COMPUTER-AIDED TEACHING

OF

DYNAMIC SYSTEM BEHAVIOR

by

Ronald C. Rosenberg

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Signature redacted

Signature of Author . . . . . .
Department of Mechanical Engineering, September 1, 1965.

Signature redacted

Certified by. . . . . .
Thesis Supervisor

Signature redacted

Accepted by . . . . . .
Chairman, Departmental Committee on Graduate Students

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Submitted to the Department of Mechanical Engineering on September 1, 1965, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

An algorithmic procedure for the analysis of linear lumped-parameter dynamic systems based on bond graph notation and a state space approach is presented. Graphical procedures for transforming electrical circuits, mechanical schematics, and other notations directly into bond graphs are given.

A Dynamic Systems Laboratory, which is based on a time-shared digital computer and implements the analysis procedure mentioned above, is presented. The Laboratory simulates the behavior of linear dynamic systems specified as bond graphs. The dynamic response variables are available as tables or plots, and rapid access response plots are generated on a display set up by a digitally-controlled analog computer.

A teaching experiment which used the Dynamic Systems Laboratory as the basis of a self-instructional system for subjects inexperienced in the behavior of dynamic systems is reported. Subjects were able to develop an understanding of first and second order behavior based on experimentally derived relations. However, the process was inefficient. A test facility, in which subjects tested and modeled unknown (black box) systems, was found to be a useful teaching tool.

Thesis Supervisor: Thomas B. Sheridan

Title: Associate Professor of Mechanical Engineering

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And, since man cannot live by ideas alone, the sponsors of this research.

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THE TEACHING OF DYNAMIC SYSTEM BEHAVIOR

1.10 Introduction

1.11 Research in the Teaching of Engineering

There is at the present time a generally recognized need for continued innovation in the teaching of engineering. Two particular factors contribute to this need -- the frequent modification of engineering curricula in the wake of rapid advances in the state of knowledge, and the increasing numbers of engineers who must be educated to a high level of competence. In studying ways to systematically improve the teaching of engineering considerable difficulty arises due to the absence of a generally accepted theory of learning upon which a theory of teaching could be based. Thus, although there are many teaching experiments being carried out currently, very few are reported upon in a form such that the various results may be related and generalizations made. One of the most widespread types of experiment is the development of new courses (see, for example, reference 33), which reflect a new organization of material. Another type of innovation involves the use of programmed instruction (5; 7; 8)*. Texts have been fully or partially programmed (10; 25), and computers and programmed instruction have been combined to make sophisticated teaching machines (1; 13; 15; 16). Other types of teaching experiments have used modified administrative arrangements for teaching courses in an effort to influence student-instructor contact to improve learning.

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* Numbers in parentheses refer to the bibliography.

1.12 Objectives in Teaching Dynamic System Behavior

In order to conduct certain studies in teaching in a specific context, the subject of lumped-parameter dynamic systems behavior was selected. This is a "core course" in many engineering curricula, including the electrical, mechanical, and chemical engineering disciplines, among others. It frequently is encountered early in the engineer's education, and serves as a basic element in his evolving view of the physical world. One of its most demanding pedagogical characteristics is that a dynamic systems course tries to make both physics and mathematics meaningful to the engineer, by ranging from the modeling of physical systems to the analysis of their behavior as derived from mathematical models.

Typical objectives of an introductory dynamics course (as reflected by homework problems and examinations) are (1) to impart a knowledge of a certain notation or notations (e.g. electrical circuit symbolism), (2) to teach mathematical manipulations within, and in forms derived from, that notation, often in the form of the analysis of equations (e.g. complex variables in linear analysis), and (3) to permit the student to model aspects of the physical world according to the concepts embodied in the notation(s).

The manner in which material is presented also influences the behavior of the student at, we might say, one higher behavioral level. Thus the noted psychologist Piaget has remarked (21):

> "'The question comes up whether to teach the structure, or to present the (subject) with situations where he is active and creates structure himself...The goal in education is not to increase the amount of knowledge, but to create the possibilities for a (subject) to invent and discover...Teaching means creating situations where structures can be discovered...'"

On the other hand, in an interesting study by Wolfe (35) the following

view is attributed to D. P. Ausubel:

> "'...I would conclude that secondary school and college students who already possess a sound, meaningful grasp of the rudiments of a discipline like mathematics can be taught this subject meaningfully, and with maximal efficiency through the method of verbal exposition, supplemented by appropriate problem solving experience; and that the use of the discovery method in these circumstances is inordinately time-consuming, wasteful, and rarely warranted.'"

In teaching dynamics, where part of the student's education involves modeling and part involves manipulation within a well-defined framework, the specification of such objectives is a very difficult task.

1.13 The Plan of This Study

At this point the reader will be introduced to the general plan of this study, which is concerned with the teaching of dynamic systems behavior and the related role of computers.

The balance of this chapter will consider the organization of material and its presentation by computer. Chapter 2 introduces a new notation for the description of dynamic systems, discusses its use in modeling and then presents an analysis procedure for linear systems based on the notation. In Chapter 3 a computer-based Dynamic Systems Laboratory is described, and some of its teaching properties are presented. The plan and results of a teaching experiment using the Dynamic Systems Laboratory are described in Chapter 4, and conclusions and recommendations for further research are stated in Chapters 5 and 6 respectively.

1.20 The Selection and Organization of Material

Many different organizations of material are in use in the area of dynamic systems. The bibliography of this thesis contains several references

in different engineering fields, (e.g. 18; 22; 23; 25) and each approach is unique. A few common factors do appear among the various examples, such as an immediate and sustained emphasis on linearity, use of a graphical notation for system description, and an analytical approach to system behavior. Generally, the degree of systems considered in depth is limited to the second order, with selected aspects of higher order behavior considered. The emphasis is placed upon signal variables (e.g. voltage, force, velocity, current) almost from the start, with energy, work, and power being given an initial introduction and then discarded.

The usual reason for the use of a linear systems approach in spite of the inherent non-linearities in physical systems is that it permits a neat, analysis-oriented body of mathematical methods to be developed and exploited. The following sections describe the motivation for the particular organization of material adopted in this study.

1.21 An Energetic Systems Basis

In order to create a strong link between the physical word and the models of it to be studied, it seemed appropriate to use as the basis of all subsequent development the concept of a dynamic system in a state of power balance. That is, the prototype system was initially isolated from its environment by defining all energy exchanges, and then the divergence theorem was invoked. The student must accept certain conservation properties for energy, but they were rather familiar to him because of his experience with the conservation of mass. One advantage of the energy approach is that it applies to all physical engineering domains simultaneously, making generalizations of certain types very easy, even for mixed systems (e.g. electro-mechanical).

Time was chosen as the "sweep" variable, rather than frequency, because of its general familiarity, and because it is most relevant to typical engineering problems outside the electrical domain.

1.22 State Space

The problem of relating a general energetic definition of a system to a particular finite set of observable quantities was handled by using the concept of state space. In developing a model one may view shortcomings in the predictions of behavior as the lack of a sufficient state description, and so add suitable variables. In addition the entire dynamic behavior may be considered as the motion of a point in the state space whose trajectory is governed by the model structure and an initial state. The net result is to provide a framework for grasping the entire system behavior, rather than relations between pairs of variables. For linear systems the selection of state variables plays a less critical role than it does for general non-linear systems, but it still retains its important attribute of displaying the interrelation of the entire system, usually in a set of physically "mixed" variables (e.g. voltages and currents).

1.23 Notations

The selection of a notation is of great importance in teaching about dynamic system behavior. For an interesting discussion of some functions of a mathematical notion in assisting or hindering understanding the reader is referred to Cajori's work, A History of Mathematical Notations (reference 19). A. N. Whitehead, as quoted by Cajori (19, p. 333),* had this to say:

---

*(19, p. 333) means reference 19, page 333.

> "...By the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism, repeated by all copy-books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them."

Figure 1.1 presents examples of several notations which are physically motivated by particular disciplines (such as electrical circuit symbols), followed by a group of abstract notations. The latter group is meant to be illustrative, not exhaustive, and includes a bond graph, which will be introduced more formally below.

Some desirable properties of a notation for dynamic systems are:

(1) that it be graphical, in order to retain and make clear the structural relations among the elements of the system;

(2) that it preserve some general relation to physics, for example, by embodying the conservation of energy law;

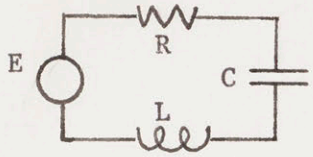
(3) that the notation be compatible with the state-space approach; and

(4) that it be transformable in some reasonable way to a set of equations.

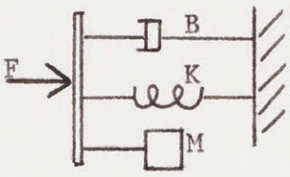
A notation which possesses all of these properties (to a greater or lesser extent) exists, and is called the bond graph notation. It was invented by Prof. H. M. Paynter of M.I.T., and many aspects of the language not discussed in this work are presented in his book The Analysis and Design of Engineering Systems (31). Chapter 2 of this study is given to a presentation and discussion of bond graphs in considerable detail, with particular emphasis on teaching applications.

Some of the advantages and disadvantages of bond graphs will be presented here, but the interested reader is urged to consider the points raised again after reading Chapter 2. Among the positive features of bond graphs

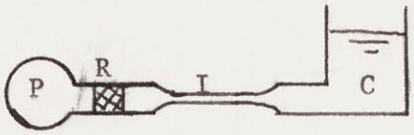
(1) <u>Physical notations</u> - schematics.



(A) electrical

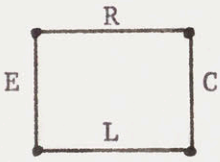


(B) mechanical

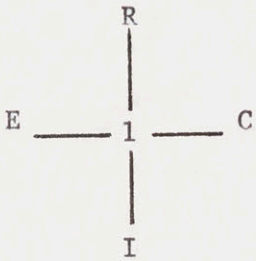


(C) fluid

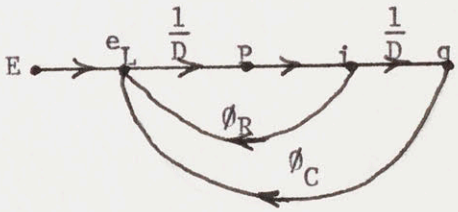
(2) <u>Abstract notations</u> - graphs and equations.



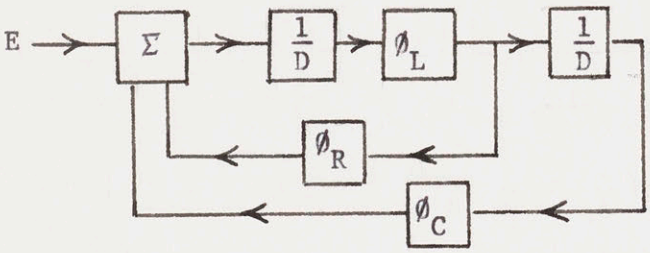
(D) circuit graph



(E) bond graph



(F) signal flow graph



(G) block diagram

$$E + e_R + e_C + e_L = 0$$

$$e_R = \phi_R(i)$$

$$e_C = \phi_C(\frac{1}{D} i)$$

$$i = \phi_L(\frac{1}{D} e_L)$$

(H) differential equations

Fig. 1.1. A Comparison of Several Notations for a Sample Problem

are (1) that it is a graphical notation (in fact, bond graphs are linear graphs in the common topological sense); (2) that the interconnectedness of elements is explicitly shown (permitting, for example, the creation of the structural dual of a system in a simple way); (3) that any bond graph will preserve a power balance, unless explicit steps are taken to destroy such a condition (for example, by deleting the influence of a variable); (4) that for predominantly passive systems the line code for the graph (see Section 3.11) is a succinct one, becoming minimal for purely passive systems; and (5) that the bond graph may be used as an aid in organizing system equations (see Sections 2.32 and 2.33).

Some disadvantages of bond graphs are (1) that the physical imagery in relation to many specific disciplines is more abstract than existing notations; (2) that, for systems which are predominantly active, bond graphs are not necessarily as efficient as signal oriented graphs (e.g. block diagrams); and (3) that bond graphs are generally unknown, and their theory is mainly undeveloped as yet.

Bond graph notation was selected for use in this thesis primarily for its energetic, structural, and succinctness properties, and considerable effort was devoted to developing procedures which would make the bond graph useful in organizing system equations.

Having described a general point of view on the selection and organization of material for teaching dynamic systems behavior at an introductory level, let us now turn our attention to the problem of presentation -- how can these concepts be communicated to a student?

1.30 The Presentation of Material

In the conventional approach to teaching dynamic system behavior there are four major forms of presentation used. These are lectures, recitations, texts, and home problems. On occasion other aids are used in conjunction with these, such as demonstrations on a computer, or the use of "breadboard" or portable laboratory kits, or partially programmed texts (25). It is typical to plan the organization of material with these forms in mind, and to seek new presentation forms in a rather specific context; e.g. "How can I show superposition more dramatically?"

In the context of dynamic systems the question may be asked -- are there other ways to present the material, such that either the level of achievement is raised, or the process of teaching is made more efficient? One such way is to consider the use of a teaching machine, particularly at the level of using a high-speed digital computer to perform many of the decision and other logic functions. In the words of Dr. J. C. R. Licklider (6, p. 216) --

> "I should like to pick up this thought about the pedagogical use of computers. The main thing, I think, that has come out of the study of teaching machines (a la Fred Skinner at Harvard), is that the only way to get knowledge and wisdom into the human brain is to put it in through a very extensive process.
>
> "Let me explain. You can take a field of mathematics and axiomatize it and get it down to a static nutshell: a beautiful kernel, a thing that a mathematician thinks is just lovely. And maybe you can get somebody to memorize it; but if he does so, he has not learned a thing. The only way to get it into the person is to blow it up into 50,000 operations and teach it from many points of view, using many examples. Finally, it gets down into his brain in a little kernel: a beautiful pearl of knowledge.
>
> "This process is just killing us. We don't have enough teachers to handle it. University professors leave universities because of it. Computing machines...can do this kind of thing rapidly, economically, patiently, and without the frustration and the unhappiness. I think the computer offers a real match to the problem of getting knowledge into human skulls."

1.31 Computer-Aided Teaching

Many roles have been assigned to computers in the field of engineering education, but we are interested in considering the applications of time-shared or multiple-access computers, such as the Project MAC* machine.

Computers have been used as aids in the classroom, as teaching machines for providing individual instruction (1; 15; 16) and as research tools in the study of the teaching process (3). An opinion on what computers should be used for is given by May (10, p. 22) when he says--

> "...Experiments in automation should concentrate on devices that maximize individualization for the single student (e.g., computer based instruction) and, for group instruction, that extend the teacher's range and the degree of student involvement (e.g., multi-media presentation systems and classroom communicators)."

Another function of potentially great value in teaching is that of simulation by computers. On this point May (op. cit., p. 15) comments--

> "One kind of teaching machine that has proved itself is the simulator of environments in which the student needs practice... Such teaching machines are very specialized. Their high cost is balanced by the still higher cost of the real thing."

1.32 A Teaching Machine Model For Dynamics

This thesis will concentrate on the development and use of a computer-based teaching machine for individual instruction. Stolurow has discussed the principal functions which a fully automated device should perform (14, p. 6-14). The discussion below follows his presentation outline, but modifies the content of some of the functions to apply specifically to a teaching machine for synamic systems.

Fig. 1.2 is adapted from Stolurow (op. cit., Fig. 2) with the addition of the simulator (for linear lumped dynamic systems) and the deletion of a

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*See reference 11, Chapter 8 for a description of the MAC system.

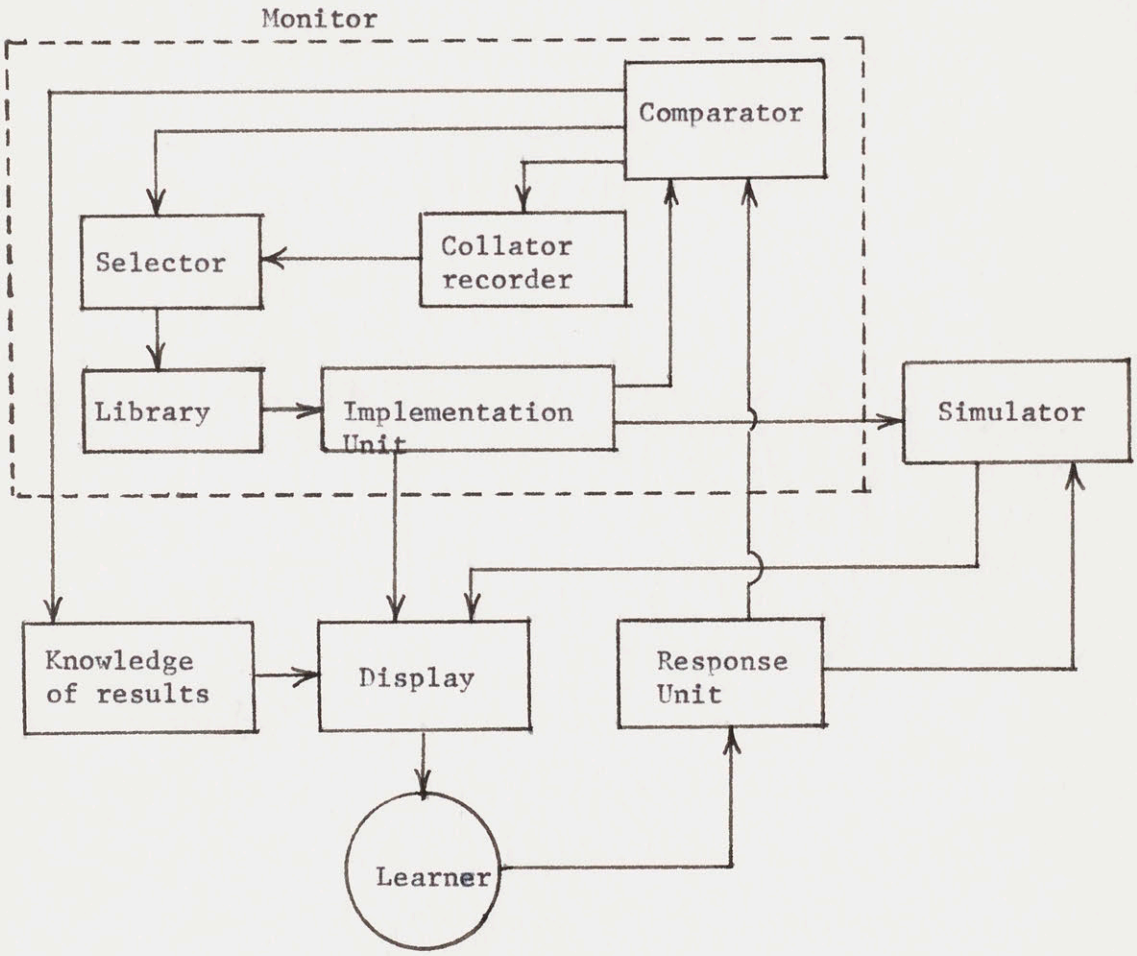


Fig. 1.2. A Teaching Machine Model for Dynamics

(After Stolurow, ref. 14, Fig. 2.)

pacer-timer function. Notice particularly the functions which are included as part of the monitor. The monitor (or teaching monitor) will be referred to in subsequent discussion.

(1) Display. The display should be capable of presenting tables of data, plots of response versus time for system variables, and various amounts of printed information, some of it in predetermined form and some of it more flexibly. An ability to present linear graphs is also desirable.

(2) Response. The primary form of input information from the student is alpha-numeric, and a keyboard type of unit is adequate. Some capability to draw linear graphs is useful, but not essential.

(3) Knowledge of Results. There are two aspects to this function. One is the form of the information, ranging from a single binary response (e.g. right/wrong) to a rather detailed assessment of the student's prior response (e.g. "The resistance value should be known to you from the steady-state data."). The other aspect is the schedule of reinforcement (i.e. how often knowledge of results is given to the student). It is reasonable to expect a new student to require fairly frequent (perhaps continuous) reinforcement, whereas an experienced student may be taught how to guide himself by a sparse, intermittent reinforcement schedule. In the realm of programmed instruction this is a most important question.

All of the preceding functions should be present in a teaching machine. Some of the following items may be provided by the learner himself.

(4) Simulator. The function of this element is to make available to both the learner and the teaching monitor the dynamic behavior of as broad a class of systems as possible.

(5) Comparator. The comparator assesses the status of the learning situation after a response has been made, on the basis of information from the

implementation unit, the simulator, and the response unit. It, in turn, communicates to the selector and collator-recorder.

(6) Collator-recorder. The function of the collator-recorder is to provide historical data to the selector, and also to the researcher for studies of the performance of the teaching machine system.

(7) Selector. The selector is the unit which determines to a significant extent how individually adaptive a teaching machine will be. For example, Smallwood (13) has developed a rather sophisticated decision structure (which is computer-based), suitable for use with a general organization of material. At the other end of the spectrum, one may have a single constant decision; e.g. go on to the next step.

(8) Library. Some bulk storage of information is required, and may be implemented by devices ranging from a book to a random-access slide projector under computer control.

(9) Implementation. The implementation unit must effect the decisions of the selector by causing appropriate material to be displayed. It may also initiate activity in the simulator, and it informs the comparator of the current status of the monitor.

There are several concepts of how to organize and present material by machine (or other techniques) in a fashion designed to achieve individual optimum learning efficiency. For example, styles range from using small units of information (e.g. single sentences), each of which requires some response, to presenting pages of text followed by set of exercises. The general type of organization is referred to as the teaching program (5; 7; 8; 9).

1.33 Implementation of the Model as the Dymanic Systems Laboratory

A realization of the general teaching machine model with limited functions has been constructed. It is called the Dynamic Systems Laboratory (or DSL). Most of the monitor functions are performed by the student. The principal development effort has been directed toward the simulator, the display, the comparator in part, and the implementation function in part. In certain cases, considerable attention has been paid to the communication channels themselves, for example, the link between the implementation unit and the simulator.

(1) Display. A standard teletype printer provides for the display of tables, plots, and short typed messages. For more rapid presentation of system behavior in the form of traces of variables versus time, a small analog computer is set up by the monitor (the implementation unit) and its variables displayed on an oscilloscope. The bulk of the textual information is contained in a guidebook, including the questions and problems.

(2) Response. The teletype keyboard is a very adequate response unit.

(3) Knowledge of Results. The simulator and guidebook both provide results, and the student provides his own confirmation.

(4) Simulator. Simulation is accomplished by a computer-based system called Enport, and is discussed in Chapter 3.

(6) Collator-recorder. A complete record of the learning data is available on the teletype logsheets for the researcher's use. The data is not available for a monitor in the DSL.

(8) Library. A guidebook used by the student served as the library.

The remaining functions -- the comparator, selector, and the essential parts of knowledge of results and implementation -- were performed by the student, who therefore served as his own teaching monitor. There were two

reasons for constructing the first version of the DSL in this way. One was to gather data on which to base the design of a completely automated (i.e. computer-controlled) teaching system, and the second was to collect information on the range of monitor behaviors exhibited by students, since this information generally becomes inaccessible when an automated monitor is provided. Chapter 4 discusses a teaching experiment which used the DSL in "open-loop" fashion (i.e. with the students serving as their own monitors).

2. THE USE OF BOND GRAPHS IN STUDYING DYNAMIC SYSTEM BEHAVIOR

This chapter will discuss bond graphs from an operational point of view. The items to be considered are definitions of the elements of bond graphs, some modeling procedures for dynamic systems using bond graphs, and an analysis procedure based on them. The reader who is interested in more detail about the physical and mathematical bases for bond graphs will find it in reference 31. I am indebted to Dr. A. MacFarlane of Queen Mary College, London, for many stimulating suggestions about the material contained herein.

2.10 An Introduction to Bond Graphs

2.11 The Power and Energy View of Systems

A system may be separated from its environment by listing all the places (or ports) at which energy may be exchanged.

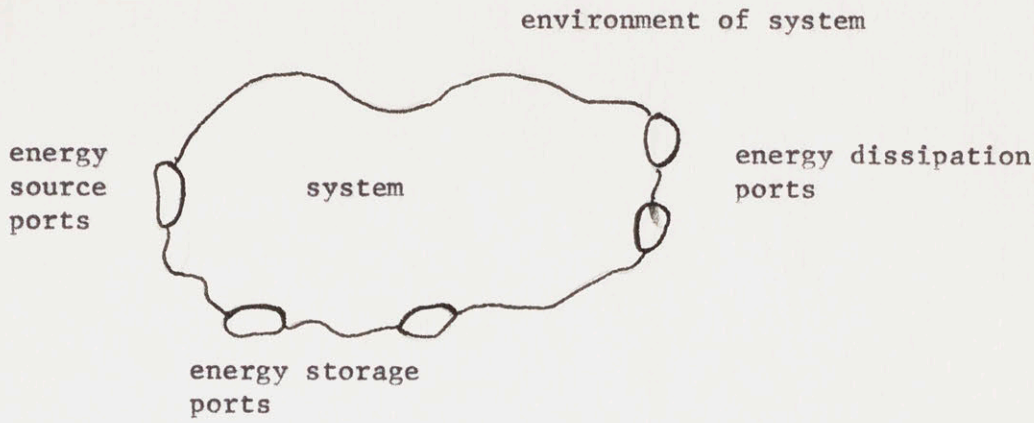
In making such a definition of a system both energy and power are useful, and their relationship is given by $P(t) = \frac{dE(t)}{dt}$. We wish to structure our system such that it is in state of power balance at each instant, by which we mean that the free energy of the system is always accounted for by considering all the port energy transactions. There are three types of transactions to be considered:

(1) energy storage -- at certain ports energy is being stored or retrieved, but the process is conservative in the long run;

(2) energy dissipation -- at some ports energy leaves the system in a manner such that it is not recoverable at those ports;

(3) energy sources -- at source ports the environment is capable of supplying energy to the system in the long run.

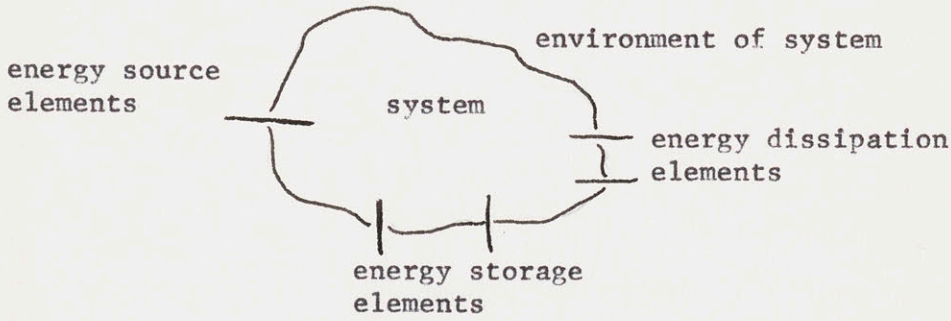
At this level a system could be defined by its energy ports as shown below.



That part of the system within the boundary is capable only of routing or transferring the energy among the various ports, so that the power balance property will hold.

2.12 The Power Bond

If instead of viewing energy exchanges (or powers) as going through ports, we show such exchanges by links (or lines) called <u>power bonds</u>, the nature of the previous figure changes graphically as shown below.



Each power bond indicates a connection between two elements, where the elements will be given precise definitions in 2.13.

Powers have been factored into signal (or power factor) quantities in various ways, depending upon both the physical discipline (e.g., mechanics, electrical circuits) and the type of medium (e.g., wave scattering variables,

lumped quantities). For the class of systems of interest in this thesis the signal variables associated with power will be taken as <u>effort</u> and <u>flow</u>. As shown in Fig. 2.1 below, effort and flow are generalized variables for common signal quantities in several fields.

Bond Graph	Network	Mechanical Translation	Mechanical Rotation	Fluid Systems
effort	voltage	force	torque	pressure
flow	current	velocity	ang. velocity	volume flow

Fig. 2.1. Some Effort-Flow Pairs

There are three quantities associated with a power bond in a bond graph, two of which are shown explicitly (effort and flow), and the third one of which is given by the relation $P(t) = e(t) \cdot f(t)$. In Fig. 2.2 the notation convention for displaying efforts and flows is shown.

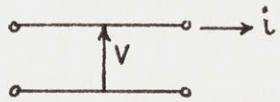
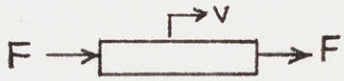
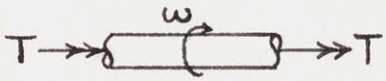
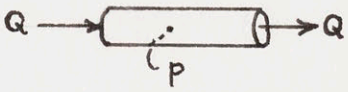
Fig. 2.2. Effort-Flow Notation Convention

Each bond displays two variables, as shown by some examples in Fig. 2.3. There are times when only one variable is significant, and the other variable is to be ignored. The step of <u>suppressing</u> a signal on a bond leads to a bond which is no longer passive. Hence it is called an <u>active</u> bond. If this step is taken, a bond graph no longer guarantees that a power balance exists within the system. To show the activation of a bond one crosses out the variable to

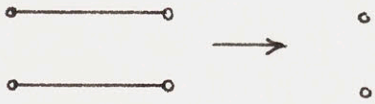
be suppressed. Thus

passive bond		active bond
$\frac{e}{f}$	suppress the flow, for example	$\frac{e}{fx}$

Further consideration will be given to the step of activation in sections 2.32 and 2.33.

An ideal electrical bond	
An ideal mechanical bond (bar)	
An ideal torsional bond (shaft)	
An ideal fluid bond (pipe)	

<u>Note</u>: At times these bonds may diminish to points or planes in the system.

e.g. 

All of the above examples are represented by the bond graph notation $\frac{e}{f}$.

Fig. 2.3. Some Examples of Bonds

2.13 Some Ideal Bond Graph Elements.

To complement the bond symbol in bond graphs we now introduce a set of node symbols which will be referred to as bond graph <u>elements.</u> Any element may be characterized in part by the number of bonds attached to it. Those for which there is one bond are called <u>one-ports,</u> since there is one energy

port or path between the element and its environment. For multi-port elements (i.e., more than one bond) the term ideal means that they obey the power balance rule. This will be stated more explicitly for each class of elements. For one-port elements the term ideal means that the generally defined energy characteristic is uniformly obeyed (i.e., no local deviations from the required property are permitted).

Before discussing the one-port relations we will find it useful to define two other variables derived from effort and flow. These are the generalized momentum, p, and the generalized displacement, q, respectively. We have

$$p = \int e dt \qquad \text{and } q = \int f dt$$

With reference to Fig. 2.1 and considering the mechanical translation column, it is seen that, with effort identified with force, p (momentum) is identified with impulse or momentum. Similarly, for flow identified with velocity, q (displacement) is identified with distance.

A. One-ports

Consider the general one-port element 'X', which is shown in a bond graph as $\frac{e}{f}$X. There are several types of useful constraints one could impose on the relation between e and f.

(1) Let $e = \phi(f)$, where ϕ is a static* (i.e., non-time dependent) function imposed by X. Provided that $\phi(e,f)$ exists only in the first and third quadrants of an e-f characteristic and that it is bi-unique** (a slightly too severe restriction; see 2.32) then the energy will be transferred into X, never to return to the system

---

* ϕ will be used consistently to represent a static function.

** This property makes the element state determined by e or f.

through that port again (the power is always positive into X). An element with such a characteristic is called a generalized <u>resistance</u>, and is written $\frac{e}{f}R$.

Examples of all the one-ports are shown in Fig. 2.5.

