A STUDY OF THE SURFACE TENSION CONTROLLED REGIME OF OIL SPREAD

by

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Chairman, Departmental Committee on Graduate Students

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ABSTRACT

The thesis investigates the spreading behaviour of a small volume oil slick moving over calm water. A theory is developed which predicts spreading rates and height variations within the slick as functions of time. Experimental results are found to agree well with the theoretical predictions; the end result, a method for obtaining an estimate of volumetric release rate at the site of a spill, is discussed.

Thesis Supervisor: David P. Hoult

Title: Associate Professor

Acknowledgments

The author wishes to express his deepest gratitude to all the wonderful people connected with the Fluid Mechanics Laboratory, students, technicians, and even professors, for their constant assistance, advice, and encouragement. Special thanks to Professor David P. Hoult for his never-ending patience in guiding one particularly unruly research assistant. Many thanks also to Miss Sara Rothchild, who prepared the manuscript, and to Lana and Edwin Lee for their invaluable help with the experiments.

Peace.

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List of Symbols

σ	surface tension - dynes/cm
w	channel width - cm
V	slick volume - cm^3
τ	shear stress - dynes/cm^2
ℓ	scaling length
μ	absolute viscosity
ρ	density - grams/cm^3
ν	kinematic viscosity - cm^2/sec
u	horizontal velocity component in water boundary layer - cm/sec
v	vertical velocity component in water boundary layer - cm/sec
t	time - sec
r	horizontal position along the slick - cm
y	vertical position in boundary layer - cm
R	non-dimensional radius
Y	non-dimensional vertical position
h	height of slick - cm
r_f	instantaneous position of slick front - cm
$\tilde{h},\tilde{u},\tilde{v}$	non-dimensional similarity variables for h, u, and v
ℓ_o^2	the product of starting length and starting height of oil for a one dimensional channel flow - cm^2

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I. Introduction

The increased occurence of major oil spillage accidents in recent years has led to intensified efforts in the area of oil pollution research. Initial phases of this work have involved the predictions of size and location of a slick, given such parameters as rate of release and physical oil properties. Only with such predictions can effective use of containment and recovery techniques be made.

In past months, studies[1,2,3] have shown that by analyzing the various forces which control the spreading phenomenon, descriptions of slick size as a function of time can be obtained. Fay[1] has shown that the effects of gravity, surface tension, inertia and viscous forces should be considered; the first two drive the slick while the last two retard motion. By looking at the magnitudes of these forces at any particular moment during a slick's propagation, one finds that two of these forces, one driving, the other retarding, control the spreading process while the other two remain dimensionally insignificant.

The spreading of an oil slick, then, is characterized by discrete regimes which depend on pairs of forces. For large slicks, Suchon[2] has found that initially, gravity-inertial effects dominate motion and at some critical transition point the gravity-viscous force couple becomes the important one. Figure 1 illustrates how slick velocity changes as this transition takes place. Thus, each spreading regime has its own particular spreading rate associated with it.

A more specialized aspect of this oil pollution problem is that of determining, by aerial observation, the volume of oil released in a small leakage. In this way, regulatory agencies can penalize offenders according to the amounts of oil which they allow to leak from their tankers.

Figure 2 shows how slick spread for this kind of a spill can be approximated by a series of one-dimensional channel flows. The problem of estimating the quantity of oil spilled, then, is reduced to one which involves determining for each for each channel the height distribution from the center line out to the leading edge, integrating this distribution over that same length and summing over all the channels. For these smaller volumes, one would expect the effects of gravity and inertia forces to be much less significant than those of surface tension and viscous forces. Thus, this type of leakage represents yet another regime of spreading.

The intent of this study, then, is to investigate this phase of oil spread by first developing a theory which can adequately describe the spreading phenomenon, and then collecting experimental data on spreading rates and height distributions to determine whether our predictions can indeed lead to accurate estimates of oil volume spilled.

II. Theory

In order to predict the motion of a thin oil film, the physics of the spreading phenomenon must be analyzed; by studying the forces which interact to cause this motion, one can obtain descriptions of spreading behaviour. In specific, it is necessary to derive a theory which can predict both:

(1) the spreading velocity of a thin oil slick, and

(2) the height distributions of oil in the slick as a function of time and position away from the source.

First, determine a scaling length, ℓ, which will be used to characterize motion in this surface tension-viscous controlled regime.

Consider a slick with dimensions as shown in Figure 3. Balancing the two forces involved (force/unit Volume):

Surface Tension = Viscous

$$\frac{\sigma w}{V} = \frac{\tau \ell w}{V}$$

where:

σ = surface tension/unit width

τ = shear stress at interface

Shear stress τ is related to the velocity gradient in the water boundary layer under the slick by:

$$\tau = \mu \frac{\partial u}{\partial y} = \rho \nu \frac{\partial u}{\partial y}$$

By order of magnitude approximation:

$$u \sim \frac{\ell}{t}$$

$$\frac{\partial u}{\partial y} \sim \frac{(\ell/t)}{\delta}$$

where δ = boundary layer thickness ~ $\sqrt{\nu t}$. Combining these terms:

$$\frac{\sigma w}{V} = \frac{\rho \nu^{1/2} \ell^2 w}{t^{3/2} V}$$

or:

$$\ell = \left(\frac{\sigma^2 t^3}{\rho^2 \nu}\right)^{1/4} \qquad \text{Scaling Length}$$

Differentiating ℓ with respect to time gives

$$\frac{d\ell}{dt} \sim \left(\frac{\sigma^2}{\rho^2 \nu}\right)^{1/4} \frac{1}{t^{1/4}} \quad ,$$

a velocity which varies like $t^{-1/4}$.

The problem of determining height distributions in the oil slick is approached by adopting the following solution strategy. Consider a thin oil slick moving in the (r - y) plane as shown in Fig. 4. To find the height as a function of position and time, solve first for the velocity distribution in the water boundary layer. The resultant expression for velocity, when substituted into the continuity equation for the oil film, yields a first order partial differential equation from which a self-similar solution for heights can be obtained. The details of the theory are as follows:

Use incompressible boundary layer analysis; assume no pressure gradients. Navier-Stokes equation for water boundary layer is the r direction:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial^2 y} \qquad (1)$$

Continuity equation for water boundary layer:

$$\frac{\partial u}{\partial r} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

Continuity equation for oil film:

$$\frac{\partial}{\partial r} (uh) + \frac{\partial h}{\partial t} = 0 \qquad (3)$$

Define non-dimensional variables:

$$R = \frac{r}{\ell} \qquad (4)$$

$$Y = \frac{y}{\sqrt{\nu t}} \qquad (5)$$

where

ℓ = scaling length

ν = viscosity

Rewriting the three equations in terms of the non-dimensional variables:

$$\frac{\partial u}{\partial t} + \left(\frac{u}{\ell} - \frac{3}{4}\frac{R}{t}\right)\frac{\partial u}{\partial R} + \left(\frac{v}{\nu t} - \frac{Y}{2t}\right)\frac{\partial u}{\partial Y} = \frac{1}{t}\ \frac{\partial^2 u}{\partial Y^2} \tag{1}$$

$$\frac{1}{\ell}\ \frac{\partial u}{\partial R} + \frac{1}{\sqrt{\nu t}}\frac{\partial v}{\partial Y} = 0 \tag{2}$$

$$\frac{1}{\ell}\frac{\partial}{\partial R}(uh) + \frac{\partial h}{\partial t}\ \frac{3}{4}\ \frac{R}{t}\ \frac{\partial h}{\partial R} = 0 \tag{3}$$

Now, look for self-similar solutions by defining variables u, v, and h in terms of non-dimensional "tilda" variables $\tilde{u}$, $\tilde{v}$, and $\tilde{h}$. Velocity u should be of form (ℓ/t). By considering equation (2) it then follows that velocity v should be of magnitude $\sqrt{\frac{\nu}{t}}$. Height h will depend on the inverse of the scaling length (as the slick spreads, the height decreases) and on the initial volume of oil which shall be called ℓ_o^2. Thus, for the three variables:

$$u = \tilde{u}\,\frac{\ell}{t} \tag{6}$$

$$v = \tilde{v}\quad\frac{\nu}{t} \tag{7}$$

$$h = \tilde{h}\,\frac{\ell_o^2}{\ell} \tag{8}$$

Writing (1), (2), and (3) in terms of the 'tilda' variables:

$$(\tilde{u} - \frac{3}{4} R) \frac{\partial \tilde{u}}{\partial R} + (\tilde{v} - \frac{Y}{2}) \frac{\partial \tilde{u}}{\partial Y} = \frac{\partial^2 \tilde{u}}{\partial Y^2} \qquad (1)$$

$$\frac{\partial \tilde{u}}{\partial R} + \frac{\partial \tilde{v}}{\partial Y} = 0 \qquad (2)$$

$$\frac{\partial}{\partial R} (\tilde{u}\tilde{h}) - \frac{3}{4} R \frac{\partial \tilde{h}}{\partial R} - \frac{3}{4} \tilde{h} = 0 \qquad (3)$$

Assume a standard similarity solution for $\tilde{v}$:

$$\tilde{v} = -g(Y) \qquad (9)$$

Substitute $\tilde{v}$ into (2) to get:

$$\tilde{u} = Rg'(Y) \qquad (10)$$

Substitute this $\tilde{u}$ into (3) to get:

$$\tilde{h} = \frac{A}{R}, \quad A = \text{constant} \qquad (11)$$

The results of this analysis are:

(1) determination of a spreading rate. Since $R = r/\ell$, then $r = R\ell$. If R_f is the constant which characterizes self-similar behavior at the slick front, then r_f, the position of the front will be

$$r_f = R_f \ell \qquad \sim t^{3/4} \qquad (12)$$

In terms of velocity,

$$u_f = \frac{d}{d_t}(r_f) \sim R_f \frac{\ell}{t} \sim t^{-1/4} \tag{13}$$

u_f = velocity of the slick front

(2) descriptions of height distribution by the expression

$$h = \tilde{h}\,\frac{\ell_o^2}{\ell} = \frac{A}{R}\,\frac{\ell_o^2}{\ell} \quad .$$

The theory as derived above presents one difficulty however. If the volume of oil is evaluated by the integral

$$V = \int_o^r h\,dr$$

and the self similar solution for h is used, the volume becomes infinite. Although the $\tilde{h}$ which the theory predicts satisfies the oil continuity relation, it does not conserve volume.

What is required, then, is a solution for $\tilde{h}$ which will satisfy both the oil continuity equation and the conservation of oil volume.

One solution which meets both these requirements is $\tilde{h} = A$ where A = constant. If the volume is evaluated with this $\tilde{h}$,

$$V = \int_0^r \bar{h} dr = \int_0^r \frac{h\ell_o^2}{\ell} dr$$

$$= \int_0^{R_f} h\ell_o^2 \, dR = \tilde{h} \, \ell_o^2 \, R_f$$

Since $V = \ell_o^2$, then $\tilde{h}R_f = AR_f = 1$.

$$\therefore \quad A = \frac{1}{R_f}$$

III. Experiments

Laboratory experiments were designed with the following objectives in mind:

(1) Determine spreading rates for oil slicks moving in the surface tension-viscous controlled regime.

(2) Study the oil height variations in a moving slick.

(3) Determine to what extent these observed quantities agree with the theoretical predictions of Section II.

Tests were initially conducted in a two foot long plexiglas tank with provision for seven observation points along its length.

An optical method of measurement, using a photocell at each observation point, was chosen to monitor simultaneously slick incidence and instantaneous slick height. Figure 6 shows the configuration of each observation station with a fiber optic transmitting constant intensity light to the photocell.

By looking at the resistance changes of each photocell as the slick passes by, arrival times and an analog signal of height can be detected; the latter can be processed to obtain information about actual heights, while the former can be used to evaluate spreading velocities (~observation radius/arrival time).

Unfortunately, the change of resistance for a conventional photocell varies exponentially with change in oil height as shown in the typical calibration curve in Fig. 7. In order to get a linear correlation of signal/height data, the logarithm of photocell resistance change must be taken; Figure 8 shows the circuit used to obtain a linear voltage output analogous to a linear change in slick height. Output voltages for all seven observation points are recorded simultaneously on magnetic film by a seven track tape recorder.

Experimental procedure is as follows:

(1) Measure net surface tension (see Appendix 1).

(2) Load a given volume of oil behind the dam; note starting length and starting height (ℓ_o^2).

(3) Start tape recorder.

(4) Allow the oil to reach stady state height (which gives a calibration voltage for each photocell).

(5) Dump another known volume of oil into the tank and record this second height (which gives another calibration voltage for each photocell).

This procedure then gives a calibration curve for each photocell and eliminates the need for careful pre-experiment checks of amplifier settings and light intensities.

The raw data recorded on magnetic tape is further processed by analog and digital computers as shown in the flow chart in Figure 9; Figure 10 traces a data signal from photocell to computer disk storage, giving approximate voltage ranges and digital amplitudes. Discrete data points can then be retrieved from disk storage and processed to produce final graphical representations of each experiment. In particular, we require two plots:

(1) height vs. time for each of the seven channels

(2) similarity variable $\tilde{h}$ vs. R according to equations (8) and (4) which give

$$\tilde{h} = hr/R\ell_o^2$$

The second plot is particularly important, because if $\tilde{h}$ data points form a coherent pattern along an h = constant line, we can conclude that the similarity solution for h proposed in Section II is indeed a valid one.

A sample of each of the two graphs required is shown in Figures 11 and 12; accompanying each computer plot is a figure which shows, in simplified form, the results of the machine graphics.

Since slick spread from end to end in the two foot tank was observed to last only about 3 seconds, a four foot tank, in which the bulk of the experimental data was collected, was later constructed.

