OPTIMIZING THE ARRANGEMENT OF TRUSS MEMBERS OR THE STIFFENERS OF PLATED PLANE PANELS UNDER INPLANE LOADS

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B.S., MASSACHUSETTS INSTITUTE OF TECHNOLOGY

(1969)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN NAVAL ARCHITECTURE

AND MARINE ENGINEERING

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

AUGUST, 1971

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Submitted to the Department of Ocean Engineering on August 16, 1971 in partial fulfillment of the requirements for the degree of Master of Science in Naval Architecture and Marine Engineering, ~~and the Naval Architect Degree~~.

ABSTRACT

Clerk Maxwell's theorem for the least weight arrangement of filamentary trusses enabled A. G. M. Michell to come to the tentative conclusion that in certain cases the least weight arrangement of the truss members or the stiffeners of an isotropic elastic continuum is obtained by following the principal strain trajectories.

Using this as a basis, the Plane Stress Approximation incorporated with the Finite Element Displacement Method are employed to determine the general state of stress of a two-dimensional planar structure through which a technique is developed to obtain the principal stress trajectories for any plane stress with an isotropic material problem.

A computer program has been written through which the displacements, the stresses, and the stress trajectories can be obtained; the results for several planar problems are in agreement with those theoretically predicted.

A comparison among several stiffener arrangements suggests that the least weight arrangement that reduces the stress level by the required amount, throughout a cross-section of a plate, is the one which adds material at the location of maximum principal stress.

THESIS SUPERVISOR: J. Harvey Evans

TITLE: Professor of Naval Architecture

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to his thesis supervisor, Professor J. Harvey Evans, for his constant guidance and motivation throughout this study.

Acknowledgement is made to the Department of Ocean Engineering for financially enabling me to conduct this investigation.

All calculations were performed by System 65/40 at the Information Processing Center at the Massachusetts Institute of Technology.

Thanks are also extended to Miss Artemis Georgantas for the preparation of the graphic artwork.

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INTRODUCTION

The present study formulates a technique which may assist in determining the minimum weight design of two-dimensional planar structures such as plated panels and trusses. Of particular interest is the determination of the minimum weight arrangement of (a) stiffeners for a plate, or (b) the members of a truss, both being loaded in their plane.

Rectangular plates or truss members of linearly elastic, isotropic and homogeneous material are examined in detail. Although instability is not considered, the present method of analysis may aid in the examination of such effects as instability and/or plasticity.

The first part of this study functions as background. The basic theorem for the design of truss-like structures established by Clerk Maxwell in 1869 is discussed and numerical applications of the theorem are made.

A. G. M. Michell in 1904 used Maxwell's theorem to arrive at the tentative conclusion that in certain cases we can find lower limits to the total material necessary to sustain given loads, and also assign the forms of frames which are most economical. The implications of his conclusion is the basis for the present study.

Michell has shown for a number of simple two-dimensional truss problems that the minimum weight arrangement of the truss members follows the lines of the "Michell Fields" which are lines that meet orthogonally at any point in the Field.

These lines are analogous to the principal strain trajectories which are curves that follow the directions of the principal strains. Since only isotropic material is considered in this study, the search for a way to obtain the principal strain trajectories of a structure has been substituted by an efficient technique by which we obtain the principal stress trajectories.

The principal stress trajectories, similar to the principal strain trajectories are lines which follow the directions of the principal stresses.

The second part of this investigation, demonstrates how one can efficiently obtain the stress trajectories by incorporating the *plane stress approximation* through the Finite (or Discrete) Element Technique.

An introduction to the Finite Element (Displacement) Method as applied to the development of a rectangular element is given. This involves the definition of the displacement functions, the development of the Element's Stiffness Matrix, and the determination of the Energy Equivalent nodal forces.

From the assembled structure's stiffness matrix and the Energy Equivalent nodal forces, the nodal displacements are obtained. Therefore, the state of stress of the total structure can be determined from the stress-displacement relationships. The magnitude and direction of the principal stresses at any nodal point (or control station) can be calculated. Profiting from the existence of the orthogonal grid provided by the Finite Elements' boundaries and from the fact that the stresses vary linearly along the interelement boundaries, a linear extrapolation method is described, through which the principal stress trajectories are obtained, (for our purposes only the compressive stress trajectories).

A computer program has been written in FØRTRAN IV for the use of the G level compiler of the IBM 360 computer model.

The program enables one to adequately model any plane stress problem subject to the above discussed limitations. By reading in the geometric parameters, the boundary conditions and the applied forces, concentrated anywhere in the structure or distributed on the boundaries, one obtains as output the displacements and the average stresses at each nodal point, (or control station) of the structure.

From the calculation of the principal stresses derived from the average nodal stresses, a plotting of the principal compressive stress trajectories is obtained, (by the aid of the IBM Calcomp Plotter).

Furthermore, the program is capable of reinforcing the structure along the principal compressive trajectories with fictitious stiffeners; that is, by increasing the thickness of the elements which are traversed by the trajectories.

Once the first trial of "stiffening" has been performed, the new plot of stress trajectories is given together with a listing of the elements to be reinforced for the second trial, and so on.

The complete description of the Program's capabilities and limitations is given in Appendix II-A.

The procedure followed by the program goes beyond the determination of the stress trajectories. However, it is not the purpose of this study to further develop the computer method of reinforcing a structure by true stiffener sizes.

It is of greater importance to demonstrate first that by adding material at the location(s) of the maximum principal stress, the required reduction of stress is achieved, resulting at the same time with the least weight addition of material for the specified stress reduction.

Part three of the present work compares four different stiffener arrangements for the optimum (least weight) reduction of the principal stresses.

Two cross-sections along the length of a centrally loaded plated plane panel (in its plane) and with fixed ends were conveniently chosen to be at the quarter length and at the midspan.

The Simple Beam Theory which is a good approximation when applied to plane panels was employed. By its aid, the maximum shear on the N.A. of the quarter length cross-section and the maximum bending stress at the edges of the midspan were calculated for the four different plate-stiffener combinations, to yield two sets of curves, one for each kind of stress. These curves describe the behavior of the three shear stresses and the three bending stresses corresponding to the three best arrangements at each of the above specified cross-sections respectively.

Part four includes all the significant results of this study. Three plane stress problems are examined in detail and their results for the nodal displacements, the average nodal stresses, and the stress trajectories are in agreement with the theoretically predicted or anticipated results.

The computer "stiffening" procedure results are included to demonstrate explicitly the maximum capability of the present computing technique.

At the end of Part four two sets of graphs illustrate that the stiffening of a plate is of least weight when the reinforcing material is added at the location of the maximum principal stress.

I BACKGROUND

A. Maxwell Structures

The first to establish the theorem that governs the design of "single purpose structures" as those structures were described by H. L. Cox[1] was Clerk Maxwell in 1869.[2] The theorem as it is stated by Maxwell is:

THEOREM--"If every one of a system of points in a plane is in equilibrium under the action of tensions and pressures acting along the lines joining the points, then if we substitute for each point a small smooth ring through which smooth thin rods of indefinite length corresponding to the lines are compelled to pass, then if to each rod be applied a couple in the plane, whose moment is equal to the product of the length of the rod between the points multiplied by the tension or pressure in the former case, and tends to turn the rod in the positive or the negative direction, according as the force was a tension or a pressure, then every one of the system of rings will be in equilibrium. For each ring is acted on by a system of forces equal to the tensions and pressures in the former case, each to each, the whole system being turned round a right angle, and therefore the equilibrium of each point is undisturbed."

LEMMA--"In any system of points in equilibrium in a plane under the action of repulsions and attractions, the sum of the products of each attraction multiplied by the distance of the points between which it acts, is equal to the sum of the products of the repulsions multiplied each by the distance of the points between which it acts."

Maxwell's Lemma takes the algebraic form:

$$\Sigma F_T L_T + \Sigma Ph = \Sigma F_C L_C + \Sigma Rh \qquad [1.1]$$

where:

F_T, F_C are the internal tensile and compressive forces, respectively.

L_T, L_C are the lengths of the truss members (ties and struts, respectively).

P, R are the external loads (applied and reactions, respectively)

h is the height of the point on which the external loads are applied.

If the reactions of the truss are not vertical, their horizontal components must be considered to create tensile or compressive moments.

Two examples of planar trusses are used to illustrate the application of both the Theorem and the Lemma. These are described in Appendix I-A.

According to Michell (4) Maxwell's Lemma takes the form:

$$\Sigma F_T L_T - \Sigma F_C L_C = C \qquad [1.2]$$

where:

$$C = \Sigma Ph + \Sigma Rh \qquad [1.2.1]$$

C is a function of applied forces and the coordinates of their points of application, and is independent of the form of the frame.

As it was mentioned in the beginning, Maxwell's Theorem and Lemma apply to "single purpose structures" only. By that it is meant that the structure will be able to support a given set of loads. However, it may or may not support another set of loads. Furthermore, the structure is:

(1) Filamentary (that is, a structure consisting entirely of normal stress carrying elements).

(2) Constructed from uniform linearly-elastic material.(5)

This Theorem is significant in that it relates the total quantity of material of any truss member with a given allowable stress, to the external pressure or tension on that member. For any truss member with a known allowable stress $+\sigma_{all.}$ or $-\sigma_{all.}$, depending on whether the member is in tension or in compression, the strength of the member will be proportional to the cross-section A of the member.

That is,

$$|F| \leq \sigma_{all.} A \quad (6) \qquad [1.3]$$

The weight of the member is

$$W = \rho g \, A\ell \qquad [1.4]$$

where ρg is the density of the material and ℓ, the length of the bar. Therefore,

$$W = \rho g \frac{F}{\sigma_{all.}} \ell \qquad [1.5]$$

or

$$V \cdot \sigma_{all.} = F\ell \qquad [1.6]$$

where V is the volume of the bar. Therefore Maxwell's Lemma becomes:

$$V_t f_t - V_c f_c = \sum_i \bar{F}_i \cdot \bar{r}_i \qquad [1.7]^*$$

which is the algebraic form in which H. L. Cox introduces Maxwell's Lemma. In equation [1.7]:

V_t, V_c are the total volumes of the material in tension and compression, respectively.

f_t, f_c are the maximum allowable tensile and compressive stresses.

$\bar{F}_i$ is the planar external vector force applied to the i^{th} node of the structure.

$\bar{r}_i$ is the vector distance of the i^{th} point from an assigned origin on the plane of the structure.

We obtain the total volume of material needed by adding the total material being in tension (all the ties) to the total material being in compression (all the struts).

Using equation [1.6] and solving for V, we obtain the following relationship which gives the minimum total volume of a truss which will adequately support a given set of external loads:[4]

$$V = \Sigma\, \ell_T \frac{F_T}{\sigma_{all._T}} + \Sigma\, \ell_c \frac{F_c}{\sigma_{all._c}} \qquad [1.8]$$

---

*Note: In equation [1.7] the volume of the end fittings is not included.

In general, $\sigma_{all._T}$ for tension is not the same as $\sigma_{all._c}$ for compression in the same structure and this is why the notation $\pm\sigma_{all.}$ is not used above. ℓ_T, ℓ_c, F_T, and F_c are as defined before.

To summarize, Maxwell's Theorem is the cornerstone in the theory of optimization of weight in structural design because it gives the relationship of the minimum weight necessary to equilibrate a given set of applied loads. However, it conveys strict limitations:

(a) In that it applies only to a given structure with a given set of loads (single purpose structure).

(b) In that it applies only to filamentary truss structures of uniform elastic material.

B. Michell Structures

A. G. M. Michell's contribution to the field of optimization of weight in structural design is made in the expansion of Maxwell's Theorem. He also deals with filamentary type structures of uniform elastic material but which are restricted in that they consist of two types of filaments or bars which meet orthogonally. This orthogonality is due to the fact that each type of the filaments, depending on whether it is a tie or a strut, follows the paths of the principal tensile or compressive strain trajectories.

Michell shows that in *certain cases* we can find (a) the global minimum or the optimum of the optima of weight solutions which equilibrate a given set of forces and (b) the configuration of the frames which gives the least weight of all the minimum weight solutions.

Starting with equation [1.8] which gives the minimum weight for a given frame and a given set of loads, he seeks to find the least weight frame out of a set of acceptable frames.

First he indicates that in order for the total volume of the structure V to be the least,

$$\Sigma\ \ell|F| \quad \text{must also be the least,}^{(4)} \qquad [1.9]$$

where ℓ is the length of any tie or strut and $|F|$ is the absolute value of any tension or compression.

Consider a region of space R, and a set of frames within R such that every frame in R equilibrates the applied loads and satisfies the given boundary conditions.

If the imposed virtual deformation on the boundary of the structure due to the external loads is being shared by every member in such a way that the corresponding elongation or contraction is

$$|\Delta\ell| \leq \varepsilon\ell \qquad [1.10]$$

where ε is an infinitesimal positive ratio, then, the virtual work done on the structure will be

$$\delta W = \Sigma\ F\ \Delta\ell \qquad [1.11]$$

or

$$|\delta W| = |\Sigma\ F\ \Delta\ell| \qquad [1.12]$$

But, mathematically, it is also true that:

$$|\Sigma\ F\ \Delta\ell| \leq \Sigma\ |F|\ |\Delta\ell| \qquad [1.13]$$

Notice, however, that the inequality sign has no physical significance here since the products of tension × elongation or compression × contraction are always positive. Therefore, equations [1.3], [1.10], and [1.13] yield

$$\Sigma\ |F|\ |\Delta\ell| \leq \sigma_{all.}\ \varepsilon\ \Sigma\ A\ell \qquad [1.14]$$

or

$$\leq \sigma_{all.}\ \varepsilon\ V \qquad [1.14.1]$$

where V is the total volume of the material of the structure.

From equations [1.12] and [1.14.1] we obtain

$$\frac{|\delta W|}{\sigma_{all.}\ \varepsilon} \leq V \qquad [1.15]$$

Since we want the least volume of the structure for the maximum use of the frame,

$$\frac{|\delta W|_{max}}{\sigma_{all.}\ \varepsilon} = V_{least} \qquad [1.16]$$

which is in complete agreement with equation [1.9].

This is easy to check. Note that if we substitute V_{least} by $\Sigma\ A\ell$, equation [1.16] becomes

$$\frac{|\delta W|_{max}}{\varepsilon} = \Sigma\ell\ \sigma_{all.}A \qquad [1.16.2]$$

Since $\sigma_{all.}$ stands for the numerical value of the allowable stress and it is positive

$$\sigma_{all.}A = |F| \qquad [1.16.3]$$

where $|F|$ is as defined in equation [1.9]. For a particular frame M which satisfies the condition of equation [1.16], equation [1.16.4] becomes

$$\frac{|\delta W|_{max}}{\varepsilon} = \Sigma\ \ell_M |F|_M \qquad [1.16.5]$$

For any other frame of the admissible frames that we compare with frame M, say frame A,

$$\frac{|\delta W|_{max}}{\varepsilon} < \Sigma\ \ell_A |F|_A \qquad [1.16.6]$$

will hold true. Therefore, frame M, M representing Michell, is the least weight frame.

To conclude, Michell has shown that there is always a least volume material that could be used in a structure to equilibrate a given set of loads. However, whether or not a structure that uses the least volume material exists for all cases is not known.(6) Michell has demonstrated the existence of such structures in a number of examples, some of which are described in Appendix I-B.

It is clear from the preceding material that the least weight structure, if it exists, must satisfy two conditions:(7)

(1) The stresses in all members are equal to the allowable stress $\sigma_{all.}$ (Equation [1.16.3])

(2) The virtual strains in each of the members of the structure are equal to $\pm\varepsilon$, where the sign is in agreement with the sign of the end loads for each

member and in no case exceed the numerical value of ε.

These conditions imply that the members of the optimum structure M must lie along the principal strain trajectories. If they do not, and they simultaneously satisfy condition (2), then points can exist on any of these members at which the directions of the corresponding principal strains will be different than the directions of the members, and their magnitude would be greater than ε.

Since the principal strain trajectories as defined by Mohr's circle form an orthogonal mesh of lines, it follows that at any node of a Michell structure, a tie comes vertical to a strut.

C. The Stress Trajectory Approach

The approach followed in this study for the optimum arrangement of two-dimensional planar structures is basically the same as the one described in the previous Sections. However, instead of obtaining the principal strain trajectories, the technique to follow obtains the principal stress trajectories.

The principal stress trajectories are continuous curves, divided into two families, each corresponding to the principal compressive and tensile stresses, and at any point in a continuous medium are tangent to the directions of the principal compressive and tensile stresses which meet at right angles; the magnitude of the principal stress varies along the paths of the trajectories, and it can be indicated in a pictorial representation either by varying their thickness or by assigning values of the principal stresses along their paths.

As recently as 1966, they were known as curves laborious to obtain, not easy to represent on paper and therefore not often used in practical stress analysis work.(12)

Today, however, the aid of the computer and the applicability of the Finite Element Technique to linear elastic problems provide the means by which one can easily and economically obtain the stress trajectories. Also, the representation on paper is done by the computer.

Two examples of stress trajectories have already been introduced in association with the Michell Field lines in Appendix I-B.

The stress trajectories have been introduced to ship structural design by Hovgaard.(13) He particularly demonstrates their importance in that they are curves indicative of the way to stiffen a plate against "wrinkling" which is a particular case of buckling.

Specifically, for high stress levels, the principal compressive stresses in a thin webbed plate like that of Figure I-B.5 can cause wrinkling. The regions of the plate where those principal stresses are totally, or to the greater percentage of their value, depended on the shear stress are more likely to wrinkle. The directions of the principal compressive stresses are at 45° to the Neutral Axis (N.A.) of the plate at the N.A. and at a distance of L/4 from either end of the plate.

Therefore, the buckling wave formed when the critical stress value is reached runs along the direction of the principal compressive stresses. Its effect however is amplified due to the principal tensile stresses which in stretching the wave in a 90° direction from that in which the wave runs, create the wrinkling waves.

It is of extreme importance to note that in the longitudinally-framed ship, such as the warship of Figure I-1, the girders run approximately along the pathways of the stress trajectories. The plating of Section FFFF according to the previous discussion will be the most vulnerable to compressive stress due to shearing. Therefore, proper stiffening along the compressive stress trajectories will strengthen it.

This author has explained how one can obtain the stress trajectories applying Simple Beam Theory to rectangular plates;[14] however, the computational method is limited to one particular example, a centrally-loaded plate fixed at the ends. A more complete method which can treat any two-dimensional planar stress problem is described in detail in the next Section.

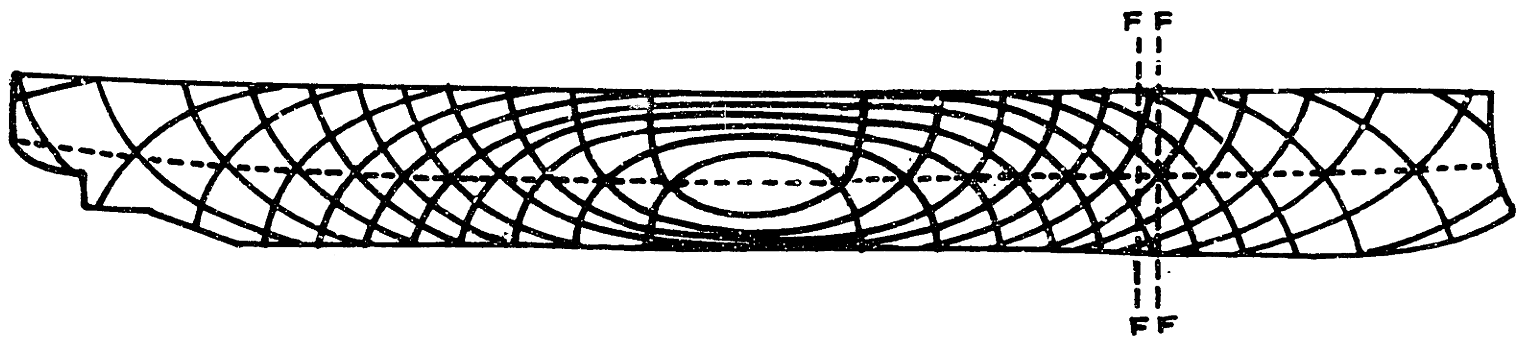


Figure I-1 (13)

The Principal Stress Trajectories in a Warship in the Hogging Condition

II THE STRESS TRAJECTORY TECHNIQUE

A. The Finite Element Approach

1. Introduction. The importance of the stress trajectories in determining the minimum weight arrangement of the truss members or the stiffeners of a plate under two dimensional loading has been explained in the previous section. However, the question still remains as to how one can easily obtain the principal stress trajectories.

The Finite Element Technique applied to the Plane Stress Approximation serves as one answer: one can arrive at the general state of stress of the structure and from Mohr's circle derive the principal stresses from which the stress trajectories will be obtained. The efficiency of this Technique is attributed to the fact that it is aided by the computer.

2. General. The Finite Element Technique is based on the fact that an elastic continuum, such as a plate or a shell, can be thought of as an assemblage of finite structural elements interconnected at a discrete number of finite nodal points, or control stations. Within each discrete or Finite Element, the behavior (displacements) is described in terms of a limited number of degrees of freedom, which are usually defined at interelement nodal stations (the nodal displacements in the present case).

In reality, however, an elastic continuum has an infinite number of degrees of freedom, but since it is impossible to treat them, the Finite Element Technique becomes an approximation to reality.

The approach to a planar stress structural problem using the Finite Element Displacement Method is similar to that used in simple Frame Structural Analysis as again the basic unknown parameters are the displacements of the nodal points. The difference between the two is that a two dimensional Finite Element can have three or more discrete number of nodal points situated on its boundaries, while the bars in Frame Analysis have only two nodes, one at each end.

The following is the general procedure for working with the Finite Element Technique:

(1) Separate the continuum into a number of Finite Elements by imaginary lines.

(2) Define a set of nodes on the boundaries of each element. The unknown displacements, q, will be defined there.

(3) Choose a function (or functions) to define uniquely the displacements (u, v) of every point within the element, in terms of the nodal displacements, q.

(4) Define the strains (ε_x, ε_y, γ_{xy}) of every point within the element in terms of the displacements,

u, v; and thus, together with the elastic properties of the material, E, ν define the stresses (σ_x, σ_y, τ_{xy}) within the element and therefore on its boundaries in terms of the strains.

(5) Determine a system of nodal equivalent forces, Q, which balance the boundary stresses and any distributed boundary loads, so that from the stiffness relationship

$$\{Q\} = [K]\{q\} \qquad [2.1]$$

We can solve for q having worked out the stiffness matrix [K].[15]

3. <u>Geometry</u>. As mentioned in the Introduction, the present Technique examines only orthogonal geometry. Therefore, the simplest manner to represent a rectangular plate is to use a rectangular element, with dimensions which are multiples of the dimensions of the plate. To simplify the element further for our purposes, the element under consideration has only four nodes, one at each corner, and has no other nodes along its four edges.

4. <u>Forces and Displacements</u>. If the Force-Displacement relationships for the individual elements are known, we can derive the properties and study the behavior of the assembled structure.

The Finite Element Technique assumes that the internal stresses which actually act along the boundaries of an element are substituted by equivalent fictitious forces on the nodes of the element. These equivalent forces, Q, will relate to the nodal displacements which are the basic unknown quantities, q.

Since the three kinds of possible inplane loads acting on a two-dimensional planar structure--namely, the forces in the x-direction, the forces in the y-direction, and the moments about the z-direction in a cartesian coordinate system--can be adequately modeled by a two-dimensional orthogonal system of forces acting on each node of the structure, each nodal point will have two degrees of freedom. Therefore, the displacements u, in the x-direction, and v, in the y-direction, within an element will have to be uniquely defined by the nodal displacements, q, as defined in Figure II-1. They are functions of the local coordinates x and y and their simplest representation for a rectangular element is the following:

$$u = \alpha_1 + \alpha_2 \frac{x}{L_x} + \alpha_3 \frac{y}{L_y} + \alpha_4 \frac{xy}{L_x L_y} \qquad [2.1.1]$$

$$v = \alpha_5 + \alpha_6 \frac{x}{L_x} + \alpha_7 \frac{y}{L_y} + \alpha_8 \frac{xy}{L_x L_y} \qquad [2.1.2]$$

where the constants α_1, α_2, α_3, α_4 can be expressed in terms of the nodal displacements, q.[15]

For each horizontal nodal displacement, we can write

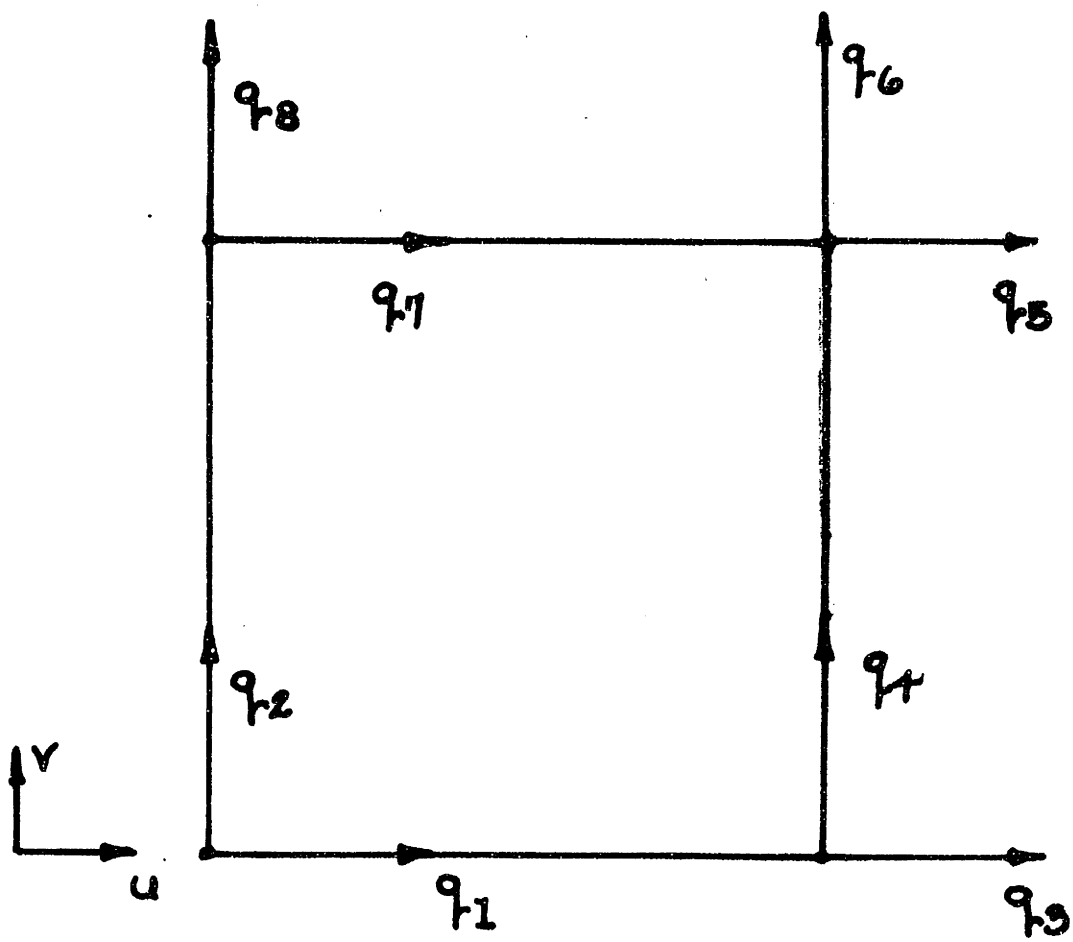


Figure II-1

The Modal Displacements of
An Element in Their Positive Directions

$$q_1 = \alpha_1 \quad [2.2.1]$$

$$q_3 = \alpha_1 + \alpha_2 \quad [2.2.2]$$

$$q_5 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \quad [2.2.3]$$

$$q_7 = \alpha_1 + \alpha_3 \quad [2.2.4]$$

Solving for α_1, α_2, α_3, and α_4, we obtain

$$\alpha_1 = q_1 \quad [2.3.1]$$

$$\alpha_2 = q_3 - q_1 \quad [2.3.2]$$

$$\alpha_3 = q_7 - q_1 \quad [2.3.3]$$

$$\alpha_4 = q_1 - q_3 + q_5 - q_7 \quad [2.3.4]$$

Substitution for α_1, α_2, α_3, and α_4 in equation [2.1.1] yields

$$u = q_1 + (q_3 - q_1)\frac{x}{L_x} + (q_7 - q_1)\frac{y}{L_y} + (q_1 - q_3 + q_5 - q_7)\frac{xy}{L_x L_y} \quad [2.4.1]$$

or, rewriting equation [2.4.1] one has

$$u = q_1\left(1 - \frac{x}{L_x} - \frac{y}{L_y} + \frac{xy}{L_x L_y}\right) + q_3\left(\frac{x}{L_x} - \frac{xy}{L_x L_y}\right) + q_5\,\frac{xy}{L_x L_y} + q_7\left(\frac{y}{L_y} - \frac{xy}{L_x L_y}\right) \quad [2.4.2]$$

Note that the above expression for u, checks with the horizontal nodal displacements of Figure II-1, if we substitute for $(x = 0,\ y = 0)$, $(x = L_x,\ y = 0)$, $(x = L_x,\ y = L_y)$, and $(x = 0,\ y = L_y)$.

Similarly, we obtain the expression for the vertical displacement:

$$v = q_2\left(1 - \frac{x}{L_x} - \frac{y}{L_y} + \frac{xy}{L_xL_y}\right) + q_4\left(\frac{x}{L_x} - \frac{xy}{L_xL_y}\right)$$

$$+ q_6\left(\frac{xy}{L_xL_y}\right) + q_8\left(\frac{y}{L_y} - \frac{xy}{L_xL_y}\right) \qquad [2.5]$$

Clearly, the two expressions for the horizontal and vertical displacements are similar because of the symmetry of the element. Note that the displacements along the interelement boundaries are compatible.

Thus, the third step of the general procedure of the Finite Element Technique has been completed, that is, that a function must be chosen to uniquely define the state of displacement within each element in terms of its nodal displacements, q.

5. <u>Strains and Stresses</u>. From Timoshenko and Goodier[16] it is clear that the strains at any point on a two-dimensional planar elastic continuum are given by

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} \qquad [2.6]$$

Also, the stresses will be given from

$$\{\sigma\} = [D]\,\{\varepsilon\} \qquad [2.7]$$

where

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad [2.8]$$

and

$$[D] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \qquad [2.9]$$

where [D] is the *elasticity matrix* for the *Plane Stress--Isotropic Material* Approximation. E is Young's Modulus, and ν is Poisson's ratio.

The relationship of strain and stress in equation [2.7] is the matrix notation of the following set of simultaneous equations:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} \qquad [2.10.1]$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} \qquad [2.10.2]$$

$$\gamma_{xy} = 2(1 + \nu)\frac{\tau_{xy}}{E} \qquad [2.10.3]$$

which explicitly explain the origin of [D].

From equation [2.6] one can obtain the total strain at any point within the element in terms of the unknown nodal displacements, q, and the x, y coordinates.

$$
\{\varepsilon\} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\[2ex] \dfrac{\partial v}{\partial y} \\[2ex] \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix}
$$

$$
= \begin{bmatrix}
\left[\frac{-1}{L_x} + \frac{y}{L_xL_y}\right] & 0 & \left[\frac{1}{L_x} - \frac{y}{L_xL_y}\right] & 0 & \left[\frac{y}{L_xL_y}\right] & 0 & \left[\frac{-y}{L_xL_y}\right] & 0 \\
0 & \left[\frac{-1}{L_x} + \frac{x}{L_xL_y}\right] & 0 & \left[\frac{-x}{L_xL_y}\right] & 0 & \left[\frac{x}{L_xL_y}\right] & 0 & \left[\frac{1}{L_y} - \frac{x}{L_xL_y}\right] \\
\left[\frac{-1}{L_y} + \frac{x}{L_xL_y}\right] & \left[\frac{-1}{L_x} + \frac{y}{L_xL_y}\right] & \left[\frac{-x}{L_xL_y}\right] & \left[\frac{1}{L_x} - \frac{y}{L_xL_y}\right] & \left[\frac{x}{L_xL_y}\right] & \left[\frac{y}{L_xL_y}\right] & \left[\frac{1}{L_y} - \frac{x}{L_xL_y}\right] & \left[\frac{-y}{L_xL_y}\right]
\end{bmatrix}
\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{Bmatrix}
$$

or

$$
\{\varepsilon\} = [B]\{q\} \qquad [2.11]
$$

Noticing the similarities in the terms of [B], one defines

$$A_1 = \frac{-1}{L_x} + \frac{y}{L_x L_y} \qquad [2.12.1]$$

$$B_1 = \frac{y}{L_x L_y} \qquad [2.12.2]$$

$$\Gamma_1 = \frac{-1}{L_y} + \frac{x}{L_x L_y} \qquad [2.12.3]$$

$$\Delta_1 = \frac{-x}{L_x L_y} \qquad [2.12.4]$$

Therefore [B] becomes:

$$[B] = \begin{bmatrix} A_1 & 0 & -A_1 & 0 & B_1 & 0 & -B_1 & 0 \\ 0 & \Gamma_1 & 0 & \Delta_1 & 0 & -\Delta_1 & 0 & -\Gamma_1 \\ \Gamma_1 & A_1 & \Delta_1 & -A_1 & -\Delta_1 & B_1 & -\Gamma_1 & -B_1 \end{bmatrix} \qquad [2.13]$$

Equation [2.11] shows the linear relationship between the strains and the nodal displacements and therefore, the [B] matrix contains linear components.

6. <u>The Stiffness Matrix</u>. The stiffness matrix is defined as follows:

$$[K] = \int_V [B]^T [D] [B] dV \qquad [2.14.1]$$

(15)

$$= \int_V [B]^T [D] [B] t dx dy \qquad [2.14.2]$$

where t is the thickness of the plate and the integral is taken over an elemental volume dV = tdxdy.

Knowing [D] and [B], one can write

$$[B]^T[D][B]t =$$

$$= \frac{Et}{1-\nu^2}\begin{bmatrix} A_1 & 0 & \Gamma_1 \\ 0 & \Gamma_1 & A_1 \\ -A_1 & 0 & \Delta_1 \\ 0 & \Delta_1 & -A_1 \\ B_1 & 0 & -\Delta_1 \\ 0 & -\Delta_1 & B_1 \\ -B_1 & 0 & -\Gamma_1 \\ 0 & -\Gamma_1 & -B_1 \end{bmatrix}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix}\begin{bmatrix} A_1 & 0 & -A_1 & 0 & B_1 & 0 & -B_1 & 0 \\ 0 & \Gamma_1 & 0 & \Delta_1 & 0 & -\Delta_1 & 0 & -\Gamma_1 \\ \Gamma_1 & A_1 & \Delta_1 & -A_1 & -\Delta_1 & B_1 & -\Gamma_1 & -B_1 \end{bmatrix} =$$

[2.15.1]

$$= \frac{Et}{1 - \nu^2} \begin{bmatrix} A_1 & A_1\nu & \frac{(1-\nu)}{2}\Gamma_1 \\ \Gamma_1\nu & \Gamma_1 & \frac{(1-\nu)}{2}A_1 \\ -A_1 & -A_1\nu & \frac{(1-\nu)}{2}\Delta_1 \\ \Delta_1\nu & \Delta_1 & -\frac{(1-\nu)}{2}A_1 \\ B_1 & B_1\nu & -\frac{(1-\nu)}{2}\Delta_1 \\ -\Delta_1\nu & -\Delta_1 & \frac{(1-\nu)}{2}B_1 \\ -B_1 & -B_1\nu & -\frac{(1-\nu)}{2}\Gamma \\ -\Gamma_1\nu & -\Gamma_1 & -\frac{(1-\nu)}{2}B_1 \end{bmatrix} \begin{bmatrix} A_1 & 0 & -A_1 & 0 & B_1 & 0 & -B_1 & 0 \\ 0 & \Gamma_1 & 0 & \Delta_1 & 0 & -\Delta_1 & 0 & -\Gamma_1 \\ \Gamma_1 & A_1 & \Delta_1 & -A_1 & -\Delta_1 & B_1 & -\Gamma_1 & -B_1 \end{bmatrix} = \frac{Et}{1 - \nu^2} [A]$$

[2.15.2]

$A_1^2 + \Gamma_1^2 \frac{(1-\nu)}{2}$	$A_1\Gamma_1\nu + A_1\Gamma_1 \frac{(1-\nu)}{2}$	$-A_1^2 + \Gamma_1\Delta_1 \frac{(1-\nu)}{2}$	$A_1\Delta_1\nu - A_1\Gamma_1 \frac{(1-\nu)}{2}$	$A_1B_1 - \Gamma_1\Delta_1 \frac{(1-\nu)}{2}$	$-A_1\Delta_1\nu + B_1\Gamma_1 \frac{(1-\nu)}{2}$	$-B_1A_1 - \Gamma_1^2 \frac{(1-\nu)}{2}$	$-A_1\Gamma_1\nu - B_1\Gamma_1 \frac{(1-\nu)}{2}$
	$\Gamma_1^2 + A_1^2 \frac{(1-\nu)}{2}$	$-A_1\Gamma_1 + \Delta_1A_1 \frac{(1-\nu)}{2}$	$\Gamma_1\Delta_1 - A_1^2 \frac{(1-\nu)}{2}$	$B_1\Gamma_1\nu - A_1\Delta_1 \frac{(1-\nu)}{2}$	$-\Gamma_1\Delta_1 + B_1A_1 \frac{(1-\nu)}{2}$	$-B_1\Gamma_1\nu - \Gamma_1A_1 \frac{(1-\nu)}{2}$	$\Gamma_1^2 - B_1A_1 \frac{(1-\nu)}{2}$
		$A^2 + \Delta_1^2 \frac{(1-\nu)}{2}$	$-A_1\Delta_1\nu - A_1\Delta_1 \frac{(1-\nu)}{2}$	$-A_1B_1 - \Delta_1^2 \frac{(1-\nu)}{2}$	$\Delta_1A_1\nu + B_1\Delta_1 \frac{(1-\nu)}{2}$	$A_1B_1 - \Gamma_1\Delta_1 \frac{(1-\nu)}{2}$	$\Gamma_1A_1\nu - B_1\Delta_1 \frac{(1-\nu)}{2}$
			$\Delta_1^2 + A_1^2 \frac{(1-\nu)}{2}$	$B_1\Delta_1\nu + A_1\Delta_1 \frac{(1-\nu)}{2}$	$-\Delta_1^2 - A_1B_1 \frac{(1-\nu)}{2}$	$-B_1\Delta_1\nu + A_1\Gamma_1 \frac{(1-\nu)}{2}$	$-\Gamma_1\Delta_1 + A_1B_1 \frac{(1-\nu)}{2}$
				$B_1^2 + \Delta_1^2 \frac{(1-\nu)}{2}$	$-\Delta_1B_1\nu - B_1\Delta_1 \frac{(1-\nu)}{2}$	$-B_1^2 + \Gamma_1\Delta_1 \frac{(1-\nu)}{2}$	$-\Gamma_1B_1\nu + B_1\Delta_1 \frac{(1-\nu)}{2}$
	(symmetric)				$\Delta_1^2 + B_1^2 \frac{(1-\nu)}{2}$	$+B_1\Delta_1\nu - \Gamma_1B_1 \frac{(1-\nu)}{2}$	$\Gamma_1\Delta_1 - B_1^2 \frac{(1-\nu)}{2}$
						$B_1^2 + \Gamma_1^2 \frac{(1-\nu)}{2}$	$B_1\Gamma_1\nu + B_1\Gamma_1 \frac{(1-\nu)}{2}$
							$\Gamma_1^2 + B_1^2 \frac{(1-\nu)}{2}$

Table II-1 The [A] Matrix of Equation [2.15.2] where [A] is the symmetric matrix shown in Table II-1.

Taking the integral of the right-hand-side of equation [2.15.2] over an elemental volume, tdxdy, essentially means to take the following integrals where t has already been taken out of the integral:

From equations [2.12.1] through [2.12.4], one has

$$\int_V A_1^2 dxdy = \int_0^{L_y}\int_0^{L_x}\left(-\frac{1}{L_x}+\frac{y}{L_xL_y}\right)^2 dxdy = \frac{L_y}{3L_x} \qquad [2.16.1]$$

$$\int_V B_1^2 dxdy = \int_0^{L_y}\int_0^{L_x}\left(\frac{y}{L_xL_y}\right)^2 dxdy = \frac{L_y}{3L_x} \qquad [2.16.2]$$

$$\int_V \Gamma_1^2 dxdy = \int_0^{L_y}\int_0^{L_x}\left(-\frac{1}{L_y}+\frac{x}{L_xL_y}\right)^2 dxdy = \frac{L_x}{3L_y} \qquad [2.16.3]$$

$$\int_V \Delta_1^2 dxdy = \int_0^{L_x}\int_0^{L_x}\left(\frac{x}{L_xL_y}\right)^2 dxdy = \frac{L_x}{3L_y} \qquad [2.16.4]$$

$$\int_V A_1 B_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(-\frac{1}{L_x} + \frac{y}{L_x L_y}\right)\left(\frac{y}{L_x L_y}\right) dxdy = \frac{-L_y}{6L_x} \qquad [2.16.5]$$

$$\int_V A_1 \Gamma_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(-\frac{1}{L_x} + \frac{y}{L_x L_y}\right)\left(-\frac{1}{L_y} + \frac{x}{L_x L_y}\right) dxdy = \frac{1}{4} \qquad [2.16.6]$$

$$\int_V A_1 \Delta_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(-\frac{1}{L_x} + \frac{y}{L_x L_y}\right)\left(\frac{-x}{L_x L_y}\right) dxdy = \frac{1}{4} \qquad [2.16.7]$$

$$\int_V B_1 \Gamma_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(\frac{y}{L_x L_y}\right)\left(-\frac{1}{L_y} + \frac{x}{L_x L_y}\right) dxdy = -\frac{1}{4} \qquad [2.16.8]$$

$$\int_V B_1 \Delta_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(\frac{y}{L_x L_y}\right)\left(\frac{-x}{L_x L_y}\right) dxdy = -\frac{1}{4} \qquad [2.16.9]$$

$$\int_V \Gamma_1 \Delta_1 dxdy = \int_0^{L_y}\int_0^{L_x} \left(-\frac{1}{L_y} + \frac{x}{L_x L_y}\right)\left(\frac{-x}{L_x L_y}\right) dxdy = \frac{L_x}{6L_y} \qquad [2.16.10]$$

$L_y/3L_x + \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_y/3L_x + \frac{L_x}{6L_y}\frac{(1-\nu)}{2}$	$\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_y/6L_x - \frac{L_x}{6L_y}\frac{(1-\nu)}{2}$	$-\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$L_y/6L_x - \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$-\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$
	$L_x/3L_y + \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$	$-\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$L_x/6L_y - \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$	$-\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_x/6L_y - \frac{L_y}{6L_x}\frac{(1-\nu)}{2}$	$\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_x/3L_y + \frac{L_y}{6L_x}\frac{(1-\nu)}{2}$
		$L_y/3L_x + \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$-\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$L_y/6L_x - \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_y/6L_x - \frac{L_x}{6L_y}\frac{(1-\nu)}{2}$	$\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$
			$L_x/3L_y + \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$	$-\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_x/3L_y + \frac{L_y}{6L_x}\frac{(1-\nu)}{2}$	$\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_x/6L_y - \frac{L_y}{6L_x}\frac{(1-\nu)}{2}$
				$L_y/3L_x + \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$-L_y/3L_x + \frac{L_x}{6L_y}\frac{(1-\nu)}{2}$	$\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$
	(symmetric)				$L_x/3L_y + \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$	$-\nu/4 + \frac{1}{4}\frac{(1-\nu)}{2}$	$L_x/6L_y - \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$
						$L_y/3L_x + \frac{L_x}{3L_y}\frac{(1-\nu)}{2}$	$-\nu/4 - \frac{1}{4}\frac{(1-\nu)}{2}$
							$L_x/3L_y + \frac{L_y}{3L_x}\frac{(1-\nu)}{2}$

Table II-2 The Stiffness Matrix [K] × $[1-\nu^2/Et]$ for One Rectangular Element

Therefore,

$$[K] = \frac{Et}{1 - \nu^2} [A'] \qquad [2.17]$$

where [A'] is shown in Table II-2.

Here ends the development of one element's stiffness matrix. The stiffness matrix for the entire structure is obtained by assembling all of the individual stiffness matrices. The inverse of the assembled stiffness matrix is used to solve for all the nodal displacements, that is, from equation [2.1].

$$\{q\} = [K]^{-1}_{total} \{Q\} \qquad [2.18]$$

where {Q} is the matrix of the equivalent nodal forces for the entire structure.

7. Equivalent Nodal Forces. As mentioned in Part 3 of this section, the Finite Element Technique deals with fictitious forces, at the nodes of each element, which are energy equivalent(15) to the internal stresses acting on its boundaries. These nodal forces, Q, have a one-to-one correspondence with the nodal displacements of the element, q.

Presently, since each node will have two degrees of freedom, two equivalent nodal forces are required. That is, for every element which has a stiffness matrix [K] as defined on Table II-2,

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{Bmatrix} = [K] \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{Bmatrix} \qquad [2.19]$$

When distributed loads act on an edge of an element, as the case is for an element on the boundary of a plate, the nodal forces of the element must be energy equivalent to the distributed external loads or the internal interelement boundary stresses.

<u>Example</u>. Let us assume the rectangular element of Figure II-2 loaded on one of its edges with a linearly distributed force of intensity f(x).

The total external work done on that element, if one allows a virtual displacement {δ} will be:

$$W_{ext.} = \int_0^{L_x} (\vec{F} \cdot \vec{\delta})\, dx \qquad [2.19]$$

where $\vec{F}$ is the vector force resultant formed by integrating the applied load f(x) over the thickness of the element. When together with the normal intensity f(x), there is a shear distribution acting along the boundary of the element, $\vec{F}$ will no longer be normal to the edge of the element.

Therefore, in general

$$\vec{F} = F_x\vec{i} + F_y\vec{j} \qquad [2.20]$$

where the $\vec{i}$ and $\vec{j}$ directions are shown in Figure II-2, and $\vec{\delta}$ is the total virtual displacement due to $\vec{F}$ and for the general case, it will be:

$$\vec{\delta} = \delta_x\vec{i} + \delta_y\vec{j} \qquad [2.21]$$

Therefore, applying equation [2.19] to the example of Figure II-2, we obtain:

$$W_{ext.} = \int_0^{L_x} F_y \; \delta_y \; dx \qquad [2.22]$$

Since F_y will equal to the intensity of the load over the elemental side dx,

$$F_y = - \left\{ \frac{[f(\ell) - f(k)]}{L_x} x + f(k) \right\} \qquad [2.23]$$

Similarly,

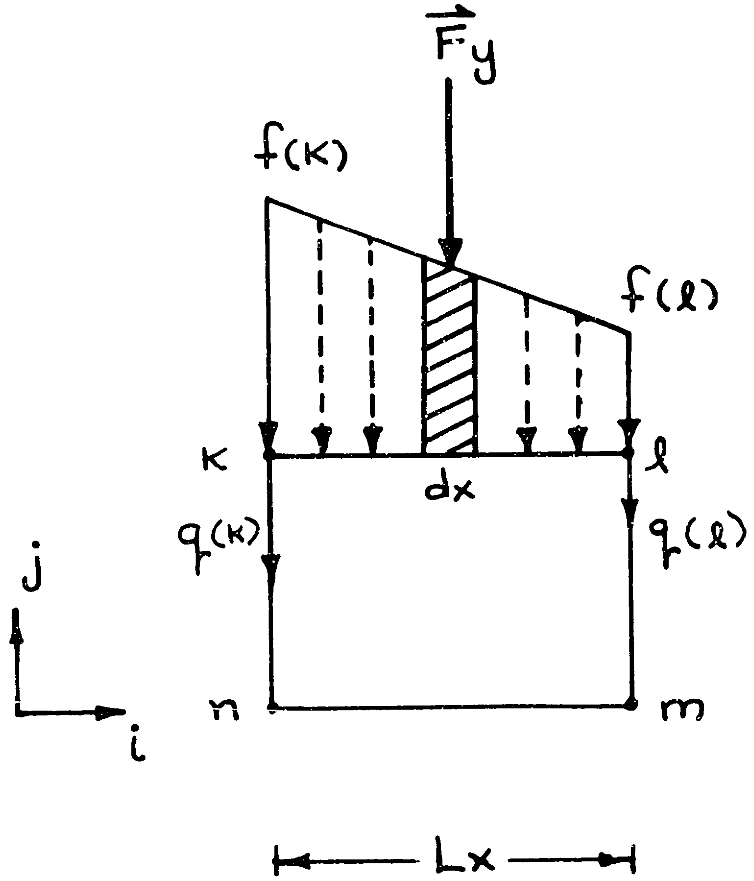
$$\delta_y = - \left\{ \frac{[q(\ell) - q(k)]}{L_x} x + q(k) \right\} \qquad [2.24]$$

where $q(\ell)$ and $q(k)$ are the nodal displacements.

Integrating to get $W_{ext.}$ in equation [2.22] we have:

$$W_{ext.} =$$

$$= \int_0^{L_x} \left\{ \frac{[f(\ell) - f(k)]}{L_x} x + f(k) \right\} \left\{ \frac{[q(\ell) - q(k)]}{L_x} x + q(k) \right\} dx =$$



NOTE: THE POSITIVE FORCES AND DISPLACEMENTS ARE POINTING AWAY FROM THE ELEMENT.

Figure II-2

An Element under a Linearly Varying Boundary Load Distribution

$$= \frac{1}{3}[f(\ell)q(\ell) - f(k)q(\ell) - f(\ell)q(k) + f(k)q(k)]L_x$$

$$+ f(k)q(k)L_x + \frac{1}{2}[f(\ell)q(k) - f(k)q(k)]L_x$$

$$+ \frac{1}{2}[q(\ell)f(k) - q(k)f(k)]L_x$$

or

$$W_{ext.} = q(k)\left\{L_x\left[\frac{f(k)}{3} + \frac{f(\ell)}{6}\right]\right\} + q(\ell)\left\{L_x\left[\frac{f(k)}{6} + \frac{f(\ell)}{3}\right]\right\} \qquad [2.25]$$

Clearly, the expressions in the brackets have units of Force and represent the equivalent nodal forces, Q, since

$$\int_x (\vec{F} \cdot \vec{\delta})dx = \sum_i q_i Q_i \qquad [2.26]$$

Therefore, the equivalent nodal forces are:

$$Q(k) = L_x \frac{f(k)}{3} + \frac{f(\ell)}{6}$$

and
$$Q(\ell) = L_x \frac{f(k)}{6} + \frac{f(\ell)}{3} \qquad [2.27]$$

When external concentrated forces $\bar{Q}$ are applied on the nodes of the assembled structure, their effects must be added to the equilibrium equation, that is,

$$[K]\{q\} = \{Q\} = \{\bar{Q}\}_{\substack{\text{nodal}\\ \text{concentrated}}} + \{Q\}_{\substack{\text{nodal}\\ \text{equivalent}}} \qquad [2.28]$$

However, the computer program which is explained in Appendix II-2 is equipped to handle either linearly distributed forces or external concentrated loads but not both at the same time.

B. Stresses and Stress Trajectories

1. General State of Stress for a Rectangular Element.

In the previous section, it is shown explicitly how one can obtain the nodal displacements for the total structure. That is:

(1) By developing the stiffness matrix for the assembled structure

(2) By defining the "force entries" (that is, the magnitude and direction of every applied force at the control point of application)

(3) By solving for the q's in equation [2.18].

According to the fourth step of the Finite Element general procedure, the strains ε_x, ε_y, and γ_{xy} of one element are defined in terms of the displacement functions u(x, y) and v(x, y) which are expressed in terms of the nodal displacements, q (equations [2.4.2], [2.5], and [2.6]).

Therefore, using the strain-displacement relationship of equation [2.11] and the stress-strain relationship of equation [2.7], one obtains

$$\{\sigma\}^e = [D][B]\{q\}^e \qquad [2.29]$$

where the superscript e refers to the stresses and nodal displacements of one element.

Multiplying the elasticity matrix [D] with [B], and substituting from equations [2.9] and [2.13] for their equals

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix}\begin{bmatrix} A_1 & 0 & -A_1 & 0 & B_1 & 0 & -B_1 & 0 \\ 0 & \Gamma_1 & 0 & \Delta_1 & 0 & -\Delta_1 & 0 & -\Gamma_1 \\ \Gamma_1 & A_1 & \Delta_1 & -A_1 & -\Delta_1 & B_1 & -\Gamma_1 & -B_1 \end{bmatrix}$$

$$= \frac{E}{1-\nu^2}\begin{bmatrix} A_1 & \Gamma_1\nu & -A_1 & \Delta_1\nu & B_1 & -\Delta_1\nu & -B_1 & -\Gamma_1\nu \\ A_1\nu & \Gamma_1 & -A_1\nu & \Delta_1 & B_1\nu & -\Delta_1 & -B_1\nu & -\Gamma_1 \\ \Gamma_1\frac{(1-\nu)}{2} & A_1\frac{(1-\nu)}{2} & \Delta_1\frac{(1-\nu)}{2} & -A_1\frac{(1-\nu)}{2} & -\Delta_1\frac{(1-\nu)}{2} & B_1\frac{(1-\nu)}{2} & -\Gamma_1\frac{(1-\nu)}{2} & -B_1\frac{(1-\nu)}{2} \end{bmatrix}$$

$$= [S]^e \qquad [2.29.1]$$

where $[S]^e$ is the element stress matrix.

Therefore, substituting for the values of A_1, B_1, Γ_1 and Δ_1 from equations [2.12.1] through [2.12.4] and multiplying $[S]^e$ by $\{q\}^e$, we obtain the stresses σ_x, σ_y, and τ_{xy} where

$$\sigma_x = \left(\frac{E}{1-\nu^2}\right)\left\{\left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_1 + \nu\left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_2 - \left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_3 + \nu\left(\frac{-x}{L_xL_y}\right)q_4 + \left(\frac{y}{L_xL_y}\right)q_5 - \nu\left(\frac{-x}{L_xL_y}\right)q_6 - \left(\frac{y}{L_xL_y}\right)q_7 - \nu\left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_8\right\} \quad [2.30.1]$$

$$\sigma_y = \left(\frac{E}{1-\nu^2}\right)\left\{\nu\left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_1 + \left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_2 - \nu\left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_3 + \left(\frac{-x}{L_xL_y}\right)q_4 + \nu\left(\frac{y}{L_xL_y}\right)q_5 - \left(\frac{-x}{L_xL_y}\right)q_6 - \nu\left(\frac{y}{L_xL_y}\right)q_7 - \left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_8\right\} \quad [2.30.2]$$

$$\tau_{xy} = \left(\frac{E}{2(1+\nu)}\right)\left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_1 + \left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_2 + \left(\frac{-x}{L_xL_y}\right)q_3 - \left(\frac{-1}{L_x} + \frac{y}{L_xL_y}\right)q_4 - \left(\frac{-x}{L_xL_y}\right)q_5 + \left(\frac{y}{L_xL_y}\right)q_6$$

$$- \left(\frac{-1}{L_y} + \frac{x}{L_xL_y}\right)q_7 - \left(\frac{y}{L_xL_y}\right)q_8 \quad [2.30.3]$$

Thus, one arrives at the general expressions for the state of stress at any point within the element.

Rearranging and redefining, equations [2.30] become

$$\sigma_x = \sigma_{x_1} + \sigma_{x_2}x + \sigma_{x_3}y \quad [2.31.1]$$

$$\sigma_y = \sigma_{y_1} + \sigma_{y_2}x + \sigma_{y_3}y \quad [2.31.2]$$

$$\tau_{xy} = \tau_{xy_1} + \tau_{xy_2}x + \tau_{xy_3}y \quad [2.31.3]$$

where

$$\sigma_{x_1} = \frac{E}{1 - \nu^2}\left[\frac{1}{L_x}(q_3 - q_1) + \frac{\nu}{L_y}(q_8 - q_2)\right] \quad [2.32.1]$$

$$\sigma_{x_2} = \frac{E}{1 - \nu^2}\left[\frac{\nu}{L_xL_y}(q_2 - q_4 + q_6 - q_8)\right] \quad [2.32.2]$$

$$\sigma_{x_3} = \frac{E}{1 - \nu^2}\left[\frac{1}{L_xL_y}(q_1 - q_3 + q_5 - q_7)\right] \quad [2.32.3]$$

$$\sigma_{y_1} = \frac{E}{1 - \nu^2}\left[\frac{\nu}{L_x}(q_3 - q_1) + \frac{1}{L_y}(q_8 - q_2)\right] \quad [2.32.4]$$

$$\sigma_{y_2} = \frac{E}{1 - \nu^2}\left[\frac{1}{L_xL_y}(q_2 - q_4 + q_6 - q_8)\right] \quad [2.32.5]$$

$$\sigma_{y_3} = \frac{E}{1 - \nu^2}\left[\frac{\nu}{L_x L_y}(q_1 - q_3 + q_5 - q_7)\right] \quad [2.32.6]$$

$$\tau_{xy_1} = \frac{E}{2(1 + \nu)}\left[\frac{1}{L_y}(q_7 - q_1) + \frac{1}{L_x}(q_4 - q_2)\right] \quad [2.32.7]$$

$$\tau_{xy_2} = \frac{E}{2(1 + \nu)}\left[\frac{1}{L_x L_y}(q_1 - q_3 + q_5 - q_7)\right] \quad [2.32.8]$$

$$\tau_{xy_3} = \frac{E}{2(1 + \nu)}\left[\frac{1}{L_x L_y}(q_2 - q_4 + q_6 - q_8)\right] \quad [2.32.9]$$

To assure the linearity of stress throughout a uniform elastic planar structure, such as a rectangular plate which is the subject of the present study, the stresses $\{\sigma\}$ at each nodal point of the structure must represent the average stress value of those stresses calculated there separately for each element common to the nodal point. The way to calculate the average stress at a nodal point due to two or four common to the nodal point elements is explained in Appendix II-A.

2. Principal Stresses and Stress Trajectories. Once the average stresses σ_x, σ_y, and τ_{xy} at all control points of the structure have been calculated, the principal stresses there can be obtained using the well-known Mohr's circle equations:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad [2.33.1]$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad [2.33.2]$$

where σ_1 is the principal tensile stress and σ_2 is the principal compressive stress.

Their directions with respect to the horizontal x-axis are given by

$$\theta = \frac{1}{2} \tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right] \qquad [2.33.3]$$

where 2θ is defined to be in the range of

$$0 \leq 2\theta \leq \pi/2 \qquad [2.33.4]$$

for $$0 \leq \frac{2\tau_{xy}}{\sigma_x - \sigma_y} < +\infty \qquad [2.33.5]$$

and $$-\pi/2 \leq 2\theta < 0 \qquad [2.33.6]$$

for $$-\infty < \frac{2\tau_{xy}}{\sigma_x - \sigma_y} < 0 \qquad [2.33.7]$$

Since the magnitudes and the directions of the principal stresses with the +x-axis can be calculated at any nodal point of the structure, the stress trajectories can easily be obtained.

