A BANDWIDTH-ADAPTIVE AUTOMATIC
PULSE POSITION TRACKER

by

CURTIS ALLEN SHIVELY

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Certified by ____________________________

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Submitted to the Department of Electrical Engineering on May 23, 1969, in partial fulfillment of the requirements for the degree of Bachelor of Science and for the degree of Master of Science.

ABSTRACT

This thesis describes the development and testing of an algorithm for adapting the bandwidth of a second-order passive sonar tracking servo to changes of its input signal and noise characteristics. This bandwidth-adaption algorithm is used to select values of bandwidth which optimize tracker performance by minimizing a mean square error cost function. This error cost function is chosen to reflect the deterioration in tracker performance due to tracker output signal error and tracker output noise variance. A procedure is developed for estimating from the tracker error signal and input noise variance the optimum bandwidth for tracking a constant acceleration input signal corrupted by stationary, wide-band noise. A controller is then designed to implement this bandwidth selection procedure so as to automatically adapt the tracker bandwidth to unpredictable changes of its input signal acceleration and input noise variance.

The performance of simulated bandwidth-adaptive and non-adaptive trackers is compared in terms of a time average root mean square error cost function. For a low frequency sinusoidal input signal corrupted by stationary noise, the bandwidth-adaptive tracker performs as well as a nonadaptive tracker having optimum constant bandwidth.

The bandwidth-adaption algorithm is used by a digital computer controller to improve the real time performance of a correlogram peak tracker responding to a sinusoidal input signal corrupted by nonstationary noise in a passive sonar system. In this application the bandwidth-adaptive tracker provides improved tracking capability under a wide variety of tracker input conditions, thus reducing the human effort required to monitor the tracker operation.

Thesis Supervisor: George C. Newton, Jr.
Title: Professor of Electrical Engineering
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I wish to dedicate this thesis to my late mother, who suddenly passed away while the thesis was in preparation.
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CHAPTER 1
Development and Testing of a Tracking Servo
Bandwidth-Adaption Algorithm

1.1 Introduction

Control theory may often be successfully applied to the problem of selecting the optimum gains in a tracking servo such as the one shown in Figure 9. This tracking servo is used in a passive sonar system to follow an input positional signal corrupted by wide-band noise. If the tracker input signal and noise may be sufficiently characterized a priori, classical control theory provides a straightforward solution to the problem of choosing optimum values for the tracking gains. This solution usually amounts to selecting the filter bandwidth of the tracker so that it passes sufficiently high frequencies of the input signal, yet blocks enough of the input noise to give tracking performance which is optimum in some sense.

However, if the tracker input signal and noise characteristics undergo a wide range of unpredictable variations in practice, it is sometimes not possible to select, a priori, a tracker bandwidth that is adequate under all input conditions. Even if the tracking servo gains or "tracking parameters" are chosen to be optimum for average or expected input properties, tracking performance may be intolerably poor when the input properties assume infrequently occurring extreme

* All figures and tables are included in Appendix G
values. Consequently, the use of classical control theory to select tracking gains which remain constant during the operation of the tracking servo may not provide optimum, or even adequate tracking capability.

Tracking performance which is optimum under a wide variety of input signal and noise conditions may be attained through the use of some relatively new concepts of input adaptive control. An input adaptive control system is one which has the capability to adjust itself to changes in the nature of its inputs, in such a manner as to maintain a desired level of performance under a wide variety of input conditions. In order to do this, the control system has to estimate some characteristics of its inputs, evaluate its own performance according to some prescribed criterion, and vary its own parameters of operation so as to maintain a nearly optimum level of performance.

A tracking servo with such input adaptive capabilities may be attained by using a computer to supervise the operation of a hardware tracker with variable tracking parameters, as shown in Figure 10. The computer supervisor or controller monitors characteristics of the tracker input signal and noise and sets the servo gains to the values it computes to be optimum for the observed input conditions. Hence, good tracker response for a wide range of inputs may be assured, if a versatile algorithm is used by the computer controller to evaluate the tracker performance and select the optimum values for the tracking parameters.
This scheme of input adaptive control might be used to improve the performance of a digital correlogram peak tracker used in passive sonar systems such as those being developed at the U.S. Naval Ordnance Laboratory (N.O.L.). In Chapter 2 it is shown that this tracker operates as a second-order servo following an input signal in the presence of wide-band noise. At present, the tracking servo gains, or tracking parameters, remain constant or are manually varied while the tracker is operating under a wide variety of input signal and noise conditions. As a result, tracking performance may be far from optimum if the tracker operator does not closely monitor changes in input conditions and vary the tracking parameters accordingly.

Pryor of N.O.L. has suggested\(^1\) that the performance of this correlogram peak tracker could be improved by automatically adapting its tracking parameters or filter bandwidth to the nature of the input correlogram peak being tracked. He proposed that the tracker bandwidth be automatically increased to reduce the tracker error when tracking rapidly varying input signals, and decreased to reduce the tracker noise output when tracking slowly varying input signals. Pryor also proposed that the digital tracker be interfaced with a general purpose digital computer, which would then be used to supervise the input adaptive control of its tracking parameters.

1.2 **Purpose of the Thesis**

The purpose of this thesis is the development of a scheme
for input adaptive control of the bandwidth of a digital correlogram peak tracking servo. In this adaptive scheme a general purpose digital computer controller is used to evaluate the tracker performance and to vary the tracker bandwidth according to a stored program adaption algorithm. The development of this adaption algorithm required the modeling, the analysis, and the simulation of the operation of the servo to be controlled. The goal of the research was the programming of the adaption algorithm so that the actual real time performance of the bandwidth-adaptive tracker could be compared with the performance of a previously used nonadaptive correlogram peak tracker.

Although the research described in this thesis was directed towards improving the performance of a particular passive sonar tracker, the results of this work may be relevant to other applications as well. Hence, for greater generality, the development of the bandwidth-adaption algorithm in this chapter is divorced from its sonar context as much as possible. Chapter 4 describes the implementation of this algorithm for the particular sonar system used to demonstrate its effectiveness in improving real time tracker performance. In Chapter 2, models are developed for the operation of the sonar tracker, and restrictions are placed upon the nature of the bandwidth-adaption algorithm by the sonar application. We describe these models and restrictions in a more general context here before developing the algorithm based upon them.
1.3 Models Assumed to Describe Tracking Process

A sampled-data servo model for a rate compensating correlogram peak tracker is shown in Figure 7. This model contains ideal samplers because the tracking servo effectively samples its input and changes its estimate of the input positional signal once every $T$ seconds. Each successive tracker output depends only on the signs of previous sample values of the tracker error signal, as indicated by the presence of the clipper in the model. The gains $\Delta$ and $\Theta$ in the model determine how changes in the tracker input are related to the clipped error signal $e_c(t)$, and hence are referred to as the tracking parameters.

The tracker input $x(t)$ consists of the sum of two independent components, the input positional signal $s_1(t)$ and an input noise $n_1(t)$. The input signal is assumed to be of low frequency in relation to the sampling frequency $1/T$, and in relation to the natural frequency of the tracking servo. The input noise is assumed to be a sample function from a non-stationary gaussian random process with zero mean and finite variance $C_1^2(t)$, which changes very slowly compared to the sampling frequency. However, it is assumed that this noise has significant frequency components extending over a wide range, including frequencies high in relation to those in the input signal. Thus the input noise has a sufficiently wide bandwidth to be effectively white noise in comparison to the input signal.
Since the tracker input signal and noise are independent, and since the clipper may be approximated by a linear gain, the tracker output may be expressed as the sum of independent signal and noise components, $s_o(t)$ and $n_o(t)$ respectively. The output noise $n_o(t)$ is assumed to be approximately gaussian with zero mean and variance $C_o^2(t)$ which depends on the tracking parameters and the variance of the input noise. This dependence is derived in Appendix C from a probabilistic analysis of the tracker operation. The output signal $s_o(t)$ represents the time varying mean of the tracker output resulting from the tracker response to the input signal $s_i(t)$. If $s_i(t)$ varies slowly relative to $n_i(t)$, $s_o(t)$ may be expressed as the output of a linear tracking servo responding to $s_i(t)$.

In this linear signal response tracker model, the clipper has been replaced by a gain $G$ which depends on the variance of the tracker input noise, but not on the input signal, as shown in Figure 8.

The analysis of the tracker signal response may be further simplified by reducing this linear, sampled-data model to an analogous continuous-time model. If the input signal frequency is much lower than the sampling frequency, the samplers and delay may be removed from the sampled-data model and the gains in the integrator modified to yield an equivalent continuous-time model.

The resulting model for the analysis of the tracker signal response is shown in Figure 9. The equivalence of this model and the sampled-data model from which it is derived may be noted by comparing the results of
the analyses in Appendices A and B. These two models have the same steady state error for constant acceleration and low frequency sinusoidal input signals, and the same impulse response within a factor of T. However, the continuous-time signal response model in Figure 9 is easier to deal with, both analytically and conceptually, and will therefore be referred to as the tracker signal response model in the development that follows.

1.4 Development of a Bandwidth-Adaption Algorithm
1.4.1 Constraints on Bandwidth-Adaption

The scheme considered in this thesis for using a digital computer to supervise the input adaptive control of tracker bandwidth is shown in Figure 10. In this scheme the computer controller monitors the tracker operation and selects optimum values for the tracking parameters according to a stored program bandwidth-adaption algorithm. The nature of this algorithm depends to a great extent upon the amount of information about the tracker inputs that is available to the controller. In the sonar tracker application an estimate of the tracker input noise variance \( C_1^2(t) \) may be readily obtained by the controller (see section 2.5). However, the tracker input signal \( s_1(t) \) is not available to the controller. Only from the tracker output \( y(t) \), the clipped error signal \( e_c(t) \), and a knowledge of the tracker dynamics can the controller obtain information about the nature of the tracker input signal. Hence, the bandwidth-adaption algorithm must be based upon
the results of previous analysis of tracker operation, in
order for the controller to use the limited information that
is available to it.

The bandwidth-adaptation algorithm must also be based upon
a criterion by which the quality of tracker performance can
be measured. A useful performance criterion is an error cost
function which reflects the deterioration in tracker perform-
ance resulting from tracker output signal error and tracker
noise output. For optimum tracking performance, the control-
ler selects the values of the tracking parameters which mini-
mize the error cost function under the observed input signal
and noise conditions. In the sonar application it is desir-
able that this error cost function assign a cost to the tracker
error in proportion to the square of the difference between
the tracker input and output signals (see section 2.5). We
therefore investigate the use of a mean square error cost
function for evaluating tracker performance.

1.4.2 Mean Square Error Cost Function

If we choose as the error cost function the expected
value of the square of the difference between the tracker
input signal \(s_1(t)\) and the tracker output \(y(t)=s_0(t)+n_o(t)\),
the resulting error cost function is

\[
E((s_1(t)-y(t))^2) = (s_1(t)-s_o(t))^2 - 2(s_1(t)-s_o(t))E(n_o(t))
+ E(n_o^2(t)) \tag{1-1}
\]

Assuming that \(E(n_o(t))=0\), the error cost function becomes
\[ E((s_1(t) - y(t))^2) = e^2(t) + C_o^2(t) \] (1-2)

where
\[ e(t) = s_1(t) - s_0(t) \quad C_o^2(t) = E(n_o^2(t)) \] (1-3)

From equation (1-1) we see that the use of a mean square error cost function equates optimum tracker performance with minimizing the sum of the tracker output noise variance and the square of the tracker error signal.

1.4.3 Optimum Bandwidth for Stationary Input Noise and Constant Input Signal Acceleration

The noise component \( C_o^2(t) \) of the error cost function depends on the tracking parameters, and also varies with time if the input noise is nonstationary. If, however, we may consider the tracker input noise to be stationary with constant variance \( C_i^2(t) = C_i^2 \), then as shown in Appendix C the tracker output noise variance \( C_o^2(t) \) may be expressed as

\[ C_o^2(t) = C_o^2 = N_q C_i^2 f_b \] (1-4)

where \( f_b \) is the half power bandwidth (in Hertz) of the tracker signal response model, and where \( N_q \) is a function only of the tracker damping ratio \( q \) and the tracker sampling period \( T \).

Thus for stationary input noise, fixed sampling period, and constant damping ratio, the tracker output noise variance \( C_o^2 \) is directly proportional to the tracker bandwidth \( f_b \).

The signal component \( e^2(t) \) of the error cost function depends upon the input signal \( s_1(t) \), and in general varies with time. Since the tracker builds up an estimate of the
input signal velocity in the integrator with gain $\Theta$, $e(t)$ approaches a steady state value of zero for input signals with constant velocity (see section 2.4). However, suppose that the tracker input signal is

$$s_i(t) = \frac{At^2}{2}; \quad t \geq 0$$

$$= 0; \quad t < 0$$  \hspace{1cm} (1-5)

corresponding to an input position moving with constant acceleration $A$. Then in the steady state, $e(t)$ approaches a constant value $e_s$ which from Appendix A is given by

$$e(t) = e_s = \frac{qA}{\gamma^2}$$  \hspace{1cm} (1-6)

where $B_q$ is a function only of the tracker damping ratio $q$. Thus for a constant acceleration input signal and constant damping ratio, the tracker error approaches a steady state value which is inversely proportional to the square of the tracker bandwidth.