IV. Results

Figure 11 shows that arrival of the slick at any observation point is easily detected because of the sharp increase in height. Using this arrival time and noting the distance from the front wall ($r = 0$), a simple l/t velocity can be evaluated. Figure 13 shows non-dimensional length vs. non-dimensional time for the experiments conducted in the longer tank. The line drawn through the data is of slope 3/4, in agreement with the prediction of Section II and the previous work of Garrett and Barger.[4]

Figure 12 and the simplified representation of Figure 12a show that the similarity variable $\tilde{h}$ is characterized by three distinct regions:

(1) an initial surge

(2) a flat region

(3) a monotonically increasing section in which $\tilde{h}$ grows in a $1/R$ fashion (R decreases with increasing time).

Part (3) indicates the end of the experiment when a steady state oil height has been attained in the tank, and thus, $\tilde{h}$, described by the relation

$$\tilde{h} = h \ell_o^2 \, r/R \quad ,$$

becomes C/R for any fixed observation radius, where C is a constant. Part (1), which corresponds to the arrival of the slick at the photocell, exhibits a surge characteristic which cannot be explained by considering surface tension and viscous forces only, and suggests that perhaps gravity and inertia are also called into play in the immediate vicinity of the slick nose. This surge, however, was found to be of short duration (passing by any one photocell in about 0.2 seconds), and is therefore of minor importance compared with the remainder of the experiment.

The region of primary concern, then, is that section of the $\tilde{h}$ curve where that variable is approximately a constant. As can be seen from the graphical results of Figure 12 and those included in Appendix II, h remains a constant within a factor of 1.5, the data centering about the value h = 0.75.

V. Errors

There were several sources of error in the experiments conducted:

(1) There is a possible timing error of about 0.2 seconds between lifting the dam and electrically marking the beginning of the experiment on the magnetic tape. This introduces some error in the ℓ/t evaluation of velocity.

(2) Gravity effects could not be totally eliminated from the experiments in the larger tank; the minimum steady state height needed to keep the photocells in their linear range (about 0.1 cm) required the use of large initial volumes. This resulted in volume-dependent velocities which is responsible for much of the scatter in the length vs. time graph in Figure 13. In general, the points below the line drawn through the data correspond to smaller initial volumes (~500 cc), while the remaining points correspond to the larger volumes (~1000 cc). A factor of 2 increase in initial volume resulted in a 27% increase in spreading rate. However, pure surface tension experiments in the two foot tank yielded spreading rates which were independent of volume.

(3) Scatter in the $\tilde{h}$ vs. R diagrams is attributed to height fluctuations caused by small amplitude, high frequency waves introduced at the lifting of the dam. The surface of the oil slick just after the start of the experiment was seen to be "rippled". Other height errors involved

the reflection of the slick after hitting the back wall of the tank causing a sudden jump in the signal recorded at observation stations 5, 6, and 7.

VI. Conclusions

A. Spreading rates

The experiments conducted have shown that physical spreading rates for this regime agree closely with those predicted by theory. The constant R_f which relates instantaneous slick length r_f to scaling length ℓ was observed to be 1.33 for pure surface tension spreading.

$$r_f = 1.33 \left[\frac{\sigma^2 t^3}{\rho 2\nu}\right]^{1/4} \tag{14}$$

That this spreading velocity remains invariant for different oil volumes (ℓ_o^2) was also verified.

B. Height distributions

It was found that the $\tilde{h}$ similarity variable is approximately a constant for a major portion of the spreading process for a small slick. According to equation (8) then,

$$h = \tilde{h}\ell_o^2/\ell$$

$$= A\ell_o^2/\ell \qquad \sim \qquad \text{Constant}/\ell$$

which tells us that <u>on the average</u>, the slick height decreases uniformly with time such that

$$h \sim t^{-3/4}$$

In other words, the slick "flattens out" as it propagates, obeying continuity ($hr_f = \ell_o^2$) throughout its motion; Figure 14 illustrates this spreading process for times $t_1 < t_2 < t_3 < t_4$.

We must remember of course that this is only a crude description of the actual phenomenon and does not account for the nose region mentioned earlier. This rough approximation, however, can be used to make an estimate of the initial volume, the problem posed in the beginning of this thesis. Using equation (8) again:

$$h = \frac{\tilde{h}\ell_o^2}{\ell}$$

$$= \frac{\tilde{h}\ell_o^2}{\left[\dfrac{\sigma^2 t^3}{\rho^2 \nu}\right]^{1/4}}$$

or:

$$\ell_o^2 = \frac{h\left[\dfrac{\sigma^2 t^3}{\rho^2 \nu}\right]^{1/4}}{\tilde{h}} \qquad (15)$$

In the above expression for ℓ_o^2, σ, ρ, and ν are known, or can be measured easily enough without the need for visiting the site of the spill. All that is required now is a reference oil height and a measurement of the time it takes the slick to reach that height. A convenient reference height is that thickness which causes "rainbows", interference fringes which can be spotted from the air.

In order to make a volume estimate for one of the one dimensional channels pictured in Figure 2, one need only measure the time it takes the slick to become colored at its outer boundary, and substitute this time into equation (15) along with the appropriate reference height (on the order of a wavelength of visible light). This resulting estimate of ℓ_o^2 (cm^2), when multiplied by the ship velocity, (cm/sec) gives the final approximate value of volume rate of release (cm^3/sec).

+ + + + + +

It should be noted here in the concluding remarks that there is still an inconsistency in the arguments presented which remains unexplained. We have seen that the similarity solution $\tilde{h}$ = constant satisfies the oil continuity equation (3) and the conservation of volume by the integral $\int_0^r h\,dr$. However, if the empirically determined value of $\tilde{h}$ is substituted into equation (3) and the resulting value for $g'(0)$ is used as an initial condition for the third order partial differential equation for g, the solution obtained is g = Constant, which is incorrect (a constant vertical velocity component in the water boundary layer). It is not known at this time how this inconsistency was introduced into the analysis of the problem, but it does not affect the validity of the experimental results or their agreement with predictions of the theory.

+ + + + + +

References

1. Fay, J. A., "The Spread of Oil Slicks on a Calm Sea", Massachusetts Institute of Technology, Fluid Mechanics Laboratory Publication 69-6.

2. Suchon, W., "An Experimental Investigation of Oil Spreading over Water", M.S. Thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology, April, 1970.

3. Hoult, D. P. and Suchon, W., "Spread of Oil in a Channel", Massachusetts Institute of Technology, Fluid Mechanics Laboratory Publication, May, 1970.

4. Garrett, W. D. and Barger, B. R., "Factors Affecting the Use of Monomolecular Surface Films to Control Oil Pollution on Water", Environmental Science and Technology, Vol. 4, No. 2, February, 1970.

5. Bartell, F. E. and Miller F. L., "A Method for the Measurement of Interfacial Tension of Liquid-Liquid Systems", Trans. Chem. Soc., Vol. 50, July, 1928.

Figures

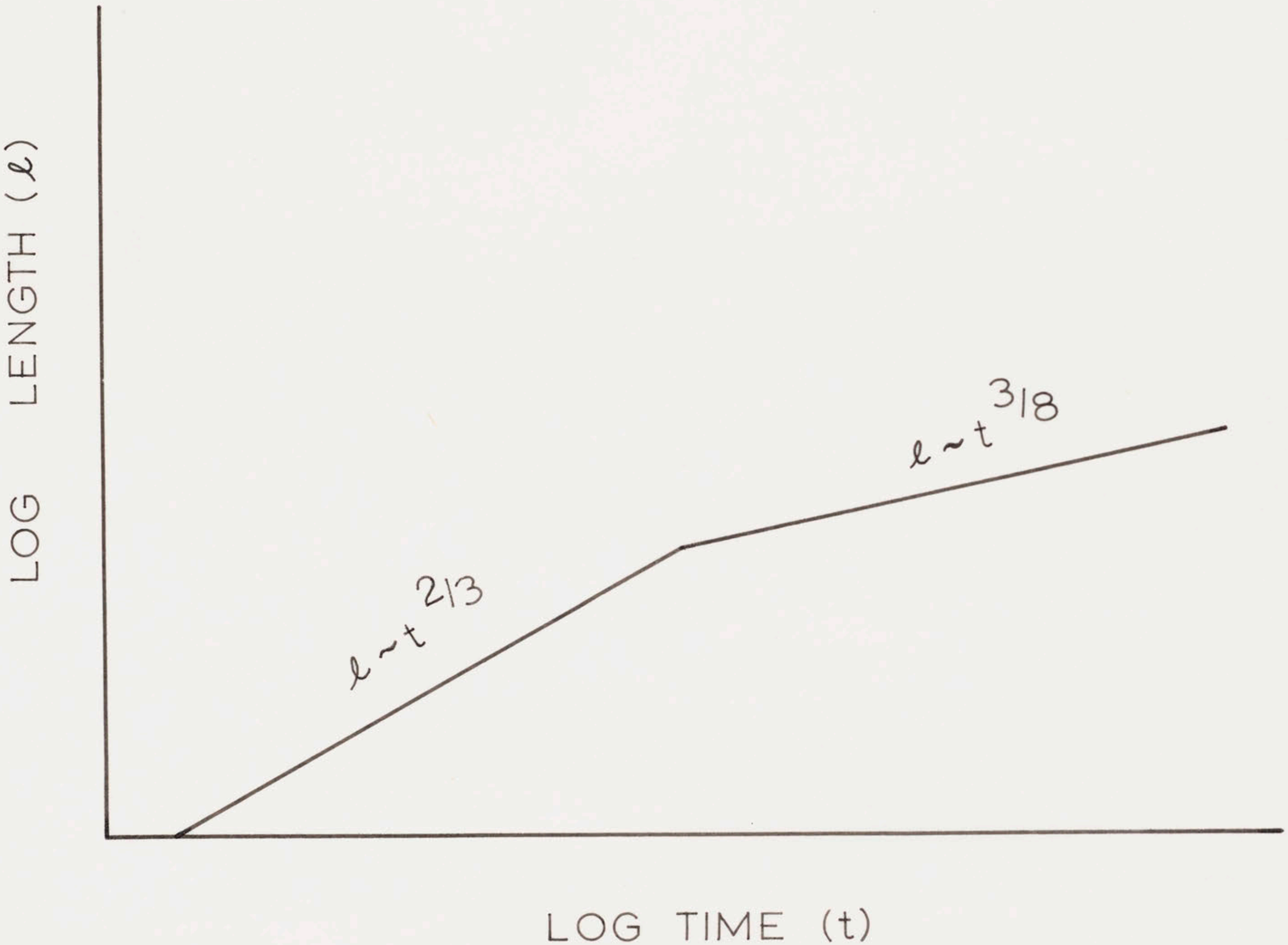


Figure 1: Transition From Gravity-Inertia to Gravity-Viscous Spreading of an Oil Slick