According to the definition of the stress trajectories given in section C of part I, the directions of the principal tensile and compressive stresses are perpendicular to each other at any point in an elastic continuum such as a plate subjected to inplane loading. These directions are tangent to the stress trajectories that pass from that point.

As long as the stress trajectories maintain a smooth curvature and have no sharp discontinuities, they can be approximated in a satisfactory manner by straight line segments between the interelement boundaries of the structure which form an equidistant orthogonal grid of lines.

Example. Let us assume the simple orthogonal grid of the structure of Figure II-3, which is a plate divided into six rectangular finite elements.

Let us assume that the geometry of the structure has been previously defined and that the average stresses σ_x, σ_y, and τ_{xy} at each control point have been already calculated.

The first questions to be asked would be:

(a) Where to start?

(b) Which family of trajectories (compressive and/or tensile) to represent?

(c) How many trajectories would sufficiently define the state of stress of the entire structure?

Since the average stresses at each nodal point are known, the most natural place to start is at a corner of the structure nodal point because to start at some point on an element boundary, or anywhere else in the structure (even at points within elements), would be a very involved problem

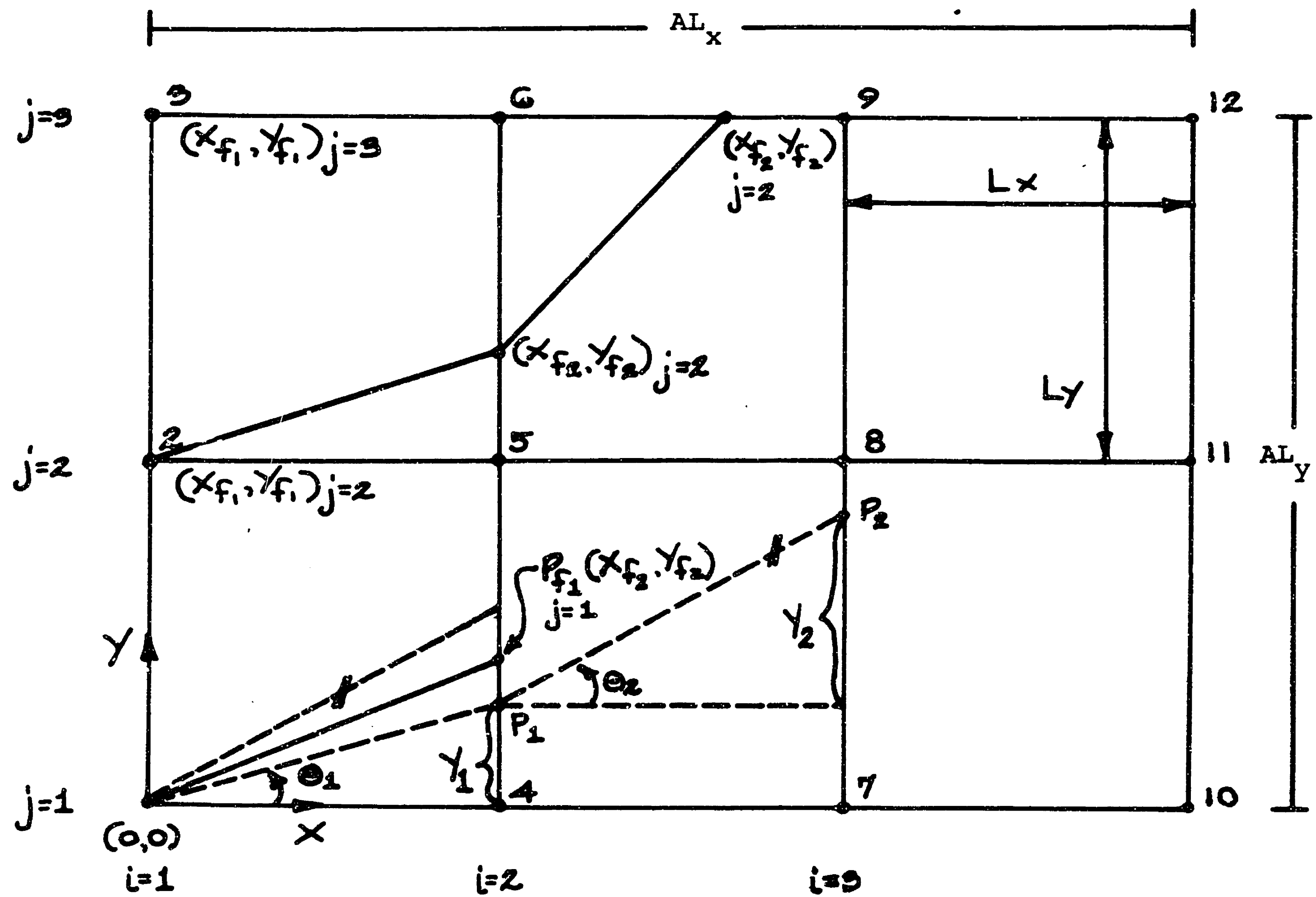


Figure II-3

Linear Extrapolation of the Directions of the Principal Stresses to Get the Stress Trajectories in a Six Element Structure and beyond the scope of the present study. Nevertheless, this can almost be achieved if we assign a large number of elements to the geometry of the structure, creating thus a very fine orthogonal grid, with more nodal points.

Furthermore, to assure that the extrapolated direction of principal stress will meet an element boundary or another control station, the structure must be "properly" oriented.

In other words, if it is decided that the starting point is the first node of the structure and that it is desired to obtain the principal compressive stress trajectories, we should not anticipate a negatively sloped principal compressive stress at the starting point because then the extrapolated slope would be out of the structure.

If, however, we do get a negative slope at the starting point, the structure should be turned at 90° so that in the present example, the new node No. 1 will be that which at the present is node No. 10.

It appears that the number of the stress trajectories to define the state of stress could be limited sufficiently to the number of control points in the y-direction. This is really a decision that depends on the designer. For our purposes, however, which is to show how one can efficiently obtain the stress trajectories, we will allow the number of trajectories to be determined by the number of control points in the y-direction, that is, in the example of Figure II-3, the number of trajectories is "three".

To summarize, the answers given to the three questions above define the following limitations to the problem of determining the stress trajectories.

(a) The calculation of the first trajectory starts at the first node of the structure which is "properly" oriented.

(b) Only the principal compressive stress trajectories are obtained in this study. The principal tensile stress trajectories can be obtained in a similar manner.

(c) The number of control points in the y-direction and on the left edge of a rectangular structure will correspond to the number of trajectories to determine the state of compressive stress of the structure.

The analytic procedure is partly explained by means of Comment cards in Subroutine TRAJEC (see Appendix II-A) but for purposes of clarification and continuity it is explained in detail below, and it is shown how it applies to the example of Figure II-3.

(1) Start at the first node of the structure where the x, y coordinates are (0, 0).

(2) Define the number of trajectories to be calculated. (that is, the number of trajectories is equal to the number of stations in the y-direction which is equal to 3).

Note that the last trajectory is actually one point, (that is, the control station No. 3 in the present example). However, its magnitude evaluated there completes the picture.

(3) Knowing the average stresses σ_x, σ_y, and τ_{xy} evaluate σ_2 (the principal compressive stress) at node No. 1.

(4) Define the number of stations along the x-direction at which the extrapolated direction of principal compressive stress will start. (Here, we have three stations, namely the interelement boundaries vertical to nodes No. 1, No. 4, and No. 7.

(5) Calculate the angle θ_1 between the positive x-axis and the direction of the principal stress at node No. 1. Knowing the element's side L_x and θ_1, we can calculate y_1 which is the difference in y between point P_1 and the ordinate of the starting point.

(6) Check if $y_1 > AL_y$. If so, specify that the trajectory in question is off the plate and in that case define its final coordinates x_f, y_f and its final σ_x, σ_y and σ_{xy} stresses. Then go to trajectory No. 2 which starts at control station No. 2. If not, continue.

(7) Calculate the stresses at the first extrapolation point P_1 which is in general between two nodes N_1 and N_2 (in our case P_1 is between nodes No. 4 and No. 5).

(8) Calculate the difference Δy_1 between the ordinates of P_1 and N_1.

(9) Interpolate between the stress values at nodes N_1 and N_2 to obtain the stresses at P_1. (No need to calculate the σ_2 at P_1 because P_1 is a temporary point).

(10) Calculate the angle θ_2 between the direction of the principal compressive stress and the +x-axis (similarly to step No. 5) and obtain y_2.

(11) Take the average between y_1 and y_2 and add it to the final y_{f_i} coordinate of the current station i to get the final y_f of the i + 1 station. (Here the final y_f of the current station is zero).

(12) Similarly to step No. 6, check if $y_{f_{i+1}} > AL_y$. If so, define the new $y_{f_{i+1}}$ final coordinate to be $y_{f_{i+1}} = AL_y$. But its $x_{f_{i+1}}$ coordinate will be at some point between x_{f_i} and $(x_{f_i} + L_x)$ which can be easily obtained by comparing similar triangles. Then go to trajectory No. 2 and start from control point No. 2. If not, continue.

(13) Evaluate the stresses at the final point $P_{f_1}(x_{f_{i+1}}, y_{f_{i+1}})$ by interpolation between the stresses at N_1 and N_2. (Here P_{f_1} is again between nodes No. 4 and No. 5).

(14) Go back to step No. 5 and repeat all the steps up to here until the trajectory is off the plate or until the trajectory passes through all stations along the x-direction.

(15) In any case continue the procedure by going to the second control point where the second trajectory commences and repeat the above steps until all the trajectories have been properly defined in terms of the x and y coordinates.

Thus, from the present example we anticipate three trajectories (subscripted j) of which $\text{trajectory}_{j=3}$ is just the magnitude of the compressive stress σ_2 at control station No. 3.

Since the final coordinates x_f, y_f of each trajectory have been properly calculated, we can easily plot them by the aid of the computer (see Appendix II-A).

To conclude, this part has shown how through applying the Plane Stress Approximation to the Finite Element Technique one can obtain the general state of stress for two-dimensional planar problems. Furthermore, this part has explicitly described and analyzed the procedure through which a good approximation of the stress trajectories can be obtained.

Appendix II-A includes a discussion of the capability as well as the complete listing of the computer program through which we obtain the displacements {q}, the stresses {σ} and the principal compressive stress trajectories. (Subroutine STIFEN which was lately added to the main program is related to the next section.

III STIFFENER ARRANGEMENTS

1. Introduction

The Stress Trajectory Technique explained in detail in Part II together with its associated computer program described in Appendix II-A may give the designer the means of properly reinforcing a plate or arranging the truss members in a two-dimensional truss problem. For the latter case one has to imagine that the initial thickness, t, of the plate is infinitesimal.

In this part different stiffener arrangements of the same cross-sectional area of reinforcing material are being compared in two distinct cross-sections of a planar structure. The results of this comparison may lead one to determine the least weight arrangement for the maximum stress reduction.

2. A Comparison between Two Stiffener Arrangements--Case I

Let us consider the panel of Figure III-1 (that is, a centrally loaded fixed ends plate) for which the Simple Beam Theory is a very good approximation.

Observing a small element at $x = L/4$ and $y = B/2$ (location A), one expects it to have a principal compressive stress σ_2 due to the shear stress τ_{xy}. At $x = L/2$, $y = B$ (location B) the principal compressive stress σ_2 will be due only to the bending stress, σ_x.

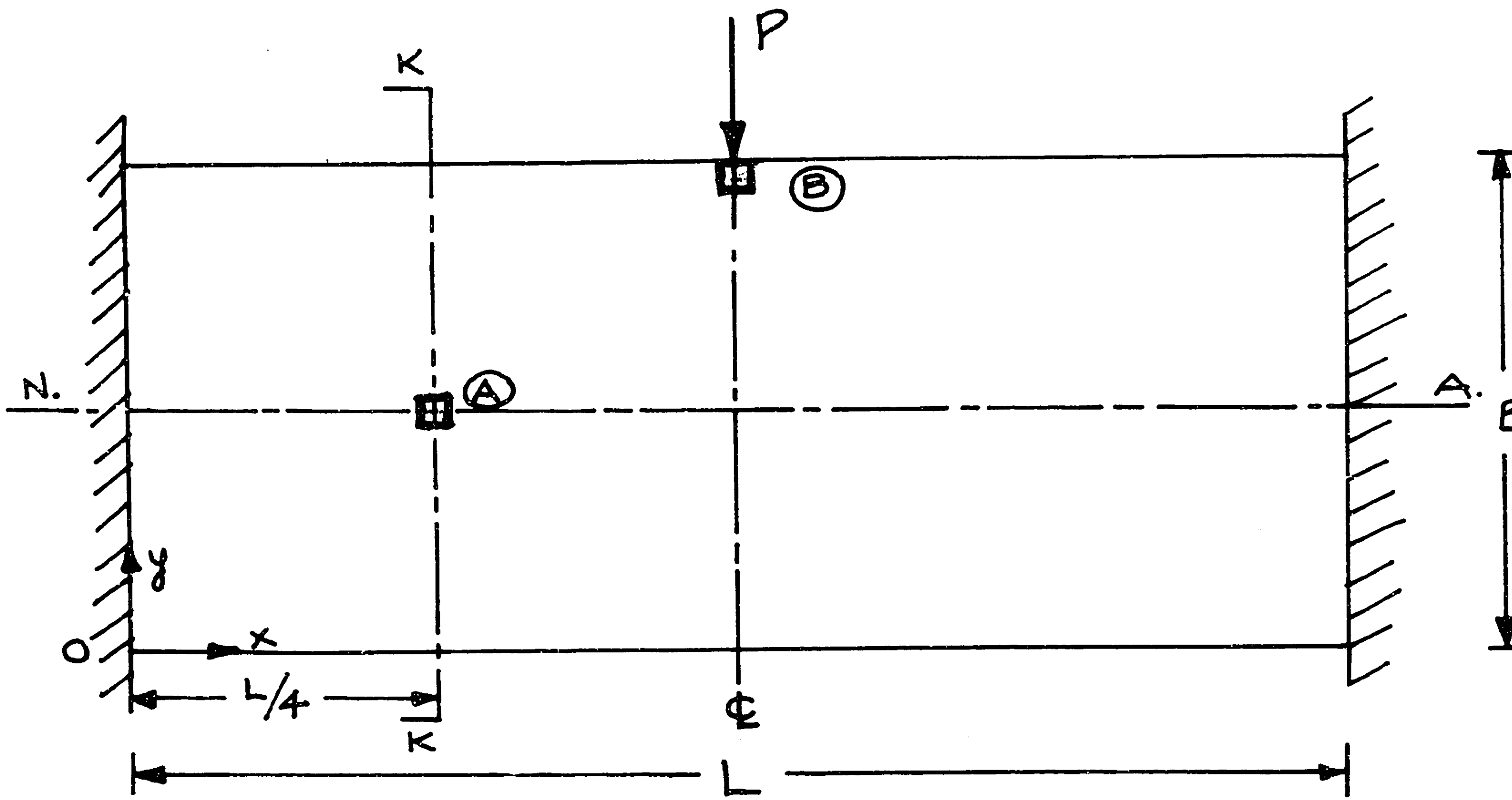


Figure III-1

A Centrally Loaded Plate

Consider the cross-section KK and question where would the best location or locations be to add reinforcing material in order to reduce the maximum shear stress $\tau_{max.}$.

The shear stress τ_{xy} is given by

$$\tau_{max.} = \frac{VQ}{It} \qquad [3.1]$$

where

V is the shear force constant throughout the cross-section at x = L/4.

Q is the first moment of all the material above or below the N.A. and taken about the N.A.

I is the moment of inertia of the cross-section.

t is the thickness of the panel.

Since V is independent of the x cross-section of the panel it must be investigated how the other three variables would be arranged to minimize $\tau_{max.}$.

Let us consider the same cross-sectional material added as shown in Figure III-2 to two panels of the same thickness, t. The first moments, Q, of half the cross-sectional area about the N.A. for A and B are given by:

$$Q_A = \frac{B^2t}{8}[1 + 4Hc(1 - H)] \qquad [3.2.1]$$

$$Q_B = \frac{B^2t}{8}(1 + 4H^2c) \qquad [3.2.2]$$

The moments of inertia, I, are:

$$I_A = \frac{B^3t}{12}[1 + c - (1 - 2H)^3c] \qquad [3.3.1]$$

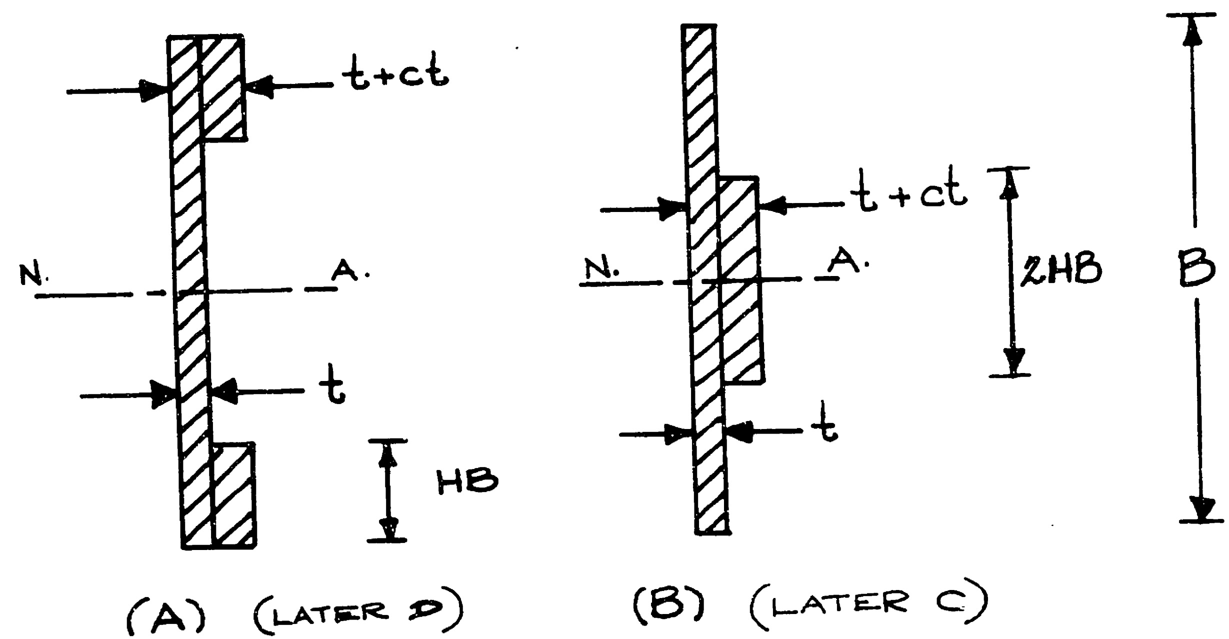


Figure III-2

Comparison between Two Stiffener Arrangements (Case I)

$$I_B = \frac{B^3 t}{12} (1 + 8cH^3) \qquad [3.3.2]$$

Therefore, the expressions for the maximum τ are

$$\tau_A = \frac{3}{2} \frac{V}{Bt} \frac{[1 + 4Hc(1 - H)]}{\{1 + c[1 - (1 - 2H)^3]\}} \qquad [3.4.1]$$

$$\tau_B = \frac{3}{2} \frac{V}{Bt} \frac{(1 + 4cH^2)}{(1 + c)(1 + 8cH^3)} \qquad [3.4.2]$$

To check the validity of equations [3.4.1] and [3.4.2] one can substitute for $c = 1$ and $H = 1/2$.

From equation [3.4.1]

$$\tau_A = \frac{3}{2} \left(\frac{V}{Bt}\right) \qquad [3.5.1]$$

and from equation [3.4.2]

$$\tau_B = \frac{3}{4} \left(\frac{V}{Bt}\right) \qquad [3.5.2]$$

The factor of two between equations [3.5.1] and [3.5.2] is due to the fact that adding material from the N.A. towards the edges of the panel the thickness of the panel is essentially doubled, while adding material from the edges towards the N.A., a cut is left.

Comparing τ_A and τ_B for $0 < c < \infty$ and $0 < H < 1/2$

$$\tau_A > \tau_B \qquad [3.6]$$

<u>Proof</u>

Substituting from equations [3.4.1] and [3.4.2]

$$\frac{1 + 4Hc(1 - H)}{1 + c[1 - (1 - 2H)^3]} \overset{?}{>} \frac{1 + 4cH^2}{(1 + c)(1 + 8cH^3)}$$

Multiplying out and rearranging, one obtains

$$c(-64H^5 + 80H^4 - 16H^3 - 4H^2 + 4H)$$
$$+ c^2 (-32H^5 + 32H^4) \overset{?}{>} 2H - 4H^2 - 1 \qquad [3.6.1]$$

where the left-hand-side of the inequality is defined

$$g(H, c) = c[-64H^5 + 64H^4 + 16H^3 - 4H^2 + 4H]$$
$$+ c^2[32H^4(1 - H)] \qquad [3.6.2]$$

Since the second term of [3.6.2] will always be positive for the specified range of H, the first term is being examined.

Factoring out and rearranging

$$c[(64H^4 - 16H^3 + 4H)(1 - H)] \overset{?}{>} 0$$

where $c > 0$

$(1 - H) > 0$

Therefore,

$$4H(16H^3 - 4H^2 + 1) \overset{?}{>} 0$$

where $4H > 0$ and $4H^2(4H - 1) > -1$

where $4H^2 > 0$,

leaving $(4H - 1) \geq 0$ for $1/4 \leq H \leq 1/2$

and $(4H - 1) < 0$ for $0 < H < 1/4$

As H becomes smaller than 1/4 and tends to 0, $4H - 1$ will tend to -1 but it will never become -1. In fact, multiplied with $4H^2$, its absolute value will become even smaller than unity but algebraically it will always be greater than -1.

Therefore,

$$4H^2(4H - 1) > -1 \qquad [3.6.3]$$

is satisfied for $0 < H \leq 1/2$ and implies that the first term of the left-hand-side of inequality [3.6.1] is always positive.

Examining the second side of inequality [3.6.1] one notices that it has a maximum at $H = 1/4$. That is, if

$$f(H) = 2H - 4H^2 - 1$$

$$f'(H) = 2 - 8H$$

and for $f'(H) = 0$, $H = 1/4$

where

$$f(1/4) = -3/4$$

See also Figure III-3 for a graphical proof by which it is shown that $g(H, c)$ does not cross $f(H)$ for any value of H within the specified range $0 < H \leq 1/2$ and for $c > 0$.

$$g'(H,c) = 4[c(-80H^4 + 80H^3 - 12H^2 - 2H + 1) + c^2(-40H^4 + 32H^3)$$

As H goes to 0 and $c > 0$

$$\lim_{H\to 0} g'(H, c) \to 4c \qquad [3.6.4]$$

which implies a positive slope there.

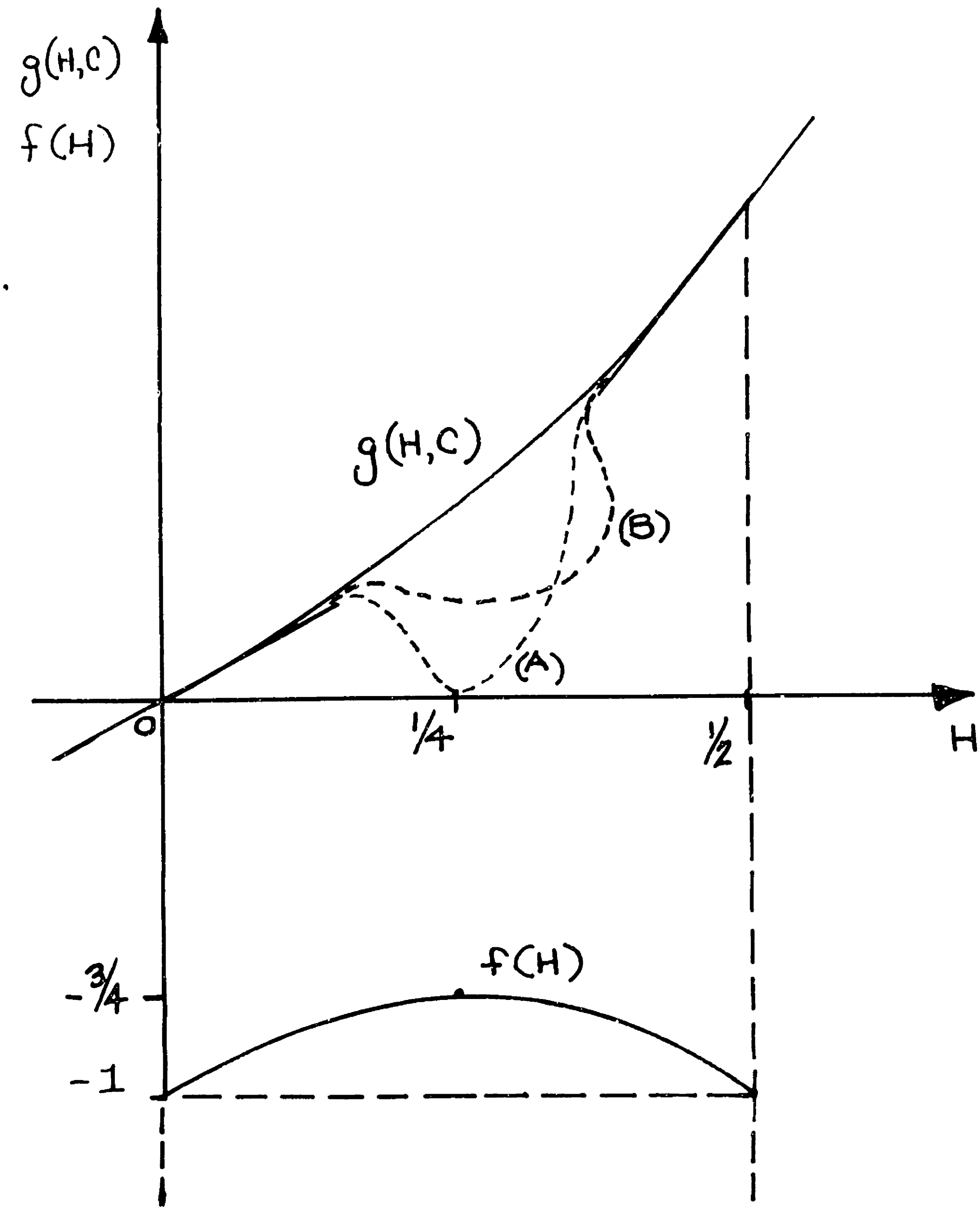


Figure III-3

A Graphical Proof That g(H, c) and f(H) of Inequality [3.6.1] have no common point.

At $H = 1/2$

$$g'(1/2, c) = 4(2c + 3/2c^2) \qquad [3.6.5]$$

which implies a more positive slope than that at $H = 0$. Thus the slopes drawn in Figure III-3 are consistent with equations [3.6.4] and [3.6.5]. The dotted lines (A) and (B) between $H = 0$ and $H = 1/2$ show two possible deviations of $g(H, c)$ from its expected smooth course of direction. However, as shown before $g(H, c)$ will always be positive. Therefore, it never meets $f(H)$ at its maximum value at $H = 1/4$

Q.E.D.

Therefore, according to [3.6] it can be concluded that arrangement B will yield a lower value of τ.

The bending stress σ_x in element B of Figure III-1 will depend on I_A, and I_B, because

$$\sigma_x = \frac{M(B/2)}{I} \qquad [3.7]$$

where M is the bending moment a $x = L/2$ which is constant throughout the cross-section there.

Comparing I_A with I_B one has

$$I_A \overset{?}{>} I_B \qquad [3.7.1]$$

substituting for their values from equations [3.3.1] and [3.3.2] one obtains

$$-12H^2 + 6H > 0 \qquad [3.7.2]$$

which is true for all values of H. When $H = 1/2$, the inequality yields

$$I_A = I_B \qquad [3.7.3]$$

which implies that the correct relationship between I_A and I_B is

$$I_A \gtrless I_B \qquad [3.8]$$

from equation [3.7] and [3.8] it is clear that

$$\sigma_B - \sigma_A \qquad [3.9]$$

which indicates that arrangement (A) will decrease the maximum principal compressive value of σ_2 which depends only on σ_x (in element B in Figure III-1) more rapidly than arrangement (B).

3. A Comparison between Two Stiffener Arrangements--Case II

As a second possible arrangement of stiffeners for cross-section KK at x = L/4 and the center line cross-section, for the reduction of $\tau_{max.}$ in the former and σ_x in the latter, consider the arrangements (A) and (B) of Figure III-4.

Following the same procedure as for Case I, the first moments, Q, are calculated to be

$$Q_A = \frac{Bt}{8} [B + 4cH(B - ct)] \qquad [3.10.1]$$

$$Q_B = \frac{B^2t}{8} (1 + 2H^2c) \qquad [3.10.2]$$

and the moments of inertia I

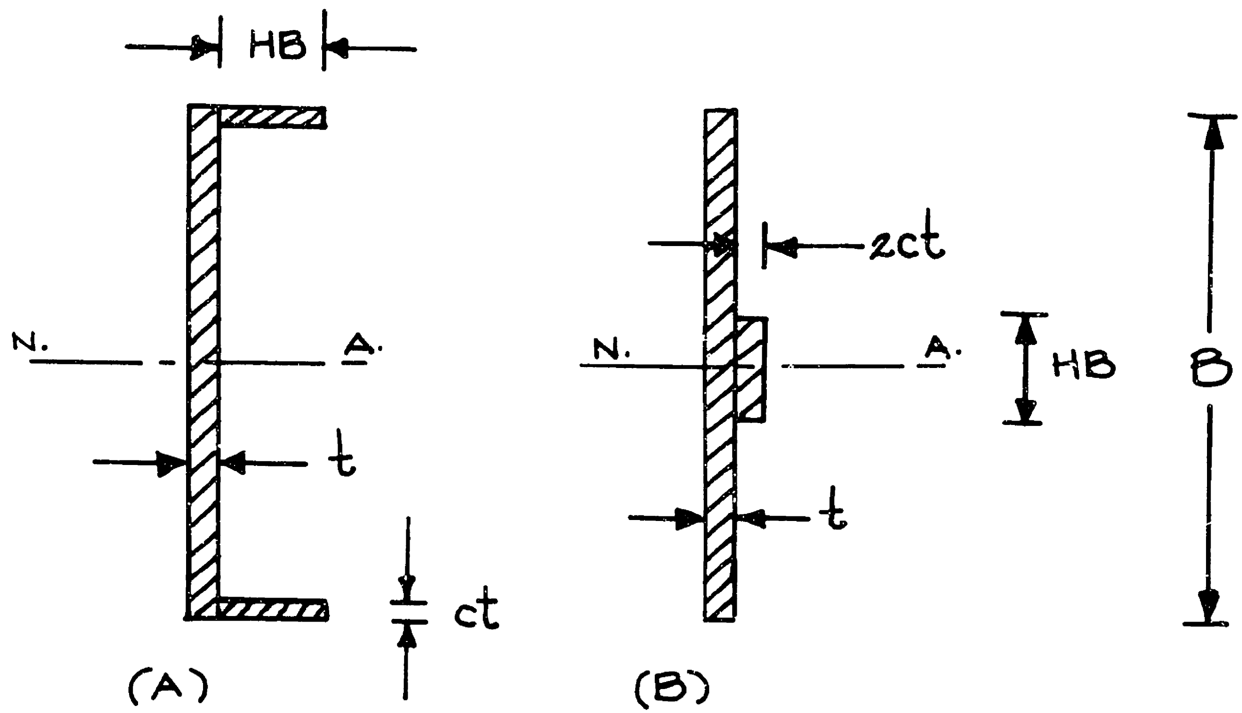


Figure III-4

Comparison between Two Stiffener Arrangements--Case II

$$I_A = \frac{B}{12} [tB^2 + HB^3 - H(B - 2ct)^3] \qquad [3.11.1]$$

$$I_B = \frac{B^3 t}{12} (1 + 2cH^3) \qquad [3.11.2]$$

From equations [3.10] and [3.11] and from the expression for τ, equation [3.1], one obtains

$$\tau_A = \frac{3}{2} \frac{V}{t^2} \frac{[\alpha + 4cH(\alpha - c)]}{\{\alpha^2 + \alpha^3 H[1 - [1 - 2c/\alpha)^3]\}} \qquad [3.12.1]$$

and

$$\tau_B = \frac{3}{2} \frac{V}{t^2} \frac{(1 + 2H^2 c)}{\alpha(1 + 2cH^3)(1 + 2c)} \qquad [3.12.2]$$

where $\alpha = B/t$.

To compare τ_A and τ_B, one must define reasonable ranges for c, H, and α.

According to the "Design Data for Tee Stiffeners" by the U. S. Navy Bureau of Ships,[19] the following ranges of stiffener sizes have been used in the past:

$$10 \leq \alpha \leq 60 \qquad [3.13.1]$$

$$10 \leq \frac{HB}{ct} \leq 30 \qquad [3.13.2]$$

where $$0 < H \leq 1 \qquad [3.13.3]$$

and $$c = 1, 2, 3. \qquad [3.13.4]$$

Non dimensionalizing τ_A and τ_B for a constant plate height B, one obtains (from equations [3.12]),

$$\bar{\tau}_A = \frac{\tau_A}{\frac{3}{2}\frac{V}{B^2}} = \frac{\alpha(1 + 4cH) - 4c^2H}{1 + H\alpha[1 - (1 - 2c/\alpha)^3]} \qquad [3.14.1]$$

and

$$\bar{\tau}_B = \frac{\tau_B}{\frac{3}{2}\frac{V}{B^2}} = \frac{\alpha(1 + 2H^2c)}{(1 + 2cH^3)(1 + 2c)} \qquad [3.14.2]$$

The most efficient way to compare $\bar{\tau}_A$ and $\bar{\tau}_B$ for the values of α, HB/ct, H, and c specified in the ranges of equations [3.13] is to write a small computer program. In doing so, it is more efficient if at the same time one compares $\bar{\tau}_A$ and $\bar{\tau}_B$ of Case II he also compares the shear stress τ_B of the best arrangement of Case I, which when non dimensionalized becomes

$$\bar{\tau}_C = \bar{\tau}_{B_I} = \frac{\alpha(1 + 4cH^2)}{(1 + c)(1 + 8cH^3)} \qquad [3.14.3]$$

where $\bar{\tau}_{B_I}$ is the $\bar{\tau}_B$ of the first case and is renamed here for purposes of clarification. The results of comparison among $\bar{\tau}_A$, $\bar{\tau}_B$, $\bar{\tau}_C$ is shown in Figures III-5 through Figure III-10 for α = 10, 20, 30, 40, 50, 60, H = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, and c = 1, 2, 3.

From these it can be observed that

$$\bar{\tau}_B < \bar{\tau}_C < \bar{\tau}_A \qquad [3.15]$$

for all c's and for the same α and H. (see section 4)

From Case I arrangement (A) was proven to be more desirable than (B) for the center line cross-section. (see Figure III-1)

Presently, in Case II, due to the results of Case I, one may anticipate that arrangement A is the most appropriate because the material is added at the upper and lower edges of the plate where the maximum principal stresses exist. This anticipated result implies that $I_A > I_B$.

Therefore, substituting for I_A and I_B from equation [3.11] into the above inequality, simplifying,

$$tB^2 + HB^3 - H(B - 2ct)^3 \overset{?}{>} B^2t(1 + 2cH^3) \qquad [3.16]$$

which by rearranging and substituting for $\alpha = B/t$ becomes

$$\alpha^2(3 - H^2) - 6\alpha c + 4c^2 \overset{?}{>} 0 \qquad [3.16.1]$$

For H = 1, inequality [3.16.1] becomes

$$\alpha^2 - 3\alpha c + 2c^2 > 0 \qquad [3.16.2]$$

or

$$(\alpha - 2c)(\alpha - c) > 0 \qquad [3.16.3]$$

Since $10 \leq \alpha \leq 60$, for $\alpha = 10$, [3.16.3] yields

$144 > 0$ for $c = 1$

$96 > 0$ for $c = 2$

and $56 > 0$ for $c = 3$.

Clearly, for any other value of $H < 1$, the first term of the left-hand-side will be more positive.

Therefore, it has been shown that $I_A > I_B$ is true for all values of α, H, and c that satisfy conditions [3.13] from which

$$\sigma_A < \sigma_B \qquad [3.17]$$

implying that arrangement (A) is better than (B) at the center line cross-section of the centrally loaded plate of Figure III-1.

To compare σ_A and σ_B of the present Case with the lowest σ_A of Case I, the small computer program which calculated $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ is employed again, which gives the results of Figures III-11 through III-16 as discussed in the next section.

For the purpose of clarification, σ_A of Case I has been renamed σ_D, while σ_C is not being compared because $\sigma_C < \sigma_D$.

The expression for σ_A, σ_B, and σ_D are

$$\bar{\sigma}_A = \frac{A}{6M/B^3} = \frac{1}{1/\alpha + H[1 - (1 - 2c/\alpha)^3]} \qquad [3.18.1]$$

$$\bar{\sigma}_B = \frac{B}{6M/B^3} = \frac{\alpha}{(1 + 2cH^3)} \qquad [3.18.2]$$

$$\bar{\sigma}_D = \frac{D}{6M/B^3} = \frac{\alpha}{1 + c[1 - (1 - 2H)^3]} \qquad [3.18.3]$$

4. <u>Results from Comparing Arrangements A, B, and C</u>

Figures III-5 and III-6 apply to a stiffener the thickness of which is equal to that of the plate (that is, c = 1). As the α ratio increases, the difference among

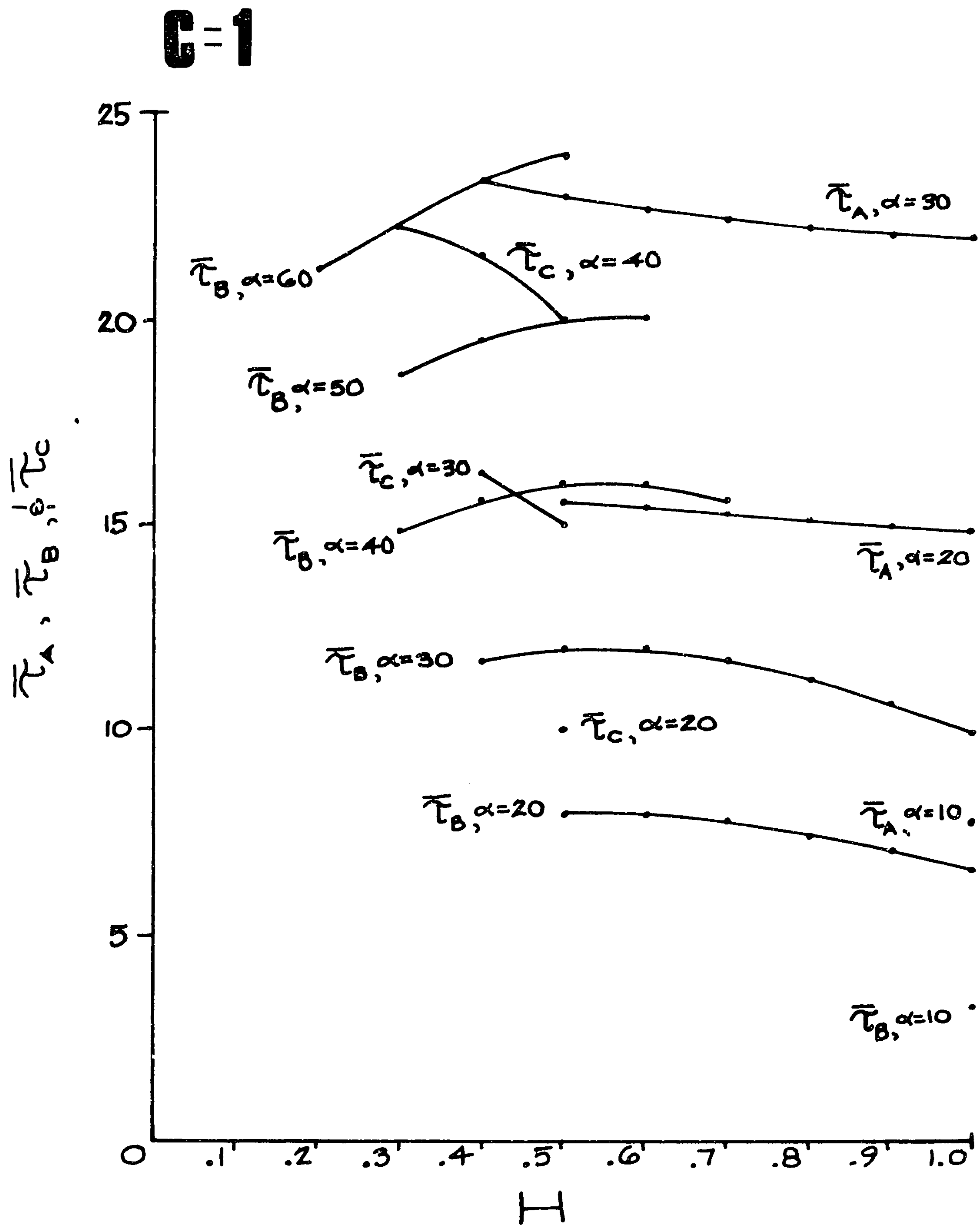

C=1
25
20
15
10
5
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\bar{\tau}_A, \alpha=30$
$\bar{\tau}_C, \alpha=40$
$\bar{\tau}_B, \alpha=60$
$\bar{\tau}_B, \alpha=50$
$\bar{\tau}_C, \alpha=30$
$\bar{\tau}_B, \alpha=40$
$\bar{\tau}_A, \alpha=20$
$\bar{\tau}_B, \alpha=30$
$\bar{\tau}_C, \alpha=20$
$\bar{\tau}_B, \alpha=20$
$\bar{\tau}_A, \alpha=10$
$\bar{\tau}_B, \alpha=10$
$\bar{\tau}_A, \bar{\tau}_B, \bar{\tau}_C$
H

Figure III-5

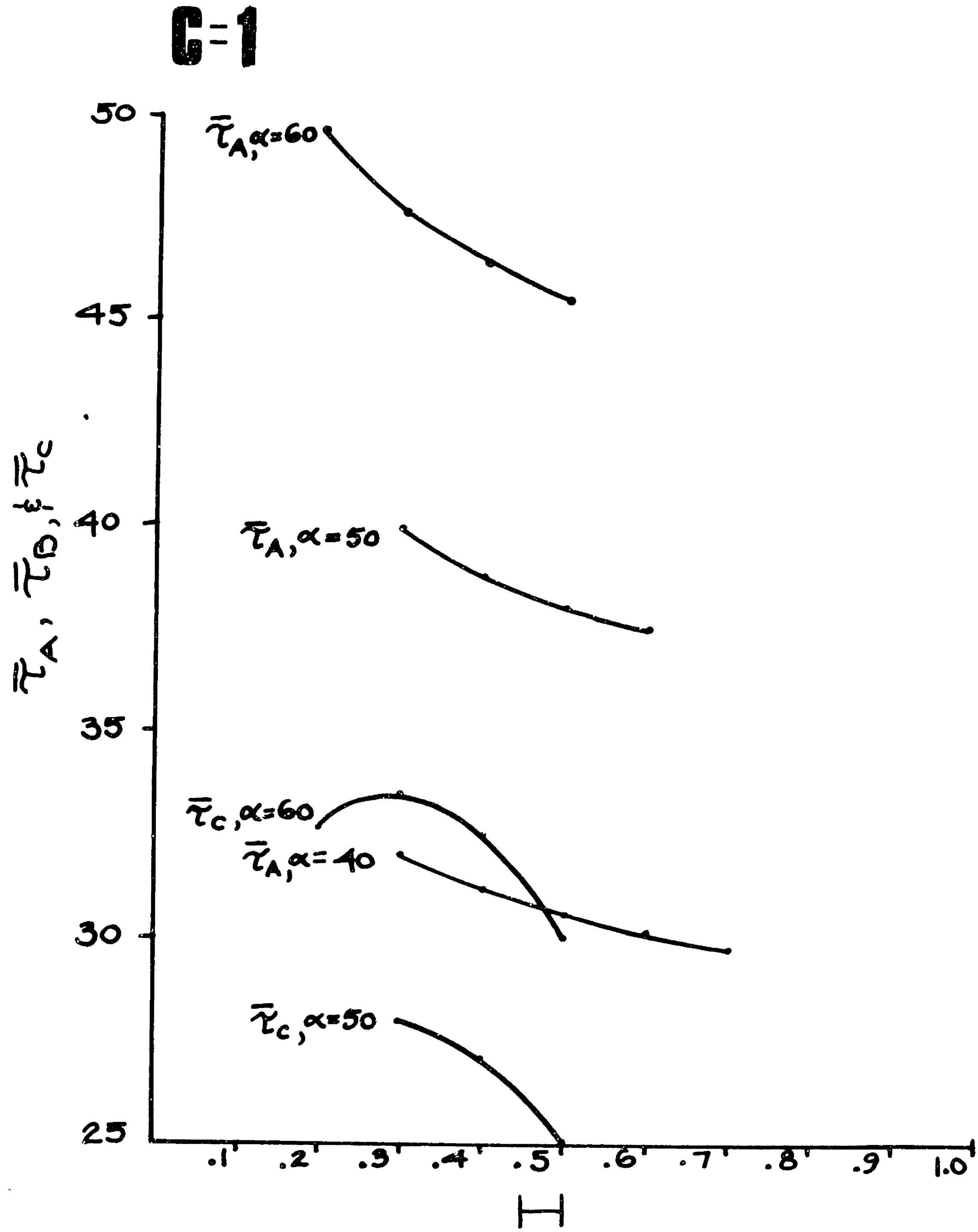

C=1
50
45
40
35
30
25
$\bar{\tau}_A, \alpha=60$
$\bar{\tau}_A, \alpha=50$
$\bar{\tau}_C, \alpha=60$
$\bar{\tau}_A, \alpha=40$
$\bar{\tau}_C, \alpha=50$
$\bar{\tau}_A, \bar{\tau}_B, \bar{\tau}_C$
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
H

Figure III-6

$\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ increases, while at the same time, the acceptable values of H decrease.

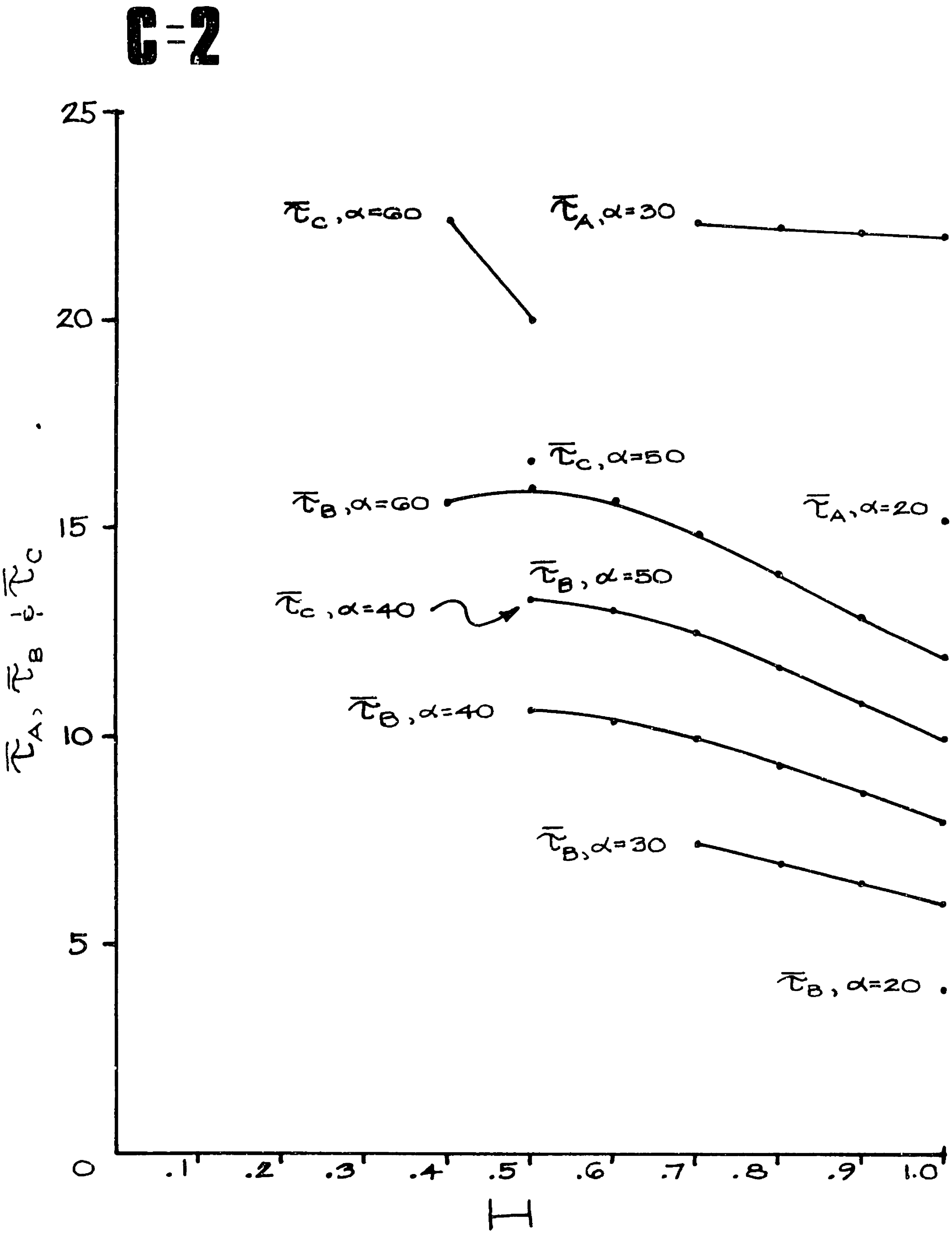
This can be interpreted as follows: The ratio α is proportional to the initial weight of the plate so that if one compares $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ for the same α and the same H, he compares the maximum nondimensional shear stresses (at the N.A. of the cross-section KK, Figure III-1) for three different stiffener arrangements of the same weight. Clearly, for any comparison among A, B, and C with a common α and H, the result is always that of the simultaneous inequalities [3.15]. Furthermore, one can see the variation of any $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ with respect to H for each α.

For the (A) arrangement, $\bar{\tau}_A$ drops with the increase of H, and the rate of dropping increases as α increases.

For the (B) arrangement and for high values of α ($\alpha = 60$, $\alpha = 50$), $\bar{\tau}_B$ increases as H increases, while for medium values of α ($\alpha = 40$, $\alpha = 30$), it exhibits a convex behavior with H increasing.

For the (C) arrangement (that is, arrangement B of Case I), $\bar{\tau}_C$ decreases with increasing H, except for $\alpha = 60$ where it exhibits convex behavior.

Figures III-7 and III-8 apply to the case where the thickness of the stiffener is double the thickness of the plate, (that is, $c = 2$). The values of $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$, for the same H, are far more spread apart than those obtained when $c = 1$.



NOTE: ⤳ ARROW INDICATES THAT ONE POINT OF $\bar{\tau}_B$ COINCIDES WITH $\bar{\tau}_C$.

Figure III-7

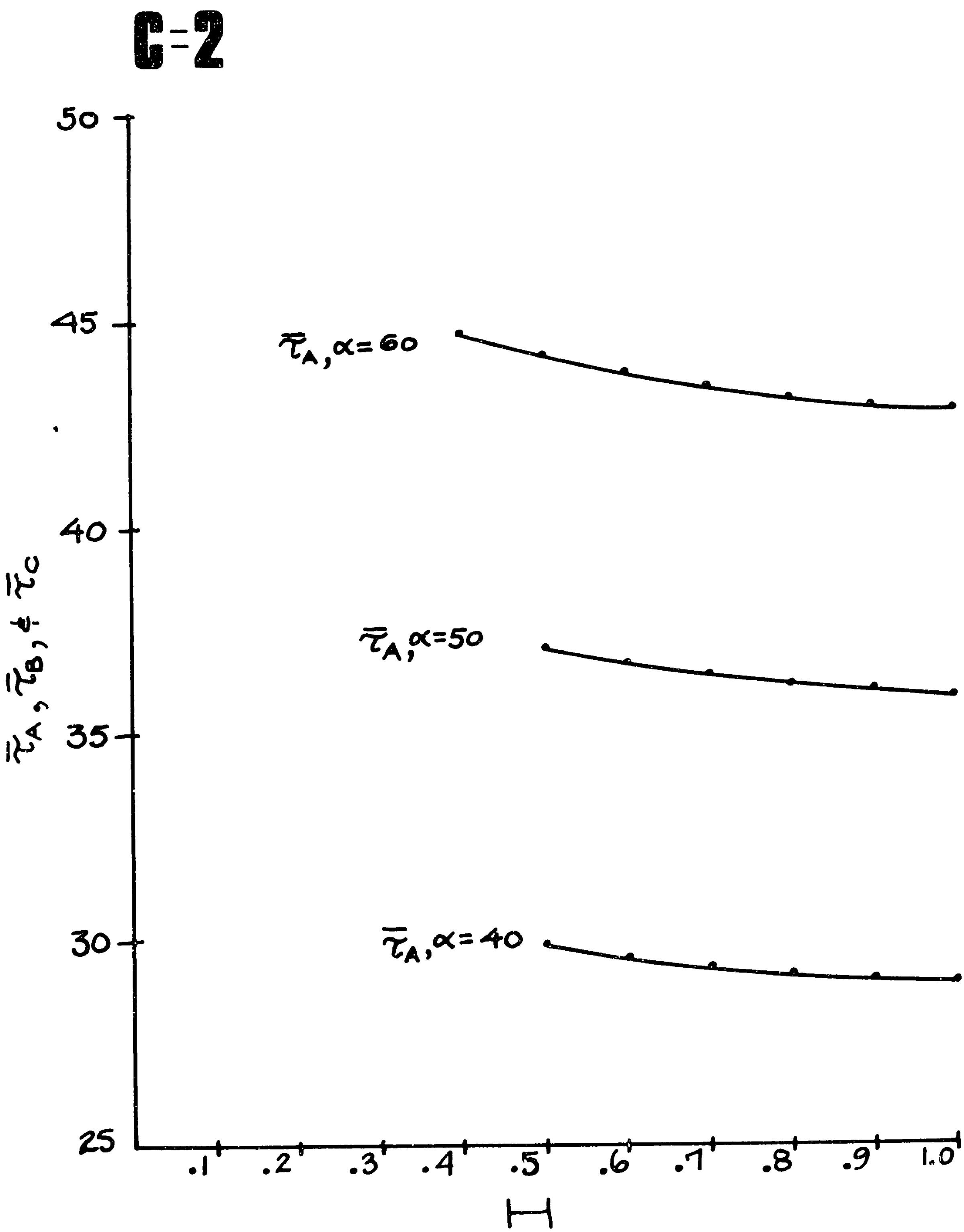

C=2
50
45
40
35
30
25
$\bar{\tau}_A, \bar{\tau}_B, \& \bar{\tau}_C$
$\bar{\tau}_A, \alpha=60$
$\bar{\tau}_A, \alpha=50$
$\bar{\tau}_A, \alpha=40$
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
H

Figure III-8

In fact, $\bar{\tau}_A$ for $\alpha = 20$ and $H = 1$ equals $\bar{\tau}_B$ for $\alpha = 60$ and $H = 0.67$. This implies that the same stress level can be attained if one uses arrangement B for $\alpha = 60$, which is lighter than arrangement A for $\alpha = 20$ by a factor of 4.07.

Furthermore, from Figures III-7 and III-8 one notices that the condition of inequality [3.13.2] is not satisfied for small α. Notice that $\bar{\tau}_C$ exists only for $\alpha = 50$, 60 and that there are no $\alpha = 10$ ratios for $\bar{\tau}_A$, $\bar{\tau}_B$.

Aside from the above differences, the behavior of stress with respect to H for each arrangement and at a particular α is the same as that for $c = 1$.

Finally, from Figures III-9 and III-10, one observes an even greater discrepancy between $\bar{\tau}_A$ and $\bar{\tau}_B$ in favor of arrangement B. Arrangement C has only one value of $\bar{\tau}_C$ for $\alpha = 60$ and is the second most favorable.

Figures III-11 through III-16 show that for any H value and for the same α, for the same weight,

$$\sigma_A < \sigma_D < \sigma_B \qquad [3.19]$$

It may be seen that for $c = 1, 2, 3$, $\bar{\sigma}_A$, $\bar{\sigma}_B$, and $\bar{\sigma}_D$ exhibit the same behavior as expected. However, as c increases, $\bar{\sigma}_A$, $\bar{\sigma}_B$, and $\bar{\sigma}_D$ come more closely together and towards $H = 1$, and smaller values of stress.

The behavior of the bending stresses $\bar{\sigma}_A$, $\bar{\sigma}_B$, and $\bar{\sigma}_D$ resembles that of the shear stresses $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$.

