

COMPUTER SIMULATIONS
OF
VARIOUS BLACKJACK STRATEGIES

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ABSTRACT

A generalized blackjack strategy evaluator has been programmed. It has shown that the game of blackjack can be beaten. It also tells what the expectations are for a given strategy.

In addition to providing support for the expectations previously suggested for imitating the dealer and for Thorp's basic strategy, the evaluator has shown the true expectation for a never bust strategy.

The evaluator has shown the high-low index to be a more accurate partial deck evaluation function than the tens ratio method. The high-low index not only had a higher expectation than the tens ratio method, but it did so with a smaller average bet size.

Now that the strategy evaluator exists, it may be modified so that it copies the rules of any given casino (or group of casinos using the same rules, as in Puerto Rico). Then different strategies may be simulated until an acceptable strategy has been found.

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TABLE OF CONTENTS

I.	TITLE PAGE	1
II.	ABSTRACT	2
III.	ACKNOWLEDGEMENTS	3
IV.	TABLE OF CONTENTS	4
V.	INTRODUCTION	5
VI.	IMPORTANCE OF MEMORY IN BLACKJACK	8
VII.	PARTIAL DECK EVALUATION	10
VIII.	COMPUTER SOLUTION	12
IX.	EVALUATED STRATEGIES	18
X.	CONCLUSIONS	27
XI.	APPENDICES	28
XII.	BIBLIOGRAPHY	60

INTRODUCTION

In the past few years there has been quite a bit of controversy concerning the casino card game of blackjack. There have been many claims of winning strategies (3,4,5,6,7) as well as counterclaims from casino owners that system players are always welcome in their clubs (6,p.65). The public generally agreed with the casino owners. After all, if someone could consistently win sizeable sums gambling, then why would he not do so to the exclusion of any other job?

This skepticism bothered me because of the structure of blackjack. In most games, the cards are shuffled after each hand. In blackjack, the cards are shuffled at the end of the deck. It is quite possible that someone with the knowledge of which cards remained to be played might be able to devise a winning strategy for the game. In order to resolve the question, I have written a blackjack strategy evaluator program which may be used to test given strategies, as well as to help devise newer, more powerful ones.

Before explaining the program and evaluating strategies, I think that an understanding of the rules of black jack is important. If the reader is not already familiar with those rules, he may refer to Appendix A

where they are listed.

As it may be seen from the rules, at various times the player is called upon to make such strategic decisions as how large his bet should be; whether to insure his hand when the dealer shows an ace; whether he should split a pair; whether he should double down; and whether he should stand on a given total. These decisions may be expressed in the form of a flow chart as in figure 1.

In general, the choice made will depend upon the cards that have been played and the dealer's up card. These decisions concerning bet size, insurance size, splitting, doubling down, and drawing comprise a player's strategy. The following questions arise. What comprises a good overall strategy? How can the player determine his chance of winning in a particular round? How can the player's expectation be changed by variations in his betting strategy? And finally, how can the expectation for a particular set of player strategies be determined? These are some of the questions which will be answered in this paper.

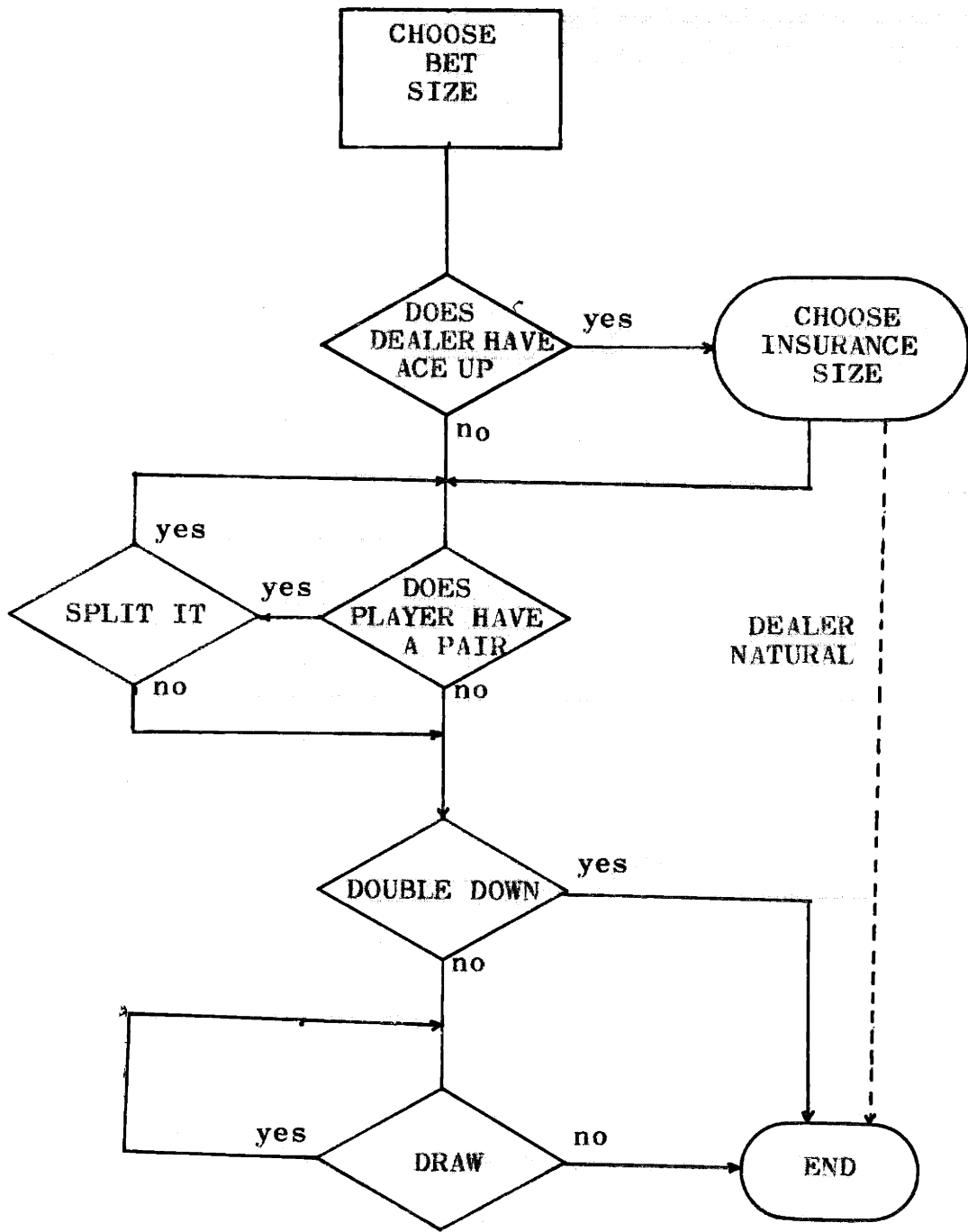


FIGURE 1 - - PLAYER DECISIONS

IMPORTANCE OF MEMORY IN BLACKJACK

Before continuing, we should understand exactly what we are looking for and why we expect to find it. As we have already seen, blackjack differs from other casino games in that the hands are not independent of one another. Since the cards are only shuffled at the end of the deck, we know that a card that has been exposed in a particular round cannot be dealt in a succeeding round. In particular, if we have observed all four aces dealt in previous rounds, then we know there will be no more naturals until the cards have been shuffled. This dependency upon the cards which have been played comprises the "memory" of the game. Thus, blackjack, unlike other casino games, disobeys the old gambler's adage that says "the cards have no memory." Once this has been observed, it is quite reasonable to expect that with a perfect memory one might be able to devise a strategy with which he could "beat the game."

Unfortunately, few of us are able to sit down amidst the chaotic surroundings of a casino gambling table and calmly memorize every card which has been dealt since the last shuffle. Even if this was a simple matter, however, there is still a question as

to just how useful this information would be. For example, if one had a list of all of the cards remaining to be played, how could this information be used in determining strategies? Clearly, if there are no tens left in the deck, then any offer to insure would be refused. In this case the insurance strategy would be changed. If the player knew the deck consisted of only five cards, namely three eights and two sevens, he would alter his strategy in that he would now bet the house limit and stand on whatever two cards he drew. The dealer would then have a total of either fourteen (7,7), fifteen (7,8), or sixteen (8,8). In any case, he would have to draw and bust. Generally, however, it is a difficult task to tell how to react to a particular partial deck composition.

PARTIAL DECK EVALUATION

We now see that a method of evaluating partial decks is at least as important as knowing what the partial decks are. If we could find a reliable method of partial deck evaluation with respect to our probability of winning a hand dealt from that partial deck we can obtain a winning strategy. This would be done by betting small amounts when our partial deck evaluation formula told us that we had a small probability of winning and betting large amounts when our partial deck evaluation formula told us that we had a large probability of winning.

Now that we know what is necessary for a winning strategy we must return from the theoretical world to reality. As we know, it is a difficult enough task to know what comprises the partial deck at any time. Yet, for a winning strategy, we must also be able to evaluate our probability of winning, given the partial deck composition. This is certainly not the easiest of tasks, and the only rational solution, short of bringing a computer along on trips to the casino, is to develop some approximate method of remembering and evaluating partial decks. Here a problem arises as to whether the particular approximate method actually

works and also whether it is better than other methods which are known to exist. Until now there has been no definitive way of answering this question.

COMPUTER SOLUTION

In order to determine how "good" a particular strategy is, I have programmed an IBM 360 computer to play the game of blackjack with the rules of Appendix A. Knowing that these rules are not strictly adhered to in all casinos, I provided enough generality in the program so that rule changes, such as not dealing to the bottom of the deck or using more than one pack of cards (both now used in Puerto Rico to make card counting more difficult) could easily be incorporated into the original program with a minimum amount of reprogramming.

