

ESSAYS ON FIRM MANAGEMENT AND CONTROL

by

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ABSTRACT

This dissertation consists of a trio of essays which investigate the effect considerations of corporate control have on the behavior of firms and the individuals that comprise them. The first essay, *Block Investment and Partial Benefits of Corporate Control*, considers the possibility that benefits of control are divided among block shareholders according to the strategic importance of these blocks in forming winning coalitions. The consequent effect on individual investment decisions and the shareholder structure within and across firms is examined. This paper predicts large investors will "create their own space" by staking out large enough blocks to deter other block investors, there will be a threshold level above which large investors are not challenged, and that the shareholder structure across firms will exhibit a particular clientele effect. These predictions are consistent with a preliminary review of empirical tendencies.

The latter two essays explore effects of personal motivations on managerial decisions. In particular, the second essay, *Corporate Conservatism, Herd Behavior and Relative Compensation*, considers how concerns for reputation may induce "conservative behavior" among managers, in which superior innovations are forgone for standard actions. We find that very high and very low ability managers will undertake superior innovations when such opportunities present themselves, while all others will forgo innovations and continue to undertake the industry standard.

The third essay, *An Agency/Control Theory of Capital Structure When Management Can Alter Debt*, demonstrates that managerial empire building tendencies together with limited entrenchment is capable of yielding rich implications for capital structure, dividend policy, and the term structure of debt. Debt serves to restrict inefficient empire building through the possibility of bankruptcy, and the potential consequences this may have for managers' control. Managers voluntarily employ debt in such a manner to credibly limit their future inefficiency, thereby preventing control challenges.

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INTRODUCTION

Utility maximizing individuals and profit maximizing firms lie at the heart of the behavioral paradigm of modern economics. However, the compatibility of these two behavioral foundations rests on tenuous grounds. Firm decisions are undertaken by many individuals with disparate personal goals. Such decision makers may care about compensation, control and reputation, among other concerns, in addition to firm profits. Even if all shareholders are primarily concerned with profits, the separation of control between shareholders and management which characterizes the modern corporation may restrict the ability of shareholders to enforce profit-maximizing decisions. Familiar agency and free-rider problems, and other facets of institutionalized entrenchment, may both limit the effective control that shareholders can exercise over management and restrict the ability of incentives or the market for corporate control to align managerial and shareholder incentives.

Such considerations have motivated research in a wide array of economic fields. Literature in areas as varied as, theory of the firm and organizations, contract theory, finance, industrial organization, labor, and accounting, among others, have all addressed questions dealing with the internal organization of the firm. The interaction of these fields of research has yielded rich theoretical implications and empirical findings on numerous topics ranging from managerial incentives to capital structure to determinants of firm scope and size.

Motivated in part by dramatic events on Wall Street in the past decade, topics relating to corporate control and governance has become a prominent component of such research on the firm. While still in a formative stage, this research has addressed the operation of the market for corporate control, managerial incentives, and the consequent effect of such considerations on other financial and product market decisions of firms. Broadly, this line of research attempts to explain firm decisions from the perspective of managerial motivations, thereby reconciling the dichotomy between individual utility maximization and firm behavior.

This dissertation consists of a trio of essays which investigate such issues. Each essay addresses an aspect of the effect considerations of corporate control may have on the behavior of firms and the individuals that comprise them. While theoretical in nature, each paper purports to describe empirically observable behavior, and hence also discusses implications and empirical predictions of the theory. The first essay considers determinants of shareholder control, and the consequent effect on individual investment decisions and the shareholder structure within and across firms. The latter two essays explore aspects of how personal motivations may affect managerial decisions. In particular, the second essay considers how concerns for reputation may induce "conservative behavior" among managers, in which superior innovations are forgone for standard actions. The third essay demonstrates that managerial empire building tendencies together with limited entrenchment is capable of yielding rich

implications for capital structure, dividend policy, and the term structure of debt.

The first essay, *Block Investment and Partial Benefits of Corporate Control*, is motivated by a simple puzzle. Investors often choose to hold sizable blocks of equity in the same firm despite familiar compelling arguments for diversification. While potential control benefits from majority or dominant blocks has received attention in the literature, most block shareholders are significantly smaller than this literature typically envisions. The average Fortune 500 firm has 10.5 shareholders holding 1% blocks of equity, but only .54 shareholders holding 10% blocks. This essay addresses this puzzle by recasting the manner in which shareholders may benefit from control. Rather than assuming that control benefits only accrue to a dominant shareholder, this paper instead considers the possibility that potential benefits of control are divisible among multiple shareholders. Moderate block shareholders can consequently form controlling coalition which divide benefits of control. Intuitively, one can envision managements' position being supported by a coalition of block shareholders, who in turn receive control benefits from management.

The control benefits a shareholder receives for a given position in a firm will depend on the distribution of other shares of the firm; in particular, on the strategic importance of the shareholder's block in forming majority coalitions. The implications of such a cooperative game among block shareholders for the shareholder structure within and across firms are

examined. This paper predicts large investors will "create their own space" by staking out large enough blocks to deter other block investors, there will be a threshold level above which large investors are not challenged, and that the shareholder structure across firms will exhibit a particular clientele effect. These predictions are consistent with a preliminary review of empirical tendencies.

The second essay, *Corporate Conservatism, Herd Behavior and Relative Compensation*, explores how concerns for reputation may lead managers to prefer standard actions over superior innovations. In the basic model, all managers can undertake some industry standard action; a few can alternatively choose to undertake a new innovation which stochastically dominates this standard action. Common components of uncertainty for managers undertaking the same action lead the market to form inferences of managerial ability based on relative performance. While the innovation is no riskier in an absolute sense than the standard action, managers who undertake the standard action are consequently evaluated with a more accurate benchmark than those who innovate, since most managers undertake the industry standard. Managers who can take the new action must consider how both the higher expected outcome and the noisier evaluation will impact reputation.

This technological structure is considered in a setting where managers have a strong aversion to low appraisals; optimal firm actions imply managers with very low appraisals are fired. Additionally, there is asymmetric information concerning managers

ability; managers know their types while firms do not. We find that very high and very low ability managers will undertake superior innovations when such opportunities present themselves, while all others will forgo innovations and continue to undertake the industry standard. Average type managers take the old action because it is important that the market evaluate them with an accurate benchmark and not confuse them with low types who firms want to fire. Very high types are less likely to be confused with low types, and their reputation benefits enough from the increase in expected outcome that they prefer the new action. Low types likely to be fired prefer both the less accurate evaluation and the higher mean associated with the new action. Thus, innovations are undertaken by the brilliant and the desperate managers, while others continue to undertake the industry standard action.

Additionally, an example demonstrates the possibility of obtaining a similar result when all managers can take the new action and firms offer optimal incentive contracts. Provided there are enough high types relative to average types, the optimal incentive schedule induces high and low types to take the new action, while average types takes the old action.

The final essay, *An Agency/Control Theory of Capital Structure When Management Can Alter Debt*, considers the role debt may play as a credible self-constraint for partially entrenched, empire-building managers. Debt serves to restrict inefficient empire building through the possibility of bankruptcy, and the potential consequences this may have for managers' control. Managers voluntarily employ debt in such a manner to credibly

limit their future inefficiency, thereby preventing control challenges. While this capital structure constrains managers' actions, it is dynamically consistent across periods for them; debt and dividends each period reflect management's optimal tradeoff between empire building ambitions and the need to retain control to realize these ambitions.

In the model, the market for corporate control can only remove a manager for sufficiently high anticipated future inefficiency. Bankruptcy is assumed to reduce this entrenchment, making a manager easier to replace. Managers desire to undertake new projects each period; with some probability a positive NPV project is available, otherwise only negative NPV projects exist. Since investments are sunk costs once undertaken, the market for corporate control cannot credibly take past inefficient investment decisions into account when evaluating the merits of a takeover. Hence, if managers are unconstrained when it is time to invest, they will always undertake whatever project is available. This, however, may lead to enough anticipated inefficiency to justify an ex ante takeover.

Such takeovers can be prevented by management by credibly committing to refrain from bad projects through the issuance of debt that leads to bankruptcy if bad projects are undertaken. Such debt can commit managers to refrain from bad projects where the market for corporate control cannot, because bankruptcy entails less entrenchment. In this manner, debt serves to make a manager's future control dependent on the present action.

In order for debt to serve as a credible commitment, the firm

must not have sufficient retained funds to avoid bankruptcy if a bad project is undertaken. Dividend policy must therefore be coordinated with leverage decisions. Thus, managers undertake sufficient debt and pay dividends that guarantee enough efficiency to thwart takeover attempts.

Term structure implications follow from the consideration of differing availabilities of good projects. The smaller the probability that positive NPV projects will be available, the greater the anticipated future inefficiency of an unconstrained manager. Therefore, when the likelihood of good future projects is low, a manager must commit to refraining from more inefficient future projects by issuing more debt in order to retain control. The optimal manner in which this is accomplished yields implications for the timing of debt payments.

All three essays aspire to add to the present understanding of the internal organization and decision making process of firms. Each examines an aspect of how control considerations impact upon the optimal actions of individuals who comprise the firm. It is hoped that these essays will serve to further present knowledge and motivate further inquiry into such topics.

Chapter 1

BLOCK INVESTMENT AND PARTIAL BENEFITS OF CORPORATE CONTROL

Abstract - Despite familiar arguments for diversification, many investors choose to hold significant blocks of equity in the same firm. While control benefits may explain majority blocks, most blocks are much smaller than what is generally considered necessary for control. This paper develops a theory whereby such blocks can confer to their holders partial benefits of control; in particular, small block shareholders can join together and form controlling coalitions. The implications of such a cooperative game among block shareholders for the shareholder structure within and across firms are examined. This paper predicts large investors will "create their own space" by staking out large enough blocks to deter other block investors, there will be a threshold level above which large investors are not challenged, and that the shareholder structure across firms will exhibit a particular clientele effect among block shareholders. These predictions are consistent with a preliminary review of empirical tendencies.

SECTION 1 INTRODUCTION

Despite theoretical recommendations for diversification, many investors choose to hold significant blocks of equity in the same firm. In Fortune 500 corporations in 1981, the average number of shareholders that held blocks of greater than 1% of a firm's equity was 10.5, with 4.7 holding blocks greater than 2%, and 1.4 holding blocks greater than 5%.¹

Furthermore, these block shareholdings exhibit regular patterns across firms not explained by standard investment theory. The largest shareholders tend to "create their own space"; their presence seems to dissuade other large shareholders from investing in their firm. Similarly, the larger the leading shareholder in a firm, the fewer smaller block shareholders are present. Also there is a tendency for firms without one very large block shareholder to have a greater number of moderate sized shareholders than those with a dominant shareholder. This paper attempts to explain such investment behavior through a model in which investors receive private benefits from partial control. In particular, block investors play an active role in the firm, attempting to form controlling coalitions which in turn can divide benefits of control.

Previous considerations of control contests have considered

¹ These numbers are for the 456 firms covered in the 1981 *CDE Stock Ownership Directory: Fortune 500*. The 44 excluded firms had either recently merged, were privately held, or were subsidiaries or cooperatives.

the battle between an incumbent and a potential raider, taking other shareholders as passive observers whose only actions are their tender or their vote. Additionally, these shareholders are typically assumed to be sufficiently dispersed so that each one individually has a negligible effect on the outcome of a control contest. This leads to the Grossman and Hart (1980) free-rider problem, whereby a superior rival cannot take over a firm at less than his public value, because nontendering shareholders would realize this value upon a successful takeover. Recent literature on control contests can be viewed as an attempt to understand the raider's motivation for a takeover, and the process through which such a contest is played out, in light of this free rider problem. Several different manners in which raiders can profit despite free-rider problems have been proposed: Grossman and Hart (1980) consider the ability of a raider to dilute a firm for private gain; Shleifer and Vishny (1986) and Hirshleifer and Titman (1990) examine a raider's ability to profit on the appreciation of shares held prior to a takeover attempt; and Grossman and Hart (1988), Harris and Raviv (1988a), Stulz (1988) and Dewatripont (1989), among others, all consider the existence of private benefits to control.

Unlike these paper, shareholders in Harris and Raviv (1988b) and Holmstrom and Nalebuff (1988) do take into account the possibility that their tender decision may be pivotal, which in turn affects equilibrium bids of the control contestants. However, beyond this function, shareholders are once again relegated to playing a passive role in the control contest; the

tender decision is the only action they can take. The possibility that shareholders and control contestants can coordinate behavior in any manner is not considered. If instead, the raider and any one of the shareholders could coordinate actions (for example, if a shareholder could secretly sell a few shares to the raider, or could promise to tender in exchange for a transfer), both could gain considerably if others played equilibrium actions.

Thus while this existing literature has advanced our understanding of the market for corporate control, it only yields a rationale for block shareholding for control contestants; it fails to address why the great majority of block shareholders choose to hold blocks significantly smaller than majority blocks, and the role these investors play in control contests. Additionally, many firms lack the majority or dominant shareholder often posited in the literature; while at the same time, taken together, block shareholders typically hold a sizable fraction of equity.² Motivated by these observations, this paper departs from the literature by considering an active role played by block shareholders as a central component in determining corporate

² Of the 10.5 shareholders holding greater than 1% of the average Fortune 500 firm's equity, only .54 of them hold greater than 10% of the firm. The median largest shareholder in these firms holds only about 9%, and the mean largest shareholder 15.4%, whereas the mean top 5 shareholders hold 28.8%. These figures are all for firms in the 1981 *CDE Stock Ownership Directory: Fortune 500*; the later two means are also stated in Shleifer and Vishny (1986). Demsetz and Lehn (1985) find similar numbers for a slightly different data set consisting of 511 firms in the *CDE Stock Ownership Directory: Energy (1980), Banking and Finance (1980) and Fortune 500 (1981)*. In their data set, the top 5 shareholders held on average 24.8% of a firms' equity, and the top 20 held 37.7%.

control. Shareholders' role as active participants in controlling coalitions in turn gives them an incentive to hold blocks.

Private benefits of control have received much attention recently. Jensen and Ruback (1983), Demsetz (1986), Grossman and Hart (1988), Harris and Raviv (1988a and b), Dewatripont (1989), Barclay and Holderness (1989), and Hart and Moore (1990), among others, all model or discuss such benefits. Additionally there exists empirical evidence suggesting that control is valuable. Lease, McConnell and Mikkelsen (1983), Levy (1983) and Zingales (1990) find that for firms with dual classes of equity identical except for their voting privileges, the class with the superior voting privilege sells at a premium, in respectively the United States, Israel, and Italy. Barclay and Holderness find that on average large premiums (greater than 20.0%) relative to post-trade market price are paid for large blocks of equity.

While much of the literature is vague on the origins of these private benefits of control, sources of these benefits mentioned include the ability of Management (or Directors) to dilute corporate funds for private benefit, synergies obtainable through mergers, favors conferred by a firm, access to inside information, perquisites of control, and power/control being valued directly.³ While previous papers, with the exception of Hart and Moore (1988), have modeled control benefits as indivisible, all these benefits, to the extent to which they exist, are plausibly shared

³ For a good discussion on the plausibility and source of private benefits, see Barclay and Holderness (1989).

by a number of individuals. This paper assumes an investor need not be a majority shareholder, or even the largest shareholder, to enjoy some benefits of control. Instead, investors of smaller size can form controlling coalitions, and divide the benefits of control amongst one another.

Such activity implies that the degree of control an investor derives from a block depends on the strategic importance of this block in forming winning coalitions. If one investor has a majority position, a moderate-sized block investor obtains no control at all; if instead other shares are held by many disperse individuals, a moderate-sized block may confer a sizable degree of control. Large investors would like to invest their money across firms in a manner that maximizes benefits from control, understanding that others are acting likewise. We consider such a game among investors and its implications for the shareholder structure across firms. This setup yields a setting through which the shareholder structure, the motivation for block shareholding, and the value of partial control can be explored. Additionally, the framework could be utilized in the future to explore related issues of the value of voting, optimal voting structure, and tradeoffs between diversification and control for investors.

We initially employ Shapley values to capture analytically the notion that the division of control benefits depends on the strategic importance of shareholdings. While Shapley values yield convenient analytic expressions for control benefits in our model, results we derive are robust to many other specifications. We proceed to give a general set of assumptions for the division of

control benefits, which when satisfied, yield the same results as Shapley values.

One may think of the following stylized story to put assumptions on the division of benefits in perspective. Suppose shareholders first choose where to invest, after which shareholders of each firm gather at an annual meeting and elect a Board of Directors. Small costs prevent all but block shareholders from attending. Shareholders present at the meeting attempt to form coalitions before the vote; nonattendeess may vote by proxy, but cannot participate in coalition formation. Coalitions reach agreements on which slate of directors to support, and what policies these directors will undertake, which implicitly specifies how spoils of control are divided. In this setting, both Shapley values or the general set of axioms for control division considered are plausible specifications which capture the notion that benefits should correspond with the strategic importance of one's votes in a coalition formation game.⁴

This model makes several strong predictions for the shareholder structure which are distinct from those of previous models of block shareholders. First, the model predicts that large investors "create their own space"; the presence of a large

⁴ Under the assumption that private benefits to be divided by the election winners are fixed, all outcomes are optimal in this simple voting game. In other circumstances, the assumption that the optimal outcome is obtained (i.e. that all carriers obtain their value) is perhaps the most problematic of the Shapley axioms.

block in a firm deters others large blocks from locating in the same firm. Second, the model predicts a clientele effect in the shareholder structure. In equilibrium there are three different types of shareholder structures: firms with one very large shareholder and no smaller block shareholders, firms with one large shareholder and many smaller block shareholders, and firms consisting of numerous small block shareholders but no dominant shareholder. Additionally, the larger the leading block shareholder, the fewer smaller block shareholders will be present in the firm. And third, the model predicts that there will be a threshold size, above which a large block will not be challenged for control.

A preliminary examination suggests that these predictions correspond with empirical tendencies among block shareholders. For the 456 firms reported in 1981 *CDE Stock Ownership Directory: Fortune 500*, there are 246 shareholders holding blocks of greater than 10% of a firm's equity and 123 holding blocks greater than 20%. Table 1 gives both the actual distribution across firms of these block shareholders and the expected distribution under a random allocation.⁵ Though only meant to be suggestive, this table supports the hypothesis that large shareholders indeed do "create their own space". For both the 10% and 20% blocks, there

⁵ All tables and diagrams appear at the end of the text.

For the random allocation, each large block is taken to have an identical chance of being located in any firm. While this could conceivably violate an allocation constraint, i.e., more than 100% of the shares in a firm are held, the possibility is remote enough to be inconsequential.

are many fewer firms with multiple large block shareholders and many more firms with a single large block shareholder than a random distribution would predict. For example, while a random distribution would predict approximately 14 firms with two or more shareholders holding 20% blocks, only 3 such firms exist. A goodness of fit chi-square test finds the difference between the actual and random distribution significant at .001 and .003 levels, for the 10% and 20% blocks respectively.⁶

Similarly, there appears to be empirical support for a clientele effect in the shareholder structure. Regressing the number of one percent block shareholders on the size of the largest block shareholder and a constant, one finds a negative coefficient for the size of the largest block, significant at a .001 level.⁷ Additionally, there is indirect evidence supporting the existence of a threshold level. Barclay and Holderness (1989) finds that blocks whose size is on the order of 25% of a firm's equity sell for a significant premium. Morck, Shleifer and Vishny

⁶ An examination of the data shows a size effect as well; there are more large blocks in small firms than large firms. This suggests that we should find more firms with multiple large blocks than under the random distribution, making the observed effect more striking.

⁷ Nor is this result driven by an allocation constraint, whereby, some firms with large leading shareholders have almost all shares held by block shareholders. The typical firm in the sample has only about 50% of its shares held by shareholders of size 1% or greater, leaving plenty of shares available for another investor desiring to hold such a block. Furthermore, regressing the number of 1% blocks normalized by the fraction of shares not held by the largest shareholder on the size of the largest shareholder - a fairly stringent test -, still yields a negative coefficient, however, only significant at a .1 level.

(1988) find the presence of such large blocks affects managerial performance. Both these papers discuss the possibility of a threshold level. Thus, a preliminary review of the shareholder structure in Fortune 500 firms finds evidence supporting the implications of the model. A careful empirical examination on the distributional structure of block shareholders and on dynamic changes in this structure would be very interesting.

In addition to this suggestive empirical evidence, there is considerable anecdotal evidence of block shareholders actively pursuing private benefits. During takeover contests, opposing sides actively recruit block shareholders. Pound (1988) finds that block shareholders generally side with management during proxy fights, and that management tries to influence the votes of money funds. Brickley, Lease, and Smith (1988) find that institutional and block shareholders vote more often than other shareholders, and institutions noted for business ties with firms in which they invest (banks, insurance companies and trusts) are more likely to vote with management than other institutions (mutual funds, foundations and public pension funds). Thus, institutions may obtain benefits to partial control through a profitable business relationship with the firm in question. Brickley, Lease and Smith also discuss the Council of Institutional Investors, a coalition of block investors formed in 1985 with the stated purposes of ensuring that companies are responsive to shareholders and seeking dialogue with management. Such examples can be interpreted as evidence that management is supported by a coalition of block shareholders who receive

benefits in return for support.

Additionally, there have been several well publicized recent instances where large equity holders have taken an active role in determining firm policy or control.⁸ One striking example is the Lockheed proxy contest. In this control contest, over one dozen institutional investors agreed to support the challenger Harold Simmons in exchange for a larger voice in the new management. Incumbent management defeated Simmons only after granting institutional investors three seats on the board and an expanded role in management.

While the primary contribution of this paper is in developing a reasonable model capable of explaining empirical evidence on block shareholding, there exist a number of other economic situations for which such a model may be applicable, in which agents with finite resources partake in a multidimensional competition. Examples include multiple patent races, advertising among competing conglomerates, and political contests between parties in multiple elective races.

Section 2 presents a general model to analyze strategic investment decisions when control benefits are divisible. Section 3 analyzes equilibria of this model, and Section 4 extends results derived under Shapley values to more general divisions of control benefits. Section 5 suggests extensions and empirical test of the model, and concludes.

⁸ For several cases where pension plan investors have played an active role, see the *Employee Benefit Research Institute*, Issue Brief, April 1990, No. 101.

Section 2 - The Model

We now consider a model of financial investment when there exist divisible control benefits. We consider an economy with J identical firms, each with a single class of equity. Total private benefits to control of each firm is 1. The next two sections consider benefits to be divisible among block shareholders according to their Shapley values in a normalized cooperative majority voting game. We initially consider Shapley values for the sake of analytical simplicity and the intuitive interpretation such a division yields (in particular, see Lemma 2 and the discussion at the end of Section 2 below). In Section 4 we extend these results to a more general specification for the division of control benefits.⁹

In order to explore the interaction between different sized investors, initially two sizes of risk neutral block investors are

⁹ Shapley values can be derived as the unique cooperative game solution which satisfies a set of three axioms. These axioms consist of a symmetry axiom, a linearity axiom, and an axiom stating that any carrier (a set containing all strategically relevant players in the game) obtains the optimal outcome. These axioms yield a unique division of surplus given by,

$$\phi_i = \sum_{\substack{T \subset N \\ i \in T}} \frac{(t-1)!(N-t)!}{n!} [v(T) - v(T - \{i\})],$$

where v represents the characteristic function for the game, N the set of all players, t the number of players in T , and ϕ_i individual i 's share of benefits. Shapley values can be given the interpretation of the expectation, taken over an equal weighting of all possible orderings of players, of the marginal addition a player makes to the set of preceding players in the ordering. See, for example, Owen (1982).

modeled. N agents are endowed with wealth n , and M with wealth m ; where wealth is given in firm-size units. We assume there is no borrowing of funds, and all wealth is invested.¹⁰ To distinguish between the two types of investors, we denote the investors of size n as "type 1 investors", and those of size m as "type 2 investors"; and we let $s = n/m$. The N type 1 investors are to be considered "very large investors" capable of "dominating" one firm, while the M type 2 investors are large enough to hold significant blocks and participate in coalitions, but not large enough to "dominate" a firm. We assume $n > m$, $N < J$, and $M \gg J$ to reflect this interpretation and to correspond with empirical observation on block investors. Furthermore, we assume the wealth of all block investors combined is strictly smaller than the entire market.

All shares not held by block investors are held by noise traders, who are assumed to be too small individually to acquire blocks and obtain any benefits of control. Noise traders play two roles in the model. First, they provide liquidity. Since they obtain no private benefits, they are willing to buy and sell at

¹⁰ Proposition 4 extends results to allow for an arbitrary number of different sizes. Allowing for risk aversion would imply that in equilibrium, instead of holding single blocks, shareholders would hold several blocks, trading off standard diversification benefits with the control related concentration benefits of the model. This will not alter any of the main implications for the shareholder structure. Similarly, allowing investors to borrow at an increasing cost will imply that investors will borrow to the point at which the marginal cost from further borrowing equals the marginal benefit of greater concentration. Then wealth levels n and m can be interpreted as wealth after such an optimal borrowing decision, and once again results concerning the shareholder structure are still valid.

the firms' public valuations (which is identical across all firms). This allows us to consider large investors' strategies solely as an allocation problem.¹¹ Large shareholders' objective is hence solely to allocate their wealth among firms to maximize the control benefits they receive.

Secondly, some noise traders vote randomly, thereby creating noise in the outcome of close control contests and smoothing the value of control to large shareholders. To illustrate the motivation for this assumption, consider the following simple example. Suppose there are three large shareholders in a firm, with Shareholders 2 and 3 each holding a .1 share of the firm's equity. If noise shareholders do not vote at all, the Shapley value of the first shareholder is $1/3$ for a share $0 < \theta \leq .2$, and 1 for $\theta > .2$. While it is natural that Shareholder 1 receives all benefits if θ is significantly larger than .2 and that benefits are split evenly if θ is much less than .2 (any two shareholders can form a winning coalition), the sharp discontinuity at $\theta = .2$ seems implausible. Furthermore, analytically, the lack of upper semi-continuity at points where any pair of coalitions has an identical number of votes leads to the nonexistence of

¹¹ Implicitly, we assume here that noise traders hold some shares of all firms. Provided that noise traders collectively comprise a "large enough" share of the market, this will be true in equilibrium. That is, in equilibrium there will not be any firm so hotly contended among block shareholders that all noise traders are driven out, while simultaneously there exist other firms which have many noise traders and could be contested relatively easily. An interesting extension to this model would be to consider the effect of dual equity classes with differential voting rights, which would serve to separate noise traders and block shareholders.

equilibrium. However, by allowing some noise traders to vote randomly, with a small net vote, continuity and existence are obtained. By adding noise to the above example, benefits to individual 1 still increase from from $1/3$ to 1 around $\theta = .2$, but they do so continuously.

Let ϕ_1^j give control benefits that investor i receives from firm j , and let 2β denote the number of noise votes¹²; the number of votes a coalition needs to win increases by β due to their participation. For a competing pair of coalitions, noise traders cast $\beta + \epsilon$ votes for one of the coalitions and $\beta - \epsilon$ for the other. We assume the random variable ϵ is distributed uniformly on support $[-m/2, m/2]$.¹³ Noise figures into the division of benefits in the following manner. Without noise, the Shapley value of a participant in a normalized simple voting game can be given by the probability over equally-weighted random orderings that she will be the swing voter; with noise, expected benefits are given by this probability taken jointly over equally-weighted random orderings and noise.

Finally, we assume that type 1 investors (who are large enough to purchase large blocks, and will do so in equilibrium) must allocate their wealth before type 2 investors.¹⁴ Type 2

¹² This can be interpreted as either all the noise traders, or some fraction of the noise traders that vote.

¹³ This specific distributional assumption is made for analytic simplicity. The reason for this assumption will become clear below.

¹⁴ This timing assumption can be modeled formally by allowing for a small transaction cost on large block transactions which aren't incurred on small block transactions.

investors (who can only purchase smaller blocks) are modeled as completely liquid, and can therefore react both to type 1 and other type 2 shareholders' investments. This timing assumption can be thought of capturing the notion that there are costs associated with obtaining large blocks of equity not present for smaller blocks.¹⁵

It will be useful to first consider control benefits for several cases which play an important role in the analysis below. These results are given in the following three lemmas.

Lemma 1 - Denote the $I \times 1$ vector of control benefits when there are I individuals in a firm of sizes s_1, s_2, \dots, s_I , and noise is distributed $\epsilon \sim U[-z, z]$, by the function $\phi(s_1, s_2, \dots, s_I, z)$. Then ϕ is homogeneous of degree 0.

Proof - Follows immediately from the definition of Shapley values and the manner in which noise is added. □

This lemma simply states that rescaling the size of all voters (including net noise voters) does not change the outcome within a firm.

The following two Lemmas characterize how benefits of control are divided up in firms with one or two large shareholders and L

¹⁵ Large tender offer premiums attest to this fact. Additionally, Barclay and Holderness find that large blocks tend to sell for a significant premium over ex-trade market price, while smaller blocks do not.

smaller identical block shareholders.

Lemma 2 - Suppose there is one individual of size x and L of size y in a firm, and the noise from noise traders' votes is distributed as $\epsilon \sim U[-y/2, y/2]$. Define s as the ratio of investors' sizes; $s = x/y$. Then the benefits to control of the investor of size x and the investors of size y , $E(\phi_1)$ and $E(\phi_2)$ respectively, are given by,

$$E(\phi_1) = \begin{cases} \frac{s}{L+1} & \text{if } s \leq L+1 \\ 1 & \text{if } s > L+1 \end{cases} \quad (1)$$

$$E(\phi_2) = \begin{cases} \frac{L+1-s}{L(L+1)} & \text{if } s \leq L+1 \\ 0 & \text{if } s > L+1 \end{cases} \quad (2)$$

Proof - From Lemma 1, we can consider a majority voting game with one individual of size s , L individuals of size 1, and noise distributed as $U[-1/2, 1/2]$. First suppose that noise traders do not vote. Let $s \leq L+1$. The Shapley value for the agent of size s is given by,

$$\phi_1 = \frac{1}{L+1} \sum_{i=0}^L I \left[\alpha \geq i > \alpha - s \right] \quad (3)$$

where I is an indicator function and $\alpha = (s+L)/2$ is the vote needed for a majority; half of all votes cast. Equation (3) states that the benefits of agent 1 are given by the probability that he will be the swing vote in a random ordering of

shareholders. Rearranging (3) yields,

$$\phi_1 = \frac{1}{L+1} \left(\text{int}(s) + I [\text{mod}(s) > \text{mod}((s+L)/2)] \right), \quad (4)$$

where $\text{int}(\cdot)$ is the nearest lower integer and $\text{mod}(\cdot)$ is modulus one of their respective arguments.

