

# The Development of Counting and the Concept of Number

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# Development of Counting and the Concept of Number

Karen Wynn

Submitted to the Department of Brain and Cognitive Sciences in September, 1990 in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in Cognitive Science.

## Abstract

Two theories of children's understanding of counting are evaluated in the first paper. The "Principles-Before" theory claims that children are born with a concept of number that consists of a set of counting principles. These define correct counting and so help children to improve their own counting skills. The "Principles-After" theory claims that children must learn how to count, and must abstract out the counting principles from examples of correct counting. To evaluate these theories, 2- and 3-year-olds' knowledge of the *cardinal word* counting principle, which states that the last number word used in a count indicates the number of items counted, was examined. It was found that children learn the cardinal word principle at roughly 3-1/2 years of age, well after they have acquired considerable skill at counting. This supports the Principles-After theory, while disconfirming the Principles-Before theory.

Children must therefore learn that counting determines the numerosity of a set, and must learn the meanings of the number words. The second paper examines how children do this, and at what age they learn the way in which the counting system represents numerosity. Results showed that at a very early stage of counting, children already know that the counting words each refer to a distinct, unique numerosity. The possibility that children learn this from the syntax of the number words is discussed. Despite this early knowledge, however, it takes children a long time (on the order of a year) to learn how the counting system represents numerosity. This suggests that children's own initial concept of number is represented quite differently from the way the counting system represents number, making it a difficult task for them to map the one onto the other. It is proposed that humans are initially equipped with an analogue representation of number, and the differences between such a representation of number and that embodied by the counting system are discussed.

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## Preface

This dissertation is composed of two papers, or chapters, that are independent, though very closely related, bodies of work. Since each was designed and written to stand on its own, I have placed the relevant tables and figures at the end of each for easy access. The second paper is built on the first, and so refers to it in the same manner as to any independent source, summarizing relevant results where appropriate. All references are placed together at the end of the document.

## Table of Contents

Preface.....	page 5
Children's Understanding of Counting.....	page 7
How Children Learn the Meanings of Number Words.....	page 61
References.....	page 128

## Children's Understanding of Counting

Because counting is one of the very earliest number-related activities, it may shed light on the young child's concept of number. Much research currently revolves around whether domain-specific knowledge of number underlies children's counting, or whether it is through general cognitive capacities that children learn to count, and thus come to understand number. The two main theories regarding this question are outlined below.

### *Current Theories of Children's Counting*

#### *'Principles-Before' Theory*

There has been growing interest over the past 15 years in a domain-specific theory of the basis of counting, developed by Gelman and colleagues (e.g. Gelman & Gallistel, 1978; Gelman & Greeno, 1987; Gelman & Meck, 1983; Gelman, Meck & Merkin, 1986; Greeno, Riley & Gelman, 1984). The claim (outlined in detail in Gelman & Gallistel, 1978) is that there are innate, number-specific principles that underlie children's ability to count. The following "How-to-count" principles define the counting procedure: The *One-to-one correspondence* principle states that there must be a one-to-one correspondence between things to be counted, and counting tags; the *Stable-Order* principle states that the counting tags must have a fixed order in which they are consistently used; and the *Cardinality* principle stipulates that the last counting tag used in a particular count represents the numerosity, or cardinality, of the collection of items counted.<sup>1</sup> I posit two further principles which specify a *lack* of constraints on the application of the counting

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<sup>1</sup>Gelman & Gallistel (1978)

procedure. *Order-Irrelevance principle*: The same result will obtain regardless of the order in which a set of entities is counted. *Abstraction principle*: Any entities can be grouped together for a count. For example, we can count the number of eyes, sneezes, and ideas in this room between 1:00 and 2:00 pm as a single count.) These principles exist before children have any experience with counting, and do two main things for children. First, they allow children to recognize different instances of counting in their environment *as counting*, not just some meaningless activity, and to group them together as a single type of activity, performed to determine the numerosity of a collection of items. Children notice that this activity honors the counting principles, and so view it as a counting activity, and identify the number words of the culture as counting tags. Second, the principles serve as guidelines for counting, so that children can initiate, monitor, and attempt to correct their own counting, and thus improve their counting skills.

The above counting principles have also been proposed (e.g. Gallistel, 1989) as the basis of numerical abilities of nonhumans. Animals may have some mechanism that assigns each element in a set a unique (nonverbal) tag in a way such that the last tag is the output of the process and represents the numerosity of the items thus "counted". It may also be that other human numerical abilities embody the counting principles in some form. For example, infants are able to discriminate small numerosities (e.g., Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981). It is possible that the process underlying this ability is a "counting" process, assigning stably-ordered mental tags (e.g., ordered memory bins) to items, with the last tag representing their numerosity. These possible roles of the counting principles are very different from the one described above, as the nature of the activity itself is quite different in the two cases – a culturally supported, consciously controlled activity in the one case, an automatic process in the other case. To distinguish these different senses of the cardinality principle, I



introduce the term "*Cardinal Word* principle". The cardinal word principle states that the last number word used in the culturally supported, oftentimes overt activity that we call "counting" represents the numerosity of the set of items counted. (We might wish to introduce the term "*Cardinal Tag* principle" to refer to the form of cardinality principle that might play a role in animals' or infants' numerical abilities.) This paper is concerned with children's knowledge of the culturally supported counting activity, and so with their knowledge of the cardinal word principle.

### *'Principles-After' Theory*

Briars and Siegler (1984), Fuson (1988), and Fuson and Hall (1983), among others, are advocates of the following account of children's counting: Young children first learn counting as a routine activity. Counting is modeled for them by parents, teachers, etc., and they begin to imitate it. At this point it is an activity without special meaning, much like a game of patty-cake. Children do not distinguish between different components of the counting routine; all components are equally essential. They must learn different routines for different counting contexts; counting objects arranged in a circle, for example, entails a different procedure than counting objects in a line. Once they have learned many such routines, children eventually generalize over all these routines, abstracting out what all have in common -- namely, the counting principles. Only after this has happened do children have principled knowledge.

On this view, how do children come to understand that counting is related to numerosity? Children cannot learn *why* we count (to determine numerosity) just by learning *how* we count. *Subitization*, the ability to very quickly and accurately determine certain numerosities without having to consciously count, may be at the root of this accomplishment. Adults and children can subitize small numerosities, up to four or five for adults and two or three for 3-, 4-, and 5-year-olds (Chi & Klahr, 1975; Schaeffer, Eggleston &

Scott, 1974; Silverman & Rose, 1980). It has been suggested (Klahr & Wallace, 1973, 1976) that by applying the initially meaningless counting routine to sets of items within the subitizing range, children come to associate words (e.g. "one", "two", "three") with those numerosities they can recognize (e.g. one, two, three), and thus come to relate the counting activity with the concept of numerosity.

Subitization is often claimed to be a non-serial (and therefore non-counting) process (e.g., Klahr & Wallace, 1973), or a *canonical pattern*-recognition process rather than a *number*-recognition process per se (e.g., Mandler & Shebo, 1982). However, the mechanism underlying the ability to determine small numerosities without conscious counting has not yet been determined. It is possible that this mechanism works by "counting" in some fashion. In this paper, the term "subitization" is used merely to describe this ability, without making any claim regarding the nature of its underlying process. To emphasize this, the term shall be placed in quotation marks whenever it is used.

### ***Generalizability of Children's Counting***

At issue in the above debate is the very young child's mental representation of the counting routine. If it is represented in terms of abstract components, such as the counting principles, as the Principles-Before theory states, then it can potentially be generalized to a wide variety of dissimilar tasks. If, however, it is represented as one or more context-specific procedures, in terms of a series of precise, concrete steps as the Principles-After theory states, then it will not be widely generalizable because different tasks require different procedures. Examining the extent to which children can generalize their counting to new contexts can therefore shed light on the form of representation of the counting routine.

It has been shown that children as young as 3-and-a-half can catch and correct genuine errors that a puppet makes in counting, in which the puppet violates one-to-one correspondence, fails to use the standard stably-ordered count list, or gives a number other than the last word used when asked, after a count, how many objects there are. More interestingly, they can distinguish these genuine errors from unusual but correct counts by the puppet which, while differing significantly from a 'normal' counting routine, remain true to the counting principles (e.g., starting a count in the middle of an array of objects, proceeding to the end of the array, then going back and counting the remaining objects; or counting every alternate object, then turning around and counting back the other way, getting the ones missed) (Gelman & Meck, 1983; Gelman et al., 1986). Thus children are applying knowledge of counting to these apparently novel counting situations. However, these results have not been consistently obtained (Briars & Siegler, 1984). Furthermore, although the "correct but unusual" ways to count were chosen to be novel to the children, it is quite possible that children may have seen things counted out of order at one time or another. In fact, the counting contexts in these experiments were quite similar to those children encounter when being shown how to count; they all consisted of counting objects arranged linearly, of roughly the same size and proximal distance, with merely the order of counting altered. Finally, the mean or median age of the younger children in these experiments was over 3-and-a-half. Some children start learning to count as early as their second birthday. It is therefore possible that these experiments reflect knowledge *learned* about counting rather than knowledge *underlying* the learning of counting.

As a more stringent test of children's counting abilities under novel circumstances, consider all the types of entities other than objects that are countable: sounds, actions, abstract entities such as thoughts or mental representations, properties of objects, etc. There have been only a few studies

of children's counting of entities other than objects. To determine how much of the list of number words their subjects could recite, Schaeffer et al. (1974) asked children to count to taps of a drum, saying a number word in time with each tap, for up to 10 taps. If the child increased the pace of saying the numbers, the experimenter increased the pace of drum tapping. The mean number of taps that their younger two groups of children (mean ages 3:5 and 3:8) counted to was about 6.5. In comparison, these children counted sets of 1 to 4 objects correctly about 74% of the time, and sets of 5 to 7 objects correctly 43% of the time. They thus appear to count drum taps and objects about equally well, which suggests that they have an abstract, generalizable representation of counting. However, it is not clear what children considered the task to be, given that they were asked to count *with* the drum taps -- were they actually counting the drum taps, or were they simply reciting a list in time to the beat of the drum?

In another set of experiments, children's counting of parts and properties of objects was examined (Shipley & Shepperson, 1990). When some of the objects in the array of, e.g., cars, to be counted were cut in half, children under about 5 years of age, when asked to count the cars, tended to count each individual item rather than each complete car. When children (mean age 3:11) were asked to count properties of objects (the number of different colors, sizes, or kinds of objects), they instead predominantly counted the individual objects, though performance improved with tutoring. Children thus showed a strong bias to count physically separate entities and/or discrete whole objects rather than parts or properties of objects. This could be because children's counting routine is usually applied to separate discrete objects, and children may represent this as an integral component to the counting routine and are therefore unable to generalize their counting to entities other than separate objects. Alternatively, it could be that young children have a general bias to interact with discrete physical objects that is

neither limited to nor derived from counting. Further experiments (Shipley & Shepperson, 1990) support the hypothesis that children have a general discrete object bias, so children's poor counting in these experiments is not necessarily due to lack of an abstract, generalizable representation of the counting routine.

It would be interesting to see whether 2- and 3-year-old children can generalize their counting to novel situations, by comparing children's counting of objects (a familiar counting context) with their counting of actions and sounds (novel counting contexts). The counting of, e.g., rings of a bell is different from counting objects arranged in a row in several important respects. Objects have material existence and can be seen, touched, and pointed to, while sounds are not visually but aurally accessible. Objects have continuous existence and are separated from each other in space, while sounds have a momentary existence and are separated from each other in time; one can therefore choose the order in which to count objects in a way that one cannot for sounds, which are already temporally ordered. Perhaps most importantly, one has perceptual access to the entire collection of objects to be counted simultaneously, while one has perceptual access to only one element at a time of the collection of sounds to be counted, and cannot anticipate the final element. Thus the procedural requirements for counting sounds (and actions) are very different from those for counting objects. If children can count sounds, then, they must be representing the counting routine at a relatively abstract level.

### *Unique Status of the Cardinal Word Principle*

However, even if it turns out that young children do have an abstract representation of the counting routine that honors principles of counting, they may not understand the relationship between counting and numerosity.

Children may be capable of representing some abstract components of the counting routine, such as one-to-one correspondence, without having any knowledge of number. Children as young as 2 years old honor one-to-one correspondence in many situations -- they can point to each picture on a page exactly once when asked to (Potter & Levy, 1968), can give one cookie to each person in the room, can name each person in a photograph exactly once while pointing, e.g., "Mummy, Daddy, me!", can put one sock on each foot or one spoon in each dish, can learn turn-taking games, etc. In some sense it appears that children *know* that they must point to each picture on the page exactly once; that they must name everybody, and that once is enough; and that everyone must get exactly one cookie. As these kinds of tasks vary so widely and occur so frequently in children's activities, it is very likely that children have a general cognitive ability to recognize and represent a one-to-one correspondence between two sets of entities. Similarly, children exhibit sensitivity to a stable ordering of entities every time they recite the alphabet, learn a stable ordering of actions such as, say, a game of patty-cake, learn the list of the days of the week or the months of the year, or learn a nursery rhyme. Children are capable of easily and quickly recognizing and representing stable orderings of words or events at a very young age. Thus, studying whether young children represent the one-to-one correspondence and stable-order principles in counting will distinguish between the Principles-Before theory and the Principles-After theory only if it turns out that children do *not* represent them; if children do represent these principles as part of counting, it could be either because they have unlearned knowledge of counting, or because of their ability to recognize and represent these principles as components of many activities.

The cardinal word principle, however, is qualitatively different from the other two How-to-count principles. It is relevant only to counting, not to other activities; this is true by definition of the cardinal word principle. An

understanding of the cardinal word principle depends on an understanding of the significance of the counting activity, that counting determines *numerosity*. Thus the ultimate test of the two theories must concentrate on whether very young children do in fact understand the cardinal word principle. It is entirely possible that young children mentally represent one-to-one correspondence as part of the counting routine, and know the correct order of the counting word list, but do not at first connect this routine with any concept of numerosity. If children understand the cardinal word principle, however, they must be granted number-specific knowledge of counting, *by definition* of the cardinal word principle.

It has been considered evidence of possession of the cardinal word principle (Gelman & Gallistel, 1978) if children (a) emphasize the last word used in a count, (b) repeat the last word used in a count, (c) state the correct numerosity of a set without counting after that set has been counted earlier, or (d) respond with the correct number word without counting when asked how many items there are. It has been found that most children as young as 2-and-a-half display one or more of these behaviors at least sometimes when counting sets of 2 or 3 items (Gelman & Gallistel, 1978). However, repeating or emphasizing the last word, or stating the correct numerosity without counting after the set has been counted previously, could result simply from imitating adult counters, who tend to emphasize, repeat, and otherwise direct attention to the last word when instructing children in counting. Emphasis of the last word could also simply be signalling the end of the routine. Just giving the correct number word when asked how many items there are could be the result of "subitization", which does not require an understanding of the cardinal word principle (e.g., Fuson & Hall, 1983). Also, children often produce wrong number words when asked how many items there are. Given that the sets were of only 2 or 3 items, and that children are more familiar with number words that refer to small numbers

and are therefore more likely to produce them, if children were responding with random number words one would expect that occasionally they would respond with the right one. Thus none of these behaviors is a clear indication of possession of the cardinal word principle.

Some studies, however, do offer more conclusive evidence of children's possession of the cardinal word principle. In one experiment, children were shown cards with different numbers of items on them and asked how many things there were on each card. They used words for larger numbers to describe larger numerosities, even if the number of items on a card was too large (up to 19 items) or the exposure time of the card too short (as little as 1 second) for the child to accurately determine the numerosity (Gelman & Tucker, 1975). These children thus apparently understood that a number word's position in the number word list is related to the number of items that word refers to, which suggests an understanding of the cardinal word principle. It has also been shown that children can correct a puppet who gives a response other than the last number word used in a count when asked, after the count, how many items there are (Gelman & Meck, 1983). In another experiment, a puppet counted a set of items twice and was asked "how many" after each count. Both times, the puppet gave the last word used for that count, but in the second count it had surreptitiously made an error, so that the last word differed in the two counts. Most of the children said the puppet's second response was wrong, even though they had not caught the puppet's surreptitious error. When asked to justify their judgement, children often expressed the belief that the answer should still be the same as the puppet gave the first time. This suggests that they understand that the last word represents the cardinality (Gelman et al., 1986). However, the youngest children in each of these three experiments was over 3 years old (median age 3:7, median age 3:7, and mean age 3:8, respectively), so all that can safely be concluded from these studies is that by roughly age



3-and-a-half, children understand the cardinal word principle.

As evidence for the claim that children do *not* start out with an understanding of the cardinal word principle, children were asked to put 1 to 7 candies in a cup from a large pile of candies, and to tap a drum 1 to 7 times (Schaeffer et al., 1974). It was found that the youngest two age groups (mean ages 3:5 and 3:8) gave the correct number of candies only about 45% of the time, and the correct number of drum taps only about 25% of the time. Children were especially poor on the larger numbers, and did not in general count aloud while responding on any of the trials. This suggests that they may have been "subitizing" to obtain the correct number when giving the correct amount, rather than using the cardinal word principle. However, it is not clear exactly what children's strategies were -- they may have been estimating the number of candies they took or drum taps they made, and just not getting the exact number. Also, these same groups of children in another task *on average* only recited the number word list to "five" or "six". Thus many of the children were sometimes being asked for more candies or taps than they could reliably count to, so one would expect them to do poorly on these numbers. These results therefore do not clearly indicate whether or not children possess the cardinal word principle.

Other evidence that children do not possess the cardinal word principle comes from a study by Fuson and Mierkiewicz (1980), who found that many children will recount a set of objects when asked "how many" following a count, rather than repeating the last word used in the count. However, children may simply view the question as a request to count, whether or not they understand the cardinal word principle. After all, one way of asking a child to count a set of things in the first place is to ask how many there are. To test this hypothesis, Schaeffer et al. (1974) asked children "how many" after covering up the set of objects the child had just counted, thus preventing the child from recounting the set. They found that most

3-year-olds did not respond with the last word used in the count, suggesting that they lacked the cardinal word principle. However, asking a "how many" question just after a child has counted a set may be pragmatically strange; after all, the child has *already* indicated how many there are, by counting. Children may take the question as an indication that their first result was wrong, and change their answer. Furthermore, hiding the objects and then asking "how many" may seem strange to the child; if the adult wants to know how many there are, why cover them up?

Three experiments were conducted to examine the generalizability of children's counting and their understanding of the cardinal word principle. Experiment 1 studies the generalizability of children's counting routine by comparing children's counting of objects, actions, and sounds. Experiments 1, 2, and 3 study children's understanding of the cardinal word principle. In Experiment 1, children's responses to "How Many?" questions following the counting of objects, actions, and sounds are examined. In Experiment 2, the same children are tested for the cardinal word principle in a different way, to rule out alternative explanations for children's performance in Experiment 1. Experiment 3 examines how children learn the meanings of the number words, and the relationship between their understanding of number words and their understanding of the cardinal word principle, using a variation of the task given in Experiment 2.

### **Experiment 1: The "Novel Entities" Study**

In this experiment, children's performance in the counting of objects (a familiar counting context) is compared to their performance in the counting of actions and sounds (novel counting contexts). Having children count objects, actions and sounds also allows an interesting test of their understanding of the cardinal word principle; children cannot construe a

"how many" question about a set of sounds or actions just counted as a request to recount, because the set is no longer available. To reduce the pragmatic effect of asking "how many" right after the child has just counted, children were introduced to a puppet who "had forgotten how to count", and were encouraged to help show the puppet how. The assumption is that the question is more natural when the child is in the role of teacher to an ignorant puppet.

## *Method*

### *Subjects*

Subjects were 8 2-and-a-half-year-olds (mean 2:7; range 2:4 - 2:8), 8 3-year-olds (mean 3:0; range 2:10 - 3:2), and 8 3-and-a-half-year-olds (mean 3:5; range 3:4 - 3:7), labelled Age I, Age II, and Age III, respectively. There were roughly equal numbers of girls and boys in each group. They were from day-care centers in the greater Boston area with a predominantly middle-class population.

### *Procedure*

There were four conditions; Object, Cave, Jump, and Sound (described below), each with four trials, one each of set sizes 2, 3, 5, and 6. The set sizes 2 and 3 were chosen to be within young children's "subitizing" range, those of 5 and 6 to be outside. The two smaller set size trials were always given first, in counterbalanced order, followed by the trials with set sizes 5 and 6, again counterbalanced. The order of conditions was counterbalanced between children within each age group. Roughly two-thirds of the trials in each condition were randomly designated as "How-many" trials, in which children were asked "how many" after counting. (It was felt that to designate all the trials as such might make the children worry too much about having

to justify their responses.)

- *Object* condition: Children were asked to count objects linearly arranged. Toy dinosaurs were used, about 4 cm long, glued to a board with about 3 cm space between each one. A different kind of dinosaur was used for each trial; within each trial, only one kind of dinosaur was used.
- *Cave* condition: Children were asked to count toy dinosaurs as they were moved from one cardboard box into another box which they could not see inside of, a "dinosaur cave" with a small hole in the lid. For each trial, the experimenter moved one dinosaur at a time from the first box into the cave, at a rate of about 2 seconds per dinosaur. This condition is procedurally similar to the counting of sounds. It thus controls for the possibility that children might perform poorly on the Sound condition not because they haven't the correct procedures at their disposal, but because they do not consider sounds as countable entities. Here, children are still counting objects which they can see and which have a permanent existence.
- *Jump* condition: Children were asked to count the jumps of a puppet (Big Bird from the children's TV show "Sesame Street"). The experimenter made the puppet jump so that each jump took one-half to 1 second, with roughly an additional second between jumps. This condition is more novel than the Cave condition; although children can see the jumps, each jump exists only for an instant in time.
- *Sound* condition: Children were asked to count sounds played on a tape recorder. Four sounds, one per trial, were used: an "elephant" roaring (actually a person's voice); a single-chime

doorbell ringing; a splash in a bathtub; and the beep of a computer. Sounds occurred about 2 seconds apart. This condition is the most novel, as children cannot even see the items they are counting.

Two seconds was used as the time between items in the Cave, Jump, and Sound conditions because in pilot studies it proved to be long enough to allow children to say a number word before the next event occurs, and short enough that children do not tend to forget their place in the list of number words.

