replicate the real asset in question.

[11]This solution follows Meehan [27].

[12]Recall that in this model we assume the C_v is constant. This would be more representative of a nuclear or coal-fired plant than of a gas or oil-fired plant. In a subsequent model, however we relax this restriction and include a stochastic variable cost.

So, suppose that a derivative security[13] exists whose value is solely a function of electricity price, P, and time, t. That is,

$$Y = F(P, t)$$

Moreover, the value of this security evolves stochastically as[14]

$$\frac{dY}{Y} = (\mu_Y - \delta_Y)\,dt + \sigma_Y\,dz_Y \tag{3.2}$$

By Itô's Lemma[15], we have that the price dynamics of Y obey the relation

$$dY = \frac{1}{2}F_{PP}(dP)^2 + F_P\,dP + F_t\,dt \tag{3.3}$$

Substituting the expression for dP from Equation 3.1 yields

$$dY = [\frac{1}{2}\sigma_P^2 P^2 F_{PP} + (\mu_s - \delta_P)PF_P + F_t]\,dt + \sigma_P P F_P\,dz_Y \tag{3.4}$$

where

$$\mu_Y Y = \mu_Y F = [\frac{1}{2}\sigma^2 P^2 F_{PP} + (\mu_s - \delta_P)PF_P + F_t]\,dt \tag{3.5}$$

$$\sigma_Y Y = \sigma_Y F = \sigma_P P F_P \tag{3.6}$$

$$dz_Y = dz_P = dz$$

Next, one may construct a three-security portfolio consisting of the derivative security, the commodity and the riskless asset. In addition, one finances the purchases with short sales and borrowings such that the net investment is zero.

---

[13]What we are postulating, in simple terms, is a derivative security whose value is a function of the price of electricity and of time in the same manner that the value of a stock option is a function of the price of the underlying stock and its own time to expiration.

[14]In the context of Merton, δ_Y represents the interim payouts on the derivative security. In the present context, δ_Y would represent payouts such as the switching cost for temporarily starting up or shutting down the plant. We make the simplifying assumption that $\delta_Y=0$.

[15]See Merton [32] and Malliaris & Brock [21] for an explanation of Itô's Lemma and an overview of the application of stochastic calculus to models in continuous-time finance

Let W_1 be the investment in the derivative security, W_2 the investment in the commodity and W_3 $(= -(W_1 + W_2))$ the investment in the riskless asset. Keeping in mind that the portfolio composition is continuously adjusted in response to price movements, the instantaneous return on the portfolio, dX, may be expressed as

$$\begin{aligned} dX &= W_1 \frac{dY}{Y} + W_2[\frac{dP}{P} + \delta_P \, dt] + W_3 r \, dt \\ &= [W_1(\mu_Y - r) + W_2(\mu_s - r)]dt + (W_1\sigma_Y + W_2\sigma_P)dz \end{aligned}$$

where the term, $W_2\delta_P \, dt$, represents the reinvestment of the convenience yield[16] on the commodity or, alternatively, the cost of financing a short position.

Now, choose a portfolio strategy such that the coefficient of the dz term is always zero. The result is that the portfolio is nonstochastic, or riskless, and that since it requires no net investment, it must, to avoid arbitrage profits, yield an expected return of zero. Denoting this strategy by '*', we have the conditions

$$\begin{aligned} W_1^*\sigma_Y + W_2^*\sigma_P &= 0 \qquad \text{(no risk)} \\ W_1^*(\mu_Y - r) + W_2^*(\mu_s - r) &= 0 \qquad \text{(no arbitrage)} \end{aligned}$$

A nontrivial solution exists if and only if

$$\frac{\mu_Y - r}{\sigma_Y} = \frac{\mu_s - r}{\sigma_P} \tag{3.7}$$

Substitute into Equation 3.7 the expressions for μ_Y and σ_Y given in Equation 3.5 and 3.6, respectively. This yields

$$\frac{\mu_s - r}{\sigma_P} = \frac{\frac{1}{2}\sigma_P^2 P^2 F_{PP} + (\mu_s - \delta_P)PF_P + F_t - rF}{\sigma_P P F_P} \tag{3.8}$$

[16]In general, both σ_P and δ_P are stochastic; however, in this analysis we make the simplifying assumption that they are constant and known with certainty. See Gibson & Schwartz [15] for an application of CCA with a stochastic convenience yield.

or

$$0 = \frac{1}{2}\sigma_P^2 P^2 F_{PP} + (r - \delta_P)PF_P + F_t - rF \tag{3.9}$$

The derivative security will satisfy this partial differential equation (PDE) subject to the boundary conditions:

$$F(0,t) = 0 \tag{3.10}$$

$$F(P,T) = \max[0, (P(T) - C_v)] \tag{3.11}$$

$$\lim_{P \to \infty} F_P(P,t) = e^{-\delta_P(T-t)} \tag{3.12}$$

The boundary condition described by Equation 3.10 is interpreted as meaning that in the event that the price, P, goes to zero, it stays there and the option becomes worthless. Equation 3.11 represents the initial condition which describes the fact that in the final time period the plant will operate if $P > C_v$, otherwise it will shut down and have no further value. Equation 3.12 describes the fact that, as P becomes very large, the rate of change of F relative to P is constant.

This problem is identical to that of a European call option on a dividend-paying stock, as given by Merton [28]. The solution is

$$F[P,t;T] = P(t)e^{-\delta_P(T-t)}N(d_1) - C_v e^{-r(T-t)}N(d_2) \tag{3.13}$$

where

T = the expiration date of the option

$N(d)$ = the cumulative standard normal distribution function

$$d_1 = [ln(\frac{P(t)}{C_v}) + (r - \delta_P + \frac{1}{2}\sigma_P^2)(T-t)]/\sigma_P\sqrt{T-t} \tag{3.14}$$

$$d_2 = d_1 - \sigma_P\sqrt{T-t} \tag{3.15}$$

Note that this result is independent of future price expectations. It relies, instead, on the parameters, σ_P and δ_P. While these values are not directly observable, they are embodied in the price histories in the spot and futures markets.[17] So, estimates of sufficient reliability may be obtained through time series analysis. The implication is that investors who may have totally divergent views of the future direction of prices, but who do agree on the estimates of σ_P and δ_P, will agree on the value of the contingent claim. Likewise, these investors will agree on the value of the real asset when we express it in terms of such contingent claims.

Note also that the variable production cost, C_v, is equivalent to the exercise price of the option. In effect, for each time period the operator holds the option to pay the variable cost and run the plant or to shut down and avoid a loss. The value of a plant is embodied in this array of sequential options, each of which differs from the others only in its maturity date. These options are acquired upon completion of the plant. Consequently, looking out from t=0, its value may be expressed as the sum over time of all such operating options

$$V(0) = \int_0^T F_i(P, t_i; T_i)\, dt$$

While this integral expression may not be directly evaluated, a numerical solution has been developed by Meehan using the technique proposed by Brennan & Schwartz [9]. First, the PDE, equation 3.9 is expressed in dimensionless form using the following change of variables[18]:

$$F(P, \tau) = C_v \cdot D(X, \tau) \tag{3.16}$$

where

$$\begin{aligned} X &= ln\left(\frac{P}{C_v}\right) \\ \tau &= T - t \end{aligned}$$

[17]Refer to McDonald & Siegal [25] for an explanation of how the convenience yield, δ, may be derived from the relationship between the spot price and the futures price.

[18]This change of variables involves, first, redefining the numeraire by normalizing P with respect to C_v, then employing the log transform.

Differentiating yields

$$\begin{aligned} F_P &= \frac{C_v}{P} D_X \\ F_{PP} &= \frac{C_v}{P^2}(D_{XX} - D_X) \\ F_t &= -F_T = -C_v \cdot D_T \end{aligned}$$

Substituting into the PDE yields

$$0 = \frac{1}{2}\sigma_P^2 D_{XX} + (r - \delta_P - \frac{1}{2}\sigma_P^2)D_X - D_T - rD \tag{3.17}$$

Subject to

$$\begin{aligned} D(X, 0) &= \max[0, (e^X - 1)] \\ \lim_{X \to -\infty} D(X, \tau) &= 0 \\ \lim_{X \to \infty} D_X(X, \tau) &= e^X e^{-\delta_P T} \end{aligned}$$

A solution is obtained using the finite difference method. The particular technique employed is the forward time-central space method of discretization which makes use of the following finite difference approximations to the partial derivatives:

$$\begin{aligned} D_X &= \frac{D_{i+1,j} - D_{i-1,j}}{2(\Delta X)} \\ D_{XX} &= \frac{D_{i+1,j} - 2D_{i,j} + D_{i-1,j}}{(\Delta X)^2} \\ D_t &= \frac{D_{i,j+1} - D_{i,j}}{\Delta t} \end{aligned}$$

Substituting these approximations into the PDE yields the following discretized equation:

$$D_{i,j+1} = a \cdot D_{i+1,j} + b \cdot D_{i,j} + c \cdot D_{i-1,j} \tag{3.18}$$

where

$$\begin{aligned}
a &= \frac{1}{2}R[\sigma_P^2 + h(r - \delta - \frac{1}{2}\sigma_P^2)] \\
b &= 1 - R\sigma^2 - m \cdot r \\
c &= \frac{1}{2}R[\sigma_P^2 - h(r - \delta - \frac{1}{2}\sigma_P^2)] \\
m &= \Delta t \qquad\qquad h = \Delta X \qquad\qquad R = \frac{m}{h^2}
\end{aligned}$$

It has been shown [2] [14] that this numerical algorithm is consistent, i.e., it converges to the correct solution, and that its stability is assured when

$$h < \sigma_P^2 |r - \delta - \frac{1}{2}\sigma_P^2| \tag{3.19}$$

and

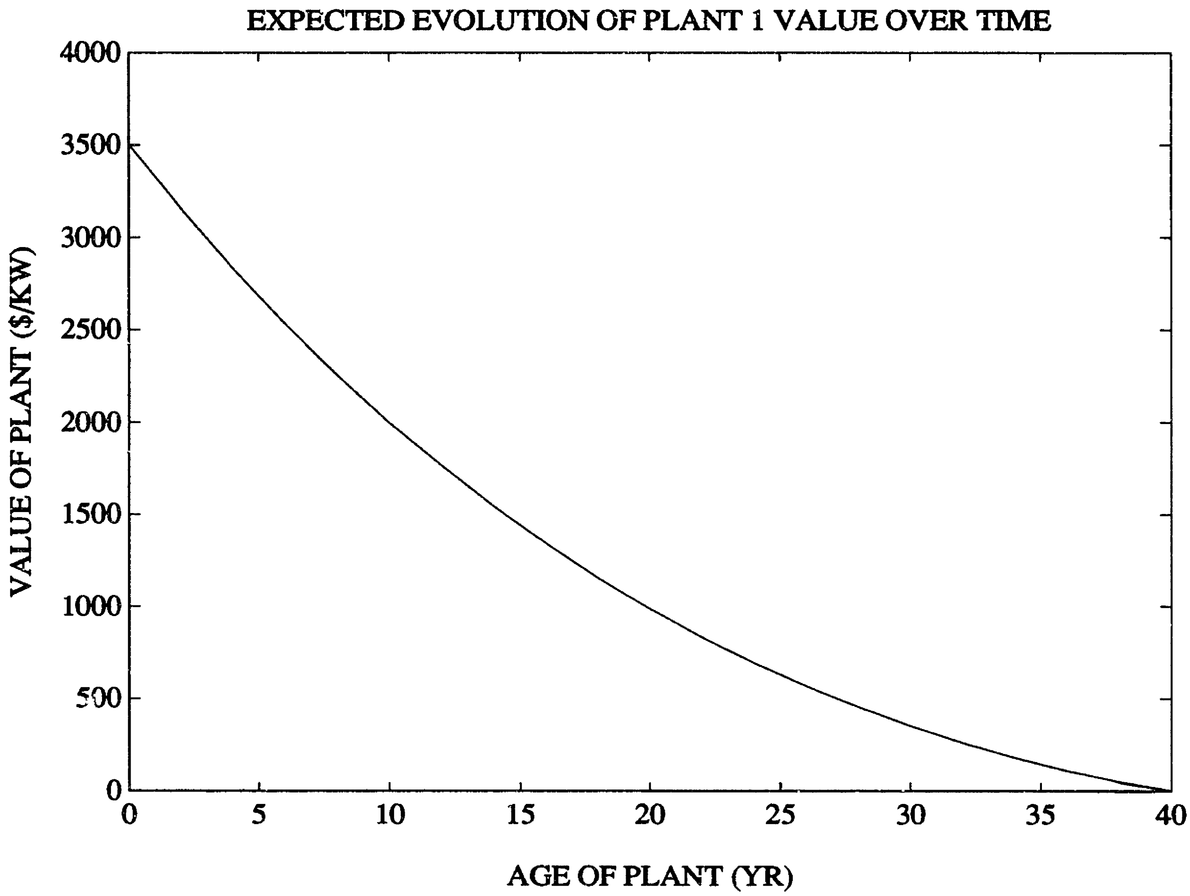
$$R < (1 - r)\sigma_P^2 \tag{3.20}$$

To obtain the solution to this part of the problem, one employs the backward dynamic programming approach. First, it is necessary to define the initial condition in terms of the schedule of all possible price outcomes at t=T, i.e., from P=0 to P equal to a number sufficiently large to approximate infinity. This schedule becomes the initial value condition. Then, the problem is solved by marching in time backward from t=T to t=0. Once the solution converges, the algorithm has computed the value of each individual operating option for each possible output price over the entire time domain. From here, we integrate numerically over time, in simple fashion, to determine the value of the asset as the summation of the operating options.

$$V_0 = \frac{\Delta t}{2} \cdot \sum_{\tau=1}^{T} \{F(P, \tau) + F(P, \tau - 1)\} \qquad \text{for each P}$$

The end result, at t=0, is a schedule of values, each one of which defines the market value of a newly completed plant as a function of the price of the output. This schedule then becomes the input to the second phase of the problem. Figure 3-1 offers an illustration of the expected evolution of the plant's value over its operating

Figure 3-1:



life, given the current output price, P(0), and the current level of volatility, assumed, in this case, to be $\sigma_P = .20$.

3.4 The Option to Invest or Delay

While the above algorithm serves to value a plant once it has been built, our objective is to determine the value of a program to invest in the construction of a plant. The difference between the two is not simply the amount of capital to be invested. As explained earlier, the fact is that time is required to complete the project and the investor has the option to delay or abandon in response to changing market conditions. As Majd & Pindyck demonstrate, the 'time to build' is an important determinant of valuation in the presence of uncertainty. Its effect on the value of a project is over and above the simple cost of funds, or AFUDC, which is all that is accounted for in a DCF analysis. Rather, it has to do with the fact that market uncertainty makes the project risky, and that during construction capital is invested irreversibly and is not productive until the completion of the project.

So, given the assumptions outlined earlier, we seek to determine the market value of an entire investment program as a function of the stochastic evolution of the value of a completed plant. Majd & Pindyck structure this as an optimal control problem. In effect, the investment program is modelled as a compound option. In each time period the investor holds the option to invest another increment of capital or to defer. The investor observes V, the market value of a completed plant, and K, the total amount of investment remaining to completion, and then chooses the optimal level of incremental investment, I. At any point in time the value of the project is defined by the optimal path of the remaining investment decisions to completion. Since each such decision is an option, or contingent claim, on the plant, we can determine each one's value by employing Merton's dynamic portfolio hedging strategy, as was performed earlier.

Following Majd & Pindyck, the contingent claim is expressed as G(V,K). The control variable defining the optimization is the rate of investment, I. The solution to the problem entails determining the control rule, $I^*(V,K)$, which optimizes the value of the investment program. We impose the constraint, $0 \leq I^*(V,K) \leq \kappa$, where κ is the maximum rate of investment per period. Majd & Pindyck point out that, under the simplifying assumptions that there are no additional costs associated with delaying investment or varying the rate of investment apart from the interest rate, the optimal level of investment will either be 0 or κ in each period.

As before, suppose that a derivative security exists whose value is solely a function of a completed plant, V, and time, t. That is,

$$Y = F(V,t)$$

The value of this security evolves stochastically as [19]

$$\frac{dY}{Y} = (\mu_Y - \delta_Y)\,dt + \sigma_Y\,dz_Y$$

To apply Itô's Lemma to the contingent claim, G(V,K), we define the change of variables

$$\begin{aligned} F(V,t) &= G(V,K) \\ \frac{dK}{dt} &= -\kappa \\ F_t &= -\kappa G_K \end{aligned}$$

Applying Merton's technique exactly as before yields the PDE

$$0 = \frac{1}{2}\sigma_V^2 V^2 G_{VV} + (r - \delta_V)VG_V - \kappa G_K - rG - \kappa \tag{3.21}$$

In addition, in the event that κ=0, the PDE is expressed as

$$0 = \frac{1}{2}\sigma_V^2 V^2 g_{VV} + (r - \delta_V)Vg_V - rg \tag{3.22}$$

The boundary conditions are:

$$G(V,0) = V(0) \tag{3.23}$$

$$\lim_{V\to\infty} G_V(V,K) = e^{\frac{-K}{\kappa}\delta_V} \tag{3.24}$$

$$g(0,K) = 0 \tag{3.25}$$

$$g(V^*,K) = G(V^*,K) \tag{3.26}$$

$$g_V(V^*,K) = G_V(V^*,K) \tag{3.27}$$

Where V^* denotes the critical value, or free boundary condition, above which it is optimal to invest in the period and below which it is optimal to not invest. The

[19]In this case $\delta_Y = \kappa$ because κ is, in effect, the interim payout on the derivative security.

boundary condition given in Equation 3.23 describes the initial condition and is interpreted as meaning that when construction is completed, the value of the investment program is equal to the market value of a completed plant. Equation 3.24 describes the fact that, as V becomes very large, the rate of change of G relative to V is constant. [20] Equation 3.25 means that when the value of the project goes to zero, so does the value of the investment program. Lastly, Equations 3.26 and 3.27 mean that the function describing the value of the investment program is continuous and differentiable from both above and below at V^*, the critical value.