Hence, if the tracker input noise is stationary with variance $C_i^2$, and if the tracker input signal is given by equation (1-6), the mean square error cost function becomes

$$e^2(t) + C_0^2(t) = \frac{B_q A^2}{f_b^4} + N_q C_i^2 f_b$$  \hspace{1cm} (1-7)

which is minimized when

$$f_b = f_{bo} = \left[ \frac{4B_q A^2}{N_q C_i^2} \right]^{1/5}$$  \hspace{1cm} (1-8)

Thus for the mean square error cost function (equation (1-1)), the optimum tracker bandwidth $f_{bo}$ for steady state tracking
of an input position moving with constant acceleration $A$ in
the presence of wide-band noise with variance $C_i^2$ is given by
equation (1-8) above.

If the bandwidth-adaption controller can obtain values
for $B_q$, $N_q$, $A$, and $C_i^2$, it can compute the optimum tracking
bandwidth using equation (1-8). Presumably it is desirable
to maintain the tracker damping ratio $q$ approximately con-
stant at some preselected value which gives a good compromise
between tracker transient response decay time and overshoot.
Thus from the preselected damping ratio, the controller can
compute values for $B_q$ and $N_q$. However, if $A$ and $C_i^2$ are not
known a priori, the controller can only estimate the optimum
value for the tracking bandwidth before the operation of the
tracker is begun.

1.4.4 Estimating Optimum Tracker Bandwidth

If this initial choice of bandwidth is sufficient to
keep the tracker from losing track during the initial error
transient, the controller can adjust the tracker bandwidth to
a more optimum value after the tracker error has stabilized.
A measure of $C_i^2$ is assumed to be available to the controller
during tracker operation. We also assume here (and later show)
that the controller can estimate $e(t)$ from the clipped error
signal $e_c(t)$. Therefore, when the tracker error has stabi-
lized to a value $e_s$, a value for $A$ can be computed using
equation (1-6) written as

$$A = \frac{e_{sf}^2}{B_q}$$  (1-9)
where $f_{bi}$ is the initial tracker bandwidth. Substituting equation (1-9) into equation (1-8) given

$$f_{bo} = \left[ \frac{4e_{s}^{2}f_{bi}^{4}}{NqC_{i}^{2}} \right]^{1/5}$$ (1-10)

Thus after the tracker error resulting from a constant acceleration input signal has stabilized, the controller can estimate the optimum bandwidth from the initial bandwidth $f_{bi}$, the tracker error $e_{s}$, and the input noise variance $C_{i}^{2}$ using equation (1-10). The controller can then adapt the tracker bandwidth by resetting the tracking parameters to values that give the tracker the estimated optimum bandwidth.

Just how optimum these new tracking parameters will be depends on the quality of the estimates of $e_{s}$ and $C_{i}^{2}$ from which the parameter values are computed. Perhaps after resetting the tracker bandwidth and allowing the tracker error to stabilize again, the controller should recompute the optimum bandwidth from the new error and the bandwidth being used. Then if this new bandwidth estimate differs much from the previous one, the controller should again adapt the tracker bandwidth, allow the error to stabilize, and estimate the optimum bandwidth. Presumably if the controller continued to repeat this bandwidth estimation and adaption cycle at sufficiently large intervals of time, the estimates of optimum tracking bandwidth would become better and would stabilize near the truly optimum value as desired.

The interval between bandwidth adaption and estimation
must be long enough to permit the newly set bandwidth to become effective, i.e. to affect the values of the tracker error signal. An adequate period for this bandwidth adaption cycle would be somewhat longer than the effective duration of the tracker transient response. If this duration is less than D seconds, the controller can compute successive estimates of optimum bandwidth at intervals of D seconds using

\[ f_{bo}(JD) = \left( \frac{4e^2(jD)f_{bo}4((j-1)D)}{NqC_i^2(jD)} \right)^{1/5} \]  

(1-11)

where \(e(jD)\) and \(C_i^2(jD)\) are estimates of the tracker error signal and input noise variance respectively at time \(t=jD\), and where \(f_{bo}((j-1)D)\) is the bandwidth computed and set at time \(t=(j-1)D\).

1.4.5 Algorithm for Adapting Tracker Bandwidth to Changes of Tracker Input Characteristics

The development of the above recursive relation for computing optimum bandwidth was based upon the assumption that the tracker input signal acceleration and noise variance were constant. However, in practice these tracker input characteristics may vary with time. Suppose that the tracker input signal acceleration and input noise variance vary with time as \(B(t)\) and \(C_i^2(t)\) respectively. Consider the case where \(B(t)\) and \(C_i^2(t)\) undergo discrete changes, infrequently enough, that after changing value each remains constant for at least several adaption periods. Under these input conditions, the controller can successfully adapt the tracker bandwidth to each
new value of $B(t)$ or $C_{12}(t)$ by repeatedly using equation (1-11) as though the change of input characteristic were to remain in effect indefinitely. Moreover, if $B(t)$ and $C_{12}(t)$ vary continuously with time, but change very little during each adaption period, we suspect that the optimum bandwidth computed by the controller from equation (1-11) will follow with a slight delay the changes of the tracker input signal acceleration and input noise variance. Thus the variable bandwidth tracker and the bandwidth-adaption controller would compromise a bandwidth-adaptive tracker having the capability to adjust its bandwidth to maintain optimum tracking performance under a wide variety of input signal and noise conditions.

In view of the above discussion we propose the following algorithm to be used by the bandwidth-adaption controller:

1. Obtain estimate of tracker input noise variance $C_{12}(t)$.
2. Estimate tracker error $e(t)$ from the clipped tracker error signal $e_c(t)$.
3. Estimate optimum tracker bandwidth $f_{bo}$ by using the present bandwidth and estimates of $e(t)$ and $C_{12}(t)$ in equation (1-11).
4. Adjust tracker bandwidth toward newly estimated $f_{bo}$ by selecting corresponding values for tracking parameters.
5. After a delay of $D$ seconds, repeat cycle beginning with step 1.

The adaption period $D$ should be longer than the effective duration of the tracker transient signal response, but short enough to allow the adapted bandwidth to follow changes of
the tracker input conditions as closely as desired.

1.4.6 Bandwidth-Adaption Controller

A block diagram of the controller used to adapt the bandwidth of the rate compensating tracker according to the above algorithm is given in Figure 11. This block diagram shows the controller passing $e_c(t)$ through an exponential filter in order to obtain an estimate of $e(t)$. Pryor of N.O.L. has shown\(^2\) that if the rate compensating tracker error is small compared to the tracker input noise, the expected value of $e_c(t)$ is $G(t)e(t)$, where the gain $G(t)$ is a function of the tracker input noise variance $C_i^2(t)$. Thus, the controller can obtain an estimate $\hat{e}(t)$ of $e(t)$ by filtering $e_c(t)$ to recover its average value $e_f(t)$, and by scaling the result inversely according to the value of $G(t)$ (see Figure 11).

The controller also passes the estimate of optimum bandwidth $f_{bo}(t)$ through an exponential filter, after using equation (1-11) to compute it from $C_i^2(t)$, $e(t)$, and the previous bandwidth $f_b(t-D)$. From the filtered bandwidth $f_b(t)$ and from $G(t)$ the controller computes the actual tracking parameters $\Delta(t)$ and $\Theta(t)$, using the relations given below Figure 11. The controller thus averages each new bandwidth estimate with previous estimates to obtain the smoothed value to which the bandwidth of the tracker is actually set. By so doing, the controller reduces the noise in the tracker output resulting from the adjustment of the tracker bandwidth, and provides a means for controlling the rate at which the tracker bandwidth
can be adapted.

An effectively exponential method of filtering $e_c(t)$ and $f_b(t)$ was used by the controller implemented during the thesis research because of the ease of doing so on a general purpose digital computer (see Appendix F). The optimum values for the time constants $1/L$ and $1/M$ in these filters depend on how rapidly the tracker input noise variance and input signal acceleration vary with time. Ideally, these time constants should be at least several adaptation periods long to provide adequate smoothing of the noise in $e_c(t)$ and $f_{b0}(t)$, yet short enough that the tracker bandwidth is properly adapted to sudden changes of the tracker input conditions. Hence these time constants $1/L$ and $1/M$ were considered to be adjustable parameters of controller operation, and their effect on the operation of the bandwidth-adaptive tracker was observed during the research (see section 1.5).

1.4.7 Model for Signal Response of Bandwidth-Adaptive Tracker

If we restrict the manner in which the tracker input conditions vary, we may develop a nonlinear model for the signal response of the bandwidth-adaptive tracker. First, assume that the input noise is stationary with variance $C_i^2(t) = C_i^2$. Then suppose that the input signal acceleration and resulting tracker error vary slowly enough that the lags in bandwidth adaption due to the filtering of $e_c(t)$ and $f_{b0}(t)$ can be neglected. If bandwidth adaption may thus be considered to be instantaneous, the tracker bandwidth $f_b(t)$ may be considered
to take on approximately the equilibrium values satisfying equation (1-11) and given by

$$f_b(t) = \left[ \frac{4\epsilon^2(t)f_b(t)}{N_q C_i^2} \right]^{1/5}$$

or

$$f_b(t) = \frac{4\epsilon^2(t)}{N_q C_i^2}$$

Thus, for stationary input noise and slowly varying input signal acceleration, the bandwidth of the adaptive tracker varies approximately in proportion to the square of the tracker error.

If the tracker bandwidth is given by equation (1-12), the tracking parameters $\Delta(t)$ and $\Theta(t)$ selected by the controller may also be considered to be functions of the tracker error signal $e(t)$. As shown in Appendix D, $\Delta(t)$ and $\Theta(t)$ vary approximately in proportion to the square and to the fourth power, respectively, of the tracker error. Therefore, we may modify the tracker signal response model in Figure 9 to include the effects of bandwidth adaption by replacing the tracking parameters $\Delta$ and $\Theta$ with nonlinear operations on $e(t)$. The resulting nonlinear model for the signal response of the bandwidth-adaptive tracker for stationary input noise and slowly varying input signal acceleration is given at the end of Appendix D and in Figure 12. The results of a computer simulation of the operation of this model responding to a sinusoidal input are discussed in section 3.3.
1.5 **A Comparison of the Performance of Simulated Bandwidth-Adaptive and Nonadaptive Trackers**

1.5.1 Introduction

After the tracker bandwidth-adaption algorithm was developed, tests were conducted to determine its usefulness. Two types of computer simulation, as well as the real time adaptive operation of a hardware tracker under computer control, were used to compare the performance of the bandwidth-adaptive tracker and the nonadaptive rate compensating tracker. The real time operation of the trackers in a passive sonar system is discussed briefly in section 1.6 and in more detail in Chapter 4. The results of a computer simulation of the operation of the adaptive tracker signal response model developed in section 1.4.7 are discussed in section 3.3. We discuss here the choice of a tracker test input signal, and present the results of computer simulations of the complete bandwidth-adaptive tracker and a rate compensating tracker.

1.5.2 Sinusoidal Test Input Signal

In the computer simulations a low frequency sinusoidal test input signal was used to compare the performance of the adaptive and nonadaptive trackers. This particular choice of tracker input signal was made for several reasons. First, sonar data recorded in the field and available for processing on the actual passive sonar system produced an approximately sinusoidal movement of the correlogram peak to be tracked, as indicated by Figure 22. Therefore, it was desirable to discover how well the adaptive tracker would respond to an exactly
sinusoidal input signal under ideal, simulated conditions for comparison with the real time performance of the adaptive sonar tracking system.

Second, as shown in Appendix A for the nonadaptive rate compensating tracker, the error resulting from a low frequency sinusoidal input signal is approximately proportional to the second derivative of the input signal, the input signal acceleration. Therefore, it was believed that a sinusoidal input signal would be useful for determining how well the controller could adapt the tracker bandwidth by estimating variations of the input signal acceleration from the clipped error signal, as proposed in section 1.4.4.

A third reason for using a sinusoidal test input signal was that tracker performance could be measured in terms of a time average square error cost function equivalent to the ensemble average square error cost function that was used in section 1.4.3 to evaluate tracker response to constant input accelerations. For stationary input noise with variance $C_1^2$ and input signal $s_1(t) = A_1 \sin (W_1 t)$, the time average square error of the rate compensating tracker is shown in Appendix A to be given by

$$\frac{(s_1(t) - y(t))^2}{2f_b} = \frac{B q^2 A_1^2 W_1^4}{2f_b} + N q C_1^2 f_b$$

and the corresponding optimum tracker bandwidth is
\[ f_{bo} = \left[ \frac{2Bq_i^2A_i^2w_i^4}{Nq_i^2} \right]^{1/5} \]  \hspace{1cm} (1-15)

A comparison of equation (1-15) with equation (1-8) reveals that the square of constant input signal acceleration \(A_i^2\) in equation (1-8) has been replaced in equation (1-15) by the time average square acceleration \(A_i^2w_i^4/2\). Thus, for the constant bandwidth tracker, the optimum bandwidth and mean square error for constant input signal acceleration are the same as the optimum bandwidth and time average square error for a sinusoidal input signal having the same average square acceleration. Therefore, it was desirable to determine if a smaller time average square error could be obtained by adapting the tracker bandwidth directly to the acceleration of the input sinusoid, than by maintaining the tracker bandwidth optimally constant during each period of the input signal.

1.5.3 Simulated Trackers and Inputs

The operation of the nonlinear, sampled-data rate compensating tracker model in Figure 7 was simulated on the DISAC general purpose digital computer at the U.S. Naval Ordnance Laboratory, using the programs described in Appendix E. Provision was made for varying the tracker bandwidth while keeping the tracker damping ratio constant. In this simulation the square of the tracker damping ratio was chosen to be equal to "one half", because this value seemed to provide a compromise between desirable qualities of the tracker transient.
response and the variance of the tracker output noise, as discussed in sections 3.1 and 3.2.

