HUNTING MOTION OF HIGH SPEED TRACKED VEHICLES

by

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ABSTRACT

The complexity and the slow speed of the trains have caused
this mode of transportation to receive very little theoretical attention.
The design of the truck is considered to be an art rather than a science.
High speed certainly demands extensive theoretical and experimental
investigations. In 1964, the Japanese were successful in applying
theoretical and experimental investigations which resulted in the develop-
ment of the "NEW TOKAIDO LINE" with top speed of 250 km/hr. From
a study of Japanese publications and being aware of a French record of
331 km/hr. for 90 seconds in 1955, arose the question, "What is the
inherent critical speed above which a four wheel truck equipped train can-
not be operated safely or provide ride quality?" The answer to this
question for such a mechanical system with so many degrees of freedom
and external parameters is too complicated to be handled analytically.
This study concerns the analysis of what is considered the most hazardous
motion in high speed train transportation, i.e. hunting. The model used
for this study consists of a rigid frame with a pair of two wheel sets
incorporating free rotation. This bogie is suspended longitudinally and
transversally from the car body. The car body is moving with constant
velocity on a smooth, straight, rigid track [page 10]. A formula has
been derived for determining the critical speed which is a function of ten
independent variables [page 18]. In order to handle the independent
variables eight non-dimensional groups were introduced [page 21].
Value has been determined for six out of seven of these groups [page 24].
By comparing and inspecting the non-dimensional groups, the effective
value of each independent variable has been determined [page 26]. For
a range of practical variables, the critical speed has been calculated by
independently changing the values in recommended directions. This
resulted in an increase in speed from 40 miles/hr. to infinity [page 29 and 30].

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carrying out the study, and in making the study a valuable learning
experience for the author.

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GENERAL

Two of the essential problems to be solved for realization of high speed trains are: safety and ride comfort. As to the safety, the problem is of two different categories.

1. The car-dynamics such as over-turn, derailment and strength of materials.

2. The operational techniques such as headway, signals, safety appliances, etc.

Similarly, the ride comfort has two different categories:

1. The dynamic qualities such as vibration, acceleration and deceleration.

2. Aesthetic qualities such as noise, illumination, air-conditioning, color, seats, etc.

Car dynamics and dynamic riding qualities are linked together, and form the dynamic study of the car and truck. Such a study is very extensive. The subject of this present investigation has application in safety from derailment and reduction of the vibration of riding quality.

Derailment: By definition, derailment is, "The phenomenon of a wheel flange riding on the rail top and falling from it". [1] The forces acting between the rail and wheel are shown as in Fig. 1.
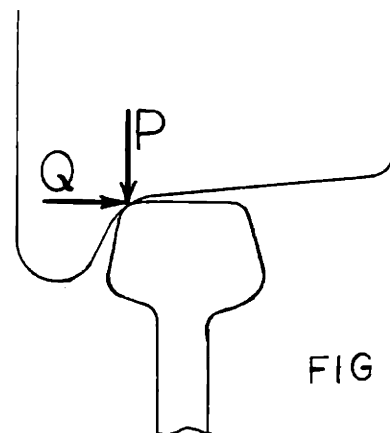
P = Vertical force

Q = Lateral force

FIG 1

As Q becomes larger, it is easier for the flange to ride over the rail. The ratio of q = Q/P is interpreted as an index of the dangerousness of derailment, and this is called "derailment quotient". The critical value of this "q" when derailment happens varies within a wide range. Experimental results show the effects of conditions of contact, such as the inclination of the inside surface of the flanges, and the frictional co-efficient between the contact surfaces. However, the critical values of "q" could not be determined by simple mathematical relations. By neglecting impact, wheel jump, and dynamic limitation. The answer would be of an emperical nature to be determined expecially on the curved tracks [1, 2, 3, 4]. The critical values of the derailment quotient on the existing cars with the speed under 120 km/hr. are small and, thus, there is no anxiety of derailment. Experimental results, however, have shown that the quotient has a tendency of increasing rapidly when the vehicle's speed gets over a critical amount for each particular car. This phenomenon is the result of the hunting (snaking) motion of the axle-wheel under the conditions of the experiment. The derailment due to hunting at high speed for 1/5 model train has shown the three following results: [2]

1.   When the hunting of axle-wheel gets sufficiently severe and the impact speed of the wheel flange against rail exceeds a critical value, the wheel jumps and the car finally derails.