Majd & Pindyck show that the analytic solution to equation 3.22 is

$$g(V) = A \cdot V^{\alpha} \tag{3.28}$$

where

$$\alpha = \{-(r - \delta_V - \frac{1}{2}\sigma_V^2) + [(r - \delta_V - \frac{1}{2}\sigma_V^2)^2 + 2r\sigma_V^2]^{\frac{1}{2}}\}/\sigma^2$$

and the coefficient, A, is determined jointly with the solution to equation 3.21 which must be solved numerically. Combining the boundary conditions 3.26 and 3.27, using equation 3.22, provides an additional relation defining the free boundary condition.

$$G(V^*, K) = \frac{V^*}{\alpha} G_V(V^*, K) \tag{3.29}$$

We point out here that, in the event that we cannot deduce σ_V from the market price dynamics of the asset[21], we can, nonetheless, derive it from P(t), F(P,t) and σ_P, all of which are known. The relationship between σ_P and σ_V is identical to that which exists between the volatility of a stock option and the volatility of the underlying stock. Cox & Rubinstein [11] derive this relationship, showing that it is

[20]Majd & Pindyck explain that while the relative rate of change is constant, it is always less than 1, meaning that the value of the investment program increases less rapidly than does the value of a completed project. This result derives from the fact that the time to build imposes an opportunity cost on the investment program.

[21]It may be that an efficient, liquid market for the asset does not exist. We would expect this to be the case relative to power plants.

linear. Adapting their solution to our problem, we have

$$\sigma_V(t) = \Omega(t) \cdot \sigma_P(t) \tag{3.30}$$

where

$$\begin{aligned}
\Omega(t) &= \frac{P(t)}{F(P,t)} \cdot N(d_1) = \text{elasticity of the call option} \\
N(d_1) &= \text{the cumulative standard normal distribution function} \\
d_1 &= \text{as defined in Equation 3.14}
\end{aligned}$$

While we see that the relationship is not constant over time, it is adequate for our purposes to assume that it is constant and, furthermore, that $\sigma_V = \sigma_P$, at all times.

The numerical solution to equation 3.21 is derived following Majd & Pindyck who utilize the forward time-central space technique proposed by Brennan & Schwartz. In order to transform the PDE, equation 3.9, into constant coefficient form, we employ the following change of variables:

$$G(V, K) = e^{-\frac{rK}{\kappa}} H(X, K) \tag{3.31}$$

where

$$X = ln(V)$$

Differentiating yields

$$\begin{aligned}
G_V &= e^{-\frac{rK}{\kappa}} \frac{1}{V} H_X \\
G_{VV} &= e^{-\frac{rK}{\kappa}} \frac{1}{V^2} (H_{XX} - H_X) \\
G_K &= e^{-\frac{rK}{\kappa}} H_K - \left(\frac{r}{\kappa}\right) e^{-\frac{rK}{\kappa}} H
\end{aligned}$$

Substituting into the PDE yields

$$0 = \frac{1}{2}\sigma_V^2 H_{XX} + (r - \delta_V - \frac{1}{2}\sigma_V^2) H_X - \kappa H_K - \kappa e^{\frac{rK}{\kappa}} \tag{3.32}$$

subject to

$$
\begin{aligned}
H(X,0) &= e^X && (3.33)\\
\lim_{X\to\infty}[e^{-X}e^{\frac{-rK}{\kappa}}H_X] &= e^{\frac{-K}{\kappa}}\delta_V && (3.34)\\
H(X^*,K) &= H_X(X^*,K)/\alpha && (3.35)
\end{aligned}
$$

Substituting the same finite difference approximations as were used earlier yields the following discretized equation:

$$
H_{i,j+1} = u * H_{i+1,j} + v * H_{i,j} + w * H_{i-1,j} - z_j \tag{3.36}
$$

where

$$
\begin{aligned}
u &= \frac{R}{2\kappa}[\sigma_V^2 + h(r - \delta_V - \frac{1}{2}\sigma_V^2)]\\
v &= 1 - \frac{R}{\kappa}\sigma_V^2\\
w &= \frac{R}{2\kappa}[\sigma_V^2 - h(r - \delta_V - \frac{1}{2}\sigma_V^2)]\\
z &= m \cdot e^{\frac{r}{\kappa}\sum_0^j m}\\
m &= \Delta K \qquad\qquad h = \Delta X \qquad\qquad R = \frac{m}{h^2}
\end{aligned}
$$

Since the explicit finite differencing scheme employed here is the same as was used in the first algorithm, the stability constraints are, likewise, identical to those given earlier:

$$
h < \sigma_V^2|r - \delta_V - \frac{1}{2}\sigma_V^2| \tag{3.37}
$$

and

$$
R < (1-r)\sigma_V^2 \tag{3.38}
$$

As before, this part of the problem also employs the backward dynamic programming method. The output from the first part becomes the initial boundary condition. As the algorithm proceeds backward in time the free boundary is defined at each time step. As was explained, this free boundary represents the schedule of critical values[22] above which it is optimal to continue to invest and below which it is optimal to defer. Once the solution converges, the result is a schedule of values which defines the worth of the investment program as a function of the current market value of the asset and the construction lead-time.

To illustrate the point, we will now apply the preceding analysis to a stylized example in which three power plant proposals having different cost characteristics are compared. In this example we consider the valuation of a unit of capacity (1 KW) of each of the plants. We ignore taxes, inflation, plant availability, capacity factor and all other sources of uncertainty, save that of output price, in order to focus on the central issue which concerns the valuation of risky projects in the presence of real operating options.[23]

The essential data are[24]:

ITEM	Plant 1	Plant 2	Plant 3
Overnight Capital Cost[25]($/KW)	1,000	2,280	2,490
Variable Operating Cost (mills/KWh)	16.1	4.6	4.6
Plant Operating Life (Yr)	40	40	40
Construction Lead-Time (Yr)	6	6	3
Levelized Busbar Cost (mills/KWh)	22.5	22.5	22.5
Cost of Capital (%)	6	6	6

Note, first, that the cost figures have been contrived such that the three plants have identical levelized busbar costs, that is, according to traditional DCF analysis which implies forecasting with certainty. Comparing Plant 1 to Plant 2, however, note

[22]Mathematically we may express the critical value, V_c, as $V_c = f(K, \sigma_V)$. That is, the critical value is a function of capital remaining to invest and asset volatility; furthermore, it is increasing in both.

[23]In this analysis we consider the capital cost, which is relevant to the option to delay investment, and the variable operating cost, which is relevant to the option to shut down temporarily. Were we to consider the option to abandon, then the fixed operating costs as well as the salvage value would be relevant.

[24]We also set the parameters $r = .03$, $\delta_P = .01$ and $\delta_V = .05$

[25]Overnight capital cost refers to what it would cost if the plant were built instantaneously, i.e., no inflation and no interest charges imposed.

that the former has a lower capital cost and a higher variable operating cost but that lead-time is invariant between the two. A comparative valuation will allow us to study the effect of capital intensity on value in isolation from other factors. With respect to capital intensity alone, this example would reflect the difference between, for instance, a low-capital cost gas-fired combustion turbine technology and a high-capital cost nuclear power technology.

On the other hand, Plant 3 has a shorter construction lead-time than Plant 2 but the capital intensity is invariant between the two. A comparative valuation will allow us to study the effect of lead-time on value in isolation from other factors. With respect to lead-time alone, this example would illustrate the difference between, for instance, a large LWR and the smaller MGR.

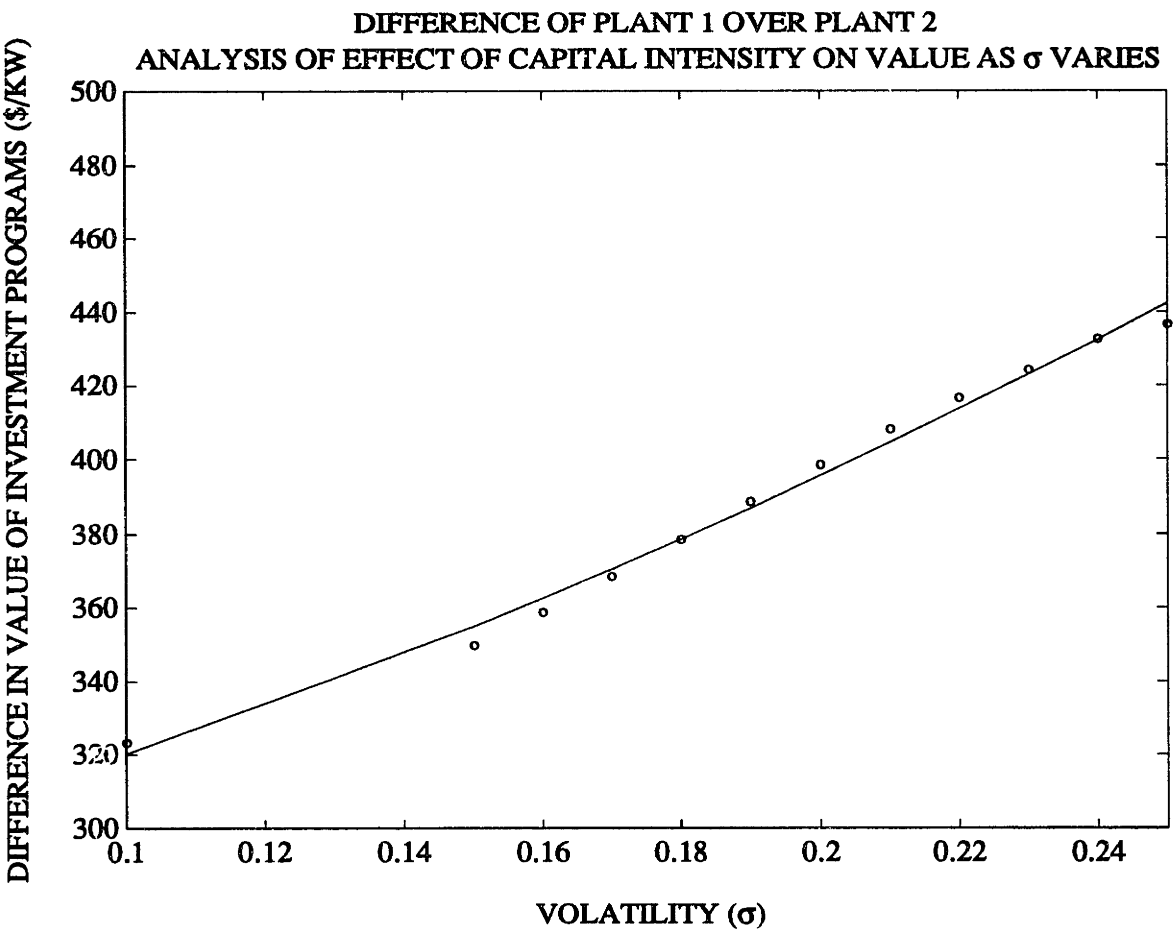
Now, judging solely on the basis of the foregoing static cost projections, one would presumably be indifferent to the three technologies. (By construction, NPV ≈ 0 for each of the programs.) However, when one performs the analysis within the CCA framework the result is quite different. We begin by comparing Plant 1 to Plant 2. We evaluate the two proposals using the options model[26] developed earlier and we allow the volatility, σ, to vary from 0.10 to 0.25. The results are illustrated in Figure 3-2.[27] When we account for market uncertainty in terms of the output price volatility, we find that Plant 1 dominates Plant 2 insofar as its ex ante investment value is concerned. That is, the rational investor will value the program to invest in the construction of Plant 1 more highly than that of Plant 2. We find, furthermore, that the relative valuation difference is an increasing function of σ. In other words, market uncertainty disadvantages the more capital-intensive project.

In this case the difference of about \$440/KW at the upper end of the volatility range represents roughly 30% of the value of Plant 2. In theory, we would expect the values of the two programs to converge as $\sigma \to 0$. In practice, we do observe this convergence to a certain extent, as shown in Figure 3-2. However, the accuracy of

[26]The Fortran program of the model is found in Appendix B.

[27]We point out that because the analysis is based on fictitious data, the absolute values of the results in \$/KW are meaningless. It is the relative valuation of the programs which is of importance and for which the analysis is valid within the context of the model and its assumptions.

Figure 3-2:



the numerical approximation degrades for $\sigma < .10$ such that no conclusions may be confidently drawn from the results when σ is very small. Fortunately, as a practical matter, we are not interested in the case of small σ. In actual financial and commodity markets, generally speaking, we observe volatilities in the neighborhood of $\sigma = .20$.[28] This being the case, we will restrict our analysis to the range, $.15 \leq \sigma \leq .25$.

In the context of CCA, we may think of the greater value of Plant 1 as deriving

[28]In general, we would expect that the dynamics of electricity price are be driven by fuel price volatility, and by changes in generation and transmission efficiencies which are a function of system-wide demand. In addition, there is the capacity component of price which does not exhibit volatility in the Wiener process sense but does contribute a jump process effect to the total dynamics. The jump occurs when there is a change in the marginal plant. This produces a sudden jump in the marginal cost of capacity. The character of the overall price volatility will be system-dependent, and its magnitude will strongly influenced by the covariance between demand and price.See Merton [30] and Cox & Ross [10] for treatments of the jump process and its impact on option pricing.

from two sources.[29] First, the shut-down option is more valuable to Plant 1 due to its higher variable cost. Second, its lower capital cost translates into less valuable options which are exercised during the construction period. In other words, since an option is worth more when in hand than when exercised, the lower capital cost of Plant 1 means less value foregone during construction. Also, since option value is an increasing function of σ, the difference in the value of the two programs widens increasing market uncertainty.

It is also instructive to study the relationship between the value of the investment program and its critical value. Recall that when the value of the project is above the critical value it is optimal to invest; when below it, it is optimal to delay. This relationship for Plant 1 and Plant 2 is shown in Figures 3-3 and 3-4, respectively. We may interpret these graphs as a further illustration of the greater relative riskiness of Plant 2.

The conclusion is that market uncertainty penalizes more severely those projects which are more highly capital intensive or have greater fixed costs as opposed to those which have low capital costs and high variable costs. This result is not surprising and, in fact, it validates investor behavior in recent years in response to higher levels of uncertainty in the market for power. This behavior has been characterized by a general aversion toward the more capital-intensive power technologies and a preference for smaller projects requiring less capital, in spite of the supposedly greater economies of scale and lower busbar costs of the former.

[29]Recall that investment program is composed of two compound options: the operating option which represents cash inflow and which grows positively with σ; and the capital expenditure option which represents cash outflow and which grows negatively with σ.

Figure 3-3:

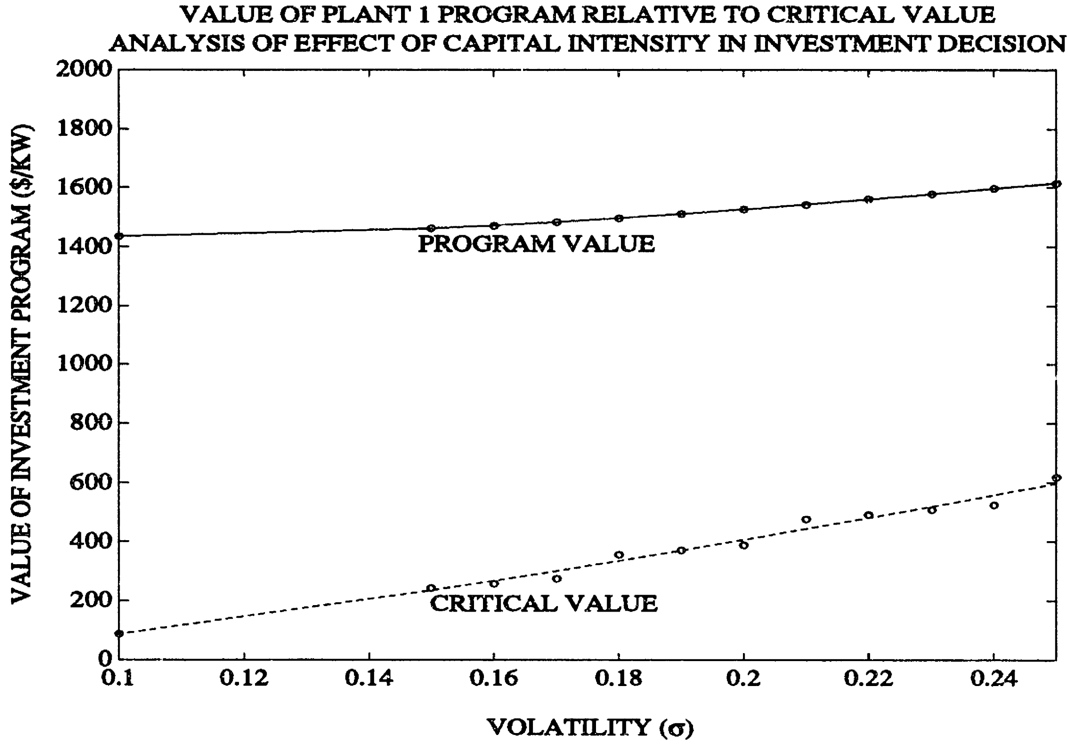
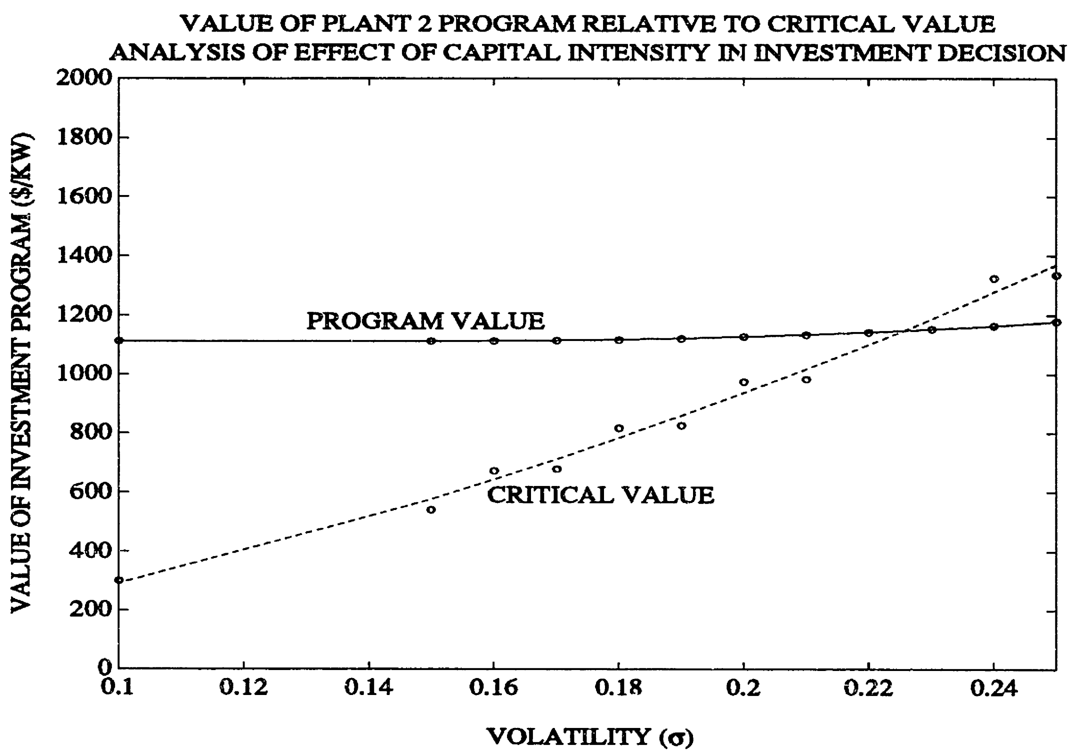