Now consider the effect of the noise traders' votes. The number of votes α needed for a majority increases by β , while the number of votes in individual 1's coalition increases by $\beta + \epsilon$. Equation (4) thus becomes,

$$E[\phi_1] = \frac{1}{L+1} E \left(\text{int}(s) + I [\text{mod}(s) > \text{mod}((s+L)/2 - \epsilon)] \right). \quad (5)$$

Since ϵ is distributed as $U[-1/2, 1/2]$, $\text{mod}((s+L)/2 - \epsilon)$ is distributed as $U[0, 1]$. Thus,

$$E[\phi_1] = \frac{1}{L+1} \left(\text{int}(s) + \text{mod}(s) \right) = \frac{s}{L+1}. \quad (6)$$

And since $\phi_1 + L\phi_2 = 1$,

$$E[\phi_2] = \frac{L+1-s}{L(L+1)}.$$

If instead $s > L+1$, individual 1 wins all votes for any realization of ϵ , and therefore $\phi_1=1$ and $\phi_2=0$. \square

Lemma 3 - Suppose there is one individual of size x_0 , one of size x_1 and L of size y in a firm, and the noise from noise traders' votes is distributed as $\epsilon \sim U[-y/2, y/2]$. Define $s_i = x_i/y$, $i=0,1$. Then the expected benefits of control to the investors of size x_i , $i=0,1$ are given by,

$$E(\phi_1) = \begin{cases} \frac{s_i(L-s_j+2) + (\sigma_{u,i}^2 - \sigma_{u,i}^2)}{(L+2)(L+1)} & \text{if } s_0 + s_1 \leq L+1 \\ \frac{(L-s_j+s_i+2)^2 - 1 + 4\sigma_{u,i}^2}{4(L+2)(L+1)} & \text{if } s_0 + s_1 > L+1, \\ & \text{and } s_0 < L+s_1+1 \\ & s_1 < L+s_0+1 \\ 1 & \text{if } s_i > L+s_j+1 \\ 0 & \text{if } s_j > L+s_i+1 \end{cases} \quad (7)$$

where $i, j = 0, 1, i \neq j$, and $\sigma_{u,i}^2$ and $\sigma_{u,i}^2$ are continuous periodic functions of $L-s_j+s_i$ and $L-s_j-s_i$, with period 2, that take values between 0 and 1/4. Their precise values are given in equation (A8) in the proof.

Proof - See Appendix A.

Note that the distributional assumption on noise implies $\text{mod}((s+L)/2-\epsilon)$ is distributed as $U[0,1]$, and therefore the indicator function in equation (5) in Lemma 2 has expectation $\text{mod}(s)$. Likewise this distributional assumption affects the expression in Lemma 3, albeit in a more complicated manner. The $\sigma_{u,i}^2$ and $\sigma_{u,i}^2$ terms are artifacts of this noise, small relative to other terms, and inconsequential to what follows. When $s_0+s_1 = L+1$, $\sigma_{u,i}^2 = 0$, and the first two expressions in equation (7) are equal; when $s_i = L+s_j+1$, $\sigma_{u,i}^2 = 0$ and the second expression in equation (7) yields 1; thus $E(\phi_1)$ is continuous for all values of s_i and s_j .

While these results are obtained for a specific distribution of noise, they can be defended under several alternative assumptions. If noise is large relative to type 2 investors (and still uniform), then once again expected Shapley values are given by the results in Lemmas 2 and 3, provided that s is not "too close" to $L+1$ in Lemma 2 and s_i is not "too close" to $L+s_j+1$ in Lemma 3. Furthermore, as L and s grows large proportionally, benefits given by Lemmas 2 and 3 approach those when there is no noise. Without noise the bracketed term in equation (5) yields either $\text{int}(s)$ or $\text{int}(s)+1$, with noise it is s . As s and L grow proportionally, this difference becomes negligible in equation (5). In the limit, as $y \rightarrow 0$ and $L \rightarrow \infty$ proportionately (and therefore $s \rightarrow \infty$), so that yL - the total size of type 2 shareholders - is constant, it can be shown that expressions (1) and (7) converge to Shapley values of shareholders with fractional shares s_i , where the L shareholders are replaced by a continuum and there is no noise.

The division of benefits specified in Lemma 2 yields a simple interpretation. When $s=1$, individual 1 is the same size as other shareholders and thus receives identical control, therefore $E(\phi_i) = 1/(L+1)$. At $s=L+1$, individual 1 always wins any vote (even with the worse possible draw of noise), hence $E(\phi_i)=1$. Between these values, $E(\phi_i)$ is linear in s . Thus between $s+1$ and $s=M+1$, marginal benefits to concentration for the large shareholder are constant. This does not, however, imply that control benefits are proportional to the fractional share of total block shareholdings one possesses. The fraction of the block shares held by

individual 1, given by $s/(L+s)$, is concave in s , and therefore $E(\phi_1)$ is convex in this fraction over the range $[0, 1/2]$. If L and s increase proportionally (which would occur if the L investors were to split up into smaller positions), benefits to the large shareholder $E(\phi_1)$ grows, and total benefits to the smaller block shareholders $LE(\phi_2)$ falls. This result is related to the superadditivity of Shapley values; one large concentrated block brings more value than the block in two pieces. Investor 1 would rather see the other individuals broken up, as this allows more possibilities to form winning coalitions.

Similar comparative statics results follow from Lemma 3. Ignoring the small σ^2 terms, $E(\phi_1)$ is once again linear in s_1 when $s_0 + s_1 \leq L+1$. When instead $s_1 + s_2 > L+1$, and therefore the two large shareholders can form a winning coalition by themselves, $E(\phi_1)$ is convex in s_1 . In both cases benefits to the large shareholders are once again convex in the fractional share of block shareholdings held by them. If $s_0 = 0$, $E(\phi_1) = \frac{s_1}{L+1}$, and if $s_0 = 1$, $E(\phi_1) = \frac{s_1}{L+2}$; which are the values from Lemma 2 when shareholder 0 doesn't exist or is replaced by an additional small block shareholder of size y .

Section 3 - Market Equilibrium

We now consider the market equilibrium with J firms, N type 1 shareholders of size n and M type 2 shareholders of size m . We

restrict attention to pure strategy subgame perfect equilibria (PSSPE) for the following reason. It is optimal for type 1 individuals to cooperate amongst one another (by staking out their own firms), and they move before type 2 individuals. Thus they have nothing to gain from concealing their strategies through mixing, and such mixing could cause type 1 shareholders to accidentally stake out the same firm, which is costly.¹⁶ Furthermore, any steady state must be a pure strategy equilibrium in type 2 shareholders' strategies since they can costlessly reinvest. Note that smoothing the control benefits through noise does not immediately guarantee the existence of a pure strategy equilibrium by the standard Debreu-Glicksberg-Fan existence result, as payoffs are not quasi-concave in strategies. Investing all wealth in either firm j or firm j' may be better than investing half of wealth in each firm. However, many such equilibria do exist, as characterized in Proposition 1.

The application of some cooperative refinement to equilibria is natural under the assumption that type 2 individuals can reinvest costlessly. Suppose we have reached a PSSPE in which a coalition of type 2 individuals could affect a Pareto improvement by jointly deviating. Such situations arise naturally in this game; it never pays for one type 2 shareholder to unilaterally challenge a larger type 1 shareholder for control of a firm, but a

¹⁶ We could formally justify considering only equilibria where type 1 shareholders play pure strategies either by allowing them to reallocate wealth at a small transaction cost or by having them move sequentially.

group of type 2 shareholders may benefit by doing so. A type 2 shareholder could costlessly test whether others will follow such a deviation by challenging a type 1 shareholder. And other type 2 shareholders could costlessly join in, until enough have deviated to make the deviations profitable. Such considerations lead to the consideration of cooperative refinements of coalition-proof Nash equilibrium (Bernheim, Peleg and Whinston (1987)), and strong equilibrium (Aumann (1959)). Given the dynamic structure of the game, however, we restrict coalitions to shareholders of the same type (who move at the same time). We refer to such refinements as class coalition-proof Nash equilibrium (CCPNE) and class strong equilibrium (CSE).¹⁷ These refinements yield a unique equilibrium.¹⁸ PSSPEs are characterized in Proposition 1, while Proposition 2 gives the unique CCPNE and CSE.

Proposition 1 states that PSSPE exist, and have a simple appealing form, which depends on the relative size s of type 1

¹⁷ In this setting, CCPNEs are equivalent with perfect coalition-proof Nash equilibria of Bernheim, Peleg and Whinston, but CSEs are less restrictive than perfect strong equilibria of Rubinstein (1980). Intuitively, we don't allow deviations by a coalition consisting of both type 1 and type 2 shareholders to break an equilibrium, because with all such coalitions, after type 1 shareholders have moved, a new coalition of type 2 shareholders (including the deviators) can improve by deviating in a manner which make the initial type 1 deviators worse off. Such dynamic considerations are built into the definition of perfect coalition-proof Nash equilibrium, but are lacking in the very stringent definition of perfect strong equilibrium, which requires that the equilibrium be strong in every subgame.

¹⁸ One may wonder how a class strong equilibrium can exist for an essential constant sum game. A CSE exists because coalitions are only considered over type 2 agents. While the overall game is constant sum, the game induced on type 2 agents by taking the play of type 1 agents as given is not.

versus type 2 shareholders.

Proposition 1 - For given values of J , N , and M , there exists values $1 = s_{N+1} < s_N < s_{N-1} < \dots < s_1 = \bar{s}$, such that for s satisfying $s_{k+1} < s \leq s_k$, PSSPE are as follows:

Each of the type 1 agents invests all wealth in a different firm. Let $J(N) \subset J$ denote the set of firms with a type 1 investor. Each type 2 investor also concentrates all wealth in one firm. Denoting the number of type 2 investors in firm j by $L(j)$,

$$L(j) = \begin{cases} M_1 & \forall j \in J \setminus J(N) \\ M_2 & \forall j \in J' \subseteq J(N) \\ 0 & \forall j \in J(N) \setminus J' \end{cases} ; \quad (8)$$

where J' (the set of firms with type 1 individuals who are challenged) is any subset of $J(N)$ of cardinality $|J'| \leq k$, and M_1 and M_2 are constants satisfying the following conditions.¹⁹

$$M_1(J-N) + M_2 J' = M \quad (9)$$

$$\phi_2^j(M_1) = \phi_2^{j'}(M_2) \quad \forall j \in J \setminus J(N), \forall j' \in J' \quad (10)$$

$$\frac{\partial \phi_2^j}{\partial L}(L(j)) \leq 0 \quad \forall j \in J \setminus J(N), \forall j' \in J' ; \quad (11)$$

where $\phi_2^j(L)$ denotes expected control benefits for a type 2 investor in firm j when there are L type 2 investors in that

¹⁹ When the meaning is clear, we let J , N and J' denote the cardinality of these sets.

firm.²⁰

While this proposition is long, what it states is intuitive. Both types of large shareholders invest all their wealth in one firm (recall that risk aversion motives for diversification have not been built into this model). This result follows from the weak convexity of payoffs in size for the large shareholder in Lemmas 2 and 3. This convexity implies that a type 1 shareholder acquiring a bigger share of a firm is challenged by fewer type 2 shareholders, thereby yielding benefits to concentration. Type 1 shareholders create stakes in firms, and the smaller type 2 shareholders react by distributing themselves across firms without type 1 shareholders and challenging a subset $J' \subset J(N)$ of firms with type 1 shareholders. In equilibrium, there are three types

²⁰ This proposition does not characterize all PSSPE for two reasons. The first is an integer problem; equilibria exist with M_1 investors in some firm $j \in J \setminus J(N)$ and M_1+1 in another firm $j' \in J \setminus J(N)$, where individuals in the former do not switch to the latter because then the latter would then have M_1+1 individuals.

More accurately, this proposition and others throughout the paper characterize equilibria to within such integer rounding.

A second reason that this proposition does not characterize all equilibria is more substantive. As is seen in the proposition, there are equilibria in which any number from 0 to k type 1 investors are challenged by type 2 investors. The fewer type 1 investors are challenged, the better off they are. Thus equilibria exist of the following nature: type 1 investors diversify their holdings, because if they do, type 2 investors play an equilibrium good for them, where no large investors are challenged; whereas if they don't, type 2 individuals play an equilibrium bad for them, where they challenge k type 1 investors. However, all such equilibria fail the class coalition-proof and class strong refinements of Proposition 2; as under these refinements, type 2 individuals always challenge the maximum k investors.

of firms; those without any dominant shareholders but rather many shareholders holding smaller blocks, those with one large shareholder who is uncontested, and those with one large shareholder who is contested by smaller block shareholders.

In equilibrium, benefits that type 2 shareholders receive must be equalized across all firms in which they do invest (up to integer differences mentioned in footnote 20). Of the N firms with type 1 shareholders, small shareholders will contest any number $|J'|$ between 0 and $k(s)$, where $k(s)$ is a decreasing function of s defined by the critical values s_{N+1}, s_N, \dots, s_1 of Proposition 1. As the ratio of sizes s grows, it becomes infeasible for type 2 investors to contest all type 1 investors; rather they can only challenge a subset of less than or equal to $k(s)$ type 1 investors.

If s is greater than \bar{s} , it follows that $k=0$, and then only $J' = \emptyset$ is an equilibrium (no type 1 investors are contested). Since type 1 investors are not challenged beyond this size, they will cease investing all their wealth in one firm. Rather they will invest $\bar{s}m$ in the first firm in which they create a stake, and then invest remaining wealth in another firm (up to $\bar{s}m$, and if they have more wealth, invest elsewhere, etc...).

Proof - First we consider the play of type 2 shareholders. Within integer deviations, in equilibrium ϕ_2^j must be equated across all firms with type 2 shareholders, and be decreasing in $L(j)$, so that no type 2 shareholders want to change firms. This implies that equations (8) through (11) must hold. From Lemma 2,

using equation (8), ϕ_2^j is given by,

$$\phi_2^j = \frac{M_2 + 1 - s}{M_2(M_2 + 1)} \quad \forall j \in J' \quad (12)$$

$$\phi_2^j = \frac{1}{M_1} \quad \forall j \in J \setminus J(N) \quad (13)$$

Equating these values, imposing (9), and solving for M_2 yields,

$$M_2 = \frac{(M+J's - (J-N+J')) \pm \sqrt{(M+J's - (J-N+J'))^2 + 4M(J-N+J')(1-s)}}{2(J-N+J')} \quad (14)$$

Two solutions to (14) exist - provided the root is real - for the following reason. As the number of type 2 shareholders challenging a type 1 shareholder increases, benefits to type 2 shareholders undergo two opposing effects. The more type 2 investors are in the firm, the better they are able to contest the type 1 shareholder, thereby increasing their joint benefits. However, there are also more type 2 shareholders to share in these contested benefits. As M_2 exceeds $s-1$, initially the first effect dominates and ϕ_2 increases with M_2 ; for higher M_2 , the second effect dominates and ϕ_2 decreases with M_2 . The changing sign of $\partial\phi_2/\partial M_2$ in equation (12) induces two solutions to equations (9) and (10).

This situation is depicted in Diagram 1. This diagram graphs the benefits to a type 2 shareholder both in a firm with a type 1 shareholder and in a firm without one, as a function of the number of type 2 shareholders in the firm. The higher curve represents benefits in a firm without a type 1 shareholder. Note the single

peaked shape of benefits in a firm with a type 1 shareholder. The two lines connecting the curves in the diagram are the values at which equation (14) is satisfied; that is, benefits are equated across firms and all type 2 shareholders have allocated wealth in some firm. The lower value (which corresponds to the negative root in equation (14)) however, is never an equilibrium, as $\partial\phi_2/\partial M_2 > 0$ for firms with a type 1 investor. This in turn would induce type 2 investors in firms without a type 1 investor to switch to firms with one. At the positive root, $\partial\phi_2/\partial M_2 \leq 0$ provided that,

$$M_2(s) \geq (s-1) + \sqrt{s(s-1)}. \quad (15)$$

In this situation, no type 2 shareholder can benefit from switching firms.

The radical in (14) decreases with s (over the relevant range), and for any $J' \leq N$ there is a critical value $\tilde{s}_{J'}$, given by,

$$\tilde{s}_{J'} = \frac{J'^2 - (M+N-J)J' + 2M(J-N+J') - 2\sqrt{M(J-N)(M+J')(J-N+J')}}{J'^2} \quad (16)$$

at which the root vanishes. For $s > \tilde{s}_{J'}$, there is no equilibrium with J' or more firms contested. If instead $s \leq \tilde{s}_{J'}$, provided equation (15) holds, the positive root of (14) will denote an equilibrium. For any given J' , it can be shown that equation (15) is not satisfied at $s = \tilde{s}_{J'}$. However, there exists a value $s_{J'} < \tilde{s}_{J'}$, such that equation (15) holds for all $s \leq s_{J'}$, where M_2 is given by the positive root of equation (14). These are the values s_N, s_{N-1}, \dots, s_1 of the proposition. Furthermore, both $s_{J'}$ and $\tilde{s}_{J'}$,

are strictly decreasing in J' .

Intuitively, if $s > \bar{s}_{j'}$, any distribution of type 2 individuals which challenges J' firms leaves so few type 2 shareholders in firms $J \setminus J(N)$ that they do better than the type 2 shareholders in firms J' . If instead $s'_j < s < \bar{s}_{j'}$, any distribution of type 2 shareholders which challenges J' firms and has enough type 2 shareholders in firms $J \setminus J(N)$ to satisfy equation (10) will consist of so few shareholders in firms of J' that $\frac{\partial \phi_2^j(L)}{\partial L}(M_2) > 0$ for $j \in J'$. This cannot be an equilibrium because type 2 shareholders will want to switch from firms without a type 1 shareholder to those with one. Thus, there exists an equilibrium with J' firms challenged if and only if $s \leq s_{j'}$.

To complete the proof, it must be shown that both type 1 and type 2 shareholders don't want to deviate by diversifying, and that type 1 shareholders don't deviate by jointly investing in the same firm. We use Lemmas 2 and 3 in Appendix B to show that diversification is unprofitable for a type 2 agent. We can easily show the desired results for type 1 shareholders, in the spirit of footnote 20, by letting type 2 shareholders play the worst equilibrium for a type 1 deviator. However, we put off showing that type 1 shareholders don't want to deviate until the proof of Proposition 2, where we further show that that no type 1 shareholder deviates when type 2 shareholders challenge the maximal number k of type 1 shareholders which they can challenge in any PSSPE, both on and off the equilibrium path. It will be seen below that this is both the optimal equilibrium for type 2

shareholders and the only equilibrium that survives the proposed coalitional refinements. □

Note the role that the timing assumption plays in this result. In equilibrium, while no type 2 shareholders want to change positions, the type 1 shareholders who are not challenged would want to shift some wealth to other firms. But if type 2 investors can respond to this move, they would then challenge the type 1 shareholders' reduced position, making such a move undesirable.

Diagrams 2 and 3 depict PSSPE for different numbers of firms challenged when $J=500$, $N=200$, $M=10000$ and $s = 5$ (Diagram 2) and $s = 8$ (Diagram 3). Points on the curves represent benefits per type 2 shareholder in the equilibria where the labeled number of firms are challenged. When $s = 5$, there exists an equilibrium where all firms are challenged. When instead $s = 8$, in the equilibrium where the maximum number of firms are challenged, the number of type 2 challengers per firm maximizes possible benefits against a type 1 shareholder of this size, to within integer roundings on $k(s)$. That is, the number of challengers is that which obtains, within integer roundings, the peak of ϕ_2^j , $j \in J(N)$, as in Diagram 3. In general, one of these two outcomes always occurs, as is stated in the following lemma.

Lemma 4 - For any given s , in the equilibrium where the maximal number of firms $k(s)$ are challenged, either $k(s) = N$, or $k(s)$ is such that to within integer roundings on k ,

$$M_2 = \underset{L}{\operatorname{argmax}} \left(\phi_2^j(L), j \in J(N) \right). \quad (17)$$

Proof - Define $M_2^* = \underset{L}{\operatorname{argmax}} \left(\phi_2^j(L), j \in J(N) \right)$, and $\phi_2^* = \max \left(\phi_2^j(L), j \in J(N) \right)$,²¹

and let $M_{2,J'}$ denote the positive root of M_2 in equation (14) when J' type 1 shareholders are challenged. This will yield a PSSPE with J' type 1 shareholders challenged if and only if equation (11) is satisfied at $M_{2,J'}$ as well. Equation (11) in turn is satisfied if and only if $M_{2,J'} \geq M_2^*$, since ϕ_2^j is single-peaked at M_2^* . Thus $M_{2,k(s)} \geq M_2^*$, since by definition there is an equilibrium with $k(s)$ type 1 shareholders challenged.

Now suppose $k(s) < N$. It can be shown using equation (14) that $M_{2,J'}$ is decreasing in J' . (This is shown below in Lemma 5 for $J' \leq k(s)$.) Furthermore, $M_{2,k(s)+1} < M_2^*$, because if not, challenging $k(s)+1$ shareholders would yield an equilibrium, contradicting the definition of $k(s)$ as the maximal number of shareholders that can be challenged in equilibrium. Thus, $M_{2,z} = M_2^*$ and $\phi_2^j(M_{2,z}) = \phi_2^*$ for some z between $k(s)$ and $k(s)+1$. That is, within integer roundings on k , $M_{2,k(s)} = \underset{L}{\operatorname{argmax}} \left(\phi_2^j(L), j \in J(N) \right)$.

□

The following Lemma, which will be useful below, states that type 2 shareholders do better, and type 1 shareholders worse, in equilibria with more type 1 shareholders challenged. Hence, the

²¹ M_2^* and ϕ_2^* depend on s ; we notationally suppress this dependence for the present proof, where s is fixed.

PSSPE with the maximal number of firms $k(s)$ challenged, is the best PSSPE for type 2 shareholders.

Lemma 5 - As the number of firms J' challenged in equilibrium increases, M_1 and M_2 decrease and ϕ_2 increases.

Proof of Lemma 5 - Equations (9), (10) and (11) must hold in all equilibria. It therefore follows that $\text{sign} \left(\frac{\partial M_1}{\partial J'} \right) = \text{sign} \left(\frac{\partial M_2}{\partial J'} \right)$. Differentiating equation (9) with respect to J' yields,

$$\frac{\partial M_1}{\partial J'}(J-N) + \frac{\partial M_2}{\partial J'}J' + M_2 = 0. \quad (18)$$

Since J' and $J-N \geq 0$, and $M_2 > 0$, this implies the sign of the partials must be negative. And since $\phi_2 = 1/M_1$, it follows that $\frac{\partial \phi_2}{\partial J'} > 0$. \square

While Proposition 1 indicates that many PSSPE exist in this game, only one survives cooperative refinements of class strong equilibrium and class coalition-proof Nash equilibrium over type 2 agents. The following proposition characterizes this equilibrium.

Proposition 2 - There exists a unique PSSPE which survives the refinement of class coalition-proof Nash equilibrium and class strong equilibrium. This equilibrium is the PSSPE characterized in Proposition 1, with the maximal number of firms k contested both on and off the equilibrium path. That is, for $s_{k+1} < s \leq s_k$, $|J'| = k$, and for $s > \bar{s} = s_1$, $J' = \emptyset$.

Proof - The proof that type 1 shareholders play as specified when type 2 shareholders are playing CSE strategies is given in Appendix B. We presently demonstrate that Proposition 2 specifies CSE strategies for type 2 shareholders.

Lemma 5 indicates that the equilibrium with the maximal number of type 1 shareholders challenged dominates all other equilibria for type 2 shareholders. Hence, if it is class strong, it is the unique CSE; all other PSSPE are not immune to a joint deviation by all type 2 shareholders to this superior equilibrium. Now suppose the equilibrium with the maximum number of firms k challenged is not class strong. Then there exists some coalition of type 2 shareholders that can Pareto improve by deviating. Any deviation involves an increase in the number of type 2 shareholders in some firm. Denote such a firm j' . Now since in equilibrium, $\frac{\partial \phi_2^j}{\partial L} < 0 \quad \forall j \in J \setminus J(N)$ and $\forall j \in J'$ (equation 11), and since $\phi_2^j(L)$ is quasi-concave for $j \in J'$, any increase in the number of type 2 shareholders in these firms decreases their benefits. Hence it must follow that $j' \in J(N) \setminus J'$. That is, a Pareto improvement among a coalition must involve challenging at least one type 1 shareholder not challenged in equilibrium. But by assumption, the maximal equilibrium number of firms k are challenged. Lemma 4 indicates that this implies either that all N type 1 shareholders are already challenged, or that the number of type 2 shareholders challenging type 1 shareholders is M_2^* (the number which maximizes possible benefits per challenger when

challenging a type 1 shareholder). In either case, it is not possible to challenge another firm and increase benefits. Hence, deviators who move to firm j' do not improve over the equilibrium. Thus the equilibrium is a class strong equilibrium.

Class strong equilibrium implies that it is class coalition-proof. Furthermore, since a deviation by all type 2 shareholders from any other PSSPE to the class strong equilibrium is a Pareto improvement, which is in turn immune to counterdeviations (since it is class strong), the CSE equilibrium is also the unique CCPNE. \square

While the characterization of PSSPE and CSE (setting $J' = k(s)$) in equation (14) appears quite unwieldy, a number of simple comparative statics results follow from this expression.

Proposition 3 -

1. As $s \rightarrow 1$, $M_2 \rightarrow \frac{(M - (J - N))}{J - N + J'}$, $M_1 \rightarrow \frac{J' + M}{J - N + J'}$, and $(M_1 - M_2) \rightarrow 1$.
2. $J = N \Rightarrow M_2 = M/J'$.
3. $M_1(M, N, J, J'; s)$ and $M_2(M, N, J, J'; s)$ are homogeneous of degree 0 in (M, N, J, J') .
4. In the CSE of Proposition 2, $\forall s$, $\frac{\partial M_1}{\partial s} > 0$, $\frac{\partial (J' M_2)}{\partial s} < 0$,
 $\frac{\partial \phi_1}{\partial s} > 0$, $\frac{\partial \phi_2}{\partial s} < 0$.
5. In the CSE of Proposition 2, besides at critical values s_N ,

s_{N-1}, \dots, s_1 where J' falls and therefore M_2 jumps up,

$$\frac{\partial M_2}{\partial s} < 0.$$

6. Comparing different PSSPE of Proposition 1, ϕ_2 increases with J' , and both M_1 and M_2 decrease with J' .

Proof - All follow immediately from equation (14) and simple algebra.

Proposition 3.1 states that as type 1 shareholders' size approaches that of type 2 shareholders, type 2 shareholders distribute themselves symmetrically across all firms they challenge (which is all firms for the CSE as $s < s_N$ and therefore $J' = N$) save for taking into account that there is already 1 shareholder present in firms of $J(N)$. When $s=1$, this results in all firms having the same number of identical sized shareholders. When there is one large type 1 shareholder in each firm, Proposition 3.2 indicates that type 2 shareholders distribute themselves evenly across all firms. Proposition 3.3 states that M_1 and M_2 are homogeneous of degree zero in the parameters. This allows us to obtain results as $M, J, N,$ and $J' \rightarrow \infty$. Propositions 3.4 and 3.5 state that as the size of type 1 shareholders grows relative to the size of type 2 shareholders; type 1 shareholders do better, type 2 shareholders do worse, the number of type 2 shareholders in any firm without a type 1 shareholder rises, the total number of type 2 shareholders challenging type 1 shareholders falls, and the number challenging

in any one firm falls except at the critical values of s beyond which one less firm is challenged (and those who were challenging this firm must reallocate across other firms). Finally, Proposition 3.6 restates Lemma 5; type 2 shareholders do better in PSSPEs in which more type 1 shareholders are challenged.

Table 2 gives some numerical results to give an intuitive feel for what the equilibria prescribe. In this table the parameters are set to $J=500$, $N=200$, and $M=10,000$, though any choice consistent with the interpretation of these parameters in the model yields similar results. The first column specifies the PSSPE by giving the number of type 1 shareholders challenged. The second column gives the critical levels s_j of Proposition 1. All these values fall within a reasonable range; if $s < 6.92$, all type 1 shareholders can be challenged in equilibrium, while none can be challenged if $s > 9.0$. Following columns give the number of type 2 shareholders challenging each type 1 shareholder, benefits of type 1 and type 2 shareholders, and the value of concentration in different PSSPE equilibria with J' firms challenged, when $s=5$ and $s=8$. The value of concentration is defined as the ratio $\phi_1/(\phi_2 s)$, the premium in benefits per share that type 1 shareholders realize over type 2 shareholders due to their concentration.

Note that as the number J' of firms challenged increases, M_2 , ϕ_1 and the value of concentration decreases, and ϕ_2 increases. When $s=8$, at most 71 type 1 shareholders can be challenged in a PSSPE. The class strong equilibria are the PSSPEs with type 2 shareholders challenging the maximal number of type 2 shareholders

possible; when $s=5$, this is all 200 (as depicted in Diagram 2), when $s=8$, this is 71 (Diagram 3).

While the assumption of only two sizes of shareholders allows us to obtain an analytic specification for equilibria in equation (14), similar results to the above propositions can be obtained under a much more general setting. The following result is a simple generalization of Proposition 2 when type 1 shareholders take on different sizes.

Proposition 4 - Suppose there are N type one shareholders with wealth x_i , and M type 2 shareholders with wealth y , where $x_i > y$, $i=1,2,\dots,N$. Then there exists a unique CSE and CCPNE given as follows:

There exists a cutoff level \bar{x} such that all type 1 shareholder with positions smaller than \bar{x} are challenged, and all those with positions greater than \bar{x} are not. Type 1 shareholders smaller than or equal to \bar{x} put all their wealth in one firm. Type 1 shareholders larger than \bar{x} put \bar{x} in their first firm, and then stake out another firm with remaining funds (if this is still greater than \bar{x} , they stake out yet another firm, etc...).

Type 2 shareholders will allocate themselves such that benefits they receive are equalized across all firms in which they invest. The larger the type 1 shareholder in any firm, the fewer type 2 shareholders will challenge that firm; type 1 shareholders beyond \bar{x} are not challenged.

While many PSSPE with different cutoff levels will typically exist, the equilibrium in this class with the maximal cutoff level

is the unique CSE.

