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A BEHAVIORAL PRINCIPLE FOR TRAFFIC NETWORKS WITH DYNAMIC ASSIGNMENT

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ABSTRACT

This paper presents a model of a transportation network in which travelers behave according to the travel budget theory of Yacov Zahavi. It differs from an earlier detailed network formulation in that the variation of travel behavior during the course of a day is represented explicitly.

1. Introduction

Yacov Zahavi observed (1979) that in developed countries throughout the world, the average traveler spends between 1.0 and 1.5 hours every day in the transportation system. Those who own cars spend about 11% of their income on travel, and those who do not spend about 4%. A wide variety of cities and countries at very different times have displayed this pattern.

Based on these observations, he developed the UMOT (Unified Mechanism of Travel) model for predicting travel behavior. He concluded that travelers have two budgets: one for time and one for money. Travelers have a maximum amount of time and money that they are willing to spend each day for transportation, and they attempt to obtain the greatest possible benefits within those constraints.

The benefits are in the form of work, shopping, school, recreation, and other activities to which people must travel. If they have more time and money in their budgets, they are willing to travel further to work (*i.e.*, to choose a residence location further from work location), or go further from home to shop, and so forth.

Zahavi used these ideas to develop a theory of urban transportation behavior and to help transportation planners. His original model did not include a detailed network representation. This was provided by Gershwin, Orlicki, and Zahavi (1981) and Gershwin, Orlicki and Platzman (1984a, b). The network representation, however, did not allow for the variation of traffic behavior during the course of a day.

The latter is of critical importance in understanding and predicting network performance. The subtle issue is that static causes may have dynamic effects. For example, Stephenson and Teply (1981) have shown experimentally that reducing the capacity of a roadway can stretch out a rush hour period.

In Section 2 of this paper, we review the static network version of Zahavi's model. Section 3 presents a new dynamic version of this work.

2. Static Assignment

The purpose of this section is to present a very brief summary of the time-invariant network analysis. Terms and concepts are defined informally here. More complete statements can be found in the references. Some definitions are repeated, suitably modified, in the section on dynamic networks.

2.1 Travel Budgets

Each traveler is assumed to have a time budget and a money budget. These budgets are assumed to vary randomly from person to person and for each person, from day to day. However, travelers are grouped into socioeconomic classes and there is a distribution function that characterizes each class.

The classification of travelers can include residence location and other attributes that affect how much they travel; all travelers of the same class have a certain range of income, a certain range of travel time budgets, a certain household size, a certain number of cars; they live in a certain neighborhood, and they have other similar attributes, as defined by the modeler.

Let $f^a(t,m)$ be the density function of budgets and D^a be the population of class

a. Let

$$U = \{(u,n) \mid u \geq t,\ n \geq m\}. \tag{2.1}$$

Then

$$D^a \int_U f^a(u,n)\ du\ dn \tag{2.2}$$

is the number of class a travelers in the system with time and money budgets greater that t and m, respectively. That is, this is the number of class a travelers who can afford a daily journey with time and money costs t and m.

2.3 Assignment Principle

A journey is a connected sequence of links and nodes in the traffic network. Because it represents a day's travel, it starts and ends at the same node.

Journeys are characterized by time and money costs and also by utility. Utility represents the desirability of a journey, and it is a measure of how many attractive potential destinations can be found along it. Journeys with many desirable workplaces, stores, theaters, scenic views, have greater utility than those with fewer. Utility is in the eye of the beholder; journeys with expensive stores are more attractive to the rich than to the poor, and *vice versa*.

Assume that there are k(a) journeys available to class a, and that they are indexed in order of increasing utility. Our basic assumption is:

Travelers choose the journey with the highest utility whose costs fall within their travel time and money budgets.

Let $X^a_{k(a)}$ be the number of travelers of class a that can afford, and thus travel on, journey k(a). Then

$$X^a_{k(a)} = D^a \int_{U^a_{k(a)}} f^a(u,n)\ du\ dn \tag{2.3}$$

where

$$U^a_{k(a)} = \{(u,n) \mid u \geq t^a_{k(a)},\ n \geq m^a_{k(a)}\}. \tag{2.4}$$

in which $t^a_{k(a)}$ and $m^a_{k(a)}$ are the costs of journey k(a) and $U^a_{k(a)}$ is the set of budgets that are greater than the costs of the journey.