Figure 3-4:



Next, we perform a lead-time sensitivity analysis on Plant 2. We vary the construction time and adjust the overnight capital cost accordingly, such that the levelized busbar cost and program NPV remain unchanged. The relevant data are:

LEAD-TIME SENSITIVITY ON Plant 2					
Overnight Capital Cost ($/KW)	2,570	2,420	2,150	2,025	1,910
Construction Lead-Time[30] (Yr)	2	4	8	10	12

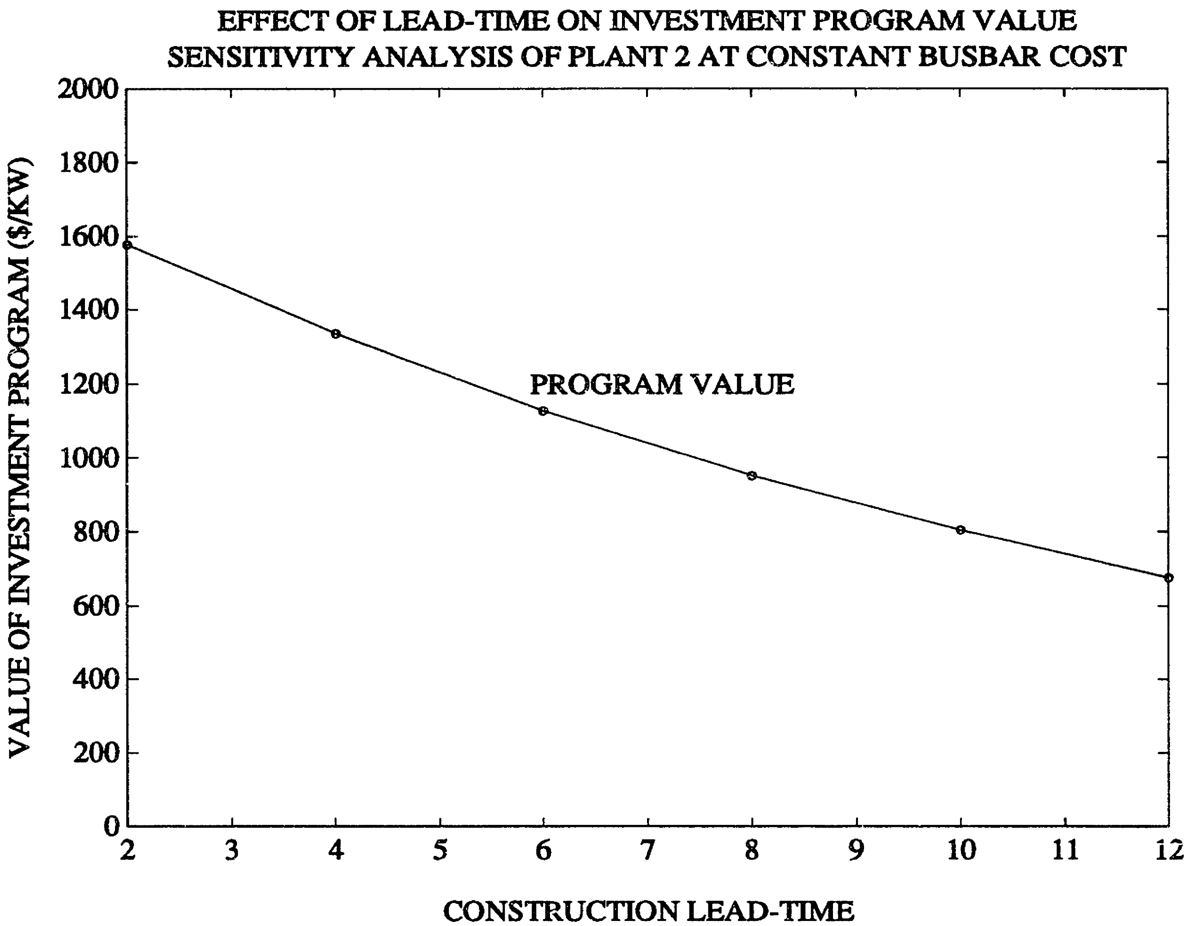
The question is, what will CCA reveal to us, that DCF analysis does not, concerning the value or cost of lead-time differential. Or, to pose the question differently, at what incremental or decremental cost would the investor be indifferent to the various proposals. Referring to Figure 3-5, what we find is that a change in lead-time is worth more than would be indicated by DCF analysis. In this analysis we hold volatility constant at $\sigma = .20$ and show the differential values of the investment program relative to the base case of 6-year lead-time. Again, this result should not be surprising. Longer lead-time projects are riskier, as investors have painfully realized in recent years. Through the use of CCA we now have a rigorous method for determining just how much more it is worth paying for shortening the lead-time of a project.

3.5 The Option to Expand

We now consider the value of flexibility in capacity expansion as it is embodied in a modular technology. We are interested in determining the incremental value that such flexibility confers to an investment program. While there are a variety of ways to structure this type of option, or a series of such options, we choose a relatively simple framework in which there are two sequential embedded options in the investment program. That is, the program engages the investor to construct one unit of capacity and, upon its completion, he acquires the right, but not the obligation, to engage in the construction of a second unit. The option on the third unit is earned in similar fashion. The option may be defined as of the European type, requiring exercise or

[30] The lead-time forecast is based on the assumption of no delay.

Figure 3-5:



forfeiture at a given point in time, or of the American type, allowing exercise at any point in time up to the expiration date. We will consider both but employ the latter.

The expansion option is readily incorporated into the framework of the algorithm developed above by simply revising the initial boundary values of the time-to-build phase of the model. Recall that according to Majd & Pindyck the initial condition is the value of a completed plant, V. After the change of variable, $X = ln\, V$, this condition is expressed as the initial boundary value of the algorithm according to Equation 3.33:

$$H(X, 0) = e^X$$

Now, when there is one embedded option to construct a second plant this boundary condition is expressed as the sum of the first plant, which is completed at that point, plus the value of the option to engage in the construction of the second plant. The value of this option is simply H(X, K), the value of a stand-alone investment program to build one plant which we analyzed in Section 3.4.

If the option were of the European type, requiring exercise or forfeiture upon completion of the first plant, its value would depend upon the relation between the value of a completed plant, V, and the critical value, V^*. Recalling that it is optimal to invest only when $V \geq V^*$, we define the value of the option, $H_1(X, K)$, as

$$\begin{aligned} H_1(X, K) &= H(X,K) && \text{for } V \geq V^* \\ H_1(H, K) &= 0 && \text{for } V < V^* \end{aligned}$$

However, we choose to define the option as a perpetual, or non-expiring, American option in which case its value is simply that of a stand-alone investment program:[31]

$$H_1(X, K) = H(X, K) \qquad \text{for all V}$$

We proceed to add this value to the original boundary condition

$$H(X, 0) = e^X$$

for all values of X along the boundary, K=0, of the mesh, and then we recompute the value of the investment program using the Majd & Pindyck algorithm exactly as we did before. The result is the value of a program to build one plant and earn the option to build a second one at any time subsequent to the completion of the first.

[31]Note that one of the initial assumptions in the Majd & Pindyck model is that there is no expiration to the opportunity to engage in the investment program.

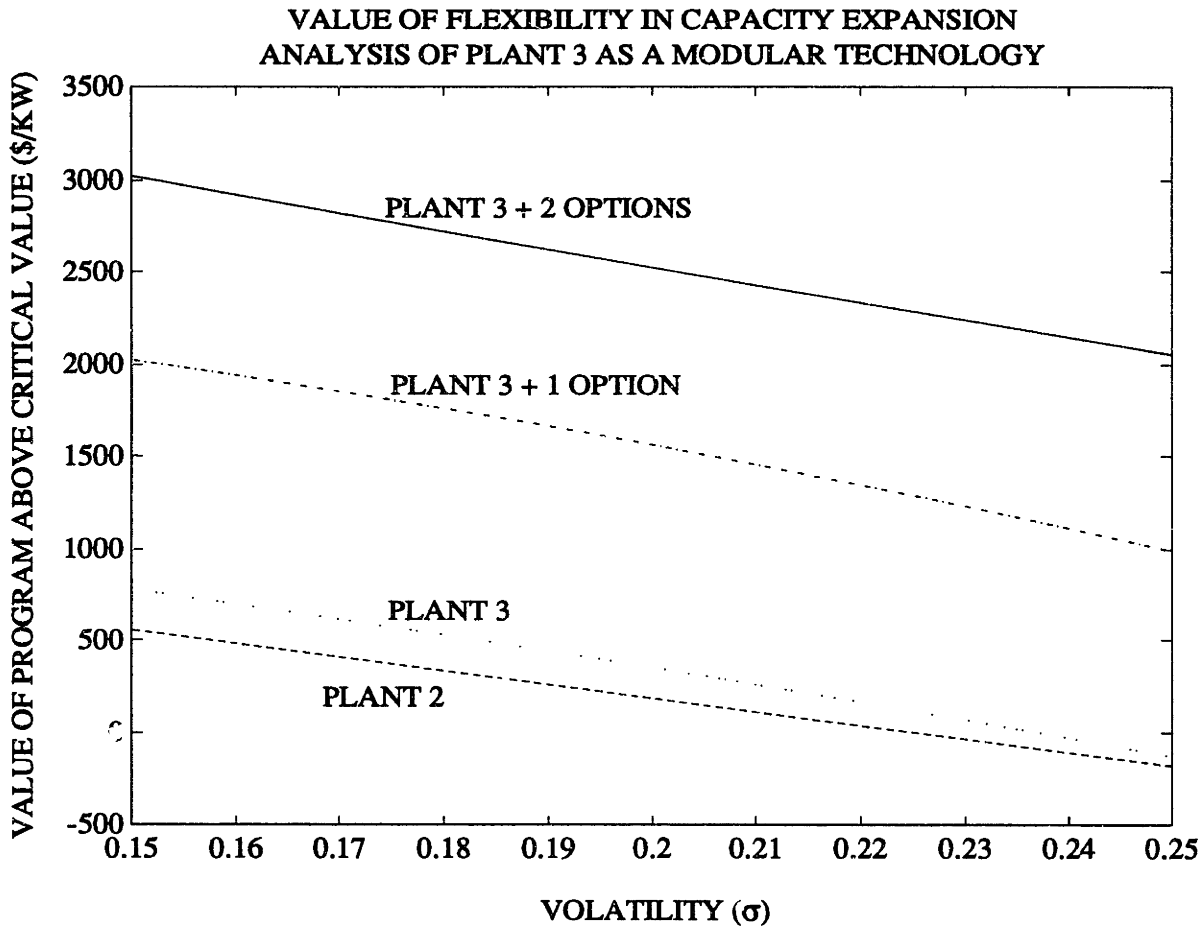
It should be obvious that we are now able to construct and evaluate an investment program with any number of concurrent and/or sequential embedded options simply by revising the initial boundary conditions of the problem as needed.

To demonstrate the value of modularity in power plant capacity expansion, we again compare Plant 2 to Plant 3 but this time we formulate the problem somewhat differently. We suppose that a firm forecasts a positive, yet uncertain, growth in demand and faces the alternative of investing in either a large, long lead-time technology (Plant 2) which offers economies of scale, or a modular technology (Plant 3) which is more costly to build but which offers short lead-time and flexibility in capacity expansion. Note that this differs from the earlier example in which we compared two technologies on the basis of lead-time alone. The critical difference is that, presently, instead of independent capacity additions, the modular technology implies that the siting costs and common facilities are all paid at the outset thereby allowing individual units of capacity to be added on a timely basis. In the context of CCA these up-front payments constitute the purchase of the expansion options. Although we have not explicitly included these costs in the analysis, doing so is a trivial matter.

So, the investor faces the decision of choosing between economies of scale and flexibility in expansion, and is in need of some basis upon which to make a rational determination. The application of CCA to this problem is illustrated in Figure 3-6. In this graph we are comparing the incremental value of each investment program above its critical value as a means of measuring the relative valuation per unit of capacity. We see, first, in the bottom two curves, a straightforward comparison between 1 KW of Plant 2 and 1 KW of Plant 3. We note that there is little difference between the two. However, when we account for the value of the expansion options which are embedded in the modular technology which Plant 3 represents, we see a significant increase in the overall value of the investment program per unit of capacity. The example illustrates, moreover, how the value of the modular investment program grows as the number of embedded expansion options increases.

We must point out, however, that the values shown must be taken with caution. On the one hand, we do not account for the expected economies of scale of which

Figure 3-6:



Plant 2 would benefit. Also, because the analysis is performed on the basis of a single KW of capacity, we do not account for the greater value of reserve capacity which would accrue to the system immediately by virtue of a large expansion project. Clearly, the value of having this capacity-in-place must be weighed against the cost of overcapacity.

On the other hand, we show the benefit which accrues to Plant 3 by virtue of its shorter lead-time and its expansion options, yet we do not account for the significantly higher capital expenditures which these features inevitably entail. Nonetheless, this example very clearly illustrates the potential incremental value that a modular technology offers by virtue of allowing flexibility in capacity expansion. Moreover, we again make the point, this source of value is not accounted for, in a rigorous manner, in the classical methods of investment analysis.

Finally, we could even argue that the foregoing analysis may even underestimate the value of flexibility because our model employs a very simplified representation of uncertainty. We assume, for example, a perfect competitive market which implies that no one competitor's actions will influence the market's dynamics. This ignores the fact that in a power market of finite size each increment of capacity expansion, at any one time, is more risky than the previous one because of the growing risk of overcapacity. Likewise, we do not consider the regulatory environment and the risk of cost disallowance in the event of excess new capacity. We conclude, then, that a more elegant representation of market uncertainty, which is beyond the scope of the present study, would not only improve the practical value of the analysis but may very well indicate that, under certain conditions, the value of flexibility is even greater than is shown above.

3.6 The Option to Operate or Shut Down when Variable Cost is Uncertain

Recall that one of the initial assumptions was that the variable production cost is constant and known with certainty. We will now relax this assumption by introducing a stochastic variable cost. This case is of particular interest in the analysis of power plant projects as it allows one to model in more realistic fashion the economics of those technologies for which the variable costs include the more highly price-volatile fossil fuels, namely, oil and gas.