The adaption of the tracker bandwidth by the controller described in section 1.4.6 was also simulated on the computer, thus permitting a comparison of bandwidth-adaptive and non-adaptive tracking performance. The adaption period D of the simulated controller was made equal to the tracker decision period T for programming simplicity and because this was nearly the case in the actual sonar tracker implementation (see section 4.4). Consequently, the rate at which the tracker bandwidth could be adapted was completely dependent upon the values of the controller filter time constants, and provision was made for varying these time constants as parameters of the adaptive tracker operation. A more detailed description of the techniques and programs used in simulating the adaptive tracker is given in Appendix E.

The input sampling rate 1/T of the simulated trackers was about 38 samples per second, giving a corresponding decision period T of about the same length (26 milliseconds) as that of the sonar tracker. The tracker input noise samples were a pseudo-random sequence of gaussian distributed numbers having nearly zero mean and known variance. The tracker input signal samples were chosen to correspond to a sinusoid of the same amplitude, 7.0 milliseconds compensating delay time, and frequency, 1/120 Hertz, as the signal provided by recorded data for tracking by the sonar tracker. Three values of the tracker input signal-to-noise ratio were simulated, to
correspond to the sonar tracker attempting to track a correlogram peak having a maximum value of either 50%, 25%, or 12.5% correlation.

1.5.4 Performance of Simulated Trackers

The error cost function used to measure tracking performance was the ratio of the square root of the time average square tracker error to the input signal amplitude. Optimum tracker performance was obtained by experimentally minimizing the error cost function, as a function of the tracker bandwidth for the rate compensating tracker, and as a function of the controller time constants, \(1/L\) and \(1/M\), for the bandwidth-adaptive tracker. Figure 13 shows experimental values of this error cost function versus tracker bandwidth for the rate compensating tracker, and versus average tracker bandwidth for the adaptive tracker. Also shown in Figure 13 are theoretical values of the error cost for the rate compensating tracker, computed by using equation (A3) of Appendix A. Table 1 compares the theoretical and experimentally observed optimum performance of the nonadaptive rate compensating tracker with the optimum performance of the simulated bandwidth-adaptive tracker.

For the rate compensating tracker the theoretical and experimental minima of the error cost function agree closely, even though the search for optimum tracker bandwidth was made by varying the tracker bandwidth in rather coarse increments of about 0.14 Hertz near the theoretically optimum values. However, for all three input signal-to-noise ratios simulated,
the experimental minima of the error cost function are larger than the theoretical ones. This uniform discrepancy probably results from the output noise component of the tracker error being larger than theoretically predicted in the range of bandwidths involved. Even so, the close agreement of these minima indicates that the results of the simulation are quite valid.

It is interesting to note from Table 1 that for each input S/N the optimum controller parameters provided an average tracker bandwidth nearly equal to the optimum bandwidth for the rate compensating tracker. For each input S/N the search for optimum controller parameters was conducted by varying the filter time constants, 1/L and 1/M, by factors of two until the error cost function appeared to be near its minimum. In the neighborhood of such a minimum, the average tracker bandwidth was observed to vary inversely as the value of 1/L, and the variance of the tracker bandwidth was observed to vary inversely as the value of 1/M. Moreover, the minimum error cost appeared to be nearly obtained by the combination of controller parameters which gave an average tracker bandwidth close to that of the optimum rate compensating tracker, and which also minimized the bandwidth variance. For the optimum controller parameters, the variance of the tracker bandwidth was small enough that the adaptive tracker was practically operating as a rate compensating tracker with optimum bandwidth.

In light of the above observation it is not surprising that the minimum values of the error cost function for the bandwidth-adaptive tracker agree very closely with those for the rate
compensating tracker. This would seem to indicate that, for a sinusoidal input signal and stationary input noise, no smaller an RMS tracker error can be attained by adapting the tracker bandwidth directly to the input signal acceleration than could be attained by maintaining the tracker bandwidth optimally constant during each period of the input signal. However, the adaptive tracker was not supplied with a priori information about the amplitude and frequency of the input sinusoid, as would be required for the selection of optimum bandwidth for constant bandwidth tracking. Moreover, the algorithm proposed in section 1.4.5 for automatically adapting the tracker bandwidth directly to unpredictable changes of input signal acceleration appears, in the case of sinusoidal input signal, to give tracking performance as good as the best that could possibly result from an a priori selection of constant tracker bandwidth for the rate compensating tracker.

In this simulation the performance of adaptive and non-adaptive trackers was compared for only a single frequency of the input signal. However, since the optimum adaptive tracker appears to operate as a rate compensating tracker with optimum bandwidth, it is reasonable to suspect that the optimum performance of the two trackers will be nearly the same over the range of frequencies for which the theoretical performance of the rate compensating tracker is valid. As long as the input signal frequency is very small compared to the average tracker bandwidth, and as long as the controller time constants are properly chosen, the bandwidth-adaptive tracker will probably
respond to sinusoidal input signals just as well, if not better than, the optimum rate compensating tracker.

1.6 Advantages Resulting from the Use of Bandwidth-Adaptive Tracking in a Passive Sonar System

To accomplish the real goal of the research, the bandwidth-adaptive tracking technique developed in section 1.4 was applied to a passive sonar tracker. The DISAC general purpose digital computer at the U.S. Naval Ordnance Laboratory was programmed to adapt the bandwidth of a digital correlogram peak tracker according to the bandwidth-adaption algorithm discussed in section 1.4.5. Chapter 4 describes in more detail the programming and operation of the computer controller, and the results of using the adaptive tracker to process recorded sonar data. We briefly discuss here the nature of the tracker inputs and some of the general advantages of using the adaptive tracker in a sonar application.

The recorded sonar data provided a tracker input signal consisting of several cycles of nearly sinusoidal correlogram peak motion corrupted by nonstationary noise with variance that underwent a variety of rapid and slow variations (see Figure 22). In addition to the desired correlogram peak, the recorded data produced an unwanted correlogram peak moving with opposite phase, resulting in an interfering signal sometimes stronger that the signal desired to be tracked (see section 4.4 and Figure 23b). Thus, the recorded sonar data provided a good test of the ability of the adaptive tracker to changes of input noise variance, as well as to changes of input
signal acceleration.

For comparison with the performance of the adaptive tracker, attempts were first made to track this recorded data with a nonadaptive rate compensating tracker having damping ratio $q=\sqrt{1/2}$. The same recorded data was processed many times, each time with different values for the tracking parameters. However, the presence of the interfering input signal and the changes of the input noise variance made it difficult (if not impossible) to find constant values for the tracker parameters that would permit the tracker to maintain track during several cycles of the input signal as desired. Since the damping and bandwidth of the rate compensating tracker depend on its input noise variance (see Appendix A), widely different values of the tracking parameters were needed to maintain constant tracker damping and to provide the proper tracking bandwidth during different sections of the recorded input data.

A bandwidth-adaptive sonar tracker was successfully used to track this same recorded data. As in the computer simulation, the adaption period of this adaptive tracker was about the same length as the tracker decision period, and provision was made for controlling the rate of tracker bandwidth adaption by selecting values for the controller filter time constants. It was soon discovered that the values of these controller parameters determine the sensitivity of the adaptive tracker to changes of its input conditions, and that the best tracker performance results when the time constants are selected to match the adaption sensitivity of the tracker to the rate of input
condition variation to be followed. Thus, by observing whether the tracker adaptation sensitivity was too great or too small when the tracker had difficulty tracking, it was possible to rather quickly arrive at values for the controller parameters that resulted in the tracker maintaining track during all of the recorded data desired to be tracked.

The main advantages of using bandwidth-adaptive tracking in a passive sonar system seem to be an improvement in tracker performance and a reduction in the human effort required to monitor the tracker operation. Since the bandwidth-adaption controller automatically performs the task of continuously selecting often crucial values of the tracking parameters, the tracker operator need only occasionally change the controller filter time constants to maintain good tracking performance under a wide variety of tracker input conditions. Moreover, this selection of controller parameters can be based on a subjective evaluation of whether the tracker adaption sensitivity is too great or too small for a given tracking situation.

1.7 Suggestions for Further Development of Bandwidth-Adaptive Tracking Technique

The next logical step in the development of this bandwidth-adaptive tracker would be to give the controller the capability to automatically vary its rate of bandwidth adaption. In the presently implemented adaptive tracker, the bandwidth-adaption period is fixed, and the rate of bandwidth adaption and the bandwidth-adaption sensitivity are varied by manually changing the controller filter time constants. The controller
itself could automatically select values for its parameters of operation by monitoring the rate of change of the estimated tracker error. When the tracker error is nearly constant, the controller could increase its filter time constants to decrease the bandwidth adaptation sensitivity and thus reduce the tracker output noise due to bandwidth fluctuations. In response to a sudden increase in the magnitude of the tracker error, the controller would decrease its filter time constants to be more sensitive to a suspected variation of tracker input condition. Further investigation is needed to determine if the added complexity of such a method of automatic variation of adaptation rate would be offset by a sufficient improvement in adaptive tracking capability.
CHAPTER 2
Passive Sonar Tracking Servos

2.1 Passive Sonar Systems

Researchers at the U.S. Naval Ordnance Laboratory have been developing passive sonar systems for tracking the position of a source of underwater acoustical noise. These sonar systems use arrays of pairs of hydrophones to detect acoustic noise emitted from the target. The time difference or delay between the arrival of the target noise at each of the two hydrophones in a pair defines a delay time for that pair of hydrophones. The value of delay time for each hydrophone pair determines a three dimensional target locus relative to that hydrophone pair (see Figure 1). For sonar tracking, the loci relative to several hydrophone pairs are combined with a knowledge of the array geometry to yield an estimate of the target depth, range, and bearing.

The delay time for a pair of hydrophones is determined by sampling and crosscorrelating the outputs of the two hydrophones. The maximum or peak of the resulting crosscorrelation function occurs for a value of delay time which exactly compensates for the delay between the target noise received at the two hydrophones. This compensating delay also determines the location of the peak in a display of the crosscorrelation function, or correlogram. Therefore, we will refer to this compensating delay as the position of the peak of the correlogram, or as the correlogram peak position.
Movements of the target or the hydrophone array cause the compensating delay to vary and thus make necessary the continual tracking of the correlogram peak position. Several types of automatic digital correlogram peak trackers have been developed at the U.S. Naval Ordnance Laboratory.\textsuperscript{2,3,9} These trackers basically operate as servo mechanisms trying to follow an input positional signal in the presence of an interfering noise. These trackers may also be modeled as filters attempting to recover an input signal corrupted by additive noise. A more detailed description of the operation of two of these correlogram peak trackers is given below.

2.2 Tracker Decision Process

Two automatic digital correlogram peak trackers used in the passive sonar system described above are the primitive step tracker\textsuperscript{2} and the rate compensating tracker.\textsuperscript{3} Both of these trackers track a correlogram peak by attempting to keep a tracking "gate" centered on it. The position of this tracking gate is the tracker output representing the tracker estimate of the correlogram peak position. Figure 2 shows the tracking gate displaced slightly from the center of a typical correlogram peak. When the tracking gate is not centered on the peak, a comparison of the correlogram areas under the two gate halves indicates to which side of the gate center the correlogram peak lies (see Figure 2). Once every T seconds the tracking logic compares the correlogram areas under the gate halves and makes a tracking "decision" to move the gate
in the direction apparently necessary to center it on the correlogram peak.

A model for this tracker decision making process is given in Figure 4. In this model \( y(t) \) corresponds to the position of the center of the tracking gate and is thus the tracker estimate of the position of the correlogram peak. Corresponding to the tracker decision is a clipped error signal \( e_C(t) \), which indicates only the sign of the difference between the input \( x(t) \) and the tracker output \( y(t) \). Since tracker decisions are made at intervals of \( T \) seconds, \( e_C(t) \) is passed through an ideal sampler in the model. The input-output relation for an ideal sampler is given in Figure 3.

In the tracker decision model, the input \( x(t) \) is expressed as the sum of the actual correlogram peak positional signal \( s_i(t) \) and an input noise \( n_i(t) \), which is independent of \( s_i(t) \). The noise term is present because the sonar system hydrophones receive uncorrelated background noise in addition to the correlated noise emitted from the target. When the outputs of a pair of hydrophones are crosscorrelated, the background noise in each output causes random fluctuations in the delay time for which the crosscorrelation maximum due to the correlated target noise occurs. The background noise thus causes the position of the correlogram peak to fluctuate randomly about an average value of delay time, \( s_i(t) \), truly related to the target locus. It has been shown\(^4\) that this correlogram peak positional noise may be modeled as wide-band gaussian noise having a standard deviation inversely related to the actual
maximum value of the crosscorrelation function, and hence to the ratio of target noise power to background noise power at the hydrophones.

2.3 **Primitive Step Tracker**

The effect of the tracking decision on the estimate of the correlogram peak position depends on the type of tracker being used. In the primitive step tracker the position estimate is changed periodically by a fixed, discrete step according to the sign of the clipped error signal \( e_c(t) \). The magnitude of this step is determined by a tracking parameter \( \Delta \) known as the position step size. Since \( \Delta \) is independent of the magnitude of the actual error in the tracker position estimate, this simple tracker will be referred to as the primitive step tracker or as the primitive tracker.

