Quantum Beauty: Real *and* Ideal

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When I was asked to talk about quantum beauty I was a little startled, because the beauty of quantum theory is something that practicing physicists, in the course of their work, rarely think about or mention. But when I gave the idea a chance, it really caught my imagination. And that's why I'm here. Quantum beauty really is a wonderful, true thing to talk about.

I'm going to sneak up on quantum beauty by putting it in historical context. The right context, I think, is a broader question:

> *Does the world embody beautiful ideas*?

That is a question that people have thought about for a long time. Its intellectual history deserves volumes and syllabi. Here, though, I want to keep things brief and entertaining, so I'll spin a simple tale of heroes:

Pythagoras and Plato intuited that the world *should* embody beautiful ideas; Newton and Maxwell demonstrated how the world *could* embody beautiful ideas, in specific impressive cases. Finally in the 20^{th} century in modern physics, and especially in quantum physics, we find a definitive answer: Yes! – the world *does* embody beautiful ideas.

Pythagorean Beauty

According to Raphael, in his *School of Athens*, this is what Pythagoras looked like (Figure 1). You see he's writing something there. You have to squint to see exactly what, and when you do basically it's gibberish, but I like to pretend that he's documenting his famous credo "All Things are Number". It's hard to know at this distance in time exactly what Pythagoras had in mind with that credo, but we can make a pretty good guess at its spirit.

Figure 1: Pythagoras, as imagined by Raphael in his *School of Athens*.

Pythagoras, obviously, was very impressed by Pythagoras' theorem. Pythagoras' theorem is a statement about right triangles. It tells you that if you erect squares on the different sides, then the sum of the areas of the two smaller squares adds up to the area of the largest square. Pythagoras' theorem is very familiar by now to most educated people, but when you really listen to its message afresh, with Pythagoras' ears so to speak, you realize that it is saying something quite startling. It is telling you that the *geometry* of objects embodies hidden *numerical* relationships. It says, in other words, that Number describes, if not yet everything, at least something very important about physical reality, namely the sizes and shapes of the objects that inhabit it.

Pythagoras is also responsible for another startling, impressive realization of the idea that "All Things are Number". In this much cruder representation (Figure 2), Pythagoras is exposing another of his great discoveries: If you pluck strings which are under tension by weights that are in simple numerical proportions (ratios of small whole numbers) and/or strings stopped so that their lengths are in simple numerical proportions, then you get tones that sound pleasant together. We say they are in harmony. This is another startling relationship bridging Things – here the world of perception, of sound – and Number.

Platonic Beauty

All this made a tremendous impression on Plato, and through Plato it was transmitted down the centuries. Plato, among many other things, is famous for the Platonic solids (Figure 3). These are solids whose faces are identical can be made out of identical regular polygons – that is, polygons having all their sides and angles equal, like squares. For later use, and also just because it's

Figure 2: Pythagoras demonstrating how Number governs musical harmony.

interesting, I'd like to say a few words about these remarkable objects. There are exactly five Platonic solids. There's the tetrahedron, which has four sides, each of which is an equilateral triangle. There's a wonderful shape, the icosahedron, built from 20 equilateral triangles coming together in groups of five. The dodecahedron, which will become our friend later in this lecture, has 12 sides, all pentagonal. There's the familiar cube, and the octahedron, and that's it – just these five shapes, no more and no less. Euclid's *Elements* ends with the construction of these Platonic solids, and many people conjecture that Euclid intended that to be an appropriate climax to his work.

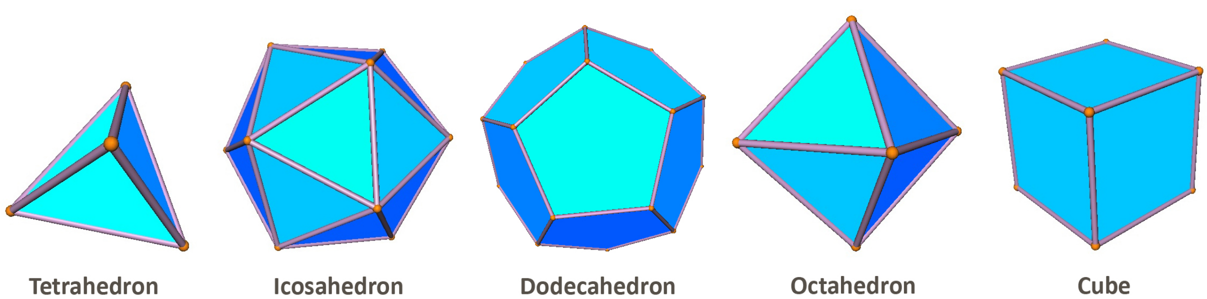

Figure 3: The five Platonic solids

That requiring symmetry can pin down a few specific, discrete possibilities for structure was a striking discovery. It inspired Plato to propose a Theory of Everything that, while primitive, is very much in the spirit of modern ideas, as we will see. Plato proposed that the different building-blocks of Nature, the elements, of which at that time there were thought to be four (Earth, Air, Fire, Water), are made out of atoms, each having the shape of one of the Platonic solids. The icosahedron is left over, and Plato proposed that it is the shape of the the Universe as a whole. I can't accept Plato's theory in detail, but the idea that there are exactly five things that you can use to construct the world, and that these things embody mathematical principles, is a remarkable kind of intuition.

Plato also described a more philosophical concept that's central to our question. This is his metaphor of the cave (Figure 4). According to this metaphor, as human beings we perceive not ultimate reality, but only a sort of projection: mere shadows on the wall of a cave. Plato's message here was that the world at its most fundamental level could embody beauty, despite paltry (or messy) appearances. One of our goals as thinkers, philosophers, or just appreciators of the world should be to imagine, from the projections we perceive, what the real thing is. Plato believed that the real thing would be more perfect, and more beautiful, and more spacious than the projection. Yet there must be a precise mapping from the underlying reality to the projection, and that's how you can test and refine your ideas about what the underlying reality is.

