DYNAMICS OF A CONDUCTING SPHERE

BETWEEN CAPACITOR PLATES

by

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ABSTRACT

Experiments are performed to attempt to verify theoretical predictions for the force and charge on a sphere resting on an infinite conducting plane in a uniform electric field. Both gases and polar and non-polar liquids are used as dielectrics. Higher fields than those predicted are required for liftoff of metallic spheres having diameter 1/16 inch. Gaseous freon and castor oil require more than twice the predicted field; non-polar liquids such as transformer oil require approximately 20% higher field than predicted.

Charge on the particle at liftoff is measured by finding the period and maximum height of oscillation of a conducting sphere in a highly viscous, slightly conducting liquid (corn oil). Measurement of charge indicates it deviates from predictions by 20%, but experimental uncertainties are approximately 30%.

A liftoff experiment is performed on a closely packed array of spheres, with liquid dielectrics. Force on this array is computed using the Maxwell stress-tensor. It is found that one sphere lifts off at a 10-20% lower field than predicted, followed by an avalanche of the remaining particles.

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Thesis Supervisor: Prof. J. R. Melcher

Acknowledgement

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ε dielectric constant of fluid

ε_o dielectric constant of air

r radius of sphere or cross-sectional circle

E electric field

F_e electric force

F_g gravitational force

ρ mass density of particle

ρ_{fluid} mass density of fluid

g gravitational constant of earth

Q charge on particle

η viscosity

R_y Reynolds number

D drag force

σ_f free surface charge

σ conductivity of liquid

ξ height

τ relaxation time of liquid

I. Statement of Problem

Two questions are examined in this thesis. The first is what

electric field is required to levitate a single conducting sphere

initially resting on an infinite plate in a uniform electric field.

Figure 1. Single particle test apparatus

Theoretical predictions for the charge and electric force on such a

sphere are given by Maxwell[6] ($Q = (1.64)4\pi\epsilon r^2 E$), and Lebedev and

Skal'skaya[5] ($Q = (1.64)4\pi\epsilon r^2 E$, $F_e = (1.37)4\pi\epsilon r^2 E^2$). Equating F_e

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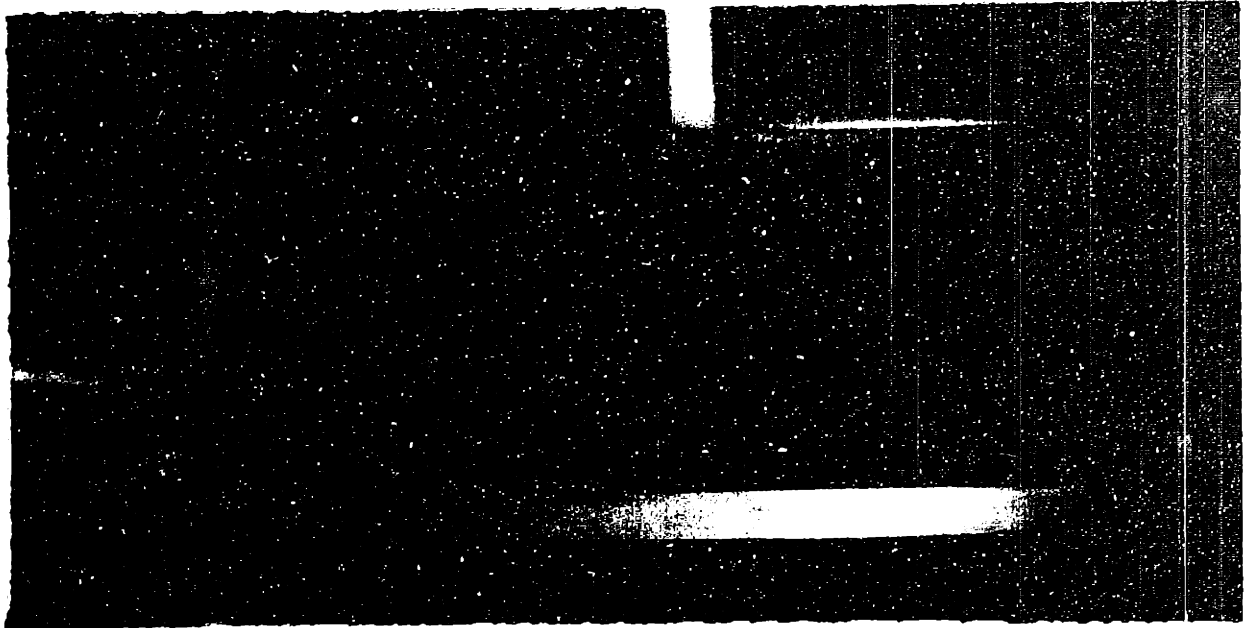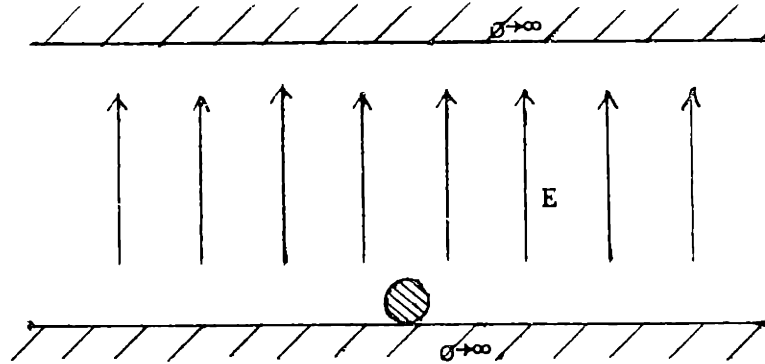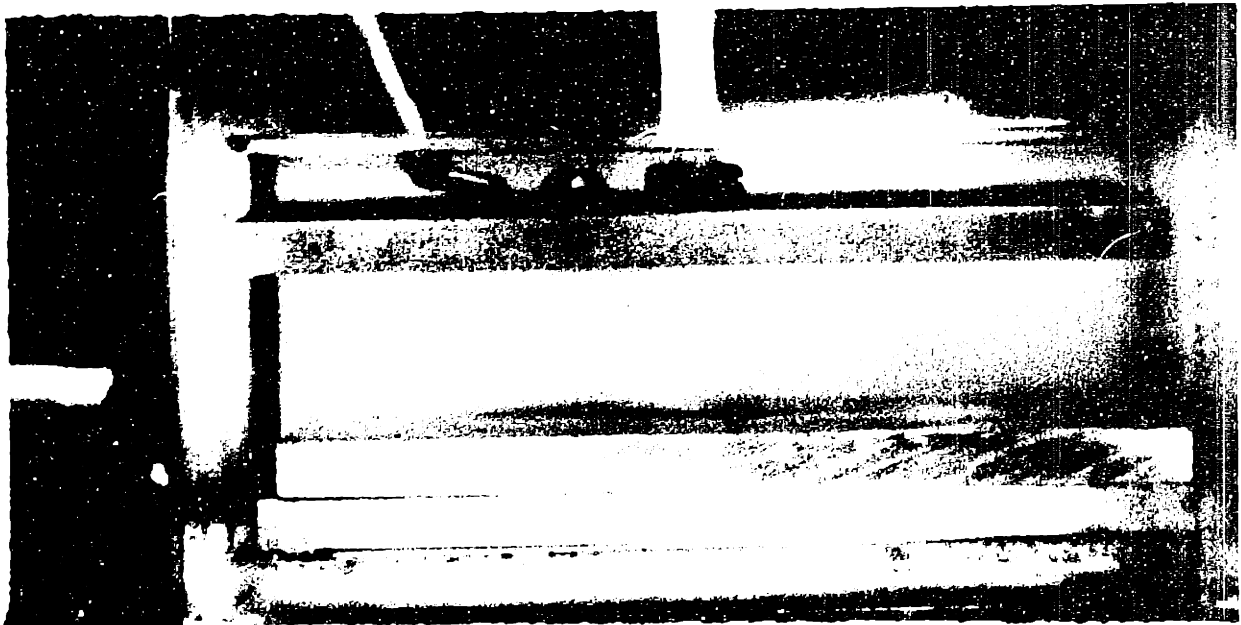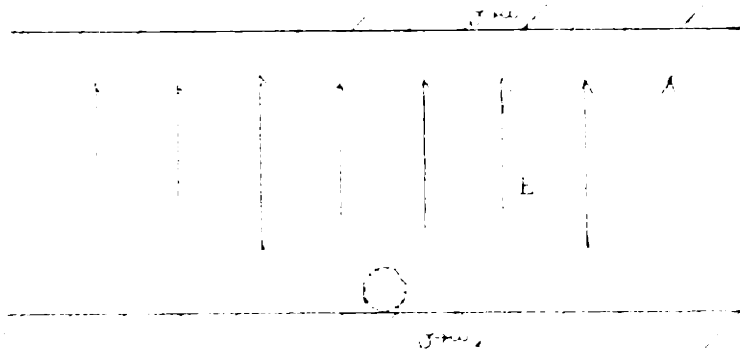

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to gravitational force on a solid sphere gives a predicted field for

particle liftoff.

$$(1.37)4\pi\epsilon r^2 E^2 = \frac{4}{3}\pi r^3(\rho - \rho_{fluid})g$$

$$E_{liftoff} = \sqrt{\frac{r(\rho - \rho_{fluid})g}{(1.37)3\epsilon}} \tag{1}$$

In addition to attempting to verify the predictions of force and charge, explorations are made into additional forces that may be present in systems with high electric fields. It is desired to determine whether these forces are present solely due to the size of the system, and negligible in microscopic systems at lower fields.

