A DESIGN CONCEPT OF A DAMPER SHIELD OF A SUPERCONDUCTING ALTERNATOR

by

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ABSTRACT

One of the biggest problems in designing a large superconducting alternator is the formation of a damper shield. The requirements to the damper shield may be classified as transient stability, dynamic stability, thermal requirement for negative sequence current, stress and deflection at a terminal fault, shielding of the field winding, and critical speed of rotor. The purpose of this thesis is to find the design of a superconducting alternator which meets these requirements and minimizes the cost.

Six penalty functions are so defined that the values of the functions may increase rapidly if the required limits are exceeded. The material cost of machine active elements multiplied by these penalty functions is assumed to be an index of a good design for the cost and the performance combined. The index called here the cost function is minimized by using the steepest descent method.

The results show that the cost, the weight and the efficiency are quite attractive in spite of a thick damper support required to absorb the strong crushing load at a terminal fault.

Thesis Supervisor: James L. Kirtley Jr.
Title: Assistant Professor of Electrical Engineering

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LIST OF SYMBOLS

Subscripts

a	armature winding
b	damper support
c	field winding core
cu	copper
e	end portion
f	field winding
i	inner or inside
m	ferromagnetic shield
o	outer or outside
p	primary damper
r	radial or rated
s	secondary damper
st	stainless steel
v	field container vessel
θ	azimuthal or tangential
0	initial
1	variation

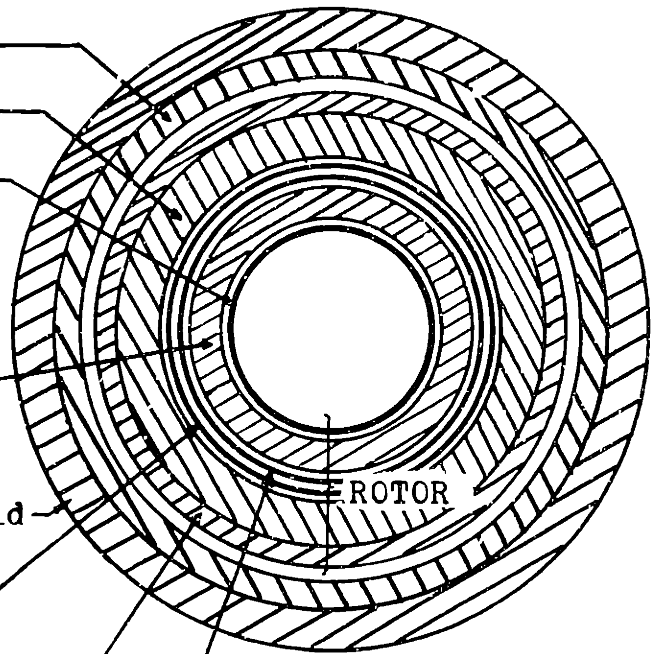


Fig.1 Schematic Cross Section

Symbols

A	field defined by eq.(3.16) (A/m)
B	field defined by eq.(3.17) (A/m)
C	field defined by eq.(3.18) (A/m), or cost ($)
c	specific cost ($/kg) or ($/kA·m)

D	field defined by eq.(3.22) (A/m)
E	field defined by eq.(3.23) (A/m) or EMF (pu) or Young's modulus (N/m^2)
F	field defined by eq.(3.24) (A/m)
H	field intensity (A/m)
I	moment of inertia of section (m^4) or current (pu)
i	current (A)
i_{ar}	rated armature current (A rms)
J	current density (A/m^2)
J_{fo}	no load field current density (A/m^2)
J_{fr}	rated load field current density (A/m^2)
J_a	armature current density (A rms/m^2)
$J_{f_{max}}$	maximum induced field current density (A $peak/m^2$)
J_{flimit}	maximum allowable field current density (A/m^2)
L	inductance (Hy)
L'	inductance / length / $turns^2$ (Hy/m)
ℓ	length (m)
M	mutual inductance (Hy)
M'	mutual inductance / length / $turns^2$ (Hy/m)
N_{at}	number of turns in armature winding
N_{ft}	number of turns in field winding
N_p	number of turns in primary damper (dummy variable)
N_s	number of turns in secondary damper (dummy variable)
nc	critical speed (rpm)
P	power or power loss (W)

R	radius (m)
t	thickness (m)
t_c	critical fault clearing time (sec)
u	damper radial deflection (m)
V_t	armature terminal voltage (pu)
W	weight (kg)
x	ratio of armature inner and outer radius $\frac{R_{ai}}{R_{ao}}$
y	ratio of field inner and outer radius $\frac{R_{fi}}{R_{fo}}$
X	reactance (pu)
T	time constant (sec)
δ	power angle (rad)
$\Delta\ell$	armature end length (m)
θ	angular displacement (rad)
θ_{wae}	armature winding angle (rad)
θ_{wfe}	field winding angle (rad)
λ	space factor
φ	phase angle at the instant of a fault (rad)
ψ	power factor angle (rad)
ρ	mass density (kg/m^3)
σ	stress (N/m^2)
σ'	conductivity (mho/m)
ω	angular velocity (rad/sec)
Z	cross section coefficient (m^3)

See also Tables 1 and 2.

I. INTRODUCTION

A. BACKGROUND

The application of superconductors to large alternators has recently been considered attractive. There are, however, a lot of problems to be solved for practical use. One of the biggest problems is the design of a damper shield. Superconducting alternators will have one or more cylindrical conducting shells in the annular space between the field and armature, rotating with the field. These shells are called, collectively, the "damper shield" in this thesis.

There are various requirements for the damper shield, and some of them conflict with one another. There are some previous papers[1,2] which show optimized design for large superconducting alternators. In these paper, however, those requirements for the damper shield were not considered.

In this thesis, the requirements for the damper shield will be summarized and an optimum design which meets the requirements and minimizes the cost will be shown. Using the results of the design, feasibility of large superconducting alternators will be discussed.

B. CONSTRUCTION OF A SUPERCONDUCTING ALTERNATOR

The principal functions of the damper shield are to shield the superconducting field winding from alternating magnetic fields, and to damp the mechanical oscillation of the rotor. In addition, in the event of a terminal fault, the damper shield should withstand strong crushing and torque loads.[3]

The time constant of the damper shield should be long enough for shielding and yet should not be too long for damping. The subtransient reactance should be large enough for limiting the stress in the damper shield at a terminal fault but should not be too large for good stability.

The double damper shield[4] was proposed as a solution to these requirements. According to this principle, the rotor would have two damper shields, arranged concentrically as shown in Fig.1. The inner damper shield, called here the primary damper, would operate at a temperature of about 20°K, have a relatively long time constant, and serve as the main shield. It shields the field winding not only from alternating fluxes due to harmonics, negative sequence current and surges, but also from heat transfer in the form of thermal radiation. The outer damper shield, called here the secondary damper, would operate at substantially room temperature, have a shorter time constant than the primary damper, and serve as an electromechanical damper. In addition, at the first instant of a terminal fault, the secondary damper would absorb strong crushing and torque loads. These loads are so strong that the secondary damper would need a relatively thick damper support.

Thus, the rotor cross section will consist of six elements. Arranged from inside to outside, they are: field winding core, superconducting field winding, field winding container vessel, primary damper, damper support, and secondary damper. Between the armature and the secondary damper, a plastic shell is provided to keep the inside vacuum for reducing the rotor windage loss. The damper support shell will serve for maintaining high vacuum for thermal insulation.

A smooth outer shell outside the armature provides a uniform boundary condition and confines the magnetic field within the machine. The shell may be of laminated iron, or of a highly conducting material. In this thesis, a laminated iron shield is assumed because of efficiency and economy.

None of the armature elements are ferromagnetic or conducting to reduce the eddy current loss, except the armature winding itself.

A proposed drawing of a 2000 MVA alternator is shown in Fig.2.

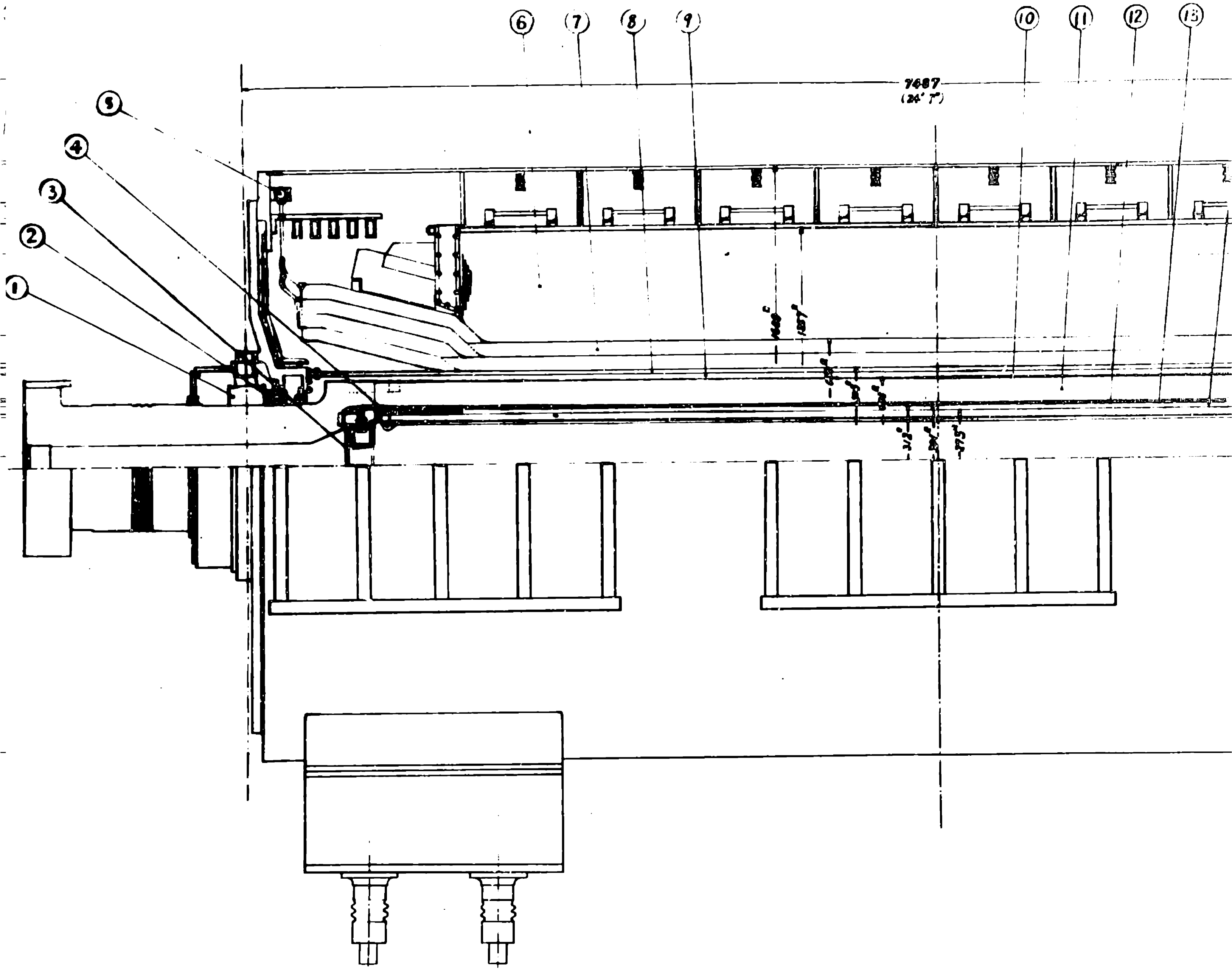
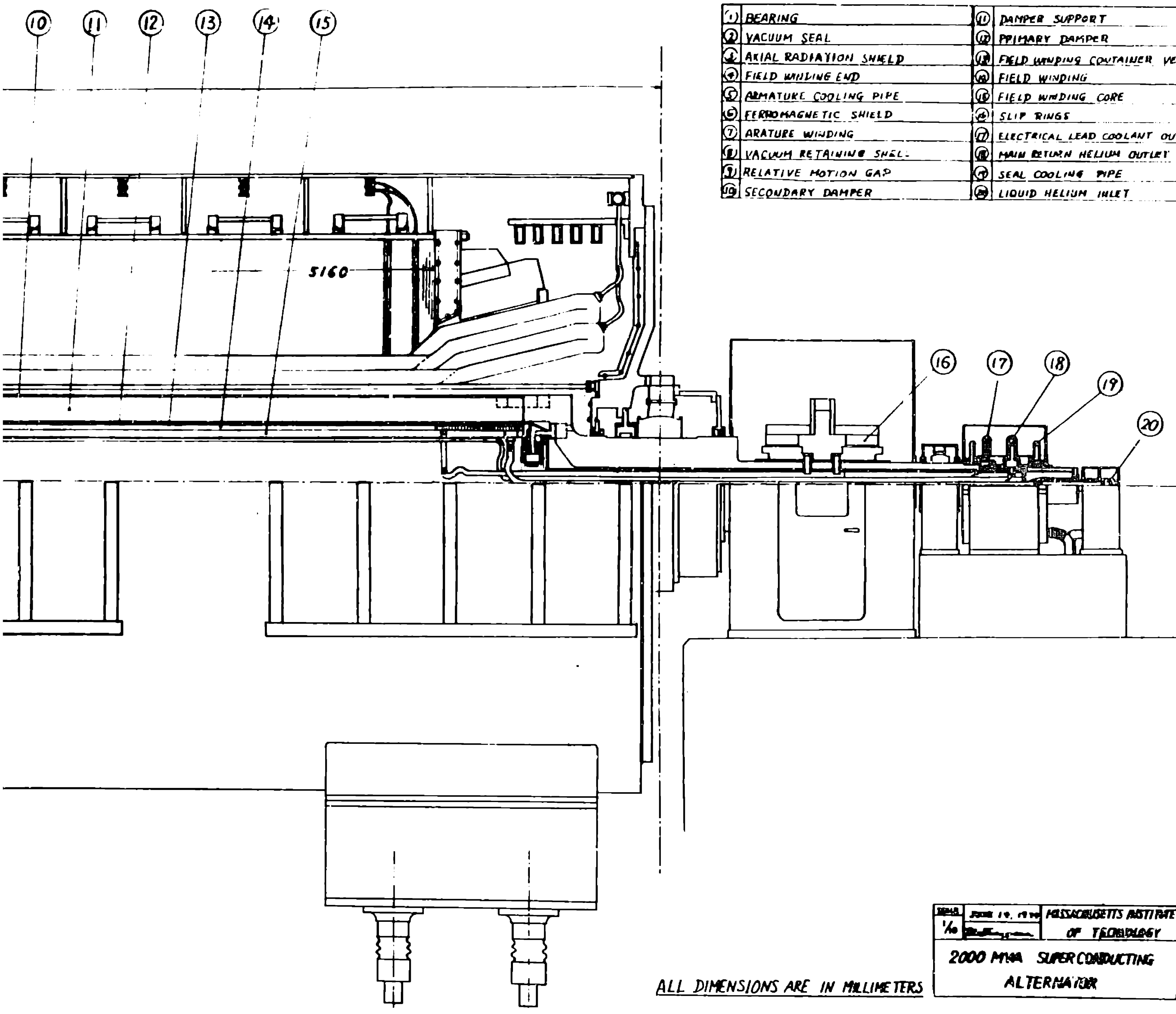


Fig.2 2000 MVA Superconducting Alternator



1	BEARING	11	DAMPER SUPPORT
2	VACUUM SEAL	12	PRIMARY DAMPER
3	AXIAL RADIATION SHIELD	13	FIELD WINDING CONTAINER VESSEL
4	FIELD WINDING END	14	FIELD WINDING
5	ARMATURE COOLING PIPE	15	FIELD WINDING CORE
6	FERROMAGNETIC SHIELD	16	SLIP RINGS
7	ARATURE WINDING	17	ELECTRICAL LEAD COOLANT OUTLET
8	VACUUM RETAINING SHELL	18	MAIN RETURN HELIUM OUTLET
9	RELATIVE MOTION GAP	19	SEAL COOLING PIPE
10	SECONDARY DAMPER	20	LIQUID HELIUM INLET

ALL DIMENSIONS ARE IN MILLIMETERS

SCALE 1/40	JUNE 19, 1970	MASSACHUSETTS INSTITUTE OF TECHNOLOGY

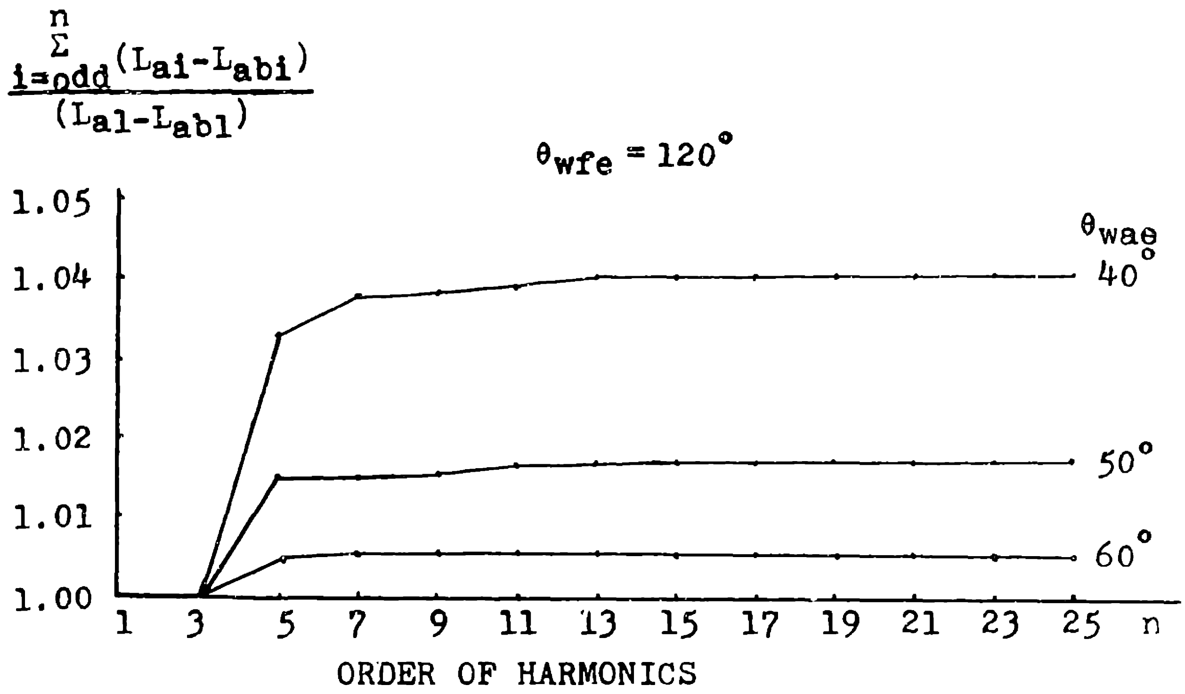
2000 MVA SUPERCONDUCTING ALTERNATOR

II. FUNDAMENTAL EQUATIONS

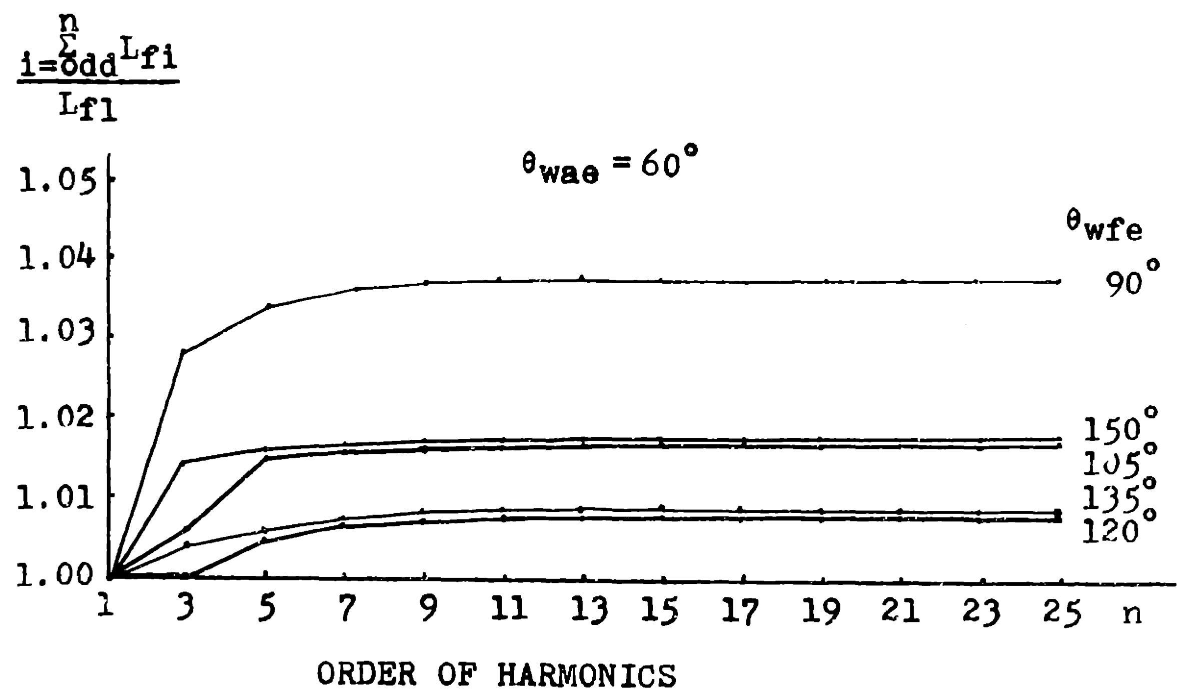
The field analysis of an air core machine has been done previously[5,6] for a machine with one damper shield. Similar methods can be easily applied to a machine with a double damper shield by using the thin shell assumption for both the primary and the secondary dampers.

