TECHNOLOGICAL AND ECONOMIC ASPECTS OF GLASS FIBER-REINFORCED CEMENT COMPOSITES

by

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ABSTRACT

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Submitted to the Department of Civil Engineering on August 23, 1974, in partial fulfillment of requirements for the degree of Master of Science in Civil Engineering.

The research on glass fiber-reinforced cement composites (GRC) has been progressively accelerated with the development of alkaline resistant glass fiber. Some of the advantages offered by glass fiber addition to cement include improved tensile properties, higher toughness and better crack resistance. From the analytical point of view, GRC is an intractable material because of the inherent variations of the fiber-reinforcing mechanisms. As a result, theoretical investigations have not satisfactorily explained actual experimental results. Furthermore, there are still several problems in both economic and technological phases.

Among those, two important problems are investigated in this thesis which are :

1) evaluation of glass fiber-reinforced cement composites in terms of cost-effectiveness and
2) corrosion properties of alkaline-resistant glass fiber-reinforced cement composites.

Linear and non-linear multiple regression analyses were used for the investigation of the experimental results. The three parameters sand/cement ratio, water/cement ratio and volume content of fiber, were chosen as independent variables. The magnitude of the influences of these parameters on flexural strength and toughness of GRC is discussed. A method of determining an optimal composite composition in terms of usage requirement on properties is presented as well as the assessment of various types of fiber-reinforced cement composites.

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TABLE OF CONTENTS

LIST OF TABLES

Chapter 3

Chapter 4

Chapter 5

LIST OF FIGURES

Figure page

Chapter 4

Chapter 5

Chapter 1

Introduction

1.1 Preface

As is commonly known, cement has for a long time played an important role in the construction world. Because of the stability in supply of this material, we can expect continued use of cement far into the future. However, cementitious materials have some shortcomings in the form of brittleness and low tensile strength that prevent them from gaining into more widespread usage. To improve these shortcomings, cement concrete has conventionally been used in conjunction with steel reinforcement. But recent trends in construction technology such as modular and mobile homes demand a new type of construction materials which meet the following needs : (1)

1) to reduce the labor cost of the placing of reinforcements
2) to decrease the thickness and weight of structural components in order to improve construction efficiency and reduce the material cost

3) to open new possibilities for prefabricated elements

4) to increase the fatigue life of structures

5) to give greater fire resistance

6) to allow for time-saving in design and construction

The fiber-reinforced cement composites which are investigated here are the key materials strongly compatible with these above requirements. By definition, fiber-reinforced cement composites are composed of some kind of discrete fibers dispersed throughout the cementitious (usually portland cement) matrix. From the technological point of view, the idea of improving the shortcomings of brittle materials such as brick and cement with fibrous reinforcement is not new. Straw has been used to reinforce sunbaked bricks and clay wall since ancient times. Asbestos cement products were the first commercialized fiber-reinforced cement composites, and have recently been widely used for modern building materials. However, the current demands in the construction field which have been accelerating the development of fiber-reinforced cement are substantially different from those of the old ages. Furthermore, the great success of fiber-reinforced plastic for the last two decades has led to a lot of knowledge both on the fiber-reinforcing mechanism and on the fabrication process of fiber-reinforced composites.

As for fibrous reinforcement, a very wide range of fiber types is currently available. Almost all types of fiber have been investigated in view of their compatibility as a reinforcing material to portland cement-based matrices. Some types of fiber have been eliminated from the list of possibilities because of poor resistance to deterioration by the highly alkaline environment of cement paste or the low cost-effectiveness as reinforcement to cement composites. Currently, steel and alkaline resistant glass fiber are very promising fibrous reinforcements because of their excellent mechanical properties, comparably low price and the possibility of stable supply to a large demand for fiber. Carbon fiber and some newly developed plastic fibers, such as fiber-B and PRD-49, hold a great potential for future research and development because of their attractive fiber properties.

Extensive work on steel and glass fiber-reinforced cement composites performed mainly in the United States and in the United Kingdom during last ten years has brought about current usage of these two materials which are being assessed as to their potentiality as new construction materials in terms of cost-effectiveness. As this moment, there are still ambiguities in the evaluation of the true effectiveness of fiber addition. So far, available literature on the subject are sufficient for an understanding of the qualitative tendencies of the improvement of properties of plain mortar and concrete by addition of fibers. But these experimental results are not always appropriate for quantitative comparison. The use of different types of fiber, form of reinforcement, fabrication method, sand/cement ratio and water/cement ratio in each experiment program causes unlimited variability in the results, and thus makes any sort of comparison extremely difficult. It is well known that the prices of fiber are much higher than that of cementitious matrices resulting in a dramatic increase of material cost. In this regard, more intensive work on cost analysis is needed.

So far, most research efforts have been devoted to finding the effect of fiber addition and not to the composition of the matrix. Consequently, there are few studies concerned with the over-all effect of fiber and matrix composition. A cost analysis study of fiber-reinforced cement composites is essential for finding the economical compositions to meet the variety of usage requirements facing the industry today.

1.2 Present Background of Research and Development on FRC

A wide range of fibers are available for use as reinforcement to cement-based matrices. The primary physical properties of the fibers currently under consideration are shown in Table 1-1.

Table 1-1 Typical Physical Properties of Fibers Currently Under Consideration for Use as Reinforcement to Cement-Based Matrices (2,3)

Properties Fiber	Specific Gravity	Tensile Strength 10^3psi	Elastic Modulus 10^6psi	Ultimate Elongation (%)	Resistance to Alkalis
Asbestos (chrysolite)	2.9	80-140	12-20	2- 3	good
Steel (carbon)	7.8	50-400	28-30	0.5-35	good
Glass fiber (alkali-resistant)	2.7	200-500	9-11	3- 4	fairly good - poor
Poly-propylene (filament)	0.9	80-110	0.7-1.0	18-25	good
Nylon (mono-filament)	1.14	120-280	0.4-0.6	14-25	good - fairly good
Carbon fiber	1.7-2.0	200-450	28-35	0.4-1.0	good

The effectiveness of reinforcement depends on the modulus of elasticity, tensile strength and elongation at the fracture of reinforced fiber. Steel and glass fibers which have a modulus greater than 10^7 psi provide improvement in impact strength and ductility as well as static and dynamic flexural strength and compression strength. Asbestos fiber also has a high modulus of elasticity and provides improvement in flexural and compressive strength. But it gives a comparably low contribution to improvement in impact strength because of its short length of fiber. The low-modulus fibers such as nylon and polypropylene provide improvement in impact strength and ductility of composite, but they often give negative effects on the other physical properties. Consequently, their application is limited.

The criteria for the selection of fiber as reinforcement to cement-based matrices should include :

1) mechanical properties
2) resistance to alkaline (long term durability)
3) cost
4) non-flammability or thermal stability
5) physical and health hazards involved in production and usage

6) stability in future supply or potential production level, and

7) ease of fabrication of composite.

The result of a qualitative evaluation of fibers in terms of these criteria is given in the following table.

criteria fiber	1	2	3	4	5	6	7
Asbestos	A	A	A	A	D	C	A
Glass*	A	B·C	C	A	B	B	A·B
Steel	A	A	C	B	B	B	C
Carbon	A	A	D	A	B	C	B
Polypropylene	C	A	B	D	A	C	A
Nylon	C	B	B	D	A	B	A

A : good
B : fairly good
C : presents some problems
D : presents severe problems

* alkaline-resistant fiber

Although asbestos is the only fiber which has been widely used for reinforcement to cement-based matrices, there is a great uncertainty about its future use because of lung cancer associated with asbestos. At this moment, glass and steel fibers seem to have a good possibility to provide an innovative material in the construction market.

The current problems involved with the use of glass and steel fibers are described below.

For glass fiber :
- The confirmation of long term deterioration of alkaline-resistant glass fibers in the high alkaline environment of cement matrices

For steel and glass fibers :
- Fiber producers efforts for reducing the cost of fiber
- Establishment of an evaluation method of economical composition
- Establishment of the fabrication method which gives the best effectiveness of fiber addition
- Establishment of experience in application for specific use

1.3 Brief Review of Theoretical Consideration for Strength of Fiber-Reinforced Cement Composites

Most theoretical investigations on fiber-reinforced cement composites developed so far are based on the assumption of an ideal, uniformly-reinforced matrix. The common methods are the rule of mixture or its modification. Two separate mechanisms have been thought to explain the increase of tensile properties by the addition of fibrous reinforcement.

1) Direct strengthening, similar to that achieved with conventional steel reinforcement
2) The ability of fibers to act as a crack arrestor, which provide improvement in the ultimate strength of the cement matrix

The common parameters on which the composite strengths depend include type of fiber, the degree of fiber alignment with stress direction (orientation of fiber), aspect ratio (or length of fiber) and fiber content, as well as the primary parameters of the strength of cement matrix, sand/cement ratio and water/cement ratio. The effects of these parameters are summarized below.

Type of Fiber

One important parameter is the type of fiber and its properties which affect the properties of the whole composite. Plastic fibers have low tensile strength, high ultimate elongation, low modulus of elasticity and poor bond with the matrix. These factors combine to give a low contribution to flexural strength, tensile strength or modulus of elasticity of the composite, but tend to have a large effect on impact strength and toughness, because of the large amount of energy absorbed in breaking and/or pulling out fibers.

Glass fibers have a fairly high modulus of elasticity, high strength, low elongation and good bond with the matrix. These combine to make a composite with high tensile and bending strength and high modulus elasticity. The impact strength and toughness are also improved, though not as much as with other fibers. Steel fibers have high strength, high modulus of elasticity, high elongation and a good bond with the matrix. These properties give steel-reinforced composites good properties all around.

Fiber Orientation and Fabrication Method

The highest improvement in mechanical properties such as flexural strength and tensile strength would come from aligning the fibers unidirectionally in the direction of greatest stress. After this, in descending order of improvement, would be a 2-dimensional orthognal array, a 2-dimensional random, and finally 3-dimensional random array. This is due to the fact that the fibers give the most reinforcement when aligned with the direction of stress.

Fabrication methods which give the best results, in decreasing order of quality, are : aligning long fibers uniaxially or biaxially, and filling in with matrix, as in conventional glass fiber - polyester composites. The next would be a method which arrays the fibers randomly, but essentially in 2 dimensions, such as the spray-suction method. The least effective method of fabrication would be casting the material as premixed, which would give it, essentially, 3-dimensional characteristics — the least efficient orientation.

Aspect Ratio and Volume Fraction of Fiber

Volume fraction influences only slightly the first crack stress. Both volume fraction and aspect ratio have a large influence on ultimate strength, both bending and tensile, and some influence on compressive strength. Ultimate strength seems to increase linearly with a increase in either V_f or aspect ratio. However, it is not possible to predict with accuracy the ultimate strength of the composite because of the combination of cracking and pull-out that occurs with failure---probably due in large part to the nonuniformity of the material.

Toughness increases significantly with V_f and aspect ratio. Toughness of fiber-reinforced cement composites is at least an order of magnitude higher than that of plain concrete. (4) It is reasonable to assume that the same factors that influence ultimate strength (V_f, aspect ratio, orientation) also influence toughness. Too high V_f and/or aspect ratio create workability problems, and voids.

Sand/Cement Ratio

Increased sand/cement ratio increases the toughness of plain mortar because of the increased microcracking and pathways for energy absorption. So it should improve the toughness of fiber-reinforced cement composite---up to a certain limit, of course. Problems include increased number of voids which would decrease ultimate strength, and decreased bonds between matrix and fiber. Increasing sand/cement also decreases workability and requires more water for easier working.

Water/Cement Ratio

As is commonly known, decreased water/cement means increased mortar strength and decreased workability. A problem in fiber-reinforced cement composites occurs with water-absorbent fibers like glass, which would tend to reduce significantly the water/cement ratio. Water-reducing agents may help, but caution must be taken to insure that no adverse chemical reactions occur.

1.4 Objectives of This Research

As is described in the previous section, glass fiber-reinforced cement composite has been getting into the final stage of research and will possibly be introduced to the market in the near future. However, there are still several problems in both economic and technological phases amongst which two important problems are investigated in this thesis :

1) Evaluation of glass fiber-reinforced cement composites in terms of cost-effectiveness
2) Corrosion properties of alkaline-resistant glass fiber-reinforced cement composite

As for problem 1), first the magnitude of the effect of fiber addition is investigated by a comparison with the effects of the other important factors on strengths. Three parameters are chosen which include volume fraction of fiber, sand/cement ratio and water/cement ratio.

Considering the fact that the addition of fiber results in a dramatic increase of cost, the magnitude of the increase of strengths should largely exceed that of the others.

Regression analysis is primary used for the analysis of the effect of each parameter on physical properties. Before determining the experimental program which is shown in Chapter 3, regression analysis is applied to some previous work to observe the potentiality of this method, and to obtain some ideas on the experiment condition and on the limits of this method. These case studies will be developed in Chapter 2.

The experiment results will be given in Chapter 4. The relative magnitude of the effect of the variables on flexural strength and toughness is evaluated, based on the regression equations obtained. Furthermore, the adaptability of the estimation through regression equations are investigated under the consideration of the valid region of the analysis. Finally in Chapter 5, a model for the selection of an economical composition under a given usage requirement will be introduced and some case study is performed based on the regression equations obtained in

Chapter 4.

For investigating problem 2), flexural bars reinforced with glass fiber were exposed to 2, 5, 8 and 11 cycles of corrosion bath (NaOH soln. pH 12.5)/ oven-drying and were tested after the indicated number of cycles. Besides, pull-out experiments were performed to observe the deterioration of bond between matrix and fiber. These results will be given in Chapter 4.

Chapter 2

Regression Analysis of the Experimental Results of Previous Work

The objectives of this chapter are to determine some experimental conditions for the main body experiment and the limits of the statistical method. The data selected for this analysis comprise two experimental studies on steel fibers, and two experimental studies on glass fiber. The main aspects of these cases are summarized in the following table below.

Case	Fiber	Matrix	Type of Mixer
1	Steel	Mortar	Food Mixer
2	Steel	Concrete	Kitchen Type Mixer Foot Tilting Mixer
3	Glass	Concrete	Drum Mixer
4	Glass	Mortar	Rotary Type Mixer

2.1 Steel Fiber-Reinforced Cement Composites

Case (1) : Tensile Test Results by A. E. Naaman (5)

The objective of Naaman's work is to clarify the causal effects of fiber volume percent and aspect ratio on the composite cracking strength and the maximum post-cracking strength. The body of his experimental program comprises tensile tests on fiber-reinforced prisms.

The Data used for regression analysis is tabulated in Table 2-1. The procedure of regression analysis is given

Table 2-1 Tensile Test Results by A. E. Naaman

	Fiber Diameter ϕ , inch	Fiber Length 1, inch	Aspect Ratio $1/\phi$	Percentage Fiber by Volume (%)	Cracking Strength (psi)	Maximum Post-cracking Strength (psi)
Variable name / Sample#	---	---	ASPR	VOLF	CRST	PCST
1	0.01	0.50	50	1	265	53.3
2	0.01	0.50	50	2	282	85
3	0.01	0.50	50	3	310	124
4	0.01	0.75	75	1	295	77
5	0.01	0.75	75	2	311	130
6	0.01	0.75	75	3	357	191
7	0.01	1.00	100	1	322	100
8	0.01	1.00	100	2	339	172
9	0.01	1.00	100	3	389	243
10	0.006	0.50	83.5	1	307	125
11	0.006	0.50	83.5	2	328	147
12	0.006	0.50	83.5	3	368	184

in the following :

Regression Model : Multiple Linear Regression

Computer Program : 1130 IBM Statistical System
Stepwise Linear Regression

	1st Run	2nd Run
Dependent variable (Variable name)	Cracking strength (CRST)	Post-cracking strength (PCST)
Independent variable (Variable name)	Fiber volume percent (VOLF)	Fiber volume percent (VOLF)
	Aspect ratio (ASPR)	Aspect ratio (ASPR)
Variable range	$1 \leq \text{VOLF} \leq 3$ (%) $50 \leq \text{ASPR} \leq 100$	

Regression equation :

$$(\text{CRST}) = a_1 + b_1(\text{VOLF}) + c_1(\text{ASPR}) + e_1$$

$$(\text{PCST}) = a_2 + b_2(\text{VOLF}) + c_2(\text{ASPR}) + e_2$$

The results of the first run of a computer program using the above equation and the data in Table 2-1 are shown in Table 2-2, 2-3 and Appendix B 1.

Table 2-2 Matrix of Correlation Coeficients
(case 1a)

Variable	CRST	VOLF	ASPR
CRST	1.0000	0.6968	0.6863
VOLF		1.0000	0.0000
ASPR			1.0000

Table 2-3 Regression Equation for Cracking Strength
(case 1a)

Variable	Coefficient	Standard Error	t-ratio
Constant	a_1^* 163.0742	Sa_1^* 8.2823	
VOLF	b_1^* 29.3750	Sb_1^* 2.9282	10.0314
ASPR	c_1^* 1.3094	Sc_1^* 0.1325	9.8823
$R^2 = 0.957$, R = 0.978 F ratio = 99.122 Degree of Freedom = 9, t-critical(α =0.05) = 2.262 Standard Error of Mean = 2.3908 Residual Standard Deviation = 8.2467			

We can examine the simple correlation matrix shown in Table 2-2 to determine whether multicollinearity does exist and to see how each independent variable is related to the cracking strength.

All statistical tests are perfectly accepted. The final equation is given by :

$$CRST = 163.1 + 29.4(VOLF) + 1.3(ASPR)$$

Our main objective for applying regression analysis to the experimental results is to clarify the magnitude of the contribution to strength with one unit increment in each independent variable. Before we evaluate how large an influence these variables have on the cracking strength we should be very careful of the units of each variable, and the reliability of the estimated coefficients, b^* and c^*, which are obtained by the least squares of the limited number of sample points. However, the coefficients of variables calculated by least squares (a^*, b^*, c^*) are unbiased estimates of the coefficients of the true regression equation (a, b, c) that is :

$$E\left[a^*\right] = a \;, \quad E\left[b^*\right] = b \;, \quad E\left[c^*\right] = c$$

where E refers to the expected value. According to the statistical evaluation, the coefficient of variables of the true regression equation would fall in the ranges given below.

$$b^* - \text{t-critical} \times S_b{}^* \leq b \leq b^* + \text{t-critical} \times S_b{}^*$$

$$c^* - \text{t-critical} \times S_c{}^* \leq c \leq c^* + \text{t-critical} \times S_c{}^*$$

where S_b^* and S_c^* denote the standard errors of estimate of variable VOLF and ASPR respectively.

Suppose we choose 0.05 as a degree of significance (α), we will find 2.262 as the value of t-critical (α = 0.05, DF=9). If we insert this value into the above equation, we can find some ranges in which the coefficients of VOLF and ASPR might fall with ninety five percent confidence level.

Table 2-4 The Magnitude of the Effect of Variables on Cracking Strength (case 1a)

Variable	Coefficient	Amount of Change in Variable	Change in Cracking Strength (psi)
VOLF	29.4 ± 6.6	1% (e.g. 1% → 2%)	29.4 ± 6.6
ASPR	1.3 ± 0.3	23 (e.g. 50 → 73)	29.9 ± 6.9

(95% confidence level)

Taking into account the unit of each variable, the effect on cracking strength of an increase of fiber content by one volume percent is equivalent to that of an increase of the aspect ratio by 23, approximately.

In the second run, maximum post-cracking strength is substituted for cracking strength of the first run as the dependent variable. The results of computation are given in Table 2-5, 2-6 and 2-7. The computer printout of the final step equation is given in Appendix B2 .

Table 2-5 Matrix of Correlation Coefficient (case 1b)

Variable	PCST	VOLF	ASPR
PCST	1.0000	0.6968	0.6863
VOLF		1.0000	0.0000
ASPR			1.0000

Table 2-6 Regression Equation for Post-cracking Strength (case 1b)

Variable	Coefficient	Standard Error	t-ratio
Constant	-93.5971		
VOLF	48.3750	5.8307	8.2966
ASPR	1.7214	0.2636	6.5303

R^2 = 0.9292, R = 0.9619 F = 55.729
Degree of Freedom = 9, t-critical(α =0.05) = 2.262
Standard Error of Mean = 2.3908
Residual Standard Deviation = 16.4919

Table 2-7 The Magnitude of the Effect on Post-Cracking Strength (case 1b)

Variable	Coefficient	Amount of Change in Variable	Change in Cracking Strength (psi)
VOLF	48.4 ± 13.2	1% (eg1% → 2%)	48.4 ± 13.2
ASPR	1.7 ± 0.6	28 (eg50 → 82)	47.6 ± 16.8

The results of statistical tests given in Table 2-6 are acceptable and show that the effect of the addition of steel fiber on post-cracking behavior is much greater than that on cracking strength.

In the original paper by Naaman, he observed a different trend of the reinforcing effect between the fibers with a 0.006 inch diameter and those with a 0.010 inch diameter. Since the regression equation merely gives the average effect of reinforcement of two different variables, VOLF and ASPR, it is difficult to predict this type of observation. The result also points out the shortcomings of aspect ratio as a parameter since fiber of vastly different lengths and diameters can have the same aspect ratio. Further discussions on the results of the application of regression analysis are developed later in this chapter.

Case (2) Flexural Strength of Steel Fiber-Reinforced Concrete (6)

This work was done by G. Williamson(Ohio River Division Lab.,Corps Engineers in 1965), and is one of the important studies done in the early stages of steel fiber-reinforced concrete development.

The main objectives of their research were to study various materials suitable for use as random fibrous reinforcements and to determine which of these materials are most effective toward improving the flexural strength and dynamic loading resistance of the concrete by the 7-, 28- and 90-day tests. The fibers used as random reinforcement consisted of Nylon, fiberglass, vinyl-coated fiberglass and steel wire.

Here, regression analysis was applied to the 28-day flexural test results of SRC. In the original paper it was pointed out that the amount of increase of the ultimate flexural strength of SRC was proportional to the percentage of wire used. However no quantitative investigations were developed there.

The materials and concrete mixes used in the experiment are shown below.