(2) Let $q = \phi(e)$. The quantity q comes from $\int f dt$, and is called the generalized <u>displacement</u>. An element with the same types of properties for $\phi(e,q)$ as given for $\phi(e,f)$ previously, (i.e., bi-uniqueness; defined in the first and third quadrants only) is called a generalized <u>capacitance</u>, and is denoted $\frac{e}{f}C$.

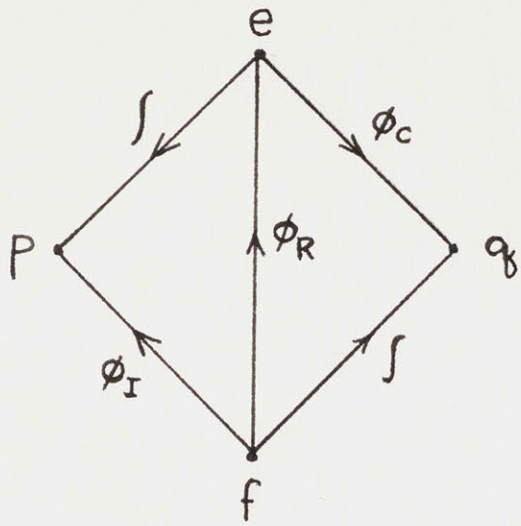
(3) Let $p = \phi(f)$. The quantity p comes from $\int e dt$, and is called the generalized <u>momentum</u>. An element for which $\phi(p,f)$ is bi-unique and exists only in the first and third quadrants of a p-f plot is called a generalized <u>inertance</u>, written $\frac{e}{f}I$.

These three element types may be represented in a tetrahedrom of state, which shows the relations described above. See Fig. 2.4.

(4) Let e(t) be a prescribed function of time, independent of f(t). Then the element X is referred to as an <u>effort source</u>, and is denoted by $\frac{e}{}E$.

(5) Let f(t) be a prescribed function of time, independent of e(t). Then the general one-port X is called a <u>flow source</u>, and is written $\frac{}{f}F$.

These five elements constitute the set of one-port elements used in this thesis. Several examples of these elements are given in Fig. 2.5.



<u>Capacitance</u>:	$q = \phi_C\ (e)$
Susceptance:	$e = \phi_S\ (q)$
<u>Inertance</u>:	$p = \phi_I\ (f)$
Fluance:	$f = \phi_F\ (p)$
<u>Resistance</u>:	$e = \phi_R\ (f)$
Conductance:	$f = \phi_G\ (e)$

Where e = effort

f = flow

$q = \int e\,dt$ = generalized displacement

$p = \int f\,dt$ = generalized momentum

and ϕ is a static function.

Fig. 2.4 Tetrahedron of State, Showing Characteristics, Resistance Capacitance, and Inertance

(after reference 31, p. 136 (figure unnumbered and untitled))

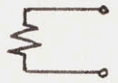
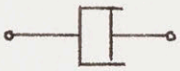
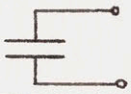
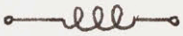
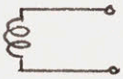
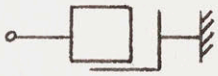
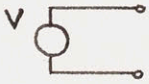
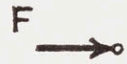
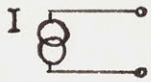
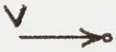
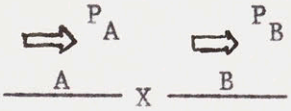
Element Type	Bond Graph	Electrical Circuit	Mechanical Translation
Variables	effort flow	voltage current	force velocity
1)Resistance	R —		
2)Capacitance	C —		
3)Inertance	I —		
4)Effort Source	E —		
5)Flow Source	F —		

Fig. 2.5. Examples of Ideal One-Port Elements

B. <u>Two-ports</u>

A two port element has two bonds. If the element is <u>ideal</u> (the only type we will consider), then the net power in at any instant vanishes.



$$P_A = P_B$$

$$e_A \cdot f_A = e_B \cdot f_B$$

Considering the case of the simplest linear two-port we may write a transmission matrix in the (acausal) form:

$$\begin{bmatrix} e_B \\ f_B \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e_A \\ f_A \end{bmatrix}$$

But, using the power constraint,

$$e_B \cdot f_B = (ae_A + bf_A)(ce_A + df_A)$$

$$= ace_A^{\ 2} + (ad + be)e_A f_A + bdf_A^{\ 2}$$

Thus, $ac = 0$, $ad + be = 1$, and $bd = 0$.

Two solutions are possible, namely,

(1) $a = 0$, $d = 0$, $c = 1/b$;

(2) $c = 0$, $b = 0$, $d = 1/a$.

(1) The first solution leads to a matrix of the following form,

$$M_{GY} = \left[\begin{array}{c|c} o & b \\ \hline 1/b & 0 \end{array}\right]$$

which is equivalent to the relations

$$e_A = b \cdot f_B \quad \text{and} \quad f_A = (1/b) \cdot e_B$$

The element with this property is called a <u>gyrator</u>, denoted

$$\text{———} \overset{GY}{\underset{b}{\cdot\cdot}} \text{———} \quad .$$

'b' is called the modulus of the gyrator.

(2) The second solution corresponds to the matrix and relations

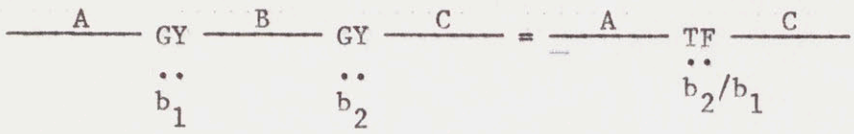
$$M_{TF} = \left[\begin{array}{c|c} a & 0 \\ \hline 0 & 1/a \end{array}\right] \qquad \text{and} \qquad \begin{array}{l} e_A = a \cdot e_B \\ f_A = (1/a) \cdot f_B \end{array}$$

Such an element is called a transformer, and written $\text{———} \overset{TF}{\underset{a}{\cdot\cdot}} \text{———}$,

with 'a' being the modulus.

Examples of transformers are numerous, including ideal levers in mechanics, ideal transformers in electrical circuits, and ideal piston-area ratios in hydraulics. Lumped parameter examples of gyrators are not so easily given, however.

An interesting property of the gyrator is given below in bond graph notation.

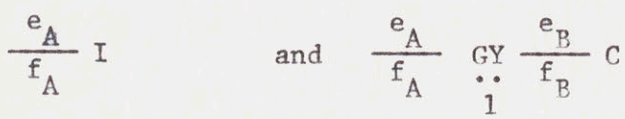


This relation may be proved using the transmission matrices given above.

It should be noted that

——— TF ———TF ——— is <u>not</u> equivalent to ——— GY ——— .

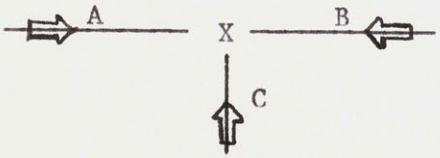
Because the gyrator relates an effort variable to a flow variable it can be viewed as a "dualizing" element. For example, the properties of the following one-ports are indistinguishable at their external bonds.



where ϕ_I and ϕ_C are the same function.*

C. <u>Three-ports</u>

The final set of elements to be considered is the pair of ideal three-port junctions. The first property of these elements is that the net sum of power into the elements vanishes instantaneously.



$$P_A + P_B + P_C = 0$$

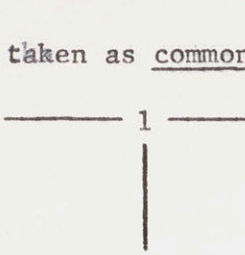
$$e_A \cdot f_A + e_B \cdot f_B + e_c \cdot f_c = 0$$

---

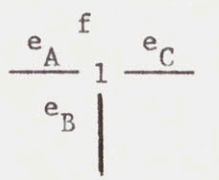
* For I: $f_A = \phi_I \left(\int e_A dt\right)$

For C-GY: $f_A = e_B = \phi_C \left(\int f_B dt\right) = \phi_C \left(\int e_A dt\right)$

The simplest property which can be imposed on such a junction element is to let one of its variables be common on all bonds. Thus, if <u>flow</u> is taken as <u>common</u>, we call the element a <u>one-junction</u>, written



; this implies the relations



or

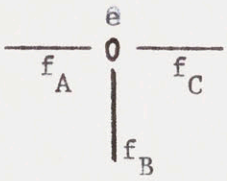
$$f_A = f_B = f_C$$

and

$$e_A + e_B + e_C = 0$$

Because such an element sums (or joins) the efforts, it is also called an <u>effort</u> junction.

In similar fashion we define a <u>zero-junction</u> to have a <u>common effort</u> on its bonds, and write



or

$$e_A = e_B = e_C$$

and

$$f_A + f_B + f_C = 0$$

Since the flows are summed by such an element, it is also referred to as a <u>flow-junction</u>.*

Fig. 2.6 presents a table of properties of the ideal multi-ports just discussed. It also gives examples of interpretation in electrical and mechanical terms. There is no need to consider any higher order multi-

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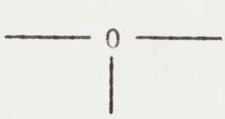
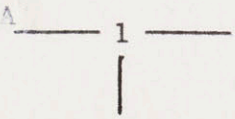
* The following mnemonics may prove useful in keeping the junctions properly defined.

A 

is like a pipe-tee which sums the flows...

A 

is like a linkage which sums the forces...

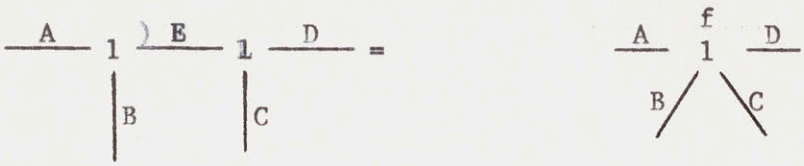
Name and Equations	Bond Graph	Electrical Circuit*	Mechanical Translation**
GYRATOR $e_A = b \cdot f_B$ $e_B = b \cdot f_A$	A GY B b	A B	?
TRANSFORMER $e_A = a \cdot e_B$ $f_B = a \cdot f_A$	A TF B a	a : 1 A B	1 : a A B
EFFORT JUNCTION $f_A = f_B = f_C$ $\sum e_i = 0$	A 1 B C	A B C (series connection)	A B C (parallel connection)
FLOW JUNCTION $e_A = e_B = e_C$ $\sum f_i = 0$	A 0 B C	A B C (parallel connection)	A B C (series connection)

* Electrical: effort = voltage, flow = current

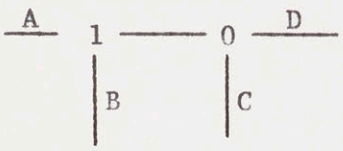
** Mechanical: effort = force, flow = velocity

Fig. 2.6. A Set of Multiport Elements

ports at this level (i.e., static, linear), since elements of the present set may be combined to make higher order ideal <u>junction structures</u>. For example, to make a four-port with a common flow simply join two 1 - junctions.



A four-port with a "richer" structure may be made by joining an 0 and a 1.

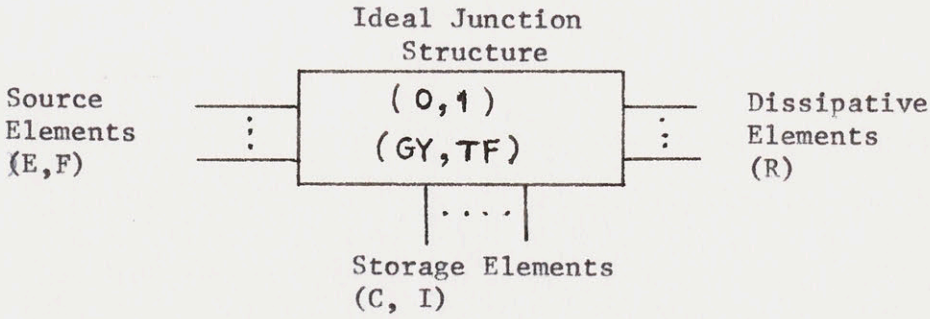


$$f_A = f_B \;;$$

$$e_C = e_D \;;$$

$$e_A f_A + e_B f_B + e_C f_C + e_D f_D = 0$$

The structure of a system in terms of bond graph elements may now be displayed as in the figure below. The only dynamic elements are the capacitances and inertances (C's and I's). Note that every element in the <u>junction structure</u> obeys the power balance relation. Thus the junction structure may be viewed as a "switchyard" for the energy, routing incident energy into appropriate paths instant by instant.



In the next major section (2.20) some procedures for modeling in terms of bond graphs will be discussed, which should assist the reader in his

understanding of this set of bond graph elements, partly by the use of analogies.

2.20 Bond Graph Modeling of Dynamic Systems.

2.21 Transformation Techniques from Schematics to Bond Graphs

In this section a routine procedure will be given (not necessarily the most elegant) for transforming a model of a dynamic system in electrical circuit, mechanical, fluid notation, or circuit graph form, into a bond graph representation.

A. Electrical Circuit Models

(1) For each vertex of the circuit which corresponds to a distinct voltage (effort) write a zero junction.

(2) Between each pair of 0 junctions for which a branch exists establish a 1 junction corresponding to the current in the branch. Attach the circuit element to that 1 junction.

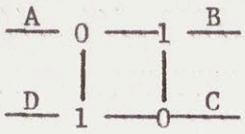
(3) To the ground voltage 0-junction assign a constant effort source, E_{GND}. If all of the circuit elements are nodic (i.e., they depend only on the difference between their terminal voltages) the ground reference may be eliminated and the graph may be simplified. The other voltages may be defined with respect to ground.

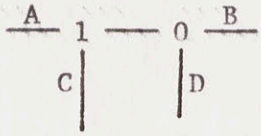
In general, elimination consists in deleting all bonds one of whose variables is at ground level, since (relative) powers are zero on such bonds ($P = e.f = E_{GND} \cdot f = 0$; $P = e \cdot f = e \cdot F_{GND} = 0$), and they do not influence the dynamics of the system in question.

If multiports are made degenerate by this process (e.g., only two bonds remain on a 0-junction), they may be elided into appropriate forms.

Certain bond graph forms may also be simplified internally, under

appropriate conditions, e.g.

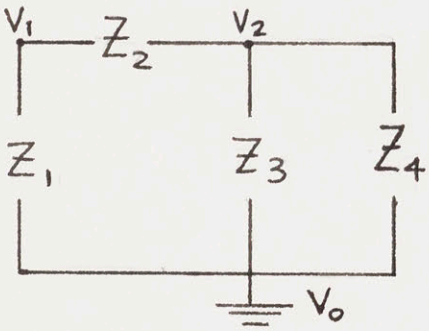


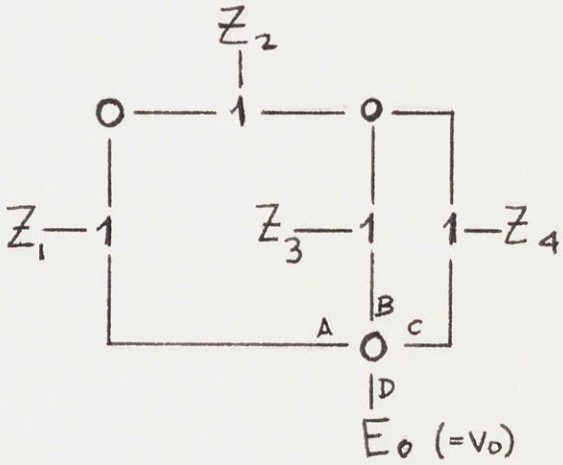


for nodic elements.

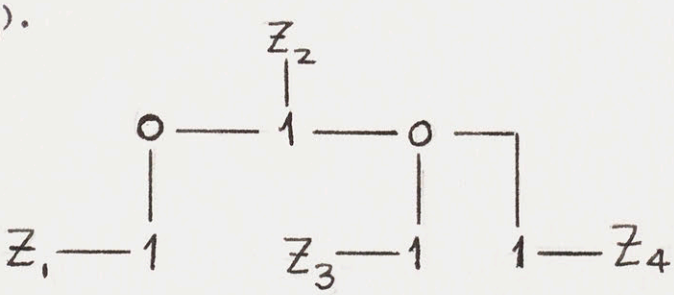
Some electrical circuit examples are given below.

Example (A1)

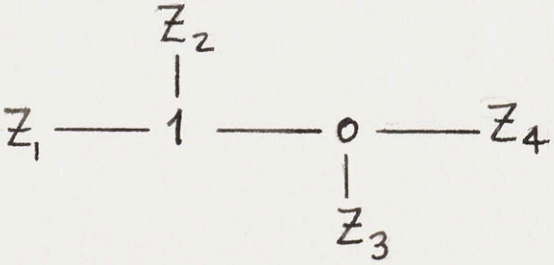




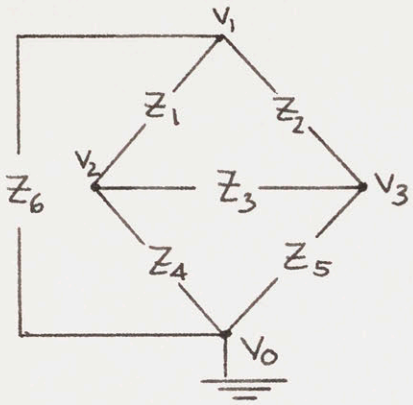
Eliminate bonds A,B,C, and D, since their efforts may be taken as zero (i.e., V_0).

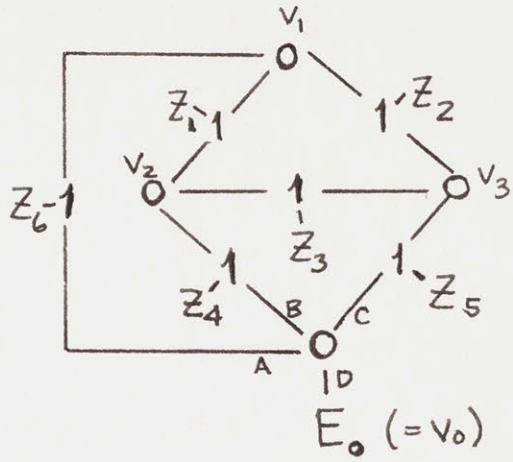


Elide the degenerate 0- and 1- junctions.

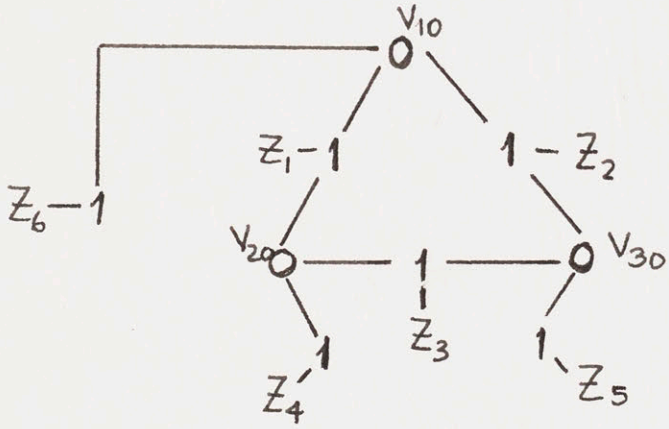


Example (A2). A Wheatstone bridge.

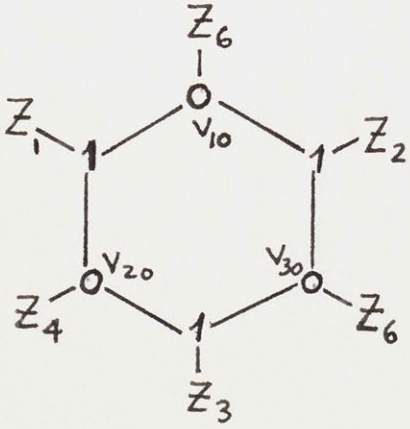




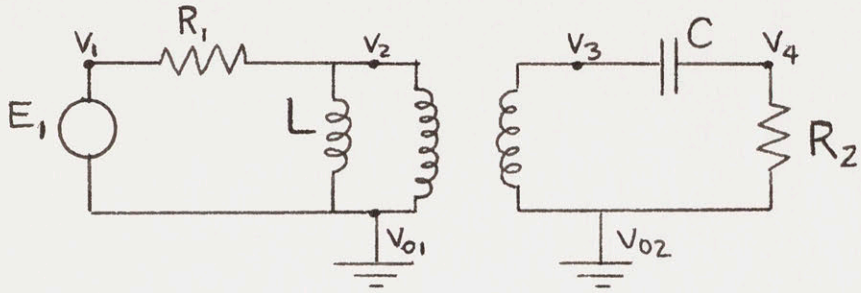
Simplify by eliminating bonds A,B,C, and **D**.

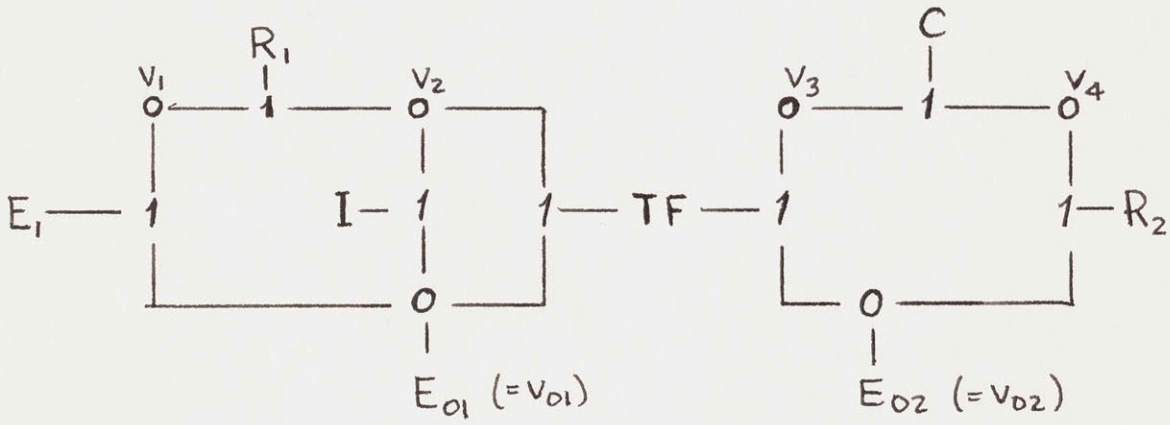


Elide the degenerate junctions, and symmetrize the graph.

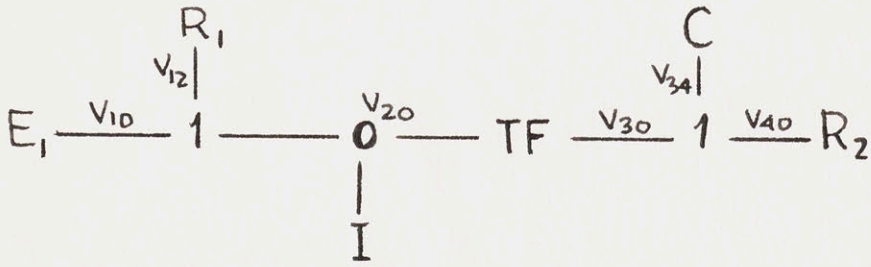


Example (A3).





If $E_{01} = E_{02}$ and all elements are nodic, the bond graph becomes



B. Mechanical Examples

It is assumed that masses are shown connected to ground as dinode circuit elements.

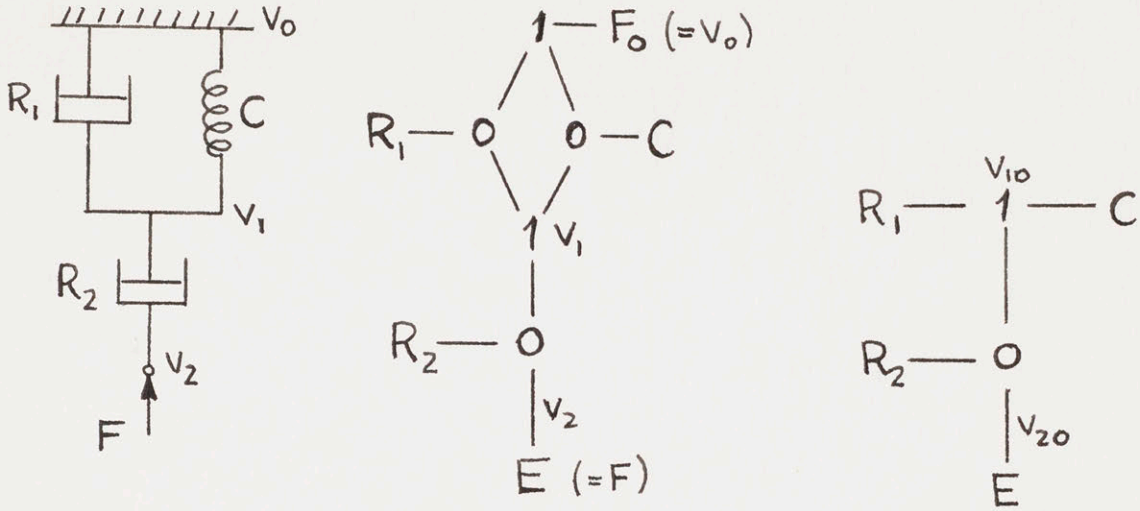
(1) For each linkage or joint which corresponds to a distinct velocity (flow) write a 1-junction.

(2) Insert a 0-junction with the element attached for each branch in the schematic.

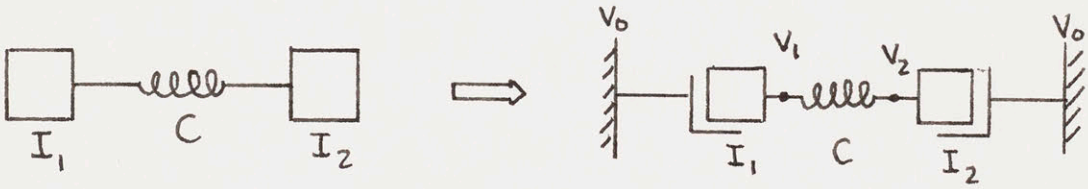
(3) To the velocity reference frame assign a constant (zero) flow source, F_0. If all elements are nodic (i.e., they depend only on the difference between their terminal velocities) the velocities may be redefined with respect to ground, and the reference junction eliminated.

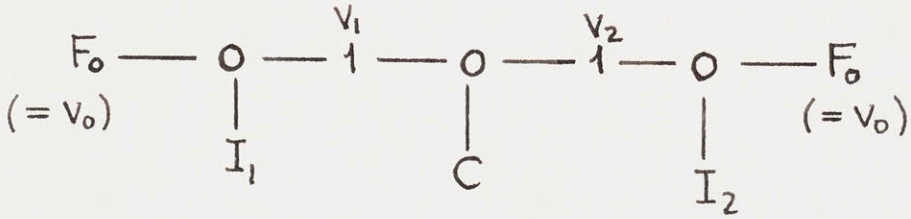
The same elimination procedures for bond graph simplification apply here, of course, as described under part A.

Example (B1)

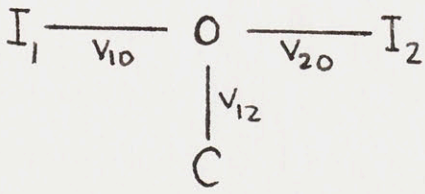


Example (B2)

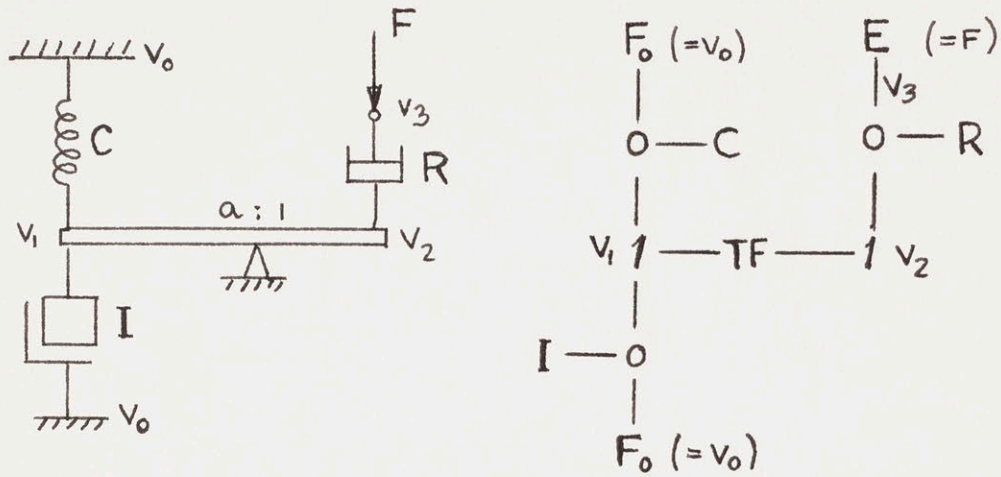




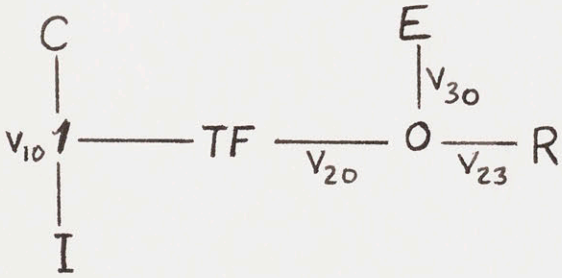
Redefine velocities and simplify



Example (B3)



Simplify



C. <u>Fluid Examples</u>

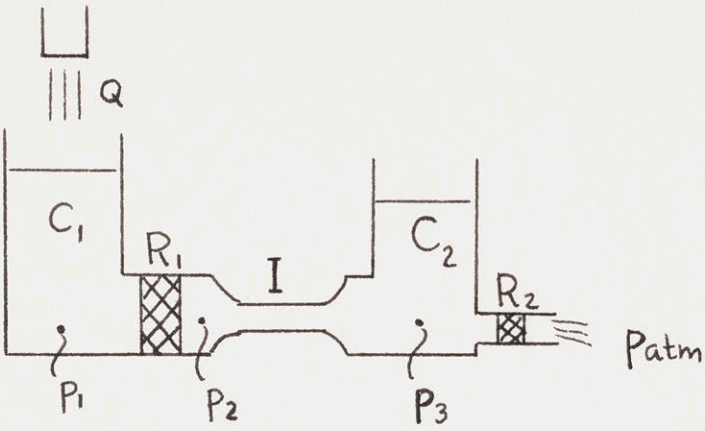
The procedure for fluid notation is very similar to the preceding examples, with pressure being identified with effort, and volume flow with flow. In brief form

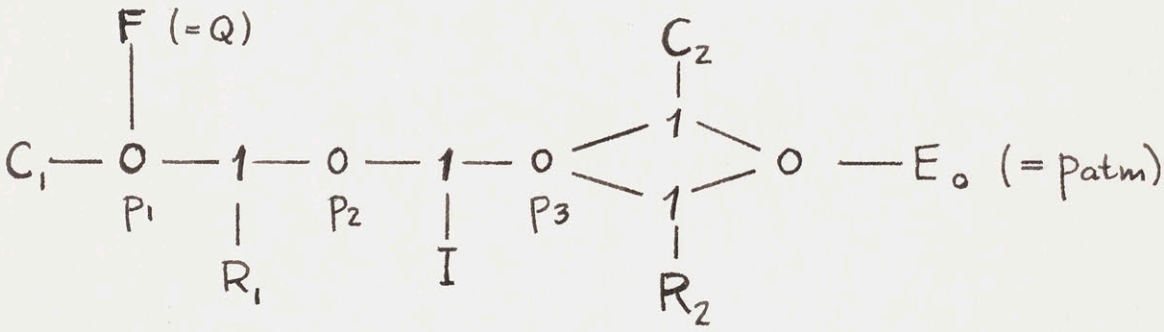
(1) Establish 0-junctions for the distinct pressures.

