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An Efficient 1-D Thermal Stratification Model for Pool-type Sodium-Cooled Fast Reactors

Cihang Lu, Zeyun Wu*, Sarah Morgan
Department of Mechanical and Nuclear Engineering, Virginia Commonwealth University, Richmond, VA 23219

James Schneider and Mark Anderson
Department of Engineering Physics, University of Wisconsin – Madison, Madison, WI 53706

Liangyu Xu and Emilio Baglietto
Department of Nuclear Science and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

Matthew Bucknor and Matthew Weathered
Argonne National Laboratory, Lemont, IL 60439

Sama Bilbao y Leon
OECD Nuclear Energy Agency
2, rue André Pascal 75775 Paris, France

ABSTRACT

Investigating thermal stratification in the upper plenum of a Sodium Fast Reactor (SFR) is currently a technology gap in SFR safety analysis. Understanding thermal stratification will promote safe operation of the SFR before its commercial deployment. Stratified layers of liquid sodium with a large vertical temperature gradient could be established in the upper plenum of an SFR during a down-power or a loss-of-flow transient. These stratified layers are unstable and could result in uncertainties for the core safety of an SFR. In order to predict the occurrence of the thermal stratification efficiently, we developed a 1-D transport model to estimate the temperature profile of the ambient fluid in the upper plenum. This model demands much less computational efforts than CFD codes and provides calculations with higher fidelity than historical system-level codes. Two flow conditions were considered separately in the current study depending on if in-vessel components are presented in the upper plenum. For the condition where in-vessel components, specifically the upper internal structure, are presented, we assumed that the impinging sodium was evenly dispersed in the ambient fluid within the distance between the bottom of the in-vessel

* Corresponding author: zwu@vcu.edu
component and the jet inlet surface. For the condition where no in-vessel components are presented, we assumed that the impinging sodium was evenly dispersed in the ambient fluid within the jet length which was determined through data-driven trainings. The newly developed 1-D model showed similar performance with the CFD model in both cases. However, due to the assumption of flat profiles of the impinging jet axial dispersion rate, non-negligible discrepancies between the 1-D prediction and the measured data were observed.

**Keywords:** Thermal stratification, Sodium fast reactor, System code, CFD, Sensitivity analysis.
I. Introduction

The SFR is one of the six Gen IV reactor designs which represent the future shape of nuclear energy. As one of the advanced nuclear reactors, the SFR uses liquid sodium as its primary coolant. Compared to light water, liquid sodium has higher heat capacity, greater thermal conductivity, and larger atomic weight. It can therefore provide an enhanced cooling ability to the reactor without thermalizing the neutrons. The fast neutron spectrum in an SFR also grants a better fuel economy because higher burnup can be achieved for both fissile and fertile materials in fast reactors. However, several key technology gaps remain to be filled to ensure a safe operation of the SFR before its commercial deployment. The thermal stratification behavior of the liquid sodium coolant is one of the key challenges.

Stratified layers of liquid sodium could be established in the upper plenum of a pool-type SFR during a down-power transient or a Protected Loss of Flow (PLOF) transient with SCRAM. In such conditions, the cooler coolant flows into the lower portion of the upper plenum while the upper portion remains hot. This established stratified layers of liquid sodium coolant with a large vertical temperature gradient. Thermal stratification could also take place during Unprotected Loss of Flow (ULOF) accidents without SCRAM, in which the hotter coolant flows into a relatively cooler upper plenum and leads to stratified layers. A schematic of thermal stratification that may exist in the upper plenum of an SFR in the event of a reactor trip is shown in Figure 1 (blue represents cooler sodium and red represents hotter sodium).

![Figure 1. A schematic of an upper plenum of an SFR [1].](image)

Being unstable, the stratified layers could result in low-frequency temperature oscillations of fairly large amplitude [2], which could further cause neutronic and thermal-hydraulic instabilities in the reactor core. The stratified layers could also impede the start of natural circulation during
the loss-of-flow accidents and introduce more uncertainties to the core safety, or result in damages of both the reactor vessel and in-vessel components, such as the Upper Instrumentation Structure (UIS), due to thermal fatigue crack growth. In order to understand the consequences that thermal stratification may cause to reactor safety accurately, an efficient yet accurate approach to predict thermal stratification is firstly desired. Remarkable efforts have therefore been made in the literature to predict the thermal stratification phenomenon with different fidelities.

Several system-level codes are capable of providing predictions of the thermal stratification phenomenon. However, these system-level codes employ 0-D or 1-D models, and can only provide approximated solutions for simple cases. For example, the Argonne National Laboratory (ANL) -developed SAS4A/SASSYS-1 uses a 0-D stratified volume model by dividing the upper plenum into three regions considering five sequential stages of the phenomenon [3]. The Institute for Radiological Protection and Nuclear Safety (IRSN) -developed DYN2B employs a 0-D zone model based on the Richardson number [4]. The Japan Atomic Energy Agency (JAEA) -developed Super-COPD utilizes a 1-D model which cannot provide a prediction as correctly as 3-D models [5], and the University of California, Berkeley (UC Berkeley) -developed BMIX++ can only be used for stable or nearly stable thermal stratifications [6].