At the beginning of the experiment, the child was introduced to Big Bird and told: "Big Bird has a problem. He's forgotten how to count, and he needs someone to show him how. Would you like to help Big Bird, and show him how to count? Can you help him count his toys?" Then the child was presented with the first trial, e.g.: "Look what Big Bird has, some dinosaurs! Can you show Big Bird how to count how many there are?" Children were frequently reminded that Big Bird did not know how to count and needed help. A trial was started over again if a child was obviously distracted in the middle, e.g., if the child started telling a story, ran off, or otherwise interrupted the trial. On the "how many" trials, children were asked after counting, "So how many dinosaurs are there/how many times did it ring?" etc. Sometimes the puppet asked the question; sometimes the experimenter did, often adding "can you tell Big Bird how many?". This was to make it seem that the object of the question was to help inform the ignorant puppet. The experiment was usually conducted over two sessions for each child (sometimes one session); sessions were typically between 1 and 3 days apart. Big Bird's problem was explained at the beginning of each session.

## *Results and Discussion*

### *Counting of novel entities*

In the early stages of counting, many children consistently use an idiosyncratic list of the counting words, e.g., "One, six, seven, eleven" (e.g., Fuson & Mierkiewicz, 1980; Gelman & Gallistel, 1978). The fact that children have such lists is consistent with the claim that children understand the stable-order principle. Even if children start out with the counting principles, they must still *learn* the number word sequence. For this reason, children who used a different list than the standard number word list were not automatically considered to be counting incorrectly. The predominant list used by a child throughout Experiments 1 and 2 was considered that child's stably-ordered list. A set was considered correctly counted if:

- the child started the count with the first number word in his or her own stably-ordered list (for almost all children, the first number word was "one");
- for sets of 3, 5, and 6 items, the trial contained no more than one of the following mistakes: double-counting or skipping an item (one-to-one correspondence mistakes), or skipping or repeating a word in the child's own stably-ordered list (stable-order mistakes);
- for sets of 2 items, the trial contained *no* one-to-one correspondence mistakes, and no more than one stable-order mistake (e.g., if the child's stably-ordered list was the standard one, a count of "one, three" or "one, one" would be considered correct; a count of "one, four" would be considered incorrect).

A 3(Age group) by 4(Condition) by 2(Set Size -- Large or Small) ANOVA

was performed. There was a significant Age effect,  $F(2, 21) = 11.017, p < .001$ . A contrast analysis on the three age groups showed a strong linear trend for older age groups to count more trials correctly (mean number of correct trials out of two was 0.67 for Age I, 1.22 for Age II, and 1.64 for Age III),  $t(21) = 4.681, p < .0001$ , one-tailed. Set Size was also significant (smaller set sizes were easier to count -- the mean for small Set Size was 1.33 and for large Set Size was 1.02),  $F(1, 21) = 13.636, p < .001$ . There was a Condition effect,  $F(3, 63) = 4.684, p < .005$ . It might be asked whether there was a trend for children to perform more poorly the more novel the condition (from Objects to the Cave condition to Jumps to Sounds). This was so. Each child had 2 trials within each set size (Large or Small) for each condition; the mean number of successful trials (out of 2) for each condition (over all children and both set sizes) was 1.42, 1.29, 1.10, and 0.90 respectively. A Newman-Keuls post-hoc means comparison showed that performance in the Object condition and performance in the Cave condition were both significantly better than performance in the Sound condition,  $p < .05$ . There was a marginally significant Age X Size interaction:  $F(2, 21) = 3.045, p = .069$ ; for older children there was less difference in performance on large versus small set sizes. Figure 1 shows the percentage of correctly counted trials out of 32 (4 per condition for 8 children) in each condition for each age group.

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 INSERT FIGURE 1 ABOUT HERE  
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The Condition effect indicates that children are worse at counting in quite novel circumstances. However, even if children do have an abstract counting routine, one would not expect them to perform *equally* in both familiar and novel contexts. Practice in applying the counting routine in different contexts will improve performance. For instance, children who have counted objects but not sounds will have had practice coordinating their words with their pointing, but will not have had practice coordinating

their words with entities that occur one after another in time. Thus, if children have an abstract, generalizable counting routine, they ought to apply it in novel contexts, but not necessarily with equal facility.

Almost all of the younger children could count the non-Objects to some extent. Only the 3 youngest children (mean age 2:5) failed to count at all in any of the non-Object trials. They seemed not to understand what was being asked of them, appearing bewildered, even though they counted and appeared comfortable during the Object condition. All of the other children counted for the non-Object trials, and all except one child counted at least 25% of the non-Object trials correctly. The percentage of correctly counted non-Object trials increases steadily with age (the 5 Age I children who counted on the non-Object trials had a mean of 38% correct non-Object trials per child; Age II children's mean was 60% per child; Age III children's mean was 79% per child). This suggests that very early on, children begin to develop an abstract, generalizable representation of the counting routine that they can apply to new counting situations. However, the fact that the 3 youngest children did not count at all for the non-Object trials suggests that children may not start out with a generalizable counting routine. But what about those children who counted at least some of the novel entities correctly, thus showing that they have developed at least the beginnings of an abstract mental representation of counting? This representation may or may not encompass an understanding of the significance of counting -- that counting determines the numerosity of a set. The "How-many" test for the cardinal word principle addresses this question.

*"How-many" cardinal word task*

If children understand the cardinal word principle, they should repeat the last number word used in a count when asked, after counting, how many items there are. There was no effect of set size on performance for any of the age groups; children were equally likely to give a cardinality (last-word)



response following the counting of a Large set size (35% overall) as a Small set size (29% overall). Therefore all results are collapsed over set size. Table 1 shows the overall proportions of "how many" responses where children repeated the last word used in counting, and the proportion of responses in the Object condition where children recounted (indicated in square brackets). The overall proportion of cardinality responses for all the non-Object conditions combined is also given. Types of responses not represented in Table 1 include, for example, saying another number word, saying several number words, saying nothing, etc.

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INSERT TABLE 1 ABOUT HERE

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Children in the younger two age groups, when asked "how many" after counting a set of physical objects, preferred recounting the set to saying the last word used in the count. In the non-Object conditions, when the opportunity to recount was not available, children gave more cardinality responses than in the Object condition, when they could recount. This makes sense; because the recounting response is no longer possible, the likelihood of other responses must go up. Yet even in the non-Object conditions, the younger two groups of children give cardinality responses only about one-quarter to one-fifth of the time, in contrast to the Age III children, who give cardinality responses well over half the time. This suggests that the older children may possess the cardinal word principle while the younger children may not.

It is helpful to look at individual data. For each child, the percentage of responses to "how many" questions which were cardinality responses was determined. The mean of individual children's percentages of cardinality responses was 20% for Age I, 21% for Age II, and 52% for Age III. There was a significant difference between the Age I and Age II children's mean percentages and the Age III children's ( $t(22) = 2.836$ ,  $p < .005$ , one-tailed).

(The same result is obtained when considering individuals' percentages for non-Object trials only; the means for Age I (35%) and Age II (24%) differ significantly from that of Age III (56%),  $t(20) = 1.923$ ,  $p < .05$ , one-tailed.) It could be asked why the younger children give any cardinality responses at all, if they do not understand the cardinal word principle. But since children generally do know to give a number word in response to a "how many" question, one would expect them to sometimes give the correct number by chance, especially since the last word in the count will be the one most recent in memory. Given this, the above results suggest that children learn the cardinal word principle at about 3-and-a-half years of age, and therefore that the younger children's ability to generalize their counting routine to the non-Object conditions is due to learned knowledge of counting rather than to innate counting principles.

It could be objected that perhaps children learn by observing adults that the correct response to a "how many" question is just to repeat the last word used, without understanding that this word refers to the numerosity of the set; older children may have just learned this better than younger children. This account would explain the results without having to credit any of the children with an understanding of the cardinal word principle. However, there is a wealth of independent evidence that 3-and-a-half-year-olds do understand the cardinal word principle (e.g., Gelman & Meck, 1983; Gelman et al, 1986; Gelman & Tucker, 1975).

Alternatively, it may be that even the younger children understand that counting is about determining the cardinality of a set, but simply do not understand that the question "how many" is asking about cardinality. There is a large body of research indicating that young children do not understand until quite late terms such as "less", "more", "the same", and other quantifiers (e.g., Carey, 1982; Clark & Clark, 1977; Donaldson & Balfour, 1968; Palermo, 1973), and it is plausible that they also have trouble with "many" and/or

"how many". Furthermore, although the experiment was designed to reduce the possible pragmatic effects of asking how many items there were immediately after the children have just counted them, this might not have been completely successful. Experiment 2 was designed to eliminate both of these concerns.

## Experiment 2: The "Give-a-Number" Study

### *Method*

#### *Subjects*

Subjects were the same 24 children who participated in Experiment 1. They were given the Give-a-number task at the end of their final session of Experiment 1. Two children, one in Age I and one in Age II, did not want to finish this task and were dropped from Experiment 2, leaving 22 subjects.

#### *Procedure*

After the final condition of Experiment 1, 15 toy dinosaurs were placed in front of the child in a pile, and the puppet and child began to play with them. The child was then asked to give the puppet a certain number of dinosaurs. Children were first asked to give 1, then 2 and 3 in counterbalanced order, and then 5 and 6, also counterbalanced. The request was of the form: "Could you give Big Bird *two* dinosaurs to play with, just give him *two* and put them here (experimenter pats a place in front of Big Bird, within easy reach of the child), can you get *two* dinosaurs for Big Bird?" The request was repeated until the child responded (usually children responded right away).

After responding, children were given "follow-up" questions. They were asked to "check and make sure" that they'd given the correct number, and were reminded how many had been asked for. Any child who did not

spontaneously count the objects was prompted to count them, e.g. "Can you count and make sure there are two?" Children who counted and obtained a different number than what they had been asked for were then prompted, e.g. "But Big Bird wanted two. How can we make it so there's two? How can we fix it so that Big Bird has two?" Children's further responses were followed up until the child seemed to be getting bored or uncomfortable.

### *Results and Discussion*

Children were divided into two groups on the basis of the initial strategies they used. "Counters" were those who, on at least four of the five trials, responded in one of the following three ways:

- They counted the items as they gave them, using their own stably-ordered list, and stopped at the number word asked for (they were allowed to make a single one-to-one correspondence mistake when counting);
- They silently gave the correct number of items *one by one*. What was considered the "correct" number of items was determined by the child's own stably-ordered list. For example, if a child regularly counted "one, three, four, five, six", then, when asked for "three dinosaurs", a correct response would be to give 2 one at a time; when asked for "six dinosaurs", a correct response would be to give 5 one at a time, as the word "six" is the fifth word in the child's list;
- They spontaneously counted the items they had grabbed and, if necessary, adjusted them (within plus or minus one) to the number asked for, according to their own stably-ordered list.

All other children were "Grabbers", and their strategy was generally to grab and give a handful all at once, or, occasionally, to give some other number of items silently one at a time.

The results are divided into three sections. First, children's initial performance on the Give-a-number task is analyzed. Second, the Grabbers' performance in Experiment 1 is compared with that of the Counters. Finally, children's responses to the follow-up questions are examined.

*Initial performance on Give-a-number task*

There were 4 Counters, all in Age III. All used the standard count word list. Every Counter gave a correct response on all five trials. In three cases Counters, when checking how many items they had given, counted a number different than that asked for; in all of these cases they either added or took away items as appropriate, and then counted again, repeating if necessary until they counted the number asked for. Thus the Counters showed a clear understanding of the cardinal word principle.

All of the children in the younger two age groups, and half those in the oldest age group, were Grabbers, and tended either to simply grab a handful of items and give them all, or to give an apparently random number of items one at a time. However, it is possible that despite their poor strategy, Grabbers could be grabbing the number asked for. Table 2 shows the number of Grabbers who gave each number when asked for 1, 2, 3, 5, and 6 items. There were only 3 Grabbers whose stably-ordered lists differed from the standard list. Two of them alternated between the correct word list, and that list with one word omitted ("two" was omitted half the time in one case, "five" in the other). On no trial did either child give a number of items that would be correct according to their list with the omission – their responses were either the correct number according to the standard list, or incorrect by both lists. The third child counted correctly up to five, but had no consistent list beyond that. He gave 5 items when asked for 6 (and 8 items when asked

for 5). Thus, these children's data do not distort the results in Table 2.

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INSERT TABLE 2 ABOUT HERE

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All Grabbers gave the correct number when asked for 1 item, while tending not to give 1 when asked for more than 1 item. Thus *one* seems to be a number readily identified by children as young as 2:4. Grabbers also gave 2 more often when asked for 2 than when asked for 3, 5, or 6. However, this was not significant, by a Wilcoxon Signed Ranks test comparing the percentage of time individual Grabbers gave 2 items when asked for 2, with the percentage of time they gave 2 when asked for 3, 5, or 6 items. Grabbers did not give 3, 5, or 6 items more often when asked for one of those numbers than when asked for other numbers.

Furthermore, analyses reveal that the younger two groups of Grabbers are not even approximating the number of items they grab, when asked for 2, 3, 5, or 6 items. The correlation between the number asked for (2, 3, 5, or 6) and the number given is  $r = .19$  (NS) for Age I, and  $r = -.09$  (NS) for Age II. It could be asked, however, whether these correlations mask an ability to distinguish between the lower numbers asked for -- it may be that these children simply fail to distinguish between 5 and 6, considering them both "larger numbers". But this is not the case. The correlations between the number asked for (for the numbers 2, 3, and 5 only) and the number given are still nonsignificant ( $r = .23$  (NS) for Age I, and  $r = -.03$  (NS) for Age II). Thus, these children are not even distinguishing smaller numbers asked for from larger numbers asked for.

The Age III Grabbers' correlation between the number asked for (2, 3, 5, or 6) and the number given is significant,  $r = .61$ ,  $p < .05$ . However, this appears to be due primarily to their responses when asked for 2 items (3 of the 4 Age III Grabbers gave 2 items when asked for 2; the fourth child gave 1

item). The correlation between the number asked for (for the numbers 3, 5, and 6) and the number given is not significant,  $r = .42$  (NS). Table 3 shows the mean number of items initially given by Grabbers and Counters of each age group.

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INSERT TABLE 3 ABOUT HERE

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The problem is not that the children could not count this high. For each Grabber, the largest correctly counted set of objects in the Object condition was determined. The mean largest correct trial for Age I was 4.5 objects, for Age II was 5.0 objects, and for Age III was 5.8 objects. Thus even the youngest group of children can count to four or five accurately, yet they fail even at giving 3 items. Thus, except for 1 and possibly 2 items, Grabbers do not give the number asked for, nor do they appear even to be approximating the number asked for. Even in those cases where Grabbers did give the correct number of items when asked for 2 or 3, they never counted the items aloud as they gave them. This suggests that they were not applying the cardinal word principle to get the right number, but were either "subitizing" the correct number, or getting the right number by chance. This suggests that most children do not understand the cardinal word principle before around 3-and-a-half years of age, contrary to what the Principles-Before theory claims.

#### *Grabbers' vs Counters' performance in Experiment 1*

If it is true that the Counters possess the cardinal word principle while Grabbers do not, this should be reflected in their performance on the How-many task in Experiment 1 -- Counters should give more cardinality responses than Grabbers. This is in fact the case. The mean percentage of individual Grabbers' cardinality responses was only 23%, while the mean percentage of Counters' cardinality responses was 71% ( $t(22) = 3.729$ ,  $p <$

.001, one-tailed). (The same result obtains when considering individuals' percentages for non-Object trials only; the Grabbers' mean (32%) differs significantly from that of the Counters' (69%),  $t(20) = 2.179$ ,  $p < .05$ , one-tailed.) Figure 2 shows the mean of individuals' percentages of cardinality responses given by Counters versus Grabbers on the How-many task in Experiment 1, broken down by age. There is no significant increase with age in the Grabbers' percentages of cardinality responses to "how many" questions. Rather, there is a sudden shift from a cardinality response rate of about one-quarter of the time to a cardinality response rate of almost three-quarters of the time, that coincides with the shift from Grabber to Counter.

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 INSERT FIGURE 2 ABOUT HERE  
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Yet another prediction can be made about Grabbers' versus Counters' performance on the How-many task in Experiment 1. If it is indeed the case that Counters understand the cardinal word principle while Grabbers do not, one would predict that Counters would be less likely to give the last word used in the count in cases where they were uncertain of the accuracy of their counting. Thus they should give cardinality responses less often after incorrect counts than after correct counts. Grabbers, on the other hand, should not understand the relationship between accuracy of counting, and likelihood that the last word in the count represents the cardinality of the set, so there should be no difference in their rate of cardinality responses following correct versus incorrect counts.

This prediction too is borne out by the data. Fourteen of the Grabbers and 3 of the 4 Counters had "how many" responses following both correct and incorrect counts. Figure 3 shows the means of these individuals' percentages of cardinality responses following correct versus incorrect counts, for Counters and Grabbers.



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INSERT FIGURE 3 ABOUT HERE

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Grabbers were actually slightly *less* likely to give cardinality responses after correct counts than after incorrect counts, though this difference is not significant. In comparison, Counters were *three times* as likely to give cardinality responses following correct counts (84% of the time) as they were following incorrect counts (28% of the time),  $t(4) = 3.503$ ,  $p < .05$ , one-tailed. This indicates that Counters understand the relationship between accuracy of counting, and likelihood that the last word in the count indicates the cardinality of the set. Grabbers appear not to appreciate this relationship, further supporting the conclusion that Grabbers do not understand the cardinal word principle while Counters do. These findings provide a firm basis for the claim that children do not understand the cardinal word principle before about 3-and-a-half years of age.

Age III Grabbers and Counters were equally successful at counting the novel entities in Experiment 1. Counters gave correct counts on 83% of the non-Object trials, while Age III Grabbers gave correct counts on 75% of the non-Object trials; this difference does not approach significance. Thus, children appear to acquire considerable skill at counting before understanding that counting determines the numerosity of a set. This is as the Principles-After theory predicts, while going contrary to the predictions of the Principles-Before theory.

*Responses to follow-up questions*

As previously stated, Counters, when prompted to check how many items they had given, always counted them and, if necessary, added or took away items as appropriate. Grabbers, on the other hand, revealed several strategies that reinforce the conclusion that they do not understand the cardinal word principle. In most cases, the experimenter succeeded in

getting Grabbers to count what they'd given. There were several ways they attempted to reconcile the discrepancy between what they had been asked for, and what they counted:

**1) Naming or Tagging the Last Item With the Number Asked For:** Seventeen responses by 9 Grabbers (4 Age I children, 3 Age II children, 2 Age III children) were to count so that a number word said (usually the last one) was the number asked for. For example, a boy who had given 2 items when asked for 6 counted them "one, six!". A girl who gave 2 items when asked for 5 counted them, "five, five!". Three responses by 2 children clearly appeared to be *naming* particular items with the number words, e.g. the following interaction between a child and the experimenter after the child was asked to give 5 objects, and had grabbed and given 3:

Experimenter: So how many are there?

Adam: (Counting the 3 objects which are in a triangular arrangement)  
One, two, five!

E: So there's five here? (pointing towards the 3 items)

A: No, *that's* five (pointing to the item he'd tagged "5"). One, two, five (counting them in the original order).

E: So there are five altogether?

A: No, one, two, five (counting them again in the original order).

E: So does Big Bird have five?

A: Yeah, this is five (pointing to the item always tagged "5" in his counting). One, two, five (counting again, in the same order as before).

E: What if you counted this way, one, two, five? (experimenter counts the objects in a different order than Adam has been doing)

A: No, *this* is five (pointing to the one he has consistently tagged "5").

E: That one's five? (pointing to the one Adam called "five").

A: Yes.

E: Why is this one five?

A: Because, one, two, five (counting them once again, in the same order).

**2) Denial:** Eight responses by 8 Grabbers (2 Age I children, 4 Age II children, 2 Age III children) were to count what they'd given, and deny that there was any discrepancy. For example, one girl gave 3 when asked for 6. When asked to "count and make sure" there were 6, she counted them correctly, saying, "one, two, three. That's six!" This is a direct violation of the cardinal word principle.

**3) Changing the Number of Items:** Eighteen responses by 10 Grabbers (2 Age I children, 7 Age II children, 1 Age III child) were to change the number of the items, in all but one case by adding more to what they'd given. In only 10 of these cases had the children first counted the number of items they'd given or indicated that they believed they'd given an incorrect number (e.g. by saying "that's not five!"). In six of these cases the children changed the number in the right direction; in the other four, the children changed the number in the wrong direction (e.g. giving and counting 4 items after being asked for 3, and then adding another item).

**4) Silence/No Justification:** Twenty-nine responses by 14 Grabbers (5 Age I children, 6 Age II children, 3 Age III children) were to remain silent, or say it was the number asked for without justification, when prompted to check what they'd given.

The first strategy is particularly interesting. It strongly suggests that some children have a sort of "last-word rule". They understand that the last number word is the answer to "how many" there are, but do not understand "how many" in the same way we do. They do not yet have the cardinal word principle, i.e., they do not yet understand that the last word indicates the numerosity of a set, or even that the last word refers to some property of the entire set as a whole rather than to a particular member of the set. They

simply have the heuristic that "the last number word in a count is 'how many' there are". Thus, if counting items the usual way does not end with the correct number word, they 'count' the items in a way that *does* end with the correct number word.

One would expect that if these Grabbers did have a "last-word rule" which states that "the last number word is 'how many' items there are", they should apply it in the How-many task in Experiment 1. In particular, they should respond, more often than the other Grabbers, with the last number word used in the count when asked "how many". The 9 Grabbers who employed Strategy 1 gave cardinality responses an average of 30% of the time, while the other 9 Grabbers gave cardinality responses on average only 16% of the time. This difference is not significant ( $t(16) = 1.206$ ,  $p = .12$ , one-tailed), but is suggestive nonetheless.

However, these 9 Grabbers did not give cardinality responses to the extent that the 4 Counters did. The difference between their mean of 30% and the Counters' mean of 71% is significant,  $t(11) = 2.639$ ,  $p < .05$ , one-tailed. This is consistent with the interpretation that these Grabbers are giving cardinality responses for a different reason than are the Counters, who appear to possess, rather than a meaningless "rule", a *principled* reason for stating the last word when asked "how many".

Fuson (1988) has also proposed that some children have such a "last-word rule", on the basis of several empirical results. For example, when asked to count very large sets, some children gave a very small number as a response to a "how many" question, if that number word was the last said in the count; e.g., a child counted a set of 26 items, "one, two, three, six, seven, eight, nine, one, two." When asked "how many", she responded, "two". In another study, after counting  $n$  soldiers, 2- and 3-year-olds were asked to choose between the last soldier and all the soldiers in response to either the question "Is this the soldier where you said  $n$ ?" or the question "Are these the

$n$  soldiers?". Children tended to choose the last soldier more often than all the soldiers in answer to both questions, often saying things like, e.g., "This one's the five soldiers", while pointing to the last one.