C=3

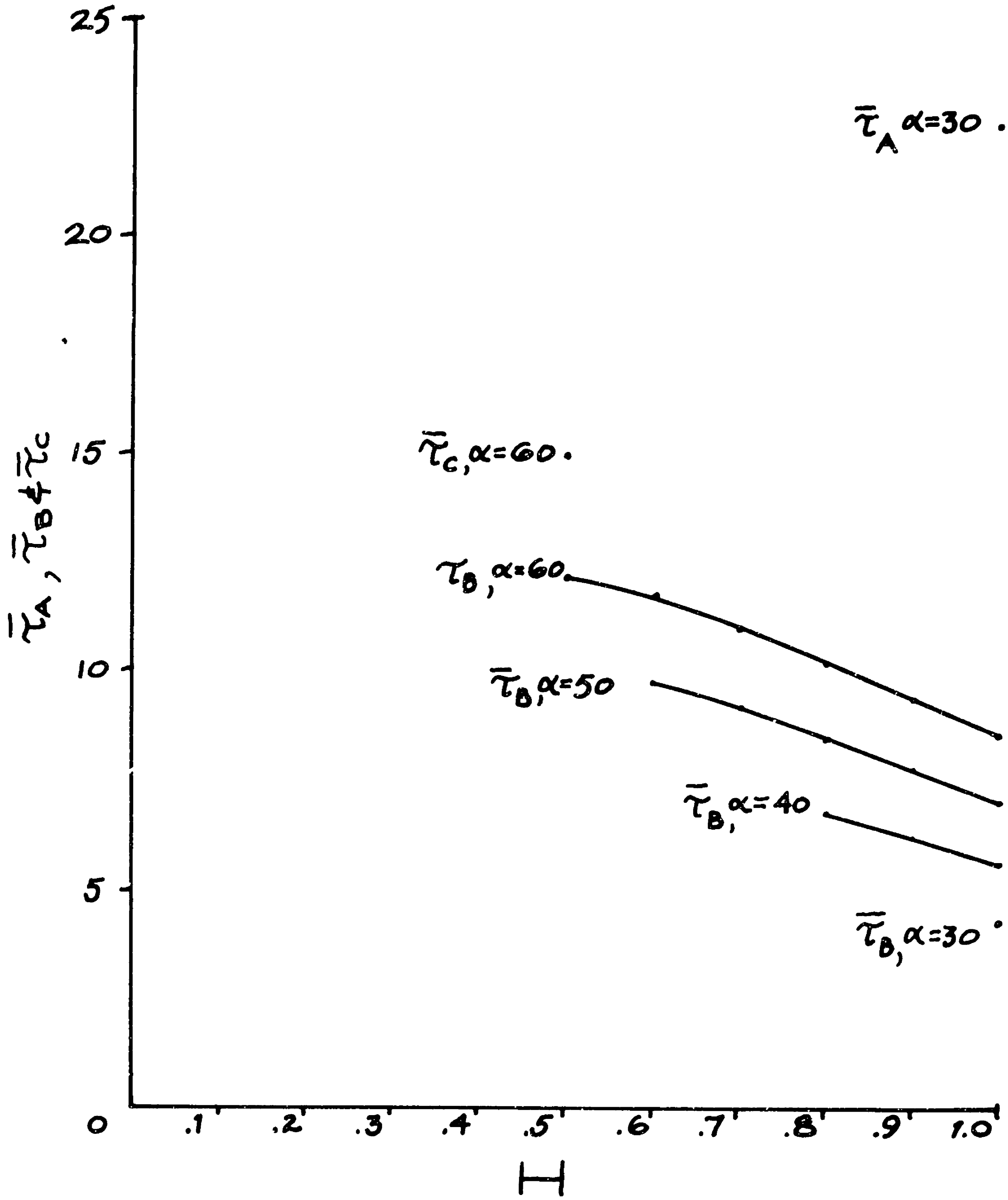


Figure III-9

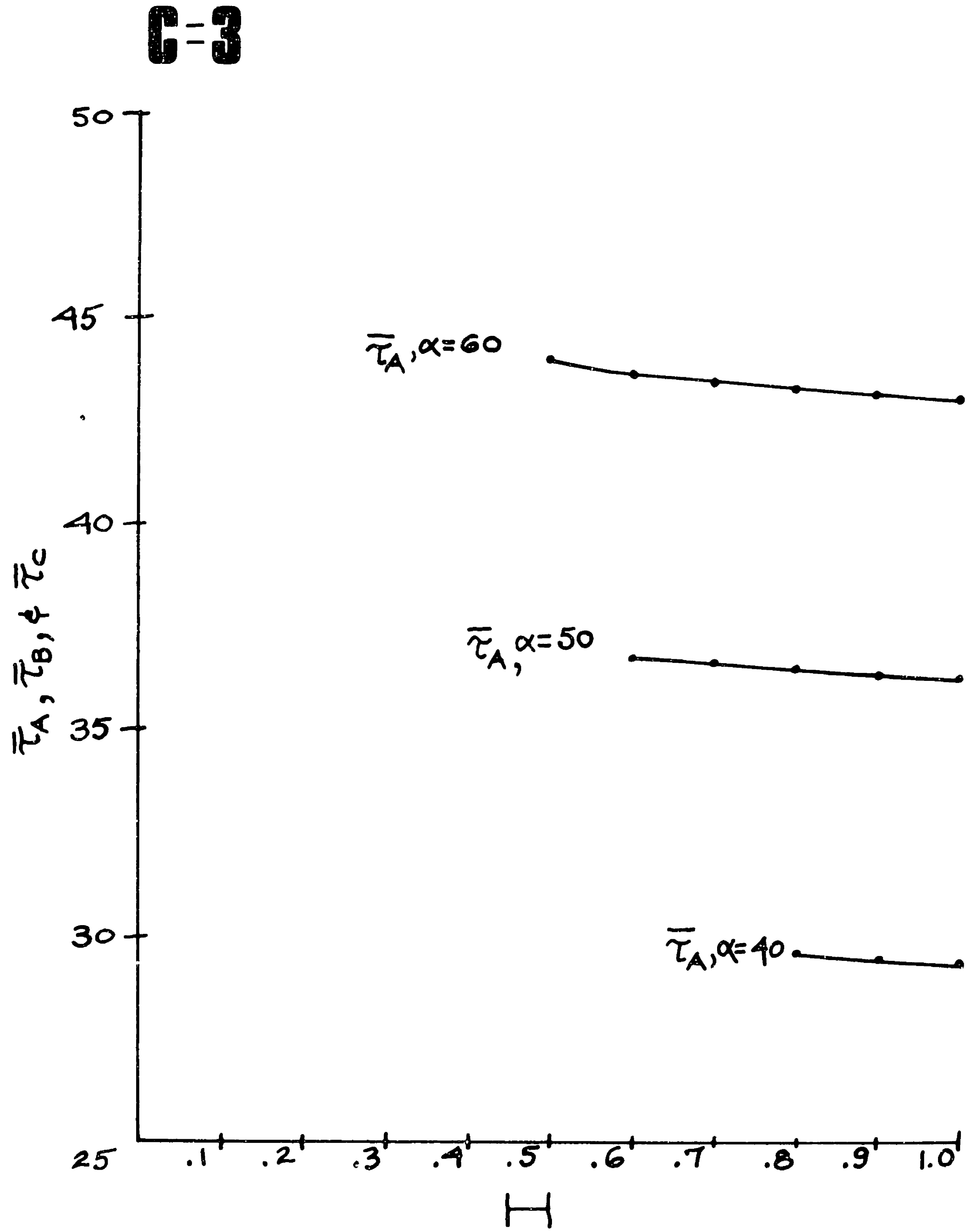


Figure III-10

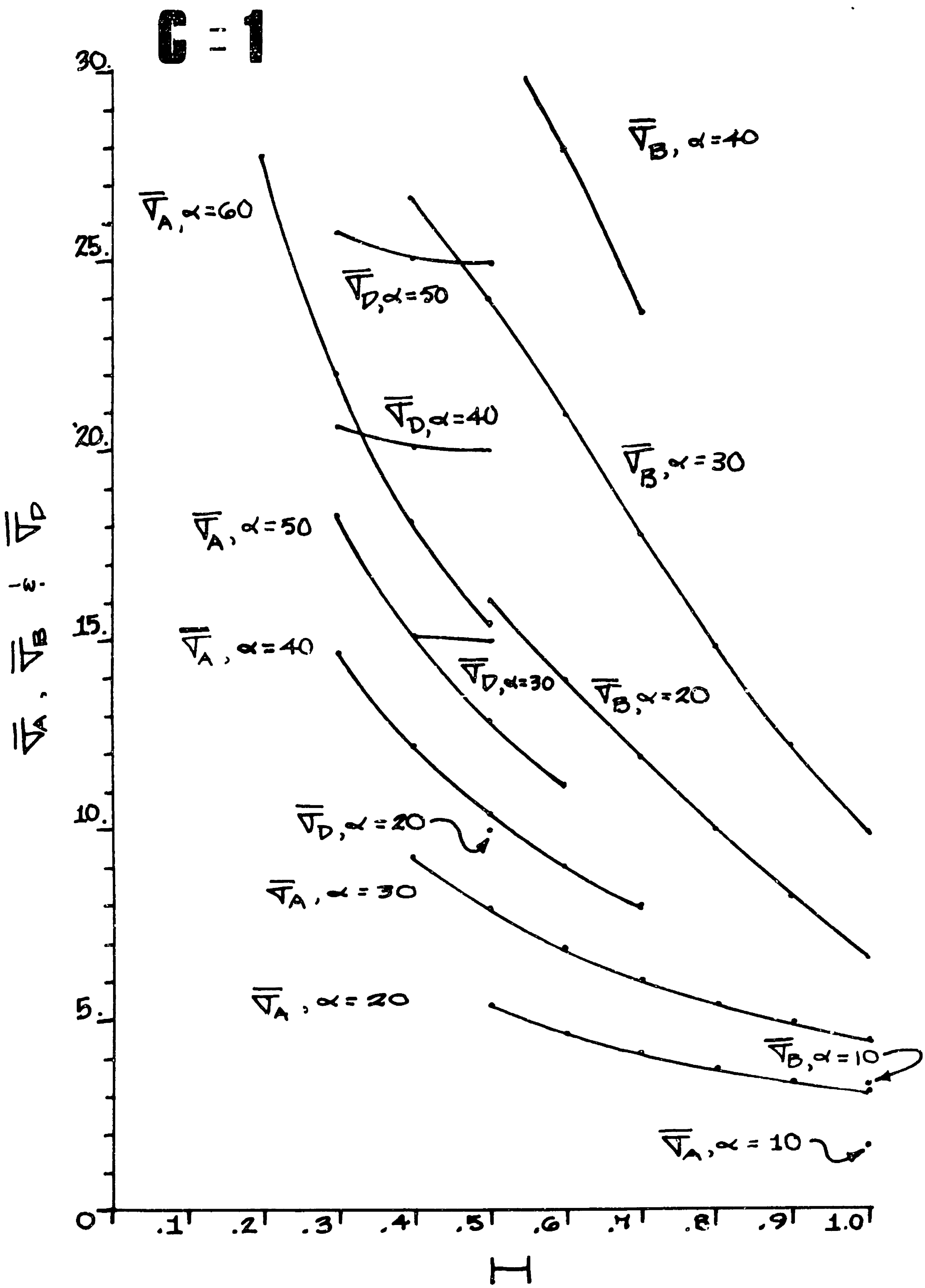


Figure III-11

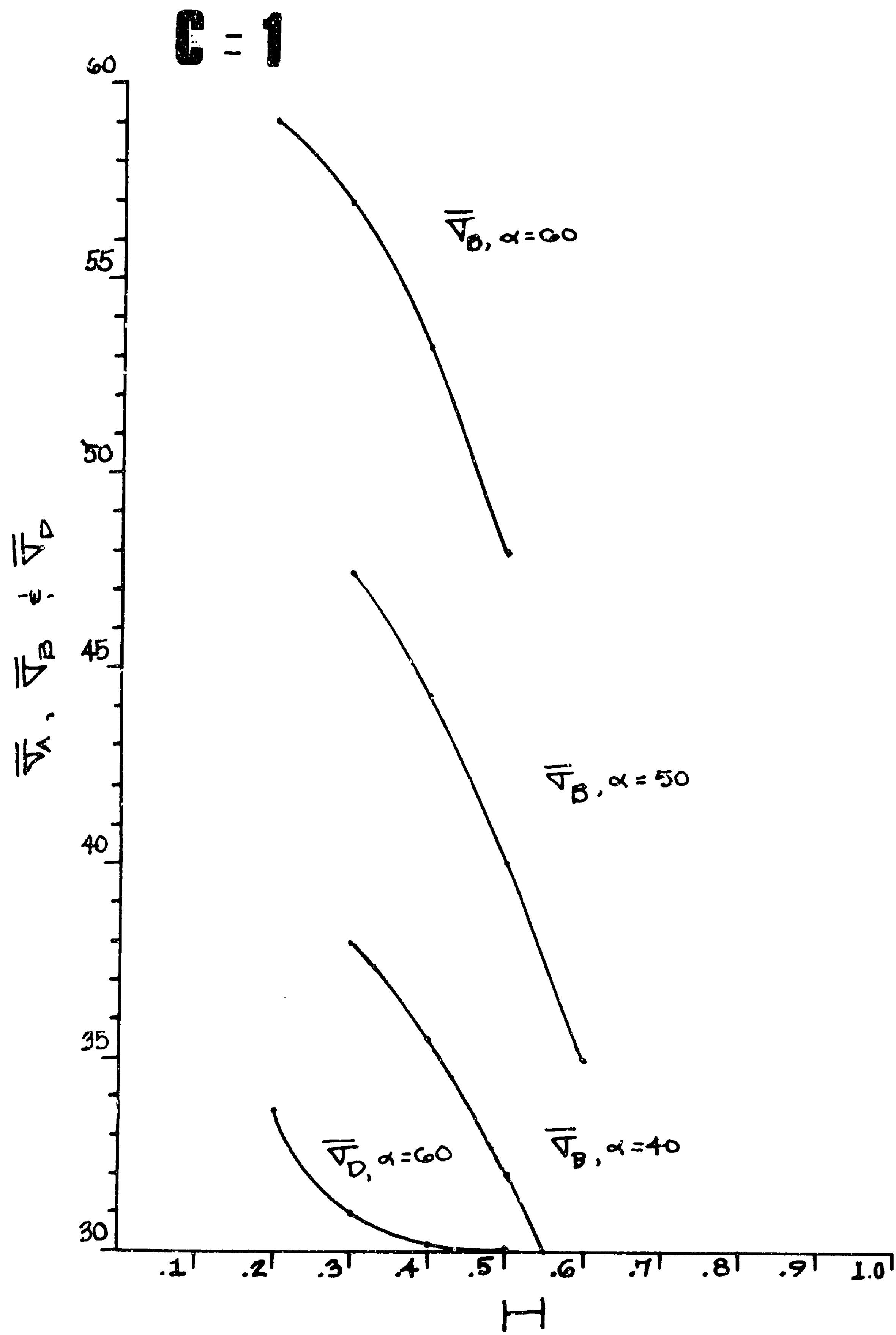


Figure III-12

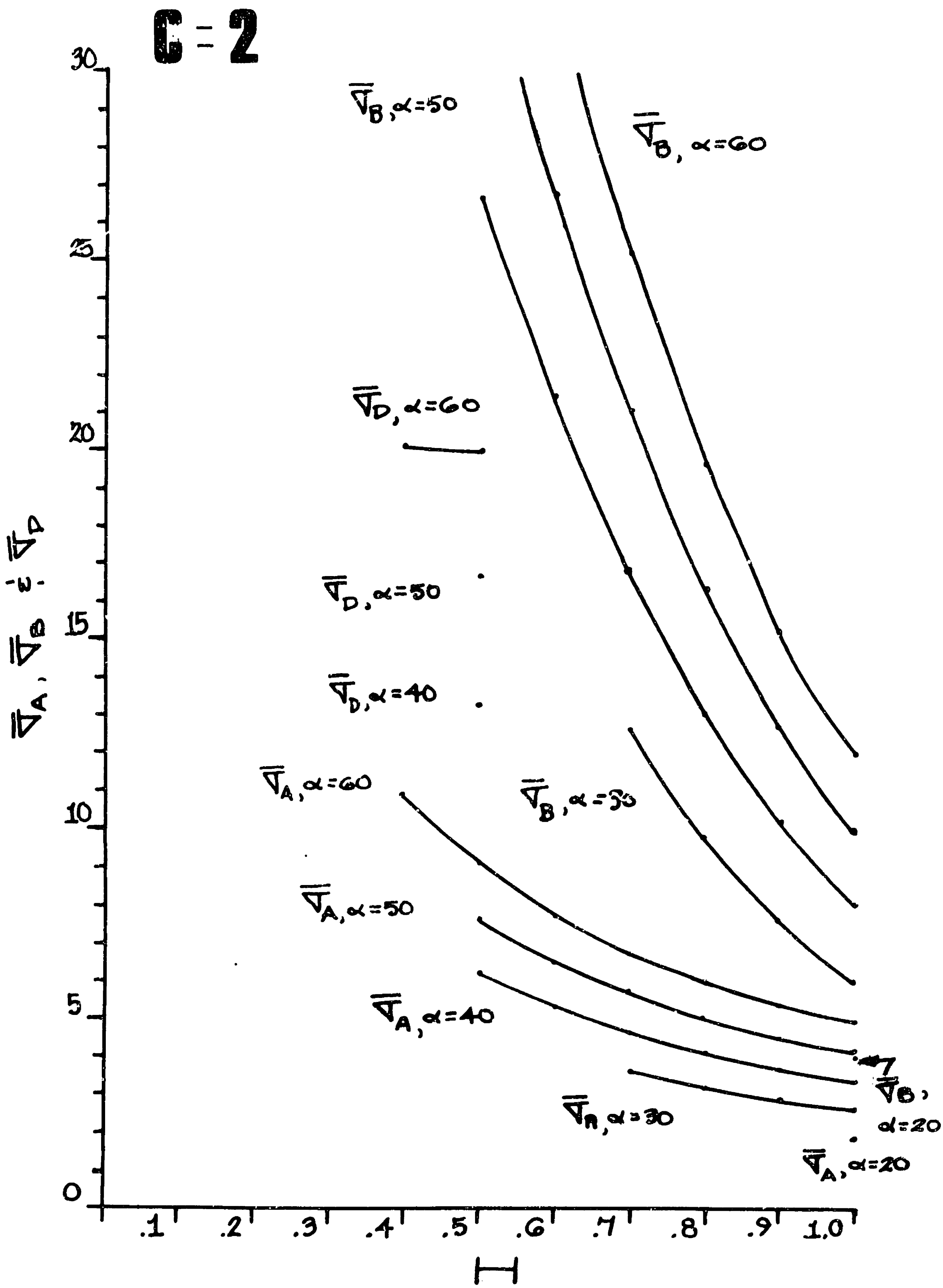


Figure III-13

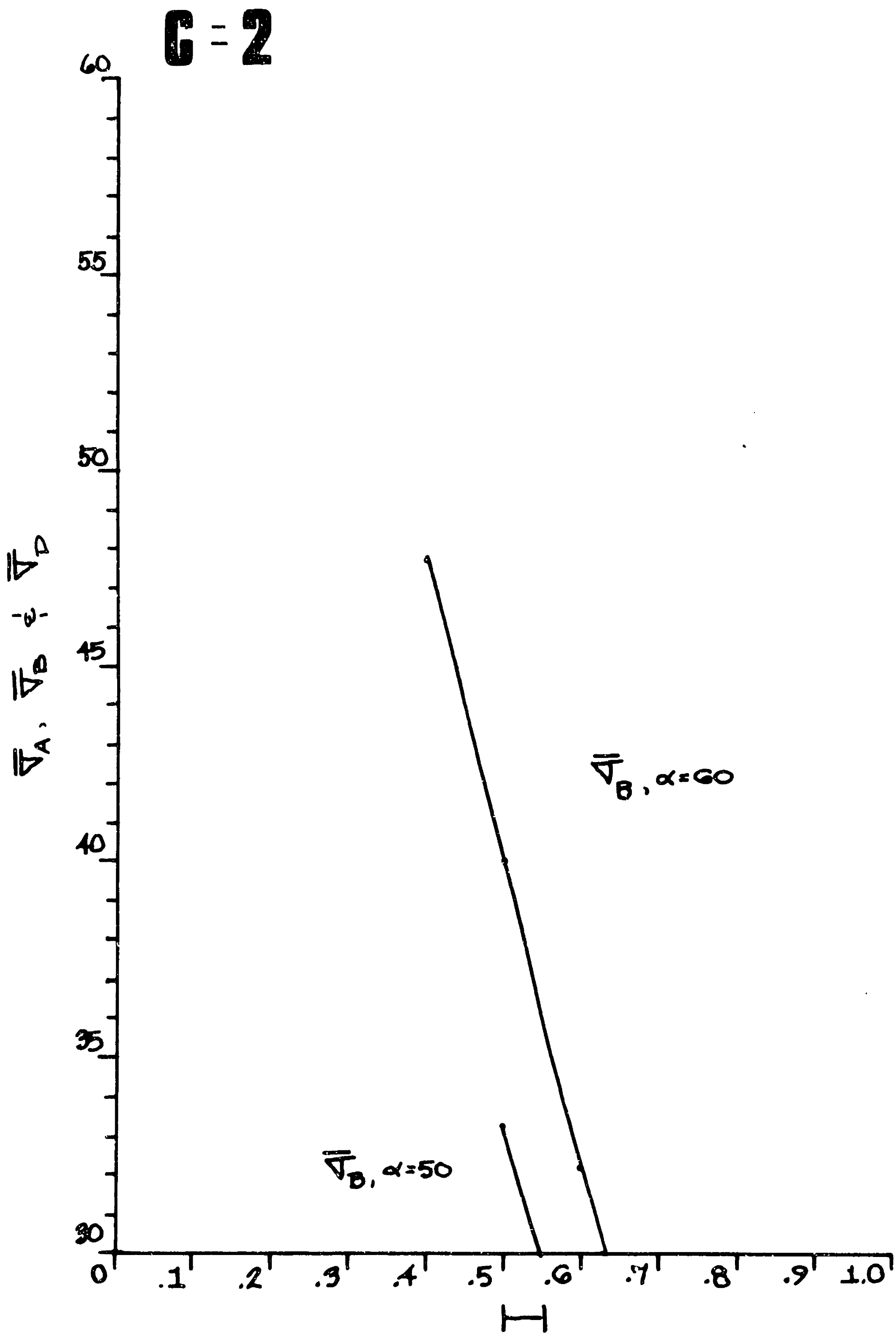


Figure III-14

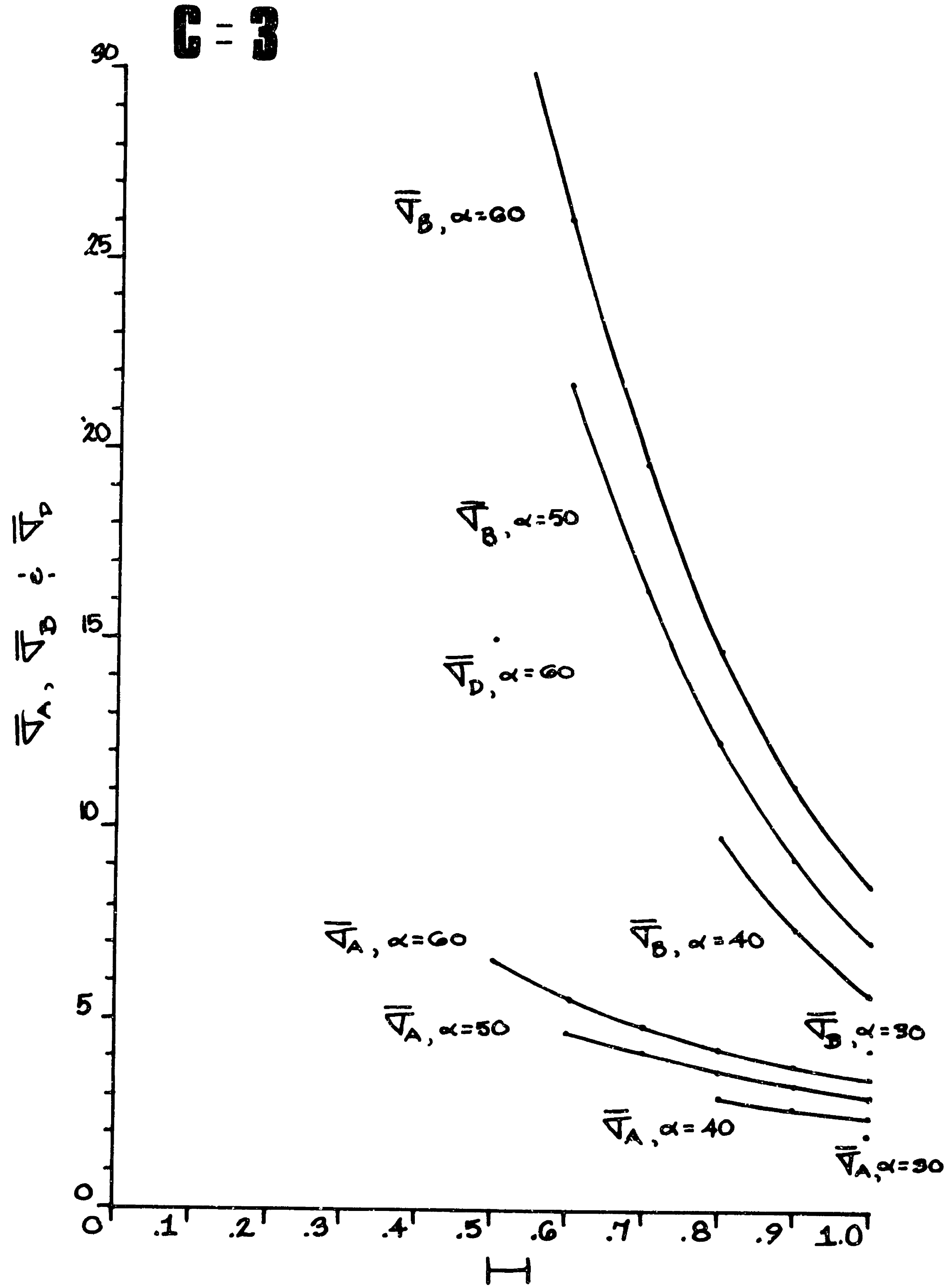


Figure III-15

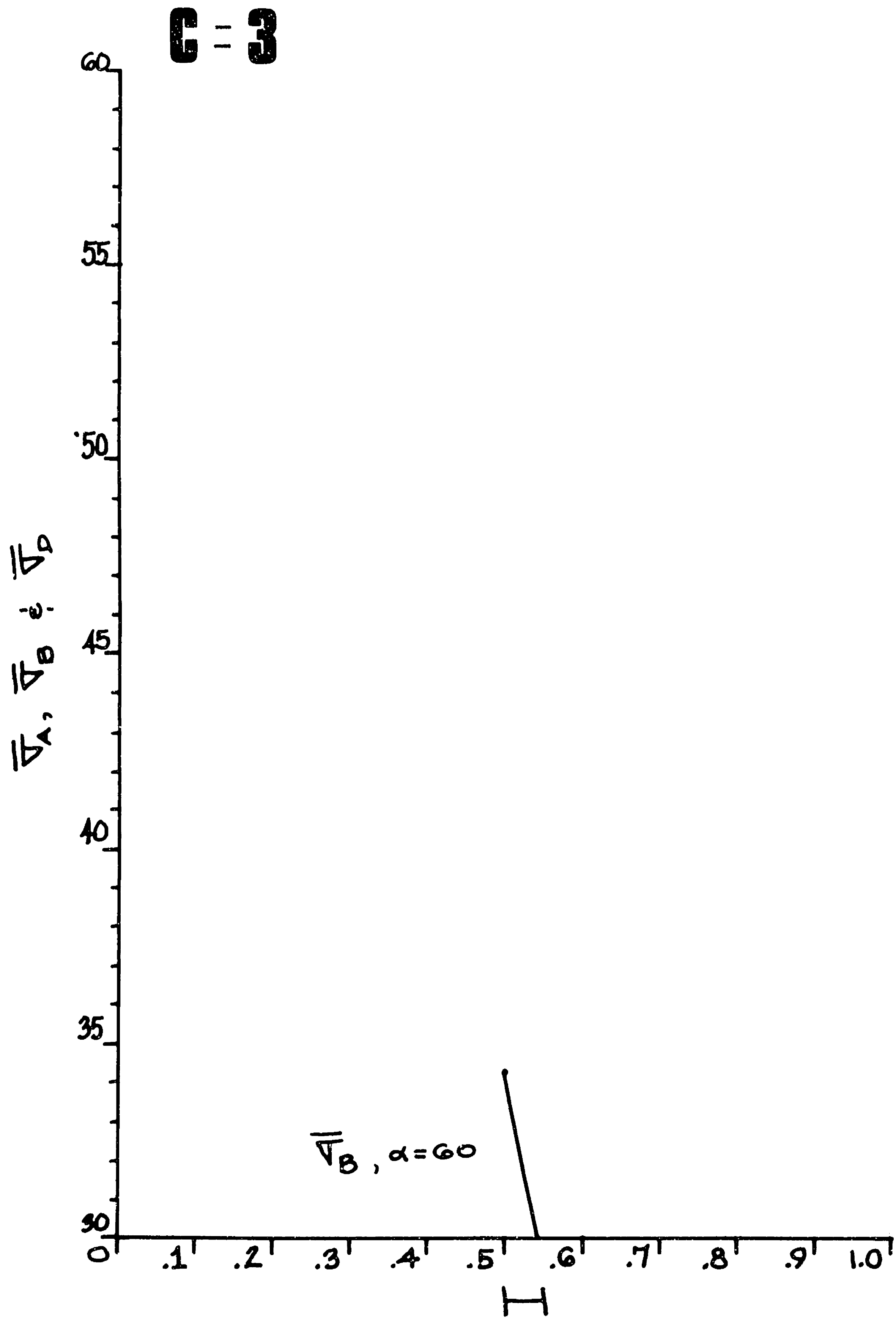


Figure III-16

Notice that for a particular α, all the $\bar{\sigma}$'s decrease with increasing H.

To summarize, in this part four different stiffener arrangements have been compared in two's. For the best three arrangements, the expressions of the bending and shear stresses for a reasonable range of values of plate height over thickness, α, and flange width over flange thickness, HB/ct, where $0 < HB \leq B$ and ct = t, 2t, 3t have been plotted against H, α, and c.

The results suggest that the maximum stress reduction for the least weight of reinforcing material is achieved when the stiffeners are added (a) vertical to the plate at the location of maximum principal compressive stress due to bending stress and (b) horizontal to the plate at the location of the maximum principal compressive stress due to shear stress.

Two more sets of curves are used in Part IV to explicitly show the behavior of all the shear stresses $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ and the bending stresses $\bar{\sigma}_A$, $\bar{\sigma}_B$, and $\bar{\sigma}_D$ as a function of a nondimensionalized weight expression.

IV RESULTS

1. Energy Equivalent Boundary Loads

Example. Let us consider the centrally loaded plate of Figure IV-1 for which

$$t = 0.4 \text{ in.}$$

$$E = 30 \quad 10^6 \text{ in.}$$

$$\nu = .25$$

Taking advantage of the existing symmetry of the load and of the boundary conditions, one needs to consider only half the plate to calculate (a) the displacements $\{q\}$, (b) the average stresses $\{\sigma\}^s_{average}$ and (c) the principal compressive stress trajectories.

Since the aspect ratio of the half plate is 2.5:1, "250" is an appropriate number of elements to be used for the modeling of the plate. These "250" elements correspond to "286" nodal points (or control stations), of which the four at the corners of the total structure as well as the two transversed by the horizontal N.A. of the plate on its vertical sides are shown in Figure IV-2.

Since the nodes for the entire structure have thus been properly numbered, the next step is to restrain nodes No. 1 through No. 11 and No. 276 through No. 286, in the x-direction due to symmetry, and node No. 6 in the y-direction, also. It must be mentioned that the restraining in the x-direction of all nodal points of the left and right-hand-sides of the left half plate as shown in Figure IV-2 contradicts with the anticipated elongation of the plate along the x-axis. This anticipated elongation on the x-axis, however, can be neglected since the deflection in the y-direction will be infinitesimal (Infinitesimal Theory).

Having thus properly defined the boundary conditions of the problem, the next step is to simulate the parabolically distributed shear load equal to P/2, as shown in Figure IV-2.

Knowing that the total applied load on that edge will be 60 lbs., all one needs to do is to find the equation of the parabola as a function of the height of the plate.

For this purpose, define (a) the τ, y axes, as shown in Figure IV-3, and (b) the equation of the parabola in its general form,

$$\tau = ay^2 + by + c \qquad [4.1]$$

for which,

(1) at $y = 0$, $\tau = 0$ gives $c = 0$ [4.1.1]

(2) at $y = 144$, $\tau = 0$ gives $b/a = -144$ [4.1.2]

Clearly, the area under the curve in Figure IV-3, the total force $F = P/2 = 60$ lbs.

Therefore,

$$F = t \int_{y_1=0}^{y_2=144} \tau dy$$

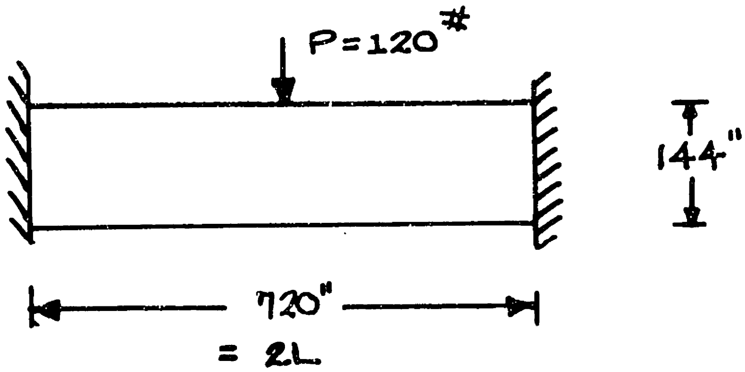


Figure IV-1

A Centrally-Loaded Plate

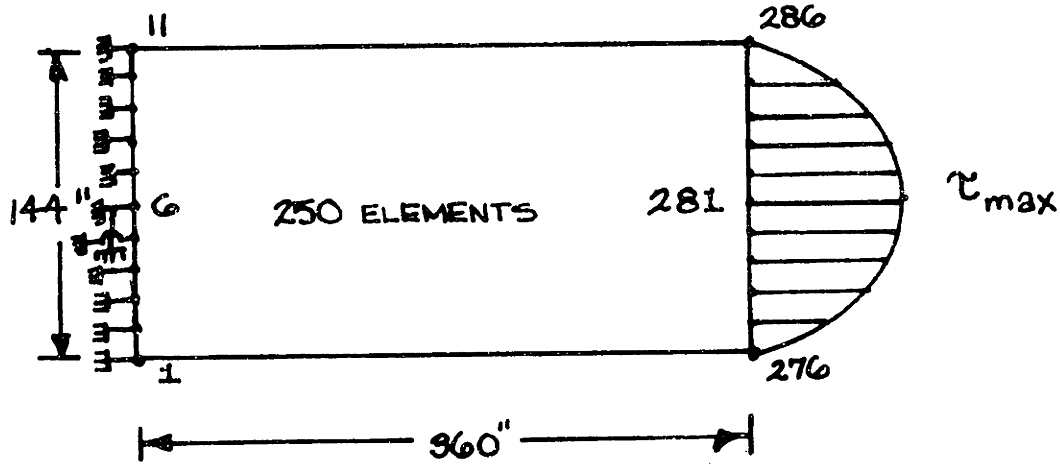


Figure IV-2

Modeling of the Left-Half Plate

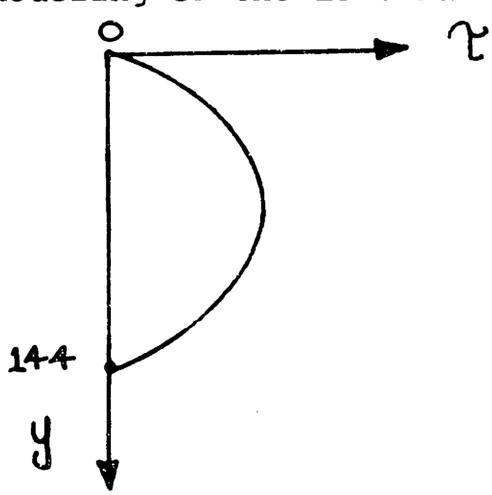


Figure IV-3

Set up of Origin for the τ vs y Distribution or

$$\int_0^{144} (ay^2 + by)dy = \frac{60}{0.4} = 150 \text{ lbs/in} \qquad [4.2.1]$$

Integrating, one gets:

$$\left. \frac{ay^3}{3} + \frac{by^2}{2} \right|_0^{144} = -\frac{(144)^3 a}{6} = 150 \qquad [4.2.2]$$

Therefore,

$$a = -\frac{900}{(144)^3} \qquad [4.3.1]$$

and

$$b = -144a = \frac{900}{(144)^2} \qquad [4.3.2]$$

Therefore equation [4.1] becomes

$$\tau = \frac{6P}{tB^2}\left[y\left(1 - \frac{y}{B}\right)\right] \qquad [4.4]$$

Solving for the y values corresponding to the nodal points of the structure at the edge of interest, one obtains the shear stress τ values in lbs/in^2, which are tabulated in Table IV-1.

Since, however, the computer program accepts the values of τ in units of (lbs/in), one must multiply column No. 3 by $t = 0.4$ in. to obtain the shear stress τ in column No. 4.

Note that the values of column No. 4 are read as described in Section 3 of Appendix II-A.

COLUMN #	1.	2.	3.	4.
	NODE	y in.	τ #/in.^{2}	τ #/in.
	286	0	0.0	0.0
	285	14.4	−.5625	−.225
	284	28.8	−1.0000	−.400
	283	43.2	−1.3125	−.525
	282	57.6	−1.5000	−.600
	281	72.0	−1.5500	−.625
	280	86.4	−1.5000	−.600
	279	100.8	−1.3125	−.525
	278	115.2	−1.0000	−.400
	277	129.6	− .5625	−.225
	276	144.0	0.0	0.0

TABLE IV-1

The Computer Input Required Values, (Column 4), of
the Shear Intensities per Node of Action and per y-Coordinate

That is,

$$\left.\begin{array}{l} TS\ (276,\ 2) = 0.0 \\ TS\ (277,\ 2) = -0.225 \\ TS\ (278,\ 2) = -0.400 \\ \quad \vdots \qquad\qquad \vdots \\ TS\ (285,\ 2) = -0.225 \\ TS\ (286,\ 2) = 0.0 \end{array}\right\} \qquad [4.5]$$

where the first dimension for every TS is the node number at which the shear intensity acts; the second dimension is defined to be 1, if the intensity acts on a horizontal side, or 2 when the shear intensity acts on a vertical side, as is the case here.

The energy equivalent nodal forces Q are obtained by using equations [2.27] as shown in Table IV-2. As noticed, nodes 282 and 280 accept the same loads due to symmetry.

The total load obtained from the above calculated energy equivalent loads is:

$$F_T = 2[(-54) + (-3.12) + (-5.64) + (-7.44) + (-8.52) + (8.88)] = 60.24 \qquad [4.6]$$

which deviates by 0.4%, from F of equation [4.2], and which is to be expected since the integration according to the computer program is approximated by straight line segments.

line	line	Force	Calculation	Result
260		Q_1	$= -(0/3. + .225/6.)(14.4) =$	$-.54$
	285	Q_2	$= -(0/6. + .225/3.)(14.4) =$	$= -3.12$
		Q_1	$= -(.225/3. + .400/6.)(14.4) =$	
259	284	Q_2	$= -(.225/6. + .400/3.)(14.4) =$	$= -5.64$
		Q_1	$= -(.400/3. + .525/6.)(14.4) =$	
258	283	Q_2	$= -(.400/6. + .525/3.)(14.4) =$	$= -7.44$
		Q_1	$= -(.525/3. + .600/6.)(14.4) =$	
257	282	Q_2	$= -(.525/6. + .600/3.)(144) =$	$= -8.52$
		Q_1	$= -(.600/3. + .625/6.)(14.4) =$	
256	281	Q_2	$= -(.600/6. + .625/3.)(14.4) =$	$= -8.88$
		Q_1	$= -(.625/3. + .600/6.)(14.4) =$	
255	280	Q_2	$= -(.625/6. + .600/3.)(14.4) =$	$= -8.52$
		Q_1	$= -(.600/3. + .525/6.)(14.4) =$	
254	279	Q_2	$= -(.600/6. + .525/3.)(14.4) =$	⋮
⋮	⋮	⋮	⋮	⋮

TABLE IV-2

The Analytical Calculations to Obtain the Energy Equivalent Nodal Forces of the Example of Figure IV-2

Notice that the above energy equivalent loads tabulated in Table IV-2 are identical to those obtained for the cantilever printout (see end of Appendix II-B) where exactly the same parabolic distribution is used on the free edge of the cantilever.

Also, notice the similarity between the approximately estimated discrete nodal forces and the computed energy equivalent nodal forces in Appendix II-B.

2. Nodal Displacements

Example 1. The boundary conditions imposed on the left half plate of Figure IV are compatible with those boundary conditions required to simulate Kirchoff's Simple Beam Theory which does not include the effects of the transverse shear deformation.

According to the Theory, in order to obtain the total deflection, δ_T, at half the length of the plate, L = 360 in. from the wall, for the centrally loaded plate, one has to apply superposition. That is,

$$\delta_T = \delta_{P_o} - \delta_{M_o} \qquad [4.7]$$

where δ_{P_o} is the deflection due to the pointing downwards load P_o, and δ_{M_o} is the upwards deflection due to the existing moment at L (see Figure IV-1).

Substituting in the well-known general expressions for the deflections, equation [4.7] becomes

$$\delta_T = \frac{P_oL^3}{3EI} - \frac{P_oL^3}{4EI} = \frac{P_oL^3}{12EI} \qquad [4.8]$$

Therefore, substituting for

P_o = 60 lbs

L = 360 in

E = 30×10^6 psi

I = 0.4×12^5 in^4

in equation [4.8] one obtains

$$\delta_T = 78.1 \times 10^{-6} \text{ in} \qquad [4.9]$$

For the centrally loaded case, the computer program gives the displacements of each node along the edge of the plate at the midspan, to be as described in column 1 of Table IV-3.

The average displacement of these center line nodal displacements is -118.7×10^{-6} in. which of course is way off from the predicted value calculated from the Simple Beam Theory.

However, for an aspect ratio of 5:1 (overall aspect ratio for the present plate), the shear deformation is considerably significant. Therefore, one must calculate the deflection due to the shear deformation and add it to the δ_T obtained from Kirchoff's Simple Beam Theory to obtain the true total deflection which we want to compare with the results of the computer program.

Accordingly, from Timoshenko's Theory of Elasticity [16] the deflection of a cantilever beam due to shear is

COLUMN 1		COLUMN 2	
NODE NUMBER	DISPLACEMENTS $\times 10^{-6}$	NODE NUMBER	DISPLACEMENTS $\times 10^{-6}$
286	−116.9	11	205.4
285	−118.0	10	205.6
284	−118.9	9	205.7
283	−119.5	8	206.0
282	−119.9	7	206.2
281	−120.1	6	206.3
280	−119.9	5	206.4
279	−119.5	4	206.4
278	−118.9	3	206.4
277	−118.0	2	206.4
276	−116.0	1	206.4

TABLE IV - 3

Results Obtained for Examples No. 1 and No. 3

$$\delta_{s_{cantilever}} = \frac{3P}{4cG} (\ell - x) \qquad [4.10]$$

where

P is the applied load at the free end of the beam

c is half the width

ℓ is the length as shown in Figure IV-4

and G is $E/2(1 + \nu)$

If we imagine the half centrally-loaded plate as two cantilever beams together as shown in Figure IV-5, the total deflection for the centrally-loaded plate at its center line would be

$$\delta_{s_{C.L.P.}} = 2\delta_{s_{cantilever}} = 2\,\frac{3(P/2)}{4cG}\,(L/2) \qquad [4.11]$$

where

P = 120 lbs.

c = 72 in

L = 360

ν = .25

Therefore, substitution of these values in [4.11] would yield

$$\delta_{s_{C.L.P.}} = 18.750 \times 10^{-6} \qquad [4.12]$$

However, since Timoshenko's formula has been derived for unity thickness one must divide δ_s of [4.12] by t = 0.4 which is the thickness of our plate.

Therefore,

$$\delta_s = 1/t \; \delta_{s_{C.L.P.}} = 46.875 \times 10^{-6} \qquad [4.13]$$

Adding the above deformation due to the effect of shear to the deformation calculated from the Simple Beam Theory,

$$\delta_{overall} = \delta_T + \delta_s \qquad [4.14]$$

$$= 78.1 \times 10^{-6} + 46.875 \times 10^{-6}$$

$$= 124.975 \times 10^{-6}$$

which when compared with the average displacement of the center line nodal displacements gives an error of 5.2% which is acceptable.

Example 2. Let us compare the displacement results as obtained for the cantilever case. The average displacement (in the y-direction) of nodes 276 through 286 (see Appendix II-B, results listed under MAIN0320 through MAIN0334) which is $\delta = -353.6 \times 10^{-6}$ with the analytical result from

$$(v)_{y=0} = \frac{Px^3}{6EI} - \frac{P\ell^2 x}{2EI} + \frac{P\ell^3}{3EI} + \frac{Pc^2}{2IG}(\ell - x) \qquad [4.15]$$

which is the general expression for the vertical deflections applied to Figure IV-4.[16]

Substitution for $x = 0$ gives

$$(v)_{y=0} = \frac{P\ell}{I}\left[\frac{\ell^2}{3E} + \frac{c^2}{2G}\right] \qquad [4.15.1]$$

Substituting for the known quantities ($t = 0.4$ in., $\nu = 0.25$, $c = 72$ in., $\ell = 360$ in., $E = 30 \times 10^6$ psi, $P = 60$ lbs.), we

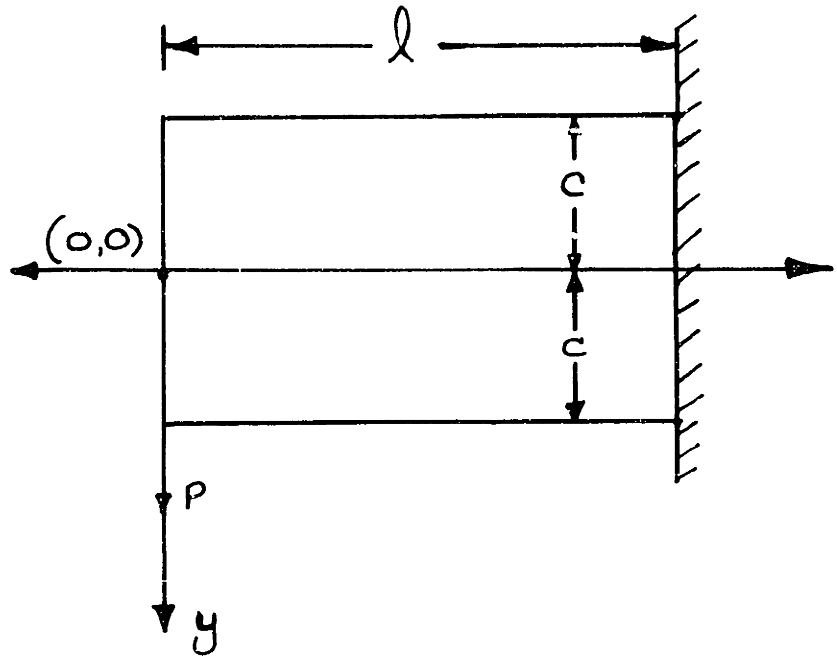


Figure IV-4

A Cantilever Plate Loaded at Its Free End with a Concentrated Load, P

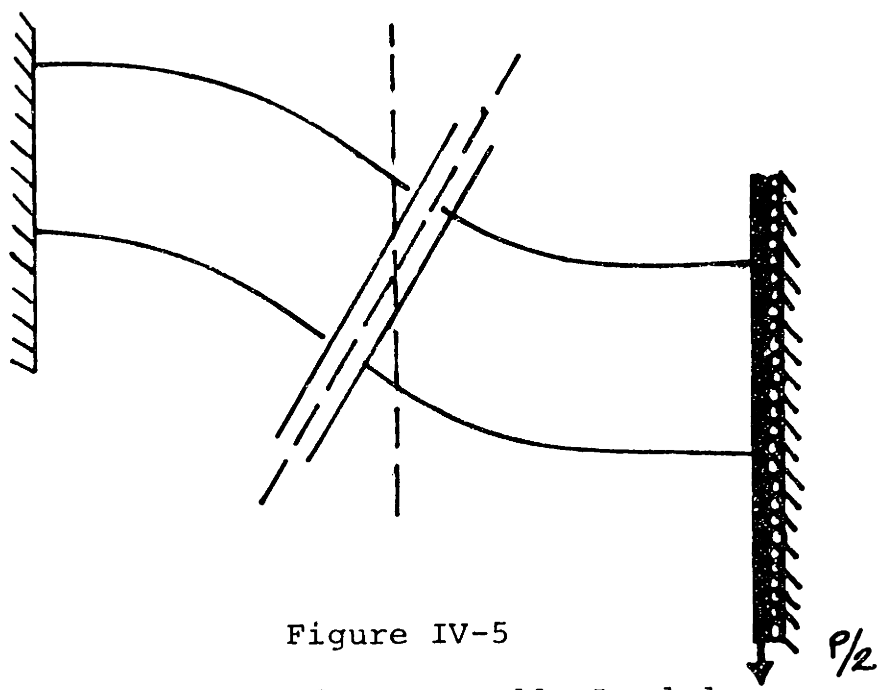


Figure IV-5

The Left Half of a Centrally Loaded Plate obtain

$$(v)_{y=0} = 359.3 \times 10^{-6} \qquad [4.15.2]$$

Therefore, the average nodal displacement at the free edge of the cantilever case as obtained by the computer program compared with the analytic solution's result gives only a 1.6% error!

<u>Example 3.</u> Consider the self-equilibrated plate of Figure IV-6.

The deflection at the center line is given by: (16)

$$\delta = \frac{5}{24} \frac{q\ell^4}{EI} \left[1 + \frac{12}{5} \frac{c^2}{\ell^2} \left(\frac{4}{5} + \frac{\nu}{2} \right) \right] \qquad [4.16]$$

For

$q = 0.1667$ lbs/in

$\ell = 360$ in

$c = 72$ in

$\nu = 0.25$

$E = 30 \times 10^6$ psi

$$\delta = 212.5 \times 10^{-6} \text{ in.} \qquad [4.17]$$

The nodal displacement values obtained from the computer run for this case are tabulated in the second column of Table IV-3, for which the average value is $\delta = 206.12 \times 10^{-6}$ in. The difference between the two values gives a 3% error.

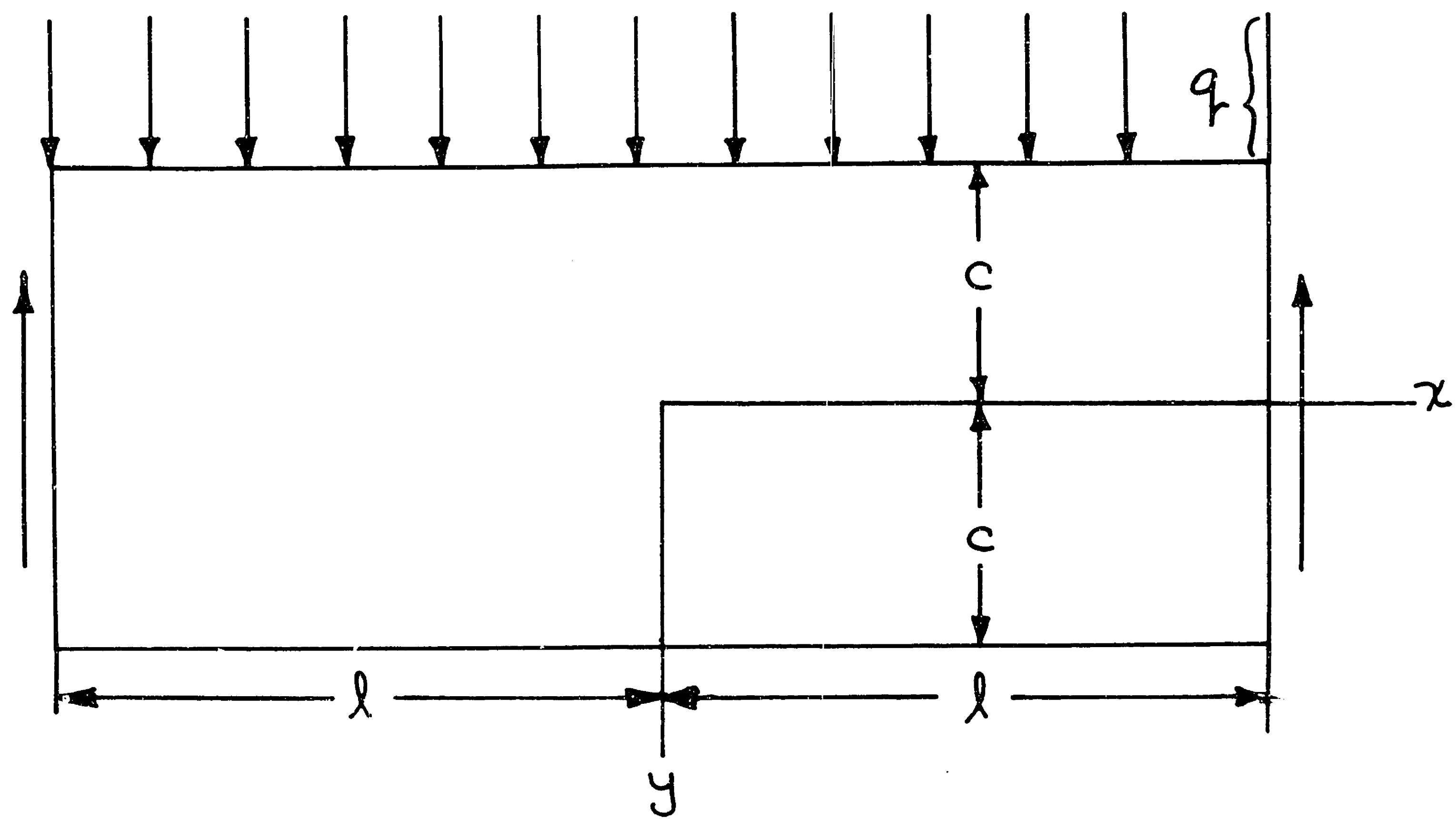


Figure IV-6

A Uniformly Loaded and Self-Equilibrated Plate with Shear Reactions at its Free Ends

From the above discussion the conclusion is that the computer program gives excellent results for the calculated nodal displacements.

It should be noted that the percentage error between the analytic results and the computed results is an overall error in which the modeling error is included (that is, when simulating Simple Beam Theory, one should be careful of which nodes to restrain, etc.).

Now, that it has been shown that the nodal displacements are in agreement with the theoretically predicted results, we may expect that the average nodal stresses are also agreeable to the theoretical expected results.

3. Average Nodal Stresses

The results for the σ_x bending stress obtained from example one of the previous section have been plotted in Figure IV-7. These results reveal that the overall behavior of the bending stress σ_x is in agreement with that expected from the Simple Beam Theory.

The small discrepancy between the maximum σ_x at the center line and the σ_x at the wall is indicative of the boundary effects. That is, the small percentage error associated with the displacements is also reflected in the stress results as it should be expected.

Aside from the fact that there is a slight assymetry about the quarter length axis of the plate, all the σ_x values

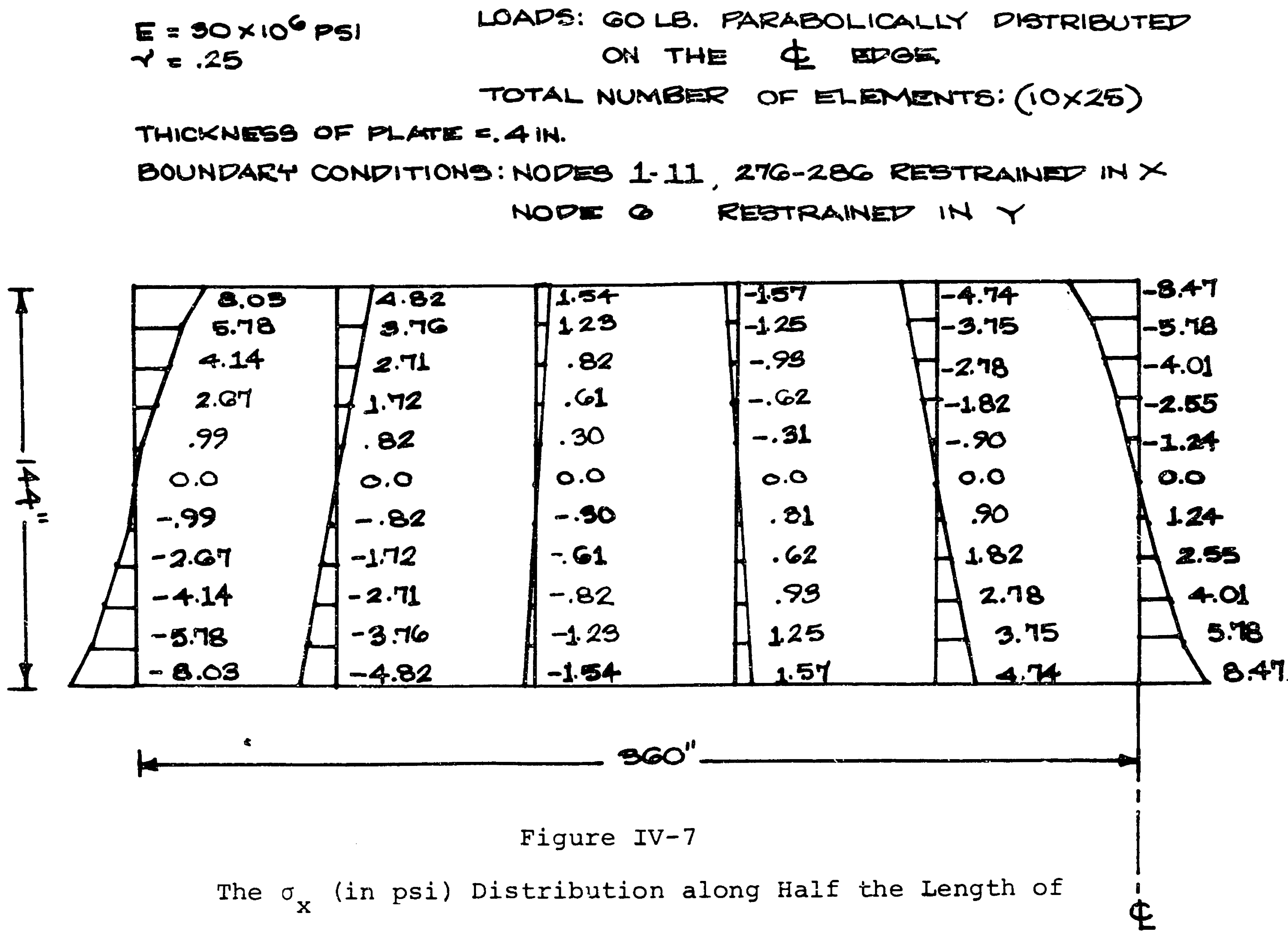


Figure IV-7

The σ_x (in psi) Distribution along Half the Length of a Centrally Loaded (in its Plane) Plate are completely symmetrical about the horizontal N.A. as it is expected from Beam Theory.

The σ_x results for the cantilever case are shown in Figure IV-8. The stress decreases with x increasing; starting from the maximum values of stress at the upper and lower edges of the plate and falling off to zero at the free edge.

The maximum value for the bending stress on the upper and lower edges of the plate at the edge of the wall is ±15.82 psi.

From Simple Beam Theory,

$$\sigma_x = \pm \frac{My}{I} = \pm \frac{PLc}{I} = \pm 15.62 \text{ psi} \qquad [4.18]$$

Therefore, it is clear that the desired values for σ_x are close to those obtained from the Simple Beam Theory which is simulated here.

From the third example of the previous section, the σ_x results are shown in Figure IV-9.

Note that the upper half plate of Figure IV-6 is in compression, and the lower half is in tension. This is confirmed by the variation of σ_x in Figure IV-9 (that is, there are no alternating stresses about the quarter length axis). The assymetry of σ_x about the horizontal N.A. at the last two sections where it is plotted, is due to the fact that the upper and lower nodes of the plate at the center line were left unrestrained in y.