The operation of the program may be understood by referring to the diagram of figure 2. This flow chart describes the game of blackjack. It is purposely left in a coarse form (i.e. settling of naturals is left out) in order to make it more understandable. There are five blocks which contain asterisks. These refer to the fact that player decisions are called for before the proper action can be taken. For example, the player must decide how much to bet before he or the dealer is dealt the initial hand. Since these decisions are dependent upon the player's strategy and not upon the evaluator program, they have been separated from the main program in the form of subprograms, or subrou-

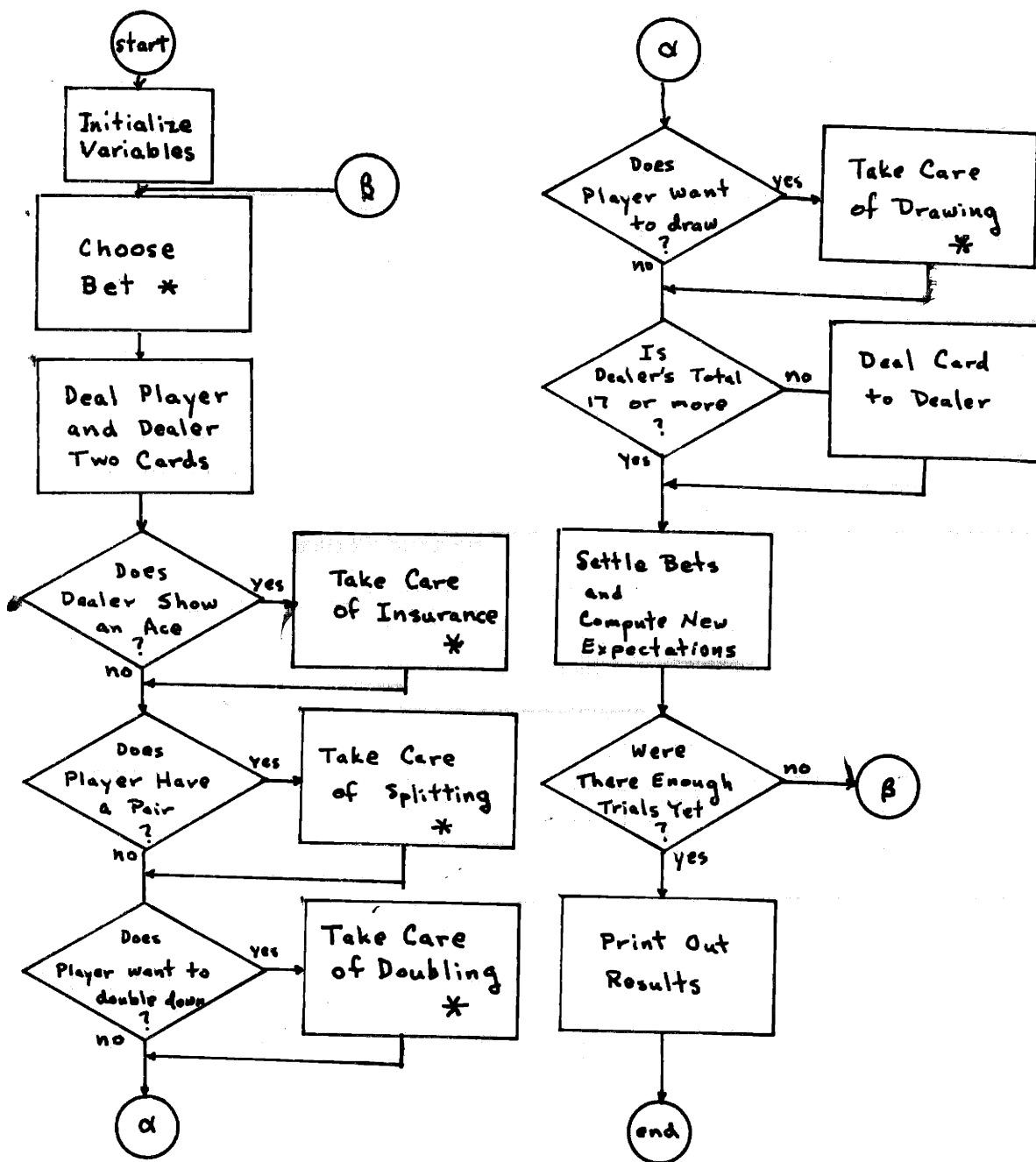


FIGURE 2 -- PROGRAM OPERATION

tines as they are called. Thus, changing the strategy (or strategies) being evaluated entails only changing the subroutine(s) associated with the respective strategy (or strategies). This provides numerous advantages. The complexity of the evaluator is absorbed in the main program, so the subroutines are very easy to write. They are also quite short. This means that a given strategy may usually be implemented as a subroutine in a time commensurate with the time it takes to describe the strategy.

Referring to figure 2, we see that there is a loop which is circled a number of times corresponding to the number of rounds of play we wish to simulate. This does not correspond to the number of hands we are simulating, as that number may be much larger due to splitting pairs. Obviously, the more rounds we simulate, the more valid our final answer will be. However, we will reach a point where additional simulations do us relatively little good. This may be seen by letting our average bet be B , the number of games simulated be G , our winnings be W , and our observed expectation $E(G)$ be:

$$E(G) = \frac{W}{G}$$

If we are to simulate one additional round, our new

expectation will be:

$$E(G+1) = \frac{W \pm B}{G+1} = \frac{W}{G+1} \pm \frac{B}{G+1}$$

Assuming that G is much greater than 1 this becomes:

$$E(G+1) \approx \frac{W}{G} \pm \frac{B}{G} = E(G) \pm \frac{B}{G}$$

$$|\Delta E| = |E(G+1) - E(G)| = \frac{B}{G}$$

Now, if we arbitrarily decide that we want $|\Delta E|$ to be less than some small number, say 0.0005, we can determine the number of rounds we need to simulate from the inequality:

$$|\Delta E| < 0.0005$$

$$\frac{B}{G} < 5 \times 10^{-4}$$

$$G > 2 \times 10^3 B$$

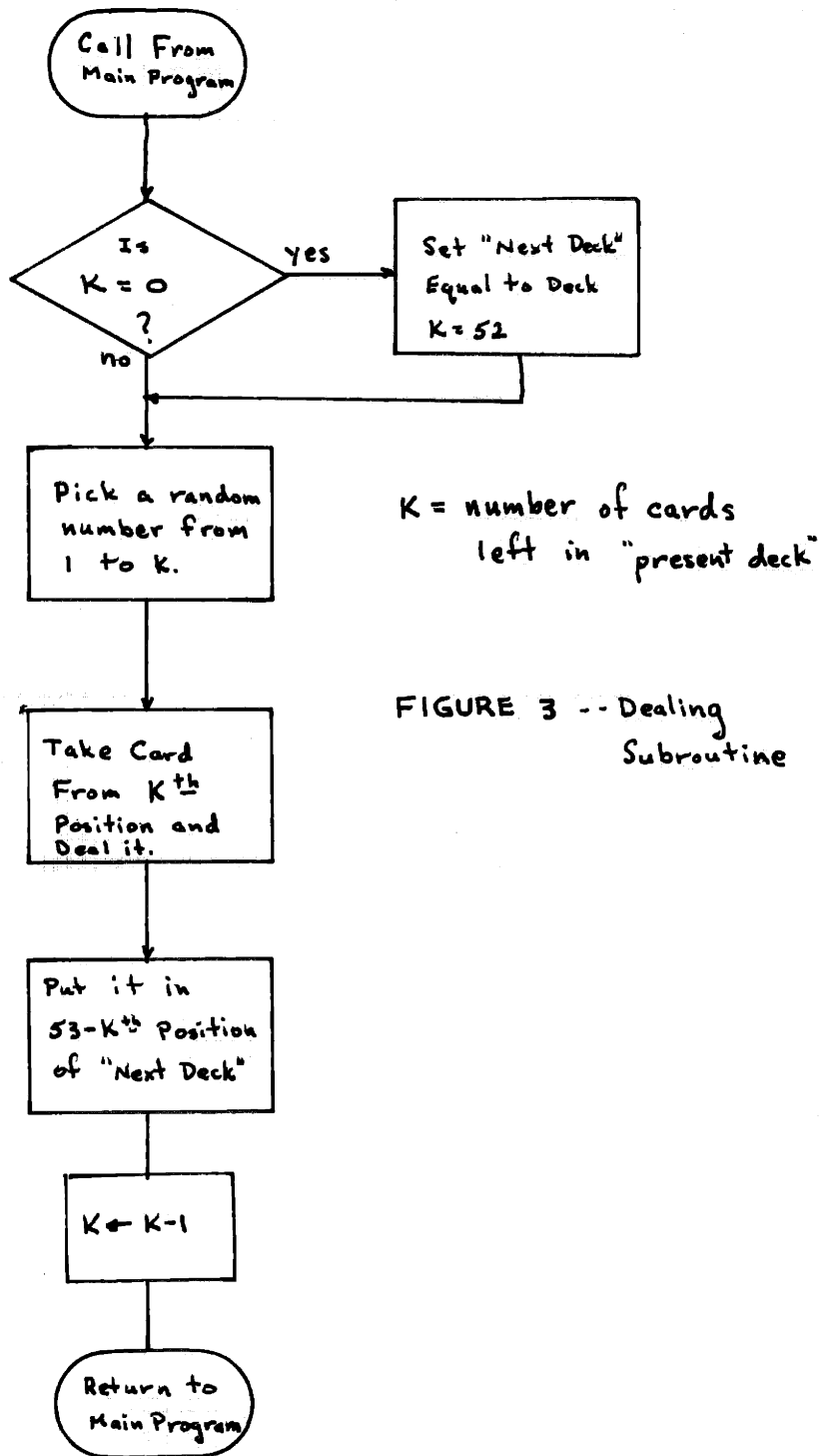
If we assume that the expectation is converging to the correct value and we use a constant bet size, B , of 2 units, the inequality tells us we should simulate at least $2 \times 10^3 \times 2$ or 4000 hands. Since we want a high probability of obtaining the expectation to within 0.0005, we will simulate 5000 rounds (which may be more than 5000 hands). When the strategy calls for variable bet sizes of from 2 to 10 units, 50,000 rounds will

be simulated.