The second question examined is what electric field is required to cause a large array of identical conducting spheres, evenly packed, to begin levitating.

Figure 2. Array test apparatus.

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3(\rho - \rho_{fluid})$$

$$\rho_{fluid} = \frac{r^3 - \rho_{fluid}}{\cdots}g \tag{1}$$

In addition to attempting to verify the predictions of force and charge, explorations are made into additional forces that may be present in contact with the electric fields. It is desired to determine whether these forces are present solely due to the size of the system, and negligible in microscopic systems at lower fields.

The second question examined is what electric field is required to cause a large array of identical conducting spheres, evenly spaced, to begin levitating.
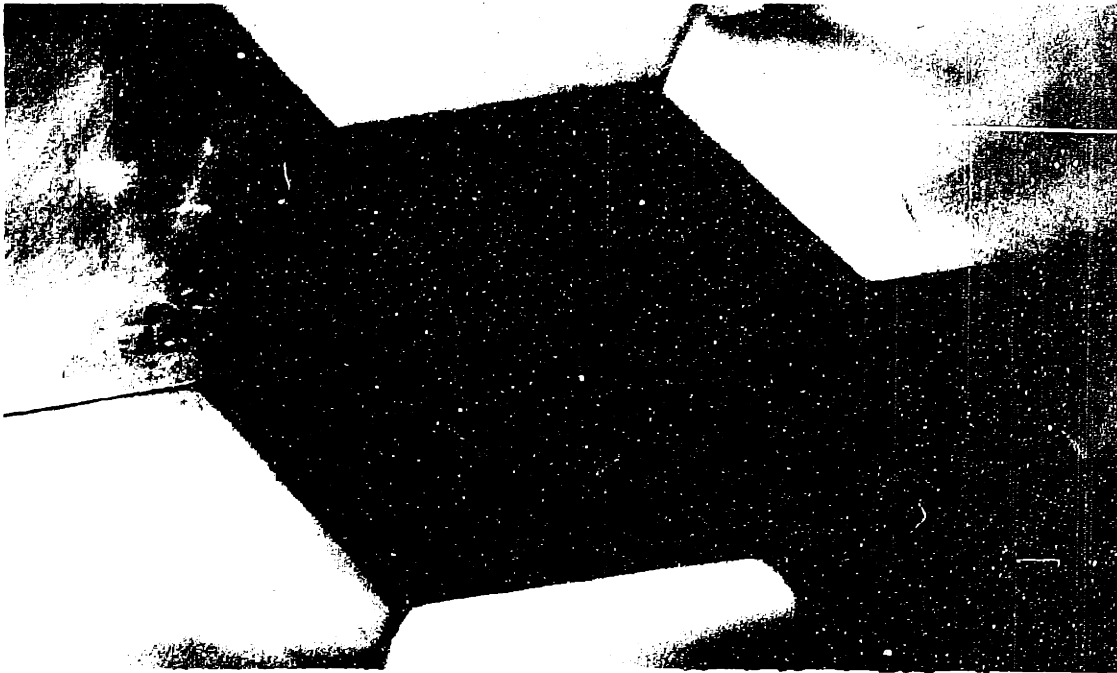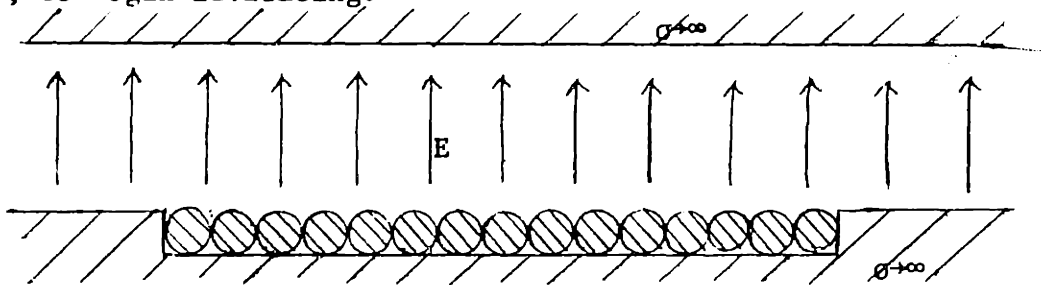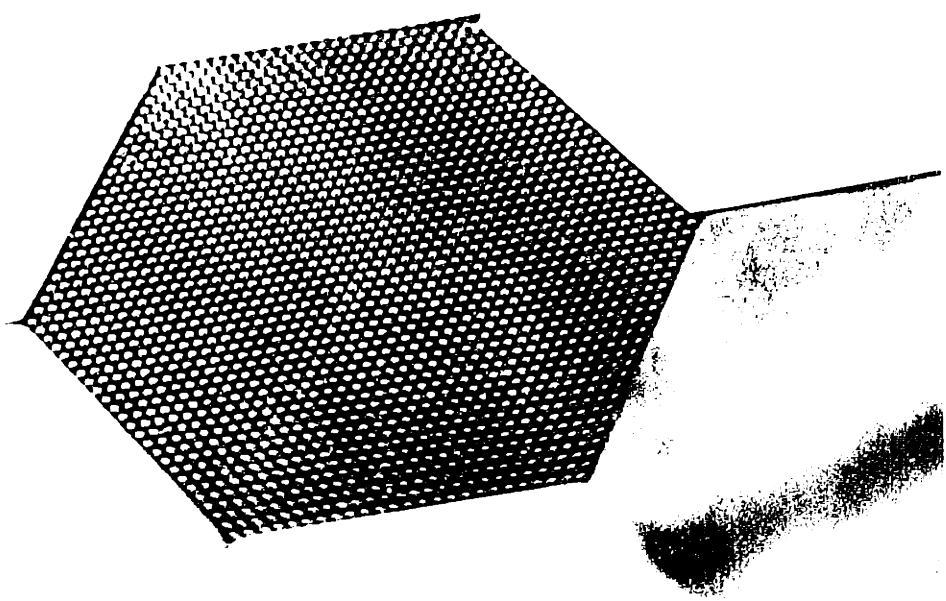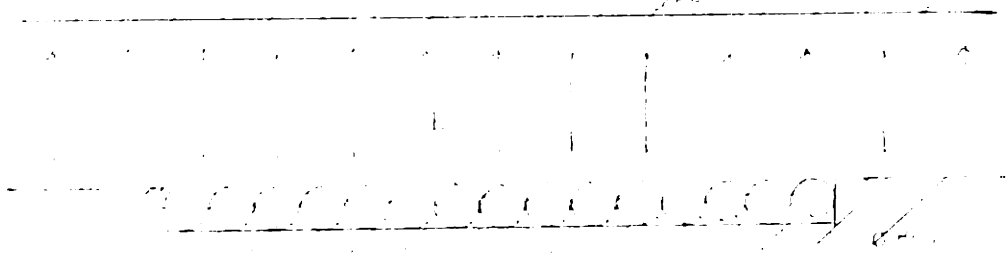

Figure 2. Array test apparatus.

Once a single sphere has left the array, the E-field formerly terminating on it now terminates on the spheres surrounding it. Although the array is no longer regular and the force cannot be computed easily, the increased field at the surface of the surrounding spheres means that at least one of them must have an increased force on it and will also leave the array. Thus the process avalanches, once the first particle is extracted.

The force on a sphere in a regular array can be computed by integrating the Maxwell stress-tensor over the area associated with the sphere. Associated with each sphere is a hexagon with all sides tangent to the largest cross-section of the sphere. Extending the sides of the hexagon downward to construct a complete box, we see that the sides lie in planes of symmetry of the array; because of the symmetry of the array, there is no electric field normal to the vertical sides of the box. The shear component of the Maxwell stress-tensor is therefore zero, and the sides of the box contribute no force to the particle. The area underneath the sphere is shielded, since the spheres resting on the plate are at the same potential; there is no field and therefore no electric stress.

Integrating the stress over the top surface of the box thus constructed gives the force on a single particle in the array.
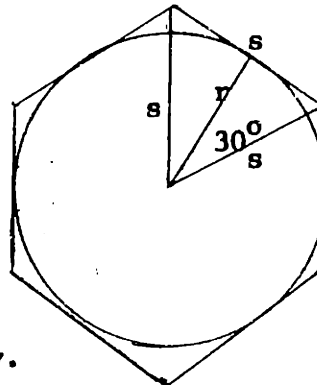
Figure 3. Geometric analysis of array.

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{s}{2r}$$

$$s = \frac{2r}{\sqrt{3}}$$

$$A_{hexagon} = 6 \times \frac{1}{2} \times rs = 6 \times \frac{1}{2} \times r \times \frac{2r}{\sqrt{3}} = 2\sqrt{3}r^2$$

Integrating the stress-tensor to compute the force gives

$$F_e = \int T_{zz} \cdot n_z dA = (\frac{1}{2}\epsilon E^2)(2\sqrt{3}r^2) = \sqrt{3}\epsilon E^2 r^2 = \frac{4}{3}\pi r^3 (\rho - \rho_{fluid})g$$

$$E_{liftoff} = \sqrt{\frac{4\pi r(\rho - \rho_{fluid})g}{\epsilon 3\sqrt{3}}} \qquad (2)$$

where z is the vertical coordinate and E is the field far from the

array.