(2) Show flows between pressure points by 1-junctions with the appropriate elements attached.

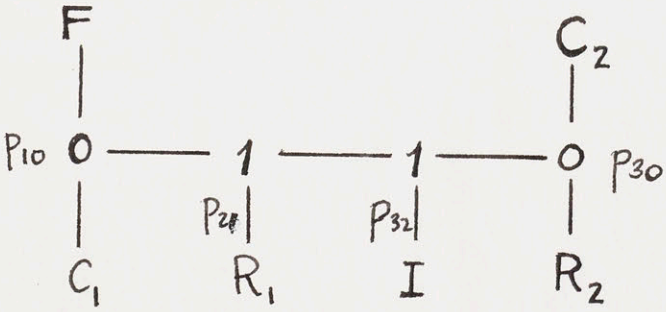
(3) Use E_o to denote atmospheric pressure. Under nodic conditions redefine all pressures relative to atmospheric and simplify the bond graph accordingly.

Example (C1)



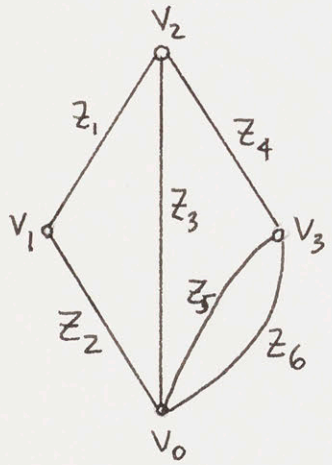


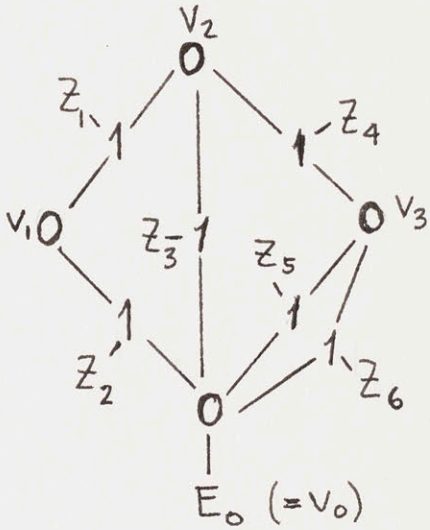
Simplify by redefining pressures



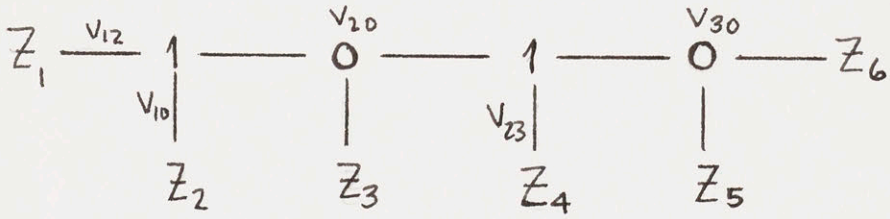
D. Circuit (or "Linear") Graphs

Use the same approach for these graphs as for electrical circuits, writing O's for vertices. A simple example should suffice to illustrate the procedure.





Simplify to



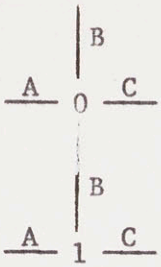
2.22 Transformation from Bond Graphs to Schematics

In transforming from a bond graph to a particular schematic the conversion of the junction elements is the key step, since they reveal the structure of the system.

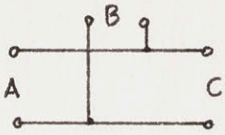
Attention must be paid to the ground or reference convention implied by the bond graph, especially in the case of certain mechanical and fluid elements which are <u>not</u> nodic.

A. Conversion to <u>Electrical Circuit Notation</u>

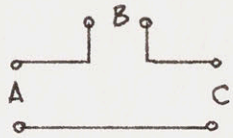
Using the effort-voltage, flow-current analogy



implies a parallel connection

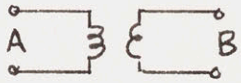


implies a series connection

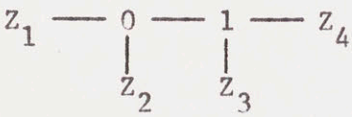


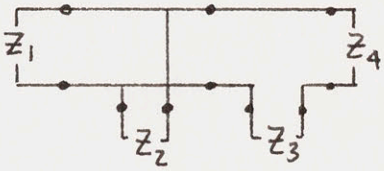
Also

implies

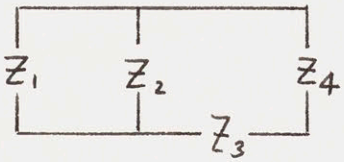


An example is given below, where the Z's are generalized impedances.





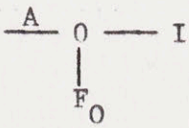
By rearranging the circuit configuration the diagram becomes



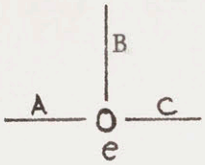
B. Conversion to <u>Mechanical Schematic Notation</u>

The analogy used will be the effort-force, flow-velocity set. Mechanical inertias must have their velocities referred to the same ground or reference. One way to show this requirement is to transform all I's in a bond graph as follows:

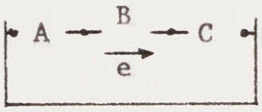
becomes

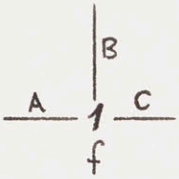


where F_0 is the ground velocity.

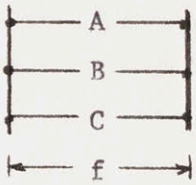


implies common force, or the sum of velocities (series)

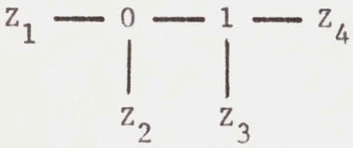


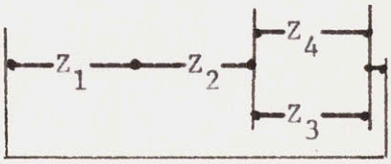


implies common velocity or the sum of forces (parallel)

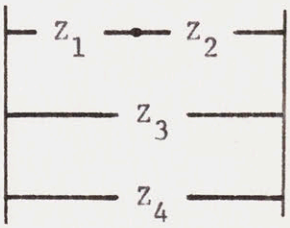


As an example, consider



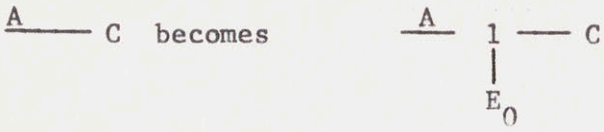


Rearranging the schematic

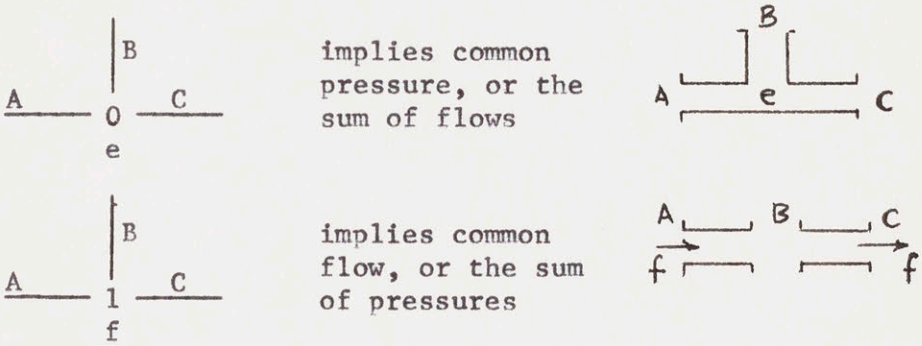


C. Conversion to Fluid Notation

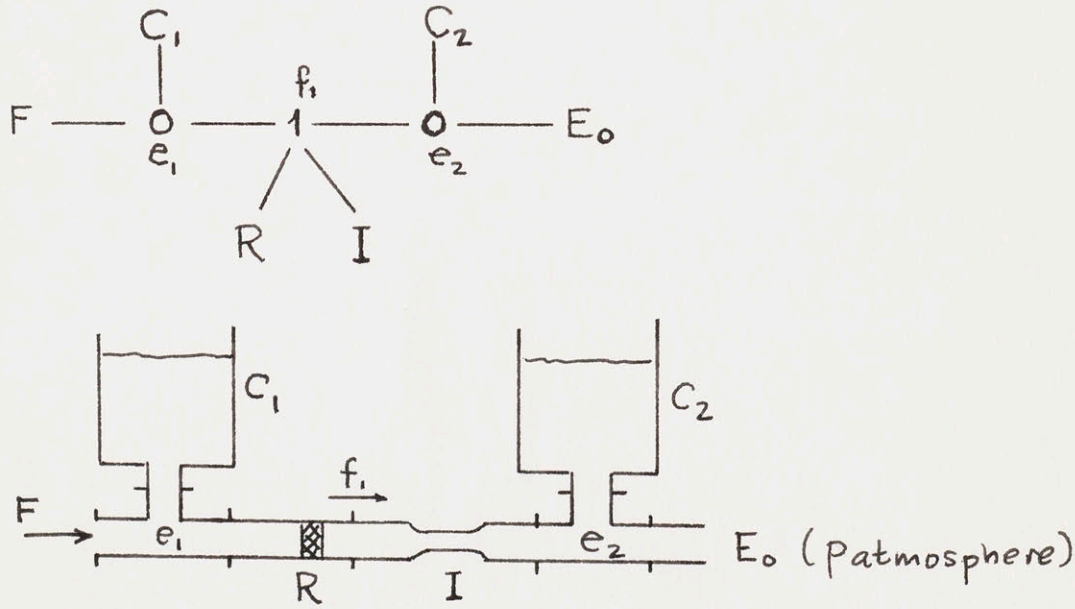
The effort-pressure, flow-volume flow analogy will be used. In this case the C elements all depend upon a pressure difference with respect to atmospheric. The C elements in the bond graph may be modified to show this constraint explicitly, as follows:



where E_0 is atmospheric pressure (or head reference level).



An example is given below:



D. Conversion to Circuit (or Linear) Graphs

The procedure is so similar to that given in A of this section (for electrical circuit notation) that the reader is referred to that discussion, keeping in mind the analogy effort-across, flow-through (or effort, two-point; flow, one-point) for the variables.

The conversion of bond graphs to other graphical notations used for computing, such as block diagrams and signal flow graphs, is deferred until section 2.33.

2.23 Direct Modeling of Physical Systems Using Bond Graphs

Modeling of physical systems in terms of any notation is very much a matter of skill and experience. No set of precise rules is known to the author which would permit a successful bond graph model to be developed for every problem. However, an indication will be given here of a point of view in modeling which takes account of certain properties of bond graphs. In general the "steps" are:

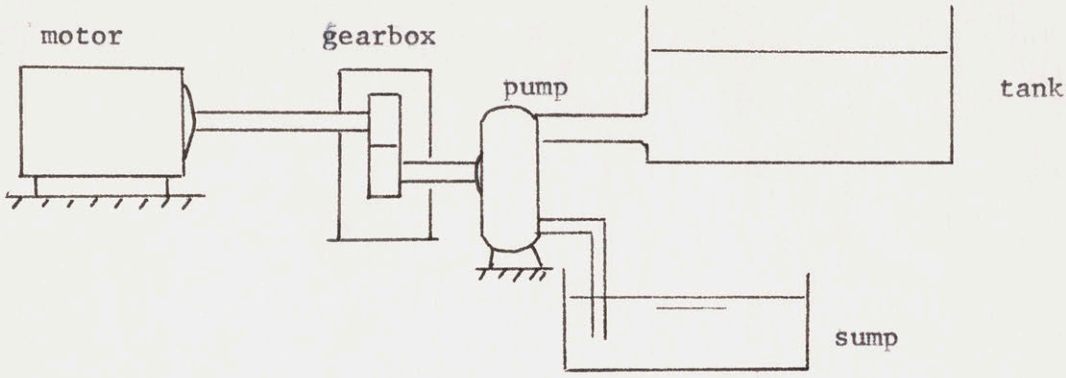
(1) Identify the energy transfer paths of interest between components, and give a localized representation (i.e., a bond with bond variables) to each.

(2) Identify the physical items or phenomena as bond graph nodes whose representatives are their common names (e.g., MOTOR, SHAFT, etc.)

(3) Continue the process, trying to reduce the "word models" to sets of ideal bond graph elements. In some cases this will not be possible (or realistic), and a particular model will become a one-port or a multi-port whose variables are related functionally, or by a table or chart (e.g., actual pump characteristics).

As a simple example, consider the system below, where the motor is

considered an ideal speed source.



A first model might be —

MOTOR —— SHAFT —— GEAR BOX —— PUMP —— PIPE —— TANK
PUMP | SUMP

If it is reasonable to assume that the shaft is a rigid, lossless transmission element, and the pipe is sufficiently short and frictionless to also be considered a perfect fluid transmitter, and if the ideal source property of the motor is shown, the next model becomes -

$F \frac{\tau_1}{\omega_1}$ GEAR BOX (n:1) $\frac{\tau_2}{\omega_2}$ PUMP $\frac{p_2}{q_2}$ TANK
$p_1 | q_1$
SUMP

Considering the gear box, assume that the only important dynamic effects are from the inertias, and that all losses are dependent upon shaft speeds only. Then the gear box model becomes

$\frac{\tau_1}{\omega_1}$ GEAR BOX $\frac{\tau_2}{\omega_2}$ $\Longrightarrow$ $\frac{\tau_1}{\omega_1}$ 1 —— TF (n) —— 1 $\frac{\tau_2}{\omega_2}$

I_1 R_1 I_2 R_2

By such procedures the entire system may be represented in bond graph notation.

2.30 Formulation of Linear System Relations

The procedure described in this section makes use of bond graphs, and it assumes that the reader has some familiarity with such graphs. If this is not true, please see sections 2.10 and 2.20 of this thesis for an introduction to the notation.

2.31 The Plan of the Formulation Procedure

The principal steps include the selection of a set of state variables suitable for specifying the dynamic behavior of the system (e.g., for computing). In general these state variables will be a mixed set of efforts and flows, often very naturally related to the physical observables of the original system. In terms of the state variable set and the set of external sources (or disturbances) the system structure will be represented in (so-called) A-matrix form, according to equation (2.1).

$$DS(t) = \underline{A} \cdot S(t) + \underline{B} \cdot X(t) \qquad (2.1)$$

The symbols have the following meanings:

D = d/dt, the time derivative operator;

S = the state vector for the system;

$\underline{A}$ = the matrix of coefficients for the homogeneous part of the system (hereafter called the A-matrix);

$\underline{B}$ = the matrix of coefficients for the forced part of the system;

X = the source or external disturbance vector.

In order to compute a particular behavior it is necessary to specify an initial state for the system, which will be denoted by S(0). Thus a homogeneous system may be characterized by the two arrays - $\underline{A}$ and S(0), with the relation of equation (2.1) implied in the form

$$DS(t) = \underline{A} \cdot S(t); \qquad S(0) \qquad (2.2)$$

With the system relations organized in form of (2.1) or (2.2) it is possible to study some system properties in terms of A directly, to analyze behavior using Laplace or exponential or other transformation techniques, or to generate a single higher order differential equation in terms of a selected variable. The reader interested in a detailed discussion of some of the uses of the A formation is referred to Tou's book, Modern Control Theory (34).

The formulation procedure consists in developing an augmented bond graph from the initial bond graph model by the use of certain organizing information (such as the directing of signal flow, or causality; the selection of sign orientations, or the power convention; and the designation of a set of bond variables to be used in formulation). This process is described in section 2.32.

The transformation of the system description from the augmented bond graph into standard (A) form (see eqs. (2.1) and (2.2)) by a graphical search procedure is explained in section 2.33.

The augmented bond graph may be converted into other, more common, computing diagrams quite directly. In section 2.34 the conversion to block diagram, and to signal flow, notation is described.

It will be the standard practice in the balance of this chapter to rely upon a single system (i.e., its bond graph) as much as possible for illustration. For the sake of clarity that system and its electrical and mechanical interpretations are presented in Fig. 2.7.

2.32 Development of the Augmented Bond Graph

There are three steps to be taken to augment a bond graph. These are:

(1) the assignment of signal flow directions (or causality);

(2) the selection of a set of bond variables, in terms of which the

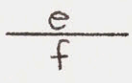
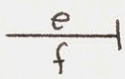
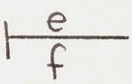
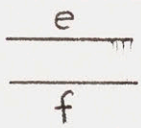
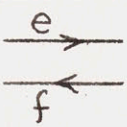
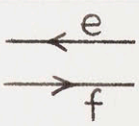
formulation will be made; and

(3) the assignment of a set of power directions, for the determination of signs in the relations.

These steps will now be described in detail.

A. Causality

Every bond has two (signal) variables associated with it. In order for computing (in any sense) to occur it is necessary to show the direction of influence (or cause and effect) of each of the two variables. This may be done with a short line perpendicular to the bond at one end, as follows:

	Acausal	Causal	
Bond			
Signal Pairs			

A possibly useful mnemonic for causal notation is that "effort pushes, flow points"; thus  . Causality is associated withtthe dependence-independence roles of the bond variables of an element. Figs. 2.8 and 2.9 show the causality constraints and corresponding linear relations for each ideal element. The one-port elements C and I have fixed causality, which in each case preserves the integral form of the defining relation. (In the case of a non-linear characteristic this constraint may be used to relax the bi-uniqueness restriction). In the linear case there is considerable freedom in writing the governing relation, and, keeping in mind the eventual standard form for system equations, it is written in derivative form. The one-port sources E

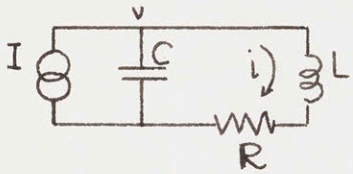
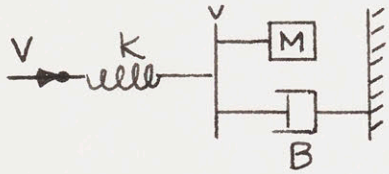
Representation	Parameters	Variables
Bond Graph F — 0 (e) — 1 (f) — I 0 — C, 1 — R	C, I, R	F, e, f
Electrical Circuit  	C, L, R	I, v, i
Mechanical Schematic  	K, M, B	V, f, v

Fig. 2.7. A Sample System - Its Representation by Bond Graph, Electrical Circuit, and Mechanical Schematic.

Element Type	Without Causality	With Causality	Linear Equations
External Disturbances	E ——	E —⊣	e(t) independent
	F ——	F ⊢—	f(t) independent
Energy Storage	C ——	C —⊣	$C \cdot De = f$
	I ——	I ⊢—	$I \cdot Df = e$
Energy Dissipation	R ——	R —⊣	$e = R \cdot f$
		R ⊢—	$f = \frac{1}{R} \cdot e$

Fig. 2.8 The Causality Constraints for Ideal One-Port Elements

Element Type	Without Causality	With Causality	Linear Equations
Two-port	A — TF (m) — B	— TF —	$e_B = m\, e_A$ $f_A = m\, f_B$
	A — GY (m) — B	— GY —	$f_B = \frac{1}{m} e_A$ $f_A = \frac{1}{m} e_B$
		— GY —	$e_B = m\, f_A$ $e_A = m\, f_B$
Junction	A — 0 — B, C	— 0 —	$e_B = e_A$ $e_C = e_A$ $f_A + f_B + f_C = 0$ *
	A — 1 — B, C	— 1 —	$f_B = f_A$ $f_C = f_A$ $e_A + e_B + e_C = 0$ *

* The algebraic signs are <u>not</u> implied by the causality, but shown for example only.

Fig. 2.9 The Causality Constraints for Ideal Multi-Ports

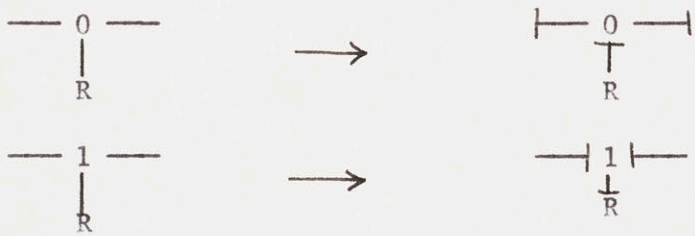
and F prescribe the e and f on their bonds, respectively. Either causality may be imposed upon R, and there is not a prior fixed causality for any of the remaining static elements. The steps in assigning causality are:

a. Introduce the causal information into the graph by using all the C, I, E, and F elements.

b. Extend the information throughout the graph, using the GY, TF, 0, and 1 constraints.

c. Check the graph to see if a complete and consistent caus-lity has been assigned. If it has, go on to part B.

d. If part of the graph has not had any causality assigned, this indicates the existence of an R field. Select an R and assign causality to its bond as follows:



(An arbitrary assignment will do, but the assignment above is more efficient for computing purposes).

Return to step b.

e. If part of the graph has had an inconsistent causality assigned (e.g., two effort inputs on a 0) this indicates the presence of a static sub-field, and must be reduced to an equivalent dynamic sub-system in order to complete causality (i.e., return to step b). The reduction may frequently be carried out by inspection.

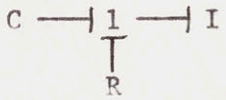
The procedure of assigning causality is illustrated for the example system

in Fig. 2.10.

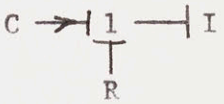
Once a bond graph has been directed causally it is possible to indicate the activation of a bond due to a suppressed signal very simply. This is done by using an arrow in the center of the bond to indicate the (active) signal direction. For example:

The original bond graph

The graph with causality



The activated graph



The last graph indicates that the flow signal on bond A has been suppressed, making bond A active.

Uses of activation will be discussed in Chapter 3.

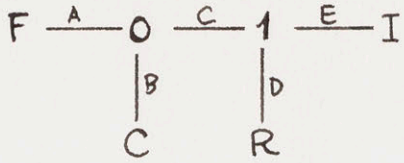
B. Selecting State Variables

The selection of state and other useful variables follows in a perfectly straight-forward way, once a complete, consistent causality has been assigned to the graph.

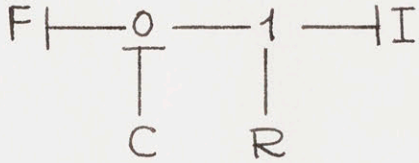
a. On each C element designate the effort, and on each I element designate the flow, variable, labelling the variables $S_1 \ldots S_N$. The array so designated will be referred to as the state vector, S(t). Note that each causally independent dynamic element has a state variable assigned to it.

b. For each E element label the effort with a subscripted X, and

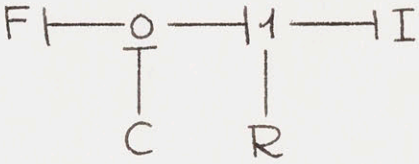
The acausal bond graph



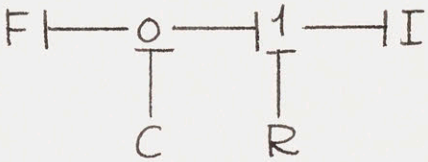
Use the F, C, and I information



Use the 0 constraint to extend causality (bond C)



Use the 1 constraint to extend causality (bond D)



Causality is complete and consistent.

Fig. 2.10 An Illustration of Assigning Causality

for each F designate the flow by an X_i, so that X(t) refers to the disturbance vector.

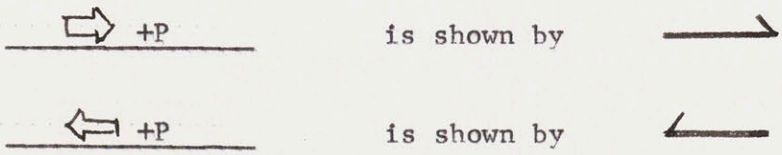
c. On each R element label the output variable (i.e., the one determined by the R) by a subscripted 't' variable. These variables are temporary and will be eliminated in the final formulation.

There are now N1 'S' (or state) variables, N2 'X' (or external) variables, and N3 't' (or temporary) variables. These variables will be used in formulating the system equations. Variables are assigned to the example system as shown in Fig. 2.11.

Fig. 2.11 Computing Variables Assigned to a Graph

C. <u>Assigning Power Directions</u>

In order to use a consistent set of signs when writing the relations for the various bond graph elements it is sufficient to display the direction of positive power on each bond. This information is indicated by a half-arrow placed at the end of the bond as follows:



There are two distinct sources for the orientation of powers in a

graph. If all of the variables of the physical system have been oriented (i.e., if they have been given reference directions) then they will indicate the orientations for the powers in the bond graph. For example,

On the other hand, there may be no prior definitions for the bond graph variables. Then the following conventions may be used to achieve a consistent orientation of powers.*

a. Show the power <u>into</u> the C, I, and R elements

b. Show the power <u>from</u> the appropriate E and F elements which are energy sources, (and <u>into</u> the E and F sinks of energy, e.g., the atmosphere);

c. Show the power <u>through</u> the two-ports GY and TF;

d. If the bond graph has a loop or loops, give each independent loop a consistent orientation (either clockwise or counter-clockwise);

e. The remaining paths or chains in the graph should each be oriented in a single direction

Fig. 2.12 shows examples of loop and chain orientations, and Fig. 2.13 indicates the (signed) relations for oriented elements.

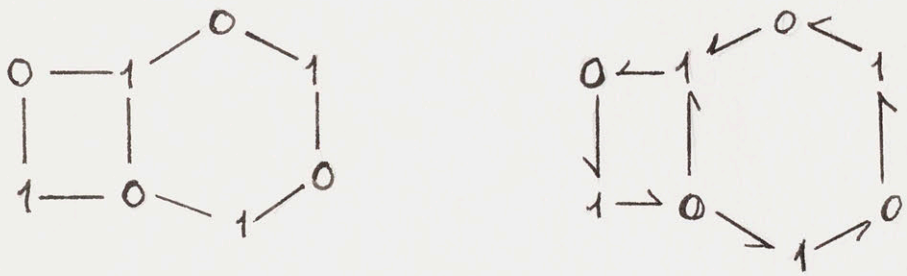
---

* Further discussion of the power orientation convention is given in Appendix A.

** No bonds in common with any other loops.

A. An example of chain orientation

B. An example of loop orientation



(The free bonds are not shown)

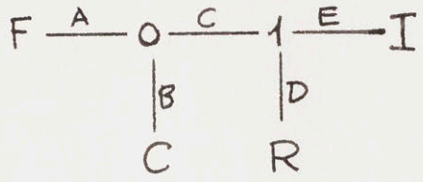
Fig. 2.12 Examples of Chain and Loop Orientation

C	$p_{in} = e \cdot f$ $C \cdot D_e = +f$
I	$p_{in} = e \cdot f$ $I \cdot Df = +e$
R	$e = R \cdot f$
A, B, C, 0	$p_A - p_B - p_C = 0$ $e_A = e_B = e_C$ $f_A - f_B - f_C = 0$
A, B, C, 1	$p_A - p_B - p_C = 0$ $f_A = f_B = f_C$ $e_A - e_B - e_C = 0$
A, B, TF, m	$p_A - p_B = 0$ $e_A = m \cdot e_B$ $f_B = m \cdot f_A$
A, B, GY, m	$p_A - p_B = 0$ $e_A = m\ f_B$ $e_B = m\ f_A$
E	$p_{out} = e \cdot f$
F	$p_{out} = e \cdot f$

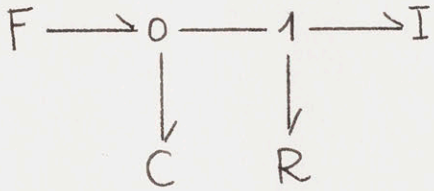
Fig. 2.13 Relations for Oriented Bond Graph Elements

The orientation conventions are applied to the example graph in Fig. 2.14 below.

The unoriented graph



Orienting the one-ports
'F' is an energy source)



Orienting the chain
(bond C)

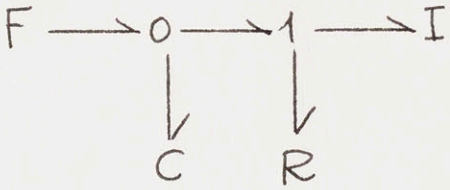


Fig. 2.14 Orienting Powers on a Bond Graph

D. <u>The Augmented Bond Graph</u>

By combining all of the previous information on a single graph, we will find that much of the difficulty in selecting variables and organizing the system equations has been eliminated. The previous discussion should have made clear that assigning causality and assigning power orientations are two independent processes. The augmented bond graph resulting from the graphs of Figs. 2.11 and 2.14 are shown in Fig. 2.15 below.

The original graph The augmented graph

Fig. 2.15 An Augmented Bond Graph

2.33 Formulation of System Equations from the Augmented Bond Graph

In general it requires two steps to achieve standard form for the system equations. In the first formulation there appear 's', 'x', and 't' variables. The 't' variables are then eliminated (in a simple way) to arrive at the desired goal. The basic element in transforming the graph to equations is the search procedure.

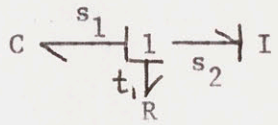
The search procedure starts with a given bond variable (e or f) which is to be expressed solely in terms of the 's', 'x', and 't' variables. In order to do this one follows the path(s) indicated in the bond graph by causal information, applying the sign information as required and terminating each path when either an acceptable variable has been found or no further paths are available.

In particular, we note that paths are terminated by the elements E, F, C, I, and R. Also, free bonds and active bonds may serve to terminate paths. The element GY transforms the search variable (e to f, or f to e) and introduces a scale factor (its modulus) but continues the path. The element TF merely introduces a scale factor and continues the path. The junction elements may introduce multiple paths, depending on the search variable. The sign information enters here. What one is doing by following the search procedure is selecting and substituting the equation relations for the various elements in a systematic fashion, to yield at the end a predetermined form. If the size

of the graph is such that many multiple paths are generated, it is always possible to introduce a few auxiliary variables at key points, use them in the formulation, and eliminate them first using the same search procedure. Unless systems are large and closely coupled this step is rarely necessary. Below is given an example of the application of the search procedure to an augmented bond graph.

The original graph

The augmented graph



(1) Find f_A: $f_A = f_B$ (path A B)

$= s_2$

(2) Find e_B: $e_B = -e_A - e_C$ (paths B A and B C)

$= -s_1 - t_1$

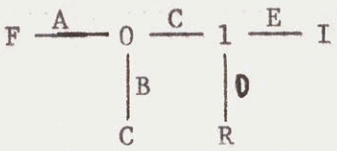
Each 's' variable and each 't' variable will require a search and will yield an equation. Fig. 2.16 indicates the form of equation for each generating element.

Element	Form	Equation	Search Variable
C ——	C ——	C·DS = f	flow
I ——	I ——	I·DS = e	effort
R ——	R —t—	(1/R)·t = f	flow
	R —— t	R·t = e	effort

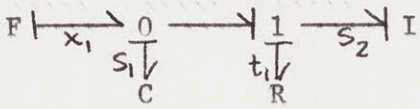
Fig. 2.16 Generating Elements for Forming System Equations

Fig. 2.17 illustrates the formulation of system equations using the search procedure for the example system.