Materials : Type of cement   Portland cement Type I
Steel fiber   Brass-plated steel wire

Concrete Mix : Three basic concrete mixes were used.

a. Neat Cement   W/C = 0.29

b. No.8 Maximum Size Aggregate
Aggregate/Cement = 2.47
W/C = 0.46

c. No.50 Maximum Size Aggregate
Aggregate/Cement = 0.55
W/C = 0.36

Table 2-8 Flexural Test Results by Army Corps

	Fiber Diameter ϕ, inch	Fiber Length ℓ, inch	Aspect Ratio ℓ/ϕ	Percentage Fiber by Volume (%)	Aggregate/ Cement Ratio	W/C Ratio (by wt)	Flexural Strength (psi)
Variable Name / Sample#	---	---	ASPR	VOLF	ABYC	WBYC	FLST
--	0.004	0.5	125	0.00	0.00	0.29	1875
1				1.00	0.00	0.29	2630
2				3.00	0.00	0.29	2815
--				0.00	2.47	0.45	1385
3				1.00	2.47	0.45	1655
4				2.09	2.47	0.60	2200
--				0.00	0.55	0.36	1525
5				1.00	0.55	0.36	2710
6				3.00	0.55	0.36	2955
--	0.10	0.5	50	0.00	0.00	0.29	2190
7				1.00	0.00	0.29	1950
8				3.00	0.00	0.29	2780
9				5.00	0.00	0.29	3495
--				0.00	2.47	0.461	1345
10				1.00	2.47	0.461	1460
11				3.00	2.47	0.461	2330
12				5.00	2.47	0.461	3210
--				0.00	0.55	0.36	1560
13				1.00	0.55	0.36	1750
14				3.00	0.55	0.36	2335
15				5.00	0.55	0.36	3060

Table 2-8 continued

	Fiber Diameter ϕ, inch	Fiber Length ℓ, inch	Aspect Ratio ℓ/ϕ	Percentage Fiber by Volume (%)	Aggregate/ Cement Ratio	W/C Ratio (by wt)	Flexural Strength (psi)
Variable Name / Sample#	---	---	ASPR	VOLF	ABYC	WBYC	FLST
--	0.004	1	250	0.00	0.00	0.29	1885
16				1.00	0.00	0.29	2670
--				0.00	2.47	0.45	1555
17				1.00	2.47	0.45	1920
18				1.70	2.47	0.49	1165
--				0.00	0.55	0.36	1745
19				1.00	0.55	0.36	2175
20				1.50	0.55	0.40	1960
--	0.10	1	100	0.00	0.00	0.29	1645
21				1.00	0.00	0.29	2485
22				3.00	0.00	0.29	3660
23				3.75	0.00	0.29	4080
--				0.00	2.47	0.45	1355
24				1.00	2.47	0.45	2020
25				1.69	2.47	0.45	2395
--				0.00	0.55	0.36	1625
26				1.00	0.55	0.36	2035
27				3.00	0.55	0.36	4225

The variables in the regression equation are shown as follows.

Dependent Variable : The 28-day Flexural Strength (FLST)

Independent Variable :

1) Water/Cement ratio	(WBYC)
2) Percentage of fiber by volume	(VOLF)
3) Aspect ratio (ℓ/ϕ)	(ASPR)
4) Aggregate/Cement ratio	(ABYC)

The size of the aggregate is not taken into account in the regression model.

$$FLST = a + b(WBYC) + c(VOLF) + d(ASPR) + e(ABYC)$$

The data used for regression analysis is tabulated in Table 2-8. As is shown in the above concrete mix program, the authors used the same W/C ratio for a given aggregate size and aggregate/cement ratio. Consequently, the correlation coefficient between (ABYC) and (WBYC) has an extremely high value (0.921). (See Table 2-9)

Table 2-9 Matrix of Correlation Coefficient (case 2)

Variable	FLST	WBYC	VOLF	ASPR	ABYC
FLST	1.000	-0.491	0.680	-0.205	-0.494
WBYC		1.000	-0.128	0.083	0.921
VOLF			1.000	-0.452	-0.136
ASPR				1.000	0.025
ABYC					1.000

So, we have to eliminate either WBYC or ABYC from our final equation based on the appropriate criteria described in Appendix A . The final results obtained from the stepwise regression analysis are given in Appendix B 3, and are summarized in Table 2-10.

Table 2-10 Regression Equation for Flexural Strength (case 2)

Variable	Coefficient	Standard Error	t-ratio
Constant	3192.8		
WBYC	-3781.2	1157.6	3.2665
VOLF	346.7	69.5	4.9804

$R^2 = 0.63$, $R = 0.79$ $F = 20.2$
Degree of Freedom = 24, t-critical($\alpha = 0.05$) = 2.064
Standard Error of Mean = 92.5
Residual Standard Deviation = 480.8

The variables, ABYC and ASPR, are eliminated as suggested by the statistical evaluation (t-test & multicollinearity). And the final equation of the two independent variables explains sixty three percent of the experimental results.

$$FLST = 3192 + 346.7(VOLF) - 3781.2(WBYC)$$

Note that aspect ratio and aggregate/cement ratio are not significant parameters within the experimental ranges of each variable.

The amounts of increase of the flexural strength with a given increment of each variable are shown in Table 2-11.

Table 2-11 The Magnitude of the Effect on Flexural Strength (case 2)

Variable	Coefficient	Amount of Change in Variable	Change in Flexural Strength (psi)
VOLF	346.7 ± 143.4	1 %	346.7 ± 143.4
WBYC	-3781.2 ± 2389.3	-0.1	378.1 ± 238.9

(Confidence level 95%)

This shows the effect of one volume percent addition of fiber on flexural strength is almost the same magnitude as that of the reduction of water/cement ratio by 0.1. But from the economical point of view, the latter is more efficient in terms of cost effectiveness where fabrication is simple.

2.2 Glass Fiber-Reinforced Cement Composites

Two cases of GRC are selected for the application of regression analysis. The first is the work done by H. N. Marsh and L. L. Clark (7), where alkaline-resistant glass fiber and a premixing-cast method, was used. The other is the work by J. Takagi (8), where he used E-glass fiber and high alumina cement, and the conventional premixing-cast method.

Case (3) : Flexural Strength of GRC (7)

In the program of this study, five parameters are taken into account : glass fiber length, amount of glass fiber, cement factor, water/cement ratio and coarse/fine aggregate ratio. Among them the following variables were used for our regression analysis :

Dependent variable	Flexural strength	(FLST)
Independent variable	1 Glass fiber length	(LENF)
	2 Amount of glass fiber	(VOLF)
Cement factor	846 lb/cu yard	
Water/Cement ratio	0.5	

Fiber : Alkaline-resistant glass fiber

Fabrication Method :

$$FLST = a + b(LENF) + c(VOLF) + d(RTAG)$$

The data used for the regression analysis is shown in Table 2-12. The correlation coefficients are given in Table 2-13

Table 2-12 Flexural Test Results by H. Marsh and L. Clark

Sample #	Flexural Strength	Sand/Cement	Percentage Fiber by Volume (%)	Length of Fiber
1	864	0.33	0.5	0.5
2	892	0.33	0.5	1
3	946	0.33	0.5	1.5
4	1070	0.33	0.5	2
5	1200	0.33	1.0	0.5
6	1140	0.33	1.0	1
7	1210	0.33	1.0	1.5
8	1020	0.33	1.0	2.0
9	1290	0.33	1.5	0.5
10	1520	0.33	1.5	1
11	1370	0.33	1.5	1.5
12	1420	0.33	1.5	2
13	1340	0.33	2.0	0.5
14	1630	0.33	2.0	1.0
15	1790	0.33	2.0	1.5
16	1480	0.33	2.0	2
17	1310	0.33	2.5	0.5
18	1640	0.33	2.5	1
19	2320	0.33	2.5	1.5
20	1800	0.33	2.5	2
Control	475	0.33	0	0

where a strong correlation between VOLF and FLST is shown.

Table 2-13 Matrix of Correlation Coefficient (case 3)

Variable	FLST	LENF	VOLF
FLST	1.000	0.203	0.837
LENF		1.000	0.000
VOLF			1.000

The computed results based on a linear regression model are given in Table 2-14.

Table 2-14 Regression Equation of Flexural Strength (case 3a)

Variable	Coefficient	Standard Error	t-ratio
Constant	a^* 584.0498		
LENF	b^* 126.8799	76.7955	1.6522
VOLF	c^* 413.3001	60.7122	6.8078

R^2 = 0.7427, R = 0.8618 F = 24.536
Degree of Freedom = 17, t-critical(α =0.05) = 2.110
Standard Error of Mean = 42.9300
Residual Standard Deviation = 191.987

According to the t-test, the variable, LENF, has a lower t-ratio than t-critical and should be eliminated from the equation. The final equation obtained is given by :

$$FLST = 742.65 + 413.30(VOLF)$$

$$(Sc^{*} = 63.5)$$

$R^2 = 0.701$, $R = 0.837$ F-ratio = 42.2
t-ratio = 6.50
Degree of Freedom = 18
t-critical($\alpha = 0.05$) = 2.101

The expected range in which the coefficient of VOLF might fall in a true regression equation is :

$$285.2 \leq b \leq 541.4$$

Looking at the Table 2-11, we can observe a tendency of increase of flexural strength with an increase of glass fiber length up to 1.5 inches. However, the specimens with 2-inch lengths of fiber give lower strengths than those which are 1.5 inches in length. In the linear regression model, it is very difficult to explain this type of deviation. It only gives the average tendency of the effect of fiber length on the strength, and the positive effect is canceled with the negative effect.

The comparative low effectiveness of reinforcement with 2-inch fiber possibly comes from difficulties in the mixing or in the fabrication method.

There may be two possible ways to eliminate this type of problem. One would be to reduce the variable range, and the other would be to use a non-linear model. In this case, the latter is not an appropriate method because there are only a few discrete values of length, even though there are twenty sample points from the statistical point of view. The additional computer run was carried for the reduced sample size. There the data of 2-inch fiber were eliminated from those for the first run.

Table 2-15 Regression Equation for Flexural Strength (case 3b)

Variable	Coefficient	Standard Error	t-ratio
Constant	a^* 403.1		
LENF	b^* 326.4	117.7	2.773
VOLF	c^* 423.0	67.9	6.230
R^2 = 0.79, R = 0.89 F = 23.3 Degree of Freedom = 12, t-critical(α =0.05) = 2.179 Standard Error of Mean = 48.0 REsidual Standard Deviation = 186.0			

$$FLST = 403.1 + 326.4(LENF) + 423.0(VOLF)$$

The resulting equation shows a better fit to the first equation in terms of overall significance, and the variable, LENF, is accepted this time. However the possible range of the coefficient of LENF is too wide to predict a reasonable value (see following table).

Variable	Changes in Strength	Amount of Change of Variable
LENF	$70.0 \leq b \leq 582.8$(psi)	1 inch (0.5"→1.5")
VOLF	$275.0 \leq c \leq 571.0$	1 % (1% → 2%)

Note: the coefficient of VOLF is not sensitive to a reduction of sample size and the entering variable, LENF.

Case (4) : Flexural Strength of GRC by J. Takagi (8)

The objective of this work is to clarify the effect of the length of randomly distributed fibers and the glass content on the flexural strength, compressive strength, tensile strength and Young's modula of fiber-reinforced mortar and concrete. Here, the results of the flexural tests are used for the regression analysis.

Material : Matrix : High Almina Cement
River Sand (Maximum Size : 0.1 ")
S/C = 2
W/C = 0.63
Fiber : E-glass chopped strand containing 200 monofilaments of 9 micron each.
Mixing = Rotary type mixer

Table 2-16 Flexural Test Results by J. Takagi

Specimen No.	Fiber Length mm (inch)	Fiber Content (% by weight)	Flexural Strength Kg/cm^2
1	3(0.12)	0.25	59.7
2	"(")	0.50	63.2
3	"(")	0.75	71.3
4	"(")	1.00	65.0
5	6(0.24)	0.25	65.4
6	"(")	0.50	67.2
7	"(")	0.75	66.5
8	"(")	1.00	65.9
9	13(0.51)	0.25	62.7
10	"(")	0.50	73.0
11	"(")	0.75	68.9
12	"(")	1.00	64.2
13	25(0.98)	0.25	67.1
14	"(")	0.50	75.3
15	"(")	0.75	77.1
16	"(")	1.00	67.1
Control		0.00	62.0

Dependent Variable   Flexural Strength   FLST

Independent Variable 1. Percentage of fiber by weight (%) WGHF

2. Fiber length   (mm) LENF

Regression Model

$$(FLST) = a + b(WGHF) + c(LENF)$$

The data used here is given in Table 2-16. The regression equation obtained through a computer run is given in Table 2-17 and Appendix B5.

Table 2-17 Regression Equation of Flexural Strength (case 4)

Variable	Coefficient	Standard Error	t-ratio
Constant	62.3		
LENF	0.30	0.12	2.500
WGHF	2.70	3.65	0.7397

$R^2 = 0.34$, $R = 0.58$ $F = 3.33$
Degree of Freedom = 13, t-critical(α =0.05) = 2.16
F-critical(α =0.05) = 3.34
Standard Error of Mean = 1.02
Residual Standard Deviation = 4.08

According to the criterion for overall significance, this equation is not sufficient to predict a reliable value. The reasons of low multiple correlation could come from :

1. Relatively short fiber lengths, some of which are

probably less than the effective length of glass fiber in cement mortar.

2. Relatively low content of fiber where the effect of fiber addition is not likely to exceed random fluctuation.

3. Linear-model is not sufficient to predict actual phenomena.

2.3 Discussion of the Case Studies

Although the multiple correlation coefficients (R^2) of the linear regression analysis obtained in the four cases are widely scattered, the results of these studies imply that there is a good possibility to apply regression analysis for investigating the strengthening mechanism of fiber-reinforced cement composites. However, care must be taken to insure the valid region of the application of regression analysis. The poor fit of the linear regression equation could be observed when experimental programs include an irrelevant variable range. This could be encountered either if the effect of the variable on the strength does not exceed stochastic and inherent experimental errors or if the composition of the mix is too harsh for the fabrication method used, (which causes a poor strength specimen). The fractural behavior of such a poor specimen is difficult to explain without a consideration of the parameters associated with the differences in structurally defective specimens.

In general, the region where consistent experimental results are expected to be obtained seems rather narrow. Consequently, the region where a straight line prediction, based on either theoretical or statistical investigation, is expected to have a good fit to the actual is also narrow.

The magnitudes of the effect of fiber addition on the strength were relatively small. In Naaman's Case, the cracking strength increased an average of 29 psi with 1 percent increase in fiber volume content which corresponded to only 12 percent of the strength of the control specimen (VF=0). And in Case 2, the increase of flexural strength with 1 volume percent increase of steel fiber content, fall in the range of 20 to 30 percent of the strength of plain mortar and concrete. Only in Case 3 where glass fiber was used as reinforcement to concrete, the increase of flexural strength with 1 percent change of fiber volume content reached 80 percent of the strength of the plain concrete specimens. The result of Case 4 does not show any significant effect of addition of glass fiber on the flexural strength of composite.

Considering the possible range of fiber volume content, the increase of strength achieved by means of conventional pre-mixing and casting methods is limited up to 2 or 3 times that of plain mortar or concrete.

Fiber length could be an important factor. However, increased length results in poorer workability and dispersion of fiber, and decreased length results in ineffective reinforcement. Therefore, the variable range in length is rather narrow (usually 0.5" to 1.5"). As a result, fiber length is not always an important variable. In this respect, aspect ratio is a variable which is more interesting than the fiber length itself, because aspect ratio can be changed over a relatively wide range by changing the fibers' diameter.

Through all the cases, the observed points are not always scattered throughout a given multi-dimensional space composed of variables. This includes the danger of systematic biases in data. In the experiment developed in a later chapter, randomization is applied for the determination of sample points.

Chapter 3

Experimental Program

3.1 Introduction

The main portion of the experiment was concerned with a regression analysis of GRC composition to determine the contribution of each parameter to the over-all performance of the composite, as well as to determine an optimum composition. Some preliminary work was done to know the limits of the values of water/cement ratio, sand/cement ratio, and volume fraction, and to have some data on physical properties as one variable was changed at a time. Then the main body of work was performed with a number of different compositions selected from a random table. Finally a series of tests were made to determine the effects of corrosive conditions on the strength and toughness of GRC.

Flexural tests were performed on specimens in all phases of the experiment, with the addition of pull out tests in the corrosion series.

3.2 Sample Preparation

All mortar was made with type III Portland cement and

20-30g Standard Ottawa Silica Sand. Glass fibers, where used, were Owens-Corning K885CA chopped strand alkaline-resistant glass, in-one inch lengths. The fibers come in bundles averaging 204 filaments, each filament diameter 0.0005 inch. The filaments are held in bundles by organic sizing (polyvinyl alcohol).

All mixing was done by hand. First, the cement and sand were weighed and mixed, then fibers added where neccesary, and mixed in until uniformly dispersed. Then the water was added and mixing continued for another 1-2 minutes, once again insuring even dispersion of fibers and a uniform mix.

Flexure specimens were cast in specially designed plexiglass flat sheet molds, 8¼ wide by 9½ in long by 3/8 in deep. For one sheet, 1.5Kg confined weight of cement and sand plus water and fibers were needed. The fresh mortar was trowelled into the mold by hand and levelled off, then given 2 minutes gentle vibration on a vibrating table to aid settling and elimination of voids. The sheets were allowed to "set up" in air for 3-4 hours to prevent water damage in the curing room. Then they were placed in the room at 100 percent humidity and 75° F for seven days, and unmolded after one day. After seven days curing, the sheets were cut into 9 strips,each one-inch wide, the ends of the sheet being discarded. The specimens were then dried in air one day and either tested, in the optimum composition experiment, or placed in a corrosive environment for the corrosion experiment.

Pull-out specimens were prepared from a standard mortar of 4 parts sand to two parts cement to one part water. They were cast in ASTM standard briquette molds. In half of each mold was placed a styrofoam-cardboard sandwich which held a single fiber strand. The fibers embedded ¼ inch. (9) The prepared mortar was then carefully trowelled into the other half mold around the protruding fiber, rodded several times, and leveled. The molds were given ½ minute gentle vibration, then placed in the curing condition for seven days. They were unmolded after the second day. After curing, the specimens dried in air one day, then half were placed in corrosion solutions and the other half were kept in air as controls. The fibers protruding on specimens kept in corrosive solutions were protected first by coating with wax to facilitate removal of the coating before testing, and "5 minute" epoxy, to prevent corrosive damage to the fibers themselves.

3.3 Experimental Conditions

The preliminary work on GRC optimum composition involved selecting a series of compositions detailed in Table 3-1. These were chosen as the extreme values for each variable in a given variable range, and to give an indication of the effect of each individual component of the mix on over-all performance. Nine specimens (one sheet) of each composition were cast, cured cut and tested.

These data indicated guidelines for the next portion of the experiment. Twenty additional compositions, given in Table 3-2, were selected from a table of random values. The ranges of each variable are :

$$0 \leq S/C \leq 3 \; , \quad 0.4 \leq W/C \leq 0.7 \text{ and } \; 1 \leq VF \leq 4(\%)$$

These also were cast, cured, cut and tested, as specimens for each composition. When all data were compiled, a series of statistical regression analyses were performed to determine the optimum composition to give the desired flexural strength, and toughness, and to develop a formula for the contribution of each component to these properties.

The corrosion portion of the experiment dealt with the effects of a corrosive environment on the fiber-matrix bond, and how this affected the material's physical properties. Table 3-3 gives the conditions to which specimens were subjected. The aklaline condition was intended to simulate the weaker alkaline condition of normal concrete over a much longer period of time.

All corrosion specimens for flexure tests were made with a standard mortar of the following composition :

$$W/C = 0.5 \ , \quad S/C = 2 \ , \quad VF = 2\% \ .$$

They were prepared the same as for the other flexure specimens. Half of them were then placed in corrosive conditions with the pull out specimens after curing and one day drying. The other half served as controls. In addition, an equal number of plain mortar (without fiber) specimens were prepared at the same time, and subjected to the same conditions as the fiber-reinforced specimens. This additional measure was taken to determine the effects of the corrosive conditions on the mortar matrix itself. Specimens were tested after the predetermined number of corrosive cycles was reached.

3.4 Testing

All testing was performed with an Instron universal testing machine. The pull-out specimens were held by an ASTM standard mortar briquette test grip and the fibers were fastened by epoxy to cardboard, which was held in a set of flat jaws. The flexure tests were performed on a 2-point roller support over a span of 7 inches with one-point center roller loading.

Test conditions are given below.

Pull-out Test

Loading rate = 0.02 "/min

Chart speed = 0.5 "/min

Full scale load = 20 lb

Flexure Test

Loading rate = 0.05 "/min

Chart speed = 1 "/min

Full scale load = 100 lb

All specimens were weighed and measured to determine their density, and the thickness and width of each at fracture were measured. A note was also made of the distance off center at which fracture occurred.

The Instron machine gives a direct Load vs. Deflection graph, from which directly were read ultimate loads for determination of ultimate flexure stress. The area under the curve was determined by an OH Planimeter and used as a measure of toughness, or the energy absorbed to complete rupture.

