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Electronic Systems Laboratory
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SECOND STATUS REPORT ON

OPTIMAL DESIGN METHODS FOR MULTIVARIABLE
SAMPLED-DATA CONTROL SYSTEMS

NASA GRANT NSG 1312

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0. SUMMARY

This status report presents a brief summary of research in progress under grant NSG 1312 for the time period 2/1/77 to 11/31/77.

1. DESCRIPTION OF RESEARCH

The principal objective of the research conducted under this grant is to investigate fundamental robustness properties of linear-quadratic-gaussian (LQG) regulators for sampled data systems. We use the word "robustness" to designate the ability of such regulators to tolerate plant uncertainties without losing stability. In classical terms, this tolerance is usually expressed in the form of gain margins and phase margins -- the amount of gain or phase uncertainty which can be tolerated without loss of stability. A control law is robust if the margins are large.

Continuous-Time Robustness Tolerances

Research conducted earlier by Safonov, Wong and Athans [1, 2] has shown that continuous time multi-variable lines-quadratic (LQ) regulators are impressively robust. They can tolerate arbitrary gain increases, gain decreases up to a factor of two (6 db), or $\pm$ 60 deg. phase variations in any combination of control channels. More general types of nonlineari-ties, time variations and dynamic uncertainties are also tolerated.

Greater Gain Tolerances

The above properties hold independent of plant dynamics or performance index selection. They are global, "guaranteed" properties which are a consequence of LQ optimality alone! It is not surprising, therefore,

that strictly greater tolerance bounds can be derived if we take plant-specific data into account. This has been accomplished recently by Wong, Athans, and Stein. We have shown that pure gain reduction strictly greater then 6 db can be tolerated by LQ regulators with state weighting matrix Q>0. Explicit formulas for the amount of reduction in excess of 6 db have been derived and examples have been used to illustrate that this amount frequently permits gain reductions all the way to zero in some or all of the control channels. The latter result is highly significant with respect to reliability and integrity issues of multivariable regulators. A paper covering these new results has been accepted for presentation at the '78 IFAC Congress in Helsinki next summer [3].

Frequency Domain Interpretation

We have also found that the original robustness results in [1] and [2] have appealing, classically-oriented frequency domain interpretations. These are based on known properties the loop transfer matrix L(s) of LQ regulators. It can be shown [4] that this matrix transfer function has a Nyquist "surface" $[L(jw)v; \ ||v|| = 1, \ 0 \leq w < \infty]$ which lies outside of a unit ball centered at $(-1+j0)I$ in complex Euclidean n-space (i.e., the distance from any point on the surface to the critical point $(-1+j0)I$ is greater than or equal to unity). Gain and phase uncertainties of the plant cause the surface to approach the critical point more closely. However, from the geometry of the unit ball, it takes at least 6 db gain reduction or $\pm$ 60 deg phase shift in some direction to actually intersect the critical point and, hence, produce instability. This interpretation

is a direct generalization of single-input LQ regulators results, where
the Nyquist surface reduces to a standard Nyquist diagram and lies outside
of a unit circle centered at (-1+j0). More details can be found in Stein's
think-piece on LQG robustness questions recently sent to NASA-LRC.

Discrete-Time Robustness Tolerances

The above extensions of continuous-time LQ robustness have provided
significant insight for our derivation and interpretation of analogous
robustness properties for sample data systems. In essence, we have now
completed these derivations. The main results are contained in Safonov's
Ph.D. Thesis [6], which was finished this summer, and in a Wong, Athans
manuscript currently in preparation [7]. The results seemed surprising
to us at first: Sample data regulators have neither the 6 db gain reduction,
nor the +∞ db gain increase, nor the $\pm$ 60 deg phase uncertainty tolerance
of their continous time counter parts. Instead, the gains (α_i) in each
control channel must lie between the following limits

$$\frac{1}{2} < \frac{1}{1+a_i} \leq \alpha_i \leq \frac{1}{1-a_i} < \infty \qquad (a_i < 1)$$

and phase uncertainties ϕ_i must be bounded by

$$\phi_i \leq 2 \sin^{-1}(a_i/2) < 60 \text{ deg}$$

Detailed expressions for these discrete gain and phase margins as well
as tolerance bounds for more general types of nonlinear, time varying
and dynamic uncertainties are contained in [6, Part IV].

Physically, these margin reductions occur when going from continuous to sampled systems because sampling involves inherent bandwidth limitations and phase delays. Using frequency domain interpretations, in fact, a completely analogous picture of the continuous time system's Nyguist surface has been developed for the sampled system. In the sampled case, the optimal loop transfer matrix has a Nyguist surface which lies outside of an ellipsoidal ball which is again centered at the critical point $(-1+j0)I$ but has maximum radius strictly less than unity [5]. This means that the sampled system's surface is potentially closer to the critical point than the continuous system's surface and, hence its margins must be smaller.