The operation of the primitive step tracker may be described by a formula relating successive tracker position estimates resulting from tracker decisions made at intervals of \( T \) seconds. Let \( X_n \) be the tracker input at time \( t=nT \) and let \( Y_n \) be the tracker position estimate, the tracker output, at time \( t=nT \). Then the input and output samples of the primitive step tracker are related by

\[
Y_{n+1} = Y_n + \Delta E_n
\]

(2-1)

where

\[
E_n = \frac{|X_n - Y_n|}{|X_n - Y_n|}
\]

(2-2)

and \( \Delta \) is the position step size. Every \( T \) seconds the primitive step tracker corrects its position estimate by a fixed
step according to the error of the previous estimate.

The operation of the primitive step tracker is analogous to the operation of a servo mechanism attempting to follow an input positional signal corrupted by noise and sampled at intervals of T seconds. From the above input-output relation it is easily seen that the analogous sampled-data servo is one in which the error signal e(t) is clipped, sampled, delayed one sample period T, and then passed through an integrator with gain $\Delta$. A nonlinear, sampled-data servo model for the primitive step tracker is given in Figure 5.

The output $y(t)$ of the primitive step tracker model is expressed as the sum of two terms, $s_o(t)$ and $n_o(t)$. The term $n_o(t)$ represents the tracker output noise resulting from the tracker response to the zero mean input noise $n_1(t)$. The term $s_o(t)$ represents the time varying mean of the tracker output, the output signal resulting from the tracker response to the input signal $s_1(t)$. It has been shown$^2$ that the clipper in the nonlinear model may be approximated by a simple gain, for analysis of the tracker signal response, if the input signal $s_1(t)$ is slowly varying compared to the input noise. The gain $G$ approximating the clipper is a function of the variance of the input noise, but does not depend on the input signal itself. For the present, we assume that the linear approximation for the clipper is valid and take the tracker output signal $s_o(t)$ to be the response of the linear primitive step tracker model in Figure 6. The validity of this model for the primitive step tracker signal dynamics
was verified during the research (see section 3.2).

The primitive step tracker provides adequate tracking for some input signals, but it has several deficiencies that may greatly reduce its effectiveness. Since the gain $G$ in the linear signal model depends on the variance of the input noise, the signal dynamics of the primitive step tracker depend on the maximum value of the correlogram peak it is trying to track. Another deficiency of the primitive step tracker is that it cannot follow a constant input velocity without a steady state error or lag proportional to the input velocity. It can easily be shown that this velocity lag can be decreased by increasing the position step size $\Delta$. However, as Pryor of N.O.L. has demonstrated, the variance of the tracker output noise increases in proportion to the position step size $\Delta$. It is possible to choose a position step size which provides a compromise between tracker output noise and velocity lag. However, such a choice may result in an unnecessarily large tracker output noise variance when the correlogram peak is moving very little.

2.4 Rate Compensating Tracker

To overcome the velocity error deficiency of the primitive step tracker, researchers at the U.S. Naval Ordnance Laboratory have developed an extension of the primitive step tracker known as the rate compensating tracker. This rate compensating tracker builds up an estimate of the rate of motion of the correlogram peak by recording the algebraic sum
of the tracking decisions in a "rate counter." After each tracking decision, a number of additional steps are added to the position estimate in proportion to the rate estimate which has been accumulated in the rate counter. The proportionality constant is given by a tracking parameter known as the rate step size. If the tracker lags a correlogram peak moving with a non-zero rate, an excess of tracking decisions occurs in the direction of the input rate, and causes a corresponding increase in the rate estimate. The rate estimate thus increases until the rate compensation has centered the tracking gate on the correlogram peak by matching its rate of motion. Then, opposite tracking decisions occur with equal probability, and the rate estimate stabilizes at a value proportional to the velocity of the correlogram peak.

The operation of the rate compensating tracker may also be described by a formula relating successive tracker position estimates occurring at intervals of $T$ seconds. If $X_n$ and $Y_n$ are the tracker input and output respectively at time $t=nT$, then for the rate compensating tracker

$$Y_{n+1} = Y_n + \Delta E_n + \Theta R_n T$$

(2-3)

where

$$E_n = \frac{X_n - Y_n}{|X_n - Y_n|} \quad R_n = \sum_{j=0}^{n} E_j$$

(2-4)

and where $\Delta$ is the position step size, $\Theta$ is the rate step size, and $T$ is the interval between tracking decisions. The quantity $R_n$ is the number in the rate counter at time $t=nT$, and $\Theta R_n$ is the actual rate estimate in delay units per second.
The quantity $\Theta_R^n T$ then is the change in the position estimate occurring between the nth and n+1st tracking decisions due to the rate estimate $\Theta_R^n$. This change in tracker position estimate occurs in addition to the simple decision step of magnitude $\Delta$.

The rate compensating tracker may also be modeled as a nonlinear, sampled-data control system. Pryor has shown\(^2\) that the rate compensation may be represented by the addition of a second integrator to the primitive step tracker model as indicated by the nonlinear rate compensating tracker model in Figure 7. The output of the rate compensating tracker may be expressed as the sum of an output noise $n_o(t)$ and an output signal $s_o(t)$ which are the tracker responses to the independent input noise $n_i(t)$ and input signal $s_i(t)$. As in the case of the primitive step tracker, we presently assume that for analysis of the tracker signal response the clipper in the nonlinear model may be approximated by a gain which depends on the input noise variance. The resulting linear model for the signal response of the rate compensating tracker in given in Figure 8. The adequacy of this model for describing the signal dynamics of the rate compensating tracker was verified during the research (see section 3.2).

The rate compensating tracker is a significant improvement of the basic primitive step tracker. Since the rate compensation closely matches the tracker output velocity to the input signal velocity, a large position step size $\Delta$ is not needed to reduce tracker lag for large input signal rates.
Thus the position step size $\Delta$ may be decreased somewhat to reduce the tracker output noise variance. However, the damping of this tracker is a function of the tracking parameters $\Delta$ and $\Theta$, and of the gain $G$ which approximates the clipper. Since the gain $G$ is a function of the variance of the input noise, the damping of the rate compensating tracker depends on the amount of noise added to the input signal it is attempting to track. Therefore, it would seem desirable to be able to estimate the tracker input noise variance and to select the tracking parameters $\Delta$ and $\Theta$ to provide the desired degree of tracker damping. That is, it is desirable to control the damping of the rate compensating tracker by adapting its tracking parameters to the amount of noise in the tracker input.

Even with controlled damping, the rate compensating tracker would still have difficulty tracking correlogram peaks moving with non-zero accelerations. It could not follow a constant input acceleration without an acceleration lag analogous to the velocity lag of the primitive step tracker. This acceleration lag could in principle be eliminated by adding to the rate compensating tracker an "acceleration counter" which builds up an estimate of the input signal acceleration and introduces a compensating acceleration in the tracker output. However, the stability of the resulting third-order servo would depend on the values of $G, \Delta, \Theta$, and the amount of acceleration compensation. Thus, input noise adaptive control of tracking parameters would be necessary just to maintain the stability of an acceleration compensating tracker.
Furthermore, the development of a scheme for the input adaptive control of tracking parameters requires the analysis of the signal and noise dynamics of the tracker to be controlled. For an acceleration compensating tracker this analysis would be more difficult and the resulting adaptive scheme probably more complicated than for the second-order rate compensating tracker. Therefore, we consider the possibility of using input adaptive control of tracking parameters to minimize the acceleration lag of the rate compensating tracker.

In Appendix A it is shown that the acceleration lag of the rate compensating tracker varies inversely as the square of the tracker bandwidth. In Appendix C it is shown that the output noise variance of the rate compensating tracker varies in proportion to the tracker bandwidth. Therefore, it would seem possible to select a tracker bandwidth which is optimum in some sense for tracking a given input signal acceleration in the presence of a known amount of input noise. This choice of optimum tracker bandwidth could be made by a bandwidth-adaption controller which somehow estimates characteristics of the tracker inputs and chooses corresponding optimum values for the tracking parameters Δ and Θ. Hence, we choose to investigate a scheme for using a bandwidth-adaption controller to adapt the parameters of the rate compensating tracker to its input signal acceleration and to its input noise variance.

2.5 Constraints on an Adaptive Tracking Scheme

Any scheme for automatically adapting the bandwidth of
the rate compensating tracker is constrained by the nature of existing digital systems which generate the crosscorrelation function and track the motion of its peak. The digital tracking logic periodically determines only the sign of the apparent tracker error, the clipped error signal \( e_c(t) \) in the tracker models. Consequently, the tracker input positional signal \( s_i(t) \) is not directly available to the bandwidth-adaptation controller. Hence, the controller must infer variations of the input signal acceleration from the clipped tracker error signal \( e_c(t) \) and from a knowledge of the tracker signal dynamics.

An estimate of the tracker input noise variance is more readily available to the controller. Since the variance of the tracker input noise depends on the maximum value of the correlogram peak being tracked, it may be estimated during the operation of the tracker from the value of the crosscorrelation at the compensating delay time corresponding to the tracker output position. This value of the correlation at the tracker estimate of the correlogram peak position may be simply converted by a digital computer controller into an estimate of the tracker input noise variance. However, the estimate thus obtained is meaningful only after the tracker has acquired the correlogram peak and is tracking it with relatively small error. Thus while the tracker is tracking well, an estimate of its input noise variance may be considered to be directly available to the bandwidth-adaptation controller.
Once the adaption controller has obtained estimates of the tracker input signal and noise characteristics, it must select the corresponding tracker bandwidth which is in some sense optimum. In the preceding section we suggested that the optimum tracker bandwidth would provide a compromise between tracker output noise variance and tracker acceleration lag. Thus the optimum tracking bandwidth could be chosen to minimize an error cost function or performance criterion which reflects the deterioration in tracker performance due to both the tracker signal error and output noise variance.

It is desirable that this performance criterion associate high relative cost with large tracker errors and lower relative cost with small tracker errors. If the tracker error becomes too large and the tracking gate becomes completely displaced from the correlogram peak, tracking decisions based upon the correlogram area under the gate are not related to the motion of the peak, and the tracker loses track of it. However, tracker errors which are small in relation to the width of the correlogram peak need not be considered as costly since the resulting slight inaccuracies in computations based upon the tracker output can usually be tolerated. Therefore, a useful tracking performance criterion might assign a cost to tracker errors in proportion to the square of the difference between the tracker input signal $s_1(t)$ and output signal $s_0(t)$.

A heavy penalty should also be associated with large noise fluctuations in the tracker output, since they increase the probability that the tracker will lose track of the peak.
Therefore, the tracker performance criterion might assign a cost to the tracker output noise in proportion to the average of the square of the random fluctuations that the noise adds to the tracker output signal. Then, an error cost function composed of a linear combination of the tracker output noise variance and the square of the tracker error signal would be a meaningful performance criterion whose minimization corresponds to optimum tracker performance. As shown in Chapter 1, the use of such a performance criterion also leads to a simple bandwidth-adaption algorithm which may be used by a bandwidth-adaption controller to select optimum tracking bandwidth on the basis of the limited information available to it in a passive sonar system.
CHAPTER 3
Simulations of Signal and Noise Responses of Bandwidth-Adaptive and Nonadaptive Trackers

3.1 Output Noise Variance of the Rate Compensating Tracker

In order to develop a tracker bandwidth-adaptation algorithm to minimize the mean square error cost function proposed in section 1.2.2, it was first necessary to derive and verify an expression relating the output noise variance of the rate compensating tracker to the tracker bandwidth $f_b$ and input noise variance $C_i^2$. In Appendix C it is shown that, for stationary gaussian input noise with variance $C_i^2$, the output noise variance $C_o^2$ of the nonlinear rate compensating tracker model (Figure 7) is given approximately by

$$C_o^2 = N_q f_b C_i^2$$

(3-1)

where, for a fixed tracker sampling period $T$, $N_q$ is a function only of the tracker damping ratio $q$, as defined by equation (C12) of Appendix C.

A plot of the ratio $N_q/T$ as a function of the square of the tracker damping ratio $q$ is given in Figure 18. From this plot it may be seen that, for a fixed sampling period $T$, $N_q$ begins to increase significantly with decreasing values of $q^2$ only after $q^2$ is made somewhat less than $1/2$. Thus, if equation (3-1) is valid, the tracker damping ratio $q$ may be decreased to a value in the neighborhood of $\sqrt{1/2}$ without significantly increasing the tracker noise output for given values
of $T$, $f_b$, and $C_1^2$.

In Appendix C it is also shown that an expression for $C_0^2$ which is equivalent to equation (3-1) is

$$C_0^2 = \frac{A}{8} C_1 \Delta \left[ 1 + \frac{1}{4q^2} \right]$$  \hspace{1cm} (3-2)

where $\Delta$ is the tracker position step size. The derivation of equations (3-2) and (3-1) in Appendix C is based upon the assumption that the tracker damping ratio $q$ is large compared to unity. Indeed, as $q$ becomes infinite in equation (3-2), the resulting tracker output variance becomes equal to that derived and verified by Pryor$^2$ for the primitive step tracker.

In order to determine the range of $q$ for which $C_0^2$ is approximately given by equations (3-1) and (3-2), the output variance of a simulated rate compensating tracker was measured experimentally. The operation of the nonlinear, sampled-data model of the rate compensating tracker (Figure 7) was simulated on a digital computer using the programs described in Appendix E. The tracker input samples were a pseudo-random sequence of nearly uncorrelated gaussian distributed numbers having nearly zero mean and known variance, thus simulating an input noise process whose bandwidth was large compared to the bandwidth of the tracker.