Another point of departure from the coarse flow chart is that the portion of the evaluator which is called upon to deal and shuffle cards is also maintained as a subroutine. This results from the fact that the algorithm used for dealing and shuffling is fairly complex and is called from various points in the main program. By writing it in this manner, each call may be written as only one statement.

Basically, the algorithm for dealing may be represented by the flow chart in figure 3. No initialization is shown because this occurs at the beginning of the main program as shown in figure 2. Each time this subroutine is called it "deals" a card, adds it to the "next deck", and eliminates it from the "present deck." As shown, when the "present deck" has been depleted of cards the "next deck" which now contains all fifty-two cards randomly ordered with respect to the "present deck" becomes the new deck.

In order to change the program to play under different rules, such as using more than one pack of cards or not dealing to the bottom of the deck, we need only change the portions of the main program corresponding to initialization and portions of this subroutine.



EVALUATED STRATEGIESI. Imitate the Dealer

The first strategy which was simulated was that of imitating the dealer. This means that the player will always bet the same amount (2 units), never insure, never split pairs, never double down, and will always draw to totals of less than 17. Many players have chosen this to be their strategy assuming that it would cancel any advantage which the house has. Unfortunately for them, the game is not entirely symmetrical in two respects. First of all, they wrongly assume that any time there is a tie they neither win or lose their bet. This is called a "push". The fallacy of this argument occurs when the player busts. As this occurs before the dealer has had an opportunity to draw, he cannot also bust and "tie" for a push. Thus, the dealer has the advantage any time the player busts in a round in which the dealer would have subsequently busted. Since the probability of a bust using the dealer's strategy is approximately equal to 0.30, the probability of both the player and the dealer busting in the same round is $(0.30)^2$ or 0.09. This represents a 9% advantage for the house.

There is one other asymmetry in the game. When

the player is dealt a natural he is paid back at the rate of 1.5 to 1, whereas the dealer only receives a payment of 1 to 1 for receiving a natural. As naturals occur approximately 4.8% of the time, the house expectation (the probability of occurrence times the payoff) is reduced by one-half this amount to 0.066 (i.e. 0.090 minus 0.024).

The above reasoning yields a figure which is in very close agreement to my derived expectation for the player using this strategy (see Appendix B1) of -0.072 (i.e. a house advantage of 0.072). The 0.006 discrepancy between the two figures may be attributed to the fact that the first figure is derived by assuming that all cards are being dealt from a full deck in order to simplify the calculations (5,p.328 and 6,p.38). This is not expected to yield a true value because conditional probabilities are neglected. The 0.072 house expectation is attributed to simulations involving full and partial decks, so I assume that it is the more accurate figure.

II. Never Bust

The second strategy which I simulated was that of never busting. With this strategy, the player has a "hard standing number" of 12 (i.e. He will stand on

hard totals of 12 or more). Since the "soft standing number" (the total which counts an ace as 11) is not clearly defined, I chose to let it be 17 as a high soft standing number is to the player's advantage. The rest of the strategy is that the player always bets the same amount and he never insures, splits, or doubles down. Using this conservative strategy, the player's simulated expectation is -0.039 (see Appendix B2).



Upon finding the above figure to be in sharp contrast with the -0.215 expectation derived by John Scarne (5,p.329), I followed through his derivation and found an error in his reasoning. He assumed that the dealer would only bust when the player had a total of less than 17, which is not true. After correcting the logical errors in Scarne's argument, his figures yield an expectation for this strategy which is quite close to the one which I have proposed.

III. Thorp's Basic Strategy

This strategy and the two which follow, rely upon the basic strategy presented by Edward Thorp (6,7). It is shown in figure 4. In the first version simulated we do not keep a record of the cards which have fallen. Also, we still maintain a constant bet



Pair Splitting

You have	Dealer Shows										
	2	3	4	5	6	7	8	9	10	A	
A,A	Split	Split	Split	Split	Split	Split	Split	Split	Split	Split	
10,10	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	
9,9	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	
8,8	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	
7,7	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	
6,6	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	
5,5	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	
4,4	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	Do Not Split	
3,3	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	
2,2	Split	Split	Split	Split	Split	Split	Split	Split	Split	Do Not Split	

 Split
 Do Not Split



Hard Doubling

You have	Dealer Shows										
	2	3	4	5	6	7	8	9	10	A	
11	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	
10	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Do Not Double Down	
9	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Double Down	Do Not Double Down	

 Double Down
 Do Not Double Down

Soft Doubling

You have	Dealer Shows				
	2	3	4	5	6
A,7	Do Not Double Down	Double Down	Double Down	Double Down	Double Down
A,6	Do Not Double Down	Double Down	Double Down	Double Down	Double Down
A,5	Do Not Double Down	Do Not Double Down	Double Down	Double Down	Double Down
A,4	Do Not Double Down	Do Not Double Down	Double Down	Double Down	Double Down
A,3	Do Not Double Down	Do Not Double Down	Double Down	Double Down	Double Down
A,2	Do Not Double Down	Do Not Double Down	Double Down	Double Down	Double Down
A,A	Do Not Double Down	Do Not Double Down	Double Down	Double Down	Double Down

 Double Down
 Do Not Double Down

Standing Numbers

You have	Dealer Shows										
	2	3	4	5	6	7	8	9	10	A	
19	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
18	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
17	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
16	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
15	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
14	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
13	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	
12	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	Do Not Stand	



 Soft Standing Number
 Hard Standing Number

FIGURE 4 -- Thorp's Basic Strategy

size and do not insure. Yet, this strategy produces a positive player expectation of 0.001 (see Appendix B3). This is the approximate expectation which was claimed for the strategy (6, p. 31).

The relevance of a positive expectation without memorizing cards should be pointed out. Essentially this means that casino countermeasures such as adding packs or not dealing to the bottom of the deck cannot adversely affect the strategy. Furthermore, now that it has been shown that a positive expectation can be derived, we can explore those strategies pertaining to betting which may increase that expectation.

IV. Basic Strategy with Tens Ratio

By now we should realize that we can increase our expectation by betting large amounts when we expect to win and betting small amounts when we expect to lose. Obviously, the more tens which remain in the deck the poorer are the chances of the dealer of not busting a hand which is in the 12 to 16 range. Also, tens are an essential component of naturals (which have the only asymmetric expectation favorable to the player). This implies that the ratio of non-tens to tens (or the tens ratio as it will be called) in the undealt portion of the deck may be used to compute an

approximate probability of a player win. When the deck is "fresh" this ratio is 36/16 or 2.25. If more tens than non-tens are dealt this ratio decreases below 2.25. The lower it falls, the more favorable the deck becomes, and consequently, the larger the bet size should be. The bets suggested by Thorp (6, p. 106) are shown in figure 5. Also, this is the first strategy which suggests insuring. The insurance option should be accepted whenever the tens ratio is below 2.00. As it may be seen from the output of this strategy (see Appendix B4), the insurance expectation was negative. This figure is probably inaccurate though, because there were not enough trials simulated to get an accurate idea of the insurance expectation. The bet expectation (all bets except insurance) was seen to rise to 0.004 from its former value (without bet variations) of 0.001. In addition, the average bet size was seen to rise from 2 units to 7.6 units. This shows that the tens ratio does provide a somewhat accurate method of predetermining the probability of winning a given round. Thus, the tens ratio may be used as a partial deck evaluation formula.

V. Basic Strategy with High Low Index

For reasons similar to those which said that

BETTING SCHEME FOR THE TEN RATIO STRATEGY

<u>RATIO</u>	<u>BET</u>
above 2.00	2
2.00 - 1.75	4
1.75 - 1.65	8
below 1.65	10

FIGURE 5

tens in the deck are good for the player, it can be argued that low cards in the deck are bad for the player. The high-low point count index (high-low index) developed by Harvey Dubner (6, p.93) evaluates the probability of a win in the next hand by keeping track of the high cards (aces and tens) and the low cards (twos through sixes). Initially there are an equal number of high and low cards in the deck. Since the high cards in the deck are "good" and the low cards in the deck are "bad", we keep track of the high-low index by adding +1 each time we see a low card fall (The deck has become "better"). Similarly, we add -1 to the high-low index each time we see a high card fall (The deck has become "worse"). This method of counting allows us to remember a single total which gives us an idea of the relative "goodness" of the partial deck (The higher the high-low index, the better the deck is.). It should be clear, however, that a high-low index of +5 is more meaningful toward the end of the deck than it would be near the beginning of the deck. The high-low index is used in the following manner to determine the bet size. If the high-low index is not positive, then the bet is 2 units. Otherwise the bet size is:

$$\text{BET} = \left\lfloor \frac{100 \times \text{high-low index}}{\text{number of cards remaining in deck}} \right\rfloor$$

Using this strategy for betting and insuring whenever the high-low index is above 8, a bet expectation of 0.012 was obtained (see Appendix B5). The insurance expectation obtained was negative. However, once again, there were not enough trials to determine the accuracy of the insurance expectation figure. The significant advantages of this strategy are that it has the highest expectation of the strategies mentioned, it is easier to use than the tens ratio, and the average bet size is 6.6 units as opposed to 7.6 units for the tens ratio partial deck evaluation function. For these reasons, the high-low index provides a powerful method of predetermining the approximate probability of winning a given hand.

CONCLUSIONS

A generalized blackjack strategy evaluator has been programmed. It has shown that the game of blackjack can be beaten. It also tells what the expectations are for a given strategy.

In addition to providing support for the expectations previously suggested for imitating the dealer and for Thorp's basic strategy, the evaluator has shown the true expectation for a never bust strategy.

The evaluator has shown the high-low index to be a more accurate partial deck evaluation function than the tens ratio method. The high-low index not only had a higher expectation than the tens ratio method, but it did so with a smaller average bet size. This explains Thorp's comment that the tens ratio method repeatedly showed "...moderately heavy losing streaks, mixed with 'lucky streaks'..."(6, p. 67).

Now that the strategy evaluator exists, it may be modified so that it copies the rules of any given casino (or group of casinos using the same rules, as in Puerto Rico). Then different strategies may be simulated until an acceptable strategy has been found.

APPENDIX ATHE RULES OF BLACKJACKNumber of Players

Blackjack has a dealer and from one to seven players.