With the improving computational power, the computational fluid dynamics (CFD) modeling of the thermal stratification phenomenon has been proven feasible. Different commercial CFD codes, including FLUENT [7], STAR-CD [8], STAR-CCM+ [8] and CFX-13 [9], have been used to perform 3-D analyses of thermal stratification in the upper plenum of the Japanese Monju reactor, and showed good results. However, being computationally expensive and time consuming, the CFD modeling is not suitable when a large number of transient calculations are needed for the reactor core safety analyses. Moreover, when whole-plant transient analyses are desired, if the CFD codes are used to model the regions of 3-D interest, they have to be coupled with the system-level codes which model the rest of the reactor circuit. Efforts have been made in the literature regarding the methodologies of coupling CFD codes with system-level codes. The coupling between the IRSN-developed system-level code CATHARE with the French Atomic Energy Commission (CEA) -developed CFD code TRIO_U [4], Idaho National Laboratory (INL) -developed system-level code RELAP5 with STAR-CCM+ [10], SAS4A/SASSYS-1 with both STAR-CD [11] and STAR-CCM+ [12], as well as several other combinations of codes have been realized. Feedback is provided mutually between the system-level code and the CFD code when a whole-plant analysis is performed, which provided good results. However, as the thermal stratification phenomenon is modeled by the CFD method, a system-level – CFD codes coupling is also computational expensive and time consuming. A status review of various thermal stratification modeling methodologies can be found in the work of Morgan et al. [13].

Therefore, a fast-running 1-D thermal stratification model with improved fidelity is indispensible for the reactor development, core safety analyses, and the reactor licensing process when a large amount of transient calculations are to be performed. The 1-D thermal stratification
model will also be easy to implement into the system-level codes to perform whole-plant transient analyses. Wilson and Bindra [14] developed a 1-D transport model based on the advection-diffusion equation to predict the thermal behavior in the upper plenum of an SFR. The flow condition considered was similar to that of a PLOF accident with a cold coolant jet entering from the bottom of the plenum. The results of their 1-D model showed reasonable comparison with CFD calculations when the mass flow rate of the impinging jet was relatively low. However, the 1-D model developed by Wilson and Bindra could only predict the coolant temperature along the centerline of the geometry, and was therefore not applicable to the cases where the center of the upper plenum is occupied by the in-vessel components. Moreover, the temperature along the centerline of the upper plenum is different from that of the ambient fluid in the upper plenum which physically contacts the reactor vessel and the in-vessel components.

In light of these observations, we developed a 1-D system-level model based on the theoretical framework established by Peterson [15] for the prediction of the thermal stratification phenomena in the pool-type SFRs with improved fidelity. The model was built upon 1-D transient governing equations. By assuming the Neumann boundary conditions, we solved the 1-D equation numerically by using center difference scheme for the spatial discretization and semi-implicit approach for the temporal discretization. The newly developed 1-D model is able to predict the temperature profile of the ambient fluid in the upper plenum of an SFR in both conditions whether there are in-vessel components presenting in the center of the plenum or not. Some preliminary results of our work were reported in the recent ANS winter meeting [16]. The current paper includes a more thorough and complete discussion of the development of our 1-D model. The rest of the paper is organized as the following: In section II, the flow conditions in which the thermal stratification may occur in a pool-type SFR are summarized, and the conditions focused in the current study are specified. In section III, the experimental facility and its CFD model used for the validation of the 1-D thermal stratification model are briefly introduced. In section IV, the 1-D thermal stratification model is described in detail, followed by numerical discretization of the newly developed 1-D thermal stratification model for computational implementation. In section V, the newly developed 1-D thermal stratification model is validated against both the CFD calculations and the experimental data obtained in the current study. In the last section, the completed efforts are summarized, and a perspective of future investigations to this project is briefly discussed.

II. Flow conditions considered

We considered three different flow conditions in the current study, as depicted in Figure 2, which may potentially lead to the stratification phenomena in the upper plenum of an SFR:

1. The impinging jets have a higher temperature (therefore a lower mass density) than that of the ambient fluid;
2. The impinging jets have a lower temperature (therefore a higher mass density) than that of the ambient fluid, and there are in-vessel components blocking the impinging jets at the inlets.

3. The impinging jets have a lower temperature (therefore a higher mass density) than that of the ambient fluid, and there are no in-vessel components in the tank.

Figure 2. Different flow conditions in which the thermal stratification phenomena may occur.

**Condition 1: \( T_{\text{jet}} > T_{\text{sf}} \)**

This situation could occur in an SFR during a ULOF accidental scenario, in which hotter sodium coolant enters the cooler upper plenum from its bottom. We currently did not focus on this condition because we were not able to experimentally observe the thermal stratification phenomenon in any of the experiments performed in the corresponding flow conditions. Therefore, we did not have any experimental data to validate our 1-D model in this condition.

**Condition 2: \( T_{\text{jet}} < T_{\text{sf}} \) with in-vessel components in the pool**

This situation could occur in an SFR during a PLOF accidental scenario, in which cooler sodium coolant enters the hotter upper plenum from its bottom. When there are in-vessel components located close to the inlet of the jet, the impinging jets will not be able to rise above the in-vessel components. The jet could therefore be considered as completely dispersed within the distance between the inlet of the jet and the in-vessel component. In our experiments, an upper instrumentation structure (UIS) was installed in the tank above the inlets to simulate the in-vessel components in the upper plenum. The vertical distance between the bottom of the UIS and the inlets was \( z_{\text{UIS}} = 5 \text{ cm} \).

**Condition 3: \( T_{\text{jet}} < T_{\text{sf}} \) with no in-vessel components in the pool**

Similar to condition 2, this situation could occur in an SFR during a PLOF accidental scenario in which cooler sodium coolant enters the hotter upper plenum from its bottom. The impinging
jets could reach a higher height as no in-vessel components presented. In the current study, we proposed a model for the prediction of the maximum height that the impinging jets could reach.