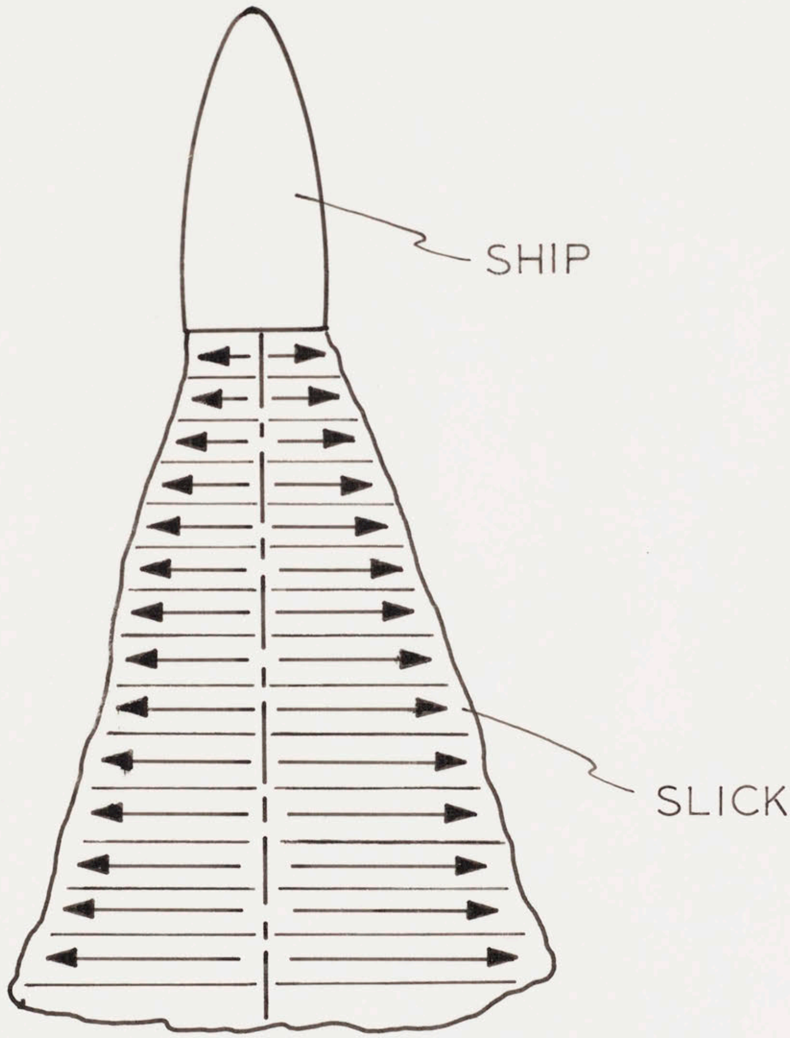


Figure 2: One Dimensional Channel Flow Approximation for a Small Oil Spill

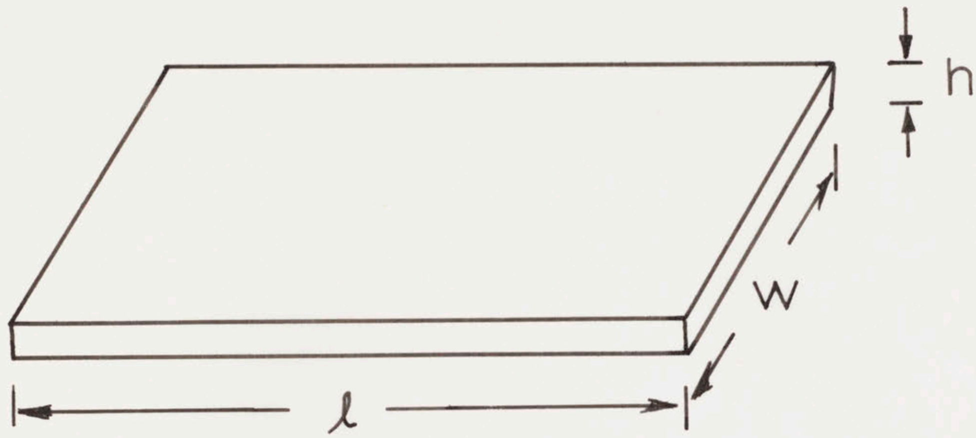


Figure 3: The Oil Slick To Be Considered

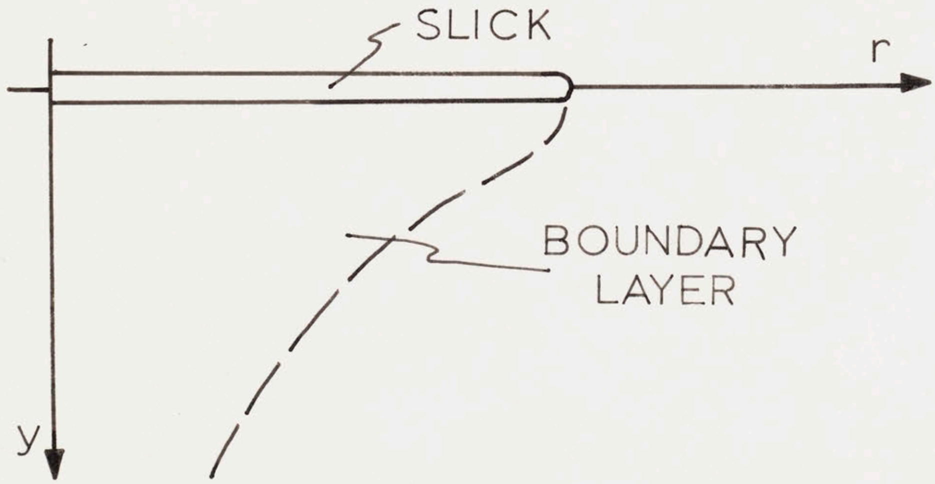


Figure 4: Slick Moving in the r-y Plane

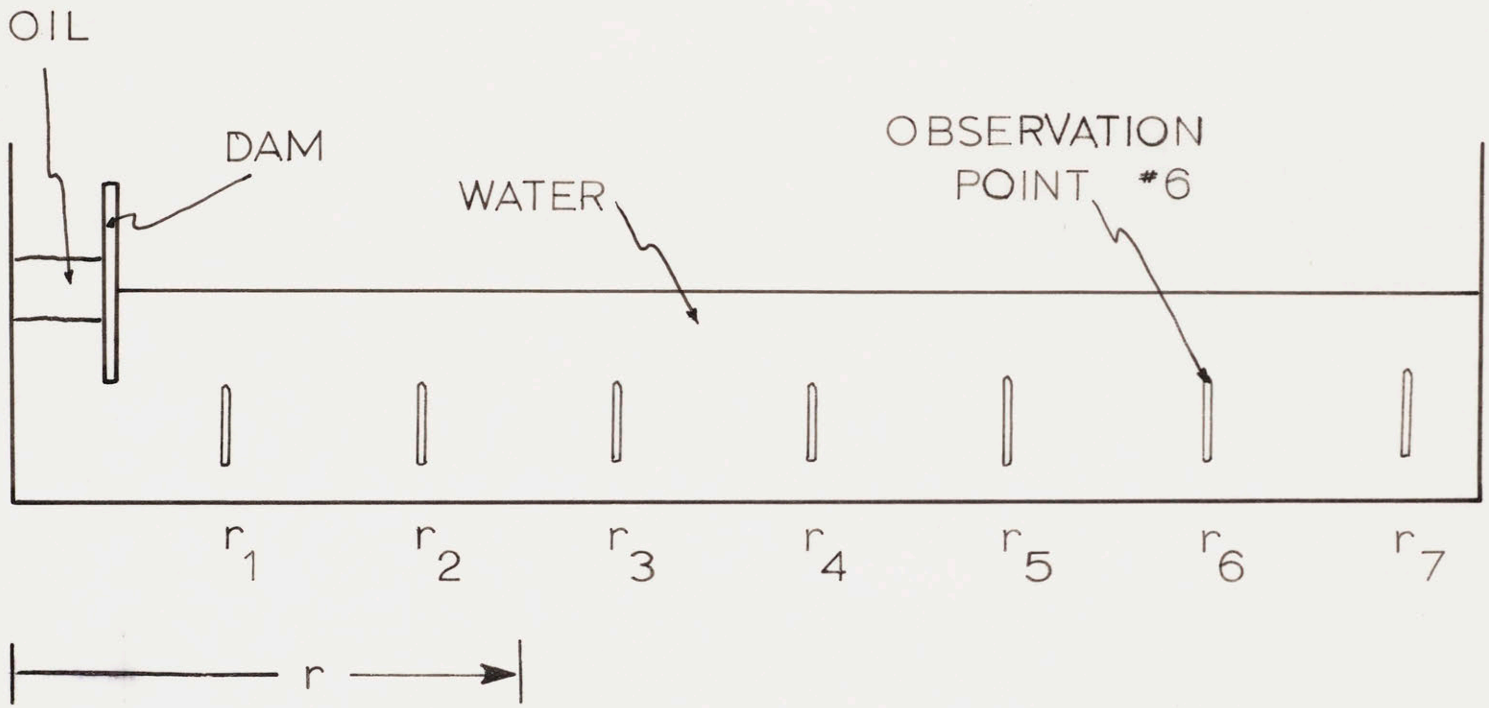


Figure 5: The Experimental Tank with Observation Stations at r_1, r_2, ..., r_7

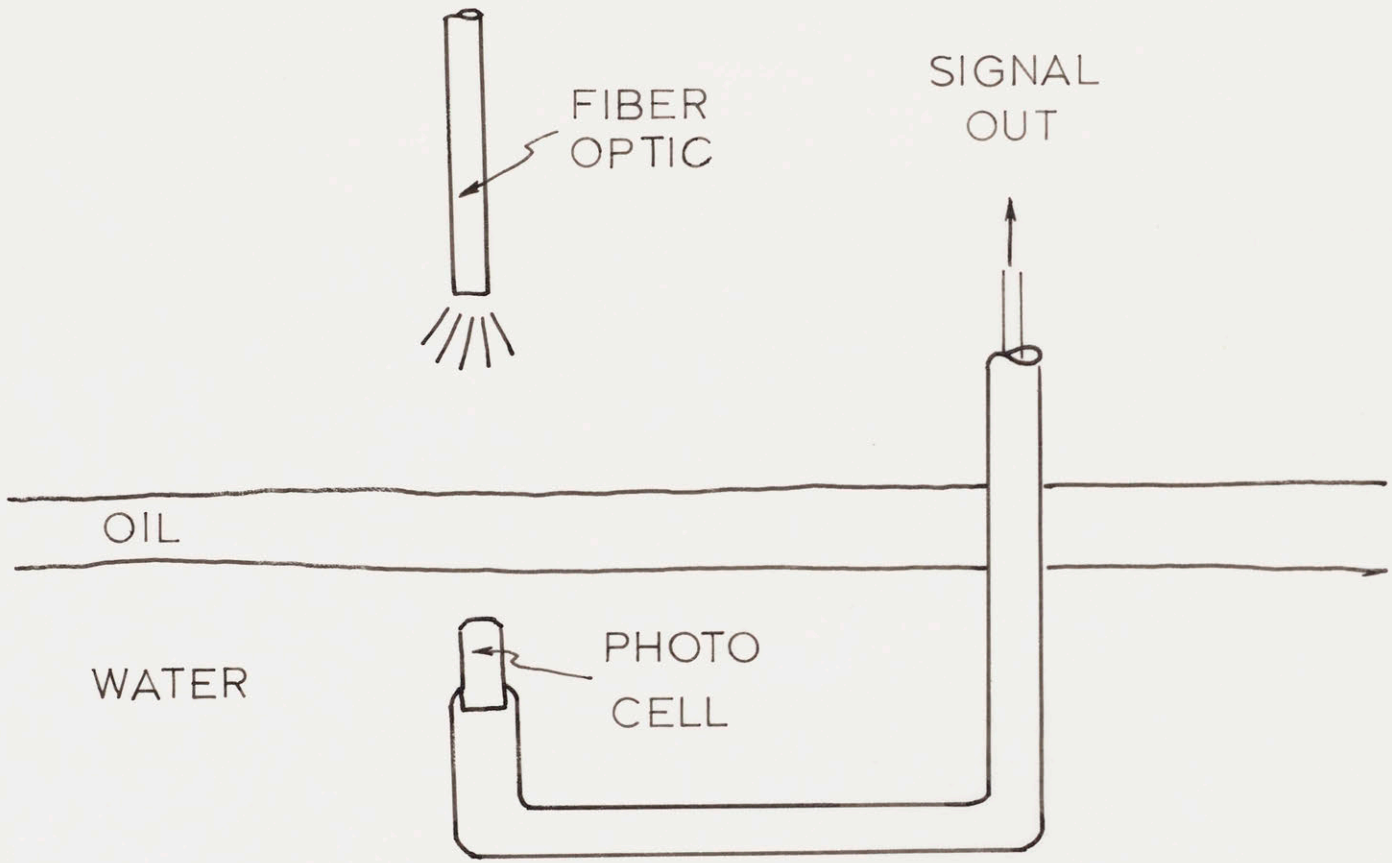


Figure 6: Detail of One Observation Station with Photocell and Fiber Optic

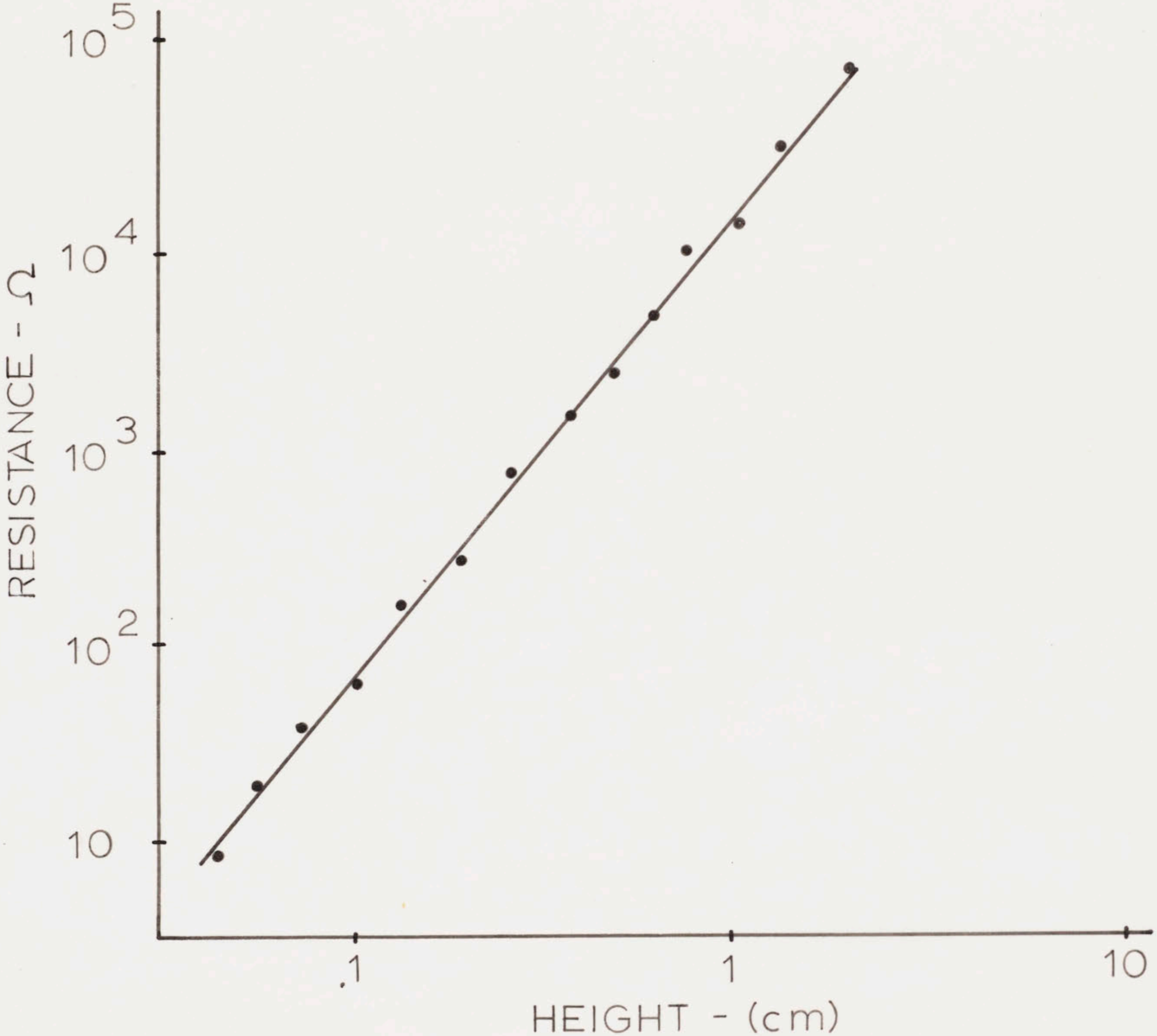


Figure 7: Typical Resistance-Oil Height Calibration Curve for a Photocell

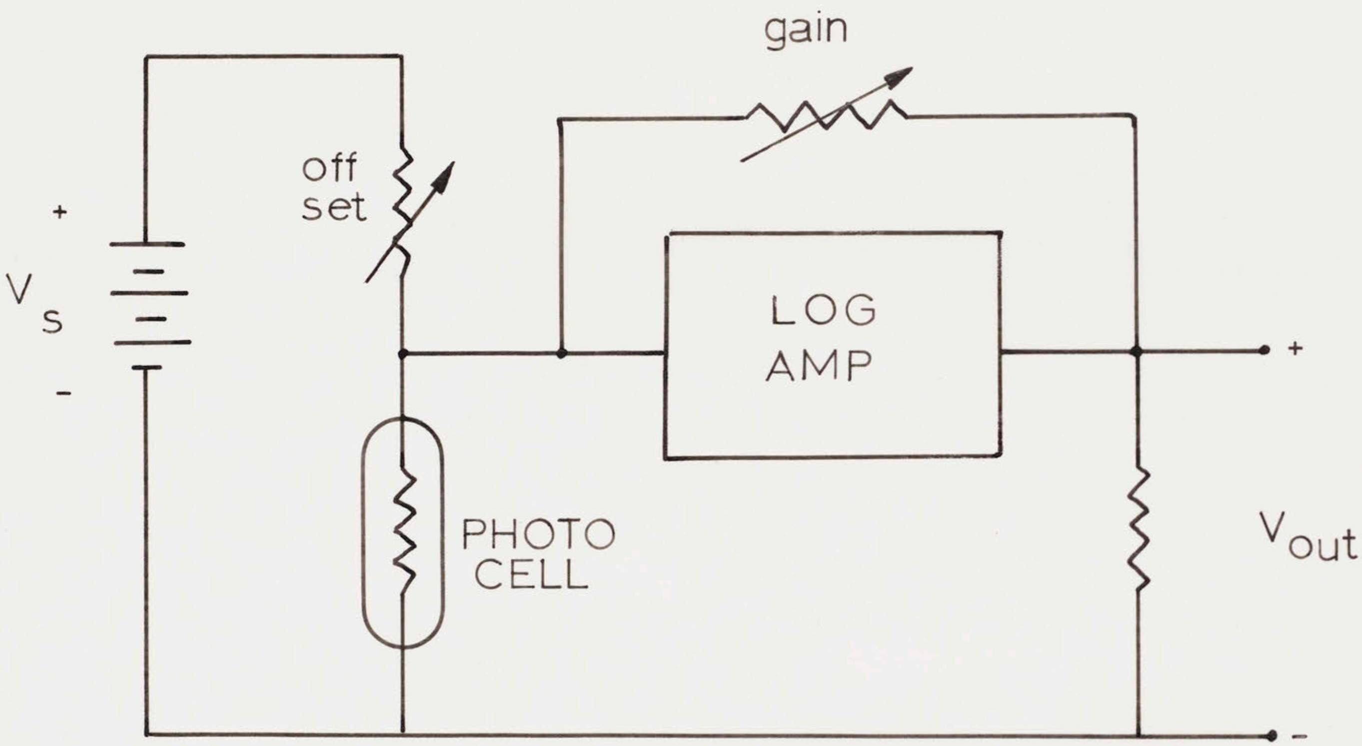