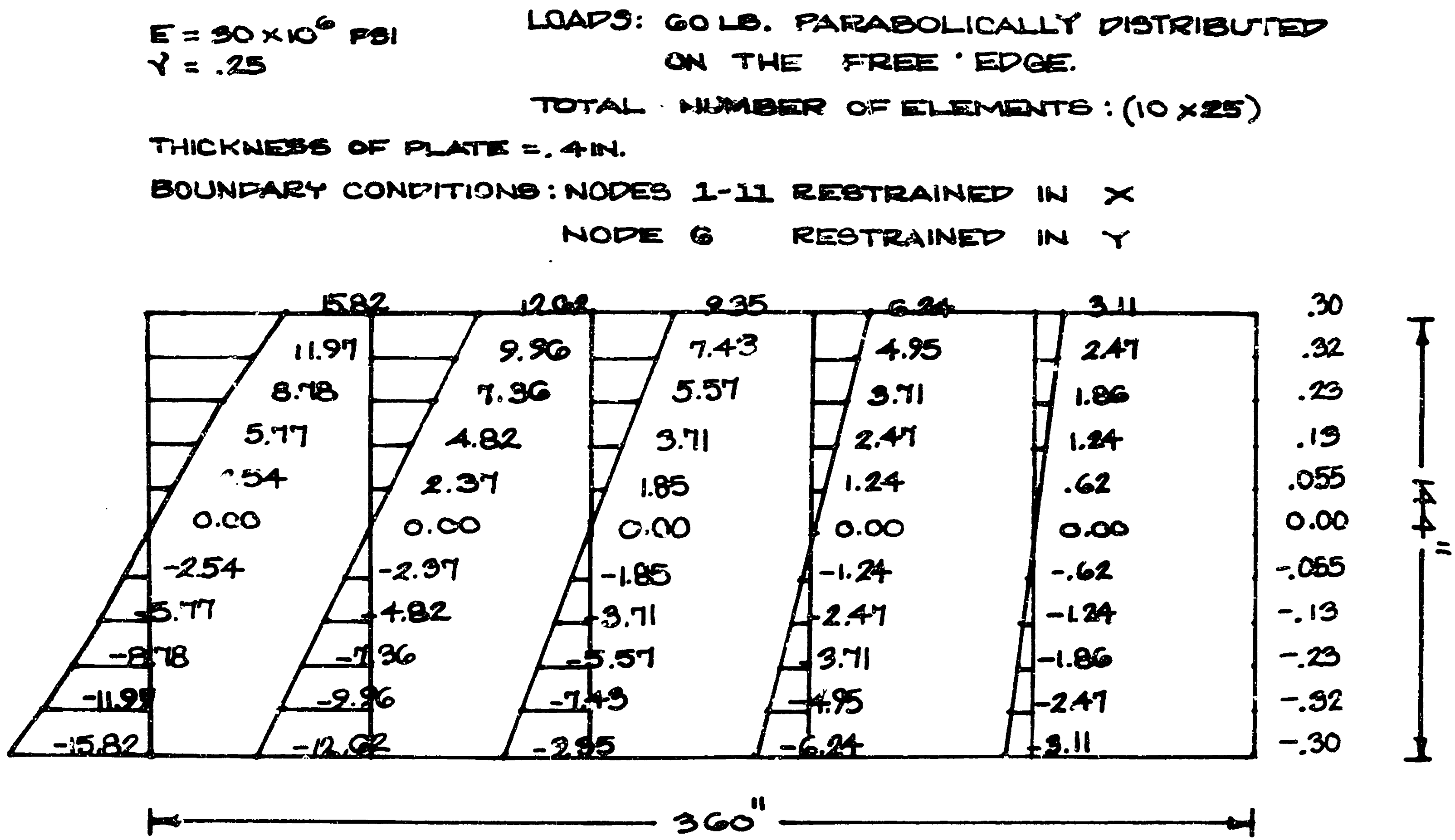


Figure IV-8

The σ_x (in psi) Distribution along the Length of a Cantilever

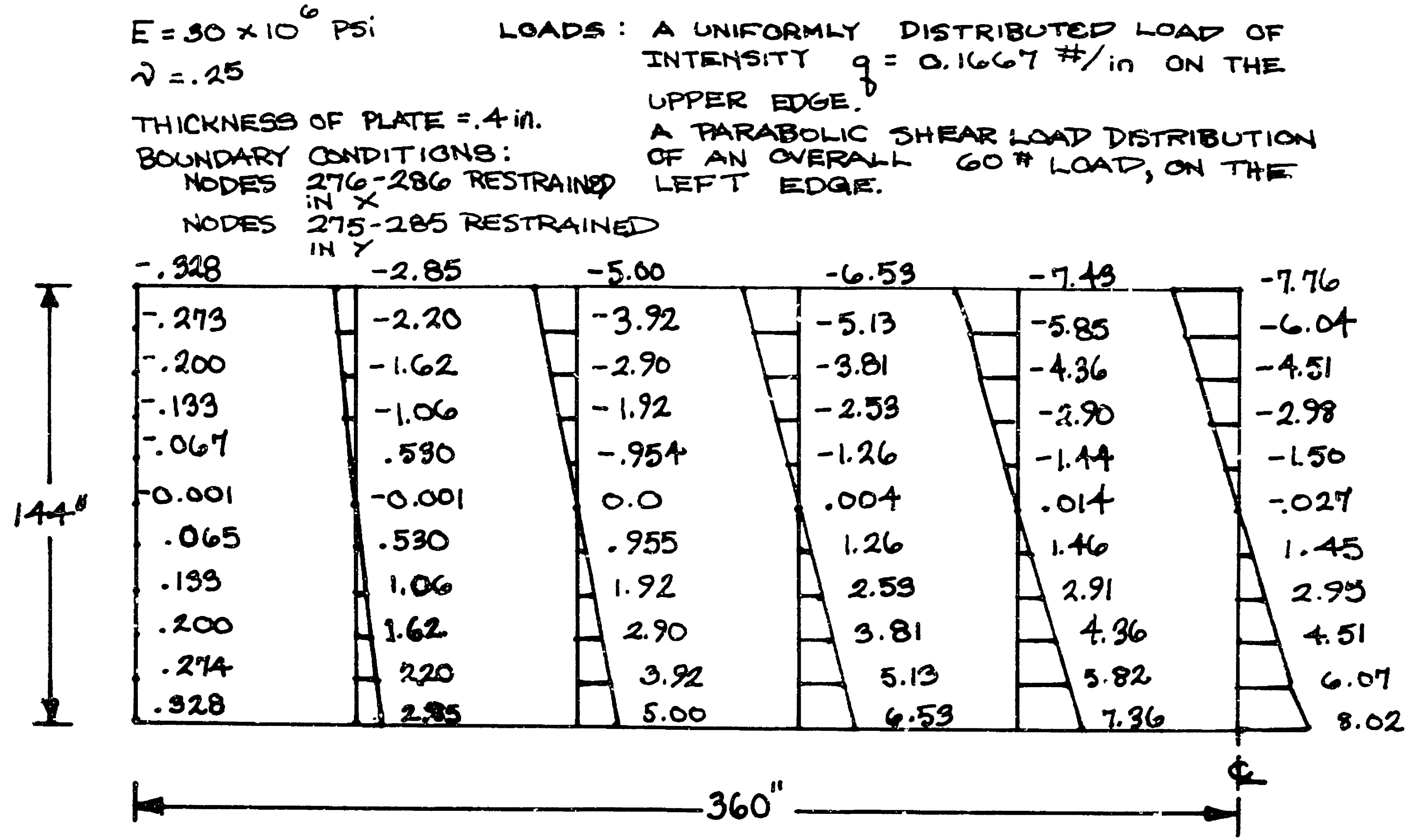


Figure IV-9

The σ_x (in psi) Distribution along the Length of a Self-Equilibrated Plate

4. Principal Compressive Stress Trajectories

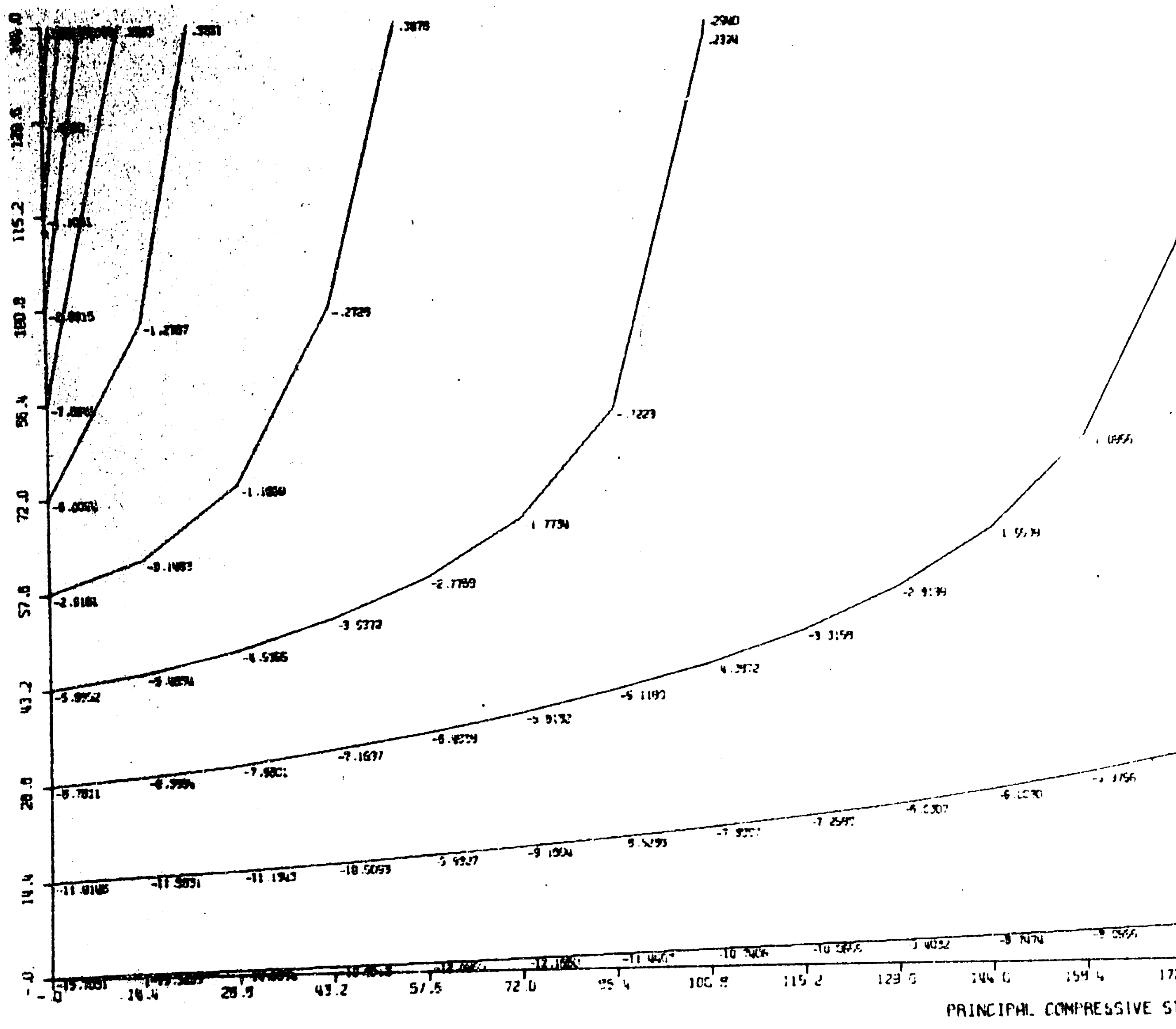
The results for the principal compressive stress trajectories of the above three examples are shown in PLATE I (see Appendix II-A) and in PLATES II, III, and IV.

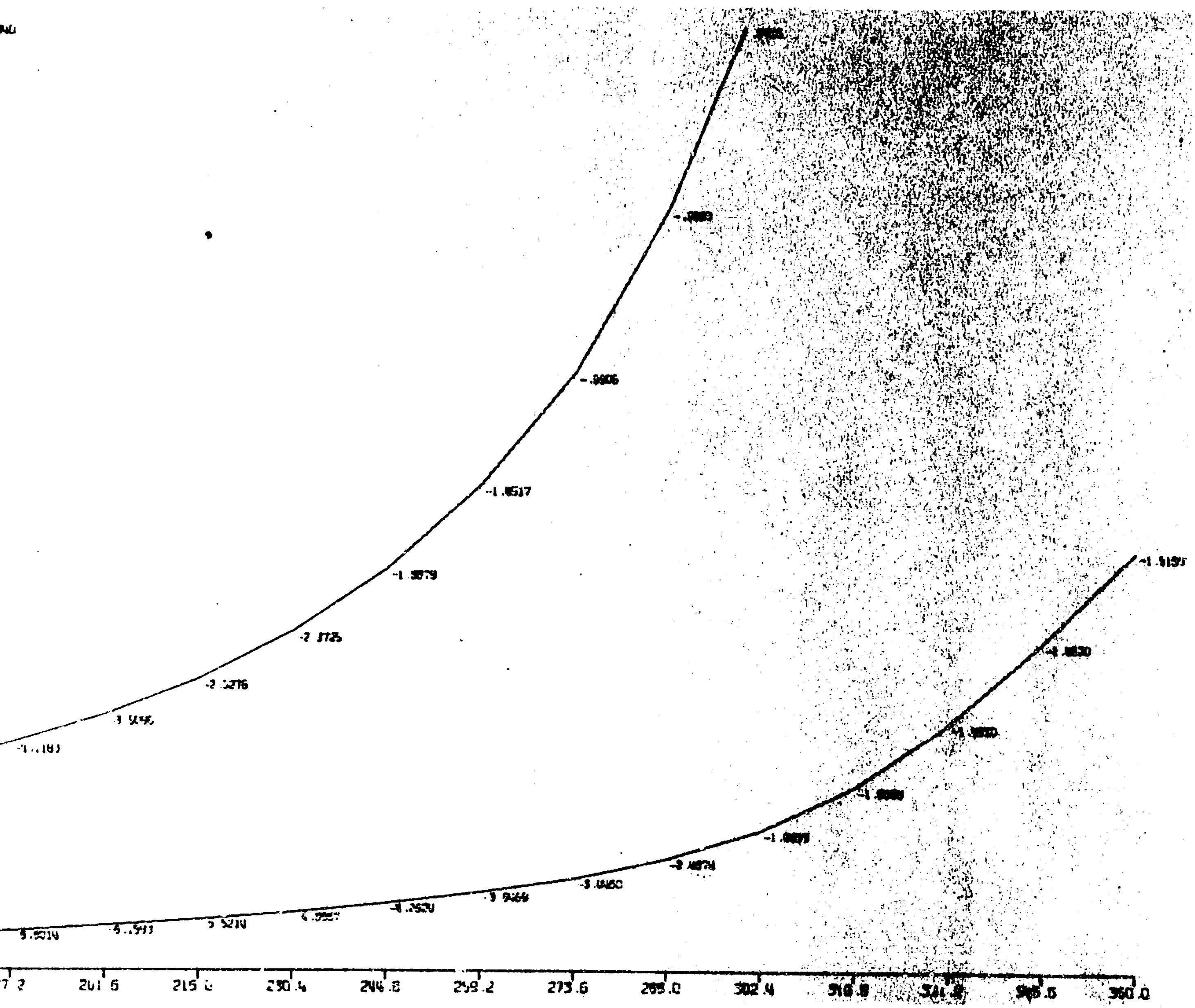
PLATE I and II are almost identical; the small differences in the principal compressive stresses, σ_2, reveal that for the case of PLATE I, the parabolic shear distribution is simulated by discrete nodal loads, while for the case of PLATE II, it is simulated by distributed boundary loads. In both cases, however, the stress trajectories are agreeable to the expectation of Figure I-B-2, in Appendix I-B.

The stress trajectories appearing in PLATE III correspond to the centrally loaded plate (left-half) and are in agreement with theoretical expectations.[14]

Notice that the directions of the trajectories along the N.A. are at 45° to the horizontal, revealing that the principal stress there is due to shear. Also, notice that at the center line and at the wall sections the principal stress direction becomes horizontal, due to the bending stress, σ_x. (There is no shear stress component there). The trajectories that start from nodal points on the left edge of the plate and above the N.A. are directed upwards, implying that they are at right angles with the principal tensile stress trajectories (which are not indicated here).

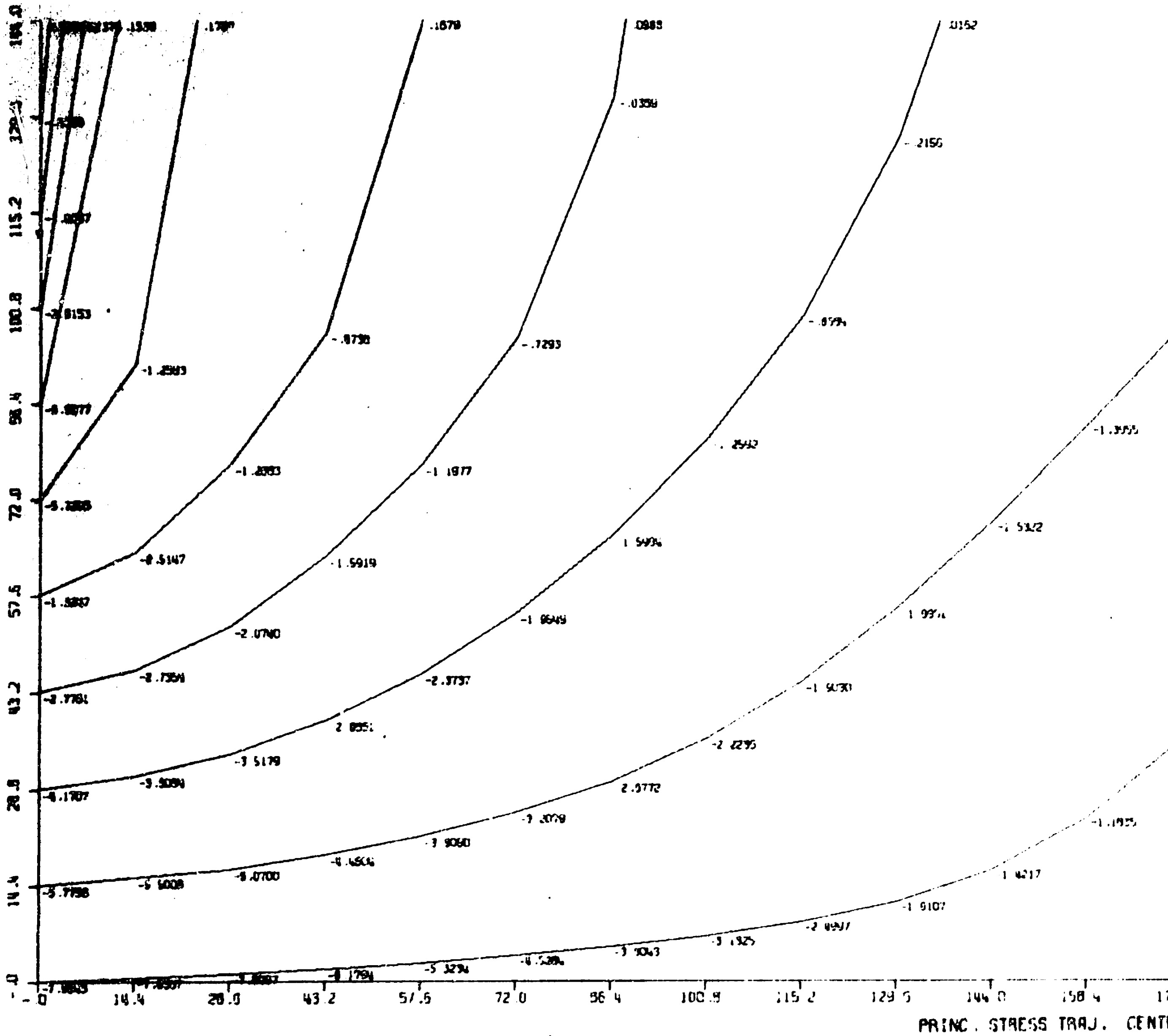
Notice also that for the maximum compressive stress trajectory that commences at nodal point 1, the magnitude


115.2
72.0
57.6
43.2
14.4
28.8
43.2
57.6
72.0
100.8
PRINCIPAL COMPRESSIVE S



JECTORIES FOR A CANTILEVER

PLATE II


PRINC. STRESS TRAJ. CENT

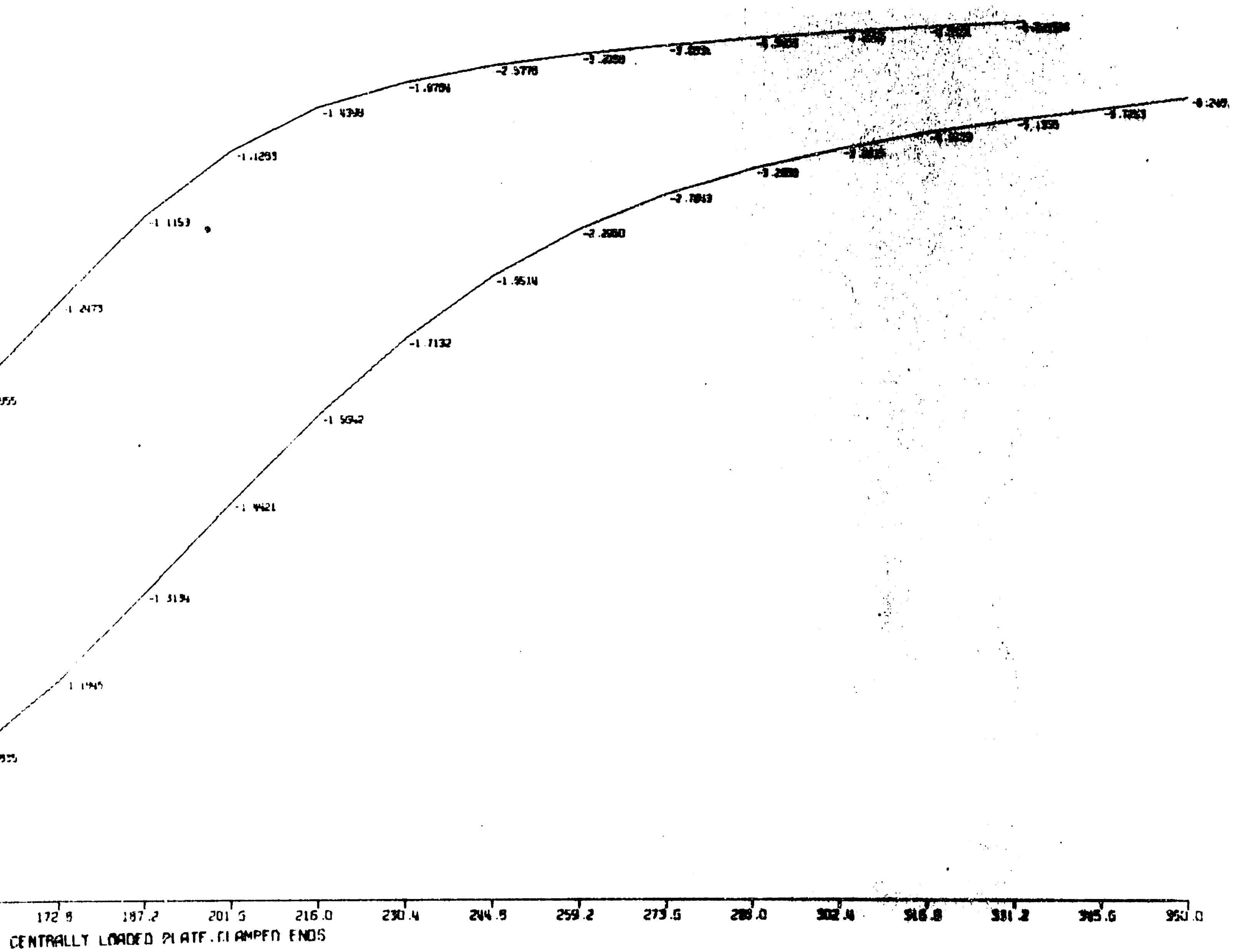
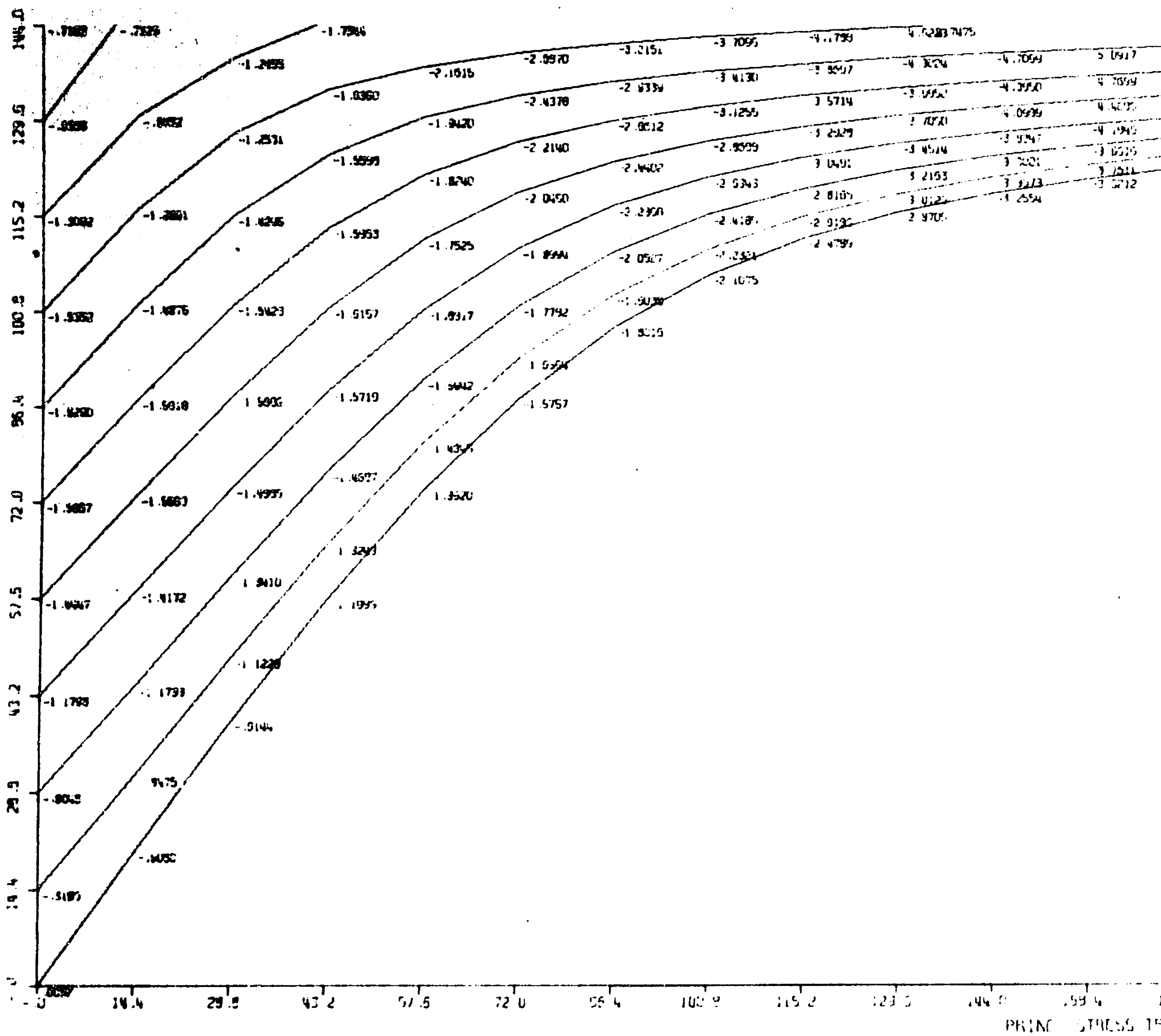

172.8
187.2
201.6
216.0
230.4
244.8
259.2
273.6
288.0
302.4
316.8
331.2
345.6
350.0
CENTRALLY LOADED PLATE, CLAMPED ENDS

PLATE III



72.0
43.2

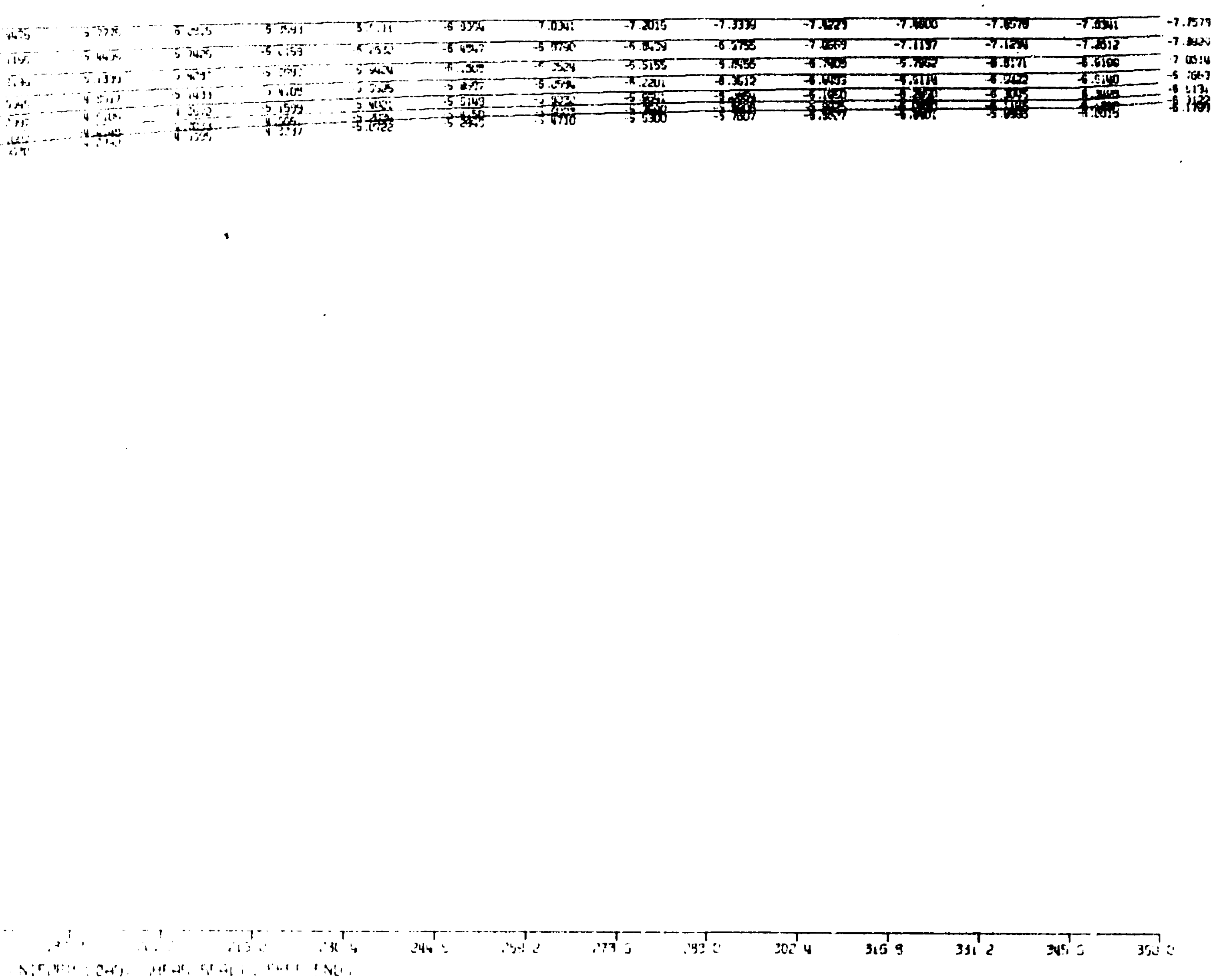


PLATE IV

of the principal stress decreases with x increasing, until it reaches the N.A. at quarter lengths and increases again in a symmetrical manner.

In PLATE IV the compressive stress trajectories are in agreement with Figure IV-9 where the compressive stresses are all above the N.A. Since a shear reaction acts along the left edge of the plate, all the trajectories are expected to be at 45° angles to the horizontal.

Notice that as the σ_x stress increases while moving towards the center line, the compressive stress trajectories also increase and tend to become more horizontal.

5. Computer Stiffening Procedure

The results in this section demonstrate the computer program's "stiffening" capabilities.

Let us divide the left-half of the centrally-loaded plate into 250 finite elements. By assigning numbers to each one of the elements starting from the lower left hand side element of the structure and counting always from bottom to top, subroutine STIFEN identifies all elements which are transversed by every stress trajectory.

The stress trajectories for this case cross all the elements which have been darkened in Figure IV-10. By setting NUM = 2 (See Appendix II-A) the computer program procedure is repeated after the identification of the elements need to be "stiffened"; the new average nodal stresses are calculated.

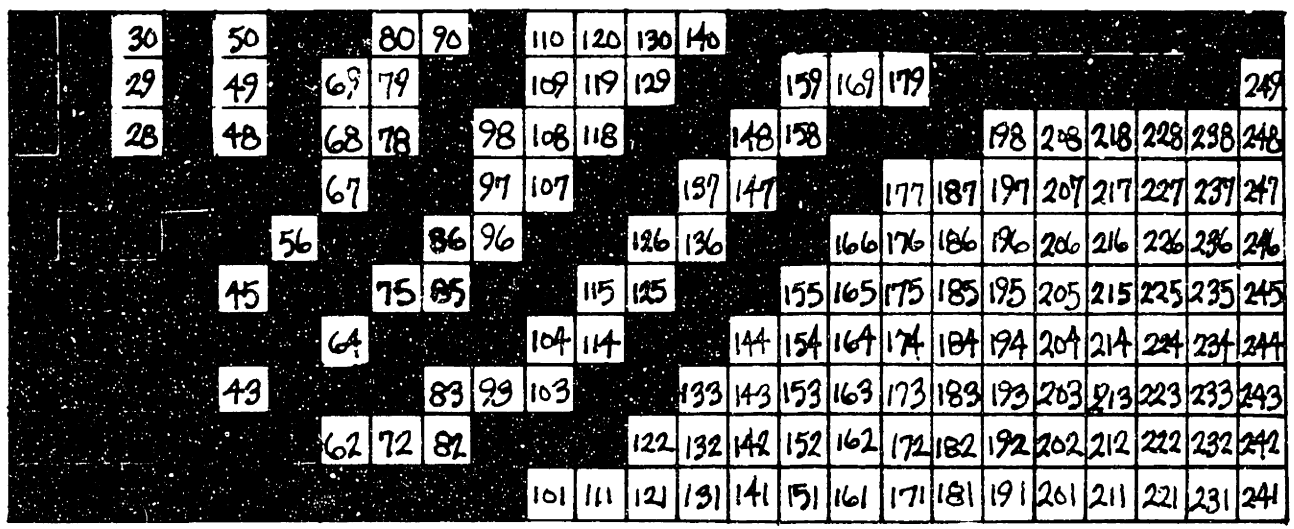


Figure IV-10

The Computer Program's "Stiffening" Arrangement (First Trial)

For the case where the "stiffening" is made by doubling the thickness of the elements, traversed by trajectories, the σ_x average nodal stresses obtained are shown in Figure IV-11.

By comparing Figures IV-7 and IV-11, one observes that the stresses for the thus "stiffened" case are lower than those of the unstiffened as it is expected.

Furthermore, the symmetric distribution of the σ_x about the horizontal N.A. of the plate is disturbed in a similar manner to that of the pathways of the stress trajectories.

Notice that the compressive $-\sigma_x$ stresses at the extreme lower left portion of the plate (first section) are decreased more than what the corresponding tensile $+\sigma_x$ stresses are in that same section. The same behavior is observed in the next two sections; when passed the quarter length of the plate, the lower stress values in a section appear on the above of the N.A. portion of the plate.

Thus, it has been explicitly shown that by assigning a new thickness value to the elements which are traversed by stress trajectories, the overall stress in the structure is reduced; in particular, it is reduced in a manner proportional to the magnitude of the compressive stress and along the principal compressive stress trajectories.

Before the repeated procedure ends, the new stress trajectories for the thus "stiffened" plate will yield a new set of elements for further "reinforcement" of the

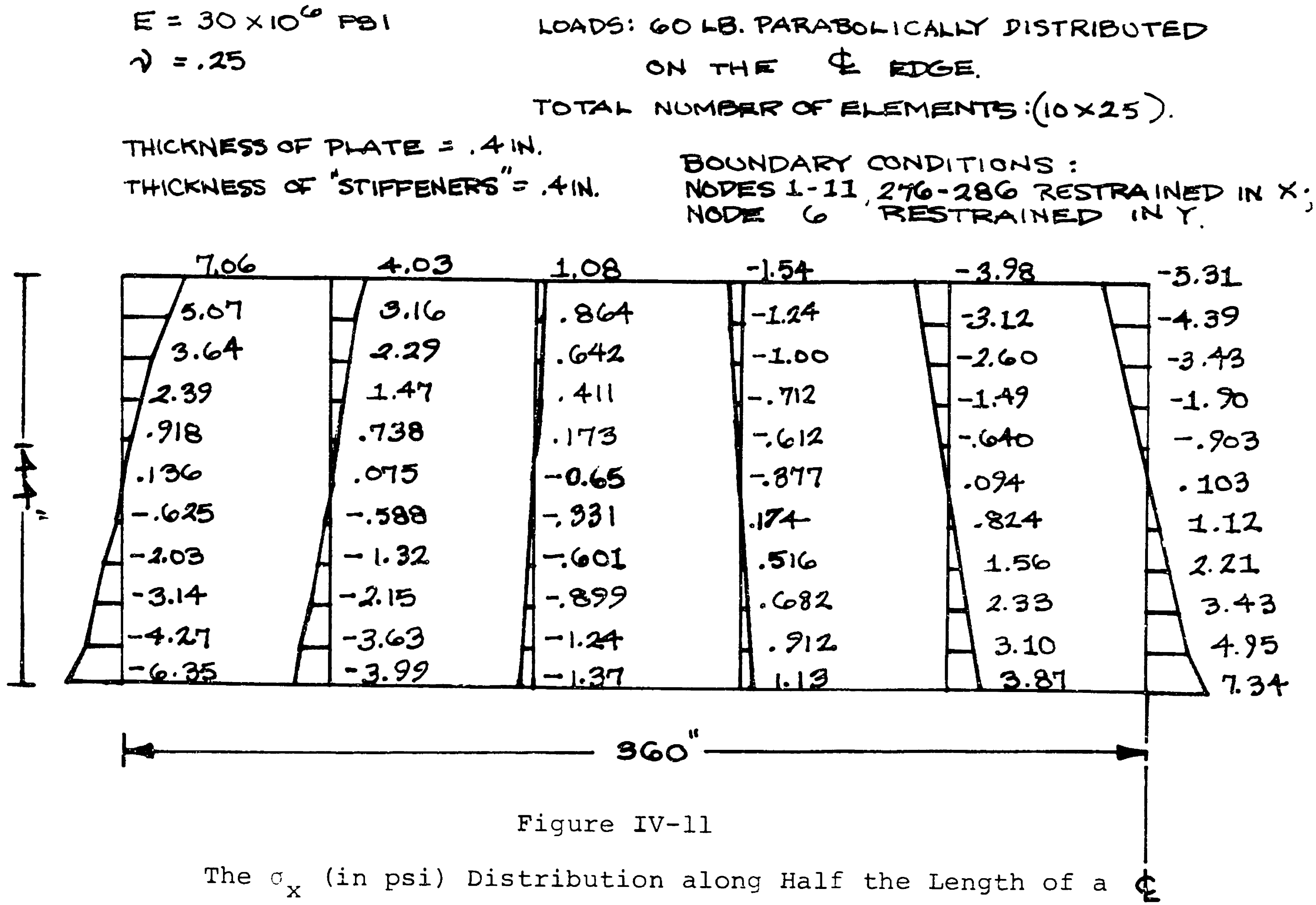


Figure IV-11

The σ_x (in psi) Distribution along Half the Length of a Centrally-Loaded Plate after It Has Been "Reinforced" structure.

Figure IV-12 illustrates the new darkened elements. If one compares the darkened elements of the first trial with those identified for a second trial in Figure IV-12, one may notice that the darkened elements to be "stiffened" the second time maintain the same pattern with those darkened elements which were "stiffened" the first time.

The difference between the two however is that the second pattern shifts slightly to the left, creating this way a steeper slope of ascent for the maximum principal stress trajectory; this is to be expected since the values of the compressive $-\sigma_x$ stresses have been lowered from the "stiffening" of the first trial.

6. Least Weight Stiffener Arrangements

In Part III, the results for $\bar{\tau}_A$, $\bar{\tau}_B$, and $\bar{\tau}_C$ vs H as well as the results for $\bar{\sigma}_A$, $\bar{\sigma}_B$, and $\bar{\sigma}_D$ vs H are discussed and it is mentioned that $\bar{\tau}_B < \bar{\tau}_C < \bar{\tau}_A$ is always true for the same H, α, and c which essentially implies the same weight. The conclusion that $\bar{\sigma}_A < \bar{\sigma}_D < \bar{\sigma}_B$ was the similar relation for the case where the principal stress σ_2 depends completely on the σ_x bending stress.

However, the Figures in the corresponding section there do not explicitly show the variation of stress with weight.

The results to follow in this section show the variation of the non dimensional shear and bending stresses with

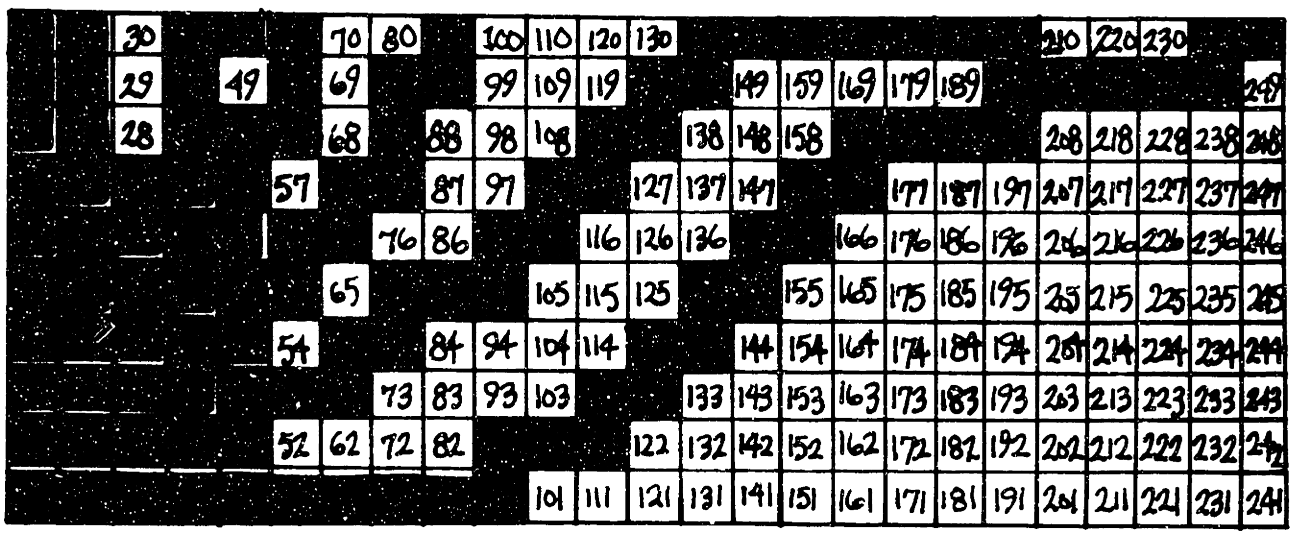


Figure IV-12

The Computer Program's "Stiffening" Arrangement (Second Trial) respect to a common non-dimensional weight parameter, A, to all arrangements A, B, C, and D where

$$A = Bt + 2cHBt \qquad [4.19]$$

where B, t, c, and H are as defined, in Part III.

Since B is constant for all the arrangements, it is appropriate to non-dimensionalize A with respect to B as it was done for the non-dimensional expression for the shear stresses and the bending stresses for all cases.

Multiplying both numerator and denominator of the right hand side of equation [4.19] by B one obtains

$$A = \frac{B^2}{\alpha} [1 + 2cH] \qquad [4.20]$$

or

$$\bar{A} = \frac{A}{B^2} = \frac{1}{\alpha} (1 + 2cH) \qquad [4.21]$$

which is the non-dimensional weight.

Therefore, plotting $\bar{\tau}_A$, $\bar{\tau}_B$, $\bar{\tau}_C$, and $\bar{\sigma}_A$, $\bar{\sigma}_B$, $\bar{\sigma}_D$ vs $\bar{A}$ one obtains the results of Figures IV-13 through IV-24 which verify the results of the Figures of Part III.

Furthermore, they show that the stress varies in a hyperbolic manner with respect to weight.

Therefore, the closest family of curves to the axes corresponds to the best stiffener arrangement; that is, the B arrangement for the case of maximum shear and the A arrangement for the case of maximum bending stress.

The conclusion one may draw from the plotted results of the following Figures is: the least weight arrangement of stiffeners to reduce the stress level in any cross-section of any planar structure is that one which adds material at the location where the principal compressive stress is maximum.

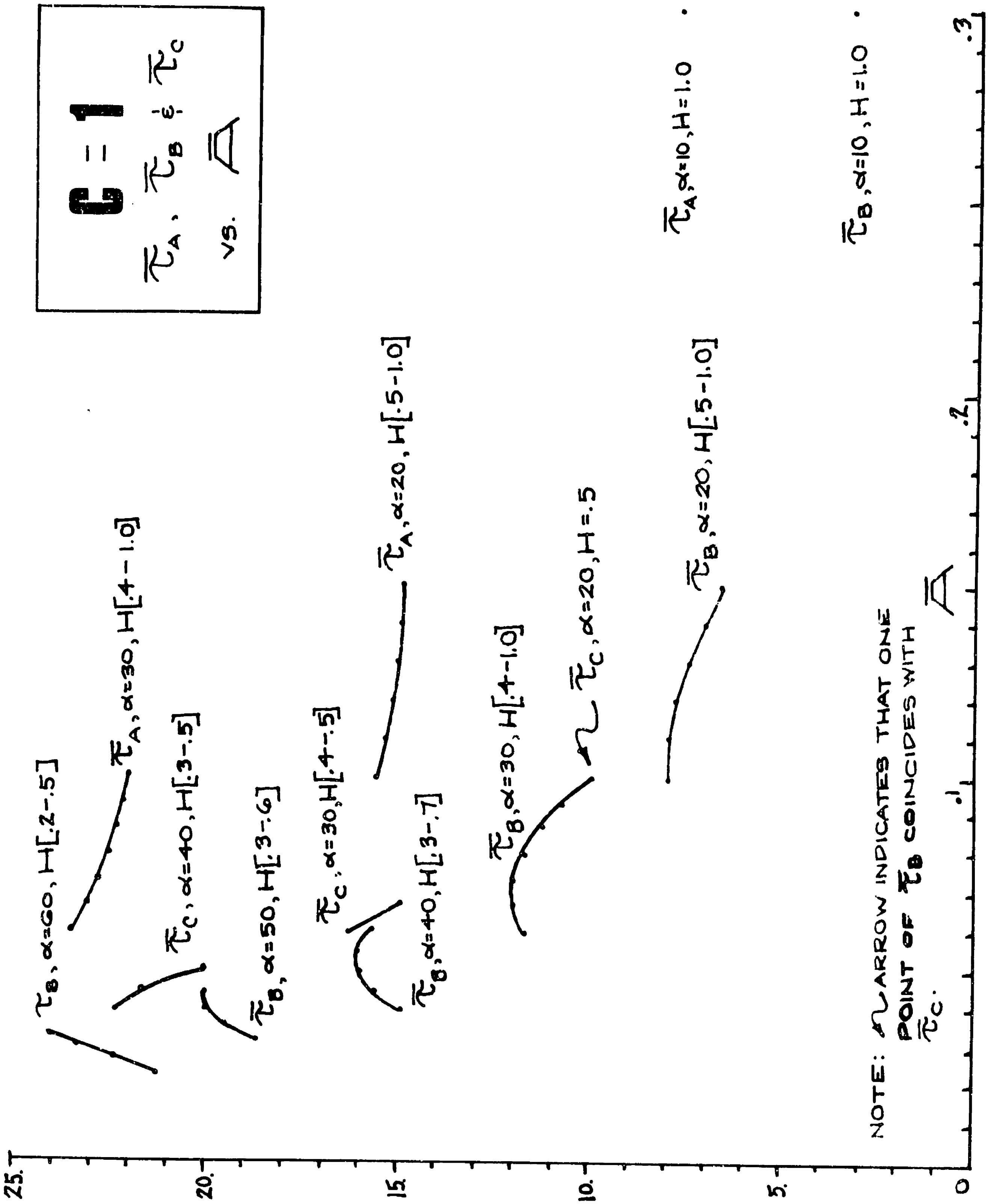

C = 1
τA, τB & τC
vs. A
τB, α=60, H[.2-.5]
τA, α=30, H[.4-1.0]
τC, α=40, H[.3-.5]
τB, α=50, H[.3-.6]
τC, α=30, H[.4-.5]
τB, α=40, H[.3-.7]
τA, α=20, H[.5-1.0]
τB, α=30, H[.4-1.0]
τC, α=20, H=.5
τB, α=20, H[.5-1.0]
τA, α=10, H=1.0
τB, α=10, H=1.0
25.
20.
15.
10.
5.
0
.1
.2
.3
A
NOTE: ARROW INDICATES THAT ONE POINT OF τB COINCIDES WITH τC.

Figure IV-13

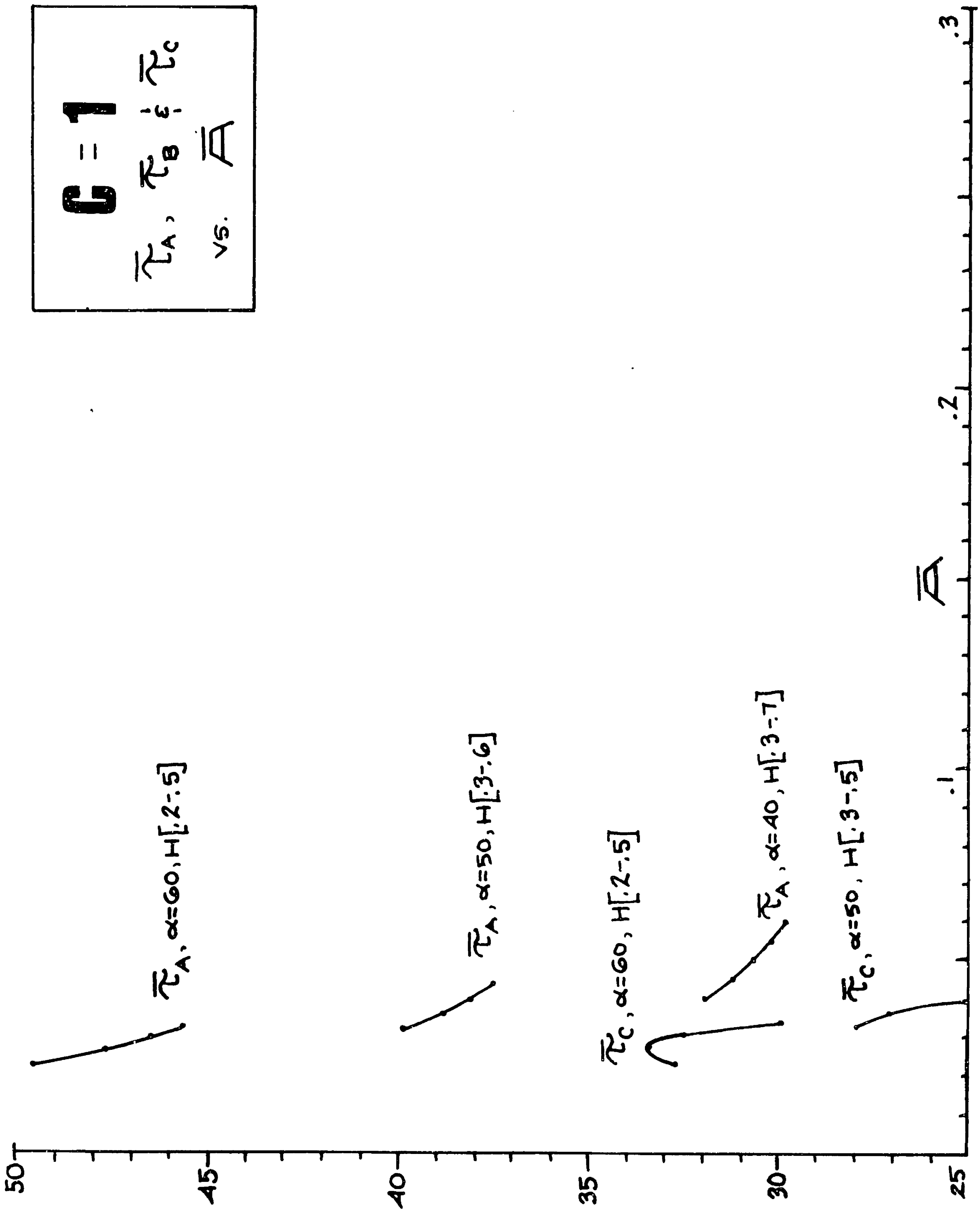
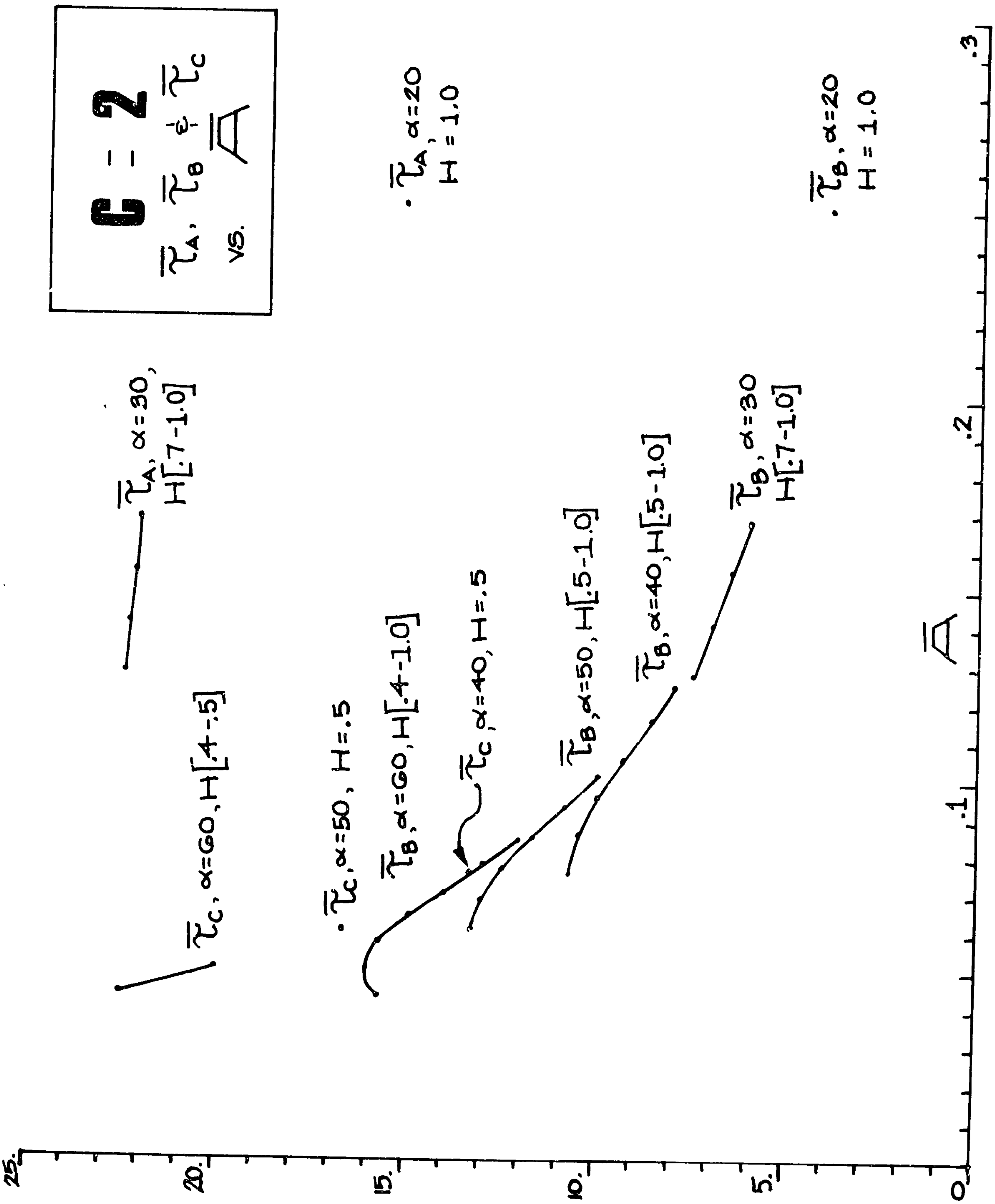
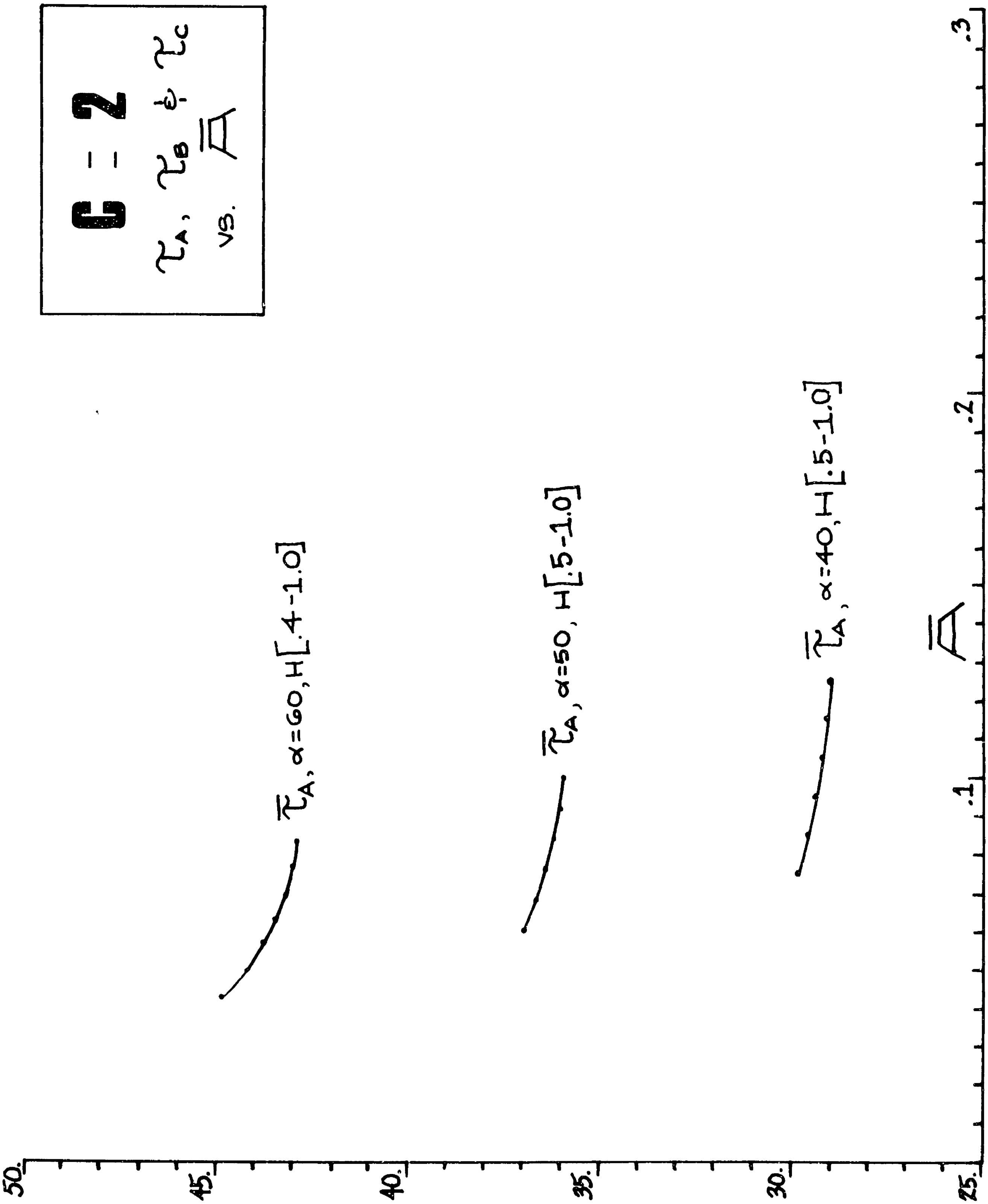

C = 1
τ̄A, τ̄B & τ̄C
vs. Ā
τ̄A, α=60, H[.2-.5]
τ̄A, α=50, H[.3-.6]
τ̄C, α=60, H[.2-.5]
τ̄A, α=40, H[.3-.7]
τ̄C, α=50, H[.3-.5]
Ā
.1
.2
.3
50
45
40
35
30
25

Figure IV-14



C = 2
τ̄A, τ̄B & τ̄C
vs. Ā
τ̄A, α=30, H[.7-1.0]
τ̄A, α=20, H=1.0
τ̄B, α=20, H=1.0
τ̄C, α=60, H[.4-.5]
τ̄C, α=50, H=.5
τ̄B, α=60, H[.4-1.0]
τ̄C, α=40, H=.5
τ̄B, α=50, H[.5-1.0]
τ̄B, α=40, H[.5-1.0]
τ̄B, α=30, H[.7-1.0]
Ā
0
.1
.2
.3
5.
10.
15.
20.
25.