As for reactances, a new quantity, X_d''', called here "sub-subtransient reactance", should be introduced, corresponding to the secondary damper. X_d''' can be calculated similarly to X_d''.[6] The formulas for calculating the reactances and time constants are shown in App.I, where only the fundamental component of the space harmonics is considered. By ignoring the higher harmonic component, X_d', X_d'' and X_d''' can be calculated straightforwardly from X_d as shown in App.I. The effect of the space harmonics in the winding distribution on inductances is demonstrated in Fig.3. As can easily be seen, higher harmonic components have a negligible effect on the inductances, and may be ignored.

For dynamic characteristics of the machine, Park's equations can be extended by adding one more damper shield circuit to the direct and to the quadrature axes, respectively. By using the thin shell assumption and X_{ad} base per-unit system,[7] we can obtain a simple form of the extended Park's equations for a double-damper machine.[8]



Armature Inductance



Field Inductance

Fig.3 Effect of Space Harmonics on Inductances

III. REQUIREMENTS FOR THE DAMPER SHIELD

Requirements for the machine affected by the damper shield may be classified as: transient stability, dynamic stability, thermal requirement for negative sequence current, stress and deflection due to a terminal fault, shielding of the field winding and critical speed of rotor. The first three requirements have been previously analyzed, and in this thesis only the results of the previous papers are summarized and transformed into easier forms for optimizing the machine design. The latter three requirements will be analyzed and formulated in this chapter.

A. TRANSIENT STABILITY

The critical fault clearing time under a fixed fault and system condition may be a criterion of transient stability of an alternator. The analysis has been done assuming a simplified fault and system condition as shown in Fig.4, and the critical fault clearing time has been given as a function of each machine parameter.(8) The results are summarized here by using a regression analysis described in Appendix II.(9) as follows:

Critical fault clearing time (in seconds)

$$t_c = 0.1339 - 0.1330\, X_d'' + 0.0005\, T_s + 0.0278\, H \quad (3.1)$$

valid for $2 \leq H \leq 5$, $T_s \leq 20$

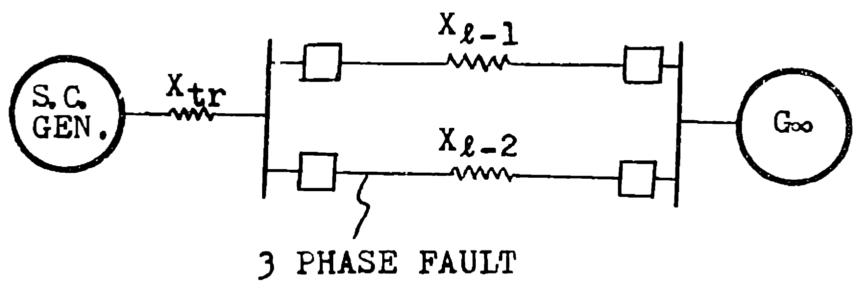


Fig.4 System Used for Transient Stability Simulation

$X_{\ell-1} = 0.2$ pu

$X_{\ell-2} = 0.1$ pu

$X_{tr} = 0.15$ pu

(pu based on generator rating)

A three phase fault is assumed to occur at the sending end of one of the two parallel circuits between the unit in question and an infinite bus. The fault is cleared by opening breakers at both ends of the circuit.

Initial operating condition is assumed the rated load at 0.85 power factor.

$$0.25 \le X_d \le 1.00 \quad , \quad 0.15 \le X_d' \le 0.60$$

$$0.125 \le X_d'' \le 0.50 \quad , \quad 0.075 \le X_d''' \le 0.30$$

$$0.5 \le T_p \le 20 \quad , \quad T_f > 10$$

Comparing the results of eq.(3.1) with the original transient stability analysis,[8] the standard error of estimate by eq.(3.1) for 25 sample cases is 0.0032 seconds.

As a reference value of the fault clearing time, 0.18 seconds, a typical value for 1000 MVA conventional machine, is chosen.

B. DYNAMIC STABILITY AND THERMAL REQUIREMENT

Dynamic stability has two factors; one is positive synchronizing torque and the other is positive damping. The former requirement is easily fulfilled because of the relatively small synchronous reactance of a superconducting machine. Positive damping, however, may be a somewhat more difficult requirement for the secondary damper.

Damping is related to the time constant of the damper shield. For the time constant of the damper shield, Einstein gave the range of 0.16 - 1.0 seconds in his thesis, [10] and 0.2 - 0.5 seconds in a recent paper,[11] as giving reasonable damping. Hamblem[12] showed that a double damper superconducting machine is stable enough for normal operation if the time constant is between 0.01 and 1.0 second. At any rate, damper resistance must not be too large (resistance limited) or too short (inductance limited) for good damping. Time constant should be on the same order as the swing angular period.

Thermal requirements for negative sequence current such as I_2^2t are another factor constraining the time constant of the secondary damper. We may assume from Luck [3] and Dagalakis[13] that superconducting alternators fulfill the thermal requirements in power systems protected with appropriate relays, if the secondary damper made of copper is not thinner than the depth of flux penetration for negative sequence current, which is about 6 mm. This thickness corresponds to a time constant of 0.1 -0.3 seconds depending on the radius.

Thus, we may choose 0.1 - 1.0 second as a rough criterion for the time constant of the secondary damper.

C. STRESS AND DEFLECTION AT TERMINAL FAULT

Suppose a thin conducting shell is so located in magnetic field with radial component H_r and tangential components inside and outside the shell are $H_{\theta i}$ and $H_{\theta o}$, respectively. The force per unit area (described by Maxwell's stress tensor) on the shell is given by:

$$\sigma_r = \frac{\mu_o}{2} \left(H_{\theta o}^2 - H_{\theta i}^2 \right) \tag{3.2}$$

$$\sigma_\theta = \mu_o \left(H_{\theta o} - H_{\theta i} \right) H_r \tag{3.3}$$

These relations can be applied to the secondary damper during a terminal three phase fault from rated load conditions by using the thin shell assumption. Because the secondary damper is a shell of good conductivity, the field inside the secondary damper does not change immediately after the fault. Considering substantially no current flows in the secondary damper before the fault, the field inside is the superposition of the fields due to the rated field current and the rated armature current.

Field outside the shell, however, is induced by the change of armature current (AC and DC components) and the damper current, in addition to the initial field before the fault.

Armature current during a three phase fault from rated load condition is given, immediately after the fault, by applying Thevenin's superposition theorem:

$$i_a = \sqrt{2}\, i_{ar} \left[\cos(\omega t + \varphi + \frac{\pi}{2} - \psi) + \frac{V_t}{X_d'''} \cos(\omega t + \varphi) - \frac{V_t}{X_d'''} \cos\varphi \right] \quad (3.4)$$

The first two components (AC) in the bracket can be represented by :

$$\frac{E'''}{X_d'''} \cos(\omega t + \varphi + \delta''')$$

where E''' is the voltage behind X_d''' shown in Fig.5 and φ

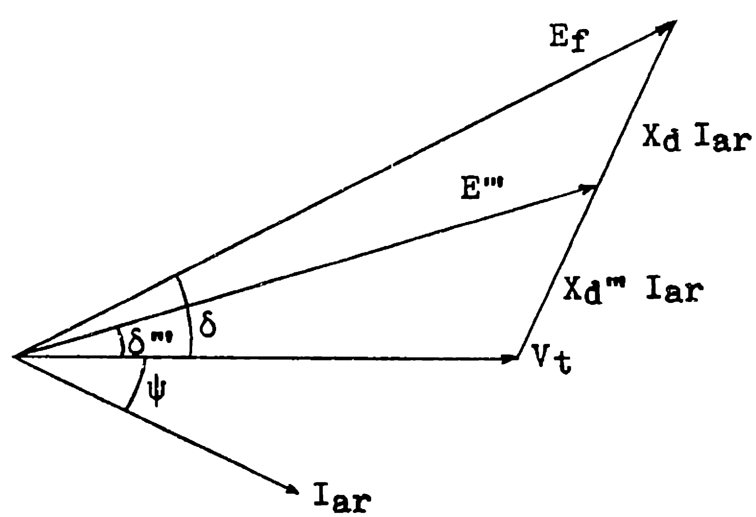


Fig.5 Vector Diagram at Rated Load Condition

is phase angle which depends on the rotor position at the instance of the fault.

Here, however, it is more convenient to divide the armature fault current into three components:
initial load current, change in AC current, and DC current, because the secondary damper current is induced only by the change in armature current and because E''' is not applicable to the DC component.

The magnitude of the fundamental component of the tangential field at the secondary damper due to the average rms armature current density J_a is given by:[6]

$$H_\theta = \frac{3\sqrt{2}}{\pi} \sin\left(\frac{\theta_{wae}}{2}\right) R_{ao} \left[1 - x + \frac{1}{3}(1 - x^3)\left(\frac{R_{ao}}{R_{mi}}\right)^2 \right] J_a \qquad (3.5)$$

From eqs. (3.4) and (3.5), tangential field at the angle θ outside the secondary damper due to the armature current immediately after the fault is:

$$H_{\theta a} = \frac{3\sqrt{2}}{\pi} \sin\left(\frac{\theta_{wae}}{2}\right) R_{ao} J_{ar} \left[1 - x + \frac{1}{3}(1 - x^3)\left(\frac{R_{ao}}{R_{mi}}\right)^2 \right]$$

$$\cdot\left[\frac{V_t}{X_d'''} \{ \cos(\omega t + \varphi - \theta) - \cos(\varphi - \theta) \} + \cos(\omega t + \varphi - \theta + \frac{\pi}{2} - \psi) \right] \qquad (3.6)$$

Tangential field inside the secondary damper due to the rated armature current, which is not affected by the fault, is given by:

$$H_{\theta a0} = \frac{3\sqrt{2}}{\pi} \sin\left(\frac{\theta_{wae}}{2}\right) R_{ao} J_{ar} \left[1 - x + \frac{1}{3}(1 - x^3) \right.$$

$$\left. \cdot \left(\frac{R_{ao}}{R_{mi}}\right)^2 \right] \cos\left(\omega t + \varphi - \theta + \frac{\pi}{2} - \psi\right) \qquad (3.7)$$

Tangential field at the secondary damper due to field current, which is constant before and immediately after the fault, is given by:

$$H_{\theta f} = \frac{2}{3\pi} \sin\left(\frac{\theta_{wfe}}{2}\right) R_s \left(\frac{R_{fo}}{R_s}\right)^3 (1 - y^3) \left\{ 1 - \left(\frac{R_s}{R_{mi}}\right)^2 \right\}$$

$$\cdot J_f \cos(\omega t + \varphi - \theta + \delta) \qquad (3.8)$$

The secondary damper current is induced so that the radial component of total field is kept constant before and after the fault. The field due to this induced current outside the secondary damper immediately after the fault is:

$$H_{\theta s} = \left[\frac{1 - \left(\frac{R_s}{R_{mi}}\right)^2}{1 + \left(\frac{R_s}{R_{mi}}\right)^2} \right] \frac{3\sqrt{2}}{\pi} \sin\left(\frac{\theta_{wae}}{2}\right) R_{ao} \left[1 - x + \frac{1}{3}(1 - x^3) \right.$$

$$\left. \cdot \left(\frac{R_{ao}}{R_{mi}}\right)^2 \right] J_{ar} \frac{V_t}{X_d''} \left[\cos(\omega t + \varphi - \theta) - \cos(\varphi - \theta) \right]$$

$$(3.9)$$

Adding up those components, we obtain the total tangential field outside the secondary damper immediately after the fault:

$$H_{\theta o} = H_{\theta a} + H_{\theta s} + H_{\theta f}$$

$$= H_{a0} \cos(\omega t + \varphi - \theta + \frac{\pi}{2} - \psi) + \frac{2H_{a1}}{1 + (\frac{R_s}{R_{mi}})^2} [\cos(\omega t$$

$$+ \varphi - \theta) - \cos(\varphi - \theta)] + H_f \cos(\omega t + \varphi - \theta + \delta)$$

(3.10)

For the inside:

$$H_{\theta i} = H_{\theta f} + H_{\theta a0}$$

$$= H_f \cos(\omega t + \varphi - \theta + \delta) + H_{a0} \cos(\omega t + \varphi - \theta + \frac{\pi}{2} - \psi)$$

(3.11)

where

$$H_{a0} = \frac{3\sqrt{2}}{\pi} \sin(\frac{\theta_{wae}}{2}) R_{ao} J_{ar} [1 - x + \frac{1}{3} (1 - x^3)$$

$$\cdot (\frac{R_{ao}}{R_{mi}})^2] \qquad (3.12)$$

$$H_{a1} = \frac{H_{ao}}{X_d''} \qquad (3.13)$$

$$H_f = \frac{2}{3\pi} \sin(\frac{\theta_{wfe}}{2}) R_s J_f (\frac{R_{fo}}{R_s})^3 (1 - y^3) [1$$

$$- (\frac{R_s}{R_{mi}})^2] \qquad (3.14)$$

To obtain the maximum value of σ_r , we may assume that the maximum σ_r occurs when the traveling wave (sum of three terms which have ωt in eq.(3.10)) comes to the same phase as the standing wave ($\cos(\varphi - \theta)$ term in eq. (3.10)), because $H_{\theta o}{}^2 \gg H_{\theta i}{}^2$, referring to Fig.6.

From eq.(3.10),

$$H_{\theta o} = \sqrt{A^2 + B^2}\cos(\omega t + \varphi - \theta + \tan^{-1}\frac{B}{A}) - C\cos(\varphi - \theta) \tag{3.15}$$

where

$$A = H_{a0}\sin\psi + \frac{2H_{a1}}{1 + (\frac{R_s}{R_{mi}})^2} + H_f\cos\delta \tag{3.16}$$

$$B = H_{a0}\cos\psi + H_f\sin\delta \tag{3.17}$$

$$C = \frac{2H_{a1}}{1 + (\frac{R_s}{R_{mi}})^2} \tag{3.18}$$

Two terms in eq.(3.15) have the same phase, when

$$\omega t = \pi - \tan^{-1}\frac{B}{A} \tag{3.19}$$

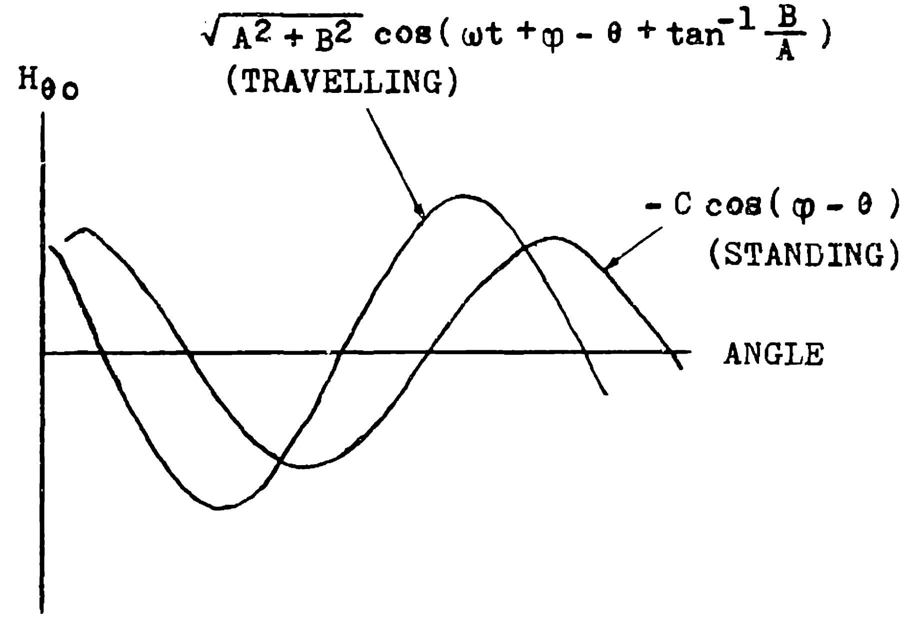
Then,

$$H_{\theta o} = F\cos(\varphi - \theta) \tag{3.20}$$

$$H_{\theta i} = E\cos(\varphi - \theta) + D\sin(\varphi - \theta) \tag{3.21}$$

where

$$D = H_{ao}\cos(\tan^{-1}\frac{B}{A} + \psi) - H_f\sin(\tan^{-1}\frac{B}{A} - \delta) \tag{3.22}$$