The Deck

One ordinary 52-card pack of playing cards is used. However, most casinos now use from two to four packs shuffled together in order to make card counting more difficult.

The Deal

Before play begins, the cards are shuffled by the dealer and cut by a player. A card is then "burned" (placed face up on the bottom of the deck). This card may or may not be shown. The dealer then deals two cards to himself and to each of the players. The dealer receives one card face up and one face deown.

Bets

The players place all bets other than insurance

before any cards are dealt. The house establishes a minimum and a maximum bet size.

Numerical Value of Cards

The player can choose either 1 or 11 to be the value of an ace. The numerical value of picture cards is 10, and the numerical value of all other cards is their face value. A hand is called "soft" if it contains an ace and that ace is counted as 11. All other hands are called "hard".

Object of the Player

The player tries to obtain a total which is greater than that of the dealer but which does not exceed 21. Hands which have exceeded 21 are said to have "busted".

Naturals

If the first two cards dealt either to the player or to the dealer consist of an ace and a 10-value card, they constitute a "natural" or "blackjack". If a player has a natural and the dealer does not, the player receives 1.5 times his original bet from the dealer. If a player does not have a natural and the dealer does, the player loses his original bet. If both

player and dealer have naturals, no money changes hands.

Drawing

The draw starts at the left of the dealer and proceeds in a clockwise fashion. A player looks at his cards and may decide to "stand"; otherwise, he can request additional cards from the dealer, which are dealt face up, one at a time. If a player busts, he immediately turns up his cards and pays his bet to the dealer.

The Dealer's Strategy

After each player has drawn, the dealer turns up his hole card. If his total is 16 or less, he must draw a card and continue to draw cards until his total is 17 or more, at which point he must stand. If the dealer receives an ace, and if counting it as 11 would bring his total to 17 or more without exceeding 21, then he must count it as 11 and stand.

Splitting Pairs

If the player's original cards are numerically identical, they are called a pair. He may choose to treat them as the initial cards in two separate hands. This is known as "splitting a pair". The original bet

goes on one of the split cards and an equal amount is bet on the other card. The player automatically receives a second card on each of the split cards. He then plays his two hands, one at a time, as though they were ordinary hands, with the following exceptions. In the case of split aces, the player receives only one card on each ace. Further, if a 10-value card falls on one of the split aces, the hand is not counted as a natural but only as ordinary 21. Similarly, if a player splits a pair of 10-value cards and then draws an ace, it counts only as an ordinary 21.

Doubling Down

After looking at his first two cards a player may elect to double his bet and draw one, and only one more card. This strategy is known as "doubling down". A player who splits any pair except aces may, after receiving an additional card on each of the split cards, double down on one or both of his hands.

Insurance

If the dealer's up card is an ace, an additional wager is allowed before the draw. After checking his cards, a player may put up an additional side bet equal at most to half his original bet. After the player

has decided whether or not to do this, the dealer checks his down card. If the dealer has a natural, the side bet wins twice its amount. If the dealer does not have a natural, the side bet is lost and the play continues.

The Settlement

If the player does not go over 21 and the dealer does, the player wins an amount equal to his original bet. If neither player nor dealer busts, the person with the higher total wins an amount equal to the original bet of the player. If the player and the dealer have the same total, not exceeding 21, no money changes hands (This is called a "push".).

APPENDIX BRESULTS OF COMPUTER SIMULATIONS

This Appendix contains copies of the computer outputs obtained for each of the strategies mentioned in the text. The format of each of the outputs is the same. The results for the last 50 rounds of the simulation are shown, so the reader is able to see how the expectations mentioned were obtained.

The ROUND column refers to the number of times the loop of figure 2 was circled. The FIRST through FOURTH PLAYER and the DEALER columns refer to the final totals which they each received (There will be more than one player only when the first player has split a pair.).

The columns for TOTAL BETS, TOTAL WINNINGS, and BET EXPECTATION do not take insurance into account. This was done because the insurance bet would have introduced inaccuracies with the number of rounds simulated.

The INSURANCE EXPECTATION is found by dividing the total amount won on insurance by the total amount bet on insurance.

GAMES PLAYED tells how many games were simulated.

The number of games simulated must be greater than, or equal to the number of rounds simulated. It will be a larger number only if the strategy simulated says to split pairs.

IMITATE THE DEALER

APPENDIX B1

ROUND *****	FIRST PLAYER *****	SECOND PLAYER *****	THIRD PLAYER *****	FOURTH PLAYER *****	DEALER *****	TOTAL BETS ****	TOTAL WINNINGS *****	INSURANCE EXPECTATION *****	BET EXPECTATION *****	GAMES PLAYED *****
4950	22	0	0	0	18	9900	-689	0.0	-0.069596	4950
4951	17	0	0	0	19	9902	-691	0.0	-0.069704	4951
4952	17	0	0	0	20	9904	-693	0.0	-0.069812	4952
4953	19	0	0	0	19	9906	-695	0.0	-0.069920	4953
4954	20	0	0	0	21	9908	-697	0.0	-0.070028	4954
4955	24	0	0	0	12	9910	-697	0.0	-0.070136	4955
4956	10	0	0	0	21	9912	-699	0.0	-0.070244	4956
4957	21	0	0	0	11	9914	-696	0.0	-0.070352	4957
4958	17	0	0	0	21	9916	-698	0.0	-0.070460	4958
4959	24	0	0	0	20	9918	-700	0.0	-0.070568	4959
4960	23	0	0	0	11	9920	-702	0.0	-0.070676	4960
4961	19	0	0	0	17	9922	-700	0.0	-0.070784	4961
4962	20	0	0	0	24	9924	-698	0.0	-0.070892	4962
4963	19	0	0	0	17	9926	-696	0.0	-0.071000	4963
4964	24	0	0	0	18	9928	-698	0.0	-0.071108	4964
4965	22	0	0	0	7	9930	-700	0.0	-0.071216	4965
4966	18	0	0	0	18	9932	-700	0.0	-0.071324	4966
4967	20	0	0	0	20	9934	-700	0.0	-0.071432	4967
4968	22	0	0	0	9	9936	-702	0.0	-0.071540	4968
4969	24	0	0	0	12	9938	-704	0.0	-0.071648	4969
4970	24	0	0	0	18	9940	-706	0.0	-0.071756	4970
4971	20	0	0	0	19	9942	-704	0.0	-0.071864	4971
4972	20	0	0	0	17	9944	-702	0.0	-0.071972	4972
4973	19	0	0	0	20	9946	-704	0.0	-0.072080	4973
4974	19	0	0	0	17	9948	-702	0.0	-0.072188	4974
4975	22	0	0	0	16	9950	-704	0.0	-0.072296	4975
4976	23	0	0	0	13	9952	-706	0.0	-0.072404	4976
4977	18	0	0	0	24	9954	-704	0.0	-0.072512	4977
4978	20	0	0	0	20	9956	-704	0.0	-0.072620	4978
4979	25	0	0	0	17	9958	-706	0.0	-0.072728	4979
4980	20	0	0	0	21	9960	-708	0.0	-0.072836	4980
4981	19	0	0	0	18	9962	-706	0.0	-0.072944	4981
4982	20	0	0	0	18	9964	-704	0.0	-0.073052	4982
4983	22	0	0	0	15	9966	-706	0.0	-0.073160	4983
4984	20	0	0	0	20	9968	-706	0.0	-0.073268	4984
4985	17	0	0	0	21	9970	-708	0.0	-0.073376	4985
4986	19	0	0	0	24	9972	-706	0.0	-0.073484	4986
4987	24	0	0	0	4	9974	-708	0.0	-0.073592	4987
4988	22	0	0	0	15	9976	-710	0.0	-0.073700	4988
4989	19	0	0	0	20	9978	-712	0.0	-0.073808	4989
4990	22	0	0	0	19	9980	-714	0.0	-0.073916	4990
4991	21	0	0	0	20	9982	-712	0.0	-0.074024	4991
4992	18	0	0	0	19	9984	-714	0.0	-0.074132	4992
4993	21	0	0	0	17	9986	-712	0.0	-0.074240	4993
4994	19	0	0	0	22	9988	-710	0.0	-0.074348	4994
4995	17	0	0	0	18	9990	-712	0.0	-0.074456	4995
4996	26	0	0	0	20	9992	-714	0.0	-0.074564	4996
4997	25	0	0	0	12	9994	-716	0.0	-0.074672	4997
4998	22	0	0	0	7	9996	-718	0.0	-0.074780	4998
4999	20	0	0	0	21	9998	-720	0.0	-0.074888	4999
5000	20	0	0	0	21	10000	-722	0.0	-0.075000	5000

