

A SPATIALLY DISTRIBUTED QUEUEING MODEL
FOR POLICE PATROL SECTOR DESIGN

by

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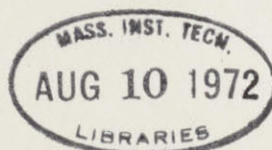
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Submitted to the Department of Electrical Engineering on February 24, 1972 in partial fulfillment of the requirements for the Degree of Master of Science

ABSTRACT

A spatially distributed queueing model is formulated as an analytic tool for studying deployment and dispatching of police patrol forces at the district level. A major application of the model is patrol sector design.

The model focuses on the two major activities of patrol forces: (1) preventive patrol and (2) response to calls for police service. It incorporates the specific travel time characteristics and spatial distribution of calls for police service which occur in the district under consideration. Output of the model includes: Average patrol car response time to all incidents, equity in distribution of response time within the district, workload balance among patrol units, and average number of dispatches that require each patrol car to leave its own sector.

The model is applied to three hypothetical districts. The effectiveness of utilizing incident location information in the dispatching process is analyzed. The trade-offs in response time vs. intersector dispatching for a simple system of overlapping sectors is studied.

A case study involving District 14 of the Boston Police Department is used to illustrate the use of the model as a decision aid in patrol sector design. Data are supplied by the Boston Police Department. The present sector configuration is compared with one alternative proposed by police personnel from District 14, and one alternative proposed by the author. Measures of system performance are calculated for each design. Because of the limited number of alternatives, additional designs should be analyzed prior to implementation.

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TITLE: Assistant Professor of Electrical Engineering and Urban
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CHAPTER I

INTRODUCTION

1.1. PURPOSE AND SCOPE

The purpose of this study is to formulate and illustrate a spatially distributed queueing model (GEOQUEUE) useful in studying police patrol operations at the district or precinct level. The two major uses of the model are the following:

1. An analytic tool used to investigate alternative methods of deploying and dispatching police patrol forces.
2. A decision aid for patrol sector design which police planners can use to calculate measures of performance which are helpful in the evaluation of proposed alternative designs.

As an analytic tool, the model will be used to analyze several simple hypothetical police districts. The effectiveness of utilizing call location information will be studied. A simple scheme of overlapping sectors will be investigated in terms of the trade-offs in response time versus intersector dispatching.

As a decision aid in patrol sector design, the model will be applied to a case study involving District 14 of the Boston Police Department. The present sector design will be compared to two alternative designs, one proposed by police personnel from District 14, the other proposed by the author.

In both of these applications, the approach is to consider several measures of system performance. These measures are:

1. Average patrol car response time to calls for police service originating in the district. Calls for service include both crime related and non-crime related incidents that require the response of a patrol car.
2. Equity in distribution of response time to different parts of the district. It will be shown that response is generally faster to incidents in the center of the district than to those occurring near the edges.
3. Workload balance among the patrol units. In this study, workload is defined as the fraction of time a patrol car is busy responding to calls for service. Since intersector dispatching is allowed, some of these calls for service may have originated outside of a patrol car's own sector.
4. Amount of intersector dispatching. This is defined as the proportion of dispatches that require a patrol car to be sent out of its own sector.
5. Balance in intersector dispatching among the patrol units. Intersector dispatching can be measured as either car specific or sector specific. Car specific intersector dispatching is the proportion of dispatches that require the car to leave its sector. Sector specific intersector dispatching is the proportion of calls for service arriving in a sector which require a non-sector car for response.

The relative importance of these measures of performance is left to the discretion of the police personnel who may use the model.

Most of the examples presented in this study involve trade-offs among these measures of performance. In each case, these trade-offs are outlined and discussed in terms of the methods that police planners should use to compare the alternatives.

1.2. RELATED RESEARCH

Recently a considerable amount of research has been directed toward the allocation of police patrol forces within a district.

In 1968, Saul Gass¹ used a heuristic political redistricting algorithm to design police patrol beats (sectors) for the City of Cleveland. The design criteria included balanced workload and crimeload, compactness, and contiguity.

As an extension of this approach, Heller, Phegley, Rother, and Schmidt² used a computer simulation to evaluate alternative designs generated by a political redistricting algorithm. The simulation used actual time series data on the arrival and servicing of calls. Output included fraction of total service time cars work in their own sectors, overall readiness (defined as the fraction of calls that experience no queueing delay), and response delay due to queueing. The method was illustrated for a police district in St. Louis.

The most comprehensive work in the area has been done by Larson.³ Among the many analytic models he developed are response time models which can be applied at the district level. He introduced the concepts of "strict center-of-mass" and "modified center-of-mass" dispatching. He also developed some of the concepts of repositioning of forces within a district.

In 1971, Carter, Chaiken, and Ignall developed an analytic method of determining optimal response areas for two fixed-position emergency units.⁴ Both response time and workload balance were considered.

1.3. GENERAL DESCRIPTION OF THE MODEL

The spatially distributed queueing model, GEOQUEUE, incorporates the specific geographical characteristics of a police district with a multiserver queueing model which considers the activities of each patrol car.

The important geographical characteristics of the district are the spatial distribution of calls for service, travel times throughout various parts of the district, and patrol sectors covered by each patrol car. In order to incorporate these characteristics into the model, the district is divided into a finite number of basic geographical areas. The spatial distribution of calls for service is specified by associating a call rate, λ_i , with basic area i . Travel times are specified by a matrix, T_{ij} , which is the travel time from basic area i to basic area j . Patrol sectors are composed of the basic geographical areas. Patrol effort within each sector is specified through the patrol allocation matrix, A . The term A_{ij} represents the proportion of the patrol time of car i spent in reporting area j .

The multiserver queueing model focuses on the activities of each patrol car. These activities are:

1. Preventive patrol.
2. Response to calls for police service.

The car is considered to be available for dispatch only when on preventive patrol. Therefore, corresponding to these functions, the patrol car is in one of two states:

1. Busy - On a call for service.
2. Free - On patrol.

The state of the whole system is specified by identifying which patrol cars are busy and which are free.

Call-for-service input to the system is modeled as a Poisson process with parameter λ_j associated with basic area j . Service time is assumed to be exponentially distributed with mean $1/\mu$ independent of time and the particular patrol car involved. The queueing discipline requires that a car always be dispatched to a call for service as long as a car is available when the call arrives. The call is answered by an outside unit (patrol supervisor or non-district car) if no car is available. Therefore, a queue of waiting calls is not allowed to form. The order of preference for dispatching a car to each basic area is specified in the dispatching matrix, P . The order is usually determined by which car is expected to be closest to the scene of an incident.

Under the above assumptions, the model can be formulated as a continuous time Markov process. The equations of detailed balance can be constructed and solved for the steady-state probabilities of each state of the system. From these probabilities, the workload of car i is determined by adding the probabilities of each state in which car i is busy. In order to calculate average travel time and amount of intersector dispatching, the average number of dispatches that involve sending each car to each basic geographical area is determined. Average travel time is then

easily calculated using the travel time matrix, T_{ij} . The amount of intersector dispatching is easily determined by examining which dispatches require cars to travel out of their own sectors.

The capabilities of the model are fairly comprehensive. Specific geographical characteristics of the police district are considered as well as the activity of each patrol car. The model is detailed enough to reflect changes in dispatching strategy and sector design. Thus, it can be useful in studying the allocation of forces within a police district.

FOOTNOTES

¹Gass, S., "On the Division of Police Districts into Patrol Beats," Paper presented at National Conference of the Association for Computing Machinery, New York, 1968.

²Heller, Phegley, Rother, and Schmidt, "Computer Design and Evaluation of Police Patrol Beats," Paper presented at the 58th Annual Meeting of the Operations Research Society of America, Detroit, Michigan, October 28-30, 1970.

³Larson, R.C., Models for the Allocation of Urban Police Patrol Forces, Technical Report No. 44, M.I.T. Operations Research Center, Cambridge, Massachusetts, 1969, Chapters 3 and 8.

⁴Carter, Chaiken and Ignall, Response Areas for Two Emergency Units, New York City - Rand Institute, R-532-NYC/HUD, March 1971.

CHAPTER II

THE MODEL: SPATIALLY DISTRIBUTED QUEUEING SYSTEM

The purpose of this chapter is to formulate a spatially distributed queueing model, GEOQUEUE. The chapter is organized into six areas: Geography, patrol car, input process, service process, queueing process, and calculation of output. In each section the relevant assumptions and approximations are discussed. The chapter is concluded with a summary of the model along with the assumptions and approximations. A summary of the definitions used in the chapter is given in Table 2.1.

2.1. GEOGRAPHY

The geographical organization of a police district is one of the fundamental factors in the spatially distributed queueing model. The important geographical considerations are:

1. Spatial distribution of incoming calls for service.
2. Travel times throughout the various parts of the district.
3. Patrol areas, or sectors, covered by each patrol car.

In order to model these characteristics, the district is divided into a finite number of basic geographical areas (sometimes referred to as basic areas). All of the geographic quantities are specified in terms of these basic areas. That is, the spatial

TABLE 2.1

SUMMARY OF DEFINITIONS

A. GENERAL

m	The number of patrol cars
n	The number of basic geographical areas
j, k	Indices denoting basic geographical area number
i	Index denoting sector number of car number
A_{ij}	Patrol allocation matrix - The proportion of car i 's patrol time spent in basic area j
λ	Average number of calls for service per unit time arriving in the district
λ_j	Average number of calls for service per unit time arriving in basic area j
$1/\mu$	Average service time for a call for service (includes travel time to the scene as well as service at the scene of the incident)

B. TRAVEL TIME QUANTITIES

(x_i, y_i)	Coordinates of patrol car i
(x_j, y_j)	Coordinates of an incident located in basic area j
PDF	Probability density function
$f_{x_i, y_i}(x_o, y_o)$	Joint PDF of x_i, y_i

TABLE 2.1

(Continued)

T_{jk}	Travel time matrix denoting average travel time from basic area j to basic area k (The exact location of cars and incidents within the basic areas are not known.)
T'_{ij}	Travel time matrix denoting average travel time from sector i to basic area j (The exact location of car within its sector is not known.)
v_x, v_y	Response speed east-west (north-south)

C. QUEUEING QUANTITIES

α, β	Indices representing possible states of the system. For a system with m patrol cars, α and β represent m digit binary numbers. Digit k corresponds to the state of car k . The digit is a "1" if the car is busy or a "0" if the car is free.
Π_α	The steady state probability of state α .
$R(\alpha, \beta)$	Transition rate from state α to state β (on the state transition diagram)
$b(\alpha)$	The number of busy cars represented by the state α . (The number of 1's in the m digit binary number represented by α .)
$P_{ij}(\alpha)$	The dispatching strategy matrix. This term is the probability that car i is assigned to a call for service in basic area j given that the state of the system is α .
$\alpha(i)$	The i^{th} digit of the binary number represented by state α . This is the state of car i .

distribution of calls for service is specified by associating a call rate, λ_i , with basic geographical area i . Travel times are specified as a matrix, T_{ij} , which is the expected travel time from basic area i to basic area j . It is assumed that exact locations within each area are not known, therefore these travel times represent averages over the areas involved. Finally, sectors are composed from the basic geographical areas.

These basic areas can be of any size depending on the desired use and accuracy of the model. For example, if the model is used in sector design, the basic areas must be small enough to allow the desired flexibility in adjusting sector boundaries. On the other hand, if it is used to investigate the use of overlapping sectors, then the basic area may be defined as the sector itself. An example of basic areas which could be used in sector design are reporting areas, the smallest area for which data are collected at the Boston Police Department.

In considering accuracy, since the basic areas can be made arbitrarily small, any degree of detail can be attained. However, one must balance the desire for detail with the limitations of the available data as well as the computational limits of storage and run time.

2.1a. Calculation of Travel Time

The average travel time from basic area i to basic area j can be calculated in any manner appropriate to the problem under consideration. Specifically, if the basic areas are small compared

to the size of the district, then travel times can be measured between particular points, or centroids, located in each area. However, if the areas are large, then travel times must be averaged over the areas involved.

An example of the first case is the sector design case study of Chapter IV. Here, the actual street network, along with experimentally observed travel speeds, was used to construct a matrix of travel times between each pair of reporting area centroids. The advantage of this method is that actual street characteristics as well as impediments to travel, such as railroad lines, can be built right into the model. The disadvantage is that a great deal of work and/or computer time is spent in the process of constructing the matrix.

An example of the second case, where basic areas are large, is the theoretical model presented in Chapter III. Here, travel times are computed by evaluating the expected value of the travel time between random car and call locations, specifically, under the following assumptions:

1. The coordinates of a car located in basic area i are (x_i, y_i) . The coordinates of a call for service location in basic area j are (x_j, y_j) .
2. Car and call locations can be modeled as independent random variables with joint PDF's $f_{x_i, y_i}(x_o, y_o)$ and $f_{x_j, y_j}(x_o, y_o)$.
3. Travel distances are measured as "right angle" distance, simulating a regular perpendicular street network running north-south and east-west.

4. Travel speed in the x direction is v_x ; travel speed in the y direction is v_y . Travel speeds are also random variables which may be dependent on travel distances and locations.

The travel time between a car located in area i and a call in area j is:

$$T_{ij} = E \left[\frac{|x_i - x_j|}{v_x} + \frac{|y_i - y_j|}{v_y} \right]$$

where the expected value is evaluated by integrating over area i and area j.¹ The integration may be a difficult task in practical situations.

In addition to the above assumptions, the model in Chapter III assumes square sectors, uniform location distributions over each sector, and deterministic travel speeds. These assumptions greatly simplify the computations while still allowing the desired flexibility in the model.

2.1b. Patrol Sectors

In the model, patrol sectors are composed of basic geographical areas. Patrol sectors are designated by specifying a patrol allocation matrix, A. The term A_{ij} represents the proportion of the patrol time of car i spent in basic area j. (For each i, $\sum_j A_{ij} = 1$.) The patrol sector of car i is thus determined by the non-zero values in row i. This system is quite flexible in that it allows non-uniform patrol within each patrol sector as well as almost any conceivable system of overlapping sectors.

2.1c. Travel Time From Sector to Basic Geographical Area

The travel time from patrol sectors to basic geographical areas is a useful quantity later in the model. Let T'_{ij} be the average travel time from sector i to area j . Then

$$T'_{ij} = \sum_{k=1}^n A_{ik} T_{kj}$$

where T_{kj} is the travel time matrix, A_{ik} is the patrol allocation matrix, and n is the number of basic areas.

2.2. THE PATROL CAR

In the model, the patrol car is considered to have two functions:

1. Preventive Patrol.
2. Response to Calls for Service.

The car is considered to be available for dispatch only when on preventive patrol. Therefore, corresponding to these two functions are two states:

1. Busy - On a Call for Service.
2. Free - On Patrol.

It is recognized that in addition to these functions, a patrol car may be busy or out of service for any one of a large number of activities, such as meal breaks, vehicle repair, or court appearance. These types of activity are not accounted for directly

in the model. However, the overall manpower available can be adjusted to account for this unavailable time.

2.3. INPUT PROCESS - CALLS FOR SERVICE

Incoming calls for service originating in each basic geographical area are modeled as a Poisson process with constant arrival rate λ_i associated with area i . Larson² has investigated call-for-service data at the Boston Police Department and found that the Poisson assumption is quite realistic. However, the arrival rate is a function of time. Wide variations in arrival rate occur as a function of time of day, day of the week, and season. Variations in arrival rate can be incorporated into the model presented here through changing the input quantities, λ_i . However, the model assumes steady state conditions. As a result, transient effects caused by relatively rapid changes in the arrival rate cannot be modeled.

One further approximation concerning the input process is that no distinction is made among different types of calls. As a result, there is no system of priorities built into the model. Priorities can be applied to queueing models. Whether or not it is computationally tractable to apply priorities to the GEOQUEUE model should be studied further.

2.4. SERVICE PROCESS

In any queueing model, the primary impact of the service process is the amount of time that a server (in this case a patrol car)

is busy. Therefore, the service process is considered to include both elements involved with responding to a call for service. These elements are:

1. Travel time to the scene of an incident.
2. Service at the scene.

In the model, service time is considered to be an exponentially distributed random variable with mean $\frac{1}{\mu}$, independent of time and type of call. According to Larson's study,³ the distribution of service time appears to be a convolution of two exponentials, one for travel time and one for service time at the scene. However, since service time is usually an order of magnitude greater than travel time,⁴ the effect of travel time on the form of the distribution can be neglected.

One further approximation is that after completing a call for service, the patrol car is assumed to return immediately to its own sector and resume preventive patrol. The time required to return is ignored.

2.5 THE QUEUEING PROCESS

The queueing aspects of the police car response system are modeled as a multiserver queue. The spatially distributed queueing model presented here is an extension of the simple M/M/C queue.⁵ Therefore, some of the overall characteristics are similar. However, the model treats the state of the system in more detail. Specifically, each server (patrol car) is considered separately, resulting in the

ability to calculate the desired quantities (e.g. travel time, workloads, and intersector dispatches) that cannot be calculated from the simpler system.

2.5a. The M/M/C Queue

In order to visualize the state space of the spatially distributed queueing model, it is helpful to consider the M/M/C queue. The M/M/C queue is a continuous Markov process characterized by the following:

1. The state of the system is the number of customers (calls for service) both in service and in the queue. If the number of customers is less than or equal to C , then the state is also the number of cars that are busy.
2. Interarrival times are mutually independent and exponentially distributed with mean $1/\lambda$.
3. Service times are mutually independent and exponentially distributed with mean $1/\mu$.

An example of the state transition diagram for the case of 3 servers is shown in Figure 2.1a. Using this model, it is easy to solve for the steady state distribution. From this, the following important quantities can be calculated:

1. Probability that a customer will experience a delay.
2. Mean waiting time given a delay.

An important modification of the above model can be made when one is interested primarily in the activity of the patrol cars

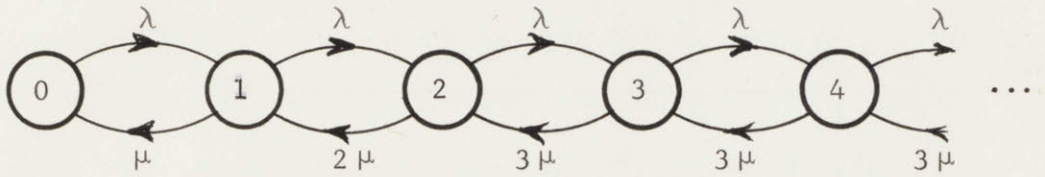


FIGURE 2.1a. State Transition Diagram of the M/M/3 Queue.

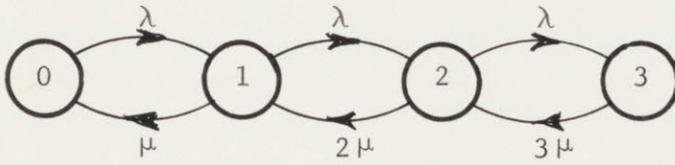


FIGURE 2.1b. State transition diagram of the M/M/3 queue where customers are served by units outside the system when it is saturated. The state of the system is characterized by the number of servers that are busy.

("servers") and not in the queue of waiting calls for service. If one of the following assumptions is made:

1. The probability of saturation is negligibly small.
2. If a call arrives when the system is saturated, it is answered by a unit other than regular district cars. (This other unit could be either a supervisory unit within the district or a patrol car from outside the district.)

then the state transition diagram can be modified as in Figure 2.1b. Notice that the state can be characterized completely by the number of patrol cars that are busy since this number is also the same as the number of customers in the system. The spatially distributed model is an expansion of this system. Therefore, the queueing of calls is ignored and the emphasis is placed on the patrol cars. However, this is not a severe restriction since it is easy to include the queue if desired.

2.5b. State Specification - Spatially Distributed System

As mentioned in Section 2.2, patrol cars are considered to be in one of two states, either "busy" or "free." Let the busy state be represented by a "1" and the free state by a "0." Then the states of system as represented in Figure 2.1b can be expanded to explicitly identify which servers are free and which are busy. For a system with m patrol cars, the expanded state is specified by an m digit binary number, where the i^{th} digit represents the state of the i^{th} patrol car. For example, consider the case of 3 patrol cars. The state representing 1 busy car is expanded to the three states 100,

010, 001, which represent, respectively, car numbers 1, 2, or 3 as busy. The expanded state transition diagram is shown in Figure 2.2a.

Another convenient way to view the state space of the expanded system is shown in Figure 2.2b. Here, the binary state representation of each patrol car is plotted on a coordinate axis in m dimensions. The states form an m dimensional hypercube with transitions occurring along the edges. This hypercube representation was originally used in communication theory, however, it is helpful in visualizing this model.⁶

An interesting observation can be made here. In the expanded system, if it is still assumed that service time is independent of the patrol car involved, then the steady state probabilities that r cars are busy is the same for both the simple and expanded systems. In other words, the aggregated behavior of both systems is the same. The expanded model merely distributes the steady state probabilities among the expanded states represented by the possible combinations of individual servers.

The next step is to find the transition rates for the expanded system. These rates are a function of the dispatching strategy which is discussed in the next section.

2.5c. Dispatching Strategy

The dispatching strategy is the method used to decide which available car to dispatch to an incoming call for service given the existing state of the system.

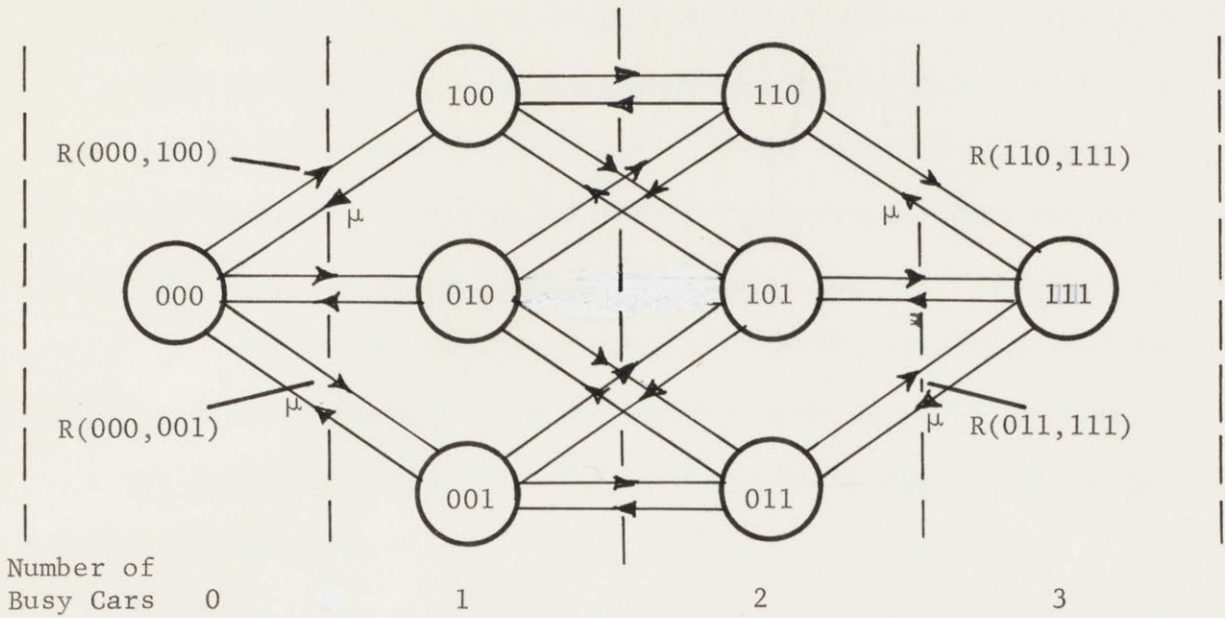


FIGURE 2.2a. State Transition Diagram of Expanded M/M/C Queue.