III. Experimental design and CFD model

The Thermal Stratification Experimental Facility (TSTF) was developed at the University of Wisconsin-Madison to provide experimental data for the validation of the 1-D model, as shown in Figure 3. Figure 4 gives a diagram of the TSTF test section. In the experiments, jets of sodium were injected into a pool of sodium from its bottom to mimic the inlet flow to the upper plenum of an SFR. More detailed descriptions of the TSTF can be found in our previous publication [1]. Twelve thermocouples were installed in the test section of the TSTF at six different axial levels for temperature measurements as indicated in Figure 4.

Two outlets at different levels were designed to examine the effects of the thermal stratification. However, only the high outlet has been used so far to generate experimental data. The temperature measurements, obtained from the 8 thermocouples located lower than the high outlet, were used for the validation of the 1-D model developed in the current study. The vertical distance between these thermocouples and the jet inlets were 16.5 cm (TC 35/29), 27.9 cm (TC 34/28), 55.9 cm (TC 33/27), and 69.9 cm (TC 32/26). All the thermocouples were installed 2.54 cm (1 inch) from the wall of the TSTF test section. The test conditions of the experiments performed are summarized in Table 1.

Figure 3. The Thermal Stratification Test Facility with description of component location.
Figure 4. Positions of the thermocouples in the TSTF.

Table 1. Test conditions of the experiments performed.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Inlet T (℃)</th>
<th>Ambient initial T (℃)</th>
<th>Flow rate (gpm)</th>
<th>Flow rate (L/s)</th>
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<tr>
<td>1</td>
<td>200</td>
<td>250</td>
<td>6</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>250</td>
<td>10</td>
<td>0.63</td>
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<td>3</td>
<td>200</td>
<td>225</td>
<td>10</td>
<td>0.63</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>300</td>
<td>1.5</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>250</td>
<td>3</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>300</td>
<td>3</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>250</td>
<td>10</td>
<td>0.63</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>300</td>
<td>10</td>
<td>0.63</td>
</tr>
</tbody>
</table>

A CFD model of the test section of the TSTF was built, and simulations were performed for all the experimental settings of the thermal stratification. The CFD model contained all geometrical details of the test section, including the thermocouples. About five million cells were used to represent the geometry of the test section, as shown in Figure 5. The CFD model was first built to
inform the design of the experimental facility, and then used for the validation of the 1-D model by comparing the calculation results with both methods. More detailed descriptions of the CFD model can be found in our previous publication [1].

Figure 5. Geometry and CFD mesh of the experimental test section with UIS.

IV. 1-D thermal stratification model

IV.A. Governing Equations

In order to provide a better idea of the 1-D model development process, the complete mathematics derivation and the numerical discretization process are shown. We focused the 1-D modeling on the ambient fluid in the upper plenum of an SFR, because it has direct contact with the heat structures. Following the work of Peterson [15], we considered the ambient fluid to be quasi-steady in our 1-D model. The radial temperature variation of the ambient fluid was neglected, and the impinging jets were considered as a heat source of the ambient fluid with negligible volumes. By using an integration technique, the governing field equations for the ambient fluid were simplified as the following, corresponding to the conservation law of mass, momentum, and energy, respectively

\[
A_{sf}(z) \frac{\partial \rho_{sf}}{\partial t} + \frac{\partial (\rho_{sf} Q_{sf})}{\partial z} = \sum_{k=1}^{N_{jet}} \rho_k Q'_k
\]

\[
\frac{\partial \rho_{sf}}{\partial z} = -\rho_{sf} g
\]

\[
A_{sf}(z) \frac{\partial (\rho_{sf} h_{sf})}{\partial t} + \frac{\partial (\rho_{sf} h_{sf} Q_{sf})}{\partial z} - A_{sf}(z) \frac{\partial}{\partial z} \left( k_{sf} \frac{\partial T_{sf}}{\partial z} \right) = \sum_{k=1}^{N_{jet}} \rho_k h_k Q'_k
\]

where

- \( A_{sf} \) is the surface area of the ambient fluid;
- \( Q_{sf} \) is the vertical volume flow rate of the ambient fluid;
\( h_{sf} \) is the enthalpy of the ambient fluid (stratified fluid);
\( k_{sf} \) is the thermal conductivity;
\( \rho_k \) is the density of the \( k^{th} \) impinging jet;
\( Q'_k \) is the volumetric dispersion rate of the \( k^{th} \) impinging jet;
\( h_k \) is the enthalpy of the \( k^{th} \) impinging jet;
\( N_{jet} \) is the number of all the impinging jets.