NEVER BUST STRATEGY

APPENDIX B2

ROUND *****	FIRST PLAYER *****	SECOND PLAYER *****	THIRD PLAYER *****	FOURTH PLAYER *****	DEALER *****	TOTAL BETS ****	TOTAL WINNINGS *****	INSURANCE EXPECTATION *****	BET EXPECTATION *****	GAMES PLAYED *****
4950	17	0	0	0	21	9900	-393	0.0	-0.039697	4950
4951	20	0	0	0	21	9902	-395	0.0	-0.039891	4951
4952	17	0	0	0	17	9904	-395	0.0	-0.039883	4952
4953	14	0	0	0	17	9906	-397	0.0	-0.040077	4953
4954	19	0	0	0	18	9908	-395	0.0	-0.039867	4954
4955	12	0	0	0	17	9910	-357	0.0	-0.040061	4955
4956	16	0	0	0	17	9912	-399	0.0	-0.040254	4956
4957	15	0	0	0	22	9914	-357	0.0	-0.040044	4957
4958	14	0	0	0	25	9916	-395	0.0	-0.039835	4958
4959	17	0	0	0	24	9918	-393	0.0	-0.039825	4959
4960	13	0	0	0	20	9920	-395	0.0	-0.039819	4960
4961	21	0	0	0	21	9922	-395	0.0	-0.039811	4961
4962	20	0	0	0	21	9924	-397	0.0	-0.040004	4962
4963	17	0	0	0	17	9926	-397	0.0	-0.039996	4963
4964	14	0	0	0	26	9928	-395	0.0	-0.039786	4964
4965	12	0	0	0	25	9930	-393	0.0	-0.039577	4965
4966	20	0	0	0	26	9932	-391	0.0	-0.039368	4966
4967	16	0	0	0	20	9934	-393	0.0	-0.039561	4967
4968	17	0	0	0	18	9936	-355	0.0	-0.039754	4968
4969	12	0	0	0	19	9938	-397	0.0	-0.039948	4969
4970	14	0	0	0	20	9940	-399	0.0	-0.040141	4970
4971	12	0	0	0	20	9942	-401	0.0	-0.040334	4971
4972	19	0	0	0	19	9944	-401	0.0	-0.040326	4972
4973	20	0	0	0	17	9946	-399	0.0	-0.040117	4973
4974	21	0	0	0	17	9948	-396	0.0	-0.039807	4974
4975	14	0	0	0	22	9950	-394	0.0	-0.039598	4975
4976	18	0	0	0	23	9952	-392	0.0	-0.039389	4976
4977	13	0	0	0	22	9954	-390	0.0	-0.039180	4977
4978	14	0	0	0	18	9956	-392	0.0	-0.039373	4978
4979	14	0	0	0	17	9958	-394	0.0	-0.039566	4979
4980	19	0	0	0	22	9960	-392	0.0	-0.039357	4980
4981	14	0	0	0	18	9962	-394	0.0	-0.039550	4981
4982	20	0	0	0	17	9964	-392	0.0	-0.039342	4982
4983	21	0	0	0	25	9966	-390	0.0	-0.039133	4983
4984	17	0	0	0	21	9968	-352	0.0	-0.039326	4984
4985	18	0	0	0	17	9970	-390	0.0	-0.039117	4985
4986	15	0	0	0	25	9972	-388	0.0	-0.038909	4986
4987	13	0	0	0	26	9974	-386	0.0	-0.038701	4987
4988	21	0	0	0	15	9976	-383	0.0	-0.038492	4988
4989	15	0	0	0	22	9978	-381	0.0	-0.038284	4989
4990	16	0	0	0	17	9980	-383	0.0	-0.038377	4990
4991	18	0	0	0	20	9982	-385	0.0	-0.038569	4991
4992	15	0	0	0	20	9984	-387	0.0	-0.038762	4992
4993	19	0	0	0	17	9986	-385	0.0	-0.038554	4993
4994	14	0	0	0	19	9988	-387	0.0	-0.038746	4994
4995	16	0	0	0	18	9990	-389	0.0	-0.038939	4995
4996	21	0	0	0	15	9992	-386	0.0	-0.038631	4996
4997	12	0	0	0	22	9994	-384	0.0	-0.038423	4997
4998	14	0	0	0	20	9996	-386	0.0	-0.038615	4998
4999	13	0	0	0	18	9998	-388	0.0	-0.038808	4999
5000	12	0	0	0	23	10000	-386	0.0	-0.038600	5000

BASIC STRATEGY

APPENDIX B3

ROUND *****	FIRST PLAYER *****	SECOND PLAYER *****	THIRD PLAYER *****	FOURTH PLAYER *****	DEALER *****	TOTAL BETS ****	TOTAL WINNINGS *****	INSURANCE EXPECTATION *****	BET EXPECTATION *****	GAMES PLAYED *****
49950	20	0	0	0	17	113516	99	0.0	0.000872	51111
49951	20	0	0	0	23	113518	101	0.0	0.000890	51112
49952	19	0	0	0	23	113520	103	0.0	0.000907	51113
49953	18	0	0	0	19	113522	101	0.0	0.000890	51114
49954	20	0	0	0	20	113524	101	0.0	0.000890	51115
49955	16	0	0	0	19	113526	99	0.0	0.000872	51116
49956	19	0	0	0	26	113528	101	0.0	0.000890	51117
49957	20	0	0	0	21	113530	99	0.0	0.000872	51118
49958	20	0	0	0	20	113532	99	0.0	0.000872	51119
49959	17	0	0	0	18	113534	97	0.0	0.000854	51120
49960	18	0	0	0	23	113536	99	0.0	0.000872	51121
49961	19	0	0	0	20	113538	97	0.0	0.000854	51122
49962	17	0	0	0	17	113540	97	0.0	0.000854	51123
49963	19	0	0	0	20	113542	95	0.0	0.000837	51124
49964	19	0	0	0	24	113544	97	0.0	0.000854	51125
49965	18	0	0	0	17	113546	99	0.0	0.000872	51126
49966	19	0	0	0	19	113548	99	0.0	0.000872	51127
49967	20	0	0	0	20	113550	99	0.0	0.000872	51128
49968	21	0	0	0	21	113552	99	0.0	0.000872	51129
49969	18	0	0	0	20	113554	97	0.0	0.000854	51130
49970	20	0	0	0	18	113556	99	0.0	0.000872	51131
49971	21	0	0	0	7	113558	102	0.0	0.000898	51132
49972	25	0	0	0	18	113560	100	0.0	0.000881	51133
49973	21	0	0	0	23	113562	102	0.0	0.000898	51134
49974	21	0	0	0	17	113564	104	0.0	0.000916	51135
49975	17	0	0	0	24	113566	106	0.0	0.000933	51136
49976	19	0	0	0	24	113568	108	0.0	0.000951	51137
49977	20	0	0	0	21	113570	106	0.0	0.000933	51138
49978	17	0	0	0	20	113574	102	0.0	0.000898	51139
49979	20	0	0	0	23	113576	104	0.0	0.000916	51140
49980	17	20	0	0	19	113582	102	0.0	0.000898	51142
49981	20	0	0	0	22	113584	104	0.0	0.000916	51143
49982	21	0	0	0	19	113588	108	0.0	0.000951	51144
49983	13	0	0	0	21	113590	106	0.0	0.000933	51145
49984	14	0	0	0	24	113594	110	0.0	0.000968	51146
49985	25	0	0	0	20	113596	108	0.0	0.000951	51147
49986	17	0	0	0	18	113598	106	0.0	0.000933	51148
49987	22	0	0	0	13	113600	104	0.0	0.000915	51149
49988	20	0	0	0	17	113602	106	0.0	0.000933	51150
49989	17	0	0	0	17	113604	106	0.0	0.000933	51151
49990	24	0	0	0	12	113606	104	0.0	0.000915	51152
49991	18	0	0	0	22	113608	106	0.0	0.000933	51153
49992	15	0	0	0	23	113610	108	0.0	0.000951	51154
49993	13	0	0	0	19	113612	106	0.0	0.000933	51155
49994	21	0	0	0	19	113614	108	0.0	0.000951	51156
49995	18	0	0	0	17	113616	110	0.0	0.000968	51157
49996	26	0	0	0	11	113618	108	0.0	0.000951	51158
49997	21	0	0	0	18	113620	111	0.0	0.000977	51159
49998	18	0	0	0	21	113622	109	0.0	0.000959	51160
49999	19	0	0	0	23	113624	111	0.0	0.000977	51161
50000	20	0	0	0	20	113626	111	0.0	0.000977	51162

BASIC STRATEGY USING TENS-RATIO FOR BETS

APPENDIX B4

ROUND *****	FIRST PLAYER *****	SECOND PLAYER *****	THIRD PLAYER *****	FOURTH PLAYER *****	DEALER *****	TOTAL BETS ****	TOTAL WINNINGS *****	INSURANCE EXPECTATION *****	BET EXPECTATION *****	GAMES PLAYED *****
45950	20	0	0	0	17	387078	1408	-0.171867	0.003638	51111
49951	20	0	0	0	23	387088	1418	-0.171867	0.003643	51112
49952	19	0	0	0	23	387090	1420	-0.171867	0.003648	51113
49953	18	0	0	0	19	387092	1418	-0.171867	0.003643	51114
49954	20	0	0	0	20	387094	1418	-0.171867	0.003648	51115
49955	16	0	0	0	19	387096	1416	-0.171867	0.003658	51116
49956	19	0	0	0	26	387098	1418	-0.171867	0.003663	51117
49957	20	0	0	0	21	387108	1408	-0.172214	0.003637	51118
49958	20	0	0	0	20	387118	1408	-0.172214	0.003637	51119
49959	17	0	0	0	18	387128	1398	-0.172561	0.003611	51120
49960	18	0	0	0	23	387138	1408	-0.172561	0.003637	51121
45561	19	0	0	0	20	387148	1398	-0.172561	0.003611	51122
49962	17	0	0	0	17	387158	1398	-0.172561	0.003611	51123
49963	19	0	0	0	20	387168	1388	-0.172561	0.003585	51124
49964	19	0	0	0	24	387178	1398	-0.172561	0.003611	51125
49965	18	0	0	0	17	387188	1408	-0.172561	0.003636	51126
49966	19	0	0	0	19	387198	1408	-0.172561	0.003636	51127
49967	20	0	0	0	20	387208	1408	-0.172561	0.003636	51128
49968	21	0	0	0	21	387218	1408	-0.172561	0.003636	51129
49969	18	0	0	0	20	387220	1406	-0.172561	0.003631	51130
49970	20	0	0	0	18	387230	1416	-0.172561	0.003637	51131
49971	21	0	0	0	7	387232	1416	-0.172561	0.003664	51132
49972	25	0	0	0	18	387242	1409	-0.172561	0.003639	51133
49973	21	0	0	0	23	387244	1411	-0.172561	0.003644	51134
49974	21	0	0	0	17	387246	1413	-0.172630	0.003649	51135
49975	17	0	0	0	24	387256	1423	-0.172630	0.003675	51136
49976	19	0	0	0	24	387266	1433	-0.172630	0.003700	51137
49977	20	0	0	0	21	387270	1429	-0.172630	0.003690	51138
49978	17	0	0	0	20	387290	1409	-0.172630	0.003638	51139
49979	20	0	0	0	23	387300	1419	-0.172630	0.003664	51140
49980	17	20	0	0	19	387306	1417	-0.172630	0.003659	51142
49981	20	0	0	0	22	387308	1419	-0.172700	0.003664	51143
49982	21	0	0	0	19	387328	1439	-0.172700	0.003715	51144
49983	13	0	0	0	21	387338	1429	-0.171790	0.003689	51145
49984	14	0	0	0	24	387358	1446	-0.171790	0.003741	51146
49985	25	0	0	0	20	387368	1439	-0.171790	0.003715	51147
49986	17	0	0	0	18	387378	1429	-0.171790	0.003689	51148
49987	22	0	0	0	13	387388	1419	-0.171790	0.003663	51149
49988	20	0	0	0	17	387390	1421	-0.171790	0.003668	51150
49989	17	0	0	0	17	387400	1421	-0.171790	0.003668	51151
49990	24	0	0	0	12	387410	1411	-0.171790	0.003642	51152
49991	18	0	0	0	22	387412	1413	-0.171790	0.003647	51153
49992	15	0	0	0	23	387414	1415	-0.171790	0.003652	51154
49993	13	0	0	0	19	387416	1413	-0.171790	0.003647	51155
49994	21	0	0	0	19	387418	1415	-0.171859	0.003652	51156
49995	18	0	0	0	17	387428	1425	-0.171859	0.003678	51157
49996	26	0	0	0	11	387438	1415	-0.171859	0.003652	51158
49997	21	0	0	0	18	387440	1418	-0.171859	0.003660	51159
49998	18	0	0	0	21	387442	1416	-0.171859	0.003655	51160
49999	19	0	0	0	23	387444	1418	-0.171859	0.003660	51161
50000	20	0	0	0	20	387446	1418	-0.171859	0.003660	51162