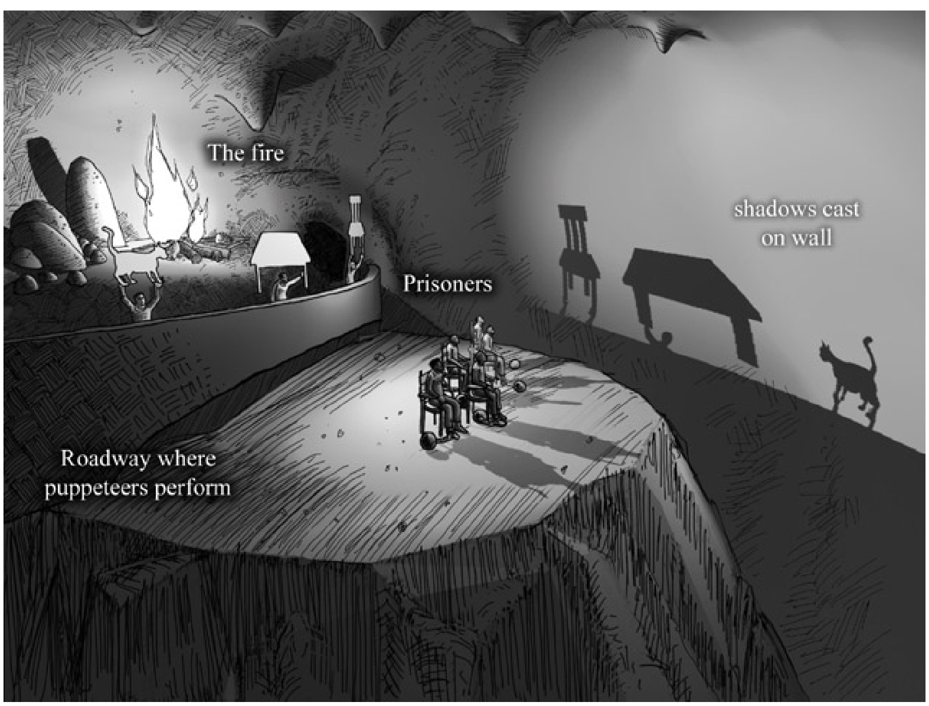

Figure 4: Plato's metaphor of the cave. Those who rely exclusively on their senses are like spectators of a shadow puppet show; they see only the projection of a richer reality.

Newtonian Beauty

The ancients gave us hopes and philosophical ideas for how beauty might be embodied in the physical world, and a few splendid examples. Those hopes and very partial insights became much more impressive when Cambridge started to get into the act. Here, from Newton's *Principia*, is my favorite diagram in the whole literature of science (Figure 5). You have to imagine someone on top of a mountain throwing a stone horizontally, harder and harder. And it's clear, intuitively, what should happen. The stone will go further and further. And eventually, it you throw it hard enough, it comes all the way back! Then you can imagine a higher mountain. There would also be closed orbits – possible motions of the thrown object – starting from those higher peaks. And you begin to see from this thought experiment, without any equation or elaborate mathematics, the emergent idea that gravity could be a universal force. The same force can be responsible for how projectiles move close to the surface of the Earth, and how the Moon orbits the Earth – or, by extension, how Galileo's moons of Jupiter orbit Jupiter, or both planets orbit the Sun. Another aspect of Newton's theory of motion, which leaps out of this diagram, is that orbiting is no different from falling. The stone, even when it's orbiting, is still falling. But because it's falling towards a moving target, so to speak, it never hits the surface. It just keeps falling without ever getting closer to the ground.

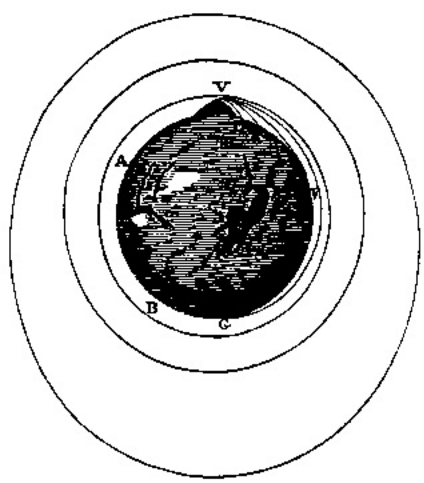

Figure 5: Newton's thought-experiment relating the motion of projectiles to the orbits of astronomical bodies.

Newton also famously introduced the idea he called "analysis and synthesis", basically what we now call "reductionism". More accurately I should not say that Newton introduced reductionism, but rather that Newton provided such impressive examples that analysis and synthesis, as a method, became the default strategy of fundamental science. The way to understand light in particular, but by an analogy potentially everything in the physical world, according to Newton, is to break it down into its most minute components (Figure 6). A prism breaks any kind of light, for example sunlight, into its component pure colors. But then to show that these beams are the fundamental components of light, you have to show that they cannot be broken down further, which Newton checked in other beautiful experiments. And that you can use them to build back up the white light, which he also did.

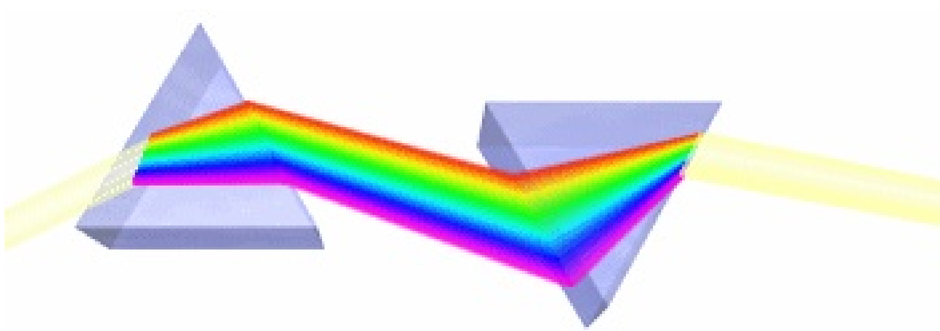

Figure 6: Breaking white light into its component parts, and reconstructing it.

This is what Newton looked like when he was making these discoveries as a young man, according to William Blake (figure 7). You see he was very fit, and most comfortable in the buff. But the most important message of this image, the message that Blake took from Newton and meant to convey, is the idea that the world is constructed according to precise constructions of a mathematical nature, and that we should be ambitious in trying to find precisely what they are. It is the message of Plato's cave, but the metaphor has been fleshed out, and there's a new confidence: We *can* get from shadows to substance, and the 'substance' we discover is mathematical law, or in other words ideal concepts.

Figure 7: Newton constructing a mathematical world, as imagined by William Blake.

Maxwellian Beauty

These are Maxwell's equations (Figure 8). First written in the 1860s, they summarized everything that was known about electricity and magnetism at that time plus one more effect, the entry in the box, which was Maxwell's original contribution. Maxwell introduced this term, which says that changing electric fields produce magnetic fields, because when he put together all the other equations describing the phenomena that were known at the time, he discovered an inconsistency. So something had to change. Now the way Maxwell came to his term was by constructing an elaborate mechanical model ("constructed", of course, in his mind!) that supported the same forces and flows of energy as the electric and magnetic fields. His model, because it was constructed according to sound Newtonian mechanics, could not be inconsistent. But when he analyzed it carefully, Maxwell saw that his imaginative mechanical model of electric and magnetic fields contained the new effect, and so working backwards he proposed that real, physical electric and magnetic fields have must have it as well. Very Platonic! And to this day, the equations Maxwell wrote down in 1864 are the foundation of our understanding of the phenomena of electricity and magnetism and much else.