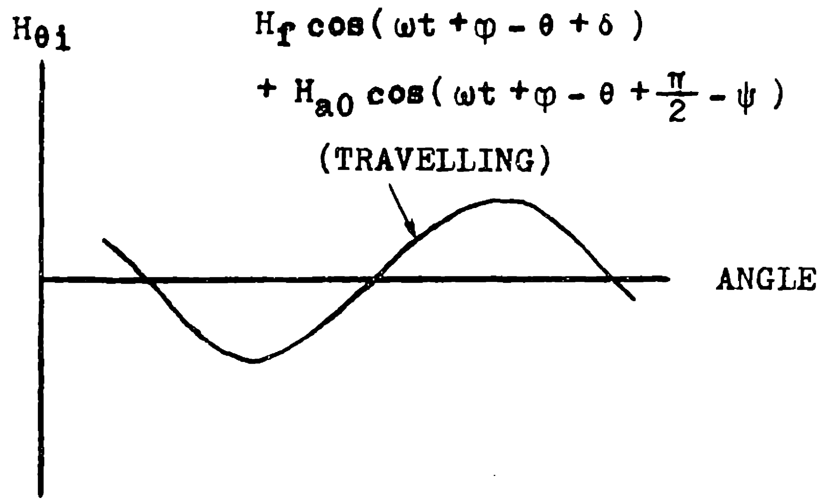


Fig.6 Tangential Flux Wave Distribution at a Three Phase Terminal Fault

$$E = -H_{ao}\sin\left(\tan^{-1}\frac{B}{A}+\psi\right) - H_f\cos\left(\tan^{-1}\frac{B}{A}-\delta\right) \tag{3.23}$$

$$F = -\sqrt{A^2+B^2} - C \tag{3.24}$$

Substituting eqs.(3.20) and (3.21) into eq.(3.2), we obtain:

$$\sigma_r = \sigma_{r1} + \sigma_{r2}\cos2(\varphi-\theta+\gamma) \tag{3.25}$$

where

$$\sigma_{r1} = \frac{\mu_o}{4}\left(F^2 - D^2 - E^2\right) \tag{3.26}$$

$$\sigma_{r2} = \frac{\mu_o}{4}\sqrt{(F^2-E^2+D^2)^2 + 4E^2D^2} \tag{3.27}$$

$$\gamma = \frac{1}{2}\tan^{-1}\frac{2ED}{F^2-E^2+D^2} \tag{3.28}$$

We assume a thin damper shell to find the bending stress due to the radial loading $\sigma_{r2}\cos2(\varphi-\theta+\gamma)$. First, we will consider concentrated force as shown in Fig.7. The radial displacement u, and the bending moment M are given by[14]:

$$\frac{d^2u}{ds^2} + \frac{u}{R^2} = -\frac{M}{EI} \tag{3.29}$$

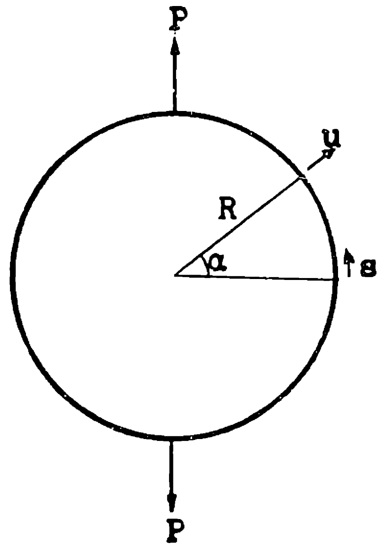


Fig.7 Concentrated Radial Force on the Damper

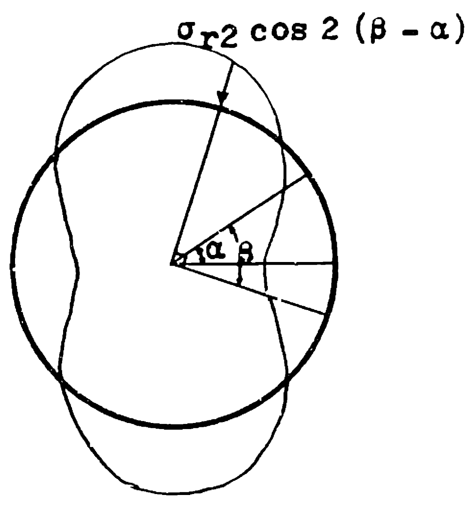


Fig.8 Distributed Radial Force on the Damper

$$M = \frac{PR}{2} \left(\cos\alpha - \frac{2}{\pi}\right) \tag{3.30}$$

For the bending stress:

$$\sigma_b = \frac{M}{Z} = \frac{PR}{2Z} \left(\cos\alpha - \frac{2}{\pi}\right) \tag{3.31}$$

The deflection can be solved:

$$u = \frac{PR^3}{\pi EI} - \frac{PR^3}{4EI}\alpha\sin\alpha - \frac{PR^3}{4EI}\cos\alpha \tag{3.32}$$

For the distributed load as shown in Fig.8, integrating eq.(3.32), deflection at angle α is:

$$u(\alpha) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sigma_{r2}\, \ell\, R \left(\frac{R^3}{EI\pi} - \frac{R^3}{4EI}\beta\sin\beta - \frac{R^3}{4EI}\cos\beta\right)$$

$$\cdot\cos 2(\beta - \alpha)\, d\beta$$

$$= \frac{4}{3}\,\frac{\sigma_{r2} R^4}{E t^3}\cos 2\alpha \tag{3.33}$$

where t is the thickness of the shell.

Similarly, the bending stress in Fig.8 is given by:

$$\sigma_b = \frac{2\sigma_{r2} R^2}{t^2}\cos 2\alpha \tag{3.34}$$

Adding the deflection and the stress due to the centrifugal force and the uniform magnetic force to eq. (3.33) and (3.34), we obtain:

$$\sigma_{total} = -\frac{\sigma_{r1} R}{t} + \frac{2\sigma_{r2} R^2}{t^2} \cos 2\alpha + \rho\,\omega^2 R^2 \qquad (3.35)$$

$$u_{total} = -\frac{\sigma_{r1} R^2}{E\,t} + \frac{4\sigma_{r2} R^4}{3\,E\,t^3} \cos 2\alpha + \frac{\rho R^3 \omega^2}{E} \qquad (3.36)$$

Maxwell's stress tensor of tangential direction has two components. One is fundamental frequency alternating torque component which is induced by the interaction between the standing tangential flux wave and the traveling radial flux wave. The other is a $\sin 2\theta$ component of shearing load induced by the interaction between the traveling tangential flux wave and the traveling radial flux wave.

Stress and deflection due to the tangential component of Maxwell's stress tensor are much smaller than those due to the radial component. In eq.(3.3), H_r is constant before and immediately after the fault, and H_r is much smaller than H_θ. Therefore;

$$\sigma_\theta << \sigma_r$$

In addition, bending stress due to the shearing load of $\sin 2\theta$ distribution is much smaller than that due to sinusoidally distributed radial load. Thus, stress and deflection due to σ_θ is not chosen as a criterion of the damper design, although it is a little optimistic estimation to ignore this factor.

Dagalakis is working for more complicated boundary conditions, and the stress and deflection are found to be a little bigger for the thick shell.[13] Here, however, the thin shell assumption is used for rough estimation.

D. MAXIMUM INDUCED CURRENT IN THE FIELD WINDING

Superconducting wire has a maximum current limit that depends on the flux density. The flux density at the field winding is kept practically constant during the transient state because of long field time constant. The field current, however, may change according to low frequency alternating MMF such as a transient power swing, from which the field winding is not sufficiently shielded by the damper shield. The frequency is so low that we do not have to consider the AC loss in the winding, but the magnitute of the total current should be within the limit.

The maximum current induced in the field winding during a transient power swing can be calculated by using the same program as the transient stability study.[8]

Einstein gave the relative value of the field current induced by a rotor angular swing without considering the damper shield.[10,11] Now we will consider the shielding effect of the damper shield and the cumulative field current rise during the fault and after the fault is cleared. Fig.9 is one of the results of the calculation for the rotor swing and the field current. The maximum field current at a condition very close to the transient stability limit is plotted as a function of some machine parameters in Fig.10.

If we set that $X_d' = 0.6\ X_d$

and $X_d'' = 0.5\ X_d$

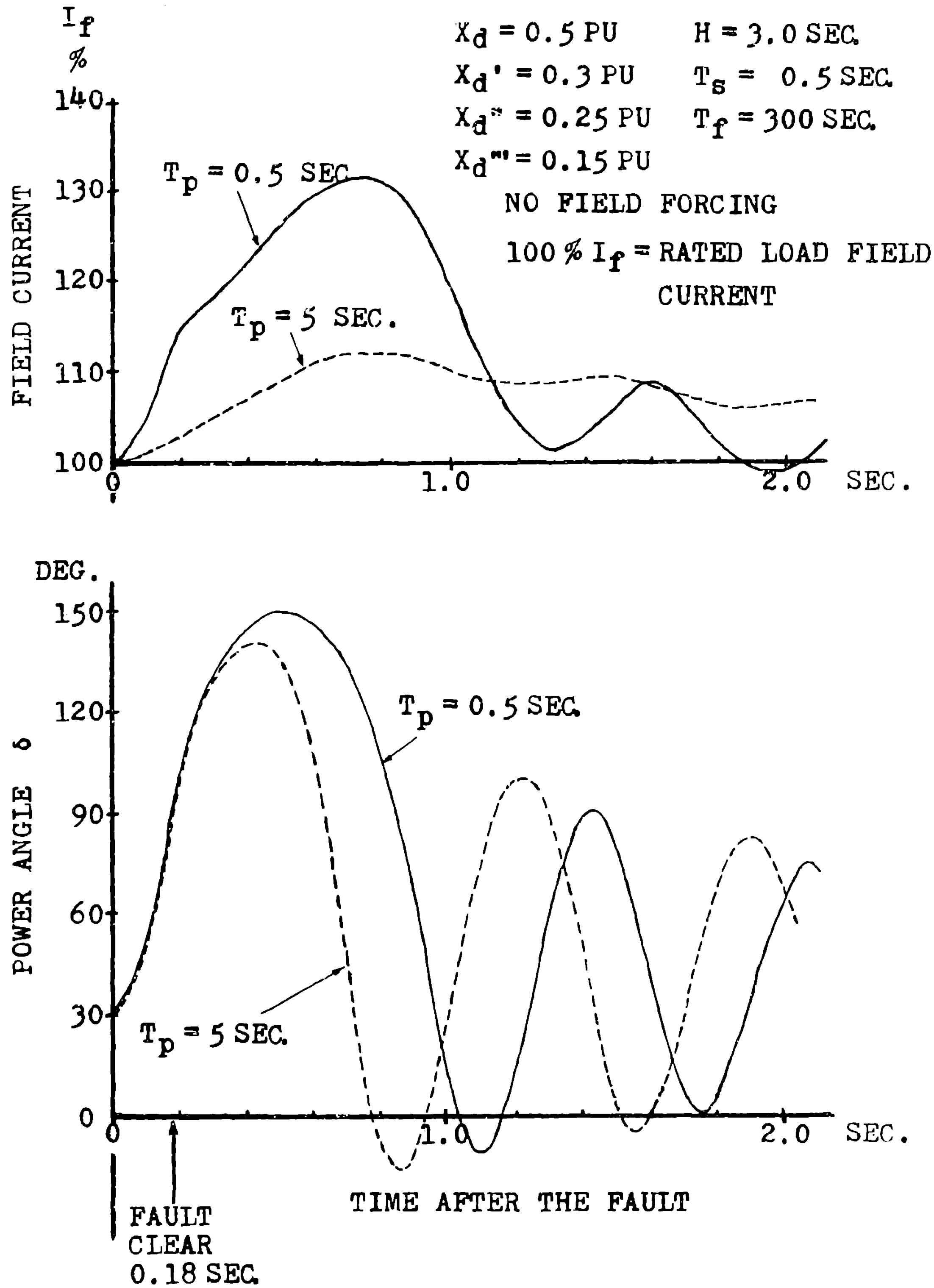
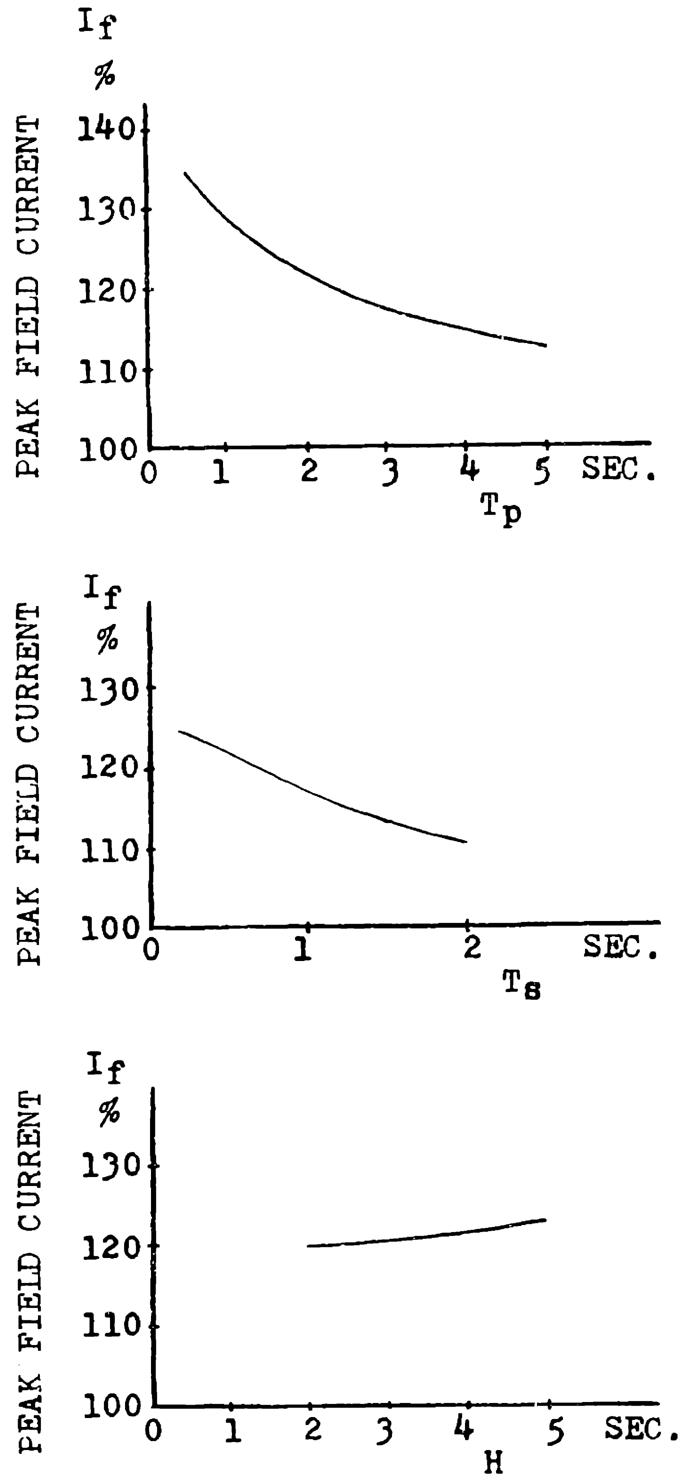


Fig.9 Rotor Swing and Field Current



100 % I_f = RATED LOAD FIELD CURRENT

NO FIELD FORCING

$X_d = 0.5$ PU
$X_d' = 0.3$ PU
$X_d'' = 0.25$ PU
$X_d''' = 0.15$ PU
$H = 3.0$ SEC.
$T_s = 0.5$ SEC.
$T_p = 2$ SEC.
$T_f = 300$ SEC.
EXCEPT FOR PARAMETER INDICATED ON ABSCISSAE.

Fig.10 Effect of Machine Parameters on Field Induced Current

which is very rough but pessimistic assumption, we obtain from the regression analysis that:

$$\frac{I_{f_{max}}}{I_{f_{rated}}} = 0.9716 + 0.3701\left(1 - e^{-\frac{1.01}{T_s}}\right)\left(1 - e^{-\frac{1.98}{T_p}}\right)$$

$$- 0.0288\, X_d'' + 0.0095\, H \qquad (3.37)$$

valid for

$2 \leq H \leq 5$

$T_s\,,\, T_p \leq 20$

$T_f > 50$

$0.125 \leq X_d'' \leq 0.5$

$X_d' \approx 0.6\, X_d$

$X_d'' \approx 0.5\, X_d$

Standard error of estimate = 0.030 for 20 samples

The effect of the inertia constant H comes from the fact that larger inertia constant reduces the swing frequency and lets the alternating flux penetrate into the field winding a little more.

E. CRITICAL SPEED OF ROTOR

One of the factors that constrain the length of the damper shield (or rotor length) is the critical speed. Considering that some conventional 3600 rpm turbine generators are operating with their first critical speed less than 1000 rpm, the critical speed may not be a very strict constraint for design. However, some factor that limits the length of the rotor is necessary for optimizing the design.

The first mode critical speed of a shaft supported at both ends is given by:[15]

$$n_c = 94.3\sqrt{\frac{EI}{\ell_b^4 m}} \quad \text{(rpm)} \qquad (3.38)$$

where E: Young's modulus of material (N/m^2)

I: Moment of inertia of the section (m^4)

m: mass per unit length (kg/m)

ℓ_b: bearing span (m)

Strictly speaking, eq.(3.39) is not applicable to a superconducting machine rotor which consists of several co-axial cylinders. However, for a rough estimation, we may extend eq.(3.39) as:

$$n_c = 94.3\sqrt{\frac{\Sigma(EI)}{\ell_b^4 \Sigma m}} \quad \text{(rpm)} \quad \Sigma \text{ for all elements} \qquad (3.39)$$

IV. DESIGN OF SUPERCONDUCTING ALTERNATORS

A. EQUATIONS FOR DESIGN

Equations for requirements to the damper shield have been summarized in previous chapter. In this chapter, some other equations which will be necessary to establish the optimum design are introduced.

Rules of Thumb and Constants

Rules of thumb used here based on Kirtley's rules of thumb for superconducting alternator design[5] are presented in Table 1. Constants used here are listed in Table 2. Most of the constants are the same or almost the same as those use in the reference[2]. However, cost of superconducting wire is based on a recent report[16], and is much lower. Characteristic curve of superconductor capability is also updated as shown in Fig.11.

