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Abstract

We present a critical evaluation of the force balance approach in predicting the departure of rapidly growing bubbles from a boiling surface. To this end, we conduct separate effect bubble growth experiments in a carefully controlled environment. We use high-speed video to quantify experimentally all the external forces acting on a growing bubble through the profile of the liquid-vapor interface. Our experimental data show that the momentum conservation equation is always rigorously satisfied, as it should, if the various forces are precisely quantified. However, based on our analysis and our observations, we come to the conclusion that force balance models cannot be either robust or accurate for the purpose to predict bubble departure. They are not robust because the rate of change of the bubble momentum, i.e., the key quantity that force balance models aim at evaluating as the sum of the external forces, is orders of magnitude smaller than each of the force terms in the momentum conservation equation throughout the entire bubble life cycle. Thus, the slightest error on one of the external forces leads to very different predictions for bubble departure. The approach is also not accurate because the analytical expressions used to estimate the external forces are riddled with questionable assumptions (e.g., on the bubble growth rate, added mass coefficient, contact line length, and contact angle) and uncertainties that are, once again, orders of magnitude larger than the rate of change of the bubble momentum itself.

Keywords

Bubble dynamics, Boiling.

1. Introduction

Predicting the departure diameter of vapor bubbles from a heated surface is a critical challenge for scientists and engineers interested in elucidating the physics of boiling heat transfer and developing boiling heat transfer models [1-3]. The departure diameter has a pivotal role in heat flux partitioning schemes, where the typical evaporation and quenching heat transfer terms depend on the departure diameter to the power three and two, respectively [3]. Simply put, small errors in the departure diameter may lead to large errors in the prediction of the boiling heat transfer rate.

The need for versatility and robustness has motivated scientists to develop mechanistic models for bubble departure, mostly leveraging the so-called 'force balance' approach. Force balance models are based on the assumption that bubble detachment occurs when the change of momentum for a bubble becomes non negligible, i.e., in the equation of motion the sum of the external forces acting on the bubble departs from equilibrium:

$$\sum F_{ext} \neq 0. \tag{1}$$

The most popular (and, to the best of the authors' knowledge, first) force balance model is arguably the one proposed by Klausner et al. [4], who conceptualized this idea in order to predict the departure of bubbles from a single isolated nucleation site in flow boiling. Under such conditions, the detachment of bubble can occur either by lift-off, i.e., the force equilibrium is broken in the direction normal to the surface, or by sliding, i.e., the force equilibrium is broken in the direction tangential to the surface. To discriminate between these two mechanisms, Eq. 1 has to be written in two directions, i.e., normal and parallel to the surface.

Klausner et al. [4] disaggregated the external forces acting on a growing bubble into surface tension, buoyancy, growth, quasi-static drag, shear-lift, hydrodynamic pressure, and contact pressure forces. This approach has set the basis for the development of many other models, offering some incremental improvements of the original formulation. For instance, Yun et al. [5] introduced the effects of vapor recondensation at the bubble tip, which is needed to capture the bubble growth rate in subcooled flow boiling conditions and, in turn, the bubble growth force. They also introduced a correlation between the bubble equivalent diameter and the bubble base diameter, which is needed to estimate the surface tension, contant pressure, and hydrodynamic pressure forces. Sugrue et al. [6] neglected re-condensation effects and introduced a different correlation between the base radius and the equivalent diameter to fit a large experimental database. Instead, vapor recondensation effects were included in two recent models by Colombo et al. [7] and Mazzocco et al. [8], where the authors also considered how the microlayer evaporation affects the bubble growth rate. In summary, there is no shortage of such models in the literature (e.g., see also Refs. [9-16]).

In this paper, we are not interested in assessing the accuracy of these models compared to the existing databases. Instead, we are interested in critically evaluating the physical basis and practical usefulness of the force balance approach.

To start this reflection, we notice that the list of modeling assumptions and coefficients required to evaluate the external forces is quite long. The growth force represents a meaningful example. First, we need a model to estimate the force itself: it is typically assumed that the growth force can be described using the Rayleigh equation for the spherical expansion of a gas bubble

$$F_{gr} = -\rho_l \pi R^2 \left(\frac{3}{2} C_s \dot{R}^2 + R\ddot{R}\right) \tag{2}$$

where ρ_l is the liquid density and C_s is a coefficient that is introduced to account for the presence of the wall (for a comprehensive nomenclature table, see the Supplementary Material, Section A). Then, we need a model to capture the bubble growth, $R(t)$, which is typically described by a heat diffusion law:

$$R(t) = b \frac{2}{\sqrt{\pi}} \frac{\rho_l}{\rho_v} Ja \sqrt{\alpha_l t} \tag{3}$$

where ρ_v is the vapor density, α_l is the liquid thermal diffusivity, and Ja is the Jacob number, defined as

$$Ja = \frac{c_{p,l}(T_{nuc} - T_{sat})}{h_{lv}} \tag{4}$$

Here, T_{sat} is the saturation temperature at the pressure of the system, T_{nuc} is the nucleation temperature, $c_{p,l}$ is the liquid specific heat, and h_{lv} is the latent heat of vaporization. b is another coefficient, which accounts for the deformation of the conduction thermal boundary layer surrounding the bubble, as the bubble expands. Plesset and Zwick [17] have derived $b = \sqrt{3}$, whereas Forster and Zuber [18] have derived $b = \pi/2$. If the liquid is subcooled, we may account for vapor re-condensation effects using