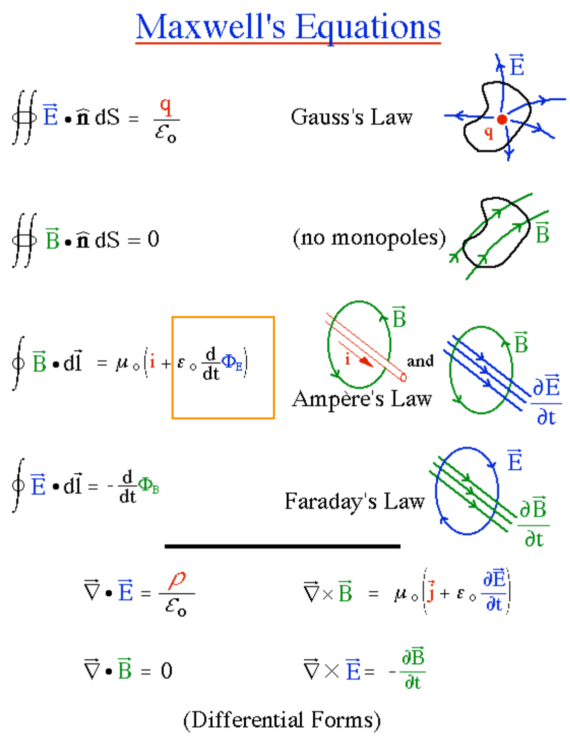

Figure 8: The Maxwell equations in integral, differential, and pictorial forms.

Heinrich Hertz, the experimentalist who verified some of the surprising predictions of Maxwell's equations twenty-five years later, and in so doing invented radio, said something about the Maxwell equations that I find beautiful and poetic, and also true and very relevant:

> One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.

What is it that makes these Maxwell equations so special, and inspired Hertz' rhapsody[1]? Three things:

- They have extremely *powerful* consequences
- They have extremely *pretty* consequences

[1] If you look in the other writings of Hertz, you find that normally he's quite restrained and professorial.

- They introduced a new paradigm of beauty into physics: *Symmetry of Equations*

Before Maxwell added his new term to the equations, which remember said that changing electric fields can produce magnetic fields – there was already a kind of dual effect, discovered experimentally by Faraday, that changing magnetic fields can produce electric fields. Putting them together, you have the possibility that you start with changing magnetic fields, they produce electric fields, which are also changing, so they produce changing magnetic fields. And the cycle can keep going, thereby producing a self-perpetuating disturbance that can move through space. Maxwell, using his equations, could calculate the speed at which those newly predicted disturbances would travel. He discovered that their speed matched the speed of light. So Maxwell, being a very clever fellow, made the leap that "We can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena." And to this day, that's our theory of what light is, at the deepest level. It's those disturbances in electric and magnetic fields, which spin out of Maxwell's equations.

Powerful indeed! But there's much more – even much more than bringing light within the purview of electricity and magnetism. For there are possibilities for disturbances of different wavelengths – in Maxwell's theory, different rates of changing electric into magnetic fields. Visible light corresponds to only a limited range of wavelengths, with Newton's beams of pure colors each representing some specific wavelength. That is basically what was known at the time of Maxwell. But the equations have more solutions. Hertz eventually produced waves of much longer wavelength, which have become famous as radio. Infrared and ultraviolet radiation, microwaves, x-rays, gamma rays – all those mainstays of modern technology and astronomy – they are all contained in Maxwell's equations.

When you solve them, you find that Maxwell's equations give you beautiful structures. I will not be able to do justice here to the beauty of essentially mathematical phenomena. But since the solutions describe tangible realities, I can show you the beautiful tangible reality instead! Here is the shadow projected by a razor blade, or anything with a sharp straight edge, when illuminated by pure light (Figure 9). And you see that the shadow is not merely the absence of light. Formal geometric reasoning, based on the crude idea that light strictly travels in straight lines, would have told you the shadow is a sharp division between darkness and light. But when we calculate the disturbances in electric and magnetic fields that Maxwell taught us light really is, we find there is a lot more structure. It is a beautiful, very precisely calculable structure that you can get from the Maxwell equations, and nowadays with bright monochromatic lasers available you can compare the prediction directly with reality. Looking at this thing you just have to say: Isn't that pretty?

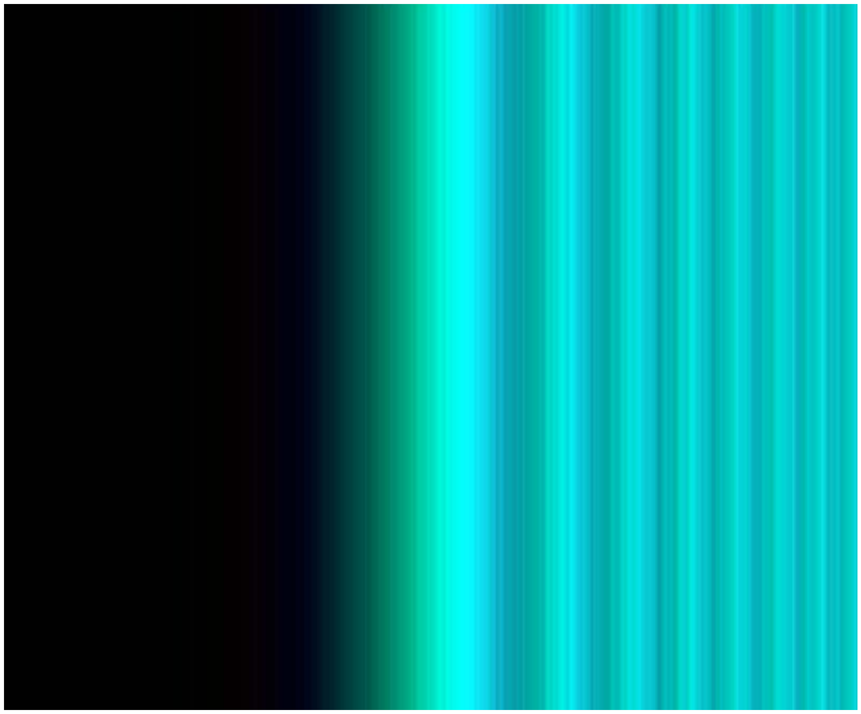

Figure 9: The shadow of a razor's edge, cast in monochromatic light, viewed at high resolution.

Most profoundly for the future of physics, Maxwell's equations introduced an essentially new idea, that had not really played a big role in science before, and which has become more and more dominant in our attempts to guess new laws of nature: the idea that *equations* can have symmetry. And moreover that the equations Nature likes, have lots of symmetry.

What does it mean, to say that equations have symmetry? While the word symmetry has various, often vague meanings in everyday life, in mathematics "symmetry" means something quite precise. Symmetry means change without change. Spelling out that Delphic formulation: We say an object is symmetric if we can make transformations on it that might have changed it but in fact do not. So for instance a circle is very symmetric because you can rotate a circle around its center, and though every point on it moves, overall it remains the same circle; whereas if you took some more lopsided shape and rotated it, you'd always get something different.

The same idea can be applied to equations. Here's a simple equation

$$x = y$$

which you can see is neatly balanced between x and y. You'd be tempted to say it is symmetric. And indeed it is, according to the mathematical definition. For if you change x into y and y into x, you get a different equation, namely

$$y = x$$

But the new equation has exactly the same content as the old one, so we've got change without change: symmetry. Whereas, say, $x = y + 2$ changes into $y = x + 2$, which is not the same thing at all, so that equation is not symmetric. Symmetry is a property that certain equations, or by extension systems of equations, have while others don't.

