May 1, 1978

ELECTRONIC SYSTEMS LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS 02139

STATUS REPORT ON CONTROL OPTIMIZATION, STABILIZATION AND COMPUTER ALGORITHMS FOR AIRCRAFT APPLICATIONS

NASA GRANT NGL-22-009-124

ESL-SR-819
M.I.T. PROJECT OSP 76265
Twenty-Third Status Report
September 15, 1977 to April 15, 1978
EDITED BY: Professor Michael Athans

SUBMITTED TO:

1) Office of Research Grants and Contracts (Code SC)
NASA Headquarters
Washington, D.C. 20546

2) NASA Scientific and Technical Information Facility
P.O. Box 8557
Baltimore/Washington International Airport
Maryland 21420

3) Mr. Brian F. Doolin
Leader, Control Theory and Applications Group
FSC: 210-3
NASA/Ames Research Center
Moffett Field, California 94035

SUMMARY

A brief description of the research carried out by faculty, staff, and students of the M.I.T. Electronic Systems Laboratory under NASA grant NGL-22-009-124 is presented. The period covered in this status report is from September 15, 1977 to April 15, 1978.

The basic scope of this grant is to carry out fundamental research in the areas of system theory, estimation, control, and algorithm development, motivated by problems of current and future aircraft. In particular, the research is to be supportive of the long range plans of NASA's Ames Research Center. The emphasis of the research over the past year has been more and more focussed on issues of reliable, robust, and fault-tolerant control system designs and the issues raised by the question of digital implementation of a control system design. In addition we have initiated some basic studies related to VSTOL aircraft.

The specific topics covered in this progress report are:

1. VSTOL Control System Studies.
2. Reliable Control System Designs With and Without Feedback Configuration.
3. Fault-Tolerant Optimal Control Systems.
4. Development of a Failure Detection Methodology
5. Sensor Redundancy, Analytical Redundancy, and Microprocessor Redundancy
6. Finite-State Dynamic Systems
7. Dynamic Compensation of Time-Invariant Linear Multivariable Systems
8. Qualitative Performance of the MMAC System

1. VSTOL CONTROL SYSTEM STUDIES

A new project was initiated during this reporting period by Professor G. Stein, M. Athans and Mr. C. McMuldroch. This project was motivated by the fact that the NASA Ames Research Center was recommended to provide technical assistance to the U.S. Navy in their VSTOL A studies.

The specific study deals with VTOL automatic hovering performance while tracking ship deck motions. Our objectives are to assess basic performance characteristics of these vehicles and to explore the applicability of LQG methodology for their controller designs. A linearized state variable model has been derived and programmed in FORTRAN, for the complete system of aircraft motion and simplified controls, ship motion, mean wind and wind disturbances. For this model and each set of performance penalties chosen, modern control theory results are used to solve a steady state linear quadratic regulator problem. For each resulting aircraft controller design, the steady state covariances of the states and outputs can be computed. Simulations of the design for aircraft initial condition, and wind and ship driving inputs can also be made. So far, reduced order models of the aircraft alone have been used to develop an understanding of its performance as a function of state penalty weights. Current activity is aimed at expanding this understanding to the longitudinal aircraft tracking ship problem in high sea states.

The aircraft modeled is the Lift/Cruise Fan shaft coupled VSTOL Research Technology Aircraft. The model, linearized at stationary hover, has twelve states for velocity, position, angular velocity, and Euler angles, as well as states modeling first order dynamics between control signals and fan thrusts. The linearized equations, and nominal parameters were derived from the following two reports: "Mathematical Model For Lift/Cruise Fan VSTOL Aircraft Simulator Programming Data" by M.P. Bland, B. Fajfar, and R.K. Konsewicz; and "Simulation Test Results for Lift/Cruise Fan Research and Technology Aircraft" by Michael P. Bland and K. Konsewicz. The first report was also used to set up the wind models, and their coupling into the aircraft motion. The mean wind consists of two first order systems driven by two independent pseudo-white noise generators. This produces a slowly varying mean wind in the horizontal plane. The disturbance model, with three Dryden filters each simplified to first order and driven by an independent pseudo-white noise, generates wind gusts in the vertical and two horizontal directions. For the ship model, three independently driven, second order, lightly damped systems are combined to give pitch, roll, and heave motions to the landing deck. The parameters for the system were chosen to approximately represent ships with the motion characteristics of destroyers in sea state five.