Figure 8: Circuit Used to Linearize Photocell Output

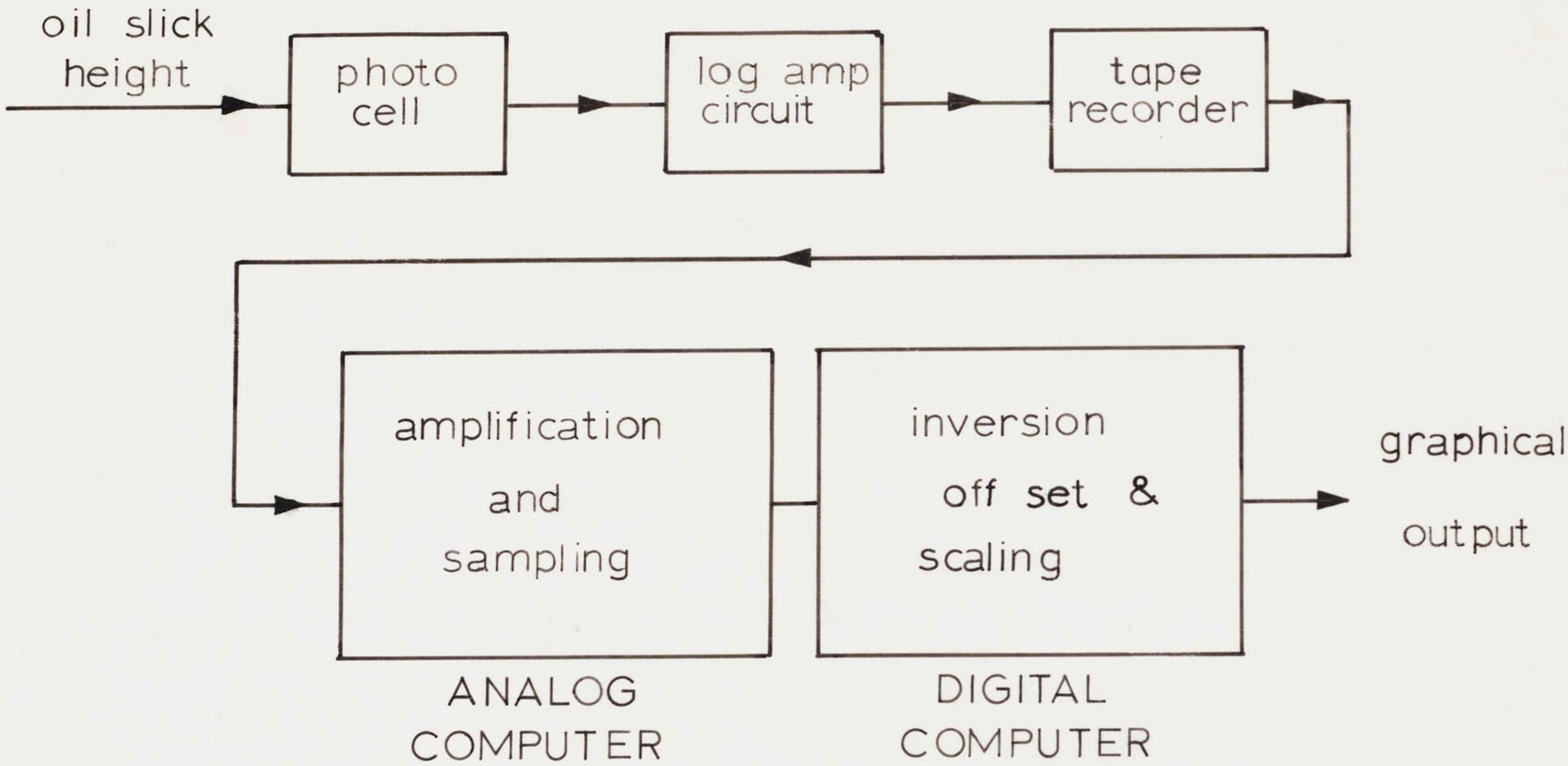
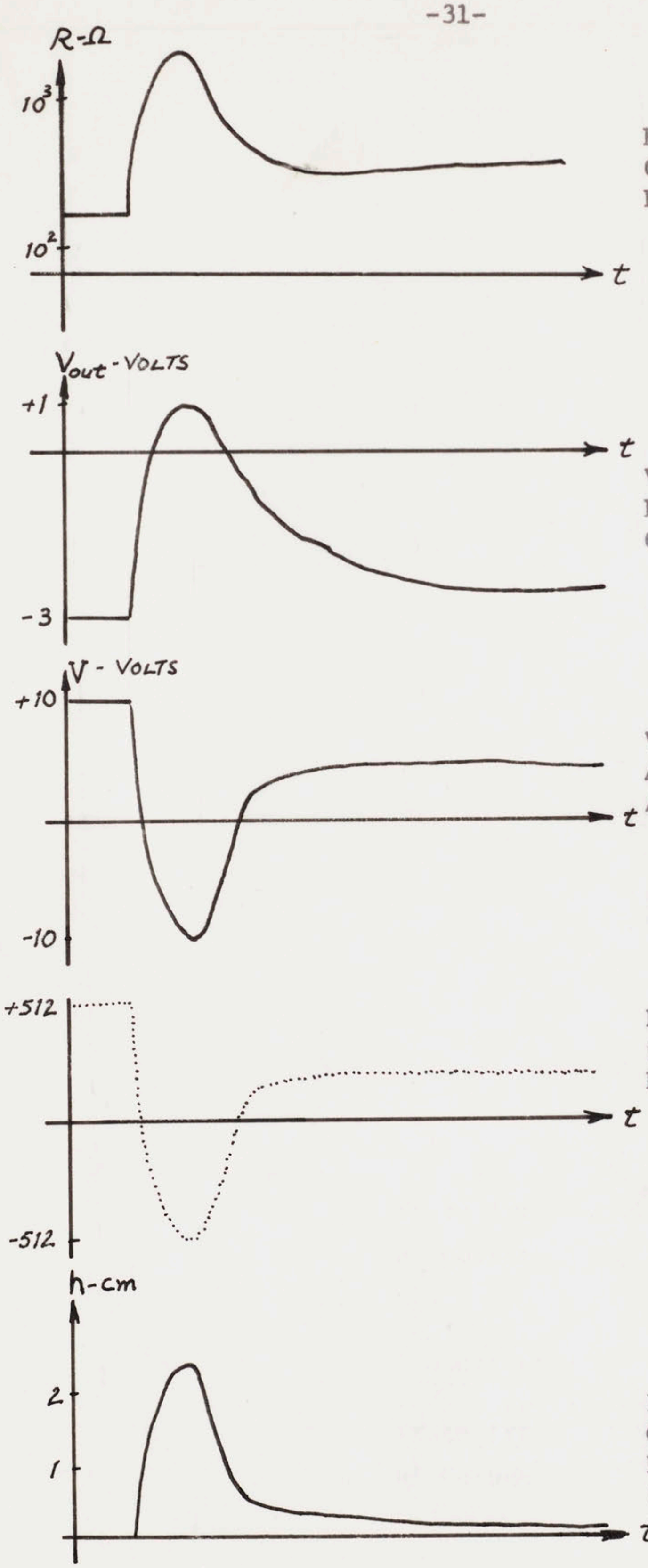


Figure 9: Flow Chart of the Data Processing from Photocell to Graphical Output



Photocell Resistance Change as the Slick Passes By

Voltage Out of the Photocell Linearization Circuit

Voltage Out of the Analog Computer and Into Analog-Digital Interface

Discrete Digital Amplitudes Stored on IBM 1130 Disk

Final Height vs. Time Graph Using Data Reduction Program (See Appendix III)

Figure 10: Signal Trace from Photocell to Graphical Output

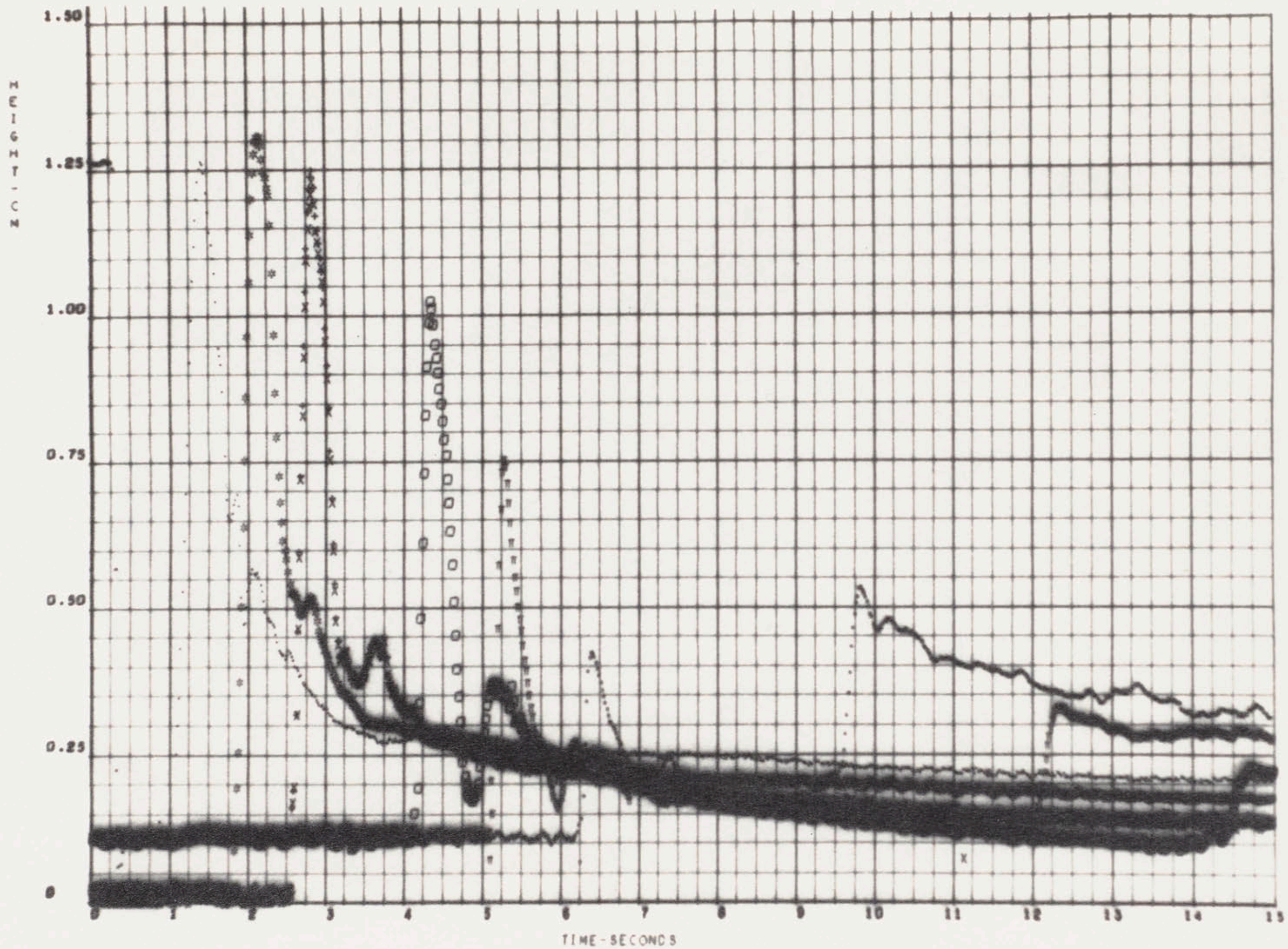


Figure 11: A Sample of an h vs. t Plot (Experiment 6)

