

ON PROMOTIONS AND ADVERTISING POLICIES:

A STRATEGIC APPROACH

by

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ABSTRACT

This thesis consists of three essays on competitive marketing strategies, each of which is described below.

The first essay, "A Competitive Rationale for Price Promotions: Evidence from the Retail Coffee Market" considers a model for price promotions due to Varian (1980). In a model with a population of informed and uninformed customers, price competition yields a static symmetric equilibrium in which each seller draws a price from a specified density function. Price data on coffee products is used to test if the sample of prices on each product could have possibly come from the theoretically specified density function. The results suggest that some markets may indeed be explained by the model. In the process of testing the model, estimates of the marginal cost for each product, the number of competing goods, the percentage of informed consumers and the ratio of fixed costs to potential demand are obtained.

The second essay, "Predicting Advertising Pulsing Policies in an Oligopoly: a Model and Empirical Test" addresses the question, should firms pulse in advertising together or at different times? It is shown that the alternative way maximizes the duopoly profits and is also the Markov perfect equilibrium of the infinite horizon game. The basic intuition for this result comes from the following fact: it is more profitable to increase consideration when the competitors consideration is lower. Evidence from several product categories seem to support this theoretical result.

The third essay, "Proprietary Information in Vertical Relationships: the Advertising Agency Case" studies the foundation for the common wisdom that two clients that compete with each other do not

like to have the same advertising agency. The problem is that once the advertising agency works for a firm it learns its private information which it can then use strategically. But in oligopolistic situations possessing more information about a competitor is not necessarily beneficial. Similarly, allowing the competitor to have more information is not necessarily detrimental. Conditions on the nature of competition and the type of private information are derived such that in equilibrium firms have different agencies. Finally, it is argued that some of the competition on the variables, for which the services of advertising agencies are relevant, satisfies the above conditions.

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Chapter 1

A COMPETITIVE RATIONALE FOR PRICE PROMOTIONS:
EVIDENCE FROM THE RETAIL COFFEE MARKET

I - INTRODUCTION

Even with homogeneous products price dispersion occurs in almost all markets. Sellers vary the price they charge with some frequency, and different sellers often charge different prices at any given moment in time. Random variations (through time and cross-section) of supply and demand do not seem to explain all this price dispersion.

Two basic approaches have been used to explain price dispersion: explaining price dispersion with static equilibria or with dynamic equilibria¹.

The first approach was explored by Butters (1977), Narasimhan (1988), Raju, Srinivasan and Lal (1990), Rao (1986), Rosenthal (1980), Salop (1977), Salop and Stiglitz (1977), Shilony (1977), Stiglitz (1979), Tellis and Wernerfelt (1987), and, Varian (1980). These authors typically get price dispersion by assuming consumers have different information levels, different brand-loyalties, or, different search costs.

Some of the work in this approach seeks to construct equilibrium static strategies (in the sense that they do not depend on the past, i.e., they only depend on the payoff relevant state variables, and the model is constructed such that the set of these variables is empty)

I am distinguishing here between dynamic and static equilibria in the following sense: both dynamic and static equilibria refer to a repeated game with an infinite horizon; however, in a dynamic equilibrium, history (past payoffs, past actions) matters in the players' equilibrium actions, while in a static equilibrium, equilibrium actions are independent of history. In the dynamic equilibrium case, history can influence the payoffs directly or indirectly through the actions of the competitors.

such that "uninformed" consumers would not learn about which stores have the lowest prices. Such strategies involve temporal price dispersion: each store intentionally varies prices through time. In market economies this fact is observed with some regularity: sales are common and well advertised in several categories of goods. Then, the equilibrium strategies of firms, for consumers never to learn, have to be mixed as shown in Narasimhan (1988), Rao (1986), Raju, Srinivasan and Lal (1990), Rosenthal (1980), Salop (1977), Shilony (1977), and, Varian (1980). The Varian model will be explained in some depth below as it will be used for empirical testing.

The second approach lets the competitors actions depend on the past outcomes or actions. These dynamic equilibria have been explored by Aghion, Bolton and Jullien (1988), Blattberg, Eppen and Lieberman (1981), Doyle (1983), Green and Porter (1984), Jeuland and Narasimhan (1985), Lal (1990), Lazear (1986), Rotemberg and Saloner (1986), Salop and Stiglitz (1982), Sobel (1984), Villas-Boas (1990), and, Wernerfelt (1988, 1990).

Some of these models are dynamic in the sense that history affects payoffs directly. These authors had then to change some of the assumptions of the basic model: inventory costs greater for sellers than for consumers (Blattberg, Eppen and Lieberman 1981, and, Salop and Stiglitz 1982), asymmetric information "against" the consumers about the quality of the goods (Doyle 1983), discrimination between high valuation-low discount factor consumers and low valuation-high discount factor ones for durable goods (Sobel 1984) or between high valuation-high holding costs consumers and low valuation-low holding

costs ones (Jeuland and Narasimhan 1985), asymmetric information "against" the sellers about the tastes of the consumers (Lazear 1986 or type of demand (Aghion, Bolton and Jullien 1988), interaction between the pricing and the advertising decision (Villas-Boas 1990).

Another set of these models are dynamic simply because the competitors actions can depend on past actions or outcomes, although history does not affect payoffs directly. This "supergame approach" has been used by Green and Porter (1984), Rotemberg and Saloner (1986) and Lal (1990).

Empirical work on the supergame approach has been done by Porter (1984) (who tests Green and Porter) and by Domowitz, Hubbard and Petersen (1986) (who test Green and Porter vs. Rotemberg and Saloner). The results of this work have been somewhat supportive of the theoretical models.

Pashigian (1988) confirms also some implications of Lazear (1986). Raju, Srinivasan and Lal (1990) present also some evidence on their results, although they do not make direct tests on the density functions.

In this study, I concentrate on testing the explanations of temporal and spatial price dispersions that rely on price discrimination between "informed" and "uninformed" consumers (or consumers with high and low search costs), and where history does not affect payoffs directly. Given the Folk theorem, we know that the set of equilibria of this type of model is very large. Picking an equilibrium over another is an empirical question which I do not address in this work. I restrict my attention to equilibria where

actions only depend on the payoff relevant state variables². This means, I concentrate on static equilibria as all the sources of dynamic equilibria are assumed away. The objective of this work is to test if the results of Varian occur at all, and not to test if the results of Varian occur always.

Coffee price data from several supermarkets is used to check whether the distribution of prices observed for each brand during a certain period, is close to the theoretical distribution derived in Varian (1980). In order to do this, a maximum likelihood estimation of the density function parameters is implemented: marginal cost, reservation price and the percentage of informed consumers are estimated.

I start by presenting the theoretical model (section II) and discuss the data that were used (section III). Then, I describe the estimation procedure (section IV), discuss the results (section V), and the most critical assumptions for model application (section VI). Finally, in section VII, the main conclusions and implications for future research are presented.

II - THE MODEL

Description

In this section I briefly review the model of Varian (1980).

²An alternative assumption is that there is free entry in the market under consideration.

There is a large number of consumers each of whom wants to buy one unit per period and has a reservation price r . There are two types of consumers: informed and uninformed³. The informed consumers buy from the seller that posts the lowest price. The uninformed consumers buy at random and uniformly through all the stores.

The number of informed consumers is $I > 0$. The number of uninformed consumers is $M > 0$. The number of sellers is n . Then, the number of uninformed consumers per store is $U = M/n$.

In each period, each seller chooses a price. If the price chosen is the lowest, that seller gets a demand $I+U$. If a seller does not have the lowest price it gets a demand U . If two or more sellers charge the lowest price each of the low price sellers gets an equal share of the informed consumers plus U .

In each period, each seller has a mixed strategy on prices (P) $f(P)$ (that can, for the moment, degenerate to a pure strategy). Sellers maximize profits given the strategies of other sellers and the behavior of the consumers.

Each seller has the same cost function with a fixed cost K per period, of producing the good, and a constant marginal cost c . Production is realized when customers arrive at the sellers so that there is never excess supply or excess demand at the level of each seller.

No good can be stored from one period to another.

³I am considering here the information difference among consumers as exogenous. An equivalent way would be to make it dependent on search costs which are different among consumers.

I, M, n, r, c, K are constant over time.

Analysis

Given the Folk theorem, the set of equilibria of this market is very large (in particular, there are dynamic equilibria where all sellers set the cooperation price - r in this case). The selection of equilibrium is a problem for which very few solutions have been presented and which I do not address at all. I simply concentrate on the equilibrium in which the actions are independent of the past (which in this case is equivalent to actions being only dependent on the payoff relevant state variables). Varian solved for this equilibrium and his analysis is summarized below.

Given the structure of the model, notice that each seller can guarantee himself at least $\pi^0 = (r-c)U - K$.

The highest demand a seller can ever have per period is $I+U$. The lowest price a firm would consider charging associated with this demanded quantity is $P^* = (K + \pi^0) / (I+U) + c$.

Given this model, and starting from the assumption of symmetric strategies, Varian (1980) proves several facts (see Varian for proofs):

Fact 1: There is no symmetric equilibrium where all sellers charge the same price.

This fact results from the advantage of underpricing the

competitors if the equilibrium was at a price above P^* . Notice then that at P^* the best response is to charge at the price r .

Fact 2: There are no prices that are charged with a probability greater than zero.

This fact results also from the intuition in Fact 1 and basically means that there are no mass points in the mixed strategy of each competitor.

So, as the probability of a tie is zero, there are only two different events for a seller: "having the lowest price" in which case it gets $\Pi_s(P) = (P-c)(I+U)-K$ and "not having the lowest price" in which case it gets $\Pi_f(P) = (P-c)U-K$.

"Having the lowest price" happens with probability $(1-F(P))^{n-1}$, where $F(P)$ is the cumulative distribution function of $f(P)$.

The seller indifference (given the mixed strategies) among the prices that are set yields,

$$\Pi_s(P)(1-F(P))^{n-1} + \Pi_f(P)[1-(1-F(P))^{n-1}] = \pi^0 \quad \forall P; f(P) > 0^4$$

which can be written as

$$1-F(P) = \left(\frac{\Pi_f(P) - \pi^0}{\Pi_f(P) - \Pi_s(P)} \right)^{1/(n-1)} \quad (1)$$

But, what happens around P^* and r ?

⁴This condition is obvious given the fact that we are considering mixed strategies: whatever price, the expected profit must be the same. Notice that if there is free entry $\pi^0 = 0$.

Notice that all the prices slightly above P^* must be charged, because otherwise a firm charging a price slightly below the lowest price being charged makes a profit above π^0 .

Notice also that all the prices slightly below r must be charged, because otherwise a firm charging r has more profit than if it was charging the highest price of the proposed equilibrium.

These comments result in the following two facts.

Fact 3: $F(P^* + \epsilon) > 0, \forall \epsilon > 0$.

Fact 4: $F(r - \epsilon) < 1, \forall \epsilon > 0$.

After some algebra we can get Fact 5.

$$\text{Fact 5: } F(P) = 1 - \left[\frac{(r-P)(P^*-c)}{(r-P^*)(P-c)} \right]^{1/(n-1)} \quad (2)$$

Notice, from (2), that $F(r) = 1$ and $F(P^*) = 0$.

Notice also that the model is completely determined as π^0 can be obtained from $(r-c)M/n = K + \pi^0$ (profit π^0 for price r) and P^* from $(P^*-c)(I+M/n) = K + \pi^0$ (profit π^0 for price P^*).

$$\text{Fact 6: } f(P) = \left(\frac{P^*-c}{r-P^*} \right)^{1/(n-1)} \frac{1}{n-1} \left(\frac{r-P}{P-c} \right)^{1/(n-1)-1} \frac{r-c}{(P-c)^2} \quad (3)$$

for $P^* \leq P \leq r$.

Notice that $f(P) > 0$, for $P^* \leq P \leq r$.

For $m=1-1/(n-1)$, and using the indicator function⁵ (3) can be written as

$$f(P) = 1[P^* \leq P \leq r] \left(\frac{P^* - c}{r - P^*} \right)^{1-m} (1-m) \left(\frac{r-P}{P-c} \right)^{-m} \frac{r-c}{(P-c)^2} \quad (4)$$

Finally, notice that

$$\lim_{P \rightarrow r} f(P) = \infty$$

so that the major part of the price density is concentrated in the high prices. A typical graph of $f(P)$ is shown in figure 1 for the case where P^* is sufficiently close to c ($P^* < r - m(r - c)/2$). The graph also shows that there is greater density in the low prices than in the intermediate prices.

I now proceed to estimate expression (4) for each good by maximum likelihood (using P^* , r , c and m as parameters). Data on prices for each good are described in the next section. The estimation procedure is explained in section IV, and the results are presented in section V.

III - THE DATA

The empirical testing of the model developed in section II uses price data on coffee products⁶. This data set was obtained from a

⁵The indicator function is represented by [expression] and takes the value one if the expression is true and the value zero if the expression is false.

⁶All data were collected and provided by Information Resources Incorporated.

panel of families from Kansas City. It is composed of prices on 198 cover packages across 6 stores (so, 1188 goods⁷ for our purpose) during 108 weeks (a price per week from mid-1985 to mid-1987). There were only enough data⁸ for 541 goods (from the original set of 1188 goods).

The 6 stores were part of three chains: chain 1 had three stores; chain 2 had two stores and chain 3 had one store.

A cover package is defined by the brand (Ex: Folgers, Tasters' Choice, Maxim, etc), the type of coffee (Ex: instant coffee, freeze dried coffee, roast instant coffee, etc) and the size of the package (Ex: 2 OZ., 4 OZ., 8 OZ., 16 OZ., etc). The physical good is defined by the brand and type of coffee.

Each brand belongs to one manufacturer (Philip Morris, Procter & Gamble, Tetley, etc.). Some manufacturers carried more than one brand.

In this sample there were 8 manufacturers which carried a total of 32 brands (the distribution of brands across manufacturers can be seen in Table I). Out of these 32 brands only 7 were not represented in all the chains.

In addition to these 32 brands each chain carried a private label brand.

⁷A good is defined here by the physical characteristics, package size and location of sale: the same physical good in different package sizes corresponds to different goods.

⁸As the data come from a panel of families, in periods when no family bought a particular good in a store, there were no available data. So, for each good, there is only a subset of the total number of weeks for which there are data. This fact might cause a bias against the goods which were priced too high.

Several types of coffee could be sold under the same brand. In this sample the total number of physical goods and cover packages being sold were respectively 87 and 136 across the three chains.

Chain 1 carried 96 out of these 136 cover packages. Chains 2 and 3 carried respectively 87 and 85 cover packages. Chain 1 was the least intensive in the private label (these results can be seen in Table II).

The size of each package ranged from 0.25 OZ. to 32 OZ. The average size was 13.1 OZ. and the standard deviation of the size was 9.2 OZ.

Table III presents some statistics on the data set.

Notice that, despite the correlations between prices of goods of different cover packages characteristics and between prices of goods with the same cover package characteristics and sold in different store chains being somewhat large, they indicate that there is some room for the stores to decide on prices. The final retail price is a combination of manufacturer and retailer decisions and this fact was not allowed for in the original model. This problem will be discussed further below.

The value of the correlation between prices of goods of the same cover package and sold in stores in the same store chain averages 0.97 and has a standard deviation of 0.01. Stores belonging to the same store chain seem to have little independence among themselves. In relation to the model presented above there are three explanations for

this fact⁹: goods of the same physical characteristics and sold in stores belonging to the same store chain have the same price because of large costs in randomizing or large costs of transmitting (advertising) the randomization output to the "informed" customers,¹⁰; another explanation is that the majority of the analyzed goods are in a dynamic equilibrium as I conjecture below; finally, as I also discuss below, the assumptions of the model might not be "fairly" satisfied.

Despite this potential failure of the cross-checking of the model, figures 2 through 8 (plots of the observed prices through time, for some of the goods) show that the model may still have explanatory power: a high price is typically charged and there are "random" price cuts.

Another potential problem shows up in these figures: the parameters of the model do not seem stable through time (it seems that there are two relatively stable subsamples). The solution that was adopted has two components.

First, notice that either continuous changes in the parameters of

⁹A fourth explanation might just be the existence of state laws that do not allow different stores of the same chain to charge different prices for the same good.

¹⁰If these costs are sufficiently high and if the market share of the chain is sufficiently small, the model can still be applied and the price of a physical good should be the same across stores of a same chain.

Additionally, I am assuming that there is an advertising expenditures decision variable which helps distinguish between "informed" and "uninformed" consumers and whose effects on profits (on revenues and costs) need not be considered in order to determine the equilibrium pricing behavior.

the model (for example, input prices fluctuate over time) or dynamic equilibrium behavior lead to positive autocorrelation in the observed prices. Only if the parameters are constant through time or the sellers play a static equilibrium will there be no correlation. This observation conducts us to try to test the model only in the goods for which it does not seem to exist much autocorrelation (i.e. for which the parameters of the market do not seem to change very much). This approach is very simple and conservative in the sense that one could always use other variables to predict the change in the parameters of the model and always get a better fit.

Secondly, one can assume that the decision-makers (on prices of goods) make a periodic "measurement" of some of the parameters. This "measurement" is costly, so that it cannot be made continuously through time, for example, it is made once a year. The decision-maker behaves as if the parameters took the values of the most recent "measurement". This type of story can support all the figures presented above. The estimations to be performed are then the ones of the values of the parameters in each of the subperiods¹¹ and the moment in time when the "measurement" is realized.

The solution adopted was to drop the subperiods of the price series where there seemed to exist some dependence through time. The criterion that was used, was to drop out of the analysis subperiods (for any good) where the regression of $P(t)$ on $P(t-1)$ showed a

¹¹I assume there is an unique "measurement" during the sample period. The timing of this event defines two subperiods: one subperiod before the "measurement" and another one after.

coefficient on $P(t-1)$ greater than 0.7 and a t-statistic on this coefficient greater than 2¹² 13. Obviously, this criterion still keeps a lot of goods that might be in a dynamic type of equilibrium (typically, in these models, $P(t)$ depends also on competitors past prices) or for which the market parameters might change too much¹⁴. Notice once more that this is a very conservative approach and that the results are biased against the model.

The use of this criterion seems to imply (given the dynamic equilibrium type of reasons) that 1) smaller manufacturers (in terms of number of cover packages offered) behave more competitively, 2) the private labels are priced as in the cooperation equilibrium, 3) chain 3 goods are priced more than chains 1 and 2 goods as in the cooperation equilibrium, 4) the percentage of "instant coffee" (the most traded type of coffee) cover packages in the set of goods that

¹²This criterion was applied after the estimation (of the moment in time the "measurement" took place) was performed.

¹³The result was that I only kept 40 out of the initial 1082 subperiods (two for each of the 541 goods; one subperiod before the "measurement" and the other one afterwards). In fact, I checked the Varian model against all subperiods: as expected 90% of the subperiods where there was autocorrelation rejected the model (in terms of the criterion explained in section V). This is an additional validation of the model.

¹⁴I am assuming that each good can be either in a dynamic or static type of equilibrium for reasons that will not be discussed in detail here (for example, if there are barriers to entry, the equilibrium is more likely to be of the dynamic type). As the model I am using only applies for the static equilibrium markets, I should not test the model against markets where the equilibrium is of the dynamic type. The same reasoning applies to changes in the market parameters. By "testing the model" I mean testing how well the model explains the observed prices in a market that can be in a static type of equilibrium and where the market parameters do not seem to change.

passes the above criterion is slightly higher than in the original sample (45% versus 40%) and 5) the occurrence of the cooperation equilibrium does not depend on the package size. These results seem to confirm the criterion that was used at least in relation to dynamic equilibrium issue. Tables IV and V present the distribution of the goods that passed the above criterion across chains and manufacturers.

IV - ESTIMATION PROCEDURE

Accepting the assumptions of the model (and the way they are "almost" satisfied) the next step is to estimate $f(P)$ for each good and subperiod¹⁵. In order to do this step, expression (4) was used. The parameters to estimate are r , P^* , c and m for each of the subperiods and the moment in time the "measurement" was realized (B).

A maximum likelihood estimation technique was used. The log of the maximum likelihood function is

$$\begin{aligned} \log(L) = & B(1-m_1)\log(P_1^*-c_1) - B(1-m_1)\log(r_1-P_1^*) + B\log(1-m_1) - \\ & - m_1 \sum_{i=1}^B \log(r_i - P_i) + (m_1-2) \sum_{i=1}^B \log(P_i - c_i) + B\log(r_1 - c_1) + V_1 + \end{aligned}$$

¹⁵An alternative way (and a more efficient one) would be to estimate the joint density of all the prices. The problem with this approach is that I would then have to define which are the sets of goods that were in direct competition. This task is not easy and straightforward.

$$+ (N-B)(1-m_2)\log(P_2^*-c_2) - (N-B)(1-m_2)\log(r_2-P_2^*) + (N-B)\log(1-m_2) -$$

$$- m_2 \sum_{i=B+1}^N \log(r_2-P_i) + (m_2-2) \sum_{i=B+1}^N \log(P_i-c_2) + (N-B)\log(r_2-c_2) + V_2$$

where N is the number of observations for the price of the good being considered, the index 1 and 2 refer to the 1st and 2nd subperiod and

$$V_1 = \sum_{i=1}^B \log (1[P_1^* \leq P_i \leq r_1])$$

$$V_2 = \sum_{i=B+1}^N \log (1[P_2^* \leq P_i \leq r_2])$$

Given B, $\log(L)$ can be expressed as the sum $\log(L_1)+\log(L_2)$, where $\log(L_i)$ is the part of $\log(L)$ that refers to subperiod i. We can then maximize $\log(L_i)$ independently of the other subperiod: this maximization gives the maximum likelihood estimators (and the standard deviations) of r_i , c_i , m_i and P_i^* . The properties of these estimators are exactly the same as if one did maximum likelihood on $\log(L_i)$.

The estimation of B was performed using the following steps: 1) take a B; 2) maximize $\log(L_1)$ and $\log(L_2)$; 3) calculate $\log(L)=\log(L_1)+\log(L_2)$; 4) if all the B's have not been checked yet, take another B and go to 2); 5) the maximum likelihood estimator of B is the one that lead to the highest $\log(L)$.

The rest of this section explains how to maximize $\log(L_i)$, the properties of the maximum likelihood estimators of r_i , c_i , m_i and P_i^* and the test of the theoretical model using these estimators.

In the rest of the paper 1) the variables L , r , c , m , P^* , n , I , U and K refer to one of the subperiods; 2) T is the number of observations in the subperiod being estimated; and 3) I assume I know B with certainty¹⁶ so that, doing maximum likelihood on $\log(L)$, is equivalent to doing maximum likelihood on $\log(L_i)$.

Derivatives in respect to r , P^* , c and m have to be calculated in order to maximize $\log(L)$. It can be easily confirmed that the maximum likelihood estimators for r and P^* are respectively the maximum and the minimum values of the series $\{P(t)\}$ in this subperiod.

Using the maximum likelihood estimates of r and P^* as the true values¹⁷, I can now proceed to the usual differentiation with respect to c and m and subsequent equalization to zero.

$$\frac{\partial \log(L)}{\partial c} = 0 \Leftrightarrow -T(1-m) \frac{1}{P^* - c} - \frac{T}{r-c} - (m-2) \sum_{i=1}^T \frac{1}{P_i - c} = 0 \quad (5)$$

$$\begin{aligned} \frac{\partial \log(L)}{\partial m} = 0 \Leftrightarrow & -T \log(P^* - c) + T \log(r - P^*) - \frac{T}{1-m} + \\ & + \sum_{i=1}^T \log(P_i - c) - \sum_{i=1}^T \log(r - P_i) = 0 \end{aligned} \quad (6)$$

¹⁶As I do not know B with certainty, the estimators of the standard deviations that are presented in the text are biased downwards.

¹⁷Assuming that the maximum likelihood estimates of r and P^* are the true values could result in an underestimation of the asymptotic variances of c and m . In fact, this is not the case, as while the maximum likelihood estimators of c and m converge to their true values at rate \sqrt{T} , the maximum likelihood estimators of r and P^* converge at rate T , as shown in Kendall and Stuart (1961), p. 424. So, in the asymptotic distribution of $\sqrt{T}(\beta - \hat{\beta})$, the maximum likelihood estimates of r and P^* can be taken as fixed numbers ($\beta' = (c, m)$ and $\hat{\beta}$ is the maximum likelihood estimator of β).

Definition: α converges at rate T^γ to its true value α iff $\text{plim } T^\gamma(\alpha - \hat{\alpha}) = 0$.

The solution of the system composed of (5) and (6) gives the maximum likelihood estimators for c and m ¹⁸ in that subperiod. These estimators¹⁹ are consistent, are asymptotically efficient relative to all other consistent uniformly asymptotically normal estimators and have an asymptotically normal distribution in the sense that

$$\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} N(0, \text{plim}(I(\beta)/T)^{-1})$$

where $\beta' = (c, m)$, $\hat{\beta}$ is the maximum likelihood estimator and

$$I(\beta) = - E \begin{bmatrix} \frac{\partial^2 L}{\partial c^2} & \frac{\partial^2 L}{\partial c \partial m} \\ \frac{\partial^2 L}{\partial c \partial m} & \frac{\partial^2 L}{\partial m^2} \end{bmatrix} \quad (7)$$

The estimation was made through numerical approximation using the iterative method of scoring (Berndt, Hall, Hall and Hausman (1974)). The estimator for $\text{lim}(I(\beta)/T)$ that was used, was

$$- \frac{1}{T} \left[\sum_{i=1}^T \left(\frac{\partial^2 \log(f(P_i | \beta))}{\partial \beta \partial \beta'} \right) \right]$$

¹⁸The reader can check that the second order conditions are satisfied given that the density of P is concentrated in values close to r and away from P^* .

¹⁹For a discussion of the properties of this estimator see Judge et al (1985). The regularity conditions that allow for the properties which are stated above are presented in this reference.

which is consistent given the regularity conditions referred to above.

After having estimated $f(P)$ for this subperiod, I tested if the data rejected $f(P)$ as the theoretical distribution from which the observation $\{P(t)\}$ of this subperiod came from²⁰. In order to do this test, the range of $f(P)$ was divided in intervals. In each interval i , f_i was defined as the proportion of P 's in that interval and Π_i the theoretical probability of P being in that interval.

It is well known that

$$Q = \sum_{i=1}^k \frac{(Tf_i - T\Pi_i)^2}{T\Pi_i} \xrightarrow{d} G \cong \chi^2(q-1)$$

²⁰As stated above, this test was exclusively realized for the subperiods where the price did not seem very dependent through time. The criterion to determine this dependence and the reasons for not doing the test for these subperiods are stated in section III.

where q is the number of intervals^{21 22} and G is some distribution. The null hypothesis is that the observed P 's come from the estimated theoretical distribution $f(P)$. The statistic takes the value zero if the fit is perfect, and is greater, the further away the real distribution of the P 's is from $f(P)$. The null hypothesis is rejected if Q is greater than the critical value on the distribution $\chi^2_{(q-1)}$ (in all the cases, considered with a 5% significance level).

V - THE RESULTS

The results can be divided in two parts: estimates of the parameters of the model and the value of the statistic Q . All of these are presented in table VI.

²¹In fact, this is not the standard case for the application of this test. In the standard case Π_i is known with certainty while in this case Π_i is estimated. If Π_i was estimated with a maximum likelihood procedure on the q intervals used in the test, Q would still be distributed asymptotically as χ^2 , but with $q-s-1$ degrees of freedom, where s is the number of estimated parameters that converge at \sqrt{T} (in this case $s = 2$). This fact is shown in Kendall and Stuart (1961), p. 425.

But here, Π_i was estimated with a maximum likelihood procedure on the T observations of the random variable (the price). In this case, Q does not have an asymptotic χ^2 distribution. "However the distribution of Q is bounded between a $\chi^2_{(q-1)}$ and a $\chi^2_{(q-s-1)}$ variable, and as q becomes large these are so close together that the difference can be ignored" (Kendall and Stuart (1961), p. 430).