The model construction program begins by computing the linear state variable model-system, input, and ouput matrices. These matrices are then reduced to yield a model with only the desired states, controls and physically meaningful outputs, for example a longitudinal motion or lateral-directional motion model. To convert the aircraft-tracking-ship deck problem into an output regulator problem, the desired linear combinations of aircraft and ship outputs are read in to compute a new output matrice. With this output matrice and the penalty weightings for each of the outputs, the design program sets up the equivalent linear, quadratic state regulator problem, which it solves for the steady state feedforward (from ship and wind states) gains. With the design complete, the steady state covariances of the states and outputs are computed, for example: the covariance of the relative aircraft-ship position or orientation, and the aircraft controller effort (which gives thrust level). Finally simple simulations with outputs and controls printed can be run for almost any integration step size and time length.

The program has been run for the reduced state cases of aircraft pitch motion, and pitch-heave motion. During these runs nominal values for the penalties on pitch rate, pitch angle, heave rate and heave position were chosen. The criteria used were that the aircraft have a bandwidth of approximately one radian per second in the heave states, and three radians per second in pitch attitude, and that each mode be well damped. With the understanding of the aircraft behavior obtained from those runs, the case of tracking ship pitch and heave motion is now under study. So far the penalties applied above have been used to generate a bench mark in response performance. The results which are aimed at, for the complete longitudinal aircraft and ship case, are graphs of root mean square relative position error, actuator values and rates; and aircraft control system bandwidths, all vs. root mean square percentage control authority used.

Currently the following trends are apparent: In pitch motion fast response is easily achievable with reasonable control authority. In heave motion the ship has a frequency near one radian per second - a bandwidth which appears safely achievable by an aircraft with a thrust to weight ratio of 1.1; thus, to track typical ship heaving motion (± 5ft) with small deviations (less than 1 ft.) the aircraft may well require thrust to weight ratios beyond the physical capabilities of 1.1 given in the simulation test report.

2. ON RELIABLE CONTROL SYSTEM DESIGNS WITH AND WITHOUT FEEDBACK RECONFIGURATIONS

Mr. J.D. Birdwell, Professor M. Athans and Dr. D.A. Castanon have continued their investigations in the area of stochastic control with special emphasis of developing a method of approach and theoretical framework which advances the state of the art in the design of reliable multivariable control systems, with special emphasis on actuator failures and necessary actuator redundancy levels.

The mathematical model consists of a linear time invariant discrete time dynamical system. Configuration changes in the system dynamics, (such as actuator failures, repairs, introduction of a back-up actuator) are governed by a Markov chain that includes transition probabilities from one configuration state to another. The performance index is a standard quadratic cost functional, over an infinite time interval.

If the dynamic system contains either process white noise and/or noisy measurements of the state, then the stochastic optimal control problem reduces, in general, to a dual problem, and no analytical or efficient algorithmic solution is possible. Thus, the results are obtained under the assumption of full state variable measurements, and in the absence of additive process white noise.

Under the above assumptions, the optimal stochastic control solution can be obtained. The actual system configuration, i.e. failure condition, can be deduced with an one-step delay. The calculation of the optimal control law requires the solution of a set of highly coupled Riccati-like matrix difference equations; if these converge (as the terminal time goes to infinity) one has a reliable design with switching feedback gains, and, if they diverge, the design is unreliable and the system cannot be stabilized unless more reliable acuators or more redundant actuators are employed. For the reliable designs, the feedback system requires a switching gain solution, that is, whenever a system change is detected, the feedback gains must be reconfigured. On the other hand, the necessary reconfiguration gains can be precomputed, from the off-line solutions of the Riccati-like matrix difference equations.

Through the use of the matrix discrete minimum principle, a sub-optimal solution can also be obtained. In this approach, one wishes to know whether or not it is possible to stabilize the system with a constant feedback gain, which does not change even if the system changes. Once more this can be deduced from another set of coupled Riccati-like matrix difference equations. If they diverge as the terminal time goes to infinity, then a constant gain implementation is unreliable, because it cannot stabilize the system. If, on the other hand, there exists an asymptotic solution to this set of Riccati-like equations then a reliable control system without feedback reconfiguration can be obtained. The implementation requires constant gain state variable feedback, and the feedback gains can be calculated off-line.

In summary, these results can be used for off-line studies relating the open loop dynamics, required performance, actuator mean time to failure, and functional or identical actuator redundancy, with and without feedback gain reconfiguration strategies.

The above results have been partially documented in a paper in the 1977 IEEE Conference on Decision and Control. Full documentation will be contained in the doctoral thesis by J.D. Birdwell, scheduled for completion in June 1978.