By defining the horizontal surface area averaged velocity
\[
\bar{u}_z(z) = \frac{q_{sf}(z)}{A_{sf}(z)} = \frac{\sum_{k=1}^{N_{jet}} Q'_k}{A_{sf}(z)} \tag{4}
\]
the mass conservation equation (1) and energy conservation equation (3) can be written as:
\[
\frac{\partial \rho_{sf}}{\partial t} + \frac{\partial (\rho_{sf} \bar{u}_z)}{\partial z} = \frac{1}{A_{sf}(z)} \sum_{k=1}^{N_{jet}} \rho_k Q'_k \tag{5}
\]
\[
\frac{\partial (\rho_{sf} h_{sf})}{\partial t} + \frac{\partial (\rho_{sf} h_{sf} \bar{u}_z)}{\partial z} - \frac{\partial}{\partial z} \left( k_{sf} \frac{\partial T_{sf}}{\partial z} \right) = \frac{1}{A_{sf}(z)} \sum_{k=1}^{N_{jet}} \rho_k h_k Q'_k \tag{6}
\]
By combing the equation (5) with (6), we obtain the non-conservative form of the energy
conservation equation as the following:
\[
\rho_{sf} \frac{\partial h_{sf}}{\partial t} + \rho_{sf} \bar{u}_z \frac{\partial h_{sf}}{\partial z} - \frac{\partial}{\partial z} \left( k_{sf} \frac{\partial T_{sf}}{\partial z} \right) + h_{sf} \frac{1}{A_{sf}(z)} \sum_{k=1}^{N_{jet}} \rho_k Q'_k = \frac{1}{A_{sf}(z)} \sum_{k=1}^{N_{jet}} \rho_k h_k Q'_k \tag{7}
\]
By using the thermodynamic relation for enthalpy change \( dh = c_p dT \), equation (7) can be written
in terms of temperature rather than enthalpy:
\[
\rho_{sf} c_p \frac{\partial T_{sf}}{\partial t} + \rho_{sf} c_p \bar{u}_z \frac{\partial T_{sf}}{\partial z} - \frac{\partial}{\partial z} \left( k_{sf} \frac{\partial T_{sf}}{\partial z} \right) = \frac{1}{A_{sf}(z)} \sum_{k=1}^{N_{jet}} (\rho Q')_k (h_k - h_{sf}) \tag{8}
\]
In the case where the inlet jet and the ambient fluid have heat capacities close to each other, and
the inlet jets are identical, equation (8) can be rewritten as:
\[
\rho_{sf} c_p \frac{\partial T_{sf}}{\partial t} + \rho_{sf} c_p \bar{u}_z \frac{\partial T_{sf}}{\partial z} - \frac{\partial}{\partial z} \left( k_{sf} \frac{\partial T_{sf}}{\partial z} \right) = \frac{N_{jet}}{A_{sf}} c_{p, jet} \rho_{jet} Q'_{jet} (T_{jet} - T_{sf}) \tag{9}
\]
Eq. (9) is the equation that we need to solve for the temperature profile of the ambient fluid, and
\( Q'_{jet} \) is the parameter that we need to calculate through closure equations.

**IV.B. Turbulence-enhanced heat transfer**

The ambient fluid may become turbulent due to the dispersion of the impinging jets, and the
heat transfer of the ambient fluid may be enhanced because of the turbulence. Shih et al. [17]
studied the relation between the turbulence-enhanced thermal diffusivity \( \alpha_{tot} \) and the static
thermal diffusivity $\alpha_c$ using direct numerical simulations (DNS). Based on the calculation results, they defined three mixing regimes according to the ratio of the turbulent Reynolds number $Re_t$ to the Richardson number $Ri$, and developed empirical correlations between $\alpha^{tot}$ and $\alpha_c$ in each mixing regime, as summarized in Table 2. The proposed correlation and its applicability for the analysis of thermal stratification were further validated by Ward et al. [18].

Table 2. Correlations between $\alpha^{tot}$ and $\alpha$ [17].

<table>
<thead>
<tr>
<th>Regime</th>
<th>$\frac{Re_t}{Ri}$</th>
<th>$\alpha^{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular</td>
<td>$Re_t &lt; 150$</td>
<td>$\alpha_c$</td>
</tr>
<tr>
<td>Transitional</td>
<td>$150 &lt; \frac{Re_t}{Ri} &lt; 1000$</td>
<td>$0.015\alpha_c \cdot \left(\frac{Re_t}{Ri}\right)^{0.5}$</td>
</tr>
<tr>
<td>Energetic</td>
<td>$1000 &lt; \frac{Re_t}{Ri}$</td>
<td>$0.015\alpha_c \cdot \left(\frac{Re_t}{Ri}\right)^{0.5}$</td>
</tr>
</tbody>
</table>

In the current study, we neglected the impact of sodium mass density and heat capacity on its thermal diffusivity and assumed the correlation between the turbulence-enhanced thermal conductivity $k_{sf}$ and the static thermal conductivity $k_c$ to be similar to that between $\alpha^{tot}$ and $\alpha_c$. The turbulence Reynolds number of the jet was defined by Jones and Launder [19] as:

$$Re_t = \frac{\rho k^2}{\mu \epsilon}$$

where $k$ was the turbulent kinetic energy, and $\epsilon$ was its dissipation rate. In the current study, we used the same estimations of these two turbulence parameters as those proposed in the work of Lai et al. [20]:

$$k = 0.01U_{jet}^2$$

$$\epsilon = \frac{2k^{3/2}}{d_{jet}}$$

where $U_{jet}$ was the entering velocity of the jets, and $d_{jet}$ was the diameter of the inlets of the jets. The Richardson number of the ambient fluid was defined as:

$$Ri_{sf} = \frac{g}{\rho_{sf,0} (\partial u_z/\partial z)^2}$$

The $\rho_{sf,0}$ was the initial density of the ambient fluid. In the current study, we assumed that the impinging jets uniformly dispersed in the ambient flow within a length of $L_{jet}$, and made the following approximations:
\[
\frac{\partial \rho_{sf}}{\partial z} = \frac{(\rho_{sf,0} - \rho_{jet})}{L_{jet}} \tag{14}
\]

\[
\frac{\partial u_z}{\partial z} = \frac{Q_{jet}}{A_{sf}} / L_{jet} \tag{15}
\]

We calculated the ratio of the turbulent Reynolds number \( Re_T \) to the Richardson number \( Ri \) for each experiment performed, and found that the ambient fluid was classified to be in the molecular regime in all the experimental settings. This implied that the impinging jets did not introduce a turbulence significant enough to enhance the heat transfer in the ambient fluid. The estimation of the jet length, \( L_{jet} \), is discussed in section V.