Machine Rating and Machine Length

Assuming the cross sectional dimensions and the current density of the field and armature windings, we can calculate the output kVA per unit length of the machine. However, the derivation of the length of the machine is not so straightforward because of armature end effects. By combining several equations given in previous work[5], machine output apparent power is given

Table 1 Rules of Thumb

Armature end turn length	$\Delta\ell = R_{ai} + R_{ao}$
Ferromagnetic shield inner radius	$R_{mi} = R_{ao}$
Damper support outer radius	$R_{bo} = R_{si}$
Rotor body length	$\ell_{rb} = \ell_{af} + \frac{3\pi}{2} R_{fo}$
Bearing span	$\ell_{br} = \ell_{rb} + 2 R_{so}$
Ferromagnetic shield length	$\ell_m = \ell_{af}$
Effective length	
Armature self inductance	$\ell_a = \ell_{af} + \Delta\ell$
All mutual inductances	$\ell_{af} = \ell_{as} = \ell_{ap} = \ell_{sp} = \ell_{sf} = \ell_{pf}$
Armature copper loss	$\ell_{at} = \ell_{af} + 2\Delta\ell$

Table 2 Constants

Average current density

Armature: $J_a = 3.0 \times 10^6$ A/m^2 (rms)

Space factor

Armature winding: $\lambda_a = 0.25$

Field winding: $\lambda_f = 0.625$

Peak flux density

Ferromagnetic shield: $B_m = 1.5$ T

Iron loss

Ferromagnetic shield: $P_m = 4$ W/kg

Conductivity

Armature: $\sigma_a = 6 \times 10^7$ mho/m

Primary damper (20°K): $\sigma_p = 5 \times 10^9$ mho/m

Secondary damper (R.T.): $\sigma_s = 6 \times 10^7$ mho/m

Damper support: $\sigma_b = 1.38 \times 10^6$ mho/m

Mass density

Average field winding: $\rho_f = 5.56 \times 10^3$ kg/m^3

Field winding core: $\rho_c = 7.8 \times 10^3$ kg/m^3

Field winding container vessel: $\rho_v = 7.8 \times 10^3$ kg/m^3

Armature middle part: $\rho_a = 4.5 \times 10^3$ kg/m^3

Armature end turns: $\rho_{ae} = 4.0 \times 10^3$ kg/m^3

Damper support: $\rho_b = 7.8 \times 10^3$ kg/m^3

Ferromagnetic shield: $\rho_m = 7.5 \times 10^3$ kg/m^3

Damper shield: $\rho_s = \rho_p = 8.9 \times 10^3$ kg/m^3

Table 2 Constants (continued)

Young's modulus

Stainless steel: $E_{st} = 2.1 \times 10^{11}$ N/m^2

Copper: $E_{cu} = 1.1 \times 10^{11}$ N/m^2

Gaps between

Armature and secondary damper: $g_{as} = 0.05$ m

Damper support and primary damper: $g_{bp} = 0.002$ m

Primary damper and field container vessel: $g_{pv} = 0.002$ m

Thickness

Field winding core: $t_c = 0.025$ m

Field container vessel: $t_v = 0.005$ m

Winding angle

Armature: $\theta_{wae} = 60^\circ$

Field: $\theta_{wfe} = 120^\circ$

Cost

Non-magnetic stainless steel: $c_{st} = 4.0$ \$/kg

Oxygen-free copper: $c_{cu} = 4.0$ \$/kg

Silicon steel core punching: $c_m = 2.0$ \$/kg

Superconducting wire(16): $c_f = (0.12 + 0.018\, B_{max} + 0.0036 \cdot B_{max}^2) \times 2$ \$/kA·m

Material

Armature, primary and secondary dampers: copper

Damper support, field winding core and field winding container vessel: stainless steel

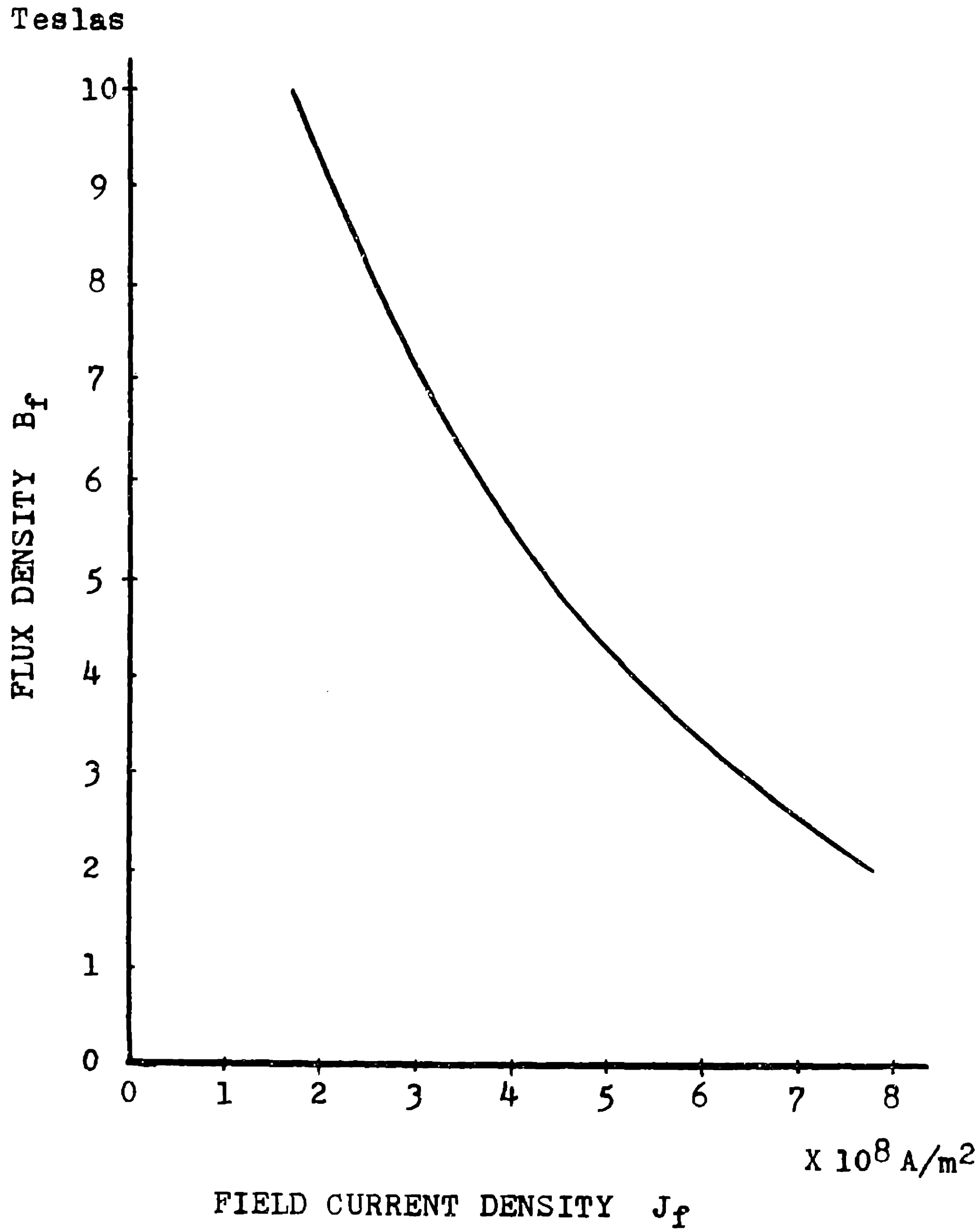


Fig.11 Characteristic Curve of a Superconductor

by:

$$P = \frac{3}{4\sqrt{2}} \omega M_{af}' J_a J_f \theta_{wae} \theta_{wfe} R_{ao}^2 R_{fo}^2 (1-x^2)(1-y^2)$$

$$\cdot\left[-X_{af} (\ell_{af} + \Delta\ell) \sin\psi + \sqrt{\ell_{af}^2 - X_{af}^2 (\ell_{af} + \Delta\ell)^2 \cdot \cos^2\psi} \right] \quad (4.1)$$

where

$$X_{af} = \frac{3\sqrt{2}\, L_a' \, J_a \, \theta_{wae} \, R_{ao}^2 (1-x^2)}{2\, M_{af}' \, J_f \, \theta_{wfe} \, R_{fo}^2 (1-y^2)} \quad (4.2)$$

$$L_a' = \frac{L_a}{\ell_a N_{at}^2} \quad (4.3)$$

$$M_{af}' = \frac{M_{af}}{\ell_{af} N_{at} N_{ft}} \quad (4.4)$$

L_a', M_{af}' and X_{af} can be calculated from the cross sectional dimensions and the current density. Armature end length $\Delta\ell$ is also a function of the cross section using the rule of thumb. Then we can solve eq.(4.1) for ℓ_{af} as a function of the machine rating, that is, kVA and power factor.

Iron and Copper Loss

A procedure to decide the radial thickness of the ferromagnetic shield is as follows.

Maximum radial flux at the inner surface of the ferromagnetic shield at the rated voltage is given by:

$$B_{mi} = \frac{4}{3\pi} \mu_o J_{fo} \sin \frac{\theta_{wfe}}{2} R_{mi} \left(\frac{R_{fo}}{R_{mi}} \right)^3 (1 - y^3) \quad (4.5)$$

If we set the maximum flux density in the ferromagnetic shield to be B_m, the radial thickness of the ferromagnetic shield is:

$$t_m = \frac{B_{mi} R_{mi}}{B_m} \quad (4.6)$$

The iron loss is given by:

$$P_m = p_m \rho_m \pi (R_{mo}^2 - R_{mi}^2) \ell_m \quad (4.7)$$

where $R_{mo} = R_{mi} + t_m$, and p_m is iron loss per unit mass at maximum flux density B_m.

Armature copper loss is given by:

$$P_a = \frac{J_a^2}{\sigma_a' \lambda_a} 3 \theta_{wae} (R_{ao}^2 - R_{ai}^2) \ell_{at} \quad (4.8)$$

Weight of Machine Active Elements

Rotor:

$$W_r = \ell_{rb} \pi \Sigma [\rho (R_o^2 - R_i^2)] \quad (4.9)$$

(Σ for all rotor elements) (ℓ_{at}, ℓ_{rb}: See Table I.)

Armature:

$$W_a = 3\ \theta_{wae}\ (R_{ao}^2 - R_{ai}^2)\ \rho_a\ \ell_{af} + 6\ \theta_{wae}\ [(2\ R_{ao} - R_{ai})^2 - R_{ai}^2]\ \rho_{ae}\ \Delta\ell \tag{4.10}$$

Ferromagnetic shield:

$$W_m = \pi\ \rho_m\ (R_{mo}^2 - R_{mi}^2)\ \ell_m \tag{4.11}$$

Total active elements:

$$W_t = W_r + W_a + W_m \tag{4.12}$$

Cost of Machine Active Elements

Superconductor:

$$C_f = \theta_{wfe}\ (R_{fo}^2 - R_{fi}^2)\ (\ell_{af} + \pi\ R_{fo})\ c_f\ J_{f limit} \tag{4.13}$$

where $J_{f limit}$ is the field maximum current density at the rated field flux density B_f in Fig.11.

Total material cost of the active elements:

$$C_t = W_{st}\ c_{st} + W_{cu}\ c_{cu} + W_m\ c_m + C_f \tag{4.14}$$

where W_{st} and W_{cu} are the weight of elements of stainless steel and that of copper, respectively. (See Table 2, Material) Small c means cost per unit mass($/kg).

B. OPTIMIZING PROCEDURE

Our purpose is to minimize the cost within the performance requirement limit stated in Chapter III. However, these limits are not very strict. We may use some penalty functions for the limits, and multiply the cost given in eq.(4.3) by these penalty functions to get an index of optimization called here the cost function. In order to find the design that minimizes the cost function, the steepest descent method will be used.

Variables to Be Optimized

If the machine rating is given, seven independent variables listed in table 3 may decide the whole machine design, using the rules of thumb and constants given in Tables 1 and 2. Starting from the initially guessed values of the seven variables, we will find the values that minimize the cost function by using the steepest descent method.

Penalty Functions

From the results of Chapter III, six penalty functions are established to constrain the design. These functions are listed in Table 4. The values of the functions increase very rapidly if the limits are exceeded.

Table 3. Variables to Be Optimized

$x_1 = R_{fi}$: Inner radius of field winding

$x_2 = t_f$: Thickness of field winding

$x_3 = t_p$: Thickness of primary damper

$x_4 = t_b$: Thickness of damper support

$x_5 = t_s$: Thickness of secondary damper

$x_6 = t_a$: Thickness of armature winding

$x_7 = J_f$: Field winding current density at rated load

Table 4. Penalty Functions

Critical fault clearing time (Transient stability)

$$f_1(t_c) = 0.95 + 0.05\left(\frac{0.19}{t_c}\right)^{15} \qquad (t_c \text{ in seconds})$$

Maximum field current density (Shielding effect)

$$f_2(J_{f_{max}}) = 0.9 + 0.1\left(\frac{J_{f_{max}}}{0.8\,J_{f_{limit}}}\right)^{15}$$

where $J_{f_{max}}$ is given by eq.(3.37), and $J_{f_{limit}}$ is a function of B_f (flux density at the field winding at the rated load condition) as shown in Fig.11.

Critical speed

$$f_3(n_c) = 0.9 + 0.1\left(\frac{n_{rated}}{n_c}\right)^3$$

Damper stress

$$f_4(\sigma_{max}) = 0.95 + 0.05\left(\frac{\sigma_{max}}{0.9\,\sigma_y}\right)^{15}$$

where σ_{max} is the maximum value of σ_{total} in eq.(3.35), and σ_y is yield strength of commonly used stainless steel that is 4.2×10^8 N/m^2.

Dynamic stability and thermal requirements

$$f_5(T_s) = 0.95 + 0.05\left(\frac{T_s}{1.0}\right)^{15} + 0.05\left(\frac{0.1}{T_s}\right)^{15}$$

Damper deflection

$$f_6(u_{max}) = 0.95 + 0.05\left(\frac{u_{max}}{0.002}\right)^{15}$$

where u_{max} is the maximum value of u_{total} in eq.(3.36).

Steepest Descent Method

The steepest descent method is a method to find the bottom of a basin by descending along the steepest slope. In this case, the basin is located in the eight-dimensional space spanned by the seven independent variables and the altitude.

The flow chart is shown in Fig.12. In this program, Δx_i ($i = 1 - 7$) is set as 5% of the initially guessed value of x_i .

Complete program list is shown in App.II .

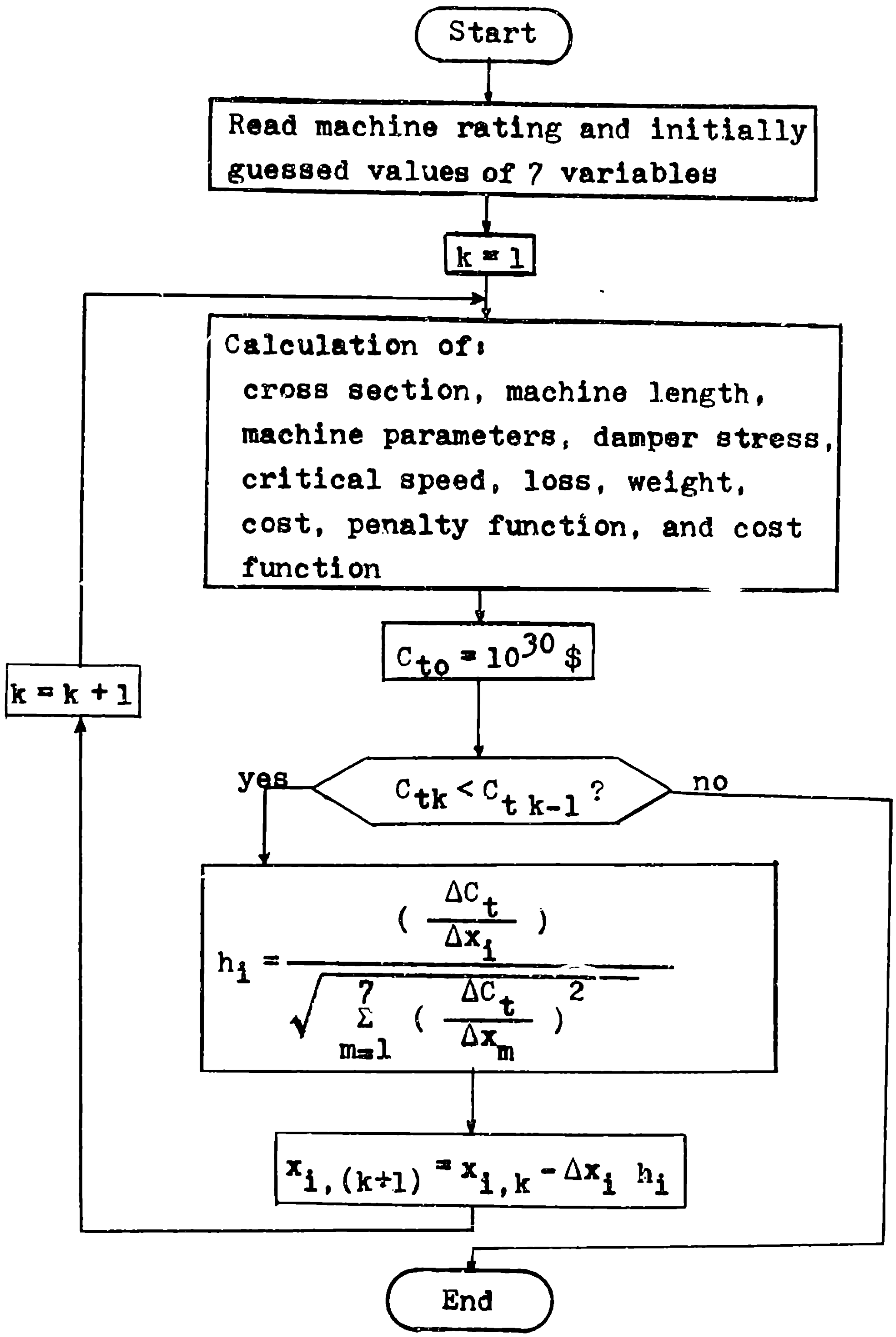


Fig.12 Flow Chart

C. RESULTS OF DESIGN

Optimized design sheets for 1000 - 10,000 MVA, 3600 rpm machines are shown in App. IV. All machines are assumed to operate at 0.85 PF. The results are summarized in Figs.13 - 22. A proposed 2000 MVA machine drawing based on the design sheet is shown in Fig.2.

As indicated by the design sheets, among the six penalty functions, functions of the critical fault clearing time, critical speed, and damper stress are conflicting. For larger machines, the damper stress is so severe that the critical speed and the critical fault clearing time decrease as a result of the thicker damper support and the longer rotor.

These designs are based on many assumptions as stated previously in this thesis, and there will be many discussions. One of them might concern the penalty functions. Obviously, different penalty functions give different optimum design. If we want a machine with small damper stress, we could get such a design by setting appropriate penalty functions.

Penalty functions shown in Table 4 seem to have rather sharp limits. In order to find the sensitivity to the sharpness, another set of penalty functions are tried.

The new penalty functions have 25% linear component such as,

$$f_1'(t_c) = 0.70 + 0.25\left(\frac{0.19}{t_c}\right) + 0.05\left(\frac{0.19}{t_c}\right)^{15} \qquad (4.15)$$

for critical fault clearing time, for example. The results for 2000 MVA machine using the new penalty functions are plotted with a sign ∇ in Fig.13 - 22. The 25% linear component in penalty functions seems to have very little effect.

Another discussion might be about the gap between the armature and the secondary damper. In this thesis, the gap has been set 0.05 m, assuming that the minimum allowable gap from a constructional point of view would give the minimum cost. In order to confirm this assumption 2000 MVA design was done with 0.075 m gap between the armature and the secondary damper. The results are shown with a sign * in Fig.13 - 22. We can see that a longer gap is not an economical choice. However, it may be a way to reduce the damper stress as suggested by Dagalakis(13).

There will be many other points to be checked or improved for more precise design.