Let X^a_i be the number of travelers of class a that can afford journey i or better. The number of travelers that can afford journey i but cannot afford any better journey is

$$x^a_i = X^a_i - X^a_{i+1} \tag{2.5}$$

Define the set of budgets that are greater than or equal to the costs of journey i to be

$$R^a_i = \{(u,n) \mid u \geq t^a_i,\ n \geq m^a_i\}. \tag{2.6}$$

The set of people who can take journey i or better are those whose budgets exceed the costs of journey i and those of journey i+1, and all those up to journey k(a). That is, it is the set of travelers whose budgets fall in set

$$U^a_i \equiv \bigcup_{j=i,\dots,k(a)} R^a_i \tag{2.7}$$

where the union is performed over all journeys j whose utility is greater than or equal to that of i. Then

$$X^a_i = D^a \int_{U^a_i} f^a(u,n)\ du\ dn. \tag{2.8}$$

To complete the picture, it is necessary to show how travel costs (t^a_i and m^a_i) depend on journey flows.

Let A^a be the link-journey incidence matrix for class a. That is,

$$A^a_{i\ell} = \begin{cases} 1 & \text{if link } \ell \text{ is in journey } i \\ 0 & \text{otherwise.} \end{cases} \tag{2.9}$$

Then if f_ℓ is the flow rate on link ℓ,

$$f_\ell = \sum_{a,j} A^a_{j\ell} x^a_j. \tag{2.10}$$

The travel costs on each link are functions of the flow on each link:

$$\left.\begin{aligned} \tau_\ell &= \tau_\ell(f_\ell) \\ \mu_\ell &= \mu_\ell(f_\ell), \end{aligned}\right\} \tag{2.11}$$

and the travel costs on a journey are simply the sums of the travel costs on the links that the journey traverses:

$$\left.\begin{aligned} t^a_j &= \sum_\ell A^a_{j\ell}\tau_\ell \\ m^a_j &= \sum_\ell A^a_{j\ell}\mu_\ell \end{aligned}\right\} \tag{2.12}$$

This completes the statement of the static assignment. The journey flows depend on the journey costs in (2.3)-(2.8); and the journey costs depend on the journey flows in (2.10)-(2.12).

3. Dynamic Assignment

In this section we translate the time-invariant assignment into one that can vary during the course of a day. To simplify notation and concepts, we only allow for a single class of travelers in the system. This means that all travelers have the same distributions of time and money to spend on traveling. It also means that they either live at the same location in the network or that they all may be allowed to change their origins (*i.e.*, their residences) equally freely.

3.1 Network Description

Let network $\mathcal{N}$ consist of nodes N and links L. A <u>location</u> is an element λ of the set

$$\mathcal{L} \equiv N \cup \{(\ell,z) \mid \ell \in L;\ 0 < z < d_\ell\} \tag{3.1}$$

where d_ℓ is the length of link ℓ. That is, a location is either a node ($n \in N$) or a point along a link (ℓ,z). A <u>journey</u> p is a function from $\mathcal{T}$ (time) to $\mathcal{L}$; that is, it is a sequence of locations $\lambda(s)$ where $s \in \mathcal{T}$. It must be continuous and consistent with the network topology, so that after leaving a node, it continuously traverses one of the links emanating from that node and then enters another node. In addition,

$$\lambda(T) = \lambda(0) \tag{3.2}$$

where T is the length of one day. Thus, a journey is a continuous path in the network starting and ending at the same point.

3.2 Assignment

An <u>assignment</u> is a probability distribution on $\mathcal{N}$ indexed by time s. For every time s, if λ is a node, $A(\lambda|s)$ is the probability of finding a traveler at location λ. If λ is a point on a link (ℓ,z), $A(\lambda|s)dz$ is the probability of finding a traveler between (ℓ,z) and $(\ell,z+dz)$.

Let D be the total population of travelers in the network. Then

$$D\ A(\lambda|s) \tag{3.3}$$

is the number of travelers at λ if λ is a node; otherwise it is the density of travelers at location λ along link ℓ.

3.3 Costs and Utility

Assume that journey p reaches the beginning of link ℓ at time s_ℓ. Let the <u>delay</u> on link ℓ be given by $\tau_\ell(s_\ell)$. Let the <u>money cost</u> of link ℓ be $\mu_\ell(s_\ell)$. These costs are incurred by travelers that arrive at link ℓ at time s_ℓ. They are functions of the distribution of travelers on that link and further downstream in the network.

One representation of these costs is

$$\left.\begin{aligned} \tau_\ell(s_\ell) &= \phi(\ell,\ DA((\ell,0)|s_\ell)) \\ \mu_\ell(s_\ell) &= \gamma(\ell,\ DA((\ell,0)|s_\ell)) \end{aligned}\right\} \tag{3.4}$$

where ϕ and γ are functions of the roadway (number of lanes, repair condition, tolls), and the arrival rate of travelers at the given time of day.