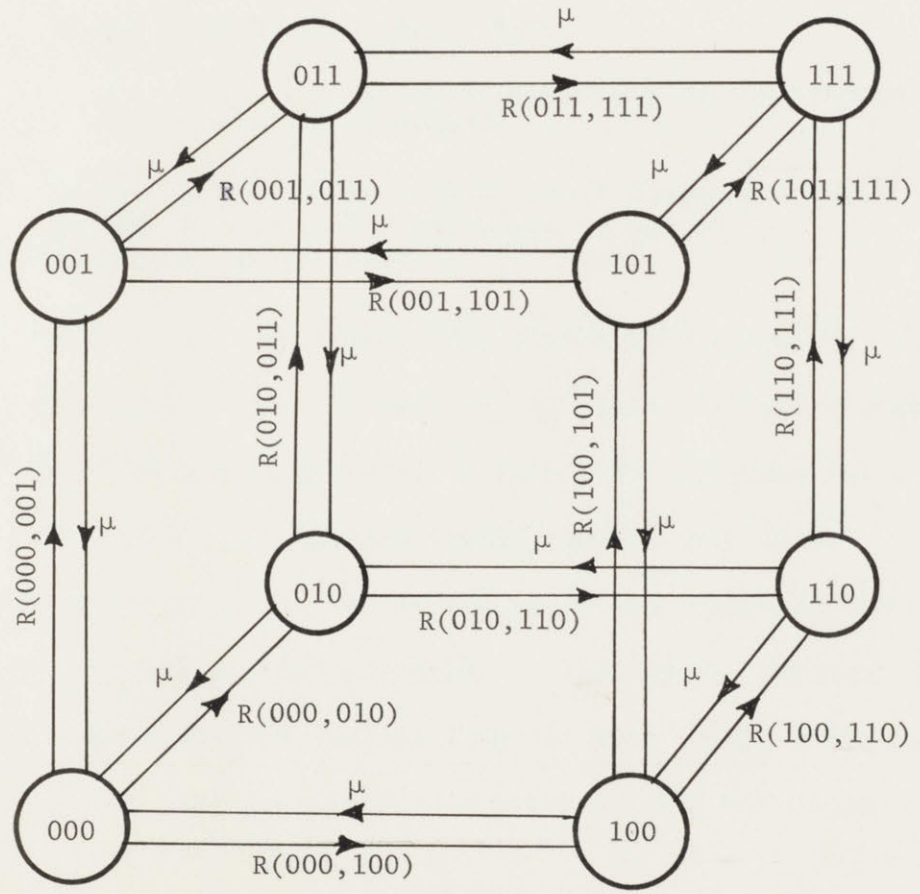


FIGURE 2.2b. State Transition Diagram in the Form of a Hypercube.

The way the strategy is incorporated into the model is through the dispatching matrix, P. An element of this matrix, $P_{ij}(\alpha)$, is the probability that car i is assigned to a call arriving in basic area j given the state α . A variety of dispatching strategies can be modeled with the system as long as they can be specified through the dispatching matrix, P.

In most of the examples of Chapters III and IV, the dispatching strategy is to minimize the expected travel time to the call which has just arrived. Using this criterion, the values of $P_{ij}(\alpha)$ are found in the following manner:

1. Fix values for α and j.
2. Find the minimum value of T'_{ij} of all cars i, which are represented as "free" in the state α .
3. If the minimum is unique, then:

$$P_{ij}(\alpha) = \begin{cases} 1 & \text{for } i \text{ corresponding to the minimum} \\ 0 & \text{otherwise} \end{cases}$$

4. If the minimum is not unique, then divide the probability equally among those values of i corresponding to ties.

In other words, the dispatching algorithm examines all possible states of free cars. Then for each state, it decides which available car to dispatch to a call arriving in each basic geographical area in order to minimize the expected travel time to that call.

This strategy is optimal in the sense that response time to the call under immediate consideration is minimized. However, if it is desired to minimize the average response time to all calls for service, then not only the present call for service but future calls

for service must be considered. In other words, it may be better in the long run to assign a car that is not closest to the present call but which would leave the remaining cars in a state in which they are better able to respond to future calls for service. This type of strategy is referred to by the author as the "total configuration strategy" because it considers desirable configurations of available patrol cars. Chaiken, Carter, and Ignall developed the concept of a "total configuration strategy" and applied it to the design of response areas for two fixed-position emergency units.⁷ Numerical results from their study indicate that response time could be improved by only 1 or 2 percent. Therefore, the use of such a strategy was not pursued in the present paper.

2.5d. Equations of Detailed Balance

Using the dispatching matrix defined in the previous section, the transition rates on the state transition diagram of Figure 2.2 are calculated as follows: Consider the state α . Define the set, $B = \{\beta_i\}$, to be the set of states, derived from α , where exactly one digit of α has been changed (either "0" to "1" or "1" to "0"). The index i is the number of the digit which has been changed. Then the transition rate from state α to state β_i is given by:

$$R(\alpha, \beta_i) = \begin{cases} \sum_{j=1}^n P_{ij}(\alpha) \lambda_j & \text{if } \alpha(i) = 0 \\ \mu & \text{if } \alpha(i) = 1 \end{cases} \quad (1)$$

where $P_{ij}(\alpha)$ is the dispatching matrix, λ_j is the call-for-service

rate of basic area j , μ is the service rate of a patrol car at an incident, and $a(i)$ is the i^{th} digit of a .

This equation can be interpreted by considering state a and car i . If car i is free ($a(i) \neq 0$), it will become busy at a rate equal to the sum of the call rates for all the basic areas to which it will be dispatched, given state a . On the other hand, if car i is already busy ($a(i) \neq 1$), it will become free at a rate μ .

Using these transition rates, the equations of detailed balance can be written:

$$\Pi_a (\lambda + \mu \cdot b(a)) = \sum_{\beta_i \in B} R(\beta_i, a) \Pi_{\beta_i} \quad (2)$$

where Π_a and Π_{β_i} are the steady-state probabilities of states a and β_i respectively, $b(a)$ is the number of busy cars represented by state a , and the other variables are defined above.

This set of simultaneous equations (one for each value of a), along with the restriction that the state probabilities sum to one, can be solved to give the steady-state probabilities of the system.

Solving these equations poses a computational problem because of the large number of state variables. For a system with m cars, there are 2^m possible states. For example, with 10 patrol cars, there are $2^{10} = 1024$ variables to solve for. In order to keep from using excessive amounts of computer time, the examples of the GEOQUEUE model were restricted to a maximum of 6 patrol cars.

2.6. OUTPUT

Given the steady-state probabilities, it is quite straightforward to calculate the desired output values. The output variables are:

1. \bar{T} , the average travel time to all incidents.
2. \bar{T}_j , the average travel time to incidents arriving in basic area j .
3. WL_i , the average workload of car i . Workload is defined as the fraction of time that the car is busy on call-for-service assignments.
4. IDO_i , the proportion of dispatches that car i makes outside its own sector. (Intersector dispatches by car i .)
5. IDI_i , the proportion of calls arriving in sector i which are answered by a car outside of the sector. (Intersector dispatches by sector.)

The easiest variable to calculate is workload. The workload of car i , WL_i , is simply:

$$WL_i = \sum_{a \in I} \Pi_a \quad (3)$$

where I is the set of states where car i is busy.

In order to calculate average travel time and average intersector dispatches, it is convenient to calculate an intermediate matrix, PD . The element PD_{ij} is the proportion of all dispatches which involve sending car i to basic area j . The PD matrix is given by:

$$PD_{ij} = \frac{1}{(1 - \Pi_\Omega)} \sum_a \Pi_a P_{ij}^{(a)} \frac{\lambda_j}{\lambda} \quad (4)$$

where the sum is over all states, and Ω is the state in which all cars are busy. Given the PD matrix, the average travel time to a call for service is simply:

$$\bar{T} = \sum_{i=1}^m \sum_{j=1}^n PD_{ij} T'_{ij} \quad (5)$$

The average travel time to basic area j is given by:

$$\bar{T}_j = \frac{\sum_{i=1}^m PD_{ij} T'_{ij}}{\sum_{i=1}^m PD_{ij}} \quad (6)$$

Intersector dispatching is also easy to calculate given the PD matrix. The average intersector dispatches out of sector i is given by:

$$IDO_i = \frac{\sum_{j \in I'} PD_{ij}}{\sum_{j=1}^n PD_{ij}} \quad (7)$$

where I' is the set of all basic areas not in sector i . Similarly, the average intersector dispatches into sector i is given by:

$$IDI_i = \frac{\sum_{k \neq i} \sum_{j \in I} PD_{kj}}{m} \quad (8)$$

$$\sum_{i=1} \sum_{j \in I} PD_{ij}$$

where I is the set of all areas included in sector i.

2.7. SUMMARY

The spatially distributed queuing model is a combination of geographical characteristics and a multiserver queueing model. The geography of a police district is characterized as follows:

1. The district is divided into a set of basic geographical areas.
2. Calls for service arrive in each area at a rate λ_i .
3. Patrol sectors and the distribution of patrol within the sector is specified by the patrol matrix, A. (A_{ij} is the proportion of car's patrol time that is spent in basic area j.)
4. Average travel times from area i to area j are specified in the travel time matrix T_{ij} .

The multiserver queueing model is characterized as follows:

1. The state of a system with m cars is specified as an m digit binary number where the i th digit is "1" if car i is busy, and "0" where car i is free.
2. The arriving calls for service at basic area j constitute a Poisson process with parameter λ_j . The overall arrival rate is λ .

3. The service times are mutually independent with mean $1/\mu$ independent of time and the particular patrol car involved.
4. The dispatching strategy is contained in the dispatch matrix $P_{ij}(a)$. ($P_{ij}(a)$ is the probability that car i is assigned to an arriving call in basic area j given state a .)
5. A car is always dispatched to a call for service as long as a car is available when the call arrives. The call is answered by an outside unit if a car is not available.

The steady state probabilities are solved using the equations of detailed balance, Equation (2). The output variables calculated are average travel time, Equation (5); average travel time to each basic area, Equation (6); workload, Equation (4); and intersector dispatches, Equations (7) and (8).

FOOTNOTES

¹For a more detailed explanation of the calculation of travel times, see Larson, R.C., Models for the Allocation of Urban Police Patrol Forces.

²Larson, R.C., Operational Study of the Police Response System, Technical Report No. 26, M.I.T. Operations Research Center, Cambridge, Massachusetts, 1967.

³Ibid., page 164.

⁴In Boston, the average travel time was measured to be between 4 and 6 minutes depending on the type of call while the average service time was between 23 and 60 minutes. From Larson, Models for the Allocation of Urban Police Patrol Forces, page 87.

⁵An M/M/C queue is one with Poisson arrivals, exponential service times, C identical servers, and infinite waiting line capacity.

⁶The use of the hypercube in modeling police patrol was first noted by Larson, Models for the Allocation of Urban Police Patrol Forces, page 124.

⁷Carter, Chaiken, and Ignall, Response Areas for Two Emergency Units.

CHAPTER III

APPLICATION OF THE MODEL

The purpose of this chapter is to demonstrate some of the capabilities of the spatially distributed queueing model as an analytic tool. A number of important topics are covered in this chapter. Each one could be the subject of much more intensive study. It is hoped that the brief examples presented here will indicate a few interesting results that will encourage further development and use of the model.

The specific topics that are studied in this chapter are as follows:

1. General relationships of the output variables (i.e. travel time, workload, and intersector dispatching) as a function of workload and the size and shape of districts.
2. Imbalance in service to different sectors of the district due to "boundary effects." That is, the characteristics of sectors near the edge of a district are different than those near the center.
3. The effectiveness of utilizing call location information.

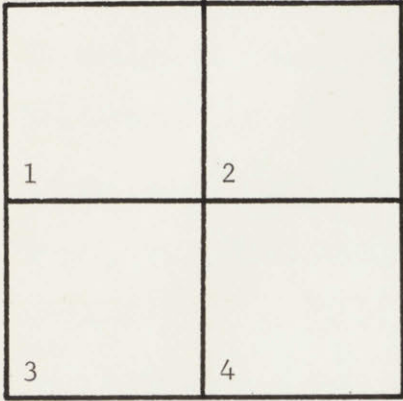
To study these topics, three specific examples of simple hypothetical police districts were used. The examples are purposely kept simple so that the results are easy to interpret. It is granted that general results are difficult to obtain from specific examples, however some general trends can be identified when comparing the

examples. Since the model takes into consideration the actual geographic organization of the district, one would expect the results to reflect this organization. This is one of the most important powers of the model. Therefore, some generality is sacrificed but a much broader range of phenomena can be analyzed.

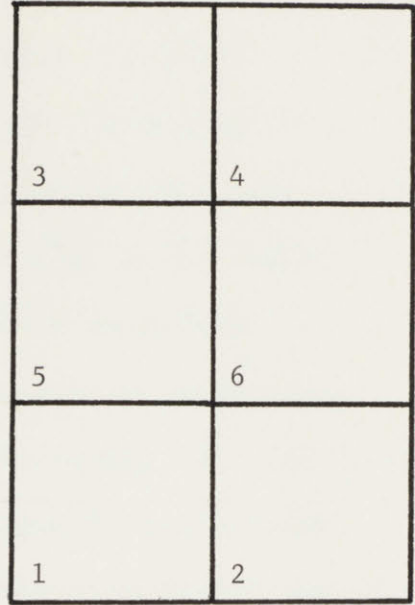
3.1. THE EXAMPLE DISTRICTS

The three example districts used in this chapter are shown in Figure 3.1. District 1 contains four sectors and four patrol cars. Districts 2 and 3 each contain 6. The reason for dealing with such small examples is the computation limits of the computer program as presently written. The author is confident that with careful programming and the utilization of the sparsity in the matrices, the program could be made to handle larger districts.

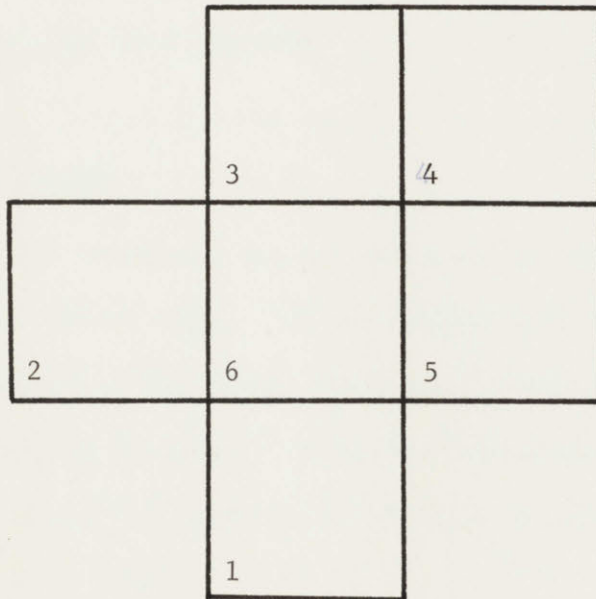
For simplicity, each district is composed of square sectors, simulating a grid system of perpendicular streets with sector boundaries aligned on the streets. In this particular case, the basic geographical areas coincide with the sectors. Later in the chapter, when call location information is utilized, the basic areas will be smaller than the sectors. In all cases throughout this chapter, calls for service arriving at a basic area are assumed to be uniformly distributed over that area. Patrol is also assumed uniform over each sector and independent of call locations.



District 1 (4 Sectors)



District 2 (6 Sectors)



District 3 (6 Sectors)

FIGURE 3.1. Three Example Hypothetical Police Districts. Sector Numbers at Lower Left Hand Corner of Each Sector.

3.1a. Spatial Organization of Example Districts

Referring to the example districts in Figure 3, it is interesting to compare their spatial organization. Districts 1 and 2 are quite compact and symmetric. These two districts are used primarily to demonstrate the effect of district size on the output variables. On the other hand, District 3 is quite asymmetric. Comparing this with District 2, the effect of shape can be studied. Note that even if call rates to each sector of District 3 are identical, sectors 1, 3, 4, and 6 should have different characteristics since each has a distinctly different geographical relationship with the other sectors. Specifically, one would expect that the workload of car 6 would be exceptionally high since it is surrounded by other sectors and is highly likely to be dispatched to an incident there if the sector car is busy. This and other intuitive ideas will be studied in the next section.

3.1b. Travel Distances

In all of the examples, travel distance to incident locations is used instead of travel time. It is assumed that both north-south and east-west speeds are the same. Therefore, travel time is directly proportional to travel distance. Since the examples are compared relative to each other, it makes no difference whether travel time or travel distance is used.

Travel distances from one basic area to another are calculated using the "right angle" distance as outlined in Section 2.1. An example of this calculation follows. Assume a coordinate system as

shown in Figure 3.2. The origin is in the lower left corner of the sector and distances are measured in sector length. Assume that a call for service has arrived. Let the coordinates of the call be designated (x_j, y_j) . Let the coordinates of the sector patrol car be (x_i, y_i) . Both car and call locations are assumed to be uniformly distributed over the sector and mutually independent. The travel distance to the incident is:

$$d_r = |x_i - x_j| + |y_i - y_j| = x_r + y_r$$

The average travel distance \bar{d}_r is given by:

$$\bar{d}_r = \bar{x}_r + \bar{y}_r$$

In other words, the expected travel distance is the sum of the expected distance in the x direction plus the expected distance in the y direction. By calculating the derived distributions for x_r and y_r and taking expected values, \bar{x}_r and \bar{y}_r are both found to be 1/3 sector length. Therefore, the average travel distance is:

$$\bar{d}_r = 1/3 + 1/3 = 2/3 \text{ (sector length)}$$

In a similar manner, the expected travel distance from a patrol car located in a sector outside of the one containing the call for service can be calculated. The results of these calculations are shown in Figure 3.3. These values were used to construct the travel distance matrix, T_{ij} , for each of the three example districts.

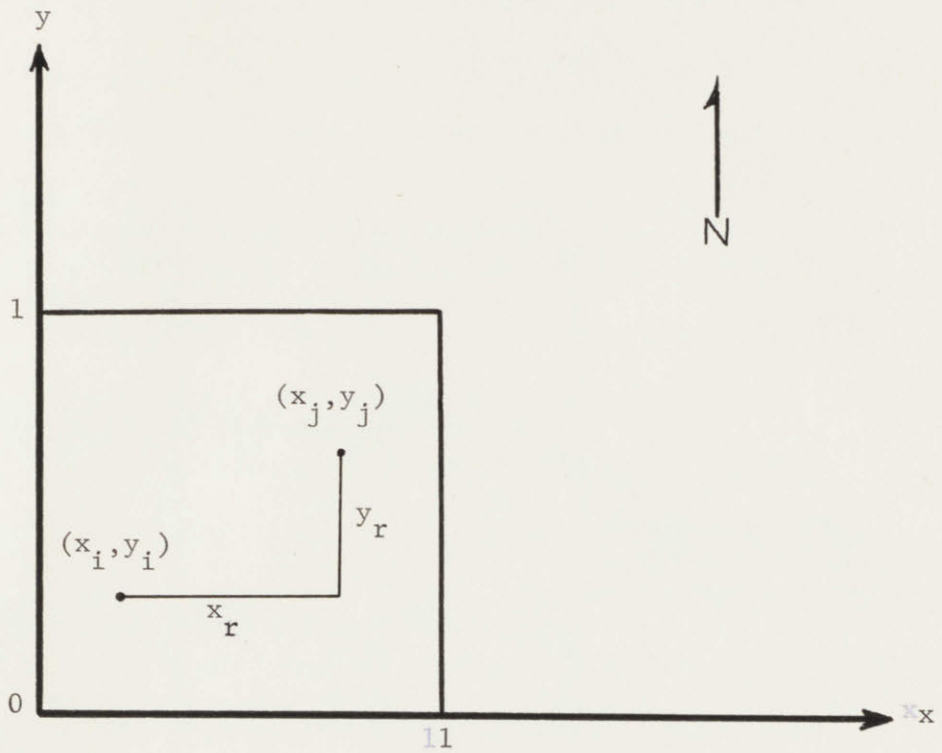


FIGURE 3.2. Travel Distance Within a Sector.

(x_i, y_i) = Car Location

(x_j, y_j) = Call Location

$x_r = x_i - x_j$ = Distance Along E-W Streets

$y_r = y_i - y_j$ = Distance Along N-S Streets

$d_r = x_r + y_r$ = Travel Distance

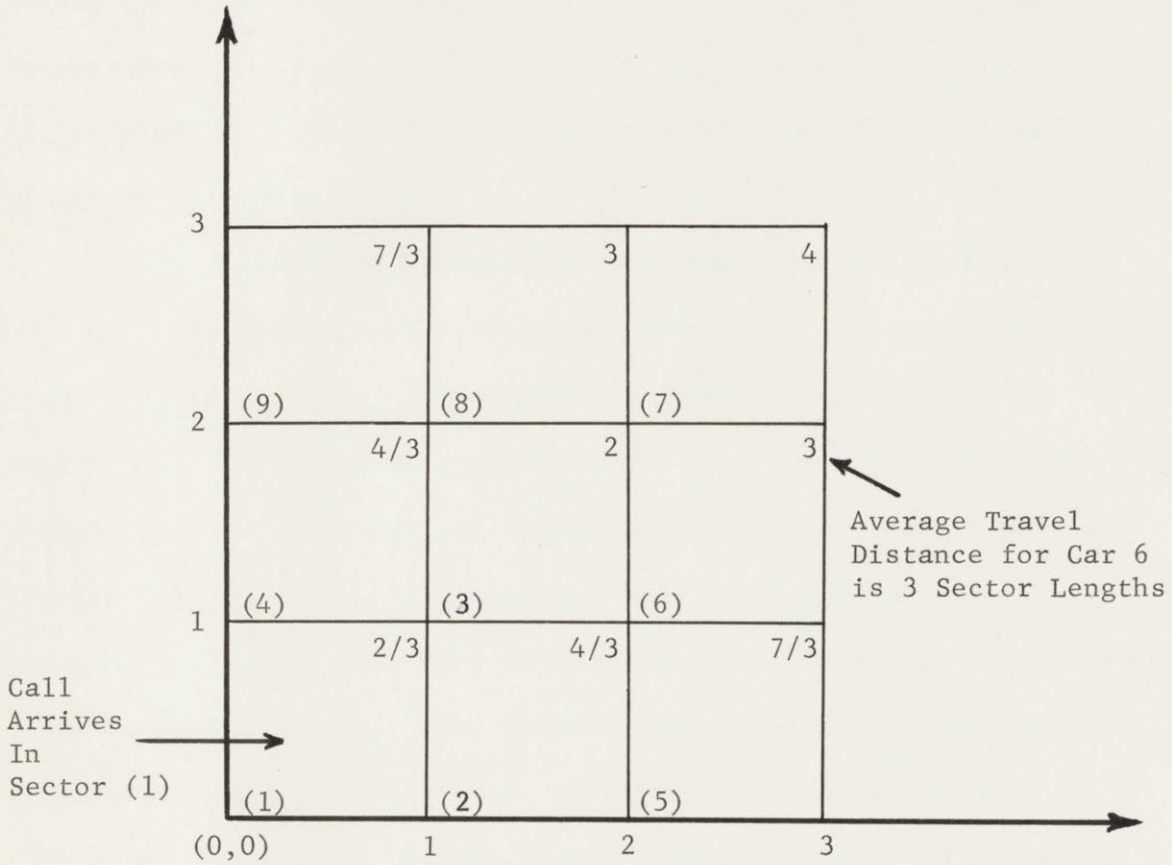


FIGURE 3.3. Average Travel Distance in Sector Lengths from Each Sector to Sector (1). Call location is uniformly distributed over Sector (1). Car location is uniformly distributed over each sector.