II. References

In 1892, James Clerk Maxwell computed that charge on a small sphere in contact with a large sphere[6] is $\frac{\pi^2 a^2}{6b}$ (a = radius of the small sphere, b = radius of the large sphere), where the sphere is at unity potential. This formula was (apparently) rederived in 1961[5] by Lebedev and Skal'skaya, for a sphere in contact with an infinite plate. Their contribution is that they also derive the force on a sphere in this situation. Most experimental interpretations have assumed F = QE, where Q is Maxwell's value, which is invalid near the surface. The difference (QE = $(1.64)4\pi\epsilon r^2 E^2$, vs F_e = $(1.37)4\pi\epsilon r^2 E^2$) is small enough that experimental error is likely to obscure the true figure. In the following, we will distinguish the force based on Maxwell's charge from the exact force by calling them the Maxwell and Lebedev-Skal'skaya forces, respectively.

T. W. Dakin and John Hughes perform the experiment with spheres in transformer oil[1] (several diameters, both aluminum and steel) and seem to obtain agreement with Maxwell's formula.* Having obtained reasonable agreement with the formula in transformer oil, they generalize it to the liftoff field in air, without actually performing experiments in air. As will be evident from experiments to be described, there are severe (and unexplained) discrepancies when the experiment is

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*The formula they attribute to Maxwell is incorrectly stated. They give $Q = \pi^2 r^2 \kappa E/(36 \times 10^{11})$ coul, E in V/cm, r in cm, κ relative dielectric constant. In these units, Maxwell's formula would be $Q = \pi^2 r^2 \kappa E/(54 \times 10^{11})$. However, the formula they present for the field necessary to lift against gravity seems to be compatible with Maxwell's formula.

performed in air (gaseous freon), and also deviations from theory using other liquids. These do not cast doubt on the validity of Maxwell's or Lebedev's and Skal'skaya's formulae, but on the relevance of their application as sole criterion for liftoff field.

Z. Krasucki[4] uses Maxwell's formula for charge to predict the conductivity of insulating liquids. Particles in contact with a surface, as described in the problem statement, acquire charge, leave the surface and travel to the opposite plate. The motion of these charged particles is responsible for the measured conductivity of insulating liquids. (The diameter of these particles in hexane is typically .1 to .01 μm.) Particle mobility is computed on the basis that Stokes drag equals electric force. Since the particles spend virtually all of their time far from the plates, $F = QE$ is valid for electric force and the formulae derived are valid. The field required for conduction to begin is relevant to the question of particle liftoff, and the data presented has a great deal of scatter at this point. Apparently conduction begins at a relatively low field strength, but the value of the conductivity appears uncertain -- indicating a high variability in the number of particles lifting off. This indicates that circumstances (vibrations of the apparatus, relative locations of the spheres) dictate liftoff if the field strength is between that consistent with Maxwell and Lebedev-Skal'skaya.

Two theses dealing with compressed gas insulation in co-axial cables[2,3] indicate that one of the major causes of breakdown is free conducting particles (FCPs). Since naturally occurring FCPs generally have highly irregular shapes, they cause ionization at high field strength when they are in contact with either conductor, and thus con-

tribute to ion current. The additional current caused by the charge
carried with FCP motion is neglible. FCPs at the electrodes were also
observed to vibrate at frequences of 5 to 50 kHz, without lifting off.
The explanation offered by Diessner[2] is that a particle (with a
diameter $\sim\frac{1}{2}$mm) charges up, leaves the electrode, and runs into the ion
cloud of opposite sign generated by corona at irregularities on its
surface. It discharges and reverses sign, then returns to the surface
to repeat the process. It is doubtful that this explanation applies
to experiments described here.

III. Summary of Experiments

A.   Liftoff of Single Particle

The first experiment performed concerns a single particle -- a conducting sphere sitting on a semi-infinite plane. The force of electric origin on the particle is evaluated by observing the field at which the particle levitates, and equating the gravitational and electrical forces at the point: equation (1) gives

$$E_{liftoff} = \sqrt{\frac{r(\rho-\rho_{fluid})g}{3(1.37)\varepsilon}}$$

Thus, the liftoff field value rises as the radius goes up. In order to have field values below those causing corona or arcing, we would like to have the particle at as low a mass density and as small as possible. To this end aluminum spheres 1/16th inch in diameter are used.

A.1. Liftoff of single particle in gaseous freon.

For a single sphere of this type in air, the predicted field for liftoff is E = 7.6kV/cm (see table I). Unfortunately, when the experiment is tried, breakdown is observed in the form of rapid appearance of light and heating of the air, before the particle lifts off (although above the predicted field strength for liftoff). Thus, the experiment is performed in gaseous Freon 117, which has a higher breakdown field, but essentially the same permittivity as air.

While performing these experiments, it becomes apparent that there is a wide scatter to the values of liftoff field, and that if the container is accidentally jarred, the sphere lifts off at a much lower field. This can perhaps be explained by the fact that the charge on the sphere on an infinite plane, times the nominal electric field,

Table I

Liftoff Field Strength for a Single Particle

Medium	ϵ/ϵ_o	$\dfrac{\rho_{fluid}}{\rho_{water}}$	Predicted Field (kV/cm)	gap (cm)	Measured V(kV)	Actual Field (kV/cm)	Error(%)
Gaseous Freon	1	–	7.607	1.11	17–18	~16	~110
Transformer Oil	2.56	.87	3.915	2.0	9.1	4.6	17.5
Corn Oil	3.1	.94	3.489	1.8	7.9	4.4	26.1
Silicone Oil	2.63	.94	3.788	1.58	6.6	3.8	.3
Castor Oil	4.67	.96	2.826	1.3	11.6	8.9	215

Liftoff Field Strength for an Array of Particles

Medium	ϵ/ϵ_o	$\dfrac{\rho_{fluid}}{\rho_{water}}$	Predicted Field (kV/cm)	gap (cm)	Measured V(kV)	Actual Field (kV/cm)	Error(%)
Transformer Oil	2.56	.87	12.34	2.0	21.5	10.75	−12.9
Corn Oil	3.1	.94	11.00	2.0	16.8	8.4	−23.6

is larger than the force on the particle while it is sitting on the plate, by a factor of 1.64/1.37 = 1.2. Alternatively, the mechanism preventing liftoff may cease to function as the particle gets far enough from the plate.

In order to try to provide controlled motion, a speaker is attached to a platform on which the apparatus rests; the platform is free to roll back and forth. The speaker is connected to line voltage (through a transformer) and can be switched on and off. With this apparatus, the experimental scatter seems to be somewhat reduced, and the liftoff field seems to be approximately 16 kV/cm.

A.2. Liftoff of Single Particle in Liquids

The same experiment is performed in a variety of liquids, including transformer oil, corn oil, silicone oil (Dow 200 series), and castor oil. Data for these experiments is presented in table I. Although there is disparity among the liquids (as compared with the predicted value), only castor oil is further from the prediction than gaseous freon.

B.   Liftoff of Array of Particles

The same experiment with a large array of identical balls is more difficult to set up and has been performed only twice. The apparatus for this experiment is two plates 19 cm in diameter and 2 cm apart. The array, 24 particles on a side (containing 1657 particles) is surrounded by plates 1/16th inch thick; thus the entire upper electrode is the same distance from both the top of the array and the lower plate; in effect, a hole is dug into the lower electrode and the spheres are set inside. The setup time for a hexagonal array of 1/16th inch

diameter spheres, 24 on a side, is on the order of 1 1/2 hours.

C. Dynamics of a Single Particle

C.1. Frequency in Gaseous Freon

Once the particle has left the lower plate and begun bouncing between the plates, increasing the voltage causes the frequency of vibration to vary, although the particle travels so rapidly that it is nearly invisible to the unaided eye. However, by connecting an oscilloscope in series with one electrode, measurements of frequency can be made by observing the interval between charge exchanges on the plate. Data for this is taken. In order to prevent the signal from being lost in the noise -- the 120 Hz ripple of the power supply -- it is necessary to place a gigantic capacitor in parallel with the experiment in order to shunt the AC signal of the power supply.

C.2. Motion in Conducting Liquid

When the experiment is conducted in liquids, the viscosity limits the frequency to typically 1/2 to 1 Hz. In fact, when the experiment is performed in slightly conducting liquids (corn oil), the sphere sometimes rises and falls without ever reaching the upper electrode, as the charge leaks off and the gravitational force overcomes the electric force. Since the motion is slow $(R_y = \frac{\rho V r}{\eta} = \frac{10^3 10^{-2} 10^{-3}}{5 \times 10^{-2}} = .2)$ the Stokes drag approximates the drag force on the sphere and its trajectory can be computed.

$$6\pi\eta r\frac{d\xi}{dt} = Q_o E e^{-t/\tau} - Mg$$

$$\frac{d\xi}{dt} = \frac{Q_o E e^{-t/\tau}}{6\pi\eta r} - \frac{Mg}{6\pi\eta r}$$

$$\xi(t) = -\frac{Q_o E \tau}{6\pi\eta r}(e^{-t/\tau} - 1) - \frac{Mgt}{6\pi\eta r}$$

Table II

Single Particle Frequency in Gaseous Freon

gap (cm)	E (kV/cm)	pulse separation (msec)	frequency (Hz)
.95	22.8	3.5	286
.95	18.6	4	250
.95	15.8	4.5	222
.95	14.0	5	200
.95	10.7	6	167
.95	9.3	7	143
.95	8.0	8.4	119
.95	6.6	10.5	95.3
.95	6.1	12.2	82
1.35	19.5	6.3	159
1.35	15.9	7	143
1.35	13.5	7.8	128
1.35	11.8	8.8	114
1.35	9.35	10.6	94.3
1.35	7.25	14.4	69.5

ξ next equals zero when

$$\frac{Q_oE\tau}{6\pi\eta r}(e^{-t/\tau} - 1) + \frac{Mgt}{6\pi\eta r} = 0$$

$$e^{-T/\tau} = 1 - \frac{Mg}{Q_oE}\frac{t}{\tau} \tag{3}$$

Equation (3) is graphed in figure 4. Find T/τ on the vertical axis. The corresponding point on the horizontal axis represents the ratio Mg/Q_oE.