Table 3-1 The Mix Program for Preliminary Experiment (Flexural Test)

Specimen No.	S/C	W/C	VF (%)	Specimen No.	S/C	W/C	VF (%)
p-1	0.00	0.4	0.5	p-11	2.00	0.4	0.0
2	0.00	0.7	0.5	12	2.00	0.7	0.0
3	0.00	0.4	4.0	13	2.00	0.4	0.5
4	0.00	0.7	4.0	14	2.00	0.4	1.0
5	3.00	0.4	0.5	15	2.00	0.4	1.5
6	3.00	0.7	0.5	16	2.00	0.4	2.0(1")
7	3.00	0.4	4.0	17	2.00	0.4	3.0
8	3.00	0.7	4.0	18	2.00	0.4	2.0(0.5")
				19	2.00	0.4	2.0(1.5")
				20	2.00	0.7	2.0
				21	1.00	0.4	2.0
				22	3.00	0.4	2.0

Table 3-2 The Random Mix Program of the Main Body Experiment (Flexural Test)

Specimen No.	S/C	W/C	VF (%)	Specimen No.	S/C	W/C	VF (%)
M-1	2.57	0.59	1.39	M-11	1.77	0.40	0.78
2	1.38	0.55	3.00	12	2.40	0.45	3.68
3	1.88	0.69	3.37	13	0.30	0.60	1.11
4	1.50	0.66	1.88	14	2.57	0.47	0.99
5	0.40	0.59	1.50	15	1.11	0.66	1.91
6	2.33	0.68	1.68	16	0.70	0.69	1.50
7	1.50	0.63	0.48	17	1.38	0.57	0.50
8	0.40	0.54	2.28	18	2.57	0.42	2.91
9	2.40	0.56	2.61	19	2.94	0.47	1.12
10	0.59	0.65	0.91	20	0.59	0.53	3.20

Table 3-3 Condition of Corrosion Experiments

Material : 1. GRC (S/C=0, W/C=0.5, VF*=2%)
* Glass : Alkaline-resistant glass fiber

2. Mortar

Corrosive Condition : NaOH soln. pH=12.5

Corrosive Cycle :

Stored in corrosive environment	48 hrs
Stored in oven at 120 F	48 hrs
Periodic cycle	96 hrs (4 days)

Number of Cycle :

Cycle	0	2	5	8	11
days*	7	15	27	39	51

* days after casting specimens

Testing : Flexural Test
Pull-out Test

Chapter 4

Experimental Results and Discussion

4.1 Preliminary Experiment

The objectives of the preliminary experiment aimed to observe :

1. whether some substantial changes are likely to occur in the region which was set up for the main experiment.
2. how flexural strength and toughness are influenced with the change of one variable at a time in the same experiment condition as the main body experiment.

The experimental results are shown in Table 4-1, where the last column, Da/Di denotes the ratio of the actual density (weight/volume) to the calculated density based on the given compositions. The specimens numbered p1 to p8 have the compositions of the eight extreme points in the variable range of the main experiment. The experimental results of specimens p11 - p23 are compared later with the estimated value calculated from the regression equation obtained by a computer run.

Table 4-1 Bending Test Results (Preliminaly Experiment)

No.	S/C	W/C	VF (%)	Flexural Strength (psi)	Toughness (Pound-inch)	Da/Di
p-1	0.00	0.4	0.5	1042	0.767	1.07
2	0.00	0.7	0.5	691	1.624	1.01
3	0.00	0.4	4.0	1858	2.740	1.06
4	0.00	0.7	4.0	1541	4.223	0.99
5	3.00	0.4	0.5	1018	0.507	0.93
6	3.00	0.7	0.5	597	0.872	0.95
7	3.00	0.4	4.0	474	1.542	0.66
8	3.00	0.7	4.0	781	2.539	0.80
11	2.00	0.4	0.0	1121	0.110	0.96
12	2.00	0.7	0.0	767	0.089	0.96
13	2.00	0.4	0.5	1253	0.689	0.97
14	2.00	0.4	1.0	1265	1.071	0.97
15	2.00	0.4	1.5	1281	1.666	0.95
16	2.00	0.4	2.0(1")	1374	2.091	0.93
17	2.00	0.4	3.0	706	1.426	0.83
18	2.00	0.4	2.0(0.5")	1353	1.448	0.92
19	2.00	0.4	2.0(1.5")	1055	1.587	0.93
20	2.00	0.7	2.0	947	2.178	0.96
21	1.00	0.4	2.0	1579	2.036	1.00
22	3.00	0.4	2.0	1308	1.392	0.85

More detailed data are given in Appendix Cl.

A drastic decrease of flexural strength and toughness was observed on the specimens which have compositions (S/C=3, W/C=0.4, VF=4%) and (S/C=3, W/C=0.7, VF=4%). As will be discussed later, the ratio Da/Di can be used for the rough evaluation of the quality of the specimens and compatibility of the material composition to a given fabrication method.

4.2 Main Body Experiment : Flexural Test Results and Regression Analysis of the Experiment Results

The first run of a computer program of Stepwise Regression Analysis is applied for the experiment results of 28 sample points including the experiment No. p1 - p8 (extreme points) and No. M1 - M20 (random composition). The experimental results of M1 - M20 are given in Table 4-2. The variables used for this analysis are listed below.

Dependent variable		Flexural Strength	(FLST)
(two separate runs)		Toughness	(TOEG)
Independent variable	1.	Sand/Cement ratio	(SBYC)
	2.	Water/Cement ratio	(WBYC)
(Linear Model 1-3)	3.	Fiber volume percent	(VOLF)
(Non-Linear 1-9)	4.	$(SBYC)^2$	(SCSQ)
Model	5.	$(WBYC)^2$	(WCSQ)
	6.	$(VOLF)^2$	(VFSQ)
	7.	ln(SBYC)	(LGSC)
	8.	ln(WBYC)	(LGWC)
	9.	ln(VOLF)	(LGVF)

Table 4-2 Flexural Test Results (Random Mix Program)

No.	S/C	W/C	VF (%)	Flexural Strength (psi)	Toughness (Pound-inch)	Da/Di
M-1	2.57	0.59	1.39	806	1.730	0.92
2	1.38	0.55	3.00	1348	3.341	0.97
3	1.88	0.69	3.37	1069	3.513	0.94
4	1.50	0.66	1.88	1174	3.803	0.95
5	0.40	0.59	1.50	1177	2.137	0.99
6	2.33	0.68	1.68	741	1.960	0.98
7	1.50	0.63	0.48	890	0.941	1.01
8	0.40	0.54	2.28	1529	3.659	0.99
9	2.40	0.56	2.61	868	2.709	0.93
10	0.59	0.65	0.91	695	1.774	0.95
11	1.77	0.40	0.78	1249	1.119	1.01
12	2.40	0.45	3.68	1379	2.699	0.82
13	0.30	0.60	1.11	861	2.052	1.05
14	2.57	0.47	0.99	1087	1.056	0.96
15	1.11	0.66	1.91	959	3.597	1.02
16	0.70	0.69	1.50	888	2.631	1.03
17	1.38	0.57	0.50	936	1.208	0.98
18	2.57	0.42	2.91	896	1.529	0.88
19	2.94	0.47	1.12	831	1.221	0.87
20	0.59	0.53	3.20	1731	4.400	0.96

More detailed data are given in Appendix C2.

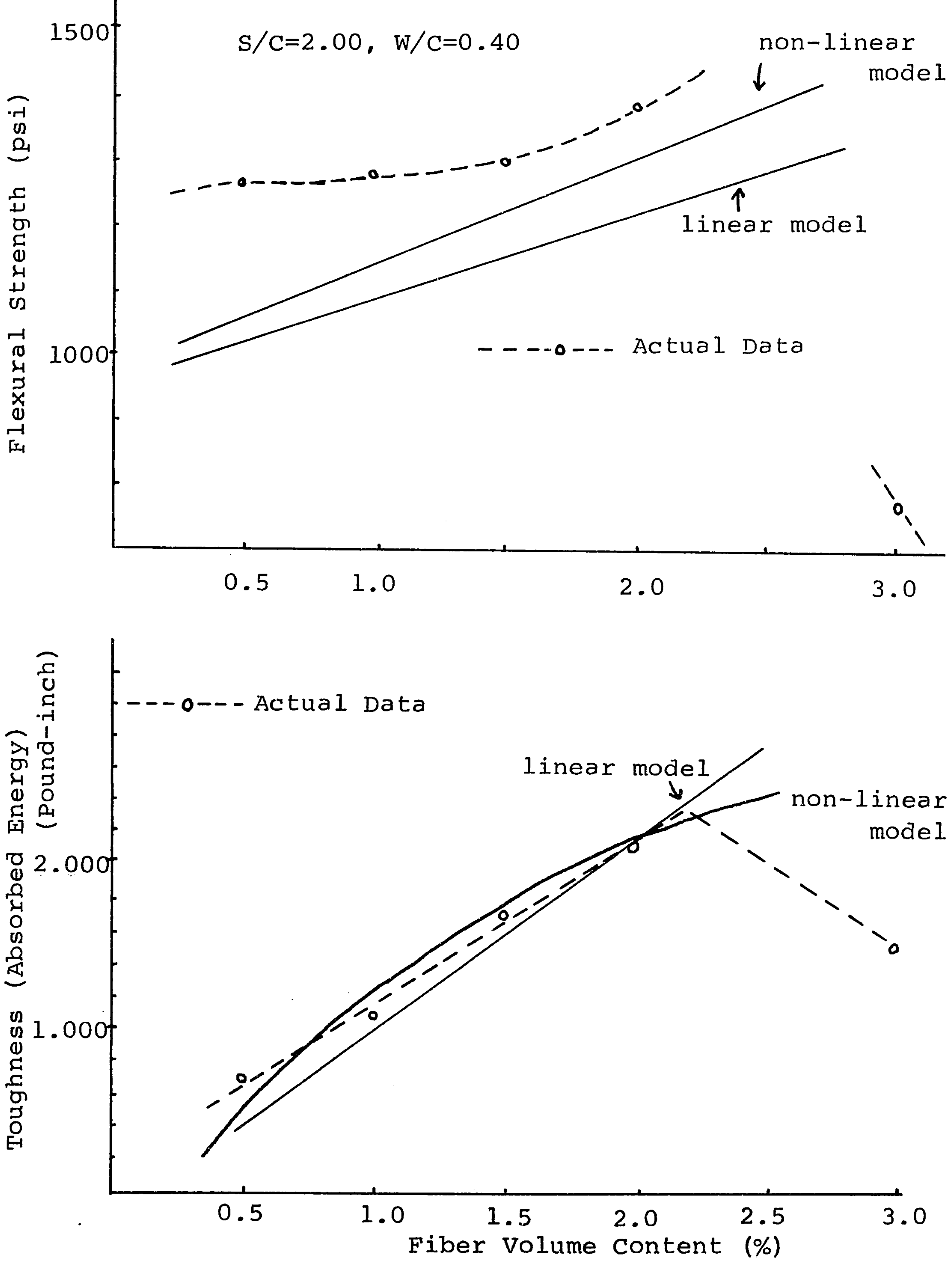
Multiple linear and non-linear models were used to analyze the influence of three parameters, S/C, W/C and VF (Volume percentage of fiber) on flexural strength and toughness. The results of computer runs both in linear and non-linear models are given in Appendix B6-B9 and are summarized in Table 4-3. The required tests of significance for these four equations are acceptable but not outstanding. Flexural strength is explained only 56 percent and 61 percent by linear equation and non-linear equation, respectively. Toughness is explained 72 percent by linear equation and 83 percent by non-linear equation. In both cases of flexural strength and toughness, the non-linear model gave a better fit than the linear model. The predicted value and the actual value are given graphically in Figure 4-1. In the figure, the actual values were picked up from the results of the preliminary experiment (Table 4-1, specimens p11 - p14 and p7). Note that these actual data were not included in those used for calculation of regression equations except specimen p7. The predicted curves both of linear and non-linear models are considerably lower than the actual in the range of 0.5 to 2 percent of fiber volume content. A large positive deviation

Table 4-3 Regression Equation for Flexural Strength and Toughness (28 sample points)

Type of Model	Dependent Variable	Independent Variable	Coefficient	Standard Error	t-ratio
Linear	Flexural Strength (FLST)	Constant	1794.3		
		SBYC	-182.0	42.6	-4.272
		WBYC	-1187.0	418.9	-2.834
		VOLF	103.0	35.7	2.885
		$R^2 = 0.56$, R = 0.75 F = 10.3 DF = 24 , t-critical(α =0.05) = 2.064 SEM = 44.9, RSD = 237.5			
	Toughness (TOEG)	Constant	-0.383		
		SBYC	-0.357	0.112	3.188
		WBYC	3.448	1.106	3.118
		VOLF	0.615	0.094	6.542
		$R^2 = 0.72$, R = 0.85 F = 20.9 DF = 24 , t-critical(α =0.05) = 2.064 SEM = 0.119, RSD = 0.627			
Non-Linear	Flexural	Constant	1447.3		
		$(SBYC)^2$	-61.5	13.0	4.731
		$(WBYC)^2$	-1180.9	360.5	-3.276
		VOLF	107.3	33.9	3.165
		$R^2 = 0.61$, R = 0.78 F = 12.3 DF = 24 , t-critical(α =0.05) = 2.064 SEM = 42.6, RSD = 225.3			
	Toughness	Constant	3.096		
		$(SBYC)^2$	-0.125	0.029	-4.310
		$\ln$(WBYC)	1.534	0.475	3.229
		$\ln$(VOLF)	1.094	0.125	8.752
		$R^2 = 0.83$, R = 0.91 F = 37.8 DF = 24 , t-critical(α =0.05) = 2.064 SEM = 0.094, RSD = 0.498			

SEM : Standard Error of Mean, RSD : Residual Standard Deviation

Fig. 4-1 Effect of Fiber Content on Flexural Strength and Toughness



was observed in the range of 3 to 4 percent of fiber content.

As was described before, the 28 specimens prepared for the statistical application included poor specimens such as p7 and p8 which have an extremely low strength. The decrease of strength of these specimens is so drastic that it is rather difficult to interpret this behavior without an investigation of the structural defects inside the specimens, which could result from an improper material composition (or improper combination of parameters) in the fabrication method used.

Limiting scope of the data limits the possibilities of finding the ranges suitable for the experiment undertaken through this study, because of the interaction between components which are difficult to predict. One common way to avoid this problem is to reduce the ranges of each variable (S/C, W/C and VF). This is a sure way to obtain a better regression equation, but might eliminate some valid points as well as invalid points.

The ratio, Da/Di, can be used as a criterion for the validity of a sample point. Da is defined as the actual density of a specimen which is calculated from the average value of the actual weight and volume of the specimens. Di denote the theoretical density based on the actual weight of each component in cement mix, which is shown in the form of :

$$Di = \frac{W_c + W_s + W_w + W_f}{\frac{W_c}{S_{Gc}} + \frac{W_s}{S_{Gs}} + W_w + \frac{W_f}{S_{Gf}}} ,$$

where W_c, W_s, W_w and W_f are the weight in the mix of cement, sand, water and fiber respectively and S_{Gc}, S_{Gs}, S_{Gw} and S_{Gf} are the specific gravity of these component materials. Actually, small amounts of air are trapped inside specimens. In most instances, the air content of mortar and concrete is of the order of 1 to 3 percent except for air-entrained cement (10). So, the actual density should be less than the theoretical. The value of Da/Di of each specimen in Table 4-1 and 4-2 are widely scattered. In some specimens, the values of Da exceed Di. This violation probably results from the experimental error of the volume of the specimens. Consequently, the discussion on Da/Di should be limited. However, it can be assumed that the specimen which has a low value of Da/Di has a loose packing. Looking at

Table 4-1 and 4-2, there are five specimens which have a value of Da/Di less than 0.9. And there was a significant gap of the value of Da/Di between these five and the remainder. According to this observation, the data of these five specimen were eliminated and the second run of a computer program of Stepwise Regression Analysis was applied to the remainder.

The results of the computer run are given in Appendix B10-B13 and are summarized in Table 4-4 . The cut off of the five low quality specimens gives a remarkable improvement of regression equation in terms of multiple correlation (R^2) and F-ratio, both of which are the measure of the overall significance of a regression equation. Although the non-linear model gives somewhat better results than the linear model in the case of flexural strength (see F-ratio), the difference between them is not significant. However, the non-linear model gives considerably better fit than the linear model for the experiment of toughness. The actual value, the estimated value based on the non-linear model, and the residual are given in Table 4-5.

Once a regression equation is obtained, the value of flexural strength and toughness for a given

Table 4-4 Regression Equation for Flexural Strength and Toughness (23 sample points)

Type of Model	Dependent Variable	Independent Variable	Coefficient	Standard Error	t-ratio
Linear	Flexural Strength (FLST)	Constant	1913.1		
		SBYC	-97.1	31.0	3.132
		WBYC	-1761.7	288.9	6.117
		VOLF	186.1	27.2	6.842
		R^2 = 0.85, R = 0.92 F = 34.7 DF = 19 , t-critical(α =0.05) = 2.093 SEM = 29.6, RSD = 142.1			
	Toughness (TOEG)	Constant	-1.0013		
		WBYC	3.1507	1.1910	2.645
		VOLF	0.8531	0.1079	7.906
		R^2 = 0.77, R = 0.88 F = 32.7 DF = 20 , t-critical(α =0.05) = SEM = 0.122, RSD = 0.586			
Non-Linear	Flexural	Constant	1533.1		
		$(\mathrm{SBYC})^2$	-32.8	10.4	3.154
		$(\mathrm{WBYC})^2$	-1547.4	254.7	6.075
		$(\mathrm{VOLF})^2$	42.8	6.2	6.903
		R^2 = 0.85, R = 0.92 F = 36.4 DF = 19 , t-critical(α =0.05) = 2.093 SEM = 29.0, RSD = 139.2			
	Toughness	Constant	0.7978		
		$(\mathrm{SBYC})^2$	-0.0796	0.0358	2.223
		$\ell n(\mathrm{WBYC})^2$	2.2855	0.9849	2.230
		$\ell n(\mathrm{VOLF})^2$	1.2821	0.1439	8.910
		R^2 = 0.87, R = 0.93 F = 47.6 DF = 19 , t-critical(α =0.05) = 2.093 SEM = 0.100, RSD = 0.480			

SEM : Standard Error of Mean, RSD : Residual Standard Deviation

Table 4-5 Predicted Value and Residuals

SP.NO	FLST (psi)	ESTST (psi)	ERROR (psi)	ERROR (%)	TOEG (lb-in)	ESTEG (lb-in)	ERROR (lb-in)	ERROR (%)
M- 1	806.	851.	-45.	-5.6	1.730	2.044	0.314	18.1
2	1348.	1379.	-31.	-2.3	3.341	3.312	-0.028	-0.8
3	1069.	1163.	-94.	-8.8	3.513	3.651	0.138	3.9
4	1174.	924.	249.	21.2	3.803	2.938	-0.864	-22.7
5	1177.	1072.	104.	8.8	2.137	2.659	0.522	24.4
6	741.	742.	-1.	-0.1	1.960	2.584	0.624	31.8
7	890.	848.	41.	4.6	0.941	1.140	0.199	21.2
8	1529.	1297.	231.	15.1	3.659	3.078	-0.580	-15.8
9	868.	1137.	-269.	-31.0	2.709	2.849	0.140	5.1
10	695.	884.	-189.	-27.2	1.774	2.139	0.365	20.6
11	1249.	1200.	48.	3.8	1.119	1.150	0.031	2.7
13	861.	1027.	-166.	-19.3	2.052	2.301	0.249	12.1
14	1087.	1008.	78.	7.1	1.056	1.340	0.284	26.9
15	959.	970.	-11.	-1.2	3.597	3.037	-0.559	-15.5
16	888.	870.	17.	1.9	2.631	2.854	0.223	8.4
17	936.	976.	-40.	-4.3	1.208	1.061	-0.146	-12.1
20	1731.	1526.	204.	11.8	4.400	3.475	-0.924	-21.0
P- 1	1216.	1298.	-82.	-6.7	0.767	0.863	0.096	12.5
2	1514.	1460.	53.	3.5	4.223	4.173	-0.049	-1.1
3	1858.	1965.	-107.	-5.7	2.740	3.482	0.742	27.1
4	691.	787.	-96.	-13.9	1.624	1.532	-0.091	-5.6
5	1018.	999.	18.	1.7	0.507	-0.091	-0.598	-118.0
6	597.	506.	90.	15.2	0.872	0.789	-0.082	-9.5

combination of variables within the ranges of each variable can be estimated. According to Table 4-4, the four regression equations are written in the form of :

Linear Model

$$FLST = 1913.1 - 97.1(S/C) - 1761.7(W/C) + 186.1(VF) \quad 4\text{-}1)$$

$$TOEG = -1.0013 + 3.1507(W/C) + 0.8531(VF) \quad 4\text{-}2)$$

Non-linear model

$$FLST = 1533.1 - 32.8(S/C)^2 - 1547.4(W/C)^2 + 42.8(VF)^2 \quad 4\text{-}3)$$

$$TOEG = 0.7978 - 0.0796(S/C)^2 + 2.2855 \ln(W/C) + 1.2821 \ln(VF) \quad 4\text{-}4)$$

The estimated values are calculated for the same combinations of variables as the specimens (p13 - p23) which were tested to observe the effect of one variable change and compared with the actual experiment results. Note: the specimens (p13 - p23) were not used for the derivation of the regression equations. The estimated line (or curve) and the actual line (or curve) of flexural strength and toughness for the given compositions are shown in Fig. 4-2 to Fig. 4-7. In general, the estimated curves align closely with the actual. However, considerable large