3.1c. Dispatching Strategy

The dispatching strategy used throughout this chapter is to assign the "expected closest" available car, where the distance matrix T_{ij} is used to determine the closest distance. Under the assumptions of the model, this strategy corresponds to having no car location information except that an available car is located within its assigned sector.

Call location information can be handled in one of two ways, depending on how the districts are divided into basic geographic areas. If the sectors are not divided further than as shown in Figure 3.1, then there is essentially no call location information except that a call is located somewhere within its reported sector.¹ However, it is possible to break the sectors into smaller basic geographical areas. The call location is then pinned down to different sections of the sector. This provides approximate call location information which can be used in selecting a car to dispatch.

In the examples of this chapter, the "closest car" strategy always results in assigning the sector car if it is available. If it is not available, then one of the others is chosen according to the expected "closest car" criterion. In case of a tie, which would occur for example when both cars in two adjacent sectors are available, then the dispatcher is assumed to be indifferent between the alternative patrol cars. He then picks one at random with equal probability of picking any of the ties.

3.2. CHARACTERISTICS OF EXAMPLES

The purpose of this section is to compare the output characteristics of the three example districts. The focus will be on two subjects:

1. The overall level of service and workload within each district.
2. The distribution of service and workload for each sector within the district. This is a question of equity of distribution as opposed to overall quality of service.

3.2a. Average Level of Service

It is interesting to examine and compare the average values of the output variables for each district as shown in Figures 3.4 to 3.6. The output quantities are average workload, average amount of intersector dispatching, and average travel distance to all calls for service. Each output variable is plotted as a function of ρ , the utilization factor.²

The effect of the assumption that no queue is allowed to form is apparent in the workload plot, Figure 3.4. If the district cars had to answer all the calls for service, then the workload would be the same as the utilization factor, ρ . However, since calls can be answered by outside units, the system never becomes saturated. As a consequence, all of the results of this chapter, near $\rho = 1$, will differ from those one would expect for a conventional **infinite** capacity queueing system.

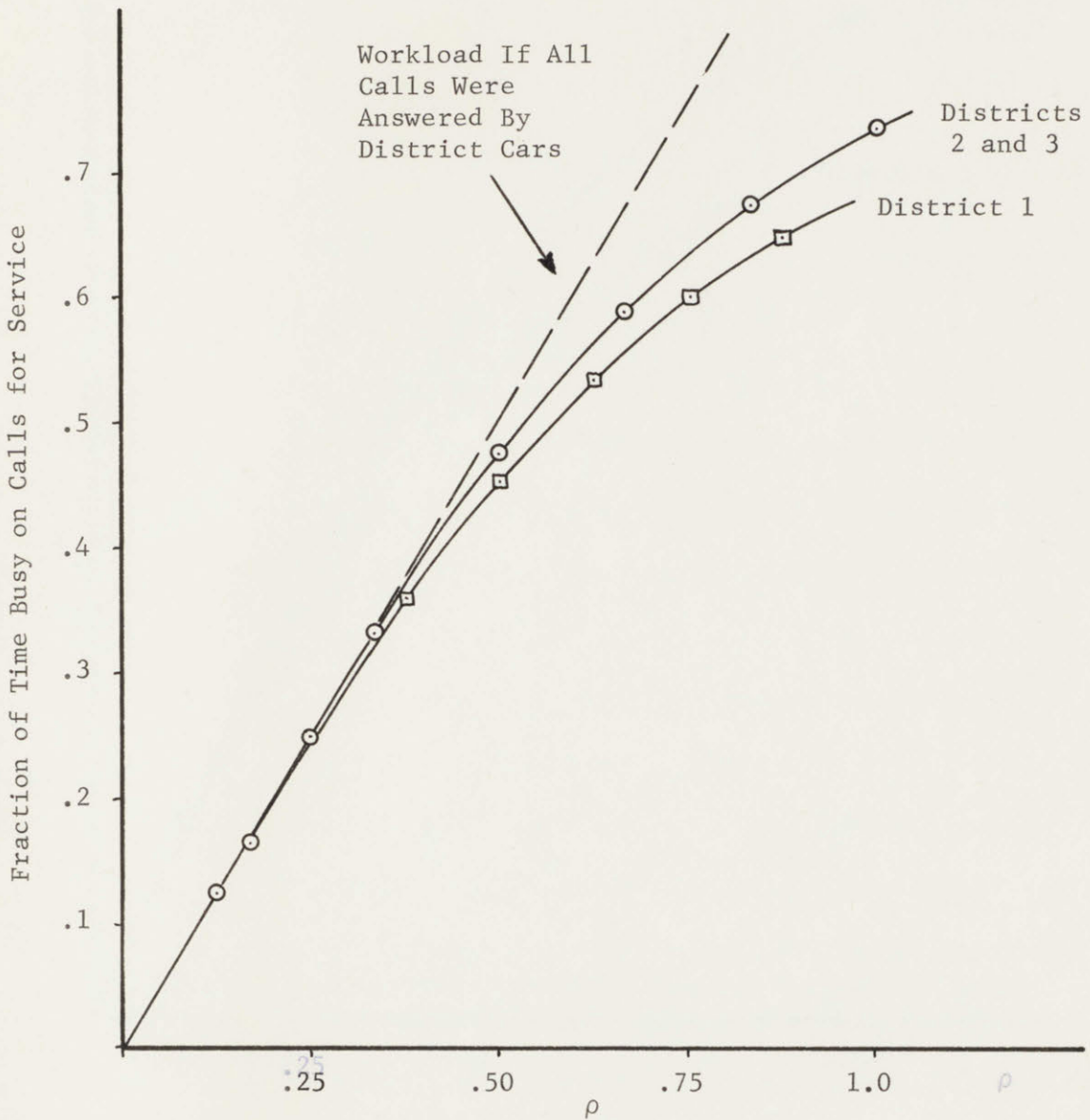


FIGURE 3.4. Average Workload As a Function of Utilization, ρ . Hypothetical Districts 1, 2, and 3.

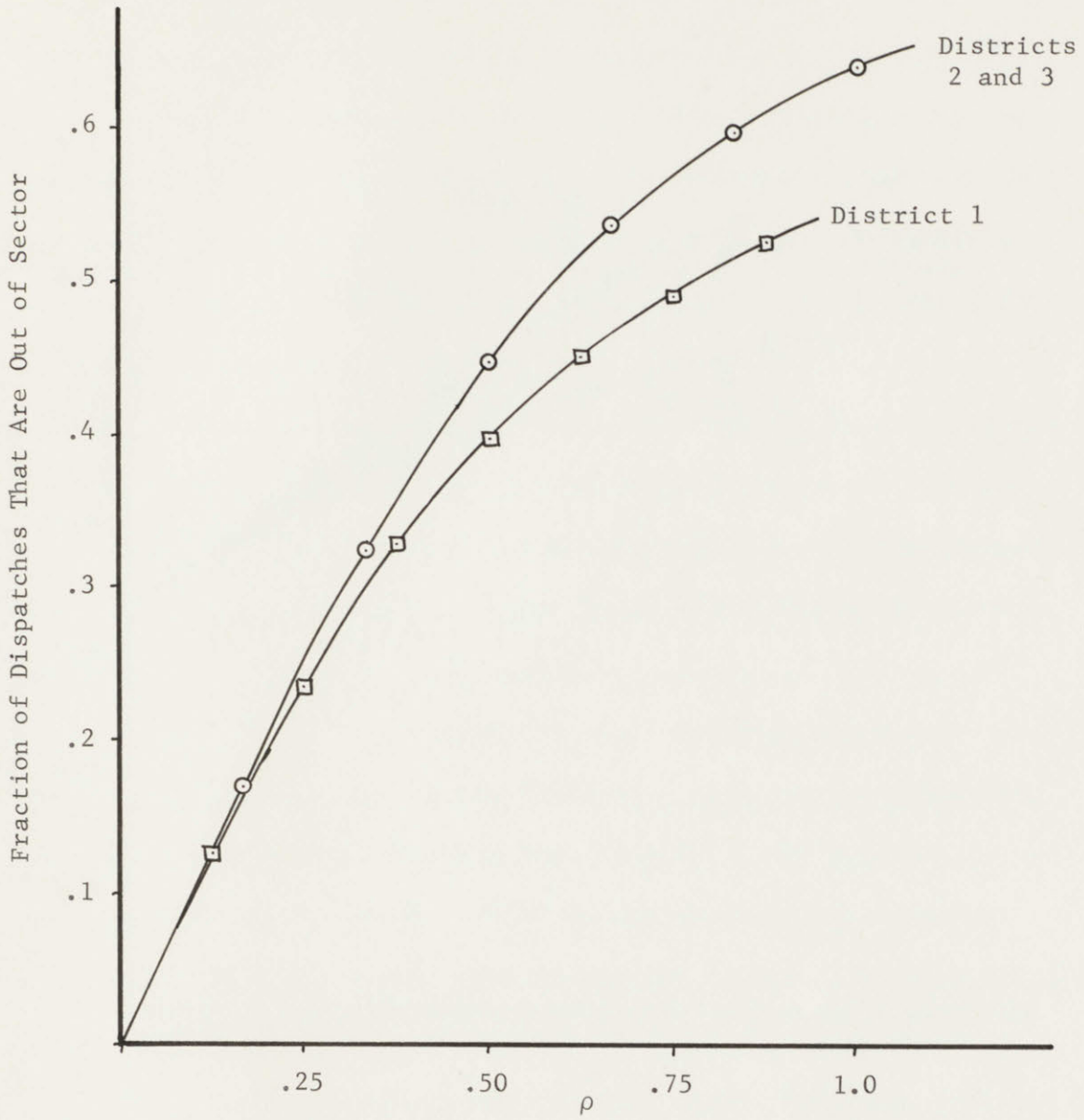


FIGURE 3.5. Amount of Intersector Dispatching as a Function of ρ . Hypothetical Districts 1, 2, and 3.

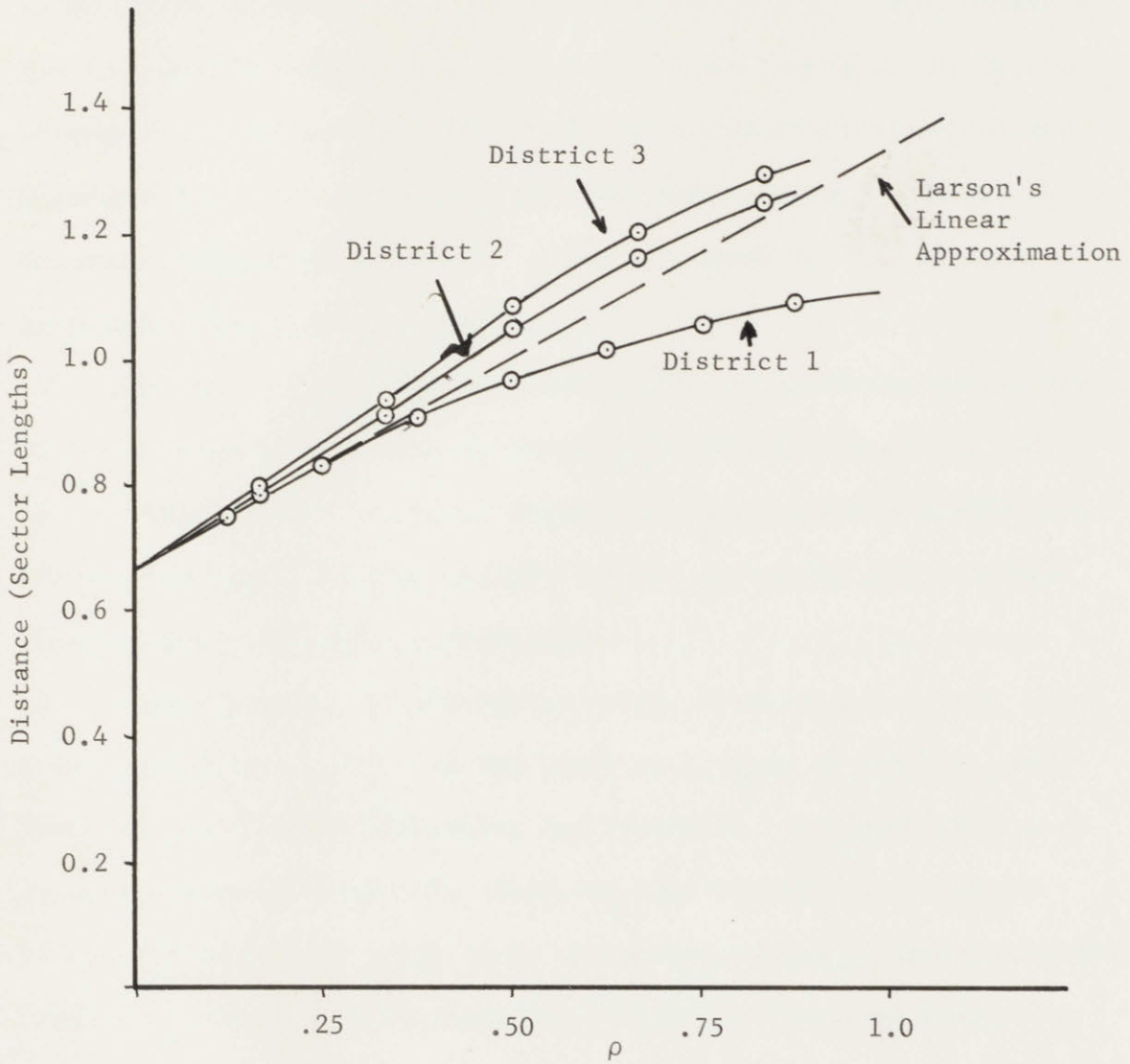


FIGURE 3.6. Average Travel Distance as a Function of ρ . Hypothetical Districts 1, 2, and 3. Straight line is Larson's approximation of travel distance for an infinite command.

The difference in size between District 1 and the other two districts is quite apparent from the graph of workloads. The smaller district has lower workload for the same value of ρ . This reflects the fact that increasing the number of servers in a queueing system increases its efficiency. Also note that workload is identical for Districts 2 and 3. From the assumptions of the model, workload is dependent only on the number of servers assigned and not on the geographic nature of the district.

The plot of the amount of intersector dispatching, Figure 3.5, is not so easy to explain. Intersector dispatching should depend on the shape of the district. There is actually a small difference, about .01 percent, between Districts 2 and 3, but for all practical purposes the difference is negligible.

Again, because of the smaller size, District 1 has less intersector dispatching. In the limit as ρ goes to infinity, the level of intersector dispatching for Districts 2 and 3 should asymptotically approach $5/6$. That is, near saturation, it should be equally likely for a car to be dispatched to any of the six sectors. Similarly, the asymptotic value for District 1 should be $3/4$.

Finally, in the next graph, average travel distance, Figure 3.6, there is a distinct difference between Districts 2 and 3. District 2 is more compact, therefore one would expect the average travel distance to be smaller, and this is indeed the case. At $\rho = 0$, the average for all three districts is $2/3$ sector length. This reflects the fact that at low utilization, all calls for service are made by the sector car. With increasing ρ , the difference in travel time increases as intersector dispatches increase.

As ρ approaches infinity, the asymptotes are 1.50, 1.66, and 1.70 for Districts 1, 2, and 3, respectively. These were calculated by similar logic as described above. Near saturation, it is equally likely for a trip to be made between any two pairs of sectors. The average travel time will thus be the average of the elements of the travel time matrix, T_{ij} . Calculating these averages results in the above asymptotic values.

It is interesting to compare the average travel time as calculated by the spatially distributed queueing model with a simple theoretical model by Larson.³ The model consists of a grid system of square sectors similar to Figure 3.3, except that it extends to infinity in all directions. It is assumed that each car is busy with probability, ρ , independent of all other cars. The average travel time to an incident can be calculated by considering an arbitrary call having arrived in one of the sectors. If the sector car is available (the probability of this is $1-\rho$), the expected travel distance is $2/3$. If the sector car is not available, it must be handled by one of the other cars. By considering each case of cars being unavailable, it is possible to construct a series expansion for the average travel time as a function of ρ . Keeping only the linear terms, the result is:

$$\bar{d} = 2/3 (1+\rho)$$

which is valid for values of ρ less than .7. The implicit assumption in this approximation is that either the sector car or one of the four adjacent cars is always available. This line is plotted in Figure 3.6

as a comparison to the numerical results. Notice the close agreement for low values of ρ , which one would expect.

It is interesting to note that travel distances for Districts 2 and 3 are worse than the infinite model due to the fact that large values of travel distance occasionally are encountered. However, District 1 performs better than the infinite model for values of ρ greater than .375. Here the effect of being able to handle a call by an outside unit takes over and dominates the effect of dispatching to a diagonal as opposed to an adjacent sector.

3.2b. Distribution of Service Within the District

The purpose of this section is to determine how the spatial orientation of sectors with respect to each other affects the distribution of workload and the distribution of service response to different parts of the district. District 3 is the primary example used in this section because of its irregularity in shape. To begin with, each sector has the same call rate or input workload. The cause of the discrepancies in distribution of actual workload and response time is the spatial relationship of the sectors within each other. Specifically, note that sectors 1 and 2 share one common side with another sector. Sectors 3, 4, and 5 each share two sides. Sector 6 shares four sides.

Figure 3.7 is a plot of the difference in workload from the average for each sector. Notice that the difference is low at small values of ρ , increases to a maximum at about $\rho = .5$, and

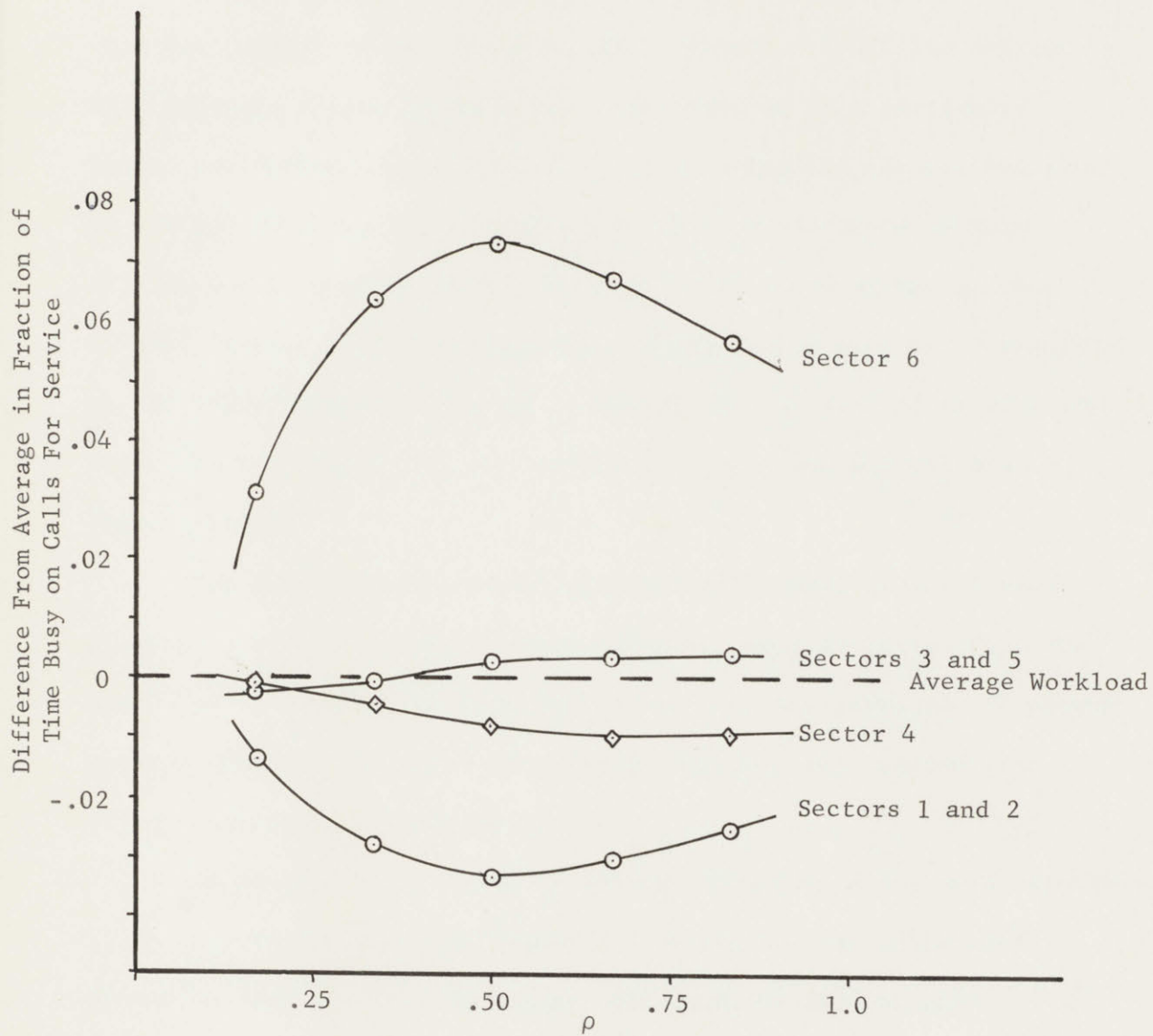


FIGURE 3.7. Difference in Workload From Average for Each Sector. Hypothetical District 3.

then decreases as ρ increases. This behavior is quite reasonable considering that at low utilization the patrol cars are busy primarily within their own sectors, whereas near saturation all are busy most of the time. This graph clearly demonstrates the fact that for typical values of utilization (about .5) call for service load does not determine workload. The workload of a particular patrol car depends not only on the calls within its sector, but also on how likely it is to be assigned to calls in adjacent sectors. If a sector is surrounded by nearby sectors, as is sector 6, then it will have an unusually high workload due to intersector dispatches. On the other hand, if a sector is relatively isolated at an edge or a corner of a district, as are sectors 1 and 2, then it will have a lower workload.

The significance of this workload difference is shown more clearly in Figure 3.8 which shows the percentage deviation from the mean. For $\rho = 1/3$, the workload of car 6 is approximately 30 percent greater than that of car 1 even though the call for service load within each sector is identical.

To evaluate the effect of spatial orientation on travel distance, it is interesting to study Figure 3.9, a plot of the percentage deviation from the mean for travel distances to each sector of District 3. Notice here that the dispersion increases monotonically with utilization as opposed to workload imbalance which has a maximum. Average travel distance to sector 1 is about 10 percent greater than the average to sector 6. Near $\rho = 1.0$, the difference increases to 20 percent.