These results indicate that before about 3-and-a-half years of age, children do not understand the cardinal word principle. Grabbers' responses to the Give-a-number task show that they have no understanding that the last number word refers to the *numerosity* of the set, or even to some property of the set as a whole.

The fact that all the children gave 1 item when asked for 1, and most gave 2 when asked for 2, suggests that children learn the meaning of the word "one" very early, followed by the word "two". It may be that in general children learn the meanings of smaller number words before larger ones, even for number words well within their counting range. At some point, children must acquire the meaning of *every* number word -- this is the adult competence. But children may learn the meanings of particular number words before making an induction about the meanings of number words in general. Experiment 3 tests this hypothesis.

### Experiment 3: "Give-a-Number" Follow-up

In this study, children are asked several times for each of a number of animals (1, 2, 3, 5, and 6), so the consistency of individual children's responses for different numerosities is obtained. If children do learn the meanings of smaller number words before those of larger number words within their counting range, then two things should occur. First, individual children should succeed when asked for a number of items up to a certain numerosity, and then fail for all higher numerosities (e.g., there should be no children who consistently succeed at 1 and 3 but not at 2). Second, different children should have different numerosities at which they start to fail. There should be some children who succeed consistently only when asked for 1,

and others who succeed consistently when asked for 1 and 2. Whether there are children who succeed when asked for 1, 2, and 3 items but not more, or even 1, 2, 3, and 5 items but not 6, depends at what point children make the general induction that *all* the counting words (within their counting range) refer to particular numerosities. Once they make this generalization, children should succeed at all the numerosities within their counting range.

This study also examines the relationship between children's understanding of the cardinal word principle and their understanding of the meanings of number words, and tests children's ability to perform some of the number-irrelevant task demands of the Give-a-number task.

## *Method*

### *Subjects*

Subjects were 18 2- and 3-year-olds (mean age 3:3; range 2:4 - 4:0) from schools and day care centers in the Greater Boston area. About half were girls. Five more children were tested but not used as subjects; one had a hearing impairment and had trouble understanding the experimenter, three stopped before the end of the experiment, and the data from one child were lost due to equipment failure.

### *Procedure*

There were three tasks given to the subjects:

- "*Give-a-number*" task: Children were asked to give 1, 2, 3, 5, and 6 animals to the puppet, and were given a sticker to put on a piece of paper after each trial to keep them motivated (each sticker was different). The goal was to determine the maximum numerosity each child could succeed at, so the exact procedure differed for different children. All children were first asked for 1 item and then for 2 items; depending on their success, they were then asked

for 3 items, or asked again for 1 or 2 items. What children were asked for on a trial depended partially on their success in the previous trial. Children who failed on a trial were next asked for a numerosity at which they had previously succeeded. This served two purposes; to determine the consistency of a child's performance on a particular numerosity, and to avoid discouraging children. All children were asked at least twice for 2 and 3 items. The experimenter concentrated on the highest number a child succeeded at reliably and on the next highest number, so children got more trials for these than for other numbers. At some point in the task, however, all children were asked for the larger numerosities at least once (usually twice). The exact number of trials depended largely on a child's willingness to keep playing. The experimenter followed up children's responses by asking questions such as, "Is that three?", "Can you count and make sure?", etc.

- "*Give-some-pigs*" task: This task is a control for some of the number-irrelevant task demands of the Give-a-number task. Children were asked to give the puppet Big Bird some pigs (or other kind of animal) from a pile containing four kinds of animals, from 4 to 10 of each kind. The kinds of animals were easily recognized and named even by 2-year-olds (pigs, dogs, dinosaurs, horses). Many of the same task demands are present in the Give-a-number task: children must (a) construct a subset of the entire pile of animals, and (b) give that subset to the experimenter. No particular animal need be included in the subset for either task; for example, any three animals are okay for the Give-a-number task, and any of the pigs are okay for the Give-some-pigs task. Children who succeed at the Give-some-pigs task but fail at the Give-a-number task cannot be

failing from an inability to construct a subset and give it to someone. They must be failing due to an inability to construct a set of a certain numerosity.

- "Count/How-many" task: Children were asked to count 3, 2, 5, and 6 linearly arranged items (the same stimuli used in the Object condition in Experiment 1), in that order. After counting each set, children were asked how many items there were.

All children received the Give-a-number task last. About half the children received the Give-some-pigs task first, half the Count/How-many task first. When given the Count/How-many task, children were told that Big Bird had "forgotten how to count" and were asked to help him count his toys. At the beginning of the Give-a-number task, children were first given a sticker to put on a piece of paper, and then told that: "The way this game goes is that Big Bird is going to ask you for a certain number of animals, and when you give him the right number, you get another sticker. So it will go like this: Big Bird is going to say, 'Can you give me *one* animal?'" This was the child's first trial. The experimenter repeated the question in different ways until the child gave one or more animals. Though told they would be given a sticker after giving "the right number", children were actually given a sticker after every trial.

## ***Results and Discussion***

Only 1 child (age 2:4) failed at the Give-some-pigs task. He also failed to count correctly on any of the four Count/How-many trials, and failed on all numerosities in the Give-a-number task. This child is not included in the following analyses. All of the other children carefully picked out some or all members of the kind of animal asked for in the Give-some-pigs task; they



never just grabbed a handful of animals.

For each of the 17 remaining children, the numerosities at which they succeeded consistently in the Give-a-number task were determined. The criterion for "consistent success" on a certain numerosity was as follows:

- On at least two-thirds of a child's trials for that numerosity, the child's final response was either the correct number according to his or her own stably-ordered count list, or the correct number plus or minus one if the child had counted aloud from the pile to the number word asked for, but had erred in the counting by either double-counting or skipping one item. Two-thirds was chosen as the criterion for success because many children were given three trials of a particular numerosity, so it is a natural cut-off point. Children's final responses were used rather than their initial responses because children occasionally corrected a wrong response, but rarely changed a response that was initially correct.
- The child responded with that number when asked for other numerosities no more than half as often, percentage-wise, as he or she did when asked for that number itself. For example, a child who gave 2 items 80% of the time when asked for 2, was scored consistently correct on 2 only if he or she gave 2 items no more than 40% of the time when asked for 1, 3, 5, and 6 items. This was to prevent children who had a preference for giving, e.g., 2 items no matter what they were asked for, from being considered to know the meaning of the word "two".

Children fell into five groups according to the numerosities at which they succeeded. Table 4 shows the number and ages of children in each group, and how high the children in each group could count (determined by

averaging children's highest correct counts in the Count/How-many task). The criterion for a correct count in the Count/How-many task was the same as that used in Experiment 1: Children had to start the count with the first element in their own stably-ordered list, and were allowed a single one-to-one correspondence or stable-order mistake on sets of 3, 5, and 6 items, and a single stable-order mistake on sets of 2 items.

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INSERT TABLE 4 ABOUT HERE

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It can be seen that each child succeeded up to a certain numerosity, and then failed for all higher ones.<sup>2</sup> Children's failures are not a result of not being able to count that high. All of the 10 children that failed at some numerosities could correctly count set sizes larger than they could correctly give when asked. (Only one child had a count word list differing from the standard count list; she omitted the word "four" from her list. She did not succeed for any of the numerosities, and when asked for "five" and "six" items, did not give 4 and 5 respectively, which would be correct by her list, but gave 1 item.) These results support the hypothesis that children learn the meanings of smaller number words before those of larger ones, even when they use those larger words capably in counting.

The pattern of children's ages also supports this hypothesis. Children who succeeded at larger numerosities are in general older than those who succeeded only at smaller numerosities. The correlation between children's

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<sup>2</sup>Three children, after giving 1 item when asked for 1, were asked to "count to make sure". Because it is an odd request to make for 1 item, children's responses to this question were not included in the analysis. If responses to this question are included, then 1 child who was successful at 2 was not consistently correct when asked for 1; for two trials she gave 1, and upon being asked to count it she added some more items and counted them all. On the other three trials in which she was asked for 1, her response was to give 1. If this child is considered as going against the hypothesis, then a total of 16 out of 17 children performed in the expected manner -- still a highly significant result.

ages in months and the highest numerosity they succeeded at is  $r = .64$  ( $t(15) = 3.218$ ,  $p < .005$ , one-tailed). Thus children appear to learn the meanings of the number words one at a time, for progressively larger numbers. However, this pattern of learning does not continue indefinitely. All 7 children who succeeded at giving 5 items also succeeded at giving 6 items, while 4 children succeeded at giving 3 items but not more. This suggests that by the time children learn the meaning of the word "five", but after they have learned the meaning of "three", they acquire the meanings of *all* the number words within their counting range.

In order to further examine children's understanding of the cardinal word principle, and its relationship to their understanding of the meanings of number words, children were divided into Counters and Grabbers on the basis of their strategies in the Give-a-number task when asked for 3 or more items. (For 2 items, many children just gave 1 in each hand, and it was impossible to determine whether they had counted them silently or not; for 1 item, almost all children just gave 1.) "Counting strategies" were to count the items while giving them and stop at the number word asked for, to silently give the correct number of items (determined by children's own stably-ordered lists) one by one, or to spontaneously count what was given and correct it if necessary to within plus or minus one of the correct number.

There was a bimodal distribution in children according to how often they performed Counting strategies. Ten of the children (mean age 3:1; range 2:7 - 3:8) applied a Counting strategy on 0% to 38% of their trials (the mean was 13%). These children were classified as Grabbers. The other 7 children (mean age 3:7; range 2:11 - 4:0) applied Counting strategies on 86% to 100% of their trials (the mean was 96%). These children were the Counters. A  $t$ -test on Grabbers' versus Counters' individual percentages of Counting strategies was significant,  $t(15) = 13.019$ ,  $p < .0001$ , two-tailed. The difference in the ages (in months) of children in the two groups was also significant

( $t(15) = 2.366, p < .05$ , one-tailed).

This sharp division of children according to their strategies, with a dramatic increase in the use of Counting strategies, indicates a sudden shift in children's approach to the task. This shift appears to occur at roughly 3-and-a-half years of age on average, though it varies from child to child (the oldest Grabber was 3:8 while the youngest Counter was 2:11). The 2 children who consistently succeeded at giving 2 items did not tend to count out items aloud from the pile when giving 2, and the 4 children who consistently succeeded at giving 3 items did not tend to count out items aloud from the pile when giving 2 or 3. These children did so, on average, only 6% of the time. This suggests that their strategy was to "subitize" in order to give the right number. In contrast, the 7 children consistently successful at giving all numbers of items, counted items aloud from the pile when asked for 2 or 3 items, on average, 45% of the time ( $t(11) = 2.666, p < .05$ , one-tailed). Thus, children appear to be abandoning one successful strategy for giving 2 or 3 items in favor of another. This suggests that a major conceptual change occurs in children's understanding of counting at this time.<sup>3</sup>

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<sup>3</sup>All the strategies employed by Grabbers in the Give-a-number task in Experiment 2 were also used by Grabbers in Experiment 3. Many children counted so that the last number word said was the number asked for, and some children named an item the number asked for. For example, one boy gave 2 when asked for 6. When asked to count them, he pointed to each of them while saying, "One, this is six". The experimenter then asked him "How many are there altogether?", to which he replied, pointing, "This is six, and this is six. They're *both* sixes!" Some children tried to "fix it" in new ways. One girl gave 2 when asked for 6. She then counted them "six, six". When asked to count them "the normal way", she counted them correctly, "one, two". She was then asked, "How can we fix it so there's six?" She picked them up, turning them around and switching their places, and put them back down saying, "Maybe this way". Some children, when asked to "fix it" to the correct number, used what could be called "magic" strategies. A boy who had given 4 when asked for 6, picked up a toy dog and touched each of the 4 animals with it, saying, "There! The dog fixed it!". Another child "tickled" each of 3 animals to "make it 5". Thus, Grabbers again show that they do not understand that counting determines the numerosity of a set. The children in these examples also did not appear to understand that they had been asked for a certain *number* of items; hence, the number-irrelevant operations they performed in efforts to fix what they'd given to what had been asked for.

There was again a relationship between being a Counter, and giving cardinality responses when asked "how many" following counting. Counters gave cardinality responses an average of 61%<sup>4</sup> of the time on the 4 Count/How-many trials, while Grabbers gave cardinality responses an average of 17% of the time ( $t(15) = 2.581, p < .05$ , one-tailed). Since there were so few incorrect counts, a comparison of Grabbers' and Counters' responses following correct versus incorrect counts could not be made.

The 7 children who were consistently correct on all the numerosities in the Give-a-number task, and thus appear to have learned the meanings of all the number words within their counting range, are all Counters. They are thus the only children who were clearly applying the cardinal word principle to obtain the number of items asked for. This suggests that children's acquisition of the cardinal word principle is intimately linked with their acquisition of the meanings of the number words beyond their "subitizing" range. It would not have to be this way. Children could learn the cardinal word principle first for those numerosities whose number words they know the meanings of, before making a general induction over these instances about the meanings of higher number words. If that were so, it would be expected that some of the children who only succeeded on numerosities of 3 or less would have counted items out aloud from the pile to obtain the correct number, when asked for 2 or 3 items. The fact that this did not occur suggests that children's acquisition of the cardinal word principle helps them to immediately acquire the meanings of *all* the number words.

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<sup>4</sup>One of the Counters appeared not to be paying attention to the task, and gave no cardinality responses; when retested at the end of the experiment, she gave 100% cardinality responses. If her later responses are used in the calculation rather than her initial ones, the Counters' percentage of cardinality responses rises to an average of 75% of the time.

## General Discussion

In this section I will consider two main issues. First, I discuss the results of this paper regarding children's understanding that counting establishes numerosity of a set, and what this means for the Principles-Before theory. Second, I discuss the possibility that, contrary to typical instantiations of the Principles-After theory, there may be some unlearned abilities and concepts underlying children's ability to learn to count. I review findings that suggest such abilities, and relate them to the results of this paper.

### *Children's Knowledge of Counting and Numerosity*

Results of the How-many and Give-a-number tasks strongly suggest that most children do not understand the cardinal word principle, or the relationship between counting and numerosity, until about 3-and-a-half years of age. These results suggest that the Principles-Before theory is incorrect -- children do not start out with a set of principles which guide their counting behavior and constitute an understanding of the significance of counting.

The conclusion that children do not understand the cardinal word principle until about 3-and-a-half may at first appear to conflict with conclusions from other studies (e.g., Gelman & Gallistel, 1978; Gelman & Meck, 1983; Gelman et al., 1986; Gelman & Tucker, 1975). However, the youngest children tested in these experiments were over 3 years old (mean or median ages were over 3-and-a-half), so their results are consistent with the claim that most children learn the cardinal word principle at roughly 3-and-a-half years of age. Results from the Schaeffer et al. (1974) study, in which children were asked to put 1 to 7 candies in a cup and to tap a drum 1 to 7 times, support the conclusion that children learn the cardinal word principle some time between their third and fourth birthdays. While the first

two groups of children (mean ages 3:5 and 3:8) succeeded on the candy placement about 45% of the time and on the drum tapping about 25% of the time, these numbers for the third group of children (mean age 4:2) are 87% and 75% respectively, a dramatic improvement.

It could be argued that the poor performance of the younger children on the Give-a-number tasks reflects performance demands, rather than competence. There are several ways children can fail at a task. They can fail due to lack of conceptual understanding (in this case, lack of knowledge of the cardinal word principle). They can fail even if they have conceptual competence, by not having learned appropriate procedures that instantiate their conceptual competence in a particular context. Different counting situations require different counting procedures in order to honor the counting principles. The counting principles are not themselves procedures; children must learn appropriate procedures for different situations. Finally, even if children have at their disposal procedures which are appropriate to a particular counting context, children might not know which of their procedures to utilize, and fail at the task (see Greeno et al., 1984, for a detailed discussion of what they term conceptual, procedural, and utilizational competences; see also Gelman & Greeno, 1987).

However, there is strong support for the claim that children's failure in the Give-a-number tasks is due to lack of knowledge of the cardinal word principle. This comes from the finding that success or failure in this task is a good predictor of several things: (a) whether a child will respond a majority of the time with the last number word used in a count when asked "how many" following counting; (b) whether a child will give the last number word more often after correct than incorrect counts when asked "how many"; and (c) whether a child will tend to count out items aloud from a pile, when asked for a number that she or he is generally successful at giving. The How-many task is procedurally very different from the Give-a-number task,

so it is unlikely that there is a procedural requirement common to both tasks with which the Grabbers were having difficulty. It is at the conceptual level that the two tasks are similar, and therefore children's failure is almost certainly due to lack of conceptual competence.

The general conclusion that young children do not have unlearned knowledge of counting at their disposal also may seem to conflict with studies suggesting that children do represent one-to-one correspondence as a component of the counting routine, and are sensitive to the stable ordering of the counting words. Children will say that a puppet has counted wrong and will often correct the puppet, when it violates one-to-one correspondence in some way (Gelman & Meck, 1983). Again, however, the median age of the youngest children for which this has been shown was over 3-and-a-half, so these results could reflect knowledge learned about counting rather than knowledge underlying counting. Furthermore, as noted above, given that young children appear to represent one-to-one correspondence as a component of so many of their daily tasks, it is plausible that they have a general ability to quickly recognize when it is part of an activity. Showing that very young children represent one-to-one correspondence as a part of the counting routine, then, does not by itself show that they did not *learn* this knowledge. It has also been found (Fuson & Mierkiewicz, 1980; Gelman & Gallistel, 1978) that children as young as 2-and-a-half use consistently ordered lists of number words when counting, even though these lists may not follow the standard order of the counting words (e.g., a child may consistently count, "one, two, six, eight, eleventeen."). This indicates that children are sensitive to the fact that counting uses a stably ordered list of words. However, as argued above, children of this age are sensitive to many other stable orderings as well, such as the alphabet. There is no evidence that children represent the (necessary) stable-ordering of the counting words any differently than the (arbitrary and nonessential) ordering of the letters of the



alphabet.

However, the results in this paper do not preclude there still being some role for an innate representation of the principles, *other* than guiding the acquisition of counting skills. For example, it is possible that the "subitization" mechanism works by counting in some fashion, and it may embody some or all of the counting principles. The ability to discriminate small numerosities, in turn, may underlie children's ability to learn that counting determines numerosity. But this would be a very different kind of role for the principles than that posited by the Principles-Before theory. Children's acquisition of skills in counting is not guided by the counting principles. Rather, children learn how to count.

### *Unlearned Components Underlying Counting and Number*

The Principles-After theory is clearly correct insofar as it claims that knowledge of the How-to-count principles does not underlie children's acquisition of counting skills. I shall now discuss a weakness with typical instantiations of this theory. With the exception of children's ability to "subitize", proponents of the Principles-After theory have typically concentrated only on what children learn. An impressive body of data has been gathered cataloguing the sequence of children's acquisition of various counting skills -- what is missing is a set of adequate explanations of *how* children learn what they learn. This may be due at least in part to a failure to investigate the very rich body of unlearned abilities and concepts children bring with them. An understanding of what these abilities and concepts might be could considerably enrich a theory of the development of counting, by providing a basis on which to construct mechanisms of *how* children are able to learn what they do. That is, it may help us to follow Gelman and colleagues in spirit, if not in particulars. Below, I review some findings that.

begin to shed light on the unlearned components that might underlie children's ability to count.

*Children's discrete object bias*

In the "Novel Entities" experiment, most of even the youngest children were able to generalize their counting routine to sounds and actions, showing that children have the ability to develop very quickly an abstract and sophisticated mental representation of the counting routine. This is suggestive of strong powers of abstraction in young children, and may point to unlearned abilities more general than knowledge of counting. Findings along the lines of those of Shipley and Shepperson (1990) could inform us on this point. They have shown in a number of experiments that children have a very strong bias to both count, and otherwise interact with, discrete, physically separate entities, rather than attached parts of objects or individual objects that have been divided into physically separate parts. They argue from these results that the "oneness" of discrete physical objects is highly salient to children, and that this may help them in learning to count. More generally, they propose that other such biases, or general predispositions, may interact in ways that facilitate mastery of the counting principles. They have suggested that the following general cognitive abilities may predispose children to abstract the one-to-one correspondence principle from examples of adult counting: the above bias to operate on discrete whole objects; a tendency to exhaust a set of things being operated on, e.g., to throw *all* the toys out of the playpen (see Potter & Levy, 1968); and an ability to match elements of one set to elements in another (see Sugarman, 1983). Further analyses of this sort could lead to an explanation of how children are able to so quickly develop an abstract representation of counting.

*Knowledge of number independent of knowledge of counting*

It has been argued by some that children must acquire the very concept

of numerosity itself (e.g., Steffe, von Glasersfeld, Richards, & Cobb, 1983). Yet the extent to which there is an unlearned concept of number, the representational form it might take, and the exact role such a concept might play in children's learning that counting determines numerosity, are all unanswered questions. There is some evidence that children as young as 2-and-a-half recognize at least small numerosities and can perform inductions over them. In one experiment, children of this age were shown two plates, one with three toy mice on it identified as the "winner" plate, and one with two mice identified as the "loser" (Gelman, 1977). The plates were covered and shuffled, and children had to identify which was the winner plate. Children could do this task. Then, surreptitiously, either the number of mice on the winner plate was reduced to two mice, or a number-irrelevant transformation was made with the winner plate such as replacing one of the mice with a soldier, or changing the spatial arrangement of the three mice. When the covers were removed after shuffling, in the number-irrelevant transformations children still chose the three-item plate as the winner, while in the number-relevant transformations, children often declared that there was no winner, and in some cases even fixed one or both plates to be winner plates by adding an extra mouse. They had thus evidently represented the number of the array.

There is also evidence that even infants have some very basic knowledge of the numerosities *two* and *three*. When habituated to many different pictures of two objects, newborn infants (Antell & Keating, 1983), 5-month-olds (Starkey & Cooper, 1980), and 10-month-olds (Strauss & Curtis, 1981) will dishabituate when shown a picture of three objects, and vice-versa. As well, when shown two pictures simultaneously, one of two objects and one of three objects, and played a sound recording of either two knocks or three knocks, 7-month-old infants show preferential looking at the picture with the same number of objects as the number of knocks heard (Starkey,

Spelke, & Gelman, 1983).