Fig. 4-2 Effect of Fiber Content on Flexural Strength

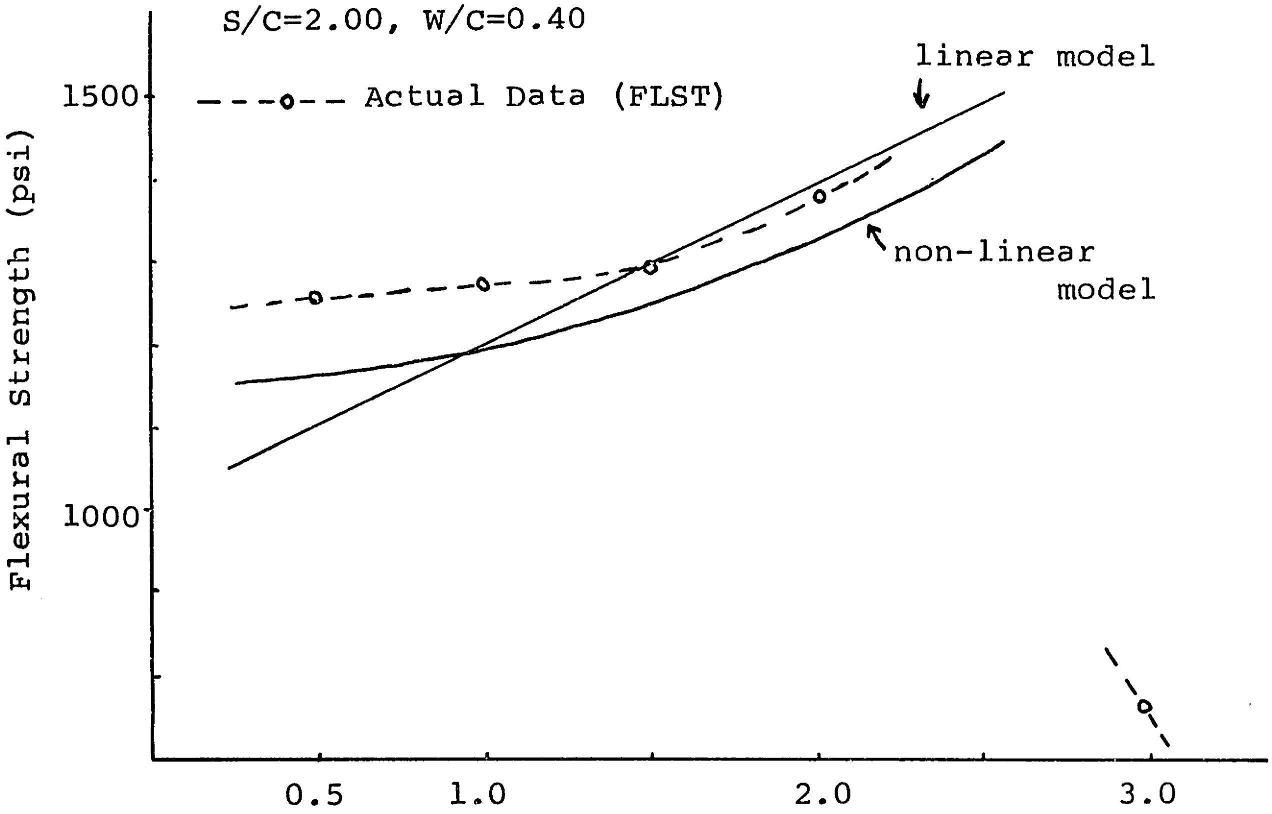


Fig. 4-3 Effect of Fiber Content on Toughness

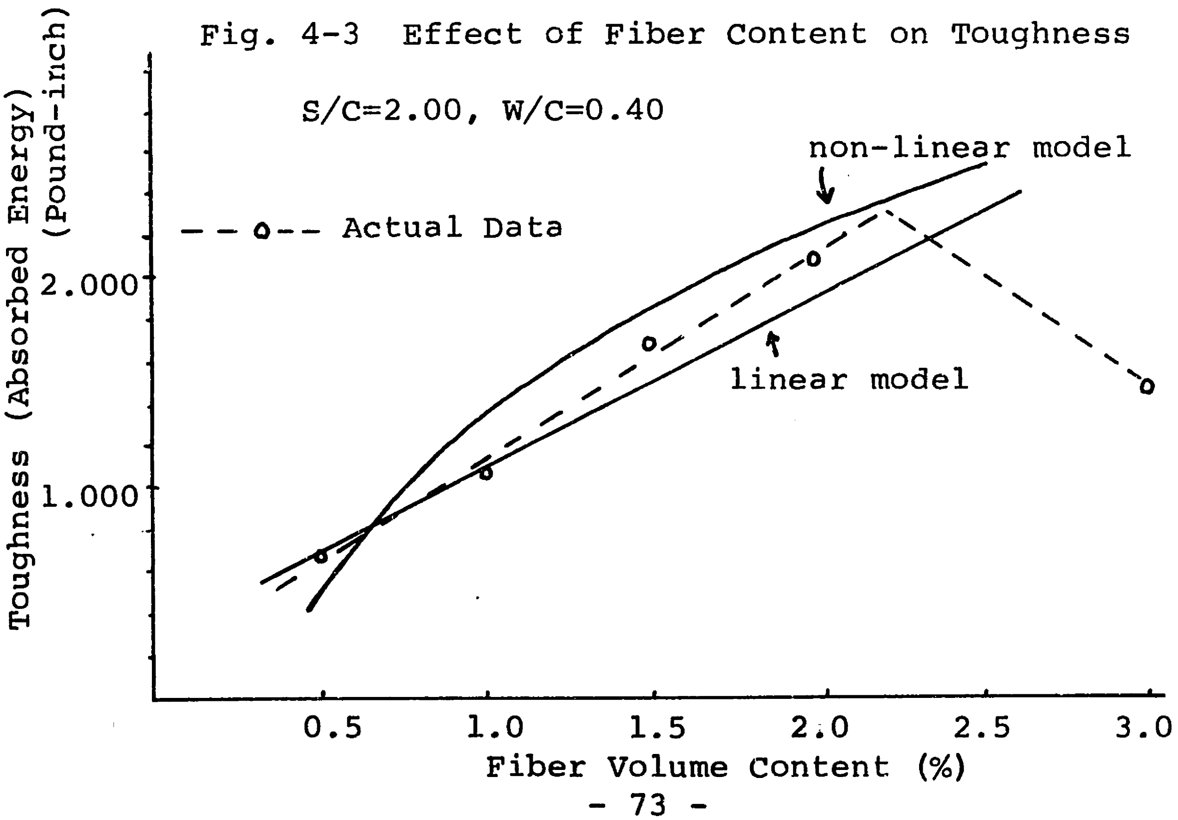
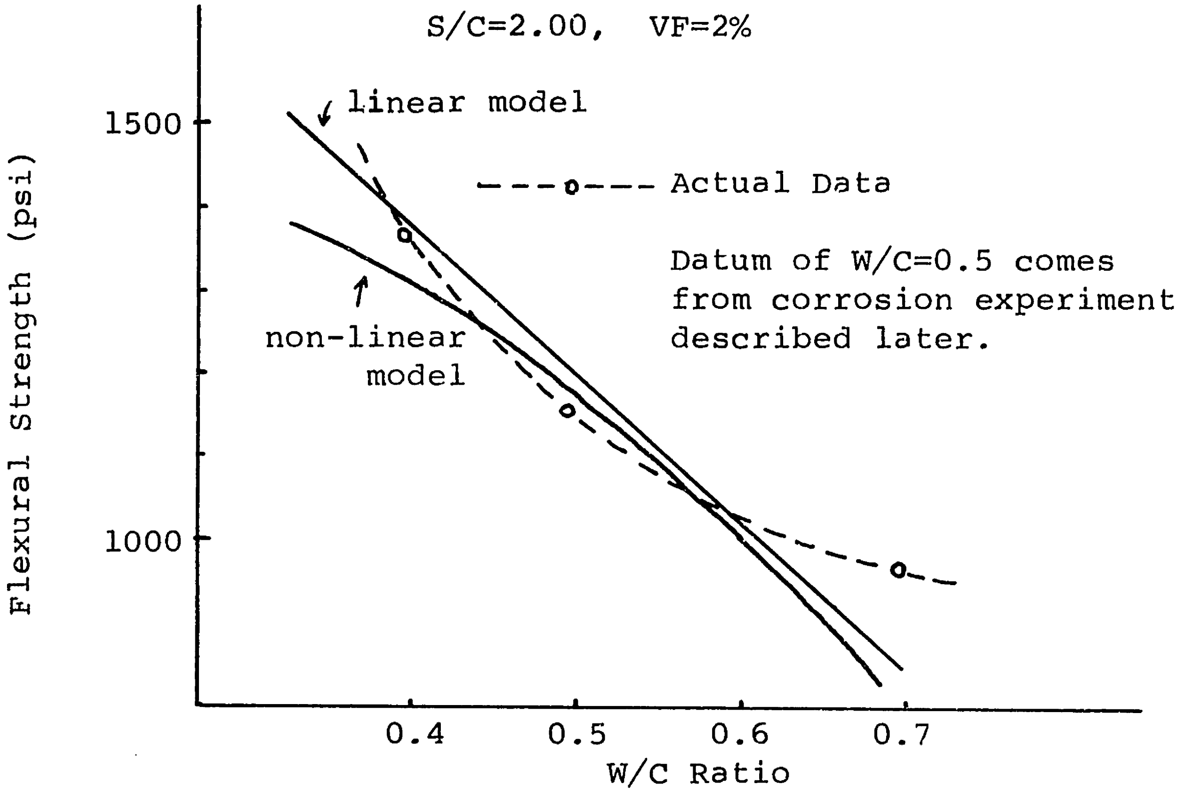


Fig. 4-4 Effect of W/C on Flexural Strength



Datum of W/C=0.5 comes from corrosion experiment described later.

Fig. 4-5 Effect of W/C on Toughness

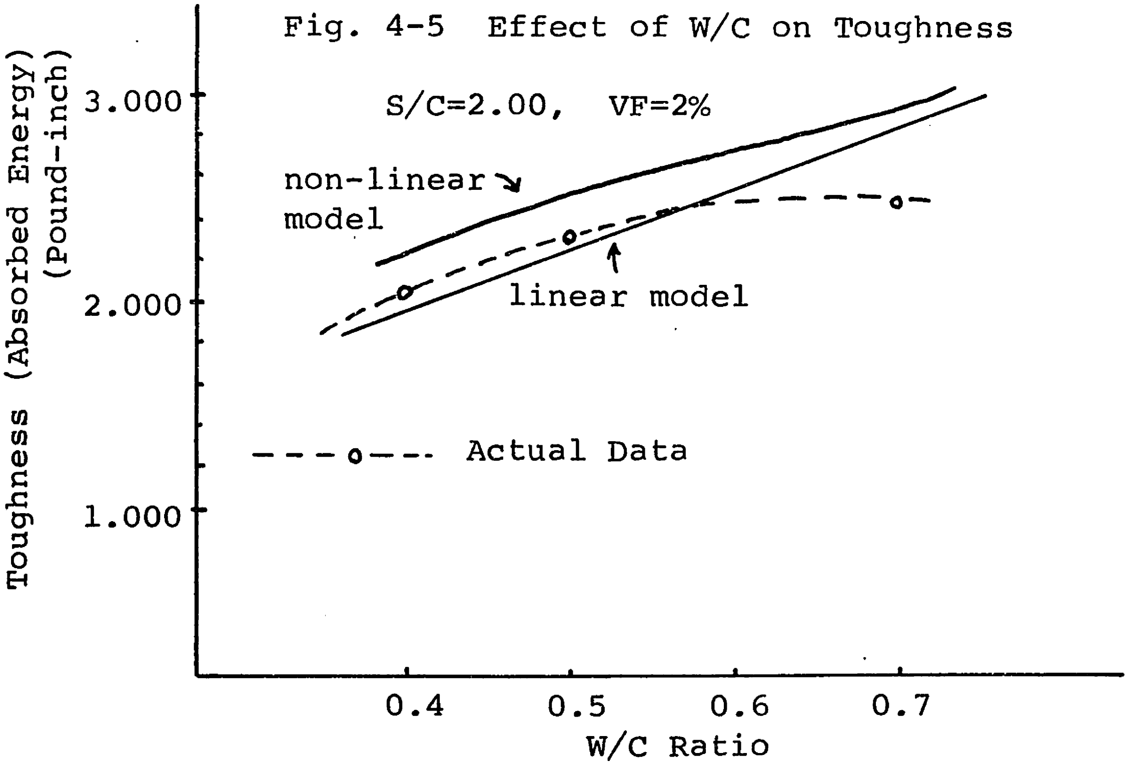


Figure 4-6 Effect of S/C on Flexural Strength

W/C=0.4, VF=2.00

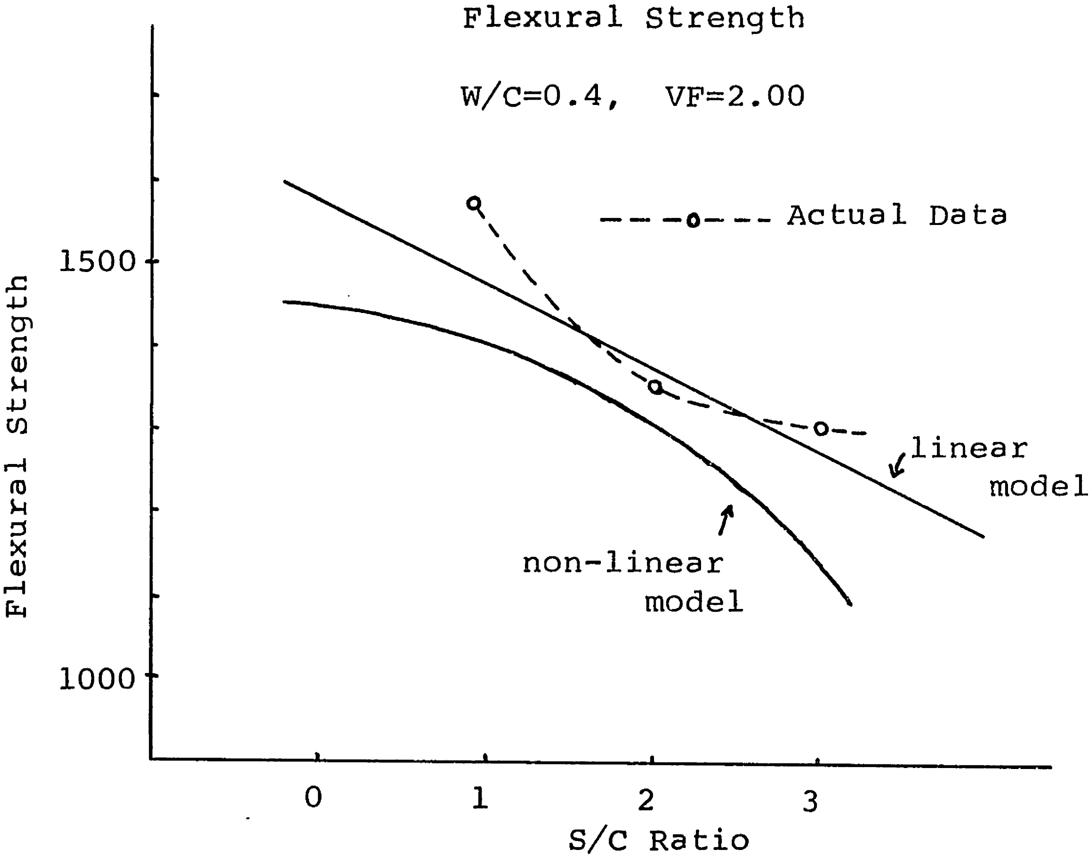
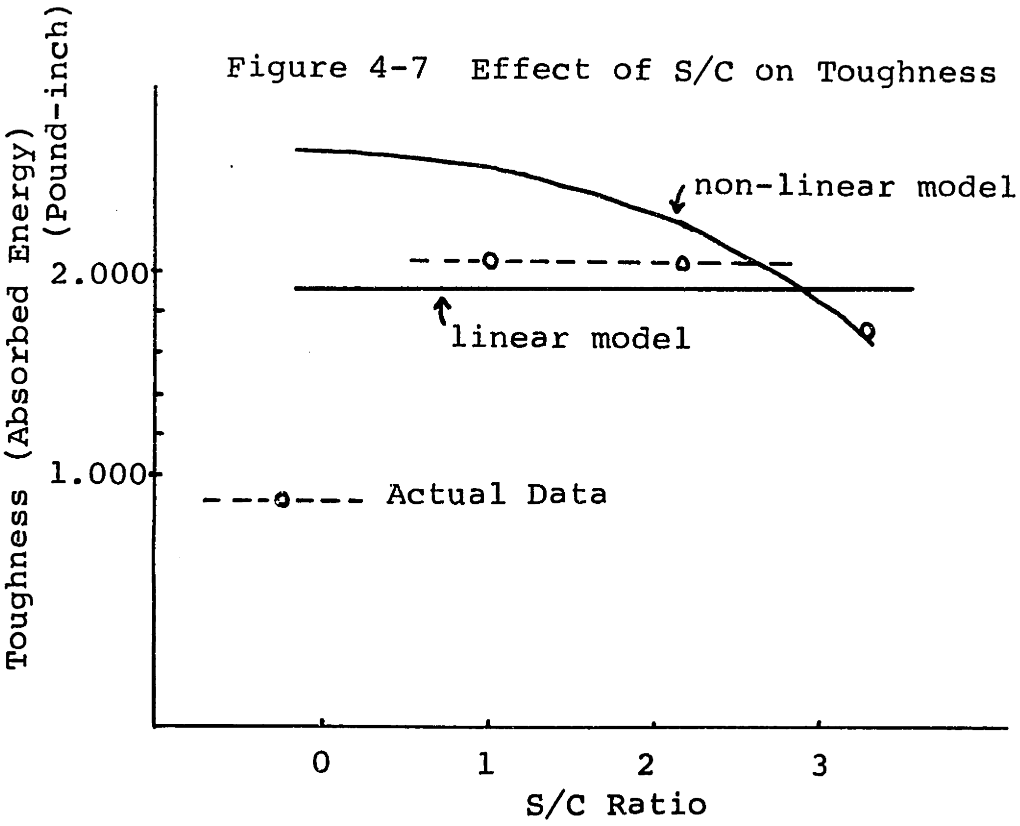


Figure 4-7 Effect of S/C on Toughness



deviations are observed around the upper and lower boundaries of each variable. No significant difference is observed between linear and non-linear models.

Further improvement of the overall significance of the regression equation may be achieved by using a higher cut-off point of Da/Di. However, considering the inherent variation of the experimental data and the limit of the statistical application, further excution of a computer is not likely to give meaningful results unless more data are added to those used.

A discussion on the effect of the three primary variables on the flexural strength and toughness of the cement composites will be developed later.

4.3 The Effect of Compactness and Uniformity of the Specimen

When the discussion is extended to the accuracy of the experiment and therefore the meaning of the statistical analysis for this study, compactness of specimen should be taken into account. There exist two different types of problems for the compactness of specimen . One problem is concerned with the loose-packed structure resulting from a too harsh composition for the fabrication method used. In this study, increased volume content of fiber and sand usually results in poorer workability which cause considerably looser-packed specimens than expected. The ratio, Da/Di, can be used as a criterion for detecting these types of specimen. The other problem belongs to the realm of stochastic error. It is the error inherent in the irreproducibility of the experiment, especially in sample preparation - e.g., the same composition of mix will not give the same compactness of specimen on two separate occasions.

Considering the first problem of compactness, regression analysis is applied for the 28 experimental data (p1 - p8, M1 - M20) to find a correlation between Da/Di and the three parameters (S/C, W/C and VF). The result is given in Appendix B14 and in Table 4-6.

Table 4-6 Regression Equation of Da/Di

Dependent Variable RTSD : (Da/Di)			
Independent Variable	Coefficient	Standard Error	t-ratio
Constant	1.0486		
SCSQ: $(S/C)^2$	-0.0166	0.0024	-6.917
VFSQ: $(VF)^2$	-0.0071	0.0014	5.071
$R^2 = 0.87$, R = 0.76 F = 39.9			
Degree of Freedom = 25, t-critical(α =0.05) = 2			
Standard Error of Mean = 0.0080			
Residual Standard Deviation = 0.0427			

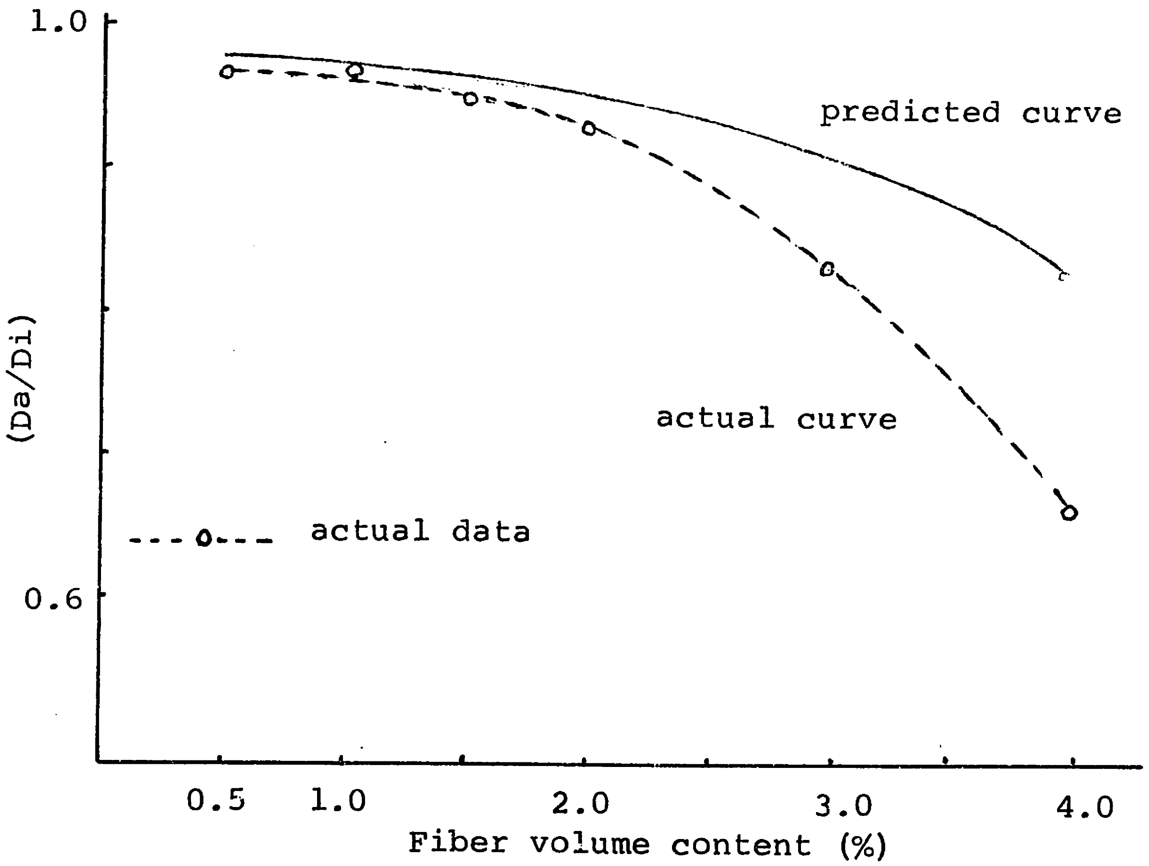
Then the regression equation is written in the form of :

$$(Da/Di) = 1.0486 - 0.0116(S/C)^2 - 0.0071(VF)^2 \qquad 4\text{-}5)$$

The equation obtained explains 76 percent of the actual value of Da/Di in terms of $(S/C)^2$ and $(VF)^2$. The actual and the predicted values of Da/Di are shown graphically in Figure 4-8.