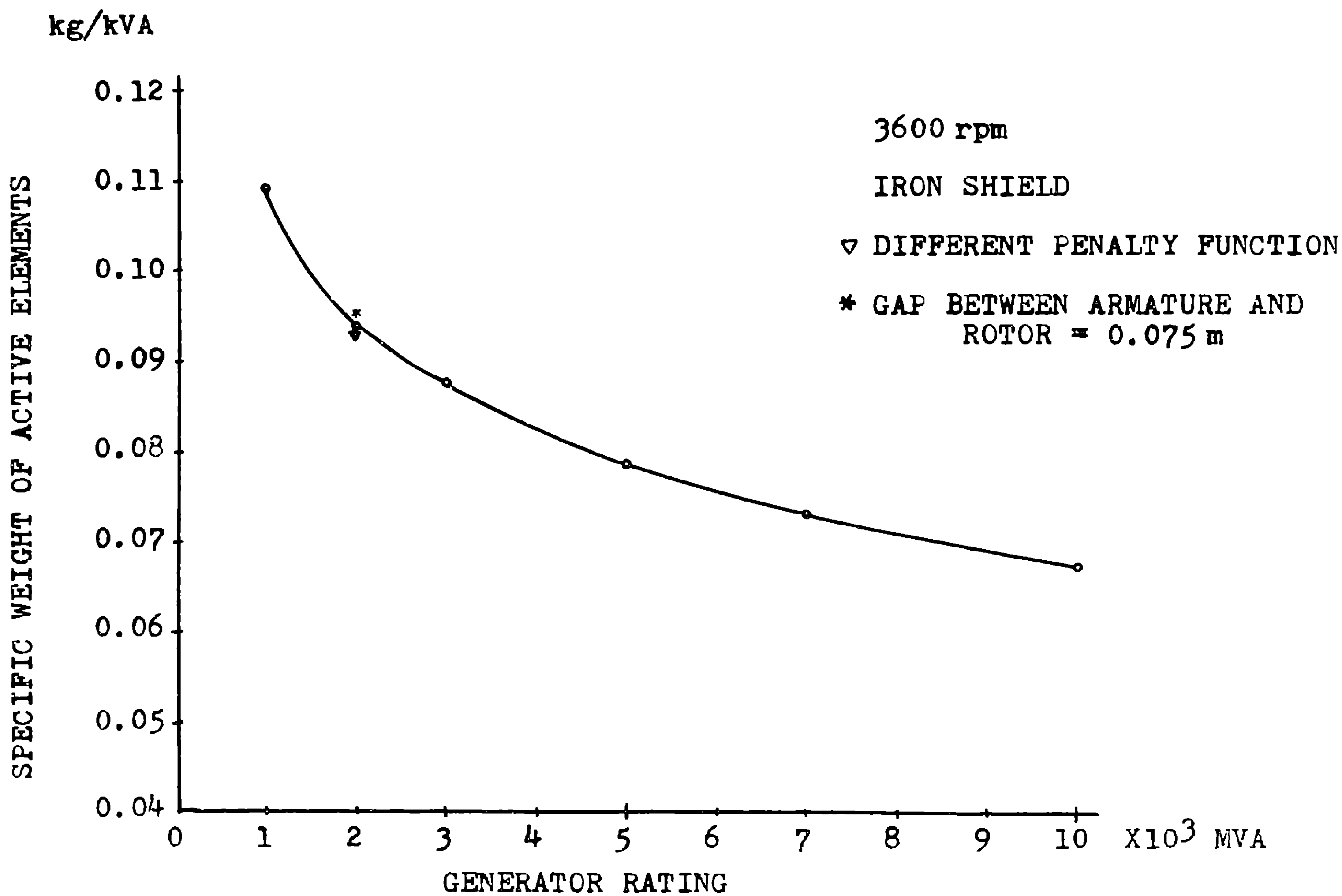


Fig.13 SPECIFIC WEIGHT OF ACTIVE ELEMENTS

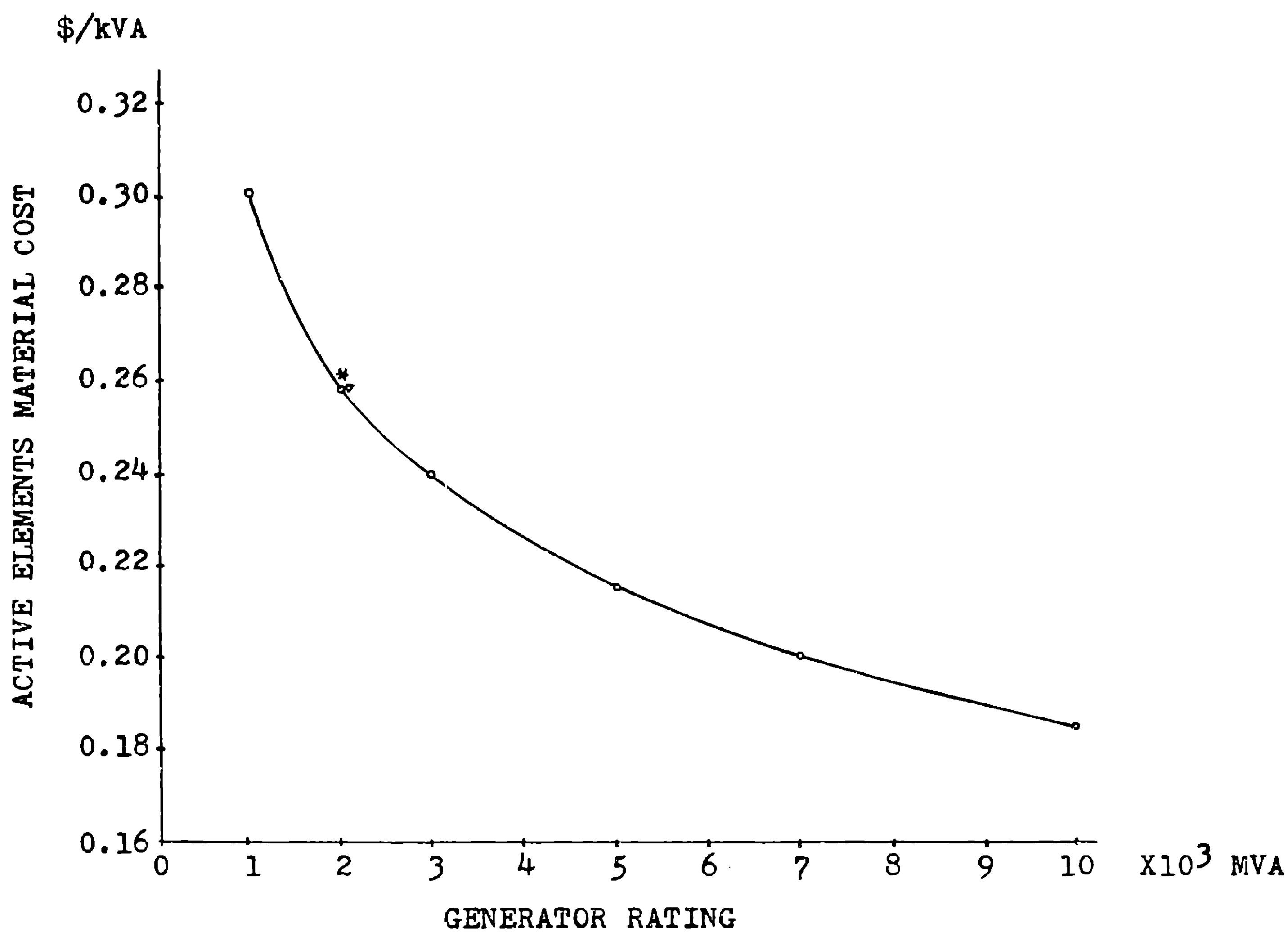


Fig.14 ACTIVE ELEMENTS MATERIAL COST

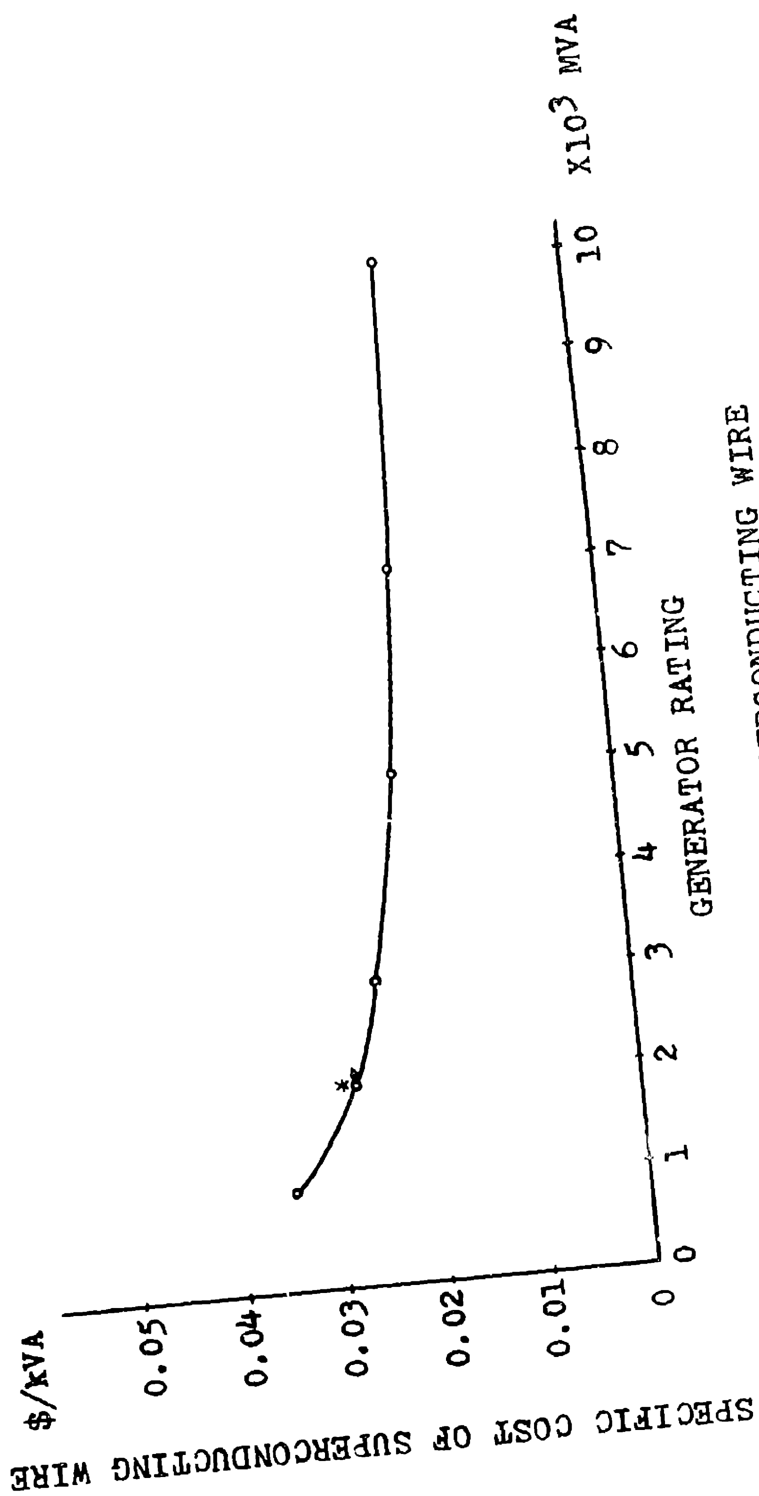


Fig.15 SPECIFIC COST OF SUPERCONDUCTING WIRE

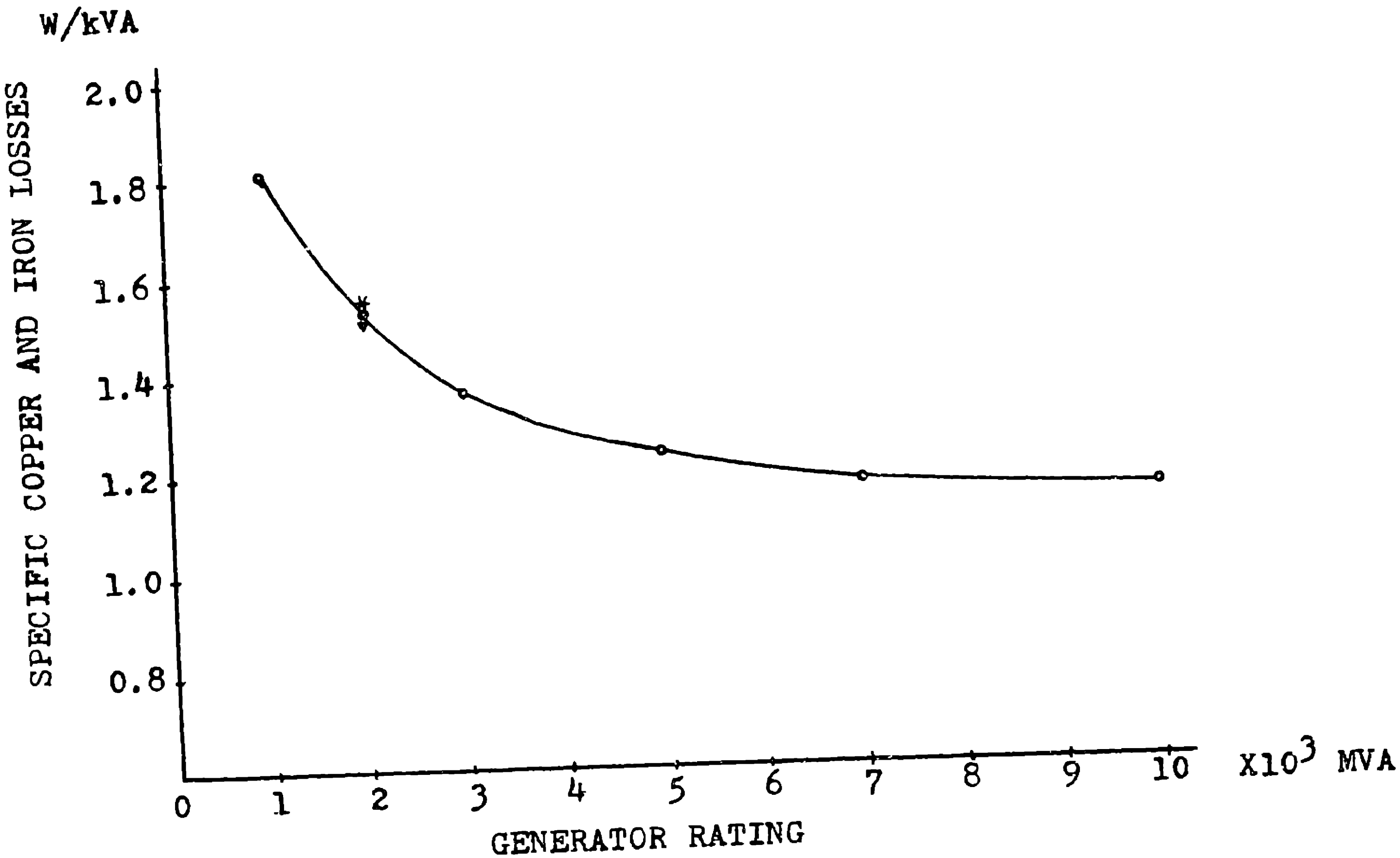


Fig.16 ARMATURE COPPER AND SHIELD IRON LOSSES / RATING

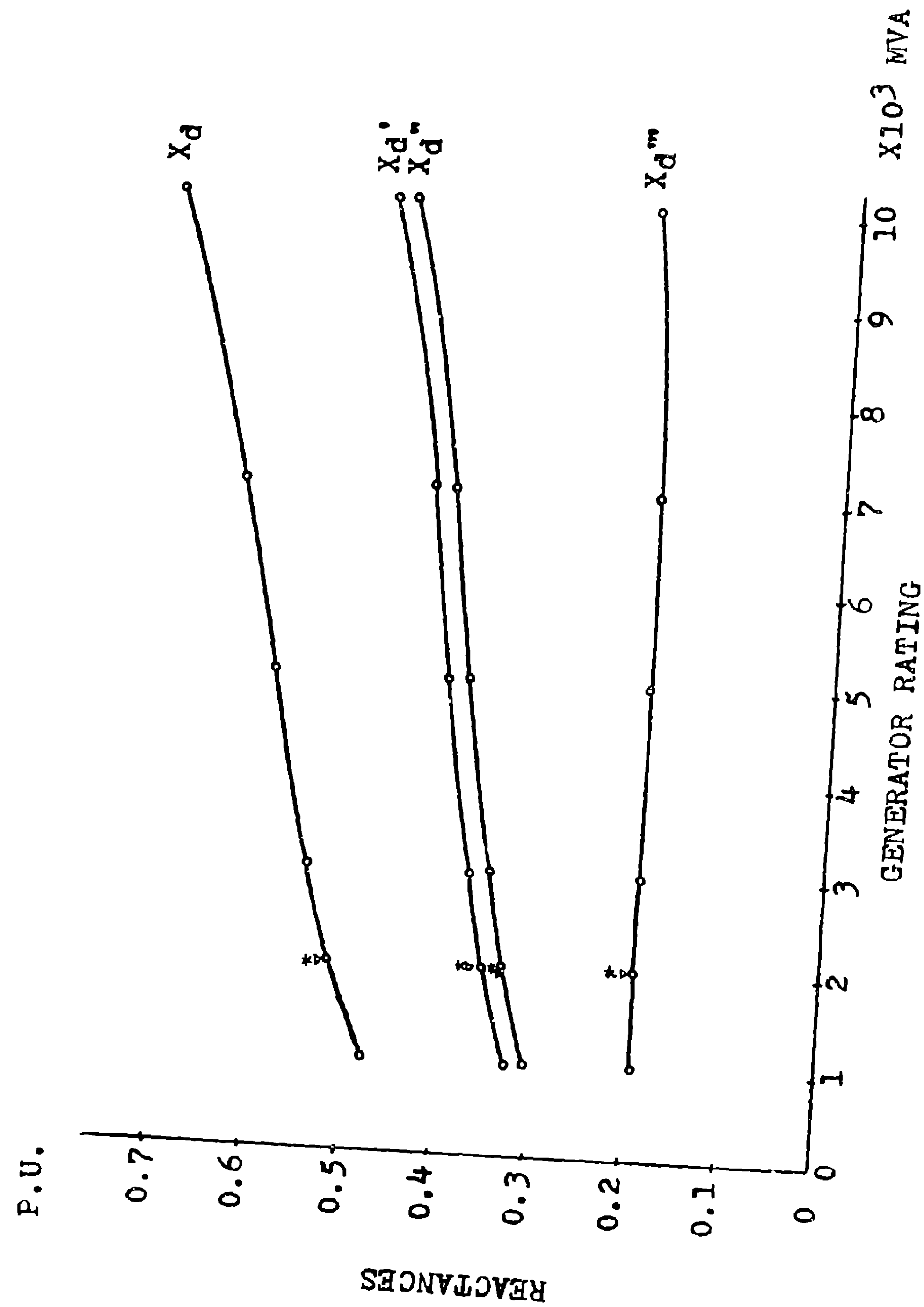


Fig.17 REACTANCES

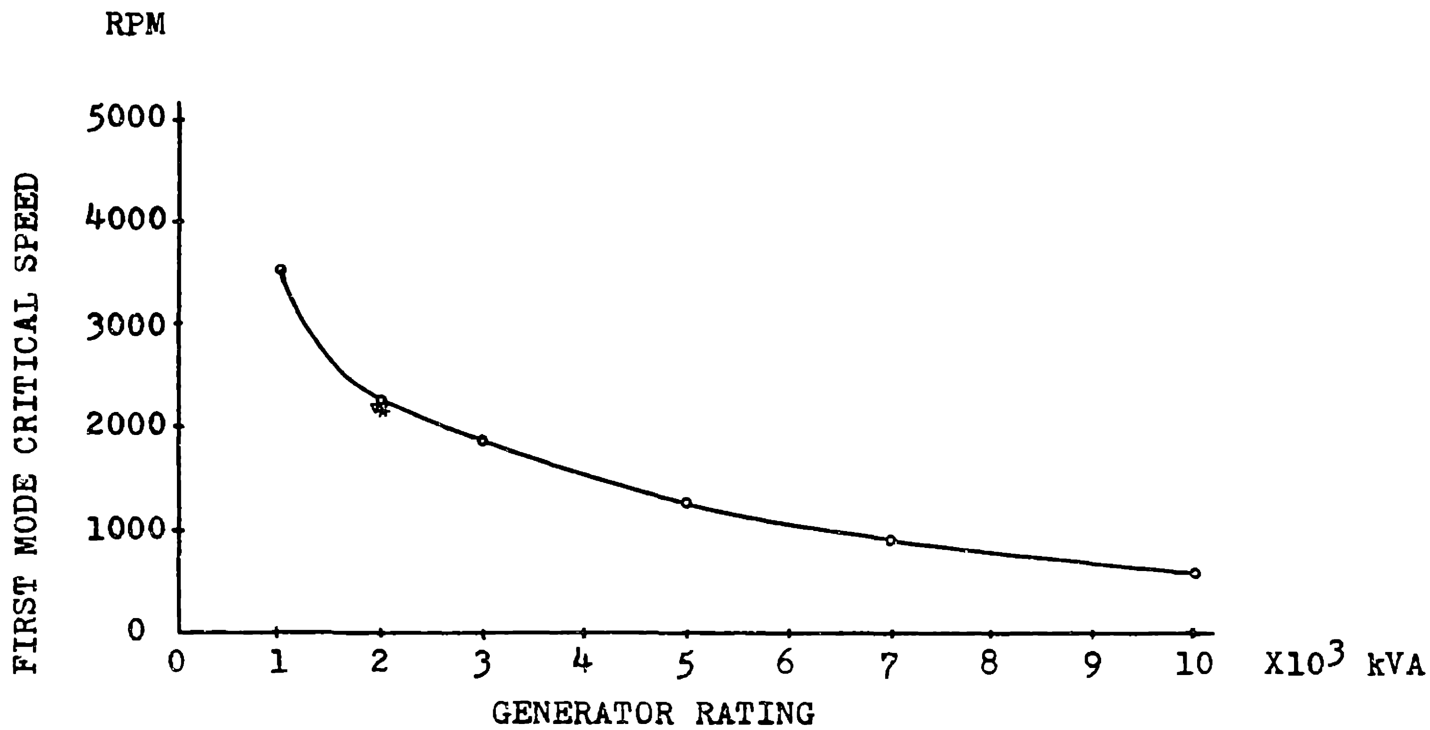


Fig.18 CRITICAL SPEED OF ROTOR

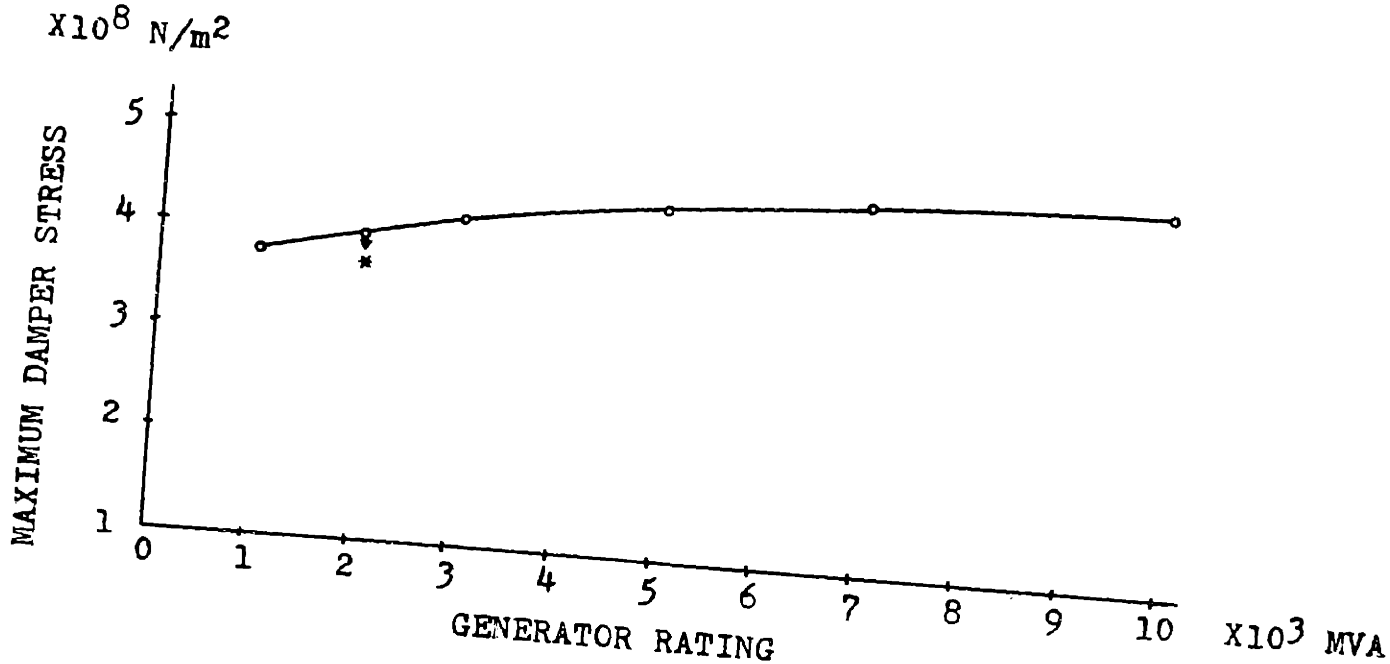


Fig.19 MAXIMUM DAMPER STRESS AT 3ϕ TERMINAL FAULT

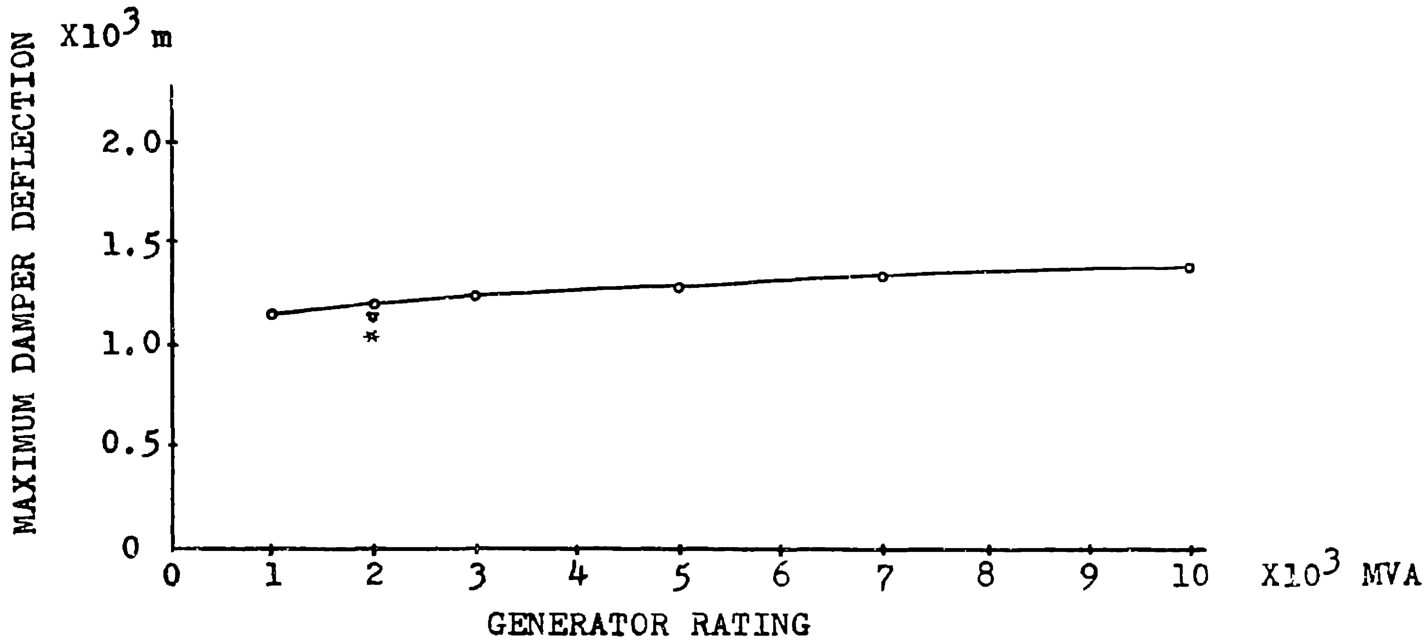


Fig.20 MAXIMUM DAMPER DEFLECTION AT 3ø TERMINAL FAULT

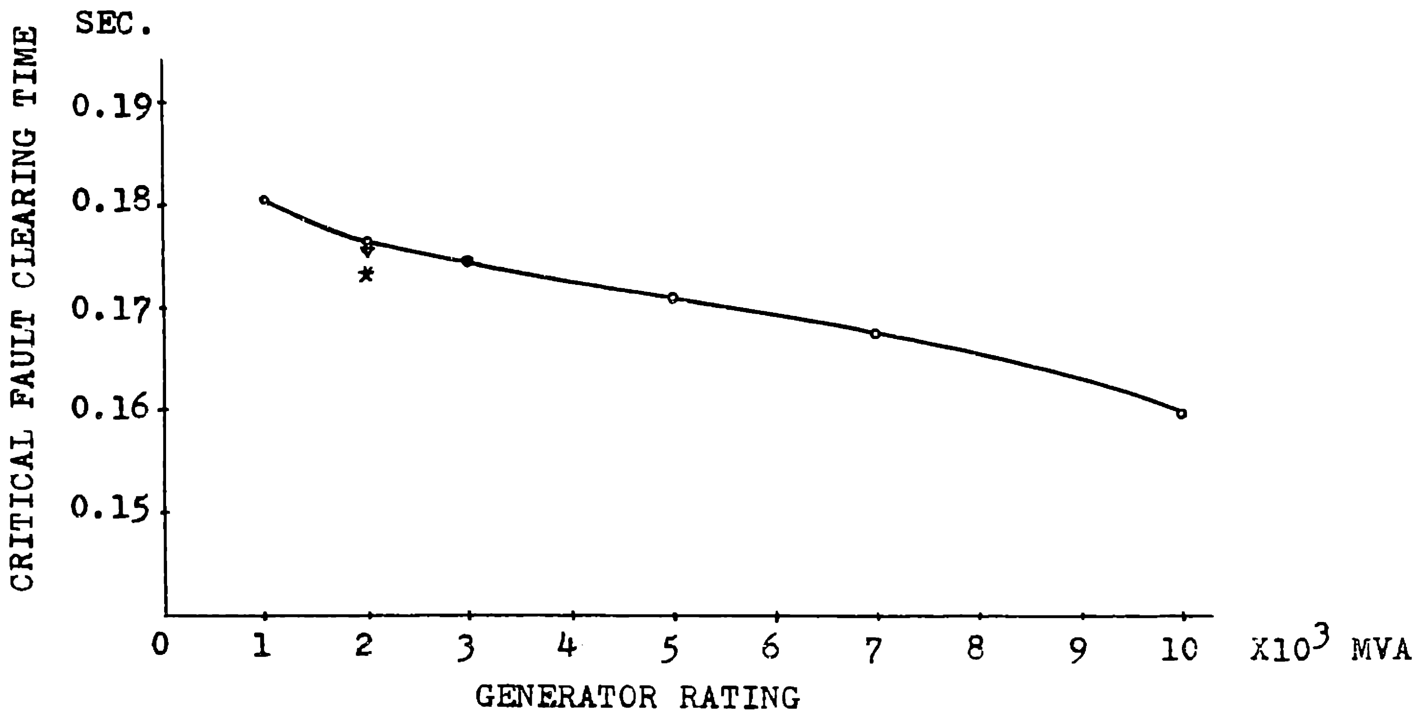


Fig.21 CRITICAL FAULT CLEARING TIME

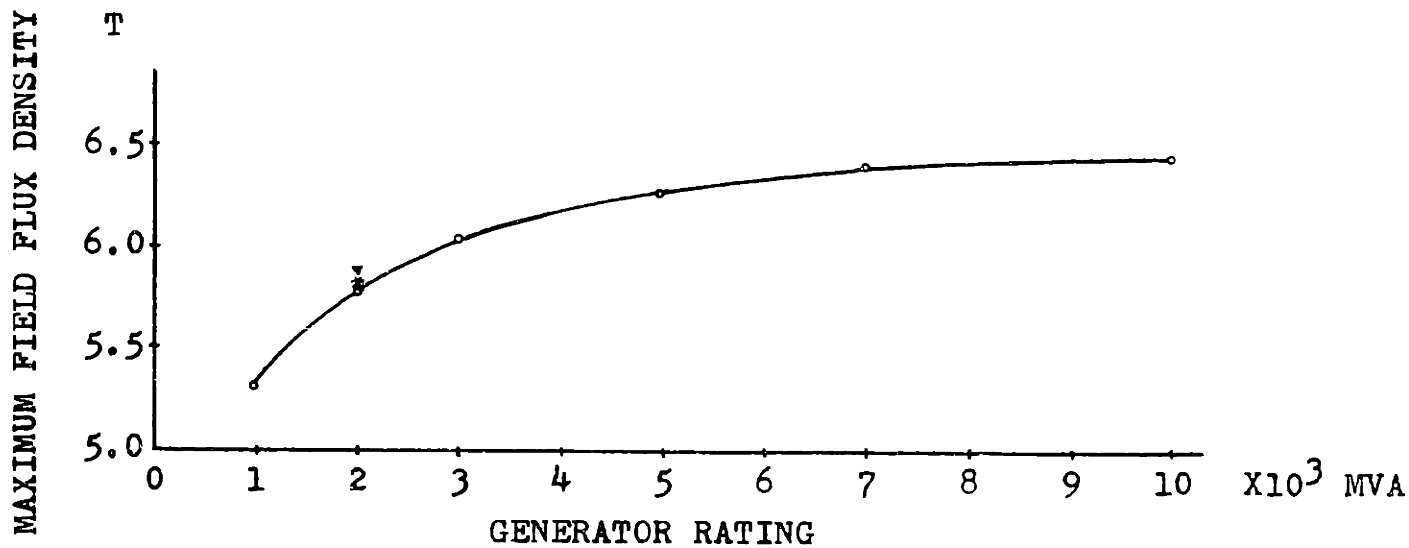


Fig.22 MAXIMUM FLUX DENSITY AT FIELD WINDING AT RATED LOAD

V. FEASIBILITY OF SUPERCONDUCTING ALTERNATOR

Prospects for large superconducting alternators are bright from the view point of weight, cost and efficiency with reference to Figs.13 - 16. Specific weight in Fig.13 is 30 - 50 % more than that given by a previous study[2] because of the thick damper support. Still, specific weight of superconducting alternators is quite attractive compared with current conventional machines, even if multiplied by two to account for bearings, enclosure, etc.

As for other factors shown in Figs.17 - 22, none of them are completely unsatisfactory, although some of them seem to be very severe, especially in larger machines.

However, we should expect many other problems to be solved before a superconducting alternator is actually constructed for an electric power utility in another decade.

VI. CONCLUSIONS

1. Requirements for the damper shield may be classified as: transient stability, dynamic stability, thermal requirements for negative sequence current, stress and deflection at a terminal fault, shielding of the field winding, and critical speed of rotor.
2. Among these requirements, damper stress, critical speed and the transient stability are conflicting especially in larger machines.
3. Optimized design sheets for 1000 - 10000 MVA machines are given by using such techniques as regression analysis, penalty functions, and the steepest descent method.
4. With reference to these design sheets, weight, cost and efficiency are quite attractive. The prospects are bright for large synchronous alternators.

APPENDIX I. REACTANCES AND TIME CONSTANTS

Inductances

