COMPUTER MODELING OF CORONA DISCHARGE PHENOMENA

BY

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ABSTRACT

This work is a numerical study of the parameters that effect the behavier of corona on coaxial lines in the steady state. The important primary processes are creation of electrons by electron neutral collision, and attachment of electrons to neutrals forming slow moving negative ions. The secondary process that this work concentrated on is photoionization in the bulk of the gas.

An iterative method is used to solve the self consistant problem involving space charges and its effect on the electric field. The current densities as well as total current are also calculated.

For both positive and negative corona, the total current per unit length below onset is very small. Within a narrow range of applied voltage at onset, the current jumps by 5 or 6 orders of magnitude. Above onset, the current increase slowly again as the voltage is raised. This type of current versus voltage curves is very similar to experimentally measured curves.

An increase in attachment or decrease in ionization coefficient raises the corona onset. An increase in photon absorption rate decreases the onset. Changes in mobility of ions only effect the current above onset.

The onset of corona for many different inner conductor radii are also calculated and plotted against Peek's formula. The onset of negative corona is always above that of positive corona.

THESIS SUPERVISOR: Gerald L Wilson
TITLE: Associate Professor of Electrical Engineering

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1. Introduction

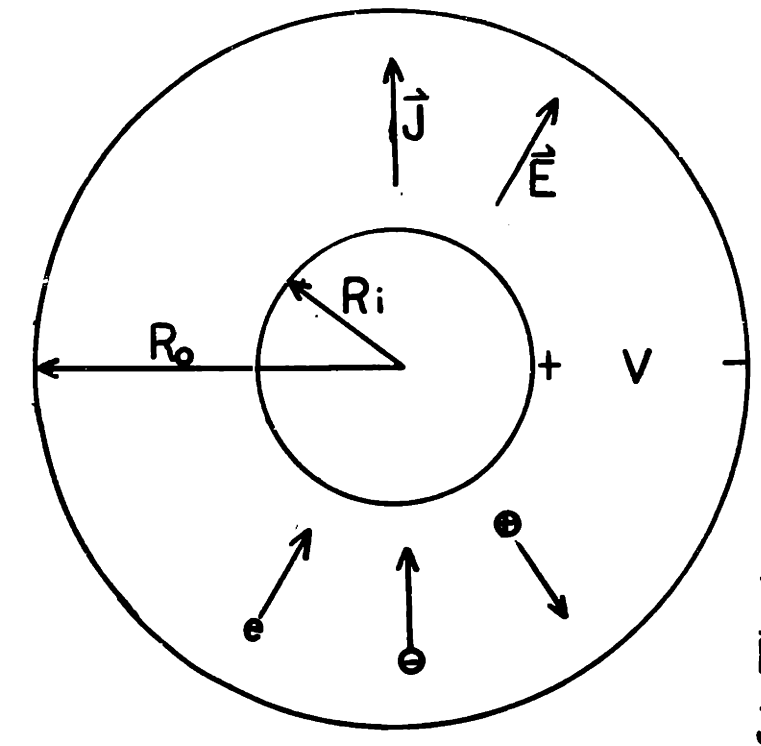
Transmission line voltages are designed at an electric field value on the surface of the conductor below the corona breakdown threshold field in dry air. When there is rain, snow or fog on the conductor, the electric field is locally increased resulting in local breakdown. Corona not only results in heavy losses for the transmission line, but also creates a disturbance both both acoustically and on a wide range of radio and television broadcast bands. To be able to increase the onset voltage and or reduce the noise, the physical processes involved must be well understood. Many experiements have been done to study corona. However it is very hard to separate one process from another. If the problem can be modelled mathematically, than it can be solved by the use of computers. If the model is correct, the results of computer programs should be in agreement with some known experiemental data. Then not only may each known processes be studied independently or concurrently with other processes, but also running the program becomes an easy way to perform an actual experiement. It can give useful data that is hard to measure.

2. Corona Characteristics

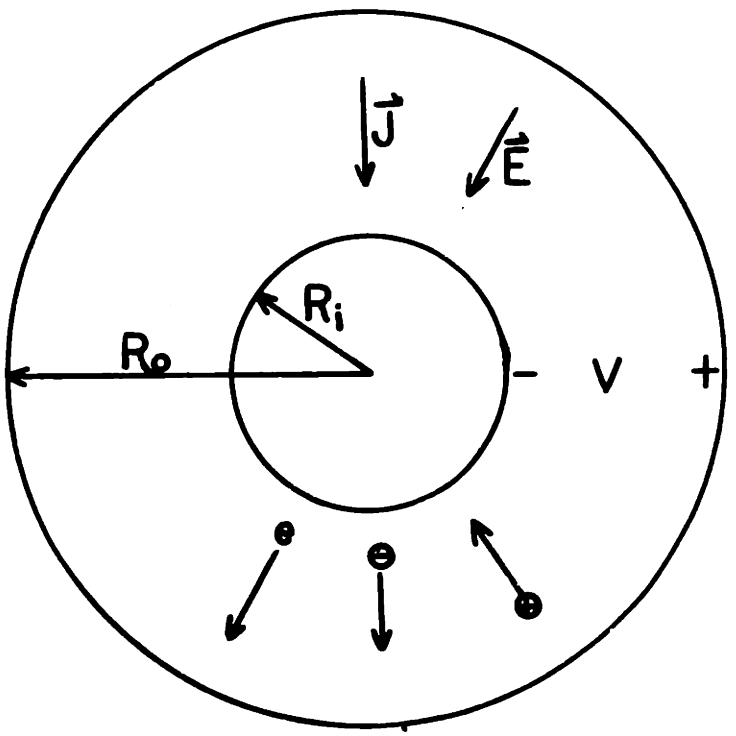
To be able to model corona, it is essential to know its characteristics. Air is a pretty good insulator in electric fields much below 30 kv/cm. Near or above that range, air becomes conductive, causing breakdown. When a voltage is applied to a curved surface, the electric field is large near the surface and decays with distance. The smaller the radius of curvature, the faster the decay. As a consequence, a voltage can be applied such that the electric field near the curved surface is high enough for breakdown and yet not high enough at increasing distances to cause complete flashover. The partial breakdown near such a highly stressed surface is called corona. Since the main interest is in understanding the problems associated with transmission lines, coronas on coaxial geometry are considered. The first trace of breakdown can be associated with a large increase in current measured and appearance of visible light near the stressed surface.

2.1 Positive (Anode) Corona

Positive corona is a name given to the type of corona that exists on the surface with higher curvature when it is charged positively with respect to ground. For the geometry of interest, consider the inner cylinder or wire as the anode. Electrons and negative ions travel toward



a) Positive corona case



b) Negative corona case

V - applied voltage

$\vec{E}$ - electric field

$\vec{J}$ - current density

R_i - radius of inner cylinder

R_o - radius of outer cylinder

e - electron

⊖ - negative ion

⊕ - positive ion

Figure 2.1 Geometry of set up

the center into stronger fields while positive ions move into weaker fields. See Figure 2.1.a.

Assuming that a positive voltage is applied to the wire for an "indefinitely" long period of time in air, three distinct modes of corona can be differentiated above threshold as a function of the applied voltage. When the voltage applied is below the threshold, no visible activity can be seen. As the voltage is raised slowly to the threshold, luminous light with the shape of a slightly branched filamentary thread can be seen in a well-darkened room. It is usually given the name burst pulses. The burst pulses become more frequent as voltage is raised and they tend to spread laterally. The burst pulses exist only in a narrow band of voltages at the threshold.[15,16]

As the voltage is raised further, the streamers tend to be self-sustaining and more frequent until the discharge becomes continuous. A steady thin glow appears very close to the surface. As the voltage raises, the glow becomes more luminous.

At even higher voltages, streamers reappear. They are more vigorous, very bright, reaching further than the burst pulses, and quite noisy acoustically. These types of streamers are called breakdown streamers. They exist along with the glow. As voltage is raised higher and higher, eventually a spark occurs resulting in a complete

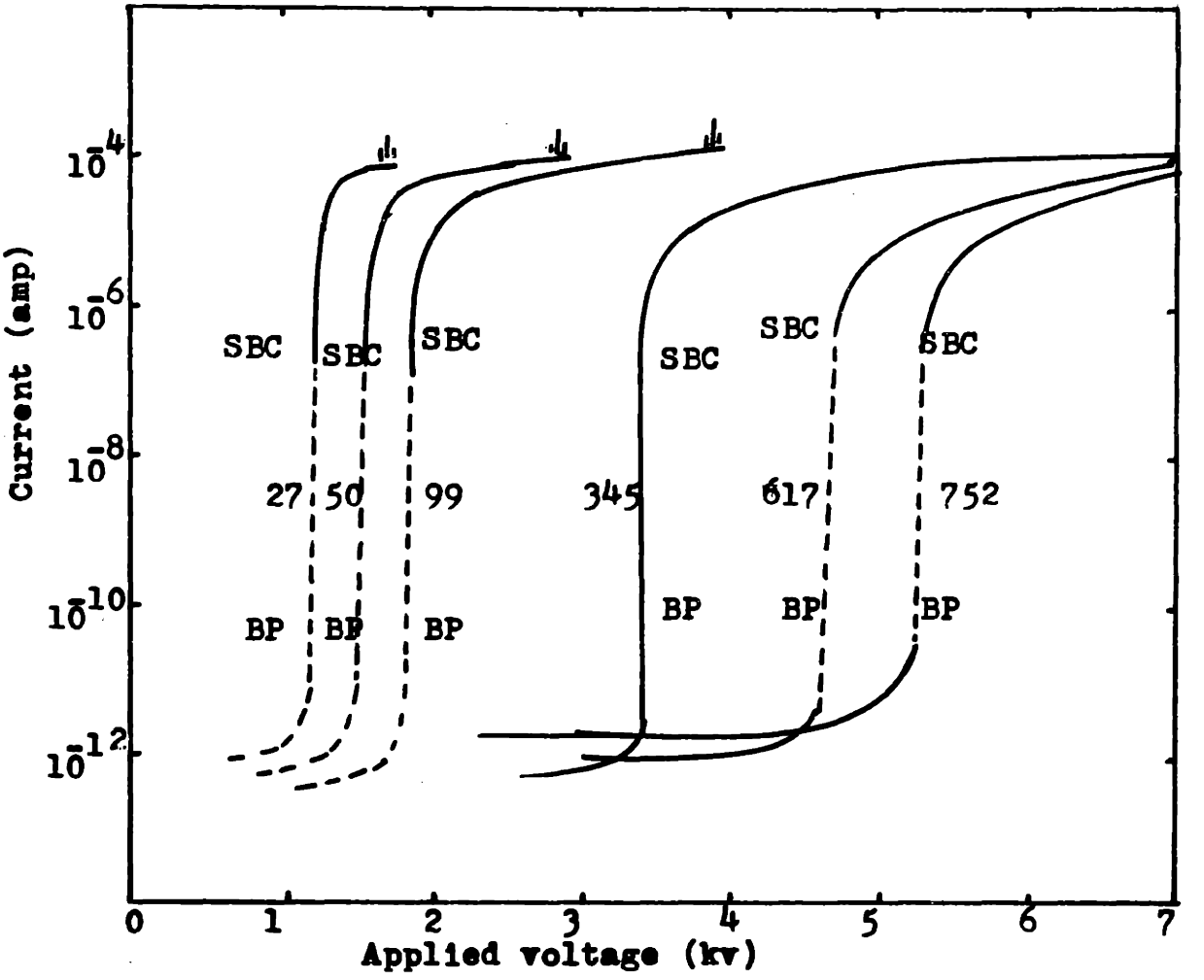


Figure 2.2 This is the current-potential curves in clean dry air with coaxial cylindrical geometry measured by Miller. The numbers indicate the pressure in mm Hg. Note the abrupt current jump to a burst pulse corona, marked by BP, and onset of steady corona, SBC. Breakdown streamers, indicated by vertical lines above the curve, could only be observed below 99 mm Hg as the potential source was not adequate. Dashed lines indicate none reproducible data.

breakdown.

The onset of corona can also be detected by the average current measured. Below the threshold the conduction current is very small generally on the order of 10^{-11} amp/cm or smaller,[12] and is due to charged pairs created by cosmic radiation and earth radioactivity. Around a small band of voltage at the threshold, the current suddenly increases by many orders of magnitude. After the onset of corona, the current increases graduately until complete breakdown. Miller measured corona onset values for a P_t wire of .174 mm diameter mounted coaxially in a 28.5 mm diameter nickel cylinder. Figure 2.2 shows a set of voltage versus current curves for air at different pressures. Note that glow is the most important mode of discharge.

2.2 Negative (Cathode) Corona

When a steady voltage applied to the central wire is negative as in Figure 2.1.b, the type of corona discharge is quite different. Because of the secondary processes involved on the cathode such as emission of electrons by positive ion impact or photoelectric emission, the resulting behaviors depend highly on the type and condition of the negative wire.

Miller used the same set up for negative corona measurements. The polished P_t wire is allowed to rest

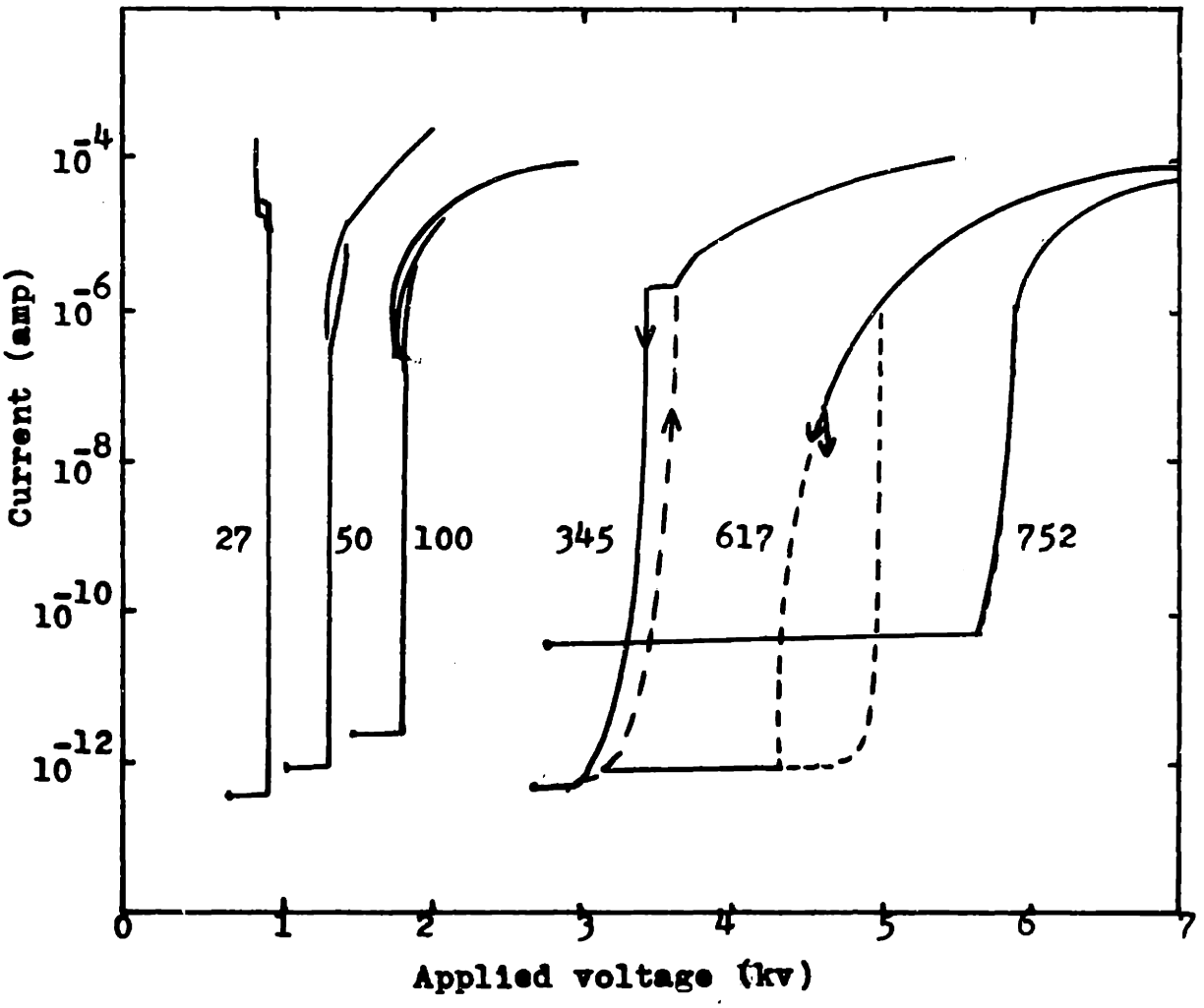


Figure 2.3 This is the current-potential curves in air with negative wire in coaxial cylindrical geometry measured by Miller. The numbers indicates the pressure in mm Hg. The curve at 752 mm Hg was made with a freshly outgassed filament. Note the varied levels of the pre-threshold currents and sharp irregular thresholds. The curves at 617 mm Hg and below represent the altered system after those at 752. Note the onset at 752 mm Hg is higher for the negative wire than that of the positive wire.

in air with no voltage. The voltage is than increased until the visual manifestation of threshold is observed as a glow along the whole length of wire. At the same time the current jump to about 10^{-5} amp. See Figure 2.3. The uniform glow is explained by the fact that the surface oxidizes in air so that the number of electrons created per ion impact, γ_i, is low. Photoelectric emission at the cathode or photoionization in air could become the dominant secondary processes. If the glow is left standing for 10 to 12 hours, than the discharge will contract to spots. This is because positive ion impact on the cathode cleaned off some oxide on the surface and effectively increased γ_i at the spot.[18,14,17]

Trichel pulses began with the appearance of spots. The frequency of the pulses increase with increasing potential at the start. However, Trichel pulses in coaxial geometry never reach a frequency of 10^6 cycles per sec as in point-to-plane geometry.

However if the wire is initially oxidized, and before the glow contractes to spots, a potential can be reached so that no spots can form at all. This can be explained by the fact that oxides form on the surface faster than can be cleared by ion impact. If spots did form, then no further uniform glow regions are experienced for voltages above this level.

2.3 Critical Corona Gradient

There are two phenomenon associated with onset of corona: a jump in current and appearance of visible light at the surface of the inner conductor. Peek collected data for corona onset by the visual method. He found that the onset for alternating current is the same as the direct current positive and direct current negative corona cases. Others such as Whitehead, Brown and Farwel found that negative corona onset occured above the positive corona onset, and alternating current corona onset is close to the onset of negative corona.[18] Miller also found the negative corona onset above positive corona onset. All the experiments mentioned above were done on very small inner conductors.

Because the voltage applied to create identical fields depends on the dimensions of the configuration, the onset can be more easily defined in terms of the electric field needed at the surface of the inner conductor. This electric field is called the critical corona gradient, E_c. It is unique for a given corona, independent of the outer radius, because below onset the space charge is so low that the electric field still maintains the K/r form. Peek fitted a formula to his data.

$$E_c = 31m\delta\left(1 + \frac{.308}{\sqrt{\delta a}}\right) \quad \text{kv/cm} \tag{1}$$

The radius of the inner conductor is 'a' in centimeters.

Pressure, p, in cm Hg and temperature, T, in C^{o} are also important. Their effect are incorporated into the variable δ.

$$\delta = \frac{3.92p}{273 + T} . \qquad (2)$$

For standard temperature and pressure $\delta = 1$. For polished wire, but oxidized, the start of the visual corona is sharp and the irrigularity factor m is unity. If the wire developed spots or is soiled, then m can be as low as .72.

3. Physical Processes

Breakdown is a term used when the current suddenly increases by many orders of magnitude even though the voltage is changed only slightly. Currents come about as results of movement of charged particles. Therefore, the first question that should be asked is where did the charged particles come from and where do they go. The primary processes that occur in air consist of ionization by cosmic radiation, ionization by electron-neutral collision, attachment of electrons to neutrals, recombination, and diffusion. However, a discharge cannot be self-sustaining unless there is at least one secondary feedback mechanism. One of the most acknowledged mechanism is emission of electrons from the cathode due to ion impact. Other important feedback processes are photoelectric emission from metal and photoionization in the bulk of the air.

3.1 Drift Velocity and Mobility

When a charged particle is placed in an electric field, it accelerates. In a gas, this charged particle will lose its kinetic energy gained during collision with the gas molecules. The average velocity travelled along the electric field lines is called the drift velocity, v. Mobility, μ, is defined as v/E, where

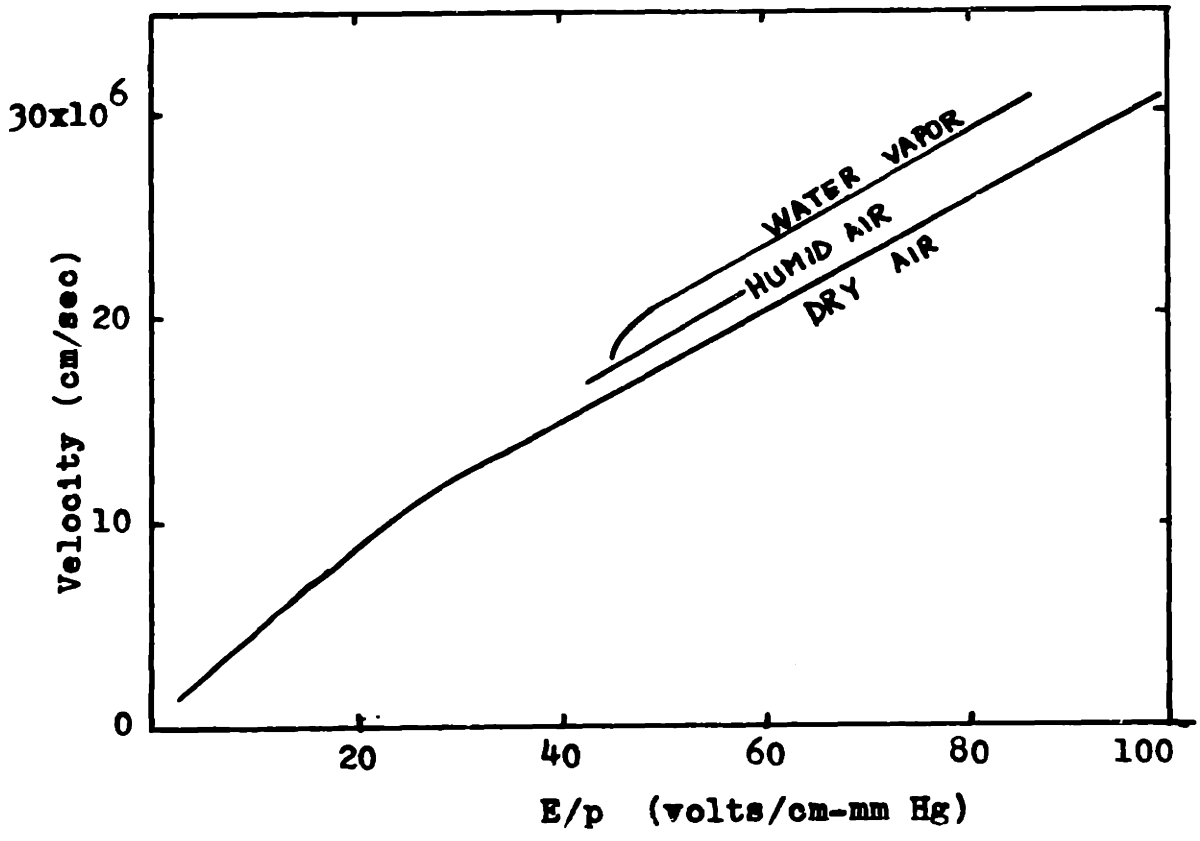


Figure 3.1 Electron drift velocity in dry air, humid air and, water vapor.

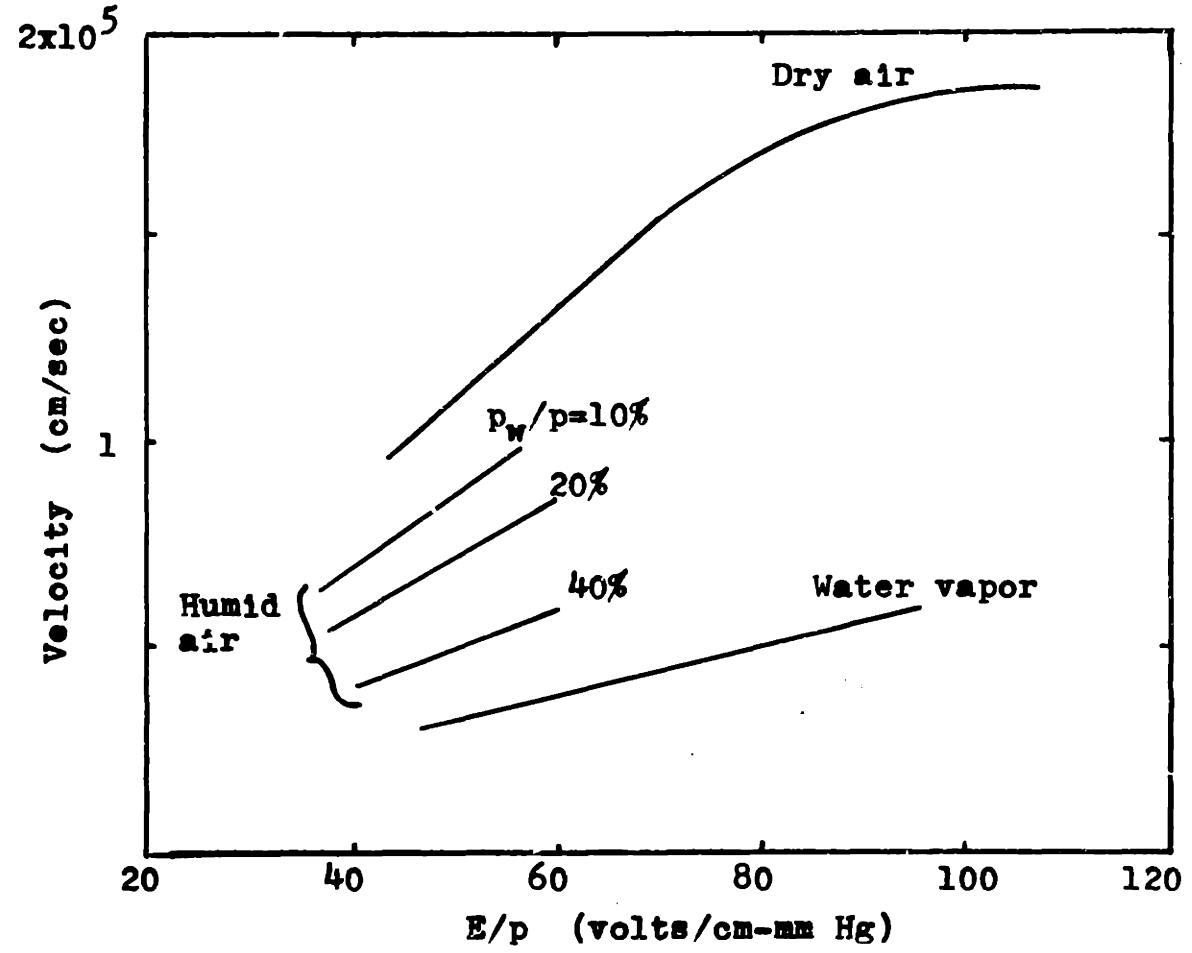


Figure 3.2 Drift velocity of positive ions in dry air, humid air, and water vapor.

E is the electric field strength. In a certain range of interest, μ can be approximated by a constant. Figure 3.1 and 3.2 respectively have plots of electron drift velocity and positive ion drift velocity in air as function of E/p, where p is the pressure.[20] The drift velocity of negative ions is assumed to be not too different from that of the positive ions.

3.2 Cosmic Radiation

Cosmic rays, usually very high energy protons, are capable of striping an electron from a gas molecule, resulting in an electron and a heavy positively charged ion. Because cosmic rays are capable of penetrating most conventional materials, free electrons are created everywhere continuously. From sea level to a few thousand feet, the number of electron-ion pairs created ranges from about 4 to 10 per cm^3/sec.[24] The current produced by cosmic radiation alone is insignificantly small. But it is important as a source of free electrons that can start other processes.

3.3 Alpha Ionization Process

One of the most important processes is ionization by electron neutral collisions. During such a collision the electron can loss all or part of its kinetic energy to the neutral. If the kinetic energy of the electron is

greater than the ionization energy, it can ionize the neutral directly producing an electron/ion pair. If the energy of the electron is less than the ionization energy but larger than required for the next allowable atomic transition, then the atom could be excited.

Before the atom emits one or more photons in the process of returning to lower states and eventually the ground state, the excited atom could collide with another electron. The atom has another chance to be ionized or move up to an even higher excited state. This is called "step ionization". When the average kinetic energy of electrons are small, "step ionization" is the predominant method of ionization.

It is possible that an atom is left in a metastable state so that the atom could not return to a lower state without violating the selection rules. The average lifetime of metastable states is on the order of 10^{-3} second, much longer then that of the normal excited states, which is only on the order of 10^{-8} seconds. Because of its long lifetime, metastable states play an important role in gaseous ionization.

Positive ions are not efficient ionizers for E/p less than 100 volts/cm-mm Hg. They are effective ionizers when their velocities are as great as those of electrons which have fallen through the minimum ionization potential.

They are unable to gain such high velocity because their mass prevents them from having accelerations as high as electrons.

3.4 Attachment of Electrons to Neutrals

Not all electrons freed remain free to ionize. Electrons can be captured by atoms lacking one or two electrons in their outer shell forming heavy slow moving negative ions. All gases that have this property are known as electro-negative gases. They are composed of atoms in the groups VII A and VI A family. In air, the attached molecules are mainly O_2^- and O^-. The reaction

$$e + O_2 \rightarrow O^- + O \qquad (1)$$

is called dissociative attachment and is predominent in molecular gases, in this case oxygen.[7] Electron energies required are of the order of 3.1 to 3.6 ev.[12] For a negative ion to be stable, so that it can exist for sometime, its total energy must be lower than its neutral ground state. Attachment is an important process because it reduces the number of electrons available as efficient ionizing agents. Also the negative ion's slow drift velocity results in accumulation of negative space charges, which can alter the local electric field to a great extent.

Negative ions can be lost through many processes

such as recombination, detachment or collision into walls. No reliable data on detachment of negative ions in air as a function of E/p has been measured die to experimental difficulties. However, it is known that the following processes

$$O^- + O_2 \rightarrow e + O_3 \qquad (2)$$

or

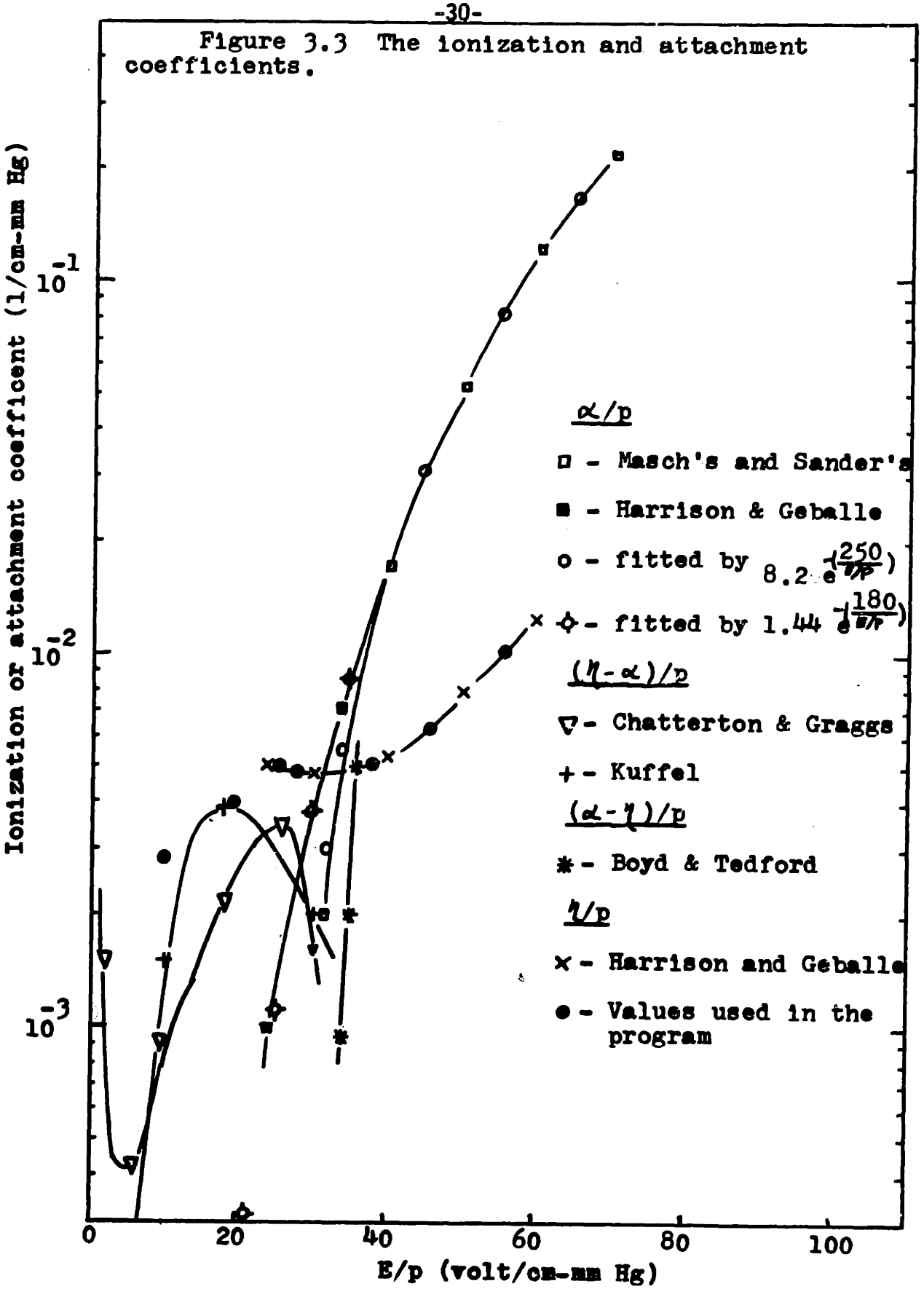
$$\rightarrow e + O_2 + O$$

occur in air. In both cases, approximately 1 ev is needed for the reaction to occur.[19] This process is probably included in the measurement of α/p since the experiments could not isolate them. However it is known that in the higher field region of E/p greater tnan 90 volts/cm-mm Hg, electrons detach and remain free.[12]

3.5 Measurements of Ionization and Attachment Coefficients

Because the growth of the electron density in a constant electric field is an expotential function of space, it is vital to know exactly kow many electrons are produced per unit length in the field direction by one electron. This data is usually known as the ionization coefficient by electron collision, Townsend's first ionization coefficient or simply as alpha, α. The effectiveness of electrons to ionize depends directly on the energy an electron can pick up in an electric

Figure 3.3 The ionization and attachment coefficients.



field between two collisions. However, alpha is not a function of the electric field alone. Pressure is proportional to the particle density and inversely proportional to the mean free path and hence inversely proportional to a power of the energy gained between successive collisions. Figure 3.3 gives a few plots of α/p versus E/p as measured by different people for air in the region of interest. Both Masch's and Sander's data differs from Harrison and Geballe's data at low values of E/p. This can be explained by the fact that Masch and Sanders both did not separate out the effect of attachment of electrons to neutrals. Otherwise their data are in fair close agreement. Boyd and Tedfor also measured the apparent primary ionization coefficient, $(\alpha-\eta)/p$, about E/p = 35 volts/cm-mm Hg when dissociation is assumed to be negligible. Their values are the smallest.

The number of electrons attached per centimeter per electron is usually called η. Figure 3.3 contains one measured curve for η/p and two curves for $\eta/p - \alpha/p$ as function of E/p. They are not in close agreement with each other. However, there is general concent that below E/p vales of about 32 volts/cm-mm Hg, η/p is greater than α/p. That means almost all of the electrons produced become attached, forming negative ions in the low E/p region.

3.6 Recombination

Recombination is a loss mechanism for all charged particles of opposite sign namely electrons with positive ions, and negative ions with positive ions. The number of recombinations per unit time must be directly proportional to the positive ion density, n^+, as well as the negative ion density, n^-. So one can write

$$dn^+ = dn^- = - \beta n^+ n^- dt \qquad (3)$$

where β is called the recombination coefficient. For air at normal temperature and pressure the ion-ion recombination coefficient is about 2.3×10^{-6} $cm^3/sec.$[2] The electron-ion recombination would be smaller because electrons have much larger velocities than ions; leaving the ion after an elastic collision as fast as it approaches. The interaction time is so short that an electron is hard to be captured.

3.7 Diffusion

Collisions of gas molecules and charged particles result in semi-random motion in an applied electric field, so that the net result is that particles tend to smooth out concentration gradients. This phenominon is known as diffusion. This effect can be measured in terms of current

$$\vec{J} = (D_e \nabla n_e + D_- \nabla n_- - D_+ \nabla n_+) e \qquad (4)$$

where D_e, D_+, D_- are respectively the diffusion coefficients of electrons, positive ions and negative ions. The diffusion coefficients are functions of concentration, average velocity and mean free path of particles of gas type 1 and gas type 2.[4] Measurements by various experimentalists gave an approximate value of .03 cm^2/sec for D_+ and .043 cm^2/sec for D_- in dry air under normal temperature and pressure. The diffusion coefficient for electrons are much higher. For a rough estimate of D_e one can use a simple relation

$$D_e = \mu \, 2/3 \, u_{av}. \qquad (5)$$

The quantity u_{av} is called the average energy. μ is the mobility. For an electron with 1 ev, D_e is on the order of 400 cm^2/sec.

3.8 Secondary Mechanisms and Their Physical Characteristics

To understand all the processes, it is instructive to visualize them in time. The clearest experiment is to use the inner conductor as the anode, and with no space charges between conductors initially. A pulse of n_o electrons are released at the cathode. As the electrons travel across the gap more electrons are created by electron neutral collision. Photons too are released when

excited atoms return to lower energy states. The total number of electrons created is

$$\exp \int_{R_1}^{R_o} (\alpha(r) - \eta(r))dr \qquad (6)$$

Most of the electron-ion pairs, and as well as photons are created in the neighborhood of the anode. A large pulse is measured due to the creation of electrons within the electron transit time across the gap, τ_e . If no secondary mechanisms present, a small residual current remains for the length of ion transit time, τ_e , due to the movement of positive and negative ions. Because of their slow mobility, the current measured is a few hundred magnitude smaller and τ_e is a few hundred times longer than that of electrons.

The avalanche of electrons, and resulting ions and photons enables three possible secondary mechanisms. 1) On arrival of positive ions at the cathode, on the average γ_i electrons can be released per positive ion. This results in one or more pulses spaced about $\tau_e + \tau_i$ apart in time. 2) If photons have energy larger than the cathode work function, γ_p electrons can be released per photon arrived. This process is characterized by one or more pulses separated τ_e apart in time. 3) Photons with enough energy can ionize gas molecules also.

This process results in current of much larger magnitude and duration. For onset streamers, the delay time is independent of triggering. For heavy ionizing streamers, the current is regular and very short.

3.9 Photoelectric Emission

Electron can be emitted from the metal surface due to an incident photon when the energy of the photon is greater than the work function of the metal. However, Huber found that with a cathode made of nickel placed in air, photoelectric emission is insignificant; it is not the mechanism responsible for breakdown. In general either photoionization in air or ion bombardment become more important than photoelectric emission.

3.10 Emission of Electrons from Metal due to Ion Impact

The phonomenon of secondary liberation of electrons from the cathode by positive ion bombardment is important in "most " corona discharges. In fact it is the predominent cause of Trichel pulses with period of the pulse roughly on the order of the ion transit time. However, it will be shown that for air, with smooth coaxial cylinders , and low E/p values at atmospheric pressure, the ion bombardment coefficient is not the cause of breakdown.

The established principle behind electron emission

by positive ion bombardment is called Auger neutralization.[5,6] When a positive ion is close to the surface of the metal, the energy level of the ion is much lower then the highest conduction bands in the metal. An electron can tunnel through to the ground state of the atom at the same time releasing the energy difference between the electron and the ground state of the atom to a second electron in the metal, which with the excess energy could overcome the work function and be detected outside. See Figure 3.4. The kinetic energy of the electron, E_k, is dependent on the ionization energy of the atom, E_i, minus the energy levels ζ and ω of the two responsible electrons below the vacuum level in the metal.

$$E_k = E_i - \zeta - \omega \, . \tag{7}$$

Therefore, the necessary condition for Auger neutralization is $E_i > 2\phi$, where ϕ is the work function of the metal. (Actually E_i should be replaced by the effective ionization energy because ionization energy is a function of distance S from the surface.)

Measurements of electron yield, γ_i, per ion created between the cathode and anode, is usually called the Townsend second coefficient. It has been found that indeed γ_i is dependent on the ionization energy of the gas but independent of the kinetic energy of the ion, if

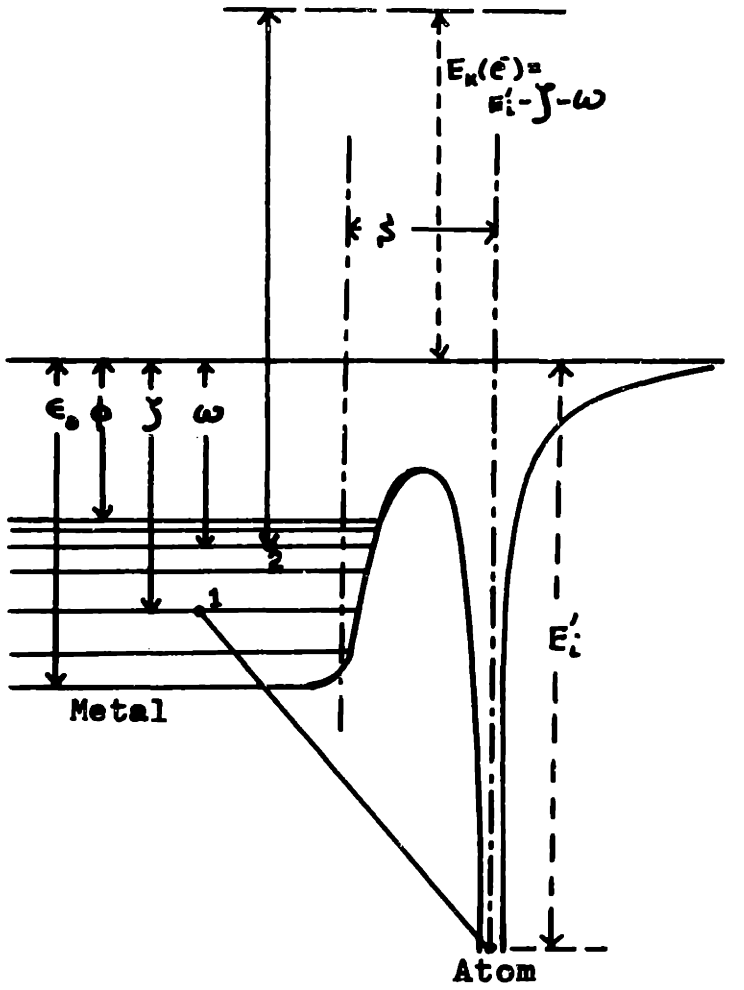


Figure 3.4 Schematic diagram illustrating Auger neutralization of an ion at a metal surface.

the metal is clean.[5,17,22] However, the γ_1 of interest has to be for air. The predominent components of air are nitrogen and oxygen. Huber studied γ_1 from a nickel cathode with different concentrations of oxygen in nitrogen. She did the experiment with positive coaxial geometry. Based on the theory of Auger neutralization that γ_1 is independent of ion kinetic energy, the same result can be applied to negative corona as well.

Huber found the following results. With pure nitrogen the average γ_1 is about .025, relatively independent of pressure and applied field when back diffusion is taken into account. Back diffusion is caused by reflection of electrons in their first few encounters with gas atoms or molecules. It is more important when pressure is high and electric field is weak; or equivalently E/p values below 100 volts/cm-mm Hg.[26] The actual measurements of γ_1 varied as a function of pressure. At 600 mm Hg the measured γ_1 is only .0006, much smaller than the adjusted value. With 5% oxygen, the value of γ_1 decreased approximately by a factor of 5. Increasing the oxygen concentration up to 20%, Huber found that the cathode indicated no secondary mechanism at all. At the same time the onset voltage is actually lowered due to another feedback mechanism. photoionization in the gas.

The effect of oxygen on nickel, as well as other metals,[17] can be associated with an increase of the work function. This is the first reason why γ_1 will decrease. Another reason is that oxygen decreases the population density of higher energy levels in the range of levels the electrons could occupy inside the metal. This results in the emission of more slow electrons. Back diffusion has a greater effect on slow electrons. Again γ_1 is decreased.