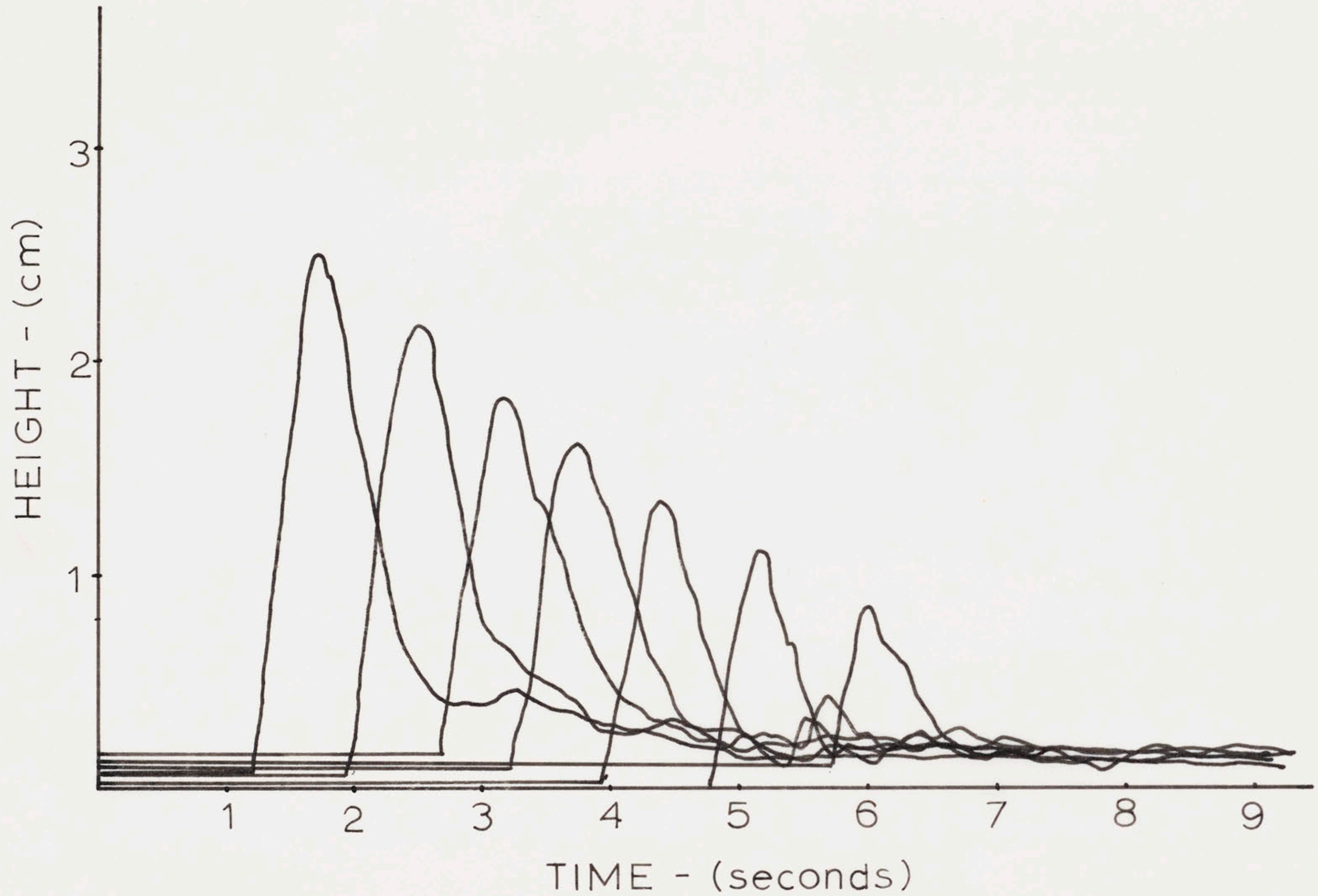
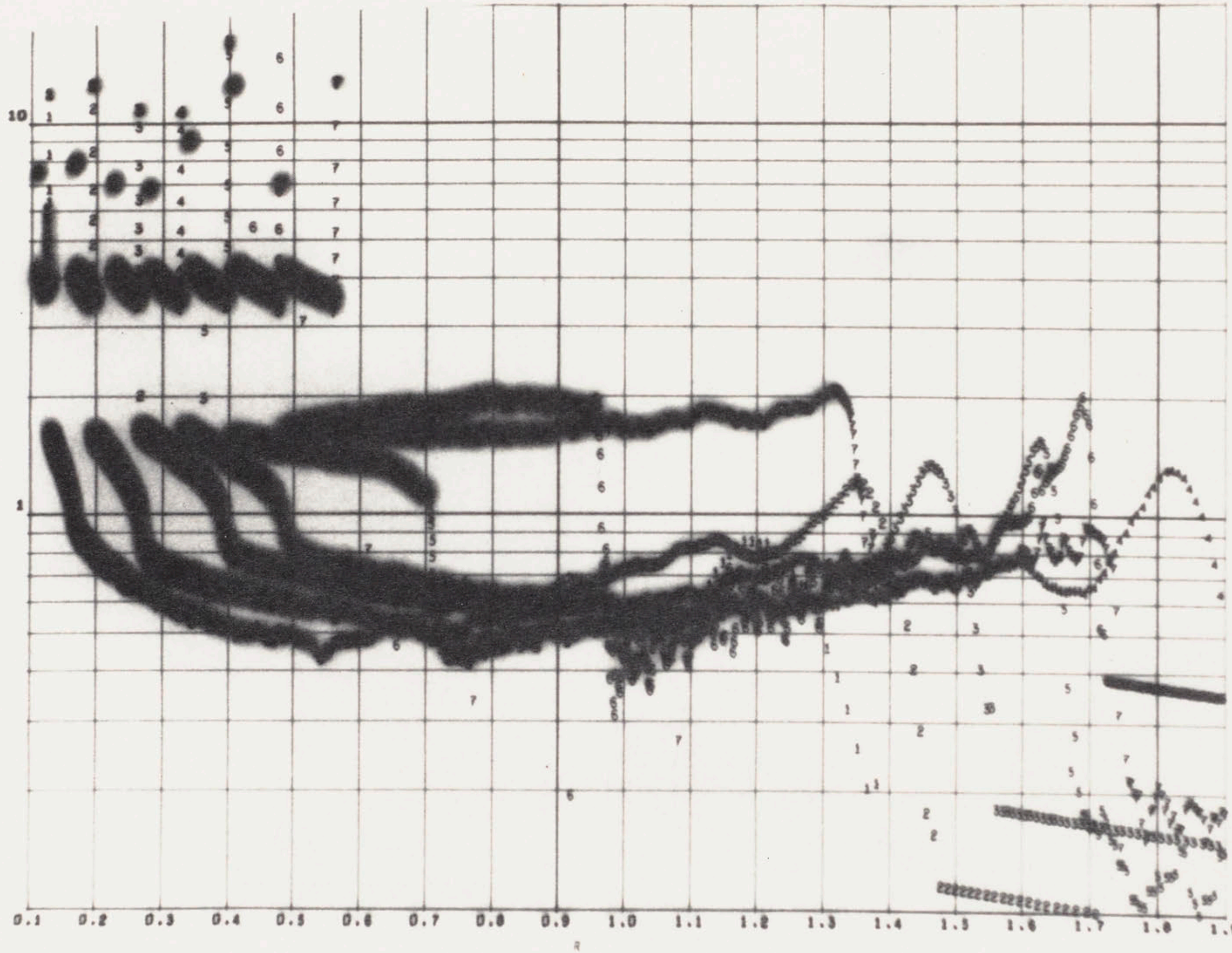


Figure 11a: Height vs. Time



(Experiment 6)