3. FAULT-TOLERANT OPTIMAL CONTROL SYSTEMS

Professor A.S. Willsky and Mr. H. Chizeck have undertaken the problem of trying to determine controllers which optimize performance prior to failure but are content to operate in a degraded mode subsequent to failure i.e. *optimal* self-reorganizing controllers, rather than robust controllers, which sacrifice some performance prior to failure to guarantee adequate performance if a failure occurs. Clearly the former system *requires* an excellent fault detection syste, while the latter may lead to significant performance loss if failures are unlikely to occur.

Since our criterion of performance depends upon the system mode, we are led to consider a "split-cost" formulation. In the simplest case, in which there are two modes, denoted by i=1,2

$$\dot{x}(t) = A_i(t) + B_i(t) + W_i(t) \tag{1}$$

where the system starts in mode 1 and may switch to mode 2 at time T. The basic optimal control problem is to solve for the optimal control u(t) assuming perfect knowledge of x(t) and the mode i(t), where the cost criterion is

$$J=E\int_0^T \{x'(t)Q_1x(t) + u'(t)R_1u(t)\}dt + \int_T^{t_f} \{x'(t)Q_2x(t) + u'(t)R_2u(t)\}dt \tag{2}$$

This problem has been solved, as have extensions to more complex problems, such as the inclusion of an arbitrary but finite set of modes. In addition, we have begun to attack the problem of the design of a useful maintenance policy, where we model the decision to perform maintenance as a switch of the system to a new model (perhaps restored to the original operating state) at some fixed maintenance cost. A solution to a simple form of this problem has been obtained, and we are continuing to analyze the nature of this formulation and its solution.

In addition to continuing our study of the problems mentioned above, we are adding complexity to the model by including measurement noise. No difficulties should be encountered if we have noisy linear observations of x and perfect knowledge of the mode. If knowledge of the mode is dropped, the problem becomes a dual control problem. However, our earlier work should allow us to study this problem at some depth. Specifically, the control laws for the problem (1) - (2) switches at failure times. If we do not observe the mode directly, this switch basically must be replaced by a failure detection system. We hope to analyze the performance of this detection system not by itself but as it effects the performance of the overall closed-loop system.

4. DEVELOPMENT OF A FAILURE DETECTION METHODOLOGY

During this time period Professor A.S. Willsky, Dr. S.B. Gershwin and Mr. E.Y. Chen have continued the development of an overall methodology for the design of systems for the detection of failures in dynamical systems. Earlier work had led to the development of a Bayes' risk formulation for determining decision rules that are optimal with respect to criteria that allow tradeoffs among key performance characteristics such as detection delay, false alarm rate, incorrect failure mode identification, etc.

Work has continued in understanding the structure of the resulting optimization problem for the calculation of the optimum decision rule and in devising alternative problem formulations. Both of these investigations have as this goal the development of relatively simple design algorithms (the full-blown Bayes' risk algorithm is definitely not simple!) that capture many of the tradeoffs of interest and that lead to decision rules with structures similar to that of the optimum Bayes' risk rule.

Related to all of this work is our continuing investigation into the development of methods for evaluating the performance of various decision rules. Several approximation methods are being pursued at present. The goal of this work and that described above for the design of useful decision rules is the development of a design package that could, perhaps, be used in an interactive mode, allowing the designer to adjust different tradeoffs by changing parameters in the cost functional and to observe the effect of these changes on the performance.

We are also continuing our examination into the design of robust failure detection systems, using the fact that dissimilar sensors can be compared if their observable subspaces overlap. Then, depending upon the uncertainty in the parts of the dynamics observed by different sensors, we can choose those sets of comparisons which lead to the least sensitivity to modelling errors. This concept is presently understood for simple examples, and extensions to more general systems remain to be explored.

Finally, Professor Willsky and Mr. Jyh-yun Wang are continuing the development of extended GLR, a system for the detection of non-additive failures. A basic algorithm has been developed, some performance analysis has been begun, and simulation tests are being planned.

5. SENSOR REDUNDANCY, ANALYTICAL REDUNDANCY, AND MICROPROCESSOR REDUNDANCY

A new project was initiated during this time period, by Professor Michael Athans, Dr. Paul K. Houpt, and Mr. Eric Helfebein. The objective of this project is to determine the appropriate tradeoffs relating to the use of analytical redundancy methods in the case of functionally and physically redundant sensors, (such as dually redundant position sensors, velocity sensors, acceleration sensors, as outlined in our latest proposal).

The eventual goal of this long range research program is to deduce appropriate measures of effectiveness for an overall failure detection and isolation system, which takes into account the failure probabilities of the physical sensors, and the failure characteristics of the microprocessor hardware that are used, using digital Kalman filtering techniques to provide analytical redundancy. What we would like to develop is an overall design methodology which will result in a hardware and software configuration that can detect and isolate failures, in both the hardware and the firmware, with a certain degree of confidence and in the presence of noise, while minimizing the probabilities of false sensor failure alarms.