Figure 14 compares theoretical and experimental values of the normalized output variance $C_0^2/C_1^2$ plotted as a function of normalized position step size $\Delta/C_1^2$ for various values of the square of the tracker damping ratio $q$. For values of $q^2$
which are even as small as 1/4, the experimental dependence of the tracker output variance upon the step size $\Delta$ agrees closely with that theoretically predicted using equation (3-2). However, for values of $q^2$ somewhat smaller than 1/4, the experimental results seem to indicate that the tracker output noise variance begins to vary in proportion to $\Delta$ raised to some power slightly greater than unity. Note from Figure 14, that for $q^2 = 1/2$, the value selected for comparing the performance of adaptive and nonadaptive trackers, the tracker output noise variance may be approximated quite well using equation (3-2) or equation (3-1).

3.2 An "Impulse Response" for the Rate Compensating Tracker

In order to establish the validity of the linear model for the signal response of the rate compensating tracker given in Figure 8, the "impulse response" of a simulated rate compensating tracker was determined experimentally. A well known procedure for experimentally obtaining the impulse response of a continuous-time linear system is to form the crosscorrelation of the system input and output when the system input is uncorrelated noise having a bandwidth that is wide compared to the system bandwidth. The resulting crosscorrelation is proportional to the impulse response of the system.\(^5\)

In Appendix B a similar relationship is shown to hold between the sampled impulse response $h_s(nT)$ of the linear sampled-data model for the rate compensating tracker, and the sampled input-output crosscorrelation $R_{xy}(nT)$ resulting when
the tracker model input samples are a sequence of uncorrelated
 gaussian distributed numbers. In particular, if the variance
 of the input noise samples is $c_i^2$, then $R_{xy}(nT)$ is given by

$$R_{xy}(nT) = c_i^2 h_s(nT)$$

(3-3)

A theoretical expression for the corresponding input-output
crosscorrelation of the nonlinear rate compensating tracker
model (Figure 7) is not so readily obtained. However, since
the linear tracker model represents a first-order linear ap-
proximation to the nonlinear tracker model, we suspect that
the input-output crosscorrelation $R_{xy}(nT)$ of the nonlinear
tracker model responding to uncorrelated noise will be given
approximately by equation (3-3) above.

To determine an "impulse response" for the rate compens-
sating tracker the operation of the nonlinear tracker model
(Figure 7) was simulated on a digital computer using the pro-
grams described in Appendix E. The tracker input samples were
the same uncorrelated gaussian distributed numbers used to
determine the tracker output variance (see section 3.1). For
convenience and speed, only the polarity coincidence correla-
tion $R_{p_{xy}}(nT)$ between the tracker input and output samples
was computed. A theoretical expression for this crosscorrela-
tion for the linear rate compensating tracker model is given
by equation (81) of Appendix B.

Figures 15 and 16 compare theoretical and experimental
values of the crosscorrelation $R_{p_{xy}}(nT)$ for two rate compen-
sating trackers having nearly the same product of bandwidth
and decision period, but different damping ratios. Each experimental point in these figures represents the average of 400 sample values of the crosscorrelation between 300 input and output samples. In both cases of damping ratio, for values of \( n \) larger than about 15, the theoretical and experimental values of the crosscorrelation \( R_{p_{xy}}(nT) \) agree closely, indicating that the impulse response and transfer function of the linear tracker model are useful for characterizing the signal response of the rate compensating tracker. Note in particular the close agreement of the theoretical and experimental values of \( n \) at which the first zero crossing occurs for both values of \( q^2 \). Note also that, for the tracker with damping ratio \( q=1/2 \), the per cent overshoot is about twice that for the tracker with damping ratio \( q=\sqrt{1/2} \), while the times required for the two transients to decay to 10% of their initial values differ by only about 15%. Thus, it appears that a value of tracker damping ratio \( q \) which is approximately equal to \( \sqrt{1/2} \) will provide a good compromise between transient response decay time and overshoot for the rate compensating tracker.

3.3 Simulation of the Adaptive Tracker Signal Response Model

During the research a model was developed for the signal response of the bandwidth-adaptive tracker responding to slowly varying input signal accelerations in the presence of stationary input noise (see section 1.4.7 and Appendix D). The operation of this model was simulated using a computer
program to solve the corresponding nonlinear differential equation relating the adaptive tracker error to the tracker input signal. For this simulation the tracker input signal was chosen to correspond to a sinusoid of the same amplitude, 7.0 milliseconds, and frequency, 1/120 Hertz, as the input signal provided for a sonar tracker by recorded sonar data (see Figure 22). By appropriately selecting the gains $K_A$ and $K_B$ in the adaptive tracker model (Figure 12), three values of the tracker input signal-to-noise ratio were simulated, corresponding to maximum values of 50%, 25%, and 12.5% for the correlogram peak being tracked by a sonar tracker.

For each input signal-to-noise ratio, Figure 17 gives a plot of the tracker bandwidth and the resulting tracker error as a function of time. Note how the tracker errors rise sharply and then flatten off smoothly, and note also the nonlinear increase in peak values of tracker error for equal increments of tracker input signal-to-noise ratio. The results of this simulation are also summarized in Table 3, which compares the average and peak values of the bandwidth of this model with the average bandwidth used by the optimum bandwidth-adaptive tracker in the complete tracker simulation discussed in section 1.5. Note that the adaptive tracker signal model gives smaller average values of bandwidth even though the peak values of its bandwidth are somewhat larger than the values of average bandwidth for the simulated bandwidth-adaptive tracker. This probably results because the bandwidth of the adaptive tracker signal model varies as the square of the
tracker error, while the bandwidth of the simulated adaptive tracker appears to remain relatively constant for sinusoidal input signals (see section 1.5).

Table 3 also compares the ratio of the square root of the time average square tracker signal error to input signal amplitude. This comparison assumes that the time average square signal error $e^2(t)$ of the bandwidth-adaptive tracker is equal to one fifth of the total time average square tracker error, as is shown to be the case for the rate compensating tracker at the end of Appendix A. This comparison seems to indicate that, for sinusoidal input signals, the adaptive tracker signal response model gives a good estimate of the resulting time average square adaptive tracker error. However, the manner in which the bandwidth and error of the bandwidth-adaptive tracker vary with time, appears to differ somewhat from that given by the adaptive tracker signal response model and shown in Figure 17.
CHAPTER 4

The Use of Bandwidth-Adaptive Tracking
In a Passive Sonar System

4.1 The Sonar Data Processing System

The bandwidth-adaptive tracking technique developed in Chapter 1 was used to improve the performance of a digital correlogram peak tracker in a sonar data processing system at the U.S. Naval Ordnance Laboratory (N.O.L.). A block diagram of this data processing system is given in Figure 19. This diagram shows a correlation processing technique used to obtain the compensating delay time for a single pair of hydrophones in a passive sonar system (see figure 1). The hydrophone outputs are recorded on magnetic tape in the field, for later processing on this system at N.O.L.

After first being filtered, the hydrophone outputs pass into a DELTIC correlator, where they are clipped and sampled, and the polarity coincidence correlation between them, \( Q_{12}(\tau) \), is generated. This raw correlation function is fed to the tracker, which attempts to follow the movement of its peak by the method described in section 2.2. The raw correlation is also filtered by a Post-Integrator to give a smoothed correlogram for display on a CRT. The tracker output position is an estimate of the compensating delay time \( \tau_c \) corresponding to the maximum of the correlation function, and appears as a spike on the displayed correlogram (see Figure 23a). The DELTIC correlator, the tracker, the Post-Integrator, and the display CRT make up an Integrated Correlation Processing
System described in reference 4.

DISAC, a general purpose digital computer, is used to monitor the operation of the Integrated Correlation Processing System. The DISAC computer is interfaced with the tracker so as to be able to read the tracker output position, and to set the tracking parameters, the tracker rate, and the tracker position. DISAC is also interfaced with the Post-Integrator, from which it can obtain either the raw or the post-integrated value of the correlation corresponding to any delay time for which it is generated by the DELTIC correlator. During the research DISAC was programmed to control the adaption of the tracker bandwidth using the algorithm developed in section 1.4.

4.2 Computations Performed by the Computer Controller

In order to carry out the adaption of tracker bandwidth, the DISAC computer was programmed to select values for the tracking parameters in a manner analogous to that used by the controller described in section 1.4.6 and shown in Figure 11. However, the inputs to that controller, $e_c(t)$ and $C_1^2(t)$ are not directly available to the DISAC computer. Consequently, the computer controller computes values for the tracking parameters from different quantities in the slightly different manner indicated in Figure 20.

The inputs to the computer controller are sample values of $P$, the tracker position, and $C$, the value of the correlation function at the tracker position, obtained at intervals of $D$ seconds. From these inputs the controller computes
values for the tracker position step size $SS$ and the tracker rate step size $RSS$, the tracking parameters corresponding directly to the gains $\Delta$ and $\Theta$ in the tracker models. Since the tracker being controlled operates basically as a primitive step tracker with an amount of rate compensation that must be selected externally, the controller uses the filtered rate step size $FRSS$ to directly compute values for the tracker rate $R$. Therefore, the controller outputs fed to the tracker every $D$ seconds are values of the filtered rate $FR$ and the filtered position step size $FSS$.

In order to compute the tracker rate and parameters, the controller must obtain an estimate of the tracker error. Although the clipped error signal $e_c(t)$ is not directly available to it, the controller can estimate the tracker error from samples of the tracker position taken at intervals of $D$ seconds. From two successive tracker position samples and a knowledge of the tracking parameters and rate being used, the controller can deduce the difference between the number of decisions made by the tracker to increase or to decrease the position estimate during the interval between position samples (see Figure 20). This difference in tracker decisions $E$ has the same value as would the algebraic sum of samples of the clipped error signal $e_c(t)$ obtained at intervals of $T$ seconds during the same interval between position samples. Thus, if the tracker made $m$ decisions during the $D$ seconds between position samples, then $E/m$ was the average value of the clipped error signal during that interval. Therefore,
the filtered difference in tracker decisions FE obtained by the computer controller in Figure 20, corresponds to m times the filtered clipped error signal $e_f(t)$ obtained by the controller in Figure 11.

In order to estimate the actual tracker error $e(t)$ and the optimum bandwidth $f_{bo}(t)$ from FE and $G(C_i^2)$ as does the controller in Figure 11, the computer controller would have to obtain an estimate of the tracker input noise variance $C_i^2(t)$. It has been shown\(^4\) that for this correlation processing system the square root of the tracker input noise variance is inversely related to the maximum value of the correlation function. Therefore, while the tracker error is small, the controller could estimate $C_i^2(t)$ from $C$, the value of the correlation at the tracker position. However, instead of first estimating $C_i^2(t)$ and then computing $e(t)$ and $f_{bo}(t)$ from $C_i^2(t)$ and from FE, the computer controller computes the tracking parameters SS and RSS directly from FE, the filtered difference in tracker decisions, and from FC, the filtered correlation at the tracker position.

Before being used in the tracking process, the parameters SS and RSS are passed through "exponential" filters, as were the correlation C and difference in tracker decisions E from which the parameters were computed. The effectively exponential method of filtering that was used by the implemented controller to average these quantities is developed in Appendix F. The time constants of these exponential filters may be used to control the rate at which tracker bandwidth is
adapted and the adaption sensitivity of the adaptive tracker to transient changes of its input conditions, as discussed in section 4.3.

Once every D seconds the controller also computes the tracker rate R, an estimate of the amount of rate or velocity compensation needed to match the velocity of the tracker input signal. Each successive rate estimate is obtained by adding the product of the filtered rate step size FRSS and the difference in tracker decisions E to the previous rate, or delayed filtered rate DFR. This method of estimating the necessary tracker rate is analogous to the procedure used by the rate compensating tracker described in section 2.4. However, the computer controller averages each new rate estimate with previous estimates to obtain the exponentially filtered rate FR which is sent to the tracker.

4.3 Using the Computer Controller Programs

Two computer programs, TRAK and CTP, were written in DISAC assembly language\textsuperscript{8} to implement the computer controller described in the preceding section. A flow diagram of TRAK, the main adaptive tracking subroutine, is given in Figure 21. Each time TRAK is called, it samples the tracker position P and corresponding correlation C, computes the difference in tracker decisions E, and sets the values of the tracker position step size SS and rate R in the manner described in the preceding section. TRAK calls CTP to perform the actual computation of the tracking parameters SS and RSS from FC and FE.
Thus, TRAK and CTP constitute an adaptive tracking subroutine package which may be called by another data processing monitor program to cause the DISAC computer to supervise the input adaptive control of the bandwidth of a digital correlogram peak tracker.

The quality of bandwidth-adaptive tracking that results from the use of TRAK and CTP depends on the proper selection of values for the time constants in the exponential filters used to average quantities in TRAK. If the adaption period $D$ between calls to TRAK is roughly the same length as the tracker decision period $T$, the filter time constants in TRAK may be made somewhat longer than $D$, and thereby used to control the rate of tracker bandwidth adaption and the sensitivity of the adaptive tracker to changes of its input signal and noise characteristics.

The time constants TS and TRS controlling the filtering of the tracker step size SS and tracker rate step size RSS, respectively, may be selected to provide a compromise between the rate at which tracker bandwidth can be adapted and the amount of noise in the tracking parameters SS and RSS. Separate time constants, TC and TE, are used in TRAK to control the filtering of the correlation $C$ and the difference in tracker decisions $E$, from which the tracking parameters are computed. Since variations in $C$ correspond to changes of the tracker input noise variance, and since variations in $E$ correspond to changes of the tracker input signal acceleration, time constants TC and TE may be independently adjusted to
control the sensitivity of the bandwidth-adaptive tracker to changes of its input noise variance or input signal acceleration, respectively. The tracker rate $R$ is also filtered with time constant $T_R$, whose value influences the ability of the controller to adjust the tracker rate in response to input signal accelerations.