Figure 12: A Sample of an $\tilde{h}$ vs. R Plot

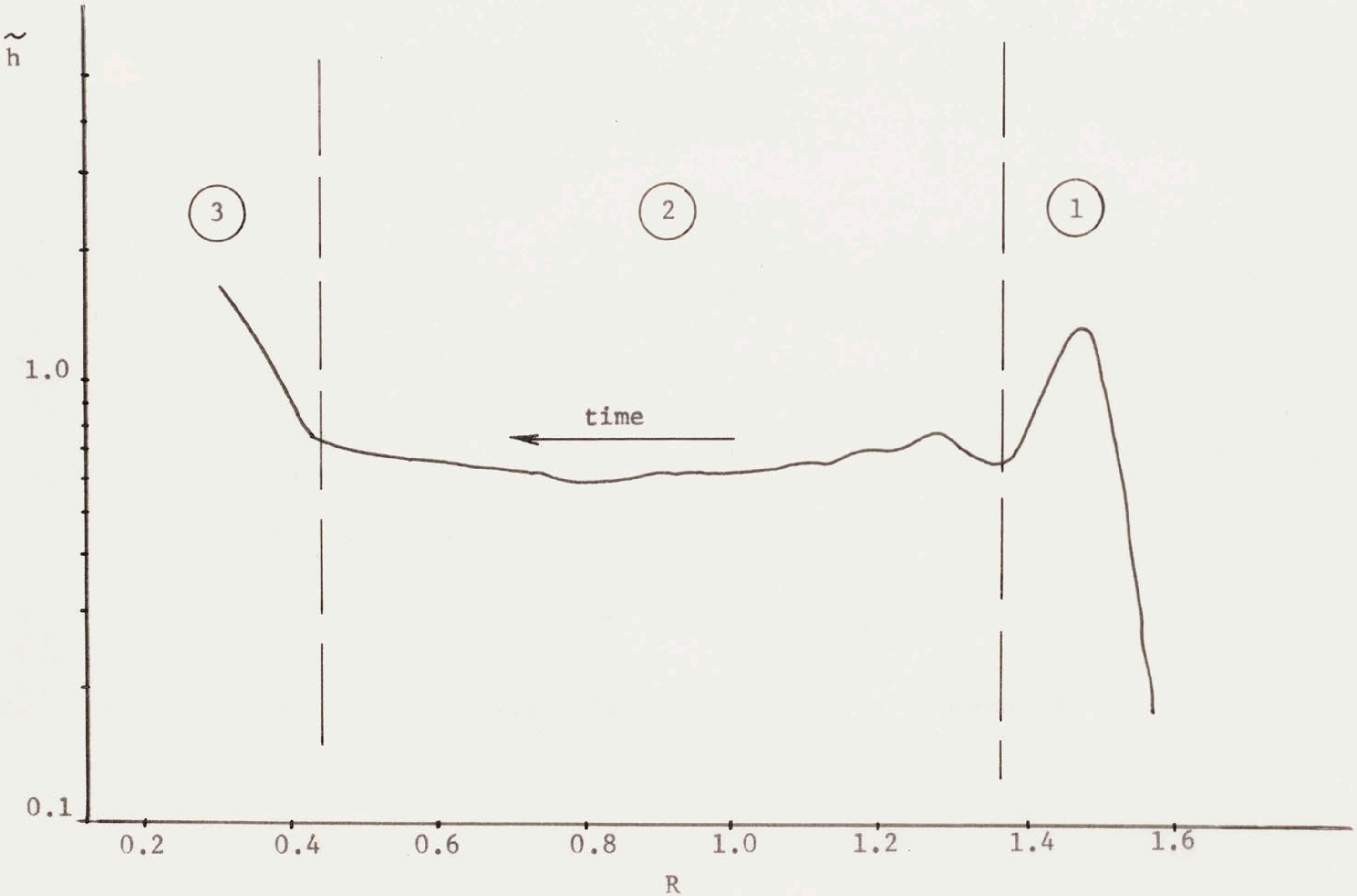


Figure 12a: $\tilde{h}$ vs. R for Channel 3 of Experiment #6

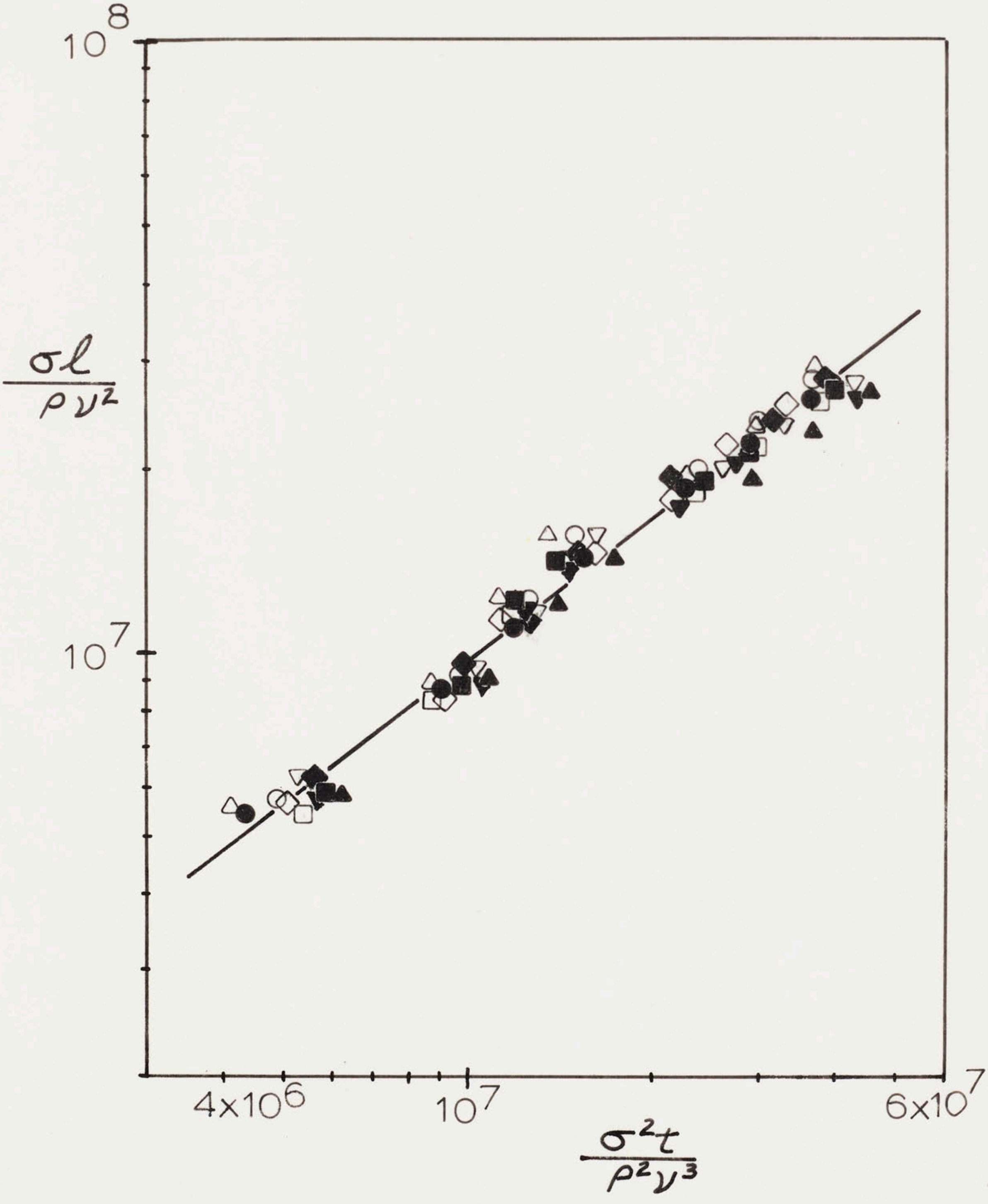


Figure 13: Non-Dimensional Length vs. Non-Dimensional Time for Surface Tension Spreading

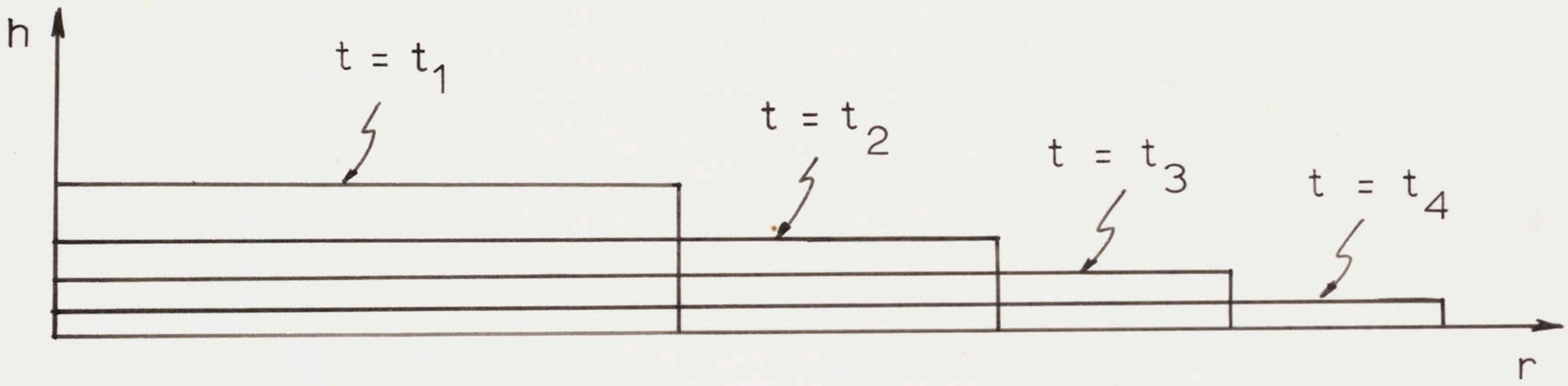


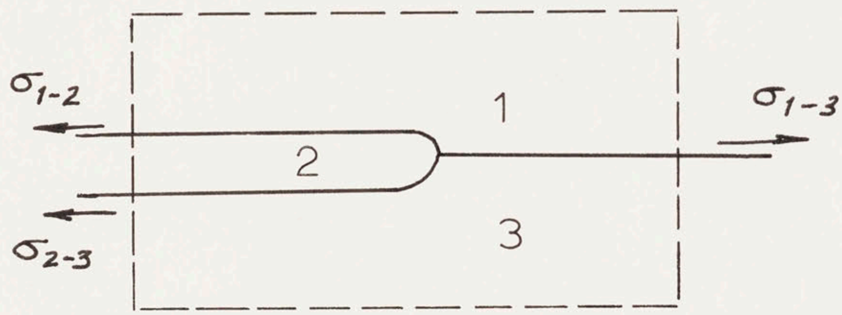
Figure 14: Approximate Height Variation with Time for the Spreading of a Small Slick

Appendix I

The Measurement of Net Surface Tension Coefficient

Since it is surface tension which controls the spreading process in this regime, it is critical to the experiments to determine accurately the net spreading coefficient for each test run.

As can be seen from the figure below, this spreading coefficient is the algebraic sum of three surface tensions:



$$\sigma_{net} = \sigma_{1-3} - \sigma_{1-2} - \sigma_{2-3}$$

It is this σ_{net} which regulates the speed of the slick nose enclosed in the control volume.

The various surface tensions were measured using precision capillary tubes according to the formula

$$\frac{gdh}{4} (\rho_a - \rho_b)$$

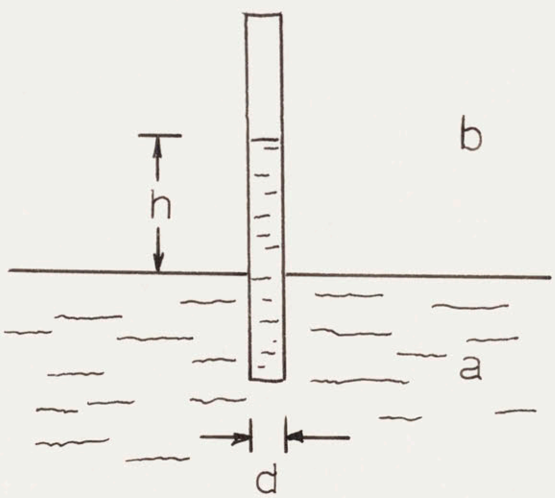
where:

g = gravitational constant

d = inner diameter of capillary tube

h = height rise above the interface

ρ = density



The liquid-liquid interfacial surface tension was measured using the procedure suggested by Bartell and Miller.[5]

Appendix II

Experimental Data

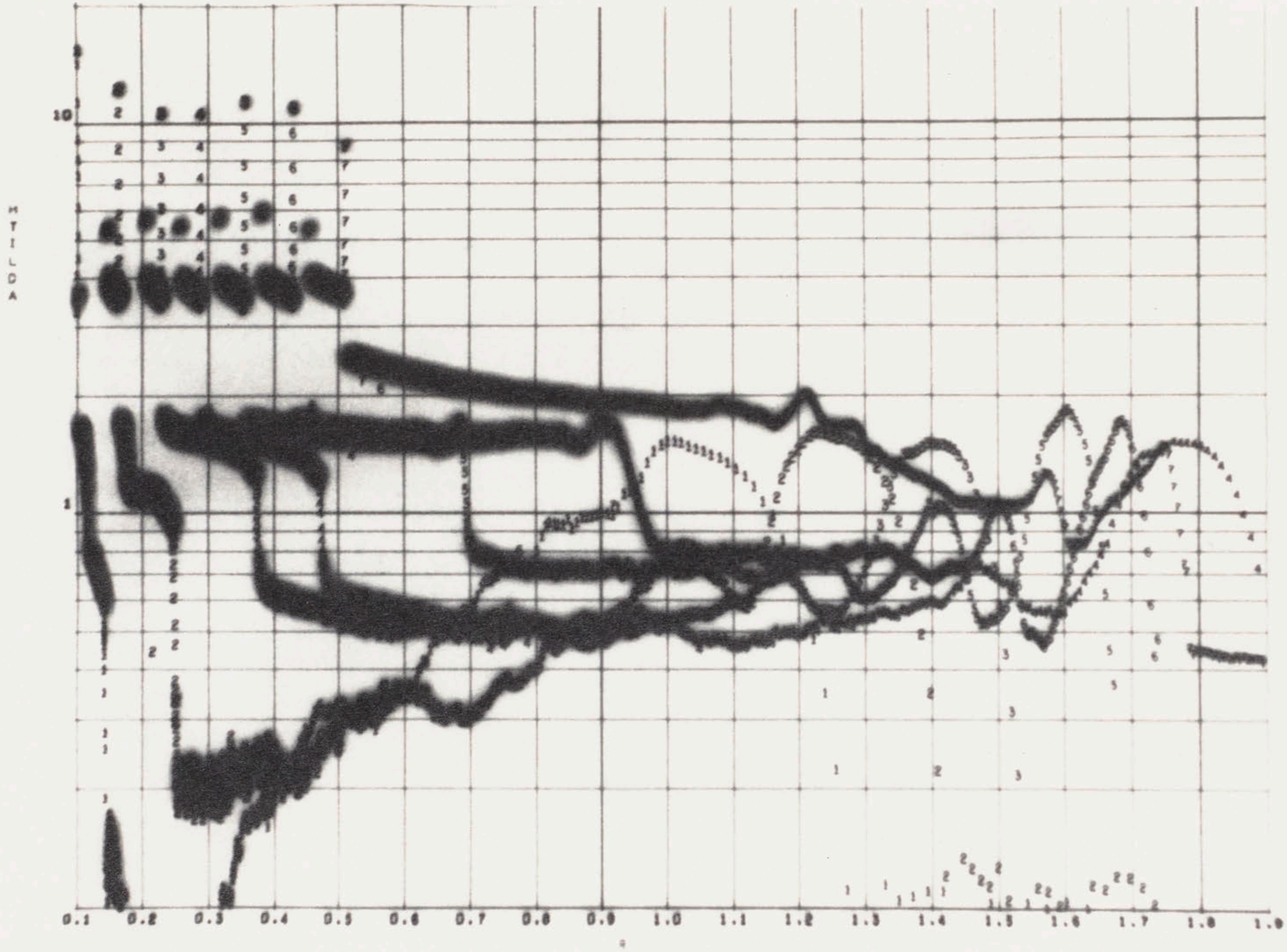
Included in this appendix are the details of each experiment conducted; surface tensions, starting lengths and heights, initial volumes, and photocell placements are listed. Also included are the computer plots of $\tilde{h}$ vs. R for each experiment.