ξ_{max} occurs when $v = 0$.

$$Q_oEe^{-t/\tau} = Mg$$

$$e^{-t/\tau} = Mg/QE$$

$$t = - \tau\ln\frac{Mg}{Q_oE}$$

At this time

$$\xi_{max} = - \frac{Q_oE\tau}{6\pi\eta r}(\frac{Mg}{Q_oE} - 1) - \frac{Mg\tau}{6\pi\eta r}\ln\frac{Mg}{Q_oE}$$

$$\frac{\xi_{max}}{\frac{Mg\tau}{6\pi\eta r}} = -1 + \frac{1}{Mg/Q_oE} - \ln\frac{Mg}{Q_oE} \tag{4}$$

Equation (4) is graphed in figure 5. Find $\dfrac{\xi_{max}}{Mg\tau/6\pi\eta r}$ on the vertical axis. The corresponding point on the horizontal axis represents the ratio Mg/Q_oE.

τ is a known property of the fluid ($\tau = \epsilon/\sigma$). By measuring T (period of oscillation) we can determine T/τ, and thus the ratio Mg/Q_oE. Also, we can compute $Mg\tau/6\pi\eta r$, measure ξ_{max}, and determine the ratio Mg/Q_oE. Working backward from this ratio, we can determine what the initial charge is, at the instant the particle lifts off.
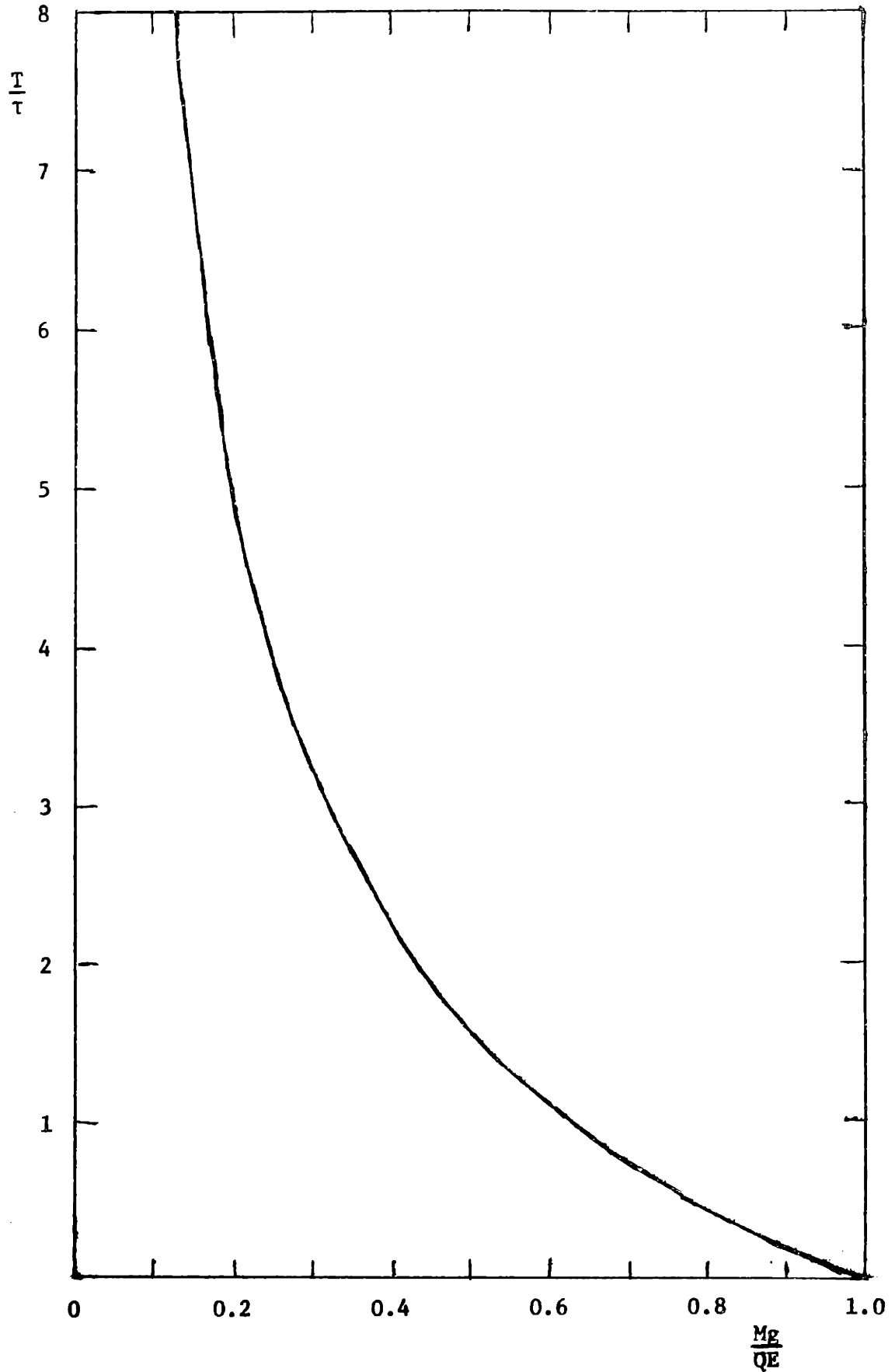
Figure 4. Charge based on trajectory period in conducting fluid. T/τ on vertical axis gives Mg/QE on horizontal axis.
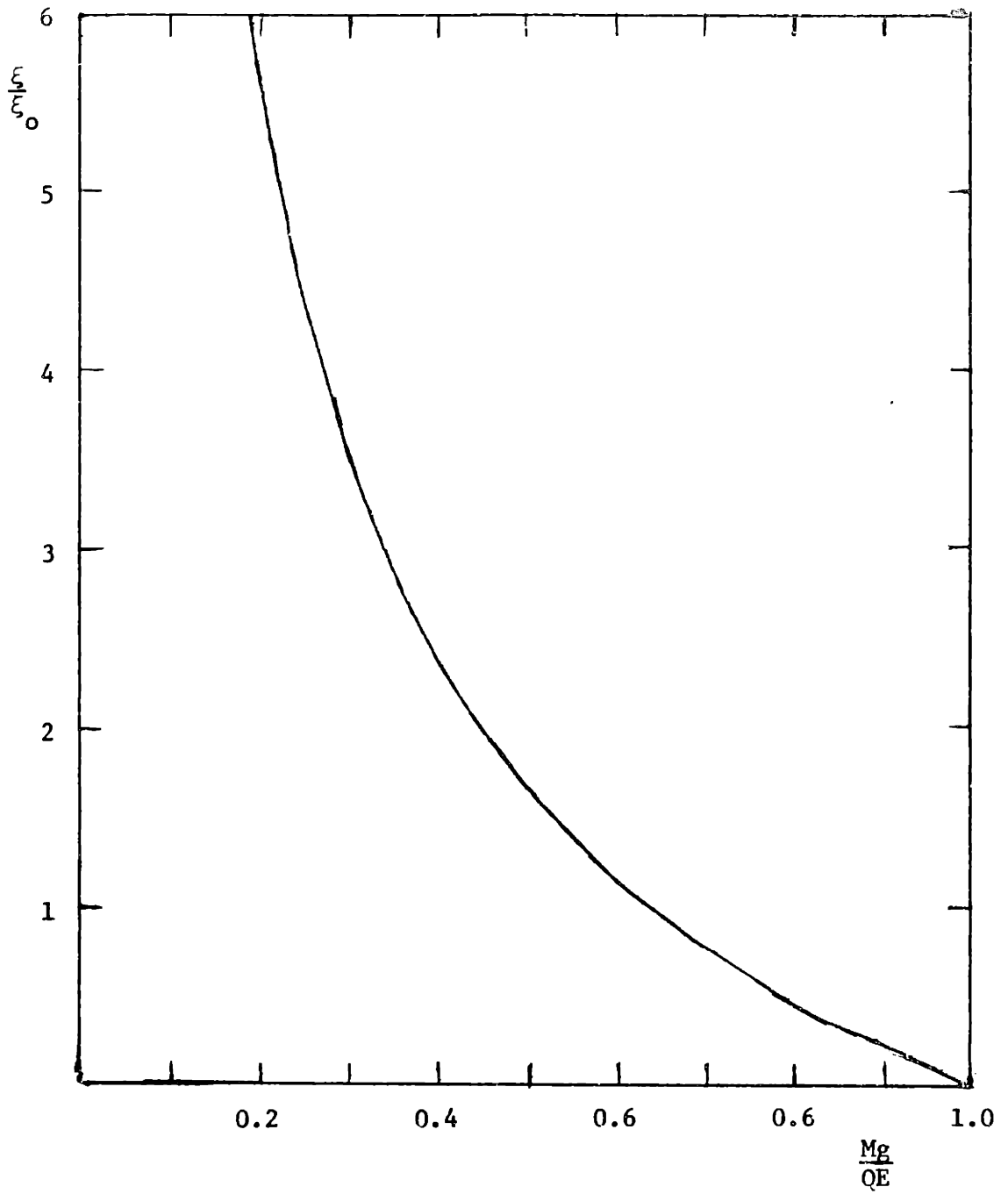
Figure 5.   Charge based on maximum trajectory height in conducting liquid. ξ_{max}/ξ_o on vertical axis gives Mg/QE on horizontal axis.

$$\xi_o = \frac{Mg\tau}{6\pi\eta R}$$

Data is taken in this method, but is not expected to be extremely accurate, since the reflexes of the experimenter become important in measuring time intervals on the order of 1.5 seconds (typical transit time). If the particle is observed for several transits, the data becomes unrelaible since the particle can be observed to rest on the lower plate before taking off again. Furthermore, at a single value of the electric field, the particle can be observed to rise to different heights and take different periods for its transit, indicating that it is not charging the same for each trial.