Zuber's modification of Eq. 3, which becomes

$$R(t) = b\left(\frac{2}{\sqrt{\pi}}\frac{\rho_l}{\rho_v}\mathrm{Ja}\sqrt{\alpha_l t} - \frac{q''_b\, t}{2\,\rho_v\, h_{lv}}\right) \tag{5}$$

Here, q''_b is the heat flux from the liquid-vapor interface to the liquid phase (e.g., due to the re-condensation of the vapor), for which we need another model (or a correlation). Therefore, for the bubble growth force alone, we need to define three physical models and two tuning parameters, b and C_s. This picture becomes even more disturbing if one is to include the evaporation of the microlayer, a physically rich and hard to predict region at the base of the bubble. A similar barrage of assumptions, models and empiricism afflicts most of the other external forces used in the force balance approach, with the exception of gravity, which is straightforward.

Given the amount of modeling assumptions and parameters, it is legitimate to wonder if "accurate" predictions from these models are the result of data overfitting, rather than the reward of a more sound physical description of the governing phenomena. Finally, force balance models are typically developed and validated against bubble departure volume data, i.e., an integral quantity describing the outcome of a complex process, instead of being validated against data for the individual forces which determine the process.

In this paper, we conduct a theoretical and experimental investigation of bubble growth phenomena, performing separate effect experiments in carefully controlled operating conditions and quantifying each of the external forces acting on a growing bubble. To simplify the analysis and come to clear conclusions, we have considered the simplest possible case of a single bubble nucleating on a horizontal surface and growing in a stagnant pool of water, as discussed hereafter.

2. Theoretical basis and modeling assumptions

The theoretical basis

Let us consider the simple case of a bubble growing in a initially stagnant liquid, on a horizontal surface (see Figure 1). In such conditions, a single isolated bubble grows symmetrically, as verified in countless experiments.

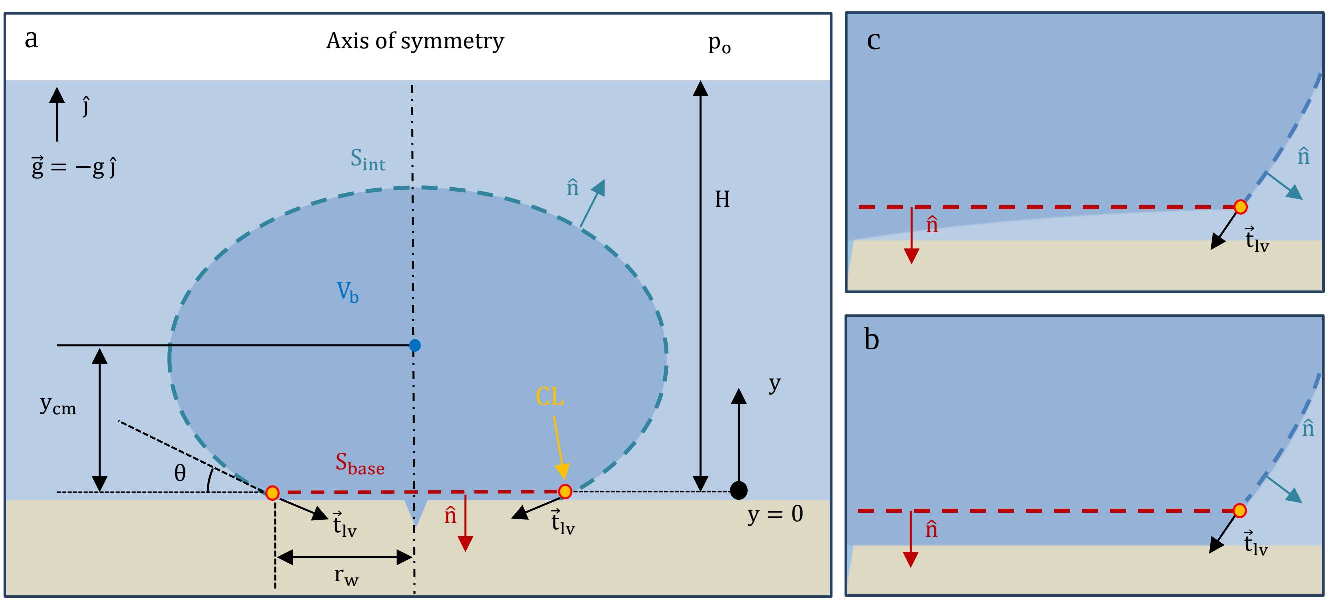

Figure 1. Domain and definitions of the analytic problem for the growth of an axisymmetric bubble on a surface.