BASIC STRATEGY USING HIGH-LOW INDEX FOR BETS

APPENDIX B5

ROUND *****	FIRST PLAYER *****	SECOND PLAYER *****	THIRD PLAYER *****	FOURTH PLAYER *****	DEALER *****	TOTAL BETS *****	TOTAL WINNINGS *****	INSURANCE EXPECTATION *****	BET EXPECTATION *****	GAMES PLAYED *****
49950	20	0	0	0	17	338490	3966	-0.021927	0.011717	51111
49951	20	0	0	0	23	338492	3968	-0.021927	0.011723	51112
49952	19	0	0	0	23	338494	3970	-0.021927	0.011728	51113
49953	18	0	0	0	19	338496	3968	-0.021927	0.011722	51114
49954	20	0	0	0	20	338498	3968	-0.021927	0.011722	51115
49955	16	0	0	0	19	338500	3966	-0.021927	0.011716	51116
49956	19	0	0	0	26	338510	3976	-0.021927	0.011746	51117
49957	20	0	0	0	21	338520	3966	-0.022412	0.011716	51118
49958	20	0	0	0	20	338530	3966	-0.022412	0.011715	51119
49959	17	0	0	0	18	338534	3962	-0.022412	0.011703	51120
49960	18	0	0	0	23	338536	3964	-0.022412	0.011709	51121
49961	19	0	0	0	20	338542	3958	-0.022412	0.011691	51122
49962	17	0	0	0	17	338552	3958	-0.022412	0.011691	51123
49963	15	0	0	0	20	338562	3958	-0.022412	0.011661	51124
49964	19	0	0	0	24	338572	3958	-0.022412	0.011690	51125
49965	18	0	0	0	17	338582	3968	-0.022412	0.011719	51126
49966	19	0	0	0	19	338592	3968	-0.022412	0.011719	51127
49967	20	0	0	0	20	338602	3968	-0.022412	0.011719	51128
49968	21	0	0	0	21	338612	3968	-0.022412	0.011718	51129
49969	18	0	0	0	20	338614	3966	-0.022412	0.011712	51130
49970	20	0	0	0	18	338616	3968	-0.022412	0.011718	51131
49971	21	0	0	0	7	338618	3971	-0.022412	0.011727	51132
49972	25	0	0	0	18	338620	3969	-0.022412	0.011721	51133
49973	21	0	0	0	23	338622	3971	-0.022412	0.011727	51134
49974	21	0	0	0	17	338624	3973	-0.022412	0.011733	51135
49975	17	0	0	0	24	338628	3977	-0.022412	0.011744	51136
49976	19	0	0	0	24	338636	3985	-0.022412	0.011768	51137
49977	20	0	0	0	21	338646	3975	-0.022412	0.011738	51138
49978	17	0	0	0	20	338650	3971	-0.022412	0.011726	51139
49979	20	0	0	0	23	338656	3977	-0.022412	0.011743	51140
49980	17	20	0	0	19	338662	3975	-0.022412	0.011737	51142
49981	20	0	0	0	22	338664	3977	-0.022412	0.011743	51143
49982	21	0	0	0	19	338672	3985	-0.022412	0.011767	51144
49983	13	0	0	0	21	338676	3981	-0.022412	0.011755	51145
49984	14	0	0	0	24	338680	3985	-0.022412	0.011766	51146
49985	25	0	0	0	20	338690	3975	-0.022412	0.011736	51147
49986	17	0	0	0	18	338700	3965	-0.022412	0.011707	51148
49987	22	0	0	0	13	338702	3963	-0.022412	0.011701	51149
49988	20	0	0	0	17	338704	3965	-0.022412	0.011706	51150
49989	17	0	0	0	17	338706	3965	-0.022412	0.011706	51151
49990	24	0	0	0	12	338708	3963	-0.022412	0.011700	51152
49991	18	0	0	0	22	338710	3965	-0.022412	0.011706	51153
49992	15	0	0	0	23	338712	3967	-0.022412	0.011712	51154
49993	13	0	0	0	19	338714	3965	-0.022412	0.011706	51155
49994	21	0	0	0	19	338720	3971	-0.022412	0.011724	51156
49995	18	0	0	0	17	338730	3981	-0.022412	0.011753	51157
49996	26	0	0	0	11	338740	3971	-0.022412	0.011723	51158
49997	21	0	0	0	18	338742	3974	-0.022412	0.011732	51159
49998	18	0	0	0	21	338744	3972	-0.022412	0.011726	51160
49999	19	0	0	0	23	338746	3974	-0.022412	0.011732	51161
50000	20	0	0	0	20	338748	3974	-0.022412	0.011731	51162

APPENDIX CSTRATEGY EVALUATOR PROGRAM

This Appendix contains a copy of the strategy evaluator program and the subroutine used for dealing.

The variables, arrays, and subroutines used and their purposes are presented in Table C1.

The operation of both routines is explained in the text. The comments on the listings should help the reader to distinguish portions of the program which correspond to different situations which occur in the game.

TABLE C1LIST OF VARIABLES AND ARRAYS

BETEX - bet expectation
EXINS - insurance expectation
IACE - flag set to 1 if an ace is dealt
IBET - total winnings
IBTOT - total amount bet
ICARD(52) - present deck
IDECK(52) - "next" deck
IDUBL - flag set to 1 if player wants to double down
INS(4) - amount of insurance bet on hand
INSTOT - total amount of all insurance bets placed
INSWON - total amount of insurance winnings
IPLAY(4,11) - cards received by player(s)
ITOT(4) - each player's total
ISPLIT - flag set to 1 if player wants to split a pair
ISTRAT - flag set to 1 if player wants to draw
IVAL - numerical value of card dealt
IWON - sum of bet and insurance winnings
IX - input to random number generator
JBET(4) - amount bet by each player on hand
K- number of cards left in present deck
KDLR(11) - dealer's cards

KTR - face value of card dealt
LTOT - dealer's total
MACE(4) - number of aces in each player's hand
NGAME - total number of games played
NP - number of player actually playing
NPLAY - total number of players in that round

LIST OF SUBROUTINES

BETTR - sets the bet value by putting it into JBET(NP)
DBLDN - tell whether or not player wants to double down
by setting the IDUBL flag
DEALA - deals cards by setting the IACE, IVAL, and
KTR variables
INSUR - sets the value of the insurance bet in INS(NP)
SPLIT - tells whether player wants to split a pair
by setting the ISPLIT flag
STRAT - tells whether player wants to draw by setting
the ISTRAT flag

```

C*****
C
C
C   THIS IS THE MAIN BODY OF THE PROGRAM.  IT ALLOWS THE GAME OF BLACKJACK TO
C   BE PLAYED ACCORDING TO THE RULES SEEN UNDER NORMAL CASINO SITUATIONS.
C
C*****
C
C   INITIALIZE VARIABLES
C
C*****
0001      COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
          2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
          3NP, K, IACE
0002      1600 FORMAT( 10X, 'FIRST', 4X, 'SECOND', 3X, 'THIRD', 4X, 'FCURTH', 13X, 'TCTA
          2L', 4X, 'TOTAL', 6X, 'INSURANCE', 8X, 'BET', 7X, 'GAMES')
0003      1601 FORMAT(2X, 'RCUAD', 3X, 4('PLAYER', 3X), 'DEALER', 4X, 'BETS', 4X, 'WINNING
          2S', 3X, 2(' EXPECTATION', 3X), 'PLAYED')
0004      1602 FORMAT(1X, I6, 5X, 4(I2, 7X), I2, 5X, 2(I6, 4X), 2(F11.6, 3X), I6)
0005      1603 FORMAT(2X, 5('**'), 3X, 5(6('**'), 3X), 1X, 4('**'), 4X, 8('**'), 2(3X, 11('**'))
          2, 3X, 6('**'), //)
0006      1604 FORMAT(1H1, '      BASIC STRATEGY', //)
0007          JKLM=50000
0008          WRITE(6,1604)JKLM
0009          WRITE(6,1600)
0010          WRITE(6,1601)
0011          WRITE(6,1603)
0012          IX=913
0013          IWON=0
0014          IBTCT=0
0015          IBET=0
0016          INSTOT=0
0017          INSWON=0
0018          NGAME=0
0019          DO 10 IC=1,13
0020             IDECK(IC)=IC
0021      10  CONTINUE
0022             DO 20 IC=1,39
0023                IDECK(IC+13)=ICECK(IC)
0024      20  CONTINUE
0025          K=0
C*****
C
C   THE MAIN LOOP STARTS HERE