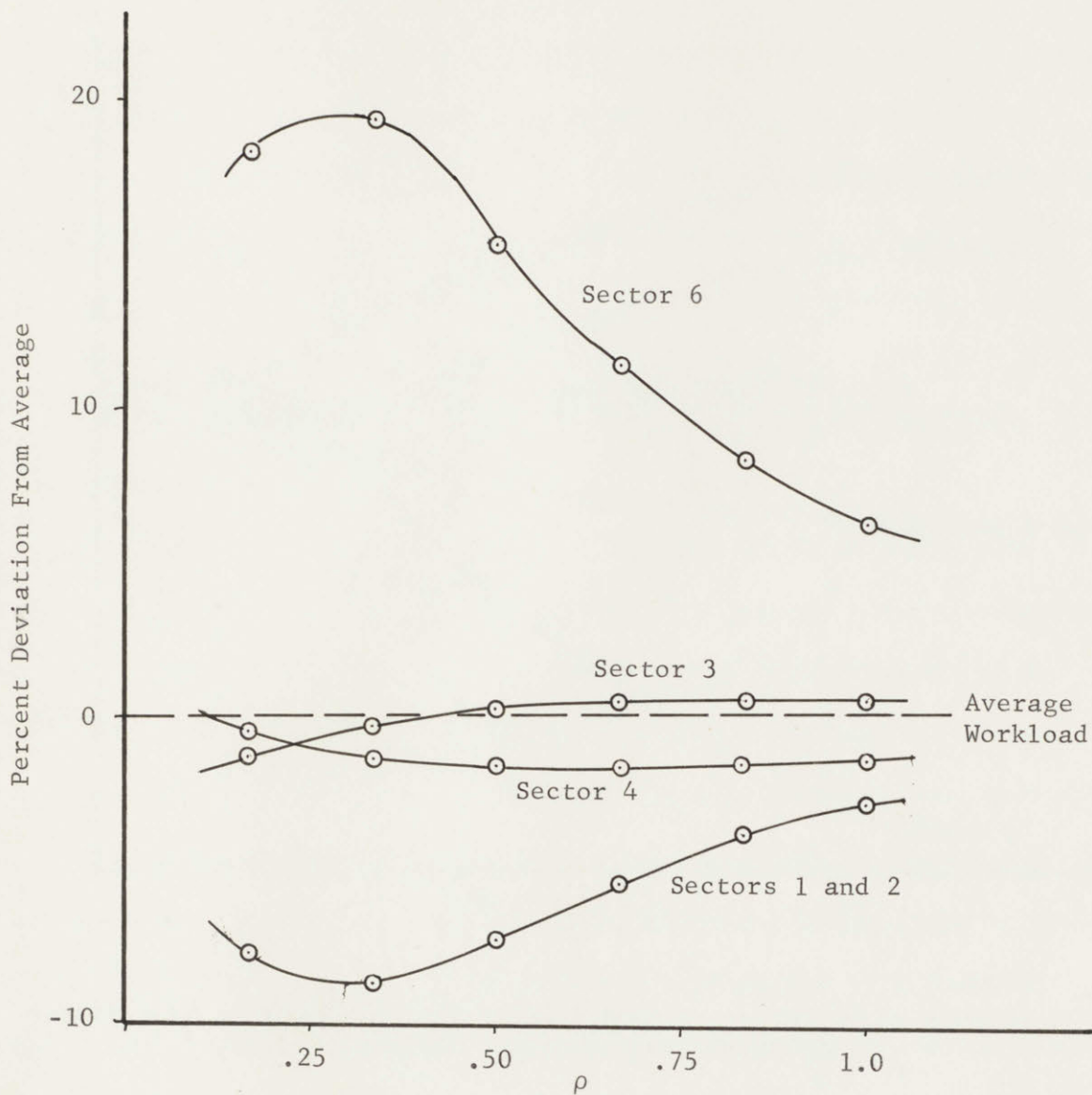


FIGURE 3.8. Percentage Deviation of Sector Workload From Average Workload. Hypothetical Districts 1, 2, and 3.

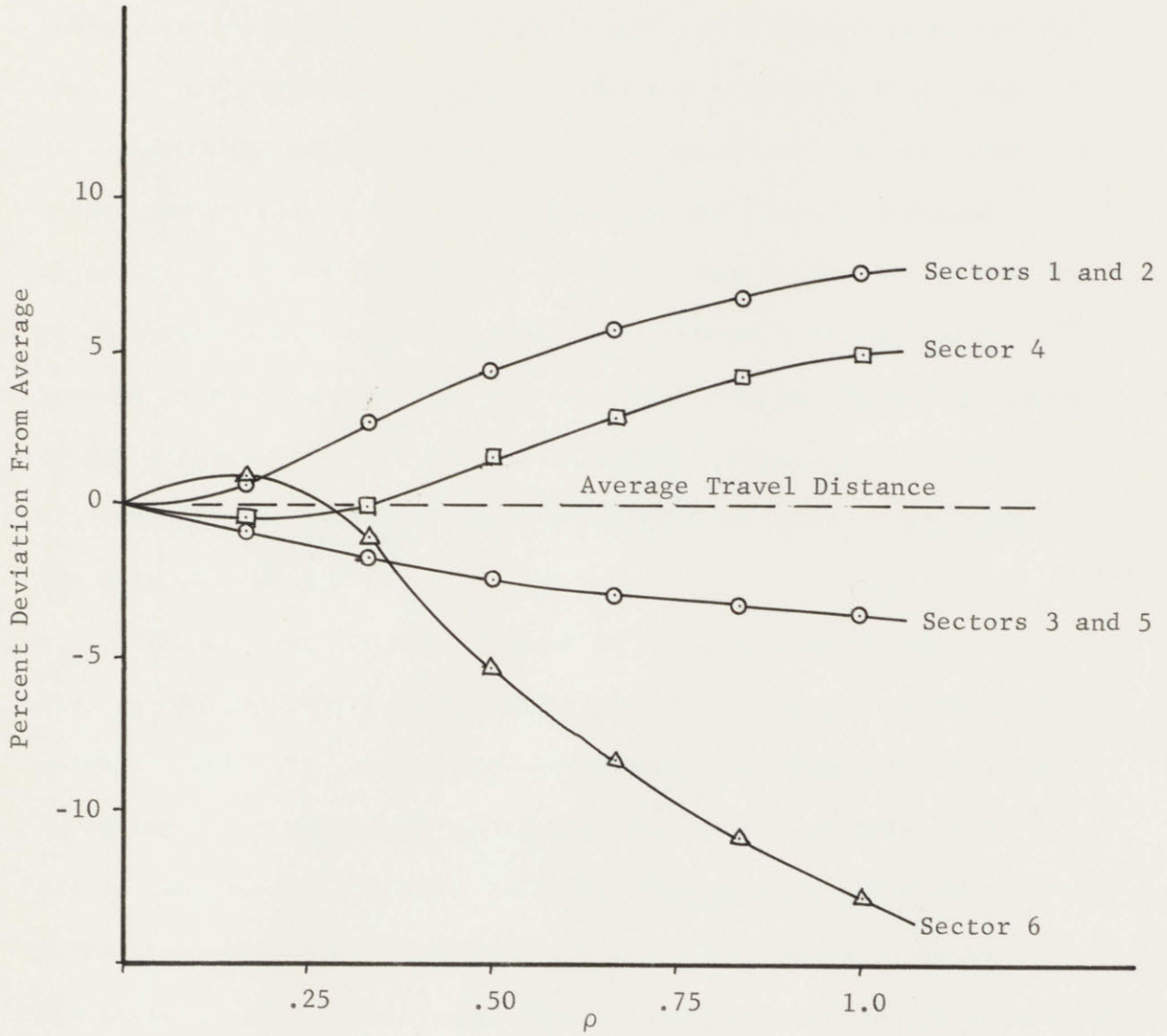


FIGURE 3.9. Travel Distance to Each Sector - Compared to Average Travel Distance. Hypothetical District 3.

The most interesting part of the graph, however, is for low values of utilization. For $\rho = 1/6$, travel time to sector 6, the most centrally located sector, is greater than for any other sector. Then, as utilization increases, it changes to the smallest value. This phenomenon apparently is caused by two conflicting elements that affect travel time. The sector 6 car has the highest workload, therefore it is more likely for a call in that sector to be answered by an out-of-sector car. However, since sector 6 is centrally located, travel distances by out-of-sector cars are relatively short. At low utilization, the effect of workload dominates. At high utilization, the effect of travel distances dominates, resulting in the observed behavior.

Finally, an attempt was made to balance workload and travel time by shifting input calls for service from sector 6 to the outlying sectors. This process is roughly equivalent to decreasing the size of sectors 6, 3, and 5 and increasing the size of sectors 1, 2, and 4. The changes in input call rate were made proportional to the percentage deviation in workload for $\rho = .5$ as shown in Figure 3.8. For example, the call rate for sector 6 was decreased by 15 percent and the call rate for sectors 1 and 2 were both increased by 7 percent. The increases and decreases were made so that the overall utilization remained .5.

The results of this attempt to balance workloads and travel times are quite striking. The results are shown in Figure 3.10. As the call rate is shifted to outlying sectors, the workload does indeed tend to equalize, except much more slowly than one might

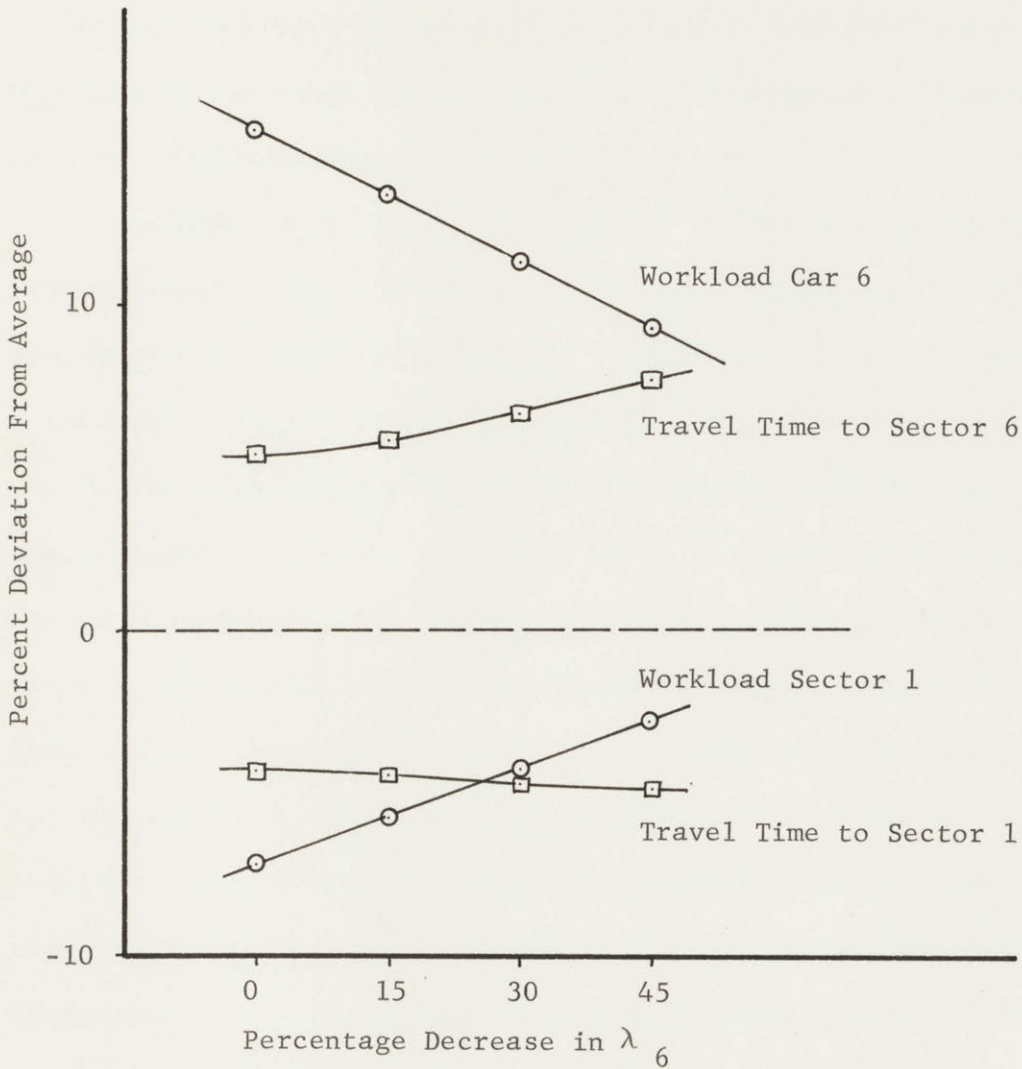


FIGURE 3.10. An Attempt to Balance the Workload by Shifting Calls for Service to Outlying Sectors. Workload balance improves, but travel time balance gets worse. Curves are plotted for $\rho = .5$, hypothetical District 3.

expect. Even when the call rate to sector 6 is reduced by 45 percent, the workload of car 6 is still far greater than the workloads of the other sectors. If the curve is extrapolated as a straight line, it appears that the call rate to sector 6 would have to be reduced almost to zero before workloads would be balanced. This demonstrates the strong influence of intersector dispatching on such a centrally located sector.

In addition to the problem of the inability to balance workloads, average travel distances become more imbalanced as calls are shifted to outlying sectors as shown in Figure 3.10. This is a fascinating problem which apparently cannot be easily resolved. One is faced with a tradeoff between balancing workloads and balancing response time to various parts of the district. There may be other ways to get around the problem, however. One way would be to redesign the sectors in such a way as to avoid such large differences between sectors. The other would be to alter the dispatching strategy so as to avoid dispatching car 6. In any practical situation, these alternatives should be more fully explored. For the present purposes, however, it was felt that the above analysis was sufficient to illustrate the problem and to point to the possibility of further research.

3.3. UTILIZATION OF CALL LOCATION INFORMATION

The purpose of this section is to demonstrate the effectiveness of utilizing call location information in making dispatching decisions. Call location information is readily available in the

form of incident addresses. The problem is to effectively utilize this information instead of assuming only that calls are located somewhere within a sector. In most large cities, dispatchers have little time to pinpoint locations on a map and carefully evaluate which of the available cars is most likely to be closest. In New York City, an on-line computer-aided dispatching system is currently being developed. Even in this system the call location information is not used. However, with a more detailed data base, it would be possible to incorporate this information into the system. The three example districts described earlier in this chapter are used to investigate the effectiveness of using this information.

The procedure for utilizing call location information in the spatially distributed queueing model is most clearly demonstrated for the case of District 1, composed of 4 square sectors. Each sector is divided into 2 basic geographical areas as shown in Figure 3.11.

Consider a call which has just arrived in basic area 1. No matter where the call is located within area 1, the closest expected car is the sector car. Similarly, sector 3 is second closest, sector 2 is third closest, and sector 4 is fourth closest. By observing Figure 3.11a, it is clear that this is the correct order of preference no matter where the call is located within area 1. Therefore, exact call location information can be modeled by specifying the basic geographical area in which a call arrives and by specifying the conditional travel time to a call given it is in that area. These conditional travel times are also shown in

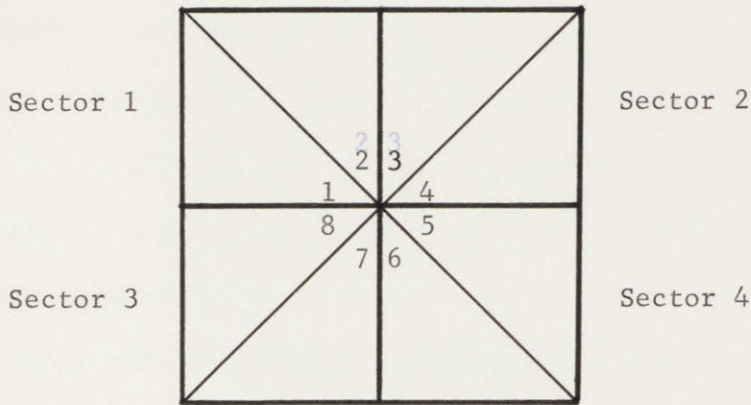


FIGURE 3.11a. District 1 Divided Into Basic Geographical Areas. For Modeling the Use of Call Location Information.

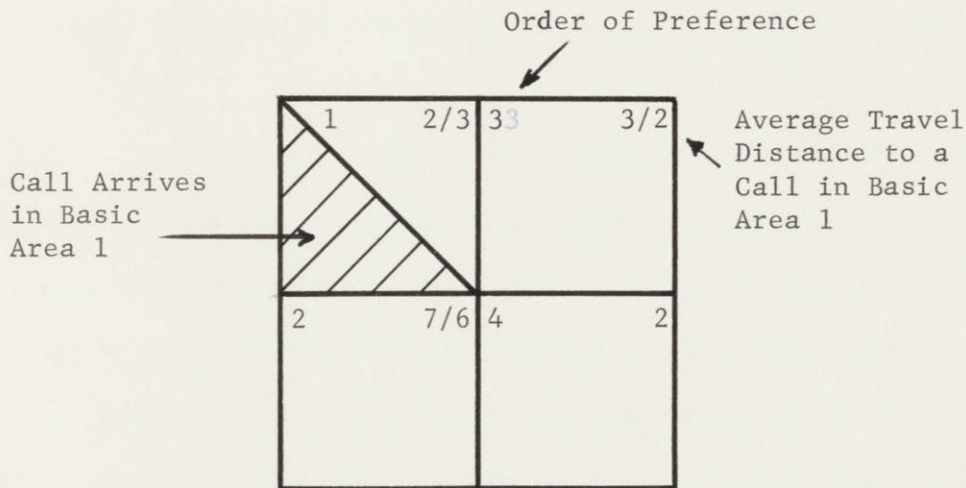


FIGURE 3.11b. Travel Distance and Order of Preference for Each Sector Car to Area 1. Order of Preference in Upper Left Corner. Travel Distance in Upper Right Corner.

Figure 3.11b. The travel distance matrix, T_{ij} , is constructed using these travel distances.

In a similar fashion, the other two districts are broken into basic areas as shown in Figure 3.12. The sectors are more finely divided because of the greater number of alternative cars available for dispatch. Actually, the division is an approximation since the basic areas do not distinguish exactly between the third and fourth choices. However, the approximation should be fairly good since there is a high probability that one of the first three choices is available.

The travel distance plots for each district using call location information are quite similar to those without call locations. The interesting quantity to compare is the difference in travel distance between runs using and not using call location information.

Figure 3.13 is a plot of the percentage reduction in travel time due to call location information. As one would expect, each curve is a unimodal function of the utilization, ρ . The maxima occur at about $\rho = 1/3$.

It is interesting to note some general trends indicated by the plots. Call location information appears to be increasingly effective with increasing district size. This is reasonable considering that there are more alternative choices in larger districts.

The effect of district shape is shown in the comparison between Districts 2 and 3. District 2, which is more compact, has a higher maximum in effectiveness. However, for high utilization, call location information is more effective in District 3, the more asymmetric district.

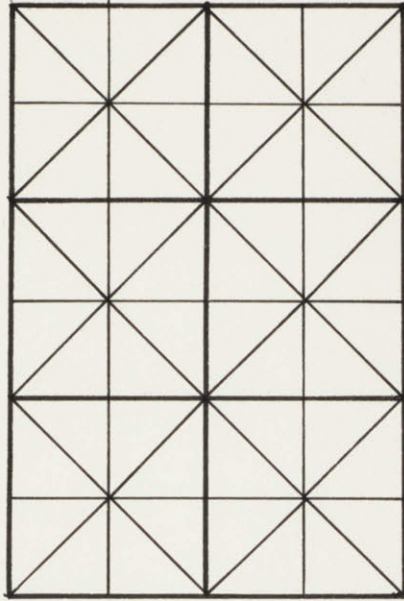


FIGURE 3.12a. District 2 Divided Into Basic Geographical Areas for Utilizing Call Location Information.

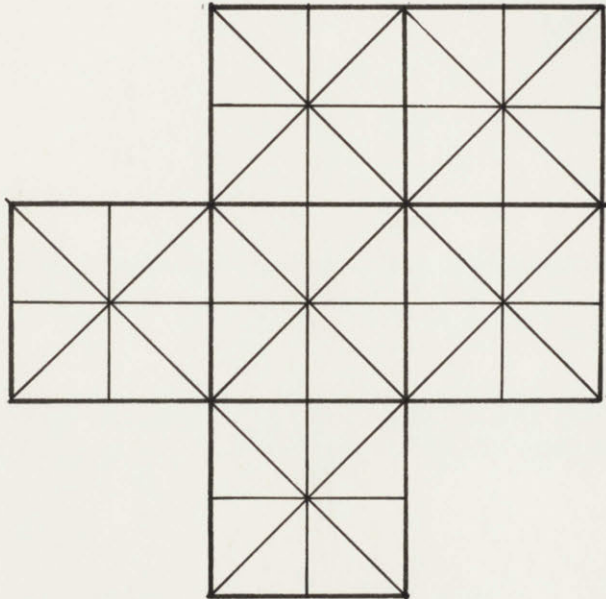


FIGURE 3.12b. District 3 Divided Into Basic Geographical Areas for Utilizing Call Location Information.



FIGURE 3.13. Percent Reduction in Average Travel Distance Using Call Location Information. Hypothetical Districts 1, 2, and 3.

Finally, note the relatively low value of maximum percentage reduction in travel distance, 3.6 percent at $\rho = 1/3$ for District 2. In some recent work,⁴ Larson has calculated the effectiveness of utilizing call location information for the infinite grid system of square sectors, with each sector divided into eight areas as in Figure 3.13. In this model, it is assumed that each car is busy with probability ρ independent of all other cars. Using the model, the maximum reduction in response time due to call location information is approximately 10 percent which occurs at $\rho = .45$. This confirms the notion that call location information is more valuable for larger districts.

3.4. OVERLAPPING SECTORS

The purpose of this last section is to outline some of the tradeoffs in response time vs. intersector dispatching involved in a simple system of overlapping sectors. The concept of overlapping sectors has been suggested as a way to avoid intersector dispatching and to increase "sector identity," the patrolman's feeling of responsibility for his own sector.

Another way to avoid intersector dispatching would be not to allow it. Under such a system, each car would have to answer all calls within its sector. This causes the problem of creating a large number of small queues, which would not form if intersector dispatching were allowed.

On the other hand, a system of overlapping sectors decreases the amount of intersector dispatching without affecting the queueing

nature of the problem. Some other advantages of overlapping sectors are that individual patrolmen become more familiar with the street pattern of a larger area which may reduce delays in response. In addition, patrol coverage is improved because patrol is not reduced to zero if only one sector car is busy.

Balancing the advantages of overlapping sectors is a definite increase in response time. To illustrate this tradeoff, Districts 1 and 2 were used as examples. Figure 3.14 shows the way sectors are overlapped. Adjacent pairs of sectors are combined to form rectangular sectors, each patrolled by 2 cars. It is recognized that there are many ways to design overlapping sectors. However, this particular design illustrates the concept.

In Figures 3.15 and 3.16 the designs with overlapping sectors are compared to the original designs with mutually exclusive sectors. Notice that for both Districts 1 and 2, intersector dispatching is almost eliminated for low values of utilization. For high values of utilization, it appears that overlapping sectors are more effective at reducing intersector dispatching in District 1 than in District 2. This is due to the fact that District 2 has 3 sectors as opposed to only 2 sectors in District 1.

As expected, average travel distance is always greater for designs with overlapping sectors. However, the difference in travel distance resulting from the alternative designs decreases as utilization increases. For District 1, the percentage increase in travel distance due to overlapping sectors ranges from 35 percent at $\rho = .125$ to only 8 percent at $\rho = .875$.



FIGURE 3.14a. Overlapping Sectors - District 2. Two Patrol Cars Cover Each Sector.