Table III

Charge Relaxation in Conducting Liquid

Corn Oil

$$\tau = \frac{\varepsilon}{\sigma} = .78 \text{ sec}^{*}$$

$$\xi_o = \frac{Mg\tau}{6\pi\eta r} = 5.27 \text{ cm}$$

E (kV/cm)	T (sec)	$\frac{T}{\tau}$	ξ_{max} (cm)	$\frac{\xi_{max}}{\xi_o}$	$\frac{Mg}{Q_o E}\binom{from}{T}$	$\frac{Mg}{Q_o E}\binom{from}{\xi_{max}}$
6.27	2.8	3.59	3.4	.645	.27	.32
6.27	2.3	2.95	3.0	.57	.317	.34
5.62	2.0	2.57	2.0	.38	.355	.40
5.62	1.5	1.93	1.5	.285	.44	.44
5.30	1.4	1.8	1.3	.247	.48	.467
5.30	1.0	1.28	0.8	.152	.56	.54
5.0	0.5	.642	0.3	.057	.74	.67
5.0	0.7	.898	0.8	.152	.66	.54

* ε established by measuring capacitance, σ determined by conductivity cell, DC measurements

IV. Discussion of Results

A. Single Particle -- Air Liftoff

Unfortunately, no proven explanation can be offered for the tremendous error in the liftoff of the sphere in gaseous freon. Liftoff requires approximately twice the E-field predicted, which indicates the electric force must be four times as great as predicted. However, since the field strength is fairly high, effects may occur which appear only at high fields.

One possible explanation is that the field at the top of the sphere, as it sits on the plate, is concentrated enough so that there is corona discharge continuously during the charging process, and thus the field there is decreased, and the actual force on the sphere is less than predicted. This is an attractive theory because the field at the top of the sphere _is_ considerable more concentrated than the uniform field. If we assume the entire charge ($Q = (1.64)4\pi\epsilon r^2 E$) is uniformly distributed over the top hemisphere, $E_{surface} = \dfrac{\sigma_{surface}}{\epsilon_o}$

$$E_{surface} = \frac{Q}{A\epsilon_o} = \frac{(1.64)4\pi r^2 E_o r^2}{2\pi r^2 r_o} = 3.28 E_o$$
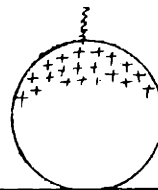
$3.28 \times 7.6 \text{kV/cm} = 25 \text{ kV/cm}$ (5)

Figure 6. Corona leakage from sphere.

This value of surface field is computed assuming uniform distribution of the charge over the top half of the sphere; in actuality, we suspect that it probably is not uniform and is more concentrated at the very top, so corona may be possible. Unfortunately, observation seems to indicate that corona is not present. When the experiment is performed in near total darkness (at 6AM, under a blackout curtain), there is no

visible corona, (although corona is observed at several other points

in the system, where wire ends are exposed. Also, when the oscilloscope

is connected in series with the experiment, no signal attributable

to corona is observed, down to a level of 10microvolts, indicating

that no currents large than 10^{-11} amps are present.

A possible explanation for the apparent reticence of the sphere

to lift off is that the sphere is bouncing microscopically ($\sim$1µm).

As the sphere leaves the plate, the charge rearranges itself (since

it is no longer constrained to be at the potential of the plate)

at the relaxation time of aluminum ($\sim$10^{-19}sec). Field amplification

occurs in the gap between the sphere and the plate. The Paschen

curve for a sphere to plane gap indicates that the field required for

breakdown decreases as the sphere gets further away. Thus, at some

point as the sphere moves away, breakdown occurs and some of the charge

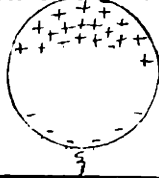leaks off the sphere. It then returns to the plate under the force

Figure 7. Charge leakage between sphere and plate.

of gravity. This theory is somewhat like that presented by Diessner[2],

in a thesis concerning compressed-gas cables. He observes FCPs ($d \approx \frac{1}{2}$mm)

oscillating at 5 - 50 kHz at the surface of the electrode. Although

he attributes particle discharge to the FCP running into its own ion

cloud as it leaves the plate, oscillations of this size particle at

a frequency of 5 kHz would require essentially invisible motion.

$$x = \frac{1}{2}at^2 = \frac{a}{2f^2}$$

$$a = \frac{F}{m} = \frac{(1.37)4\pi\epsilon_0 f^2 E^2}{4\pi r^3 \rho/3} = \frac{3(1.37)\epsilon_0 E^2}{r\rho}$$

$$E \sim 10 \text{ v/m}$$

$$a \sim \frac{4 \times (9 \times 10^{-12}) \times (10^{12})}{(10^{-3}) \times (2.7 \times 10^{3})} \cong 13 \text{m/sec}$$

$$x \cong \frac{13}{2 \times (5 \times 10^{3})^{2}} \cong .26 \times 10^{-6} \text{m} \cong \frac{1}{4}\mu\text{m} \tag{6}$$

When the field reaches a high enough value, not enough charge leaks off before the particle gets out of the range for continuation of breakdown and it continues across the gap to the other plate. Further experiments would be necessary to verify this theory.

B.   Liquid Liftoff -- Single Particle

The sphere does not lift off as far from the predicted field value in non-conducting liquids as it does in freon, but the difference is significant (the field required is typically about 20% higher than predicted). In highly insulating liquids like transformer oil and silicone oil, space charge could be present in the bulk, terminating some of the field. It can be seen in Table I that the error is greatest in the most conducting liquids (castor oil, corn oil), and least in the most insulating liquids (transformer oil, silicone oil). In conducting liquids like corn oil and castor oil, a conductivity gradient could exist near the electrodes (perhaps over the entire surface of the conductor, which would mean it covers the sphere as well). The conductivity gradient and ambient field would imply a charge layer which might shield the sphere. The force on a _conducting_ sphere in a conductivity gradient is in the direction of _lower_ conductivity (see Appendix A-I). If a conductivity gradient exists near conducting surfaces, we would expect the conductivity to be higher near the plates because of ions

leaking from the metallic surfaces. In an attempt to verify this, measurements of average conductivity are made with decreasing plate spacing (down to 1mm). Average conductivity is observed to increase as plate spacing is decreased. This might imply zones of higher conductivity near the surfaces. However, the reverse is possible; evidence
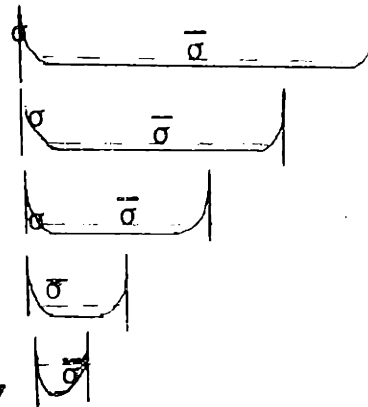
Figure 8. Average conductivity with decreasing plate spacing.

for reduced conductivity near the surfaces is offered by the motion of small air bubbles in the fluid near the conducting sphere. Air bubbles can be considered to be insulating spheres in a conducting liquid; in a conductivity gradient they should move toward the region of greater conductivity (see Appendix A-II). The observed behavior of these air bubbles is that they approach the plate, but veer off, travel parallel to the plate, follow the surface of the sphere (closely, but not touching it) and then shoot off the tip of the sphere. Thus it may be that conductivity is reduced near the electrodes and the sphere (all metal surfaces). Further experiments on this subject are necessary.

C.   Array Liftoff

The fact that the array begins to levitate at a substantially lower field strength than predicted is subject to several possible interpretations. It may be explained by an irregularity in the array. The predicted field for liftoff of the array is based on the area associated with a particle being at an absolute minimum. [The area associated with a sphere is that which is essentially

shielded by the sphere on the lower plate. For a regular array, it is a hexagon with all sides tangent to the largest cross-section of the sphere, as discussed in Section I.] If the array is not perfectly formed in any fashion, the area associated with at least on particle in the array must be increased, and the charge and force on that sphere will be greater than predicted; this particle will lift off at a lower field than predicted.