2.   The height of the wheel jump is proportional to the square of impact speed of the wheel flange. The derailment occurs when the jumping height exceeds the flange height.

3.   The side trust caused by the impact of the wheel against the track is proportional to the impact speed.

The prevention of hunting is one of the big steps in the realization of high speed trains. The problem is quite old but has not been completely worked out. When this is achieved, we can assume that the high speed train is safe from derailment. However, the residual car-vibrations must be controlled to secure ride comfort.

It is impossible, in advance, to estimate the car vibrations. Theory and experiment should fill this gap.

From a theoretical point of view, the nature of car-vibration is both forced vibrations, which is caused by the irregularity of the rails, and the self-excited vibrations, which can happen on rails that are perfectly flat and straight. The former is usually expected to reach a constant amplitude after the car speed exceeds some value [1, 5, 6]. The latter usually makes a sudden appearance at the critical speed, and increases rapidly. There is no self-excited vibration in the vertical direction, but on the other hand, the lateral vibration includes hunting of the axle-wheel, and the motion is fundamentally of the self-excited nature. Up to several hundred miles per hour, it is relevant to mention that the dynamic characteristic of the track shows no significant importance in this present study [7, 8].

Information about ride quality can be found in the references.

THE GENERAL MOTION OF A RAILWAY VEHICLE

The simple vehicle case has ten degrees of freedom on a smooth and straight track, in which the vehicle consists of a body of four wheel sets incorporated without longitudinal and lateral play, and suspended by springs which govern the vertical displacement of each wheel set with regard to the body. With uniform rotations of wheels, six degrees of freedom are possible. The movement of such a system is described by assuming a point of reference which is attached to the body, in such a way that the point is situated on the plane through the axis of revolution of the wheel sets, when the vehicle is in the center of the track. The displacements of this point with regard to a fixed coordinate system are designated by X (in longitudinal direction), Y (in the transverse direction), and Z (in the vertical direction), and calling θ, $\curlywedge$, ϕ, and ω the angles of rotation about the respective axis of this system. The angle of rotation of wheel sets $\underline{i}$ with respect to its axis of revolution is ω_i. Then, we have these motions:

The Symmetrical Movement
- The recoiling movement (X)
- The jumping movement (Z)
- The pitching movement ($\curlywedge$)
- The rotation of the wheel sets (ω_i)

The Lateral Movement
- The transverse movement (Y)
- The rolling movement (θ)
- The hunting movement (ϕ)

With the assumption of rigid tracks, these co-ordinates completely determine the movement. In the general case, coupling exists between the equations for lateral and symmetrical movements. Such a system is not a train, and a better mathematical model is: A car body equipped with two, four-wheeled trucks with three degrees of freedom (X, Y, φ) for each truck, and six degrees of freedom for the car body. Such a system would have a total of twelve degrees of freedom. It is possible that some of the motions can be neglected for further simplifications, but still it would be too complicated to give a closed solution, and a physical picture. Other researchers have made further assumptions based on experience and intuition. As the first important approximation, it was assumed that the hunting of the truck could be considered independent of car motion. Thus, the problem is simplified to hunting of the truck.

HUNTING OF FOUR WHEELED HIGH SPEED TRUCK

Since the 1920's, few people have worked out theoretical investigations on simplified models for trucks and locomotives [5, 9 - 20]. Their idealized models were quite far from the older conventional bogies. The reports of these investigations actually did not contain any specific recommendations. Some formula and suggestions of a more empirical nature are available [6, 21, 22].