Figure IV-15



C = 2
τA, τB & τC
vs. Ā
τ̄A, α=60, H[.4-1.0]
τ̄A, α=50, H[.5-1.0]
τ̄A, α=40, H[.5-1.0]
Ā
.1
.2
.3
25.
30.
35.
40.
45.
50.

Figure IV-16

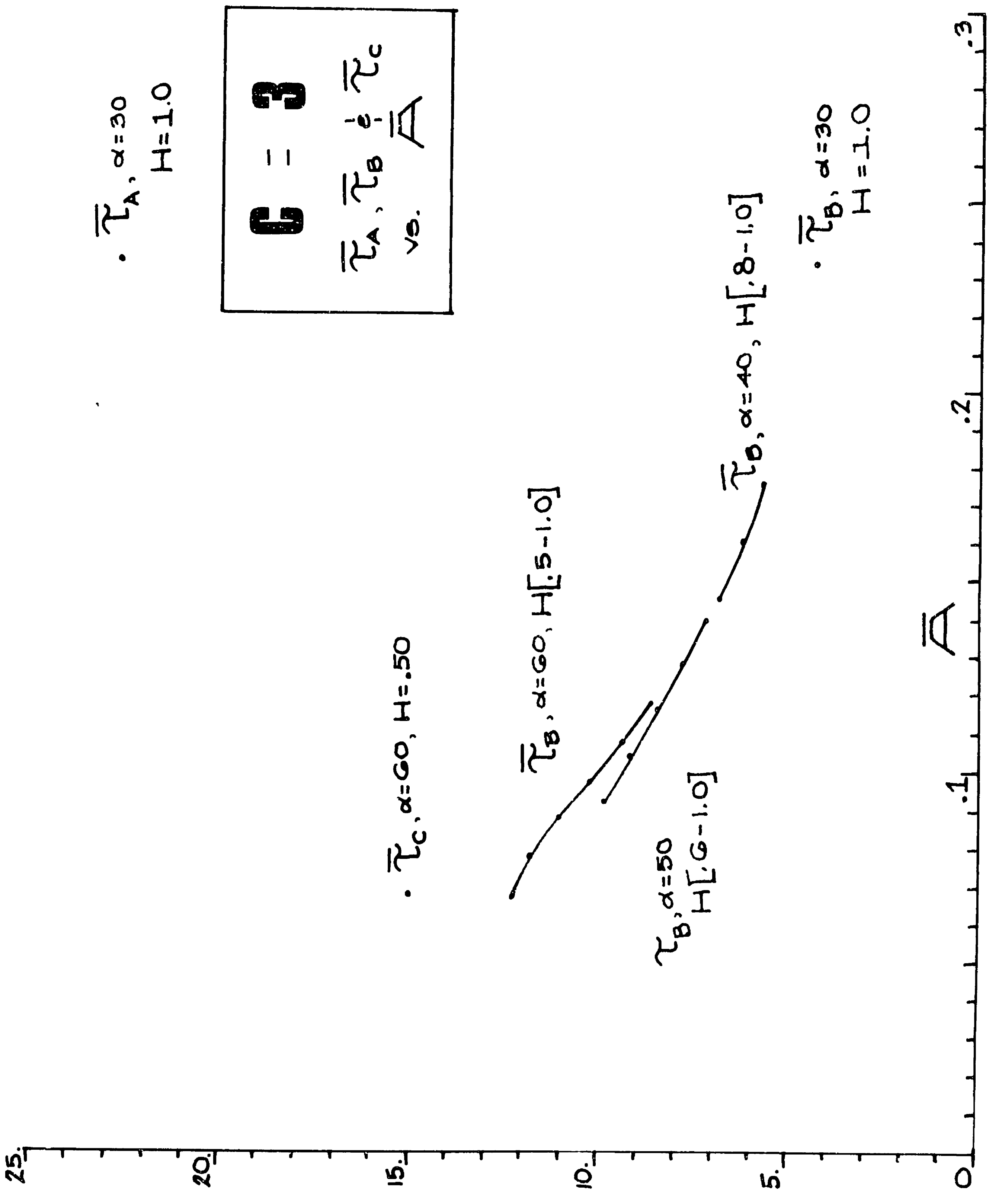

C = 3
τ̄A, τ̄B & τ̄C vs. Ā
τ̄A, α=30 H=1.0
τ̄B, α=30 H=1.0
τ̄B, α=40, H[.8-1.0]
τ̄B, α=60, H[.5-1.0]
τ̄C, α=60, H=.50
τB, α=50 H[.6-1.0]
Ā
0
5.
10.
15.
20.
25.
.1
.2
.3

Figure IV-17

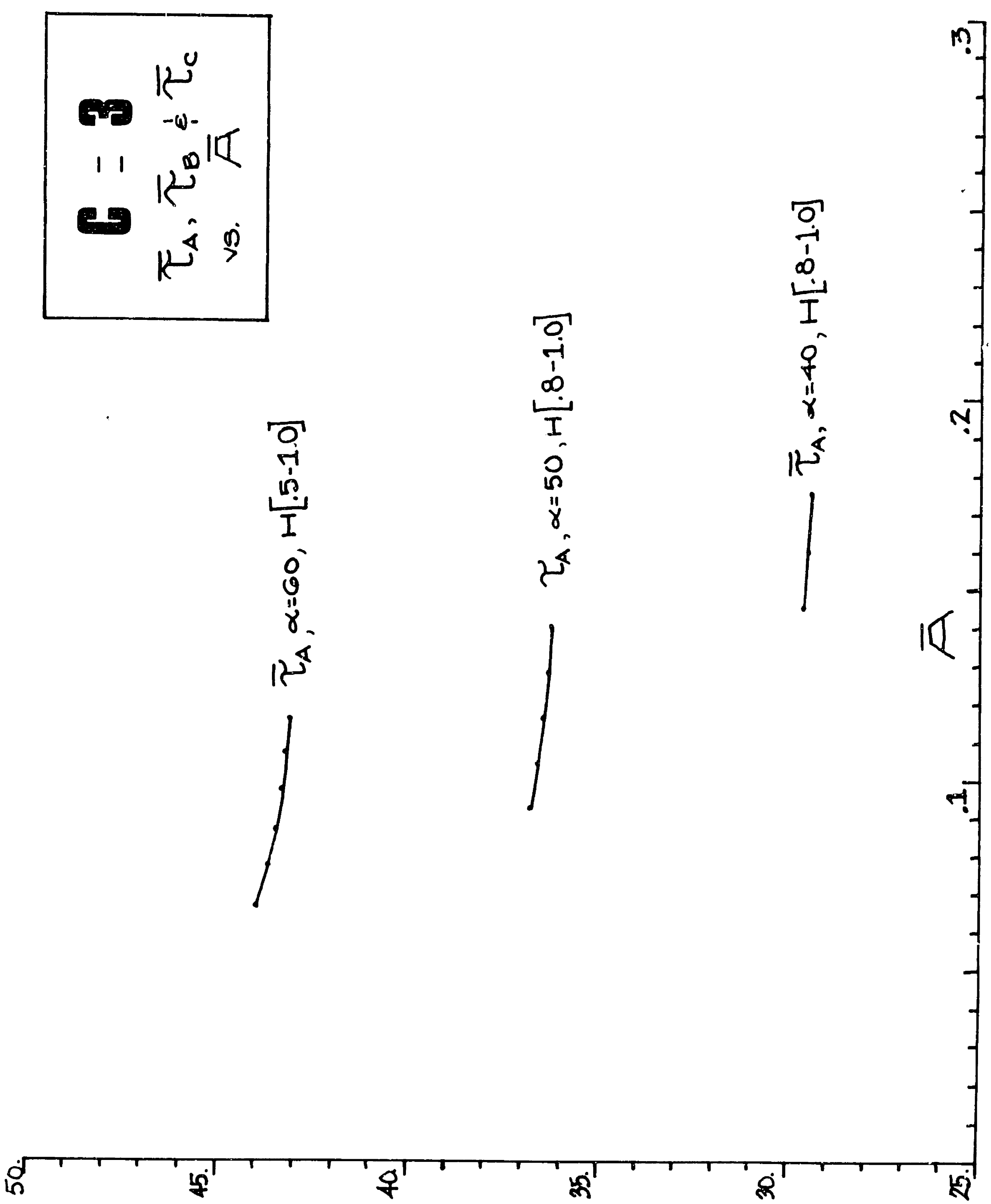

C - 3
τA, τB & τC vs. A
τA, α=60, H[.5-1.0]
τA, α=50, H[.8-1.0]
τA, α=40, H[.8-1.0]
A
.1
.2
.3
50.
45.
40.
35.
30.
25.

Figure IV-18

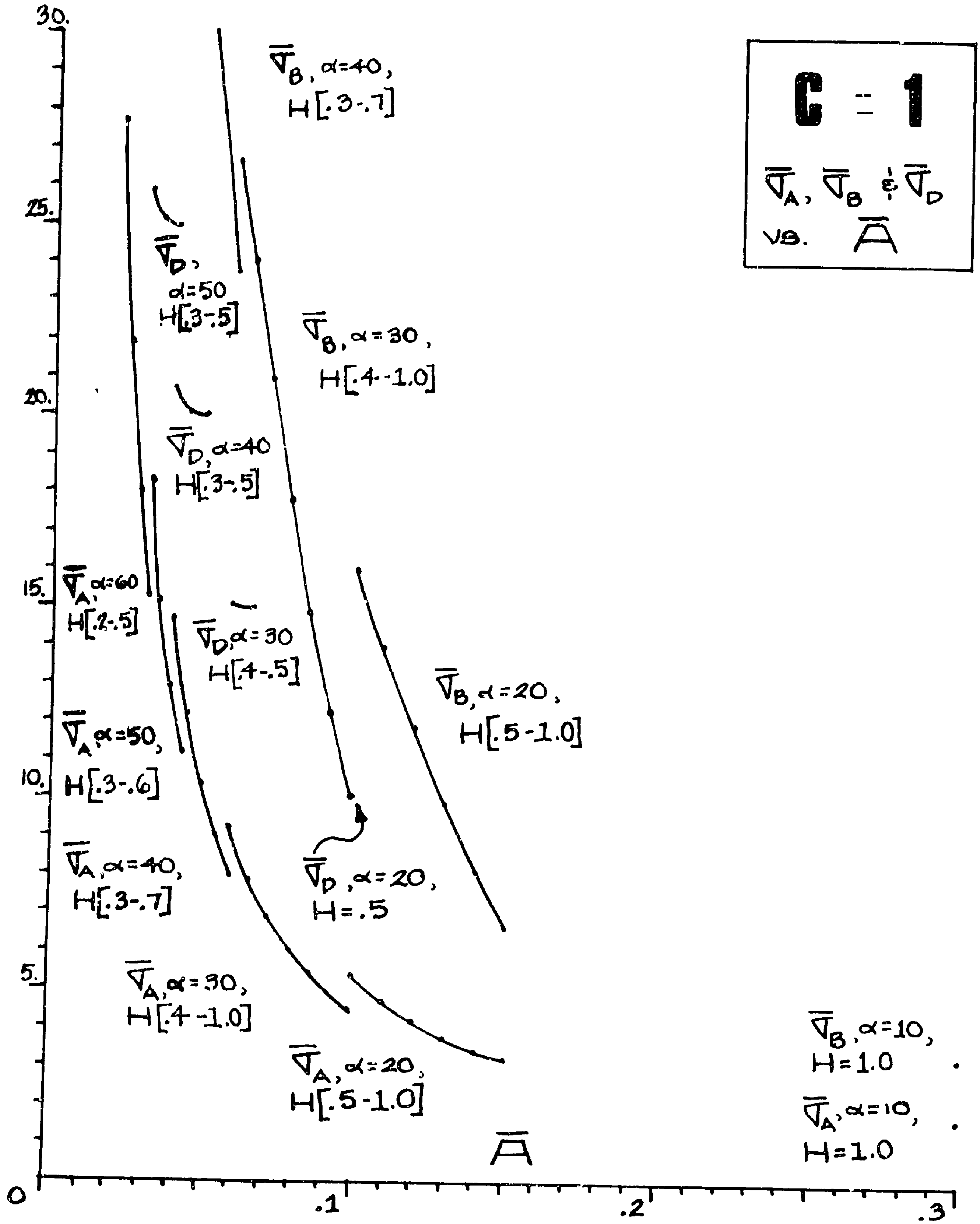

C = 1
$\bar{\nabla}_A$, $\bar{\nabla}_B$ & $\bar{\nabla}_D$
vs. $\bar{A}$
$\bar{\nabla}_B$, α=40, H[.3-.7]
$\bar{\nabla}_D$, α=50 H[.3-.5]
$\bar{\nabla}_B$, α=30, H[.4-1.0]
$\bar{\nabla}_D$, α=40 H[.3-.5]
$\bar{\nabla}_A$, α=60 H[.2-.5]
$\bar{\nabla}_D$, α=30 H[.4-.5]
$\bar{\nabla}_B$, α=20, H[.5-1.0]
$\bar{\nabla}_A$, α=50, H[.3-.6]
$\bar{\nabla}_A$, α=40, H[.3-.7]
$\bar{\nabla}_D$, α=20, H=.5
$\bar{\nabla}_A$, α=30, H[.4-1.0]
$\bar{\nabla}_A$, α=20, H[.5-1.0]
$\bar{\nabla}_B$, α=10, H=1.0
$\bar{\nabla}_A$, α=10, H=1.0
$\bar{A}$
30.
25.
20.
15.
10.
5.
0
.1
.2
.3

Figure IV-19

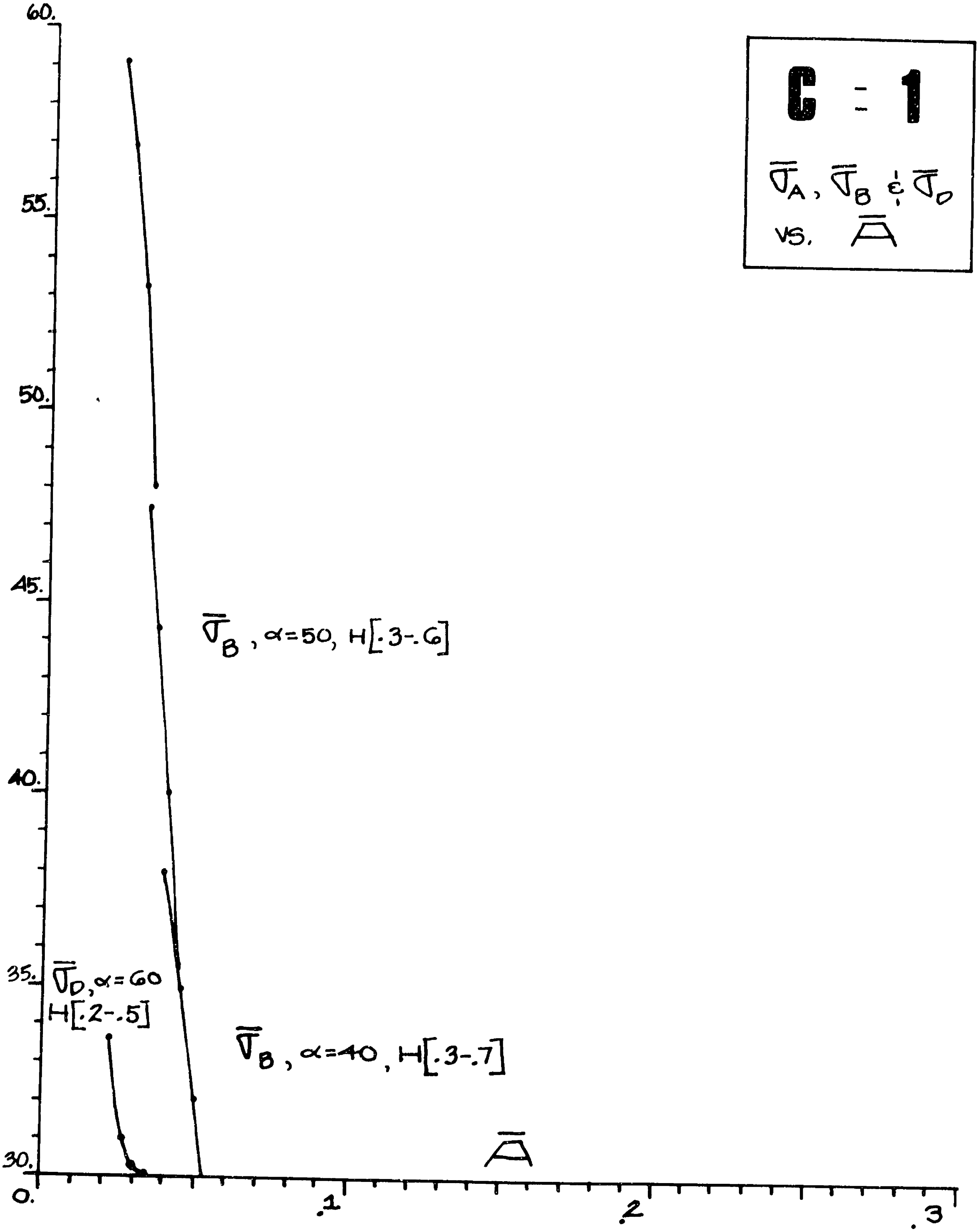
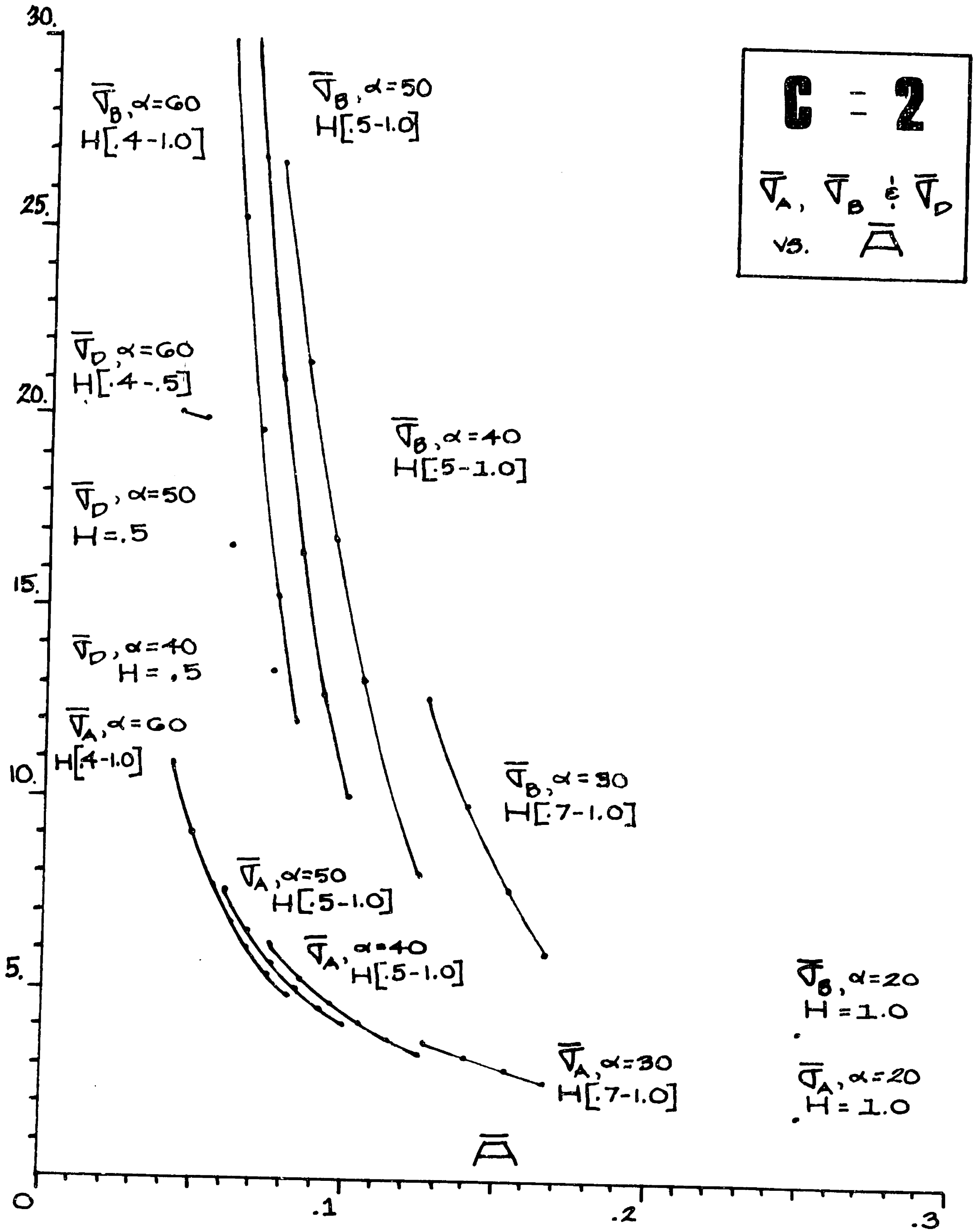

C = 1
$\bar{\sigma}_A$, $\bar{\sigma}_B$ & $\bar{\sigma}_D$ vs. $\bar{A}$
$\bar{\sigma}_B$, α=50, H[.3-.6]
$\bar{\sigma}_D$, α=60 H[.2-.5]
$\bar{\sigma}_B$, α=40, H[.3-.7]
$\bar{A}$
60.
55.
50.
45.
40.
35.
30.
0.
.1
.2
.3

Figure IV-20



C = 2
$\bar{\nabla}_A$, $\bar{\nabla}_B$ & $\bar{\nabla}_D$ vs. $\bar{A}$
$\bar{\nabla}_B$, α=60 H[.4-1.0]
$\bar{\nabla}_B$, α=50 H[.5-1.0]
$\bar{\nabla}_D$, α=60 H[.4-.5]
$\bar{\nabla}_B$, α=40 H[.5-1.0]
$\bar{\nabla}_D$, α=50 H=.5
$\bar{\nabla}_D$, α=40 H = .5
$\bar{\nabla}_A$, α=60 H[.4-1.0]
$\bar{\nabla}_B$, α=30 H[.7-1.0]
$\bar{\nabla}_A$, α=50 H[.5-1.0]
$\bar{\nabla}_A$, α=40 H[.5-1.0]
$\bar{\nabla}_B$, α=20 H = 1.0
$\bar{\nabla}_A$, α=30 H[.7-1.0]
$\bar{\nabla}_A$, α=20 H = 1.0
$\bar{A}$
30.
25.
20.
15.
10.
5.
0
.1
.2
.3

Figure IV-21

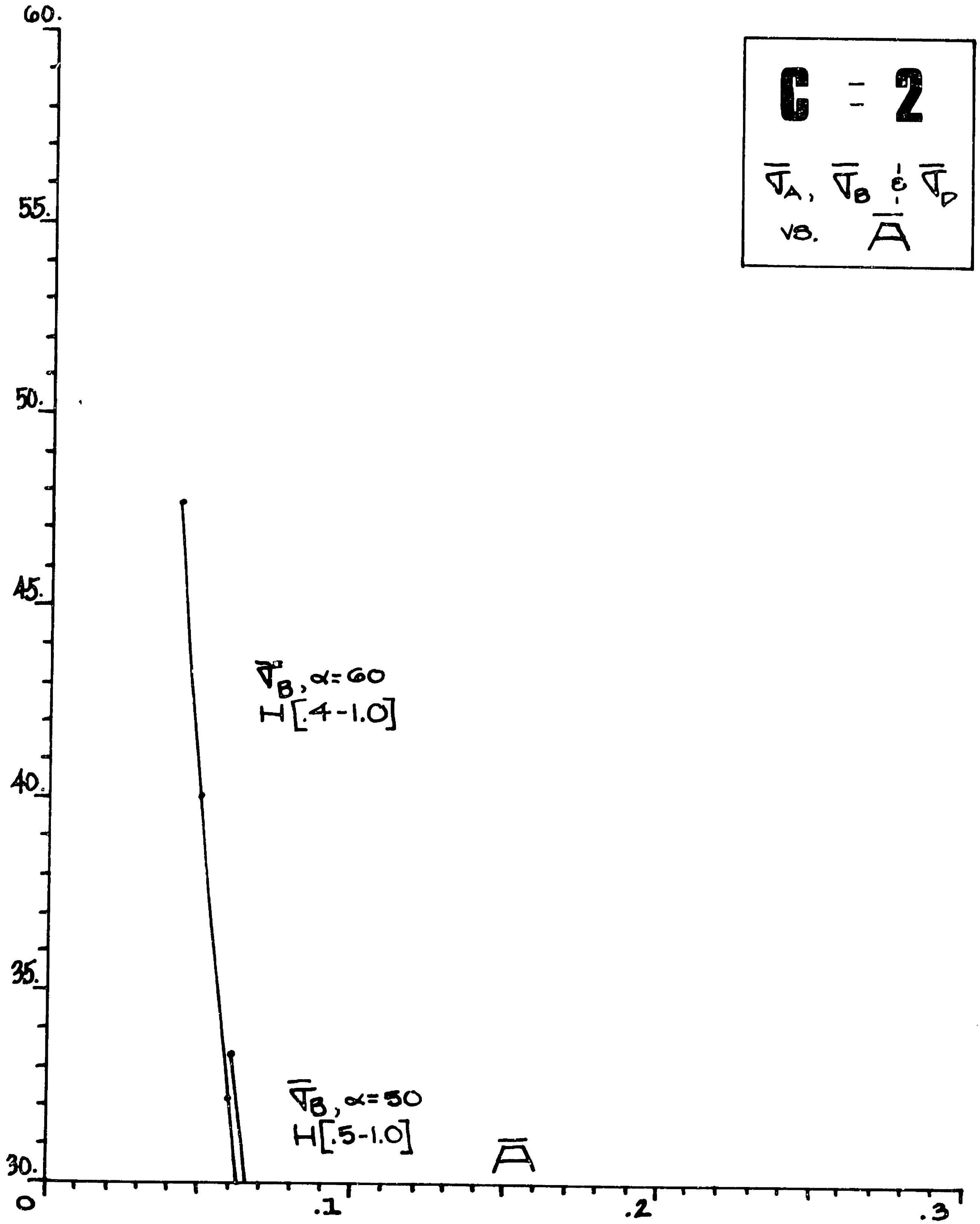

C = 2
$\bar{\sigma}_A$, $\bar{\sigma}_B$ & $\bar{\sigma}_D$
vs. $\bar{A}$
60.
55.
50.
45.
40.
35.
30.
$\bar{\sigma}_B$, α=60
H[.4-1.0]
$\bar{\sigma}_B$, α=50
H[.5-1.0]
$\bar{A}$
0
.1
.2
.3

Figure IV-22

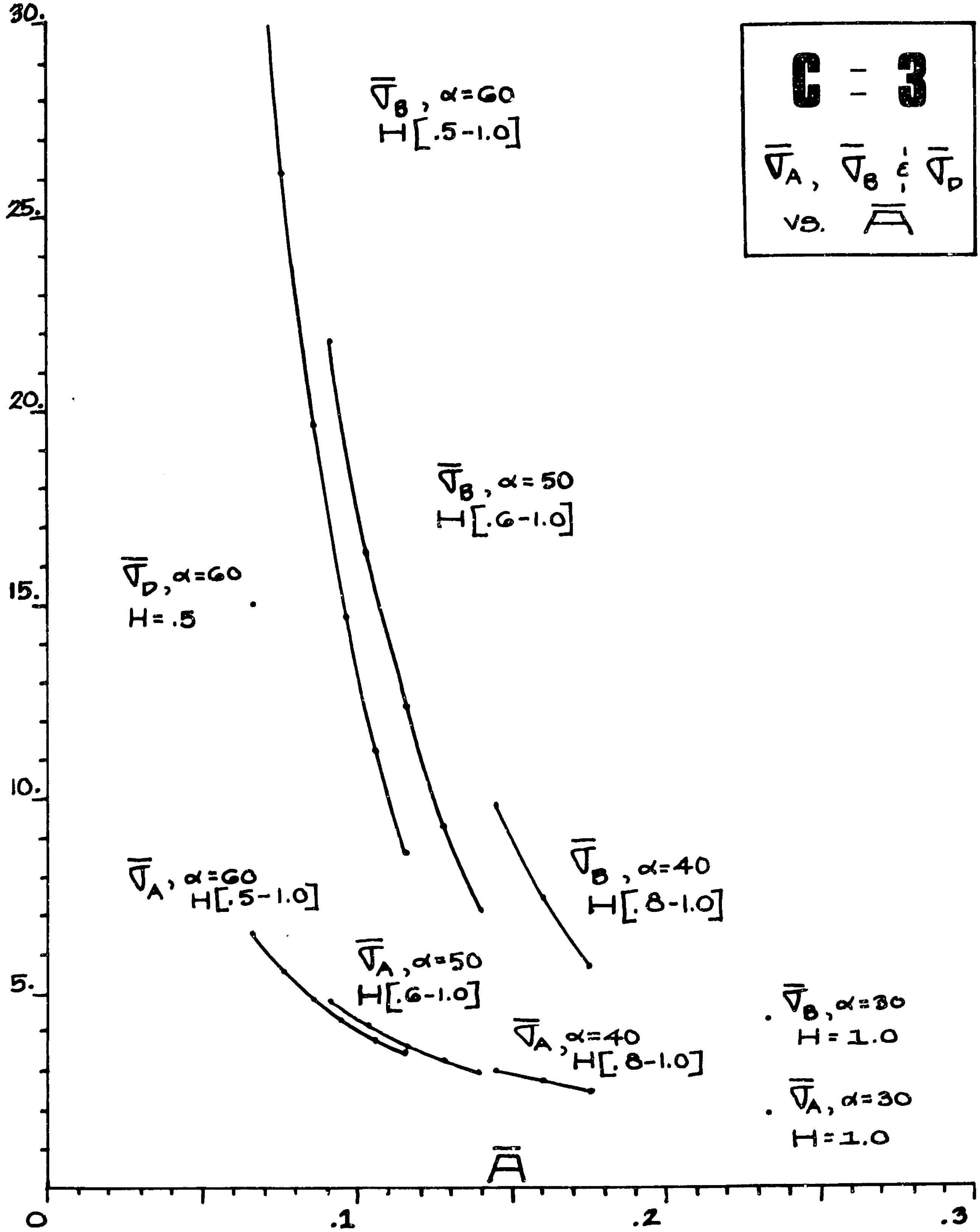

C = 3
$\bar{\nabla}_A$, $\bar{\nabla}_B$ & $\bar{\nabla}_D$ vs. $\bar{\bar{A}}$
$\bar{\nabla}_B$, α=60 H[.5-1.0]
$\bar{\nabla}_B$, α=50 H[.6-1.0]
$\bar{\nabla}_D$, α=60 H=.5
$\bar{\nabla}_B$, α=40 H[.8-1.0]
$\bar{\nabla}_A$, α=60 H[.5-1.0]
$\bar{\nabla}_A$, α=50 H[.6-1.0]
$\bar{\nabla}_A$, α=40 H[.8-1.0]
$\bar{\nabla}_B$, α=30 H=1.0
$\bar{\nabla}_A$, α=30 H=1.0
30.
25.
20.
15.
10.
5.
0
.1
.2
.3
$\bar{\bar{A}}$

Figure IV-23

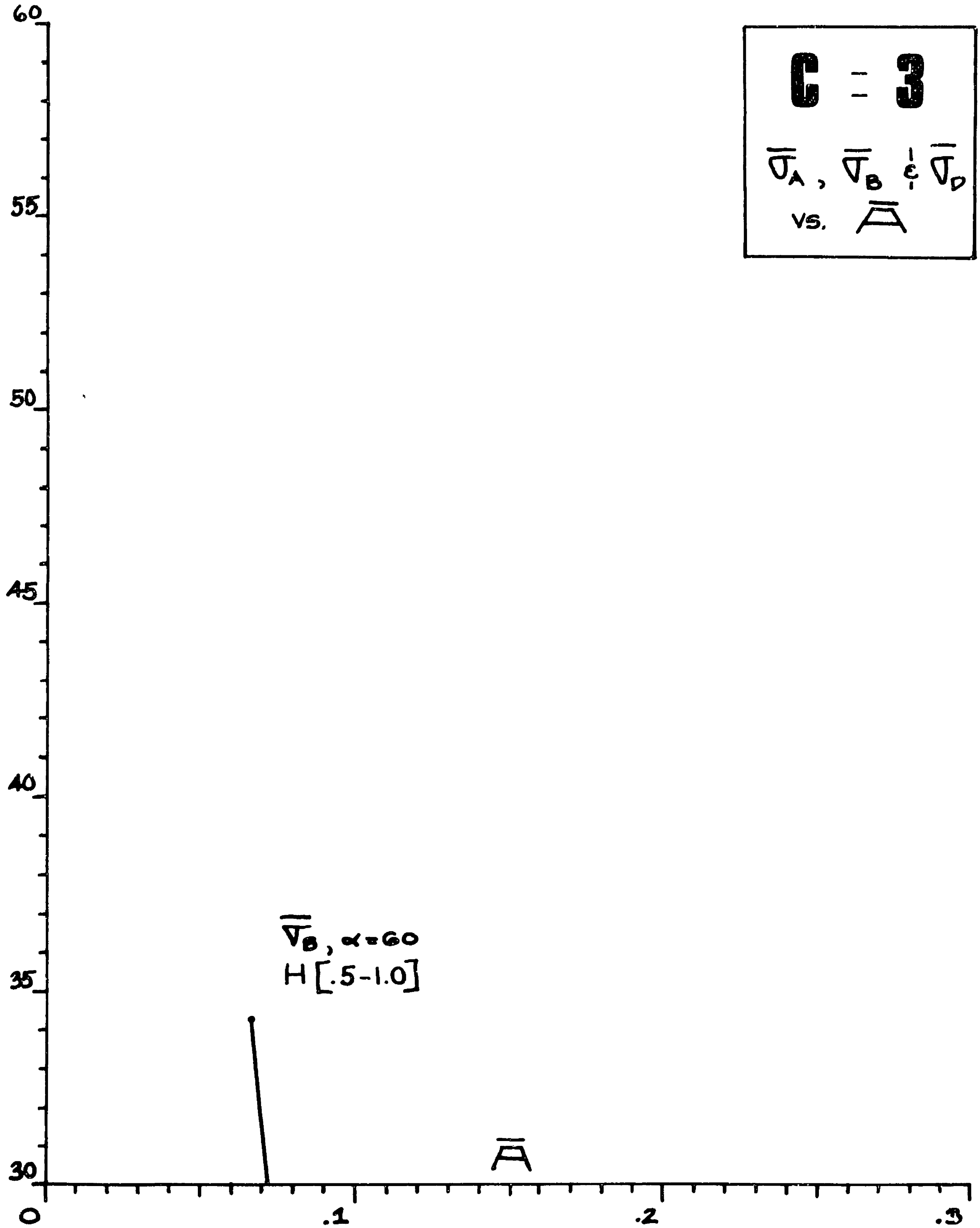

C = 3
$\bar{\sigma}_A$, $\bar{\sigma}_B$ & $\bar{\sigma}_D$
vs. $\bar{A}$
$\bar{\sigma}_B$, α=60
H [.5-1.0]
$\bar{A}$
60
55
50
45
40
35
30
0
.1
.2
.3

Figure IV-24

CONCLUSIONS

It may be concluded from the results obtained for the three classical cases used to illustrate the Stress Trajectory Technique that the present study has fulfilled its purpose within the limitations originally defined.

Specifically, for any two-dimensional isotropic material plane stress problem, the general state of stress can be described in detail by determining the state of stress, first. From the general state of stress it has been shown how the principal stress trajectories can be efficiently obtained.

The fact that only compressive stress trajectories are obtained at the present, implies that although no mention is made of instability, eventually the investigator of optimum arrangement structures aims to arrive at the stage that he can handle problems in which instability is a factor.

This would involve an optimum sizing of the particular truss members or plate stiffeners, in order to resist buckling, as well as a particular spacing of the individual frames.

In this work, however, although instability is ignored, implicitly it is being considered. When one refers to a tie for example, only the maximum allowable tensile stress σ_T is of importance. In the case of a strut, however, the designer has to take into account the σ_{cr} stress as well as the maximum allowable compressive stress σ_c.

In reducing σ_c by adding material on the location where the principal compressive stress is a maximum, we efficiently increase σ_{cr}.

It has been tentatively shown that the arrangement which adds the reinforcing material on the location of the principal stress in every cross section throughout the planar structure is the least weight arrangement.

If one observes the texture of a sea shell he can justify its light structure by noticing that its "ribs" are in alignment with the compressive "Michell lines" (or the stress trajectories) of the lower arc of the structure of Figure I-B.3; where the radially spaced spokes could be carrying the hydrostatic pressure that the shell carries when the animal in it is alive and the shell stands in the upright position on the bottom of the sea.

The optimization (least weight) of structural framework is a relatively new field ever since men left the caves and started building houses and boats.

The study of the weight minimization in structural design assisted by the Finite Element Technique will efficiently arrive at revolutionary results for structural design; especially, with the increasing demand for composite materials, which are being used in nuclear submarine structural design, and where the particular matrices of the structure can be analyzed only by filamentary-type structures such as those of Maxwell's or Michell's.

APPENDICES

Appendix I-A

APPLICATION OF MAXWELL'S THEOREM AND LEMMA

Example 1.

Let us consider a truss problem,[3] the solution of which is given in Figure I-A.1. The external loads are 10^K, 12^K, 12^K, and 8^K, as shown. The reactions can be easily obtained either by a graphical solution or algebraically; their magnitudes are 6^K, 21.9^K and 18.1^K, and their directions are as shown.

Using either Bow's notation[3] or applying equilibrium of forces at each node, the tensions and the compressions acting along the rods have been found and written--the + sign indicating tension and the - sign compression.

(a) Applying Maxwell's Theorem, the equilibrium of the "small smooth rings", as the nodes are described, is undisturbed and the forces are as shown in Figure I-A.2. The internal tensile forces and the external applied loads have turned 90° counterclockwise, while all the internal compressive forces and the reactions have turned 90° clockwise.

(b) If we now apply Maxwell's Lemma to Figure I-A.2, we should have:

Σ[external and internal loads creating moments clockwise]
= Σ[external and internal loads creating moments counterclockwise] [I-A.1]

which is another way of interpreting equation [1.1].

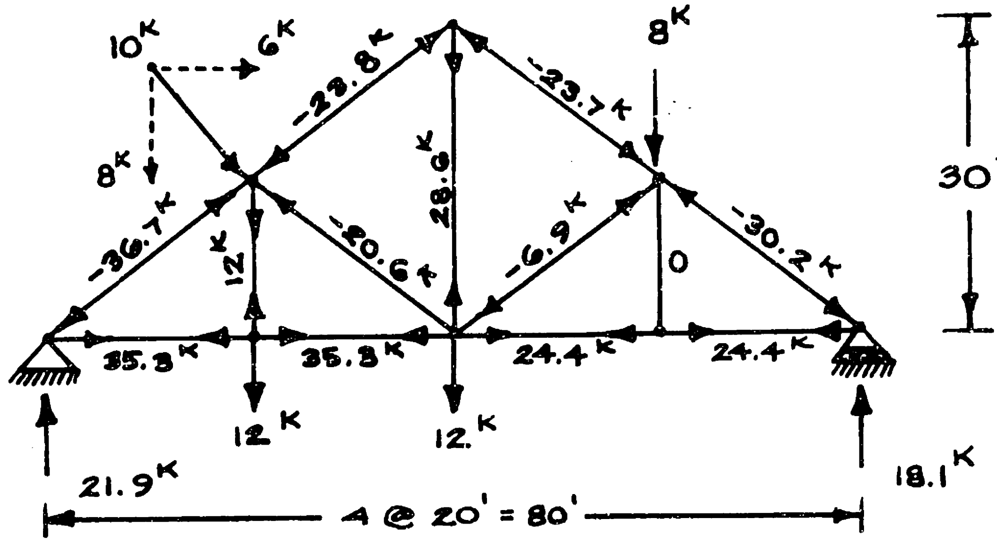


Figure I-A.1

Truss of Example 1

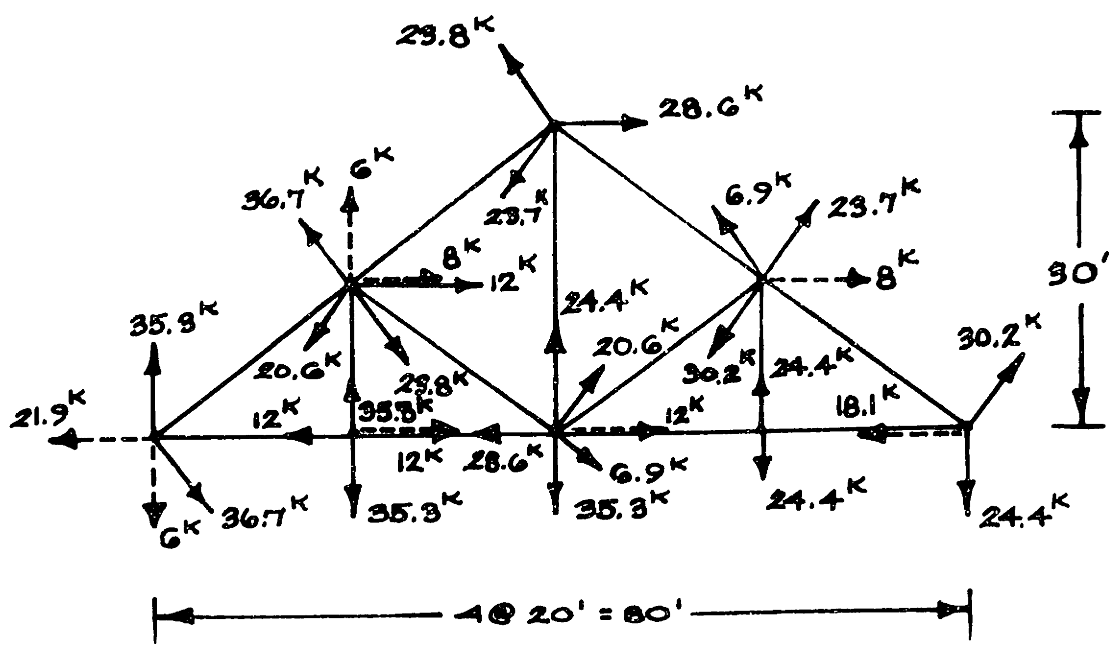


Figure I-A.2

Maxwell's Equivalent Equilibrium

Therefore:

$$\text{Left-hand-side of equation [1.1]} = \overbrace{8^K \times 15' + 8^K \times 15'}^{\text{applied vertical forces}}$$

$$\overbrace{+ 35.3^K \times 20' + 35.3^K \times 20' + 24.4^K \times 20' + 24.4^K \times 20' + 28.6^K \times 30'}^{\text{internal tensile forces}} = 240^{K \cdot FT} + 3426^{K \cdot FT} = 3,666^{K \cdot FT}$$

Note that only the vertical component of the 10^K external applied force is considered part of the summation of the clockwise moments.

$$\text{Right-hand-side of equation [1.1]} = \overbrace{6^K \times 20'}^{\text{horizontal reactions}}$$

$$\overbrace{+ 36.7^K \times 25' + 23.8^K \times 25' + 30.2^K \times 25' + 20.6^K \times 25' + 6.9^K \times 25'}^{\text{internal compressive forces}} = 120^{K \cdot FT} + 3,547.5^{K \cdot FT}$$

$$= 3,667.5^{K \cdot FT}$$

Therefore, the two sides of equation [I-A.1] are equal as they should be according to Maxwell's Lemma. Their small discrepancy in the fourth significant digit is due to the round-off error in the calculation of the internal loads and the reactions.

Example 2.

Consider the truss of Figure I-A.3a with the applied external load vertical to the direction of bar CB.

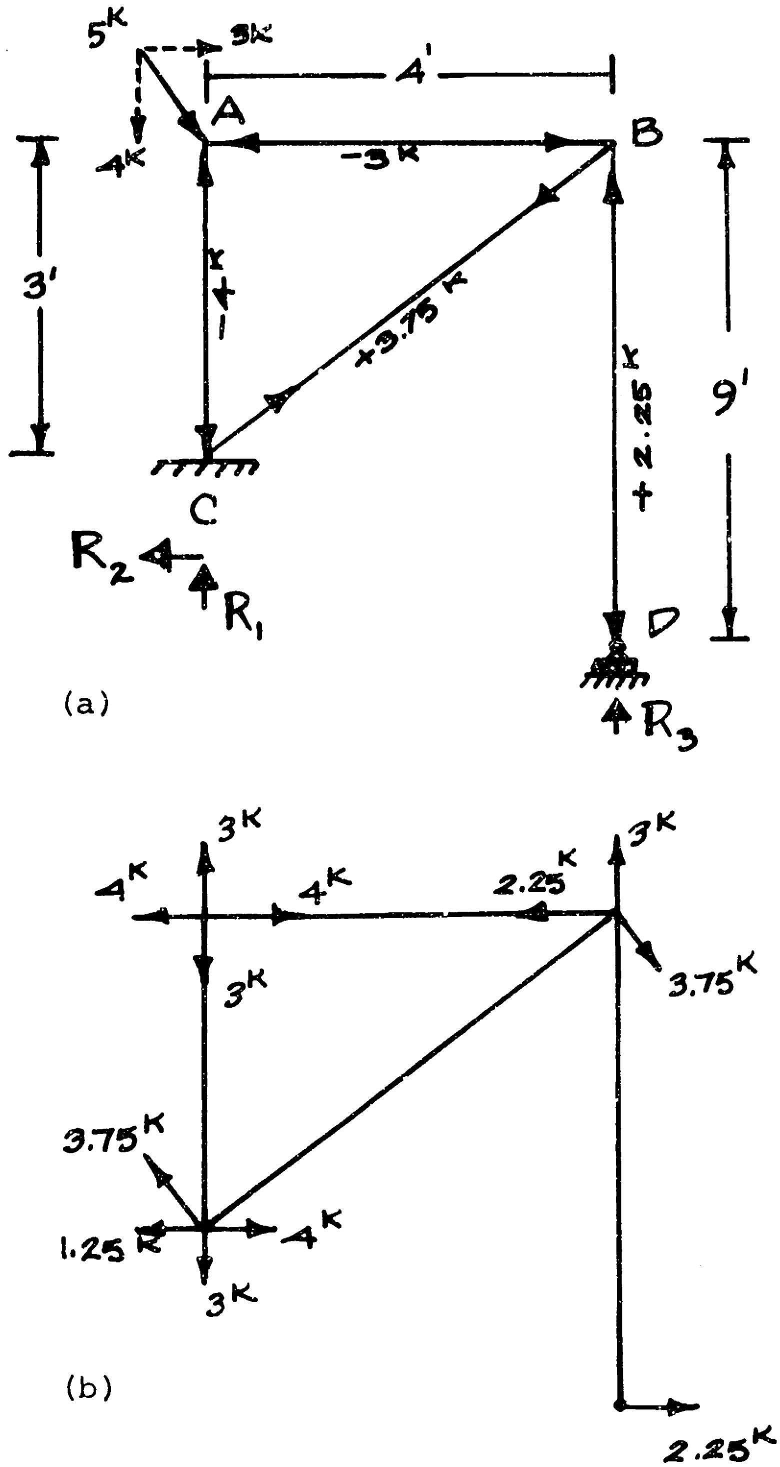


Figure I-A.3

(a) Truss of Example 2

(b) Maxwell's Equivalent Equilibrium

Applying equilibrium of forces and moments, we find the force reactions to be:

$$R_1 = 1.25^k$$
$$R_2 = 3^K$$
$$R_3 = 2.25^K$$

Equilibrium at each of the nodes yields the internal tensile and compressive forces as shown.

(a) Applying Maxwell's Theorem, all the forces turn 90° and create moments in the clockwise or counterclockwise direction depending on whether they are tensile or compressive, respectively, and applied or reactions, respectively (Figure I-A.3b).

(b) According to Maxwell's Lemma or equation [1.1]:

$$\text{Left-hand-side of equation [1.1]} = \overbrace{4^K \times 3'}^{\text{applied vertical forces}} \overbrace{+\ 3.75^K \times 5'}^{\text{internal tensile forces}} = 12^{K\cdot FT} + 18.75^{K\cdot FT} = 30.75^{K\cdot FT}$$

Right-hand-side of equation [1.1] =

$$= \overbrace{-2.25^K \times 6'}^{\text{horizontal reactions}} + \overbrace{4^K \times 3' + 3^K \times 4' + 2.25^K \times 9'}^{\text{internal compressive forces}}$$

$$= -13.50^{K\cdot FT} + 24^{K\cdot FT} + 20.25^{K\cdot FT} = 30.75^{K\cdot FT}$$

Therefore, the left-hand-side = the right-hand-side in equation [1.1].

Notice that the first term has a negative sign to account for the height since 0 height is considered at point C.

Appendix I-B

APPLICATIONS OF THE MICHELL STRUCTURES

Example 1.

Let us assume that we have two points a distance, AB, apart and a load, P, vertical to AB and pointed at, A, which must be equilibrated by a reaction R, and a moment, M, at the other end, B.

We now want to build the optimum (least weight) structure that would equilibrate these loads. Michell provides the solution that appears in Figure I-B.1. The minimum frame is formed by two symmetric equiangular spirals about AB, which have their origin at B and intersect orthogonally at A, together with all the smaller spirals enveloped inside these two.

The circle of radius, r, about point B justifies the fact that there cannot be an infinite amount of material at B. We can imagine according to the interpretation given by R. L. Barnett(7) that the cantilever is supported on a shaft of radius, r.

The least necessary volume is given by

$$V_{least} = PL \cdot \log\frac{L}{r} \left(\frac{1}{\sigma_{all._T}} + \frac{1}{\sigma_{all._C}}\right) \qquad [I\text{-}B.1]$$

where L = (AB).

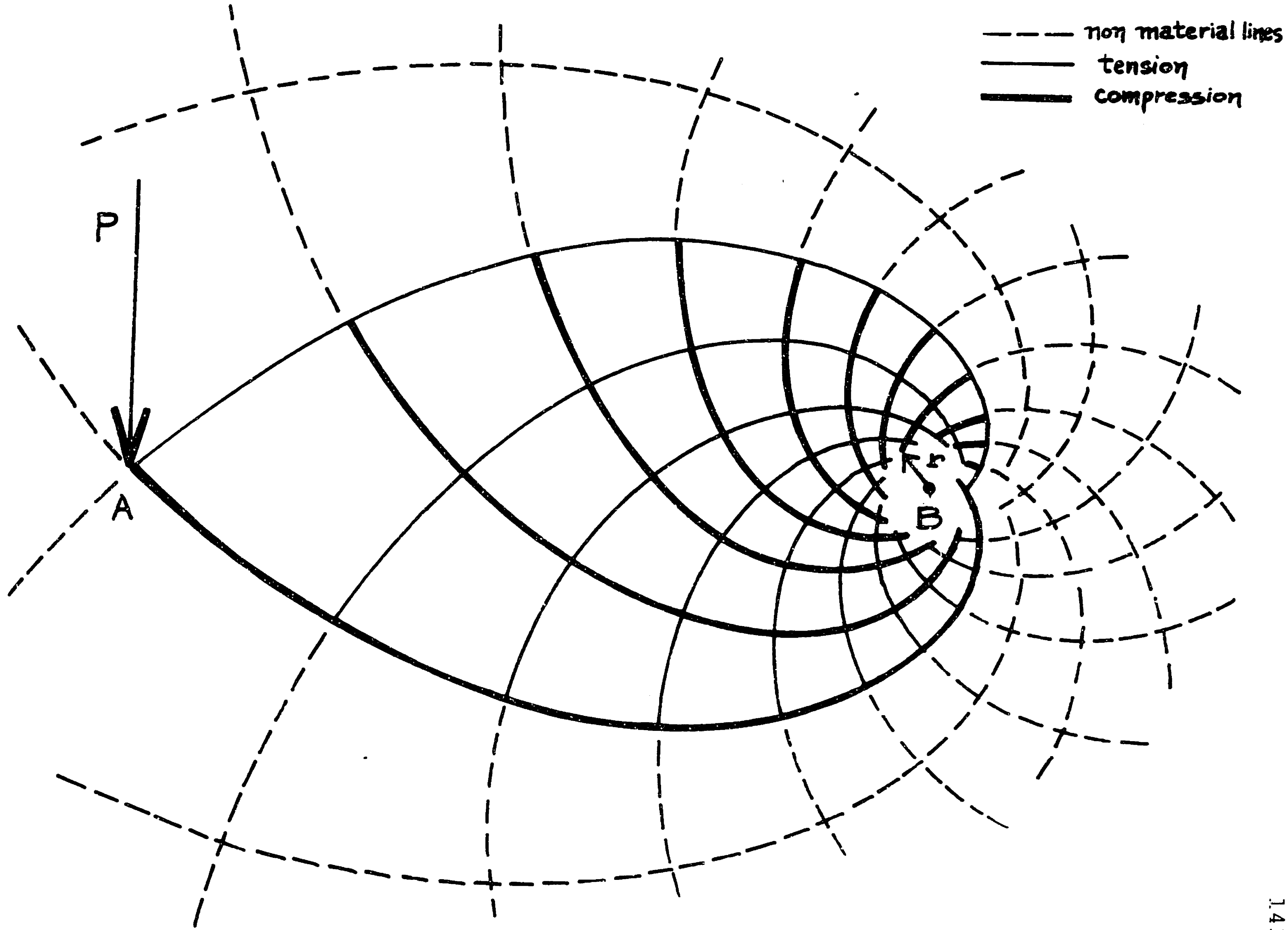


Figure I-B.1 Michell Field: Layout for a Cantilever

A comparison between the Michell Field of Figure I-B.1 and the principal stress trajectories of Figure I-B.2 reveals their similarity. Figure I-B.2 has been reproduced here for convenience from the accompanying figure of Problem 7.55 of Crandall and Dahl.(8)

An expansion of the first example of the Michell Field is given in detail by D. M. Richards and H. S. Y. Chan.(9)

Example 2.

Another least-weight structure arrangement given by Michell and expanded by H. L. Cox,(10) and W. S. Hemp(6) is the following:

Consider that we want to build a structure to support three point loads, their points lying in a straight line so that point C is at the midspan of AB. In seeking the optimum structural arrangement, Michell provides the solution shown in Figure I-B.3.

The least volume of material for the centrally loaded beam is

$$V_{least} = P\frac{L}{2}\,(1/2 + \pi/4)\left(\frac{1}{\sigma_{all._T}} + \frac{1}{\sigma_{all._C}}\right) \qquad \text{[I-B.2]}$$

where L/2 = AC = CB.

Maxwell could have solved for the least volume if he knew the arrangement *a priori*. Applying equation [1.8] in Figure I-B.3, we have

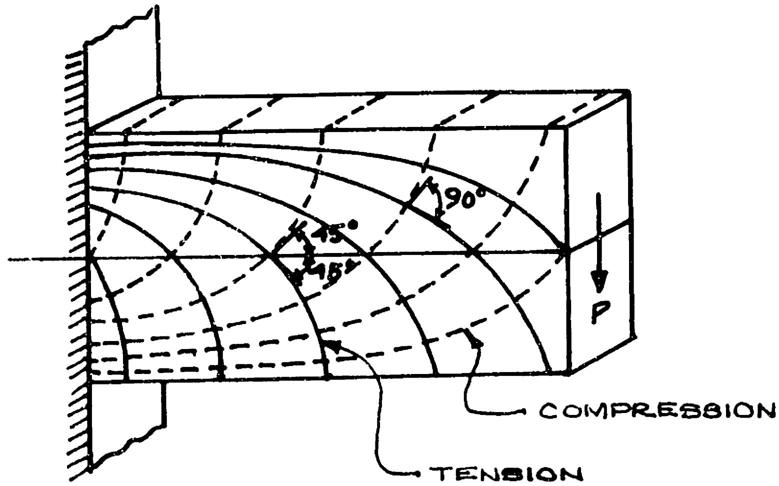


Figure I-B.2 The Principal Stress Trajectories for a Cantilever Appearing in Crandall and Dahl.[8]

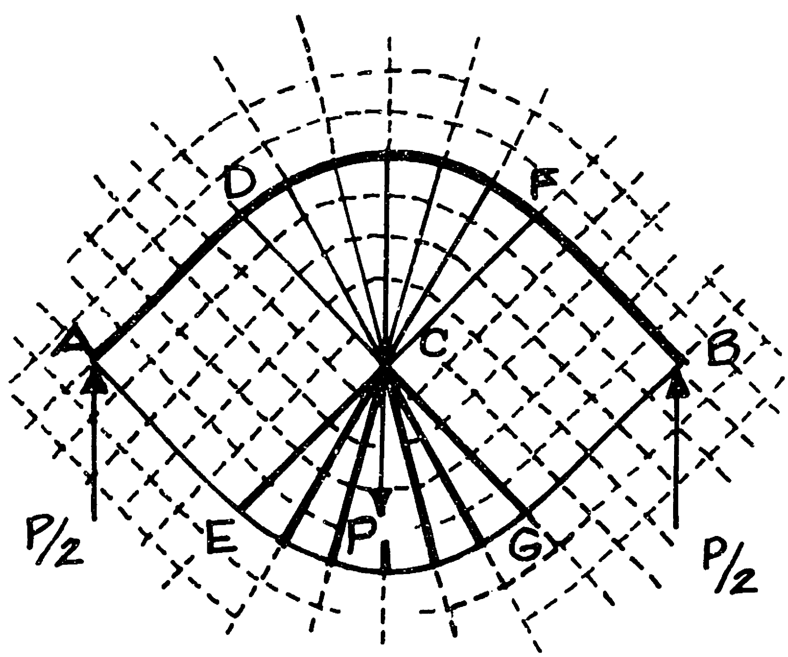


Figure I-B.3 Michell Field: Layout for a Centrally Loaded Beam

$$V_{least} = [(AE + EG + GB)\frac{P}{2} \cos 45° + (DF)\frac{P}{2} \cos 45°] \frac{1}{\sigma_{all._T}}$$

$$+[(AD + DE + FB)\frac{P}{2} \cos 45° + (EG)\frac{P}{2} \cos 45°] \frac{1}{\sigma_{all._c}}$$

$$=[(\frac{L}{2\sqrt{2}} + \frac{\pi L}{4\sqrt{2}} + \frac{L}{2\sqrt{2}}) \frac{P}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\pi L}{4\sqrt{2}} \cdot \frac{P}{2} \cdot \frac{\sqrt{2}}{2}]$$

$$\cdot \left(\frac{1}{\sigma_{all._T}} + \frac{1}{\sigma_{all._c}} \right)$$

$$= \frac{LP}{2} (\frac{1}{2} + \frac{\pi}{4}) \left(\frac{1}{\sigma_{all._T}} + \frac{1}{\sigma_{all._c}} \right)$$

Q.E.D.

