SYSTEM IDENTIFICATION APPLIED TO MANEUVERING TRIALS

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ABSTRACT

This thesis shows the application of two particular approaches of system identification to the maneuvering trials of a surface ship. The model reference contouring and the extended Kalman filtering are used to identify the hydrodynamic coefficients of the Mariner class hull form.

The mathematical model representing the ship dynamics is first developed. The concept of parametric identifiability is presented and the techniques which will be used are described. The scheme adopted to conduct the identification studies is then presented. The computation steps for the implementation of the identification approaches to the mathematical model are detailed.

The results of the identification procedure are analyzed. They give a good idea about the identifiability of the hydrodynamic coefficients, particularly the linear coefficients. The use of simpler models is shown to produce better results. The conclusions of the studies express some general rules about the application techniques, specially extended Kalman filtering to the maneuvering studies.

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LIST OF SYMBOLS

$\dot{a}$	Dot over a letter means it's time derivative $\frac{d}{dt}$
$\underline{a}$	Underlined letter means it is a column vector
$\bar{a}$	Overlined letter refers to its mean value $E[x] = \bar{x}$
A^T	Indicates the transpose of matrix A
A^{-1}	Indicates the inverse of the matrix A
$E[a]$	Refers to the mean value of the random variable a
σ	Standard deviation of a Gaussian random variable
$N(m,s)$	Normally distributed or Gaussian random variable of mean m and variance s.
(x,y,z,ϕ,θ,ψ)	Ocean vehicle linear and angular coordinates
(X,Y,Z,K,M,N)	Ocean vehicle forces and moments (sometimes called simply force)
(u,v,w,p,q,r)	Ocean vehicle six degrees of freedom in linear and angular velocities
(x_G,y_G,z_G)	Coordinates of the ocean vehicle center of gravity
(I_x,I_y,I_z)	Moment of inertia
$\underline{F}$	Force on vehicle
$\underline{G}$	Moment on vehicle
u	Vehicle surge velocity (ft/sec) relative to a coordinate system moving with the vehicle.
$\underline{u}$	General ocean vehicle input vector for state space models
v	Vehicle sway velocity (ft/sec)
$\underline{v}$	Measurement noise vector for state space models.

$\underline{w}$	Vehicle input noise vector for state space models.
r	Vehicle yaw velocity (rad/sec)
δ	Rudder deflection
$\delta(\overline{\tau})$	Dirac delta function
η	Propeller angular velocity
$\underline{x}$	General ocean vehicle state vector
$\underline{z}$	Vehicle output vector
$\underline{p}$	Vehicle parameter vector
t	Time, seconds
$\underline{f}$	System structure vector in state space models
$\underline{h}$	Measurement structure vector in state space models
H	Linear measurement structure $\underline{z} = Hx$
$\underline{g}$	Parameter structure vector in state space models
Q	Discrete process or input noise covariance
R	Discrete measurement noise covariance
$\underline{p}^*$	The optimum or best value of the parameter vector $\underline{p}$
C	Scalar cost functional
($\underline{P}$A1,$\underline{P}$A2)	Notation for 2 generic parameters used in the identification studies
A(I)	Notation for the hydrodynamic coefficients in the state equations according Table A-4 (Appendix 1)
LP1	Index of the first parameter A(I) in the parameter vector $\underline{p}$
$\underline{e}$	Measurement error vector

E	Error covariance matrix used in Kalman filtering
$\hat{\underline{x}}$	Estimated value of the state vector $\underline{x}$
F	Gradient matrix of an ocean vehicle, $F = \partial \underline{f}/\partial \underline{x}$
K	Kalman filter gain matrix
$\underline{P}$ST1	Starting value of the (LP1)th parameter in Kalman filtering identification
$\underline{P}$CV1	Estimated variance of the (LP1)th parameter in Kalman filtering identification
nmE ℓ	Exponential representation for the number $nm \cdot 10^{\ell}$
X_u	Notation for the hydrodynamic derivative $\frac{\partial x}{\partial u}$
δt	Time increment

LIST OF APPENDICES

CHAPTER 1

1. INTRODUCTION

The studies in this thesis are concerned with the application of system identification techniques to the maneuvering trials of a surface ship. The primary objective is the identification of the motion parameters, but it is expected that the work may also provide some information about the use of the identification techniques. The first Chapter presents an introduction to the problem of system identification. The application of the approach specifically in the area of ship design and control is described. This Chapter presents also an outline of the whole thesis detailing the several steps in which the work was divided.

1.1. Introduction to System Identification

When considering control and simulation studies of ship dynamics with the use of a mathematical model, an important aspect is the establishment of the proper form of the equations as well as the appropriate numerical values of the various parameters in these equations. The precise knowledge of these coefficients is of significative importance for the naval architect to predict the behavior of a given ocean vehicle. This information is also necessary in the development of the vehicle control system.

At the present time, the main method of determining the various hydrodynamic force and moment coefficients (hydrodynamic coefficients) in a desired model for a particular type of marine craft is by means of captive model tests in a towing tank complemented by the mathematical analysis of the experimental data in order to provide the required coefficients. Considering the computational and data reduction equipment required as ancillary elements of the measuring devices as well as the time and expense required to obtain the desired parameter values by these means, other methods that may reduce the effort required for determination of the hydrodynamic coefficients then become more attractive.

There is a general approach that can be applied to find the hydrodynamic coefficients. This approach which is used to determine the values of the various parameters in the mathematical model of a dynamic system has been developed recently as part of modern control theory. This procedure is known as system identification which in the present case is a means of determining the numerical value of the parameters that enter into the state equations of the mathematical model that represents the vehicle dynamics. These values are considered to be the appropriate values representing the vehicle dynamics when they are obtained from a number of different trajectories of the vehicle motions.

In a broader sense, system identification is closely related with modeling in the extent they are concerned with the determination of the model structure. In the strict sense, that system identification is being considered in the work, the model structure is assumed known and the problem is to find the value of the parameters. In this condition, it is also known as parametric identification.

Basically, the parametric identification approach consists in obtaining responses of a vehicle by measuring the trajectories following different types of disturbances. With the formulated mathematical model values for the unknown parameters are then sought so that the solutions to the dynamic equations give a best fit to the data, where the best is, in general, defined by minimizing the mean square error between the solution of the equations using these coefficients and the actual data record itself. This approach can be applied to data from both full scale trajectory observations, like in the work of Goodman, et. al. [12] and model scale trajectory observations, as it was done by Reis [7].

In the studies of this thesis, a mathematically simulated ocean vehicle (the Mariner class ship) with a fixed and known set of parameter is used to generate a noisy vehicle input-output data.

The same deterministic model structure , but with different or unknown parameters is then used in the identification procedures in attempts to determine the original or "true" set of parameters used in the data generation. In this form, parametric identification was applied by Hayes [2] and Goodman[12]. The application of the techniques is concerned not only in identifying the true values of the parameters, but also in determining their identifiability.

1.2. Thesis Outline

This thesis is related to the use of parametric identification techniques to determine the motion parameters for the horizontal maneuvers of a surface ship. According to the problem formulation the application of the systems identification approach requires the use of a mathematical model representing the vehicle under study. In the particular case of the present work a stochastic model with the same structure, taken as the system, is used to simulate the maneuvering trials. Specific methods of parametric identification will be applied to the mathematical model in order to determine the true value of the various parameters. A scheme to investigate the identifiability characteristics of the various coefficients should be designed. In the next chapters each aspect of the identification problem will be treated.

Chapter 2 is concerned with the development of the mathematical representation of the vehicle. The equations of motion for the horizontal maneuver of a surface ship are derived and put in the form of state equations. This form is very convenient because it permits to use all the techniques of the modern control theory. Two types of uncertainty representing process and measurement noise are added to the model. Some constraints are imposed on the noise characteristics. The stochastic model thus obtained will be used to simulate the ship maneuvers, generating the input-output data which is utilized to identify the parameters. A linear version of the complete mathematical model is derived to be used in the identification studies. The hydrodynamic coefficientsattached to the model belong to the Mariner class hull form, and are given in the literature.

In Chapter 3 a more detailed formulation of the parametric identification problem is given, and some of the available approaches of system identification are listed. Two techniques, the model reference and the extended Kalman filtering which will be utilized in their work are described. All the steps necessary to the implementation of these approaches are detailed. Finally the concept of identifiability of a parameter is presented.

Chapter 4 described the procedure selected to conduct the identification studies. The scheme designed divides the work in three parts. The first part is a preliminary investigation of the relative importance of some hydrodynamic coefficients. Parameters of negligible influence on the system behavior are eliminated from the model. The second part consists in the identification of the linear coefficients. The linear model derived in Chapter 2 is utilized in this phase. Finally, using the complete model the nonlinear coefficients are analysed. The computation steps for application of the techniques for both models are described.

The results of the identification studies are presented in Chapter 5. A large amount of information is obtained from the analysis of these results. Chapter 6 presents the general conclusions of the thesis. These conclusions concern not only to the identifiability of the various parameters analysed, but also to the scheme used in this study and to the techniques of parametric identification employed. Complementing the text a series of appendices is included. The first appendix presents the hydrodynamic coefficients for the Mariner class hull form. All the other apprendices are used to present listing of the computer programs.

CHAPTER 2

2. DEFINITION OF THE MODEL

The techniques of system identification will be applied to the maneuvering trials of a ship. The ultimate objective of this Chapter is to obtain the state equations for the mathematical model that will be used in the identification process. The equations of motion for a general ocean vehicle are presented at the beginning but the equations are developed specifically for the horizontal maneuvering of a surface ship. The equations are then put in the form of state space equations which is appropriate to the analysis to be carried out. Up to this point a deterministic model for the ship motion has been considered. However, due to the fact that neither theoretical nor experimental analysis can completely determine the structure of the vehicle equations or of the measurement function, two forms of uncertainty or noise are introducted - process and measurement noise. In both cases the uncertainties are modeled as stochastic processes added to the deterministic model, and the mathematical model for the identification studies is defined.

2.1 Equations of Motion

In order to simulate the overall motion of an ocean vehicle we need to develop the correspondent mathematical model.

The model for a dynamic system consists of two parts: equation structure and initial conditions. For an ocean vehicle, the equation structure for overall motion usually consists of sets of differential equations and the initial conditions represent the values of the variable in the differential equations at a beginning time of interest to the observer. Once the equation structure is known and the initial conditions are fixed, the system can be simulated by solving equations in some way for a specified input.

The equation structure for a general ocean vehicle is presented in several works in the literature, particularly in references [1] and [3]. Therefore, the development of equation structure in this thesis does not go into deep details. There are two basic types of dynamic structure to be developed for the mathematical model of an ocean vehicle: the rigid body structure and the hydrodynamic structure. The rigid body structure is derived from the application of Newton's law. The hydrodynamic structure is a collection of terms that represent the properties of the vehicle, properties of its motion and properties of the fluid through which the vehicle is moving.

The equations of motion are, therefore, developed from the following equation [2].

$$\text{Rigid Body Structure} = \text{Hydrodynamic Structure} \qquad (2.1)$$

$$N = I_z\dot{r} + (I_y - I_x)pq + m\left[x_G(\dot{v} + ru - pw) - y_G(\dot{u} - qw - rv)\right] \tag{2.9}$$

In this thesis only horizontal manuevers of surface ship will be considered. Thus, not all of the above equations will be taken into account. If it is assumed that motion in the horizontal plane does not excite any rolling only the equations for X, Y and N must be considered. This assumption is not always valid, specifically for tight manuevers when a coupling with roll, motion is verified. This is due to the deck-keel assymetries. In the present case, however, there is no major concern to the point because the choice of the model does not affect the identification studies. Besides the literature does not present any reference to the value of the coefficients necessary to develop the coupling model. Therefore, only equations 2.4, 2.5, and 2.9 will be carried throughout this thesis. Some simplification can be introduced into those equations once the motion is reduced to the horizontal plane: $p = q = w = 0$. Furthermore, with the origin of the coordinates system taken in the longitudinal plane of symmetry, $y_G = 0$ in practically all the cases.

The rigid body structure for the mathematical model can be represented then by the following equations:

2.1.1. Rigid Body Structure

The overall motion of an ocean vehicle modeled as a rigid body motion must satisfy Newton's law expressed by equations 2.2 and 2.3 for a system of coordinates with origin in the center of gravity.

$$\frac{d}{dt}\text{ (Momentum)} = \underline{F} \tag{2.2}$$

$$\frac{d}{dt}\text{ (Angular Momentum)} = \underline{M} \tag{2.3}$$

Each one of the above vector equations produces three scalar equations. The six degree of freedom rigid body equations as derived by Abkowitz [1] for a system of axes fixed in the ship, with origin not necessarily at the center of gravity, but which are the principal axes of inertia, are:

$$X = m\left[\dot{u} + qw - rv - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] \tag{2.4}$$

$$Y = m\left[\dot{v} + ru - pw - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] \tag{2.5}$$

$$Z = m\left[\dot{w} + pv - qu - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq - \dot{p})\right] \tag{2.6}$$

$$K = I_x\dot{p} + (I_z - I_y)qr + m\left[y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)\right] \tag{2.7}$$

$$M = I_y\dot{q} + (I_x - I_z)rp + m\left[z_G(\dot{u} + qw - rv) - x_G(\dot{w} + pv - qu)\right] \tag{2.8}$$

$$X = m\left[\dot{u} - rv - x_G r^2\right] \tag{2.10}$$

$$Y = m\left[\dot{v} + ru + x_G \dot{r}\right] \tag{2.11}$$

$$N = I_z \dot{r} + m x_G (\dot{v} + ru) \tag{2.12}$$

2.1.2. Hydrodynamic Structure

The overall motion of an ocean vehicle through a fluid results in and from forces and moments that are functionsof the properties of the body, motion, and fluid [1]. This is represented by the hydrodynamic structure of the mathematical model as shown by the following equation:

Hydrodynamic Structure = F (body, motion, fluid) (2.13)

The hydrodynamic structure for the purpose of this thesis will be considered as one only function of all the variables involved in the problem. Once the vehicle is specified and for a motion in a given fluid the hydrodynamic structure becomes a function only of the body motion:

Hydrodynamic Structure =

$$f(\underbrace{x_0, y_0, z_0, \phi, \theta, \psi}_{\text{orientation parameters}}; \underbrace{u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}}_{\text{motion parameters}}; \underbrace{\delta, \dot{\delta}, \ddot{\delta}, n, \dot{n}, \ldots}_{\text{control surface parameters}}) \tag{2.14}$$

It is possible to break the hydrodynamic structure into hydrodynamic forces and effector forces as it was done by Hayes [2]. This procedure is somehow arbitrary. However, as it will be seen later, there is a coupling between the state variables and the effector variable which would be lost with the fractioning of the hydrodynamic structure.

For the purposes of this thesis, the control surface parameter will resume to the rudder deflection. It will be assumed that the only important forces and moments acting on the ship are produced by the rudder deflection δ, and that the forces and moments produced on the ship as a result of $\dot{\delta}$ and $\ddot{\delta}$ are negligible. It is also assumed that there is no dependency on the orientation parameters, which is always true when the ship does not operate in restricted water.

The hydrodynamic structure is represented by the following set of equations:

$$X = X(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \tag{2.15}$$

$$Y = Y(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \tag{2.16}$$

$$Z = Z(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \tag{2.17}$$

$$K = K(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \tag{2.18}$$

$$M = M(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \quad (2.19)$$

$$N = N(u,v,w,p,q,r,\dot{u},\dot{v},\dot{w},\dot{p},\dot{q},\dot{r},\delta) \quad (2.20)$$

If horizontal motion is considered,the equations are reduced to

$$X = X(u,v,r,\dot{u},\dot{v},\dot{r},\delta) \quad (2.21)$$

$$Y = Y(u,v,r,\dot{u},\dot{v},\dot{r},\delta) \quad (2.22)$$

$$N = N(u,v,r,\dot{u},\dot{v},\dot{r},\delta) \quad (2.23)$$

2.2 State Space Representation for Ocean Vehicles

The state space representation of a dynamic system is very convenient because in this form the wealth of the powerful, organized, and practical results from modern control theory can be applied to the understanding of the system.

The state of the system as defined in reference (4) is the minimum set of number $x_1(to)$, $x_2(to) \cdots x_n(to)$, which is combination with the input to the system $\underline{u}(t)$ for $t \geq to$, is sufficient to determine the behavior of the system for all time $t > to$.

The usual representation for state variables and inputs is through the vectors.

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \qquad (2.24) \qquad\qquad \underline{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \qquad (2.25)$$

A dynamic system which can be represented by states and state equations is called a state determined dynamic system. The order of the system is referred as the number of states necessary to determine the system. The state equations consist usually of n first order differential or difference equations. These equations can be time dependent as (2.26) or time independent as (2.26a)

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t) \qquad (2.26)$$

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}) \qquad (2.26a)$$

Usually a measurement function is defined to express the observed output of the dynamic system.

$$\underline{z}(t) = \underline{h}(\underline{x},t) \tag{2.27}$$

or

$$\underline{z}(t) = \underline{h}(\underline{X}) \tag{2.27a}$$

For a general ocean vehicle six are the state variables, or the primary state variables - the vehicle velocities for the mathematical model used in this thesis three states define the system - the linear velocities u and v, and the angular velocity r that is

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} u(t) \\ v(t) \\ r(t) \end{bmatrix} \tag{2.28}$$

The control vector is reduced to a scalar

$$\underline{u}(t) = \delta(t) \tag{2.29}$$

The state space representation of the hydrodynamic strucutre for the mathematical model is

$$\underline{X}_{hidr} = \underline{X}_{hidr}(\underline{x}, \underline{\dot{x}}, \underline{u}) \tag{2.30}$$

2.3 Taylor Series Expansion of the Hydrodynamic Structure

In order to take advantage of the state space representation for ocean vehicle it is necessary to define explicity the hydrodynamic **structure** in terms of the state variables, their time derivatives, and the control vector.

There is one method that can be generally applied to specify the hydrodynamic strucutre of an ocean vehicle. It is by expanding (2.30) in a Taylor series about the nominal values of the state $\underline{x}_o$, the state time derivative $\underline{\dot{x}}_o$ and the control vector $\underline{u}_o$. It is necessary afterwards to specify by some means the coefficients of the Taylor series. Once the structure is established the results of theoretical or experimental investigations can be used to determine those coefficients.

The Taylor series expansion of (2.30) about $\underline{x}_o$, $\underline{\dot{x}}_o$, δ is given by

$$\underline{X}_{hidr} = \underline{X}_{hidr}(\underline{x}_o, \underline{\dot{x}}_o, \delta_o) + \sum_{i=1}^{3} \frac{\partial \underline{X}_{hidr}}{\partial x_i}\Bigg|_{(\underline{x}_o, \underline{\dot{x}}_o, \delta_o)} \Delta x_i +$$

$$\sum_{i=1}^{3} \frac{\partial \underline{X}_{hidr}}{\partial \dot{x}_i}\Bigg|_{(\underline{x}_o, \underline{\dot{x}}_o, \delta_o)} \Delta \dot{x}_i + \frac{\partial \underline{X}_{hidr}}{\partial \delta}\Bigg|_{(\underline{x}_o, \underline{\dot{x}}_o, \delta)} \Delta \delta +$$

$$\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^2 X_{hidr}}{\partial x_i \partial x_j}\Bigg|_{(\underline{X}_0,\, \underline{\dot{X}}_0,\, \delta_0)} \Delta x_i \Delta x_j + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^2 X_{hidr}}{\partial \dot{x}_i \partial \dot{x}_j}\Delta \dot{x}_i \Delta \dot{x}_j +$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^2 X_{hidr}}{\partial x_i \partial \dot{x}_j}\Delta x_i \Delta \dot{x}_j + \sum_{i=1}^{3}\frac{\partial^2 X_{hidr}}{\partial x_i \partial \delta}\Delta x_i \Delta \delta +$$

$$\sum_{i=1}^{3}\frac{\partial^2 X_{hidr}}{\partial \dot{x}_i \partial \delta}\Delta \dot{x}_i \Delta \delta + \frac{1}{2}\frac{\partial^2 X_{hidr}}{\partial \delta^2}\Delta \delta^2 +$$

$$\frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial x_j \partial x_k}\Delta x_i \Delta x_j \Delta x_k + \frac{1}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\partial^3 X_{hidr}}{\partial \dot{x}_i \partial \dot{x}_j \partial \dot{x}_k}\Delta \dot{x}_i \Delta \dot{x}_j \Delta \dot{x}_k +$$

$$\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial x_j \partial \dot{x}_k}\Delta x_i \Delta x_j \Delta \dot{x}_k + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial \dot{x}_j \partial \dot{x}_k}\Delta x_i \Delta \dot{x}_j \Delta \dot{x}_k +$$

$$\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial x_j \partial \delta}\Delta x_i \Delta x_j \Delta \delta + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^3 X_{hidr}}{\partial \dot{x}_i \partial \dot{x}_j \partial \delta}\Delta \dot{x}_i \Delta \dot{x}_j \Delta \delta +$$

$$\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial \dot{x}_j \partial \delta}\Delta x_i \Delta \dot{x}_j \Delta \delta + \frac{1}{2}\sum_{i=1}^{3}\frac{\partial^3 X_{hidr}}{\partial x_i \partial \delta^2}\Delta x_i \Delta \delta^2 +$$

$$\frac{1}{2}\sum_{i=1}^{3}\frac{\partial^3 X_{hidr}}{\partial \dot{x}_i \partial \delta^2}\Delta \dot{x}_i \Delta \delta^2 + \frac{1}{6}\frac{\partial^3 X_{hidr}}{\partial \delta^3}\Delta \delta^3 + \qquad (2.31)$$

$$O(\Delta X)^4$$

Terms up to the third order were considered in the expansion. All the derivatives are evaluated at the nominal point although the notation was omitted after the first terms.

For the mathematical model used in this thesis, the nominal point as given by

$$X_o = \begin{bmatrix} u_o \\ 0 \\ 0 \end{bmatrix} \quad ; \quad \dot{X}_o = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad ; \quad \delta_o = 0 \qquad (2.32)$$

Since all the variables have nominal values equal zero, except for u, the change in value for all these variables can be written in the form

$$\Delta \text{ (variable)} = \text{variable} \qquad (2.33)$$

In almost all the computational work of this thesis, the notation u will be equivalent to Δu.

The notation for the hydrodynamic derivatives presented by Abkowitz [1] will be used. The equation (2.31) gives three scalar equations. The longitudinal force is taken as an example to illustrate the notation to be used.

$$X = X_o + X_u \Delta u + X_v v + X_r r + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{r}} \dot{r} + X_\delta \delta + \frac{1}{2} X_{uu} (\Delta u)^2 + X_{uv} \Delta u\, v + X_{ur} \Delta u\, r + X_{u\dot{u}} \Delta u\, \dot{u} + X_{u\dot{v}} \Delta u\, \dot{v} + X_{u\dot{r}} \Delta u\, \dot{r} + X_{u\delta} \Delta u\, \delta + \frac{1}{2} X_{vv} v^2 + X_{vr} v r + X_{v\dot{u}} v \dot{u} + X_{v\dot{v}} v \dot{v} + X_{v\dot{r}} v \dot{r} + X_{v\delta} v \delta + \frac{1}{2} X_{rr} r^2 + \ldots \quad (2.34)$$

Considering only terms up to the third degree more than 50 terms appear in the x equation. It would be a very difficult task to evaluate each coefficient and the numerical solution of the differential equations in a digital computer would be almost impossible. Fortunately, many of the coefficients in the Taylor expansion can be proved by theoretical considerations to be zero while others are sufficiently small to be neglected.

2.4. Hydrodynamic Coefficients

The hydrodynamic .ucture of the mathematical model was expanded in Taylor series. The large number of hydrodynamic coefficients can be greatly reduced if a detailed analysis of the physical problem is carried out. The conclusions presented in references (1),(3), and (7) are reproduced below.

a. Symmetry considerations demonstrate that the X equation should be an even function of the parameters, v, r, $\dot{v}$, $\dot{r}$, δ. Similarly the Y and N equations are odd functions of the same parameters. Consequently, odd terms in v, r, $\dot{v}$, $\dot{r}$, δ are eliminated from the x-equation and even terms in the same parameters are eliminated from the Y and N equations. In the same way odd terms in Δu, $\dot{u}$ are eliminated from the Y and N equations.

b. As another consequence of body symmetry, Y_u, Y_{uu}, Y_{uuu}, $Y_{\dot{u}}$ and the corresponding derivatives in the moment equation, N_u, N_{uu}, N_{uuu}, $N_{\dot{u}}$ are all zero.

c. The nature of acceleration forces eliminate other terms. According to Abkowitz no second or higher order acceleration terms can be expected. This is based on the assumption that there is no significant interaction between viscous and inertia properties of the fluid and that acceleration forces calculated from potential theory give only linear terms, when applied to submerged bodies.

d. All terms representing cross-coupling between acceleration and velocity parameters are zero or negligible small. This is supported by Abkowitz based on reasons similar to those just given and verified experimentally.

If all these simplifications are applied, one ends up with the following equations for the hydrodynamic structure:

$$X = X_o + X_u \Delta u + \frac{1}{2} X_{uu} (\Delta u)^2 + \frac{1}{6} X_{uuu} (\Delta u)^3 + \frac{1}{2} X_{vv} v^2 + \frac{1}{2} X_{rr} r^2 + X_{\delta\delta} \delta^2 + X_{\dot{u}} \dot{u} + \frac{1}{2} X_{vvu} v^2 \Delta u + \frac{1}{2} X_{rru} r^2 \Delta u + \frac{1}{2} X_{\delta\delta u} \delta^2 \Delta u + X_{vr} v r + X_{r\delta} r\delta + X_{v\delta} v\delta + X_{vru} v r \Delta u + X_{v\delta u} v\delta \Delta u + X_{r\delta u} r\delta \Delta u. \tag{2.35}$$

$$Y = Y_o + Y_v v + \frac{1}{6} Y_{vvv} v^3 + \frac{1}{2} Y_{vrr} v r^2 + \frac{1}{2} Y_{v\delta\delta} v\delta^2 + Y_{vu} v \Delta u + \frac{1}{2} Y_{vuu} v \Delta u^2 + Y_r r + \frac{1}{6} Y_{rrr} r^3 + \frac{1}{2} Y_{rvv} r v^2 + \frac{1}{2} Y_{r\delta\delta} r\delta^2 + Y_{ru} r \Delta u + \frac{1}{2} Y_{ruu} r \Delta u^2 + Y_\delta \delta + \frac{1}{6} Y_{\delta\delta\delta} \delta^3 + \frac{1}{2} Y_{\delta vv} \delta v^2 + \frac{1}{2} Y_{\delta rr} \delta r^2 + Y_{\delta u} \delta \Delta u + Y_{\delta uu} \delta \Delta u^2 + Y_{vr\delta} v r \delta + Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} \tag{2.36}$$

$$N = N_o + N_v v + \frac{1}{6} N_{vvv} v^3 + \frac{1}{2} N_{vrr} v r^2 + N_{v\delta\delta} v\delta^2 + N_{vu} v \Delta u + \frac{1}{2} N_{vuu} v \Delta u^2 + N_r r + \frac{1}{6} N_{rrr} r^3 + \frac{1}{2} N_{rvv} r v^2 + \frac{1}{2} N_{r\delta\delta} r\delta^2 + N_{ru} r \Delta u + \frac{1}{2} N_{ruu} r \Delta u^2 + N_\delta \delta + \frac{1}{6} N_{\delta\delta\delta} \delta^3 + \frac{1}{2} N_{\delta vv} \delta v^2 + \frac{1}{2} N_{\delta rr} \delta r^2 + N_{\delta u} \delta \Delta u + \frac{1}{2} N_{\delta uu} \delta \Delta u^2 + N_{vr\delta} v r \delta + N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} \tag{2.37}$$

2.5. State Equations and Measurement Function for the Mathematical Model.

The rigid body structure and the hydrodynamic structure were developed for the mathematical model. If the resultant expressions (2.10), (2.11), (2.12) and (2.35), (2.36), (2.37) are introduced in equation (2.1) the state equations can be derived. This substitution leads to the following equations:

$$(m - X_{\dot{u}})\dot{u} = f_1(u, v, r, \delta) \qquad (2.38)$$

$$(m - Y_{\dot{v}})\dot{v} + (mx_G - Y_{\dot{r}})\dot{r} = f_2(u, v, r, \delta) \qquad (2.39)$$

$$(mx_G - N_{\dot{v}})\dot{v} + (I_z - N_{\dot{r}})\dot{r} = f_3(u, v, r, \delta) \qquad (2.40)$$

where $f_1(u,v,r,\delta)$, $f_2(u,v,r,\delta)$, and $f_3(u,v,r,\delta)$ are given by

$$f_1(u,v,r,\delta) = X_u\Delta u + \frac{1}{2}X_{uu}\Delta u^2 + \frac{1}{6}X_{uuu}\Delta u^3 + \frac{1}{2}X_{vv}v^2 + (\frac{1}{2}X_{rr} + mx_G)r^2 + \frac{1}{2}X_{\delta\delta}\delta^2 + \frac{1}{2}X_{vvu}v^2\Delta u + \frac{1}{2}X_{rru}r^2\Delta u + \frac{1}{2}X_{\delta\delta u}\delta^2\Delta u + (X_{vr} + m)vr + X_{v\delta}v\delta + X_{vru}vr\Delta u + X_{v\delta u}v\delta\Delta u + X_{r\delta u}r\delta\Delta u$$

$$f_2(u,v,r,\delta) = Y_0 + Y_v v + \frac{1}{6}Y_{vvv}v^3 + \frac{1}{2}Y_{vrr}vr^2 + \frac{1}{2}Y_{v\delta\delta}v\delta^2 + Y_{vu}v\Delta u + \frac{1}{2}Y_{vuu}v\Delta u^2 + (Y_r - mu)r + \frac{1}{6}Y_{rrr}r^3 + \frac{1}{2}Y_{rvv}rv^2 + \frac{1}{2}Y_{r\delta\delta}r\delta^2 + Y_{ru}r\Delta u + \frac{1}{2}Y_{ruu}r\Delta u^2 + Y_\delta\delta + \frac{1}{6}Y_{\delta\delta\delta}\delta^3 + \frac{1}{2}Y_{\delta vv}\delta v^2 + \frac{1}{2}Y_{\delta rr}\delta r^2 + Y_{\delta u}\delta\Delta u + \frac{1}{2}Y_{\delta uu}\delta\Delta u^2 + Y_{vr\delta}vr\delta$$

$$f_3(u,v,r,\delta) = N_o + N_v v + \frac{1}{6} N_{vvv} v^3 + \frac{1}{2} N_{vrr} v r^2 + \frac{1}{2} N_{v\delta\delta} v \delta^2 + N_{vu} v \Delta u + \frac{1}{2} N_{vuu} v \Delta u^2 + (N_r - m x_G u) r + \frac{1}{6} N_{rrr} r^3 + \frac{1}{2} N_{rvv} r v^2 + \frac{1}{2} Y_{r\delta\delta} r \delta^2 + Y_{ru} r \Delta u + \frac{1}{2} Y_{ruu} r \Delta u^2 + Y_\delta \delta + \frac{1}{6} Y_{\delta\delta\delta} \delta^3 + \frac{1}{2} Y_{\delta vv} \delta v^2 + \frac{1}{2} Y_{\delta rr} \delta r^2 + Y_{\delta u} \delta \Delta u + \frac{1}{2} Y_{\delta uu} \delta \Delta u^2 + Y_{vr\delta} v r \delta$$

The equations (2.38) to (2.40) can be further modified to give the state equations:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1(u,v,r,\delta)/(m - X_{\dot{u}}) \\ \dfrac{(I_z - N_{\dot{r}}) \cdot f_2(u,v,r,\delta) - (m x_G - Y_{\dot{r}}) f_3(u,v,r,\delta)}{(m - Y_{\dot{v}})(I_z - N_{\dot{r}}) - (m x_G - N_{\dot{v}})(m x_G - Y_{\dot{r}})} \\ \dfrac{(m - Y_{\dot{u}}) f_3(u,v,r,\delta) - (m x_G - N_{\dot{v}}) f_2(u,v,r,\delta)}{(m - Y_{\dot{v}})(I_z - N_{\dot{r}}) - (m x_G - N_{\dot{v}})(m x_G - Y_{\dot{r}})} \end{bmatrix} \quad (2.41)$$

In order to complement the structure of the state space formulation, a measurement function will be defined. It is assumed that all the state variables are observable according to the criterion of modern control theory [4]. The measurement function adopted for the mathematical model is a linear time invariant one

$$\underline{z} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \quad (2.42)$$

Two comments must be made about the state equations obtained for the mathematical model. The functions $f_1(u, v, r, \delta)$, $f_2(u, v, r, \delta)$, and $f_3(u, v, r, \delta)$ contain cross-coupling terms between the state variables and the control variable. These coefficients, some of which may be significant would not have appeared if the hydrodynamic structure had been broken into hydrodynamic forces and effector forces.

The hydrodynamic coefficients in the state equations (2.41) are in the dimensional form. These coefficients are given in the literature usually in the non-dimensional form. Most of these coefficients are obtained from model tests and to be applied to the prototype it is better to use them as non-dimensional parameters. If they are used the variables in the state equations must be redefined. However, for the purposes of the thesis the dimensional form is more convenient and will be employed throughout.

2.6 Mathematical Model with Uncertain Structure

The state equations and the measurement function developed for the mathematical model are deterministic. Nevertheless, in any practical case both the dynamic process and the measurement process are disturbed by noise. It is necessary, in consequence, to incorporate these uncertainties in the mathematical model.

The process noise, represented by the vector $\underline{w}$ and the measurement noise, represented by the vector $\underline{v}$ are added to the state space formulations of the dynamic system. A very strong assumption is made that the noise processes are coupled linearly into the dynamic of the system. In these conditions, the stochastic model for a generic dynamic system is characterized by the following equations

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}) + G\underline{w} \quad (2.48)$$

$$\underline{z} = \underline{h}(\underline{x}) + D\underline{v} \quad (2.49)$$

For the mathematical model used in this thesis the resultant equations are:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1(u,v,r,\delta)/(m - X_{\dot{u}}) \\ \dfrac{(I_z - N_{\dot{r}}) f_2(u,v,r,\delta) - (m x_G - Y_{\dot{r}}) f_3(u,v,r,\delta)}{(m - Y_{\dot{v}})(I_z - N_{\dot{r}}) - (m x_G - N_{\dot{v}})(m x_G - Y_{\dot{r}})} \\ \dfrac{(m - Y_{\dot{v}}) f_3(u,v,r,\delta) - (m x_G - N_{\dot{v}}) f_2(u,v,r,\delta)}{(m - Y_{\dot{v}})(I_z - N_{\dot{r}}) - (m x_G - N_{\dot{v}})(m x_G - Y_{\dot{r}})} \end{bmatrix} \quad (2.45)$$

$$\underline{Z} = \underline{X} + \underline{V} \tag{2.46}$$

where the matrices $\underline{D}$ and $\underline{G}$ are taken as identity matrices.

For the purposes of the identification studies the noise vectors $\underline{w}_n$ and $\underline{v}_n$ will be assumed to be uncorrelated, discrete, zero mean, Gaussian - white noise processes. They are described by the following equations:

$$\bar{\underline{W}} = E[\underline{W}] = \bar{\underline{V}} = E[\underline{V}] = \underline{0} = \bar{\underline{W}}_n = \bar{\underline{V}}_n \tag{2.47}$$

$$E[\underline{W}(t)\, \underline{W}^T(t+\tau)] = Q_c\, \delta(\tau)\,; \quad Q \approx Q_c\, \delta t \text{ for discrete } \underline{W}_n \tag{2.48}$$

$$E[\underline{V}(t)\, \underline{V}^T(t+\tau)] = R_c\, \delta(\tau)\,; \quad R \approx R_c\, \delta t \text{ for discrete } \underline{V}_n \tag{2.49}$$

$$E[\underline{W}\, \underline{V}^T] = [0] \tag{2.50}$$

The equations (2.45) to (2.50) define completely the stochastic model which will be used in the identification studies.

2.7 Linear Model for the Ship Maneuvering

The mathematical model developed to describe the dynamics of the ocean is highly non linear. It is, however, of certain interest for the objective of the present work to use a simpler model in the identification studies. For some types of meneuvers the linear model is perfectly appropriate and will be employed for a first analysis of the hydrodynamic coefficients.

The linear state equations can be derived from the general model developed in the previous sections. If only the linear terms are picked up in the equations (2.10) to (2.12) and (2.35) to (2.37) the linear equations obtained are

$$(m - X_{\dot{u}})\dot{u} = X_0 + X_u \Delta u \tag{2.51}$$

$$(m - Y_{\dot{v}})\dot{v} + (m x_G - Y_{\dot{r}})\dot{r} = Y_0 + Y_v v + (Y_r - m u_0) r + Y_\delta \delta \tag{2.52}$$

$$(m x_G - N_{\dot{v}})\dot{v} + (I_z - N_{\dot{r}})\dot{r} = N_0 + N_v v + (N_r - m x_G u_0) r + N_\delta \delta \tag{2.53}$$

It is noticed that even for the linear case there is a coupling between the Y and N equations, but what is important is that the first equation is completely decoupled from the others. For the identification studies the first equation will be omitted in the linear model and the state equations are reducted to:

$$
\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} =
\begin{bmatrix}
\dfrac{(I_z - N_{\dot{r}})Y_v - (m x_G - Y_{\dot{r}})N_v}{f_1} & \dfrac{(I_z - N_{\dot{r}})(Y_r - m u_0) - (m x_G - Y_{\dot{r}})(N_r - m x_G u_0)}{f_1} \\
\dfrac{-(m x_G - N_{\dot{v}})Y_v + (m - Y_{\dot{v}})N_v}{f_1} & \dfrac{-(m x_G - N_{\dot{v}})(Y_r - m u_0) + (m - Y_{\dot{v}})(N_r - m x_G u_0)}{f_1}
\end{bmatrix}
\begin{bmatrix} v \\ r \end{bmatrix} +
$$

$$
\begin{bmatrix}
\dfrac{(I_z - N_{\dot{r}})Y_\delta - (m x_G - Y_{\dot{r}})N_\delta}{f_1} \\
\dfrac{-(m x_G - N_{\dot{v}})Y_\delta + (m - Y_{\dot{v}})N_\delta}{f_1}
\end{bmatrix} \delta +
\begin{bmatrix}
\dfrac{(I_z - N_{\dot{r}})Y_0 - (m x_G - N_{\dot{r}})N_0}{f_1} \\
\dfrac{-(m x_G - N_{\dot{v}})Y_0 + (m - Y_{\dot{v}})N_0}{f_1}
\end{bmatrix}
\qquad (2.54)
$$

$$
f_1 = \left[(I_z - N_{\dot{r}})(m - Y_{\dot{v}}) - (m x_G - N_{\dot{v}})(m x_G - Y_{\dot{r}})\right] \qquad (2.55)
$$

An important simplification is brought by the linear assumption, the state variables are decoupled from the control variable. There appears also constant terms in the state equations which express the possibility of existing non zero force and moments of the nominal or equilibrium state. This is particularly true for single screw ships where even for zero rudder deflection there are some efforts applied to the ship.

2.8 Hydrodynamic Coefficients for the Mathematical Model.

The mathematical model for the identification studies is almost complete. At this point one needs only to select a ship whose hydrodynamic coefficients will be used in the identification procedure.

There is not available in the literature much data about the hyrodynamic derivatives for exisitng ships. One of the few cases that is well documented is the Mariner class ship. There are some sources (5), (6), (13) which present the complete set of the hydrodynamic coefficients for this ship. In the Appendix 1 the coefficients are presented first in the non-dimensional form as given in the literature and next in the dimensional form. The details of dimensionalization are also covered in this appendix.

The Chapter described the whole development of the mathematical model for the horizontal maneuvering of a surface ship. This model will be employed later in the identification studies. In the next chapter the methods of identification are outlined and in Chapter 4 the procedure adopted to identify the parameters of the mathematical method is decribed.

CHAPTER 3

3. TECHNIQUES OF SYSTEM IDENTIFICATION

The previous chapter was dedicated to the development of a mathematical model for the horizontal maneuver of a surface ship. The model was obtained and its structure is characterized by the presence of the hydrodynamic coefficients. These coefficients are the parameters to be studied in the identification process. In this chapter, the concept of parametric identification is first presented and some of the techniques for system identification are listed. The next sections are used to present and discuss the equations for the problem of parametric identification using model reference contour and extended Kalman filtering techniques. And finally the criteria of identifiability of parameters is presented.

3.1 Definition of System Identification

This section starts with the presentation of the basic definitions of system identification. The basic foundation underlining system identification as a means of representing the system dynamics has the same degree of validity as any method of dynamic analysis (12). It may be applied indistinctly to any type of system, either deterministic or stochastic.

Parametric identification is the determination of a set of parameters or coefficients of a dynamic system mathematical model of known structure using measurement of the actual system's behavior with the ultimate aim of having the model be the mathematical equivalent of the system.

The dynamic system behavior may be determined from a full-scale trial with the prototype, from a model test run, or alternately from a computer simulation of the actual vehicle. This last approach will be used in the present work.

The main interest of system identification in the thesis is related for stochastic systems. The general nonlinear stochastic parametric identification problem is defined [2] by the following formulation.

Given

i) state equation

$$\dot{x} = f(\underline{x}, \underline{u}, \underline{p}, \underline{w}, t) \tag{3.1}$$

where $\underline{p}$ is the parameter vector $(n_p \times 1)$

with $\dot{p} = g(p)$; $\underline{p}(T_0) \equiv \underline{p}_0$ (3.2)

ii) measurement function

$$\underline{z} = \underline{h}(\underline{x}, \underline{w}, \underline{p}, \underline{v}, t) \tag{3.3}$$

iii) cost functional

$$c = c(\underline{z}, \underline{z}_m) \; ; \quad c \geqslant 0 \tag{3.4}$$

where C is a scalar cost functional representing a measure of closeness between the system output $\underline{Z}$ and the mathematical model output $\underline{Z}_m$; known structure $C \geq 0$.

Using

$\underline{u}(t)$, $\underline{Z}(t)$, $\underline{f}$, $\underline{g}$, $\underline{h}$, $\underline{x}(to)$; $\underline{P}o$, C

Find

$\underline{p}(t)$ to minimize C

The problem is very complex and has no completely general solution. Any solution technique to be applied to this kind of problem must in general be tailored to the positive semi-definite cost functional and to be specific types of structural nonlinearities.

The problem of identification of the ocean vehicle described by the mathematical model derived in Chapter 2 is considerably less complex. The following assumptions are applied to the general problem:

i) Model structure and measurement structure are invariant

ii) Model and measurement noises are linear and enter the system directly

iii) The structure of the measurement function $\underline{h}$ is simply the vehicle states $\underline{x}$ with linear measurement noise.

iv) The cost functional is a weighted integral of the square of the difference between model and the system.

v) The parameters are not states, but are constants to be evaluated.

With these assumptions, the formulation of the parametric identification problem for the ocean vehicle is presented below:

Given

i) state equation

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, \underline{p}) + \underline{w} \qquad (3.5)$$

where $\dot{\underline{p}} = 0$

ii) measurement equation

$$\underline{z} = \underline{x} + \underline{v} \qquad (3.6)$$

iii) cost functional

$$c = \int_{t_0}^{T_f} (\underline{z} - \underline{z}_m)^T R_n^{-1} (\underline{z} - \underline{z}_m)\, dt \qquad (3.7)$$

where

$c = c(\underline{u}, \underline{p})$; $c \geqslant 0$

R_n — weighting matrix

Using

$\underline{u}(t)$, $\underline{z}(t)$, $\underline{f}$, $\underline{x}(to)$ $\underline{P}(to)$, to, C

Find

$\underline{p}$ to minimize C.

3.2. Parametric Identification Techniques

Most of the available techniques of parameter identification have been developed for linear dynamic systems [11]. Some of those techniques, assuming linearized equations of motion to be a valid description of the behavior of an ocean vehicle may eventually be applied to determine the ship motion parameters.

There are other approaches that have already been applied for non linear models and specifically for identification of ocean vehicles. Two of these techniques will be used in this thesis. The model reference and the extended Kalman filtering approaches. These approaches were used by Hayes [2] and Reis [7].

The model reference technique assumes a mathematical model [*] for the system and comparing the output of both model and system to the same input, searches for the strucure of the model which minimizes a function of the errors between the two outputs.

The Kalman filtering technique essentially converts the identification problem into an estimation problem. The parameters are taken as states in an "augmented" state space. An extension of the Kalman filter for nonlinear systems is then used to estimate the states.

* This mathematical model should not be confused with the mathematical model developed in Chapter 2, which is the "true" representation of the system itself.

Two other approaches were used by Goodman et al. [12] for the identification of ocean vehicle parameters - an iterative method and a sequential method.

The first method applies primarily to deterministic systems and is essentially a generalization of a Newtonian iteration procedure. The differential equations of motion of the vehicle whether it as linear or nonlinear, are used together with additional variables that represent the unknown coefficients in these equations. The coefficients themselves are the actual variables that are sought in this system identification procedure, and different techniques are used within the course of the analysis. Solutions are necessary for all the variables starting with estimated initial conditions, where the variables include the state variables of the system as well as the coefficients themselves. Errors between the calculated state variables and the actual measured trajectory data itself are determined , and the modification of the unknown coefficients are obtained in this procedure. These new values are inserted again, solutions obtained, modified coefficient values found, and these are inserted again with the method repeated, i.e. and iterative procedure.

The other method is designed specifically for stochastic systems and is based on modern control theory - maximum principle, two-point boundary value problem, invariant imbedding and sequential estimation. The basic technique is applied to problems that are generally nonlinear. Using continuous time histories of the observed output measurements the task is then to obtain optimal estimates of the state variables and also various parameters, by a procedure that is based in minimizing an integral of the sum of weighted square of residual errors. The errors are the difference between the observed data and the actual desired system outputs (i.e. eliminating the measurement noise), and also the difference between the nominal trajectory of the system and the assumed form of the equation representation (i.e. eliminating the noisy input excitation and achieving a proper representation of the basic system dynamics). In this case the unknown parameters are also added as additional variables in the complete dynamic representation.

These two approaches were applied with satisfactory results for identification of ocean vehicle parameters using data generated on a computer as well as from full scale tests [12]. Although recognizing the merits of these techniques it was decided to limit the identification studies in this thesis to the approaches that are described in the next sections - model reference and Kalman filtering.

3.3. Model Reference Identification

The model reference identification is a general procedure that runs the model with the same inputs as to the system, for a large

Step 4 Calculate a new value of p by some decision and the modification algorithm

Step 5 Branch to Step 2 and continue until complete or until C(p) is minimum.

The sketch of the approach is very clear and does not require additional explanation. The process can be implemented in a on-line identification but the off-line process is more realistic for the ocean vehicle identification applications.

Some considerations should be made about the application of the technique. The values and ranges of p must be specified to avoid unstable or perhaps singular solutions.

Structural errors make the results of the identification process meaningless with very large values for C(p) even for the optimum p (p^*). This would happen also if the level of noise is considerably high.

The process of decision defined in step 4 could be some kind of gradient algorithm. Nevertheless in this thesis the process of variation of the parameter values will simply cover a specified range with constant increment (systematic search).

One of the practical limitations of the model reference approach is that it permits the identification of a maximum of two parameters at once. It is because a pictorial representation of the output is necessary for the purposes of identification. Actually the optimum $\underline{p}$ and the minimum value of C(p) do not provide sufficient information about the identifiability of the parameters. A picture of the function within the range specified for the parameters is by far a better information to the understanding of the system identifiability. Conditioned to produce a picture of the cost function the model reference approach is limited to identify one parameter - cost function curve or two parameters - cost function contouring - at once.

In some cases it may be desirable or necessary to greatly accentuate the minimum value of $C(p^*)$. This can be accomplished by contouring the natural logarithm $\log_\ell C(\underline{p})$ vs. $\underline{p}$. This procedure will be used in this thesis.

Sometimes it may become convenient in order to provide a better visualization of the identification results to plot slices of the cost function along each parameter. This procedure will be also adopted in the present work.

3.4 Extended Kalman Filtering

Kalman filtering is essentially a linear technique with a firm theoretical foundation developed to estimate the state of a linear dynamic system subject to a noisy process. This technique when applied to the identification of a nonlinear system loses its theoretical foundation but in some cases works extremely well.

There is a large amount of information about Kalman filter in the literature. The analytical formulation of Kalman filter for the state estimation of linear system is presented in texts of modern control theory as Bryson-Ho [10], Sage [13], etc. The details of application of Kalman filter to nonlinear systems may be found on the mentioned references but is very well described by Brock [8]. Finally the steps of utilization of Kalman filtering for identification purposes are presented by Hayes [2], Reis [7], etc. It is not necessary, therefore, to present a detailed description of this technique. Only the basic ideas will be presented in this section.

It was previously mentioned that the Kalman filtering approach converts the identification problem into a state estimation problem. All the parameters that we want to identify must be state variables. There is, in consequence, an augmentation of the state space according the following scheme:

$$\underline{x}^a = \left|\frac{\underline{x}}{\underline{p}}\right| \tag{3.8}$$

where

$\underline{x}$ is a $n_s * 1$ vector

$\underline{p}$ is a $n_p * 1$ vector

$\underline{x}^a$ the augmented state vecotr $(n_s+n_p)*1$

(In the following, $\underline{x}$ will be used with the meaning of $\underline{x}^a$)

Then, given

(i) $$\dot{\underline{x}} = \underline{f}(\underline{x}, t) + G\underline{w} \qquad (3.9)$$

where

$$\underline{f} = \begin{bmatrix} \underline{f} \\ \underline{g} \end{bmatrix} \; ; \; \underline{g} = \underline{0} \quad (n_p \quad 1) \qquad (3.10)$$

(ii) $$\underline{z} = H\underline{x} + D\underline{v} \qquad (3.11)$$

where H, D as well as G are taken as identity matrices of order n_s

(iii) Cost Functional

$$c = \int_{t_0}^{t_f} (\underline{z} - \underline{z}_m)^T R_n^{-1} (\underline{z} - \underline{z}_m) dt \qquad (3.12)$$

using $\underline{z}(t)$, $\underline{f}$, $\underline{x}(t_0)$, t_0, c

find $x(t)$ to minimize c

There is no general solution to this problem, particularly due to the nonlinear characteristic of the dynamic system. It is, however, possible to find a general solution for the linear system.

The Kalman filtering is an approach that provide a solution for the estimation problem of a linear system. The basic technique for linear systems require rigid assumptions about the form of $\underline{v}$ and $\underline{w}$ and knowledge of the numerical characteristics of these noises. Specifically, $\underline{w}_n$ and $\underline{v}_n$ are assuemd to be zero mean uncorrelated white noise processes as defined in Chapter 2, with assumed or known process noise covariance Q and measurement noise R. In these conditions the Kalman filter can be proved to be the optimum estimator of the given linear system.

The linear Kalman filter is also valid for nonlinear systems as long as it can be shown that the errors in the estimate of the state variables can be approximated by a linear system. The theoretical considerations about the extension of Kalman filter to nonlinear system is presented by Brock [8].

The computational steps of the extended Kalman filtering technique applied to the mathematical model of the ocean vehicle is described in the sequence.

Step 1 Collect or generate noisy data $\underline{z}$ and inputs $\underline{u}$

$$\dot{\underline{x}} = f(\underline{x}, t) + \underline{w} \qquad (3.13)$$

$$\underline{z} = H\underline{x} + \underline{v}$$ (solved discretly for $\underline{z}_n$)

Step 2 Propagate the estimate state $\hat{\underline{x}}$ to t_n

$$\dot{\hat{\underline{x}}} = \underline{f}(\hat{\underline{x}}, t)$$
$$\underline{z}_m = H\hat{\underline{x}} \qquad (3.14)$$

Step 3 Propagate the error covariance matrix E to t_n

$$\dot{E} = FE + EF^T + Q \qquad (3.15)$$

where

$$Q = E\left[\underline{w}_n \underline{w}_n^T\right] \qquad (3.16)$$

$$E = E\left[\underline{e}\ e^T\right] \; ; \qquad (3.17)$$

$\underline{e}$ = state estimate error; carat ^ denotes estimate.

$$F = \frac{\partial \underline{f}(\hat{\underline{x}}, t)}{\partial \hat{x}} \qquad (3.18)$$

Step 4 Calculate the Kalman filter gain matrix k at t_n

$$K = EH^T(HEH^T + R)^{-1} \qquad (3.19)$$

where

$$R = E\left[\underline{v}_n \underline{v}_n^T\right] \qquad (3.20)$$

Step 5 Update E to E' at t_n

$$E' = E - KHE \qquad (3.21)$$

Step 6 Set $\hat{\underline{x}}'$ and E' as initial conditions for propagation equations at t_n and return to Step 2.