Maxwell's equations, it turns out, have an enormous amount of symmetry. There are several families of transformations you can make on Maxwell's equations, that change their form but not their overall content. You can change space into time, if you do it in the right way. That possibility is the essence of the theory of special relativity – which, historically, arose from thinking about Maxwell's equations. You can also change electric fields into magnetic fields, if you do it in the right way.

In modern physics we have learned to work toward truth in the opposite direction, reversing Maxwell's path. Instead of using experiments to infer equations, and then finding to our delight and astonishment that the equations have a lot of symmetry, we propose equations with enormous symmetry and then check to see whether Nature uses them. It has been an amazingly successful strategy.

The Quantum Beauty of Matter

With that background, we are well prepared to appreciate the beauty of the quantum world.

The quantum description of atoms realizes Pythagoras' vision uncannily. The mathematics of electrons in atoms is exactly the same mathematics that people developed to describe musical instruments. The equations for vibrations of the air within a woodwind instrument, specifically, bear a strong family resemblance to the equations we use to describe the motion of electrons in atoms. When the woodwind is sounding a pure tone, inside it there is a space-dependent pattern of density and pressure that oscillates in time. We call it a standing wave. In the quantum atom, the nature of the waves that vibrate is a little (actually, a lot) more abstract. They are waves of probability amplitude, and the things that oscillate one into the other are the real and imaginary parts of the wave functions. It would require a long digression to spell all that out from scratch, so suffice it to say that the equations are basically the same. In the early days of quantum mechanics, when physicists were developing these equations and learning how to solve them, one of the main textbooks on the mathematics of quantum theory was actually Lord Rayleigh's classic textbook

The Theory of Sound.

Let's consider for example the simplest atom – the hydrogen atom – and ask what the electron wave patterns look like when the electron is in one of its stable states (Figure 10).

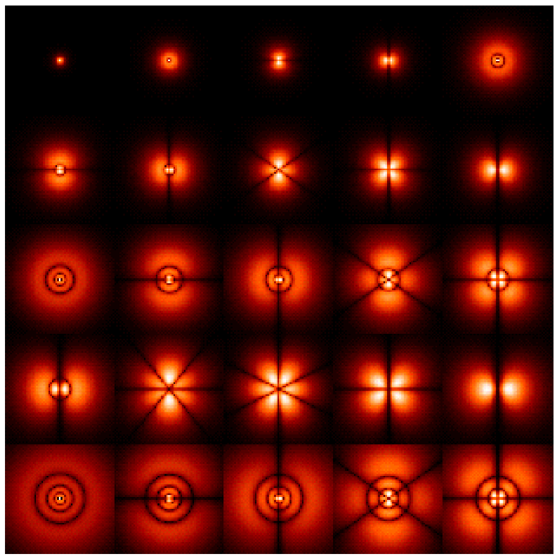

Figure 10: Two-dimensional projections of the wave functions for stationary states of electrons in hydrogen.

These so-called stationary states are the analogue of pure tones for the musical instrument. This picture shows two-dimensional sections of what the stationary states look like. I think you will agree they are esthetically attractive. And the more you know, the more profoundly attractive they become, as you understand that they are the solution of precise mathematical equations, whose consequences have been checked down to the ninth decimal place. Nor do the two-dimensional projections do them justice; they have a lot of internal spatial structure. Here's a cut-away view of one particular orbital (Figure 11). It's wonderful to think that inside us, as our substance, we find structure that is fabulously intricate and pretty to visualize, that embodies precise equations whose consequences we can check in fantastic detail.

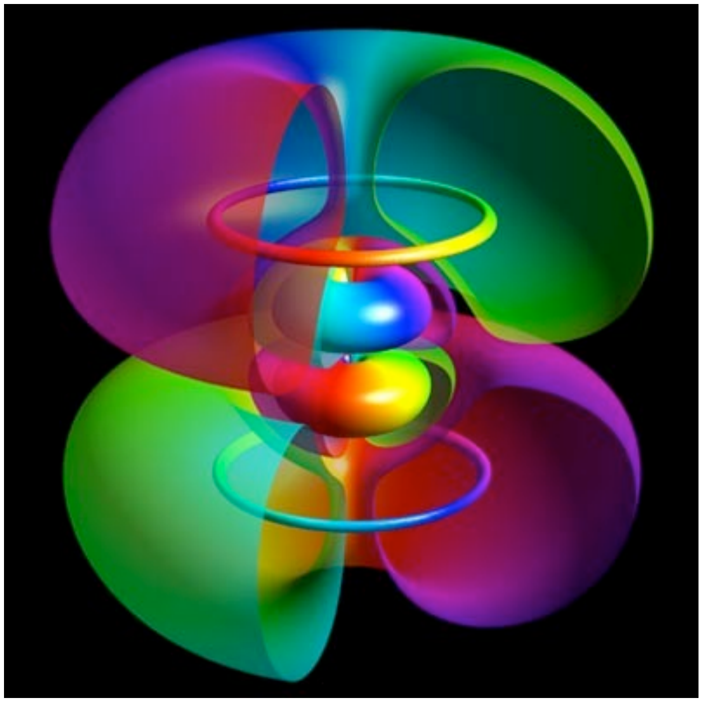

Figure 11: Cross-section showing the internal structure of one particular stationary state. (For experts: (421).)

In the limited time available I will not be able to do anything remotely approaching full justice to the beauty of quantum mechanics revealed in the description of matter and the many phenomena of chemistry. I will just give you a taste by way of a sampler from one particularly relatively simple, yet dramatic, corner of the subject, namely the chemistry of pure carbon.

In a hydrogen atom there is one proton and one electron. That electron is bound to the proton and forms patterns that are centered on that proton. If you have a situation with many nuclei – many concentrations of positive charges that are attractive to electrons – the electrons find efficient ways to play the field. Since all the electrons are the same, and (since we're dealing with pure carbon) all the nuclei are the same, and arranged in a regular crystal-like (*i.e.* symmetric) pattern, we will find the same tricks used over and over again.

The electrons arrange their wave functions according to simple principles that follow directly from the equations of quantum theory. Each carbon atom has four electrons to dispose of. There are two especially symmetric ways for these four electrons to arrange themselves, and these, as you might have guessed, turn out to be the most efficient arrangements. One way is for three of them to point symmetrically to the vertices of an equilateral triangle. To be more precise, the wave functions are partly concentrated on a central carbon atom, but the region of high density is shaped like a little cigar, and the cigars point out in different directions, toward the vertices of an equilateral triangle. The payoff is that there can be another carbon nucleus at the other end of the cigar, and then the electron gets to flirt both with its original mate and with that other one. Three electrons from each carbon atom reach out that way, toward the corners of an equilateral triangle, and the fourth wanders off into the transverse direction. That is one efficient arrangement. The other is to have the four electrons in orbits that pointing toward the vertices of a regular tetrahedron – one of Plato's ideally symmetric arrangements, the Platonic solid with four vertices, you might recall.

Nature exploits these efficient arrangements with exuberant flair and, I'd say, a certain jovial humor. I'll show you some pictures that demonstrate those points quite convincingly. But first I want to emphasize that there are equations and experiments that go with the pretty pictures. One of the nice things about quantum beauty is the better you understand it, the more beautiful it seems. (With other kinds of beauty, like the beauty of magic and ceremony, that is not always the case, unfortunately.)