Notice that the terms $(DF)\frac{P}{2} \cos 45°$, $(EG)\frac{P}{2} \cos 45°$ represent the volume of all the radial members of the arcs $\overset{\frown}{DF}$ and $\overset{\frown}{EG}$ respectively. Since buckling is not considered here, the necessary material is independent of the number of spokes. Therefore, instead of taking the summation of the products of each of the spokes in quadrants DCF and ECG times their associated tensile and compressive loads which are not known, we consider the statically equivalent case: $\overset{\frown}{DF}$ and $\overset{\frown}{EG}$ are in tension and in compression respectively, from which we obtain $(DF)\frac{P}{2} \cos 45°$ and $(EG)\frac{P}{2} \cos 45°$.

If any design constraint is imposed on the second Michell example, the necessary volume will be greater than that of equation [I-B.2].

Example 2.1. Let us consider the second example again but with the constraint that all of the necessary material to equilibrate the three point loads must lie on either side of AB. Michell's solution to that problem is directly applicable to the problem of the optimum stiffening arrangement of a bicycle wheel. (See Figure I-B.4)

The least volume structure for half the wheel is easily obtained by applying equation [1.8] as before:

$$V_{least} = P \cdot \frac{L}{2} \cdot \frac{\pi}{2} \left(\frac{1}{\sigma_{all._T}} + \frac{1}{\sigma_{all._C}} \right) \qquad [I\text{-}B.3]$$

Comparing the unconstrained V_{least} with that of the constrained one we see that

$$\frac{[V_{least}] \text{ constrained}}{[V_{least}] \text{ unconstrained}} = \frac{2\pi}{2 + \pi} \qquad [I\text{-}B.4]$$

which implies that any imposed constraint on the arrangement of the truss members or the stiffeners for a Michell type structure increases the least possible weight of the unconstrained structure.

Furthermore, this last example illustrates well the similarity of the Michell fields with the principal stress trajectories. Although Figure I-B.5 shows reverse loading from that of Figure I-B.4, the similarity is clear. The difference being that load P is radially transmitted by the principal compressive trajectories, and the reactions are being carried by the concentric semicircular principal tensile trajectories.

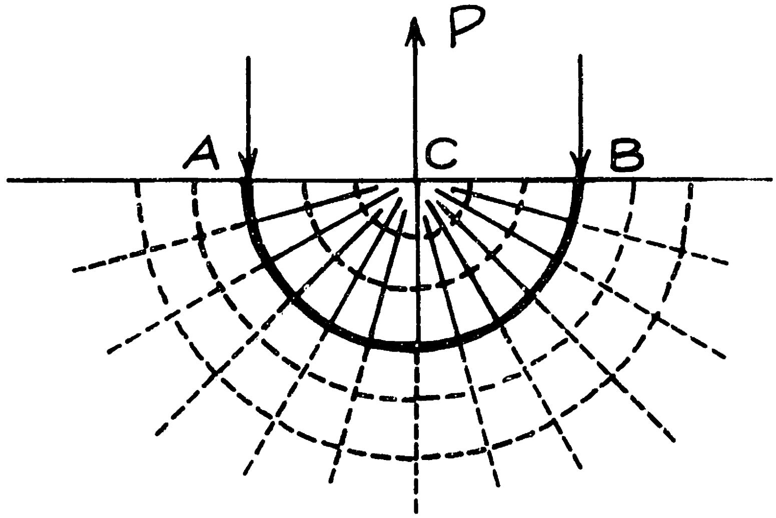


Figure I-B.4

Michell Field: Layout of a Centrally Loaded Beam, Whose Frame Lies Completely in the Semiplane of AB

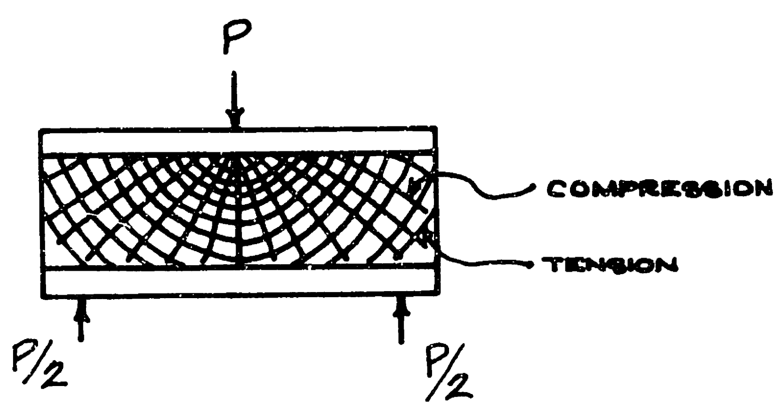


Figure I-B.5

Principal Stress Trajectories in an "I" Beam by A. W. Hendry[11]

Appendix II-A

THE COMPUTATIONAL PROCEDURE

1. Introduction

The computer program listed and explained here is written in FØRTRAN IV, for the G level compiler of the IBM 360 computer model.

Its objectives, some of which have been already introduced in Part II are:

(1) To calculate the nodal displacements {q} from the stiffness relationship of equation [2.1]. For this calculation:

(a) The element stiffness matrix [K], as defined in Table II-2, must be assembled to give the stiffness matrix $[K]_{total}$ for the total structure.

(b) The "force entries" {Q} must be defined

(i) either from the applied concentrated forces $\{\bar{Q}\}$ or

(ii) from the distributed boundary loads which are converted to energy equivalent nodal forces.

(2) To calculate the average stresses at each node. This calculation involves:

(a) The use of equations [2.31] and [2.32] in which the stress σ_x, σ_y, and τ_{xy} are functions of the nodal displacements, q.

(b) The calculation of the average stresses at each control point which is the common point of two or four elements.

(3) To determine the points in terms of the x, y coordinates as well as the principal compressive stress at each point and for each compressive stress trajectory according to the analytical and detailed explanation given in the last section of Part II.

(4) To plot the principal compressive stress trajectories. That is, to connect the points of each stress trajectory by straight lines and print the value of σ_2 at each point.

(5) To reduce the stress level in the structure of a plane stress problem by increasing the thickness of each Finite Element which lies on any principal compressive stress trajectory.

Furthermore, for this purpose the program should provide the option of using one or more trajectories and not necessarily all of them. The reason is that a closely spaced number of trajectories can result in a doubling of the thickness of the original unreinforced structure. It must, for example, be capable of increasing the thickness of the elements associated with three out of nine stress trajectories, every three trajectories, that is, j = 3, j = 6, j = 9 (where j is the trajectory number as defined in Figure II-3) without increasing the thickness of the elements associated with the other stress trajectories.

2. General Description--Macro Flowchart

The complete program consists of the following Main Program (MAIN) and subprograms:

a. MAIN:

(1) The geometry of the structure is defined. This involves the total number of elements to be used as well as the overall dimensions of the entire structure, the initial thickness of the structure, Young's Modulus of Elasticity and Poisson's Ratio.

Each nodal point of the structure is assigned a number starting from the lower left corner up and counting always from bottom to top.

Each element is thus defined in terms of the four nodal points (see computer printout in the Results Section). Also, each element's side may be thus defined in terms of the adjacent nodal points.

(2) The Boundary Conditions of the structure must be defined. That involves the reading in of the total number of restrained nodes in each direction as well as the particular node numbers which are restrained in the corresponding direction. (Look at the printout results for the example of a cantilever in the Results section.)

The variable NØDEN, (see Nomenclature and Computer printout) is also defined in MAIN to be used later in subroutine STRESS for the calculation of the average stresses per node.

Once the Geometry and the Boundary Conditions for the entire structure have been defined, the procedure commences.

An option is given as to how many times it is desired to go through the entire procedure. That is, if NUM = 1, the procedure stops, at the most after it has accomplished the determination of all the elements which are transversed by all the stress trajectories. If NUM = 2, 3, 4, the entire procedure is repeated until a new plot of stress trajectories is obtained, corresponding to the trajectories of the reinforced structure. At this point, the program may either stop or continue until it has accomplished the determination of all the elements which are crossed through by all the new stress trajectories (see Results Section).

b. <u>STFMTX</u>:

In subroutine STFMTX the element's stiffness matrix is defined. Actually, all the components of BIGK(I,J) must be multiplied by the thickness THIC to give the exact components of the stiffness matrix [K] as defined by equation [2.17] and Table II-2.

c. <u>INIT</u>:

Subroutine INIT is a complete codified package of subroutines for the solution of a general set of linear algebraic equations of the form

$$[A]\{U\} = \{Q\} \qquad [II\text{-}A,1]$$

where [A] corresponds to the assembled stiffness matrix $[K]_{total}$, {U} is the column matrix which contains the unknown displacements, {q}, and {Q} is the column matrix which contains the "force entries" per degrees of freedom as they are stored in subroutine FØRCER.

INIT is the name of the complete package of the following subroutines: INIT, SETUP, LØADER, RSTRN, and SØLVER, each of which is a step to obtain the solution of the simultaneous equations.[17]

Thus, INIT satisfies the first objective of the program which is to calculate the two displacements, q, per node for the entire structure.

d. FORCER:

In FØRCER, the applied concentrated forces or the distributed load intensities per node of action on the boundaries of the structure are read in. In the first case they are directly stored in the one-dimensional array FØRCE. Before they are destroyed, since the calculated q's are also stored in FØRCE, they are stored in two dummy dimensional arrays FSTØRX and FSTØRY. Before the Main program calls SØLVER, where the nodal displacements q are obtained, the two forces per node column matrix {Q}, or FØRCE in this case, are printed out under the title "Force Entries Two Per Node in Order," (see computer printout for the cantilever case which is loaded on its free end by a parabolic distribution of discrete nodal forces).

In the second case, the load intensities of a distributed load per node of action are converted to energy equivalent concentrated forces and then stored in FØRCE. Similarly to the first case, before the energy equivalent nodal forces are destroyed, they are stored in QDUMMY which is a dummy one-dimensional array. Before the displacements q are calculated for this case the equivalent forces of the one-dimensional array FØRCE are printed out under the same title as that of the first case, (see computer printout for the cantilever case which is loaded on its free end by a parabolic distribution of load).

e. STRESS

Subroutine STRESS uses the displacements {q} to solve for the stresses $\{\sigma\}^e$, (equation [2.31] and [2.32]) at the four nodes of each element.

Starting thus with the first element, STRESS calculates the average stresses $\{\sigma\}^s_{average}$ at each node of the entire structure, (see the listing of STRESS for details).

f. TRAJEC

Subroutine TRAJEC uses the average stresses $\{\sigma\}^s_{average}$ to obtain the x, y coordinates of the principal compressive stress trajectories as well as the magnitude of the principal compressive stresses at each point (x, y) of all the trajectories.

g. PLTTRJ:

It uses the results of TRAJEC to plot the stress trajectories and includes instructions for the IBM 360 CALCØMP Plotter.(18)

h. STIFEN:

This subroutine identifies the element numbers transversed by each trajectory. These elements may be used in the Main to reduce the stress level of the entire structure if their thickness is properly increased. Thus, the structure may be stiffened along the paths of the stress trajectories.

The following Figure II-A.1 is a Macro-Flowchart which lends better understanding to the program's architecture.

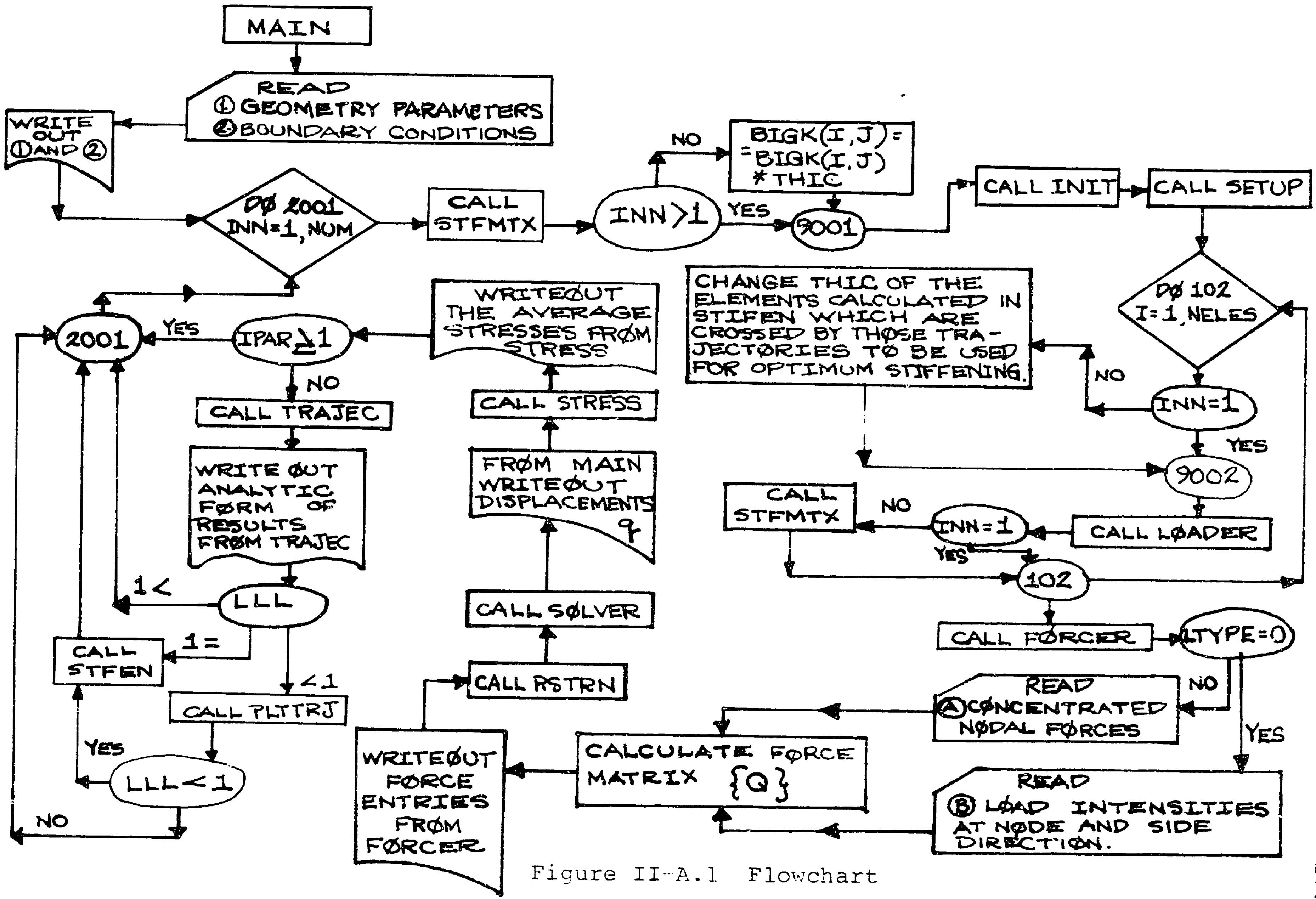


Figure II-A.1 Flowchart

3. Nomenclature and Program's Specifications

ALENX(1):

A single dimension variable indicating the length of the (rectangular) element in the x-direction. If there are more than one size of rectangular elements, then the dimension of ALENX will correspond to that number of sizes.

ALENY(1):

The same as above, but corresponding to the y-direction.

ALX:

The total length of the structure in the x-direction.

ALXX:

The length of one element in the x-direction.

ALY:

The total length of the structure in the y-direction.

ALYY:

The length of one element in the y-direction.

BIGK (IØCT, IØCT):

A doubled-dimensioned variable in which all the components of the one element stiffness matrix [K] (over the thickness t of the structure) are stored.

BIGL( ):

A single array defined in subroutines INIT and SETUP which is used for the storage of the lower triangle of the coefficient matrix. BIGL includes the diagonal terms.[17]

BIGU( ):

A single array in subroutine SETUP that is used for storage of the upper triangle of the coefficient matrix, and which does not include the diagonal terms.[17]

DET:

The determinant of the constrained stiffness matrix of the assembled structure calculated in SØLVER (see also ITEST).[17]

DX:

An argument of AXIS1[18] called from PLTTRJ indicating the number of length units in the x-direction per actual inch of plot paper to be printed out for the number labeling along the x-axis.

DY:

The same applies as in DX, but corresponds instead to the y-direction.

E:

Young's Modulus.

FØRCE(IFØRCE):

A single array corresponding to the column matrix {q} where the displacements for the entire structure are stored after they have been calculated in SØLVER and printed out from the Main program. The array must be at least the number of degrees of freedom of the total structure in length. Before the solution {q} is stored in FØRCE. FØRCE contains the "force entries" of the entire structure, (that is, for programming economy purposes, the same memory space is used twice to store the known "force entries" and after solving for the q's to store them there). This can be achieved by means of the JORDAN Reduction Method for the solution of linear algebraic simultaneous equations.

FØRVAL(IDSTR):

. A work vector in FØRCER dimensioned at least equal to the number of nodes of the entire structure. (see also FSTØRX, FSTØRY).

FS(IXF1):

A single array, argument of SCLGPH[18] called by PLTTRJ and used for storage for the y-coordinates of the number of points per trajectory to be plotted.

FSTØRX(IDSTR):

The same definition applies as in FØRVAL above.

FSTØRY(IDSTR):

The same definition applies as in FØRVAL above.

FT(IT):

A work vector used in PLTTRJ which stores the maximum number of the y-coordinates of all the stress trajectories to be plotted.

HEADNG(15):

A variable of subroutine PLTTRJ in which the heading title for the plot of the principal compressive stress trajectories, is stored in an alpha-numeric field in

15 groups of four letters per group giving a 60-character range for the title.

I1KELE:

A dummy dimension variable corresponding to a maximum anticipated number of elements through which one stress trajectory passes. (see KELEM)

I2KELE:

A dummy dimension variable corresponding to the number of stress trajectories which are to be used for the reinforcing of the structure. (see KELEM)

IDIR:

A dummy dimension variable, equal to "2" in the present study, but may correspond to the number of sides in different directions that define the boundaries of a structure.

IDNØDE:

A dummy dimension variable corresponding to the number of elements in a structure. (see also NØDE)

IDST1:

A dummy dimension variable corresponding to the maximum number of stations along the x-direction on which the stresses $\{\sigma\}$ are defined for the determination of the stress trajectories.

IDST2:

A dummy dimension variable corresponding to the number of stress trajectories and is used as a second index to distinct the stresses $\{\sigma\}$ at the same station among the different trajectories that pass through that station.

IDSTR:

A dummy dimension variable corresponding to the number of nodes of the entire structure and used in many subscripted variables.

IFØRCE:

A dummy dimension variable equal to the number of degrees of freedom of the entire structure. (see also FØRCE)

IIPØIN:

A dummy dimension variable which is equal to at least the number of trajectories of the structure. (see also IPØINT)

ILINUM:

A dummy dimension variable corresponding to twice the number of the total line segments on the boundaries of the structure.

IMPØIN:

A dummy dimension variable corresponding to the number of stress trajectories. (see also MPØINT)

INN:

The index of the big DØ loop which goes from 1 to the number of times it is desired to go through the entire procedure.

INØDEN:

A dummy dimension variable corresponding to the number of nodes of the entire structure. (see also NØDEN)

IØCT:

A dummy dimension variable corresponding to the number of vertical or horizontal components of the stiffness matrix for one element.

IPAR:

A code variable which, if zero, the program goes through the calculation of the stress trajectories; if one, the program stops with the calculation of the average stresses at each node of the structure.

IPØINT(IIPØIN):

A single dimension variable which stores the number of y-coordinates per trajectory.

IT:

A dummy dimension variable equal to the total number of the x, y-coordinates of all the stress trajectories which at the most will equal the number of nodes of the structure.

ITEST:

A code which is defined to be zero for no calculation of the determinant in subroutine SØLVER.[17]

IVCT(4):

A single array used in the Main program and subroutines INIT, LOADER as an array of master node numbers for the element and is number of nodes/element in length.[17]

IWØRK( ):

A work vector at least equal to the number of degrees of freedom of the total structure and is used in subroutines INIT and SETUP.[17]

IXF1:

A dummy dimension variable corresponding to the number of stations along the x-direction on which the y-final coordinates of the stress trajectories are calculated.

IXF2:

A dummy dimension variable equal to the number of stress trajectories.

JSTFN:

The number of trajectories associated with the stiffening of the structure. There can be used less trajectories then these plotted for the best stiffening of the structure.

JTEST:

A code variable which equals zero for no dumping of pointer data in SETUP.[17]

JWØRK( ):

A work vector equal at least to the number of degrees of freedom of the total structure used in INIT and SETUP.

KELEM(I1KELE, I2KELE):

A two-dimensional array which defines the element number associated with each stress trajectory.

KKWØRK(4):

A single array used in INIT and LØADER as a vector which contains the number of degrees of freedom at each node of the element. It must be dimensioned according to the number of nodes per element.

KWØRK(4):

A work vector in LØADER at least the number of nodes per element long.

KEXPX:

An argument of AXIS1 called from PLTTRJ which specifies the exponent to be printed on the label of the x-scale, (that is, if the label is desired in the form N × 10 KEXPX where N is the number to be printed out.)

KEXPY:

Same as KEXPX, but defines the exponent of the y-scale.

LIL:

A code variable which, if equal to zero, the program goes through STIFFEN after having plotted the stress trajectories. If equal to 1, it skips the subroutine.

LINUM(ILINUM):

A one-dimension array variable corresponding to the number of line segments which are subjected to boundary loads; that is, if both normal and shear distributed loads are exerted only on one line segment on the boundary of the structure LINUM could be dimensioned "2". The dimension ILINUM corresponds to twice the maximum possible number of boundary line segments under distributed loads.

LLL:

A code variable which, if equal to zero, the program plots the stress trajectories; if equal to one, skips plotting and goes through STIFEN; if greater than one, the program stops.

LTYPE:

A code variable in FØRCER which, if zero, the distributed boundary loads are converted to the *energy equivalent* nodal forces and stored in FØRCE, that is, in matrix {Q}; if it is equal to one, the *concentrated* nodal forces are stored in FØRCE.

MARCH:

The number equal to the j^{th} trajectory from which the stiffening of the structure, by increasing the thicknesses of the element associated with each trajectory, commences. (see Main and subroutine STIFEN)

MPØINT(IMPØIN):

A single dimension variable which stores the number of elements associated with each trajectory, the thickness of which must be increased for the stiffening of the structure.

MTAP:

Is a READ input code variable and should equal "five" if the program runs in the IBM 360 computer.

NBD1:

Total number of restrained nodes in the x-direction.

NBC2:

Total number of restrained nodes in the y-direction.

NCØL:

A single array variable used in INIT; a vector of column numbers of the first non zero entry in each row of the assembled matrix. It must be dimensioned at least the number of degrees of freedom of the entire structure.[17]

NCR:

Total number of restrained nodes in both x and y directions.

NCRT( ):

A single dummy dimension array (used in MAIN) in which the nodes constrained in the x-direction and the y-direction are stored beginning with these in the x-direction.[17]

NDIGX:

An argument of AXIS1 called from PLTTRJ and representing the number of digits after the decimal point to be printed on the label of the x-scale.

NDIGY:

The same definition as for NDIGX, applying instead to the y-scale.

NDØF:

The number of degrees of freedom of the entire structure.

NELEM:

The current number of elements used to assign the node numbers associated with each element.

NELES:

The total number of elements of the entire structure.

NFØRX:

Total number of concentrated forces acting on the nodes of the structure in the x-direction.

NFØRY:

Same definition as for NFØRX, but applying to the y-direction.

NLIN:

Total number of line segments between nodes in the structure.

NLINE(NLIN, IDIR):

A double dimension array which gives the two nodal points for each line segment of the structure. (that is, if for line segment 1, NLINE(1, 1) is 1; and NLINE(1, 2) is 2.

NNØDEN:

Total number of nodal points associated with the boundary line segments of the structure on which the intensity of a normal distributed load on the boundary is specified.

NNØDES:

The same as NNØDEN, but referring to a shear distributed load.

NØ:

(In subroutine FØRCER) indicates whether the load intensity (in the case of distributed loads) is on a horizontal side of the structure, in which case it is equal to one; or on a vertical side, where it is equal to two.

NØDAT:

Specifies the node at which the intensity of a distributed load is applied.

NØDE(, 4):

A two-dimensional array whose first dimension corresponds to the number of elements of a structure, and the second to the number of nodes per element. Notice that in the example printout for a cantilever, the two-dimensional array has a heading: "NØDE(4)".

NØDEN( ):

A one-dimensional array which specifies the number of elements which are common to every nodal point of the structure per nodal point.

NØDES:

Total number of nodes or control points of the structure.

NØDLS(, 4):

A double array variable that identifies numerically the sides of each element. (That is, for element No. 1 of a 250-element structure, the left vertical side is No. 1; the lower horizontal is No. 261; the right vertical is No. 11; and the upper horizontal is No. 262. The second dimension refers to the four sides of each rectangular element.

NØDST( ):

A work vector in INIT equal to at least the number of degrees of freedom of the total structure in length.

NØDX(IDSTR):

A one-dimension variable that stores the node number of the structure on which the concentrated load acts in the x-direction. IDSTR represents the maximum possible dimension.

NPAGES:

The first READ input variable of PLTTRJ and refers to the number of "pages" on which the trajectories will be plotted. In the present case, since we request one plot of the total number of stress trajectories, NPAGES = 1. However, if in the future the principal tensile stress trajectories need to be plotted on a different "page" or graph, NPAGES will be "2", etc.

NSS:

The total number of line segments in the y-direction.

NSTAS:

The number of stations or control points in the y-direction.

NSTAT:

The number of stations in the x-direction.

NSTRN( ):

An output code which, when defined to be "6", the results of the program are printed out; when "7", the results are punched in computer cards.

NU:

Poisson's ratio.

NUM:

The upper limit of IIN. If defined to be greater than "1", the program will go through the entire procedure twice or more times; it is expected that each time it must pass through subroutine STIFEN.

NUMLNN:

An integer read as INPUT in subroutine FØRCER which corresponds to the line segments on the boundaries of the structure which are subjected to normal distributed loads.

NUMLNS:

Read as input in the same card with NUMLNN, and which corresponds to the number of line segments on the boundaries of the structure which are subjected to shear distributed loads.

QDUMMY(IFØRCE):

A dummy column vector where the "force entries" are stored from FØRCE so that when the solution for the displacements, q, will be stored in FØRCE, the "force entries" will not be destroyed.

RHO:

The ratio by which the thickness of the elements to be used for the reinforcement of the structure will increase. The elements are identified in STIFEN, but the reinforcement involves computations in the Main.

S(IXF1):

A single array argument of SCLGPH[18] called by PLTTRJ and used for storage for the x-coordinates of the number of points per trajectory to be plotted.

SIGMA2(IDST1, IDST2):

Represents the principal compressive stress calculated at each point of the stress trajectories along the x-direction.

STRX(IDSTR):

Indicates the average σ_x stress at each node of the structure.

STRY(IDSTR):

The average σ_y as above.

STRXY(IDSTR):

The average σ_{xy} as above.

STRX1:

(1) As defined in STRESS corresponds to equation [2.32.1].

(2) As defined in TRAJEC corresponds to the interpolated value of stress for point P_1 (see Figure II-3).

STRX2:

The σ_{x_2} according to equation [2.32.2].

STRX3:

The σ_{x_3} according to equation [2.32.3].

STRY1:

(1) In STRESS it is defined according to equation [2.32.4].

(2) In TRAJEC it represents the corresponding σ_{y_1} for point 1.

STRY2:

Defined according to equation [2.32.5].

STRY3:

Defined according to equation [2.32.6].

STRXY1:

(1) In STRESS it is defined according to equation [2.32.7].

(2) In TRAJEC it represents the associated σ_{xy_1} for the intermediate extrapolation point P_1.

STRXY2:

Defined according to equation [2.32.8].

STRXY3:

Defined according to equation [2.32.9].

STX(IDST1, IDST2):

Corresponds to the σ_x stress at each (x, y) point for each trajectory.

STY(IDST1, IDST2):

Same as STX but corresponds to σ_y.

STXY(IDST1, IDST2):

Same as STY but corresponds to σ_{xy}.

T(IT):

A dummy one-dimension array used in PLTTRJ which stores the maximum number of the x-coordinates of all the stress trajectories.

THIC:

The thickness of the plate t.

TIT(IT):

A single dimension dummy array used for the storage of all the σ_2 (SIGMA2) principal stresses from which their values are printed out adjacent to the y-coordinates of all plotted stress trajectories in PLTTRJ.

TN(IDSTR, IDIR):

A two-dimension array which stores the intensities of a normal distribution of load; the first dimension refers to the node number at which the intensity is applied and the second to the direction number of the boundary side of the structure. (see NØDAT, NØ)

TS(IDSTR, IDIR):

The same definition applies as in TN except that the array stores the intensities of a shear distribution of load.

X:

(1) In STRESS it represents the length of an element in the x-direction.

(2) In PLTTRJ it represents the number of inches the origin is set away from the left side of the page.

XF(IXF1, IXF2):

Two-dimensional array which stores the final x-coordinates for each trajectory.

Y:

(1) In STRESS it represents the length of an element in the y-direction.

(2) In PLTTRJ it represents the number of inches the origin is set from the bottom of the page.

YF(IXF1, IXF2):

The same as in XF, but storing the y-coordinates for each trajectory.

YY:

Equal to $E/(1 - \nu^2)$.

YXZERØ:

Used as an argument in subroutine SCLGPH which is called from PLTTRJ, it represents the value on the y-axis at the origin.

ZZ:

Equal to $(1 - \nu)/2$.