The sequence is repeated until the end of the process.

The Kalman filtering approach produces as outputs not only the estimate of the states but also the estimate of the error covariance. The meaning of these values will be discussed in the next section.

3.5 Identifiability of Parameters

The identification techniques described in the two last sections will be applied to the mathematical model developed in Chapter 2. The only concern in this study is the identification of $\underline{p}$. It is assumed that the model structure is sufficiently accurate.

In this thesis the parameters for the mathematical model of the Mariner class ship are studied for their identifiability characteristics by using a known set of parameters and a computer simulation with added noises to generate the data for use in the model reference and Kalman filtering techniques. The identifiability studies will be concerned with finding the original set of parameters used in the vehicle simulation rather than with the single problem of minimizing $C(\underline{p})$ as defined in section 1.

Some general concepts of identifiability that will be employed in the next chapters are presented here.

A parameter p_i belonging to the vector $\underline{p}$ will be termed identifiable if one or more values of p_i^* may be found from simulated data. The identifiability of p_i will refer to the ease with which one or more values of p_i^* may be found in the model reference contours and to the accuracy with which it may be determined by extended Kalman filtering. Special care should be taken about the value obtained for the parameters, even if they produce trajectories that match the measured values quite well. Sometimes a parameter that has only a small influence on the particular motion data being analyzed is sought by the system identification technique. In that case very little information related to that parameter is contained in the data, and the value determined by the procedure is spurious and could sometimes contaminate other parameter values.

The choice of the input is also of some importance; it may have a major effect on how well the system identification can be performed. It is generally very difficult to determine which is the best input to be used in a identification study. The best input is certainly function of the structure and the true value of the parameters [14].

The basic considerations applied by the system modeler to the model reference contours are judgments with regard to the slopes, shapes, and minimum values around the known or true values of the parameters used to generate the data.

The analysis of the cost function contour, and the other additional plots (slices of the function along one parameter) may lead to some kind of conclusions. The most significant are that both parameters are identifiable, or that one is identifiable but the other is not, or that both parameters are unidentifiable. Most of the information obtained from model reference identification are qualitative.

Extended Kalman filtering results in the "augmented" state trajectories $\underline{x}(t)$ and their error covariance E(t). In the case of unidentifiable parameters, the parametric states in $\underline{x}(t)$ may not converge to steady state values or may become unstable. In some cases the states may reach steady-state values which are biased away from the true value of parameters; and at the same time, the corresponding covariances E(t) may be very small, saying that the filter has a high degree of confidence in an erroneous value of a parameter. This may be due to the relative unimportance of the co-efficient, as it was already pointed out.

The identifiability of the parameter p is judged by how closely the random variable $p = N(p_f, E_f)$, where f denotes final estimate, corresponds to the known value of p* used in the vehicle simulation. The identification of p is highly dependent upon the initial values $N(P_o, E_o)$ used in the Kalman filter and some qualitative judgments of identifiability may be based on how much closer $N(P_f, E_f)$ is to the true value than $N(P_o, E_o)$ was prior to the Kalman filtering.

Model reference contouring requires a great deal more computation work than does extended Kalman filtering, but actually provides more information. The Kalman filtering is expected to be a more efficient technique for system identification since it uses the noise characteristic in its estimation of the parameters.

The chapter has presented the basic concepts of parametric identification, and some of the techniques used to handle this problem. Two approaches were described at the level of details necessary to understand the computational steps and analyse the identification results. In the next chapter the procedure used to identify the parameters of the mathematical model of the Mariner class ship is described. Finally in Chapter 5 the results of the identification studies are presented.

CHAPTER 4

4. IDENTIFICATION OF HYDRODYNAMIC COEFFICIENTS FOR THE MARINER CLASS SHIP

The mathematical model for the horizontal maneuvering of a surface ship was derived in Chapter 2. The complete nonlinear model was developed and a linear version was considered. Both models will be employed in the identification studies. In Chapter 3, the concept of parametric identification was presented. Two approaches of system identification were described and the computation steps for their application were listed. The idea of identifiability of parameters in the sense it will be used in this thesis was defined. The present Chapter describes the procedure employed in the identification process. All the analytical and computation details involved in each phase of the identification studies are discussed.

4.1. Phases of the Identification Process

The hydrodynamic coefficients inserted in the mathematical model belong to the Mariner class ship. The number of coefficients given in the literature for the ship (see Appendix I) is about 30. It must be understood that not all the coefficients have the same importance. This is particularly true for some specific maneuvers. It is necessary, therefore, to design a procedure to identify the coefficients.

It would be possible to carry the identification studies with all the coefficients, but the scheme exhibits a series of drawbacks:

1. Some parameters may be negligible in a relative sense for every type of maneuver. If these parameters are included in the study, no information related to themselves will be obtained, and at the same time the identification of the other coefficients may be degraded.
2. Parameters are, in general, only identifiable if they are in some sense excited by the vehicle effector or are in some manner coupled into the vehicle equations of motion for a specific manuever. Thus, the sea trial manuevers must be designed to excite specific parameters of interest and the model structure must be selected specifically for the pertinent input and parameters being utilized in order to have any chance of identifying the true parameters from noisy data. As more knowledge is gained about parameter identifiability, more sophisticated models are employed [2].

The points above mentioned suggest that some coefficients may be omitted in the identification study, while different models may be chosen to identify the other parameters. In this case, appropriate inputs should be selected for each model. The scheme adopted breaks the identification studies of this thesis into three parts:

1. Preliminary analysis -- the influence of some coefficients that are apparently negligible is investigated. If a particular coefficient shows very little effect on the mathematical model trajectory for different kinds of manuever, it is eliminated from further identification study.
2. Identification of linear parameters -- as it was previously mentioned some types of ship manuever can be described perfectly well by the linear model. It was then decided to use this simpler model to identify the linear coefficients. These coefficients may eventually be studied later with the nonlinear model.
3. Identification of nonlinear parameters - the linear coefficients have already been studied. In this phase the identification will be primarily conducted for the nonlinear parameters. In order to check the procedure adopted, some of the linear coefficients are studied with this model.

4.2. Preliminary Analysis

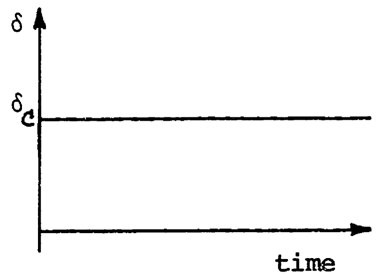
The coefficients of minor importance will not be included in the mathematical models to be used in the identification studies since they can produce spurious information, contaminating the identification of other parameters. The process chosen to eliminate the negligible coefficients is by simulating full-scale ship manuevers with the deterministic models developed in Chapter 2.

As the first step, all the coefficients that are suspected to be negligible in a relative sense are separated. This was done partially by an analysis of their values (see Appendix I) and also by following references presented in the literature [3,5]. A group of 8 parameters was selected for the phase of comparative studies.

$$X_{uuu},\ X_{v\delta},\ Y_{vvv},\ Y_{v\delta\delta},\ Y_{o},\ N_{vvv},\ N_{v\delta\delta},\ N_{o}$$

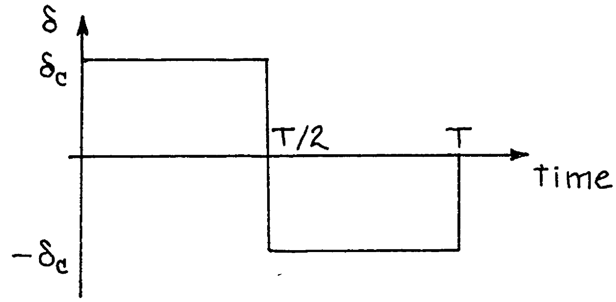
The importance of these coefficients is verified for two completely different types of manuever; one in which the linear model is expected to be valid, and another where the nonlinearities are likely to be significantly important. These manuevers are:

1. Step deflection of the rudder, simulating a turning circle with large radius - linear manuever



rudder deflection $\{\delta_c = 5$ degrees
$\{\delta_c = 10$ degrees

2. A zig-zag like manuever with a large rudder deflection exciting a tight ship manuever



rudder deflection $\{\delta_c = 25$ degrees
$\{\delta_c = 30$ degrees

The period of all runs was limited to 180 seconds. It was assumed that this period is sufficiently large to permit an analysis of the ship trajectory. In all the cases, the time lag for rudder deflection is neglected. It is believed that this set of manuevers is sufficient to explore all the dynamic behavior of the ship.

For each kind of manuever, a group of trials was run, firstly with mathematical model including all the coefficients. A standard trajectory is then obtained. Next some of the coefficients under study were omitted and the resultant trajectory is compared with the standard trajectory. The analysis of these trajectories determines whether the coefficients should be neglected or not in the final model.

The computer program used in the preliminary analysis is shown in Appendix 2. The equations of motion (see deterministic model - Chapter 2) was solved using the subroutine DYSYS (Dynamical System Simulation) developed by Department of Mechanical Engineering. This program solves the differential equations using Runge-Kutta method of fourth order.

The results of the preliminary analysis are shown in Chapter 5.

4.3. Identification Studies

The identification studies are divided into two parts: the first with the linear mathematical model and the second with the nonlinear model. Both models will not include the coefficients considered of negligible importance by the preliminary analysis.

It was primarily intended to use the two identification approaches simultaneously with each model. Most of the information obtained with one technique could be used to help the understanding of the results got with the other approach. It was not possible, however, to work in parallel with the two approaches. Some computation problems involved with application of extended Kalman filtering delayed the work. On the other hand, the model reference identification was working nicely, so it was decided to first complete the studies with the approach. Something was probably lost with the procedure, but even so the scheme produced good overall results.

One point is common to the two approaches is the level of noise to be used to generate the trial data. It was decided to adopt with slight changes, the same criteria employed by Hayes [2] and others. The amount of noise $\underline{W}_n$ and $\underline{V}_n$ are expressed as percentage values -- $\%\underline{W}$, and $\%\underline{V}$. The process noise percentage $\%\underline{W}_i$, where W_i is one element of the vector $\underline{W}_n$ means that the standard deviation ($\sigma W_i = \sqrt{Q_i}$) of W_i is that percentage of the maximum value of the correspondent elements of the vector $\dot{\underline{x}}$ evaluated by equation (4.1) for a given input. The measurement noise percentage $\%V_i$ is one element of the vector $\underline{V}_n$, means that the standard deviation ($\sigma V_i = \sqrt{R_i}$) of V_i is that percentage of maximum value of the correspondent element in vector $\underline{x}$ obtained by integrating equation (4.1), for a given input.

$$\dot{x} = f(\underline{x}, \underline{u}, t) \tag{4.1}$$

These are convenient definitions for simulation studies, but they are quite arbitrary and must be interpreted properly for a given maneuver. If $\underline{x}$ or $\dot{\underline{x}}$ is at small values for most of the manuever and then assumes its maximum value only for a short period during the manauver, then the associated $\underline{w}$ or $\underline{v}$ noise has a much greater effect upon the overall system uncertainty that it does if $\dot{\underline{x}}$ or $\underline{x}$ is at or near its maximum values for most of the maneuver

It was intended to use the same kind of input with both techniques. The linear model was actually run with the same input, but for the nonlinear model, different inputs were used with each approach. It is believed that the proper input specification is more critical to the extended Kalman filtering than to the model reference contour. For the later method, a large step deflection of the rudder was utilized. For the Kalman filtering a zig-zag like maneuver was believed to give better results.

Actually, different types of input should be used for a more complete investigation. However, the computation time required specially for model reference identification did not permit a more extensive analysis. Nevertheless, the types of maneuver employed seem to excite reasonably well the system dynamics, and the results are relatively good.

The identification study consists basically in the solution of the state equations for different conditions. It is understood that the numerical method employed to solve the equations might have some influence upon the results of the identification. This is particularly true for the Kalman filtering when besides the state equation there is error covariance matrix equation to be solved. It was intended to use the same numerical method with both approaches. However, the difficulties found with the computer implementation of Kalman filtering forced some modifications in the original scheme.

The second order Runge Kutta method was used with the model reference technique while a more precise numerical process, the fourth order Runge-Kutta method was employed with Kalman filtering. It is not unlikely that the same results could be obtained by Kalman filtering identification using the other method of integration.

The points discussed above constitute generalities that apply to the two identification approaches. The particularities of each technique, as well as the scheme of their application for the two models are presented in the next sections.

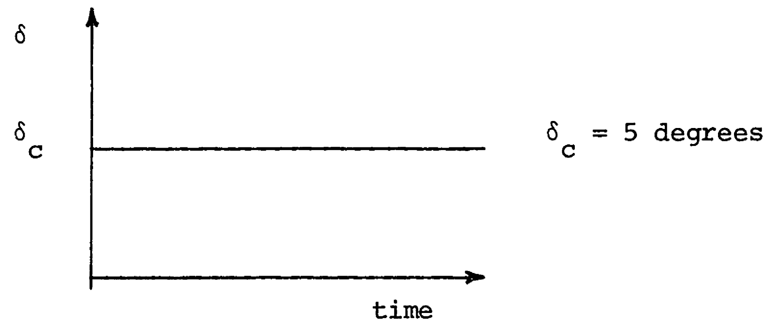
4.4. Identification of Linear Parameters

The first part of the identification study will be concerned with the linear model for horizontal manuever of a surface ship. The identifiability of the linear coefficients of the Mariner class ship will be investigated.

The first step in the identification process is to define in which conditions is the linear model expected to represent well the ship maneuver. It is generally accepted that for small maneuvers that do not involve large changes in velocities or accelerations the linear version of the mathematical model is a good representation of the ship behavior.(*) In this condition, it was felt that few types of input (rudder deflection) would be appropriate to excite only the linear dynamics of the system. Actually for the linear model there is not much choice. Although some variant could be used, the input

* Provided the ship is dynamically stable

employed for the identification of the linear parameters is a step rudder deflection shown below



The details of application of each technique are presented in the sequence.

4.4.1. Model Reference Identification

The model reference technique was described in Chapter 3 and the mathematical models were derived in Chapter 2. The state equations for the model as well as the basic scheme for the identification are repeated here. The sequence of steps is discussed below and is self-explanatory. The notation used for the hydrodynamic coefficients is the same that appears in the computer programs, and is presented in Appendix I.

Step 1

Generate the sea trial data

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \frac{1}{(A(11)A(4) - A(5)A(10))} \begin{bmatrix} A(6)v + A(7)r + A(8)\delta + A(9) \\ A(12)v + A(13)r + A(14)\delta + A(15) \end{bmatrix} + G \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad (4.2.)$$

where $w_1 = N(0, w_{\dot{v}})$; $w_{\dot{v}}$ - maximum value of $\dot{v}$ for the same manuever with the deterministic model.

$w_2 = N(0, w_{\dot{r}})$; $w_{\dot{r}}$ - maximum value of $\dot{r}$ for the same manuever with the deterministic model.

$$\begin{bmatrix} z_v \\ z_r \end{bmatrix} = \begin{bmatrix} v \\ r \end{bmatrix} + D \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad (4.3.)$$

where $v_1 = N(0, v_v)$; v_v - maximum value of v for the same manuever with the deterministic model.

$v_2 = N(0, v_r)$; v_r - maximum value of r for the same manuever with the deterministic model.

Store $\begin{bmatrix} Z_v \\ Z_r \end{bmatrix}$ at discrete time

Step 2

Select parameters to be identified, PA1, PA2 with respective indexes. LP1, LP2.

Define range of variation for each parameter and incremental value.

Set A(LP1) = minimum value of PA1

A(LP2) = minimum value of PA2

Step 3

Solve the equations for the deterministic model with the estimated values of parameters LP1 and LP2.

$$\begin{bmatrix} \dot{v}_m \\ \dot{r}_m \end{bmatrix} = \frac{1}{A(4)A(11) - A(5)A(10)} \begin{bmatrix} A(6)v + A(7)r + A(8)\delta + A(9) \\ A(12)v + A(13)r + A(14)\delta + A(15) \end{bmatrix} \quad (4.4)$$

$$\begin{bmatrix} z_{mv} \\ z_{mr} \end{bmatrix} = \begin{bmatrix} v_m \\ r_m \end{bmatrix} \quad (4.5)$$

Store $\begin{bmatrix} z_{mv} \\ z_{mr} \end{bmatrix}$ at discrete times

Step 4

Evaluate the cost funtion or performance index

$$c = \log\left[\sum^{n}\left[(z_v - z_{mv})^2 + (z_r - z_{mr})^2\right]\right] \quad (4.6)$$

Store C

Step 5

Change the value of parameters

A(LP2) = A(LP2) + increment 2

return to step 3.

When A(LP2) > maximum value of PA2, increment parameters PA1.

A(LP1) = A(LP1) + increment 1

return to step 3.

When A(LP1) > maximum value of PA1 go to step 6.

Step 6

Plot the cost function contour, and additional outputs.

This scheme was used to identify all the linear coefficients. Different values of G and D, representing different degrees of noise were used in the identification process.

The computer programs for identification of linear coefficients using model reference contour are listed in Appendix 4 with all the subroutines necessary. The subroutines plot and contour used in this thesis represent slight modified versions of the programs developed by Hayes [2].

The results of the identification studies are presented in Chapter 6.

4.4.2 Extended Kalman Filtering

Unlike the model reference technique the application of Kalman filtering approach for identification of the hydrodynamic coefficients presented a series of difficulties most of them related to computer implementation. It was understood that due to the relative complex formulation of Kalman filtering much care would be required in tailoring the computer program. The pecularities of the error covariance matrix, of the gain matrix K, and the heterogeneous composition of the state vector predicted eventual troubles. And it did happen. All the details of application of the technique are discussed here.

The first decision to make is about the number of parameters to be identified simulataneously. As an initial idea it would be theoretically possible to identify all the parameters at once, it is just a question of state augmentation. It is, however, quite unlikely that the method works, with a large number of parameters, specially if some of these parameters are of relatively little importance. The filter will probably reproduce the trajectory quite well but the accuracy in the parameters identification would be small. As it was reported by Goodman [12] more accurate values for unknown coefficients can be predicted for a simple system than for a system with large number of coefficients.

It was decided by these reasons to test the technique by identifying two parameters at once. The results of this test were satisfactory and suggested that a larger number of parameters could be identified simultaneously and for the later runs the number of coefficients was set as four.

The sequence of computation steps is outlined below. Some of the computation problems related to the implementation of the method are next described. The notation utlizied here is the same used in the computer programs.

Step 1 (Same as in the model reference identification) Generate the sea trial data.

$$\begin{bmatrix} v \\ r \end{bmatrix} = \frac{1}{(A(11)A(4) - A(10)A(5))} \begin{bmatrix} A(6)v + A(7)r + A(8)\delta + A(9) \\ A(12)v + A(13)r + A(14)\delta + A(15) \end{bmatrix} + G \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \qquad (4.7)$$

$$\begin{bmatrix} z_v \\ z_r \end{bmatrix} = \begin{bmatrix} v \\ r \end{bmatrix} + D \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (4.8)$$

Step 2 Select parameters to be identified PA1, PA2, PA3 PA4 with respective indexes LP1, LP2, LP3, LP4. Definte initial estimates for the states, VST, RST, and the parameters, PST1, PST2, PST3, PST4.

Define initial covariances for the states VCV, VCR and the parameters, PCV1, PCV2, PCV3, PCV4.

Step 3 Set the initial estimate for the "augmented" state vector

$$\hat{\underline{X}}_0 = \begin{bmatrix} VST \\ RST \\ PST1 \\ PST2 \\ PST3 \\ PST4 \end{bmatrix} \qquad (4.9)$$

Set the initial value for the error covariance matrix

$$E_0 = \begin{bmatrix} VCV & & & & & \\ & RCV & & & 0 & \\ & & PCV1 & & & \\ & & & PCV2 & & \\ & 0 & & & PCV3 & \\ & & & & & PCV4 \end{bmatrix} \qquad (4.10)$$

Step 4 Propagate the state estimate

$$\begin{bmatrix} \hat{v} \\ \hat{r} \\ P\hat{A}1 \\ P\hat{A}2 \\ P\hat{A}3 \\ P\hat{A}4 \end{bmatrix} = \frac{1}{(A(11)A(4) - A(10)A(5))} \begin{bmatrix} A(6)v + A(7)r + A(8)\delta + A(9) \\ A(12)v + A(13)r + A(14)\delta + A(15) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (4.11)$$

$$\begin{bmatrix} z_{vm} \\ z_{vr} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{H} \begin{bmatrix} \hat{v} \\ \hat{r} \\ P\hat{A}1 \\ P\hat{A}2 \\ P\hat{A}3 \\ P\hat{A}M \end{bmatrix} \tag{4.12}$$

<u>Step 5</u> Propagate the error covariance matrix

$$\dot{E} = FE + EF^T + Q \tag{4.13}$$

where $F = \dfrac{\partial \underline{f}(\underline{x}, u, t)}{\partial \underline{x}}$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} = \frac{1}{(A(4)A(11) - A(10)A(5))} \begin{bmatrix} A(6)v + A(7)r + A(8)\delta + A(9) \\ A(12)v + A(13)r + A(14)\delta + A(15) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underline{f} \tag{4.14}$$

$$F = \begin{bmatrix} \partial f_1/\partial\hat{v} & \partial f_1/\partial\hat{r} & \partial f_1/\partial P\hat{A}1 & \partial f_1/\partial P\hat{A}2 & \partial f_1/\partial P\hat{A}3 & \partial f_1/\partial P\hat{A}4 \\ \partial f_2/\partial\hat{v} & \partial f_2/\partial\hat{r} & \partial f_2/\partial P\hat{A}1 & \partial f_2/\partial P\hat{A}2 & \partial f_2/\partial P\hat{A}3 & \partial f_2/\partial P\hat{A}4 \\ & & & 0 & & \\ & & & 0 & & \\ & & & 0 & & \\ & & & 0 & & \end{bmatrix} \tag{4.15}$$

$$Q = \begin{bmatrix} Q(1) & & & & \\ & Q(2) & & & 0 \\ & & 0 & & \\ 0 & & & 0 & \\ & & & & 0 \\ & & & & & 0 \end{bmatrix}; \quad \begin{matrix} Q(1) = E\left[w_1 w_1^T\right] \\ Q(2) = E\left[w_2 w_2^T\right] \end{matrix} \tag{4.16}$$

Step 6   Calculate the Kalman filter gain

$$K = EH^T(HEH^T + R)^{-1} \tag{4.17}$$

where

$$R = \begin{bmatrix} R(1) & & & & \\ & R(2) & & & 0 \\ & & 0 & & \\ 0 & & & 0 & \\ & & & & 0 \\ & & & & & 0 \end{bmatrix}; \quad \begin{matrix} R(1) = E\left[v_1 v_1^T\right] \\ R(2) = E\left[v_2 v_2^T\right] \end{matrix} \tag{4.18}$$

Step 7   Update the state estimate

$$\hat{x}' = \hat{x} - K(z_n - z_m) \quad \text{with } z_n \text{ at time } t_n \tag{4.19}$$

Step 8   Update the error covariance estimate

$$E' = E - KHE \tag{4.20}$$

Step 9 Store values of state and error covariance estimates.

Step 10 Set x' and E' as initial conditions, for propagation equations and return to Step 3. The sequence is repeated until the end of the process.

Different values of D and G were used in the identification process representing different degrees of noise.

Based on the results and comments of Hayes [2] the values of the Q and R matrices used with the Kalman filter were changed, for the same amount of noise in the trial data. It corresponds to informing the filter that there is more noise than the one actually experienced, and has the objective of tuning the filter.

A second and a third pass of trial data through the filter were sometimes employed in order to improve the accuracy of the identification process.

Although requiring less computation time than model reference approach the relatively long program necessary to implement Kalman filtering made it impossible to write a single computer program due to the core limitation of the computer unit used. The program was broken into 3 parts to adjust to the computer capacity.

The problem of core limitation brought also other problems. It was felt after some trials that the specific characteristics of the Kalman filter equations required the utilization of double precision variables to improve the accuracy of the results. But the limitations on core capacity did not permit this alternative.

The option left was to divide all the coefficients in the equations of motion by the term

CR = (A(4) . A(11) - A(10) . A(5))

The coefficients are then given by:

$$A(n) = A(n)/CR \tag{4.21}$$

At the same time the error covariances, PCV_i were divided by CR^2.

The new program produced better results as it is shown in Chapter 5. The only disadvantage of this procedure is that the mass and inertia parameters could not be identified.

The computer programs with all the subroutines used for extended Kalman filtering identification are shown in Appendix 5. The subroutines are specially tailored for the identification of 4 parameters, but little change is required to handle a larger number of parameters.

4.5 Identification of the Non-linear Parameters

The linear parameter with exception of the coefficients in x equation, were identified using the linear model. The use of the complete mathematical model for the horizontal maneuver of the Mariner class vessel is primarily concerned with the identification of the nonlinear coefficients. Nevertheless, some of the linear parameters already identified will be studied again to investigate eventual difference in identifiability.

In order to identify nonlinear coefficients the input, rudder deflection law, is selected to produce large maneuvers. The inputs used with the two approaches are different but both cause very tight nonlinear maneuvers.

4.5.1. Model Reference - Identification

The same procedure described in 4.4.1. was used to identify the nonlinear coefficients. Essentially there is modification only in the state and measurement equations.

The input employed is a step rudder deflection at 35 degrees. The equations to be inserted in the model reference identification scheme defined in 4.4.1 are:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{f_u}{A(1)} \\ \dfrac{A(11)f_v - A(5)f_r}{f_{vr}} \\ \dfrac{A(4)f_r - A(10)f_v}{f_{vr}} \end{bmatrix} + G \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} \tag{4.22}$$

where

$$f_u = A(2)u + A(16)u^2 + A(17)u^3 + A(18)v^2 + A(19)r^2 + A(20)\delta^2 + A(21)vr + A(22)v\delta \tag{4.23}$$

$$f_v = A(9) + A(6)v + A(7)r + A(8)\delta + A(26)\delta^3 + A(27)rv^2 + A(28)\delta v^2 \tag{4.24}$$

$$f_r = A(15) + A(12)v + A(13)r + A(14)\delta + A(31)\delta^3 + A(32)rv^2 + A(33)\delta v^2 \tag{4.25}$$

$$f_{vr} = A(4)\cdot A(11) - A(10)\cdot A(5) \tag{4.26}$$

$$\begin{bmatrix} z_u \\ z_v \\ z_r \end{bmatrix} = \begin{bmatrix} u \\ v \\ r \end{bmatrix} + D \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{4.27}$$

The elements of the noise vectors are defined in the same way as in 4.4.1. The correspondent equations for the deterministic model as derived from (4.2.2) and (4.2.7). The cost function is given by:

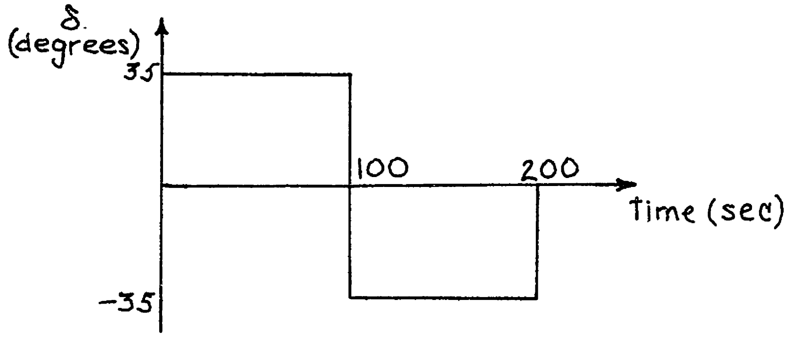
$$c = \log\left[\sum^{n}\left[(z_u - z_{mu})^2 + (z_v - z_{mv})^2 + (z_r - z_{mr})^2\right]\right] \qquad (4.28)$$

The computer programs for identification of the non-linear parameters using model reference approach are listed in appendix 6. The results are presented in the next chapter.

4.5.2 Extended Kalman Filtering

The same basic procedure described in section 4.4.2 are used to identify the nonlinear equations. The equations for the mathematical model used to generate the sea trial data are shown in section 4.5.1.

The input applied to the system is a zig-zag like law for the rudder deflection



The computer programs used for identification of the non-linear parameters using extended Kalman filter approach are listed in Appendix 7. The results are presented in the next chapter.

This chapter presented the general procedure utilized in the identification studies of this thesis. The several phases in which the study was divided are described in a wealth of details that help to understand the computer programs employed with the identification approaches. The results of the identification studies are presented and analyzed in the next chapter. The Chapter 6 presents general conclusions about the identification study and recommendations for future work.

CHAPTER 5

RESULTS OF THE IDENTIFICATION STUDIES

The previous chapters of this thesis were arranged in a logical sequence and prepared the basis for the understanding of the parametric identification results. In Chapter 2, the problem of horizontal maneuvering for a surface ship was modelled. A stochastic mathematical model was developed including process and measurement noises. A linear version of this model was derived to be used in the identification studies. In Chapter 3, the concept of parametric identification as it is considered in the present work is introduced, and the approaches for system identification were described. In Chapter 4 it was shown how the identification techniques are applied to the mathematical models. The whole procedure used in the identification studies is described. The details for computer implementation are discussed and some of the difficulties found are analyzed. This chapter will present all the results obtained in the identification procedure. These results are discussed under the criteria of parametric identifiability presented in Chapter 3. The chapter is divided into 5 sections, the first of which is reserved for the results of the preliminary analysis. All the other sections are concerned with identification itself.

5.1. Results of the Preliminary Analysis

As it was explained in Chapter 4 the preliminary analysis has the objective of determining which of the known hydrodynamic coefficients for the Mariner class ship can be neglected in the final model. The analysis consists in comparing the sea trial data, generated by ship maneuvering simulation.

The coefficients which importance was tested are $1/6\ X_{uuu}$(A4), $X_{r\delta}$(A9), $1/6\ Y_{vvv}$(B4), $1/2\ Y_{v\delta\delta}$(B10), Y_o(B0), $1/6\ N_{vvv}$(C4) $1/2\ N_{v\delta\delta}$(C9) and N_o(C10).

The ship maneuvers were simulated using the mathematical model. In order to investigate the importance of the different coefficients several runs were conducted in which some of the coefficients were omitted in the mathematical model. The analysis of the generated ship trajectories led to the following conclusions, which are applied in the conditions given below

a- small maneuvers, corresponding to small rudder deflection (step deflection was used).

1. The coefficients B4, B10, C4, C10 can be neglected.
2. Eventually, A4 and perhaps A9 might be neglected.
3. The coefficients B0 and C0 present too large an influence to be neglected.

b- tight manuevers, corresponding to large rudder deflections (zig-zag like maneuvers were simulated).

1. With some minor error B4, B10, C4 and C10 can be neglected. It is, however, clear that for this condition, the influence of the mentioned parameters is larger than for small maneuvers.
2. The other four coefficients cannot be disregarded.

As a consequence of this analysis, the coefficients B4, B10, C4, and C10 were eliminated from the mathematical models used in the identification studies.

5.2. Model Reference Identification of Linear Parameters

All the coefficients of the linear model with the exception of Y_o (A19) and N_o (A15) were studied in the identification process. A large number of runs were conducted and the principal results are presented in figures and tables.

The length of all the sea trial in 188 seconds, and for the kind of input selected for the linear model, this period is sufficiently long to ensure that the system reaches steady state.

Figure 5.1 shows the sea trial data for the noiseless process. Curve 1 is for the sway velocity, v, (ft/sec) and curve 2 is for the yaw velocity r (degree/sec). The data from these curves is used by the model reference approach to identify the parameters.

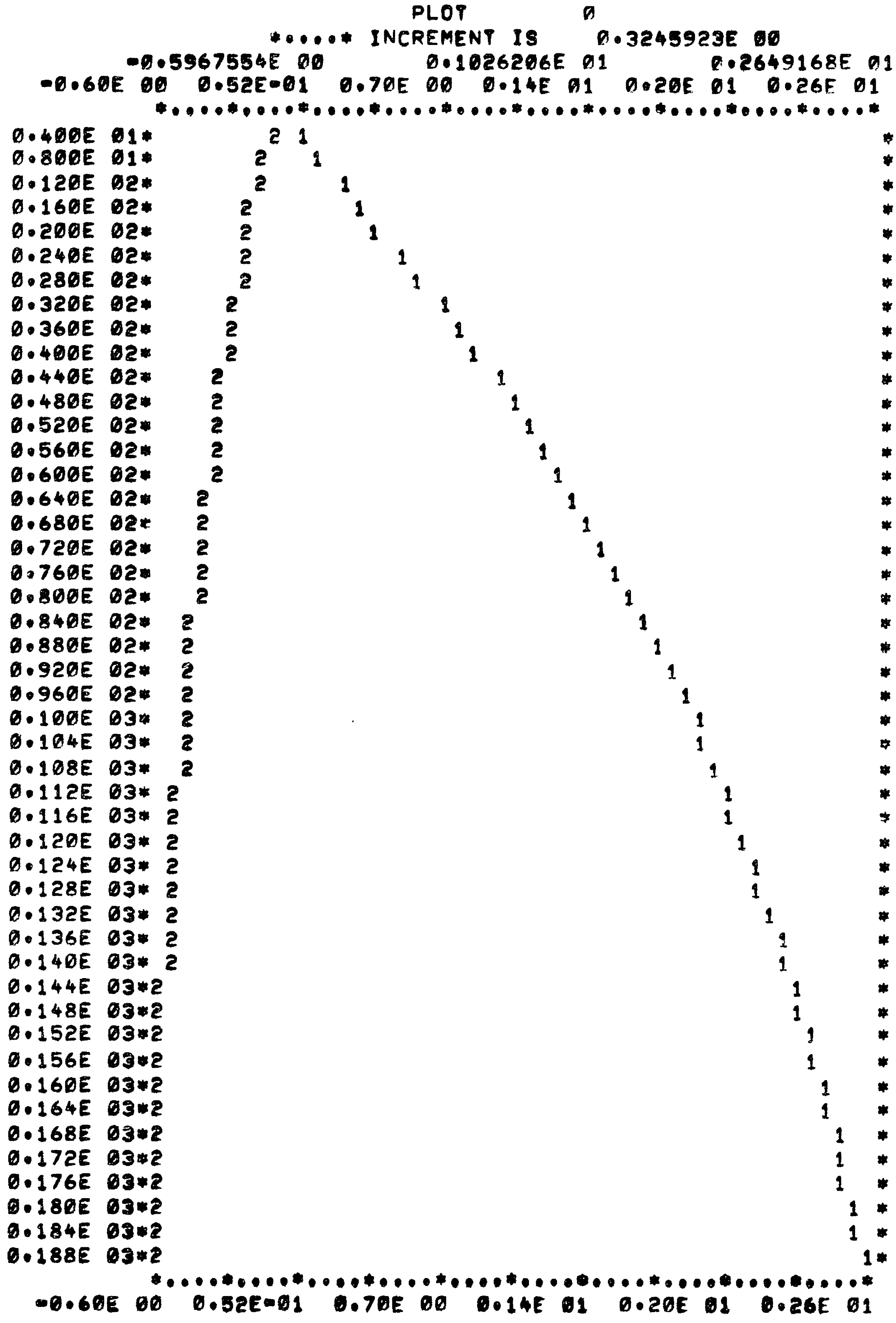


FIGURE 5.1. Sway velocity V(1) and yaw velocity r(2) for a noiseless process.

The two first parameters to be studied are $(I_z-N_{\dot{r}})$ (A(11)) and N_δ (A(14)). The contours for the zero noise sea trial are presented in Figures 5.2. and 5.3. In Figure 5.2. the x-range corresponds to the range of the parameter $\underline{P}$A1 and the y-range corresponds to the range of the parameter $\underline{P}$A2. The detailed values of $\underline{P}$A1 and $\underline{P}$A2 are printed along the edges of the contour with x running vertically and y, horizontally in Figure 5.3. The horizontal values of $\underline{P}$A2 are set such the last number in the exponential corresponds to the numerical value for that * location on the axis. At the top of the contour, the left,center and right values of $\underline{P}$A2* are given with greater accuracy than the axis values. The z-domain values correspond to the minimum and maximum values of C(p) over the contour. The contours in Fig. 5.3. run from the minimum or 1-value to the maximum or M-value in linear increments DZ. The z-domain descriptions in Figure 5.2. correspond to the 21 numbers and letters 1 through M used in the contour.

The reader can see from Figure 5.3. that the minimum point of C(p) corresponds to values of $\underline{P}$A1 = $\underline{P}$A1* and $\underline{P}$A2 = $\underline{P}$A2*, as it would be exptected from a noiseless process. The use of a logarithmic cost function served to greatly remark the optimum point. It is noticed that all the other contour points have a much higher value for c(p). In Figure 5.2. it may be seen that C(p*) = -22.18 which in a linear scale corresponds practically to zero. It may be seen also that the maximum value of the cost function, $C_{max}(p)$ is 3.322.

```
*****************************************************
*            CONTOUR   1  PARAMETERS                *
* X RANGE : 0.2370E 11 TO  0.4402E 11 DX= 0.4417E 09*
* Y RANGE :-0.1694E 09 TO -0.9124E 08 DY= 0.1564E 07*
* Z DOMAIN:-0.2220E 02 TO  0.3822E 01 DZ= 0.1301E 01*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH   *
* NO. 1-0.2218E 02 NO. 8-0.1308E 02 NO.15-0.3970E 01*
* NO. 2-0.2088E 02 NO. 9-0.1178E 02 NO.16-0.2669E 01*
* NO. 3-0.1958E 02 NO.10-0.1047E 02 NO.17-0.1368E 01*
* NO. 4-0.1828E 02 NO.11-0.9174E 01 NO.18-0.6741E-01*
* NO. 5-0.1698E 02 NO.12-0.7873E 01 NO.19 0.1233E 01*
* NO. 6-0.1568E 02 NO.13-0.6572E 01 NO.20 0.2534E 01*
* NO. 7-0.1438E 02 NO.14-0.5271E 01 NO.21 0.3822E 01*
*****************************************************
```

FIGURE 5.2. Numerical Values of Contours in Figure 5.3.

CONTOUR 1

****** INCREMENT IS 0.7820275E 07

-0.1694394E 09 -0.1303380E 09 -0.9123664E 08

-0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08

0.237E 11	M	M	M	M	M	M	M	M	M	M	M	M	M
0.255E 11	M	M	M	M	M	M	M	M	M	M	M	L	L
0.272E 11	M	M	M	M	M	M	M	L	L	L	L	L	L
0.290E 11	M	M	L	L	L	L	L	L	L	L	L	K	K
0.308E 11	L	L	L	L	L	L	K	K	K	K	K	J	J
0.325E 11	L	K	K	K	K	K	J	J	J	H	H	G	H
0.339E 11	K	J	J	J	H	G	1	G	H	J	J	J	K
0.356E 11	H	G	H	H	J	[illegible]	J	K	K	K	K	K	L
0.374E 11	J	K	K	K	K	K	K	L	L	L	L	L	L
0.392E 11	K	L	L	L	L	L	L	L	L	L	L	L	M
0.409E 11	L	L	L	L	L	L	M	M	M	M	M	M	M
0.427E 11	M	M	M	M	M	M	M	M	M	M	M	M	M
0.440E 11	M	M	M	M	M	M	M	M	M	M	M	M	M

-0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08

FIGURE 5.3. $(I_z - N_{\dot{r}})$ and N_δ contours, no noise, linear model.

In the particular case of Figure 5.3. only one point of minimum is found in the contour. It is, however, possible to find several points of minimum. Such points may be situated one close to the others in a certain region or may be situated one in a different region of the contour. In the first case, the parameters are identified with values corresponding to the center of the region. The second case is typical of noisy data and even in this case, the parameters are identified. Nevertheless, it is unlikely that the identified values are the true ones.

It would be desired to increase the identification accuracy by the use of a larger number of levels for C(p), and also a smaller increment in the parameter's range. This is not feasible, however, due to the increase in computation time.

A better insight in the shape of the contours in Figure 5.3. may be gained by plotting sets of 5 equally spaced slices of these contours along each axis. Figure 5.4. shows slices of the cost function along the y axis and Figure 5.5 shows slices of the cost function along the x axis. These plots are more helpful with noisy trials when the contour does not exhibit a very remarkable minimum.

The sea trial data was generated for different conditions of noise. The set of plots and contours from Figure 5.6 through Figure 5.18 were used in the identification studies of the parameters $(I_z-N_{\dot{r}})$ and N_δ.

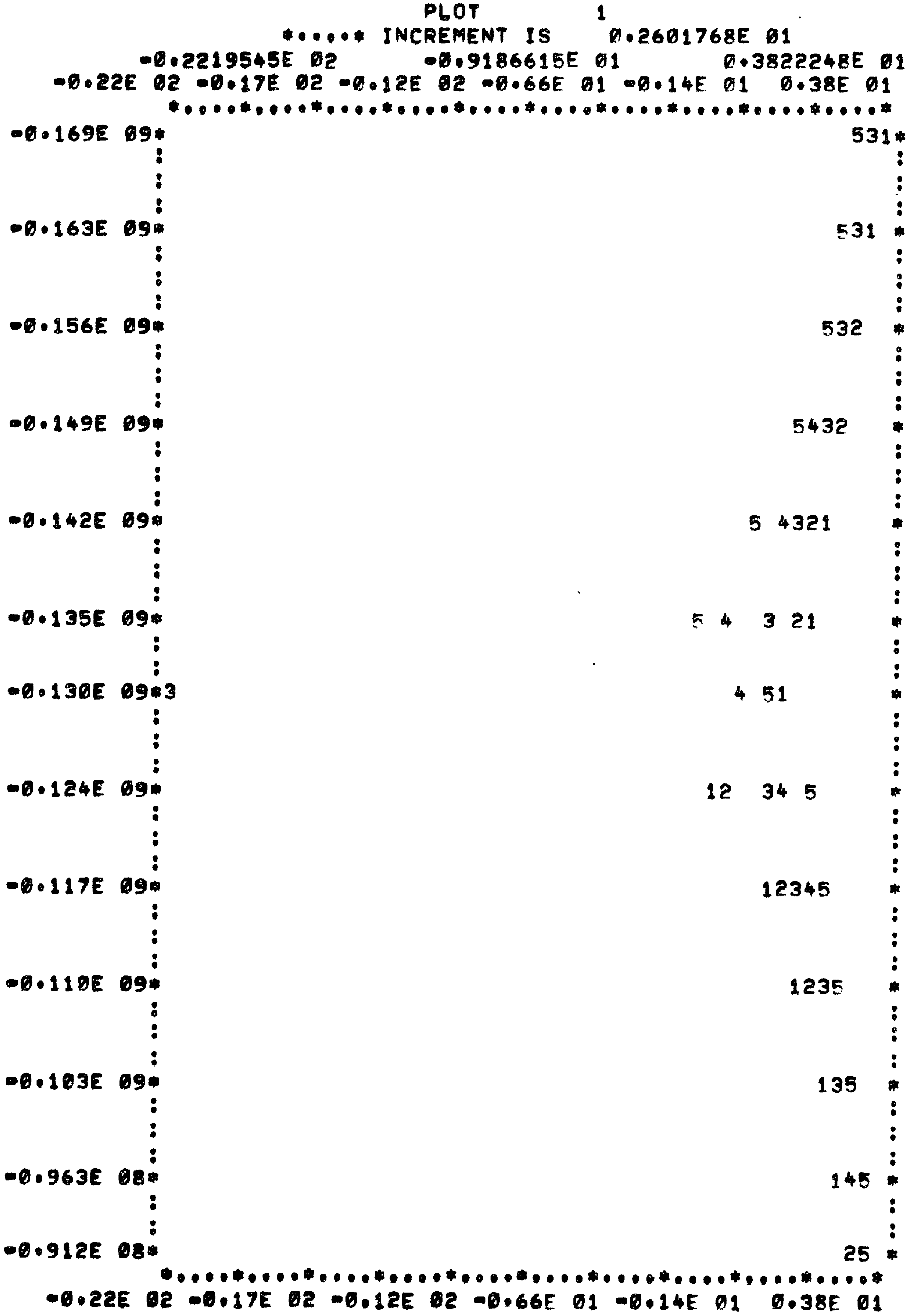


FIGURE 5.4. Five horizontal slices of Figure 5.3 (top to bottom)

```
                              PLOT          1
                     *.....* INCREMENT IS     0.2601768E 01
        -0.2219545E 02          -0.9186615E 01            0.3822248E 01
   -0.22E 02 -0.17E 02 -0.12E 02 -0.66E 01 -0.14E 01  0.38E 01
             *....*....*....*....*....*....*....*....*....*....*
0.237E 11*                                              34  25 1*
         :                                                      :
0.255E 11*                                             3   4 251 *
         :                                                      :
0.272E 11*                                            3    4 251 *
         :                                                      :
0.290E 11*                                           3     4 251 *
         :                                                      :
0.308E 11*                                         3         4251 *
         :                                                      :
0.325E 11*                                      3           4   5 *
         :                                                      :
0.339E 11*3                                                 4   5 *
         :                                                      :
0.356E 11*                                      3           4 15 *
         :                                                      :
0.374E 11*                                         3         24 15 *
         :                                                      :
0.392E 11*                                           3      2 415 *
         :                                                      :
0.409E 11*                                            3     2 415 *
         :                                                      :
0.427E 11*                                             3 2   415 *
         :                                                      :
0.440E 11*                                             3 2   415 *
          *....*....*....*....*....*....*....*....*....*....*
   -0.22E 02 -0.17E 02 -0.12E 02 -0.66E 01 -0.14E 01  0.38E 01
```

FIGURE 5.5. Five Vertical Slices of Figure 5.3. (left to right).

Before the description of those figures, one point related to the noise level must be explained. The degree of process noise is the same for the two state equations. (W_1 and W_2 in equation (4.2)). The level of measurement noise, however, is different for the two variables observed (V_1 and V_2 in equation (4.3)). There was one error in the preparation of the data and the maximum value of the velocity, v, in the noiseless manuevering trial, which is used to generate noisy data was taken 10 times **larger** than its actual value. Thus, all the noisy data used in the identification of the linear model are affected by this error. The measurement noise will, therefore, be represented by two numbers, each one associated to one observed variable.

Figure 5.6 shows the noisy sea trial data for 1% $\underline{W}$ and (5., 0.5)% $\underline{V}$. The resulting contour is shown in Figures 5.7 and 5.8. From these figures it is noticed that although the identified values of $\underline{P}$A1 and $\underline{P}$A2 are still $\underline{P}$A1* and $\underline{P}$A2*, the minimum is much less remarkable. The closeness of the model to the system is C(p*) = 1.247 compared to C($\underline{p}$*) = -22.18 for the noiseless case. At the same time, the maximum value of C(p) changed slightly from 3.822 to 3.878. This fact may be observed more clearly in Figures 5.9 and 5.10.

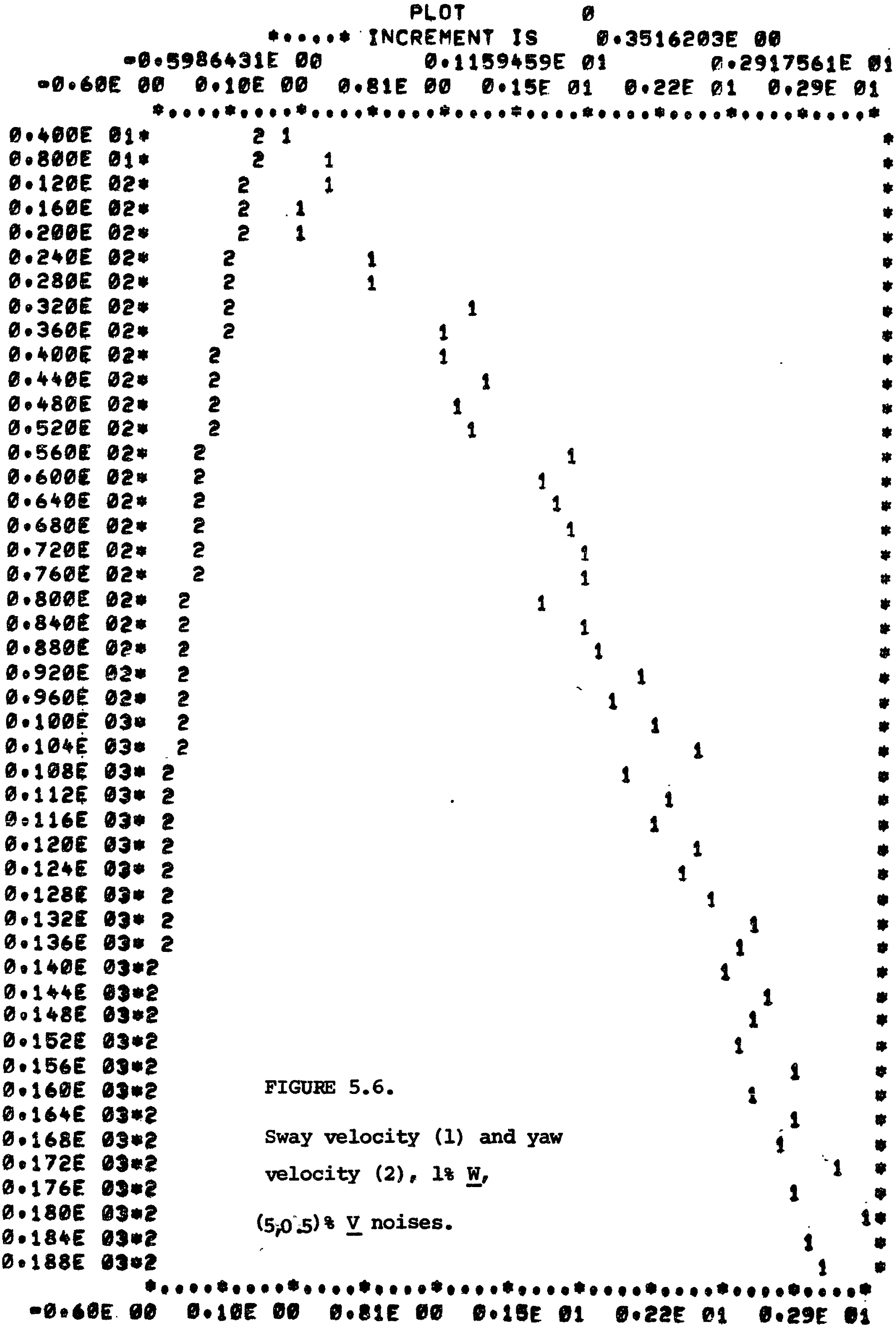


FIGURE 5.6.

Sway velocity (1) and yaw velocity (2), 1% W, (5,0.5)% V noises.

```
*****************************************************
*              CONTOUR   2  PARAMETERS              *
* X RANGE : 0.2370E 11 TO  0.4402E 11 DX= 0.4417E 09*
* Y RANGE :-0.1694E 09 TO -0.9124E 08 DY= 0.1564E 07*
* Z DOMAIN: 0.1246E 01 TO  0.3878E 01 DZ= 0.1316E 00*
* Z DOMAINS FOR THE CONTOURS :MAX VALUES FOR EACH   *
* NO. 1 0.1247E 01 NO. 8 0.2168E 01 NO.15 0.3089E 01*
* NO. 2 0.1379E 01 NO. 9 0.2300E 01 NO.16 0.3221E 01*
* NO. 3 0.1510E 01 NO.10 0.2431E 01 NO.17 0.3353E 01*
* NO. 4 0.1642E 01 NO.11 0.2563E 01 NO.18 0.3484E 01*
* NO. 5 0.1773E 01 NO.12 0.2695E 01 NO.19 0.3616E 01*
* NO. 6 0.1905E 01 NO.13 0.2826E 01 NO.20 0.3747E 01*
* NO. 7 0.2037E 01 NO.14 0.2958E 01 NO.21 0.3878E 01*
*****************************************************
```

FIGURE 5.7. Numerical Values of Contours in Figure 5.8.

```
                         CONTOUR         2
                  *.....* INCREMENT IS    0.7820275E 07
       -0.1694394E 09       -0.1303380E 09        -0.9123664E 08
  -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
            *....*....*....*....*....*....*....*....*....*....*
0.237E 11*M    M    L    L    K    K    J    J    H    H    G    G    F*
         :                                                            :
0.255E 11*K    K    J    J    H    H    G    F    F    E    D    D    C*
         :                                                            :
0.272E 11*H    G    G    F    E    E    D    C    C    B    A    A    9*
         :                                                            :
0.290E 11*E    D    D    C    B    B    A    9    8    8    7    6    6*
         :                                                            :
0.308E 11*B    A    9    9    8    7    6    6    5    4    4    3    3*
         :                                                            :
0.325E 11*7    6    6    5    4    3    3    2    2    2    2    2    2*
         :                                                            :
0.339E 11*4    3    2    2    2    2    1    2    2    2    3    3    4*
         :                                                            :
0.356E 11*2    2    2    2    2    3    3    4    4    5    6    6    7*
         :                                                            :
0.374E 11*3    4    4    5    6    6    7    7    8    9    9    A    A*
         :                                                            :
0.392E 11*7    7    8    9    9    A    A    B    B    C    C    D    D*
         :                                                            :
0.409E 11*B    B    C    C    D    D    E    E    E    F    F    G    G*
         :                                                            :
0.427E 11*E    E    F    F    G    G    G    H    H    H    J    J    J*
         :                                                            :
0.440E 11*H    H    H    J    J    J    K    K    K    L    L    L    L*
            *....*....*....*....*....*....*....*....*....*....*
  -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
```

Figure 5.8. $(I_z - N_r)$ and N_δ contours, 1% $\underline{W}$ (5,05)% $\underline{V}$ noises, linear model.