Proof (Sketch) - Suppose there does not exist a cutoff level \bar{x} . Then there exists two type 1 investors i, i' , $x_i > x_{i'}$, with i challenged and i' not challenged. But this cannot be class strong, because the collection of shareholders challenging i could affect a Pareto improvement by challenging i' instead. Thus any CSE must involve a cutoff level.

The higher the cutoff level, the more firms are challenged. Suppose cutoff levels of both \bar{x}_1 and \bar{x}_2 yield PSSPE, with $\bar{x}_1 > \bar{x}_2$. Challenging more firms in equilibrium implies there are less type 2 shareholders challenging in each firm, and therefore benefits for type 2 shareholders are greater. Hence, only the \bar{x}_1 equilibrium can be a CSE, as the \bar{x}_2 equilibrium is not immune to a deviation by all type 2 shareholders to the \bar{x}_1 equilibrium.

To show the equilibrium with the maximal cutoff level is class strong, suppose there exists some Pareto improving deviation among type 2 shareholders. Just as in the proof of Proposition 2, since in any PSSPE $\frac{\partial \phi_2^j(L)}{\partial L} < 0$ for all firms with type 2 shareholders, this deviation must involve challenging some type 1 shareholder not previously challenged. However, in a manner analogous to Lemma 4, one can show that in the prescribed equilibrium, either all type 1 shareholders are challenged, or the largest type 1 shareholder actually challenged is challenged by the number of shareholders that maximizes possible benefits per challenger against such a shareholder. (If this were not true, a

higher cutoff level in which one more type 1 shareholder was challenged would be a PSSPE). This implies however, that in the equilibrium, benefits for type 2 shareholders are greater than could ever be obtained from challenging a larger (previously unchallenged) shareholder. Hence such a deviation cannot be Pareto improving.

Type 1 shareholders smaller than \bar{s} don't want to diversify for precisely the same reason as in Proposition 2. And type 1 shareholders larger than the cutoff level want to keep only enough in their first firm to ensure they won't be challenged, and invest the remainder elsewhere. \square

Such an equilibrium is depicted in Diagram 4. Higher benefit curves correspond to firms with smaller type 1 shareholders. In this diagram, type 1 shareholders are challenged if and only if $s \leq 5$. A shareholder of size $s=5$ is challenged by the number of type 2 shareholders which maximizes benefits per challenger against such a shareholder.

The CSE of this proposition is similar to that of Proposition 2, with the added effects that the number of challengers a type 1 shareholder faces decreases with the size of his position, and that type 1 shareholders are not challenged at all if their positions are larger than a threshold level \bar{x} . Similar results could likewise be obtained under further generalization allowing type 2 shareholders to take on arbitrary sizes smaller than the type 1 shareholders.

Section 4 - Robustness to Other Specifications

While Shapley values lead to simple equilibrium divisions of control benefits and analytic characterizations of equilibria, the propositions do not depend on this explicit formulation. Here we show that provided the division of control benefits satisfies several general assumptions, Propositions 1, 2 and 4 follow. Thus this section serves both to give more generality to the results and to highlight what aspects of the Shapley values are important.

Thus we presently drop the previous assumptions of Shapley values and the specification of noise voting. Instead we make the following general assumptions on the division of control benefits.

Assumption 1 - Symmetry. For any permutation π of the I shareholders in a firm, $\forall i$,

$$\phi_i(s_1, \dots, s_1, \dots, s_I) = \phi_{\pi(i)}(s_{\pi(1)}, \dots, s_{\pi(i)}, \dots, s_{\pi(I)}).$$

Assumption 2 - Individual Rationality and Optimality. $\phi_i \geq 0$,

$$\sum_i \phi_i = 1.$$

Assumption 3 - Homogeneity.²² $\phi = (\phi_1, \dots, \phi_I)$ is homogeneous of degree 0 in $s = (s_1, \dots, s_I)$.

²² This assumption need only hold over a relevant range of potential sizes and is mainly made for convenience. Without this assumption, all results would hold, but in a slightly less general form; Propositions would have to be stated in terms of the sizes of both types of shareholders n and m instead of the ratio $s = n/m$.

Assumption 4 - Monotonicity. If $s_i \geq s_k$, then $\phi_i(s_1, \dots, s_i, \dots, s_k, \dots, s_I) \geq \phi_k(s_1, \dots, s_i, \dots, s_k, \dots, s_I)$, and if $s_i \geq s'_i$, then $\phi_i(s_1, \dots, s_i, \dots, s_I) \geq \phi_i(s_1, \dots, s'_i, \dots, s_I)$.

Assumption 5 - Unanimity - There exists a $\gamma \geq 1$ s.t. $\phi_i = 1$ if $s_i > \gamma \sum_j s_j$.

Assumption 6 - Convexity 1 - $\phi_i(s_1, \dots, s_i, \dots, s_N)$ is convex in s_i over the domain $\{(s_1, \dots, s_N) \mid \phi_i(s_1, \dots, s_N) < 1\}$.

Assumption 7 - Convexity 2 - Consider a firm with 1 individual of size s_1 and L individuals of size $s_2 < s_1$. Let $s = s_1/s_2$, and let $\Phi^1(s, L)$ be benefits to an individual of size s_1 in such a firm. Then Φ^1 is twice continuously differentiable, strictly convex in L , and $(\Phi_s^1 - L\Phi_L^1) > 0$ over the domain $\{(s, L) \mid \Phi^1(s, L) < 1\}$.²³

These assumptions are sufficient to show Propositions 1, 2 and 4 hold. The proof of these results follow along the lines of those in Section 3, albeit, with considerably more notational complexity. These proofs are given in Appendix C.

All these assumptions yield intuitive interpretations, and we

²³ Subscripts represent partial derivatives. We take $\Phi^1(s, L)$ to be defined on the domain \mathbb{R}_+^2 . More accurately, we could define $\Phi^1(s, L)$ on $\mathbb{R}_+ \times \mathbb{Z}_+$, and state this assumption as, there exists an extension of $\Phi^1(s, L)$ to \mathbb{R}_+^2 with the above properties.

argue that they are all natural given a coalition formation story. Assumptions 1-5 are straightforward and relatively innocuous; it is the two convexity assumptions which have some power.²⁴

The first of these convexity assumptions is related to superadditivity. Superadditivity implies the marginal benefit of a block is greater when combined with another block rather than when held separately. Assumption 6 states that marginal benefit from an incremental share increases with the size of the block it is being added to.

The second convexity assumption roughly states that in a firm with one large block and a number of smaller blocks, over the range where the large shareholder does not receive all benefits, the greater this large shareholder's benefits, the more these benefits decrease with an increase in the number of challengers. This seems quite plausible; a shareholder who receives almost all benefits has more to lose from an additional challenger than one who no longer receives many benefits. Specifically, the first part of Assumption 7 states that, over the relevant range, increasing the number of challengers hurts a large shareholder more the fewer the number of challengers. Thus, if a type 1 shareholder is 4 times as large as type 2 shareholders, increasing

²⁴ However, Beja and Gilboa (1990) argue that in multi-stage cooperative games, even monotonicity may be an unsatisfactory condition. Large individuals may be left out of coalitions in the initial stage because of their ability to dominate a coalition in a latter stage. This result however, depends on a rather particular commitment structure; one must be able to commit to remaining in the winning first-stage coalition, yet one cannot commit in the first stage to any division of benefits among members of this coalition, or even to dispose of one's own votes.

the number of challengers from 5 to 6 is likely to decrease the former's benefits by more than increasing the number from 15 to 16. The second part of assumption 7 states that over the relevant range, $\Phi_{L_s}^1$ is bounded by Φ_s^1/L . $\Phi_{L_s}^1 \leq 0$ implies that the larger the leading shareholder, the more an increase in the number of challengers hurts this shareholder. For example, increasing the number of challengers from 15 to 16 affect a leading shareholder that is 12 times as big as the smaller challengers more than one who is 4 times as big. $\Phi_{L_s}^1$ need not be less than 0 to satisfy Assumption 7; rather, it only has to be bounded by the rate that Φ^1 increases with size, normalized by the number of small shareholders L .

Thus while there no doubt exist reasonable specifications for the division of control benefits that don't satisfy these assumptions (such as all benefits go to the largest shareholder), all these assumptions are natural and plausible given a coalitional story underlying the division of control benefits. Additionally, it should be emphasized that these assumptions are sufficient but not necessary conditions to obtain the results of the propositions.

Section 5 - Conclusion

Under the assumption of divisible control benefits, this paper develops a model of corporate control which both justifies block investment and yields rich new implications for the

shareholder structure across firms. In particular, the model predicts that large block shareholders will "create their own space" in the sense that their presence in a firm will deter other block investors; the shareholder structure of block investors in corporations will exhibit a clientele effect; and there will be a threshold size beyond which large investors will not be challenged. All these predictions are supported by a preliminary review of empirical evidence. Further empirical tests however, are needed to substantiate these claims.

Many further empirical assumptions follow either directly from the model or from relaxing several assumptions. Allowing for different sized firms yielding different benefits will imply that firms with greater benefits per size will attract more block shareholders. This could be tested to the extent that one can exogenously identify firms or industries with high private benefits. Demsetz and Lehn propose that professional sports teams and the communications industry are likely to yield high private benefits to control; and these indeed are two industries with high ownership concentration. Similarly, if a significant share of private benefits are derived from selling blocks at premiums to control contestants, one would also expect to see greater shareholder concentration in industries undergoing such contests. This theory also predicts that in firms with multiple classes of equity, there will be greater concentration in the ownership of classes with superior voting privileges. Another prediction is that number of small blocks in a firm will respond to changes in the size of the leading shareholder, yielding simple dynamic

implications. Thus, beyond implications for the shareholder structure considered in this paper, there exist a rich set of empirical predictions which could serve to distinguish this model of block shareholding from others.

While this model is able to capture certain aspects of block shareholding, it ignores elements important to any complete theory of corporate control. Most significantly, there is little role for management. In the model, management is taken to be an extension of a supporting coalition of shareholders, while realistically, agency problems and managerial entrenchment are likely to play a role in any complete theory of corporate control. One possible extension is to model management as a participant in the control game, where management receives benefits either due to support from their own private holdings, noise traders, firm controlled funds, or institutional entrenchment. At the cost of more complexity, it would also be interesting to explore the interaction between the role of block shareholders in this model, and the role of monitoring management, as in Shleifer and Vishny. This could be accommodated for in the setting of this model by considering public and private benefits which vary with the shareholder structure.

Appendix A - Proof of Lemma 3

Proof - From Lemma 1, we can consider a majority voting game with one individual of size s_0 , one of size s_1 , L individuals of size 1, and noise distributed as $U[-1/2, 1/2]$. First suppose there is no noise, and let $s_0 \leq L+s_1+1$, $s_1 \leq L+s_0+1$. The Shapley value of agent 1 (of size s_1) is given by,

$$\phi_1 = \frac{1}{L+2} \left[\sum_{i=0}^{L+1} \left(\frac{i}{L+1} \phi_1(1,i) + \frac{L+1-i}{L+1} \phi_1(2,i) \right) \right]; \quad (A1)$$

where $\phi_1(1,i)$ and $\phi_1(2,i)$ are the payoffs to agent 1 for appearing in the i th position of a random ordering of all block shareholders from 0 to $L+1$, respectively after/before agent 0 has appeared. The weights are the probabilities of such an ordering. $\phi_1(1,i)$ and $\phi_1(2,i)$ equal one in their respective cases if agent 1 is the swing voter, otherwise they are 0. That is,

$$\begin{aligned} \phi_1(1,i) &= I \left[(i-1) + s_0 \leq \alpha < (i-1) + s_0 + s_1 \right] ; & (A2) \\ \phi_1(2,i) &= I \left[i \leq \alpha < i + s_1 \right] \end{aligned}$$

where α , half of all votes cast, is given by $(L+s_1+s_2)/2$.

Rearranging terms yields,

$$\begin{aligned} \phi_1(1,i) &= I \left[\frac{L-s_0+s_1+2}{2} \geq i > \frac{L-s_0-s_1+2}{2} \right] & (A3) \\ \phi_1(2,i) &= I \left[\frac{L+s_0+s_1}{2} \geq i > \frac{L+s_0-s_1}{2} \right] \end{aligned}$$

Noting that the contributions from when agent 0 appears before and

after agent 1 are symmetric, we obtain,

$$\phi_1 = \frac{2}{L+2} \left[\sum_{i=0}^{L+1} \left(\frac{i}{L+1} \right) I \left[\frac{L-s_0+s_1+2}{2} \geq i > \frac{L-s_0-s_1+2}{2} \right] \right]. \quad (A4)$$

And now adding noise,

$$E[\phi_1] = \frac{2}{(L+1)(L+2)} E \left[\sum_{i=0}^{L+1} i I \left[\frac{L-s_0+s_1+2}{2} - \epsilon \geq i > \frac{L-s_0-s_1+2}{2} - \epsilon \right] \right]. \quad (A5)$$

Define the random variables,

$$\begin{aligned} \bar{u}_1 &= \text{int} \left[\frac{L-s_0+s_1+2}{2} - \epsilon \right] \\ \underline{u}_1 &= \text{int} \left[\frac{L-s_0-s_1+2}{2} - \epsilon \right] \end{aligned} \quad (A6)$$

\bar{u}_1 and \underline{u}_1 have means of $\frac{L-s_0+s_1+1}{2}$ and $\frac{L-s_0-s_1+1}{2}$ respectively.

Letting $\bar{\mu}_1 = \text{mod} \left[\frac{L-s_0+s_1}{2} \right]$, and $\mu_1 = \text{mod} \left[\frac{L-s_0-s_1}{2} \right]$, one can show

that the variances of \bar{u}_1 and \underline{u}_1 are given respectively by,

$$\sigma_{\bar{u},1}^2 = \begin{cases} (\bar{\mu}_1 + 1/2)(1/2 - \bar{\mu}_1) & \text{if } \bar{\mu}_1 \leq 1/2 \\ (\bar{\mu}_1 - 1/2)(3/2 - \bar{\mu}_1) & \text{if } \bar{\mu}_1 > 1/2 \end{cases}; \quad (A7)$$

$$\sigma_{\underline{u},1}^2 = \begin{cases} (\mu_1 + 1/2)(1/2 - \mu_1) & \text{if } \mu_1 \leq 1/2 \\ (\mu_1 - 1/2)(3/2 - \mu_1) & \text{if } \mu_1 > 1/2 \end{cases}$$

which can be expressed as,

$$\sigma_{\bar{u},1}^2 = |\bar{\mu}_1 - 1/2| (1 - |\bar{\mu}_1 - 1/2|) \quad (A8)$$

$$\sigma_{\underline{u},1}^2 = |\mu_1 - 1/2| (1 - |\mu_1 - 1/2|)$$

Now if $s_0 + s_1 \leq L+1$, then for all realizations of ϵ , $\underline{u}_1 \geq 0$ and

$\bar{u}_1 \leq L+1$, and therefore equation (A5) yields the weighted sum from u_1+1 to \bar{u}_1 , given by,

$$E[\phi_1] = \frac{2}{(L+1)(L+2)} E \left[\sum_{i=\bar{u}_1+1}^{\bar{u}_1} i \right] \quad (A9)$$

$$= \frac{1}{(L+1)(L+2)} E \left[\bar{u}_1(\bar{u}_1+1) - u_1(u_1+1) \right] \quad (A10)$$

$$= \frac{1}{(L+1)(L+2)} \left[\left[\left(\frac{L-s_0+s_1+1}{2} \right)^2 + \left(\frac{L-s_0+s_1+1}{2} \right) + \sigma_{u,1}^2 \right] - \left[\left(\frac{L-s_0-s_1+1}{2} \right)^2 + \left(\frac{L-s_0-s_1+1}{2} \right) + \sigma_{u,1}^2 \right] \right] \quad (A11)$$

$$= \frac{1}{(L+1)(L+2)} \left[s_1 + 1/4 \left(4s_1(L-s_0+1) \right) + (\sigma_{u,1}^2 - \sigma_{\bar{u},1}^2) \right] \quad (A12)$$

$$= \frac{s_1(L-s_0+2) + (\sigma_{u,1}^2 - \sigma_{\bar{u},1}^2)}{(L+1)(L+2)} \quad (A13)$$

If instead $s_0+s_1 > L+1$, then for all realizations of ϵ , $u_1 \leq 1$; and $s_1 \leq L+1+s_0$ ensures that $\bar{u}_1 \leq L+1$. Therefore, equation (A5) yields,

$$E[\phi_1] = \frac{2}{(L+1)(L+2)} E \left[\sum_{i=0}^{\bar{u}_1} i \right] \quad (A14)$$

$$= \frac{1}{(L+1)(L+2)} E \left[\bar{u}_1(\bar{u}_1+1) \right] \quad (A15)$$

$$= \frac{1}{(L+1)(L+2)} \left[\left[\left(\frac{L-s_0+s_1+1}{2} \right)^2 + \left(\frac{L-s_0+s_1+1}{2} \right) + \sigma_{u,1}^2 \right] \right] \quad (A16)$$

$$= \frac{(L-s_0+s_1+2)^2 + 4\sigma_{u,1}^2 - 1}{4(L+1)(L+2)} \quad (A17)$$

Finally, if $s_1 > L+s_0+1$, then individual 1 wins all votes for any realization of ϵ , and therefore $\phi_1=1$, $\phi_0=0$; and if $s_0 > L+s_1+1$, individual 0 wins all such votes and therefore $\phi_1=0$, $\phi_0=1$. Of course the derivation for $E[\phi_0]$ is identical. \square

Appendix B - Proofs For Proposition 1 and 2

Proposition 1

Proof that type 2 shareholders hold only one firm in equilibrium

In equilibrium, a type 2 shareholder will receive benefits of

$$\phi_2^* = \frac{1}{M_1} = \frac{M_2 + 1 - s}{M_2(M_2 + 1)} . \quad (B1)$$

In order to show that a type 2 shareholder i cannot benefit by diversifying, it is sufficient to show that by deviating and holding $m_j < m$ in any firm j , investor i obtains benefits of less than or equal $(m_j/m)\phi_2^*$ in this firm. Suppose first that $j \in J/J(N)$. Then, since this firm will consist of M_1 individuals of size m and one of size m_j , Lemma 2 implies that,

$$\phi_1^j = \frac{(m_j/m)}{M_1 + 1} < \frac{(m_j/m)}{M_1} = (m_j/m)\phi_2^* . \quad (B2)$$

If instead $j \in J'$, then j will consist of one individual of size $n=sm$ (the type 1 investor), one of size m_j and M_2 of size m . From Lemma 3,

$$\phi_1^j = \frac{(m_j/m)(M_2 - s + 2) + (\sigma_{u,1}^2 - \sigma_{u,1}^2)}{(M_2 + 2)(M_2 + 1)} < \frac{(m_j/m)(M_2 - s + 1)}{(M_2 + 1)M_2} = (m_j/m)\phi_2^* , \quad (B3)$$

where the middle equality follows from equations (15) and (A8).

□

Proposition 2

Proof that type 1 shareholders hold only 1 firm in equilibrium

Suppose some type 1 shareholder i deviates by holding positions in two firms in equilibrium (the proof for greater than two firms follows by induction). Call these firms j_1 and j_2 , and the shares i holds in these firms normalized by m , s_1^1 and s_1^2 , where $s_1^1 + s_1^2 = s$. In the CSE, type 2 shareholders respond by challenging both these firms (as these positions will be smaller than other type 1 shareholder positions s). We will show that payouts to i are higher in the old equilibrium than under this deviation. It is obviously sufficient to show this is the case even when i is one of the shareholders challenged by type 2 shareholders in the initial equilibrium (the toughest case).

Some notation is needed. Let ϕ_2 and $\bar{\phi}_2$ denote control benefits of type 2 shareholders in respectively the initial equilibrium and under the deviation. Let $L(x)$ and $\bar{L}(x)$ be the number of type 2 shareholders in a firm with a type 1 shareholder of size x in the initial equilibrium and after the proposed deviation. $L(x)$ is defined for $x \in (0, s)$ and $\bar{L}(x)$ is defined for $x \in (0, s_1, s_2, s)$. Extend $L(x)$ to $[0, s]$ by defining $L(x)$ as the positive root of,

$$\phi_2 = \frac{1}{L(0)} = \frac{L(x) + 1 - x}{L(x)(L(x) + 1)} ; \quad (B4)$$

similarly extend $\bar{L}(x)$ to $[0, s]$ replacing ϕ_2 and $L(0)$ above with $\bar{\phi}_2$ and $\bar{L}(0)$. It is clear from Lemma 2 that $L(x)$ and $\bar{L}(x)$ give the number of type 2 shareholders in a firm with a type 1 shareholder of the size x such that type 2 shareholders obtain control benefits given by ϕ_2 or $\bar{\phi}_2$. Furthermore, $L(x)$ and $\bar{L}(x)$ are continuous on $[0, s]$.

Solving for $M(x)$ in (B4) yields,

$$L(x) = \frac{-(L(0)-1) + \sqrt{(L(0)-1)^2 - 4(x-1)L(0)}}{2L(0)} . \quad (B5)$$

Thus,

$$L''(x) = -2L(0) \left[(L(0)-1)^2 - 4(x-1)L(0) \right]^{-3/2} < 0. \quad (B6)$$

Therefore $L(x)$ is concave over $[0, s]$, and hence, $\forall s_1, s_2$ such that

$$s_1 + s_2 = s, \quad s_1, s_2 > 0,$$

$$L(s_1) + L(s_2) > L(0) + L(s) . \quad (B7)$$

Now suppose first that all type 1 shareholders are not challenged in the proposed equilibrium. Then from Lemma 4, the number of type 2 shareholders who challenge a type 1 shareholder is given by $M_2^* = \underset{L}{\operatorname{argmax}} \phi_2^j(L)$, and $\phi_2 = \max_L \phi_2^j(L)$. Similarly, a result analogous to Lemma 4 holds when type 1 shareholders have shareholdings of different sizes (as under the deviation). In particular, in the equilibrium where type 2 shareholders are challenging the maximal number of firms possible, the largest type 1 shareholder actually contested is challenged by the number of type 2 shareholders that maximizes their individual benefits versus a shareholder of this size. Since in this case, provided that more than 2 firms are challenged²⁵, the largest challenged

²⁵ If not, the proof follows along the following lines. By following the prescribed equilibrium a type 1 shareholder is very unlikely to be challenged, while if this shareholder deviates by diversifying, this shareholder will hold the smallest positions of any type 1 shareholder and will be challenged in all of them. This deviation can then be shown to be worse than the proposed equilibrium.

shareholder will once again be of size s , it follows that once again $\bar{\phi}_2 = \max_M \phi_2^j(M)$. Hence,

$$\phi_2 = \bar{\phi}_2; \quad \text{and} \quad \bar{L}(s_1) = L(s_1), \quad \bar{L}(s_2) = L(s_2); \quad (\text{B8})$$

and therefore, from (B7) it follows that,

$$\bar{L}(s_1) + \bar{L}(s_2) > L(0) + L(s). \quad (\text{B9})$$

Benefits in both the proposed equilibrium and under the deviation for individual i are given by the total benefits from the two firms in which i invests net the benefits type 2 shareholders receive from these firms. Hence, defining $\phi_{1,i}$ and $\bar{\phi}_{1,i}$ as the benefits for the deviating type 1 individual i under the proposed equilibrium and the deviation, it follows that,

$$\phi_{1,i} = 1 - \phi_2(L(s)) - 2 - \phi_2(L(0) + L(s)) \quad (\text{B10})$$

$$\bar{\phi}_{1,i} = 2 - \bar{\phi}_2(\bar{L}(s_1) + \bar{L}(s_2)). \quad (\text{B11})$$

The second equality in (B10) is obtained by noting that type 2 shareholders receive all benefits in any firm without a type 1 shareholder. Equations (B8), (B9), (B10), and (B11) together imply that $\phi_{1,i} > \bar{\phi}_{1,i}$, which was to be shown.

The proof is similar when instead the initial equilibrium is such that all type 1 shareholders are challenged. Only a sketch is given here for brevity. In this case, it can once again be shown that (B9) holds. Intuitively this follows from (B7) which indicates that benefits of ϕ_2 can be obtained by more type 2 shareholders in firms j_1 and j_2 than before the deviation, and therefore greater benefits are obtained by the same number of type 2 shareholders in these firms. This implies that the number of type 2 shareholders in these firms will increase (hence (B9)), the

number in all other firms will decrease, and therefore $\bar{\phi}_2 > \phi_2$. Then once again, (B9), (B10), (B11) and $\bar{\phi}_2 > \phi_2$ together imply that $\phi_{1,i} > \bar{\phi}_{1,i}$. \square

What is going on in this proof is quite simple. When individual i splits her wealth, it becomes easier for type 2 shareholders to challenge these two firms than one in which i has all her wealth and one in which there are no type 1 shareholders. This implies that the same number of type 2 shareholders in these two firms can obtain greater benefits if i splits her wealth. And this in turn induces more type 2 shareholders into these firms, weakly raising their benefits relative to individual i not diversifying. With more type 2 shareholders challenging these two firms and their benefits at least as high as before the deviation, individual i must be doing worse.

The proof that two different type 1 shareholders don't hold blocks in the same firm is almost identical and therefore only a brief sketch is provided. Basically, one shows that a fixed number of type 2 shareholders can do better in two firms if one contains two large block shareholders and the other none, than when each firm contains one large block shareholder. Thus, a pair of firms with two large investors will attract more type 2 shareholders, who will be weakly better off, if the large investors jointly invest in one of the firms than if they create individual stakes in the two firms. This in turn implies that the type 1 shareholders in these two firms will be worse off, thereby

ruling out such a deviation.

Appendix C - Proofs for Section 4

Here we show that Propositions 1, 2, and 4 in the paper follow from Assumptions 1 - 7. First, we note that $\Phi^2(s, \cdot)$ is single-peaked.

Using symmetry and optimality, it follows that $\Phi^2(s, L) = (1 - \Phi^1(s, L))/L$. Since for any strictly concave function $f(x)$ defined on \mathbb{R}^+ , $f(x)/x$ is strictly quasiconcave,²⁶ it follows from assumption 7 that $\Phi^2(s, L)$ is strictly quasiconcave in L . Furthermore since assumption 4 implies that for L small $\Phi^2(s, L) = 0$, and symmetry implies that $\lim_{L \rightarrow \infty} \Phi^2(s, L) = 0$, it follows that $\Phi^2(s, \cdot)$ is single-peaked.

Now we proceed to give the proofs for Proposition 1, 2 and 4.

Proposition 1 - First note that Assumption 6 ensures that in equilibrium type 2 shareholders will put all wealth in one firm. Given this observation, it immediately follows that equations (9)-(11) are necessary and sufficient conditions to characterize PSSPE strategies for type 2 shareholders. Also, equations (10) and (11) and the quasiconcavity of $\Phi^2(s, L)$ together imply that all type 1 shareholders who are challenged are done so

²⁶ For any $x \neq y$ such that $f(x)/x = f(y)/y$, and $z = \lambda x + (1-\lambda)y$, $0 < \lambda < 1$, strict concavity implies that $f(z)/z > \frac{\lambda f(x) + (1-\lambda)f(y)}{\lambda x + (1-\lambda)y} = f(x)/x$.

by the same number of type 2 shareholders. Thus equation (8) holds as well. As in Section 3, we will show below that type 1 shareholders don't diversify when type 2 shareholders are playing CSE strategies.

Now we show there exists $s_N < s_{N-1} < \dots < s_1$ such that for $s \leq s_k$, there exists an equilibrium with a subset $J' \subset J(N)$ of type 1 shareholders challenged if and only if $|J'| \leq k$. Define $M_2^*(s) = \operatorname{argmax}_L \Phi^2(s, L)$, $\phi_2^*(s) = \max_L \Phi^2(s, L)$ and $M_1^*(s) = 1/\phi_2^*(s)$. That is, $M_2^*(s)$ is the number of type 2 shareholders challenging a type 1 shareholder of size s that maximizes their individual benefits, $\phi_2^*(s)$ are the benefits received when $M_2^*(s)$ shareholders challenge, and $M_1^*(s)$ is the number of type 2 shareholders in a firm without a type 1 shareholder necessary to receive this level of benefits.

Now for a fixed s ,²⁷ first suppose that J' is such that $(J-N)M_1^* + J'M_2^* > M$. Then no equilibrium exists with J' firms challenged; because for equation (9) to hold, either $M_2 < M_2^*$ and then $\frac{\partial \phi_2^j}{\partial L}(M_2) > 0$ for $j \in J'$, violating (11), or $M_1 < M_1^*$, in which case $\phi_2^{j'} > \phi_2^*(s)$ for $j' \in J/J(N)$, violating (10).

If instead,

$$(J-N)M_1^*(s) + J'M_2^*(s) \leq M, \quad (C1)$$

we show there exists an equilibrium with J' type 1 shareholders challenged. Let $\bar{\Phi}^2: [M_2^*, \infty) \rightarrow (0, \phi_1^*]$ be defined by the restriction of $\Phi^2(s, \cdot)$ to the domain $[M_2^*, \infty)$. Since $\Phi(s, \cdot)$ is single-peaked at

²⁷ Notationally, we suppress the functional dependence of M_1^* , M_2^* , ϕ_2^* and Φ^2 on s while holding s fixed.

M_2^* , $\bar{\Phi}^2(L)$ is a continuous strictly decreasing function of L . Now define the homeomorphism $M_2: [M_1^*(s), \infty) \rightarrow [M_2^*(s), \infty)$ by $M_2(L) = \bar{\Phi}^2^{-1}(1/L)$. If there exists $L \geq M_1^*(s)$ type 2 shareholders in firms without a type 1 shareholder, $M_2(L)$ yields the unique number of type 2 shareholders in firms with a type 1 shareholder such that equations (10) and (11) are satisfied. Thus (M_1, M_2) denotes an equilibrium with J' firms challenged iff $M_1 \geq M_1^*(s)$, $M_2 = M_2(M_1)$, and

$$(J-N)M_1 + J'M_2 = M. \quad (C2)$$

$M_2(L)$ is continuous and strictly increasing since $\bar{\Phi}^2$ is continuous and strictly decreasing. Also, since $M(M_1^*) = M_2^*$, by assumption, $(J-N)M_1^* + J'M_2(M_1^*) \leq M$. For L large enough, $(J-N)L + J'M_2(L) > M$. Thus since $(J-N)L + J'M(L)$ is also strictly increasing in L , there exists a unique value $L = M_1$ such that $(J-N)M_1 + J'M_2(M_1) = M$, thereby yielding an equilibrium.