4. Computer Program Listing

```
C**********************************************************************  MAIN0001
C**********************************************************************  MAIN0002
C**********************************************************************  MAIN0003
C                             MAIN PROGRAM                              MAIN0004
C**********************************************************************  MAIN0005
C**********************************************************************  MAIN0006
C**********************************************************************  MAIN0007
C                                                                       MAIN0008
C     LIST EXPLICIT STATEMENTS                                          MAIN0009
C                                                                       MAIN0010
      REAL NU                                                           MAIN0011
C                                                                       MAIN0012
C     LIST COMMON STATEMENTS                                            MAIN0013
C                                                                       MAIN0014
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,  MAIN0015
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP        MAIN0016
C                                                                       MAIN0017
C     LIST DIMENSION STATEMENTS                                         MAIN0018
C                                                                       MAIN0019
      DIMENSION NODLS(250,4),NODST(572),KWORK(4),BIGU(23500),BIGL(23500) MAIN0020
     1,IWORK(572),JWORK(572),FORCE(572),NODE(250,4),IVCT(4),NSTRN(100), MAIN0021
     2NCOL(572),NCRT(100),KKWRK(4),ALENX(1),ALENY(1),BIGK(8,8),         MAIN0022
     3NODEN(300),MPOINT(11),KELEM(50,11),STRX(286),STRY(286),STRXY(286), MAIN0023
     4STX(26,11),STY(26,11),STXY(26,11),XF(26,11),YF(26,11),T(286),     MAIN0024
     5FT(286),IPOINT(26),S(26),FS(26),NODX(286),FORVAL(286),FSTORX(286), MAIN0025
     6NODY(286),FSTORY(286),SIGMA2(26,11),TIT(286),NLINE(535,2),        MAIN0026
     7 LINUM(140),TN(286,2),TS(286,2)                                   MAIN0027
C                                                                       MAIN0028
C     LIST FORMAT STATEMENTS                                            MAIN0029
C                                                                       MAIN0030
    2 FORMAT(10I5)                                                      MAIN0031
    4 FORMAT(8F10.4)                                                    MAIN0032
   22 FORMAT ( ' NUMBER OF SEQUENCE = ',I5,/, ' NUMBER OF ELEMENTS      MAIN0033
     2 = ', I5 , /,' NUMBER OF NODES =',  I5 ,  /,' NSS= ', I5, /,      MAIN0034
     3 ' NUMBER OF LINES = ' ,I5)                                       MAIN0035
   80 FORMAT(50I1)                                                      MAIN0036
```

```
   88 FORMAT(20I3)
  105 FORMAT(10E10.4)
  400 FORMAT(I4,5F10.4,I4)
  859 FORMAT ( ' DETERMINANT = ' ,E14.7)
 1101 FORMAT(2I5)
 1102 FORMAT(16I5)
 1103 FORMAT(1X,20I4)
 1150 FORMAT(1X,20I3)
 1352 FORMAT ( ' ******BOUNDARY CONDITIONS *************** ' )
 4000 FORMAT(4F10.4)
 4001 FORMAT(3F10.4)
 4052 FORMAT(1X,8E12.4)
 4444 FORMAT(6I5,F10.3)
 4848 FORMAT(16I5)
 5855 FORMAT (1X,8F14.6)
 5866 FORMAT( '  NODE        DISP. X      DISP. Y    '  )
 6050 FORMAT ( ' THICKNESS = ' , E14.6,/, ' MODULUS = ' , E14.6,/,
     2 ' POISSON RATIO = ' , E14.6)
 6799 FORMAT ( ' ALENX       ALENY ' )
 8888 FORMAT(I5,2E12.4)
C
C     DEFINE INPUT/OUTPUT CODES FOR THE IBM 360 COMPUTER
C
      MTAP=5
      NTAP=7
C
C     DEFINE JTEST=0 FOR NO DUMPING OF POINTER DATA IN
C     SOUBROUTINE SETUP
C     DEFINE ITEST=0 FOR NO CALCULATION OF THE DETERMINANT
C     IN SUBROUTINE SOLVER
C
      JTEST=0
      ITEST=1
C
C     LIST OF READ STATEMENTS
C
```

```
      READ(MTAP,4848)IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,
     1IDST1,IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT
      READ(MTAP,4444)IPAR,MARCH,JSTFN,LOC,LLL,LIL,RHO
      READ (MTAP,2) NUM,NSTAS,NSTAT
      READ(MTAP,4) THIC,E,NU
      READ (MTAP,4)  ALX,ALY
C
C     CALCULATE NUMBER OF  ELEMENTS  IN  T (X-DIRECTION)
C     CALCULATE NUMBER OF  ELEMENTS  IN  S (Y-DIRECTION)
C
      NSTAT1=NSTAT-1
      NSTAS1=NSTAS-1
C
C     NSS  IS  THE  MAXIMUM  NUMBER  OF  LINES  OF  ELEMENTS  IN  S
C
      NSS=NSTAS1*NSTAT
C
C     NODES  IS  THE  TOTAL  NUMBER  OF  NODES  OF  THE  STRUCTURE
C
      NODES=NSTAS*NSTAT
C
C     CALCULATE  THE  TOTAL  NUMBER  OF  DEGREES  OF  FREEDOM  OF  THE  STRUCTURE
C
      NDOF=NODES*2
C
C     CALCULATE  THE  TOTAL  NUMBER  OF  LINES  OF  THE  STRUCTURE
C
      NLIN=NSS+NSTAT1*NSTAS
C
C     TOTAL  NUMBER  OF  FINITE  ELEMENTS
C
      NELES=NSTAT1*NSTAS1
C
C     LENGTH  OF  EACH  ELEMENT  IN  X  AND  Y  DIRECTIONS
C
      ALYY=ALY/NSTAS1
```

```
      ALXX=ALX/NSTAT1
      ALENX(1)=ALXX
      ALENY(1)=ALYY
C
C     OUTPUT
C
      WRITE (NTAP,22) NUM,NELES,NODES,NSS,NLIN
      WRITE (NTAP,6050) THIC,E,NU
      WRITE(NTAP,6799)
      WRITE(NTAP,105) ALENX(1),ALENY(1)
      DO 50 NTN=1,NSTAT1
      DO 50 NSN=1,NSTAS1
      NTN1=NTN-1
      NSN1=NSN-1
      NELEM=NSTAS1*NTN1+NSN
C
C     CALCULATE THE FOUR NODE NUMBERS OF EACH ELEMENT CORRESPONDING TO
C     THE STRUCTURE NODE NUMBERS
C
      NODE(NELEM,1)=NSTAS*NTN1+NSN
      NODE(NELEM,2)=NSTAS*NTN +NSN
      NODE(NELEM,3)=NSTAS*NTN +NSN+1
      NODE(NELEM,4)=NSTAS*NTN1+NSN+1
C
C     CALCULATE THE FOUR EDGE NUMBERS OF EACH ELEMENT CORRESPONDING
C     TO THE LINE NUMBERS OF THE STRUCTURE
C
      NODLS(NELEM,1)=NSS+NSTAS*NTN1+NSN
      NODLS(NELEM,3)=NODLS(NELEM,1)+1
      NODLS(NELEM,2)=NSTAS1*NTN+NSN
      NODLS(NELEM,4)=NSTAS1*NTN1+NSN
C
C     CALCULATE THE TWO NODE NUMBERS OF EACH LINE CORRESPONDING
C     TO THE NODE NUMBERS OF THE STRUCTURE
C
      NLINE(NODLS(NELEM,1),1)=NODE(NELEM,1)
```

```
      ALXX=ALX/NSTAT1                                                    MAIN0109
      ALENX(1)=ALXX                                                      MAIN0110
      ALENY(1)=ALYY                                                      MAIN0111
C                                                                        MAIN0112
C     OUTPUT                                                             MAIN0113
C                                                                        MAIN0114
      WRITE (NTAP,22) NUM,NELES,NODES,NSS,NLIN                           MAIN0115
      WRITE (NTAP,6050) THIC,E,NU                                        MAIN0116
      WRITE(NTAP,6799)                                                   MAIN0117
      WRITE(NTAP,105) ALENX(1),ALENY(1)                                  MAIN0118
      DO 50 NTN=1,NSTAT1                                                 MAIN0119
      DO 50 NSN=1,NSTAS1                                                 MAIN0120
      NTN1=NTN-1                                                         MAIN0121
      NSN1=NSN-1                                                         MAIN0122
      NELEM=NSTAS1*NTN1+NSN                                              MAIN0123
C                                                                        MAIN0124
C     CALCULATE THE FOUR NODE NUMBERS OF EACH ELEMENT CORRESPONDING TO   MAIN0125
C     THE STRUCTURE NODE NUMBERS                                         MAIN0126
C                                                                        MAIN0127
      NODE(NELEM,1)=NSTAS*NTN1+NSN                                       MAIN0128
      NODE(NELEM,2)=NSTAS*NTN +NSN                                       MAIN0129
      NODE(NELEM,3)=NSTAS*NTN +NSN+1                                     MAIN0130
      NODE(NELEM,4)=NSTAS*NTN1+NSN+1                                     MAIN0131
C                                                                        MAIN0132
C     CALCULATE THE FOUR EDGE NUMBERS OF EACH ELEMENT CORRESPONDING      MAIN0133
C     TO THE LINE NUMBERS OF THE STRUCTURE                               MAIN0134
C                                                                        MAIN0135
      NODLS(NELEM,1)=NSS+NSTAS*NTN1+NSN                                  MAIN0136
      NODLS(NELEM,3)=NODLS(NELEM,1)+1                                    MAIN0137
      NODLS(NELEM,2)=NSTAS1*NTN+NSN                                      MAIN0138
      NODLS(NELEM,4)=NSTAS1*NTN1+NSN                                     MAIN0139
C                                                                        MAIN0140
C     CALCULATE THE TWO NODE NUMBERS OF EACH LINE CORRESPONDING          MAIN0141
C     TO THE NODE NUMBERS OF THE STRUCTURE                               MAIN0142
C                                                                        MAIN0143
      NLINE(NODLS(NELEM,1),1)=NODE(NELEM,1)                              MAIN0144
```

```
      NLINE(NODLS(NELEM,1),2)=NODE(NELEM,2)                             MAIN0145
      NLINE(NODLS(NELEM,2),1)=NODE(NELEM,2)                             MAIN0146
      NLINE(NODLS(NELEM,2),2)=NODE(NELEM,3)                             MAIN0147
      NLINE(NODLS(NELEM,3),1)=NODE(NELEM,4)                             MAIN0148
      NLINE(NODLS(NELEM,3),2)=NODE(NELEM,3)                             MAIN0149
      NLINE(NODLS(NELEM,4),1)=NODE(NELEM,1)                             MAIN0150
      NLINE(NODLS(NELEM,4),2)=NODE(NELEM,4)                             MAIN0151
   50 CONTINUE                                                          MAIN0152
C                                                                       MAIN0153
C     BOUNDARY CONDITIONS                                               MAIN0154
C                                                                       MAIN0155
      WRITE (NTAP,1352)                                                 MAIN0156
      READ(MTAP,1102) NBC1                                              MAIN0157
      IF ( NBC1 .EQ. 0 ) GO TO 701                                      MAIN0158
      READ(MTAP,1102)      (NCRT(IX),IX=1,NBC1)                         MAIN0159
      WRITE(NTAP,1150) NBC1,(NCRT(IX), IX=1,NBC1)                       MAIN0160
  701 READ(MTAP,1102) NBC2                                              MAIN0161
      IF ( NBC2 .EQ. 0 ) GO TO 702                                      MAIN0162
      READ(MTAP,1102)      (NCRT(IX+NBC1), IX=1,NBC2)                   MAIN0163
      WRITE(NTAP,1150)NBC2,(NCRT(IX+NBC1), IX=1,NBC2)                   MAIN0164
  702 NX12=NBC1+NBC2                                                    MAIN0165
      NCR=NBC1+NBC2                                                     MAIN0166
      WRITE(NTAP,1150)(NCRT(IX),IX=1,NCR)                               MAIN0167
      IF ( NBC1 .EQ. 0 ) GO TO 801                                      MAIN0168
      DO 1780 IX=1,NBC1                                                 MAIN0169
 1780 NSTRN(IX)=(NCRT(IX)-1)*2+1                                        MAIN0170
  801 CONTINUE                                                          MAIN0171
      IF ( NBC2 .EQ. 0 ) GO TO 802                                      MAIN0172
      DO 1781 IX=1,NBC2                                                 MAIN0173
 1781 NSTRN(IX+NBC1)=(NCRT(IX+NBC1)-1)*2+2                              MAIN0174
  802 CONTINUE                                                          MAIN0175
C                                                                       MAIN0176
C     NODEN                                                             MAIN0177
C                                                                       MAIN0178
      DO 60 NSN=1,NSTAS                                                 MAIN0179
      DO 60 NTN=1,NSTAT                                                 MAIN0180
```

```
      NTN1=NTN-1                                                        MAIN0181
      NSN1=NSN-1                                                        MAIN0182
      NN=NSTAS*NTN1+NSN                                                 MAIN0183
   60 NODEN(NN)=4                                                       MAIN0184
      DO 61 I=1,NSTAS                                                   MAIN0185
      NODEN(I)=2                                                        MAIN0186
      J=I+NSTAT1*NSTAS                                                  MAIN0187
   61 NODEN(J)=2                                                        MAIN0188
      DO 11 J=1,NSTAT                                                   MAIN0189
      K=NSTAS*(J-1)+1                                                   MAIN0190
      NODEN(K)=2                                                        MAIN0191
      K=NSTAS*(J-1)+NSTAS                                               MAIN0192
   11 NODEN(K)=2                                                        MAIN0193
      NODEN(1)=1                                                        MAIN0194
      NODEN(NSTAS)=1                                                    MAIN0195
      NODEN(NODES)=1                                                    MAIN0196
      NODEN(NSTAS*NSTAT1+1)=1                                           MAIN0197
C                                                                       MAIN0198
C     DO 2001 GOES THROUGH THE WHOLE PROGRAM ONCE ORE MORE THAN ONCE    MAIN0199
C     DEPENDING ON WHETHER IT IS DESIRED TO SEE THE EFFECTS OF          MAIN0200
C     THE REINFORCING STIFFENERS OR NOT                                 MAIN0201
C                                                                       MAIN0202
      DO 2001 INN=1,NUM                                                 MAIN0203
C                                                                       MAIN0204
C     IM=1 SINCE ALL ELEMENTS HAVE THE SAME GEOMETRY AND HENCE STIFFENES.MAIN0205
C                                                                       MAIN0206
      IM=1                                                              MAIN0207
C                                                                       MAIN0208
C     YY AND ZZ ARE CONSTANTS USED IN OBTAINING THE ELEMENTS OF         MAIN0209
C     THE STIFFENESS MATRIX                                             MAIN0210
C                                                                       MAIN0211
      YY=E/(1.-NU**2)                                                   MAIN0212
      ZZ=(1.-NU)/2.                                                     MAIN0213
      CALL STFMTX(BIGK,YY,ZZ,IM,NU,ALENX,ALENY)                         MAIN0214
      IF(INN.NE.1) GO TO 9001                                           MAIN0215
      DO 9000 I=1,8                                                     MAIN0216
```

```
      DO 9000 J=I,8                                                     MAIN0217
      BIGK(I,J)=BIGK(I,J)*THIC                                          MAIN0218
 9000 BIGK(J,I)=BIGK(I,J)                                               MAIN0219
 9001 KKWRK(1)=2                                                        MAIN0220
      KKWRK(2)=2                                                        MAIN0221
      KKWRK(3)=2                                                        MAIN0222
      KKWRK(4)=2                                                        MAIN0223
      DO 101 I=1,NELES                                                  MAIN0224
      IVCT(1)=NODE(I,1)                                                 MAIN0225
      IVCT(2)=NODE(I,2)                                                 MAIN0226
      IVCT(3)=NODE(I,3)                                                 MAIN0227
      IVCT(4)=NODE(I,4)                                                 MAIN0228
      CALL INIT ( NDOF,NCOL,IVCT,KKWRK,IWORK,4,NODST,I,NELES,BIGL)      MAIN0229
  101 CONTINUE                                                          MAIN0230
      CALL SETUP (BIGU,BIGL,FORCE,IWORK,JWORK,JTEST,6)                  MAIN0231
      DO 102 I=1,NELES                                                  MAIN0232
      IF(INN.EQ.1) GO TO 9002                                           MAIN0233
      ATHIC=THIC                                                        MAIN0234
      DO 899 JTRAJ=MARCH,JSTFN,LOC                                      MAIN0235
      MELEM=MPOINT(JTRAJ)                                               MAIN0236
      DO 898 IELEM=1,MELEM                                              MAIN0237
      IF(I-KELEM(IELEM,JTRAJ))898,821,898                               MAIN0238
  898 CONTINUE                                                          MAIN0239
  899 CONTINUE                                                          MAIN0240
      GO TO 820                                                         MAIN0241
  821 ATHIC=RHO*THIC                                                    MAIN0242
  820 DO 9020 K=1,8                                                     MAIN0243
      DO 9020 J=K,8                                                     MAIN0244
      BIGK(K,J)=BIGK(K,J)*ATHIC                                         MAIN0245
 9020 BIGK(J,K)=BIGK(K,J)                                               MAIN0246
 9002 KKWRK(1)=2                                                        MAIN0247
      KKWRK(2)=2                                                        MAIN0248
      KKWRK(3)=2                                                        MAIN0249
      KKWRK(4)=2                                                        MAIN0250
      IVCT(1)=NODE(I,1)                                                 MAIN0251
      IVCT(2)=NODE(I,2)                                                 MAIN0252
```

```
      IVCT(3)=NODE(I,3)                                                 MAIN0253
      IVCT(4)=NODE(I,4)                                                 MAIN0254
      CALL LOADER ( 8,4, BIGK,KKWRK,IVCT,KWORK)                         MAIN0255
      IF(INN.EQ.1) GO TO 102                                            MAIN0256
      CALL STFMTX(BIGK,YY,ZZ,IM,NU,ALENX,ALENY)                         MAIN0257
  102 CONTINUE                                                          MAIN0258
      CALL FORCER(FORCE, INN,NDOF,NODX,FORVAL,FSTORX,NODY,FSTORY,       MAIN0259
     1 ALENX,ALENY,NLINE,NSS,QDUMMY,NODES,NSTAS,NSTAT,NSTAS1,NSTAT1,    MAIN0260
     2 LINUM,TN, TS)                                                    MAIN0261
      CALL RSTRN (NSTRN,NCR)                                            MAIN0262
      CALL SOLVER (DET,ITEST)                                           MAIN0263
      WRITE(NTAP,859) DET                                               MAIN0264
      WRITE (NTAP,5866)                                                 MAIN0265
      DO 7777 KK=1,NODES                                                MAIN0266
      K2=(KK-1)*2+1                                                     MAIN0267
      K3=KK*2                                                           MAIN0268
 7777 WRITE(NTAP,8888) KK,FORCE(K2),FORCE(K3)                           MAIN0269
C    FORCE CALCULATION FROM THE DISPLACEMENTS GOES HERE                 MAIN0270
      CALL STRESS(NELES,NODE,FORCE,ALXX,ALYY,NODES,NU,ZZ,YY,NODEN,STRX, MAIN0271
     1 STRY,STRXY)                                                      MAIN0272
      IF(IPAR-1)13,2001,2001                                            MAIN0273
   13 CALL TRAJEC(STRX,STRY,STRXY,ALXX,ALYY,ALY,STX,STY,STXY,XF,YF,     MAIN0274
     2NSTAS,NSTAT1,T,FT,IPOINT,MPTS,TIT,SIGMA2)                         MAIN0275
      IF(LLL-1)7,8,2001                                                 MAIN0276
    7 CALL PLTTRJ(T,FT,IPOINT,NSTAS,MPTS,S,FS,TIT)                      MAIN0277
      IF(LIL-1)8,2001,2001                                              MAIN0278
    8 CALL STIFEN(XF,YF,NSTAS,NSTAS1,ALXX,ALYY,IPOINT,MPOINT,KELEM)     MAIN0279
 2001 CONTINUE                                                          MAIN0280
      CALL EXIT                                                         MAIN0281
      END                                                               MAIN0282
```

```
      SUBROUTINE STFMTX(BIGK,YY,ZZ,IM,NU,ALENX,ALENY)
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP
      REAL NU
      DIMENSION BIGK(IOCT,IOCT),ALENX(1),ALENY(1)
      BIGK(1,1)=YY*(ALENY(IM)/(3.*ALENX(IM))+ (ALENX(IM)/(3.*ALENY(IM)))
     1*ZZ)
      BIGK(1,2)=YY*((NU+1.)/8.)
      BIGK(1,3)=YY*(-ALENY(IM)/(ALENX(IM)*3.)+(ALENX(IM)/(ALENY(IM)*6.))
     1*ZZ)
      BIGK(1,4)=YY*((3.*NU-1.)/8.)
      BIGK(1,5)=YY*(-ALENY(IM)/(6.*ALENX(IM))-(ALENX(IM)/(ALENY(IM)*6.))
     1*ZZ)
      BIGK(1,6)=YY*(-(NU+1.)/8.)
      BIGK(1,7)=YY*(ALENY(IM)/(6.*ALENX(IM))-(ALENX(IM)/(3.*ALENY(IM)))
     1*ZZ)
      BIGK(1,8)=YY*((-3.*NU+1.)/8.)
      BIGK(2,2)=YY*(ALENX(IM)/(ALENY(IM)*3.0)+(ALENY(IM)/(ALENX(IM)*3.0)
     1)*ZZ)
      BIGK(2,3)=YY*((-3.*NU+1.)/8.)
      BIGK(2,4)=YY*(ALENX(IM)/(ALENY(IM)*6.0)-(ALENY(IM)/(ALENX(IM)*3.0)
     1)*ZZ)
      BIGK(2,5)=YY*(-(NU+1.0)/8. )
      BIGK(2,6)=YY*(-ALENX(IM)/(ALENY(IM)*6.0)-(ALENY(IM)/(ALENX(IM)*6.0
     1))*ZZ)
      BIGK(2,7)=YY*(3.*NU-1.0)/8.
      BIGK(2,8)=YY*(-ALENX(IM)/(ALENY(IM)*3.0)+( ALENY(IM)/(ALENX(IM)*
     1 6.0))*ZZ)
      BIGK(3,3)=YY*(ALENY(IM)/(ALENX(IM)*3.)+(ALENX(IM)/(ALENY(IM)*3.))*
     1ZZ)
      BIGK(3,4)=YY*(-(NU+1.0)/8.)
      BIGK(3,5)=YY*(ALENY(IM)/(ALENX(IM)*6.)-(ALENX(IM)/(3.*ALENY(IM)))*
     2ZZ)
      BIGK(3,6)=YY*(3.*NU-1.0)/8.
      BIGK(3,7)=YY*(-ALENY(IM)/(ALENX(IM)*6.)-(ALENX(IM)/(ALENY(IM)*6.)
     3)*ZZ)
```

```
      BIGK(3,8)=YY*(NU+1.0)/8.                                          STFM0037
      BIGK(4,4)=YY*(ALENX(IM)/(3.*ALENY(IM))+(ALENY(IM)/( ALENX(IM)*3.0)STFM0038
     1)*ZZ)                                                             STFM0039
      BIGK(4,5)=YY*(-3.*NU+1.0)/8.                                      STFM0040
      BIGK(4,6)=YY*(-ALENX(IM)/(ALENY(IM)*3.0)+(ALENY(IM)/(ALENX(IM)*6.0STFM0041
     1))*ZZ)                                                            STFM0042
      BIGK(4,7)=YY*(NU+1.0)/8.                                          STFM0043
      BIGK(4,8)=YY*(-ALENX(IM)/(ALENY(IM)*6.0)-(ALENY(IM)/(ALENX(IM)*6.0STFM0044
     1))*ZZ)                                                            STFM0045
      BIGK(5,5)=YY*(ALENY(IM)/(ALENX(IM)*3.0)+(ALENX(IM)/(ALENY(IM)*3.0)STFM0046
     4)*ZZ)                                                             STFM0047
      BIGK(5,6)=YY*(NU+1.0)/8.                                          STFM0048
      BIGK(5,7)=YY*(-ALENY(IM)/(ALENX(IM)*3.0)+(ALENX(IM)/(ALENY(IM)*6.0STFM0049
     5))*ZZ)                                                            STFM0050
      BIGK(5,8)=YY*(3.0*NU-1.0)/8.                                      STFM0051
      BIGK(6,6)=YY*(ALENX(IM)/(ALENY(IM)*3.0)+(ALENY(IM)/(ALENX(IM)*3.0 STFM0052
     1))*ZZ)                                                            STFM0053
      BIGK(6,7)=YY*(-3.0*NU+1.0)/8.                                     STFM0054
      BIGK(6,8)=YY*(ALENX(IM)/(ALENY(IM)*6.0)-(ALENY(IM)/(ALENX(IM)*3.0)STFM0055
     1)*ZZ)                                                             STFM0056
      BIGK(7,7)=YY*(ALENY(IM)/(ALENX(IM)*3.0)+(ALENX(IM)/(ALENY(IM)*3.0)STFM0057
     1)*ZZ)                                                             STFM0058
      BIGK(7,8)=YY*(-(NU+1.0)/8.)                                       STFM0059
      BIGK(8,8)=YY*(ALENX(IM)/(ALENY(IM)*3.0)+(ALENY(IM)/(ALENX(IM)*3.0 STFM0060
     1))*ZZ)                                                            STFM0061
      RETURN                                                            STFM0062
       END                                                              STFM0063
```

```
      SUBROUTINE FORCER(FORCE,INN,NDOF,NODX,FORVAL,FSTORX,NODY,FSTORY,
     1 ALENX,ALENY,NLINE,NSS,QDUMMY,NODES,NSTAS,NSTAT,NSTAS1,
     2NSTAT1,LINUM,TN,TS)
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP
      DIMENSION FORCE(IFORCE),NODX(IDSTR),FORVAL(IDSTR),FSTORX(IDSTR),
     1NODY(IDSTR),FSTORY(IDSTR),ALENX(1),ALENY(1),QDUMMY(IFORCE),
     2 LINUM(ILINUM),TN(IDSTR,IDIR),TS(IDSTR,IDIR),NLINE(NLIN,IDIR)
    1 FORMAT(I5)
    2 FORMAT(I5,F10.4)
   11 FORMAT(I6,3X,F10.4)
  101 FORMAT(1X, 6F10.4)
   10 FORMAT(' FORCXNODE      VALUE   ')
   21 FORMAT(' FORCYNODE      VALUE   ')
  100 FORMAT(//,8X,'  FORCE ENTRIES TWO PER NODE IN ORDER ',//)
   15 FORMAT(10I5)
   19 FORMAT(2I5,F10.4)
C
C     IDENTIFY CODE FOR THE TYPE OF LOADING
C
      READ(5, 1)LTYPE
      IF(LTYPE.EQ.0)GO TO 1001
C
C     LTYPE DIFFERENT THAN ZERO : DISCRETE NODAL LOADS
C
      IFOR2=IFORCE
      CALL ERASE(FORCE,IFOR2)
      IF(INN.GT.1)GO TO 5
      READ(5,1)NFORX
      READ(5,1)NFORY
      NFOR=NFORX+NFORY
      IF(NFORX.EQ.0) GO TO 20
      DO 3 I=1,NFORX
      READ(5,2) NODX(I),FORVAL(I)
      FSTORX(NODX(I))=FORVAL(I)
    3 CONTINUE
```

```
      WRITE(NTAP,10)                                               FORC0037
      WRITE(NTAP,11)(NODX( I),FSTORX(NODX(I)),I=1,NFORX)           FORC0038
   20 CONTINUE                                                     FORC0039
      IF(NFORY.EQ.0) GO TO 5                                       FORC0040
      DO 7 I=1,NFORY                                               FORC0041
      READ(5,2)NODY(I),FORVAL(I)                                   FORC0042
      FSTORY(NODY(I))=FORVAL(I)                                    FORC0043
    7 CONTINUE                                                     FORC0044
      WRITE(NTAP,21)                                               FORC0045
      WRITE(NTAP,11)(NODY( I),FSTORY(NODY(I)),I=1,NFORY)           FORC0046
    5 CONTINUE                                                     FORC0047
      IF(NFORX.EQ.0)GO TO 110                                      FORC0048
      DO 4 I=1,NFORX                                               FORC0049
      J=(NODX(I)-1)*2+1                                            FORC0050
    4 FORCE(J)=FSTORX(NODX(I))                                     FORC0051
  110 CONTINUE                                                     FORC0052
      IF(NFORY.EQ.0) GO TO 111                                     FORC0053
      DO 60 I=1,NFORY                                              FORC0054
      J=(NODY(I)-1)*2+2                                            FORC0055
   60 FORCE(J)=FSTORY(NODY(I))                                     FORC0056
  111 CONTINUE                                                     FORC0057
      IF(INN.GT.1) RETURN                                          FORC0058
      WRITE(NTAP,100)                                              FORC0059
      WRITE(NTAP,101)(FORCE(I),I=1,NDOF)                           FORC0060
      RETURN                                                       FORC0061
C                                                                  FORC0062
C      LTYPE EQUALS 0 : BOUNDARY LINE LOADS                        FORC0063
C                                                                  FORC0064
 1001 NSTAS2=NSTAS-2                                               FORC0065
      NSTAT2=NSTAT-2                                               FORC0066
      NOD250=NSTAT1*NSTAS1                                         FORC0067
      NOD276=NODES-NSTAS1                                          FORC0068
      NOD277=NODES-NSTAS1+1                                        FORC0069
      NOD284=NODES-2                                               FORC0070
      NOD285=NODES-1                                               FORC0071
      IFOR2=IFORCE                                                 FORC0072
```

```
      CALL ERASE(FORCE,IFOR2)                                           FORC0073
      IF(INN.LE.1) GO TO 1040                                           FORC0074
      DO 37 IRIS=1,NDOF                                                 FORC0075
   37 FORCE(IRIS)=QDUMMY(IRIS)                                          FORC0076
      IF(INN.GT.1) RETURN                                               FORC0077
 1040 READ(5,15)NUMLNN,NUMLNS,NNODEN,NNODES                             FORC0078
      IF(NUMLNN.EQ.0) GO TO 210                                         FORC0079
      DO 81 I=1,NNODEN                                                  FORC0080
   81 READ(5,19)NODAT,NO,TN(NODAT,NO)                                   FORC0081
      READ(5,15)(LINUM(I),I=1,NUMLNN)                                   FORC0082
  210 IF(NUMLNS.EQ.0)GO TO 23                                           FORC0083
      DO 82 I=1,NNODES                                                  FORC0084
   82 READ(5,19)NODAT,NO,TS(NODAT,NO)                                   FORC0085
      READ(5,15)(LINUM(I+NUMLNN),I=1,NUMLNS)                            FORC0086
   23 CONTINUE                                                          FORC0087
      LINTOT=NUMLNN+NUMLNS                                              FORC0088
      DO 1010 I=1,LINTOT                                                FORC0089
      LIN=LINUM(I)                                                      FORC0090
      J1=NLINE(LIN,1)                                                   FORC0091
      J2=NLINE(LIN,2)                                                   FORC0092
      IF(LIN.GT.NSS)GO TO 26                                            FORC0093
C     SIDE 2 VARYING                                                    FORC0094
      IF(LIN.GE.1.AND.LIN.LE.NSTAS1)ISIGN=-1                            FORC0095
      IF(LIN.GT.NOD250.AND.LIN.LE.NSS)ISIGN=1                           FORC0096
      K2=(J1-1)*2+1                                                     FORC0097
      K1=K2+1                                                           FORC0098
      K4=(J2-1)*2+1                                                     FORC0099
      K3=K4+1                                                           FORC0100
      ALEN=ALENY(1)                                                     FORC0101
      GO TO 25                                                          FORC0102
   26 CONTINUE                                                          FORC0103
C     SIDE 1 VARYING                                                    FORC0104
      DO 800 KA=1,NSTAT1                                                FORC0105
  800 IF(LIN.EQ.(NSS+NSTAS*KA))ISIGN=1                                  FORC0106
      DO 802 KB=1,NSTAT2                                                FORC0107
  802 IF(LIN.EQ.(NSS+NSTAS*(KB-1)+1))ISIGN=-1                           FORC0108
```

```
      K1=(J1-1)*2+1                                                     FORC0109
      K2=K1+1                                                           FORC0110
      K3=(J2-1)*2+1                                                     FORC0111
      K4=K3+1                                                           FORC0112
      ALEN=ALENX(1)                                                     FORC0113
   25 CONTINUE                                                          FORC0114
C                                                                       FORC0115
C     NORMAL FORCES PER BOUNDARY LINE                                   FORC0116
C     ALONG VERTICAL OR HORIZONTAL EDGES                                FORC0117
C                                                                       FORC0118
      IF(I.GT.NUMLNN)GO TO 1000                                         FORC0119
C                                                                       FORC0120
C     CHECK THE VERTICAL SIDE ( ALL THE IFS BELLOW )                    FORC0121
C                                                                       FORC0122
      DO 50 KK=2,NSTAS2                                                 FORC0123
      IF(J1.EQ.KK) GO TO 400                                            FORC0124
   50 CONTINUE                                                          FORC0125
      DO 61 KK=NOD277,NOD284                                            FORC0126
      IF(J1.EQ.KK) GO TO 400                                            FORC0127
   61 CONTINUE                                                          FORC0128
      IF(J1.EQ.1.AND.J2.EQ.2)GO TO 400                                  FORC0129
      IF(J1.EQ.NSTAS1.AND.J2.EQ.NSTAS)GO TO 400                         FORC0130
      IF(J1.EQ.NOD276.AND.J2.EQ.NOD277)GO TO 400                        FORC0131
      IF(J1.EQ.NOD285.AND.J2.EQ.NODES)GO TO 400                         FORC0132
      QDUMMY(K2)=(TN(J1,1)/3.+TN(J2,1)/6.)*ALEN*ISIGN                   FORC0133
      QDUMMY(K4)=(TN(J1,1)/6.+TN(J2,1)/3.)*ALEN*ISIGN                   FORC0134
      GO TO 40                                                          FORC0135
  400 QDUMMY(K2)=(TN(J1,2)/3.+TN(J2,2)/6.)*ALEN*ISIGN                   FORC0136
      QDUMMY(K4)=(TN(J1,2)/6.+TN(J2,2)/3.)*ALEN*ISIGN                   FORC0137
   40 FORCE(K2)=FORCE(K2)+QDUMMY(K2)                                    FORC0138
      FORCE(K4)=FORCE(K4)+QDUMMY(K4)                                    FORC0139
      GO TO 1010                                                        FORC0140
 1000 CONTINUE                                                          FORC0141
C                                                                       FORC0142
C     SHEAR FORCES PER BOUNDARY LINE                                    FORC0143
C     ALONG VERTICAL OR HORIZONTAL EDGES                                FORC0144
```

```
C                                                            FORC0145
C                                                            FORC0146
C     CHECK THE VERTICAL SIDE ( ALL THE IFS BELOW )          FORC0147
C                                                            FORC0148
      DO 500 LL=2,NSTAS2                                     FORC0149
      IF(J1.EQ.LL) GO TO 4000                                FORC0150
  500 CONTINUE                                               FORC0151
      DO 600 LL=NOD277,NOD284                                FORC0152
      IF(J1.EQ.LL) GO TO 4000                                FORC0153
  600 CONTINUE                                               FORC0154
      IF(J1.EQ.1.AND.J2.EQ.2) GO TO 4000                     FORC0155
      IF(J1.EQ.NSTAS1.AND.J2.EQ.NSTAS) GO TO 4000            FORC0156
      IF(J1.EQ.NOD276.AND.J2.EQ.NOD277)  GO TO 4000          FORC0157
      IF(J1.EQ.NOD285.AND.J2.EQ.NODES) GO TO 4000            FORC0158
      QDUMMY(K1)=(TS(J1,1)/3.+TS(J2,1)/6.)*ALEN*ISIGN        FORC0159
      QDUMMY(K3)=(TS(J1,1)/6.+TS(J2,1)/3.)*ALEN*ISIGN        FORC0160
      GO TO 14                                               FORC0161
 4000 QDUMMY(K1)=(TS(J1,2)/3.+TS(J2,2)/6.)*ALEN*ISIGN        FORC0162
      QDUMMY(K3)=(TS(J1,2)/6.+TS(J2,2)/3.)*ALEN*ISIGN        FORC0163
   14 FORCE(K1)=FORCE(K1)+QDUMMY(K1)                         FORC0164
      FORCE(K3)=FORCE(K3)+QDUMMY(K3)                         FORC0165
 1010 CONTINUE                                               FORC0166
      DO 17 LIA=1,NDOF                                       FORC0167
   17 QDUMMY(LIA)=FORCE(LIA)                                 FORC0168
      WRITE(NTAP,100)                                        FORC0169
      WRITE(NTAP,101)(FORCE(I),I=1,NDOF)                     FORC0170
      RETURN                                                 FORC0171
      END                                                    FORC0172
```

```
      SUBROUTINE  STRESS(NELES,NODE,FORCE,ALXX,ALYY,NODES,NU,ZZ,YY,NODEN,
     2STRX,STRY,STRXY)
      REAL NU
      COMMON/A/  ICNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP
      DIMENSION NCDE(IDNODE,4),FCKCE(IFORCE),NODEN(INODEN),STRX(ICSTR),
     1STRY(IDSTR),STRXY(IDSTR)
      X=ALXX
      Y=ALYY
      IDSTR2=IDSTR
      CALL ERASE(STRX,IDSTR2,STRY,IDSTR2,STRXY,IDSTR2)
      WRITE(NTAP,1457)
 1457 FORMAT(//,' NODE(4) ',//)
      WRITE(NTAP,1200)((NODE(I,J),J=1,4),I=1,NELES)
 1200 FORMAT(4I5)
      WRITE(NTAP,780)X,Y
  780 FORMAT( //,' X=',E12.4,' Y=',E12.4,//)
      WRITE(NTAP,777)
  777 FORMAT(//,' NODEN',//)
      WRITE(NTAP,778)(NODEN(II),II=1,300)
  778 FORMAT(1X,20I3)
      WRITE(NTAP,779)NU,ZZ,YY
  779 FORMAT(// ' NU=',E12.4,' ZZ=',E12.4,' YY=',E12.4)
      DO 7 IM=1,NELES
      N1=NODE(IM,1)
      N2=NODE(IM,2)
      N3=NODE(IM,3)
      N4=NODE(IM,4)
      I1=(N1-1)*2+1
      I2=(N1-1)*2+2
      I3=(N2-1)*2+1
      I4=(N2-1)*2+2
      I5=(N3-1)*2+1
      I6=(N3-1)*2+2
      I7=(N4-1)*2+1
      I8=(N4-1)*2+2
```

```
      Q1=FORCE(I1)                                                      STRE0037
      Q2=FORCE(I2)                                                      STRE0038
      Q3=FORCE(I3)                                                      STRE0039
      Q4=FORCE(I4)                                                      STRE0040
      Q5=FORCE(I5)                                                      STRE0041
      Q6=FORCE(I6)                                                      STRE0042
      Q7=FORCE(I7)                                                      STRE0043
      Q8=FORCE(I8)                                                      STRE0044
      STRX1=((1./X)*(Q3-Q1)+(1./Y)*(Q8-Q2)*NU)*YY                       STRE0045
      STRX2=((NU/(X*Y))*(Q2-Q4+Q6-Q8))*YY                               STRE0046
      STRX3=((1./(X*Y))*(Q1-Q3+Q5-Q7))*YY                               STRE0047
      STRY1=((NU/X)*(Q3-Q1)+(1./Y)*(Q8-Q2))*YY                          STRE0048
      STRY2=((1./(X*Y))*(Q2-Q4+Q6-Q8))*YY                               STRE0049
      STRY3=((NU/(X*Y))*(Q1-Q3+Q5-Q7))*YY                               STRE0050
      STRXY1=(ZZ*((1./Y)*(Q7-Q1)+(1./X)*(Q4-Q2)))*YY                    STRE0051
      STRXY2=((ZZ/(X*Y))*(Q1-Q3+Q5-Q7))*YY                              STRE0052
      STRXY3=((ZZ/(X*Y))*(Q2-Q4+Q6-Q8))*YY                              STRE0053
      STRX(N1)=STRX(N1)+(1./NODEN(N1))*STRX1                            STRE0054
      STRX(N2)=STRX(N2)+(1./NODEN(N2))*(STRX1+STRX2*X)                  STRE0055
      STRX(N3)=STRX(N3)+(1./NODEN(N3))*(STRX1+STRX2*X+STRX3*Y)          STRE0056
      STRX(N4)=STRX(N4)+(1./NODEN(N4))*(STRX1         +STRX3*Y)         STRE0057
      STRY(N1)=STRY(N1)+(1./NODEN(N1))*(STRY1)                          STRE0058
      STRY(N2)=STRY(N2)+(1./NODEN(N2))*(STRY1+STRY2*X)                  STRE0059
      STRY(N3)=STRY(N3)+(1./NODEN(N3))*(STRY1+STRY2*X+STRY3*Y)          STRE0060
      STRY(N4)=STRY(N4)+(1./NODEN(N4))*(STRY1         +STRY3*Y)         STRE0061
      STRXY(N1)=STRXY(N1)+(1./NODEN(N1))*(STRXY1)                       STRE0062
      STRXY(N2)=STRXY(N2)+(1./NODEN(N2))*(STRXY1+STRXY2*X)              STRE0063
      STRXY(N3)=STRXY(N3)+(1./NODEN(N3))*(STRXY1+STRXY2*X+STRXY3*Y)     STRE0064
      STRXY(N4)=STRXY(N4)+(1./NODEN(N4))*(STRXY1          +STRXY3*Y)    STRE0065
    7 CONTINUE                                                          STRE0066
      WRITE(NTAP,8)                                                     STRE0067
    8 FORMAT(//,' AVERAGE STRESSES PER NODE',//)                        STRE0068
      WRITE(NTAP,9)                                                     STRE0069
    9 FORMAT(//,'     NODE        STRESS-X       STRESS-Y       STRESS-X STRE0070
     1Y',//)                                                            STRE0071
      WRITE(NTAP,10)(I,STRX(I),STRY(I),STRXY(I),I=1,NODES)              STRE0072
```

```
10   FORMAT(1X,I5,3E16.5)                                    STRE0073
     RETURN                                                  STRE0074
     END                                                     STRE0075
```

```
      SUBROUTINE TRAJEC(STRX,STRY,STRXY,ALXX,ALYY,ALY,STX,STY,STXY,
     2XF,YF,NSTAS,NSTAT1,T,FT,IPOINT,MPTS,TIT,SIGMA2)
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP
      DIMENSION STRX(IDSTR),STRY(IDSTR),STRXY(IDSTR),XF(IXF1,IXF2),
     1YF(IXF1,IXF2),STX(IDST1,IDST2),STY(IDST1,IDST2),STXY(IDST1,IDST2),
     2T(IT),FT(IT),IPOINT(IIPOIN),SIGMA2(IDST1,IDST2),TIT(IT)
C
      WRITE(    6,1051)
 1051 FORMAT('1 ANALYTIC PROCEDURE FOR OBTAINING THE STRESS TRAJECTORIES
     1',/////)
C
C
C     START AT THE FIRST NODE OF THE STRUCTURE
C
C     DO LOOP FOR THE TRAJECTORY NUMBER
C
      J=0
      DO 10 II=1,NSTAS
C
C     DEFINE INITIAL COORDINATES XF, YF, AND INITIAL STRESSES
C
      XF(1,II)=0.0
      YF(1,II)=(II-1)*ALYY
      STX(1,II)=STRX(II)
      STY(1,II)=STRY(II)
      STXY(1,II)=STRXY(II)
C
C     CALCULATE THE INITIAL PRINCIPAL COMPRESSIVE STRESS SIGMA2
C
      SIGMA2(1,II)=(STX(1,II)+STY(1,II))/2.-SQRT(STXY(1,II)**2+
     1  (STX(1,II)-STY(1,II))**2/4.)
C
C     WRITE THE INITIAL STRESSES OUT
C
      WRITE(    6,40)II,XF(1,II),II,YF(1,II),II,STX(1,II),II,STY(1,II),
```

```
     1II,STXY(1,II),II,SIGMA2(1,II)                                     TRAJ0037
   40 FORMAT(//,1X,' XF(1,,',I3,')=',F8.3,' YF(1,,',I3,')=',            TRAJ0038
     1F8.3,//,1X,' STX(1,,',I3,')=',E12.4,' STY(1,,',I3,')=',E12.4,    TRAJ0039
     2' STXY(1,',I3,')=',E12.4,' SIGMA2(1,',I3,')=',E12.4)              TRAJ0040
C                                                                       TRAJ0041
C     DO LOOP FOR THE NUMBER OF STATIONS ALONG THE X-DIRECTION          TRAJ0042
C                                                                       TRAJ0043
      DO 11 I=1,NSTAT1                                                  TRAJ0044
C                                                                       TRAJ0045
C     DEFINE NS TO BE THE INDEX OF THE NEXT ADJACENT                    TRAJ0046
C     STATION ALONG THE X-DIRECTION                                     TRAJ0047
C                                                                       TRAJ0048
      NS=I+1                                                            TRAJ0049
C                                                                       TRAJ0050
C     CALCULATE THETA1,Y1                                               TRAJ0051
C                                                                       TRAJ0052
      IF(I .GT. 1) GO TO 5                                              TRAJ0053
      X1=STRX(II)-STRY(II)                                              TRAJ0054
      THETA1=0.5*ATAN(2.*STRXY(II)/X1)                                  TRAJ0055
  200 IF(THETA1)151,152,152                                             TRAJ0056
  152 IF(X1)160,112,112                                                 TRAJ0057
  160 THETA1=THETA1+3.1416                                              TRAJ0058
      GO TO 112                                                         TRAJ0059
  151 IF(X1)170,112,171                                                 TRAJ0060
  170 THETA1=THETA1+3.1416                                              TRAJ0061
      GO TO 112                                                         TRAJ0062
  171 THETA1=THETA1+1.5708                                              TRAJ0063
  112 Y1=ALXX*ABS(TAN(THETA1))                                          TRAJ0064
      GO TO 6                                                           TRAJ0065
    5 X1=STX(I,II)-STY(I,II)                                            TRAJ0066
      THETA1=0.5*ATAN(2.*STXY(I,II)/X1)                                 TRAJ0067
      GO TO 200                                                         TRAJ0068
C                                                                       TRAJ0069
C     CHECK IF Y1 IS GREATER THAN THE LENGTH OF THE                     TRAJ0070
C     STRUCTURE IN THE Y DIRECTION, I.E. Y1 GREATER THAN ALY            TRAJ0071
C                                                                       TRAJ0072
```

```
    6 IF((Y1+YF(I,II))-ALY)12,12,14
   14 WRITE(    6,15)II
   15 FORMAT(//,1X,' TRAJECTORY(',I3,') IS OFF THE PLATE')
C
C        IN THAT CASE DEFINE XF,YF,STX,STY,STXY
C
      XX=ALXX*(ALY-YF(I,II))/Y1
      XF(NS,II)=XF(I,II)+XX
      YF(NS,II)=ALY
      WRITE(    6,41)NS,II,XF(NS,II),NS,II,YF(NS,II)
   41 FORMAT(//,1X,' XF(',I3,',',I3,')=',F8.3,' YF(',I3,',',
     1I3,')=',F8.3)
      NODER=I*NSTAS+1
      GO TO 87
   12 CONTINUE
C
C        CALCULATE STRESSES AT P1 WHICH IS THE FIRST EXTRAPOLATION POINT
C        THIS POINT IS IN GENERAL BETWEEN TWO NODES, SAY N1 AND N2
C
C        DEFINE NODER (NODE NUMBER ALONG X-AXIS ONLY)
C
      NODER=I*NSTAS+1
      DUMN=(YF(I,II)+Y1)/ALYY
      NNODE=DUMN
C
C        CALCULATE N1 AND N2
C
      N1=NODER+NNODE
      N2=N1+1
C
C        CALCULATE THE DIFFERENCE DIFY1 BETWEEN P1 AND N1
C
      DIFY1=(Y1+YF(I,II))-ALYY*NNODE
C
C        EVALUATE THE TEMPORARY STRESSES AT P1 BY INTERPOLATION
C
```

```
      STRX1=STRX(N1)+(STRX(N2)-STRX(N1))*DIFY1/ALYY
      STRY1=STRY(N1)+(STRY(N2)-STRY(N1))*DIFY1/ALYY
      STRXY1=STRXY(N1)+(STRXY(N2)-STRXY(N1))*DIFY1/ALYY
C
C     CALCULATE THETA2 AND Y2 AT P1
C
      X2=STRX1-STRY1
      THETA2=0.5*ATAN(2.*STRXY1/X2)
      IF(THETA2)191,192,192
  192 IF(X2)165,116,116
  165 THETA2=THETA2+3.1416
      GO TO 116
  191 IF(X2)210,116,211
  210 THETA2=THETA2+3.1416
      GO TO 116
  211 THETA2=THETA2+1.5708
  116 Y2=ALXX*ABS(TAN(THETA2))
C
C     TAKE AVERAGE SLOPE
C
      YF(NS,II)=(Y1+Y2)/2.+YF(I,II)
C
C     CHECK IF YF(NS,II) IS GREATER THAN ALY
C
      IF(YF(NS,II)-ALY)22,22,24
   22 XF(NS,II)=I*ALXX
      WRITE(   6,16)NS,II,XF(NS,II),NS,II,YF(NS,II)
   16 FORMAT(//,1X,' XF(',I3,',',I3,')=',F8.3,' YF(',I3,',',I3,')=',
     1F8.3)
C
C     CALCULATE THE STRESSES AT THE FINAL POINT PF1
      DUMN=YF(NS,II)/ALYY
      NNODE=DUMN
      N1=NODER+NNODE
      N2=N1+1
C
```

```
C     CALCULATE THE DIFFERENCE DIFY2 BETWEEN
C     THE FINAL POINT AND N1
C
      DIFY2=YF(NS,II)-ALYY*NNODE
C
C     EVALUATE STRESSES AT FINAL POINT
C
      STX(NS,II)=STRX(N1)+(STRX(N2)-STRX(N1))*DIFY2/ALYY
      STY(NS,II)=STRY(N1)+(STRY(N2)-STRY(N1))*DIFY2/ALYY
      STXY(NS,II)=STRXY(N1)+(STRXY(N2)-STRXY(N1))*DIFY2/ALYY
      SIGMA2(NS,II)=(STX(NS,II)+STY(NS,II))/2.-SQRT(STXY(NS,II)**2+
     1 (STX(NS,II)-STY(NS,II))**2/4.)
C
C     WRITE THE STRESSES OUT
C
      WRITE(   6,43)NS,II,STX(NS,II),NS,II,STY(NS,II),NS,II,STXY(NS,II),
     1NS,II,SIGMA2(NS,II)
   43 FORMAT(//,1X,' STX(',I3,',',I3,')=',E12.4,
     1' STY(',I3,',',I3,')=',E12.4,' STXY(',I3,',',I3,')=',E12.4,
     2 ' SIGMA2(',I3,',',I3,')=',E12.4)
      GO TO 11
   24 XX=ALXX*(ALY-YF(I,II))/(YF(NS,II)-YF(I,II))
      XF(NS,II)=(I-1)*ALXX+XX
      YF(NS,II)=ALY
      WRITE(   6,44) NS,II,XF(NS,II),NS,II,YF(NS,II)
   44 FORMAT(//,1X,'XF(',I3,',',I3,')=',F8.3,' YF(',I3,',',I3,')='
     1,F8.3)
   87 NN1=NODER-1
      NN2=NN1+NSTAS
      STX(NS,II)=STRX(NN1)+(STRX(NN2)-STRX(NN1))*XX/ALXX
      STY(NS,II)=STRY(NN1)+(STRY(NN2)-STRY(NN1))*XX/ALXX
      STXY(NS,II)=STRXY(NN1)+(STRXY(NN2)-STRXY(NN1))*XX/ALXX
      SIGMA2(NS,II)=(STX(NS,II)+STY(NS,II))/2.-SQRT(STXY(NS,II)**2+
     1 (STX(NS,II)-STY(NS,II))**2/4.)
      WRITE(   6,43)NS,II,STX(NS,II),NS,II,STY(NS,II),NS,II,STXY(NS,II),
     1 NS,II,SIGMA2(NS,II)
```

```
      GO TO 100                                                         TRAJ0181
   11 CONTINUE                                                          TRAJ0182
  100 J=J+1                                                             TRAJ0183
      IPOINT(J)=NS                                                      TRAJ0184
   10 CONTINUE                                                          TRAJ0185
      MPTS=0                                                            TRAJ0186
      DO 85 L=1,NSTAS                                                   TRAJ0187
   85 MPTS=MPTS+IPOINT(L)                                               TRAJ0188
      NPOINS=0                                                          TRAJ0189
      K=0                                                               TRAJ0190
      DO 91 II=1,NSTAS                                                  TRAJ0191
      NPOINS=IPOINT(II)                                                 TRAJ0192
      DO 92 I=1,NPOINS                                                  TRAJ0193
      JJ=I+K                                                            TRAJ0194
      T(JJ)=XF(I,II)                                                    TRAJ0195
      FT(JJ)=YF(I,II)                                                   TRAJ0196
      TIT(JJ)=SIGMA2(I,II)                                              TRAJ0197
   92 CONTINUE                                                          TRAJ0198
      K=IPOINT(II)+K                                                    TRAJ0199
   91 CONTINUE                                                          TRAJ0200
      RETURN                                                            TRAJ0201
      END                                                               TRAJ0202
```

```
      SUBROUTINE PLTTRJ(T,FT,IPOINT,NPLOTS,MPTS,S,FS,TIT)               PLTT0001
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,  PLTT0002
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MJAP        PLTT0003
      DIMENSION HEADNG(15),T(IT),FT(IT),IPOINT(IIPOIN),S(IXF1),FS(IXF1) PLTT0004
     1  ,TIT(IT)                                                        PLTT0005
C     INITIALIZE PLOTTER ROUTINES                                       PLTT0006
C     LABEL WITH USER IDENTIFICATION                                    PLTT0007
C     SPECIAL INSTRUCTIONS FOR CALCOMP OPERATOR                         PLTT0008
      CALL NEWPLT('M8707','8129','WHITE ','BLACK')                      PLTT0009
C     DEFINE A NEW ORIGIN                                               PLTT0010
      CALL PLOT1(4.0,0.0,-3)                                            PLTT0011
      READ(5,900)NPAGES                                                 PLTT0012
  900 FORMAT(I3)                                                        PLTT0013
C     FOR EACH PAGE                                                     PLTT0014
      DO 500 KPAGE=1,NPAGES                                             PLTT0015
C     FORCE CORRECT POSITIONINING OF PEN                                PLTT0016
      CALL PLOT1(0.0,10.5,3)                                            PLTT0017
      CALL PLOT1(0.,-.5,3)                                              PLTT0018
      CALL PLOT1(4.0,0.0,-3)                                            PLTT0019
C     LABEL EACH PAGE                                                   PLTT0020
      READ(5,901)(HEADNG(J),J=1,15)                                     PLTT0021
  901 FORMAT(15A4)                                                      PLTT0022
      CALL SYMBL5(-2.,0.,-.12,HEADNG,90.,60)                            PLTT0023
C     READ IN SCALING INFORMATION FOR THIS PAGE                         PLTT0024
      READ(5,902)        X,Y,YXZERO,DX,DY,NDIGX,KEXPX,NDIGY,KEXPY       PLTT0025
C     DEFINE AN ORIGIN FOR THIS PAGE                                    PLTT0026
      CALL PLOT1(X,Y,-3)                                                PLTT0027
      READ(5,555)XX1,YY1,XA,YA                                          PLTT0028
  555 FORMAT(4F10.2)                                                    PLTT0029
      DDX=DX/XA                                                         PLTT0030
      DDY=DY/YA                                                         PLTT0031
      XLNGTH=XX1-X                                                      PLTT0032
      YLNGTH=YY1-Y                                                      PLTT0033
C     DRAW AXES                                                         PLTT0034
      CALL AXIS1(-X,0.,'      ',4,XLNGTH,0.,0.0,DX,NDIGX,KEXPX,XA)      PLTT0035
  902 FORMAT(3X,5E12.6,4I3)                                             PLTT0036
```

```
      CALL AXIS1(0.,-Y,'     ',4,YLNGTH,90.,YXZERO,DY,NDIGY,KEXPY,YA)
C     FOR EACH PLOT
      KSYMBL=0
      LL=0
      DO 400 KPLOT=1,NPLOTS
      K=IPOINT(KPLOT)
      DO 20 JJ=1,K
      NPTS=K
      LL=LL+1
      S(JJ)=T(LL)
   20 FS(JJ)=FT(LL)
C     PLOT ONE SET OF POINTS
      CALL SCLGPH(S,FS, NPTS,.000,KSYMBL,0.,DDX,YXZERO,DDY)
      LK=LL-K
      DO 300 JJ=1,K
      LK=LK+1
      PRINTX=S(JJ)/DX+.05
      PRINTY=FS(JJ)/DY-.1
      CALL NUMBR1(PRINTX,PRINTY,.08,TIT(LK),0.,4)
  300 CONTINUE
      IF(LL-MPTS)400,400,401
  400 CONTINUE
C     SET ORIGIN BACK TO PREVIOUS POSITION
  401 CALL PLOT1(-X,-Y,-3)
C     SPACE AHEAD FOR THE NEXT PAGE
      CALL PLOT1(27.0,0.0,-3)
  500 CONTINUE
      CALL ENDPLT
      WRITE(NTAP,910)
  910 FORMAT(////16H PLOTS COMPLETED)
      RETURN
      END
```

```
      SUBROUTINE STIFEN(XF,YF,NSTAS,NSTAS1,ALXX,ALYY,IPOINT,MPOINT,
     1KELEM)
      COMMON/A/ IDNODE,INODEN,IFORCE,IMPOIN,I1KELE,I2KELE,IDSTR,IDST1,
     1 IDST2,IXF1,IXF2,IT,IIPOIN,ILINUM,NLIN,IDIR,IOCT,NTAP,MTAP
      DIMENSION XF(IXF1,IXF2),YF(IXF1,IXF2),IPOINT(IIPOIN),
     1KELEM(I1KELE,I2KELE),MPOINT(IMPOIN)
      M=0
      DO 10 J=1,NSTAS
      KK=0
      NPOINS=IPOINT(J)
      NPOIN1=NPOINS-1
      DO 11 I=1,NPOIN1
      NS=I+1
      IF(I-1)45,45,46
   45 NYINC=J-1
      GO TO 500
   46 NYINC=YF(I,J)/ALYY
  500 NARCH=(I-1)*NSTAS1+1+NYINC
      IF(XF(NS,J)-(I*ALXX))12,13,13
   12 NFINAL=(I-1)*NSTAS1+NSTAS1
      DO 20 K=NARCH,NFINAL
      KK=1+KK
   20 KELEM(KK,J)=K
      GO TO 100
   13 NN=YF(NS,J)/ALYY
      NFINAL=(I-1)*NSTAS1+1+NN
      DO 30 K=NARCH,NFINAL
      KK=1+KK
   30 KELEM(KK,J)=K
   11 CONTINUE
  100 M=M+1
      MPOINT(M)=KK
   10 CONTINUE
      DO 40 JJ=1,NSTAS
      WRITE(NTAP,199)JJ
  199 FORMAT(' FOR THE ',I2,' TRAJECTORY')
```

```
      MELEM=MPOINT(JJ)                                              STIF0037
      DO 50 II=1,MELEM                                              STIF0038
      WRITE(NTAP,200)II,JJ,KELEM(II,JJ)                             STIF0039
  200 FORMAT( ' KELEM(',I2,' , ',I2,')=',I3)                        STIF0040
   50 CONTINUE                                                      STIF0041
   40 CONTINUE                                                      STIF0042
      RETURN                                                        STIF0043
      END                                                           STIF0044
```

Appendix II-B

LISTING OF COMPUTER PROGRAM'S PRINTOUT FOR THE CANTILEVER CASE

```
ANALYTIC TABULATION OF RESULTS FOR THE CASE OF A PLATE WHICH IS LOADED

IN ITS PLANE SIMILARLY TO A CANTILEVER BEAM.

EACH ONE OF THE DISCRETE LOADS IS APPLIED AT EACH OF THE NODES OF THE

FREE EDGE.

THE MAGNITUDE OF THE DISCRETE LOADS HAS A PARABOLIC PROFILE ALONG THE

EDGE OF THE PLATE.



 NUMBER OF SEQUENCE =     1

 NUMBER OF ELEMENTS       =   250

 NUMBER OF NODES =  236

 NSS=   260

 NUMBER OF LINES =   535

 THICKNESS =    0.400000E 00

 MODULUS =   0.300000E 08

 POISSON RATIO =   0.250000E 00

 ALENX     ALENY
0.1440E 020.1440E 02
```

```
******BOUNDARY CONDITIONS ***************

 11   1   2   3   4   5   6   7   8   9  10  11

  1   6

  1   2   3   4   5   6   7   8   9  10  11   6




THE FIRST NUMBER IN THE FIRST TWC LINES INDICATES THE NUMBER OF NODES

FIXED IN THE X ANC Y DIRECTICNS RESPECTIVELY. THE NUMBERS FOLLOWING IN

THESE FIRST TWO LINES CORRESPOND TO THE INDIVIDUAL NODES WHICH ARE FIXED

IN THE X AND THE Y DIRECTIONS. THE THIRD LINE LISTS THE NODES WHICH ARE

FIXED IN BOTH DIRECTIONS IN ORDER.
```

FORCYNODE	VALUE
277	-3.0000
278	-6.0000
279	-8.0000
280	-8.5000
281	-9.0000
282	-8.5000
283	-8.0000
284	-6.0000
285	-3.0000

FORCE ENTRIES TWO PER NODE IN ORDER

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0

FORCE ENTRIES TWO PER NODE IN ORDER CONTINUED FORC0037

FORC0038

0.0	0.0	0.0	0.0	0.0	0.0	FORC0039
0.0	0.0	0.0	0.0	0.0	0.0	FORC0040
0.0	0.0	0.0	0.0	0.0	0.0	FORC0041
0.0	0.0	0.0	0.0	0.0	0.0	FORC0042
0.0	0.0	0.0	0.0	0.0	0.0	FORC0043
0.0	0.0	0.0	0.0	0.0	0.0	FORC0044
0.0	0.0	0.0	0.0	0.0	0.0	FORC0045
0.0	0.0	0.0	0.0	0.0	0.0	FORC0046
0.0	0.0	0.0	0.0	0.0	0.0	FORC0047
0.0	0.0	0.0	0.0	0.0	0.0	FORC0048
0.0	0.0	0.0	0.0	0.0	0.0	FORC0049
0.0	0.0	0.0	0.0	0.0	0.0	FORC0050
0.0	0.0	0.0	0.0	0.0	0.0	FORC0051
0.0	0.0	0.0	0.0	0.0	0.0	FORC0052
0.0	0.0	0.0	0.0	0.0	0.0	FORC0053
0.0	0.0	0.0	0.0	0.0	0.0	FORC0054
0.0	0.0	0.0	0.0	0.0	0.0	FORC0055
0.0	0.0	0.0	0.0	0.0	0.0	FORC0056
0.0	0.0	0.0	0.0	0.0	0.0	FORC0057
0.0	0.0	0.0	0.0	0.0	0.0	FORC0058
0.0	0.0	0.0	0.0	0.0	0.0	FORC0059
0.0	0.0	0.0	0.0	0.0	0.0	FORC0060
0.0	0.0	0.0	0.0	0.0	0.0	FORC0061
0.0	0.0	0.0	0.0	0.0	0.0	FORC0062
0.0	0.0	0.0	0.0	0.0	0.0	FORC0063
0.0	0.0	0.0	0.0	0.0	0.0	FORC0064
0.0	0.0	0.0	0.0	0.0	0.0	FORC0065
0.0	0.0	0.0	0.0	0.0	0.0	FORC0066
0.0	0.0	0.0	0.0	0.0	0.0	FORC0067
0.0	0.0	0.0	0.0	0.0	0.0	FORC0068
0.0	0.0	0.0	0.0	0.0	0.0	FORC0069
0.0	0.0	0.0	0.0	0.0	0.0	FORC0070
0.0	0.0	0.0	0.0	0.0	0.0	FORC0071
0.0	0.0	0.0	0.0	0.0	0.0	FORC0072

FORCE ENTRIES TWO PER NODE IN ORDER CONTINUED FORC0073

FORC0074

0.0	0.0	0.0	0.0	0.0	0.0	FORC0075
0.0	0.0	0.0	0.0	0.0	0.0	FORC0076
0.0	0.0	0.0	0.0	0.0	0.0	FORC0077
0.0	0.0	0.0	0.0	0.0	0.0	FORC0078
0.0	0.0	0.0	0.0	0.0	0.0	FORC0079
0.0	0.0	0.0	0.0	0.0	0.0	FORC0080
0.0	0.0	0.0	0.0	0.0	0.0	FORC0081
0.0	0.0	0.0	0.0	0.0	0.0	FORC0082
0.0	0.0	0.0	0.0	0.0	0.0	FORC0083
0.0	0.0	0.0	0.0	0.0	0.0	FORC0084
0.0	0.0	0.0	0.0	0.0	0.0	FORC0085
0.0	0.0	0.0	0.0	0.0	0.0	FORC0086
0.0	0.0	0.0	0.0	0.0	0.0	FORC0087
0.0	0.0	0.0	0.0	0.0	0.0	FORC0088
0.0	0.0	0.0	0.0	0.0	0.0	FORC0089
0.0	0.0	0.0	0.0	0.0	0.0	FORC0090
0.0	0.0	0.0	0.0	0.0	0.0	FORC0091
0.0	0.0	0.0	0.0	0.0	0.0	FORC0092
0.0	0.0	0.0	0.0	0.0	0.0	FORC0093
0.0	0.0	0.0	0.0	0.0	0.0	FORC0094
0.0	0.0	0.0	0.0	0.0	0.0	FORC0095
0.0	0.0	0.0	0.0	0.0	0.0	FORC0096
0.0	0.0	0.0	0.0	0.0	0.0	FORC0097
0.0	0.0	0.0	0.0	0.0	0.0	FORC0098
0.0	0.0	0.0	0.0	0.0	0.0	FORC0099
0.0	0.0	0.0	0.0	0.0	0.0	FORC0100
0.0	0.0	0.0	0.0	0.0	0.0	FORC0101
0.0	0.0	0.0	0.0	0.0	0.0	FORC0102
0.0	0.0	0.0	0.0	0.0	0.0	FORC0103
0.0	0.0	0.0	0.0	0.0	0.0	FORC0104
0.0	0.0	0.0	0.0	0.0	0.0	FORC0105
0.0	0.0	0.0	0.0	0.0	0.0	FORC0106
0.0	0.0	0.0	0.0	0.0	0.0	FORC0107
0.0	0.0	0.0	0.0	0.0	0.0	FORC0108

```
      FORCE  ENTRIES  TWO  PER  NODE  IN  ORDER CONTINUED                                FORC0109
                                                                                         FORC0110
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0111
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0112
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0113
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0114
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0115
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0116
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0117
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0118
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0119
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0120
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0121
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0122
0.0         0.0         0.0         0.0         0.0         0.0                          FORC0123
0.0        -3.0000      0.0        -6.0000      0.0        -8.0000                       FORC0124
0.0        -8.5000      0.0        -9.0000      0.0        -8.5000                       FORC0125
0.0        -8.0000      0.0        -6.0000      0.0        -3.0000                       FORC0126
0.0         0.0                                                                          FORC0127
```

THE DETERMINANT IS THAT OF THE CCNSTRAINEC STIFFNESS MATRIX CF THE ASSEMBLED STRUCTURE.

IF THE DETERMINANT IS POSITIVE THE STRUCTURE IS PROPERLY CONSTRAINED.

IF THE DETERMINANT IS NEGATIVE OR ZERO THE STRUCTURE IS A MECHANISM.

DETERMINANT = C.9148679E 03

THE FOLLOWING IS A LISTING OF THE DISPLACEMENTS PER NODE IN CRDER

NODE	DISP. X	DISP. Y
1	0.0	-0.1187E-04
2	0.0	-0.1014E-04
3	0.0	-0.8616E-05
4	0.0	-0.7040E-05
5	0.0	-0.4823E-05
6	0.0	0.0
7	0.0	-0.4823E-05
8	0.0	-0.7040E-05

NODE	DISP. X	DISP. Y	CONTINUED	MAIN0037
				MAIN0038
9	0.0	-0.8616E-05		MAIN0039
10	0.0	-0.1014E-04		MAIN0040
11	0.0	-0.1187E-04		MAIN0041
12	-0.7550E-05	-0.1286E-04		MAIN0042
13	-0.5795E-05	-0.1117E-04		MAIN0043
14	-0.4339E-05	-0.9730E-05		MAIN0044
15	-0.3072E-05	-0.8391E-05		MAIN0045
16	-0.2026E-05	-0.7175E-05		MAIN0046
17	-0.6639E-10	-0.7284E-05		MAIN0047
18	0.2026E-05	-0.7175E-05		MAIN0048
19	0.3072E-05	-0.8391E-05		MAIN0049
20	0.4339E-05	-0.9730E-05		MAIN0050
21	0.5795E-05	-0.1117E-04		MAIN0051
22	0.7550E-05	-0.1286E-04		MAIN0052
23	-0.1488E-04	-0.1576E-04		MAIN0053
24	-0.1145E-04	-0.1416E-04		MAIN0054
25	-0.8496E-05	-0.1289E-04		MAIN0055
26	-0.5816E-05	-0.1194E-04		MAIN0056
27	-0.3032E-05	-0.1147E-04		MAIN0057
28	-0.1310E-09	-0.1127E-04		MAIN0058
29	0.3032E-05	-0.1147E-04		MAIN0059
30	0.5816E-05	-0.1194E-04		MAIN0060
31	0.8496E-05	-0.1289E-04		MAIN0061
32	0.1144E-04	-0.1416E-04		MAIN0062
33	0.1488E-04	-0.1576E-04		MAIN0063
34	-0.2186E-04	-0.2038E-04		MAIN0064
35	-0.1686E-04	-0.1887E-04		MAIN0065
36	-0.1243E-04	-0.1774E-04		MAIN0066
37	-0.8275E-05	-0.1698E-04		MAIN0067
38	-0.4166E-05	-0.1655E-04		MAIN0068
39	-0.1846E-09	-0.1644E-04		MAIN0069
40	0.4166E-05	-0.1655E-04		MAIN0070
41	0.8275E-05	-0.1698E-04		MAIN0071
42	0.1243E-04	-0.1774E-04		MAIN0072

NODE	DISP. X	DISP. Y	CONTINUED	MAIN0073
				MAIN0074
43	0.1686E-04	-0.1887E-04		MAIN0075
44	0.2186E-04	-0.2038E-04		MAIN0076
45	-0.2843E-04	-0.2649E-04		MAIN0077
46	-0.2202E-04	-0.2507E-04		MAIN0078
47	-0.1619E-04	-0.2401E-04		MAIN0079
48	-0.1068E-04	-0.2330E-04		MAIN0080
49	-0.5322E-05	-0.2290E-04		MAIN0081
50	-0.2392E-09	-0.2277E-04		MAIN0082
51	0.5321E-05	-0.2290E-04		MAIN0083
52	0.1068E-04	-0.2330E-04		MAIN0084
53	0.1619E-04	-0.2401E-04		MAIN0085
54	0.2202E-04	-0.2507E-04		MAIN0086
55	0.2843E-04	-0.2649E-04		MAIN0087
56	-0.3462E-04	-0.3393E-04		MAIN0088
57	-0.2692E-04	-0.3259E-04		MAIN0089
58	-0.1979E-04	-0.3157E-04		MAIN0090
59	-0.1303E-04	-0.3088E-04		MAIN0091
60	-0.6472E-05	-0.3047E-04		MAIN0092
61	-0.2901E-09	-0.3035E-04		MAIN0093
62	0.6471E-05	-0.3047E-04		MAIN0094
63	0.1303E-04	-0.3088E-04		MAIN0095
64	0.1979E-04	-0.3157E-04		MAIN0096
65	0.2692E-04	-0.3259E-04		MAIN0097
66	0.3462E-04	-0.3393E-04		MAIN0098
67	-0.4046E-04	-0.4258E-04		MAIN0099
68	-0.3157E-04	-0.4131E-04		MAIN0100
69	-0.2324E-04	-0.4033E-04		MAIN0101
70	-0.1530E-04	-0.3965E-04		MAIN0102
71	-0.7595E-05	-0.3926E-04		MAIN0103
72	-0.3201E-09	-0.3913E-04		MAIN0104
73	0.7594E-05	-0.3926E-04		MAIN0105
74	0.1530E-04	-0.3965E-04		MAIN0106
75	0.2324E-04	-0.4033E-04		MAIN0107
76	0.3157E-04	-0.4131E-04		MAIN0108

NODE	DISP. X	DISP. Y	CONTINUED	MAIN0109
				MAIN0110
77	0.4046E-04	-0.4258E-04		MAIN0111
78	-0.4598E-04	-0.5236E-04		MAIN0112
79	-0.3598E-04	-0.5116E-04		MAIN0113
80	-0.2653E-04	-0.5023E-04		MAIN0114
81	-0.1748E-04	-0.4957E-04		MAIN0115
82	-0.8677E-05	-0.4918E-04		MAIN0116
83	-0.3483E-09	-0.4906E-04		MAIN0117
84	0.8676E-05	-0.4918E-04		MAIN0118
85	0.1748E-04	-0.4957E-04		MAIN0119
86	0.2653E-04	-0.5023E-04		MAIN0120
87	0.3598E-04	-0.5116E-04		MAIN0121
88	0.4598E-04	-0.5236E-04		MAIN0122
89	-0.5119E-04	-0.6321E-04		MAIN0123
90	-0.4015E-04	-0.6207E-04		MAIN0124
91	-0.2965E-04	-0.6119E-04		MAIN0125
92	-0.1955E-04	-0.6056E-04		MAIN0126
93	-0.9709E-05	-0.6019E-04		MAIN0127
94	-0.3756E-09	-0.6007E-04		MAIN0128
95	0.9708E-05	-0.6019E-04		MAIN0129
96	0.1955E-04	-0.6056E-04		MAIN0130
97	0.2965E-04	-0.6119E-04		MAIN0131
98	0.4015E-04	-0.6207E-04		MAIN0132
99	0.5119E-04	-0.6321E-04		MAIN0133
100	-0.5610E-04	-0.7506E-04		MAIN0134
101	-0.4408E-04	-0.7399E-04		MAIN0135
102	-0.3259E-04	-0.7316E-04		MAIN0136
103	-0.2151E-04	-0.7257E-04		MAIN0137
104	-0.1069E-04	-0.7221E-04		MAIN0138
105	-0.3920E-09	-0.7210E-04		MAIN0139
106	0.1069E-04	-0.7221E-04		MAIN0140
107	0.2150E-04	-0.7257E-04		MAIN0141
108	0.3259E-04	-0.7316E-04		MAIN0142
109	0.4407E-04	-0.7399E-04		MAIN0143
110	0.5610E-04	-0.7506E-04		MAIN0144

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NODE       DISP. X     DISP. Y          CONTINUED                                        MAIN0145
                                                                                         MAIN0146
111 -0.6071E-04 -0.8786E-04                                                              MAIN0147
112 -0.4776E-04 -0.8686E-04                                                              MAIN0148
113 -0.3535E-04 -0.8608E-04                                                              MAIN0149
114 -0.2335E-04 -0.8552E-04                                                              MAIN0150
115 -0.1161E-04 -0.8519E-04                                                              MAIN0151
116 -0.4284E-09 -0.8508E-04                                                              MAIN0152
117  0.1161E-04 -0.8519E-04                                                              MAIN0153
118  0.2335E-04 -0.8553E-04                                                              MAIN0154
119  0.3535E-04 -0.8608E-04                                                              MAIN0155
120  0.4776E-04 -0.8686E-04                                                              MAIN0156
121  0.6071E-04 -0.8787E-04                                                              MAIN0157
122 -0.6502E-04 -0.1016E-03                                                              MAIN0158
123 -0.5121E-04 -0.1006E-03                                                              MAIN0159
124 -0.3794E-04 -0.9989E-04                                                              MAIN0160
125 -0.2507E-04 -0.9937E-04                                                              MAIN0161
126 -0.1247E-04 -0.9906E-04                                                              MAIN0162
127 -0.4429E-09 -0.9896E-04                                                              MAIN0163
128  0.1247E-04 -0.9906E-04                                                              MAIN0164
129  0.2507E-04 -0.9938E-04                                                              MAIN0165
130  0.3794E-04 -0.9990E-04                                                              MAIN0166
131  0.5121E-04 -0.1006E-03                                                              MAIN0167
132  0.6502E-04 -0.1016E-03                                                              MAIN0168
133 -0.6904E-04 -0.1161E-03                                                              MAIN0169
134 -0.5443E-04 -0.1152E-03                                                              MAIN0170
135 -0.4035E-04 -0.1145E-03                                                              MAIN0171
136 -0.2668E-04 -0.1141E-03                                                              MAIN0172
137 -0.1327E-04 -0.1138E-03                                                              MAIN0173
138 -0.4638E-09 -0.1137E-03                                                              MAIN0174
139  0.1327E-04 -0.1138E-03                                                              MAIN0175
140  0.2668E-04 -0.1141E-03                                                              MAIN0176
141  0.4035E-04 -0.1145E-03                                                              MAIN0177
142  0.5443E-04 -0.1152E-03                                                              MAIN0178
143  0.6904E-04 -0.1161E-03                                                              MAIN0179
144 -0.7275E-04 -0.1314E-03                                                              MAIN0180
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NODE	DISP. X	DISP. Y	CONTINUED	
				MAIN0181
				MAIN0182
145	-0.5740E-04	-0.1306E-03		MAIN0183
146	-0.4258E-04	-0.1300E-03		MAIN0184
147	-0.2817E-04	-0.1295E-03		MAIN0185
148	-0.1402E-04	-0.1292E-03		MAIN0186
149	-0.4738E-09	-0.1292E-03		MAIN0187
150	0.1402E-04	-0.1292E-03		MAIN0188
151	0.2816E-04	-0.1295E-03		MAIN0189
152	0.4258E-04	-0.1300E-03		MAIN0190
153	0.5740E-04	-0.1306E-03		MAIN0191
154	0.7275E-04	-0.1314E-03		MAIN0192
155	-0.7618E-04	-0.1474E-03		MAIN0193
156	-0.6014E-04	-0.1467E-03		MAIN0194
157	-0.4464E-04	-0.1461E-03		MAIN0195
158	-0.2953E-04	-0.1457E-03		MAIN0196
159	-0.1470E-04	-0.1454E-03		MAIN0197
160	-0.5C93E-09	-0.1454E-03		MAIN0198
161	0.1470E-04	-0.1454E-03		MAIN0199
162	0.2953E-04	-0.1457E-03		MAIN0200
163	0.4463E-04	-0.1461E-03		MAIN0201
164	0.6014E-04	-0.1467E-03		MAIN0202
165	0.7618E-04	-0.1474E-03		MAIN0203
166	-0.7930E-04	-0.1641E-03		MAIN0204
167	-0.6264E-04	-0.1634E-03		MAIN0205
168	-0.4651E-04	-0.1629E-03		MAIN0206
169	-0.3078E-04	-0.1625E-03		MAIN0207
170	-0.1533E-04	-0.1623E-03		MAIN0208
171	-0.5239E-09	-0.1622E-03		MAIN0209
172	0.1532E-04	-0.1623E-03		MAIN0210
173	0.3078E-04	-0.1625E-03		MAIN0211
174	0.4651E-04	-0.1629E-03		MAIN0212
175	0.6264E-04	-0.1634E-03		MAIN0213
176	0.7930E-04	-0.1641E-03		MAIN0214
177	-0.8213E-04	-0.1813E-03		MAIN0215
178	-0.6490E-04	-0.1807E-03		MAIN0216

NODE	DISP. X	DISP. Y	CONTINUED	
				MAIN0217
				MAIN0218
179	-0.4821E-04	-C.1803E-03		MAIN0219
180	-0.3191E-04	-0.1799E-03		MAIN0220
181	-0.1589E-04	-0.1797E-03		MAIN0221
182	-0.5384E-09	-0.1797E-03		MAIN0222
183	C.1589E-04	-0.1797E-03		MAIN0223
184	0.3191E-04	-0.1799E-03		MAIN0224
185	0.4821E-04	-0.1803E-03		MAIN0225
186	0.6490E-04	-0.1807E-03		MAIN0226
187	0.8213E-04	-0.1813E-03		MAIN0227
188	-0.8466E-04	-0.1991E-03		MAIN0228
189	-0.6692E-04	-0.1986E-03		MAIN0229
190	-0.4972E-04	-0.1982E-03		MAIN0230
191	-0.3293E-04	-0.1979E-03		MAIN0231
192	-0.1640E-04	-0.1977E-03		MAIN0232
193	-0.5821E-09	-0.1977E-03		MAIN0233
194	0.1640E-04	-0.1977E-03		MAIN0234
195	0.3293E-04	-0.1979E-03		MAIN0235
196	0.4972E-04	-0.1982E-03		MAIN0236
197	0.6692E-04	-0.1986E-03		MAIN0237
198	0.8466E-04	-0.1991E-03		MAIN0238
199	-0.8689E-04	-0.2174E-03		MAIN0239
200	-0.6871E-04	-0.2169E-03		MAIN0240
201	-0.5106E-04	-0.2166E-03		MAIN0241
202	-0.3382E-04	-0.2163E-03		MAIN0242
203	-0.1684E-04	-0.2162E-03		MAIN0243
204	-0.6257E-09	-C.2161E-03		MAIN0244
205	0.1684E-04	-0.2162E-03		MAIN0245
206	0.3382E-04	-0.2163E-03		MAIN0246
207	0.5106E-04	-0.2166E-03		MAIN0247
208	0.6871E-04	-0.2169E-03		MAIN0248
209	0.8689E-04	-0.2174E-03		MAIN0249
210	-0.8883E-04	-0.2361E-03		MAIN0250
211	-0.7026E-04	-0.2357E-03		MAIN0251
212	-0.5223E-04	-0.2354E-03		MAIN0252

NODE	DISP. X	DISP. Y	CONTINUED	
				MAIN0253
				MAIN0254
213	-0.3459E-04	-0.2352E-03		MAIN0255
214	-0.1723E-04	-0.2350E-03		MAIN0256
215	-0.7858E-09	-0.2350E-03		MAIN0257
216	0.1723E-04	-0.2350E-03		MAIN0258
217	0.3459E-04	-0.2352E-03		MAIN0259
218	0.5222E-04	-0.2354E-03		MAIN0260
219	0.7026E-04	-0.2357E-03		MAIN0261
220	0.8883E-04	-0.2361E-03		MAIN0262
221	-0.9046E-04	-0.2552E-03		MAIN0263
222	-0.7157E-04	-0.2548E-03		MAIN0264
223	-0.5321E-04	-0.2546E-03		MAIN0265
224	-0.3525E-04	-0.2544E-03		MAIN0266
225	-0.1756E-04	-0.2543E-03		MAIN0267
226	-0.9168E-09	-0.2542E-03		MAIN0268
227	0.1756E-04	-0.2543E-03		MAIN0269
228	0.3525E-04	-0.2544E-03		MAIN0270
229	0.5321E-04	-0.2546E-03		MAIN0271
230	0.7157E-04	-0.2548E-03		MAIN0272
231	0.9046E-04	-0.2552E-03		MAIN0273
232	-0.9180E-04	-0.2745E-03		MAIN0274
233	-0.7264E-04	-0.2743E-03		MAIN0275
234	-0.5402E-04	-0.2740E-03		MAIN0276
235	-0.3579E-04	-0.2739E-03		MAIN0277
236	-0.1783E-04	-0.2738E-03		MAIN0278
237	-0.9750E-09	-0.2738E-03		MAIN0279
238	0.1783E-04	-0.2738E-03		MAIN0280
239	0.3579E-04	-0.2739E-03		MAIN0281
240	0.5401E-04	-0.2740E-03		MAIN0282
241	0.7264E-04	-0.2743E-03		MAIN0283
242	0.9180E-04	-0.2745E-03		MAIN0284
243	-0.9283E-04	-0.2941E-03		MAIN0285
244	-0.7347E-04	-0.2939E-03		MAIN0286
245	-0.5465E-04	-0.2937E-03		MAIN0287
246	-0.3621E-04	-0.2936E-03		MAIN0288

NODE	DISP. X	DISP. Y	CONTINUED	
				MAIN0289
				MAIN0290
247	-0.1804E-04	-0.2936E-03		MAIN0291
248	-0.9459E-09	-0.2935E-03		MAIN0292
249	0.1804E-04	-0.2936E-03		MAIN0293
250	0.3621E-04	-0.2936E-03		MAIN0294
251	0.5465E-04	-0.2937E-03		MAIN0295
252	0.7347E-04	-0.2939E-03		MAIN0296
253	0.9283E-04	-0.2941E-03		MAIN0297
254	-0.9356E-04	-0.3139E-03		MAIN0298
255	-0.7407E-04	-0.3137E-03		MAIN0299
256	-0.5510E-04	-0.3136E-03		MAIN0300
257	-0.3652E-04	-0.3136E-03		MAIN0301
258	-0.1819E-04	-0.3135E-03		MAIN0302
259	-0.8586E-09	-0.3135E-03		MAIN0303
260	0.1819E-04	-0.3135E-03		MAIN0304
261	0.3652E-04	-0.3136E-03		MAIN0305
262	0.5510E-04	-0.3136E-03		MAIN0306
263	0.7407E-04	-0.3137E-03		MAIN0307
264	0.9356E-04	-0.3139E-03		MAIN0308
265	-0.9396E-04	-0.3337E-03		MAIN0309
266	-0.7444E-04	-0.3336E-03		MAIN0310
267	-0.5539E-04	-0.3336E-03		MAIN0311
268	-0.3669E-04	-0.3336E-03		MAIN0312
269	-0.1827E-04	-0.3336E-03		MAIN0313
270	-0.7567E-09	-0.3336E-03		MAIN0314
271	0.1827E-04	-0.3336E-03		MAIN0315
272	0.3669E-04	-0.3336E-03		MAIN0316
273	0.5538E-04	-0.3336E-03		MAIN0317
274	0.7444E-04	-0.3337E-03		MAIN0318
275	0.9396E-04	-0.3337E-03		MAIN0319
276	-0.9406E-04	-0.3534E-03		MAIN0320
277	-0.7455E-04	-0.3536E-03		MAIN0321
278	-0.5547E-04	-0.3537E-03		MAIN0322
279	-0.3675E-04	-0.3538E-03		MAIN0323
280	-0.1830E-04	-0.3537E-03		MAIN0324

NODE	DISP. X	DISP. Y	CONTINUED	
				MAIN0325
				MAIN0326
281	-0.6092E-09	-0.3537E-03		MAIN0327
282	0.1830E-04	-0.3537E-03		MAIN0328
283	0.3675E-04	-0.3538E-03		MAIN0329
284	0.5547E-04	-0.3537E-03		MAIN0330
285	0.7455E-04	-0.3536E-03		MAIN0331
286	0.9406E-04	-0.3534E-03		MAIN0332
				MAIN0333
				MAIN0334

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NODE(4)    ( I.E. THE NODE NUMBERS PER ELEMENT, COUNTERCLOCKWISE )     STRE0001
                                                                       STRE0002
                                                                       STRE0003
   1   12   13    2                                                    STRE0004
   2   13   14    3                                                    STRE0005
   3   14   15    4                                                    STRE0006
   4   15   16    5                                                    STRE0007
   5   16   17    6                                                    STRE0008
   6   17   18    7                                                    STRE0009
   7   18   19    8                                                    STRE0010
   8   19   20    9                                                    STRE0011
   9   20   21   10                                                    STRE0012
  10   21   22   11                                                    STRE0013
  12   23   24   13                                                    STRE0014
  13   24   25   14                                                    STRE0015
  14   25   26   15                                                    STRE0016
  15   26   27   16                                                    STRE0017
  16   27   28   17                                                    STRE0018
  17   28   29   18                                                    STRE0019
  18   29   30   19                                                    STRE0020
  19   30   31   20                                                    STRE0021
  20   31   32   21                                                    STRE0022
  21   32   33   22                                                    STRE0023
  23   34   35   24                                                    STRE0024
  24   35   36   25                                                    STRE0025
  25   36   37   26                                                    STRE0026
  26   37   38   27                                                    STRE0027
  27   38   39   28                                                    STRE0028
  28   39   40   29                                                    STRE0029
  29   40   41   30                                                    STRE0030
  30   41   42   31                                                    STRE0031
  31   42   43   32                                                    STRE0032
  32   43   44   33                                                    STRE0033
  34   45   46   35                                                    STRE0034
  35   46   47   36                                                    STRE0035
  36   47   48   37                                                    STRE0036
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NODE(4) CCNTINUED