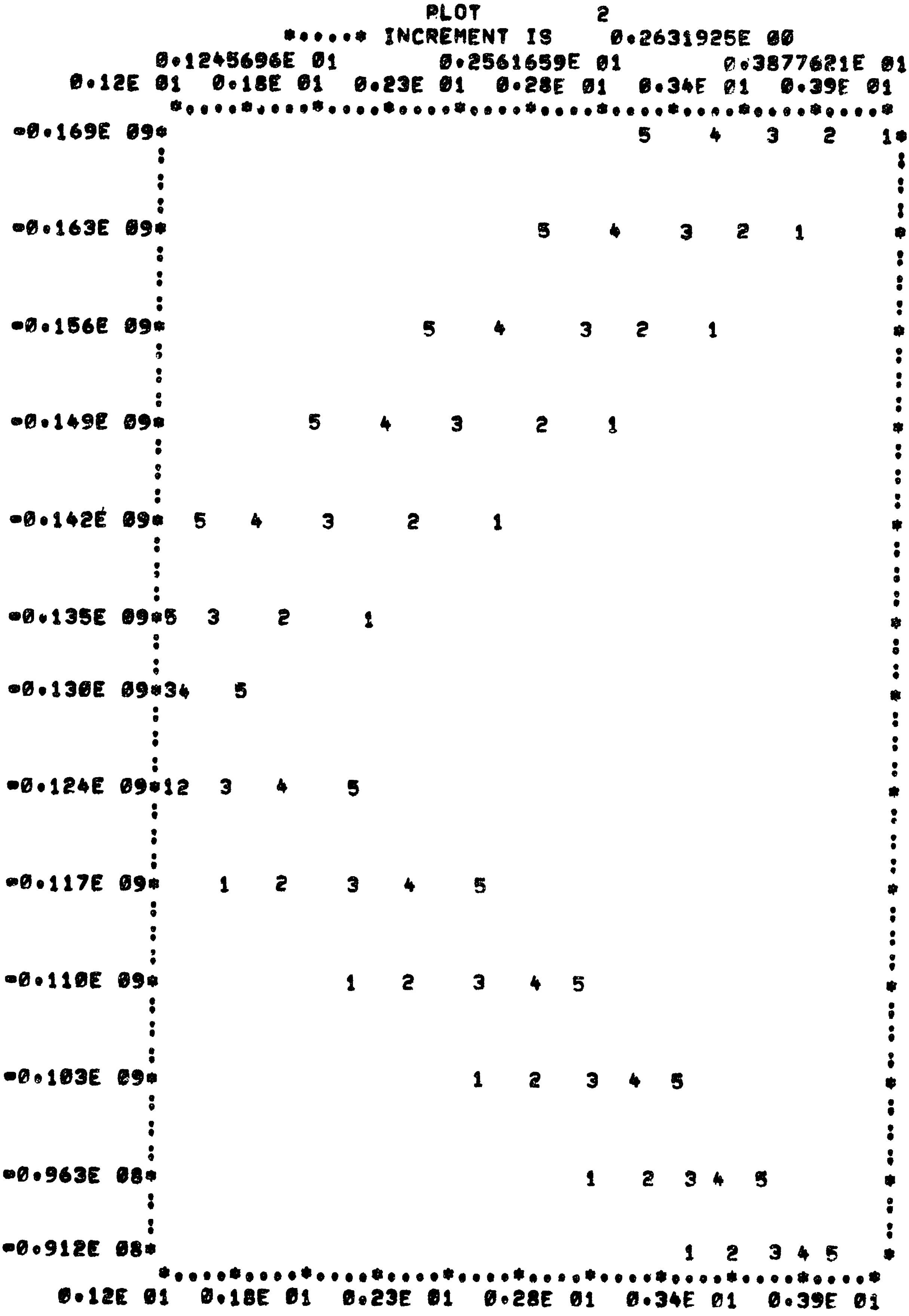


FIGURE 5.9. Five Horizontal Slices of Figure 5.8 (top to bottom)

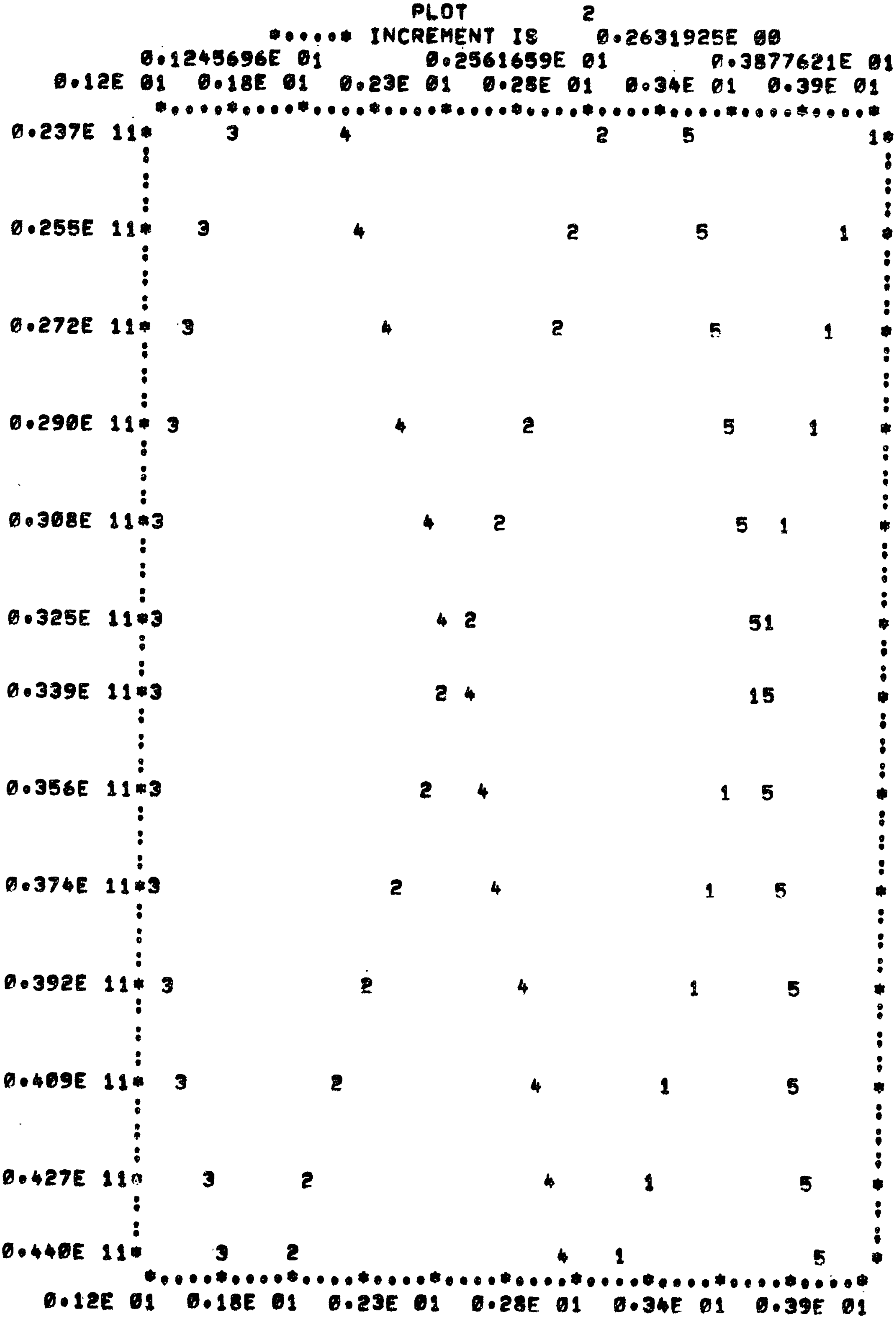


FIGURE 5.10. Five Vertical Slices of Figure 5.8 (left to right).

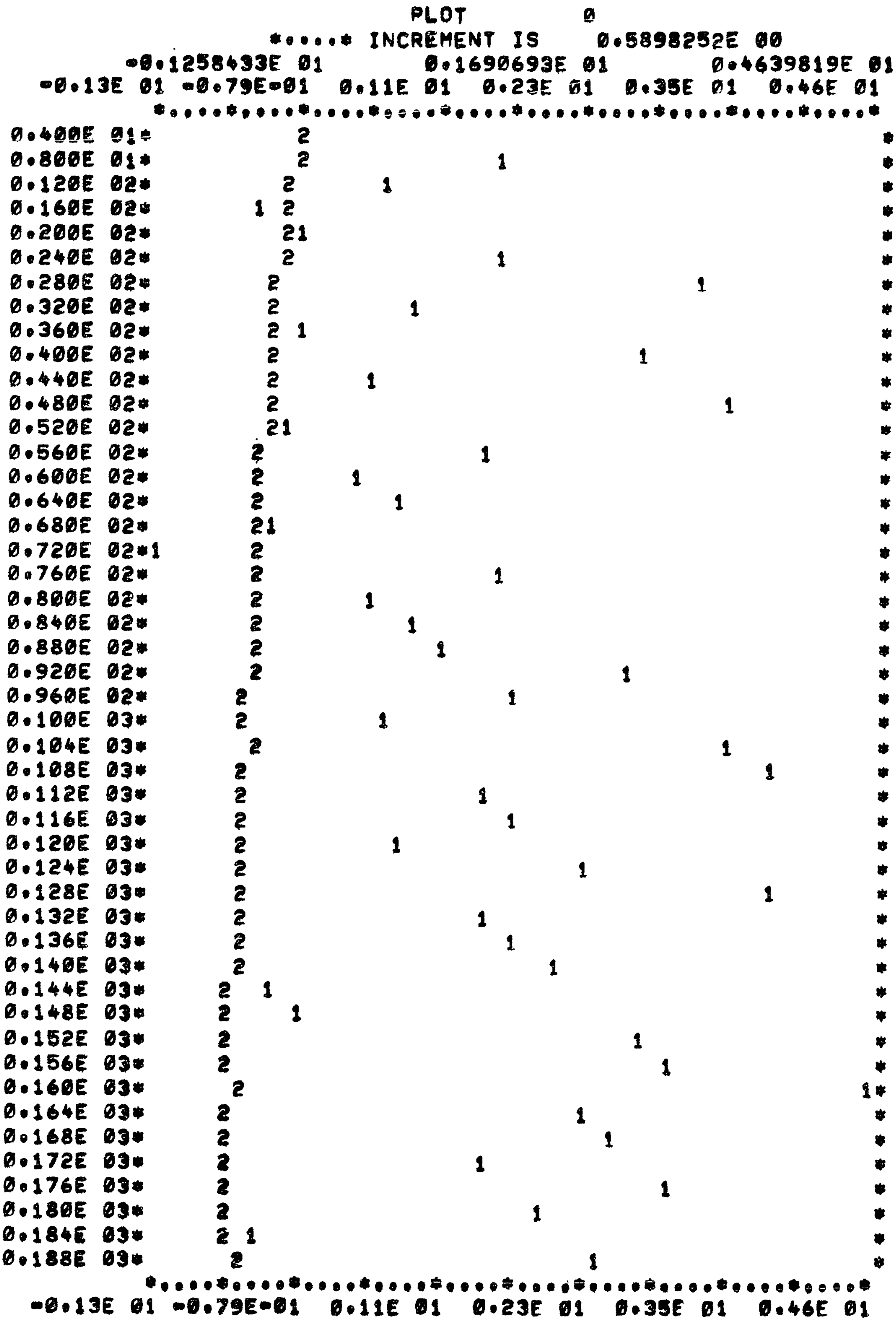


FIGURE 5.11. Sway velocity (1) and yaw velocity (2), 1% W (50,5)% V noises, linear model.

Figure 5.11 shows the noisy sea trial data for 1% $\underline{w}$ (50.,5.)% $\underline{V}$. Comparing Figures 5.11 and 5.1, it is noticed the effect of the high degree of measurement noise. The resulting contour presented in Figure 5.12 shows that the $\underline{w}$ and $\underline{v}$ noises greatly affect the identifiability of the parameters. The most significant difference is that the minimum values of the noisy contour are shifted away from the known "true" parameters to a new pair of values. This shift is most likely due to the biases generated by the two noise processes and has the effect of making the parameters unidentifiable if 88% or better accuracy is required. In addition, if 88% accuracy is acceptable, then the closeness of the model to the system is C (p*) = 5.164 whereas it was C(p*) = 22.18 for the noiseless case. It may be seen from Figure 5.12 that the parameter $\underline{P}$A2 (N_δ) is more sensitive to the noises effect while the parameter $\underline{P}$A1 (I_z-N_r) is still well identifiable. This fact is shown clearly in Figures 5.13 and 5.14.

The same basic identifiability characteristics are exhibited by the contours shown in Figures 5.16 and 5.18. They are respectively the resulting contours for the 10% $\underline{W}$, (5., 0.5)% $\underline{V}$ noisy sea trial data shown in Figure 5.15 and 10% $\underline{W}$, (50., 5)% $\underline{V}$ noisy sea trial data shown in Figure 5.17. In the first case, the identifiability of the parameters is only slightly affected,but in the second case, it is quite degraded.

CONTOUR 3

..... INCREMENT IS 0.7820275E 07

-0.1694394E 09 -0.1303380E 09 -0.9123664E 08

	-0.17E 09		-0.15E 09		-0.14E 09		-0.12E 09		-0.11E 09		-0.91E 08	
0.237E 11	M	L	K	J	H	G	F	E	D	D	C	B	A
0.255E 11	H	G	F	E	D	C	C	B	A	9	9	8	8
0.272E 11	D	C	B	A	A	9	8	8	7	7	6	6	5
0.290E 11	9	9	8	7	7	6	6	5	5	4	4	4	3
0.308E 11	6	6	5	5	5	4	4	3	3	3	3	2	2
0.325E 11	4	4	3	3	3	3	2	2	2	2	2	2	2
0.339E 11	3	2	2	2	2	2	2	2	2	2	2	2	2
0.356E 11	2	2	2	1	1	2	2	2	2	2	2	2	2
0.374E 11	2	2	2	2	2	2	2	2	2	3	3	3	3
0.392E 11	2	2	2	3	3	3	3	4	4	4	4	5	5
0.409E 11	3	4	4	4	5	5	5	6	6	6	7	7	7
0.427E 11	5	6	6	7	7	7	8	8	9	9	9	A	A
0.440E 11	8	9	9	9	A	A	B	B	C	C	D	D	E

-0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08

FIGURE 5.12. (I_z-N_r) and N_δ contours, 1% W, (50,5)% V noise, linear model.

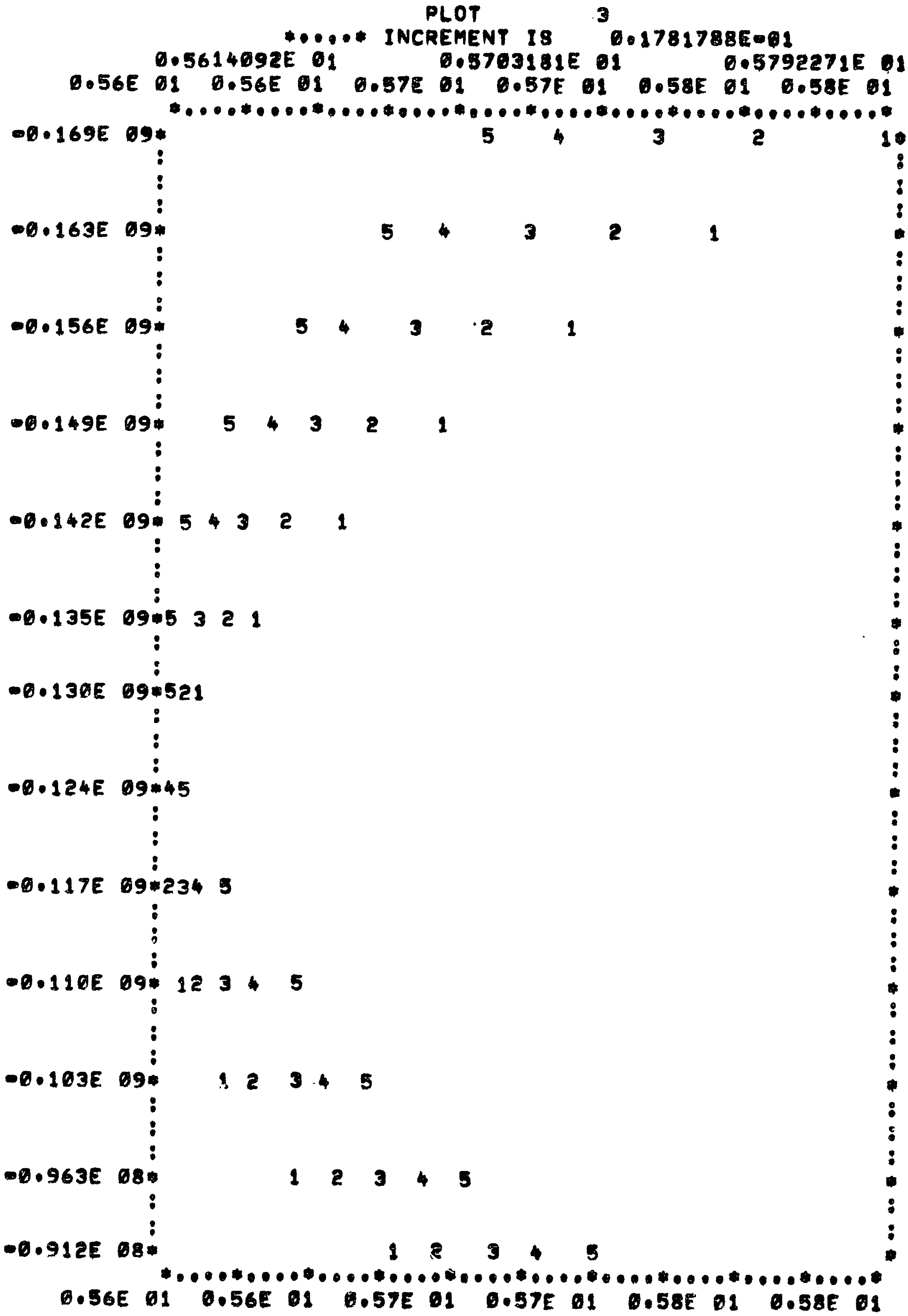


FIGURE 5.13. Five horizontal slices of Figure 5.12 (top to bottom)

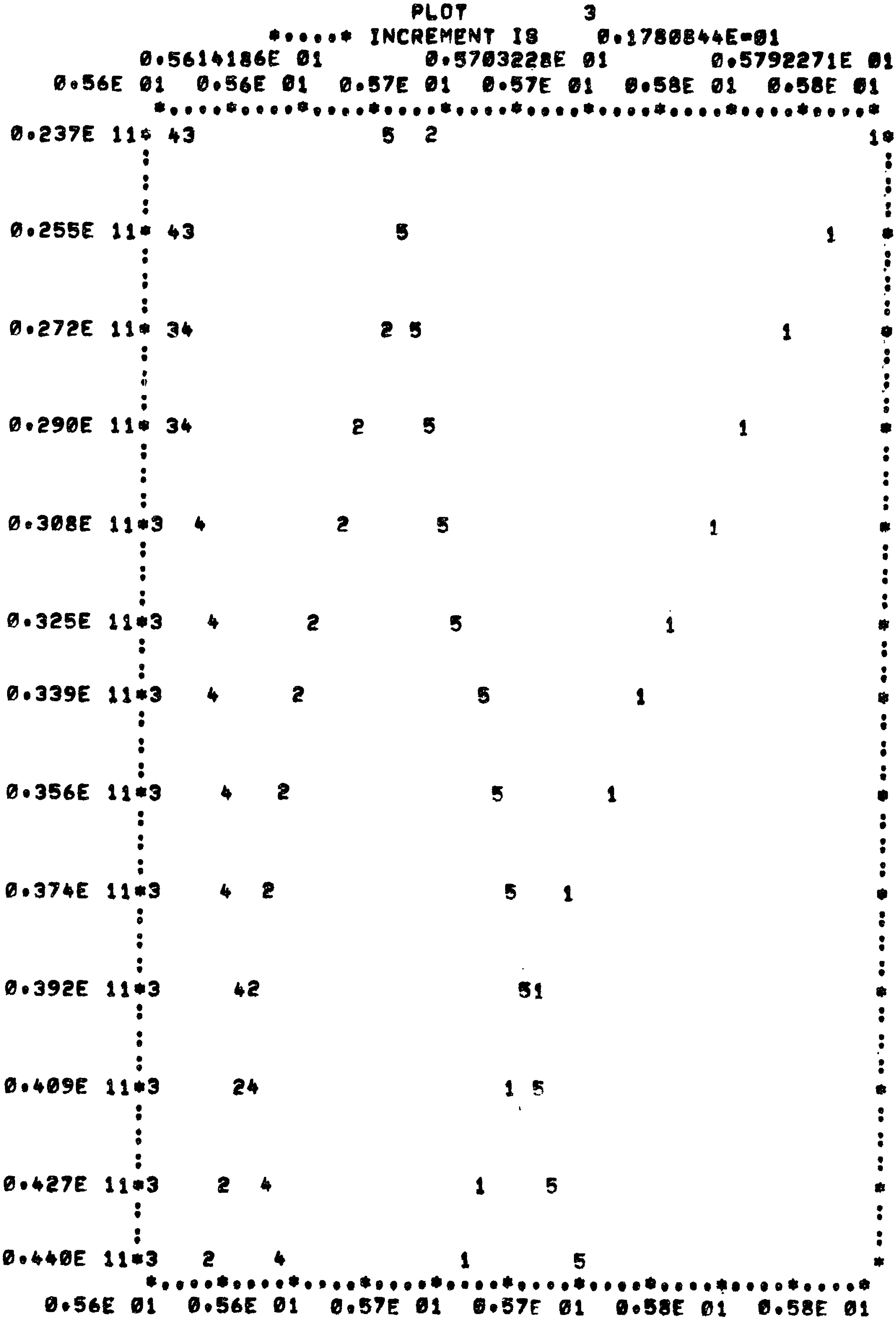


FIGURE 5.14. Five Vertical Slices of Figure 5.12 (left to right)

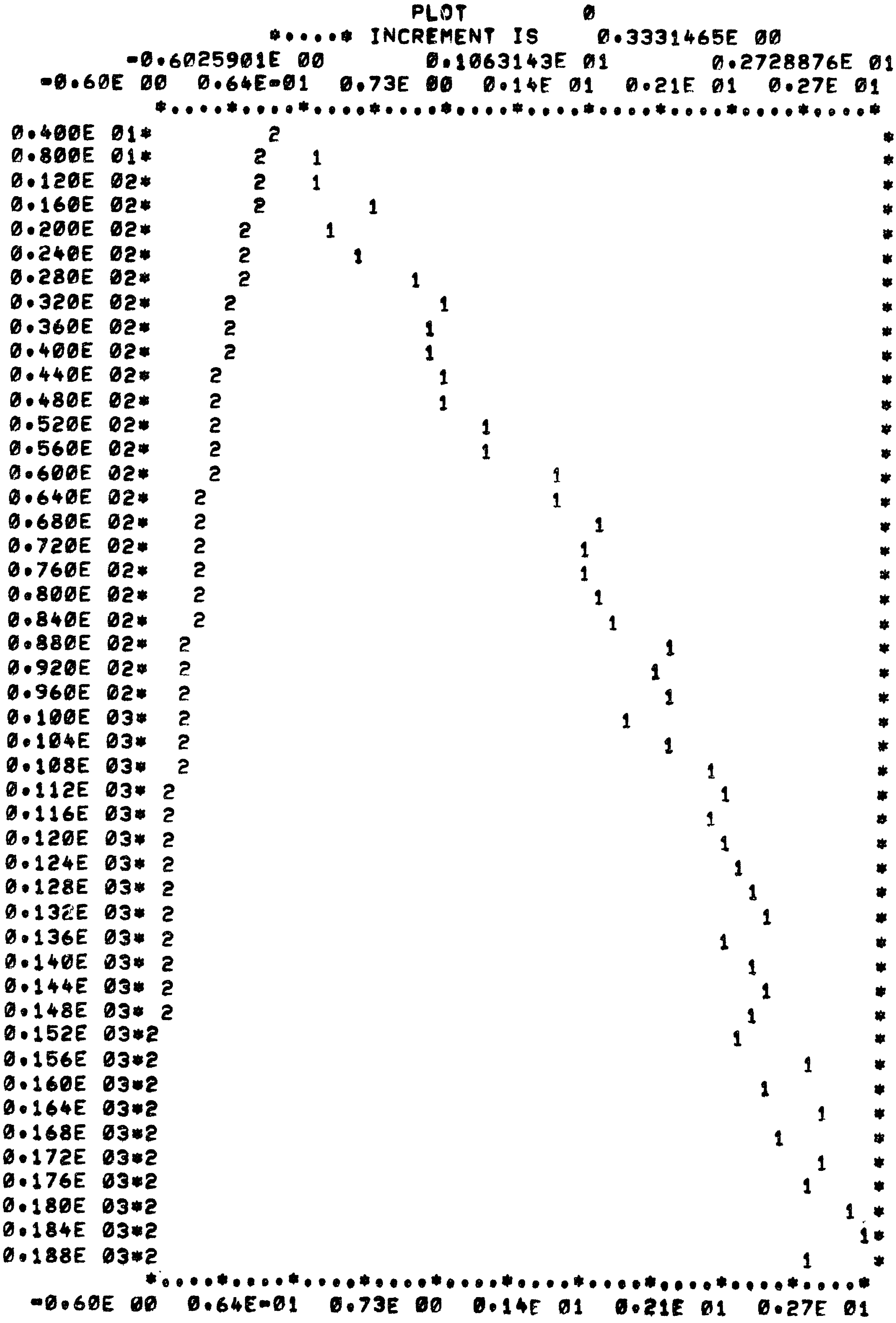


FIGURE 5.15. Sway Velocity (1) and yaw velocity (2). 10% W (5.,0.5)% V noise, linear model.

```
                         CONTOUR          4
                   *.....* INCREMENT IS    0.7820275E 07
          -0.1694394E 09       -0.1303380E 09       -0.9123664E 08
    -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
               *....*....*....*....*....*....*....*....*....*....*....*....*
0.237E 11*M    M    M    L    L    K    K    J    J    H    H    G    G*
         :                                                            :
0.255E 11*L    K    K    J    J    H    G    G    F    F    E    E    D*
         :                                                            :
0.272E 11*H    H    G    G    F    F    E    D    D    C    B    B    A*
         :                                                            :
0.290E 11*F    E    E    D    C    C    B    A    9    9    8    7    7*
         :                                                            :
0.308E 11*C    B    A    A    9    8    7    7    6    5    5    4    3*
         :                                                            :
0.325E 11*8    8    7    6    5    5    4    3    3    2    2    2    2*
         :                                                            :
0.339E 11*5    4    3    3    2    2    2    1    2    2    2    2    3*
         :                                                            :
0.356E 11*2    2    2    2    2    2    3    3    4    4    5    5    6*
         :                                                            :
0.374E 11*3    3    4    4    5    5    6    6    7    8    8    9    9*
         :                                                            :
0.392E 11*6    7    7    8    8    9    9    A    A    B    C    C    C*
         :                                                            :
0.409E 11*A    A    B    B    C    C    D    D    D    E    E    F    F*
         :                                                            :
0.427E 11*D    D    E    E    F    F    F    G    G    G    H    H    H*
         :                                                            :
0.440E 11*G    G    G    H    H    H    J    J    J    K    K    K    K*
               *....*....*....*....*....*....*....*....*....*....*....*....*
    -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
```

FIGURE 5.16 $(I_z - N_r)$ and N_δ contours 10% $\underline{W}$, (5., 0.5)% $\underline{V}$ noises, linear model.

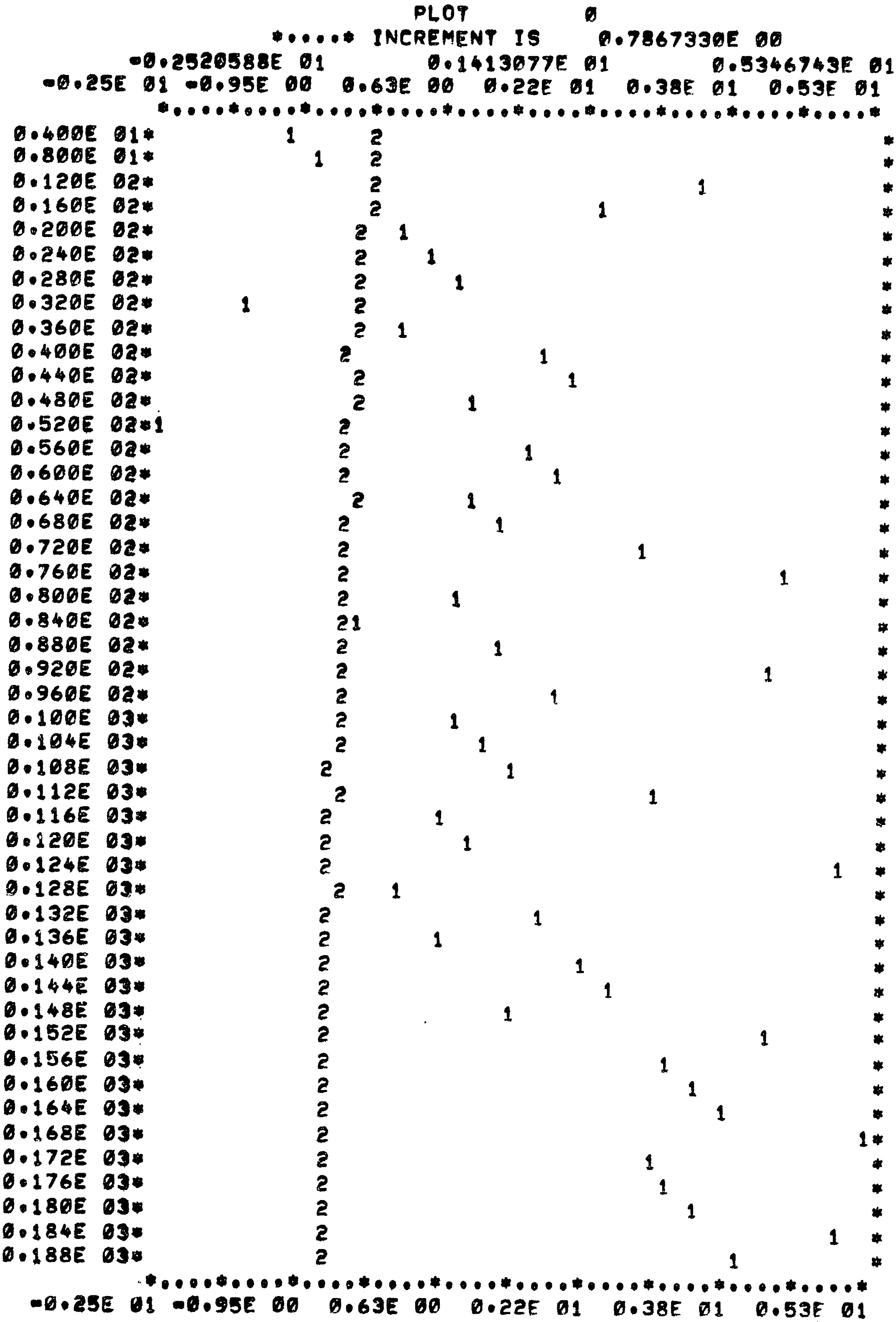


FIGURE 5.17. Sway Velocity (1) and yaw velocity (2). 10% $\underline{W}$ (50, 0.5)% $\underline{V}$ noises; linear model.

```
                              CONTOUR          5
                      *.....* INCREMENT IS    0.7820275E 07
          -0.1694394E 09        -0.1303380E 09        -0.9123664E 08
   -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
              *....*....*....*....*....*....*....*....*....*....*....*....*
0.237E 11*A    9    8    7    7    6    6    5    5    4    4    4    3*
         :                                                             :
         :                                                             :
0.255E 11*7    6    6    5    5    4    4    4    3    3    3    2    2*
         :                                                             :
         :                                                             :
0.272E 11*5    4    4    4    3    3    3    2    2    2    2    2    2*
         :                                                             :
         :                                                             :
0.290E 11*3    3    3    2    2    2    2    2    2    2    2    1    1*
         :                                                             :
         :                                                             :
0.308E 11*2    2    2    2    2    2    2    2    2    2    2    2    2*
         :                                                             :
         :                                                             :
0.325E 11*2    2    2    2    2    2    2    2    2    2    2    3    3*
         :                                                             :
0.339E 11*2    2    2    2    2    3    3    3    3    3    4    4    4*
         :                                                             :
         :                                                             :
0.356E 11*3    3    3    3    4    4    4    4    5    5    5    6    6*
         :                                                             :
         :                                                             :
0.374E 11*4    5    5    5    5    6    6    6    7    7    8    8    8*
         :                                                             :
         :                                                             :
0.392E 11*6    7    7    7    8    8    9    9    9    A    A    B    B*
         :                                                             :
         :                                                             :
0.409E 11*9    9    A    A    A    B    B    C    C    D    D    E    E*
         :                                                             :
         :                                                             :
0.427E 11*C    C    D    D    E    E    F    F    G    G    H    H    J*
         :                                                             :
0.440E 11*F    G    G    H    H    J    J    K    K    L    L    M    M*
              *....*....*....*....*....*....*....*....*....*....*....*....*
   -0.17E 09 -0.15E 09 -0.14E 09 -0.12E 09 -0.11E 09 -0.91E 08
```

FIGURE 5.18 (I_z-$N_{\dot{r}}$) and $N_{\dot{\delta}}$ contours; 10% $\underline{W}$, (50., 5.) % $\underline{V}$ noises; linear model.

The essential results of all the contours previously discussed and presented in Table 5.1. There are some conclusions relative to the identifiability of parameters that can be drawn from this table. The coefficient $(I_z - N_{\dot{r}})$ in any case is identified with a better accuracy than N_δ, expressing it is more identifiable. For the highest degrees of noise, 10% $\underline{W}$ and (50., 5.)% $\underline{V}$ $(I_z - N_{\dot{r}})$ cannot be identified to within ±85% accuracy and N_δ cannot be identified to within ± 75% accuracy.

TABLE 5.1

Noise			$\underline{PA1} \equiv (I_z - N_{\dot{r}})$	$PA2 = N_\delta$			
%W	% V		$\underline{PA1}^*$=33.86E 9	PA2*=-13.03E 7	C($\underline{p}$*)	$C_{max}(\underline{p})$	Comment
0	0	0	33.86E 9	-13.03E 7	-22.18	3.322	Very gd. ident.
1	5	0.5	33.86E 9	-13.03E 7	1.247	3.378	Good
1.	50	5	35.60E 9	-14.65E 9	5.6414	5.792	
10	5	0.5	33.86E 9	-12.38E 9	1.105	3.944	
10	50	0.5	29.00E 9	- 9.48E 9	5.828	6.002	

```
                              CONTOUR          1
                      *.....* INCREMENT IS    0.2965381E 05
          0.3459610E 06         0.4942301E 06         0.6424990E 06
    0.35E 06  0.41E 06  0.46E 06  0.52E 06  0.58E 06  0.64E 06
              *.....*.....*....@.....*.....*.....*.....*.....*.....*.....*.....*
0.159E 07*K    L    L    L    L    M    M    M    M    M    M    M    M*
0.170E 07*K    K    K    L    L    L    L    M    M    M    M    M    M*
0.182E 07*K    K    K    K    L    L    L    L    M    M    M    M    M*
0.194E 07*L    K    J    J    K    L    L    L    L    L    M    M    M*
0.206E 07*L    L    K    J    J    K    K    L    L    L    L    M    M*
0.218E 07*L    L    L    K    J    H    K    K    L    L    L    L    M*
0.227E 07*M    L    L    L    K    J    1    J    K    L    L    L    L*
0.238E 07*M    M    L    L    L    K    K    H    J    K    K    L    L*
0.250E 07*M    M    M    L    L    L    K    K    J    J    K    K    L*
0.262E 07*M    M    M    M    L    L    L    K    K    J    J    K    K*
0.274E 07*M    M    M    M    M    L    L    L    L    K    J    J    K*
0.286E 07*M    M    M    M    M    M    L    L    L    L    K    K    J*
0.294E 07*M    M    M    M    M    M    M    L    L    L    L    K    K*
              *.....*.....*....@.....*.....*.....*.....*.....*.....*.....*.....*
    0.35E 06  0.41E 06  0.46E 06  0.52E 06  0.58E 06  0.64E 06
```

FIGURE 5.19 $(m-Y_v)$ and Y_δ contours no noise; linear model.

The same study carried out for the coefficients $(I_z - N_{\dot{r}}), N_\delta$ was applied to the other linear parameters. The essential results of the contours are presented in tabular form. Only the contour for the noiseless sea trial data are shown.

Figure 5.19 shows contour of C(p) for the parameters $(m - Y_{\dot{v}})$ (PA1) and Y_δ (PA2), corresponding to the noiseless sea trial data of Figure 5.1. The identifiability of both parameters for zero noise data is also very remarkable. The essential results of this and other contours generated from noisy sea trial data are presented in Table 5.2. In general, the coefficient $(m - Y_{\dot{v}})$ is identified with better accuracy than Y_δ. For the highest degrees of noise, the parameters cannot be identified if 80% or better accuracy is required.

TABLE 5.2

Noise % W	%	V	PA1 ≡ $(M - Y_{\dot{v}})$ PA1* = 22.65E 5	PA2 ≡ Y_δ PA2* = 49.42 E 4	C(p*)	C_{max}(p)	Comment
0	0	0	22.65E 5	49.42 E 4	-30.69	4.142	
1	10	1	22.65E 5	49.42 E 4	2.397	4.281	
1.	100	10	27.40E 5	64.00 E 4	7.086	7.145	
10.	10	1	24.40E 5	53.20 E 4	2.610	4.305	
10.	100	10	23.80E 5	44.40 E 4	7.236	7.281	

```
                              CONTOUR          1
                    *.....* INCREMENT IS    0.5864103E 06
         -0.1270556E 08        -0.9773511E 07        -0.6841459E 07
   -0.13E 08 -0.12E 08 -0.10E 08 -0.92E 07 -0.80E 07 -0.68E 07
              *....*....*....@....*....*....*....*....*....*....*
-0.241E 08*M      M      M      L      L      K      H      K      L*
          :                                                         :
-0.226E 08*M      M      M      L      L      K      J      K      L*
          :                                                         :
-0.212E 08*M      M      L      L      K      H      K      K      L*
          :                                                         :
-0.197E 08*M      L      L      K      J      J      K      L      L*
          :                                                         :
-0.185E 08*M      L      L      K      1      K      K      L      L*
          :                                                         :
-0.171E 08*L      L      K      J      J      K      L      L      L*
          :                                                         :
-0.156E 08*L      K      J      J      K      K      L      L      L*
          :                                                         :
-0.142E 08*K      J      H      K      K      L      L      L      L*
          :                                                         :
-0.130E 08*J      J      K      K      L      L      L      L      L*
              *....*....*....@....*....*....*....*....*....*....*
   -0.13E 08 -0.12E 08 -0.10E 08 -0.92E 07 -0.80E 07 -0.68E 07
```

FIGURE 5.20 (Y_r-mu) and N_v contours, no noise, linear model.

Figure 5.20 and Table 5.3 are concerned to the identification of the parameters (Y_r-mu) (PA1) and N_v (PA2). Figure 5.20 shows the contour of C(**p**) for the noiseless sea trial data of Figure 5.1. The contours of C(**p**) are plotted using a smaller number of points than it was used in the previous contours. It was decided to decrease the number of points in order to reduce the computation time.

Table 5.3. presents the essential results of the model reference contours for (Y_r-mu) and N_v.

For the 1% W, (50.5)% V noisy sea trial data there are serveral points of minimum in the contour. In this case, as mentioned before, the pairs of values identified do not give any information about the true parameter values.

TABLE 5.3.

Noise			PA1≡ (Y_r-mu)	PA2 ≡ N_v			
% W	%	V	PA1*=18.51 E 6	PA2*=-97.73 E 5	C(p*)	C_{max}(p)	Comment
0	0	0	-18.51 E 6	-97.73 E 5	-30.70	7.052	
1	5	0.5	-18.51 E 6	-97.73 E 5	1.249	7.050	
1.	50.	5.			5.616	7.287	Many points
10.	5.	0.5	-18.51 E 6	-97.73 E 5	1.123	7.063	
10.	50.	5.	-22.60 E 6	-104.80 E 5	5.827	7.207	

```
                              CONTOUR          1
                        ****** INCREMENT IS    0.1950618E 09
            -0.4226339E 10        -0.3251030E 10        -0.2275721E 10
     -0.42E 10 -0.38E 10 -0.34E 10 -0.31E 10 -0.27E 10 -0.23E 10
                *....*....*....@....*....*....*....*....*....*....*
-0.106E 06*K       K      K      K      K       J      H      F      J*
-0.996E 05*K       K      K      K      J       J      F      H      K*
-0.932E 05*K       K      K      J      J       G      G      J      K*
-0.868E 05*K       K      J      J      H       F      J      K      K*
-0.815E 05*K       K      J      H      1       H      K      K      L*
-0.751E 05*J       J      H      D      H       J      K      L      L*
-0.688E 05*J       H      D      H      J       K      L      L      M*
-0.624E 05*G       G      J      K      K       L      L      M      M*
-0.571E 05*H       J      K      K      L       L      M      M      M*
                *....*....*....@....*....*....*....*....*....*....*
     -0.42E 10 -0.38E 10 -0.34E 10 -0.31E 10 -0.27E 10 -0.23E 10
```

FIGURE 5.21. Y_v and $(N_r - mx_g u)$ contours, no noise, linear model.

The next series of runs was conducted to study the identifiability of the coefficients Y_v (PA1) and $(N_r - mx_G u)$ (PA2). Figure 5.21 shows the contours of C(p) for the noiseless sea trial data of Figure 5.1. The essential results of model reference contours for these parameters are presented in Table 5.4.

It is noticed from Table 5.4. that for the most of the noisy cases, there is more than one minimum for the cost function. The values identified, however, seem to be biased by the noise and do not give information about the true values. The analysis of the corresponding contours shows that there is an approximately linear relation between Y_v and $(N_r - mx_G u)$. The identification of the parameters is not obtained within an accuracy greater than 80%.

TABLE 5.4.

Noise % W	%	V	PA1 ≡ Y_v PA1*=-81.51E 3	PA2≡$(N_r - mx_G u)$ PA2*=-32.51 E 8	C(p*)	$C_{max}(p)$	Comment
0	0	0	-81.51E 3	-32.51 E 8	-19.41	7.783	
1	5.	0.5			1.249	7.781	Many points
1	50.	5.			5.615	7.908	Many points
10	5.	0.5	-75.15E 3	-35.02 E 8	1.115	7.790	
10	50.	5.			5.828	7.883	Many points

```
                          CONTOUR          1
                    *.....* INCREMENT IS    0.1053598E 07
         -0.2282811E 08       -0.1756013E 08        -0.1229213E 08
   -0.23E 08 -0.21E 08 -0.19E 08 -0.17E 08 -0.14E 08 -0.12E 08
               *....*....*....*....*....*....*....*....*....*
-0.865E 07*M     M     M     M     M     M     M     M     M*
-0.813E 07*M     M     M     M     M     M     L     L     L*
-0.761E 07*M     M     L     L     L     L     L     K     K*
-0.709E 07*L     L     L     K     K     K     J     H     H*
-0.665E 07*K     K     J     H     1     H     J     K     K*
-0.613E 07*H     H     J     K     K     K     L     L     L*
-0.561E 07*K     K     L     L     L     L     L     M     M*
-0.509E 07*L     L     L     M     M     M     M     M     M*
-0.466E 07*M     M     M     M     M     M     M     M     M*
               *....*....*....*....*....*....*....*....*....*
   -0.23E 08 -0.21E 08 -0.19E 08 -0.17E 08 -0.14E 08 -0.12E 08
```

FIGURE 5.22. $(mx_G - Y_r)$ and $(mx_G - N_v)$ contours, no noise, linear model.

The essential data for the identification of $(mu_G - Y_{\dot{r}})$ (PA1) and $(mu_G - N_{\dot{v}})$ (PA2) are presented in Table 5.5. Figure 5.22 shows the contour of C(p) for the noiseless trial data of Figure 5.1. It is noticed from Table 5.5 that in most of the noisy cases, the identification of the parameters is not satisfactory. The coefficients are unidentifiable even for 10% W and (50.5) %V noisy data if 70% or better accuracy is required. These results are very poor if they are compared with the results obtained for the other parameters. It may be concluded that at least for the input selected, the coefficients $(mu_G - Y_{\dot{r}})$ and $(mu_G - N_{\dot{r}})$ exhibit identifiability characteristics poorer than the other linear parameters.

TABLE 5.5.

Noise % W	%	V	PA1 = $(mu_G - Y_{\dot{r}})$ PA1*=-66.52 E 5	PA2 = $(mu_G - N_{\dot{v}})$ PA2*=-17.56 E 6	C(p*)	C_{max}(p)	Comment
0	0	0	-66.52 E 5	-17.56 E 6	-29.22	8.500	
1.	5.	0.5	-56.10 E 5	-18.02 E 6	1.245	1.260	
1.	50.	5.	-86.50 E 5	-23.10 E 6	5.615	5.616	
10.	5.	0.5	-86.50 E 5	-23.10 E 6	1.104	1.155	
10.	50.	5.	-46.60 E 5	-12.29 E 6	5.838	5.841	

The results of model reference identification of the linear coefficients for the Mariner ship were presented in a series of figures and tables. The identification of most parameters is considerably biased for high degree of noise. In general, the identification of the parameters for noisy data is not good but it is probably caused by the high and heterogeneous measurement noise. It is not believed that the identifiability of the parameters may have been degraded by the choice of the input.

5.3. Extended Kalman Filtering for the Linear Model.

The extended Kalman filter technique was applied to the identification of all the linear hydrodynamic coefficients of the Mariner class ship. Unlike the model reference technique, the application of the approach required a larger number of decisions and involved some difficulties. Correspondingly, much more information was obtained concerning not only the identifiability of the parameters, but also some aspects of application of extended Kalman filtering.

The error made in the preparation of the data for the model reference identification was repeated here. The measurement noise, in consequence, is heterogeneous being 10 times larger for the observed variable v than for the variable r.

The first set of trials were conducted with a program that identifies simultaneously only two parameters while the others are kept constant. The coefficients intially studied were Y_v and Y_δ. The results of the extended Kalman filtering pass over the 1% $\underline{W}$, (5., 0.5)% $\underline{V}$ noisy data of Figure 5.6 are shown in the plots of Figures 5.23 through 5.26, and Table 6. These plots show how the noise is filtered out of the primary states v and r and how the parameter values for Y_v and Y_r are arrived at by the filter. Table 5.6. presents the true values for the parameters Y_v(6) Y_δ(8), the initial vaues (S.V) and the final values (F.V.) of the parameters estimate. These results show that the values identified by the filter have a very high accuracy. It was not considered necessary to make another pass of the filter over the sea trial data.

A series of extended Kalman filter runs with this simple program were made and the corresponding plots generated for the studies in this section. The results of these runs are described in tabular form. Table 5.7 shows the effect of the standard deviation (variance) in the initial estimate of the parameters. It is observed that larger is the variance in the initial estimate higher is the accuracy in the identified parameters.

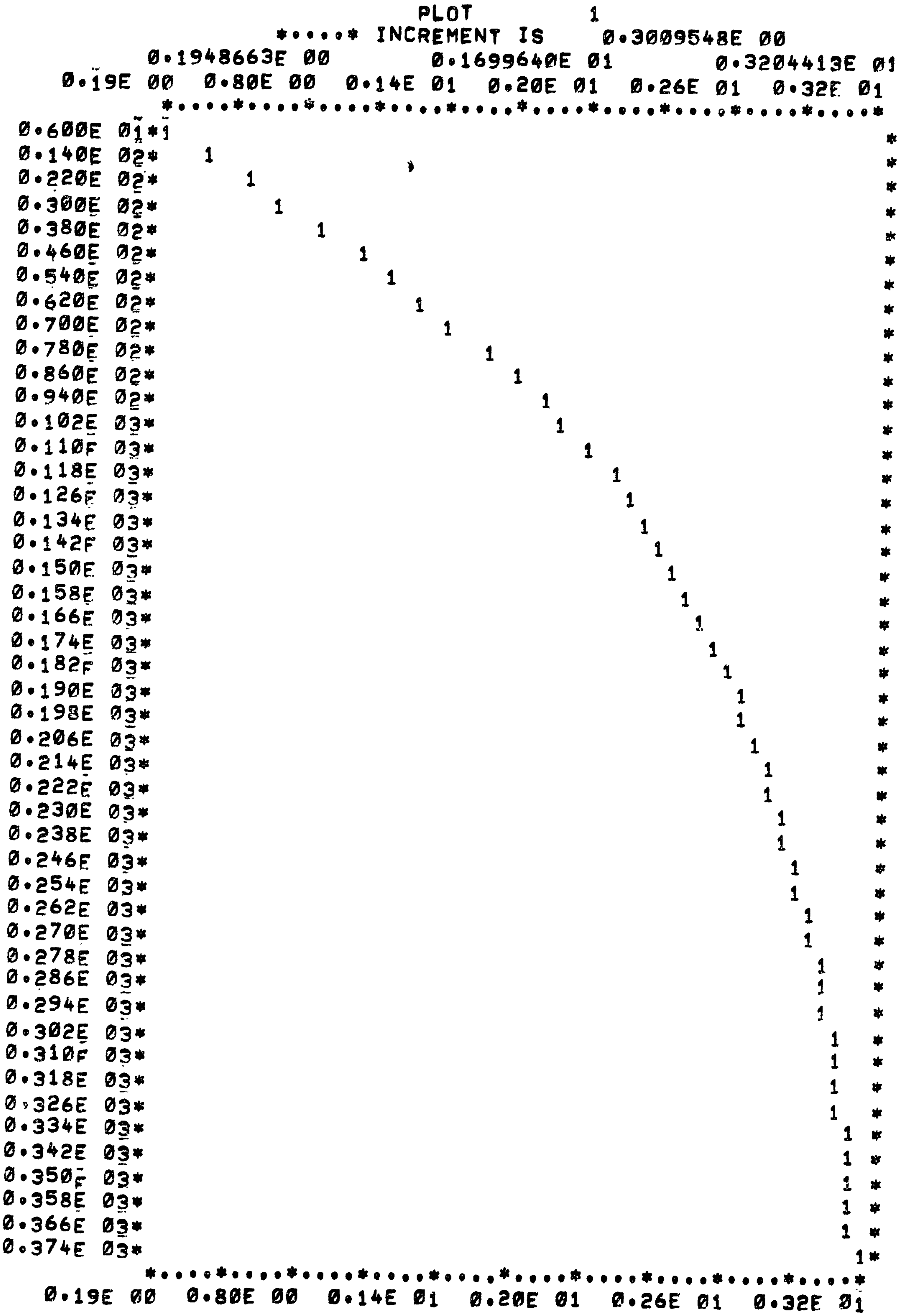


FIGURE 5.23. Kalman Filter Sway Velocity for Figure 5.6.