This is responsible for avalanching. If any one particle in the entire array lifts off, the rest follow as discussed in Section I. In an experiment with several hundred particles, it is possible that one particle is jostled or charged by a dust particle in motion. Although they are not necessarily visible, any small conducting particle will be in motion, at lower fields than the experiment, but by the identical mechanism. Thus it is possible that, at a lower field than predicted for liftoff of the array, a microscopic particle delivers a jolt or a charge to a single particle in the array, causing the avalanche to initiate.

If the microscopic vibration theory is accepted for single particle liftoff, it is apparent that it need not apply to the array. Although the liftoff field strength is considerable higher for the array than for the single particle, the charge on a single particle in the array is less, by a factor of

$$\frac{2\sqrt{3}r^2 \varepsilon E_{array}}{(1.64)4\pi\varepsilon r^2 E_{single}} = .168 \frac{E_{array}}{E_{single}}$$

$$\frac{E_{array}}{E_{single}} \approx 3 \quad \text{for liftoff}$$

$$\frac{Q_{array}}{Q_{single}} \approx .5 \quad \text{at predicted liftoff fields} \qquad (7)$$

Therefore the field at the immediate surface of the particle in the array is less by this factor, and breakdown need not occur at all.

If a conductivity gradient exists near the metal surfaces, it should raise the field strength for liftoff of a perfectly formed array by about 20%, as it does for single particle liftoff. More experiments with arrays are necessary to determine whether the liftoff field is highly dependent on how carefully the array is set up and how the liftoff field depends on liquid conductivity.

D.   Charge on Particle

The charge on a sphere can be computed from the trajectory experiments in a conducting liquid; Figure 9 presents data based on Table 3 from both trajectory period and maximum height measurements. Unfortunately, in this experiment the scatter in the time measurement and variation in the height of the trajectory is considerable (about 30%); therefore we cannot draw any firm conclusions about the charge. Dimensional analysis would lead us to expect a -2 power dependence of $Mg/Q_o E$ on E. However, the data in Figure 9 seems to best fit -3.25 ($Mg/Q_o E \propto E^{-3.25}$). Assuming -2 power dependence gives us approximately $Q_o = (1.98) 4\pi \epsilon r^2 E$. The variation present in the measurement of the time and height of the sphere in this experiment indicates a variation in initial charging at a given field strength. This would be possible if there is a boundary layer of varying conductivity present that is not uniform across the plate; depending on where the sphere sets down, it might be in a region of much lower or only slightly lower conductivity, and shielding would vary.
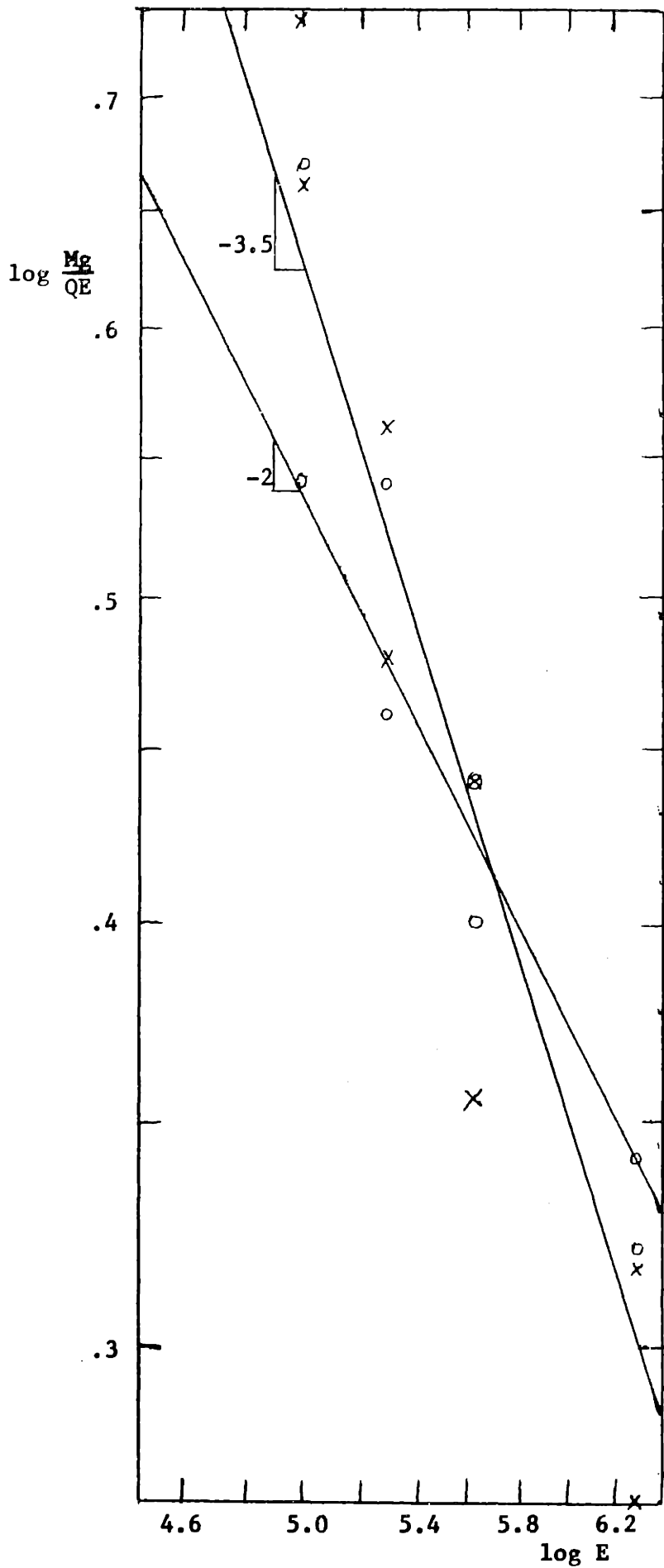
Figure 9. Log (Mg/QE) vs. log (Electric Field); Data from conducting fluid trajectory experiments.
x -- data from period measurement
o -- data from height measurement

E. Frequency in Air

Figure 10, which presents the data from table II, indicates there are two regions of frequency dependence. Since $f = 0$ at some critical field strength (where $Q_o E < Mg$), a logarithmic plot of frequency vs. field strength will have varying slope for small values of field strength, and become essentially vertical at the critical field. For large field strengths, frequency can be computed on the assumption that the work done by the electric field per cycle equals the mechanical losses associated with striking the walls. When a ball strikes a plate with kinetic energy $\frac{1}{2}mv^2$, it bounces back with kinetic energy $(v)\frac{1}{2}mv^2$, due to loss associated with the impact. For this system, v has been measured to be approximately .7. The energy lost in a collision is $(1 - v)\frac{1}{2}mv^2$.

$$\delta W/cycle = 2dQ_o E = 2(1 - v)\frac{1}{2}mv^2_{avg}$$

$$f = \frac{v_{avg}}{2d} = \frac{1}{2d}\sqrt{\frac{2Q_o Ed}{m(1 - v)}} = \sqrt{\frac{Q_o E}{2dm(1 - v)}}$$

If we assume $Q_o = (1.64)4\pi\epsilon r^2 E$

$$f = E\sqrt{\frac{(1.64)4\pi\epsilon r^2}{2dm(1 - v)}} \tag{8}$$

However, figure 10 indicates that $f \propto E^{.7}$ in the high field region. This implies the model is incomplete. Some other process significantly affects the frequency. It is not viscous drag since drag is almost negligible.[*]

$$R_y = \frac{\rho v r}{\eta} \approx \frac{(1)(1)(10^{-3})}{2 \times 10^{-5}} \approx 50$$

$$D = (1.0)(\frac{1}{2}\rho v^2)(\pi r^2) \approx (1)(\frac{1}{2})(1)(1)^2(\pi)(10^{-3})^2 \approx 2 \times 10^{-6} nt$$