We take a control volume indicated by the dashed lines in Figure 1.a. The closed boundary of this control volume consists of two parts. The first part (blue line), S_{int}, is a curved surface conformal with the liquid-vapor interface and infinitesimally displaced within the liquid domain. The second part (red line), S_{base}, is a flat surface cutting the vapor phase right above the solid surface (Figure 1.b) or the liquid microlayer, if it is present (Figure 1.c). In axisymmetric conditions, the intersection between S_{int} and S_{base} defines a circumference corresponding to the contact line of the control volume, CL, which coincides with the bubble apparent contact line (if there is a liquid microlayer in contact with the surface) or the actual contact line (if the microlayer has entirely evaporated or did not form to begin with). The outward-pointing vector normal to the interface is $\hat{n}$. $\hat{t}_{lv}$ is the vector tangential to S_{int} along CL, and directed towards the solid wall.

In axisymmetric conditions, the pressure of the liquid on S_{int} at a vertical distance y from S_{base}, i.e., $y = 0$, can be expressed as

$$p_l(y) = p_0 + \rho_l\, g\, (H - y) + p_{l,h}(y) \tag{6}$$

where g is the gravity, ρ_l is the liquid density, H is the depth of the water pool, p_0 is the pressure at the free surface, and $p_{l,h}$ is the hydrodynamic component of the pressure in the liquid. The integral momentum balance for our bubble control volume is

$$\rho_v V_b \frac{d^2 y_{cm}}{dt^2}\hat{j} = \oint_{CL} \sigma\, \hat{t}_{lv} dL + \iiint_{V_b} \rho_v\, \vec{g}\, dV - \iint_{S_{int}} (p_o + \rho_l g(H - y))\, \hat{n}\, dS - \iint_{S_{int}} p_{l,h}(y)\, \hat{n}\, dS - \iint_{S_{base}} p_v\, \hat{n}\, dS \tag{7}$$

where ρ_v is the vapor density, V_b is the bubble volume, σ is the liquid-vapor surface tension, y_{cm} is distance of the bubble center of mass from the surface, and p_v is the pressure of the vapor inside the bubble. Note that, to obtain this equation and in the following developments, we make use of the following assumptions:

- Uniform vapor pressure, p_v.
- Uniform vapor and liquid densities, ρ_v and ρ_l, with $\rho_v \ll \rho_l$.
- Uniform vapor-liquid surface tension, σ.
- Negligible mass transfer effects at the liquid-vapor interface (i.e., negligible vapor recoil effect on the surface curvature, asymmetric evaporation effect and rocket effect).
- Negligible viscous stresses compared to normal stresses at the liquid-vapor interface.

Under these assumptions, Eq. 7 is an exact formulation of the conservation of momentum for the bubble. The term on the left hand side of Eq. 7 represents the rate of change of the bubble momentum. The first and second terms on the right hand side represent surface tension force and bubble weight. The third and fourth terms describe the pressure force acting on the liquid-vapor interface, S_{int}, whereas the last term represents the reaction force at the bubble base. We can manipulate Eq. 7 to obtain a more convenient formulation (for the mathematical derivation, see the Supplementary Material, Section B):

$$\rho_v V_b \frac{d^2 y_{cm}}{dt^2}\hat{j} = \oint_{CL} \sigma\, \hat{t}_{lv} dL + \iiint_{V_b} (\rho_v - \rho_l)\vec{g} dV - \iint_{S_{base}} \sigma\, C_0\, \hat{n}\, dS - \iint_{S_{int}} (p_{l,h}(y) - p_{l,h,0})\hat{n}\, dS \tag{8}$$

Here, C_0 is the curvature of the liquid-vapor interface, S_{int}, at the contact line. Similarly, $p_{l,h,0}$ is the hydrodynamic pressure at the contact line. The second term on the right hand side expresses the net buoyancy force, given by the sum of weight and Archimede's forces. The third term is the so-called contact pressure force, which represents the reaction force at the bubble base. Hydrodynamic effects (e.g., the deviation of the pressure stress from its hydrostatic profile) are captured by the last integral.

Practically, in our pool boiling condition, this term represents the dynamic force exerted by the liquid on the expanding bubble.

The modeling assumptions

Eq. 8 sets the stage for force balance models. In the case of pool boiling, only the vertical component, i.e., in the $\hat{j}$ direction, is relevant. The sum of the external forces acting on the bubble is:

$$\sum F_{ext} = F_b + F_\sigma + F_{cp} + F_{gr} \tag{9}$$

Here, F_b is the buoyancy force:

$$F_b = \iiint_{V_b} (\rho_v - \rho_l)\, \vec{g} \cdot \hat{j}\, dV = (\rho_l - \rho_v)\, g\, V_b = (\rho_l - \rho_v)\, g\, \frac{4}{3}\pi R^3 \tag{10}$$

where R is the radius of a sphere of volume V_b, and it is therefore called equivalent (spherical) radius.