$$L_a' = \frac{L_a}{\ell_a N_{at}^2} = \frac{16\,\mu_o \sin^2 \frac{\theta_{wae}}{2}}{3\pi\,\theta_{wae}^2\,(1 - x^2)^2} \Big[1 - 4x^3 + 3x^4 + \frac{2}{3}(1 - x^3)^2 \cdot \Big(\frac{R_{ao}}{R_{mi}}\Big)^2 \Big] \tag{A.1}$$

$$L_f' = \frac{L_f}{\ell_f N_{ft}^2} = \frac{16\,\mu_o \sin^2 \frac{\theta_{wfe}}{2}}{3\pi\,\theta_{wfe}^2\,(1 - y^2)^2} \Big[1 - 4y^2 + 3y^4 + \frac{2}{3}(1 - y^3)^2 \cdot \Big(\frac{R_{fo}}{R_{mi}}\Big)^2 \Big] \tag{A.2}$$

$$M_{af}' = \frac{M_{af}}{\ell_{af} N_{at} N_{ft}} = \frac{32\,\mu_o \sin\frac{\theta_{wfe}}{2} \sin\frac{\theta_{wae}}{2}}{3\pi\,\theta_{wfe}\,\theta_{wae}\,(1 - y^2)(1 - x^2)} \Big(\frac{R_{fo}}{R_{ao}}\Big) \cdot (1 - y^3) \Big[1 - x + \frac{1}{3}(1 - x^3)\Big(\frac{R_{ao}}{R_{mi}}\Big)^2 \Big] \tag{A.3}$$

$$L_p' = \frac{L_p}{\ell_p N_p^2} = \frac{\mu_o \pi}{8} \Big[1 + \Big(\frac{R_p}{R_{mi}}\Big)^2 \Big] \tag{A.4}$$

$$L_s' = \frac{L_s}{\ell_s N_s^2} = \frac{\mu_o \pi}{8} \Big[1 + \Big(\frac{R_s}{R_{mi}}\Big)^2 \Big] \tag{A.5}$$

$$M_{ap}' = \frac{M_{ap}}{\ell_{ap} N_a N_p} = \frac{2\,\mu_o \sin\frac{\theta_{wae}}{2}}{\theta_{wae}\,(1 - x^2)} \frac{R_p}{R_{ao}} \Big[1 - x + \frac{1}{3}(1 - x^3) \cdot \Big(\frac{R_{ao}}{R_{mi}}\Big)^2 \Big] \tag{A.6}$$

$$M_{as}' = M_{ap}' \frac{R_s}{R_p} \tag{A.7}$$

Reactances

$$X_a = \frac{1.5\sqrt{2}\; L_a' \, J_a \, \theta_{wae} \, R_{ao}^2 \, (1 - x^2)\, \ell_a}{M_{af}' \, J_f \, \theta_{wfe} \, R_{fo}^2 \, (1 - y^2)\, \ell_{af}} \tag{A.8}$$

$$X_d = \frac{X_a}{-X_a \sin\psi + \sqrt{1 - X_a^2 \cos^2\psi}} \tag{A.9}$$

$$X_d' = X_d \left(1 - \frac{M_{af}'^{\,2}}{L_a' L_f'} \frac{\ell_{af}}{\ell_a}\right) \tag{A.10}$$

$$X_d'' = X_d \left(1 - \frac{M_{ap}'^{\,2}}{L_a' Lp'} \frac{\ell_{af}}{\ell_a}\right) \tag{A.11}$$

$$X_d''' = X_d \left(1 - \frac{M_{as}'^{\,2}}{L_a' L_s'} \frac{\ell_{af}}{\ell_a}\right) \tag{A.12}$$

Time Constants

$$T_p = \frac{\mu_o}{2} \sigma_p' \, t_p R_p \left[1 + \left(\frac{R_p}{R_{mi}}\right)^2\right] \tag{A.13}$$

$$T_s = \frac{\mu_o}{2} \sigma_s' \, t_s R_s \left[1 + \left(\frac{R_s}{R_{mi}}\right)^2\right] + \frac{\mu_o}{2} \sigma_b' \, t_b R_b \left[1 + \left(\frac{R_b}{R_{mi}}\right)^2\right] \tag{A.14}$$

APPENDIX II . MULTIPLE REGRESSION ANALYSIS

Suppose y is to be regressed on a number of variables or "regressors". For the purpose of simplification, let us assume y is a function of two variables, x and z in the form of:

$$y = f(x, z) \qquad (A.15)$$

where $f(x, z)$ may contain some coefficients, say α, β, and γ. Our purpose is to find the values of α, β and γ which minimize:

$$u = \sum_{i}^{n} [y_i - f(x_i, z_i)]^2 \qquad (A.16)$$

where n is the sample size.