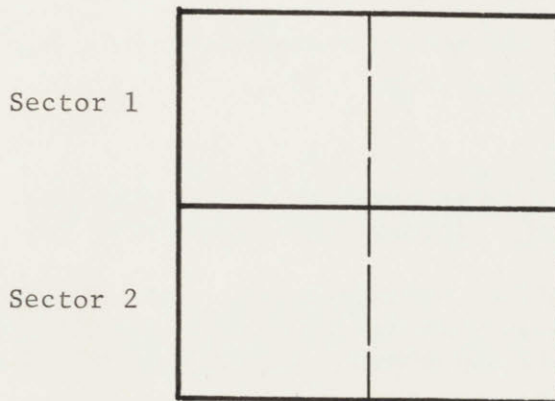


FIGURE 3.14b. Overlapping Sectors - District 1. Two Patrol Cars Cover Each Sector.

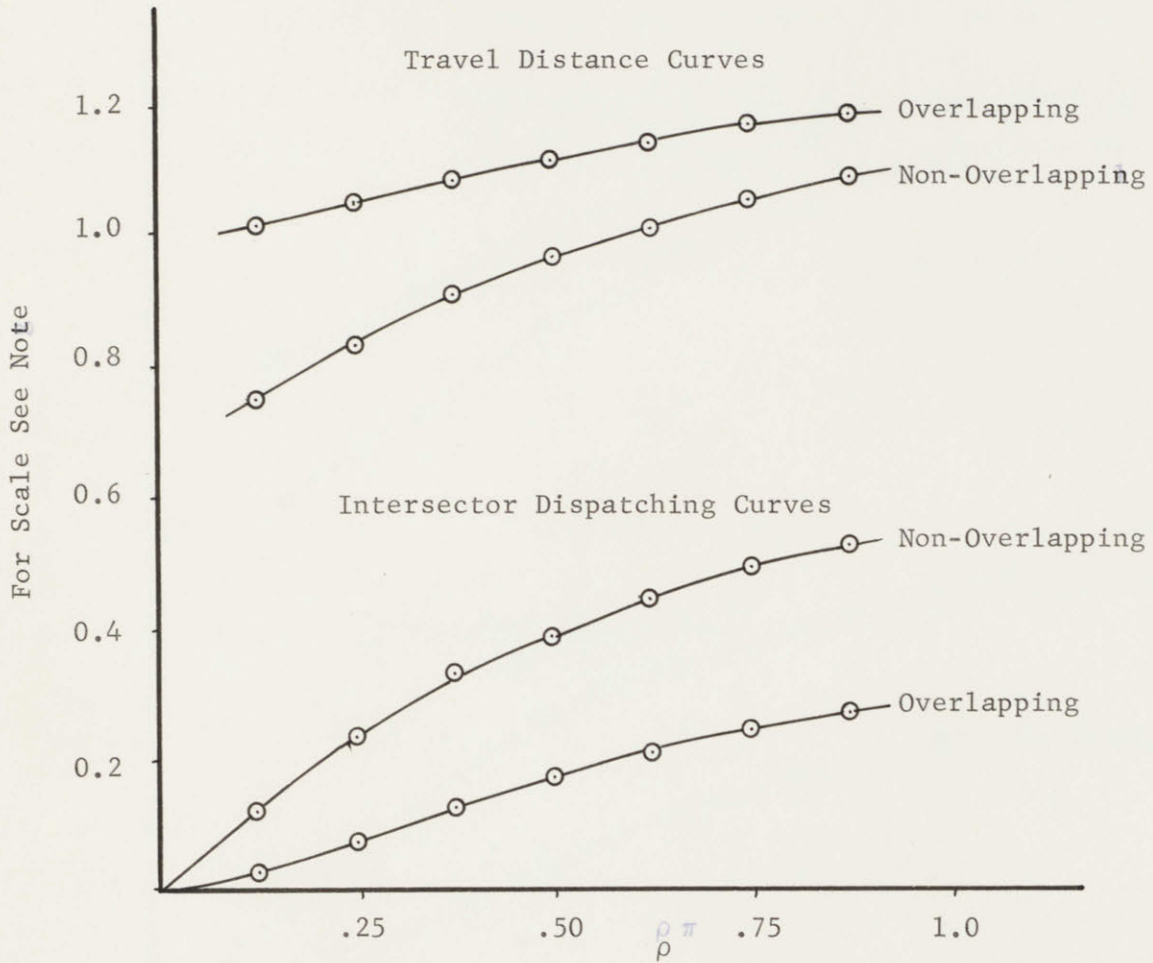


FIGURE 3.15. Overlapping vs. Non-Overlapping Sectors. Travel Time and Intersector Dispatching. District 1.

NOTE: The vertical scale is interpreted as sector lengths for travel distance curves. It is interpreted as fraction of dispatches that are intersector for intersector dispatching curves.

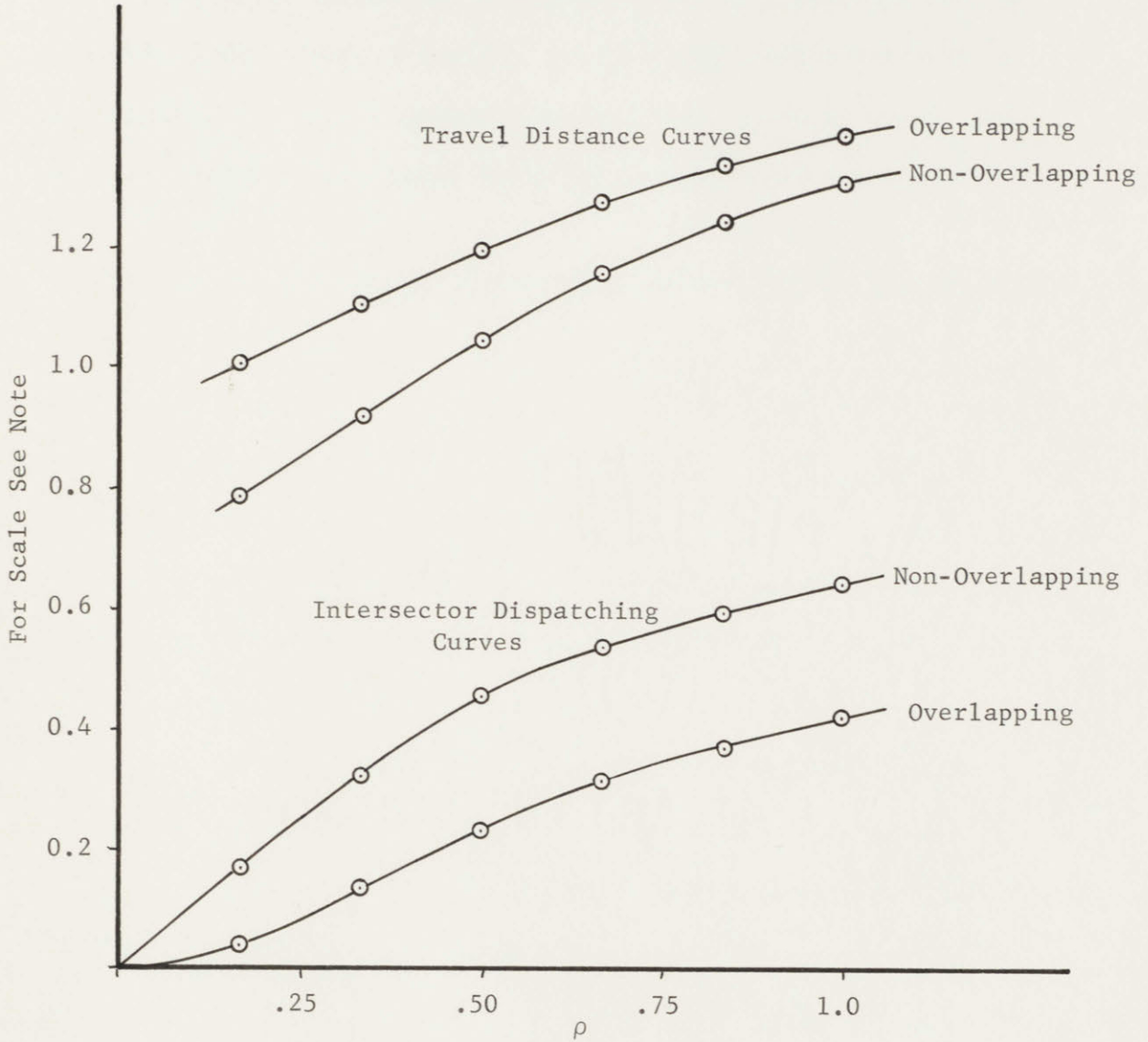


FIGURE 3.16. Overlapping vs. Non-Overlapping Sectors. Travel Time and Intersector Dispatching. District 2.

NOTE: The vertical scale is interpreted as sector lengths for travel distance curves. It is interpreted as fraction of dispatches that are intersector for intersector dispatching curves.

Clearly, when considering overlapping sectors, a choice must be made between reduced intersector dispatching and increased travel time. Such a choice deserves considerable attention by police personnel.

This brief example is not intended to be a thorough analysis of overlapping sectors. However, it does demonstrate one more of the capabilities of the spatially distributed queueing model, and indicates another area which could be studied more thoroughly.

FOOTNOTES

- ¹This corresponds to "strict center-of-mass" dispatching as defined by Larson. Utilizing call location information is referred to as "modified center-of-mass" dispatching.
- ²The utilization factor is defined as $\rho = \lambda / \mu m$ where λ is the total call-for-service rate, μ is the service rate, and m is the number of patrol cars.
- ³See Larson, Models for the Allocation of Urban Police Patrol Forces, page 133.

CHAPTER IV

SECTOR DESIGN CASE STUDY

This chapter is a case study in the use of the spatially distributed queueing model, GEOQUEUE, as a decision aid for patrol sector design. When applied to sector design, the model is used to help evaluate alternative designs. Police planners must provide the alternative designs as input to the model. In this way, important subjective factors can automatically be taken into account before the design is tested. The model then provides estimates of important measures of performance such as the average response time and workload balance that will result from that design. Without such a model (either analytic or simulation), there is no way to calculate these measures of performance.

Because of time limitations, only three alternative sector designs were tested. While these designs clearly demonstrate the use of the model, police personnel may wish to examine alternative designs prior to implementation.

The district used in the case study was Police District 14 in Boston, which covers the Brighton area of the city (see Figure 4.1). This district was suggested by Mr. Steven Rosenberg, Director of the Planning and Research Division of the Boston Police Department, since he had been considering changing the sector design by increasing the number of sectors from four to six. At present, the district

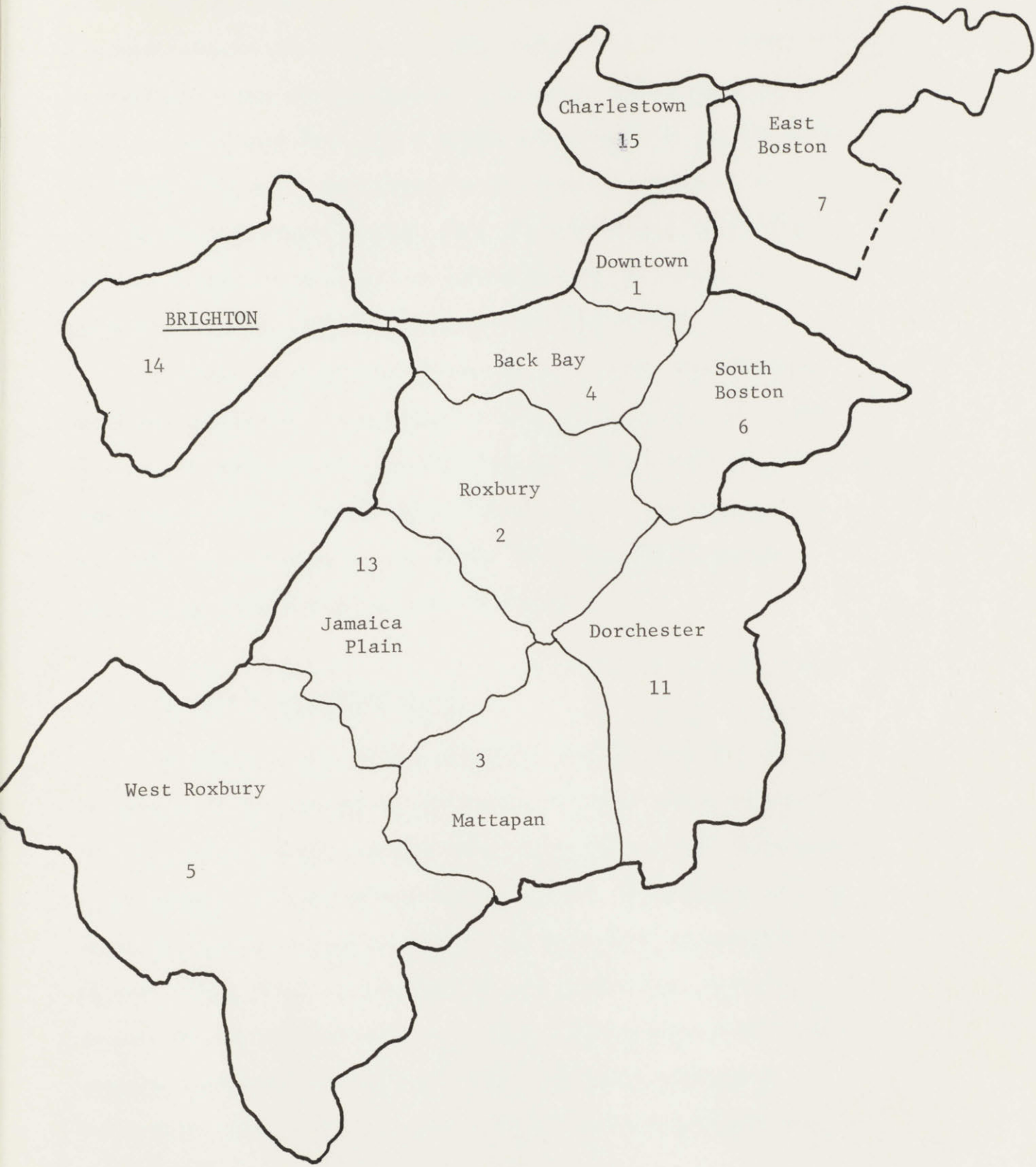


FIGURE 4.1. Police District Map - City of Boston.

is divided into four large sectors. However, because of the high workload, six patrol cars are usually assigned to the district. In addition to the sector cars, at least one station wagon, which makes all ambulance runs, and a patrol supervisor car are available for dispatch if the sector cars are all busy. In addition to considering alternative designs with six sectors, Mr. Rosenberg was interested in comparing the performance of the system as presently designed with that attained with six sectors.

In order to accomplish these two objectives, the GEOQUEUE model was applied to the existing sector design as well as to two alternative designs each with six sectors. One alternative six-sector design was developed by police personnel from District 14. The other was developed by the author in an attempt to improve the response time resulting from the design.

4.1. PRINCIPLES OF SECTOR DESIGN

In order to see how the spatially distributed queueing model is used in sector design, it is helpful to consider the subjective criteria and the administrative constraints that affect sector design. It is recognized that subjective criteria are an important part of sector design and cannot be ignored simply because an analytic model is available. There is no substitute for experience, therefore someone who is familiar with the district should propose and help evaluate alternative designs. The model provides additional information that could then be incorporated into the evaluation.

Some of the subjective criteria and administrative constraints used in sector design are the following:

1. Sector boundaries should coincide with main streets. This creates double patrol in areas where it is needed. It also clearly outlines patrol areas.
2. Impediments to travel, such as limited access highways, railroads, and parks must be considered. (This can also be incorporated in the model through the travel time matrix.)
3. State highways which are not patrolled but can be used for travel must be considered.
4. The character of the neighborhood should be considered. It is desirable to have a sector cover a fairly cohesive neighborhood so that the patrolman can identify with it. On the other hand, an attempt is made to share trouble spots among sectors.
5. Sectors should not have "peculiar" shapes. Compactness helps reduce travel time within the sector. It also makes a sector map easier to read.

Besides these subjective criteria, there are several other problems that complicate the process of sector design. First, the call-for-service data are not perfect. Mistakes are made by the patrolman in filling out reports and by clerks in the process of tabulating statistics. Second, the demand for police service is time-dependent, but the sector design is fixed. This problem can be approached by testing designs under different input conditions. However, unless the department is willing to use multiple sector designs, some compromise must be made in deciding which design to implement. Third, the level of patrol force assigned to the district may vary, depending on the shift and the absentee rate.

Again, various manpower levels can be tested using the model. From this, designs can be evaluated in terms of how well they adapt to changing manpower levels.

4.2. DISTRICT 14: BRIGHTON

In order to understand the problem of sector design in Brighton, it is helpful to know something about the general nature of the district. Brighton has a population of 63,653,¹ has an area of 4.446 square miles, and contains 66.3 road miles. Figure 4.1 is an outline of the City of Boston showing the relative location of Brighton. The district is fairly isolated from the rest of the city. Therefore, the assumption that the patrol cars do not interact with other districts is a good one. Some of the main features of the district are shown in Figure 4.2.

Brighton is primarily a residential area, with a high proportion of students from the local universities. Some light industry is located along the Penn Central Railroad. The main commercial districts are located in the southeast corner along Commonwealth Avenue, Brighton Avenue, Harvard Avenue, and Cambridge Street. Harvard Stadium and Harvard Business School are located in the northwest corner. Parts of Boston College and Boston University are located in Brighton, as shown on the map.

The residential areas of the district have distinctly different characteristics. The northwest section is made up of individual one and two family houses. The southern section along Commonwealth Avenue is almost solid apartment buildings. This is where most of

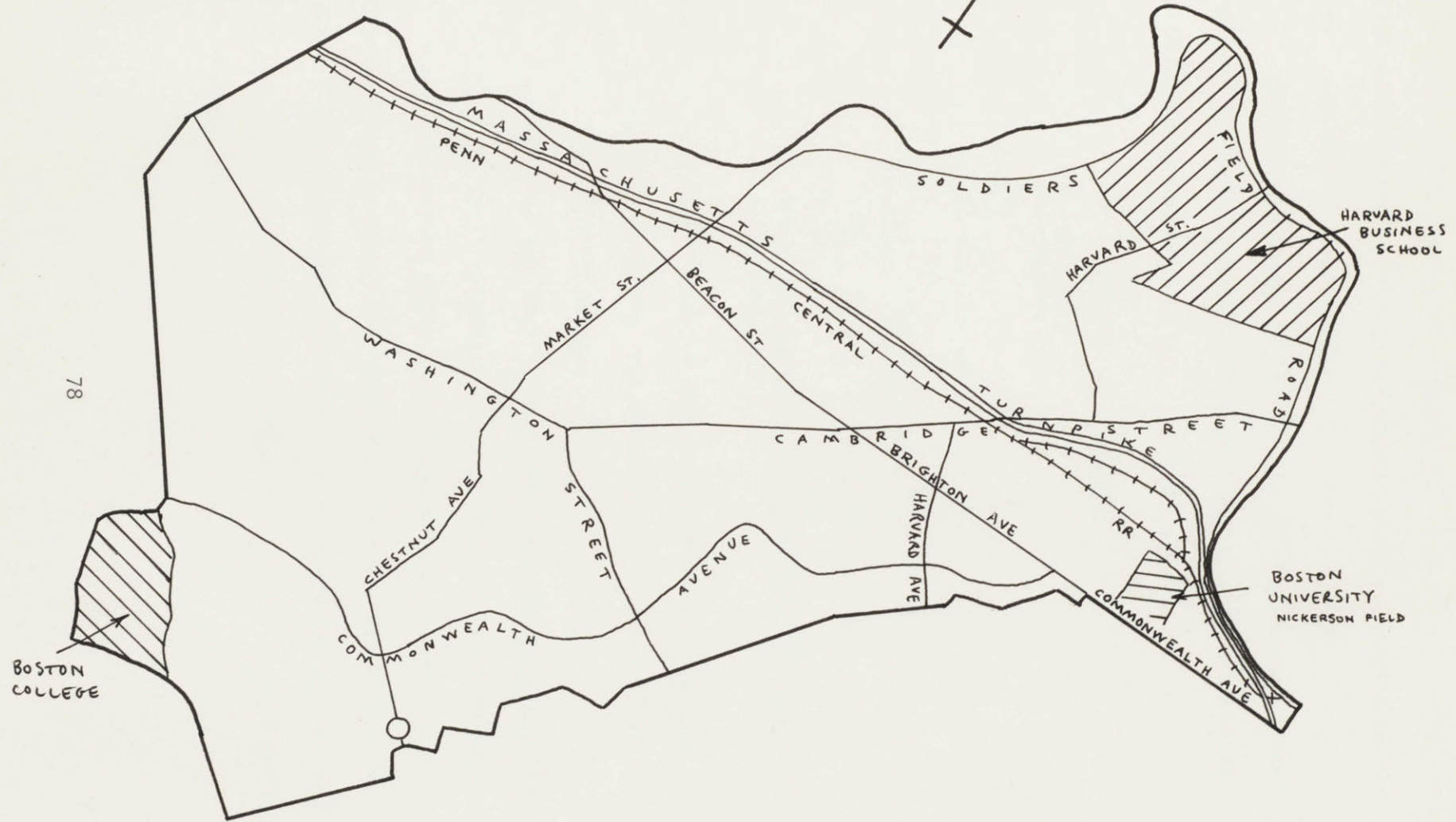
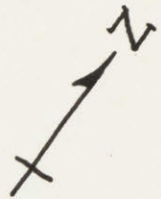


FIGURE 4.2. Major Streets of Brighton.

the students live. Other parts of the city have scattered apartments and individual houses. A large subsidized housing project is located at Washington Street and Commonwealth Avenue.

To get an idea of the nature of crime in Brighton, a listing of the Part I crimes,² recorded from January to November 1971 are given in Table 4.1 below:

<u>Crime</u>	<u>Number</u>
Murder	1
Manslaughter	2
Rape	13
Robbery	81
Aggravated Assault	34
Burglary	973
Larceny over \$50	108
Larceny under \$50	80
Auto Theft	<u>1,894</u>
Total	3,186

Table 4.1. Part I Crimes Reported in Brighton, January to November 1971.

Burglary and auto theft comprise almost 90 percent of the Part I crimes in Brighton. Burglary represents about 30 percent and auto theft represents about 59 percent.

From examining the Part I crime data recorded by reporting area, Appendix 2, it is apparent that most of this crime is located in the densely populated areas along Commonwealth Avenue. It should be noted that the people living here are often the crime victims and are not necessarily the criminals. A predominance of the call-for-service load also originates in this area. This is an important factor in sector design.

On Friday and Saturday nights, much of the activity in the district is generated by the licensed establishments along Harvard Avenue. According to Mr. Rosenberg, Harvard Avenue is becoming Boston's new "Combat Zone," a term usually applied to lower Washington Street in Downtown Boston. It is thus important to have adequate patrolling along Harvard Avenue late Friday and Saturday nights. It is also desirable to use Harvard Avenue as a sector boundary to obtain double patrol along it.

4.3. APPLICATION OF THE MODEL

This section describes how the spatially distributed queueing model was applied to Brighton. The input data and dispatching strategy are discussed as well as the assumptions inherent in the model.

4.3a. Input Data

Travel time data are incorporated into the model in the form of the travel time matrix. The elements of the matrix are the average travel times required between each pair of reporting areas in the district. Ideally, the matrix is constructed element by element by examining the actual street network and calculating the travel times. Data for this approach are available on computer files under the Urban Transportation Planning System 360.³ The 1963 Highway Network File for Eastern Massachusetts which is on this system contains travel speed data for all the major streets in Eastern Massachusetts. The speeds are given between each pair of intersections for the network.