3.11 Photoionization

In Huber's experiment, photoionization in the gas is a more important secondary mechanism than ion bombardment, or photoelectric emission. Gas can be ionized not only by electron neutral collisions, but also by photons of high enough energy. Experiments show that photons of 3.87 ev can ionize some gases while photons of 9.9 ev can ionize almost all gases; even though most gases have ionization energies greater than 9.9 ev. This can be explained by the process of step ionization and photo emission from dust and moisture.

Photons are emitted when an excited gas molecule returns to a lower energy state or during the process of ion-ion recombination. The same photon may be absorbed by another atom and leave it in an excited state. If $h\nu$ of the photon is greater than the excitation energy

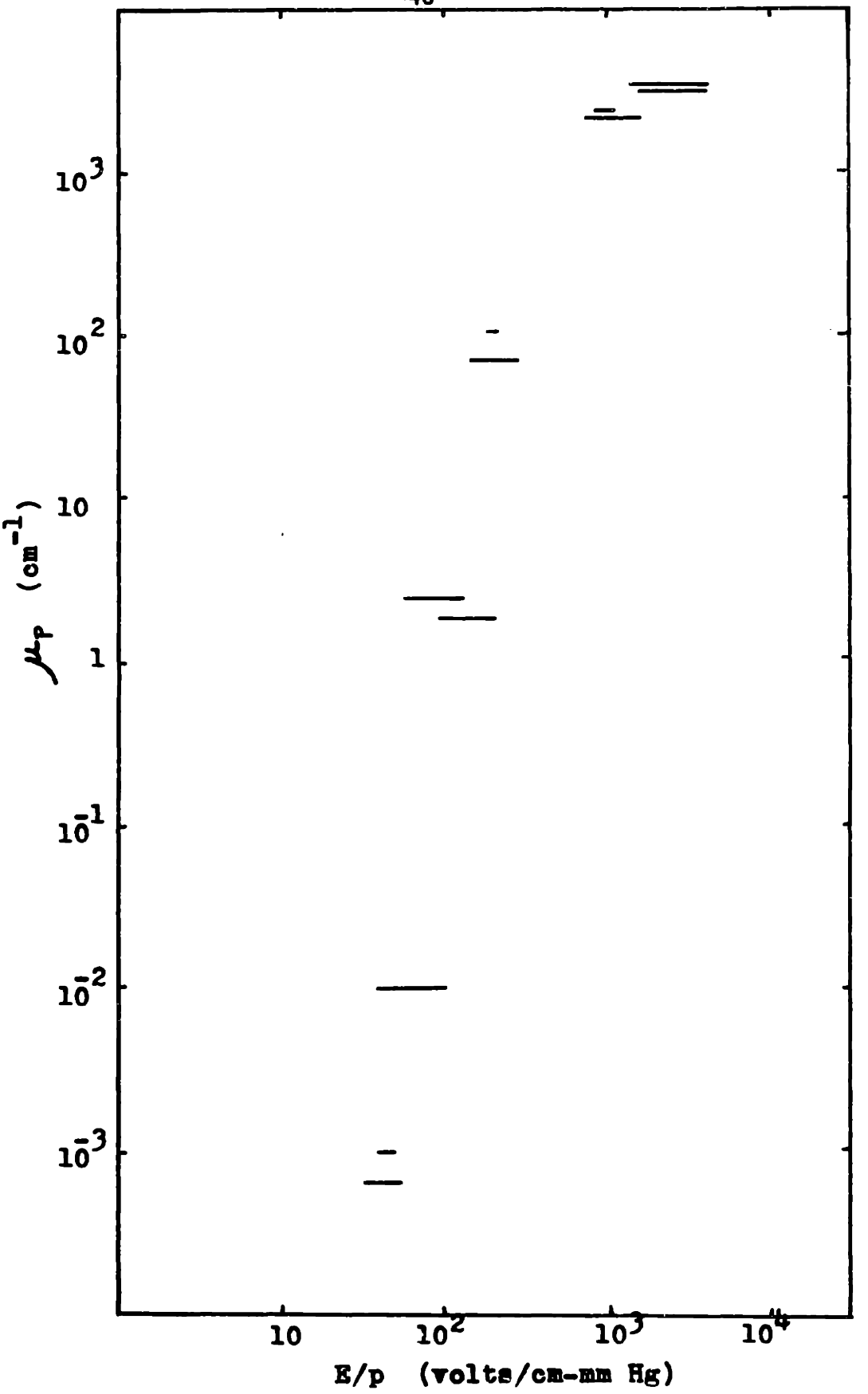


Figure 3.5 The rate of photon absorption, μ_p.

of the atom, the excess energy may be emitted as a photon.

$$A^* \rightleftharpoons A + h\nu \tag{8}$$

is a reversible process. The effect in the presence of oxygen is shorter ionization steps and greater number of total steps.

Let δ be the number of photons of frequency of interest produced by one electron moving for one centimeter. One can define photon efficiency per ionizing collision, Q, as

$$Q = \delta/\alpha = \text{photons/electron created.} \tag{9}$$

Q is found to vary only from 10^{-2} to 10^{-3} for corona discharge in air.[19] The rate of photon absorption, μ_p, in air in an interval Δx is proportional to Δx and the photon intensity, I.

$$dI = -\mu_p I\, dx \tag{10}$$

Figure 3.5 gives some measurements of μ_p as function of E/p. Both Q and μ_p are averaged values of many frequencies and different transitions that take place in air.

4. Model for Positive Corona without Feedback Process

4.1 Assumptions

In forming a methematical model, it is necessary to isolate the problem and make appropriate assumptions. As mentioned before, coaxial geometry is picked for analysis. For this set up, experimental results showed that a glow is the dominent mode of discharge for positive corona, and it could be the dominent mode of negative corona discharge as well if the surface condition is right. This leads to two simplifying assumptions, which will be very good for glow discharge but poor for streamers and Trichel pulses. First, space charge distributions can be made independent of angle and position along the axis, so that all other variables too are independent of the angle and position along the length of the wire. Second, all processes are in the steady state. The net result is that all variables become a function of radius alone. The vector quantities E and J have the form

$$\vec{E} = E(r)\hat{i}_r \qquad (1)$$

and

$$\vec{J} = J(r)\hat{i}_r .$$

4.2 Equation with Just Ionization and Attachment Processes

A rather simple model to start with involves only two processes: the production of electron/ion pairs by electron-neutral collisions denoted by α, and attach-

ment of electrons to neutrals forming heavy negative ions denoted by η . The equations involved are:

$$\vec{J}_e = - \rho_e \mu_e \vec{E} \tag{2}$$

$$\vec{J}_- = - \rho_- \mu_- \vec{E} \tag{3}$$

$$\vec{J}_+ = + \rho_+ \mu_+ \vec{E} \tag{4}$$

$$\nabla \cdot \epsilon_o \vec{E} = \rho_+ + \rho_- + \rho_e \tag{5}$$

$$\vec{E} = -\nabla V \tag{6}$$

$$\nabla \cdot \vec{J}_e = - (\alpha - \eta) J_e \tag{7}$$

$$\nabla \cdot \vec{J}_- = - \eta J_e \tag{8}$$

$$\nabla \cdot \vec{J}_+ = \alpha J_e \tag{9}$$

where the subscripts e, -, + stand for the species : electron, negative ion and positive ion respectively. Equations 2 through 4 express the fact that the current density, J, is related to the electric field E, the mobility, μ , and space charge density ρ . The sign of the charge is include in ρ . Equations 5 and 6 calculate the electric field in terms of the free charge density and in terms of the imposed voltage V. Equations 7 through 9 express the creation of charged particles by the ionization and attachment mechanisms.

A derivation for the electron continuity equation in coaxial cylindrical geometry is given below. See Figure 4.1 for reference. The net gain of electrons/sec-axial length in a gring of radius r to r+Δr is

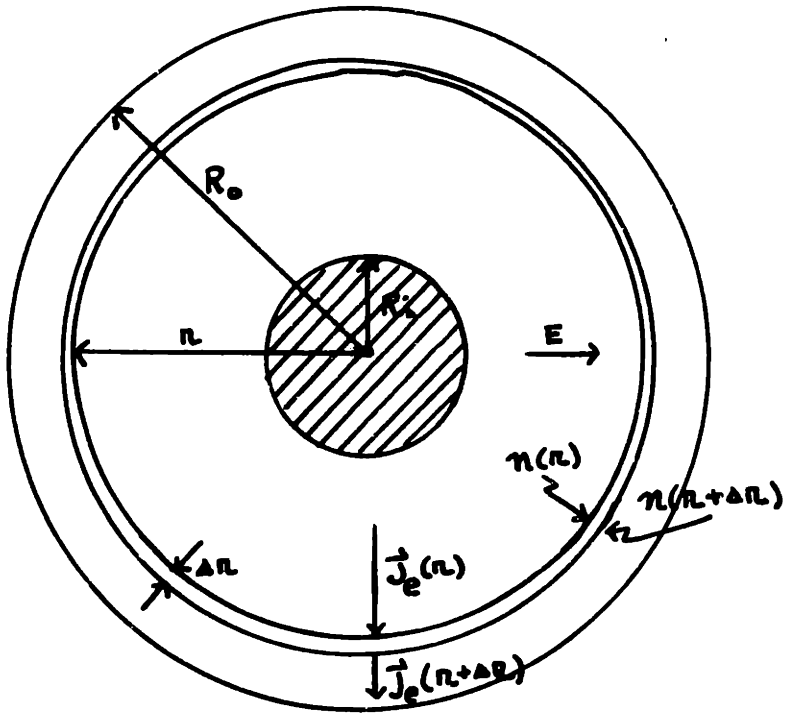


Figure 4.1 The diagram used in the explaination of the derivation for the electron continuity equation for positive corona.

$$(r+\Delta r)\, J_e(r+\Delta r) - r\, J_e(r) \tag{10}$$

This expression can be rewritten in the limit $\Delta r \to 0$ as

$$d(rJ_e) = (\nabla \cdot \vec{J}_e)\, r dr \tag{11}$$

using equation 1, equation 9 can also be rewritten as

$$-q \mu_e E \left[r\, n_e(r) - (r+\Delta r)\, n_e(r+\Delta r) \right] \tag{12}$$

using

$$J_e = -\rho_e \mu_e E = -(-q)\, n_e \mu_e E \tag{13}$$

where $(-q)$ is the charge of an electron and n_e is electron density. The difference in n_e is equal to the number of electrons created minus number of electrons attached. as

$$r\, n_e(r) - (r+\Delta r)\, n_e(r+\Delta r) \tag{14}$$
$$= r\, n_e(r+\Delta r)(\alpha - \eta)\Delta r$$

which to first order becomes

$$r\, dr\, (\nabla \cdot \vec{J}_e) = -q \mu_e E\, (\alpha - \eta)\, n_e\, r dr \tag{15}$$
$$= \mu_e E \rho_e (\alpha - \eta)$$

$$\nabla \cdot \vec{J}_e = -(\alpha - \eta) J_e. \tag{16}$$

By similar arguments one can get the rest of the continuity equations.

The boundary conditions are

$$J_e(R_o) = J_{ec}, \tag{17}$$
$$J_-(R_o) = J_{-o},$$

and $$J_+(R_1) = 0,$$

where R_1 is the radius of the inner cylinder and R_o is the radius of the outer cylinder. Actually, the outer

cylinder is only an imaginary cylinder.

4.3 An Exact Expression of Current for Unperturbed Electric Field

An exact solution can be derived if space charges are assumed to be unimportant. The electric field will remain in a K/r form wher K can be related to the applied voltage by

$$V = K \ln(R_o/R_i). \tag{18}$$

The total current per centimeter in the z direction can be found.

Equation 6 to 8 can be rewritten in the following form

$$\frac{d}{dr}(rJ_e) = -(\alpha - \eta)(rJ_e) \tag{19}$$

$$\frac{d}{dr}(rJ_-) = -\eta(rJ_e) \tag{20}$$

$$\frac{d}{dr}(rJ_+) = \alpha(rJ_e) \tag{21}$$

with the equation 17 as boundary conditions.

Equation 19 can be solved independently. The solution is

$$J_e(r) = \frac{R_o}{r} J_{eo} \exp \int_r^{R_o} (\alpha(r') - \eta(r'))\, dr' \tag{22}$$

where $0 < R_i < r < r' < R_o$

Substitute $J_e(r)$ into right handside of equation 20. The negative ion current density can be integrated directly.

$$J_-(r) = \frac{R_o}{r} J_{eo} \int_r^{R_o} \eta(r'') \exp\left[\int_{r''}^{R_o} (\alpha(r') - \eta(r'))dr'\right] dr''$$

$$+ \frac{R_o}{r} J_{-o} \qquad (23)$$

where $0 < R_i < r < r'' < r' < R_o$

Similarly the current due to the movement of positive ions can be found.

$$J_+(r) = \frac{R_o}{r} J_{eo} \int_{R_i}^{r} \alpha(r'') \exp\left[\int_{r''}^{R_o} (\alpha(r') - \eta(r'))dr'\right] dr'' \qquad (24)$$

where $0 < R_i < r'' < r' < r < R_o$

The total current, I, perunit length of z should be a constant independent of r because

$$\nabla \cdot \vec{J}_e + \nabla \cdot \vec{J}_- + \nabla \cdot \vec{J}_+ = 0. \qquad (25)$$

I is defined as

$$I = 2\pi r (J_e + J_- + J_+) = \text{constant} \qquad (26)$$

To derive I, $J_e(r)$ has to be rewritten in another form by substituting the origional J_e as in equation 22 back into equation 19 One finds

$$J_e(r) = - \frac{R_o}{r} J_{eo} \int_{R_o}^{r} (\alpha(r'') - \eta(r'')) \exp\left[\int_{r''}^{R_o} (\alpha(r') - \eta(r')dr'\right] dr'' \qquad (27)$$

now simply add J_e, $J_-$and J_+, (28)

$$I = 2\pi R_o \left\{J_{-o} + J_{eo} \int_{R_i}^{R_o} \alpha(r'') \exp\left[\int_{r''}^{R_o} (\alpha(r') - \eta(r'))dr'\right] dr''\right\}$$

I is found to be indeed independent of radius r.

4.4 Some Numerical Results

Equation 28 cannot be integrated directly without making more assumptions. It is reasonable to take R_o as the position α and η cross and it is at

$$E/p = 32 \text{ volts/cm-mm Hg}, \tag{29}$$

since all experiments found that the radius of the outer cylinder is unimportant. Using

$$E = K/r, \tag{30}$$

$$R_o = K/32p. \tag{31}$$

Letting

$$(\alpha - \eta)/p = A \exp(-\frac{B}{E/p}) \tag{32}$$

with $A = 8.2$ 1/cm-mm Hg (33)

and $B = 250$ volts/cm-mm Hg

I can be integrated. See plot of the equation 32 in Figure 3.3 .

$$I = \frac{2\pi K}{p}\left\{J_{-o} + J_{eo}\int_{R_1}^{R_o} Ap \exp(-\frac{Bpr''}{K})\exp\int_{r''}^{R_o} Ap \exp(-\frac{Bpr'}{K})dr'\, dr''\right\} \tag{34}$$

$$= \frac{\pi K}{p}\left\{J_{-o} + J_{eo}\left[\exp(\frac{AK}{B}(\exp(-\frac{BpR_1}{K}) - \exp(-\frac{B}{32})) - 1\right]\right\} \tag{35}$$

A plot of current as a function of the electric field

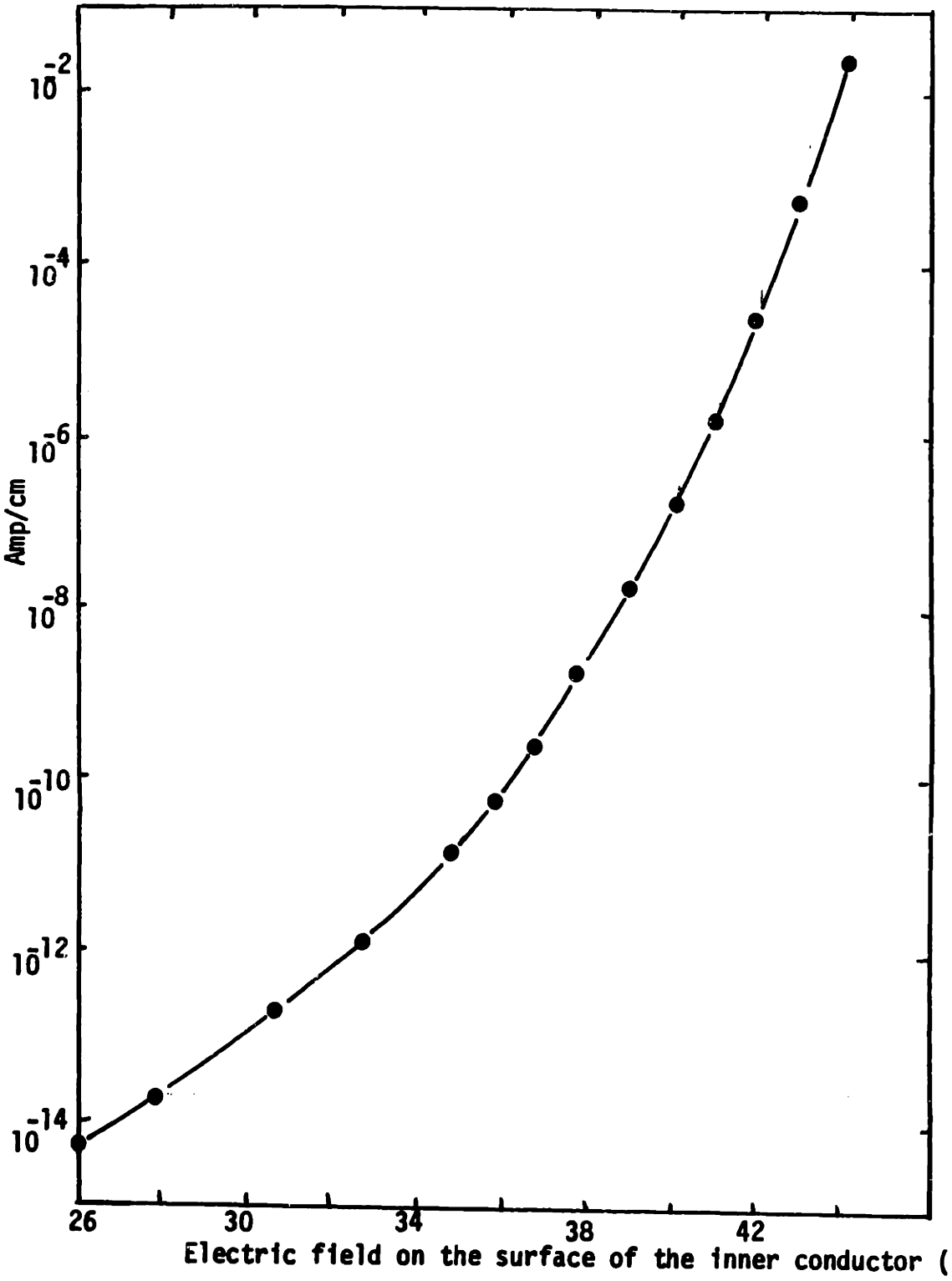


Figure 4.2 Total current as a function of the applied voltage with $J_{eo} = 1 \times 10^{-15}$ amp/cm.

at the surface of a 1.5 cm radius inner cylinder is shown in Figure 4.2 . J_{-o} is taken to be zero for simplicity. J_{eo} is arbitrarily taken to be $1x10^{-15}$ amp/cm since on log scale, the effect of multiplying J_{eo} to the whole equation is simply achieved by scaling the whole curve. At this point, wee are really interested in its shape and not its absolute value.

The current as a function of electric field is simply the result of Townsend's electron avalench. The Peek's value of electric field for breakdown on a 1.5 cm radius inner conductor is 38.8 kv/cm. The curve include the case of 38.8 kv/cm electric field. However, it does not look like typical current versus voltage graphs near the threshold of breakdown.

4.5 Space Charges

The space charges can be estimated. For simplicity, η is assumed to be identically zero. and

$$\alpha/p = (\alpha - \eta)/p. \tag{35}$$

So the total current I now is due to movement of electrons and positive ions only.

$$\rho_e(r) = - \frac{R_o J_{eo}}{K_e} \exp (AK/B (\exp(-Bpr/K) - \exp(-BpR_o/K)) \tag{36}$$

$$\rho_+(r) = \frac{R_o J_{eo}}{K_+} \exp (AK/B (\exp(-BpR_1/K) - \exp(-Bpr/K)) \tag{37}$$

The electron charge density, $\rho_e(r)$, is large near the surface of the inner conductor and the positive ion charge

density, $\rho_+(r)$, is larger further away from center. However, $\rho_e(r)$ and $\rho_+(r)$ are of equal magnitude very close to the surface of the inner conductor because

$$\mu_+ \ll \mu_e. \tag{38}$$

Their effect on the electric field for a fixed voltage is to lower the electric field on the surface of the inner conductor and raise it further away.

4.6 Cosmic Radiation

The boundary conditions now control the magnitude of the total current. The true source of initial free electrons comes from cosmic radiation. The effect of cosmic radiation can be included very easily. Assume cosmic rays creat on the order of 10 charged pairs/cm^3/sec, then the continuity equations look like

$$\nabla\cdot\vec{J}_e = -(\alpha - \eta)J_e - 10q \tag{39}$$

$$\nabla\cdot\vec{J}_- = -\eta J_e \tag{40}$$

$$\nabla\cdot\vec{J}_+ = \alpha J_e + 10q \tag{41}$$

The solution of equation 39 looks like

$$J_e(r) = -\frac{10q}{rS(r)}\int_{R_o}^{r} r'S(r')dr' + \frac{R_o J_{eo} S(R_o)}{rS(r)} \tag{42}$$

where $$S(r) = \exp\left(\int(\alpha(r'') - \eta(r''))dr''\right). \tag{43}$$

The second term on the right handside is the same as without cosmic radiation. The negative ion and positive ion current densities both have one additional term also.

$$J_-(r) = \frac{10q}{r}\int_{R_o}^{r}\frac{\eta(r")}{S(r")}\int_{R_o}^{r"} r' S(r')\, dr'\, dr" \qquad (44)$$

$$- \frac{R_o J_{eo} S(R_o)}{r}\int_{R_o}^{r}\frac{(r")}{S(r")}\, dr" + \frac{R_o J_{-o}}{r} .$$

$$J_+(r) = -\frac{10q}{r}\int_{R_i}^{r}\frac{\alpha(r")}{S(r")}\int_{R_o}^{r"} r' S(r')dr' \qquad (45)$$

$$+ \frac{R_o J_{eo} S(R_o)}{r}\int_{R_i}^{r}\frac{(r")}{S(r")}\, dr" + \frac{10q}{r}\int_{R_i}^{r} r"\, dr"$$

$$I = 2\pi(10q)\int_{R_i}^{R_o} r"\, dr" \qquad (46)$$

$$+ 2\pi(10q)\int_{R_i}^{R_o}\frac{(r")}{S(r")}\int_{r"}^{R_o} r'S(r')\, dr'\, dr"$$

$$+ 2\pi R_o J_{eo} S(R_o)\int_{R_i}^{R_o}\frac{(r")}{S(r")}\, dr" + 2\pi R_o J_{eo} .$$

The first term on the right handside of the expression for total current expresses the current produced by cosmic radiation without multiplication, and it is insignificant compared to the following terms. The second term gives the current produced due to Twonsend avalanche but with electrons initially produced by cosmic radiation. The third and fourth terms are the same as without cosmic radiation.

If the same assumptions are made here as in section 4.4,

$$I = \pi(10q)\,(R^2 - R_1^2) \qquad (47)$$

$$+ 2\pi(10q)\int_{R_1}^{R_o} Ap\,\exp(-\frac{Bpr''}{K})\,\exp(\frac{AK}{B}\exp(-\frac{Bpr''}{K})\;x$$

$$\left[\int_{r''}^{R_o} r'\,\exp(-\frac{AK}{B}\exp(-\frac{Bpr'}{K})\,dr'\right]\,dr''$$

$$+ 2\pi R_o J_{eo}(\,\exp(\frac{AK}{B}\,(\exp(-\frac{BpR_1}{K}) - \exp(-\frac{B}{32}) - 1\,)$$

$$+ 2\pi R_o J_{-o}\,.$$

The second term of equation 47 has to be integrated numerically. Figure 4.3 is a plot of the result- the total current as a function of electric field with just cosmic radiation and Townsend avalanche. This curve has the same shape as the curve given by the third term of equation 47 . If one defines J_{eo} as the current necessary to make the second term the same as the third term, J_{eo} is found to be fairly constant, ranging from $1.9\text{x}10^{-15}$ amp/cm at 27 kv/cm to $54\text{x}10^{-15}$ amp/cm at 45 kv/cm.

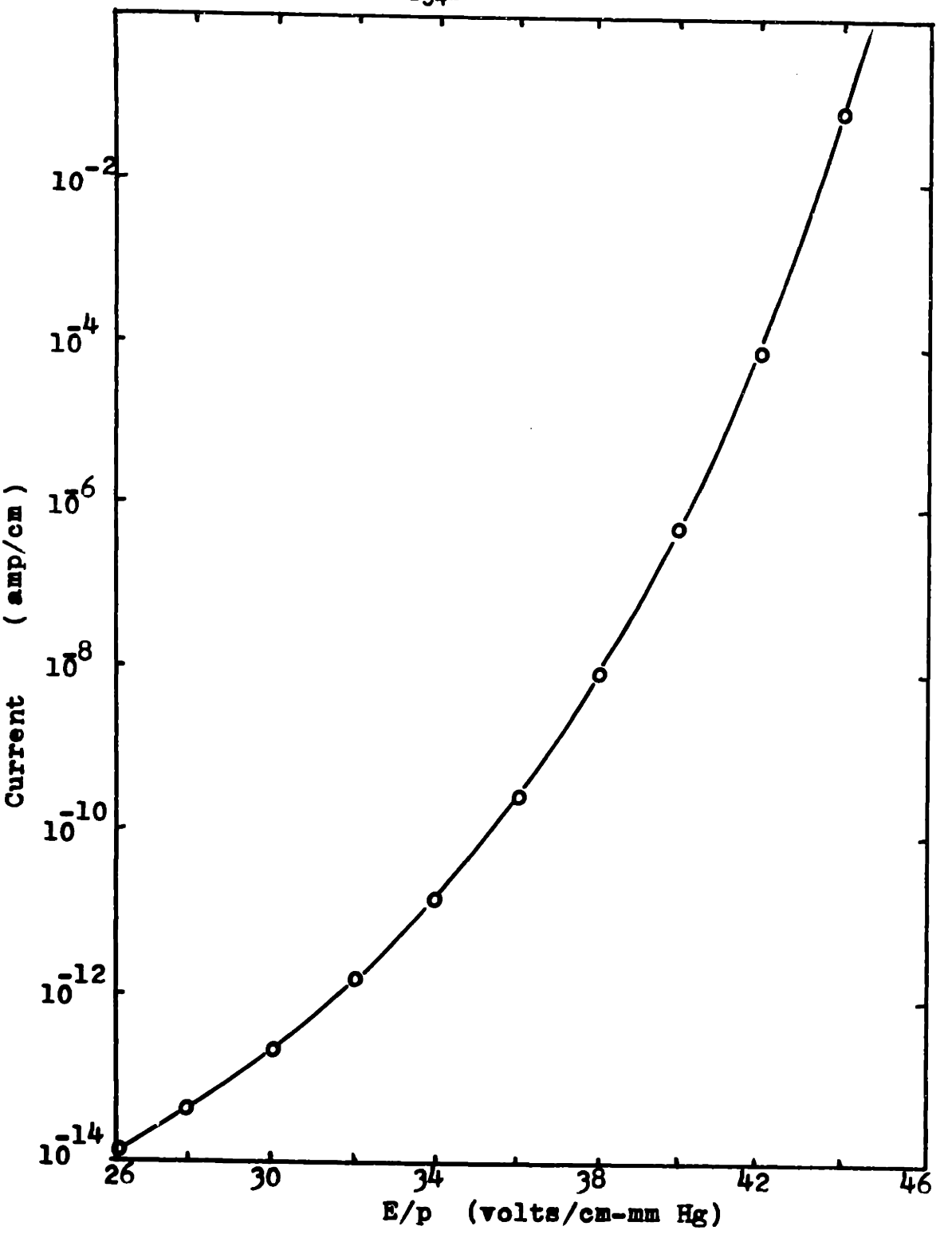


Figure 4.3 The total current per cm axial length as a function of the electric field at the surface of the inner conductor with cosmic radiation.

5. Positive Corona with Photoionization

5.1 Justification for Photoionization

A large change in current of many orders of magnitude within a small range of voltage was found to be unabtainable based simply on Townsend avalanche. A feedback mechanism is necessary. For now, consider only a smooth polished wire placed in air. Ion bombardment and photoelectric emission are not found to be the feedback mechanism, due to the difference in time delay and magnitude of current involved in breakdown; at the same time those facts support photoionization in air as the important process. Additional information can support this theory. First of all both visible and ultraviolet light are detected at and above the voltage that the current increases by many orders of magnitude.[18] Secondly, the threshold for negative corona is the same or slightly higher than the critical electric field of positive corona.

5.2 Assumptions

A few assumptions are made in the process of making a methematical model. Photons are assumed to have equal probability in travelling in any direction

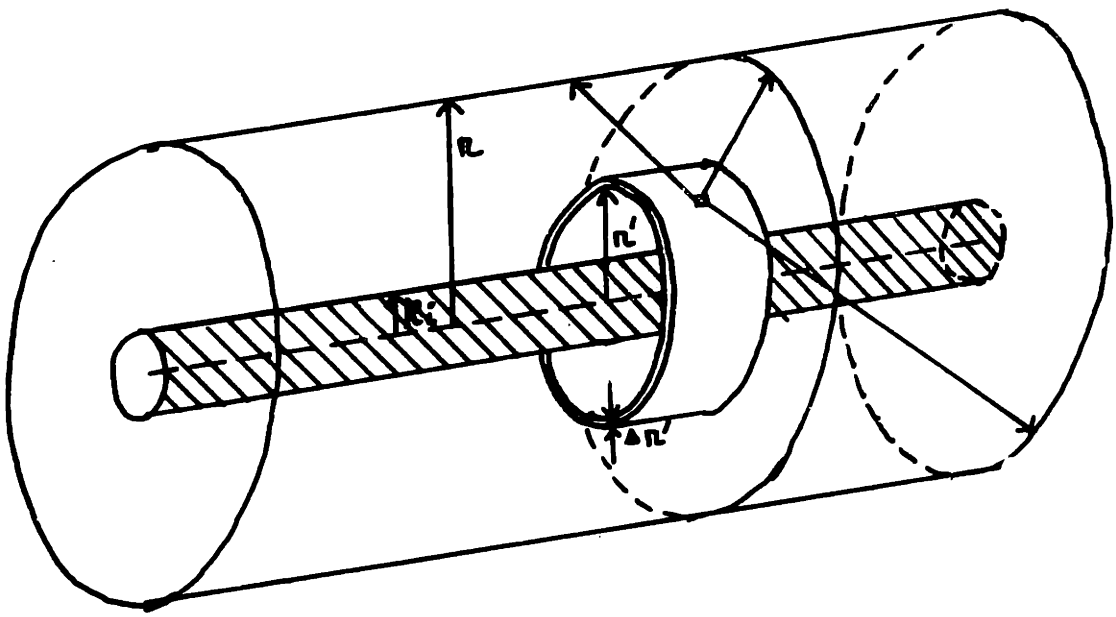


Figure 5.1 Illustration of some possible directions a ray emitted on the ring of radius r' could travel, and be absorbed on the cylinder of radius r, where $r > r'$.

in a straight line from the point of creation. Most of the Townsend electrons are created near the inner wire surface. For every electron created, Q photons are created. Consequently most of the photons are also created near the surface of inner wire. Visual observation confirms this fact.[18] Therefore, one can assume that all photons that reach a ring of radius r come from within the ring of radius r'. See Figure 5.1. The assumption as a whole is good, even though the numbers will not be accurate very close to the surface of the inner conductor, because the electrons created further away are more important. They create more electrons on their way in due to Townsend avalanche, then electrons created near the inner cylinder surface.

5.3 Derivation of the Photoionization term

The contribution to the number of electrons created due to photons is formulated below. For references see Figure 5.1 and 5.2. The total number of electrons passing through a ring of unit length in the axial direction defined by radius r' per second is

$$2\pi r' J_e(r') \qquad (1)$$

The number of photons created at r' per unit length in the axial direction is

$$2\pi Q \alpha(r') r' J_e(r') \qquad (2)$$

Therefore for a ring $\Delta r'$ thick, the number of photons created is

$$2\pi Q \alpha(r') r' J_e(r') \Delta r'. \tag{3}$$

Not all the photons created in the ring of radius r' $\Delta r'$ thick can reach an infinitly long cylinder of radius r greater than r'. Part of the photons run into the inner wire; part are absorbed in the intermediate space. The ones that do reach the cylinder of radius r have to travel different distances to get there. Let K be the percentage of photons arriving at radius r per photon emitted from a point P on radius r'. Then the number of photons that actually reach the ring of radius r is

$$K 2\pi Q \alpha(r') r' J_e(r') \Delta r' \tag{4}$$

where $$K = \frac{r}{\pi} \int_0^{\theta_{max}} \int_0^{\infty} \frac{(r - r'\cos\theta)\exp(-\mu D)}{D^3} dz\, d\theta \tag{5}$$

$$|D| = (r^2 + r'^2 - 2rr'\cos\theta + z^2)^{\frac{1}{2}} \tag{6}$$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{rr'}(R_1^2 - ((r^2 - R_1^2)(r'^2 - R_1^2))^{\frac{1}{2}})\right] \tag{7}$$

The derivation of K is in Appendix 1. This is too complicated an expression to evaluate. An approximate expression in the form

$$K \cong G \exp(-\mu_{eff}(r-r')) \tag{8}$$

is simple to understand and easy to use. G, the geometric

factor, is the number of electrons not blocked by the inner wire, under the assumption that nothing is absorbed by air. μ_{eff} is the effective μ_p when approximating a three dimensional system by a one dimensional system.

$$G = \frac{\theta_{max}}{2\pi} + \frac{1}{\pi} \tan^{-1}\left(\left(\frac{r + r'}{r - r'}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right) \qquad (9)$$

See Appendix 3 for calculation of μ_{eff}.

So the total number of photons produced in a ring of radius r' thickness $\Delta r'$ per unit axial length can result in

$$2\pi \mu_p Q G \alpha(r') r' J_e(r') \exp(-\mu_{eff}(r-r'))\, \Delta r' \Delta r \qquad (10)$$

electrons produced at a cylinder of radius r and thickness Δr. Summing up the effect of all the photons created from within the cylinder of radius r, one gets

$$2\pi \mu_p Q \int_{R_1}^{r} G \alpha(r') r' J_e(r') \exp(-\mu_{eff}(r-r')\, dr'\, \Delta r \qquad (11)$$

as the number of electrons created in thickness Δr per unit length in z-direction.

$$\begin{aligned} & J_e(r + \Delta r) 2\pi (r + \Delta r) - J_e(r) 2\pi r \qquad (12) \\ &= 2\pi r \Delta r (\nabla \cdot J_e) \\ &= -2\pi r (\alpha(r) - \eta(r)) J_e(r) \Delta r - 10q 2\pi r \Delta r \\ &\quad - 2\pi \mu_p Q \int_{R_1}^{r} G \alpha(r') r' J_e(r') \exp(-\mu_{eff}(r-r'))\, dr' \Delta r \quad . \end{aligned}$$

Now the continuity equations can be replaced by

$$\frac{d}{dr}(rJ_e) = -(\alpha(r) - \eta(r))(rJ_e) - 10qr - Ph \qquad (13)$$

$$\frac{d}{dr}(rJ_-) = -\eta(r)(rJ_e) \qquad (14)$$

$$\frac{d}{dr}(rJ_+) = \alpha(r)(rJ_e) + 10qr + Ph \qquad (15)$$

where

$$Ph = \mu_+ Q \int_{R_1}^{r} G\,\alpha(r')\, r' J_e(r') \exp(-\mu_{eff}(r-r')\, dr' \qquad (16)$$

5.4 Analytical Prediction of Corona Onset

A very much simplified picture might help to understand photoionization and give a rough estimate of critical electric field required for breakdown as a function of the radius of the inner conductor. One can assume that instead of photoionization occuring in the interelectrode region in air, photions could only ionize at a ring of thickness Δr, radius R_o, where E/p is 32 volts/cm-mm Hg. This particular radius R_o is picked because if an electron is created by photon, it can produce the most number of new electrons in an avalanche, yet not in such a low field region that attachment is dominent. Further assume the geometric factor G to be a constant

and no decay in photon intensity between the place of emission and arrival at radius R_o.

Then a self sustaining discharge requires

$$R_o J_{eo} = G\mu_p Q \int_{R_1}^{R_o} \alpha(r') r' J_e(r')\, dr' \qquad (17)$$

If one takes the expression for α as

$$\alpha/p = A \exp(-\frac{B}{E/p}) \qquad (18)$$

Then the solution for $J_e(r)$ is equation 4.22. Substituting them into equation 18 above, then

$$1 = G\mu_p Q \left[\exp(\frac{AK}{B}(\exp(-\frac{BpR_1}{K}) - \exp(-\frac{B}{32}) - 1 \right] \qquad (19)$$

For a given $G\mu_p Q$, a set of critical electric fields as a function of inner conductor radii can be calculated numerically. Pick $G\mu_p Q$ so that the critical electric field of a 1.5 cm radius conductor coincides with Peek's value. When

$$A = 8.2 \quad \text{1/cm-mm Hg} \qquad (20)$$
$$B = 250 \quad \text{volts/cm-mm Hg}$$

then $G\mu_p Q$ is found to be 1.4×10^{-6}. Figure 5.2 shows plots of both curves. The agreement is not very good. If instead,

$$A = 50 \quad \text{1/cm-mm Hg} \qquad (21)$$
$$B = 335 \quad \text{volts/cm-mm Hg}$$

are used, the critical electric field curve is found to

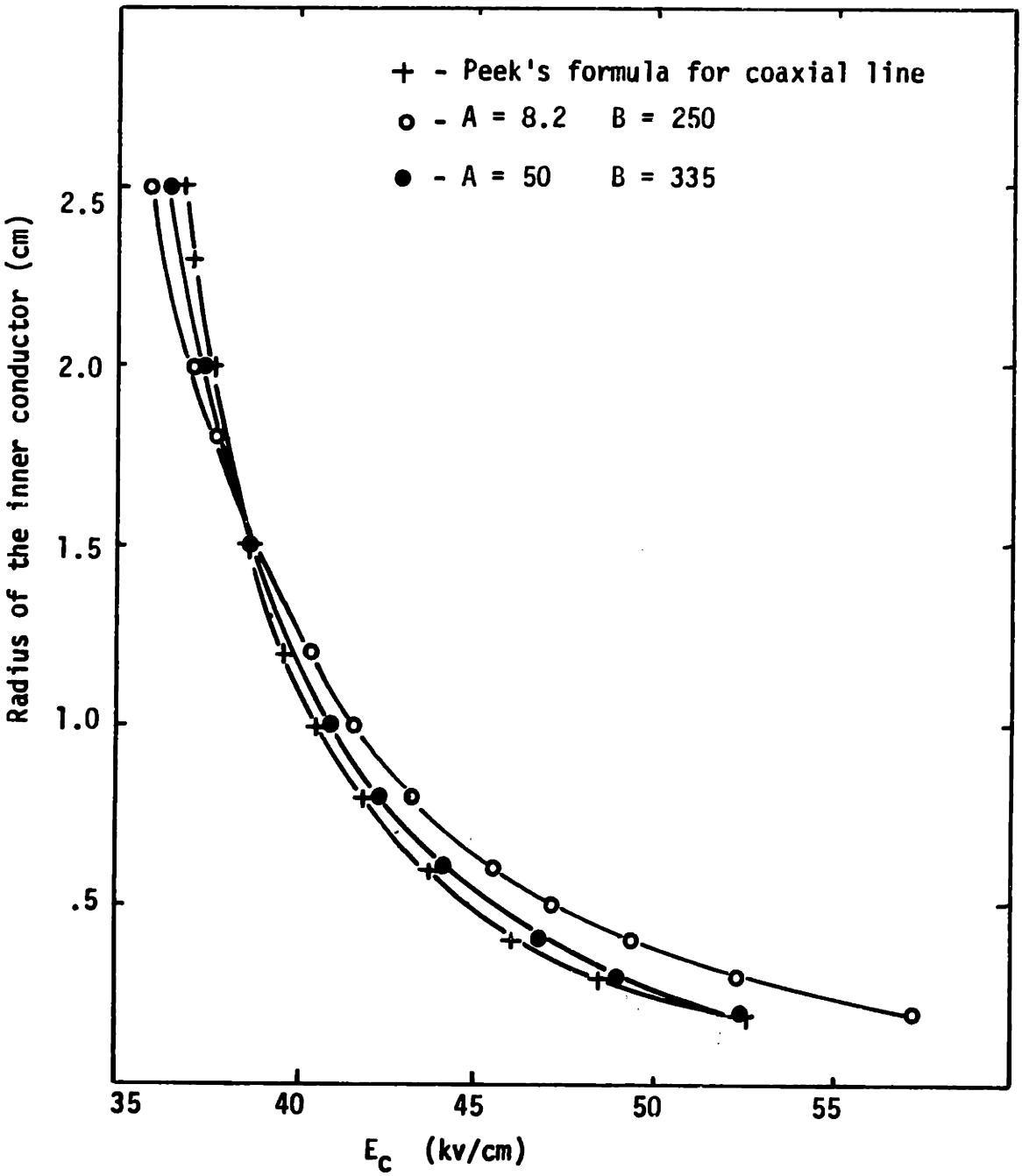


Figure 5.2 Analytic calculation of critical corona onset gradient and comparison against Peek's formula.

be much closer to Peek's formula. The values in equation 21 produces a much steeper α/p as function of E/p. This seems to indicate the importance of attachment, because $\alpha - \eta$ for E/p greater than 32 volts/cm-mm Hg is a curve similar to α but steeper.

5.5 The Full Set of Equations Used

Collecting all the equations involved, they are

$$\vec{J}_e = - v_e(E)\rho_e \tag{22}$$

$$\vec{J}_- = - \mu_- \vec{E} \rho_- \tag{23}$$

$$\vec{J}_+ = \mu_+ \vec{E} \rho_+ \tag{24}$$

$$\vec{E} = -\nabla V \tag{25}$$

$$\nabla \cdot \epsilon_o \vec{E} = \rho_+ + \rho_- + \rho_e \tag{26}$$

$$\nabla \cdot \vec{J}_e = -(\alpha - \eta) J_e - 10q - Ph \tag{27}$$

$$\nabla \cdot \vec{J}_- = - \eta J_e \tag{28}$$

$$\nabla \cdot \vec{J}_+ = \alpha J_e + 10q + Ph \tag{29}$$

where

$$Ph = \mu_+ Q \int_{R_1}^{r} G(r')\, \alpha(r')\, r' J_e(r') \exp(-\mu_{eff}(r-r'))dr' \tag{30}$$

$$G(r') = \frac{\theta_{max}}{2\pi} + \frac{1}{\pi}\tan^{-1}\left(\left(\frac{r + r'}{r - r'}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right) \tag{31}$$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{rr'}\left(R_1^2 - ((r'^2 - R_1^2)(r^2 - R_1^2))\right)\right] \tag{32}$$

α, η and $v_e(E)$ will have to be fitted to experi-

mental data.

Marsh and Sander's data can be fitted quite well by

$$A = 8.2 \quad 1/\text{cm-mm Hg} \qquad (33)$$

and $B = 250$ volt/cm-mm Hg .

Harrison and Geballe's data is probably more accurate for regions of $E/p < 40$ volts/cm-mm Hg, since they separated out the effect of attachment. One set of A and B, however, will not fit the whole range of E/p for Harrison and Geballes's data. For $E/p \geq 40$ volts/cm-mm Hg, A and B are the same as in equation 5.33. For $E/p < 40$ volts/cm-mm Hg,

$$A = 1.44 \quad 1/\text{cm-mm Hg} \qquad (34)$$

and $B = 180$ volts/cm-mm Hg

fit better.