IV.C. Numerical Discretization

We used the standard staggered scheme with uniform mesh size, as depicted in Figure 6, for the discretization of the governing equations. Field variables such as density \((\rho)\), pressure \((P)\), enthalpy \((h)\) and temperature \((T)\) were defined at the mesh center, and flow variables such as velocities \((u)\) and volumetric flow rates \((Q\) and \(Q')\) were defined at the mesh edge. The standard control volume approach was used for the spatial discretization, and the semi-implicit approach was used for the temporal discretization.

![Figure 6. Mesh scheme used in the control volume method.](image)

The first order spatial derivative was approximated by the upwind scheme and the second order spatial derivative was approximated by the center difference scheme. The discretized form of Eq. (9) for the mesh-average temperature of the ambient fluid at the mesh \( i \) in the time step \( n+1 \) can be written as:

\[
\rho_i c_{p,i} \frac{\partial T_i}{\partial t} + \rho_i c_{p,i} \bar{u}_z,i \frac{T_i - T_{i-1}}{\Delta z_i} - \frac{2}{\Delta z_i} k_i \left[ \frac{T_{i+1} - T_i}{\Delta z_{i+1} + \Delta z_i} - \frac{T_i - T_{i-1}}{\Delta z_i + \Delta z_{i-1}} \right] = \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,i} \left( T_{jet} - T_i \right) \tag{16}
\]

By using the semi-implicit time discretization approach, equation (16) became:
\[
\rho_l^n c_{p,l} T_{i-1}^{n+1} - T_i^n \Delta t_n = \rho_l^n c_{p,l} \bar{u}_{z,i} \frac{T_{i+1}^{n+1} - T_i^{n+1}}{\Delta z_i} - \frac{2}{\Delta z_i} k_i^n \left[ \frac{T_{i+1}^{n+1} - T_{i+1}^{n}}{\Delta z_{i+1} + \Delta z_i} - \frac{T_i^{n+1} - T_i^{n}}{\Delta z_i + \Delta z_{i-1}} \right]
\]
\]

in which all the thermal-hydraulics parameters were calculated in the previous time step, and the temperature at the new time step were obtained by solving the equation sets. Equation (17) can further be expressed in the form of matrices:

\[
A_{i,i-1} T_{i-1}^{n+1} + A_{i,i} T_i^n + A_{i,i+1} T_{i+1}^{n+1} = B_i
\]

where

\[
A_{i,i-1} = -\left( \rho_l^n c_{p,l} \frac{1}{\Delta z_i} + \frac{2}{\Delta z_i} k_i^n \frac{1}{\Delta z_i + \Delta z_{i-1}} \right)
\]

\[
A_{i,i} = \rho_l^n c_{p,l} \frac{1}{\Delta t_n} + \rho_l^n c_{p,l} \bar{u}_{z,i} \frac{1}{\Delta z_i} + \frac{2}{\Delta z_i} k_i^n \left[ \frac{1}{\Delta z_{i+1} + \Delta z_i} + \frac{1}{\Delta z_i + \Delta z_{i-1}} \right]
\]

\[
A_{i,i+1} = -\frac{2}{\Delta z_i} k_i^n \frac{1}{\Delta z_{i+1} + \Delta z_i}
\]

\[
B_i = \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,i} T_{jet} + (\rho_l^n c_{p,l} \frac{1}{\Delta t_n} - \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,i}) T_i^n
\]

For the inlet mesh, positioned from \( z_i = 0 \) to \( z_2 = \Delta z_i \), we used the Neumann boundary condition and assumed \( T_0 = T_1 \). Equation (17) became:

\[
\rho_l^n c_{p,1} \frac{T_1^{n+1} - T_1^n}{\Delta t_n} - \frac{2}{\Delta z_1} k_1^n \left[ \frac{T_2^{n+1} - T_1^{n+1}}{\Delta z_2 + \Delta z_1} \right] = \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,1} (T_{jet} - T_1^n)
\]

Writing it into the matrix form of the system equations

\[
A_{1,1} T_1^{n+1} + A_{1,2} T_2^{n+1} = B_1
\]

where

\[
A_{1,1} = (\rho_l^n c_{p,1} \frac{1}{\Delta t_n} + \frac{2}{\Delta z_1} k_1^n \frac{1}{\Delta z_2 + \Delta z_1})
\]

\[
A_{1,2} = -\frac{2}{\Delta z_1} k_1^n \frac{1}{\Delta z_2 + \Delta z_1}
\]

\[
B_1 = \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,1} T_{jet} + (\rho_l^n c_{p,1} \frac{1}{\Delta t_n} - \frac{N_{jet}}{A_{sf}} c_{p,jet} \rho_{jet} Q'_{jet,1}) T_1^n
\]

For the outlet mesh, positioned from \( z_{N+1} = L \) to \( z_N = z_{N+1} - \Delta z_N \), we also used the Neumann boundary condition and assumed \( T_{N+1} = T_N \). In this case, equation (17) became:
\[
\rho_N^n c_p^N \frac{T_N^{n+1} - T_N^n}{\Delta t_n} + \rho_N^n c_p^N \overline{u}_{z,N}^n \frac{T_N^{n+1} - T_N^{n+1}}{\Delta z_N} + \frac{2}{\Delta z_N} k_N^n \left[ \frac{T_N^{n+1} - T_N^{n+1}}{\Delta z_N + \Delta z_{N-1}} \right] = \frac{N_{\text{jet}}}{A_{sf}} c_p \rho \rho_{\text{jet}} Q'_{\text{jet,N}} (T_{\text{jet}} - T_N^n)
\]