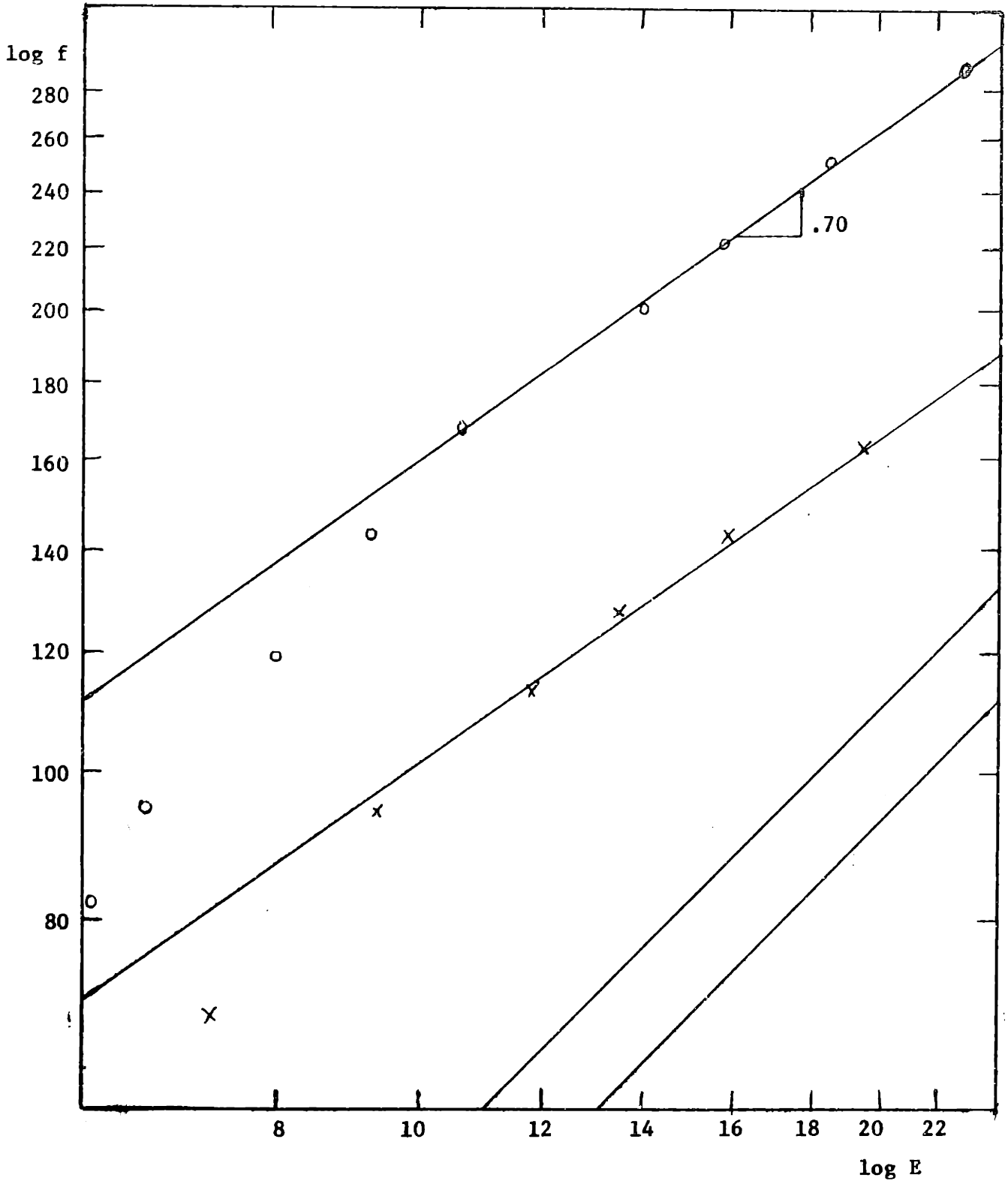[*]Batchelor, G.K., _An Introduction to Fluid Dynamics_, p. 341.

Figure 10. log(frequency) vs. log(Electric field); experiment performed in gaseous freon. Top line for gap = .95cm, lower line for gap = 1.35cm. Bottom two lines are predicted for respective gaps.

$$F_e = (1.64)4\pi\epsilon r^2 E^2 \cong (20)(9 \times 10^{-12})(.6 \times 10^{-6})(10^{12}) \cong 10^{-4} \text{nt}$$

$$\frac{D}{F_e} \cong .02 \qquad\qquad (9)$$

The error is not that the coefficient in the charge equaltion (1.64) is wrong, since chaning it does not affect the power law dependence, but merely shifts the curve upward or downward..

The best explanation that can be offered is that as the field increases, the charge does not increase proportionally to it. This constitutes additional evidence that there is a field-strength dependent mechanism controlling the charging.

V. Suggestions for further experiments

In order to verify the existence and shielding effect of the conductivity gradient, we would like to control the geometry even more than at present: ideally, the simplest geometry is the easiest to understand. To do this, the lower electrode in a condenser containing a polar dielectric might have a hole cut in it, an a highly conducting liquid (like Mercury) fed in from a tube on the bottom so that it is even with the plate. By controlling the pressure at the other end of the tube to keep the surface level with the plate, one could calculate the force of electric origin on the lower plate. If there is shielding present, the force should be less than a simple integration of $\sigma_f \vec{E} \cdot d(\text{Area})$.

To verify microscopic particle vibration before liftoff in gaseous freon, it may be necessary to immensely magnify the particle as it rests on the plate, so that motions on the order of a micron (twice the wavelength of visible light) become visible. This would require special apparatus and great patience.

Appendix A

I.   Force on a Conducting Sphere in a Conductivity Gradient
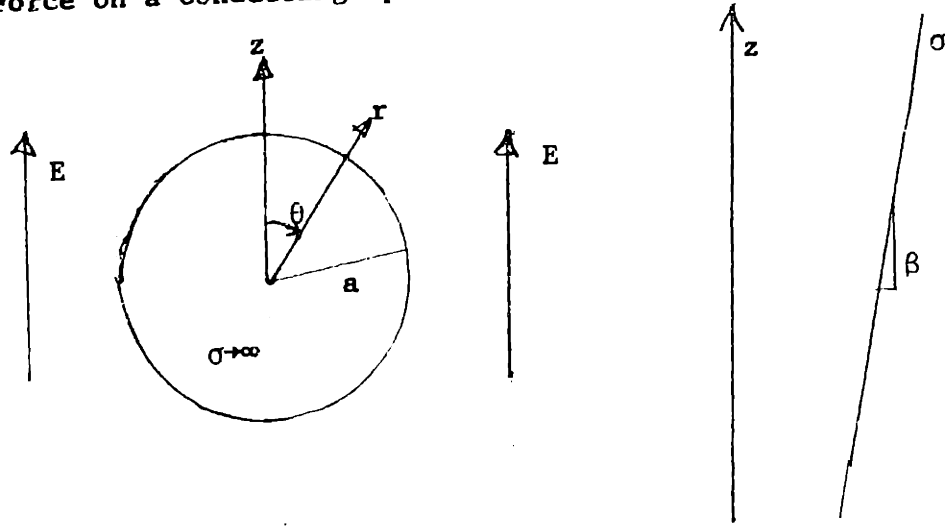

Figure 11.   Geometry of sphere in a conductivity gradient.

Given an imposed electric field and conductivity gradient, the Maxwell stress-tensor is used to compute the force on the sphere. Since the sphere is conducting, $\vec{E}$ is normal to the sphere every-where: $E_\theta(r{=}a) = 0$.

$$F_z = \oint \tau_z \, da$$

$$\tau_z = T_{rr}\cos\theta - T_{r\theta}\sin\theta$$

$$T_{rr} = \frac{1}{2}\epsilon E_r^{\,2}$$

$$T_{r\theta} = \epsilon E_r E_\theta = 0$$

$$\tau_z = \frac{1}{2}\epsilon E_r^{\,2}\cos\theta$$

$$F_z = \int_0^\pi (\frac{1}{2}\epsilon E_r^{\,2}\cos\theta)(2\pi a\sin\theta)a\,d\theta \tag{1}$$

We must have:

Conservation of Charge $\nabla\cdot\vec{J} = 0$ $\tag{2}$

Electrostatic System $\qquad \nabla \times \vec{E} = 0$ (3)

$$\vec{J} = \sigma \vec{E} \qquad (4)$$

Assume a conductivity gradient, increased conductivity upwards.

Let $\qquad \sigma = \sigma_0 (1 + \beta z) \qquad (\beta \ll 1/a)$

$$\vec{E} = -\nabla \Phi$$

Using (2)

$$0 = \nabla \cdot \vec{J} = \nabla \cdot (\sigma \vec{E}) = \sigma \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \sigma = \sigma(-\nabla^2 \Phi) - \nabla \Phi \cdot \nabla \sigma \quad (5)$$

Using (5)

$$\nabla^2 \Phi = -\nabla \Phi \cdot \frac{\nabla \sigma}{\sigma} = -(\beta \hat{z}) \cdot \nabla \Phi \qquad \text{to first order} \quad (6)$$

Let $\qquad \Phi = \Phi_0 + \beta \Phi_1$

$$\nabla^2 (\Phi_0 + \beta \Phi_1) = -(\beta \hat{z}) \cdot \nabla(\Phi_0 + \beta \Phi_1) \qquad (6a)$$

There is no zero-order free charge

$$\nabla^2 \Phi_0 = 0 \qquad (6b)$$

Using (6a)

$$\beta \nabla^2 \Phi_1 = -\beta \hat{z} \cdot \nabla \Phi_0 \overset{0}{\uparrow} - \beta^2 \hat{z} \cdot \nabla \Phi_1 \qquad (6c)$$

$$\nabla^2 \Phi_1 = -(\hat{z}) \cdot \nabla \Phi_0 \qquad (6d)$$

The zeroth order field near a conducting sphere is

$$\Phi_0 = -E_0 (r - \frac{a^3}{r^2}) \cos \theta \qquad (7)$$

$$\nabla \Phi_0 = -E_0 [\hat{r}(1 + \frac{2a^3}{r^3} \cos \theta + \hat{\theta}(1 - \frac{a^3}{r^3})(-\sin \theta)]$$

$$\nabla^2 \Phi_1 = -z \cdot \nabla \Phi_0 = +E_0 [(1 + \frac{2a^3}{r^3}) \cos^2 \theta + (1 - \frac{a^3}{r^3}) \sin^2 \theta]$$

$$= E_0 [1 + \frac{a^3}{r^3}(2\cos^2 \theta - \sin^2 \theta)]$$

$$= E_0 [1 + \frac{a^3}{2r^3}(3\cos 2\theta + 1)] \qquad (8)$$

Let $\Phi_1 = A_1 r^2 (1 + \cos2\theta) + \frac{A_2}{r} \cos2\theta$

Then $\nabla^2 \Phi_1 = 4A_1 - \frac{2A_2}{r^3}(3\cos2\theta + 1)$

$A_1 = \frac{E_o}{4}$ $\qquad$ $A_2 = \frac{-E_o a^3}{4}$

A homogenous part must be added to keep the surface at zero potential.