²²The choice of the number of intervals has impact on the null hypothesis being rejected or not. For all the estimations that were made the number of intervals chosen was the highest integer less than or equal to $N/5$.

The estimates of the parameters of the model take very acceptable values in terms of the market being studied.

The estimates of marginal cost are reasonable across all goods: the profit margin²³ takes values between 0.12 and 0.85 (but 90% are below 0.5) for the high prices that are charged (prices close to r , which ones are the prices that happen with greater probability).

Within each good the price variation (maximum minus minimum price) ranges between \$.40 and \$1.49, without any significant positive correlation between price level and absolute variation.

The estimates of n ($m=1-1/(n-1)$) look also reasonable across all goods and range between 3 and 8.

Notice also that the asymptotic standard deviations of the estimators of c and m are relatively small which shows that we have relatively good estimates of c and n .

From the estimated parameters I can derive some relations between the other parameters of the model. This capacity shows the explanatory power that exist in a simple price series once one is willing to make the (hopefully) right assumptions.

I can estimate the percentage of potential demand for a good that is informed and the ratio of fixed cost plus profit per week and potential demand²⁴. These estimates look also acceptable at first sight.

²³ Profit margin is here defined as $(P-c)/P$.

²⁴ Notice that

$$\frac{I}{I+U} = \frac{r-P^*}{r-c} \text{ and } \frac{K+\pi^0}{I+U} = P^* - c$$

The percentage of "informed" customers ranges between 87% and 12%, but the most part of these values are above 50%. These surprising high values might result from the in-store search - which is not allowed for in the Varian model.

There does not seem to be any other pattern in the relation among price dispersion, percentage of "informed customers", ratio of fixed costs to potential demand, manufacturer and package size in the results presented in Table VI.

Finally, the χ^2 test on the estimated distribution does not let reject the model for about 85% of the studied subperiods. This figure looks very good given the assumption on the movement of $f(P)$ through time and the fact that several goods (and their subperiods) which are in a dynamic equilibrium may have been included in the analysis. The results seem very supportive of the model and its assumptions as able to explain static equilibrium price dispersion.

VI - DISCUSSION OF ASSUMPTIONS

While applying the model presented in section II to the coffee data several problems arise. We have already discussed in section I the existence of other well known reasons for price cuts.

Another set of issues relates to the definition of goods. The model applies to an equivalent physical good sold by different independent sellers. The target data do not match this on three counts: 1) the relation between cost characteristics and reservation price varies across goods (the net dollar benefit at the price P_i^* -

lowest price with possible non-negative profits for firm i -, i.e., $r_i - P_i^*$, varies across goods), 2) in several cases the same seller makes decisions on different goods and 3) in several goods, decisions are taken by two decision makers (the manufacturer and the store).

Partial explanations for using the model in face of these problems are the following: 1) even though the $r_i - P_i^*$'s might be different across goods in the same submarket, every seller believes they are equal, because, if not, the seller with the largest $r_i - P_i^*$ would surely reduce the competitors demand to just the uninformed consumers (by charging $P_i^* + \epsilon$), which is unlikely to have happened given the observed strong interaction among the different competitors; 2) in the cases where a seller makes decisions on several goods in a submarket, the impact on the model equilibrium is likely to be small as the number of competitors is relatively large; and, 3) decisions that are taken by several decision makers might, under some incentive schemes (which are assumed to exist and to be implemented), be the same as the ones taken by a unique decision maker.

Notice that these potential problems are biasing the results against the model so that positive outcomes of the test do really seem to support the predictions of the model.

VII - CONCLUSION

Price data on coffee products is used to test whether the sample

of prices on each product could have possibly come from the density function specified in the equilibrium of the Varian (1980) model. The results were relatively supportive of the model.

In the process of testing the model, estimates of the marginal cost for each product, the number of perceived competing goods, the percentage of informed demand to potential demand and the ratio of fixed costs to potential demand were obtained.

Goods that do not conform with this model are, either in a market where there is some kind of monopolization (in these cases the equilibrium is dynamic, and, in these types of equilibria the cooperation outcome could be expected), or in a market where its parameters are changing too much.

Being one of the first attempts at testing price dispersion results, this work is still incomplete but, without any doubt, very promising. A natural continuation of this line of research would be to try to include the parameter changes into the model, to operationalize and confirm the explanatory power of the dynamic equilibrium type of models of price dispersion which were cited above, and, to try to identify the factors that influence equilibrium selection. Another interesting line of work is also to test the asymmetric firms results as derived, for example, in Raju, Srinivasan and Lal (1990).

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TABLE I
MARKET AND BRANDS SHARES

Manufacturer	# Brands in Sample	National Market Share
1	11	22.8%
2	2	11.7%
3	5	4.5%
4	2	3.7%
5	1	2.2%
6	1	1.8%
7	1	1.0%
8	1	0.8%

Note: The national market share is for 1989
and was obtained from Dialnet.

TABLE II
 PHYSICAL GOODS AND COVER PACKAGES
 ACROSS CHAINS

Chain	# Physical Goods	# Cover Packages	# Private Label Cover Packages
1		96	1
2		87	11
3		85	17
Total	87	136	29

TABLE III
PRICE VARIABILITY AND CORRELATION BETWEEN PRICES

	Range	Average	Standard Deviation
Prices	\$.89-\$7.25		
Coefficient of variation of the prices (mean/ standard deviation)		21.45%	11.88%
Correlation between prices of cover packages from different manufacturers and sold in different stores		0.52	0.45
Correlation between prices of the same cover package across stores belonging to different chains		0.74	0.30
Correlation between prices of the same cover package across stores of the same chain		0.97	0.01

TABLE IV
DISTRIBUTION OF TESTED GOODS ACROSS CHAINS

% of original cover
packages that pass the
criterion for cooperation

Chain 1	13.5%
Chain 2	14.9%
Chain 3	3.5%

TABLE V
DISTRIBUTION OF TESTED GOODS ACROSS MANUFACTURERS

Manufacturer	% of original goods that pass the criterion for cooperation
1	3.0%
2	10.3%
3	8.1%
4	18.2%
5	28.6%
6	33.3%
7	25.0%
8	33.3%
Private Labels	6.9%

TABLE VI

RESULTS

Good	Number Obs.	Number Obs T.	r	c	P*	m	n	χ^2	d.f.	Crit. Value	$\frac{I}{I+U}$	$\frac{K+\pi^0}{I+U}$
13B	43	102	2.49	1.20 (.319)	1.59	0.78 (.04)	5.6	10.7	7	14.1	0.701	0.38
14B	43	99	2.49	1.20 (.324)	1.59	0.77 (.04)	5.3	11.1	7	14.1	0.696	0.39
27B	67	104	2.49	1.50 (.111)	1.68	0.77 (.04)	5.3	18.9	12	21.0	0.817	0.18
28B	67	104	2.49	1.52 (.103)	1.68	0.78 (.03)	5.6	17.8	12	21.0	0.837	0.16
29B	29	106	2.49	0.23 (4.91)	1.99	0.54 (.13)	3.2	3.7	4	9.5	0.222	1.76
33B	67	104	2.49	0.80 (.437)	1.48	0.73 (.04)	4.8	21.3	12	21.0	0.599	0.68
34B	59	106	2.49	1.56 (.085)	1.68	0.78 (.04)	5.5	12.8	10	18.3	0.868	0.12
36A	10	57	3.99	2.74 (1.22)	3.29	0.81 (.07)	6.4	0.1	1	3.8	0.560	0.55
37A	10	60	3.99	2.74 (1.22)	3.29	0.81 (.07)	6.4	0.1	1	3.8	0.560	0.55
69B	27	104	4.55	3.62 (.103)	3.78	0.71 (.07)	4.4	12.9	4	9.5	0.826	0.16
73B	19	108	4.57	3.78 (.204)	3.89	0.64 (.18)	3.8	1.4	2	6.0	0.864	0.11
112B	19	67	5.17	4.32 (.234)	4.49	0.69 (.10)	4.2	1.0	2	6.0	0.798	0.17
113B	19	68	5.17	2.23 (4.57)	4.29	0.53 (.15)	3.1	0.2	2	6.0	0.300	2.06
121B	13	72	2.99	1.68 (2.38)	2.59	0.68 (.10)	4.2	1.2	1	3.8	0.306	0.91
155A	39	95	5.98	4.38 (.213)	4.69	0.82 (.03)	6.6	10.5	7	14.1	0.807	0.31

TABLE VI (continuation)

Good	Number Number		RESULTS							Crit. Value	I $\overline{I+U}$	K+ π^0 $\overline{I+U}$
	Obs.	Obs	T.	r	c	* P	m	n	χ^2			
156A	20	91	5.98	4.44	4.79	0.85	7.6	1.3	3	7.8	0.775	0.35
				(.424)		(.04)						
156B	72	91	5.98	3.53	4.49	0.48	3.2	48.6	13	22.4	0.609	0.96
				(.666)		(.10)						
157A	10	109	5.97	4.17	5.39	0.80	5.9	0.2	1	3.8	0.323	1.22
				(4.74)		(.08)						
172B	27	105	4.59	2.48	3.59	0.50	3.0	5.6	4	9.5	0.474	1.11
				(1.34)		(.14)						
218A	20	60	4.36	1.16	3.97	0.37	2.6	8.9	3	7.8	0.122	2.81
				(10.2)		(.18)						
219A	20	65	3.89	2.81	3.09	0.79	5.9	4.7	3	7.8	0.741	0.28
				(.271)		(.05)						
363B	24	83	3.69	0.56	2.98	0.76	5.1	1.6	3	7.8	0.227	2.42
				(9.75)		(.07)						
366B	23	101	3.69	1.18	2.98	0.77	5.3	1.2	3	7.8	0.284	1.80
				(6.22)		(.07)						
416B	19	108	2.99	1.11	2.59	0.42	2.7	4.8	2	6.0	0.213	1.48
				(3.56)		(.18)						
417B	18	107	2.99	1.25	2.59	0.44	2.8	4.8	2	6.0	0.231	1.34
				(3.25)		(.18)						
444A	40	108	5.49	4.81	5.19	0.73	4.7	7.1	7	14.1	0.443	0.38
				(.541)		(.06)						
458A	20	64	7.25	5.29	5.99	0.44	2.8	2.4	3	7.8	0.644	0.70
				(.754)		(.19)						
464B	27	106	4.39	3.34	3.69	0.69	4.2	1.3	4	9.5	0.668	0.35
				(.697)		(.35)						
465B	49	108	4.39	2.47	3.29	0.81	6.3	4.5	8	15.5	0.575	0.81
				(.775)		(.03)						
481B	19	108	2.99	1.11	2.59	0.42	2.7	4.8	2	6.0	0.213	1.48
				(3.56)		(.18)						

TABLE VI (continuation)
RESULTS

Good	Number Obs.	Number Obs T.	r	c	P*	m	n	X ²	d.f.	Crit. Value	$\frac{I}{I+U}$	$\frac{K+\pi^0}{I+U}$
482B	19	108	2.99	1.11 (3.56)	2.59 (.18)	0.42	2.7	4.8	2	6.0	0.213	1.48
497B	46	105	2.99	1.61 (.332)	1.99 (.04)	0.80	6.1	4.2	8	15.5	0.724	0.38
505B	24	83	3.39	2.56 (.173)	2.78 (.07)	0.73	4.7	7.2	3	7.8	0.735	0.22
507B	29	88	3.19	1.78 (.929)	2.49 (.05)	0.79	5.7	3.2	4	9.5	0.496	0.71
515B	42	101	2.39	0.37 (3.01)	1.68 (.09)	0.66	4.0	13.4	7	14.1	0.352	1.31
516B	44	103	2.39	1.07 (.968)	1.68 (.08)	0.68	4.1	14.3	7	14.1	0.538	0.61
522A	20	91	2.99	0.88 (.605)	2.19 (.04)	0.76	5.1	1.8	3	7.8	0.454	0.73
530A	30	103	3.49	1.52 (2.26)	2.99 (.26)	0.03	2.0	25.0	5	11.1	0.255	1.47
531A	30	103	3.49	2.47 (.244)	2.99 (.23)	0.00	2.0	27.6	5	11.1	0.490	0.52
538B	59	108	3.99	1.91 (.579)	2.68 (.31)	0.80	5.9	13.9	10	18.3	0.629	0.77

NOTE: A and B refer respectively to the 1st and 2nd subperiod; Number Obs. is the number of observations in that subperiod and Number Obs T. is the total number of observations for that good; r is reservation price, c is marginal cost, P* is the minimum price charged, n is the number of firms, $m=1-1/(n-1)$, I is the number of informed consumers, U is the number of uninformed consumers that will buy that good and K is the fixed cost per week of producing that good; r, c, P* and K are in dollars; these are the sets of goods that are the same cover package and are sold in different stores: (13,14), (27,28,29), (33,34), (36,37), (69,73), (112,113), (155,156,157), (416,417), (464,465), (481,482), (515,516), (530,531); these are the sets of goods that are the same cover package and

TABLE VI (continuation)

RESULTS

NOTE (continuation): are sold in the same chain of stores: (13,14), (27,28), (33,34), (112,113), (156,157), (464,465), (481,482), (515,516), (530,531); these are the sets of goods which are produced by the same manufacturer: (13,14,121) by manufacturer 4, (27,28,29,33,34,36,37) by manufacturer 7, (69,73,112,113) by manufacturer 2, (155,156,157,172) by manufacturer 3, (363,366,416,417,444,458,464,465,481,482) by manufacturer 1, (497) by manufacturer 6, (507) by manufacturer 8, (515,516,522,530,531,538) by manufacturer 5, and (218,219) are private labels; asymptotic standard deviations are in brackets; the critical value presented is for the 5% significance level.

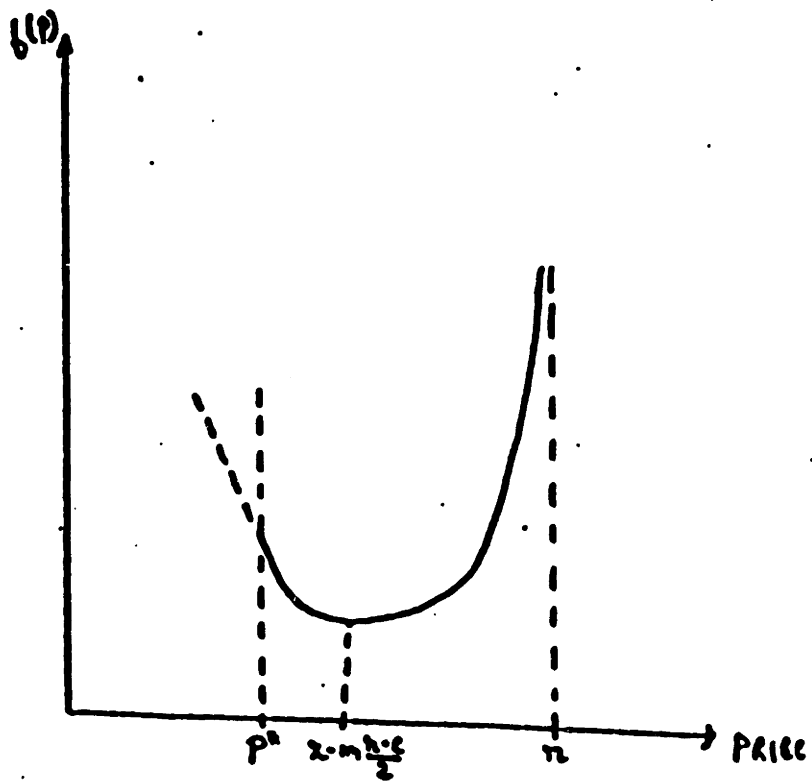


FIG. 1

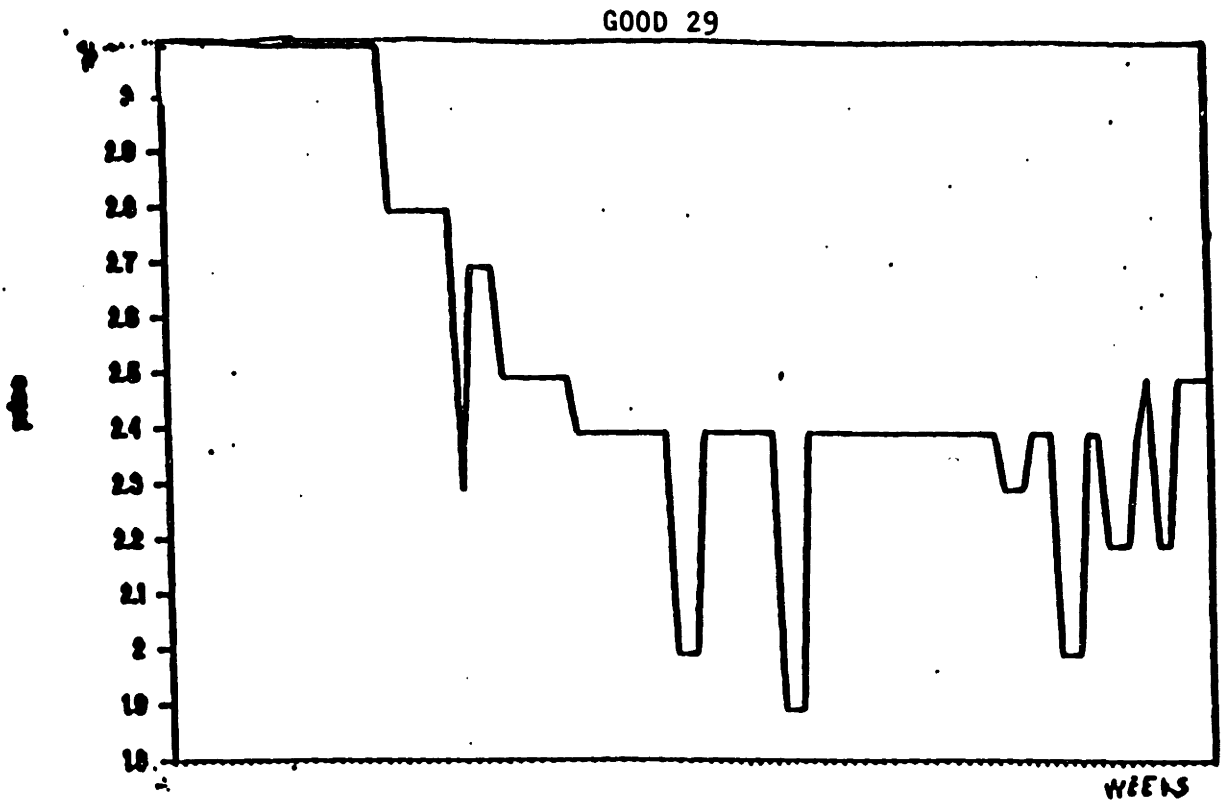


FIG. 2

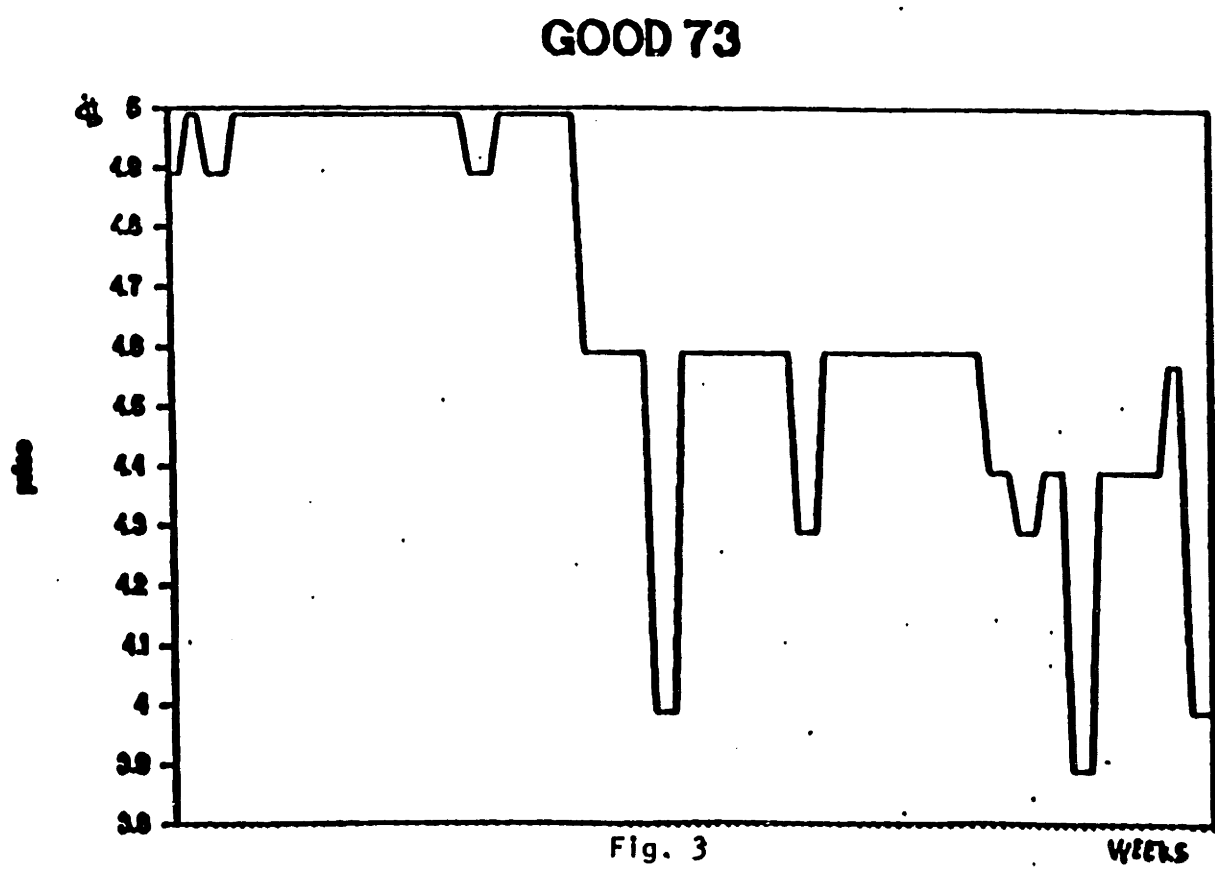


Fig. 3

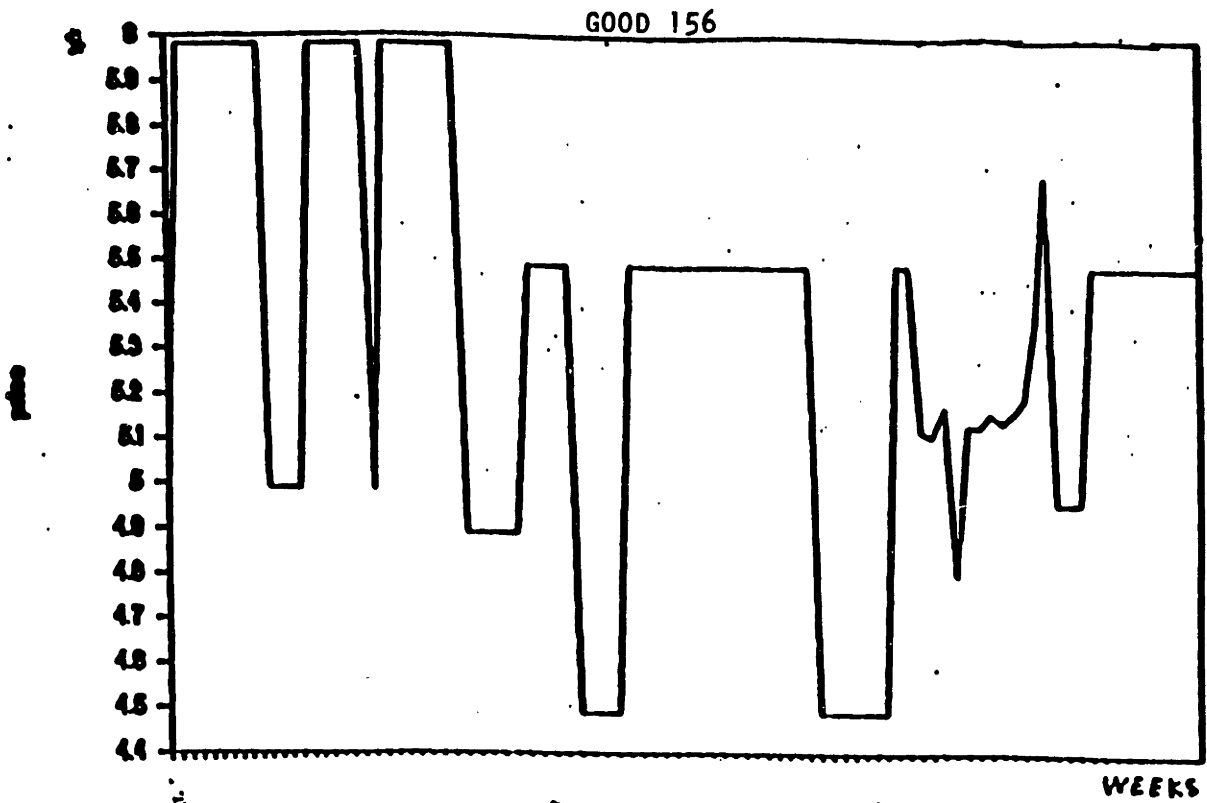


Fig. 4

GOOD 157

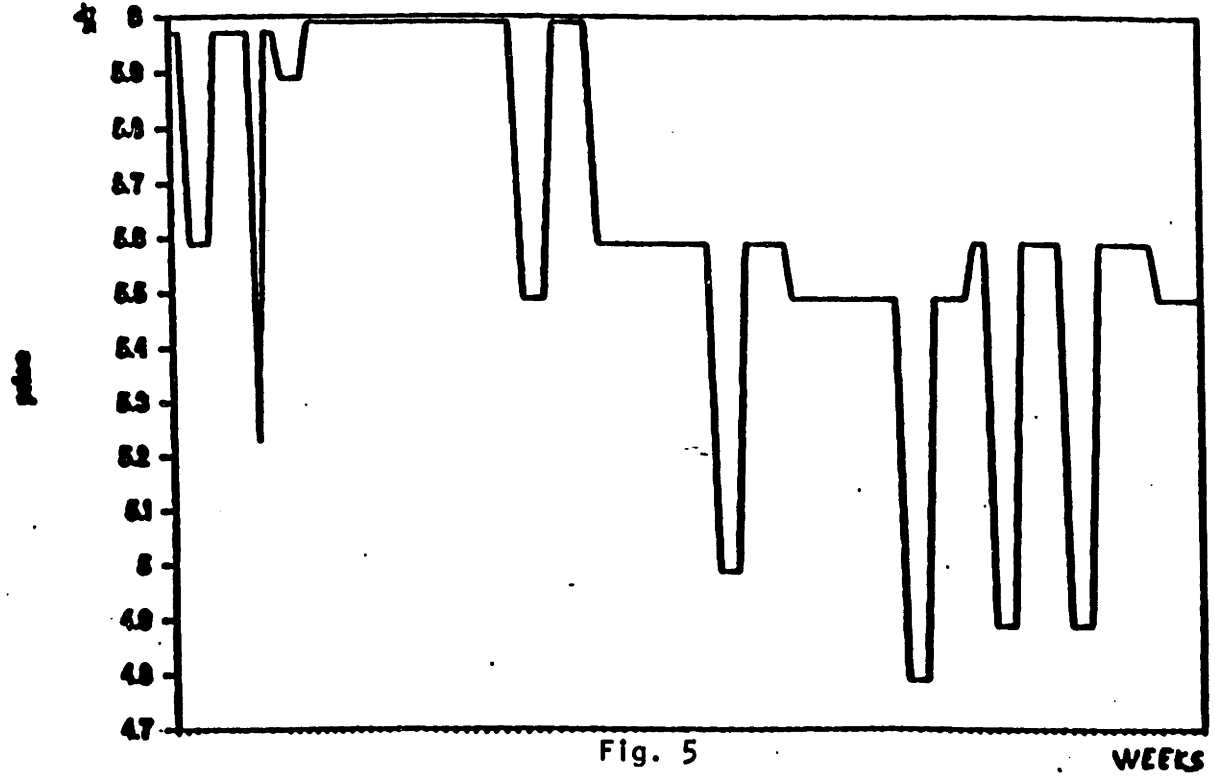


Fig. 5

GOOD 538

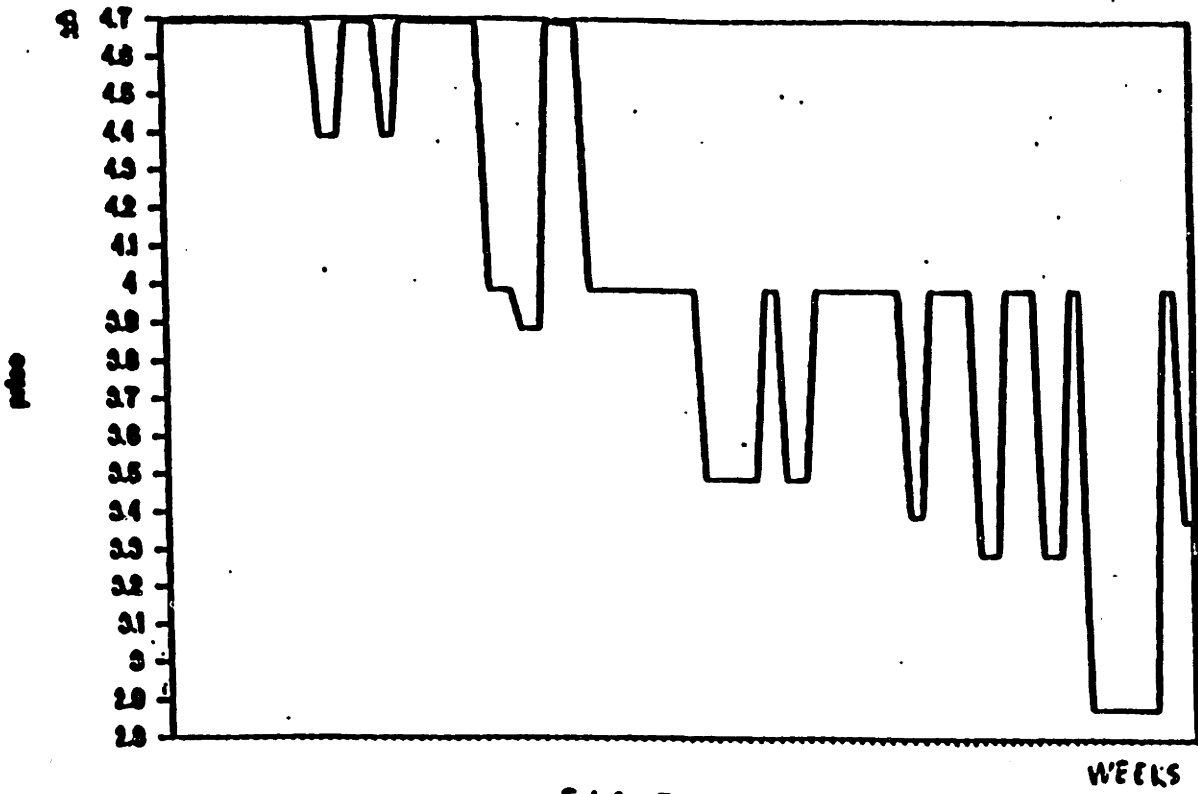


FIG. 8

Chapter 2

PREDICTING ADVERTISING PULSING POLICIES IN AN OLIGOPOLY:

A MODEL AND EMPIRICAL TEST

I - INTRODUCTION

When selling a product, a firm is restricted to the consumers that are informed (aware) about the existence of that product and consider buying it: the demand for the product that firm sells is only the fraction of total potential demand that considers buying the product.