For the attachment coefficient, no good explaination exists for the discripencies between experimental results. For $E/p \geq 26$ volts/cm-mm Hg, again Harrison and Geballe's η/p curve is used as a reference. It is fitted by

$$\eta/p = 9.15(10^{-6})\left(\frac{E}{p}\right)^2 - .000567\left(\frac{E}{p}\right) + .0135. \qquad (35)$$

For $E/p < 26$ volts/cm-mm Hg, Kuffel's data is used as a reference.

$$\eta/p = 1.545(10^{-4})(E/p) + 1.14(10^{-3}). \qquad (36)$$

Variations of η on corona is also studied later.

$$v_e(E) = 2.5(10^5)(E/p) + 5(10^6) \qquad (37)$$

for $E/p \geq 25$ volts/cm-mm Hg. And

$$v_e(E) = 4.5(10^5)(E/p) \qquad (38)$$

for $E/p < 25$ volts/cm-mm Hg.

$v_e(E)$ is used for electron drift velocity insteady of using the concept of mobility, because mobility associated with measured data is not a constant as E/p is changed for $E/p > 25$ volts/cm-mm Hg. See Figure 3.1 for electron velocity as measured by Ryzko.

5.6 Numerical Method

Because the equations are nonlinear and coupled, an iterative method is used. For the first iteration only, the electric field is assumed to have a K/r form and $\mu_p = 0$. Equation 5.27 is solved first. The results of $J_e(r)$ is used in calculating J_- and J_+. All the integrations are done by a fourth-order Runge-Kutta method. From J_e, J_- and J_+, the space charges are calculated using equations 5.22 to 5.23. Equations 5.25 and 5.26 are used to calculate the new electric field. Then the photoionization term is calculated. Equation 5.27 is solved again with the new electric field and the new photoionization term. This process repeats until the solution of total current density changes only by .5% .

5.7 Method Used in the Calculation of the Electric Field

Once the charge distribution is known, and the voltage between the conductors specified, the electric field distribution can be calculated. The equations used in the program are derived below. Trap zoidal rule is used for integration.

Define $r_j = R_1 + j(\Delta r)$, where j is an integer. All the charges within the radius r_j can be written as

$$\int_{r_o}^{r_j} (\rho_e(r) + \rho_-(r) + \rho_+(r)) r 2\pi dr + \rho_o \qquad (39)$$

where ρ_o is the total charge on the inner conductor. Also define

$$\text{charge}(j) = \int_{r_o}^{r_j} (\rho_e(r) + \rho_-(r) + \rho_+(r)) r dr \ . \qquad (40)$$

Then the Gauss's Law can be written as

$$2\pi r_j E(r_j) = (\rho_o + 2\pi \text{charge}(j))/\epsilon_o \ . \qquad (41)$$

Since

$$2\pi r_o E(r_o) = \rho_o/\epsilon_o \ , \qquad (42)$$

$$E(r_j) = E(r_o)\frac{r_o}{r_j} + \frac{\text{charge}(j)}{\epsilon_o(r_j)} \ . \qquad (43)$$

Equation 5.25 can be rewritten in the integral form.

$$V(r_o) - V(r_N) = \int_{r_o}^{r_N} \vec{E} \cdot d\vec{r} \qquad (44)$$

where $r_N = R_o = R_1 + N(\Delta r)$. (45)

Since the difference in voltage is important and not the absolute value, $V(r_N)$ can be set to zero. Using the trapezoidal rule for integration,

(46)

$$\text{charge}(j) = \Delta r \left[r_o(\rho_e(r_o) + \rho_-(r_o) + \rho_+(r_o))/2 + \sum_{k=1}^{j-1} r_k(\rho_e(r_k) + \rho_-(r_k) + \rho_+(r_k)) + r_j(\rho_e(r_j) + \rho_-(r_j) + \rho_+(r_j))/2 \right] .$$

$$V(r_o) = \Delta r \left[\frac{E(r_o)}{2} + \sum_{k=1}^{N-1} E(r_k) + \frac{E(r_N)}{2} \right] \quad (47)$$

$$= \Delta r \left[(\tfrac{1}{2} + \sum_{k=1}^{N-1} \frac{r_o}{r_k} + \frac{r_o}{2r_N})E(r_o) + \frac{1}{\epsilon_o} \sum_{j=1}^{N-1} \frac{\text{charge}(j)}{r_j} + \frac{1}{\epsilon_o} \frac{\text{charge}(N)}{2r_N} \right]$$

$$E(r_o) = \frac{\frac{V(r_o)}{\Delta r} - \left[\frac{1}{\epsilon_o} \sum_{j=1}^{N-1} \frac{\text{charge}(j)}{r_j} + \frac{1}{\epsilon_o} \frac{\text{charge}(N)}{2r_N} \right]}{\frac{1}{2} + \sum_{k=1}^{N-1} \frac{r_o}{r_k} + \frac{r_o}{2r_N}} \quad (48)$$

$E(r_o)$ is the electric field on the surface of the inner conductor. Once that is known, the electric field everywhere can be found.

$$E(r_o) = \frac{1}{\epsilon_o} \frac{\text{charge}(j)}{r_j} + \frac{r_o}{r_j} E(r_o) \qquad (49)$$

6. Results for Anode Corona

6.1 A Detailed Analysis of One Special Case

A set of numerically computed current as a function of voltage curves are shown in Figure 6.1. There are two common features in these results: an inner conductor of 1.5 cm radius and with photoionization as the only secondary mechanism. For clarity, a detailed analysis of a specific case is described below.

The outer radius is 4.5 cm. The mobility of positive and negative ions are 2.5 cm^2/sec-volt. Q is .005 based on data from Raether.[19] μ_p is .001 cm^{-1} based on data from Figure 3.5.

$$\alpha/p = 1.44 \exp(-\frac{180}{E/p}) \quad \text{for } E/p \geq 40 \text{ volts/cm-mm Hg;} \tag{1}$$

$$\alpha/p = 8.2 \exp(-\frac{250}{E/p}) \quad \text{for } E/p < 40 \text{ volts/cm-mm Hg.} \tag{2}$$

The primary sources of electrons are 10 electron-ion pairs created per second per cm^3 by cosmic radiation, and no electrons coming through at the boundary of the outer radius. Recombination is neglected. This will be called the standard case because most of the others are variations from this one.

The horizontal axis of Figure 6.1 is the electric field at the surface of the inner conductor, E_o, when the voltage is applied and space charge is non existant.

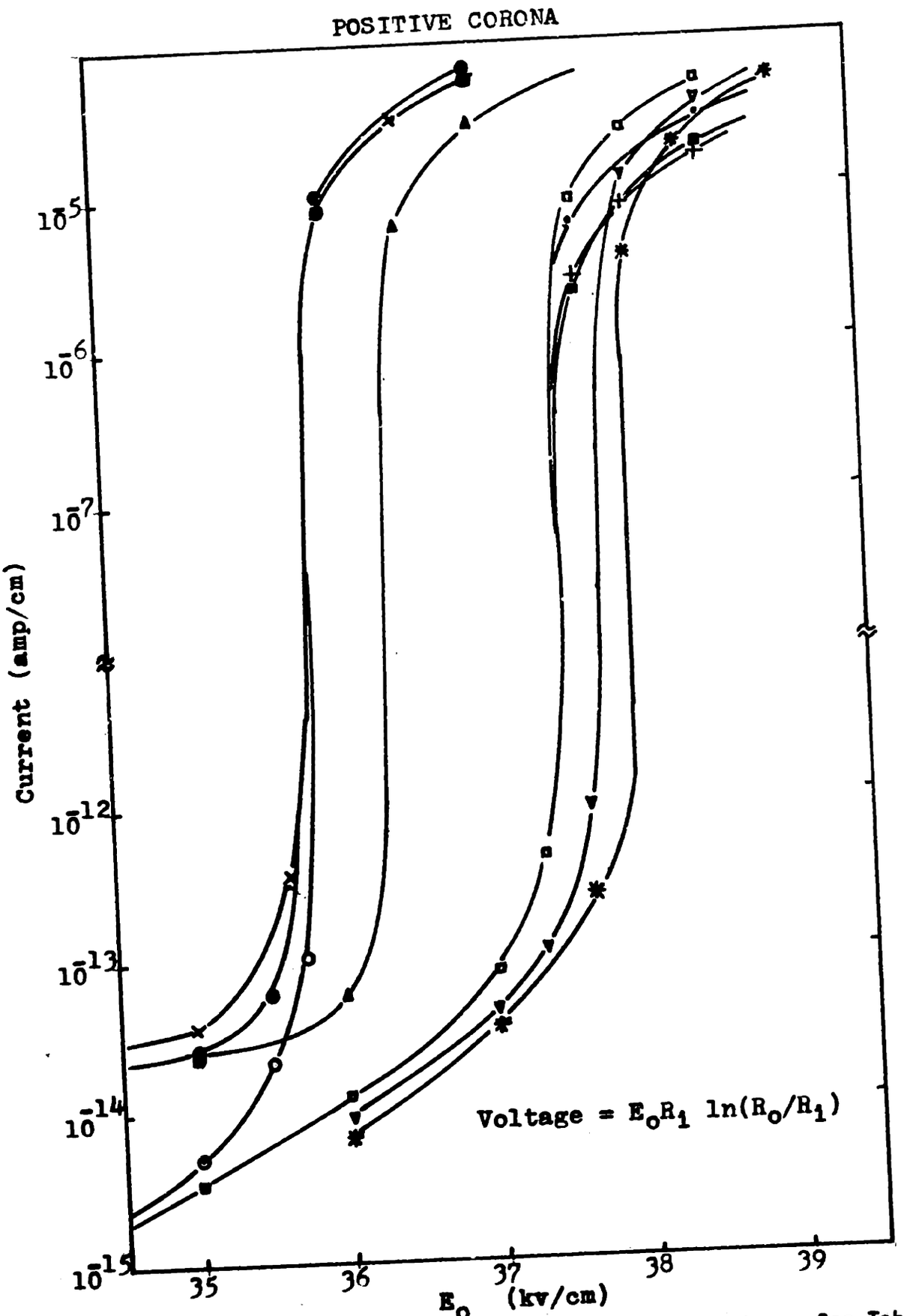


Figure 6.1 Sensitivity tests for 1.5cm inner conductor. See Table 6.1 for reference.

Table 6.1

Explaination for the Figure 6.1

1. -□- Standard case with on without recombination term

The following are the same as the standard case with the exceptions specified below.

2. -o- $\mu_p = .01\ cm^{-1}$.

3. -x- $\mu_p = .01\ cm^{-1}$. $J_{eo} = 1x10^{-15}\ amp/cm^2$.
No cosmic radiation.

4. -Δ- $\mu_p = .01\ cm^{-1}$. $J_{eo} = 1.x10^{-15}\ amp/cm^2$.
No cosmic radiation.
$\alpha/p = 8.2exp(-250p/E)$ for $E/p < 40$ volts/cm-mm Hg

5. -●- $\mu_p = .01\ cm^{-1}$. $J_{eo} = 1x10^{-15}\ amp/cm^2$.
No cosmic radiation. $\Delta r = .005$ cm
$\alpha/p = 8.2exp(-250p/E)$ for $E/p < 40$ volts/cm-mm Hg

6. -▽- η is increased by 10% for all E/p

7. -+- $\mu_- = .8\ cm^2/sec/volt$
$\mu_+ = .8\ cm^2/sec/volt$

8. -·- $R_o = 5.5$ cm

9. -■- $R_o = 7.5$ cm

10. -✱- α is decreased by 5% for all E/p

From field theory one knows that the electric field without space charge in this case has the form

$$E = K/r. \tag{3}$$

Therefore, the voltage applied between cylinders is

$$\text{voltage} = K \ln(R_o/R_1) \tag{4}$$
$$= E_o R_1 \ln(R_o/R_1)$$

To plot current against voltage is inconvient because the curves cannot be compared when the outer cylinder's dimensions are changed.

For the standard case, when E_o is below 37.5 kv/cm, the total current is below 10^{-12} amp/cm. The current decreases steadly as voltage is decreased. When the electric field at R_1 is slightly above 37.5 kv/cm. the current jumps above 10^{-6} amp/cm, and increases slowly again as voltage is increased. The voltage versus current curves are very similar to the ones measured by Miller in Figure 2.2. The electric field at which the current changes by many orders of magnitude is designated as the onset of positive corona for a given sized inner conductor. For all the cases tested, the onset is very sharp.

Consider the case E_o = 37 kv/cm, below onset. Figure 6.2 shows the current densities as a function of position. For most of the region in between conductors, the current is carried by positive ions. The electron and negative ion current densities are roughly the same

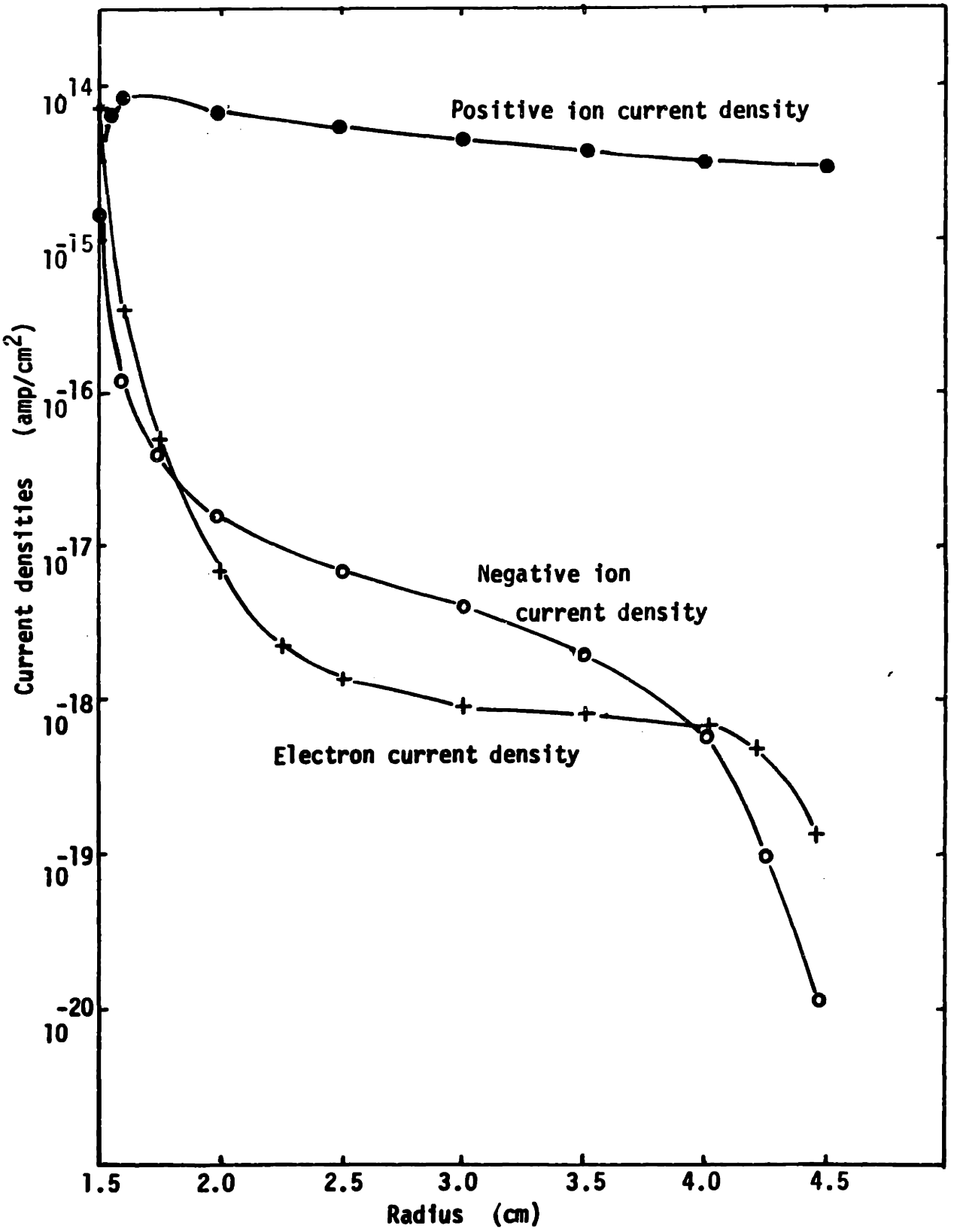


Figure 6.2 Current densities for E_0 = 37 kv/cm standard case.

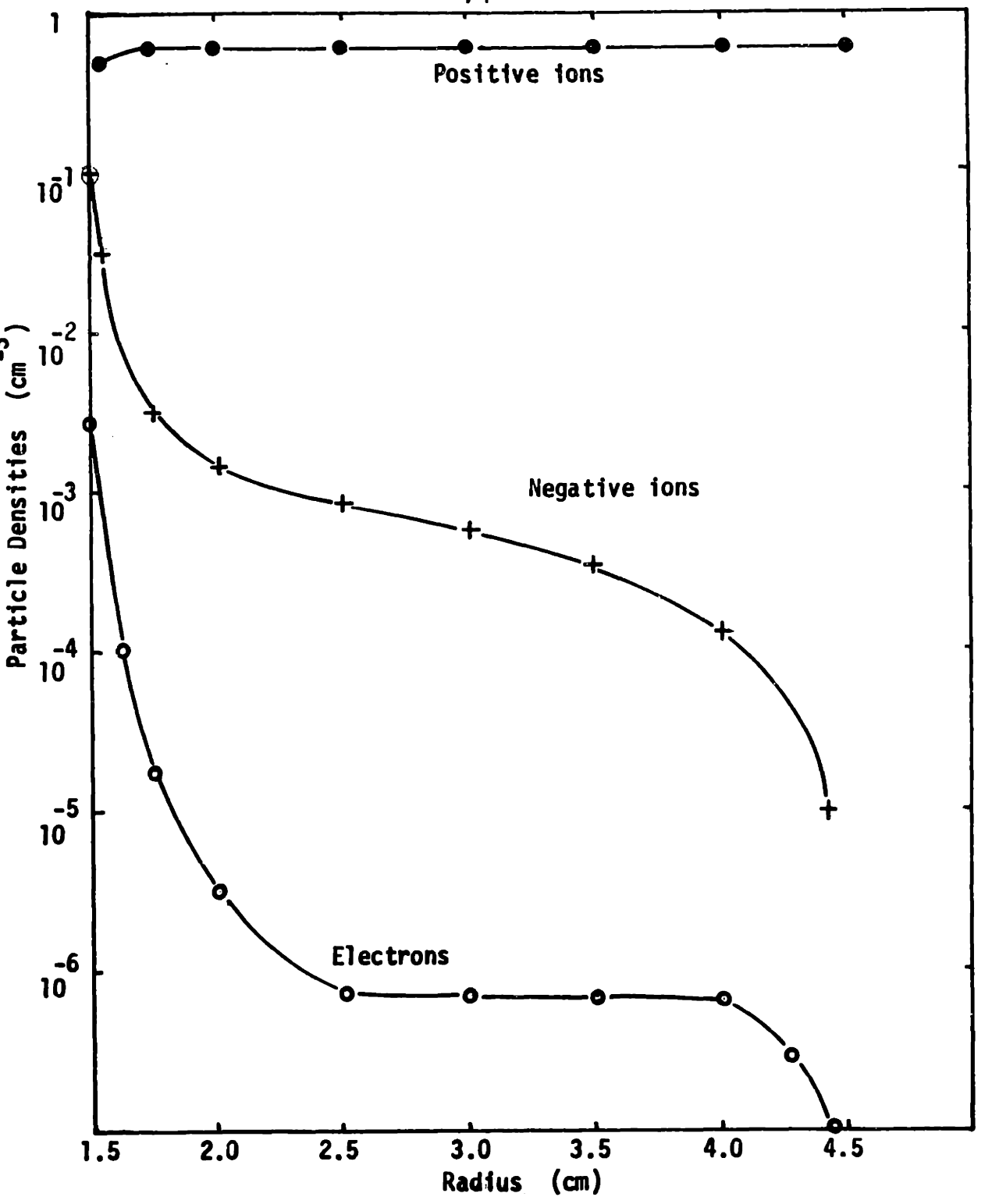


Figure 6.3 Particle densities for E_0 = 37 kv/cm standard case.

order of magnitude. There are two regions of interest: ionization versus attachment dominated. Figure 6.4 shows the values of α and η as a function of position. For this case, they cross around 2.35 cm from the center. At distances greater than 2.35 cm the electron current density stayed about the same. The negative ion current density increased as radius decreased. It could not increase by too much since free electrons are created too slowly. Attachment of electrons to neutrals provides a check on the growth of electrons. When electrons eventually drifted into the region of $\alpha > \eta$, their number starts to increase sharply.

Figure 6.3 shows the charge densities. Negative ions are important only in a very small region roughly 1 mm thick on the surface of anode, while the rest of the space between conductors is occupied by positive ions. Electrons are insignificant because of their large velocity. Compare the case above to the results of higher applied electric field, E_o = 38 kv/cm, above the onset. The shape of the current densities and charge densities are roughly the same as the previous case. See Figures 6.5 and 6.6. However, they are many orders of magitude larger. Figure 6.4 showes that η as a function of radius barely changed. α changed only slightly also. This slight change provides a gain greater than one in the feedback loop, resulting in changing the magnitudes of current

Figure 6.4 Values of α and η as a function of position for two different applied voltages.

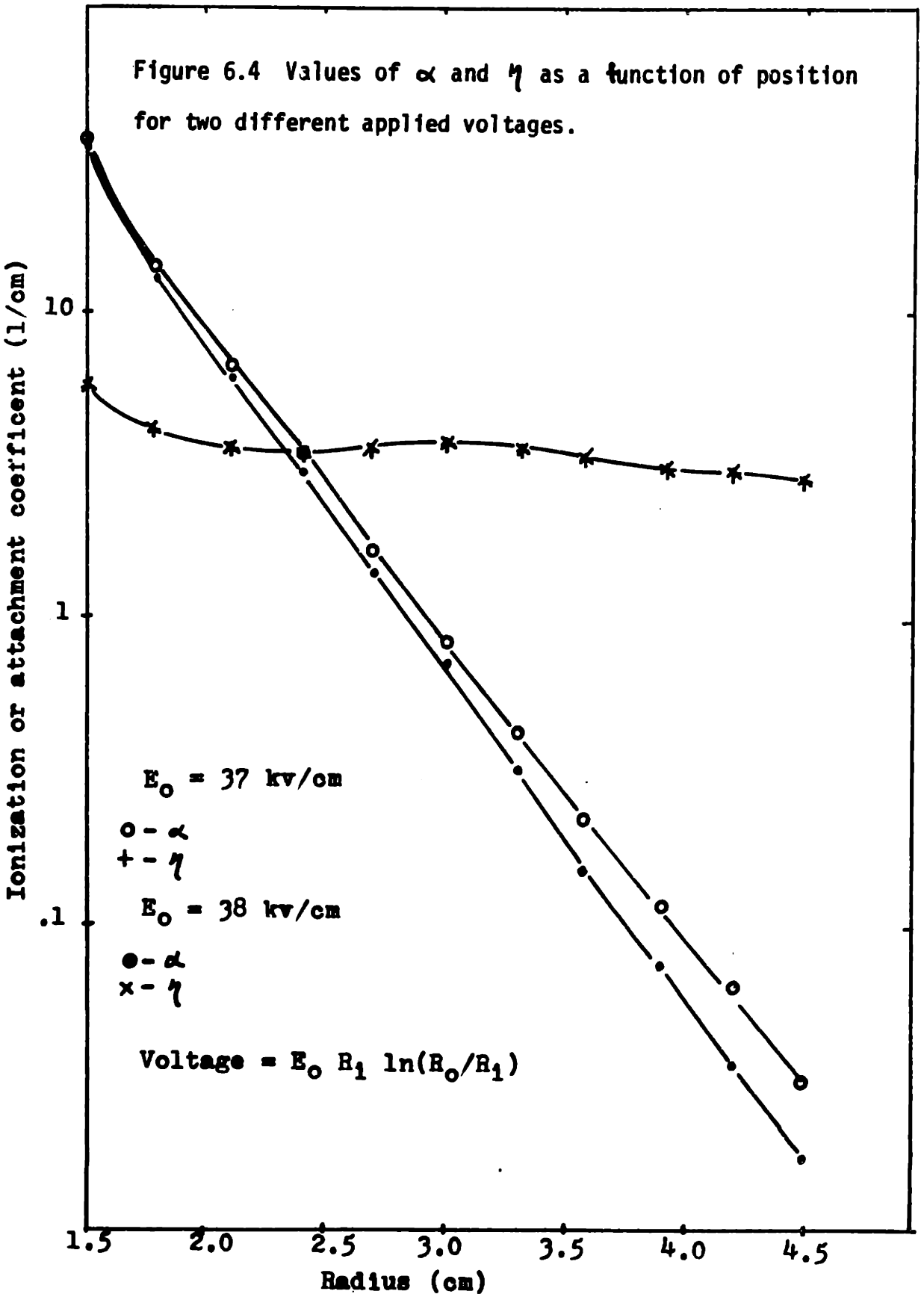
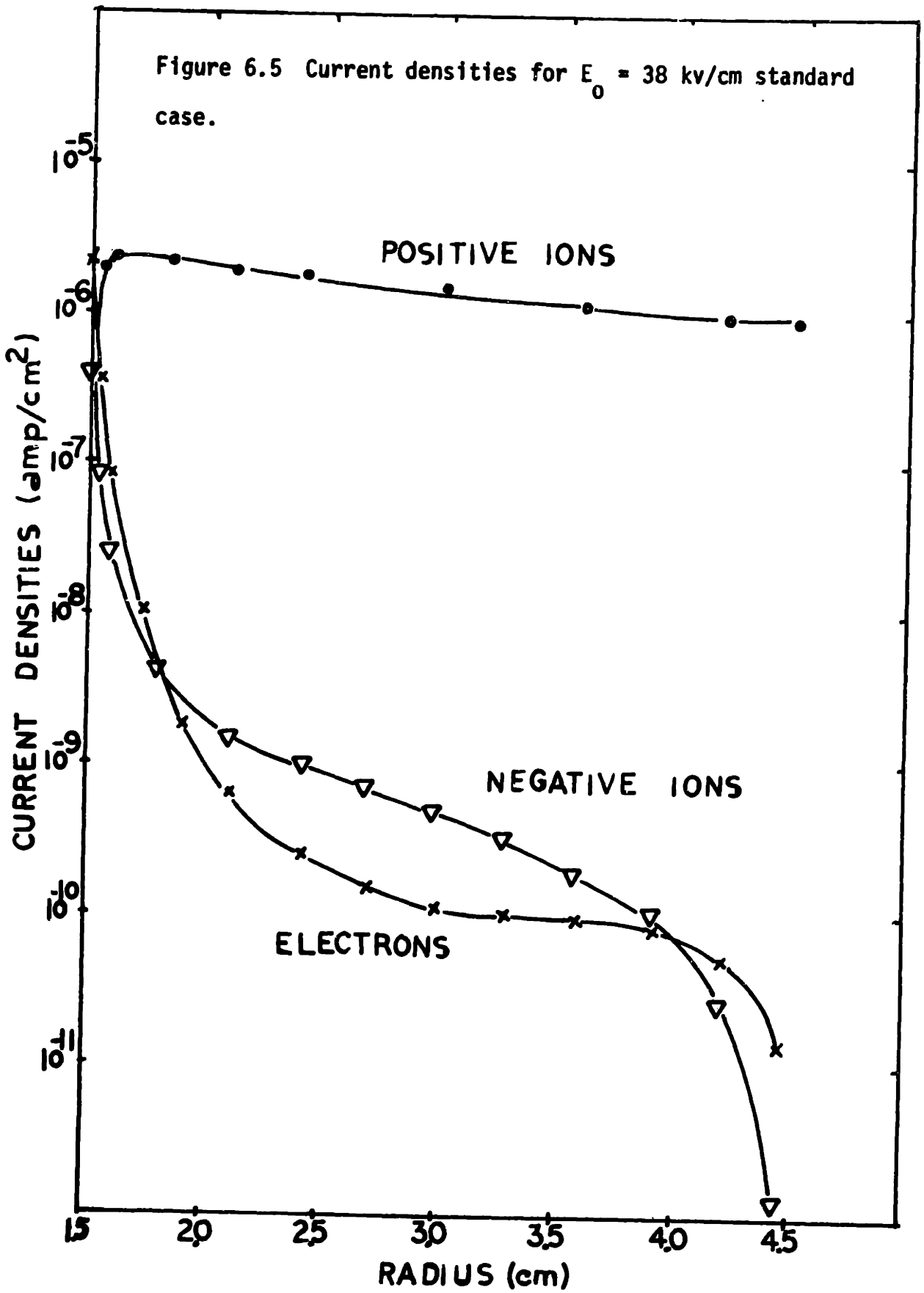


Figure 6.5 Current densities for E_0 = 38 kv/cm standard case.



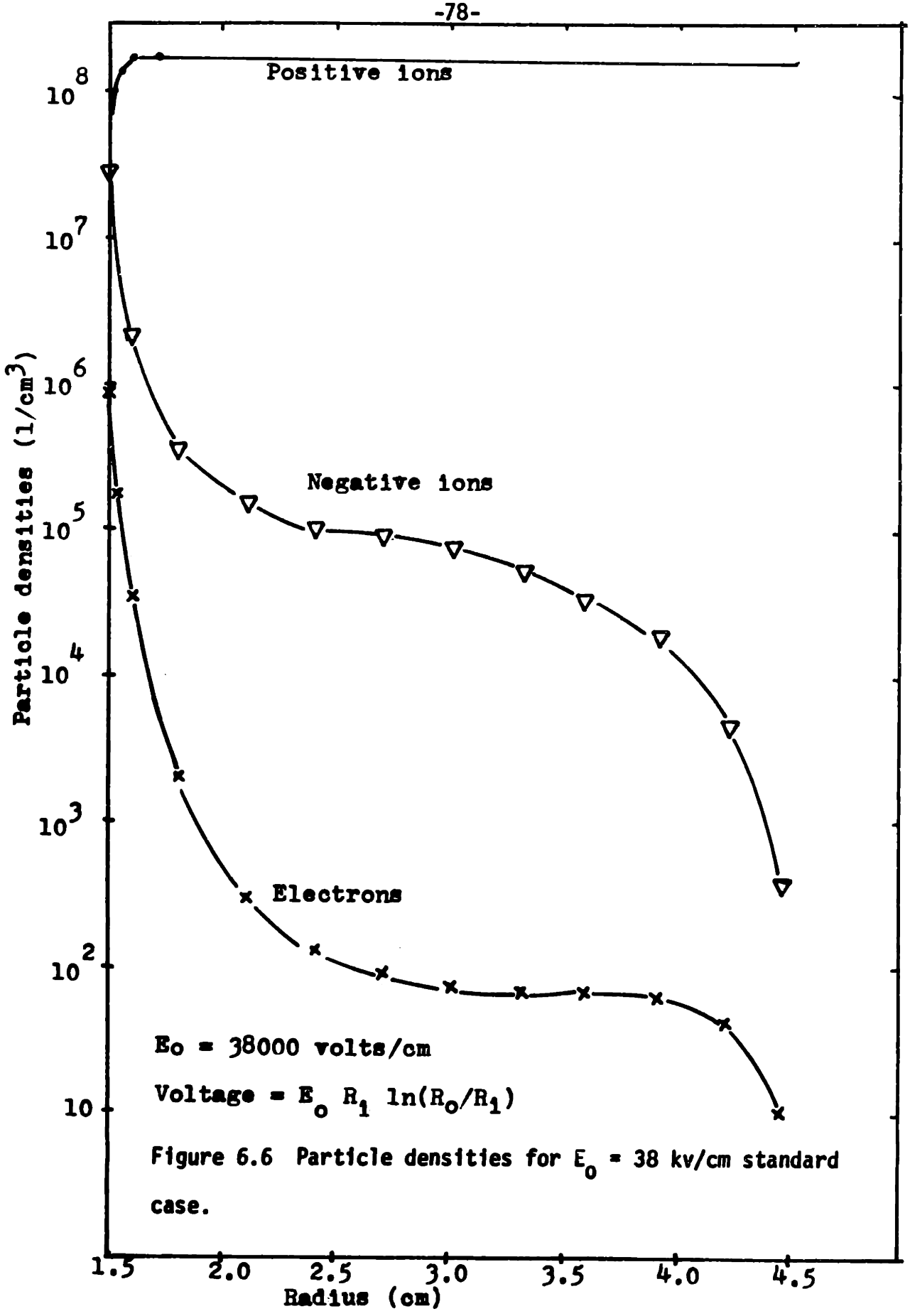


Figure 6.6 Particle densities for E_0 = 38 kv/cm standard case.

densities by amny orders of ten.

The fact that the currents stayed at some finite value after onset means that the space charge has modified the electric field to such an extent the current could not grow unbounded. Below onset the charge densities are too low. The electric field remained as a function of K/r. Figure 6.7 shows the change of the electric field from the K/r field as a function of position for four separate applied voltages. The decrease of electric field near the conductor is known as choking. The deviation is larger for higher applied voltage.

6.2 Electron Current Density Boundary Condition

The standard case had zero electron current density as a boundary condition on the outer radius. If a small electron current density is assumed, the onset and the current densities above onset are unchanged. The current densities below onset level off to a relatively constant value as voltage is decreased. For comparison, see cases 2 and 3 in Figure 6.1.

6.3 Numerical Accuracy

The numerical method used is accurate only when the grid spacing is made very small. The main source of error introduced is in the integration in the high E/p region near the conductor, and possibly in the calculation of the electric field.

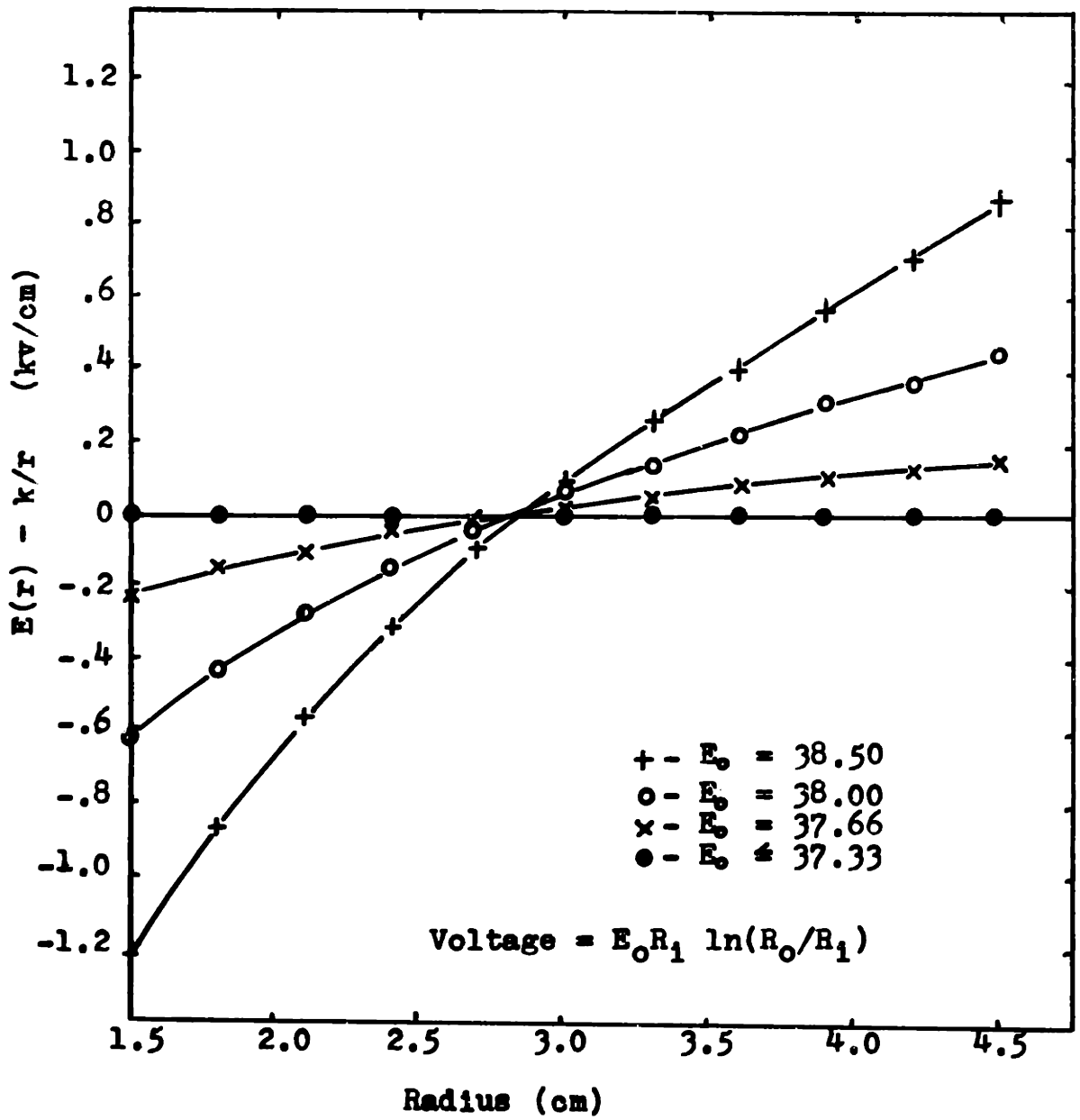


Figure 6.7 Deviation of the electric field from the K/r field as a function of position for different applied voltages.

The current is a very sensitive function of α, and α, in turn, is a very sensitive function of the electric field. Trapazoidal rule is used for numerical integration for the calculation of the electric field and it is only a first order method. This is the only method usable without undue labor.

The test case for accuracy was not performed on the standard case but with the following changes

$$\alpha/p = 8.2 \exp(-\frac{250}{E/p}) \qquad (5)$$

for the whole E/p region, and $\mu_p = .01$ cm^{-1}. For an applied voltage of 60973 volts, equivalent ot an E_o of 37 kv/cm at 1.5 cm radius, the calculated voltage is 120 volts higher than the applied for grid size of .01 cm. When the grid size is decreased by half to .005 cm, the error in voltage calculation also decreased by half to 60 volts above the applied. For the .01 cm grid size, the electric field is in error by at most one place in the fifth digit.

The critical onset electric field calculated is approximately at 36.3 kv/cm for the .005 cm grid size case. However, the .01 cm grid size will be used in all other calculations for a 1.5 cm radius anode because decreasing the grid size by half more than doubled the cost of a calculation.

The way to increase accuracy at the same cost is to have variable grid sizes in r, by making a transform-

ation. Let

$$r = f(x). \tag{6}$$

Pick an appropriate function f, such that equal spacing in x will produce more concentrated grids in r in the region of higher E/p. See Figure 6.8. For this study, however, the main objective was devoted to examining the effect of different parameters. The program in its present state will provide the necessary information.

6.4 Rate of Convergence and Stability*

When the applied voltage results in a surface electric field on the inner conductor at a few kv/cm below the critical onset value, the convergence rate is very fast, requiring only three or four iterations. The rate of convergence becomes slower as the critical electric field is reached. See Figure 6.9. The worst situations occur when E_o is within 1/4 kv/cm above the critical electric field. The climb is slow because the gain is only slightly greater than 1. Sometimes two or three hundred iterations are needed for convergence. It is for these cases, that a relaxation parameter becomes useful. Normally, the relaxation parameter is set to 1. Making it larger than 1, effectively increases the constant Q and

* Material in this section is true for both positive and negative corona cases.

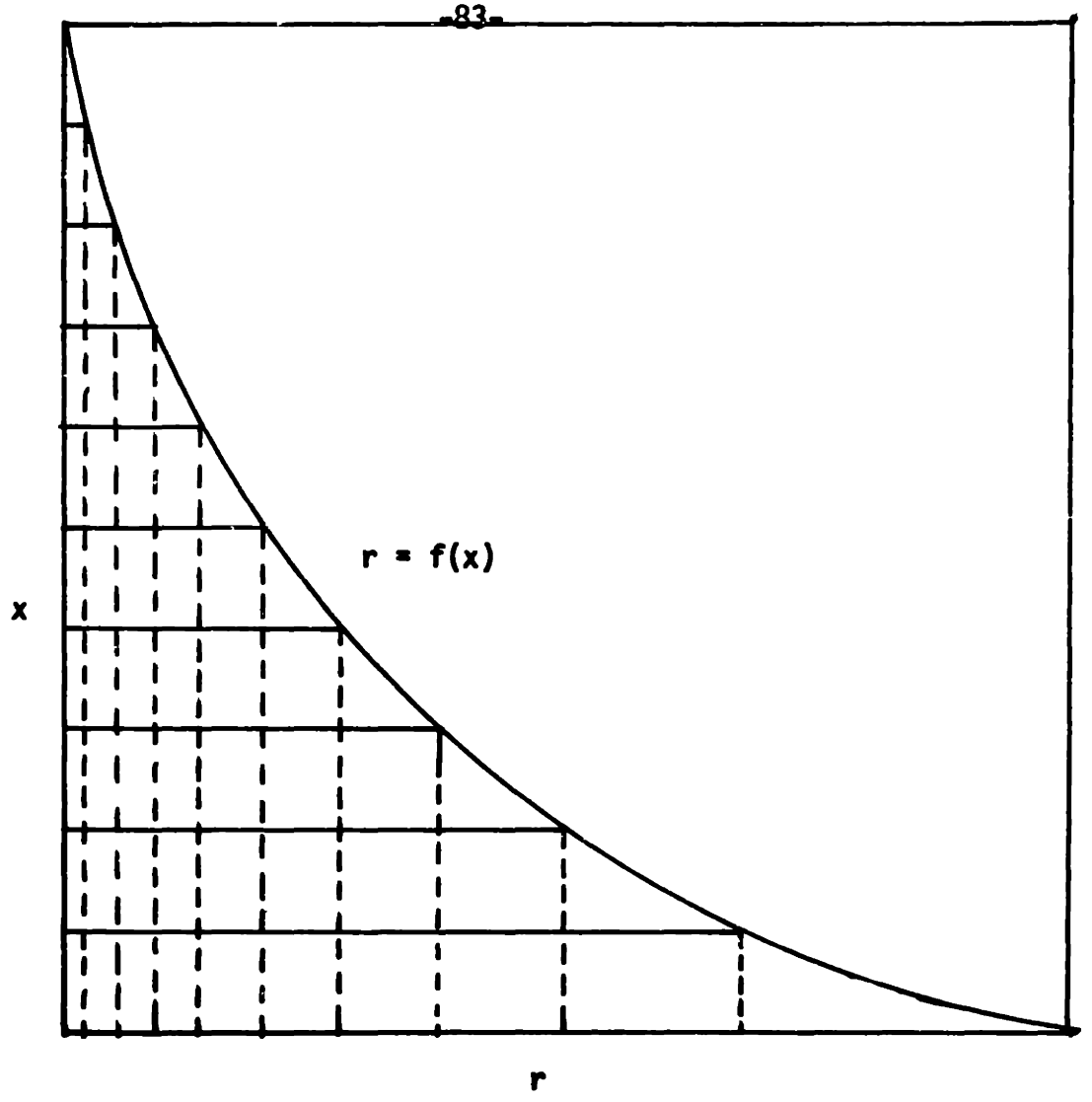


Figure 6.8 With the transformation, r = f(x), equal spacing in x will give variable spacing for r.