Finally consider the effect of increasing s . By assumption 4, raising s leads $\phi_2^*(s)$ to fall and therefore $M_1^*(s)$ increases. Since $M_2^*(s) = \arg\max_L \Phi^2(s, L)$, it follows that,

$$M_2^* \Phi_L^1(s, M_2^*) - (1 - \Phi_s^1(s, M_2^*)) = 0. \quad (C3)$$

Differentiating implicitly with respect to s yields,

$$\frac{dM_2^*}{ds} = \frac{\Phi_s^1 - L\Phi_{Ls}^1}{L\Phi_{LL}^1} \geq 0; \quad (C4)$$

where the inequality follows from assumption 7. Since both M_1^* and M_2^* increase with s , and an equilibrium with J' firms challenged exists iff (C1) holds, it follows the cutoff levels s_j , beyond which J' firms cannot be challenged in an equilibrium is a decreasing function of J' . \square

Lemmas 4 and 5 follow immediately from this proof. The above proof show that there exists an equilibrium with J' firms challenged iff equation (C1) holds. Thus for a given s , the maximum number of firms that can be challenged in an equilibrium is,

$$\min \left(N, \text{int} \left(\frac{M - (J-N)M_1^*(s)}{M_2^*(s)} \right) \right).$$

And when $\frac{M - (J-N)M_1^*(s)}{M_2^*(s)}$ firms are challenged, then $M_2 = M_2^*(s)$, which is what Lemma 4 states.

Lemma 5 follows because as J' increases, the left hand side of (C2) rises, and therefore M_1 and $M_2 = M_2(M_1)$ must fall so that (C2) holds. Consequently, ϕ_2 rises.

Proposition 2 - Provided that type 1 shareholders do not diversify when type 2 shareholders play CSE strategies, Proposition 2 follows from Lemmas 4 and 5, and $\phi_2(s, \cdot)$ single-peaked, in the identical manner as in Section 3. Thus we only need show that type 1 shareholders don't diversify given CSE strategies of type 2 shareholders.

Let $s_1 + s_2 = s$, $s_1, s_2 > 0$, and let ϕ_2 , $\tilde{\phi}_2$, $L(x)$ and $\tilde{L}(x)$ be defined as in Appendix B, with $L(x)$ and $\tilde{L}(x)$ extended by $\phi^2 - \phi^2(x, L(x))$ and $\tilde{\phi}^2 - \phi^2(x, \tilde{L}(x))$ instead of equation (B4). Assumption (6) implies that,

$$\phi^1(0, L(s)) + \phi^1(s, L(s)) \geq \phi^1(s_1, L(s)) + \phi^1(s_2, L(s)). \quad (C5)$$

And since $\Phi^1(0, \cdot) = 0$ and $L(0) > L(s_1)$, it follows that,

$$\Phi^1(0, L(0)) + \Phi^1(s, L(s)) > \Phi^1(s_1, L(0)) + \Phi^1(s_2, L(s)). \quad (C6)$$

Now suppose $\bar{\phi}_2 \leq \phi_2$. Equation (C6) implies that the same number of type two shareholders do better in the two firms when the type 1 shareholder diversifies than when the type 1 shareholder holds just 1 firm (the type 1 shareholder does worse so the type 2 shareholders must be doing better). Thus, $\bar{\phi}_2 \leq \phi_2$ requires more type 2 shareholders in these firm; that is,

$$\bar{L}(s_1) + \bar{L}(s_2) > L(0) + L(s). \quad (C7)$$

$\bar{\phi}_2 \leq \phi_2$ also implies that for all x , $\bar{L}(x) \geq L(x)$, and in particular, $\bar{L}(0) \geq L(0)$ and $\bar{L}(s) \geq L(s)$. But these two relationships together with equation (C7) yield a contradiction; equation (9) can't hold for both the proposed equilibrium and the deviation.

Thus $\bar{\phi}_2 > \phi_2$ must be true. This in turn implies that $\bar{L}(0) < L(0)$ and $\bar{L}(s) < L(s)$. In order for equation (9) to hold for both cases, (C7) must still be true. Therefore, under the deviation, the two firms in which the deviator holds a stake have more type 2 shareholders than before the deviation, and these type 2 shareholders are receiving higher benefits. This implies that the type 1 deviator must be doing worse under the deviation. \square

And finally, using the above results, Proposition 4 follows by a proof identical to that in Section 3.

Control: Benefits for Type 2 Investors

Diagram 1 - Equilibrium

$$s = 5$$

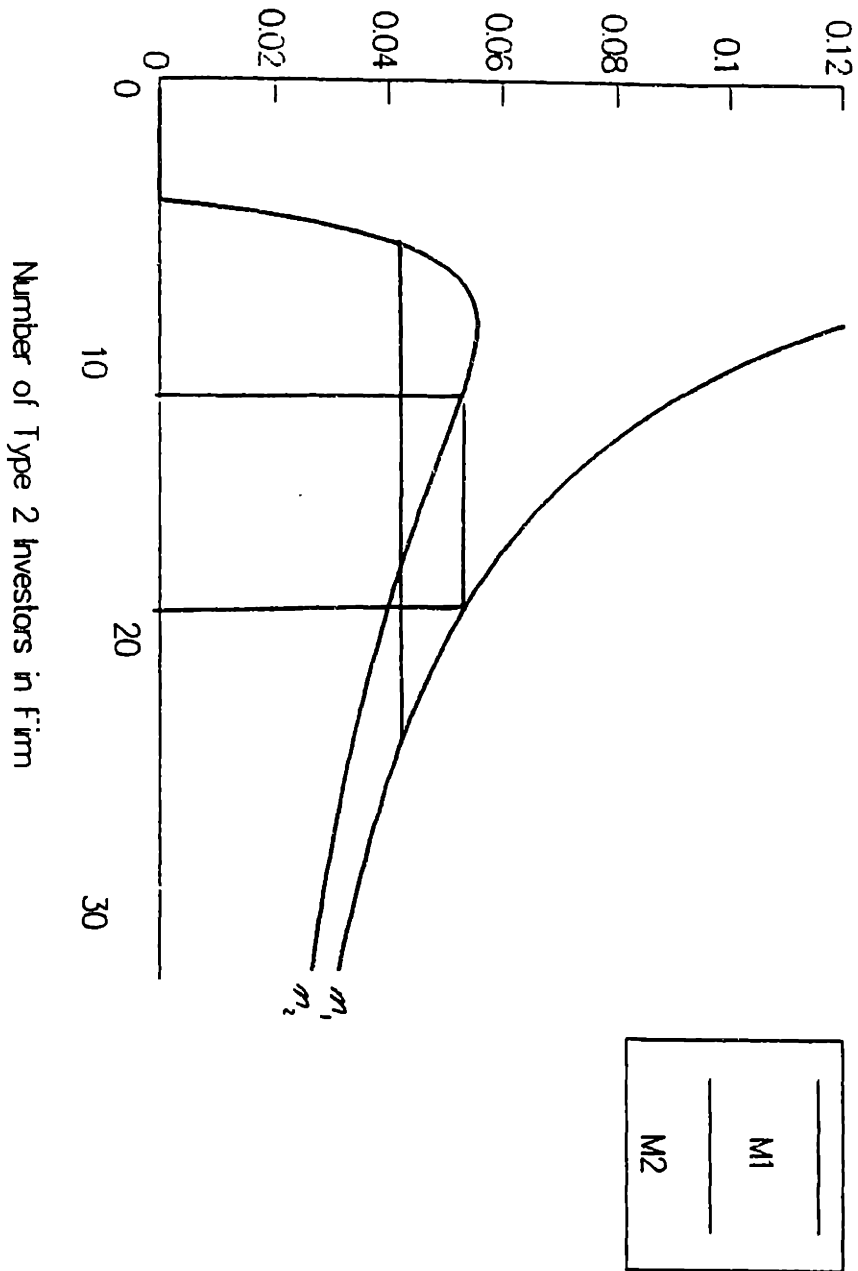


Diagram 2 – Strong Equilibrium

s=5 All Firms Challenged

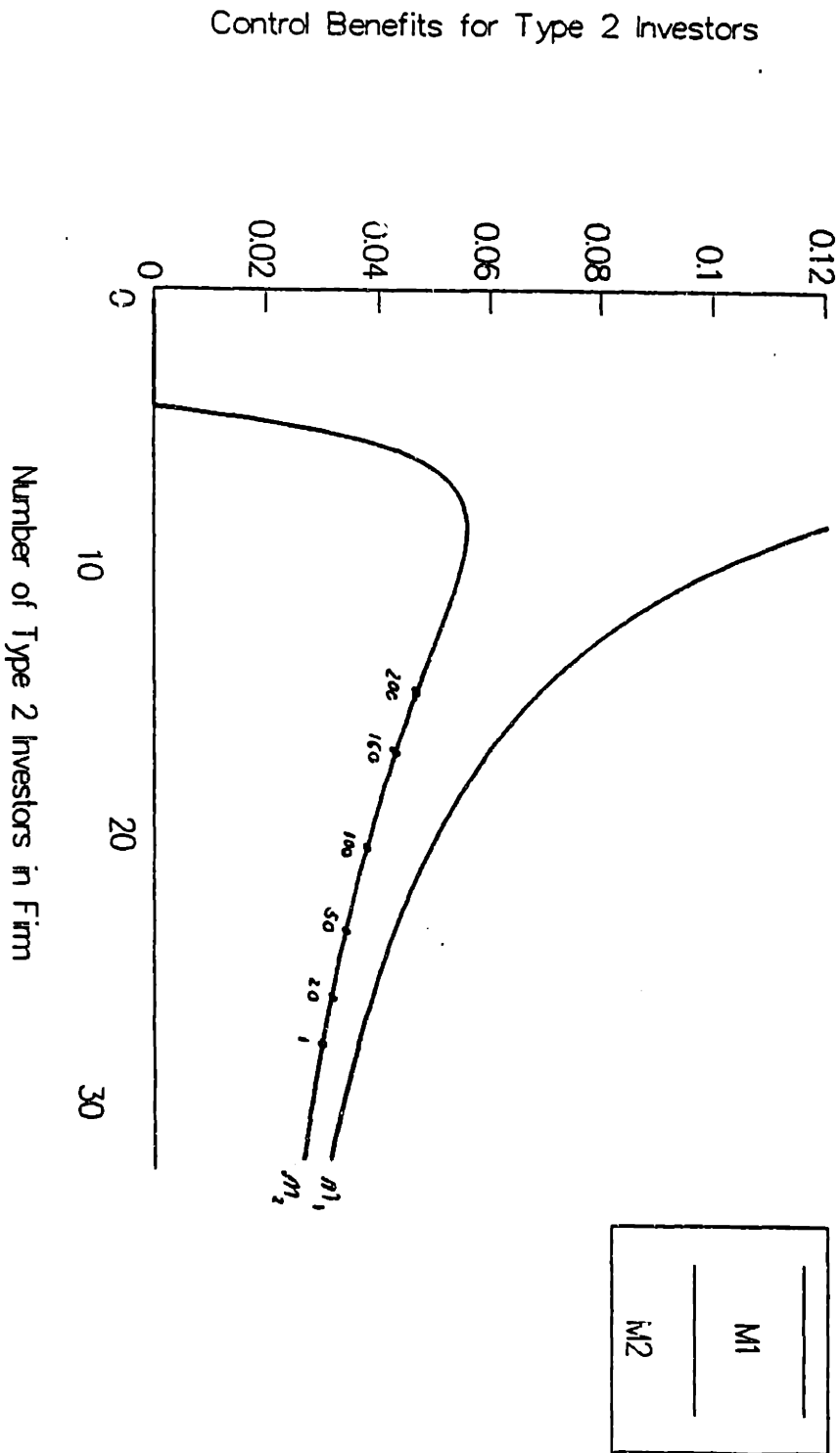
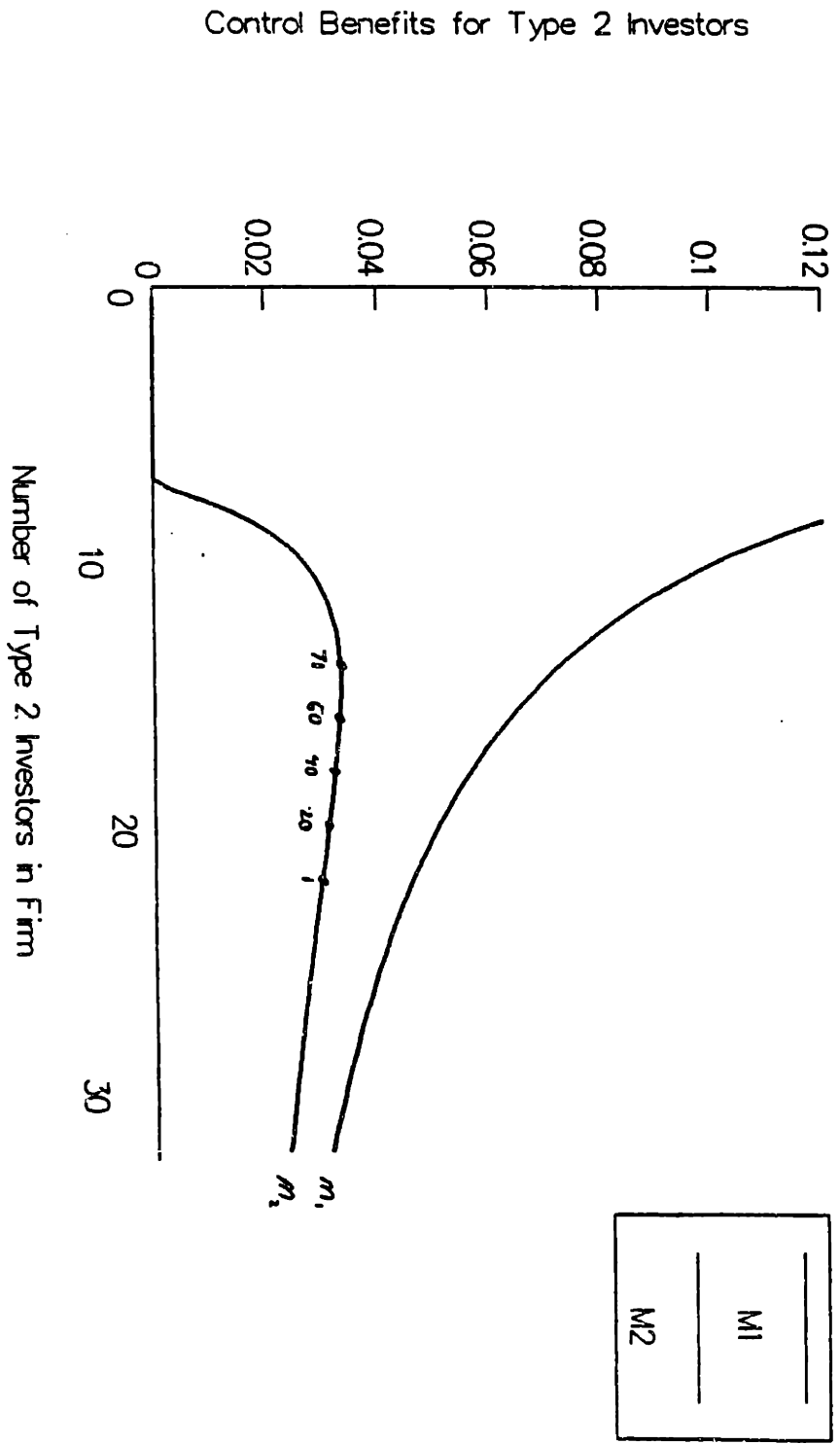


Diagram 3 – Strong Equilibrium

s=8 Not All Firms Challenged



Control Benefits for Type 2 Investors

Diagram 4 – Strong Equilibrium
 Different Sized Type 1 Shareholders

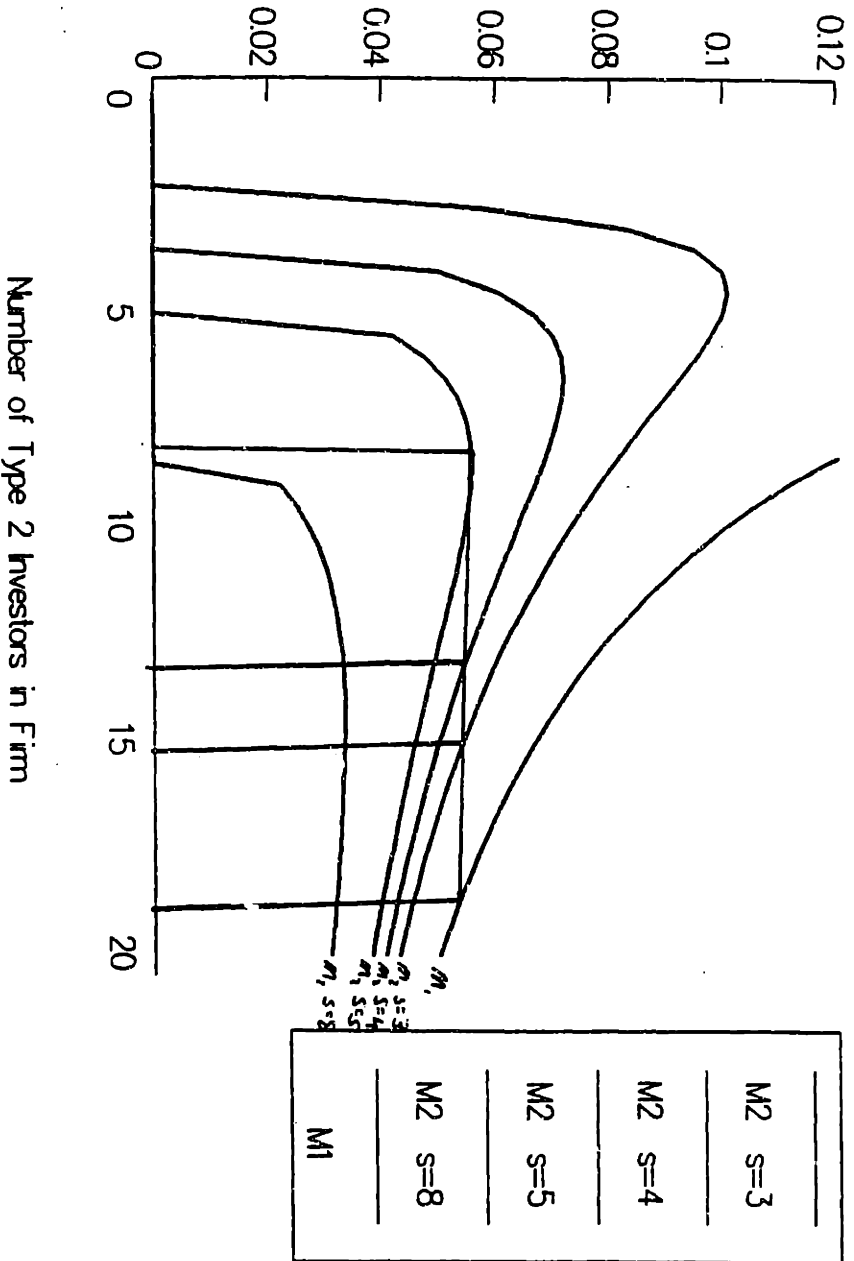


Table 1 - Distribution of Large Block Shareholders

Blocks > 10%		
Number of Blocks in Firm	Number of Firms (Actual)	Number of Firms Under Random Choice
1	178	143.7
2	28	38.7
3	4	6.9
4	0	0.9
5	0	0.1

Goodness of Fit $\chi^2(3) = 16.14$
(.001)

Blocks > 20%		
Number of Blocks in Firm	Number of Firms (Actual)	Number of Firms Under Random Choice
1	117	94.1
2	3	12.6
3	0	1.1
4	0	0.1

Goodness of Fit $\chi^2(3) = 14.51$
(.003)

Table 2 - Example of PSSPE and Strong Equilibrium

J= 500 N=200 M=10000

J'	$\bar{s}_{J'}$	s = 5				s=8			
		M_2	ϕ_1	ϕ_2	VOC	M_2	ϕ_1	ϕ_2	VOC
1	9.00	27.4	.996	.0300	6.62	21.4	.997	.0300	4.14
20	8.58	25.7	.919	.0302	5.81	19.6	.939	.0312	3.76
40	8.34	24.1	.840	.0306	5.06	17.8	.885	.0323	3.43
60	8.12	22.7	.763	.0309	4.40	15.8	.842	.0332	3.18
80	7.92	21.5	.689	.0312	3.81	NE			
100	7.72	20.3	.617	.0316	3.28	NE			
120	7.54	19.3	.547	.0319	2.81	NE			
140	7.38	18.4	.480	.0322	2.38	NE			
160	7.22	17.5	.416	.0326	2.00	NE			
180	7.07	16.7	.354	.0330	1.64	NE			
200	6.92	16.0	.294	.0333	1.33	NE			

NE - No Equilibrium. When s=8, k, the greatest number of firms that can be challenged in equilibrium is 71. In this case, $M_2=14.2$, $\phi_1=.832$, $\phi_2=.0334$, and the value of concentration = 3.12.

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Chapter 2

Corporate Conservatism, Herd Behavior and Relative Compensation

Abstract - This paper develops a model in which asymmetric information on managerial ability and concern for reputation lead managers to refrain from undertaking innovations which stochastically dominate an industry standard. Common components of uncertainty in outcome lead the market to form inferences of managerial ability based on relative performance. Managers who undertake the industry standard are in turn evaluated with a more accurate benchmark than those innovating. Discontinuities in compensation when performance is low (due, for example, to firing) or risk aversion lead managers to have differing valuations of an accurate benchmark, depending on type. We find that very high and very low ability managers will undertake superior innovations when such opportunities present themselves, while all others will forgo innovations and continue to undertake the industry standard. Thus innovations are undertaken by the brilliant and the desperate, while others will remain with the herd.

Section 1 - Introduction

Many observers have noted a bureaucratic corporate mind set which sometimes seems to stifle the adoption of new ideas and innovations. Rules of thumb used to guide economic activity often appear to possess a stubborn resilience to change, lasting far beyond their usefulness. Summarizing Edwards (1955), the classic text Scherer (1980) discusses the possibility that large firms are "inordinately slow responding to changes in demand and in eliminating inefficient or wasteful operations."¹ More recently, in *Made in America*, the summary of the findings of the MIT Commission on Industrial Productivity, Dertouzos, Lester and Solow write, "In industry after industry the Commission's studies have found managers and workers so attached to the old way of doing things that they cannot understand the new economic environment."²

Examples of herd behavior fall into this pattern of corporate conservatism. Scharfstein and Stein (1990) relate a herding interpretation of the 1987 stock market crash whereby most money managers believed the market was overpriced and more likely to fall than rise, but didn't sell for fear of being the only manager out of the market in the event of a further rise. Similarly, the failure of the American steel industry to adopt new technologies of the basic oxygen furnace, continuous casting and computer controls, and the American automobile industry to adapt rapidly to changing demand in its market, are dramatic examples of such

¹ Scherer (1980), p 345.

² *Made in America* (1989), p. 46.

conservatism.³

Such sluggishness is often attributed to rather vague notions of corporate bureaucracy, herd mentality, and lack of managerial initiative. Thus, one often hears such behavior rationalized with phrases as "the path of least resistance", "stifled creativity", "lack of inspiration", and "bureaucratic mentality". More concrete conventional explanations focus on risk aversion and moral hazard. New actions are not taken because they involve an increased expenditure of effort or are riskier than the old action. However, in many cases, these explanations are not very convincing. For the pre-crash herding story above, allocating assets in bonds or real estate doesn't seem to require more effort than allocating them in the stock market, and for the ostensible beliefs, would have involved less risk. However, while new actions may not involve any more absolute risk, they are likely to be riskier relative to the outcome of other managers taking the standard action. While removing funds from the market prior to the crash may have lowered absolute risk for money managers, it most certainly would have increased risk in their performance relative to other money managers.

This paper builds on this insight to give an explanation for corporate conservatism. Out of regard for reputation and career concerns, most managers have an incentive not to deviate from the herd; to take standard actions so the market will have an accurate benchmark with which to evaluate them. Managers follow others because reputation induces aversion to risk in relative outcome.

³ See *Made in America*, Industry Studies A and G.

As with risk aversion and moral hazard, this explanation pins excessive conservatism on a misalignment of managerial and ownership objectives. This misalignment stems from managers' concerns about reputation, together with the inability to bond managers to a firm for life. We consider a model of asymmetric information; managers' know their ability, the market doesn't. Systematic stochastic components in outcome lead the market to draw inferences on managerial ability through relative performance. This reputation in turn affects the future market value of managers.

In such a situation, it is reasonable that benefits from reputation are concave in relative performance, at least over some range. This naturally follows either from risk aversion, or the existence of discontinuities in compensation for poor relative performance, such as getting fired. Either of these effects will lead some managers to dislike increased variance in relative performance. We consider risk neutral managers in a model where firings induce discontinuities, and find that many managers will turn down superior innovations that lead to a higher expected reputation because they also increase variance in relative performance and therefore reputation. In remarks at the end, we will discuss how managerial risk aversion induces similar results.

The basic framework is as follows. We consider an industry with many firms and managers. All managers can take one action/project, the industry standard. Some managers are given the opportunity to instead take a superior action (in the sense of first order stochastic dominance). One can think of these

managers as having original ideas, or as the first with access to a new superior technology. The action taken is nonverifiable and initially it will be considered unobservable.

Outcome depends on a project specific stochastic component, the manager's type, and idiosyncratic manager-specific noise. A higher relative outcome leads the market to infer a higher type, increasing compensation the following period. However, despite leading to a lower average relative outcome, some managers may prefer the old action because the market has a benchmark with which to compare outcome under this action, thereby leading to a more accurate inference.⁴ A precise evaluation is valued most highly by average managers who want to differentiate themselves from bad managers which the firm wants to fire. Good managers also value such an evaluation, but not as highly; as they are less likely to be mistaken for bad managers. Bad managers, however, would rather have a noisy evaluation, on the chance that a good draw could save their job.

A manager deciding whether to undertake the new action must compare these consequences of a less precise evaluation with the higher expected inference induced by the new project's stochastic dominance. We find average managers choose to take the old action, while very high and very low ability managers take the new action. Thus, innovative activity is undertaken by brilliant

⁴ This is similar to a comment in Holmstrom (1982b, p. 338), which briefly mentions that firms may have an incentive to choose projects more correlated with the market than otherwise, in order to have a yardstick by which to evaluate outcome. In the present model, it is managers instead of firms who have an incentive to choose the same action so that there will be an accurate yardstick to which they can be compared.

managers, and desperate managers in danger of being fired; other managers will prefer to take the standard action.

This paper follows Holmstrom (1982a), Holmstrom and Ricart i Costa (1986) and Scharfstein and Stein (1990), in examining implications of a misalignment in managerial and ownership objectives attributable to managerial reputation and career concerns. Holmstrom (1982a) considers how career concerns influence managerial effort, while Holmstrom and Ricart i Costa (1986) examine how reputational concerns induce a divergence in investment preferences of ownership and management. Scharfstein and Stein (1990) extend reputational concerns to a two manager model, and show such considerations may lead a manager to inefficiently ignore private information and copy another manager. Our paper derives a new implication of reputational concerns; we show how in a multi-manager model with asymmetric information, reputation may lead to stifled originality and corporate conservatism.

Both this paper and Scharfstein and Stein are concerned with how conflicts between ownership objectives and managerial concerns for reputation can lead to herd-type behavior. However, while superficially similar, these papers explores altogether different motivations for and aspects of herd behavior. Scharfstein-Stein is a model explaining why managers with uncertain information are likely to follow a leader. In this sense, it is related to a number of other recent herding papers, which examine information structures which lead to "follow the leader" behavior. Hendricks and Kovenock (1989) consider herding in exploratory investment;

Froot, Scharfstein and Stein (1989) herding in information acquisition; Seabright (1989) herding in investment due to bounded rationality in the formation of higher order beliefs, and Banerjee (1990) herding in search. Our model considers a flip side to such herding behavior; why no manager chooses to be the leader and break away from the herd when aware of a superior alternative. It will be worthwhile to briefly compare the setup of this model and Scharfstein and Stein to clarify how they differ.

Foremost among differences is the information structures of the two papers. Whereas Scharfstein and Stein is driven by the informational content of signals received, this paper instead relies on asymmetric information. The Scharfstein and Stein model consists of two managers who do not know their own types. Good managers receive correlated informative signals, bad ones get uncorrelated noise signals; and managers move sequentially. This in turn leads to an incentive to herd; holding the outcome of a manager fixed, the manager's posterior is higher if the other manager took the same action. This is because managers who received the same signal are more likely to be good types. The present paper instead starts with many managers who have private information about their own types and access to an "industry standard action", and asks which managers take a superior action if given the opportunity to do so. Managers must in turn take into account the likely outcome of different actions and the subsequent consequences on their reputation, given their type.

These different information structures lead to different factors in reputation formation. In *Scharfstein and Stein*,

reputation depends on absolute performance and relative action. A manager's reputation is enhanced both by a good (absolute) outcome, and by undertaking the same project as the other manager. This effect of relative action seems plausible when outcome is bad; it captures the notion that it is better to fail together than alone. However, it seems rather counterintuitive when outcome instead is good. Given a good outcome, a manager's reputation is better when the other manager also obtain the good outcome. It seems more plausible, however, that a manager is inferred to be better if she does well and beats the competition than if she does well together with the competition.

Our information structure instead implies that reputation depends on a manager's relative performance, and the absolute action, if observable. A manager obtains a higher reputation through outperforming other managers. Thus holding a manager's outcome fixed, reputation is higher the worse other managers do. Furthermore, since reputation does not depend on relative action, there is no bias toward herding independent of project outcome. In the Scharfstein and Stein setting, if all managers were to take the action they considered best for the firm, and if the market were to evaluate managers prior to any outcome, different actions would imply lower evaluations. In our model, optimal managerial actions would convey no information about types prior to observing outcome. Thus whereas the Scharfstein and Stein story implies that a money manager would hurt his reputation by pulling out of the market when others remained in it, our model instead maintains that this reputation would only be harmed if the market

consequently did well.

Note further that the model in this paper does not require restrictions on communication to prevent Pareto improving information sharing. Herding models such as Scharfstein and Stein and Banerjee implicitly assume through the sequential structure of the game that such communication is impossible. First movers must move without the latter movers' information, despite the desire of the latter to share this information. If such communication were possible, everyone would credibly reveal information and the first best would be attained. In contrast, in the present model there are no informative signals to be shared; and furthermore, the dependence of reputation on relative performance would make managers unwilling to share such private information.