The original graph



The augmented graph



(1) $C \cdot Ds_1 = +f_B$ (generating element)

$= +x_1 - f_c$ (paths B A and B C)

$C \cdot Ds_1 = +x_1 - s_2$ (path C E)

---

(2) $I \cdot Ds_2 = + e_E$ (generating element)

$= + e_C - t_1$ (paths E C and E D)

$I \cdot Ds_2 = + s_1 - t_1$ (path C B)

---

(3) $\frac{1}{R} \cdot t_1 = + f_D$ (generating element)

$\frac{1}{R} \cdot t_1 = + s_2$ (path D E)

Reorganizing the equations -

(1a) $Ds_1 = - \frac{1}{C}s_2 + \frac{1}{C}x_1$

(2a) $Ds_2 = \frac{1}{I}s_1 - \frac{1}{I}t_1$

(3a) $t_1 = + Rs_2$

Fig. 2.17 An Illustration of the Formulation Procedure

The final step in arriving at standard form for the equations is to eliminate the 't' variables which appear in algebraic (i.e., static) relations. In matrix notation

(1) After the search

$$DS = \underline{M}_1 \cdot S + \underline{M}_2 \cdot X + \underline{M}_3 \cdot T \tag{2.3}$$

$$T = \underline{M}_4 \cdot S + \underline{M}_5 \cdot X + \underline{M}_6 \cdot T \tag{2.4}$$

(2) After eliminating T

$$\begin{aligned} DS = &\left[\underline{M}_1 + \underline{M}_3 \cdot (\underline{I} - \underline{M}_6)^{-1} \cdot \underline{M}_4\right] \cdot S \\ &+ \left[\underline{M}_2 + \underline{M}_3 \cdot (\underline{I} - \underline{M}_6)^{-1} \cdot \underline{M}_5\right] \cdot X \end{aligned} \tag{2.5}$$

In standard notation this is

$$DS = \underline{A} \cdot S + \underline{B} \cdot X$$

By referring to Fig. 2.17 it may be seen that the appropriate result for the example system is -

(1b) $$Ds_1 = -\frac{1}{C} s_2 + \frac{1}{C} x_1$$

(2b) $$Ds_2 = \frac{1}{I} s_1 - \frac{R}{I} s_2$$

and the associated coefficient arrays are

$$\underline{A} = \left[\begin{array}{c|c} 0 & -\frac{1}{C} \\ \hline \frac{1}{I} & -\frac{R}{I} \end{array}\right] \quad \text{and} \quad \underline{B} = \left[\begin{array}{c} \frac{1}{C} \\ \hline 0 \end{array}\right]$$

There are variations possible in the procedure presented here, which become useful as the analyst gains familiarity with the basic search technique. One modification is intended to limit the number of 't' variables according to the following rule:

When assigning causality only introduce a 't' variable on an R when

additional causal information must be introduced; that is, when underconstraining of causality has occurred (see 2.32.A).

The modified assignment procedure is illustrated in Fig. 2.18 below.

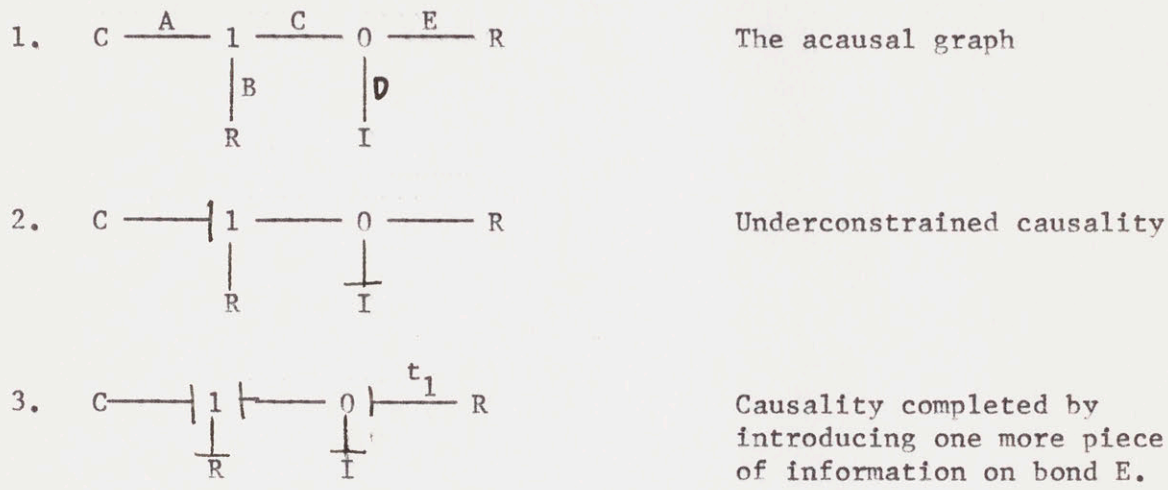
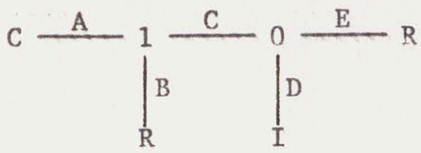


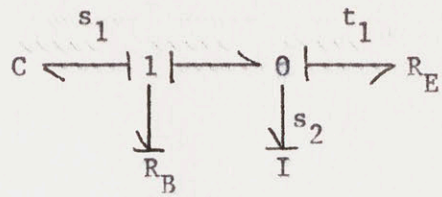
Fig. 2.18 An Application of the Modified Causality Assignment Procedure

The search procedure must be modified to account for the (missing) 't' variables. The effect is merely to continue the search path "through" the R element, with the appropriate modifications to the expression. Application of the modified search procedure is demonstrated in the following example.

The original graph



The augmented graph



(1a) $C \cdot Ds_1 = f_A = f_B = \frac{1}{R_B}$; $e_B = \frac{1}{R_B}(-s_1 - e_C)$

(1b) $C \cdot Ds_1 = -\frac{1}{R_B} s_1 - \frac{1}{R_B} \cdot t_1$

(2a) $I \cdot Ds_2 = e_d$

(2b) $I \cdot Ds_1 = t_1$

(3a) $\frac{1}{R_E} \cdot t_1 = f_C - s_2 = f_B - s_2$

(3b) $\frac{1}{R_E} \cdot t_1 = -\frac{1}{R_B} s_1 - s_2 - \frac{1}{R_B} t_1$

After eliminating t_1 and putting equations (1b) and (2b) into standard form, the A-matrix is found to be

$$\underline{A} = \left[\begin{array}{c|c} -\dfrac{1}{C(R_B + R_E)} & \dfrac{R_E}{C(R_B + R_E)} \\ \hline \dfrac{R_E}{I(R_B + R_E)} & -\dfrac{R_B R_E}{I(R_B + R_E)} \end{array}\right]$$

With some practice the analyst can trace lengthy paths through an augmented graph and produce well-ordered relations in a very efficient fashion.

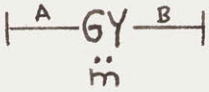
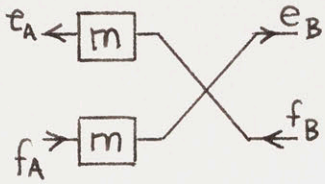
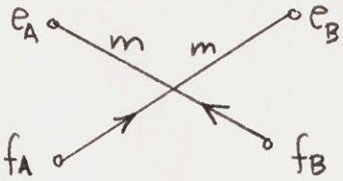
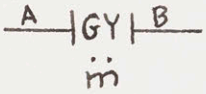
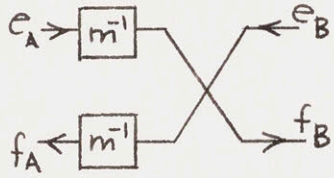
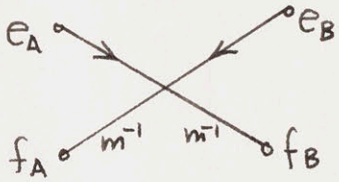
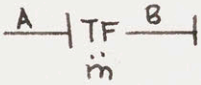
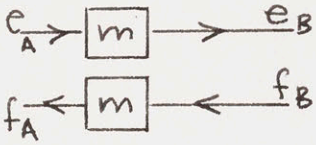
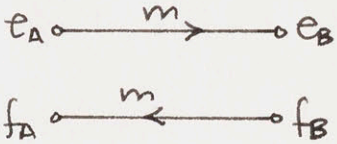
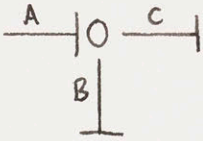
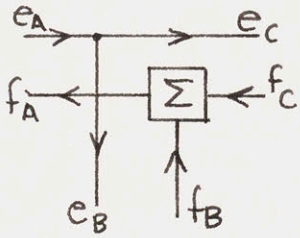
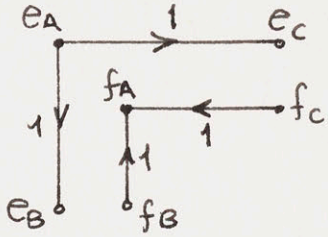
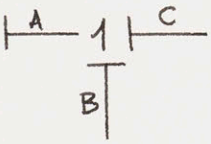
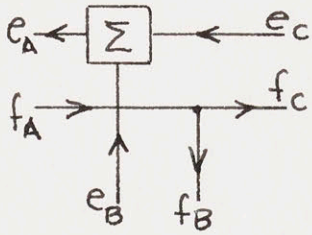
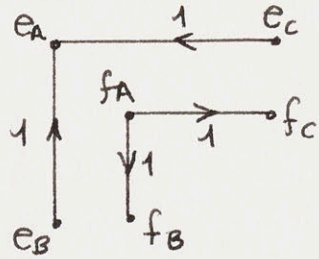
2.34 Formulation of Other Computing Diagrams from the Augmented Bond Graph

The transformation of augmented bond graphs into block diagram, and signal flow graph, form will be presented in this section in brief fashion. The principal sources of reference are Figs. 2.14 and 2.20, showing the equivalents

for causally determined bond graph elements. The general procedure is to split the bond into a pair of directed signal branches for the block diagram, or a pair of signal nodes for the flow graph. In writing signs for the summation relations use is made of the power orientations (see Fig. 2.13). The same example will be used in this section as shown in Fig. 2.7 of section 2.31.

Bond Graph Element (Causal)	Block Diagram Equivalent	Signal Flow Graph Equivalent
C	$\frac{1}{CD}$; e ; f	$\frac{1}{CD}$; e ; f
I	$\frac{1}{ID}$; e ; f	$\frac{1}{ID}$; e ; f
R	R ; e ; f	R ; e ; f
R	$\frac{1}{R}$; e ; f	$\frac{1}{R}$; e ; f
E	E ; f	E
F	e ; F	F

Fig. 2.19 One-Port Bond Graph Equivalents in Block Diagram and Signal Flowgraph Form

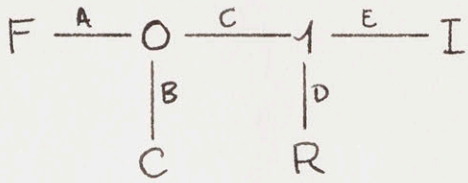
Bond Graph Element (causal)	Block Diagram Equivalent	Signal Flow Graph Equivalent	
			
			
			
			*
			*

* Signs for the relations must be found from the power orientations, in general.

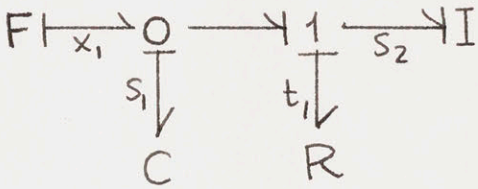
Fig. 2.20 Multi-Port Bond Graph Equivalents in Block Diagram and Signal Flowgraph Form

(1) Transformation into a Block Diagram

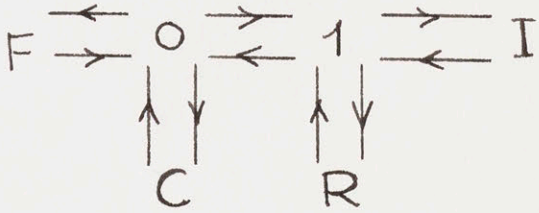
a. The bond graph



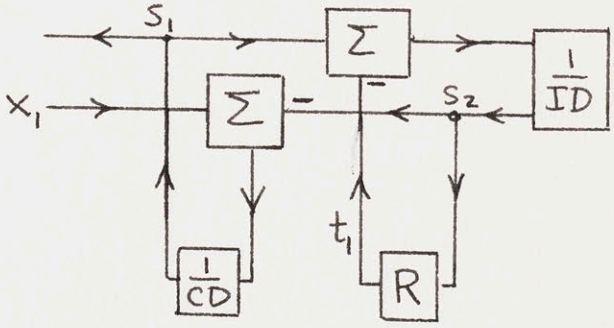
b. The augmented bond graph



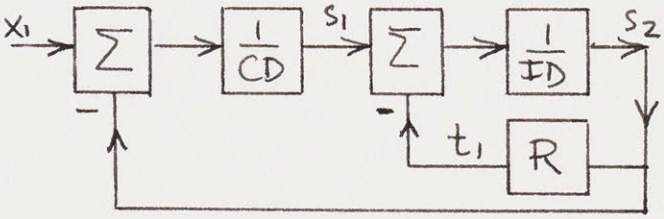
c. The directed signal pairs (e above, f below convention)



d. Insertion of the function blocks

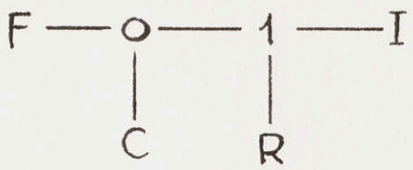


e. Simplification of the arrangement

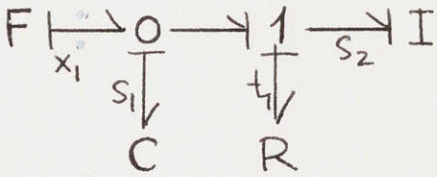


(2) Transformation into a <u>Signal Flow Graph</u>

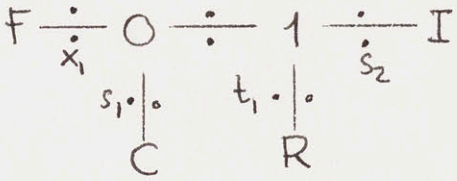
a. The bond graph



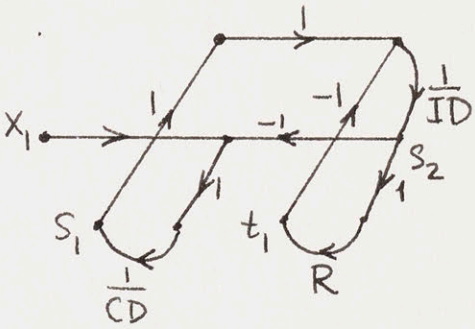
b. The augmented bond graph



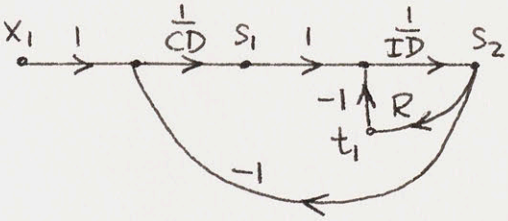
c. The node pairs (e and above, f below convention)



d. Insertion of the connecting branches showing functions



e. Simplification of the arrangement



3. THE DYNAMIC SYSTEMS LABORATORY

The Dynamic Systems Laboratory has as its main purpose the simulation of the behavior of dynamic systems. The Laboratory consists of computer programs, a time-shared digital computer, and associated terminal equipment. Communication with the Laboratory is carried out using a teletype, with a special oscilloscope display for viewing system behavior. A picture of the terminal equipment is shown in Fig. 3.1.

3.10 A General Description

The Dynamic Systems Laboratory (or DSL, as it will henceforth be called) is capable of simulating the time behavior of linear, lumped-parameter dynamic systems. These are systems which may be represented by constant coefficient ordinary differential equations. Another way to describe the class of simulatable systems is shown in Fig. 3.2, where the allowable elements of the bond graph set, as well as analogous electrical and mechanical elements, are presented.

3.11 System Specification

In order to specify a linear dynamic system, the following information must be given:

1. System structure.
2. Parameters, or values of the elements.
3. The initial conditions or initial state of the system.

In the DSL the description of a dynamic system is made in terms of a bond graph. Figure 3.3 shows a complete specification of a sample system with an electrical circuit interpretation.

The structure of a system is communicated by using a simple line-coding technique. For example, to line-code this graph

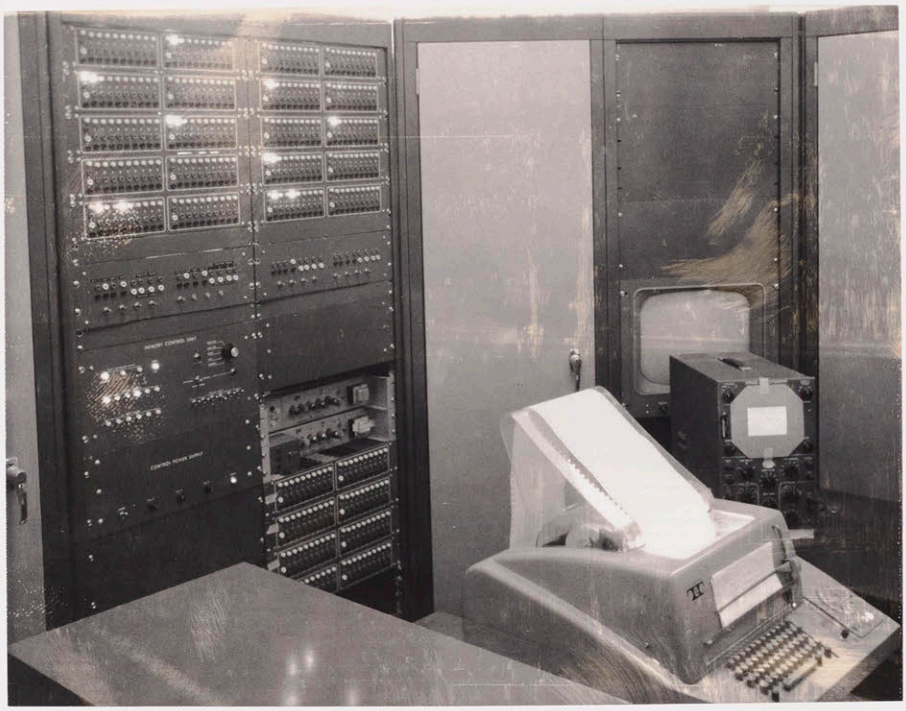


Fig. 3.1. The Terminal Equipment

Element	Bond Graph	Electrical Network	Mechanical Schematic
Energy Storage	C		
	I		
Energy Dissipation	R		
Ideal Gyrator	GY		(gyroscope)
Ideal Transformer	TF		
Flow Junction	0		
Effort Junction	1		
Effort Source	E	V	FORCE
Flow Source	F	I	VELOCITY

Fig. 3.2. The Set of Admissible Elements for the DSL

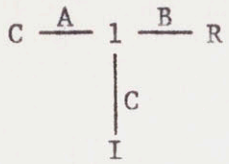
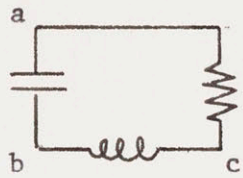
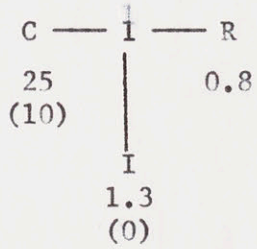
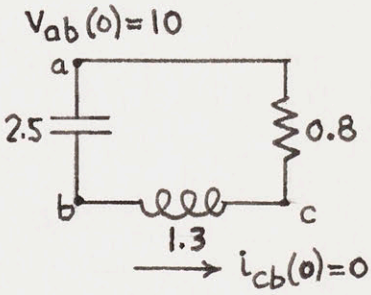
Information	Bond Graph	Electrical Circuit
System Structure		
Parameters	CA = 2.5 IC = 1.3 RB = 0.8	C = 2.5 L = 1.3 R = 0.8
Initial Conditions	$e_A(0) = 10$ $f_C(0) = 0$	$V_{ab}(0) = 10$ $i_{cb}(0) = 0$
Sampling Interval	DT = 1	DT = 1
A Complete Statement		

Fig. 3.3 Information Required for System Simulation

```
C —— 1 —— I
       |
       R
```

one labels the bonds (uniquely but arbitrarily)

```
C —A— 1 —C— I
       |B
       R
```

forms the node-bond groups

```
C A
1 A B C
I C
R B
```

and puts them in a line, separated by commas.

C A, 1 A B C, I C, R B .

There are several advantages to this coding procedure, which has been adapted from suggestions given in (31). The bond labeling process names the bonds (and their associated variables, e and f), and simultaneously distinguishes the attached nodes. Hence, there is no need for additional subscripting. The line code has minimum information for a graph with no active bonds. This means that for passive or predominantly passive systems the code is a succinct one. The coding procedure is very easy to learn and use, and is an economical way for a person to impart graphical information to the computer.

The parameters are associated with their particular elements as shown in Fig. 3.4. The definitions for C, I, and R elements are in accord with usage in electrical circuit and mechanical schematic notation. The element

Parameter	Element	Definition
Capacitance	C ——	$C \cdot De = f$
Inertance	I ——	$I \cdot Df = e$
Resistance	R ——	$e = R \cdot f$
Gyrator Modulus	$\underline{\quad A \quad}$ GY m $\underline{\quad B \quad}$	$e_A = m \cdot f_B$ $e_B = m \cdot f_A$
Transformer Modulus	$\underline{\quad A \quad}$ TF m $\underline{\quad B \quad}$ (TFAB)	$e_A = m \cdot e_B$ $f_B = m \cdot f_A$

Fig. 3.4

Definitions of Parameters

GY (gyrator) has its modulus defined in the resistive sense, while the modulus of the element TF (transformer) depends on the order of bond reference in its name. The DSL is a numerical device, and the user may adopt any set of consistent units he wishes.

The initial state of the system is specified in terms of the state variables, which are defined by the simulator after the graph has been processed.

As a matter of convenience, the user may specify a time interval, DT, to the simulator. This interval is the smallest step in time for which the digital results are available. Since the total number of results which can be stored are limited, increasing the step size means increasing the total time of behavior available. It also results in a coarser resolution for output.

The part of the DSL which carries out the simulations has been named ENPORT, a title which derives from the fact that one takes an energy port view of systems when using bond graphs. The original conception of the ENPORT system was due to Prof. H. M. Paynter. Figure 3.5 shows an example of system specification using ENPORT. Figure 3.5 shows that the state variables were selected as e_D and f_C. We shall soon observe that any system variables are available to us for inspection, however.

3.12 System Behavior

There are three principal forms in which to observe system behavior. These are tables, digital graphs on the teletype, and analog graphs on the display scope. The variables and quantities whose behavior may be examined are:

1. All efforts and flows in the system (of which the state variables

```
READY...

STRUCTURE *

THE GRAPH, PLEASE...

C A, I C, R B, 1 A B C.

 ST VBL  1 IS EFFORT ON CA
 ST VBL  2 IS  FLOW  ON IC

READY...

PARAMETERS

CA   = 2.
IC   = 1.
RB   = .5

READY...

DT
   .5
TMAX =      249.500

READY...

INITIAL

  S1 = 10.
  S2 = -5.

READY...
```

* All user statements are underlined.

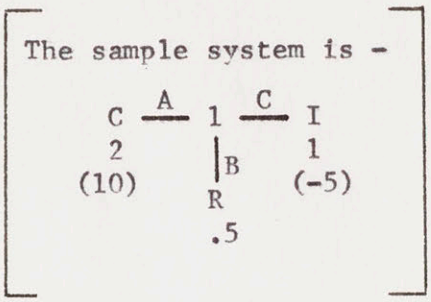


Fig. 3.5. An Example of System Specification

are a sub-set).

2. All momenta and displacements (i.e., the time integrals of efforts and flows, respectively) in the system.
3. All powers on the bonds.
4. All energies associated with the capacitances and inertances.
5. The cumulative energy transfer on any bond (useful for studying dissipative behavior of resistances, for example).

When using the oscilloscope display only the state variables may be examined.

There are two types of time constraints put on the available results, depending upon whether a digital or analog output format is used. In the digital case, results are limited to a fixed number of points because of computer memory storage requirements. Two relations express the implicit limitations:

KMAX = NØ.WØRDS / NSV

TMAX = KMAX * DT

where

KMAX = number of stages of solution

NØ.WØRDS = number of words of computer storage available for results

NSV = number of state variables

TMAX = time range of simulations

DT = sampling interval

The results may be examined in intervals which are integer multiples of the selected interval (DT).

The limitations on the display time arise from two sources. The first

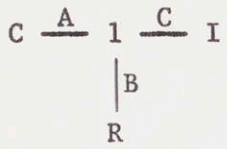
is that the digital solution must exist so that automatic set-up and scaling procedures can be used (e.g., finding the maximum value of a state variable). The second limitation is due to the fact that the analog computer gains (or coefficients) have a maximum and a minimum value which can be set.

The output facilities are exhibited by application to the problem specified in Fig. 3.5. Figure 3.6 illustrates the three types of output requests -- DISPLAY, HPLOT, and TABLE. The 'S' variables refer to the state variables, and the 'EN' variables refer to the energies of CA and IC.

3.13 Bond Activation

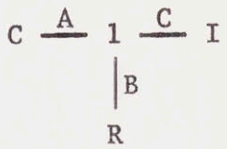
When it becomes appropriate to show pure signal coupling in a bond graph rather than power bonding, the information may be communicated in either (or both) of two ways.

In the first case one simply modifies the input line-code to show signals which are to be suppressed. For example,



Input:

C A*, 1 A B C , R B, I C.

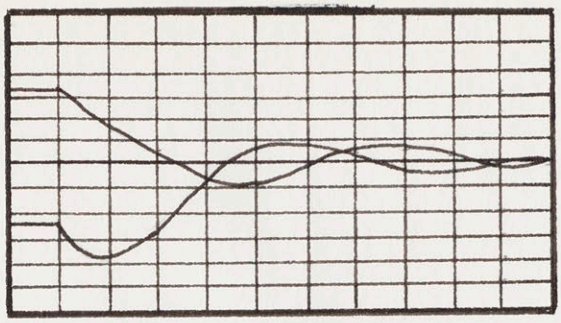


The convention is: an asterisk before the bond suppresses the effort, an asterisk after the bond suppresses the flow. This convention depends on the causality (i.e., the signal polarization) of the graph.

Another approach is to build the graph with complete bonds and then suppress the required signals by specifying the bond--signal pairs.

Fig. 3.7 shows an annotated example of the use of ENPORT.

```
DISPLAY
0.,15.

DISPLAY SCALES
 VARIABLE    FULL SCALE RANGE
      1          .125E 02
      2          .100E 02
←←!]X>:SF;YC7J>6MB↑↑↑
```



The display screen is shown about three-sevenths actual size.

```
HPLOT
0.,15.,.5
S(1) S(2)

 .10000E 02                 1...S(1)
     .                      2...S(2)
     . 1
     .   1
     .
     .     1
     .
     .
     .       1
     .                        2 2 2 2 2
     .         1                        2 2
     . . . . . . . . . . 2 . . . . . . 1 1 2 1 1 1 1 1 1 . . . . 2 2 2
     .           1     2         1 1       2 2 2 2 2 2 2 2 2 2 2 1 1 1
     .             1           1
     .               2     1 1
     .                 1 1
     . 2             2
     . 2
     .           2
     .   2
     .         2
     .     2 2            TIME FROM  0. TO  15. IN INTERVALS OF  .500 .
-.10000E 02
```

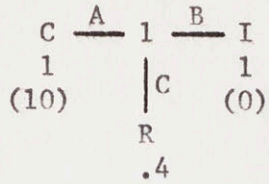
```
TABLE
0.,15.,3.
S(1) S(2) EN(1) EN(2)
```

TIME	S(1)	S(2)	EN(1)	EN(2)
0.	.10000E 02	-.50000E 01	.25000E 02	.12500E 02
3.000	-.18982E 01	-.47730E 01	.90078E 00	.11391E 02
6.000	-.15107E 01	.29276E 01	.57052E 00	.42855E 01
9.000	.99704E 00	-.46459E-01	.24852E 00	.10792E-02
12.000	-.41444E-01	-.63560E 00	.42940E-03	.20199E 00
15.000	-.20673E 00	.25166E 00	.10685E-01	.31667E-01

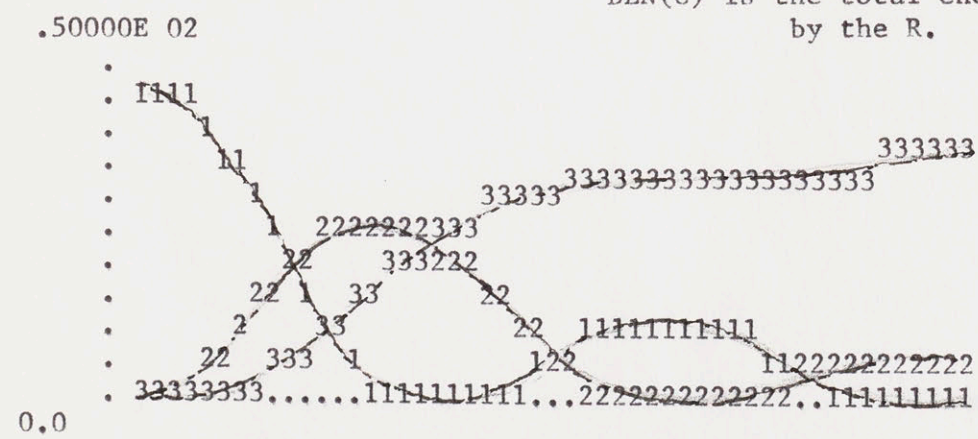
Fig. 3.6. Some Output Requests

```
STRUCTURE

THE GRAPH, PLEASE...

C A, I B, R C, 1 A B C.

 ST VBL  1 IS EFFORT ON CA
 ST VBL  2 IS  FLOW  ON IB

READY...

PAR
SCALE
1.
RC
.4
Q

READY...

INIT
CA
10.
Q

READY...

DT
.1
TMAX =      49.900

READY...

HPLOT
0.,5.,.1
EN(1) EN(2)DEN(C)
```

The sample system is



Note.
EN(1) is the energy in the C.
EN(2) is the energy in the I.
DEN(C) is the total energy dissipated by the R.



TIME FROM 0. TO 5.000 IN INTERVALS OF .100 .

Fig. 3.7. An Illustration of the Use of Enport

3.14 A Brief Command List

A partial list of commands, which indicates some of the DSL capabilities, is included here. A description of the complete DSL command set is given in Appendix B.