Trucks on the "NEW TOKAIDO LINE" in Japan have received theoretical and experimental treatment [1, 2, 23]. Also, by studying the new developments in bogie designs such as General Steel's passenger car truck, "THE GENERAL 70" [24], and Budd Company's pioneer No. III design, [25, 26], the following model is proposed, limited by simpler mathematical manipulations, as in Fig. II from which many designs could be idealized to it.

The Mathematical Model: This model contains the following assumptions:

FIG 2
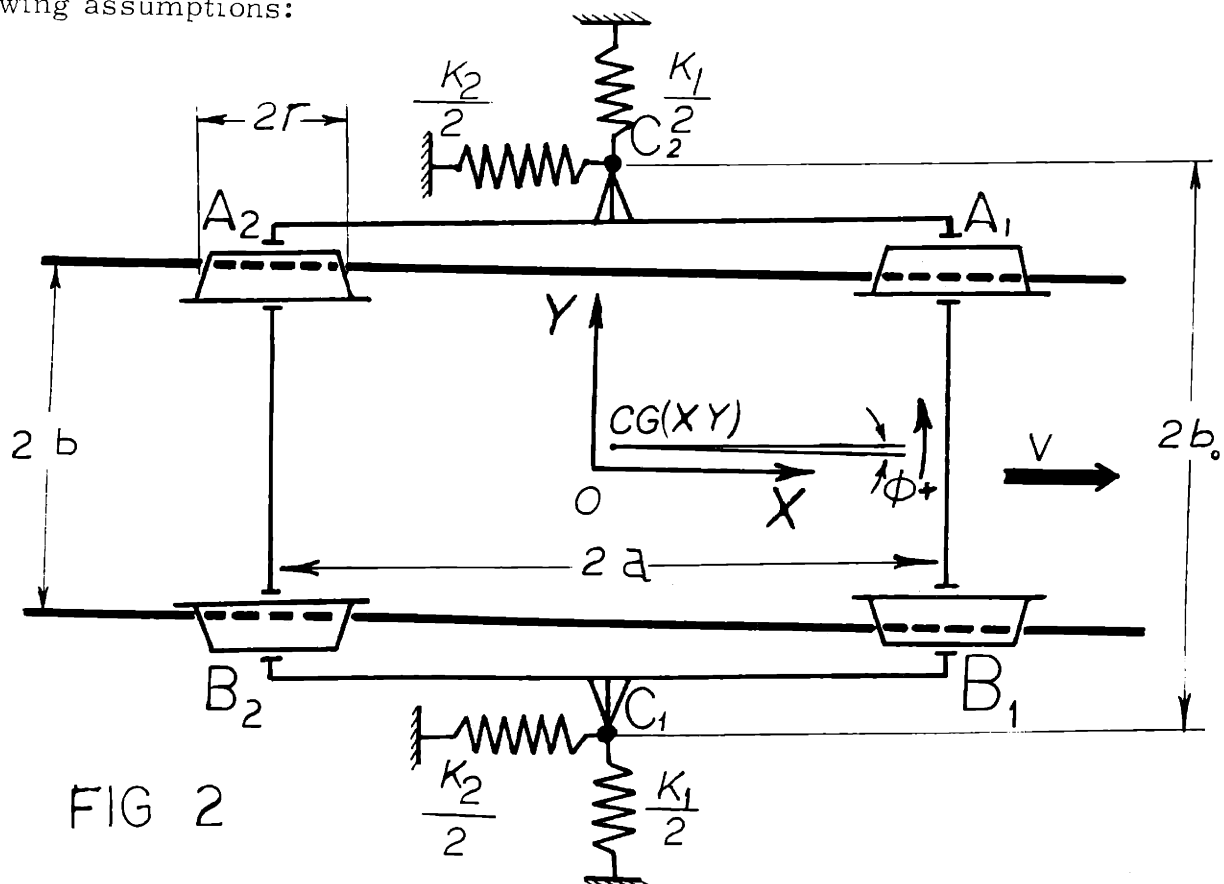
1. The car moves with constant velocity v.