```

```

C
C
C*****
0026      DO 100 IXYZ=1,JKLM
0027      DO 105 I=1,4
0028      ITOT(I)=0
0029      IBUST(I)=0
0030      MACE(I)=0
0031      IBJ(I)=0
0032      INS(I)=0
0033      JBET(I)=0
0034      DC 105 J=1,11
0035      IPLAY(I,J)=0
0036      105 CONTINUE
0037      NPLAY=1
0038      NP=NPLAY
0039      LTOT=0
0040      LACE=0
0041      LBJ=0
C*****
C
C
C      CHOOSE PLAYER'S BET
C
C
C*****
0042      CALL BETTR
0043      IBTOT=IBTOT+JBET(NP)
C*****
C
C
C      DEAL PLAYER TWO CARDS
C
C
C*****
0044      DO 110 I=1,2
0045      CALL DEALA
0046      ITOT(NP)=ITOT(NP)+IVAL
0047      IPLAY(NP,I)=KTR
0048      IF(IVAL-11)110,120,110
0049      120 MACE(NP)=MACE(NP)+1
0050      110 CONTINUE
0051      IF(ITOT(NP)-21)130,140,150
0052      140 IBJ(NP)=999
0053      GO TO 130
0054      150 ITOT(NP)=ITOT(NP)-10
0055      MACE(NP)=MACE(NP)-1
C*****

```

```

C
C
C   DEAL DEALER TWO CARDS
C
C*****
0056   130  DO 170 J=1,2
0057       CALL DEALA
0058       LTOT=LTOT+IVAL
0059       KDLR(J)=KTR
0060       IF(IVAL-1)170,180,170
0061   180  LACE=LACE+1
0062   170  CONTINUE
C*****
C
C
C   CHECK FOR INSURANCE AND NATURALS
C
C*****
0063       IF (LTOT-21)190,200,210
0064   200  LBJ=999
0065       GO TO 190
0066   210  LTOT=LTOT-10
0067       LACE=LACE-1
0068   190  IF (KDLR(1)-1)230,220,230
0069   220  CALL INSUR
0070       INSTOT=INSTOT+INS(NP)
0071   230  IF(LBJ)240,240,250
0072   240  INSWON=INSWON-INS(NP)
0073       IF(IBJ(NP))260,260,270
0074   270  IBET=IBET+(3*JBET(NP))/2
0075       GO TO 680
0076   250  INSWON=INSWON+2*INS(NP)
0077       IF(LBJ-IBJ(NP))680,680,280
0078   280  IBET=IBET-JBET(NP)
0079       GO TO 680
C*****
C
C
C   CHECK FOR SPLITTING PAIRS
C
C*****
0080   260  NP=1
0081   290  IF(IPLAY(NP,1)-IPLAY(NP,2))300,310,300
0082   310  CALL SPLIT
C*****

```

```

C
C
C   SPLIT IF ISPLIT IS SET TO 1
C
C*****
0083   IF (ISPLIT-1)300,320,300
0084   320 NPLAY=NPLAY+1
0085       NPLAYP=NPLAY+1
0086       JPL=NPLAY-NP
0087       DO 330 KAS=1,JPL
0088           I=NPLAYP-KAS
0089           IPLAY(I,1)=IPLAY(I-1,1)
0090           IPLAY(I,2)=IPLAY(I-1,2)
0091           JBET(I)=JBET(I-1)
0092           IBTCT=IBTCT + JBET(I)
0093   330 CONTINUE
0094       CALL DEALA
0095           IPLAY(I-1,2)=KTR
0096           IF (IPLAY(I-1,1)-1)331,332,331
0097   331 IF (IPLAY(I-1,1)-10) 334,335,335
0098   334 ITOT(I-1)=IPLAY(I-1,1)
0099       GO TO 333
0100   335 ITOT(I-1)=10
0101       GO TO 333
0102   332 ITOT(I-1)=11
0103       MACE(I-1)=1
0104   333 ITOT(I-1)=IVAL+ITOT(I-1)
0105       IF (IVAL-11)340,350,340
0106   350 MACE(I-1)=MACE(I-1)+1
0107   340 CALL DEALA
0108       IPLAY(I,2)=KTR
0109       IF (IPLAY(I,1)-1)341,342,341
0110   341 IF (IPLAY(I,1)-10) 344,345,345
0111   344 ITOT(I)=IPLAY(I,1)
0112       GO TO 343
0113   345 ITOT(I)=10
0114       GO TO 343
0115   342 ITOT(I)=11
0116       MACE(I)=1
0117   343 ITOT(I)=IVAL+ITOT(I)
0118       IF (IVAL-11)290,370,290
0119   370 MACE(NP)=MACE(NP)+1
0120       GO TO 290
0121   300 IF (NP-NPLAY)300,390,390
0122   380 NP=NP+1
0123       GO TO 290
C*****
C

```

```

C
C   SEE IF A PLAYER WANTS TO DOUBLE DOWN
C
C
C*****
0124 390 DO 531 NP=1,NPLAY
0125   CALL DBLON
0126   IF (ICUBL-1)400,410,400
0127 410 IBTGT=IBTCT + JBET(NP)
0128   JBET(NP)=2*JBET(NP)
0129   CALL DEALA
0130   IPLAY(NP,3)=KTR
0131   ITCT(NP)=ITOT(NP)+IVAL
0132   IF (IACE) 441,441,440
0133 440 MACE(NP)=MACE(NP)+1
0134 441 IF (ITOT(NP)-21) 531,531,442
0135 442 IF (MACE(NP)) 490,490,443
0136 443 ITCT(NP)=ITOT(NP)-10
0137   MACE(NP)=MACE(NP)-1
0138   GO TO 531
C*****
C
C   CHECK IF PLAYER WOULD LIKE TO DRAW
C
C
C*****
0139 400 KNP=2
0140 460 CALL STRAT
0141   IF (ISTRAT-1)531,450,531
0142 450 CALL DEALA
0143   KNP=KNP+1
0144   ITCT(NP)=ITCT(NP)+IVAL
0145   IPLAY(NP,KNP)=KTR
0146   IF (IACE)470,470,480
0147 480 MACE(NP)=MACE(NP)+1
0148 470 IF (ITCT(NP)-21) 460,460,500
0149 500 IF (MACE(NP))490,490,510
0150 510 MACE(NP)=MACE(NP)-1
0151   ITGT(NP)=ITCT(NP)-10
0152   GO TO 460
0153 450 IBUST(NP)=1
0154 531 CONTINUE
C*****
C
C   SEE IF DEALER MUST DRAW
C

```

```

C
C*****
0155      KBUST=0
0156      DO 591 NP=1,NPLAY
0157      KBUST=KBUST+IBUST(NP)
0158      591  CCINUE
0159      IF (KBUST-NPLAY)592,600,600
0160      592  KD=2
0161      590  IF (LTOT-16)580,580,570
0162      580  CALL DEALA
0163      IF (IACE)593,593,594
0164      594  LACE=LACE+1
0165      593  LTOT=LTCT+IVAL
0166      KD=KD+1
0167      KDLR(KD)=KTR
0168      GO TO 590
0169      570  IF (LTOT-21)620,620,610
0170      610  IF (LACE)630,630,640
0171      640  LACE=LACE-1
0172      LTCT=LTCT-10
0173      GO TO 590
C*****
C
C
C      ALL PLAYER(S) HAVE BUSTED
C
C
C*****
0174      600  DO 601 NP=1,NPLAY
0175      IBET=IBET-JBET(NP)
0176      601  CONTINUE
0177      GO TO 680
C*****
C
C
C      DEALER HAS BUSTED
C
C
C*****
0178      630  DO 631 NP=1,NPLAY
0179      IF (IBUST(NP)) 632,632,633
0180      632  IBET=IBET+JBET(NP)
0181      GO TO 631
0182      633  IRET=IBET-JBET(NP)
0183      631  CONTINUE
0184      GO TO 680
C*****
C

```



```

C
C   NEITHER HAS BUSTED
C
C
C*****
0185   620  DD 68C NP=1,NPLAY
0186       IF (IBUST(NP))625,625,624
0187   625  IF ((ITOT(NP)-LTOT) 624,680,622
0188   622  IBET=IBET+JBET(NP)
0189       GO TO 680
0190   624  IBET=IBET-JBET(NP)
0191   680  CONTINUE
0192       NGAME=NGAME+NPLAY
0193       BET=IBET
0194       BTOT=IBTCT
0195       BETEX=BET/BTCT
0196       RINS=INSTCT
0197       BNS=INSWON
0198       IF (INSTCT) 681,681,682
0199   682  EXINS = BNS / BINS
0200       GO TO 683
0201   681  EXINS=0.
0202       IF (200-JKLM+IXY7) 100,683,683
0203   683  WRITE(6,16C2)IXYZ,(ITOT(NP),NP=1,4),LTCT,IBTCT,IBET,EXINS,BETEX,NG
2AME
0204   100  CONTINUE
0205       END

```

```

C*****
C
C
C   THIS SUBROUTINE WILL DEAL A CARD WHENEVER ONE IS REQUESTED.  IT WILL
C   CONTINUALLY REMOVE ONE CARD AT A TIME FROM A SHUFFLED DECK UNTIL THAT
C   DECK HAS BECOME DEPLETED.  AT THAT TIME IT WILL START DEALING FROM A
C   FRESHLY SHUFFLED DECK.
C
C*****
0001      SUBROUTINE DEALA
0002      CCMCN ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KCLR(11), ITCT(4), JBET(4
          2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDOBL, ISTRAT, IBUST(4), IBJ(4),
          3NP, K, IACE
0003          IACE=0
0004          IF(K)910,510,920
0005      910  DO 900 IK=1,52
0006          ICARD(IK)=IDECK(IK)
0007      900  CONTINUE
0008          K=52
0009          NUM=51
0010      920  CALL RANDU(IX, IX, YFL)
0011          ANUM=NUM
0012          N=ANUM*YFL
0013          N=N+1
0014          KTR=ICARD(N)
0015          IDECK(K)=ICARD(N)
0016          ICARD(N)=ICARD(K)
0017          GO TO(930,540,540,540,940,940,940,940,940,940,950,560,970),KTR
0018      930  TR=1
0019          IVAL=11
0020          IACE=1
0021          GO TO 990
0022      950  TR=2
0023          GO TO 980
0024      960  TR=3
0025          GO TO 980
0026      970  TR=4
0027      980  IVAL=10
0028          INDEX=2
0029          GO TO 990
0030      940  IVAL=KTR
0031          INDEX=1
0032      990  NUM=NUM-1
0033          K=K-1
0034          RETURN
0035          END

```

APPENDIX DSTRATEGY SUBROUTINES

This Appendix contains the listings of all the strategy subroutines used in the simulations broken up as follows:

- D1. All of the strategies for imitating the dealer and never busting except for the drawing strategy.
- D2. Drawing strategy for imitating the dealer.
- D3. Drawing strategy for never busting.
- D4. All the strategies for Thorp's basic strategy, tens ratio, and high-low except for betting and insuring.
- D5. Betting and insuring for Thorp's basic strategy.
- D6. Betting and insuring for tens ratio.
- D7. Betting and insuring for high-low.