Using these travel speeds, the travel time matrix could be calculated by hand, or generated by computer.⁴ If the spatially distributed queueing model were used regularly for sector design, it certainly would be worth the effort to construct an accurate matrix. However, since the present study is primarily illustrative, a major simplification was made. Instead of calculating each element of the matrix (there are $69 \times 69 = 4,761$ elements), x and y coordinates of the approximate center of each reporting area were taken from a map of the district. The coordinate system was oriented so that the x-axis was parallel to Cambridge Street (see Figure 4.2). Travel distances between each pair of reporting areas were calculated using the "right angle" metric, the sum of the x distance and the y distance. Travel time was then calculated using the constant travel speed of 17 miles per hour.⁵ The actual value of the speed is not critical, however, since the same speed was used for each sector design, and the designs were compared relative to each other.

Call-for-service data used as input to the model were actual data from the Boston Police files supplied by Mr. Rosenberg. The data for District 14 were collected over the period December 1970 to November 1971 (see Appendix 2 for a listing of the data). The data are aggregated over a whole year, thus obscuring variations as a function of time. In order to model various levels of workload, it was assumed that the spatial distribution of calls for service was independent of the overall call rate. The utilization factor, ρ , was then given different values reflecting the various

workloads that might arise. This process was the same as that used in the hypothetical examples of Chapter III.

The input data for specifying the patrol sectors and the required patrol effort for each reporting area were modified slightly from that described in Chapter II to make it easier to alter sector designs. Instead of defining sectors through the patrol allocation matrix, A_{ij} , sectors were defined by a list of reporting area numbers. For example, sector 2 might be defined by the numbers 31, 32, 37, 38, and 42. These numbers correspond to the reporting areas that compose sector 2. If one wished to modify sector 2, reporting area numbers could be simply added to or subtracted from this list.

The patrol effort allocated to each reporting area is specified by assigning a patrol weight, pw_i , to area i . The sector car is then assumed to patrol within its sector in proportion to the specified patrol weight. For lack of a better measure, the patrol weights used were the call-for-service load for each reporting area. Ideally, the patrol weights should be based on the number of "suppressible" crimes per area. However, these data were not available.

4.3b. Dispatching Strategy

The dispatching strategy assumed in the model was closest expected car with approximate call location information. (This corresponds to "Modified Center of Mass Strategy" in Larson's terminology.⁶) Specifically, the dispatcher is assumed to know the reporting area of each call for service. He then dispatches the available car whose expected travel time to the incident is a minimum. Available cars are assumed to be on patrol within their

own sectors. The probability of being in any reporting area within the sector is proportional to the specified patrol weight associated with that reporting area.

This strategy corresponds to a very careful dispatcher who spends the time to utilize call location information, or to an automated system which is based on this strategy. It is interesting to note that under this strategy, the sector car may not always be the first choice to respond to a call within its sector. For example, consider alternative sector design number 2 of Figure 4.4. If a call arrives in reporting area 782 which is located in the southeast corner of sector 5, the expected travel time of car 2 to the incident is less than that of car 5 even though the call is in sector 5. In this case, the order of preference for the assignment of cars is 2, 5, 3, 4, 1, 6. The dispatching strategy could be modified to require that the sector car always be dispatched when it is available. This modification would certainly be more realistic in terms of present practices. However, the modification was not made in the analysis of the alternative sector designs. It is important to keep this in mind when reviewing the results of the next section.

Finally, recall that in its present form, the model does not allow a queue of waiting calls to form. It is assumed that the district supervisor or the district wagon will respond to calls for service when the sector cars are all busy. Thus, the probability of a queue forming is small.

4.4. COMPARISON OF ALTERNATIVE DESIGNS

The three alternative designs compared by the spatially distributed queueing model are shown in Figures 4.3 to 4.5. The designs are labeled alternatives 1, 2, and 3. Alternative 1 is the present sector design with 4 sectors. The two extra cars assigned are assumed to patrol district-wide. Alternatives 2 and 3 are designs containing 6 sectors each. Alternative 2 was proposed by Patrolman Arthur Doyle and approved by Captain Rachalski of the District 14 station house. Alternative 3 was a modification of Design 2 developed by the author in an attempt to reduce system response time. All three designs provide double patrol along Harvard Avenue, which is desirable.

By examining the call-for-service data, it is clear that the greatest number of incidents occur along Commonwealth Avenue which runs through the southern section of the district. Design 3 is an attempt to concentrate a large number of patrol cars in that vicinity to increase the probability of having an available car close to expected incidents. The sectors of Design 2 have much more regular shapes. However, Design 3 does concentrate the patrol cars where the demand is located.

When considering the results of the model as applied to Design 1, it is important to see how the two district-wide cars are selected for dispatching under the present strategy. Each district-wide car is assumed to patrol the whole district, patrolling each reporting area in proportion to its patrol weight, pw_i . The whole district essentially serves as the sector of a district-wide car. Using the patrol weights and the travel time matrix, it is possible



FIGURE 4.3. Alternative Design 1, Existing Sector Design. Two extra cars are assumed to patrol district-wide.

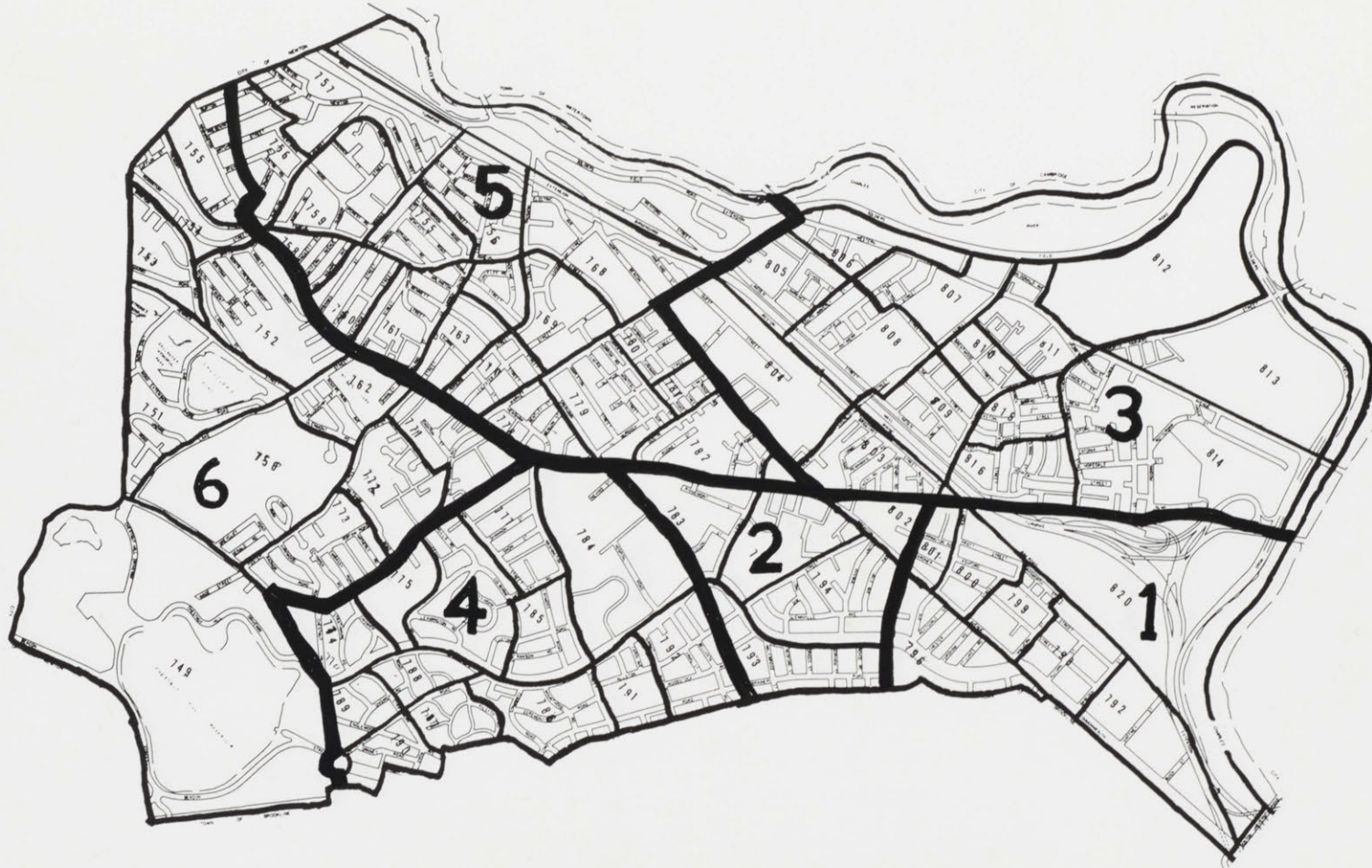


FIGURE 4.4. Alternative Design 2, Six Sectors. Proposed by Personnel From District 14.

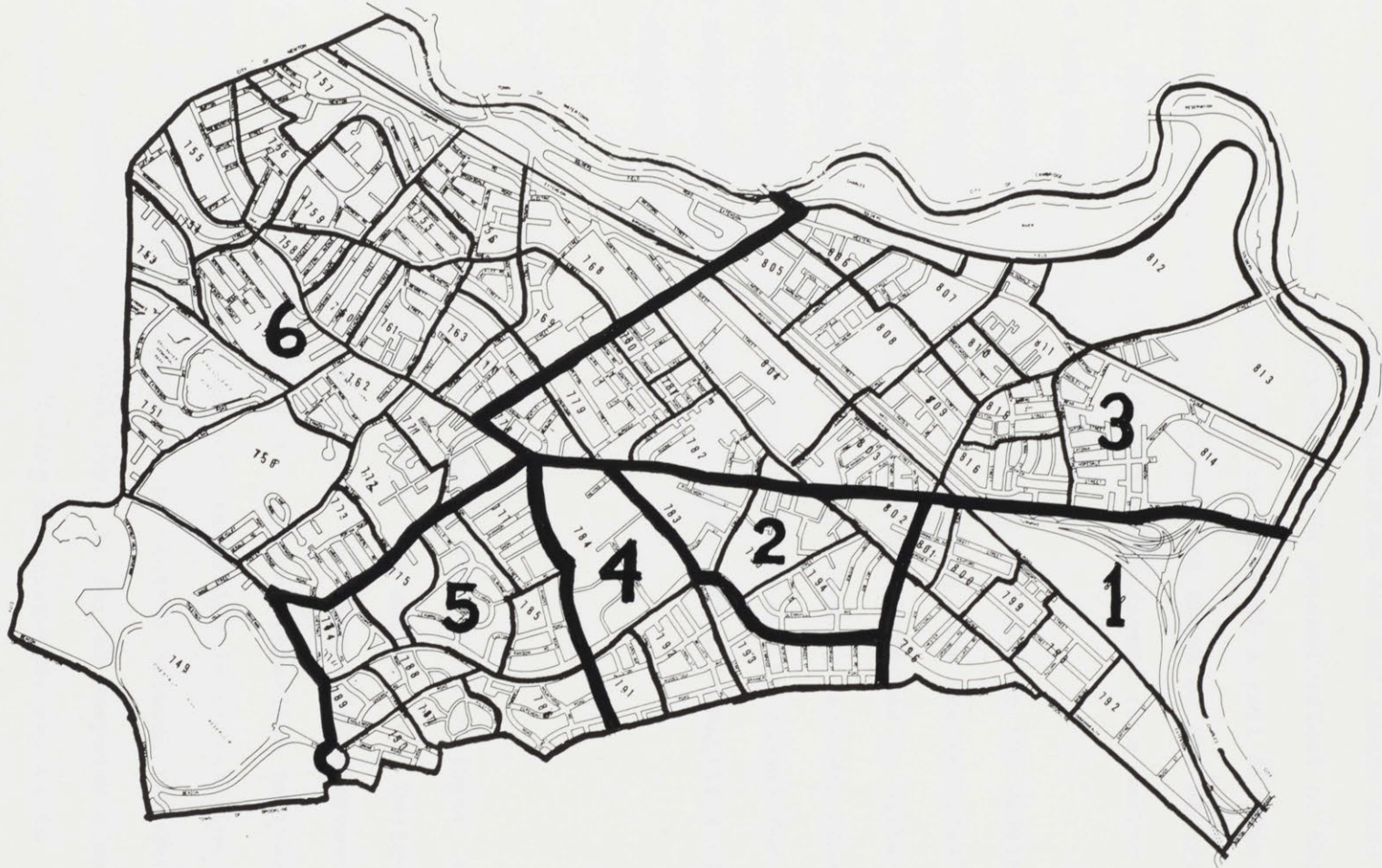


FIGURE 4.5. Alternative Design 3, Six Sectors. Proposed by Author.

to calculate the expected travel time of a district-wide car to each reporting area in the district. These expected travel times are used exactly the same way as those calculated for the sector cars in determining which car to dispatch. The general effect of this procedure is that the sector car of the sector in which a call arrives and perhaps an adjacent sector car will be dispatched ahead of the district-wide cars. However, a district-wide car is preferred to a sector car which is a long distance from an incident.

For example, consider reporting area 782 in the middle of the district (Figure 4.3). The order in which cars are selected is 3, 2, 5 or 6, 1, 4. The district-wide cars are preferred to either cars 1 or 4 since, on the average, the district-wide cars are closer to that reporting area. When a district-wide car is chosen for dispatch, the selection between 5 and 6 is random with a probability of .5 for choosing either one.

4.4a. Response Time

Plots of average travel time as a function of input call-for-service load are shown in Figure 4.6. As anticipated, Design 3 has the best average response time of the three alternatives. At $\rho = .5$, Design 3 results in a 4 percent reduction in travel time over Design 2 and a 17 percent reduction in travel time over Design 1. The greatest difference is clearly between the design with 4 sectors and the two with six. As far as response time is concerned, it appears that allowing district-wide patrol is detrimental to system performance. The basic reason for this increase

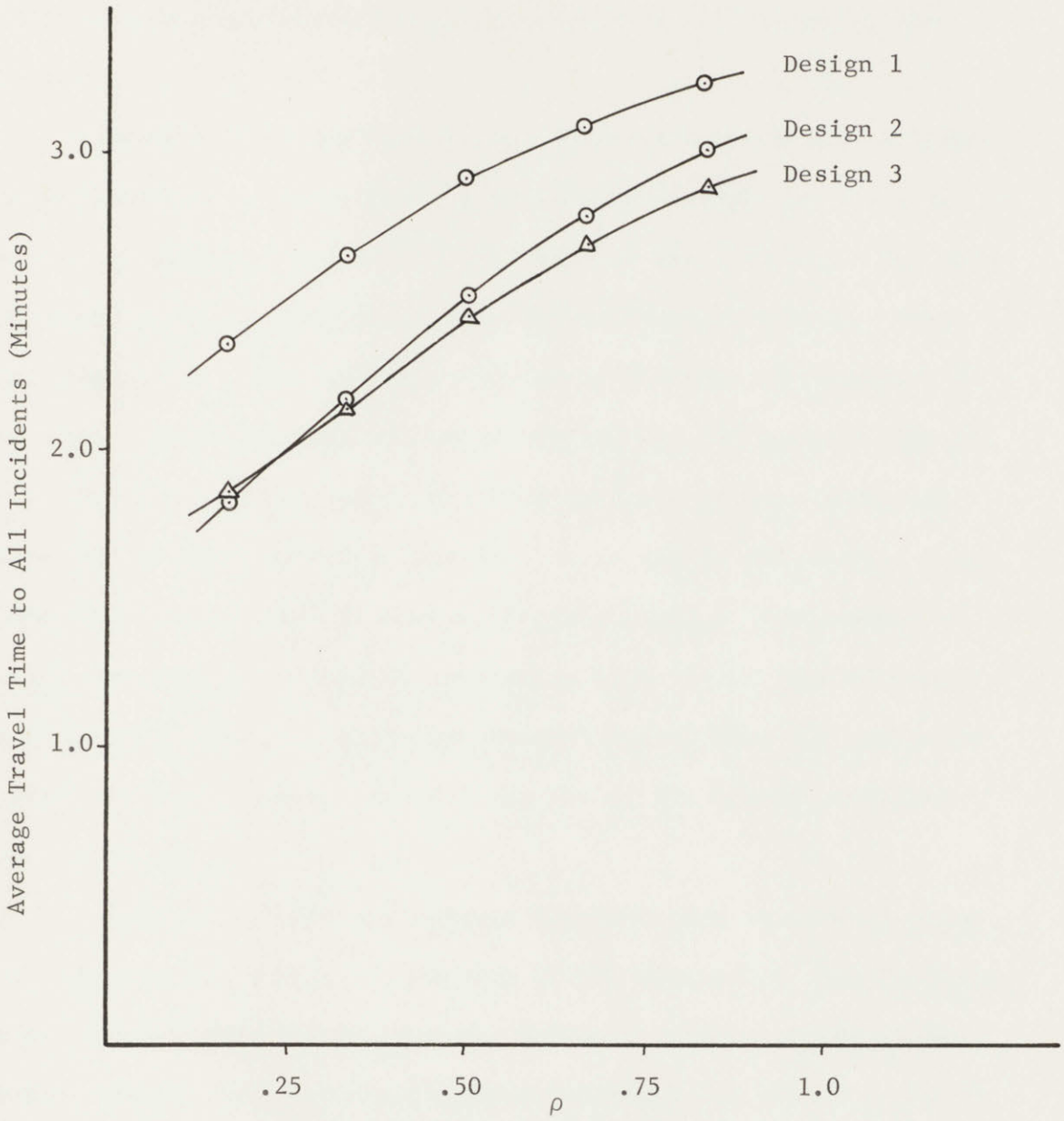


FIGURE 4.6. Average Travel Time Resulting From Designs 1, 2, and 3.

in response time is that the dispatcher has no vehicle location information for the district-wide cars. Therefore, he is often likely to choose a car which is not closest to the scene of the incident.

Besides examining the average response time to all incidents, it is interesting to compare the alternative designs on the basis of equity of response time to various parts of the district. For this purpose, three representative reporting areas were chosen. These are numbers 751, 784, and 796. (Refer to the maps of Figures 4.3 to 4.5.) Reporting area 751 is located at the far western edge of the district. It is typical of other outlying areas. Reporting area 784 is more centrally located. It is one of the busiest areas since it contains Fidelis Housing Project where a large number of calls for service originate. Reporting area 796 is located along a densely populated area of Commonwealth Avenue near the southeast corner of the district. It also has one of the highest call-for-service rates.

Figure 4.7 shows the average response time to each of these reporting areas resulting from each of the alternative sector designs. Note that for each reporting area, Design 1 results in the highest travel time. The comparison between Designs 2 and 3 is more interesting, however. Design 3 results in a higher travel time to the outlying area but a lower time to the most centrally located reporting area. This is clearly a result of the higher concentration of patrol cars in central areas of the district. These graphs are typical of the comparison between Designs 2 and 3. Design 2 generally

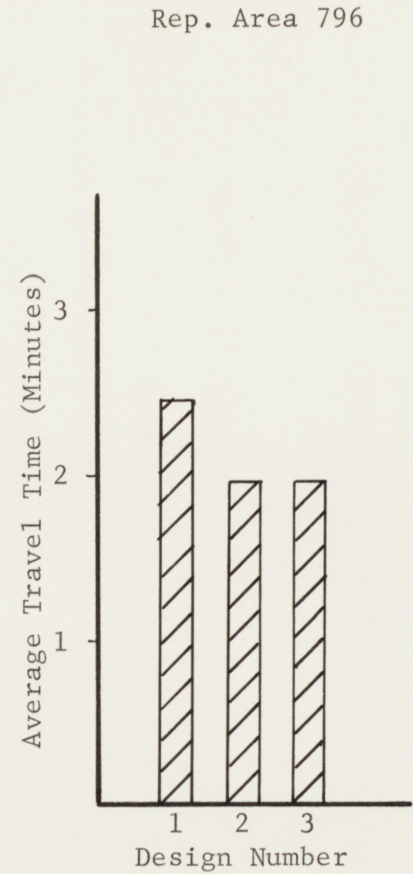
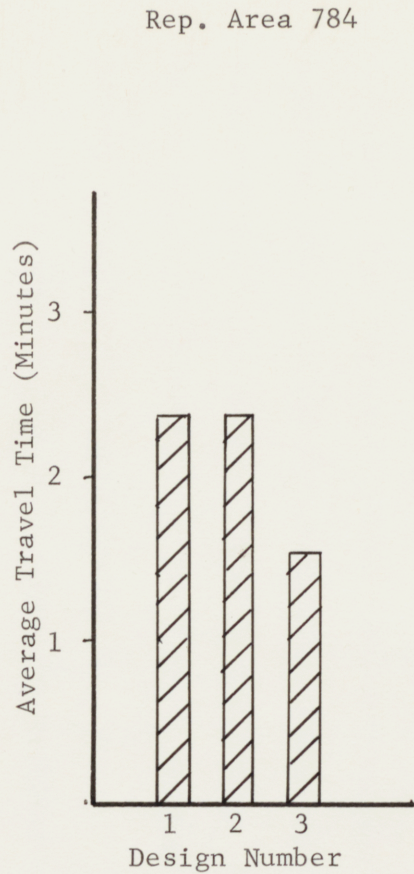
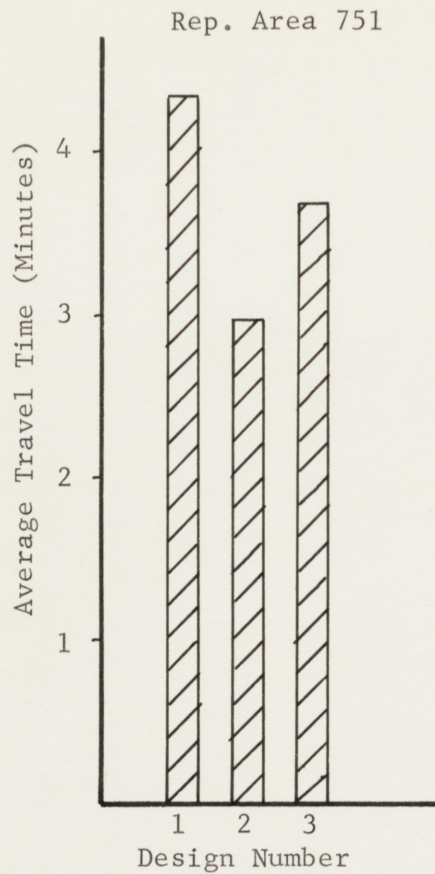


FIGURE 4.7. Average Travel Times to Reporting Areas 751, 784, and 796 Resulting From Alternative Sector Designs 1, 2, and 3. (Utilization = .5 for All Areas.)

provides a more even distribution in response time to all parts of the district, while Design 3 provides fast response to centrally located areas at the expense of outlying areas. Of course, most of the incidents occur in the central areas, which is the reason that the response time to all incidents is better under Design 3.

4.4b. Workload Balance

Figures 4.8 to 4.10 show how the call-for-service load and workload are divided among the sector cars in each of the three sector designs. For example, examine the call-for-service graph in Figure 4.8a. This graph is for Design 1, the existing system containing four sectors. If the call-for-service load were equally balanced, then each sector would have 25 percent of the total as shown by the "equal balance" line on the graph. However, the load is not equally balanced. Sectors 1 and 3 receive less than 25 percent while 2 and 4 receive more.