2. The car body has a very large mass com-
    pared to the mass of the truck.

3. The vertical spring between the car body,
    and the truck is infinitely soft.

4. The load is uniform on all four wheels.

5. Infinitely soft journal spring between
    wheels and truck frqm (no vertical move-
    ment for the frame).

6. Neither longitudinal nor lateral play bet-
    ween the wheel set.

7. The vehicle is symmetrical and wheels have
    the same nominal radius and coning.

8. Rigid and smooth track, with no irregularity.

9. The track is straight.

10. Longitudinal and lateral spring's stiffness
    k_1, k_2 = Constant (the damping on k_1, k_2
    would add to laborand they are not instructive
    for the purpose of this work).

11. Small parasitic motions.

Before becoming involved with the treatment, it is rele-
vant to mention we see under the above assumption that the ω of the
wheels is constant (v = rω), and the system assumes three degrees
of freedom (X, Y, ϕ). Practically, some of these assumptions are
difficult to reach. A mathematical model with no spring connection to
the body for the trucks, and locomotives have been proposed before.

Analytical results could be found in [5, 10, 11, 12, 13, 14, 15]. The result for a four-wheel truck indicates stable or unstable motion for cylindrical or connical wheels for all speeds respectively and this is as far as can be deduced. However, because of the flange impacts, the motion of the system is non-linear even for the case of cylindrical wheels. Any disturbance would cause flange impact and the system again should be treated nonlinearly [15, 16, 19, 20]. By considering the suspensions effect of a car body, we will determine the effect of each individual member of the truck.

Forces acting on the Wheel: We assume that the tangential forces between wheel and rail are proportional to the so-called creep which implicates that the propulsion forces are exerted from the wheels, and the motion of the wheel is not a simple rolling [27].

$$F = f \cdot \frac{\text{velocity of creep}}{\text{velocity of roll}} \qquad (1)$$

Where f = creepage coefficient. This relation was first derived by F. W. Carter [28] based on a treatment of theory elasticity [29]. Another treatment by theory of elasticity gave the same answer [30]. In these treatments, f is found to be:

$$f = 3500 \, [rLw]^{1/2} \, [1 + (1-q)^{1/2}]$$

where:

f = creepage coefficient

r = radius of the wheel

L = a linear magnitude representing the

effective length of the contact area

transverse to the rail in inches

q = the ratio of the tangential force to the maximum that could be sustained without skidding or bodily slipping

W = load on the wheel

From the measurements, it appears that this law holds sufficiently exact when the values of creep are less than about 0.2 [19]. We see that the creepage coefficient increases twice as much when the tractive effort decreases to zero, and q and L are a matter of conjecture. The usual assumption of L = 1 is reasonable and recent experimental results agree with that [31].

So, we can write,

$$f = A[r \cdot w]^{1/2}$$

and "A" could be found experimentally. B. S. Cain [5, 12] has tested and given:

$$f = 3500 \, [r \cdot w]^{1/2} \, \text{lb.}$$

W = load on the wheel lb.

r = radius of the wheel inches

For our numerical example, we shall use this relation. Even though the above-mentioned treatment has been derived for forces acting in longitudinal direction, experimental results show that this law is equally applicable to transverse direction [5, 9].

Calculations: For Fig. II, we distinguish the following:

M   =   Mass of the truck

I   =   Movement of inertia of the truck about

the vertical axis through CG

R   =   Radius of gyration

f   =   Creepage coefficient

v   =   Velocity of the truck, uniform

$2b$ =   Gage

$2b_o$ =   Side Bearer

$2a$ =   Wheel Base

k_1 =   Transversal spring constant

k_2 =   Longitudinal spring constant

r   =   Nominal radius of the wheel

λ =   Wheel Cone

L^2 = $a^2 + b^2$

ω =   Angular velocity of wheel $\omega = \dfrac{v}{r}$

The bogies C.G. assume the small x, y displacements and rotate through the small angle ϕ with respect to moving coordinates X, Y. We assume positive forces and movement along the sense of coordinates.