F_σ is the surface tension force:

$$F_\sigma = \oint_{CL} \sigma\, \hat{t}_{lv} \cdot \hat{j}\, dL = -2\, \pi\, r_w\, \sigma \sin\theta \tag{11}$$

where θ is the contact angle and r_w is the bubble base radius. In force balance models, the bubble base radius is often considered constant [1] or assumed to scale with the equivalent radius [2-4], i.e., $r_w = f(R)$. For instance, Klausner et al. [1] used $d_w = 2r_w = 0.09$ mm. Yun et al. [2] assumed that $r_w = 0.067$ R. Sugrue et al. [3] assumed $r_w = 0.025$ R.

F_{cp} is the contact pressure force:

$$F_{cp} = -\iint_{S_{base}} \sigma\, C_0\, \hat{n} \cdot \hat{j}\, dS = \pi\, r_w^2\, C_0\, \sigma \tag{12}$$

However, for modeling purposes, the curvature of the liquid-vapor interface at the contact line, C_0, is often approximated without much justification as $2/5R$ (e.g., see [4-11]), and this term reads:

$$F_{cp} = \pi\ f(R)^2 \frac{2\,\sigma}{5\,R} \tag{13}$$

The bubble growth force, F_{gr}, is usually modeled using Eq. 2. Several authors take $C_s = 1$ (i.e., the same coefficient as an ideal spherical expansion). However, values as large as 20/3 can be found in literature (e.g., see Refs. [19,20]) to account for the presence of the wall.

Thus, the sum of the external forces can be obtained by adding Eqs. 2, 10, 11, and 13. One also needs to postulate a bubble growth law, $R(t)$, which can be expressed using Mikic's model (e.g., in Klausner et al. [4]) or, neglecting the inertia-limited growth phase, using Eq. 3 with $b = \pi/2$ (e.g., Sugrue et al. [6] and Yun et al. [5]). Note that b values as diverse as 0.48 and 24 can be found in literature for pool boiling conditions (e.g., see the analysis presented in Ref. [20]).

The exact definition of the forces and their models are summarized in Table 1. Precisely, the "Definition" column lists the exact formulations of the forces, analytically derived from the integrals on the right hand side of the momentum balance, i.e., Eq. 8. Instead, the "Model" column lists their approximate formulations, i.e., those used in the force balance models. The last column summarize the modeling assumptions necessary to evaluate these approximate formulations Once again, even for the simplest possible case of bubble growth in pool boiling, on a horizontal surface, normal to gravity, force balance models require many modeling assumptions (e.g., for the growth and contact pressure forces, or the bubble base radius, let alone the contact angle) and several tunable parameters (e.g., C_s, b, and the coefficient of proportionality between the bubble base radius and the equivalent radius).

Table 1. Definition and modeling of the external forces.

Force	**Definition**	**Model**	**Notes (for modeling purposes)**
Buoyancy, F_b	$(\rho_l - \rho_v)\, g\, V_b$	$(\rho_l - \rho_v)\, g\, \frac{4}{3} \pi R^3$	R is the equivalent spherical radius of V_b
Surface tension, F_σ	$-2\, \pi\, r_w\, \sigma \sin\theta$	$-2\, \pi\, f(R)\, \sigma \sin\theta$	Need to prescribe the relationship f(R) between the bubble base radius, r_w, and R, and the contact angle, θ.
Contact pressure, F_{cp}	$\pi\, r_w^2\, C_0\, \sigma$	$\pi\, f(R)^2 \frac{2\,\sigma}{5\,R}$	Need to prescribe the relationship f(R) between the bubble base radius, r_w, and R. Approximate C_0 with $2/5$ R.
Growth, F_{gr}	$-\iint_{S_{int}} \left(\sigma\left(C_0 - C(y)\right) + \rho_l\, g\, y\right) \hat{n} \cdot \hat{j}\, dS$	$-\rho_l\, \pi\, R^2 \left(\frac{3}{2} C_s \dot{R}^2 + R\ddot{R}\right)$	Need to prescribe C_s and R(t), possibly including b.

Note that the definition of the growth force shown in Table 1 was obtained by observing that, using the Young-Laplace equation, the hydrodynamic pressure difference can be expressed exactly as

$$p_{l,h}(y) - p_{l,h,0} = \sigma\left(C_0 - C(y)\right) + \rho_l\, g\, y, \tag{14}$$

and thus

$$F_{gr} = -\iint_{S_{int}} \left(p_{l,h}(y) - p_{l,h,0}\right) \hat{n} \cdot \hat{j}\, dS = -\iint_{S_{int}} \left(\sigma\left(C_0 - C(y)\right) + \rho_l\, g\, y\right) \hat{n} \cdot \hat{j}\, dS. \tag{15}$$

The key controversy

Examples of application of force balance models (i.e., with the formulations in the "Model" column of Table 1) to our typical experimental conditions are shown in Figure 2 using the modeling assumptions and parameters proposed by Klausner et al. [1] (left) and Sugrue et al. [3] (right), and a contact angle of 55 degrees.