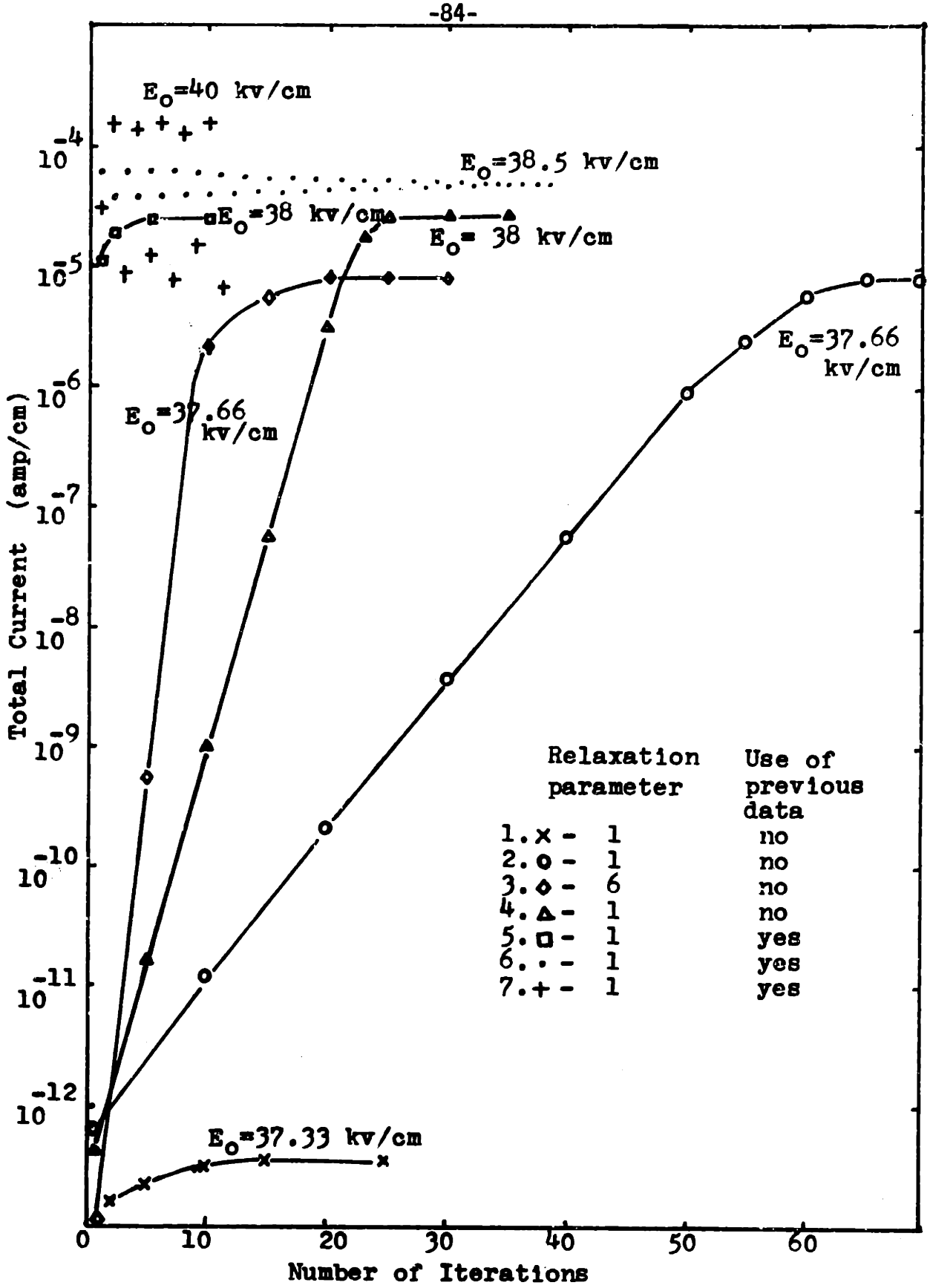


Figure 6.9 Rate of convergence illustrated by the total current at the end of each iteration for the standard case.

and lowers the onset. So in effect, it is the same as moving out of the region of slow convergence. When the current reaches an estimated value, the parameter is set back to 1. Comvergence again becomes slow. Case 2 in Figure 6.9 shows that more than 70 iterations are needed for convergence. At the same applied voltage with the relaxation parameter set at 6 and the current estimate set at 10^{-6} amp/cm, the number of iterations required is reduced by more than half.

Another way to speed up convergence is to use data from the previous case. There are two plots of E_o = 38 kv/cm in Figure 6.9. One started when there is no initial space charges. The number of iterations to reach the final value is large. The other case used the space charge distribution from E_o = 37.667 kv/cm. The results converged in a few iterations.

When the applied voltage is increased even higher so that E_o is in a very narrow neighborhood of 1 kv/cm above the critical electric field, the rate of convergence again becomes very slow. This time the cause is due to problems associated with stability. Look at the case E_o = 38.5 kv/cm in Figure 6.9. The current oscillates about the final value. The reason is that associated with an initially large current there is a large positive space charge distribution, which in turn reduces the electric field. The reduced electric field gives a smaller current

density and space charge density. The electric field near the anode goes back toward the value of unperturbed electric field. The process repeats. Each time the current and the electric field get a little closer to the final value.

If the applied voltage is increased to an even larger value, the solution diverges. The equations try to over stabilize the situation. The current oscillates further and further apart as the iterations are increased. See the case E_o = 39 kv/cm in Figure 6.9.

6.5 Effect of R_o

Peek observed that the size of the outer conductor did not change the critical onset values. This is confirmed by changing the radius of the outer cylinder from 4.5 cm, to 5.5 cm and to 7.5 cm. See Figure 6.1. However, the size of the outer cylinder does effect the electric field and the magnitude of the current after onset. Figure 6.10 is a graph of the current densities when the outer cylinder radius is 7.5 cm. The shape of the curves are very similar to the standard case. Figure 6.11 shows the effect of space charges on the electric field for different outer cylinder radii with voltages set such that E_o is 38.5 kv/cm. Increasing the radius of the outer cylinder decreases the current above onset. It is consistant with the electric field, because

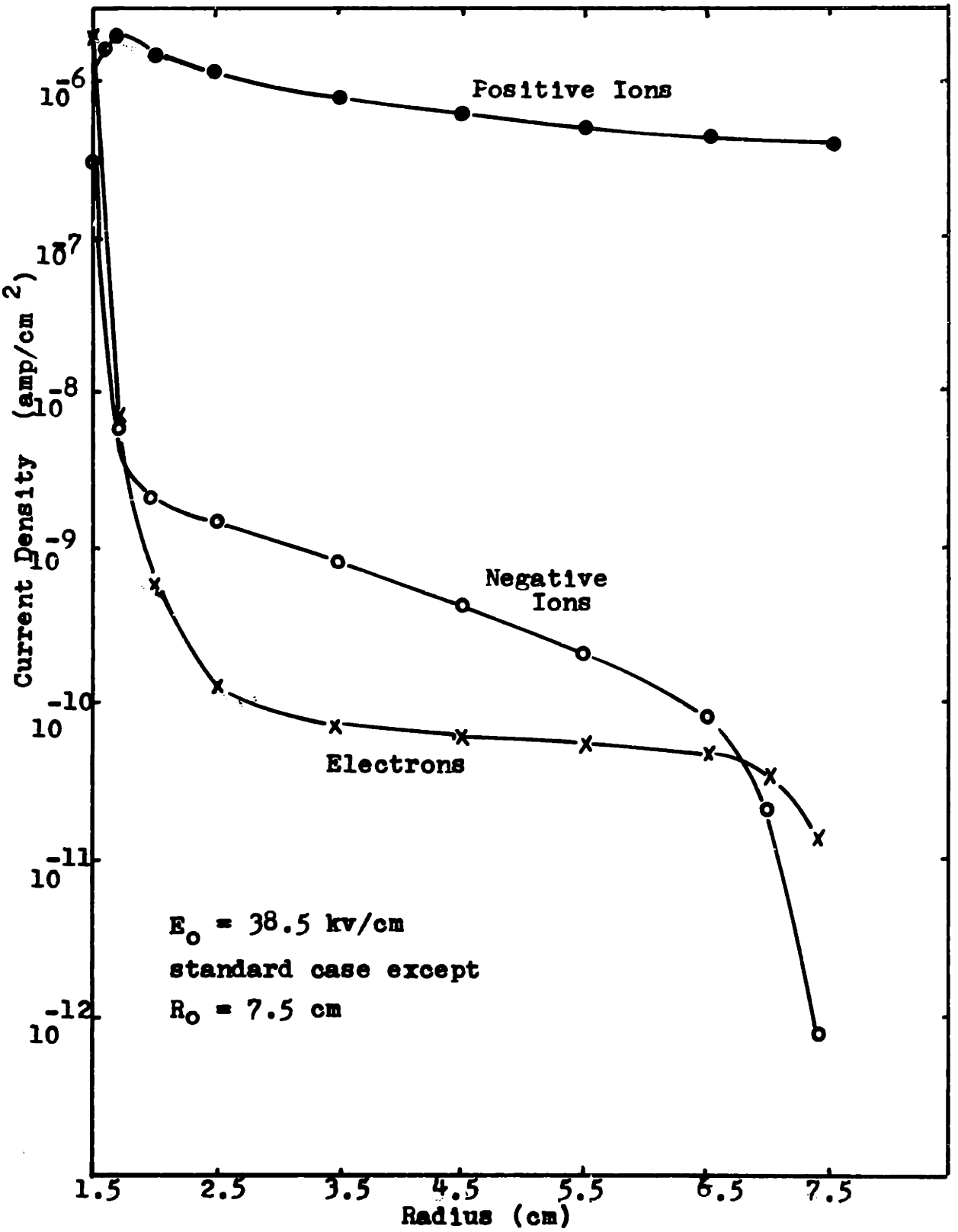


Figure 6.10 Current density when the radius of the outer cylinder is changed to 7.5 cm.

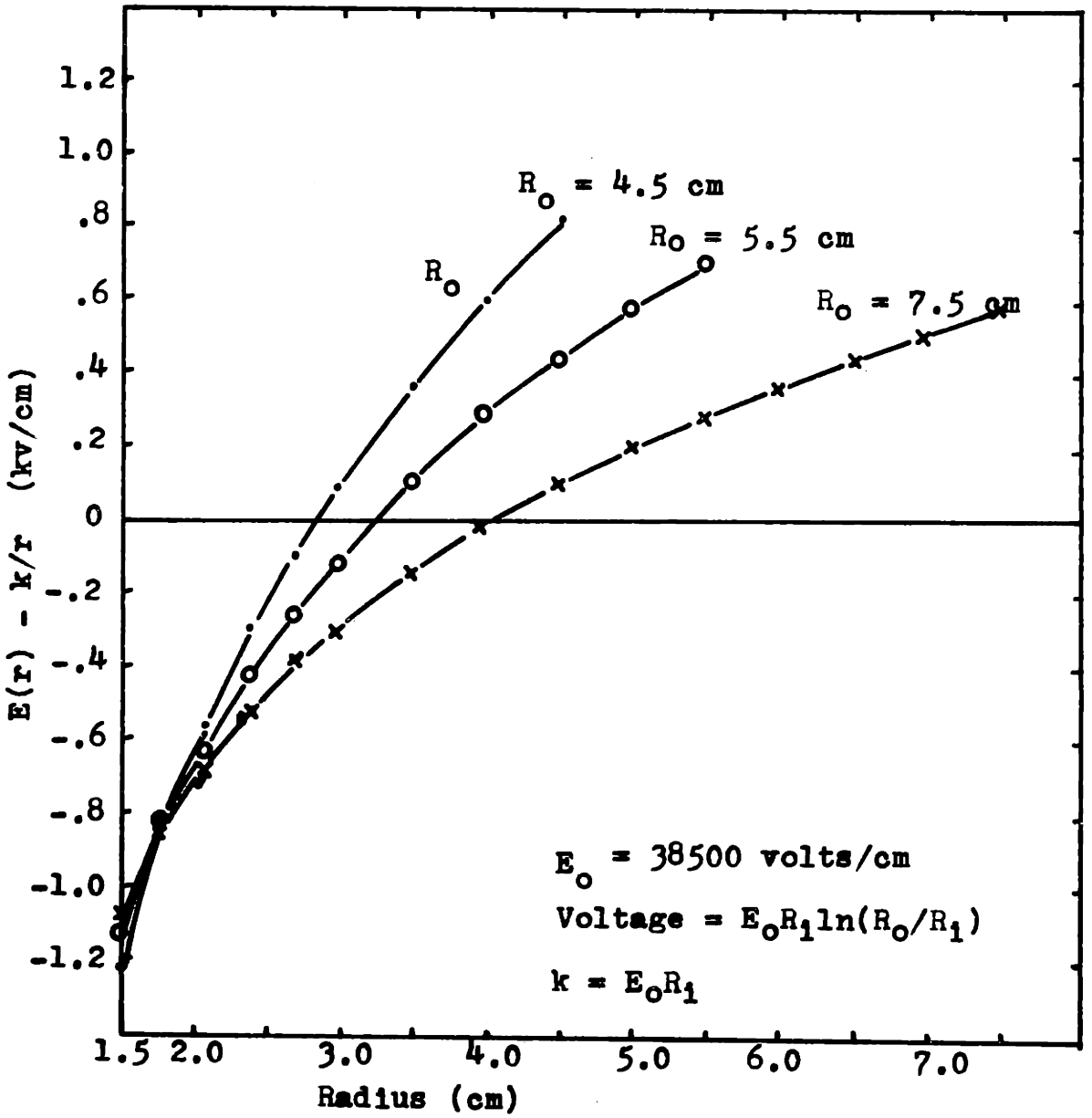


Figure 6.11 The effect of the dimension of the outer cylinder on the electric field.

larger outer cylinder radii results in a larger region where the electric field fell below the K/r field near the anode.

6.6 Modifying Alpha

The value of α is very important and its effects show up in almost every aspect. Let alpha decreases by 5% for all E/p. The onset increases by about ½ kv. This is expected without even going through the calculations, because decreasing the α curve by 5% is the same as moving the curve to the higher E/p region, in this case to the right, by approximately ½ kv.

In the region of $E/p < 40$ volts/cm-mm Hg, the data on α measured by different experimentalists do not have good agreement. One would like to see how important are these variations. The first case was done with the parameters of the standard case except $\mu_p = .01$ cm^{-1}. The onset is about 35.8 kv/cm. Then α is fitted by

$$8.2p \exp(-250p/E) \qquad (7)$$

For all E/p. This lowers the value of α for the region $E/p < 40$ volts/cm-mm Hg. With μ_p still set at .01 cm^{-1}, the onset increases to 36.3 kv/cm. There is about half a kilovolt difference. In section 6.12, another effect as a result of modifying α will also be pointed out.

6.7 Modified η

Attachment is not only a process of increasing the negative ion current density. It also effectively decreases α. If attachment is increased by 10% for all the E/p values, the onset increases from 37.5 kv/cm of the standard case to 37.75 kv/cm. See case 6 of Figure 6.1.

Compare the current densities of Figure 6.12 to Figure 6.5. They both have the same applied voltage. The increase in attachment did not increase the negative ions by any noticable amount relative to electron current density. The effect is just moving all current densities lower due to a net decrease in free electrons.

The experimental variations on η are more than $\pm 10\%$ of the fitted curve. That means the onset could be more than $\frac{1}{2}$ kv/cm off just due to uncertainties in η. This error is on the same order of magnitude as the others observed so far.

6.8 Modifying μ_p and Q

The absoption rate of photons by gas molecules is not a very well defined quantity. It depends on the energy of the photons, which in turn depends on the value of E/p and the type of gase molecules. If μ_p is increased by a factor of 10 to .01 cm^{-1}, the critical onset field changes from 37.5 kv/cm of the standard case to 36.3 kv/cm. See

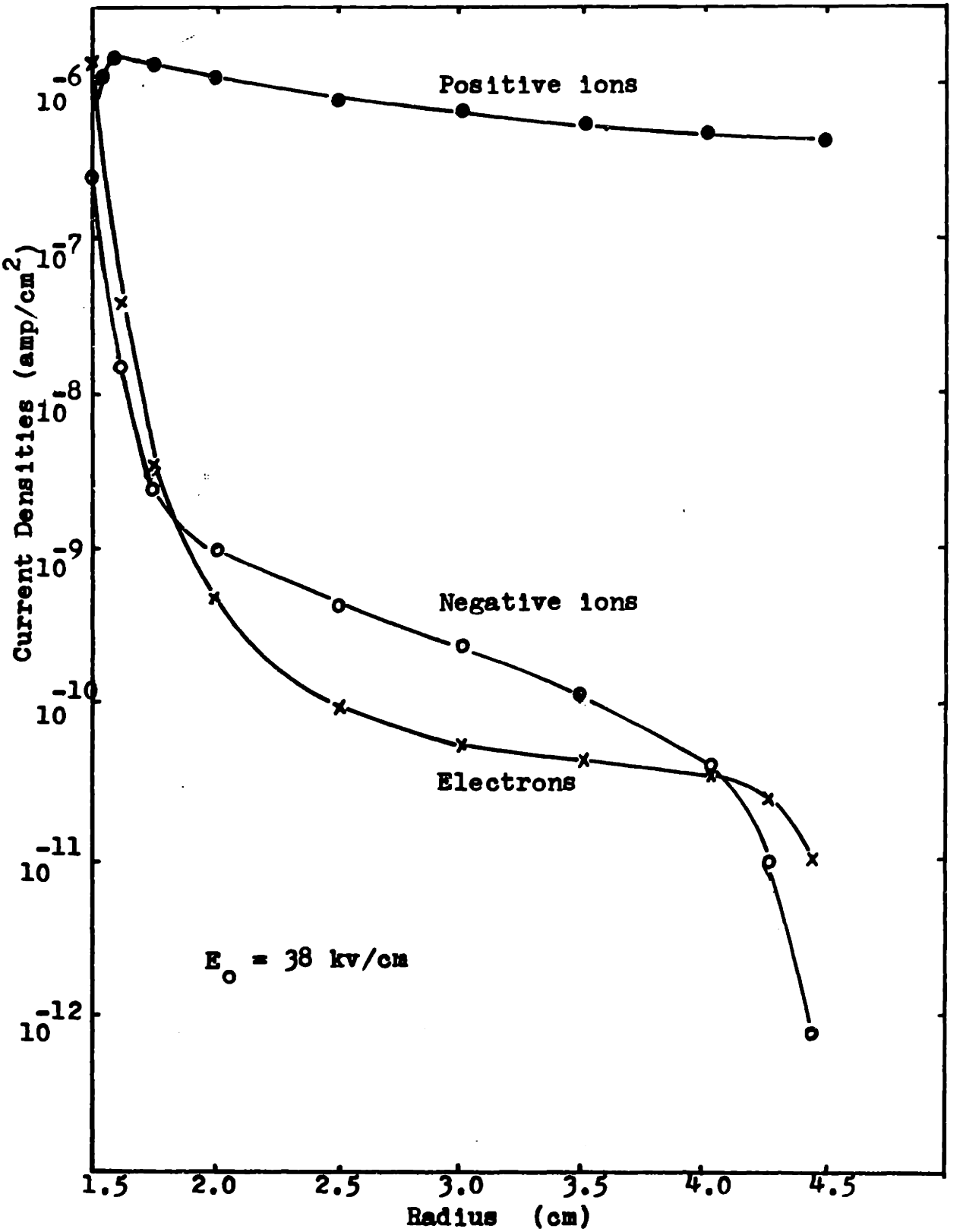


Figure 6.12 The effect of increasing the attachment coefficient for all E/p by 10% on the current densities.

Figure 6.1. For the large variations in μ_p as shown in Figure 3.5, this can be one of the most important factor on the critical onset electric field.

Figure 6.13 shows the calculated photoionization term as a function of position. The shape between μ_p=.01 cm^{-1} and μ_p=.001 cm^{-1} is not very different and stayed relatively constant over most of the region. The reason is that the photon density decays according to

$$\exp(-\mu_p r) \qquad (8)$$

and $\mu_p r \ll 1$ in both cases. If μ_p is a larger number, the photoionization will have a noticable peak close to the anode.

6.9 Mobility

The velocities of charged particles are a function of the humidity in air. The change of velocity of the electron is unimportant because its charge density is many orders smaller than both positive and negative ions.

In dry air the mobility of positive ions is 2.85 cm^2/sec-volt. The mobility decreases steadily for humid air. For water vapor the ion mobility is as low as .8 cm^2/sec-volt. See Figure 6.1 for the result when mobility of both positive and negative ions are set to .8 cm^2/sec-volt. The critical onset field did not change, but the current after onset decreased in value. A smaller mobility means a greater charge density, which in turn modifies the

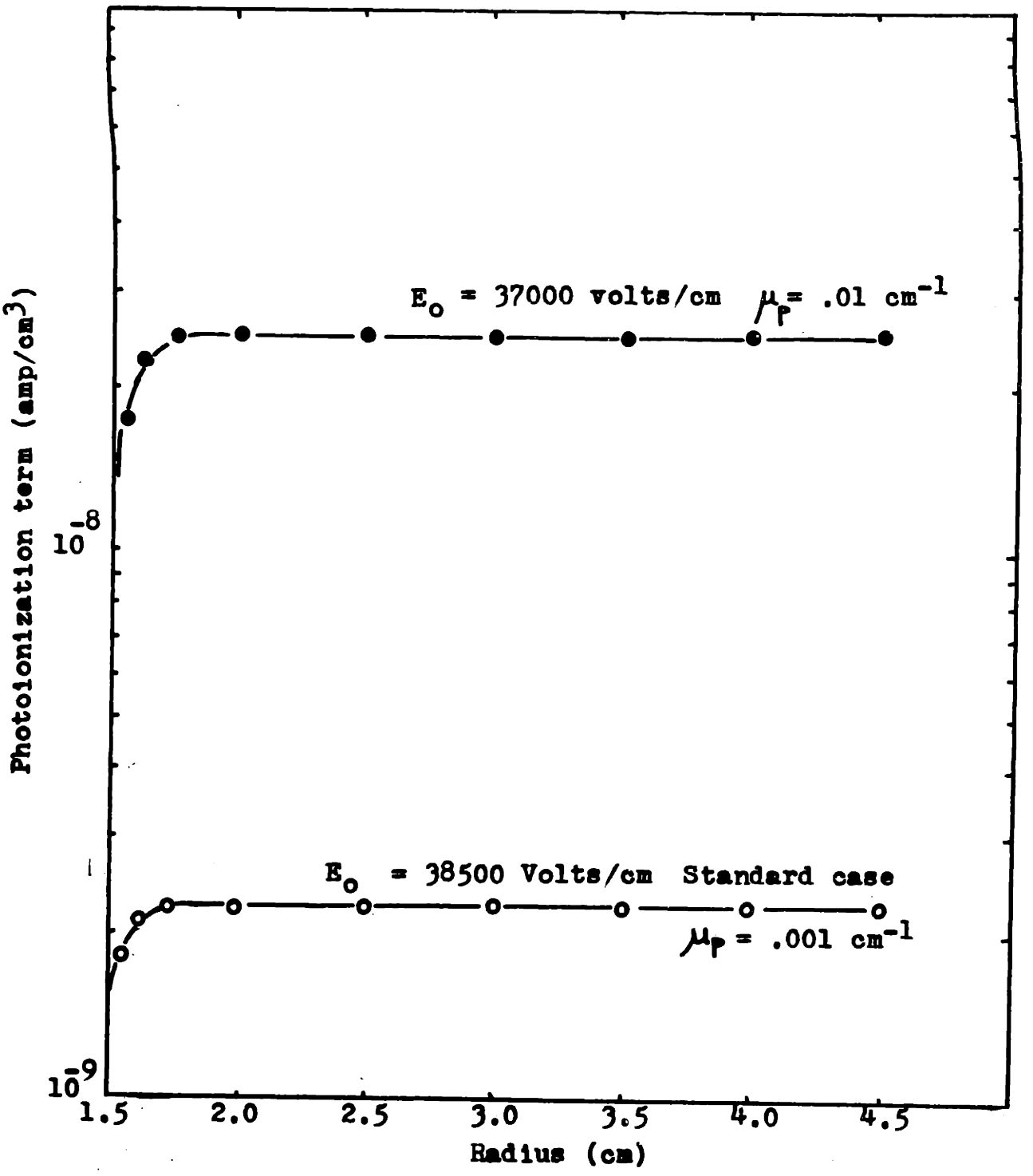


Figure 6.13 The photoionization term as a function of position.

electric field more. The final solution shows that the electric field differed from the case mobility is 2.5 cm^2/sec-volt only in the sixth significant figure. However, it is enough to reduce the total current by 60% for some voltages.

6.10 Recombination

When there is recombination, the continuity equations become

$$\nabla \cdot \vec{J}_e = -(\alpha - \eta) J_e - 10q - Ph \qquad (9)$$

$$\nabla \cdot \vec{J}_- = -\eta J_e + \beta \rho_- \rho_+ / q \qquad (10)$$

$$\nabla \cdot \vec{J}_+ = \alpha J_e + 10q + Ph - \beta \rho_- \rho_+ / q \qquad (11)$$

where β is the recombination constant of positive and negative ions.

$$\beta = 2.3 \times 10^{-6} \quad cm^3/sec. \qquad (12)$$

Figure 6.14 has plots of each of the terms on the right hand side of the continuity equations for the standard case with E_o= 38 kv/cm. The recombination term is much smaller than all other terms. Numerical results in Figure 6.1 confirm that recombination is unimportant.

6.11 Diffusion

Diffusion, another primary process mentioned, will also be proven to be unimportant. The current due to diffusion has the form

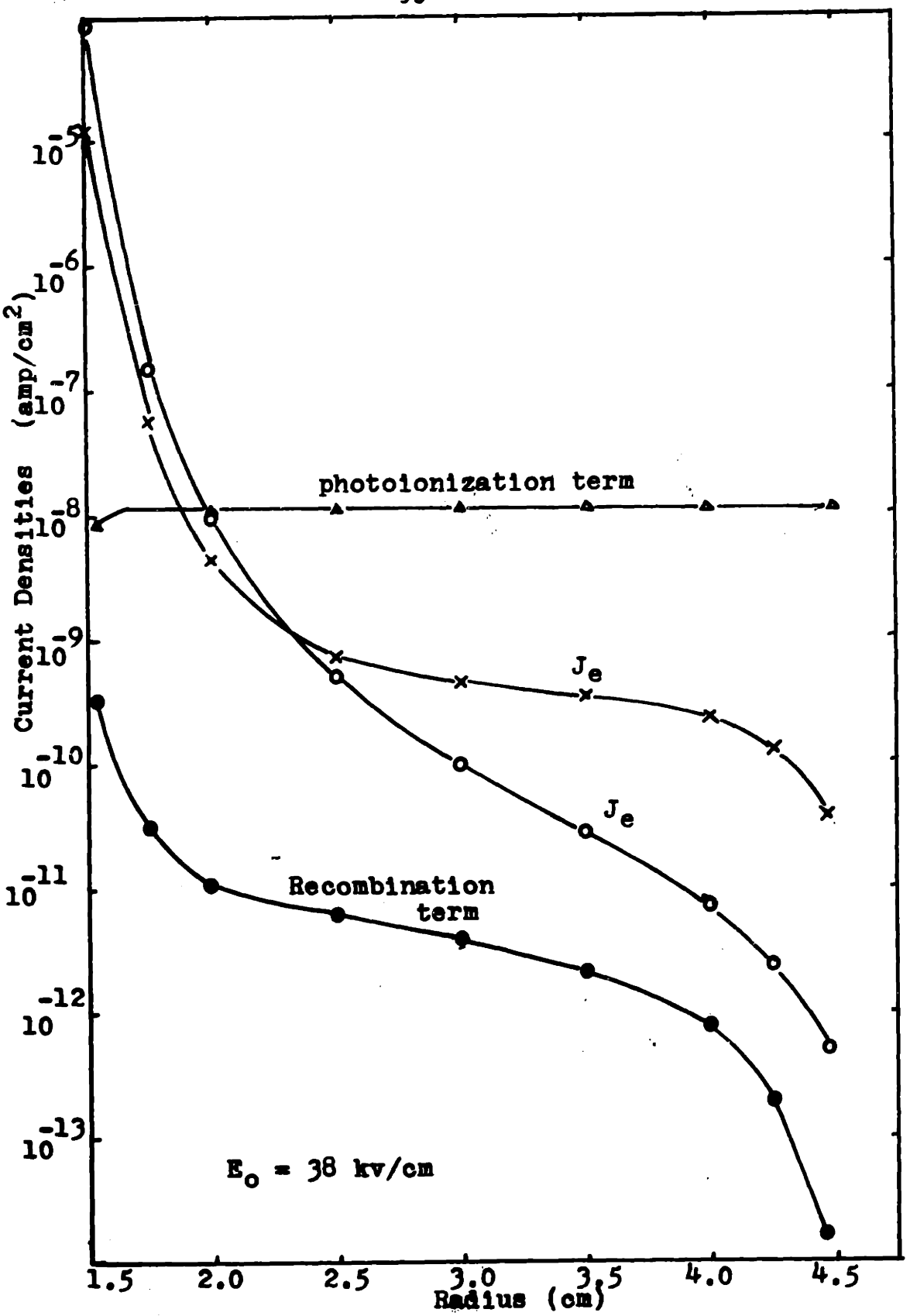


Figure 6.14 Comparing the recombination term to other terms.

$$\vec{J} = -(D_e \nabla \rho_e + D_+ \nabla \rho_+ + D_- \nabla \rho_-). \qquad (13)$$

The steepest gradients of all the charge densities occur at the surface of the anode. With

$$D_e = 400 \quad cm^2/sec \qquad (14)$$
$$D_- = .043 \quad cm^2/sec$$
$$D_+ = .03 \quad cm^2/sec$$

The diffusion current at the largest is only about 5×10^{-11} amp/cm length of wire, for the standard case at $E_o = 38$ kv/cm. This is much smaller than the calculated total current, 2.5×10^{-5} amp/cm of length.

6.12 Variations on Anode Size and Comparison of Results to Peek's Formula

A set of voltage versus current curves for 2.0 cm, 1.0 cm, .5 cm and .2 cm anode radius with all other input parameters the same as the standard case, are plotted in Figures 6.18, 6.17, 6.16 and 6.15 respectively. Their onset values are plotted with Peek's formula in Figure 6.19. The agreement is poor. In section 5.4, the calculations showed that the curves will agree better if α had a steeper slope as function of E/p. With μ_p fixed at .01 cm^{-1} one case was tested for .2 cm radius anode with

$$\alpha/p = 8.2 \exp(-250p/E). \qquad (15)$$

Along with the point at 1.5 cm radius, extrapolations show that the general shape agrees with Peek's formula

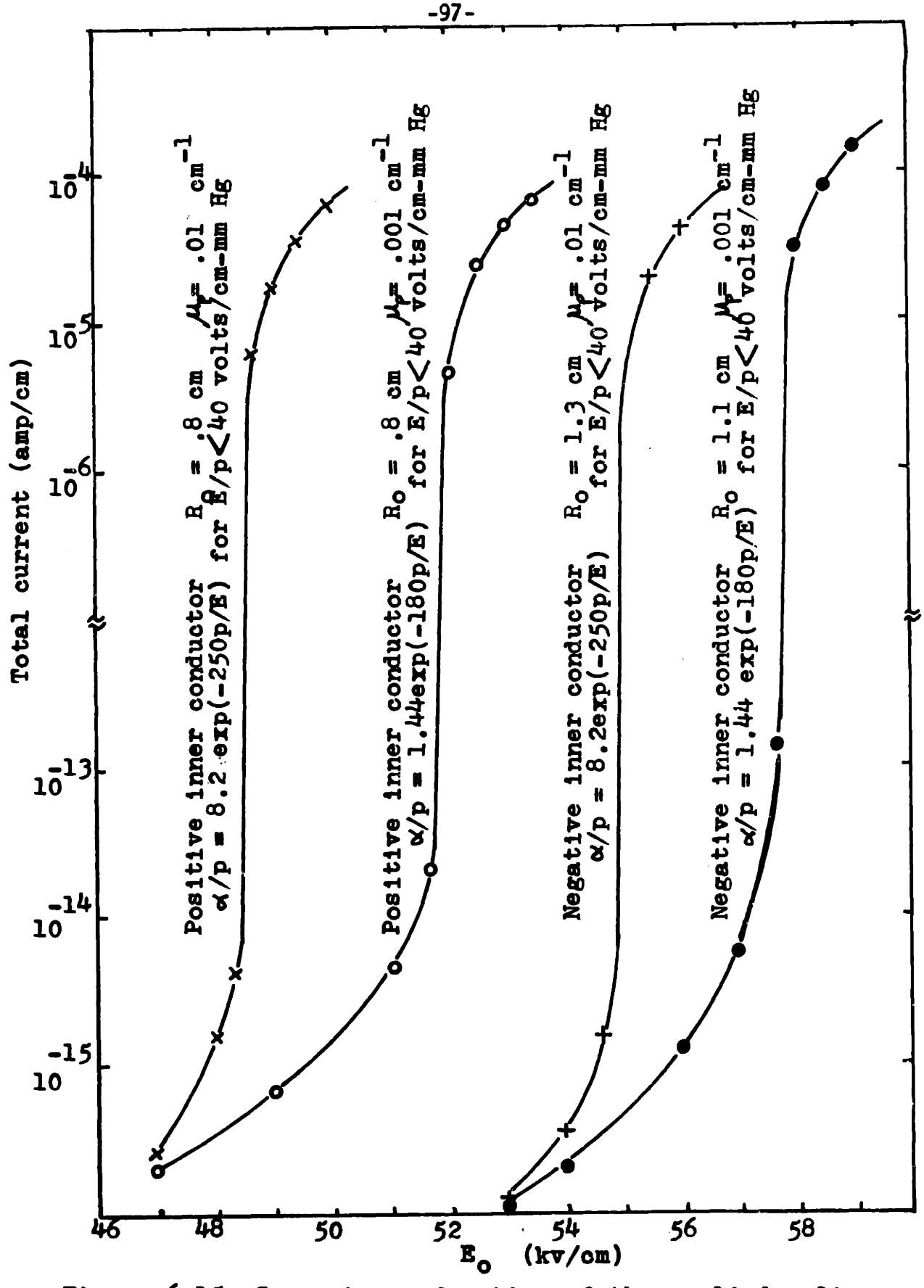


Figure 6.15 Current asa function of the applied voltage when the radius of the inner conductor is .2 cm.

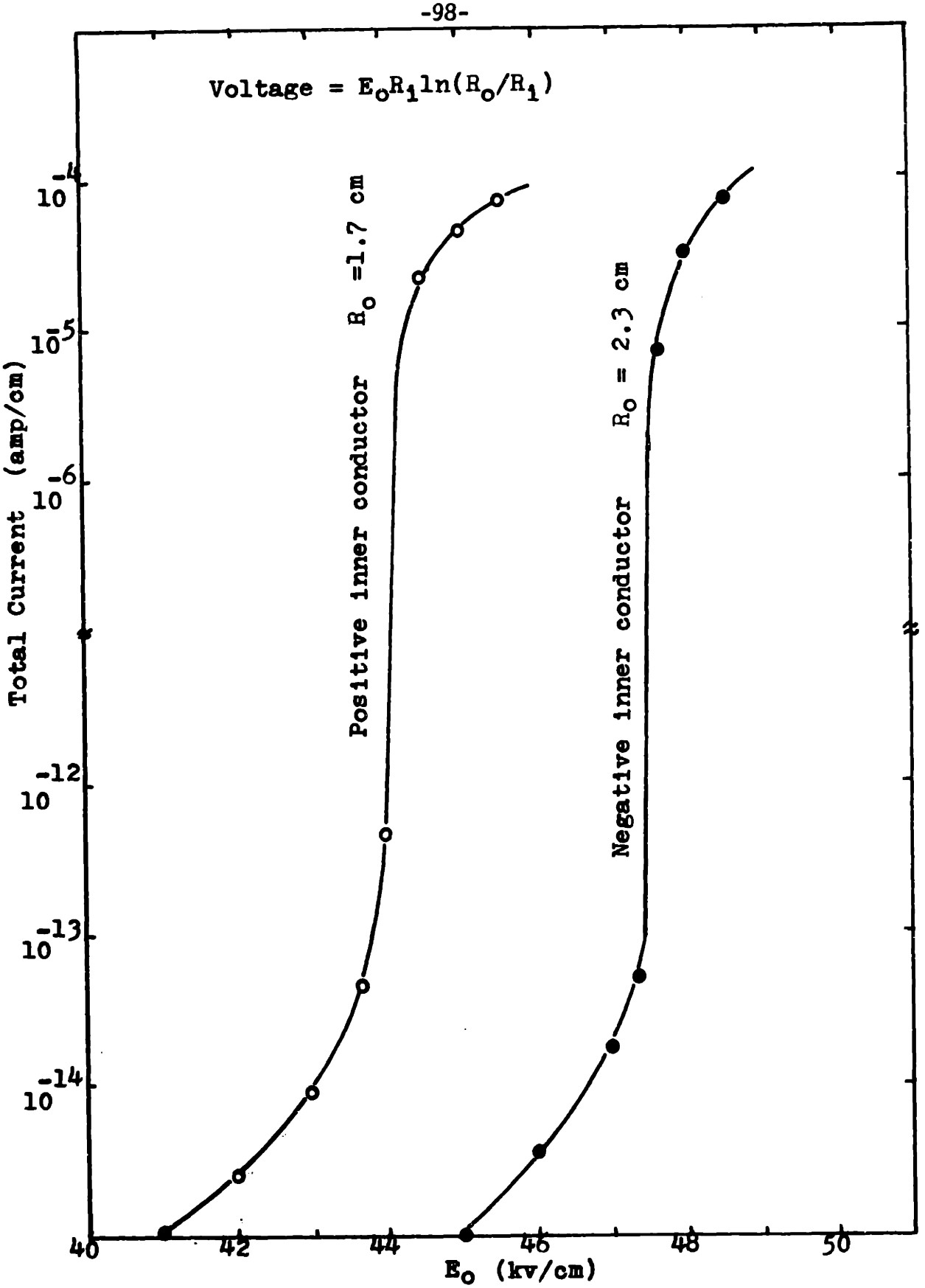


Figure 6.16 Current as a function of the applied voltage when the radius of the inner conductor is .5 cm.

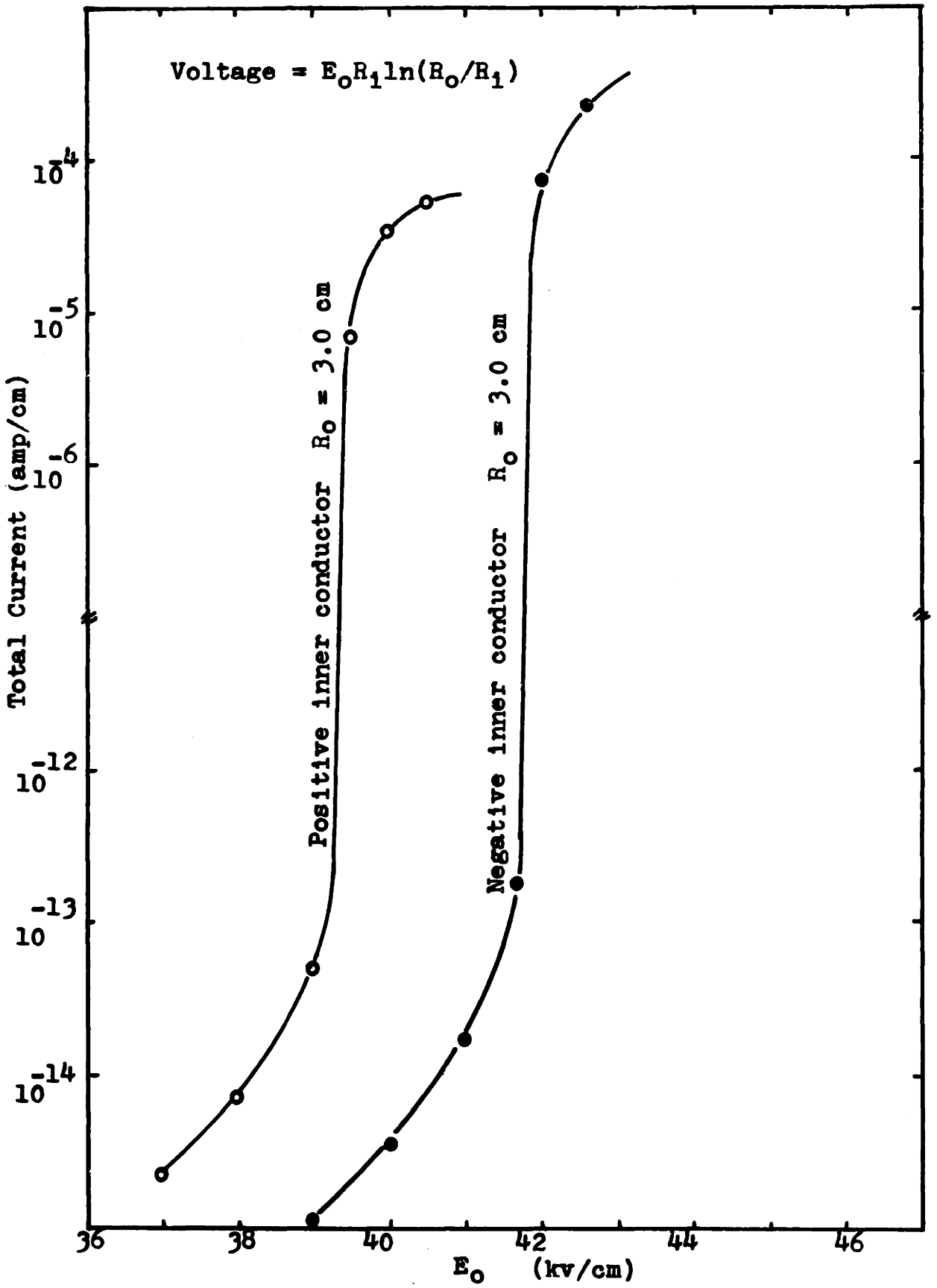


Figure 6.17 Current as a function of the applied voltage when the radius of the inner conductor is 1.0 cm.

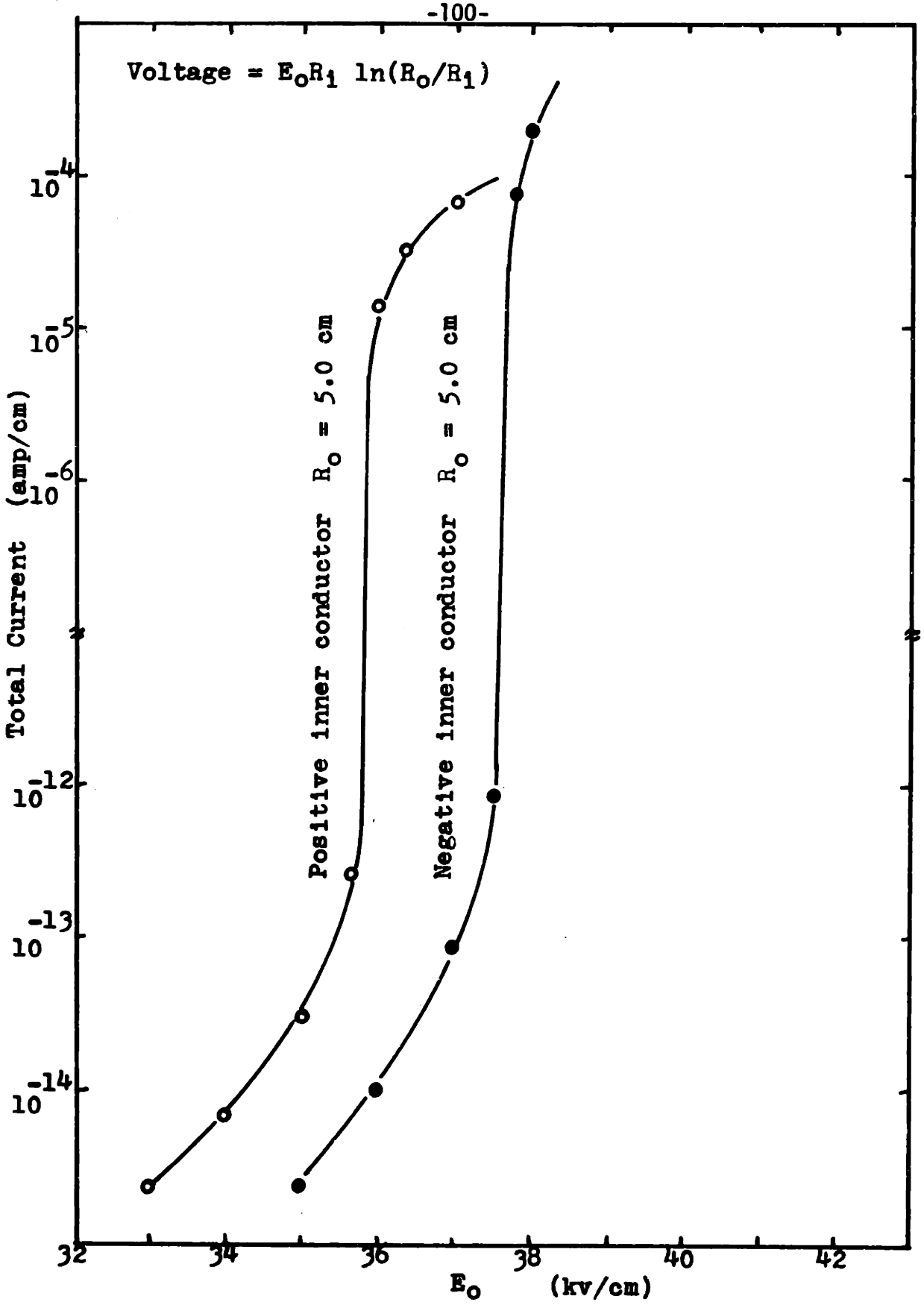


Figure 6.18 Current as a function of the applied voltage when the radius of the inner conductor is 2.0 cm

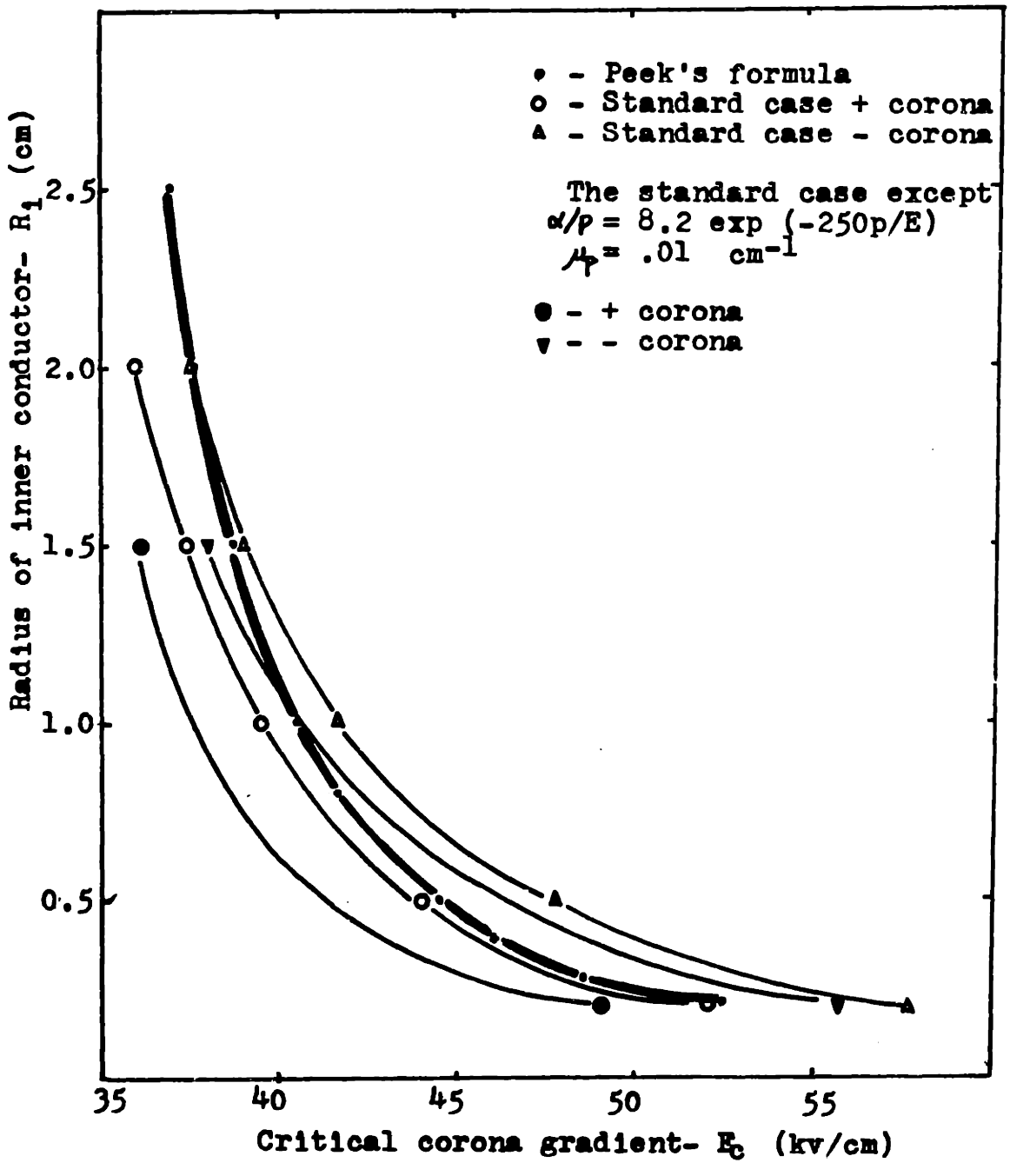


Figure 6.19 Comparison of results to Peek's Formula

but lower values for critical onset field.