Also in contrast to the continuous-time case, sample data robustness properties do not appear to be characterizable in a form which is independent of plant and performance index information. The Safonov results above, for example, depends on the discrete Riccati matrix solution and the B matrix through the parameters a_i [6]. We have found, however, that by using additional plant and performance index information, it is still possible to derive strictly greater gain tolerances analogous to the continuous results in Wong, Athans, and Stein [3]. These are being documented in [7]. Also in [7], we have shown that for discrete-time systems which are obtained from continuous-time by standard discretization, the continous-time robustness tolerances are recovered asymptotically as sample rate tends to infinity (i.e., the higher the sample rate the less loss of margins). This result is intuitively reasonable and promises to shed light on the fundamental sample rate selection issue in digital flight control.

LQG Robustness Tolerances

As the next step in our efforts to understand general robustness

properties of optimal discrete regulators, we "back-tracked" a bit to

examine the robustness properties of LQG regulators in continuous time

(i.e., LQ regulators which include Kalman filters for state reconstruction).

The motivation here was the recognition that if bandwidth limitations

and phase delays due to sampling cause margin reduction in discrete-time,

then similar reductions should occur in continuous-time regulators with

Kalman filters. This proved to be true. General robustness conditions for

LQG systems were derived and show that

1) LQG regulators generally have smaller margins than
   LQ regulators

2) LQG margins equal LQ margins if the Kalman filter "knows"
   the plant's uncertainties

3) LQG margins approach LQ margins asymptotically as the ratio
   of magnitudes of control eigenvalues to filter eigenvalues
   tends to zero.

These results are currently being documented. They are important

because they confirm that the common mechanism of bandwith limitation

is responsible for loss of margin in both continuous and sampled systems.

Implicit ways to improve the situation are also evident.

Given Items (1)-(3) above, one may still harbor the conviction that

LQG regulators provide reasonable margins, albeit smaller that the LQ

designs. One may even hope that some minimum lower limit on LQG margins

might be "guaranteed" as a consequence of LQG optimality. Unfortunately,

this latter hope was recently shattered by a counterexample due to

Doyle [8] which exhibits arbitrarily small gain tolerances in a technically

legitimate LQG design.

Further Research

We believe that the results described above represent a substantial improvement in our understanding of the basic robustness properties of continuous and discrete-time LQ regulators. This understanding puts us in an excellent position to address the next phase of research -- robustness of hybrid regulators. Here we intend to view the digital controller with prefilters at its inputs and hold operations at its outputs as a continuous-time control element interacting with the continuous-time plant. The control element is hybrid in the sense that it involves internal sampling, digital filtering, and digital control calculations. Its interaction with the "outside world" however, occurs in continuous time. This system description should come closest to a true representation of actual digital flight control implementations.

In addition to the hybrid robustness research, we also plan to pursue further research directions suggested by the results cited so far. These include development of greater tolerance regions for phase uncertainties and nonlinearities (the present results in [3] and [7] are limited to gain variations only), development of weight selection procedures to increase margins, and design methods to assure good margins in LQG designs.

2. INTERACTIONS

The research results summarized above were discussed in various stages of completion with NASA personnel at the following meetings:

Prof. Stein at Langley	May 2, 1977
Prof. Athans at Langley	June 6,7, 1977
Prof. Stein at Langley	June 21, 1977
Dr. Armstrong at MIT	Nov. 16, 1977

3. PERSONNEL, BUDGET AND SCHEDULES

The following have participated in the research activities since the last status report was issued:

Prof. M. Athans
Prof. G. Stein
Dr. M. Safonov
Mr. P.K. Wong
Mr. W.H. Lee
Mr. J. Carrig
Mr. D. Castonon

As of Nov. 30, 1977, approximately $53,000 were expended from the initial grant amount of $59,800. We believe these expenditures are consistent with research accomplishments.

Two schedule changes of the grant were approved, extending the current phase of research to Dec. 15, 1977. We are also happy to report that the grant has been continued for the next year, effective Dec. 16, 1977 through Oct. 15, 1978, with total funding raised to $99,000. This continuation will allow us to seriously address the research questions outlined above.

4. REFERENCES

1. M.G. Safonov and M. Athans, "Gain and Phase Margins for Multi-Loop LQ Regulators", IEEE Trans. Auto-Control, April 1977.

2. P.K. Wong and M. Athans, "Closed Loop Structural Stability for Linear Quadratic Optimal Systems", IEEE Trans. Auto. Control, February, 1977.

3. P.K. Wong, M. Athans, and G. Stein, "Structural Reliability and Robustness Properties of Linear Quadratic Multivariable Regulators," to be presented at IFAC 1978.

4. Kwakernaak, H., and R. Sivan, <u>Linear Optimal Control Systems</u>, Wiley-Interscience; New York, 1972.

5. G. Stein, "An Overview of Robustness Issues in Modern Digital Control", June 1977.

6. M.G. Safonov, "Robustness and Stability Aspects of Stochastic Multi-variable Feedback System Design", Ph.D. Thesis, MIT, Sept. 1977.

7. P.K. Wong, G. Stein, and M. Athans, "Robustness Properties of Sampled Data LQ Regulators," in preparation.

8. J.C. Doyle, "Guaranteed Margins for LQG Regulators", submitted for publication <u>IEEE Trans. Auto. Control.</u>