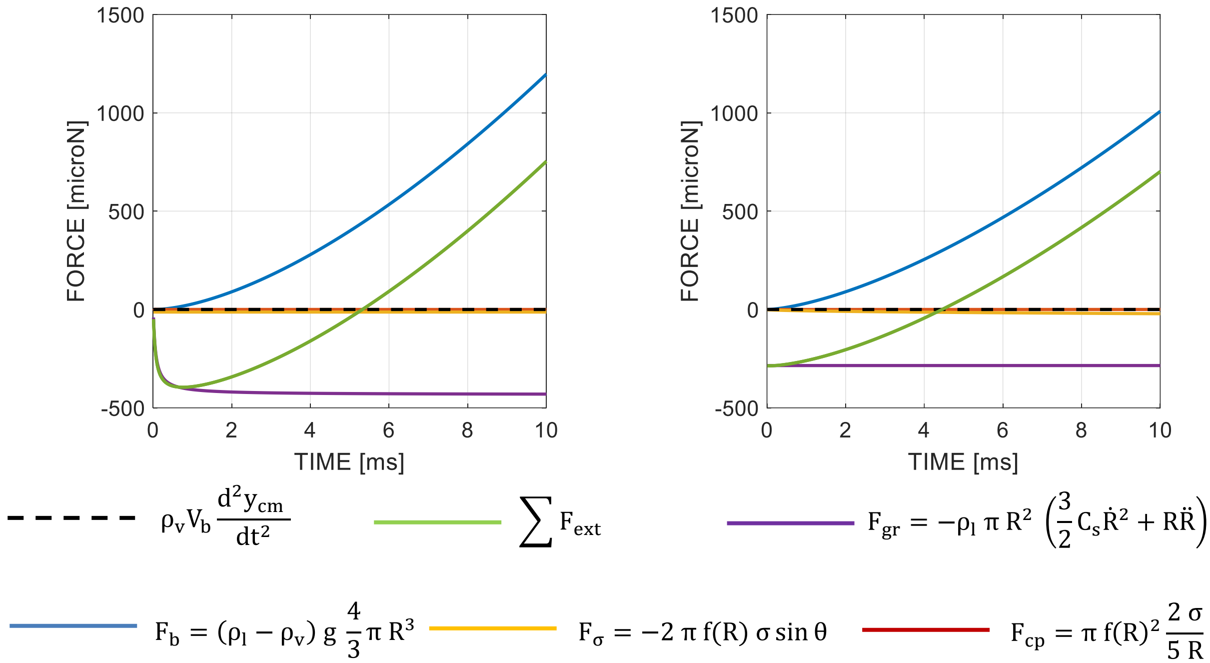

Figure 2. Temporal trend of the modelled external forces acting on a bubble growing in pool boiling conditions, with a nucleation temperature of 113 ˚C at atmospheric pressure, using the modeling assumption of Klausner et al. [1] (left) and Sugrue et al. [3] (right).

The curves in Figure 2 are obtained for a nucleation temperature of 113 °C, i.e., the typical bubble nucleation temperature in our experiment. Note that detaching forces are positive, and adhesion forces are negative. However, while the two models use different assumptions for the bubble growth law and the bubble base diameter, they both suggest that growth and buoyancy are the most important forces, whereas contact pressure and surface tension are practically negligible. Thus, bubble detachment should be determined by a competition of the buoyancy force (promoting detachment) and the growth force (opposing detachment).

It is important to note that the resultant of the forces, as shown in Figure 2, is a few hundred μN and negative during the first phase of the bubble growth. This is surprising, as the rate of change of the vapor bubble momentum, which should be equal to the sum of the external forces, is always in the order of a few μN before the bubble departure. Simply put, the description of the forces used in force balance models does not satisfy the conservation of momentum! It is not clear why the resultant should be negative, and two to three orders of magnitude larger than the rate of change of the bubble momentum.

To the best of the authors' knowledge, a similar concern was first raised by Van der Geld in 1996 [21], discussing the work of Mitrovic [22]. Thorncroft et al. [23] addressed Van der Geld's criticism in 2001, arguing that the violation of momentum balance may be due to the absence (among the external forces) of an additional reaction force at the liquid-vapor-solid triple contact line. This additional reaction force is attributed to London-Van der Waals interactions, and should be positive (of the same order of the other forces) and proportional to the contact line length, i.e., it becomes null at the moment of detachment. As models do not account for this force, according to this reasoning, bubble detachment should occur at the moment when the sum of the "classical" external forces that we have discussed (i.e., buoyancy, surface tension, contact pressure, and growth) changes sign from negative to positive. This assumption, while not always clearly stated, seems to be at the basis of other force balance models appearing in the literature (e.g., see Refs. [5-16]). However, experimental investigations of quasi-static gas bubbles ejection from orifices (e.g., see Refs. [24-26]) have revealed that the resultant of the “classical” forces (for negligible growth forces) prior to detachment is practically null. This observation suggests that the issue could be in the description of the external forces, and in particular the growth force, rather than due to the absence of an additional London-Van der Waals reaction force.