Writing it into the matrix form of the system equations

\[
A_{N,N-1} T_{N-1}^{n+1} + A_{N,N} T_N^{n+1} = B_N
\]

where

\[
A_{N,N-1} = -\rho_N^N c_p^N \overline{u}_{z,N}^n \frac{1}{\Delta z_N} - \frac{2}{\Delta z_N} k_N^n \frac{1}{\Delta z_N + \Delta z_{N-1}}
\]

\[
A_{N,N} = \rho_N^N c_p^N \frac{1}{\Delta t_n} + \rho_N^N c_p^N \overline{u}_{z,N}^n \frac{1}{\Delta z_N} + \frac{2}{\Delta z_N} k_N^n \frac{1}{\Delta z_N + \Delta z_{N-1}}
\]

\[
B_N = \rho_N^N c_p^N \frac{T_N^n}{\Delta t_n}
\]

V. Training and validation of the 1-D model

V.A. Sensitivity analysis

In order to choose appropriate time and space steps to solve equation (17), we analyzed the sensitivity of the ambient fluid temperature profile to both steps. The experimental setting #1, with an initial ambient fluid temperature of 250 ℃, a jet temperature of 200 ℃, and a jet volumetric flow rate of 6 gallons per minute (gpm), was used for the sensitivity analysis. We chose to investigate the sensitivity of the temperature profile, at an elapsed time of 100 s, to both the time and the space steps, because the sensitivity at this elapsed time is the most eminent compared to others, as shown in Figure 7.

Figure 7. 1-D-model-predicted ambient fluid temperature profile at different elapsed times for experimental setting #1.
We first analyzed the sensitivity of the ambient fluid temperature profile to the time step. A comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different time steps is shown in Figure 8. The space step used for these calculations was \( \Delta z = 1 \ cm \), and the times step used varied from \( \Delta t = 5 \ s \) to \( \Delta t = 0.01 \ s \). It was observed that a \( \Delta t = 0.01 \ s \) was sufficiently small such that the temperature profile calculated converged.

![Figure 8. Comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different time steps.](image)

Using the temperature profile calculated with \( \Delta t = 0.01 \ s \) as the reference, the maximum temperature error caused by using larger time steps was calculated. In Figure 9, the y-axis shows the error caused by using different time steps, and x-axis shows the relative finesse of the time step defined by:

\[
\text{relative finesse of the time step} = \frac{0.01 \ s}{\Delta t}
\]  

The time step \( \Delta t = 0.5 \ s \) was chosen for all the future calculations as it required a relatively shorter calculation time and guaranteed an acceptable error smaller than 1 °C.
We also analyzed the sensitivity of the ambient fluid temperature profile to the space step. A comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different space steps is shown in Figure 10. The space step used for these calculations was $\Delta z = 1 \text{ cm}$ and the times step used varied from $\Delta t = 5 \text{ s}$ to $\Delta t = 0.01 \text{ s}$. It was observed that a $\Delta t = 0.01 \text{ s}$ was sufficiently small such that the temperature profile calculated converged.

Figure 9. Comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different time steps.

Figure 10. Comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different space steps.
Using the temperature profile calculated with $\Delta z = 0.25 \text{ cm}$ as the reference, the maximum temperature error caused by using larger time steps was calculated. In Figure 11, the y-axis shows the error caused by using different time steps, and x-axis shows the relative finesse of the time step defined by:

$$\text{relative finesse of the time step} = \frac{0.25\text{cm}}{\Delta z}$$

(34)

Figure 11. Comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with different space steps.

We chose a space step of $\Delta z = 1 \text{ cm}$ for the future calculations, such that the combination of both time and space steps would at most introduce an error of 1 °C to the calculated temperature profile. A comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with $\Delta t = 0.5 \text{ s}$ and $\Delta z = 1 \text{ cm}$ to that calculated with $\Delta t = 0.01 \text{ s}$ and $\Delta z = 0.25 \text{ cm}$ is shown in Figure 12.
Figure 12. Comparison of the ambient fluid temperature profile at 100 s elapsed time calculated with $\Delta t = 0.5 \, s$ and $\Delta z = 1 \, cm$ to that calculated with $\Delta t = 0.01 \, s$ and $\Delta z = 0.25 \, cm$.

**V.B. Condition 2: $T_{jet} < T_{sf}$ with in-vessel components in the pool**

The experiments used for the validation of the 1-D model for this condition were experiments #1, #2 and #3. In all the three experiments, we had an Upper Instrumentation Structure (UIS) installed in the test section, the bottom of which was about 5 cm away from the inlet surfaces of the jets. The jets hit the UIS after entering through the inlets and were not able to rise above the UIS before dispersing in the ambient fluid, as shown in Figure 13. Therefore, for the 1-D model, we assumed that the impinging sodium was evenly dispersed in the ambient fluid within the distance between the bottom of the UIS and the jet inlet surface ($z_{UIS}$). The jet length $L_{jet} = z_{UIS}$, and the impinging sodium volumetric dispersion rate was $Q'_{jet} = Q_{jet}/N_{jet}L_{jet}$. 
The calculation of the ambient fluid temperature distribution for experiment #1 was completed in less than 6 seconds by using the newly developed 1-D model. The temperature of the ambient fluid predicted by the 1-D model at different axial locations is shown in Figure 14, in comparison with both the CFD predictions and the experimental data. At the beginning of the experiment, a steady state was established in the test section, and the ambient fluid had a uniform temperature. Because of the injection of the cooler sodium, the temperature of the ambient fluid closer to the jet inlets started to decrease. Through the accumulation of the cooler jets dispersed, the temperature of the ambient fluid would eventually converge to that of the impinging jets. The time-averaged percentage differences between the predicted axial temperatures and the experimental data were compared, as shown in Figure 15. Both the 1-D model and the CFD method under-predicted the temperature of the ambient fluid. The CFD method provided slightly better predictions than the 1-D model, but the accuracy of both methods were overall very similar at all the axial locations.