A firm can increase the number of consumers that have the product it sells in their consideration set by spending in advertising. If the advertising expenditures are too low, consideration fades down¹.

The effect of advertising on consideration has been the object of intensive empirical research (Ebbinghaus (1913), Strong (1914), Rao (1970), Zielske (1974), Ackoff and Emshoff (1975), Simon (1982), Simonson and Winer (1990) to cite only some examples). One of the "stylized facts" that came out of this stream of research is that advertising affects consideration through an S-shaped response function: for small levels of advertising there are increasing marginal returns on consideration; for high levels of advertising these returns are decreasing.

Several authors have then used this "stylized fact" to justify the use of pulsing policies in advertising in the real world (Sasieni (1971 and 1989), Lodish (1971), Mahajan and Muller (1986), Feinberg

¹ Throughout all the paper I use the word consideration for the fraction of the number of consumers that have the firm's product in their consideration set, i.e., consider buying the firm's product.

(1988)). Looking at a monopolist these authors note that: 1) it is never optimal for the firm to advertise at the convex part of the response function and 2) for some range of the parameters it might be optimal for the firm to pulse (to alternate between advertising zero and the efficient amount of advertising. In figure 1 efficient amount of advertising (u) is determined by the tangency between a line coming through the origin and the response function ($g(u)$). The efficient amount of advertising is denoted by \bar{u} .

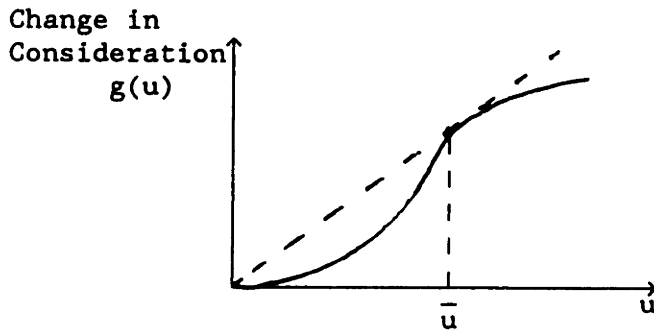


Fig. 1: The S-Shaped Advertising Response Function

But, what happens in a duopoly? We would expect to observe uneven advertising policies, but should we observe synchronous or staggered advertising rates by the duopolists?

This problem of the timing of the advertising expenditures in a competitive situation has been recognized by several authors. Among others, Wells, Burnett and Moriarty (1989) note that "a number of variables affect a timing strategy: consumer needs, the use cycle of the product and the degree of usage and competitive actions" (p.221).

This timing problem can be specially important in the relancing of a product: "In crowded product segments the brand's share of voice can have a powerful influence on aperture opportunity. Share of voice strategies can shift timing and even media selection" (Wells, Burnett and Moriarty, p.231).

In this paper it is shown that for a large discount factor the competitors should take turns at advertising. This is consistent with the well known fact in the advertising industry that competitors, if pulsing, should not advertise at the same time².

The intuition for this result comes from the specificities of the competition for consideration: the gains from having a greater consideration level are larger if the consideration level of the competitor is smaller. Then, the incentive for a firm to invest in advertising (in order to increase its own consideration level) is larger when the consideration level of the competitor is smaller. This results in an equilibrium where firms advertise in alternative periods.

Section II replicates the result of Mahajan and Muller (1986) for the monopolist in a simple model, and, in Section III, a simplified version (with all the main effects) is analyzed for the duopoly case. Section IV presents the result for the oligopoly case. Section V reports some empirical evidence, and, Section VI concludes.

²For example, the Hendry Corporation recommends to only pulse in alternate periods (if pulsing at all).

II - THE MONOPOLY

There is a single seller in the market being analysed. This single seller cares about the present discounted value of profits till infinity. Time is discrete³. Demand and cost conditions are stable over time. The monopolist sells only one product (fixed forever) and can use a set of variables (the marketing-mix variables) in order to manipulate demand. In each period the monopolist chooses the values of these variables. One of these variables is advertising expenditures (u).

Advertising expenditures do not affect demand directly but only through consideration (C). The other variables in the marketing-mix (vector X: price, quality of product, location, sales force, etc) affect demand directly. Furthermore, the values of the variables in X in period t affect only demand in period t (i.e. demand in period t+i, with $i \neq 0$, is independent of X in period t).

Consideration affects demand as only the fraction of the customers that consider buying the product are potential customers for that firm.

³The previous literature on this topic (Sasieni (1971), Feinberg (1988)) has typically considered time to be continuous. This brings additional complexities, though not fundamental to obtain the major insights we are looking for in this work. This paper does not pretend to solve the complexities of using continuous time ("chattering can be the optimal policy", see Feinberg (1988)), but its main contribution is in the analysis of advertising pulsing behavior in a duopoly.

Advertising affects consideration according to an S-shaped response function (figure 1). In the case the monopolist does not advertise in a certain period (zero advertising expenditures), the consideration level will fall in that period.

In each period, the timing of events is the following one: 1) the monopolist observes the consideration level; 2) decides whether or not to advertise; 3) if it advertises, the consideration increases to 1 - otherwise the consideration takes the value it had at the beginning of the period; 4) given the consideration level, the monopolist decides upon the other marketing variables; 5) the monopolist receives the payoff corresponding to the values of the realized advertising expenditures, consideration level and other marketing variables; and, 6) the consideration depreciates by some amount.

Two simplifying assumptions are made on the advertising response function: 1) the curve is sufficiently S-shaped such that it only makes sense to spend 0 or \bar{u} in advertising⁴; and, 2) \bar{u} is so

⁴ In order to be precise let us define an S-Shaped function and what it means for a function to be more S-Shaped than another one. $g(u)$ is an S-Shaped function iff 1) $g(u)$ is continuous, $\forall u \geq 0$, 2) $g(0)=0$; 3) $g(u)$ is weakly increasing in u , $\forall u$; 4) $\exists u^*$, such that $g(u)$ is convex for $u \leq u^*$, and concave for $u \geq u^*$; and, 5) $\lim_{u \rightarrow +\infty} g'(u) = 0$.

Consider $g_1(u)$ and $g_2(u)$, two S-Shaped functions, such that there is one line that passes through the origin and is tangent at both $g_1(u)$ and $g_2(u)$ at \bar{u} , and $g_1(\bar{u})=g_2(\bar{u})$. $g_1(u)$ is more S-Shaped than $g_2(u)$ if $\int_0^{\bar{u}} g_1(u) du + \int_{\bar{u}}^{+\infty} [g_1(u) - g_1(\bar{u})] du < \int_0^{\bar{u}} g_2(u) du + \int_{\bar{u}}^{+\infty} [g_2(u) - g_2(\bar{u})] du$.

The most S-Shaped function is $g(u)$, such that, 1) $g(u)=0$, $\forall u < \bar{u}$; and 2)

powerful that when \bar{u} is spent the consideration reaches its maximum level (say 1). These assumptions allow us to worry only about two values of advertising expenditures (0 and \bar{u}), and about, when to spend in advertising. However, they do not change the nature of the problem (the first assumption is typically a result of the literature on S-shaped reaction functions and the 2nd one simplifies the type of pulsing that results from the model).

Let us now define some notation: C_t is the consideration level in period t ; $f(C_{t-1})$ is the consideration level in the beginning of period t , with $f()$, such that, $f(x) \leq x, \forall x$.

Furthermore, the following variables and relations are defined: D_t takes only values zero or one (one if the monopolist is advertising in period t , zero if not); $C_t = D_t + (1-D_t)f(C_{t-1})$; \bar{u} is the cost of advertising if different from zero; $\pi(C)$ is period profit for the monopolist given that it has consideration C (this function incorporates the optimal decisions of the monopolist on the vector X); $V_t^M(f(C_{t-1}))$ is the net present value of the profits of the monopolist at the beginning of period t , if it has consideration C_{t-1} in period $t-1$ (as the conditions of the problem do not change through time we have that $V_t^M(.) = V_k^M(.) = V^M(.), \forall t, k$); δ is the discount factor ($0 < \delta < 1$); $r(C)$ is the optimal policy of the monopolist given C at the beginning of the period (it takes the value 1 if the monopolist advertises \bar{u} ,

$g(u) = g(\bar{u}), \forall u \geq \bar{u}$. Assuming a response function like this one is a sufficient condition for only to make sense to spend 0 or \bar{u} in advertising (but response functions close to this one give the same result).

and zero if it advertises zero).

Furthermore,

$$V^M(C) = \text{Max}_{0 \leq r \leq 1} r(\pi(1) + \delta V(f(1)) - \bar{u}) + (1-r)(\pi(C) + \delta V(f(C))) \quad (1)$$

A solution to the problem of the monopolist are the functions $r(C)$ and $V^M(C)$ that, respectively, maximize and satisfy (1).

The model presented above is too general (and complex to solve). In order, to answer the questions we are interested in, it is enough to consider a simplified version where the monopolist advertises at least every two periods. This allows us to restrict our attention to only two states of the world: 1) the monopolist having advertised 1 period ago; and, 2) the monopolist having advertised 2 periods ago.

Assumptions F and D below attain this objective without loss of generality.

Assumption F: $f(C_{t-1}) = \text{Max}(C_{t-1} - 1/2, 0)$

Assumption D: If a firm stays a whole period with zero consideration level it can not sell this product anymore.

Assumption F simplifies the number of consideration levels that are possible: complete consideration, 50% consideration and zero consideration. Assumption D guarantees us that zero consideration is a reflection barrier: once zero consideration is reached the monopolist

advertises or exits the market (if the net present value of staying in is negative, which is assumed away).

Given the structure of the problem (and parameters that allow the monopolist to stay in the market), the monopolist has just to decide between advertising every period and advertising every two periods.

The two states of the world to be considered are: $(C=1/2)$ and $(C=0)$.

In order to compute the payoffs of the monopolist in each state of the world, let us further assume that the only other marketing-mix variable (besides advertising) is price (P) . Demand (Q) for the product the monopolist produces is

$$Q = C(2 - P).$$

Production costs are zero⁵.

The profit of the monopolist (exclusive of the advertising expenditures) is

$$\pi = P \cdot Q$$

The optimal price (whatever C) is $P^* = 1$. Then, $\pi(1) = 1$, $\pi(1/2) = 1/2$, and $\pi(0) = 0$.

We can now compute the payoffs in each of the states of the world.

⁵ Having positive marginal production costs or different coefficients in the linear demand function does not change the results and complicates unnecessarily the analysis. An appropriate redefinition of quantities and money can transform the problem into the one treated above.

In state C=1/2 the payoffs are:

State A=1/2

D=1	$1 + \delta V^M(1/2) - \bar{u}$
D=0	$1/2 + \delta V^M(0)$

Notation: D=1(=advertise); D=0(no advertising)

In state C=0 the payoffs are:

State C=0

D=1	$1 + \delta V^M(1/2) - \bar{u}$
D=0	0

Proposition 1: If $\pi(1) - \pi(1/2) < \bar{u}/\delta$, the optimal policy for the monopolist is $r(1/2)=0$ and $r(0)=1$; if $\pi(1) - \pi(1/2) > \bar{u}/\delta$ the optimal policy for the monopolist is $r(1/2)=r(0)=1$.

This proposition simply says that if the efficient amount of

advertising (\bar{u}) is small (large), or the cost of low consideration is large (small), then the monopolist advertises in every period (in every two periods). In the example presented above, for δ close to 1, if $\bar{u} > 1/2$ the monopolist advertises every two periods; if $\bar{u} < 1/2$, the monopolist advertises in every period.

This result is exactly the same one of Mahajan and Muller (1986): if the efficient advertising expenditures (\bar{u}) are high enough it is optimal for the monopolist to pulse.

III - THE DUOPOLY

As in the monopolist case, firms compete in an infinite horizon dynamic game in discrete time. In each period, a firm decides 1) whether or not to advertise (to advertise at \bar{u} or at zero) and 2) the value of the other market variables (i.e. price, quantity, location, product quality, sales force, etc ...). There is no uncertainty.

The analysis is restricted to Markov strategies: firms strategies at period t are functions only of the payoff-relevant state of the world (in this case, the consideration levels of both firms at the beginning of the period) at period t but, in any other way, independent of the history until t .

The restriction to Markov strategies rules out all types of perfect equilibria where the strategies are function of non-payoff relevant state variables. In particular, it rules out the supergame approach to oligopoly (examples in Friedman (1977), Green and Porter (1984), Brock and Scheinkman (1985) and Rotemberg and Saloner (1986)).

As pointed out by Maskin and Tirole (1988) there are several advantages of the Markov approach over the supergame one: 1) in the supergame literature a firm reacts to non-payoff relevant state variables only because other firms do so (moreover, a firm reacts to whatever it did in the past) and this "bootstrap" characteristic of the equilibria might not represent realistically business behavior; 2) "...supergame equilibria rely on the infinity of repetitions..." while the results we present are extendable to long but finite horizons; and 3) "... the supergame approach is plagued by an enormous number of equilibria..." while in this model we have at most two equilibria.

In each period, the timing of events is the following one: 1) firms observe the vector of consideration levels (one for each firm), 2) they decide simultaneously whether or not to advertise; 3) if firm i advertises the consideration level of that firm increases to 1 in that period - otherwise its consideration takes the value it had at the beginning of the period; 4) given the consideration levels, the firms decide upon the other marketing variables; 5) the firms receive the payoffs of that period, corresponding to the advertising expenditures, consideration levels and other marketing variables; and 6) the consideration of each firm depreciates by some amount.

As in the monopolist case, it is assumed: 1) advertising (if done) costs \bar{u} and allows the consideration to jump to its maximum (say 1); 2) in the periods of zero advertising the consideration fades down.

As in the monopolist case let us define the following notation: C_{it} is the consideration level of firm i in period t ; $f(C_{it-1})$ is the

consideration level of firm i in the beginning of the period t , with $f(\cdot)$, such that, $f(x) \leq x, \forall x$.

Furthermore, the following variables and relations are defined: D_{it} takes only values zero or one (one if firm i advertises in period t , zero if not); $C_{it} = D_{it} + (1 - D_{it})f(C_{it-1})$; $\pi^i(C_i, C_j)$ is the period profit for firm i (exclusive of advertising expenditures) given that firm i has consideration C_i and firm j consideration C_j (this function incorporates the game - i.e. it is the Nash equilibrium of that game - in the other variables the firms are deciding upon - i.e. price, quantity, etc. These other variables do not affect the future payoffs of either firm. Given the Markov strategies, $\pi^i(\cdot)$ exists); $V_t^i(f(C_{it-1}), f(C_{jt-1}))$ is the net present value of the profits of firm i at the beginning of period t , if products i and j had respectively consideration C_{it-1} and C_{jt-1} in period t (as the conditions of the problem do not change through time we have that $V_t^i(\cdot) = V_k^i(\cdot) = V^i(\cdot), \forall t, k$).

In order to further simplify our task we assume that the firms are symmetric. Then, $\pi^i(\cdot) = \pi^j(\cdot) = \pi(\cdot)$.

In order to formalize the equilibrium of the game we define the following additional variables: $r(C_1, C_2)$ is the probability with which firm 1 advertises in a certain period if the consideration levels of firms 1 and 2 at the beginning of that period are respectively C_1 and C_2 ; $s(C_2, C_1)$ is the probability with which firm 2 advertises in a certain period if the consideration levels of firms 2 and 1 at the beginning of that period are respectively C_2 and C_1 .

$L_i(D_i, D_j)$ represents the payoffs for firm i for every possible

pair of actions of firms 1 and 2.

$$L_1(1,1) = \pi(1,1) + \delta V^1(f(1), f(1)) - \bar{u} \quad \text{Firm 1 and 2 advertise}$$

$$L_1(1,0) = \pi(1, C_j) + \delta V^1(f(1), f(C_j)) - \bar{u} \quad \text{Only firm i advertises ,}$$

$$L_1(0,1) = \pi(C_i, 1) + \delta V^1(f(C_i), f(1)) \quad \text{Only firm j advertises}$$

$$L_1(0,0) = \pi(C_i, C_j) + \delta V^1(f(C_i), f(C_j)) \quad \text{Noone advertises}$$

Then,

$$V^1(C_1, C_2) = \text{Max}_{0 \leq r \leq 1} r(sL_1(1,1) + (1-s)L_1(1,0)) + (1-r)(sL_1(0,1) + (1-s)L_1(0,0)) \quad (2)$$

and similarly for firm 2 in order to calculate s.

The equilibrium is then characterized by $r(C_1, C_2)$ (which solves the maximization in (2)), $s(C_2, C_1)$, $V^1(C_1, C_2)$ (which satisfies (2)) and $V^2(C_2, C_1)$.

As in the monopoly case, let us consider a simplified version of this model, where firms advertise at least every two periods. This allows to restrict our attention to only four states of the world: 1) firms 1 and 2 having advertised 1 period ago; 2) firms 1 and 2 having advertised 2 periods ago; 3) firm 1 having advertised 1 period ago and firm 2 having advertised 2 periods ago; and, 4) firm 2 having advertised 1 period ago and firm 1 having advertised 2 periods ago.

With the restriction of the analysis to strategies where the

duopoly subsists over time⁶ and with assumptions F and D above, this objective is attained without loss of generality.

The four states to be considered are: $(C_1=1/2, C_2=1/2)$, $(C_1=1/2, C_2=0)$, $(C_1=0, C_2=1/2)$ and $(C_1=0, C_2=0)$.

In each period there are four types of consumers: 1) consumers that consider both products; 2) consumers that consider product 1 only; 3) consumers that consider product 2 only; and, 4) consumers that do not consider either product. We assume probabilistic independence (the consideration probability of product i is independent of whether product j is considered; see Silk and Urban (1978) and Hauser and Wernerfelt (1989) on the use of this assumption).

In order to compute the payoffs of the duopolists in each state of the world, let us further assume that the only other marketing-mix variable (besides advertising) is price (P). If all the consumers consider both products, demand for firm i is:

$$Q_i = \begin{cases} \text{Max}[0, 1 - P_i + \beta P_j] & \text{if } P_i < 1/\beta, P_j < 1/\beta \\ \text{Max}[0, 2 - P_i] & \text{if } P_j \geq 1/\beta, P_i < 1/\beta \\ 0 & \text{if } P_i \geq 1/\beta \end{cases}$$

with $0 < \beta < 1$. If all the consumers consider product i only, demand for

⁶In the equilibria in which the duopoly subsists over time, the participation constraint for each firm must be satisfied: the equilibrium payoff for each firm must be greater than (or equal to) zero.

firm i is⁷

$$Q_i = \text{Max}[0, 2 - P_i]$$

In the general case (when there are four types of consumers in the market) demand for firm i is (if $P_i, P_j < 1/\beta$)

$$Q_i = C_i(1-C_j)[2-P_i] + C_i C_j \text{Max}[0, 1-P_i+\beta P_j]$$

Production costs are zero⁸.

The profit for firm i (exclusive of the advertising expenditures) is

$$\pi^i = P_i Q_i$$

In each period, there is a Bertrand game in prices given the consideration levels. Each firm $\text{Max}_{P_i} \pi^i$ given C_i, C_j and P_j .

We can then compute the Nash equilibrium of the price game and obtain the function $\pi^i(C_i, C_j)$.

⁷Notice that, if all consumers are aware of both products, total market demand is $Q_i+Q_j = 2 - (1-\beta)(P_i+P_j)$. Total potential demand is then equal to 2 (when $P_i=P_j=0$). Then, if all consumers are only aware of product i , the total potential demand for product i must not exceed 2.

⁸The same comment as in the case of the monopolist applies to generalizations of the model to production costs different than zero or general coefficients in the demand function (see footnote 5).

In state (1/2,1/2) the game to be played is (in matrix form):

		Firm 2	
		$D_2=1$	$D_2=0$
F i r m 1	$D_1=1$	$(\pi(1,1)+\delta V^1(1/2,1/2)-\bar{u},$ $\pi(1,1)+\delta V^2(1/2,1/2)-\bar{u})$	$(\pi(1,1/2)+\delta V^1(1/2,0)-\bar{u},$ $\pi(1/2,1)+\delta V^2(0,1/2))$
	$D_1=0$	$(\pi(1/2,1)+\delta V^1(0,1/2),$ $\pi(1,1/2)+\delta V^2(1/2,0)-\bar{u})$	$(\pi(1/2,1/2)+\delta V^1(0,0),$ $\pi(1/2,1/2)+\delta V^2(0,0))$

Notation: $D_i=1$ (=advertise); $D_i=0$ (=no advertising)

The games to be played in states (1/2,0), (0,1/2) and (0,0) can be also represented by similar matrices.

In order to analyse the duopoly case, let us first derive one result on the payoff function.

Result 1: $\pi(1,1/2)-\pi(1/2,1/2) > \pi(1,1)-\pi(1/2,1)$

Proof: See Appendix I.

Result 1, does not depend on Assumption F (on the allowed consideration levels being only 1, 1/2 and zero), but has a very simple interpretation: the benefits for a firm from increasing its own consideration are greater when the competitor's consideration is lower (for a discussion of a similar result see Hauser and Wernerfelt (1989)).

Using this result we can now proceed to analyse the duopoly case.

Proposition 2: $\pi(1,1) - \pi(1/2,1) > \bar{u}$ is a necessary and sufficient condition for the strategies $r(A_1, A_2) = s(A_1, A_2) = 1, \forall A_1, A_2$, to be a Markov perfect equilibrium.

Proof: See Appendix I.

This proposition simply shows that if \bar{u} is small enough or $\pi(1,1) - \pi(1/2,1)$ is large enough, the unique equilibrium is, as in the monopoly case, for both firms to advertise in every period. In general, in cases where \bar{u} is small or the penalty of lagging behind is large, firms advertise more often.

But this is not the most interesting situation, and from now on we will always assume $\pi(1,1) - \pi(1/2,1) < \bar{u}$ (i.e., both firms advertising in every period is not an equilibrium).

We can then derive the main result of the paper.

Proposition 3: Under reasonable conditions (i.e. $\pi(1,1) - \pi(1/2,1) < \bar{u}$, δ close to 1 and \bar{u} large), the only Markov perfect equilibria in pure strategies are characterized by alternations in advertising, i.e., are characterized by the following strategies:

i) $r(1/2, 1/2) = 1, s(1/2, 1/2) = 0; r(1/2, 0) = 0, s(0, 1/2) = 1; r(0, 1/2) = 1,$
 $s(1/2, 0) = 0; r(0, 0) = 1, s(0, 0) = 1.$

or, ii) $r(1/2, 1/2) = 0, s(1/2, 1/2) = 1; r(1/2, 0) = 0, s(0, 1/2) = 1; r(0, 1/2) = 1.$

$$s(1/2,0)=0; r(0,0)=1, s(0,0)=0.$$

Proof: See Appendix I.

The importance of Proposition 3 is that it says that in a duopoly, firms should alternate the pulsing in advertising if the efficient amount of advertising is large enough.

The basic intuition for Proposition 3 is the following one: firm i has a smaller increase in profit due to a raise in its consideration when the consideration of firm j is higher; then, if firm j is advertising, firm i should not also do so, and we obtain the staggered equilibrium.

The condition $\pi(1,1)-\pi(1/2,1)<\bar{u}$, guarantees us that advertising in every period is not an equilibrium. The condition " δ close to 1" basically says that there is a $\bar{\delta}$, such that, for any δ greater than $\bar{\delta}$, the proposition holds. The condition " \bar{u} large" is of this same type, but \bar{u} is also constrained to satisfy the participation constraint ($\bar{u}<\pi(1/2,1)+\pi(1,1/2)$ for δ close to 1). These conditions on δ and \bar{u} are stronger than the ones that are actually necessary for the proposition to hold⁹.

⁹The conditions on δ and \bar{u} that are necessary for Proposition 3 to hold are

$$\delta[\pi(1,1)-\pi(1/2,1)]-[1,1/2]<-\bar{u}(1-\delta)$$

$$[\pi(1,1)-\pi(1/2,1)]-\delta[\pi(1,1)-\pi(1,1/2)]<\bar{u}$$

$$[\pi(1,1)-\pi(1/2,1)]-\delta[\pi(1/2,1)-\pi(1/2,1/2)]<\bar{u}$$

$$[\pi(1,1)-\pi(1/2,1)]-\delta[\pi(1,1/2)-\pi(1/2,1/2)]<\bar{u}(1-\delta)$$

δ close to 1 and \bar{u} close to $\pi(1,1)+\pi(1/2,1/2)<\pi(1/2,1)+\pi(1,1/2)$

Notice also that these optimal advertising policies incorporate optimal pricing policies in each period. The evolution through time of the prices charged by the firms is positively correlated with the advertising expenditures: a firm charges a high price when it advertises and cut prices when the competitor advertises. This feature of the model is consistent with the widely observed phenomenon of firms promoting their products when the competitors launch important advertising campaigns. Figure 2 presents the behavior of prices and advertising expenditures through time. Proposition 4 summarizes the result.

satisfy these inequalities.