3. Discussion

We conduct a series of experiments to investigate the growth and departure of single isolated bubbles in horizontal pool boiling conditions, using high-speed video and infrared thermometry. In particular, we focused on the first bubble growing over the boiling surface after a stepwise increase in the heat flux. This first bubble nucleates in an environment where the liquid is practically at uniform temperature (with exception of the conduction thermal boundary layer) and stagnant, i.e., there are no recirculation flows induced by the growth and detachment of previous bubbles or by temperature gradients in the boiling cell. The bubble profile from the high-speed video images is processed to extract the quantities necessary to quantify all the external forces according to their analytical definition (i.e., in the “Definition” column of Table 1). Precisely, we can measure bubble volume, V_b, base radius, r_w, contact angle, θ, base curvature, C_0, and the local curvature, $C(y)$, on the liquid-vapor interface. The details of the experimental apparatus and the post-processing methodology are discussed in the Supplementary Material (Section C).

Figure 3 provides an overview of the type of experimental results that we obtain for each bubble. Similar results for other bubbles are shown in the Supplementary Material (Section D). The plots shown in Figure 3 refer to a bubble nucleating at approximately 113℃, and growing with an initial volumetric growth rate of 7.8 mm^3/s. This volumetric growth rate is evaluated from a linear interpolation of the measured bubble volume vs. time during the first ~20% of the bubble growth (where this trend is quite linear). The bubble profile is shown in the top sequence for a few significant instants. Below, from top to bottom, the three subplots show the time-evolutions of measured equivalent and base radius, base curvature and contact angle, and the resulting forces, respectively. Note that we distinguish between apparent and actual bubble base radius. The apparent bubble base radius is obtained from the high-speed video, and coincides with the contact line, CL, of our control volume. Instead, the actual bubble base radius defines the liquid-vapor-solid triple contact line, and is tracked using the infrared camera (see the Supplementary Material, Section C). The heater surface in the radial region between the actual and the apparent bubble radius is covered by the microlayer. Here, the microlayer entirely evaporates after approximately 9.5 ms, and from then on, the apparent contact line tracked through the high-speed video image coincides with the actual liquid-vapor-solid contact line. One may note that the bubble attains its maximum radial expansion after approximately 6 ms. This is the time when the bubble base starts to shrink and the detachment process begins. However, as the microlayer has completed its evaporation, and the apparent contact line meets the actual contact line, the bubble base shrinking stops. The bubble rises and the (actual) contact angle starts to increase. The shrinking of the bubble base restarts after the contact angle has reached approximately 55°, and eventually the bubble detaches from the surface. Beyond these empirical observations, the first key finding is that the sum of the external forces (green line) reported in the bottom subplot is the same (within experimental uncertainties) as the rate of change of the bubble momentum (black dashed line) throughout the bubble life cycle, as it should be. This result (and all the additional examples shown in the Supplementary Material, Section D) demonstrates that there is no additional London-Van der Waals type reaction force to consider in the force balance, if the external forces are correctly quantified. However, these results have an even more profound implication. One may note that the rate of change of the bubble momentum and the resultant of the external forces are in the order of a few μN throughout the entire bubble lifecycle. The bubble base starts to shrink after approximately 6 ms, but the rate of change of its momentum is still small. Practically, we expect force balance models to predict a change of the order of a μN by substrating external forces of the order of hundreds or throusands of μN. Even assuming that the models for the external forces were physically correct, the slightest error on any of these terms would result in drastically different prediction of the onset of detachment and consequently the departure diameter. It did not escape our attention that a similar conclusion was reached by Kumar et al. [27] in a very reacent theoretical work about the quasi-steady detachment of droplets and bubbles. Noteworthy, our work fill the critical knowledge gap identified in that work, i.e., about the detachment in highly dynamic conditions, with a theoretical and experimental analysis.

One may note that such conclusion is in contradiction with the praised robustness of force balance models. To support our reasoning, we show in Figure 4 a comparison between the modeled and measured forces (with exception of the buoyancy force, for which there are no specific modeling assumptions to compare). Precisely, we have re-plotted the measured forces from Figure 3, together with the modeled forces, i.e., calculated using the formulations in the "Model" column of Table 1. In calculating the modeled forces, we have used several levels of empiricism. For instance, in Figure 4 (top), the solid orange line represents the measured contact pressure force (same as in Figure 3). The grey curve shows the force estimates obtained with the model formulation, using the assumptions of Sugrue et al. [6] for f(R) and $b = \pi/2$ for the growth law, $R(t)$. As one can see, the model is orders of magnitude inaccurate. However, even calculating the "Model" force using the measured $R(t)$ and taking f(R) equal to the measure bubble base radius (i.e., for the lowest degree of empiricism, dashed orange line), the predictions of the model are at least one order of magnitude far from the measured values. This suggest that the submodel typically used for the base curvature, i.e., $C_0 = 2/5R$, is not accurate. Similar observations can be made by analyzing the bubble growth force (Figure 4, middle) and the surface tension force (Figure 4, bottom) analyses. In summary, whatever the force, the conclusion that we can draw from the comparison of measured and modeled forces is straightfoward. The models and assumptions used to quantify the external forces are grossly inaccurate, with differences between the modeled and measured forces as large as the magnitude of the force itself (i.e., two to three orders of magnitude larger than the rate of change of the bubble momentum). In brief, force balance models are not accurate because the description of the forces is not accurate. Moreover, even if the description of the forces were accurate, they would not be robust, because they need to precisely quantify a small change in a small quantity by substracting quantities that are two to three orders of magnitude larger.