				STRE0037
				STRE0038
37	48	49	38	STRE0039
38	49	50	39	STRE0040
39	50	51	40	STRE0041
40	51	52	41	STRE0042
41	52	53	42	STRE0043
42	53	54	43	STRE0044
43	54	55	44	STRE0045
45	56	57	46	STRE0046
46	57	58	47	STRE0047
47	53	59	48	STRE0048
48	59	60	49	STRE0049
49	60	61	50	STRE005C
50	61	62	51	STRE0051
51	62	63	52	STRE0052
52	63	64	53	STRE0053
53	64	65	54	STRE0054
54	65	66	55	STRE0055
56	67	68	57	STRE0056
57	68	69	58	STRE0057
58	69	70	59	STRE0058
59	70	71	60	STRE0059
6C	71	72	61	STRE006C
61	72	73	62	STRE0061
62	73	74	63	STRE0062
63	74	75	64	STRE0063
64	75	76	65	STRE0064
65	76	77	66	STRE0065
67	78	79	68	STRE0066
68	79	80	69	STRE0067
69	80	81	70	STRE0068
7C	81	82	71	STRE0069
71	82	83	72	STRE007C
72	83	84	73	STRE0071
73	84	85	74	STRE0072

NODE(4) CONTINUED STRE0073

74	85	86	75	STRE0074
75	86	87	76	STRE0075
76	87	88	77	STRE0076
78	89	90	79	STRE0077
79	90	91	80	STRE0078
80	91	92	81	STRE0079
81	92	93	82	STRE0080
82	93	94	83	STRE0081
83	94	95	84	STRE0082
84	95	96	85	STRE0083
85	96	97	86	STRE0084
86	97	98	87	STRE0085
87	98	99	88	STRE0086
89	100	101	90	STRE0087
90	101	102	91	STRE0088
91	102	103	92	STRE0089
92	103	104	93	STRE0090
93	104	105	94	STRE0091
94	105	106	95	STRE0092
95	106	107	96	STRE0093
96	107	108	97	STRE0094
97	108	109	98	STRE0095
98	109	110	99	STRE0096
100	111	112	101	STRE0097
101	112	113	102	STRE0098
102	113	114	103	STRE0099
103	114	115	104	STRE0100
104	115	116	105	STRE0101
105	116	117	106	STRE0102
106	117	118	107	STRE0103
107	118	119	108	STRE0104
108	119	120	109	STRE0105
109	120	121	110	STRE0106
111	122	123	112	STRE0107
				STRE0108

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NODE(4)                                  CCNTINUED                STREC109
                                                                  STREC11C
  112   123   124   113                                           STRE0111
  113   124   125   114                                           STRE0112
  114   125   126   115                                           STRE0113
  115   126   127   116                                           STREC114
  116   127   128   117                                           STRE0115
  117   128   129   118                                           STRE0116
  118   129   130   119                                           STREC117
  119   130   131   120                                           STREC118
  120   131   132   121                                           STRE0119
  122   133   134   123                                           STREC12C
  123   134   135   124                                           STRE0121
  124   135   136   125                                           STRE0122
  125   136   137   126                                           STREC123
  126   137   138   127                                           STRE0124
  127   138   139   128                                           STRE0125
  128   139   140   129                                           STRE0126
  129   140   141   130                                           STRE0127
  130   141   142   131                                           STREC128
  131   142   143   132                                           STRE0129
  133   144   145   134                                           STRE0130
  134   145   146   135                                           STRE0131
  135   146   147   136                                           STRE0132
  136   147   148   137                                           STRE0133
  137   148   149   138                                           STRE0134
  138   149   150   139                                           STRE0135
  139   150   151   140                                           STRE0136
  140   151   152   141                                           STRE0137
  141   152   153   142                                           STRE0138
  142   153   154   143                                           STREC139
  144   155   156   145                                           STRE014C
  145   156   157   146                                           STRE0141
  146   157   158   147                                           STREC142
  147   158   159   148                                           STRE0143
  148   159   160   149                                           STREC144
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NODE(4)                                    CONTINUED                          STRE0145
                                                                              STRE0146
  149  160  161  150                                                          STRE0147
  150  161  162  151                                                          STRE0148
  151  162  163  152                                                          STRE0149
  152  163  164  153                                                          STRE0150
  153  164  165  154                                                          STRE0151
  155  166  167  156                                                          STRE0152
  156  167  168  157                                                          STRE0153
  157  168  169  158                                                          STRE0154
  158  169  170  159                                                          STRE0155
  159  170  171  160                                                          STRE0156
  160  171  172  161                                                          STRE0157
  161  172  173  162                                                          STRE0158
  162  173  174  163                                                          STRE0159
  163  174  175  164                                                          STRE0160
  164  175  176  165                                                          STRE0161
  166  177  178  167                                                          STRE0162
  167  178  179  168                                                          STRE0163
  168  179  180  169                                                          STRE0164
  169  180  181  170                                                          STRE0165
  170  181  182  171                                                          STRE0166
  171  182  183  172                                                          STRE0167
  172  183  184  173                                                          STRE0168
  173  184  185  174                                                          STRE0169
  174  185  186  175                                                          STRE0170
  175  186  187  176                                                          STRE0171
  177  188  189  178                                                          STRE0172
  178  189  190  179                                                          STRE0173
  179  190  191  180                                                          STRE0174
  180  191  192  181                                                          STRE0175
  181  192  193  182                                                          STRE0176
  182  193  194  183                                                          STRE0177
  183  194  195  184                                                          STRE0178
  184  195  196  185                                                          STRE0179
  185  196  197  186                                                          STRE0180
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NODE(4)                                   CONTINUED                STRE0181
                                                                   STRE0182
  186  197  198  187                                               STRE0183
  188  199  200  189                                               STRE0184
  189  200  201  190                                               STRE0185
  190  201  202  191                                               STRE0186
  191  202  203  192                                               STRE0187
  192  203  204  193                                               STRE0188
  193  204  205  194                                               STRE0189
  194  205  206  195                                               STRE0190
  195  206  207  196                                               STRE0191
  196  207  208  197                                               STRE0192
  197  208  209  198                                               STRE0193
  199  210  211  200                                               STRE0194
  200  211  212  201                                               STRE0195
  201  212  213  202                                               STRE0196
  202  213  214  203                                               STRE0197
  203  214  215  204                                               STRE0198
  204  215  216  205                                               STRE0199
  205  216  217  206                                               STRE0200
  206  217  218  207                                               STRE0201
  207  218  219  208                                               STRE0202
  208  219  220  209                                               STRE0203
  210  221  222  211                                               STRE0204
  211  222  223  212                                               STRE0205
  212  223  224  213                                               STRE0206
  213  224  225  214                                               STRE0207
  214  225  226  215                                               STRE0208
  215  226  227  216                                               STRE0209
  216  227  228  217                                               STRE0210
  217  228  229  218                                               STRE0211
  218  229  230  219                                               STRE0212
  219  230  231  220                                               STRE0213
  221  232  233  222                                               STRE0214
  222  233  234  223                                               STRE0215
  223  234  235  224                                               STRE0216
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NODE(4)                         CONTINUED                          STRE0217
                                                                   STRE0218
 224  235  236  225                                                STRE0219
 225  236  237  226                                                STRE0220
 226  237  238  227                                                STRE0221
 227  238  239  228                                                STRE0222
 228  239  240  229                                                STRE0223
 229  240  241  230                                                STRE0224
 230  241  242  231                                                STRE0225
 232  243  244  233                                                STRE0226
 233  244  245  234                                                STRE0227
 234  245  246  235                                                STRE0228
 235  246  247  236                                                STRE0229
 236  247  248  237                                                STRE0230
 237  248  249  238                                                STRE0231
 238  249  250  239                                                STRE0232
 239  250  251  240                                                STRE0233
 240  251  252  241                                                STRE0234
 241  252  253  242                                                STRE0235
 243  254  255  244                                                STRE0236
 244  255  256  245                                                STRE0237
 245  256  257  246                                                STRE0238
 246  257  258  247                                                STRE0239
 247  258  259  248                                                STRE0240
 248  259  260  249                                                STRE0241
 249  260  261  250                                                STRE0242
 250  261  262  251                                                STRE0243
 251  262  263  252                                                STRE0244
 252  263  264  253                                                STRE0245
 254  265  266  255                                                STRE0246
 255  266  267  256                                                STRE0247
 256  267  268  257                                                STRE0248
 257  268  269  258                                                STRE0249
 258  269  270  259                                                STRE0250
 259  270  271  260                                                STRE0251
 260  271  272  261                                                STRE0252
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NODE(4)                              CONTINUED                                STRE0253
                                                                              STRE0254
 261  272  273  262                                                           STRE0255
 262  273  274  263                                                           STRE0256
 263  274  275  264                                                           STRE0257
 265  276  277  266                                                           STRE0258
 266  277  278  267                                                           STRE0259
 267  278  279  268                                                           STRE0260
 268  279  280  269                                                           STRE0261
 269  280  281  270                                                           STRE0262
 270  281  282  271                                                           STRE0263
 271  282  283  272                                                           STRE0264
 272  283  284  273                                                           STRE0265
 273  284  285  274                                                           STRE0266
 274  285  286  275                                                           STRE0267
                                                                              STRE0268
                                                                              STRE0269
REWRITE THE DIMENSIONS OF THE ELEMENT FOR CHECKING PURPOSES                   STRE0270
                                                                              STRE0271
 X=   0.1440E 02 Y=  0.1440E 02                                               STRE0272
                                                                              STRE0273
                                                                              STRE0274
                                                                              STRE0275
                                                                              STRE0276
                                                                              STRE0277
                                                                              STRE0278
                                                                              STRE0279
                                                                              STRE0280
                                                                              STRE0281
                                                                              STRE0282
                                                                              STRE0283
                                                                              STRE0284
                                                                              STRE0285
                                                                              STRE0286
                                                                              STRE0287
                                                                              STRE0288
```

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NODEN INDICATES THE NUMBER OF ELEMENTS PER NODE IN ORDER                                    STRE0289
                                                                                             STRE0290
 NODEN                                                                                       STRE0291
                                                                                             STRE0292
                                                                                             STRE0293
   1  2  2  2  2  2  2  2  2  2  1  2  4  4  4  4  4  4  4  4                                STRE0294
   4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4                                STRE0295
   4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4                                STRE0296
   4  4  4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4                                STRE0297
   4  4  4  4  4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  2                                STRE0298
   4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4  4  4  4                                STRE0299
   2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4  4                                STRE0300
   4  4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4                                STRE0301
   4  4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4                                STRE0302
   4  4  4  4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  2  4                                STRE0303
   4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4  4  4  4  2                                STRE0304
   2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4  4  4                                STRE0305
   4  2  2  4  4  4  4  4  4  4  4  4  2  2  4  4  4  4  4  4                                STRE0306
   4  4  4  2  2  4  4  4  4  4  4  4  4  4  2  1  2  2  2  2                                STRE0307
   2  2  2  2  2  1                                                                          STRE0308
                                                                                             STRE0309
REWRITE NU AND WRITE ZZ, YY FOR CHECKING PURPOSES                                            STRE0310
                                                                                             STRE0311
 NU=  0.2500E 00 ZZ=  0.3750E 00 YY=  0.3200E 08                                             STRE0312
                                                                                             STRE0313
                                                                                             STRE0314
                                                                                             STRE0315
                                                                                             STRE0316
                                                                                             STRE0317
                                                                                             STRE0318
                                                                                             STRE0319
                                                                                             STRE0320
                                                                                             STRE0321
                                                                                             STRE0322
                                                                                             STRE0323
                                                                                             STRE0324
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AVERAGE STRESSES PER NODE STRE0325

STRE0326
STRE0327
STRE0328
STRE0329

NODE	STRESS-X	STRESS-Y	STRESS-XY	STRE0330
				STRE0331
1	-0.15817E 02	-0.35259E 00	-0.82947E 00	STRE0332
2	-0.11975E 02	0.39103E 00	-0.85944E 00	STRE0333
3	-0.87827E 01	0.10301E 01	-0.92883E 00	STRE0334
4	-0.57741E 01	0.25077E 01	-0.11261E 01	STRE0335
5	-0.25470E 01	0.66964E 01	-0.19605E 01	STRE0336
6	-0.14305E-03	-0.21935E-04	-0.60700E 01	STRE0337
7	0.25467E 01	-0.66964E 01	-0.19605E 01	STRE0338
8	0.57738E 01	-0.25077E 01	-0.11261E 01	STRE0339
9	0.87824E 01	-0.10301E 01	-0.92884E 00	STRE0340
10	0.11975E 02	-0.39104E 00	-0.85947E 00	STRE0341
11	0.15817E 02	0.35258E 00	-0.32950E 00	STRE0342
12	-0.15595E 02	-0.37191E 00	-0.16243E 00	STRE0343
13	-0.11847E 02	0.29887E 00	-0.33833E 00	STRE0344
14	-0.86692E 01	0.72518E 00	-0.64555E 00	STRE0345
15	-0.57526E 01	0.12235E 01	-0.10760E 01	STRE0346
16	-0.30611E 01	0.38790E 00	-0.14 92E 01	STRE0347
17	-0.14243E-03	-0.26688E-04	-0.30060E 01	STRE0348
18	0.30608E 01	-0.38795E 00	-0.14892E 01	STRE0349
19	0.57524E 01	-0.12235E 01	-0.10760E 01	STRE0350
20	0.86699E 01	-0.72521E 00	-0.64557E 00	STRE0351
21	0.11847E 02	-0.29889E 00	-0.33834E 00	STRE0352
22	0.15595E 02	0.37191E 00	-0.16243E 00	STRE0353
23	-0.15004E 02	-0.40571E 00	-0.26958E 00	STRE0354
24	-0.11497E 02	0.12273E 00	-0.55039E 00	STRE0355
25	-0.83754E 01	0.22270E 00	-0.99062E 00	STRE0356
26	-0.53866E 01	0.13113E 00	-0.13009E 01	STRE0357
27	-0.21920E 01	0.14870E 00	-0.14842E 01	STRE0358
28	-0.13493E-03	-0.47982E-04	-0.12868E 01	STRE0359
29	0.21918E 01	-0.14879E 00	-0.14843E 01	STRE0360

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STRE0361
					STRE0362
30	0.53864E 01	-0.13119E 00	-0.13009E 01		STRE0363
31	0.83751E 01	-0.22275E 00	-0.99064E 00		STRE0364
32	0.11497E 02	-0.12277E 00	-0.55041E 00		STRE0365
33	0.15004E 02	0.40569E 00	-0.26957E 00		STRE0366
34	-0.14216E 02	-0.41456E 00	-0.30799E 00		STRE0367
35	-0.11015E 02	0.69582E-03	-0.62179E 00		STRE0368
36	-0.80168E 01	-0.28738E-01	-0.10574E 01		STRE0369
37	-0.50776E 01	-0.36607E-01	-0.12889E 01		STRE0370
38	-0.23936E 01	-0.33600E-01	-0.13133E 01		STRE0371
39	-0.12805E-03	-0.62406E-04	-0.13203E 01		STRE0372
40	0.23934E 01	0.33488E-01	-0.13133E 01		STRE0373
41	0.50773E 01	0.36518E-01	-0.12889E 01		STRE0374
42	0.80165E 01	0.28604E-01	-0.10575E 01		STRE0375
43	0.11015E 02	-0.76789E-03	-0.62180E 00		STRE0376
44	0.14216E 02	0.41459E 00	-0.30803E 00		STRE0377
45	-0.13395E 02	-0.39332E 00	-0.30397E 00		STRE0378
46	-0.10488E 02	-0.37617E-01	-0.61269E 00		STRE0379
47	-0.76809E 01	-0.67026E-01	-0.10404E 01		STRE0380
48	-0.49720E 01	-0.80078E-01	-0.12639E 01		STRE0381
49	-0.24150E 01	-0.53128E-01	-0.13499E 01		STRE0382
50	-0.12498E-03	-0.61512E-04	-0.13613E 01		STRE0383
51	0.24148E 01	0.52977E-01	-0.13499E 01		STRE0384
52	0.49718E 01	0.79913E-01	-0.12639E 01		STRE0385
53	0.76807E 01	0.66880E-01	-0.10405E 01		STRE0386
54	0.10488E 02	0.37342E-01	-0.61273E 00		STRE0387
55	0.13394E 02	0.39291E 00	-0.30406E 00		STRE0388
56	-0.12623E 02	-0.35956E 00	-0.28636E 00		STRE0389
57	-0.99602E 01	-0.32366E-01	-0.58458E 00		STRE0390
58	-0.73644E 01	-0.55976E-01	-0.10120E 01		STRE0391
59	-0.48259E 01	-0.63652E-01	-0.12666E 01		STRE0392
60	-0.23784E 01	-0.42585E-01	-0.13887E 01		STRE0393
61	-0.10589E-03	-0.86963E-04	-0.14239E 01		STRE0394
62	0.23782E 01	0.42444E-01	-0.13887E 01		STRE0395
63	0.48257E 01	0.63457E-01	-0.12666E 01		STRE0396

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STREC397
					STREC398
64	0.73641E 01	0.55606E-01	-0.10120E 01		STREC399
65	0.99599E 01	0.31954E-01	-0.58459E 00		STRE0400
66	0.12623E 02	0.35921E 00	-0.28638E 00		STRE0401
67	-0.11915E 02	-0.32844E 00	-0.27147E 00		STRE0402
68	-0.94433E 01	-0.20434E-01	-0.56096E 00		STRE0403
69	-0.70288E 01	-0.34812E-01	-0.99161E 00		STRE0404
70	-0.46422E 01	-0.40087E-01	-0.12702E 01		STRE0405
71	-0.23034E 01	-0.26962E-01	-0.14209E 01		STRE0406
72	-0.10502E-03	-0.17774E-03	-0.14675E 01		STRE0407
73	0.23031E 01	0.26650E-01	-0.14209E 01		STRE0408
74	0.46420E 01	0.39849E-01	-0.12703E 01		STRE0409
75	0.70285E 01	0.34552E-01	-0.99164E 00		STRE0410
76	0.94430E 01	0.19981E-01	-0.56092E 00		STRE0411
77	0.11915E 02	0.32783E 00	-0.27142E 00		STRE0412
78	-0.11251E 02	-0.30384E 00	-0.26218E 00		STRE0413
79	-0.89345E 01	-0.11016E-01	-0.54625E 00		STRE0414
80	-0.66767E 01	-0.18934E-01	-0.97872E 00		STRE0415
81	-0.44297E 01	-0.21958E-01	-0.12729E 01		STRE0416
82	-0.22061E 01	-0.14900E-01	-0.14406E 01		STRE0417
83	-0.93520E-04	-0.14472E-03	-0.14947E 01		STRE0418
84	0.22059E 01	0.14590E-01	-0.14406E 01		STRE0419
85	0.44295E 01	0.21657E-01	-0.12729E 01		STRE0420
86	0.66765E 01	0.18619E-01	-0.97871E 00		STRE0421
87	0.89343E 01	0.10640E-01	-0.54622E 00		STRE0422
88	0.11250E 02	0.30336E 00	-0.26207E 00		STRE0423
89	-0.10610E 02	-0.28339E 00	-0.25728E 00		STRE0424
90	-0.84318E 01	-0.54373E-02	-0.53840E 00		STRE0425
91	-0.63141E 01	-0.94026E-02	-0.97160E 00		STRE0426
92	-0.41998E 01	-0.10979E-01	-0.12742E 01		STRE0427
93	-0.20958E 01	-0.75713E-02	-0.14514E 01		STRE0428
94	-0.88602E-04	-0.17375E-03	-0.15095E 01		STRE0429
95	0.20956E 01	0.72640E-02	-0.14514E 01		STRE0430
96	0.41996E 01	0.10629E-01	-0.12742E 01		STRE0431
97	0.63140E 01	0.89660E-02	-0.97153E 00		STRE0432

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	STRE0433
					STRE0434
98	0.84317E 01	0.50815E-02	-0.53834E 00		STRE0435
99	0.10610E 02	0.28310E 00	-0.25724E 00		STRE0436
100	-0.99809E 01	-0.26548E 00	-0.25502E 00		STRE0437
101	-0.79331E 01	-0.25954E-02	-0.53465E 00		STRE0438
102	-0.59464E 01	-0.43536E-02	-0.96805E 00		STRE0439
103	-0.39603E 01	-0.50570E-02	-0.12747E 01		STRE0440
104	-0.19784E 01	-0.34746E-02	-0.14568E 01		STRE0441
105	-0.82329E-04	-0.11158E-03	-0.15170E 01		STRE0442
106	0.19782E 01	0.31582E-02	-0.14567E 01		STRE0443
107	0.39602E 01	0.47038E-02	-0.12747E 01		STRE0444
108	0.59463E 01	0.40184E-02	-0.96794E 00		STRE0445
109	0.79330E 01	0.21513E-02	-0.53455E 00		STRE0446
110	0.99808E 01	0.26496E 00	-0.25497E 00		STRE0447
111	-0.93561E 01	-0.24835E 00	-0.25412E 00		STRE0448
112	-0.74364E 01	-0.12694E-02	-0.53303E 00		STRE0449
113	-0.55764E 01	-0.20210E-02	-0.96647E 00		STRE0450
114	-0.37161E 01	-0.22719E-02	-0.12750E 01		STRE0451
115	-0.18574E 01	-0.16333E-02	-0.14593E 01		STRE0452
116	-0.72081E-04	-0.78753E-04	-0.15205E 01		STRE0453
117	0.18573E 01	0.13674E-02	-0.14593E 01		STRE0454
118	0.37160E 01	0.18824E-02	-0.12749E 01		STRE0455
119	0.55763E 01	0.16212E-02	-0.96635E 00		STRE0456
120	0.74363E 01	0.89389E-03	-0.53295E 00		STRE0457
121	0.93561E 01	0.24800E 00	-0.25403E 00		STRE0458
122	-0.87329E 01	-0.23173E 00	-0.25382E 00		STRE0459
123	-0.69407E 01	-0.77638E-03	-0.53246E 00		STRE0460
124	-0.52054E 01	-0.99838E-03	-0.96589E 00		STRE0461
125	-0.34698E 01	-0.98239E-03	-0.12751E 01		STRE0462
126	-0.17347E 01	-0.71450E-03	-0.14603E 01		STRE0463
127	-0.79941E-04	-0.17148E-03	-0.15220E 01		STRE0464
128	0.17345E 01	0.38518E-03	-0.14604E 01		STRE0465
129	0.34697E 01	0.72634E-03	-0.12750E 01		STRE0466
130	0.52053E 01	0.63890E-03	-0.96576E 00		STRE0467
131	0.69406E 01	0.31966E-03	-0.53237E 00		STRE0468

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STRE0469
					STRE0470
132	0.87329E 01	0.23136E 00	-0.25371E 00		STRE0471
133	-0.81101E 01	-0.21510E 00	-0.25374E 00		STRE0472
134	-0.64454E 01	-0.54605E-03	-0.53229E 00		STRE0473
135	-0.48341E 01	-0.64297E-03	-0.96571E 00		STRE0474
136	-0.32227E 01	-0.42865E-03	-0.12752E 01		STRE0475
137	-0.16113E 01	-0.35426E-03	-0.14609E 01		STRE0476
138	-0.86948E-04	-0.21867E-03	-0.15227E 01		STRE0477
139	0.16112E 01	0.57772E-04	-0.14609E 01		STRE0478
140	0.32225E 01	0.26144E-03	-0.12752E 01		STRE0479
141	0.48340E 01	0.29129E-03	-0.96566E 00		STRE0480
142	0.64453E 01	0.12177E-03	-0.53222E 00		STRE0481
143	0.81102E 01	0.21484E 00	-0.25364E 00		STRE0482
144	-0.74875E 01	-0.19875E 00	-0.25367E 00		STRE0483
145	-0.59502E 01	-0.53382E-03	-0.53234E 00		STRE0484
146	-0.44627E 01	-0.53383E-03	-0.96578E 00		STRE0485
147	-0.29751E 01	-0.37608E-03	-0.12755E 01		STRE0486
148	-0.14876E 01	-0.36742E-03	-0.14612E 01		STRE0487
149	-0.11082E-03	-0.25509E-03	-0.15231E 01		STRE0488
150	0.14874E 01	-0.13600E-03	-0.14612E 01		STRE0489
151	0.29750E 01	0.27090E-04	-0.12754E 01		STRE0490
152	0.44626E 01	0.23869E-03	-0.96574E 00		STRE0491
153	0.59501E 01	0.18221E-03	-0.53223E 00		STRE0492
154	0.74875E 01	0.19843E 00	-0.25356E 00		STRE0493
155	-0.68647E 01	-0.18249E 00	-0.25376E 00		STRE0494
156	-0.54550E 01	-0.57830E-03	-0.53247E 00		STRE0495
157	-0.40913E 01	-0.60654E-03	-0.96604E 00		STRE0496
158	-0.27275E 01	-0.43264E-03	-0.12757E 01		STRE0497
159	-0.13638E 01	-0.27847E-03	-0.14615E 01		STRE0498
160	-0.95595E-04	-0.17551E-03	-0.15234E 01		STRE0499
161	0.13636E 01	-0.10492E-03	-0.14615E 01		STRE0500
162	0.27274E 01	0.24438E-04	-0.12756E 01		STRE0501
163	0.40911E 01	0.17807E-03	-0.96592E 00		STRE0502
164	0.54549E 01	0.10940E-03	-0.53236E 00		STRE0503
165	0.68644E 01	0.18190E 00	-0.25367E 00		STRE0504

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	STRE0505
					STRE0506
166	-0.62416E 01	-0.16587E 00	-0.25381E 00		STRE0507
167	-0.49597E 01	-0.61464E-03	-0.53257E 00		STRE0508
168	-0.37197E 01	-0.61055E-03	-0.96628E 00		STRE0509
169	-0.24798E 01	-0.31137E-03	-0.12760E 01		STRE0510
170	-0.12400E 01	-0.26077E-03	-0.14618E 01		STRE0511
171	-0.10496E-03	-0.29904E-03	-0.15237E 01		STRE0512
172	0.12398E 01	-0.12112E-03	-0.14617E 01		STRE0513
173	0.24797E 01	-0.28148E-04	-0.12759E 01		STRE0514
174	0.37196E 01	0.68903E-04	-0.96617E 00		STRE0515
175	0.49596E 01	0.16196E-03	-0.53249E 00		STRE0516
176	0.62412E 01	0.16551E 00	-0.25370E 00		STRE0517
177	-0.56182E 01	-0.14934E 00	-0.25382E 00		STRE0518
178	-0.44644E 01	-0.62275E-03	-0.53276E 00		STRE0519
179	-0.33482E 01	-0.67526E-03	-0.96654E 00		STRE0520
180	-0.22322E 01	-0.46095E-03	-0.12763E 01		STRE0521
181	-0.11162E 01	-0.22641E-03	-0.14620E 01		STRE0522
182	-0.68862E-04	-0.32246E-04	-0.15239E 01		STRE0523
183	0.11160E 01	-0.14944E-03	-0.14619E 01		STRE0524
184	0.22320E 01	-0.14137E-03	-0.12761E 01		STRE0525
185	0.33481E 01	0.10931E-03	-0.96637E 00		STRE0526
186	0.44642E 01	0.19422E-03	-0.53263E 00		STRE0527
187	0.56181E 01	0.14915E 00	-0.25376E 00		STRE0528
188	-0.49944E 01	-0.13287E 00	-0.25406E 00		STRE0529
189	-0.39688E 01	-0.52164E-03	-0.53312E 00		STRE0530
190	-0.29768E 01	-0.43666E-03	-0.96700E 00		STRE0531
191	-0.19847E 01	-0.11733E-03	-0.12766E 01		STRE0532
192	-0.99260E 00	-0.16178E-03	-0.14622E 01		STRE0533
193	-0.14950E-03	-0.23443E-03	-0.15240E 01		STRE0534
194	0.99229E 00	-0.25050E-03	-0.14620E 01		STRE0535
195	0.19845E 01	-0.18986E-03	-0.12763E 01		STRE0536
196	0.29765E 01	0.10931E-03	-0.96667E 00		STRE0537
197	0.39688E 01	0.19823E-03	-0.53282E 00		STRE0538
198	0.49947E 01	0.13257E 00	-0.25380E 00		STRE0539
199	-0.43702E 01	-0.11623E 00	-0.25422E 00		STRE0540

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STRE0541
					STRE0542
200	-0.34727E 01	-0.37206E-03	-0.53348E 00		STRE0543
201	-0.26053E 01	-0.20224E-03	-0.96753E 00		STRE0544
202	-0.17376E 01	0.23432E-03	-0.12769E 01		STRE0545
203	-0.86911E 00	0.48497E-03	-0.14623E 01		STRE0546
204	-0.16568E-03	0.18594E-03	-0.15239E 01		STRE0547
205	0.86879E 00	-0.36778E-03	-0.14620E 01		STRE0548
206	0.17372E 01	-0.57794E-03	-0.12766E 01		STRE0549
207	0.26052E 01	-0.37178E-03	-0.96710E 00		STRE0550
208	0.34730E 01	0.82254E-05	-0.53319E 00		STRE0551
209	0.43709E 01	0.11616E 00	-0.25410E 00		STRE0552
210	-0.37452E 01	-0.99713E-01	-0.25455E 00		STRE0553
211	-0.29764E 01	-0.27502E-03	-0.53411E 00		STRE0554
212	-0.22341E 01	0.24237E-03	-0.96818E 00		STRE0555
213	-0.14909E 01	0.81234E-03	-0.12771E 01		STRE0556
214	-0.74609E 00	0.72758E-03	-0.14618E 01		STRE0557
215	-0.25055E-03	0.21023E-03	-0.15234E 01		STRE0558
216	0.74569E 00	-0.63054E-03	-0.14618E 01		STRE0559
217	0.14906E 01	-0.11155E-02	-0.12769E 01		STRE0560
218	0.22342E 01	-0.85688E-03	-0.96798E 00		STRE0561
219	0.29768E 01	-0.51327E-03	-0.53400E 00		STRE0562
220	0.37459E 01	0.99062E-01	-0.25452E 00		STRE0563
221	-0.31178E 01	-0.82416E-01	-0.25555E 00		STRE0564
222	-0.24797E 01	0.48090E-03	-0.53553E 00		STRE0565
223	-0.18640E 01	0.11074E-02	-0.96932E 00		STRE0566
224	-0.12453E 01	0.17906E-02	-0.12771E 01		STRE0567
225	-0.62340E 00	0.13097E-02	-0.14611E 01		STRE0568
226	-0.21017E-03	-0.52482E-04	-0.15223E 01		STRE0569
227	0.62305E 00	-0.13986E-02	-0.14611E 01		STRE0570
228	0.12450E 01	-0.23444E-02	-0.12771E 01		STRE0571
229	0.18639E 01	-0.21221E-02	-0.96918E 00		STRE0572
230	0.24800E 01	-0.11277E-02	-0.53546E 00		STRE0573
231	0.31186E 01	0.82133E-01	-0.25553E 00		STRE0574
232	-0.24854E 01	-0.64012E-01	-0.25739E 00		STRE0575
233	-0.19830E 01	0.16936E-02	-0.53766E 00		STRE0576

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STRE0577
					STRE0578
234	-0.14964E 01	0.25586E-02	-0.97044E 00		STRE0579
235	-0.10008E 01	0.27567E-02	-0.12762E 01		STRE0580
236	-0.50057E 00	0.14592E-02	-0.14598E 01		STRE0581
237	-0.16166E-03	-0.52547E-03	-0.15212E 01		STRE0582
238	0.50021E 00	-0.22757E-02	-0.14598E 01		STRE0583
239	0.10006E 01	-0.33065E-02	-0.12763E 01		STRE0584
240	0.14962E 01	-0.33388E-02	-0.97030E 00		STRE0585
241	0.19831E 01	-0.23929E-02	-0.53756E 00		STRE0586
242	0.24864E 01	0.63786E-01	-0.25733E 00		STRE0587
243	-0.18416E 01	-0.42265E-01	-0.25983E 00		STRE0588
244	-0.14894E 01	0.28375E-02	-0.54028E 00		STRE0589
245	-0.11333E 01	0.31043E-02	-0.96918E 00		STRE0590
246	-0.75523E 00	0.20655E-02	-0.12741E 01		STRE0591
247	-0.37523E 00	-0.43258E-03	-0.14594E 01		STRE0592
248	-0.64614E-04	-0.74380E-03	-0.15225E 01		STRE0593
249	0.37495E 00	-0.84884E-03	-0.14594E 01		STRE0594
250	0.75497E 00	-0.31003E-02	-0.12742E 01		STRE0595
251	0.11330E 01	-0.43898E-02	-0.96934E 00		STRE0596
252	0.14895E 01	-0.37834E-02	-0.54042E 00		STRE0597
253	0.18426E 01	0.42039E-01	-0.25989E 00		STRE0598
254	-0.11766E 01	-0.19124E-01	-0.26284E 00		STRE0599
255	-0.10094E 01	-0.11323E-03	-0.53751E 00		STRE0600
256	-0.77068E 00	-0.25264E-02	-0.96491E 00		STRE0601
257	-0.50325E 00	-0.91556E-02	-0.12721E 01		STRE0602
258	-0.24583E 00	-0.90707E-02	-0.14647E 01		STRE0603
259	-0.11314E-03	-0.12410E-02	-0.15285E 01		STRE0604
260	0.24567E 00	0.68475E-02	-0.14647E 01		STRE0605
261	0.50296E 00	0.71426E-02	-0.12724E 01		STRE0606
262	0.77026E 00	0.48100E-03	-0.96512E 00		STRE0607
263	0.10096E 01	-0.10429E-02	-0.53757E 00		STRE0608
264	0.11779E 01	0.18950E-01	-0.26344E 00		STRE0609
265	-0.51954E 00	0.69040E-02	-0.21725E 00		STRE0610
266	-0.51042E 00	-0.22830E-01	-0.52644E 00		STRE0611
267	-0.38930E 00	-0.43724E-01	-0.97207E 00		STRE0612

NODE	STRESS-X	STRESS-Y	STRESS-XY	CONTINUED	
					STRE0613
					STRE0614
268	-0.25307E 00	-0.42168E-01	-0.12934E 01		STRE0615
269	-0.12245E 00	-0.20910E-01	-0.14721E 01		STRE0616
270	-0.46299E-04	-0.12242E-02	-0.15392E 01		STRE0617
271	0.12246E 00	0.18002E-01	-0.14722E 01		STRE0618
272	0.25288E 00	0.39209E-01	-0.12935E 01		STRE0619
273	0.38915E 00	0.41263E-01	-0.97231E 00		STRE0620
274	0.51012E 00	0.19605E-01	-0.52659E 00		STRE0621
275	0.52020E 00	-0.10619E-01	-0.21786E 00		STRE0622
276	-0.30197E 00	-0.38011E 00	-0.17064E 00		STRE0623
277	-0.32874E 00	-0.37225E 00	-0.52169E 00		STRE0624
278	-0.23629E 00	-0.26304E 00	-0.98878E 00		STRE0625
279	-0.13255E 00	-0.70245E-01	-0.13219E 01		STRE0626
280	-0.55782E-01	0.98225E-02	-0.14750E 01		STRE0627
281	-0.12492E-03	-0.17289E-02	-0.15440E 01		STRE0628
282	0.55865E-01	-0.12955E-01	-0.14752E 01		STRE0629
283	0.13226E 00	0.66049E-01	-0.13218E 01		STRE0630
284	0.23587E 00	0.25809E 00	-0.98938E 00		STRE0631
285	0.32803E 00	0.36795E 00	-0.52226E 00		STRE0632
286	0.30261E 00	0.37639E 00	-0.17066E 00		STRE0633

[illegible]

```
STX( 21,, 1)= -0.2398E-01 STY( 21,, 1)= [illegible] STXY( 21,, 1)= [illegible] SIGMA2( 21, 1)= -0.2512E-01

XF( 22,, 1)= 372.46  YF( 22,, 1)= [illegible]

STX( 22,, 1)= -0.174[illegible] STY( 22,, 1)= [illegible] STXY( 22,, 1)= - .75[illegible] SIGMA2( 22, 1)= -0.2[illegible]21E-01

XF( 23,, 1)= 310.[illegible]  YF( 23,, 1)= [illegible]

STX( 23,, 1)= - .114[illegible] STY( 23,, 1)= [illegible] STXY( 23,, 1)= - .[illegible] SIGMA2( 23, 1)= - .1634[illegible]-01

XF( 24,, 1)= 331.26  YF( 24,, 1)= 37.71[illegible]

STX( 24,, 1)= - .[illegible] STY( 24,, 1)= - .655[illegible]E-0[illegible] STXY( 24,, 1)= - .115[illegible] SIGMA2( 24, 1)= -0.1499E-01

XF( 25,, 1)= 345.66[illegible]  YF( 25,, 1)= [illegible]

STX( 25,, 1)= -0.15[illegible] STY( 25,, 1)= - .32[illegible]E-01 STXY( 25,, 1)= - .[illegible] SIGMA2( 25, 1)= -0.1492E-01

XF( 26,, 1)= 360.30[illegible]  YF( 26,, 1)= 63.667

STX( 26,, 1)= - .3117[illegible]E-01 STY( 26,, 1)= .671[illegible]E-02 STXY( 26,, 1)= - .1[illegible] SIGMA2( 26, 1)= -0.1515E-01

XF( 1,, 2)= [illegible]  YF( 1,, 2)= 1[illegible]
STX( 1,, 2)= -0.1159[illegible]-02 STY( 1,, 2)= [illegible] STXY( 1,, 2)= - .[illegible] SIGMA2( 1, 2)= -0.12[illegible]-02

XF( 2,, 2)= 14.46  YF( 2,, 2)= 15.114

STX( 2,, 2)= -0.115[illegible] STY( 2,, 2)= [illegible] STXY( 2,, 2)= - .[illegible] SIGMA2( 2, 2)= - .117[illegible]

XF( 3,, 2)= 23.86  YF( 3,, 2)= 15.[illegible]

STX( 3,, 2)= - .1122E-02 STY( 3,, 2)= .1317[illegible] STXY( 3,, 2)= - .[illegible] SIGMA2( 3, 2)= - .1125E-02

XF( 4,, 2)= 43.20  YF( 4,, 2)= 16.332

STX( 4,, 2)= -0.1[illegible] STY( 4,, 2)= - .3562E-02 STXY( 4,, 2)= -0.[illegible] SIGMA2( 4, 2)= -0.1[illegible]

XF( 5,, 2)= 57.86[illegible]  YF( 5,, 2)= 17.513

STX( 5,, 2)= -0.[illegible]E-01 STY( 5,, 2)= - .4[illegible]57E-01 STXY( 5,, 2)= - .7[illegible] SIGMA2( 5, 2)= -0.3332E-01

XF( 6,, 2)= 72.65  YF( 6,, 2)= 13.573
```

[illegible]

STX(17,, 2)= -0.1600E 01 STY(17,, 2)= -0.3281E-03 STXY(17,, 2)= -0.1381E 01 SIGMA2(17, 2)= -0.2396E 01

XF(18,, 2)= 244.80 YF(18,, 2)= 6 .923

STX(18,, 2)= - .7636E 00 STY(18,, 2)= - .1730E-03 STXY(18,, 2)= - .1470E 1 SIGMA2(18, 2)= -0.1037E 01

XF(19,, 2)= 259.2[illegible] YF(19,, 2)= 73.734

STX(19,, 2)= [illegible].1029E 00 STY(19,, 2)= [illegible].12[illegible]E-[illegible] STXY(19,, 2)= - .1517E 1 SIGMA2(19, 2)= -[illegible].1466E 1

XF(20,, 2)= 273.60[illegible] YF(20,, 2)= [illegible].434

STX(20,, 2)= [illegible].5751E [illegible] STY(20,, 2)= -[illegible].776[illegible]E-[illegible] STXY(20,, 2)= - .14[illegible] 01 SIGMA2(20, 2)= -[illegible].1[illegible] 01

XF(21,, 2)= 288.[illegible] YF(21,, 2)= 115.922

STX(21,, 2)= [illegible].1859E 01 STY(21,, 2)= -[illegible].[illegible]7[illegible]E-[illegible] STXY(21,, 2)= -[illegible].[illegible]76E [illegible] SIGMA2(21, 2)= - .3342E [illegible]

TRAJECTORY(2) IS OFF THE PLATE

XF(22,, 2)= 299.[illegible]21 YF(22,, 2)= 14[illegible].

STX(22,, 2)= [illegible].26[illegible]E 01 STY(22,, 2)= .[illegible]73[illegible]E- 1 STXY(22,, 2)= - .257[illegible]E 0[illegible] SIGMA2(22, 2)= [illegible].416[illegible]E- 1

XF(1,, 3)= [illegible].0 YF(1,, 3)= [illegible]

STX(1,, 3)= -0.8783E 1 STY(1,, 3)= .1[illegible]E 01 STXY(1, 3)= -[illegible] 00 SIGMA2(1, 3)= -[illegible].887[illegible]E 01

XF(2,, 3)= 14.4[illegible] YF(2,, 3)= 3[illegible].[illegible]11

STX(2,, 3)= -0.8424E 01 STY(2,, 3)= [illegible].7[illegible]7[illegible] STXY(2,, 3)= - .[illegible] SIGMA2(2, 3)= -[illegible].8474E 01

XF(3,, 3)= 28.8[illegible] YF(3,, 3)= 31.451

STX(3,, 3)= -[illegible].792[illegible] 01 STY(3,, 3)= .2[illegible] STXY(3,, 3)= - .1[illegible] 01 SIGMA2(3, 3)= -[illegible].7[illegible]E 01

XF(4,, 3)= 43.2[illegible] YF(4,, 3)= 33.4[illegible]

STX(4,, 3)= -[illegible].7[illegible]E 01 STY(4,, 3)= -[illegible].31[illegible]E- 1 STXY(4,, 3)= - .1133E 1 SIGMA2(4, 3)= - .7236E 01

XF(5,, 3)= 57.6[illegible] YF(5,, 3)= 35.[illegible]

STX(5,, 3)= -0.6345E 01 STY(5,, 3)= -[illegible].736[illegible]E- 1 STXY(5,, 3)= - .115[illegible]E 01 SIGMA2(5, 3)= -[illegible].654[illegible]E 01

XF(6,, 3)= 72.[illegible] YF(6,, 3)= [illegible].[illegible]4

X(3,, 4)= 23.[illegible] Y(3,, 4)= [illegible].623

STX(3,, 4)= -.[illegible] 01 STY(3,, 4)= [illegible].1377[illegible] STXY(3,, 4)= -.1[illegible] 1 SIGMA2(3, 4)= -.4591[illegible] 01

X(4,, 4)= 43.2[illegible] Y(4,, 4)= [illegible]3.345

STX(4,, 4)= -0.3[illegible] 1 STY(4,, 4)= -.[illegible] 1 STXY(4,, 4)= -.1[illegible] 1 SIGMA2(4, 4)= -0.3574E 01

X(5,, 4)= 57.[illegible] Y(5,, 4)= 53.2[illegible]

STX(5,, 4)= -0.2145E 01 STY(5,, 4)= -.471[illegible] 1 STXY(5,, 4)= -0.1[illegible] 1 SIGMA2(5, 4)= -0.2805E 01

X(6,, 4)= 72.[illegible] Y(6,, 4)= 67.45[illegible]

STX(6,, 4)= -0.[illegible]75[illegible] STY(6,, 4)= -.1[illegible] 1 STXY(6,, 4)= -.[illegible] 01 SIGMA2(6, 4)= -0.1771E 01

X(7,, 4)= [illegible] Y(7,, 4)= [illegible]

STX(7,, 4)= -0.1[illegible] 01 STY(7,, 4)= .[illegible] 1 STXY(7,, 4)= -.1[illegible] 1 SIGMA2(7, 4)= -0.[illegible]

X(8,, 4)= 100.[illegible] Y(8,, 4)= 1[illegible]1.[illegible]

STX(8,, 4)= 0.1[illegible] 02 STY(8,, 4)= [illegible] STXY(8,, 4)= -.[illegible] SIGMA2(8, 4)= 0.2345E [illegible]

[illegible]STORY(4) IS OFF THE PLATE

X(9,, 4)= 1[illegible] Y(9,, 4)= 1[illegible]

STX(9,, 4)= 0.1125[illegible] 2 STY(9,, 4)= [illegible] STXY(9,, 4)= -.2[illegible] SIGMA2(9, 4)= 0.297[illegible]

X(1,, 5)= [illegible] Y(1,, 5)= 57.[illegible]

STX(1,, 5)= -0.2547E 01 STY(1,, 5)= [illegible] 1 STXY(1, 5)= -.15[illegible] 01 SIGMA2(1, 5)= -.2[illegible]E 01

X(2,, 5)= 14.4[illegible] Y(2,, 5)= 62.523

STX(2,, 5)= -0.1[illegible] 01 STY(2,, 5)= 0.252[illegible] STXY(2,, 5)= -.[illegible] 1 SIGMA2(2, 5)= -0.313[illegible] 01

X(3,, 5)= 28.8[illegible] Y(3,, 5)= 7[illegible].653

STX(3,, 5)= 0.2523[illegible] STY(3,, 5)= -0.1715E-01 STXY(3,, 5)= -.1[illegible] 1 SIGMA2(3, 5)= -0.1199E 01

X(4,, 5)= 43.2[illegible] Y(4,, 5)= 1[illegible]

[illegible]

TRAJECTORY 51 IS OFF THE PLATE

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

TRAJECTORY 61 IS OFF THE PLATE

[illegible]

[illegible]

[illegible]

[illegible]

TRAJECTORY 71 IS OFF THE PLATE

[illegible]

[illegible]

[illegible]

[illegible]

TRAJECTORY 81 IS OFF THE PLATE

[illegible]

[illegible]

[illegible]

[illegible]

TRAJECTORY 91 IS OFF THE PLATE

```
XF( 2,, 8)= 5.769 YF( 2,, 8)= 144.[illegible]

STX( 2,, 8)= 0.1573E 02 STY( 2,, 8)= [illegible] STXY( 2,, 8)= -[illegible] SIGMA2( 2, 8)= 0.3398E 03

XF(1,, 9)= [illegible] YF(1,, 9)= 115.[illegible]
STX(1,, 9)= [illegible].6782E 1 STY(1,, 9)= -[illegible] STXY(1, 9)= -[illegible] SIGMA2(1, 9)= -[illegible].1117E C1

TRAJECTORY( 9) IS OFF THE PLATE

XF( 2,, 9)= 2.702 YF( 2,, 9)= 14[illegible].

STX( 2,, 9)= [illegible].157[illegible] 02 STY( 2,, 9)= [illegible] STXY( 2,, 9)= -[illegible] SIGMA2( 2, 9)= [illegible].3241E [illegible]

XF(1,, 10)= [illegible] YF(1,, 10)= 1[illegible]
STX(1,, 10)= [illegible].11[illegible] 02 STY(1,, 10)= -[illegible] STXY(1, 10)= -[illegible] SIGMA2(1, 10)= -0.4[illegible]E 0[illegible]

TRAJECTORY( 10) IS OFF THE PLATE

XF( 2,, 10)= [illegible] YF( 2,, 10)= 14[illegible]

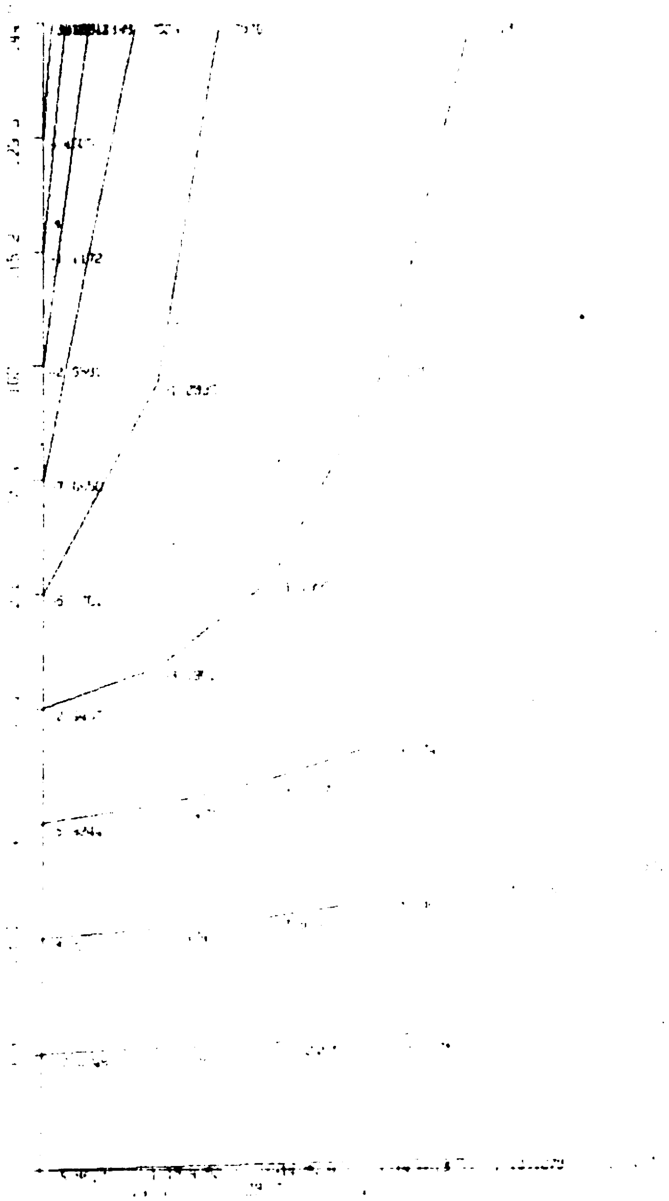
STX( 2,, 10)= [illegible].156[illegible] 02 STY( 2,, 10)= [illegible] STXY( 2,, 10)= -[illegible] SIGMA2( 2, 10)= 0.3143E 0[illegible]

XF(1,, 11)= [illegible] YF(1,, 11)= 14[illegible].
STX(1,, 11)= [illegible].15[illegible]2E 02 STY(1,, 11)= [illegible] STXY(1, 11)= -[illegible] SIGMA2(1, 11)= [illegible]

TRAJECTORY( 11) IS OFF THE PLATE

XF( 2,, 11)= [illegible] YF( 2,, 11)= 14[illegible]

STX( 2,, 11)= 0.15[illegible]2E 02 STY( 2,, 11)= [illegible] STXY( 2,, 11)= -[illegible] SIGMA2( 2, 11)= [illegible].3[illegible]42E 0[illegible]
```



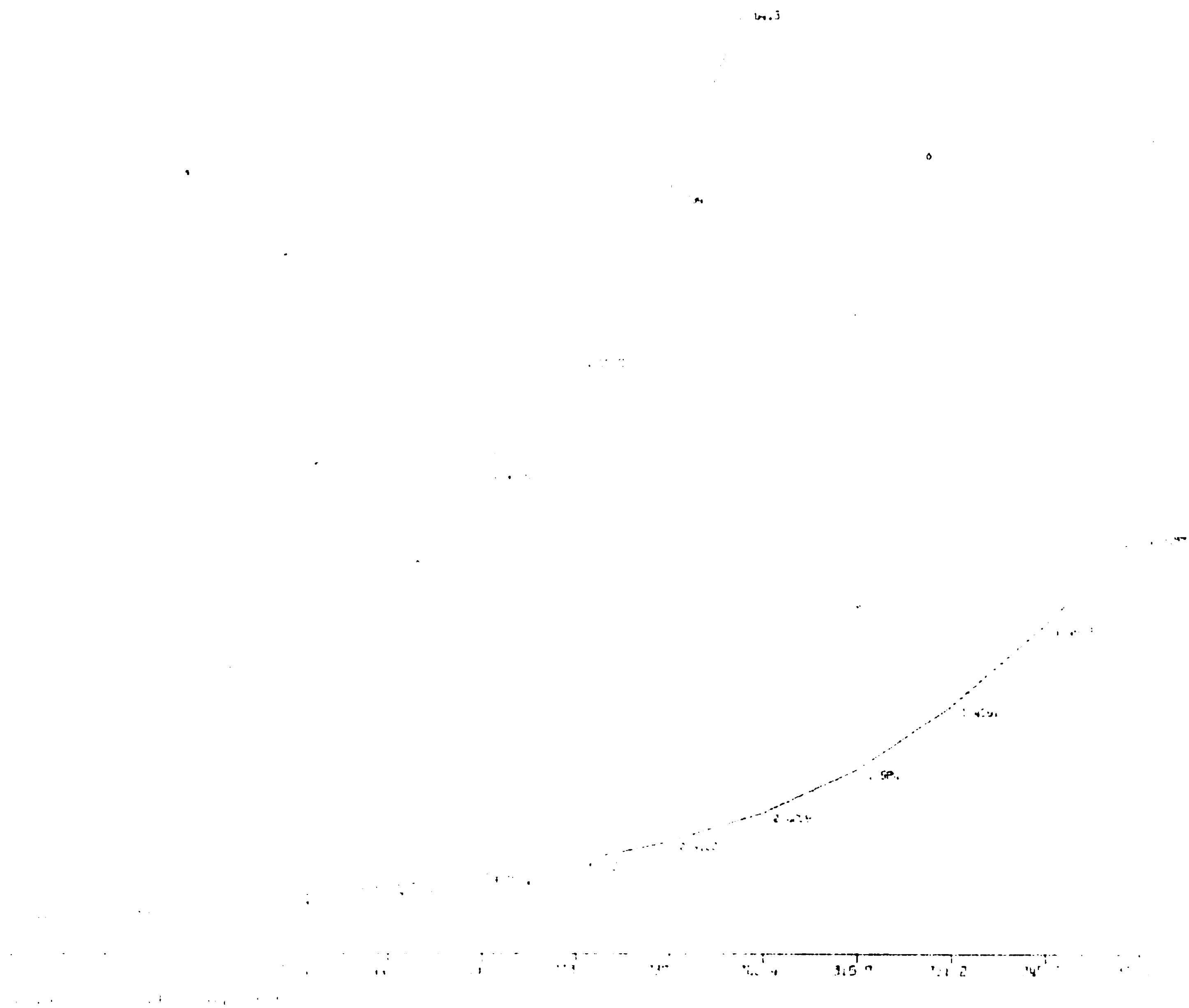


PLATE I

```
THE FOLLOWING IS A MESSAGE CODE THAT THE JOB HAS BEEN COMPLETED          PLTT0001
                                                                         PLTT0002
 PLOTS COMPLETED                                                         PLTT0003
                                                                         PLTT0004
```

```
THE FOLLOWING LISTING OF FORCE ENTRIES PER NCDE REFERS TO THE PREVIOUS   FORC0001
CASE OF THE CANTILEVER PLATE, BUT WHICH IS NCW LOADED WITH A BOUNDARY    FORC0002
SHEAR PARABOLIC CISTRIBUTION RATHER THAN WITH DESCRETE NODAL FORCES.     FORC0003
                                                                         FORC0004
                                                                         FORC0005
                                                                         FORC0006
     FORCE ENTRIES TWO PER NODE IN CRDER                                 FORC0007
                                                                         FORC0008
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0009
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0010
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0011
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0012
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0013
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0014
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0015
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0016
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0017
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0018
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0019
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0020
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0021
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0022
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0023
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0024
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0025
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0026
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0027
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0028
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0029
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0030
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0031
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0032
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0033
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0034
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0035
   0.0      0.0      0.0      0.0      0.0      0.0                      FORC0036
```

```
     FORCE  ENTRIES  TWO  PER  NODE  IN  ORDER CONTINUED                    FORC0037
                                                                            FORC0038
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0039
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC004C
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0041
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0042
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0043
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0044
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0045
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0046
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0047
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0048
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0049
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0050
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0051
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0052
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0053
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0054
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0055
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0056
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0057
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0058
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0059
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0060
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0061
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0062
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0063
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0064
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0065
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0066
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0067
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0068
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0069
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0070
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0071
   0.0       0.0       0.0       0.0       0.0       0.0                    FORC0072
```

FORCE ENTRIES TWO PER NODE IN ORDER CONTINUED FORC0073

						FORC0074
0.0	0.0	0.0	0.0	0.0	0.0	FORC0075
0.0	0.0	0.0	0.0	0.0	0.0	FORC0076
0.0	0.0	0.0	0.0	0.0	0.0	FORC0077
0.0	0.0	0.0	0.0	0.0	0.0	FORC0078
0.0	0.0	0.0	0.0	0.0	0.0	FORC0079
0.0	0.0	0.0	0.0	0.0	0.0	FORC0080
0.0	0.0	0.0	0.0	0.0	0.0	FORC0081
0.0	0.0	0.0	0.0	0.0	0.0	FORC0082
0.0	0.0	0.0	0.0	0.0	0.0	FORC0083
0.0	0.0	0.0	0.0	0.0	0.0	FORC0084
0.0	0.0	0.0	0.0	0.0	0.0	FORC0085
0.0	0.0	0.0	0.0	0.0	0.0	FORC0086
0.0	0.0	0.0	0.0	0.0	0.0	FORC0087
0.0	0.0	0.0	0.0	0.0	0.0	FORC0088
0.0	0.0	0.0	0.0	0.0	0.0	FORC0089
0.0	0.0	0.0	0.0	0.0	0.0	FORC0090
0.0	0.0	0.0	0.0	0.0	0.0	FORC0091
0.0	0.0	0.0	0.0	0.0	0.0	FORC0092
0.0	0.0	0.0	0.0	0.0	0.0	FORC0093
0.0	0.0	0.0	0.0	0.0	0.0	FORC0094
0.0	0.0	0.0	0.0	0.0	0.0	FORC0095
0.0	0.0	0.0	0.0	0.0	0.0	FORC0096
0.0	0.0	0.0	0.0	0.0	0.0	FORC0097
0.0	0.0	0.0	0.0	0.0	0.0	FORC0098
0.0	0.0	0.0	0.0	0.0	0.0	FORC0099
0.0	0.0	0.0	0.0	0.0	0.0	FORC0100
0.0	0.0	0.0	0.0	0.0	0.0	FORC0101
0.0	0.0	0.0	0.0	0.0	0.0	FORC0102
0.0	0.0	0.0	0.0	0.0	0.0	FORC0103
0.0	0.0	0.0	0.0	0.0	-0.5400	FORC0104
0.0	-3.1200	0.0	-5.6400	0.0	-7.4400	FORC0105
0.0	-8.5200	0.0	-8.8800	0.0	-8.5200	FORC0106
0.0	-7.4400	0.0	-5.6400	0.0	-3.1200	FORC0107
0.0	-0.5400					FORC0108

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