Run #	Initial Volume	Starting Length	Starting - Height	Net Surface Tension	Placement of Photocell #1 *
1	930 cm^3	7.9 cm	3.9 cm	26.1 $\frac{dynes}{cm}$	21.5 cm
2	930	7.9	3.9	26.4	21.0
3	696	7.9	2.9	23.5	23.0
4	600	8.3	2.4	24.8	25.4
5	700	10.4	2.2	22.9	25.2
6	750	7.5	3.3	24.3	22.3
7	500	10.0	1.64	24.1	23.9

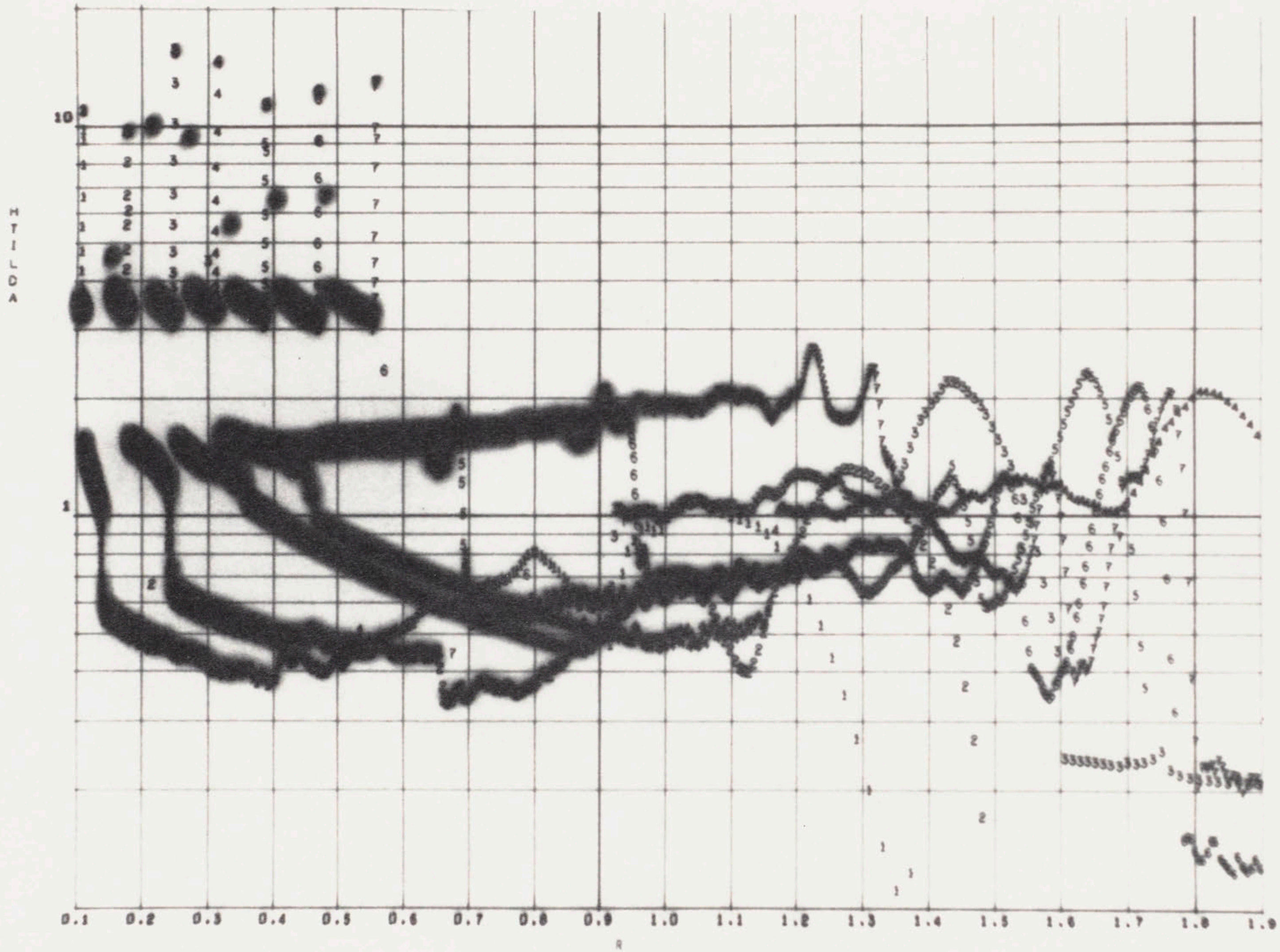
Run #	Initial Volume	Starting L Length	Starting Height	Net Surface Tension	Placement of Photocell #1
8	500 cm^3	5.0 cm	3.3 cm	23.6 $\frac{dynes}{cm}$	25.0 cm
9	500	9.0	1.8	23.2	24.5

*

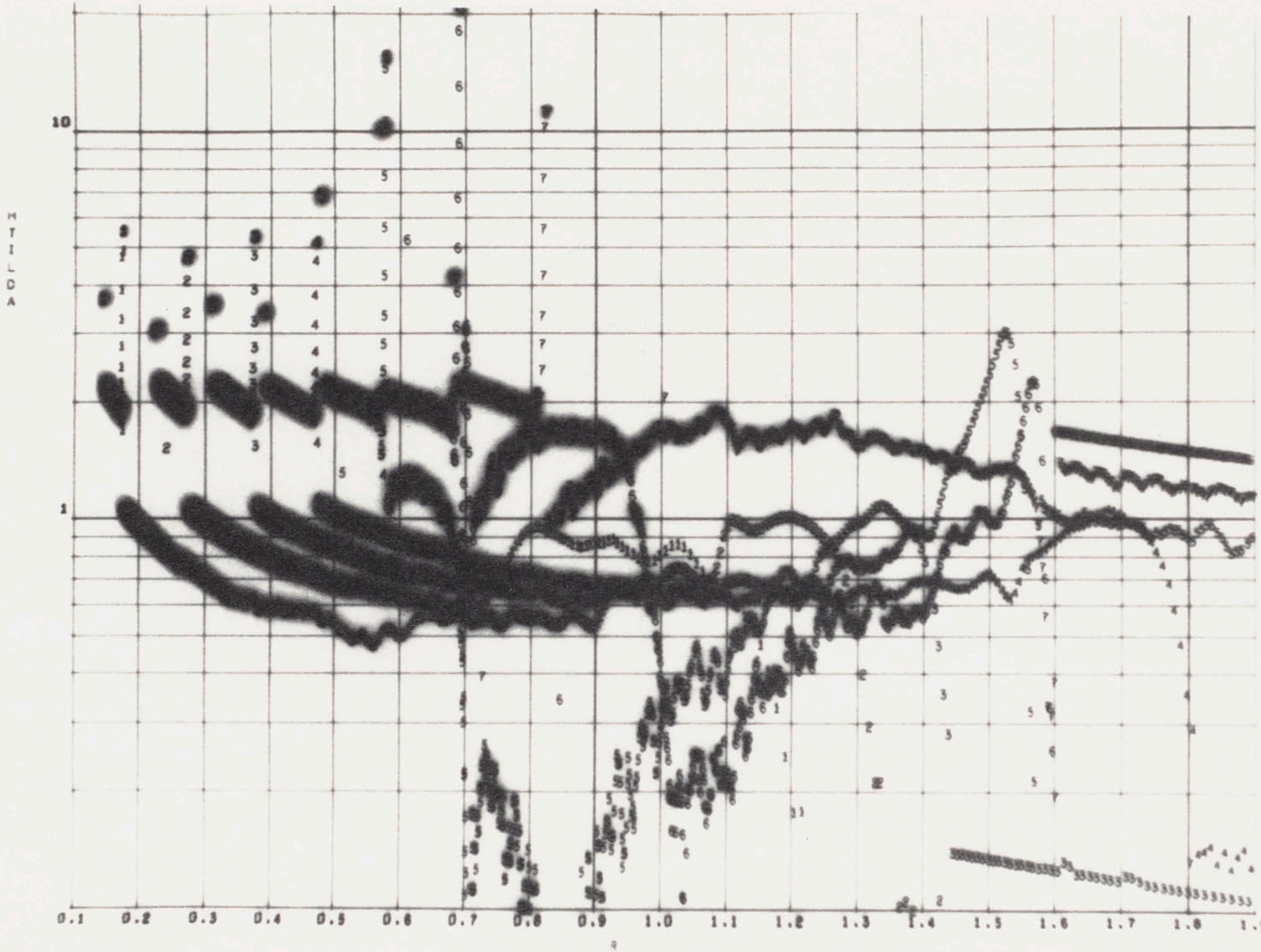
For the experiments described here, distances between photocells was fixed. To get placement of photocells 2,3,4,5,6, and 7, add to photocell #1 placement successively 13.0, 13.5, 12.7, 14.1, 15.5, and 16.7 cm.