One of the most difficult modelling issues is to deduce scientific ways of evaluating failure rates of the firmware. One can distinguish between intermittent software failures, in which errors in addition and multiplication operations take place only at the infrequent intervals of time, versus microprocessor failures which result in numerical calculations that are incorrect. Our current method of attack is to examine the estimation accuracy of the state variable reconstruction than can be obtained by different configurations of digital Kalman filters. These distinct configurations are defined by the specific sensors acting as inputs to the Kalman filters. This in turn translates into different addition and multiplication software requirements. We are attempting to formulate a basic approach of assigning failure probability to these addition and multiplication operations. This is not necessarily the case when an entire microprocessor chip is used to implement the digital Kalman filter, but we feel that some fundamental studies are needed in assigning failure characteristics to arithmetic operations and carrying out tradeoffs with respect to the failure characteristics of each individual physical sensor. In our opinion, if we understand these individual hardware versus software tradeoffs, we can extend the results to the more realistic case of tradeoff studies between sensor hardware and microprocessor hardware, that implement the necessary Kalman filters for analytical redundancy.

The research is at the present time in its formative stages, and we do not expect any definitive methodological or theoretical results until the end of the current grant.

6. FINITE-STATE DYNAMIC SYSTEMS

When a continuous dynamic system is interfaced to a finite-state sequential machine (digital computer or microprocessor) in a feedback configuration, several exciting control possibilities arise. For the most part, existing approximate methods for the design of digital control systems completely fail to exploit such possibilities, or exploit them inefficiently and incompletely. The consequences include lower bandwidth and reduced reliability than might be achieved with other design methods.

Professor Johnson has taken an important first step in developing such a design method by introducing the notion of a finite-state dynamic system. The hybrid feedback system just mentioned is one example of such a system. The major technical difficulty which has been overcome is how to define the notion of state for such a system. We have discovered one notion which works. The state of a finite-state dynamic system consists of

(a) the continuous system "state"

(b) the finite sequential system "state"

(c) a part of the state which involves a record of past transition times of the finite-state system.

It is this last key element which was previously missing. A correct definition of this part of the state is nontrivial because of the need to assure that hybrid feedback systems are well-defined. We expect that this will open the door to a rigorous design method for systems involving asynchronous logic, decision functions, relays, etc., as well as high-speed microprocessor based controllers.

7. DYNAMIC COMPENSATION OF TIME-INVARIANT LINEAR MULTIVARIABLE SYSTEMS

7.1 Continuous-Time, Continuous-State Systems:

A new formulation of the steady-state linear-quadratic-Gaussian problem which includes steady-state response tracking and disturbance rejection, is under development by Professor Johnson. Inclusion of these features is quite important in reliability evaluation, where loss of steady-state stracking or disturbance rejection are usually second only to failure detection and/or guarantee of transient stability. In this research, we are attempting to incorporate our previous insights about the algebraic structure of the reduced-order compensator problem (Blanvillain and Johnson; 1978) into a broader framework. The optimal compensator gains are characterized by a set of nonlinear algebraic equations. We are in the process of checking the limiting cases corresponding to standard LQG problems, while dealing with the singularity of output-feedback previously described by Platzman and Johnson (IEEE Trans. Auto. Control, Oct. 1976). We expect to unify several results on the reduced-order compensation problem as a result of this research.

7.2 Discrete-Time Continuous-State Systems:

There is an equivalence relation between deterministic finite-dimensional system in state-space form and finite-dimensional systems in autoregressive (ARMA) form. From the earlier work of Llorens (E.E. Thesis, 1976), we know that the optimum minimum-order observer-based compensator for the discrete time plant with no observation noise has the same z-transfer function independent of the plant realization. On the other hand, it is quite difficult to find the optimum compensator transfer function in autoregressive form for a plant which is in autoregressive form (even in the pure minimum-variance case, this involves spectral factorization).[1] In the research of Mainemer, and Professor Johnson they undertook to show how the forementioned equivalence relation could be used for the design of ARMA compensators and to see whether it was possible to devise a direct method of finding the optimum gains of the corresponding ARMA compensator. We were successful in the former task, but only partially successful in the latter objective.

Autoregressive models offer some attractive possibilities for adaptive and/or reliable control design, so it would be desirable to possess a direct (possibly even on-line) compensator design method. This possibility can be realized in the single-input, single-output case

---

[1] The solution is known for minimum-variance control in the signle-input, single-output case (see K.J. Astrom, Stochastic Control, 1973).

by the self-tuning regulator of Astrom and Wittenmark. Several technical issues remain to be worked through before the multi-input multi-output version of this problem can be solved, particularly when constraints on actuator excursions and rates must be imposed.