At contact points A_1, A_2, B_1, B_2

$$A_1 \begin{bmatrix} x + a - b\phi \\ y + a\phi + b \end{bmatrix} \qquad r_{A_1} = r + \lambda\,(y+a\phi)$$

$$B_1 \begin{bmatrix} x + a + b\phi \\ y + a\phi - b \end{bmatrix} \qquad r_{B_1} = r - \lambda\,(y+a\phi)$$

$$A_2 \begin{bmatrix} x - a - b\phi \\ y - a\phi + b \end{bmatrix} \qquad r_{A_2} = r + \lambda\,(y-a\phi)$$

$$B_2 \begin{bmatrix} x - a + b\phi \\ y - a\phi - b \end{bmatrix} \qquad r_{B_2} = r - \lambda\,(y-a\phi)$$

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Using Equation (1):

$$F_{A_1} \begin{cases} -f\left[\dfrac{\dot{x}-b\dot{\phi}}{v} - \dfrac{\lambda(y+a\phi)\omega}{v}\right] \\[3em] -f\left[\dfrac{\dot{y}+a\dot{\phi}}{v} - \phi\right] \end{cases} \tag{2}$$

$$F_{B_2} \begin{cases} -f\left[\dfrac{\dot{x}+b\dot{\phi}}{v} + \dfrac{\lambda(y+a\phi)\omega}{v}\right] \\[3em] -f\left[\dfrac{\dot{y}+a\dot{\phi}}{v} - \phi\right] \end{cases} \tag{3}$$

$$F_{A_2} \begin{cases} -f\left[\dfrac{\dot{x}-b\dot{\phi}}{v} - \dfrac{\lambda(y-a\phi)\omega}{v}\right] \\[3em] -f\left[\dfrac{\dot{y}-a\dot{\phi}}{v} - \phi\right] \end{cases} \tag{4}$$

$$F_{B_2} \begin{cases} -f\left[\dfrac{\dot{x}+b\dot{\phi}}{v} + \dfrac{\lambda(y-a\phi)\omega}{v}\right] \\[3em] -f\left[\dfrac{\dot{y}-a\dot{\phi}}{v} - \phi\right] \end{cases} \tag{5}$$

Forces due to springs on the truck:

$$F_{C_1} \begin{cases} -\dfrac{k_2}{2}(x+b_o\phi) \\[3ex] -\dfrac{k_1}{2}y \end{cases} \tag{6}$$

$$F_{C_2} \begin{cases} -\dfrac{k_2}{2}(x-b_o\phi) \\[3ex] -\dfrac{k_1}{2}y \end{cases} \tag{7}$$

For this system, Newton's equations are:

$$M\ddot{x} + \frac{4f}{v}\dot{x} + k_2 x = 0 \tag{8}$$

$$M\ddot{y} + \frac{4f}{v}\dot{y} + k_1 y - 4f\phi = 0 \tag{8}$$

$$I\ddot{\phi} + \frac{4f}{v}(a^2+b^2)\dot{\phi} + k_2 b_o^2 \phi + \frac{4fb\lambda y}{r} \tag{10}$$

We see that the x direction motion is not coupled with the other two and is a damped motion.

Assuming for the motion,

$$y = a_1 e^{st} \tag{11}$$

$$\phi = a_2 e^{st} \tag{12}$$

the characteristic equation will fall off [32]

$$s^4 + s^3 \cdot \frac{1}{IM} \cdot \frac{4f}{v}(ML^2+I) + s^2 \frac{1}{IM}[16\frac{f^2}{v^2}L^2 + k_2 b_o^2 M + k_1 I]$$

$$+ s\frac{1}{IM}\frac{4f}{v}[k_2 b_o^2 + k_1 L^2] + \frac{1}{IM}(k_1 k_2 b_o^2 + \frac{16f^2}{r}\lambda b) = 0 \qquad (13)$$

which could be written as:

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_o = 0$$

The following two conditions for stability should be satisfied simultaneously [32]:

I = All coefficients should be positive

$$II = A_1 A_2 A_3 > A_1^2 + A_3^2 A_o \qquad (14)$$

By inspection, we see that the first condition is satisfied and the second condition gives:

$$\frac{1}{v^2} > \frac{M}{16\cdot r}[-\frac{(k_2 b_o^2 - k_1 R^2)^2}{A^2 W(L^2+R^2)(k_2 b_o^2 + k_1 L^2)} + \frac{16\lambda b(L^2 + R^2)}{L^2(k_2 b_o^2 + k_1 L^2)}] \qquad (15)$$

This equation could be used to determine the critical speed of the particular design. As we see, there are two terms in the bracket; one, which is negative, and the other, which is positive. If there is a design that can be created to make the first term numerically larger than the second term, then we have stability for all speeds. Here, we shall find the properties to minimize the right hand side of Equation 15. From the dimensional form, we can deduce:

1. M = Mass of the truck to be minimum

2. r = Radius of the wheel to be made large

3. To make the first term of some significance,

    W should be as small as possible, which

    indicates a light weight car is desirable.

Wheel Cone: For $\lambda = 0$ (Eq. 15) shows that stability exists for all speeds. This result can be found also in 7 and 12. In this case, for better treatment, the system should be treated as non-linear, because more likely, any disturbance will cause flange impact. For a treatment when $k_1 = k_2 = 0$ (see 16). The cylindrical wheels have been known and experienced some years ago. Prototypes with cylindrical wheels, for any disturbance, finally after flange impacts, come to a smooth running with leading and trailing flanges against the same rail. The speed and properties of the truck merely determined whether it approached this condition smoothly or by a decreasing series of flange impacts; for these wheels the flange wear had been such a problem that the cylindrical wheel was considered impractical. However, the result is that λ should be made as small as possible.

Equation 15 has the following variables:

1. v = Uniform speed of the car

2. f^2 = Creepage coefficient

   = $A^2 \cdot W \cdot r$

3. M = Mass of the truck

4. R = Radius of gyration about the vertical axis at CG

5. b = Half the gage width

6. a = Half the base length

7. b_o = Half the side bearer

8. r = Wheel nominal radius

9. λ = Wheel cone (dimensionless)

10. k_1 = Transverse spring constant

11. k_2 = Longitudinal spring constant

We introduce the following dimensionless group:

1. $\dfrac{1}{V} = \dfrac{Mv^2}{16\,\mathit{fL}}$

2. $K_1 = \dfrac{k_1 L}{f}$

3. $K_2 = \dfrac{k_2 b_o}{f}$

4. $\xi = \dfrac{b_o}{L}$

5. $\eta = \dfrac{R}{L}$

6. $\zeta = \dfrac{b}{L}$

7. $\gamma = \dfrac{r}{L}$

8. $\lambda = \lambda$

Equation 15 in terms of these   dimensionless groups:

$$V > -\frac{K_2\,\xi - K_1\,\eta^2}{(K_2\,\xi + K_1)(\eta^2 + 1)} + \frac{16\lambda\zeta(\eta^2 + 1)}{\gamma(K_1 + K_2\,\xi)} \tag{16}$$

Experimental results [2] show that a wide b_o, large k_2 is desirable. We assume that for the designs,

$$k_2 b_o^2 > k_1 R^2 \tag{17}$$

then it follows that,

$$K_2 \xi > K_1 \eta^2 \tag{18}$$

From the non-dimensional form again we can deduce that:

1. M should be "small" in favor of large V

also,

$$\frac{\partial V}{\partial \lambda} = \frac{16 \zeta(\eta^2 + 1)}{\gamma(K_1 + K_2 \xi)} > 0 \tag{19}$$

indicates that small λ is desirable.