These studies suggest that young children do have some basic concept of number, or at least of smaller numbers. This in turn suggests that the child's task in counting is one of mapping already existing concepts of *oneness*, *twoness*, and *threeness* with the number words "one", "two", and "three", and with the counting activity. Indeed it has been suggested that children learn the meanings of number words by associating them with "subitized" numerosities (Klahr & Wallace, 1973, 1976; Schaeffer et al., 1974). Results from the Give-a-number tasks indicate that children map smaller numbers onto their number words before achieving such a mapping for larger numerosities. This is plausible. The word "one" occurs much more frequently than other number words, and in many special contexts (e.g. "I want another one", "give your brother one of those", "get me the little one", "which one do you want", etc.). It may even be that children first learn the word "one", not as a number word but rather as a pronoun that picks out a single individual, similar to "he", "she", or "it". Children then map the word "two" onto its corresponding numerosity, followed by the word "three". Further evidence that children learn the meanings of number words sequentially comes from the "subitization" literature; when shown small numerosities and asked to tell "how many" there are, children's rate of correct response decreases as the number increases (e.g., Gelman & Tucker, 1975; Silverman & Rose, 1975). Results of the Give-a-number task in Experiment 3 indicate that, after acquiring the meanings of the words "one", "two", and "three" in sequence, children acquire the meanings of *all* the number words within their counting range, in conjunction with their acquisition of the cardinal word principle.

It appears that the development of children's understanding of counting is complex and piecemeal. Infants may have an innate concept of numerosity, or at least of the numerosities *one*, *two*, and *three*, which they

must map onto the correct number words. Children first map the word "one" onto its numerosity, achieving this next for the word "two" and then the word "three". Counting starts out as a meaningless activity, something like a game of patty-cake, from which children abstract certain properties earlier, others later, perhaps aided by unlearned abilities such as those suggested by Shipley & Shepperson (1990). One property that some children may learn is that the last number word used in a count is the answer to "how many" items there are, even though they do not at first understand that "how many" refers to *numerosity*. At roughly 3-and-a-half years of age, most children come to understand that the last number word used in a count represents the numerosity of the items counted, and use this knowledge to learn the meanings of all the number words in their counting range.

TABLE 1

*Proportion of cardinality and recount responses in Experiment 1*

Age Group	Object	Cave	Jump	Sound	Total non-Object	Total
Age I (2:7)	.06 [.11]	.25	.33	.00	.23	.16
N	8	6	5	4	6	8
Age II (3:0)	.12 [.47]	.19	.27	.24	.24	.21
N	8	7	8	8	8	8
Age III (3:5)	.36 [.36]	.67	.53	.57	.59	.55
N	8	8	8	8	8	8
<i>Total</i>	.16 [.31]	.39	.37	.34	.37	.

Recount responses indicated by square brackets, N = no. children contributing to each cell

**TABLE 2**

*No. Grabbers giving 1, 2, 3, 5, and 6 items  
in Experiment 2, by no. asked for*

No. Given	No. Asked For				
	1	2	3	5	6
1	18	1	1	0	0
2	0	13	8	7	8
3	0	1	5	5	3
5	0	0	3	0	2
6	0	1	0	1	0
Other	0	2	1	5	5

**TABLE 3**

*Mean no. objects first given by Grabbers and Counters in  
Experiment 2 (standard deviations given in brackets)*

Age Group	Number asked for			
	2	3	5	6
Age I Grabbers	2.6 (1.5)	2.9 (1.1)	3.4 (2.1)	3.3 (2.0)
Age II Grabbers	3.6 (2.4)	2.9 (1.2)	3.3 (2.2)	2.9 (0.9)
Age III Grabbers	1.7 (0.5)	2.7 (1.7)	4.7 (2.1)	4.5 (2.1)
Age III Counters	2.0 (0.0)	3.0 (0.0)	5.2 (0.3)	6.0 (0.0)

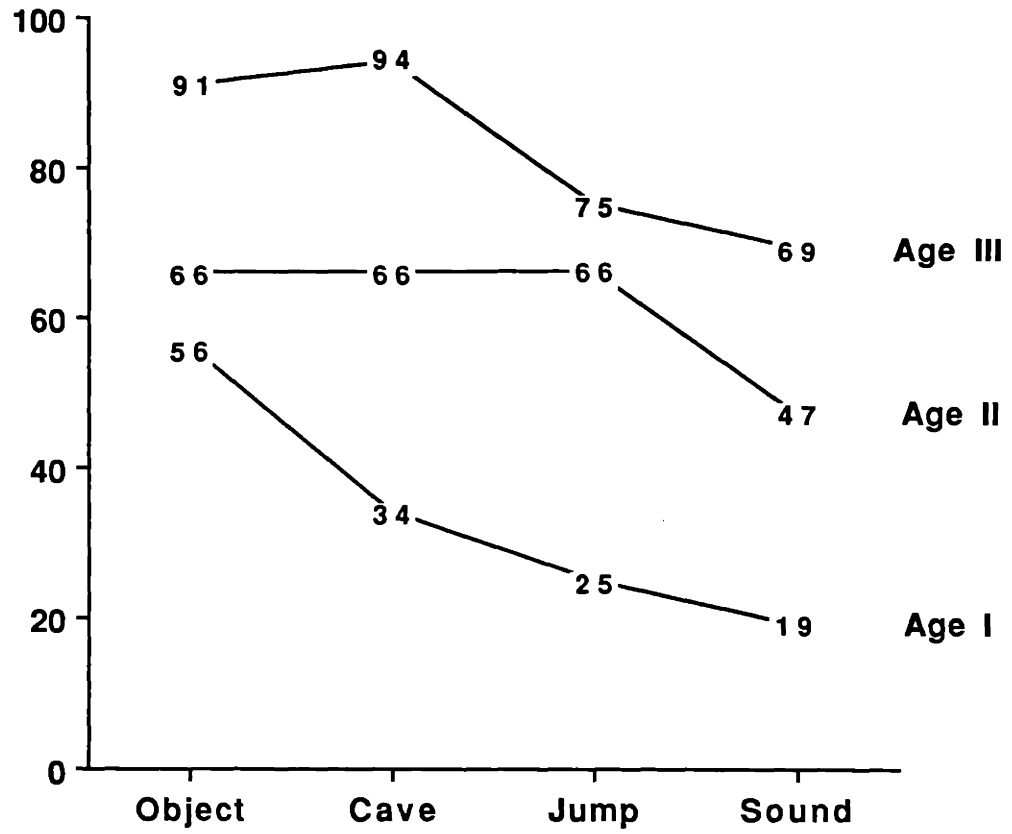


TABLE 4

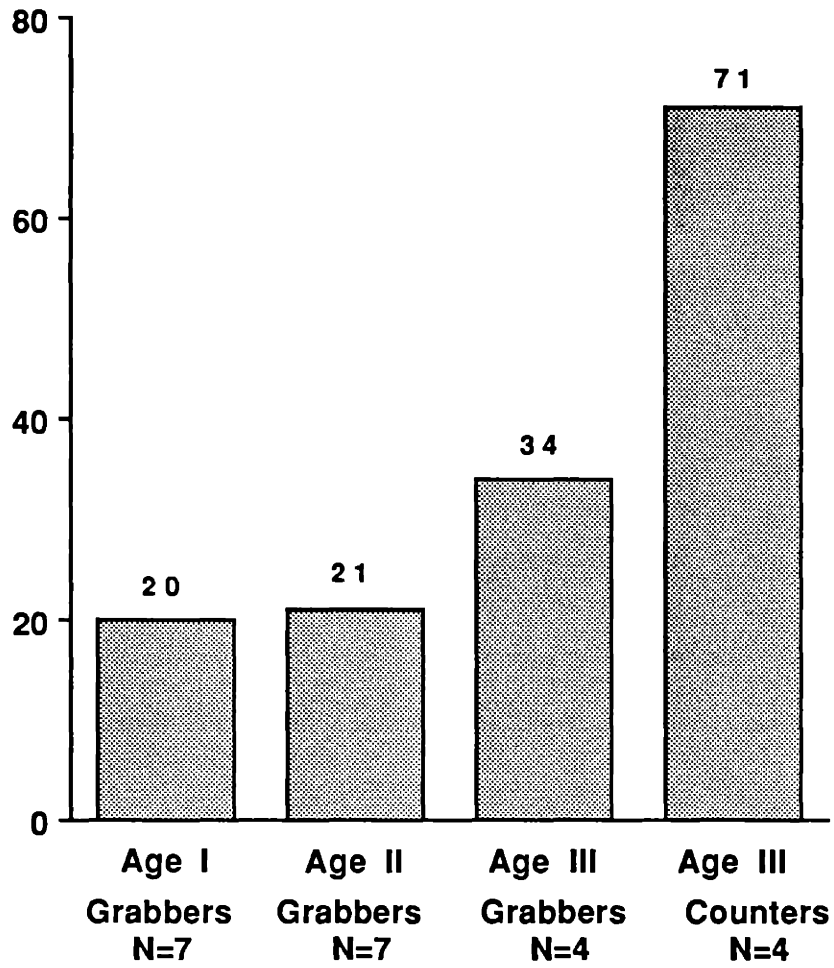
*Patterns of success in Give-a-number task in Experiment 3*

Success Pattern					No. of Children	Mean Age	Counting Ability	
1	2	3	5	6			Mean	Range
-	-	-	-	-	1	2:8	3.00	(3 - 3)
+	-	-	-	-	3	3:0	4.67	(3 - 6)
+	+	-	-	-	2	2:11	4.50	(3 - 6)
+	+	+	-	-	4	3:5	5.75	(5 - 6)
+	+	+	+	+	7	3:7	6.00	(6 - 6)

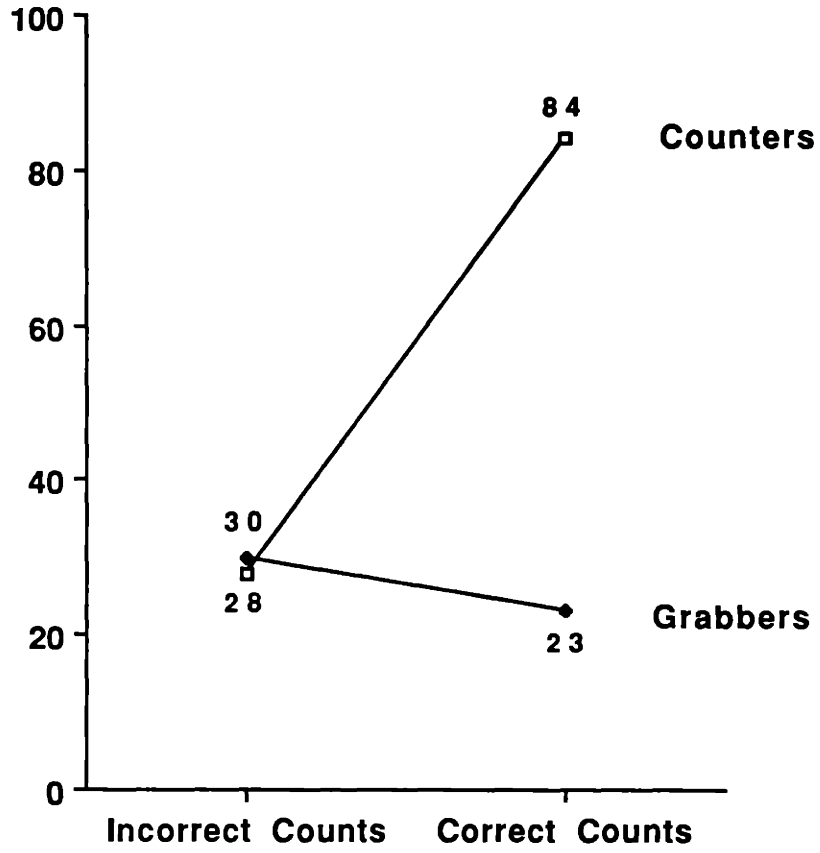
(Note: "+" indicates success on a numerosity; "-" indicates failure.)

**Percent Correct Counts in Experiment 1**

**Mean % Cardinality Responses of Grabbers  
vs Counters in "How many" Task**



**Mean % Cardinality Responses Following  
Correct vs Incorrect Counts**



# How Children Learn the Meanings of Number Words

## Introduction

It appears to be a real mystery as to how children learn the meanings of number words. In this task children are faced with the problems inherent to any word-learning task -- from an infinity of logically possible meanings, they must somehow infer the correct meaning of a word. This problem is especially acute when it comes to number words and other words not denoting physical objects. There is no physical object, or property of a physical object, which corresponds to, for example, the number *three*. When we speak of "red apples", each individual apple has the property "red", but when we speak of "three apples", none of the individual apples has the property "three". Yet when we count, we assign a number word to each item, so the child sees an item labelled "one", another "two", another "three", etc.

We do know that children and infants are sensitive to numerosity. Young infants are able to discriminate small numerosities; when habituated to pictures of three objects, they will dishabituate when shown a picture of two objects, and vice-versa (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981), and under some conditions will dishabituate when shown a picture of four objects and vice-versa (Strauss & Curtis, 1981). Furthermore, they are able to match a number of sounds heard with a visual display depicting the same number of items (Starkey, Spelke, & Gelman, 1983; in press). There is also evidence that children as young as 2-1/2 years old can recognize small numerosities and perform inductions over them. In one experiment, children of this age were shown two plates, one with three toy mice on it identified as the "winner" plate, and one with two mice identified as the "loser" (Gelman, 1977). Children were capable of identifying the winner plate, and sometimes even changed a two-item plate to be a winner plate by adding an extra mouse, showing that they had represented

the number of the array. Given these studies, we do know that children have the *concept* of numerosity that they must have in order to learn the meanings of the number words. The problem, then, is in mapping these concepts onto words.

This task is made more difficult for children by the fact that number words are unmistakably assigned to individual items when counting, yet the number words do not refer to individual items, or to properties of individual items, but rather to properties of *sets* of items. Of course, children do not only hear number words in the context of counting; they also hear number words uttered in sentences, with all the accompanying contextual and syntactic cues.

One helpful aspect to hearing a word used in a sentence is that the situational cues are available. It has been shown that children can (at least sometimes) learn about the meanings of words when those words are contrasted with known words in the same domain. For example, when asked to "get the tray; not the red one, the chromium one", at least some children will infer that the word "chromium" refers to a *color* (since it was contrasted with "red", which the children already knew, or at least knew was a color), and will sometimes even pick out the color of the opposing tray as the referent of the word (Carey & Bartlett, 1978). But this only helps when children *already* know the meaning of one of the words in a domain. It will not explain how children learn their first word or words belonging to that domain.

It could be that their syntactic status as quantifiers tells children that the number words refer to properties of *collections* of entities, not of individual entities. But when number words are used in sentences, their syntax, at least at first glance, would seem to support the hypothesis that they refer to properties of individual objects, since number words usually take the same position as adjectives (e.g., "see the *big* dogs" versus "see the *three* dogs").

Thus, if the syntax of the number words helps children to determine their meanings, it must be through their possession of a unique configuration of several syntactic properties, not of some single syntactic property, and as such will entail sophisticated knowledge on the part of the child.

Discovering how children solve this word-learning task may inform us about inherent word-learning and/or general learning strategies possessed by children. It may also tell us something about the initial nature of the concept of number, since the task children have in learning the number words is that of mapping their own existing concept of number onto the counting system. In this paper the child's number-word-learning strategies will be studied indirectly, by investigating the pattern of children's knowledge acquisition. By looking at children's developing understanding of the meanings of number words, an account can be constructed of the sorts of inductions they make along the way, which will help suggest a mechanism (or mechanisms) by which children attain this knowledge. This study also addresses the question of how children's acquisition of the number words relates to their mental representation of number.

### **Some theories of number word meaning acquisition**

#### *"Counting Principles" Theory*

Gelman and colleagues (e.g., Gelman & Gallistel, 1978; Gelman & Meck, 1984; Gelman, Meck, & Merkin, 1986; Gelman & Greeno, 1989) have proposed that young children possess an innate concept of number consisting of a set of counting principles that define correct counting. The three "how-to-count" principles are as follows: The *one-to-one correspondence* principle states that items to be counted must be put into one-to-one correspondence with members of the set of number tags that are used to count with (e.g., a set of number words); the *stable-order* principle states that the number tags must have a fixed order in which they are consistently used;

and the *cardinality* principle states that the last number tag used in a count represents the cardinality of the items counted. Children do not possess innate knowledge of the number *words*, of course; they must learn the number words of their language, and map them onto their own innately given mental number tags. But this task is made easy for children by the fact that number words operate on the same principles as their mental number tags; they have a fixed order in which they are consistently used, and they are applied to items in one-to-one correspondence. This allows children to very early on identify the linguistic, culturally supported counting activity *as counting* (i.e., as the same kind of activity as their own innate, non-linguistic counting activity), and so to map the number words onto their mental number tags. Thus the meanings of the number words are acquired easily and rapidly by children; each number word holds the same meaning as does its corresponding number tag, namely a specific, unique numerosity determined by its ordinal position in the number word/tag list. Once children have mapped the number words onto their mental number tags, the counting principles also allow children to develop their skills in the overt counting activity, by serving as guidelines for correct counting so that children can monitor their counting performance.

However, there is converging evidence that children do not learn the meanings of the number words until well after they have acquired considerable skill at counting (Fuson, 1988; Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; see especially Chapter 1). That is, children know how to count quite well before learning that counting determines the numerosity of a set. It thus appears that children do not start out with unlearned counting principles that give them the meanings of the number words and guide the acquisition of their counting skills. Rather, children must learn how to count, and must learn the meanings of the number words by some means other than their correspondence to a set of mental counting tags.



*"Different Contexts" Theory*

Fuson and colleagues (e.g., Fuson, 1988; Fuson & Hall, 1983; Fuson, Richards, & Briars, 1982; and Fuson & Mierkiewicz, 1980) have argued that there are different contexts in which the number words have different meanings; they further argue that children sequentially acquire these different meanings, learning each number word at first as several different context-dependent words. These different meanings gradually come to be interrelated, and for the adult comprise a system of closely connected meanings for that word. In particular, there are the *sequence*, *counting*, and *cardinal* "meanings", or contexts, of number words. Sequence meanings occur in contexts in which the number words are recited in sequence, but are not used to count actual items or to refer to the numerosity of some set of items. The "meaning" of a number word used in this context is that it comprises part of this sequence; the words have no referents. The number words each take on a *counting* meaning when used to count a group of items. The "meaning" of number words used in this context is their successive assignment to items in a one-to-one correspondence, and the referent of a number word is the item it is paired with; thus, the referent of a particular number word differs with each count, in the same way the referent of a pronoun differs from sentence to sentence. A number word is used in a *cardinal* context when it is used to describe the cardinality, or numerosity, of a set of discrete objects or events; in this context, the referent of a number word is the numerosity of the set of events.<sup>5</sup>

The claim that children first acquire the sequence meaning is supported

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<sup>5</sup>Fuson & Hall (1983) also posit several other contexts of number words; a *measure* context, in which the number words refer to numbers of units along a dimension into which some entity has been divided; an *ordinal* context, in which a number word refers to the position of an entity in a well-defined ordered set with a specified initial point; and a *non-numerical* context, in which number words are used as labels or codes, for example, numbered players of a football team, telephone numbers, and so on.

by Fuson and colleagues as follows: In children's earliest productions of the number word list, they treat the words as an unbreakable string, a sequence of sounds with no intrinsic meaning, recited by rote. At this level, the number words are at first not even segmented in the string (Fuson et al., 1982). When children acquire the counting meanings of the number words, it is claimed, the words come to have reference to the objects with which they are paired during a count, without the child understanding that this pairing has anything to do with determining the numerosity of the items counted. The evidence that children go through such a stage in their understanding of the number words is that children can count correctly (can segment the number words and assign each one to a unique object) before knowing that the last number word indicates the numerosity of the counted set (Fuson & Mierkiewicz, 1980; Chapter 1). Finally, children also acquire the cardinal meaning of a number word, in which the number word refers to the numerosity of "a well-defined set of discrete objects or events" when used in appropriate contexts (Fuson & Hall, 1983 p. 58).

Fuson and colleagues' account conflates the *meaning* of a term with the *context* in which that term is used. But there is a great difference between the function a particular word plays in an algorithm or routine, and the meaning of that word. The "counting meaning" level of understanding was proposed in light of observations of children's production of the number words, and so is descriptive of their external behavior; but it is not sufficient to look only at children's production of a word to learn what they *infer* from the word. What is needed is evidence concerning the role the number words play at this stage in the child's conceptual system. It does not follow that the *meaning* imputed to a number word by children at this level of understanding is literally the object to which it is assigned, just because in the process of counting they do indeed assign one number word to each object.

Little research has been done to determine what information children

infer from the number words. However, there is some tentative empirical evidence against the claim that before children possess the full adult meanings of number words, they believe the referent of a number word is the object to which it is assigned. This comes from a previous study conducted by myself (in Chapter 1) on 2-1/2- to 3-1/2-year-olds. In what I called the "Give-a-number" task (Experiments 2 and 3), children were asked to give a puppet a certain number of items ranging from 1 to 6, from a pile of toy animals. The structure of the questions was "Could you give Big Bird three animals?" and variations. If children who have not yet acquired the cardinal meanings of the number words consider the meaning of the word "three" (for example) to be an object to which it is assigned during a count, then they presumably should count some objects and, when they get to "three", give the puppet the item labelled "three". Alternatively, they might choose to give the puppet a single animal while labelling it "three". But children did not behave this way. Instead, the younger children tended to simply give the puppet a handful of animals, almost never by counting them; the 3-1/2-year-olds tended to count items from the pile as they gave them and to stop at the number word asked for, thus giving the correct number. Even children who were just learning to count almost *never* gave the puppet a single item, when more than one item was asked for (of 40 children, only 2 did this). In contrast, children *always* gave the puppet a single item when asked for one item. This suggests that children impute a different meaning to number words than their assignment to an object. It might be argued that the syntactic cues of plurality or singularity ("Give Big Bird three *animals*/one *animal*") were what caused children to give one when asked for one, and more than one otherwise. However, independent studies have shown that children of this age have not mastered the English plural markers (e.g., Brown, 1973). It thus appears that children are sensitive to the fact that the number words other than "one" refer to more than a single item even before

they know singular/plural syntax.