Here is an arrangement in which you have the cigar-orbitals making the equilateral triangle kind of connections everywhere, over and over again (Figure 12). This the so-called buckyball, or buckminsterfullerene molecule, which is a dodecahedron-like object. A literal dodecahedron isn't quite flat enough; the four neighboring carbon atoms do not lie in a plane, and the favorable orbital arrangement is too compromised to be stable. But you can flesh out a dodecahedron with as many hexagons as you like, it turns out, consistent with the rule that each carbon reaches out to three near neighbors. The buckyball is a particularly stable from that contains 60 carbon atoms, forming exactly 12 pentagons (as in the parent dodecahedron) and the rest hexagons. Its existence was a great discovery that won the Nobel prize in chemistry for the discoverers.

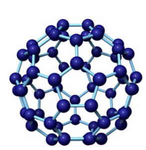

Figure 12: The C_{60} molecule, also known as the "buckyball".

I should mention that these buckyballs, and their relatives, are components of soot – the black rubbish that's left over when you burn carbon inefficiently. You see there is great beauty hidden in soot, when you view it with the mind's eye. We must imagine Plato smiling.

You can keep going with these ideas. Here is a molecule constructed that's been made with 540 carbon atoms, obeying the same basic principles, and making a very impressive, large sphere (Figure 13). You can try to put things inside the sphere, and transport them around in that nice organic cage. This could have practical applications, for instance in drug delivery, as you can keep an active molecule sequestered until it arrives where it's wanted.

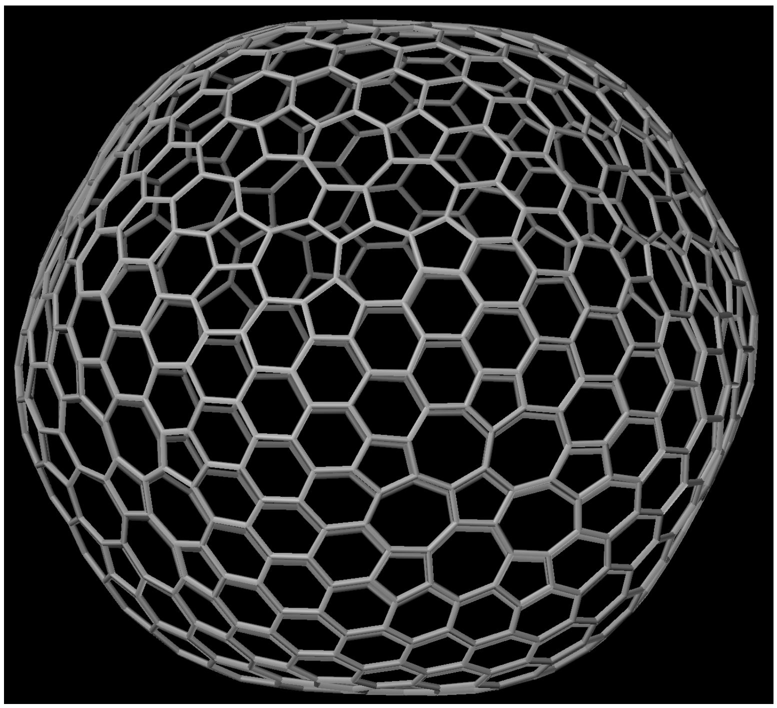

Figure 13: A giant molecule of pure carbon, with near-planar bonding.

The ultimate in this kind of planar symmetry is a true two-dimensional flat plane, extending those equilateral triangle arrangements indefinitely (Figure 14). This describes the substance called graphene. Graphene was first isolated in the pure form just a few years ago, in 2004. That achievement was recognized with the most recent (2010) Nobel prize in physics, awarded to Andre Geim and Konstantin Novoselov. Remarkably, the properties of this material had been worked out in great detail long before, in the late 1940s. The definiteness of the equations of quantum theory, and the simple symmetry of the solutions relevant to graphene, allowed physicists to predict the properties of graphene in advance, and to make rapid progress toward exploiting it once it was isolated.

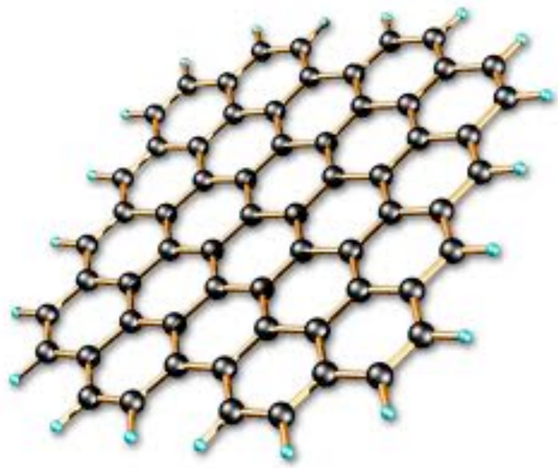

Figure 14: The ultimate in planar carbon: graphene.

You can also fold up ribbons of graphene, to make tubes. These are called nanotubes. Because you can start with ribbons of different width, and also introduce twists before reconnecting, nanotubes come in many varieties. Nanotubes that are only slightly different geometrically turn out to have extremely different properties – some conduct electricity beautifully while others are insulators, for example. The world of nanotubes is a wonderful playground both for mathematics and for technology.

The three-dimensional symmetric arrangement – where the electrons reach out to the different vertices of a tetrahedron – makes diamond (Figure 15). The reason diamonds are so hard is that they embody an extremely efficient way for electrons to do what they want. And because the electrons are so satisfied, it is very difficult to persuade them to break up their network.

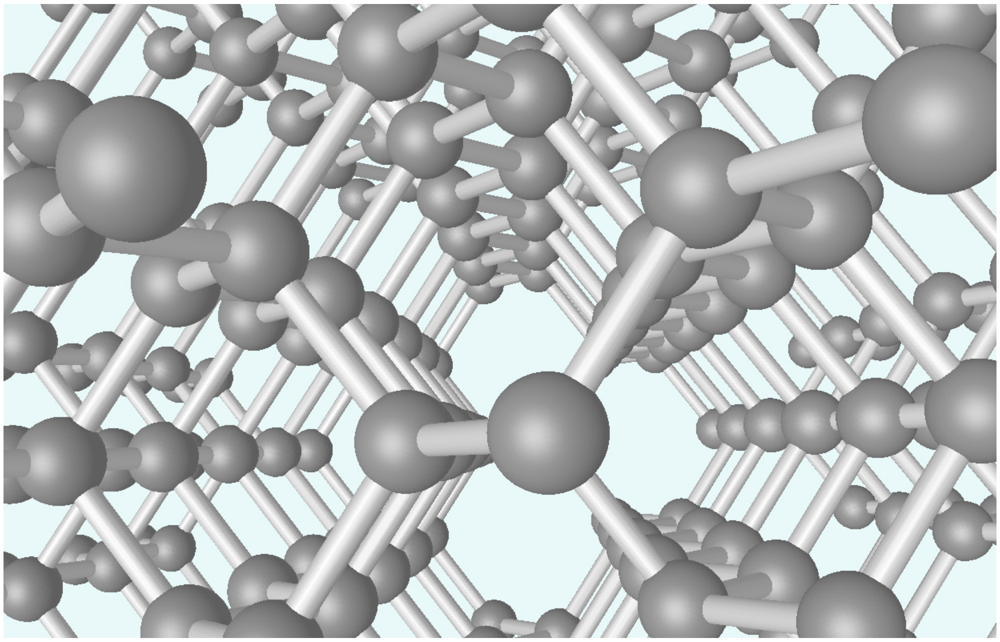

Figure 15: Diamond: A crystal of pure carbon with tetrahedral bonding.