We also have,

3. $$\frac{\partial V}{\partial \zeta} = \frac{16(\eta^2 + 1)}{\gamma(K_1 + K_2 \xi)} > 0 \tag{20}$$

this gives,

$$\zeta = \frac{b}{L} \quad \text{should be "small"}$$

and,

$$1 - \zeta^2 = \frac{a^2}{L^2} \quad \text{becomes "large"}$$

4. $\dfrac{\partial V}{\partial \gamma} = -\dfrac{16\zeta(\eta^2+1)}{\gamma^2(K_1+K_2\dot{\xi})} < 0$ (21)

$\gamma = \dfrac{r}{L}$ should be "large".

With the above assumption that,

$$K_2\,\xi - K_1\,\eta^2 > 0$$

we get,

5. $\dfrac{\partial V}{\partial \eta} > 0$, and so   (22)

$\eta = \dfrac{R}{L}$ should be "small"   (23)

6. $\dfrac{\partial V}{\partial \xi} < 0$, and so

$\xi = \dfrac{b_o}{L}$ should be "large

7. $\dfrac{\partial V}{\partial K_2} < 0$, and so   (25)

$K_2 = \dfrac{k_2 b_o}{f}$ to be "large"

$\partial V/\partial K_1$ does not give its sign and should be determined for specific design. We tabulate the results:

1. M = Mass of the truck to be made   "small"

2. W = Load on the wheel to be made   "small"

3. r = Radius of the wheel to be made   "large"

4. λ = Wheel cone to be made   "small"

5. ζ = $\dfrac{b}{L}$ to be made   "small"

6. γ = $\dfrac{r}{L}$ to be made   "large"

7. ξ = $\dfrac{b_o}{L}$ to be made   "large"

8. η = $\dfrac{R}{L}$ to be made   "small"

9. K_2 = $\dfrac{k_2 b_o}{f}$ to be made   "large"

10. K_1 = $\dfrac{k_1 L}{f}$ to be determined for specific design

In order to find the desirable dimensions for "L", we can find that:

$$\left|\left(\frac{\partial V}{\partial \zeta}\right) / \left(\frac{\partial V}{\partial \gamma}\right)\right| = \frac{\gamma}{\zeta} = \frac{r}{b} < 1 \qquad (26)$$

and also from:

$$\left|\left(\frac{\partial V}{\partial \xi}\right) / \left(\frac{\partial V}{\partial \gamma}\right)\right| = \left| \frac{1}{128} \frac{R \cdot r^2 \cdot L^3 [k_2 b_o^2 - k_1 R^2]}{f[R^2 + L^2]^3 \lambda \cdot b} [k_1 R^2 + k_2 b_o^2 + 2k_1 L^2] \right.$$

$$\left. + \frac{2rR}{R^2 + L^2} \right. \qquad (27)$$

By only inspecting the second part of this relation for a large range of practical design, we can deduce that,

$$\left|\left(\frac{\partial V}{\partial \eta}\right) / \left(\frac{\partial V}{\partial \gamma}\right)\right| > 1.$$

From these   and a numerical inspection, we deduce that "L" should be made "large".

THE CONCLUSIONS

From the above results, we draw the following conclusion for all the properties:

1. M = Mass of the truck should be made "small"

2. W = Weight on the wheel should be made "small"

3. r = Radius of the wheel should be made "large"

4. λ = Wheel coning, should be made "small"

5. L = $(a^2+b^2)^{1/2}$ should be made "large"

6. b = Gage, should be made "small"

7. b_o = Longitudinal spring arm should be made "large"

8. R = Radius of gyration about a vertical axis through C.G. should be made "small"

9. k_2 = Longitudinal spring constant should be made "large"

10. k_1 = Numerical inspection in a range of practical variables showes that, k_1 should be made "large"

NUMERICAL EXAMPLE

In prevention of hunting and based on previous results, a distinction between a good and bad design is made. However, it should be known that, for some of the dimensions, a compromise should be made for the other considerations. For example, a wide gage is recommended for safety from overturn, while for hunting prevention a narrower gage is desirable. From the study of typical truck, the best and the worst value of the variables, which are practical, are tabulated.