If α is assumed to be correct. A curve similar to Peek's could also be obtained if μ_p is allowed to have larger values for smaller anode. Figure 3.5 shows that μ_p is indeed a strongly increasing function of E/p, and supports the above assumption.

7. Model for Negative Corona

7.1 Derivation of Equation without Feedback

All the assumptions stated in the beginning of the anode corona section hold for the cathode corona case as well. The geometry is the same, but the polarity of the voltage is switched. Consequently

$$\vec{E}(r) = E(r)\hat{1}_r = -|E(r)|\,\hat{1}_r \tag{1}$$

$$\vec{J}(r) = J(r)\hat{1}_r = -|J(r)|\,\hat{1}r \tag{2}$$

where $|E(r)|$ and $|J(r)|$ are magnitudes of electric field and of current densities respectively.

Without any kind of feedback mechanisms, the derivation of the continuity equations follow the same approach as in the positive corona case. See Figure 7.1.

$$\begin{aligned} r\Delta r(\nabla\cdot\vec{J}_e) &= (r+\Delta r)|J_e(r+\Delta r)|\,(-\hat{1}_r)\cdot(\hat{1}_{n1}) \\ &\quad + r|J_e(r)|\,(-\hat{1}_r)\cdot(\hat{1}_{n2}) \\ &= r|J_e(r)| - (r+\Delta r)|J_e(r+\Delta r)| \end{aligned} \tag{3}$$

$$\begin{aligned} |J_e(r)| &= |-\rho_e(r)\,\mu_e E(r)| \\ &= q\,n_e(r)\,\mu_e|E(r)| \end{aligned} \tag{4}$$

To first order

$$r\Delta r(\nabla\cdot\vec{J}_e) = q\,\mu_e|E(r)|(r\,n_e(r) - (r+\Delta r)n_e(r+\Delta r)) \tag{5}$$

Due to Townsend ionization and attachment processes,

$$(r+\Delta r)n_e(r+\Delta r) - r\,n_e(r) = (\alpha(r) - \eta(r))n_e(r)r\Delta r.$$

So (6)

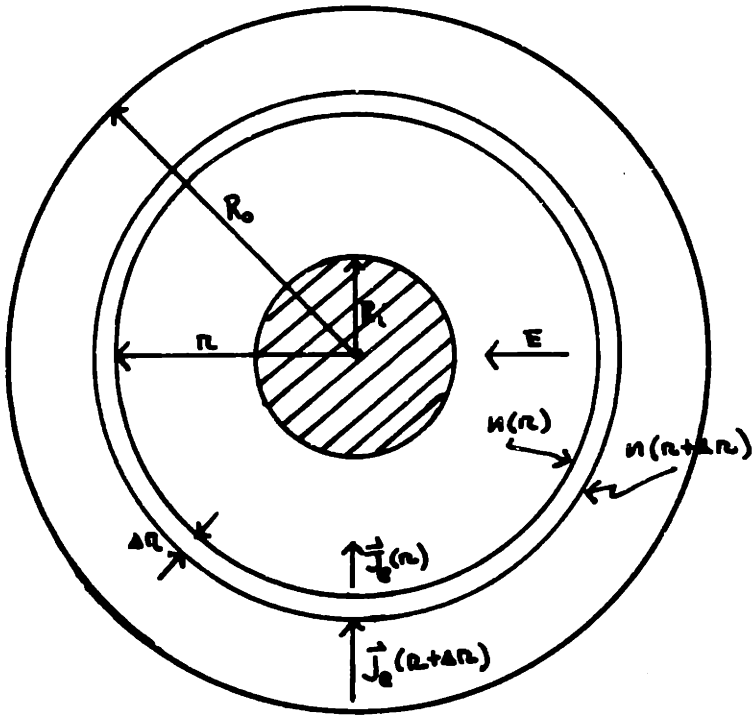


Figure 7.1 The diagram used in the explaination of the derivation for the electron continuity equation for negative corona.

$$r\Delta r(\nabla\cdot\vec{J}_e) = -q\mu_e|E(r)|(\alpha(r) - \eta(r))n_e(r)r\Delta r \tag{7}$$
$$= -(\alpha(r) - \eta(r))|J_e(r)|r\Delta r.$$

The continuity equations are, therefore,

$$\nabla\cdot\vec{J}_e = -(\alpha - \eta)|J_e|, \tag{8}$$
$$\nabla\cdot\vec{J}_- = -\eta|J_e|, \tag{9}$$
$$\nabla\cdot\vec{J}_+ = \alpha|J_e|, \tag{10}$$

and these equations are also applicable to the positive corona case.

The boundary conditions for electron and negative current densities has to be set at the surface of the inner conductor and positive ion current density set at the outer cylinder.

Rewriting the current densities in the form

$$\vec{J} = J(r)\hat{i}_r, \tag{11}$$

the continuity equations reduce to

$$\frac{d}{dr}(rJ_e) = (\alpha - \eta)(rJ_e), \tag{12}$$

$$\frac{d}{dr}(rJ_-) = \eta(rJ_e), \tag{13}$$

$$\frac{d}{dr}(rJ_+) = -\alpha(rJ_e). \tag{14}$$

Boundary conditions can be written as

$$J_e(R_i) = J_{eo}, \tag{15}$$
$$J_-(R_i) = 0, \tag{16}$$
$$J_+(R_o) = J_{+o}. \tag{17}$$

7.2 The Exact Solution

The current densities can be solved easily.

$$J_e(r) = \frac{R_i J_{eo}}{r} \exp \int_{R_i}^{r} (\alpha(r') - \eta(r'))\, dr' \qquad (18)$$

where $0 < R_i < r' < r < R_o$.

$$J_-(r) = \frac{R_i J_{eo}}{r} \int_{R_i}^{r} \eta(r'') \exp\left(\int_{R_i}^{r''} (\alpha(r') - \eta(r'))dr' \right) dr'' \qquad (19)$$

where $0 < R_i < r' < r'' < r < R_o$

$$J_+(r) = \frac{R_i J_{eo}}{r} \int_{r}^{R_o} \alpha(r'') \exp\left(\int_{R_i}^{r''} (\alpha(r') - \eta(r'))dr' \right) dr'' + \frac{R_o J_{+o}}{r} \qquad (20)$$

where $0 < R_i < r < r' < r'' < R_o$.

Note that the current densities are not the same as the positive corona case. Even though at first glance, they look very similar. The electron current density is not necessarily a monitonically increasing function of radius. If r is taken far enough so that attachment dominates, $J_e(r)$ will start to grow slower and finally decrease. However, $J_-(r)$ is a monitonically increasing function of radius and $J_+(r)$ increases as radius decreases.

The total current

$$I = 2\pi \left[R_o J_{+o} + R_i J_{eo} \int_{Ri}^{R_o} (r'') \exp \int_{R_i}^{r''} (\alpha(r') - \eta(r'))dr'dr'' \right]. \qquad (21)$$

The same type of approximation as for the positive corona case are employed. Take R_o to where α is the same as η. Let

$$\alpha/p = A \exp(-\frac{B}{E/p}) \tag{22}$$

and set J_{+o} and attachment coefficient to zero.

$$I = 2\pi R_1 J_{eo} \left[\exp(\frac{AK}{B}(\exp(-\frac{BpR_1}{K}) - \exp(-\frac{B}{32})) - 1 \right] \tag{23}$$

This is the same as the positive corona case except for the constant in front.

7.3 With Cosmic Radiation

$$\frac{d}{dr}(rJ_e) = (\alpha - \eta)(rJ_e) - 10qr. \tag{24}$$

$$\frac{d}{dr}(rJ_-) = \eta(rJ_e). \tag{25}$$

$$\frac{d}{dr}(rJ_+) = -\alpha(rJ_e) + 10qr. \tag{26}$$

The solutions are

$$J_e(r) = \frac{-10q}{rS(r)} \int_{R_1}^{r} r'S(r')dr' + \frac{R_1 J_{eo} S(R_1)}{rS(r)}. \tag{27}$$

$$J_-(r) = \frac{-10q}{r} \int_{R_1}^{r} \frac{\eta(r'')}{S(r'')} \int_{R_1}^{r''} r'S(r')dr'\, dr'' + \frac{R_1 J_{eo} S(R_1)}{r} \int_{R_1}^{r} \frac{\eta(r'')}{S(r'')}\, dr''. \tag{28}$$

$$J_+(r) = \frac{10q}{r}\int_{R_o}^{r}\frac{\alpha(r'')}{S(r'')}\int_{R_o}^{r''} r'S(r')dr'\,dr'' + \frac{J_{+o}R_o}{r} \qquad (29)$$

$$- \frac{R_1 J_{eo} S(R_1)}{r}\int_{R_o}^{r}\frac{\alpha(r'')}{S(r'')}dr'' + \frac{10q}{r}\int_{R_o}^{r} r''dr''$$

where $S(r) = \exp\int(\alpha(r') - \eta(r'))dr'$. (30)

The total current

$$I = -2\pi(10q)\int_{R_1}^{R_o} r''\,dr'' \qquad (31)$$

$$+ 10q\int_{R_1}^{R_o}\frac{\alpha(r'')}{S(r'')}\int_{R_1}^{r''} r'S(r')dr'dr''$$

$$+ 2\pi R_1 J_{eo} S(R_1)\int_{R_1}^{R_o}\frac{\alpha(r'')}{S(r'')}dr'' + 2\pi R_o J_{+o}.$$

Now make the same assumptions as in section 4.4 when calculating total current. Then

$$I = -(10q)(R_o^2 - R_1^2) \qquad (32)$$

$$+ 10q\int_{R_1}^{R_o} Ape^{xp}(-\frac{Bpr''}{K})\exp(-\frac{AK}{B}\exp(-\frac{Bpr''}{K}))\cdot$$

$$\int_{R_1}^{r''} r'\exp(\frac{Ak}{B})\exp(-\frac{Bpr'}{K})\,dr'\,dr''$$

$$+ 2\pi R_1 J_{eo}\left[\exp(\frac{AK}{B}(\exp(-\frac{BpR_1}{K}) - \exp(-\frac{B}{J2}) - 1\right] + 2\pi R_o J_{+o}.$$

7.4 Photoionization Term

The photioionization term to be added into the continuity equation cannot be just the term added for positive corona. For negative corona, the distribution of electron creation as a function of radius is

$$\alpha(r)J_e(r) = \frac{R_i J_{eo}}{r}\alpha(r)\exp\int_{R_i}^{r}(\alpha(r')-\eta(r'))dr' \qquad (33)$$

It is no longer true that most ionization by alpha process occur at the inner cylinder surface; and that implies the maximum photon creation too has moved away from the inner cylinder surface. At the same time the important secondary electrons are the ones photoionized near the cathode surface. Therefore the creation of electrons by photons coming from outside the cylinder of radius r will have to be taken into account also. See Appendix 2.

Now the continuity equations with cosmic radiation and the photionization term looks like

$$\frac{d}{dr}(rJ_e) = (\alpha-\eta)(rJ_e) - 10q - Ph, \qquad (34)$$

$$\frac{d}{dr}(rJ_-) = \eta(rJ_e), \qquad (35)$$

$$\frac{d}{dr}(rJ_+) = -\alpha(rJ_e) + 10q + Ph, \qquad (36)$$

where

$$Ph = \mu_p Q\int_{R_i}^{r} G(r')\alpha(r')|J_e(r')|r'\exp(-\mu_{eff}(r-r'))dr' \qquad (37)$$

$$+ \mu_p Q \int_r^{R_o} G'(r,r') \alpha(r') \left|J_e(r')\right| r' \exp(-\mu_{eff}(r'-r))dr'$$

where

$$G'(r,r') = 1 - \frac{1}{\pi}\cos^{-1}\left(\frac{2r^2 - r'^2}{r'^2}\right) + GG(r'), \qquad (38)$$

$$GG(r') = \frac{\theta_{max}}{2\pi} - \frac{1}{\pi}\tan^{-1}\left[\left(\frac{r'+r}{r'-r}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right], \qquad (39)$$

$$G(r') = \frac{\theta_{max}}{2\pi} + \frac{1}{\pi}\tan^{-1}\left[\left(\frac{r+r'}{r-r'}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right]. \qquad (40)$$

$G'(r,r')$ is a function of both r and r'. Therefore, for each new grid point r_j, the second integral has to be evaluated completely from scratch. One way to get around this difficulty is to try to take the r dependence outside the integral. This can be accomplished by approximating

$$1 - \frac{1}{\pi}\cos^{-1}\left(\frac{2r^2 - r'^2}{r'^2}\right) \qquad (41)$$

by a polynomial of r/r'. A straight line is good enough for purposes here. One possible choice is

$$.93\left(\frac{r}{r'}\right) - .112 \qquad (42)$$

See Figure 7.2 for comparison. The photionization term now looks like

$$Ph = \mu_p Q \int_{R_1}^{r} G(r') \alpha(r') \left|J_e(r')\right| r' \exp(-\mu_{eff}(r-r'))dr' \qquad (43)$$

$$+ \mu_p Q \int_r^{R_o} GG(r')\,\alpha(r')\left|J_e(r')\right| r' \exp(-\mu_{eff}(r'-r))\,dr'$$

$$- .112\mu_p Q \int_r^{R_o} \alpha(r')\left|J_e(r')\right| r' \exp(-\mu_{eff}(r'-r))\,dr'$$

$$+ .93\mu_p Qr \int_r^{R_o} \alpha(r')\left|J_e(r')\right| \exp(-\mu_{eff}(r'-r))\,dr'$$

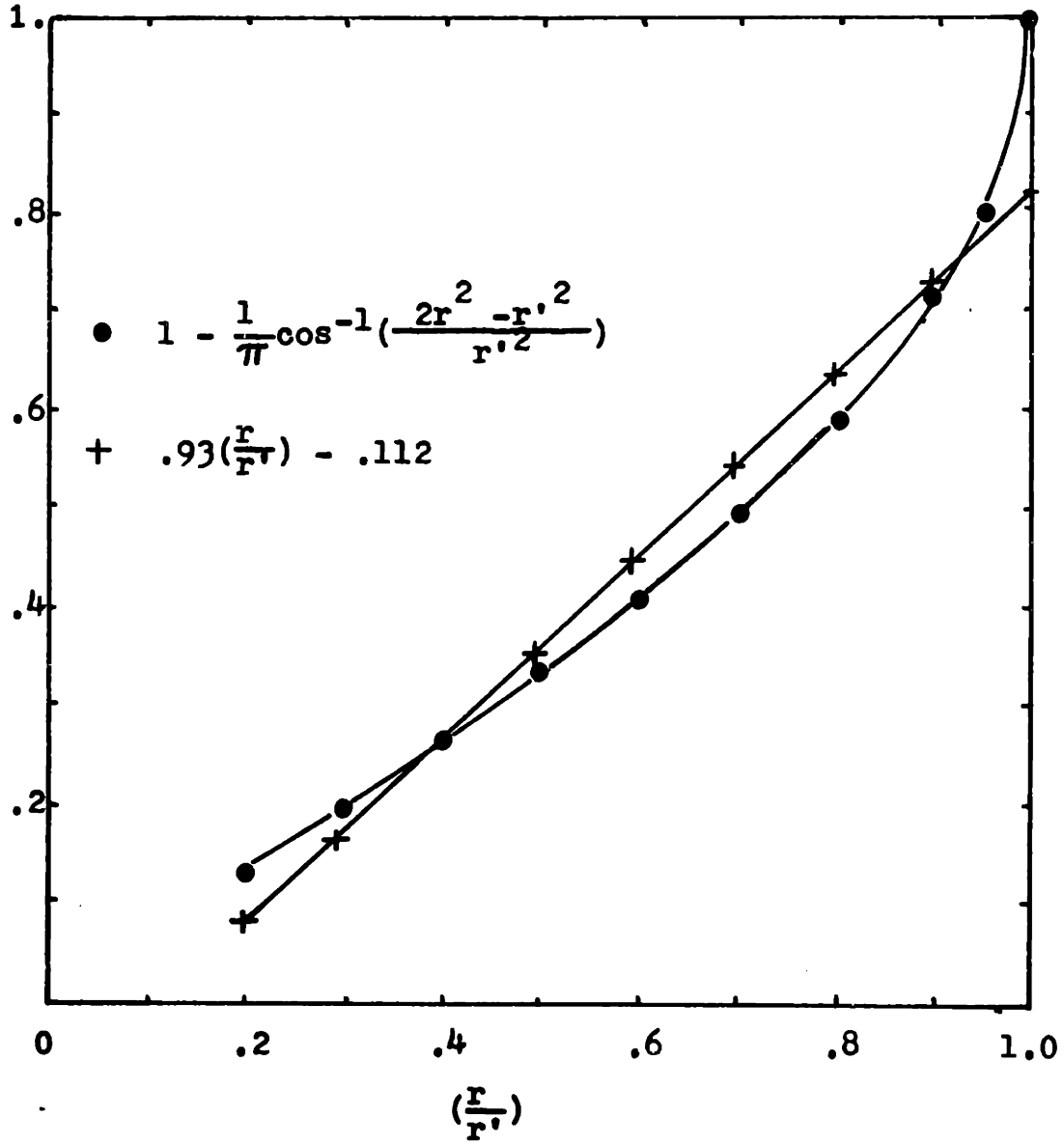


Figure 7.2 Approximating a term of the geometric factor by a linear function of $(\frac{r}{r'})$

8. Results for Negative Corona

8.1 A Detailed Analysis of the Standard Case

The shape of the voltage versus current curve has the basic characteristics as the curves for positive corona. The critical onset electric field for the standard case is at E_o approximately equal to 39.4 kv/cm, higher than that of the positive corona. This phenomenon is expected if photoionization is the only secondary mechanism. All secondary electrons for positive corona could be effective ionizers. While for negative corona only the secondary electrons produced in the region $\alpha > \eta$ could be effective ionizers because electrons are moving from a high electric field into a low electric field.

Figure 8.2 shows the current density for the case E_o = 39 kv/cm, below the onset. Starting with no electrons at the surface of the cathode, the electron current density increases very sharply. The peak occurs where $\alpha = \eta$. In the region where α is important, the positive ion current density dominates, while in the region where attachment is greater than α, the negative ions dominate.

When the voltage is increased above onset until E_o is 40 kv/cm, the current density still has the same shape except the magitude has increased by factor of 10^9. In general for the identical configuration, the current for negative corona is larger than that of positive corona

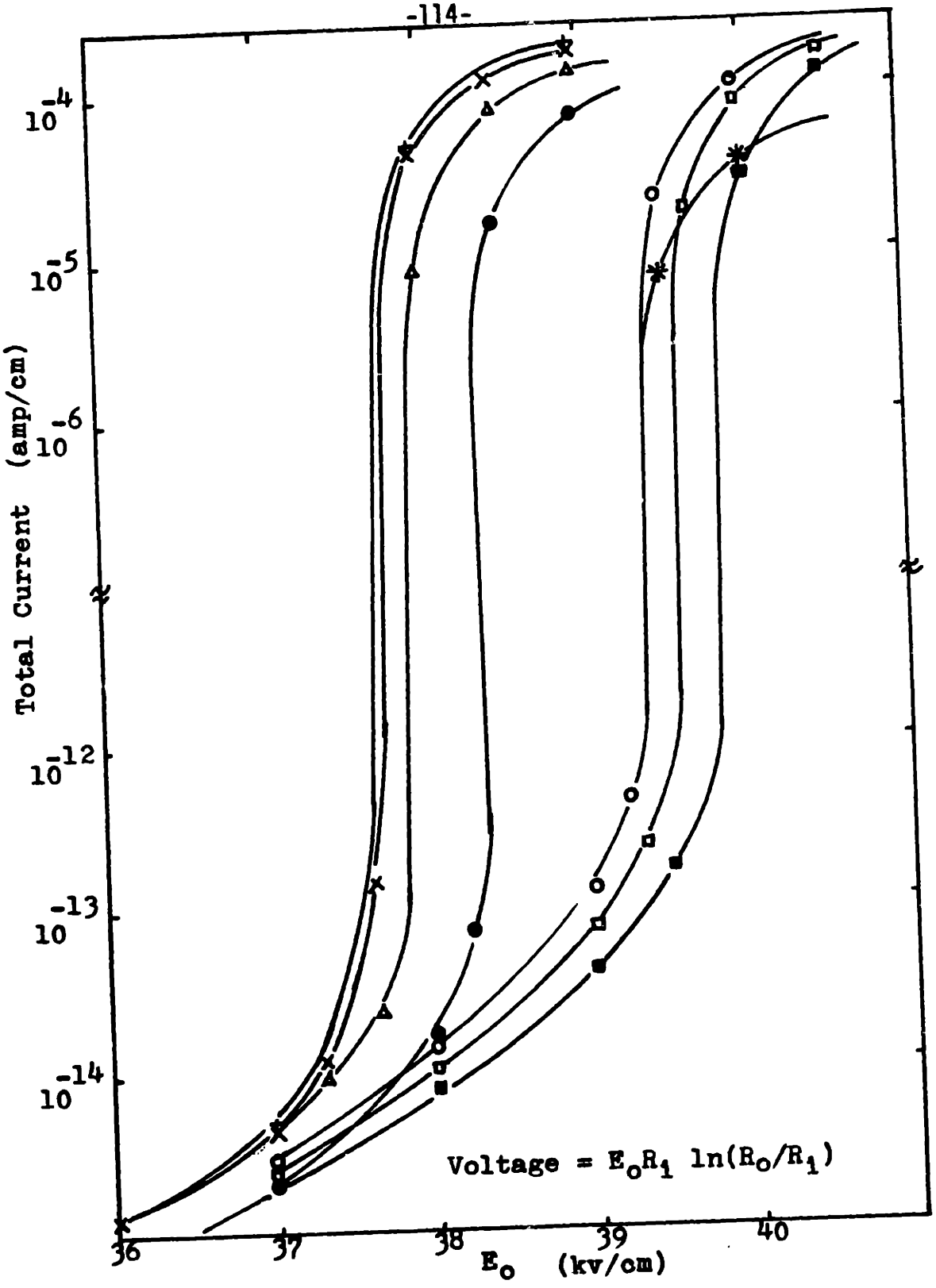


Figure 8.1 Sensitivity tests when the applied voltage on the 1.5 cm inner conductor is negative with respect to the outer conductor. See Table 8.1 for reference.

Table 8.1

Explaination for the Figure 8.1

1. -o- Standard case with or without recombination term

The following are the same as the standard case with the exception specified below.

2. -■- α is decreased by 5% for all E/p

3. -□- η is increased by 10% for all E/p

4. -△- $\mu_p = .01 \text{ cm}^{-1}$

5. -●- $\mu_p = .01 \text{ cm}^{-1}$. /p=8.2exp(-250p/E) for E/p 40 volts/cm-mm Hg

6. -✱- $\mu_- = .8 \text{ cm}^2/\text{sec/volt}$

$\mu_+ = .8 \text{ cm}^2/\text{sec/volt}$

7. -×- $\gamma_i = .0001$

8. -+- $\gamma_p = .001$

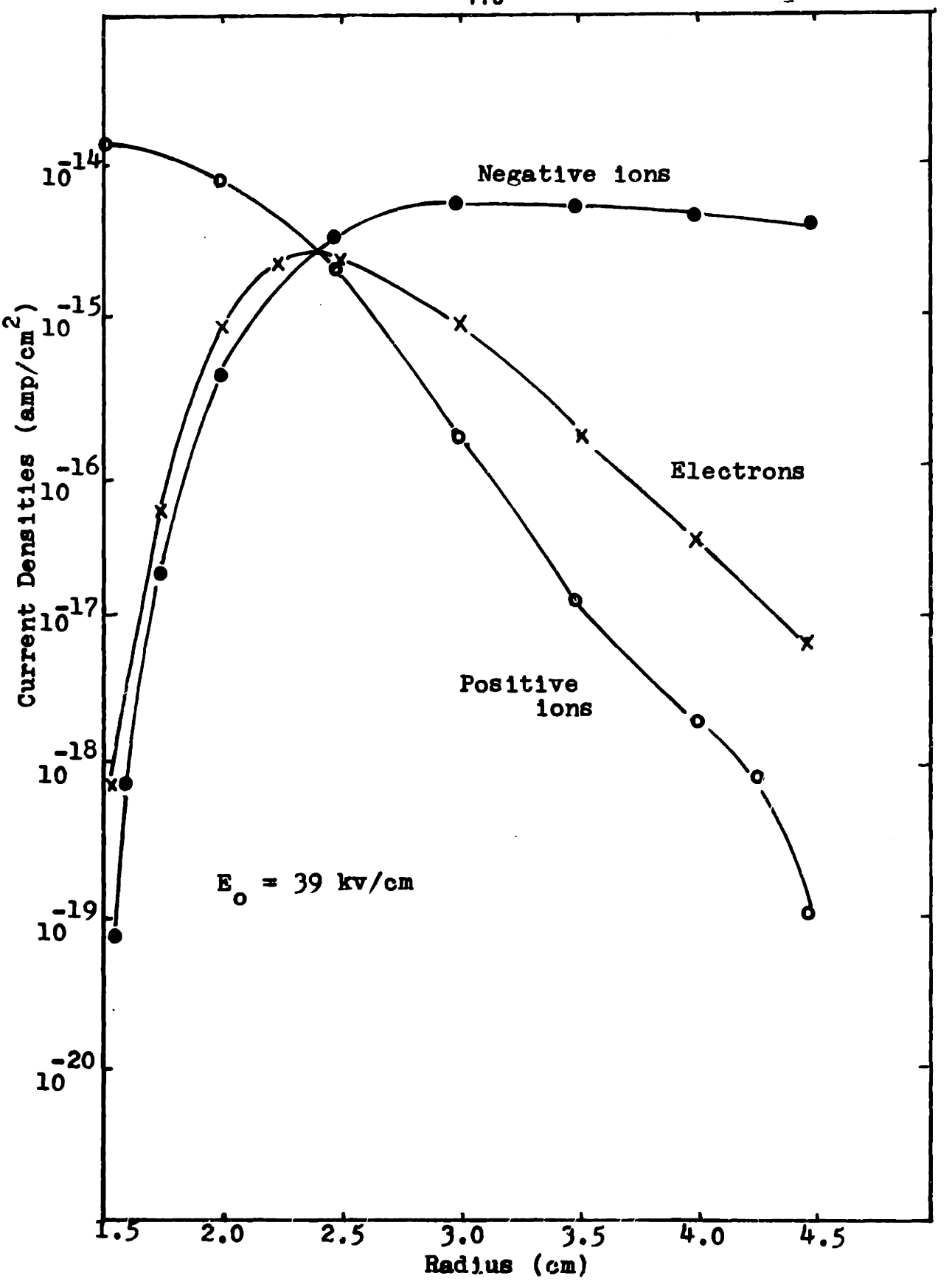


Figure 8.2 Current densities for E_o = 39 kv/cm standard case.

above the onset. See Figure 8.3.

The corresponding charge densities for E_o = 39 kv/cm are shown in Figure 8.4. Both positive ions and negative ions are important in modifying the electric field. Again electron charge density is insignificant.

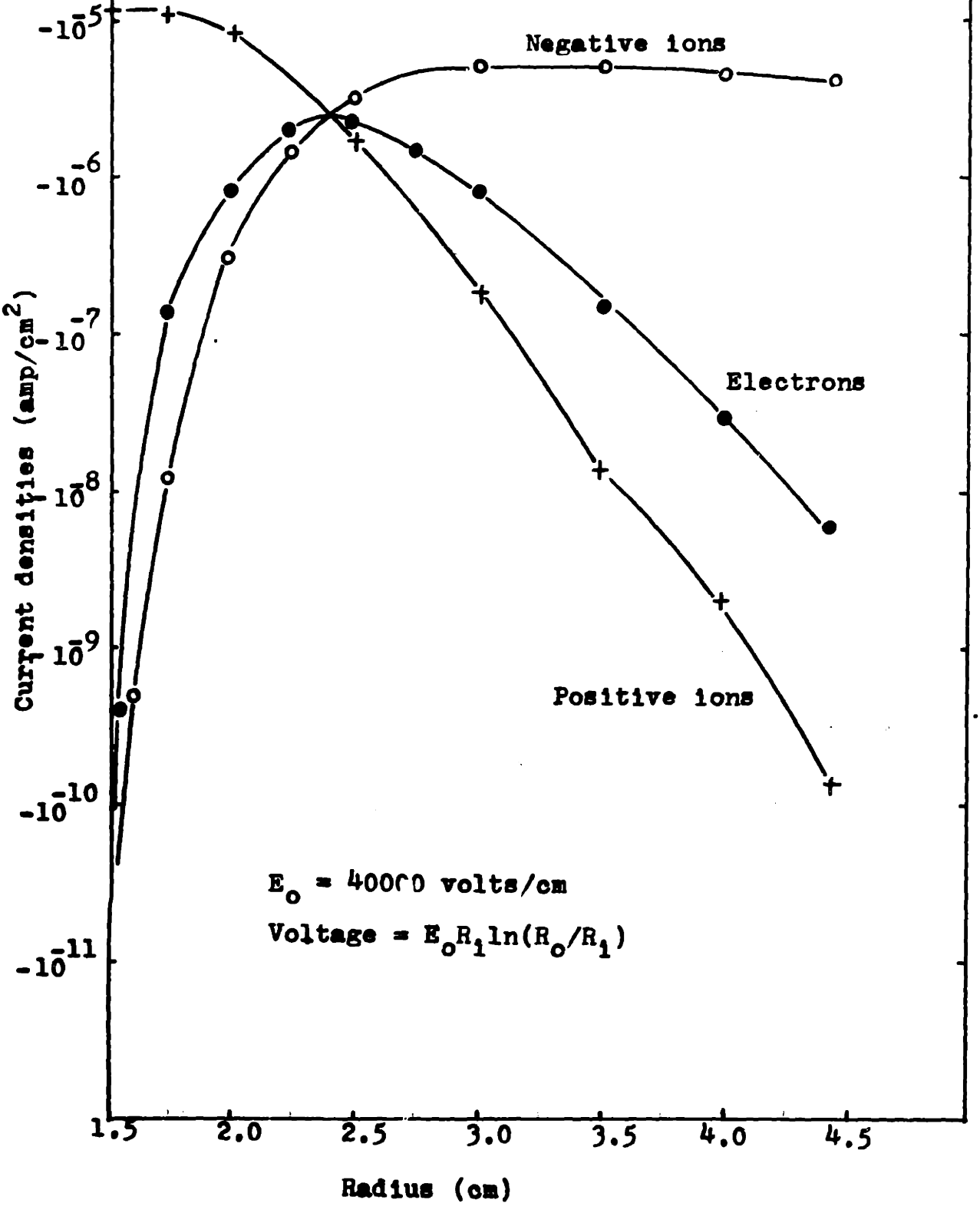
Figure 8.6 shows the effect of space charges on the electric field. Below onset the charge densities are so small that the electric field is unaltered from its K/r form. Above onset, the electric field near the center falls below the unperturbed K/r field and it deviates further until a minimum is reached approximately at where $\alpha = \eta$. This phenominon is caused by the layer of positive space charge on the surface of the cathode. At distances far from the cathode, the electric field starts to increase and eventually becomes larger than the K/r field. Increases in voltage results in an increase in deviation from the K/r field. This is known as choking similar to the positive corona case.

The deviation of the electric field is reflected on the plots of α and η as a function of radius in Figure 8.7.

8.2 Sensitivity Tests

A set of sensitivity tests similar to the positive corona case are performed for the negative corona also.

Figure 8.3 Current densities for E_o = 40 kv/cm standard case.



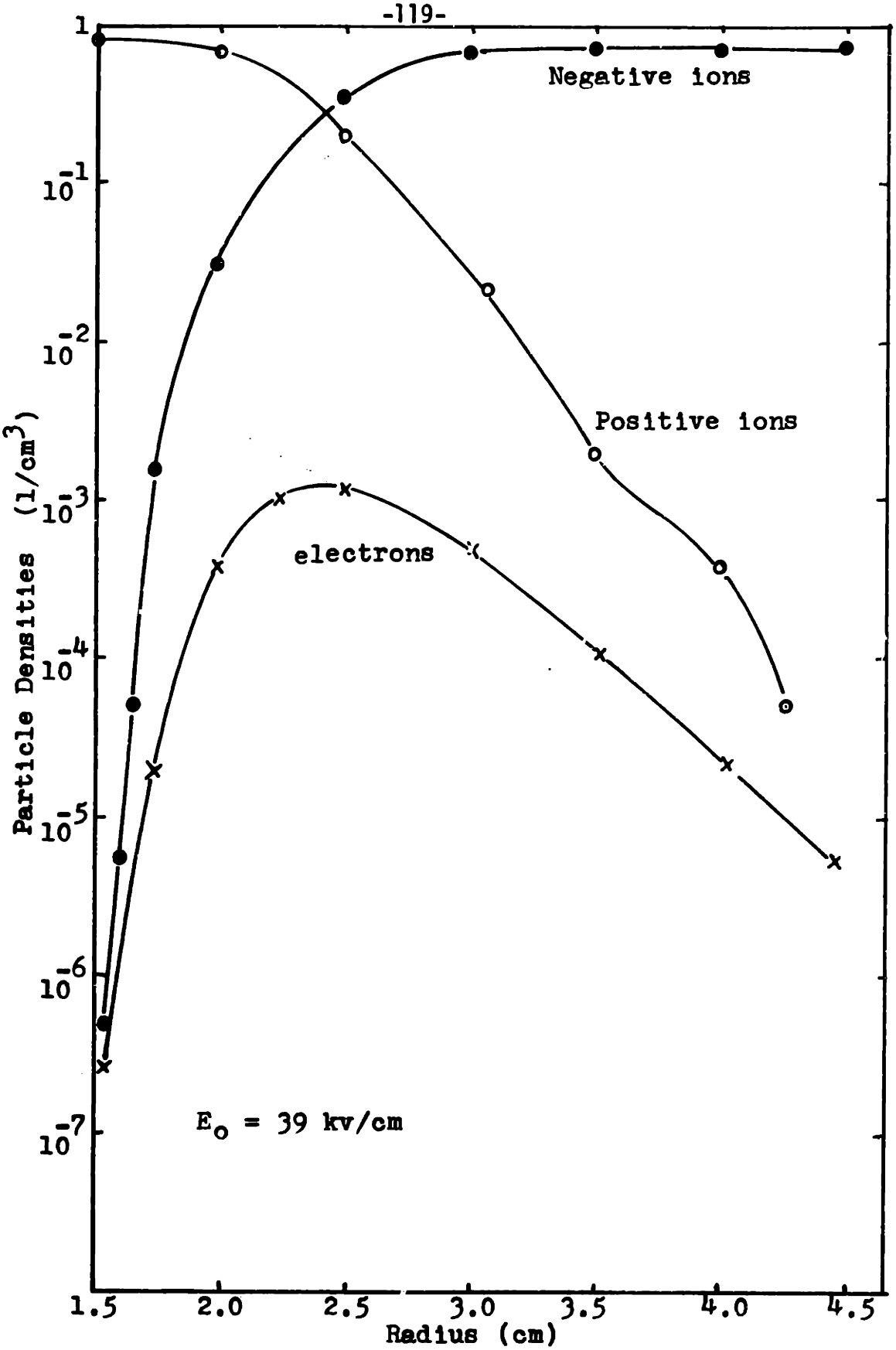


Figure 8.4 Particle densities for E_o = 39 kv/cm standard case.

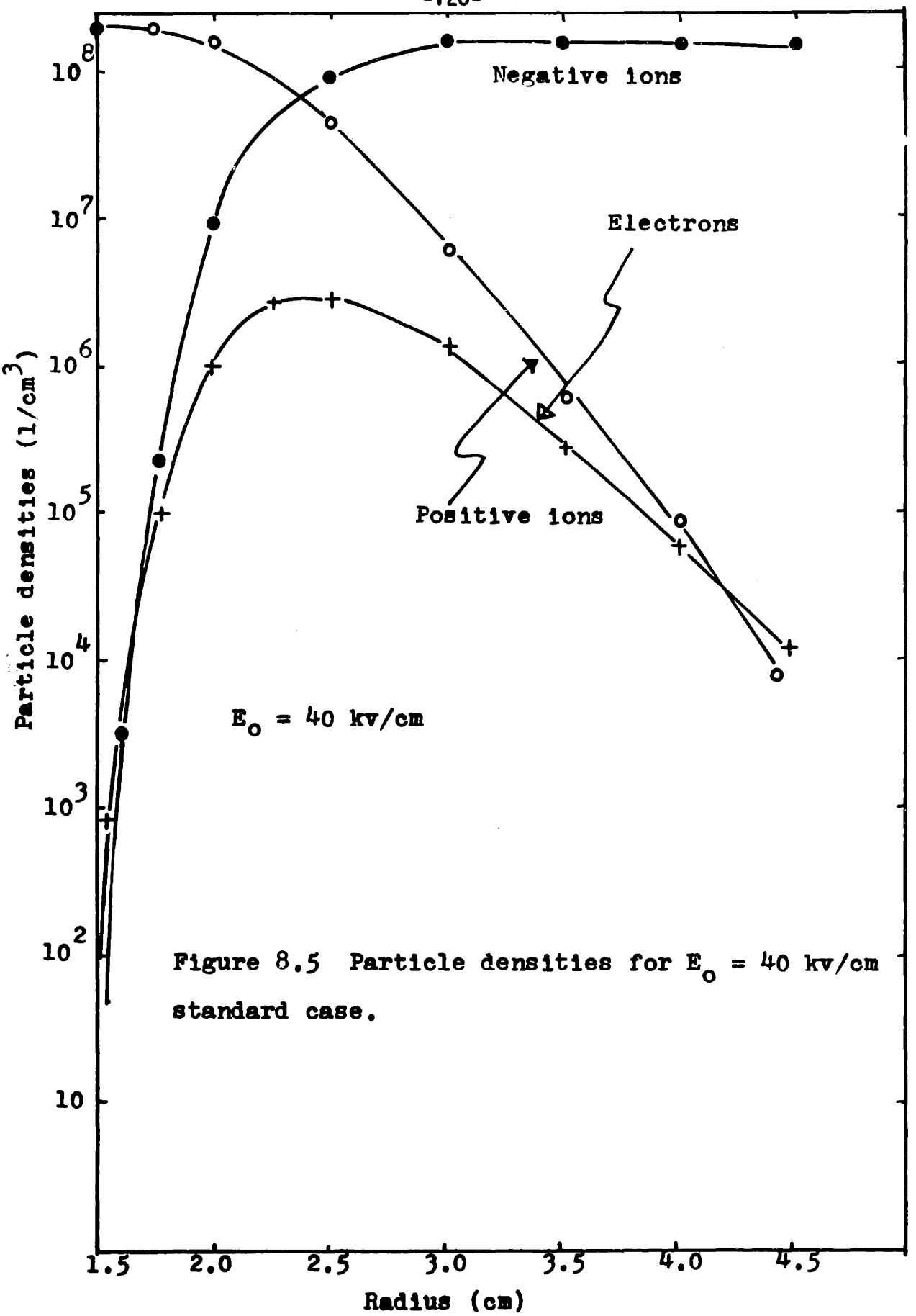


Figure 8.5 Particle densities for E_o = 40 kv/cm standard case.

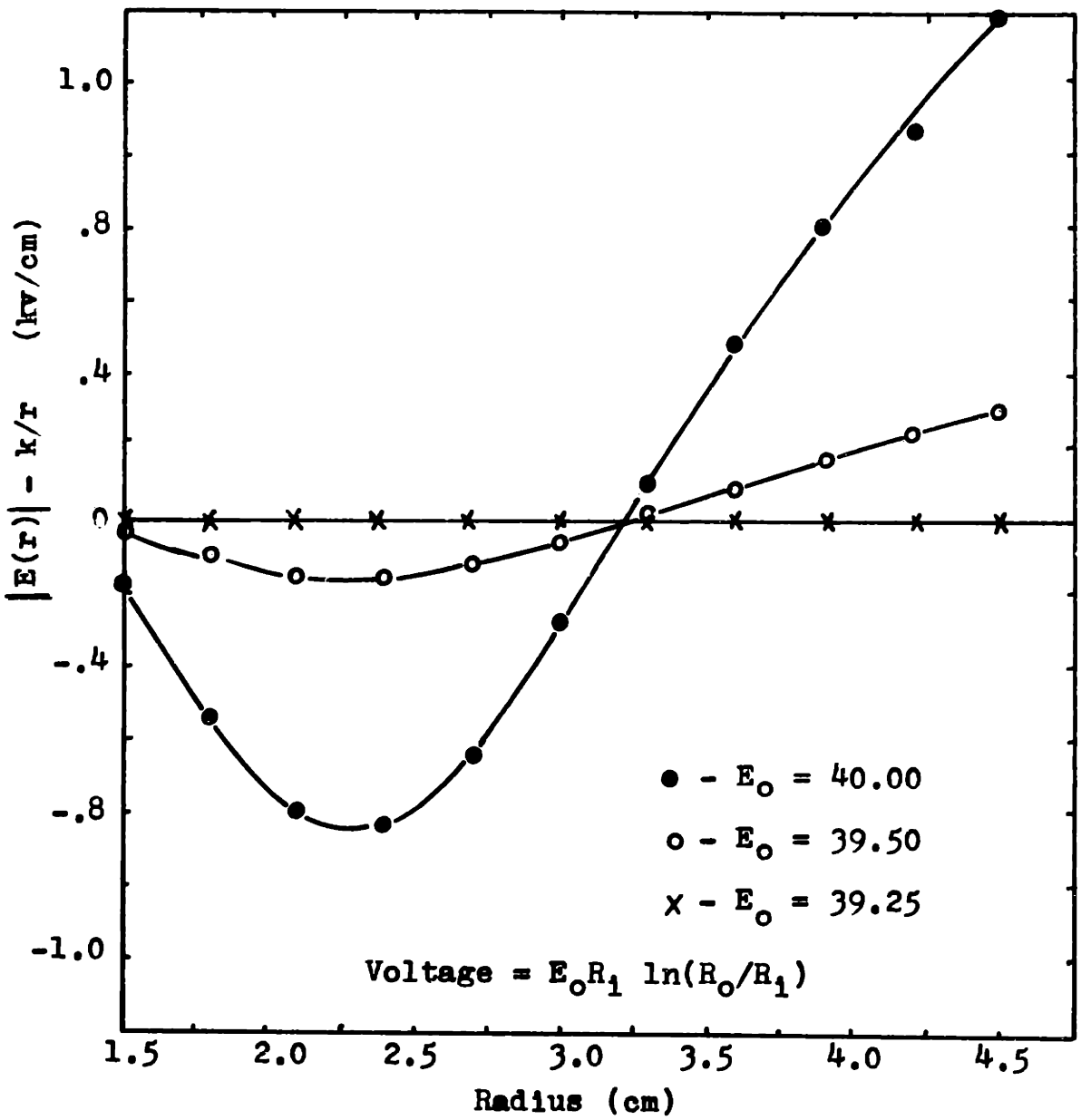


Figure 8.6 Deviation of the electric field from the k/r field as a function of position for different applied voltages.