The following analysis borrows from Fischer [13], who first solved the option valuation problem when the exercise price is uncertain, as well as from McDonald & Siegal [25], who applied this model to a single period investment valuation. We begin by assuming the the stochastic behavior of both the output price and the variable cost may be described as diffusion processes. That is, the output price dynamics obey

Equation 3.1 while the variable production cost follows a similar stochastic process

$$dC = \alpha_C C\, dt + \sigma_C C\, dz_C \tag{3.39}$$

where

$$\begin{aligned}
\alpha_C &= \mu_x - \delta_C = \text{expected growth rate of variable cost} \\
\sigma_C &= \text{instantaneous standard deviation of variable cost} \\
\mu_x &= \text{equilibrium return on a financial asset having the same} \\
&\quad\ \text{market covariance as the variable cost} \\
\delta_C &= \text{convenience yield, or opportunity cost, which accrues} \\
&\quad\ \text{to the holder of the commodity in question}
\end{aligned}$$

Following Merton, as before, suppose that a derivative security exists whose value is solely a function of electricity price, P, variable cost, C, and time, t

$$Y = F(P,\, C,\, t)$$

and that the value of this security evolves stochastically as

$$\frac{dY}{Y} = \alpha_Y dt + \sigma_Y dz_Y \tag{3.40}$$

By Itô's Lemma, we have that the price dynamics of Y obey the relation

$$dY \;=\; \frac{1}{2}F_{PP}(dP)^2 + F_{PC}dP\, dC + \frac{1}{2}F_{CC}(dC)^2 + F_P\, dP + F_C\, dC + F_t\, dt$$

Substituting the expression for dP from Equation 3.1 and for dC from Equation 3.39 yields

$$dY \;=\; \{\frac{1}{2}\sigma_P^2 P^2 F_{PP} + \rho_{PC}\sigma_P\sigma_C P\, C\, F_{PC} + \frac{1}{2}\sigma_C^2 C^2 F_{CC}\}\, dt + \sigma_P P\, F_P\, dz_P + \sigma_C C\, F_C\, dz_C$$

where

$$dz_P \cdot dz_C = \rho_{PC}\, dt$$
$$\rho_{PC} = \text{the coefficient of correlation between the price dynamics of P and C}$$

Once again, we employ the dynamic hedging strategy to derive the following PDE:

$$\frac{1}{2}\sigma_P^2 P^2 F_{PP} + \rho_{PC}\sigma_P\sigma_C P\, C\, F_{PC} + \frac{1}{2}\sigma_C^2 C^2 F_{CC} + (r-\delta_P)P\, F_P + (r-\delta_C)C\, F_C - rF + F_t = 0 \quad (3.41)$$

subject to the boundary conditions

$$\begin{aligned} F(P, C, T) &= \max[0, P(T) - C(T)] \\ F(0, C, t) &= 0 \\ F(P, 0, t) &= P \\ \lim_{P\to\infty} F_P(P, C, t) &= e^{-\delta_P(T-t)} \\ \lim_{C\to\infty} F_C(P, C, t) &= 0 \end{aligned}$$

In the case of a single option, the analytic solution to this problem is given by Fischer [13]. In our case, however, we are modelling the operation of a power plant as a compound option and must again resort to a numerical solution. Similar to what was done in Section 3.3, we first employ the following change of variables:[32]

$$X = ln\left(\frac{P}{C_0}\right) \qquad Y = ln\left(\frac{C}{C_0}\right) \qquad \tau = T - t$$

such that

$$F(P, C, t) = C_0 \cdot G(X, Y, \tau)$$

[32]We normalize with respect to C_0, the variable operating cost at t=0.

and

$$
\begin{aligned}
F_P &= \frac{C_0}{P} G_X & F_{PP} &= \frac{C_0}{P^2}(G_{XX} - G_X) \\
F_C &= \frac{C_0}{C} G_Y & F_{CC} &= \frac{C_0}{C^2}(G_{YY} - G_Y) \\
F_t &= -C_0 \cdot G_\mathcal{T} & F_{PC} &= \frac{C_0}{PC} G_{XY}
\end{aligned}
$$

The PDE, tranformed into constant coefficient form, is

$$
\frac{1}{2}\sigma_P^2 G_{XX} + \rho_{PC}\sigma_P\sigma_C G_{XY} + \frac{1}{2}\sigma_C^2 G_{YY} + (r - \delta_P - \frac{1}{2}\sigma_P^2) G_X + (r - \delta_C - \frac{1}{2}\sigma_C^2) G_Y - rG - G_\mathcal{T} = 0 \tag{3.42}
$$

subject to

$$
\begin{aligned}
G(X, Y, 0) &= \max[0, e^X - e^Y] \\
\lim_{X \to -\infty} G(X, Y, \tau) &= 0 \\
\lim_{Y \to -\infty} G(X, Y, \tau) &= e^X \\
\lim_{X \to \infty} G_X(X, Y, \tau) &= e^X e^{-\delta_P \mathcal{T}} \\
\lim_{Y \to \infty} G_Y(X, Y, \tau) &= 0
\end{aligned}
$$

To solve the PDE numerically, we employ an alternating direct implicit (ADI) algorithm developed by McKee & Mitchell [26] which applies specifically to equations having a mixed derivative term. First, we express Equation 3.42 as

$$
a \cdot G_{XX} + 2b \cdot G_{XY} + c \cdot G_{YY} + d \cdot G_X + e \cdot G_Y - rG - G_\mathcal{T} = 0 \tag{3.43}
$$

where

$$
\begin{aligned}
a &= \frac{1}{2}\sigma_P^2 \\
2b &= \rho_{PC}\sigma_P\sigma_C \\
c &= \frac{1}{2}\sigma_C^2
\end{aligned}
$$

$$\begin{aligned} d &= r - \delta_P - \frac{1}{2}\sigma_P^2 \\ e &= r - \delta_C - \frac{1}{2}\sigma_C^2 \end{aligned}$$

We also define the discretization operators as

$$\begin{aligned} \delta_X^2 &= G_{i+1,j} - 2G_{i,j} + G_{i-1,j} & \delta_y^2 &= G_{i,j+1} - 2G_{i,j} + G_{i,j-1} \\ H_X &= G_{i+1,j} - G_{i-1,j} & H_Y &= G_{i,j+1} - G_{i,j-1} \\ R &= \frac{\Delta\tau}{\Delta X^2} = \frac{\Delta\tau}{\Delta Y^2} & S_X &= \frac{\Delta\tau}{2\Delta X} \qquad S_Y = \frac{\Delta\tau}{2\Delta Y} \end{aligned}$$

Equation 3.43 becomes

$$\begin{aligned} &\{1 + A + BH_X + CHY + D\delta_X^2 + E\delta_Y^2 + FH_XH_Y + K_1\delta_X^2\delta_Y^2\}\, G^{n+1} \\ = \; &\{1 + (A - r\Delta\tau) + (B + dS_X)H_X + (C + eS_Y)HY + (D + aR)\delta_X^2 \\ &+(E + cR)\delta_Y^2 + (F + \frac{1}{2}bR)H_XH_Y + K_2\delta_X^2\delta_Y^2\}\, G^n \end{aligned}$$

Setting

$$A = B = C = F = 0 \qquad K_1 = DE$$

yields

$$\begin{aligned} (1 + D\delta_X^2)(1 + E\delta_Y^2)\, G^{n+1} \;=\; &\{1 - r\Delta\tau + dS_XH_X + eS_YHY + (D + aR)\delta_X^2 \\ &+(E + cR)\delta_Y^2 + \frac{1}{2}bRH_XH_Y + K_2\delta_X^2\delta_Y^2\}\, G^n \end{aligned}$$

To employ ADI we split the equation as follows

$$\begin{aligned} (1 + D\delta_X^2)\, G^{n+\frac{1}{2}} \;=\; &\{1 - r\Delta\tau + dS_XH_X + eS_YHY + (D + aR)\delta_X^2 + cR\delta_Y^2 \\ &+\frac{1}{2}bRH_XH_Y + K_2\delta_X^2\delta_Y^2\}\, G^n \end{aligned}$$

$$(1 + E\delta_Y^2)\, G^{n+1} = G^{n+\frac{1}{2}} + E\delta_Y^2\, G^n$$

We further define

$$D = \frac{1}{f} - \frac{1}{2}aR \qquad E = \frac{1}{f} - \frac{1}{2}cR \qquad K_2 = (\frac{1}{f} + \frac{1}{2}aR)\cdot(\frac{1}{f} + \frac{1}{2}cR)$$

This allows the system of two equations to be expressed in terms of a single parameter, f, as follows

$$\begin{aligned} \{1 + (\frac{1}{f} - \frac{1}{2}cR)\delta_X^2\}\, G^{n+\frac{1}{2}} &= \{1 - r\Delta\tau + (dS_X)H_X + eS_Y)H_Y + (\frac{1}{f} + \frac{1}{2}aR)\delta_X^2 \\ &\quad + cR\delta_Y^2 + \frac{1}{2}bRH_XH_Y + \frac{r}{f}(a+c)\delta_X^2\delta_Y^2\}\, G^n \end{aligned}$$

$$\{1 + (\frac{1}{f} - \frac{1}{2}cR)\delta_Y^2\}\, G^{n+1} = G^{n+\frac{1}{2}} + (\frac{1}{f} - \frac{1}{2}cR)\delta_Y^2\, G^n$$

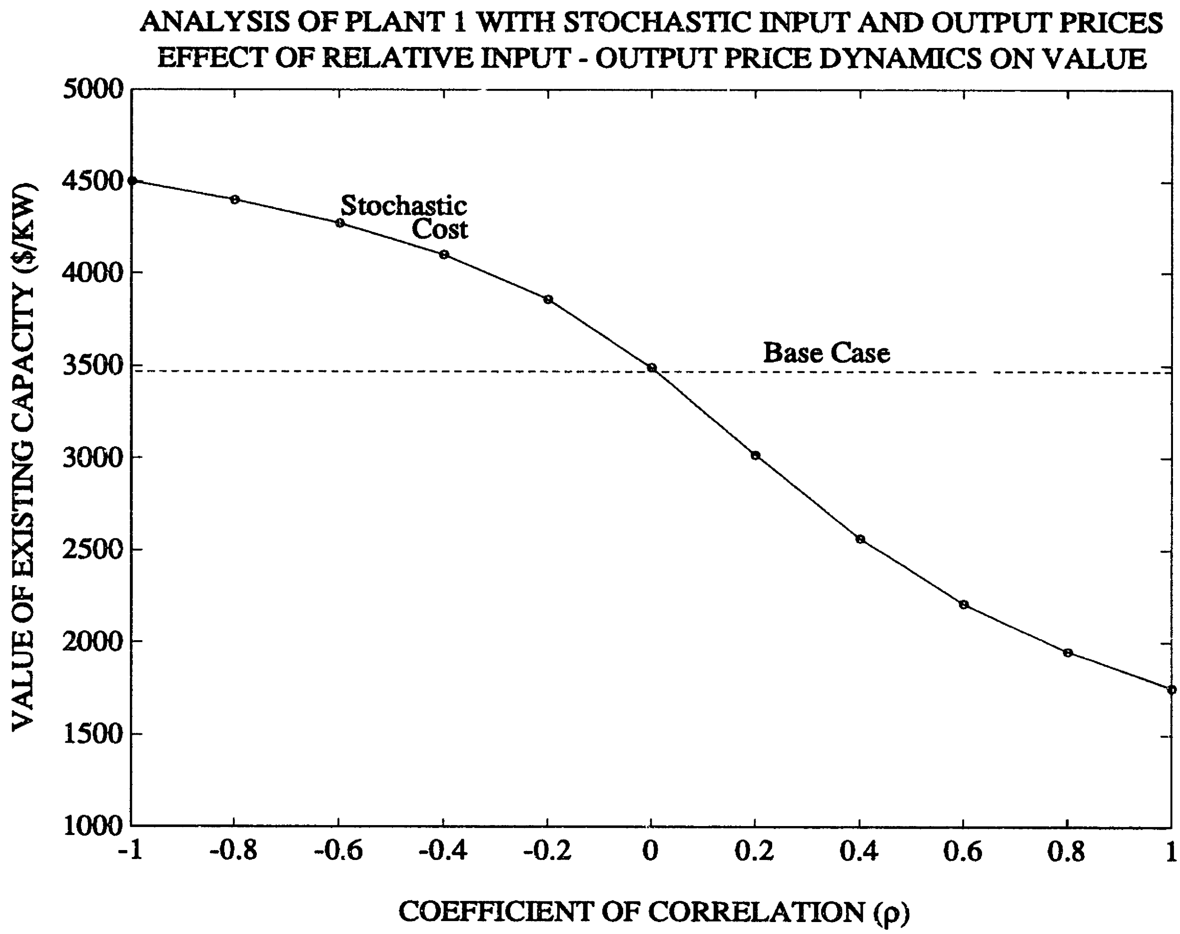
McKee & Mitchell show that the algorithm is second-order accurate in X and Y, first-order accurate in time and unconditionally stable for $f < 0$ or $f \geq 4$. In the present analysis it was found that high values of f accelerated convergence; so a value of $f = 100$ was used.[33]

We construct a straightforward example employing the algorithm to demonstrate how the relative price dynamics of input and output affect valuation. Specifically, we compare the Plant 1 base case, for which the variable operating cost is constant, against an identical Plant 1 which has stochastic variable operating cost. In the former we hold $\sigma_P = .20$; in the latter we hold $\sigma_P = .20$ and $\sigma_C = .20$. In addition, our convenience yields estimates are $\delta_P = .01$ and $\delta_C = .01$. Lastly, we allow ρ_{PC} to vary from -1.0 to 1.0.

Referring to Figure 3-7, we see quite clearly how the market value of an existing plant is influenced by the correlation between the input and output price dynamics. To some these results may seem counterintuitive because the negative price correlation produces greater profit variance which may lead one to conclude that the asset is riskier and worth less than in the case of positive price correlation. For example, in a DCF analysis framework one may wish to apply a higher discount rate to the

[33]The Fortran code for this algorithm is found in Appendix C.

Figure 3-7:



cash flow with greater variance thereby lowering the valuation. Recall, however, that the value of an option is an increasing function of variance. Because there is an option to shut down the plant will never operate at a loss. So, the bottom end of the profit probability distribution is truncated leaving only the positive outcomes whose expected value increases with variance. Consequently, in the CCA framework we arrive at a result which is directly opposite to that of DCF analysis.

It is significant, furthermore, to explain how the system-dependent nature of ρ affects relative valuation. Recall that we earlier described how the marginal price of electricity is, in part, tied to the cost of the fuel which is consumed at the margin in the power system in question. As a rule, we would expect this fuel to be one of the fossil fuels. This being the case, were we to value a "same fuel" plant and an MGR in the context of the system, we would find the "same fuel" plant, with positive correlation, to be at an economic disadvantage relative to the MGR, with zero correlation. This simple example serves to illustrate the value of fuel diversification, a principle which is widely recognized but generally not explicitly quantified.

3.7 The Option to Switch Inputs or Outputs

Finally, we consider the case in which there is an option to choose from between two inputs and the parallel case in which there is an option on either of two outputs. Our analysis follows the work of Stulz [41] who has derived the analytical expression for the single European option on the minimum or maximum of two risky assets. This type of option is of particular relevance to two classes of problems which may arise in power plant investment decisions. First, there is the case of equipping a fossil fuel-fired plant to accommodate fuel switching. This option is valued as one on the minimum of two risky assets. Second, there is the case of a plant which is equipped to produce two outputs, most often electricity and steam, and for which there is some margin of flexibility as to the mix of the final outputs. This option is valued as one on the maximum of two risky assets.

The second one is particularly relevant to cogenerators as well as to the MGR and other advanced, high-temperature technologies. For example, certain conceptual designs of future MGR's incorporate the ability to produce both electricity and high-quality steam for industrial processes. While the steam may be used to produce additional electricity, it may also be used directly in an application requiring that it be at high pressure and high temperature. We may readily imagine that the technology can be configured to allow for a certain degree of switching between the two outputs. We may also envision that the value of the steam will be tied to the value of the product for which it is being used, for example, a petroleum-based chemical or plastic. If the steam is used in secondary oil recovery, then its value would be tied to the price of crude oil itself. Under any of these scenarios, as well as others, that unit of switchable capacity will be valued according to CCA.

Stulz shows that the option on the maximum of two risky assets satisfies the PDE:

$$
\begin{aligned}
0 &= \frac{1}{2}\sigma_{P_1}^2 P_1^2 F_{P_1P_1} + \rho_{P_1P_2}\sigma_{P_1}\sigma_{P_2} P_1 P_2 F_{P_1P_2} + \frac{1}{2}\sigma_{P_2}^2 P_2^2 F_{P_2P_2} \\
&\quad +(r-\delta_{P_1})P_1 F_{P_1} + (r-\delta_{P_2})P_2 F_{P_2} - rF + F_t
\end{aligned}
\tag{3.44}
$$

subject to the boundary conditions

$$
\begin{aligned}
F(P_1, P_2, T) &= \max\{0, \max[P_1(T), P_2(T)] - C_v]\} \\
F(P_1, 0, t) &= F(P_1, t) \\
F(0, P_2, t) &= F(P_2, t) \\
F(0, 0, t) &= 0 \\
\lim_{P_1 \to \infty} F_{P_1}(P_1, P_2, t) &= e^{-\delta_{P_1}(T-t)} \\
\lim_{P_2 \to \infty} F_{P_2}(P_1, P_2, t) &= e^{-\delta_{P_2}(T-t)}
\end{aligned}
$$

where the two state variables are the prices of the two outputs, P_1 and P_2, respectively. In this case, the variable operating cost, C_v, which represents the exercise price, is assumed to be constant.