Figure 4-8 Predicted Value and Actual Data of Da/Di

~~Flexural Strength~~ (S/C=2.00, W/C=0.4)



The actual value shows a more drastic drop of the curve than the predicted value. As was mentioned in the previous section, the value of Da/Di itself is not accurate in this study. However, the similarity of the shape of the curve between the actual and the predicted implies that the regression equation could be used as a good criterion for detecting an unsuitable composition to a fabrication method, and therefore unacceptable specimen to the analysis. Note that the water/cement ratio does not appear in the equation, but that the amount of water is largely dependent on the sand/cement ratio.

There is no sure way to predict the slight variation of compactness resulting from the inherent irreproducibility of the experiment. However, the stochastic error is probably reduced in large scale production facilities with modern equipment and tight quality control.

As a whole, the result of this study described above suggests that regression analysis can be effectively used for the prediction of the actual behaviors of fiber-reinforced cement composites, within the limits of the variable boundaries.

4.4 The Result of the Corrosion Experiment

The corrosion of fiber-reinforced cement composite generally is thought to be an over-all effect resulting from a separate deterioration of mortar, fiber, or bond between mortar and fiber.

The object of the experiments in this study is to investigate the over-all deterioration of alkaline-resistant glass fiber-reinforced mortar in a harsh corrosive environment created over a short period of time. (NaOH soln. pH 12.5) Flexural tests were performed both with glass fiber-reinforced mortar (S/C=2, W/C=0.5, VF=2%) and with plain mortar (S/C=2, W/C=0.5, VF=0%), after a predetermined number of corrosive cycles were reached.

Ultimate flexural strength and toughness (the area under the curve of Load vs. Deflexion graph) were calculated. The results of flexural tests are given in Appendix C-3 and also are given graphically in Figure 4-9 and 4-10. Both plain mortar and GRC specimens did not show any significant change in flexural strength within the time units of the experiment. However, GRC specimens show a dramatic decrease in toughness when exposed to the corrosive medium. As is suggested in Chapter 4, flexural

Figure 4-9 Flexural Strength of GRC with Corrosive Cycles

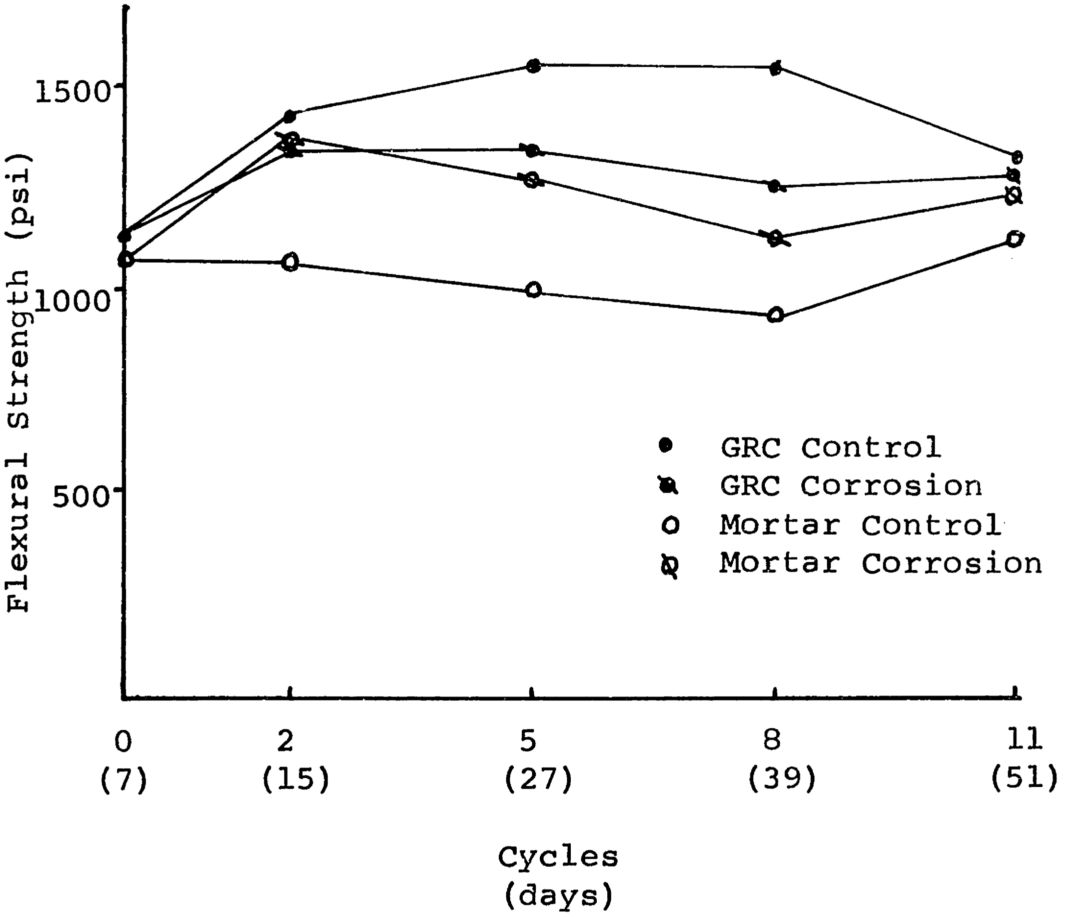
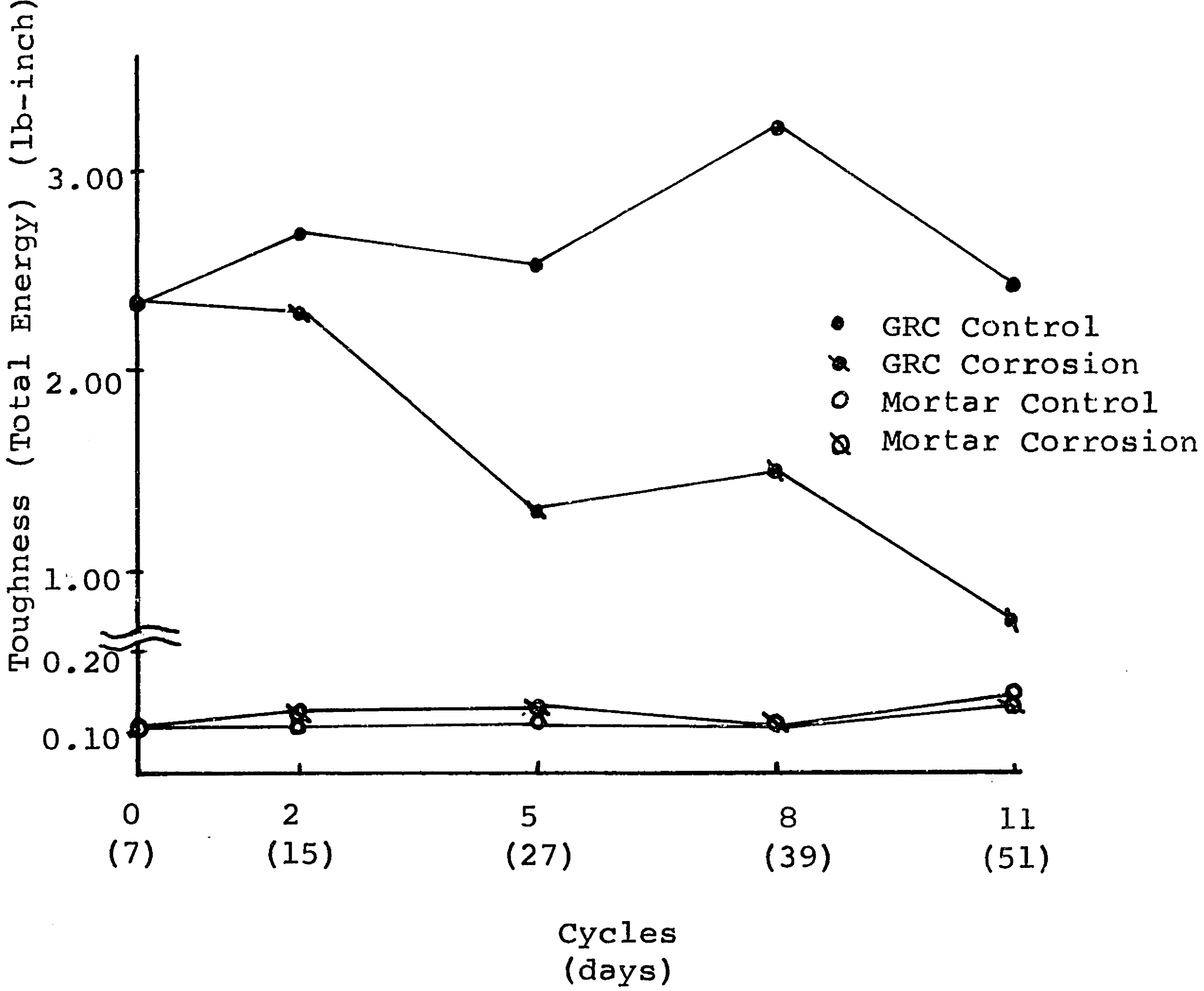


Figure 4-10 Toughness of GRC with Corrosive Cycles



strength is less dependent on the fiber addition than toughness. Consequently, flexural strength would not be greatly influenced by a deterioration of fiber and bond. However, since toughness is much more sensitive to fiber content any decrease in fiber or bond strength would have a significant effect on the toughness.

The purpose of the pull-out test is to investigate the deterioration of bond between the reinforcing fiber and the matrix. However, the results of these tests were inconclusive. The strands exhibited a general tendency of breaking instead of pulling out. Some broke clearly, with little or no pull-out, others pulled out partially and then broke, and still others broke partially and pulled out. Only a very few strands actually pulled out intact. In this experiment, corrosion seemed to have no effect on this tendency of fibers to break, and did not seem to weaken the matrix-fiber bond significantly. (Ref.AppendixC4)

4.5 Discussion of the Experimental Results

Although slight differences in the magnitude of the effect of each variable are observed between linear and non-linear models, both behave almost in the same way.

This made it difficult to assess which model is more relevant.

In investigating the strengthening mechanism of glass fiber-reinforced mortar, a non-linear regression equation could accurately predict the variation of the strength with variable change throughout the entire range. However the linear model gives a better fit to the actual except in the range close to the boundary. (see Figure 4-2 - Figure 4-7) Linear regression is also easy to understand and to compare with the theoretical prediction.

The magnitude of the effect of the three primary variables on flexural strength and toughness based on the linear model are summarized in the following table.

Variable	Change in Variables	Change in Flexural Strength	Change in Toughness (lb-in)
SBYC	+1	decrease 32 - 162 psi	noneffective
WBYC	+0.1	decrease 116 - 236 psi	increase 0.315
VOLF	+1%	increase 129 - 243	increase 0.8531

The increase of flexural strength and toughness with the increase of 1 volume percent of fiber content (e.g. 1% to 2% or 2% to 3%) corresponds to about 20 and 1000 percent of that of plain mortar, respectively (cf Table 4-1). For flexural strength, this magnitude is almost the same as the one observed in Case 2, Chapter 2. The change of flexural strength with one discrete unit change of water/cement ratio and volume content of fiber are almost same. Similar results were also pointed out in Case 2. The effect of sand/cement ratio is relatively small on flexural strength and is negligible on toughness. (According to the non-linear model, a small negative effect is observed.)

The table also shows that increased water/cement ratio gives a decreased flexural strength but increased in toughness. The flexural strength behavior is identical to the one observed in plain mortar and concrete, while the toughness behavior is not. The negative effect of sand/cement ratio on toughness observed in the non-linear model is also opposite to the plain mortar behavior. These two contradictions could be explained with a consideration of other factors such as dispersion of fiber and fiber-matrix bond associated with S/C and W/C.

Da/Di is an interesting index which can be used as a criterion of an improper composition in the fabrication method used. The prediction based on the non-linear equation of Da/Di can not sufficiently explain the severe drop of the actual data which occur at Da/Di value less than 0.9. But it is possible to get an approximate region where the experiment is performed soundly, by the use of an appropriate cut-off point with consideration of the gap between the predicted and the actual curve.

Chapter 5

Evaluation of Material Cost and Choice of Composition

5.1 Derivation of Cost Function

Assume that a product of cement composite is designed for a specic use. Usually a product is offered in the same depth, width and length through a production lot. The total volume of the products, V, is given in the form of

$$V = v_c + v_s + v_w + v_f + v_a \quad , \qquad 5\text{-}1)$$

where v denotes the absolute volume of each component in the product and the subscripts, c, s, w, f and a refer to cement, water, fiber and air respectively. Although the air content changes with the change of composition, it contributes only a few percent of the total volume, except in extremely harsh compositions or in air-entraint cement composites.(10) As the objective of this study is to find the most economical composition which is significantly different from the others, the air content can be ignored. This assumption will reduce considerably the effort of formulation of the cost function of fiber-reinforced cement composites, and still be likely to give a good approximation of the material cost. The volume of each component material is given by the weight of the material corresponding to a given volume divided by the specific gravity of the material, S_G,. Therefore Eq. 5-1) is given by,

$$V = v_c + v_s + v_w + v_f \qquad 5\text{-}2)$$

$$= \frac{W_c}{S_{Gc}} + \frac{W_s}{S_{Gs}} + \frac{W_w}{S_{Gw}} + \frac{W_f}{S_{Gf}} \quad ,$$

where subscripts, c, s, w and f refer cement, sand, water and fiber respectively.

Now, if the material cost, u, of an ingredient of the composite is given in terms of dollars per gram and the weight of each ingredient for a product which has a certain volume, v, is given in terms of grams, the total material cost of the product, Z, is represented by

$$Z = U_cW_c + U_sW_s + U_wW_w + U_fW_f \quad . \qquad 5\text{-}3)$$

Define X_1, X_2 and X_3 as three primary variables, sand/cement ratio, water/cement ratio and volume fraction of fiber respectively, which were used in the previous chapter. Note X_1, X_2 and X_3 are given in the form of

$$X_1 = \frac{W_s}{W_c}, \quad X_2 = \frac{W_w}{W_c} \quad \text{and} \quad X_3 = \frac{v_f}{V} = \frac{W_f}{S_{Gf} \cdot V}$$

respectively.

Then, the total volume and the total cost of cement composite are described in terms of the three variables by inserting X_1, X_2 and X_3 into Eq. 5-2) and 5-3).

$$V = W_c \left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}} X_1 + \frac{1}{S_{Gw}} X_2 \right) + VX_3 \qquad 5\text{-}4)$$

$$Z = W_c \left(U_c + U_s X_1 + U_w X_2 \right) + U_f \cdot S_{Gf} \cdot VX_3 \qquad 5\text{-}5)$$

From Equation 5-4), W_c is given by

$$W_c = \frac{\left(1 - X_3\right) V}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}} X_1 + \frac{1}{S_{Gw}} X_2 \right)} \qquad 5\text{-}6)$$

Then the total cost, Z, is described in the form of :

$$Z = V \left[\frac{\left(1 - X_3\right)\left(U_c + U_s X_1 + U_w X_2\right)}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}} X_1 + \frac{1}{S_{Gw}} X_2 \right)} + U_f \cdot S_{Gf} \cdot X_3 \right] \qquad 5\text{-}7)$$

Assuming $S_{Gw}=1$, and $U_w=0$, the Equation 5-7), is given by

$$Z = V\left[\frac{\left(1 - X_3\right)\left(U_c + U_s X_1\right)}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}}X_1 + X_2\right)} + U_f \cdot S_{Gf} \cdot X_3\right] \qquad 5\text{-}8)$$

Even if the total volume of a product is not given, it is still possible to estimate the cost per unit volume for a given combination of the three variables, by using Z', $= \frac{Z}{V}$, instead of Z in Equation 5-8).

5.2 Formulation of Non-Linear Programming Model for the Optimal Composition of FRC with a Given Fiber

Suppose we are now concerned with the selection of a material for a specific use. An optimization problem comes from the requirement for finding a composition which has the minimum cost under a given performance requirement.

The material cost per unit volume is given by :

$$Z' = \frac{Z}{V} = \frac{\left(1 - X_3\right)\left(U_c + U_s X_1\right)}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}}X_1 + X_2\right)} + U_f \cdot S_{Gf} \cdot X_3$$

The model is to minimize Z', which is subject to the several constraints developed below.

a) Constraint on Properties Requirement

Suppose flexural strength and toughness are chosen as criteria for the material selection, and the lower limit of these properties is determined quantitatively based on the performance requirement for the particular use.

According to Equation 4-1) - 4-4), the property requirements are given by

1) linear equations

$$Y_1 = a_1 + b_1X_1 + c_1X_2 + d_1X_3 \rangle L_1$$

$$Y_2 = a_2 + c_2X_2 + d_2X_3 \rangle L_2$$

or 2) non-linear equations

$$Y_1 = a_1' + b_1'X_1^2 + c_1'X_2^2 + d_1'X_3^2 \rangle L_1$$

$$Y_2 = a_2' + b_2'X_1^2 + c_2'\ln X_2 + d_2'\ln X_3 \rangle L_2$$

where, Y_1 and Y_2 denote flexural strength and toughness respectively and X_1, X_2 and X_3, sand/cement ratio, water/cement ratio and volume fraction of fiber , and a_1, a_1', a_2'..... denote positive or negative coefficients obtained from the regression equation. L_1 and L_2 are defined as the lower limits of flexural strength and toughness determined from the usage requirement. In this study, only regression analysis is used for presenting the relationship between physical properties and the three important parameters. If a theoretical equation can be used for the explanation of the relationship, it is always a better way than the experimental description (including regression equation), because theoretical equation can directly clarify the causal effects between parameters and physical properties.

b) Constraints on the Range of Variables

Not only a regression equation but also a theoretical equation usually have a restricted region where the application of these equations can be used.

These constraints are usually expressed by

$$L_i \leq X_i \leq U_i \qquad \text{(for } i=1,2.\ldots.n\text{)}$$

where X_i denotes the i th variable in n variables which are used as independent variables, and U_i and L_i denote the upper and the lower limit of the i th variable respectively. These constraints merely give a valid range of variables in the equation of physical properties and the parameters given above. So, if it is difficult to figure the range of variables, other appropriate constraints can be used as substitutes of valid range constraints.

In this study, Da/Di is one of the substitutes for the constrains on the range of variables. Assume Y_3 as the ratio of the actual density of a product to the theoretical density of the product, Da/Di.

non-linear model

$$Y_3 = a_3 + b_3X_1^2 + c_3X_3^2 > L_3$$

5.3 Case Study : Estimation of Properties and Cost of GRC

Instead of solving the non-linear programming, the estimated value of flexural strength, toughness and Da/Di are calculated for various combinations of the variables. The material cost for each combination is also calculated. Actually this result is more meaningful than that of non-linear programming, because at the same time it gives several alternatives as well as the optimal composition.

Unit price and specific gravity of ingredients of the composite used in this study are given in Table 5-1. The equations used are given below (Ref. Chapter 4),

Flexural Strength

$$\mathrm{FLST} = 1533.1 - 32.8(S/C)^2 - 1547.4(W/C)^2 + 42.8(VF)^2$$

Toughness

$$TOEG = 0.7978 - 0.0796(S/C)^2 + 2.2855 \cdot \ln(W/C) + 1.2821 \cdot \ln(VF)$$

Da/Di

$$(Da/Di) = 1.0486 - 0.0116(S/C)^2 - 0.0071(VF)^2$$

Assuming that the volume of a glass fiber-reinforced cement sheet (V), is 27000cm^3 (8' x 4' x 3/8").

Then, total material cost ($/Sheet) is given by :

$$Z = 27000\left[\frac{(1-X_3)(3.25 \times 10^{-5} + 2.50 \times 10^{-6}X_1)}{(0.32 + 0.38X_1 + X_2)} + 4.84 \times 10^{-3}X_3\right]$$

(Note $X_3 = VF/100$)

Table 5-1 Unit Price and Specific Gravity of Materials Used

	Unit Price		Specific Gravity (S_G) (g/cm^3)	$1/S_G$
	$/Kg	$/g		
Portland cement Type I	3.53×10^{-2}	3.52×10^{-5}	3.15	0.32
Sand	2.50×10^{-3}	2.50×10^{-6}	2.66	0.38
Water	0		1.00	1.00
Glass fiber* (alkali-resistant)	1.8	1.8×10^{-3}	2.69	0.37

Source : ENR December 20, 1973 p.57

* Glass fiber : Estimated price

The computed results for given combinations of variables are given in Table 5-2.

Suppose the minimum requirements for flexural strength, toughness and Da/Di are given as follows :

$FLST \geq 1700$

$TOEG \geq 3.000$

$Da/Di \geq 0.90$

Six possibilities are easily found in Table 5-2, which are summarized in the following table.

Alternatives	S/C	W/C	VF	FLST	TOEG	Da/Di	Cost ($)
1	0.00	0.40	3.50	1809	3.318	0.96	5.82
2	0.00	0.40	4.00	1970	3.489	0.93	6.47
3	0.00	0.50	4.00	1830	3.719	0.93	6.31
4	1.00	0.40	3.50	1777	3.238	0.94	5.45
5	1.00	0.40	4.00	1937	3.409	0.91	6.10
6	1.00	0.50	4.00	1797	3.639	0.91	6.02

The most economical combination of variables in the six alternatives with the given constraints is alternative 4.

Actually this is only a simple evaluation method for a product. There are many methods for evaluating the economical composition, based on the four primary equations used above in Table 5-2.