7.3 Discrete-time, Discrete-State Case:

The results of K. Davis concerning feedback systems evolving on integer fields, and the problems of approximating continuous systems by integer systems of higher order, were discussed in our previous progress report. The thesis was completed in August, 1977. We plan future research in this area, as it has important implications for system aggregation (or disaggregation!) and implementation of controllers on integer machines.

8. QUALITATIVE PERFORMANCE OF THE MMAC SYSTEM

Professor A.S. Willsky, Dr. S.B. Gershwin, Professor G.S. Stein, and Mr. C.S. Greene have continued their investigation of the performance of the MMAC method. Work has consisted both of simulations of some simple two-dimensional examples involving an unstable plant and of analysis of the simple example and of more general cases. From the simulations we have been able to deduce certain directions of analysis that are presently being pursued. One of the most striking properties of the MMAC method is the behavior of the probabilities for the hypothesized models -- under many conditions they are essentially piecewise constant, and thus the overall closed-loop system is piecewise linear with "switches" every once in a while. This behavior can be studied analytically, and efforts are continuing in this direction to characterize the several types of overall system behavior than can occur --

i.e. limit cycles, eventual stability, instability. In addition, if the initial conditions of the sytem are sufficiently small, this oscillating behavior of the probabilities is avoided, and the system response is far smoother. We are presently working on a characterization of this "domain of attractions" -- i.e. the set of initial state disturbances under which the system exhibits a smooth response.

The results of our study have been extremely useful in allowing us to gain insight into the nature of this adaptive feedback system. Using this insight, we have proposed several modifications to the MMAC design, such as the use of only a finite window of data for the probability calculations, in order to improve closed-loop system response. These modifications are also being studied via analysis and simulations.

PERSONNEL

During this time period the following have contributed to the research effort.

Faculty

Professor Michael Athans, Principal Investigator
Professor Alan S. Willsky, Principal Co-Investigator
Professor Sanjoy K. Mitter
Professor Timothy L. Johnson
Professor Nils R. Sandell
Professor Gunter Stein

ESL Research Staff

Dr. Paul K. Houpt
Dr. David Castanon

Graduate Students

Mr. J.D. Birdwell
Mr. H. Chizeck
Mr. C.S. Greene
Mr. C. McMuldroch
Mr. E. Helfenbein
Mr. C. Mainemer
Mr. P. Wong
Mr. E.Y. Chow
Mr. J.Y. Wang
Mr. P. Moreney

REFERENCES FOR NASA AMES GRANT

The publications below have appeared during this reporting period. Copies have been transmitted to NASA in accordance with grant rules. The research reported in these publications was supported in whole or in part by NASA Grant NGL22009124.

1. M.G. Safonov and M. Athans, "Robustness and Computational Aspects of Nonlinear Stochastic Estimators and Regulators," Proc. IEEE Conference on Decision and Control, December 1977.

2. P.K. Wong, M. Athans, and G. Stein, "Structural Reliability and Robustness Properties of Optimal Linear-Quadratic Multivariable Regulators," accepted 1978 IFAC Congress, Helsinki, Finland, June 1978.

3. J.S.H. Liu and T.L. Johnson, "Tensor Relaizations for Large-scales Systems," Proc. Lawrence Symposium on Systems and Decision Sciences, October 1977.

4. T.L. Johnson, "Finite-State Compensators for Continuous Process," accepted 1978 IFAC Congress, Helsinki, Finland, June 1978.

5. B. Llorens, "On the Optimal Minimal-Order Observer-Based Compensator and the Limited State Variable Feedback Controller," M.I.T Electronic Systems Laboratory, Report No. ESL-R-732, September 1976.

6. A.S. Willsky, "Relationships Between Digital Signal Processing and Control and Estimation Theory," to appear as an invited paper in Proc. IEEE.

7. P.J. Blanvillain and T.L. Johnson, Invariants of Optimal Minimal-Order Observer-based Compensators", IEEE Trans. Auto. Control, Vol. AC23, No. 3 (to appear, June, 1978).

8. P.J. Blanvillain and T.L. Johnson, "Specific-Optimal Control With a Dual Minimal-Order Observer-based Compensator", Int. J. Control (to appear).

9. C.I. Mainemer, "Design of a Compensator for an ARMA Model of a Discrete-Time System", S.M. Thesis, M.I.T., Dept. of Electrical Engineering (to appear as an ESL report).