ADVERTISING AND PRICES EVOLUTION

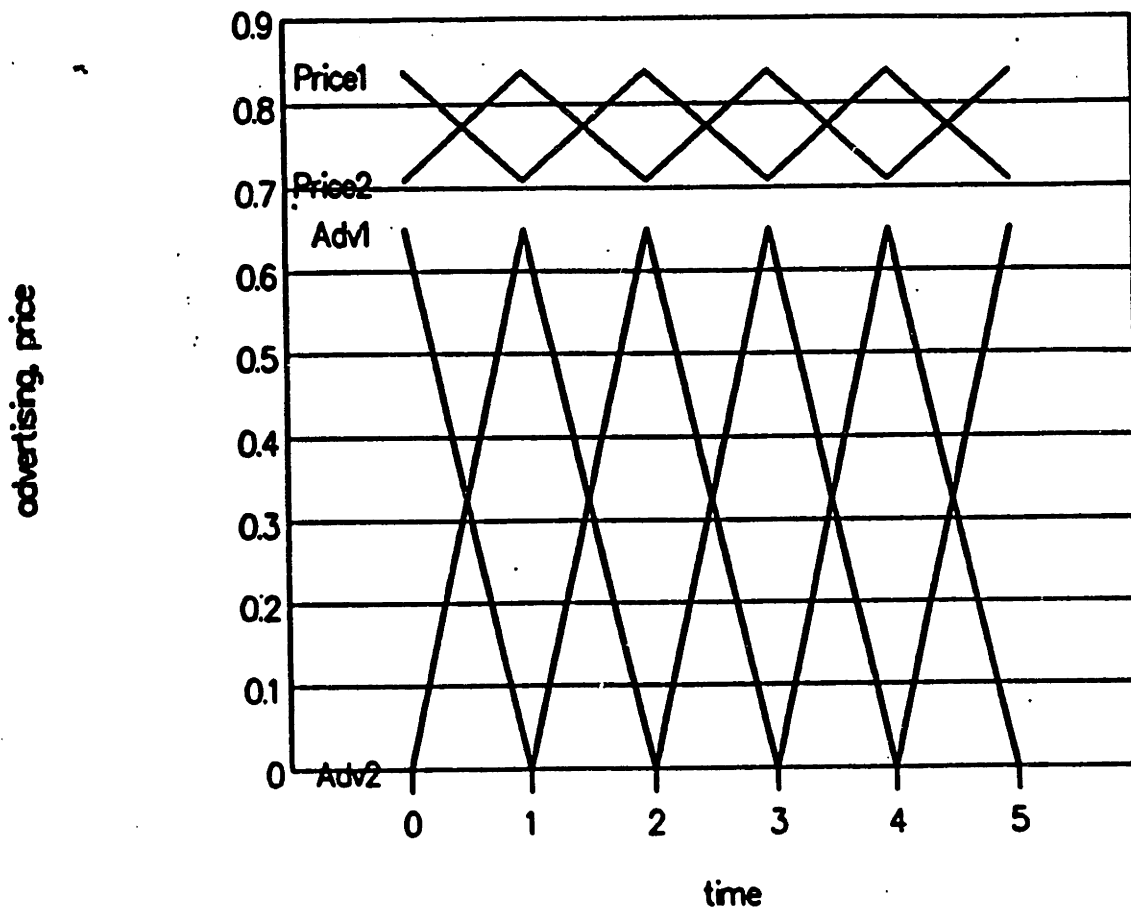


fig. 2

Proposition 4: Under the conditions of Proposition 3, the prices and the advertising expenditures of each firm are positively correlated in a duopoly.

Proposition 3 does not say anything in relation to mixed strategies, i.e. there might exist equilibria in mixed strategies which have firms advertising in a synchronized way. Proposition 4 below shows that even if we allow for symmetric mixed strategies, the system settles down in finite time to a path with alternation.

Proposition 5: Under reasonable conditions (i.e. $\pi(1,1) - \pi(1/2,1) < \bar{u}$, δ close to 1 and \bar{u} large), the unique Markov perfect equilibrium in symmetric strategies settles down in finite time in a path with alternation of advertising, i.e., is characterized by the following strategies: $r(1/2,1/2) = s(1/2,1/2) = p$ with $0 < p < 1$; $r(1/2,0) = s(0,1/2) = 0$; and, $r(0,1/2) = s(1/2,0) = r(0,0) = s(0,0) = 1$.

Proof: See Appendix I.

As in Proposition 3, the conditions " δ close to 1" and " \bar{u} large" are stronger than the ones needed to prove this proposition (the proof in the Appendix shows these conditions).

IV - THE OLIGOPOLY CASE

Let us now consider the oligopoly case. There are N firms in

the market, which are assumed to subsist overtime. Firms are symmetric. Assumptions F and D hold, such that, every firm advertises at least every two periods. The payoff (exclusive of advertising expenditures) per period for each firm i is $\bar{\pi}^i(C_1, \dots, C_N)$.

The equilibrium of this large game is a function for each firm i : $\bar{r}_i(C_1, \dots, C_N)$ which is the probability with which firm i advertises if the the consideration levels at the beginning of that period are C_1, \dots, C_N . Corresponding to these strategies, there is a value fuction for each firm i , $\bar{V}^i(C_1, \dots, C_N)$, that gives the net present value of the profits of firm i at the beginning of a period in which the consideration levels of the N firms are C_1, \dots, C_N .

Let us further assume that N and \bar{u} are large. Let us define α_t as the fraction of firms that have consideration zero at the beginning of period t ; $(1-\alpha_t)$ of the firms have consideration $1/2$ (given Assumptions F and D, there are only firms with two consideration levels at the beginning of each period: $1/2$ and 0). If \bar{u} is large enough, in equilibrium, firms advertise every two periods (intuition from the previous Section). Given N large, if firm i deviates (or not) from its equilibrium strategy in period t , $\alpha_{t+1} = 1 - \alpha_t$.

Given N large, the payoff per period (exclusive of advertising expenditures) for firm i can be written as a function of the consideration level of firm i and the average consideration (\bar{C}): $\pi^i(C_i, \bar{C})$ (and given that the α fraction of firms that have awareness zero at the beginning of the period advertise we have $\bar{C} = \frac{1}{2}(1+\alpha)$). Furthermore, the strategy function ($\bar{r}_i(\cdot)$) and the value function ($\bar{V}^i(\cdot)$) can be also written as a function of C_i and \bar{C} : $r_i(C_i, \bar{C})$ and

$v^i(C_i, \bar{C})$. Notice that the arguments of $\pi^i(\cdot)$ in period t are C_i and \bar{C} during period t (C_{it} and $\bar{C}_t = \frac{1}{2}(1+\alpha_t)$), while the arguments of $r_i(\cdot)$ and $v^i(\cdot)$ in period t are C_i and \bar{C} in the beginning of period t ($f(C_{it-1})$ and $\frac{1}{2}(1+\alpha_t)$).

We can then generalize Proposition 3: there is a unique stable equilibrium (one in which the actions repeat themselves at least every two periods) with $\alpha_t = 1/2$, $\forall t$. Proposition 5 gives the result.

Proposition 6: Given N and \bar{u} large and δ close to 1, the oligopoly case has an unique equilibrium in which the actions repeat themselves at least every two periods. Furthermore, in this equilibrium $\alpha_t = 1/2$, $\forall t$ (half the firms advertise in the odd periods and half the firms advertise in the even periods).

Proof: See Appendix I.

V - EMPIRICAL EVIDENCE

The basic prediction of the model presented in this paper is that there should exist a negative correlation in the advertising expenditures of the products in the same category.

But in the real world there are several factors that are not present in the model and that contradict its predictions (instead of the predicted negative correlation, these factors can explain positive correlations).

First, there are seasonality factors. For several reasons, in some product categories there are strong seasonal effects which extend into the advertising expenditures. To the extent that these seasonal effects are common across brands in the same product category, we should expect positive correlations in the advertising expenditures.

Second, in every product category there are trends that extend into the advertising expenditures and that affect all brands. Also, for this reason, we should expect positive correlations in the advertising expenditures.

Finally, the model postulates a Markov behavior (firms' strategies are only function of payoff-dependent state variables). In the supergame literature, firms are not restricted to this behavior. One could then construct models where firms punish each other during recessions (as in Green and Porter (1984)) by increasing the advertising expenditures; or where firms can cooperate less during booms (as in Rotemberg and Saloner (1986)) by advertising more than the colluded oligopoly. In both cases, one should expect advertising expenditures to be positively correlated.

When analysing the data on advertising expenditures one can control for the first factor (by introducing seasonal dummies) and the second factor (by simply detrending the data) but the third factor is difficult to account for.

So, finding negative correlation in the data on advertising expenditures should be interpreted as a support to the model presented above (although there might be other explanations as changes in consumer preferences, wrong definitions of segments, etc.)

In the next sub-sections the data that were used for the empirical test is described, the methodology of the test is presented and, finally, the results are reported and discussed.

V.1. DATA

Data on eight consumer product categories and one service category were used to empirically test the theoretical results presented above. The consumer product categories included: Liquid Deodorizing Cleaners, Automatic Dishwashing Detergents, Toilet Soaps, Glass Cleaners, Hand Dishwashing Detergents, Skin Care Lotions, Pain Relievers, and, Sleeping Aid Products. The service category was Credits Cards.

The number of brands chosen for analysis in each product category ranged from 3 to 12. Brands of each product category were selected based upon the share of voice¹⁰ held by the respective brand and upon how strategically each brand seemed to behave. . The number of brands in each product category can be seen in Table I.

The data was provided by a leading advertising agency and consisted of advertising expenditures in Network Television (NBC, ABC and CBS) per brand, per month, between January 1988 and December 1989 (24 months). For the product categories that were chosen, advertising expenditures in Network Television account for about 75% of total

¹⁰ Share of voice of brand i is the advertising expenditures of brand i divided by the total advertising expenditures of the product category.

advertising expenditures (other advertising channels are magazines, newspaper supplements, spot television, network radio, outdoors and cable TV networks).

V.2. METHODOLOGY

One of the implications of our theoretical model is that there is a negative correlation between any brand advertising expenditures and the advertising expenditures of the rest of the brands in that product category (if one controls for trend and seasonal effects, which are not included in the model).

Furthermore, one must account for the relative size effect of the brands competing in a certain product category: the theoretical model considered only the case of brands of the same size (share of voice) while the data presented some strong asymmetries in the relative size of the share of voice of the brands that were considered strategic players. The approach that was followed was to assume that the effect of advertising expenditures on consideration varied across brands: in particular it was assumed that this effect was smaller for large firms than for small ones. In order to test the model, the correct solution was then to consider that the theoretical model held for standardized advertising expenditures.

After standardizing every series¹¹, we have the following model

¹¹If (X_{it}) is the series of advertising expenditures for brand i , the standardized series (X'_{it}) is constructed by computing

$$(3) S_{it} - \sum_{j=1}^{12} \delta_{tj} \gamma_j - \alpha_i T_t = \theta \sum_{\substack{h=1 \\ h \neq i}}^N (S_{ht} - \sum_{j=1}^{12} \delta_{tj} \gamma_j - \alpha_h T_t) + \epsilon_{it}$$

for $i=1, \dots, N$ (N is the number of brands in the product category) and $t=1, \dots, 24$ (24 months of data per brand). S_{it} is the standardized version of the advertising expenditures of brand i at time t . δ_{tj} is the element (t, j) of the matrix δ of dimensions (24×12) . The matrix δ includes the dummy variables to compute the seasonal effects: $\delta_{tj}=1$ if $t=j$ or $t=j+12$, and $\delta_{tj}=0$ otherwise. γ_j is the seasonal effect of month j . Notice that the seasonal effects are considered equal across brands.

T_t is the generic element of the vector T of dimension 24. This vector accounts for the trend in advertising expenditures ($T_t=t$). α_i is the trend effect for brand i . Notice that the trend effects are different across brands (such that share changes during the period under analysis are allowed).

The LHS (Left Hand Side) of the above expression represents the advertising expenditures net of the seasonal and trend effects. The RHS (Right Hand Side) is composed of the parameter θ times the sum across the other brands in the product category of the advertising expenditures net of the seasonal and trend effects plus an error term

$$X'_{it} = \frac{X_{it} - \bar{X}_i}{\sigma_i}, \text{ where } \bar{X}_i = \sum_{t=1}^{24} \frac{X_{it}}{24} \text{ and } \sigma_i = \left(\sum_{t=1}^{24} \frac{(X_{it} - \bar{X}_i)^2}{23} \right)^{1/2}$$

ϵ_{it} . θ is assumed to be constant across brands¹² and is the parameter where the model will be tested upon: the theoretical model predicts θ to be negative. ϵ_{it} is assumed to be normally distributed with $E[\epsilon_{it}] = 0$, $V_{i,t}$, $E[\epsilon_{it}^2] = \sigma^2$, $V_{i,t}$. Furthermore it is assumed that $E[\epsilon_{it}\epsilon_{jt}] = 0$, $V_{i,t,j,r,i \neq j}$ or $t \neq r$.

The expression above can be transformed into

$$(4) S_{it} = \sum_{j=1}^{12} \delta_{tj} \gamma'_j + \alpha'_i T_t + \theta \sum_{\substack{h=1 \\ h \neq i}}^N S_{ht} + \epsilon_{it}, \quad i=1, \dots, N; t=1, \dots, 24$$

where $\gamma'_j = \gamma_j [1 - (N-1)\theta]$ and $\alpha'_i = \alpha_i - \sum_{\substack{h=1 \\ h \neq i}}^N \theta \alpha_h$.

Estimating (4) (or (4) jointly with all the other brands) by ordinary least squares would lead us to biased estimates of the parameters due to errors in variables: the variable $\sum_{\substack{h=1 \\ h \neq i}}^N S_{ht}$ is correlated with the error term ϵ_{it} .

In fact, (4) can be interpreted as a system of simultaneous equations (one equation for each brand), which can be estimated through a Maximum Likelihood Estimation (MLE) method (see Appendix 11 for the construction of the Likelihood function $\mathcal{L}(\gamma, \alpha, \theta, \sigma^2)$, where γ is a vector of dimension 12 where γ'_j is the generic element and α is a vector of dimension N where α'_i is the generic element).

¹²Given that all the series were standardized assuming θ to be constant across brands is the logical assumption. Furthermore, this assumption allows us to have greater power in the statistical tests of the model.

Maximizing $L(\gamma, \alpha, \theta, \sigma^2)$, ($L(\cdot) = \log \mathcal{L}(\cdot)$) with respect to γ , α , θ and σ^2 yields the MLE of these parameters ($\hat{\gamma}$, $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\sigma}^2$). Following Theil (1971, p. 524), these estimators are consistent. Furthermore, $\sqrt{N}(\hat{\beta} - \beta)$ distributes asymptotically as a normal of mean zero and variance V , where

$$V = - \left[E \left[\frac{\partial^2 L(\beta)}{\partial \beta^2} \right] \right]^{-1} \quad \text{and} \quad \beta^T = [\gamma \quad \alpha \quad \theta \quad \sigma^2]$$

(where the superscript T refers to the transpose operation). A

consistent estimator of V , is $\hat{V} = - \left[\frac{\partial^2 L(\hat{\beta})}{\partial \hat{\beta}^2} \right]^{-1}$.

Then, one can also show that $\sqrt{N}(\hat{\beta} - \beta)$ distributes asymptotically as a vector of normal random variables of mean zero and variance \hat{V} . V has dimensions $(14+N) \times (14+N)$ and β has dimension $14+N$.

Testing the theoretical model is then testing if $\hat{\theta}$ is statistically different from zero and negative.

The maximization of $L(\cdot)$ was implemented using the method of Gill et al. (1984).

V.3. RESULTS

Table 1 presents the results for the nine product categories under study. $\hat{\theta}$ is positive for 1 of the nine product categories (Pain Relievers). $\hat{\theta}$ is statistically different from zero and negative at the 0.5% significance level for three product categories (Toilet Soaps, Glass Cleaners, and Skin Care Lotions), at the 5% significance level for 4 product categories (the previous three ones plus Automatic

Dishwashing Detergents), and at the 10% significance level for 6 product categories (the previous four ones plus Liquid Deodorizing Cleaners and Hand Dishwashing Detergents).

The results look supportive of the theoretical model and seem to show the existence of advertising "wars" in the product category Pain Relievers (as explained above).

TABLE 1

Product Category	Number of Brands	$\hat{\theta}$	$\hat{\sigma}_{\theta}$	LL
Liquid Deod. Cleaners	3	-0.147	0.099	-14.44
Automatic Dishw. Det.	4	-0.152	0.079	-33.00
Toilet Soaps	7	-0.211	0.047	-55.70
Credit Cards	4	-0.093	0.075	-33.61
Glass Cleaners	3	-0.436	0.103	8.34
Hand Dishwashing Det.	3	-0.137	0.099	-18.62
Skin Care Lotions	7	-0.147	0.049	-55.25
Pain Relievers	12	0.252	0.014	-117.32
Sleeping Aid Products	4	-0.085	0.074	-37.14

LL is the value at the optimum of LogL plus $24N\log\sqrt{2\pi}$.

VI - CONCLUDING REMARKS

It was shown that competitors in a duopoly (or oligopoly) should advertise in alternate periods (if the only role of advertising is to increase consideration). The basic intuition for this result is that, when a firm is raising its consideration level, it has greater benefits if the other firm's consideration is lower. Furthermore, this equilibrium, maximizes industry profits.

The evidence from several product categories look supportive of the model. The results shown in this paper can be applied to problems where there exists a similar type of framework: in a model where

consumers care for novelty, should firms introduce new products at the same time or not?; in a model of technology adoption, should firms adopt technologies synchronously or not?; in model of investment in additional capacity, should firms invest at the same time or not?

APPENDIX I

Proof of Result 1: Result 1 is equivalent to

$$\frac{1 + \frac{5}{4}\beta}{4 - \frac{\beta^2}{2}} > \frac{\beta}{4 - \beta^2} + \frac{1}{2} \frac{1 + \frac{3}{4}\beta}{4 - \frac{\beta^2}{4}}$$

which is implied, for $0 < \beta < 1$, by

$$\frac{1 + \frac{5}{4}\beta}{4 - \frac{\beta^2}{4}} > \frac{\frac{1}{2} + \frac{11}{8}\beta}{4 - \beta^2}$$

In this expression, $\beta=1$ maximizes (RHS-LHS) and $\text{Max(RHS-LHS)}=5/8-9/14 < 0$. Q.E.D.

Proof of Proposition 2: First, note that if $(D_1=1, D_2=1)$ is a Nash equilibrium in state 1, then it is a Nash equilibrium in states 2 and 3. Then, in order to prove the proposition we just need to show that $(1,1)$ being an equilibrium in state $(1/2, 1/2)$ (given the equilibrium strategies) is equivalent to $\pi(1,1) - \pi(1/2,1) > \bar{u}$. For this we need

$$(A1) \quad \pi(1,1) + \delta V^1(1/2, 1/2) - \bar{u} > \pi(1/2,1) + \delta V^1(0, 1/2), \quad \text{and}$$

$$(A2) \quad \pi(1,1) + \delta V^2(1/2,1/2) - \bar{u} > \pi(1/2,1) + \delta V^2(0,1/2).$$

But, given the equilibrium strategies we know that

$$\pi(1,1) + \delta V^1(1/2,1/2) - \bar{u} = V^1(1/2,1/2)$$

which results in

$$V^1(1/2,1/2) = \frac{\pi(1,1) - \bar{u}}{1 - \delta}, \quad \forall i.$$

But, in states (1/2,0) and (0,1/2), using (A1), (A2) and Assumption D, we get that

$$V^1(1/2,0) = V^1(0,1/2) = \pi(1,1) + \delta V^1(1/2,1/2) - \bar{u}$$

Then

$$V^1(0,1/2) = V^1(0,1/2) = V^1(1/2,1/2)$$

and then (A1) and (A2) are equivalent to $\pi(1,1) - \pi(1/2,1) > \bar{u}$.

Furthermore, given Assumption D, in state (0,0) we get

$$V^1(0,0) = \pi(1,1) + \delta V^1(1/2,1/2) - \bar{u} = V^1(1/2,1/2)$$

Q.E.D.

Proof of Proposition 3: Here and in the proof to Proposition 4 we define the following variables in order to simplify notation: $\pi_1 = \pi(1,1)$, $\pi_2 = \pi(1,1/2)$, $\pi_3 = \pi(1/2,1)$ and $\pi_4 = \pi(1/2,1/2)$. First, we check that these strategies constitute a perfect equilibrium (we check for (i) as (ii) is just the symmetric case). We do this by checking that the prescribed strategies are a Nash equilibrium in each of the possible states of the world. Then, we show that no other strategies constitute a perfect equilibrium.

Given the postulated equilibrium strategies we have:

$$V^1(1/2,0) = \frac{\pi_3 + \delta\pi_2 - \delta\bar{u}}{1 - \delta^2}, \forall i$$

$$V^1(0,1/2) = \frac{\pi_2 + \delta\pi_3 - \bar{u}}{1 - \delta^2}, \forall i$$

$$V^1(1/2,1/2) = \frac{\pi_2 + \delta\pi_3 - \bar{u}}{1 - \delta^2}$$

$$V^2(1/2,1/2) = \frac{\pi_3 + \delta\pi_2 - \delta\bar{u}}{1 - \delta^2}$$

$$V^1(0,0) = \frac{\pi_1(1-\delta^2) + \delta\pi_2 + \delta^2\pi_3 - \bar{u}(1+\delta-\delta^2)}{1 - \delta^2}$$

$$V^2(0,0) = \frac{\pi_1(1-\delta^2) + \delta\pi_3 + \delta^2\pi_2 - \bar{u}}{1 - \delta^2}$$

Now, in order for (1,0) to be a Nash equilibrium in state (1/2,1/2) we need that

$$\pi_2 + \delta V^1(1/2,0) - \bar{u} > \pi_4 + \delta V^1(0,0)$$

and

$$\pi_3 + \delta V^1(0,1/2) > \pi_4 + \delta V^2(1/2,1/2) - \bar{u}.$$

These conditions are respectively equivalent to

$$(A3) \quad \delta(\pi_1 - \pi_3) - (\pi_2 - \pi_4) < -\bar{u}(1-\delta)$$

and

$$(A4) \quad (\pi_1 - \pi_3) + \delta(\pi_1 - \pi_2) < \bar{u}.$$

Condition (A3) is equivalent to

$$\delta[\pi(1,1) - \pi(.5,1)] - [\pi(1,.5) - \pi(.5,.5)] < -\bar{u}(1-\delta)$$

which is true for δ close to 1 (using Result 1).

In order to confirm that condition (A4) is satisfied one can take the largest possible \bar{u} (which is close to $e+f$ for δ close to 1)¹³ and δ close to 1. Then condition (A4) becomes $d < e+f$ which is satisfied given Result 1 and $g > 0$.

In state $(0, 1/2)$, in order for $(1, 0)$ to be the Nash equilibrium condition (A4) must hold.

In order to complete the proof that these strategies constitute an equilibrium we just need to show that $(0, 1)$ is a Nash equilibrium in state $(1/2, 0)$. Given Assumption D, $(0, 1)$ is a Nash equilibrium in state $(1/2, 0)$ if

$$\pi_3 + \delta V^i(0, 1/2) > \pi_1 + \delta V^i(1/2, 1/2) - \bar{u}$$

which is equivalent to $\pi(1, 1) - \pi(1/2, 1) < \bar{u}$, which is assumed above.

Finally, we just need to prove that (i) and (ii) constitute the only Markov perfect equilibria in pure strategies. For this to be true we first need to prove that $(0, 0)$ is not a Nash equilibrium in state $(1/2, 1/2)$ (given that by Proposition 2, $(1, 1)$ can not be).

Suppose not, i.e., $(0, 0)$ is a Nash equilibrium in state $(1/2, 1/2)$. Then

$$V^i(0, 0) = \frac{\pi_1 + \delta \pi_4 - \bar{u}}{1 - \delta^2}, \quad \forall i$$

$$V^i(1/2, 1/2) = \frac{\pi_4 + \delta \pi_1 - \delta \bar{u}}{1 - \delta^2}, \quad \forall i.$$

¹³This results from the participation constraint on each firm: the net present value of future profits must be greater than zero.

Furthermore,

$$(A5) \quad \pi_4 + \delta V^i(0,0) > \pi_2 + \delta V^2(1/2,0) - \bar{u}$$

and

$$(A6) \quad \pi_4 + \delta V^i(0,0) > \pi_2 + \delta V^1(1/2,0) - \bar{u}.$$

In order to check (A5) and (A6), we need to know the values of $V^2(1/2,0)$ and $V^1(1/2,0)$. For this, we need to consider four cases in respect to the equilibria in states $(1/2,0)$ and $(0,1/2)$: 1) $(1,1)$ is a Nash equilibrium in states $(1/2,0)$ and $(0,1/2)$; 2) $(0,1)$ and $(1,0)$ are respectively the Nash equilibria in states $(1/2,0)$ and $(0,1/2)$; 3) $(1,1)$ is the Nash equilibrium in state $(1/2,0)$ and $(1,0)$ is the Nash equilibrium in state $(0,1/2)$; and 4) $(0,1)$ is the Nash equilibrium in state $(1/2,0)$ and $(1,1)$ is the Nash equilibrium in state $(0,1/2)$.

In case 1, we have

$$V^i(1/2,0) - V^i(0,1/2) - V^i(0,0) = \frac{\pi_1 + \delta\pi_4 - \bar{u}}{1 - \delta^2}, \quad \forall i$$

and

$$(A7) \quad V^i(1/2,0) > \pi_3 + \delta V^i(0,1/2).$$

(A7) is in this case equivalent to

$$(\pi_1 - \pi_3) - \delta(\pi_3 - \pi_4) > \bar{u}$$

Given δ close to 1, \bar{u} large and Result 1, this condition implies $\pi_2 + \pi_3 - 2\pi_3 > \pi_2 + \pi_3$, which is never satisfied.

In case 2, we have

$$V^i(1/2,0) = \frac{\pi_3 + \delta\pi_2 - \delta\bar{u}}{1 - \delta^2}, \quad \forall i$$

$$V^i(0,1/2) = \frac{\pi_2 + \delta\pi_3 - \bar{u}}{1 - \delta^2}, \quad \forall i$$

but then condition (A5) becomes the opposite of condition (A3), which is true given Result 1.

In case 3, we have

$$V^1(1/2,0) = V^1(0,0)$$

$$V^1(0,1/2) = \pi_2 + \delta V^1(0,0) - \bar{u}$$

But then, in order for (1,1) to be a Nash equilibrium in state (1/2,0) we would need

$$V^1(1/2,0) > \pi_3 + \delta V^1(0,1/2)$$

which is equivalent to

$$(\pi_1 - \pi_3) - \delta(\pi_2 - \pi_4) > \bar{u}(1 - \delta)$$

which is not true given Result 1 and δ close to 1. (A similar argument works for case 4). Q.E.D.