The Quantum Beauty of Unification

I could go on with many, many other beautiful examples from chemistry, but I want to conclude by describing a current frontier of understanding – or maybe misunderstanding – in our exploration of quantum beauty. As I hinted briefly before, the big idea is that in theoretical physics we have gone from *observing* symmetry in equations that experiments lead us to, to *proposing* equations with enormous symmetry, and asking Nature's verdict on them. In other words we are trying to use beauty actively, as our guiding principle, rather than as something we observe passively. We have become artists, but artists with a difference: We voluntarily submit our work to a hopefully loving but in any case unimpeachable critic, Mother Nature.

Consider again the dodecahedron. It is perhaps most familiar as a calendar. It is very convenient for that use, having 12 sides that are nice and equal, so you can fit a month on each one. A dodecahedron is interesting to look at, and by now it's becoming our friend. And so if we stumbled on this unlabeled object (Figure 16) on the internet, we'd recognize what it is, or rather what it's meant to be. Look, it has 12 pentagons – 12 regular pentagons, partially connected. Obviously, this is an object that is meant to be folded up into a dodecahedron.

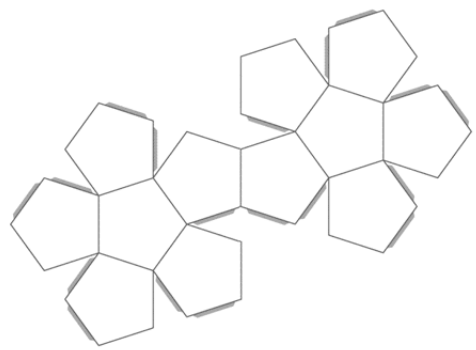

Figure 16: An object that's begging to be folded into a dodecahedron.

But suppose some evil spirit erased part of unfolded dodecahedron, to make this mystery object. (Figure 17). Now it gets harder to recognize what it should be. Most people, perhaps not having thought about dodecahedra recently, wouldn't know what to make of that peculiar thing. But if you remembered about regular solids and dodecahedra you might say well, it is very special that we have pentagons, and that they are connected in this particular way, so I guess what this is meant to be is a dodecahedron, but somebody has erased part of it. Brilliant deduction! Keep that in mind.

And now we turn to the Standard Model of particle physics. It describes an enormous wealth of facts – hard, quantitative realities about the physical world – in a very compact set of equations. I will not remotely be able to do justice to what these equations are, nor their details. But I hope you'll trust me that almost everything we know about physics is encoded honestly in Figure 18, if you know how to decode it. And the decoding does not require putting in extra information or fudge factors or anything of that sort, but just spelling out the logic of the symbols.

A central pillar of the Standard Model is the idea of gauge symmetry. Gauge symmetry is a principle that was discovered in connection with the Maxwell equations, and has been vastly generalized

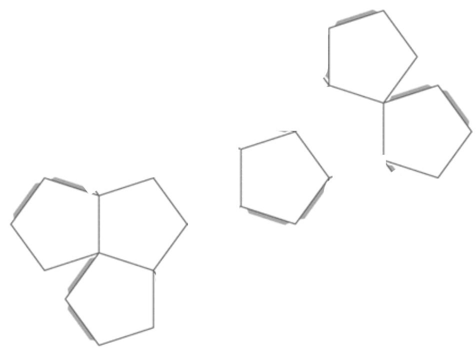

Figure 17: When the object is partially obscured, its meaning is less obvious.

since then. For present purposes, the important point is that equations of the Standard Model have symmetries that can change many of its particles one into another. In our diagram, all the particles within each bracket are related by symmetries of the equations.

Thus, although there are a lot of particles that go into this description of physics, in a strong sense many of them are just different aspects of the same particle. If two particles are related by symmetry – if one is transformed into the other – they really should not be thought of as separate, independent elements of reality. Symmetry tells you that as those particles transform one into the other, the content of the equations has to stay the same. If the transformed equations are going to have the same content, the particle that an existing particle transforms into had better also exist, and have equivalent properties. If you have one, and symmetry, you have the other.

The Standard Model is a very powerful, very compact framework. It would be difficult, as I said, to exaggerate its precision, its power, and – when you spell it out properly – its beauty. But physicists are not satisfied. Just because the Standard Model is so close to Nature's last word, we should judge it by high standards, and try to reconstruct its hidden beauties (remembering the lesson of Plato's cave).

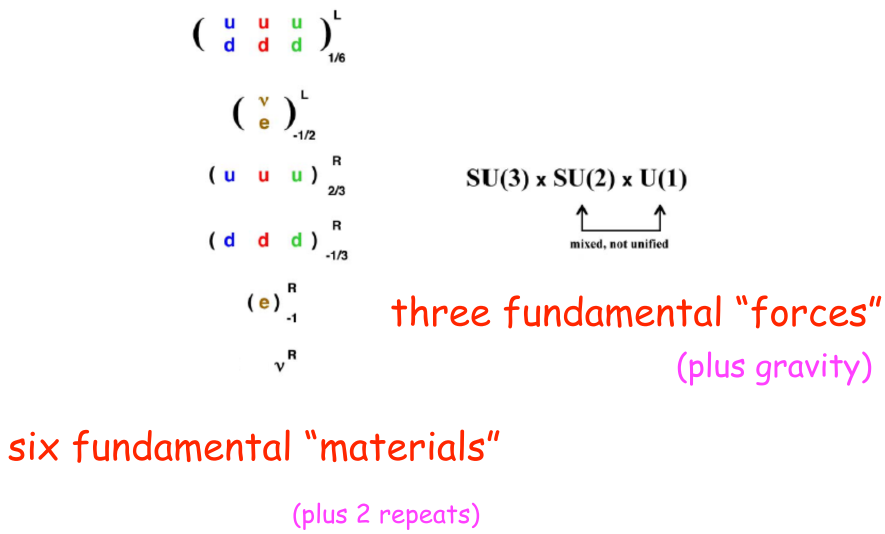

Figure 18: Structure of the Standard Model of fundamental physics.

Scrutinized in that spirit, the Standard Model challenges us to do better. It contains three mathematically similar but independent forces: the strong, weak, and electromagnetic interactions. (Gravity is a fourth force, superficially of a very different character from the others; we will come back to it shortly.) We would like to have one unifying force that really underlies everything in a coherent description of nature. Three (or four) is more than one, so we're not there yet. Even worse, even after declaring different particles related by symmetry to be a single entity, we are left with six unrelated "fundamental" entities, and six is also definitely more than one.