4.4 Results of Tracking Recorded Sonar Data

The adaptive tracking subroutines TRAK and CTP described in the preceding section were executed by the DISAC computer to determine if the bandwidth-adaptive tracking algorithm developed in Chapter 1 could be used to advantage in the processing of recorded sonar data by the system described in section 4.1. A monitor program was used to call the subroutine TRAK about sixteen times each second, making the tracker bandwidth adaptation period $D$ approximately 2.3 times the tracker decision period $T$ of about 27 milliseconds.

The tracker input signal provided by the recorded sonar data consisted of several cycles of approximately sinusoidal correlogram peak motion with amplitude 7.0 milliseconds compensating delay time and frequency $1/120$ Hertz (see Figure 22). The tracker input noise was nonstationary with variance that sometimes remained nearly constant for almost a full period of the input signal, and at other times changed by as much as a factor of four over a period of several seconds. In addition to the desired correlogram peak the recorded data produced an unwanted correlogram peak moving with opposite phase,
giving an interfering tracker input signal sometimes stronger than the signal desired to be tracked (see Figure 23b.)

The performance of a primitive step tracker (rate compensating tracker with infinite damping) and of a bandwidth-adaptive tracker responding to a portion of this recorded sonar data was recorded in photographs of the system output display. Figures 23a thru 23g show a time sequence of two nearly identical correlograms resulting from the separate crosscorrelation of the outputs of two hydrophone pairs. As the sequence begins in Figure 23a, the tracker output positions appear as spikes atop the desired correlogram peaks at the left, while the unwanted correlogram peaks are starting to appear at larger values of delay time to the right. Note the drastic change between Figure 32a and Figure 23b in the relative heights of desired and unwanted peaks, and thus in the relative signal-to-noise ratios of signals to be tracked and to be ignored.

As the desired correlogram peaks begin to move to the right, i.e. to occur for larger delay times, both the primitive step tracker (upper trace) and the bandwidth-adaptive tracker (lower trace) appear to be tracking well. However, when the desired and unwanted correlogram peaks cross in Figures 23d thru 23f, the primitive step tracker loses track of the desired peak and begins to track the stronger input signal corresponding to the position of the unwanted peak. In Figure 23g the primitive step tracker is tracking the unwanted peak, while the bandwidth-adaptive tracker is still
tracking the desired correlogram peak.

One reason for the success of the bandwidth-adaptive tracker was that, unlike the primitive step tracker, it had built up an estimate of the rate of motion of the desired peak, an estimate whose sign could not be reversed during the short time while the two peaks were crossing. Unsuccessful attempts were made to select values of tracking parameters for a rate compensating tracker that would keep its damping ratio constant and give it the ability to maintain track during the drastic change in its input noise variance that occurred even before the two peaks crossed. However, it was experimentally possible to "fine tune" the adaption sensitivity of the adaptive tracker so that it properly decreased its bandwidth in response to the drastic increase of its input noise variance, and yet retained too narrow a bandwidth to acquire the large but rapidly moving unwanted correlogram peak when the two correlogram peaks crossed.

A cycle of input signal accompanied by noise with approximately constant variance was selected for comparing the performance of trackers in terms of an error cost function. The error cost function used was the ratio of the square root of the average square estimated tracker error ($\hat{e}(t)$ in Figure 11) to the input signal amplitude. Table 2 compares the values of this error cost function for two types of adaptive trackers with that of a primitive step tracker and the theoretically optimum rate compensating tracker. It was also desired to evaluate the performance of a nonadaptive rate compensating
tracker with damping ratio \( q = \sqrt{1/2} \). However, the tracker input
noise variations and the interfering input signal made it
impossible to experimentally find values for the tracking
parameters that would keep the tracker properly damped and yet
provide the bandwidth necessary for tracking the desired cor-
relogram peak. The primitive step tracker also had some dif-
iculty tracking the desired input signal, and as a result it
gave the largest error cost of all trackers tested.

Smaller tracker error costs were attained by two types
of bandwidth-adaptive rate compensating trackers with damping
ratio \( q = \sqrt{1/2} \). A rate compensating tracker whose parameters
were adapted only to its input noise in order to maintain
constant damping and bandwidth achieved a smaller error cost
than did the primitive step tracker. The smallest error cost
was attained by an adaptive tracker whose bandwidth was adap-
ted to its input signal acceleration, as well as to its input
noise variance. The optimum values of the filter time con-
stants used in this adaptive tracker are given at the end of
Appendix F. The minimum error cost attained by this tracker
was not as small as that predicted for the optimum rate com-
ensating tracker tracking a sinusoidal input signal in the
presence of stationary input noise. However, the adaptive
sonar tracker was tracking an only approximately sinusoidal
signal in the presence of nonstationary noise and an inter-
fering input signal.
Appendix A

Analysis of Linear, Continuous-Time Model for Signal Response of Rate Compensating Tracker

\[ G(C_{1}^2) = \frac{2}{\sqrt{\pi} C_{1}^2} ; C_{1}^2 = E(n_{1}^2(t)) \]

Rate Compensating Tracker Signal Response Model

From the above model we find the transfer function \( H(S) \)

\[ H(S) = \frac{O(S)}{I(S)} = \frac{\frac{G\Delta}{T} + \frac{G\Theta}{T}}{S^2 + \frac{GAB}{T} + \frac{G\Theta}{T}} = \frac{2qW_n S + W_n^2}{S^2 + 2qW_n S + W_n^2} \]

where

\[ q^2 = \frac{G\Delta^2}{4\Theta T} \quad W_n^2 = \frac{G\Theta}{T} \quad (A1) \]

The corresponding impulse response \( h_s(t) \) is easily shown to be

\[ h_s(t) = \frac{W_n}{\sqrt{1-q^2}} e^{-qW_n t} \sin(\sqrt{1-q^2} W_n t) ; \quad \tan(\Theta) = \frac{2q\sqrt{1-q^2}}{1-2q^2} \quad (A2) \]
If $f_b$ is the tracker half-power bandwidth, then $W_b = 2\pi f_b$ satisfies
\[
\left| \frac{H(jW_b)}{H(j0)} \right|^2 = \left| \frac{2j\omega_n W_b + W_n^2}{W_n^2 - W_b^2 + 2j\omega_n W_b} \right|^2 = \frac{1}{2}
\]
or
\[W_b^4 - 2(1 + 2q^2)W_n^2W_b^2 - W_n^4 = 0\]

\[W_b^2 = W_n^2 \left[ 1 + 2q^2 + \sqrt{1 + (1 + 2q^2)^2} \right]
\]
giving
\[f_b^2 = \frac{W_b^2}{4\pi^2} = B_q W_n^2; \quad B_q = \frac{1}{4\pi^2} \left[ 1 + 2q^2 + \sqrt{1 + (1 + 2q^2)^2} \right]
\]
at $q^2 = \frac{1}{2}$, $f_b^2 = 0.112 W_n^2$

From the preceding model the transfer function for $e(t)$ is
\[
\frac{E(s)}{I(s)} = \frac{s^2}{s^2 + \frac{G_a S}{T} + \frac{G_b S}{T}} = \frac{s^2}{s^2 + 2qW_n s + W_n^2}
\]

Thus the steady state error $e_s$ for $s_1(t) = \frac{At^2}{2} u_{-1}(t)$ is
\[
e_s = \lim_{S \to 0} \left[ \frac{A}{s^2} \frac{E(s)}{I(s)} \right] = \lim_{S \to 0} \left[ \frac{A}{s^2 + 2qW_n s + W_n^2} \right] = \frac{A}{W_n^2}
\]
or in terms of the tracker bandwidth $f_b$
\[
e_s = \frac{B_q A}{f_b^2}
\]
For $s_1(t) = A_1 \sin(W_1 t)$ and $C_1^2(t) = C_1^2$ the transfer function for $e(t)$ becomes
\[
\frac{E(jW_1)}{I(jW_1)} = \frac{-W_1^2}{W_n^2 + 2jQW_1W_n - W_n^2} = \frac{-W_1^2/W_n^2}{1 + 2jQW_1/W_n - W_1^2/W_n^2}
\]

If \( W_1 \ll W_n \)

\[
\frac{E(jW_1)}{I(jW_1)} \equiv \frac{W_1^2}{W_n^2} \quad e(t) \equiv -\frac{1}{W_1^2} \sin(W_1t)
\]

The time average square error cost function is

\[
(s_1(t)-y(t))^2 = (s_1(t)-s_0(t))^2 - 2(s_1(t)-s_0(t))n_0(t) + n_0^2(t)
\]

Assuming that

\[
2(s_1(t)-s_0(t))n_0(t) = 0
\]

the time average square error cost function becomes

\[
(s_1(t)-y(t))^2 = e^2(t) + C_o^2 \quad ; \quad C_o^2 = n_0^2(t)
\]

Thus if \( s_1(t) = A_1 \sin(W_1t) \) and \( C_o^2 = N_q C_1^2 f_b \) (see Appendix C)

the time average square error cost function is

\[
(s_1(t)-y(t))^2 = \frac{A_1^2 W_1^4}{2W_n^4} + N_q C_1^2 f_b = \frac{B_q A_1^2 W_1^4}{2f_b^4} + N_q C_1^2 f_b
\]

which is minimized when

\[
f_b = f_{bo} = \left[ \frac{2B_q A_1^2 W_1^4}{N_q C_1^2} \right]^{1/5} = \left[ \frac{4e^2(t)f_b^4}{N_q C_1^2} \right]^{1/5}
\]

The resulting minimum time average square error is
\[
\min (s_i(t) - y(t))^2 = 5 \left[ \frac{B_q^2 A_i^2 W_i}{2 f_{bo}} \right]^{1/4} = 5e^2(t)
\]

A useful quantity for comparing the performance of simulated trackers and the actual sonar tracker is

\[
\min (s_i(t) - y(t))^2 / A_i^2 = 5 \left[ \frac{B_q^2 W_i}{2 f_{bo}} \right]^{1/4} = 5 \left[ \frac{B_{Nq}^2 C_i W_i}{16 A_i} \right]^{1/5}
\] (A3)
Appendix B

Analysis of Linear, Sampled-Data Model for Signal Response of Rate Compensating Tracker

![Z-Transform Model of Rate Compensating Tracker](image)

The above Z-transform model of the rate compensating tracker is derived from the sampled-data model in Figure 8 by removing the ideal samplers and replacing the integrators with corresponding Z-transforms. The relationship between a sampled time function $r(nT)$ and its Z-transform is given in Figure 3. The Z-transforms for a few selected time functions are given at the end of this appendix. For a thorough discussion of the use of Z-transform theory in the analysis of sampled-data systems see reference 5.

From the above model we find the transfer function

$$H(Z) = \frac{O(Z)}{I(Z)} = \frac{Z(G\Delta + G\Theta T) - G\Delta}{Z^2 - 2Z \left[ 1 - \frac{G\Delta}{2} - \frac{G\Theta T}{2} \right] + 1 - G\Delta}$$

We note that $H(Z)$ may be written in the form
\[ H(Z) = \frac{C_1(Z-e^{-aT}\cos(WT)) + C_2Ze^{-aT}\sin(WT)}{Z^2 - 2Ze^{-aT}\cos(WT) + e^{-2aT}} \]

where
\[ e^{-2aT} = 1-G\Delta \quad e^{-aT}\cos(WT) = 1 - \frac{G\Delta}{2} - \frac{G\Theta}{2} \]

and \( C_1 \) and \( C_2 \) are appropriately chosen. Thus from the last entry in the Z-transform table we see that the corresponding tracker sampled impulse response \( h_{ss}(nT) \) is of the form

\[ h_{ss}(nT) = De^{-a(n-1)T}\sin(W(n-1)T+\Theta)u_{n-1}((n-1)T) \]

where
\[ D^2 = C_1^2 + C_2^2 \quad \tan(\Theta) = \frac{C_1}{C_2} \]

If \( G\Delta << 1 \), then \( e^{-2aT} = 1-2aT \approx 1-G\Delta \), giving

\[ a = \frac{G\Delta}{2T} \quad \cos(WT) = \frac{1 - \frac{G\Delta}{2} - \frac{G\Theta}{2}}{(1-G\Delta)^{\frac{1}{2}}} \approx 1 - \frac{G\Theta}{2} \left[ 1 - \frac{G\Delta^2}{2G\Theta} \right] \]

Assuming that \( WT << 1 \), gives \( \cos(WT) \approx 1 - \frac{(WT)^2}{2} \), and thus

\[ W^2 = \frac{G\Theta}{T} \left[ 1 - \frac{G\Delta^2}{2G\Theta} \right] \]

Making use of the definitions of "\( W_n \)" and "\( q \)" given in equation (A1) of Appendix A we have

\[ a = qW_n \quad W = W_n\sqrt{1-q^2} \]

If similar approximations are used to solve for \( C_1 \) and \( C_2 \) above, the resulting sampled impulse response \( h_{ss}(nT) \) is
\[ h_{ss}(nT) = \frac{W_n T}{\sqrt{1-q^2}} e^{-qW_n(n-1)T} \sin(W_n \sqrt{1-q^2} (n-1)T + \theta) u_{-1}((n-1)T) \]

where

\[ \tan(\theta) = \frac{2q\sqrt{1-q^2}}{1-2q^2} \]

Thus if \( qW_n T \) and \( W_n \sqrt{1-q^2} T \) are small compared to "1", the impulse response \( h_{ss}(nT) \) for the sampled-data tracker model is approximately equal to "T" times the impulse response \( h_s(t) \) of the continuous-time tracker model given in equation (A2) of Appendix A.