Table 5-2 Prediction of Properties and Cost of Fiber-Reinforced Cement Composites

S/C	W/C	VF (%)	FLST (psi)	TOEG (ln-in)	Material Cost (8'x 4'x3/8")	Da/Di
0.00	0.40	0.50	1296.	0.823	$ 1.96	1.04
0.00	0.40	1.00	1328.	1.711	$ 2.60	1.04
0.00	0.40	1.50	1381.	2.231	$ 3.25	1.03
0.00	0.40	2.00	1456.	2.600	$ 3.89	1.02
0.00	0.40	2.50	1553.	2.886	$ 4.54	1.00
0.00	0.40	3.00	1670.	3.120	$ 5.18	0.98
0.00	0.40	3.50	1809.	3.318	$ 5.82	0.96
0.00	0.40	4.00	1970.	3.489	$ 6.47	0.93
0.00	0.50	0.50	1156.	1.053	$ 1.80	1.04
0.00	0.50	1.00	1188.	1.941	$ 2.44	1.04
0.00	0.50	1.50	1241.	2.461	$ 3.09	1.03
0.00	0.50	2.00	1316.	2.830	$ 3.73	1.02
0.00	0.50	2.50	1412.	3.116	$ 4.38	1.00
0.00	0.50	3.00	1530.	3.350	$ 5.02	0.98
0.00	0.50	3.50	1669.	3.547	$ 5.67	0.96
0.00	0.50	4.00	1830.	3.719	$ 6.31	0.93
0.00	0.60	0.50	984.	1.282	$ 1.67	1.04
0.00	0.60	1.00	1016.	2.171	$ 2.32	1.04
0.00	0.60	1.50	1070.	2.691	$ 2.96	1.03
0.00	0.60	2.00	1145.	3.060	$ 3.61	1.02
0.00	0.60	2.50	1241.	3.346	$ 4.25	1.00
0.00	0.60	3.00	1359.	3.579	$ 4.90	0.98
0.00	0.60	3.50	1498.	3.777	$ 5.55	0.96
0.00	0.60	4.00	1658.	3.948	$ 6.19	0.93
0.00	0.70	0.50	782.	1.512	$ 1.57	1.04
0.00	0.70	1.00	814.	2.401	$ 2.22	1.04
0.00	0.70	1.50	867.	2.920	$ 2.86	1.03
0.00	0.70	2.00	942.	3.289	$ 3.51	1.02
0.00	0.70	2.50	1039.	3.575	$ 4.16	1.00
0.00	0.70	3.00	1156.	3.809	$ 4.80	0.98
0.00	0.70	3.50	1295.	4.007	$ 5.45	0.96
0.00	0.70	4.00	1456.	4.178	$ 6.09	0.93
1.00	0.40	0.50	1263.	0.743	$ 1.57	1.03
1.00	0.40	1.00	1295.	1.632	$ 2.22	1.02
1.00	0.40	1.50	1349.	2.152	$ 2.87	1.01
1.00	0.40	2.00	1423.	2.521	$ 3.51	1.00
1.00	0.40	2.50	1520.	2.807	$ 4.16	0.98
1.00	0.40	3.00	1637.	3.040	$ 4.81	0.96
1.00	0.40	3.50	1777.	3.238	$ 5.45	0.94
1.00	0.40	4.00	1937.	3.409	$ 6.10	0.91
1.00	0.50	0.50	1123.	0.973	$ 1.50	1.03
1.00	0.50	1.00	1155.	1.862	$ 2.14	1.02
1.00	0.50	1.50	1208.	2.381	$ 2.79	1.01
1.00	0.50	2.00	1283.	2.750	$ 3.44	1.00
1.00	0.50	2.50	1380.	3.036	$ 4.08	0.98
1.00	0.50	3.00	1497.	3.270	$ 4.73	0.96
1.00	0.50	3.50	1636.	3.468	$ 5.37	0.94
1.00	0.50	4.00	1797.	3.639	$ 6.02	0.91

Table 5-2 continued

S/C	W/C	VF (%)	FLST (psi)	TOEG (ln-in)	Material Cost (8'x 4'x 3/8")	Da/Di
1.00	0.60	0.50	952.	1.203	$ 1.43	1.03
1.00	0.60	1.00	984.	2.091	$ 2.08	1.02
1.00	0.60	1.50	1037.	2.611	$ 2.72	1.01
1.00	0.60	2.00	1112.	2.980	$ 3.37	1.00
1.00	0.60	2.50	1208.	3.266	$ 4.02	0.98
1.00	0.60	3.00	1326.	3.500	$ 4.66	0.96
1.00	0.60	3.50	1465.	3.697	$ 5.31	0.94
1.00	0.60	4.00	1626.	3.869	$ 5.96	0.91
1.00	0.70	0.50	749.	1.432	$ 1.37	1.03
1.00	0.70	1.00	781.	2.321	$ 2.02	1.02
1.00	0.70	1.50	835.	2.841	$ 2.67	1.01
1.00	0.70	2.00	910.	3.210	$ 3.31	1.00
1.00	0.70	2.50	1006.	3.496	$ 3.96	0.98
1.00	0.70	3.00	1124.	3.730	$ 4.61	0.96
1.00	0.70	3.50	1263.	3.927	$ 5.26	0.94
1.00	0.70	4.00	1423.	4.098	$ 5.90	0.91
2.00	0.40	0.50	1165.	0.504	$ 1.39	0.98
2.00	0.40	1.00	1197.	1.393	$ 2.03	0.97
2.00	0.40	1.50	1250.	1.913	$ 2.68	0.96
2.00	0.40	2.00	1325.	2.282	$ 3.33	0.95
2.00	0.40	2.50	1421.	2.568	$ 3.97	0.93
2.00	0.40	3.00	1539.	2.802	$ 4.62	0.91
2.00	0.40	3.50	1678.	2.999	$ 5.27	0.89
2.00	0.40	4.00	1839.	3.170	$ 5.91	0.86
2.00	0.50	0.50	1024.	0.734	$ 1.34	0.98
2.00	0.50	1.00	1057.	1.623	$ 1.98	0.97
2.00	0.50	1.50	1110.	2.143	$ 2.63	0.96
2.00	0.50	2.00	1185.	2.511	$ 3.28	0.95
2.00	0.50	2.50	1281.	2.798	$ 3.93	0.93
2.00	0.50	3.00	1399.	3.031	$ 4.57	0.91
2.00	0.50	3.50	1538.	3.229	$ 5.22	0.89
2.00	0.50	4.00	1699.	3.400	$ 5.87	0.86
2.00	0.60	0.50	853.	0.964	$ 1.30	0.98
2.00	0.60	1.00	885.	1.852	$ 1.94	0.97
2.00	0.60	1.50	939.	2.372	$ 2.59	0.96
2.00	0.60	2.00	1014.	2.741	$ 3.24	0.95
2.00	0.60	2.50	1110.	3.027	$ 3.89	0.93
2.00	0.60	3.00	1228.	3.261	$ 4.53	0.91
2.00	0.60	3.50	1367.	3.459	$ 5.18	0.89
2.00	0.60	4.00	1527.	3.630	$ 5.83	0.86
2.00	0.70	0.50	651.	1.193	$ 1.26	0.98
2.00	0.70	1.00	683.	2.082	$ 1.91	0.97
2.00	0.70	1.50	736.	2.602	$ 2.55	0.96
2.00	0.70	2.00	811.	2.971	$ 3.20	0.95
2.00	0.70	2.50	907.	3.257	$ 3.85	0.93
2.00	0.70	3.00	1025.	3.491	$ 4.50	0.91
2.00	0.70	3.50	1164.	3.688	$ 5.14	0.69
2.00	0.70	4.00	1325.	3.860	$ 5.79	0.86

Table 5-2 continued

S/C	W/C	VF (%)	FLST (psi)	TOEG (ln-in)	Material Cost (8'x 4'x⅜")	Da/Di
3.00	0.40	0.50	1001.	0.106	$ 1.27	0.89
3.00	0.40	1.00	1033.	0.995	$ 1.92	0.89
3.00	0.40	1.50	1086.	1.515	$ 2.57	0.88
3.00	0.40	2.00	1161.	1.884	$ 3.22	0.87
3.00	0.40	2.50	1257.	2.170	$ 3.86	0.85
3.00	0.40	3.00	1375.	2.404	$ 4.51	0.83
3.00	0.40	3.50	1514.	2.601	$ 5.16	0.81
3.00	0.40	4.00	1675.	2.772	$ 5.81	0.78
3.00	0.50	0.50	860.	0.336	$ 1.24	0.89
3.00	0.50	1.00	893.	1.225	$ 1.89	0.89
3.00	0.50	1.50	946.	1.745	$ 2.54	0.88
3.00	0.50	2.00	1021.	2.113	$ 3.18	0.87
3.00	0.50	2.50	1117.	2.400	$ 3.83	0.85
3.00	0.50	3.00	1235.	2.633	$ 4.48	0.83
3.00	0.50	3.50	1374.	2.831	$ 5.13	0.81
3.00	0.50	4.00	1535.	3.002	$ 5.77	0.78
3.00	0.60	0.50	689.	0.566	$ 1.21	0.89
3.00	0.60	1.00	721.	1.454	$ 1.86	0.89
3.00	0.60	1.50	775.	1.974	$ 2.51	0.88
3.00	0.60	2.00	850.	2.343	$ 3.15	0.87
3.00	0.60	2.50	946.	2.629	$ 3.80	0.85
3.00	0.60	3.00	1064.	2.863	$ 4.45	0.83
3.00	0.60	3.50	1203.	3.061	$ 5.10	0.81
3.00	0.60	4.00	1363.	3.232	$ 5.75	0.78
3.00	0.70	0.50	487.	0.795	$ 1.18	0.89
3.00	0.70	1.00	519.	1.684	$ 1.83	0.89
3.00	0.70	1.50	572.	2.204	$ 2.48	0.88
3.00	0.70	2.00	647.	2.573	$ 3.13	0.87
3.00	0.70	2.50	743.	2.859	$ 3.78	0.85
3.00	0.70	3.00	861.	3.093	$ 4.42	0.83
3.00	0.70	3.50	1000.	3.290	$ 5.07	0.81
3.00	0.70	4.00	1161.	3.462	$ 5.72	0.78

5.4 The Other Optimization Model

In the case where material cost is the strongest restriction, the optimal model is completely different from the one described above.

The objective function should be a function which explains the total effects on the properties selected as criteria for a specific use. (If required, weight should be used to clarify the importance of criteria.) Objective function, E, might be given in the form of :

$$E = W_1Y_1 + W_2Y_2 \quad ,$$

where the notation of Y_1 and Y_2 is the same as before and W_1 and W_2 denote the normalized weight ($W_1 + W_2 = 1$). Instead of the estimated value of flexural strength (Y_1) or toughness (Y_2), it is also possible to use a relative score showing some levels of each property.

For example

$$Y_1 = a_1 + b_1X_1 + c_1X_2 + d_1X_3$$

Score

1 if $L_1 \leq Y_1 \leq L_2$
2 if $L_2 \leq Y_1 \leq L_3$
⋮ ⋮
10 if $L_{10} \leq Y_1$

The objective of this model is to maximize E, subject to

$$Z' \leq C$$

and the constraints on the valid range of variables, where Z' is the cost function developed before and C denote the maximum cost allowed. Although the experiment in this study was devoted to flexural strength and toughness, the results obtained imply that there is a good possibility of representing the other important properties in terms of S/C, W/C and VF. However, there might not be any difference in the method of the formulation of the optimization model.

5.5 Cost Evaluation of Fiber-Reinforced Cement Composite with Different Fibers

In the previous section, the cost per unit volume, Z', is given by

$$Z' = (1-X_3)\frac{(U_c + U_s X_1)}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}}X_1 + X_2\right)} + U_f \cdot S_{Gf} \cdot X_3$$

Suppose X_1 = constant and X_2 = constant.

Z' is given by simple linear function :

$$Z' = c(1-X_3) + U_f \cdot S_{Gf} \cdot X_3$$

$$Z' = c + (U_f \cdot S_{Gf} - c)X_3$$

where $$c = \frac{U_c + U_s X_1}{\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}} X_1 + X_2} = \text{constant} .$$

According to the final equation of Z', it is possible to figure out the aspects of increasing cost with the addition of various fibers which have different unit costs and specific gravities.

Suppose $X_1 = S/C = 0$ and $X_2 = W/C = 0.4$.

Then the constant c is given by

$$c = \frac{3.25 \times 10^{-5}}{(0.32 + 0.4)} = 4.51 \times 10^{-5}$$

Therefore

$$Z' = 4.51 \times 10^{-5} + (U_f \cdot S_{Gf} - 4.51 \times 10^{-5}) X_3$$

The value of $U_f \cdot S_{Gf}$ of the various fibers is given in the following table.

Fiber	Form(length)	U_f($/g)	S_{Gf}(g/cm^3)	$U_f \cdot S_{Gf}$
Asbestos (chrysotile)	Fiber	0.4×10^{-3}	3.20	1.28×10^{-3}
Glass fiber (alkaline-resistant)	Chopped strand Glass mat	1.8×10^{-3}	2.69	4.84×10^{-3}
Steel (brass coated)	Fiber (1", round)	0.6×10^{-3}	7.80	4.68×10^{-3}
Nylon-66	Monofilament (250d)	2.5×10^{-3}	1.14	2.85×10^{-3}
Carbon fiber	Fiber	20×10^{-3}	1.60	3.2×10^{-2}

Source : Mirerals Yearbook 1972, Modern Textile March,1974

In Fig. 5-1, the aspects of unit cost increase of several fiber-reinforced cements are shown graphically with the increase of fiber volume percentage. Based on the material cost of typical asbestos cement products which includes 15 to 30 weight percent of asbestos fiber,* comparable fiber volume percentage of the other fiber-reinforced cement

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* Assuming that S/C=0 and W/C=0.4, then, 15 to 30 percent of fiber by weight correspond to approximately 10 to 20 percent of fiber by volume.

If V_f and w_f are the volume fraction and weight fraction of fiber then :

$$w_f = \frac{W_f}{W_c + W_s + W_w + W_f} = \frac{W_f}{W_c(1 + X_1 + X_2) + W_f} ,$$

$$W_f = \frac{w_f}{(1 - w_f)}(1 + X_1 + X_2)W_c \qquad 5\text{-}1)^*$$

$$V_f = \frac{\frac{W_f}{S_{Gf}}}{W_c\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}}X_1 + X_2\right) + \frac{W_f}{S_{Gf}}} \qquad 5\text{-}2)^*$$

From 5-1)* and 5-2)* :

$$V_f = \frac{\frac{w_f}{(1 - w_f)}(1 + X_1 + X_2)\frac{1}{S_{Gf}}}{\left(\frac{1}{S_{Gc}} + \frac{1}{S_{Gs}}X_1 + X_2\right) + \frac{w_f}{(1 - w_f)}(1 + X_1 + X_2)\frac{1}{S_{Gf}}}$$

Suppose $X_1=0$, $X_2=0.4$, $S_{Gc}=3.15$, $S_{Gs}=2.66$, $S_{Gw}=1.0$, $S_{Gf}=3.2$.

For $w_f=0.30$, $V_f=0.204$ and for $w_f=0.15$, $V_f=0.096$.

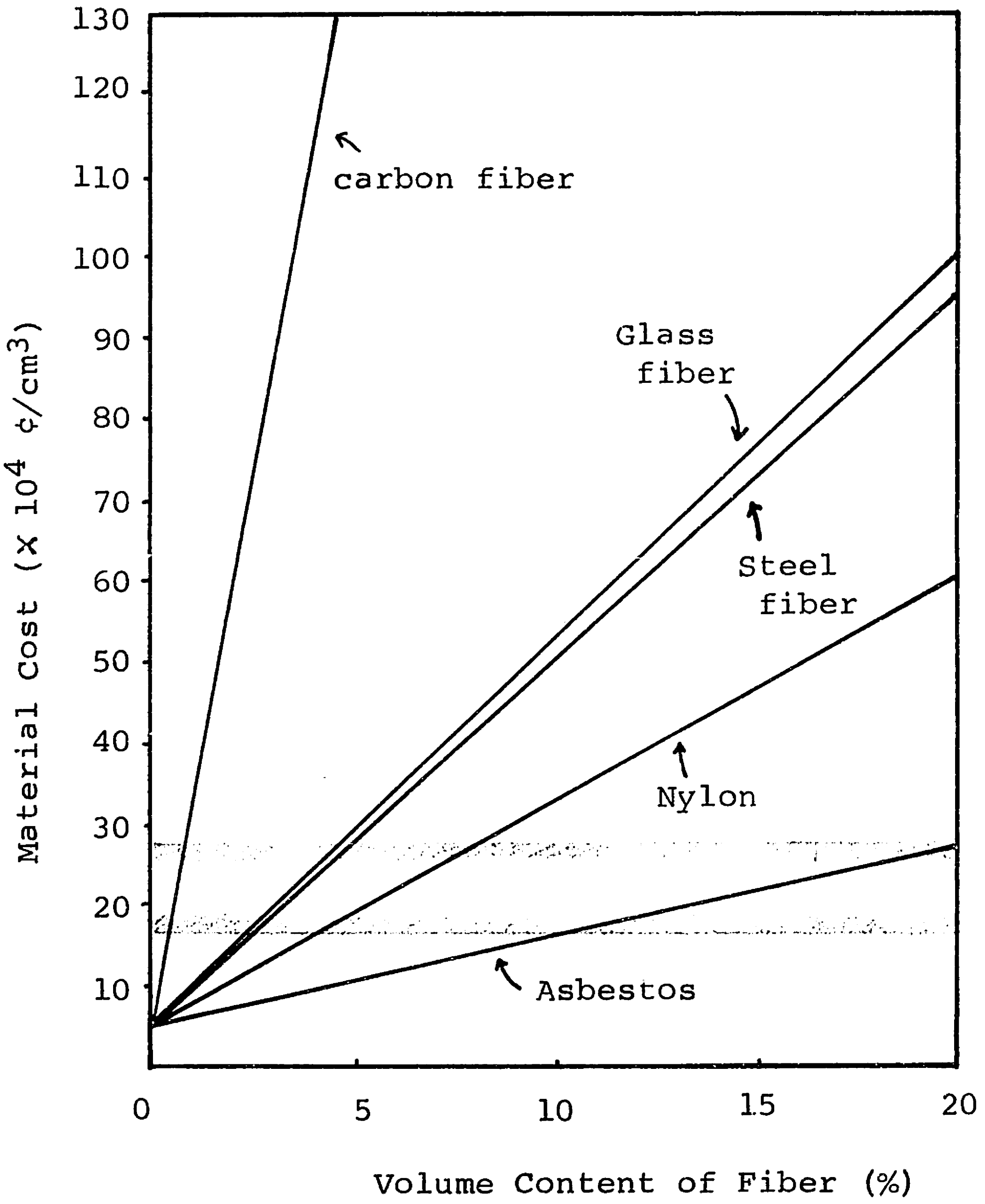

Figure 5-1 Material Cost of Various Types of Fiber-Reinforced Cement Composites with Volume Content of Fiber

products are estimated in such a way that the total material cost falls in the range of the asbestos cement product. The results of the estimation are given in the following table.

Fiber	Percentage of Fiber by Volume (%)
Asbestos	10 - 20
Glass fiber	2.4 - 4.8
(Shopped strand)	
(Glass mat)	
Steel fiber	2.5 - 5.0
Nylon fiber	4.4 - 8.8
Carbon fiber	

As asbestos cement products are only one FRC which is now on the market, its cost is an important target for new fiber-reinforced cement composites. If the properties of glass fiber-reinforced cement (GRC) with less than 4.8 percent of fiber are assumed to exceed those of asbestos cement products with 20 volume percent of fiber, glass fiber has a good chance to enter construction materials market. Some properties of various types of fiber-reinforced cement composites selected from previous publications are summarized in Table 5-3 . Glass and steel fiber have almost the same cost function, but so far there are

Table 5-3 Typical Physical Properties of Fiber Reinforced Cement Composites

Fiber	Type of Cement Mix Proportioning Fabrication	Volume Content (%)	Strengths of Composites *			Ref.
			Tensile (psi)	Flexural (psi)	Impact (lbf/in^2)	
Asbestos	Portland Cement Type I W/C=0.158 - 0.328 Excess water removed by suction 252 days	2.6 ~ 16.5	--	2430 ~ 4870	7.4 ~ 27.7	13
E-glass (Mat)	High Almina Cement S/C=0 Lay-up (4 layers of glass strand mat)	6.7	2300	4460	--	11
E-glass (Strands)	Portland Cement Type I S/C=0 , W/C=0.13 - 0.28 Spray-up method	4.5 ~ 12.7 (wt%)	980 ~ 2120	-- (119 ~ 231) %	-- (125 ~1450) %	12
Zr-glass* (Strands)	Portland Cement Type I S/C=0, W/C=0.3 Spray-up and suction	3.6	1300	3520	--	11
Zr-glass (Strands)	Portland Cement Type I S/C=0, W/C=0.4 Premixing and cast	2		1374 (123%)	--	This study

* Stress figures are quoted in lbf/in^2 or as % of control.

Table 5-3 continued

Fiber ℓ/ϕ	Type of Cement Mix Proportioning Fabrication	Volume Content (%)	Strength of Composites			Ref.
			Tensile (psi)	Flexural (psi)	Impact (lbf/in^2)	
Steel 1"/0.01"	Portland Cement Type I S/C=0, W/C=0.29 Mixer blending 28 day	3.0	--	3660 (222%)	--	6
Steel 1"/0.01"	Portland Cement Type I S/C=0.55, W/C=0.36 Mixer blending	3.0	--	4225 (260%)	--	
Nylon 1.5"/0.01"	Portland Cement Type I A/C=2.47, W/C=0.46 Mixer blending	3.0	--	1290 (95%)	--	
Poly-propylene	Portland Cement S/C=4.0, W/C=0.5 Mixer blending	1.2	--	-- (110%)	--	14
Carbon	Portland Cement Type I S/C=0 , W/C=0.28-0.30 lay up	3.7	3780 (500%)	--	20.4 ~25.8	15

some differences in target product and application fields between them. However, it is highly possible that they will compete with each other in the future when their uses are extended. Nylon and polypropylene fibers are less expensive but lack the required positive reinforcing properties. Carbon fibers are too expensive as reinforcement to cement at this moment.