This is done with calculus by setting the partial derivatives of the function u with respect to α, β and γ equal to zero.

For example, if y is a linear function of x and z, the relation is of the form:

$$y = f(x, z) = \alpha + \beta x + \gamma z \qquad (A.17)$$

Then,

$$u = \sum_{i} (y_i - \alpha - \beta x_i - \gamma z_i)^2 \qquad (A.18)$$

By setting the partial derivatives equal to zero, we obtain:

$$\alpha = \overline{y} \quad \text{(mean value of } y) \tag{A.19}$$

$$\sum_i x_i y_i = \beta \sum_i x_i^2 + \gamma \sum_i x_i z_i \tag{A.20}$$

$$\sum_i z_i y_i = \beta \sum_i x_i z_i + \gamma \sum_i z_i^2 \tag{A.21}$$

These equations may be solved for α, β and γ.

An index of the goodness of fit is a standard error of estimate given by:

$$\sigma = \left(\sum_i^n e_i^2 / \nu \right)^{\frac{1}{2}} \tag{A.22}$$

where e_i is the error for i th sample, and ν is the degree of freedom given by:

$\nu = n -$ (number of coefficients to be estimated)

We may find the confidence level of the estimate by using t-distribution.

APPENDIX.III. IBM-1130 PROGRAM LIST

```
C**** SUPERCONDUCTING ALTERNATOR DESIGN      MIKE FURUYAMA  RM.9-414 X.3-3360
      DIMENSION V(7), S(7), D(7), F(8)
      EQUIVALENCE (V(1),RFI),(V(2),THF),(V(3),THP),(V(4),THB),(V(5),THS)
     1,(V(6),THA),(V(7),AJF)
      DATA        FRQ , AJA       , SFA , SGA        , SFF
     1/           60.0, 3000000., 0.25, 60000000., 0.625/
      DATA CNDP   , CNDS   , TF ,CNDB, RHM , RHA , RHAE
     1/  5000000.,60000000.,300.,1380000., 7500.,4500.,4000./
      DATA GAS , GBP   , GPV  , THC   , THV   , SGY
     1/     0.05, 0.002, 0.002, 0.025, 0.005, 42000./
      DATA EST      ,ECP , THWAE, THWFE, BSAT , PM
     1/     2.10E11 ,1.1E11 , 60.0 , 120.0 , 1.5 , .004/
      DATA RHCP   , RHST , CTST, CTCP , CTM
     1/     8900.0, 7800.0, 4.0 , 4.0 , 2.0 /
      THWAE=THWAE/57.3
      THWFE=THWFE/57.3
      PI=3.14159
      UM=4.0E-7*PI
      READ(2,700) PWR,PF,VLT,RFI,THF,THP,THB,THS,THA,AJF
      FCO=1.0E20
      DO 3 K=1,7
    3 D(K)=0.05*V(K)
C**** RATING
      OMG=2.0*PI*FRQ
      POLE=2.0
      RPM=FRQ *60.0
      CRA=PWR*1000.0/VLT/1.73205
      IDSG=0
   10 DO 50 J=1,8
      IF(J-2) 45,35,25
   25 V(J-2)=V(J-2)-D(J-2)
   35 V(J-1)=V(J-1)+D(J-1)
C**** CROSS SECTION
   45 RFO=RFI+THF
      RVI=RFO
      RVO=RVI+THV
      RPI=RVO+GPV
      RPO=RPI+THP
      RBI=RPO+GBP
      RBO=RBI+THB
      RSI=RBO
      RSO=RSI+THS
      RAI=RSO+GAS
      RAO=RAI+THA
      RMI=RAO
      RCO=RFI
      RCI=RCO-THC
C**** LENGTH OF MACHINE
      X=RAI/RAO
      Y=RFI/RFO
      XX=RAO/RMI
      YY=RFO/RMI
      C1=16.0     *UM          /(  PI*THWAE*THWAE*(1.0-X*X)**2)
      C2=16.0     *UM          /(  PI*THWFE*THWFE*(1.0-Y*Y)**2)
      C21=32.0*UM/(  PI*THWFE*THWAE*(1.0-Y*Y)*(1.0-X*X))
```

```
      C10=    THWAE*0.5
      C11=    THWFE*0.5
      SC10=SIN(C10)
      SC11=SIN(C11)
      C3=-3.0
      C4=-1.0
      C5=3.0
      C6=C4/C5
      C12=2.0
      C7=C1*SC10      **2/C3*(C4-C5*X**4+4.0*X**C5+2.0*C6*(1.0-X**C5)**2
     1   *XX**C12)
      C9=C2*SC11      **2/C3*(C4-C5*Y**4+4.0*Y**C5+2.0*C6*(1.0-Y**C5)**2
     1   *YY**C12)
      C22=C21*SC10*SC11         /(-C3   )*(1.0-Y**C5)*(RFO/RAO)        *
     1    (1.0-X**(-C4)-C6*(1.0-X**C5)*XX**C12)
      ELA=    C7
      ELS=ELA*1.5
      ELF=    C9
      EMAS=    C22
      XAA=ELS*1.41421*AJA*THWAE*RAO*RAO*(1.0-X*X)/(EMAF*AJF*THWFE*RFO*
     1    RFO*(1.0-Y*Y))
      PBB=0.375*1.41421*OMG*EMAF*AJA*AJF     *THWAE*THWFE*RAO*RAO*RFO*RFO
     1    *(1.0-X*X)*(1.0-Y*Y)*0.001
      CNTH=ABS(PF)
      SNTH=SQRT(1.0-PF*PF)*PF/CNTH
      C31=PBB*PBB*(1.0-XAA*XAA)
      ELAE= RAO+RAI
      C32=PBB*XAA*PWR*SNTH+PBB*PBB*XAA*XAA*ELAE
      C33=PWR*PWR+2.0*PBB*XAA*PWR*SNTH*ELAE+(PBB*XAA*ELAE)**2
      ELAF=( C32+SQRT(C32*C32+C31*C33))/C31
      ELAS=ELAF+ELAE
      ELRB=ELAF+RFO *1.5*PI
      ELBR=ELRB+2.0*RSO
      ELAT=ELAS+ELAE
C**** MACHINE PARAMETERS
      XA=XAA*ELAS/ELAF
      C41=  (-XA*SNTH+SQRT(1.0-XA*XA*CNTH*CNTH))
      XD=XA/C41
      XD1=XD*(1.0-1.5*EMAF*EMAF/ELS/ELF*ELAF/ELAS)
      RP=(RPO+RPI)*0.5
      RS=(RSO+RSI)*0.5
      ELP= PI*UM*(1.0+(RP/RMI)**2       )/8.0
      ELSS=PI*UM*(1.0+(RS/RMI)**2       )/8.0
      EMAP=2.0*UM*SC10     *(RP/RAO)    *(1.0-X+1.0
     1     /3.0     *(1.0-X**3        )*(RAO/RMI)**2)
     2     /(   THWAE*(1.0-X*X)         )
      EMAS=EMAP*(RS/RP)
      XD11=XD*(1.0-1.5*EMAP*EMAP/ELS/ELP*ELAF/ELAS)
      XD111=XD*(1.0-1.5*EMAS*EMAS/ELS/ELSS*ELAF/ELAS)
      TP=0.5*UM*CNDP*THP*RP*(1.0+(RP/RMI)**2       )*1000.
      TS=0.5*UM*CNDS*THS*RS*(1.0+(RS/RMI)**2       )
      RB=(RBO+RBI)/2.0
      TB=0.5*UM*CNDB*THB*RB*(1.0+(RB/RMI)**2)
      TSB=TS+TB
      SCR=1.0/XD
```

```
C**** MAXIMUM FLUX EDNSITY AT FIELD WINDING
      BMX=2.0*UM/PI*AJF*RFI*SC11     /Y*(1.0-Y+0.333333*(1.0-Y*Y*Y)*
     1    (RFO/RMI)**2)
C**** INERTIA CONSTANT
      AIF=RFO**4-RFI**4
      AIB=RBO**4-RBI**4
      AIS=RSO**4-RSI**4
      AIC=RCO**4-RCI**4
      AIV=RVO**4-RVI**4
      AIP=RPO**4-RPI**4
      AIST=AIC+AIV+AIB
      AICP=AIP+AIS
      H=0.3E-3/PWR*ELBR*(RHST*AIST+RHCP*AICP+RHCP*SFF*AIF)*PI*OMG*OMG+3.
C**** DAMPER STRESS
      RA=(RAO+RAI)/2.0
      RF=(RFO+RFI)/2.0
      RSB=(RSO+RBI)/2.0
      THSB=RSO-RBI
      ZZ=RS/RMI
      WW=RP/RMI
      SNDL=XA*CNTH
      CNDL=SQRT(1.0-SNDL*SNDL)
      BA0=4.2426*UM/PI*SC10*RAO*(1.0-X+(1.0-X*X*X)/3.0*XX*XX)*AJA
      BA1=BA0/XD111
      BF0=0.666667*UM/PI*SC11*RS*(RFO/RS)**3*(1.-Y*Y*Y)*(1.0-ZZ*ZZ)*AJF
      DELT=ATAN(SNDL/CNDL)
      THET=ATAN(SNTH/CNTH)
      CCC=2.0*BA1/(1.0+ZZ*ZZ)
      AAA=BA0*SNTH+CCC+BF0*CNDL
      BBB=BA0*CNTH+BF0*SNDL
      BOT=SQRT(AAA*AAA+BBB*BBB)+CCC
      ATBA=ATAN(BBB/AAA)
      EEE=BF0*COS(ATBA-DELT)+BA0*SIN(ATBA+THET)
      DDD=BF0*SIN(ATBA-DELT)-BA0*COS(ATBA+THET)
      FR1=(BOT*BOT-EEE*EEE-DDD*DDD)/UM/4.0E4
      FR2=SQRT((BOT*BOT-EEE*EEE+DDD*DDD)**2+(EEE*DDD)**2*4.0)/UM/4.0E4
      FR=FR1+FR2
      STMB=FR2*2.0*RSB*RSB/THSB/THSB
      STMU=-FR1*RSB/THSB
      STMG=STMB-STMU
      VLB=RBO*RBO-RBI*RBI
      VLS=RSO*RSO-RSI*RSI
      RHSB=(VLB*RHST+VLS*RHCP)/(VLB+VLS)
      STCF=RHSB   *RSB*RSB*OMG*OMG*1.0E-4
      STB=STMB+STCF+STMU
      ESB=(EST*AIB+ECP*AIS)/(AIB+AIS)
      DFMB=0.666667*FR2*2.0E4/ESB/THSB**3*RSB**4
      DFMU=STMU*RSB/ESB*1.0E4
      DFMG=DFMB-DFMU
      DFCF=STCF*RSB/ESB*1.0E4
      DFB=DFMB+DFCF+DFMU
C**** CRITICAL SPEED
      C42=PI/4.0*(EST*AIST+ECP*AICP)
      WTS=RHST   *PI*(RCO*RCO-RCI*RCI+RVO*RVO-RVI*RVI+VLB)
      WTC=RHCP   *PI*(RPO*RPO-RPI*RPI+VLS)
```

```
      WTF=RHCP   *PI*(RFO*RFO-RFI*RFI)*SFF
      ENC=94.3/ELBR/ELBR*SQRT(C42      /(WTS+WTC+WTF))
C**** LOSS AND EFFICIENCY
      AJO=AJF*C41
      BRM=4.0*UM*AJO*SC11/3.0/PI*RMI*YY**3*(1.0-Y**3)
      THM=BRM*RMI/BSAT
      RMO=RMI+THM
      WTM=RHM*PI*(RMO*RMO      -RMI*RMI)*ELAF
      PLM=PM*WTM
      PLA=3.0*AJA*AJA*RAO*RAO*(1.0-X*X)*THWAE/SGA/SFA*1.0E-3*ELAT
      PLT=PLM+PLA
      PLK=PLT/PWR*1000.
      EFF=PWR/(PWR+PLT/PF)
C**** WEIGHT OF MACHINE ACTIVE ELEMENTS
      WTR=ELRB*(WTS+WTC+WTF)
      WTA=3.0*THWAE*((RAO* RAO-RAI*RAI)*RHA*ELAF+((2.0*RAO-RAI)**2-RAI*
     1   RAI)*RHAE*2.0*ELAE)
      WTT=WTR+WTA+WTM
      WTK=WTT/PWR
C**** COST
      AJFLT=1.46E10/(BMX+8.4)-6.4E8
      CTF=(0.12+0.018*BMX+0.0036*BMX*BMX)*AJFLT*2.0E-3
      CSF=CTF*THWFE*(RFO*RFO-RFI*RFI)*(ELAF+PI*RFO)
      CFK=CSF/PWR
      CST=(CTST*WTS+CTCP*WTC)*ELRB+CTM*WTM+CSF+WTA*CTCP
      CTK=CST/PWR
C**** PENALTY FUNCTION
      CFCT=0.1339-0.1330*XD11+0.0005*TS+0.0278*H
      AJFR=0.9716+0.3701*(1.0-EXP(-1.01/TS)) *(1.0-EXP(-1.98/TP))
     1       +0.0288*XD11+0.0095*H
      AJFMX=AJF*AJFR
      PFTS=0.95+0.05*(0.19/CFCT)**15
      PFFC=0.90+0.10*(AJFMX/0.8/AJFLT)**15
      PFSS=0.95+0.05*(STB/0.9/SGY)**15
      PFDD=0.95+0.05*(DFB/0.002)**15
      PFDS=0.95+0.05*TSB**15+0.05*(0.1/TSB)**15
      PFCS=0.9+0.1*(RPM/ENC)**3
C**** COST FUNCTION
      FC=CST*PFTS*PFFC*PFSS*PFDS*PFCS*PFDD
      IF(J-1) 49,49,50
   49 CONTINUE
      IDSG=IDSG+1
      IPAGE=1
      WRITE(3,701) IDSG,IPAGE
      WRITE(3,703)
      WRITE(3,705)
      WRITE(3,707) PWR
      WRITE(3,709) PF
      WRITE(3,711) VLT
      WRITE(3,713) CRA
      WRITE(3,715) FRQ
      WRITE(3,717) RPM
      WRITE(3,719) POLE
      WRITE(3,703)
      WRITE(3,721)
```

```
WRITE(3,723) RAO
WRITE(3,725) RAI
WRITE(3,726) THA
WRITE(3,727) ELAF
WRITE(3,729) AJA
WRITE(3,730) ELAS
WRITE(3,731) SFA
WRITE(3,733) SGA
WRITE(3,703)
WRITE(3,741)
WRITE(3,743) RFO
WRITE(3,745) RFI
WRITE(3,746) THF
WRITE(3,747) ELAF
WRITE(3,751) AJF
WRITE(3,753) SFF
WRITE(3,755) BMX
WRITE(3,757) AJFMX
WRITE(3,759) AJFR
WRITE(3,703)
WRITE(3,761)
WRITE(3,763) RPI
WRITE(3,765) RPO
WRITE(3,767) THP
WRITE(3,769) CNDP
WRITE(3,703)
IPAGE=2
WRITE(3,701) IDSG,IPAGE
WRITE(3,703)
WRITE(3,781)
WRITE(3,783) RSI
WRITE(3,785) RSO
WRITE(3,787) THS
WRITE(3,789) FR
WRITE(3,791) CNDS
WRITE(3,703)
WRITE(3,801)
WRITE(3,803) RBI
WRITE(3,805) RBO
WRITE(3,807) THB
WRITE(3,809) STCF
WRITE(3,811) STMG
WRITE(3,813) STB
WRITE(3,815) DFCF
WRITE(3,817) DFMG
WRITE(3,819) DFB
WRITE(3,703)
WRITE(3,821)
WRITE(3,823) XD
WRITE(3,825) XD1
WRITE(3,827) XD11
WRITE(3,829) XD111
WRITE(3,831) TF
WRITE(3,833) TP
WRITE(3,835) TSB
```

```
      WRITE(3,837) H
      WRITE(3,703)
      WRITE(3,841)
      WRITE(3,843) CFCT
      WRITE(3,845) SCR
      WRITE(3,703)
      WRITE(3,941)
      WRITE(3,943) RMO
      WRITE(3,945) RMI
      WRITE(3,947) THM
      WRITE(3,703)
      IPAGE=3
      WRITE(3,701) IDSG,IPAGE
      WRITE(3,703)
      WRITE(3,851)
      WRITE(3,852) ELBR
      WRITE(3,853) ENC
      WRITE(3,703)
      WRITE(3,881)
      WRITE(3,883) PLM
      WRITE(3,885) PLA
      WRITE(3,887) PLT
      WRITE(3,889) PLK
      WRITE(3,891) EFF
      WRITE(3,703)
      WRITE(3,901)
      WRITE(3,903) WTR
      WRITE(3,905) WTA
      WRITE(3,907) WTM
      WRITE(3,909) WTT
      WRITE(3,911) WTK
      WRITE(3,703)
      WRITE(3,861)
      WRITE(3,863) PFTS
      WRITE(3,865) PFCS
      WRITE(3,867) PFSS
      WRITE(3,873) PFDD
      WRITE(3,869) PFDS
      WRITE(3,871) PFFC
      WRITE(3,703)
      WRITE(3,921)
      WRITE(3,923) CSF
      WRITE(3,925) CST
      WRITE(3,927) CFK
      WRITE(3,929) CTK
      WRITE(3,931) FC
      WRITE(3,703)
   50 F(J)=FC
      V(7)=V(7)-D(7)
      SUM=0.0
      IF(F(1)-FCO) 60,60,100
   60 DO 70 K=1,7
      S(K)=(F(K+1)-F(1))/F(1)
   70 SUM=SUM+S(K)*S(K)
      DO 80 K=1,7
```

```
      V(K)=V(K)-S(K)/SQRT(SUM)*D(K)
      IF(V(K)) 75,75,80
   75 V(K)=0.0001
   80 CONTINUE
      FCO=F(1)
      GO TO 10
  100 CALL EXIT
  700 FORMAT(F9.0,8F7.0,F11.0)
  701 FORMAT(1H1,10X,40H SUPERCONDUCTING ALTERNATOR DESIGN SHEET,5X ,4H
     1NO.,I2,4X,5H PAGE,I2)
  703 FORMAT(/72H -------------------------------------------------------
     1----------------/)
  705 FORMAT(13H ** RATING **)
  707 FORMAT(5X,46H RATED POWER (KVA) ...........................,F10.0)
  709 FORMAT(5X,46H POWER FACTOR ................................,F10.3)
  711 FORMAT(5X,46H LINE TO LINE VOLTAGE (V) ....................,F10.0)
  713 FORMAT(5X,46H ARMATURE CURRENT (A) ........................,F10.0)
  715 FORMAT(5X,46H FREQUENCY (HZ) ..............................,F10.1)
  717 FORMAT(5X,46H SPEED (RPM) .................................,F10.0)
  719 FORMAT(5X,46H NUMBER OF POLES .............................,F10.0)
  721 FORMAT(15H ** ARMATURE **)
  723 FORMAT(5X,46H OUTER RADIUS OF ARMATURE WINDING (M) ........,F10.4)
  725 FORMAT(5X,46H INNER RADIUS OF ARMATURE WINDING (M) ........,F10.4)
  726 FORMAT(5X,46H RADIAL THICKNESS OF ARMATURE WINDING (M) ....,F10.4)
  727 FORMAT(5X,46H LENGTH OF ARMATURE MIDDLE PART (M) ..........,F10.4)
  729 FORMAT(5X,46H CURRENT DENSITY OF ARMATURE WINDING (A/M2) ..,F10.0)
  730 FORMAT(5X,46H ARMATURE TOTAL EFFECTIVE LENGTH (M) .........,F10.4)
  731 FORMAT(5X,46H ARMATURE WINDING SPACE FACTOR ...............,F10.4)
  733 FORMAT(5X,46H ARMATURE WINDING CONDUCTIVITY (MHOS/M) ......,F10.0)
  741 FORMAT(20H ** FIELD WINDING **)
  743 FORMAT(5X,46H OUTER RADIUS OF FIELD WINDING (M) ...........,F10.4)
  745 FORMAT(5X,46H INNER RADIUS OF FIELD WINDING (M) ...........,F10.4)
  746 FORMAT(5X,46H RADIAL THICKNESS OF FIELD WINDING (M) .......,F10.4)
  747 FORMAT(5X,46H LENGTH OF FIELD WINDING MIDDLE PART (M) .....,F10.4)
  751 FORMAT(5X,46H AVERAGE CURRENT DENSITY (A/M2) ..............,F10.0)
  753 FORMAT(5X,46H FIELD WINDING SPACE FACTOR ..................,F10.4)
  755 FORMAT(5X,46H MAX.FLUX DENSITY IN FIELD WINDING (WB/M2) ...,F10.4)
  757 FORMAT(5X,46H MAX.CURRENT DENSITY AT TRANSIENT (A/M2) .....,F10.0)
  759 FORMAT(5X,46H MAX.CURRENT RATIO AT TRANSIENT ..............,F10.4)
  761 FORMAT(21H ** PRIMARY DAMPER **)
  763 FORMAT(5X,46H INNER RADIUS OF PRIMARY DAMPER (M) ..........,F10.4)
  765 FORMAT(5X,46H OUTER RADIUS OF PRIMARY DAMPER (M) ..........,F10.4)
  767 FORMAT(5X,46H RADIAL THICKNESS OF PRIMARY DAMPER (M) ......,F10.4)
  769 FORMAT(5X,46H PRIMARY DAMPER CONDUCTIVITY (MHOS/MM) .......,F10.0)
  781 FORMAT(23H ** SECONDARY DAMPER **)
  783 FORMAT(5X,46H INNER RADIUS OF SECONDARY DAMPER (M) ........,F10.4)
  785 FORMAT(5X,46H OUTER RADIUS OF SECONDARY DAMPER (M) ........,F10.4)
  787 FORMAT(5X,46H RADIAL THICKNESS OF SECONDARY DAMPER (M) ....,F10.4)
  789 FORMAT(5X,46H MAX.RADIAL FORCE AT SHORT CIRCUIT (N/CM2) ...,F10.1)
  791 FORMAT(5X,46H SECONDARY DAMPER CONDUCTIVITY (MHOS/M) ......,F10.0)
  801 FORMAT(21H ** DAMPER SUPPORT **)
  803 FORMAT(5X,46H INNER RADIUS OF DAMPER SUPPORT (M) ..........,F10.4)
  805 FORMAT(5X,46H OUTER RADIUS OF DAMPER SUPPORT (M) ..........,F10.4)
  807 FORMAT(5X,46H RADIAL THICKNESS OF DAMPER SUPPORT (M) ......,F10.4)
  809 FORMAT(5X,46H CENTRIFUGAL STRESS AT RATED SPEED (N/CM2) ...,F10.0)
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811 FORMAT(5X,46H MAGNETIC STRESS AT SHORT CIRCUIT (N/CM2) .....,F10.0)
813 FORMAT(5X,46H TOTAL STRESS AT SHORT CIRCUIT (N/CM2) ........,F10.0)
815 FORMAT(5X,46H CENTRIFUGAL DEFLECTION AT RATED SPEED (M) ....,F10.5)
817 FORMAT(5X,46H MAGNETIC DEFLECTION AT SHORT CIRCUIT (M) .....,F10.5)
819 FORMAT(5X,46H TOTAL DEFLECTION AT SHORT CIRCUIT (M) ........,F10.5)
821 FORMAT(25H ** MACHINE PARAMETERS **)
823 FORMAT(5X,46H SYNCHRONOUS REACTANCE (PU) ...................,F10.4)
825 FORMAT(5X,46H TRANSIENT REACTANCE (PU) .....................,F10.4)
827 FORMAT(5X,46H SUBTRANSIENT REACTANCE (PU) ..................,F10.4)
829 FORMAT(5X,46H SUB-SUBTRANSIENT REACTANCE (PU) ..............,F10.4)
831 FORMAT(5X,46H FIELD TIME CONSTANT (SEC) ....................,F10.4)
833 FORMAT(5X,46H PRIMARY DAMPER TIME CONSTANT (SEC) ...........,F10.4)
835 FORMAT(5X,46H SECONDARY DAMPER TIME CONSTANT (SEC) .........,F10.4)
837 FORMAT(5X,46H INERTIA CONSTANT INCLUDING TURBINE (SEC) .....,F10.4)
841 FORMAT(16H ** STABILITY **)
843 FORMAT(5X,46H CRITICAL FAULT CLEARING TIME (SEC) ...........,F10.4)
845 FORMAT(5X,46H SHORT CIRCUIT RATIO (PU) .....................,F10.4)
851 FORMAT(30H ** NATURAL FREQUENCY **       )
852 FORMAT(5X,46H BEARING SPAN (M) .............................,F10.4)
853 FORMAT(5X,46H CRITICAL SPEED OF ROTOR (RPM) ................,F10.1)
861 FORMAT(23H ** PENALTY FUNCTION **)
863 FORMAT(5X,46H TRANSIENT STABILITY ..........................,F10.4)
867 FORMAT(5X,46H DAMPER STRESS ................................,F10.4)
865 FORMAT(5X,46H CRITICAL SPEED ...............................,F10.4)
869 FORMAT(5X,46H DYNAMIC STABILITY ............................,F10.4)
871 FORMAT(5X,46H FIELD CURRENT LIMIT ..........................,F10.4)
873 FORMAT(5X,46H DAMPER DEFLECTION ............................,F10.4)
881 FORMAT(26H ** LOSS AND EFFICIENCY **)
883 FORMAT(5X,46H IRON LOSS (KW) ...............................,F10.4)
885 FORMAT(5X,46H ARMATURE COPPER LOSS (KW) ....................,F10.4)
887 FORMAT(5X,46H IRON AND COPPER LOSS (KW) ....................,F10.4)
889 FORMAT(5X,46H IRON AND COPPER LOSS PER KVA (W/KVA) .........,F10.4)
891 FORMAT(5X,46H EFFICIENCY (PU) ..............................,F10.4)
901 FORMAT(13H ** WEIGHT **)
903 FORMAT(5X,46H ACTIVE PART OF ROTOR (KG) ....................,F10.0)
905 FORMAT(5X,46H ACTIVE PART OF ARMATURE (KG) .................,F10.0)
907 FORMAT(5X,46H OUTER FERROMAGNETIC SHIELD (KG) ..............,F10.0)
909 FORMAT(5X,46H TOTAL ACTIVE PART (KG) .......................,F10.0)
911 FORMAT(5X,46H ACTIVE PART WEIGHT PER KVA (KG/KVA) ..........,F10.6)
921 FORMAT(11H ** COST **)
923 FORMAT(5X,46H FIELD WINDING ($) ............................,F10.0)
925 FORMAT(5X,46H ACTIVE PART MATERIAL COST ($) ................,F10.0)
927 FORMAT(5X,46H FIELD WINDING COST PER KVA ($/KVA) ...........,F10.5)
929 FORMAT(5X,46H ACTIVE PART MATERIAL COST PER KVA ($/KVA) ....,F10.5)
931 FORMAT(5X,46H COST FUNCTION ($) ............................,F10.0)
941 FORMAT(33H ** OUTER FERROMAGNETIC SHIELD **)
943 FORMAT(5X,46H OUTER RADIUS OF FERROMAGNETIC SHIELD (M) .....,F10.4)
945 FORMAT(5X,46H INNER RADIUS OF FERROMAGNETIC SHIELD (M) .....,F10.4)
947 FORMAT(5X,46H THICKNESS OF FERROMAGNETIC SHIELD (M) ........,F10.4)
    END
// XEQ
2000000. 0.85   29000. 0.26   0.028  0.002  0.138  0.006  0.155  260000000.
// *ENDJOB
```