However, while we focus our analysis on the simplest possible case of a single bubble nucleating in a stagnant pool of water, it is natural to expect that the same conclusions could be drawn, and would be even more drastic, for other operating conditions, e.g., flow boiling, and in the presence of multiple bubbles on the surface.

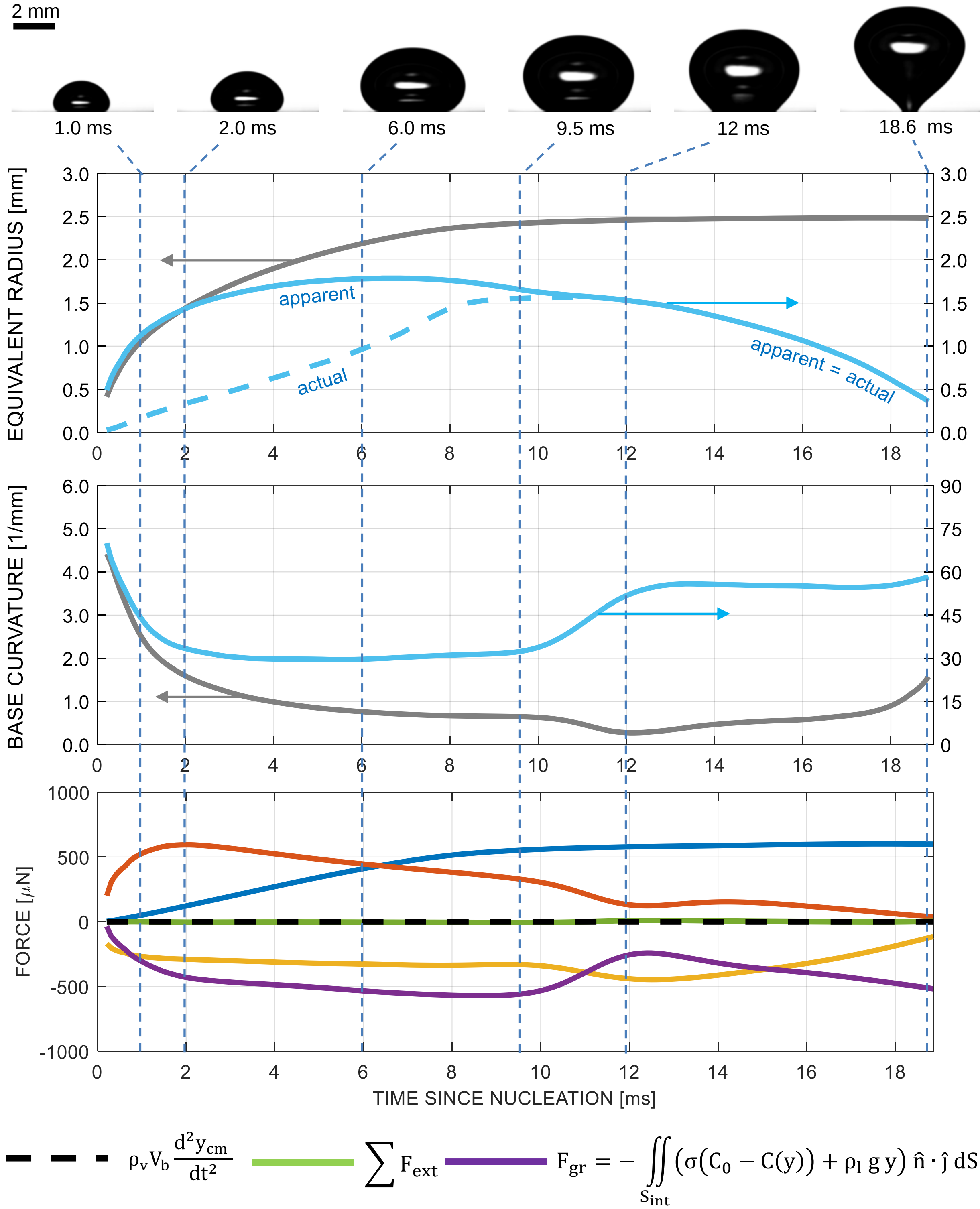

Figure 3. Measured bubble parameters and force breakdown for a bubble nucleating at approximately 113℃ at tmospheric pressure and with an intial volumetric growth rate of 7.8 mm^3/s.