A simple discreet example illustrating the basic intuition of the model is presented in Section 2. The basic setup and assumptions of the model are introduced in Section 3. Section 4 analyzes this model for the case where very few managers can take the new action, which simplifies inference and allows incentive contracts to be ignored. These issues are considered in an example in Section 5. Section 6 discusses extensions and robustness to alternative specifications, and Section 7 considers applications and concludes.

Section 2 - An Example

This section considers a simple example in which asymmetric

information and reputational considerations lead to excessive conservative behavior. We consider a model with two periods and many firms. In the first period, all firms draw a manager from a large pool of potential managers with type t distributed over the set $T = (-2, 0, 2)$. The probability of types -2 and 2 are $.2$, the probability of type 0 is $.6$. All firms and managers are risk neutral. In the first period, all managers can take a standard action, action 0 , and a small fraction $p \approx 0$ can also take an original action, action 1 . The action taken is unobservable to the market, and the ability to take this new action is uncorrelated with the manager's type. Furthermore, a manager who undertakes the new action in the first period does not have a greater ability to take a new action in the second period than one who does not.⁵ For this example, we will take p to be small enough so that the probability that more than one manager can take the new action is negligible relative to the probability that only one manager has this choice. This assumption ensures there will be no benchmark to compare a new action against. Alternatively, one could interpret new actions as manager specific innovations (as we will in Section 5).

If a manager of type t undertakes the old action, the outcome of the project is given by $\tilde{x}(0; t) = \tilde{\mu} + \tilde{\epsilon}_1 + t$; where $\tilde{\mu}$ is a random variable common to the outcome of everyone who takes action 0 , and $\tilde{\epsilon}_1$ is manager specific noise. The outcome of a new project is $\tilde{x}(1; t) = \tilde{\nu} + \tilde{\epsilon}_1 + t$. $\tilde{\mu}$ takes on values $(-1, 1)$ with probability

⁵ One can consider the new action as an innovation which managers will all be able to copy if undertaken by one in the first period.

1/2 each, $\tilde{\nu}$ takes on the value 1 with a .6 probability and -1 with a .4 probability. For all i , $\tilde{\epsilon}_i$ is drawn from $(-1,1)$ with probability 1/2 each. $\tilde{\mu}$, $\tilde{\nu}$, and $(\epsilon_i)_i$ are jointly independent. This technological structure implies that the outcome for a manager of type t will be an element of $(t-2,t,t+2)$. Note that outcome under the new action first order stochastically dominates that under the old action.

The outcome x for each manager is observed by the market.⁶ Since almost all managers are taking the standard action, firms can accurately deduce μ from the average outcome. A key assumption, discussed in Section 3, is that there exists a minimum managerial wage greater than a manager's best nonmanagerial option. This wage floor, can be thought of as managerial perks together with an income constraint which prohibits paying a manager less than 0. For this example we set the wage floor at 4. The nonmanagerial outside option is normalized to 0.⁷ Managers also cannot be contractually bound to a firm, and therefore second period wage must be at least the market value of the manager's reputation. Firms can fire their manager after the first period, and hire a new manager at cost $c=1/2$. Finally, we assume firms make take it or leave it offers to managers, and therefore possess all bargaining power over any bilateral surplus due to hiring cost c .

⁶ Throughout this paper we will denote a random variable and its realization by the same symbol with and without a tilde respectively.

⁷ Note that we are assuming that this nonmanagerial option is independent of managerial ability.

This setup implies a simple structure for managerial compensation. As long as p is small enough (which we assume for this example), it is easy to show that any incentive contract based on output x , that will induce managers who can to take the new action, will cost more than the benefits attained through this new action. Intuitively, since all managers must be paid at least 4 for all outcomes, any incentive contract that induces the new action must do so through an additional bonus for at least one outcome that is more likely under the new action. But these bonuses are also enjoyed by managers taking the old action who are of high type or who happen to get a good draw of nature. If p is small enough, the cost of these bonuses exceeds the benefits from getting the few managers who can to undertake the new action.

Also note that since the wage floor dominates all managers' outside options, firms cannot screen through contracts when selecting a manager; all manager will accept any contract that satisfies the wage floor. Thus in the first period, all firms will pay their manager the minimum payment of 4.

Since the second period is the final period and there are no further benefits from reputation building, all managers will take the optimal action at this time. First period action therefore yields no information about second period action. Consequently, risk neutrality and the linear dependence of outcome on type implies that the relative value of a manager to a firm in the second period is simply given by the expectation $E(t|x, \mu)$ of the manager's posterior. The value of a newly drawn replacement manager is $E(t) - 0$. Thus, taking the wage floor into account, a

firm which fired its manager would be willing in the second period to bid up to $4 + E(t|x,\mu)$ for a manager with expected type $E(t|x,\mu) \geq 0$. The value of this manager to his first period firm will exceed this by $1/2$ due to the hiring cost. This initial firm would therefore be willing to outbid other firms, though it would only have to do so by an arbitrarily small amount. Thus managers will be rehired by their original firms at market value when $E(t|x,\mu) \geq 0$. If instead $-1/2 \leq E(t|x,\mu) < 0$, outside firms would prefer a newly drawn manager, while the initial employer would rehire this manager at the wage floor of 4 (since hiring costs exceed expected gain in type from a new manager). And when $E(x) < -1/2$, the initial firm will fire its manager and draw a new one. Second period compensation is therefore given by,

$$P(E(t|x,\mu)) = \begin{cases} E(t|x,\mu) + 4 & \text{if } E(t|x,\mu) \geq 0 \\ 4 & \text{if } -1/2 \leq E(t|x,\mu) < 0. \\ 0 & \text{if } E(t|x,\mu) < -1/2 \end{cases} \quad (1)$$

We now consider the market's inference for a manager with outcome x_1 when average market performance is μ . Since $p \approx 0$, if this data is consistent with a manager having taken action 0, the market will place almost all weight on action 0 having been taken. (Taking these approximations as exact will not affect the results here). There are four possible equally likely outcomes of (μ, x_1) for a manager of type $t=0$ undertaking action 0, given by $((-1, -2), (-1, 0), (1, 0), (1, 2))$. Similarly, the outcome for managers of types $t=2$ and $t=-2$ are equally distributed over $((-1, 0), (-1, 2), (1, 2), (1, 4))$ and $((-1, -4), (-1, -2), (1, -2), (1, 0))$ respectively. If any of these outcomes is observed for a given manager, the manager's expected type follows from simple Bayesian

updating. If instead the outcomes $(\mu, x_1) = (-1, 4)$ is observed, the market will realize the new action was taken by a manager of type 2 (as this is the only way x_1 can equal 4 when $\mu = -1$). Similarly, $(\mu, x_1) = (1, -4)$ indicates a manager of type $t=-2$ undertook the new action. The market's inferred expected type for the manager, and subsequent Period 2 compensation P_2 that follows from equation (1) is given below in Table 1.

Table 1 Expected Type and Compensation

(μ, x)	$E(t)$	P_2	(μ, x)	$E(t)$	P_2
$(-1, -4)$	-2	0	$(1, -4)$	-2	0
$(-1, -2)$	-1/2	4	$(1, -2)$	-2	0
$(-1, 0)$	1/2	4	$(1, 0)$	-1/2	4
$(-1, 2)$	2	6	$(1, 2)$	1/2	4.5
$(-1, 4)$	2	6	$(1, 4)$	2	6

We now consider what action managers with the ability to take the new action will choose given these inferences. Table 2 below gives the probability of outcomes (μ, x_1) under the old and new action for a manager of type t . For a given manager t under the old action, since $\tilde{x}_1(t) = \tilde{\mu} + \tilde{\epsilon}_1 + t$, there are four equally likely outcomes of (μ, x_1) corresponding to the four possible draws of $(\tilde{\mu}, \tilde{\epsilon}_1)$. Under the new action, since $\tilde{x}_1(t) = \tilde{\nu} + \tilde{\epsilon}_1 + t$ is independent of $\tilde{\mu}$, all outcomes $(\mu, x_1) \in (-1, 1) \times (t-2, t, t+2)$ are possible. The probability of each of these outcomes given in Table 2 follows immediately from our distributional assumptions and independence.

Table 2 - Probability of Outcomes for Manager t Under Old and New Actions

		Old Action		New Action		
		μ		μ		
		-1	1	-1	1	
x	$t-2$.25	0	$t-2$.1	.1
	t	.25	.25	t	.25	.25
	$t+2$	0	.25	$t+2$.15	.15

Note that while under the new action expected outcome $E(x)$ is higher, the variance of relative outcome $\bar{x}_1 - \bar{\mu}$ is higher as well. Now consider a manager of type 0. From Tables 1 and 2 it follows that expected second period compensation for this manager is 4.25 under the old action and 4.1 under the new action. Thus managers of type 0 will choose the old action.

This result is central to our story. While the new action is no riskier than the old action in absolute terms, it is riskier compared to the market average, which consists of many managers taking the old action. The new action consequently increases both the mean and the variance of the market's expected posterior beliefs about the manager. Managers of type 0 dislike the increased variance because a low market inference leads to firing. In particular, if $(\mu, x_1) = (1, -2)$, the managers' expected type will be inferred to be low enough to warrant firing. This outcome never happens under the old action, but happens with probability .1 under the new action. The potential cost of getting fired

exceeds benefits from higher mean reputation under the new action for this manager, and therefore the old action is chosen.

Now consider managers of type $t=2$. From Tables 1 and 2, expected benefits are 5.25 under the old action and 5.275 under the new action. Thus these managers would take the new action if it is available. Their higher type makes the occurrence of a poor enough relative performance to justify firing under the new action less likely than for a manager of type 0.⁶ Consequently, benefits from higher mean reputation under the new action exceed the costs of potentially being firing for managers of type 2, and the new action is chosen.

Finally, from Tables 1 and 2 it is seen that managers of type $t=2$ obtain expected Period 2 compensation of 2 under the old action and 2.275 under the new action, and will therefore also choose the new action. These managers have a good chance of getting fired under the old action, as their low type leads to poor relative performance. Thus they benefit from the increased variance as well as the increased mean of the markets' expected inference under the new action. A good draw of $\tilde{\nu}$ for such a manager together with a bad draw of $\tilde{\mu}$ for the market will make relative performance respectable and prevent firing.

Thus we find that average types take the old action while high and low types take the new action when it is available to them. Average types take the old action because it is important that the market evaluate them with an accurate benchmark and not

⁶ In fact, in this example, a manager of type $t=2$ never gets fired. This is not needed for the result however, as is seen in Section 4.

confuse them with low types who firms would like to fire. High types are less likely to be confused with low types, and benefit enough from the higher mean under the new action that they undertake it. Low types take the new action as they prefer both the less accurate evaluation and a higher mean evaluation.

In the following two sections we present and analyze a model which extends the intuition of the above example to a more general setting. In the numerical example above, outcome under different types have nonidentical supports. Furthermore there is a highest and lowest type; which implies the market will always underestimate the type of the highest manager and overestimate that of the lowest. Our model shows that results are not dependent on such a setting. In the model, types and outcomes are normally distributed; thus the support of outcomes for all types and actions is the entire real line, and every type has others both superior and inferior to it. In Section 5, we consider an example where actions are observable, and all managers are endowed with the ability to take a new action, thereby making incentive contracts beneficial to firms.

Section 3 - The Model

We consider a two period model with asymmetric information. All firms and managers are risk neutral. There are many firms, and an even larger pool of potential managers. Managers know their types, the market has a common prior for managers. The market updates its beliefs based on first period performance.

Managers in turn take into account how different actions are likely to effect their reputation, which determines their market value in the second period. As in Holmstrom and Ricart i Costa, firms can assume managers will take the optimal action in the second period since reputation has no further consequences.

In the first period, all managers can take action 0, which we will refer to as the "industry standard" or the "old action". This action can be thought of as the unique optimal action in the past. Some managers obtain an original idea which they can pursue in lieu of action 0. The probability that a manager obtains such an idea is independent of type. In Section 4, we can interpret the new action to be the same action for different managers; in Section 5, for reasons discussed in that section, action 1 will be considered manager specific. The outcome under action 1 stochastically dominates outcome under action 0. New actions played by any manager in the first period can be copied by all managers in the second period; thus taking a creative action in the first period does not convey any information on the ability to do so in the next period. The main question we will be concerned with is which type managers will take the new action, and which will pass it up.

Profits of the old project when undertaken by manager i of type t_i are given by $\bar{x}_i(0; t_i) = \bar{\mu} + \bar{\epsilon}_i + t_i$, where the random variable $\bar{\mu}$ is a systematic component of action 0 common to all managers undertaking the old project, and $\bar{\epsilon}_i$ is noise specific to manager i . The outcome of a new project undertaken by manager i is $\bar{x}_i(1; t_i) = \bar{\nu} + \bar{\epsilon}_i + t_i$, where $\bar{\nu}$ is the systematic component of

the new project. Thus a manager faces both project-specific and idiosyncratic personal noise.

The realization x_i for each firm is observed accurately by the market, noise is not observed. In Section 4 the action taken by a manager is unobservable. Neither μ nor ν is observed directly, though the market will be able to deduce μ from the average outcome.⁹ In Section 5, where all managers can take a new action, we will consider observable but noncontractable actions and will assume new projects are manager specific, and therefore cannot be used to evaluate others undertaking new projects. In both cases, the conceptual difference between the old and new action is that there exists a more accurate benchmark under the former to evaluate managers.

We assume that $\tilde{\mu} \sim N(a, \sigma^2/2)$; $\tilde{\nu} \sim N(a+b, \sigma^2/2)$, with $b > 0$; and $\tilde{\epsilon}_i \sim N(0, \delta^2) \forall i$; and that $\tilde{\mu}$, $\tilde{\nu}$, and $(\tilde{\epsilon}_i)_i$ are all jointly independent; where $N(\cdot, \cdot)$ represents a normal distribution with mean and variance given by its arguments. Potential managers are distributed according to $t \sim N(0, \tau^2)$. Firms make all offers to managers, and therefore, conditional on satisfying compensation constraints described below, a firm realizes any bilateral surplus between itself and a manager.

The technological structure and distributional assumptions are similar to that of Holmstrom (1982a). In Holmstrom, output is a function of type, individual noise, and effort; with type and noise normally distributed. Information is symmetric, both the

⁹ We assume there are enough firms so that μ can be observed accurately. A noisy inference of μ would needlessly complicate the analysis without yielding any new insights.

market and manager use performance to make identical inferences on the manager's type. In this model, output instead depends on type, individual noise, the project chosen, and project specific noise. Managers know their types and choose projects accordingly. While there is no disutility of effort, managers must take into account the effect project choice will have on reputation.

Firms face a hiring/training/search cost of c in replacing a manager. This cost can perhaps best be thought of as learning costs for a new manager. We impose this cost to reflect the empirical observation of low manager turnover; without this cost, the entire bottom half of all managers would be replaced each period with new managers. With this cost, a manager will be fired only if the market infers her expected type to be less than $-c$.

We further assume that firms must pay managers at least a fixed amount $F > 0$ in all periods. This assumption deserves some comment. Analytically, it generates a discontinuity in second period compensation in relative performance. The marginal manager with expected type $-c$ receives F , while a manager with expected type slightly below $-c$ is fired and receives 0. This discontinuity plays a key role in the results; the possibility of being fired induces average managers to forgo innovations and instead play the industry standard.

There are several reasons why we model this firing discontinuity rather than risk aversion. First, this story is more in line with our intuitive notion of corporate conservatism. Under this notion, managers forgo "rocking the boat" out of fear of losing their job, future promotions or some other

discontinuity. Echoing this sentiment is the well known market aphorism, "No manager gets fired for buying IBM computers." It is such a motivation which we attempt to capture. Furthermore, empirical evidence suggests fired managers are compensated less in future jobs.¹⁰ The reader not satisfied with these justifications, however, hopefully will take comfort in the discussion of Section 6, where we argue that decreasing absolute risk aversion will generate similar results. Here we will be content to give several possible explanations for the minimum compensation of F .¹¹

1. F represents managerial perks enjoyed regardless of salary, and monetary compensation must be at least 0. Intuitively, one can consider managing a firm to be a coveted position even at no compensation.

2. A fired manager must face search costs F to obtain a new nonmanagerial job (and obtain her reservation value of 0). Under this interpretation F is a cost to managers when fired, and not a payment by the firm.

3. Firing leads to unmodeled reputation costs. For instance, firing may convey information to a nonindustry sector which cannot observe the manager's outcome, and may therefore

¹⁰ For example, using PSID data from 1969 to 1985, layoffs are associated with approximately a 17 percent decline in wages among male managers who were employed the following year. It is interesting to note that the decline is much smaller - only about 6 percent - for all males laid off. Of course, in considering only managers who were employed the next period, these figures may greatly underestimate the cost of a layoff. I thank Ed Glaeser for providing these figures.

¹¹ Note that while a rigid downward wage is derived in Harris and Holmstrom (1982) and Holmstrom and Ricart i Costa (1986), these model also predict no firings. The rigidity serves to insure risk averse managers against a low reputation.

lower the manager's outside reputation.

4. An unmodeled efficiency wage story is responsible for the floor of F .

Note that the wage floor makes it impossible for firms to screen which managers it hires. Trivially, any contract satisfying the wage floor requirement is superior to the outside option of 0 for all types of potential managers.

We follow Harris and Holmstrom (1982) among others, in further assuming management cannot be contractually committed to working against their will. This assumption is consistent with legal constraints prohibiting involuntary servitude. Thus, managers must be paid at least the market value of their reputation in the second period. Note also that the wage floor F restricts the use of bonding as a device to tie managers to the firm in the second period.

Managers have zero initial wealth and are constrained from borrowing; otherwise managerial ownership of the firm would obtain optimal actions. This assumption and the restriction of involuntary servitude can be considered assumptions which prohibit integration. The firm can't own the manager, and the manager can't own the firm.

Section 4 - Infrequent Innovative Actions

We now consider a continuous pool of firms and potential managers, where only a 0-measure subset of managers can take a new

action, which is unobservable. One can think of a situation where the new action is truly a creative innovation which only occurs to a given manager in a given period with a very small probability. When this probability is very small, the market can virtually ignore the possibility that a new unseen improvement was undertaken when forming inferences about managers' types. The above assumption can be thought of as the limiting case of this situation, where the probability of a specific manager innovating goes to 0. Hence the market will evaluate all managers by the industry standard.

Furthermore, firms will not offer incentive contracts. Firms will either pay managers the minimum required given the compensation constraints of the wage floor and market competition, or they will fire their manager if his expected type is below $-c$. Given a minimum required compensation above a manager's reservation value, any additional incentives based on performance would be costly and would yield negligible expected benefits, since practically no managers can take the new action. Such a contract would mainly compensate managers for having high types or obtaining fortunate draws of noise rather than providing an incentive to take the superior action.

Managerial incentives depend on the learning process by which reputation is updated. The market observes outcome $x_i = \mu + t + \epsilon_i$ or $x_i = \nu + t + \epsilon_i$, depending on the action taken, and infers μ from the market average.¹² The distributional assumptions yield a

¹² Note that μ (or more precisely \bar{x}_i) is a sufficient statistic for $\{x_i\}_i$ with respect to t_i for all i , in the sense of Holmstrom

simple normal learning model (see for example DeGroot, Chapter 9), in which the market's posterior of manager's i 's type is distributed according to,

$$(t|x_1, \mu) \sim N \left(\frac{\tau^2}{\tau^2 + \delta^2} (x_1 - \mu), \frac{\delta^2 \tau^2}{\tau^2 + \delta^2} \right). \quad (2)$$

Linear technology and risk neutrality implies that the expected value of a manager to a firm will be given by the expectation of this posterior. Consequently, the manager's market value in Period 2 is $\frac{\tau^2}{\tau^2 + \delta^2}(x_1 - \mu)$; the relative outcome weighted by the information to signal ratio.

Firms will fire managers with expected type below $-c$, the expected type of a newly drawn manager net hiring costs. Thus all managers whose relative performance in the first period is below the cutoff $-\frac{\tau^2 + \delta^2}{\tau^2} c$, which we will denote p_1 , will be fired by their firm after the first period. All firms having so fired their manager would be willing to bid up to the expected type plus F for a manager with positive expected type rather than drawing a new manager. However, such managers' first period firms will value their manager at this market valuation plus c . Consequently, they will outbid the market by an arbitrarily small amount (which we can ignore) and retain their manager.¹³ Additionally, a firm will retain their initial manager at the minimum wage of F if expected type is between $-c$ and 0 . Thus second period managerial compensation is given by,

1982b.

¹³ Since firms make all offers, managers cannot capture some of the hiring cost c . This assumption about bargaining is only made for concreteness, and is of no consequence to our results.

$$P(x, \mu) = \begin{cases} 0 & \text{if } (x - \mu) < -\frac{\tau^2 + \delta^2}{\tau^2} c = p_1 \\ F & \text{if } -\frac{\tau^2 + \delta^2}{\tau^2} c \leq (x - \mu) < 0 \\ F + \frac{\tau^2}{\tau^2 + \delta^2} (x - \mu) & \text{if } (x - \mu) \geq 0 \end{cases} \quad (3)$$

Note that second period compensation is equivalent to the minimum payment plus an option on relative performance with an exercise price of 0;¹⁴ together with the possibility of being fired and receiving 0. If relative outcome is taken to be contractible, this could be written into a first period contract, which would consist of fixed wage F in both periods, a performance incentive package (which compensates managers relative to the industry average) contingent on the manager remaining with the firm, together with a clause allowing the firm to terminate the contract after the first period. Such performance incentive contracts are indeed observed in practice.

We now consider the implications this second period compensation has on the first period actions of a manager who can play action 1. Expected second period compensation under both the standard and the new action are given in the following lemma.

Lemma 1 - For a manager of type t , expected second period benefits under action 0 and action 1, denoted by $P(0, t)$ and $P(1, t)$ respectively, are given by,

¹⁴ Alternatively, one can consider this an option on human capital as in Holmstrom and Ricart i Costa.

$$P(0, t) = E_{x, \mu} [P(x, \mu; t)] = F \Phi \left(\frac{t - p_1}{\delta} \right) + \frac{\delta r^2}{\delta^2 + r^2} \int_{-\infty}^{t/\delta} \Phi(z) dz, \quad (4)$$

$$P(1, t) = E_{x, \mu, \nu} [P(x, \mu; t)] \\ = F \Phi \left(\frac{t + b - p_1}{(\sigma^2 + \delta^2)^{1/2}} \right) + \frac{r^2}{\delta^2 + r^2} (\sigma^2 + \delta^2)^{1/2} \int_{-\infty}^{(t+b)/(\sigma^2 + \delta^2)^{1/2}} \Phi(z) dz, \quad (5)$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution.

Proof - For a manager of type t , $(\bar{x} - \bar{\mu}) \sim N(t, \delta^2)$. From equation (3), it follows that expected second period compensation $P(0, t)$ under action 0 is given by,

$$F \int_{p_1}^{\infty} \frac{1}{\sqrt{2\pi}\delta} e^{-1/2((y-t)/\delta)^2} dy + \int_0^{\infty} \left[\frac{r^2}{\delta^2 + r^2} y \right] \left(\frac{1}{\sqrt{2\pi}\delta} e^{-1/2((y-t)/\delta)^2} \right) dy \quad (6)$$

$$= F \left[\Phi \left(\frac{t - p_1}{\delta} \right) \right] + \frac{r^2}{\delta^2 + r^2} \int_{-t/\delta}^{\infty} (\delta z + t) \frac{1}{\sqrt{2\pi}\delta} e^{-1/2z^2} dz \quad (7)$$

$$= F \left[\Phi \left(\frac{t - p_1}{\delta} \right) \right] + \frac{r^2}{\delta^2 + r^2} \left[t \Phi(t/\delta) + \delta \phi(t/\delta) \right], \quad (8)$$

where $\phi(\cdot)$ is the density function for the standard normal distribution. Equation (4) follows immediately from (8) and the relationship,

$$x \Phi(x) + \phi(x) = \int_{-\infty}^x \Phi(z) dz \quad (9)$$

Expression (5) is derived in an identical manner upon noting that

under action 1, $(\tilde{x}-\tilde{\mu}) \sim N(t+b, \sigma^2 + \delta^2)$.

□

Note from equation (3) that when action 0 is played, systematic project uncertainty $\tilde{\mu}$ does not affect the managers' reputation. Rather, only the manager's relative outcome, $\tilde{x}-\tilde{\mu}=t+\tilde{\epsilon}$, is relevant. It therefore follows that σ^2 , which indexes systematic project variance, does not enter into expected compensation in equation (4). Systematic project noise, however, will matter for a manager who instead plays action 1, as $\tilde{x}-\tilde{\mu} = \tilde{\nu}-\tilde{\mu}+t+\tilde{\epsilon}$. Then, for instance, either a good draw of $\tilde{\mu}$, or a bad draw of $\tilde{\nu}$, will lower the manager's relative performance. Hence under action 1, variance in relative performance increases by σ^2 , even though the absolute variance is given by $\sigma^2/2 + \delta^2$ for either action.

Expressions (4) and (5) have a simple interpretation. The first term in each is the minimum payment F times the probability that the manager's relative outcome will be above p_1 for the given action; that is, F times the probability the manager will not be fired. The argument of the cumulative distribution function is the number of standard deviations in the manager's relative outcome by which expected outcome exceeds the cutoff p_1 . The second term in the two expressions is the expected additional compensation over F from enhanced reputation when relative outcome $x-\mu$ is positive. This term increases in type, and in the information to signal ratio. These two equations differ only through changes in the mean and standard deviation of the manager's relative outcome, which increase from t to $t+b$ and from δ to $(\sigma^2 + \delta^2)^{1/2}$ respectively, in going from action 0 to action 1.

Note, however, that δ does not change between the two expressions insofar as it enters as part of the information to signal ratio, as this is the slope of reputational benefits in equation (3) which all managers face.

These changes lead to four differences between the two equations, as the higher relative mean and relative variance under action 1 enters both terms of equation (5). Each of these differences has a natural interpretation.

1. $b > 0$ increases the probability that relative outcome p_1 will be obtained, thereby leading to an increase in the first term.
2. $b > 0$ also increases $E[\max(\bar{x}-\bar{\mu}, 0)]$, which is proportional to reputational rents given by the second term. Hence $b > 0$ also generates an increase in the second term.
3. The higher relative variance affects the probability that relative outcome p_1 will be obtained. For all types $t > p_1 - b$ (and recall that $p_1 < -c$), the higher variance leads to a greater chance of being fired, and therefore a decrease in the first term. For types $t < p_1 - b$, higher variance reduces the probability of being fired, and therefore the first term increases. Intuitively, the first group consists of all types not likely to be fired, and therefore, by increasing their relative variance, the probability they are fired increases. The second group consists of managers likely to be fired; a higher relative variance increases the chance that a good draw of nature will save them.
4. Outside of the firing effect of point 3, managers like more relative noise; managers are risk neutral, and compensation increases linearly in relative outcome above 0, while it is

bounded below by 0 for very bad draws. Hence higher relative variance leads to an increase in the second term.

A manager with the ability to take both actions chooses the action which maximizes expected second period compensation. Comparing expressions (4) and (5) yields the following theorem.

Theorem 1 - Provided that,

$$F > \frac{\tau^2}{\delta^2 + \tau^2} (\sigma^2 + \delta^2)^{1/2} \frac{\phi\left(\frac{p_1 + \delta}{(\sigma^2 + \delta^2)^{1/2}}\right)}{\phi(1)}, \quad (10)$$

$\exists \bar{b}(F) > 0$, s.t. $\forall b < \bar{b}$, $\exists t_1 < t_2 < t_3 < t_4$ s.t.,

$\forall t < t_1$ action 1 is preferred to action 0

$\forall t_2 < t < t_3$ action 0 is preferred to action 1

$\forall t > t_4$ action 1 is preferred to action 0

When instead $b \geq \bar{b}$, all types prefer the new action. Also, $\bar{b}(F)$ is strictly increasing when F satisfies equation (10). Furthermore, letting $T(b)$ represent types who prefer action 1 to action 0 for a given b , if $b_1, b_2 < \bar{b}$, then $T(b_1) \subseteq T(b_2)$ iff $b_1 \leq b_2$.

Proof - See Appendix.

In fact, for all $F, b > 0$, there exists a t_1 and t_4 such that all types below t_1 and above t_4 prefer the new action. The above theorem states that provided F is sufficiently large and b is small enough, there will also be an interval of types in between that prefer the old action. This interval grows as b falls.

While the theorem does not indicate what types (t_1, t_2) and (t_3, t_4) will choose, simulations indicate that for most parameter values, $t_1 = t_2$ and $t_3 = t_4$; that is, all types in the interval (t_1, t_4) prefer the old action.

Additionally, for small values of b , $t_1 \approx p_1$ and $t_4/r \gg 0$. Thus provided the new action does not dominate the old action by too much, only very high types, and types below the cutoff level will take the new action when available. Very high types take the new action because it is very unlikely they will be fired, as their type more than compensates for most low draws, and the new action yields a higher expected outcome and therefore enhances their reputation. Low type take the new action because they are likely to be fired if they play the same action as everyone else; the best they can do is take the new action and hope for a high relative draw. Hence the innovators will be the brilliant and the desperate, and all others will go along with the herd.

Inequality (10) in the theorem is only a sufficient condition to ensure that $T(0) \neq \mathbb{R}$, and in fact simulations indicate that results of the theorem hold for significantly lower values of F . $\phi\left(\frac{p_1 + \delta}{(\sigma^2 + \delta^2)^{1/2}}\right)$ is bounded above by $\phi(0)$, and the ratio $\phi(0)/\phi(1)$ is approximately equal to 1.65. Thus if δ , σ , and r are equivalent, a sufficient condition for inequality (10) to be satisfied is $F > 1.17r$. From equation (A15) in the appendix, we find that a sufficient condition for the mean type, type 0, to prefer the new action is given by,

$$F > \left(\frac{r^2}{\delta^2 + r^2} \right) \frac{(\sigma^2 + \delta^2)^{1/2} \phi(0)}{(-p_1/\delta) \phi(-p_1/\delta)} . \quad (11)$$

Welfare Implications

The action chosen by the manager has both financial and informational consequences. The financial effect is simple; action 1 increases expected outcome over action 0 by b . Additionally however, the action chosen affects the likelihood that a firm will mistakenly fire a manager with type above $-c$, or not fire a manager with type below $-c$. There is social value for the more accurate information associated with the old action, as this will make an inefficient firing decision less probable. We define $L(t)$ as the social loss due to less accurate information when action 1 is played instead of action 0. The following theorem characterizes the nature and magnitude of this informational loss.