			Good	Bad
w	=	weight of the truck	2,500 lb.	15,000 lb.
W_1	=	total weight on eight wheels	60,000 lb.	150,000 lb.
r	=	radius of the wheel	18 in.	10 in.
λ	=	wheel cone	1/60	1/10
b	=	half the track gage	28 in.	28 in.
b_o	=	half the side bearer	60 in.	30 in.
R	=	radius of gyration	20 in.	50 in.
k_1	=	transverse spring constant	10,000 lb./in.	2,500 lb./in.
k_2	=	longitudinal spring constant	50,000 lb./in.	3,000 lb./in.

The above figures are to clarify the discussion mentioned herein, and to justify it for today's trucks which could be idealized to fall into this range. Also, to show how it is possible to achieve high speed through the recommended better design. Some of these figures may require a major design study. For example, small mass of the truck and its relative largeness with respect to the mass of the car body to fulfill our idealization may suggest other mode of propulsion. The value for the variables has not been confined on today's practice. By no means should these figures be looked upon as the limits.

In the following table, we have listed some numerical results. The following comments should be made:

1. Up to Column 8, it is shown how the design of heavy trains could be improved, which has also been partly a matter of experience before. In these results, the first term of Equation 15 is of no significance.

2. Weight of the car body has been a matter of controversy; in these results, it has been shown how these have been possible, and the use of Equation 15 will show how this controversy could have happened.

3. In Columns 8 - 11, the concept of high-speed rolling stock has been implied through the advantages of the light weight car and large b_o, k_2, k_1, and small R, λ.

TABLE OF NUMERICAL EXAMPLES

		1	2	3	4	5	6
w lb.	=	15,000	-	-	-	-	-
W_1 lb.	=	140,000	-	120,000	-	-	-
a in.	=	30	-	-	40	50	-
b in.	=	28	-	-	-	-	-
b_o in.	=	0	-	-	-	-	-
R in.	=	20	-	-	25 in.	30	-
r in.	=	16	-	-	-	18	-
k_1 lb./in.	=	2,500	-	-	3,000	-	7,000
k_2 lb./in.	=	10,000	-	-	-	-	-
λ	=	1/10	1/20	-	-	-	-
$\dfrac{1}{v^2}\left(\dfrac{\text{sec.}^2}{\text{in.}}\right)$	=	2×10^{-6}	1.003×10^{-6}	-	6.02×10^{-7}	3.922×10^{-7}	1.68×10^{-7}
v mile/hr.	=	40	56.7	56.7	73	90.8	138.8

TABLE OF NUMERICAL EXAMPLES (CONT'D)

		7	8	9	10	11
w	lb. $=$		2,500	–	–	–
W_1	lb. $=$	–	60,000	–	–	–
a	in. $=$	–	–	–	–	–
b	in. $=$	–	–	–	–	–
b_o	in. $=$	30	–	50	–	–
R	in. $=$	–	–	–	20	–
r	in. $=$	–	–	–	–	–
k_1	lb./in. $=$	10,000	–	–	–	–
k_2	lb./in. $=$	–	–	–	–	50,000
λ	$=$	–	–	1/40	–	–
$\dfrac{1}{v^2}$	$\left(\dfrac{\text{sec.}^2}{\text{in.}}\right) =$	0.923×10^{-7}		5.30×10^{-9}	4.39×10^{-9}	-0.434×10^{-8}
v	mile/hr. $=$	187	187	780	857	∞

SUMMARY

It has been shown that a high speed truck could be realized. Further safety and ride comfort, theoretical and experimental, study would determine the exact dimensions of such a truck. The main question of "What is the inherent critical speed above which a four-wheel truck equipped train cannot be operated safely or provide the ride quality?" has not been answered. It has been demonstrated, however, that it is possible, through a new concept of bogie design, to prevent the hunting motion of the truck. Hence, the trains with four-wheel trucks show promise for High Speed Ground Transportation.

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