Similarly, Figure 4.8b shows the workload balance for Design 1 at a utilization of .5. In this figure, if the workload were equally balanced among the four sector-cars and the two district-wide cars, then each would have 1/6 of the load. Clearly, the two district-wide cars, numbers 5 and 6, have lower workloads than the sector cars. This is to be expected since the district-wide cars are usually dispatched only when the sector cars are busy. In order to bring these workloads up to the others, the dispatcher could occasionally dispatch the district-wide cars even when they are not expected to be closest to an incident.

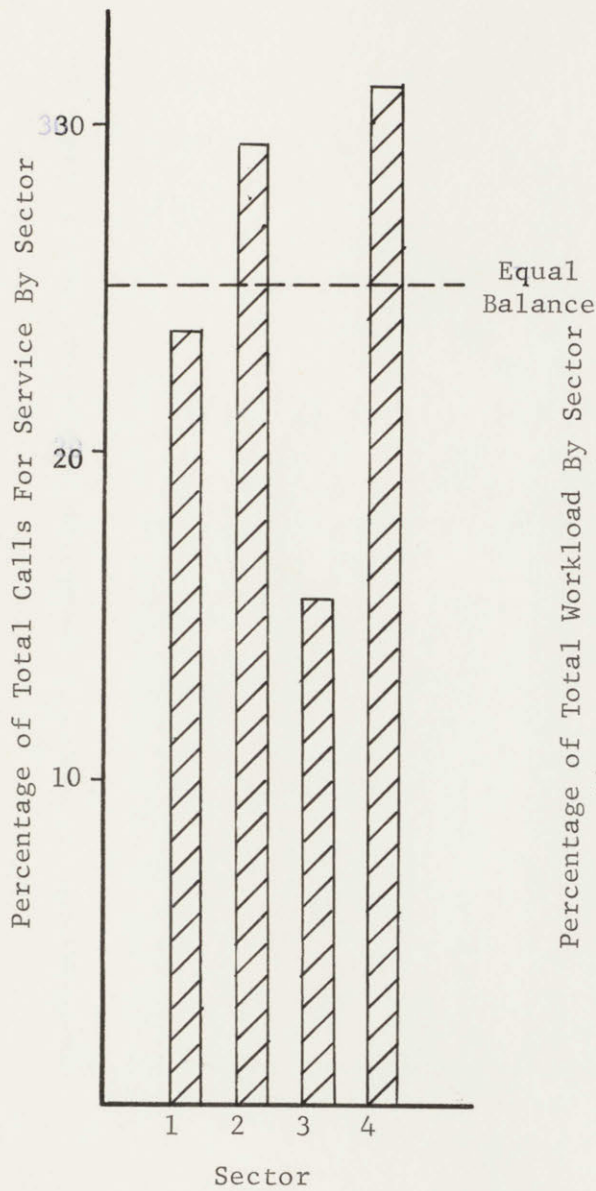


FIGURE 4.8a. Call-For-Service Balance, Design 1.

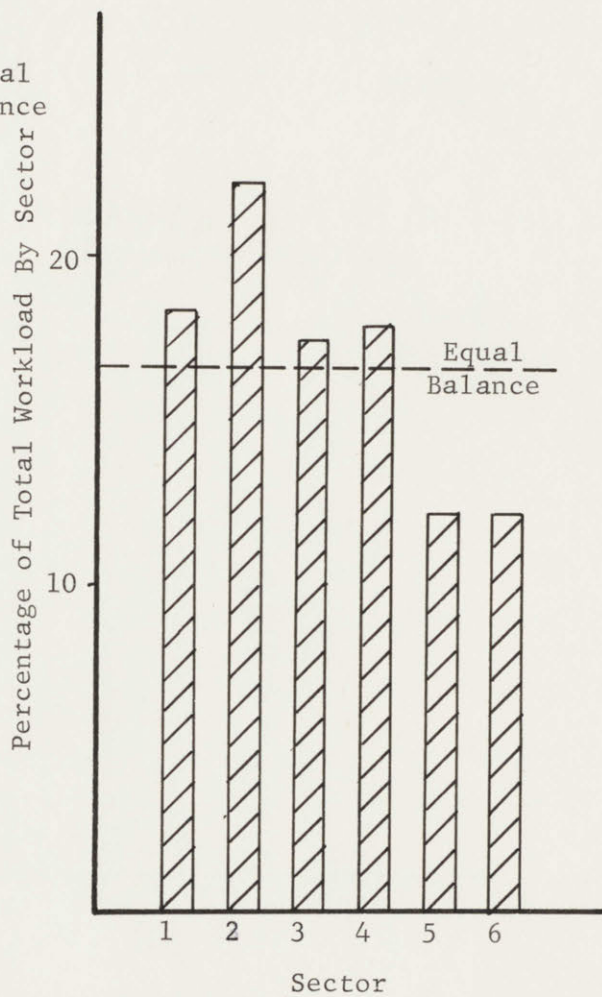


FIGURE 4.8b. Workload Balance, Design 1. (Utilization = .5)

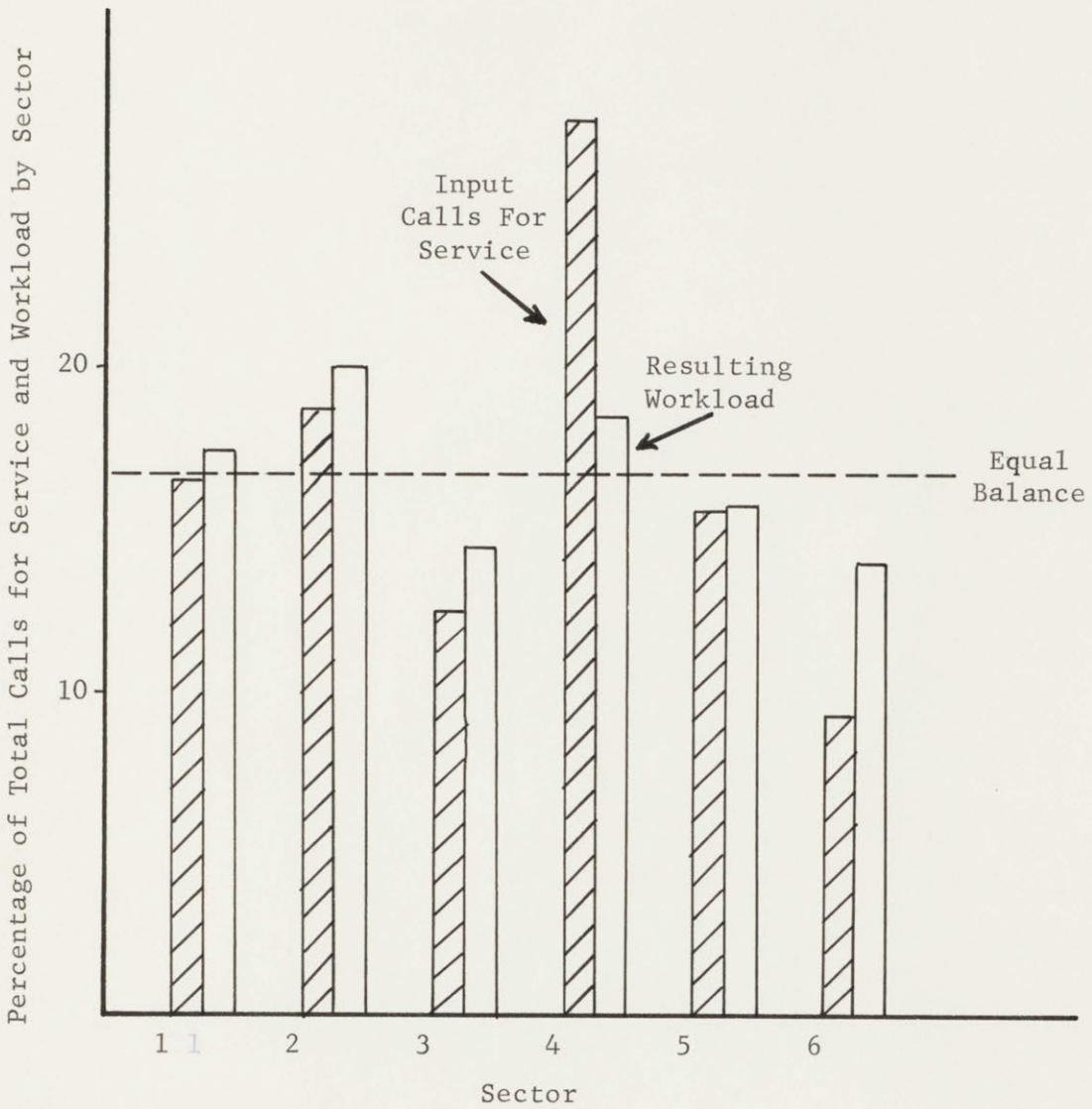


FIGURE 4.9. Call-For-Service Balance and Workload Balance, Design 2. (Utilization = .5)



FIGURE 4.10. Call-For-Service Balance and Workload Balance, Design 3. (Utilization = .5)

Figures 4.9 and 4.10 are similar to Figure 4.8 except that call-for-service load and workload are plotted on the same graph. All of the graphs demonstrate the complex relationship among call-for-service load, workload, and sector design. For example, consider sector 4 in Design 2 (Figure 4.9). The call-for-service load is 70 percent above equal balance level, but the workload is only 12 percent above equal balance. The reason for this is that at a utilization rate of .5, half of the calls for service arriving in sector 4 are answered by cars outside of the sector. In fact, it appears that many of them were answered by Car 6 since its workload is substantially above its call-for-service load. This is quite reasonable considering that sector 6 is adjacent to sector 4 in this design (see Figure 4.4).

In general, the effect of this intersector dispatching into busy sectors is to even the workload among the patrol cars. As utilization increases, one would expect workloads to become more even. At saturation, the workload is perfectly even because each car is always busy.

There are a few exceptions to this general trend of equalizing workloads, however. Consider sector 2 of Design 2. Here, the workload is more out of balance than the call-for-service load. The reason for this is that there are several reporting areas with extremely high call-for-service rates on both sides of sector 2. Since this car is close to these incidents, it will be first in line to be dispatched out of its sector if either car 1 or car 4 are busy. This same effect is even more dramatic for sector 2 of

Design 3 (Figure 4.10). Car 2 is required to make a large number of intersector dispatches to adjacent reporting areas with high call-for-service rates.

4.4c. Intersector Dispatching

To confirm the relationship between intersector dispatching and workload, examine the plot of intersector dispatches out of sector, shown in Figure 4.11. Car number 2 of Design 3 does indeed exhibit a high level of intersector dispatching. Almost 80 percent of its dispatches are made out of its sector. This may seem somewhat high. However, note that the call-for-service rate into the sector is quite low and since the car is busy approximately 50 percent of the time, half of the calls that actually do arrive in the sector are answered by a car outside the sector.

The other sectors in Figure 4.11 have more reasonable values of intersector dispatching. Note the low value for sector 6 of Design 3 and sector 4 of Design 1. Both of these sectors are quite large and have high call-for-service rates. In each case, the sector car is kept busy answering calls in its own sector.

The most striking difference among the three designs is the low overall level of intersector dispatching in Design 1. Figure 4.12 shows this relationship even more clearly. As expected, the level of intersector dispatching is a monotonically increasing function of utilization. Also as expected, Design 1 has the least amount of intersector dispatching for all values of utilization. This is

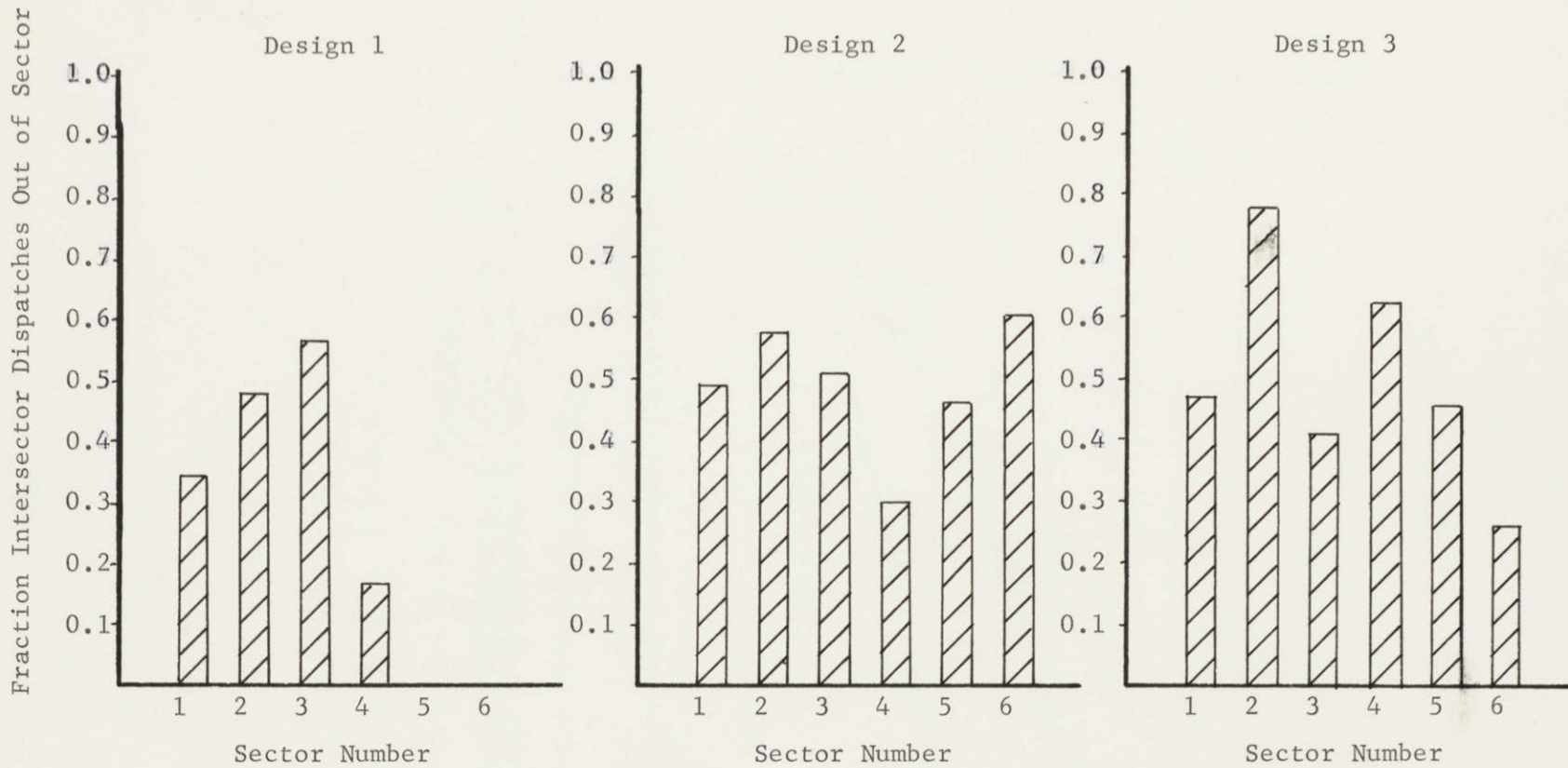


FIGURE 4.11. Intersector Dispatches Out of Each Sector, Resulting From Sector Designs 1, 2, and 3. (Utilization = .5)

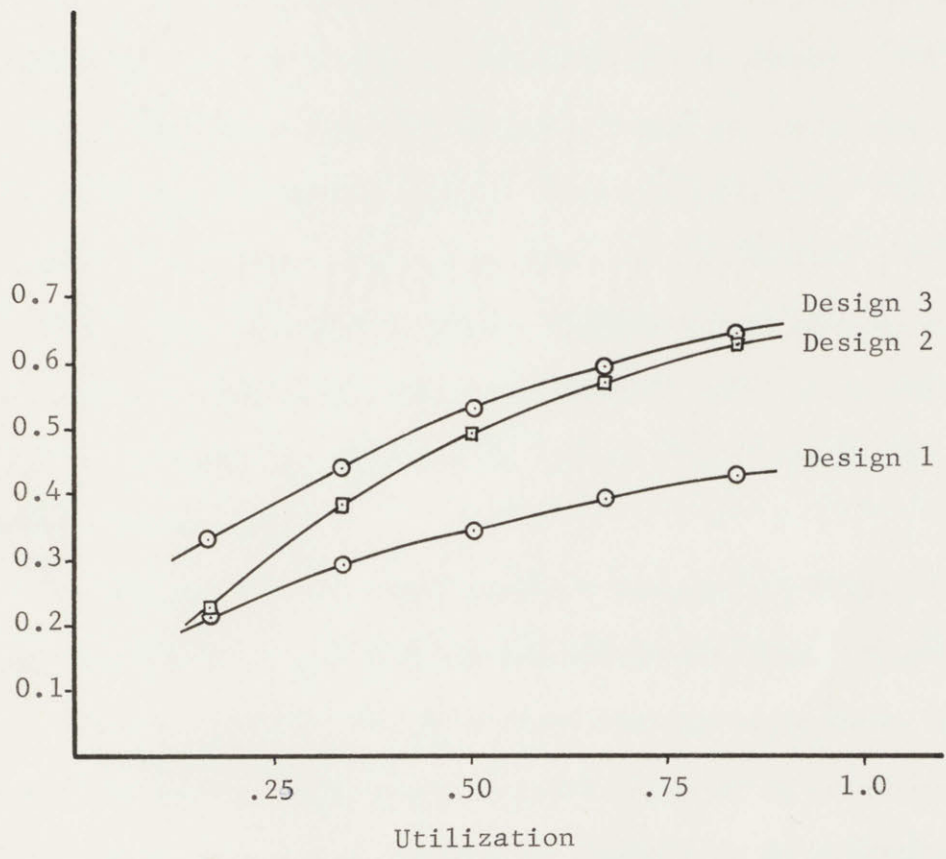


FIGURE 4.12. Total Intersector Dispatching as a Function of Utilization. (The District-Wide Cars in Design 1 Are Not Included.)

because the district-wide cars answer calls which normally would require a sector car to be dispatched outside its own sector.

4.4d. Probability of No Patrol on Harvard Avenue

As mentioned earlier, one of the reasons for aligning sector boundaries on main streets was to provide double patrol. One of the interesting quantities that the GEOQUEUE model can calculate is the probability that two adjacent sector cars are both busy. Calculating this quantity for cars 1 and 2 will give the probability that there is no sector car available to patrol Harvard Avenue between Cambridge Street and Commonwealth Avenue. This is difficult to calculate because the probability that one car is busy is dependent on whether adjacent cars are busy.

To see why this is true, consider Design 2 of Figure 4.4. Assume that car 1 is busy and all the others are free. Since car 2 is the closest available car to most of the reporting areas in sector 1, it will be dispatched to calls originating in either sector 1 or its own sector, sector 2. Therefore, in a small interval of time, car 2 is approximately twice as likely to be dispatched as any of the other remaining cars. Similarly, if both cars 1 and 2 are busy, then car 3 is most likely to be the next car dispatched. The result of this process is that busy cars tend to be found clumped together in adjacent sectors.

In order to calculate the probability that cars 1 and 2 are simultaneously busy, it is necessary to consider the effect of clumping. Let $P(1)$ and $P(2)$ be the unconditional probabilities that

cars 1 and 2 are busy. Let $P(2 | 1)$ be the conditional probability that car 2 is busy, given car 1 is busy. From the results of the model for Design 2 at a utilization of .5, $P(1) = .492$, $P(2) = .568$, and $P(2 | 1) = .651$. The probability that both car 1 and car 2 are busy is $P(2 | 1) \cdot P(1) = .320$. If one makes the simplifying assumption that the busy probabilities are independent, then the joint probability would be calculated $P(2) \cdot P(1) = .279$, a 12.5 percent underestimate of the true value. This underestimate is typical of the results one obtains under the independence assumption.

Comparing the correct figure for Design 2 (.320) with that calculated from the other two designs, Design 1 resulted in the highest probability that cars 1 and 2 were both busy (.357), and Design 3 resulted in the lowest probability (.284). Of course, one must remember that in the case of Design 1, one of the district-wide cars could occasionally be assigned to patrol Harvard Avenue.

4.5. CONCLUSIONS

It is difficult to draw strong conclusions from the brief results discussed here. Further modifications in the designs should be tested and evaluated according to the subjective criteria mentioned earlier in the chapter before one is implemented.

To summarize some of the important characteristics of each design, Design 1 resulted in the least amount of intersector dispatching, however it resulted in the poorest average response time to all incidents. In addition, the workloads of the two district-wide cars were well below the equal balance level. However, this

may not be a problem depending on the purpose of the district-wide cars. The differences between Designs 2 and 3 are somewhat less dramatic. Design 3 resulted in slightly better response time but was within 4 percent of the value for Design 2 for utilization values of .5 or less. In each case, workloads were balanced within 20 percent of the equal workload level. However, Design 2 has less peculiar shaped sectors, which may be an important consideration.

In implementation, police planners must make the choice among alternative sector designs, using the above type of reasoning as an aid in evaluating the alternatives.

FOOTNOTES

¹1970 Census.

²Under the F.B.I. classification system, Part I crimes consist of: Murder, manslaughter, rape, robbery, aggravated assault, burglary, larceny, and auto theft.

³Urban Transportation Planning System 360, U.S. Department of Transportation, Federal Highway Administration, Bureau of Public Roads, June 1970.

⁴The Concord Research Corporation, Burlington, Massachusetts has a computer program that could be modified to calculate the desired matrix. A considerable amount of time would have been required to make the modification.

⁵The value 17 was chosen after examining selected speeds from the Eastern Massachusetts Highway Network File. A sample of the speeds is given in Appendix II, Section 2.2.

⁶Larson, Models for the Allocation of Urban Police Patrol Forces, page 106.

CHAPTER V

CONCLUSIONS AND FURTHER RESEARCH

5.1. CONCLUSIONS

The spatially distributed queueing model, or GEOQUEUE, has been formulated. It has been shown to be a useful analytic technique for studying police patrol operations at the district level. The model is of fine enough detail to consider the activities of each individual patrol car. In addition, it has the capability to consider the specific geographical characteristics of the police district. The model appears to be potentially useful for two purposes:

1. As an analytic tool for investigating alternative methods of deploying and dispatching police patrol forces.
2. As a decision aid for designing police patrol sectors.

As an analytic tool, the model was used to analyze several simple hypothetical police districts. The functional relationships of patrol car response time, workload balance, and amount of intersector dispatching were studied. It was shown that because of intersector dispatching, equal balance in call-for-service loads does not necessarily result in equal workloads. Because of the finite size of districts, response time is not equally distributed to all parts of the district. Utilizing call location information was shown to be effective in reducing average response time, although

the magnitude of the improvement (less than 4 percent) was surprisingly small. Overlapping sectors were shown to be a means of reducing the amount of intersector dispatching, however this must be traded-off against increased response time.

As a decision aid in police patrol sector design, the model was applied to a case study involving District 14 of Boston. The present sector design was compared to two alternative designs, one proposed by police personnel from District 14, the other proposed by the author. The designs were compared on the basis of: Average travel time to all incident locations, equity in distribution of response time to various sections of the district, patrol car workload balance, and average level of intersector dispatching. The case study illustrates the capabilities of the model in calculating measures of performance which are useful in evaluating alternative sector designs.