The temperature of the ambient fluid predicted by the 1-D model as a function of the elapsed time is shown in Figure 16, in comparisons with both the CFD predictions and the experimental data. It can be seen that the temperature predicted by the 1-D model decreased faster than the experimental data. This is because the mixing of the jet and the ambient fluid took time, while the 1-D model assumed that the dispersion process was instantaneous. The axial-location-averaged percentage differences between the predicted axial temperatures and the experimental data were compared, as shown in Figure 17. The 1-D model provided slightly better predictions than the CFD method during the first 75 s. The differences between the predicted temperatures and the measurement data decreased after 150 s because the temperature of the ambient fluid in the whole test section converged to that of the impinging jets.
Figure 14. Comparison of the predicted temperature with experimental data at different axial locations for experiment #1.

Figure 15. Time-averaged percentage difference between the predicted temperature and the experimental data for experiment #1.
Figure 16. Comparison of the predicted temperature with experimental data at different elapsed time for experiment #1.

Figure 17. Axial-location-averaged difference between the predicted temperature and the experimental data for experiment #1.

The differences between the predicted axial temperatures and the experimental data at different measurement locations are shown respectively in Figure 18. Both the 1-D model and the CFD model gave a larger under-prediction of the temperature at a higher location, when the measurement location was farther away from the jet inlets. The difference between the CFD and experimental results is attributed to the limitations of the Eddy diffusivity assumption, which is
known to be not fully applicable for very low Prandtl number fluids. The larger discrepancy between the 1-D prediction and the experimental data revealed the fact that our estimation of \( Q'_{jet} \) introduced more errors to the predicted temperature at higher locations. The applicability of Shih et al.’s correlation for the eddy thermal diffusivity could potentially be another explanation for the observed temperature discrepancies, which requires further investigation in the future.

![Figure 18. Difference between the predicted temperature and the experimental data at different measurement locations for experiment #1.](image)

The maximum errors of the predictions for experiments #1~#3 are summarized in Table 3. Overall, the CFD and the 1D model had similar performance, and the maximum error of the predicted temperature was proportional to the difference between the initial temperature of the ambient fluid and that of the impinging jets.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Inlet T (°C)</th>
<th>Initial T (°C)</th>
<th>( \Delta T ) (°C)</th>
<th>Flow rate (gpm)</th>
<th>Max error CFD (°C)</th>
<th>Max error 1D (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>250</td>
<td>-50</td>
<td>6</td>
<td>-21.7</td>
<td>-21.0</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>250</td>
<td>-50</td>
<td>10</td>
<td>-18.2</td>
<td>-20.5</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>225</td>
<td>-25</td>
<td>10</td>
<td>-8.8</td>
<td>-9.6</td>
</tr>
</tbody>
</table>

**V.C. Condition 3:** \( T_{jet} < T_{sf} \) with no in-vessel components in the pool

When there is no UIS installed in the tank, the impinging jets can rise without hitting any in-vessel components, as shown in Figure 19. It was more complicated to predict the \( Q'_{jet} \) in this condition, because we needed first to predict the \( L_{jet} \) due to the absence of the UIS. After entering
the tank, the forced jet will decelerate due to the stopping force. It will either completely disperse in the ambient fluid or partially exits from the outlet of the tank before dispersing in the ambient fluid.

![Temperature distribution](image)

**Figure 19.** CFD-calculated temperature distribution at 180 s elapsed time for experiment #5.

In the current study, we considered that the stopping force of each jet consisted of two components, namely the surface force and the body force. The surface force was proportional to the density of the ambient fluid, the surface area of the jet inlet, and the velocity squared of the impinging jet:

\[
F_D = -C_D \rho_{sf} A_{jet} v_{jet}^2
\]  

where \(C_D\) was the drag coefficient. The body force consisted of the gravity force and the buoyancy force, and can be expressed as:

\[
F_g = -(\rho_{jet} - \rho_{sf}) V_{jet}
\]

Therefore, for a hypothetical cylindrical jet with a length \(L_{jet}\), the resultant acceleration of the jet was:

\[
\frac{dv_{jet}}{dt} = \frac{F_D + F_g}{\rho_{jet} A_{jet} L_{jet}} = -\left(\frac{C_D v^2 \rho_{sf}}{L_{jet} \rho_{jet}} + \frac{\rho_{jet} - \rho_{sf}}{\rho_{jet}}\right)
\]

and

\[
dv_{jet} = -(C \frac{v^2 \rho_{sf}}{\rho_{jet}} + \frac{\rho_{jet} - \rho_{sf}}{\rho_{jet}}) dt
\]
by noting $C = \frac{C_D}{L_{jet}}$.

By numerically integrating the velocity of the jet over time, we obtained the maximum height that the jet reached in the ambient fluid (jet length $L_{jet}$). We further put forward a data-driven training process to find the coefficient $C$ that best fits our experimental data, as shown in Figure 20. During the training process, we gradually varied the coefficient $C$, and calculated the corresponding $L_{jet}$. Then we calculated the volumetric dispersion rate of the impinging jet by assuming that the jet was evenly dispersed in the ambient fluid within the jet length ($Q'_{jet} = Q_{jet} / N_{jet} L_{jet}$) and obtained the ambient fluid temperature profile. We compared the temperature predicted at both the highest (TC 29/35) and the lowest (TC 26/32) measurement locations with those obtained from the experiments, and found the coefficient $C$ that provided the best fit of the 1-D prediction to the experimental data.

Figure 20. Diagram of the training process.