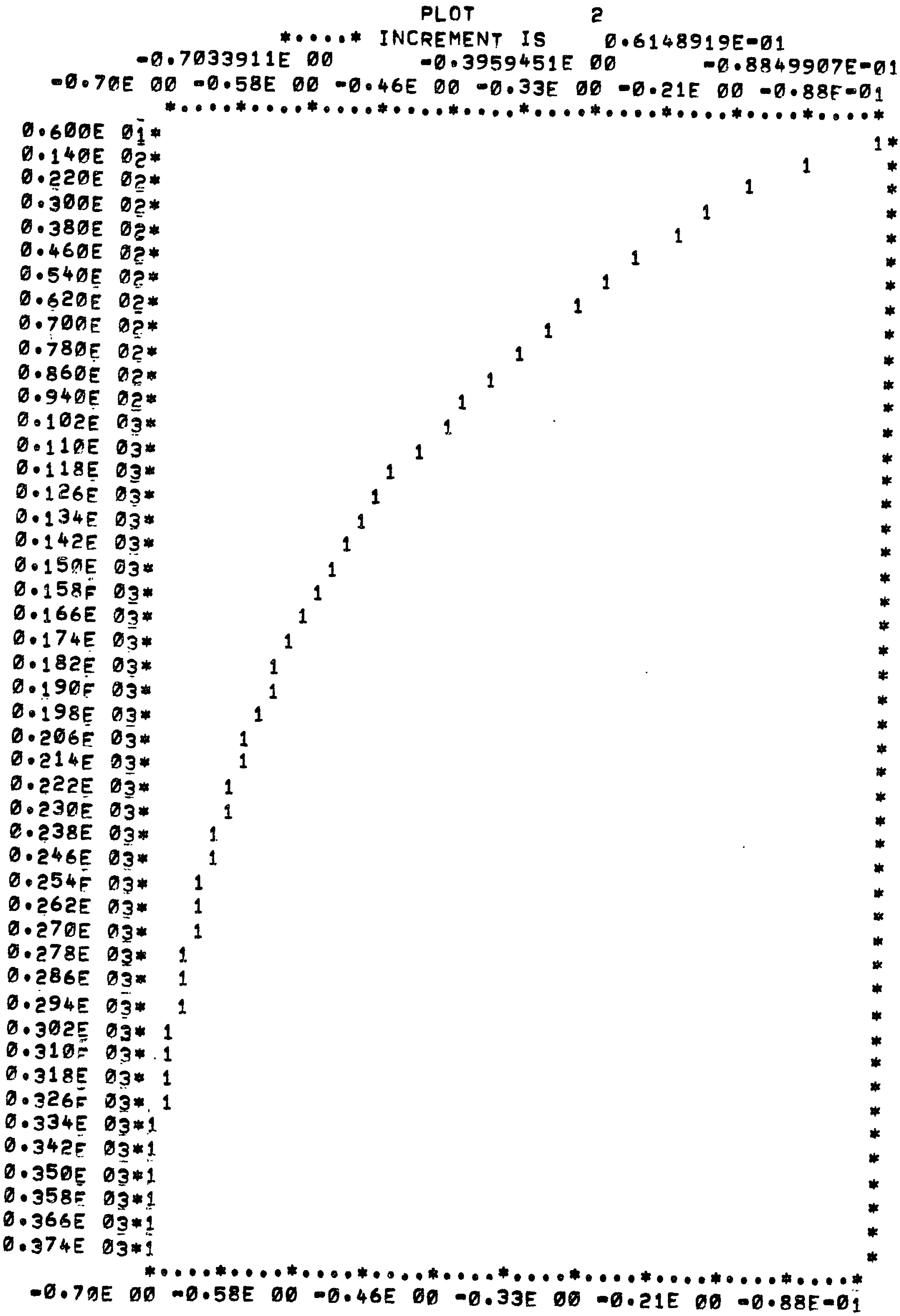


FIGURE 5.24. Kalman Filter Yaw Velocity for Figure 5.6.

```
                                 PLOT          3
                       *.....* INCREMENT IS    0.2971837E 04
          -0.8882856E 05        -0.7396938E 05          -0.5911020E 05
   -0.89E 05 -0.83E 05 -0.77E 05 -0.71E 05 -0.65E 05 -0.59E 05
          *.....*.....*.....*.....*.....*.....*.....*.....*.....*.....*
0.600E 01*                                                          1 *
0.140E 02*                                                           1*
0.220E 02*                               1                            *
0.300E 02*                  1                                         *
0.380E 02*1                                                           *
0.460E 02*            1                                               *
0.540E 02* 1                                                          *
0.620E 02*   1                                                        *
0.700E 02*  1                                                         *
0.780E 02*              1                                             *
0.860E 02*                1                                           *
0.940E 02*                  1                                         *
0.102E 03*                1                                           *
0.110E 03*                  1                                         *
0.118E 03*                  1                                         *
0.126E 03*                 1                                          *
0.134E 03*                1                                           *
0.142E 03*               1                                            *
0.150E 03*               1                                            *
0.158E 03*               1                                            *
0.166E 03*              1                                             *
0.174E 03*              1                                             *
0.182E 03*              1                                             *
0.190E 03*              1                                             *
0.198E 03*              1                                             *
0.206E 03*              1                                             *
0.214E 03*              1                                             *
0.222E 03*             1                                              *
0.230E 03*              1                                             *
0.238E 03*              1                                             *
0.246E 03*             1                                              *
0.254E 03*              1                                             *
0.262E 03*              1                                             *
0.270E 03*              1                                             *
0.278E 03*              1                                             *
0.286E 03*              1                                             *
0.294E 03*              1                                             *
0.302E 03*              1                                             *
0.310E 03*              1                                             *
0.318E 03*              1                                             *
0.326E 03*              1                                             *
0.334E 03*              1                                             *
0.342E 03*              1                                             *
0.350E 03*              1                                             *
0.358E 03*              1                                             *
0.366E 03*              1                                             *
0.374E 03*              1                                             *
          *.....*.....*.....*.....*.....*.....*.....*.....*.....*.....*
   -0.89E 05 -0.83E 05 -0.77E 05 -0.71E 05 -0.65E 05 -0.59E 05
```

FIGURE 5.25. Identification of Y_v for Figure 5.6.

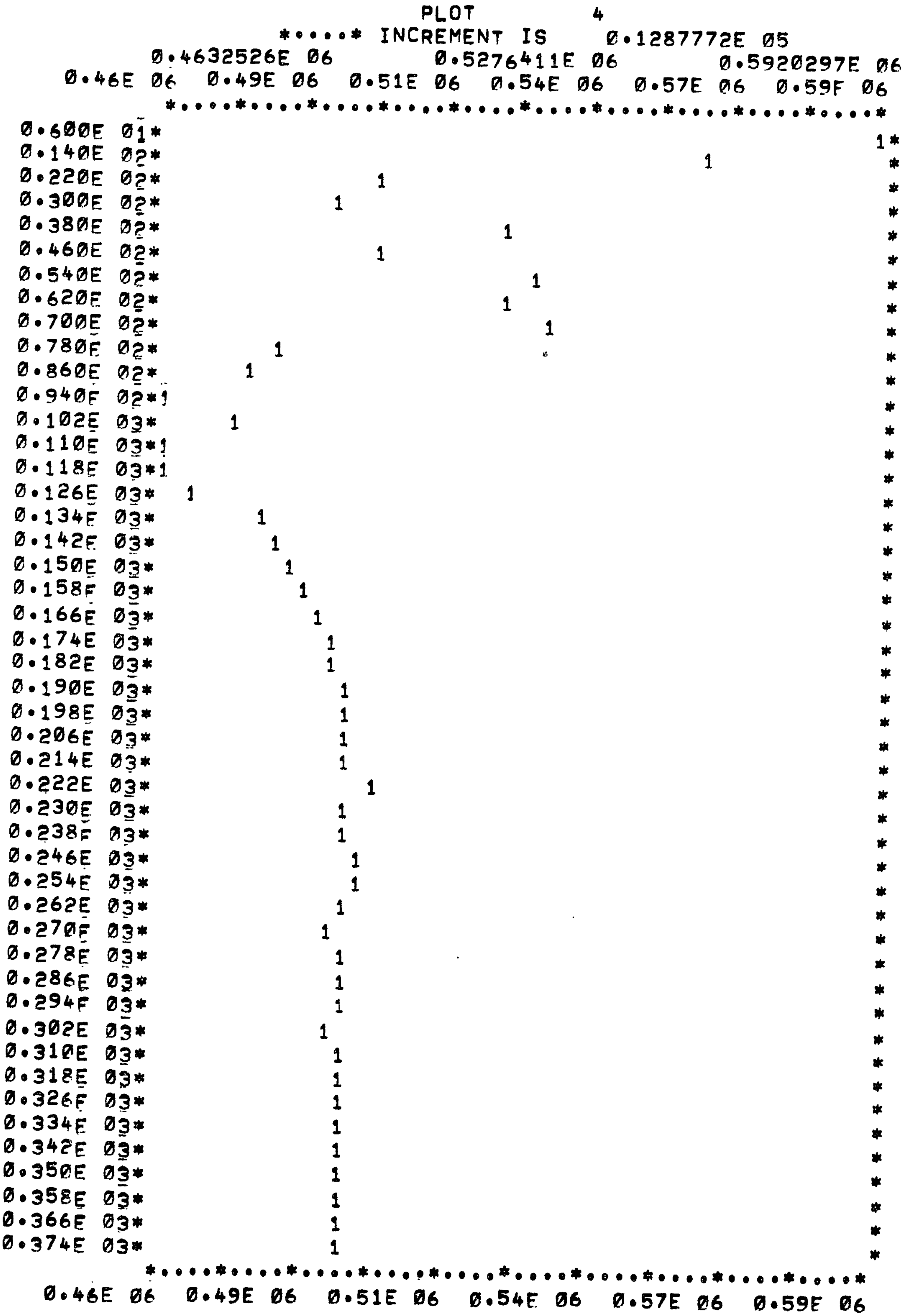


FIGURE 5.26. Identification of Y_{δ} for Figure 5.6.

TABLE 5.6.

PARAMETRIC IDENTIFICATION USING KALMAN FILTER

NP = 6	TRUE VALUE =	-0.81515E 05		
SV =	-0.57060E 05	+ OR -	0.24454E 05	
FV =	-0.81618E 05	+ OR -	0.14387E 03	
NP = 8	TRUE VALUE =	0.49423E 06		
SV =	0.64000E 06	+ OR -	0.14827E 06	
FV =	0.49787E 06	+ OR -	0.41291E 04	

LINEAR MODEL

TABLE 5.7.

Noise: $\% \underline{W} = 1$; $\% \underline{V} = (5, 0.5)$		
	Y_v	Y_δ
True Value	-81.515 E 3	49.423 E 4
Initial Estimate	-57.060 E 3 ± 24.454 E 3	64.000 E 4 ± 14.827 E 4
Final Value	-81.618 E 3 ± 14.387 E 1	49.787 E 4 ± 41.291 E 2
Initial Estimate	-57.060 E 3 ± 81.515 E 2	64.000 E 4 ± 49.423 E 3
Final Value	-81.688 E 3 ± 14.534 E 1	49.998 E 4 ± 41.797 E 2

Hayes [2] pointed out in his work that it might be necessary to tune the filter. For given amounts of process and measurement noises expressed by Q_n and R_n, the value of the corresponding values Q and R which are used in the filter may need to be adjusted for the best filter convergence over the sea trial length. Table 5.8 shows the results of the investigation conducted with the purpose of adjusting the filter. It is noticed from this table that the best filter convergence occurs for $Q = Q_n$ and $R = R_n$. It seems to be a logical conclusion and will be used throughout this chapter.

TABLE 5.8.

Noise:	% $\underline{W}$ = 10;	%V = (50,5.)
	Y_v	Y_δ
True Value	-81,515E 3	49.423 4
Initial Estimate	-57.060E 3 1 24.454E 3	64.000E ± 14.827E 4
Final Value		
Q = [illegible] R = R_n	-82.747E 3 ± 13.785E 2	53.800E 4 ± 39.654E 3
Q = $5Q_n$ R = $5R_n$	-82.942E 3 1 26.456E 2	54.245E 4 ± 75.133E 3
Q = $10Q_n$ R = $10R_n$	-83.163E 3 ± 32.891E 3 2	54.903E ± 92.120E 3

For the low level of noise contained in the data of Figure 5.6 it was considered not necessary to run the filter again over the sea trial data. For a higher degree of noise when a single pass does not produce good accuracy, it is important to check whether any improvement is obtained with other passes. Table 5.9 shows the results of 3 passes of the filter over the 10%$\underline{w}$ (50,5)% $\underline{v}$ noisy of Figure 5.17[*]. With the second pass the investigation of both parameters is improved. The third pass although improves the identifiction of Y_v degrades the identification of Y_δ. Furthermore the estimate of the parameter Y_δ becomes unstable. It was decided therefore to disregard the results of the third pass unless a longer sea trial length is used. It is recognized that the usefulness of multiple passes must be investigated in each specific application of extended Kalman filtering.

*The final estimate of the parameters for one pass is taken as the initial estimate for the following pass.

TABLE 5.9.

Noise: % $\underline{W}$ = 10 % $\underline{V}$ = (50., 5.)		
	Y_v	Y_δ
True Value	-81.515 E 3	49.423 E 4
Initial Estimate	-57.060 E 3 ± 24.454 E 3	64.000 E 4 ± 14.827 E 4
First Pass	-82.747 E 3 ± 13.785 E 2	53.800 E 4 ± 39.654 E 3
Second Pass	-82.440 E 3 ± 81.107 E 1	52.507 E 4 ± 23.244 E 3
Third Pass	-82.440 E 3 ± 76.140 E 3	53.387 E 4 ± 23.324 E 3

The overall results of parametric identification using extended Kalman filter with two parameters were very satisfactory, better than the results obtained with the model reference approach. They suggest that more parameters could be identified simultaneously. All the other runs employed computer programs that permits the identification of 4 parameters at once.

A set of runs was conducted to study the identifiability of the parameters Y_v, $(Y_r\text{-mu})$, N_v, $(N_r\text{-mx}_G u)$. The results of the extended Kalman filtering pass over the 1% W,(15., 0.5) % V noisy data of Figure 5.6 are presented in Table 5.10. It is noticed that the parameters Y_v(6), $(Y_r\text{-mu})$ (7), N_v(12), $(N_r\text{-mx}_G u)$(13) are identified with a reasonably good accuracy. There are previous results about the identifiability of the coefficient Y_v, obtained with the 2 parameters program. The comparison of Tables 5.6 and 5.10 shows that there is less accuracy in the identification of Y_v in the present case. It cannot be concluded that this fact is caused by the increase in the number of parameters handled simultaneously. There was a reduction in the sea trial length (376 sec. to 188 sec) and the change may be responsible for lower accuracy in the identification results.

Table 5.11 shows a comparison of the parametric identification results obtained with different lengths of sea trial data. It must be pointed out that a longer sea trial does not mean a larger number of measurements. If this were the case, certainly a better filter performance would occur for the longer trial since the filter accuracy is proportional to the number of observations. However, in any condition the number of computation steps is constantly given the limitation on core capacity. Thus, for the longer run the time step is increased.

TABLE 5.10

PARAMETRIC IDENTIFICATION USING KALMAN FILTER

NP = 6 TRUE VALUE = -0.81515E 05

SV = -0.57060E 05 + OR - 0.24454E 05

FV = -0.82420E 05 + OR - 0.20761E 04

NP = 7 TRUE VALUE = -0.18508E 08

SV = -0.12955E 08 + OR - 0.55525E 07

FV = -0.18792E 08 + OR - 0.47552E 06

NP = 12 TRUE VALUE = -0.97735E 07

SV = -0.68414E 07 + OR - 0.29321E 07

FV = -0.97644E 07 + OR - 0.45851E 06

NP = 13 TRUE VALUE = -0.32510E 10

SV = -0.22757E 10 + OR - 0.97531E 09

FV = -0.32549E 10 + OR - 0.10968E 09

LINEAR MODEL

From Table 5.11, it can be concluded that for a constant number of observations, it is better to increase the length of the trial data than to have a smaller time step.

TABLE 5.11

Noise: % $\underline{W}$ = 10; % $\underline{V}$ = (50., 5)				
	Y_v	$(Y_r\text{-mu})$	N_v	$(N_r\text{-mx}_G u)$
True Value	-81.515 E 3	-18.508 E 6	-97.735 E 5	-32.510 E 8
Initial Est.	-57.060 E 3± 24.454 E 3	-12.955 E 6± 55.525 E 3	-68.414 E 5± 29.321 E 5	-22.752 E 8± 97.581 E 7
F.Value T=188sec	-72.934 E 3± 12.896 E 3	-16.760 E 6± 31.644 E 5	-79.064 E 5± 15.481 E 5	-28.062 E 8± 37.366 E 7
F.Value T=376sec	-79.796 E 3± 10.585 E 3	-18.052 E 6± 28.213 E 5	-93.683 E 5± 12.959 E 5	-31.376 E 8± 32.591 E 7

The results of the extended Kalman filtering pass over the 10% W, (50., 5.) % V noisy data of Figure 5.17 are shown in the plots of Figures 5.27 through 5.32. These plots show how the noise is filtered out of the primary states (compare Figures 5.27 and 5.28 to Figure 5.17) and how the parameter values for Y_δ, $(Y_r\text{-mu})$, N_v, $(N_r\text{-mx}_G u)$ are arrived at by the filter.

A second pass over the trial data improves the accuracy in the parametric identification, and the results are shown in Table 5.12.

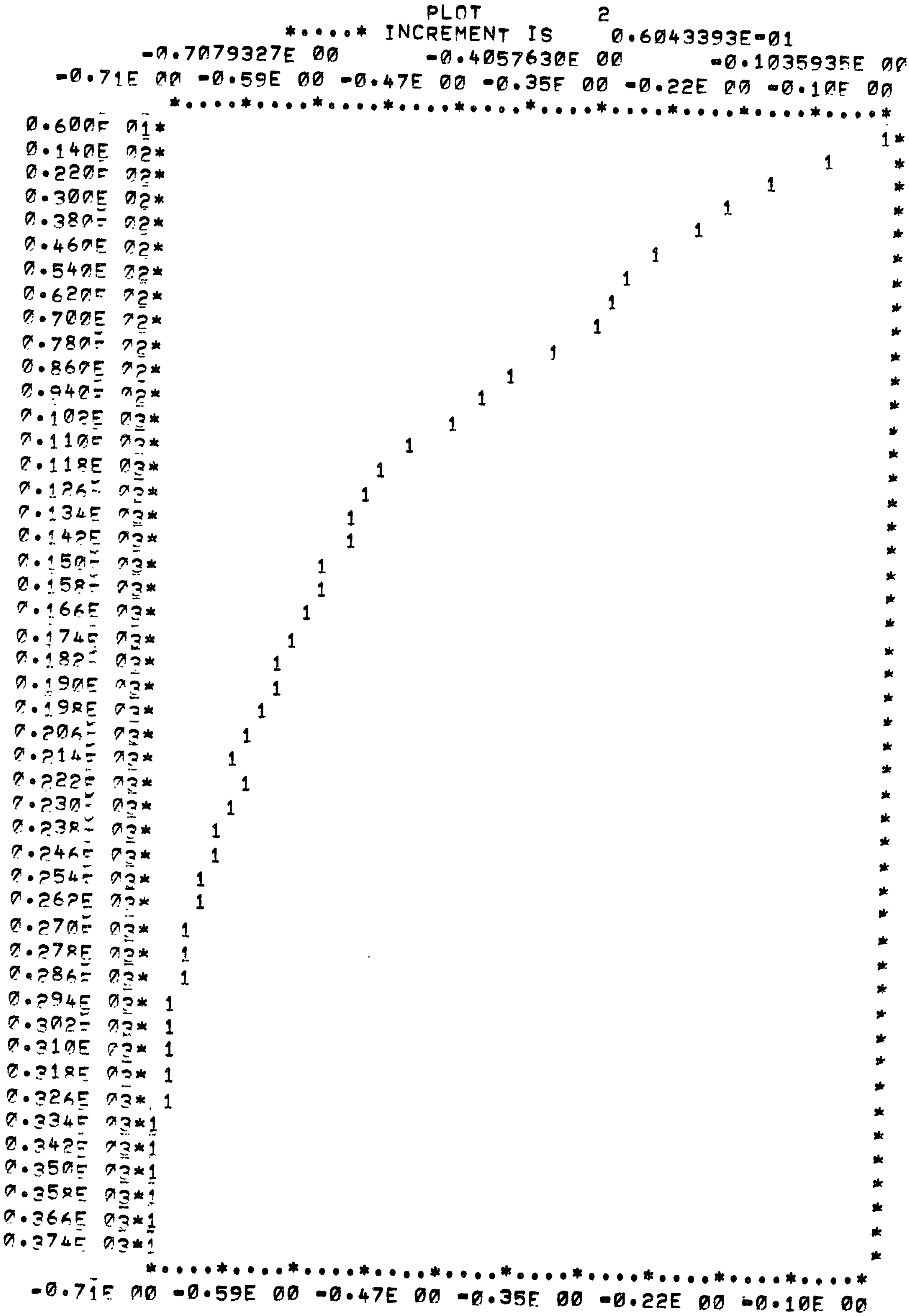


FIGURE 5.28. Kalman Filter Yaw Velocity For Figure 5.17.

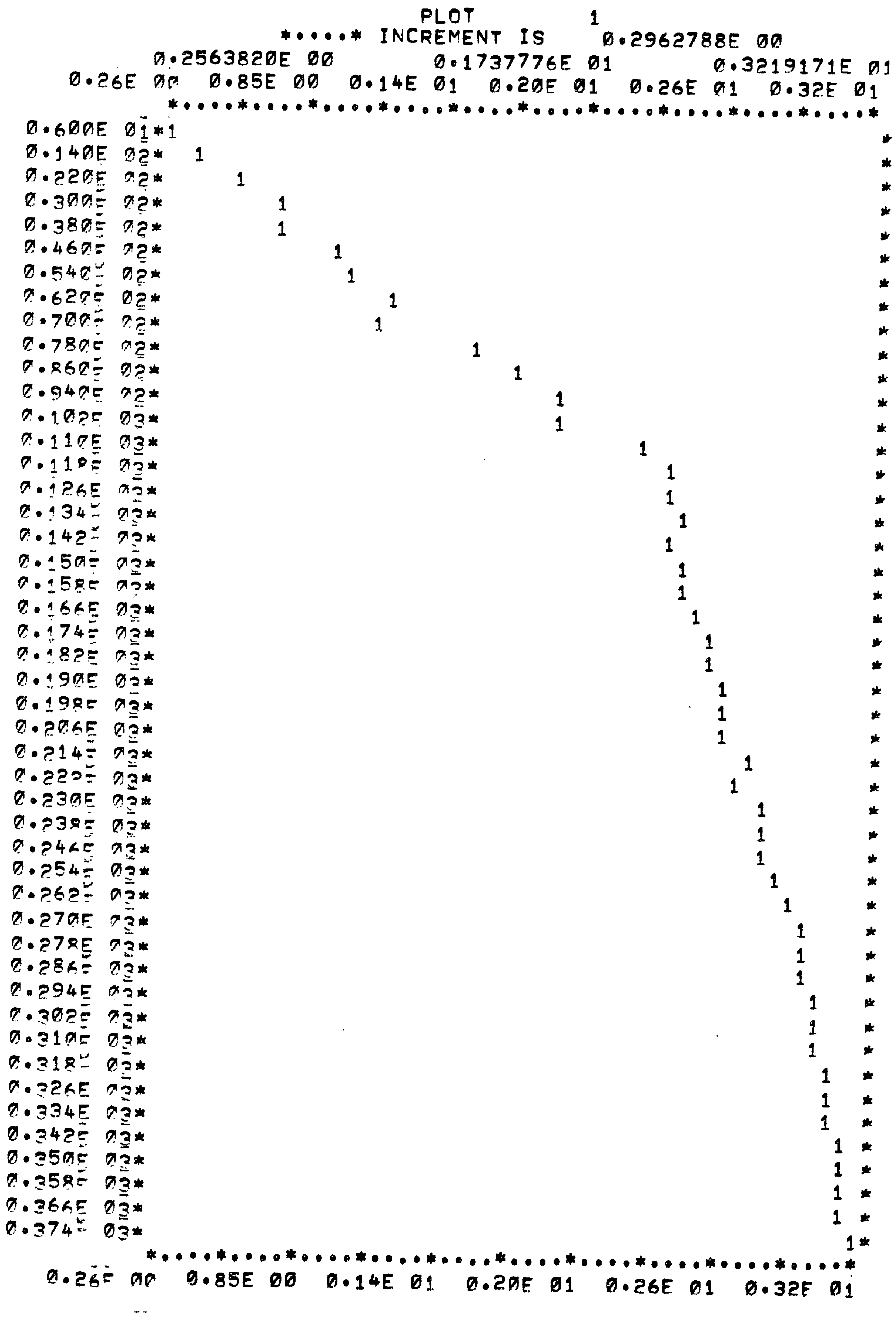


FIGURE 5.27. Kalman Filter Sway Velocity For Figure 5.17.

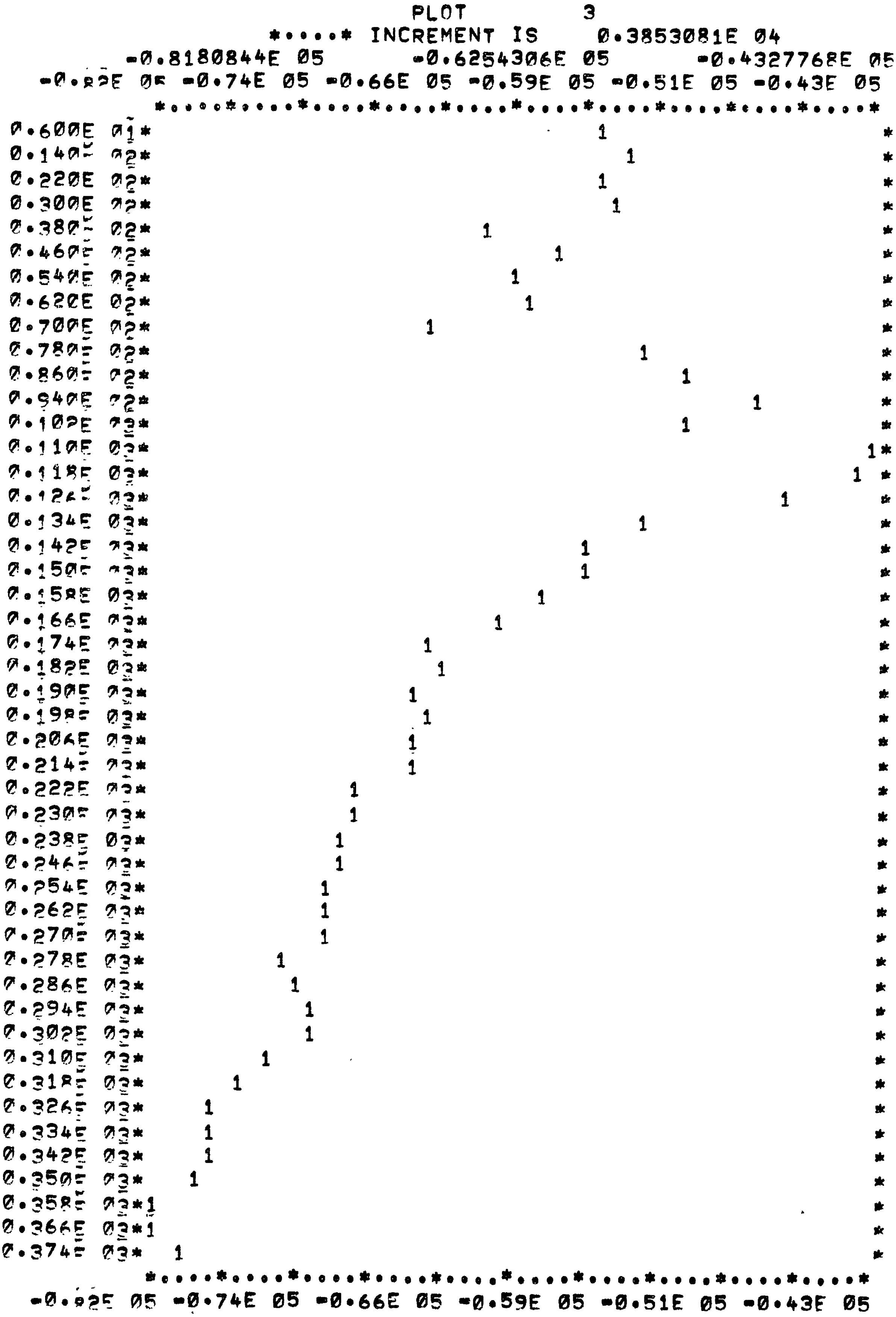


FIGURE 5.29. Identification of Y_v For Figure 5.17.

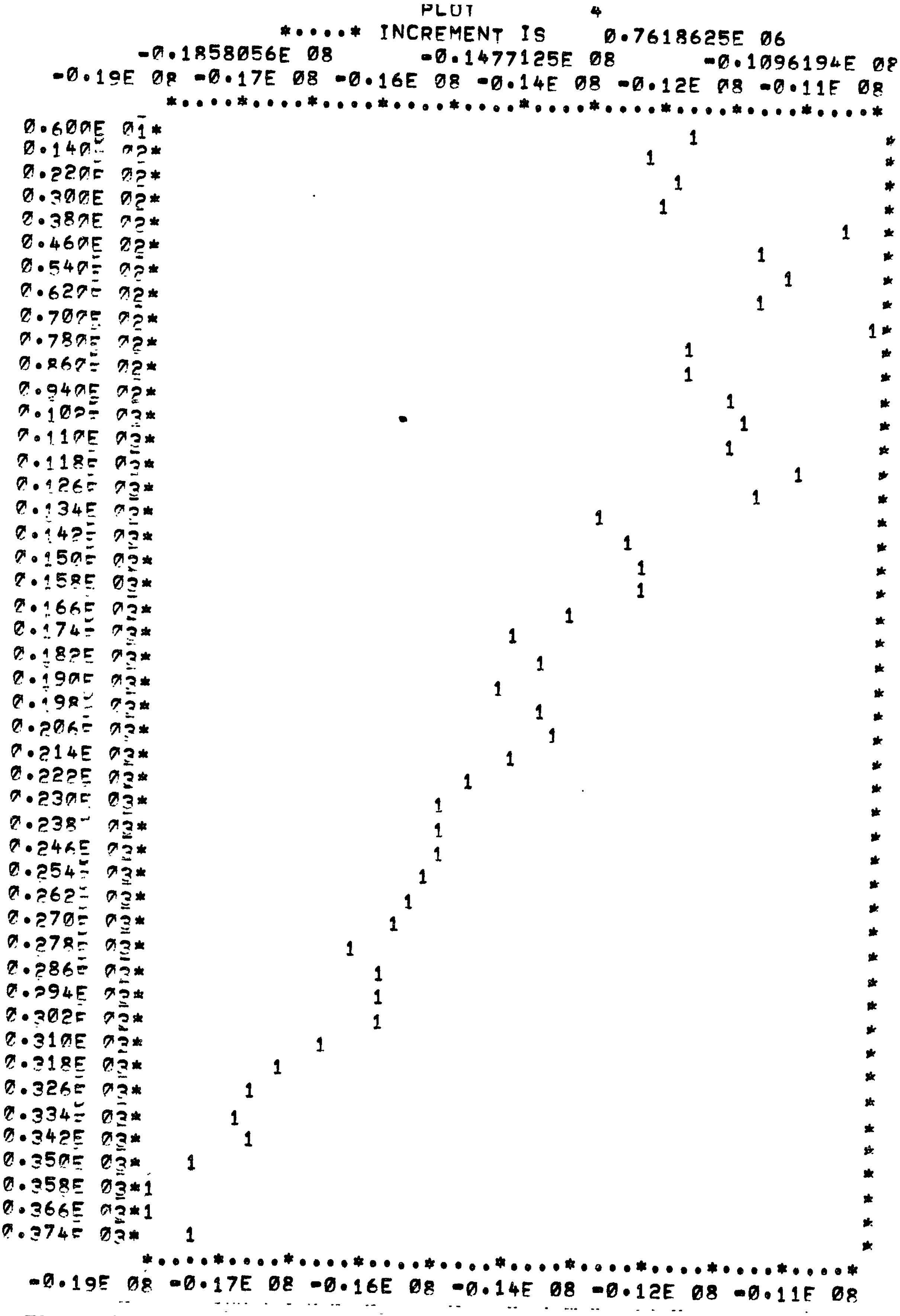


FIGURE 5.30. Identification of $(Y_r - m_u)$ for Figure 5.17.

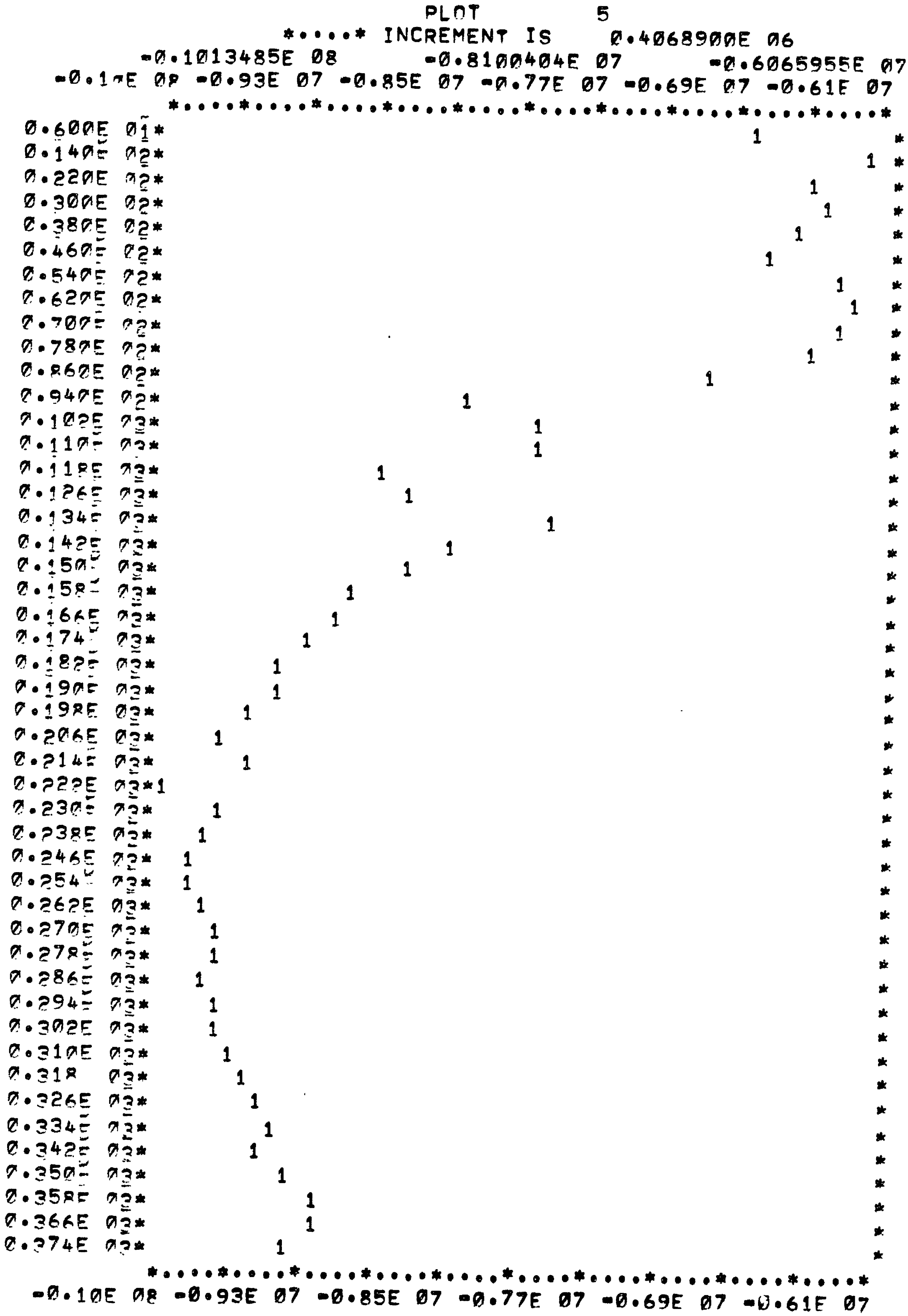


FIGURE 5.31. Identification of N_v for Figure 5.17.

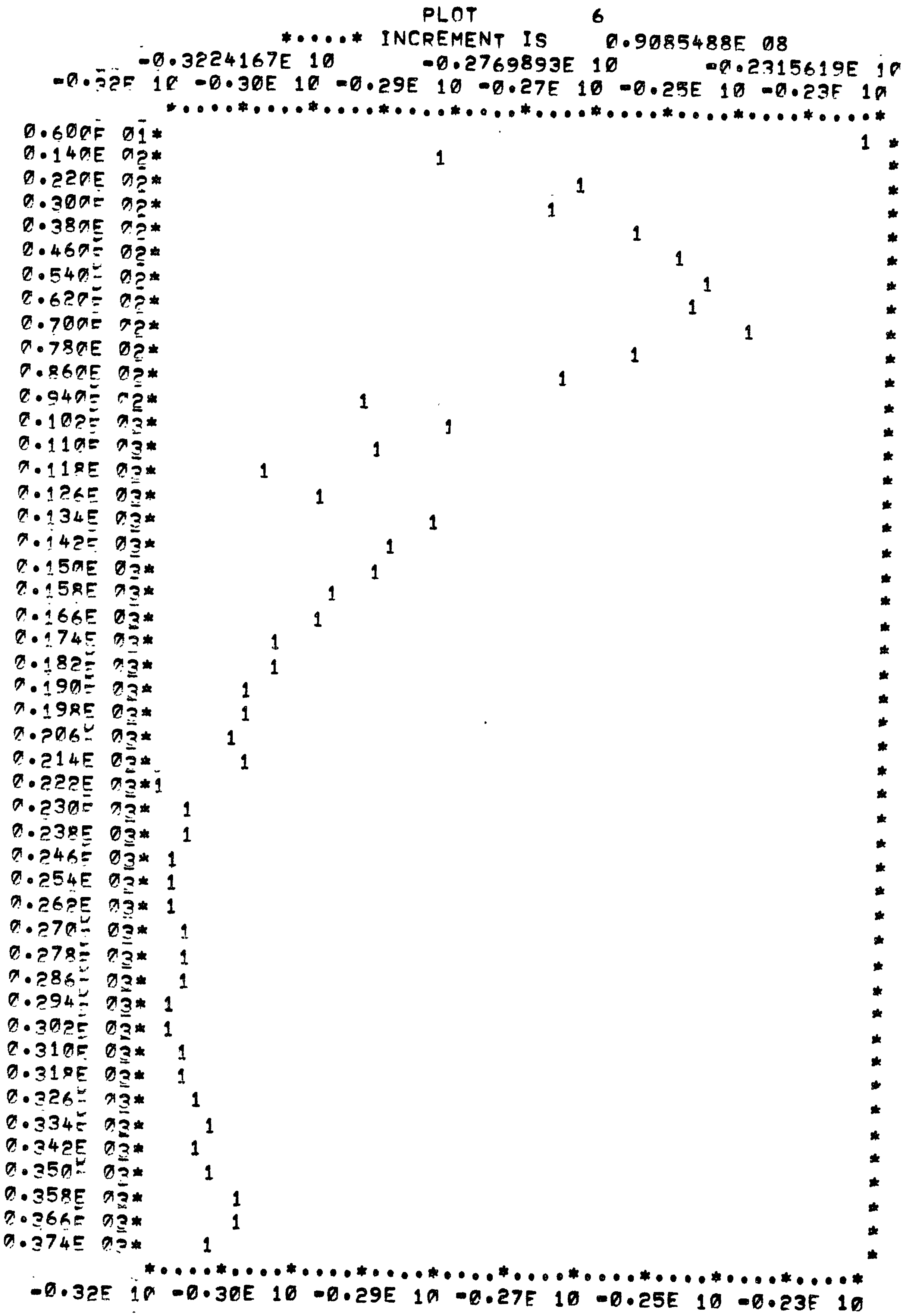


Figure 5.32. Identification of (N_r - $mx_G u$) for Figure 5.17.

TABLE 5.12

Noise:	%$\underline{w}$ = 10;	% $\underline{v}$ = (50., 5)		
	Y_v	$(Y_r\text{-mu})$	N_v	$(N_r\text{-m}x_G u)$
True Value	-81,515E 3	-18.508E 6	-97.735E 3	-32.510E 6
Initial Estimate	-57.060E 3 ± 24.454E 3	-12.955E 6 ± 55.525E 5	-68.414E 5 ± 29.321E 5	-22.752E 8 ± 97.531E 7
First Pass	-79.796E 3 ± 10.585E 3	-18.052E 6 ± 28.213E 5	-93.683E 5 ± 12.959E 5	-31.376E 8 ± 32.591E 7
Second Pass	-81.642E 3 ± 64.705E 2	-18.677E 6 ± 18.061E 5	-93.987E 5 ± 84.216E 4	-31.651E 8 ± 20.228E 7

The very nice results obtained so far indicate that the extended Kalman filtering approach is of great effectiveness to identify very noisy trial data. It is important, however, to realize that the results of Kalman filtering depend greatly on the characteristics of the parameters. If the coefficients do not have much influence on the dynamic system behavior the accuracy in the parametric identification is low. This fact is very well illustrated by the results shown on Table 5.13. The hydrodynamic coefficients studied in this case are Y_v, $(Y_r\text{-mu})$, Y_δ and Y_o. Even for the low level of noise considered the identification is very poor compared to the previous cases. This is a good example of how a parameter of minor importance, Y_o can degrade the filter performance contaminating the identification of other parameters.

TABLE 5.13

Noise: % W 1.; % V (5., 0.5)				
	Y_v	$(Y_r^{\cdot}-mu)$	Y_δ	Y_o
True Value	-81.515 E 3	-18.508 E 6	49.423 E 4	-64.042 E 2
Initial Est.	-57.060 E 3± 24.454 E 3	-17.955 E 6± 55.525 E 5	64.000 E 4± 14.827 E 4	-44.830 E 2± 19.213 E 2
F.Value T=188sec	-74.895 E 3± 11.637 E 3	-16.493 E 6± 34.340 E 5	51.447 E 4± 59.777 E 3	-47.245 E 2 ± 19.034 E 2
F.Value T=376sec	-69.163 E 3± 11.252 E 3	-14.814 E 6± 33.473 E 5	54.086 E 4± 60.622 E 3	-46.738 E 4 ± 19.035 E 2

In the next series of runs, it was investigated the identifiability of the parameters Y_v, Y_δ, N_v and N_δ.
The results of the extended Kalman filtering identification for this set of coefficients are presented in Table 5.14.

The analysis of results presented in Table 5.9 and 5.14 indicates that the identification of the parameter Y_δ is better in the first case. It means that a smaller number of parameters permits a more accurate identification.

TABLE 5.14

	Y_v	Y_δ	N_v	N_δ
True Value	-81.515E 3	49.423E 4	-97.735E 5	-13.034E 7
Initial Estimate	-57.060E 3 ± 24.454E 3	64.000E 4± 14.827E 4	-68.414E 5± 29.321E 5	-91.236E 6 ± 39.101E 6
Noise	% $\underline{w}$ = 1		% $\underline{v}$ = (5.,0.5)	
First Pass	-83.863E 3 ± 40.634E 2	61.427E 4± 85.220E 3	-98.775E 5± 62.240E 4	-12.214E 7 ± 15.198E 6
Second Pass	-83.232E 3 ± 23.329E 2	58.904E 4± 60.314E 3	-99.493E 5± 34.357E 4	-11.997E 7 ± 83.188E 5

Four of the linear hydrodynamic coefficients have not already been studied; they are $(m-Y_{\dot{v}})$, $(mx_G-Y_{\dot{r}})$ $(mx_G-N_{\dot{r}})$, and $(I_z-N_{\dot{r}})$. As it was explained in Chapter 4 a computer program to handle these parameters must use double precision variables, which was not possible in the present case due to the limited computer capacity. Anyway the Kalman filtering approach was used to identify these coefficients. The poor accuracy of the results is certainly the consequence of numerical imprecisions in the computation. Table 5.15 presents the results of identification studies for the mentioned parameters.

TABLE 5.15.

	$(m-Y_{\dot{v}})$	$(mx_G-Y_{\dot{r}})$	$(mx_G-N_{\dot{v}})$	$(I_z-N_{\dot{r}})$
True Value	22.650 E 5	-66.527 E 5	-17.560 E 6	33.861 E 9
Initial Est.	29.450 E 5 ± 67.951 E 4	-46.569 E 5 ± 19.958 E 5	-12.292 E 6± 52.680 E 5	43.900 E 9± 10.158 E 9
	Noise: % W = 1 ; % V = (5., 0.5)			
Final Value	22.807 E 5 ± 44.506 E 3	-46.847 E 5 ± 19.948 E 5	-13.208 E 6± 47.180 E 5	34.413 E 9± 21.263 E 8
	Noise: % $\underline{w}$ = 10; % $\underline{v}$ = (50.,5.)			
Final Value	21.280 E ± 20.092 E 4	-46.401 E 5 ± 19.957 E 5	-11.962 E 6 ± 52.583 E 5	38.725 E 9± 81.845 E 8

One additional table is included to complete the identification studies of this section. Table 5.16 confirms the statement that numerical imprecisions in the computations degrade the identifiability of the parameters. The coefficients shown in this table were identified with better accuracy using the other program (see Table 5.10).

TABLE 5.16

Noise: $\% \underline{w} = 1$; $\% \underline{v} = (5., 0.5)$				
	Y_v	$(Y_r - mu)$	N_v	$(N_r - mx_G u)$
True Value	-81.5.5E 3	-18.508E 6	-97.735E 5	-32.510E 8
Initial Estimate	-57.060E 3 ± 24.454E 3	-12.955E 6 ± 55.525E	-68.414E 5 ± 29.321E 5	-22.752E 8 ± 97.531E 7
Final Value	-80.037E 3 ± 29.167E 2	-18.183E 6 ± 73.428E 4	-92.796E 5 ± 57.862E 4	-31.292E 8 ± 14.725E 7

5.4 Model Reference Identification of the Non Linear Parameters

in section 5.2 the results of model reference identification were presented and discussed. A smaller number of runs were carried out for the identification of the nonlinear coefficients since the computation time increases enormously.

It was decided to increase the sea trial length (376 seconds) to permit the vehicle to reach steady state condition. In order to keep constant the number of computation steps the time step was proportionally increased.

The first parameters to be analyzed with this model are the linear coefficients $(m - X_{\dot{u}})$ and X_u which are not included in the linear model. The results of the noiseless sea trial maneuvering are shown in Figures 5.33 to 5.35. These figures present the plots of the state variables, u, v and r, respectively. The contours for the noiseless conditions are shown in Figure 5.36.

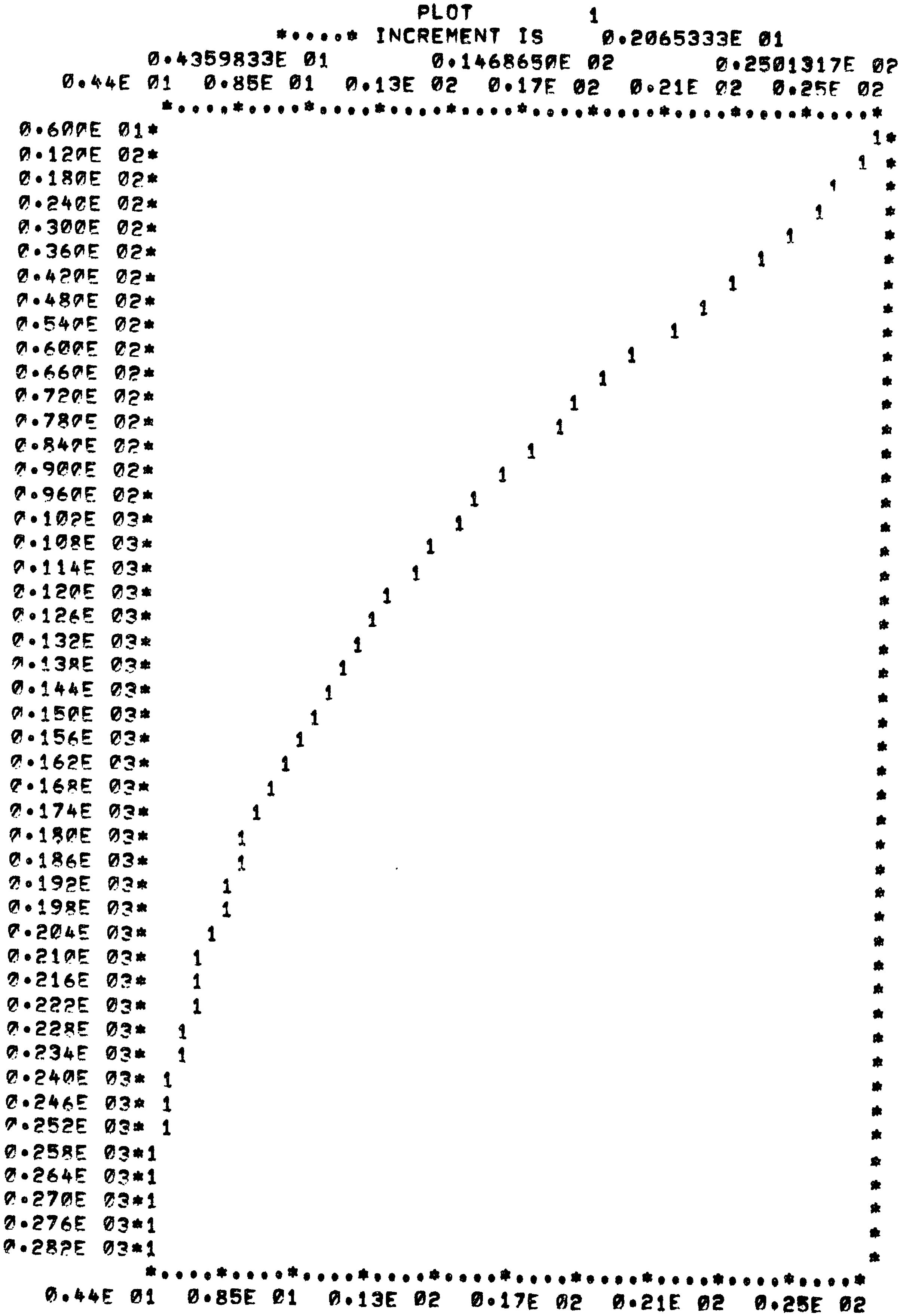


FIGURE 5.33. Surge Velocity, No Noise, Nonlinear Model.

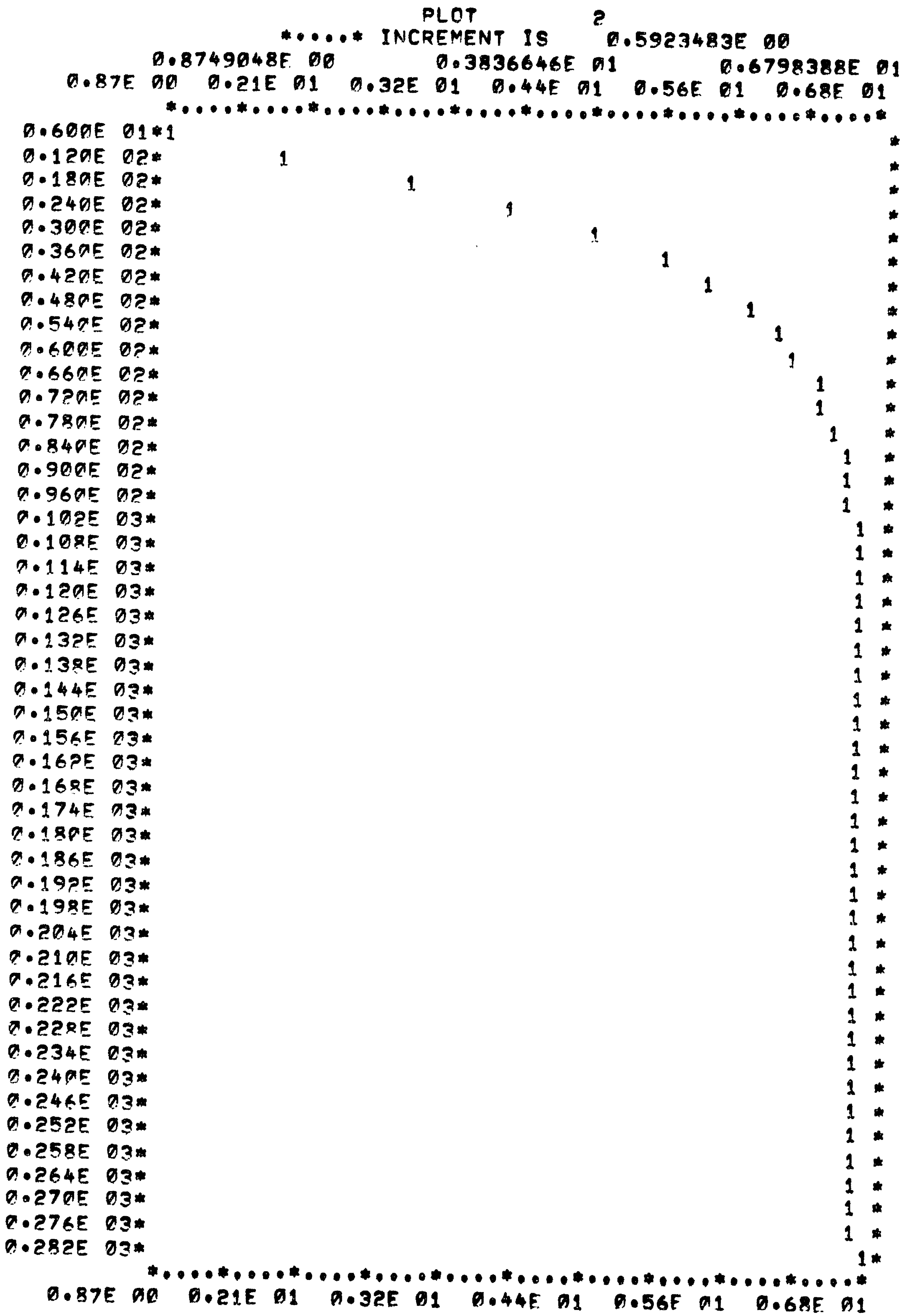


FIGURE 5.34. Sway Velcoity. No Noise, Nonlinear Model.