Theorem 2 - The informational loss for a manager of type t is given by,

$$L(t) = -(t+c) \left[\Phi \left(\frac{t+b-p_1}{(\sigma^2 + \delta^2)^{1/2}} \right) - \Phi \left(\frac{t-p_1}{\delta} \right) \right]. \quad (12)$$

$L(t) > 0$ iff either,

$$t > \max(-c, \bar{t}) \quad \text{or} \quad t < \min(-c, \bar{t}); \quad (13)$$

where \bar{t} is the value which equates the arguments of the two standard normal cumulative distribution functions in equation (12), and is given in equation (A9) in the Appendix. Furthermore, as $t \rightarrow \pm \infty$, $L(t) \rightarrow 0$.

Proof - See Appendix.

Action 0 generally leads to a more accurate inference on type

and therefore less mistakes in the firing decision for most types. The increased variance in relative performance under the new action makes high types more likely to be fired and low types less likely. However there exists an interval of types either directly above or below $-c$ for which the new action leads to a more efficient firing decision. Type \bar{t} is the type in the middle, who is fired with the same probability under either action. If $\bar{t} > -c$, types between $-c$ and \bar{t} are less likely to be inefficiently fired under the new action. Intuitively, the upward bias in the inference induced by $b > 0$ makes the market less likely to mistakenly fire these types. If instead $\bar{t} < -c$, types between \bar{t} and $-c$ are more likely to be efficiently fired under the new action. In this case, the higher variance in the market's inference increases the chance that such managers' outcomes will fall below the threshold p_1 .

Thus, provided that t is not between $-c$ and \bar{t}_1 , there will be a positive information loss associated with the new action. In general, either $\bar{t} > -c$ or $\bar{t} < -c$ is possible. It is easy to show that $\bar{t} \geq c$ if and only if,

$$c \leq b \frac{\tau^2}{\sigma^2} \left(1 + \left(1 + \frac{\sigma^2}{\delta^2} \right)^{1/2} \right). \quad (14)$$

This condition holds if we take manager specific noise to be small relative to project variance and the dispersion of types, that is if $\delta \ll \sigma, \tau$, and if b and c are comparable in size. Furthermore, under these parameter assumptions, a Taylor expansion yields,

$$\bar{t} + c \approx b\delta/\sigma. \quad (15)$$

Thus when $\delta \ll \sigma$, and b is not too large, the range $[-c, \bar{t}_1]$ for

which $L(t) \leq 0$ is small.

Hence for most values of t , there is an information loss associated with action 1. However, provided b is not too small, one can show from equations (10) that this loss will be at least an order of magnitude smaller than the financial gain b . Thus the firm will still prefer that all managers who can play action 1 do so.

All these considerations have a negligible impact on social welfare, however, if new actions are only available with a very low probability. Much more significant for welfare are losses related to the dissemination of new ideas. If we interpret action 1 as a new creative innovation possibly known only to one manager, which can be copied by all other managers in the second period only if it is undertaken by a manager in the know, then the second period welfare loss associated with an untaken creative action will be on the order of bN , where N is the total number of firms. Neither the manager nor her firm internalizes the benefits she bestows on other firms by introducing the innovation. This is similar to the welfare losses explored by Banerjee, which consist of lost potential welfare from searches not carried out.

Even if b was uncertain and had a negative expectation, there are welfare losses when the new action goes untaken, as there is social value to experimentation. If the action turns out successful, other firms can copy it in the second period. Our theory predicts many managers might not even undertake a creative action which dominates the standard action; managers are even less likely to explore such an action when the expected improvement is

negative. Finally, it is worth noting that insofar as managerial investment decisions are being made strictly with regard to reputation, they also do not take into account the market's value for diversification.

Section 5 - An Example Where All Managers Have a New Action Available

Even if action is observable, outcome under the new action may still be a noisier indicator of type than under the old action. Consider the setting of the previous section, in which only a few managers can take the new action. With observable actions, the market would no longer use the inappropriate benchmark μ to evaluate managers who took the new action. However, if ν is not observed by the market, there is no benchmark to use for such managers. Thus under the new action, a manager's reputation is subject to systematic project risk which is eliminated through the relative performance benchmark under the old action. Of course, when action is observable, the market's inference will also depend on the action and on who takes take this action in equilibrium.

Note further that when action is observable there is no longer any gain in reputation from the stochastic dominance of the new project; when the new action is taken, the market expects a higher outcome.¹⁵ Thus while the variance of the market's

¹⁵ Of course, if we instead assume that taking the new action informs the market about the ability to innovate again in the following period, innovations would increase expected reputation.

evaluation is still higher under the new action, this is no longer accompanied by a higher mean reputation. Consequently, we will find that average and high types who dislike increased variance in reputation will only take the new action when induced to through incentives contracts. Such incentive contracts are only beneficial for firms when a significant fraction of managers can take the new action.

In this section we therefore consider an example in which all managers have the ability to undertake a new observable but noncontractable action. One can think of a situation where during the first period managers may come up with their own new ideas for improvements, but it is not clear prior to this period what in particular this will entail. Thus, the adoption of a new procedure is observable, but it is not ex-ante contractible. While contracts cannot be written on the action taken, this action and the incentive contract selected by the manager may signal information about type, thereby affecting reputation.

The following example demonstrates the possibility of obtaining a result similar to Proposition 1, in which low and high types take the new action but average types don't, even when actions are observable and optimal incentive contracts are offered. Intuitively, the existence of a wage floor implies that incentive contracts must consist of bonuses in addition to minimum compensation; the wage floor restricts the use of offsetting punishments and rewards in incentives. In order to induce the new

We have ignored the issue of building a reputation for creativity by assuming that in the second period all can mimic first period creative actions, and there are no further innovations to be made.

action among average and high type managers, an incentive contract must give bonuses for outcomes that are more likely for these types under the new action than the old action. However, offering a contract that induces average types to take the new action makes it more costly to induce high types to take the new action as well. This follows both because reputation costs for undertaking the new action are greater for high types since the market may confuse them with average types, thereby lowering second period compensation, and because high types may be able to benefit from bonuses intended for average types when undertaking the old action. This in turn implies that high types will have to be given further incentives to undertake the new action when average types are also induced to do so.

Thus we will find that it may be optimal for firms to ignore average types, and only give high types sufficient incentives to take the new action. Not surprisingly, whether it is optimal to only give incentives for high types or both average and high types will depend on the prior distribution of types. The more average types there are, the more important it is to induce them to take the new action as well. Low types always take the new action, as they prefer the associated higher variance in reputation.

In keeping with the conceptual distinction between the old and new action of previous sections, in which the old action has a more accurate benchmark, we assume that the old action is intrinsically easier to evaluate than the new action.¹⁶ We assume

¹⁶ Instead taking the ability to evaluate a project to be endogenously determined by the number of managers who undertake the project, as in Section 4, would yield a coordination story.

that there is no manager specific noise and μ is observable, and therefore under the old action type is perfectly revealed. Outcome under the new action is manager specific; that is, outcome depends on type and independent draws $(\tilde{\nu}_i)_i$. These extreme assumptions are made for simplicity; we could obtain similar results as long as the benchmark to evaluate the old action is more accurate than that for the new one. Thus, adding manager specific noise or allowing for imperfect correlation in the draws of $(\tilde{\nu}_i)_i$ would not alter the basic results.

The setup of the example we consider is similar to that in Section 2. Outcome under the old action is given by $\tilde{x}_i = \tilde{\mu} + \tau$, where $\tilde{\mu}$ takes on the values of -2, 0 and 2 with probabilities of .25, .5 and .25 respectively. (We eliminate the noise $\tilde{\epsilon}_i$ of the Section 2 example, and collapse the draw $\tilde{\mu} + \tilde{\epsilon}_i$ into a draw for just $\tilde{\mu}$.) Outcome under the new action for manager i is given by $\tilde{x}_i = \tilde{\nu}_i + \tau$, where for all i , $\tilde{\nu}_i$ takes on the values of 0, 2 and 4 with probabilities of .25, .5 and .25 respectively. $\tilde{\mu}$ and $(\tilde{\nu}_i)_i$ are jointly independent. Thus the value to the firm of the new action

The more managers that undertake the new action the more accurate it's benchmark, thereby making it more desirable for average and high type managers. As with other herding stories, there is a positive informational externality associated with undertaking an action. This paper has little insight to add such coordination stories besides to note that they are indeed a plausible consequence of a reputation building model such as ours.

One could rationalize our assumption of an intrinsically more accurate benchmark for the old action by positing that by virtue of its familiarity, the market knows how to evaluate outcomes under the industry standard better than under the new action. For example, the market may have accurate indices to evaluate a manager's performance in the stock market, while lacking a good benchmark to evaluate a manager investing in Eastern Europe.

relative to the old, given by the mean improvement, is 2.¹⁷ Table 4 below gives the probability that a manager of type t will obtain outcome x when undertaking the new action.

Table 4 - Probability of Outcome x Under the New Action

		x				
		-2	0	2	4	6
t	-2	.25	.5	.25		
	0		.25	.5	.25	
	2			.25	.5	.25

The market observes which action is chosen, the project specific component μ for the old action, and each managers' outcome x_1 , and therefore can accurately deduce type when the old action is undertaken. Since ν_1 is not observed, the market must infer expected type when the new action is chosen. In general this inference will depend on outcome x_1 , the incentive contract chosen by i and equilibrium actions. We let $q \leq 1/2$ denote the prior probability a manager is of type -2 and 2; the probability a

¹⁷ If instead outcome under the new action was as in the example of Section 2, one can show it would not be in the firm's interest to offer any incentive contracts. The expected improvement in outcome under the new action of .2 in this example is not sufficient to compensate for the cost of benefits necessary to induce the new action. Thus we presently consider a new action that represents a much larger improvement on the old action. Conversely, if the improvement was this large in Section 2, all managers who could take the new action would do so. Intuitively, when action is observable, a much larger associated improvement is necessary to induce the new action, as managerial reputation does not benefit from the higher mean of the new action. Arguably however, it is precisely the actions which implement major improvements that are likely to be observable.

manager is of type 0 is $1-2q$. We set minimum compensation at $F=4$ and firing costs at $c=1/2$ as in the example of Section 2. Since outcome is still additive in type and participants are risk neutral, equation (1) in Section 2 once again yields the manager's second period market value for a given posterior. From equation (1), it follows that when action 0 is taken and type is therefore accurately revealed, second period compensation is given by 0, 4 and 6 for types -2, 0 and 2 respectively.

Firms can introduce incentive contracts, based on outcome x_1 , to induce managers to take the new action.¹⁸ However, these incentive contracts must satisfy the wage floor; they cannot rely on punishments that lead to wages less than F for some outcomes. Thus we will adopt the convention of describing an incentive contract by the "bonus" paid for outcome x_1 ; that is, by wage net F under the outcome. We assume the incentive contract chosen by a manager is observable to the market and therefore this choice may signal information which affects reputation.

In order to compute the optimal schedule of incentive contracts, it will be useful to first compute the market's inferences and consequent second period compensation when a set $S \subseteq (-2,0,2)$ of managers taking the new action are pooled together under the same incentive contract. Table 5 gives the market's inference for expected type and the subsequent second period compensation P_2 from equation (1) when managers of type S are

¹⁸ Incentive contracts based on relative performance have no advantage over those based solely on absolute performance provided there is no common uncertainty; i.e. provided the \tilde{v}_1 terms are uncorrelated. See for example Holmstrom (1982b).

pooled together and outcome x is obtained under the new action. These inferences follow from Bayesian updating. (All possible off the equilibrium path outcomes, for instance when $S = (-2,0)$ and $x_1=6$, could only have been obtained by a single type and are therefore attributed to this type with probability 1.)

Table 5 - Market Inference and Compensation Under the New Action

S	(-2,0,2)		(-2,0)		(-2,2)		(0,2)	
x	<u>E(t)</u>	<u>P₋₂</u>	<u>E(t)</u>	<u>P₋₂</u>	<u>E(t)</u>	<u>P₋₂</u>	<u>E(t)</u>	<u>P₋₂</u>
-2	-2	0	-2	0	-2	0	-2	0
0	-4q	0	-4q	0	-2	0	0	4
2	0	4	$-2q/(2-3q)$	*	0	4	$2q/(2-3q)$ $4+2q/(2-3q)$	
4	4q	4+4q	0	4	2	6	4q	4+4q
6	2	6	2	6	2	6	2	6

Note that we have implicitly assumed $q \geq 1/8$ to obtain second period compensation of 0 when $E(t) = -4q$. The case of $q < 1/8$ can be handled separately to obtain results consistent with those that we find. Compensation when $E(t) = -2q/(2-3q)$ is given by 4 if $q < 2/7$, otherwise it equals 0. When compensation in this state is given by 4, it is less costly to induce type 0 to take the new action. Hence setting compensation to 4 in this state places a lower bound on the cost of inducing the set $S = (-2,0)$ to undertake the new action. We will work with this lower bound on cost and find that inducing types $(-2,0)$ to take the new action is

always dominated by an incentive contract that induces all types $(-2,0,2)$ to take the new action. Table 6, which gives expected second period compensation for managers undertaking the new action when pooled together with all managers of set S, follows from Tables 4 and 5.

Table 6 - Expected Second Period Market Compensation Under the New Action

	S	$(-2,0,2)$	$(-2,0)$	$(-2,2)$	$(0,2)$
t					
-2		1	1	1	$3 + q/(2(2-3q))$
0		$3+q$	3	3.5	$4 + q + q/(2-3q)$
2		$4.5+2q$	4.5	5.5	$4.75 + 2q + q/(2-3q)$

Now we consider the least costly incentive schedules which induce given sets of managers to take the new action. We then compare benefits net costs for optimally inducing a given set of managers to take the new action across all sets to find the optimal incentive schedule. This analysis is rather detailed and consequently, for the sake of brevity, we will be somewhat sketchy. The reader uninterested in the details presented can skip to Table 7 and the following result which summarizes the analysis. Note that since managers of type -2 need no incentives to take the new action, we only need consider sets S which include this type.¹⁹

¹⁹ It is easy to show that it is not beneficial for firms to

First consider singleton $S = \{-2\}$. This set can be induced to take the new action with no incentive contracts at all. Types $t=-2$ will be indifferent between taking the old action which reveals type or signaling type under the new action; both manners lead to 0 second period compensation.²⁰ A firm's maximal expected benefits from the new action when no incentive contracts are offered is therefore given by $2q$ (the proportion of type -2 investors is q , and the new action increases expected outcome by 2).

Next we consider the cheapest incentive schedule that induces types $S=\{-2,2\}$ to take the new action and type 0 to take the old action. First we consider a pooling contract, where a single incentive contract is offered that will induce both these types to take the new action. Since from the third column of Table 6, types -2 obtain in expectation 1 when pooled with type 2 managers as opposed to 0 under the old action, they need no further incentive to take the new action. Also from the third column of Table 6, type 2 managers must be compensated for the expected reputation loss of .5 under the new action (since expected second period compensation is 5.5 under the new action and 6 under the old action). This can be accomplished with a bonus of 2 either for outcomes $x=4$ or $x=6$; as the probability that either of these

offer incentive contracts that would induce such managers not to take the new action.

²⁰ While getting types $t=-2$ to take the new action with no incentive contracts is razor edged, and with just a little manager specific noise they would take the old action, we will show that this no incentives option is always dominated by some schedule of incentives and therefore is inconsequential.

outcomes is obtained under the new action exceeds that under the old action by .25. The former option dominates the latter for the firm because the bonus has to be paid with probability of .25 when $x=6$ and with probability .5 when $x=4$.²¹ This bonus will never be paid to type -2 or type 0 managers, as they cannot achieve the outcome $x=6$. Thus expected payments under this incentive contract is given by $2(1/4)q = .5q$. Note that when only this incentive contract is offered, type 0 managers will take the old action.

Also note that the firm cannot induce types $S = (-2, 2)$ to take the new action any cheaper with a pair of separating incentive contracts. Since type -2 gains 1 by pooling with type 2 when the latter takes the new action, types -2 must be paid at least this much through their incentive contract to make such separation incentive compatible.²² This cost alone exceeds the .5q cost of the pooling contract.

Thus the least cost manner for a firm to induce managers of type $(-2, 2)$ to take the new action and those of type 0 to take the old action is by offering a single incentive contract, which pays a bonus of 2 for outcomes of $x=6$. The probability the firm's manager undertakes the new action will be $2q$, and the bonus of 2 will have to be paid to types 2 with probability $1/4$. Thus the expected benefit from the new action being undertaken net the

²¹ Note that linearity together with restrictions against punishments (negative bonuses) implies that the firm cannot improve on this with an incentive contract that compensates for both $x=4$ and $x=6$.

²² Note that the no punishment condition is crucial for this result. Otherwise the firm could offer a contract with a bonus for $x=6$ and a large punishment for $x=-2$ which would be accepted by managers of type 2 and rejected type -2 managers.

costs of the incentive contract is given by $2(2q) - 2(1/4)q = 3.5q$.

Next we consider the optimal incentive schedule inducing types $S = (-2, 0)$ to undertake the new action and type 2 to take the old action. First consider a single pooling incentive contract. This incentive contract must yield type 0 at least additional benefits of 1 from undertaking the new action rather than the old action in order to compensate for lost second period compensation. (The second column of Table 6 indicates that with such a contract, expected second period compensation for type 0 is given by 3, whereas it equals 4 under the old action.) This can be accomplished with a bonus of 4 either for outcome $x=4$ or $x=2$, since the probability that type 0 obtains either outcome increases by .25 under the new action. If a bonus is given when $x=4$, it must be paid with probability $.25(1-q)$; it is paid with probability $.25(1-2q)$ to managers of type 0 taking the new action and with probability $.25q$ to managers of type 2 taking the old action. If instead the bonus is paid for an outcome of $x=2$ it will have to be paid with frequency $.5(1-q)+.25q$; with probability $.5(1-2q)$ to managers of type 0 taking the new action, with probability $.5q$ to managers of type 2 taking the old action, and with probability $.25q$ to managers of type -2 taking the new action. It therefore follows that the bonus when $x=4$ is less costly than that when $x=2$.

Once again it can be shown that this pooling contract dominates any separating contracts which induces types -2 and 0 to take the new action and types 2 to take the old action.

Intuitively, in order to induce type -2 not to pool with type 0 but to still take the old action, an incentive contract must both yield benefits that are 1 greater when not pooling than when pooling with type 0 and that keeps the new action more attractive than the old action. This involves giving a bonus of at least 4 for either the outcome $x=0$ or $x=2$. This bonus, however, would be enjoyed by managers of type 0 taking the old action, which will imply that they will need further inducement to undertake the new action. It can therefore be shown that all such schemes will be more costly to the firm than the above pooling incentive contract.

Thus the least cost manner for a firm to induce managers of type $(-2,0)$ to take the new action and those of type 2 to take the old action is by offering a single incentive contract, which pays a bonus of 4 for outcomes of $x=4$. Provided only types -2 and 0 undertake the new action, market beliefs for outcome x under the new action are given by the second column of Table 5, which in turn justify these types taking the new action. The subsequent probability a firm's manager undertakes the new action will be $1-q$, and the bonus for outcome $x=4$ will have to be paid to types 0 with probability $1/4$ and to types 2 (who undertake the old action) with probability $1/4$ as well. Thus the expected benefit from the new action being undertaken net costs of the incentive contract is given by $2(1-q) - 1/4(4)(1-q) = 1-q$.

Finally we consider the optimal incentive schedule that induces all types $(-2,0,2)$ to undertake the new action. First we consider the least cost pooling incentive contract. When all types are pooled together, the first column of Table 6 indicates

that the cost in reputation of the new action relative to the old is $1-q$ for types 0 and $1.5 - 2q$ for types 2. Types 0 will therefore take the new action given a bonus of $4(1-q)$ for either $x=4$ or $x=2$. Once again it can be shown that the bonus for $x=4$ is superior to the bonus for $x=2$. Such a bonus however yields managers of type 2 an expected payment of $(1-q)$ if they were to take the old action. Thus, to induce type 2 managers to take the new action, a bonus must be given for $x=6$ such that the total expected bonus for such managers under the incentive contract when the new action is taken is given by $6 - (4.5+2q) + (1-q) = 2.5 - 3q$. (The manager could get $6+(1-q)$ under the old action, and obtains expected second period benefits of $4.5 + 2q$ under the new action). Under this new action, type 2 managers obtain the bonus of $4(1-q)$ with probability .5. Since $x=6$ occurs with probability .25, it follows that a bonus of $(1/.25)\left((2.5 - 3q) - 2(1-q)\right) = 2(1-2q)$ must be paid when $x = 6$.

Thus the least costly single incentive contract that induces all types to take the new action is one which gives a bonus of $4(1-q)$ when $x=4$ and $2(1-2q)$ when $x=6$. Such a contract will induce all three types of managers to take the new action. Types 0 will receive the bonus of $4(1-q)$ with probability .25, and types 2 will receive the bonus of $4(1-q)$ with probability .5 and the bonus of $2(1-2q)$ with probability .25. Thus the total expected cost of the incentive contracts is given by,

$$(1-2q)(1/4)(4(1-q)) + q\left(1/2(4(1-q)) + 1/4(2(1-2q))\right) = 1-.5q-q^2 \quad (16)$$

Thus the expected benefit from the new action being undertaken net costs of the incentive contract is given by $2-(1-.5q-q^2)=q^2+.5q+1$.

Furthermore, this pooling contract is the cheapest incentive schedule which will induce all types to take the new action. That is, it is less costly for firms than all semi-pooling and separating contracts that induce all types to take the new action. For the sake of brevity, we will just sketch the argument here.

First note that any semi-pooling or separating incentive schedule that induces all types to take the new action for which types -2 and 0 do not pool will be very costly. Even if types -2 are pooling with types 2, they will be inferred to be of type -2 and receive 0 second period compensation if $x=-2$ or $x=0$. If instead a type -2 manager deviates and accepts the contract of type 0 managers, his type will be inferred to be at least 0 when $x=0$ (it could be greater if type 2 managers are pooling with type 0 managers). This leads to an increase in expected period 2 compensation of at least 2, since outcome $x=0$ is obtained with probability $1/2$ for type -2 under the new action and compensation is at least 4 when $E(t) \geq -1/2$. Thus to support the outcome where types -2 and 0 are separated will require giving managers of type -2 a bonus of at least 4 when $x = 0$ or 8 when $x = 2$. But this in turn implies that managers of type 0 must also be given a large bonus in order to induce them to take the new action rather than the old action together with the incentive contract intended for managers of type -2. Similarly, this will imply that managers of type 2 will also have to be given a large bonus. The sum cost of these bonuses greatly exceeds the cost of the pooling contract above.²³

²³ A more sophisticated argument shows the same is true when

Thus the only possible incentive contracts which induce all managers to take the new action still to be considered are semi-pooling pairs of contracts in which types 0 and -2 pool and type 2 separates. It can be shown that the least cost manner that an incentive compatible incentive schedule attains this end is with one contract that pays a bonus of 6 when $x=4$ and another contract which pays a bonus of 6 when $x=6$. With this pair, types -2 and 0 will take the new action and the former contract, and types 2 the new action and the latter contract. The expected cost of such incentives is given by $6(1-2q)(1/4) + 6q(1/4) = 1.5(1-q)$. And since for all $q \leq 1/2$, $1.5(1-q) > 1 - .5q - q^2$, it follows that the full pooling contract is a more efficient incentive schedule than this semi-pooling schedule. Thus the full pooling contract is the optimal schedule which induces all managers to take the new action.

Finally, the overall optimal incentive schedule is found by comparing net benefits for these least cost incentive schedules that induce each of the possible sets S to take the new action. These net benefits are summarized in Table 7.

Table 7 - Net Benefits Under the Least Cost Incentive Schedules Which Induce S to Take the New Action

S	Cost	Benefits	Net Benefits
$\{-2\}$	0	$2q$	$2q$
$\{-2, 0\}$	$1-q$	$2(1-q)$	$1-q$
$\{-2, 2\}$	$.5q$	$4q$	$3.5q$
$\{-2, 0, 2\}$	$1 - .5q - q^2$	2	$q^2 + .5q + 1$

only some fraction of type -2 managers pool with type 0 managers.

Note that net benefits for the optimal incentive schedule which induces $S = (-2, 2)$ dominates offering no incentive contracts at all (which induces $S = (-2)$), and the optimal incentive schedule which induces $S = (-2, 0, 2)$ dominates that which induces $S = (-2, 0)$. Thus the only incentive schedules that are ever optimal are the least cost schedules that induce either $(-2, 2)$ or $(-2, 0, 2)$ to take the new action. The range of q over which each of these schedules is optimal follows immediately from Table 7 and is summarized below.

Result 1 - When $q < q^* = (3 - \sqrt{5})/2 \approx .382$, it is optimal to induce $S = (-2, 0, 2)$ to take the new action, and when $q > q^*$, it is instead optimal to induce $S = (-2, 2)$ to take the new action. In both cases this is optimally accomplished with a single incentive contract. The least cost manner of inducing $S = (-2, 0, 2)$ to take the new action is through a single incentive contract which gives bonuses of $4(1-q)$ when $x=4$ and $2(1-2q)$ when $x=6$. The least cost manner of inducing $S = (-2, 2)$ to take the new action is through a single incentive contract giving a bonus of 2 when $x=6$.

The intuition for this result is as follows. Type 2 managers can be induced to take the new action by paying a bonus when $x=6$ that exactly compensates for reputation costs. This bonus does not induce a need to further compensate other types since they can never obtain outcome $x=6$. Furthermore, the presence of type 2 managers taking the new action improves the reputation of other types taking the new action. Thus, provided that the cost in

reputation for type 2 managers undertaking the new action is less than the expected benefit to the firm from the new action (which is always true in this example), firms will always want to give sufficient incentives for these managers to take the new action. The presence of these managers taking the new action implies that managers of type -2 will always take the new action as well.

Then the only question that remains is whether it is worthwhile for a firm to offer incentives that induce managers of type 0 to undertake the new action as well. The presence of these managers taking the new action will mean managers of type 2 will have to be compensated more to take the new action. This higher compensation is necessary both because type 0 managers lower the market's inference for type 2 managers when taking the new action, and because when undertaking the old action, type 2 managers can realize the bonuses that are used to induce types 0 to take the new action. When $q > q^*$, indicating relatively many type 2 managers and relatively few type 0 managers, this tradeoff is not worthwhile, and the firm is better off ignoring types 0. If instead $q < q^*$, there are enough type 0 managers relative to type 2 managers that it is optimal to also give type 0 managers incentives to innovate.

Thus in this example, whether a firm prefers giving bonuses which induce only high type managers to take the new action, or whether it instead prefers bonuses that induce both average and high type managers to take new actions, depends on the distribution of types. (In both cases, low type managers will take the new action without any incentives to do so.) We find

that if there are enough high type managers relative to average type managers, firms will offer incentives that get high and low types to take the new action and average types to take the old action. While firms would like average type managers to take the new action as well, they face a large reputation cost because of the possibility they will be confused with low type managers and consequently get fired. Compensating for this cost through incentives would imply greater incentives are also needed to get high types to take the new action, which becomes too costly for the firm.

Section 6 - Extensions

While the firing discontinuity plays a considerable role in the above analysis, similar results can be obtained by replacing this assumption with decreasing absolute risk aversion among managers. With no discontinuity, second period compensation is given by $\max(E(t|x, \mu), 0)$. Bad managers will like an increase in the variance of relative performance because their outside opportunity puts a lower bound of 0 on compensation.²⁴ This is substantially no different from above; a manager likely to get fired, discontinuity or not, will benefit from increased relative performance variance, as this increases the probability that he will be saved by a good draw. Average and good managers, unlikely

²⁴ The possibility that an outside opportunity limits the incentives for herd behavior is mentioned in Scharfstein and Stein.

to get fired, will dislike this increase in relative variance due to risk aversion. However, since the new action increases variance in relative outcome by a fixed amount σ^2 for all types, decreasing absolute risk aversion implies that average types will be more averse to this increase than high types. All managers will of course benefit in reputation from the higher mean of the new action. Therefore, under suitable parameter assumptions, the net effect will yield low and high types willing to take the new action, and average types taking the old action. This is of course the same preferences that the discontinuity induced.

Extending this model to multiple periods would imply that at any given time there are managers with different career lengths. An interesting question is how managerial seniority will affect innovativeness. The longer a manager's career, the more precise the market's inference of her type will be; precision of the market's posterior in the model of Sections 3 and 4 after n periods of managing is given by $1/r^2 + n/\delta^2$. The greater precision implies that future performance will have less impact on the manager's reputation. With unobservable actions, this will work both ways; the manager will care less both about benefits to reputation from the higher mean outcome and costs from higher variance in relative outcome. While initially the effect is ambiguous, as a manager's career grows long and the market's precision grows large, both costs and benefits go to 0, but costs fall off quicker than benefits. Thus, while the manager will become nearly indifferent between the two action, she will have a slight preference for the new action. If instead action is

observable, the manager will have a slight preference for the old action, but low powered incentive schemes could induce the manager to take the new action.

The above discussion assumes the manager's ability stays constant over her career. Alternatively, one could consider managerial ability subject to random normal shocks each period as in Holmstrom (1982a). Under such an assumption, the precision of the market's inference will eventually reach a steady state. Whether the number of types willing to innovate increases with seniority is ambiguous and depends on specific parameters. Another reasonable model for evolving ability over time is that a manager's ability is constant for a fixed period, and then falls in one period, with a decay rate increasing with n . Such a process corresponds to a manager losing ability with old age. This process would lead managers to be more conservative over time, as the older the manager, the more the market will attribute a poor relative performance to a drop in ability. Thus while the market might take a wait and see attitude after one period of poor performance for a young manager, it will consider an old manager with such a performance likely to be "over the hill", and will fire this manager. The effect of changing ability on a manager's appetite for innovations appears to be an interesting topic for future research.