Note the Equation 3.44 is exactly the same type of PDE as Equation 3.41. Only the boundary conditions have changed. Consequently, we may employ the ADI algorithm developed in Section 3.6 to solve the present compound option valuation problem. We begin by making the following change of variables

$$
X = ln\left(\frac{P_1}{C_v}\right) \qquad Y = ln\left(\frac{P_2}{C_v}\right) \qquad \tau = T - t
$$

to convert the equation to dimensionless, constant-coefficient form, such that

$$
F(P_1, P_2, t) = C_v \cdot G(X, Y, \tau)
$$

and

$$\begin{aligned} F_{P_1} &= \frac{C_v}{P_1} G_X & F_{P_1P_1} &= \frac{C_v}{P_1^2}(G_{XX} - G_X) \\ F_{P_2} &= \frac{C_v}{P_2} G_Y & F_{P_2P_2} &= \frac{C_v}{P_2^2}(G_{YY} - G_Y) \\ F_t &= -C_v \cdot G_T & F_{P_1P_2} &= \frac{C_v}{P_1P_2} G_{XY} \end{aligned}$$

The Transformed PDE is identical to Equation 3.42

$$\begin{aligned} 0 &= \frac{1}{2}\sigma_{P_1}^2 G_{XX} + \rho_{P_1P_2}\sigma_{P_1}\sigma_{P_2} G_{XY} + \frac{1}{2}\sigma_{P_2}^2 G_{YY} \\ &+(r - \delta_{P_1} - \frac{1}{2}\sigma_{P_1}^2) G_X + (r - \delta_{P_2}\frac{1}{2}\sigma_{P_2}^2) G_Y - rG - G_T \end{aligned} \quad (3.45)$$

subject to

$$\begin{aligned} G(X, Y, 0) &= \max\{0, \max[e^X, e^Y] - 1\} \\ \lim_{Y\to-\infty} G(X, Y, \tau) &= G(X, \tau) \\ \lim_{X\to-\infty} G(X, Y, \tau) &= G(Y, \tau) \\ \lim_{X\to\infty} G_X(X, Y, \tau) &= e^X e^{-\delta_{P_1}\tau} \\ \lim_{Y\to\infty} G_Y(X, Y, \tau) &= e^Y e^{-\delta_{P_2}\tau} \end{aligned}$$

We find, however, that it is more efficient to solve directly for the option on the minimum of two assets, and then to determine the value of the option on the maximum of two assets by employing the formula, given by Stulz

$$MX(P_1, P_2, \tau) = F(P_1, \tau) + F(P_2, \tau) - MN(P_1, P_2, \tau)$$

where

$$\begin{aligned} MX(P_1, P_2, \tau) &= \text{option on the maximum of } P_1 \text{ and } P_2 \\ F(P_i, \tau) &= \text{option on asset } P_i \\ MN(P_1, P_2, \tau) &= \text{option on the minimum of } P_1 \text{ and } P_2 \end{aligned}$$

The contingent claim, $MN(P_1, P_2, \tau)$, satisfies Equation 3.44 subject to the following boundary conditions after the change of variables to X and Y:

$$\begin{aligned} G(X, Y, 0) &= \max\{0, \min[e^X, e^Y] - 1\} \\ \lim_{Y \to -\infty} G(X, Y, \tau) &= 0 \\ \lim_{X \to -\infty} G(X, Y, \tau) &= 0 \\ \lim_{X \to \infty} G_X(X, Y, \tau) &= 0 \\ \lim_{Y \to \infty} G_Y(X, Y, \tau) &= 0 \end{aligned}$$

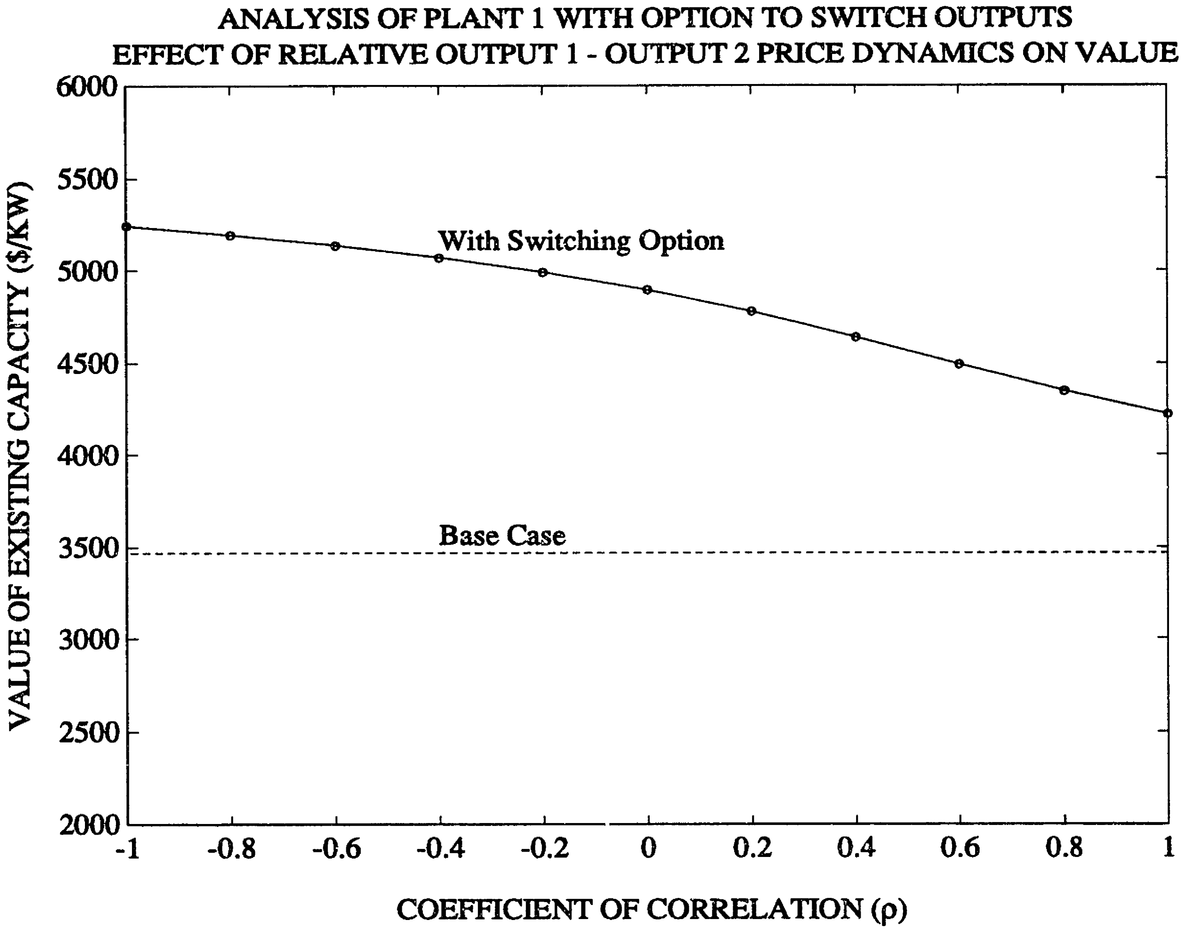
Once the boundary conditions are defined, the solution proceeds exactly as in Section 3.6.[34] To illustrate the value of the switching option we again construct a simple example in which we compare a unit of capacity of the Plant 1 base case against an identical unit of capacity but which has the option to switch outputs. In the former we hold $\sigma_{P_1} = .20$; in the latter we begin with $P_1 = P_2$ and we hold $\sigma_{P_1} = \sigma_{P_2} = .20$. In addition, our convenience yields estimates are $\delta_{P_1} = \delta_{P_2} = .01$. Lastly, we again allow $\rho_{P_1P_2}$ to vary from -1.0 to 1.0.

Referring to Figure 3-8, we see that this option is of significant value. We note also that the value varies inversely with the coefficient of correlation between the two assets in question. The reason for this follows a similar argument to that which was presented relative to the option in Section 3.6. Again, we point out that we have not incorporated into the analysis the additional costs which would invariably be required to equip a plant to exercise switching. Nonetheless, this illustration of the incremental value embodied in the option to switch outputs will, hopefully, motivate decision-makers to think of future power plants as more than just a 'one-product' asset, and to find creative ways of extracting more value from the significant amount of capital invested in these plants.

Briefly, to summarize this chapter, we have proposed a simple model of market uncertainty and demonstrated how CCA allows us to gain a better understanding of

[34]The Fortran code for this problem is found in the Appendix D.

Figure 3-8:



the true economic value of capital investment programs and long-lived assets when real options exist.

Chapter 4

Policy Implications and Recommendations

The principal theme of this study has been that DCF analysis is an inadequate method of valuing capital investment programs and capital assets in an environment characterized by uncertainty and in which there exists any number of options regarding future managerial decisions. Hopefully, the point has been sufficiently illustrated in the previous chapter.[1] The immediate fallout of the analysis is that levelized busbar cost is an inappropriate measure of economic value. Furthermore, recalling that the value of an option is an increasing function of volatility and time to expiration, we realize that the greater is the level of uncertainty and the more distant in time is the planning horizon, then the more inadequate becomes any evaluation based on busbar cost estimates.[2] Lastly, the crux of the issue is that a preoccupation with busbar costs leads us to frame the problem incorrectly while CCA teaches us how to think about the future.

Not only do we recommend that CCA be applied to long-term R&D programs, such as the MGR, but that it be incorporated into the regulatory framework which governs the capacity expansion and technology selection decisions of the electric power

[1] The literature on the application of CCA to issues of corporate finance and strategy is rich and growing. The interested reader is referred to Sanchez [37] and to Mason & Merton [22].

[2] This is because the value of the options being ignored is of greater importance.

industry. The traditional practice of focusing on the levelized busbar cost, or revenue requirement, ignores the value of real options and, consequently, skews the investment decision-making process. While we do recognize that some jurisdictions have adopted various point-scoring systems in an attempt to account for other factors of potential value, we caution that all of these schemes are subject to the bias of individual preferences, be they of the investor or the regulator. In contrast, CCA, as we have shown, leads to a result which is independent of individual preferences.

Much along the same lines, we recommend that the above arguments be extended to the notion of avoided cost. In practice, the avoided cost, as it is employed in planning and financial decision-making, is generally estimated on the basis of static price forecasts, similar to levelized busbar cost estimation. In fact, however, as we have mentioned earlier, the avoided cost is a stochastic variable, tied to marginal fuel costs as well as to factors specific to the power system. Presently, and increasingly, the avoided cost concept is being applied to decisions concerning fuel purchase agreements, conservation programs, etc. Yet, the analyses performed to inform such decisions do not rigorously account for its stochastic nature. Hopefully, the foregoing treatment of CCA is sufficient to demonstrate that it provides the correct framework for addressing this issue.

As we have alluded to earlier, the logic of CCA also has important implications relative to R&D strategy. To begin with, Since R&D projects are characterized by very high levels of uncertainty and distant time horizons, we must recognize that valuations based on DCF analysis have virtually no information content. Furthermore, it is impossible to ascertain that such analyses are or are not biased. CCA overcomes this limitation since its results are independent of individual preferences as well as of expectations of future events.

On the other hand, however, when it is a new or emerging product market or technology at issue, we often cannot apply CCA in the manner as was done in this study[3]. Nonetheless, CCA provides the appropriate framework for learning how to think about R&D. To elaborate on this point, we reiterate an argument made in

[3]Recall that the quantitative analysis was dependent on the ability to observe market prices.

Section 3.6. That is, under DCF analysis high levels of uncertainty increase the sense of riskiness and generally lead one to apply a higher discount rate, or a higher contingency cost item, thereby reducing the NPV of a project. Under CCA, however, the value of an R&D program is an increasing function of uncertainty just as is the case for a call option[4]. This provides the theoretical motivation for high-risk R&D. The reality of the market corroborates our argument insofar as we do observe that high-risk industries are characterized by higher levels ot R&D expenditures. To drive home the point, we cite the CEO of Merck, Dr. P. Roy Vagelos, who recently explained,

> "The odds against getting a compound to market have been cited, for some years now, as 10,000 to 1....In 1975, the year I joined Merck, the chief executive officer was concerned that for some time the company had introduced few important new medicines in the United States, despite having spent approximately $500 million dollars on R&D in the previous 10 years. But he did not cut back. Instead, he increased the R&D budget....The result of the company's persistence - the paradox of the high-risk pharmaceutical business is that the route to success is to invest more - was the introduction of a number of important new products..." [42]

Employing the CCA framework, we can visualize the R&D process as a compound option problem, similar to the methodology developed in the previous chapter. That is, we decide whether or not to invest an increment of capital, some degree of uncertainty is resolved over the time period, we revise the expected value of the technology, we again decide whether or not to invest another increment, and so forth. We should, furthermore, visualize, the R&D effort as a compound option not only in time but also in space. By this we refer to the fact that R&D not only creates options on the

[4]Recall that the upper end of the probability distribution extends out with increasing variance thereby increasing the possibility of a big payoff. At the same time, since the program will never be worth less than zero, the bottom end of the probability distribution is truncated leaving only the positive outcomes whose expected value increases with variance.

technology in question, but also creates options on subsidiary, or spin-off, technologies which appear along the way. In addition, we should understand that these options are a store of value, until they expire, independent of whether or not the technology is ever brought to market. Recognizing the value of these options on the future leads us, finally, to understand that keeping them alive and creating new ones requires a long-term commitment to the R&D effort.

So, insofar as DOE budget policy is concerned, we recommend long-term commitments to programs in place of the current practice of providing funding on the basis of annual budget approvals. As a general rule, we should recognize that technological and market uncertainty are resolved slowly over time. Increasing the level of funding does not often speed the process by and of itself. On the other hand, cutting funding often allows options to expire. Certainly, we should accept the idea that moderate, level funding of R&D on complex technologies is preferable to the strategy that was pursued following the oil price rise of 1979 when projects were hastily ramped up, and then just as hastily abandoned when the price of oil subsequently dropped.

While some may find it difficult to justify dedicating capital to the development of an esoteric nuclear technology when the prices of oil and gas are so low and the public is so adamantly against nuclear power, we argue, first, that similar to an option the value of the R&D is a function of market uncertainty, i.e., price volatility, and is independent of our beliefs as to the future direction of fuel prices; second, the value of the R&D is a function of the options which it creates for the future, regardless of whether or not those options are ever exercised.

Our final recommendation concerns the issue of regulatory accommodation which refers to the degree to which the NRC will recognize that the MGR is different from the existing stock of nuclear power plants. As we saw in Chapter 2, the eventual cost of an MGR plant will vary greatly depending on the degree of accommodation. So, assuming that the MGR does fulfill its promises of inherent safety, etc., we would seek to promote the differentiation of the technology. To accomplish this, we would recommend that the regulation of the MGR be divorced as much as possible from that of the LWR. It would be ideal, for example, to have an entirely separate set of regulations for each of the two technologies and to have independent groups responsible for inspection and enforcement. If the MGR does meet our expectations relative to safety, there is no reason why it should be burdened by the costs of LWR regulation.

Appendix A

Cost Assumptions

ITEM	UNIT COST[1]
Labor (10 hrs/day)	\$30/hr
Labor (around the clock)	\$40/hr
Concrete (nuclear grade)	\$90/yd^3
Concrete (non-nuclear grade)	\$60/yd^3
Reinforcing Steel (nuclear grade)	\$700/ton
Reinforcing Steel (non-nuclear grade)	\$600/ton
Graphite (machined and finished)	\$18/lb*
Reactor Vessel ($2\frac{1}{4}$ Cr-1 Mo, finished)	\$10/lb*
Primary Helium Duct ($2\frac{1}{4}$ Cr-1 Mo, finished)	\$7,000/ft*
Inventory Control Vessel ($2\frac{1}{4}$ Cr-1 Mo, finished)	\$10/lb*
High-Pressure Turbine	\$600/kWe*
Low-Pressure Turbine	\$500/kWe*
Generator	\$100/kWe*
Control Rod	\$200,000/unit*
KLAK	\$200,000/unit*
Fuel	\$20/pebble*
Construction Engineering	\$120/hr
Construction Management	\$150/hr
Operating Personnel	\$100,000/yr
Cooling Tower	\$8 million/unit*

[1] Those items denoted by '*' are based on non-citable estimates each of which was obtained via a telephone survey of one or more manufacturers.