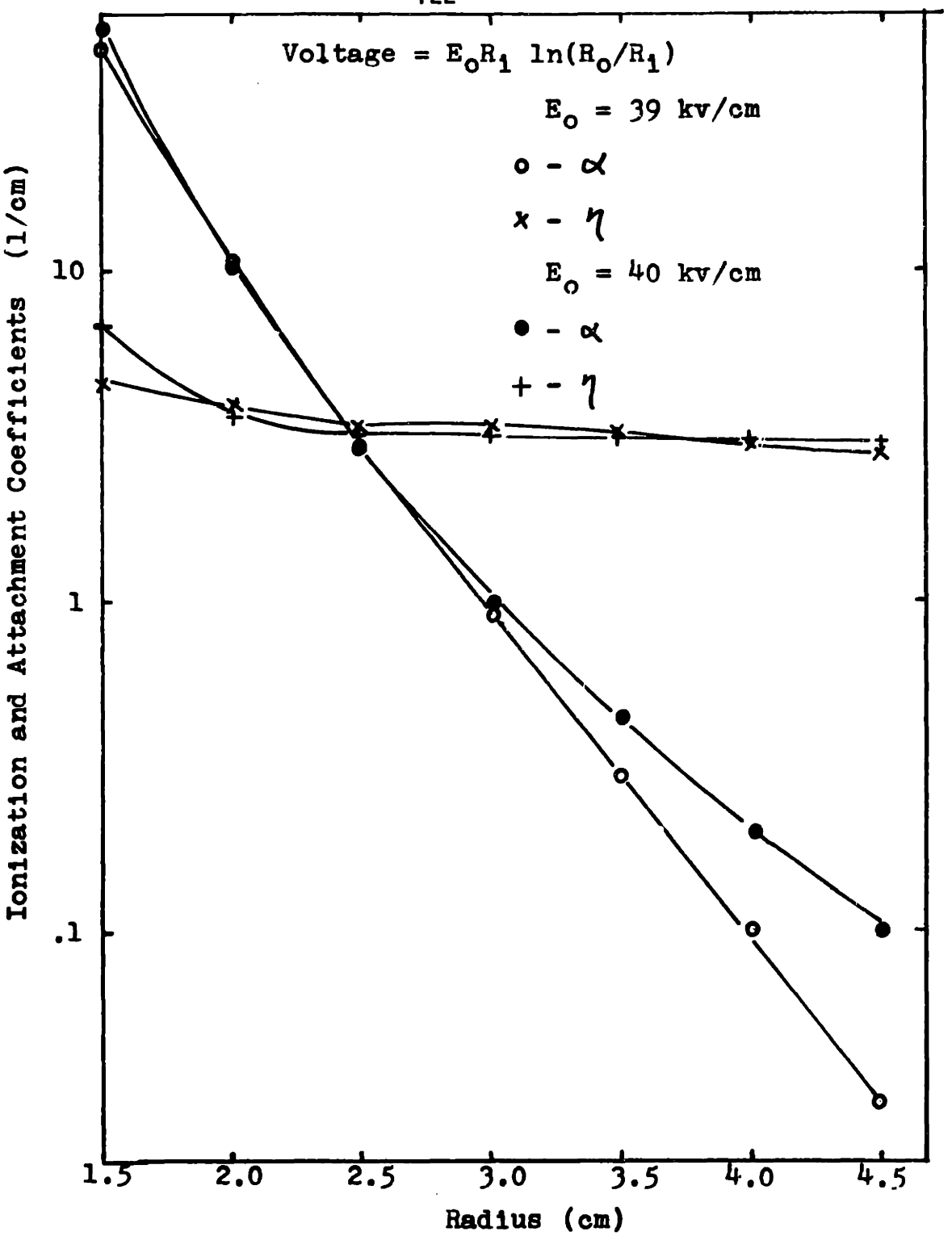


Figure 8.7 Values of α and η as a function of position for two different applied voltages.

Their curves are shown in Figure 8.1. First, consider the decrease in α by 5%. The onset increases by about $\frac{1}{2}$ kv/cm, Then the value of α in the region E/p = 40 volts/cm-mm Hg is allowed to vary from

$$\alpha/p = 1.44 \exp(-\frac{180}{E/p}) \qquad (1)$$

to

$$\alpha/p = 8.2 \exp(-\frac{250}{E/p}) \qquad (2)$$

with μ_p= .01 cm^{-1} for both cases. The critical onset electric field is at 37.9 kv/cm-mm Hg for the first α used. The critical onset field is increased to 38.5 kv/cm for the second α.

Similar to the positive corona case, when μ_p is changed from .001 cm^{-1} to .01 cm^{-1}, the onset dropped from 39.4 kv/cm to 37.9 kv/cm. The photoionization term for the standard case with E_o = 40 kv/cm at the center electrode, when there is no space charges, is shown in Figure 8.8. If μ_p is a much larger number, the photoionization term will have a distinct peak between the two conductors. For changes in ion mobilities, recombination coefficient, and attachment coefficient, the effects are exactly the same as the positive case.

8.3 Variations on the Size of the Outer Conductor

If the size of the outer conductor is many times the

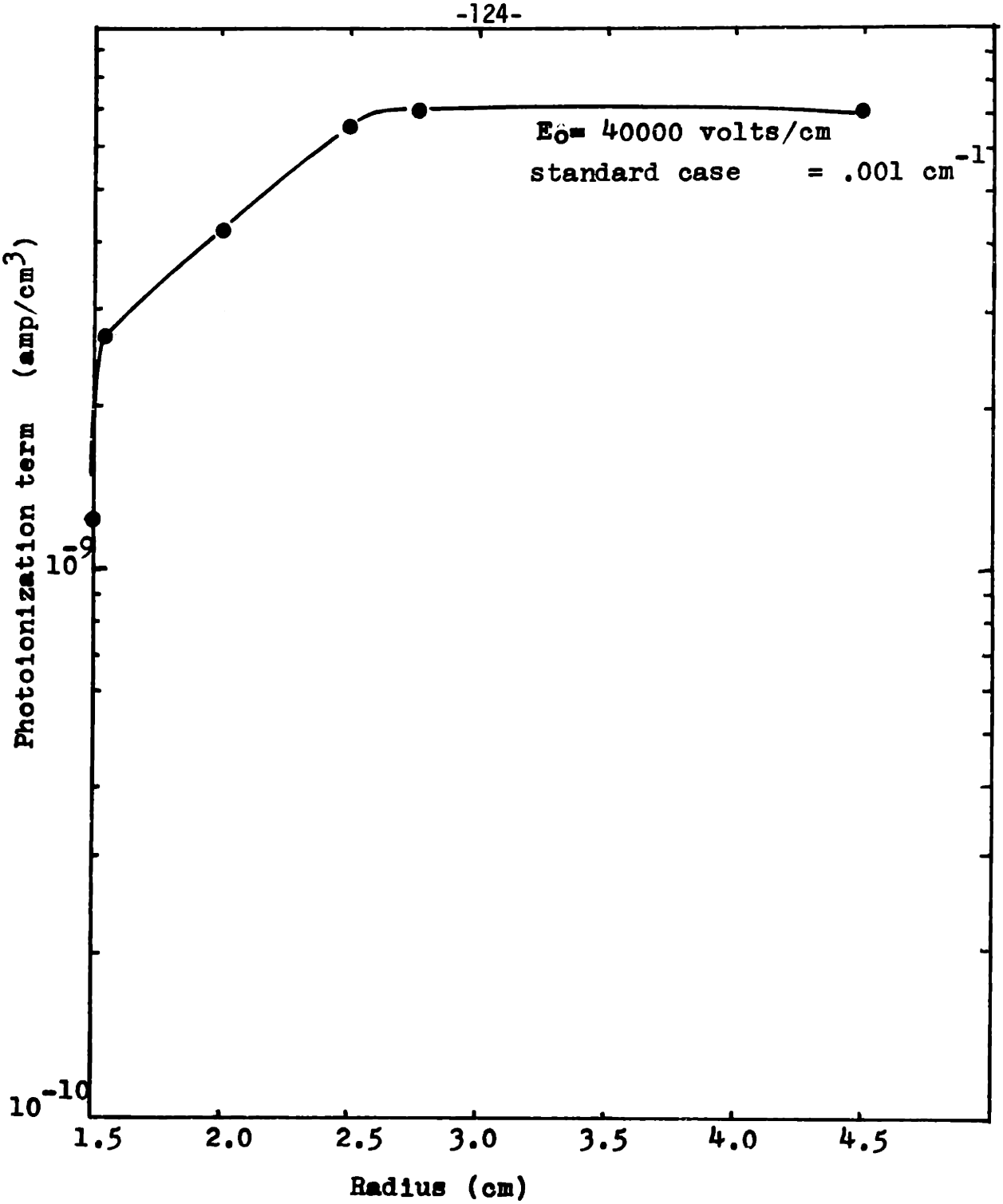


Figure 8.8 The photoionization term as a function of position.

distance from the inner conductor to the peak of electron current density, then the change in the outer cylinder radius only changes the current above onset. The electric field will be decreased near the conductor and increased further away from the K/r field as described in section 6.5.

If the outer conductor is made too small, the electric field at the surface of the cathode could actually be increased above the K/r field. If the deviation is not too large as in the case shown in Figure 8.9, the situatation is still stable. If the outer conductor is further decreased in size, the large increase of the electric field on the surface of the cathode produces a larger current, and the larger current increases the electric field more near the cathode. This is similar to a complete breakdown. The effect is opposite to choking. It could not happen in the positive corona case, unless the outer cylinder is only a few mm away from the inner cylinder.

8.4 Ion Bombardment and Photoelectric Emission.

Negative Corona is noted to occur on or above the positive corona onset for a polished and oxidized wire. γ_1 can at most be .00001 if the onset is to occur at Peek's prediction for a 1.5 cm radius cathode. Case 7 in Figure 8.1 is a current as a function of voltage curve

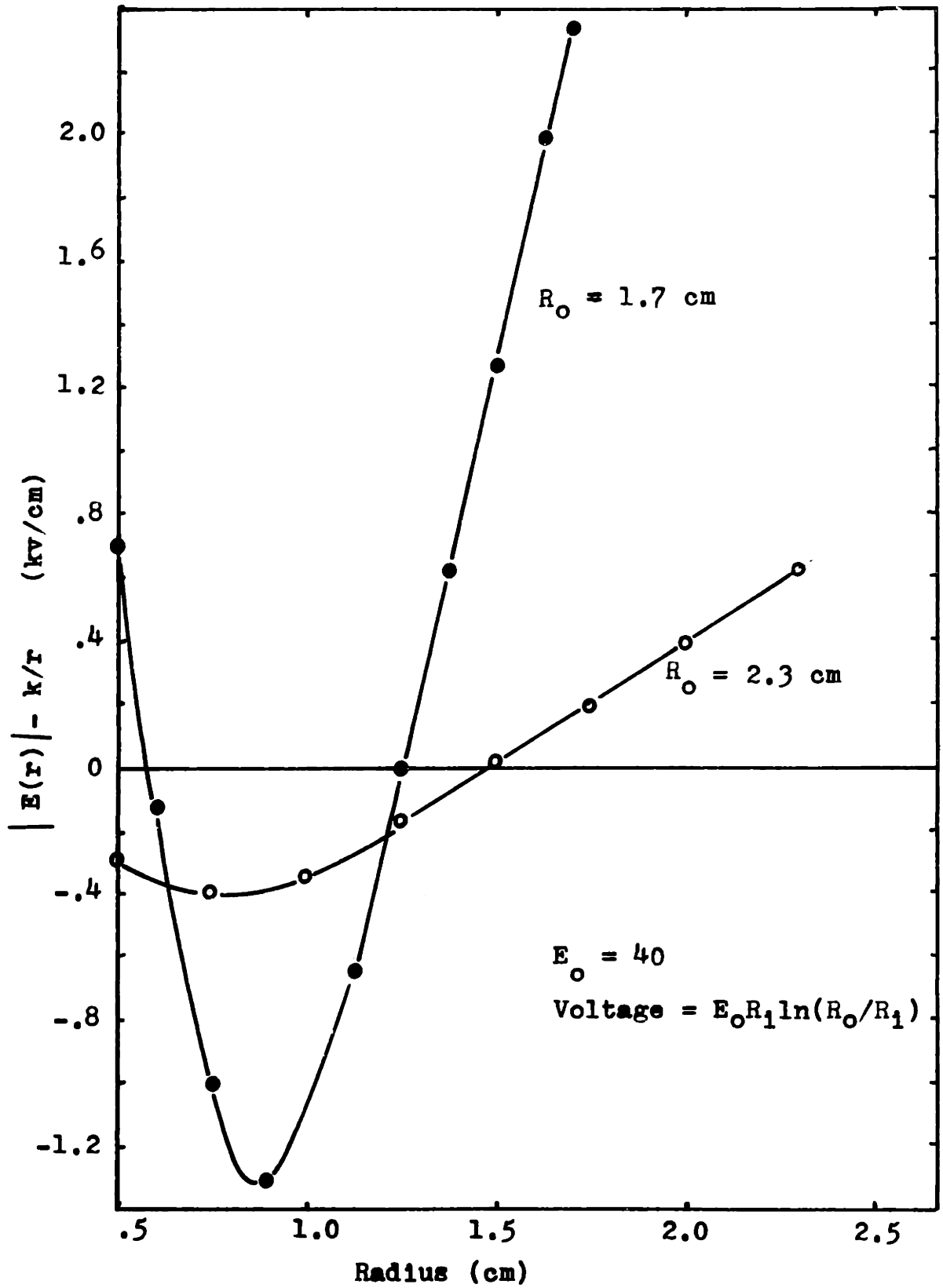


Figure 8.9 The effect of the dimension of the outer cylinder on the electric field.

when γ_i = .0001. The critical onset field is at 37.9 kv/cm. The current densities are shown in Figure 8.10. There is no marked difference in current distribution between the photoionization case and the case with ion impact.

Similarly if photoelectric emision is the only secondary feedback mechanism, γ_p has to be less than .0001.

There is no way to distinguish the three different secondary mechanixms using steady state measurements. Possibly, measurements of the time lag for an applied current impulse is one method to distinguish the three secondary processes.

8.5 Variations on Cathode Size and Comparison of Results to Peek's Formula

For all size inner conductors tested, the onset for negative corona is higher than positive as shown in Figures 6.15, 6.16, 6.17 and 6.18 in the positive corona section. The shape of the critical onset electric field as function of radius in Figure 6.1 again is different from Peek's formula, but the same as analytical predictions in Figure 5.2 . By changing α in the low E/p region, however, did not have as big a change on the shape of the onset curve. The reason is the electrons move from high field into low E/p region. Changes in μ_p as a function of E/p could twist the curve the correct way. However, the correctness

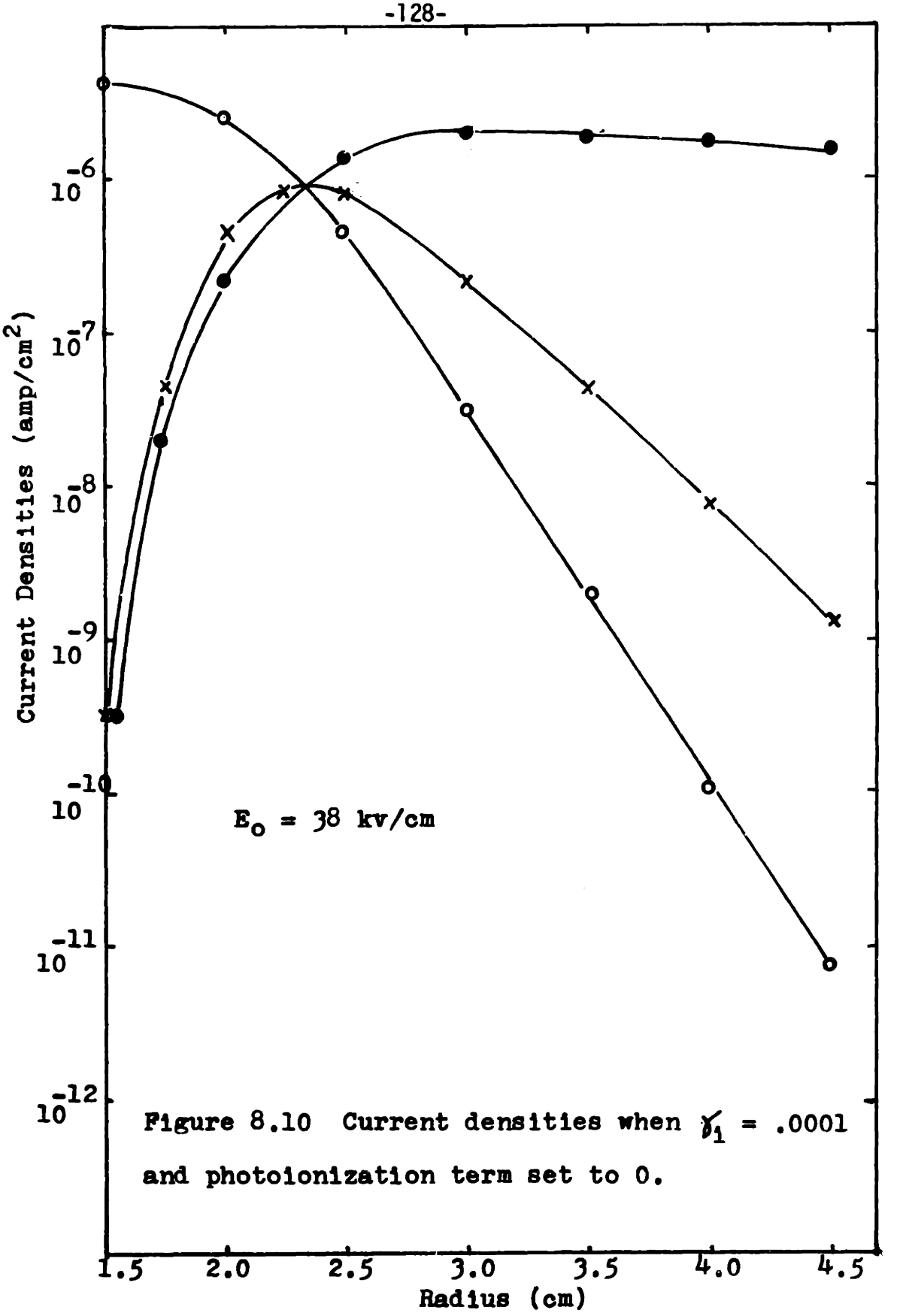


Figure 8.10 Current densities when γ_1 = .0001 and photoionization term set to 0.

of Peek's formula for direct current measurements in negative corona is very questionable.

9. Conclusions

The model used can generate a total current as a function of voltage similar to observations. The important primary processes are ionization by electron neutral collisions and attachment of electrons to neutrals. Recombination and diffusion are insignificant. Changes of ion mobilities only effect the current above onset. Changes in electron velocity only changes the electron charge densities, while the current densities are not effected at all. Both the positive and the negative corona can be produced when the only secondary process included is photoionization in the bulk of the gas. The negative corona onset then is always above the positive corona onset. There is no way to differentiate the type of secondary process using this model.

The model can be imporved if the photon abserption rate, μ_p, is made to be a function of E/p. A better approximation for the geometric factor for negative corona will also improve the accuracy of onset.

The program too can be improved, such as a second order method for evaluation of the electric field. To increase the accuracy at minimum cost, variable grid sizes should be allowed.

The next logical step would be to include time depen-

dence. The equations used in this assume that the average time from electron neutral collision to emission of a photon and eventual absorption is short compared to the characteristic distance over the electron velocity. The continuity equations for the case with time dependence become

$$\frac{\partial \rho_e}{\partial t} = -\nabla\cdot\vec{J}_e - (\alpha - \eta)|J_e| - 10q - Ph \qquad (1)$$

$$\frac{\partial \rho_-}{\partial t} = -\nabla\cdot\vec{J}_- - \eta|J_e| \qquad (2)$$

$$\frac{\partial \rho_+}{\partial t} = -\nabla\cdot\vec{J}_+ + \alpha|J_e| + 10q + Ph \qquad (3)$$

The greatest difficulty associated with the model when time is incorporated is the difference in the velocity of electrons versus the ions in an electric field.

Appendix 1

Derivation of Geometric Factor for Positive Corona

This section is devoted to deriving an expression for the percentage of photons created at a point P located r' away form the axis that reach an imaginary cylinder of radius r, for $r > r'$.

Consider a small area dA on the imaginary cylinder with radius r. See Figure A1.2. The solid angle $d\Omega$ subtended by dA as seen at point P is

$$d\Omega = \frac{\cos\gamma}{D^2}\, dA \qquad (1)$$

where D = distance between point P and dA,

and γ = angle between the vector D and the normal vector of dA.

$d\Omega/4\pi$ is the percentage of rays from point P that get to area dA when there is no absorption of photons in the intermittant space. With absorption, the actual percentage of rays leaving P and arriving at dA is decreased to

$$\frac{d\Omega}{4\pi} \exp(-\mu_p D) \qquad (2)$$

Summing over all the area S an cylinder with radius r, where rays from point P could reach, the percentage of photon, K arriving is

$$K = \iint_S \exp(-\mu_p D)\, \frac{\cos\gamma}{4\pi D^2}\, dA \qquad (3)$$

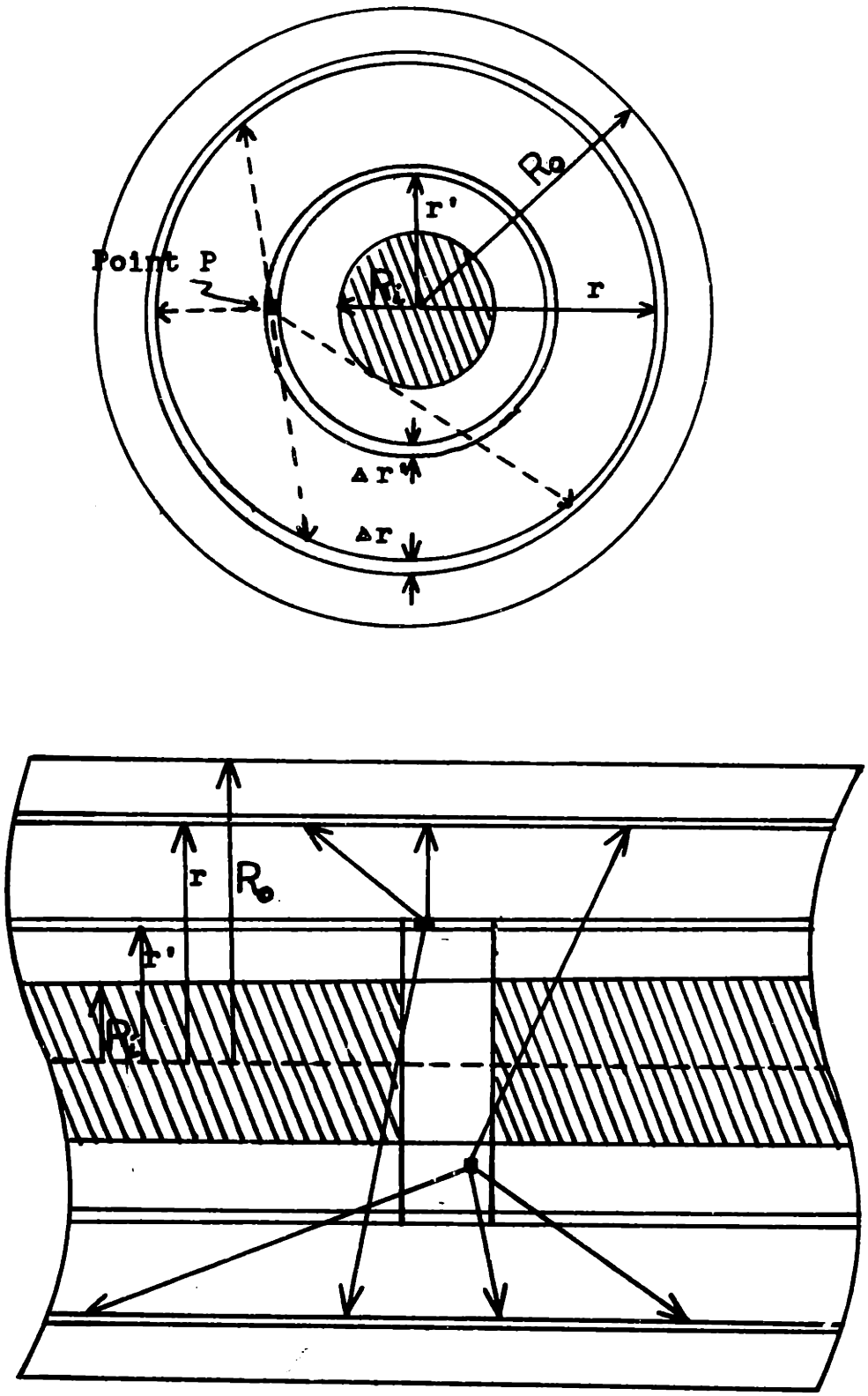


Figure A1.1 Two different view points for the derivation of geometric factor of positive corona.

$$K = 4 \int_0^{\infty} \int_0^{\theta_{max}} \frac{\cos\gamma}{4\pi D^2} \exp(-\mu_\gamma D)\, r\, d\theta\, dz. \quad (4)$$

The expressions for θ_{max}, γ and D will be derived below.

Consider the configuration shown in Figure A1.2 . dA is located at (x,y,z) and the point P is at (r',0,0). The results derived below are very general. The distance D between point P and dA is

$$|D| = ((x - r')^2 + y^2 + z^2)^{\frac{1}{2}} \quad (5)$$

$$= (r'^2 + r^2 - 2rr'\cos\theta + z^2)^{\frac{1}{2}}$$

The definition of γ is

$$\cos\gamma = \hat{i}_n \cdot \hat{i}_D \quad (6)$$

where $\hat{i}_n = (x\hat{i}_x + y\,\hat{i}_y)/r$ (7)

and $\hat{i}_D = ((x-r')\,\hat{i}_x + y\,\hat{i}_y + z\,\hat{i}_z)/D$ (8)

$$\cos\gamma = (x(x-r') + y^2)/(rD) \quad (9)$$

$$= (r - r'\cos\theta)/D$$

The last unknown is θ_{max}. θ_{max} is the angle between the x-axis and a line that includes the origion and a point B on the circle with radius r. Point B is picked in such a way that the line connecting points B and P is tangent to the inner cylinder with radius R_1; and the point of contact is called C.

$$x_c^2 + y_c^2 = R_1^2 \quad (10)$$

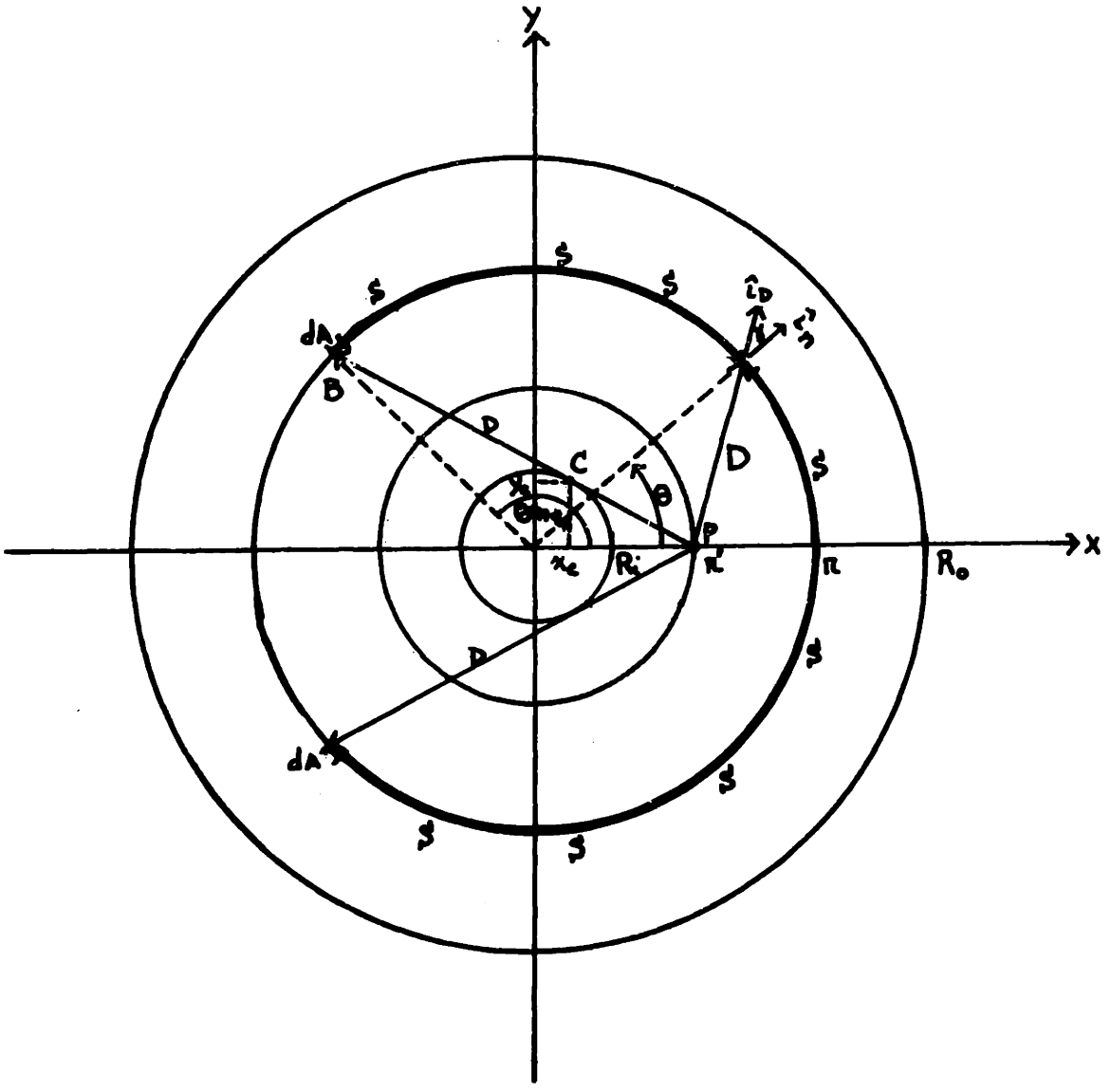


Figure A1.2 Definition of variables used in the derivation.

Definition of the slope of the tangent at point C is

$$\frac{dy}{dx} = \frac{-x_c}{(R_1^2-x_c^2)^{\frac{1}{2}}} \tag{11}$$

The triangle made of points C, P and $(x_c, 0)$ also can give an expression for tangent.

$$\frac{dy}{dx} = \frac{-y_c}{r'-x_c} = \frac{-(R_1^2 - x_c^2)^{\frac{1}{2}}}{r' - x_c} \tag{12}$$

Solve for x_c.

$$x_c = \frac{R_1^2}{r'} \tag{13}$$

Now the equation of the tangent is known.

$$y = \frac{R_1}{(1-(R_1/r')^2)^{\frac{1}{2}}} \left(1 - \frac{x}{r'}\right) \tag{14}$$

Solving for the intersection of this equation with the circle

$$x^2 + y^2 = r^2 \tag{15}$$

gives the coordinates of the point B.

$$x_B = \frac{1}{r'}\left(R_1^2 - ((r^2 - R_1^2)(r'^2 - R_1^2))^{\frac{1}{2}}\right) \tag{16}$$

So θ_{max} can now be found.

$$\cos\theta_{max} = \frac{x_B}{r} \tag{17}$$

$$= \frac{1}{r\,r'}\left(R_1^2 - ((r^2 - R_1^2)(r'^2 - R_1^2))^{\frac{1}{2}}\right)$$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{r\,r'}\left(R_1^2 - ((r^2 - R_1^2)(r'^2 - R_1^2))^{\frac{1}{2}}\right)\right] \tag{18}$$

Finally the expression for K is completely determined.

$$K = \frac{r}{\pi}\int_0^{\infty}\int_0^{\theta_{max}} \frac{(r - r'\cos\theta)\exp(-\mu_\nu D)}{D^3}\, d\theta\, dz \qquad (19)$$

where $D = (r^2 + r'^2 - 2r\, r'\cos\theta + z^2)^{\frac{1}{2}}$. (20)

Unfortunately the above expression cannot be integrated. An expression that is simplier to use in photoionization calculations is

$$K \cong G \exp(-\mu_{eff}(r-r')) \qquad (21)$$

where $G = \frac{r}{\pi}\int_0^{\theta_{max}}\int_0^{\infty} \frac{r - r'\cos\theta}{D^3}\, dz\, d\theta$ (22)

and μ_{eff} is calculated from equation 21 for each point after K is integrated numerically.

G can be integrated; first over the variable z giving

$$G = \frac{r}{\pi}\int_0^{\theta_{max}} \frac{r - r'\cos\theta}{(r^2 + r'^2 - 2rr'\cos\theta)}\, d\theta \qquad (23)$$

$$= \frac{r}{2\pi r'}\int_0^{\theta_{max}} \frac{d\theta}{\left(\frac{r^2 + r'^2}{2rr'} - \cos\theta\right)}$$

$$- \frac{1}{2\pi}\int_0^{\theta_{max}} \frac{\cos\theta\, d\theta}{\left(\frac{r^2 + r'^2}{2rr'} - \cos\theta\right)} \cdot$$

The second term can be written in another form, (24)

$$- \frac{1}{2\pi}\left[-\theta\Big|_0^{\theta_{max}} + \frac{r^2 + r'^2}{2rr'}\int_0^{\theta_{max}} \frac{d\theta}{\left(\frac{r^2 + r'^2}{2rr'} - \cos\theta\right)}\right]$$

which when combined with the first term of G, gives

$$G = \frac{\theta_{max}}{2\pi} + \frac{1}{2\pi}\left(\frac{r}{r'} - \frac{r^2 + r'^2}{2rr'}\right)\int_0^{\theta_{max}} \frac{d\theta}{\frac{r^2 + r'^2}{2rr'} - \cos\theta}$$

$$= \frac{\theta_{max}}{2\pi} + \frac{1}{2\pi r'} \frac{r^2 - r'^2}{2r} \frac{2}{\left(\frac{r^2 + r'^2}{2rr'} - 1\right)^{\frac{1}{2}}} \cdot$$

$$\left. \frac{\tan^{-1}\left(\frac{r^2 + r'^2}{2rr'} - 1\right)^{\frac{1}{2}} \tan\left(\frac{\theta}{2}\right)}{\frac{r^2 + r'^2}{2rr'} - 1} \right|_0^{\theta_{max}}$$

$$= \frac{\theta_{max}}{2\pi} + \frac{1}{2\pi} \tan^{-1}\left(\frac{r + r'}{r - r'}\right)\tan\left(\frac{\theta_{max}}{2}\right) \qquad (25)$$

Numerical calculations show that G is not a function of r, which is very reasonable since G is really the percentage of rays emitted from r' that are not blocked by the inner cylinder of radius R_1. The limiting case

$$r' = R_1 \qquad (26)$$

gives

$$G = .5. \qquad (27)$$

This checks with expectations.

Appendix 2

Derivation of Geometric Factor for Negative Corona

For negative corona the percentage of photons created at a point P located r away from the axis that reach an imaginary cylinder of radius r', for $r > r'$, is also of importance.

This time consider a small area dA on the imaginary cylinder with radius r'. See Figure A2.1 . The arguments in deriving K is identical with that in appendix 1.

$$K = 4 \int_0^{\infty} \int_0^{\theta_{max}} \frac{\cos\gamma \exp(-\mu D)}{4 \pi D^2} r' \, d\theta \, dz \qquad (1)$$

where $\theta_{max} = \cos^{-1}\left[\frac{1}{rr'}(R_1^2 - ((r^2 - R_1^2)(r'^2 - R_1^2))^{\frac{1}{2}}) \right]$

and $D = (r^2 + r'^2 - 2rr'\cos\theta + z^2)^{\frac{1}{2}}$

The same as before. However, the expression for $\cos\gamma$ is changed. See Figure A2.2.

$$\hat{i}_D = (x - r')\hat{i}_x + y\hat{i}_y + z\hat{i}_z \qquad (2)$$

so $$\cos\gamma = \frac{x - r'}{D} = \frac{r\cos\theta - r'}{D} \qquad (3)$$

Note that $\cos\gamma$ changes sign when

$$\cos\theta = \frac{r'}{r} \qquad (4)$$

This can be explained by the fact that rays from point

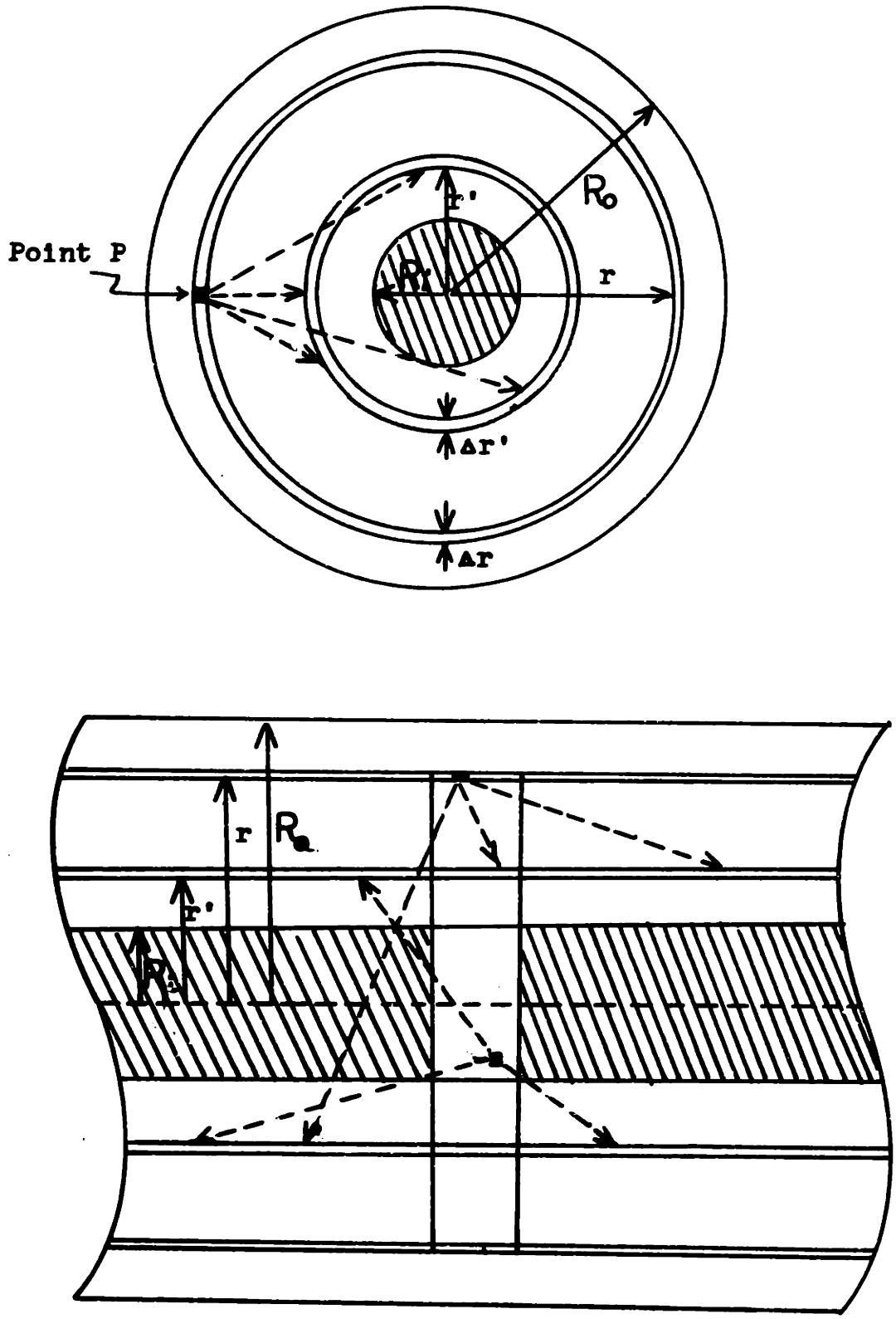


Figure A2.1 Two different view points for the derivation of geometric factor of negative corona.

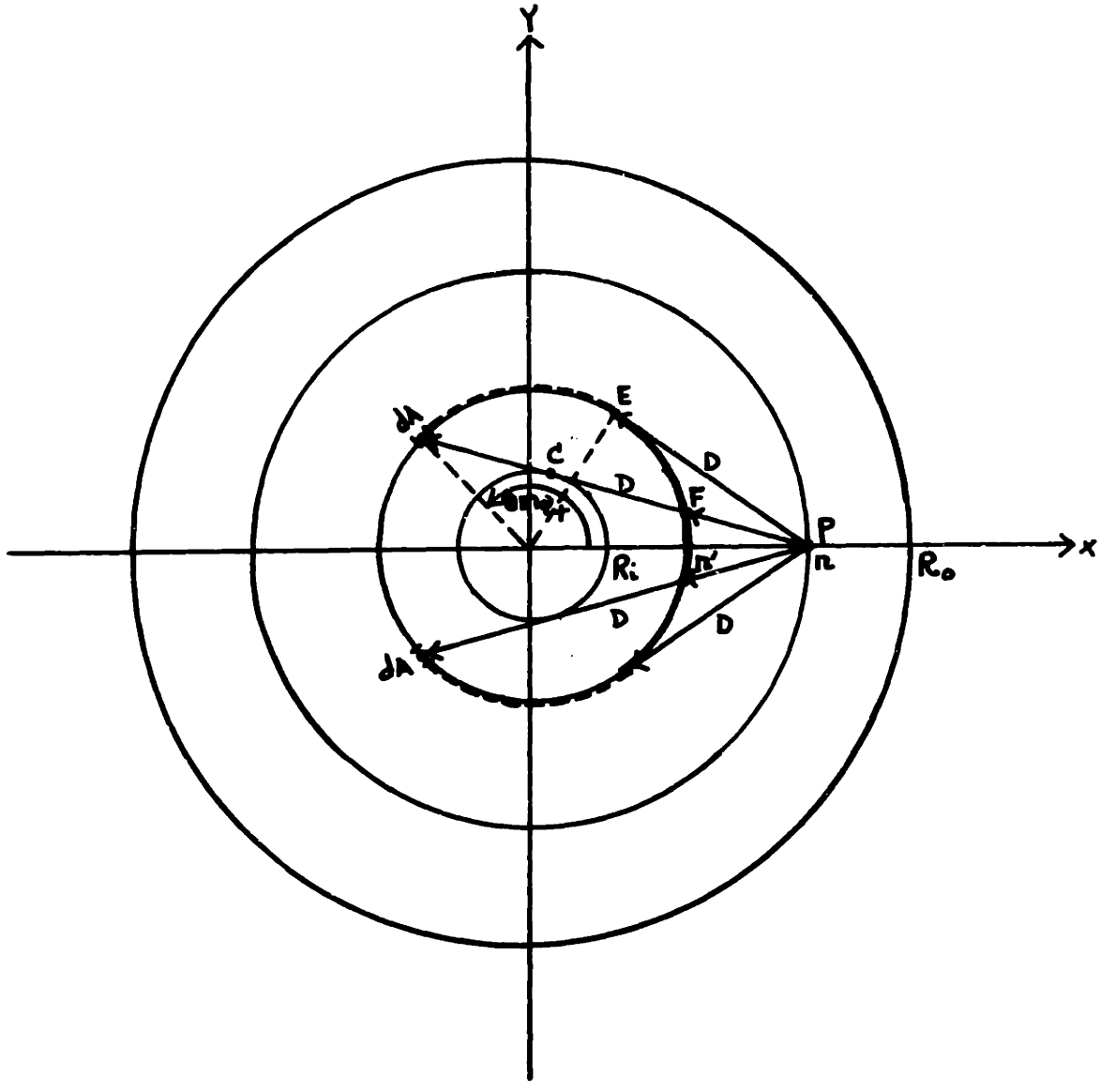


Figure A2.2 Definition of variables used in the derivation.