However, in apparent contradiction with this conclusion, when asked to count the items they had given, about half the children in the Give-a-number task in Experiment 2 labelled the last item in the count with the number word asked for when that was not the next count word they should have said. For example, when asked for 5 objects one girl gave 3; when she was then asked to count them to make sure how many there were, she counted them "one, two, five". This kind of response could have occurred because children thought the referent of the number word was a single item, and since the puppet had asked for, say, "five animals", they knew they had to label one of the animals "five". However, it is more likely that children simply knew that the last number word in their count should correspond to the word asked for, without knowing why -- this would explain why they tended to label the *last* item in the count with the number word asked for, rather than just labelling any one of the items. Two children occasionally *named* one of the items they had given with the number word asked for, clearly assigning a single item as the referent of the number word. This would seem to suggest that, at least for these children, the meaning of the number word was either the item to which it was assigned, or some aspect of that item. However, these two children did so very seldom, and they *always* honored the distinction between being asked for one item versus more than one, suggesting that at some level they knew that number words (other than "one") did not refer to a single item. Clearly, more research is needed to resolve the question of what meaning children ascribe to a number word before acquiring the adult understanding.

## **Focus of the study**

### *Background Research*

I previously showed (Chapter 1) that by roughly 3-1/2 years of age, children understand the counting system and know the cardinal meanings of

the number words within their counting range. They can use counting to retrieve a specified number of objects from a pile. As well, when asked, immediately after counting a collection of items, "how many" items there are, they respond with the last number word they said in the count. This shows that by roughly 3-1/2 years of age, children learn the cardinal *word* principle (in contrast to the *cardinality* principle), which states that the last number *word* in a count represents the numerosity of the items counted. But there are several stages along the way to this knowledge. Children appear to learn the meanings of smaller number words before those of larger ones within their counting range -- children can give 1 item from a pile when asked before they can give 2, can give 2 items before they can give 3, and can give 3 items before they can give higher numbers. They then appear to make some kind of general induction to simultaneously acquire the meanings of all the number words beyond "three" (or possibly "four") that are within their counting range. However, before they can give larger numbers, children successful at giving these smaller numbers virtually *never use counting* to do so; they still do not know that counting is the general solution to the task. They appear instead to have directly mapped particular small numbers onto their correct number words, and so can succeed at giving those numbers, but have no general solution. As soon as they learn the way in which counting determines the numerosity of a set of items (that is, that the last number word used in a count represents the numerosity of the items counted -- the cardinal word principle), they acquire the meanings of the rest of the number words within their counting range, and so can give any number of items they are asked for.

Two major questions arise from the above work; they are the focus of this paper and are discussed below.

*Components of meaning of number words*

First, there is the question of how children's understanding of the meaning of a number word develops; whether there is a stage at which children believe the number words refer to individual items, and if children's knowledge of a particular number word comes in all at once or if children acquire different aspects of the adult's meaning of a number word at different times. Acquisition of the full meaning of a number word may not come all in one piece; there may be different components of meaning (as opposed to different usage contexts) that children acquire at different times. There are (at least) three essential components to understanding the full meanings of the number words:

1. The knowledge that a number word refers to a *numerosity*, regardless of *which* numerosity it refers to;
2. The knowledge of the precise numerosity a word refers to.
3. The knowledge that it is a word's position in the list that determines the precise numerosity it refers to (the cardinal word principle).

This last component specifies the *way* in which the number words are assigned their meanings, rather than specifying some aspect of meaning of individual words. One need not have knowledge of this last component in order to know the meaning of a particular number word; indeed, the evidence is that children acquire knowledge of the meanings of small number words before learning the cardinal word principle. However, at some point every person must induce the general rule for how number words attain their precise meanings; otherwise we could never acquire the meanings of literally an infinite number of number words. Determining when children learn these three different components of meaning can help us

in understanding *how* children come to understand the meanings of the number words, by suggesting a possible path of inductions by which the child attains full knowledge.

*Time span of acquisition of number words*

Second, we do not know how long it takes children to learn the full cardinal meanings of the number words. My previous findings show that children first learn the word "one", then "two", then "three", before making a general induction, but they do not show how stable each of these stages is within a child, because the data are cross-sectional and there is a lot of individual variation -- some children have acquired the cardinal word principle before their third birthday, others nearing their fourth have yet to learn it. The present study examines how quickly children who have acquired the meanings of one or two of the number words come to learn the others, and to learn the cardinal word principle.

Understanding the full meanings of all the number words in general comes down to understanding the *way in which counting represents numerosity* -- that it is a word's position in the count word list that determines the numerosity it refers to. Knowing how long it takes children to understand this can then give us an idea of how easy or difficult a task it is for children to understand the counting system. This in turn may reflect on the nature of young children's initial representation of number, since the child's task in learning the counting system is to map her own concept of number, whatever it may be, onto the counting system. If this entails a very long, protracted period of learning, that will suggest that it is difficult for children to learn the counting system. This would then suggest that the counting system may entail a very different form of representation of number than the one the very young child possesses. Conversely, if it's a relatively short process, that would suggest that the counting system embodies a representation of number similar in form to the child's own, making it a simple task for the

child to map her own representation of number onto the counting system.

This study investigates how long individual children know some of the number words before going on to learn the rest and learn the cardinal word principle, and when and how children learn different components of the meanings of number words.

## **The Experiment**

### **Subjects**

Twenty 2- and 3-year-old children were tested. Fourteen (8 boys, 6 girls) were tested over a period of about 8 months, with 5 to 8 weeks between each session; their mean age at the start of the study was 3;2, range 2;6 - 4;2. The remaining 6 children (3 boys, 3 girls) were tested for a period covering 2 months, with 1 month between sessions; their mean age at the start of the study was 2;11, range 2;7 - 3;3. This second group of children was added on late in the school year, so could not be followed up as long as the first group of children. Seven additional children were tested at least once, but not included as subjects; four because their attention span was too short for them to sit through the Phase 2 task described below, two because they did not satisfy the criteria necessary for participating in Phase 2, and one because he was not available for testing beyond the first session and so did not contribute to the longitudinal aspect of the study.

### **Procedure**

There were two phases of testing for each session, typically given one to four days apart:



**Phase 1:***"Give-a-number" task*

Children were given the "Give-a-number" task, to determine which number words they knew the meanings of. They were asked several times to give a puppet 1 to 5 items, and the highest number word they could succeed at consistently was determined. Children were given a sticker after each trial to keep them motivated. The goal was to determine the maximum numerosity each child could succeed at, so the exact procedure differed for different children. All children were first asked for 1 item and then for 2 items; depending on their success, they were then asked for 3 items, or asked again for 1 or 2 items. What children were asked for on a trial depended partially on their success in the previous trial; children who failed on a trial were next asked for a numerosity at which they had previously succeeded. This served two purposes; to determine the consistency of a child's performance on a particular numerosity, and to avoid discouraging children. The experimenter concentrated on the highest number a child succeeded at reliably and on the next highest number, so children got more trials for these than for other numbers. The exact number of trials a child got depended on his or her consistency on a particular numerosity, and in some cases the child's willingness to keep playing. The experimenter followed up children's responses by asking questions such as, "Is that three?", "Can you count and make sure?", to give children every opportunity to correct their responses and exhibit knowledge of a number word.

*"How-many" task*

Children were asked to count sets of 2 to 6 items, once for each set size, and were asked, immediately after counting, "how many" items there were. This tested children's knowledge of the cardinal word principle -- whether they know that the last number word they used in the count indicates how many items there are. For this task, items were plastic toy animals about 7cm

long, glued about 3cm apart to a board in a linear arrangement. Items were homogeneous within a set, but different across sets.

### *Color control task*

As part of a control for Phase 2, children were also tested for knowledge of the color words "red", "blue", "green", and "yellow", or of the words "top" and "bottom". They were shown 4 balls simultaneously, one of each color, and were twice asked to point to the ball of each color (in randomized order). Children who spontaneously named the colors correctly (as many did) were asked to point to each of the colors only once, since production of the color words is a more definitive indication of knowledge than simply correct identification. Those children failing the color task were given a test of their knowledge of the words "top" and "bottom", by being shown 4 cards, each with 2 pictures on it, and being asked, for each one, to point to the picture on the top, or the picture on the bottom (twice the top, twice the bottom). Nineteen of the children knew the meanings of the four color words; the remaining child knew "top" and "bottom". Two additional children did not know either the four color words or both "top" and "bottom", and for this reason were excluded from the study.

### **Phase 2:**

Only children who could give at least 1 item when asked, and who knew the four color words or the words "top" and "bottom", participated. Children were given a "Point-to-x" task, in which they were shown cards, each with 2 pictures with different numbers of items on them, and were asked to point to the picture with a particular number of items. The items on a particular card were the same kind in both pictures, e.g., dogs in both pictures or balloons in both pictures, but of different colors, e.g., red dogs in one picture and blue dogs in the other. Items differed from card to card.

To test whether children knew the *precise* numerosity a word refers to,

children were shown that numerosity paired with a numerosity one larger, and asked to point to the picture with that number of items. For example, to test if a child knew the precise meaning of the word "four", she was shown a picture of 4 balloons paired with one of 5 balloons and asked, "Can you show me the four balloons?" Children will only be able to do this successfully if they know the precise meaning of the number word "four". To test whether they knew that a particular word refers to a *numerosity*, even if they did not know *which* numerosity the word refers to, children were given a picture of that numerosity, paired with a picture of 1 item. Since all the children knew that the word "one" refers to a single item, then if they knew that, for example, the word "five" refers to a numerosity, they should infer that it does *not* refer to a single item since they already have a word for the numerosity *one*.<sup>6</sup> They should therefore be able to choose the correct picture by a process of elimination. In all of the pairs where one picture was of only a single item, the pictures depicted either sheep or fish, which have the same singular and plural syntax, so children were given no syntactic cues as to which picture to choose (e.g., "Can you show me the three fish/the one fish?").

### General structure of study

Children were placed into one of four groups on the basis of their performance on the Give-a-number task. Children who could give only a single item consistently were placed into Group 1; those who could give up to 2 items were placed into Group 2; those who could give up to 3 items went into Group 3; and those who could give up to 4 or more items went into Group 4. Each group was tested on different number pairs, as shown in Table 1. For example, if a child could give up to 2 items, she was tested on the pairs (2, 3), (3, 4), (1, 3), (1, 4); so it was tested whether the child knew the

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<sup>6</sup>This reasoning assumes that children possess the *principle of contrast*, which says that no two words have the same meaning. See Clark, 1988 for discussion.

*precise* meanings of the words "two" and "three", and whether the child knew *partial* meanings of the words "three" and "four" (whether the child knew they are *number* words).

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INSERT TABLE 1 ABOUT HERE

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For each number pair, the child was asked twice to point to the smaller numerosity, twice to point to the larger (e.g., "can you show me the three flowers?"). The child was also asked, twice for each pair, to point to the picture of a certain color (red, yellow, green or blue), or to point to the top or bottom picture if that child had failed the color task, e.g., "can you show me the yellow sheep?"/"can you show me the bottom sheep?". This served three purposes: (1) to prevent children who may not have known that, e.g., "five" is a number word from inducing that it must be, which might happen if all the questions were about numerosity and children were to pick up on this; (2) to help prevent children from becoming discouraged, by giving them questions they definitely knew the answer to; and (3) to reveal whether children were attending to the task and performing to the best of their abilities, since if so, they should get all these questions right. Finally, children were asked, twice for each number pair, a question with a nonsense word, e.g., "Can you show me the blicket elephants?", to measure any possible bias to point to a particular one of the pictures in any given pair. So, altogether, children were given 8 questions for each number pair: 4 number questions, 2 color (or top/bottom) questions, and 2 nonsense questions, all worded identically except for the crucial word. There were 2 cards for each number pair, each picturing different items, for a total of 8 cards (only 6 cards for Group 1). This was to prevent children from becoming bored, which might happen if

there were only 4 (or 3) cards, with 8 questions for each one. Thus, for each of the 8 (or 6) cards, children were asked to make four judgements; 2 number questions (once for the larger number, once for the smaller), 1 nonsense question, and 1 color (or top/bottom) question, for a total of 32 (or 24) questions altogether.<sup>7</sup>

Each child was followed up until she could give up to 5 items when asked in the "Give-a-number" task, succeeded on at least 75% of the questions for each of the number pairs in the "Point-to-x" task, and gave last-word responses a majority of the time on the How-many task, or until the end of the school year.

### Scoring criteria

For the Give-a-number task, the following criterion was used to determine the highest number children could consistently succeed at giving:

- On at least two-thirds of a child's trials for that numerosity, the child's response was either the correct number according to his or

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<sup>7</sup>The order of presentation of cards and questions was randomized in the following manner: For each card, the questions to be asked about that card, and the order in which they were asked, were held constant. Cards were shuffled at the beginning of a session, and then were gone through in a first pass, asking the child the first question for each card. The cards were then shuffled and gone through a second time, asking the child the second question for each card; shuffled again and gone through a third time, asking the third question for each card; then shuffled and gone through a fourth time, during which the final set of questions was asked. After each pass through the "deck", children were given a sticker. Thus, although for each individual card the question order did not vary from one session to the next, for no trial did one question predict the next question to be asked, nor were two questions about the same card asked consecutively. Across different cards, different question types (larger number, smaller number, color, nonsense) were counterbalanced to be first, second, third, or fourth questions, so that for each pass through the cards, a child was asked equal proportions of each question type. Questions for the two cards for any given number pair were ordered such that the question for the smaller numerosity came before that for the larger numerosity on one card, and after that for the larger numerosity on the other.

her own stably-ordered count list,<sup>8</sup> or the correct number plus or minus one if the child had counted aloud from the pile to the number word asked for, but had erred in the counting by either double-counting or skipping one item or by repeating or skipping one number word; and

- The child responded with that number when asked for higher numerosities no more than half as often, percentage-wise, as he or she did when asked for that number itself. For example, a child who gave 2 items 80% of the time when asked for 2, was scored consistently correct on 2 only if he or she gave 2 items no more than 40% of the time when asked for 3, 4, and 5 items. This was to prevent children who had a preference for giving, e.g., 2 items no matter what they were asked for, from being considered to know the meaning of the word "two".

In the Point-to-x task in Phase 2, children were considered to give a correct response if they pointed to the correct picture; *or* if they counted the items in one or both pictures in order to answer a number question, but made an error when counting the wrong picture such that the outcome of the count was the number they had been asked to show, and then chose that picture for their response (this only happened 10 times out of the total of 368 number questions given of the Point-to-x task where both pictures were of more than 1 item). If a child pointed to both pictures, the experimenter asked the child

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<sup>8</sup>It has been pointed out (Fuson & Mierkiewicz, 1980; Gelman & Gallistel, 1978) that children sometimes have a stably-ordered list of the counting words that differs from the standard list, usually in the consistent omission of one or more number words. For example, a child may consistently count "one, two, five, six, ...". In this case, when asked to give, for example, 5 items, he child would be considered correct if she gave 3 items, since "five" is the third word in her stably-ordered list. As it turned out, all 20 children in this study used the standard count word list.

to pick only one of the pictures. Two children, on one session each, insisted on always picking both pictures; they seemed to have a strong desire to touch the second picture as well, after having made their response. In these cases, children's first choices were used, since their performance on the color questions suggested that their first choices were their responses to the questions (their first choices were almost always the correct answer for the color questions). In three sessions (of a total of 58) children were miscategorized by the Experimenter into the wrong Group level, and hence tested with the wrong number pairs in the Point-to-x task; data for the Point-to-x task on these sessions were thrown out.

In the How-many task, children were considered to give a "last-word" response if they gave the last number word they had used in the count, regardless if they had counted correctly or if that word was the correct answer. In the Color control task, if children failed at one of the two questions for a given color, they were asked that question a third time (after some intervening questions). Children were considered to know the meaning of a color word if they got two questions for that color correct out of the two or three total questions for that word (chance was 25% since there were 4 possible answers). They were considered to know the words "top" and "bottom" if they got both questions for each word correct.

The experimenter scored children's responses at the time of testing. Children's sessions were also videotaped, and then transcribed and scored later by an independent judge. Inter-scorer agreement was 97% for the Give-a-number task, 97% for the Point-to-x task, and 93% for the How-many task.

## Results and Discussion

The Results section is broken down into five general parts. A preliminary section describes children's performance on the control questions in the Point-to-x task. Next, children's acquisition of the different components of meaning of number words is examined. Third, children's performance on the Give-a-number task is compared with that on the Point-to-x task. Fourth, the time span of acquisition for individual children is discussed. Finally, children's performance on the How-many task is discussed in relation to where they stand in the acquisition process.

Table 2 shows the basic pattern of results for the Give-a-number task over the duration of the study: The number of children that were classified into each group in their initial session and their mean ages; the total number of children observed at each Group level at least once during the study and the total number of sessions for which data was obtained for each Group level (brackets indicate the number of sessions for which there are data for the Point-to-x task as well); and the number of children who made the transition into each Group level and the mean ages at which the transition was made. One child was observed at the Group 3 level in one session, but in successive sessions was back in Group 2 (this could be either because she obtained tentative knowledge of the word "three" but then lost it after a month, or because she gave the correct number of items by chance when asked for 3). For the analyses, she was considered to be in Group 3 for that session, but was not considered to have "*moved* into Group 3", since she did not stay there. Thus, her session at the Group 3 level does not contribute to the two final columns of Table 2.



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INSERT TABLE 2 ABOUT HERE

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### **Performance on control questions**

In the nonsense questions, children showed no preference to point to a particular one of the pictures on any given card. There were 30 cards used in the experiment (6 for the Group 1 children, 8 for each of the other 3 groups); a series of *t*-tests (one for each card), adjusted by the Bonferroni technique for the number of *t*-tests performed, showed there to be no significant preference for one of the pictures over the other in any of the cards. Nor was there any preference on the nonsense questions to choose a particular *number* over the other number in a number pair, when Bonferroni-adjusted *t*-tests were conducted over number pairs rather than over individual cards. Nor, finally, did children have any preference on the nonsense questions to point to the larger or the smaller number of items, when a *t*-test was conducted on the number of times children chose the larger numerosity as opposed to the smaller numerosity, across all cards and all number pairs. These results indicate that the children had no intrinsic biases affecting their responses that might contaminate the results of the questions asked about number.

In addition, individual children on average asked the experimenter for confirmation of their response (e.g., by saying "this one?" while pointing, or by asking before responding "which one is that?", etc.) on 13% of the nonsense questions, while doing this on only 2% of the color and number questions (there was no significant difference between color and number questions, so they are collapsed for this analysis). For each child, the difference between these two percentages was computed. A *t*-test showed they were significantly greater than zero; the mean difference was 11

percentage points ( $t(19) = 4.149, p < .0001$ ). In fact, for only 1 of the 20 children was there a higher proportion of confirmation requests on the color and number questions than on the nonsense questions (8% versus 5%). Children clearly distinguished between the nonsense words and the other words, indicating firstly that they were paying attention to the experimenter's questions, and secondly that even number words they do not know the full meanings of are not considered in the same light as nonsense words are.

In the color questions, children each made at most one mistake out of the 8 (or 6, for Group 1) such questions they were asked in each session. Altogether, only 12 color mistakes were observed on the 392 color questions asked in the Point-to-x task. Thus, children were clearly paying attention to the task and giving answers that reflected their knowledge.

### **Acquisition of components of meaning**

Do children know that the number words refer to numerosities before they know precisely *which* numerosities they refer to? For each of the Groups, individual children's percentages of correct responses on each of the relevant number pairs (the pairs  $(1, n+1)$  and  $(1, n+2)$ , where  $n$  is the Group number of the child in that session) were determined. For this analysis, the percentage of correct responses expected if children do not know that the larger words are number words is 75%, because all children knew the meaning of the word "one" and so should succeed on the "show me the one ..." questions whether or not they knew that the larger words were *number* words. "Show me the one ..." questions made up half the number questions for these pairs. If children do not know that the larger words are number words, they should be expected to succeed on half of the larger-number questions by chance, for a total expected success rate of 75%. As it turned out, for both pairs in every Group children succeeded significantly more

often than this, as shown in Figure 1.<sup>9</sup> These results show that children know that the number words refer to numerosities, before they know the precise meanings of all the number words. For example, well before a child is able to identify a picture of 3 items when it is paired with one of 4 items, she can identify a picture of 3 items when it is paired with a picture of only 1 item.

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INSERT FIGURE 1 ABOUT HERE

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What is really surprising is just how soon children seem to have this knowledge. Even the very youngest children, who could not successfully give even 2 items when they were asked to, and who could not even point out a picture of 2 items when it was paired with 3 items, *could* identify the picture of 2 and 3 items, when each of these was paired with 1 item. So even at a very early stage of counting, children know that the counting words refer to *numerosities*.

It could be argued that the results from this part of the Point-to-x task might be explained *without* having to credit children with knowledge that the words other than "one" are number words. Children might know just enough about the counting words to know that these words can only apply in the context of a plurality of entities (Fuson, personal communication). For example, a child might take the word "four" to refer to some property an entity possesses when it is in the presence of other entities (something like

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<sup>9</sup>The *t*-values are as follows: For Group 1, for the pair (1, 2),  $t(5) = 8.301$ ,  $p < .0001$ , one-tailed; for the pair (1, 3),  $t(5) = 2.764$ ,  $p < .02$ , one-tailed. For Group 2, all children performed perfectly on the pair (1, 3); for the pair (1, 4),  $t(8) = 44.000$ ,  $p < .0001$ , one-tailed. For Group 3, for the pair (1, 4), all children performed perfectly; for the pair (1, 5),  $t(2) = 5.250$ ,  $p < .05$ , one-tailed. For Group 4, for both pairs  $t(10) = 21.917$ ,  $p < .0001$ , one-tailed.

"one in a bunch", for example). When asked to "show the four sheep", then, and shown a picture of 4 sheep together with a picture of 1 sheep, a child with such an interpretation of the word "four" would be able to choose the correct picture, but without having knowledge that "four" refers to a *numerosity*.