$$\Phi_1 = \frac{E_o r^2}{4}(1 + \cos 2\theta) - \frac{E_o a^3}{4r}\cos2\theta + \frac{A_1}{r}$$

At $r = a$ $\quad (\frac{E_o a^2}{4} + \frac{A_1}{a}) + (\frac{E_o a^2}{4} - \frac{E_o a^2}{4})\cos2\theta = 0$

$$A_1 = - \frac{E_o a^3}{4}$$

Thus $\qquad \Phi_1 = \frac{E_o}{4}(r^2 - \frac{a^3}{r})(1 + \cos2\theta)$ $\hfill (9)$

From (7) $\quad \Phi_0 = -E_o(r - \frac{a^3}{r^2})\cos \theta$

$$E_{ro} = - \frac{\partial \Phi_0}{\partial r} = E_o(1 + \frac{2a^3}{r^3})\cos \theta = 3E_o\cos \theta \hfill (10)$$

$$E_{r_1} = - \frac{\partial \Phi_1}{\partial r} = - \frac{E_o}{4}(2r + \frac{2a^3}{r^2})(1 + \cos2\theta)$$

$$= - E_o a(1 + \cos2\theta) \hfill (11)$$

From (1) $\quad F_z = \pi a^2 \epsilon \int_0^\pi (E_o + \beta E_1)^2 \sin \theta \cos \theta \, d\theta$

$$= \pi a^2 \epsilon \int_0^\pi (E_o^2 + 2\beta E_o E_1)\sin \theta \cos \theta \, d\theta$$

$$= \pi a^2 \epsilon \int_0^\pi (9E_o^2\cos^2\theta + 2\beta \cdot 3E_o\cos \theta(-E_o a(1 + \cos2\theta)))\sin \theta\cos \theta \, d$$

$$= \pi a^2 \epsilon E_o^2 [9\int_0^\pi \cos^3\theta \sin \theta \, d\theta - 6\beta a\int_0^\pi \cos^4\theta \sin \theta \, d\theta]$$

$$= \pi a^2 \epsilon E_o^2 [- \frac{9}{4}[\cos^4\theta]_0^\pi + \frac{6}{5}\beta a[\cos^5\theta]_0^\pi]$$

$$F_z = \pi a^2 \epsilon E_o^2 \left[-\frac{9}{4}(1 - 1) + 6\beta a(-1 - 1) \right]$$

$$= \pi a^2 \epsilon E_o^2 (-12\beta a) = -12\pi\beta a^3 \epsilon E_o^2 \qquad (12)$$

Since conductivity increases upward ($\beta > 0$), and the force on the sphere is downward ($F_z < 0$), the conducting sphere moves toward the region of lower conductivity.

Appendix A

II. Force on an Insulating Sphere in a Conductivity Gradient.

As with the conducting sphere and figure 11, the Maxwell stress-tensor is used to compute the force; in this case, the insulating sphere imposes $E_r = o$ at $r = a$.

$$F_z = \oint \tau_z \, da$$

$$\tau_z = T_{rr}\cos\theta - T_{r\theta}\sin\theta$$

Since $E_r(r = a) = 0$,

$$T_{r\theta} = \epsilon E_r E_\theta = 0$$

$$T_{rr} = \epsilon E_r^2 - \frac{1}{2}\epsilon[E_r^2 + E_\theta^2] = -\frac{1}{2}\epsilon E_\theta^2$$

$$\tau_z = -\frac{1}{2}\epsilon E_\theta^2\cos\theta$$

$$F_z = \int_0^\pi (-\frac{1}{2}\epsilon E_\theta^2\cos\theta)2\pi a\sin\theta\, a\, d\theta \tag{13}$$

Equations (2) thru (6d) of Appendix A-I also apply to the bulk in this case.

The zeroth order solution near an insulating sphere in a uniform field is

$$\Phi_0 = -E_o(r + \frac{a^3}{2r^2})\cos\theta \tag{14}$$

$$\nabla\Phi_0 = -E_o[(1 - \frac{a^3}{r^3})\cos\theta\,\hat{r} - (1 + \frac{a^3}{2r^3})\sin\theta\,\hat{\theta}] \tag{15}$$

$$\hat{z}\cdot\nabla\Phi_0 = -E_o[(1 - \frac{a^3}{r^3})\cos^2\theta + (1 + \frac{a^3}{2r^3})\sin^2\theta\,]$$

$$= -E_o[(\cos^2\theta + \sin^2\theta) + \frac{a^3}{2r^3}(\sin^2\theta - 2\cos^2\theta)]$$

$$= -E_o[1 + \frac{a^3}{2r^3}(1 - 3\cos^2\theta)]$$

$$\hat{z} \cdot \nabla \Phi_0 = -E_o [1 + \frac{a^3}{2r^3}(1 - \frac{3}{2}(1 + \cos 2\theta))] \qquad (16)$$

Using equation (6d)

$$\nabla^2 \Phi_1 = E_o [1 - \frac{a^3}{4r^3}(3\cos 2\theta + 1)]$$

Let

$$\Phi_1 = A_1 r^2 (1 + \cos 2\theta) + \frac{A_2}{r}\cos 2\theta \qquad (17)$$

Then

$$\nabla^2 \Phi_1 = 4A_1 - \frac{2A_2}{r^3}(3\cos 2\theta + 1)$$

$$A_1 = \frac{E_o}{4} \qquad A_2 = \frac{E_o a^3}{8}$$

We now need to add a homogenous part to Φ_1, so that we can make $E_r(r = a) = 0.$

$$\Phi_1 = \frac{E_o a^3}{8r}\cos 2\theta + \frac{E_o r^2}{4}(1 + \cos 2\theta) + \frac{A_1}{r^2}(3\cos 2\theta + 1) + \frac{A_2}{r}$$

$$\frac{\partial \Phi_1}{\partial r} = -\frac{E_o a^3}{8r^2}\cos 2\theta + \frac{E_o r}{2}(1 + \cos 2\theta) - \frac{2A_1}{r^3}(3\cos 2\theta + 1) - \frac{A_2}{r^2}$$

At $r = a$

$$(-\frac{E_o a}{8} + \frac{E_o a}{2} - \frac{6A_1}{a^3})\cos 2\theta + (\frac{E_o a}{2} - \frac{2A_1}{a^3} - \frac{A_2}{a^2}) = 0$$

$$A_1 = \frac{E_o a^4}{16} \qquad A_2 = \frac{3}{8}a^3 E_o$$

$$\Phi_1 = \frac{E_o a^3}{8r}\cos 2\theta + \frac{E_o r^2}{4}(1 + \cos 2\theta) + \frac{E_o a^4}{16r^2}(3\cos 2\theta + 1) + \frac{3a^3 E_o}{8r}$$

At $r = a$

$$\Phi_1 = \frac{9}{16}E_o a^2 \cos 2\theta + \frac{11}{16}E_o a^2$$

$$\Phi_0 = -\frac{3}{2}E_o a \cos \theta$$

$$E_{\theta_0} = -\frac{1}{a}\frac{\partial \Phi_0}{\partial \theta} = -\frac{3}{2}E_o \sin \theta$$

$$E_{\theta_1} = -\frac{1}{a}\frac{\partial \Phi_1}{\partial \theta} = \frac{9E_o a}{8}\sin 2\theta$$

$$E_\theta = E_{\theta_0} + \beta E_{\theta_1}$$

$$F_z = 2\pi a^2 \int_0^\pi (-\tfrac{1}{2}\epsilon(E_{\theta_0} + \beta E_{\theta_1})^2 \cos\theta)\sin\theta\, d\theta$$

$$= -\pi a^2 \epsilon \int_0^\pi (\tfrac{9}{4}E_o^2 \sin^2\theta + 2\beta(-\tfrac{27}{16}E_o^2 a \sin\theta \sin2\theta))\sin\theta\cos\theta\, d\theta$$

$$= -\tfrac{9}{4}\pi a^2 \epsilon E_o^2 [\int_0^\pi \sin^3\theta \cos\theta\, d\theta - 3\beta a \int_0^\pi \sin^3\theta \cos^2\theta\, d\theta]$$

$$= -\tfrac{9}{4}\pi a^2 \epsilon E_o^2 [\ [\tfrac{\sin^4\theta}{4}]_0^\pi + 3\beta a[\tfrac{\cos^3\theta}{3} - \tfrac{\cos^5\theta}{5}]_0^\pi\ ]$$

$$= -\tfrac{9}{4}\pi a^2 \epsilon E_o^2\ [\ \tfrac{1}{4}\ [\ 0 - 0\ ] + 3\beta a[-\tfrac{1}{3} - \tfrac{1}{3} - (-\tfrac{1}{5} - \tfrac{1}{5})]]$$

$$= -\tfrac{9}{4}\pi a^2 \epsilon E_o^2 \cdot 3\beta a[\tfrac{2}{5} - \tfrac{2}{3}] = \tfrac{9}{5}\pi\beta a^3 \epsilon E_o^2 \qquad (20)$$

Since σ increases upward ($\beta > 0$), and the force on the sphere is upward ($F_z > 0$), the insulating sphere moves toward a region of higher conductivity.

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