Proof of Proposition 5: First, we prove that the strategies described in Proposition 4 constitute an equilibrium. Using these strategies, we have

$$V(1/2,0) = \frac{\pi_3 + \delta\pi_2 - \delta\bar{u}}{1 - \delta^2} = V_2$$

$$V(0,1/2) = \frac{\pi_2 + \delta\pi_3 - \bar{u}}{1 - \delta^2} = V_1$$

$$V(0,0) = \pi_1 + \delta V(1/2,1/2) - \bar{u}$$

$$V(1/2,1/2) = V$$

Now, given that in state (1/2,1/2) each firm plays a mixed strategy.

each firm must be indifferent between advertising \bar{u} and zero advertising.

$$p(\pi_1 + \delta V - \bar{u}) + (1-p)V_1 = pV_2 + (1-p)(\pi_4 + \delta\pi_1 + \delta^2 V - \delta\bar{u})$$

which solving for V becomes

$$(A8) \quad V = \frac{pV_2 - (1-p)V_1 + (1-p)(\pi_4 + \delta\pi_1 - \delta\bar{u}) - p\pi_1 + p\bar{u}}{p\delta - (1-p)\delta^2}$$

Furthermore we know that

$$V = p\pi_1 + p\delta V - p\bar{u} + (1-p)V_1$$

which solving for V becomes

$$(A9) \quad V = \frac{(1-p)V_1 + p\pi_1 - p\bar{u}}{1 - p\delta}$$

Finally, (0,1) and (1,0) must be respectively the Nash equilibria in states (1/2,0) and (0,1/2). These conditions are equivalent to

$$V(0,0) < V(1/2,0)$$

which is equivalent in this case to

$$(A10) \quad V < \frac{V_2 - \pi_1 + \bar{u}}{\delta}$$

The equilibrium strategies stated in Proposition 4 constitute an equilibrium if there is a V and a p that satisfy (A8), (A9) and (A10).

To confirm this we need some more results.

Result 2: $\text{sign} \left(\frac{dV}{dp} \Big|_{A1} \right) = \text{sign} \left(- \frac{\delta[\delta V_2 - V_1 - \pi_4]}{[p\delta - (1-p)\delta^2]^2} \right) = \text{sign}(\pi_4 + V_1 - \delta V_2) =$

$$= \text{sign}(\pi_2 + \pi_4 - \bar{u}) = \text{sign}(-\pi_j^1), \text{ for } \bar{u} \text{ close to}$$

$$\pi_2 + \pi_3 \text{ or } \bar{u} \text{ close to } \pi_1 + \pi_4.$$

Result 3: $\text{Sign} \left(\frac{dV}{dp} \right)_{A2} = \text{Sign} \left(\frac{(\delta-1)V1 + \pi1 - \bar{u}}{(1-p\delta)^2} \right) < 0$, given Result 1

and \bar{u} close to $\pi1+\pi4$ or \bar{u} close to $\pi2+\pi3$.

Result 4: $V_{A1}(p=0) = \frac{V1 - \pi4 - \delta\pi1 + \delta\bar{u}}{\delta^2}$; $V_{A1}(p=1) = \frac{V2 - \pi1 + \bar{u}}{\delta}$

Result 5: $V_{A2}(p=0) = V1$; $V_{A2}(p=1) = \frac{\pi1 - \bar{u}}{1 - \delta}$

Result 6: $V_{A1}(p=1) > V_{A2}(p=1)$, given Result 1 and \bar{u} close to $\pi1+\pi4$;
 $\text{Sign}(V_{A1}(p=0) - V_{A2}(p=0)) = \text{Sign}(\pi2+\pi3-\pi1-\pi4)$

Result 7: $V2 > V_{A2}(p=1)$; $V2 < V_{A1}(p=1)$, given Result 1 and \bar{u} close to $\pi1+\pi4$ or \bar{u} close to $\pi2+\pi3$.

Result 8: $\text{sign}(V1-V2) = \text{sign}(\pi2-\pi3-\bar{u}) < 0$, for \bar{u} close to $\pi2+\pi3$.

Result 9: $\lim_{p \rightarrow \frac{\delta}{1+\delta}} |V_{A1}| = \infty$

Then, one can draw (A8) and (A9) in the (p,V) space. Figure A1 and A2 show respectively the cases where $\pi_j^1 < 0$ and $\pi_j^1 > 0$. In both cases there is an unique solution p^* which satisfies condition (A3).

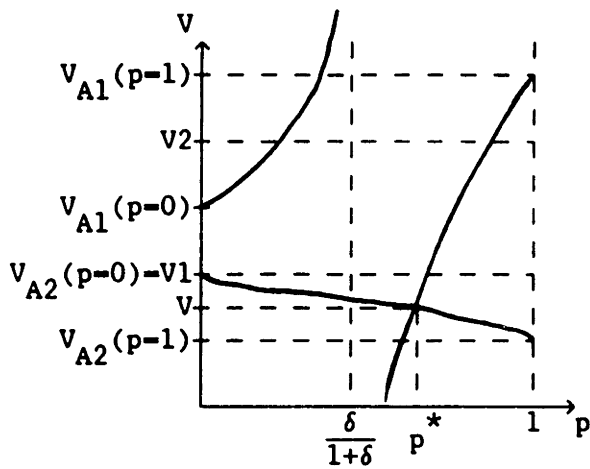


Figure A1: $\pi_j^i < 0$

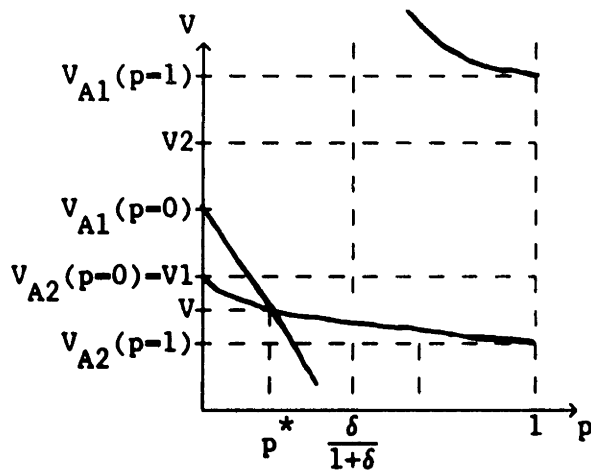


Figure A2: $\pi_j^i > 0$

Now, we just need to prove that there is no other symmetric strategies equilibrium other than the one defined in Proposition 4.

In order to prove this I need to prove that 1) play mixed strategies in state (1/2,1/2) and (1,1) in both states (1/2,0) and (0,1/2) is not an equilibrium (Result 10), that 2) play (0,0) in state (1/2,1/2) and mixed strategies in states (1/2,0) and (0,1/2) is not an equilibrium (Result 17) and, that 3) play mixed strategies in states (1/2,1/2), (1/2,0) and (0,1/2) is not an equilibrium (Result 18).

Result 10: Given the conditions of Proposition 4, to play (non-degenerate) mixed strategies in state (1/2,1/2) and (1,1) in both states (1/2,0) and (0,1/2) is not an equilibrium.

Proof: Suppose there is an equilibrium with these strategies, where both firms mix in state (1/2,1/2) with probability p (of playing 1).

Then

$$V(0,0) = \pi_1 + \delta V(1/2,1/2) - \bar{u}$$

$$V(1/2,0) = V(0,1/2) = V(0,0)$$

$$V = V(1/2,1/2).$$

In order for the firms to be willing to mix in state (1/2,1/2) we have:

$$\begin{aligned} p(\pi_1 + \delta V) + (1-p)(\pi_2 + \delta \pi_1 + \delta^2 V - \delta \bar{u}) - \bar{u} = \\ = p(\pi_3 + \delta \pi_1 + \delta^2 V - \delta \bar{u}) + (1-p)(\pi_4 + \delta \pi_1 + \delta^2 V - \delta \bar{u}) \end{aligned}$$

which solving for V becomes

$$(A11) \quad V = \frac{\bar{u}(1-p\delta) + p\pi_3 + p\pi_1(\delta-1) + (1-p)(\pi_1-\pi_2)}{p\delta(1-\delta)}$$

Furthermore,

$$V = p(\pi_1 + \delta V) + (1-p)(\pi_2 + \delta \pi_1 + \delta^2 V - \delta \bar{u}) - \bar{u}$$

which solving for V becomes

$$(A12) \quad V = \frac{p\pi_1 - \bar{u} + (1-p)\delta(\pi_1 - \bar{u}) + (1-p)\pi_2}{1 - p\delta - (1-p)\delta^2}$$

Is there a (p,V) that solve (A11) and (A12) with $0 < p < 1$?

In order to solve for this we need some additional results.

Result 11: $\text{Sign}\left(\frac{dV}{dp}\right)_{A4} = \text{Sign}(\pi_2 - \pi_4 - \bar{u}) = \text{Sign}(-\pi_4 - \pi_3) < 0$, for \bar{u}

close to $\pi_2 + \pi_3$ and δ close to 1.

Result 12: $\text{Sign}\left(\frac{dV}{dp}\right)_{A5} = \text{Sign}(\pi_1 - \pi_2) = \text{sign}(\pi_j^i)$

Result 13: $V_{A4}(p=1) = \frac{\pi_3 - (\pi_1 - \bar{u})(1-\delta)}{\delta(1-\delta)}$; $\lim_{p \rightarrow 0} |V_{A4}| = \infty$

Result 14: $V_{A5}(p=0) = \frac{\pi_2 + \delta\pi_1 - \bar{u}(1+\delta)}{1 - \delta^2}$; $V_{A5}(p=1) = \frac{\pi_1 - \bar{u}}{1 - \delta}$

Result 15: $\text{Sign}(V_{A4}(p=1) - V_{A5}(p=1)) = \text{Sign}(\bar{u} + \pi_3 - \pi_1) > 0$.

Result 16: $\text{Sign}(V_{A4}(p=1) - V_{A5}(p=0)) = \text{Sign}(2\bar{u} + 2\pi_3 - \pi_1 - \pi_2) = \text{Sign}(4\pi_3 + \pi_2 - \pi_1) > 0$

for δ close to 1, \bar{u} close to $\pi_2 + \pi_3$ and given Result 1.

Given these results one can draw figures A3 and A4 (respectively for

$\pi_j^i < 0$ and $\pi_j^i > 0$) which show that there is no (p, V) that satisfy (A11) and (A12). Q.E.D.

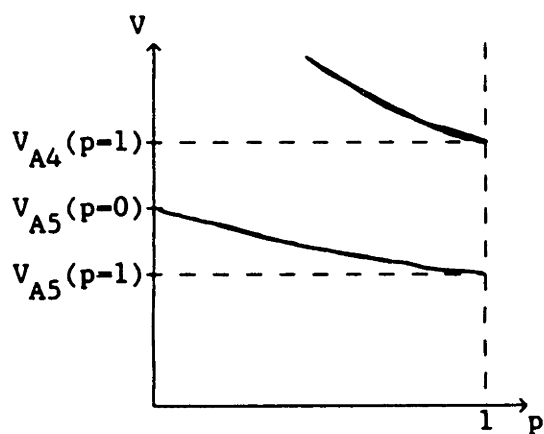


Figure A3: $\pi_j^i < 0$

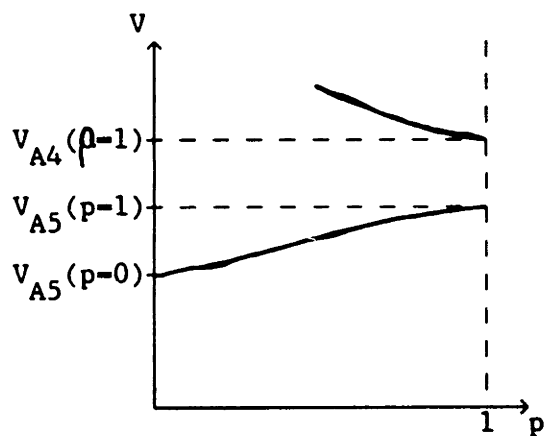


Figure A4: $\pi_j^i > 0$

Result 17: Given the conditions of Proposition 4, to play $(0,0)$ in state $(1/2, 1/2)$ and mixed strategies in states $(1/2, 0)$

and (0,1/2) does not constitute an equilibrium.

Proof: Suppose these strategies constitute an equilibrium, being p the probability of firm 1 advertising in state (1/2,0), and, of firm 2 advertising in state (0,1/2). Then,

$$V(0,0) - V(0,1/2) = \frac{\pi_1 + \delta\pi_4 - \bar{u}}{1 - \delta^2}$$

$$V(1/2,1/2) = \frac{\pi_4 + \delta\pi_1 - \delta\bar{u}}{1 - \delta^2}$$

$$V(1/2,0) = \frac{\pi_1 + \delta\pi_4 - \bar{u}}{\delta(1-\delta)^2} - \frac{\pi_3}{\delta}$$

$$V(1/2,0) = p \left(\frac{\pi_1 + \delta\pi_4 - \bar{u}}{1 - \delta^2} \right) + (1-p) (\pi_2 + \delta V(0,1/2) - \bar{u})$$

and solving for p with δ close to 1 results in

$$p = \frac{2(\pi_2 + \pi_3 - \pi_1 - \pi_4)}{2\pi_2 - \pi_1 - \pi_4}$$

and for \bar{u} close to $\pi_1 + \pi_4 < \pi_2 + \pi_3$ (if \bar{u} close to $\pi_2 + \pi_3$, $p > 1$ or $p < 0$)

$$p = \frac{\pi_2 + \pi_3 - \pi_1 - \pi_4}{\pi_2 - \pi_1 - \pi_4} > 1$$

so that play (0,0) in state (1/2) and mixed strategies in states (1/2,0) and (0,1/2) cannot be an equilibrium. Q.E.D.

Result 18: There is no symmetric equilibrium where firms play (non-degenerate) mixed strategies in states (1/2,1/2).

(1/2,0) and (0,1/2).

Proof: Suppose this equilibrium exists. Define $V_1=V(1/2,1/2)$, $V_2=V(1/2,0)$, $V_3=V(0,1/2)$, $V_4=V(0,0)$, p_1 =probability of firm 1 playing 1 in state (1/2,1/2) and p_2 =probability of firm 1 playing 1 in state (1/2,0), and, of firm 2 playing 1 in state (0,1/2). Then

$$V_4 = \pi_1 + \delta V_1 - \bar{u}$$

$$V_2 = \pi_1 + \delta V_1 - \bar{u} = \pi_3 + \delta V_3$$

$$V_3 = p_2(\pi_1 + \delta V_1 - \bar{u}) + (1-p_2)(\pi_2 + \delta V_2 - \bar{u})$$

But, given that both firms mix in state (1/2,1/2) they must be indifferent between playing 1 and 0, and then

$$p_1(\pi_1 + \delta V_1 - \bar{u}) + (1-p_1)(\pi_1 + \delta V_2 - \bar{u}) = p_1(\pi_3 + \delta V_3) + (1-p_2)(\pi_4 + \delta V_4)$$

which can be written in terms only of V_2 and we can then obtain

$$\pi_2 - \pi_4 = \bar{u}$$

which does not hold in general. Q.E.D.

Proof of Proposition 6: Let us consider a stable equilibrium in which firms advertise every two periods and there is a fraction α of firms with consideration zero in the beginning of the odd periods (and a fraction $1-\alpha$ of firms with consideration zero in beginning of the even periods). Assume first $\alpha \neq 0$. Consider an odd period. Firms that have consideration zero will advertise surely (given Assumption D). Firms that have consideration 1/2 may advertise or not: they may deviate from equilibrium by advertising (and joining the other group of firms; we are considering only one period deviations).

If such a firm follows the equilibrium strategy it gets

$$\frac{\pi(\frac{1}{2}, \frac{1}{2}(1+\alpha)) + \delta\pi(1, 1-\frac{1}{2}\alpha) - \delta\bar{u}}{1 - \delta^2}$$

If it deviates it gets

$$\pi(1, \frac{1}{2}(1+\alpha)) - \bar{u} + \delta \frac{\pi(\frac{1}{2}, 1-\frac{1}{2}\alpha) + \delta\pi(1, \frac{1}{2}(1+\alpha)) - \delta\bar{u}}{1 - \delta^2}$$

For this firm not to deviate we need the payoff from the equilibrium strategy to be greater than the payoff from the deviation, which is equivalent to

$$(A13) [\pi(1, \frac{1}{2}(1+\alpha)) - \pi(\frac{1}{2}, \frac{1}{2}(1+\alpha))] - \delta[\pi(1, 1-\frac{1}{2}\alpha) - \pi(\frac{1}{2}, 1-\frac{1}{2}\alpha)] < \bar{u}(1-\delta)$$

Consider now an even period. In order for a firm that has awareness $1/2$ at the beginning of the period not to deviate (i.e. advertise), an argument similar to the above applies, from which follows condition (A14).

$$(A14) [\pi(1, 1-\frac{1}{2}\alpha) - \pi(\frac{1}{2}, 1-\frac{1}{2}\alpha)] - \delta[\pi(1, \frac{1}{2}(1+\alpha)) - \pi(\frac{1}{2}, \frac{1}{2}(1+\alpha))] < \bar{u}(1-\delta)$$

Consider $\alpha > \frac{1}{2}$. Then, $\frac{1}{2}(1+\alpha) > 1-\frac{1}{2}\alpha$, and given δ close to 1 and the correspondent to Result 1 in the oligopoly case (it is more profitable to raise the consideration when the average consideration of the competitors is smaller), (A13) is satisfied but (A14) is not. But if $\alpha < \frac{1}{2}$, a similar argument applies, and (A14) is satisfied and (A13) is

not. But if $\alpha = \frac{1}{2}$, conditions (A13) and (A14) are equivalent to condition (A15)

$$(A15) \quad \pi(1, \frac{3}{4}) - \pi(\frac{1}{2}, \frac{3}{4}) < \bar{u}$$

which is satisfied for \bar{u} large enough (and the participation constraint¹⁴ can be satisfied).

We can then conclude that if in equilibrium, in every period, there is at least one firm advertising, then, half the firms advertise in the odd periods and the other advertise in the even ones.

In order to complete the proof, we just need to show that all the firms advertising in the odd periods (without loss of generality) and no firm advertising in the even periods is not an equilibrium.

Consider an even period. If a firm follows the equilibrium strategy (not advertise) it gets

$$\frac{\pi(\frac{1}{2}, \frac{1}{2}) + \delta\pi(1, 1) - \delta\bar{u}}{1 - \delta^2}$$

If it advertises it gets

$$\pi(1, \frac{1}{2}) - \bar{u} + \delta \frac{\pi(\frac{1}{2}, 1) + \delta\pi(1, \frac{1}{2}) - \delta\bar{u}}{1 - \delta^2}$$

In order for this firm not to deviate, it must be that what it gets by following the equilibrium strategy is greater than what it gets by

¹⁴For δ close to 1 the participation constraint is $\pi(1, \frac{3}{4}) + \pi(\frac{1}{2}, \frac{3}{4}) > \bar{u}$.

deviating. This condition is equivalent to (A6) with $\alpha=0$. But by the reasons presented above, (A6) is not satisfied for $\alpha < \frac{1}{2}$ which is the case. Then, all the firms advertising in the odd periods and no firm advertising in the even periods can not be an equilibrium. Q.E.D.

APPENDIX II

(4) can be interpreted as a system of simultaneous equations (one equation for each brand)

$$(A16) \left\{ \begin{array}{l} S_{1t} - \sum_{j=1}^{12} \delta_{tj} \gamma'_j + \alpha'_1 T_t + \theta \sum_{\substack{h=1 \\ h \neq 1}}^N S_{ht} + \epsilon_{1t} \\ S_{2t} - \sum_{j=1}^{12} \delta_{tj} \gamma'_j + \alpha'_2 T_t + \theta \sum_{\substack{h=1 \\ h \neq 2}}^N S_{ht} + \epsilon_{2t} \\ \vdots \\ S_{Nt} - \sum_{j=1}^{12} \delta_{tj} \gamma'_j + \alpha'_N T_t + \theta \sum_{\substack{h=1 \\ h \neq N}}^N S_{ht} + \epsilon_{Nt} \end{array} \right.$$

for $t=1, \dots, 24$.

This system of equations can be estimated through a Maximum Likelihood Estimation (MLE) method. The likelihood function for this system is

$$L(\gamma, \alpha, \theta, \sigma^2) = \prod_{t=1}^{24} \prod_{i=1}^N \frac{1}{\sqrt{2\pi} \sigma} \text{Exp} \left(-\frac{1}{2} \frac{\epsilon_{it}^2}{\sigma^2} \right)$$

where γ is a vector of dimension 12 where γ'_j is the generic element and α is a vector of dimension N where α'_i is the generic element. Changing variables from ϵ_{it} to S_{it} for $i=1, \dots, N$ and $t=1, \dots, 24$, one has to compute the Jacobian of ϵ_{it} as a function of S_{it} for $i=1, \dots, N$ and $t=1, \dots, 24$. Notice that $\frac{\partial \epsilon_{it}}{\partial S_{jr}} = 0$, for $t \neq r$.

Furthermore, $\frac{\partial \epsilon_{it}}{\partial S_{jt}} = \frac{\partial \epsilon_{ir}}{\partial S_{jr}}$, $\forall i, j, t, r$. Then, let us

compute the Jacobian J of dimension $(N \times N)$ and with generic element $\frac{\partial \epsilon_{it}}{\partial S_{jt}}$, which is independent of t . J is then equal to

$$J = \begin{bmatrix} 1 & -\theta & -\theta & \dots & -\theta \\ -\theta & 1 & -\theta & \dots & -\theta \\ -\theta & -\theta & 1 & \dots & -\theta \\ \vdots & \vdots & -\theta & \ddots & \vdots \\ \vdots & \vdots & \vdots & & 1 & -\theta \\ -\theta & -\theta & -\theta & \dots & -\theta & 1 \end{bmatrix}$$

The likelihood function can be then transformed into

$$L(\gamma, \alpha, \theta, \sigma^2) = \prod_{t=1}^{24} \text{ABS}(|J|) \prod_{i=1}^N \frac{1}{\sqrt{2\pi} \sigma} \text{Exp}\left(-\frac{1}{2} \left(S_{it} - \theta \sum_{\substack{j=1 \\ j \neq i}}^N S_{jt} - \sum_{j=1}^{12} \delta_{jt} \gamma'_j - \alpha_i T_t \right)^2 / \sigma^2 \right)$$

where $|J|$ is the determinant of J and $\text{ABS}(x)$ is the absolute value

of x . Notice that $|J| = 1 - \sum_{i=1}^N \theta^i (i-1) C_{N-1}^N$, where $C_k^N = \frac{N!}{k!(N-k)!}$.

Finally, the log-likelihood function is

$$L(\gamma, \alpha, \theta, \sigma^2) = \log L = N \log[\text{ABS}(|J|)] - 24N \log(\sqrt{2\pi}) - 24N \log \sigma - \frac{1}{2\sigma^2} \sum_{t=1}^{24} \sum_{i=1}^N \left(S_{it} - \theta \sum_{\substack{j=1 \\ j \neq i}}^N S_{jt} - \sum_{j=1}^{12} \delta_{jt} \gamma'_j - \alpha_i T_t \right)^2$$

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Chapter 3

PROPRIETARY INFORMATION IN VERTICAL RELATIONSHIPS:

THE ADVERTISING AGENCY CASE

I - INTRODUCTION

Two clients that compete with each other do not typically "like" to have the same advertising agency. This type of conflict is documented, for example, by the recommendation of the Committee on Client Service of the American Association of Advertising Agencies (1979, pp. 2-3) on the client-agency account conflicts:

"The practical client-agency policy on account conflicts is (such that) ...Under such a policy, an agency would not handle products which are directly competitive for more than one client..."

The conflicts can arise, as pointed by Siman (1989, p. 6) from two types of reasons: 1) the client desired "confidentiality of plans, strategy and proprietary information"¹, and, 2) the client desired "exclusivity of [the] agency services and talent". In this paper we concentrate on the first type of reasons.

While this reasoning seems obvious, it is noteworthy that the structure of the Japanese advertising industry is very different from the American one: the Japanese industry is much more concentrated (i.e. one firm dominates the market almost completely). We here

¹An example of this type of reasons at work is the lawsuit described in the Rossin Greenberg Seronick & Hill Inc. case study, Smith 1989. This case presents a former client (Lotus) suing an advertising agency because it wanted to use "confidential information" on work for one competitor (Microsoft Corporation).

present some possible explanations for this difference. In the process of doing so, we clarify the relevant incentive issues.

From a theoretical perspective, the set of reasons presented above is also much less obvious than it would appear at first sight. In fact, it is well known that, in competitive situations, possessing more information (about a competitor) can be detrimental for a firm, or, similarly, that allowing the competitor to have more information can be beneficial. In the real world, these possible outcomes are widely observed, for example, the existence of Trade Associations as a way for competitors to share some information.

Such organizations can transmit to the rivals some signal (with pre-committed variance) of their private information. This type of approach is the one taken in Vives (1984), Gal-Or (1985, 1986), and others. The differences with our work is that in theirs the trade association does not act strategically while in this case the advertising agency does. Furthermore, they assume that the trade association reports all the information it receives to both rivals, while an advertising agency can potentially discriminate in the dissemination of information.

Their findings can be summarized in the following statements:
1) if the private information of the rivals is on a common value and the competition is Bertrand (Cournot), the equilibrium in dominant strategies is (not) to share information; 2) if the private information of the rivals is on private values (costs) and the competition is Cournot (Bertrand), the equilibrium in dominant strategies is (not) to share information. The results in Section IV

allows one to understand better the peculiarities of some of these results.

The conflict of interests is presented here for the simplest framework as two rivals (the clients, the firms) being served by a same advertising agency. Each client requires the services (either in creating the advertising copy or in placing the advertisements) of an advertising agency in order to execute its business (in order to remain in business the firms have to advertise and they are unable or very inefficient at creating advertising copies or at placing advertisements). Each firm has private information of some sort on its product (positioning in the consumers perceptual map of its product, costs of repositioning its product, etc.)². Once the advertising agency works for a firm it learns its private information (proprietary information in Siman's terminology, which can translate in private information about strategies or plans).

Assume that the market outcome for a firm is better if it knows the private information of its competitor. Then, when the advertising agency is in possession of the private information of both rivals it has an incentive (and is unable to commit not to) to use them to improve its situation. This is done by selling (to one or the two firms) the private information of the competitor (the sale can involve money transfers, or any other type of transfers as promise of future contracts, perks, employment contracts, etc.; as The Economist puts it

²In Stole (1990) the perspective is exactly the opposite one: the agency has private information and the two clients (principals) contract simultaneously with it.

(1990, p. 4), "agencies have been like over paid butlers"). After the information sale the firms choose their market actions.

So, if the expected market outcome for each firm in the case they are served by two different agencies is greater than the expected market outcome if they are served by the same agency (and one of them gets the private information of the other one), firms will prefer to hire two different agencies.

But if this was the only factor influencing the advertising industry we would expect to observe a very fragmented industry with an agency per firm (or an in-house agency) as most of the products are substitutes in some degree. In fact, this is not the case: the four largest firms in the U.S. account for 23% of the revenues (The Economist, June 9, 1990, Advertising Age, October 26, 1985). The advertising industry is composed by some very large firms as shown in table I.

TABLE I
TOP TEN AGENCIES WORLDWIDE 1989

	Total Revenues (\$Millions)
Dentsu	1,316
Saatchi & Saatchi	890
Young & Rubicam	865
Backer Spielvogel	760
McCann-Erickson	716
Ogilvy & Mather	700
BBDO	657
J. Walter Thompson	626
Lintas	593
Hakuhodo	586

Source: Advertising Age, The Economist, June 9, 1990

The explanation for this concentration is that there are some advantages of an advertising agency being large, in terms of costs structure (i.e. economies of scale, Schmalensee, Silk and Bojanek 1983, though the evidence seems contradictory; economies of scope, Silk and Berndt 1990), or in terms of better services to the clients (i.e. "creative talent available to the client" (Siman, 1989, p.8), need for a "comprehensive service" (The Economist, 1990, Survey, p. 8), etc.). In the next section some more discussion on these advantages is presented.

In any case, the advantages for the large advertising agencies

seem to have gotten more important recently, given the evidence on the merger activity in the advertising industry in the mid-80's.

Then, the adoption by the two rivals of the same advertising agency or two different ones depends on the trade-offs of these two effects: the private information effects and the effect that gives advantages to the large agencies. Given that the costs structure of the American firms are likely to be similar to one of the Japanese firms, a probable explanation for the difference between the dynamics of each market, might be the limited amount of competition among Japanese firms (as it is made clear in section VI).