A. Structure Commands

STRUCTURE	interprets the line code, builds the internal structure, and selects state variables.
SUPPRESS	permits suppression of selected signals
RESTORE	used to restore suppressed signals

B. Input Commands

INPUT	provides an automatic guide for all required input about a system, other than the line code
PARAMETERS	provides an automatic guide for parameter values
INITIAL	provides an automatic guide for the initial conditions in terms of state variables
PAR	permits (manual) input of selected parameters
INIT	permits (manual) input of selected initial conditions
DT	permits specification of the sampling interval
AUTO = STRUCTURE + INPUT	a completely automatic input guide

C. Output Commands

TABLE	prints results in table form
HPLØT	prints results in graphical or plotted form
DISPLAY	presents results on oscilloscope

3.20 Laboratory Aspects of the Dynamic Systems Laboratory

It is appropriate to ask at this point, In what sense is the Dynamic Systems Laboratory a laboratory? Clearly a person does not put on his shop coat and start to assemble equipment and instruments. Many of the actions he takes in using the DSL, however, are analogous to ones he would take in any physical laboratory. In the sense that the reasoning and thought patterns used, as interpreted from external evidence, are in the experimental vein, the person is acting as if he were in a laboratory. Three particular examples will be discussed in the following section, but excerpts from some trial material presented to freshmen subjects in a teaching experiment may illustrate the point.*

A. The subject was asked to construct and explore the behavior of a system unfamiliar to him.

"2. Build the following system in the DSL.

A.

$F \xrightarrow{A} C$	Structure
10. 1.	Parameters
(0)	Initial condition
1 = DT	Sampling interval

B. Determine from experimental evidence the relation between e(t) and F.

C. Determine from experimental evidence the relation between e(t), F, and e(0)."

In the problem above, F represented a source of constant flow (=10 units).

B. Another common way to use the DSL was to ask a question in the following way.

---

* The material is discussed more fully in Chapter 4.

"Predict and verify the value of e(t) at t = 3 for this system:

$$F \xrightarrow{B} R$$

22. 2.

DT = 1 "

C. The subject may also be asked to construct quantitative relations from careful observation and experimentation. For example --

"F. So far we have found that there is a characteristic type of response for e(t) in the C —R system, which is

$$e_A(t) = e_A(0) \cdot e^{-t}$$

Now the question is -- how does the value of C influence the time behavior?

Would you expect the system to lose its energy faster or slower if C were larger?

Make a prediction, and then conduct some experiments to find the effect of C (See clue 3.3 if you need to)."

This section was taken from a chapter attempting to develop quantitative understanding of first order system behavior using a guided experimental approach.

3.21 Constructing the System Response

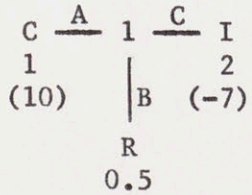
The three basic "instruments" of the DSL are the oscilloscope trace, the tables of digital data, and the quantized (or digital) graphs of behavior versus time. Although the scope display is automatically set to have maximum resolution, it is difficult to answer accurately questions about behavior like "what is the initial slope?", "what is the peak value of the

effort?", and "at what time did the variable cross zero?." The display is best suited to give a good qualitative insight into system behavior.

In the use of tables, which are listed with five significant figures, the step size (DT) plays a role in determining the interpolation accuracy. In addition there exists under certain conditions some computing "noise"; that is, when a student expects two results to be identical, they will disagree on occasion in the last digit. Since noise or uncertainty is an essential property of real measurement processes, a certain amount of computer noise may be a useful phenomenon.

The digital plot presents a response characteristic which has been sampled in two directions -- the time dimension and the magnitude dimension. Selection of an appropriate sampling interval for making the plot is important in obtaining reliable information. A finer interval is generally more reliable. In the interests of efficiency, however, it is useful to choose as coarse an interval as possible. Learning to make this balanced choice a priori (i.e., before any plot for a system is taken) helps a student gain experience in estimating the dominant response characteristics of dynamic systerm. Figure 3.8 shows the effect of too large an interval, and the results of using a more appropriate sampling interval for plotting the same system response.

The coordinated use of the various instruments also presents a worthwhile challenge. Consider the problem of estimating the <u>peak magnitude</u> of the <u>flow on bond C</u> for the following system.



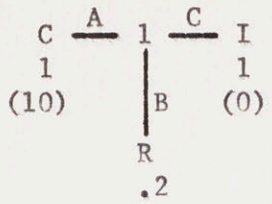
$v(0) = 10$

$i(0) = -7$

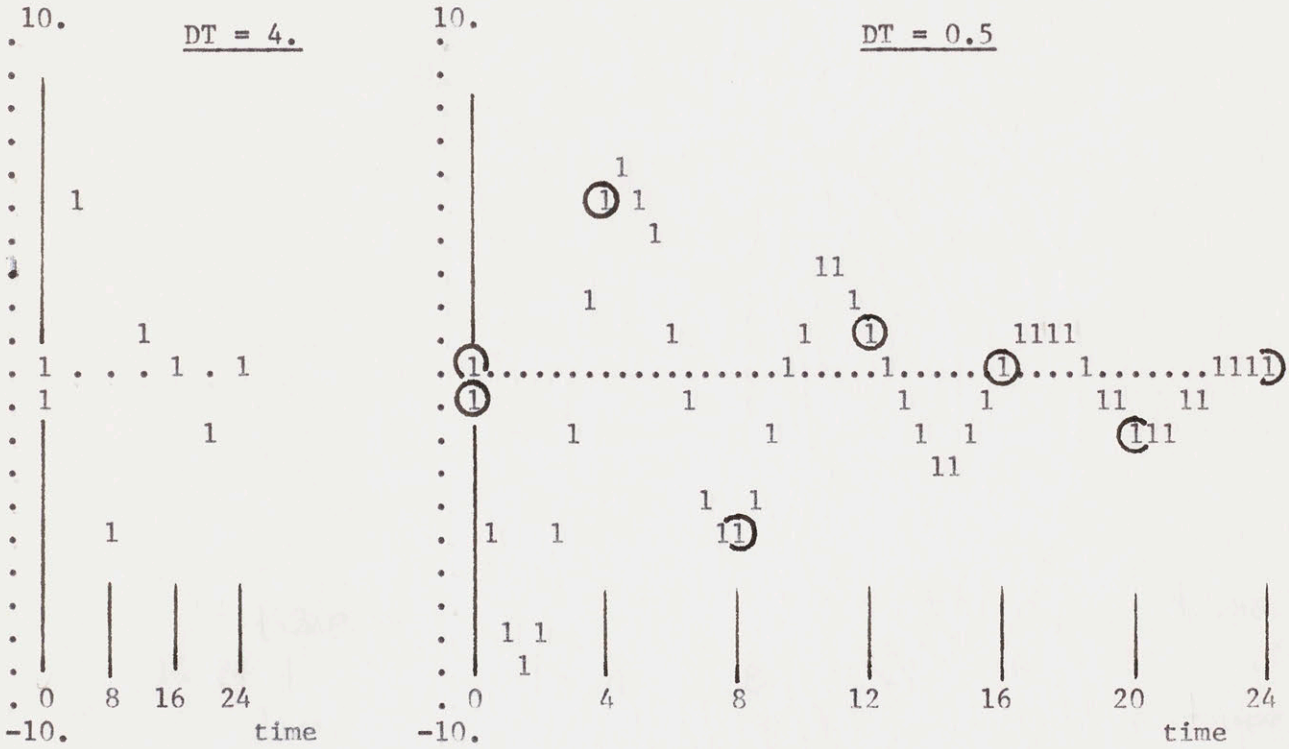
The Bond Graph

An Electrical Circuit Equivalent

The sample system is



Both plots are from t=0. to t=24. for the variable f(C).



A. A misleading plot - DT is too large.

B. A satisfactory plot - DT is appropriate.

Fig. 3.8. Influence of the Sampling Interval on the Digital Plot

Assume the system has been constructed in the DSL. A scan of the display tells us to concentrate in the initial time region to find the peak value. The digital plot tells us to look in the region between T = 0.5 and T = 1.0, as shown below.

```
                    .10000E 02
HPLOT                   .
0.,4.,.5                . 1
E(A) F(C)               .                                  1...E(A)
                        .                                  2...F(C)
                        .
                        .   1
                        .
                        .
                        .     1           2          TIME FROM  0. TO  4.000
                        .
                        . . . . . . . . 2 .           IN INTERVALS OF  .500 .
                        .
                        .       1     2
                        .           2
                        .         1
                        .         2
                        . 2
                        . 2     2   1     1
                        .   2-2       111
                        .
                   -.10000E 02
```

It is now advisable to select a finer interval and examine a table in the region of interest.

```
TABLE                TIME       F(C)
.5, 1., .05
F(C)                 .500      -.80804E 01
                     .550      -.81286E 01
                     .600      -.81662E 01
                     .650      -.81931E 01
                     .700      -.82095E 01
                     .750      -.82155E 01
                     .800      -.82113E 01
                     .850      -.81969E 01
                     .900      -.81724E 01
                     .950      -.81382E 01
                    1.000      -.80942E 01
```

The table indicates that $f_c(.750) = .822$ as the maximum magnitude.

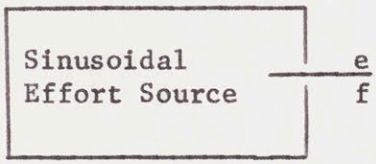
Clearly, as the behavior of dynamic systems considered becomes more complex, there is an increase in the challenge to use the instruments wisely.

3.22 Some Uses of Activation

In addition to using bond activation as a direct modelling device (e.g., as an aid in representing a vacuum tube amplifier), there are some other applications of general interest.

A. The Construction of Ideal Sources

Suppose a student wishes to test a system by driving it with a sinusoidal effort (e.g., voltage) and he wishes the source to be ideal. That is, he would like to construct a system with the following property:



$e(t) = E_0 \sin \omega t$
independent of $f(t)$

This may be done in the following way. It is known that an I — C system will oscillate and that the frequency depends on $\sqrt{IC}$. Furthermore, the magnitude of the effort can be controlled by the initial conditions and the ratio $\sqrt{I/C}$ if necessary. All that is missing is a way to transfer the desired effort to another system. Steps "a" through "c" show the process involved.

a. Design the source (select the parameters and initial state) — C — I

b. Insert a bond coupling for the desired variable — C —A— 0 —B— I, 0 —C— C

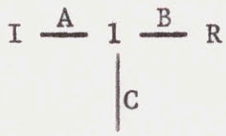
c. Activate the bond and the source is ready — C —A— 0 —B— I, 0 ↓C $e_C(t) = E_0 \sin \omega t$

This means that rather than use ideal mathematical function sources directly, the student may call upon his previous experience with dynamic system behavior in the design and construction of sources.

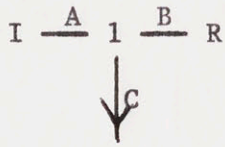
One more brief example will show the potential of this approach. Suppose it is desired to pulse a system with a flow. A first order system response with a fast time constant with respect to the driven system will serve as an approximation to an impulse source.

a. Design the source (a flow pulse)

b. Insert the bond coupling for the flow

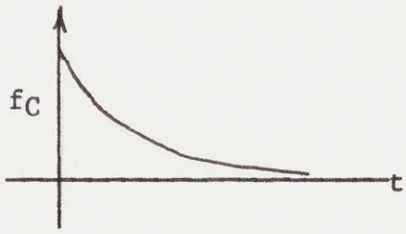


c. Activate the bond



$$f_C(t) = f_A(0) \cdot e^{-t/\tau}$$

The characteristic is



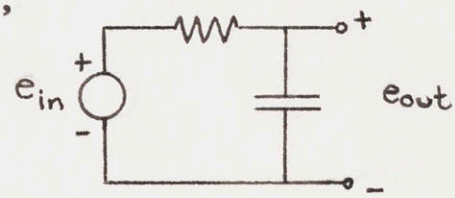
B. <u>Instrumentation</u>

An ideal measurement is one in which taking the data does not affect the measured system. Therefore, any effort or flow can be measured ideally by inserting one of the ideal instruments shown below.

a. Bond containing variable to be measured

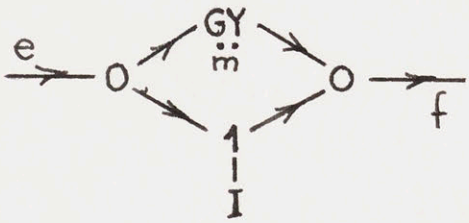
b. Insert the appropriate instrument

Although such a process is implied by the user being able to request as output a variable like $e_C(t)$, the insertion of an activated zero or one junction as shown above emphasizes the measurement process, or the variable(s) in question. The reader familiar with electrical circuit practice will notice the analogy to the use of open circuit terminal pairs for calling attention to output variables; for example,



C. Functional Modelling

There are occasions when it is desirable to represent part of a system in terms of functional operations on the variables. This is a common way to design controllers in dynamic systems. Suppose it was required to generate a flow which was proportional to an effort and the integral of the effort. The activated graph below will serve as such a device.



$$f(t) = \frac{1}{m} \cdot e(t) + \frac{1}{I}\int e \cdot dt$$

Such an approach to developing controllers has several advantages. It preserves the common notation for the entire system. The representation may suggest practical ways to realize the control device. And, in general, it is easy to distinguish between the passive part of the system and the largely active part which corresponds to the control system.

3.23 The Black Box Facility

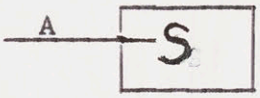
The black box facility in the DSL enables a user to request an unknown

system to be constructed (or, more accurately, to be retrieved from storage, where it was put by its originator so that he can examine its behavior. The step of deducing what a system must be from observed behavior is an important intermediate one in the complete analysis-design spectrum. Several examples taken from the pilor teaching material are shown below.

In the case where there is the least prior knowledge about the system the student asks for a particular black box and is informed of the number of available bonds. He may then use a test source to determine the conjugate bond variable behavior and from the evidence try to develop an equivalent system.

An example from the pilot material is simply:

"SYSTEM NUMBER 7

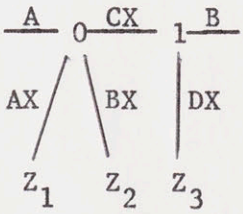


What is S?"

The black box has one bond (named "A").

Partial information may be given about an unknown system in a number of ways. In the following example the free bonds and the connection structures are shown and the field elements must be found.

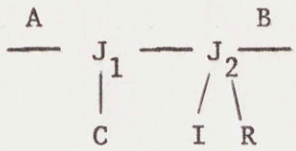
"SYS. NØ. 8



Find each Z_i (either C, I, or R) and its value."

In this example the field elements are shown, and the connection elements are to be found.

"SYS. NØ. 10



J_1 and J_2 are to be found.
(They are either 0 or 1) "

Another way to stage access to information about the black box system is to limit the output information available. A brief list of some of the available data about a system will show the possible variations.

a. Connection structure (0, 1, GY, TF)

b. Field elements (C, I, R)

c. Parameters

d. Initial conditions

e. Activity of the bonds

f. Response of selected variables

g. The A matrix (the array of coefficients for a first order equation formulation)

3.30 The Simulation Procedure

The heart of the Dynamic Systems Laboratory is ENPORT, which carries out the simulation of dynamic system behavior. The particular steps which are taken by the computer in the simulation process are listed below and presented in Fig. 3.9 in relation to the information required for the operstions and the information available after their completion.

The steps are:

1. to represent the system structure or topology.

2. to assign a computing causality to the structure.

3. to select a set of state variables.

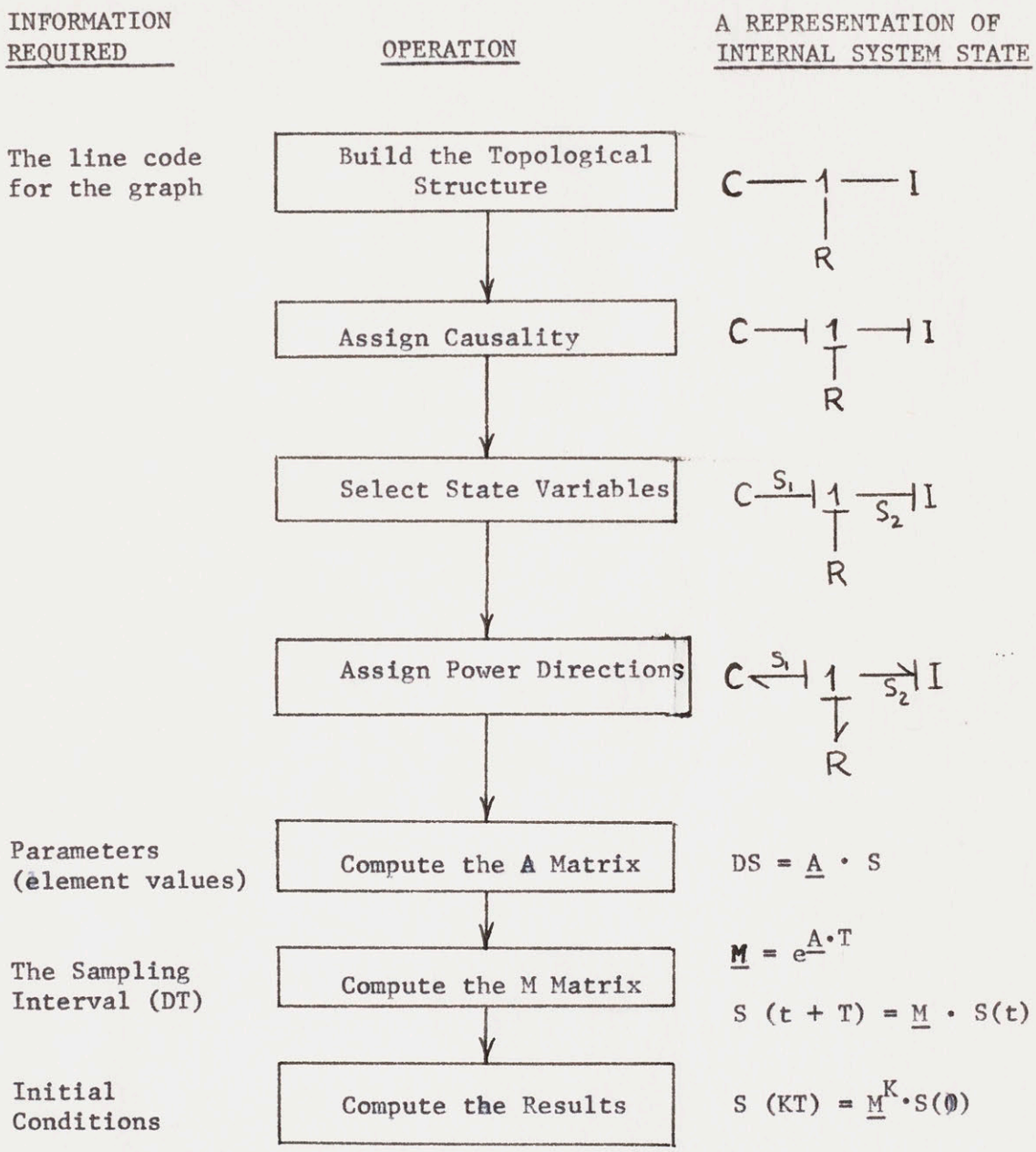


Fig. 3.9. The Main Sequence of Operations for Simulation in ENPORT

4. to assign a sign convention.

5. to transform the system representation into the form

$$D\,S(t) = \underline{A} \cdot S\,(t) \qquad \text{(Eq. 3.1)}$$

where $D = d/dt$

S = the state vector

$\underline{A}$ = the array of coefficients of S

6. to compute the matrizant for $\underline{A}$ according to

$$\underline{M} = e^{\underline{A}T} \qquad \text{(Eq. 3.2)}$$

where $\underline{M}$ = the matrizant

$e = 2.7183...$, and

T = a prescribed sampling interval

7. to compute the system response, according to

$$S(KT) = \underline{M}^{K} \cdot S\,(0) \qquad \text{(Eq. 3.3)}$$

Appendix E contains more details about the programming of the system.

3.40 The Equipment of the Dynamic Systems Laboratory

The terminal equipment of the DSL is organized around a teletype which is a remote station of a time-shared digital computer. Figure 3.10 shows a plan view of the terminal equipment, which includes the teletype, a local digital memory unit, an analog computer, and a display unit. The communication links for this equipment and the central computer are shown in Fig. 3.11. This figure shows that the teletype, local memory, and associated analog computer and display are all under digital computer program control.

The main components will be discussed briefly in the next three sections, but the interested reader is referred to Appendix C for details about the design and operation. Some indication of costs is also given there.

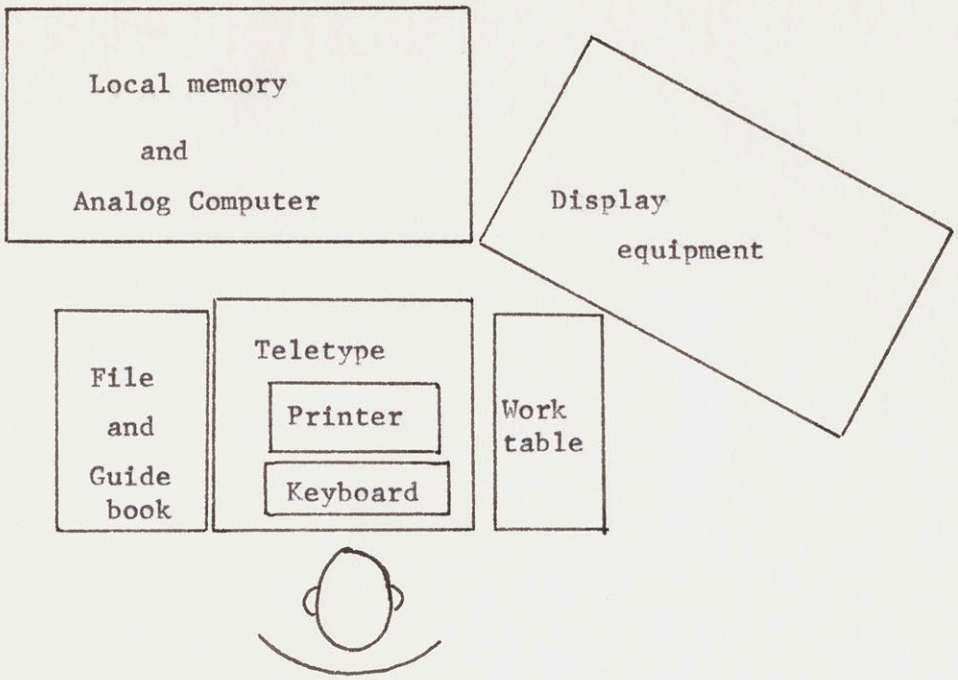


Fig. 3.10. Plan View of the Terminal Equipment

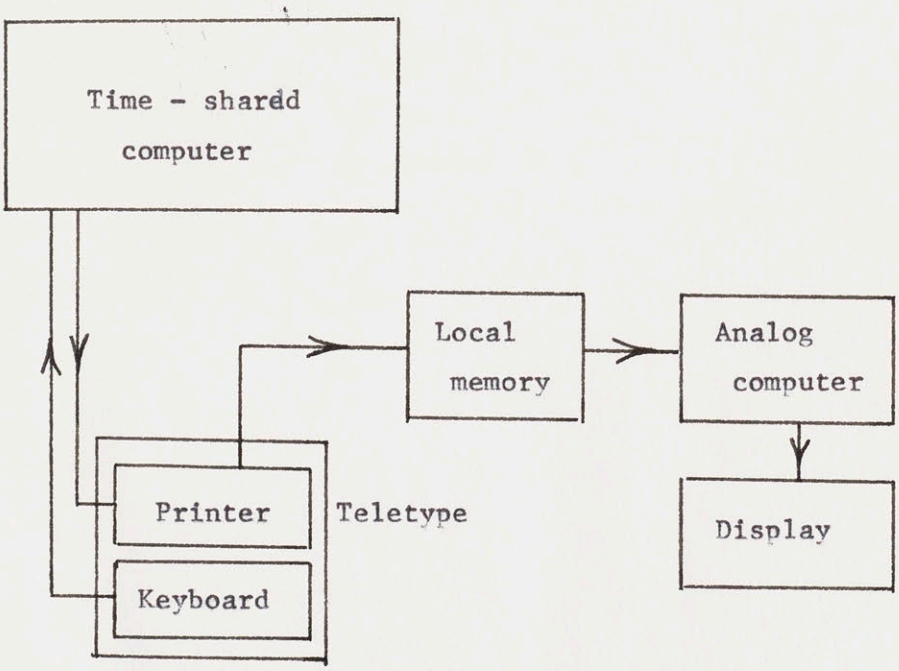


Fig. 3.11. Communication Links for the Terminal Equipment

3.41 The Teletype and Time-Shared Computer

The teletype of the DSL is one of many remote stations associated with the time-shared computer of Project MAC at M.I.T. The basis of the time-sharing concept is that, although each of several users is receiving very rapid sequential access to the computer, the individual user feels that he has complete access. The actual service characteristics of the time-sharing system varied considerably in the course of a twenty-four hour period, and some of the observed effects on subjects are discussed in the next chapter.

The functions provided by the teletype were to permit user input by the keyboard, to supply computer output by the printer, to maintain a complete record of all communications on the logsheet, and to permit direct program communication with the local memory unit without user intervention. The last function required a minor modification to the otherwise standard teletype.

3.42 The Local Memory Unit

The local memory unit was designed and constructed to provide access to information from the computer program in a form useful for controlling other equipment.

The unit has thirty-two addressable words, each containing nine bits (switches) plus a sign bit (a two position switch). It requires a group of three teletype characters to select and store information in a word. The first character is used as the address and the first five bits of each of the next two characters are used to fill the ten bits of the addressed word. The memory was designed to match teletype transmission rates (about six characters per second) and to minimize the cost in so doing.

As may be seen in Fig. 3.1, each bit of the memory is provided with a light. This feature has proven extremely valuable both in maintaining the

equipment and in providing to the student an additional source of information about his particular dynamic system. This feature will be discussed further in Section 3.43.

3.43 The Analog Computer and Display

By setting up an analog computer by digital computer in a form corresponding to the dynamic system being studied, the state variables of the original system could be displayed on an oscilloscope. The display graphed the state variables versus time and was exceedingly flexible in time and magnitude range, due to an automatic digital set-up and scaling program written by R. A. Humphrey, following (36).

The information transmitted for display generation was a minimum, since only the structural and parametric data, plus the initial conditions, had to be sent. If only a minor change in the dynamic system was made (e.g., to a parameter or initial condition) then only the modified information had to be transmitted. Since a second order system might require six data words plus a few control characters to be sent, the time behavior could be displayed in less than four seconds. A closely coupled third order system might require about seven seconds.

The reader may recall that the dynamic system is cast in the form $D\mathbf{S} = \underline{\mathbf{A}} \cdot \mathbf{S}$ at one point in the simulation process. This implies that each state variable may be found from a relation of the form

$$S_i = S_i(0) + \int_0^t \left(\sum_{j=1}^{N} a_{ij} S_j \right) dt \qquad \text{(Eq. 3.4)}$$

where N is the number of state variables. Figure 3.11 shows an analog block diagram which realizes this relation for $N \leq 4$.

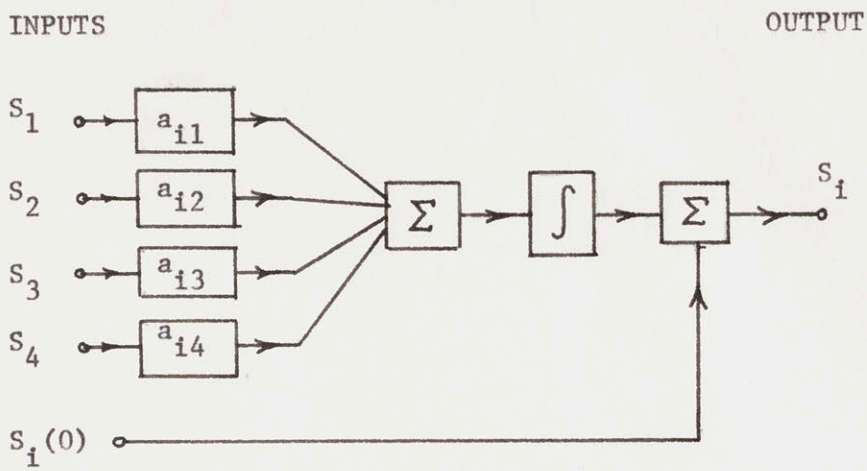
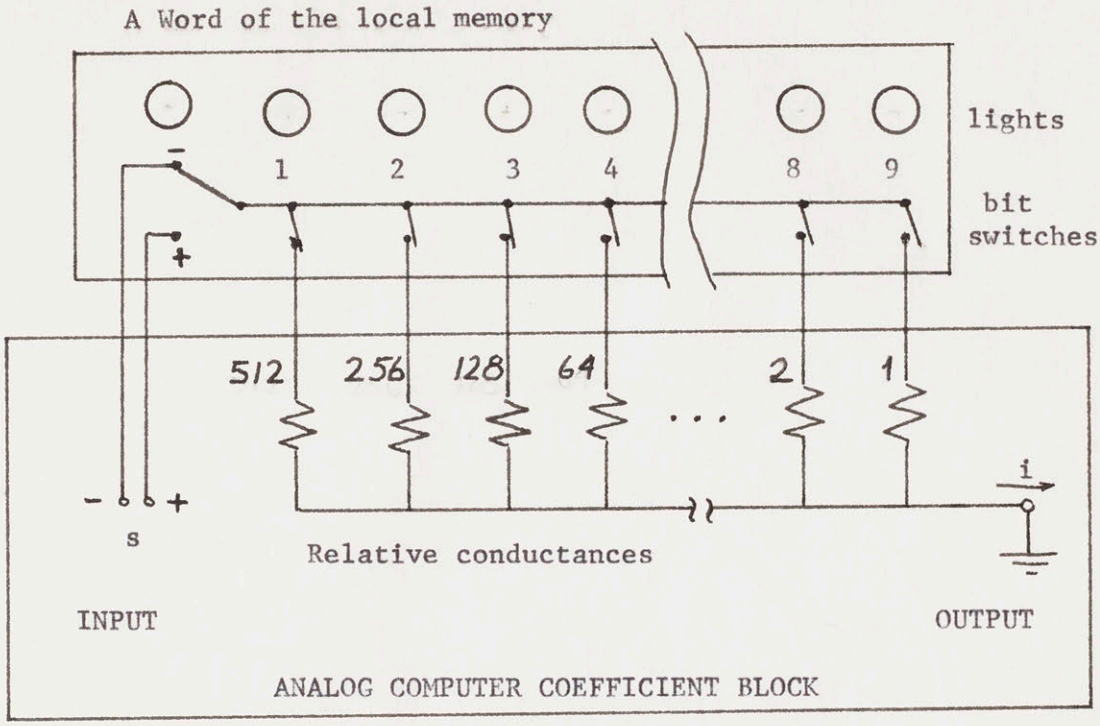


Fig. 3.12. Analog Integrator Unit Block Diagram.

The analog computer consisted of four integrator units of the form shown in Fig. 3.12, realized with the aid of operational amplifiers. The deisgn was patterned after the Philbrick K5-U units (see Appendix C).

The information required for setting up such an analog computer is the set of coefficients a_{ij} and the initial conditions, $S_i(0)$. The way in which the memory words were used to digitally set the coefficients a_{ij}, and hence control the analog set-up, is shown in Fig. 3.13. From this figure it may be seen that the light patterns of words were directly interpretable as the coefficients, showing most clearly the sign and relative magnitude of the various entries. In fact, the words corresponding to particular coefficients were spatially arranged to show the transposed A directly, as depicted below.



INPUT	DATA BITS	OUPUT
voltage	switch positions	current, i
s	0 000 000 000	0 s
s	1 000 000 001	-1 s
s	0 000 000 011	3 s
s	1 110 000 000	-768 s

Fig. 3.13. Technique Used to Set the Analog Coefficients

a_{11}	a_{21}	a_{31}	a_{41}
a_{12}	a_{22}	a_{32}	a_{42}
a_{13}	a_{23}	a_{33}	a_{43}
a_{14}	a_{24}	a_{34}	a_{44}
$S_1(0)$	$S_2(0)$	$S_3(0)$	$S_4(0)$

Each column may be read as an equation of the form of Eq. 3.4.