From the preceding Z-transform model the transfer function for the tracker error \( e(nT) \) is found to be

\[ \frac{E(Z)}{I(Z)} = \frac{Z^2 - 2Z + 1}{Z^2 - 2Z \left(1 - \frac{\Delta \Theta}{2} - \frac{\Theta T}{2}\right) + 1 - \Delta} \]

If the tracker input signal is the constant acceleration \( s_1(nT) = \frac{A(nT)^2}{2} \), then the steady state tracker error \( e_{ss} \) is

\[ e_{ss} = \lim_{Z \to 1} \left[ A T Z(Z+1) \right] \frac{Z^2 - 2Z + 1}{2(Z-1)^2 \left(Z^2 - 2Z \left(1 - \frac{\Delta \Theta}{2} - \frac{\Theta T}{2}\right) + 1 - \Delta \right)} \]

\[ = \frac{AT}{\Theta = \frac{A}{W_n^2}} \]

When the tracker sampled input signal is \( s_1(nT) = A_1 \sin(W_1 nT) \) the Z-transform of the sampled tracker error \( e(nT) \) is

\[ E(Z) = \left[ A_1 Z \sin(W_1 T) \right] \frac{Z^2 - 2Z + 1}{Z^2 - 2Z \cos(W_1 T) + 1} \frac{Z^2 - 2Z \left(1 - \frac{\Delta \Theta}{2} - \frac{\Theta T}{2}\right) + 1 - \Delta} \]
which can be expressed in the form

\[
E(Z) = \frac{C_3(Z^2 - Z\cos(W_1T)) + C_4Z\sin(W_1T)}{Z^2 - 2Z\cos(W_1T) + 1} + T(Z)
\]

where \(T(Z)\) corresponds to the transient component of the sampled error signal \(e(nT)\). The first term in the above equation corresponds to the steady state error signal of form

\[
e_s(nT) = A_0\sin(W_1nT+\phi) ; A_0^2 = C_3^2 + C_4^2 \quad \frac{\tan(\phi)}{C_3} = \frac{C_3}{C_4}
\]

Of interest is the steady state time average square error

\[
e_s^2(nT) = \frac{A_0^2}{2} = \frac{C_3^2 + C_4^2}{2}
\]

If we assume that \(aT < < 1\) and \(W_1T < < 1\), a great deal of tedious algebraic manipulation yields

\[
e_s^2(nT) = \frac{A_1^2W_1}{2W_n} \left[ 1 - G\Delta \frac{W_1^2}{W_n^2} \right]
\]

Therefore, if \(W_1 < < W_n\), the time average square tracker signal error is given approximately by

\[
e_s^2(nT) = \frac{A_1^2W_1}{2W_n}
\]

which is the same as that derived in Appendix A for the continuous-time rate compensating tracker signal model.

The sampled input-output crosscorrelation \(R_{xy}(nT)\) of the linear, sampled-data model of the rate compensating tracker is

\[
R_{xy}(nT) = E[x(jT)y((j+n)T)] = \sum_{k=0}^{\infty} h_{ss}(mT)E[x(jT)x((j+n-k)T)]
\]

If the input samples \(x(jT)\) are uncorrelated gaussian random
variables with variance \( C_i^2 \), then we have

\[
E[x(jT)x((j+n-k)T)] = C_i^2 \quad ; \; k = n
\]
\[
0 \quad ; \; k \neq n
\]

and the sampled crosscorrelation \( R_{xy}(nT) \) becomes

\[
R_{xy}(nT) = C_i^2 \cdot h_{ss}(nT)
\]

If the crosscorrelation is taken between only the polarities of the input and output samples, the resulting sampled polarity-coincidence crosscorrelation \( R_{pxy}(nT) \) is (reference 7)

\[
R_{pxy}(nT) = \frac{2}{\pi} \sin^{-1} \left[ \frac{C_i h_{ss}(nT)}{E[y^2(kT)]} \right]
\]

To proceed further we evaluate \( E[y^2(kT)] \) as

\[
E[y^2(kT)] = \sum_{p=0}^{\infty} \sum_{r=0}^{\infty} h_{ss}(pT)h_{ss}(rT)E[x((k-p)T)x((k-r)T)]
\]
\[
= C_i^2 \sum_{p=0}^{\infty} h_{ss}^2(pT)
\]

By using an integral approximation to the summation above, we find that the output variance \( E[y^2(kT)] \) of the linear sampled-data tracker signal model is approximately equal to \( \frac{2}{\pi} \) times that of the nonlinear tracker as derived in Appendix C

\[
E[y^2(kT)] \approx \frac{2C_i^2 f_b N q}{\pi}
\]

Thus the sampled, polarity-coincidence input-output crosscorrelation of the linear, sampled-data tracker signal model is

\[
R_{pxy}(nT) = \frac{2}{\pi} \sin^{-1} \left[ \frac{C_i h_{ss}(nT)}{(2f_b N q)^{\frac{1}{2}}} \right]
\] (B1)
Z-Transform Table

\[ r(nT) \]

\[ R(Z) = \sum_{n=0}^{\infty} r(nT)Z^{-n} \]

\[ u_{-1}(nT) = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; n < 0 \end{cases} \]

\[ u_{-1}((n-1)T) = \begin{cases} 1 & ; n \geq 1 \\ 0 & ; n < 1 \end{cases} \]

\[ \frac{A(nT)^2}{2} u_{-1}(nT) \]

\[ \frac{A^2T^2Z(Z+1)}{2(Z-1)^3} \]

\[ \sin(WnT+\varnothing)u_{-1}(nT) \]

\[ \frac{\sin(\varnothing)(Z^2-Z\cos(WT)) + \cos(\varnothing)Z\sin(WT)}{Z^2 - 2Z\cos(WT) + 1} \]

\[ e^{-a(n-1)T} \sin(W(n-1)T+\varnothing)u_{-1}((n-1)T) \]

\[ \frac{\sin(\varnothing)(Z-e^{-aT}\cos(WT)) + \cos(\varnothing)Ze^{-aT}\sin(WT)}{Z^2 - 2Ze^{-aT}\cos(WT) + e^{-2aT}} \]
Appendix C

Derivation of Output Noise Variance of Rate Compensating Tracker

In Section 2.4 it is shown that, if \( T \) is the tracker decision period, successive outputs \( Y_n \) of the rate compensating tracker are related to successive tracker inputs \( X_n \) and the tracking parameters \( \Delta \) and \( \Theta \) by

\[
Y_{n+1} = Y_n + \Delta E_n + \Theta TR_n ; \quad E_n = \frac{X_n - Y_n}{|X_n - Y_n|} \quad R_n = \sum_{j=0}^{n} E_j
\]

or

\[
Y_{n+1} = Y_n + (\Delta + \Theta T)E_n + \Theta TR_{n-1} \quad (C1)
\]

We wish to find the tracker output variance \( E[Y^2] = C_{oo}^2 \) when the tracker inputs \( X_n \) are sample values of independent identically distributed gaussian random variables with zero mean and variance \( C_1^2 \). We first find the conditional variance

\[
C_{oc}^2 = E[Y_{n+1}^2 | Y_n = Y_o]
\]

and then take its expectation over all values of "\( Y_o \)" to obtain the desired result. Using equation \( (C1) \) we have

\[
C_{oc}^2 = Y_o^2 + 2\Delta TV_{o}R_{n-1} + \Theta^2 T^2 R_{n-1}^2 + (\Delta + \Theta T)^2 E[E_n^2 | Y_n = Y_o]
\]

\[ + 2(\Delta + \Theta T)(Y_o + \Theta TR_{n-1})E[E_n | Y_n = Y_o] \quad (C2)
\]

In order to proceed further we must find \( E[E_n | Y_n = Y_o] \) and \( E[E_n^2 | Y_n = Y_o] \). If \( Y_o \) is small compared to \( C_1 \), we have

\[
P\{E_n = 1 | Y_n = Y_o\} \approx \frac{1}{2} - \frac{Y_o}{\sqrt{2\pi C_1^2}} \quad P\{E_n = -1 | Y_n = Y_o\} \approx \frac{1}{2} + \frac{Y_o}{\sqrt{2\pi C_1^2}}
\]
and thus
\[ E\left[ E_n^2 \mid Y_n = Y_0 \right] = 1 \quad E\left[ E_n \mid Y_n = Y_0 \right] = -G Y_0 \quad \text{G} = \frac{2}{\sqrt{\pi c_1}} \] (C3)

The use of equations (C3) in equation (C2) gives
\[ C_{oc}^2 = (1 - 2G(\Delta + \Theta T))Y_o^2 + (\Delta + \Theta T)^2 + 2\Theta T(1 - G(\Delta + \Theta T))Y_o R_{n-1} \]
\[ + \Theta^2 T^2 R_{n-1} \] (C4)

At this point it is difficult to proceed without relating "\( R_{n-1} \)" to "\( Y_o \)." In order to do this we consider the case where the tracker has a damping ratio \( q \) which is large enough that \( q^2 = \frac{GA^2}{\Theta T} \ll 1 \). Since in practice \( \Delta \) is usually chosen so that \( G\Delta \ll 1 \), it results for very large damping that \( \frac{\Theta T}{\Delta} = \frac{GA}{4\Delta^2} \ll 1 \). We suspect that under these conditions the effect of the tracker rate estimate \( R_{n-1} \) is of secondary importance and that the tracker output position \( Y_n \) is approximately given by
\[ Y_n = Y_o = \Delta \sum_{j=0}^{n-1} E_j = \Delta R_{n-1} \]
and therefore we may relate "\( R_{n-1} \)" to "\( Y_o \)" by
\[ R_{n-1} = \frac{Y_o}{\Delta} \] (C5)

Substituting equation (C5) into equation (C4) and making use of the assumptions above that "\( G\Delta \)" and "\( \frac{\Theta T}{\Delta} \)" may be neglected compared to "\( 1 \)" the conditional variance \( C_{oc}^2 \) becomes
\[ C_{oc}^2 = E\left[ Y_{n+1}^2 \mid Y_n = Y_0 \right] = Y_o^2 - 2G Y_o \left[ \frac{1 - \frac{1}{4\Delta^2}}{q^2} \right] + \Delta^2 \] (C6)

The expectation of equation (C6) over all values of "\( Y_o \)" is
\[
E[Y_{n+1}^2] = E_{y_0}[Y_0^2] - 2G\Delta \left[ 1 - \frac{1}{4q^2} \right] E_{y_0}[Y_0^2] + \Delta^2
\] (C7)

The stationarity of the tracker output noise process gives
\[
E_{y_0}[Y_0^2] = E[Y_0^2] = E[Y^2] = E[Y_{n+1}^2] = C_o^2
\] (C8)

Thus from equations (C7) and (C8) the tracker output noise variance is given by
\[
C_o^2 = \frac{\Delta}{2G \left[ 1 - \frac{1}{4q^2} \right]}
\] (C9)

However, we have assumed that the damping ratio \(q\) is large compared to unity and therefore equation (C9) may be written
\[
C_o^2 \approx \frac{\Delta}{2G} \left[ 1 + \frac{1}{4q^2} \right] = \frac{\Delta}{8} C_i \Delta \left[ 1 + \frac{1}{4q^2} \right] ; q^2 \gg 1
\] (C10)

It is useful to express \(C_o^2\) in terms of the tracker bandwidth \(f_b\). First making use of equations (A1) of Appendix A we have from equation (C10)
\[
C_o^2 \approx \frac{\pi T q C_i 2 \bar{W}_n}{2} \left[ 1 + \frac{1}{4q^2} \right] ; q^2 \gg 1
\] (C11)

From Appendix A we may also relate \(f_b\) and \(\bar{W}_n\) by
\[
f_b = B_q \frac{\pi T q C_i 2 \bar{W}_n}{2} \left[ 1 + \frac{1}{4q^2} \right]^{\frac{1}{2}}
\]

Thus for large damping ratio \(q\) the output noise variance of the rate compensating tracker may be expressed as
\[
C_o^2 \approx N_q C_i^2 f_b ; N_q = \frac{\pi T q}{2B_q} \left[ 1 + \frac{1}{4q^2} \right]
\] (C12)
Appendix D

Development of Signal Response Model for Adaptive Tracker

\[ h(u) = L e^{-Lu} \]

\[ e(t) \rightarrow \hat{e}(t) \rightarrow B(e_q, G) \]

\[ G(C_1^2) \]

\[ \Delta(f_b, G) \]

\[ \Delta(t) \rightarrow \Theta(f_b, G) \]

\[ \Theta(t) \]

\[ f_b(t-D) \]

\[ \text{Delay } D \]

\[ B(e_q, G) = \frac{e_q}{G} \]

\[ G(C_1^2) = \frac{2}{\sqrt{\pi C_1^2}} \]

\[ \Delta(f_b, G) = \frac{2Tqf_b}{GBq^{1/4}} \]

\[ F(\dot{e}, f_b, C_1^2) = \left[ \frac{4e^2f_b}{N_q C_1^2} \right]^{1/5} \]

\[ \Theta(f_b, G) = \frac{Tf_b^2}{GBq} \]

Input-Output Relations of Controller

Suppose \( C_1^2(t) = C_1^2 \), and \( e(t) \) varies slowly enough that \( f_b(t) \) assumes approximately the equilibrium values satisfying

\[ f_b(t) = \left[ \frac{4e^2(t)f_b^{14}(t)}{N_q C_1^2} \right]^{1/5} \]

or

\[ f_b(t) = \frac{4e^2(t)}{N_q C_1^2} = \frac{2\pi C^2 e^2(t)}{N_q} \quad ; \quad G^2 = \frac{2}{\pi C_1^2} \]
Then using
\[ \Delta(t_b, G) = \frac{2Tq f_b}{N_q B_q^{1/4}} \]
\[ \Theta(t_b, G) = \frac{Tq^2}{G B_q} \]
we have
\[ \Delta(t) = \frac{4\pi TqG^2e^2(t)}{N_q B_q^{1/4}} \]
\[ \Theta(t) = \frac{4\pi^2 Tq^2 G^4 e^4(t)}{N_q^2 B_q} \]

In the tracker signal response model in Figure 9, the quantities \( G\Delta e(t) \) and \( G\Theta e(t) \) become
\[ G\Delta e(t) = \frac{4\pi TqG^2 e^3(t)}{N_q B_q^{1/4}} = K_\Delta e^3(t) ; K_\Delta = \frac{4\pi TqG^2}{N_q B_q^{1/4}} = \frac{8Tq}{N_q B_q^{1/4} C_1^2} \]
\[ G\Theta e(t) = \frac{4\pi^2 Tq^4 G^4 e^4(t)}{N_q^2 B_q} = K_\Theta e^5(t) ; K_\Theta = \frac{4\pi^2 Tq^4 G^4}{N_q^2 B_q} = \frac{16T}{N_q^2 B_q C_1^4} \]

The resulting model for the signal response of the adaptive tracker is shown below.