Appendix B

Program #1

```
c     Program #1
c
      real p,vc,life,r,delta,delta2,tol,sig
      real fval(0:150),vcritx,sigma(20)
      real xinv,xinvr,hval(0:150),plant(20),proj(20)
      real ecrit1x,ecrit2x,eval1(0:150)
      real eval2(0:150),proj1(20),proj2(20)
      integer imax,nmax
c
c    INPUT AND OUTPUT FILES
c
c     open(unit=15, file='fval')
c     open(unit=25, file='hval')
      open(unit=26, file='eval1')
      open(unit=27, file='eval2')
c     open(unit=35, file='vcrit')
c     open(unit=36, file='ecrit1')
c     open(unit=37, file='ecrit2')
c     open(unit=45, file='optdata1')
c     open(unit=46, file='optdata2')
      open(unit=47, file='optdata3')
      open(unit=55, file='sigdata1')
c     open(unit=65, file='outdata1')
```

```
c      open(unit=66, file='outdata2')
      open(unit=67, file='outdata3')
c
c     VARIABLE DEFINITIONS
c
c     p = price, or annualized revenue ($)
c     vc = variable cost, or annualized variable operating cost ($)
c     life = planned operating life of plant (years)
c     r = annualized riskless interest rate (%/yr)
c     delta = annualized commodity storage yield (%/yr)
c     delta2 = annualized dividend yield on plant (%/yr)
c     xinv = overnight capital cost of plant ($)
c     xinvr = rate of capital investment in plant construction ($/yr)
c     tol = error tolerance for convergence check
c     jmax = number of grid point along the time axis
c     imax = number of grid point along the price axis
c     nmax = limit on the maximum number of iterations
c
c     sigma = annualized standard deviation of commodity price
c
c     READ IN SIGMA
c
      read (55,*) (sigma(i), i=1,15)
c
c      goto 300
c
c     MAIN PROGRAM
c
c     READ IN PROBLEM DATA - PLANT #1
c
      read (45,*) p,vc,life,r,delta,delta2,xinv,xinvr,tol,
     *     nmax
c
      do 103 i=4,15
        sig=sigma(i)
        print *, sig
```

```
c
c     SET INITIAL CONDITIONS
c
      do 105 k=0,imax
        fval(k)=0.0
        hval(k)=0.0
 105  continue
c
      call operate(p,vc,life,r,delta,sig,imax,nmax,tol,fval,k0)
c
c     CURRENT PLANT VALUE
c
      plant(i)=fval(k0)
c
      call build(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx)
c
c     CURRENT PROJECT VALUE - BASE
c
      proj(i)=hval(k0)

      call expand1(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx,eval1,ecrit1x)
c
c     CURRENT PROJECT VALUE - ONE OPTION
c
      proj1(i)=eval1(k0)

      call expand2(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,eval1,ecrit1x,eval2,ecrit2x)
c
c     CURRENT PROJECT VALUE - TWO OPTIONS
c
      proj2(i)=eval2(k0)

      write (65,104) sigma(i),plant(i),proj(i),vcritx,
```

```
     *        proj1(i),ecrit1x,proj2(i),ecrit2x,k0
 104     format (f4.2,7f9.2,2i4)
c
 103  continue
c
c
c    MAIN PROGRAM
c
c    READ IN PROBLEM DATA - PLANT #2
c
      read (46,*) p,vc,life,r,delta,delta2,xinv,xinvr,tol,nmax
c
      do 203 i=4,15
        sig=sigma(i)
        print *, sig
c
c    SET INITIAL CONDITIONS
c
        do 205 k=0,imax
          fval(k)=0.0
          hval(k)=0.0
 205     continue
c
        call operate(p,vc,life,r,delta,sig,imax,nmax,tol,fval,k0)
c
c    CURRENT PLANT VALUE
c
        plant(i)=fval(k0)
c
        call build(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *        tol,fval,hval,vcritx)
c
c    CURRENT PROJECT VALUE - BASE
c
        proj(i)=hval(k0)
c
```

```
      call expand1(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *        tol,fval,hval,vcritx,eval1,ecrit1x)
c
c     CURRENT PROJECT VALUE - ONE OPTION
c
      proj1(i)=eval1(k0)
c
      call expand2(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *        tol,fval,eval1,ecrit1x,eval2,ecrit2x)
c
c     CURRENT PROJECT VALUE - TWO OPTIONS
c
      proj2(i)=eval2(k0)
c
      write (66,204) sigma(i),plant(i),proj(i),vcritx,
     *        proj1(i),ecrit1x,proj2(i),ecrit2x,k0
 204    format (f4.2,7f9.2,2i4)
c
 203  continue
c
 300  continue
c
c     MAIN PROGRAM
c
c     READ IN PROBLEM DATA - PLANT #3
c
    read (47,*) p,vc,life,r,delta,delta2,xinv,xinvr,tol,nmax
c
    do 303 i=4,15
      sig=sigma(i)
      print *, sig
c
c     SET INITIAL CONDITIONS
c
      do 305 k=0,imax
        fval(k)=0.0
```

```
         hval(k)=0.0
 305     continue
c
      call operate(p,vc,life,r,delta,sig,imax,nmax,tol,fval,k0)
c
c   CURRENT PLANT VALUE
c
      plant(i)=fval(k0)
c
      call build(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx)
c
c   CURRENT PROJECT VALUE - BASE
c
      proj(i)=hval(k0)
c
      call expand1(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx,eval1,ecrit1x)
c
c   CURRENT PROJECT VALUE - ONE OPTION
c
      proj1(i)=eval1(k0)
c
      call expand2(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,eval1,ecrit1x,eval2,ecrit2x)
c
c   CURRENT PROJECT VALUE - TWO OPTIONS
c
      proj2(i)=eval2(k0)
c
      write (67,304) sigma(i),plant(i),proj(i),vcritx,
     *     proj1(i),ecrit1x,proj2(i),ecrit2x,k0
 304  format (f4.2,7f9.2,2i4)
c
 303  continue
c
```

```
c      close(unit=15)
c      close(unit=25)
      close(unit=26)
      close(unit=27)
c      close(unit=35)
c      close(unit=36)
c      close(unit=37)
c      close(unit=45)
c      close(unit=46)
      close(unit=47)
      close(unit=55)
c      close(unit=65)
c      close(unit=66)
      close(unit=67)
      stop
      end
c
c  SUBROUTINE TO CALCULATE THE VALUE OF AN EXISTING PLANT
c  HAVING THE OPTION TO TEMPORARILY SHUT DOWN AND WHEN
c  THE VARIABLE OPERATING COST IS NONSTOCHASTIC
c
      subroutine operate(p,vc,life,r,delta,sig,imax,nmax,tol,fval,k0)
c
      real fval(0:150),x,x0,delx,dely,xmax,xmin,emax,enum
      real a,b,c,g(0:150,0:3000),f(0:150,0:3000)
      real life,sig,sigsq,dr,delta,p,vc,pnorm
c
c  NORMALIZE USING VARIABLE COST AS NUMERAIRE
c
c  USE 10*P/VC AS UPPER BOUNDARY OF MESH (P=INFINITY)
c
      pnorm=p/vc
      xmax=log(10.0*pnorm)
      xmin=log(.10*pnorm)
      x0=log(pnorm)
      sigsq=sig*sig
```

```
c
c   SET MESH DIMENSIONS TO CONFORM TO STABILITY CONSTRAINTS
c
c   USE I=K0 TO INDEX LOG OF CURRENT PRICE AND
c   CURRENT VALUE OF PLANT
c
      delx=sigsq/abs(r-delta-.5*sigsq)
      imax=nint(((xmax-xmin)/delx+100.0)/10.0)*10
      if(imax.lt.100)then
        imax=100
      endifif
      delx=(xmax-xmin)/(float(imax-1))
      k0=imax-nint((xmax-x0)/delx)
c
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      jmax=nint((life/dely)/10.0)*10
      if(jmax.lt.500)then
        jmax=500
      endifif
      dely=life/float(jmax)
      dr=dely/(delx*delx)
c
      print *, imax,jmax
c
c   INITIALIZE
c
      do 11 i=0,imax
        do 10 j=0,jmax
          f(i,j)=0.0
 10     continue
 11   continue
c
c   SET BOUNDARY CONDITIONS
c
c   INITIAL VALUE
c
```

```
      x=xmin
      do 15 i=1,imax
        if ((exp(x)-1.0).gt.0) then
          f(i,0)=exp(x)-1.0
        else
          f(i,0)=0.0
        endifif
        x=x+delx
 15   continue
        print *, f(1,0),f(10,0),f(20,0),f(imax,0)
        print *, exp(x)
c
c     LOWER BOUNDARY (P=0)
c
      do 20 j=0,jmax
        f(0,j)=0.0
 20   continue
c
c     FINITE DIFFERENCE ALGORITHM
c
      a=.5*dr*(sigsq-delx*(r-delta-.5*sigsq))
      b=1.0-dr*sigsq-dely*r
      c=.5*dr*(sigsq+delx*(r-delta-.5*sigsq))
      do 100 k=1,nmax
c
        do 40 j=1,jmax
          realj=j
          do 50 i=1,imax-1
            f(i,j)=a*f(i-1,j-1)+b*f(i,j-1)+c*f(i+1,j-1)
 50         continue
          f(imax,j)=f(imax-1,j)+delx*exp(xmax)
     *        *exp((-delta*realj)*dely)
 40     continue
c
c     ERROR CHECK
c
```

```
      enum=0.0
      emax=0.0
      do 70 j=0,jmax
         do 60 i=0,imax
            if (abs(f(i,j)-g(i,j)).gt.tol) then
               enum=enum+1
            endifif
            if (abs(f(i,j)-g(i,j)).gt.emax) then
               emax=abs(f(i,j)-g(i,j))
            endifif
60       continue
70    continue
      print *, enum,emax,f(imax,jmax),f(imax-1,jmax),f(1,jmax),k0
c
c        ITERATION COUNT
c
      if (enum.eq.0) then
         goto 110
      endifif
c
c   SECONDARY STORAGE MATRIX
c
      do 90 j=0,jmax
         do 80 i=0,imax
            g(i,j)=f(i,j)
80       continue
90    continue
100   continue
110   continue
c
c   OUTPUT - VALUE OF COMPLETED PLANT
c
     do 120 i=0,imax
       fval(i)=0.0
120  continue
     do 140 j=1,jmax
```

```
      do 130 i=0,imax
         realj=j
         fval(i)=fval(i)+vc*(f(i,j)+f(i,j-1))*dely/2.0
130   continue
140 continue
c     write(15,150)(fval(i), i=0,imax)
c 150 format(8f10.2)
c
      return
      end
c
c     SUBROUTINE TO CALCULATE THE VALUE OF THE PROGRAM
c     TO INVEST IN THE CONSTRUCTION OF A PLANT WHEN THE
c     PROJECT REQUIRES TIME AND THERE IS AN OPTION TO DELAY
c
      subroutine build(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx)
      real fval(0:150),h(0:150,0:3000),hval(0:150),sig,sigsq
      real dr,delx,dely,xmax,xmin,emax,enum,delta2,x0
      real a,u,v,w,g(0:150,0:3000),vcrit(0:3000),val
      real vcr,alpha,xinv,xinvr,tau,vcritx
c
c     USE I=K0 TO INDEX LOG OF CURRENT VALUE OF PLANT
c     AND CURRENT VALUE OF INVESTMENT PROGRAM
c
c     DUE TO CHANGE OF VARIABLES AN INDEX SHIFT IS REQUIRED
c
      xmax=log(fval(imax))
      xmin=log(fval(1))
      x0=log(fval(k0))
      delx=(xmax-xmin)/(float(imax-1))
      xmax=x0+(float(imax)/2)*delx
      xmin=x0-(float(imax)/2-1)*delx
c
c     SET TIME MESH DIMENSION TO CONFORM TO STABILITY CONSTRAINTS
c
```

```
      sigsq=sig*sig
      tau=xinv/xinvr
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      jjmax=nint((tau/dely)/10.0)*10
      if(jjmax.lt.500)then
        jjmax=500
      endifif
      dely=tau/float(jjmax)
      dr=dely/(delx*delx)
      alpha=(sqrt(2*r*sigsq+(r-delta2-.5*sigsq)**2)
     *     -(r-delta2-.5*sigsq))/sigsq
c
      print *, imax,jjmax
c
c     INITIALIZE
c
      do 11 i=0,imax
        do 10 j=0,jjmax
          h(i,j)=0.0
          g(i,j)=0.0
 10     continue
 11   continue
c
c     SET BOUNDARY CONDITIONS
c
c     INITIAL VALUE
c
      do 300 i=1,imax
        h(i,0)=exp(xmin+float(i-1)*delx)
c         fval(i)=h(i,0)
 300  continue
      print *, fval(imax), h(imax,0)
c
c     LOWER BOUNDARY (PLANT VALUE=0)
c
        vcrit(0)=0.0
```

```
      do 310 j=0,jjmax
        h(0,j)=0.0
310   continue
c
c     DYNAMIC PROGRAMMING ALGORITHM
c
      u=.5*dr*(sigsq-delx*(r-delta2-.5*sigsq))
      v=1.0-dr*sigsq
      w=.5*dr*(sigsq+delx*(r-delta2-.5*sigsq))
      do 400 k=1,nmax
        do 330 j=1,jjmax
          realj=j
c
c     BOUNDARY CONDITION AT HMAX AS GIVEN BY MAJD & PINDYCK
c     IN THE APPENDIX OF 'TIME TO BUILD'
c
          h(imax,j)=w*2*delx*exp(xmax)*exp((r-delta2)*dely*
     *       realj)+v*h(imax,j-1)+(u+w)*h(imax-1,j-1)-
     *       dely*xinvr*exp(r*(dely*(realj-1)))
          do 325 i=imax-1,1,-1
            reali=i
            h(i,j)=u*h(i-1,j-1)+v*h(i,j-1)+w*h(i+1,j-1)-
     *         dely*xinvr*exp(r*(dely*(realj-1)))
            if (h(i,j).gt.(h(i+1,j)/
     *         (alpha*delx+1.0)+delx/10.0))then
              goto 325
            endifif
            vcrit(j)=h(i,j)
            vcr=exp(xmax-(float(imax)-reali)*delx)
            a=(h(i,j)*exp(-r*realj*dely))
            do 315 l=i-1,1,-1
              reall=l
              val=exp(xmax-(float(imax)-reall)*delx)
              h(l,j)=exp(r*realj*dely)*a*(val/vcr)**alpha
315         continue
            goto 330
```

```
325       continue
330     continue
c
c    ERROR CHECK
c
        enum=0
        emax=0
        do 350 j=0,jjmax
          do 340 i=0,imax
            if (abs(h(i,j)-g(i,j)).gt.tol) then
              enum=enum+1
            endifif
            if (abs(h(i,j)-g(i,j)).gt.emax) then
              emax=abs(h(i,j)-g(i,j))
            endifif
340       continue
350     continue
        print *, enum,emax,h(imax,jjmax),h(imax-1,jjmax),h(k0,jjmax)
c
c    ITERATION COUNT
c
        if (enum.eq.0) then
          goto 410
        endifif
c
c    SECONDARY STORAGE MATRIX
c
        do 370 j=0,jjmax
          do 360 i=0,imax
            g(i,j)=h(i,j)
360       continue
370     continue
400   continue
410   continue
c
c    OUTPUT - VALUE OF INVESTMENT PROGRAM
```

```
c
      do 420 i=0,imax
        j=jjmax
        hval(i)=exp(-r*tau)*h(i,j)
 420  continue
c      write(25,430)(hval(i), k=i,imax)
c 430  format(8f10.2)
c
c     OUTPUT - CRITICAL VALUE
c
      vcritx=exp(-r*tau)*vcrit(jjmax)
c
c      do 440 j=1,jjmax
c         realj=j
c         vcrit(j)=exp(-r*realj*dely)*vcrit(j)
c 440  continue
c      write(35,450)(vcrit(j), j=1,jjmax)
c 450  format(8f10.2)
c
      return
      end
c
c     SUBROUTINE TO CALCULATE THE VALUE OF THE PROGRAM
c     TO INVEST IN THE CONSTRUCTION OF A MODULAR PLANT WHEN
c     THERE IS ONE EMBEDDED OPTION TO EXPAND CAPACITY
c
      subroutine expand1(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
     *     tol,fval,hval,vcritx,evall,ecrit1x)
      real fval(0:150),h(0:150,0:3000),hval(0:150),sig,sigsq
      real dr,delx,dely,xmax,xmin,x0,emax,enum,delta2
      real a,u,v,w,g(0:150,0:3000),val,evall(0:150)
      real alpha,xinv,xinvr,tau,vcritx
      real zcr1,ecrit1(0:3000),alpha,ecrit1x
c
c     USE I=K0 TO INDEX LOG OF CURRENT VALUE OF PLANT
c     AND CURRENT VALUE OF INVESTMENT PROGRAM
```

```
c
c   DUE TO CHANGE OF VARIABLES AN INDEX SHIFT IS REQUIRED
c
c   BOUNDARY VALUE OF INVESTMENT PROGRAM INCLUDES
c   ONE AMERICAN OPTION TO EXPAND
c
      xmax=log(fval(imax)+hval(imax))
      xmin=log(fval(1)+hval(1))
      x0=log(fval(k0)+hval(k0))
      delx=(xmax-xmin)/(float(imax-1))
      xmax=x0+(float(imax)/2)*delx
      xmin=x0-(float(imax)/2-1)*delx
c
c   SET TIME MESH DIMENSION TO CONFORM TO STABILITY CONSTRAINTS
c
      sigsq=sig*sig
      tau=xinv/xinvr
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      jjmax=nint((tau/dely)/10.0)*10
      if(jjmax.lt.500)then
        jjmax=500
      endifif
      dely=tau/float(jjmax)
      dr=dely/(delx*delx)
      alpha=(sqrt(2*r*sigsq+(r-delta2-.5*sigsq)**2)
     *     -(r-delta2-.5*sigsq))/sigsq
c
      print *, imax,jjmax
c
c   INITIALIZE
c
      do 630 i=0,imax
        do 620 j=0,jjmax
          h(i,j)=0.0
          g(i,j)=0.0
620     continue
```

```
 630  continue
c
c   SET BOUNDARY CONDITIONS
c
c   THE INITIAL VALUE CONDITION IS THE
c   SUM OF THE VALUE OF A PLANT PLUS
c   THE VALUE OF THE OPTION TO EXPAND
c
      do 640 i=1,imax
c          h(i,0)=exp(xmin+float(i-1)*delx)+xopt1(i)
        h(i,0)=exp(xmin+float(i-1)*delx)
 640  continue
c
      ecrit1(j)=0.0
      do 650 j=0,jjmax
        h(0,j)=0.0
 650  continue
c
c   DYNAMIC PROGRAMMING ALGORITHM
c
      u=.5*dr*(sigsq-delx*(r-delta2-.5*sigsq))
      v=1.0-dr*sigsq
      w=.5*dr*(sigsq+delx*(r-delta2-.5*sigsq))
      do 700 k=1,nmax
        do 730 j=1,jjmax
          realj=j
          h(imax,j)=w*2*delx*exp(xmax)*exp((r-delta2)*dely*
     *         realj)+v*h(imax,j-1)+(u+w)*h(imax-1,j-1)-
     *         dely*xinvr*exp(r*(dely*(realj-1)))
          do 725 i=imax-1,1,-1
            reali=i
            h(i,j)=u*h(i-1,j-1)+v*h(i,j-1)+w*h(i+1,j-1)-
     *           dely*xinvr*exp(r*(dely*(realj-1)))
            if (h(i,j).gt.(h(i+1,j)/
     *           (alpha*delx+1.0)+delx/10.0))then
              goto 725
```

```
          endifif
          ecrit1(j)=h(i,j)
          zcr1=exp(xmax-(float(imax)-reali)*delx)
          a=(h(i,j)*exp(-r*realj*dely))
          do 715 l=i-1,1,-1
            reall=l
            val=exp(xmax-(float(imax)-reall)*delx)
            h(l,j)=exp(r*realj*dely)*a*(val/zcr1)**alpha
 715      continue
          goto 730
 725    continue
 730  continue
c
c     ERROR CHECK
c
      enum=0
      emax=0
      do 750 j=0,jjmax
        do 740 i=0,imax
          if (abs(h(i,j)-g(i,j)).gt.tol) then
            enum=enum+1
          endifif
          if (abs(h(i,j)-g(i,j)).gt.emax) then
            emax=abs(h(i,j)-g(i,j))
          endifif
 740    continue
 750  continue
c
c     ITERATION COUNT
c
      if (enum.eq.0) then
        goto 710
      endifif
c
c     SECONDARY STORAGE MATRIX
c
```

```
      do 770 j=0,jjmax
        do 760 i=0,imax
          g(i,j)=h(i,j)
760     continue
770   continue
700 continue
710 continue
c
c   OUTPUT - VALUE OF INVESTMENT PROGRAM
c
    do 780 i=0,imax
      j=jjmax
      eval1(i)=exp(-r*tau)*h(i,j)
780 continue
    write(26,785)(eval1(i), i=0,imax)
785 format(8f10.2)
c
c   OUTPUT - CRITICAL VALUE
c
    ecrit1x=exp(-r*tau)*ecrit1(jjmax)
c
c     do 790 j=1,jjmax
c        realj=j
c        ecrit1(j)=exp(-r*realj*dely)*ecrit1(j)
c 790       continue
c     write(36,795)(ecrit1(j), j=1,jjmax)
c 795  format(8f10.2)
c
    return
    end
c
c   SUBROUTINE TO CALCULATE THE VALUE OF THE PROGRAM
c   TO INVEST IN THE CONSTRUCTION OF A MODULAR PLANT WHEN
c   THERE ARE TWO EMBEDDED OPTIONS TO EXPAND CAPACITY
c
    subroutine expand2(xinv,xinvr,r,delta2,sig,imax,nmax,k0,
```

```
     *     tol,fval,eval1,ecrit1x,eval2,ecrit2x)
c
      real fval(0:150),h(0:150,0:3000),eval1(0:150)
      real dr,delx,dely,xmax,xmin,x0,emax,enum,delta2
      real a,u,v,w,g(0:150,0:3000),ecrit2(0:3000),val
      real zcr2,alpha,xinv,xinvr,tau,sig,sigsq
      real ecrit1x,alpha,ecrit2x,eval2(0:150)
c
c     USE I=K0 TO INDEX LOG OF CURRENT VALUE OF PLANT
c     AND CURRENT VALUE OF INVESTMENT PROGRAM
c
c     DUE TO CHANGE OF VARIABLES AN INDEX SHIFT IS REQUIRED
c
c     BOUNDARY VALUE OF INVESTMENT PROGRAM INCLUDES
c     TWO AMERICAN OPTIONS TO EXPAND
c
      xmax=log(fval(imax)+eval1(imax))
      xmin=log(fval(1)+eval1(1))
      x0=log(fval(k0)+eval1(k0))
      delx=(xmax-xmin)/(float(imax-1))
      xmax=x0+(float(imax)/2)*delx
      xmin=x0-(float(imax)/2-1)*delx
c
c     SET TIME MESH DIMENSION TO CONFORM TO STABILITY CONSTRAINTS
c
      sigsq=sig*sig
      tau=xinv/xinvr
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      jjmax=nint((tau/dely)/10.0)*10
      if(jjmax.lt.500)then
        jjmax=500
      endifif
      dely=tau/float(jjmax)
      dr=dely/(delx*delx)
      alpha=(sqrt(2*r*sigsq+(r-delta2-.5*sigsq)**2)
     *     -(r-delta2-.5*sigsq))/sigsq
```

```
c
      print *, imax,jjmax
c
c     INITIALIZE
c
      do 820 i=0,imax
        do 810 j=0,jjmax
          h(i,j)=0.0
          g(i,j)=0.0
810     continue
820   continue
c
c     SET BOUNDARY CONDITIONS
c
c     THE INITIAL VALUE CONDITION IS THE
c     SUM OF THE VALUE OF A PLANT PLUS
c     THE VALUE OF AN INVESTMENT PROGRAM WITH
c     TWO EMBEDDED OPTIONS TO EXPAND
c
      do 830 i=1,imax
c         h(i,0)=exp(xmin+float(i-1)*delx)+xopt2(i)
        h(i,0)=exp(xmin+float(i-1)*delx)
830   continue
c
      ecrit2(j)=0.0
      do 840 j=0,jjmax
        h(0,j)=0.0
840   continue
c
c     DYNAMIC PROGRAMMING ALGORITHM
c
      u=.5*dr*(sigsq-delx*(r-delta2-.5*sigsq))
      v=1.0-dr*sigsq
      w=.5*dr*(sigsq+delx*(r-delta2-.5*sigsq))
      do 850 k=1,nmax
        do 880 j=1,jjmax
```

```
        realj=j
        h(imax,j)=w*2*delx*exp(xmax)*exp((r-delta2)*dely*
     *     realj)+v*h(imax,j-1)+(u+w)*h(imax-1,j-1)-
     *     dely*xinvr*exp(r*(dely*(realj-1)))
        do 870 i=imax-1,1,-1
          reali=i
          h(i,j)=u*h(i-1,j-1)+v*h(i,j-1)+w*h(i+1,j-1)-
     *       dely*xinvr*exp(r*(dely*(realj-1)))
          if (h(i,j).gt.(h(i+1,j)/
     *       (alpha*delx+1.0)+delx/10.0))then
            goto 870
          endifif
          ecrit2(j)=h(i,j)
          zcr2=exp(xmax-(float(imax)-reali)*delx)
          a=(h(i,j)*exp(-r*realj*dely))
          do 860 l=i-1,1,-1
            reall=l
            val=exp(xmax-(float(imax)-reall)*delx)
            h(l,j)=exp(r*realj*dely)*a*(val/zcr2)**alpha
860       continue
          goto 880
870     continue
880   continue
c
c   ERROR CHECK
c
      enum=0
      emax=0
      do 900 j=0,jjmax
        do 890 i=0,imax
          if (abs(h(i,j)-g(i,j)).gt.tol) then
            enum=enum+1
          endifif
          if (abs(h(i,j)-g(i,j)).gt.emax) then
            emax=abs(h(i,j)-g(i,j))
          endifif
```

```
890         continue
900      continue
c
c     ITERATION COUNT
c
       if (enum.eq.0) then
          goto 910
       endifif
c
c     SECONDARY STORAGE MATRIX
c
       do 930 j=0,jjmax
          do 920 i=0,imax
             g(i,j)=h(i,j)
920         continue
930      continue
850   continue
910   continue
c
c     OUTPUT - VALUE OF INVESTMENT PROGRAM
c
      do 940 i=0,imax
         j=jjmax
         eval2(i)=exp(-r*tau)*h(i,j)
940   continue
      write(27,950)(eval2(i), i=0,imax)
950   format(8f10.2)
c
c     OUTPUT - CRITICAL VALUE
c
      ecrit2x=exp(-r*tau)*ecrit2(jjmax)
c
c      do 960 j=1,jjmax
c         realj=j
c               ecrit2(j)=exp(-r*realj*dely)*ecrit2(j)
c 960       continue
```

```
c           kmax=k-1
c     write(37,970)(ecrit2(j), j=1,jjmax)
c 970  format(8f10.2)
c
      return
      end
```