Chapter 6

Conclusion and Recommendation

1. Regression analysis can effectively be used for the investigation of fiber-reinforcing mechanisms and for the prediction of actual behavior. However, a careful consideration should be given to the boundaries of the experimental region.

2. In the conventional pre-mixing method, the range where a sound specimen is expected to be obtained is relatively narrow. The effect of fiber addition on tensile and flexural strength is relatively small, while that on toughness is extremely large even in the conventional premixing method.

If the direct strengthening is of primary importance, the conventional pre-mixing method is not suitable from the cost-effectiveness point of view.

3. The fabrication method is one of the most important factors controlling the strength of cement composites. Spray suction and laminate methods of fabrication could give a much more attractive reinforcing effect and a wider composition range than the pre-mixing.

4. The ratio of the actual density of specimen to the calculated based on the mix composition, (Da/Di), serves as a practical criterion for a sound composition of the mix and ensures a sound specimen.

5. Neither plain mortar nor GRC specimens showed any significant change in flexural strength during the short life of the corrosion experiment, while GRC specimens showed a dramatic decrease in toughness. This deterioration does not seem to come from a degradation of either mortar or the fibers, but from the weakened fiber and matrix bond.

6. It is possible that glass fiber will compete with asbestos fiber in the future, from the cost-effectiveness point of view. Steel fiber cement composites are in the same range as glass fiber-reinforced cement composites in terms of cost effectiveness.

Bibliography

1. Wang, L. R., "Structural Materials and Testings for Low-Cost Housing", Proceedings of the Second International Symposium on Lower-cost Housing Problems, Department of Civil Engineering, University of Missouri April 1972, P.199

2. Lankard, D. R., "Fiber-Reinforced Cement and Concrete Composites" The 76th Annual Meeting of the American Ceramic Society, Chicago, Illinois, April 28 1974, p.3

3. "Fiber-reinforced Cement Composites", the Materials Technology Division of The Concrete Society, Technical Report 51.067 July 1973, P.6-11

4. A.C.I. Committee 544, "State-of-the-Art Report on Fiber-Reinforced Concrete" American Concrete Institute Journal, Nov., 1973, P.735

5. Naaman, A. E., "Statistical Theory of Strength for Fiber-Reinforced Concrete", Thesis presented to the Massachusetts Institute of Technology, at Cambridge, Mass., in 1972, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

6. Williamson, G. R., "Fibrous Reinforcement for Portland Cement Concrete" US Clearing House for Federal Scientific and Technical Information, 1965. AD 465999

7. March, H. N. and Clark, L. L., "Glass Fibers in Concrete" International Conference of Fiber Reinforced Concrete, American Concrete Institute Fall Convention, Ottawa, Oct., 1973

8. Takagi, J., "Some Properties of Glass Fiber-Reinforced Concrete" International Conference of Fiber-Reinforced Concrete, American Concrete Institute Fall Convention, Ottawa, Oct. 1973

9. Stamm, John Andrews, "Tensile Properties of Portland Cement Concrete with Alkali Resistant Glass Fiber Reinforcement" Masters Thesis, M. I. T., Cambridge, Mass., Jan. 1974 P.18

10. Concrete Information "Proportioning Concrete Mixtures" Portland Cement Association, P.7

11. Allen, H. G. "Strength and Stiffness of Two Glass-Fiber-Reinforced Cement Laminates" Journal of Composite Materials. Vol. 5. April 1971. p.194-207

12. Majumdar, A. J. and Ryder, J. F. "Glass-Fiber Reinforcement of Cement Products. Glass Technology Vol.9, No.3 June 1968, P.78-84

13. Krenchel, H. "Fiber Reinforcement" Copenhagen, Akademisk Forlag, 1964.

14. Nanda, V. K. and Hannant,D.J. "Fiber-Reinforced Concrete" Concrete Building and Concrete Products. Vol. 44, No. 10 October 1969, P.179-181

15. Ali, M. A. and Majumdar, A. J. and Rayment, D. L. , "Carbon Fiber Reinforcement of Cement" Cement and Composite Research. Vol.2, 1972 P.201-212

16. Draper, N. R. "Applied Regression Analysis" John Wiley & Sons, Inc. 1966 P.171-177

TABLE OF APPENDICES

Appendix C Experimental Results

Appendix A

Multiple Regression Analysis : General Concepts and Computational Procedure

1. Definition and Procedure of Linear Regression Analysis

The term "Regression Analysis" refers to the methods by which estimates are made of the values of a variable (dependent variable : Y) from a knowledge of the values of one or more other variables (independent variables : X_i, $i=1,2.....n$), and to the measurement of the errors involved in this estimating process. We begin with the case of a simple linear regression analysis (two variables : Y and X_1). The term "linear" means that the relationship between Y and X_1 is specified by a linear equation of the form

$$Y = a + bX_1 + \text{error} \quad .$$

Regression Analysis is a procedure whereby one unique equation is selected from the infinite number of possible equations according to the criterion of least square.

Multiple regression is the extension of a simple two-variable regression in order to take into account the effect of more than one independent variable Xi (i=2,3.....n) on the dependent variable Y. It is obviously the appropriate technique when we want to investigate the effects on Y of several variables simultaneously. Even if we are interested in the effect of only one variable on Y, better results are usually obtained by including the other variables influencing Y in a multiple regression analysis, for two reasons :

1. To reduce stochastic error
2. To eliminate bias that might result if we were just to ignore an uncontrolled variable that substantially effects Y.

Although regression analysis described above was devoted exclusively to the linear model, we often encounter cases where non-linear models give much better results than a linear model. Two common types of non-linear equation are shown here :

1) Polynominal Regression

A polynominal is an equation of the form :

$$Y = a + bX + cX^2 + dX^3 \ldots\ldots \text{ (one independent variable)}$$

$$Y = a + b_1X_1 + c_1X_1^{\;2} + d_1X_1^{\;3} + \ldots\ldots\ldots$$

$$\ldots + b_2X_2 + c_2X_2^{\;2} + d_2X_2^{\;3} + \ldots\ldots\ldots$$

(more than one independent variable)

Let the observed variable X be transformed as follows :

$$Z_m = X^m \quad , \quad m = 1,2......$$

Then the original equation can be written in the linear form :

$$Y = a + bZ_1 + cZ_2 + dZ_3 + ...$$

2) Log-linear Regression

Another common type of equation is one in the form :

$$Y = AX_1{}^bX_2{}^cX_3{}^d \; ....$$

If we take the logarithms of all observations on all the variables, i.e. :

$$\begin{aligned} Y^* &= \ln Y \\ X_1{}^* &= \ln X_1 \\ X_2{}^* &= \ln X_2 \\ &\vdots \end{aligned}$$

then the original equation can be expressed in the linear form :

$$Y^* = a + bX_1{}^* + cX_2{}^* + dX_3 + ...,$$

where a = lnA.

As can be observed in the above two examples, most non-linear regression equations can be transferred to linear forms.

In this thesis, multiple linear regression equations are used primarily. Non-linear regression analysis is applied for only the final stage of the analysis of the experimental results. There, a mixed-type regression model is used for the analysis :

Dependent Variable	FLST (Flexural Strength)
Independent Variable	S/C, $(S/C)^2$, $\ln(S/C)$
	W/C, $(W/C)^2$, $\ln(W/C)$
	VF, $(VF)^2$, $\ln(VF)$

These nine variables result from taking three different forms of each of the three important parameters (sand/cement ratio, water/cement ratio and volume content of fiber.) Obviously there are strong multicollinearities among the three variables within each of the following sets of variables.

S/C, $(S/C)^2$, $\ln(S/C)$, W/C, $(W/C)^2$, $\ln(W/C)$
and VF, $(VF)^2$, $\ln(VF)$

The final regression equation is composed of the best variable from each set.

2. Stepwise Regression Procedure (16)

Stepwise regression procedure does not provide a true least squares solution for the variables included in the final equation. In this procedure, first a regression in the X variable most correlated with Y has been fitted, and then, the residuals are calculated. These residuals are now considered as response values and regressed against the X which is the most correlated with this new response. The process is continued until the variable does not offer any significant improvement in the goodness of fit. (F-ratio criterion)

Stepwise regression estimate is rather smaller, in absolute value than the least squares estimate, but it has the following advantage. Suppose one is given a set of X'^{s} highly correlated with each other. The selection procedures will show first that X is most highly correlated with Y. At the next stage, all other X-variables not in regression are adjusted for the one chosen. Thus, if a variable is highly correlated with the first variable, it might be rejected as a possible variable.

Appendix B

Computer Printouts of Stepwise Regression Analysis

Appendix B1 Cracking Strength of SRC (case 1a)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 CRST
RESIDUAL STANDARD DEVIATION      8.2467
STANDARD ERROR OF THE MEAN       2.3806
MULTIPLE R                       0.9782
MULTIPLE RSQR                    0.9569


VARIABLE ENTERED                   ASPR



VARIABLE            B - COEF   STD ERROR OF B          PARTIAL-R         BETA-COEF    STD ERROR OF BETA

  ASPR                1.3087       0.1318                0.9572            0.6865             0.0691
  VOLF               29.3750       2.9156                0.9584            0.6968             0.0691


CONSTANT           163.0664


                              ANALYSIS OF VARIANCE TABLE

               SOURCE               D.F.      SUM OF SQUARES   MEAN SQUARE         F

               MEAN                  1         0.12500E 07      0.12500E 07
               REGRESSION            2         0.13604E 05      0.68020E 04     0.10001E 03
               ERROR                 9         0.61208E 03      0.68008E 02

JOB COMPLETED
```

Appendix B2 Post-Cracking Strength of SRC (case 1b)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                PCST
RESIDUAL STANDARD DEVIATION    16.4919
STANDARD ERROR OF THE MEAN      4.7608
MULTIPLE R                      0.9619
MULTIPLE RSQR                   0.9252


VARIABLE ENTERED                  ASPR



VARIABLE           B - COEF    STD ERROR OF B        PARTIAL-R         BETA-COEF    STD ERROR OF BETA

  ASPR               1.7214          0.2636           0.9086            0.5948            0.0911
  VOLF              48.3750          5.8307           0.9404            0.7559            0.0911


CONSTANT          -93.5971


                              ANALYSIS OF VARIANCE TABLE

                SOURCE           D.F.     SUM OF SQUARES   MEAN SQUARE          F

                MEAN              1        0.22168E 06     0.22168E 06
                REGRESSION        2        0.30315E 05     0.15157E 05    0.55729E 02
                ERROR             9        0.24478E 04     0.27198E 03

JOB COMPLETED
```

Appendix B3 Flexural Strength of SRC (case 2)

```
DEPENDENT VARIABLE                   FLST
RESIDUAL STANDARD DEVIATION      480.8295
STANDARD ERROR OF THE MEAN        92.5357
MULTIPLE R                         0.7917
MULTIPLE RSQR                      0.6268


VARIABLE ENTERED                     WBYC


VARIABLE   B - COEF    STD ERROR OF B   PARTIAL-R   BETA-COEF   STD ERROR OF BETA

 WBYC    -3781.2431     1157.5534        -0.5547     -0.4107          0.1257
 VOLF      346.0970       69.4908         0.7129      0.6262          0.1257


CONSTANT   3192.8188


                  ANALYSIS OF CARIANCE TABLE

       SOURCE         D.F.    SUM OF SQUARES   MEAN SQUARE         F

       MEAN             1      0.17183E 09     0.17183E 09
       REGRESSION       2      0.93202E 07     0.46601E 07    0.20156E 02
       ERROR           24      0.55487E 07     0.23119E 06
```

Appendix B 4 Flexural Strength of GRC (case 3)

```
REGRESSION ANALYSIS

 DEPENDENT VARIABLE                    FLST
 RESIDUAL STANDARD DEVIATION       191.9874
 STANDARD ERROR OF THE MEAN         42.9297
 MULTIPLE R                          0.8618
 MULTIPLE RSQR                       0.7427

VARIABLE ENTERED                       LENF


VARIABLE       B - COEF    STD ERROR OF B      PARTIAL-R     BETA-COEF   STD ERROR OF
                                                                                 BETA
 LENF          126.8800        76.7950           0.3719        0.2032         0.1230
 VOLF          413.3001        60.7117           0.8553        0.8374         0.1230


CONSTANT      584.0496

                    ANALYSIS OF VARIANCE TABLE

         SOURCE         D.F.     SUM OF SQUARES    MEAN SQUARE        F

         MEAN             1       0.37133E 08       0.37133E 08
         REGRESSION       2       0.18087E 07       0.90439E 06   0.24536E 02
         ERROR           17       0.62660E 06       0.36859E 05
```

Appendix B5 Flexural Strength of GRC (case 4)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 FLST
RESIDUAL STANDARD DEVIATION      4.0823
STANDARD ERROR OF THE MEAN       1.0205
MULTIPLE R                       0.5818
MULTIPLE RSQR                    0.3385


VARIABLE ENTERED                   WGHF


VARIABLE         B - COEF    STD ERROR OF B       PARTIAL-R       BETA-COEF    STD ERROR OF BETA

  WGHF             2.6999        3.6513             0.2008          0.1667            0.2255
  LENF             0.2979        0.1205             0.5653          0.5574            0.2255


CONSTANT         62.2871


                     ANALYSIS OF VARIANCE TABLE

          SOURCE            D.F.      SUM OF SQUARES   MEAN SQUARE        F

          MEAN               1        0.72845E 05      0.72845E 05
          REGRESSION         2        0.11090E 03      0.55453E 02     0.33274E 01
          ERROR             13        0.21665E 03      0.16665E 02

JOB COMPLETED
```

Appendix B6 Flexural Strength of GRC (Linear Model - Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                  FLST
RESIDUAL STANDARD DEVIATION     237.4824
STANDARD ERROR OF THE MEAN       44.8799
MULTIPLE R                        0.7498
MULTIPLE RSQR                     0.5622


VARIABLE ENTERED                   WBYC



VARIABLE          B - COEF    STD ERROR OF B        PARTIAL-R         BETA-COEF    STD ERROR OF BETA

 VOLF            102.9678         35.7478             0.5068            0.3899           0.1353
 SBYC           -182.0384         42.5832            -0.6574           -0.5884           0.1376
 WBYC          -1187.0651        418.8743            -0.5007           -0.3905           0.1378


CONSTANT          1794.2529


                          ANALYSIS OF VARIANCE TABLE

           SOURCE           D.F.     SUM OF SQUARES    MEAN SQUARE         F

           MEAN              1        0.30582E 08      0.30582E 08
           REGRESSION        3        0.17385E 07      0.57950E 06     0.10275E 02
           ERROR            24        0.13535E 07      0.56397E 05

JOB COMPLETED
```

Appendix B7 Toughness of GRC (Linear Model - 28 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 TOEG
RESIDUAL STANDARD DEVIATION      0.6271
STANDARD ERROR OF THE MEAN       0.1185
MULTIPLE R                       0.8503
MULTIPLE RSQR                    0.7230


VARIABLE ENTERED                  WBYC



VARIABLE          B - COEF   STD ERROR OF B        PARTIAL-R        BETA-COEF   STD ERROR OF BETA

 VOLF              0.6150       0.0944              0.7992           0.7015          0.1076
 SBYC             -0.3568       0.1124             -0.5435          -0.3473          0.1094
 WBYC              3.4479       1.1062              0.5367           0.3415          0.1096


CONSTANT          -0.3830


                         ANALYSIS OF VARIANCE TABLE

        SOURCE             D.F.      SUM OF SQUARES  MEAN SQUARE        F

        MEAN                1        0.13681E 03     0.13681E 03
        REGRESSION          3        0.24652E 02     0.82174E 01     0.20889E 02
        ERROR              24        0.94410E 01     0.39337E 00

JOB COMPLETED
```

Appendix B8 Flexural Strength of GRC
(Non-linear Model - 28 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 FLST
RESIDUAL STANDARD DEVIATION    225.3420
STANDARD ERROR OF THE MEAN      42.5856
MULTIPLE R                       0.7783
MULTIPLE RSQR                    0.6058


VARIABLE ENTERED                   VOLF


VARIABLE          B - COEF   STD ERROR OF B       PARTIAL-R        BETA-COEF     STD ERROR OF BETA

 VOLF             107.3003       33.9486            0.5421           0.4064             0.1285
 SCSQ             -61.4869       13.0045           -0.6944          -0.6220             0.1315
 WCSQ           -1180.8378      360.5802           -0.5557          -0.4305             0.1314


CONSTANT         1447.2778


                      ANALYSIS OF VARIANCE TABLE

          SOURCE            D.F.     SUM OF SQUARES   MEAN SQUARE        F

          MEAN               1        0.30582E 08     0.30582E 08
          REGRESSION         3        0.18733E 07     0.62445E 06    0.12297E 02
          ERROR             24        0.12186E 07     0.50779E 05
```

Appendix B9 Toughness of GRC (Non-linear Model - 28 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 TOEG
RESIDUAL STANDARD DEVIATION      0.4980
STANDARD ERROR OF THE MEAN       0.0941
MULTIPLE R                       0.9085
MULTIPLE RSQR                    0.8254


VARIABLE ENTERED                 LGWC


VARIABLE        B - COEF   STD ERROR OF B      PARTIAL-R      BETA-COEF   STD ERROR OF BETA

 LGVF            1.0944       0.1247            0.8730         0.7483          0.0853
 SCSQ           -0.1251       0.0288           -0.6631        -0.3814          0.0878
 LGWC            1.5342       0.4754            0.5501         0.2835          0.0878


CONSTANT        3.0956


                      ANALYSIS OF VARIANCE TABLE

         SOURCE              D.F.     SUM OF SQUARES   MEAN SQUARE          F

         MEAN                  1       0.13681E 03     0.13681E 03
         REGRESSION            3       0.28140E 02     0.93802E 01     0.37821E 02
         ERROR                24       0.59524E 01     0.24801E 00
```

Appendix B10 Flexural Strength of GRC
(Linear Model - 23 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                  FLST
RESIDUAL STANDARD DEVIATION     142.0929
STANDARD ERROR OF THE MEAN       29.6284
MULTIPLE R                        0.9195
MULTIPLE RSQR                     0.8455


VARIABLE ENTERED                  SBYC


VARIABLE          B - COEF    STD ERROR OF B         PARTIAL-R        BETA-COEF    STD ERROR OF BETA

 VOLF            186.1357        27.2024               0.8434           0.6422                0.0938
 SBYC            -97.1102        31.0090              -0.5834          -0.2935                0.0937
 WBYC          -1761.7482       288.8966              -0.8135          -0.5511                0.0903


CONSTANT          1913.0920


                          ANALYSIS OF VARIANCE TABLE

          SOURCE             D.F.      SUM OF SQUARES   MEAN SQUARE         F

          MEAN                 1        0.26961E 08      0.26961E 08
          REGRESSION           3        0.21004E 07      0.70015E 06     0.34677E 02
          ERROR               19        0.38361E 06      0.20190E 05

JCB COMPLETED
```

Appendix B11 Toughness of GRC
(Linear Model - 23 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                  TOEG
RESIDUAL STANDARD DEVIATION       0.5859
STANDARD ERROR OF THE MEAN        0.1221
MULTIPLE R                        0.8853
MULTIPLE RSQR                     0.7837


VARIABLE ENTERED                   WBYC


VARI/BLE          B - COEF   STD ERROR OF B        PARTIAL-R        BETA-COEF    STD ERROR OF BETA

 VOLF              0.8531        0.1079             0.8702           0.8233           0.1042
 WBYC              3.1507        1.1910             0.5091           0.2756           0.1042


CONSTANT          -1.0013


                         ANALYSIS OF VARIANCE TABLE

          SOURCE                D.F.     SUM OF SQUARES  MEAN SQUARE        F

          MEAN                   1       0.11921E 03     0.11921E 03
          REGRESSION             2       0.24889E 02     0.12444E 02    0.36244E 02
          ERROR                 20       0.68671E 01     0.34335E 00
```

Appendix B12 Flexural Strength of GRC
(Non-linear Model - 23 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                FLST
RESIDUAL STANDARD DEVIATION   139.2323
STANDARD ERROR OF THE MEAN     29.0319
MULTIPLE R                      0.9228
MULTIPLE RSQR                   0.8517


VARIABLE ENTERED                SCSQ



VARIABLE        B - COEF   STD ERROR OF B      PARTIAL-R       BETA-COEF   STD ERROR OF BETA

 SCSQ          -32.8136        10.4121         -0.5859         -0.2918           0.0925
 WCSQ        -1547.3630       254.7293         -0.8124         -0.5386           0.0886
 VFSQ           42.8156         6.2163          0.8449          0.6354           0.0922


CONSTANT        1533.1013


                      ANALYSIS OF VARIANCE TABLE

        SOURCE             D.F.     SUM OF SQUARES   MEAN SQUARE        F

        MEAN                 1       0.26961E 08     0.26961E 08
        REGRESSION           3       0.21157E 07     0.70525E 06   0.36380E 02
        ERROR               19       0.36832E 06     0.19385E 05
```

Appendix B13 Toughness of GRC
(Non-linear Model - 23 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                 TOEG
RESIDUAL STANDARD DEVIATION      0.4800
STANDARD ERROR OF THE MEAN       0.1000
MULTIPLE R                       0.9285
MULTIPLE RSQR                    0.8621


VARIABLE ENTERED                  SCSQ



VARIABLE          B - COEF    STD ERROR OF B         PARTIAL-R          BETA-COEF    STD ERROR OF BETA

 LGVF               1.2821          0.1439             0.8981             0.7987               0.0896
 WBYC               2.2855          0.9849             0.4699             0.1999               0.0861
 SCSQ              -0.0796          0.0358            -0.4538            -0.1982               0.0892