So we would like to do better. It's as if we have been presented with that partial realization of a dodecahedron, with something erased. The mathematics of the possible symmetries of objects in space led us to a few Platonic solids, and let us infer an underlying dodecahedron from partial, distorted evidence. Can we do something similar with the gauge symmetry we find in the equations of fundamental physics?

Needless to say, I would not be leading you down this path, unless we could. There are only a few possibilities for symmetries that perfect the gauge symmetry of the Standard Model, just as there are only a few Platonic solids. We can try them out, and see whether any of them fit the bill.

And one of the possible symmetries seems to fit the known particles and extend the structure of the Standard Model most beautifully (Figure 19). (For experts: It is based on the group $SO(10)$ and its spinor representation.) If you expand the equations this way, then all the forces can be transformed one into the other, as can all the particles. So, as I just argued, we really have just one force and one particle. Awesome!

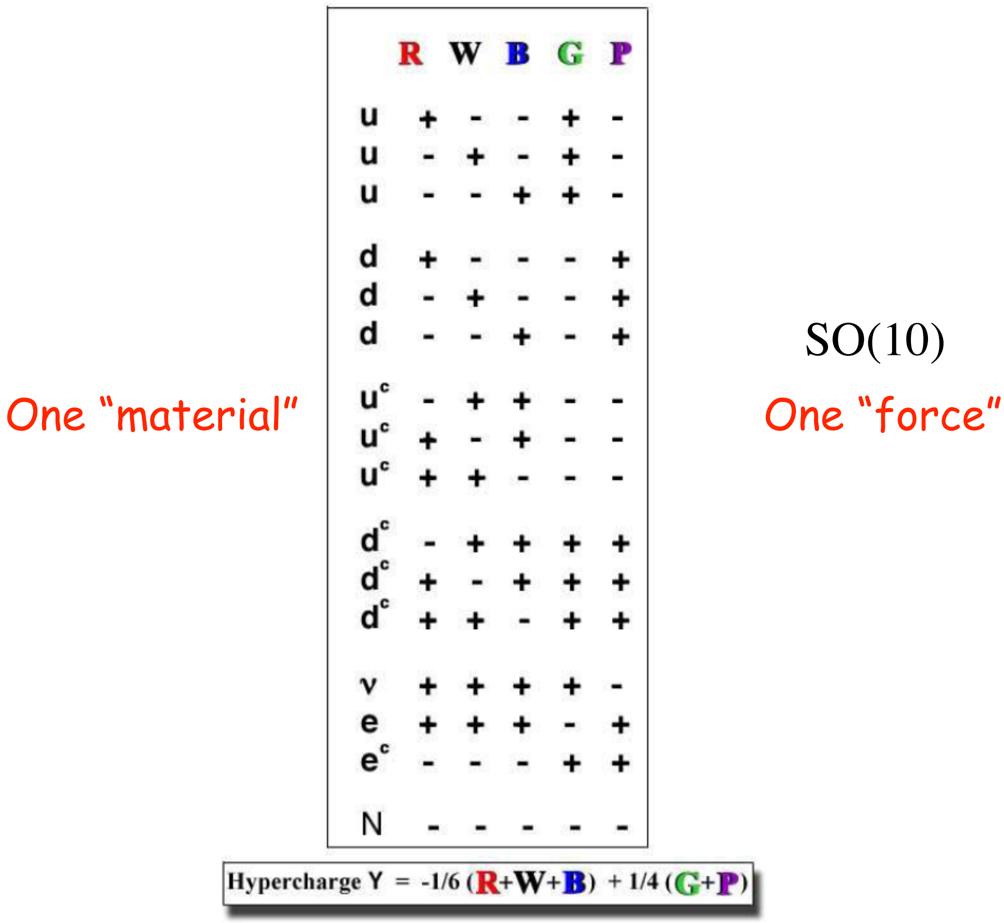

Figure 19: A unified theory of the building blocks of matter and of the standard model forces.

But when you examine this idea a little more closely, it seems to have a fatal flaw. If we are going to have symmetry among the different forces, then those forces have to have the same strength. As we observe them, however, they do not. The strong interaction really is stronger than the other interactions. The three interactions are definitely *not* equal in strength.

Having come this far, however, we should not give up too easily. Maybe here too we need to heed the lesson of Plato's cave, and think beyond the superficial appearance of things.

A great insight of the last part of the twentieth century is that what we ordinarily perceive as empty space appears, in our fundamental description of nature, to be far from empty. It is as if we are fish who have finally realized that they're immersed in a medium – water, of course – that they have taken for granted until now. We have learned that you can get a better description of nature by recognizing that you are in a medium that has its own properties.

This is what your eyes would see if they could have resolution in time of 10^{-24} seconds and resolution in space of 10^{-14} centimeters – really, really small and really, really fast (Figure 20). But for your convenience in viewing, I've blown it up and taken a snapshot. This picture, based on hard calculation, shows fluctuations in the energy and gluon fields in quantum chromodynamics (QCD), our theory of quarks and gluons. Since QCD has been tested quantitatively with almost incredible rigor, it is as certain as anything can be in science that this picture accurately depicts what is happening in the microworld.

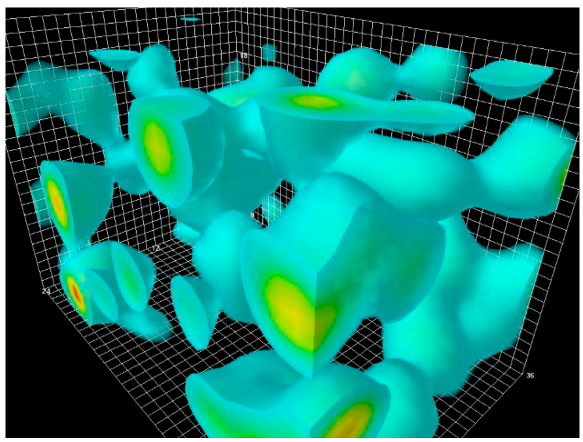

Figure 20: The fluctuating distribution of gluon field energy in space, viewed at high spatial and time resolution. The brightest colors are regions of highest energy density. Regions of very low energy density have been made completely devoid of color, allowing access to view through "empty" regions of space.

Just as water distorts the perception of fish, the medium of space distorts our vision of fundamental processes. Now we realize that correcting for that distortion might be a very good thing to do. The forces as we observe them do not appear amenable to unification; but to see things as they really are, more basically, we need to strip away the distorting effects of the medium.

We can do that with the stroke of a pen. There is no problem to calculate the requisite corrections, and thereby to get a cleared-up view of what's happening at much shorter distances. (Figure 21). As you can see, this strategy almost works – the three lines representing the strengths of the different forces almost meet in a point – but not quite. (The width of the lines indicates the experimental and theoretical uncertainties.)