As anyone who has faced the set-up and scaling problems associated with analog computers knows, the potential of a digitally-controlled unit is great. The DSL display provides a "variable width window" with maximum resolution for each variable over the entire time range of the computed solution.

4. EVALUATION OF THE DYNAMIC SYSTEMS LABORATORY

There are many dimensions to the evaluation of a system like the DSL. This thesis is concerned primarily with the use of the DSL for self-teaching, and an experiment concerned with this application is discussed in section 4.20. Several observations have general applicability to computer-based systems, however, and these are presented in section 4.10 under the category of communication between the user and the machine. Finally, another application of the DSL, which was explored in a minor way, is discussed in section 4.30.

4.10 Communications and Reliability.

4.11 Information Exchange between the User and the Dynamic Systems Laboratory.

The influence of the time-sharing system (TSS) service rate was found to be a subtle, but at times important, factor in the use of the DSL for teaching. Four communication states are defined below, and the sensitivity of the user to service delays in each state is indicated in relative terms. The states are:

a. the input state; in this state, the user is imparting a relatively large volume of information to the DSL (e.g., the line code for a graph). The main limit on communication is the user's input rate, which is how fast he can read and type the data. As long as there is an adequate input buffer, the TSS service characteristics do not materially affect this state.

b. the output state; in this state, the user is waiting for, or receiving, a large volume of information. The presentation rate of the output information was limited principally by teletype transmission rates (six

characters per second), not by TSS service characteristics. It was to relieve this bottle neck in information presentation that the analog-generated display was built. It was observed that general user interest was maintained because the information was continually being generated, so that a considerable time delay to completion was acceptable in the output state (see Section 4.22B).

c. the dialogue state; in this state, the user wishes to exchange small amounts of information with the DSL at his rate. Under these conditions, the user is quite sensitive to TSS service characteristics. For an experienced user of the system, consistent delays of five seconds or more in response were tolerable but quite annoying. Typically, for delays consistently exceeding ten to twelve seconds, users found the system unacceptable. When possible, they adjusted their strategy to minimize interactions; otherwise they stopped using the DSL. Under these conditions, users sensed the system as "fighting" them, rather than aiding them.

Another aspect of long delays in service was that they induced confusion and errors when inexperienced persons operated the DSL. A typical reaction when a long delay occurred was "What did I do wrong?" (usually nothing), followed by some erratic or inefficient behavior.

When the information exchange rate in the dialogue state was set by the user, he tended to feel in rapport with the system.

d. the inactive state; in this state the user is exchanging no information with the TSS by his choice (e.g., he is thinking). To the TSS, it could be an important state, if it were identified, because the user could be dropped from a high level of interaction to a lower one temporarily. It would be possible to have a user indicate his intention of entering this state, and it may be relatively predictable (e.g., after a sequence

of output requests).

4.12 Graphical Input by Keyboard.

The technique of line-coding bond graphs (which applies to any linear graph in modified form) was taught to six freshmen in about ten minutes by the author. No difficulty in either comprehending or applying the technique has been noted, although there are a few consistent coding and typing errors which occur. Table 4.1 summarizes the data on line coding of bond graphs for users.

One interesting fact is that the incidence of error in some cases is surprisingly high (e.g. 21%). However, when each subject was asked whether he felt the coding procedure bothered him, the uniform reply was that it did not cause any difficulty. Another point about the data is that every error encountered in examining the subjects' complete records is classifiable as one of the six types listed. This enabled the DSL to provide a helpful set of diagnostics for interpreting coding errors.

4.13 Reliability.

Even after extensive testing it is possible for a complex computer-based system like the DSL to contain errors. Reliability was increased in the DSL by providing two types of redundancy. The command set contains alternative ways to accomplish the same request (e.g., the similar commands PAR and PARAMETER). The bond graph element set also contains redundant elements. For example, the author once found it necessary to replace a transformer (TF) with a pair of gyrators (GY-GY) during a classroom demonstration, due to a DSL error in processing the element TF. The output requests provide a certain degree of overlap in presenting response information.

ITEM	SUBJECT				ITEM TOTAL
	1	2	3	4	
1. Total No. Graphs	91	62	67	95	315
2. Avg. Graph Size*	3.5	2.9	3.3	3.3	-
3. Errors - type a	3	00	3	8	14 (29)**
b	1	2	1	2	6 (12)
c	3	0	1	4	8 (16)
d	2	0	5	1	8 (16)
e	2	1	1	2	6 (12)
f	1	0	2	3	6 (12)
4. Total No. Errors	12	3	13	20	48
5. Error Ratio - % (item 4/item 1)	13	5	19	21	15 (avg)

* The size of a graph is defined as the number of nodes.

** Percent by type of error in ( ).

Error types.

a. Spaces omitted between some characters.

b. Period omitted at end of code.

c. Node group omitted.

d. Illegal node type.

e. Bond labeling error.

f. Comma omitted.

TABLE 4.1. Summary of Errors in the Line Coding of Bond Graphs.

4.20 The DSL as the Basis of a Self-Instructional System.

In order to assess the role of the DSL in a self-instructional system, a pilot teaching experiment was conducted. The experiment was designed to yield as much information as possible relevant to the planning of a completely automated teaching system (i.e., one with a computer-controlled monitor). Particular objectives were: (1) to have the student develop a close association between a bond graph (representing a linear lumper-parameter model) and its dynamic behavior, without use of an intermediate analytical formalism involving equations; (2) to have the student study the time response of first- and second-order systems, making considerable use of experimental data, so that higher order system behavior could be developed in a similar fashion; and (3) to investigate the behavior of students when they acted as their own teachers (or monitors).

4.21 The Plan of the Experiment.

A. The Material

The subject matter was the time response of first- and second-order linear dynamic systems described in bond graph notation. There was no attempt to teach modelling of physical systems, and no explicit development in terms of differential equations, complex variables, transfer functions, or other standard techniques. The material was designed to make a rather complete use of the DSL.

The list of session titles of the pilot material is:

1. Familiarization with the Use of ENPORT.
2. Investigation of Single Field Elements Systems.
3. Investigation of C-R System Behavior.
4. Investigation of I-R System Behavior.

5. The System 'FORC'.
6. Recap of First Order System Behavior.
7. Investigation of Simple Oscillating Systems.
8. Simple Oscillators with Resistive Losses.
9. Quantitative Study of Two State Variable Systems with Dissipation
   Part 1. Underdamped.
10. Quantitative Study of Two State Variable Systems with Dissipation
   Part 2. Overdamped.
11. Superposition.
12. Use of the TEST Facility.
13. Some Test Problems.

Figs. 4.1 through 4.4 show examples of the pilot material. Each example indicates a different use of the DSL in presenting the material, ranging from a qualitative investigation of system behavior (Fig. 4.1) to the decoding of unknown (or partially known) systems (Fig. 4.4).

B. The Subjects.

Subjects used in the experiment were all M.I.T. freshmen. None had had any formal experience with an organized approach to system dynamics, although one student had had extensive laboratory experience as an electronic technician during previous summers, and had been advanced in calculus to the second year (differential equations). They were selected primarily on the basis of their willingness to complete the experiment, and their lack of formal training in system dynamics. (In fact, six subjects started the experiment but two were unable to finish even half of the sessions, due to scholastic difficulties.) All subjects were paid at the standard rate for M.I.T. freshmen.

2. Observations on the general behavior of C - I systems.

In this part emphasis is placed upon general observations which will be valid for any initial conditions and parameters of a linear C - I system. Quantitative relations will be considered in part 3.

```
C —— I
1.    2.
(10.) (0.)
```

a. How does the energy behave?

b. How does the <u>power</u> relate to the <u>energy</u> behavior?

You may find the output request 'DEN' helpful. It gives the <u>energy</u> <u>transferred</u> on a bond. For example,

```
HPLOT
0.,10.,1.
DEN(A) ........
```

c. Characterize the 'e' and 'f' behavior, relative to one another, and relative to the energy and power.

d. Demonstrate that your observations apply to any C - I system, regardless of initial conditions of parameter values, by a few experiments.

(This example is Session 7.2 of the pilot material.)

Fig. 4.1. Problem 1 - The Qualitative Investigation of System Behavior

2. In this session you will discover a quantitative relation which governs the behavior of linear-element C - R systems.

A. Build this system

C —A— R		structure
1.	1.	parameters
(1.)		initial conditions

0.1 = DT

Notice the type of behavior it shows for the state variable, $e_A(t)$.

B. Predict the type of steady state (e.g., static, stationary, quasi-stationary), and demonstrate the steady state energy (or power) limit.

C. Study the e(t) response carefully and observe that there is a characteristic constant which will tell you all about the curve.

(If you don't make any progress, try looking at clue 3.1. <u>If you use the clue</u>, say so on your log-sheet, please.)

D. How can this constant be used to predict e(t) from e(0) ?

(See clue 3.2.)

E. Now observe that if you vary the initial condition of e_A, but <u>divide</u> the response curve by the initial value, then there is only <u>one</u> curve of this (normalized) type.

Prove this to yourself for at least two initial conditions.

(This example is Session 3.2 of the pilot material.)

Fig. 4.2. Problem 2 - The Discovery and Formulation of Quantitative Relations

1. Investigate the system (sometimes called 'FORC')

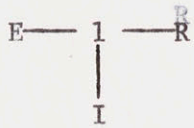
2. Consider the influence of the additional factor, F, a constant source of f.

A. How does the behavior relate to the system C - R ?

B. Can you predict the steady state right from the bond graph (for any values of F, R, and C) ? Verify your prediction with evidence.

C. Describe the steady state of all C's in a stationary steady state system. (Remember, all flows must become constant).

3. Generalize your results above to and make a prediction for some example.



(This example is Session 5 of the pilot material.)

Fig. 4.3. Problem 3 - Prediction and Verification of System Behavior.

3. Some unknown systems.

SYS. NO. 5 — A —— S — What is S ?

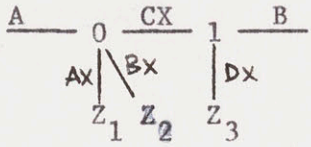
SYS. NO. 6 — A —— S — What is S ?

SYS. NO. 7 — A —— S — What is S ?

4. Somemmore unknown systems.

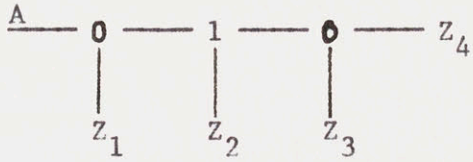
For these systems partial information is given below.

SYS. NO. 8



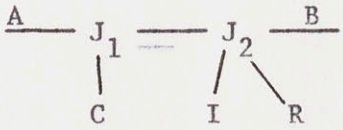
Find each Z_i (either C, I, or R) and its value. You may look at all state variables as outputs.

SYS. NO. 9



Same as above. Again you may examine each state variable by number reference (s(1)).

SYS. NO. 10



J_1 and J_2 are to be found. (They are either 0 or 1)

(This material was taken from Session 13.)

Fig. 4.4. Problem 4 - Testing of Unknown Systems.

C. The Plan of Instruction.

Students were given two initial lectures (each about one-and-a-half hours long) on the evergy basis of bond graphs, the concept of state space, the defining characteristics of the ideal bond graph elements, and some technical aspects of using the DSL, such as line-coding of graphs. A subsequent lecture was given, after session 7, on the A-matrix representation of a linear dynamic system. The rest of the teaching was done by the students, using the DSL in conjunction with the written material.

Students were requested not to discuss their experiences with anyone else, or to read about anything directly related to the pilot material. They were encouraged to use the logsheets as their complete records, and to write anything they wished on them.

This experimental plan led to a rather large volume of data of an individual nature, which is analyzed in the next section.

4.22 The Results.

The results of the teaching experiment are discussed in four parts in this section, and are supported by data from Appendix D. Achievement is concerned with what the subjects learned. Behavior, the second part, considers the tactics adopted by subjects in learning from the DSL. Subjective Reactions presents students' reactions to various aspects of the teaching experiment, and the final part, Time Costs, sums up the data on the actual (or student) and computer time used in the experiment.

In this section references will be made to problems 1, 2, 3, and 4, which are shown in Figs. 4.1 through 4.4, respectively. The problems are also referred to by type as -- qualitative investigation, quantitative investigation, prediction and verification, and test systems, respectively.

A. Achievement --What the Subjects Learned.

The data of Table 4.2, Item 4 (Test Systems), indicate that three out of four subjects were able to achieve a respectable score in decoding unknown systems. The scores referred to are, in order -- 33/39, 11/31, 30/39, and 25/39. The test systems were presented to the subject as the last session of the material. Fig. 4.4 indicates the type of problems presented. The subject was able to test the unknown system with any bond graph fragment (or partial graph) he wished. The most common test system was a constant source element (either effort or flow), whose value was varied. Students were not instructed in testing techniques. For each test system the subject arrived at a graph and a set of parameters which he felt were correct (i.e., equivalent to the test system). (In only one case did a subject construct a system of his own to observe its behavior.

In order to score a problem, each unknown element or junction and each parameter was worth one point, if identified. In Appendix D, Part 2, is shown the complete set of predicted graphs for the test systems. The other scores in Table 4.2 indicate the success of subjects on selected problems presented in the development of material. They are similar to exercises at the end of a chapter in a text. A brief characterization of the subject matter is given in the table, but in Appendix D, Part 1, the reference problems are reproduced in full. Points were awarded on an element and parameter basis, as described above for the test systems.

In relation to the use of experimental data in developing a quantitative relation for a system, two subjects had difficulty with a problem of this type, although they struggled through to an answer. However, when the same system appeared in a problem section both subjects again failed to predict the system behavior correctly.

ITEM (session reference)**	Score Basis	SUBJECT SCORES 1	2	3	4
1) First order systems (sess. 6 - a, b, d, f)	4	4	3	3	4
2) Conservative Second Order (7.4 - a, g, h)	6	6	4	5	5
3) Underdamped Second Order (9.4.B - 2, 3, 4)	6	6	2	4	5
4) Test Systems (See Fig. 4.4)	39	33	11*	30	25
TOTAL	55	49	20*	42	39

* Indicates subject's score based on 31, not 39; total based on 47.

** These will be found in Appendix D, part 1. (Table is discussed in text; section 4.22.)

TABLE 4.2. Composite Achievement Scores for Selected Material.

B. Behavior:

This section is concerned with tactics adopted by the students in learning from the DSL. Subjects showed wide variations in behavior under the freedom of the experimental plan. However, as Tables 4.3 and 4.4 show, there are many similarities in the tactics used. Items 1 through 4 of that table refer to the number of changes made by subjects to particular values. Any time any value was changed a new experiment was counted. If no asterisk is used it means that the cell entry shows the number of times a single item value was changed. An approximate measure of the amount of output information taken is given by items 5, 6, 7, and 8.

In making qualitative investigations of system behavior the subjects used the display and plotted output in preference to tables. This is shown in Table 4.3 for the first problem (see Fig. 4.1). This approach permitted subjects to formulate certain aspects of the observed behavior in their own terms, before a (teacher) pre-planned quantitative form for the development of the system was undertaken. For example, on observing a conservative second order system one subject wrote -- "The energy and the flow are in phase, but the frequency of the energy is twice the frequency of the flow." He then showed this to be true independent of parameter values and initial conditions. On the other hand, when a subject misinterpreted a response he sometimes formulated an incorrect impression of behavior, which made it harder for him to develop a quantitative relation. For example, one subject misinterpreted information from a plot of first order system response and assumed an incorrect relation between "speed of response" (not yet named the time constant) and the parameter values. Qualitative investigations used a considerable amount of subject time, since they relied heavily on plotted output (see Table 4.4, Item 5 for problem 1), and plots are the slowest form

ITEM	Qualitative Investigation Problem 1				Quantitative Investigation Problem 2			
	1	2	3	4	1	2	3	4
1) No. of experiments	3	3	4	1	3	3	3	2
2) No. of initial cond. mods.	1*	2*	2	0	2	2	2	1
3) No. of parameter mods.	1*	1*	1	0	0	0	0	0
4) No. of source mods.	-	-	-	-	-	-	-	-
5) No. of displays	1	1	6	**	1	0	1	1
6) No. of tables	0	1	0	5	5	0	4	11***
7) No. of plots	10	6	13	4	3	6	1	1
8) No. of A-matrices	0	0	0	0	0	1	0	0

ITEM	Prediction and Verification Problem 3 (5.2)				Prediction and Verification Problem 3 (5.3)			
	1	2	3	4	1	2	3	4
1) No. of experiments	14	8	3	13	2	1	7	1
2) No. initial cond. mods.	8	1	0	7	1	0	0	0
3) No. parameter mods.	9	12*	1	4	2	0	5	0
4) No. of source mods.	5	3	1	3	1	0	1	0
5) No. of displays	3	0	3	2	0	0	1	0
6) No. of tables	25	0	2	27	3	0	7	0
7) No. of plots	2	8	2	0	0	1	1	1
8) No. of A-matrices	0	0	0	0	1	0	7	0

* Several simultaneous changes made.

** The display was not working.

*** The subject was experimenting with the DSL command 'TABLE'.

TABLE 4.3. Summary of Tactics for Selected Problems.

ITEM	SYS. 5				SYS. 6				SYS. 7			
	1	2	3	4	1	2	3	4	1	2	3	4
1) No. experiments	3	3	4	5	3	2	1	5	4	2	2	5
2) No. displays	0	0	0	0	0	0	1	1	0	0	0	0
3) No. tables	1	4	5	10	6	2	2	9	2	2	5	11
4) No. plots	3	2	0	0	2	2	1	2	7	2	2	0
5) No. A-matrices	1	0	0	0	1	1	1	0	1	0	0	0

ITEM	SYS. 8				SYS. 9				SYS. 10			
	1	2	3	4	1	2	3	4	1	2	3	4
1) No. experiments	2	1	3	4	3	*	6	2	1	1	2	3
2) No. displays	0	0	0	2	0	-	0	0	0	0	0	2
3) No. tables	4	1	10	7	0	-	8	7	2	0	1	2
4) No. plots	8	1	3	0	5	-	3	0	3	1	2	0
5) No. A-matrices	2	0	1	0	1	-	1	0	1	0	0	0

* Subject 2 did not try test system 9.

TABLE 4.4. Summary of Tactics on Test Systems.

of response communication. An average plot requires about two minutes to complete, and so the ten plots taken by Subject I, for example, required twenty minutes just for communication.

In making quantitative studies of system behavior subjects relied on tables of results as the most important form of response data. One subject for whom this was not true had considerable difficulty in deriving consistent quantitative relations for data developed from the plotted results. In several cases he was almost totally unsuccessful and had to read the clues and have the entire development of the relations presented to him. The process of constructing relations from tables of data was time-consuming for the subjects. For twenty-five average size tables, for example, the time for communication alone was about half an hour. Subjects did show a considerable range of emotion on such problems, from enthusiasm (e.g., "I'm right!!!") to frustration (e.g., "I don't get it. Let's see what the clue says."). One subject, when unable to piece together several scattered results, took the data home with him (with permission) and derived a formulation from it. Other subjects collected a considerable amount of data and then ignored the DSL to write long records of derivation on their logsheet.

Prediction and verification problems were used in two ways. In one case subjects were presented with some results or relations and asked to verify them with experimental data. They uniformly showed a lack of interest in doing this, typically running one "form" experiment to show the validity. When the prediction involved results about which the subject was not certain (e.g., relations derived by himself) verification became of considerable interest. Because access to the clues and to experimental evidence was under the subject's control, variations in the significance of prediction arose. Two subjects consistently made explicit predictions before verification, one

subject sometimes did, and one subject almost never did. The latter subject was also the lowest achiever (see Table 4.2).

The test system facility produced exceptional interest on the subjects' parts as recorded on the logsheets (e.g., "Nice going, Howie!", "I'm right, I'm right, I'm right!"). When lack of success was consistent the interest flagged and relatively little serious attempt was made to decode the subsequent test systems (see Table 4.4, subject 2). Tables and plots received rather balanced use in decoding, the tables producing specific data for parameter values, and the plots suggesting the dynamic nature of the elements. Subjects evolved their own test strategies after a few trials. For example, three realized that by putting a resistance element on the test bond(s), the passivity properties of the system could be studied. One subject unsuccessfully tried to build a composite test graph made up of a flow source, an effort source, and a resistance, and be specializing values achieve the particular type of test element he required.

C. Subjective Reactions of Students.

The data in this section are derived from logsheets or are interpretations of subject remards in response to a final interview.

The subject who had had prior experience with electronic circuits reported that toward the end of the material he was thinking straight in bond graphs. Another subject indicated that in the beginning he used springs and masses to assist him (he said bond graphs were too abstract to think about).

The subject with electronics experience had no difficulty relating the material to reality (i.e., physical systems). The other subjects all noted the relation of the material to their freshman physics course, involving

mechanical oscillations. One subject said, "I can't see a real world relation -- (this material is) like a higher math course -- my physics course is just about the same."

All subjects indicated that the DSL was easy to use, but that the occasional prolonged service delays and forced terminations of sessions were extremely frustrating. One subject reported that he "enjoyed coding the graphs", and no one felt it a drawback. Being good M.I.T. freshmen, the subjects indicated that they found working at all hours of the day and night somewhat amusing ("18 straight hours at Tech -- what would my mother say?"), but there was a pronounced preference for day and early evening time on the DSL sing-up sheet, despite the fact that computer service was much worse at those times than later at night.

Although the subjects felt that it was interesting to do experiments (e.g., "Kind of fun to experiment -- doing a whole bunch of them and watching.") they uniformly agreed that the approach was inefficient, and that texts were more useful to them ("I prefer to plod along paths set about before.").

The test systems feature was given enthusiastic support, even though one subject did not do very well. It was perceived more or less as a game and a challenge.

D. Time Costs.

The time data for the four freshmen students are summarized in Table 4.5. Item 2 indicates a wide range in average session length, from about one to two hours. As shown by item 2 the most efficitnt subject (in terms of machine use) had about twice the use ratio as the least efficient subject. Under current time-sharing system operation about thirty subjects could be

ITEM	SUBJECTS			
	1	2	3	4
1. Number of sessions*	20	18	12	23
2. Avg. student time/session (minutes)	88	65	133	75
3. Max. student time/session (minutes)	204	151	235	176
4. Min. student time/session (minutes)	24	29	36	30
5. Max. computer use/session (%)**	5.0	3.8	11.3	11.8
6. Min. computer use/session (%)**	0.8	1.1	1.0	1.5
7. Total student time (hrs.)	29.4	19.6	26.7	28.6
8. Total computer time (min.)	47.4	22.4	62.1	66.0
9. Average use (%)**	2.7	119	3.9	3.8
10. Number of Forced Terminations	6	6	1	3

* A session refers to one sitting at the DSL.

** All percents are ratio $\frac{\text{computer time}}{\text{student time}} \times 100.$

Note. Item 4 excludes sessions terminated for reasons outside of the user's control.

TABLE 4.5. A Summary of the Time Data.

taught simultaneously. For all but one subject the number of forced terminations (item 10) is very high, and the effect on the preceding time data must be kept in mind.

4.23 Conclusions.

The results presented in the previous section lead to the following conclusions on the use of the DSL as the basis of a self-instructional system:

(1) Three of four subjects successfully associated dynamic responses with bond graph models on the basis of relations derived from experimental evidence. One subject was unable to achieve a clear association. This conclusion is based upon the test system results and several observations on the imagery used by the students.

(2) For students to make effective use of the DSL as a laboratory they must be able to learn from experimental data. They must either be monitored or show evidence of being able to monitor themselves satisfactorily. This conclusion is based on the wide spectrum of monitor characteristics displayed by the subjects, ranging from persistence and carefulness to laxness in the resolution of contradictions.

(3) The response characteristics of linear first- and second-order systems was not self-taught efficiently by being based on qualitative and quantitative observations. Furthermore, in one case the subject formulated incorrect impressions of system response, and the subsequent confusion was not effectively dispelled.

(4) The DSL proved effective for prediction and verification problems, provided the subject made a reasoned prediction beford examining the data. In cases when a guess was made the procedure of gathering experimental evidence

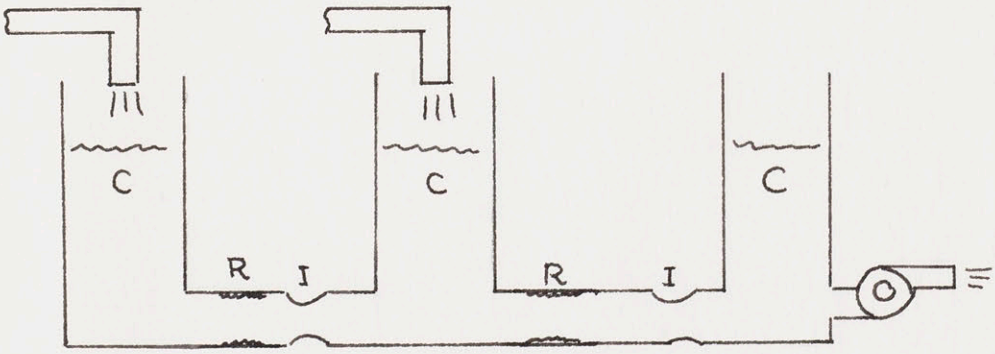
was inefficient.

(5) The test systems facility was useful for three purposes -- it increased the subjects' understanding of system behavior; it served as a vehicle for learning strategies for testing unknown systems; and it was used to measure achievement by both the teacher (i.e., the author, in this case) and the students.

4.30 An Application of the DSL in the Classroom.

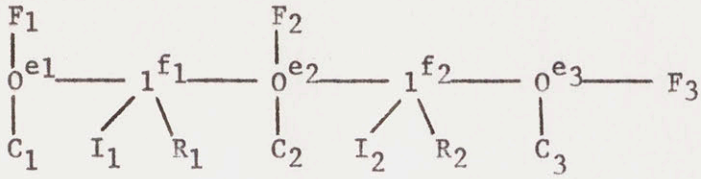
The DSL was provided as an on-line teaching aid in the classroom, in a course for graduate engineering students entitled "Analysis and Design of Engineering Systems".* The students were able to observe the response of systems by means of a large television monitor. A fairly complex theme problem was used as the continuing element in the development of the material. It is described in this section for the general interest of the reader.

The physical problem involved a sewer system which had to be controlled in an appropriate way in the presence of inflow distrubances. A representation of the physical system is --



* M.I.T. Course 2.161 in the Mechanical Engineering Department, taught by Prof. H. M. Paynter and assisted by the author, in the fall term, 1964.

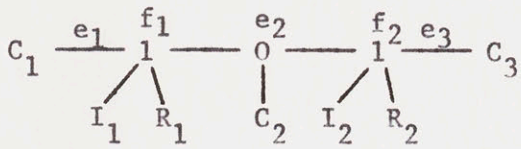
A bond graph model of the physical system is --



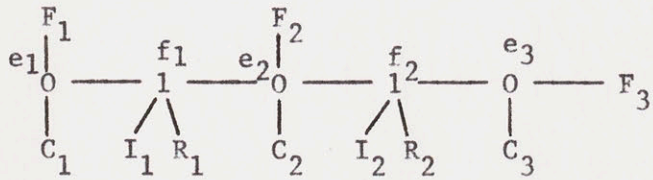
The three efforts on the zeroes are the levels, and the two flows on the one junction are the fluid flows in the pipes. The problem was discussed as a design and control task -- how should the C's be sized, and how should F_3 be manipulated in the presence of a certain class of disturbances (F_1 and F_2) to maintain e_1, e_2, and e_3 within predetermined limits?

Over a period of several weeks the following sequence of studies was conducted, with the corresponding results of each study being made available to the students in ditto form shortly after each class.

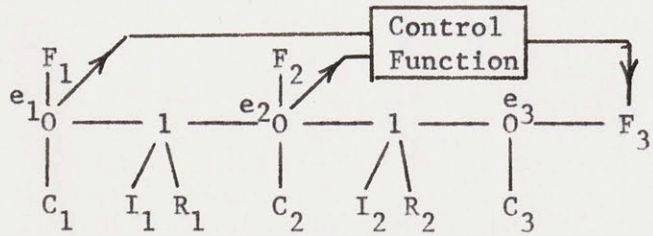
Phase 1. The passive system.



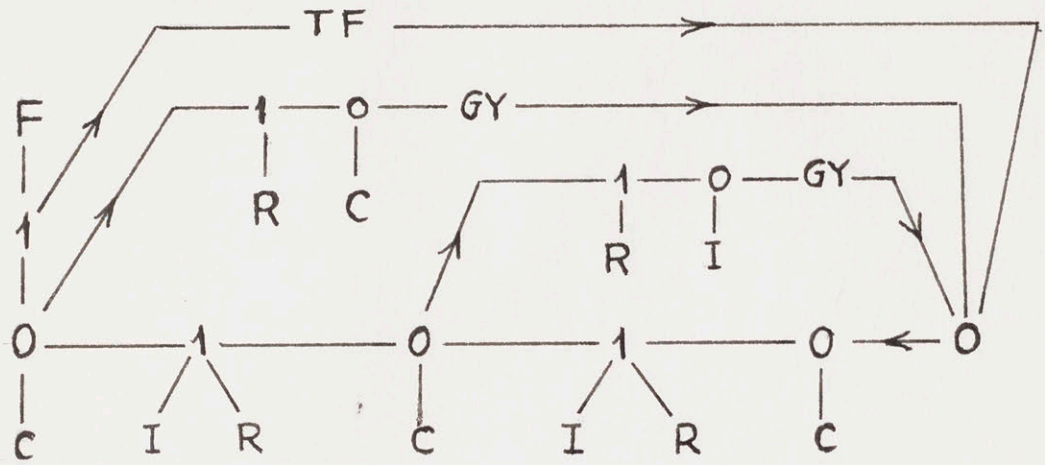
Phase 2. The disturbed system with simple control (i.e., set $F_3 = \frac{1}{2}(F_1 + F_2)$)



Phase 3. Insertion of instruments and functional control.



Phase 4. Realization of the Control Functions as Active Elements.



A more realistic investigation of the physical system was hampered by two factors about the DSL -- the linearity of the bond graph elements, and the size limit for bond graphs.

The following conlusions are based on the author's observations in the class, plus discussion with some students.

(1) Use of the DSL as a class room aid is feasible, based on cooperation between the lecturer and the operator of the DSL. Its principal use is to provide dynamic response information on a particular system in real time, creating the opportunity for a flexible lecture format.

(2) Students can be involved to a significant extent in design and control types of discussions, or in other predictive situations. The answers can come from the DSL, as representing the physical world, rather than from the instructor. To quote a student -- "In 2.161 I felt it (the DSL) was especially worthwhile, having seen the system in operation with one set of "C"'s (for example) to see what happened when you changed them." The "C"'s refer to the sewer control problem discussed previously.

The remaining two chapters present a set of conclusions about the problem of computer-aided teaching, and offer some suggestions of avenues for further research.

5. CONCLUSIONS

(1) An algorithmic procedure for the analysis of linear dynamic systems based on the bond graph notation and the state space approach has been developed (see Section 2.30). The principal advantage of the procedure is that it permits the complete pre-ordering of the system equations by graphical operations. The equations are then written in a standard form using a search technique on the augmented bond graph.