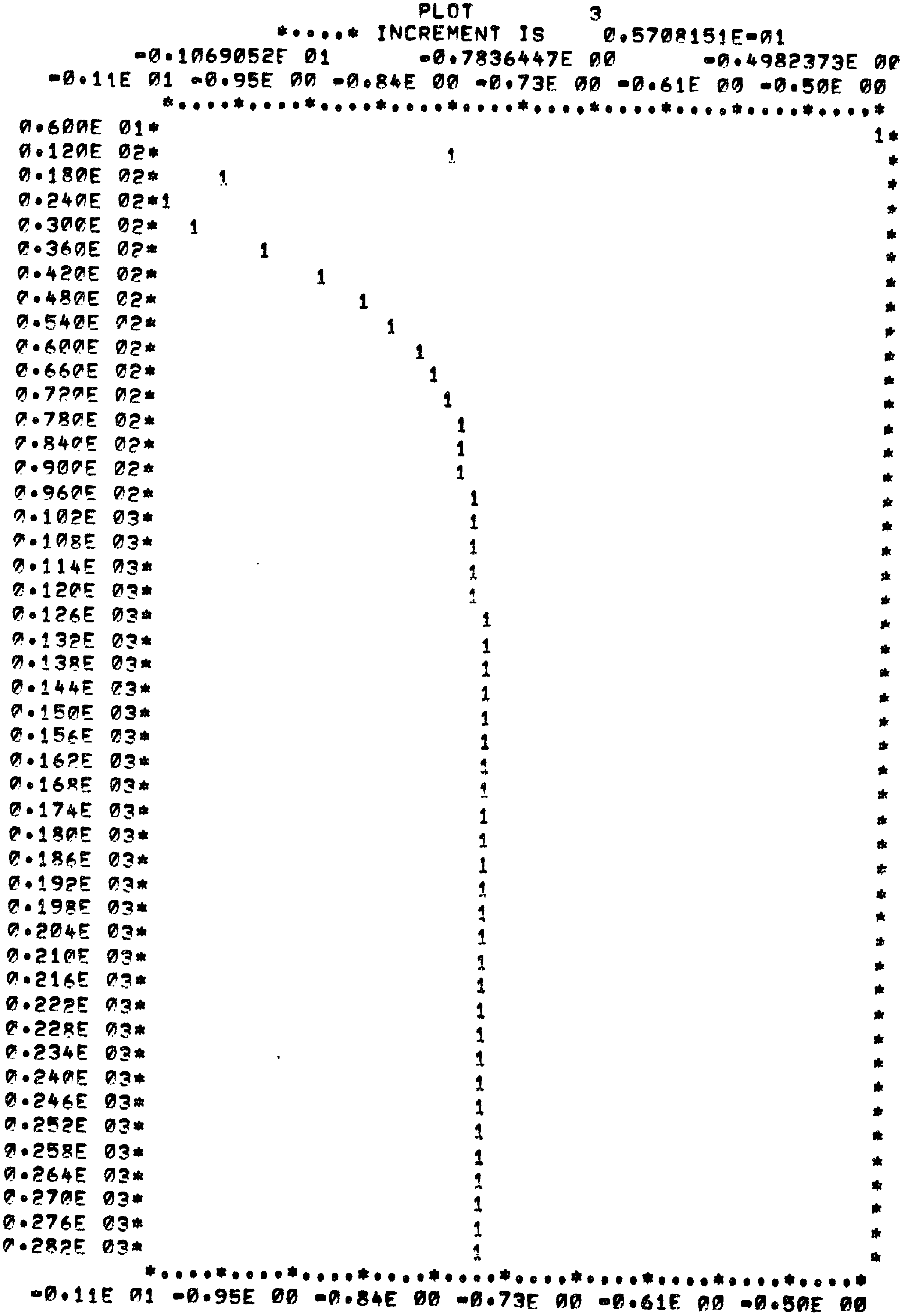


FIGURE 5.35. Yaw Velocity, No Noise, Nonlinear Model.

```
                                   CONTOUR           1
                           *.....* INCREMENT IS    0.5057749E 03
                  -0.1095848E 05         -0.8429602E 04          -0.5900727E 04
            -0.11E 05 -0.99E 04 -0.89E 04 -0.79E 04 -0.69E 04 -0.59E 04
                  *....*....*....@....*....*....*....*....*....*....*
 0.861E 06*L      L      L      L      L      L      M      M      M*
 0.958E 06*L      K      K      K      L      L      L      M      M*
 0.105E 07*L      K      K      K      K      L      L      L      M*
 0.115E 07*L      K      K      H      J      K      L      L      L*
 0.123E 07*L      L      K      J      1      J      K      L      L*
 0.133E 07*M      L      L      K      J      H      K      K      L*
 0.142E 07*M      M      L      L      K      J      J      K      L*
 0.152E 07*M      M      M      L      L      K      K      K      K*
 0.160E 07*M      M      M      M      L      L      L      K      K*
                  *....*....*....@....*....*....*....*....*....*....*
            -0.11E 05 -0.99E 04 -0.89E 04 -0.79E 04 -0.69E 04 -0.59E 04
```

FIGURE 5.36. $(m-X_u)$ and X_u Contours, No Noise, Nonlinear Model.

Figures 5.37 through 5.39 shows the 10% w, 10% v noisy trial data. The resulting contour is given in Figure 5.40. The parameters are not identifiable if a 80% or better accuracy is required. The essential results of all the contours generated to analyse the coefficients $(m-X_u)$ and X_u are presented in Table 5.17. There are significant differences between these results and those obtained with the linear model. It is not possible,however, to establish comparisons about the identifiability of the parameters.

TABLE 5.17

Noise %w	Noise %v	PA1 ≡ $(m-X_a)$ PA1* = 12.31E 5	PA2 ≡ X_u PA2* = 84.29E 2	$C(p^*)$	$C_{max}(p)$	Comment
0	0	12.31E 5	-84.29E 2	-17.90	8.598	
1	1	12.31E 5	-84.29E 2	13.11	8.603	
5	5	12.31E 5	-77.97E 2	5.394	8.843	
10	10	9.58E 5	-10.35E 3	7.132	8.861	
15	15	12.31E 5	-89.70E 2	7.855	8.785	
20	20	13.30E 5	-59.20E 2	8.101	9.432	

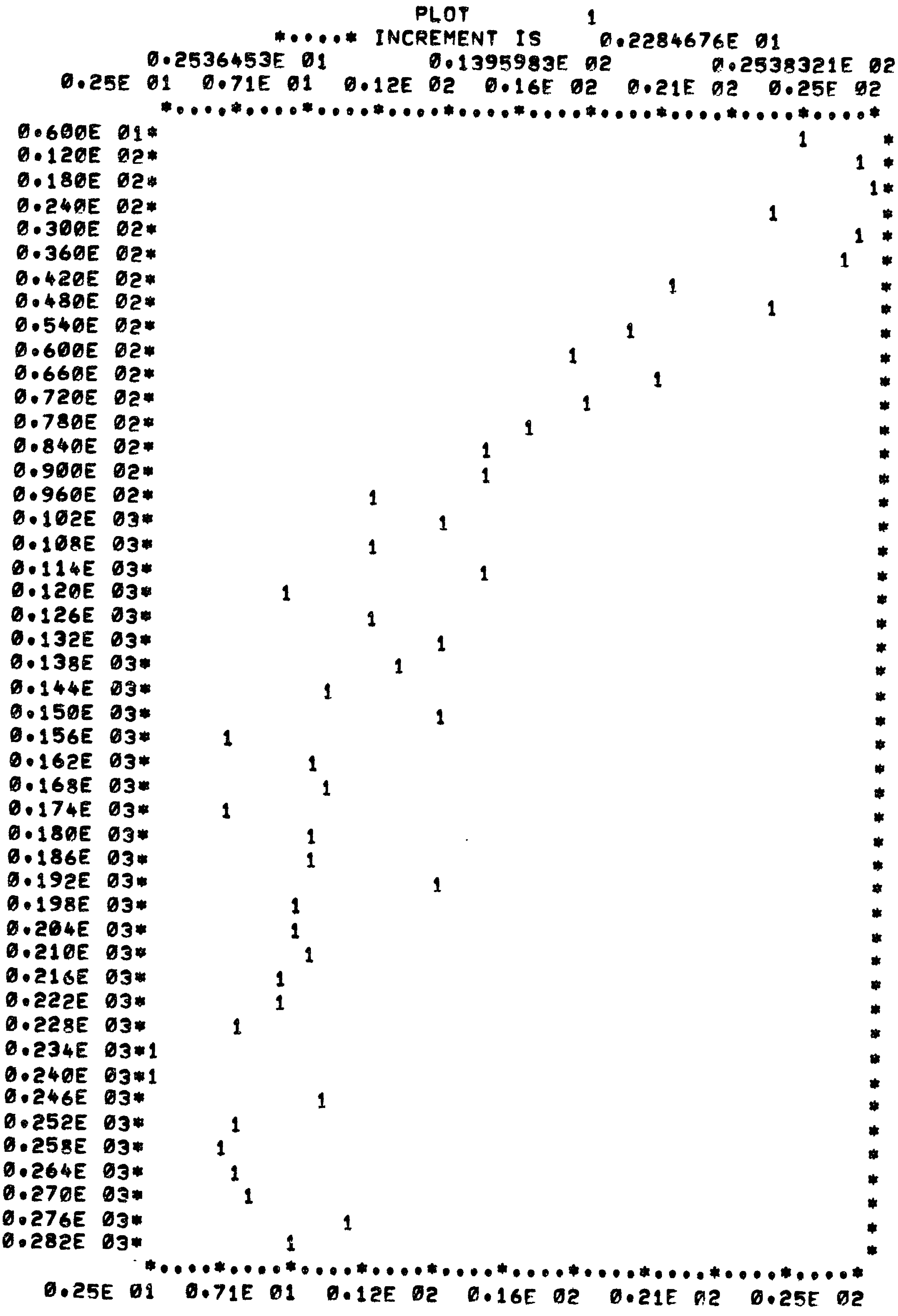


FIGURE 5.37. Surge Velocity, 10% $\underline{w}$, 10% $\underline{v}$ noises, Nonlinear Model.

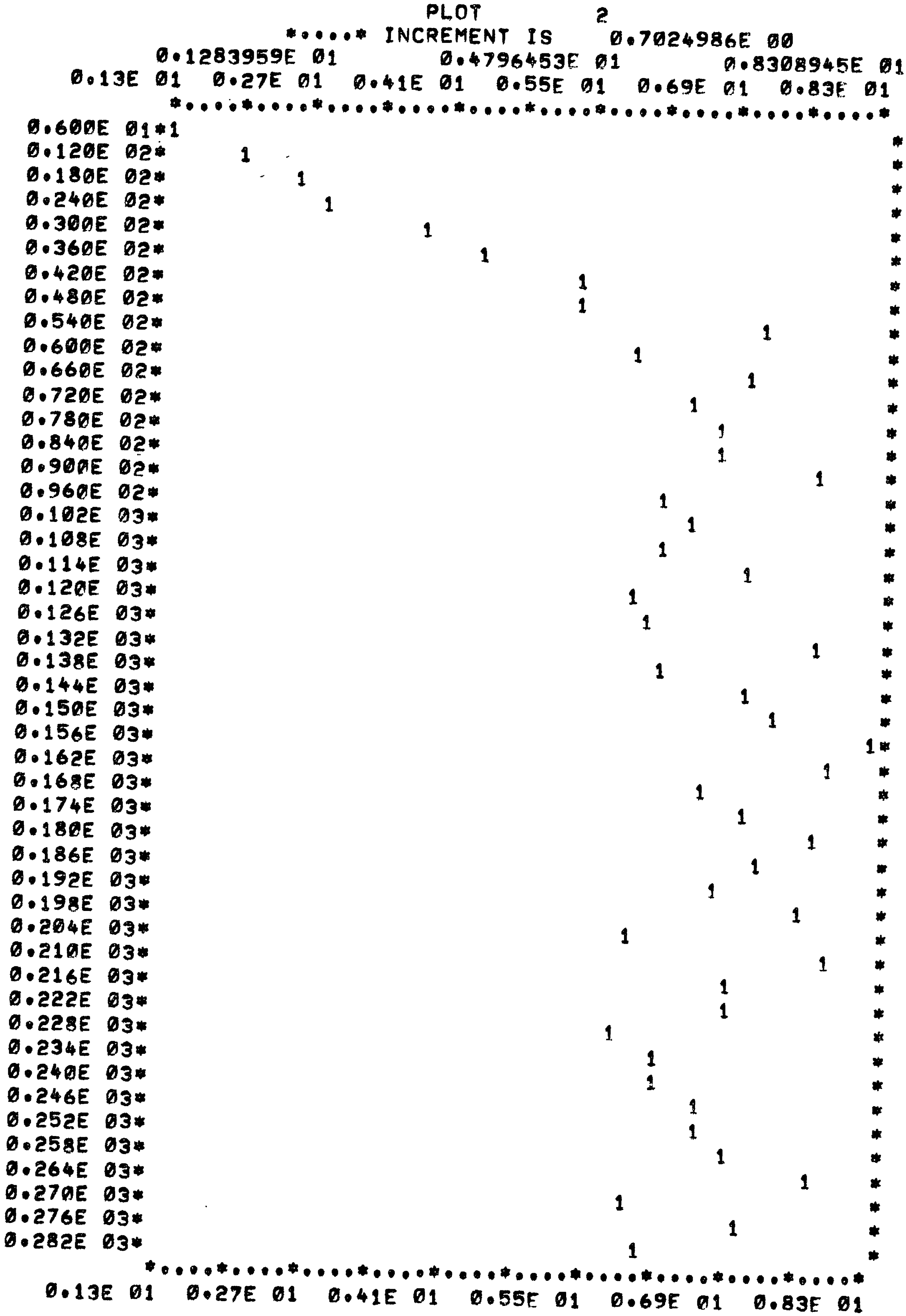


FIGURE 5.38. Sway Velocity, 10% $\underline{w}$, 10% $\underline{v}$ Noises; Nonlinear Model.

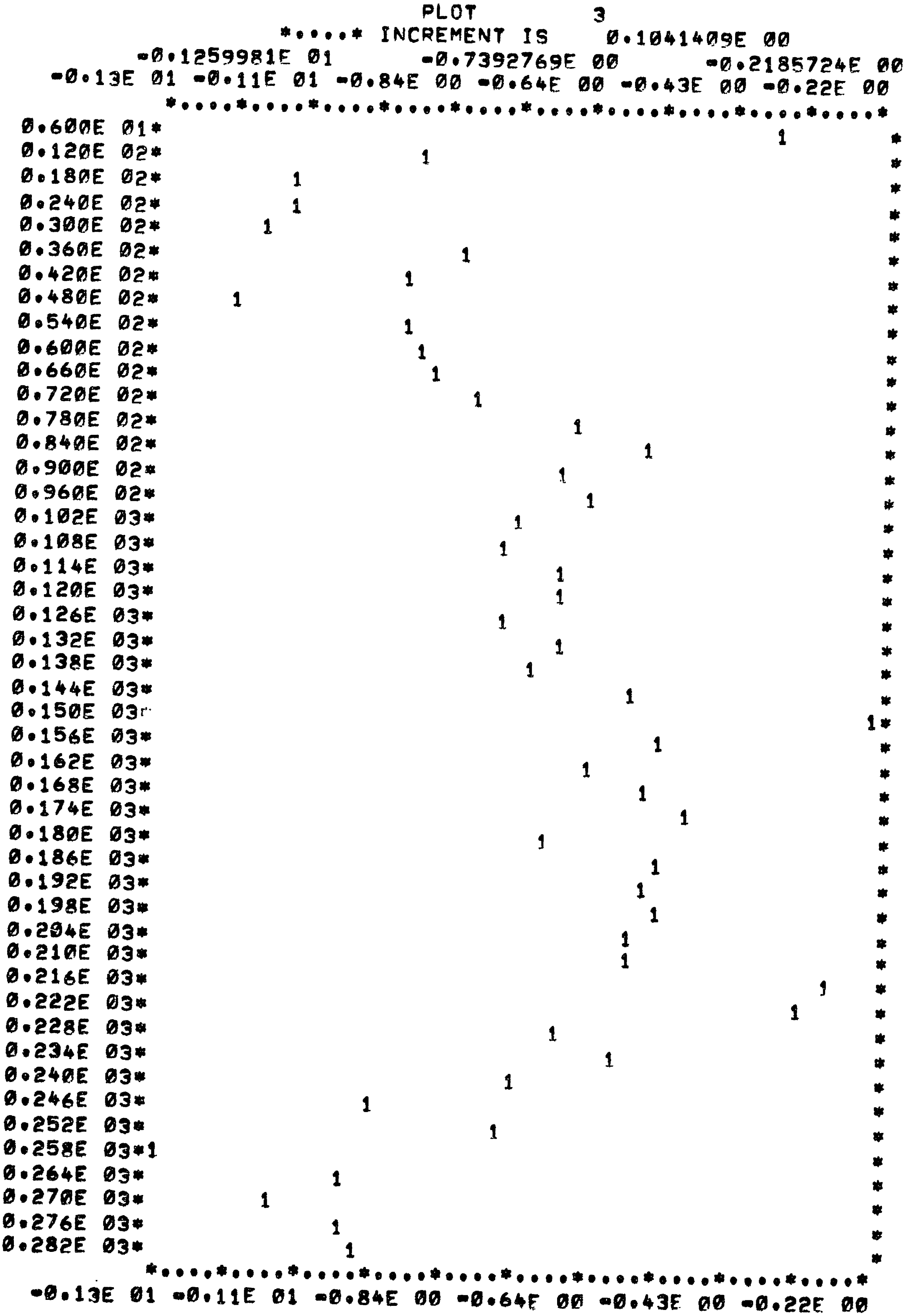


FIGURE 5.39. Yaw Velocity, 10% $\underline{w}$, 10% $\underline{v}$, Noises; Nonlinear Model.

CONTOUR 1

..... INCREMENT IS 0.5057749E 03

-0.1095848E 05 -0.8429602E 04 -0.5900727E 04

-0.11E 05 -0.99E 04 -0.89E 04 -0.79E 04 -0.69E 04 -0.59E 04

0.861E 06	2	2	3	4	6	8	A	C	E
0.958E 06	2	1	2	3	5	7	9	B	C
0.105E 07	4	2	2	2	3	5	7	9	B
0.115E 07	6	4	2	2	3	4	6	8	A
0.123E 07	9	6	4	3	3	4	5	7	9
0.133E 07	C	9	7	5	4	4	5	6	8
0.142E 07	F	D	A	8	6	6	5	6	7
0.152E 07	J	G	D	B	9	7	7	7	7
0.160E 07	M	K	G	E	C	A	9	8	8

-0.11E 05 -0.99E 04 -0.89E 04 -0.79E 04 -0.69E 04 -0.59E 04

FIGURE 5.40. $(m-X_u)$ and X_u Contours; 10% $\underline{w}$, 10% $\underline{v}$ Noises, Nonlinear Model.

In order to investigate possible differences in the parametric identification when the complete mathematical model is utilized, the coefficients $(m-Y_{\dot{v}})$ and Y_{δ} were studied again. Table 5.18 presents the essential results of the model reference contours for these parameters. It is noticed that there is no appreciable change (see also Table 5.2). Although the lack of comparable noise levels make difficult the analysis.

TABLE 5.18

Noise %w	%v	PA1 $\equiv (m-Y_{\dot{v}})$ PA1* = 22.65E 7	PA2 $\equiv Y_{\delta}$ PA2* = 49.42E 4	C(p*)	C_{max}(p)	Comment
0	0	22.65E 5	49.42E 4	-21.06	6.969	
1	1	22.65E 5	49.42E 4	1.830	7.019	
10	10	22.65E 5	38.82E 4	6.650	7.446	

A large number of runs was conducted to study the identifiability of the nonlinear coefficients and the correspondent contours were generated. Almost all the nonlinear parameters were analysed. The essential results of these contours are presented next in tabular form.

The overall conclusion that can be drawn from the analysis of the Tables 5.19 through 5.23 is that the identifiability of the nonlinear coefficients is much smaller than that of the linear parameters. In most cases even for a low degree of noise- 1%w, 1%v - the parameters are not identifiable if 90% or better accuracy is required and the accuracy is further reduced if large amounts of noise are considered.

TABLE 5.19

Noise %w	%v	PA1 ≡ 1/2 X_{uu} PA1*= 12.48 E 1	PA2 ≡ 1/6 X_{uuu} PA2* = 1.129	C(p*)	C_{max}(p)	Comment
0	0	12.48E 1	-1.129	-17.81	4.469	
1	1	16.20E 1	-1.040	2.199	4.702	In General
1	10	8.74E 1	-1.472	7.048	7.210	Very Poor
10	1	8.74E 1	-1.472	3.381	4.882	Identif.
10	10	16.20E 1	-0.955	7.016	7.128	

TABLE 5.20

Noise %w	%v	PA1 ≡ 1/2X_{vv} PA1*= 24.91E 2	PA2 ≡ 1/2 $X_{\delta\delta}$ PA2*= 16.86 E 4	C(p*)	C_{max} (p)	Comment
0	0	-24.91E 2	-16.86E 4	-22.15	7.697	
1	1	-21.05E 2	-18.18E 4	2.229	7.657	
10	10	-30.45E 2	-15.54E 4	7.051	8.291	

TABLE 5.21

Noise %w %v	PA1 ≡ $1/2Y_{\delta vv}$ PA1*= 33.08E 2	PA2 ≡ $1/2N_{\delta vv}$ PA2*=-16.86E 4	C(p*)	C_{max}(p)	Comment
0 0	33.08E 2	-71.64E 4	-22.86	5.533	
1 1	35.70E 2	-71.64E 4	-21.91	5.675	
10 10	23.20E 2	-77.20E 4	7.059	7.300	

TABLE 5.22

Noise %w %v	PA1 ≡ $1/6\ Y_{\delta\delta\delta}$ PA1* =-16.00E9	PA2 ≡ $1/2Y_{rvv}$ PA2*=-16.75E7	C(p*)	C_{max}(p)	Comment
0 0	-16.00E 4	88.86E 3	-21.31	4.581	
1 1	-18.30E 4	62.05E 3	2.135	4.833	
10 10	-16.00E 4	12.05E 4	7.072	7.203	

TABLE 5.23

Noise %w %v	PA1 ≡ 1/6 $N_{\delta\delta\delta}$ PA1*=42.25E 6	PA2 ≡ 1/2N_{rvv} PA2*=88.86E 3	C(p*)	C_{max}(p)	Comment
0 0	42.25E 6	-16.75E 7	-22.03	7.618	
1 1	42.25E 6	-15.40E 7	2.139	7.666	
10 10	45.40E 6	-12.85E 7	7.064	7.912	

It was shown that the identifiability characteristics of the parameters $(m - Y_{\dot{v}})$ and Y_δ are not significantly affected when the nonlinear model is used for the identification studies. There are then two possible explanations for the poor identifiability characteristics of the nonlinear coefficients. The first explanation is related to the input selected for the studies. It is possible that this input does not excite conveniently the nonlinear dynamics of the system. It would be reasonable in further works to test this hypothesis. The other explanation is based on the relative small importance of the nonlinear coefficients, and also to the accuracy in the determining the true values.

5.5. Extended Kalman filtering Identification of the Nonlinear Parameters

The results of Kalman filtering identification of the linear parameters, are presented in Section 5.3 were very significant. Some important conclusions were drawn from the analysis of these results. Thus, it was found out that some coefficients are identifiable with great accuracy even for large amounts of noise in the sea trial data. Others are less identifiable and finally some are practically unidentifiable, in the conditions investigated. On the other hand, the existence of some general rules for the performance of the extended Kalman filter was noticed. The numerical problems created by the impossibility of using double precision variables was also pointed out. The identification studies of the nonlinear model utilizes the same kind of computer program which caused that problem. Actually the other approach could be employed in this case with results eventually satisfactory. However, it would not be possible to study the two remaining linear coefficients, $(m-X_{\dot{u}})$ and X_u.

From the conclusions drawn with the study of the linear model it is not expected that the application of extended Kalman filtering to the nonlinear model can give results as good as those obtained in the identification of the linear coefficients. The numerical imprecisions will be stressed out due to the nonlinearities in the state equations. The handling of a more complex system also makes the identification less accurate.

The first set of parameters studied with the nonlinear model were $(m-x_{\dot{u}})$, x_u, $(1/2x_{uu})$, and $(1/2x_{rr}+mx_G)$ Figure 5.41 to 5.43 shows the 1% $\underline{w}$, 1%$\underline{v}$ noisy sea trial data to be used in the extended Kalman filter studies. These figures show the plots of the primary state variables, u, v, and r, respectively. The results of the extended Kalman filtering pass over the noisy data are shown in the plots of Figures 5.44 through 5.50. These plots show how the noise is filtered out of the primary states and how many parameters values for $(m-x_{\dot{u}})$, $x_v(1/2x_{uu})$, and $(1/2x_{rr}+mx_G)$ are arrived at by the filter.

The overall results of the identification are shown in Table 5.24 for the parameters $(m-x_{\dot{u}})$(1), x_u(2), $1/2x_{uu}$(16) and $(1/2x_{rr}+mx_G)$(19). The accuracy although not good is not unexpected. The only remarkable fact is the complete unidentifiability of the parameter $(1/2x_{rr}+mx_G)$. It is quite likely that this parameter is degrading the filter performance and generating their biased values for the other parameters.

Given the quality of these results, not surprising due to the reasons already explained; given also the poor results of identification of nonlinear coefficients using the model reference

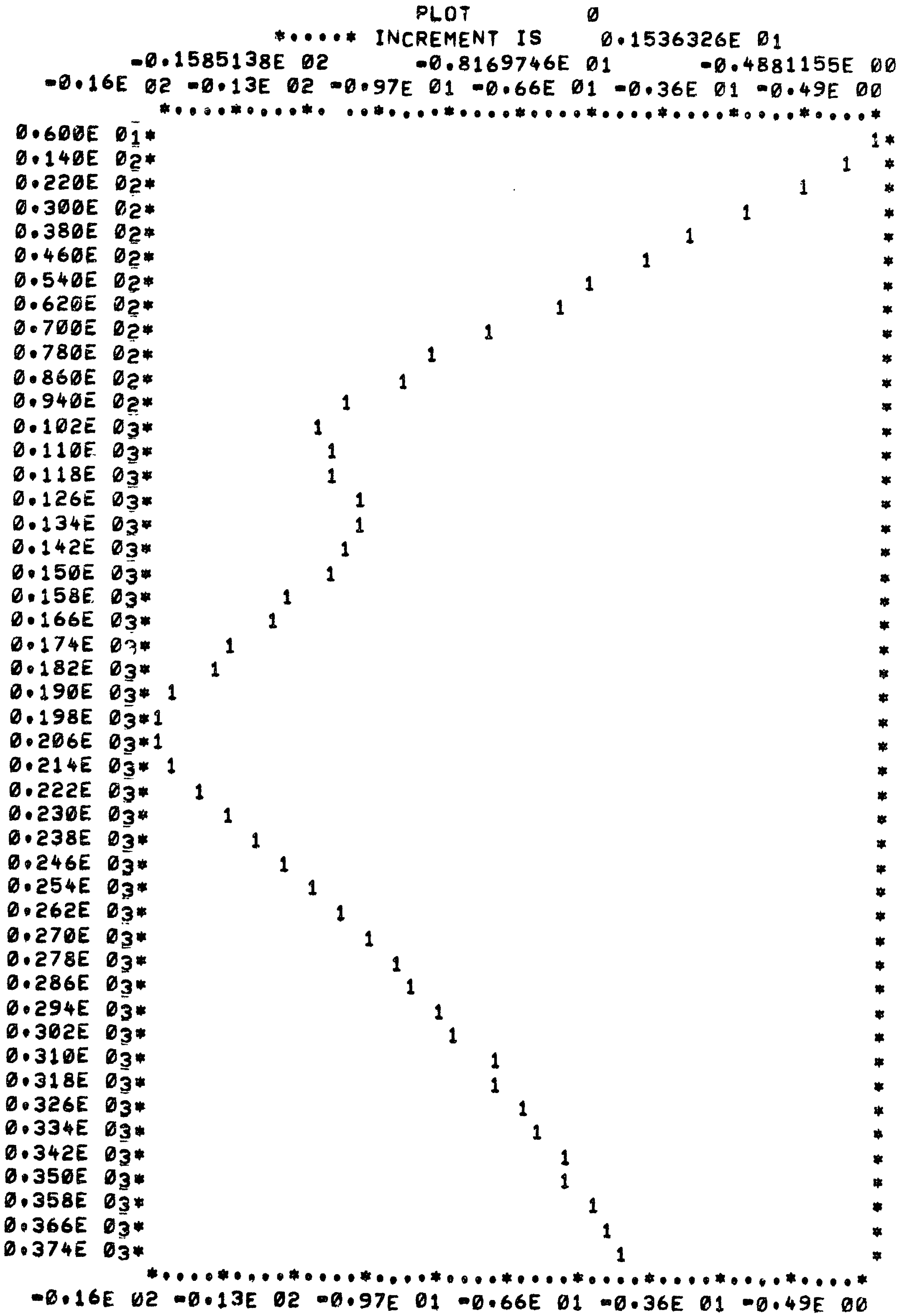


FIGURE 5.41 Surge Velocity, 1% $\underline{w}$, 1% $\underline{v}$ Noises; Nonlinear Model.

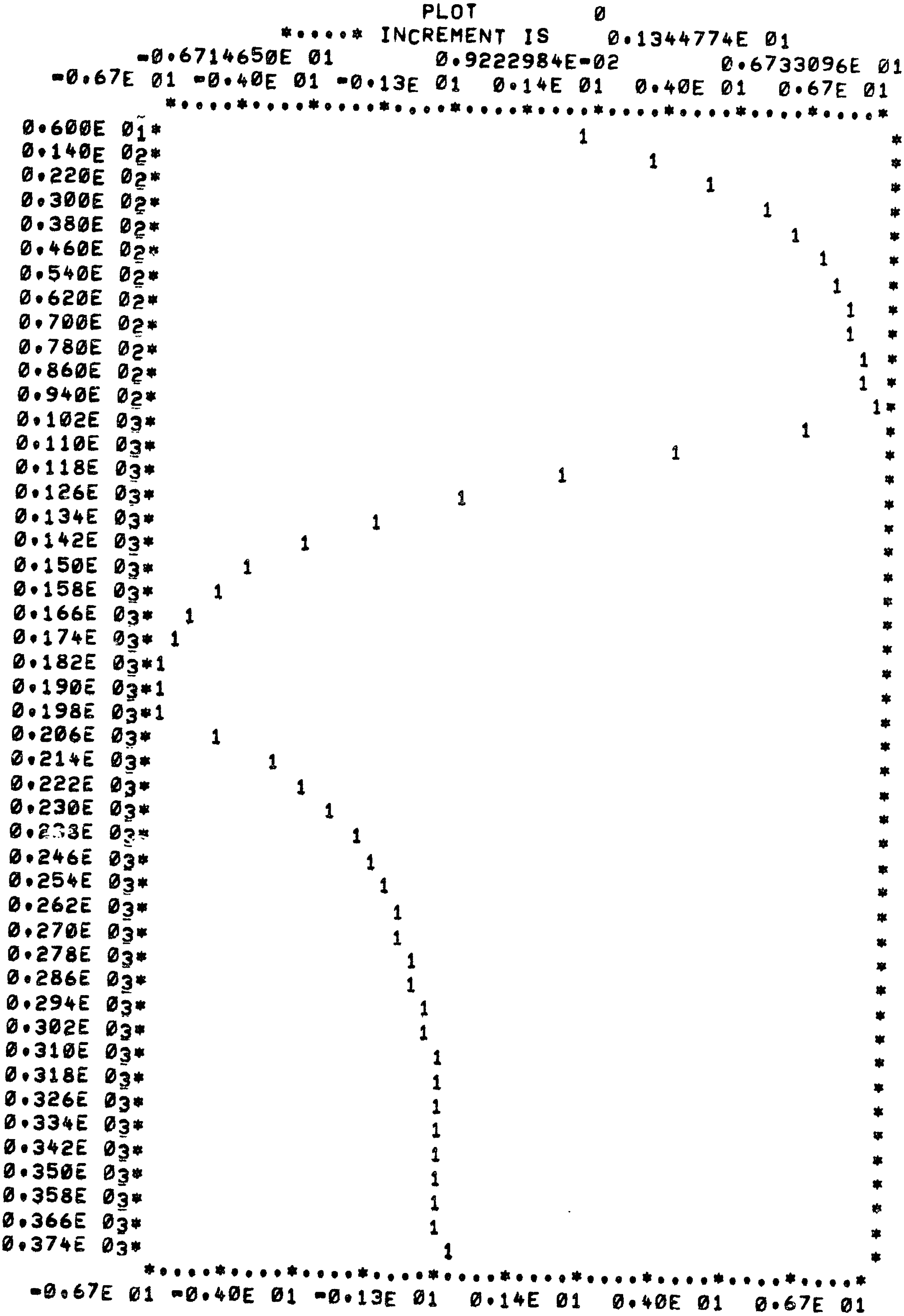


FIGURE 5.42 Sway Velocity; 1%w, 1%v Noises; Nonlinear Model

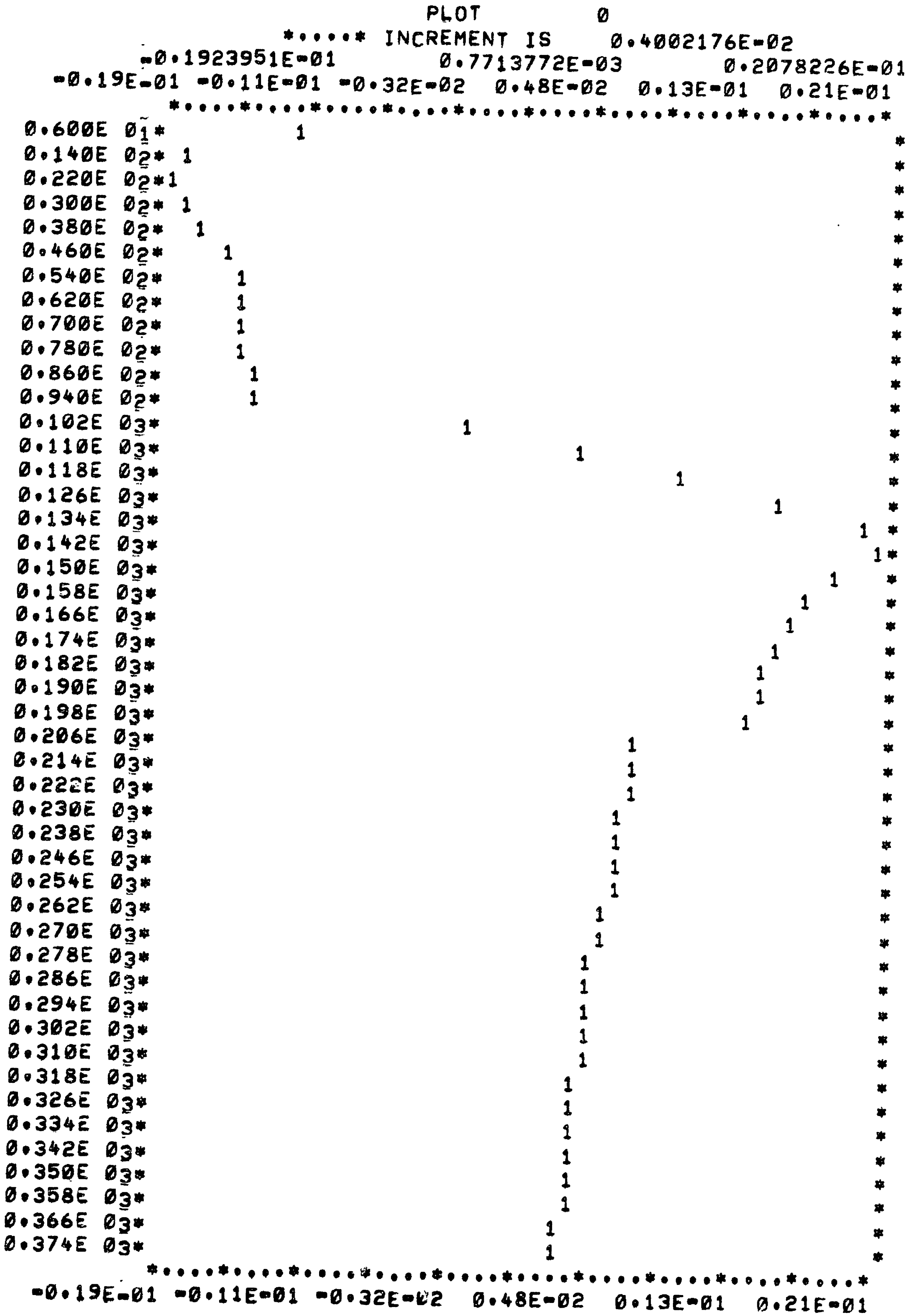


FIGURE 5.43. Yaw Velocity; 1%w, 1%v Noises; Nonlinear Model.

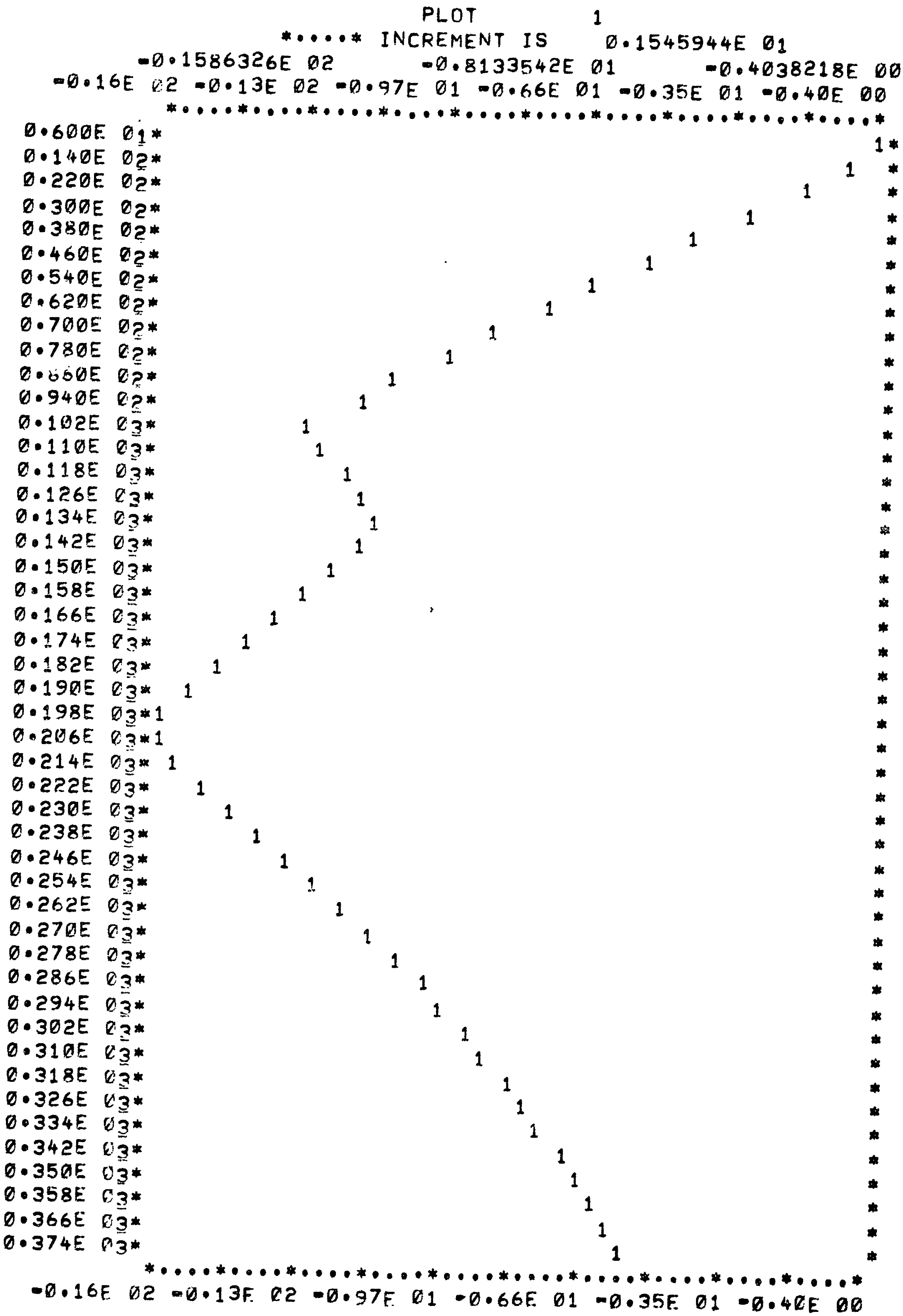


FIGURE 5.44. Kalman Filter Surge Velocity, For Figure 5.41.

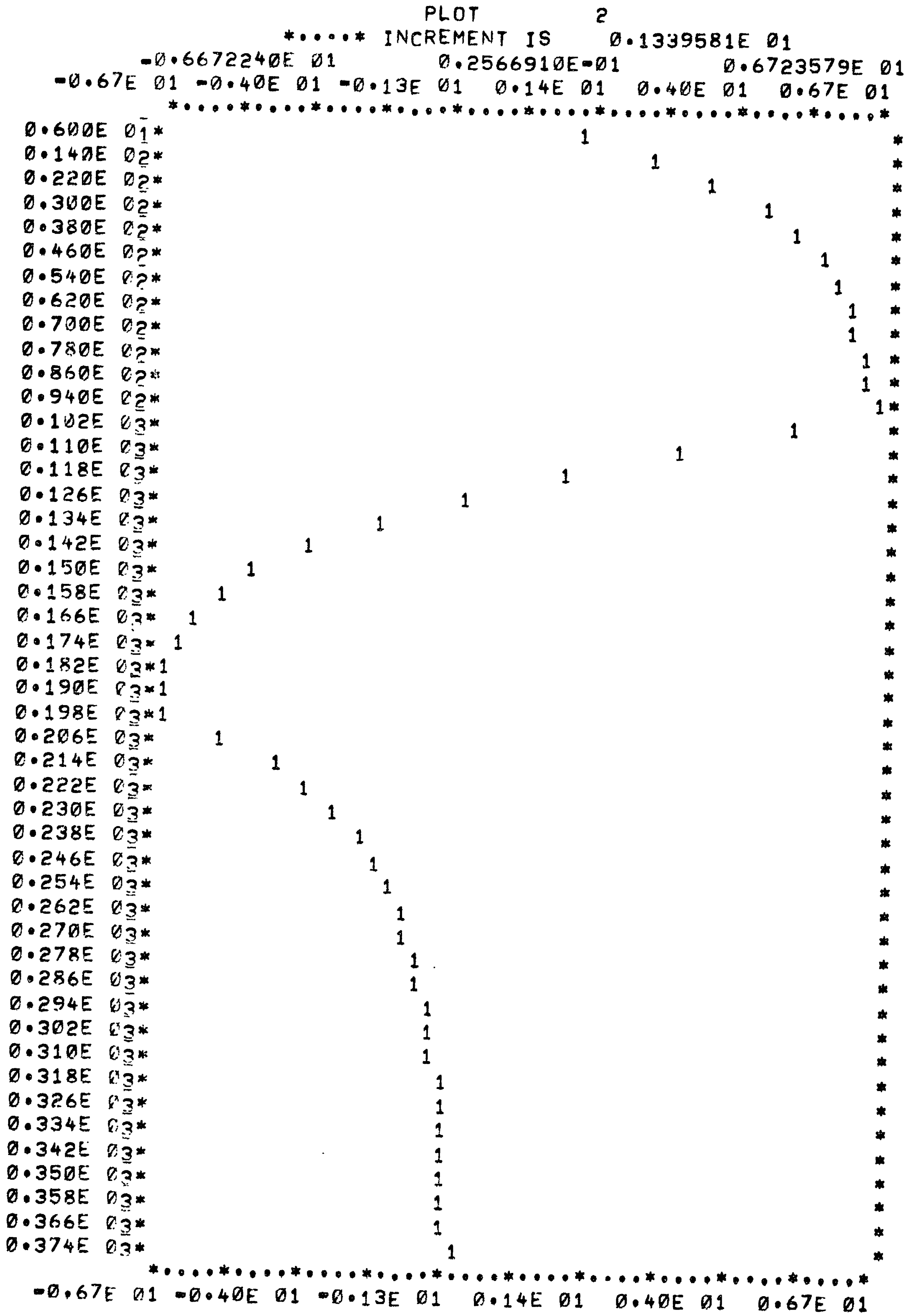


FIGURE 5.45. Kalman Filter Sway Velocity For Figure 5.42.

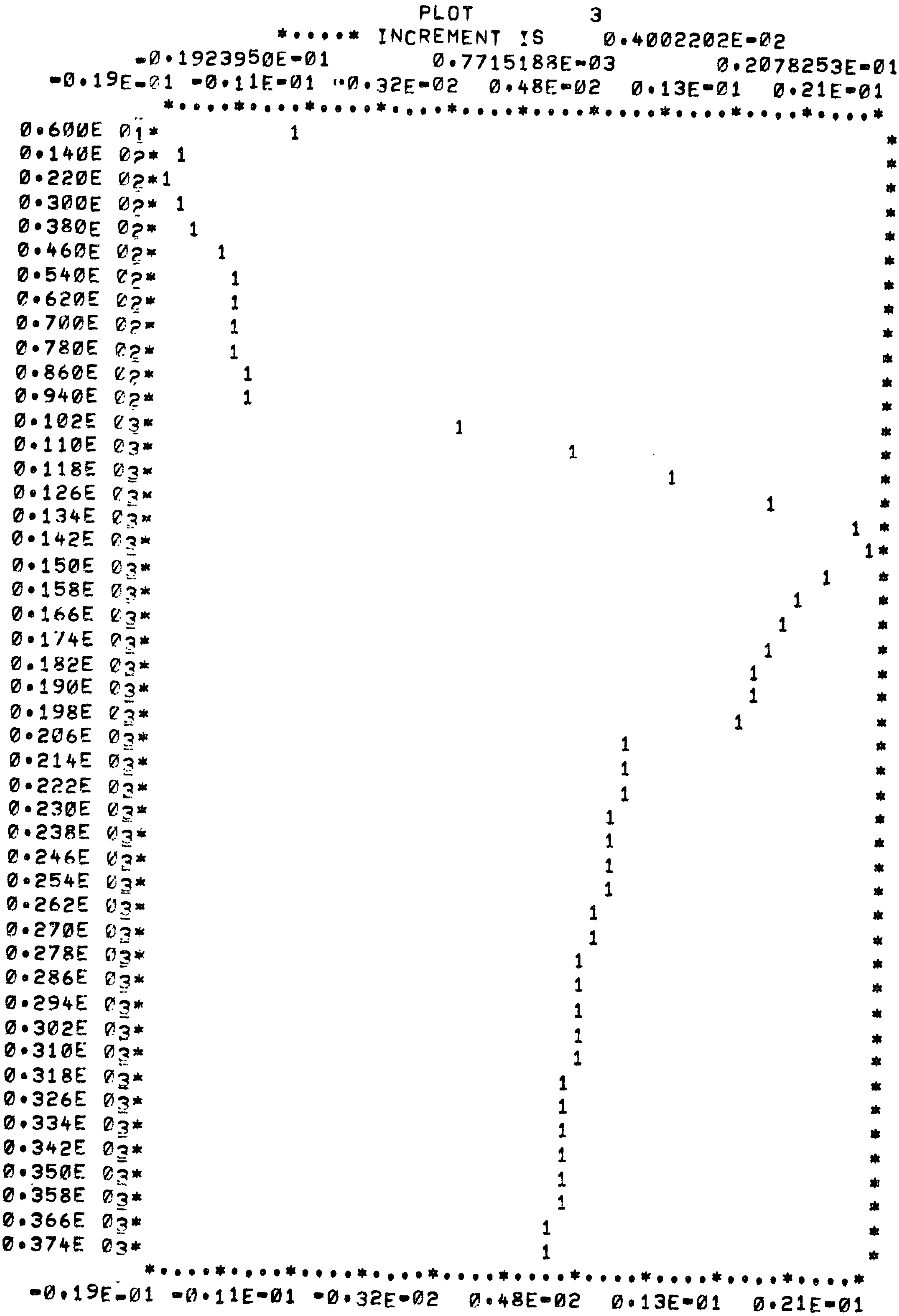


FIGURE 5.46. Kalman Filter Yaw Velcoity for Figure 5.43.

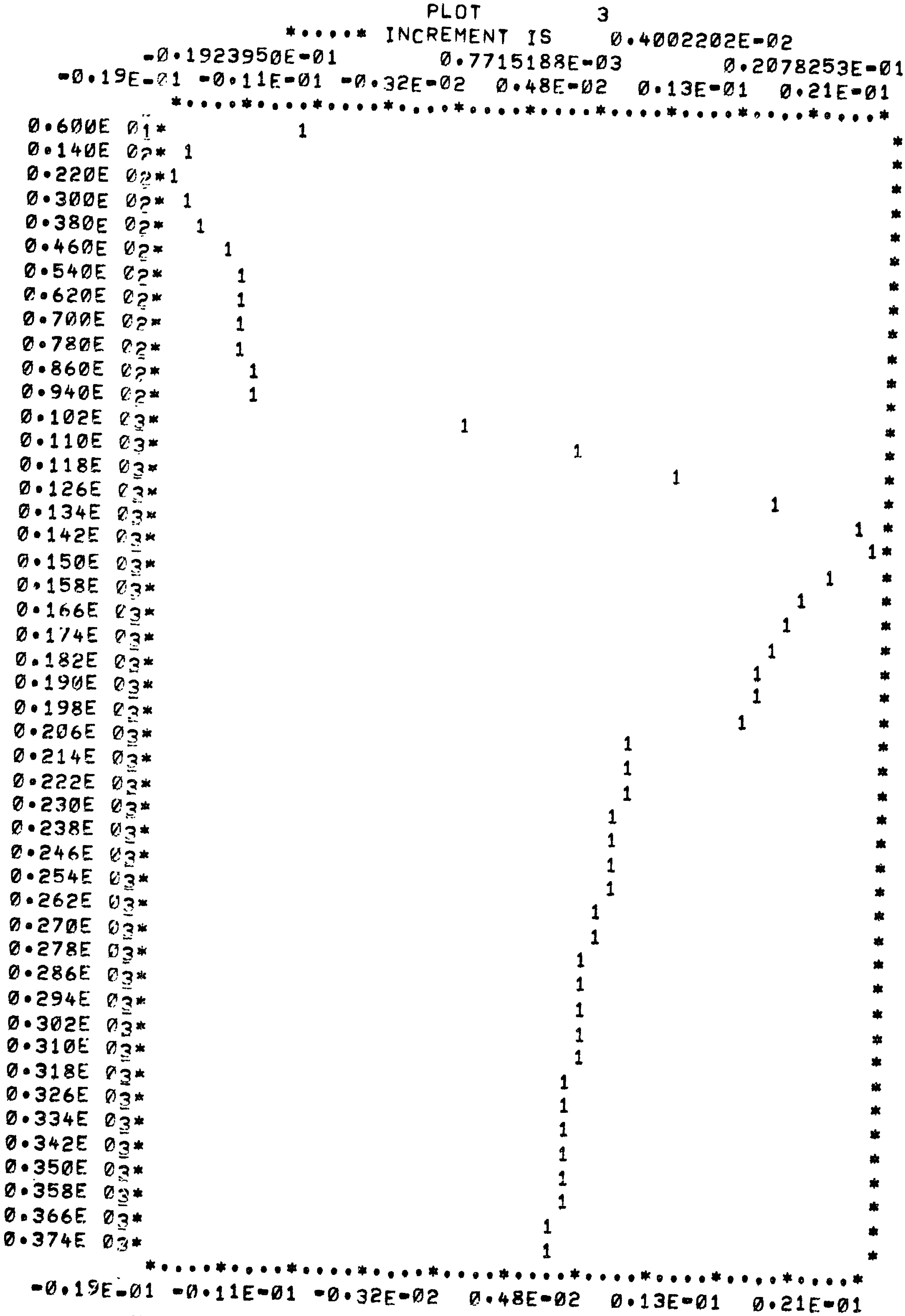


FIGURE 5.46. Kalman Filter Yaw Velcoity for Figure 5.43.