Appendix C

Program #2

```
c     Program #2
c
      real p,sigma,vc,life,r,delta1,delta2,tol,f
      real alphas,alphap,alphax,alphac
      real fval(0:200,0:200),rho(20),plant(20)
      integer imax,nmax,kmax,jmax
c
c    INPUT AND OUTPUT FILES
c
c     open(unit=15, file='fval')
      open(unit=45, file='optdata6')
      open(unit=55, file='rhodata1')
      open(unit=65, file='outdata6')
c
c    VARIABLE DEFINITIONS
c
c    p = price, or annualized revenue ($)
c    sigma = annualized standard deviation of commodity price
c    vc = variable cost, or annualized variable operating cost ($)
c    sigma2 = annualized standard deviation of operating cost
c    life = planned operating life of plant (years)
c    r = annualized riskless interest rate (%/yr)
c    alphas = market-required rate of return on output (%/yr)
```

```
c     alphap = expected rate of return on output (%/yr)
c     delta1 = alphas - alphap = convenience yield, or ROR shortfall (%/yr)
c     alphax = market-required rate of return on input (%/yr)
c     alphac = expected rate of return on input (%/yr)
c     delta2 = alphax - alphac = convenience yield, or ROR shortfall (%/yr)
c     f = ADI parameter
c     tol = error tolerance for convergence check
c     kmax = number of grid point along the time axis
c     imax = number of grid point along the price axis
c     jmax = number of grid point along the cost axis
c     nmax = limit on the maximum number of iterations
c
c     rho = coefficient of correlation between price movements of p & vc
c
c     READ IN RHO
c
      read (55,*) (rho(i), i=1,11)
c
c     MAIN PROGRAM
c
c     READ IN PROBLEM DATA
c
      read (45,*) p,sigma,vc,sigma2,life,r,f,alphas,alphap,alphax,
     *     alphac,tol,kmax,imax,nmax
c
c      do 110 i=1,11
        rho1=rho(i)
c
        do 20 k=0,200
          do 10 l=0,200
          fval(k,l)=0.0
 10      continue
 20   continue
c
        delta1=alphas-alphap
        delta2=alphax-alphac
```

```
      call operate(p,sigma,vc,sigma2,life,r,delta1,delta2,
     *       rho1,f,kmax,imax,nmax,tol,fval,jmax,kx,ky)
c
c     CURRENT PLANT VALUE
c
      plant(i)=fval(kx,ky)
c
c     OUTPUT
c
c        write(15,100)((fval(k,l), l=0,jmax), k=0,imax)
c 100       format(8f10.2)
c
      write (65,105) rho(6),plant(i),kx,ky
c
 105    format (2f10.2,2i4)
c
 110  continue
c
c      close(unit=15)
      close(unit=45)
      close(unit=55)
      close(unit=65)
      stop
      end
c
c     SUBROUTINE TO CALCULATE THE VALUE OF AN EXISTING PLANT
c     HAVING THE OPTION TO TEMPORARILY SHUT DOWN AND WHEN
c     THE VARIABLE OPERATING COST IS STOCHASTIC
c
      subroutine operate(p,sigma,vc,sigma2,life,r,delta1,delta2,
     *     rho1,f,kmax,imax,nmax,tol,fval,jmax,kx,ky)
      real x,x0,y,y0,delx,dely,xmax,xmin,ymax,ymin
      real dt,dr,sx,sy,pnorm,emax,enum,fval(0:200,0:200)
      real h(0:200,0:200),g(0:200,0:200),gg(0:200,0:200)
      real aa(0:200),bb(0:200),cc(0:200),dd(0:200)
      real a,b,c,d,e,a1,a2,a3,a4,a5,a6,g1(0:200,0:200)
```

```
      real delta1,delta2,p,vc,life,sigma,sigma2,f,rho1,r
c
c     DEFINE PARAMETERS
c
c     NORMALIZE USING CURRENT VARIABLE COST AS NUMERAIRE
c
      pnorm=p/vc
c
      xmax=log(10*pnorm)
      xmin=log(.10*pnorm)
      x0=log(pnorm)
      delx=(xmax-xmin)/(float(imax-1))
      kx=imax-nint((xmax-x0)/delx)
      ymax=log(10.0)
      y0=0.0
      dely=delx
      ymin=log(.10)
      jmax=nint((ymax-ymin)/dely)+1
      ky=jmax-nint((ymax-y0)/dely)
      dt=life/float(kmax)
      dr=dt/(delx*delx)
      sx=dt/(2*delx)
      sy=dt/(2*dely)
      a=.5*sigma*sigma
      b=.5*rho1*sigma*sigma2
      c=.5*sigma2*sigma2
      d=r-delta1-a
      e=r-delta2-c
      a1=1/f-.5*dr*a
      a2=1/f+.5*dr*a
      a3=dr*c
      a4=.5*dr*b
      a5=dr/f*(a+c)
      a6=1/f-.5*dr*c
c
c     SET BOUNDARY CONDITIONS
```

```
c
c   INITIAL CONDITION
c
      do 50 j=1,jmax
        do 40 i=1,imax
          reali=i-1
          realj=j-1
          x=xmin+reali*delx
          y=ymin+realj*dely
          if ((exp(x)-exp(y)).gt.0.0) then
            g(i,j)=exp(x)-exp(y)
          else
            g(i,j)=0.0
          endifif
 40     continue
 50   continue
      print *, g(1,1),g(10,10),g(20,20),g(imax,jmax)
      print *, exp(x),exp(y),d,sx
c
c   X=0 BOUNDARY
c
      do 60 j=0,jmax
        g(0,j)=0.0
 60   continue
c
c   Y=0 BOUNDARY
c
      do 80 i=1,imax
        reali=i
        g(i,0)=exp(xmin+delx*(reali-1))
 80   continue
c
c   INITIALIZE PLANT VALUE
c
      do 100 j=0,jmax
        do 90 i=0,imax
```

```
        fval(i,j)=0.0
90    continue
100  continue
c
     realkmax=kmax
c
c    ADI ALGORITHM FOR MIXED DERIVATIVE EQUATION
c
     do 350 k=1,kmax
       realk=k
c
c    UPDATE SECONDARY STORAGE MATRIX
c
         do 120 j=0,jmax
           do 110 i=0,imax
             g1(i,j)=g(i,j)
110        continue
120      continue
c
c    OUTER CONTROL LOOP
c
       do 300 m=1,nmax
c
c    UPDATE CONVERGENCE CONTROL MATRIX
c
         do 150 j=0,jmax
           do 140 i=0,imax
             h(i,j)=g(i,j)
140        continue
150      continue
c
c    X SWEEP
c
         do 180 j=1,jmax-1
           do 160 i=0,imax
             if (i.eq.0) then
```

```
            bb(i)=0.0
            dd(i)=1.0
            aa(i)=0.0
            cc(i)=g(i,j)
          endifif
          if (i.eq.imax) then
            bb(i)=-1.0
            dd(i)=1.0
            aa(i)=0.0
            cc(i)=delx*exp(xmax)*exp(-delta1*(realk*dt))
          endifif
          if ((i.gt.0).and.(i.lt.imax)) then
            bb(i)=a1
            dd(i)=1-2*a1
            aa(i)=a1
            cc(i)=g(i,j)+d*sx*(g(i+1,j)-g(i-1,j))+
     *           e*sy*(g(i,j+1)-g(i,j-1))-r*dt*g(i,j)+
     *           a2*(g(i+1,j)-2*g(i,j)+g(i-1,j))+
     *           a3*(g(i,j+1)-2*g(i,j)+g(i,j-1))+
     *           a4*(g(i+1,j)-g(i-1,j))*(g(i,j+1)-g(i,j-1))+
     *           a5*(g(i+1,j)-2*g(i,j)+g(i-1,j))*
     *           (g(i,j+1)-2*g(i,j)+g(i,j-1))
          endifif
c
 160      continue
        il=0
        iu=imax
        call tridiag(il,iu,bb,dd,aa,cc)
        do 170 i=0,imax
          gg(i,j)=cc(i)
 170      continue
 180    continue
c       print *, gg(10,10)
c
c    Y SWEEP
c
```

```
      do 210 i=1,imax-1
        do 190 j=0,jmax
          reali=i
          if (j.eq.0) then
            bb(j)=0.0
            dd(j)=1.0
            aa(j)=0.0
            cc(j)=g(i,j)
          endifif
          if (j.eq.(jmax)) then
            bb(j)=-1.0
            dd(j)=1.0
            aa(j)=0.0
            cc(j)=0.0
          endifif
          if ((j.gt.0).and.(j.lt.jmax)) then
            bb(j)=a6
            dd(j)=1-2*a6
            aa(j)=a6
            cc(j)=gg(i,j)+a6*(g(i,j+1)-2*g(i,j)+g(i,j-1))
          endifif
c
 190      continue
        il=0
        iu=jmax
        call tridiag(il,iu,bb,dd,aa,cc)
        do 200 j=0,jmax
          g(i,j)=cc(j)
 200      continue
 210    continue
c         print *, g(10,10)
c
c   ERROR CHECK
c
      enum=0.0
      emax=0.0
```

```
        do 270 j=0,jmax
          do 260 i=0,imax
            if (abs(g(i,j)-h(i,j)).gt.tol) then
              enum=enum+1
            endifif
            if (abs(g(i,j)-h(i,j)).gt.emax) then
              emax=abs(g(i,j)-h(i,j))
            endifif
260       continue
270     continue
        print *, enum,emax,g(1,1),g(imax,jmax),kx,ky,k
c
c   ITERATION COUNT
c
        if (enum.eq.0) then
          goto 310
        endifif
c
300   continue
310 continue
c
c   NUMERICAL INTEGRATION TO UPDATE COMPOUND VALUE
c   AS SUM OF OPTION VALUES OVER TIME
c
      do 330 j=0,jmax
        do 320 i=0,imax
        fval(i,j)=fval(i,j)+vc*(g(i,j)+g1(i,j))*dt/2.0
320   continue
330 continue
c
350 continue
c
    return
    end

c    SUBROUTINE TO DIAGONALIZE TRIDIAGONAL MATRIX
```

```
c
      subroutine tridiag(il,iu,bb,dd,aa,cc)
      real aa(0:200),bb(0:200),cc(0:200),dd(0:200)
      lp=il+1
      do 10 i=lp,iu
        r=bb(i)/dd(i-1)
        dd(i)=dd(i)-r*aa(i-1)
        cc(i)=cc(i)-r*cc(i-1)
10    continue
      cc(iu)=cc(iu)/dd(iu)
      do 20 i=lp,iu
        j=iu-i+il
        cc(j)=(cc(j)-aa(j)*cc(j+1))/dd(j)
20    continue
      return
      end
```