The hypothesis that children are succeeding on this task because they know the words are *number* words (the "number word" hypothesis) makes a different prediction than does this alternative "individual-in-a-group" hypothesis regarding how children's acquisition of the full meaning of one number word will affect their construal of other number words they do not yet know the full meanings of. If children know that each of the number words refers to a specific, unique numerosity, then they will restrict the meanings of the number words so that no two refer to the same numerosity (by the principle of contrast; see Clark, 1988). If a child knows the meaning of the word "two", for example, that child will not consider the words "three", "four", and so on to be applicable to a set of two items, even if she doesn't know precisely what numerosities "three", "four", etc. each pick out. Such a child will only be content to have these words describe numerosities she does not already have a name for. Since she doesn't know which numerosities these words *do* refer to, she will be equally likely to point to the picture of 3 items as she is to point to the picture of 4 items when asked to show which of two pictures is "the four ..."; her understanding of the word "four" places no restrictions at this point on whether it can refer to 3 as opposed to 4 items, since all she knows is that it must refer to some numerosity other than *one* or *two*. But once she acquires the full meaning of the word "three", she will no longer be willing to allow "four" to refer to 3 items. That is, her new knowledge of the word "three" *also* gives her knowledge of the word "four"; she now knows one more numerosity that "four" *doesn't* refer to. Similarly, a child knowing the full meaning of only the word "one" will be equally likely

to point to the 2 items as to the 3 items when shown a pair of pictures of 2 versus 3 items and asked to point to the picture with 3 items. But once a child has acquired the full meaning of the word "two" she will no longer be willing to do so -- she restricts her space of the possible candidate meanings for "three" based on her knowledge of other number words, including "two".

In contrast, consider a child who believes that the number words she does not yet know the full meanings of all refer to some property of an entity in a group of items, or simply knows that these words' satisfaction conditions entail a plurality of entities. Again, she starts off knowing the full meaning of only the word "one", and so will be equally likely to point to the picture of 2 items as to the picture of 3 items when asked to show the 3 items. When she learns the full meaning of the word "two", however, there will be *no motivation for her to further restrict the space of possible meanings of the word "three"*; she will still be happy in her belief that "three" can be satisfied when there is more than one item present. She should then still be equally likely to point to the picture of 2 items as to the picture of 3 items when asked to show the 3 items (though she will of course now always pick the picture of 2 items when asked to show the 2 items).

The critical test to determine which of these hypotheses is correct is to look at the children who only know the meaning of the words up to "two" as determined by their performance on the Give-a-number task, and see what they do when asked to show the 3 items of a pair of 2 versus 3 items in the Point-to-x task. If they know that "three" is a *number* word, they should not point to 2 items; if the "individual-in-a-group" hypothesis is true and they simply believe that "three" refers to some property of an entity in a group of entities, or that it requires a plurality of entities in order to be satisfied, they should be equally likely to point to both pictures (recall that children have no general bias to point to the larger picture as opposed to the smaller one). Similarly, children who only know the meaning of the words up to "three"

(as determined by their performance in the Give-a-number task) should never point to the 3 items when asked to show the 4 items and shown a pair of 3 versus 4 items, if they know that "four" refers to a specific numerosity. (It is no use looking at the children who only know the meaning of the word "one", since, by either hypothesis, they are presumed to know that the higher number words only apply in a context of a plurality of entities; so they should in any case never point to the picture of 1 item when asked to show the 2 items.)

For this analysis, however, we cannot include *all* the children who were classified into Groups 2 and 3. This is because, in the Give-a-number task, one of the criteria for crediting a child with knowledge of the word "three" (for example) in the first place was that a child did *not* generally give 3 items when asked for more than 3. Thus a child who always gave 3 items when asked for 3, but who also gave 3 items frequently when asked for more than 3, would not be credited with understanding the meaning of the word "three" and would be classified into Group 2. But by the "individual-in-a-group" hypothesis, this child *could* know the precise meaning of the word "three", and simply think that the words "four" and "five" could apply to a group of *any* plurality of items, including three (this could account for her performance on the Give-a-number task). Such a child would of course never point to the picture of 2 items when asked to show the 3 items, since she knows the meaning of "three". Including this child in the analysis would then spuriously make the results look more like what the "number word" hypothesis predicts. For this analysis, then, we must only look at those children who by *any* standards did not know the meaning of the word "three" – that is, those children who were classified into Group 2 who in the Give-a-number task *never* gave 3 when asked for 3. Similarly, we must include only those children classified into Group 3 who certainly did not know the meaning of the word "four", that is, who never gave 4 items when

asked for 4.

There were a total of 25 sessions with Point-to-x data (over 10 different children) in which children were classified as belonging to Group 2 or Group 3. Six of these sessions were excluded on the basis of the above concern; that is, children in these 6 sessions tended to give the next-higher numerosity of items both when asked for it and also when asked for larger numbers. This left 19 sessions (over 9 children) relevant to the analysis. Each individual child's percentage of correct responses was determined over all sessions when shown the pair  $(n, n+1)$  (where  $n$  is the Group number of the child) and asked to indicate the  $n+1$  items (there were two such questions for each session). A  $t$ -test was then performed on these percentages to see if they were significantly greater than the chance rate of 50% (the "number word" hypothesis predicts that they should be). This was indeed the case; the mean of children's percentages of correct responses was 88%, well above chance ( $t(8) = 5.969, p < .0001$ ). That is, children in Group 2 know not to point to a picture of 2 items when asked to show 3 items, and children in Group 3 know not to point to a picture of 3 items when asked to show 4 items.

This shows that children (at least by the time they understand the word "two") must be understanding the counting words as referring to *numerosities*; this explains why then children would restrict the possible meanings of higher number words once they learn the meaning of a particular number word. This same argument also counters another possible interpretation of the results -- that children believe that the words other than "one" refer simply to any *plurality*, like "several" or "bunch of". Again, if that were the case, there would be no apparent motivation for children to update the meaning of, e.g., "three" from meaning "more than one" to meaning "more than two" once they learn the full meaning of "two". The most explanatory interpretation is that children do *not* take the counting words to refer to any plurality, nor do they take them as referring to some property of an entity in

a group of entities. By the time they have learned the meaning of the word "two", and possibly by the time they have learned the meaning of the word "one", they have already determined that *the counting words each refer to a specific, unique numerosity*. This knowledge is what motivates them to restrict the reference of each number word so that it does not overlap with the reference of any other number word for which they have determined the full meaning.

### **Comparison of performance on Give-a-number and Point-to-x tasks**

If both the Give-a-number task and the Point-to-x task are valid indicators of the number words a child knows the full meaning of, children's performance on the first task should predict their performance on the second; in the Point-to-x task they should be able to precisely identify the number that they can consistently succeed at giving in the Give-a-number task, but not a higher number. For example, children who can successfully give up to 2 items in the Give-a-number task should be able to reliably identify the picture of 2 items when it is paired with one of 3 items in the Point-to-x task, but *not* the picture of 3 items when it is paired with one of 4 items.

To test if children succeeded more often than chance at identifying pictures of the numbers they could give in the Give-a-number task, within each Group level individual children's percentages of correct responses for the relevant number pairs were determined. For children in Groups 3 and lower, the relevant pair was  $(n, n+1)$ , where  $n$  is the largest number a child could give in the Give-a-number task. For children in Group 4, the relevant pairs were  $(4, 5)$  and  $(5, 6)$  -- since children in this Group could generally give *all* the numbers asked for in the Give-a-number task, they should be expected to succeed at identifying both *four* and *five*. For the relevant number pairs, children's percentages of correct responses on *all* the number questions was evaluated. For example, for a child who could give only up to 2 items in the Give-a-number task, the relevant number pair was  $(2, 3)$ , and the child's



responses both when asked to show the 2 items and when asked to show the 3 items contributed to the percentage. It was reasoned that a child who knows the meaning of the word "two" should be able to not only get the questions for 2 items correct, but also the questions for 3 items by a process of elimination, since she knows already that "three" is a *number* word and should therefore know that it refers to some numerosity other than *two*. A *t*-test was performed on each Group's percentages to see if children succeeded at identifying the number of items they could give in the Give-a-number task more often than the chance rate of 50%. To test whether children failed at identifying higher numbers than they could give in the Give-a-number task, only those sessions for Groups 3 and lower are relevant, since the children in Group 4 could give all the numbers asked for in the Give-a-number task. For this analysis, the relevant number pair for each Group is  $(n+1, n+2)$ . Within each Group, individual children's percentages of correct responses for all number questions on the relevant pair were determined, and a *t*-test was computed to see if these percentages differed significantly from the chance rate of 50%.

The breakdown of these analyses by Group level is shown in Figure 2. As can be seen, each group succeeded more often than chance on the number pairs that tested knowledge of the highest number they could give in the Give-a-number task, while performing at chance on the pairs testing knowledge of higher numbers.<sup>10</sup> Thus, children are not succeeding at recognizing larger numbers in the Point-to-x task than they are capable of giving in the Give-a-number task, though they are succeeding at the numbers they *can* give. This correlation is assurance that both the Give-a-number task

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<sup>10</sup>For Group 1's performance on the pair (1, 2),  $t(5) = 18.183$ ,  $p < .0001$ , one-tailed. For Group 2's performance on (2, 3),  $t(8) = 10.921$ ,  $p < .0001$ , one-tailed. For Group 3's performance on (3, 4),  $t(2) = 4.000$ ,  $p < .05$ , one-tailed. For Group 4's performance on (4, 5),  $t(10) = 6.516$ ,  $p < .0001$ , one-tailed; for their performance on (5, 6),  $t(10) = 3.491$ ,  $p < .005$ , one-tailed.

and the Point-to-x task really are valid indicators of the number words a child knows the precise meanings of.

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INSERT FIGURE 2 ABOUT HERE

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### **Sequence and time span of acquisition for individual children**

Table 3 shows a "time line" for each child, indicating the Group level each child was classified into for each session tested, and for approximately how long a child had been in the study at each session.

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INSERT TABLE 3 ABOUT HERE

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First of all, it can be asked if the results of the Give-a-number task support my previous finding (Chapter 1) that children learn the meanings of smaller number words sequentially, and then learn the meanings of the remaining number words within their counting range all at once, when they learn the cardinal word principle. Two predictions follow from this hypothesis. First, that children who succeed only at giving smaller numbers have different numbers at which they start to fail, but children who succeed at giving larger numbers succeed at giving *all* the larger numbers. Second, that children who succeed at giving only smaller numbers do not know the cardinal word principle, while those succeeding at giving larger numbers do know the cardinal word principle.

1. *Do children learn the meanings of larger number words all at once?*

Some children knew the full meaning only of the word "one" (i.e., could only successfully give 1 item when asked), some knew the full meaning of "one" and "two" (could give 1 or 2 items when asked), and some knew "one", "two", and "three" (could give up to 3 items when asked, but not more). But every child who could give up to 4 items when asked (there were 11 different children observed at this stage) could *also* give 5 items when asked,<sup>11</sup> even the 5 children who had just moved into Group 4 from a lower group. That is, every child who knew the full meaning of "four" also knew the full meaning of "five". Furthermore, of these 11 children, 8 were also asked to give 6 items at the end of the Give-a-number task, including 4 of the 5 children who had just moved up to Group 4. Six of the 8 succeeded in giving 6 items, including 3 of the 4 that had just moved into Group 4 from lower Groups (these three came from Groups 1, 2, and 2). The 2 who failed used counting in attempting to get the correct number, but made mistakes in their counting and so gave the wrong number, and then got lost in a swamp of recounting and adjusting (in the correct directions) which was hindered by further mistakes in counting, until they finally just gave up. The fact that every child who knew the meaning of the word "four" also knew the meaning of the word "five", and most or all (depending on one's interpretation of the two failures) also knew the word "six", *even those who had in the previous session only known the meanings of the words up to "one" or "two"*, indicates that children do learn the full meanings of the larger number words simultaneously, while learning the

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<sup>11</sup>There was one exception to this. One child (*not* one of the children who had just moved into Group 4) could give up to 4 items but failed to consistently give 5 items. She used counting to give the number asked for, and once gave 5 correctly, but on other trials made mistakes in her counting and, though she did adjust in the right direction, failed to adjust the final number to exactly what was asked for and so failed by the criterion of success laid out. The fact that she consistently used counting to solve the Give-a-number task, and adjusted in the correct direction, suggests that she did know the counting system, and that her difficulty with giving 5 was one of performance rather than of competence.

meanings of smaller number words sequentially.

*2. Is acquisition of the larger number words due to the cardinal word principle?*

If acquisition of the cardinal word principle is what gives children knowledge of the meanings of the larger number words, then children who know the meanings of the larger number words should know the cardinal word principle, while the other children should not. To test this, we can examine children's strategies in the Give-a-number task. In particular, if the 11 children who know the meanings of all the number words (Group 4 children) know the cardinal word principle, they should use counting when giving items from the pile, while the 14 children in the other three Groups should not. Figure 3 shows the mean percentages of counting aloud from the pile by children, when asked for small (2 or 3) and large (4 or more) numbers of items. Since there were no significant differences in amount of counting from the pile between any of the lower Groups, results are collapsed across them.

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 INSERT FIGURE 3 ABOUT HERE  
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When asked for small numbers of items (2 or 3), Group 4 children counted aloud on an average of 42% of their trials, while lower-Group children on average counted aloud on only 11%,  $t(23) = 2.266$ ,  $p < .02$ , one-tailed. When asked for larger numbers (4 or more) of items, Group 4 children counted aloud on an average of 70% of their trials, while children in the lower Groups did so only 5% of the time on average,  $t(21) = 6.034$ ,  $p <$

.0001, one-tailed.<sup>12</sup> In addition, on those few occasions when children in the lower Groups did count aloud (only 5 children did so), they usually did not stop at the number word asked for, indicating that their counting was not due to knowledge of the cardinal word principle. On average, they stopped at the number word asked for only 40% of the time on those few trials in which they counted items from the pile, while the 9 children in Group 4 who counted aloud (the tenth always gave the correct number of items silently one at a time, perhaps counting them in his head) stopped at the number word asked for 85% of the time,  $t(13) = 3.583$ ,  $p < .005$ , one-tailed.

It thus appears, corroborating my Chapter 1 results, that it is the acquisition of the cardinal word principle that allows the child to determine the meanings of the larger number words. This makes sense; there is a wealth of independent evidence that infants and children have a means, other than consciously counting, of precisely determining the numerosities of sets of three or fewer items, but not higher numerosities (Antell & Keating, 1983; Chi & Klahr, 1975; Silverman & Rose, 1980; Starkey & Cooper, 1980; Strauss & Curtis, 1981). That is, the only means children could have of determining the exact quantity of a set of more than three items is to count it; and they do not possess this means until they acquire an understanding of how counting determines the numerosity of a set. The only way for them then to learn the precise meanings of the larger number words is to understand counting.

The longitudinal data reveal that not all children appear to make this general induction at the same point in the number word sequence, disconfirming my conjecture in Chapter 1 that all children make it at the

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<sup>12</sup>Two of the children in the lower Groups were not asked for larger numbers because they very consistently failed to give even three items; the analysis of counting for the larger-number questions therefore includes only 12 of the 14 children observed at these lower Group levels.

same point, after learning the meaning of the word "three". Though no children learned the full meaning of the word "four" or higher before learning the counting system, some children apparently learned it even sooner. Not all the children were observed to have gone through a stage in which they were in Group 3 -- that is, at which they knew the meanings of the words up to "three" but not higher. Three children were observed at the Group 2 level in one session, and then at the Group 4 level in the next, having learned the cardinal word principle already. There was even one child who went straight from Group 1 to Group 4. It appears that different children may follow different learning paths when acquiring the cardinal word principle. Of course, it could be that these children did go through all stages, but that some went through entire stages between two consecutive sessions. At the least, then, we can say that some children very quickly pass through these stages, while others go for months at each stage.

#### *Time span of acquisition*

The time span of acquisition appears to be an extended one. Of the 6 children who were first observed at the Group 1 level, 4 were followed for up to 8 months each; only one of these achieved an understanding of the counting system in that time and had gone on to be classified in Group 4 (he did so in 5 months, and was not observed at any of the intermediate Group levels). The other 3 had in all that time moved only to Group 2, being observed for an average of 4 months before doing so (3 months, 4 months, and 5 months respectively; and it is unknown how long they had already been at the Group 1 level before the study began). The 2 who were only followed up for 2 months did not change groups. Of the 6 children who were first observed at the Group 2 level, 4 were followed for up to 8 months each; of these, 3 made the transition to Group 4, taking an average of 4 months each to do so (2 months, 5 months, and 5 months respectively; again, it is not known how long they had already been at the Group 2 level before

the study began). Interestingly, none of these 4 children was observed at the Group 3 level. The fourth child had just made the transition to Group 3 after *8 full months* at the Group 2 level. Of the 2 children initially observed at the Group 3 level, one made the transition in the second session to Group 4, while the other was observed for only 2 months and did not change groups in that time.

From this variable data, it can be estimated that children spend at least 8 to 10 months on average at the Group 1 and Group 2 levels of knowledge, before moving on to the Group 3 and/or finally the Group 4 level, though the time span of individual stages appears to vary extensively from child to child. That is, it appears to take in the neighborhood of a year on average for children, *after* already knowing the meaning of at least one number word, *and* knowing that the number words refer to numerosities, to learn the cardinal word principle -- that is, to understand the way in which the counting system represents numerosity.

### **Performance on the How-many task**

How does children's performance on the How-many task change as their performance on the Give-a-number and Point-to-x tasks reaches the Group 4 level? In particular, are children more likely to give the last word used in the count when asked "how many" following counting once they have acquired the cardinal word principle than before they have acquired it? One would expect this to hold if the How-many task is a reliable indicator of children's understanding of the cardinal word principle; but there are reasons why it might not be so. First of all, children will often recount a set of items when asked "how many" following counting. Recounting the items in such a situation is, in fact, one quite adequate means of indicating their numerosity -- indeed, one way of asking a child to count some items in the first place is to ask how many there are. Thus children may perfectly well understand that the last number word indicates the numerosity of the set, but prefer to

indicate how many there are by counting again instead. On the other hand, when counting is modeled for children by parents, teachers, and so on, the last number word in the count is often emphasized and repeated. Adults will often count items and then say "So how many are there? There are *three!*" Children might then learn to give the last number word in response to a "how many" question, but without knowing that it indicates the numerosity of the items counted.

In Chapter 1, I found a significant difference in the percentage of last-word responses following counting between those children who knew the cardinal word principle, and those who had yet to learn it -- the former gave more last-word responses than the latter. This result was first obtained in Experiment 1, and then replicated in Experiment 3. To see if those results replicate in this study, the relevant analysis is to look at children's performance on the How-many task for their initial session only, since my previous experiments were not longitudinal -- each child was tested only once. Individual children's percentages of responses which were last-word responses were computed; the means for each group are shown in Figure 4.

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INSERT FIGURE 4 ABOUT HERE  
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As can be seen, children who understood the counting system -- those children in Group 4 -- did give more last-word responses than did the other children. Their mean of 57% is significantly larger than the mean for the other three Groups combined (16%),  $t(18) = 2.441$ ,  $p < .02$ , one-tailed. Thus, the finding that children who understand the counting system (as determined by their performance on the Give-a-number task) tend to give more last-word responses when asked "how many" following counting is a



robust one.

However, when individual children are looked at over time, it becomes evident that the How-many task should not be considered a *defining* criterion of a child's understanding of the counting system. Five of the 14 different children observed in Groups 1 to 3 began giving last-word responses a majority of the time *before* entering Group 4. The predominant pattern (4 of the 5 children) was to give 0% to 20% last-word responses for the first 3 or 4 sessions, and then to suddenly start giving 60% to 100% last-word responses in all successive sessions. These children thus appeared to have learned a "last-word *rule*" (that is, to respond with the last word without understanding its significance) at some point before acquiring an understanding of the cardinal word principle. This accords with previous findings (e.g., Fuson, 1988; Chapter 1) that some children learn that the last number word is important in some way before learning *how* it is so. Conversely, 4 of the 11 children observed at the Group 4 level did *not* give last-word responses a majority of the time -- in fact, did not give *any* last-word responses, but instead consistently recounted the set when asked "how many". So some children who have acquired an understanding of the cardinal word principle prefer to indicate the numerosity of a set by counting it, even if they have already just done so.

What about those children observed both at lower Group levels and then later at the Group 4 level -- did their percentage of last-word responses on the How-many task increase when they acquired an understanding of the counting system? There were 5 children who were followed through their entry into the Group 4 level of knowledge. Each of their percentages of last-word responses immediately before and after entering the Group 4 level was determined, and the difference between these two percentages was calculated. A *t*-test was then performed on these differences to see if they were significantly greater than zero. This was indeed the case; their mean

percentage of last-word responses before entering Group 4 was only 8%, while their mean percentage after entering Group 4 was 44%, for a mean increase of 36 percentage points ( $t(4) = 2.250, p < .05$ , one-tailed). These children on average, then, did increase their percentage of last-word responses significantly after learning the counting system. However, this result was due entirely to 3 of the 5 children, who followed the same pattern as did those children *not* in Group 4 who had acquired a "last-word rule". Acquiring an understanding of the counting system, then, did seem to precipitate these three children's increase in last-word response rates. The other two children, however, continued to give no last-word responses at all, preferring instead to recount the items in response to every "how many" question, as they had done in previous sessions.<sup>13</sup>

The How-many task is therefore not as reliable an indicator of whether individual children understand the counting system as are the Give-a-number and Point-to-x tasks. Although a high proportion of last-word responses is correlated with an understanding of the counting system, and in some children does seem to be precipitated by their acquisition of the cardinal word principle, it can also occur independently. Strong conclusions should therefore not be made about children's understanding of counting on the basis of this task alone.

### Summary

In sum, then, the results of this study show (a) that children learn the number words sequentially up to the words "two" or "three", and then

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<sup>13</sup>It is possible that these two children had experienced the How-many task enough times with the experimenter that they had made a routinized game of it, and so continued to give the same kind of response they had given before learning the counting system; they were each tested at lower Group levels for 3 sessions before entering Group 4. In contrast, the 3 children whose percentages increased had been tested for only 1 or 2 sessions before reaching Group 4 and had perhaps not become as accustomed to responding in a certain way.

acquire the cardinal word principle, which allows them to determine the meanings of the larger number words; (b) that children know that the number words refer to specific *numerocities* at a very early stage of counting (that is, by the time they know the meanings of the words "one" or "two"); and (c) that, despite this early knowledge, it takes children a surprisingly long time (on the order of a year) to learn how the counting system represents number.

## General Discussion

### Components of meaning of the number words

It's an interesting question how children come to know so early that the counting words are *number* words. However children are managing this is the key to how they are solving the learning problem described in the introduction -- how they are able to learn that the number words refer to some property of *sets* of entities, rather than to some property of individual entities. Knowing that children learn this so early can help point to how children are in fact achieving this.