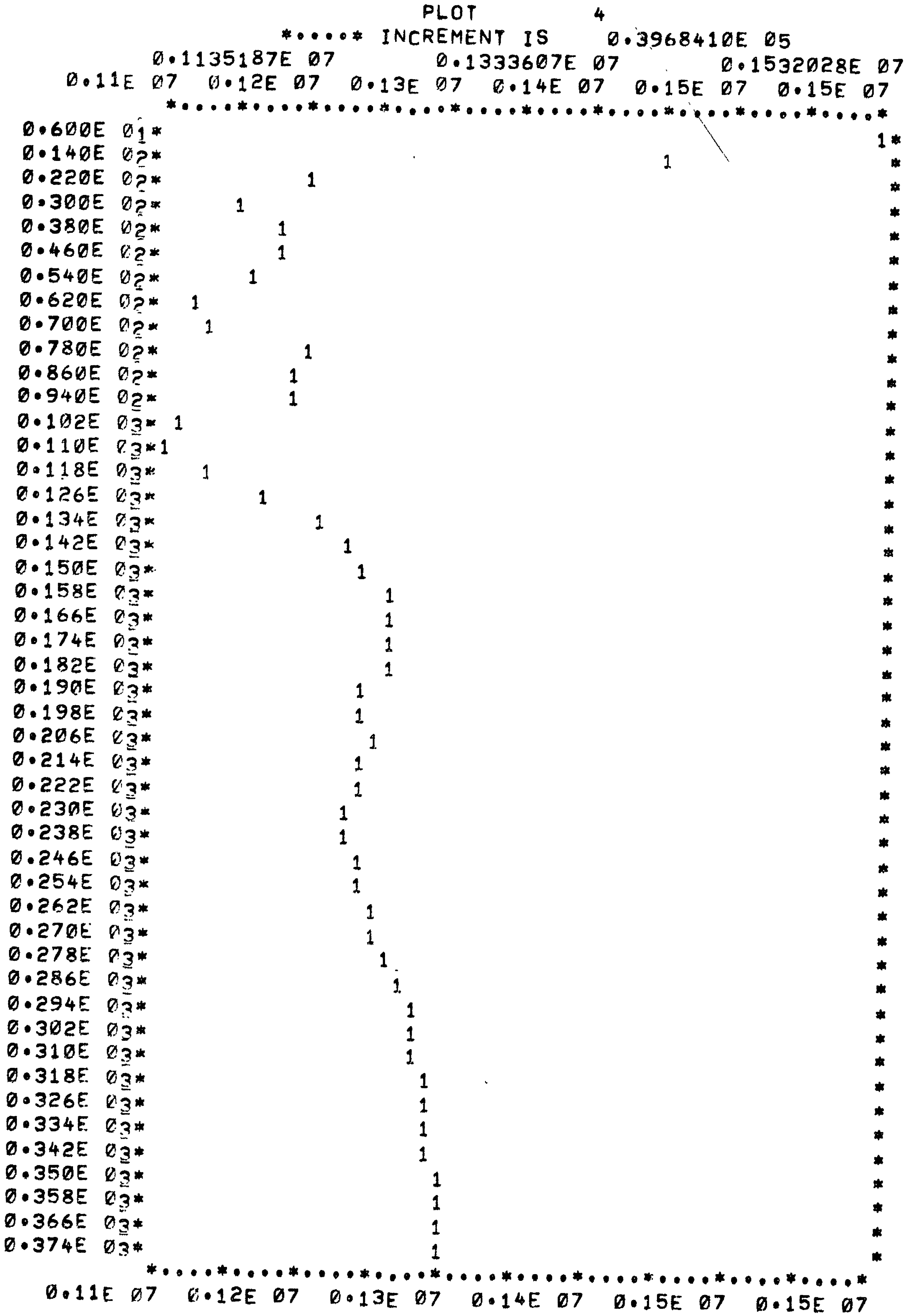


FIGURE 5.47. Identification of $(m-X_u)$, 1% $\underline{w}$, 1% $\underline{v}$ Noises, Nonlinear Model

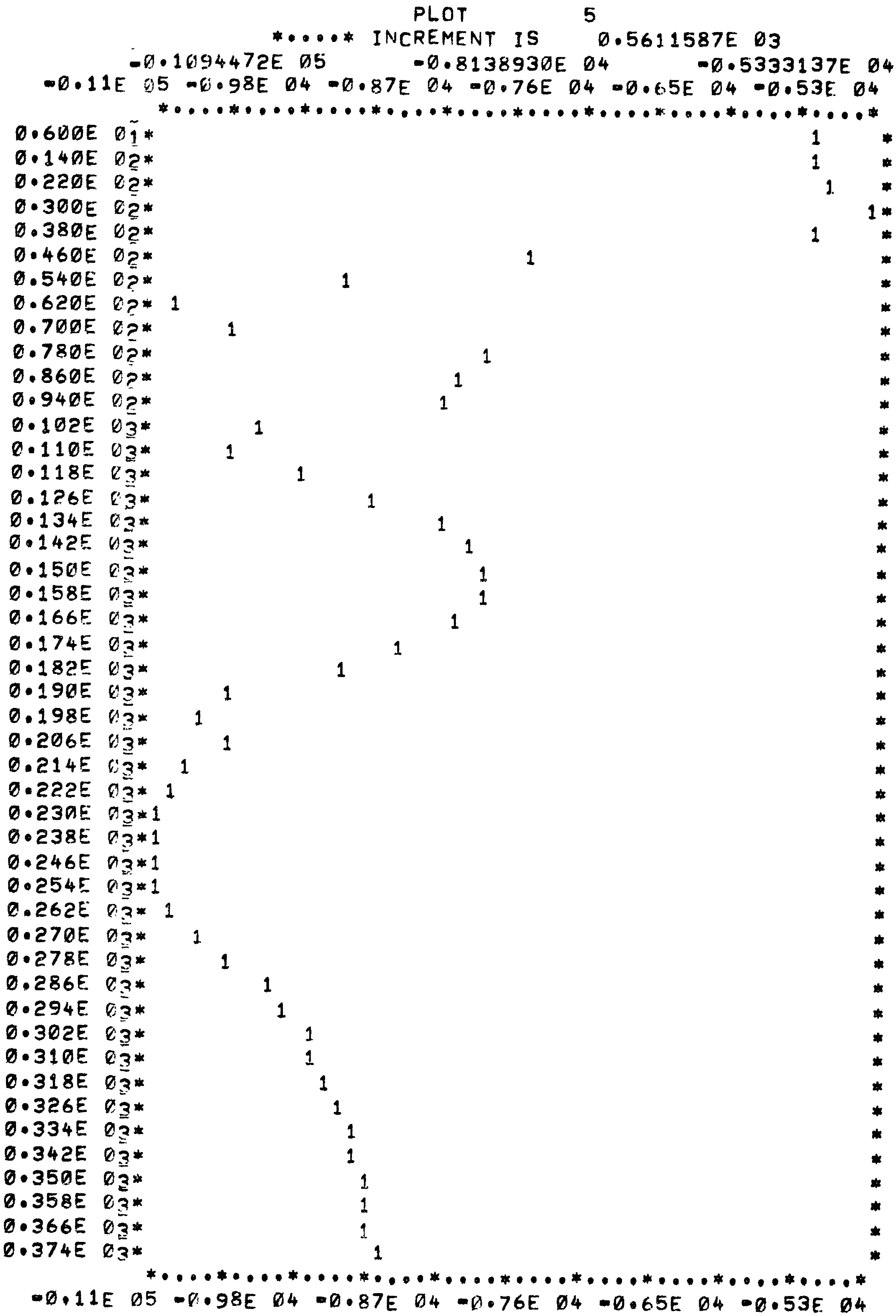


FIGURE 5.48. Identification of X_u; 1% $\underline{w}$, 1% $\underline{v}$ Noises, Nonlinear Model

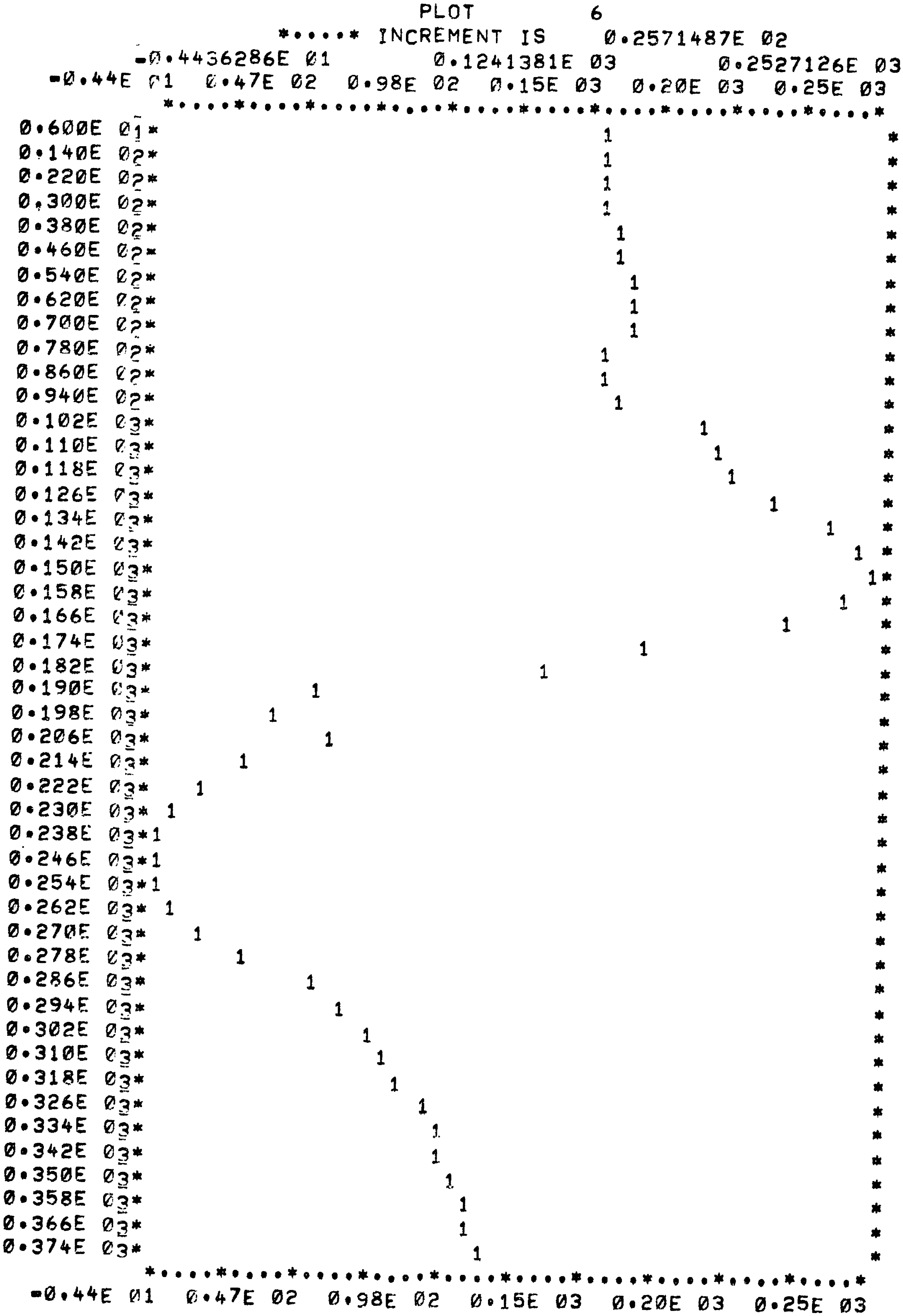


FIGURE 5.49. Identification of $1/2X_{uu}$; 1% $\underline{w}$, 1% $\underline{v}$ Noises.

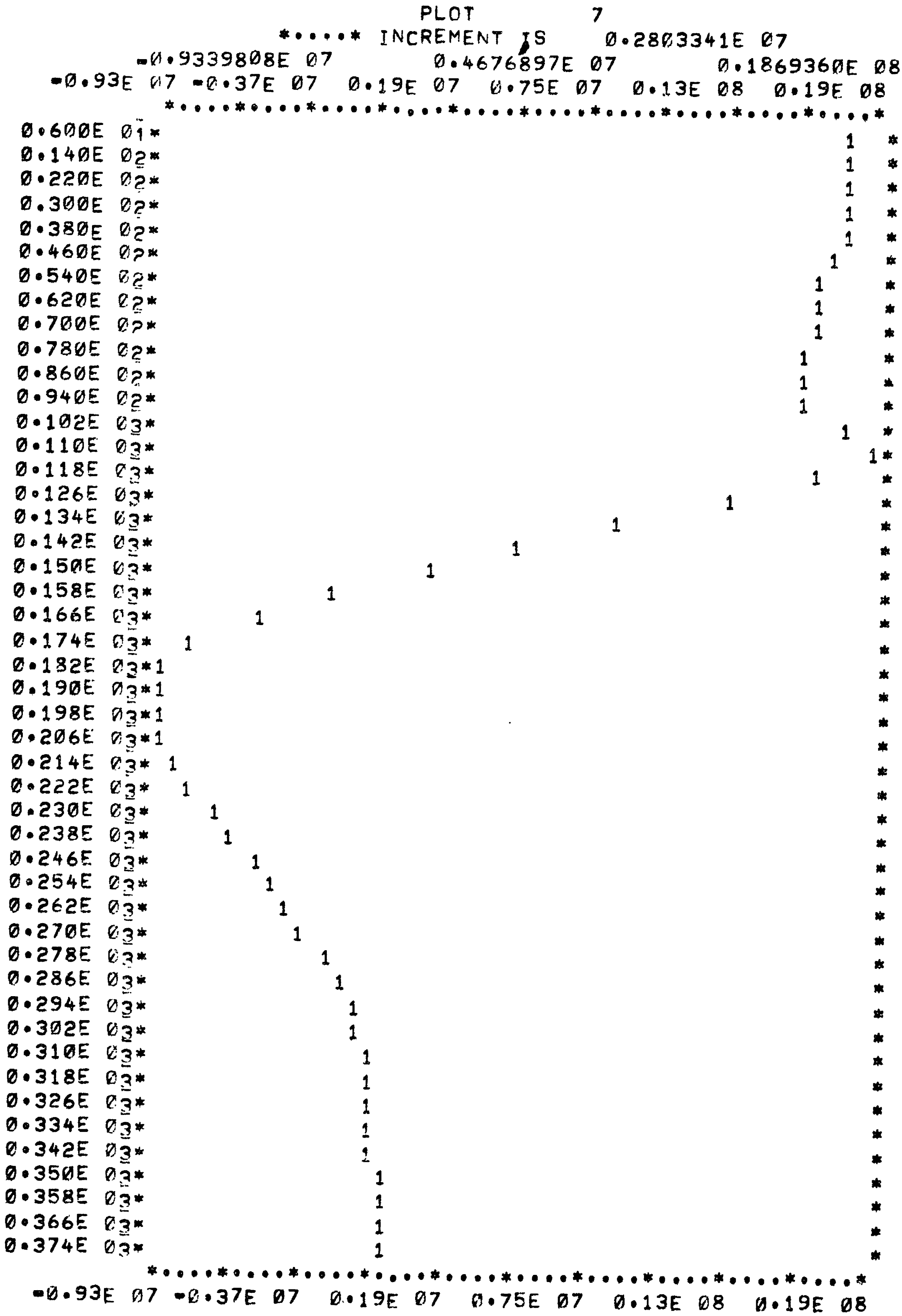


FIGURE 5.50. Identification of ($1/2\ X_{rr} + m x_G$); 1%w, 1%v Noises

TABLE 5.19

PARAMETRIC IDENTIFICATION USING KALMAN FILTER

NP = 1 TRUE VALUE = 0.12307E 07

SV = 0.16000E 07 + OR - 0.36920E 06

FV = 0.12962E 07 + OR - 0.48503E 04

NP = 2 TRUE VALUE = -0.84296E 04

SV = -0.58000E 04 + OR - 0.25289E 04

FV = -0.91243E 04 + OR - 0.11387E 03

NP = 16 TRUE VALUE = 0.12486E 03

SV = 0.16000E 03 + OR - 0.37458E 02

FV = 0.11575E 03 + OR - 0.88861E 01

NP = 19 TRUE VALUE = 0.13924E 08

SV = 0.18000E 08 + OR - 0.41773E 07

FV = -0.18182E 06 + OR - 0.32770E 07

NON LINEAR MODEL

technique, it was decided to not investigate any other nonlinear parameter. Nevertheless, it was decided to test the extended Kalman filtering with the study of a set of coefficients already analysed. Table 5.24 shows the overall results of extended Kalman filtering identification of Y_r, $(Y_r\text{-mu})$, N_v and $(N_r - mx_G u)$.

TABLE 5.25

	Y_v	$(Y_r\text{-mu})$	N_v	$(N_r\text{-}mx_G u)$
True Value	-81.515E 3	-18.503E 6	-97.735E 3	-32.510E 6
Initial Estimate	-57.060E 3± 24.454E 3	-12.955E 6± 55.525E 5	-68.414E 5± 29.321E 5	-22.752E 8± 97.531E 7
Noise	%$\underline{w}$ = 1;	% $\underline{v}$ = 1		
First Pass	-74.474E 3± 58.675E 1	-16.460E 6± 25.019E 4	-10.157E 6± 17.706E 5	-29.193E 8± 65.307E 7
Second Pass	-74.063E 3± 35.997E 1	-16.308E 6± 15.533E 4	-10.399E 6± 23.007E 5	-30.689E 8± 56.739E 7
Noise	%$\underline{w}$ = 10;	% $\underline{v}$ = 10		
Final Value	-65.965E 3± 37.550E 2	-14.067E 6± 16.484E 5	-74.459E 6± 23.007E 5	-30.689E 8± 56.739E 7

The results of this table are not unexpected and they confirm the belief that the complexity of the model added to the unsatisfactory accuracy in the numerical computation cannot provide a good parametric identification. It is recognized, however, that once the numerical problems are solved the accuracy of extended Kalman filter identification will be greatly improved.

This Chapter presented a large amount of information about the identifiability of the hydrodynamic coefficients of the Mariner class hull form. Also, some aspects of the identification techniques were discussed in face of the results obtained. Essentially most of the results are reasonably good and agree with the basic principles of ship hydrodynamics and system identification. The Chapter 6 presents the general conclusions of the identification studies developed in this thesis.

CHAPTER 6

6. CONCLUSIONS

The results of the identification studies for the hydrodynamic coefficients of the Mariner class hull form developed in this thesis were presented in Chapter 5. The analysis of these results permit one to draw some specific conclusions which were expressed along that chapter. In the present chapter all those conclusions as well as others are presented in a ordinated sequence and formulated in a broader sense. They are related not only with the identifiability of the hydrodynamic coefficients but to the scheme adopted and the identification approaches used. It is believed that a considerable amount of information was obtained from this study. Nevertheless some specific points need further investigations. There are some questions that could not be answered due to some difficiences to implement the scheme adopted. These points will be mentioned and suggestions for further work will be indicated.

6.1 Scheme of Identification

The scheme adopted for the identification studies although deserving some criticism was, in general, quite good. The idea of breaking the study in parts proved to be very right and was responsible for the successful overall results obtained.

The preliminary analysis served to eliminate from the model coefficients of negligible influence on the vehicle behavior. The importance of this decision was realized along the identification studies.

The analysis of a simpler model at the beginning, the linear model, provided most of the results obtained in this thesis. Actually, the studies developed with the model gave a good idea about the identifiability of all the linear coefficients.

On the other hand, the analysis of the complete nonlinear model did not provide such wealth of results. The identifiability of the nonlinear coefficients cannot be definitely established. Exactly at this point some criticism may be put on the scheme. It was not given the necessary attention to the question of selecting the input. If it is true that the choice was very limited for the linear model that was not the case of the nonlinear model. Although it is not sure that the input function employed is not a proper one for the identification of the ship parameters a detailed investigation of other kind of inputs should have been done. Eventually, a different input could provide a more accurate identification of the parameters.

If more results relative to the identifiability of the nonlinear coefficients were not obtained it was mainly because the proposed scheme could not be followed entirely. The proper implementation of the extended Kalman filtering was not possible by the constraint imposed by the computer capacity. This suggests that further investigation should be conducted in a computer facility more appropriate to handle the program.

6.2. Identifiability of the Hydrodynamic Coefficients

The identification scheme employed to study the coefficients although not perfect provided some useful information about the identifiability of the several parameters. The results obtained, especially for linear coefficients permit one to draw some definite conclusions about their identifiability characteristics.

There are a set of coefficients that are identifiable with high accuracy even for high degree of noise. As it would be expected they are the most important motion parameters for the horizontal maneuver of a surface ship. Actually Y_v, $(Y_r\text{-mu})$, N_v, $(N_r\text{-mx}_G\text{u})$, $(\text{m-y}_{\dot{v}})$, and $(I_z\text{-}N_{\dot{r}})$ could be identified with good accuracy in different noise conditions. The large number of results obtained particularly for the 4 first parameters give an adequate idea of their identifiability. The two other coefficients were tested using a computer program that does not provide an appropriate numerical accuracy. Even so the identifiability characteristics of those parameters were satisfactory.

Two other linear coefficients $(m-X_{\dot{u}})$ and X_u may be included in the group of the identifiable parameters. They were not investigated more extensively due to the limitations already mentioned. It is, however, quite likely that these coefficients may exhibit good identifiability characteristics if tested in proper conditions. Another set of coefficients $(mx_G-Y_{\dot{r}})$ and $(mx_G-N_{\dot{v}})$, on the other hand, exhibited relatively poor identifiability. Even recognizing that they were not investigated in adequate conditions the low identification accuracy was very conclusive.

The identifiability characteristcs of the parameters Y_δ and N_v are in an intermediate position between those of the first and the second set of coefficients. At this point a question arises concering the type of input selected. Would it be possible that with a different maneuvering trial these coefficients could be identified with a better accuracy. The answer to this question is a suggestion for future investigation.

The coefficients Y_o and N_o were kept in the mathematical model since the preliminary analysis indicated that they have some influence on the ship dynamic behavior. These coefficients, however, showed very poor identifiability characteristics and should not be included in the identification studies.

Almost nothing can be said about the nonlinear coefficients. The only results available were obtained using model reference technique, and they do not indicate good identifiability characteristics. Nevertheless, since there was not a careful investigation of the input function no definitive conclusion can be expressed. There are, however, some clues that the nonlinear coefficients exhibit poor identifiability characteristics where the sea trial data contain any amount of noise. Future investigation on this area is also recommended. In particular, it is believed that the proper application of the extended Kalman filtering could bring a better insight into the problem.

6.3. Techniques of Parametric Identification

The identification studies of this thesis were conducted by the use of two approaches of system identification. According to the designed scheme the two approaches would have been used in parallel. Following this plan the model reference technique would provide the first information about the good or poor identifiability of the several parameters. **Then,** extended Kalman filtering would be used to process the noisy trial data and give a more precise idea about the accuracy within which each coefficient is identified. Also, according to this plan much of the information got with the linear model would be employed with the more complex nonlinear model. Actually, the plan was followed only in part. By the reasons already pointed out the two techniques were not applied simultaneously and something was lost in this procedure. Thus, the model reference identification of the nonlinear coefficients was carried out without knowing the results of application of extended Kalman filtering to the nonlinear model. This was an error because it did not permit to apply the concept confirmed later that a parameter of minor importance can bias the identification of other parameters. Perhaps each nonlinear term should be studied in pair with a linear parameter already investigated. This is, however, only a hypothesis and it remains to be tested.

Some conclusions are now expressed concerning the attributes of each system identification approach. The model reference technique is essentially a comparison technique to determine the best set of parameters for a mathematical model which causes it to behave similar to the vehicle. As a general approach of system identification it is not specifically tailored to process noisy trial data, and fails to give a good indication of the true value of the parameters. The accuracy obtained with the technique is particularly low when the sea trial data contains large amount of noise. Although the conditions of application of the two approaches were not the same, the model reference identification was, in general, worse than that given by extended Kalman filtering. Resuming, it seems that the technique may be very useful when the uncertainty is very low, being more appropriate for determinstic systems.

As it was expected the extended Kalman filtering being a data processing technique that uses known characteristics of the noise in the vehicle data produced better results. The accuracy obtained in the identification of some parameters was really very good and, in general, at least satisfactory.

The implementation of the extended Kalman filtering is more difficult and requires a careful judgment of the relative importance of the various coefficients in the mathematical model. It was learnt that as a general rule the identification is better when a smaller number of parameters is investigated. There are also reasons to expect that the accuracy is better the simpler the mathematical model being used.

Although it was not tried to identify more than 4 coefficients at once, there is no reason why the extended Kalman filtering technique could not be used to study a larger number of parameters. It must be kept in mind, however, first that parameters with small influence in the system behavior should not be included in the analysis, otherwise they will bias, the identification of the other coefficients. Therefore, it is necessary to have some knowledge beforehand about the importance of the motion parameters. Secondly, it should be accepted that even if all the non important parameters are separated there is a loss in the identification accuracy when a larger number of parameters is handled.

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APPENDIX I

Hydrodynamic Coefficients

The hydrodynamic coefficients for the Mariner class hull form are presented in several references with values in general, differents. The values presented in Table A 1 in the nondimensionalized form are given by Strom-Tejsen [5]. The hydrodynamic coefficients to be inserted in the mathematical model must be dimensionalized. Table A.2 shows the dimensionalizing coefficients and Table A.3 presents the numerical values for the coefficients in the dimensionalized form. Finally, Table A.4 shows the correspondence between the usual notation of the coefficients on that employed to write the model equations.

The values of the variables that enter into the dimensionalizing coefficients are:

$$\rho = 1.9905 \text{ lbf sec.}^2/\text{ft}^4$$

$$l = 528.01 \text{ ft.}$$

$$u = 25.317 \text{ ft/sec.}$$

TABLE A.1

	NONDIMENSIONALIZED HYDRODYNAMIC COEFFICIENTS					
	X Equation		Y Equation		N Equation	
Var.	Parameter	Value $*10^{-5}$	Parameter	Value $*10^{-5}$	Parameter	Value$*10^{-5}$
$\dot{u}$	$(m-X_{\dot{u}})$	840.0				
v			$(m-Y_{\dot{u}})$	1546.0	$(mx_{\dot{G}}-N_{\dot{v}})$	-22.7
$\dot{r}$			$(mx_G-Y_{\dot{r}})$	-8.6	$(I_z-N_{\dot{r}})$	82.9
Δu	X_u	-120.0				
Δu^2	v_2X_{uu}	45.0				
Δu^3	$1/6\ X_{uuu}$	-10.3				
v			Y_v	1160.4	N_v	-263.5
v^2	$1/2\ X_{vv}$	-898.0				
v^3			$1/6\ Y_{vvv}$	-8087.2	$1/6\ N_{vvv}$	1636.1
r			(Y_r-m_u)	-499.0	(N_r-mx_{Gu})	-166.0
r^2	$(1/2X_{rr}+mx_G)$	18.0				
r^3			$1/6Y_{rrr}$	0.0	$1/6\ N_{rrr}$	0.0

TABLE A.1 (CONTINUED)

	X Equation		Y Equation		N Equation	
Var.	Parameter	Value $*10^{-5}$	Parameter	Value $*10^{-5}$	Parameter	Value $*10^{-5}$
δ			Y_{δ}	277.9	N_{δ}	-138.8
δ^2	1/2 $X_{\delta\delta}$	-94.8				
δ^3			1/6 $Y_{\delta\delta\delta}$	-90.0	1/6 $N_{\delta\delta\delta}$	45.0
v^r	$(X_{vr}+m)$	798.0				
$v\delta$	$X_{v\delta}$	93.2				
rv^2			1/2 Y_{rvv}	15,356.0	1/2 N_{rvv}	-5,483.0
δv^2			1/2 $Y_{\delta vv}$	1,189.6	1/2 $N_{\delta vv}$	-489.0
$v\delta^2$			1/2 $Y_{v\delta\delta}$	-3.8	1/2 $N_{v\delta\delta}$	12.5
O			Y_o	-3.6	N_o	2.8

TABLE A 2

DIMENSIONALIZING COEFFICIENTS						
	X Equation		Y Equations		N Equations	
Var.	Parameters	N.D. Coeff.	Parameters	N.D. Coeff.	Parameters	N.D. Coeff.
$\dot{u}$	$(m-X_{\dot{u}})$	$1/2\ \rho\ell^3$				
$\dot{v}$			$(m-Y_{\dot{v}})$	$1/2\ \rho\ell^3$	$(mx_G-N_{\dot{v}})$	$1/2\ \rho\ell^4$
$\dot{r}$			$(mx_G-Y_{\dot{r}})$	$1/2\ \rho\ell^4$	$(I_z-N_{\dot{r}})$	$1/2\ \rho\ell^5$
Δu	X_u	$1/2\ \rho\ell^2 u$				
Δu^2	$1/2X_{uu}$	$1/2\ \rho\ell^2$				
Δu^3	$1/6\ X_{uuu}$	$1/2\ \rho\ell^2/u$				
v			Y_v	$1/2\ \rho\ell^2 u$	N_v	$12/\ \rho\ell^3 u$
v^2	$1/2\ X_{vv}$	$1/2\ \rho\ell^2$				
v^3			$1/6\ Y_{vvv}$	$1/2\ \rho\ell^2/u$	$1/6\ N_{vvv}$	$1/2\ \rho\ell^4 u$
r^2	$(1/2X_{rr}+mx_G)$	$1/2\ \rho\ell^4$				

TABLE A 2 (CONTINUED)

	X Equation		Y Equations		N Equations	
Var.	Parameters	N.D. Coeff.	Parameters	N.D. Coeff.	Parameters	N.D. Coeff.
r^3			$1/6\ Y_{rrr}$	$1/2\ \rho\ell^s/u$	$1/6N_{rrr}$	$1/2\ \rho\ell^6/u$
δ			Y_δ	$1/2\ \rho\ell^2u^2$	N_v	$1/2\ \rho\ell^3u^2$
δ^2	$1/2X_{\delta\delta}$	$1/2\ \rho\ell^2u^2$				
δ^3			$1/6\ Y_{\delta\delta\delta}$	$1/2\ \rho\ell^2u^2$	N_v	$1/2\ \rho\ell^3u^2$
v^r	$X_{vr}+m$	$1/2\ \rho\ \ell^3$				
$v\delta$	$X_{v\delta}$	$1/2\ \rho\ell^2u$				
rv^2			$1/2\ Y_{rvv}$	$1/2\ \rho\ell^3/u$	$1/2\ N_{rvv}$	$1/2\ \rho\ell^4/u$
δv^2			$1/2\ Y_{\delta vv}$	$1/2\ \rho\ell^2$	$1/2\ N_{\delta vv}$	$1/2\ \rho\ell^3$
$v\delta^2$			$1/2\ Y_{v\delta\delta}$	$1/2\ \rho\ell^2u$	$1/2\ N_{v\delta\delta}$	$1/2\ \rho\ell^3u$
0			Y_o	$1/2\ \rho\ell^2u^2$		$1/2\ \rho\ell^3u^2$

TABLE A 3

DIMENSIONALIZED HYDRODYNAMIC COEFFICIENTS						
	X Equation		Y Equation		N Equation	
Var.	Parameter	Value $*10^{-3}$	Parameter	Value $*10^{-3}$	Parameter	Value $*10^{-5}$
$\dot{u}$	$(m-X_{\dot{u}})$	1230.68				
$\dot{v}$			$(m-Y_{\dot{v}})$	2265.04	$(mx_G-N_{\dot{v}})$	-175.60
$\dot{r}$			$(mx_G-Y_{\dot{r}})$	-6652.70	$(I_z-N_{\dot{r}})$	338,608.0
Δu	X_u	-8.429				
Δu^2	$1/2\ X_{uu}$	0.1248				
Δu^3	$1/6\ X_{uuu}$	-0.00113				
v			Y_v	-81.515	N_v	-97.735
v^2	$1/2\ X_{vv}$	-2.492				
v^3			$1/6\ Y_{vvv}$	-0.8853	$1/6\ N_{vvv}$	0.947
r			(Y_r-mu)	-18,508.4	$(N_r-mx_G u)$	-32,510.3
r^2	$(1/2\ X_{rr}+mx_G)$	13,924.3				

TABLE A 3 (CONTINUED

	X Equation		Y Equation		N Equation	
Var.	Parameter	Value $*10^{-3}$	Parameter	Value $*10^{-3}$	Parameter	Value $*10^{-5}$
r^2	$(1/2\ X_{rr}+mx_G)$	13,924.3				
r^3						
δ			Y_δ	494.23	$-N_\delta$	-1,303.3
δ^2	$1/2\ X_{\delta\delta}$	-168.59				
δ^3			$1/6\ Y_{\delta\delta\delta}$	-160.06	$1/6\ N_{\delta\delta\delta}$	422.56
vr	$(X_{vr}+m)$	1,169.15				
$v\delta$	$X_{v\delta}$	6.547				
rv^2			$1/2\ Y_{rvv}$	88.863	$1/2\ N_{rvv}$	-1,675.3
δv^2			$1/2\ Y_{\delta vv}$	3.308	$1/2\ N_{\delta vv}$	-7.164
$v\delta^2$			$1/2\ Y_{v\delta\delta}$	-0.2669	$1/2\ N_{v\delta\delta}$	4.636
0			Y_o	-6.404	N_o	26.293

TABLE A 4

NOTATION FOR THE EQUATIONS

	X Equation		Y Equation		N Equation	
Var.	Parameter	Symbol	Parameter	Symbol	Parameter	Symbol
$\dot{u}$	$(m-X_{\dot{u}})$	A(1)				
$\dot{v}$			$(m-Y_{\dot{v}})$	A(4)	$(mx_G-N_{\dot{v}})$	A(10)
$\dot{r}$			$(mx_G-Y_{\dot{r}})$	A(5)	$(I_z-N_{\dot{r}})$	A(11)
Δu	X_u	A(2)				
Δu^2	$1/2\ X_{uu}$	A(16)				
Δu^3	$1/6\ X_{uuu}$	A(17)				
v			Y_v	A(6)	N_v	A(12)
v^2	$1/2\ X_{vv}$	A(18)				
v^3			$1/6\ Y_{vvv}$	A(25)	$1/6\ N_{vvv}$	A(30)
r			(Y_r-mu)	A(7)	$(N_r-mx_G u)$	A(13)
r^2	$(1/2\ X_{rr}+mx_G)$	A(19)				
r^3			$1/6\ Y_{rrr}$		$1/6\ N_{vvv}$	

TABLE A 4 (CONTINUED)

Var.	X Equation Parameter	X Equation Symbol	Y Equation Parameter	Y Equation Symbol	N Equation Parameter	N Equation Symbol
δ			Y_δ	A(8)	N_δ	A(14)
δ^2	$1/2\ X_{\delta\delta}$	A(20)				
δ^3		.	$1/6\ Y_{\delta\delta\delta}$	A(26)	$1/6\ N_{\delta\delta\delta}$	A(31)
v_r	$(X_{vr}+m)$	A(21)				
$v\delta$	$X_{v\delta}$	A(22)				
rv^2			$1/2\ Y_{rvv}$	A(27)	$1/2\ N_{rvv}$	A(32)
δv^2			$1/2\ Y_{\delta vv}$	A(28)	$1/2\ N_{\delta vv}$	A(33)
$v\delta^2$			$1/2\ Y_{v\delta\delta}$	A(29)	$1/2\ N_{v\delta\delta}$	A(34)
0			Y_o	A(9)	N_o	A(15)

APPENDIX 2

```
C *******************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C ******* PRELIMINARY ANALYSIS ******
C
C *******************************************************
C
C
      SUBROUTINE EQSIM
      COMMON T,DT,Y(2Ø),F(2Ø),STIME,FTIME,NEWDT,IFWRT,N
      IF (NEWDT) 1,2,2
    1 CONTINUE
      KI=8
      KO=5
      READ (KI,5) A1,A2,A3,A5,A6,A7
      READ (KI,5) A8,B1,B2,B3,B5,B6
      READ (KI,5) B7,B8,B9,C1,C2,C3
      READ (KI,6) C5,C6,C7,C8,C9
      READ (KI,7) A4,A9,B4,B1Ø
      READ (KI,7) BØ,C4,C1Ø,CØ
      READ (KI,8) DI
    5 FORMAT (6E13.4)
    6 FORMAT (5E13.4)
    7 FORMAT (4E13.4)
    8 FORMAT (F1Ø.3)
      DI = DI/57.296
      UØ = 25.317
    2 CONTINUE
      Y(7) = DI
      G1 = 1./(B1*C2-B2*C1)
      G2 = A2*Y(1)+A3*Y(1)**2.+A4*Y(1)**3.+A5*Y(2)**2.+A6*Y(3)**2.+A7*Y(
```

```
17)**2.+A8*Y(2)*Y(3)+A9*Y(2)*Y(7)
 G3 = B0+B3*Y(2)+B4*Y(2)**3.+B5*Y(3)+B6*Y(7)+B7*Y(7)**3.+B8*Y(3)*Y(
12)**2.+B9*Y(7)*Y(2)**2.+B10*Y(2)*Y(3)**2.
 G4 = C0+C3*Y(2)+C4*Y(2)**3.+C5*Y(3)+C6*Y(7)+C7*Y(7)**3.+C8*Y(3)*Y(
12)**2.+C9*Y(7)*Y(2)**2.+C10*Y(2)*Y(3)**2.
 F(1) = 1./A1*G2
 F(2) = G1*(C2*G3-B2*G4)
 F(3) = G1*(B1*G4-C1*G3)
 F(4) = Y(3)
 F(5) = (Y(1)+U0)*COS(Y(4))-Y(2)*SIN(Y(4))
 F(6) = (Y(1)+U0)*SIN(Y(4))+Y(2)*COS(Y(4))
 Y(8) = F(1)
 Y(9) = F(2)
 Y(10) = F(3)
 RETURN
 END
```

```
C
C
C ********************************************************
      SUBROUTINE EQSIM
      COMMON T,DT,Y(20),F(20),STIME,FTIME,NEWDT,IFWRT,N
      IF (NEWDT) 1,2,2
    1 CONTINUE
      KI=8
      KO=5
      READ (KI,5) A1,A2,A3,A5,A6,A7
      READ (KI,5) A8,B1,B2,B3,B5,B6
      READ (KI,5) B7,B8,B9,C1,C2,C3
      READ (KI,6) C5,C6,C7,C8,C9
      READ (KI,7) A4,A9,B4,B10
      READ (KI,7) B0,C4,C10,C0
      READ (KI,8) DI
    5 FORMAT (6E13.4)
    6 FORMAT (5E13.4)
    7 FORMAT (4E13.4)
    8 FORMAT (F10.3)
      DI = DI/57.296
      U0 = 25.317
    2 CONTINUE
      IF (T-90.0) 3,3,4
    3 Y(7) = D1
      GO TO 10
    4 Y(7) = -D1
   10 CONTINUE
      G1 = 1./(B1*C2-B2*C1)
      G2 = A2*Y(1)+A3*Y(1)**2.+A4*Y(1)**3.+A5*Y(2)**2.+A6*Y(3)**2.+A7*Y(
     17)**2.+A8*Y(2)*Y(3)+A9*Y(2)*Y(7)
      G3 = B0+B3*Y(2)+B4*Y(2)**3.+B5*Y(3)+B6*Y(7)+B7*Y(7)**3.+B8*Y(3)*Y(
```

```
12)**2.+B9*Y(7)*Y(2)**2.+B10*Y(2)*Y(3)**2.
 G4 = C0+C3*Y(2)+C4*Y(2)**3.+C5*Y(3)+C6*Y(7)+C7*Y(7)**3.+C8*Y(3)*Y(
12)**2.+C9*Y(7)*Y(2)**2.+C10*Y(2)*Y(3)**2.
 F(1) = 1./A1*G2
 F(2) = G1*(C2*G3-B2*G4)
 F(3) = G1*(B1*G4-C1*G3)
 F(4) = Y(3)
 F(5) = (Y(1)+U0)*COS(Y(4))-Y(2)*SIN(Y(4))
 F(6) = (Y(1)+U0)*SIN(Y(4))+Y(2)*COS(Y(4))
 Y(8) = F(1)
 Y(9) = F(2)
 Y(10) = F(3)
 RETURN
 END
```

APPENDIX 3

```
      SUBROUTINE PLOT(NO,A,N,M,NS)
C
C     ******************************************************
C
C        SUBROUTINE PLOT
C
C        PURPOSE
C           PLOT SEVERAL CROSS VARIABLES Y VERSUS A BASE
C           VARIABLE X IN A FORMAT SUITABLE FOR THESIS USE
C
C        USAGE
C           CALL PLOT(NO,A,N,M,NS)
C
C        DESCRIPTION OF PARAMETERS
C           NO - PLOT NUMBER OF .LE.  3 DIGITS
C           A  - MATRIX OF DATA TO BE PLOTTED. MUST BE IN
C                STANDARD SINGLE COLUMN FORM. FIRST COLUMN
C                REPRESENTS BASE VARIABLE AND SUCCESSIVE
C                COLUMNS ARE THE CROSS VARIABLES (MAXIMUM IS
C                NINE). BASE VARIABLE IS VERTICAL.
C           N  - NUMBER OF ROWS IN MATRIX A. N MUST BE
C                .LE.  47
C           M  - NUMBER OF COLUMNS IN MATRIX A. M MUST BE
C                .LE   10
C           NS - CODE FOR SORTING THE BASE VARIABLE DATA IN
C                ASCENDING ORDER
C                  0   SORTING IS NOT NECESSARY(ALREADY IN
C                      ASCENDING ORDER)
C                  1   SORTING IS NECESSARY
C
C     ******************************************************
C
```

```
      DIMENSION OUT(51),YPR(6),IANG(9),A(1),YPT(3)
      INTEGER*2 OUT,IANG,BLANK
      DATA IANG /'1 ','2 ' ,'3 ','4 ','5 ','6 ','7 ','8 ','9 '/
      DATA BLANK /'  '/
C     FORMAT STATEMENTS FOR THESIS USE
    1 FORMAT(1H1,27X,7H  PLOT ,I8)
    2 FORMAT(1H ,E10.3,'*',51A1,'*')
    3 FORMAT(1H ,10X,':',51X,':')
    4 FORMAT(1H ,18X,' *....* INCREMENT IS  ',E15.7)
    5 FORMAT(1H ,8X,E15.7,5X,E15.7,5X,E15.7)
    7 FORMAT(1H ,11X,36H*....*....*....*....*....*....*....*,
     115H....*....*....*)
    8 FORMAT(1H ,3X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,
     1E9.2)
    9 FORMAT(1H )
      NL=47
      NTH=51
      NLL=NL
      IF(NS)16,16,10
C     SORTING ROUTINE
   10 DO 15 I=1,N
      DO 14 J=I,N
      IF(A(I)-A(J))14,14,11
   11 L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=A(L)
      A(L)=A(LL)
   12 A(LL)=F
   14 CONTINUE
```

```
   15 CONTINUE
   16 CONTINUE
C     FIND BASE AND CROSS VARIABLE SCALES
      XSCAL=(A(N)-A(1))/(FLOAT(NLL-1))
      M1=N+1
      YMAX = -1.0E37
      YMIN = 1.0E37
      M2=M*N
      DO 40 J=M1,M2
      IF (A(J) .GT.YMAX) YMAX= A(J)
      IF (A(J) .LT. YMIN) YMIN = A(J)
   40 CONTINUE
      YSCAL=(YMAX-YMIN)/50.0
      IF(YSCAL.EQ.0.)YSCAL=1.0E-37
      YPR(1)=YMIN
      DO 90 KN=1,4
      YPR(KN+1)=YPR(KN)+YSCAL*10.0
   90 CONTINUE
      YPR(6)=YMAX
      YPT(1)=YMIN
      YSTAR=YSCAL*5.0
      YPT(2)=YMIN+YSCAL*25.0
      YPT(3)=YMAX
C     PRINT HEADING AND CROSS VARIABLE SCALE
      WRITE(5,1)NO
      WRITE(5,4)YSTAR
      WRITE(5,5)(YPT(IP),IP=1,3)
      WRITE(5,8)(YPR(IP),IP=1,6)
      WRITE(5,7)
C     FIND BASE VARIABLE PRINT POSITION
      XB=A(1)
      L=1
```

```
      MY=M-1
      I=1
      XEPS=XSCAL/FLOAT(2*(NLL-1))
   45 F=FLOAT(I-1)
      XPR=XB+F*XSCAL
      XDIF = A(L)-XPR-XEPS
      IF(XDIF)50,50,70
C     FIND CROSS VARIABLES
   50 DO 55 IX=1,NTH
      OUT(IX) = BLANK
   55 CONTINUE
      DO 60 J=1,MY
      LL=L+J*N
      JP = ((A(LL)-YMIN)/YSCAL)+1.0
      OUT(JP)=IANG(J)
   60 CONTINUE
C     PRINT LINE AND CLEAR, OR SKIP
      WRITE(5,2)XPR,(OUT(IZ),IZ=1,NTH)
      L=L+1
      GO TO 80
   70 WRITE(5,3)
   80 I=I+1
      IF(I-NLL)45,84,86
   84 XPR=A(N)
      GO TO 50
C     PRINT BOTTOM AND CROSS VARIABLE SCALE
   86 WRITE(5,7)
      WRITE(5,8)(YPR(IP),IP=1,6)
      WRITE(5,9)
      RETURN
      END
```

```
      SUBROUTINE CONTUR(NO,X,Y,7,N,M,NS)
C
C     ****************************************************
C
C        SUBROUTINE CONTUR
C
C        PURPOSE
C           PLOT SEVERAL CONTOURS OF A Z VARIABLE VERSUS TWO
C           BASE VARIABLES X AND Y IN A FORM SUITABLE FOR
C           THESIS USE
C
C        USAGE
C           CALL CONTUR(NO,X,Y,Z,N,M,NS)
C
C        DESCRIPTION OF PARAMETERS
C           NO - CONTOUR NUMBER OF .LE.  3 DIGITS
C           X  - N VECTOR OF BASE VARIABLES, VERTICAL
C           Y  - M VECTOR OF BASE VARIABLES, HORIZONTAL
3           Z  - N*M VECTOR OF CONTOUR VARIABLES STORED
C                IN COLUMNWISE VECTOR FORM
C           N  - NUMBER OF ROWS IN Z. N MUST BE .LE.  47
C           M  - NUMBER OF COLUMNS IN Z. M MUST BE .LE.  51
C           NS - CODE FOR DATA SORTING IN ASCENDING ORDER
C                   Ø - NO SORTING
C                   1 - SORT X
C                   2 - SORT Y
C                   3 - SORT BOTH X AND Y
C
C                SUBROUTINE CONTUR HAS TWO OUTPUTS
C                   1 - BOX LISTING THE CONTOUR PARAMETERS
C                       ON ONE PAGE
C                   2 - CONTOUR PLOTTED ON THE NEXT PAGE
```

```
C
C     ********************************************************
C
      DIMENSION X(1),Y(1),Z(1)
      DIMENSION OUT(51),YPR(6),YPT(3)
      DIMENSION IANG(21),ZD(21)
      INTEGER*2 OUT,IANG,BLANK
      DATA BLANK /'  '/
      DATA IANG /'1 ','2 ','3 ','4 ','5 ','6 ','7 ','8 ','9 ','A ','B ',
     1'C ','D ','E ','F ','G ','H ','J ','K ','L ','M '/
C     FORMAT STATEMENTS FOR THESIS USE
    1 FORMAT(1H1,27X,7HCONTOUR,1X,I8)
    2 FORMAT(1H ,E10.3,'*',51A1,'*')
    3 FORMAT(1H ,10X,':',51X,':')
    4 FORMAT(1H1,6X,36H************************************,
     115H***************)
    5 FORMAT(1H ,5X,'*',12X,7HCONTOUR,1X,I3,12H  PARAMETERS,
     116X,'*')
    6 FORMAT(1H ,5X,'*',10H X RANGE :,E11.4,4H TO ,E11.4,
     14H DX=,E11.4,'*')
    7 FORMAT(1H ,11X,36H*....*....*....@....*....*....*....*,
     115H....*....*....*)
    8 FORMAT(1H ,3X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,E9.2,1X,
     1E9.2)
    9 FORMAT(1H ,5X,'*',10H Y RANGE :,E11.4,4H TO ,E11.4,
     14H DY=,E11.4,'*')
   13 FORMAT(1H,5X,'*',10H Z DOMAIN:,E11.4,4H TO ,E11.4,
     14H DZ=,E11.4,'*')
   17 FORMAT(1H ,5X,'*',27H Z DOMAINS FOR THE CONTOURS,
     121H :MAX VALUES FOR EACH,3X,'*')
   20 FORMAT(1H ,5X,'*',4H NO.,I2,E11.4,4H NO.,I2,
     1E11.4,4H NO.,I2,E11.4,'*')
```

```
   23 FORMAT(1H ,6X,36H************************************,
     115H***************)
   6Ø FORMAT(1H ,18X,' *....* INCREMENT IS  ',E15.7)
   61 FORMAT(1H ,8X,E15.7,5X,E15.7,5X,E15.7)
      NL=47
      NTH=51
      ZCON=1.99
      NCONT=21
      NLL=NL
C     SORTING ROUTINES
      NSKIP=Ø
      IF(NS-1)1Ø5,1Ø1,1Ø2
  1Ø2 IF(NS-2)1Ø5,1Ø3,1Ø4
  1Ø4 NSKIP=1
C     SORT X
  1Ø1 DO 15 I=1,N
      DO 14 J=I,N
      IF(X(I)-X(J))14,14,11
   11 F=X(I)
      X(I)=X(J)
      X(J)=F
      L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=Z(L)
      Z(L)=Z(LL)
      Z(LL)=F
   12 CONTINUE
   14 CONTINUE
   15 CONTINUE
```

```
      IF(NSKIP)105,105,103
C     SORT Y
  103 DO 25 I=1,M
      DO 24 J=I,M
      IF(Y(I)-Y(J))24,24,21
   21 F=Y(I)
      Y(I)=Y(J)
      Y(J)=F
      L=(I-1)*N
      LL=(J-1)*N
      DO 22 K=1,N
      L=L+1
      LL=LL+1
      F=Z(L)
      Z(L)=Z(LL)
      Z(LL)=F
   22 CONTINUE
   24 CONTINUE
   25 CONTINUE
C     FIND BASE VARIABLE SCALES
  105 XSCAL=(X(N)-X(1))/(FLOAT(NLL-1))
      YSCAL=(Y(M)-Y(1))/(FLOAT(NTH-1))
C     FIND CONTOUR VARIABLE SCALE
      ZMIN = 1.0E37
      ZMAX = -1.0E37
      M1=1
      M2=N*M
      DO 40 J=M1,M2
      IF(Z(J).GT.ZMAX)ZMAX=Z(J)
      IF(Z(J).LT.ZMIN)ZMIN=Z(J)
   40 CONTINUE
C     LINEAR INTERPOLATION FOR NCONT CONTOURS
```

```
      ZSCAL=(ZMAX-ZMIN)/FLOAT(NCONT-1)
      IF(ZSCAL.EQ.Ø.)ZSCAL=1.ØE-37
C     DEVELOP AND PRINT CONTOUR PARAMETER BOX
      XMIN=X(1)
      XMAX=X(N)
      YMIN=Y(1)
      YMAX=Y(M)
      WRITE(5,4)
      WRITE(5,5)NO
      WRITE(5,6)XMIN,XMAX,XSCAL
      WRITE(5,9)YMIN,YMAX,YSCAL
      WRITE(5,13)ZMIN,ZMAX,ZSCAL
      WRITE(5,17)
      ZD(1)=ZMIN+(2.Ø-ZCON)*ZSCAL
      NZCAL=NCONT-2
      DO 18 IZ=1,NZCAL
      ZD(IZ+1)=ZD(1)+FLOAT(IZ)*ZSCAL
   18 CONTINUE
      ZD(NCONT)=ZMAX
      IP=7
      DO 19 IZ=1,IP
      N1=IZ
      N2=N1+IP
      N3=N2+IP
      WRITE(5,2Ø)N1,ZD(N1),N2,ZD(N2),N3,ZD(N3)
   19 CONTINUE
      WRITE(5,23)
C     CALCULATE THE Y SCALE VARIABLES
      YPR(1)=Y(1)
      DO 9Ø KN=1,4
      YPR(KN+1)=YPR(KN)+YSCAL*1Ø.Ø
   9Ø CONTINUE
```

```
      YPR(6)=Y(M)
      YPT(1)=YMIN
      YPT(2)=YMIN+YSCAL*25.Ø
      YPT(3)=YMAX
      YSTAR=YSCAL*5.Ø
C     PRINT HEADING
      WRITE(5,1)NO
      WRITE(5,6Ø)YSTAR
      WRITE(5,61)(YPT(IP),IP=1,3)
      WRITE(5,8)(YPR(IP),IP=1,6)
      WRITE(5,7)
C     FIND THE X SCALE PRINT POSITION
      XB=X(1)
      L=1
      I=1
      XEPS=XSCAL/FLOAT(2*(NLL-1))
      YEPS=YSCAL/FLOAT(2*(NTH-1))
   45 F=FLOAT(I-1)
      XPR=XB+F*XSCAL
      XDIF=X(L)-XPR-XEPS
      IF(XDIF)5Ø,5Ø,7Ø
C     FIND THE Y SCALE PRINT POSITION
   5Ø DO 55 IX=1,NTH
      OUT(IX)=BLANK
   55 CONTINUE
      K=1
      LM=1
      YB=Y(1)
   35 G=FLOAT(K-1)
      YP=YB+G*YSCAL
   37 YDIF=Y(LM)-YP-YEPS
      IF(YDIF)3Ø,3Ø,31
```

```
C     FIND CONTOUR POSITION AND MAGNITUDE
   3Ø JZ=(LM-1)*N+L
      JP=IFIX(((Z(JZ)-ZMIN)/ZSCAL)+ZCON)
      OUT(K)=IANG(JP)
      LM=LM+1
   31 K=K+1
      IF(K-NTH)35,34,36
   34 YP=Y(M)
      GO TO 37
C     PRINT THE LINE
   36 WRITE(5,2)XPR,(OUT(IZ),IZ=1,NTH)
      L=L+1
      GO TO 8Ø
C     SKIP THE LINE
   7Ø WRITE(5,3)
   8Ø I=I+1
      IF(I-NLL)45,84,86
   84 XPR=X(N)
      GO TO 5Ø
C     PRINT BOTTOM AND Y VARIABLE SCALE
   86 WRITE(5,7)
      WRITE(5,8)(YPR(IP),IP=1,6)
      RETURN
      END
```