The GEOQUEUE model is proposed as an alternative to simulation models which can be used to calculate many of the same quantities. Simulation models are more powerful in many respects because a wider variety of queueing disciplines, including priorities, can be considered. However, simulations typically require long run times and the significance of the results is sometimes difficult to determine. Therefore, since the GEOQUEUE model can be used to calculate many of the important measures of system performance, it may be adequate for many purposes, including sector design. Also, since the model is analytic, it may provide a basis for future research.

5.2. FURTHER RESEARCH

In the formation of the GEOQUEUE model, two important features of police operations were not included:

1. Patrol car down time was not explicitly considered. As an approximation, the overall manpower level could be reduced. In practice, down time due to such things as meals and vehicle repair can be scheduled to have a minimum effect on the system.
2. A priority structure for different types of calls was not included in the model. This eliminated alternatives such as stacking of low priority calls when the sector car is busy. Priority structures can be applied to simple queueing systems. Whether or not it would be computationally tractable to apply priorities to GEOQUEUE is a question that should be investigated.

Implementation of one or both of these features would greatly increase the power of the model for studying police operations.

Several interesting areas of future research are apparent after studying the GEOQUEUE model. Some of these areas are outlined below:

1. The applications of the GEOQUEUE model to other urban emergency services, such as fire and ambulance service, could be considered. Both of these service systems share common properties with the police. Specifically, all are affected by the spatial distribution of calls for service as well as travel time characteristics to various parts of the city. All are affected by queueing phenomena because of the random nature of calls for service. There are obvious differences, such as fixed vehicle locations for fire and ambulance units as opposed to continually changing locations of police cars. With appropriate modifications, the GEOQUEUE model might be applied to problems of deployment and dispatching shared with these other urban emergency systems.

2. The "total configuration" dispatching strategy, mentioned in Chapter II, should be pursued. The objective of this type of strategy is to minimize the average response time to all calls for service. The closest expected car strategies minimize response time only to the incident under immediate consideration. On the other hand, the "total configuration" strategy anticipates future calls and attempts to leave the system in states that facilitate response to these calls.
3. The concept of repositioning may be suitable to study using the GEOQUEUE model as a basis. Repositioning involves the dynamic reallocation of patrol cars to areas which have been depleted of patrol cars. Using the same type of procedure as with the "total configuration" strategy, it may be possible to evaluate the state of the system with respect to its ability to respond to future calls for service. Patrol cars would then be repositioned to depleted areas on the basis of improving system readiness to respond to future calls.
4. Approximations for the GEOQUEUE model could significantly reduce the computational requirements of the model. Although the model is fairly simple, it does consider the state of each individual patrol car. With m patrol cars, there are 2^m states of the system. Solving the simultaneous equations for the state probabilities becomes a major computational task for moderately large m . (For $m = 10$, there are $2^m = 1024$ state probabilities.)

In conclusion, important issues of implementation deserve further attention. If the GEOQUEUE model is to be used for police patrol sector design, further critical thought must be devoted to how the measures of system performance, as calculated by the model, should be used to evaluate alternative sector designs. Also, the mechanics of the model should be refined to provide an on-line capability for quickly evaluating modifications in design. It is hoped that the GEOQUEUE model will continue to be developed and eventually become a standard tool for patrol sector design.

APPENDIX I

COMPUTER PROGRAM FOR THE

SPATIALLY DISTRIBUTED QUEUEING MODEL

```
GEOQUEUE: PROCEDURE OPTIONS (MAIN);  
  DCL (N,M,R,C) FIXED BIN,  
  ERROR EXTERNAL CHAR(1),  
  MLSQ ENTRY;  
  LOOP: GET LIST(M,R);  
    N=2**M;  
    C=15-M;  
    CALL CALCVAL;  
    GO TO LOOP;  
  RETURN;
```

```

CALCVAL: PROC;
  DCL (PA(N,M,R),Q(N,N),A(N,N),L(R),PI(N),T(M,R),RO) FLOAT BIN,
      (TR(R,R),B(N,1),SEC(M,R),IRO,SRO) FLCAT BIN,
      (PAT(R),X(R),Y(R),SPEED,TOTCAL) FLOAT BIN,
      H FIXED BIN,
      TITLE CHAR(50),
      STATE BIT(M),
      COUNT RETURNS (FIXED BINARY),
      (RUNNUM,NUM,ONE,KA) FIXED BIN;
  DCL (TC,K,I,J,KB) FIXED BIN,
      (MIN,FRAC,SUM) FLOAT BIN,
      (ALPHA,BETA) BIT(15) ALIGNED;
  ONE=1;
  START: CALL READATA;
        CALL GENPA;
        CALL GENQ;
        DO RUNNUM=1 TO NUM;
        CALL GENEQNS;
        CALL MLSQ (A,B,N,N,ONE);
          IF ERROR $\neq$ '0' THEN
            PUT SKIP LIST('ERROR',ERROR,'IN SIM EQNS');
          PI(*)=B(*,1);
        CALL CALCOUT;
        RC = RC+IRO;
        END; /* LOOP ON RUNNUM */
  GC TO START;
  RET: RETURN;

```

```

READATA: PROC;
DCL COMM CHAR(5);
LOOP: GET LIST(COMM);
    IF COMM='START' THEN DO;
        GET LIST(NUM,RO,IRO);
        RETURN;
    END;
    IF COMM='TITLE' THEN GET LIST(TITLE);
    IF COMM='LAMDA' THEN GET LIST(L(*));
    IF COMM='T' THEN GET LIST(T);
    IF COMM='TR' THEN GET LIST(TR);
    IF COMM='SEC' THEN GET LIST(SEC);
    IF COMM='RSEC' THEN DO;
        GET LIST(SEC);
        CALL GENTS;
    END;
    IF COMM='JOB' THEN GO TO RET;
    IF COMM='LAM' THEN DO;
        GET LIST(TOTCAL);
        GET LIST(L);
        DO J=1 TO R;
            L(J)=L(J)/TOTCAL;
        END;
        PAT=L;
    END;
    IF COMM='S' THEN DO;
        GET LIST (I,H);
        SEC(I,*)=0E0;
        DO K=1 TO H;
            GET LIST(J);
            SEC(I,J)=1E0;
        END; /* LOOP ON K */
        CALL CALCSEC(I);
    END;
    IF COMM='C' THEN DO;
        GET LIST(I,J,K);

```

III


```

SEC(J,I)=0E0;          /* MOVE AREA I FROM SEC J TO SEC K */
SEC(K,I)=1E0;
CALL CALCSEC(J);
CALL CALCSEC(K);
END;
IF COMM='ST' THEN DO;
  GET LIST(NUM,RO,IRO);
  CALL GENTS;
  RETURN;
  END;
IF COMM='TX' THEN DO;
  DO I=1 TO R;
    GET LIST(X(I),Y(I));
  END;
  DO I=1 TO R;
  DO J=1 TO R;
    TR(I,J)=(ABS(X(I)-X(J))+ABS(Y(I)-Y(J)))/SPEED;
  END;      /* LOOP ON I */
  END;      /* LOOP ON J */
  END;
IF COMM='SPEED' THEN GET LIST(SPEED);
IF COMM='PAT' THEN GET LIST(PAT);
  GO TO LOOP;
END READATA;

```

```
GENTS: PROC;  
  DO I=1 TO M;  
    DO J=1 TO R;  
      SUM=0E0;  
      DO K=1 TO R;  
        SUM=SUM+SEC(I,K)*TR(K,J);  
      END;      /* LOOP ON K */  
      T(I,J)=SUM;  
    END;      /* LOOP ON J */  
  END;      /* LOOP ON I */  
END GENTS;
```

```
CALCSEC: PROC(I);  
DCL (I,J) FIXED BIN;  
SUM=0E0;  
DO J=1 TO R;  
    IF SEC(I,J)≠0E0 THEN SUM=SUM+PAT(J);  
END; /* LOOP ON J */  
DO J=1 TO R;  
    IF SEC(I,J)≠0E0 THEN SEC(I,J)=PAT(J)/SUM;  
END; /* LOOP ON J */  
END CALCSEC;
```

```

GENPA:  PROCEDURE;                /* GENERATE PA */
      PA=OE0;
      Q=OE0;
      DO K=1 TO N;
          KA=K-1;
          ALPHA=KA;

      /* CALCULATE PA(K,I,J) */
      DO J=1 TO R;
          MIN=1E10;
          DO I=1 TO M;              /* FIND MIN T(I,J) & NUM OF TIES */
              IF SUBSTR(ALPHA,C+I,1)='0'B THEN
                  IF T(I,J)<MIN
                      THEN DO;
                          MIN=T(I,J);
                          TC=1;
                          END;
                      ELSE IF T(I,J)=MIN
                          THEN TC=TC+1;
                  END;              /* LOOP ON I */
              FRAC=1E0/TC;
              DO I=1 TO M;          /* INSERT VALUE IN PA */
                  IF T(I,J)=MIN & SUBSTR(ALPHA,C+I,1)='0'B
                      THEN PA(K,I,J)=FRAC;
                  END;              /* LOOP ON I */
              END;                  /* LOOP ON J */
          END;                      /* LOOP ON K */
      END GENPA;

```

```

GENQ: PROC;
  /* CALCULATE Q(I,J) */
  DO K=1 TO N;
    KA=K-1;
    ALPHA=KA;
    DO I=1 TO M;
      SUM=0E0;
      BETA=ALPHA;
      IF SUBSTR(BETA,C+I,1)='0'B THEN DO;
        SUBSTR(BETA,C+I,1)='1'B;
        KB=BETA+1;
        DO J=1 TO R;
          SUM=SUM+PA(K,I,J)*L(J);
        END;
        Q(K,KB)=SUM;
      END; /* LOOP ON J */
    END; /* LOOP ON I */
  END; /* LOOP ON K */
END GENQ;

```

```

GENEQNS: PROC;
  A=0E0;    B=0E0;

  /* LAST ROW */
  A(N,*)=1E0;
  B(N,1)=1E0;

  /* REST OF MATRIX */
  DO K=1 TO (N-1);
    KA=K-1;
    ALPHA=KA;
    A(K,K)=- (RO+COUNT(ALPHA));
    DO I=1 TO M;
      BETA=ALPHA;
      IF SUBSTR(ALPHA,C+I,1)='0'B
        THEN DO;
          SUBSTR(BETA,C+I,1)='1'B;
          KB=BETA+1;
          A(K,KB)=1;
          END;
        ELSE DO;
          SUBSTR(BETA,C+I,1)='0'B;
          KB=BETA+1;
          A(K,KB)=RO*Q(KB,K);
          END;
        END;
      /* LOOP ON I */
    /* LOOP ON K */
  END;
END GENEQNS;

```

```

CALCOUT: PROC;
  DCL (PD(M,R),SUM,WL(M),MWL,MISD,TAV,VAR,SWL,UBWL) FLOAT BIN,
  (INPUT(M),DINPUT(M),PINPUT(M)) FLOAT BIN,
  (TCAR(M),TSEC(M),TREP(R),DENOM,DCAR(M),DSEC(M),DREP(R),
  DWL(M),PWL(M),DISDO(M),PISDO(M),DISDI(M),PISDI(M),
  DTSEC(M),PTSEC(M),DCEP(M,M),PDEP(M,M),
  ISDO(M),ISDI(M),PINSEC(M),DEP(M,M),MAX,MIN,PINT(M)) FLCAT BIN;

```

```

/* CALC PD(I,J) */
DO I=1 TO M;
DO J=1 TO R;
  SUM=0E0;
DO K=1 TO N;
  SUM=SUM+PI(K)*PA(K,I,J)*L(J);
END; /* LOOP ON K */
PD(I,J)=SUM/(1E0-PI(N));
END; /* LOOP ON J */
END; /* LOOP ON I */

```

```

118 /* CALC TRAVEL TIMES */
TAV=0E0;
DO I= 1 TO M;
  SUM=0E0;
  DENOM=0E0;
  DO J=1 TO R;
    SUM=SUM+PD(I,J)*T(I,J);
    DENOM=DENOM+PD(I,J);
  END;
  DCAR(I)=DENOM;
  TCAR(I)=SUM/DENOM;
  TAV=TAV+SUM;
END; /* LOOP ON I */
DO J=1 TO R;
  SUM=0E0;
  DENOM=0E0;
  DO I=1 TO M;
    SUM=SUM+PD(I,J)*T(I,J);

```

```

        DENOM=DENOM+PD(I,J);
        END;          /* LOOP ON I */
    DREP(J)=DENOM;
    TREP(J)=SUM/DENOM;
    END;          /* LOOP ON I */
DO I=1 TO M;
    PINT(I)=0E0;
    SUM=0E0;
    DENOM=0E0;
    DO J=1 TO R;
        IF SEC(I,J)≠0E0 THEN DO;
            SUM=SUM+TREP(J)*DREP(J);
            DENOM=DENOM+DREP(J);
            PINT(I)=PINT(I)+PD(I,J);
            END;
        END;          /* LOOP ON J */
    DSEC(I)=DENOM;
    TSEC(I)=SUM/DENOM;
    END;          /* LOOP ON I */

/* FIND % DISP SEC(I) TO SEC(J) */
DO I=1 TO M;
    PINSEC(I)=0E0;
    DO J=1 TO R;
        IF SEC(I,J)≠0E0 THEN DO K=1 TO M;
            IF SEC(K,J)≠0E0 THEN PINSEC(I)=PINSEC(I)+PD(K,J);
            END;
        END;          /* LOOP ON K */
    END;          /* LOOP ON J */
END;          /* LOOP ON I */

/* CALC % INTERSECTOR DISPATCHES */
SUM=0E0;
DO I=1 TO M;
    ISDO(I)=1E0-PINT(I)/DCAR(I);
    ISDI(I)=1E0-PINSEC(I)/DSEC(I);
    SUM=SUM+PINT(I);

```



```

        END;          /* LOOP ON I */
MISD=1EO-SUM;

/* CALC WORKLOAD FOR CAR(I) */
SUM=0EO;
WL=0EO;
DO K=1 TO (2**(M-1));
    KA=K-1;
    ALPHA=KA;
    DO I=1 TO M;
        BETA=SUBSTR(ALPHA,2,C+I-1)||'1'B||SUBSTR(ALPHA,C+I+1);
        KB=BETA+1;
        WL(I)=WL(I)+PI(KB);
    END;          /* LOOP ON I */
END;          /* LOOP ON K */

/* MEAN WORKLOAD & UNBALANCE */
MWL=0EO;
SUM=0EO;
DO I=1 TO M;
    MWL=MWL+WL(I);
    SUM=SUM+WL(I)**2;
END;          /* LOOP ON I */
MWL=MWL/M;
VAR=(SUM-M*MWL*MWL)/(M-1);
SWL=SQRT(VAR);
UBWL=SWL/MWL;

/* AGGREGATE INPUT CALLS */
INPUT=0EO;
DO I=1 TO M;
DO J=1 TO R;
    IF SEC(I,J)≠0EO THEN INPUT(I)=INPUT(I)+L(J);
END;          /* LOOP ON J */
END;          /* LOOP ON I */

```

```

/* CALC DEPENDENCE OF STATES */
DEP=0E0;
DO I=1 TO M;
DO J=1 TO M;
IF I=J THEN DO;
  IF I>J THEN DO;
    MAX=I;    MIN=J;    END;
  ELSE DO;
    MAX=J;    MIN=I;    END;
SUM=0E0;
DO K=0 TO 2**(M-2)-1;
  ALPHA=K;
  BETA=SUBSTR(ALPHA,2,C+MIN)||'1'B||SUBSTR(ALPHA,C+MIN+2);
  ALPHA=SUBSTR(BETA,2,C+MAX-1)||'1'B||SUBSTR(BETA,C+MAX+1);
  KA=ALPHA+1;
  SUM=SUM+PI(KA);
END; /* LOOP ON K */
DEP(I,J)=SUM/WL(J);
END; /* LOOP ON DO */
END; /* LOOP ON J */
END; /* LOOP ON I */

```

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```

/* CALC DIFF & % DIFF FOR OUTPUT */
DO I=1 TO M;
  DWL(I)=WL(I)-MWL;
  PWL(I)=DWL(I)/MWL;
  DISDO(I)=ISDO(I)-MISD;
  PISDO(I)=DISDO(I)/MISD;
  DISDI(I)=ISDI(I)-MISD;
  PISDI(I)=DISDI(I)/MISD;
  DTSEC(I)=TSEC(I)-TAV;
  PTSEC(I)=DTSEC(I)/TAV;
  DINPUT(I)=INPUT(I)-1E0/M;
  PINPUT(I)=DINPUT(I)*M;
DO J=1 TO M;
  DDEP(I,J)=DEP(I,J)-WL(I);

```

```
      PDEP(I,J)=DDEP(I,J)/WL(I);
END;      /* LOOP ON J */
      END; /* LOOP ON I */
```

```
/* PRINT OUTPUT */
PUT LIST ('POLICE CAR DISPATCHING') PAGE;
PUT LIST ('PROBLEM TITLE',TITLE) SKIP;
PUT LIST ('RUN NUMBER',RUNNUM) SKIP;
PUT LIST ('NUMBER OF CARS',M) SKIP;
PUT LIST ('NUMBER OF REP-AREAS',R) SKIP;
PUT LIST ('RO',RO) SKIP;
PUT LIST ('RO/M',RO/M) SKIP;
PUT LIST('AVE TRAVEL TIME=',TAV) SKIP(3);
PUT LIST ('PROB OF SATURATION',PI(N)) SKIP;
PUT LIST('AVE WORKLOAD=',MWL) SKIP;
PUT LIST('S.D. WORKLOAD=',SWL) SKIP;
PUT LIST('UNBAL WORKLOAD=',UBWL) SKIP;
PUT LIST ('AVE INTERSECTOR DISPATCHES',MISD) SKIP;
PUT LIST('AGGREGATED INPUT CALLS') SKIP(3);
PUT LIST(INPUT) SKIP;
PUT LIST(DINPUT) SKIP(2);
PUT LIST(PINPUT) SKIP(2);
PUT LIST('WORKLOAD CAR(I)') SKIP(3);
PUT LIST(WL) SKIP;
PUT LIST(DWL) SKIP(2);
PUT LIST(PWL) SKIP(2);
PUT LIST('INTERSECTOR DISPATCHES CAR(I)') SKIP(3);
PUT LIST(ISDC) SKIP;
PUT LIST(DISDC) SKIP(2);
PUT LIST(PISDC) SKIP(2);
PUT LIST ('INTERSECTOR DISPATCHES SEC(I)') SKIP(3);
PUT LIST (ISDI) SKIP;
PUT LIST(DISDI) SKIP(2);
PUT LIST(PISDI) SKIP(2);
PUT LIST ('AVE TRAVEL TIME TO SEC(I)') SKIP(3);
PUT LIST (TSEC) SKIP;
```

```
PUT LIST(DTSEC) SKIP(2);
PUT LIST(PTSEC) SKIP(2);
PUT LIST ('AVE TRAVEL TIME TO REP AREA(J)') SKIP(3);
PUT LIST (TREP) SKIP;
PUT LIST ('AVE TRAVEL TIME BY CAR(I)') SKIP(3);
PUT LIST (TCAR) SKIP;
PUT LIST('STATE DEPENDENCE') PAGE;
PUT LIST(DEP) SKIP;
PUT LIST(DDEP) SKIP(4);
PUT LIST(PDEP) SKIP(4);
END CALCOUT;
```

```
COUNT: PROC(STR) RETURNS (FIXED BIN);  
DCL STR BIT(15) ALIGNED,  
(I,J,CNT) FIXED BIN;  
CNT=0;  
DO I=(16-M) TO 15;  
    J=SUBSTR(STR,I,1);  
    CNT=CNT+J;  
END;  
RETURN(CNT);  
END COUNT;  
END CALCVAL;  
END POLDIS;
```

APPENDIX II

DATA USED FOR THE SECTOR

DESIGN CASE STUDY IN DISTRICT 14, BRIGHTON

2.1. CALL FOR SERVICE AND CRIME DATA FOR DISTRICT 14

Incidents by Reporting Area - District 14

December 1970 Through November 1971

<u>R.A.</u> ¹	<u>Part I</u> ²	<u>Part II</u> ³	<u>Part III</u> ⁴	<u>Total</u>
749	63	9	157	229
750	41	6	157	204
751	35	8	134	177
752	24	10	144	178
753	3	5	52	60
754	12	8	90	110
755	17	26	158	201
756	2	66	92	100
757	14	13	146	173
758	26	11	142	179
759	26	10	108	144
760	20	13	152	185
761	16	9	108	133
762	11	9	92	112
763	10	8	65	83
764	10	6	54	70
765	23	6	108	137
766	9	14	70	93
767	21	11	113	145
768	39	35	322	396
769	7	10	94	111
770	88	14	84	106
771	11	26	208	245
772	5	7	72	84
773	47	11	208	266
774	56	18	256	330
775	23	11	188	222
776	58	14	289	361
777	20	7	92	119
778	42	44	302	388

APPENDIX II

(Continued)

<u>R.A.</u> ¹	<u>Part I</u> ²	<u>Part II</u> ³	<u>Part III</u> ⁴	<u>Total</u>
779	21	15	194	230
780	33	20	168	221
781	23	7	120	150
782	15	6	122	143
783	62	17	184	263
784	102	76	877	1,055
785	83	11	256	350
786	101	29	358	488
787	57	17	170	244
788	1166	22	362	500
789	81	10	291	382
790	84	28	313	425
791	56	10	263	329
792	177	46	588	811
793	231	69	914	1,214
794	1811	93	977	1,251
795	126	41	653	820
796	272	70	924	1,266
797	130	40	278	448
798	115	26	329	470
799	1044	24	341	469
800	47	17	222	286
801	61	37	311	409
802	39	11	190	240
803	50	22	287	359
804	21	8	120	149
805	16	6	114	136
806	18	11	107	136
807	28	8	128	164
808	6	2	76	84
809	10	3	125	138
810	8	6	83	97
811	19	10	156	185
812	62	37	231	330
813	18	4	44	66
814	18	6	167	191
815	31	14	158	203
816	29	13	206	248
District Totals	3,350	1,281	15,665	20,292

Notes:

1. Abbreviation for Reporting Area.
2. Part I crimes include: Murder, manslaughter, forcible rape,

APPENDIX II

(Continued)

robbery, aggravated assault, burglary, larceny involving \$50 and over, auto theft, simple assault, and larceny involving less than \$50.

3. Part II crimes include: Other assaults, arson, forgery and counterfeiting, fraud, embezzlement, stolen property (buying, receiving, possessing), vandalism, weapons (carrying, possessing, etc.), prostitution and commercial vice, other sex offences, narcotic drug laws, gambling, offenses against the family and children, driving under the influence of liquor, liquor laws, drunkenness, disorderly conduct, vagrancy, and all other state or local offenses not included in Part I and above-mentioned crimes.
4. The Part III category is used by the Boston Police Department to designate all non-crime related calls for service.
5. The above data were supplied by the Boston Police Department for use in this study.

2.2. SELECTED TRAVEL SPEEDS IN BRIGHTON

The data are taken from the 1963 Highway Network File for Eastern Massachusetts. Speeds are for peak rush-hour periods.

The speeds represent typical values for each street.

<u>Street</u>	<u>Speed</u>
Everett	18
Harvard	10-18
Cambridge	115 20
Lake	18
Commonwealth	20-22
Washington	12-15
Lincoln	15
No. Beacon	15
Arlington (Faneful)	12
Hobart	15
Brock	18
Chestnut Hill Ave.	14
Market St.	14-16
Kenrick	18
Western	18
Soldiers Field	27-35
Warren	15
Nonantum	25
Chestnut Hill Driveway	15

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