One possibility is that children induce from the syntax of the number words that they pertain to quantity. This is known as syntactic bootstrapping, a theory developed by Gleitman and colleagues (e.g., Gleitman, 1990). Studies have shown (e.g., Naigles, 1990) that children as young as 18 months can infer some semantic properties of novel words from the syntactic frames they occur in. These studies showed that children infer a novel verb to refer to either a transitive or intransitive action depending on whether it is presented in a transitive or intransitive frame. Closer to home, children have also been shown to infer the meaning of a novel noun in part on the basis of whether it appears with a determiner (the article "a") or not (Katz, Baker, & Macnamara, 1974). In this case, children who were shown a doll and told "This is Wug" took the word to be the proper name of the doll,

while children told "This is a wug" took the word to refer to the kind of doll.<sup>14</sup> Thus it is possible that the number words' syntactic status as determiners may give children information about their semantics. If this is so, it raises intriguing questions about the knowledge 2-year-olds may have of the semantic properties of determiners. The hypothesis that young children use syntactic properties of the number words to determine their semantics hinges on four assumptions:

1. There is a reliable relationship between the syntax and the semantics of determiners -- that is, determiners must have some group of syntactic properties not generally shared by other parts of speech, and they must also have a common semantics at least at some level, so that it is possible to predict (with at least a fair degree of success -- the correlation need not be 100%) the semantics of a word simply by knowing that it has the syntax of determiners.
2. Children are sensitive to these syntactic properties by 2-1/2 years of age -- that is, they must be able to pick out the class of determiners from their syntax.
3. Two-and-a-half-year-olds are also sensitive to the semantics of at least some determiners, so they have the information necessary to infer the semantics of a novel determiner from its syntax.
4. Two-and-a-half-year-olds are able to determine that number words belong to the syntactic class of Determiner.

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<sup>14</sup>The same distinction did not occur when children were shown decorated blocks instead of dolls, however; in this case, they always took the novel word to refer to the kind of item, showing that children are sensitive to the conditions in which proper names can apply.

Only some of these assumptions are relatively undisputed at present. It is not my purpose here to put forth an airtight argument in favor of this particular syntactic bootstrapping hypothesis. Rather, I merely suggest that this is a plausible place to initially look for the answer to why very young children know that each of the counting words refers to a distinct, unique numerosity. Below, I briefly discuss what evidence there is to date for each of the above four points.

1. *Determiners share a common syntax and a common semantic role*

This assumption seems well-founded. First of all, determiners have a defining set of syntactic properties. They can only come before nouns or adjectives, not after them. Some can also appear as noun phrases in their own right, unlike adjectives (e.g., "I want *three*", "I want *that*"). Also unlike adjectives, a determiner (and only a determiner) *must* appear before a singular count noun, for example, "*A/the/this/that/some/one/another/no* dog bit the mailman".<sup>15</sup>

Second, determiners by and large do share a common semantic role. Determiners are intricately linked up to quantification; the majority of determiners obviously pertain to quantity, such as "some", "all", "both", "many", "much", "little" (as in "a little bit"), "few/a few", "lots", "another", and the number words. Furthermore, "a", one of the most common determiners in young children's utterances, can only be used to pick out a single individual (similarly "another"), while "these", "those", "both", "all", and "a few" can only be applied to a plurality of individuals. If, then, children are able to infer semantics of determiners from their syntax, quantity would seem a highly salient semantic aspect.

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<sup>15</sup>Of course, not all determiners can fill this role -- those that are limited to plural count or mass nouns ("all", "both", "many", "much", "these/those", "few/a few", "lots", etc.) obviously cannot precede a singular count noun.

## 2. *Children's sensitivity to the syntax of determiners*

This assumption too has strong independent evidence. Gordon (1987) and Valian (1986) have shown that even 2-year-olds have knowledge of the syntactic category *Determiner*. Valian (1986) analyzed speech transcripts of 6 children, and found that children as young as 24 months of age honored the word ordering of determiners relative to nouns, adjectives, or pronouns; in children's utterances, determiners never followed these parts of speech.

Gordon (1987) further found that not only do children preserve the word ordering of determiners relative to adjectives, but they also distinguish between the two categories, contrary to the claim, proposed by Brown & Bellugi (1964), that children consider both adjectives and determiners to belong to a single category, "modifier".<sup>16</sup> Gordon analyzed speech transcripts of 2 children and found that, for children as young as 24 months, in utterances where the noun was either a mass noun or plural count noun (when a determiner is optional), the production of an adjective decreased the likelihood of production of a determiner, but in utterances where the noun was a singular count noun (when a determiner is obligatory), production of an adjective did *not* decrease the likelihood of production of a determiner. The children evidently knew that a determiner (but not an adjective) is necessary before singular count nouns, and not before plural count or mass nouns. If young children considered adjectives and determiners to belong to the same category "modifier", then when they learned that singular count nouns cannot appear alone but must be accompanied by a modifier, either an adjective or a determiner should do the trick. But the presence of an

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<sup>16</sup>Note that simply preserving an ordering between two sets of words does not mean that they belong to different syntactic categories; among adjectives, some always come before others. We always say "the sweet little black puppy", never "the little black sweet puppy" or "the black little sweet puppy"; see Bever, 1970.

adjective in the child's utterance does not reduce likelihood of a determiner when a determiner is obligatory; ergo, the child does not consider adjectives to be fulfilling the same linguistic function as determiners. Thus, children clearly are sensitive to the syntax of determiners.

### *3. Children's sensitivity to the semantics of determiners*

This claim too has some support. What must be shown is that there are determiners that 2-1/2-year-olds know pertain to quantity and so could use as a basis for inferring that other, unknown, words, possessing the same syntactic properties, also pertain to quantity.

Gordon's (1987) paper discussed above shows that children are sensitive to the relationship of determiners to quantity; children knew that a determiner was obligatory before singular count nouns, but not before plural count or mass nouns.

In one experiment, Soja (1990) showed that children understood some of the quantificational semantics of the determiners "a" and "some". There were two conditions relevant to this discussion. In the "matching syntax" condition, children were shown a novel object and told "this is a blicket". In the "conflicting syntax" condition, children were shown the same object and told "this is some blicket" and "this is blicket". Then, children were shown two new displays; one of a novel object with the same shape as the original but of a different substance, and one with three pieces of the same substance as the original object, where the pieces had a haphazard shape. Children were asked, in neutral syntax, "which is the blicket?" In both cases there was a tendency to pick the individual object. However, children in the conflicting syntax condition had a greater tendency to choose the three pieces than children in the matching syntax condition. There are two possible explanations for this. Either in the conflicting syntax condition the presence of the word "some" caused them to choose the several pieces more than they otherwise would have, in which case we can conclude that they knew that

"some" is used when referring to pluralities or substances; or in the matching syntax condition, the presence of the word "a" caused them to choose the individual object more than they otherwise would have, in which case we can conclude that they knew that "a" is used when referring to singular individual entities. Both explanations may be true. Either way, these results show that for one or the other or possibly both of the determiners "a" and "some", children are aware of the aspects of quantification that apply to them.

Furthermore, data obtained by Soja in several experiments (Soja, 1990; Soja, Carey, & Spelke, in press; Soja, personal communication) suggests even more strongly that young children are sensitive to the singularity/plurality restrictions of certain determiners. She elicited quasi-spontaneous utterances of 96 2- and 2-1/2-year-olds by reading them stories during which they were asked such questions as "what is this?" or "what is that green stuff?" about pictures of various singular count, plural count, and mass noun items depicted in the stories.<sup>17</sup> It was found that when children's replies included a singular count noun, 32% of the time they accompanied the noun with the determiner "a", while in their responses with a plural count or mass noun, the determiner "a" accompanied the noun only 3.7% of the time. The children evidently knew that "a" describes *single* items, not pluralities (or substances). A similar story obtains for the words "this", "that", and "some", though they were in general more rarely produced; in children's utterances containing a singular count noun, 1.5% (a total of 118 out of 7798 utterances) were accompanied by the determiner "this" or "that", while only 0.5% (18) of the 3257 utterances containing a plural count noun or mass noun were accompanied by either of these determiners. In addition, 5% (70 out of 1410)

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<sup>17</sup>I am very grateful to Nancy Soja for sharing this data with me. Data for 72 of the children are presented in Soja et al. (in press); data for another 12 children are presented in Soja (1990); data for the remaining 12 children are unpublished. Data for all 96 children are collapsed together for the discussion here.



of their utterances containing a plural count noun were accompanied by the determiner "some" (as well as 4% of their utterances containing a mass noun), compared to only 0.5% of their utterances containing a singular count noun. These percentages suggest that these 2- and 2-1/2-year-olds knew the semantics of "a", "this", "that", and "some" – which ones are restricted to single items, and which to pluralities of items and substances.

It might be objected that these numbers are a result of the fact that a determiner is obligatory for singular count nouns, but not for plural count or mass nouns. Thus, children were more likely to accompany their singular count utterances with "a", "this", or "that" than their plural count or mass noun utterances, not because they knew that these determiners select for singularity, but because they know they must put *some* determiner there. It must first be noted that this cannot in any case explain the results for "some", since they were ten times *more*, not less, likely to use "some" with plurals than with singular count nouns. But there is a stronger reason to reject this explanation, and that is that children were *equally* likely to use the determiner "the" whether before a singular count, a plural count, or a mass noun. Ten percent of children's singular count nouns were accompanied by "the", in comparison to 9% of their plural count nouns and 7% of their mass nouns. Thus, they showed no general tendency to use a neutral determiner with one kind of noun as opposed to another kind. We must then assume that any preference to use the selective determiners with certain kinds of nouns and not others is due to a sensitivity to the kinds of nouns they select for.

These data indicate that children as young as 2 years of age know for several determiners that they pertain to quantity, and so they have a basis upon which they could infer that novel words with the same syntactic properties as these words might also refer to quantity.

#### 4. *Children's knowledge that the number words are determiners*

The above studies either did not include number words at all, or did not separate out the number words from other determiners, so it cannot be concluded at this point that young children know that the number words are determiners. But they do know the syntactic properties of determiners, and it is plausible that, in hearing number words produced according to all the syntactic constraints of determiners, children categorize these words as determiners. Like all determiners, number words always come before nouns and adjectives, and can appear as NPs (e.g., "I want *three*", "I want *that*"). "One" can take the place of the determiner in constructions in which children know a determiner is obligatory (e.g., "*one* dog"). Children may be able to capitalize on these regularities and infer the grammatical category of the number words.

Even once all the above points have been supported, there is still another question -- how do children know not only that the number words pertain to quantity, but that each one, unlike other determiners, refers to a specific, unique numerosity? In this task, both the syntax of number words and the contexts in which they are used may play a crucial role. They are first of all used to describe pluralities of items, never a single item (except for the word "one", for which the opposite holds). Second, they are never applied to mass nouns such as substances, but only to count nouns. This could tell children that they have something to do with discrete items. If children already know from their syntax that they pertain to quantity, this could then tell them that the number words relate to the quantity of discrete items. This, of course, does not yet distinguish them from determiners such as "many", "a few", "fewer" and "both", all of which also apply only to the quantity of discrete items. Here, only context will be able to distinguish them. Imagine a case where a child is sitting at the table, and takes some cookies off a plate. A parent then leans over and says "No, you're not allowed to have *three*

cookies, just *two*", while removing one cookie from the child's plate. Here, the child has seen one number word applied to a set of discrete entities, and has also seen a (very salient) operation performed on the set effecting the smallest possible change in the numerosity, followed by the application of a *different* number word. Situations such as this might help the child infer that even for very small changes in numerosity, new number words must apply, and therefore that they do not apply to vague, indeterminate amounts as do words such as "many", "a few", and "some". Learning this for a few of the number words would likely allow the child to infer this about all the number words in general; there is independent evidence that 2-1/2-year-olds know that the counting words all belong to a single class. For example, when asked "how many" of something there are, they will almost always respond with a number word or words rather than with other words (e.g., Gelman & Gallistel, 1978; Fuson, 1988; Chapter 1). Furthermore, they know the list of number words for counting (up to some limit); the number words occur together in this list so children might reasonably be expected to infer that general properties of some of the number words might apply to the rest as well.

### **Time span of knowledge acquisition**

The findings of this study suggest that children do have difficulty learning the counting system. It appears to take them almost a full year or perhaps longer, after knowing that the number words refer to numerosities, to learn *which* words refer to *which* numerosities; that is, to learn how the counting system represents numerosity. This suggests that children may have difficulty in mapping their own representation of numerosity onto the counting activity, and indicates that young children's concept of number may take quite a different form than the way number is represented in the counting system. Below, I discuss one possible way in which infants' and young children's representation of number could be different from that

embodied by the counting system.

There is a wealth of evidence that animals represent number, and that their representation may be embodied in an analogue form. If this is indeed the case, it is quite likely that humans, too, are equipped with such a representation of number. Counting, however, represents number in a crucially different way than do analogue mechanisms; this could make it a difficult process to map an analogue concept of number onto counting.

To fully explore this possibility, it is worth going, in some detail, into the literature on numerical abilities of animals. I shall first summarize some of the most intriguing and suggestive empirical results, then describe an empirically supported model of the animals' quantification mechanism which embodies an analogue representation of numerosity, and finally discuss the ways in which such a mechanism differs from counting in its representation of number. If animals as disparate as rats, pigeons, raccoons, parrots, canaries, and chimpanzees (the kinds in the experiments below) possess such a representation of numerosity, it is reasonably likely that humans also come equipped with a similar such representation. This would then lend plausibility to the claim that children's difficulty in learning the counting system may be due to differences between an inherent analogue representation of number and the explicit representation of the counting system.

#### *Animals' numerical abilities*

Many fascinating results have been obtained in studies of animals' sensitivity to number (for a stimulating and detailed overview and discussion of the literature, see chapter 10 of Gallistel, 1990; see also Davis & Perusse, 1988). First of all, rats are able to determine the number of presses they have pressed on a lever, up to at least 24 presses, as in the following experiment: In a paradigm developed by Mechner (1958), rats were trained to press two levers in order to obtain a food reward, with the following reward

schedule. On some percentage of the trials, after  $n$  presses on Lever A occur, a press on Lever B will yield a food reward, with a fixed penalty for pressing Lever B too early (the counter on Lever A is reset to zero). On the remaining trials, the  $n+1^{\text{th}}$  press on Lever A will yield the reward. The rat, of course, does not know which kind of trial it is running on any given trial. By varying the percentage of trials on which switching to Lever B is necessary to obtain the reward, one can vary the extent to which the rat is willing to gamble that it might be pressing Lever B too early (when nearly all the rewards come by making an  $n+1^{\text{th}}$  press on Lever A, the rat will make quite certain that it has pressed more than  $n$  times on Lever A before switching to Lever B; when the reward is always obtained by pressing Lever B after  $n$  presses on Lever A, the rat will be more willing to risk an early switch to Lever B in the hope of avoiding useless presses on Lever A). Of most interest to us are the rats' responses on the trials when they must switch to Lever B in order to obtain the reward, because these trials reveal their judgements about when they have pressed a sufficient number of times on Lever A. On these trials, it turns out that the rats' median number of presses on A is  $n$  plus some *constant* number of presses, independent of the value of  $n$ . The percentage of the time they must switch levers to get the reward determines how many extra presses they give; when they must switch to Lever B 50% of the time, rats' median number of presses on A were 3 more than  $n$ , regardless of whether  $n$  was 4 or 24 (a higher percentage of rewards on Lever B yielded a lower number of presses on A, and vice-versa). This shows that rats can determine quite accurately when  $n$  presses have been achieved. Furthermore, the results show that the rats are not simply *timing* how long they press on Lever A, and then going for Lever B after a fixed amount of time has passed. When rats are trained to press for a specific amount of time on Lever A before going to Lever B, their response is to press some extra *proportion* of the required time interval. That is, they will press, say, 10%

longer than necessary (the actual percentage will vary with the percentage of time they must switch levers to obtain the reward); if the required time is 10 seconds they will press for 1 additional second on Lever A, whereas if the required time is 50 seconds they will press for an additional 5 seconds. If, then, the rats were using a timing strategy when rewarded for a specified *number* of presses on A before going to B, their responses should tend to be (roughly) some *proportionate* number greater than *n*, not a constant number greater. As well, when rats' deprivation before being run was varied, rats pressed much more quickly with increased deprivation, significantly decreasing the time spent on a trial under these conditions, but making no appreciable change in the number of presses (Mechner & Guevrekian, 1962).

Rats have also been trained (Davis & Bradford, 1986) to turn down the third, fourth, or fifth tunnel on the left in a maze, and will do so even when the spatial configuration of the maze is different each time, where the distance between the tunnels, and the number of corners that must be turned before the 5th tunnel on the left is reached, is varied. Again, rats could not simply be running for a fixed length of time before turning left, or determining the extent to which they felt fatigued by the run, etc. They must have been encoding the numerosity of the tunnels on the left in order to succeed at the task. Furthermore, training effects were extremely robust -- following inactive periods of 12 or in some cases even 18 months, successful performance was still found.

Pigeons have been trained to determine the number of pecks given to a key, for up to 50 pecks (the most tested). In this experiment, there were three keys, and the pigeon was to peck at the center key until its light went out and the other two keys lit up. The experimenter determined when this happened -- either after 50 pecks, or after some smaller number of pecks that varied. To obtain a reward, the pigeon had to peck the right key if it had given 50 pecks at the center key, and the left key if it had given a smaller number of pecks at

the center key. It was found that the pigeons did not fall below 60% correct in their choice of key until the alternate number reached 47; that is, they were capable of discriminating 47 pecks from 50 pecks just under 60% of the time. They discriminated 40 pecks from 50 pecks about 90% of the time (Rilling & McDiarmid, 1965). Further experiments (Rilling, 1967) revealed that the pigeons were using number of pecks as the determinant, not time interval of pecking that elapsed before the center key's light went off.

Raccoons have been shown to discriminate between small numerosities of objects, as in the following experiment (Davis, 1984): A raccoon was repeatedly presented with as many as 5 plexiglass cubes, each containing between 1 and 5 objects. In the training phase, these objects were first grapes, then raisins, and finally toy bells. The raccoon was trained to always identify and open the cube containing 3 such items. Non-numerical cues such as size, stimulus density, odor, and location of the target cube were controlled for. The raccoon was able to learn this task, and the results easily transferred to novel stimuli (small metal balls). This study is interesting because it involves an animal quantifying over a set of simultaneously presented discrete objects, rather than over events. There are two main differences that fall out from this. First, the pigeons' pecks and the rats' bar-presses are all temporally sequential, physical events of the animal; similarly, in the Davis and Bradford (1986) experiment where the rats had to always turn down, say, the 5th tunnel, it could have been the 5th "reaching-a-tunnel-on-the-left" event that was the relevant one. But the ability to reliably distinguish a group of 3 objects from groups of 1 to 5 objects cannot rest in any way on some internal event-state of the raccoon itself; it must rest solely on the properties of the groups of items (their numerosities in particular). Second, the pigeon and rat experiments described above may have tapped a sensitivity to the *ordinality* of an event -- its position in a sequence of events -- while this experiment showed a sensitivity to the *cardinality* of a set of entities. At any rate, this

experiment shows that animals' ability to determine numerosity applies to more than just a single form of presentation of entities.

Animals other than raccoons have also been shown to distinguish different numerosities of simultaneously presented objects. Matsuzawa (1985) trained a chimpanzee to associate Arabic number symbols with discrete numerosities of objects, for the numbers 1 through 6. When presented with a set of a certain numerosity, the chimp had to pick out the correct symbol. The chimp was trained on red pencils, and the results generalized to several other items, blue gloves being one example.

An ability to discriminate different numerosities of objects has been shown in birds as well as in mammals. Pepperberg (1987) trained an African Grey Parrot to say the appropriate number word when presented with up to 5 objects; the parrot was shown to succeed when presented with novel objects as well as trained objects. In a most interesting experiment, Pastore (1961) trained canaries to select an object based on its ordinal position in an array. Out of 10 partitioned cubicles along a runway, the canaries had to walk along the runway and choose the cubicle that held either the fourth or the fifth aspirin. The ordinal position of the cubicle containing the relevant aspirin varied from trial to trial, to rule out any regularity of distance from the starting point as the basis of the birds' choices. Even more impressively, to control for the possibility of the birds' using rhythm as the basis of their judgements rather than ordinal position, different numbers of aspirins were placed into each cubicle on different trials as well. Thus, both the space between baited cubicles and the number of baits per cubicle varied. The birds were clearly succeeding on the basis of ordinal position in a visual series.

Humans, too, have some automatic, unconscious means of determining the absolute and relative frequencies of events. Studies have shown this for different entities, including letters, words, colors, and even different kinds of



lethal events (e.g., Attneave, 1951; Hasher & Zacks, 1979; Hintzman, 1969; Lichtenstein, Slovic, Fischhoff, Layman, & Combs, 1978; Lund, Hall, Wilson, & Humphreys, 1983; Shapiro, 1969). It seems likely that we are born with a mechanism that computes event frequency (Hasher & Zacks, 1979). If so, this would plausibly be the same kind of mechanism that underlies other animals' numerical abilities.

*An analogue model of the mechanism underlying animals' numerical abilities*

These results indicate that all manner of vertebrates have some mechanism that determines numerosity. For rats and pigeons in particular, an ability to determine quite high numerosities (up to at least 24) has been shown. There is some very interesting data indicating that this quantification mechanism may embody an analogue representation of number. Meck & Church (1983) suggest that the *same* mechanism underlies both animals' ability to determine numerosity, and their ability to measure duration. Briefly, their proposed mechanism (based on a model for measurement of temporal intervals developed by Gibbon, 1981) works as follows: a pacemaker puts out pulses at a constant rate, which can be passed into an accumulator by the closing of a mode switch. In its counting mode, every time an entity is experienced that is to be counted, the mode switch closes for a fixed interval, passing energy into the accumulator. Thus the accumulator fills up in equal increments, one for each entity counted. In its timing mode, the switch remains closed for the duration of the temporal interval, passing energy into the accumulator continuously at a constant rate. The final value in the accumulator can be passed into working memory, and there compared with previously stored accumulator values. In this way the animal can evaluate whether a number of events (e.g., runs through a maze, presses of a lever, tunnels on the left side of a maze, etc.) or the duration of an interval is more, less, or the same as a previously stored number or duration that is associated with some outcome, such as a reward. The mechanism has several

switches and accumulators, and so can operate in both modes simultaneously, and time or count several temporal durations or sets of stimuli simultaneously.