Appendix D

Program #3

```
c     Program #3
c
      real p1,sigma1,p2,vc,sigma2,delta1,delta2
      real alphas1,alphap1,alphas2,alphap2,life,r,tol,f
      real fval(0:100,0:100),rho(20),plant(20)
      real p1val,p2val,f1val(0:100),f2val(0:100)
      integer imax,nmax,kmax,jmax
c
c     INPUT AND OUTPUT FILES
c
c     open(unit=15, file='fval')
      open(unit=45, file='optdata5')
      open(unit=55, file='rhodata1')
      open(unit=65, file='outdata5')
c
c     VARIABLE DEFINITIONS
c
c     p1 = price, or annualized revenue of output #1 ($)
c     sigma1 = annualized standard deviation of commodity price, p1
c     p1val = option value of plant producing only output #1
c     p2 = price, or annualized revenue of output #2 ($)
c     sigma2 = annualized standard deviation of commodity price, p2
c     p2val = option value of plant producing only output #2
```

```
c     vc = variable cost, or annualized variable operating cost ($)
c     life = planned operating life of plant (years)
c     r = annualized riskless interest rate (%/yr)
c     alphas1 = market-required rate of return on output #1 (%/yr)
c     alphap1 = expected rate of return on output #1 (%/yr)
c     delta1 = alphas1 - alphap2 = convenience yield, or ROR shortfall (%/yr)
c     alphas2 = market-required rate of return on output #2 (%/yr)
c     alphap2 = expected rate of return on output #2 (%/yr)
c     delta2 = alphas1 - alphap2 = convenience yield, or ROR shortfall (%/yr)
c     f = ADI parameter
c     tol = error tolerance for convergence check
c     kmax = number of grid point along the time axis
c     imax = number of grid point along the p1 axis (x)
c     jmax = number of grid point along the p2 axis (y)
c     nmax = limit on the maximum number of iterations
c
c     rho = coefficient of correlation between price movements of p & vc
c
c     READ IN RHO
c
      read (55,*) (rho(i), i=1,11)
c
c     READ IN PROBLEM DATA
c
      read (45,*) p1,sigma1,p2,sigma2,vc,life,r,f,alphas1,alphap1,
     *     alphas2,alphap2,tol,kmax,imax,jmax,nmax
c
c     MAIN PROGRAM

      do 110 i=1,11
        rho1=rho(i)
c
          do 20 k=0,100
            do 10 l=0,100
              fval(k,l)=0.0
 10         continue
```

```
 20     continue
c
        delta1=alphas1-alphap1
        delta2=alphas2-alphap2
c
        call plant1(p1,vc,life,r,delta1,sigma1,imax,nmax,tol,
     *    f1val,k0)
c
        p1val=f1val(k0)
c
        call plant2(p2,vc,life,r,delta2,sigma2,jmax,nmax,tol,
     *    f2val,m0)
c
        p2val=f2val(m0)
c
        call operate(p1,sigma1,p2,sigma2,vc,life,r,delta1,delta2,
     *        rho1,f,kmax,imax,jmax,nmax,tol,p1val,p2val,kx,ky)
c
c     CURRENT VALUE OF PLANT HAVING THE OPTION TO PRODUCE THE MORE
c     VALUABLE OF EITHER OUTPUT #1 OR OUTPUT #2
c
        plant(i)=p1val+p2val-fval(kx,ky)
c
c     OUTPUT
c
c     write(15,100)((fval(k,l), l=0,jmax), k=0,imax)
c 100  format(8f10.2)
c
        write (65,105) rho(i),plant(i),kx,ky
c
 105  format (2f10.2,2i4)
c
 110  continue
c
c      close(unit=15)
      close(unit=45)
```

```
      close(unit=55)
      close(unit=65)
      stop
      end
c
c     SUBROUTINE TO CALCULATE THE VALUE OF AN OPTION ON
c     THE MINIMUM OF TWO RISKY ASSETS
c
      subroutine operate(p1,sigma1,p2,sigma2,vc,life,r,delta1,delta2,
     *     rho1,f,kmax,imax,jmax,nmax,tol,fval,kx,ky)
c
      real x,x0,y,y0,delx,dely,xmax,xmin,ymax,ymin,sx,sy,p1,p2
      real dt,dr,p1norm,p2norm,emax,enum,fval(0:100,0:100)
      real h(0:100,0:100),g(0:100,0:100),gg(0:100,0:100)
      real aa(0:100),bb(0:100),cc(0:100),dd(0:100)
      real a,b,c,d,e,a1,a2,a3,a4,a5,a6,g1(0:100,0:100)
      real delta1,delta2,vc,life,sigma1,sigma2,f,rho1,r
      integer imax,jmax,kmax,nmax
c
c     DEFINE PARAMETERS
c
c     NORMALIZE USING VARIABLE COST AS NUMERAIRE
c
      p1norm=p1/vc
      p2norm=p2/vc
c
      xmax=log(10.0*p1norm)
      xmin=log(.10*p1norm)
      x0=log(p1norm)
      delx=(xmax-xmin)/(float(imax-1))
      kx=imax-nint((xmax-x0)/delx)
      ymax=log(10.0*p2norm)
      ymin=log(.10*p2norm)
      y0=log(p2norm)
      dely=delx
      jmax=nint((ymax-ymin)/dely)+1
```

```
      ky=jmax-nint((ymax-y0)/dely)
      dt=life/float(kmax)
      dr=dt/(delx*delx)
      sx=dt/(2*delx)
      sy=dt/(2*dely)
      a=.5*sigma1*sigma1
      b=.5*rho1*sigma1*sigma2
      c=.5*sigma2*sigma2
      d=r-delta1-a
      e=r-delta2-c
      a1=1/f-.5*dr*a
      a2=1/f+.5*dr*a
      a3=dr*c
      a4=.5*dr*b
      a5=dr/f*(a+c)
      a6=1/f-.5*dr*c
c
      print *, imax,jmax
c
c     SET BOUNDARY CONDITIONS
c
c     INITIAL CONDITION
c
      g(0,0)=0.0
c
      do 20 j=1,jmax
        realj=j-1
        y=ymin+realj*dely
        if (exp(y)-1.0.gt.0.0) then
          g(0,j)=exp(y)-1.0
        else
          g(0,j)=0.0
        endifif
 20   continue
c
      do 30 i=1,imax
```

```
      reali=i-1
      x=xmin+reali*delx
      if (exp(x)-1.0.gt.0.0) then
        g(i,0)=exp(x)-1.0
      else
        g(i,0)=0.0
      endifif
30    continue
c
    do 50 j=1,jmax
      do 40 i=1,imax
        reali=i-1
        realj=j-1
        x=xmin+reali*delx
        y=ymin+realj*dely
        if (x.gt.y) then
          g(i,j)=exp(x)-1.0
        else
          g(i,j)=exp(y)-1.0
        endifif
        if(g(i,j).le.0.0)then
          g(i,j)=0.0
        endifif
40      continue
50    continue
    print *, g(1,1),g(10,10),g(20,20),g(imax,jmax)
    print *, exp(x),exp(y),d,sx
c
c     INITIALIZE PLANT VALUE
c
    do 100 j=0,jmax
      do 90 i=0,imax
        fval(i,j)=0.0
90      continue
100   continue
c
```

```
      realkmax=kmax
c
c     ADI ALGORITHM FOR MIXED DERIVATIVE EQUATION
c
      do 350 k=1,kmax
        realk=k
c
c     UPDATE SECONDARY STORAGE MATRIX
c
          do 120 j=0,jmax
            do 110 i=0,imax
              g1(i,j)=g(i,j)
 110        continue
 120      continue
c
c     X=0 BOUNDARY
c
      do 60 j=0,jmax
        g(0,j)=0.0
 60   continue
c         print *, g(0,jmax),k
c
c     Y=0 BOUNDARY
c
      do 80 i=0,imax
        g(i,0)=0.0
 80   continue
c         print *, g(imax,0),k

c     OUTER CONTROL LOOP
c
        do 300 m=1,nmax
c
c     UPDATE CONVERGENCE CONTROL MATRIX
c
          do 150 j=0,jmax
```

```
          do 140 i=0,imax
            h(i,j)=g(i,j)
 140      continue
 150    continue
c
c    X SWEEP
c
        do 180 j=1,jmax-1
          do 160 i=0,imax
            if (i.eq.0) then
              bb(i)=0.0
              dd(i)=1.0
              aa(i)=0.0
              cc(i)=g(i,j)
            endifif
            if (i.eq.imax) then
              bb(i)=-1.0
              dd(i)=1.0
              aa(i)=0.0
              cc(i)=0.0
            endifif
            if ((i.gt.0).and.(i.lt.imax)) then
              bb(i)=a1
              dd(i)=1-2*a1
              aa(i)=a1
              cc(i)=g(i,j)+d*sx*(g(i+1,j)-g(i-1,j))+
     *             e*sy*(g(i,j+1)-g(i,j-1))-r*dt*g(i,j)+
     *             a2*(g(i+1,j)-2*g(i,j)+g(i-1,j))+
     *             a3*(g(i,j+1)-2*g(i,j)+g(i,j-1))+
     *             a4*(g(i+1,j)-g(i-1,j))*(g(i,j+1)-g(i,j-1))+
     *             a5*(g(i+1,j)-2*g(i,j)+g(i-1,j))*
     *             (g(i,j+1)-2*g(i,j)+g(i,j-1))
            endifif
c
 160      continue
          il=0
```

```
        iu=imax
        call tridiag(il,iu,bb,dd,aa,cc)
        do 170 i=0,imax
          gg(i,j)=cc(i)
170     continue
180   continue
c       print *, gg(10,10)
c
c   Y SWEEP
c
      do 210 i=1,imax-1
        do 190 j=0,jmax
          if (j.eq.0) then
            bb(j)=0.0
            dd(j)=1.0
            aa(j)=0.0
            cc(j)=g(i,j)
          endifif
          if (j.eq.(jmax)) then
            bb(j)=-1.0
            dd(j)=1.0
            aa(j)=0.0
            cc(j)=0.0
          endifif
          if ((j.gt.0).and.(j.lt.jmax)) then
            bb(j)=a6
            dd(j)=1-2*a6
            aa(j)=a6
            cc(j)=gg(i,j)+a6*(g(i,j+1)-2*g(i,j)+g(i,j-1))
          endifif
c
190     continue
        il=0
        iu=jmax
        call tridiag(il,iu,bb,dd,aa,cc)
        do 200 j=0,jmax
```

```
            g(i,j)=cc(j)
 200        continue
 210      continue
c         print *, g(10,10)
c
c   ERROR CHECK
c
        enum=0.0
        emax=0.0
        do 270 j=0,jmax
          do 260 i=0,imax
            if (abs(g(i,j)-h(i,j)).gt.tol) then
              enum=enum+1
            endifif
            if (abs(g(i,j)-h(i,j)).gt.emax) then
              emax=abs(g(i,j)-h(i,j))
            endifif
 260        continue
 270      continue
        print *, enum,emax,g(1,1),g(imax,jmax),kx,ky,k
c
c   ITERATION COUNT
c
        if (enum.eq.0) then
          goto 310
        endifif
c
 300    continue
 310  continue
c
c   NUMERICAL INTEGRATION TO UPDATE COMPOUND VALUE
c   AS SUM OF OPTION VALUES OVER TIME
c
      do 330 j=0,jmax
        do 320 i=0,imax
        fval(i,j)=fval(i,j)+vc*(g(i,j)+g1(i,j))*dt/2.0
```

```
320     continue
330  continue
c
350  continue
c
     return
     end

c      SUBROUTINE TO DIAGONALIZE TRIDIAGONAL MATRIX
c
      subroutine tridiag(il,iu,bb,dd,aa.cc)
      real aa(0:100),bb(0:100),cc(0:100),dd(0:100)
      lp=il+1
      do 10 i=lp,iu
         r=bb(i)/dd(i-1)
         dd(i)=dd(i)-r*aa(i-1)
         cc(i)=cc(i)-r*cc(i-1)
 10   continue
      cc(iu)=cc(iu)/dd(iu)
      do 20 i=lp,iu
         j=iu-i+il
         cc(j)=(cc(j)-aa(j)*cc(j+1))/dd(j)
 20   continue
      return
      end
c
c
c     SUBROUTINE TO CALCULATE THE VALUE OF THE PLANT
c     PRODUCING ONLY OUTPUT #1
c
      subroutine plant1(p1,vc,life,r,delta1,sigma1,imax,nmax,tol,
     *     f1val,k0)
c
      real f1val(0:100),x,x0,delx,dely,xmax,xmin,emax,enum
      real a,b,c,g(0:100,0:1500),f(0:100,0:1500)
      real life,sigma1,sigsq,dr,delta1,p1,vc,pnorm
```

```
c
c   NORMALIZE USING VARIABLE COST AS NUMERAIRE
c
c   USE 10*P/VC AS UPPER BOUNDARY OF MESH (P=INFINITY)
c
      pnorm=p1/vc
      xmax=log(10.0*pnorm)
      xmin=log(.10*pnorm)
      x0=log(pnorm)
      sigsq=sigma1*sigma1
c
c   SET MESH DIMENSIONS TO CONFORM TO STABILITY CONSTRAINTS
c
c   USE I=K0 TO INDEX LOG OF CURRENT PRICE AND
c   CURRENT VALUE OF PLANT
c
      delx=(xmax-xmin)/(float(imax-1))
      k0=imax-nint((xmax-x0)/delx)
c
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      kmax=nint((life/dely)/10.0)*10
      if(kmax.lt.1000)then
        kmax=1000
      endifif
      dely=life/float(kmax)
      dr=dely/(delx*delx)
c
c   INITIALIZE
c
      do 11 i=0,imax
        do 10 j=0,kmax
          f(i,j)=0.0
 10     continue
 11   continue
c
c   SET BOUNDARY CONDITIONS
```

```
c
c    INITIAL VALUE
c
      x=xmin
      do 15 i=1,imax
        if ((exp(x)-1.0).gt.0) then
          f(i,0)=exp(x)-1.0
        else
          f(i,0)=0.0
        endifif
        x=x+delx
 15   continue
        print *, f(1,0),f(10,0),f(20,0),f(imax,0)
        print *, exp(x-delx)
c
c    LOWER BOUNDARY (P=0)
c
      do 20 j=0,kmax
        f(0,j)=0.0
 20   continue
c
c    FINITE DIFFERENCE ALGORITHM
c
      a=.5*dr*(sigsq-delx*(r-delta1-.5*sigsq))
      b=1.0-dr*sigsq-dely*r
      c=.5*dr*(sigsq+delx*(r-delta1-.5*sigsq))
      do 100 k=1,nmax
c
        do 40 j=1,kmax
          realj=j
          do 50 i=1,imax-1
            f(i,j)=a*f(i-1,j-1)+b*f(i,j-1)+c*f(i+1,j-1)
 50         continue
          f(imax,j)=f(imax-1,j)+delx*exp(xmax)
     *         *exp((-delta1*realj)*dely)
 40     continue
```

```
c
c   ERROR CHECK
c
      enum=0.0
      emax=0.0
      do 70 j=0,kmax
        do 60 i=0,imax
          if (abs(f(i,j)-g(i,j)).gt.tol) then
            enum=enum+1
          endifif
          if (abs(f(i,j)-g(i,j)).gt.emax) then
            emax=abs(f(i,j)-g(i,j))
          endifif
60        continue
70      continue
    print *, enum,emax,f(imax,kmax),f(imax-1,kmax),f(1,kmax),k0
c
c        ITERATION COUNT
c
      if (enum.eq.0) then
        goto 110
      endifif
c
c   SECONDARY STORAGE MATRIX
c
      do 90 j=0,kmax
        do 80 i=0,imax
          g(i,j)=f(i,j)
80        continue
90      continue
100  continue
110  continue
c
c   OUTPUT - VALUE OF COMPLETED PLANT
c
    do 120 i=0,imax
```

```
      f1val(i)=0.0
120 continue
    do 140 j=1,kmax
      do 130 i=0,imax
        f1val(i)=f1val(i)+vc*(f(i,j)+f(i,j-1))*dely/2.0
130   continue
140 continue
    return
    end

c
c   SUBROUTINE TO CALCULATE THE VALUE OF THE PLANT
c   PRODUCING ONLY OUTPUT #2
c
    subroutine plant2(p2,vc,life,r,delta1,sigma1,jmax,nmax,tol,
   *     f2val,m0)
c
    real f2val(0:100),x,x0,delx,dely,xmax,xmin,emax,enum
    real a,b,c,g(0:100,0:1500),f(0:100,0:1500)
    real life,sigma2,sigsq,dr,delta2,p2,vc,pnorm
c
c   NORMALIZE USING VARIABLE COST AS NUMERAIRE
c
c   USE 10*P/VC AS UPPER BOUNDARY OF MESH (P=INFINITY)
c
    pnorm=p2/vc
    xmax=log(10.0*pnorm)
    xmin=log(.10*pnorm)
    x0=log(pnorm)
    sigsq=sigma2*sigma2
c
c   SET MESH DIMENSIONS TO CONFORM TO STABILITY CONSTRAINTS
c
c   USE I=K0 TO INDEX LOG OF CURRENT PRICE AND
c   CURRENT VALUE OF PLANT
c
```

```
      delx=(xmax-xmin)/(float(jmax-1))
      m0=jmax-nint((xmax-x0)/delx)
c
      dely=(delx*delx*(1.0-r)/sigsq)*.90
      kmax=nint((life/dely)/10.0)*10
      if(kmax.lt.1000)then
        kmax=1000
      endifif
      dely=life/float(kmax)
      dr=dely/(delx*delx)
c
c     INITIALIZE
c
      do 11 i=0,jmax
        do 10 j=0,kmax
          f(i,j)=0.0
 10     continue
 11   continue
c
c     SET BOUNDARY CONDITIONS
c
c     INITIAL VALUE
c
      x=xmin
      do 15 i=1,jmax
        if ((exp(x)-1.0).gt.0) then
          f(i,0)=exp(x)-1.0
        else
          f(i,0)=0.0
        endifif
        x=x+delx
 15   continue
        print *, f(1,0),f(10,0),f(20,0),f(jmax,0)
        print *, exp(x-delx)
c
c     LOWER BOUNDARY (P=0)
```

```
c
      do 20 j=0,kmax
        f(0,j)=0.0
 20   continue
c
c     FINITE DIFFERENCE ALGORITHM
c
      a=.5*dr*(sigsq-delx*(r-delta2-.5*sigsq))
      b=1.0-dr*sigsq-dely*r
      c=.5*dr*(sigsq+delx*(r-delta2-.5*sigsq))
      do 100 k=1,nmax
c
        do 40 j=1,kmax
          realj=j
          do 50 i=1,jmax-1
            f(i,j)=a*f(i-1,j-1)+b*f(i,j-1)+c*f(i+1,j-1)
 50       continue
          f(jmax,j)=f(jmax-1,j)+delx*exp(xmax)
     *          *exp((-delta1*realj)*dely)
 40     continue
c
c     ERROR CHECK
c
        enum=0.0
        emax=0.0
        do 70 j=0,kmax
          do 60 i=0,jmax
            if (abs(f(i,j)-g(i,j)).gt.tol) then
              enum=enum+1
            endifif
            if (abs(f(i,j)-g(i,j)).gt.emax) then
              emax=abs(f(i,j)-g(i,j))
            endifif
 60       continue
 70     continue
      print *, enum,emax,f(jmax,kmax),f(jmax-1,kmax),f(1,kmax),m0
```

```
c
c          ITERATION COUNT
c
       if (enum.eq.0) then
         goto 110
       endifif
c
c     SECONDARY STORAGE MATRIX
c
       do 90 j=0,kmax
         do 80 i=0,jmax
           g(i,j)=f(i,j)
 80        continue
 90      continue
 100  continue
 110  continue
c
c     OUTPUT - VALUE OF COMPLETED PLANT
c
     do 120 i=0,jmax
       f2val(i)=0.0
 120  continue
     do 140 j=1,kmax
       do 130 i=0,jmax
         f2val(i)=f2val(i)+vc*(f(i,j)+f(i,j-1))*dely/2.0
 130    continue
 140  continue
     return
     end
```

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