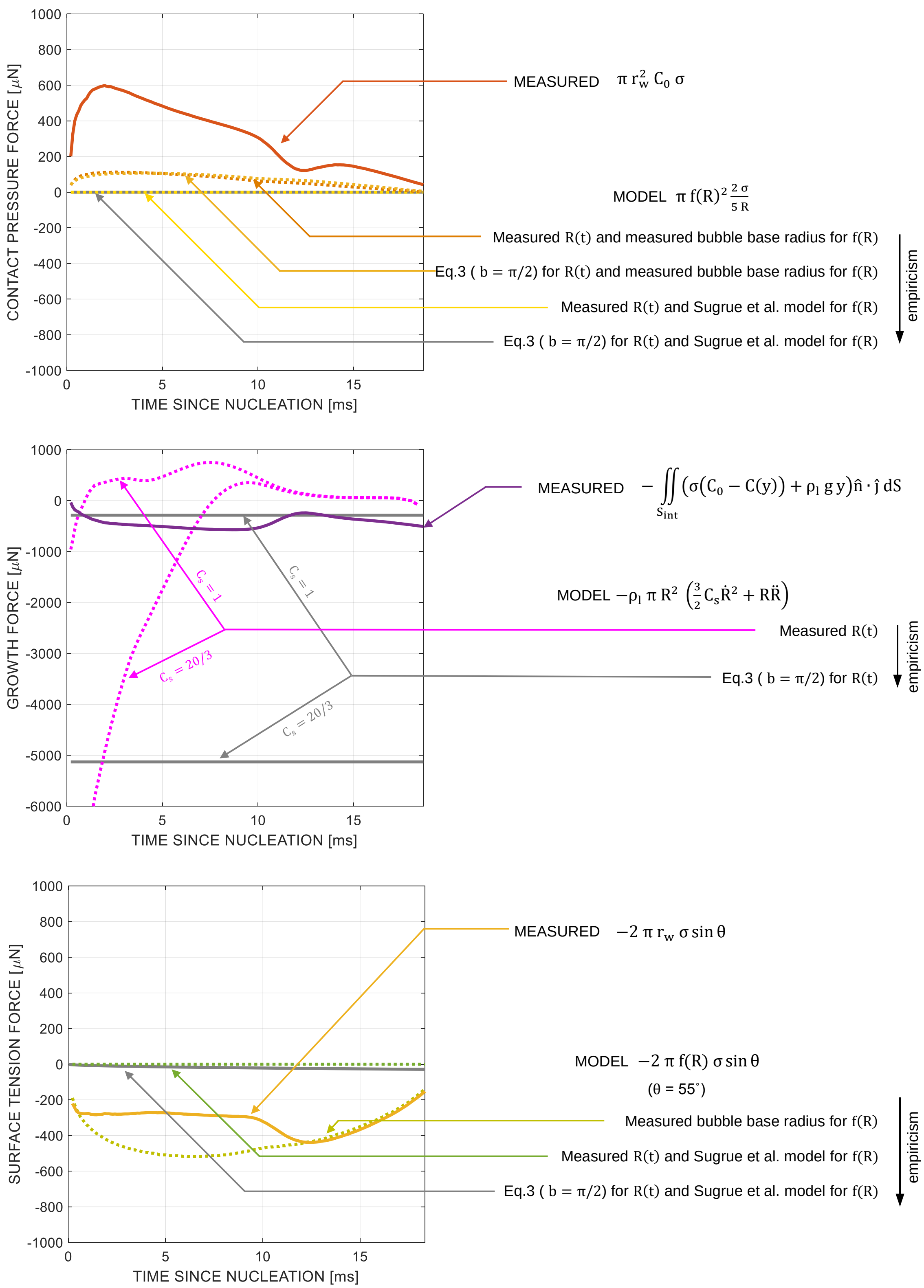

Figure 4. Comparison between measured and modeled forces, including the sensitivity of the modeled forces to physical assumptions and coefficients.

Conclusions

The force balance is arguably the most popular mechanistic approach to predict the departure of bubbles from a boiling surface. Force balance models have become increasingly sophisticated over the years, trying to provide richer descriptions of the physical phenomena associated with the growth of bubbles, and capture with better accuracy the bubble departure size. However, with richer models, the degree of empiricism also increases, as these models require many assumptions and coefficients to be calibrated. This is a potential issue, as it is not clear whether improved accuracy is the reward for a better description of the physical phenomena, or the result of overfitting.

In this paper, we conduct a separate effect, theoretical and experimental analysis of the bubble growth process. We measure every single force acting on bubbles growing at different rates and verify the balance of the forces experimentally. These results demonstrate that there is no need to invoke the addition of a London-Van der Waals type reaction force for the momentum balance to be satisfied, in contradiction with what is implicitly assumed in many force balance models. Also and importantly, we come to the conclusion that the force balance approach is neither accurate nor robust in predicting the departure of rapidly growing bubbles nucleating on a hot surface. The approach is not robust because it relies on predicting a small change of a small quantity, i.e., the rate of change of the bubble momentum, by substracting quantities, i.e., the external forces, that are two to three orders of magnitude larger. Moreover, models used to estimate the external forces are riddled with uncertainties that can be two to three orders of magnitude larger that the rate of change of the bubble momentum. This work suggests that new approaches to predicting bubble departure should be explored, particularly if one wishes to predict the efficiency of boiling phenomena near the boiling crisis, when the idea of "bubble" also loses its significance, as bubbles massively merge together even before they depart from the surface.

Supplementary Material

See Supplementary Material for nomenclature table, mathematical derivations (from Eq. 7 to Eq. 8), a complete description of the experimental apparatus, its operation and the associated post-processing techniques, and additional experimental results.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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