The issue of two rivals using the same advertising agency is similar in many aspects to the issue of using the same accounting firm, the same strategic consulting firm or the same general supplier. The real world is rich in stories where two rival firms do not want to use the same firm. For example, Bain & Co. makes a case of never working for two firms in the same industry. Stories on several types of suppliers are also very common (the close relation of Japanese car producers and their suppliers can be interpreted in this sense).

The basic contribution of this work is to provide some understanding on the conflicts of interests problems in the corporate service relationships. In particular, it is analyzed under which conditions the private information type of reasons can justify the observed will of rivals not to share the same advertising agency.

In section II, some anecdotal facts on the advertising industry are presented, and the explanations that industry observers have given to these facts are discussed.

In section III, an example of the type of competition under analysis is presented. In section IV the basic model for the private information problem is discussed. Section V presents a characterization of payoff functions such that the common wisdom on information transfers goes through, Section VI studies the strategic behavior of the advertising agency, and, Section VII presents some concluding remarks and discusses directions for further research in this area.

II - ABOUT THE ADVERTISING INDUSTRY

The conflict of interests problem is a very well known one in the advertising industry: it is always an issue when an advertising agency works for two firms which compete in a certain market.

This problem was already well known by around World War I. As described in Pope (1983, pp.163-165), at that time "it was well established that agencies would not handle competitor accounts" (Siman, p. 6).

Another important feature of the advertising industry is the relative concentration on the buyer side (on top of the also relative concentration on the supplier side, as described above and will be further discussed below). The top eight advertisers in the U.S. account for 20% of the market (see Table II) and the top ten clients billings are typically a large share in an advertising agency revenues.

TABLE II

TOP EIGHT ADVERTISERS IN THE U.S. (\$Millions) IN 1988

	Spending
Philip Morris	2,058.2
Procter & Gamble	1,506.9
General Motors	1,294.0
Sears, Roebuck	1,045.2
RJR Nabisco	814.5
McDonald's	728.3
Pepsico	712.3
Kellog	683.1
Total	44,211.0

Source: The Economist and Advertising Age.

The client conflict problem would push advertising agencies to be the smaller they can be: one agency per firm in the limit. As described above this is not observed in the real world: the advertising industry is a relatively concentrated one. What type of forces can drive this result?

One possible explanation is economies of scale: the average cost of the advertising services decline with the dimension of the advertising agency. The economies of scale could come from several types of factors: 1) creative teams can not be hired in fractions, 2) a large advertising agency has more bargaining power with the media such that it can get lower advertising rates, 3) some type of overhead costs, etc. Some anecdotal evidence supports this type of reasons:

given the typical 15% commission arrangement between advertising agencies and clients, "agencies complain that they are underpaid by small accounts, and clients are suspicious of being overcharged for large campaigns" (Schmalensee, Silk and Bojanek, 1983, p.454).

Nevertheless, empirical evidence has been contradicting this explanation for the existence of large advertising agencies. Schmalensee, Silk and Bojanek find some evidence of economies of scale, but not strong enough to explain the observed concentration: "over 200 U.S. agencies in 1977 apparently were large enough to explore essentially all economies of scale" (p. 453). Another source of evidence against the economies of scale explanation comes from the yearly study of Spicer & Oppenheim (an accountancy firm) on the profitability of the British advertising agencies. The best performers have been medium-sized agencies, and according to The Economist "there are few economies of scale for a British agency with billings above \$100m" (p. 5).

A second explanation also connected to the costs structure of advertising agencies is the existence of economies of scope. This fact would explain why the advertising industry is so concentrated and has been empirically confirmed by Silk and Berndt (1990).

A third explanation for the existence of large advertising firms is a pressure on growth. This pressure can come from the client's growth (though some clients use the services of more than one agency: Procter & Gamble and Unilever use four core agencies, Gillette use two, etc.) or the people inside the agency. This last type of pressure is best summarized by Millman (1988):

"Agency growth is important to the agency and the client. New challenges and diversity constantly stimulate creative people to do their best work" (p. 46).

Finally, another type of pressure on growth is "the ever-rising ceiling": "for an agency to remain hot, it has to grow". This type of argument has some signaling flavor to it and surely deserves further study.

A fourth explanation for the existence of large advertising agencies was hinted in the introduction: large agencies have a lot of knowledge about "how to advertise" (the common value part of the private information of each firm) and this is cherished by the potential clients. As a chairman of a top advertising agency puts it, "clients now appreciate size in agencies".

In this work the focus is on understanding why rival competitors do not like to be served by the same agency, and not, on the opposite market force (i.e. why rival competitors do not mind - or like - to be served by the same agency). In order to simplify the analysis, the economies of scale explanation for the existence of large agencies is used to create some trade-off between large and small agencies. In future research, the economies of scale explanation can be substituted by the type of explanation relying on the existence of private information on common values or even other types of explanation.

The client conflicts can arise at several degrees of direct

competition. According to this criterion, Siman (1989) classifies client conflicts in three classes: 1) Direct Product Competition: an agency A handling product N of clients X and Y; 2) Indirect Product Competition: an agency A handling products N and M of respectively clients X and Y, and N and M are partial substitutes (Example: toothpaste and mouthwash); 3) Conglomerate Competition: an agency A handling product N of client X and product M of client Y, N and M are not substitutes or complements in any degree, client Y also sells product N which is handled by agency B, and products N and M are in the same organizational division of client Y (Example: N is toothpaste and M is soup).³

All these types of conflicts occurred in the mid-eighties with the wave of mergers in the advertising industry. The merge of two agencies caused the merged agency to handle products that were rivals in the product market. For example, agency A handles product N_1 which competes with product N_2 which is handled by agency B. There is no client conflict. Once, agencies A and B merge, agency A+B handles both products N_1 and N_2 , which compete in the product market. Table III

³One must also consider the existence of intermediary situations between one advertising agency for two rivals or two advertising agencies all together. In fact, some advertising agencies try to guarantee "some" confidentiality to their clients by dividing the agency in several subsidiaries (the "umbrella concept"), by using separate offices, or, by employing different teams. These methods are in fact a continuum between the two extreme situations (one agency versus two agencies) and trade-off some confidentiality to the benefits of having a large agency: an agency divided in subsidiaries might protect better the confidentiality of clients than an agency not divided, though less than the two agencies solution; an agency divided in subsidiaries might collect to a lesser extent the benefits of being large than an agency not divided, though to a greater extent than the two agencies solution.

presents some of the conflicts that occurred in the mid-eighties.

TABLE III

ACCOUNT CONFLICTS IN THE MID-EIGHTIES

1. Conflicts after the BBDO-Doyle Dane Bernbach-Needham Harper merger.

Product N ₁	Product N ₂	Agency N ₁	Agency N ₂	Resolution
GENERAL MILLS 1-Betty Crocker (cereals)	QUAKER OATS Cereals	DDB	BBDO	BBDO resigned \$20 million Quaker's account in July 86
2-Cakes, Mixes and frostings	PILLSBURY Cakes, bread, mixes and pie crusts.	DDB and Needham	BBDO	Pillsbury resigned \$20 million account in 86
3-Cereals	NABISCO Ready-to-eat	Needham	DDB	Nabisco resigned \$20 million account in August 86.
CAMPBELL Frozen dinner products	H.J.HEINZ Ore-Ida Line of frozen food	BBDO	Needham	Agency continued with both accounts
STROH Old Milwaukee	ANHEUSER-BUSH Budweiser Michelob Light	Needham	BBDO	Stroh resigned \$100 million account in May 86
HONDA Automobiles	CHRYSLER Dodge cars & trucks	Needham	BBDO	Honda resigned \$55 million account in May 86
SEAGRAM Whiskey and Rums	NATIONAL DISTILLERS Gin, Vodka, Rum	DDB	BBDO	Agency continued with both accounts
BRISTOL-MYERS Tickle and Excedrin	GILLETTE Razors, antiperspirants	DDB	BBDO	Agency continued with both accounts

2. Conflicts after the Saatchi & Saatchi-Ted Bates Worldwide Merger:

Product N ₁	Product N ₂	Agency N ₁	Agency N ₂	Resolution
MARS M&M, gums, candy bars.	1-ROWNTREE candy 2-CADBURY SCHEPPES mixers & carbonated drinks	Bates Saatchi	Saatchi Saatchi	Mars resigned \$100 million account in July 86
GENERAL MILLS cereals	QUAKER OATS cereals	Bates	Saatchi (Backer Spielvogel)	Saatchi sold subsidiary that held this account
WARNER-LAMBERT Trident, Sugarless gum	LIFE SAVERS Buble yum gum, Breath savers, Carefree sugarless gum	Bates	Saatchi	Warner-Lambert resigned \$64 million account in July 86
COMMODORE	IBM Entry systems & Service Device	Bates	Saatchi	Commodore resigned \$110 million account
BAVARIA ST. PAUL Beer (Germany)	HOLSTEN Beer (Germany)	Bates	Saatchi	Holsten resigned \$1 million account in October 86
PHILIP MORRIS Miller brewing	ANHEUSER-BUSH Michelob	Saatchi	Bates	Anheuser-Bush resigned \$38 million account in June 86
PHILIP MORRIS Parliament. Cigarettes	RJR Wistom-Salem	Saatchi	Bates	RJR resigned \$50 million account in June 86
BMW Automobiles	JAGUAR Automobiles	Bates	Saatchi	Bates subsidiary (Scholz & Friends) issued confidentiality agreements in August 86

AMERICAN
CYANAMID
Breck Shampoo
& Old Spice

HELENA CURTIS
Industry Suave
Haircare Line

Bates

Saatchi

American Cyanamid
resigned account
January 87

Source: Siman, 1989.

III - AN EXAMPLE

Consider the following type of competition between two firms. Both firms have a certain product with certain physical characteristics. These physical characteristics would naturally position the product away from the center of the market. Both firms then advertise (i.e., reposition) these products in a way such that it appeals more to the center of the market. When doing that they have to consider the following trade-off: the closer they are to the center of the market the better off they are (higher profits) but it is costly to move in that direction. Furthermore, the physical characteristics of the product are private information of each firm (θ_i) and can be seen as the distance to the center of the market without any advertising expenditures. The action of a firm (a_i) is the positioning it elects.

The benefits of taking a certain positioning are greater the closer the firm's positioning is to the center of the market and the further away the competitor positioning is. Representing by a_i the closeness to the center of the market of firm i , we can define these benefits for firm i as

$$a_i - \gamma_1 a_j$$

The costs of taking a certain positioning are positively related to the distance between the original positioning (i.e. the physical characteristics, θ_i) and the final positioning (i.e. the action that was taken, a_i). This distance is defined by $(a_i - \theta_i)$.

The marginal costs of changing the positioning for firm i are increasing in the distance between the initial and the final positioning⁴. This means that there is an optimal value for the final positioning (which is not the center of the market, i.e., $\pi_{11} < 0$), and, that the firm takes a higher final positioning the higher is its initial one (i.e. $\pi_{13} > 0$). Another way of expressing this idea is that it is relatively easy to reposition small distances, but it is very costly to do it for large ones. This component of the costs of repositioning could then be expressed as

$$\gamma_2 (a_i - \theta_i)^2$$

Furthermore, the costs of changing the positioning for firm i are higher the closer is the final positioning of firm j to the center of the market. The idea is that it is somewhat more costly for a firm to reposition towards the center of the market if the competitor is already there (it is more difficult to cut across the clout of competition). Notice that this means that if the final positioning of firm j is large, firm i final positioning will be further away from the center of the market, i.e. the actions of firm i and j are strategic substitutes ($\pi_{12} < 0$). Furthermore, the effects on these costs of increases in the final positioning of firm j (a_j) are smaller when

⁴This is similar to the notion of "stickiness" of the original positions as mentioned in Hauser (1988).

the physical characteristics of firm i (θ_i) are closer to the center of the market. This means that $\pi_{23} > 0$. This component of the costs of repositioning are then

$$\gamma_3(a_i - \mu\theta_i)a_j$$

Then, the payoff function is

$$\pi^i(a_i, a_j, \theta_i) = a_i - \gamma_1 a_j - \gamma_2 (a_i - \theta_i)^2 - \gamma_3 (a_i - \mu\theta_i)a_j$$

Notice then that:

$$\pi_{11} = -2\gamma_2 < 0 \quad (\text{the second order conditions on the optimization on } a_i \text{ are satisfied})$$

$$\pi_{12} = -\gamma_3 < 0$$

$$\pi_{13} = 2\gamma_2 > 0$$

$$\pi_{23} = \mu\gamma_3 > 0$$

The signs of these cross-derivatives play a key role in the next sections.

IV - THE BASIC MODEL

In this section the basic model for the analysis of the conflict of interests problem is presented, i.e., we introduce the advertising agency (or agencies) and its strategic behavior.

As in the example above two firms⁵ are considered to be competing in a certain market. In order to produce and sell in this market firms have to use an outside advertising agency. The services of the advertising agency are only available in a fixed amount (either

⁵The general oligopoly case is presented in Appendix.

a firm uses the services and has positive profits or does not use it and has zero profits).

The payoff (profit) for firm i ($i=1,2$) is $\pi^i(a_i, a_j; \theta_i) - P_i - R_i$. π^i is a function $\mathbb{R}^3 \rightarrow \mathbb{R}$, A_i is the space of available actions for firm i , $a_i \in A_i$ is the action taken by firm i , $j=3-i$, A_j is the space of available actions for firm j , $a_j \in A_j$ is the action taken by firm j . π^i can be the function defined in the previous section, a_i can be the final positioning selected by firm i . Θ_i is the space of possible types of firm i . θ_i is the type of firm i in each realization. In terms of the example above, θ_i is the initial positioning of firm i (the physical characteristics of its product) which is its private information. Both firms have some prior cumulative distribution $F_i(\theta_i)$ over the space Θ_i . θ_i and θ_j are independent. Firm i has private information on the realization of θ_i and so does firm j over θ_j . Finally, P_i is the price paid to the advertising agency for its services, and, R_i are the rents the advertising agency is able to extract from firm i based on the private information it has on firm i and the other firms it works for.

The profit of an advertising agency is $\pi_A = \sum_{i \in \Psi} P_i - K + \sum_{i \in \Psi} R_i$, where Ψ is the set of firms the advertising agency A is serving (in this case it can only be $\{1\}$, $\{2\}$ or $\{1,2\}$), K is the fixed cost of operating the advertising agency (see the section II for a discussion on the existence of this fixed cost and its role in the analysis), and R_i are the rents the agency is able to extract from client i by exploiting the private information it had acquired. The marginal cost of servicing a new client are assumed zero without loss of generality.

Introducing these costs would only complicate the analysis without giving any further intuition on the problem.

The timing of events proceeds as follows: 1) the clients decide simultaneously on which agencies to use (and it is assumed they are able to coordinate on being served by the same agency or by different ones⁶); 2) the clients make take-it-or-leave-it offers P_i and P_j ; 3) the agency accepts or not to serve the firm (or firms); 4) θ_i and θ_j are realized; 5) the agency (or agencies) learns (learn) about θ_i and θ_j ; 6) the agency (or agencies) exploits (exploit) the private information they acquired in the previous stage; and, finally, 7) the clients decide on a_i and a_j .

The clients are assumed to have the bargaining power when dealing with the agency or agencies (as there is a pool of agencies available to them - that exist or can start up from nothing). If the clients are served by different agencies $P_i + R_i = P_j + R_j = K$. If the clients are served by the same agency it is assumed that $P_i = P_j = \frac{1}{2} [K - R_i - R_j]$, which is the Nash bargaining solution⁷. P_i and P_j can not be contingent on the market outcome (i.e. the values of R_i , R_j , π^i , or π^j). This assumption rules out incentives contracts: the

⁶The assumption is that the contract between the client and the agency can have a clause where it allows the agency to work for other clients or not.

⁷The clients are served by the same agency and they have to fund the losses the agency will have $(K - R_i - R_j)$. (Notice that R_i and R_j can be random variables at this stage, but as it will be shown $R_i + R_j$ is not). If the clients bargain on the funding of this project through alternating offers with some discount factor that tends to one at the same speed, one gets the Nash bargaining solution as the bargaining outcome.

realistic assumption behind this restriction is that at the time the contract is signed between the agency and the client there is so much uncertainty about π^i and π^j (being the agency risk averse), such that incentives contracts are never optimal⁸ (an alternative assumption is simply that π^i and π^j are not contractible, i.e., not verifiable).

We are looking for outcomes which are subgame perfect. In stage 7) several information structures can exist: 1) firms 1 and 2 know only about its private information; 2) firm 1 knows about the private information of firm 2 but not viceversa; 3) firm 2 knows about the private information of firm 1 but not viceversa; and, 4) firms 1 and 2 know about each others private information. Cases 2 and 3 are exactly symmetric, so that one just needs to examine one of them (it firms 1 and 2 are symmetric to begin with).

Case 1: Firms 1 and 2 know only about its private information.

In this case we are looking for the functions $a_1^*(\theta_1)$, of the form $\theta_1 \rightarrow A_1$, and $a_2^*(\theta_2)$, of the form $\theta_2 \rightarrow A_2$, such that

$$\begin{aligned} E(\pi^1[a_1(\theta_1^*), a_2(\theta_2^*); \theta_1] / \theta_1) &\geq \\ &\geq E(\pi^1[a_1, a_2^*(\theta_2); \theta_1] / \theta_1), \quad \forall a_1, \theta_1 \end{aligned}$$

$$E(\pi^2[a_2^*(\theta_2), a_1(\theta_1^*); \theta_2] / \theta_2) \geq$$

⁸The introduction of incentives contracts in this framework would complicate the analysis even more. It is our conjecture that the flavor of the results does not change very much once one allows for incentives contracts (specially if we restricted the incentives contracts to be only contingent on the performance of the client and not the performance of the competitor).

$$\geq E(\pi^2[a_2, a_1^*(\theta_1); \theta_2] / \theta_2), \forall a_2, \theta_2$$

i.e., $a_1^*(\theta_1)$ and $a_2^*(\theta_2)$ characterize the Nash equilibrium of the game in stage 7) if both firms know only about its own private information.

For later use let us define $V_1^* = E(\pi^1[a_1^*(\theta_1), a_2^*(\theta_2); \theta_1])$, and,

$$V_2^* = E(\pi^2[a_2^*(\theta_2), a_1^*(\theta_1); \theta_2])^9.$$

Case 2: Firm 1 knows about the private information of firm 2, but not viceversa:

In this case, we assume firm 1 knows about θ_1 , and θ_2 , while firm 2 knows only about θ_2 . In this case we are looking for the functions $a_1^1(\theta_1, \theta_2)$, of the form, $\theta_1 \times \theta_2 \rightarrow A_1$, and, $a_2^1(\theta_2)$, of the form, $\theta_2 \rightarrow A_2$, such that,

$$\begin{aligned} E(\pi^1[a_1^1(\theta_1, \theta_2), a_2^1(\theta_2); \theta_1] / \theta_1, \theta_2) &\geq \\ &\geq E(\pi^1[a_1, a_2^*(\theta_2); \theta_1] / \theta_1, \theta_2), \forall a_1, \theta_1, \theta_2 \end{aligned}$$

$$\begin{aligned} E(\pi^2[a_2^1(\theta_2), a_1^1(\theta_1, \theta_2); \theta_2] / \theta_2) &\geq \\ &\geq E(\pi^2[a_2, a_1^1(\theta_1, \theta_2); \theta_2] / \theta_2), \forall a_2, \theta_2 \end{aligned}$$

For later use let us define $V_1^1 = E(\pi^1[a_1^1(\theta_1, \theta_2), a_2^1(\theta_2); \theta_1])$, and, $V_2^1 = E(\pi^2[a_2^1(\theta_2), a_1^1(\theta_1, \theta_2); \theta_2])$.

Case 3 (in which firm 2 knows about the private information of firm 1 but not viceversa) is in everything symmetric to case 2. We can

⁹ In order to simplify the analysis it is assumed that the equilibria considered in cases 1 through 4 are unique and in pure strategies.

then get $a_1^2(\theta_1)$, $a_2^2(\theta_2, \theta_1)$, V_1^2 , and, V_2^2 .

Case 4: Firms 1 and 2 know about each other's private information.

In this case firm 1 knows about θ_1 , and, θ_2 , and firm 2 knows about θ_2 , and, θ_1 . We are looking for the functions $a_1^{**}(\theta_1, \theta_2)$, of the form $\theta_1 \times \theta_2 \rightarrow A_1$, and, $a_2^{**}(\theta_2, \theta_1)$, of the form $\theta_2 \times \theta_1 \rightarrow A_2$, such that,

$$\begin{aligned} E(\pi^1[a_1^{**}(\theta_1, \theta_2), a_2^{**}(\theta_2, \theta_1); \theta_1] / \theta_1, \theta_2) &\geq \\ &\geq E(\pi^1[a_1, a_2^{**}(\theta_2, \theta_1); \theta_1] / \theta_1, \theta_2), \quad \forall a_1, \theta_1, \theta_2 \end{aligned}$$

$$\begin{aligned} E(\pi^2[a_2^{**}(\theta_2, \theta_1), a_1^{**}(\theta_1, \theta_2); \theta_2] / \theta_2, \theta_1) &\geq \\ &\geq E(\pi^2[a_2, a_1^{**}(\theta_1, \theta_2); \theta_2] / \theta_2, \theta_1), \quad \forall a_2, \theta_2 \end{aligned}$$

For later use let us define $V_1^{**} = E(\pi^1[a_1^{**}(\theta_1, \theta_2), a_2^{**}(\theta_2, \theta_1); \theta_1])$ and $V_2^{**} = E(\pi^2[a_2^{**}(\theta_2, \theta_1), a_1^{**}(\theta_1, \theta_2); \theta_2])$.

When there is an agency per client, there is no way of the private information of client i being transmitted to client j ($i \neq j$)¹⁰. So, in the case the clients choose different agencies they have expected payoffs $V_1^* - K$ for firm 1, and, $V_2^* - K$ for firm 2, as the agencies can not make money on the private information of their clients ($R_1 - R_2 = 0$; $P_1 - P_2 = -K$).

When both clients choose the same agency, the agency can decide

¹⁰The results presented here can also be derived for the case where transmission across agencies is possible but costly.

either 1) to transmit no information to any of the clients (Case 1 above), or 2) to transmit the information it has to only one of the clients (Cases 2 or 3 above), or 3) to transmit the information it has to both clients (Case 4 above).

The condition that more information about the competitors is better than less can be expressed by the conditions

$$\begin{aligned} (1) \quad & V_i^i > V_i^* && i=1,2 \\ (2) \quad & V_i^j < V_i^{**} && j=3-i \end{aligned}$$

The condition that the firm is worse off when the competitors have more information can be expressed by the conditions

$$\begin{aligned} (3) \quad & V_i^j < V_i^* && i=1,2 \\ (4) \quad & V_i^{**} < V_i^i && j=3-i \end{aligned}$$

These conditions mean that the advertising agency can extract some rents from its clients. The nature of the transfers may not be exclusively monetary as discussed in the introduction.

We now proceed in the next section to characterize the functions $\pi^i(\cdot)$ and $\pi^j(\cdot)$, such that conditions (1) through (4) are satisfied.

V - THE NATURE OF COMPETITION

In this section we try to characterize the payoff functions such that more information about the competitor is better and such that the more information the competitor has the worse off is the firm (conditions (1) through (4) above).

Let us first generalize the model to allow different degrees of

informativeness. Each firm's type is composed of two elements: one element affects its payoff directly (θ_i in the previous section, which will be referred to as the intrinsic type, and, which is in the example above the initial positioning of firm i) and the other element is an imperfect signal of the intrinsic type of the competitor (s_j , which will be referred to as the signal). s_j is an imperfect signal of θ_j (so that, $s_j \in \theta_j$). s_j is not observed by firm j, such that the action taken by firm i is (and can only be) a function of both θ_i and s_j - $a_i(\theta_i, s_j)$. In terms of the example above, the action is the final positioning selected by the firm.

Furthermore, it is assumed that s_j is correct with probability p_i , and is completely uninformative with probability $1-p_i$.

Notice then that p_i is the degree with which firm i is well informed about firm j. If $p_i=0$, firm i is completely uninformed about firm j; if $p_i=1$, firm i is completely informed about firm j. In terms of the cases described in the previous section, case 1 has $p_1=p_2=0$, case 2 has $p_1=1$ and $p_2=0$, case 3 has $p_1=0$ and $p_2=1$, and, case 4 has $p_1=p_2=1$.

Furthermore, we can now define the equilibrium outcome for firm i when the degrees of informativeness of the signal for firms i and j are respectively p_i and p_j as $V_i(p_i, p_j)$.

If having more information about a competitor is better for a firm we must have

$$(5) \quad \frac{\partial V_i(p_i, p_j)}{\partial p_i} > 0 \quad \forall p_i, p_j$$

If a competitor having more information is worse for a firm we

must have

$$(6) \quad \frac{\partial V_i(p_i, p_j)}{\partial p_j} < 0 \quad \forall p_i, p_j$$

In this section we characterize the functions $\pi^i(\cdot)$ and $\pi^j(\cdot)$, such that conditions (5) and (6) are satisfied.

The analysis is done under the following assumptions:

Assumption 1 (A1): θ_1 and θ_2 are independent.

Assumption 2 (A2): $\pi^1(\cdot)$ and $\pi^2(\cdot)$ are quadratic.

Assumption 3 (A3): $\pi_{13}^i > 0$, $\forall i$.

Assumption 1 guarantees the private value nature of the private information. Assumption 2 can be generalized for the cases in which the payoff functions are reasonable approximated by Taylor expansions of the second order, i.e., the results consider only the second order effects. Finally, Assumption 3 depends simply on how one defines the private information variable (θ_i) - how one defines the direction in which the variable increases.

In the next sub-section we consider the continuous intrinsic types case and apply it to the example presented above. In sub-section V.2 we relate these results to the Trade Association literature. The results for the two intrinsic types case are presented in the appendix.

V.1 - THE CONTINUOUS INTRINSIC TYPES CASE

In this subsection we consider θ_i to be distributed in the continuous support $\theta_i = [\underline{\theta}_i, \bar{\theta}_i]$, with cumulative distribution $F_i(\theta_i)$, $i=1,2$.

If player i has the intrinsic type θ_i and receives the signal s_j , his expected payoff is

$$\begin{aligned} & p_i p_j \pi^i[a_i, a_j(s_j, \theta_i), \theta_i] + p_i(1-p_j) \int \pi^i(a_i, a_j(s_j, s_i), \theta_i) dF_i(s_i) + \\ & + p_j(1-p_i) \int \pi^i(a_i, a_j(\theta_j, \theta_i), \theta_i) dF_j(\theta_j) + \\ & + (1-p_i)(1-p_j) \iint \pi^i(a_i, a_j(\theta_j, s_i), \theta_i) dF_j(\theta_j) dF_i(s_i) \end{aligned}$$

where a_i is the action taken by firm i , and, $a_j(\theta_j, s_i)$ is the action player j takes if she has intrinsic type θ_j and receives the signal s_i .

We are trying to characterize the equilibrium strategies of both players, i.e., the equilibrium functions $a_1(\theta_1, s_2)$ and $a_2(\theta_2, s_1)$.