(2) A Dynamic Systems Laboratory, which is based on a time-shared digital computer and implements the procedure described above, has been created for the purpose of teaching dynamic system behavior. The Laboratory simulates the behavior of linear lumped-parameter dynamic systems, which are specified by the student in bond graph notation (see Section 2.10). All of the dynamic response variables (e.g., voltages, displacements, powers) are available in the form of tables or plots, and high speed response plots are generated on an oscilloscope display set up by a special analog computer. Thus a close temporal association between the description of a dynamic system and a display of its response characteristics is available to the student.

(3) The principal application of the Dynamic Systems Laboratory has been in an experiment on the teaching of introductory dynamic systems behavior, in which the Laboratory served as the basis of a self-instructional system (see Section 4.21). The DSL was used to develop a general conceptual framework for explaining the behavior of classes of dynamic systems based on qualitative investigation and quantitative formulation of experimental data by the students. This is in contrast to the usual lecture and textbook

style of straightforward exposition for concept presentation. Such an application of the DSL was found to be inefficient for all the subjects, and ineffective for one. For studying the behavior of systems in detail the pattern of thoughtful prediction followed by experimental verification proved effective and motivating to the subjects. However, when guesses, rather than reasoned predictions, were made the verification procedure became quite inefficient. A test facility, in which the subjects tested and synthesized unknown (or black box) systems, was found to be valuable for several purposes, which included the measurement of achievement, the increase of understanding of system behavior by the subjects, and the motivation of subjects.

(4) The DSL was used as a classroom teaching aid, and assisted the lecturer in conducting a flexible design and control study of a large, complex dynamic system (see Section 4.30). In addition there was an increase in student participation in the class discussions.

(5) The primary role of the Dynamic Systems Laboratory in a fully automated instructional system should be to provide a responsive environment for applications of the basic concepts, progressing from analysis to prediction and verification to testing and synthesis. A secondary role, that of teaching students good experimental technique, seems promising, but will require further investigation.

6. RECOMMENDATIONS FOR FUTURE RESEARCH

6.10 Improvements to the Dynamic Systems Laboratory

The present version of the Dynamic Systems Laboratory is limited in several ways. Each of the following sections discusses a feature which has posed a conceptual, rather than a simple developmental, problem in the continued evolution of the DSL.

6.11 Improved Manipulation of Graphs.

The value of the line coding technique now used for bond graphs (or any linear graphs) will be increased when two additional features can be provided by the DSL. These are:

a. A coding technique for building repeated fragments from a prototype (this would be valuable in a study of the lumping effect on transmission line modeling, for example).

b. An ability of the DSL to make identifications of graphs or fragments, with respect to a library of graphs. Chemists have a major interest in this problem for compound identification, but some reasonable limits for dynamic system identification, which involves far simpler graphs, could be set. Based on a very limited investigation, it appears that use of the Shimbel algorithm may provide a workable solution (see ref. 17).

6.12 Controlled Accuracy of Computed Results.

At the present time there seems to be no general criterion available for predicting the truncation error effects in expanding the so-called M-matrix from the A-matrix, according to the relation $\underline{M} = e^{\underline{A}T}$, where T plays an important role in the expansion. In the DSL, what is really desired is some way to guarantee the accuracy of the final solution, found as

$S(KT) = S(0)\underline{M}^K$, within predetermined limits. This work will be particularly useful when larger, more complex systems are simulated, with the wide range of dynamic behaviors which they will include.

6.13 Nonlinear System Simulation.

The most effective modification to the current DSL would be to introduce the ability to carry out some nonlinear system simulation. There are at least two ways, not necessarily exclusive, in which this problem may be approached. Nonlinearities may be specified in functional form directly, and the DSL would make a piecewise linear representation of them. Or a specific nonlinear element type (e.g., particularized as an ideal diode) may be included in the allowable element set and certain classes of nonlinearity could be modeled by its use. In either case the problem of piecewise linear computation arises.

6.14 Display Capabilities.

If one can envision a time when numerous remote time-sharing stations are available for teaching purposes (in the sense of this study) then some standard unit for displaying dynamic system response would be desirable. It would be possible to duplicate the approach used in this work and equip each station with its own local memory and analog computer. However, just as the digital computer benefits by being time-shared so might the analog computer and its memory. A systems study which relates the various communication, storage, computation, and display requirements for a proposed class of use would be of immense value, and might lead to new design specifications for critical elements. This is especially true when one is trying to develop an economical unit for teaching applications, which should be inexpensive and

reliable, but does not have to be too fast or precise.

6.20 Applications of the Dynamic Systems Laboratory in Teaching.

6.21 Automated Instruction.

The results of the teaching experiment described in section 4.20 suggest that the Dynamic Systems Laboratory be used in a prediction and verification role, and to provide testing and synthesis experience for the student. In order to interpret a student's predictions and evaluate his test system models the teaching monitor must be able to compare and identify graphs and graph fragments.

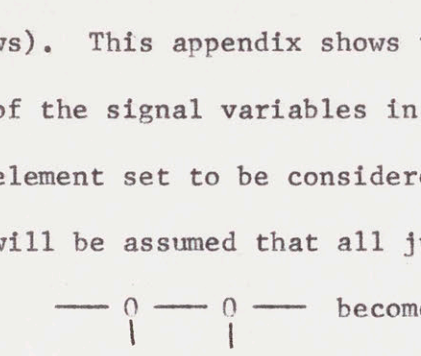
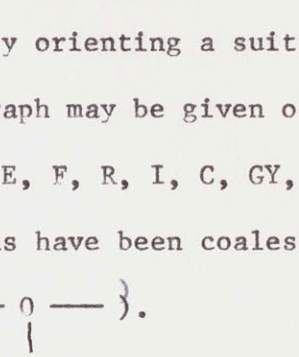
If the teaching monitor is to give instruction at the level of testing strategies and efficient experimental procedures it must be able to interpret the sequence of responses the student makes and comment appropriately. For example, if a given relation should be apparent after two parameter changes and the student has made three or four such changes, a comment could be given. Efficient use of instruments might be taught by a monitor capable of measuring the approximate amount of information output (e.g., the size and number of tables requested), and responding appropriately (e.g., You have taken two hundred data points -- the result can be obtained from twelve.).

The planning and writing of a comprehensive program for teaching dynamic systems behavior would be assisted materially if the monitor itself were capable of generating examples and problems as the need arose. Certain characteristics of appropriate problems at the particular point in the teaching program could be specified by the programmer (e.g., underdamped second-order system), and the monitor would expand the specification into a particular system.

6.22 Advanced Training in Modeling.

It is possible to control and operate a physical system (e.g., a generalized rotating electrical machine) through the Dynamic System Laboratory, using only the keyboard. If the physical system is properly instrumented the task for the student is to build a bond graph model, using the DSL, which matches the real system behavior as well as possible.

Appendix A. Notes on the Power Convention for Bond Graphs.

(1) In a bond graph with B bonds there are 2B <u>signal variables</u> (i.e., efforts and flows). This appendix shows that, by orienting a suitable set of powers, all of the signal variables in the graph may be given orientations. The bond graph element set to be considered is $\{E, F, R, I, C, GY, TF, 0, 1\}$. Furthermore it will be assumed that all junctions have been coalesced where possible (e.g.,  becomes ).

(2) The definitions of the ideal n-ports ($n \geq 2$) place constraints upon the number of arbitrary signal variables orientations associated with each type. In particular, the <u>common modulus</u> constraint for a two-port restricts the orientation of <u>one</u> signal variable. The <u>common variable</u> constraint for an n-port junction places (n - 1) constraints on the signal variables. These conventions mean that the total number of orientation constraints, C, on a bond graph is

$$C = \sum_{i=1}^{m} (i - 1) \cdot n_i \tag{A.1}$$

where n_i = number of i-ports,

m = highest i-port index, and

C = total number of signal constraints.

(These symbols will be used throughout this Appendix.)

(3) There is no way, in directing the powers, to give a <u>unique</u> orientation to the initial signal variable (e.g., "up or down", "tension or compression"). Hence the number of signal variables to be oriented in a bond graph is

$$\begin{aligned} ASV &= 2B - 1 - C \\ &= \sum_{i=1}^{m} (i \cdot n_i) - 1 - \sum_{i=1}^{m} (i - 1) \cdot n_i \end{aligned}$$

$$ASV = \sum_{i=1}^{m} (1 \cdot n_i) - 1$$

$$ASV = N - 1 \qquad (A.2)$$

where ASV = number of arbitrary signal variables, and

N = number of nodes in the graph.

We also note for future reference that the number of bonds in the graph, B, is given by

$$B = N - 1 + L \qquad (A.3)$$

where B = number of bonds, and

L = the number of loops.

The excess of bonds over the number of signals to be oriented (ASV) is equal to the number of loops.

$$B - ASV = (N - 1 + L) - (N - 1) = L \qquad (A.4)$$

In particular this result indicates that all but one of the signal variables of a bond graph with no loops (a <u>tree</u>) may be oriented by giving each bond an arbitrary power direction.

(4) A graph with L loops may be made into a tree by removing one bond from each loop. The set of L bonds so removed is called a <u>chord</u> set. Now if the tree powers are oriented, all of the arbitrary signal variables are oriented (except the initial one). Therefore the powers of the chord set must be defined by the signal variables already defined, because there are no arbitrary signal variables remaining. In summary, we conclude that

a. The bonds of any tree of a bond graph may be oriented arbitrarily with respect to power.

b. If a set of tree powers of a bond graph is oriented, the powers of the corresponding chord set are constrained.

(5) A technique for indicating the constraints implied by a tree power orientation set will now be described. The discussion will relate only to loops of a graph, since no restrictions are necessary for paths not in loops.

a. Give an element in the loop a (+ or −) sign to indicate the choice of the arbitrary signal variable orientation (e.g., an e on a 0, an f on a 1).

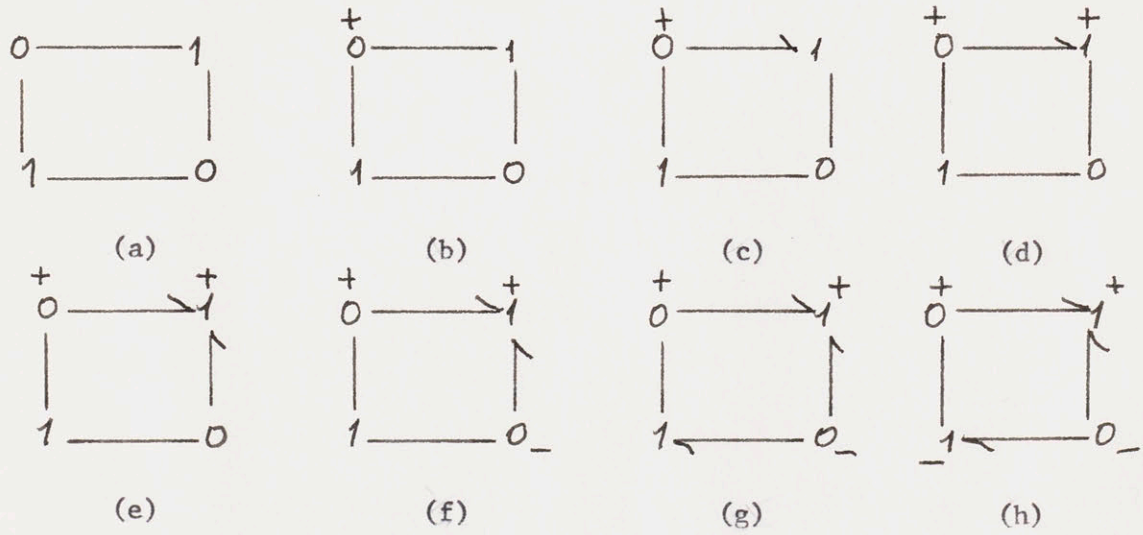
b. Orient a power on a bond incident to the signed node. Select a consistent loop definition of difection for the relation +e · +f = +P, and apply it to orient the next node.

c. Use the node constraint to extend the signal orientation to the incident bonds.

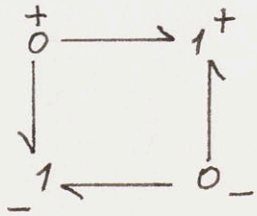
Return to step b. (until the last node in the loop has been signed).

D. The remaining bond in the loop <u>must</u> be oriented in conformity to the e · f = P definition for the graph.

An example follows:



Observe that the remaining bond already has its e and f defined. The power direction is constrained.



APPENDIX B

A User's Guide to Enport

1. Introduction

This user's guide assumes the following experience on the part of the reader:

a. that he is familiar with the M.I.T. Compatible Time Sharing System, including procedures like 'LØGIN', 'LØGØUT', etc. (otherwise see ref. 20);

b. that he is somewhat acquainted with the bond graph notation (otherwise see ref. 30 or Chapter 2 of this study).

2. General Enport Characteristics

Because Enport is constantly undergoing revision, the version referred to in this Appendix will be the system of August 1965. General properties of this version are:

a. the maximum bond graph size is limited to 25 nodes and/or 25 bonds;

b. the maximum number of state variables for a system is 15;

c. the allowable set of bond graph elements is (C, I, R, E, F, GY, TF, 0, and 1) <u>plus</u> signal suppression.

d. the following restrictions apply to the element characteristics:

(1) C, I, and R are <u>linear</u>;

(2) E and F are constant;

(3) GY and TF have constant moduli;

e. the maximum magnitude of any variable must be less than 10^{38};

f. there is a constraint on the results available at one time due to storage requirements, given by

$$\frac{TMAX}{DT} * NSV \leq 1000$$

where TMAX is the upper time limit on the currently available results,

DT is the sampling interval, and

NSV is the number of state variables.

3. The Command Organization

The command system is organized to print the message 'READY...' when it is prepared to receive the next command. This point may be reached <u>at any time</u> in Enport by giving the "INTERRUPT' signal (see ref. 20). The major command groups by type of function are (1) the structure group, (2) the data input group, (3) the output group, and (4) the auxiliary group.

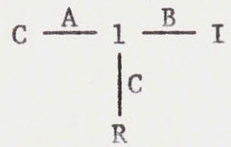
4. The Structure Commands

The purpose of these commands is to permit the creation and manipulation of bond graph structures, prior to numerical processing.

COMMAND	FUNCTION	ASSOCIATED DATA
GRAPH	to specify a graph	line code (1)
CAUSAL	to assign computing causality to a graph	-----
SIGN	to assign a power convention to a graph	-----
SUPPRESS	to suppress signals on selected bonds	signal-bond pairs (2)
RESTORE	to restore suppressed signals	signal-bond pairs (2)
STRUCTURE	= GRAPH + CAUSAL + SIGN + SUPPRESS	see above) (3)

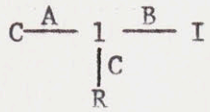
(1) Line code. The line code is a sequential collection of node-bond groups, separated by commas, and terminated by a period. Up to four lines may be used. Spaces must be observed, although extra spaces are always permitted.

example. C A, 1 A B C, I B, R C.



(2) Signal-bond pairs. These are sets of (effort-bond) or (flow-bond) pairs, which describe the particular signals to be used.

example. signal...F (flow)
bond...C
refers to the flow on bond C.



(3) Use STRUCTURE as the standard command for specifying a system.

5. The Data Input Commands

The purpose of these commands is to allow the specification of numerical data associated with the system under study.

COMMAND	FUNCTION	ASSOCIATED DATA
INPUT	provides an automatic guide for all input data	sampling interval, parameter, initial conditions
PARAMETERS	provides an automatic guide for all parameters	parameter values
INITIAL	provides an automatic guide for all initial conditions	state variable initial conditions
PAR	provides for manual input of selected parameters	(1)
INIT	provides for manual input of selected initial conditions	(2)
DT	input of sampling interval	(3)

(1) PAR...input format by example.

```
PAR
SCALE
  3.          sets all parameters = 3.
RB
  1.          sets RB = 1.
Q             quit PAR
```

SCALE and its value bay be omitted ifddesired;
any number of parameters may be specified;
data is processed sequentially, so running corrections may be made;

(2) INIT...input format by example.

```
INIT
SCALE
  0.              sets all state variables to 0.
CA
  10.             sets state variable for CA to 10.
Q                 quit INIT
```

(3) DT...input format by example.

```
DT
  0.2             sets DT = 0.2
```

6. The Output Commands

The purpose of the output commands is to present system responses and to check on the system status.

COMMAND	FUNCTION	ASSOCIATED DATA
DISPLAY	presents a scope display of state variable response (only in Room 1-115, M.I.T.)	t_1, t_2 (1)
HPLØT	plots responses versus time on teletype	t_1, t_2, dt (2) v_1 ... v_4 (3)
TABLE	prints results in tabular form on teletype	t_1, t_2, dt (2) v_1 ... v_4 (3)

(1) Results will be displayed from time t_1 to time t_2 (if possible).

(2) Results will be presented from time t_1 to time t_2 in intervals of dt.

(3) Up to four output variables may be requested at one time, from the following set:

E (bond)	-	effort on a bond;
F (bond)	-	flow on a bond;
P (bond)	-	momentum on a bond;
Q (bond)	-	displacement on a bond;
PWR (bond)	-	instantaneous power on a bond;
DEN (bond)	-	cumulative energy transfer on a bond;
S (I)	-	state variable I, where I is an integer;
EN (I)	-	the energy associated with state variable I, where I is an integer.

example. HPLØT (or TABLE)
0., 8., 2.
S(2) PWR(B) EN(5) Q(D)

The status check commands are:

COMMAND	FUNCTION - to print the
NLIST	node list
BLIST	bond list
IMX	node-bond incidence matrix
AMX	A-matrix
MMX	M-matrix
PRDT	sampling interval, DT
PRPAR	parameter values
PRINIT	initial conditions
CØDE	graph line code
PLIST	abstracted node list by parts

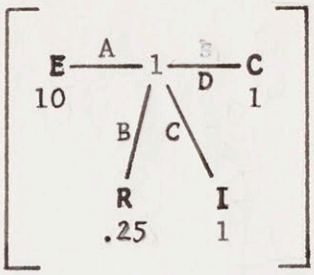
7. Auxiliary Commands

These commands provide auxiliary functions, some of which are very important or useful. These are starred (*).

COMMAND	FUNCTION
* ARITH	evaluates arithmetic expressions
* QUIT	ends ENPORT control
DELETE	deletes current system
DISTANCE	computes the distance matrix
DATA	follows 'READ DATA' convention (MAD) for certain variables (1)
COMPUTE	computes results
* SET TAPE	sets a working tape number
* FILE	files a system on tape record
LISTG	lists tape records by remark line
READ	prints a system record
* SET UP	retrieves and runs a system from tape
* TEST	retrieves and runs a 'black-box' system
CLEAR	clears the display memory

(1) See the author for clarification regarding use of this command.

8. An Example of the Use of Enport.




```
RESUME ENPORT
W 1457.9
EXECUTION.

READY...

STRUCTURE *

THE GRAPH, PLEASE...

E A, R B, I C, C D, 1 A B C D.

 STATE VBL  1 IS EFFORT ON CD
 STATE VBL  2  IS FLOW  ON IC

READY...

PARAMETERS

CD   = 1.
IC   = 1.
RB   = .25
EA   = 10.


READY...

INITIAL

  S1 = 0.
  S2 = 0.


READY...

DT
.25
TMAX =     83.000

READY...

DISPLAY
0.,15.

DISPLAY SCALES
 VARIABLE    FULL SCALE RANGE
      1          .200E 02
      2          .100E 02
      3          .125E 01
--!]PG:?A;AØ(DUIJ '''

READY...
```

* indicates that all user requests are underlined.

(line code for the graph)

(initial conditions of system in terms of the state variables.)

(the sampling interval)

(the time range of the computed results)

(user requests an oscilloscope display from t=0. to t=15. for the state vbls)

(20.)
(10.)
(1.25)

(this information was used to generate the display.)

```
READY...

HPLOT
0.,20.,.5
S(1) S(2)

      1...S(1)
      2...S(2)

  .16637E 02
      .
      .      111
      .         1
      .     1    1        1111
      .                  1    1      1111111
      .    2      1     1      11  11       11111
      .   212      1   1         11
      .              111
      .  21  2         22
      .                2  2          2
      . .1....2......2....2.......22.222......222
      . 2            2      22   22      222222
      .        2              222
      .         2  2
      .          22
      .
      .
      .
      .
      .
      .
 -.16637E 02

TIME FRØM   0.    TØ  20.000 IN INTERVALS OF    .500 .

READY...

TABLE                                   (examining the steady state)
80.,83.,1.
S(1) S(2)

    TIME     S(1)          S(2)

  80.000    .10000E 02  -.33721E-03    (10. and about-.000337)
  81.000    .99999E 01  -.38955E-03
  82.000    .99997E 01  -.11227E-03
  83.000    .99997E 01   .19613E-03


READY...
```

```
PAR
RB                                        (increasing the resistance)
2.
Q

READY...

HPLOT
0.,10.,.25
S(1) S(2)

      1...S(1)
      2...S(2)

 .99950E 01
      .
      .                 111111111111111111111111
      .             1111
      .           11
      .          1
      .        11
      .       1
      .   222222
      .     1   222
      .  2 1       2222
      . 211............2222222222222222222222222

0.0

TIME FROM   0.    TO  10.000 IN INTERVALS OF     .250 .


READY...

DT                                        (a finer sampling interval)
.1
TMAX =      33.200

READY...

TABLE                                     (examining the initial response)
0.,.5,.1
S(1) S(2)

    TIME       S(1)         S(2)
   0.        .00000E 00   .00000E 00
    .100     .46788E-01   .90484E 00
    .200     .17523E 00   .16375E 01
    .300     .36936E 00   .22225E 01
    .400     .61552E 00   .26813E 01
    .500     .90204E 00   .30327E 01

READY...
```

Appendix C. The Terminal Equipment

Part A of this appendix discusses the functions of, and gives schematics for, the major non-standard components of this equipment. The economics of the design, including costs for this design and some suggestions for reducing the cost, are presented in Part B.

Part A. The Major Components.

The two main parts to the terminal equipment are the digital memory and the analog computer. Digital information from the teletype unit is stored in the memory by the logic unit. This information is available for general use (e.g., to control switches,,etc.).

An analog computer with four integrators is wired into a completely connected four by four array, for solving equations of the form $DS = \underline{A} \cdot S \; ; \; S(0)$. The coefficients of the analog units are set by data contained in the memory. The computed results are displayed on an oscilloscope.

(1) The teletype unit.

The teletype is a model 35 KSR which has been modified to provide a set of switches to read out the eight-bit code data. Of the eight bits available the logic and memory use a set of six bits for which a full count (0 to 63 in binary form) exists. The input circuit to the logic unit is

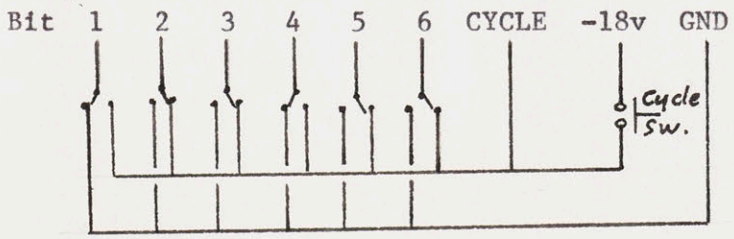


Fig. C1 shows an overall plan for the entire terminal. The teletype output to the logic (through the junction box) is by cable a.

The memory may be loaded directly from the keyboard of the teletype, which is useful for testing purposes.

(2) The logic unit.

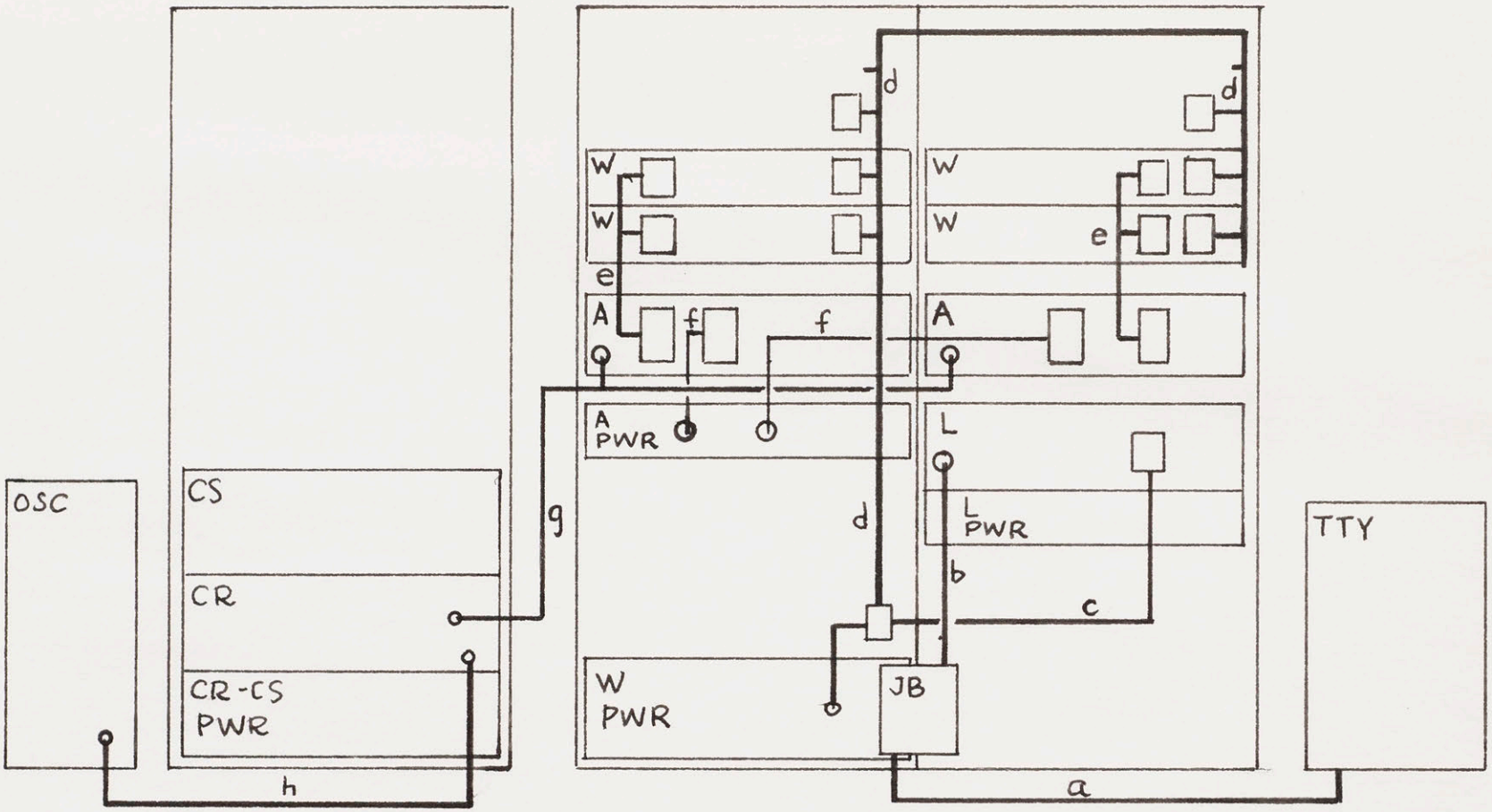
Input to the logic unit is teletype data. Depending upon its state, the logic unit routes the input data to one of three outputs to memory, or blocks it.

State	Function
START	admit all input data for processing (stable until changed);
STOP	to ignore all input data (stable until changed);
ADDRESS	to route input data for use as word address;
LEFT	to route input data for use in left (five) bits of addressed word;
RIGHT	to route input data for use in right (five) bits of addressed word.

There are three control characters for the logic unit, which set the states START and STOP, and reset the cycle counter to ADDRESS.

The cycle counter automatically advances the internal state (or function) from ADDRESS to LEFT to RIGHT (and so on), one step for each set of input data (i.e. six bits).

Fig. C2 shows the logical plan of the logic unit in some detail. In addition to the teletype (external) input, the unit is equipped with a set of toggle and control switches, which are used for test purposes.



A - analog unit

CR, CS - analog and display controls

JB - junction box

L - logic unit

OSC - oscilloscope

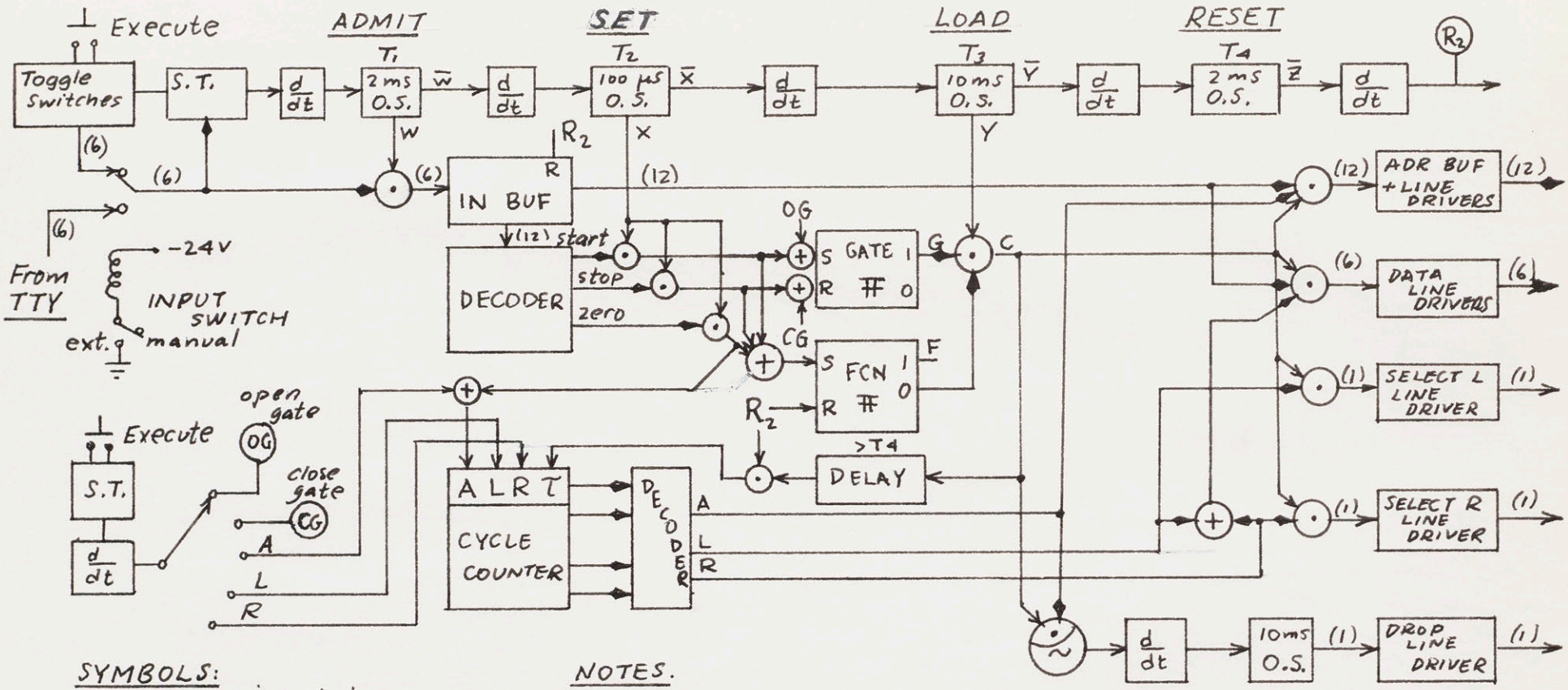
PWR - power supply

TTY - teletype

W - memory word unit

All connecting cables are labeled with lower case letters (a to h).