Assumptions on the distribution of types and the linear dependence of technology on type are linked. For any distribution of types with the real line as its support, we can monotonically rename types so they will be normally distributed. However, this

of course will alter the relationship between type and output. In particular, if after this renaming, technology is concave in type, there will be less innovations. This is true even when action is observable, as Jensen's inequality implies that a manager is worth more to a firm the more precise the firm's posterior holding the mean fixed. Likewise, convex technology will imply that more types will innovate.²⁵

Results of Section 4 imply that symmetric information is likely to lead to less innovations. If managers are unaware of their own types, they will compare benefits from the two actions over the distribution of ability. Since most types prefer the old action when b is not too large, a manager unaware of his type will stick with the old action as well. Only new projects with very high expected returns, or that are highly correlated in outcome with old actions, are likely to be adopted. However, in a multiperiod context this effect may be somewhat mitigated. After several periods, managers who have established themselves to be good will be more willing to undertake new innovations.

Section 7 - Conclusion

This paper examines how in creating a divergence between managerial and ownership investment preferences, reputation and

²⁵ See Rosen (1981) for situations where market competition may imply convex returns in type, giving rise to a "superstar effect". It is notable that professions that are likely to exhibit such an effect, such as the performing arts, are often considered environments which foster creativity and innovations.

career concerns may lead to managerial aversion to innovations, even when they stochastically dominate standard actions. Management shuns innovations because this will lead to a greater disparity between their performance and others in the industry; the down-side risk of which may lead to being fired.

This theory has several noteworthy implications. Lessening managerial concern for reputation may spur the adoption of more innovations. Thus, Japanese innovative superiority over the last several decades may be partially attributable to aspects of their corporate culture which bind managers to firms for life. If managers do not receive outside offers commensurate with their reputation, there will be less of a conflict between firm objectives and reputation building. This interpretation of Japanese innovativeness is in line with the observation that recent Japanese innovative superiority over American firms has mainly been in the process of implementation, not discovery.

This theory also has implications for the types of innovations that are likely to be adopted. Managers will be more willing to undertake projects which are easy to evaluate, or are closely related to ongoing projects, rather than more original complicated projects. For example, Dertouzos, Lester and Solow write that the design cycle in the automobile industry, "encouraged the companies to keep basic designs in production as long as possible with occasional cosmetic 'face lifts.'"²⁶ Additionally, original, hard to evaluate innovations are more likely to be introduced by entrepreneurial manager-owned firms, as

²⁶ *Made in America*, p. 178.

has been the case in the semiconductor industry. Large firms are more likely to wait for others before adopting new innovations. This yields predictions for "follow the leader" type herding in the adoption of superior technology. Only after some firms have adopted a new superior technique will others will be willing to do so as well. The first innovators are providing a public good by giving others a benchmark with which to be compared.

Finally, it is worth noting that while we have focused on the interpretation of corporate innovations, the model is relevant for a much broader scope of principle-agent activity. Governmental stagnation may in part be due to an unwillingness to take actions which increase variance in relative outcome. A bad outcome under a new action would be attributed to bad governance and lead to a consequent loss of power; while a bad outcome under the old action may be rationalized as "beyond government control", as evidenced by similar bad outcomes in other jurisdictions. This idea could be used to explain herding in laws across states and nations. Similarly, this model could be applied to the sometimes slow adoption of innovations and herding in teaching, the strategies of sports teams, academic research, and numerous other areas where agents are concerned with their reputation as well as the outcome.

Appendix

Proof of Theorem 1

From equations (4) and (5), the benefits from the new action compared to the old action for a manager of type t are given by,

$$\begin{aligned}
 P(1,t) - P(0,t) = & \\
 & F \left[\Phi \left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}} \right) - \Phi \left(\frac{t-p_1}{\delta} \right) \right] + \tag{A1} \\
 & \frac{r^2}{\delta^2+r^2} \left[(\sigma^2+\delta^2)^{1/2} \int_{-\infty}^{(t+b)/(\sigma^2+\delta^2)^{1/2}} \Phi(z) dz - \delta \int_{-\infty}^{t/\delta} \Phi(z) dz \right].
 \end{aligned}$$

We will define $V(t) = P(1,t) - P(0,t)$, $V_1(t)$ as the first term in expression (A1) and $V_2(t)$ as the second term.

The following relationship for the normal distribution will be helpful in signing expression (A1);

$$\frac{d}{da} \left[a \int_{-\infty}^{b/a} \Phi(z) dz \right] = \phi(b/a). \tag{A2}$$

First we will show that $\forall t, V_2(t) > 0$. We can rewrite $V_2(t)$ as,

$$V_2(t) = \frac{r^2}{\delta^2+r^2} \left[(\sigma^2+\delta^2)^{1/2} \int_{-\infty}^{(t+b)/(\sigma^2+\delta^2)^{1/2}} \Phi(z) dz - \delta \int_{-\infty}^{t/\delta} \Phi(z) dz \right] - \tag{A3}$$

$$\frac{r^2}{\delta^2+r^2} \left[(\sigma^2+\delta^2)^{1/2} \int_{-\infty}^{t/(\sigma^2+\delta^2)^{1/2}} \Phi(z) dz - \delta \int_{-\infty}^{t/\delta} \Phi(z) dz + (\sigma^2+\delta^2)^{1/2} \int_{t/(\sigma^2+\delta^2)^{1/2}}^{(t+b)/(\sigma^2+\delta^2)^{1/2}} \Phi(z) dz \right].$$

Then using (A2),

$$V_2(t) = \frac{r^2}{\delta^2 + r^2} \left[\int_{\delta}^{(\sigma^2 + \delta^2)^{1/2}} \phi\left(\frac{t}{z}\right) dz + (\sigma^2 + \delta^2)^{1/2} \int_{t/(\sigma^2 + \delta^2)^{1/2}}^{(t+b)/(\sigma^2 + \delta^2)^{1/2}} \Phi(z) dz \right]. \quad (A4)$$

And since $b, \sigma > 0$, both terms in the brackets are strictly positive. Thus $\forall t$,

$$V_2(t) > 0. \quad (A5)$$

We now proceed to show that for any positive b and F , as $t \rightarrow \pm \infty$, the new action is preferred. First note from (A4) that,

$$\lim_{t \rightarrow \infty} V_2(t) = \frac{r^2}{\delta^2 + r^2} \left[(\sigma^2 + \delta^2)^{1/2} \left(b / (\sigma^2 + \delta^2)^{1/2} \right) \right] = \frac{r^2}{\delta^2 + r^2} b. \quad (A6)$$

And since $\lim_{t \rightarrow \infty} V_1(t) = 0$, we obtain,

$$\lim_{t \rightarrow \infty} V(t) = \frac{r^2}{\delta^2 + r^2} b > 0. \quad (A7)$$

Now note that $V_1(t)$ is positive iff,

$$\frac{t+b-p_1}{(\sigma^2 + \delta^2)^{1/2}} > \frac{t-p_1}{\delta}; \quad (A8)$$

which is equivalent to,

$$t \leq p_1 + \frac{\delta b}{(\sigma^2 + \delta^2)^{1/2} - \delta} = \bar{t}. \quad (A9)$$

Expressions (A5) and (A9) together imply that,

$$\forall t \leq p_1 + \frac{\delta b}{(\sigma^2 + \delta^2)^{1/2} - \delta}, \quad V(t) > 0. \quad (A10)$$

Equations (A7) and (A10) ensure the existence of values t_1 and t_4 of the theorem for any $F, b > 0$. We now show that when F satisfies equation (10) and b is small enough, there exists some \bar{t}

such that $V(\bar{t}) > 0$. Then by continuity, there will be an interval $t_2 < \bar{t} < t_3$ for which $\forall t \in (t_2, t_3)$, $V(t) > 0$. We first show that such a \bar{t} exists when $b=0$.

First note from equation (A4) that when $b=0$,

$$V_2(t) = \frac{r^2}{\delta^2 + r^2} \left[\int_{\delta}^{(\sigma^2 + \delta^2)^{1/2}} \phi\left(\frac{t}{z}\right) dz \right]. \quad (A11)$$

It therefore follows that $\forall t$,

$$V_2(t) \leq \frac{r^2}{\delta^2 + r^2} \left((\sigma^2 + \delta^2)^{1/2} - \delta \right) \phi\left(\frac{t}{(\sigma^2 + \delta^2)^{1/2}}\right). \quad (A12)$$

At $b=0$, $V_1(t)$ can be rewritten as,

$$V_1(t) = -F \left[\int_{(t-p_1)/(\sigma^2 + \delta^2)^{1/2}}^{(t-p_1)/\delta} \phi(z) dz \right]; \quad (A13)$$

and therefore, provided that $t \geq p_1$,

$$V_1(t) \leq -F \phi\left(\frac{t-p_1}{\delta}\right) \left[\frac{(t-p_1) \left((\sigma^2 + \delta^2)^{1/2} - \delta \right)}{(\sigma^2 + \delta^2)^{1/2} \delta} \right]. \quad (A14)$$

Hence from (A12) and (A14), for $t \geq p_1$, a sufficient condition for type t to prefer the old action to the new one is,

$$F \phi\left(\frac{t-p_1}{\delta}\right) \left[\frac{(t-p_1)}{(\sigma^2 + \delta^2)^{1/2} \delta} \right] > \frac{r^2}{\delta^2 + r^2} \phi\left(\frac{t}{(\sigma^2 + \delta^2)^{1/2}}\right). \quad (A15)$$

The left hand side of (A15) is maximized at $t = p_1 + \delta$. Evaluating this equation there, we find that a sufficient condition to ensure the old action is preferred by this type is,

$$(F/(\sigma^2+\delta^2)^{1/2}) \phi(1) > \frac{r^2}{\delta^2+r^2} \phi\left(\frac{p_1 + \delta}{(\sigma^2+\delta^2)^{1/2}}\right); \quad (A16)$$

that is,

$$F > \frac{r^2}{\delta^2+r^2} (\sigma^2+\delta^2)^{1/2} \frac{\phi\left(\frac{p_1 + \delta}{(\sigma^2+\delta^2)^{1/2}}\right)}{\phi(1)}. \quad (A17)$$

We now show the existence of \bar{b} of the theorem. To do so we first show that $\forall F$, if b is large enough, all types would prefer the new action. We have already seen that for $t < \bar{t}$, it is always true that $V(t) > 0$ (equation(A10)). If instead $t \geq \bar{t}$, since $\bar{t} > p_1$, equation (A13) implies that,

$$V_1(t) \geq -F \phi\left(\frac{t-p_1}{(\sigma^2+\delta^2)^{1/2}}\right) \left[\frac{(t-p_1)\left((\sigma^2+\delta^2)^{1/2} - \delta\right)}{(\sigma^2+\delta^2)^{1/2} \delta} \right]. \quad (A18)$$

Thus $V_1(t)$ is bounded below by,

$$V_{-1}(t) = -F \phi(1) \left[\frac{\left((\sigma^2+\delta^2)^{1/2} - \delta\right)}{\delta} \right]. \quad (A19)$$

And from equation (A5),

$$V_2(t) > \frac{r^2}{\delta^2+r^2} \left[(\sigma^2+\delta^2)^{1/2} \int_{t/(\sigma^2+\delta^2)^{1/2}}^{(t+b)/(\sigma^2+\delta^2)^{1/2}} \Phi(z) dz \right]; \quad (A20)$$

which implies that,

$$V_2(t) > \frac{r^2}{\delta^2+r^2} b \Phi\left(\frac{\bar{t}}{(\sigma^2+\delta^2)^{1/2}}\right). \quad (A21)$$

Therefore, provided that

$$b > \left(\frac{r^2 + \delta^2}{\delta^2} \right) \frac{-V_1(t)}{\Phi \left(\frac{\bar{t}}{(\sigma^2 + \delta^2)^{1/2}} \right)}, \quad (\text{A22})$$

it follows from equations (A19) and (A21) that $V(t) > 0$ for all $t \geq \bar{t}$.

Now let $\bar{b}(F) = \inf(b | \forall t, V(t; F, b) > 0)$, where $V(t; F, b)$ represents $V(t)$ for given values of F and b . The continuity of V then implies that,

$$\min_t V(t; F, \bar{b}(F)) = 0. \quad (\text{A23})$$

And since for all t , $V(t; F, b)$ is strictly increasing in b , when $b > \bar{b}(F)$, $V(t) > 0 \forall t$ (all types strictly prefer the new action); and when $b < \bar{b}(F)$, letting $\tilde{t} = \underset{t}{\operatorname{argmin}} V(t; F, \bar{b}(F))$, $V(\tilde{t}; F, \bar{b}(F)) < 0$.

By continuity, there exists an interval $(t_2, t_3) \ni \tilde{t}$, such that $t \in (t_2, t_3)$ implies $V(t, F, \bar{b}(F)) < 0$.

Since $\forall t V_2(t) > 0$, $V(t; F, b) \leq 0$ implies that $V_1(t; F, b) < 0$. It then follows from the definition of $V_1(t)$, that $\forall t$ such that $V(t; F, b) \leq 0$, $V(t; F, b)$ is decreasing in F . This fact, equation (A23) and $V(t; F, b)$ strictly increasing in $b \forall t$, together imply that that $\bar{b}(F)$ is strictly increasing in F .

Also by continuity of V , together with equations (A7) and (A10), it follows that $\forall F$ that satisfy equation (10) and $b < \bar{b}(F)$, there exists some t such that $V(t; F, b) = 0$. Then, since $\forall t V(t; F, b)$ is strictly increasing in b , it follows that for $b_1, b_2 < \bar{b}(F)$, $T(b_1) \subseteq T(b_2) \iff b_1 \leq b_2$. \square

Proof of Theorem 2

For a manager of type $t > -c$, the informational loss is given by the amount t exceeds $-c$, times the increased probability the manager will be mistakenly fired under action 0 as opposed to action 1. Hence,

$$L(t) = (t+c) \left[\Phi \left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}} \right) - \Phi \left(\frac{t-p_1}{\delta} \right) \right],$$

which is equation (10).

Similarly, for a manager with $t < c$, the informational loss from action 1 relative to action 0 is the amount c exceeds t , times the increased probability that the manager will not be fired. This is given by,

$$L(t) = (-c-t) \left[\left(1 - \Phi \left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}} \right) \right) - \left(1 - \Phi \left(\frac{t-p_1}{\delta} \right) \right) \right],$$

which is once again identical to (10).

The bracketed term in equation (10) is positive iff inequality (A9) holds. The term $t+c$ is positive iff $t \geq -c$. Thus $L(t) > 0$ if $t > \max(-c, \bar{t})$ or $t < \min(-c, \bar{t})$, and $L(t) < 0$ if $\min(-c, \bar{t}) < t < \max(-c, \bar{t})$.

A straightforward application of l'Hôpital's rule implies that as $t \rightarrow -\infty$, both $(t+c) \left[\Phi \left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}} \right) \right]$ and $(t+c) \left[\Phi \left(\frac{t-p_1}{\delta} \right) \right] \rightarrow 0$, and therefore $L(t) \rightarrow 0$.

When $t \rightarrow \infty$,

$$0 \geq \lim_{t \rightarrow \infty} (t+c) \left[\Phi \left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}} \right) - \Phi \left(\frac{t-p_1}{\delta} \right) \right]$$

$$\begin{aligned}
&= \lim_{t \rightarrow \infty} -(t+c) \int_{(t+b-p_1)/(\sigma^2+\delta^2)^{1/2}}^{(t-p_1)/\delta} \phi(x) dx & (A26) \\
&\geq \lim_{t \rightarrow \infty} -(t+c) \phi\left(\frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}}\right) \left(\frac{t-p_1}{\delta} - \frac{t+b-p_1}{(\sigma^2+\delta^2)^{1/2}}\right) = 0;
\end{aligned}$$

where the final relationship once again follows from l'Hôpital's rule. Hence $\lim_{t \rightarrow \infty} L(t) = 0$ as well. \square

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Chapter 3

AN AGENCY/CONTROL THEORY OF CAPITAL STRUCTURE WHEN MANAGEMENT CAN ALTER DEBT

Abstract - This paper develops a model in which debt serves to constrain inefficient investments of empire building managers due to the consequent control implications of bankruptcy. Capital structure is voluntarily chosen by management, as a credible constraint which ensures sufficient efficiency to prevent takeover challenges. Thus, capital structure is derived as the optimal response of partially entrenched empire-building managers to control considerations; managers trade off empire building ambitions with the need to retain the empire to realize these ambitions. Such capital structure is dynamically consistent; in the model, managers are free to readjust leverage each period. A policy of dividend payments coordinated with capital structure decisions follows naturally, unlike in related cash-flow models. The model also yields implications for debt level and term structure as a function of outside investment opportunities.

Section 1 - Introduction

Recent years have seen a proliferation of theories explaining why, despite Modigliani and Miller (1958), capital structure is relevant. Motivated by anecdotal evidence, one active line of research proposes that debt serves to constrain management from pursuing personal goals at the expense of value maximization. Such agency theories of capital structure date to Jensen and Meckling (1976). Under this view, the effective separation of ownership and control which characterizes the modern corporation gives management scope to pursue a personal agenda. Debt is seen as a device which either constrains managerial choices, or better aligns managerial incentives with shareholders, thereby mitigating agency problems.

Recent manifestations of this view have focused on two related manners in which debt may constrain managers, both of which take empire building ambitions of management as the primary agency conflict to be resolved.¹ First, debt may force managers to liquidate inefficient operations; this possibility is considered by Williamson (1988) and Harris and Raviv (1990a). Second, by restricting the availability of free cash flow, debt limits managers' ability to undertake inefficient projects. This latter view is associated with Jensen (1986), and underlies models

¹ The desire for empire building can be reconciled with profit maximizing behavior through evidence linking managerial compensation with corporate size as well as performance; for example, see Murphy (1985). We however have little problem with the primitive assumption that some managers pursue empires and associated power for their own sake.

of Stulz (1990), Hart and Moore (1990) and Hart (1991).² In such papers, debt is typically taken to be set ex ante in a manner which maximizes firm value. Optimal capital structure is determined by trading off benefits of managerial access to funds for good investment opportunities with costs of such access to bad ones. These models yield a theory of capital structure that captures common perceptions of debt serving to restrict managerial excesses.³

However, one significant drawback of these agency cost models of debt is their reliance on the ex ante presence and ex post absence of a "discipliner", who initially imposes debt that optimally constrains management. This discipliner may take the form of a creditor from whom an entrepreneur must borrow to invest, the market when an entrepreneur wants to maximize proceeds upon taking a firm public, or a raider who forces management of a public corporation to implement optimal capital structure to avert a takeover. Debt is set ex ante to constrain the managers' future investment decisions. This constraint is useful because the

² In addition to these two recent agency cost theories of capital structure, a related line of research deserves mention. In particular, debt combined with the ability of creditors to liquidate upon default is derived as the most efficient manner for an entrepreneur to raise capital when verifying returns is costly in Townsend (1979) and Gale and Hellwig (1985). Hart and Moore (1989) explores efficiency properties of debt contracts when the threat of liquidation is the only means a creditor has to enforce payment and renegotiation is possible.

³ For an excellent summary of these as well as other models of capital structure and security design, the reader is referred to the survey papers of Harris and Raviv (1990b) and (1990c). Also see Jensen (1989) for a strong view on how such agency problems can explain widespread developments in the financial structure of corporate America.

discipliner is no longer in a position to exert any pressure later when investment decisions are made.

This view, however, conflicts with common perceptions of leverage decisions being in the domain of standard managerial actions. Managers regularly undertake capital decisions without any apparent external threat, and they appear capable of reversing these decisions. What is to stop a debt-constrained manager from swapping equity for debt when the discipliner is no longer present?⁴ And why would ostensibly cash-constrained managers ever voluntarily choose to pay out dividends?

This paper attempts to provide answers to these questions. It presents a model for which in each period, *managers voluntarily set debt to restrict themselves*. While similar in some manners to cash flow theories of debt, *the mechanism of managerial restraint is the potential loss of control which may be associated with bankruptcy* rather than a lack of cash necessary to undertake investments. Under this view, debt constrained managers don't refrain from bad projects because they lack cash on hand to start up such projects, but rather, because allocating limited cash flow to these projects increases the chance of future bankruptcy. By considering the optimal leveraging decision of managers rather than entrepreneurs, we obtain a dynamically consistent model of debt and dividend policy; managers have no incentive to reverse

⁴ This in general would be possible if the value of the corporation without the class of debt to be repurchased (taking into account the managerial actions this new capital structure will induce) is at least as great as the promised debt payment to this class. This condition is likely to met in practice in many corporations with moderate levels of debt.

leverage enforced upon them and do have incentives to pay dividends.

A key assumption in the model (to be discussed below) is that managers are less entrenched under bankruptcy than otherwise. While this might seem to bias management against issuing debt, we show that quite the contrary may occur. Some managers voluntarily choose debt, using the threat of bankruptcy as a means to credibly commit to forgo bad investments, thereby preventing a takeover. This model will be seen to yield rich implications for the level and term-structure of debt, the interconnection of debt and dividend policy, and the interaction between debt and control.

The basic setup is as follows. We consider empire building managers who enjoy both remaining in control and undertaking new investments. Each period, with some probability depending on the publicly known type of the manager, a good (positive NPV) new investment is available; otherwise, only a bad (negative NPV) investment can be undertaken. We assume managerial entrenchment is such that first, managers can only be removed by takeover or upon bankruptcy, and second, takeovers can only succeed if there is a sufficiently large gain to the raider.⁵ The bankruptcy

⁵ One can think of this assumption as a simple manifestation of Grossman and Hart (1980) free rider problems. For example, consider a simplified setting of Shleifer and Vishny (1986), whereby free-rider problems force a raider to pay the post takeover value of the firm (known here with certainty) to effect a successful takeover. The raider can however profit on the appreciation of a block held prior to the takeover attempt. Such benefits must be compared with a cost incurred mounting a takeover attempt. Then the raider will only attempt a takeover if the future profits of the firm are far enough below its profits without the manager that the appreciation on the raider's prior shares offset the takeover cost.

procedure is assumed to short-circuit this entrenchment; in the event of bankruptcy, a manager will be replaced if there is any gain to doing so.⁶

In our setup, takeover threats cannot serve to commit managers to forgo inefficient investments; the consequences of investments in each period are sunk and therefore do not affect future takeover activity. It is not credible to claim one will attempt a takeover only if a manager takes an inefficient action in the previous period; as benefits from a takeover do not depend on these past consequences. Given unique Nash equilibria in every subgame, it follows that future equilibrium play is independent of this previous action.⁷ The motivation for a raider to take over a

The assumption that the market for corporate control only reacts to inefficiencies which are sufficiently large is perhaps the simplest and most natural manner to model managerial entrenchment. Indeed, this assumption is consistent with widespread empirical evidence which indicates that the typical premium paid in a successful hostile takeover is at least 30 percent. See for example, Bradley, Desai and Kim (1988) or Jarrell, Brickley and Netter (1988). The amount of inefficiency needed for a successful takeover can be considered a measure of managerial entrenchment.

⁶ This is consistent with the interpretation of bankruptcy proceedings as a manner to coordinate creditors and avoid free-rider problems. This view is pervasive in the legal literature. See for example, Jackson (1986).

While we model bankruptcy as costless, results of the model are obtained provided that the marginal cost of removing a manager is lower under bankruptcy. Thus, results go through if either bankruptcy entails less overall costs than the costs incurred in a takeover attempt, or if bankruptcy costs consist of fixed costs automatically incurred upon default and independent of whether or not the manager is removed. While in practice bankruptcy costs may be substantial, by legally organizing creditors, bankruptcy does appear to make the removal of a manager simpler.

⁷ This is true besides for a nongeneric set of types and nodes at which a raider is indifferent between attempting a takeover or not. The uniqueness of Nash equilibria in each subgame depends on

firm remains the same regardless of whether a good or bad action was taken.

However, debt can commit managers to refrain from bad investment decisions where the takeover mechanism cannot, because bankruptcy entails less entrenchment, which in turn leads to the manager's removal under less future equilibrium inefficiency. Thus, debt may serve to make a manager's future control dependent on the investment decision taken. By employing debt in this manner, managers who would otherwise face takeovers can credibly commit to enough efficiency to make such action unwarranted. The amount of debt necessary to prevent a takeover depends on the availability of good projects and the degree of managerial entrenchment. Thus, we will find that managers who are more likely to have good future projects available will be able to indulge in more bad projects.

In order to highlight the distinction between this model and cash flow models, we make the assumption that new projects require no initial investment. While extreme, we argue such an assumption is likely to provide a more realistic setting than one in which investment costs take the form of an initial lump-sum payment. Suppose that instead of requiring an up front lump-sum investment, a new project requires a steady stream of cash infusions. Likewise, returns y from other firm projects accrue throughout the period. Then, provided that managerial use of these funds cannot be restricted within the period, the assumption of no initial

the finite life of the firm.

investment is equivalent to assuming that the stream of cash infusions needed to fund the project is dominated by the rate of accrual of y . This setting is likely to be realistic at least for large firms contemplating moderate new projects. It should be clear how our assumption serves to change the nature in which debt constrains managers; new projects are forgone not because necessary cash to fund them is lacking, but rather, because by committing existing cash to fund these projects, managers increase the probability of bankruptcy.

Similar to cash flow models, there are both benefits and costs of debt to shareholders. Though the mechanism of constraint differs, the benefits of debt in both models is that it serves to constrain managers from building empires through inefficient projects. The costs of debt, however, differ between these two stories. In cash flow models, excessive debt may force a manager to forgo good projects for lack of necessary start up cash. Such models must address why managers cannot renegotiate debt or raise cash through a new firm when good projects are available. Free rider problems and asymmetric information on project quality can be employed to address these issues.

In our model, such issues do not arise; all good projects are always undertaken within the initial firm. Rather, debt may be costly because when excessive, debt loses its ability to constrain managers from undertaking bad projects. Debt can only serve this function when the probability of bankruptcy is affected by undertaking a bad project; if bankruptcy is imminent regardless of what investments are undertaken, a manager will never refrain from

bad projects.

Thus despite costless bankruptcy in our model, it would not be in the interest of an initial entrepreneur to set unlimited debt upon going public. While unlimited debt would necessitate reorganization each period and thereby prevent entrenchment (since we assume managers can be costlessly removed in bankruptcy), this would lead empire building managers to always undertake bad projects. This follows because the managerial retention decision cannot credibly depend on previous investment choices, but rather, only on future play. Thus optimally, entrepreneurs would not want infinite debt; only through the existence of some limited entrenchment will managers have incentives to refrain from bad projects. While in practice bankruptcy costs no doubt also restrict the issuance of excessive debt, ignoring such costs allows this model to isolate a heretofore unexamined cost of debt.

While the question of how much entrenchment an entrepreneur should optimally allow management in such a setting is of interest, we leave such issues for future consideration.⁸ Rather, we consider managers with an unrestricted ability to alter capital structure (market permitting, of course) without excessive initial debt. Therefore, capital structure is determined by managers' optimal response to control concerns. While the market for corporate control will force management to be cognizant of shareholders' value, entrenchment will allow management to set

⁸ Entrenchment could conceivably be limited through infinite initial debt, limitations on a manager's ability to alter capital structure, or prohibitions on various takeover defenses in a corporate charter.

debt so that only some inefficient investments are impeded.

Note that this model combines agency and control theories of debt. While often treated separately in the literature, these two aspects of debt are tightly interrelated. Without agency conflicts, control would not be an issue; incompetent management would voluntarily step down. Conversely, agency problems are exacerbated by managerial entrenchment. In practice, managers have a personal stake in retaining control, though the market for corporate control does appear to limit some managerial excesses. This paper combines these elements by modeling control interests as both the motivation for issuing debt and the mechanism through which debt credibly constrains investments. Management voluntarily chooses debt to maintain control in the face of potential takeovers, and debt is credible because of the takeover implications of default. This debt in turn deters takeovers because it leads to more efficient managerial investment decisions.

Managerial costs of bankruptcy play a key role in this model, similar to capital structure models of Ross (1977) and Diamond (1984) among others. However, unlike these papers, costs are endogenous to the model; costs only consist of forgone control benefits if replaced, and the replacement decision is optimal. Thus this model does not rely on incredible market reactions to bankruptcy using inefficient ex-post penalties or arbitrarily assigned reputation costs. When it is not optimal to remove a manager, the manager faces no costs to bankruptcy; when it is, costs are simply given by forgone control benefits.

The implications of this model are many. First, this paper yields implications for the level and term structure of debt. Specific predictions can be made for how debt changes in response to external changes in the market for control; for example, the emergence of a potential raider, or a change in the shareholder composition. Additionally, it makes strong predictions on joint debt and dividend policy, which sharply differentiate this model from cash flow models. These predictions plausibly cast some light on the well known dividend puzzle. Furthermore, the model yields implications relating capital structure to profitability. The model is also capable of explaining common dynamic responses to takeover attempts.

The next section considers the basic model. Section 3 discusses implications and empirical evidence. Section 4 discusses work in progress extending the model to stochastic environments and concludes.

Section 2 - The Model

We consider a firm which exists for three periods. The net returns from present assets are y each period; after the third period the assets have no value. In addition to these returns the manager has the opportunity to undertake a single additional investment each period. Such projects require no initial investment. All participants are risk neutral, and for convenience, we set the interest rate to 0.

In each period, with probability t , the manager will have a "good" new investment available. This probability is uncorrelated across periods. If no good project is available, the manager can undertake a "bad" project. The availability of a good project is learned by the manager and the market right before the decision to invest is made.⁹ The investment decision is taken to be noncontractable. Net returns to the good and bad projects are denoted by \bar{r}_1 and \bar{r}_2 respectively. In this paper, we take the outcome of both good and bad investments to be deterministic. In particular, we let $r_1 = 1 < y$ and $r_2 = -1$. Current work in progress discussed in Section 4 considers the implications of stochastic returns. The variable t , which is public information, parameterizes managers' types; managers of higher type are more likely to have good investment opportunities in any period.¹⁰

Each period empire building managers get utility $A > 0$ from running firms and utility $B > 0$ from undertaking new additional projects. Furthermore, we assume such concerns dwarf others for empire builders, and therefore cannot be overridden with incentive

⁹ Assuming instead that the market only learns about the quality of projects after they are undertaken makes no difference.

¹⁰ It is important to emphasize that type is public information; this is not a model driven by asymmetric information. Rather, different types are considered to show how dynamic considerations induce debt structure that depends on the availability of good projects. Thus the outcome for any type manager does not depend on the existence of other types (except insofar as a potential replacement manager defines the firm's outside option). For a discussion of this outside option, interpretations of type in this model, and why firms may have low type managers in the first place, see footnote 13 below.

contracts.¹¹ Inefficient managers can only be replaced by a takeover or upon bankruptcy. Entrenchment is taken to restrict takeovers to cases where they will increase firm value by at least 1.¹² If removed, the replacement manager is assumed to have no access to any new investments, either good or bad.¹³

¹¹ Alternatively, we can take outcome as noncontractable. Firm securities may only have a limited ability to affect managers' incentives if managers can undo such compensation through market transactions. The extent to which managers will want to undo such compensation will depend on a number of factors, including risk aversion and the informativeness of the market. Instead of modeling such issues here, we take such compensation either to be infeasible or inconsequential relative to empire building benefits.