P. intersect the cylinder of radius r' twice except at the point of tangency, E.

Again assume

$$K \cong G' \exp(-\mu_{eff}(r - r')) \tag{5}$$

where G' is a new geometric factor

$$G' = \frac{r'}{\pi} \int_0^{\theta_{max}} \int_0 \frac{|r\cos\theta - r'|\, d\theta\, dz}{(r^2 + r'^2 - 2rr'\cos\theta + z^2)^{3/2}}$$

$$= \frac{r'}{\pi} \int_0^{\theta_{max}} \frac{|r\cos\theta - r'|\, d\theta}{r^2 + r'^2 - 2rr'\cos\theta} \tag{6}$$

$$= \frac{1}{\pi} 2\tan^{-1}\left(\frac{r + r'}{r - r'}\right) - \cos^{-1}\left(\frac{r'}{r}\right)$$

$$- \frac{1}{2\pi}\left[2 \tan^{-1}\left(\left(\frac{r + r'}{r - r'}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right) - \theta_{max}\right]$$

The second term is independent of variable r' because it is the percentage blocked by the inner cylinder. Further rearrangement of terms gives

$$G' = 1 - \frac{1}{\pi} \cos^{-1}\left(\frac{2r'^2 - r^2}{r^2}\right) \tag{7}$$

$$- \frac{1}{2\pi}\left[2 \tan^{-1}\left(\left(\frac{r + r'}{r - r'}\right)\tan\left(\frac{\theta_{max}}{2}\right)\right) - \theta_{max}\right]$$

Appendix 3

Calculating an Approximate Value for μ_{eff}

$$K \cong G \exp(-\mu_{eff}(r-r')) \tag{1}$$

μ_{eff} is approximated by the average of many exact values of effective photon absorption rate, when reducing a three dimensional problem into two dimensions.

$$\mu_{eff} \cong \frac{1}{N} \sum_{i=1}^{N} \frac{1}{(r_i - r')} \ln\left(\frac{G(r',r_i)}{K(r',r_i)}\right) \tag{2}$$

where $r_i = R_i + i\Delta r,$ (3)

$r' = R_i,$ (4)

and $r_N = R_o.$ (5)

r' is assigned to be R_i because most of the photons are emitted near the surface of inner conductor. G and K are dexcribed by equations A1.25 and A1.19 respectively. K has to be integrated numerically. To make the integration bounded, let

$$z = \tan\varphi. \tag{6}$$

$$dz = (1 + \tan^2\varphi)d\varphi. \tag{7}$$

Then (8)

$$K = \frac{r}{\pi}\int_0^{\pi/2}\int_0^{\theta_{max}} \frac{(r - r'\cos\theta)\exp(-\mu_t D)}{D^3}(1 + \tan^2\varphi)d\varphi\, d\theta$$

where $D = (r^2 + r'^2 - 2rr'\cos\theta + \tan^2\varphi)^{\frac{1}{2}}$ (9)

For $\mu_t = .01\ cm^{-1}$

$$\mu_{eff} \cong 1.8 \mu_p . \qquad (10)$$

Because μ_p is a very small number, variations of μ_{eff} from μ_p is not large. The error produced in the photoionization ter: is small.

Appendix 4

Program Listing for Positive Corona

To use the following program, the data should be inputed in the order specified below.

1. N - This is the number of divisions. When end points are included. There are N+1 grid points.
2. SAVE - Input 'YES' if the data from previous case is saved for use in the present case. Otherwise input 'NO'.
3. CARD_IN - Input 'YES' if there are any input data. Note that SAVE and CARD_IN cannot both be 'YES' at the same time.
4. CARD_OUT- Input 'YES' if data from this case is desired to be outputed on cards when iterations did not converge in the specified limit. Otherwise input 'NO'.
5. R_INNER - Input the radius of the inner conductor in units of centimeters.
6. DELTA_R - Input the grid size in centimeters.
7. INCR - For every print out of data associated with a grid point, INCR - 1 grid points are skipped. For example, if the input is 6, every 6th grid point will be printed starting at the inner cylinder.

8. M - The Maximum number of iterations allowed.

9. OUT_NUM - When the number of iterations performed become greater than M - OUT_NUM the data of that iteration will be printed. The input should be within the following limits:

$$1 \leqslant \text{OUT\_NUM} \leqslant M.$$

10. E0 - Absolute value of the electric field at the surface of the inner conductor when no space charges present. This is same as specifying the voltage applied between conductors.

$$\text{Voltage} = E0\ R_I\ \ln(R_o/R_1)$$

11. ENTERJE - Input the boundary condition for the electron current density at the outer radius in units of amp/cm^2. For positive corona this quantity should be greater than or equal to zero.

12. MOB_P_I - Input the mobility of positive ions in units of cm^2/sec-volt.

13. MOB_N_I - Input the mobility of the negative ions in units of cm^2/sec-volt.

14. A(1) - The constant A used in α for E/p = 40 volts/cm-mm Hg.

15. B(1) - The constant B used in η for E/p = 40 volts/cm-mm Hg.

16. A(2) - The constant A used in α for E/p = 40 volts/cm-mm Hg.

17. B(2) - The constant B used in η for E/p = 40 volts/cm-mm Hg.

18. PHOTOMU - This is one over the distance a photon travels before it decays to 1/e of its origional value. The units of PHOTOMU is in cm^{-1}.

19. EFF - The number of photons created in the frequency of interest for every electron created by electron-neutral collision.

20. MOD_ETA - This is a constant by which the whole attachment curve is multiplied by, so as to increase or decrease η. If η is not to be changed from the fitted values, Then set MOD_ETA to 1.

21. OVER - This is the relaxation parameter. If OVER is set greater than 1, it effectively increase the value of EFF to OVER*EFF, and it is used when iterations converges too slowly. Normally set over to 1.

22. EST - This is the estimated current for the specified voltage. If OVER is 1, EST can be set to any value. The units of EST is in amp/cm^2.

23. RECOMB - This is the recombination coefficient for

positive and negative ions with units in cm^3/sec.

24. PAIRS - Input the number of electron-ion pairs created by cosmic radiation percm^3 per second.

If CARD_IN is 'YES' then the charge density data cards should be placed after the input data PAIRS. To terminate the run, place the following card at the very end of the input data.

-1 'NO' 'NO' 'NO'

```
 SS: PROCEDURE OPTIONS (MAIN);                                          POS-0001
/*************************************************************************/ POS-0002
/* *** POSITIVE CORONA *** ANODE CORONA ***                            */ POS-0003
/* THIS PROGRAM CALCULATES THE STEADY STATE CHARGE DENSITY             */ POS-0004
/* WHEN THE SOURCE IS ELECTRON CURRENT DENSITY AT THE OUTER            */ POS-0005
/* BOUNDARY AND PHOTOIONIZATION IN THE INTERIOR.  PREVIOUS             */ POS-0006
/* RESULTS WILL BE USED IF ANY.  OVERSHUT PARAMETER IS USED            */ POS-0007
/* FOR FASTER CONVERGENCE.  RUNGE-KUTTA METHOD IS USED IN THE          */ POS-0008
/* INTEGERATION.                                                       */ POS-0009
/*************************************************************************/ POS-0010
                                                                        POS-0011
   DECLARE (R_INNER,  /* RADIUS OF INNER CYLINDER                     */ POS-0012
      OVER,           /* ADJUSTED OVERSHUT PARAMETER.                 */ POS-0013
      R_OLD,          /* INNER RADIUS                                 */ POS-0014
      MOD_ETA,        /* FACTOR BY WHICH ETA IS CHANGED               */ POS-0015
      DELTA_R) FIXED (7,4),                                             POS-0016
     (N,              /* ACTUAL NUMBER OF DIVISIONS.                  */ POS-0017
      INCR,           /* OUTPUT INCREMENT.                            */ POS-0018
      OUT_NUM,        /* # OF TIMES OUTPUT WILL BE PRINTED            */ POS-0019
      X,              /* GRID PT. E/P=40                              */ POS-0020
      Y,              /* GRID PT. E/P=25                              */ POS-0021
      I, J, K, CASE, M, T) FIXED(15,0) BINARY,                          POS-0022
     (SAVE,           /* SAVING PREVIOUS DATA?                        */ POS-0023
      CARD_IN,        /* ANY INPUT DATA                               */ POS-0024
      CARD_OUT)       /* OUTPUT DATA ON CARDS                         */ POS-0025
       CHARACTER(3),                                                    POS-0026
     (SUM1, SUM2,                                                       POS-0027
      PAIRS,          /* ELECTRON/ION PAIRS/CC BY COSMIC RAY          */ POS-0028
      COSMIC,         /* PAIRS*1.602E-19                              */ POS-0029
      RECOMB,         /* RECOMBINATION COEFFICIENT                    */ POS-0030
      A(2),B(2),AA(2),BB(2),   /* USED IN ALPHA                       */ POS-0031
      EO,             /* VALUE OF E                                   */ POS-0032
      TEMP, TEMP1, TEMP2,                                               POS-0033
      VOLT,           /* APPLIED VOLTAGE                              */ POS-0034
      ENTERJE,        /* DIFFERENT BOUNDARY CONDITIONS                */ POS-0035
      R_JE,           /* R(K) * J_E(K)                                */ POS-0036
```

```
      EST,              /* ESTIMATE OF FINAL  TOTAL CURRENT.        */   POS-0037
      R_J_N,            /* R * J_N_I                                */   POS-0038
      R_J_P,            /* R * J_P_I                                */   POS-0039
      K0, K1, K2, K3,                                                   POS-0040
      K0P, K1P, K2P, K3P,                                               POS-0041
      K0N, K1N, K2N, K3N,                                               POS-0042
      KK,               /* KK/R(0) IS E(0) FOR NO SPACE CHG         */   POS-0043
      MOB_N_I,          /* MOBILITY OF NEGATIVE IONS                */   POS-0044
      MOB_P_I,          /* MOBILITY OF POSITIVE IONS                */   POS-0045
      PHOTOMU,          /* PHOTON IONIZATION COEFFICIENT            */   POS-0046
      ABS_MU,           /* PHOTON ABSORPTION COEFFICIENT *1.86       */   POS-0047
      EFF,              /* NUMBER OF PHOTONS CREATED PER ELECTRON
                           CREATED                                  */   POS-0048
                                                                        POS-0049
      INT,                                                              POS-0050
      PI,               /* CONSTANT                                 */   POS-0051
      OLD_CHG, NEW_CHG,                                                 POS-0052
      I_DIFF,           /* CHANGE OF TOTAL I BETWEEN ITERATIONS     */   POS-0053
      R_OUT, ANGLE,     /* FOR CALCULATING GEOMETRIC FACTOR         */   POS-0054
      EPSILON) BINARY FLOAT,                                            POS-0055
/*****************************************************************/     POS-0056
/* THE SIZE OF THE FOLLOWING ARRAYS IS NOT FIXED.  THEY WILL BE  */     POS-0057
/* ALLOCATED WHEN N IS KNOWN.                                    */     POS-0058
/*****************************************************************/     POS-0059
     (R(*)) FIXED (8,5) CONTROLLED,                                     POS-0060
     (CHARGE(*),        /* TOTAL CHARGE WITHIN A CERTAIN RADIUS.    */   POS-0061
      J_P_I(*),         /* POSITIVE ION CURRENT DENSITY             */   POS-0062
      J_N_I(*),         /* NEGATIVE ION CURRENT DENSITY             */   POS-0063
      J_E(*),           /* ELECTRON CURRENT DENSITY                 */   POS-0064
      E(*),             /* ELECTRIC FIELD                           */   POS-0065
      CURRENT(*),       /* TOTAL CURRENT.                           */   POS-0066
      VOLTAGE(*),       /* VOLTAGE AS DEFINED FROM R(0)             */   POS-0067
      ALPHA(*),         /* NUMBER OF ELECTRONS CREATED /CM          */   POS-0068
      ETA(*),           /*  NUMBER OF ELECTRONS ATTACHED/CM         */   POS-0069
      ALPHA2(*), ETA2(*),                                               POS-0070
      PHOTO_ELE(*),     /* SOURCE DUE TO PHOTOIONIZATION            */   POS-0071
      PHOTO_ELE2(*),                                                    POS-0072
```

```
      G(*),             /* GEOMETRIC FACTOR                          */
      TOTAL_CUR(*),     /* VALUE OF SUM OF ALL THE CURRENT.          */
      KP(*),            /* KOP+2*K1P+2*K2P+K3P                       */
      KN(*),            /* KON+2*K1N+2*K2N+K3N                       */
      P_I_CHG_D(*),     /* POSITIVE ION CHARGE DENSITY               */
      N_I_CHG_D(*),     /* NEGATIVE ION CHARGE DENSITY               */
      ELE_CHG_D(*))     /* ELECTRON CHARGE DENSITY                   */
      BINARY FLOAT CONTROLLED;

/*****************************************************************/
/* THE PROCEDURE E_FIELD CALCULATES THE ELECTRIC FIELD WHEN THE   */
/* VOLTAGE AND CHARGE DENSITIES ARE KNOWN BETWEEN THE TWO         */
/* ELECTRODES.  SUM2 IS CALCULATED IN THE NO_SAVE DO LOOP OF THE  */
/* MAIN PROCEDURE.                                                */
/*****************************************************************/
 E_FIELD: PROCEDURE;
   ALLOCATE CHARGE (0:N);
   VOLT = KK * LOG(R(N)/R(0));
   SUM1 =0;
   CHARGE(0) = 0;
   OLD_CHG = R(0) * DELTA_R * (ELE_CHG_D(0) +
             N_I_CHG_D(0) + P_I_CHG_D(0) )/2;
   DO J = 1 TO N;
      NEW_CHG = R(J) * DELTA_R * ( ELE_CHG_D(J) +
                N_I_CHG_D(J) + P_I_CHG_D(J) ) /2;
      CHARGE(J) = (OLD_CHG+ NEW_CHG) + CHARGE(J-1);
      OLD_CHG = NEW_CHG;
      SUM1 = SUM1 + CHARGE(J) / R(J);
   END;
   SUM1 = SUM1 - CHARGE (N) /2/R(N);
   E(0)   = (VOLT       /DELTA_R-SUM1/EPSILON) /
            (R(0) * (SUM2 + .5 /R(N) ) + .5);
   X = 0;
   Y = N;
   DO J = 1 TO N;
      E(J)   = ( CHARGE(J) /EPSILON+ R(0)*E(0)  )/R(J);
```

```
      IF E(J) >= 30400 THEN X=J;                                          POS-0109
         ELSE IF E(J) >= 19000 THEN Y =J;                                 POS-0110
   END;                                                                   POS-0111
   FREE CHARGE;                                                           POS-0112
   END E_FIELD;                                                           POS-0113
                                                                          POS-0114
/*********************************************************************/  POS-0115
/* THE PROCEDURE OUT IS CALLED WHEN OUTPUT IS DESIRED.               */  POS-0116
/* THE FIRST DO LOOP CALCULATE THE VOLTAGE AS A FUNCTION OF          */  POS-0117
/* POSITION ONCE THE ELECTRIC FIELD DISTRIBUTION IS KNOWN.  IT       */  POS-0118
/* PROVIDES A CHECK ON THE ACCURACY OF THE ELECTRIC FIELD.           */  POS-0119
/*********************************************************************/  POS-0120
OUT: PROCEDURE;                                                           POS-0121
        ALLOCATE VOLTAGE(0:N);                                            POS-0122
         VOLTAGE (N) =0;                                                  POS-0123
        DO J = N-1 TO 0 BY -1;                                            POS-0124
            VOLTAGE(J) = VOLTAGE(J+1) + E(J) * DELTA_R;                   POS-0125
        END;                                                              POS-0126
                                                                          POS-0127
                                                                          POS-0128
/*********************************************************************/  POS-0129
/* NOW OUTPUT THE VALUES ASSOCIATED WITH INPUT DATA.                 */  POS-0130
/*********************************************************************/  POS-0131
        PUT LIST ('CASE'||CASE||'       '||I||'TH. ITERATION.      '      POS-0132
                  ||'DELTA_R = '|| DELTA_R||'    OVERSHUT ='||           POS-0133
                  OVER||'  RECOMBINATION ='||RECOMB) PAGE;                POS-0134
        PUT LIST (' PHOTO_MU ='||PHOTOMU||'  EFF ='||EFF||                POS-0135
                  '  MOD_ETA='||MOD_ETA) SKIP;                            POS-0136
         PUT LIST ('MOB_P_I='||MOB_P_I||'    MOB_N_I='||MOB_N_I           POS-0137
                  ||'  VELOCITY OF ELECTRON  IS 4.5E5*E/P '||              POS-0138
                  'FOR E/P<25.  IT IS 2.5E5*E/P+5.E6 FOR '||              POS-0139
                  'E/P>=25.') SKIP;                                       POS-0140
        PUT LIST ('AT R ='||R(0)||'CM  J_P_I = 0.        '||              POS-0141
                  'VOLTAGE = '||VOLT||'  E = '||KK/R(0)||                 POS-0142
                  ' (WHEN FREE OF SPACE CHARGES) ') SKIP;                 POS-0143
        PUT LIST ('AT R ='||R(N)||'CM  J_N_I = 0.        '||              POS-0144
```

```
                    'VOLTAGE = 0.     J_E = '||J_E(N)) SKIP;                    POS-0145
/*****************************************************************/             POS-0146
/* THE NEXT 3 PUT EDIT STATEMENTS PRINT OUT THE HEADINGS FOR THE  */             POS-0147
/* LIST OF DATA.                                                  */             POS-0148
/*****************************************************************/             POS-0149
        PUT EDIT ('RADIUS', 'E FIELD', 'VOLTAGE ', 'POSITIVE',                  POS-0150
                 'NEGATIVE', 'ELECTRON', 'POSITIVE', 'NEGATIVE',                POS-0151
                 'ELECTRON','TOTAL','ALPHA', 'ETA', 'PHOTO_ELE')                POS-0152
                 ( A(6), X(1), A(7), X(2), A(9), X(3), A(8), X(3),              POS-0153
                 A(8), X(3), A(8), X(2), A(8), X(4), A(8), X(4),                POS-0154
                 A(8),X(3), A(5), X(4), A(5), X(4), A(3), X(4), A(9))           POS-0155
                 SKIP(2);                                                       POS-0156
        PUT EDIT ('ION CHARGE', 'ION CHARGE', 'CHARGE',                         POS-0157
                 'ION CURRENT', 'ION CURRENT', 'CURRENT', 'CURRENT')            POS-0158
                 ( X(28), A(10), X(1), A(10), X(1), A(6), X(4),                 POS-0159
                 A(11), X(1), A(11), X(1), A(7), X(4),A(7)) SKIP;               POS-0160
        PUT EDIT ('DENSITY', 'DENSITY', 'DENSITY', 'DENSITY',                   POS-0161
                 'DENSITY', 'DENSITY')                                          POS-0162
                 ( X(28), A(7), X(4), A(7), X(4), A(7), X(3),                   POS-0163
                 A(7), X(5), A(7), X(5), A(7), X(6) ) SKIP;                     POS-0164
/*****************************************************************/             POS-0165
/* THIS SECTION ACTUALLY OUTPUT THE DATA . ONLY ONE OUT OF A      */             POS-0166
/* SPECIFIED NUMBER OF GRID POINTS IS ACTUALLY PRINTED TO SAVE    */             POS-0167
/* PRINTING COST.  THE VARIABLE INCR TAKES CARE OF THE SPECIFIED  */             POS-0168
/* NUMBER.                                                        */             POS-0169
/*****************************************************************/             POS-0170
        DO J=0 TO N BY INCR;                                                    POS-0171
          PUT EDIT (R(J), E(J), VOLTAGE(J), P_I_CHG_D(J),                       POS-0172
                 N_I_CHG_D(J), ELE_CHG_D(J), J_P_I(J), J_N_I(J),                POS-0173
                 J_E(J), CURRENT(J), ALPHA(J), ETA(J), PHOTO_ELE(J))            POS-0174
                ( F(6,4), E(11,4), E(10,3), X(1), E(10,3), X(1),                POS-0175
                 E(10,3), X(1), E(10,3), E(11,4), E(11,4),                      POS-0176
                 E(11,4), E(11,4), E(9,2), E(9,2), E(9,2))SKIP;                 POS-0177
        END;                                                                    POS-0178
        FREE VOLTAGE;                                                           POS-0179
   END OUT;                                                                     POS-0180
```

```
/*****************************************************************/
/* DATA ARE INPUTED AND VALUES INITIALIZED IN THE NEXT SECTION   */
/* BEFORE THE II DO LOOP.                                        */
/*****************************************************************/
   EPSILON= 8.854E-14;       /* COULOMB /VOLTS /CM   */
   PI  = 3.14159;
   CASE = 0;
   GET LIST (N, SAVE, CARD_IN, CARD_OUT);
   R_OLD = -1;
/*****************************************************************/
/* THE CONDITION THAT THE PROGRAM WILL CONTINUE IS THAT THE VALUE */
/* OF N INPUTED IS GREATER THAN ZERO.  THE MAIN PORTION OF       */
/* THE PROGRAM IS LOCATED IN THE CC DO LOOP.                     */
/*****************************************************************/
CC: DO WHILE (N>0);
    CASE = CASE +1;
    GET LIST (R_INNER, DELTA_R, INCR, M, OUT_NUM, E0, ENTERJE,
              MOB_P_I, MOB_N_I, A(1), B(1), A(2), B(2), PHOTOMU,
              EFF, MOD_ETA, OVER, EST, RECOMB, PAIRS);
    ALLOCATE TOTAL_CUR(0:M);
    KK = E0*R_INNER;
    TOTAL_CUR(*) = 0;
    TOTAL_CUR(0) = -1;
    I_DIFF = 10;
    COSMIC = PAIRS * 1.602E-19;
    AA(1) = A(1)*760;
    AA(2) = A(2)*760;
    BB(1) = B(1)*760;
    BB(2) = B(2)*760;
/*****************************************************************/
/* IF NO DATA IS SAVED, MOST OF THE VARIABLES WILL HAVE TO BE    */
/* INITIALIZED IN THE NO_SAVE DO LOOP.  ELECTRIC FIELD WITHOUT   */
/* SPACE CHARGES IN THE FORM OF KK/R IS CALCULATED.  IF DATA IS  */
/* SAVED, THEN N AND INNER AND OUTER CONDUCTORS ARE ASSUMED TO   */
/* BE THE SAME.                                                  */
```

POS-0181 POS-0182 POS-0183 POS-0184 POS-0185 POS-0186 POS-0187 POS-0188 POS-0189 POS-0190 POS-0191 POS-0192 POS-0193 POS-0194 POS-0195 POS-0196 POS-0197 POS-0198 POS-0199 POS-0200 POS-0201 POS-0202 POS-0203 POS-0204 POS-0205 POS-0206 POS-0207 POS-0208 POS-0209 POS-0210 POS-0211 POS-0212 POS-0213 POS-0214 POS-0215 POS-0216

```
/*****************************************************************/                POS-0217
    IF SAVE = 'NO ' THEN                                                            POS-0218
NO_SAVE: DO;                                                                        POS-0219
      ALLOCATE R(0:N), J_E(0:N), E(0:N), ELE_CHG_D(0:N),                            POS-0220
        P_I_CHG_D(0:N), N_I_CHG_D(0:N), J_N_I(0:N), J_P_I(0:N),                     POS-0221
        ALPHA(0:N), ETA(0:N), PHOTO_ELE(0:N), CURRENT(0:N);                         POS-0222
      R(0) = R_INNER;                                                               POS-0223
      SUM2 = 0;                                                                     POS-0224
      X = 0;                                                                        POS-0225
      Y = N;                                                                        POS-0226
      DO K = 1 TO N-1;                                                              POS-0227
        R(K) = R(K-1) + DELTA_R;                                                    POS-0228
        SUM2 = SUM2 + 1./R(K);                                                      POS-0229
        E(K) = KK/R(K);                                                             POS-0230
        IF E (K) >= 30400 THEN X = K;                                               POS-0231
           ELSE IF E(K) >= 19000 THEN Y = K;                                        POS-0232
      END;                                                                          POS-0233
      E(0) = KK/R(0);                                                               POS-0234
      R(N) = R(N-1) + DELTA_R;                                                      POS-0235
      E(N) = KK/R(N);                                                               POS-0236
/*****************************************************************/                POS-0237
/* THE FOLLOWING DO LOOP INPUT DATA FROM PREVIOUS RUNS SO AS TO   */                POS-0238
/* CONTINUE THE ITERATION PROCESS IF DESIRED.                     */                POS-0239
/*****************************************************************/                POS-0240
      IF CARD_IN ='YES' THEN                                                        POS-0241
      DO;                                                                           POS-0242
        GET LIST ((ELE_CHG_D(K) DO K =0 TO N));                                     POS-0243
        GET LIST ((P_I_CHG_D(K) DO K =0 TO N));                                     POS-0244
        GET LIST ((N_I_CHG_D(K) DO K =0 TO N));                                     POS-0245
      END;                                                                          POS-0246
/*****************************************************************/                POS-0247
/*  THE FOLLOWING SECTION CALCULATES THE GEOMETRIC FACTOR.        */                POS-0248
/*****************************************************************/                POS-0249
      IF (CASE ¬=1) & (R_OLD ¬= R_INNER) THEN FREE G;                               POS-0250
      IF R_OLD ¬= R_INNER THEN                                                      POS-0251
      DO;                                                                           POS-0252
```

```
      ALLOCATE G(0:N);                                                    POS-0253
      R_OUT =10;                                                          POS-0254
      DO K =0 TO N;                                                       POS-0255
         TEMP = (R(0)**2 - SQRT( (R(K)**2-R(0)**2) *                      POS-0256
                (R_OUT**2 - R(0)**2) ) )/R_OUT/R(K);                      POS-0257
         ANGLE = PI/2 - ATAN(TEMP/SQRT(1-TEMP**2));                       POS-0258
         G(K) = (ANGLE/2 + ATAN( (R_OUT+R(K)) * TAN(ANGLE/2)              POS-0259
                /(R_OUT-R(K)) ))/PI;                                      POS-0260
      END;                                                                POS-0261
    END;                                                                  POS-0262
  END NO_SAVE;                                                            POS-0263
                                                                          POS-0264
/*****************************************************************/       POS-0265
/*  THE ELECTRON CURRENT DENSITY DISTRIBUTION IS INITIALIZED IF   */       POS-0266
/* PREVIOUS DATA IS  AVAILABLE .  PHOTOIONIZATION TERM CAN BE     */       POS-0267
/* CALCULATED IF THE ELECTRON CURRENT DENSITY IS KNOWN.           */       POS-0268
/*****************************************************************/       POS-0269
  IF SAVE = 'YES' | CARD_IN = 'YES' THEN                                  POS-0270
  DO;                                                                     POS-0271
    CALL E_FIELD;                                                         POS-0272
    DO K = 0 TO Y;                                                        POS-0273
      J_E(K) = -(5.E6*329*E(K))*ELE_CHG_D(K);                             POS-0274
    END;                                                                  POS-0275
    DO K = Y+1 TO N;                                                      POS-0276
      J_E(K) = -592 * E(K) * ELE_CHG_D(K);                                POS-0277
    END;                                                                  POS-0278
  END;                                                                    POS-0279
                                                                          POS-0280
                                                                          POS-0281
/*****************************************************************/       POS-0282
/* THE ITERATION PROCESS IS PERFORMED WITHIN THE II DO LOOP.      */       POS-0283
/* TO GUARD AGAINST THE UNSTABLE CASE, THE MAXIMUM NUMBER OF      */       POS-0284
/* ITERATIONS ALLOWED IS M.  HOWEVER IF THE TOTAL CURRENT STOPPED */       POS-0285
/* INCREASING AND THE DIFFERENCE BETWEEN PRESENT AND PREVIOUS     */       POS-0286
/* TOTAL CURRENT IS WITHIN .5 PERCENT OF THE PRESENT TOTAL        */       POS-0287
/* CURRENT, AND THE OVERSHUT PARAMETER( OVER) IS SET AT 1, THEN   */       POS-0288
```

```
/* THE ITERATION PROCESS STOPS.  THE OVERSHUT PARAMETER IS TO     */
/* INCREASE THE GROWTH RATE. IF THE TOTAL CURRENT REACHED THE     */
/* ESTIMATED TOTAL CURRENT, THE OVERSHUT PARAMETER IS SET TO 1.   */
/* WHEN OVER IS NOT 1, THE PROGRAM IS ACTUALLY SOLVING A          */
/* DIFFERENT PROBLEM.                                             */
/*****************************************************************/
I1: DO I = 1 TO M WHILE ((I_DIFF>0 | I_DIFF<-.005)| OVER¬=1);
      IF TOTAL_CUR(I-1) >= EST THEN OVER =1;
      ALLOCATE ALPHA2(0:N), ETA2(0:N), PHOTO_ELE2(0:N);
      J_N_I(N) =0;
      J_E(N) = ENTERJE;
/*****************************************************************/
/* ALPHA AND ETA ARE CALCULATED.  T=1 IS FOR THE CASE E/P>=40 IN  */
/* THE CALCULATION OF ALPHA.                                      */
/*****************************************************************/
      T=1;
      DO K = 0 TO N;
        IF K=X THEN T=2;
        EO = E(K);
        ALPHA(K) = AA(T)*EXP(-BB(T)/EO);
        IF K <= Y THEN
        ETA(K) =(1.203E-8*EO*EO -.000567*EO + 10.12)*MOD_ETA;
                ELSE ETA(K) = (1.545E-4*EO + .866)*MOD_ETA;
      END;
/*****************************************************************/
/* ALPHA2 AND ETA2 ARE FOR THE VALUES OF ALPHA AND ETA HALF WAY   */
/* BETWEEN THE GRID POINTS.  THIS IS NECESSARY FOR RUNGE-KUTTA    */
/* INTEGRATION.                                                   */
/*****************************************************************/
      T=1;
      DO K = 2 TO N-1;
        IF K=X THEN T=2;
        EO = (-E(K-2)-E(K+1)+9*(E(K-1)+E(K)))/16;
        ALPHA2(K) = AA(T) * EXP( -BB(T)/EO);
        IF K <= Y THEN
        ETA2(K)=(1.203E-8*EO*EO -.000567*EO + 10.12)*MOD_ETA;
```

```
                ELSE ETA2(K) = (1.545E-4*E0 + .866)*MOD_ETA;                    POS-0325
      END;                                                                      POS-0326
      E0 = (5*E(N) +E(N-3)+15*E(N-1)-5* E(N-2) )/16;                            POS-0327
      ALPHA2(N) = AA(T) * EXP (-BB(T)/E0);                                      POS-0328
      IF E0 >= 19000 THEN                                                       POS-0329
      ETA2 (N) =(1.203E-8*E0*E0 -.000567*E0 + 10.12)*MOD_ETA;                   POS-0330
                ELSE ETA2(N) = (1.545E-4*E0 + .866)*MOD_ETA;                    POS-0331
      E0 = (5*E(0)+E(3)+E(1)*15-5*E(2))/16;                                     POS-0332
      IF E0 >= 30400 THEN T=1;                                                  POS-0333
                     ELSE T=2;                                                  POS-0334
      ALPHA2(1) = AA(T) * EXP( -BB(T)/E0);                                      POS-0335
      IF E0 >= 19000 THEN                                                       POS-0336
      ETA2 (1) =(1.203E-8*E0*E0 -.000567*E0 + 10.12)*MOD_ETA;                   POS-0337
                ELSE ETA2(1) = (1.545E-4*E0 + .866)*MOD_ETA;                    POS-0338
                                                                                POS-0339
/*****************************************************************/             POS-0340
/* IF THERE IS NO PREVIOUS DATA USED IN THE FIRST ITERATION,     */             POS-0341
/* PHOTOIONIZATION IS ASSUMED NOT TO BE PRESENT SINCE ELECTRON   */             POS-0342
/* CHARGE DENSITY IS NOT KNOWN.                                  */             POS-0343
/*****************************************************************/             POS-0344
   IF I =1 & SAVE ='NO '& CARD_IN='NO ' THEN                                    POS-0345
     DO;                                                                        POS-0346
      PHOTO_ELE(*) =0;                                                          POS-0347
      PHOTO_ELE2(*) = 0;                                                        POS-0348
      P_I_CHG_D(*) =0;                                                          POS-0349
      N_I_CHG_D(*) =0;                                                          POS-0350
     END;                                                                       POS-0351
                                                                                POS-0352
/*****************************************************************/             POS-0353
/* CALCULATE PHOTOIONIZATION TERM.                               */             POS-0354
/*****************************************************************/             POS-0355
     ELSE DO;                                                                   POS-0356
      PHOTO_ELE(0), PHOTO_ELE2(0) = 0;                                          POS-0357
      TEMP = PHOTOMU * EFF* CVER;                                               POS-0358
      ABS_MU = PHOTOMU * 1.8;                                                   POS-0359
      K0 = ALPHA(0) * R(0) * J_E(0) * EXP( ABS_MU * R(0) ) /8;                  POS-0360
```

```
      INT=0;                                                              POS-0361
      DO K = 1 TO N;                                                      POS-0362
         TEMP1 = (R(K)+R(K-1))/2;                                         POS-0363
         K1 = ALPHA2 (K) *(J_E(K)+J_E(K-1) ) * TEMP1 *                    POS-0364
              (G(K)+G(K-1)) * EXP(ABS_MU*TEMP1)/16;                       POS-0365
         INT = INT + K0 +K1;                                              POS-0366
         PHOTO_ELE2(K) = INT       * TEMP*EXP(-ABS_MU*TEMP1);             POS-0367
         K0 = ALPHA(K) * R(K)*J_E(K)*G(K)*EXP(ABS_MU*R(K))/4;             POS-0368
         INT = INT + K0 +K1;                                              POS-0369
         PHOTO_ELE(K) = INT * TEMP * EXP (-ABS_MU * R(K));                POS-0370
      END;                                                                POS-0371
     END;                                                                 POS-0372
                                                                          POS-0373
/*********************************************************************/   POS-0374
/* RUNGE-KUTTA METHOD IS USED FOR INTEGRATION BECAUSE IT IS SELF   */     POS-0375
/* STARTING AND GENERALLY RELIABLE METHOD.  HOWEVER, IT IS         */     POS-0376
/* PROBABLLY MORE EXPENSIVE THAN SOME OTHER METHODS. THE           */     POS-0377
/* PROCESSES INCLUDED ARE ELECTRON AVALANCHE, ATTACHMENT, COSMIC   */     POS-0378
/* RADIATION, PHOTOIONIZATION, AND RECOMBINATION.                  */     POS-0379
/*********************************************************************/   POS-0380
    ALLOCATE KP(0:N), KN(0:N);                                            POS-0381
      R_JE = ENTERJE  * R(N);                                             POS-0382
      TEMP1=0;                                                            POS-0383
      DO K = N TO 1 BY -1;                                                POS-0384
        KON = ETA(K)*R_JE + TEMP1;                                        POS-0385
        KOP = ALPHA(K)*R_JE + PHOTO_ELE(K) + COSMIC*R(K) +                POS-0386
              TEMP1;                                                      POS-0387
        TEMP = DELTA_R *(KOP-KON)/2 + R_JE;                               POS-0388
        TEMP2=RECOMB*(P_I_CHG_D(K)+P_I_CHG_D(K-1))*(R(K)+R(K-1))*         POS-0389
              (N_I_CHG_D(K) + N_I_CHG_D(K-1))/8/1.602E-19;                POS-0390
        K1N = TEMP * ETA2(K) + TEMP2;                                     POS-0391
        K1P = TEMP*ALPHA2(K) + PHOTO_ELE2(K) + TEMP2 +                    POS-0392
              COSMIC*(R(K)+R(K-1))/2;                                     POS-0393
        TEMP = DELTA_R *(K1P-K1N)/2 + R_JE;                               POS-0394
        K2N = TEMP * ETA2(K) + TEMP2;                                     POS-0395
        K2P = TEMP*ALPHA2(K) + PHOTO_ELE2(K) + TEMP2 +                    POS-0396
```

```
            COSMIC*(R(K)+R(K-1))/2;                                    POS-0397
         TEMP = DELTA_R * (K2P-K2N) + R_JE;                            POS-0398
         TEMP1 = RECOMB * P_I_CHG_D(K-1) * N_I_CHG_D(K-1)              POS-0399
                 * R(K-1)/1.602E-19;                                   POS-0400
         K3N = TEMP * ETA(K-1) + TEMP1;                                POS-0401
         K3P = TEMP*ALPHA(K-1) + PHOTO_ELE(K-1) + TEMP1 +              POS-0402
               COSMIC *R(K-1);                                         POS-0403
         KP(K) =(K0P+ 2*K1P + 2 *K2P + K3P)*DELTA_R/6;                 POS-0404
         KN(K)=(K0N+2*K1N+2*K2N+K3N)*DELTA_R /6;                       POS-0405
         R_JE = R_JE + KP(K) - KN(K);                                  POS-0406
       END;                                                            POS-0407
      FREE ALPHA2, ETA2, PHOTO_ELE2;                                   POS-0408
                                                                       POS-0409
/*****************************************************************/    POS-0410
/* KP AND KN ARE THE NET GAIN IN POSITIVE AND NEGATIVE CURRENT    */    POS-0411
/* DENSITIES RESPECTIVELY BETWEEN TWO GRID POINTS.  THE CURRENT   */    POS-0412
/* DENSITIES ARE THEN CALCULATED FROM THEM DIRECTLY.              */    POS-0413
/*****************************************************************/    POS-0414
      R_J_N=0;                                                         POS-0415
      R_JE = ENTERJE * R(N);                                           POS-0416
      DO K = N-1  TO 0 BY -1;                                          POS-0417
         R_JE = R_JE + KP(K+1) - KN(K+1);                              POS-0418
         J_E(K) = R_JE/R(K);                                           POS-0419
         R_J_N = R_J_N + KN(K+1);                                      POS-0420
         J_N_I(K) = R_J_N/R(K);                                        POS-0421
      END;                                                             POS-0422
      R_J_P = 0;                                                       POS-0423
      J_P_I(0) =0;                                                     POS-0424
      DO K = 1 TO N;                                                   POS-0425
         R_J_P = R_J_P + KP(K);                                        POS-0426
         J_P_I(K) = R_J_P/R(K);                                        POS-0427
      END;                                                             POS-0428
      FREE KP, KN;                                                     POS-0429
                                                                       POS-0430
/*****************************************************************/    POS-0431
/* THE ELECTRON, POSITIVE AND NEGATIVE CHARGE DENSITIES, AND THE  */    POS-0432
```

```
/* TOTAL CURRENT IS CALCULATED.                                          */
/************************************************************************/
      DO K = 0 TO Y;
         ELE_CHG_D(K) = -J_E(K)/(5.E6 + 329*E(K));
      END;
      DO K = Y+1 TO N;
         ELE_CHG_D(K) = -J_E(K)/592/E(K);
      END;
      DO K = 0 TO N;
        N_I_CHG_D(K) = -J_N_I(K)/MOB_N_I/E(K);
        P_I_CHG_D(K) = J_P_I(K)/MOB_P_I/E(K);
        CURRENT(K) = (J_E(K) + J_P_I(K) + J_N_I(K)) *2*PI * R(K);
      END;
        I_DIFF = (CURRENT(0)-TOTAL_CUR(I-1))/CURRENT(0);
        TOTAL_CUR(I) = CURRENT(0);

/************************************************************************/
/* A NEW SET OF CHARGE DENSITIES IS NOW KNOWN.  SO A NEW ELECTRIC       */
/* DISTRIBUTION IS CALCULATED, AND THE PROCESS REPEATES.                */
/************************************************************************/
        CALL E_FIELD;

/************************************************************************/
/* THE DATA IS OUTPUTED IF THE PROCESS IS ALREADY WITHIN A              */
/* SPECIFIED  (OUT_NUM) POINT OF PRINTING OR WHEN THE DIFFERENCE        */
/* BETWEEN ITERATIONS IS VERY SMALL.                                    */
/************************************************************************/
       IF I>(M-OUT_NUM) | ( (I_DIFF<.00001)&(I_DIFF>-.0055) ) THEN
          CALL OUT;
   END II;

/************************************************************************/
/* OCCASIONALLY MORE ITERATIONS ARE NEEDED FOR CONVERGENCE THAN         */
/* THE SPECIFIED.  YET ONE DOES NOT WANT TO WASTE THE PRESENT           */
/* EFFORT.  SO ELECTRON, POSITIVE ION AND NEGATIVE ION CHARGE           */
/* DENSITIES ARE OUTPUTED ON CARDS IF ITERATION DID NOT CONVERGE        */
```

```
POS-0433
POS-0434
POS-0435
POS-0436
POS-0437
POS-0438
POS-0439
POS-0440
POS-0441
POS-0442
POS-0443
POS-0444
POS-0445
POS-0446
POS-0447
POS-0448
POS-0449
POS-0450
POS-0451
POS-0452
POS-0453
POS-0454
POS-0455
POS-0456
POS-0457
POS-0458
POS-0459
POS-0460
POS-0461
POS-0462
POS-0463
POS-0464
POS-0465
POS-0466
POS-0467
POS-0468
```

```
/* AND IF INPUT SPECIFICALLY NOTED  CARD_OUT='YES'.                    */          POS-0469
/*************************Q*************************************************/          POS-0470
   IF CARD_OUT = 'YES' THEN                                                         POS-0471
   IF I>M THEN                                                                      POS-0472
   DO;                                                                              POS-0473
   PUT FILE (PUNCH) EDIT (( ELE_CHG_D(K) DO K = 0 TO N))                            POS-0474
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                            POS-0475
   PUT FILE (PUNCH) EDIT (( P_I_CHG_D(K) DO K = 0 TO N))                            POS-0476
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                            POS-0477
   PUT FILE (PUNCH) EDIT (( N_I_CHG_D(K) DO K = 0 TO N))                            POS-0478
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                            POS-0479
   END;                                                                             POS-0480
                                                                                    POS-0481
   R_OLD = R_INNER;                                                                 POS-0482
   GET LIST (N, SAVE, CARD_IN, CARD_OUT);                                           POS-0483
   IF SAVE ='NO ' THEN FREE R, J_E, E, ELE_CHG_D, N_I_CHG_D,                        POS-0484
      P_I_CHG_D, J_N_I, J_P_I, ALPHA, ETA, PHOTO_ELE, CURRENT;                      POS-0485
                                                                                    POS-0486
/*****************************************************************/              POS-0487
/* THE CURRENT IN TERMS OF ITERATIONS ARE OUTPUTED SO THAT THE    */              POS-0488
/* RATE OF CONVERGENCE IS KNOWN.  ALSO ADDITIONAL INPUT           */              POS-0489
/* PARAMETERS ARE PRINTED FOR REFERENCE.                          */              POS-0490
/*****************************************************************/              POS-0491
   DO J = 1 TO I-1 BY 3;                                                            POS-0492
      PUT LIST('I='||J||'   CURRENT='||TOTAL_CUR(J)||                               POS-0493
        '   I='||J+1||'   CURRENT='||TOTAL_CUR(J+1)||                               POS-0494
        '   I='||J+2||'   CURRENT='||TOTAL_CUR(J+2) ) SKIP;                         POS-0495
   END;                                                                             POS-0496
   PUT LIST ('A(1) ='||A(1)||'  B(1) ='||B(1)|| '  A(2) ='||                       POS-0497
             A(2)||'  B(2) ='||B(2) ) SKIP(2);                                      POS-0498
   PUT LIST (' THE INITIAL SOURCE ARE '||PAIRS||' CHARGED'||                        POS-0499
             ' PAIRS PRODUCED BY COSMIC RADIATION PER CC PER'||                      POS-0500
             ' SEC, AND ELECTRON CURRENT DENSITY OF    '||                          POS-0501
             ENTERJE||' AMPS PER CM**2 AT THE OUTER RADIUS.')SKIP;                  POS-0502
   FREE TOTAL_CUR;                                                                  POS-0503
   END CC;                                                                          POS-0504
```

```
STOPP: END SS;
```

POS-0505

Appendix 5

Program Listing for Negative Corona

To use the negative corona program, the input data is in the same order as positive corona case from 1 to 24. The ENTERJE now should be a quantity less than or equal to zero. In addition, two more parameters are needed.