Evidence that the same mechanism underlies both animals' timing processes and their counting processes comes from several sources. First of all, there is a striking similarity in the psychophysical functions for the discrimination of duration and the discrimination of numerosity. Second, amphetamine, known to increase rats' estimates of duration, increases their measure of numerosity by the same factor, strongly suggesting that it is the same mechanism being affected. The effect would be caused in the model by amphetamine causing an increase in the rate of pulse generation by the pacemaker, leading to a proportionate increase in the final value of the accumulator regardless of the mode in which it was operating. Third, both numerical and duration discriminations transferred to novel stimuli equally strongly, when rats were trained on auditory stimuli and then tested on mixed auditory and cutaneous stimuli. Finally, there are results from an elegant experiment which directly tested the claim that both numerical discriminations and duration discriminations are based on the same variable -- the output value of the accumulator.

The reasoning for this experiment went as follows: if the animal's decision is based on the result of a comparison of the final value of the accumulator with a previously stored value of the accumulator, then, since the accumulator simply spits out a "fullness value", regardless of whether it is in timing or counting mode, one might expect there to be immediate transfer from making an evaluation on the basis of the output of the timing process, to making an evaluation on the basis of the output of the counting process, so long as the final output value of the accumulator in the two cases is identical. For example, a count that yielded the same final value in the accumulator as a previously trained duration might be responded to as if it

were that duration. Their previous results had shown that rats habitually both counted and timed the stimuli they were exposed to. Rats were initially trained to press one of two levers when the shorter of two sounds (2 seconds) was heard, and the other lever when the longer (4 seconds) sound was heard. On previous pilot studies, Meck and Church had determined that the length of time for which the switch was closed for each entity when in the counting mode was 200 milliseconds. This should mean then that the value of the accumulator after counting 10 events should equal the value resulting from timing 2 seconds (10 events times 200msec per event = 2sec); and the value after counting 20 events should equal that resulting from timing 4 seconds (20 times 200msec = 4sec). Accordingly, in the test phase they gave rats, instead of a single sound stimulus of either 2 or 4 seconds' duration, a series of either 10 or 20 sound stimuli of 1 second's duration each (for a total of either 20 or 40 seconds' duration). The prediction held; when exposed to the sequence of 10 sounds, the rats chose the lever for the 2-second duration, while preferring the other lever in response to the sequence of 20 sounds. (Note that for both these sequences, the total duration was closer to that of the 4-second sound than that of the 2-second sound, so if the rats were choosing on the basis of duration, they should have chosen the 4-second lever for both sequences.) Meck and Church concluded that it was indeed the same mechanism underlying both counting and timing processes in rats.

The data supporting the proposed animal mechanism strongly suggest that at least rats, and most plausibly mammals or vertebrates in general (we know pigeons, chimpanzees, raccoons, parrots, and canaries must also have some representation of number) are innately equipped with some kind of analogue representation of number. So too may infants be equipped with an analogue "counting" mechanism. One might ask why, if infants possess a similar mechanism for determining numerosity, can they discriminate only up to 3 versus 4 at most, while in many of the experiments reviewed above

animals showed an ability to determine much higher numerosities. This could well be due to the different nature of the experiments, rather than to any difference in infants' versus animals' representation of number. In the infant experiments, infants were presented with a simultaneous visual display of objects (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981). In experiments in which animals were presented with similar stimuli, notably the experiments with the chimpanzee (Matsuzawa, 1985), the canaries (Pastore, 1961), the parrot (Pepperberg, 1987), and the raccoon (Davis, 1984), the numbers involved were quite small, going up to four or five at most. Infants have not been tested on their ability to encode precise numerosities of sequential events, other than for very small sets of sounds (Starkey et al., 1983, in press) -- it is in experiments involving such events that animals' impressive sensitivity to larger numbers has been shown.

Thus there is no conflict between the results found with human infants and those found with animals; humans may well initially possess the same kind of representation for number as do the animals in the above experiments. Below, I will discuss the ways in which such a representation of number differs from the way in which counting represents number, and therefore why it could be so difficult for children to learn the counting system.

*Differences between analogue representation of number and counting*

When in the counting mode, Meck and Church's (1983) proposed mechanism embodies the counting principles in the following way: there is a one-to-one correspondence between entities to be counted and increments of the accumulator; the states of the accumulator are arrived at in a stable order from count to count (the accumulator always reaches the level of two increments before it does that of three increments); and the final state of the accumulator represents the numerosity of the items counted. In this sense, it is a counting mechanism.

Nonetheless, this mechanism represents numerosity in a very different way than do the number words in linguistic counting. The final increment of the analogue mechanism cannot by itself represent a numerosity; it is the *entire fullness of the accumulator* that represents the numerosity of the items counted. In linguistic counting, the final word alone represents the numerosity of the items counted; it can do this because it is distinguishable from the other words.<sup>18</sup> Thus, in the proposed animal mechanism, number is inherently embodied in the *structure* of the representation, which is itself a magnitude value (the output value of the accumulator), while in counting number is not inherently represented in the structure of each individual symbol.

Each number word obtains its precise meaning by virtue of its ordinal position in the number word list. Thus the ordinality of the number words is the key to the system of their representation of number. They represent cardinalities with a system that does not directly reproduce, but is analogous to, the inherent relationships among the numerosities. The numerosities bear relationships to each other in their *cardinality* that are exactly analogous to the relationships their linguistic symbols bear to each other in their *ordinality*. The numerosity *ten* is five times larger than the numerosity *two*; its linguistic symbol occurs five times later in the number word list than does that for the numerosity *two*. The numerosity *seven* is three units larger than the numerosity *four*; the linguistic symbol for *seven* occurs three positions later in

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<sup>18</sup>Of course, all the words together could also represent the numerosity of the items counted; we can also use linguistic counting as an analogue machine – but then there would be no need to order the words stably from count to count, or to have them differ from each other. Consider the story of the ancient Mesopotamian shepherds who kept a pile of pebbles, as many pebbles as sheep, and put one pebble aside for each sheep that came into the fold at night to determine if all the sheep were there or if some were missing. There was no need to order the pebbles in a consistent fashion; the entire pile together represented the number of sheep in the flock. Similarly, no individual pebble represented any particular numerosity (other than the numerosity 1).

the number word list than the linguistic symbol for *four*. In an analogue representation, the symbols for the numerosities bear exactly the *same* relationship to each other as do the numbers themselves; the representation for *ten* is itself five times larger than the representation for *two*; the representation for *seven* is three units larger than the representation for *four*. What children must learn when they learn the counting system is that specific quantities are represented by symbols that do not themselves bear the same magnitudinal relations to each other as do the quantities they represent. Children must implicitly make the analogy between the magnitudinal relationships of their own representations of numerosities, and the positional relationships of the number words. Discovering an analogical relationship between two sets of symbols is surely not a trivial process. The only wonder is that it takes children *only* a year to achieve this.

It is crucial to note that this proposal is quite distinct from claims that have been made (Cooper, 1984; Strauss & Curtis, 1984) that infants do not know that the numerosities themselves are ordered (e.g., they do not know that *one* is smaller than *two*, *two* smaller than *three*). They may know all this perfectly well. In fact, if infants represent each numerosity in a way that preserves that numerosity's own magnitude relative to other numerosities, as the proposed animal mechanism does, then there is strong reason to think that they *do* possess ordinal knowledge of numbers, since their representation of number inherently encodes the information that *one* is less than *two*, and so on. But just knowing the precise numerical relationships that numerosities bear to each other, and hence knowing that they can be ordered in terms of their sizes, is very different from *representing* or defining each numerosity by a system that works on an analogy to ordering the numerosities in terms of their sizes. Indeed, knowing that the numbers can be ordered would seem to be a prerequisite to coming to grips with such a system.

### Summary: The learning story

So how do children learn the meanings of the number words, and learn the counting system? Very early on, children learn that the counting words refer to numerosities. I have attempted to sketch out how the syntax of the number words could help children achieve this; children know the syntax of determiners, and know that at least some determiners pertain to quantity. This gives them a basis on which to infer that novel words possessing the syntax of determiners, such as the number words, might *also* pertain to quantity. Because we appear to either be born with, or else develop very early on, the ability to discriminate numerosities up to 3 or 4 (Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey et al., 1983, in press; Strauss & Curtis, 1981), children have a means, *other than* linguistically counting, of precisely quantifying small sets of items. This means could well be an analogue mechanism for determining numerosity. Because they already know that the words "one", "two", and "three" refer to *numerosities*, having an independent, precise quantification mechanism for small numbers makes it possible for children to directly map these words onto their correct numerosities. It is "simply" a matter of observing the environment in order to make the correct mapping; they learn to associate each word with their mental representation of its numerosity. But it takes a long while (about a year) for the child to recognize the pattern behind this mapping -- that each successive word in the number word list goes with a value of the quantification mechanism resulting from one successive increment. This could be due to the differences between the two kinds of representations -- children must discover the analogy between the cardinal relationships of their own initial representations of numerosities, and the ordinal relationships of the number words.

As to precisely how this is accomplished, the results of this experiment did not shed new light beyond what was hypothesized in Chapter 1 (also by

Klahr & Wallace, 1973; Schaeffer, Eggleston, & Scott, 1974); namely that once children have some of the smaller numerosities mapped onto the correct number words, when counting sets of items with these numerosities, they come to notice that the last word in the count is reliably the word for the numerosity of the set. Once they notice this regularity, they understand the cardinal word principle, and so learn how the counting system represents numerosity. This gives them a means of precisely quantifying larger numerosities, and so allows them to determine the meanings of the other number words within their counting range.



TABLE 1

*Pairs children were tested on*

Phase 1: Number Can Give In Give-a-Number task	Phase 2: Pairs Tested On In Point-to-x task
1	(1,2) (2,3) (1,3)
2	(2,3) (3,4) (1,3) (1,4)
3	(3,4) (4,5) (1,4) (1,5)
4+	(4,5) (5,6) (1,5) (1,6)

**TABLE 2**  
*Results of Give-a-number Task*

Group #	Initial # Children	Mean Ages	Total # Children	No. of Sessions	# Kids Who	
					Moved Into Group	Mean Ages
1	6	2;9	6	13 (12)		
2	6	2;11	9	24 (21)	3	3;1
3	2	3;2	4	5 (4)	1	3;1
4	6	3;6	11	16 (15)	5	3;3

Bracketed numbers indicate the numbers of sessions for which there is also data for the Point-to-x task.

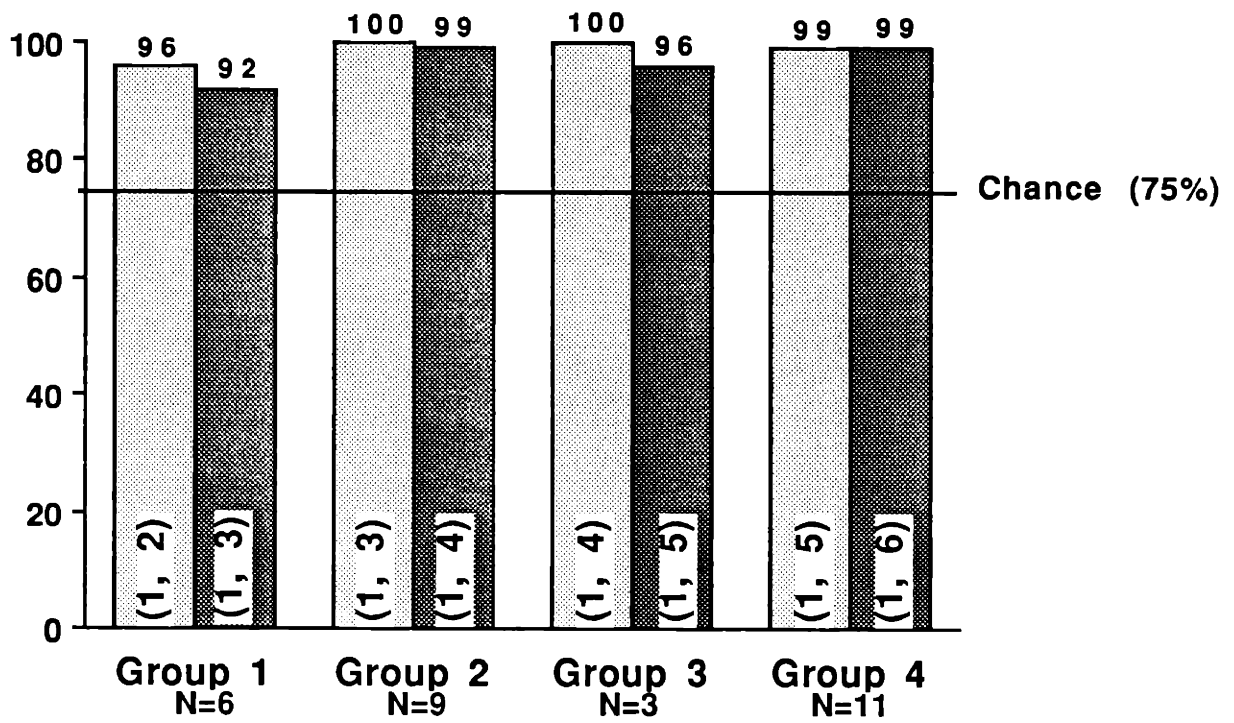
TABLE 3

*Time line: Group levels of each child for each session*

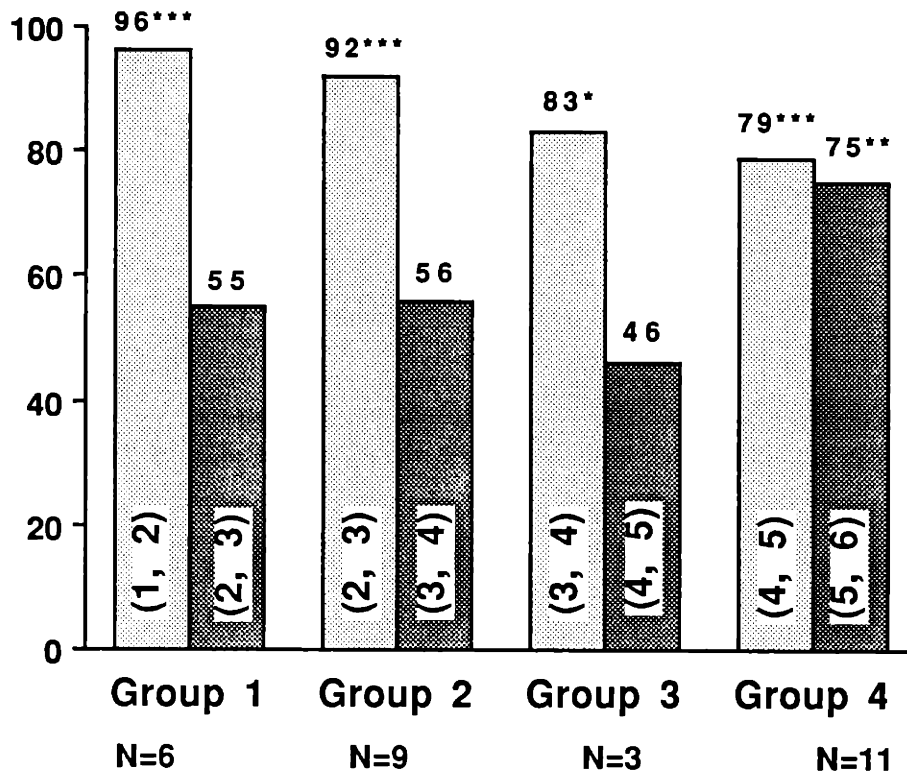
Child	Initial	Months Along in Study							
	Age	1	2	3	4	5	6	7	8
Zohar	2;6	1	-	1	-	3	2	2	
Sarah	2;7	1	1						
Jeremy	2;8	1	-	-	1	-	4		
Ashley	2;10	1	1	-	1	-	2	2*	
Johan	2;10	1	1						
Jesse	3;0	1*	1	-	2	-	2	2	2
Anna	2;6	2	2	-	2	-	2	2	3
Dana	2;9	2	2						
Max	2;11	2	2*	-	2	-	4	4	4
Dennis	2;11	2	-	4					
Allison	3;0	2	2*						
Alasdair	3;3	2	2	-	2	-	4	4	4*
Michael	3;1	3*	4						
Oliver	3;3	3	3						
Geordie	3;0	4	4						
Lawrence	3;0	4							
Johanna	3;7	4							
Lydia	3;8	4							
David	3;10	4							
Yukako	4;2	4							

\*Asterisks indicate sessions without data for the Point-to-x task.

Mean % correct responses on number questions  
in Point-to-x task on (1, n+1) and (1, n+2) pairs



Mean % correct responses on number questions  
in Point-to-x task on (n, n+1) and (n+1, n+2) pairs

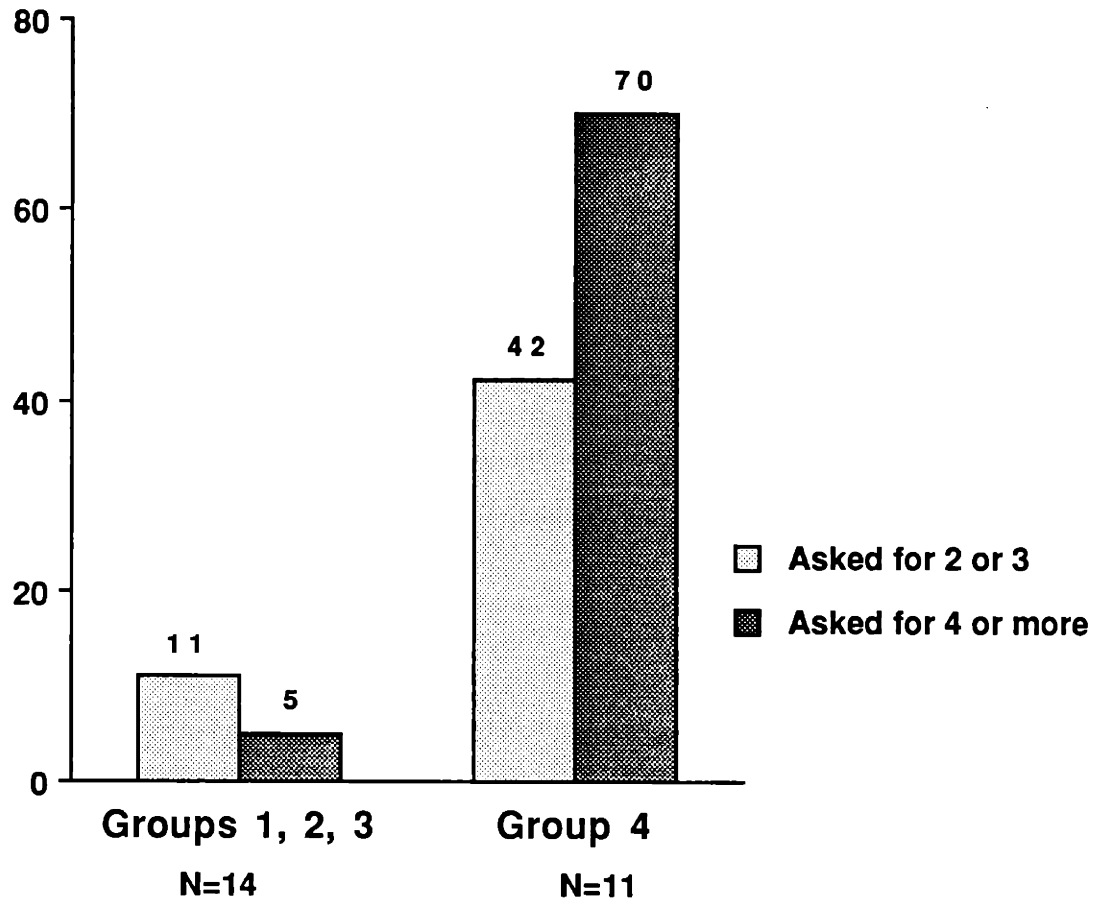


\*significantly greater than chance (50%) at .05 level, one-tailed

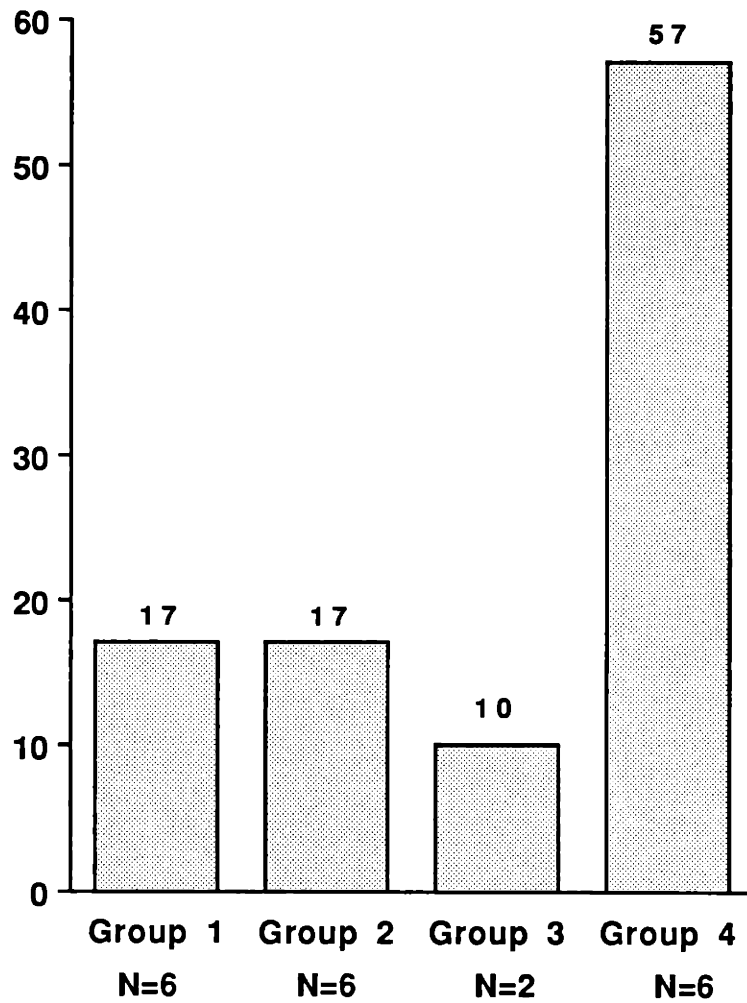
\*\*significant at .005 level, one-tailed

\*\*\*significant at .0001 level, one-tailed

**Mean % of trials on which children counted  
aloud from pile in Give-a-number task**



**Mean % last-word responses on  
How-many task in Initial session**



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