Let $w_\ell(s_\ell)$ be the <u>utility</u> of traveling on link ℓ when a traveler arrives at the start of that link at time s_ℓ. It is a function of the time of day that the traveler reaches the link, but not of the traffic conditions on the link.

Travel time (t(p)), cost (m(p)), and utility (w(p)) of a journey are the sums of the corresponding quantities for each link of journey p. These quantities are all positive or zero and are given by

$$\left.\begin{aligned} t(p) &= \sum_{\lambda(s_\ell)=(\ell,0)} \tau_\ell(s_\ell) \\ m(p) &= \sum_{\lambda(s_\ell)=(\ell,0)} \mu_\ell(s_\ell) \\ w(p) &= \sum_{\lambda(s_\ell)=(\ell,0)} w_\ell(s_\ell) \end{aligned}\right\} \tag{3.5}$$

Let ℓ_1 and ℓ_2 be two links such that ℓ_2 follows the node that ℓ_1 enters. If $\lambda(s)$ is a journey that arrives at ℓ_1 at s_{ℓ_1} and at ℓ_2 at s_{ℓ_2}, then for the journey to be consistent, the delay satisfies

$$\tau_{\ell_1}(s_{\ell_1}) = s_{\ell_2} - s_{\ell_1}. \tag{3.6}$$

3.4 Assignment Principle

Each traveler in the network is assumed to have a pair of <u>budgets</u>. He is not allowed to take a journey whose travel time exceeds his travel time budget or whose whose money cost exceeds his money budget. Travelers differ in having different pairs of budgets.

Let $f(\cdot,\cdot)$ be the density function of time and money budgets. Then, $Df(t,m)dtdm$ is the number of travelers whose time budgets are between t and t+dt and whose money budgets are between m and m+dm.

Assume that all the journeys in the network are ranked in order of utility. Then travelers behave according to the following principle:

Each traveler chooses the journey with the highest utility within both his budgets.

Let p^* be the journey with the highest utility. The number of travelers that take p^* is

$$X(p^*) = D \int_{R_{p^*}} f(u,n)\ du\ dn \tag{3.7}$$

where

$$R_{p^*} \equiv \{(u,n) \mid u \geq t(p^*),\ n \geq m(p^*)\} \tag{3.8}$$

is the set of budgets that are greater than the costs of journey p^*.

Define R_p as the set of budgets that exceed the costs of journey p:

$$R_p \equiv \{(u,n) \mid u \geq t(p),\ n \geq m(p)\} \tag{3.9}$$

The set of budgets of travelers who can afford journey p or better is given by

$$U_p \equiv \bigcup_{w(p')\geq w(p)} R_{p'} \tag{3.10}$$

where the union is performed over all journeys p' whose utility is greater than or equal to that of p.

The number of people who take journey p or one with a higher utility is then

$$X(p) = D \int_{U_p} f(u,n)\ du\ dn. \tag{3.11}$$

Define

$$dX(p) = \frac{dX}{dw}\ dw(p) \tag{3.12}$$

as the number of travelers who take a journey with utility between w(p) and w(p) + dw.

All that remains is to evaluate the costs of each journey. The costs depend on $DA(\lambda|s)$, as defined above.

Let $I(p,\lambda,s)$ be an indicator function which tells if a journey has passed through a given point at a given time.

$$I(p,\lambda,s) = \begin{cases} 1 & \text{if journey } p \text{ passes through location } \lambda \text{ at time } s; \\ 0 & \text{otherwise.} \end{cases} \tag{3.13}$$

The number of travelers that pass through location λ at time s is given by

$$D\ A(\lambda|s) = D \int_0^{w(p^*)} I(p,\lambda,s)dX(p) + D\ I(p^*,\lambda,s)X(p^*)$$

so that

$$A(\lambda|s) = \int_0^{w(p^*)} I(p,\lambda,s)dX(p) + I(p^*,\lambda,s)X(p^*) \tag{3.14}$$

Equation (3.14), finally, completes the assignment principle. The assignment A determines the costs and utilities through (3.4) and (3.5); the costs and utilities determine the distribution of travelers on journeys through (3.7)-(3.11); and the distribution of travelers defines the assignment through (3.12)-(3.14).

4. Conclusion

Considerable work remains before this principle can be used to help transportation planners. The dynamic formulation must be extended to include multiple classes of travelers. A computational algorithm must be developed to deal with the great complexity and data requirements of this model. Numerical experiments must be performed to show that the formulation behaves according to intuition. Most important, the model must be compared with the behavior of an actual transportation system.

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