CONSTANT          0.7978


                        ANALYSIS OF VARIANCE TABLE

            SOURCE              D.F.      SUM OF SQUARES    MEAN SQUARE          F

            MEAN                  1         0.11921E 03     0.11921E 03
            REGRESSION            3         0.27378E 02     0.91262E 01     0.39605E 02
            ERROR                19         0.43781E 01     0.23043E 00
```

Appendix B14 Da/Di of GRC (Non-linear Model - 28 Sample Points)

```
REGRESSION ANALYSIS

DEPENDENT VARIABLE                RTSD
RESIDUAL STANDARD DEVIATION     0.0427
STANDARD ERROR OF THE MEAN      0.0080
MULTIPLE R                      0.8726
MULTIPLE RSQR                   0.7615


VARIABLE ENTERED                  VFSQ


VARIABLE          B - COEF    STD ERROR OF B        PARTIAL-R        BETA-COEF    STD ERROR OF BETA

 SCSQ             -0.0166         0.0024            -0.8087          -0.6750            0.0981
 VFSQ             -0.0071         0.0014            -0.7053          -0.4884            0.0981


CONSTANT          1.0486


                        ANALYSIS OF VARIANCE TABLE

          SOURCE               D.F.      SUM OF SQUARES   MEAN SQUARE         F

          MEAN                  1         0.25326E 02     0.25326E 02
          REGRESSION            2         0.14598E 00     0.72992E-01    0.39926E 02
          ERROR                25         0.45704E-01     0.18281E-02
```

Appendix C1 Flexural Test Results of the Preliminary Experiment

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
p - 1		1051	0.970	2.07
		1150	1.125	2.13
	S/C=0.00	1059	0.600	2.06
		1076	0.600	2.12
	W/C=0.40	937	0.465	2.07
		1175	0.850	2.17
	VF =0.50%	920	0.750	2.10
		1076	0.750	2.16
		934	0.795	2.03
	Aver.	1042	0.767	2.10
	SD	93	0.201	0.04
2		942	1.350	1.71
	S/C=0.00	685	1.765	1.72
		726	2.095	1.79
	W/C=0.70	688	1.750	1.72
		805	2.015	1.74
	VF =0.50%	472	1.270	1.68
		517	1.130	1.57
	Aver.	691	1.624	1.70
	SD	160	0.377	0.06

Appendix C1 continued

Specimen No.	**Material Composition**	**Flexural Strength (psi)**	**Toughness (Pound-inch)**	Density
p - 3	**S/C=0.00**	**2654**	**4.426**	**2.19**
		2071	**3.500**	**2.02**
	W/C=0.40	**1454**	**1.430**	**2.07**
		2072	**2.890**	**2.14**
	VF =4.00%	**1656**	**2.350**	**2.12**
		1242	**1.850**	**2.08**
	Aver.	**1858**	**2.740**	**2.10**
	SD	**511**	**1.104**	**0.05**
4	**S/C=0.00**	**1418**	**3.610**	**1.73**
		1553	**4.795**	**1.66**
	W/C=0.70	**1449**	**4.010**	**1.68**
		1563	**6.455**	**1.75**
	VF =4.00%	**1587**	**2.250**	**1.67**
	Aver.	**1514**	**4.223**	**1.70**
	SD	**75**	**1.551**	**0.04**
5		**1135**	**0.585**	**2.27**
		1072	**0.675**	**2.29**
	S/C=3.00	**1041**	**0.430**	**2.18**
		1093	**0.350**	**2.22**
	W/C=0.40	**1167**	**0.600**	**2.22**
		1017	**0.500**	**2.25**
	VF =0.50%	**994**	**0.525**	**2.25**
		837	**0.325**	**2.22**
		803	**0.575**	**2.18**
	Aver.	**1018**	**0.507**	**2.23**
	SD	**124**	**0.118**	**0.03**

Appendix C1 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
p - 6		656	1.085	2.07
	S/C=3.00	690	1.175	2.13
		586	0.675	2.10
	W/C=0.70	492	0.750	2.07
		621	0.600	2.14
	VF =0.50%	517	0.750	2.09
		621	1.075	2.09
	Aver.	597	0.872	2.10
	SD	71	0.231	0.02
7		502	1.440	1.60
	S/C=3.00	599	1.700	1.79
		567	2.160	1.83
	W/C=0.40	483	1.170	1.49
		428	1.825	1.50
	VF =4.00%	418	1.700	1.58
		321	0.810	1.49
	Aver.	474	1.543	1.61
	SD	94	0.446	0.14
8	S/C=3.00	849	2.550	1.73
		833	2.950	1.75
	W/C=0.70	694	1.600	1.78
		744	1.750	1.87
	VF =4.00%	786	3.850	1.76
	Aver.	781	2.539	1.78
	SD	63	0.920	0.05

Appendix C1 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
p-11		1090	0.075	2.25
		1197	0.097	2.20
	S/C=2.00	1091	0.105	2.24
		1232	0.100	2.20
	W/C=0.40	1163	0.125	2.28
		1056	0.095	2.21
	VF=0.00%	1035	0.170	2.23
		1261	0.100	2.31
		964	0.130	2.28
	Aver.	1121	0.110	2.24
	SD	98	0.027	0.03
p-12		562	0.100	1.90
		729	0.100	1.96
	S/C=2.00	727	0.100	2.02
		807	0.080	2.03
	W/C=0.70	843	0.100	2.07
		763	0.080	2.02
	VF=0.00%	931	0.090	2.10
		813	0.100	2.06
		728	0.060	2.03
	Aver.	767	0.089	2.02
	SD	102	0.014	0.05
p-13		1104	0.975	2.23
		1349	0.875	2.36
	S/C=2.00	1386	0.565	2.26
		1205	0.470	2.24
	W/C=0.40	1444	1.075	2.30
		1181	0.550	2.21
	VF=0.50%	1279	0.675	2.22
		1308	0.625	2.33
		1017	0.395	2.20
	Aver.	1253	0.689	2.26
	SD	138	0.234	0.05

Appendix C1 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
		1558	1.410	2.28
p-14	S/C=2.00	1483	0.925	2.25
		1187	0.935	2.26
	W/C=0.40	1199	0.875	2.22
		1272	0.825	2.26
	VF =1.00%	1272	1.100	2.26
		1274	1.275	2.31
		1199	0.800	2.25
		945	1.500	2.27
	Aver.	1265	1.071	2.26
	SD	177	0.263	0.02
		1540	1.725	2.21
15		1056	1.500	2.20
	S/C=2.00	1396	1.675	2.15
		1406	2.960	2.26
	W/C=0.40	1281	1.675	2.23
		1197	1.300	2.21
	VF =1.50%	1112	0.720	2.27
		1298	1.600	2.27
		1246	1.840	2.15
	Aver.	1281	1.666	2.22
	SD	151	0.588	0.04
		1163	1.635	2.19
16		1090	1.775	2.19
	S/C=2.00	1012	1.250	2.26
		1558	1.380	2.22
	W/C=0.40	1599	4.000	2.18
		1509	2.675	2.13
	VF =2.00%	1575	2.450	2.10
		1490	1.565	2.24
	Aver.	1374	2.091	2.19
	SD	242	0.918	0.05

Appendix C1 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
p-17		661	1.300	1.89
		1091	3.250	1.82
	S/C=2.00	925	1.900	1.95
		573	1.020	1.94
	W/C=0.40	488	1.200	1.83
		751	1.540	1.79
	VF =3.00%	593	0.600	2.26
		789	1.365	2.08
		482	0.665	2.06
	Aver.	706	1.426	1.96
	SD	204	0.795	0.15
18		1093	1.150	2.17
	S/C=2.00	1262	1.180	2.18
		714	0.870	1.97
	W/C=0.40	1342	0.925	2.21
		1675	1.730	2.23
	VF =2.00%	1542	1.300	2.22
		1903	2.820	2.21
		1311	1.610	2.12
	Aver.	1353	1.448	2.16
	SD	361	0.629	0.08
19		1279	1.500	2.20
	S/C=2.00	843	0.750	2.15
		1325	2.625	2.18
	W/C=0.40	897	2.750	2.19
		928	1.050	2.11
	VF =2.00%	1060	0.850	2.18
	Aver.	1055	1.587	2.17
	SD	204	0.891	0.03

Appendix C1 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
p-20		1358	3.450	2.05
		962	2.740	1.92
	S/C=2.00	1022	3.500	1.98
		961	1.500	2.17
	W/C=0.70	901	1.780	2.12
		1052	1.475	2.07
	VF=2.00%	889	2.460	2.02
		751	1.700	2.03
		625	1.000	1.97
	Aver.	947	2.178	2.04
	SD	204	0.900	0.07
21		1756	1.245	2.19
	S/C=1.00	1618	2.575	2.18
		1658	2.200	2.21
	W/C=0.40	1318	1.580	2.27
		1847	3.835	2.17
	VF=2.00%	1519	1.470	2.23
		1334	1.350	2.26
	Aver.	1579	2.036	2.22
	SD	201	0.929	0.03
22		1242	1.750	1.97
		1090	1.320	1.97
	S/C=3.00	1336	0.880	2.02
		1296	0.775	2.00
	W/C=0.40	1498	1.550	2.09
		1590	1.600	2.16
	VF=2.00%	1628	1.600	2.09
		1255	1.750	2.10
		836	1.310	2.01
	Aver.	1308	1.392	2.04
	SD	249	0.357	0.06

Appendix C2 Flexural Test Results of the Body Experiment

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
		1124	3.155	2.13
M-1	S/C=2.57	1005	2.125	2.15
		562	2.025	2.04
	W/C=0.59	669	0.850	2.01
		776	0.815	2.02
	VF =1.39%	591	1.195	2.04
		1012	2.180	2.11
		710	1.500	2.10
	Aver.	806	1.730	2.07
	SD	213	0.795	0.05
		1411	2.500	2.12
2	S/C=1.38	1415	2.050	2.09
		1345	3.830	2.02
	W/C=0.55	1563	4.950	2.14
		1173	3.300	2.07
	VF =3.00%	1181	3.420	1.99
	Aver.	1348	3.341	2.07
	SD	150	1.020	0.05
		1115	2.440	2.05
3	S/C=1.88	1279	4.550	2.03
		885	3.995	2.00
	W/C=0.69	936	4.910	1.96
		1114	3.560	1.92
	VF =3.37%	1083	1.625	1.95
	Aver.	1069	3.513	1.98
	SD	141	1.263	0.04

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-4		1378	5.410	1.93
	S/C=1.50	1279	4.125	1.93
		1367	5.750	2.02
	W/C=0.66	1082	2.245	2.03
		1242	4.870	1.92
	VF =1.88%	1082	2.110	1.94
		791	2.115	1.98
	Aver.	1174	3.803	1.96
	SD	207	1.620	0.04
5		1377	3.065	1.90
	S/C=0.40	972	1.820	1.94
		1199	1.175	1.82
	W/C=0.59	931	1.690	1.89
		1377	3.230	1.83
	VF =1.50%	972	1.300	1.81
		1414	2.685	1.85
	Aver.	1177	2.137	1.86
	SD	216	0.844	0.04
6		799	1.120	2.15
		656	1.750	2.05
	S/C=2.33	596	2.125	2.03
		669	2.170	2.13
	W/C=0.68	871	2.465	2.17
		799	2.130	2.20
	VF =1.68%	757	2.750	2.07
		624	1.065	2.06
		897	2.070	2.15
	Aver.	741	1.960	2.11
	SD	109	0.563	0.06

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-7		908	0.325	2.12
		872	0.550	2.15
	S/C=1.50	981	0.570	2.12
		908	1.680	2.11
	W/C=0.63	981	1.075	2.12
		828	1.210	2.07
	VF =0.48%	931	1.230	2.08
		872	0.980	2.12
		727	0.855	2.08
	Aver.	890	0.941	2.11
	SD	79	0.418	0.02
8		1484	2.950	1.95
	S/C=0.40	1378	3.860	1.95
		1968	6.715	1.95
	W/C=0.54	2034	5.135	1.92
		1378	2.850	1.92
	VF =2.28%	1181	1.655	1.95
		1279	2.455	1.88
	Aver.	1529	3.659	1.93
	SD	336	1.740	0.02
9	S/C=2.40	1022	3.775	2.08
		880	1.155	2.09
	W/C=0.56	691	1.635	2.07
		751	3.745	2.12
	VF =2.61%	1022	4.500	2.11
		841	1.450	2.16
	Aver.	868	2.709	2.10
	SD	136	1.454	0.03

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-10	S/C=0.59	727	3.040	1.73
		727	2.000	1.82
	W/C=0.65	587	0.740	1.82
		743	1.120	1.90
	VF =0.91%	654	1.635	1.81
		734	2.115	1.76
	Aver.	695	1.774	1.81
	SD	61	0.810	0.05
11		1126	0.680	2.25
		1655	2.000	2.31
	S/C=1.77	1558	2.210	2.39
		1101	1.185	2.34
	W/C=0.40	1209	0.630	2.30
		1115	0.690	2.27
	VF =0.78%	1224	0.990	2.36
		1211	1.040	2.33
		1038	0.650	2.34
	Aver.	1249	1.119	2.32
	SD	212	0.594	0.04
12		824	1.375	1.98
	S/C=2.40	1435	3.620	1.92
		1083	1.830	1.90
	W/C=0.45	1683	3.730	1.88
		1741	3.390	1.96
	VF =3.68%	1943	3.835	1.92
		947	1.120	1.93
	Aver.	1379	2.699	1.93
	SD	433	1.202	0.03

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-13		727	1.805	1.97
	S/C=0.30	766	1.815	2.01
		890	2.145	1.97
	W/C=0.60	920	1.470	1.95
		908	3.255	1.92
	VF =1.11%	1018	3.120	1.96
		799	0.760	1.88
	Aver.	861	2.052	1.95
	SD	101	0.887	0.04
		1359	0.985	2.26
14	S/C=2.57	961	1.200	2.21
		749	1.350	2.15
	W/C=0.47	853	0.880	2.22
		1148	0.710	2.19
	VF =0.99%	1308	1.285	2.34
		1173	1.060	2.26
		1148	0.980	2.26
	Aver.	1087	1.056	2.24
	SD	214	0.214	0.05
		787	2.750	2.02
15	S/C=1.11	1050	3.775	2.05
		1104	4.640	2.07
	W/C=0.66	964	1.265	2.17
		987	6.350	1.99
	VF =1.91%	828	2.940	2.07
		994	3.465	2.03
	Aver.	959	3.597	2.06
	SD	114	1.597	0.05

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-16		1072	1.570	1.87
		787	2.670	1.88
	S/C=0.70	872	2.285	2.00
		945	3.515	1.98
	W/C=0.69	763	1.720	1.97
		828	2.450	1.95
	VF =1.50%	837	1.975	1.92
		774	2.300	1.98
		1112	5.195	1.99
	Aver.	888	2.631	1.95
	SD	128	1.118	0.04
17		754	0.725	2.05
		1035	1.460	2.15
	S/C=1.38	853	1.455	2.08
		1017	1.350	2.08
	W/C=0.57	918	1.840	2.04
		966	1.070	2.10
	VF =0.50%	1035	0.650	2.07
		918	0.625	2.07
		931	1.700	2.09
	Aver.	936	1.208	2.08
	SD	91	0.459	0.03
18		982	1.150	2.10
	S/C=2.57	863	2.365	1.98
		1017	1.875	2.15
	W/C=0.42	982	1.485	1.98
		843	1.300	2.01
	VF =2.91%	859	1.165	2.33
		729	1.365	2.10
	Aver.	896	1.529	2.09
	SD	102	0.442	0.12

Appendix C2 continued

Specimen No.	Material Composition	Flexural Strength (psi)	Toughness (Pound-inch)	Density
M-19		964	1.435	2.26
	S/C=2.94	843	0.875	2.07
		881	1.190	2.19
	W/C=0.47	803	1.365	2.13
		704	1.230	2.10
	VF =1.12%	774	1.030	2.15
		845	1.425	2.22
	Aver.	831	1.221	2.16
	SD	82	0.210	0.06
20		1500	2.775	1.99
	S/C=0.59	2363	3.330	1.92
		1377	6.150	1.97
	W/C=0.53	1634	3.365	2.14
		2001	5.800	1.80
	VF =3.20%	1622	5.965	1.81
		1620	3.420	1.92
	Aver.	1731	4.400	1.93
	SD	337	1.488	0.11

Mortar Control

Appendix C3 Corrosion Experiment: Flexural Test Results

Cycle	Flexural Strength (psi)	Toughness (Pound-inch)	Cycle	Flexural Strength (psi)	Toughness (Pound-inch)
0	1174	0.080	8	1150	0.120
	1150	0.060		1080	0.140
	981	0.110		941	0.100
	1094	0.135		828	0.130
	1154	0.080		836	0.100
	953	0.115		836	0.110
	1043	0.100		880	0.110
	1094	0.150		767	0.090
	1173	0.075		1018	0.100
Aver.	1091	0.100	Aver.	926	0.111
SD	82	0.029	SD	130	0.016
2	920	0.115	11	1267	0.200
	997	0.090		941	0.120
	1073	0.100		1226	0.205
	1188	0.100		1104	0.190
	1073	0.105		1160	0.170
	1112	0.150		861	0.105
	1188	0.125		1322	0.125
	1112	0.080		1259	0.160
	1035	0.075		999	0.150
Aver.	1078	0.104	Aver.	1127	0.158
SD	86	0.023	SD	161	0.036
5	1070	0.095			
	897	0.125			
	1000	0.090			
	931	0.150			
	966	0.100			
	1139	0.125			
	1070	0.100			
	897	0.100			
	1000	0.130			
Aver.	997	0.112			
SD	83	0.020			

Appendix C3 continued
Mortar Corrosion

Cycle	Flexural Strength (psi)	Toughness (Pound-inch)	Cycle	Flexural Strength (psi)	Toughness (Pound-inch)
2	1468	0.150	8	1093	0.070
	1358	0.125		1093	0.125
	1380	0.125		1082	0.100
	1311	0.140		1226	0.110
	1311	0.110		1181	0.110
	1495	0.115		1017	0.100
	1457	0.175		1126	0.090
	1342	0.125		1148	0.135
	1199	0.120			
			Aver.	1121	0.104
Aver.	1369	0.131			
			SD	64	0.020
SD	93	0.020			
5	1150	0.120	11	1054	0.205
	1317	0.115		1371	0.150
	1252	0.105		1303	0.165
	1468	0.120		1227	0.135
	1214	0.150		1188	0.160
	1303	0.100		1227	0.150
	1342	0.175		1227	0.120
	1317	0.120		1215	0.150
	1028	0.105		1073	0.165
Aver.	1266	0.123	Aver.	1209	0.155
SD	125	0.024	SD	99	0.023

GRC Control

Appendix C3 continued

Cycle	Flexural Strength (psi)	Toughness (Pound-inch)	Cycle	Flexural Strength (psi)	Toughness (Pound-inch)
0	1051	1.675	8	1622	3.975
	1279	1.950		1358	2.600
	1155	2.325		1745	1.920
	1124	3.465		1799	2.480
	1030	2.565		1199	1.975
	936	2.390		1490	3.550
	1618	3.865			
	1167	1.220	Aver.	1536	2.749
	1155	1.790	SD	231	0.839
Aver.	1168	2.360			
SD	194	0.850			
2	1706	3.850	11	1176	0.900
	1542	2.570		1163	2.650
	1725	4.315		981	2.300
	1410	2.800		1403	3.000
	1181	1.620		1417	3.000
	1246	1.490		927	1.350
	1252	2.130		1115	1.800
				2363	4.450
Aver.	1438	2.682			
SD	224	1.073	Aver.	1318	2.431
				456	1.114
5	1575	1.815			
	1443	2.165			
	1563	1.500			
	1725	2.840			
	1293	3.200			
	1690	2.600			
	1561	3.295			
Aver.	1550	2.487			
SD	146	0.686			

Appendix C3 continued

Cycle	Flexural Strength (psi)	Toughness (Pound-inch)
2	1808	3.535
	1378	2.870
	1279	2.615
	1050	1.770
	1246	1.310
	1181	2.590
	1378	1.410
Aver.	1331	2.299
SD	239	0.825
5	1147	1.025
	1568	1.440
	977	1.825
	1193	0.835
	1325	0.750
	1093	1.090
	1557	2.020
Aver.	1266	1.283
SD	228	0.491
8	1345	1.020
	1590	2.050
	1406	2.325
	1476	1.250
	1181	0.900
	1163	1.250
	1536	0.850
	1762	1.920
	1499	1.900
Aver.	1440	1.496
SD	191	0.554
11	1255	0.815
	1138	0.740
	1372	1.150
	1346	0.880
	1150	0.710
	1252	0.525
	1268	0.400
	1285	0.775
	1342	0.500
Aver.	1267	0.721
SD	82	0.226

Appendix C 4 Corrosion Experiments : Pull-out Test Results

0 Cycle Control

Specimen No.	Maximum Stress (LBS)	Remarks
1	3.4	Pulled out of Intact
2	5.3	"
3	5.0	Strands broken
4	6.9	"
5	5.8	"
6	9.2	"
7	6.8	"
8	7.2	"
9	5.2	"
10	4.8	"

2 Cycle Control

Specimen No.	Maximum Stress (LBS)	Remarks
1	4.4	Strands broken
2	5.4	Strands split out and broken
3	5.1	Strands broken
4	7.9	"
5	6.0	"
6	4.5	Strands split out and broken
7	6.7	"
8	6.2	Strands broken
9	4.4	"
10	7.9	"

Appendix C 4 continued

2 Cycle Corrosion

Specimen No.	Maximum Stress (LBS)	Remarks
1	6.2	Strands split out and broken
2	4.8	Strands broken
3	5.6	Strands pulled out of Intact
4	3.1	Strands split out and broken
5	7.6	"
6	3.4	"
7	5.0	"
8	6.8	Strands broken
9	5.1	"
10	5.3	Strands split out and broken

14 Cycle Corrosion

Specimen No.	Maximum Stress (LBS)	Remarks
1	0.5	Strands broken
2	3.6	"
3	6.2	Strands split out and broken
4	1.1	Strands broken
5	5.9	"
6	1.3	"
7	4.2	Strands split out and broken
8	0.7	Strands broken
9	4.0	Strands split out and broken