APPENDIX 4

```
C ******************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C ****** MODEL REFERENCE CONTOUR -- LINEAR MODEL ******
C
C
C ****  THIS PROGRAM USES THE MODEL REFERENCE APPROACH
C      TO IDENTIFY THE LINEAR HYDRODYNAMIC COEFFICIENTS
C      OF THE MARINER CLASS HULL FORM ****
C
C ******************************************

      DIMENSION VEV(188),VER(188),XMV(188),XMR(188),XVP(141)
      DIMENSION PAR1(13),PAR2(13),PI(169)
      DIMENSION P(83),GW(4),DV(4)
      DIMENSION A(40)
      DIMENSION INX(6),PMS(12)
      DIMENSION Z(100)
      DIMENSION WN(3),VN(3)
      COMMON /PNGW/ G
      COMMON /PNDV/ D
      KI = 8
      KO = 5
      READ (KI,10) (A(I),I = 1,15)
   10 FORMAT (5E12.4)
      READ (KI,448) (PMS(J),J = 1,8)
  448 FORMAT (4E13.5)
      READ (KI,444) (INX(I),I = 1,4)
  444 FORMAT (4I6)
      READ (KI,15) (DV(J),J = 1,3)
      READ (KI,15) (GW(J),J = 1,3)
```

```
   15 FORMAT (3F10.3)
      READ (KI,20) LP1,LP2
   20 FORMAT (2I5)
      READ (KI,200) NGW,NDV
  200 FORMAT (2I4)
      PA1 = A(LP1)
      PA2 = A(LP2)
      RP1 = .30*ABS(PA1)
      RP2 = .30*ABS(PA2)
      DP1 = .075*ABS(PA1)
      DP2 = .075*ABS(PA2)
      PS1 = PA1-RP1
      PS2 = PA2-RP2
      NP1 = 9
      NP2 = 9
      IP1 = NP1-1
      IP2 = NP2-1
      PAR1(1) = PS1
      PAR2(1) = PS2
      DO 100 J = 1,IP1
      PAR1(J+1) = PAR1(J)+DP1
  100 CONTINUE
      DO 105 J = 1,IP2
      PAR2(J+1) = PAR2(J)+DP2
  105 CONTINUE
      D5 = 57.296
      NS = 0
      NO = 1
      NPL = 47
C *********
C     INITIAL CONDITIONS
      TI = 0.0
```

```
      XVI = 0.0
      XRI = 0.0
C *********
      N = 47
      H = 1.0
C *********
C     FIRST DO LOOPS IN THE IDENTIFICATION PROCESS
C     DIFFERENT LEVELS OF PROCESS AND MEASUREMENT NOISE
C *********
      KWG = 1
      DO 110 IGW = 1,NGW
      G = GW(IGW)
      DO 115 IDV = 1,NDV
      IF (KWG.EQ.2) GO TO 810
      D = DV(IDV)
      A(LP1) = PA1
      A(LP2) = PA2
C *********
C     GENERATE THE SEA TRIAL DATA
      CALL VRP(H,TI,XVI,XRI,K,N,A,VEV,VER,INX,PMS)
C *********
      DO 120 K = 1,NPL
      KT = K+NPL
      KJ = KT+NPL
      KK = 4*K
      XVR(K) = TI+FLOAT(KK)*H
      XVR(KT) = VEV(KK)
      XVR(KJ) = D5*VER(KK)
  120 CONTINUE
C *********
C     PLOT THE SEA TRIAL DATA
      CALL PLOT(0,XVR,NPL,3,0)
```

```
C *********
C     MAIN LOOP IN THE SYSTEM IDENTIFICATION PROCESS
C     VARIATION OF PARAMETERS
C *********
      P1 = PS1
      DO 125 IM = 1,NP1
      P2 = PS2
      DO 130 IN = 1,NP2
C *********
C     GENERATE THE MODEL DATA
      CALL VRM(H,TI,XVI,XRI,K,N,A,P1,P2,XMV,XMR,LP1,LP2)
C *********
      NC = (IM-1)*NP1+IN
      NM = 4*N
      PI(NC) = 0.0
      DO 135 IP = 1,NM
      XDFV = VEV(IP)-XMV(IP)
      XDFR = VER(IP)-XMR(IP)
      PI(NC) = PI(NC)+(XDFV*XDFV+XDFR*XDFR)*H
  135 CONTINUE
      PI(NC) = ALOG(PI(NC))
      P2 = PAR2(IN+1)
  130 CONTINUE
      P1 = PAR1(IM+1)
  125 CONTINUE
C *********
C     PLOT CONTOUR OF THE COST FUNCTION
      CALL CONTUR(NO,PAR1,PAR2,PI,NP1,NP2,NS)
C *********
C     BETTER VIZUALIZATION
C     PLOT 3 SLICES OF PERFORMANCE INDEX ALONG 2'ND PARAMETER
C *********
```

```
      DO 140 I = 1,NP2
      NI1 = NP2+I
      NI2 = 4*NP2+I
      NI3 = 7*NP2+I
      NP4 = NP2+I
      NP5 = 2*NP2+I
      NP6 = 3*NP2+I
      Z(I) = PAR2(I)
      Z(NP4) = PI(NI1)
      Z(NP5) = PI(NI2)
      Z(NP6) = PI(NI3)
  140 CONTINUE
      CALL PLOT(NO,Z,NP2,4,Ø)
C *********
C     PLOT 3 SLICES OF PERFORMANCE INDEX ALONG 1'ST PARAMETER
C *********
      I = Ø
      DO 145 J = 1,NP1
      I = I+1
      IL = (J-1)*NP2
      Z(I) = PAR1(J)
      NP4 = NP2+I
      IL1 = IL+2
      Z(NP4) = PI(IL1)
      NP5 = 2*NP2+I
      IL2 = IL+5
      Z(NP5) = PI(IL2)
      NP6 = 3*NP2+I
      IL3 = IL+8
      Z(NP6) = PI(IL3)
  145 CONTINUE
      CALL PLOT(NO,Z,NP1,4,Ø)
```

```
      NO = NO+1
      IF (IGW.EQ.1) GO TO 815
  810 CONTINUE
      KWG = 1
  115 CONTINUE
  815 CONTINUE
      KWG = 2
  110 CONTINUE
  180 CONTINUE
      STOP
      END
```

```
      SUBROUTINE VRP(H,TI,XVI,XRI,K,N,A,VEV,VER,IN,P)
      DIMENSION VEV(1),VER(1),IN(1),P(1),A(1)
      DIMENSION VN(2),WN(2)
      COMMON /PNDV/D
      HM = H/2.0
      XS = 1.0
      NVAR = 2
      XV = XVI
      XR = XRI
      T = TI
      NN = 4*N
      DO 500 J = 1,NN
      UD = U(T,XS)
      DO 333 IVAR = 1,NVAR
      CALL WNOI(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  333 CONTINUE
      DO 334 IR = 1,NVAR
      IVAR = IR+NVAR
      CALL WNOI(IN,P,IVAR,NVAR,W)
      VN(IR) = W
  334 CONTINUE
      WM = WN(1)
      Y1 = HM*FSV(T,XV,XR,UD,A,WM)
      WM = WN(2)
      Y2 = HM*FSR(T,XV,XR,UD,A,WM)
      TM = T+HM
      XV1 = XV+Y1
      XR1 = XR+Y2
      UD = U(TM,XS)
      DO 335 IVAR = 1,NVAR
      CALL WNOI(IN,P,IVAR,NVAR,W)
```

```
      WN(IVAR) = W
  335 CONTINUE
      WM = WN(1)
      XV = XV+H*FSV(TM,XV1,XR1,UD,A,WM)
      WM = WN(2)
      XR = XR+H*FSR(TM,XV1,XR1,UD,A,WM)
      VM = VN(1)
      VEV(J) = XV+D*VM
      VM = VN(2)
      VER(J) = XR+D*VM
      T = T+H
  500 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION U(T,XS)
C *********
C     STEP DEFLECTION OF RUDDER;5 DEGREES
C     TIME LAG NEGLECTED
C *********
      U = 5./57.296
      RETURN
      END
```

```
SUBROUTINE WNOI(IN,P,IVAR,NVAR,W)
DIMENSION IN(1),P(1)
IX = IN(IVAR)
AM = P(IVAR)
L = IVAR+2*NVAR
S = P(L)
CALL GAUSS(IX,S,AM,W)
IN(IVAR) = IX
RETURN
END
```

```
      REAL FUNCTION FSV(T,XV,XR,U,A,W)
      DIMENSION A(4Ø)
      COMMON /PNGW/ G
      FSV = 1./(A(11)*A(4)=A(1Ø)*A(5))*(A(11)*(A(9)+A(6)*XV+A(7)*XR+A(8)
     1*U)=A(5)*(A(15)+A(12)*XV+A(13)*XR+A(14)*U))+G*W
      RETURN
      END
```

```
      REAL FUNCTION FSR(T,XV,XR,U,A,W)
      DIMENSION A(4Ø)
      COMMON /PNGW/ G
      FSR = 1./(A(11)*A(4)-A(1Ø)*A(5))*(A(4)*(A(15)+A(12)*XV+A(13)*XR+A(
     114)*U)-A(1Ø)*(A(9)+A(6)*XV+A(7)*XR+A(8)*U))+G*W
      RETURN
      END
```

```
      SUBROUTINE VRM(H,TI,XVI,XRI,K,N,A,P1,P2,XMV,XMR,LP1,LP2)
      DIMENSION XMV(1),XMR(1),A(1)
      A(LP1) = P1
      A(LP2) = P2
      HM = H/2.
      XS = 1.
      XV = XVI
      XR = XRI
      T = TI
      NN = 4*N
      DO 1510 J = 1,NN
      UD = U(T,XS)
      Y1 = HM*FDV(T,XV,XR,UD,A)
      Y2 = HM*FDR(T,XV,XR,UD,A)
      TM = T+HM
      XV1 = XV+Y1
      XR1 = XR+Y2
      UD = U(TM,XS)
      XV = XV+H*FDV(TM,XV1,XR1,UD,A)
      XR = XR+H*FDR(TM,XV1,XR1,UD,A)
      T = T+H
      XMV(J) = XV
      XMR(J) = XR
 1510 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION FDV(T,XV,XR,U,A)
      DIMENSION A(40)
      FDV = 1./(A(11)*A(4)-A(10)*A(5))*(A(11)*(A(9)+A(6)*XV+A(7)*XR+A(8)
     1*U)-A(5)*(A(15)+A(12)*XV+A(13)*XR+A(14)*U))
      RETURN
      END
```

```
 REAL FUNCTION FDR(T,XV,XR,U,A)
 DIMENSION A(40)
 FDR = 1./(A(11)*A(4)-A(10)*A(5))*(A(4)*(A(15)+A(12)*XV+A(13)*XR+A(
114)*U)-A(10)*(A(9)+A(6)*XV+A(7)*XR+A(8)*U))
 RETURN
 END
```

APPENDIX 5

```
C ****************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 2 - TAILORED TO IDENTIFY ALL THE PARAMETERS
C
C     THIS PROGRAM WAS DIVIDED IN 3 PARTS TO FIT THE
C     COMPUTER CAPACITY
C
C *****  FIRST PART --GENERATES THE SEA TRIAL DATA  *****
C
C ****************************************************
C
      DIMENSION ZV(188),ZR(188),TS(188),US(188),VEIV(94),VEIR(94)
      DIMENSION A(15),AI(15),ASD(15),PMS(12),INX(12),Q(2),R(2)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
```

```
      COMMON /EFN/ B
      COMMON /GUP/ EG
      KI = 8
      KO = 5
      READ (KI,10) (INX(I),I = 1,4)
   10 FORMAT (4I6)
      READ (KI,11) D,G
   11 FORMAT (2F10.6)
      READ (KI,12) (PMS(J),J = 1,8)
   12 FORMAT (4E13.5)
      READ (KI,13) (A(I),I=1,15)
   13 FORMAT (5E12.4)
      READ (KI,14) LP1,LP2,LP3,LP4
   14 FORMAT (4I5)
      READ (KI,17) (AI(I),I = 1,15)
   17 FORMAT (5E13.4)
      READ (KI,17) (ASD(I),I = 1,15)
      READ (KI,19) VST,RST
   19 FORMAT (2F10.4)
      READ (KI,20) VCV,RCV
   20 FORMAT (2E10.3)
      READ (KI,21) KS,N,H
   21 FORMAT (2I4,F10.2)
C *********
C     INITIAL CONDITIONS
C *********
      TI = 0.0
      XVI = 0.0
      XRI = 0.0
      PA1 = A(LP1)
      PA2 = A(LP2)
      PA3 = A(LP3)
```

```
      PA4 = A(LP4)
C *********
C     GENERATE SEA TRIAL DATA
      CALL RKL(H,TI,XVI,XRI,N,A,ZV,ZR,US,TS,INX,PMS)
C *********
      NPL = 47
      D5 = 57.296
      DO 101 I = 1,NPL
      L = KS*I
      NL1 = I+NPL
      VEIV(I) = TS(L)
      VEIR(I) = TS(L)
      VEIV(NL1) = ZV(L)
      VEIR(NL1) = ZR(L)
  101 CONTINUE
C *********
C     PLOT SEA TRIAL DATA
      CALL PLOT(0,VEIV,NPL,2,0)
      CALL PLOT(0,VEIR,NPL,2,0)
C *********
      PST1 = AI(LP1)
      PST2 = AI(LP2)
      PST3 = AI(LP3)
      PST4 = AI(LP4)
C *********
C     SET INITIAL STATE ESTIMATE
C *********
      XHT(1) = VST
      XHT(2) = RST
      XHT(3) = PST1
      XHT(4) = PST2
      XHT(5) = PST3
```

```
      XHT(6) = PST4
C *********
C     SET EXAGERATED NOISE PARAMETERS
      PW = 1.Ø
      QW = 1.Ø
C *********
      DO 55 IR = 1,2
      IA = IR+4
      IV = IA+2
      Q(IR) = QW*G**2.*(PMS(IA))**2.
      R(IR) = PW*D**2.*(PMS(IV))**2.
   55 CONTINUE
C *********
C     INITIALIZE EHAT MATRIX
C *********
      DO 1Ø2 J = 1,36
      EHT(J) = Ø.Ø
  1Ø2 CONTINUE
      PSD1 = ASD(LP1)
      PSD2 = ASD(LP2)
      PSD3 = ASD(LP3)
      PSD4 = ASD(LP4)
      PCV1 = PSD1**2.
      PCV2 = PSD2**2.
      PCV3 = PSD3**2.
      PCV4 = PSD4**2.
C *********
C     SET INITIAL ERROR COVARIANCES
C *********
      EHT(1) = VCV
      EHT(8) = RCV
      EHT(15) = PCV1
```

```
      EHT(22) = PCV2
      EHT(29) = PCV3
      EHT(36) = PCV4
C *********
C     CALL 2'ND PART OF PROGRAM
C *********
      CALL LINK ('HLBR2 ')
      END
```

```
      SUBROUTINE RKL(H,TI,XVI,XRI,N,A,ZV,ZR,US,TS,IN,P)
      DIMENSION ZV(1),ZR(1),TS(1),US(1),A(1),IN(1),P(1)
      DIMENSION WN(2),VN(2)
      COMMON /RKI/D
      XS = 1.0
      T = TI
      XV = XVI
      XR = XRI
      HM = H/2.
      NVAR = 2
      DO 300 IJ = 1,N
      TM = T+HM
      TN = T+H
      UD = U(T,XS)
      DO 333 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  333 CONTINUE
      WL = WN(1)
      YV1= H*FSV(T,XV,XR,UD,A,WL)
      WL = WN(2)
      YR1= H*FSR(T,XV,XR,UD,A,WL)
      XX1 = XV+.5*YV1
      XX2 = XR+.5*YR1
      UD = U(TM,XS)
      DO 335 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  335 CONTINUE
      WL = WN(1)
      YV2 = H*FSV(TM,XX1,XX2,UD,A,WL)
      WL = WN(2)
```

```
      YR2 = H*FSR(TM,XX1,XX2,UD,A,WL)
      XX1 = XV+.5*YV2
      XX2 = XR+.5*YR2
      DO 336 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  336 CONTINUE
      WL = WN(1)
      YV3 = H*FSV(TM,XX1,XX2,UD,A,WL)
      WL = WN(2)
      YR3 = H*FSR(TM,XX1,XX2,UD,A,WL)
      XX1 = XV+YV3
      XX2 = XR+YR3
      UD = U(TN,XS)
      DO 337 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  337 CONTINUE
      WL = WN(1)
      YV4 = H*FSV(TN,XX1,XX2,UD,A,WL)
      WL = WN(2)
      YR4 = H*FSR(TN,XX1,XX2,UD,A,WL)
      XV = XV+1./6.*(YV1+2.*YV2+2.*YV3+YV4)
      XR = XR+1./6.*(YR1+2.*YR2+2.*YR3+YR4)
      DO 334 IR = 1,NVAR
      IVAR = IR+NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      VN(IR) = W
  334 CONTINUE
      VL = VN(1)
      ZV(IJ) = XV+D*VL
      VL = VN(2)
```

```
      ZR(IJ) = XR+D*VL
      US(IJ) = UD
      TS(IJ) = T
      T = T+H
  300 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION U(T,XS)
C *********
C     STEP DEFLECTION OF RUDDER --5 DEGREES
C     TIME LAG NEGLECTED
C *********
      U = 5./57.296
      RETURN
      END
```

```
SUBROUTINE WNO(IN,P,IVAR,NVAR,W)
DIMENSION IN(1),P(1)
IX = IN(IVAR)
LW = IVAR+2*NVAR
AM = P(IVAR)
S = P(LW)
CALL GAUSS(IX,S,AM,W)
IN(IVAR) = IX
RETURN
END
```

```
REAL FUNCTION FSV(T,XV,XR,U,A,W)
DIMENSION A(15)
COMMON /PKI/G
FSV = 1./(A(11)*A(4)-A(10)*A(5))*(A(11)*(A(9)+A(6)*XV+A(7)*XR+A(8)
1*U)-A(5)*(A(15)+A(12)*XV+A(13)*XR+A(14)*U))+G*W
RETURN
END
```

```
      REAL FUNCTION FSR(T,XV,XR,U,A,W)
      DIMENSION A(15)
      COMMON /PKI/G
      FSR = 1./(A(11)*A(4)-A(10)*A(5))*(A(4)*(A(15)+A(12)*XV+A(13)*XR+A(
     114)*U)-A(10)*(A(9)+A(6)*XV+A(7)*XR+A(8)*U))+G*W
      RETURN
      END
```

```
C ****************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 2 - TAILORED TO IDENTIFY ALL THE PARAMETERS
C
C ***** 2'ND PART --PROGRAM HLBR2  ******
C     USES KALMAN FILTER TO PROCESS THE SEA TRIAL DATA
C
C ****************************************************


      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),US(188),TS(188)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION A(15),Q(2),R(2),HZ(2)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /EFN/ B
```

```
      COMMON /GUP/ EG
C *********
C     DEFINE H MATRIX : Z = HX
C *********
      HZ(1) = 1.0
      HZ(2) = 1.0
C *********
C     INITIAL SETUP
C     SET KALMAN FILTER INCREMENT,STARTING INDEX,AND STOPPING INDEX
C *********
      KST = 1
      KFN = 1
      KB = KST+KFN
      MH = 1
      KFIM = 47
C *********
C     PROCESS THE SEA TRIAL DATA
C *********
      USV = US(1)
      XS = 1.0
      UZ = U(TI,XS)
      US(1) = UZ
      DT = H
      CALL PROP(DT,US,A,Q,1)
      CALL GAIN(HZ,R)
      CALL UPDT(ZV,ZR,HZ,1)
      US(1) = USV
      JLI = 2
C *********
C     BEGIN ITERATIONS FOR FILTERING
C *********
      DO 104 IM = KB,N
```

```
      NH = IM-1
      JLI = JLI+1
      LL = JLI-1
      DT = TS(IM)-TS(NH)
C *********
C     PROPAGATE THE STATE AND ERROR COVARIANCE MATRIX FOR A TIME STEP DT
      CALL PROP(DT,US,A,Q,IM)
C *********
C     COMPUTE THE KALMAN FILTER GAIN
      CALL GAIN(HZ,R)
C *********
C     UPDATE THE STATE AND ERROR COVARIANCE MATRIX
      CALL UPDT(ZV,ZR,HZ,IM)
C *********
      IF (LL.LT.4) GO TO 377
C *********
C     STORE VALUES OF STATE AND ERROR COVARIANCE MATRIX FOR PLOTTING
      CALL STORB(TS,MH,KFIM)
C *********
      MH = MH+1
      JLI = 1
  377 CONTINUE
  104 CONTINUE
C *********
C     CALL 3'RD PART OF PROGRAM
      CALL LINK ('HLBR3 ')
C *********
      END
```

```
      REAL FUNCTION U(T,XS)
C *********
C     STEP DEFLECTION OF RUDDER =-5 DEGREES
C     TIME LAG NEGLECTED
C *********
      U = 5./57.296
      RETURN
      END
```

```
      SUBROUTINE PROP(H,US,A,Q,I)
      DIMENSION US(2),A(1),Q(1)
      DIMENSION XHT(6),EHT(36),XBAR(6),EBAR(36)
      DIMENSION EJ(36),E1(36),E2(36),E3(36),E4(36),E5(36)
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      XV = XHT(1)
      XR = XHT(2)
      A(LP1) = XHT(3)
      A(LP2) = XHT(4)
      A(LP3) = XHT(5)
      A(LP4) = XHT(6)
      DO 3 J = 1,36
      EJ(J) = EHT(J)
    3 CONTINUE
      UV = US(I)
      HM= H/2.
      YV1= H*FKV(XV,XR,UV,A)
      YR1= H*FKR(XV,XR,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XV+.5*YV1
      XY2 = XR+.5*YR1
      XHT(1) = XY1
      XHT(2) = XY2
      DO 4 J = 1,36
      E2(J) = E1(J)*H
      EHT(J) = HM*E1(J)+EJ(J)
    4 CONTINUE
```

```
      UV = (UB(I)+UB(I+1))/2.
      YV2 = H*FKV(XY1,XY2,UV,A)
      YR2 = H*FKR(XY1,XY2,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XV+.5*YV2
      XY2 = XR+.5*YR2
      XHT(1) = XY1
      XHT(2) = XY2
      DO 6 J = 1,36
      E3(J) = H*E1(J)
      EHT(J) = EJ(J)+HM*E1(J)
    6 CONTINUE
      YV3 = H*FKV(XY1,XY2,UV,A)
      YR3 = H*FKR(XY1,XY2,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XV+YV3
      XY2 = XR+ YR3
      XHT(1) = XY1
      XHT(2) = XY2
      DO 7 J = 1,36
      E4(J) = H*E1(J)
      EHT(J) = EJ(J)+E1(J)*H
    7 CONTINUE
      YV4 = H*FKV(XY1,XY2,UV,A)
      YR4 = H*FKR(XY1,XY2,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      DO 8 J = 1,36
      E5(J) = H*E1(J)
    8 CONTINUE
```

```
      XBAR(1) = XV+1./6.*(YV1+2.*YV2+2.*YV3+YV4)
      XBAR(2) = XR+1./6.*(YR1+2.*YR2+2.*YR3+YR4)
      XBAR(3) = XHT(3)
      XBAR(4) = XHT(4)
      XBAR(5) = XHT(5)
      XBAR(6) = XHT(6)
      DO 5 J = 1,36
      EBAR(J) = EJ(J)+1./6.*(E2(J)+2.*E3(J)+2.*E4(J)+E5(J))
    5 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION FKV(XV,XR,U,A)
      DIMENSION A(15)
      FKV = 1./(A(11)*A(4)-A(1Ø)*A(5))*(A(11)*(A(9)+A(6)*XV+A(7)*XR+A(8)
     1*U)-A(5)*(A(15)+A(12)*XV+A(13)*XR+A(14)*U))
      RETURN
      END
```

```
      REAL FUNCTION FKR(XV,XR,U,A)
      DIMENSION A(15)
      FKR = 1./(A(11)*A(4)-A(10)*A(5))*(A(4)*(A(15)+A(12)*XV+A(13)*XR+A(
     114)*U)-A(10)*(A(9)+A(6)*XV+A(7)*XR+A(8)*U))
      RETURN
      END
```

```
      SUBROUTINE EFNT1(A,U)
      DIMENSION B(36),A(15),X(6)
      COMMON /EFN/ B
      COMMON /STRV/ X
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      DO 399 J = 1,36
      B(J) = Ø.Ø
  399 CONTINUE
      C2 = 1./(A(4)*A(11)-A(5)*A(1Ø))
      C5 = A(9)+A(6)*X(1)+A(7)*X(2)+A(8)*U
      C6 = A(15)+A(12)*X(1)+A(13)*X(2)+A(14)*U
      B(1) = C2*(A(11)*A(6)-A(5)*A(12))
      B(2) = C2*(A(4)*A(12)-A(1Ø)*A(6))
      B(7) = C2*(A(11)*A(7)-A(5)*A(13))
      B(8) = C2*(A(4)*A(13)-A(1Ø)*A(7))
      NPA = LP1-3
      IPA = 13
      IPB = 14
      NPR = 2
   1Ø CONTINUE
      GO TO(11,12,13,14,15,16,17,18,19,2Ø,21,22),NPA
   11 B(IPA) = -C2**2.*A(11)*(A(11)*C5-A(5)*C6)
      B(IPB) =-C2**2.*A(11)*(A(4)*C6-A(1Ø)*C5)+C2*C6
      GO TO 25
   12 B(IPA) = C2**2.*A(1Ø)*(A(11)*C5-A(5)*C6)-C2*C6
      B(IPB) = C2**2.*A(1Ø)*(A(4)*C6-A(1Ø)*C5)
      GO TO 25
   13 B(IPA) = C2*A(11)*X(1)
      B(IPB) = -C2*A(1Ø)*X(1)
      GO TO 25
   14 B(IPA) = C2*A(11)*X(2)
      B(IPB) = -C2*A(1Ø)*X(2)
```

```
      GO TO 25
   15 B(IPA) = C2*A(11)*U
      B(IPB) = -C2*A(10)*U
      GO TO 25
   16 B(IPA) = C2*A(11)
      B(IPB) = -C2*A(10)
      GO TO 25
   17 B(IPA) = C2**2.*A(5)*(A(11)*C5-A(5)*C6)
      B(IPB) = C2**2.*A(5)*(A(4)*C6-A(10)*C5)-C2*C5
      GO TO 25
   18 B(IPA) = -C2**2.*A(4)*(A(11)*C5-A(5)*C6)+C2*C5
      B(IPB) = -C2**2.*A(4)*(A(4)*C6-A(10)*C5)
      GO TO 25
   19 B(IPA) = -C2*A(5)*X(1)
      B(IPB) = C2*A(4)*X(1)
      GO TO 25
   20 B(IPA) = -C2*A(5)*X(2)
      B(IPB) = C2*A(4)*X(2)
      GO TO 25
   21 B(IPA) = -C2*A(5)*U
      B(IPB) = C2*A(4)*U
      GO TO 25
   22 B(IPA) = -C2*A(5)
      B(IPB) = C2*A(4)
   25 CONTINUE
      IF (NPR.GT.2) GO TO 30
      NPR = NPR+1
      NPA = LP2-3
      IPA = 19
      IPB = 20
      GO TO 10
   30 CONTINUE
```

```
      IF (NPR.GT.3) GO TO 35
      NPR = NPR+1
      NPA = LP3-3
      IPA = 25
      IPB = 26
      GO TO 10
   35 CONTINUE
      IF (NPR.GT.4) GO TO 40
      NPR = NPR+1
      NPA = LP4-3
      IPA = 31
      IPB = 32
      GO TO 10
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EFNT2(EB,Q)
      DIMENSION EB(36),Q(2)
      DIMENSION B(36),E(36)
      COMMON /COV/ E
      COMMON /EFN/ B
      DO 6Ø1 I = 1,36
      EB(I) = Ø.Ø
 6Ø1  CONTINUE
      EB(1) = EB(1)+2.*(B(1)*E(1)+B(7)*E(2)+B(13)*E(3)+B(19)*E(4)+B(25)*
     1E(5)+B(31)*E(6))+Q(1)
      EB(2) = EB(2)+B(2)*E(1)+B(8)*E(2)+B(14)*E(3)+B(2Ø)*E(4)+B(26)*E(5)
     1+B(32)*E(6)+E(2)*B(1)+E(8)*B(7)+E(14)*B(13)+E(2Ø)*B(19)+E(26)*B(25
     2)+E(32)*B(31)
      EB(3) = EB(3)+E(3)*B(1)+E(9)*B(7)+E(15)*B(13)+E(21)*B(19)+E(27)*B(
     125)+E(33)*B(31)
      EB(4) = EB(4)+E(4)*B(1)+E(1Ø)*B(7)+E(16)*B(13)+E(22)*B(19)+E(28)*B
     1(25)+E(34)*B(31)
      EB(5) = EB(5)+E(5)*B(1)+E(11)*B(7)+E(17)*B(13)+E(23)*B(19)+E(29)*B
     1(25)+E(35)*B(31)
      EB(6) = EB(6)+E(6)*B(1)+E(12)*B(7)+E(18)*B(13)+E(24)*B(19)+E(3Ø)*B
     1(25)+E(36)*B(31)
      EB(7) = EB(2)
      EB(8) = EB(8)+2.*(B(2)*E(2)+B(8)*E(8)+B(14)*E(9)+B(2Ø)*E(1Ø)+B(26)
     1*E(11)+B(32)*E(12))+Q(2)
      EB(9) = EB(9)+E(3)*B(2)+E(9)*B(8)+E(15)*B(14)+E(21)*B(2Ø)+E(27)*B(
     126)+E(33)*B(32)
      EB(1Ø) = EB(1Ø)+E(4)*B(2)+E(1Ø)*B(8)+E(16)*B(14)+E(22)*B(2Ø)+E(28)
     1*B(26)+E(34)*B(32)
      EB(11) = EB(11)+E(5)*B(2)+E(11)*B(8)+E(17)*B(14)+E(23)*B(2Ø)+E(29)
     1*B(26)+E(35)*B(32)
      EB(12) = EB(12)+E(6)*B(2)+E(12)*B(8)+E(18)*B(14)+E(24)*B(2Ø)+E(3Ø)
     1*B(26)+E(36)*B(32)
```

```
EB(13) = EB(3)
EB(14) = EB(9)
EB(19) = EB(4)
EB(20) = EB(10)
EB(25) = EB(5)
EB(26) = EB(11)
EB(31) = EB(6)
EB(32) = EB(12)
RETURN
END
```

```
      SUBROUTINE GAIN(H,R)
      DIMENSION E(36),EG(12)
      DIMENSION H(2),R(2)
      DIMENSION E2(12),E3(12),E4(4)
      COMMON /PRUG/ E
      COMMON /GUP/ EG
      DO 1 J=1,6
      L=J+6
      E2(J)=E(J)*H(1)
      E2(L)=E(L)*H(2)
    1 CONTINUE
      E3(1) = H(1)*E2(1)+R(1)
      E3(2) = H(2)*E2(2)
      E3(3) = H(1)*E2(7)
      E3(4) = H(2)*E2(8)+R(2)
      C = E3(1)*E3(4)-E3(2)*E3(3)
      E4(1) = E3(4)
      E4(2) = -E3(2)
      E4(3) = -E3(3)
      E4(4) = E3(1)
      DO 7 I = 1,4
      E4(I) = E4(I)/C
    7 CONTINUE
      DO 2 IJ = 1,2
      DO 3 II = 1,6
      LL = II+6*(IJ-1)
      EG(LL) = 0.0
      DO 4 K = 1,2
      LI = II+6*(K-1)
      LJ = K+2*(IJ-1)
      E5 = E2(LI)*E4(LJ)
      EG(LL) = EG(LL)+E5
```

```
4 CONTINUE
3 CONTINUE
2 CONTINUE
  RETURN
  END
```

```
      SUBROUTINE UPDT(ZV,ZR,H,IM)
      DIMENSION EB(36),XB(6),EG(12)
      DIMENSION ZV(1),ZR(1),EH(36),XH(6)
      DIMENSION EL(2),XD(6),EA(36),EC(36),Z(2),H(2)
      COMMON /PRUP/ XB
      COMMON /GUP/ EG
      COMMON /PRUG/ EB
      COMMON /STRV/ XH
      COMMON /COV/ EH
      Z(1) = ZV(IM)
      Z(2) = ZR(IM)
      EL(1) = XB(1)*H(1)
      EL(2) = XB(2)*H(2)
      DO 11 IK = 1,6
      XD(IK) = 0.0
      DO 12 JK = 1,2
      L = IK+(JK-1)*6
      E7 = EG(L)*(Z(JK)-EL(JK))
      XD(IK) = XD(IK)+E7
   12 CONTINUE
      XH(IK) = XB(IK)+XD(IK)
   11 CONTINUE
      DO 13 J = 1,6
      I = 2*(J-1)+1
      K = 2*(J-1)+2
      L = 6*(J-1)+1
      LJ = 6*(J-1)+2
      EA(I) = EB(L)*H(1)
      EA(K) = EB(LJ)*H(2)
   13 CONTINUE
      DO 14 KJ = 1,6
      DO 15 IL = 1,6
```

```
      LK = 6*(KJ-1)+IL
      EC(LK) = Ø.Ø
      DO 16 KK = 1,2
      LN = IL+6*(KK-1)
      LM = KK+2*(KJ-1)
      E8 = EG(LN)*EA(LM)
      EC(LK) = EC(LK)+E8
   16 CONTINUE
      EH(LK) = EB(LK)-EC(LK)
   15 CONTINUE
   14 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STORB(T,MH,K)
      DIMENSION T(1)
      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION XH(6),EH(36)
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
      COMMON /STRV/ XH
      COMMON /COV/ EH
      I = MH
      L = 4*I
      D = T(L)
      VP(I) = D
      RP(I) = D
      PP1(I) = D
      PP2(I) = D
      PP3(I) = D
      PP4(I) = D
      N = I+K
      VP(N) = XH(1)
      RP(N) = 57.296*XH(2)
      PP1(N) = XH(3)
      PP2(N) = XH(4)
      PP3(N) = XH(5)
      PP4(N) = XH(6)
      IF (I.LT.K) GO TO 100
      EV = SQRT(ABS(EH(1)))
      ER = SQRT(ABS(EH(8)))
      EP1 = SQRT(ABS(EH(15)))
      EP2 = SQRT(ABS(EH(22)))
      EP3 = SQRT(ABS(EH(29)))
      EP4 = SQRT(ABS(EH(36)))
  100 CONTINUE
```

```
RETURN
END
```

```
C ****************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 2 - TAILORED TO IDENTIFY ALL THE PARAMETERS
C
C ***** 3'RD PART -- PROGRAM HLBR3 *****
C     PRINTS THE OUTPUT OF EXTENDED KALMAN FILTERING
C
C ****************************************************


      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),TS(188),US(188)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION A(15)
      DIMENSION Q(2),R(2)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
```

```
      COMMON /EFN/ B
      COMMON /GUP/ EG
      KO = 5
      K = 47
      KP = 94
      NS = 0
      N = 1
      M = 2
C *********
C     PLOT THE KALMAN FILTER PRIMARY STATES
C *********
      CALL PLOT(N,VP,K,M,NS)
      N = N+1
      CALL PLOT(N,RP,K,M,NS)
      N = N+1
C *********
C     PLOT THE PARAMETERS IDENTIFIED BY THE KALMAN FILTER
C *********
      CALL PLOT(N,PP1,K,M,NS)
      N = N+1
      CALL PLOT(N,PP2,K,M,NS)
      N = N+1
      CALL PLOT(N,PP3,K,M,NS)
      N = N+1
      CALL PLOT(N,PP4,K,M,NS)
      WRITE (KO,557)
      WRITE (KO,555) LP1,PA1,PST1,PSD1,PP1(KP),EP1
      WRITE (KO,556) LP2,PA2,PST2,PSD2,PP2(KP),EP2
      WRITE (KO,556) LP3,PA3,PST3,PSD3,PP3(KP),EP3
      WRITE (KO,556) LP4,PA4,PST4,PSD4,PP4(KP),EP4
      WRITE (KO,558)
  555 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE  =',2X,E13.5//8X,'SV  =',
```

```
   12X,E13.5,'   + OR =   ',E13.5//8X,'FV   =',2X,E13.5,'   + OR =   ',E13.
   25)
556 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE   =',2X,E13.5//8X,'SV  =',
   12X,E13.5,'   + OR =   ',E13.5//8X,'FV   =',2X,E13.5,'   + OR =   ',E13.
   25)
557 FORMAT (1H1,//5X,'PARAMETRIC IDENTIFICATION USING KALMAN FILTER')
558 FORMAT (////20X,'  LINEAR MODEL  ',//////////)
    STOP
    END
```

```
C *************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 1 -- DON'T PERMIT TO IDENTIFY THE
C     COEFFICIENTS OF VDOT AND RDOT
C
C     THIS PROGRAM WAS DIVIDED IN 3 PARTS TO FIT THE
C     COMPUTER CAPACITY
C
C *****  FIRST PART --GENERATES THE SEA TRIAL DATA  *****
C
C *************************************************
C
      DIMENSION ZV(188),ZR(188),TS(188),US(188),VEIV(94),VEIR(94)
      DIMENSION A(15),AI(15),ASD(15),PMS(12),INX(12),Q(2),R(2)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION AR(15)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
```

```
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /EFN/ B
      COMMON /GUP/ EG
      COMMON /CRD/ CR,CR1,CR2,CR3
      COMMON /AAR/ AR
      KI = 8
      KO = 5
      READ (KI,10) (INX(I),I = 1,4)
   10 FORMAT (4I6)
      READ (KI,11) D,G
   11 FORMAT (2F10.6)
      READ (KI,12) (PMS(J),J = 1,8)
   12 FORMAT (4E13.5)
      READ (KI,13) (A(I),I=1,15)
   13 FORMAT (5E12.4)
      READ (KI,14) LP1,LP2,LP3,LP4
   14 FORMAT (4I5)
      READ (KI,17) (AI(I),I = 1,15)
   17 FORMAT (5E13.4)
      READ (KI,17) (ASD(I),I = 1,15)
      READ (KI,19) VST,RST
   19 FORMAT (2F10.4)
      READ (KI,20) VCV,RCV
   20 FORMAT (2E10.3)
      READ (KI,21) KS,N,H
   21 FORMAT (2I4,F10.2)
C *********
C     INITIAL CONDITIONS
C *********
      TI = 0.0
      XVI = 0.0
```

```
      XRI = 0.0
      PA1 = A(LP1)
      PA2 = A(LP2)
      PA3 = A(LP3)
      PA4 = A(LP4)
C *********
C     GENERATE SEA TRIAL DATA
      CALL RKL(H,TI,XVI,XRI,N,A,ZV,ZR,US,TS,INX,PMS)
C *********
      NPL = 47
      D5 = 57.296
      DO 101 I = 1,NPL
      L = KS*I
      NL1 = I+NPL
      VEIV(I) = TS(L)
      VEIR(I) = TS(L)
      VEIV(NL1) = ZV(L)
      VEIR(NL1) = ZR(L)
  101 CONTINUE
C *********
C     PLOT SEA TRIAL DATA
      CALL PLOT(0,VEIV,NPL,2,0)
      CALL PLOT(0,VEIR,NPL,2,0)
C *********
      CR = A(11)/(A(11)*A(4)-A(10)*A(5))
      CR = 1./CR
      CR1 = A(5)/A(11)
      CR2 = A(4)/A(11)
      CR3 = A(10)/A(11)
      DO 38 I = 1,15
      AR(I) = A(I)/CR
   38 CONTINUE
```

```
      PST1 = AI(LP1)/CR
      PST2 = AI(LP2)/CR
      PST3 = AI(LP3)/CR
      PST4 = AI(LP4)/CR
C *********
C     SET INITIAL STATE ESTIMATE
C *********
      XHT(1) = VST
      XHT(2) = RST
      XHT(3) = PST1
      XHT(4) = PST2
      XHT(5) = PST3
      XHT(6) = PST4
C *********
C     SET EXAGERATED NOISE PARAMETERS
      PW = 1.0
      QW = 1.0
C *********
      DO 55 IR = 1,2
      IA = IR+4
      IV = IA+2
      Q(IR) = QW*G**2.*(PMS(IA))**2.
      R(IR) = PW*D**2.*(PMS(IV))**2.
   55 CONTINUE
C *********
C     INITIALIZE EHAT MATRIX
C *********
      DO 102 J = 1,36
      EHT(J) = 0.0
  102 CONTINUE
      PSD1 = ASD(LP1)/CR
      PSD2 = ASD(LP2)/CR
```

```
      PSD3 = ASD(LP3)/CR
      PSD4 = ASD(LP4)/CR
      PCV1 = PSD1**2.
      PCV2 = PSD2**2.
      PCV3 = PSD3**2.
      PCV4 = PSD4**2.
C *********
C     SET INITIAL ERROR COVARIANCES
C *********
      EHT(1) = VCV
      EHT(8) = RCV
      EHT(15) = PCV1
      EHT(22) = PCV2
      EHT(29) = PCV3
      EHT(36) = PCV4
C *********
C     CALL 2'ND PART OF PROGRAM
C *********
      CALL LINK ('HLBR2 ')
      END
```

```
C ****************************************************************
C      PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C      TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 1 -- DON'T PERMIT TO IDENTIFY THE
C      COEFFICIENTS OF VDOT AND RDOT
C
C ***** 2'ND PART --PROGRAM HLBR2  ******
C      USES KALMAN FILTER TO PROCESS THE SEA TRIAL DATA
C
C ****************************************************************

      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),US(188),TS(188)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION A(15),Q(2),R(2),HZ(2)
      DIMENSION AR(15)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
```

```
      COMMON /PRUG/ EBAR
      COMMON /EFN/ B
      COMMON /GUP/ EG
      COMMON /CRD/ CR,CR1,CR2,CR3
      COMMON /AAR/ AR
C *********
C     DEFINE H MATRIX : Z = HX
C *********
      HZ(1) = 1.0
      HZ(2) = 1.0
C *********
C     INITIAL SETUP
C     SET KALMAN FILTER INCREMENT,STARTING INDEX,AND STOPPING INDEX
C *********
      KST = 1
      KFN = 1
      KB = KST+KFN
      MH = 1
      KFIM = 47
C *********
C     PROCESS THE SEA TRIAL DATA
C *********
      USV = US(1)
      XS = 1.0
      UZ = U(TI,XS)
      US(1) = UZ
      DT = H
      CALL PROP(DT,US,AR,Q,1)
      CALL GAIN(HZ,R)
      CALL UPDT(ZV,ZR,HZ,1)
      US(1) = USV
      JLI = 2
```

```
C *********
C     BEGIN ITERATIONS FOR FILTERING
C *********
      DO 104 IM = KB,N
      NH = IM-1
      JLI = JLI+1
      LL = JLI-1
      DT = TS(IM)-TS(NH)
C *********
C     PROPAGATE THE STATE AND ERROR COVARIANCE MATRIX FOR A TIME STEP DT
      CALL PROP(DT,US,AR,Q,IM)
C *********
C     COMPUTE THE KALMAN FILTER GAIN
      CALL GAIN(HZ,R)
C *********
C     UPDATE THE STATE AND ERROR COVARIANCE MATRIX
      CALL UPDT(ZV,ZR,HZ,IM)
C *********
      IF (LL.LT.4) GO TO 377
C *********
C     STORE VALUES OF STATE AND ERROR COVARIANCE MATRIX FOR PLOTTING
      CALL STORB(TS,MH,KFIM)
C *********
      MH = MH+1
      JLI = 1
  377 CONTINUE
  104 CONTINUE
C *********
C     CALL 3'RD PART OF PROGRAM
      CALL LINK ('HLBR3 ')
C *********
      END
```

```
      REAL FUNCTION FKV(XV,XR,U,A)
      DIMENSION A(15)
      COMMON /CRD/ CR,CR1,CR2,CR3
      FKV = A(9)+A(6)*XV+A(7)*XR+A(8)*U-CR1*(A(15)+A(12)*XV+A(13)*XR+A(1
     14)*U)
      RETURN
      END
```

```
      REAL FUNCTION FKR(XV,XR,U,A)
      DIMENSION A(15)
      COMMON /CRD/ CR,CR1,CR2,CR3
      FKR = CR2*(A(15)+A(12)*XV+A(13)*XR+A(14)*U)-CR3*(A(9)+A(6)*XV+A(7)
     1*XR+A(8)*U)
      RETURN
      END
```

```
      SUBROUTINE EFNT1(A,U)
      DIMENSION B(36),A(15),X(6)
      COMMON /EFN/ B
      COMMON /STRV/ X
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /CRD/ CR,CR1,CR2,CR3
      DO 399 J = 1,36
      B(J) = 0.0
  399 CONTINUE
      B(1) = A(6)-CR1*A(12)
      B(7) = A(7)-CR1*A(13)
      B(2) = CR2*A(12)-CR3*A(6)
      B(8) = CR2*A(13)-CR3*A(7)
      NPA = LP1-3
      IPA = 13
      IPB = 14
      NPR = 2
   10 CONTINUE
      GO TO(11,12,13,14,15,16,17,18,19,20,21,22),NPA
   11 B(IPA) = 0.0
      B(IPB) = 0.0
      GO TO 25
   12 B(IPA) = 0.0
      B(IPB) = 0.0
      GO TO 25
   13 B(IPA) = X(1)
      B(IPB) = -CR3*X(1)
      GO TO 25
   14 B(IPA) = X(2)
      B(IPB) = -CR3*X(2)
      GO TO 25
   15 B(IPA) = U
```

```
      B(IPB) = -CR3*U
      GO TO 25
   16 B(IPA) = 1.
      B(IPB) = -CR3
      GO TO 25
   17 B(IPA) = 0.0
      B(IPB) = 0.0
      GO TO 25
   18 B(IPA) = 0.0
      B(IPB) = 0.0
      GO TO 25
   19 B(IPA) = -CR1*X(1)
      B(IPB) = CR2*X(1)
      GO TO 25
   20 B(IPA) = -CR1*X(2)
      B(IPB) = CR2*X(2)
      GO TO 25
   21 B(IPA) = -CR1*U
      B(IPB) = CR2*U
      GO TO 25
   22 B(IPA) = -CR1
      B(IPB) = CR2
   25 CONTINUE
      IF (NPR.GT.2) GO TO 30
      NPR = NPR+1
      NPA = LP2-3
      IPA = 19
      IPB = 20
      GO TO 10
   30 CONTINUE
      IF (NPR.GT.3) GO TO 35
      NPR = NPR+1
```

```
      NPA = LP3-3
      IPA = 25
      IPB = 26
      GO TO 10
   35 CONTINUE
      IF (NPR.GT.4) GO TO 40
      NPR = NPR+1
      NPA = LP4-3
      IPA = 31
      IPB = 32
      GO TO 10
   40 CONTINUE
      RETURN
      END
```

```
SUBROUTINE STORB(T,MH,K)
DIMENSION T(1)
DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
DIMENSION XH(6),EH(36)
COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
COMMON /STRV/ XH
COMMON /COV/ EH
COMMON /CRD/ CR,CR1,CR2,CR3
I = MH
L = 4*I
D = T(L)
VP(I) = D
RP(I) = D
PP1(I) = D
PP2(I) = D
PP3(I) = D
PP4(I) = D
N = I+K
VP(N) = XH(1)
RP(N) = 57.296*XH(2)
PP1(N) = XH(3)*CR
PP2(N) = XH(4)*CR
PP3(N) = XH(5)*CR
PP4(N) = XH(6)*CR
IF (I.LT.K) GO TO 100
EV = SQRT(ABS(EH(1)))
ER = SQRT(ABS(EH(8)))
EP1 = SQRT(ABS(EH(15)))*(CR)
EP2 = SQRT(ABS(EH(22)))*(CR)
EP3 = SQRT(ABS(EH(29)))*(CR)
EP4 = SQRT(ABS(EH(36)))*(CR)
```

```
100 CONTINUE
    RETURN
    END
```

```
C ****************************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- LINEAR MODEL ****
C **** VERSION 1 -- DON'T PERMIT TO IDENTIFY THE
C     COEFFICIENTS OF VDOT AND RDOT
C
C ***** 3'RD PART -- PROGRAM HLBR3 *****
C     PRINTS THE OUTPUT OF EXTENDED KALMAN FILTERING
C
C ****************************************************************

      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),TS(188),US(188)
      DIMENSION EHT(36),XHT(6),XBAR(6),EBAR(36),B(36),EG(12)
      DIMENSION A(15)
      DIMENSION Q(2),R(2)
      DIMENSION AR(15)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZVRTU/ ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EV,ER,EP1,EP2,EP3,EP4
```

```
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /EFN/ B
      COMMON /GUP/ EG
      COMMON /CRD/ CR,CR1,CR2,CR3
      COMMON /AAR/ AR
      KO = 5
      PST1 = PST1*CR
      PST2 = PST2*CR
      PST3 = PST3*CR
      PST4 = PST4*CR
      PSD1 = PSD1*CR
      PSD2 = PSD2*CR
      PSD3 = PSD3*CR
      PSD4 = PSD4*CR
      K = 47
      KP = 94
      NS = 0
      N = 1
      M = 2
C *********
C     PLOT THE KALMAN FILTER PRIMARY STATES
C *********
      CALL PLOT(N,VP,K,M,NS)
      N = N+1
      CALL PLOT(N,RP,K,M,NS)
      N = N+1
C *********
C     PLOT THE PARAMETERS IDENTIFIED BY THE KALMAN FILTER
C *********
      CALL PLOT(N,PP1,K,M,NS)
      N = N+1
```

```
      CALL PLOT(N,PP2,K,M,NS)
      N = N+1
      CALL PLOT(N,PP3,K,M,NS)
      N = N+1
      CALL PLOT(N,PP4,K,M,NS)
      WRITE (KO,557)
      WRITE (KO,555) LP1,PA1,PST1,PSD1,PP1(KP),EP1
      WRITE (KO,556) LP2,PA2,PST2,PSD2,PP2(KP),EP2
      WRITE (KO,556) LP3,PA3,PST3,PSD3,PP3(KP),EP3
      WRITE (KO,556) LP4,PA4,PST4,PSD4,PP4(KP),EP4
      WRITE (KO,558)
  555 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE  =',2X,E13.5//8X,'SV  =',
     12X,E13.5,'  + OR =  ',E13.5//8X,'FV  =',2X,E13.5,'  + OR =  ',E13.
     25)
  556 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE  =',2X,E13.5//8X,'SV  =',
     12X,E13.5,'  + OR =  ',E13.5//8X,'FV  =',2X,E13.5,'  + OR =  ',E13.
     25)
  557 FORMAT (1H1,//5X,'PARAMETRIC IDENTIFICATION USING KALMAN FILTER')
  558 FORMAT (////20X,'  LINEAR MODEL  ',///////////)
      STOP
      END
```