APPENDIX.IV. SUPERCONDUCTING ALTERNATOR DESIGN SHEET NO. 6 PAGE 1

** RATING **						
RATED POWER (KVA)	1000000.	2000000.	3000000.	5000001.	7000001.	10000002.
POWER FACTOR	0.850	0.850	0.850	0.850	0.850	0.850
LINE TO LINE VOLTAGE (V)	26000.	29000.	31000.	33000.	34000.	35000.
ARMATURE CURRENT (A)	22205.	39817.	55872.	87477.	118860.	164950.
FREQUENCY (HZ)	60.0	60.0	60.0	60.0	60.0	60.0
SPEED (RPM)	3600.	3600.	3600.	3600.	3600.	3600.
NUMBER OF POLES	2.	2.	2.	2.	2.	2.
** ARMATURE **						
OUTER RADIUS OF ARMATURE WINDING (M)	0.5994	0.6575	0.7249	0.7676	0.8035	0.8390
INNER RADIUS OF ARMATURE WINDING (M)	0.4663	0.5045	0.5622	0.5888	0.6126	0.6314
RADIAL THICKNESS OF ARMATURE WINDING (M)	0.1331	0.1529	0.1627	0.1788	0.1909	0.2075
LENGTH OF ARMATURE MIDDLE PART (M)	3.4507	5.1599	5.7952	8.2756	10.2620	13.4430
CURRENT DENSITY OF ARMATURE WINDING (A/M2)	3000000.	3000000.	3000000.	3000000.	3000000.	3000000.
ARMATURE TOTAL EFFECTIVE LENGTH (M)	4.5165	6.3220	7.0825	9.6321	11.6780	14.9140
ARMATURE WINDING SPACE FACTOR	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
ARMATURE WINDING CONDUCTIVITY (MHOS/M)	60000008.	60000008.	60000008.	60000008.	60000008.	60000008.
** FIELD WINDING **						
OUTER RADIUS OF FIELD WINDING (M)	0.2875	0.3009	0.3306	0.3365	0.3482	0.3555
INNER RADIUS OF FIELD WINDING (M)	0.2620	0.2726	0.2991	0.3035	0.3147	0.3219
RADIAL THICKNESS OF FIELD WINDING (M)	0.0254	0.0283	0.0315	0.0329	0.0335	0.0336
LENGTH OF FIELD WINDING MIDDLE PART (M)	3.4507	5.1599	5.7952	8.2756	10.2620	13.4430
AVERAGE CURRENT DENSITY (A/M2)	249470016.	247050016.	232980032.	236570016.	235340032.	237130016.
FIELD WINDING SPACE FACTOR	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250
MAX.FLUX DENSITY IN FIELD WINDING (WB/M2)	5.3274	5.7700	6.0553	6.3431	6.3994	6.4274
MAX.CURRENT DENSITY AT TRANSIENT (A/M2)	305480064.	301800064.	281870016.	286340032.	284100032.	286240064.
MAX.CURRENT RATIO AT TRANSIENT	1.2245	1.2216	1.2098	1.2103	1.2071	1.2071
** PRIMARY DAMPER **						
INNER RADIUS OF PRIMARY DAMPER (M)	0.2945	0.3079	0.3376	0.3435	0.3552	0.3625
OUTER RADIUS OF PRIMARY DAMPER (M)	0.2965	0.3099	0.3396	0.3455	0.3572	0.3645
RADIAL THICKNESS OF PRIMARY DAMPER (M)	0.0020	0.0020	0.0020	0.0020	0.0020	0.0020
PRIMARY DAMPER CONDUCTIVITY (MHOS/MM)	5000001.	5000001.	5000001.	5000001.	5000001.	5000001.

SUPERCONDUCTING ALTERNATOR DESIGN SHEET NO. 6 PAGE 2

(MVA)	1000	2000	3000	5000	7000	10000
** SECONDARY DAMPER **						
INNER RADIUS OF SECONDARY DAMPER (M)	0.4103	0.4485	0.5062	0.5328	0.5566	0.5754
OUTER RADIUS OF SECONDARY DAMPER (M)	0.4163	0.4545	0.5122	0.5388	0.5626	0.5814
RADIAL THICKNESS OF SECONDARY DAMPER (M)	0.0059	0.0059	0.0060	0.0059	0.0060	0.0060
MAX.RADIAL FORCE AT SHORT CIRCUIT (N/CM2)	3010.7	3962.2	4321.6	5181.3	5631.1	5763.7
SECONDARY DAMPER CONDUCTIVITY (MHOS/M)	60000008.	60000008.	60000008.	60000008.	60000008.	60000008.
** DAMPER SUPPORT **						
INNER RADIUS OF DAMPER SUPPORT (M)	0.2985	0.3119	0.3416	0.3475	0.3592	0.3665
OUTER RADIUS OF DAMPER SUPPORT (M)	0.4103	0.4485	0.5062	0.5328	0.5566	0.5754
RADIAL THICKNESS OF DAMPER SUPPORT (M)	0.1117	0.1365	0.1645	0.1853	0.1973	0.2088
CENTRIFUGAL STRESS AT RATED SPEED (N/CM2)	14279.	16396.	20379.	21889.	23675.	25026.
MAGNETIC STRESS AT SHORT CIRCUIT (N/CM2)	32433.	34083.	32608.	33932.	35439.	34562.
TOTAL STRESS AT SHORT CIRCUIT (N/CM2)	37624.	39888.	42180.	43886.	46427.	46957.
CENTRIFUGAL DEFLECTION AT RATED SPEED (M)	0.00025	0.00030	0.00042	0.00047	0.00053	0.00057
MAGNETIC DEFLECTION AT SHORT CIRCUIT (M)	0.00107	0.00107	0.00106	0.00105	0.00113	0.00110
TOTAL DEFLECTION AT SHORT CIRCUIT (M)	0.00116	0.00118	0.00126	0.00127	0.00137	0.00139
** MACHINE PARAMETERS **						
SYNCHRONOUS REACTANCE (PU)	0.4777	0.5210	0.5432	0.5911	0.6337	0.7172
TRANSIENT REACTANCE (PU)	0.3236	0.3538	0.3708	0.4058	0.4333	0.4912
SUBTRANSIENT REACTANCE (PU)	0.3087	0.3372	0.3542	0.3875	0.4141	0.4697
SUB-SUBTRANSIENT REACTANCE (PU)	0.1992	0.1953	0.1947	0.1933	0.1970	0.2129
FIELD TIME CONSTANT (SEC)	300.0000	300.0000	300.0000	300.0000	300.0000	300.0000
PRIMARY DAMPER TIME CONSTANT (SEC)	2.3124	2.3744	2.6035	2.6074	2.6839	2.7157
SECONDARY DAMPER TIME CONSTANT (SEC)	0.1840	0.2103	0.2532	0.2741	0.2918	0.3052
INERTIA CONSTANT INCLUDING TURBINE (SEC)	3.1538	3.1487	3.1807	3.1758	3.1793	3.1785
** STABILITY **						
CRITICAL FAULT CLEARING TIME (SEC)	0.1805	0.1766	0.1752	0.1707	0.1672	0.1598
SHORT CIRCUIT RATIO (PU)	2.0930	1.9191	1.8407	1.6916	1.5780	1.3941
** OUTER FERROMAGNETIC SHIELD **						
OUTER RADIUS OF FERROMAGNETIC SHIELD (M)	1.1617	1.2572	1.4053	1.4527	1.4973	1.5084
INNER RADIUS OF FERROMAGNETIC SHIELD (M)	0.5994	0.6575	0.7249	0.7676	0.8035	0.8390
THICKNESS OF FERROMAGNETIC SHIELD (M)	0.5622	0.5996	0.6803	0.6850	0.6937	0.6694

SUPERCONDUCTING ALTERNATOR DESIGN SHEET NO. 6 PAGE 3

(MVA)	1000	2000	3000	5000	7000	10000
** NATURAL FREQUENCY **						
BEARING SPAN (M)	5.6383	7.4873	8.3780	10.9390	13.0290	16.2820
CRITICAL SPEED OF ROTOR (RPM)	3527.0	2177.0	1961.5	1206.2	899.6	588.0
** LOSS AND EFFICIENCY **						
IRON LOSS (KW)	322.0600	558.4200	791.5800	1186.3000	1543.9001	1990.9001
ARMATURE COPPER LOSS (KW)	1493.1001	2508.0004	3304.5004	5024.7002	6672.4004	9426.4004
IRON AND COPPER LOSS (KW)	1815.2001	3066.4003	4096.1005	6211.1005	8216.3008	11417.0019
IRON AND COPPER LOSS PER KVA (W/KVA)	1.8152	1.5332	1.3653	1.2422	1.1737	1.1417
EFFICIENCY (PU)	0.9978	0.9981	0.9983	0.9985	0.9986	0.9986
** WEIGHT **						
ACTIVE PART OF ROTOR (KG)	13141.	22420.	32420.	49547.	65508.	89513.
ACTIVE PART OF ARMATURE (KG)	15474.	24720.	32426.	47103.	61056.	83752.
OUTER FERROMAGNETIC SHIELD (KG)	80515.	139600.	197890.	296590.	385970.	497740.
TOTAL ACTIVE PART (KG)	109130.	186740.	262740.	393240.	512540.	671010.
ACTIVE PART WEIGHT PER KVA (KG/KVA)	0.109130	0.093374	0.087581	0.078649	0.073220	0.067101
** PENALTY FUNCTION **						
TRANSIENT STABILITY	1.0571	1.0990	1.1174	1.1986	1.2879	1.6158
CRITICAL SPEED	1.0063	1.3521	1.5181	3.5583	7.5480	23.8480
DAMPER STRESS	0.9966	1.0620	1.2090	1.4193	2.0419	2.2445
DAMPER DEFLECTION	0.9500	0.9500	0.9500	0.9500	0.9501	0.9502
DYNAMIC STABILITY	0.9500	0.9500	0.9500	0.9500	0.9500	0.9500
FIELD CURRENT LIMIT	0.9211	0.9599	0.9490	1.0382	1.0445	1.0753
** COST **						
FIELD WINDING ($)	34391.	55735.	75989.	109510.	140230.	184150.
ACTIVE PART MATERIAL COST ($)	305180.	516040.	720950.	1074700.	1399900.	1848600.
FIELD WINDING COST PER KVA ($/KVA)	0.03439	0.02786	0.02532	0.02190	0.02003	0.01841
ACTIVE PART MATERIAL COST PER KVA ($/KVA)	0.30518	0.25802	0.24031	0.21495	0.19999	0.18486
COST FUNCTION ($)	268990.	705620.	1265200.	6097101.	26201004.	155210016.

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