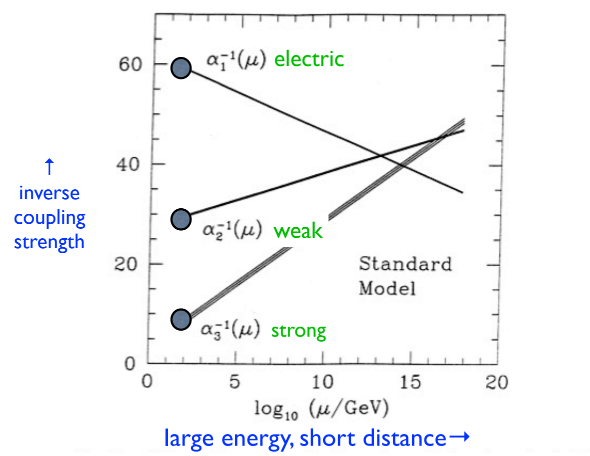

Figure 21: Near-miss for unification, after making corrections to account for fluctuating fields in space.

Now if we followed the famous philosopher Karl Popper we could be happy at this point. Karl Popper taught that the goal of science was to produce falsifiable theories, and now we have produced a theory that is not merely falsifiable, but outright false. What could be better than that?

But of course that is not the way we think about it. We had a beautiful idea that seemed promising, and almost worked. Such ideas are precious, so we should not give up too easily. In the spirit of youth and audacity, let me suggest[2] that maybe we have not been quite bold enough.

In our hypothetical unification, we have united the description of the basic ingredients of matter, including electrons and quarks. They have very different properties, but in the unified theory, they are different aspects of a single entity, viewed from different but equivalent perspectives. Similarly the forces, or if you like the force-transmitting particles such as photons and gluons, got unified.

[2]As I first did in 1981, when "youth" was a better fit.

We got down to one kind of force-transmitting entity and one kind of matter-making entity. But that is still two things, and two is more than one. So the obvious audacious question is: Can we connect those two remaining kinds of things – particles and forces – by some sort of symmetry?

Well, for a long time that was thought to be difficult or impossible, and then just impossible. But in the 1970s several physicists, led by Julius Wess and Bruno Zumino, developed another powerful unifying idea: supersymmetry (SUSY). The central idea of supersymmetry is that there are new quantum dimensions, beyond the familiar dimensions of space-time. A quantum dimension is radically different from a conventional dimension. (For experts: while the coordinates of ordinary space are ordinary real numbers, the coordinates of quantum dimensions are Grassman numbers, which satisfy $xy = -yx$.) If a particle moves off into a quantum dimension it does not change its position, in the usual sense; instead it changes into a different kind of particle! Matter-making kinds of particles turn into force-transmitting particles, and *vice versa*. More technically, we say that fermions transform into bosons, and *vice versa*, when they step into superspace. So if supersymmetry is right, then parallel to the ordinary plane of existence we have the super world, where electrons turn into selectrons, a spin zero version of electrons; gluons turn into gluinos, a sort of matter version of gluons; and so forth. And now by flipping those two planes, if the equations of the world are supersymmetric, we will get different equations where the partner particles have changed one into the other. But if those equations have symmetry – in other words, if supersymmetry is a true feature of the physical world – the new equations will have the same content as the old ones.

That next level of unification doesn't come for free. We have had postulate the existence of a new world: the world where we arrive when we step into superspace. We have to make bigger equations that make use of that new world and make it symmetric with the part of space we already know. We need those new selectrons, and gluinos, and so forth.

Those new particles also exist in fluctuating form, as virtual particles, stirring up the structure of space, further distorting our vision. We have to re-calculate our corrections, to take account of these additional distortions. And here a wonderful surprise emerges (Figure 22). Once you put in those additional corrections, the different interactions really do come together and unify accurately[3].

[3] If you complete the equations in a fully consistent manner, the interactions will not un-unify at still shorter distances. Once they come together, they stick together.

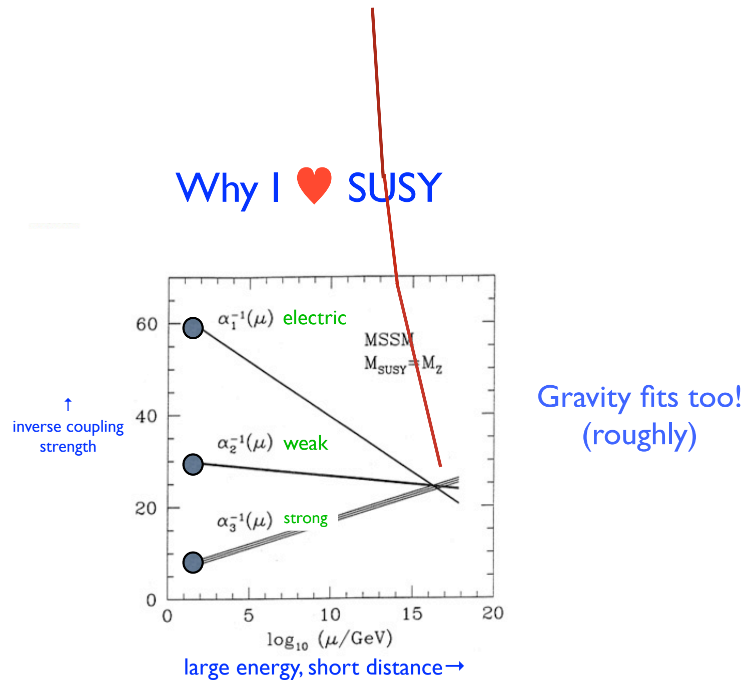

Figure 22: When we include the effect of the hypothetical particles required by supersymmetry, we find accurate unification.

And as an unexpected bonus to this, gravity – which started out horrendously weaker than the other forces, so on my plot it would start way outside the known universe – comes roaring in at extremely high energies, and manages also to unify with the other interactions pretty nearly.

So that, to my view, is how the frontier of our approach to understanding the deep structure of physical reality – realizing the dreams of Pythagoras and Plato, building on the insights of Newton and Maxwell – looks today. I think you will agree that the prospects are tantalizing.

But can we explore that frontier, not only with our minds, but physically? We can, and we will. If SUSY's new particles are going to support enough fluctuations to enable unification, they cannot be too heavy. If they exist, and are light enough to do the job, they will be produced and detected at new Large Hadron Collider – a fantastic undertaking at the CERN laboratory, near Geneva, just now coming into operation. There will be a trial by fire.

Will the particles SUSY requires reveal themselves?

If not, we will have the satisfaction of knowing we have done our job, according to Popper, by producing a falsifiable theory and showing that it is false.

But another possibility is that when we analyze the pictures of what's been produced at the LHC, we will discover that while most of it (by far) is stuff that has already incorporated in the Standard Model, once in a while, maybe once in a billion or once in a trillion events, there will be something additional, that does not conform to the Standard Model – another kind of particle. You have to sift through a lot of hay to find the needle in the haystack, so you have to be very good at recognizing hay, and it's a tough business, but after a lot of analysis we might decode that through their properties the new particles are speaking to us, announcing "I'm a selectron", "I'm a gluino", and so forth. And then our visions of unification will have reached a new level of truth and vindication, and a new world will have been discovered. Soon we will know, one way or the other.