APPENDIX 6

```
C ******************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C ***** MODEL REFERENCE CONTOUR -- NONLINEAR MODEL *****
C
C
C ****  THIS PROGRAM USES THE MODEL REFERENCE APPROACH
C     TO IDENTIFY THE NONLINEAR HYDRODYNAMIC COEFFICIENTS
C     OF THE MARINER CLASS HULL FORM ****
C
C ******************************************
      DIMENSION VEU(188),VEV(188),VER(188),XMU(188),XMV(188),XMR(188),XU
     1T(94),XVT(94),XRT(94)
      DIMENSION PAR1(13),PAR2(13),PI(169)
      DIMENSION GW(5),DV(5)
      DIMENSION A(40)
      DIMENSION INX(6),PMS(12)
      DIMENSION Z(100)
      DIMENSION WN(3),VN(3)
      COMMON /PNGW/G
      COMMON /PNDV/ D
      COMMON /PMTH/ MTH
      KI = 8
      KO = 5
      READ (KI,10) (A(I),I = 1,36)
   10 FORMAT (6E13.4)
      READ (KI,11) (PMS(J),J = 1,12)
   11 FORMAT (6E13.4)
      READ (KI,12) (INX(I),I = 1,6)
   12 FORMAT (6I6)
      READ (KI,15) (DV(J),J = 1,4)
      READ (KI,15) (GW(J),J = 1,4)
```

```
   15 FORMAT (4F10.3)
      READ (KI,16) LP1,LP2
   16 FORMAT (2I4)
      READ (KI,18) NGW,NDV
   18 FORMAT (2I4)
      PA1 = A(LP1)
      PA2 = A(LP2)
      RP1 = .30*ABS(PA1)
      RP2 = .30*ABS(PA2)
      DP1 = .075*ABS(PA1)
      DP2 = .075*ABS(PA2)
      PS1 = PA1 -RP1
      PS2 = PA2-RP2
      NP1 = 9
      NP2 = 9
      IP1 = NP1-1
      IP2 = NP2-1
      PAR1(1) = PS1
      PAR2(1) = PS2
      DO 100 J = 1,IP1
      PAR1(J+1) = PAR1(J)+DP1
  100 CONTINUE
      DO 105 J = 1,IP2
      PAR2(J+1) = PAR2(J)+DP2
  105 CONTINUE
      D5 = 57.296
      UO = 25.317
      NS = 0
      NO = 1
      NPL = 47
C ****
C     INITIAL CONDITIONS
```

```
C *********
      TI = 0.0
      XUI = 0.0
      XVI = 0.0
      XRI = 0.0
      H = 6.0
      MTH = 1
      N = 47
C *********
C     FIRST DO LOOP IN THE IDENTIFICATION PROCESS
C     DIFFERENT LEVELS OF NOISE
C *********
      DO 110 IGW = 1,NGW
      G = GW(IGW)
      IDV = IGW
      D = DV(IDV)
      A(LP1) = PA1
      A(LP2) = PA2
C *********
C     GENERATE THE SEA TRIAL DATA
      CALL UVRS(H,TI,XUI,XVI,XRI,K,N,A,VEU,VEV,VER,INX,PMS)
C *********
      DO 120 K = 1,NPL
      KD = K+NPL
      KK = MTH*K
      XUT(K) = TI+FLOAT(KK)*H
      XUT(KD) = UO+VEU(KK)
      XVT(K) = TI +FLOAT(KK)*H
      XVT(KD) = VEV(KK)
      XRT(K) = TI+FLOAT(KK)*H
      XRT(KD) = D5*VER(KK)
  120 CONTINUE
```

```
C *********
C      PLOT THE SEA TRIAL DATA
       CALL PLOT(01,XUT,NPL,2,0)
       CALL PLOT(02,XVT,NPL,2,0)
       CALL PLOT(03,XRT,NPL,2,0)
C *********
C      MAIN LOOP IN THE SYSTEM IDENTIFICATION PROCESS
C      VARIATION OF PARAMETERS
C *********
       P1 = PS1
       DO 125 IM = 1,NP1
       P2 = PS2
       DO 130 IN = 1,NP2
C *********
C      GENERATE THE MODEL DATA
       CALL UVRM(H,TI,XUI,XVI,XRI,K,N,A,P1,P2,XMU,XMV,XMR,LP1,LP2)
C *********
       NC = (IM-1)*NP1+IN
       PI(NC) = 0.0
       DO 135 MP = 1,N
       IP = MP
       XDFU = VEU(IP)-XMU(IP)
       XDFV = VEV(IP)-XMV(IP)
       XDFR = VER(IP)-XMR(IP)
       PI(NC) = PI(NC)+(XDFU*XDFU+XDFV*XDFV+XDFR*XDFR)*H
   135 CONTINUE
       PI(NC) = ALOG(PI(NC))
       P2 = PAR2(IN+1)
   130 CONTINUE
       P1 = PAR1(IM+1)
   125 CONTINUE
C *********
```

```
C     PLOT CONTOUR OF THE COST FUNCTION
      CALL CONTUR(NO,PAR1,PAR2,PI,NP1,NP2,NS)
C *********
C     BETTER VIZUALIZATION
C     PLOT 3 SLICES OF PERFORMANCE INDEX ALONG 2'ND PARAMETER
C *********
      DO 140 I = 1,NP2
      NI1 = NP2+I
      NI2 = 4*NP2+I
      NI3 = 7*NP2+I
      NP4 = NP2+I
      NP5 = 2*NP2+I
      NP6 = 3*NP2+I
      Z(I) = PAR2(I)
      Z(NP4) = PI(NI1)
      Z(NP5) = PI(NI2)
      Z(NP6) = PI(NI3)
  140 CONTINUE
      CALL PLOT(NO,Z,NP2,4,0)
C *********
C     PLOT 3 SLICES OF PERFORMANCE INDEX ALONG 1'ST PARAMETER
C *********
      I = 0
      DO 145 J = 1,NP1
      I = I+1
      IL = (J-1)*NP2
      Z(I) = PAR1(J)
      NP4 = NP2+I
      IL1 = IL+2
      Z(NP4) = PI(IL1)
      NP5 = 2*NP2+I
      IL2 = IL+5
```

```
      Z(NP5) = PI(IL2)
      NP6 = 3*NP2+I
      IL3 = IL+8
      Z(NP6) = PI(IL3)
  145 CONTINUE
      CALL PLOT(NO,Z,NP1,4,0)
      NO = NO+1
  115 CONTINUE
  110 CONTINUE
      STOP
      END
```

```
      SUBROUTINE UVRS(H,TI,XUI,XVI,XRI,K,N,A,VEU,VEV,VER,IN,P)
      DIMENSION VEU(1),VEV(1),VER(1),IN(1),P(1),A(1)
      DIMENSION WN(3),VN(3)
      COMMON /PNDV/ D
      COMMON /PMTH/ MTH
      HM = H/2.0
      XS = 1.0
      NVAR = 3
      XU = XUI
      XV = XVI
      XR = XRI
      T = TI
      NN = MTH*N
      DO 500 J = 1,NN
      UD = U(T,XS)
      DO 333 IVAR = 1,NVAR
      CALL WNNL(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  333 CONTINUE
      DO 334 IR = 1,NVAR
      IVAR = IR+NVAR
      CALL WNNL(IN,P,IVAR,NVAR,W)
      VN(IR) = W
  334 CONTINUE
      WM = WN(1)
      Y1 = HM*FNLU(T,XU,XV,XR,UD,A,WM)
      WM = WN(2)
      Y2 = HM*FNLV(T,XU,XV,XR,UD,A,WM)
      WM = WN(3)
      Y3 = HM*FNLR(T,XU,XV,XR,UD,A,WM)
      TM = T+HM
      XU1 = XU+Y1
```

```
      XV1 = XV+Y2
      XR1 = XR+Y3
      UD = U(TM,XS)
      DO 335 IVAR = 1,NVAR
      CALL WNNL(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  335 CONTINUE
      WM = WN(1)
      XU = XU+H*FNLU(TM,XU1,XV1,XR1,UD,A,WM)
      WM = WN(2)
      XV = XV+H*FNLV(TM,XU1,XV1,XR1,UD,A,WM)
      WM = WN(3)
      XR = XR+H*FNLR(TM,XU1,XV1,XR1,UD,A,WM)
      VM = VN(1)
      VEU(J) = XU+D*VM
      VM = VN(2)
      VEV(J) = XV+D*VM
      VM = VN(3)
      VER(J) = XR+D*VM
  500 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION U(T,XS)
C *********
C     STEP DEFLECTION OF RUDDER :35 DEGREES
C     TIME LAG NEGLECTED
C *********
      U = 35./57.296
      RETURN
      END
```

```
SUBROUTINE WNNL(IN,P,IVAR,NVAR,W)
DIMENSION IN(1),P(1)
IX = IN(IVAR)
AM = P(IVAR)
L = IVAR+2*NVAR
S = P(L)
CALL GAUSS(IX,S,AM,W)
IN(IVAR) = IX
RETURN
END
```

```
      REAL FUNCTION FNLU(T,XU,XV,XR,U,A,W)
      DIMENSION A(40)
      COMMON /PNGW/ G
      FNLU = 1./A(1)*(A(3)+A(2)*XU+A(16)*XU**2.+A(17)*XU**3.+A(18)*XV**2
     1.+A(19)*XR**2.+A(20)*U**2.+A(21)*XV*XR+A(22)*XV*U)+G*W
      RETURN
      END
```

```
      REAL FUNCTION FNLV(T,XU,XV,XR,U,A,W)
      DIMENSION A(40)
      COMMON /PNGW/ G
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+A
     1(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNLV = F4*(A(11)*F2-A(5)*F3)+G*W
      RETURN
      END
```

```
      REAL FUNCTION FNLR(T,XU,XV,XR,U,A,W)
      DIMENSION A(40)
      COMMON /PNGW/ G
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+A
     1(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNLR = F4*(A(4)*F3-A(10)*F2)+G*W
      RETURN
      END
```

```
      SUBROUTINE UVRM(H,TI,XUI,XVI,XRI,K,N,A,P1,P2,XMU,XMV,XMR,LP1,LP2)
      DIMENSION XMU(1),XMV(1),XMR(1),A(1)
      COMMON /PMTH/ MTH
      A(LP1) = P1
      A(LP2) = P2
      HM = H/2.0
      XS = 1.0
      XU = XUI
      XV = XVI
      XR = XRI
      T = TI
      NN = MTH*N
      DO 300 J = 1,NN
      UD = U(T,XS)
      Y1 = HM*FNDU(T,XU,XV,XR,UD,A)
      Y2 = HM*FNDV(T,XU,XV,XR,UD,A)
      Y3 = HM*FNDR(T,XU,XV,XR,UD,A)
      XU1 = XU+Y1
      XV1 = XV+Y2
      XR1 = XR+Y3
      TM = T+HM
      UD = U(TM,XS)
      XU = XU+H*FNDU(TM,XU1,XV1,XR1,UD,A)
      XV = XV+H*FNDV(TM,XU1,XV1,XR1,UD,A)
      XR = XR+H*FNDR(TM,XU1,XV1,XR1,UD,A)
      T = T+H
      XMU(J) = XU
      XMV(J) = XV
      XMR(J) = XR
  300 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION FNDU(T,XU,XV,XR,U,A)
      DIMENSION A(40)
      FNDU = 1./A(1)*(A(3)+A(2)*XU+A(16)*XU**2.+A(17)*XU**3.+A(18)*XV**2
     1.+A(19)*XR**2.+A(20)*U**2.+A(21)*XV*XR+A(22)*XV*U)
      RETURN
      END
```

```
      REAL FUNCTION FNDV(T,XU,XV,XR,U,A)
      DIMENSION A(40)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+
     1A(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNDV = F4*(A(11)*F2-A(5)*F3)
      RETURN
      END
```

```
      REAL FUNCTION FNDR(T,XU,XV,XR,U,A)
      DIMENSION A(40)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+A
     1(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNDR = F4*(A(4)*F3-A(10)*F2)
      RETURN
      END
```

APPENDIX 7

```
C ****************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- NONLINEAR MODEL ****
C
C     THIS PROGRAM WAS DIVIDED IN 3 PARTS TO FIT THE
C     COMPUTER CAPACITY
C
C *****  FIRST PART --GENERATES THE SEA TRIAL DATA  *****
C
C ****************************************************
C
      DIMENSION ZV(188),ZR(188),TS(188),US(188),VEIV(94),VEIR(94)
      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZU(188),VEIU(188),UP(94)
      DIMENSION EHT(49),XHT(7),XBAR(7),EBAR(49),B(49),EG(21)
      DIMENSION A(36),AI(36),ASD(36),PMS(12),INX(12),Q(3),R(3)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZUVRT/ ZU,ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ UP,VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EU,EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
```

```
      COMMON /EFN/ B
      COMMON /GUP/ EG
      COMMON /INPUT/ DI
      KI = 8
      KO = 5
      READ (KI,10) (A(I),I = 1,36)
      READ (KI,10) (AI(I),I = 1,36)
      READ (KI,10) (ASD(I),I = 1,36)
   10 FORMAT (6E13.4)
      READ (KI,11) (PMS(J),J = 1,12)
   11 FORMAT (6E13.4)
      READ (KI,12) (INX(I),I = 1,6)
   12 FORMAT (6I6)
      READ (KI,13) D,G
   13 FORMAT (2F10.6)
      READ (KI,14) LP1,LP2,LP3,LP4
   14 FORMAT (4I5)
      READ (KI,19) UST,VST,RST
   19 FORMAT (3F10.5)
      READ (KI,20) UCV,VCV,RCV
   20 FORMAT (3E10.3)
      READ (KI,21) KS,N,H
   21 FORMAT (2I4,F10.2)
      READ (KI,27) DI
   27 FORMAT (F10.3)
C *********
C     INITIAL CONDITIONS
      TI = 0.0
      XVI = 0.0
      XRI = 0.0
      XUI = 0.0
      PA1 = A(LP1)
```

```
      PA2 = A(LP2)
      PA3 = A(LP3)
      PA4 = A(LP4)
C *********
C     GENERATE SEA TRIAL DATA
      CALL RKL(H,TI,XUI,XVI,XRI,N,A,ZU,ZV,ZR,US,TS,INX,PMS)
      NPL = 47
      D5 = 57.296
      DO 101 I = 1,NPL
      L = KS*I
      NL1 = I+NPL
      VEIU(I) = TS(L)
      VEIV(I) = TS(L)
      VEIR(I) = TS(L)
      VEIV(NL1) = ZV(L)
      VEIU(NL1) = ZU(L)
      VEIR(NL1) = ZR(L)
  101 CONTINUE
C *********
C     PLOT SEA TRIAL DATA
      CALL PLOT(0,VEIU,NPL,2,0)
      CALL PLOT(0,VEIV,NPL,2,0)
      CALL PLOT(0,VEIR,NPL,2,0)
      PST1 = AI(LP1)
      PST2 = AI(LP2)
      PST3 = AI(LP3)
      PST4 = AI(LP4)
C *********
C     SET INITIAL STATE ESTIMATE
      XHT(1) = UST
      XHT(2) = VST
      XHT(3) = RST
```

```
      XHT(4) = PST1
      XHT(5) = PST2
      XHT(6) = PST3
      XHT(7) = PST4
C *********
C     SET EXAGERATED NOISE PARAMETERS
      PW = 1.0
      QW = 1.0
      DO 55 IR = 1,3
      IA = IR+6
      IV = IA+3
      Q(IR) = QW*G**2.*(PMS(IA))**2.
      R(IR) = PW*D**2.*(PMS(IV))**2.
   55 CONTINUE
C *********
C     INITIALIZE EHAT MATRIX
      DO 102 J = 1,49
      EHT(J) = 0.0
  102 CONTINUE
      PSD1 = ASD(LP1)
      PSD2 = ASD(LP2)
      PSD3 = ASD(LP3)
      PSD4 = ASD(LP4)
      PCV1 = PSD1**2.
      PCV2 = PSD2**2.
      PCV3 = PSD3**2.
      PCV4 = PSD4**2.
C *********
C     SET INITIAL ERROR COVARIANCES
      EHT(1) = UCV
      EHT(9) = VCV
      EHT(17) = RCV
```

```
      EHT(25) = PCV1
      EHT(33) = PCV2
      EHT(41) = PCV3
      EHT(49) = PCV4
C *********
C     CALL 2'ND PART OF PROGRAM
      CALL LINK ('HLBR2 ')
      END
```

```
      SUBROUTINE RKL(H,TI,XUI,XVI,XRI,N,A,ZU,ZV,ZR,US,TS,IN,P)
      DIMENSION ZV(1),ZR(1),TS(1),US(1),A(1),IN(1),P(1)
      DIMENSION ZU(1)
      DIMENSION WN(3),VN(3)
      COMMON /RKI/D
      XS = 1.0
      T = TI
      XU = XUI
      XV = XVI
      XR = XRI
      HM = H/2.
      NVAR = 3
      DO 300 IJ = 1,N
      TM = T+HM
      TN = T+H
      UD = U(T,XS)
      DO 333 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  333 CONTINUE
      WL = WN(1)
      YU1 = H*FNLU(T,XU,XV,XR,UD,A,WL)
      WL = WN(2)
      YV1 = H*FNLV(T,XU,XV,XR,UD,A,WL)
      WL = WN(3)
      YR1 = H*FNLR(T,XU,XV,XR,UD,A,WL)
      XX1 = XU+.5*YU1
      XX2 = XV+.5*YV1
      XX3 = XR+.5*YR1
      UD = U(TM,XS)
      DO 335 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
```

```
      WN(IVAR) = W
  335 CONTINUE
      WL = WN(1)
      YU2 = H*FNLU(TM,XX1,XX2,XX3,UD,A,WL)
      WL = WN(2)
      YV2 = H*FNLV(TM,XX1,XX2,XX3,UD,A,WL)
      WL = WN(3)
      YR2 = H*FNLR(TM,XX1,XX2,XX3,UD,A,WL)
      XX1 = XU+.5*YU2
      XX2 = XV+.5*YV2
      XX3 = XR+.5*YR2
      DO 336 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  336 CONTINUE
      WL = WN(1)
      YU3 = H*FNLU(TM,XX1,XX2,XX3,UD,A,WL)
      WL = WN(2)
      YV3 = H*FNLV(TM,XX1,XX2,XX3,UD,A,WL)
      WL = WN(3)
      YR3 = H*FNLR(TM,XX1,XX2,XX3,UD,A,WL)
      XX1 = XU+YU3
      XX2 = XV+YV3
      XX3 = XR+YR3
      UD = U(TN,XS)
      DO 337 IVAR = 1,NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      WN(IVAR) = W
  337 CONTINUE
      WL = WN(1)
      YU4 = H*FNLU(TN,XX1,XX2,XX3,UD,A,WL)
      WL = WN(2)
```

```
      YV4 = H*FNLV(TN,XX1,XX2,XX3,UD,A,WL)
      WL = WN(3)
      YR4 = H*FNLR(TN,XX1,XX2,XX3,UD,A,WL)
      XU = XU+1./6.*(YU1+2.*YU2+2.*YU3+YU4)
      XV = XV+1./6.*(YV1+2.*YV2+2.*YV3+YV4)
      XR = XR+1./6.*(YR1+2.*YR2+2.*YR3+YR4)
      DO 334 IR = 1,NVAR
      IVAR = IR+NVAR
      CALL WNO(IN,P,IVAR,NVAR,W)
      VN(IR) = W
  334 CONTINUE
      VL = VN(1)
      ZU(IJ) = XU+D*VL
      VL = VN(2)
      ZV(IJ) = XV+D*VL
      VL = VN(3)
      ZR(IJ) = XR+D*VL
      US(IJ) = UD
      TS(IJ) = T
      T = T+H
  300 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION U(T,XS)
      COMMON /INPUT/ DI
      D = DI/57.296
      IF(T-100.) 3,4,4
    3 U = D
      RETURN
    4 IF (T-200.) 5,6,6
    5 U = -D
      RETURN
    6 U = 0.0
      RETURN
      END
```

```
SUBROUTINE WNO(IN,P,IVAR,NVAR,W)
DIMENSION IN(1),P(1)
IX = IN(IVAR)
LW = IVAR+2*NVAR
AM = P(IVAR)
S = P(LW)
CALL GAUSS(IX,S,AM,W)
IN(IVAR) = IX
RETURN
END
```

```
      REAL FUNCTION FNLU(T,XU,XV,XR,U,A,W)
      DIMENSION A(36)
      COMMON /PKI/G
      FNLU = 1./A(1)*(A(3)+A(2)*XU+A(16)*XU**2.+A(17)*XU**3.+A(18)*XV**2
     1.+A(19)*XR**2.+A(20)*U**2.+A(21)*XV*XR+A(22)*XV*U)+G*W
      RETURN
      END
```

```
      REAL FUNCTION FNLV(T,XU,XV,XR,U,A,W)
      DIMENSION A(36)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+A
     1(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNLV = F4*(A(11)*F2-A(5)*F3)+G*W
      RETURN
      END
```

```
      REAL FUNCTION FNLR(T,XU,XV,XR,U,A,W)
      DIMENSION A(36)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+A
     1(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FNLR = F4*(A(4)*F3-A(10)*F2)+G*W
      RETURN
      END
```

```
C ************************************************************
C      PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C      TRIALS
C
C **** EXTENDED KALMAN FILTERING -- NONLINEAR MODEL ****
C
C ***** 2'ND PART --PROGRAM HLBR2  ******
C      USES KALMAN FILTER TO PROCESS THE SEA TRIAL DATA
C
C ************************************************************


      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),US(188),TS(188)
      DIMENSION ZU(188),UP(94)
      DIMENSION EHT(49),XHT(7),XBAR(7),EBAR(49),B(49),EG(21)
      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94)
      DIMENSION A(36),Q(3),R(3),HZ(3)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZUVRT/ ZU,ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ UP,VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EU,EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
```

```
      COMMON /EFN/ B
      COMMON /GUP/ EG
      COMMON /INPUT/ DI
C *********
C     DEFINE H MATRIX : Z = HX
      HZ(1) = 1.0
      HZ(2) = 1.0
      HZ(3) = 1.0
C *********
C     INITIAL SETUP
C     SET KALMAN FILTER INCREMENT,STARTING INDEX,AND STOPPING INDEX
      KST = 1
      KFN = 1
      KB = KST+KFN
      MH = 1
      KFIM = 47
C *********
C     PROCESS THE SEA TRIAL DATA
      USV = US(1)
      XS = 1.0
      UZ = U(TI,XS)
      US(1) = UZ
      DT = H
      CALL PROP(DT,US,A,Q,1)
      CALL GAIN(HZ,R)
      CALL UPDT(ZU,ZV,ZR,HZ,1)
      US(1) = USV
      JLI = 2
C *********
C     BEGIN ITERATIONS FOR FILTERING
      DO 104 IM = KB,N
      NH = IM-1
```

```
      JLI = JLI+1
      LL = JLI-1
      DT = TS(IM)-TS(NH)
C *********
C     PROPAGATE THE STATE AND ERROR COVARIANCE MATRIX FOR A TIME STEP DT
      CALL PROP(DT,US,A,Q,IM)
C *********
C     COMPUTE THE KALMAN FILTER GAIN
      CALL GAIN(HZ,R)
C *********
C     UPDATE THE STATE AND ERROR COVARIANCE MATRIX
      CALL UPDT(ZU,ZV,ZR,HZ,IM)
      IF (LL.LT.4) GO TO 377
C *********
C     STORE VALUES OF STATE AND ERROR COVARIANCE MATRIX FOR PLOTTING
      CALL STORB(TS,MH,KFIM)
      MH = MH+1
      JLI = 1
  377 CONTINUE
  104 CONTINUE
C *********
C     CALL 3'RD PART OF PROGRAM
      CALL LINK ('HLBR3 ')
      END
```

```
      REAL FUNCTION U(T,XS)
      COMMON /INPUT/ DI
      D = DI/57.296
      IF(T=100.) 3,4,4
    3 U = D
      RETURN
    4 IF (T=200.) 5,6,6
    5 U = =D
      RETURN
    6 U = Ø.Ø
      RETURN
      END
```

```
      SUBROUTINE PROP(H,US,A,Q,I)
      DIMENSION US(2),A(1),Q(1)
      DIMENSION  XHT(7),EHT(49),XBAR(7),EBAR(49)
      DIMENSION EJ(49),E1(49),E2(49),E3(49),E4(49),E5(49)
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /SZRV/ XHT
      COMMON /COV/ EHT
      COMMON /PZAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      XU = XHT(1)
      XV = XHT(2)
      XR = XHT(3)
      A(LP1) = XHT(4)
      A(LP2) = XHT(5)
      A(LP3) = XHT(6)
      A(LP4) = XHT(7)
      DO 3 J = 1,49
      EJ(J) = EHT(J)
    3 CONTINUE
      UV = US(I)
      HM= H/2.
      YU1 = H*FKU(XU,XV,XR,UV,A)
      YV1 = H*FKV(XU,XV,XR,UV,A)
      YR1 = H*FKR(XU,XV,XR,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XU+.5*YU1
      XY2 = XV+.5*YV1
      XY3 = XR+.5*YR1
      XHT(1) = XY1
      XHT(2) = XY2
      XHT(3) = XY3
```

```
      DO 4 J = 1,49
      E2(J) = E1(J)*H
      EHT(J) = HM*E1(J)+EJ(J)
    4 CONTINUE
      UV = (US(I)+US(I+1))/2.
      YU2 = H*FKU(XY1,XY2,XY3,UV,A)
      YV2 = H*FKV(XY1,XY2,XY3,UV,A)
      YR2 = H*FKR(XY1,XY2,XY3,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XU+.5*YU2
      XY2 = XV+.5*YV2
      XY3 = XR+.5*YR2
      XHT(1) = XY1
      XHT(2) = XY2
      XHT(3) = XY3
      DO 6 J = 1,49
      E3(J) = H*E1(J)
      EHT(J) = EJ(J)+HM*E1(J)
    6 CONTINUE
      YU3 = H*FKU(XY1,XY2,XY3,UV,A)
      YV3 = H*FKV(XY1,XY2,XY3,UV,A)
      YR3 = H*FKR(XY1,XY2,XY3,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      XY1 = XU+YU3
      XY2 = XV+YV3
      XY3 = XR+YR3
      XHT(1) = XY1
      XHT(2) = XY2
      XHT(3) = XY3
      DO 7 J = 1,49
```

```
      E4(J) = H*E1(J)
      EHT(J) = EJ(J)+E1(J)*H
    7 CONTINUE
      YU4 = FKU(XY1,XY2,XY3,UV,A)*H
      YV4 = H*FKV(XY1,XY2,XY3,UV,A)
      YR4 = H*FKR(XY1,XY2,XY3,UV,A)
      CALL EFNT1(A,UV)
      CALL EFNT2(E1,Q)
      DO 8 J = 1,49
      E5(J) = H*E1(J)
    8 CONTINUE
      XBAR(1) = XU+1./6.*(YU1+2.*YU2+2.*YU3+YU4)
      XBAR(2) = XV+1./6.*(YV1+2.*YV2+2.*YV3+YV4)
      XBAR(3) = XR+1./6.*(YR1+2.*YR2+2.*YR3+YR4)
      XBAR(4) = XHT(4)
      XBAR(5) = XHT(5)
      XBAR(6) = XHT(6)
      XBAR(7) = XHT(7)
      DO 5 J = 1,49
      EBAR(J) = EJ(J)+1./6.*(E2(J)+2.*E3(J)+2.*E4(J)+E5(J))
    5 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION FKU(XU,XV,XR,U,A)
      DIMENSION A(36)
      FKU   = 1./A(1)*(A(3)+A(2)*XU+A(16)*XU**2.+A(17)*XU**3.+A(18)*XV**2
     1.+A(19)*XR**2.+A(20)*U**2.+A(21)*XV*XR+A(22)*XV*U)
      RETURN
      END
```

```
      REAL FUNCTION FKV(XU,XV,XR,U,A)
      DIMENSION A(36)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+
     1A(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FKV  = F4*(A(11)*F2-A(5)*F3)
      RETURN
      END
```

```
      REAL FUNCTION FKR(XU,XV,XR,U,A)
      DIMENSION A(36)
      F2 = A(9)+A(6)*XV+A(7)*XR+A(8)*U+A(26)*U**3.+A(27)*XR*XV**2.+A(28)
     1*U*XV**2.
      F3 = A(15)+A(12)*XV+A(13)*XR+A(14)*U+A(31)*U**3.+A(32)*XR*XV**2.+
     1A(33)*U*XV**2.
      F4 = 1./(A(4)*A(11)-A(10)*A(5))
      FKR  = F4*(A(4)*F3-A(10)*F2)
      RETURN
      END
```

```
      SUBROUTINE EFNT1(A,U)
      DIMENSION B(49),A(36),X(7)
      COMMON /EFN/ B
      COMMON /STRV/ X
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      DO 399 J =1,49
      B(J) = 0.0
  399 CONTINUE
      C2 = 1./(A(4)*A(11)-A(5)*A(10))
      C5 = A(9)+A(6)*X(2)+A(7)*X(3)+A(8)*U+A(26)*U**3.+A(27)*X(3)*X(2)**
     12.+A(28)*U*X(2)**2.
      C6 = A(15)+A(12)*X(2)+A(13)*X(3)+A(14)*U+A(31)*U**3.+A(32)*X(3)*X(
     12)**2.+A(33)*U*X(2)**2.
      B(1) = 1./A(1)*(A(2)+2.*A(16)*X(1)+3.*A(17)*X(1)**2.)
      B(2) = 0.0
      B(3) = 0.0
      B(8) = 1./A(1)*(A(18)*2.*X(2)+A(21)*X(3)+A(22)*U)
      D1 = A(6)+A(27)*X(3)*X(2)*2.+2.*A(28)*X(2)*U
      D2 = A(13)+2.*A(32)*X(3)*X(2)+2.*A(33)*X(2)*U
      B(9) = C2*(A(11)*D1-A(5)*D2)
      B(10) = C2*(A(4)*D2-A(10)*D1)
      B(15) = 1./A(1)*(A(19)*X(3)*2.+A(21)*X(2))
      D3 = A(7)+A(27)*X(2)**2.
      D4 = A(13)+A(32)*X(2)**2.
      B(16) = C2*(A(11)*D3-A(5)*D4)
      B(17) = C2*(A(4)*D4-A(10)*D3)
      NPA = LP1
      NPR = 2
      IPA = 22
      IPB = 23
      IPC = 24
   10 CONTINUE
```

```
      GO TO(11,12,13,14,15,16,17,18,19,20,21,22,23,24,55,26,27,28,29,30,
     131,32,33,34,35,36,37,38,39,40,41,42,43),NPA
   11 B(IPA) =-1./A(1)**2.*(A(2)*X(1)+A(16)*X(1)**2.+A(17)*X(1)**3.+A(18
     1)*X(2)**2.+A(19)*X(3)**2.+A(20)*U**2.+A(21)*X(2)*X(3)+A(22)*X(2)*U
     2)
      B(IPB) = Ø.Ø
      B(IPC) = Ø.Ø
      GO TO 25
   12 B(IPA) = X(1)/A(1)
      B(IPB) = Ø.Ø
      B(IPC) = Ø.Ø
      GO TO 25
   13 B(IPA) = Ø.Ø
      B(IPB) = Ø.Ø
      B(IPC) = Ø.Ø
      GO TO 25
   14 B(IPA) = Ø.Ø
      B(IPB) = -C2**2.*A(11)*(A(11)*C5-A(5)*C6)
      B(IPC) =-C2**2.*A(11)*(A(4)*C6-A(1Ø)*C5)+C2*C6
      GO TO 25
   15 B(IPA) = Ø.Ø
      B(IPB) = C2**2.*A(1Ø)*(A(11)*C5-A(5)*C6)-C2*C6
      B(IPC) = C2**2.*A(1Ø)*(A(4)*C6-A(1Ø)*C5)
      GO TO 25
   16 B(IPA) = Ø.Ø
      B(IPB) =  C2*A(11)*X(2)
      B(IPC) = -C2*A(1Ø)*X(2)
      GO TO 25
   17 B(IPA) = Ø.Ø
      B(IPB) = C2*A(11)*X(3)
      B(IPC) = -C2*A(1Ø)*X(3)
      GO TO 25
```

```
18 B(IPA) = 0.0
   B(IPB) = C2*A(11)*U
   B(IPC) = -C2*A(10)*U
   GO TO 25
19 B(IPA) = 0.0
   B(IPB) = C2*A(11)
   B(IPC) = -C2*A(10)
   GO TO 25
20 B(IPA) = 0.0
   B(IPB) = C2**2.*A(5)*(A(11)*C5-A(5)*C6)
   B(IPC) = C2**2.*A(5)*(A(4)*C6-A(10)*C5)-C2*C5
   GO TO 25
21 B(IPA) = 0.0
   B(IPB) = -C2**2.*A(4)*(A(11)*C5-A(5)*C6)+C2*C5
   B(IPC) = -C2**2.*A(4)*(A(4)*C6-A(10)*C5)
   GO TO 25
22 B(IPA) = 0.0
   B(IPB) = -C2*A(5)*X(2)
   B(IPC) = C2*A(4)*X(2)
   GO TO 25
23 B(IPA) = 0.0
   B(IPB) = -C2*A(5)*X(3)
   B(IPC) = C2*A(4)*X(3)
   GO TO 25
24 B(IPA) = 0.0
   B(IPB) = -C2*A(5)*U
   B(IPC) = C2*A(4)*U
   GO TO 25
55 B(IPA) = 0.0
   B(IPB) = -C2*A(5)
   B(IPC) = C2*A(4)
   GO TO 25
```

```
26 B(IPA) = X(1)**2./A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
27 B(IPA) = X(1)**3./A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
28 B(IPA) = X(2)**2./A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
29 B(IPA) = X(3)**2./A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
30 B(IPA) = U**2./A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
31 B(IPA) = X(2)*X(3)/A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
32 B(IPA) = X(2)*U/A(1)
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
33 B(IPA) = 0.0
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
```

```
34 B(IPA) = 0.0
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
35 B(IPA) = 0.0
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
36 B(IPA) = 0.0
   B(IPB) = -C2*U**3.*A(10)
   B(IPC) = C2*U**3.*A(11)
   GO TO 25
37 B(IPA) = 0.0
   B(IPB) = -C2*X(3)*X(2)**2.*A(10)
   B(IPC) = C2*X(3)*X(2)**2.*A(11)
   GO TO 25
38 B(IPA) = 0.0
   B(IPB) =-C2*U*X(2)**2.*A(10)
   B(IPC) = C2*U*X(2)**2.*A(11)
   GO TO 25
39 B(IPA) = 0.0
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
40 B(IPA) = 0.0
   B(IPB) = 0.0
   B(IPC) = 0.0
   GO TO 25
41 B(IPA) = 0.0
   B(IPB) = C2*U**3.*A(4)
   B(IPC) = -C2*U**3.*A(5)
   GO TO 25
```

```
42 B(IPA) = 0.0
   B(IPB) = C2*X(3)*X(2)**2.*A(4)
   B(IPC) = -C2*X(3)*X(2)**2.*A(5)
   GO TO 25
43 B(IPA) = 0.0
   B(IPB) = C2*U*X(2)**2.*A(4)
   B(IPC) = -C2*U*X(2)**2.*A(5)
25 CONTINUE
   IF (NPR.GT.2) GO TO 60
   NPR = NPR+1
   NPA = LP2
   IPA = 29
   IPB = 30
   IPC = 31
   GO TO 10
60 CONTINUE
   IF (NPR.GT.3) GO TO 65
   NPA = LP3
   IPA = 36
   IPB = 37
   IPC = 38
   GO TO 10
65 CONTINUE
   IF (NPR.GT.4) GO TO 70
   NPA = LP4
   IPA = 43
   IPB = 44
   IPC = 45
70 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE EFNT2(EB,Q)
      DIMENSION EB(49),Q(3),B(49),E(49)
      COMMON /COV/ E
      COMMON /EFN/ B
      DO 601 I = 1,49
      EB(I) = 0.0
  601 CONTINUE
      EB(1) = EB(1)+2.*(B(1)*E(1)+B(8)*E(2)+B(15)*E(3)+B(22)*E(4)+B(29)*
     1E(5)+B(36)*E(6)+B(43)*E(7))+Q(1)
      EB(2) = EB(2)+B(2)*E(1)+B(9)*E(2)+B(16)*E(3)+B(23)*E(4)+B(30)*E(5)
     1+B(37)*E(6)+B(44)*E(7)+E(2)*B(1)+E(9)*B(8)+E(10)*B(15)+E(11)*B(22)
     2 +E(12)*B(29)+E(13)*B(36)+E(14)*B(43)
      EB(3) = EB(3)+B(3)*E(1)+B(10)*E(2)+B(17)*E(3)+B(24)*E(4)+B(31)*E(5
     1)+B(38)*E(6)+B(45)*E(7)+E(3)*B(1)+E(10)*B(8)+E(17)*B(15)+E(18)*B(2
     22)+E(19)*B(29+E(20)*B(36)+E(21)*B(43)
      EB(4) = EB(4)+E(4)*B(1)+E(11)*B(8)+E(18)*B(15)+E(25)*B(22)+E(26)*B
     1(29)+E(27)*B(36)+E(28)*B(43)
      EB(5) = EB(5)+E(5)*B(1)+E(12)*B(8)+E(19)*B(15)+E(26)*B(22)+E(33)*B
     1(29)+E(34)*B(36)+E(35)*B(43)
      EB(6) = EB(6)+E(6)*B(1)+E(13)*B(8)+E(20)*B(15)+E(27)*B(22)+E(34)*B
     1(29)+E(41)*B(36)+E(42)*B(43)
      EB(7) = EB(7)+E(7)*B(1)+E(14)*B(8)+E(21)*B(15)+E(28)*B(22)+E(35)*B
     1(29)+E(42)*B(36)+E(49)*B(43)
      EB(8) = EB(2)
      EB(9) = EB(9)+2.*(B(2)*E(2)+B(9)*E(9)+B(16)*E(10)+B(23)*E(11)+B(30
     1)*E(12)+B(37)*E(13)+B(44)*E(14))+Q(2)
      EB(10) = EB(10)+B(3)*E(2)+B(10)*E(9)+B(17)*E(10)+B(24)*E(11)+B(31)
     1*E(12)+B(38)*E(13)+B(45)*E(14)+E(3)*B(2)+E(10)*B(9)+E(17)*B(16)+E(
     218)*B(23)+E(19)*B(30)+E(20)*B(37)+E(21)*B(44)
      EB(11) = EB(11)+E(4)*B(2)+E(11)*B(9)+E(18)*B(16)+E(25)*B(23)+E(26)
     1*B(30)+E(27)*B(37)+E(28)*B(44)
      EB(12) = EB(12)+E(5)*B(2)+E(12)*B(9)+E(19)*B(16)+E(26)*B(23)+E(33)
```

```
1*B(30)+E(34)*B(37)+E(35)*B(44)
 EB(13) = EB(13)+E(6)*B(2)+E(13)*B(9)+E(20)*B(16)+E(27)*B(23)+E(34)
1*B(30)+E(41)*B(37)+E(42)*B(44)
 EB(14) = EB(14)+E(7)*B(2)+E(14)*B(9)+E(21)*B(16)+E(28)*B(23)+E(35)
1*B(30)+E(42)*B(37)+E(49)*B(44)
 EB(15) = EB(3)
 EB(16) = EB(10)
 EB(17) = EB(17)+2.*(B(3)*E(3)+B(10)*E(10)+B(17)*E(17)+B(24)*E(18)+
1B(31)*E(19)+B(38)*E(20)+B(45)*B(21))+Q(3)
 EB(18) = EB(18)+E(4)*B(3)+E(11)*B(10)+E(18)*B(17)+E(25)*B(24)+E(26
1)*B(31)+E(27)*B(38)+E(28)*B(45)
 EB(19)= EB(19)+E(5)*B(3)+E(12)*B(10)+E(19)*B(17)+E(26)*B(24)+E(33)
1*B(31)+E(34)*B(38)+E(35)*B(45)
 EB(20) = EB(20)+E(6)*B(3)+E(13)*B(10)+E(20)*B(17)+E(27)*B(24)+E(34
1)*B(31)+E(41)*B(38)+E(42)*B(45)
 EB(21) = EB(21)+E(7)*B(3)+E(14)*B(10)+E(21)*B(17)+E(28)*B(24)+E(35
1)*B(31)+E(42)*B(38)+E(49)*B(45)
 EB(22) = EB(4)
 EB(23) = EB(11)
 EB(24) = EB(18)
 EB(29) = EB(5)
 EB(30) = EB(12)
 EB(31) = EB(19)
 EB(36) = EB(6)
 EB(37) = EB(13)
 EB(38) = EB(20)
 EB(43) = EB(7)
 EB(44) = EB(14)
 EB(45) = EB(21)
 RETURN
 END
```

```
      SUBROUTINE GAIN(H,R)
      DIMENSION H(3),R(3),E2(21),E3(9),EG(21),E(49)
      DIMENSION E4(9)
      COMMON /PRUG/ E
      COMMON /GUP/ EG
      DO 1 J = 1,7
      L = J+7
      LL = L+7
      E2(J) = E(J)*H(1)
      E2(L) = E(L)*H(2)
      E2(LL) = E(LL)*H(3)
    1 CONTINUE
      E3(1) = H(1)*E2(1)+R(1)
      E3(2) = H(2)*E2(2)
      E3(3) = H(3)*E2(3)
      E3(4) = H(1)*E2(8)
      E3(5) = H(2)*E2(9)+R(2)
      E3(6) = H(3)*E2(10)
      E3(7) = H(1)*E2(15)
      E3(8) = E2(16)*H(2)
      E3(9) = H(3)*E2(17)+R(3)
      C1 = E3(5)*E3(9)-E3(6)*E3(8)
      C2 = E3(2)*E3(9)-E3(3)*E3(8)
      C3 = E3(2)*E3(6)-E3(3)*E3(5)
      D1 = E3(4)*E3(9)-E3(6)*E3(7)
      D2 = E3(1)*E3(9)-E3(3)*E3(7)
      D3 = E3(1)*E3(6)-E3(3)*E3(4)
      F1 = E3(4)*E3(8)-E3(5)*E3(7)
      F2 = E3(1)*E3(8)-E3(2)*E3(7)
      F3 = E3(1)*E3(5)-E3(2)*E3(4)
      D = E3(1)*C1-E3(4)*C2+E3(7)*C3
      E4(1) = C1/D
```

```
      E4(2) = -C2/D
      E4(3) = C3/D
      E4(4) = -D1/D
      E4(5) = D2/D
      E4(6) = -D3/D
      E4(7) = F1/D
      E4(8) = -F2/D
      E4(9) = F3/D
      DO 2 IJ = 1,3
      DO 3 II = 1,7
      LL = II+7*(IJ-1)
      EG(LL) = 0.0
      DO 4 K = 1,3
      LI = II+7*(K-1)
      LJ = K+(IJ-1)*3
      E5 = E2(LI)*E4(LJ)
      EG(LL) = EG(LL)+E5
    4 CONTINUE
    3 CONTINUE
    2 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE UPDT(ZU,ZV,ZR,H,IM)
      DIMENSION EB(49),XB(7),EG(21),ZV(1),ZU(1),ZR(1),EH(49),XH(7)
      DIMENSION EL(3),XD(7),EA(49),EC(49),Z(3),H(3)
      COMMON /PRUP/ XB
      COMMON /GUP/ EG
      COMMON /PRUG/ EB
      COMMON /STRV/ XH
      COMMON /COV/ EH
      Z(1) = ZU(IM)
      Z(2) = ZV(IM)
      Z(3) = ZR(IM)
      EL(1) = XB(1)*H(1)
      EL(2) = XB(2)*H(2)
      EL(3) = XB(3)*H(3)
      DO 11 IK = 1,7
      XD(IK) = 0.0
      DO 12 JK = 1,3
      L = IK+(JK-1)*7
      E7 = EG(L)*(Z(JK)-EL(JK))
      XD(IK) = XD(IK)+E7
   12 CONTINUE
      XH(IK) = XB(IK)+XD(IK)
   11 CONTINUE
      DO 13 J = 1,7
      I = 3*(J-1)+1
      K = 3*(J-1)+2
      M = 3*(J-1)+3
      L = 7*(J-1)+1
      LJ = 7*(J-1)+2
      LL = 7*(J-1)+3
      EA(I) = EB(L)*H(1)
      EA(K) = EB(LJ)*H(2)
```

```
      EA(M) = EB(LL)*H(3)
   13 CONTINUE
      DO 14 KJ = 1,7
      DO 15 IL = 1,7
      LK = 7*(KJ-1)+IL
      EC(LK) = 0.0
      DO 16 KK = 1,3
      LN = IL+7*(KK-1)
      LM = KK+3*(KJ-1)
      E8 = EG(LN)*EA(LM)
      EC(LK) = EC(LK)+E8
   16 CONTINUE
      EH(LK) = EB(LK)-EC(LK)
   15 CONTINUE
   14 CONTINUE
      RETURN
      END
```

```
SUBROUTINE STORB(T,MH,K)
DIMENSION UP(94)
DIMENSION T(1)
DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
DIMENSION XH(7),EH(49)
COMMON /OUTP1/ UP,VP,RP,PP1,PP2,PP3,PP4
COMMON /OUTP2/ EU,EV,ER,EP1,EP2,EP3,EP4
COMMON /STRV/ XH
COMMON /COV/ EH
I = MH
L = 4*I
D = T(L)
UP(I) = D
VP(I) = D
RP(I) = D
PP1(I) = D
PP2(I) = D
PP3(I) = D
PP4(I) = D
N = I+K
UP(N) = XH(1)
VP(N) = XH(2)
RP(N) = XH(3)
PP1(N) = XH(4)
PP2(N) = XH(5)
PP3(N) = XH(6)
PP4(N) = XH(7)
IF (I.LT.K) GO TO 100
EU = SQRT(ABS(EH(1)))
EV = SQRT(ABS(EH(9)))
ER = SQRT(ABS(EH(17)))
EP1 = SQRT(ABS(EH(25)))
```

```
      EP2 = SQRT(ABS(EH(33)))
      EP3 = SQRT(ABS(EH(41)))
      EP4 = SQRT(ABS(EH(49)))
  100 CONTINUE
      RETURN
      END
```

```
C ****************************************************
C     PARAMETRIC IDENTIFICATION APPLIED TO MANEUVERING
C     TRIALS
C
C **** EXTENDED KALMAN FILTERING -- NONLINEAR MODEL ****
C
C ***** 3'RD PART -- PROGRAM HLBR3 *****
C     PRINTS THE OUTPUT OF EXTENDED KALMAN FILTERING
C
C ****************************************************


      DIMENSION VP(94),RP(94),PP1(94),PP2(94),PP3(94),PP4(94)
      DIMENSION ZV(188),ZR(188),TS(188),US(188)
      DIMENSION UP(94),ZU(188),EHT(49),XHT(7),XBAR(7),EBAR(49),B(49),EG(
     121),A(36),Q(3),R(3)
      COMMON /PRM/PST1,PST2,PST3,PST4,PSD1,PSD2,PSD3,PSD4
      COMMON /PRAM/LP1,LP2,LP3,LP4,PA1,PA2,PA3,PA4
      COMMON /STRV/ XHT
      COMMON /COV/ EHT
      COMMON /EKF/ H,N
      COMMON /MDSTR/A,Q,R
      COMMON /ZUVRT/ ZU,ZV,ZR,US,TS
      COMMON /TINIT/ TI
      COMMON /RKI/D
      COMMON /PKI/G
      COMMON /OUTP1/ UP,VP,RP,PP1,PP2,PP3,PP4
      COMMON /OUTP2/ EU,EV,ER,EP1,EP2,EP3,EP4
      COMMON /PRUP/ XBAR
      COMMON /PRUG/ EBAR
      COMMON /EFN/ B
      COMMON /GUP/ EG
```

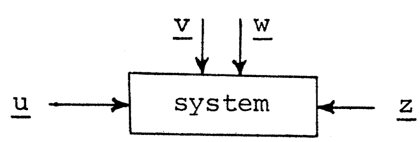
```
      COMMON /INPUT/ DI
      KO = 5
      K = 47
      KP = 94
      NS = 0
      N = 1
      M = 2
C *********
C     PLOT THE KALMAN FILTER PRIMARY STATES
      CALL PLOT(N,UP,K,M,NS)
      N = N+1
      CALL PLOT(N,VP,K,M,NS)
      N = N+1
      CALL PLOT(N,RP,K,M,NS)
      N = N+1
C *********
C     PLOT THE PARAMETERS IDENTIFIED BY THE KALMAN FILTER
C *********
      CALL PLOT(N,PP1,K,M,NS)
      N = N+1
      CALL PLOT(N,PP2,K,M,NS)
      N = N+1
      CALL PLOT(N,PP3,K,M,NS)
      N = N+1
      CALL PLOT(N,PP4,K,M,NS)
      WRITE (KO,557)
      WRITE (KO,555) LP1,PA1,PST1,PSD1,PP1(KP),EP1
      WRITE (KO,556) LP2,PA2,PST2,PSD2,PP2(KP),EP2
      WRITE (KO,556) LP3,PA3,PST3,PSD3,PP3(KP),EP3
      WRITE (KO,556) LP4,PA4,PST4,PSD4,PP4(KP),EP4
      WRITE (KO,558)
  555 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE  =',2X,E13.5//8X,'SV  =',
```

```
   12X,E13.5,'  + OR =  ',E13.5//8X,'FV  =',2X,E13.5,'  + OR =  ',E13.
   25)
556 FORMAT (////8X,'NP = ',I3,5X,'TRUE VALUE  =',2X,E13.5//8X,'SV  =',
   12X,E13.5,'  + OR =  ',E13.5//8X,'FV  =',2X,E13.5,'  + OR =  ',E13.
   25)
557 FORMAT (1H1,//5X,'PARAMETRIC IDENTIFICATION USING KALMAN FILTER')
558 FORMAT (////20X,' NON LINEAR MODEL ',//////////)
    STOP
    END
```

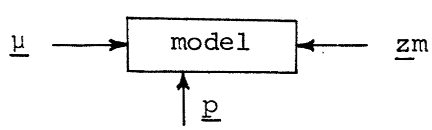
number of different parameter vectors $\underline{p}$ and then selects the specific $\underline{p}$ which results in the model output $\underline{z}_m$ which is closest to the original system output $\underline{z}$.

The model reference approach is well discussed in the literature and reference [2] presents a good explanation of the general configuration and the computation steps. In this section only a brief description of the basic procedure is presented. The formulation of the approach is shown below [2].

Step 1 Collect or generate noisy data $\underline{z}$ and inputs $\underline{u}$



Step 2 Using the inputs $\underline{u}$ run the model for a fixed set of parameters $\underline{p}$



Step 3 Calculate the cost function, or performance index [$\underline{p}$]