APPENDIX D1

```

C*****
C
C
C   STRATEGY SUBROUTINES
C
C*****
C
C   BET WILL ALWAYS BE 2 UNITS
C
C*****
0001  SUBROUTINE BETTR
0002  COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITCT(4), JBET(4
      2), INS(4), IPLAY(4, 11), MACE(4), ISPLIT, IDOUBLE, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACF
0003  JBET(NP)=2
0004  RETURN
0005  END

C*****
C
C
C   NEVER INSURE
C
C*****
0001  SUBROUTINE INSUR
0002  COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4, 11), MACE(4), ISPLIT, IDOUBLE, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003  INS(NP)=0
0004  RETURN
0005  END

C*****
C
C
C   NEVER DOUBLE DOWN
C
C*****
0001  SUBROUTINE CBLDN
0002  COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4, 11), MACE(4), ISPLIT, IDOUBLE, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003  IDOUBLE=0
0004  RETURN
0005  END

C*****
C
C
C   NEVER SPLIT
C
C*****
0001  SUBROUTINE SPLIT
0002  COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITCT(4), JBET(4
      2), INS(4), IPLAY(4, 11), MACE(4), ISPLIT, IDOUBLE, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACF
0003  ISPLIT=0
0004  RETURN
0005  END

```

APPENDIX D2

```

C*****
C
C
C   IMITATE THE DEALER
C
C*****
0001  SUBROUTINE STRAT
0002  CCMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KOLR(11), ITCT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), TSPLIT, ICUBL, ISTRAT, IAUST(4), IBJ(4),
      3NP, K, IACE
0003  IF (ITOT(NP)-16) 1020,1020,1030
0004  1020 ISTRAT=1
0005  RETURN
0006  1030 ISTRAT=0
0007  RETURN
0008  END

```

APPENDIX D3

```
C*****
C
C   NEVER BUST
C
C*****
0001   SUBROUTINE STRAT
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4, 11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   IF (MACE(NP)) 1020, 1020, 1030
C     STOP DRAWING AT HARD 12
0004   1020 IF (ITOT(NP)-12) 1040, 1050, 1050
C     STOP DRAWING AT SCFT 17
0005   1030 IF (ITOT(NP)-18) 1040, 1050, 1050
0006   1040 ISTRAT=1
0007   RETURN
0008   1050 ISTRAT=0
0009   RETURN
0010   END
```

APPENDIX D4

```

C*****
C
C
C   BASIC DOUBLING DOWN STRATEGY
C
C
C*****
0001   SUBROUTINE DBLDN
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KCLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDOUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   IF (MACE(NP)) 2000,2000,2011
C     HARD DOUBLING
0004   2000 IF (ITOT(NP)-11) 2010,2050,2060
0005   2010 IF (ITOT(NP)-10) 2030,2020,2060
0006   2020 IF (KCLR(1) -10) 2051,2060,2060
0007   2030 IF (ITOT(NP)- 9) 2060,2040,2060
0008   2040 IF (KCLR(1)-7) 2051,2060,2060
C     SOFT DOUBLING
0009   2011 IF (KCLR(1)-7) 2070,2060,2060
0010   2070 IDUM=KCLR(1)
0011   GO TO (2060,2072,2073,2074,2075,2075),IDUM
0012   2072 IF (ITOT(NP)-17) 2060,2050,2060
0013   2073 IF (ITOT(NP)-17) 2060,2050,2052
0014   2074 IF (ITOT(NP)-13) 2060,2050,2052
0015   2075 IF (ITOT(NP)-12) 2060,2050,2052
0016   2051 IF (KCLR(1)-1) 2060,2060,2050
0017   2052 IF (ITOT(NP)-19) 2050,2060,2060
C     DOUBLE DOWN
0018   2050 IDOUBL=1
0019   RETURN
C     DON'T DOUBLE DCWN
0020   2060 IDOUBL=0
0021   RETURN
0022   END

```

```

C*****
C
C
C   BASIC STRATEGY FOR SPLITTING PAIRS
C
C
C*****
0001   SUBROUTINE SPLIT
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KCLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDOUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   IDM=IPLAY(NP,1)
0004   GO TO (1070,1080,1080,1081,1090,1080,1082,1070,1083,1090,1090,1090
      2,1090),IDM
0005   1080 IF (KCLR(1)-8) 1071,1090,1090
0006   1081 IF (KCLR(1)-5) 1090,1071,1090
0007   1082 IF (KCLR(1)-9) 1071,1090,1071
0008   1083 IF (KCLR(1)-9) 1084,1071,1090
0009   1084 IF (KCLR(1)-7) 1071,1090,1071
0010   1071 IF (KCLR(1)-1) 1070,1090,1070
C     SPLIT
0011   1070 ISPLIT=1
0012   RETURN
C     DON'T SPLIT
0013   1090 ISPLIT=0
0014   RETURN
0015   END

```

```

C*****
C
C
C   BASIC STRATEGY FOR DRAWING
C
C*****
0001   SUBROUTINE STRAT
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   IF (MACE(NP)) 1020,1020,1030
C     HARD STANDING
0004   1020 JDUM=KDLR(1)
0005   GO TO (1021,1022,1022,1023,1023,1023,1021,1021,1021,1021,1021,1021
      2,1021),JDUM
0006   1021 IF (ITOT(NP)-17) 1040,1050,1050
0007   1022 IF (ITOT(NP)-13) 1040,1050,1050
0008   1023 IF (ITOT(NP)-12) 1040,1050,1050
C     SOFT STANDING
0009   1030 IF (KDLR(1)-9) 1031,1032,1032
0010   1031 IF (ITOT(NP)-18) 1040,1050,1050
0011   1032 IF (ITOT(NP)-19) 1040,1050,1050
C     DRAW A CARD
0012   1040 ISTRAT=1
0013   RETURN
C     DON'T DRAW ANY MORE CARDS
0014   1050 ISTRAT=0
0015   RETURN
0016   END

```


APPENDIX D 5

```

C*****
C
C
C   STRATEGY SUBROUTINES
C
C*****
C
C
C   BET WILL ALWAYS BE 2 UNITS
C
C*****
0001   SUBROUTINE BETTR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, ICUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   JBET(NP)=2
0004   RETURN
0005   END

C*****
C
C
C   NEVER INSURE
C
C*****
0001   SUBROUTINE INSUR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE
0003   INS(NP)=0
0004   RETURN
0005   END

```

APPENDIX D6

```

C*****
C
C
C   STRATEGY SUBROUTINES
C
C
C*****
C
C   TENS-RATIO USED TO DETERMINE BET SIZE
C
C*****
0001   SUBROUTINE BETTR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITCT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE, TR
0003       IF (TR-2.00) 3003,3003,3000
0004   3003 IF (TR-1.65) 3005,3005,3006
0005   3006 IF (TR-1.75) 3004,3004,3002
0006   3000 JBET(NP)=2
0007       RETURN
0008   3002 JBET(NP)=4
0009       RETURN
0010   3004 JBET(NP)=8
0011       RETURN
0012   3005 JBET(NP)=10
0013       RETURN
0014       END

C*****
C
C
C   TENS-RATIO USED TO DETERMINE INSURANCE SIZE
C
C*****
0001   SUBROUTINE INSUR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITCT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IDUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE, TR
0003       IF (TR-2.00) 4000,4000,4010
0004   4000 INS(NP)=JBET(NP)/2
0005       RETURN
0006   4010 INS(NP)=0
0007       RETURN
0008       END

```

APPENDIX D7

```

C*****
C
C
C   STRATEGY SUBROUTINES
C
C
C*****
C
C   HIGH-LOW INDEX DETERMINES BET SIZE
C
C*****
0001   SUBROUTINE BETTR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IOUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE, IFL, JHL
0003   IF (IHL) 3000,3000,3010
0004   3010 IF (K) 3000,3000,3020
0005   3020 HL=IHL
0006   HK=K
0007   BETS= 50. * HL / HK
0008   JHL=BETS
0009   IF (JHL-5) 3030,3030,3040
0010   3030 JBET(NP)=2 * JHL
0011   RETURN
0012   3040 JBET(NP)=10
0013   RETURN
0014   3000 JBET(NP)=2
0015   RETURN
0016   END

```

```

C*****
C
C
C   HIGH-LOW INDEX DETERMINES INSURANCE SIZE
C
C
C*****
0001   SUBROUTINE INSUR
0002   COMMON ICARD(52), IDECK(52), NUM, IX, KTR, IVAL, KDLR(11), ITOT(4), JBET(4
      2), INS(4), IPLAY(4,11), MACE(4), ISPLIT, IOUBL, ISTRAT, IBUST(4), IBJ(4),
      3NP, K, IACE, IFL, JHL
0003   IF (JHL*2-8) 4000,4000,4010
0004   4000 INS(NP)=0
0005   RETURN
0006   4010 INS(NP)=JBET(NP) /2
0007   RETURN
0008   END

```

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