Under A2 we can see that the FOC for each firm is

$$\begin{aligned} (7) \quad a_i(\theta_i, s_j) &= \frac{\pi_{13}^i}{-\pi_{11}^i} \theta_i + \frac{b_1^i}{-\pi_{11}^i} + \frac{\pi_{12}^i}{-\pi_{11}^i} \left\{ p_i p_j a_j(s_j, \theta_i) + \right. \\ & \left. + p_i(1-p_j) \int a_j(s_j, s_i) dF_i(s_i) + p_j(1-p_i) \int a_j(\theta_j, \theta_i) dF_j(\theta_j) + \right. \end{aligned}$$

$$+ (1-p_i)(1-p_j) \left\{ \int \int a_j(\theta_j, s_i) dF_i(s_i) dF_j(\theta_j) \right\}, \quad i=1,2, j=3-i$$

where the notation for the payoff function is

$$\begin{aligned} \pi^i(a_i, a_j, \theta_i) = & b_1^i a_i + b_2^i a_j + b_3^i \theta_i + \pi_{11}^i a_i^2/2 + \pi_{12}^i a_i a_j + \pi_{13}^i a_i \theta_i + \\ & + \pi_{22}^i a_j^2/2 + \pi_{23}^i a_j \theta_i + \pi_{33}^i \theta_i^2/2, \quad i=1,2, j=3-i \end{aligned}$$

We are looking for the equilibrium strategies $a_i(\theta_i, s_j)$ and $a_j(\theta_j, s_i)$, that satisfy (7). If we restrict the analysis to affine strategies of the form

$$a_i(\theta_i, s_j) = \alpha_0^i + \alpha_1^i \theta_i + \alpha_2^i s_j \quad i=1,2, j=3-i$$

we can then construct the equilibrium strategies by noticing that in equilibrium

$$(8) \quad \alpha_1^i = \frac{\pi_{13}^i / (-\pi_{11}^i)}{1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_j^2}$$

$$(9) \quad \alpha_2^i = \frac{(\pi_{12}^i \pi_{13}^j) / (\pi_{11}^i \pi_{11}^j)}{1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_i^2} p_i$$

$$(10) \quad \alpha_0^i \left(1 - \frac{\pi_{12}^i}{\pi_{11}^i} \frac{\pi_{12}^j}{\pi_{11}^j} \right) = \frac{b_1^i}{-\pi_{11}^i} + \frac{b_1^j \pi_{12}^i}{\pi_{11}^i \pi_{11}^j} +$$

$$\frac{\pi_{12}^i}{-\pi_{11}^i} \left((1-p_j^2) \frac{(\pi_{12}^j \pi_{13}^i) / (\pi_{11}^i \pi_{11}^j)}{1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_j^2} E\theta_i + (1-p_i) \frac{\frac{\pi_{13}^j}{-\pi_{11}^j} \left(1 + \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_i \right)}{1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_i^2} E\theta_j \right)$$

where E is the expected value operator.

Notice that from (8), (9), and (10) one can derive results on the behavior of the equilibrium strategies (assuming $|\pi_{11}^i| > |\pi_{12}^i|$, $\forall i$, which is the stability condition in the complete information case)

Result 1: The equilibrium action taken by a firm is increasing in its increasing type (i.e. $\frac{\partial a_i}{\partial \theta_i} > 0$) if and only if the marginal returns from its actions are increasing in its intrinsic type (i.e. $\pi_{13}^i > 0$).

Proof: It follows directly from (8). ■

Result 2: The equilibrium action taken by a firm is increasing in its signal (i.e. $\frac{\partial a_i}{\partial s} > 0$) if the actions taken by both firms are

strategic complements (i.e., $\pi_{12}^j > 0$); the equilibrium action taken by a firm is decreasing in its signal if the actions taken by both firms are strategic substitutes (i.e., $\pi_{12}^j < 0$).

Proof: It follows directly from (9). ■

We can now characterize the equilibrium actions when the information structures change (i.e. p_1 and p_2).

Proposition 1: (own degree of information) Under A1, A2, and A3, more information about a competitor makes a firm's action more sensitive to the signal it receives about the competitor intrinsic type (i.e., the support of the actions of the firm is larger). Furthermore, the expected value of the actions of the firm remains unchanged.

Proof: Differentiating $a_i(\theta_i, s_j)$ with respect to p_i yields

$$\frac{\partial a_i(\theta_i, s_j)}{\partial p_i} = \frac{\frac{\pi_{13}^j}{-\pi_{11}^j} \left(1 + \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_i \right)}{\left(1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_i \right)^2} \frac{\pi_{12}^i}{-\pi_{11}^i} (s_j - E\theta_j)$$

which coupled with Result 2 lets us conclude that the firm becomes more sensitive to the signal it receives. Furthermore notice that

$$E \frac{\partial a_i(\theta_i, s_j)}{\partial p_i} = 0. \quad \blacksquare$$

The intuition for this result is very simple. Notice that if we assume π_{13} to be positive (i.e. the marginal return of the action is increasing in the intrinsic type) we know that the actions are increasing in the intrinsic type (by Result 1). Then, if for example we further consider $\pi_{12} > 0$ (i.e. the actions of the firms are strategic complements, the marginal return of the action is increasing in the competitor's action), we get the direct effect of p_1 on the high signals to be positive and on the low signals to be negative. When a firm has more information about a competitor and has a signal that the intrinsic type of the competitor is high (low), the firm trusts more that the competitor is choosing a high (low) action (given $\pi_{13} > 0$) which will make the firm choose a higher (lower) action (for the strategic substitutes case the argument is exactly the same with some changes of sign).

The basic meaning of Proposition 1 is that when the firm has more information it will react more to its signal.

The result for the competitor degree of information is presented in the next proposition.

Proposition 2: (competitor degree of information) Under A1, A2, and A3, the competitor having more information makes a firm's equilibrium actions more sensitive to its intrinsic type (i.e., the support of the actions of the firm is larger). Furthermore, the expected value of the actions of the firm remains unchanged.

Proof: Differentiating $a_i(\theta_i, s_j)$ with respect to p_j yields

$$\frac{\partial a_i(\theta_i, s_j)}{\partial p_j} = \frac{\frac{2\pi_{13}^i}{-\pi_{11}^i} \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_j}{\left(1 - \frac{\pi_{12}^j}{-\pi_{11}^j} \frac{\pi_{12}^i}{-\pi_{11}^i} p_j\right)^2} (\theta_i - E\theta_i)$$

which coupled with Result 1 lets us conclude that the firm becomes more sensitive to its intrinsic type. Furthermore, notice that

$$E \frac{\partial a_i(\theta_i, s_j)}{\partial p_j} = 0. \quad \blacksquare$$

The intuition for Proposition 2 is the following. Consider the case where a_1 and a_2 are strategic complements (for the strategic substitutes case the argument is very similar): when the competitor has more information, the firm knows that the competitor will trust more its signal, i.e., the competitor will have a higher (lower) action when the signal is high (low); but as the signal is informative the firm is better off raising (lowering) its action when its intrinsic type is high (low), given that it is more probable that the signal received by the competitor is high (low) and the actions are strategic complements (the argument for the strategic substitutes case is exactly the same with some changes of sign).

The basic meaning of Proposition 2 is that when the competitor has more information the firm will react more to its intrinsic type.

Now that we established the behavior of the equilibrium actions as a function of the information structures we can now proceed to characterize the behavior of the equilibrium payoffs.

Differentiating $V_i(p_i, p_j)$ with respect to p_i yields

$$(11) \frac{\partial V_i(p_i, p_j)}{\partial p_i} = V(\theta_j) \cdot (\pi_{12}^i \alpha_1^j \alpha_2^i + p_i \pi_{12}^i \alpha_2^i r^j + \pi_{22}^i \alpha_1^j r^j)$$

where $r^i = \frac{\partial a_i(\theta_i, s_j)}{\partial p_j} \frac{1}{(\theta_i - E\theta_i)}$, and $V(\theta_j)$ is the variance of the random variable θ_j .

Proposition 3 gives the result for the effect in the equilibrium payoff of changes in the quality of information of the signal.

Proposition 3: (own degree of information) Given A1, A2, A3, $\pi_{22}^i > 0$, and, $|\pi_{12}^i| < |\pi_{11}^i|$, more information about a competitor makes the equilibrium payoff of a firm increase.

Proof: It follows directly from (11). ■

The most interesting feature of Proposition 3 is that it requires the payoff function to be convex in the action of the opponent. The explanation for this result is quite obvious: given that the firm has more information the competitor reacts more to its intrinsic type, i.e., there is more uncertainty in the actions of the competitor, and, more uncertainty is better if the payoff function is convex in the argument subject to the increase in uncertainty.

And what about the effect of an increase in the quality of the information of the competitor on the firm's equilibrium payoff?

Differentiating $V_i(p_i, p_j)$ with respect to p_j yields

$$(12) \frac{\partial V_i(p_i, p_j)}{\partial p_j} = V(\theta_i) \cdot (\pi_{12}^i \alpha_1^i \alpha_2^j + \pi_{23}^i \alpha_2^j + p_j \pi_{12}^i \alpha_1^i q^j + p_j \pi_{23}^i q^j + \pi_{22}^i \alpha_2^j q^j)$$

$$\text{where } q^i = \frac{\partial a_i(\theta_i, s_j)}{\partial p_i} \frac{1}{(\theta_j - E\theta_j)}$$

Proposition 4 gives the result.

Proposition 4: (competitor degree of information) Given A1, A2, A3, $\pi_{22}^i > 0$, $|\pi_{12}^i| < |\pi_{11}^i|$, and, $\pi_{12}^i > 0$ ($\pi_{12}^i < 0$), the competitor having more information makes the equilibrium payoff of a firm decrease only if $\pi_{23}^i < 0$ ($\pi_{23}^i > 0$).

Proof: It follows directly from (12). ■

The interesting feature of Proposition 4 is that it says that the sign of π_{23} is a necessary condition for the equilibrium payoff of a firm to decrease when the competitor's degree of information is raised. Consider the $\pi_{12} > 0$ case (the $\pi_{12} < 0$ case is very similar). The basic explanation for the result is the following. When the competitor has more information, she reacts more to its signal, i.e., plays an higher (lower) action when the signal is high (low), which is when it is more likely that the intrinsic type of the firm is high (low). If $\pi_{23} < 0$, we have then that the effect of the higher signal is smaller than the effect of the lower one, and the firm, in expected value, ends up in a worse situation.

Another important question that can be answered by this type of analysis is whether more information for both competitors (at the same time) increases or decreases the equilibrium payoff of each one. In order to do this one has to compute $\frac{\partial V_i(p_i, p_j)}{\partial p_i}$ restricting p_j to be equal to p_i .

Assuming symmetry in the payoff functions (i.e. $\pi^i(\cdot) = \pi^j(\cdot)$) for simplicity, this computation yields:

$$(13) \frac{\partial V_i(p_i, p_j)}{\partial p_i} \Big|_{p_i=p_j} = V(\theta_i) \cdot (\pi_{12} \alpha_1 \alpha_2 + \pi_{23} \alpha_2 + p \pi_{12} \alpha_1 q + p \pi_{23} q + \pi_{22} \alpha_2 q) + \\ + V(\theta_j) \cdot (\pi_{12} \alpha_1 \alpha_2 + p \pi_{12} \alpha_2 r + \pi_{22} \alpha_1 r)$$

where r and q are exactly the ones from expressions (11) and (12), and the scripts for the firms were dropped (given $\pi^i(\cdot) = \pi^j(\cdot)$ and $p_i = p_j$). Notice that (13) is exactly the sum of the own degree of information effect and the competitor's degree of information effect. We can then derive the result for the change in the degree of information of both firms at the same time.

Proposition 5: (degree of information of both firms) Given A1, A2, A3, $\pi_{22} > 0$, $|\pi_{12}| < |\pi_{11}|$, and, $\pi_{12} > 0$ ($\pi_{12} < 0$), both competitors having more information makes the equilibrium payoff of a firm decrease only if $\pi_{23}^i < 0$ ($\pi_{23}^i > 0$). Furthermore, if this is the case, the more information a competitor has the worse off the firm is.

Proof: It follows directly from (13), and the relation of (13) with (11) and (12).■

Notice that, from Proposition 5, both players having more information can have a negative impact on the equilibrium payoffs only if the more information the competitor has, the worse off is the firm. But there may be situations (i.e., values of the parameters) such that the more information the competitor has, the worse off is the firm ((12) being negative), but both competitors having more information to have a positive impact on the equilibrium payoff ((13) being positive).

These results state conditions under which the common wisdom on information structures is derived: more information makes a firm better off and the competitor having more information makes a firm worse off.

The next question that one should address is whether these conditions are satisfied in the type of competition firms engage in when they hire an advertising agency.

Notice that the example of section III satisfies the above conditions: $\pi_{13} > 0$, $\pi_{22} = 0$, $\pi_{12} < 0$, and, $\pi_{23} > 0$ (i.e. the conditions of Propositions 3, 4 and 5). In that example we can then be in a situation such that more information makes a firm better off, and, the competitor having more information makes the firm worse off.

Given the lower degree of competition in the Japanese markets, π_{23} would be smaller in Japan than in the United States (moving towards the center of the market is not much more costly if the

competitor is already there). Then, the costs of the competitor having more information are lower and it is more likely that the same agency serves a larger number of competitors.

V.2 - DISCUSSION OF RELATED LITERATURE

In light of the propositions stated above it is relatively easy to understand some of the literature on Trade Associations. This literature has looked at price and quantity competition and "the results obtained have been shown to be very sensitive to particular specifications of the model" (Vives, 1990, p.413). The propositions stated above allow us to understand what changes are the crucial ones in each of the specifications that was analyzed in that literature. Furthermore, the results here do not depend on the normality of the random variables (as it is the case for most of the papers on that literature).

For example in Gal-Or (1986), Bertrand competition (substitute goods) with uncertain costs is considered. We have demand functions of the type

$$Q_i = a - bP_i + dP_j$$

where Q_i is demand and P_i is the price charged by firm i . Then the payoff function is

$$\pi^i = (a - bP_i + dP_j)(P_i - c_i)$$

where c_i is the marginal costs of firm i which are private information.

We then have

$$\pi_{11} = -b < 0$$

$$\pi_{12} = d > 0$$

$$\pi_{13} = b > 0$$

$$\pi_{22} = 0$$

$$\pi_{23} = -d < 0$$

We are in the conditions of Proposition 4 and, the competitor having more information hurts the firm (i.e. in the Gal-Or model, the equilibrium in dominant strategies is not to share information).

Consider now the same structure but with Cournot competition as in Gal-Or (1986). The inverse demand function is

$$P_i = a - bQ_i - dQ_j$$

and the payoff function is

$$\pi_i^i = (a - bQ_i - dQ_j + \theta_i) Q_i$$

where $\theta_i = c_i$. We then have

$$\pi_{11} = -b < 0$$

$$\pi_{12} = -d < 0$$

$$\pi_{13} = 1 > 0$$

$$\pi_{22} = 0$$

$$\pi_{23} = 0$$

Then Proposition 4 tells us that the competitor having more information yields benefits to the firm (which is exactly the result in Gal-Or).

As a last example consider Bertrand competition with private information on the demand intercept (as in Sakai 1986), i.e., quality of the product. The demand function is then

$$Q_i = \theta_i - bP_i + dP_j$$

where θ_i is private information of firm i . The payoff function is

$$\pi^i = (\theta_i - bP_i + dP_j)(P_i - c)$$

where c is the marginal cost.

We then have

$$\pi_{11} = -b < 0$$

$$\pi_{12} = d > 0$$

$$\pi_{13} = 1 > 0$$

$$\pi_{22} = 0$$

$$\pi_{23} = 0$$

which yields (under Proposition 4) that the competitor having more information leaves the firm better off.

Finally, notice that these results are all for information on private values and it would be interesting to have a similar type of results for the common values case.

VI - THE ADVERTISING AGENCY AS A STRATEGIC AGENT

In this section we try to examine the equilibrium of the industry assuming the industry is under the common wisdom in terms of information structure: a) more information is better for a firm, and, b) the competitor having more information is worse for the firm. The

conditions for these results were derived in the previous section. Furthermore, these results are equivalent to conditions (1) through (4) in Section IV, if we restrict p_1 and p_2 to take only the values 0 or 1.

It was also assumed in Section IV that incentive contracts at the time of the choice of advertising agency were not allowed (due to too much uncertainty for example). But, after the uncertainty is resolved and the private information is learned incentive contracts might be possible (it is our assumption that this is the case; the situation in which incentive contracts are never possible is presented in appendix).

The problem that remains to be solved from the Section III is what happens when both competitors select the same agency (or it is not specified in the contracts that the agency cannot deal with the competitor).

If incentive contracts are allowed when the private informations are revealed, the agency can select to sell the information to just one firm, or, to both firms as the outcomes are completely deterministic for the firm that has all the information.

If the agency selects to sell to both firms it gets $R_1 + R_2 - 2(V_i^{**} - v_i^j)$ as explained in the appendix.

If the agency selects to sell to just one firm it is able to get $v_i^i - v_i^j$ from the firm it sells the information to and zero from the other one.

Can we say anything about the relative size of the two alternatives the agency has? It is easily seen that

$V_i^i - V_i^j - 2(V_i^{**} - V_i^j) = V_i^* - V_i^{**}$, such that the agency sells the information to only one firm if the private information outcome is superior to the complete information one. Proposition 6 presents the result.

Proposition 6: The agency sells the information to one firm only, if and only if the private information outcome is superior to the complete information one (i.e., $V_i^* > V_i^{**}$).

The revenue from selling the information is then equal to the $\text{Max}(V_i^i - V_i^j, 2(V_i^i - V_i^*))$. Foreseeing these revenues, the firms in stage 2 offer $P_1 = P_2 = \frac{K}{2} - \frac{1}{2} \text{Max}(V_i^i - V_i^j, 2(V_i^i - V_i^*))$, such that the profits of the agency remain at zero¹¹. If the agency selects to sell the information to both firms the expected profit for each firm is $V_i^{**} - \frac{K}{2}$; if the agency selects to sell the information to only one firm the expected profit for each firm is

$$\frac{V_i^i + V_i^j}{2} - \frac{K}{2}$$

Foreseeing these events the firms compare the separate agencies outcome with the common agency outcome. The direction of the comparison is stated in Proposition 13.

Proposition 7: The firms choose different agencies in the first stage if and only if $K < V_i^* - V_i^{**}$. Furthermore, if the firms accept the same agency (i.e., $K > V_i^* - V_i^{**}$) the agency only sells the information to one

¹¹It is assumed that the probability of firm i buying the information from the agency is equal to the one for firm j.

of them.

Notice that the condition for the existence of a common agency is more restrictive than in the incentive contracts never possible case. The possibility of incentive contracts at the time the private information is learned makes the firms avoid the common agency situation. Figure 1 presents the result graphically.

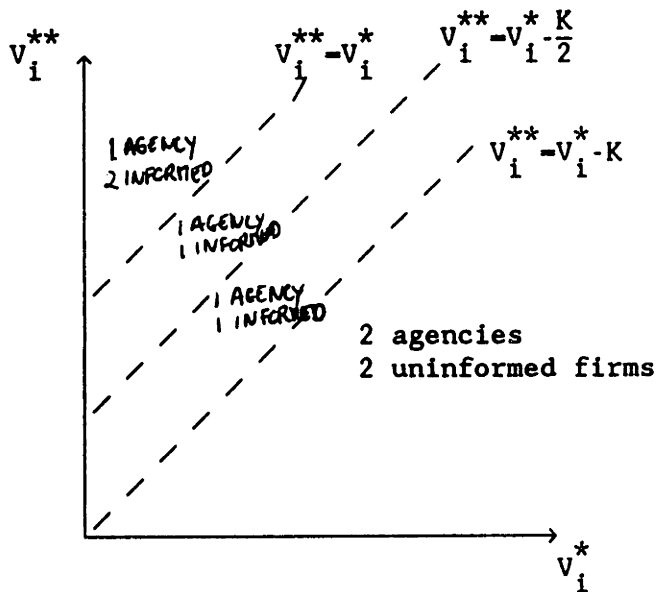


Fig. 1

Finally, notice that the existence of incentive contracts allowed us to obtain asymmetric situations as a possible outcome: the advertising agency sells the private information to only one of the competitors, which is the case for which there seems to exist more anecdotal support.

In terms of the American-Japanese dichotomy, given the lower

degree of competition in the Japanese markets, the absolute value of $V_i^* - V_i^{**}$ is smaller in Japan, and, then it is more likely that the industry settles down in the one agency outcome.

VII - CONCLUDING REMARKS

This work tries to be a first step towards understanding why firms do not like to leak information to competitors through a common advertising agency.

General conditions on the payoff functions are derived such that more information makes a firm better off and the competitor having more information makes the firm worse off. In particular, it is noticed that the sign of the second order cross derivative of the payoff function between the action of the competitor and the object of the private information plays a crucial role in the analysis.

Advertising agencies when deciding their policies in terms of account conflict must be aware of the nature of competition their potential clients could engage in.

Further work is anyway needed to understand the behavior on information transfers under common values as well as some relaxation of the quadratic assumption. Furthermore, some empirical work is needed to characterize the nature of the competition clients typically engage in (with respect to the private information advertising agencies apprehend when working with their clients).

APPENDIX

1. THE TWO INTRINSIC TYPES CASE

It is assumed that $\theta = (\underline{\theta}, \bar{\theta})$, $\underline{\theta} < \bar{\theta}$, and, that $\text{Prob}[\theta_1 = \underline{\theta}] = \text{Prob}[\theta_2 = \underline{\theta}] = \frac{1}{2}$.

For simplicity of notation let us also define $a_i(\theta_i, s_j)$ as the action taken by firm i when its intrinsic type is θ_i and its signal is s_j . Furthermore, $\bar{a}_i^+ = a_i(\bar{\theta}, \bar{\theta})$, $\bar{a}_{i+}^- = a_i(\bar{\theta}, \underline{\theta})$, $\underline{a}_i^+ = a_i(\underline{\theta}, \bar{\theta})$, and, $\underline{a}_{i+}^- = a_i(\underline{\theta}, \underline{\theta})$, i.e., the upper or under-bar reflects the intrinsic type and the upper or under-+ reflects the signal about the competitor intrinsic type.

A strategy for firm i is the vector $\bar{a}_i' = (\bar{a}_i^+, \bar{a}_{i+}^-, \underline{a}_i^+, \underline{a}_{i+}^-)$ ¹². Let us now investigate the optimal decision for firm 1. In the case its intrinsic type is $\bar{\theta}$ and its signal is $\bar{\theta}$, its expected payoff is

$$\begin{aligned} & \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi(\bar{a}_1^+, \bar{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi(\bar{a}_1^+, \bar{a}_{2+}^-, \bar{\theta}) + \\ & + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi(\bar{a}_1^+, \underline{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi(\bar{a}_1^+, \underline{a}_{2+}^-, \bar{\theta}). \end{aligned}$$

Notice that the above expression includes the fact that $\text{Prob}(\theta_2 = \bar{\theta} / s_1 = \bar{\theta}) = \frac{1}{2} + \frac{1}{2}p_1$, and, $\text{Prob}(s_2 = \bar{\theta} / \theta_1 = \bar{\theta}) = \frac{1}{2} + \frac{1}{2}p_2$. When firm 1 optimizes its expected payoff we get the following FOC (first order

¹² P' is the matrix transpose of the matrix P .

conditions):

$$\begin{aligned} & \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi_1(\bar{a}_1^+, \bar{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi_1(\bar{a}_1^+, \bar{a}_{2+}, \bar{\theta}) + \\ & + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi_1(\bar{a}_1^+, \underline{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi_1(\bar{a}_1^+, \underline{a}_{2+}, \bar{\theta}) = 0 \end{aligned}$$

We can now compute the direct effect (without strategic interactions) of p_1 on \bar{a}_1^+ . Implicitly differentiating the FOC we get

$$\begin{aligned} \frac{\partial \bar{a}_1^+}{\partial p_1} = \frac{1}{2\Delta} \left\{ \left(\frac{1}{2} + \frac{1}{2}p_2\right) [\pi_1(\bar{a}_1^+, \bar{a}_2^+, \bar{\theta}) - \pi_1(\bar{a}_1^+, \underline{a}_2^+, \bar{\theta})] + \right. \\ \left. + \left(\frac{1}{2} - \frac{1}{2}p_2\right) [\pi_1(\bar{a}_1^+, \bar{a}_{2+}, \bar{\theta}) - \pi_1(\bar{a}_1^+, \underline{a}_{2+}, \bar{\theta})] \right\} . \end{aligned}$$

where

$$\begin{aligned} -\Delta = & \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi_{11}(\bar{a}_1^+, \bar{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} + \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi_{11}(\bar{a}_1^+, \bar{a}_{2+}, \bar{\theta}) + \\ & + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} + \frac{1}{2}p_2\right)\pi_{11}(\bar{a}_1^+, \underline{a}_2^+, \bar{\theta}) + \left(\frac{1}{2} - \frac{1}{2}p_1\right)\left(\frac{1}{2} - \frac{1}{2}p_2\right)\pi_{11}(\bar{a}_1^+, \underline{a}_{2+}, \bar{\theta}) < 0 \end{aligned}$$

for the second order conditions to hold.

Furthermore, under A2, we get that the direct effect is

$$\frac{\partial \bar{a}_1^+}{\partial p_1} = \frac{\pi_{12}}{2\Delta} \left(\left(\frac{1}{2} + \frac{1}{2}p_2\right)(\bar{a}_2^+ - \underline{a}_2^+) + \left(\frac{1}{2} - \frac{1}{2}p_2\right)(\bar{a}_{2+} - \underline{a}_{2+}) \right)$$

Furthermore, one can perform the above computations for the effects of p_1 and p_2 on every element of the vector \bar{a}_1 .

This results in the following proposition.

Proposition 8: (direct effects) Given $\pi_{12} > 0$ and A3, more information about a competitor makes a firm increase (decrease) its actions if the signal it has about the competitor intrinsic type is high (low); the competitor having more information makes a firm increase (decrease) its actions if its intrinsic type is high (low).

Notice also that the sign of $(\bar{a}_1^+ - \underline{a}_1^+) - (\bar{a}_{i+} - \underline{a}_{i+})$ depends on the sign of the second order cross derivative of the function $a_i(\theta_i, s_i)$. But, under A2, $\pi(\cdot)$ is quadratic and we have that $a_i(\theta_i, s_i)$ is linear. We are then able to say that

$$\frac{\partial \bar{a}_i'}{\partial p_i} = \frac{\pi_{12}}{2\Delta} (\bar{a}_j^+ - \underline{a}_j^+) [1 \quad -1 \quad 1 \quad -1]$$

and

$$\frac{\partial \bar{a}_i'}{\partial p_j} = \frac{\pi_{12}}{2\Delta} (\bar{a}_j^+ - \underline{a}_j^+) [1 \quad 1 \quad -1 \quad -1]$$

But, do the results from Proposition 8 extend to the equilibrium values? Proposition 9 gives the result for the quality of information of the firm (which actions are under analysis), and, Proposition 10 gives the result for the quality of information of the competitor.

Proposition 9: (own degree of information) Given $\pi_{12} > 0$, A2 and A3, more information about a competitor makes a firm in equilibrium increase (decrease) its actions if the signal it has about the competitor intrinsic type is high (low).