In the current study, we used the experimental setting #5 and #6 to train our 1-D model, and obtained the best fitting coefficient $C = 4.3$. We then used the data acquired in experimental settings #4, #7 and #8 for the validation process. Comparisons of the 1-D-model and the CFD-
model predictions to the experimental data as a function of the elapsed time are shown in Figure 21 and Figure 22 for experimental settings #4 and #7 respectively. The 1-D model had similar performance as the CFD model for experimental setting #4 while it did not work as well as the CFD model in experimental setting #7 and #8.

Figure 21. Comparison of the predicted temperature with experimental data at different elapsed time for experiment #4.

Figure 22. Comparison of the predicted temperature with experimental data at different elapsed time for experiment #7.

The maximum errors of the predictions for experiments #4 through #8 are summarized in Table 4. The 1-D model had similar performance as the CFD model at low jet flow rates. However, the 1-
D model does not work as well as the CFD model at high jet flow rates. This is because when the temperature at a higher location is of interest, the assumption that “the impinging jet was evenly dispersed in the ambient fluid within the jet length” would introduce more error to the temperature prediction.

Table 4. Summary of errors of the predicted temperature for experiments #4–#8.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Inlet T (°C)</th>
<th>Initial T (°C)</th>
<th>ΔT (°C)</th>
<th>Flow rate (gpm)</th>
<th>Ljet (cm)</th>
<th>Max error CFD (°C)</th>
<th>Max error 1-D (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>200</td>
<td>300</td>
<td>-100</td>
<td>1.5</td>
<td>27.8</td>
<td>-21.7</td>
<td>-23.0</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>250</td>
<td>-50</td>
<td>3</td>
<td>49.0</td>
<td>-11.4</td>
<td>-10.4</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>300</td>
<td>-100</td>
<td>3</td>
<td>42.7</td>
<td>-28.0</td>
<td>-26.2</td>
</tr>
<tr>
<td>7</td>
<td>200</td>
<td>250</td>
<td>-50</td>
<td>10</td>
<td>78.0</td>
<td>-3.6</td>
<td>-7.8</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>300</td>
<td>-100</td>
<td>10</td>
<td>72.8</td>
<td>-5.7</td>
<td>-12.1</td>
</tr>
</tbody>
</table>

VI. Summary and future work

We developed a 1-D system-level model for the prediction of the thermal stratification phenomena in the pool-type SFRs with improved fidelity. Both flow conditions focused on in the current study correspond to the down-power transients or the PLOF accidents with SCRAM in an SFR, where cooler coolant flows into the lower portion of the upper plenum. The equation set to be solved for the prediction of the temperature profile of the ambient fluid was:

\[
\rho_n \cdot c_p \cdot \frac{T_i^{n+1} - T_i^n}{\Delta t} + \rho_n \cdot c_p \cdot \overline{u} \cdot n \cdot \frac{T_i^{n+1} - T_i^{n-1}}{\Delta z_i} - \frac{2}{\Delta z_i} \cdot k_i \cdot \left[ \frac{T_i^{n+1} - T_i^{n-1}}{\Delta z_i + \Delta z_{i-1}} \right] = \frac{N_{jet}}{A_{sf}} \cdot c_{p,jet} \cdot \rho_{jet} \cdot Q'_{jet,i} \cdot (T_{jet} - T_i^n) \]  

(39)

For the flow conditions considered in the current study, the time step \( \Delta t = 0.5 \) s and the space step \( \Delta z = 0.1 \) cm were chosen with the help of sensitivity analyses. The volumetric dispersion rate of the impinging jet, \( Q'_{jet} \), was modeled differently in the 1-D model depending on if there was in-vessel components present in the upper plenum.

For the experiments with UIS installed, we assumed that the jet length \( L_{jet} = z_{UIS} \), and the impinging sodium was evenly dispersed in the ambient fluid within the jet length \( Q'_{jet} = Q_{jet} / N_{jet} \cdot L_{jet} \). The performance of the newly developed 1-D model was proved to be similar with that of the CFD model.

For the experiments without UIS installed, we put forward a training process to find the best-fit \( L_{jet} \), and considered \( Q'_{jet} = Q_{jet} / N_{jet} \cdot L_{jet} \). The 1-D model showed similar performance with CFD calculations when the impinging jet flow rate was low, but the performance of the 1-D model was not as good as the CFD model when the jet flow rate was high. It is noted that for the flow
conditions without UIS, the newly developed 1-D model was only trained with two experimental data sets, and validated against three other experimental sets. The performance of the 1-D model is expected to be improved, and become convincing when more experimental data sets are available for both the training and the validation process.

Due to the assumption of a quasi-steady flow, the radial temperature variation of the ambient fluid was neglected in our 1-D model. Moreover, despite the similar performance showed by the newly developed 1-D model with that of the CFD model, non-negligible discrepancies between the 1-D prediction and the measured data were observed due to the simplification that we made by assuming a flat profile of the impinging jet axial dispersion rate, \( Q_{jet}' \). Several experimental studies have been performed in the literature, and some non-dimensional numbers have been proved to impact the thermal stratification phenomenon [21-23], including the Reynolds number (Re), the Prandtl number (Pr), the Peclet number (Pe), and the Richardson number (Ri) of the impinging jets. Future improvements can be made to the newly developed 1-D thermal stratification by investigating impinging jet axial dispersion rates correlated to these non-dimensional numbers. Additionally, the eddy thermal diffusivity assumption employed in the computational models could be another uncertainty source that may cause the non-negligible discrepancies. A follow-up sensitivity study on the eddy diffusivity can be performed to explore its influence on the 1-D model predictions. Moreover, the current model does not consider the heat loss through solid structures (either the vessel wall or the UIS inside the plenum), future efforts can also be made to integrate the conjugate heat transfer effects between the fluid and solid into the current model.

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