![Model for Signal Response of Adaptive Tracker](image-url)
Appendix E

Computer Programs for Simulating the Operation of Bandwidth-Adaptive and Rate Compensating Trackers

Several computer subroutines were written in DISAC assembly language to simulate the operation of the rate compensating tracker discussed in section 2.4 and the operation of the bandwidth-adaption controller discussed in section 1.4. One basic subroutine, TRAKSIM, was used to simulate the operation of the nonlinear, sampled-data rate compensating tracker model in Figure 7. For simulated bandwidth-adaptive tracking, the tracking parameters used by TRAKSIM were repeatedly computed by three other subroutines, RESC, FERS, and CNP, thus simulating the operations performed by the bandwidth-adaption controller in Figure 11. A functional description of each subroutine is given below, and a diagram showing how the subroutines process common quantities to simulate tracker operation is given at the end of this appendix.

TRAKSIM

Each time TRAKSIM is called, it makes a single tracking decision based upon the sign of the difference between the current tracker input sample \( X_n \) and the current value of the tracker output position \( Y_n \). As a result of this tracking decision, TRAKSIM computes the new values \( Y_{n+1} \) and \( V_n \) of the tracker position and rate estimates, respectively, using

\[
Y_{n+1} = Y_n + \Delta_n E_n + TV_n \quad ; \quad E_n = \frac{X_n - Y_n}{|X_n - Y_n|} \quad \quad V_n = \sum_{j=0}^{n} \Theta_j E_j \quad (E1)
\]
where $\Delta_n$ and $\Theta_n$ are the current values assigned to the tracking parameters, and where $T$ is the length of the tracker decision period being simulated. For nonadaptive rate compensating tracking, the values of $\Delta$ and $\Theta$ are preselected by the programmer and remain constant while TRAKSIM is repeatedly called to simulate the tracker response to a sequence of sample values of the input $X$. For bandwidth-adaptive tracking RESC, FERS, and CNP are called following TRAKSIM to estimate the tracker error and to compute new optimum values for the tracking parameters $\Delta$ and $\Theta$.

RESC

Each time RESC is called, it determines the most recent value of the clipped tracker error signal $E_n$ according to the sign of the tracker error producing the preceding tracker decision. RESC can do this because it has access to the current tracker rate estimate $V_n$ and the new tracker output position estimate $Y_{n+1}$, and because it has stored internally the previous tracker output $Y_n$. From equation (E1) above we have that $E_n$ is given by

$$E_n = \frac{Y_{n+1} - Y_n}{T V_n}.$$ 

FERS

Each time FERS is called, it averages the current value of the clipped tracker error $E_n$ with the previous value of the filtered tracker error $E_{fn}$ to obtain the new filtered error $E_{fn+1}$. The averaging procedure used is described in
Appendix F, and is effectively exponential with time constant $1/L$ chosen by the programmer.

CNP

Each time CNP is called, it computes new optimum values for the tracking parameters $\Delta$ and $\Theta$ from their previous values, from the current value of the filtered error $E_f n$, and from the input noise variance $C_i^2$ and tracker damping ratio $q$ specified by the programmer. This computation is analogous to the computations performed by the controller in Figure 11, except that the intermediate computations of actual tracker error $e(t)$ and bandwidth $f_b(t)$ are bypassed. The newly computed parameter values are averaged with previous parameter values to obtain the filtered tracking parameters $\Delta_f$ and $\Theta_f$ used to implement the next tracking decision. The averaging procedure used is the exponential filtering algorithm developed in Appendix F, with time constant $1/M$ that may be chosen by the programmer.
Appendix F

Exponential Filtering in Computer Controller Subroutines

The adaptive tracking subroutine TRAK and the controller simulation subroutines FERS and CNDA use an effectively exponential method of "filtering" some quantity, or averaging an "unfiltered" value of that quantity obtained every D seconds with previous values of the quantity. The new filtered value $X_{nf}$ is obtained from the old filtered value $X_{of}$ and the newly obtained unfiltered value $X_n$ using

$$X_{nf} = aX_n + (1-a)X_{of} ; 0 < a < 1 \quad (F1)$$

or equivalently

$$X_{nf} = X_{of} + a(X_n - X_{of})$$

To demonstrate the exponential nature of this filtering algorithm let $X_f(kD)$ be the filtered value of $X$ at time $t=kD$ and let $X_n(kD)$ be the new unfiltered value of $X$ obtained at time $t=kD$. Then (F1) becomes

$$X_f((k+1)D) = aX_n((k+1)D) + (1-a)X_f(kD) \quad (F2)$$

The recursive use of (F2) yields

$$X_f(kD) = \sum_{m=0}^{\infty} X_n((k-m)D)a(1-a)^m = \sum_{m=0}^{\infty} X_n((k-m)D)h(mD)$$

where

$$h(mD) = a(1-a)^m$$

Thus the result of this averaging procedure is equivalent to
to the sampled output $X_p(kD)$ of a linear filter with sampled impulse response $h(mD)$ responding to an input sampled at times $t=(k-m)D$ with values $X_n((k-m)D)$. If $a<<1$, $h(mD)$ becomes

$$h(mD) = a(1-a)^m \cong a(1-ma) \cong ae^{-ma} = ae^{-mD/T} ; T = D/a$$

and therefore this averaging procedure becomes equivalent to passage through an exponential filter with effective time constant $T=D/a$.

The filtering procedure described above is used in TRAK to reduce the noise in computed quantities and to provide a means for adjusting the rate at which these quantities can undergo significant changes. Shown below is a table giving each filtered quantity, the corresponding time constant, and the value of each time constant found to be optimum for tracking the input signal shown in Figure 22.

**TRAK Exponential Filter Time Constant Table**

<table>
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<th>Filtered Quantity</th>
<th>Corresponding Time Constant TX</th>
<th>Optimum TX</th>
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<td>C</td>
<td>TC</td>
<td>1.0 Sec.</td>
</tr>
<tr>
<td>E</td>
<td>TE</td>
<td>0.5 Sec.</td>
</tr>
<tr>
<td>R</td>
<td>TR</td>
<td>0.125 Sec.</td>
</tr>
<tr>
<td>RSS</td>
<td>TRS</td>
<td>1.0 Sec.</td>
</tr>
<tr>
<td>SS</td>
<td>TS</td>
<td>1.0 Sec.</td>
</tr>
</tbody>
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### Appendix G

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Figure 1. Passive Sonar Technique for Finding Target Bearing Angle (a)

A comparison of the correlogram areas under the two gate halves indicates that the peak lies in the direction of the larger area. Thus for the gate displaced to the right, as shown, the tracking decision will be to move the gate in the direction of the larger gate half area, or to the left.

Figure 2. Tracking Gate Displaced from Center of Correlogram Peak
Ideal Sampler

\[ r(t) \rightarrow r^*(t) \]

\[ R(S) \rightarrow R^*(S) \]

\[ r^*(t) = \sum_{n=0}^{\infty} r(nT)u_0(t-nT) \]

\[ R^*(S) = \sum_{n=0}^{\infty} r(nT)e^{-nTS} \]

\[ R(Z) = \sum_{n=0}^{\infty} r(nT)z^{-n} ; z = e^{-TS} \]

**Figure 3.** Input-Output Relations of Ideal Sampler

\[ x(t) = s_1(t) + m_1(t) + \sum_i e(t) \rightarrow e_c(t) \rightarrow y(t) \]

**Figure 4.** Model of Tracker Decision Process
Figure 5. Nonlinear, Sampled-Data Model of Primitive Step Tracker
Figure 6. Linear, Sampled-Data Model for Signal Response of Primitive Step Tracker
Figure 7. Nonlinear, Sampled-Data Model of Rate Compensating Tracker
$$G(C_1^2) = \frac{2}{\sqrt{\pi} C_1^2}; \quad C_1^2 = \mathbb{E}(n_1^2(t))$$

Figure 8. Linear, Sampled-Data Model for Signal Response of Rate Compensating Tracker
\[ G(c_1^2) = \frac{2}{\sqrt{\pi} c_1^2}; \quad c_1^2 = E(n_1^2(t)) \]

**Figure 9. Rate Compensating Tracker Signal Response Model**

**Figure 10. Scheme for Adaptive Control of Tracker Bandwidth**
Figure 11. Input-Output Relations of Controller

Figure 12. Model for Signal Response of Adaptive Tracker
Figure 13: Error Cost Versus Bandwidth for Simulated Trackers Responding to Sinusoid

\[ s_1(t) = A_t \sin(W_t t) \]  where \( A_t = 7.0 \) Milliseconds and \( W_t / 2\pi = 1/120 \) Hertz
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Tracker position in units of 812.5 microseconds

Figure 22. Tracker Estimates of Input Signal Provided by Recorded Sonar Data
1.3 Milliseconds Delay Time/Div.

a. $t=0$ Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
1.3 Milliseconds Delay Time/Div.

b. $t=5$ Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
1.3 Milliseconds Delay Time/Div.

c. $t=10$ Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
1.3 Milliseconds Delay Time/Div.

d. t=15 Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
1.3 Milliseconds Delay Time/Div.

f. $t=18$ Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
1.3 Milliseconds Delay Time/Div.

g. t=23 Seconds

Figure 23. Time Sequence of Correlogram Peak and Tracker Positions Comparing Performance of Adaptive and Nonadaptive Sonar Trackers
TABLE 1

Performance of Bandwidth-Adaptive Tracker (BAT) and Rate Compensating Tracker (RCT) Responding to Sinusoid $s_1(t) = A_1 \sin(\omega_1 t)$ where $A_1 = 7.0$ Milliseconds and $\omega_1 / 2\pi = 1/120$ Hertz

<table>
<thead>
<tr>
<th>Sonar Conditions Simulated</th>
<th>Theoretically Optimum RCT</th>
<th>Optimum Simulated RCT</th>
<th>Optimum Simulated BAT</th>
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<tbody>
<tr>
<td></td>
<td>Maximum % Correlation</td>
<td>Input S/N db</td>
<td>RMS Total Error $\frac{f_b}{A_1}$ Hertz</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td>50</td>
<td>60.6</td>
<td>$3.0 \times 10^{-4}$</td>
<td>1.26</td>
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<tr>
<td>25</td>
<td>54.6</td>
<td>$5.2 \times 10^{-4}$</td>
<td>.97</td>
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<tr>
<td>12.5</td>
<td>48.6</td>
<td>$9.1 \times 10^{-4}$</td>
<td>.72</td>
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</table>

TABLE 2

Performance of Sonar Trackers Responding to Sinusoid $s_1(t) = A_1 \cos(\omega_1 t)$ where $A_1 = 7.0$ Milliseconds and $\omega_1 / 2\pi = 1/120$ Hertz
Average Maximum Per Cent Correlation=36% and Input S/N=58db

<table>
<thead>
<tr>
<th>RMS Signal Error $\frac{A_1}{A_1}$</th>
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<th>Theoretical Optimum RCT</th>
<th>Optimum BAT (Noise Adaptive Only)</th>
<th>Optimum BAT (Signal and Noise Adaptive)</th>
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<tbody>
<tr>
<td>1.55x10^-3</td>
<td>1.59x10^-4</td>
<td>1.70x10^-4</td>
<td>6.25x10^-4</td>
<td>2.74x10^-4</td>
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</tbody>
</table>
TABLE 3

Performance of Bandwidth-Adaptive Tracker Signal Response Model (BATSRM) Responding to Sinusoid \( s_i(t) = A_i \sin(\omega_i t) \) where \( A_i = 7.0 \) Milliseconds and \( \omega_i / 2\pi = 1/120 \) Hertz

<table>
<thead>
<tr>
<th>Sonar Conditions Simulated</th>
<th>Simulated BATSRM</th>
<th>Simulated BAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum % Correlation</td>
<td>Input S/N</td>
<td>RMS Signal Error</td>
</tr>
<tr>
<td>50</td>
<td>60.6</td>
<td>( 5.7 \times 10^{-5} )</td>
</tr>
<tr>
<td>25</td>
<td>54.6</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>12.5</td>
<td>48.6</td>
<td>( 1.7 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