Proof: The proof of this proposition is simply the implicit

differentiation of the system of FOC of each of the problems of both players with respect to p_i . Define $\bar{a}_i^* = (\bar{a}_i^{+*}, \bar{a}_{i+}^*, \bar{a}_i^{-*}, \bar{a}_{i+}^{-*})$ as the vector of equilibrium actions of player i . Differentiating the FOC for player i gives

$$\begin{bmatrix} da_i^{+*} \\ da_{i+}^{-*} \\ da_i^{-*} \\ da_{i+}^* \end{bmatrix} = \frac{\pi_{12}}{-\pi_{11}} (a_j^{+*} - a_j^{-*}) dp_i \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} + \frac{\pi_{12}}{-\pi_{11}} A \begin{bmatrix} da_j^{+*} \\ da_{j+}^{-*} \\ da_j^{-*} \\ da_{j+}^* \end{bmatrix}$$

where

$$A = \begin{bmatrix} \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) \\ \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) \\ \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) \\ \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) & \left(\frac{1}{2} \frac{1}{2} p_i\right) \left(\frac{1}{2} \frac{1}{2} p_j\right) \end{bmatrix}$$

Differentiating the FOC of player j gives

$$\begin{bmatrix} da_j^{+*} \\ da_{j+}^{-*} \\ da_{-j}^{+*} \\ da_{-j+}^* \end{bmatrix} = \frac{\pi_{12}}{-\pi_{11}} (a_i^{+*} - a_{i+}^{-*}) dp_i \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} + \frac{\pi_{12}}{-\pi_{11}} A' \begin{bmatrix} da_i^{+*} \\ da_{i+}^{-*} \\ da_{-i}^{+*} \\ da_{-i+}^* \end{bmatrix}$$

Substituting we then have

$$\begin{bmatrix} da_i^{+*} \\ da_{i+}^{-*} \\ da_{-i}^{+*} \\ da_{-i+}^* \end{bmatrix} = \frac{\pi_{12}}{-\pi_{11}} [(a_j^{+*} - a_{-j}^{+*}) + \frac{\pi_{12}}{-\pi_{11}} (a_i^{+*} - a_{-i}^{+*}) p_i] dp_i (I - \frac{\pi_{12}}{-\pi_{11}} A.A')^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

where I is the identity matrix. We want to characterize the vector

$$P = (I - \frac{\pi_{12}}{-\pi_{11}} A.A')^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

Take now $|\pi_{12}| < |\pi_{11}|$ (equivalent to stability in the complete information case) and define (for simplicity of notation) $\alpha = \frac{\pi_{12}}{-\pi_{11}}$, and $r_i = 1 - 2\left(\frac{1}{2} + \frac{1}{2}p_i\right)\left(\frac{1}{2} - \frac{1}{2}p_i\right)$. Notice that $\frac{1}{2} \leq r_i \leq 1, \forall p_i$. We then have

$$\left(I - \frac{\pi_{12}}{-\pi_{11}} A.A'\right) = \begin{bmatrix} a & b & d & c \\ b & a & c & d \\ d & c & a & b \\ c & d & b & a \end{bmatrix}$$

where $a = 1 - \alpha r_i r_j$, $b = -\alpha r_j (1 - r_i)$, $c = -\alpha (1 - r_i)(1 - r_j)$, and $d = -\alpha r_i (1 - r_j)$. Notice then that $(I - \alpha A.A')^{-1}$ has the form

$$X = \begin{bmatrix} x_1 & x_2 & x_4 & x_3 \\ x_2 & x_1 & x_3 & x_4 \\ x_4 & x_3 & x_1 & x_2 \\ x_3 & x_4 & x_2 & x_1 \end{bmatrix}$$

Then, the vector $P = (x_1 + x_4 - x_2 - x_3) \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$. Then, in order to

prove the proposition we just need to show that $x_1 + x_4 - x_2 - x_3 > 0$.

Inverting the matrix $(I - \alpha A.A')$ by partitions one is able to show that

$$x_1 + x_4 - x_2 - x_3 = \frac{(a+b)(a-b+c-d)}{(a-b)(a^2-b^2-c^2-d^2) + 2cd(a+b)}$$

Notice now that

$$a + b = 1 - \alpha r_j > 0$$

$$a - b + c - d = 1 - \alpha(2r_i - 1)(2r_j - 1) > 0$$

$$a - b = 1 - \alpha r_j [r_i - (1-r_i)] > 0$$

$$cd = \alpha^2 r_i (1 - r_i)(1 - r_j)^2 > 0$$

In order to conclude the proof we just need to show that

$(a^2 - b^2 - c^2 - d^2) > 0$. In order to do that, notice that

$$\frac{\partial(a^2 - b^2 - c^2 - d^2)}{\partial r_i} = -2\alpha r_j (1 - \alpha[r_i - (1-r_i)]r_j) - 2\alpha^2 [r_i - (1-r_i)](1-r_j) > 0$$

Then the

$$\text{Min}_{r_i, r_j} (a^2 - b^2 - c^2 - d^2) = (a^2 - b^2 - c^2 - d^2)|_{r_i=r_j=1} = (1-\alpha)^2 > 0$$

Proposition 10: (competitor degree of information) Given π_{12} , A2 and A3, the competitor having more information makes a firm in equilibrium increase (decrease) its actions if its intrinsic type is high (low).

Proof: Using the notation of the proof of Proposition 9 we have

$$\begin{bmatrix} da_i^{+*} \\ da_{i+}^{-*} \\ da_{-i}^{+*} \\ da_{-i+}^{*} \end{bmatrix} = \frac{\pi_{12}}{-\pi_{11}} \left[(a_j^{+*} - a_j^{+*}) + \frac{\pi_{12}}{-\pi_{11}} (a_i^{+*} - a_i^{+*}) p_j \right] dp_j (I - \frac{\pi_{12}}{-\pi_{11}} A' \cdot A)^{-1} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Everything is the same as in the proof of Proposition 9 except that $b = -\alpha r_i (1 - r_j)$, $d = -\alpha r_j (1 - r_i)$, and that the condition to be satisfied is $x_1 + x_2 - x_3 - x_4 > 0$. We have that

$$x_1 + x_2 - x_3 - x_4 = \frac{(a - b)(a + b + c + d)}{a(a^2 - b^2 - c^2 - d^2 - 2cd) + b(a^2 - b^2 - c^2 - d^2 + 2cd)}$$

Using the information of the proof of Proposition 9 plus

$$a + b + c + d = 1 - \alpha > 0$$

and noticing that

$$\frac{\partial (a^2 - b^2 - c^2 - d^2 - 2cd)}{\partial r_i} = -2\alpha [r_2(1 - \alpha r_1 r_2) + \alpha(1 - r_2)^2] - 2\alpha^2(1 - r_1)(2r_2^2 - 1) < 0$$

which implies that

$$\text{Min}_{r_i} (a^2 - b^2 - c^2 - d^2 - 2cd) = (a^2 - b^2 - c^2 - d^2 - 2cd) \Big|_{r_i=1} = (1 - \alpha) [1 - \alpha r_j + \alpha(1 - r_j)] > 0$$

we can then show that $x_1 + x_2 - x_3 - x_4 > 0$.

■

Now that the behavior of the equilibrium actions are characterized as a function of the information structure we can

proceed to study the behavior of the equilibrium payoffs.

Proposition 11 gives the result for the effect in the equilibrium payoff of changes in the own quality of information of the signal.

Proposition 11: (own degree of information) Given $\pi_{12}, \pi_{22} > 0$, A2, A3, and, $|\pi_{12}| < |\pi_{11}|$, more information about a competitor makes the equilibrium payoff of a firm increase.

Proof: Differentiating the expected payoff ($E\pi$) of the firm with respect to p_i (the own degree of information) yields (using the envelop theorem):

$$\frac{dE\pi}{dp_i} = \frac{\partial E\pi}{\partial p_i} + \frac{\partial E\pi}{\partial a_j} \frac{\partial a_j^*}{\partial p_i}$$

$\frac{\partial E\pi}{\partial p_i}$ is the direct effect and is equal to

$$\begin{aligned} \frac{\partial E\pi}{\partial p_i} = & \frac{1}{8} \left(\frac{1}{2} + \frac{1}{2} p_j \right) \left[\int_{a_{i+}^*}^{a_i^{+*}} \int_{a_{j+}^*}^{a_j^{+*}} \pi_{12}(x, y, \bar{\theta}) dy dx + \int_{a_{i+}^*}^{a_i^{+*}} \int_{a_{j+}^*}^{a_j^{+*}} \pi_{12}(x, y, \underline{\theta}) dy dx \right] + \\ & + \frac{1}{8} \left(\frac{1}{2} - \frac{1}{2} p_j \right) \left[\int_{a_{i+}^*}^{a_i^{+*}} \int_{a_{j+}^*}^{a_j^{+*}} \pi_{12}(x, y, \bar{\theta}) dy dx + \int_{a_{i+}^*}^{a_i^{+*}} \int_{a_{j+}^*}^{a_j^{+*}} \pi_{12}(x, y, \underline{\theta}) dy dx \right] \end{aligned}$$

which is greater than zero given $\pi_{12}, \pi_{13} > 0$.

$\frac{\partial E\pi}{\partial \bar{a}_j} \frac{\partial \bar{a}_j^*}{\partial p_i}$ is the strategic effect and is equal to

$$\begin{aligned} & \frac{\partial E\pi}{\partial \bar{a}_j} \frac{\partial \bar{a}_j^*}{\partial p_i} \left\{ \left(\frac{1}{2} + \frac{1}{2} p_i \right) \left(\frac{1}{2} + \frac{1}{2} p_j \right) \left[\int_{\bar{a}_{i+}^*}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^*, \bar{\theta}) dx + \int_{\bar{a}_j^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^*, x, \bar{\theta}) dx \right] + \right. \\ & + \left(\frac{1}{2} + \frac{1}{2} p_i \right) \left(\frac{1}{2} + \frac{1}{2} p_j \right) \left[\int_{\bar{a}_{i+}^*}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^*, \bar{\theta}) dx + \int_{\bar{a}_j^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^*, x, \bar{\theta}) dx \right] + \\ & + \left(\frac{1}{2} + \frac{1}{2} p_i \right) \left(\frac{1}{2} - \frac{1}{2} p_j \right) \left[\int_{\bar{a}_{i+}^*}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^*, \bar{\theta}) dx + \int_{\bar{a}_j^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^*, x, \bar{\theta}) dx \right] + \\ & + \left(\frac{1}{2} + \frac{1}{2} p_i \right) \left(\frac{1}{2} - \frac{1}{2} p_j \right) \left[\int_{\bar{a}_{i+}^*}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^*, \bar{\theta}) dx + \int_{\bar{a}_j^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^*, x, \bar{\theta}) dx \right] + \\ & + \left(\frac{1}{2} - \frac{1}{2} p_i \right) \left(\frac{1}{2} + \frac{1}{2} p_j \right) \left[- \int_{\bar{a}_{i+}^*}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^*, \bar{\theta}) dx + \int_{\bar{a}_j^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^*, x, \bar{\theta}) dx \right] + \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{1}{2} - \frac{1}{2}p_i\right)\left(\frac{1}{2} + \frac{1}{2}p_j\right) \left[- \int_{\underline{a}_{i+}^*}^{\underline{a}_{i+}^{+*}} \pi_{12}(x, \underline{a}_{j+}^*, \underline{\theta}) dx + \int_{\underline{a}_{j+}^*}^{\underline{a}_{j+}^{-*}} \pi_{22}(\underline{a}_{i+}^*, x, \underline{\theta}) dx \right] + \\
& + \left(\frac{1}{2} - \frac{1}{2}p_i\right)\left(\frac{1}{2} - \frac{1}{2}p_j\right) \left[- \int_{\underline{a}_{i+}^*}^{\underline{a}_{i+}^{+*}} \pi_{12}(x, \underline{a}_{j+}^*, \bar{\theta}) dx + \int_{\underline{a}_{j+}^*}^{\underline{a}_{j+}^{-*}} \pi_{22}(\underline{a}_{i+}^*, x, \bar{\theta}) dx \right] + \\
& + \left(\frac{1}{2} - \frac{1}{2}p_i\right)\left(\frac{1}{2} - \frac{1}{2}p_j\right) \left[- \int_{\underline{a}_{i+}^*}^{\underline{a}_{i+}^{+*}} \pi_{12}(x, \underline{a}_j^{+*}, \underline{\theta}) dx + \int_{\underline{a}_j^{+*}}^{\underline{a}_j^{-*}} \pi_{22}(\underline{a}_{i+}^*, x, \underline{\theta}) dx \right] \Bigg\}
\end{aligned}$$

which is greater than zero for $\pi_{12}, \pi_{22} > 0$, and π_{12} relatively invariant in its arguments.

And what about the effect on a firm of an increase in the degree of information of the competitor? Proposition 12 gives the result.

Proposition 12: (competitor degree of information) Given $\pi_{12}, \pi_{22} > 0$, A2, A3, and, $|\pi_{12}| < |\pi_{11}|$, and, the competitor having more information makes the equilibrium payoff of a firm decrease only if $\pi_{23} < 0$.

Proof: Differentiating the expected payoff ($E\pi$) of the firm with respect to p_j (the competitor degree of information) yields (using the

envelop theorem):

$$\frac{dE\pi}{dp_j} = \frac{\partial E\pi}{\partial p_j} + \frac{\partial E\pi}{\partial \bar{a}_j} \frac{\partial \bar{a}_j}{\partial p_j}$$

$\frac{\partial E\pi}{\partial p_j}$ is the direct effect and is equal to

$$\begin{aligned} \frac{\partial E\pi}{\partial p_j} = & \frac{1}{8} \left(\frac{1}{2} + \frac{1}{2} p_1 \right) \left\{ \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{a_{-i}^{+*}}^{a_i^{-*+}} \pi_{12}(x, y, \bar{\theta}) dx dy + \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{\theta}^{\bar{\theta}} \pi_{23}(a_{-i}^{+*}, x, y) dx dy + \right. \\ & + \int_{a_{-j+}^{*+}}^{a_j^{*+}} \int_{a_{-i+}^{*+}}^{a_{i+}^{-*}} \pi_{12}(x, y, \bar{\theta}) dx dy + \left. \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{\theta}^{\bar{\theta}} \pi_{23}(a_{-i+}^{*+}, x, y) dx dy \right\} + \\ & + \frac{1}{8} \left(\frac{1}{2} - \frac{1}{2} p_1 \right) \left\{ \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{a_{-i+}^{*+}}^{a_{i+}^{-*}} \pi_{12}(x, y, \bar{\theta}) dx dy + \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{\theta}^{\bar{\theta}} \pi_{23}(a_{-i+}^{*+}, x, y) dx dy + \right. \\ & + \int_{a_{-j+}^{*+}}^{a_j^{*+}} \int_{a_{-i}^{+*}}^{a_i^{-*+}} \pi_{12}(x, y, \bar{\theta}) dx dy + \left. \int_{a_{j+}^{-*}}^{a_j^{-*+}} \int_{\theta}^{\bar{\theta}} \pi_{23}(a_{-i}^{+*}, x, y) dx dy \right\} \end{aligned}$$

which is less than zero only if $\pi_{23} < 0$, given that $\pi_{12} > 0$.

$$\frac{\partial E\pi}{\partial \bar{a}_j} \frac{\partial \bar{a}_j}{\partial p_j} \text{ is the strategic effect and is equal to}$$

$$\frac{\partial E\pi}{\partial \bar{a}_j} \frac{\partial \bar{a}_j^{+*}}{\partial p_j} \left(\frac{1}{2} + \frac{1}{2} p_i\right) \left(\frac{1}{2} - \frac{1}{2} p_j\right) \left\{ \int_{\bar{a}_i^{+*}}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_j^{+*}, \bar{\theta}) dx + \right.$$

$$+ \int_{\bar{a}_{j+}^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_i^{+*}, x, \bar{\theta}) dx + \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\bar{a}_i^{+*}, \bar{a}_{j+}^{+*}, x) dx +$$

$$+ \int_{\bar{a}_{i+}^{+*}}^{\bar{a}_{i+}^{+*}} \pi_{12}(x, \bar{a}_j^{+*}, \bar{\theta}) dx + \int_{\bar{a}_{j+}^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^{+*}, x, \bar{\theta}) dx + \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\bar{a}_{i+}^{+*}, \bar{a}_{j+}^{+*}, x) dx \left. \right\} +$$

$$+ \left(\frac{1}{2} - \frac{1}{2} p_i\right) \left(\frac{1}{2} - \frac{1}{2} p_j\right) \left\{ - \int_{\bar{a}_i^{+*}}^{\bar{a}_i^{+*}} \pi_{12}(x, \bar{a}_{j+}^{+*}, \bar{\theta}) dx + \right.$$

$$+ \int_{\bar{a}_{j+}^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_i^{+*}, x, \bar{\theta}) dx - \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\bar{a}_i^{+*}, \bar{a}_{j+}^{+*}, x) dx -$$

$$- \int_{\bar{a}_{i+}^{+*}}^{\bar{a}_{i+}^{+*}} \pi_{12}(x, \bar{a}_{j+}^{+*}, \bar{\theta}) dx + \int_{\bar{a}_{j+}^{+*}}^{\bar{a}_j^{+*}} \pi_{22}(\bar{a}_{i+}^{+*}, x, \bar{\theta}) dx - \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\bar{a}_{i+}^{+*}, \bar{a}_{j+}^{+*}, x) dx \left. \right\} +$$

$$\begin{aligned}
& + \left(\frac{1}{2} - \frac{1}{2}p_i\right)\left(\frac{1}{2} + \frac{1}{2}p_j\right) \left\{ \int_{\frac{-i+}{-i+}}^{\frac{-i+}{i+}} \pi_{12}(x, \frac{-i+}{j+}, \bar{\theta}) dx + \right. \\
& + \int_{\frac{-i+}{j+}}^{\frac{-i+}{j+}} \pi_{22}(\frac{-i+}{i+}, x, \bar{\theta}) dx + \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\frac{-i+}{i+}, \frac{-i+}{j+}, x) dx + \\
& + \int_{\frac{-i+}{-i+}}^{\frac{-i+}{i+}} \pi_{12}(x, \frac{-i+}{j+}, \bar{\theta}) dx + \int_{\frac{-i+}{-j+}}^{\frac{-i+}{j+}} \pi_{22}(\frac{-i+}{i+}, x, \bar{\theta}) dx - \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\frac{-i+}{i+}, \frac{-i+}{j+}, x) dx \left. \right\} + \\
& + \left(\frac{1}{2} - \frac{1}{2}p_i\right)\left(\frac{1}{2} - \frac{1}{2}p_j\right) \left\{ - \int_{\frac{-i+}{-i+}}^{\frac{-i+}{i+}} \pi_{12}(x, \frac{-i+}{j+}, \bar{\theta}) dx + \right. \\
& + \int_{\frac{-i+}{-j+}}^{\frac{-i+}{j+}} \pi_{22}(\frac{-i+}{i+}, x, \bar{\theta}) dx - \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\frac{-i+}{i+}, \frac{-i+}{j+}, x) dx + \\
& - \int_{\frac{-i+}{-i+}}^{\frac{-i+}{i+}} \pi_{12}(x, \frac{-i+}{j+}, \bar{\theta}) dx + \int_{\frac{-i+}{-j+}}^{\frac{-i+}{j+}} \pi_{22}(\frac{-i+}{i+}, x, \bar{\theta}) dx - \int_{\bar{\theta}}^{\bar{\theta}} \pi_{23}(\frac{-i+}{i+}, \frac{-i+}{j+}, x) dx \left. \right\}
\end{aligned}$$

which is less than zero only if $\pi_{23} < 0$, given $\pi_{12}, \pi_{22} > 0$, and, π_{12}, π_{22} , and, π_{23} relatively invariant in its arguments.

2. THE INCENTIVE CONTRACTS NEVER POSSIBLE CASE

Notice that in this case, as incentive contracts are not possible at any time, the agency has to sell the private information to both firms, as it can not commit not to sell the information to firm i , after having sold it to firm j . The agency can make a take-it-or-leave-it offer to both clients of R_1 and R_2 . R_1 and R_2 are chosen such that both clients are willing to pay this price for the information. Assuming, that the agency is not able to communicate truthfully whether it already sold the information to one of the clients or not, the clients are playing a simultaneous-move game with matrix form⁵⁴:

		Client 2	
		Not Buy	Buy
Client 1	Not Buy	V_1^*, V_2^*	$V_1^2, V_2^2 - R_2$
	Buy	$V_1^1 - R_1, V_2^1$	$V_1^{**} - R_1, V_2^{**} - R_2$

Assuming symmetry (i.e., $V_1^* = V_2^*$, $V_1^{**} = V_2^{**}$, $V_1^1 = V_2^1$, and $V_1^2 = V_2^2$), $R_1 - R_2 = \text{Min}(V_1^1 - V_1^*, V_1^{**} - V_1^j)$, guarantees that (Buy, Buy) is an equilibrium

⁵⁴In fact, given that the agency is not able to reveal truthfully R_i to firm j , it is like both clients and the advertising agency were playing a simultaneous move game. Each client decides on \bar{R}_i such that it buys the private information of firm j if and only if $R_i \leq \bar{R}_i$, and, the advertising agency decides on R_1 and R_2 .

(the unique one) in dominant strategies⁵⁵. Notice that the values of $R_1=R_2$ are positive given conditions (1) and (2). Notice also that $V_i^i - V_i^* = V_i^{**} - V_i^j$ as (11) is not a function of p_j .

As the clients foresee this hold-up problem in stage 2), they are only willing to pay the agency $P_1=P_2=\frac{K}{2} \cdot (V_i^{**} - V_i^j)$. The profits of the agency are then zero. The profit of each client is

$$V_i^{**} - \frac{K}{2} \quad i=1,2; j=1+1[i=1]$$

Finally, the clients choose whether or not to have the same agency depending on the comparison between what their payoff is if they are served by different agencies ($V_i^* - K$), and, they are served by the same agency ($V_i^{**} - \frac{K}{2}$).

From this discussion one can derive the following proposition:

Proposition 13: 1. The complete information outcome being worse for the clients than the private information one (i.e. $V_i^{**} < V_i^*$) is a necessary condition for the clients to choose different agencies. 2. $V_i^{**} - V_i^* + \frac{K}{2} \leq 0$ is a necessary and sufficient condition for the clients to choose different agencies.

Notice that according to Proposition 5 the condition $V_i^{**} - V_i^* + \frac{K}{2} \leq 0$ is more likely to occur the larger is $|\pi_{23}|$ and the smaller is $|\pi_{12}|$, given that they are of opposite signs.

⁵⁵ Notice that $R_1=R_2=V_i^{**} - V_i^j$ guarantees (Buy,Buy) to be a Nash equilibrium, but (Not Buy,Not Buy) can also be one if $V_i^{**} - V_i^j > V_i^i - V_i^*$. We consider in this work that the agency implements the sale of private information in dominant strategies.

3. THE OLIGOPOLY CASE

In this section the extension of the model of the previous section to the oligopoly case is presented. Assume there are N firms (potential clients) in the market. Firms have to use the service of an advertising agency to be able to sell their product.

There is one dimension under which the N firms are located in a circle: firm 1 is located between firm N and firm 2, firm 2 is located between firm 1 and firm 3, firm 3 is located between firm 2 and firm 4, etc.

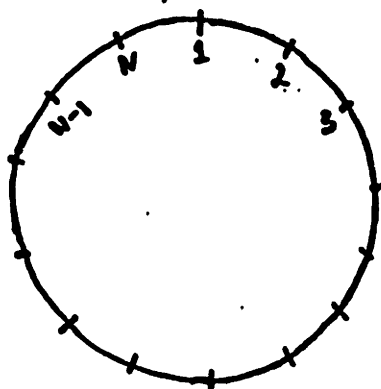


fig. 2

Firms compete relatively more with the firms they are closer to, i.e., firm 1 competes more with firms 2 and N , than with firms 3 and $N-1$. The distance between two neighbor firms is the same whatever the neighbors, i.e., $d_{12} = d_{N-1} = d_{23}$, etc. where d_{ij} is the distance

between firm i and firm j .

The intensity of the competition between two firms is captured by a function $f: \mathbb{R}^+ \times \mathbb{R}^+$, which argument is the distance between the two firms and which is strictly decreasing and concave in this argument.

A firm's i type (which is its private information, i.e. θ_i) is a vector of dimension N where element j represents the part of its type that is relevant in the competition with firm j . (For firm i , element i is an irrelevant parameter). The prior distribution on the firms' types are such that all types are independent, all elements of a firm's type are independent and the marginal distributions of θ_{ij} $\forall i, j$ are equal.

Firm's i action (i.e. a_i) is a vector of dimension N where element j represents the part of its action that is relevant in the competition with firm j . (For firm i , element i is an irrelevant parameter).

Finally, firm's i payoff is the sum of its payoffs in the competition with each of the other firms. The payoff that results from the competition between i and j is given by $f(d_{ij}) \times \pi(a_{ij}, a_{ji}, \theta_{ij})$, where $\pi(\cdot)$ is the function characterized in section III. a_{ij} is the action taken by firm i with respect to firm j ; θ_{ij} is the part of firm's i intrinsic type that matters in the relation with firm j .

Let us now define $f(d_{ij}) \times V_{ij}^*$ as the Nash equilibrium payoff for firm i of its relation with firm j if θ_{ij} remains private information of firm i and θ_{ji} remains private information of firm j (define $\delta_{ij}^* = 1$ if this event happens and $\delta_{ij}^* = 0$ otherwise). Given the definition of payoffs $V_{ij}^* = V_{rs}^* \forall i, j, r, s$

Furthermore, let us define $f(d_{ij}) \times V_{ij}^i$ as the Nash equilibrium payoff for firm i from its relation with firm j if θ_{ij} remains private information of firm i and θ_{ji} is no more private information of firm j (define $\delta_{ij}^i = 1$ if this event happens and $\delta_{ij}^i = 0$ otherwise), $f(d_{ij}) \times V_{ij}^j$ as the Nash equilibrium payoff for firm i from its relation with firm j if θ_{ij} is no more private information of firm i and θ_{ji} remains private information of firm j (define $\delta_{ij}^j = 1$ if this event happens and $\delta_{ij}^j = 0$ otherwise), and, $f(d_{ij}) \times V_{ij}^{**}$ as the Nash equilibrium payoff for firm i from its relation with firm j if no firm has any private information (define $\delta_{ij}^{**} = 1$ if this event happens and $\delta_{ij}^{**} = 0$ otherwise).

Notice that, given the definition of payoffs $V_{ij}^i = V_{rs}^r$, $V_{ij}^j = V_{rs}^s$, and, $V_{ij}^{**} = V_{rs}^{**}$, $\forall i, j, r, s$.

The total payoff for firm i is then defined as

$$\sum_{\substack{j=1 \\ j \neq i}}^N f(d_{ij}) [\delta_{ij}^i V_{ij}^i + \delta_{ij}^j V_{ij}^j + \delta_{ij}^{**} V_{ij}^{**}]$$

There are m advertising agencies in the market (m is determined endogenously as it is described below). In order to operate each advertising agency has fixed costs K.

Let us consider the case in which firms would each have its own agency if $K=0$. Following Proposition 11, this would be the case if $V_{ij}^{**} < V_{ij}^*$. Define the gain from having different advertising agencies $G = V_{ij}^* - V_{ij}^{**}$.⁵⁶

Let us now try to construct the equilibrium number of agencies

⁵⁶ It is assumed in this section that incentive contracts are not possible at any time. Everything generalizes if this assumption is relaxed.

and which firms are being served by which agency. Let us restrict ourselves to the symmetric equilibrium, i.e. all agencies serve the same number of firms.

First of all, note that given the concavity of $f(\cdot)$, the firms served by a certain agency are at most P/n close, where P is the perimeter of the circle and n is the number of firms served by each agency.

Consider now the decision about the number of advertising agencies - m ($m=N/n$). The cost per firm of having an additional advertising agency is

$$(14) \quad - \frac{K_m}{N} + \frac{K(m+1)}{N}$$

while the benefit is

$$(15) \quad 2G \left(\sum_{j=1}^{N/(2m)-1} f\left(j \frac{Pm}{N}\right) - \sum_{j=1}^{N/(2(m+1))-1} f\left(j \frac{P(m+1)}{N}\right) \right)$$

assuming m , n , and N large enough such that the integer problems are of minor importance.

Notice that while the cost per firm of having an additional advertising agency is independent of m , the benefit is decreasing in m as $f(\cdot)$ is a concave function. m can be computed by finding the m that makes (14) equal (15).

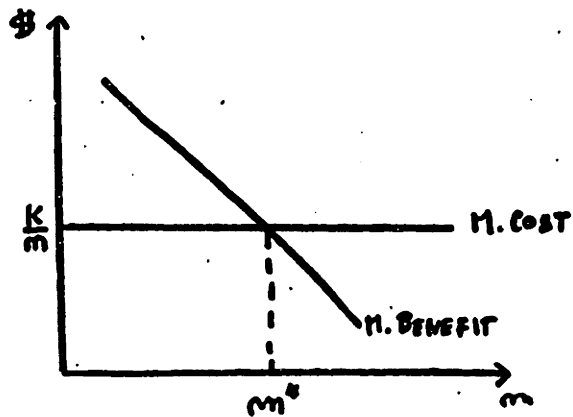


Fig. 3

Notice that as the extent of economies of scale increases (i.e. increase in K) the equilibrium number of agencies (i.e. m) decreases, as the marginal cost function goes up.

Furthermore, when the gain from having different agencies (i.e. G) goes up, the equilibrium number of agencies declines, as the marginal benefit curve goes up.

The model would then predict that one should observe in the market advertising agencies working with several and very unrelated firms.

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