25. PHOTO_E - This is the number of electrons emitted at the cathode per photon arrived, γ_p.
26. ION_B - This is the number of electrons emitted at the cathode per positive ion arrived, γ_i.

```
 SS: PROCEDURE OPTIONS (MAIN);                                          NEG-0001
/*****************************************************************/     NEG-0002
/* THIS PROGRAM CALCULATES THE STEADY STATE CHARGE DENSITY        */     NEG-0003
/* FOR NEGATIVE CORONA WHEN THE SOURCE IS ELECTRON CURRENT        */     NEG-0004
/* DENSITY AT THE INNER BOUNDARY, PHOTO EMISSION, ION BOMBARDMENT */     NEG-0005
/* AT THE CATHODE, AND OR PHOTOIONIZATION IN THE INTERIOR.        */     NEG-0006
/* PREVIOUS RESULTS WILL BE USED IF ANY. OVERSHUT PARAMETER IS    */     NEG-0007
/* USED FOR FASTER CONVERGENCE.  RUNGE-KUTTA METHOD IS USED FOR   */     NEG-0008
/* THE INTEGRATION.                                               */     NEG-0009
/*****************************************************************/     NEG-0010
                                                                         NEG-0011
   DECLARE (R_INNER,  /* RADIUS OF INNER CYLINDER                  */     NEG-0012
      OVER,           /* ADJUSTED OVERSHUT PARAMETER.              */     NEG-0013
      R_OLD,          /* INNER RADIUS                              */     NEG-0014
      MOD_ETA,        /* FACTOR BY WHICH ETA IS CHANGED            */     NEG-0015
      DELTA_R) FIXED (7,4),                                              NEG-0016
     (N,              /* ACTUAL NUMBER OF DIVISIONS.               */     NEG-0017
      INCR,           /* OUTPUT INCREMENT.                         */     NEG-0018
      OUT_NUM,        /* # OF TIMES OUTPUT WILL BE PRINTED         */     NEG-0019
      X,              /* GRID PT. E/P=40                           */     NEG-0020
      Y,              /* GRID PT. E/P=25                           */     NEG-0021
      I, J, K, CASE, M, T) FIXED(15,0) BINARY,                           NEG-0022
     (SAVE,           /* SAVING PREVIOUS DATA?                     */     NEG-0023
      CARD_IN,        /* ANY INPUT DATA                            */     NEG-0024
      CARD_OUT)       /* OUTPUT DATA ON CARDS                      */     NEG-0025
       CHARACTER(3),                                                     NEG-0026
     (SUM1, SUM2,                                                        NEG-0027
      PAIRS,          /* ELECTRON/ION PAIRS/CC BY COSMIC RAY       */     NEG-0028
      COSMIC,         /* PAIRS*1.602E-19                           */     NEG-0029
      RECOMB,         /* RECOMBINATION COEFFICIENT                 */     NEG-0030
      A(2),B(2),AA(2),BB(2),   /* USED IN ALPHA                    */     NEG-0031
      E0,             /* VALUE OF E                                */     NEG-0032
      TEMP, TEMP1, TEMP2,                                                NEG-0033
      PHOTO_E,        /* PHOTO EMISSION COEFFICIENT                */     NEG-0034
      ION_B,          /* ION BOMBARDMENT COEFFICIENT               */     NEG-0035
      INT1, INT2, INT3,                                                  NEG-0036
```

```
      VOLT,              /* VOLTAGE APPLIED BETWEEN CYLINDERS          */   NEG-0037
      ENTERJE,           /* DIFFERENT BOUNDARY CONDITIONS              */   NEG-0038
      R_JE,              /* R(K) * J_E(K)                              */   NEG-0039
      EST,               /* ESTIMATE OF FINAL  TOTAL CURRENT.          */   NEG-0040
      R_J_N,             /* R * J_N_I                                  */   NEG-0041
      R_J_P,             /* R * J_P_I                                  */   NEG-0042
      KK1, KK2, KK3,                                                       NEG-0043
      K0, K1, K2, K3,                                                      NEG-0044
      K0P, K1P, K2P, K3P,                                                  NEG-0045
      K0N, K1N, K2N, K3N,                                                  NEG-0046
      KK,                /* KK/R(0) IS E(0) FOR NO SPACE CHG           */   NEG-0047
      MOB_N_I,           /* MOBILITY OF NEGATIVE IONS                  */   NEG-0048
      MOB_P_I,           /* MOBILITY OF POSITIVE IONS                  */   NEG-0049
      PHOTOMU,           /* PHOTON IONIZATION COEFFICIENT              */   NEG-0050
      ABS_MU,            /* PHOTON ABSORPTION COEFFICIENT *1.86        */   NEG-0051
      EFF,               /* NUMBER OF PHOTONS CREATED PER ELECTRON
                            CREATED                                    */   NEG-0052
      PI,                /* CONSTANT                                   */   NEG-0053
      OLD_CHG, NEW_CHG,                                                    NEG-0054
      I_DIFF,            /* CHANGE OF TOTAL I BETWEEN ITERATIONS       */   NEG-0055
      R_OUT, ANGLE,      /* FOR CALCULATING GEOMETRIC FACTOR           */   NEG-0056
      EPSILON) BINARY FLOAT,                                               NEG-0057
/*************************************************************/            NEG-0058
/* THE SIZE OF THE FOLLOWING ARRAYS IS NOT FIXED.  THEY WILL BE  */         NEG-0059
/* ALLOCATED WHEN N IS KNOWN.                                    */         NEG-0060
/*************************************************************/            NEG-0061
     (R(*)) FIXED (8,5) CONTROLLED,                                        NEG-0062
     (CHARGE(*),        /* TOTAL CHARGE WITHIN A CERTAIN RADIUS.      */   NEG-0063
      J_P_I(*),         /* POSITIVE ION CURRENT DENSITY               */   NEG-0064
      J_N_I(*),         /* NEGATIVE ION CURRENT DENSITY               */   NEG-0065
      J_E(*),           /* ELECTRON CURRENT DENSITY                   */   NEG-0066
      E(*),             /* ELECTRIC FIELD                             */   NEG-0067
      CURRENT(*),       /* TOTAL CURRENT.                             */   NEG-0068
      VOLTAGE(*),       /* VOLTAGE AS DEFINED FROM R(0)               */   NEG-0069
      ALPHA(*),         /* NUMBER OF ELECTRONS CREATED /CM            */   NEG-0070
      ETA(*),           /*  NUMBER OF ELECTRONS ATTACHED/CM           */   NEG-0071
                                                                           NEG-0072
```

```
      ALPHA2(*), ETA2(*),                                                    NEG-0073
      PHOTO_ELE(*),   /* SOURCE DUE TO PHOTOIONIZATION              */      NEG-0074
      PHOTO_ELE2(*),                                                         NEG-0075
      G(*),           /* GEOMETRIC FACTOR                           */      NEG-0076
      G1(*),        /* GEOMETRIC FACTOR FOR EXTERIOR                */      NEG-0077
      TOTAL_CUR(*),   /* VALUE OF SUM OF ALL THE CURRENT.           */      NEG-0078
      KP(*),          /* KOP+2*K1P+2*K2P+K3P                        */      NEG-0079
      KN(*),          /* KON+2*K1N+2*K2N+K3N                        */      NEG-0080
      P_I_CHG_D(*),   /* POSITIVE ION CHARGE DENSITY                */      NEG-0081
      N_I_CHG_D(*),   /* NEGATIVE ION CHARGE DENSITY                */      NEG-0082
      ELE_CHG_D(*))   /* ELECTRON CHARGE DENSITY                    */      NEG-0083
      BINARY FLOAT CONTROLLED;                                               NEG-0084
                                                                             NEG-0085
/***************************************************************/            NEG-0086
/* THE PROCEDURE E_FIELD CALCULATES THE ELECTRIC FIELD WHEN THE   */         NEG-0087
/* VOLTAGE AND CHARGE DENSITIES ARE KNOWN BETWEEN THE TWO         */         NEG-0088
/* ELECTRODES.  SUM2 IS CALCULATED IN THE NO_SAVE DO LOOP OF THE  */         NEG-0089
/* MAIN PROCEDURE.                                                */         NEG-0090
/***************************************************************/            NEG-0091
 E_FIELD: PROCEDURE;                                                         NEG-0092
   ALLOCATE CHARGE (0:N);                                                    NEG-0093
   VOLT =-KK * LOG(R(N)/R(0));                                               NEG-0094
   SUM1 =0;                                                                  NEG-0095
   CHARGE(0) = 0;                                                            NEG-0096
   OLD_CHG = R(0) * DELTA_R * (ELE_CHG_D(0) +N_I_CHG_D(0) +                  NEG-0097
             P_I_CHG_D(0) )/2;                                               NEG-0098
   DO J = 1 TO N;                                                            NEG-0099
      NEW_CHG = R(J) * DELTA_R * ( ELE_CHG_D(J) +                            NEG-0100
                N_I_CHG_D(J) + P_I_CHG_D(J) ) /2;                            NEG-0101
      CHARGE(J) = (OLD_CHG+ NEW_CHG) + CHARGE(J-1);                          NEG-0102
      OLD_CHG = NEW_CHG;                                                     NEG-0103
      SUM1 = SUM1 + CHARGE(J) / R(J);                                        NEG-0104
   END;                                                                      NEG-0105
   SUM1 = SUM1 - CHARGE (N) /2/R(N);                                         NEG-0106
   E(0)    = (VOLT       /DELTA_R-SUM1/EPSILON) /                            NEG-0107
             (R(0) * (SUM2 + .5 /R(N) ) + .5);                               NEG-0108
```

```
   X = 0;                                                                    NEG-0109
   Y = N;                                                                    NEG-0110
   DO J = 1 TO N;                                                            NEG-0111
      E(J)   = ( CHARGE(J) /EPSILON+ R(0)*E(0)  )/R(J);                      NEG-0112
      IF -E(J) >= 30400 THEN X=J;                                            NEG-0113
         ELSE IF -E(J) >= 19000 THEN Y =J;                                   NEG-0114
   END;                                                                      NEG-0115
   FREE CHARGE;                                                              NEG-0116
   END E_FIELD;                                                              NEG-0117
                                                                             NEG-0118
/*************************************************************************/  NEG-0119
/* THE PROCEDURE OUT IS CALLED WHEN OUTPUT IS DESIRED.                   */  NEG-0120
/* THE FIRST DO LOOP CALCULATE THE VOLTAGE AS A FUNCTION OF              */  NEG-0121
/* POSITION ONCE THE ELECTRIC FIELD DISTRIBUTION IS KNOWN.  IT           */  NEG-0122
/* PROVIDES A CHECK ON THE ACCURACY OF THE ELECTRIC FIELD.               */  NEG-0123
/*************************************************************************/  NEG-0124
OUT: PROCEDURE;                                                              NEG-0125
        ALLOCATE VOLTAGE(0:N);                                               NEG-0126
         VOLTAGE (N) =0;                                                     NEG-0127
        DO J = N-1 TO 0 BY -1;                                               NEG-0128
            VOLTAGE(J) = VOLTAGE(J+1) + E(J) * DELTA_R;                      NEG-0129
        END;                                                                 NEG-0130
                                                                             NEG-0131
/*************************************************************************/  NEG-0132
/* NOW OUTPUT THE VALUES ASSOCIATED WITH INPUT DATA.                     */  NEG-0133
/*************************************************************************/  NEG-0134
        PUT LIST ('CASE'||CASE||'      '||I||'TH. ITERATION.     '           NEG-0135
                  ||'DELTA_R = '|| DELTA_R||'    OVERSHUT ='||               NEG-0136
                  OVER||'  RECOMBINATION ='||RECOMB) PAGE;                   NEG-0137
        PUT LIST ('PHOTO_MU ='||PHOTOMU||'   EFF ='||EFF||                   NEG-0138
           '   PHOTO EMISSION ='||PHOTO_E||'   ION BOMBARDMENT '             NEG-0139
           ||'COEFFICIENT ='||ION_B) SKIP;                                   NEG-0140
         PUT LIST ('MOB_P_I='||MOB_P_I||'   MOB_N_I='||MOB_N_I               NEG-0141
                  ||'  VELOCITY OF ELECTRON  IS 4.5E5* E/P'||                NEG-0142
                  'FOR E/P<25.  IT IS 2.5E5*E/P+5.E6 FOR '||                 NEG-0143
                  'E/P>=25.') SKIP;                                          NEG-0144
```

```
      PUT LIST ('AT R ='||R(0)||'CM  J_N_I = 0.  '||                    NEG-0145
               'VOLTAGE = '||VOLT||'  E = '||KK/R(0)||                  NEG-0146
               ' (WHEN FREE OF SPACE CHARGES)  ')SKIP;                  NEG-0147
      PUT LIST ('AT R ='||R(N)||'CM  J_P_I = 0.       '||               NEG-0148
               'VOLTAGE = 0.    MOD_ETA ='||MOD_ETA)SKIP;               NEG-0149
/*****************************************************************/     NEG-0150
/* THE NEXT 3 PUT EDIT STATEMENTS PRINT OUT THE HEADINGS FOR THE  */     NEG-0151
/* LIST OF DATA.                                                  */     NEG-0152
/*****************************************************************/     NEG-0153
      PUT EDIT ('RADIUS', 'E FIELD', 'VOLTAGE ', 'POSITIVE',            NEG-0154
               'NEGATIVE', 'ELECTRON', 'POSITIVE', 'NEGATIVE',          NEG-0155
               'ELECTRON','TOTAL','ALPHA', 'ETA', 'PHOTO_ELE')          NEG-0156
               ( A(6), X(1), A(7), X(2), A(9), X(3), A(8), X(3),        NEG-0157
               A(8), X(3), A(8), X(2), A(8), X(4), A(8), X(4),          NEG-0158
               A(8),X(3), A(5), X(4), A(5), X(4), A(3), X(4), A(9))     NEG-0159
               SKIP(2);                                                 NEG-0160
      PUT EDIT ('ION CHARGE', 'ION CHARGE', 'CHARGE',                   NEG-0161
               'ION CURRENT', 'ION CURRENT', 'CURRENT', 'CURRENT')      NEG-0162
               ( X(28), A(10), X(1), A(10), X(1), A(6), X(4),           NEG-0163
               A(11), X(1), A(11), X(1), A(7), X(4),A(7)) SKIP;         NEG-0164
      PUT EDIT ('DENSITY', 'DENSITY', 'DENSITY', 'DENSITY',             NEG-0165
               'DENSITY', 'DENSITY')                                    NEG-0166
               ( X(28), A(7), X(4), A(7), X(4), A(7), X(3),             NEG-0167
               A(7), X(5), A(7), X(5), A(7), X(6) ) SKIP;               NEG-0168
/*****************************************************************/     NEG-0169
/* THIS SECTION ACTUALLY OUTPUT THE DATA . ONLY ONE OUT OF A      */     NEG-0170
/* SPECIFIED NUMBER OF GRID POINTS IS ACTUALLY PRINTED TO SAVE    */     NEG-0171
/* PRINTING COST.  THE VARIABLE INCR TAKES CARE OF THE SPECIFIED  */     NEG-0172
/* NUMBER.                                                        */     NEG-0173
/*****************************************************************/     NEG-0174
      DO J=0 TO N BY INCR;                                              NEG-0175
        PUT EDIT (R(J), E(J), VOLTAGE(J), P_I_CHG_D(J),                 NEG-0176
               N_I_CHG_D(J), ELE_CHG_D(J), J_P_I(J), J_N_I(J),          NEG-0177
               J_E(J), CURRENT(J), ALPHA(J), ETA(J), PHOTO_ELE(J))      NEG-0178
              ( F(6,4), E(11,4), E(10,3), X(1), E(10,3), X(1),          NEG-0179
               E(10,3), X(1), E(10,3), E(11,4), E(11,4),                NEG-0180
```

```
                E(11,4), E(11,4), E(9,2), E(9,2), E(9,2))SKIP;                  NEG-0181
        END;                                                                    NEG-0182
        FREE VOLTAGE;                                                           NEG-0183
   END OUT;                                                                     NEG-0184
                                                                                NEG-0185
/*********************************************************************/         NEG-0186
/* DATA ARE INPUTED AND VALUES INITIALIZED IN THE NEXT SECTION     */          NEG-0187
/* BEFORE THE II DO LOOP.                                          */          NEG-0188
/*********************************************************************/         NEG-0189
   EPSILON= 8.854E-14;       /* COULOMB /VOLTS /CM    */                       NEG-0190
   PI  = 3.14159;                                                               NEG-0191
   CASE = 0;                                                                    NEG-0192
   GET LIST (N, SAVE, CARD_IN, CARD_OUT);                                       NEG-0193
   R_OLD = -1;                                                                  NEG-0194
/*********************************************************************/         NEG-0195
/* THE CONDITION THAT THE PROGRAM WILL CONTINUE IS THAT THE VALUE  */          NEG-0196
/* OF N INPUTED IS GREATER THAN ZERO.  THE MAIN PORTION OF         */          NEG-0197
/* THE PROGRAM IS LOCATED IN THE CC DO LOOP.                       */          NEG-0198
/*********************************************************************/         NEG-0199
CC: DO WHILE (N>0);                                                             NEG-0200
    CASE = CASE +1;                                                             NEG-0201
    GET LIST (R_INNER, DELTA_R, INCR, M, OUT_NUM, EO, ENTERJE,                  NEG-0202
              MOB_P_I, MOB_N_I, A(1), B(1), A(2), B(2), PHOTOMU,                NEG-0203
              EFF, MOD_ETA, OVER, EST, RECOMB, PAIRS, PHOTO_E,                  NEG-0204
              ION_B);                                                           NEG-0205
    ALLOCATE TOTAL_CUR(0:M);                                                    NEG-020o
    TOTAL_CUR(*) = 0;                                                           NEG-0207
    TOTAL_CUR(0) =  1;                                                          NEG-0208
    KK =EO*R_INNER;                                                             NEG-0209
    I_DIFF = 10;                                                                NEG-0210
    COSMIC = PAIRS * 1.602E-19;                                                 NEG-0211
    AA(1) = A(1)*760;                                                           NEG-0212
    AA(2) = A(2)*760;                                                           NEG-0213
    BB(1) = B(1)*760;                                                           NEG-0214
    BB(2) = B(2)*760;                                                           NEG-0215
/*********************************************************************/         NEG-0216
```

```
/* IF NO DATA IS SAVED, MOST OF THE VARIABLES WILL HAVE TO BE     */
/* INITIALIZED IN THE NO_SAVE DO LOOP.  ELECTRIC FIELD WITHOUT    */
/* SPACE CHARGES IN THE FORM OF KK/R IS CALCULATED.  IF DATA IS   */
/* SAVED, THEN N AND INNER AND OUTER CONDUCTORS ARE ASSUMED TO    */
/* BE THE SAME.                                                   */
/*****************************************************************/
    IF SAVE = 'NO ' THEN
NO_SAVE: DO;
      ALLOCATE R(0:N), J_E(0:N), E(0:N), ELE_CHG_D(0:N),
        P_I_CHG_D(0:N), N_I_CHG_D(0:N), J_N_I(0:N), J_P_I(0:N),
        ALPHA(0:N), ETA(0:N), PHOTO_ELE(0:N), CURRENT(0:N);
      J_P_I(0) =0;
      R(0) = R_INNER;
      SUM2 = 0;
      X = 0;
      Y = N;
      DO K = 1 TO N-1;
        R(K) = R(K-1) + DELTA_R;
        SUM2 = SUM2 + 1./R(K);
        E(K) =-KK/R(K);
        IF -E(J) >= 30400 THEN X=K;
           ELSE IF -E(K) >= 19000 THEN Y =K;
      END;
      E(0) =-KK/R(0);
      R(N) = R(N-1) + DELTA_R;
      E(N) =-KK/R(N);
/*****************************************************************/
/* THE FOLLOWING DO LOOP INPUT DATA FROM PREVIOUS RUNS SO AS TO   */
/* CONTINUE THE ITERATION PROCESS IF DESIRED.                     */
/*****************************************************************/
      IF CARD_IN ='YES' THEN
      DO;
        GET LIST ((ELE_CHG_D(K) DO K =0 TO N));
        GET LIST ((P_I_CHG_D(K) DO K =0 TO N));
        GET LIST ((N_I_CHG_D(K) DO K =0 TO N));
        J_P_I(0) = P_I_CHG_D(0) * E(0) *  MOB_P_I;
```

NEG-0217
NEG-0218
NEG-0219
NEG-0220
NEG-0221
NEG-0222
NEG-0223
NEG-0224
NEG-0225
NEG-0226
NEG-0227
NEG-0228
NEG-0229
NEG-0230
NEG-0231
NEG-0232
NEG-0233
NEG-0234
NEG-0235
NEG-0236
NEG-0237
NEG-0238
NEG-0239
NEG-0240
NEG-0241
NEG-0242
NEG-0243
NEG-0244
NEG-0245
NEG-0246
NEG-0247
NEG-0248
NEG-0249
NEG-0250
NEG-0251
NEG-0252

```
      END;                                                                  NEG-0253
/*********************************************************************/     NEG-0254
/*  THE FOLLOWING SECTION CALCULATES THE GEOMETRIC FACTOR.           */     NEG-0255
/*********************************************************************/     NEG-0256
      IF (CASE ¬=1) & (R_OLD ¬= R_INNER) THEN FREE G, G1;                   NEG-0257
      IF R_OLD ¬= R_INNER THEN                                              NEG-0258
      DO;                                                                   NEG-0259
        ALLOCATE G(0:N), G1(0:N);                                           NEG-0260
        R_OUT =10;                                                          NEG-0261
        DO K =1 TO N;                                                       NEG-0262
           TEMP = (R(0)**2 - SQRT( (R(K)**2-R(0)**2) *                      NEG-0263
                  (R_OUT**2 - R(0)**2) ) )/R_OUT/R(K);                      NEG-0264
           ANGLE = PI/2-ATAN(TEMP/SQRT(1-TEMP**2));                         NEG-0265
           TEMP1 = ATAN((R_OUT +R(K))*TAN(ANGLE/2)/(R_OUT-R(K)) );          NEG-0266
           G(K) = (ANGLE/2 + TEMP1)/PI;                                     NEG-0267
           TEMP = R(0)/R(K);                                                NEG-0268
           ANGLE = PI/2-ATAN(TEMP/SQRT(1-TEMP**2));                         NEG-0269
           TEMP1 = ATAN((R(K)+R(0))*TAN(ANGLE/2)/(R(K)-R(0)));              NEG-0270
           G1(K) = (ANGLE/2-TEMP1)/PI;                                      NEG-0271
        END;                                                                NEG-0272
        G1(0) =-.5;                                                         NEG-0273
        G(0) = .5;                                                          NEG-0274
      END;                                                                  NEG-0275
    END NO_SAVE;                                                            NEG-0276
                                                                            NEG-0277
/*********************************************************************/     NEG-0278
/*  THE ELECTRON CURRENT DENSITY DISTRIBUTION IS INITIALIZED IF      */     NEG-0279
/* PREVIOUS DATA IS  AVAILABLE .  PHOTOIONIZATION TERM CAN BE        */     NEG-0280
/* CALCULATED IF THE ELECTRON CURRENT DENSITY IS KNOWN.              */     NEG-0281
/*********************************************************************/     NEG-0282
    IF SAVE = 'YES' | CARD_IN = 'YES' THEN                                  NEG-0283
    DO;                                                                     NEG-0284
      CALL E_FIELD;                                                         NEG-0285
      DO K = 0 TO Y;                                                        NEG-0286
        J_E(K) =-(-5.E6+329*E(K))*ELE_CHG_D(K);                             NEG-0287
      END;                                                                  NEG-0288
```

```
      DO K = Y+1 TO N;                                                    NEG-0289
        J_E(K) = -592 * E(K) * ELE_CHG_D(K);                              NEG-0290
      END;                                                                NEG-0291
    END;                                                                  NEG-0292
                                                                          NEG-0293
                                                                          NEG-0294
/*****************************************************************/       NEG-0295
/* THE ITERATION PROCESS IS PERFORMED WITHIN THE I1 DO LOOP.      */       NEG-0296
/* TO GUARD AGAINST THE UNSTABLE CASE, THE MAXIMUM NUMBER OF      */       NEG-0297
/* ITERATIONS ALLOWED IS M.  HOWEVER IF THE TOTAL CURRENT STOPPED */       NEG-0298
/* INCREASING AND THE DIFFERENCE BETWEEN PRESENT AND PREVIOUS     */       NEG-0299
/* TOTAL CURRENT IS WITHIN .5 PERCENT OF THE PRESENT TOTAL        */       NEG-0300
/* CURRENT, AND THE OVERSHUT PARAMETER( OVER) IS SET AT 1, THEN   */       NEG-0301
/* THE ITERATION PROCESS STOPS.  THE OVERSHUT PARAMETER IS TO     */       NEG-0302
/* INCREASE THE GROWTH RATE. IF THE TOTAL CURRENT REACHED THE     */       NEG-0303
/* ESTIMATED TOTAL CURRENT, THE OVERSHUT PARAMETER IS SET TO 1.   */       NEG-0304
/* WHEN OVER IS NOT 1, THE PROGRAM IS ACTUALLY SOLVING A          */       NEG-0305
/* DIFFERENT PROBLEM.                                             */       NEG-0306
/*****************************************************************/       NEG-0307
I1: DO I = 1 TO M WHILE ((I_DIFF>0 | I_DIFF<-.005)| OVER¬=1);             NEG-0308
      IF TOTAL_CUR(I-1) <= EST THEN OVER =1;                              NEG-0309
      ALLOCATE ALPHA2(0:N), ETA2(0:N), PHOTO_ELE2(0:N);                   NEG-0310
      J_N_I(0) =0;                                                        NEG-0311
/*****************************************************************/       NEG-0312
/* ALPHA AND ETA ARE CALCULATED.  T=1 IS FOR THE CASE E/P>=40 IN  */       NEG-0313
/* THE CALCULATION OF ALPHA.                                      */       NEG-0314
/*****************************************************************/       NEG-0315
      T=1;                                                                NEG-0316
      DO K = 0 TO N;                                                      NEG-0317
        IF K=X THEN T=2;                                                  NEG-0318
        E0 =-E(K);                                                        NEG-0319
        ALPHA(K) = AA(T)*EXP(-BB(T)/E0);                                  NEG-0320
        IF K <= Y THEN                                                    NEG-0321
        ETA(K) =(1.203E-8*E0*E0 -.000567*E0 + 10.12)*MOD_ETA;             NEG-0322
                ELSE ETA(K) = MOD_ETA * (1.545E-4*E0+.866);               NEG-0323
      END;                                                                NEG-0324
```

```
      T=1;                                                                  NEG-0325
/*********************************************************************/     NEG-0326
/* ALPHA2 AND ETA2 ARE FOR THE VALUES OF ALPHA AND ETA HALF WAY      */     NEG-0327
/* BETWEEN THE GRID POINTS.  THIS IS NECESSARY FOR RUNGE-KUTTA       */     NEG-0328
/* INTEGRATION.                                                      */     NEG-0329
/*********************************************************************/     NEG-0330
      DO K = 2 TO N-1;                                                      NEG-0331
        IF K=X THEN T=2;                                                    NEG-0332
        E0 =-(-E(K-2)-E(K+1)+9*(E(K-1)+E(K)))/16;                           NEG-0333
        ALPHA2(K) = AA(T) * EXP( -BB(T)/E0);                                NEG-0334
        IF K <= Y THEN                                                      NEG-0335
        ETA2(K)=(1.203E-8*EC*E0 -.000567*E0 + 10.12)*MOD_ETA;               NEG-0336
                ELSE ETA2(K) = MOD_ETA * (1.545E-4*E0 +.866);               NEG-0337
      END;                                                                  NEG-0338
      E0 =-(5*E(N) +E(N-3)+15*E(N-1)-5* E(N-2) )/16;                        NEG-0339
      ALPHA2(N) = AA(T) * EXP (-BB(T)/E0);                                  NEG-0340
      IF E0>= 19760 THEN                                                    NEG-0341
      ETA2 (N) =(1.203E-8*EC*E0 -.000567*E0 + 10.12)*MOD_ETA;               NEG-0342
                ELSE ETA2(N) = MOD_ETA * (1.545E-4*E0+.866);                NEG-0343
      E0 =-(5*E(0)+E(3)+E(1)*15-5*E(2))/16;                                 NEG-0344
      IF E0 >= 30400 THEN T=1;                                              NEG-0345
                     ELSE T=2;                                              NEG-0346
      ALPHA2(1) = AA(T) * EXP( -BB(T)/E0);                                  NEG-0347
      IF E0 >= 19760 THEN                                                   NEG-0348
      ETA2 (1) =(1.203E-8*EC*E0 -.000567*E0 + 10.12)*MOD_ETA;               NEG-0349
                ELSE ETA2(1) = MOD_ETA * (1.545E-4*E0+.866);                NEG-0350
                                                                            NEG-0351
/*********************************************************************/     NEG-0352
/* IF THERE IS NO PREVIOUS DATA USED IN THE FIRST ITERATION,         */     NEG-0353
/* PHOTOIONIZATION IS ASSUMED NOT TO BE PRESENT SINCE ELECTRON       */     NEG-0354
/* CHARGE DENSITY IS NOT KNOWN.                                      */     NEG-0355
/*********************************************************************/     NEG-0356
   IF I =1 & SAVE ='NO '& CARD_IN='NO ' THEN                                NEG-0357
     DO;                                                                    NEG-0358
      PHOTO_ELE(*) =0;                                                      NEG-0359
      PHOTO_ELE2(*) = 0;                                                    NEG-0360
```

```
      N_I_CHG_D(*) =0;                                                    NEG-0361
      P_I_CHG_D(*) =0;                                                    NEG-0362
     END;                                                                 NEG-0363
                                                                          NEG-0364
/*******************************************************************/     NEG-0365
/* CALCULATE PHOTOIONIZATION TERM.                                 */     NEG-0366
/*******************************************************************/     NEG-0367
     ELSE IF PHOTOMU ¬=0 THEN                                             NEG-0368
     DO;                                                                  NEG-0369
      PHOTO_ELE2(0) = 0;                                                  NEG-0370
      TEMP = PHOTOMU * EFF* OVER;                                         NEG-0371
      ABS_MU = PHOTOMU * 1.8;                                             NEG-0372
      INT1 =0;                                                            NEG-0373
      INT2 =0;                                                            NEG-0374
      INT3 =0;                                                            NEG-0375
      K1 = ALPHA(N)*J_E(N)* EXP(-ABS_MU*R(N))/8;                          NEG-0376
      K2 = R(N)*K1;                                                       NEG-0377
      K3 = G1(N)*K2;                                                      NEG-0378
      DO K = N-1 TO 0 BY -1;                                              NEG-0379
         TEMP1 = (R(K)+R(K+1))/2;                                         NEG-0380
         KK1 = ALPHA2(K+1) * (J_E(K) + J_E(K+1))                          NEG-0381
               * EXP(-ABS_MU*TEMP1)/8;                                    NEG-0382
         KK2 = TEMP1*KK1;                                                 NEG-0383
         KK3 = (G1(K)+G1(K+1))*KK2/2;                                     NEG-0384
         INT1 = INT1 + K1 + KK1;                                          NEG-0385
         INT2 = INT2 + K2 + KK2;                                          NEG-0386
         INT3 = INT3 + K3 + KK3;                                          NEG-0387
         PHOTO_ELE2(K+1) = (INT3-.112*INT2+.93*TEMP1*INT1)                NEG-0388
                           * EXP(ABS_MU*TEMP1);                           NEG-0389
         K1 = ALPHA(K)*J_E(K)*EXP(-ABS_MU*R(K))/4;                        NEG-0390
         K2 = R(K)*K1;                                                    NEG-0391
         K3 = G1(K)*K2;                                                   NEG-0392
         INT1 = INT1 + K1 + KK1;                                          NEG-0393
         INT2 = INT2 + K2 + KK2;                                          NEG-0394
         INT3 = INT3 + K3 + KK3;                                          NEG-0395
         PHOTO_ELE(K) = (INT3-.112*INT2+.93*R(K)*INT1)                    NEG-0396
```

```
                        * EXP(ABS_MU*R(K));                                  NEG-0397
      END;                                                                   NEG-0398
      PHOTO_ELE(0) = TEMP * PHOTO_ELE(0)/2;                                  NEG-0399
      K0 = ALPHA(0)*R(0)*J_E(0)*EXP(ABS_MU*R(0))/8;                          NEG-0400
      INT1 = 0;                                                              NEG-0401
      DO K = 1 TO N;                                                         NEG-0402
         TEMP1 = (R(K)+R(K-1))/2;                                            NEG-0403
         K1 = ALPHA2(K)*(J_E(K)+J_E(K-1))*TEMP1*                             NEG-0404
              (G(K)+G(K-1))*EXP(ABS_MU*TEMP1)/16;                            NEG-0405
         INT1 = INT1 +K0 + K1;                                               NEG-0406
         PHOTO_ELE2(K) = TEMP * (PHOTO_ELE2(K) + INT1 *                      NEG-0407
                         EXP(-ABS_MU*TEMP1));                                NEG-0408
         K0 = ALPHA(K)*R(K)*J_E(K)*G(K)*EXP(ABS_MU*R(K))/4;                  NEG-0409
         INT1 = INT1 +K0 + K1;                                               NEG-0410
         PHOTO_ELE(K) = TEMP* (INT1*EXP(-ABS_MU*R(K)) +                      NEG-0411
                        PHOTO_ELE(K));                                       NEG-0412
      END;                                                                   NEG-0413
     END;                                                                    NEG-0414
     ELSE IF I=1 THEN PHOTO_ELE(*),PHOTO_ELE2(*) =0;                         NEG-0415
    ALLOCATE KP(0:N), KN(0:N);                                               NEG-0416
                                                                             NEG-0417
/*****************************************************************/         NEG-0418
/* RUNGE-KUTTA METHOD IS USED FOR INTEGRATION BECAUSE IT IS SELF  */         NEG-0419
/* STARTING AND GENERALLY RELIABLE METHOD.  HOWEVER, IT IS        */         NEG-0420
/* PROBABLLY MORE EXPENSIVE THAN SOME OTHER METHODS. THE          */         NEG-0421
/* PROCESSES INCLUDED ARE ELECTRON AVALANCHE, ATTACHMENT, COSMIC  */         NEG-0422
/* RADIATION, PHOTOIONIZATION, AND RECOMBINATION.                 */         NEG-0423
/*****************************************************************/         NEG-0424
      IF PHOTOMU =0 THEN R_JE = ENTERJE*R(0) + ION_B*J_P_I(0);               NEG-0425
      ELSE                                                                   NEG-0426
      R_JE = ENTERJE * R(0) + ION_B * J_P_I(0)                               NEG-0427
           + PHOTO_E*PHOTO_ELE(0)/PHOTOMU;                                   NEG-0428
      TEMP1=0;                                                               NEG-0429
      DO K = 0 TO N-1;                                                       NEG-0430
        KON = ETA(K)*R_JE - TEMP1;                                           NEG-0431
        KOP = ALPHA(K)*R_JE + PHOTO_ELE(K) - COSMIC*R(K) -                   NEG-0432
```

```
            TEMP1;                                                          NEG-0433
        TEMP = DELTA_R *(KOP-KON)/2 + R_JE;                                 NEG-0434
        TEMP2=RECOMB*(P_I_CHG_D(K)+P_I_CHG_D(K+1))*(R(K)+R(K+1))*           NEG-0435
               (N_I_CHG_D(K) + N_I_CHG_D(K+1))/8/1.602E-19;                 NEG-0436
        K1N = TEMP * ETA2(K+1) - TEMP2;                                     NEG-0437
        K1P = TEMP*ALPHA2(K+1) + PHOTO_ELE2(K+1) - TEMP2 -                  NEG-0438
              COSMIC*(R(K)+R(K+1))/2;                                       NEG-0439
        TEMP = DELTA_R *(K1P-K1N)/2 + R_JE;                                 NEG-0440
        K2N = TEMP * ETA2(K+1) - TEMP2;                                     NEG-0441
        K2P = TEMP*ALPHA2(K+1) + PHOTO_ELE2(K+1) - TEMP2 -                  NEG-0442
              COSMIC*(R(K)+R(K+1))/2;                                       NEG-0443
        TEMP = DELTA_R * (K2P-K2N) + R_JE;                                  NEG-0444
        TEMP1 = RECOMB * P_I_CHG_D(K+1) * N_I_CHG_D(K+1)                    NEG-0445
               * R(K+1)/1.602E-19;                                          NEG-0446
        K3N = TEMP * ETA(K+1) - TEMP1;                                      NEG-0447
        K3P = TEMP*ALPHA(K+1) + PHOTO_ELE(K+1) - TEMP1 -                    NEG-0448
              COSMIC*R(K+1);                                                NEG-0449
        KP(K) =(KOP+ 2*K1P + 2 *K2P + K3P)*DELTA_R/6;                       NEG-0450
        KN(K)=(KON+2*K1N+2*K2N+K3N)*DELTA_R /6;                             NEG-0451
        R_JE = R_JE + KP(K) - KN(K);                                        NEG-0452
      END;                                                                  NEG-0453
     FREE ALPHA2, ETA2, PHOTO_ELE2;                                         NEG-0454
                                                                            NEG-0455
/*****************************************************************/       NEG-0456
/* KP AND KN ARE THE NET GAIN IN POSITIVE AND NEGATIVE CURRENT   */       NEG-0457
/* DENSITIES RESPECTIVELY BETWEEN TWO GRID POINTS.  THE CURRENT  */       NEG-0458
/* DENSITIES ARE THEN CALCULATED FROM THEM DIRECTLY.             */       NEG-0459
/*****************************************************************/       NEG-0460
     R_J_N=0;                                                               NEG-0461
      IF PHOTOMU =0 THEN R_JE = ENTERJE*R(0) + ION_B*J_P_I(0);              NEG-0462
      ELSE                                                                  NEG-0463
      R_JE = ENTERJE * R(0) + ION_B * J_P_I(0)                              NEG-0464
           + PHOTO_E*PHOTO_ELE(0)/PHOTOMU/EFF;                              NEG-0465
     J_E(0) = R_JE /R(0);                                                   NEG-0466
     DO K = 1 TO N ;                                                        NEG-0467
        R_JE = R_JE + KP(K-1) - KN(K-1);                                    NEG-0468
```

```
      J_E(K) = R_JE/R(K);                                              NEG-0469
      R_J_N = R_J_N + KN(K-1);                                         NEG-0470
      J_N_I(K) = R_J_N/R(K);                                           NEG-0471
   END;                                                                NEG-0472
   R_J_P = 0;                                                          NEG-0473
   J_P_I(N) =0;                                                        NEG-0474
   DO K = N-1 TO 0 BY -1;                                              NEG-0475
      R_J_P = R_J_P + KP(K);                                           NEG-0476
      J_P_I(K) = R_J_P/R(K);                                           NEG-0477
   END;                                                                NEG-0478
   FREE KP, KN;                                                        NEG-0479
                                                                       NEG-0480
/*******************************************************************/  NEG-0481
/* THE ELECTRON, POSITIVE AND NEGATIVE CHARGE DENSITIES, AND THE   */  NEG-0482
/* TOTAL CURRENT IS CALCULATED.                                    */  NEG-0483
/*******************************************************************/  NEG-0484
   DO K = 0 TO Y;                                                      NEG-0485
      ELE_CHG_D(K) = -J_E(K)/(-5.E6+ 329*E(K));                        NEG-0486
   END;                                                                NEG-0487
   DO K = Y+1 TO N;                                                    NEG-0488
      ELE_CHG_D(K) = -J_E(K)/592/E(K);                                 NEG-0489
   END;                                                                NEG-0490
   DO K = 0 TO N;                                                      NEG-0491
     N_I_CHG_D(K) = -J_N_I(K)/MOB_N_I/E(K);                            NEG-0492
     P_I_CHG_D(K) = J_P_I(K)/MOB_P_I/E(K);                             NEG-0493
     CURRENT(K) = (J_E(K) + J_P_I(K) + J_N_I(K)) *2*PI * R(K);         NEG-0494
   END;                                                                NEG-0495
     I_DIFF = (CURRENT(0)-TOTAL_CUR(I-1))/CURRENT(0);                  NEG-0496
     TOTAL_CUR(I) = CURRENT(0);                                        NEG-0497
                                                                       NEG-0498
/*******************************************************************/  NEG-0499
/* A NEW SET OF CHARGE DENSITIES IS NOW KNOWN.  SO A NEW ELECTRIC  */  NEG-0500
/* DISTRIBUTION IS CALCULATED, AND THE PROCESS REPEATES.           */  NEG-0501
/*******************************************************************/  NEG-0502
      CALL E_FIELD;                                                    NEG-0503
                                                                       NEG-0504
```

```
/*****************************************************************/          NEG-0505
/* THE DATA IS OUTPUTED IF THE PROCESS IS ALREADY WITHIN A        */          NEG-0506
/* SPECIFIED  (OUT_NUM) POINT OF PRINTING OR WHEN THE DIFFERENCE  */          NEG-0507
/* BETWEEN ITERATIONS IS VERY SMALL.                              */          NEG-0508
/*****************************************************************/          NEG-0509
      IF I>(M-OUT_NUM) | ( (I_DIFF<.00001)&(I_DIFF>-.0055) ) THEN              NEG-0510
         CALL OUT;                                                           NEG-0511
   END II;                                                                   NEG-0512
                                                                             NEG-0513
/*****************************************************************/          NEG-0514
/* OCCASIONALLY MORE ITERATIONS ARE NEEDED FOR CONVERGENCE THAN   */          NEG-0515
/* THE SPECIFIED.  YET ONE DOES NOT WANT TO WASTE THE PRESENT     */          NEG-0516
/* EFFORT.  SO ELECTRON, POSITIVE ION AND NEGATIVE ION CHARGE     */          NEG-0517
/* DENSITIES ARE OUTPUTED ON CARDS IF ITERATION DID NOT CONVERGE  */          NEG-0518
/* AND IF INPUT SPECIFICALLY NOTED  CARD_OUT='YES'.               */          NEG-0519
/*****************************************************************/          NEG-0520
   IF CARD_OUT ='YES' THEN                                                   NEG-0521
   IF I>M THEN                                                               NEG-0522
   DO;                                                                       NEG-0523
      PUT FILE (PUNCH) EDIT ((ELE_CHG_D(K) DO K = 0 TO N))                   NEG-0524
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                     NEG-0525
      PUT FILE (PUNCH) EDIT ((P_I_CHG_D(K) DO K = 0 TO N))                   NEG-0526
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                     NEG-0527
      PUT FILE (PUNCH) EDIT ((N_I_CHG_D(K) DO K = 0 TO N))                   NEG-0528
      ( COLUMN(2), (5)(E(13,5), X(1) ));                                     NEG-0529
   END;                                                                      NEG-0530
                                                                             NEG-0531
/*****************************************************************/          NEG-0532
/* THE CURRENT IN TERMS OF ITERATIONS ARE OUTPUTED SO THAT THE    */          NEG-0533
/* RATE OF CONVERGENCE IS KNOWN.  ALSO ADDITIONAL INPUT           */          NEG-0534
/* PARAMETERS ARE PRINTED FOR REFERENCE.                          */          NEG-0535
/*****************************************************************/          NEG-0536
   R_OLD = R_INNER;                                                          NEG-0537
   GET LIST (N, SAVE, CARD_IN, CARD_OUT);                                    NEG-0538
   IF SAVE ='NO ' THEN FREE R, J_E, E, ELE_CHG_D, N_I_CHG_D,                 NEG-0539
      P_I_CHG_D, J_N_I, J_P_I, ALPHA, ETA, PHOTO_ELE, CURRENT;               NEG-0540
```

```
   DO J = 1 TO I-1 BY 3;                                                    NEG-0541
      PUT LIST('I='||J||'   CURRENT='||TOTAL_CUR(J)||                       NEG-0542
        '   I='||J+1||'   CURRENT='||TOTAL_CUR(J+1)||                       NEG-0543
        '   I='||J+2||'   CURRENT='||TOTAL_CUR(J+2) ) SKIP;                 NEG-0544
   END;                                                                     NEG-0545
   PUT LIST ('A(1) ='||A(1)||'  B(1) ='||B(1)|| '  A(2) ='||               NEG-0546
             A(2)||'  B(2) ='||B(2) ) SKIP(2);                              NEG-0547
   PUT LIST (' THE INITIAL SOURCE ARE '||PAIRS||' CHARGED'||                NEG-0548
             ' PAIRS PRODUCED BY COSMIC RADIATION PER CC PER'||              NEG-0549
             ' SEC, AND ELECTRON CURRENT DENSITY OF    '||                   NEG-0550
             ENTERJE||' AMPS PER CM**2 AT THE INNER RADIUS.')SKIP;          NEG-0551
   FREE TOTAL_CUR;                                                          NEG-0552
   END CC;                                                                  NEG-0553
STOPP: END SS;                                                              NEG-0554
```

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