Fig. C1 Diagram of Terminal Equipment, Showing Function Units and their Connections



SYMBOLS:
S.T. = Schmidt trigger
O.S. = one shot
GATE FF = gate flip-flop
FCN FF = function flip-flop
⊙ = AND gate
⊕ = OR gate
→ = level signal
→ = pulse signal
~ = inverter

NOTES.
A = address
L = left
R = right
START = admit data
STOP = inhibit data
ZERO = cycle to A
τ = toggle input
d/dt = pulse shaper for the one-shots

Fig. C2 The Logic Unit for the Memory

(3) The memory words.

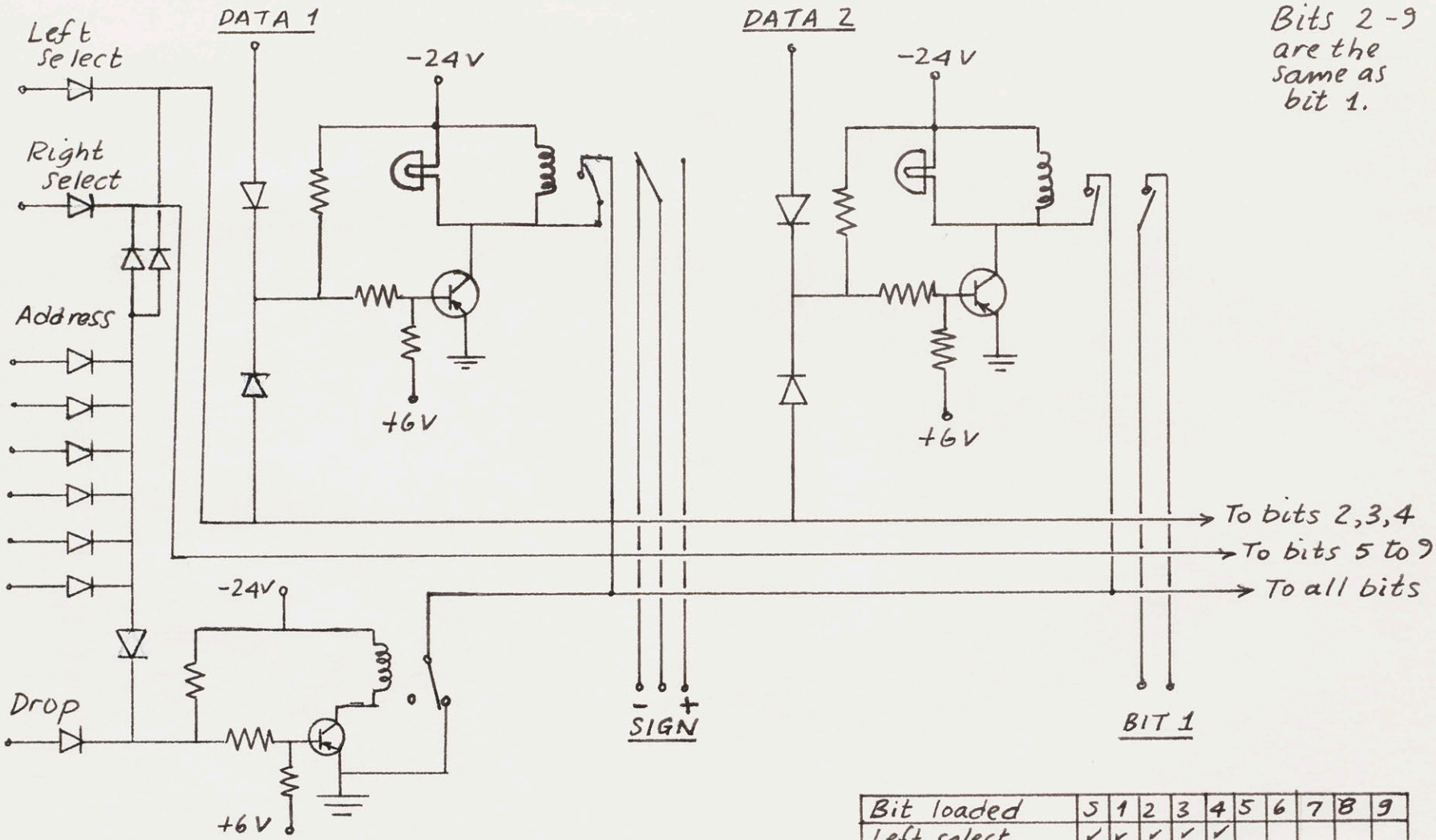
Each memory unit contains two words, whose addresses differ in the final bit. A word contains a two pole sign bit and nine single pole data bits. The bits are self-latching. When a word is addressed, its contents are dropped in preparation for loading the left five bits (S, 1 to 4), and then the right five bits (5 to 9). A schematic of the word circuit is shown in Fig. C3, describing the sign bit and bit 1. Fig. C4 presents a top view of a memory unit, showing two words. Each word has a row of bit coils and a pair of reed switches, and the circuit is printed on the bottom.

Output from the word is available at the front panel in pairs of terminals, and in the back on a connector. In addition, the state of each bit is shown by a light.

Each word unit is interchangeable with the others, since all the addressing is done by the wiring in cable d. Although only thirty-two words have been constructed, the system is designed to allow sixty-one, or sixty-four if the logic control characters are redesigned.

(4) The analog computer.

A schematic for the typical integrator circuit (there are four) is shown in Fig. C5. The integrators are packaged two to a chassis, so only two analog units (A) are shown in Fig. C1. Associated with each integrator are five transconductors, four of which serve as coefficients for the input signals (s_i), and one of which is used to set the initial condition. The amplifiers (K2W,K2P) are Philbrick components (see Part B). The computer is operated at a cycle rate of twenty repeats per second, and is controlled by the switch S.



Bit loaded	S	1	2	3	4	5	6	7	8	9
Left select	✓	✓	✓	✓	✓					
Right select						✓	✓	✓	✓	✓
Data line	1	2	3	4	5	1	2	3	4	5

Fig. C 3 Schematic of Memory Word

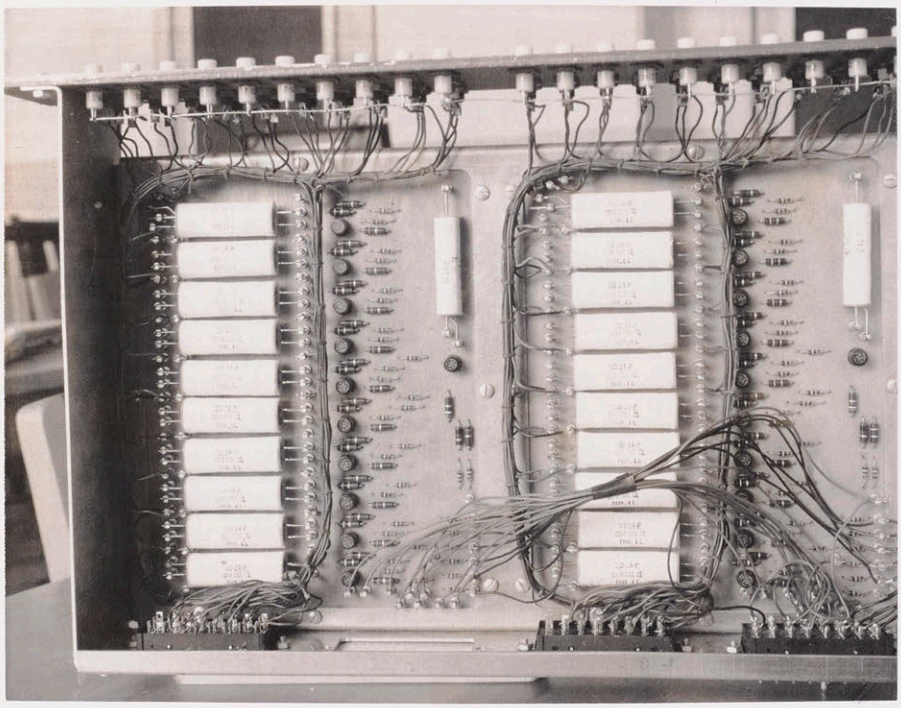
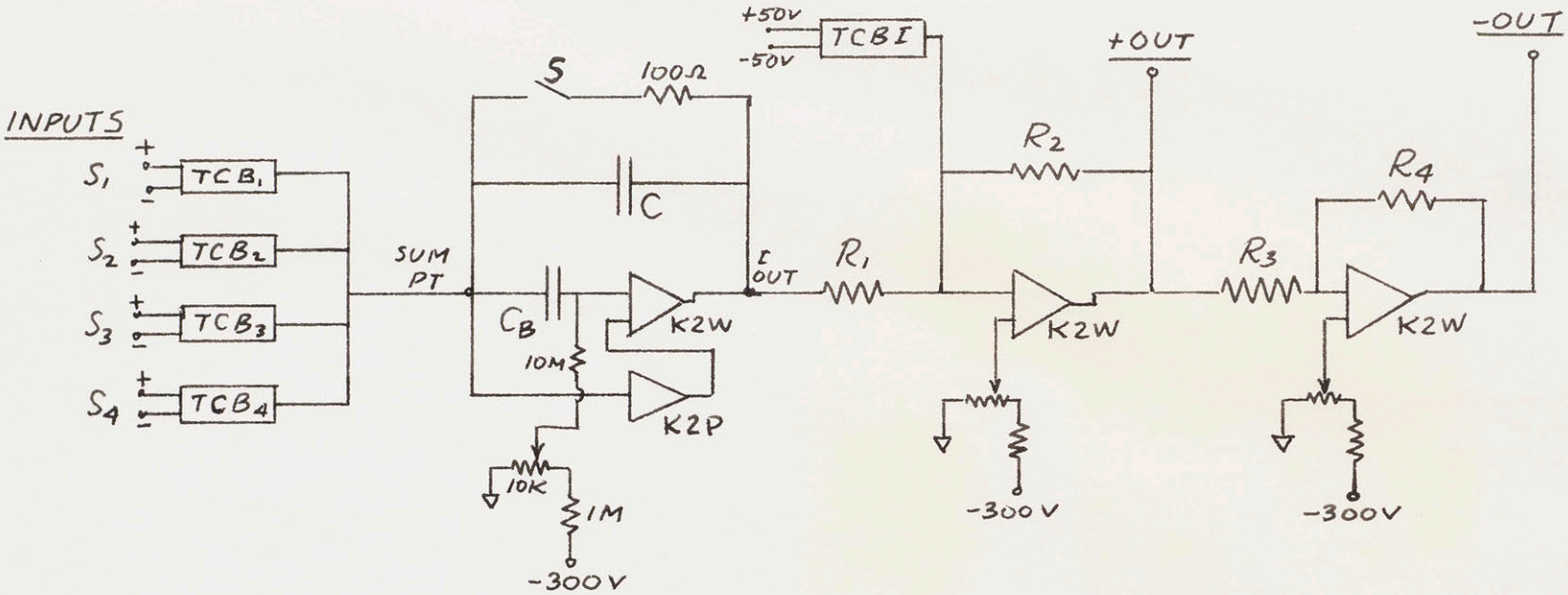


Fig. C4 Top view of a memory unit,

containing two words.



C - integrating capacitor

C_B - blocking capacitor

R_1 to R_4 - inverter and adder resistors

TCB - transconductor block

TCBI - transconductor block for initial condition

S - the cycle switch (closed is reset to zero)

Amplifier functions:

First stage - integrator (K2P stabilized)

Second stage - adds the initial condition

Third stage - inverts the positive signal

Fig. C5 Schematic of an Integrator Unit

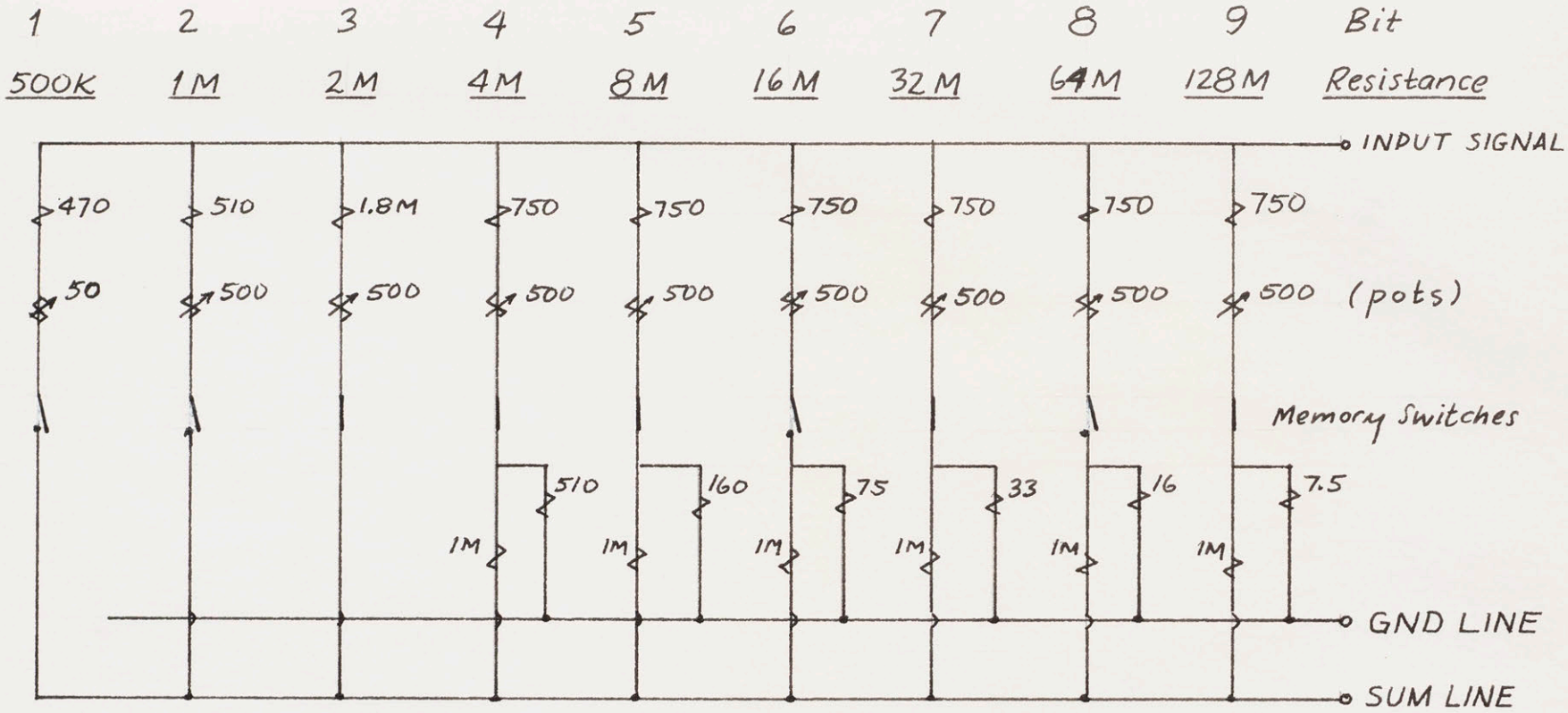
(5) The transconductors.

Each transconductor has its effective conductance set by an associated word of memory, as shown by Fig. C6. There are twenty blocks. Input to each block is by cable type e (see Fig. C1).

It was observed that there is a serious degrading effect on the analog computer due to the long, packed cables (e). The longest one presently is about two feet in length. Any design which shortens the cables considerably improves the computation accuracy.

(6) The CS and CR - for analog display and control.

Reference sheets are available from the Philbrick Company on the CR and CS units. They are used to generate a relay control signal for the computer set-run cycle, synchronized with a sweep signal for the oscilloscope. In addition, the CR unit generates voltage and time reference lines, and superimposes four signals on the grid.



All resistance values are K ohms unless specified otherwise. M = megs.

Fig. C6 Transconductor Schematic

Part B. Equipment Costs.

Detailed information on the Philbrick components (the CR and CS display units, and the K2W and K2P amplifiers) may be obtained by writing to:

Philbrick Researches, Inc.
Allied Drive
Dedham, Mass. 02026

Detail drawings of circuits and chassis units for the special equipment described in this appendix is available as the drawing series EPL - 20065 , from

Engineering Projects Laboratory

Room 3 - 158
M.I.T.
Camb., Mass. 02139

The principal items, and their approximate costs, are shown below. Not shown is a considerable amount of labor required for assembling the equipment.

ITEM	AMOUNT
1. Logic unit (CCC high speed series)	$ 1300.
2. Memory (32 ten-bit words)	2700.
3. Analog computation (four integrators)	500.
4. CR, CS display unit (estimated)	1000.
5. Other equipment (powersupplies, racks, etc.)	1000.
TOTAL	$5500.

The major ways in which savings could be effected are:

a. Use lower speed logic components, and simplify the design;

b. Use fewer bits per word in the memory ($1/2^9 = .2\%$);

c. Replace the CR, CS display control with a simpler unit.

Appendix D. Problems and Data from the Pilot Experiment.

Part 1. Selected Problems from the Pilot Material.

A. From Session 6 -

By obtaining the behavior of either of the following systems, you should be able to answer all of the questions of Part 2 below.

```
C ——— R          I ——— R
1.    1.         1.    1.
(1.)             (1.)
```

2. By using only the results of one of the systems above (plus your knowledge of scaling for first-order systems), answer these questions.

a. When will the effort reach 90% of its path to the steady-state value?

```
C ——— R
2.    3.
(30.)
```

b. If flow at t = 2. is 20., what was the initial condition?

```
C ——— R
.5    2.
(?)
```

d. When will the flow be (approximately) zero?

```
E ——— 1 ——— R
100.  |     5.
      I
      10.
      (-20.)
```

f. Draw a bond graph of a simpler system which allows you to find the steady-state flow in this system. What is that flow?

```
E ——— 1 ——— R
(E_o) |     (R_o)
      |
      I
      (I_o)
```

B. From Session 7 -

4. Use of T_o and R_o to predict aspects of system behavior.

Recall that $T_o = \sqrt{IC}$ and $R_o = \sqrt{I/C}$.

By generating data for only this system, you should be able to answer all the questions below.

C ——→ I	
1.	1.
(1.)	(0.)

(See clue 7.4 for answers if you must)

a. What is f_{max} for this system?

C —— I	
2.	.5
(20.)	(0.)

g. For this system

$R_o = 2.$
$e(t_A) = 15.$
$f(t_A) = 8.$

Find e(0.).

C —— I	
6.	?
(?)	(10.)

h. Assume that e(t) and f(t) maintain the same relative time relation as in the unit system.

If e(3.) = 10.
and f(3.) = 6.
what can you say about I, C, R_o, and/or T_o?

C —— I	
?	?
(20.)	(0.)

C. From Session 9.4 -

B. Some examples.

1. Show that the result we have just found is consistent with the behavior of C——I.

What is Q for C——I? (Clue 9.2)

2. If $s_1(0) = 0$ and $s_2(0) = -5.$, what are A, B, and ϕ ?

```
C——1——I
1.  |   4.
    R
```

(Clue 9.3)

3. Draw the decay envelope for s_1 and s_2.

```
C——1——I
1.  |   1.
(20)|  (0)
    R
    .5
```

$$\underline{A} = \begin{bmatrix} 0. & 1. \\ -1. & -.5 \end{bmatrix}$$

(Clue 9.4)

4. Predict when the total system energy will be at about 37% (1/e) of its initial value. You may verify your prediction by experimental evidence.

```
C——0——I
1.  |   1.
(10)|  (0)
    R
    10.
```

$$\underline{A} = \begin{bmatrix} -.1 & -1. \\ 1. & 0. \end{bmatrix}$$

(Clue 9.5)

Part 2. Predicted Results for Test Systems.

From Session 13 -

Test Sys.	5	6	7
Answer	—1— E 15. I 2. R 1.	—0— 1— R -.1 C 1. I 1.	—1— 0— C 1.5 R .1 R 3.
Subj. 1	—1— F 15. C -4/30 R 2.	—0— 1— R -.1 C 1. I 1.	—1— 0— C 1.5 R .1 R 3.
Subj. 2	—1— I 120. R .6	— 0 — C 1.7 I 48. I	— 0 — C -14. R -3.
Subj. 3	—0— 1— I 2. E 15. R 1.	— 0 — C 1. R -.1 I 1.	—1— 0— C 5. R .1 R 3.
Subj. 4	—1— 1— R 1. F 15. I 2.	—0— 1— R I C (IC = 1.)	—1— I 1.6 R 3.1

Note. All initial conditions are zero.

Part 2. (cont'd)

From Session 13 -

Test Sys.	8	9	10
Answer	given 0 — 1 C R I 1.4 .4 2.2	given 0 — 1 — 0 — I R C I 1. .8 2 4	0 — 1 C I R given
Subj. 1	0 — 1 C R I 1.4 .4 2.2	0 — 1 — 0 — R R C I	0 — 1 C I R
Subj. 2	0 — 1 I I C .4	(did not try this)	0 — 1 C I R
Subj. 3	0 — 1 I R R .4	0 — 1 — 0 — C R R I 1 4 .8	0 — 1 C I R
Subj. 4	0 — 0 — 1 C R R 18 1000 .4	0 — 1 — 0 — R C I I $\sqrt{IC}=7$ 1.	0 — 1 C I R

Note. All initial conditions are zero.

Appendix E. A Guide to the Programming of Enport.

The purpose of this appendix is to present in some detail the operations performed by the digital computer in carrying out a complete simulation of a linear dynamic system. Figure 3.9 (page 105) outlines the major steps to be taken. Part (9) of this appendix has a note on the availability of the actual programs.

(1) Representing the bond graph internally.

The bond graph is represented in the computer by a matrix and two associated lists :

IMX - the incidence matrix, which indicates the connections of nodes and bonds;

NLIST - the node list, which is ordered by type according to C, I, R, E, F, GY, TF, 0, 1, respectively;

BLIST - the bond list.

The line code for a graph is processed in two passes.

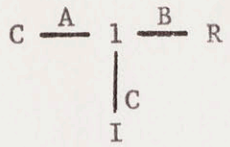
Pass 1. The node types are counted, the BLIST is set up, and certain types of errors are detected (e.g. illegal node types, missing punctuation).

Pass 2. The NLIST is set up in order (with the nodes named by their attached bonds), and <u>ones</u> are placed in the IMX to indicate the connection of bonds (associated with columns) to nodes (associated with rows).

Example.

C A,R B,1 A B C,I C.

The line code.



The bond graph.

Pass 1 completed.

(also node types counted)

NLIST	BLIST
-	A
-	B
-	C

IMX

0	0	0
0	0	0
0	0	0

Pass 2 completed.

(Nlist goes with rows, BLIST goes with columns of IMX)

NLIST
CA
IC
RB
1ABC

BLIST
A
B
C

IMX

1	0	0
0	0	1
0	1	0
1	1	1

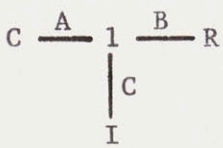
Thus IMX(3,2) shows that node RB is connected to bond B.

At this point the IMX is checked by rows to ensure the proper port counts for the nodes, and by columns to ensure that each bond has two ends.

(2) Assigning causality.

Causality is encoded in the IMX by showing, for each bond, a 3 corresponding to the effort-determining end, and a 6 corresponding to the flow-determining end. The sequence of operations is exactly the same as that referred to in section 2.32.A (pages 58 - 63). Thus a zero junction should have only one 3 in its row, for example.

Example.



IMX

	A	B	C
CA	1	0	0
IC	0	0	1
RB	0	1	0
1ABC	1	1	1

(a) Introduce the causal information.

6	0	0
0	0	3
0	1	0
1	1	1

(b) Extend the information on the bonds.

(e.g. on bonds A and C)

6	0	0
0	0	3
0	1	0
3	1	6

IMX

(c) Use the node constraints.

(e.g. on row 1ABC)

6	0	0
0	0	3
0	1	0
3	3	6

IMX

(d) Return to step (b) and repeat sequence until all causal information has been exhausted. (e.g. bond B completed)

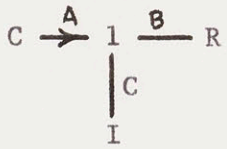
6	0	0
0	0	3
0	6	0
3	3	6

IMX

(e) Check the IMX by rows to ensure that all causal constraints have been met by node type; check by columns to ensure that bond forms are appropriate. Static sub-fields can be discovered at this point, for example.

To show the activation of a bond it is only necessary to subtract 1 from the 3 or 6 corresponding to the signal to be suppressed.

Example.



5	0	0
0	0	3
0	6	0
3	3	6

Flow on bond A has been suppressed.

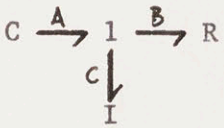
(3) Assigning state variables.

Once causality has been completed successfully, selection of state variables is simple. The NLIST has been ordered to make C's the first set, and I's the second set. State variables are chosen corresponding to the efforts on the C bonds, and to the flows on I bonds. The state variable index is identical with the position of the node in NLIST.

(4) Assigning power directions.

Power direction information is encoded in the IMX by introducing a plus and minus for each bond. The plus end indicates that power is positive into the node, and the minus sign indicates that power is positive out of the node. Clearly, the process of directing the bonds is independent of causality.

Example.



	A	B	C
CA	-1	0	0
IC	0	0	1
RB	0	1	0
1ABC	1	-1	-1

(acausal)

or

-6	0	0
0	0	3
0	6	0
3	-3	-6

(causal)

Since chains and loops always involve multi-port elements, routes are followed in the IMX corresponding to such paths in the graph, starting with an arbitrary multi-port.

(5) The search procedure.

An augmented bond graph (i.e. one with causality, power directions, and computing variables assigned) corresponds to an IMX with signed 3,6, and 0 entries (and 2's and 5's if there are active bonds).

The purpose of the search procedure is to trace a route through the IMX, defining a particular bond variable solely in terms of s, t , and x variables. This means that paths are terminated by one of four conditions:

(a) a C or I row is reached, which corresponds to a state variable (row n is state variable n);

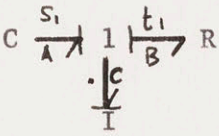
(b) an R row is reached, which corresponds to a t variable (the nth R is t_n);

(c) an E or F row is reached, corresponding to an x variable (see part (8) for the treatment of E's and F's); or

(d) an active bond is found, indicating signal suppression.

Basically, a path is traced in the IMX by scanning a row for the desired type of signal input(s), using the bond to reach the next node, and proceeding according to node type.

Example.



Find the effort on bond C.

	A	B	C	
CA	-6	0	0	
IC	0	0	3	IMX
RB	0	6	0	
1ABC	3	-3	-6	

In the table below I refers to the row and J to the column.

	Step		Search Variable Coefficient	Temporary Search Results				Cumulative Search Results			
	I	J	$\pm k$	s_1	s_2	t_1		s_1	s_2	t_1	
Start	2	3	+ 1	0	0	0					
	4	3	+ 1	0	0	0					
	4	1	+ 1	0	0	0					
End on CA	1	1	+ 1	1	0	0	→	1	0	0	
New start	4	2	- 1	0	0	0		↓			
End on RB	3	2	- 1	0	0	-1	→	1	0	-1	result

The result is $e_C = s_1 - t_1$.

(6) Formulation of the A-matrix (AMX).

In order to formulate the AMX for a system from the IMX each C and I is used to generate a line (row) of the form $C \cdot Ds = f$, or $I \cdot Ds = e$. In addition, each R yields a relation of the form $R \cdot t = e$, or $\frac{1}{R} \cdot t = f$. The search procedure is used to evaluate each e or f required in terms of s and t variables. The result is a set of equations (actually two matrices - AMX and RMX) of the form

$$\begin{aligned} D \cdot \underline{S} &= \underline{M}_{11} \cdot S + \underline{M}_{12} \cdot T \\ T &= \underline{M}_{21} \cdot S + \underline{M}_{22} \cdot T \end{aligned} \qquad (E.1)$$

The second equation is solved for T in terms of S, and a new matrix, RMX, is defined.

$$T = (\underline{I} - \underline{M}_{22})^{-1} \underline{M}_{21} \cdot S$$

$$T = \underline{R} \cdot S \ , \qquad \text{where R is RMX.}$$

The first equation of (E.1) is solved by eliminating T, and a new matrix, $\underline{A}$ or AMX, is defined.

$$
\begin{aligned}
DS &= \underline{M}_{11} \cdot S + \underline{M}_{12} \cdot T \\
&= \underline{M}_{11} \cdot S + \underline{M}_{12} \cdot \underline{R} \cdot S \\
&= \left[\underline{M}_{11} + \underline{M}_{12} \cdot \underline{R}\right] S \\
DS &= \underline{A} \cdot S \qquad \text{(E.2)}
\end{aligned}
$$

The search procedure may be used to express any bond variable in terms of state variables, by first finding $(\text{search vbl}) = \sum a_i \cdot s_i + \sum b_i \cdot t_i$, and then using RMX to eliminate the T variables, finishing with $(\text{search vbl}) = \sum c_i \cdot s_i$.

(7) Computing the response.

Starting with the AMX, a matrizant (MMX) is computed by expanding the relation $MMX = e^{AMX \cdot DT}$, or $MMX = \underline{I} + AMX \cdot DT + \frac{(AMX \cdot DT)^2}{2!} \ldots$.

First, DT is checked to ensure that $DT \cdot (a_{ij})_{max} \leq 1$.

A finite number of terms is used in the expansion, terminated by the conservative criterion that, for every term in the matrix, the new term added must be less than .0001 of the present term.

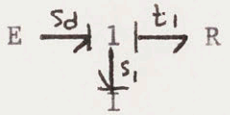
Once MMX has been obtained, the results are computed at intervals of DT by the relation $S(k \cdot DT) = MMX \cdot S((k-1)DT)$, $k = 1,2,3,4\ldots$

or $S(k \cdot DT) = MMX^k \cdot S(0)$, $k = 0,1,2\ldots$.

(8) A Note on the Treatment of Sources.

The current version of Enport is limited to external disturbances (E's and F's) which have constant value. It is then possible to treat all such sources by adding a single state variable to the system (hence one more column to AMX), and by adjusting the coefficient of the dummy state variable for each row to reflect the total effect of the external sources. Also, a relation for the dummy state variable, s_d , is written in the form $Ds_d = 0$, and $s_d(0) = 1$. This ensures that s_d stays constant at unity.

Example.



$$\left.\begin{aligned} I \cdot Ds_1 &= E\, s_d - t_1 \\ Ds_d &= 0 \\ \frac{1}{R} \cdot t_1 &= s_1 \end{aligned}\right\} \quad \text{which becomes} \quad \begin{aligned} Ds_1 &= -\frac{R}{I} s_1 + \frac{E}{I} s_d \\ Ds_d &= 0 \\ \text{and} \quad s_d(0) &= 1. \end{aligned}$$

(9) Comments.

The programs for Enport were written to operate in the Compatible Time-Sharing System used at M.I.T. by the Computation Center and Project MAC. The source programs are in MAD and FAP, and total about 2500 instructions. Core storage is on the order of 17,000 K.

Listings and/or punched cards are available from the author upon request. Please write to:

Ronald C. Rosenberg
M.I.T. 1 - 123
Camb., Mass. 02139

A note. The current listings and cards will not be accompanied by flowcharts or abstracts of the routines. A significant modification to the internal organization of the system will take place this year, and the new version will be thoroughly documented.

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