¹² See footnotes 5 and 6 above for a defense of these assumptions. The amount of entrenchment is chosen to be on scale with the inefficiencies of one bad project. Changing this amount together with a renaming of types leads to similar implications for the debt structure. Since we make no assumption on the distribution of types, this is without loss of generality.

¹³ This assumption simplifies the analysis. One can consider the new manager as an outsider who has the ability to oversee the firm's current operations but not initiate any new operations. Results are identical if the new manager were to have ability $t=1/2$. Drawing a new manager from a distribution of types would lead to similar results, albeit, with a more complicated analysis.

The question may arise as to how firms ever get stuck with managers worse than potential replacements. We give the following stories to justify such an occurrence. First, interpreting t as characterizing a firm's investment opportunities rather than managerial ability, consider changes in t over a firm's lifetime. Suppose there exist both empire building managers and bureaucratic managers; the latter being defined as those who get disutility from additional projects. This managerial "ambition" is public information. Further suppose that initially, when a firm starts up, there are a lot of good investment projects (corresponding to $t > 1/2$) and therefore firms hire empire builders. Over time, outside opportunities may change, and t may fall to less than $1/2$, upon which the best outside option is a bureaucrat, who will refrain from new investments. However, without sufficient future inefficiency, the empire building manager is entrenched. This interpretation is consistent with familiar anecdotal stories on firm lifecycles, in which initially, when there exist many good growth opportunities, a daring entrepreneurial manager is best for a firm, and later, a more cautious bureaucrat is superior.

Similarly, suppose t characterizes managerial type, which is

We assume the following outcome of bankruptcy proceedings. Debt and equity agree to an efficient reorganization; meaning, in the present context, an efficient managerial retention decision is made. Debt is converted to equity at such a rate that the efficiency gains due to reorganization are split evenly between the shareholders and creditors.¹⁴ After reorganization, the firm continues operations as before.

Timing in each period follows the timeline in Diagram 1 below. At the beginning of each period, the manager makes financial decisions; whether to issue or repurchase debt, and what dividends to pay. Debt is priced by the market at the competitive (0 interest) rate. Next, the market for corporate control operates; the firm undergoes a takeover if otherwise there will be

initially unknown (to the manager and the market). Ambition (i.e. empire builder or bureaucrat) is known. When the firm first hires a manager, there are enough good industry investment opportunities that a randomly drawn empire builder is superior to a bureaucratic manager. Over time, through observation of projects available, the manager's type is learned. Provided that at this time the industry's environment has changed so that a bureaucrat is superior to a randomly drawn empire builder, the setting will be consistent with the model. Firms will have entrenched empire builders with known types, some superior to and some inferior to the best outside option of a bureaucrat.

In the setting of both these stories, an entrepreneur would like to design a charter so that institutional entrenchment depended on t . In particular, a firm would like its manager's entrenchment to lessen in the first story when the firm's outside opportunities worsen, and in the second story, when bad information is learned about a manager's type. However these variables are likely to be hard to specify in a corporate charter.

¹⁴ The particular specification of how the surplus is split is inconsequential to what follows; we choose an even split only for concreteness. This particular division is only relevant when outcome is stochastic and bankruptcy occurs in equilibrium. The important content of the reorganization process is the efficient retention decision.

sufficient inefficiency.¹⁵ The availability of a good project is then learned, after which the manager decides whether to undertake a single new project - a good one if available, otherwise a bad one. The firm's returns are then realized, and debt is serviced. Provided all obligations to debt are met, the period ends. If not, the firm goes into bankruptcy, and is reorganized as described above. We consider three such periods of this game. After the final period, all retained earnings are paid out, and the firm value is 0.¹⁶

Manager Undertakes Capital Structure Decisions	Market for Corporate Control	Availability of Good Project Learned	Investment Decision	Returns Realized and Debt Serviced	Bankruptcy Proceedings in the Event of Default
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Diagram 1
Timeline for each Period

It will be useful to define the following notation. Let D_i represent the firm's debt obligation in period i , and $w(D_i)$ the

¹⁵ The rationale for placing capital structure decisions prior to the market for corporate control is to capture the notion that managers under takeover threat have the opportunity to adjust capital structure as a potential defense. In practice, increasing leverage is one of the most common responses to control challenges.

¹⁶ The possibility that managers can also borrow between the investment decision and when payment to creditors is due is discussed at the end of this section.

market's valuation of this debt. d_i represents dividends paid out at time i , L_i and \bar{L}_i retained earnings in period i respectively before and after capital structure decisions are made. Thus, if a firm only issues debt in the period it is due, $\bar{L}_i = L_i + w_i(D_i) - d_i$. We define $\bar{D}_i = D_i - \bar{L}_i$, and will refer to this as "net debt"; net debt gives the amount of money that will have to be raised through investments in a given period to avoid bankruptcy. $v(t, \bar{L}_i, D^i)$ and $\bar{v}_i(\bar{L}_i, D^i)$ represent the value of equity in period i after the market for corporate control has operated, for a firm with capital structure (\bar{L}_i, D^i) , where $D^i = (D_j)_{j \geq i}$, for respectively a firm with a manager of type t , and with its manager replaced. Finally, let $V_i(t, \bar{L}_i, D^i) = v_i(t, \bar{L}_i, D^i) - \bar{v}_i(\bar{L}_i, D^i)$; the value of manager t to a firm relative to a replacement manager, for the given capital structure.¹⁷

We consider the actions of managers $0 \leq t \leq 1$ when their type is known publicly. Initially, we only consider the issuance of short term debt - that is, debt due at the end of the period it is issued. We will show below that this is without loss of generality; any outcome managers can attain through general debt can likewise be achieved through a sequence of short term debt. We solve for subgame perfect equilibrium play, as usual, working backwards.

¹⁷ The dependence of the value of debt w on the current period, on t , and on D^i is notationally suppressed. In equilibrium, given perfect foresight and a deterministic environment, this value will be constant over the lifetime of the debt. We will also employ the shorthand $v_i(t)$, \bar{v}_i , and $V_i(t)$ for $v_i(t, \bar{L}_i, D^i)$, $\bar{v}_i(\bar{L}_i, D^i)$, and $V_i(t, \bar{L}_i, D^i)$ when capital structure is clear.

First, suppose we have reached the beginning of period 3, and manager t is still in control. Since this is the final period, the manager has nothing to lose by undertaking the investment regardless of whether it is good or bad. Period 3 debt therefore serves no purpose, thus $D_3=0$. The value of equity will be given by retained earnings plus expected third period earnings. Since with probability t the manager will have a good third period project yielding a net return of 1, with probability $(1-t)$ a bad project yielding a net return -1 , and assets in place earn y , expected third period earnings are given by $y + (2t-1)$. The value of equity when the manager remains in control is therefore $v_3(t, \bar{L}_3, 0) = \bar{L}_3 + y + (2t-1)$. If instead the manager is removed in Period 3 by a takeover, since the replacement does not undertake new projects, the value of equity is $\bar{v}_3(\bar{L}_3, 0) = \bar{L}_3 + y$. And because the inefficiency of any manager is less than 1 (for all t , $V_3 = v_3 - \bar{v}_3 = 2t-1 \geq -1$), no manager will be removed by a third period takeover.

Now consider the second period. First suppose the firm is bankrupt at the end of this period. Third period play implies a replacement manager is superior to managers of type $t < 1/2$, since $V_3 = 2t-1 < 0$ for $t < 1/2$. Therefore managers will be replaced in bankruptcy if $t < 1/2$.

Now we consider the investment decision and the market for control in this second period. First suppose that $D_2 = 0$, therefore precluding second period bankruptcy. In this case, all managers will invest in all second period projects, since we have shown there will be no third period takeover. Thus, if the manager is not removed in Period 2, there will be two periods of

unconstrained managerial investment, and therefore equity value will be given by $v_2(t, \bar{L}_2, 0, 0) = \bar{L}_2 + 2y - 2(1-2t)$. If instead the manager is removed, equity will be valued at $\bar{v}_2(t, \bar{L}_2, 0, 0) = \bar{L}_2 + 2y$. A takeover occurs provided that $V_2 = v_2 - \bar{v}_2 = -2(1-2t) < -1$; that is, whenever $t < 1/4$. If a manager is of type $t \geq 1/4$, no takeover will occur, and therefore there is no reason to issue debt.¹⁸

While in the absence of debt managers $t < 1/4$ will therefore face a takeover, we will show they can prevent a second period takeover by issuing debt which credibly commits them to refrain from bad projects in the second period. This can be accomplished by choosing dividend and debt policy so that the firm will go bankrupt if and only if the manager undertakes a bad project in Period 2. This is achieved by setting capital structure so that $y-1 < \bar{D}_2 \leq y$. One simple manner to do such is by issuing debt of $y-1 < D_2 \leq y$, and paying out all proceeds from this issue and all retained earnings as a dividend, so that $d_2 = w(D_2) + L_2$ and therefore $\bar{L}_2 = 0$.

¹⁸ At this time, managers of type $1/4 \leq t < 1/2$ are strictly adverse to issuing debt (they would be removed in the event of a Period 2 bankruptcy and therefore debt would restrict them from undertaking bad second period projects that they are able to undertake in the absence of debt), while managers of type $t \geq 1/2$ are indifferent (they are retained even under Period 2 bankruptcy). We assume that when an empire building manager is indifferent between issuing debt or not, the manager refrains from doing so. This could be formalized in several simple manners. First, provided the bankruptcy process leads the manager to incur some small cost even if retained, say, from the hassle of the proceedings, such managers would never choose debt. Alternatively, if there was some very small chance that the market would misperceive the manager's type, and accidentally remove the manager in the event of bankruptcy when it was inefficient to do so, once again indifferent managers would never issue debt.

Given $y-1 < \bar{D}_2 \leq y$, the manager will refrain from investing in Period 2 if only bad projects are available. To see this, recall that upon second period bankruptcy the manager will be removed, and therefore undertaking the bad project would lead to benefits of $A+B$ from period 2 and 0 in period 3. If instead the manager chooses to refrain from investing in a bad project, the firm would not go bankrupt, and as seen above, the manager would not be removed by takeover in the third period. Hence the manager would be free in the third period to undertake any investment, and therefore the manager's continuation payoff is $2A + B$. Thus, the manager of type $t < 1/4$, having chosen capital structure which leads to bankruptcy if and only if a bad project is undertaken, would refrain from investing in bad second period projects. Good projects would still be undertaken when available, as they will not lead to default.

Finally, we show that this commitment deters a second period takeover. First, note that since the manager refrains from bad projects, the firm never defaults on this debt, and therefore $w(D_2) = D_2$. Now under such a capital structure, if there is no takeover, the manager only invests in good projects in the second period, and invests in any third period project. Consequently, the value of equity, given by second and third period earnings less net debt is $v_2(t, \bar{L}_2, D_2, 0) = 2y + (3t-1)\bar{L}_2 - w(D_2) = 2y + (3t-1)\bar{D}_2$. If instead the manager is removed, no new projects are undertaken, and the value of equity is instead $\bar{v}_2(\bar{L}_2, D_2, 0) = 2y + \bar{L}_2 - w(D_2) = 2y - \bar{D}_2$. Since $V_2(t) = v_2(t) - \bar{v}_2 = 3t-1 \geq -1$, no takeover occurs at the beginning of period 2.

Furthermore, such financing decisions are optimal for manager $t < 1/4$. If the manager could credibly commit to any investment strategy, The least costly manner for such a manager to avoid a Period 2 takeover would be to refrain from bad investments for one period only, while undertaking all other new investments. This is precisely what such debt achieves.

Intuitively, managers of type $t < 1/4$ avoid takeovers by using debt to credibly commit to not undertaking bad Period 2 investments. Given that such debt implies there will only be one period of inefficiency, no takeover will occur at the beginning of Period 2. When instead $t \geq 1/4$, expected inefficiency from the manager undertaking all projects for both periods is less than 1, and therefore no takeover can occur. Hence, there is no reason for these latter type managers to constrain themselves from undertaking bad Period 2 projects.

The above argument is summarized in the following proposition.

Proposition 1 - Starting from Period 2, the unique¹⁹ subgame perfect continuation equilibrium is as follows.

All managers of type $t \geq 1/4$ undertake no debt. In both the second and third periods they invest in all possible projects. There is no takeover and (trivially) no bankruptcy.

¹⁹ It is to be understood that all outcomes with the same economic consequences (i.e. investment and retention decisions) are identified with the same equilibrium. Thus, equilibria with different choices of net debt in the range $y-1 < \bar{D}_2 \leq y$ for $t < 1/4$, all which have the same effect, are considered the same. Also, we assume that managers choose to issue no debt when indifferent, for reasons given in footnote 18 above.

Managers of type $t < 1/4$ undertake Period 2 net debt (debt net retained earnings) \bar{D}_2 such that, $y-1 < \bar{D}_2 \leq y$. One simple manner of achieving this is by issuing debt D_2 such that $y-1 < D_2 \leq y$, and distributing all retained earnings and proceeds from debt. These managers only undertake good projects in Period 2, turning down bad projects. In equilibrium no bankruptcy occurs, and debt is therefore priced by the market at its face value. In the third period, these managers issue no new debt and undertake all investment opportunities.

In the off the equilibrium path event of second period bankruptcy, a manager is replaced if $t < 1/2$.

In equilibrium, the value of manager t to a firm relative to a replacement at the beginning of period 2 is given by,

$$V_2(t, \bar{L}_2, D_2(t), 0) = \begin{cases} 4t-2 & t \geq 1/4 \\ 3t-1 & t < 1/4 \end{cases}, \quad (1)$$

where $D_2(t)$ is equilibrium debt specified above.

Now consider the first period. If manager t is still in place when Period 2 is reached, Proposition 1 gives equilibrium play. Equation (1) indicates that at this time, a replacement manager will be superior to a manager of type $t < 1/2$. Such managers will therefore be removed in reorganization if there is a bankruptcy at the end of Period 1. If instead $t \geq 1/2$, the manager will be of greater value than a replacement, and will therefore not be removed under bankruptcy.

Now first consider capital structure so that $\bar{D}_1 < y-1$; that is, either no debt is due in Period 1, or retained earnings are large enough to pay off debt even if the bad investment is undertaken. In this case, the value of the manager relative to a replacement is given by $V_2(t)$ of equation (1) plus one period of

uninhibited investment (in Period 1), since the manager cannot be removed by bankruptcy in Period 1. Thus,

$$V_1(t, \bar{L}_1, D_1, D_2, 0) = \begin{cases} 6t-3 & t \geq 1/4 \\ 5t-2 & t < 1/4 \end{cases} \quad (2)$$

The manager would be removed by a takeover in the first period if $V_1(t) < -1$, which occurs if $1/4 \leq t < 1/3$ or $t < 1/5$. For all other managers, there is no need to issue first period debt; even when they don't, $V_1(t) \geq -1$, and therefore they will not be replaced. Hence managers of type $1/5 \leq t < 1/4$ and $t > 1/4$ will not issue debt, will undertake all first period investments, and play according to Proposition 1 in subsequent periods.

Now consider the managers of type $1/4 \leq t < 1/3$ and $t < 1/5$. We show that by issuing net Period 1 debt that leads to bankruptcy if and only if a bad project is undertaken, they can avoid a takeover. In particular, suppose first period capital decisions are such that $y \geq \bar{D}_1 > y-1$. One simple manner to achieve this is by issuing debt $y \geq D_1 > y-1$, and paying out all proceeds of the issue and all retained earnings in a dividend. With such debt, these managers will choose to only undertake good first period projects, refraining from bad ones. This follows because undertaking a bad project leads to bankruptcy, and subsequent removal during reorganization since equation (1) indicates that $V_2(t) < 0$ for all such types. Instead, refraining from bad projects and only undertaking good ones upon availability ensures no first period bankruptcy, and there will be no subsequent second period takeover since $V_2(t) > -1$. Thus by setting capital structure so that only good projects will be undertaken in the

first period, the value of these managers relative to a replacement is given by equation (1) plus the value of one period in which only good projects are accepted; that is,

$$V_1(t, \bar{L}_1, D_1, D_2, 0) = \begin{cases} 5t-2 & 1/4 \leq t < 1/3 \\ 4t-1 & t < 1/5 \end{cases} \quad (3)$$

And since for $1/4 \leq t < 1/3$ and $t < 1/5$ it then follows that $V_1(t) > -1$, no takeover will occur in Period 1. Thus, issuing such debt prevents first period takeovers for managers of type $1/4 \leq t < 1/3$ and $t < 1/5$, at the minimal cost of restricting one period of bad investments. Since only good projects will be undertaken in Period 1, all debt will be paid off with certainty, and therefore $w_1(D_1) = D_1$.

These results, combined with Proposition 1, are summarized below.

Proposition 2 - The unique subgame perfect equilibrium is as follows:

For $t < 1/5$, the manager will issue net debt of $y-1 < \bar{D}_1 \leq y$ in both the first and second periods. In these two periods, the manager will only undertake good projects, turning down bad projects. In the third period the manager will invest in all projects. The manager will not be replaced and there will be no bankruptcy. In the off the equilibrium path event of bankruptcy, the manager would be replaced in the first and second period.

For $1/5 \leq t < 1/4$, the manager will issue net debt $y-1 < \bar{D}_2 \leq y$ in the second period only. In this second period, the manager will only undertake good projects, turning down bad projects. In the first and third period the manager will invest in all projects. There will not be a takeover and there will be no bankruptcy. In the off the equilibrium path event of bankruptcy, the manager would be replaced in both the first and the second

period.

For $1/4 \leq t < 1/3$, the manager will issue net debt $y-1 < \bar{D}_1 \leq y$ in the first period only. In this first period, the manager will only undertake good projects, turning down bad projects. In the second and third period the manager will invest in all projects. There will not be a takeover and there will be no bankruptcy. In the off the equilibrium path event of bankruptcy, the manager would be replaced in the first period and second period.

For $1/3 \leq t$, the manager will not issue debt in any period. The manager will undertake all projects in all periods. There will not be a takeover and there will be no bankruptcy. In the off the equilibrium path event of bankruptcy, the manager would be replaced in both the first and second periods if $t < 1/2$, and never if $t \geq 1/2$.

There is no bankruptcy in equilibrium, and therefore the market value of all equilibrium debt is given by its face value.

In equilibrium, the value of the manager to the firm at the beginning of Period 1 is given by,

$$V_1(t, L_1, D_1(t), D_2(t), 0) = \begin{cases} 6t - 3 & 1/3 \leq t \\ 5t - 2 & 1/5 \leq t < 1/3, \\ 3t - 1 & t < 1/5 \end{cases} \quad (4)$$

where $D_1(t)$ and $D_2(t)$ are equilibrium debt issues of managers as specified above.

Notice the somewhat surprising result whereby managers $1/4 \leq t < 1/3$ will undertake debt due in the first period, while lower types $1/5 \leq t < 1/4$ will not. The types in the latter range must have debt in Period 2 to avoid a takeover at that time, while the higher types cannot be removed at this point. Thus, by virtue of the fact they will only take good second period investments, the latter types cannot be constrained in the first period, whereas

the former are constrained by first period takeover threats since they cannot be prevented from undertaking bad second period projects.

Proposition 2 indicates that all possible timing combinations of debt may be seen. The worst types of managers, with type $t < 1/5$ will undertake both Period 1 and Period 2 debt. Types $1/5 \geq t > 1/4$ will undertake only Period 2 debt. Types $1/4 \geq t > 1/3$ will undertake only Period 1 debt, and finally, if $t \geq 1/3$, no debt at all is issued.

Observe that management cannot improve its outcome by issuing debt that is not due until a future period. Each type of manager in equilibrium constrains investment in bad projects for the minimal number of periods required to prevent a takeover. It does not matter when this debt is issued, but only when it is due; if not issued until the period necessary to deter a takeover, the manager will voluntarily issue it at that time. Thus, even if undertaking debt due in a future period credibly constrains a manager - and it only does so if it is set high enough that the manager can't affect an equity for debt swap later -, there will be no gain to the manager in doing so, as the manager must be constrained from at least as many bad projects along any equilibrium path as is in Proposition 2 to avert a takeover.

Thus, there is no gain to management from debt not due the period issued. However, note that there is also no loss to issuing debt due in a future period. All debt is accurately anticipated; it makes no difference when equilibrium debt is issued, it only matters when it is due. Thus, all equilibrium

debt can be issued in the first period.²⁰ We can therefore interpret Period one debt as short-term debt and Period two debt as long-term debt. Hence, the model naturally yields a theory of the term structure of corporate debt. This will be discussed further in the next section.

Now suppose that management has the ability to issue more debt between the investment decision and when payment on debt is due. Then no longer would net debt $y-1 < \bar{D} < y$ necessitate bankruptcy if a bad project is undertaken. In particular, the manager may be able to borrow against future earnings to avoid bankruptcy. But without being able to credibly commit to bankruptcy when a bad project is undertaken, a manager will not be able to avert imminent takeovers. However, this commitment can be immediately resuscitated by writing covenants in the initial debt that restrict future debt.²¹ Even if such covenants are not possible, it can be shown that there is a debt package the manager can initially issue that will satisfy requirements of Proposition 2 while containing enough long term debt that no one would be willing to loan more to the firm later (as a creditor junior to existing debt), thereby obtaining the optimal outcome for the manager.

Note that Proposition 2 implies that dividend policy and debt policy are crucially interlinked. Indeed, all capital structure

²⁰ If there was some fixed cost each time debt is issued, then the result would be strict; all debt will be issued in one period.

²¹ Note that these covenants must prevent all debt, including debt junior to existing debt, in order to ensure the manager will not be able to avoid bankruptcy by borrowing.

implications are for net debt; that is, debt net retained earnings. In order for debt to credibly restrict managers from undertaking bad projects, retained earnings must not be large enough to pay off debt if a bad investment is undertaken. Of course, managers voluntarily choose to both have debt and pay out earnings through dividends, because only in doing so can they avoid takeovers. This differs sharply from cash flow models of debt, in which managers would never voluntarily pay out dividends.

The extension of this model to any finite number of periods is obvious. It should be clear from the above analysis that results will be similar. For any possible term structure, there will be a range of types $T \subset [0,1]$ which will undertake such debt. Also, the greater the availability of new projects t , the fewer periods of debt will be necessary.

As will be discussed in Section 3, this story of debt is both consistent with many stylized facts on capital structure and the perception that while debt serves to constrain management, managers voluntarily chose debt. However, several important elements in a realistic portrayal of debt are absent from the above results. In the equilibrium above, bankruptcy never occurs, and all firms have the same debt levels per period of debt. Different debt levels and bankruptcy, however, are simply accommodated for by extending the model to an environment with stochastic outcomes to investment projects. This work in progress is discussed in the conclusion.

Section 3 - Discussion

Debt structure in this model arises as an optimal response of managers to concerns both for empire building and retaining control of their empire. Debt serves to credibly restrict managerial empire building ambitions at times where they would otherwise lead to a takeover. This is accomplished through the threat of bankruptcy, and the associated loss of managerial entrenchment. In effect, a manager effectively makes the following "speech" in issuing debt: "We all know that unchecked my empire building desires will lead to enough inefficiency to warrant a takeover. I wish I could just convince you that I will not be inefficient this period, because with one less period of inefficiency, a takeover will not be warranted. However, we all realize that because such inefficiency is a sunk cost, if I convince you not to attempt a takeover this period, I will be inefficient, as then future inefficiency will not be high enough to warrant a takeover. Thus, I will credibly commit to drop my entrenched status if inefficient this period, by issuing debt so that an inefficient choice leaves me at the mercy of the bankruptcy court. Once I so commit, it is no longer in your interest to attempt a takeover."

The better a manager's investment opportunities, the more the market will tolerate empire building. Managers of type $t \geq 1/2$ are not restricted at all by control concerns; they are better than a replacement even when undertaking all potential projects. Extending the model to many periods, managers of type $t < 1/2$ will

all undertake some debt; the higher the type, the fewer the number of periods for which they will issue debt. Additionally, allowing for several different degrees of bad projects (i.e., sometimes the manager has a moderately bad project available when a good one isn't, but sometimes only a very bad project is available), would lead to the result that lower types are restricted from more extreme bad projects than others. Thus our story yields an explanation for cross sectional variation in the term structure, frequency, and size of a firms' debt obligations.²²

Since managers with better investment opportunities need less debt to avert a takeover, the model also predicts that firms with lower debt are likely to be more profitable.²³ Interpreting t as characterizing firms' investment opportunities rather than managerial types, we can explain documented time variations in firms' capital structure as well. Firms in new rapidly expanding industries, for which many good new investments are likely to be available, should have less debt than other firms; conversely, firms in contracting industries should have a lot of debt. When the market is booming and more investment opportunities are presumably available, firms will decrease leverage; when the market is falling firms will increase leverage. Likewise, when a firm is prospering, provided this is indicative of the

²² Further implications for size of debt are obtained by considering stochastic outcomes to projects. This is discussed briefly in the conclusion.

²³ For evidence to this effect, see Titman and Wessels (1988) and Friend and Lang (1988).

availability of good marginal projects, leverage should decrease.²⁴

The model also yields an explanation as to why increased leverage is an effective takeover defense. If the acquisition or expansion of a foothold by a potential raider decreases managerial entrenchment, managers would respond by increasing leverage.²⁵ Likewise, other takeover activity, which either signals lower costs to takeovers or indicates that some fraction of a fixed takeover cost has already been expended, will also lead to increases in leverage. Accompanying such increases, the model predicts there will be a decrease in the number of bad projects undertaken. In practice, restructuring through the divestiture of unprofitable divisions typically goes hand in hand with a takeover defense of increasing leverage.

This model can also explain empirical findings on the market's reaction to debt and equity issues.²⁶ If unanticipated changes in capital structure are in response to changes in an external control threat, we should expect to see a firm's value increase with leverage, as this is indicative of the emergence of a potential raider who will tolerate less inefficiency. Conversely, equity issues indicate greater managerial

²⁴ For evidence that leverage decreases with a firm's growth opportunities, see Kim and Sorensen (1986) and Titman and Wessels (1988). Marsh (1982) and Asquith and Mullins (1986) find firms are much more likely to issue equity in rising markets and after experiencing abnormal price appreciation.

²⁵ Indeed, Friend and Land (1988) and Gonedes, Lang and Chikaonda (1988) have found that firms with more disperse shareholders typically have lower leverage.

²⁶ See for example Masulis (1983) and Asquith and Mullins (1986). Smith (1986) surveys this literature.

entrenchment, thereby leading to a decrease in firm value.

Thus, this model can simultaneously explain the market's positive reaction to increased leverage and the negative correlation between firm profitability and leverage. Additionally, it explains why leverage serves as a useful takeover defense and is generally associated with restructuring.

While some of these implications can be obtained from similar cash-flow explanations of capital structure, the models differ sharply in their implications for dividends. Since this model derives capital structure as the optimal dynamically consistent policy for an entrenched management rather than as the optimal ex ante policy for an entrepreneur constraining managers, it is able to make sense of empire building managers voluntarily choosing to pay dividends. Debt only serves its function for managers in this model if it is simultaneously coordinated with dividend policy. Managers, in order to avert challenges for control, voluntarily issue debt and pay dividends. In a deterministic setting these dividends can be implicitly promised in advance; there will be no incentive for a manager to ever renege on such a promise. Conversely, in a model where debt is set ex ante by an entrepreneur not present to constrain managers come investment time, managers should never voluntarily pay dividends. A strong prediction of our model is that firms with high debt levels also pay out a large fraction of their earnings (after servicing debt) in dividends. This differs sharply from cash flow implications, whereby high debt is indicative of a great need to restrict empire building, and therefore should be associated with no dividend

payments.

In yielding a motivation for dividend payments, this model may also shed some light on the celebrated dividend puzzle.²⁷ This puzzle addresses why most firms pay dividends despite well known tax disadvantages for individual investors. The model yields explanations both for why firms pay out dividends instead of retaining earnings (to credibly commit managers to refrain from bad projects) and why dividends are used to disgorge money rather than debt (too much debt will constrain managers excessively). However, it is silent on why firms choose to distribute earnings to shareholders through dividends instead of a share repurchase plan.²⁸

Section 4 - Conclusion

Agency theory models in which debt serves to constrain managers are among the most promising explanations of capital structure. However, if managers are free to make capital structure decisions, they will often be able to undo capital structure constraints imposed on them by an initial entrepreneur. Also, if debt serves to constrain managers' access to free cash, we should never expect to see managers voluntarily making dividend

²⁷ See Black (1976) for a clear statement of this puzzle.

²⁸ However, it is possible that if such repurchases were undertaken regularly, they would be ruled to be dividends for tax purposes.

payments. We instead derive a dynamically consistent theory of capital structure, by considering managerial optimality rather than shareholder optimality as the determinant of capital structure. Capital structure constrains managers in a manner which is optimal to them at the beginning of each period. A policy of dividend payments coordinated with debt follows as an implication of our model.

Debt in our model restricts managers due to the possibility of bankruptcy, which is undesirable for managers because it implies a loss of entrenchment. Nonetheless, managers find it useful to employ debt; debt serves as a voluntary self-constraint, which allows managers to avert control challenges. The model is seen to yield numerous implications consistent with the empirical evidence on capital structure, takeovers, and the market for control. The model also makes a start at a theory of security design, insofar as it explains the use of different security term structures.

While it may seem unnatural that no bankruptcy occurs in equilibrium, this is no longer true when stochastic project outcomes are considered. Current work in progress finds results translate quite naturally to such an environment, with additional and more precise implications for debt level. Managers still use debt to restrict themselves from undertaking bad projects when a takeover is otherwise imminent; however, when debt is issued, good projects (which are still always undertaken when available in equilibrium) can lead to bankruptcy with a bad draw of nature. Thus managers set debt which minimizes the probability of such

bankruptcy subject to debt being sufficient to credibly restrict bad projects. This yields precise implication for net debt rather than the ranges in the propositions of this paper. The amount of debt needed to credibly restrict bad projects depends on forgone control benefits in bankruptcy, which in turn rely both on the amount of future self-constraint that will be necessary (as in the deterministic case) and additionally on the likelihood of future bankruptcy. Such work appears to be a promising avenue for additional empirical implications.

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