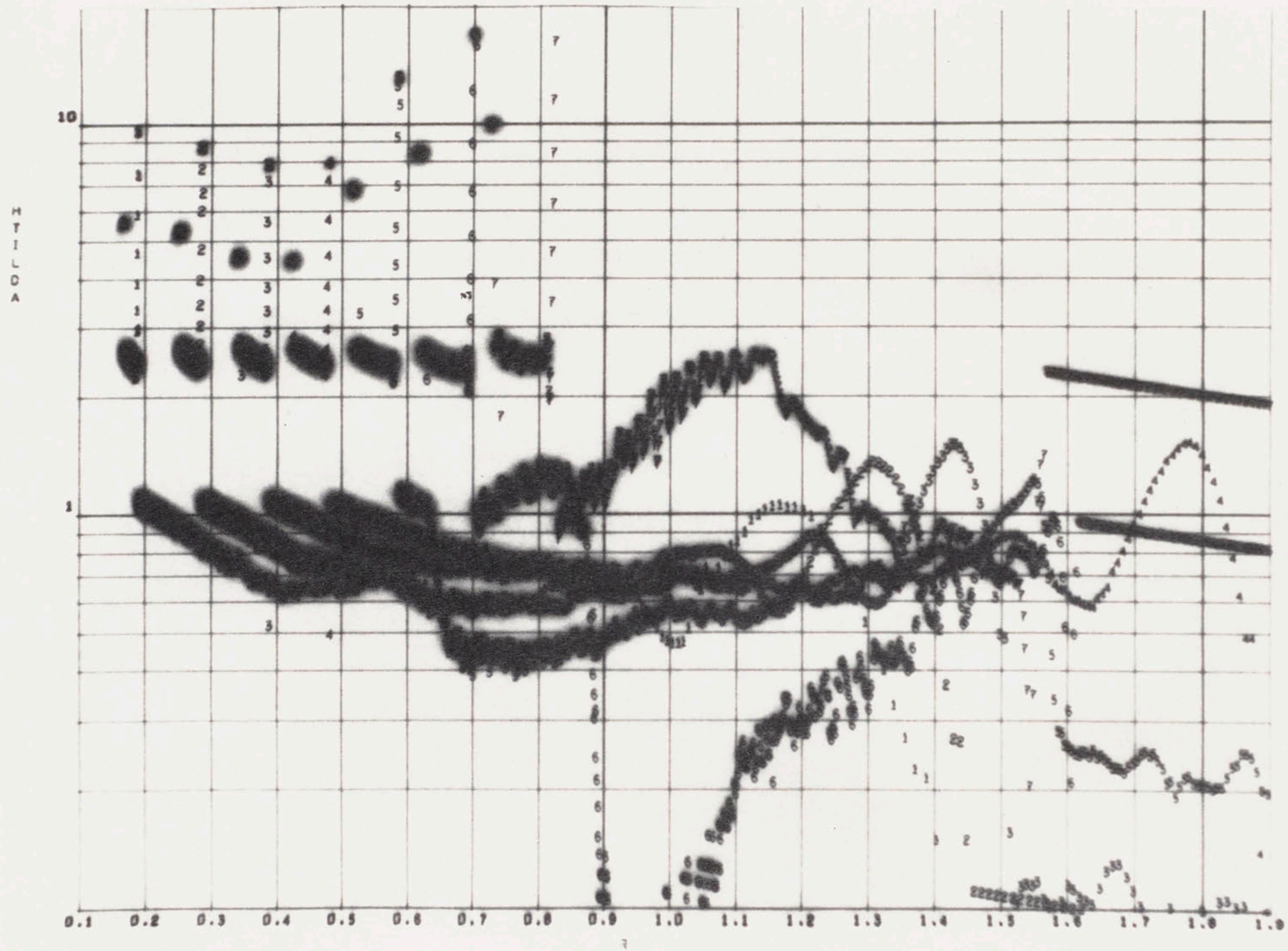
Experiment 1



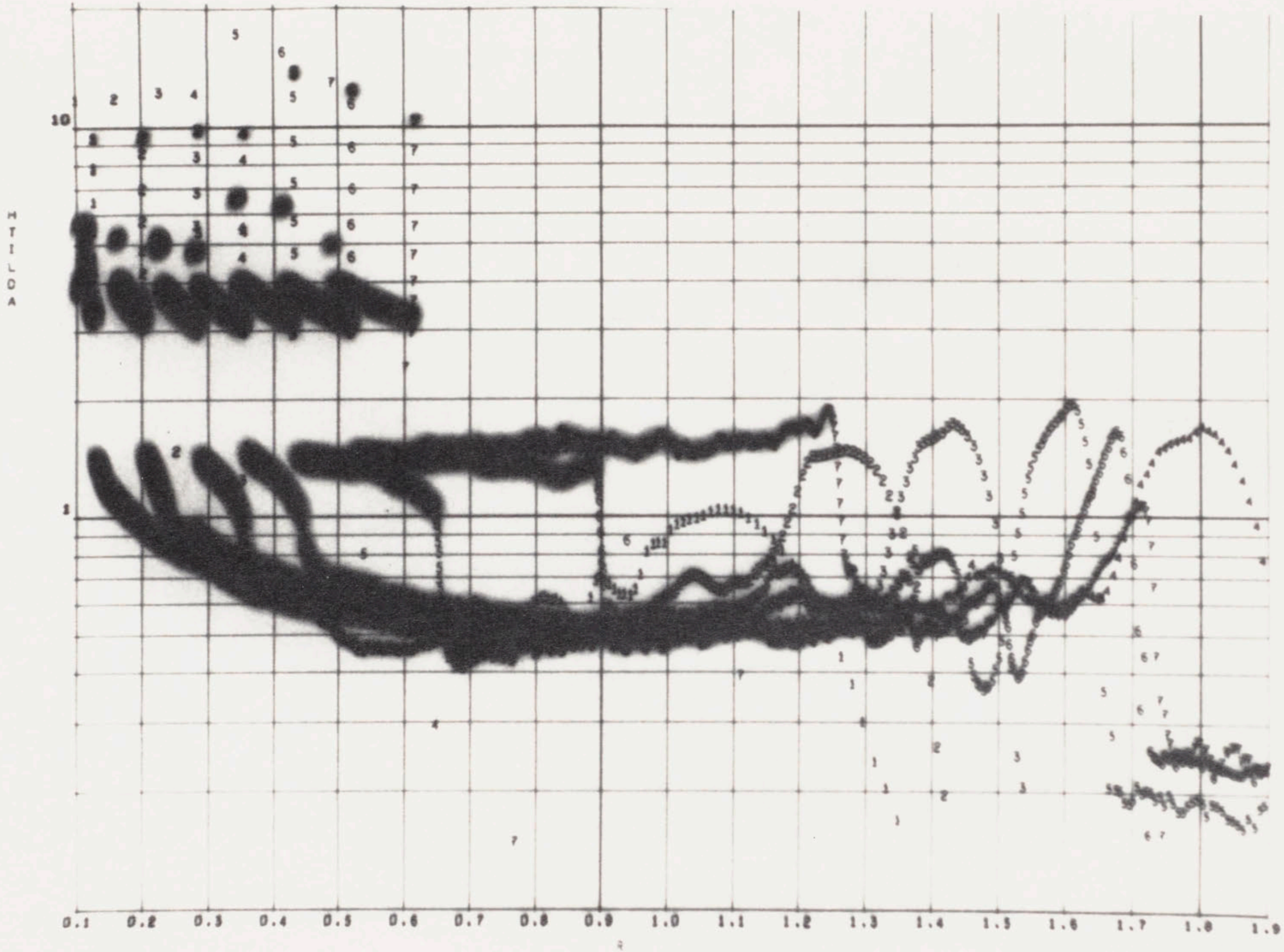
Experiment 2



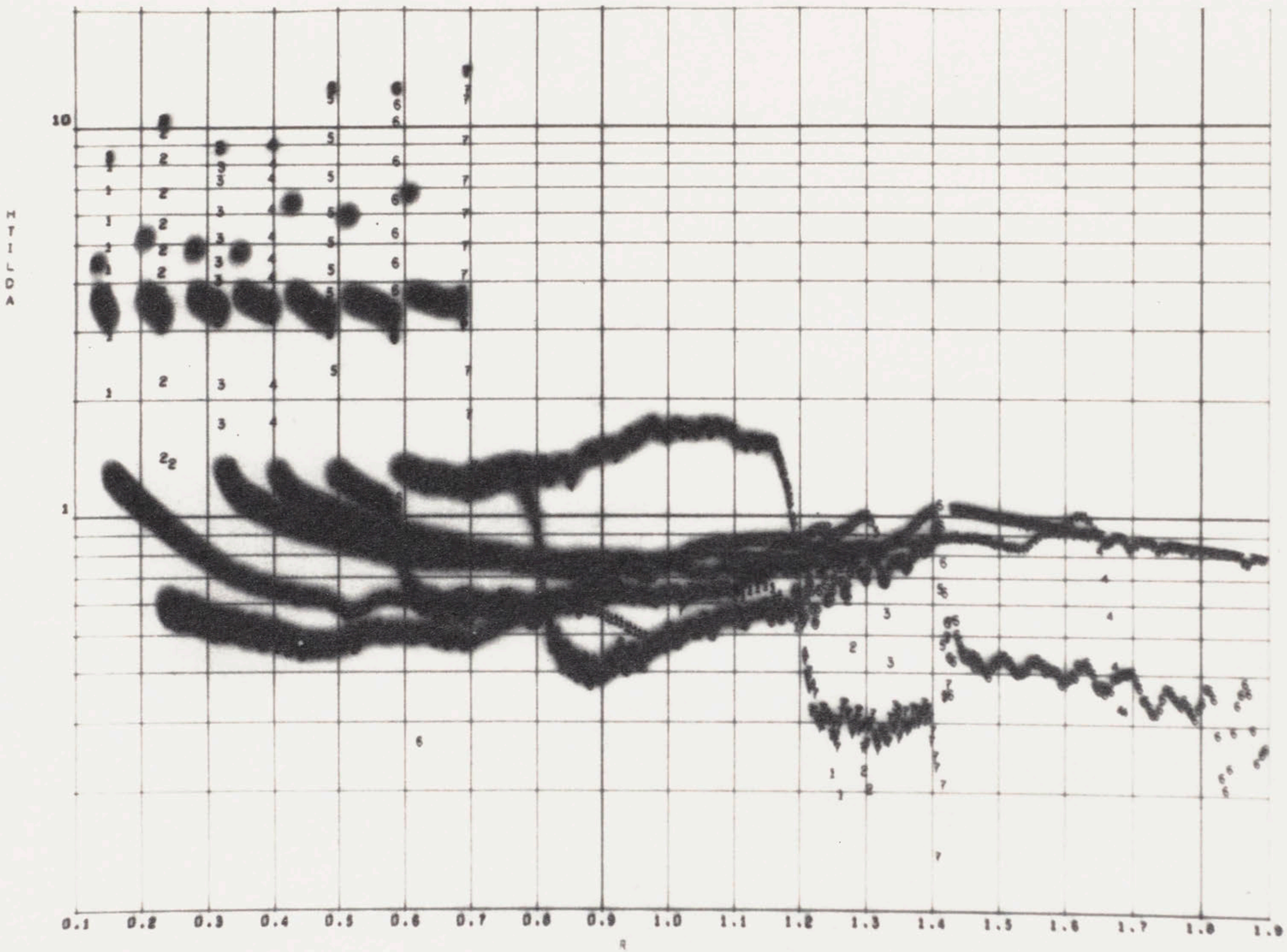
Fxperiment 3



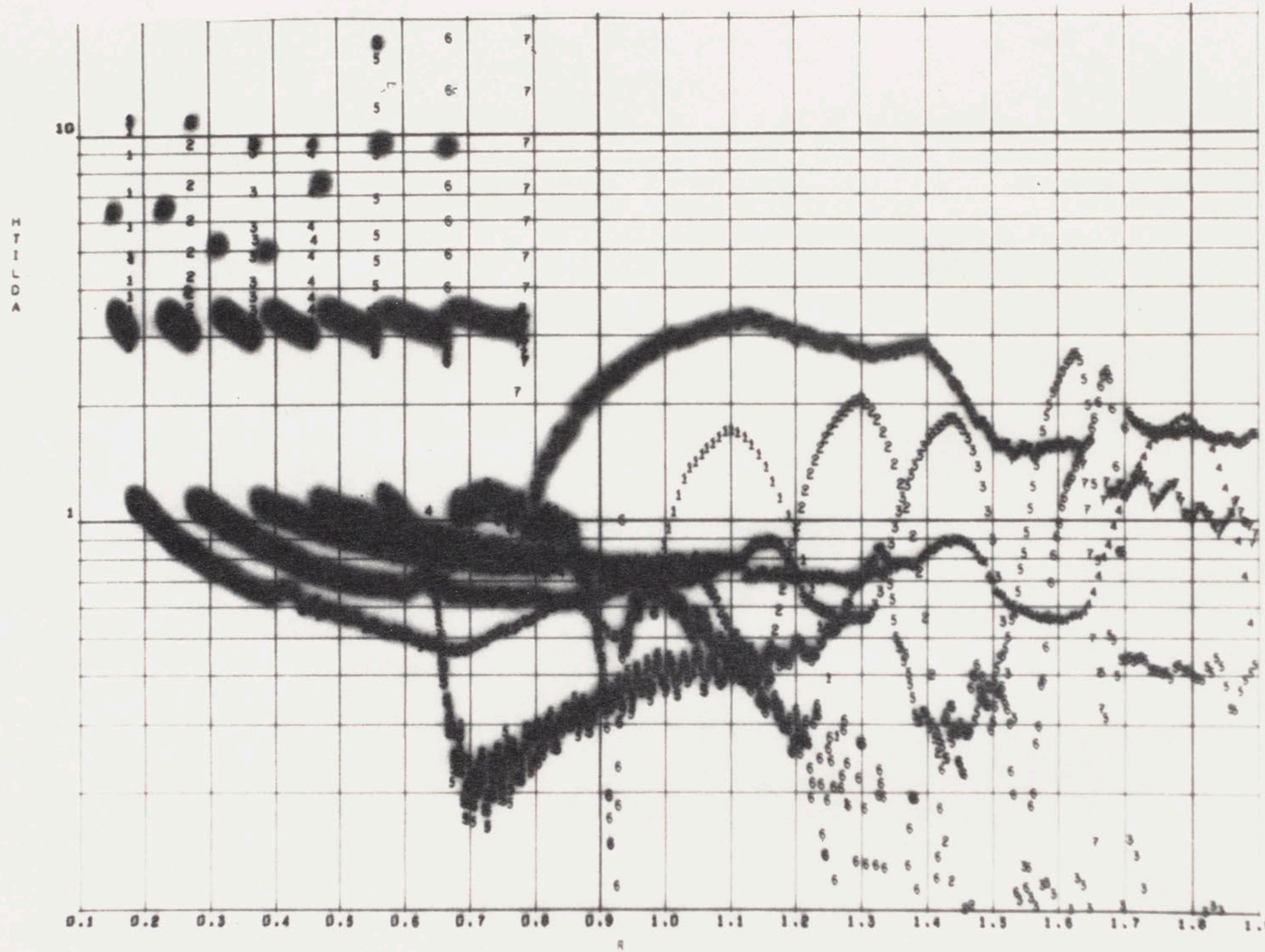
Experiment 4



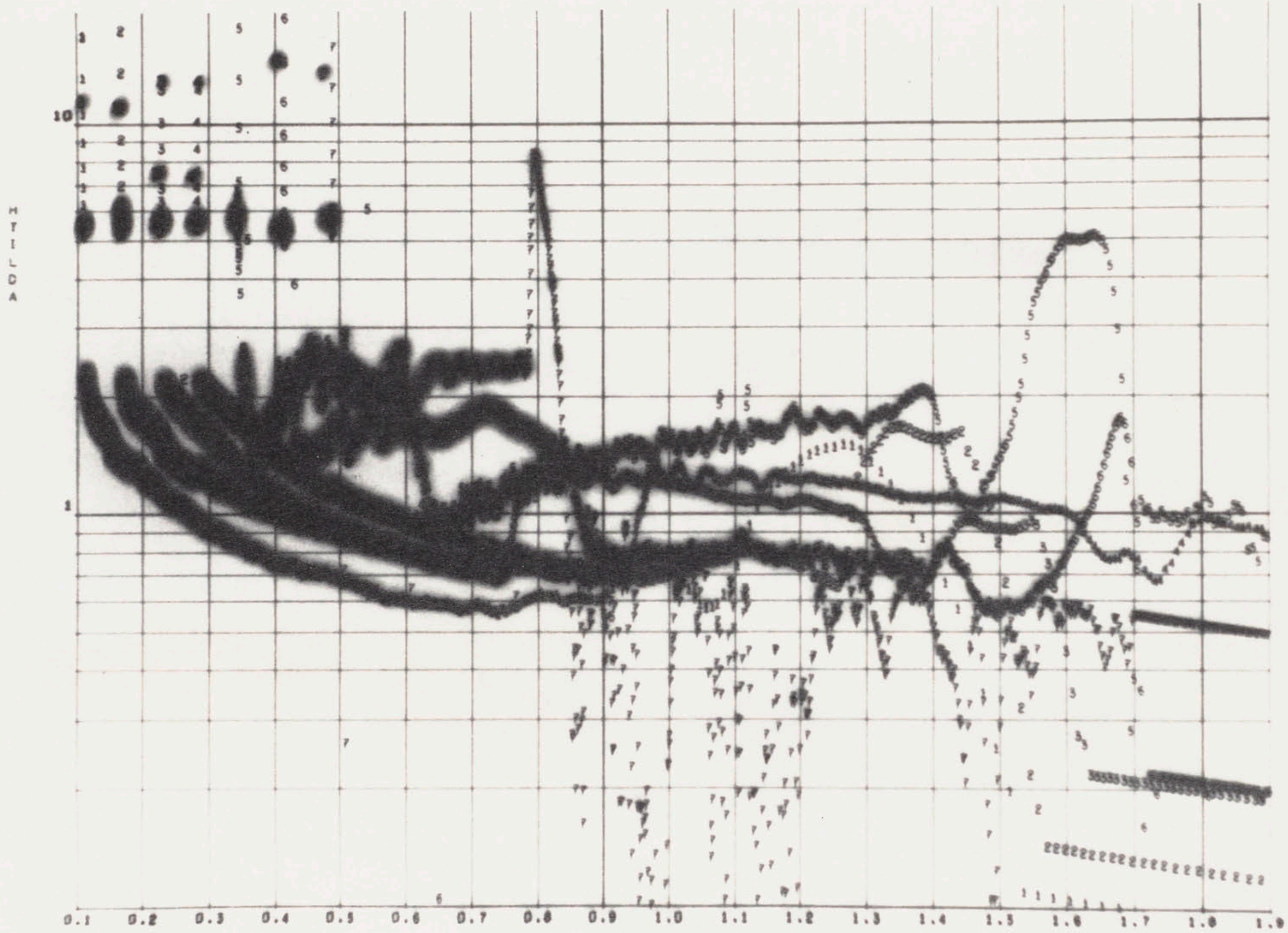
Experiment 5



Experiment 7



Experiment 8



Experiment 9

Appendix III

The Data Reduction Program

As described in Section III, experiments are finally stored on IBM 1130 disk in the form of discrete data points ranging from +512 to -512 in digital amplitude. These points are retrievd from disk storage and punched on cards, 16 points, or exactly .32 seconds of the experiment per card. (Time elapsed between sampled points is 0.02 seconds.)

The program included in this appendix takes each card and converts the digital amplitudes into physical oil height by using two calibration levels and two calibration oil heights.

```
      REAL LO,NU
      INTEGER C1(2000),C2(2000),C3(2000),C4(2000),C5(2000),C6(2000)
      INTEGER C7(2000),CHANL,V11,V21,V31,V41,V51,V61,V71,V12,V22,V32
      INTEGER V42,V52,V62,V72
      DIMENSION H1(2000),H2(2000),H3(2000),H4(2000),H5(2000),H6(2000)
      DIMENSION H7(2000),T(2000),HITE1(2000),HITE2(2000),HITE3(2000)
      DIMENSION HITE4(2000),HITE5(2000),HITE6(2000),HITE7(2000)
      DIMENSION DIST(7),HITE(7,2000),PROF(30,7),NS(30),R(7,2000)
      DIMENSION HTILDA(7,2000)
C     RATE=SAMPLING FREQUENCY
      RATE=50.0
C     T=TIME ARRAY
C     CONSTRUCT THE TIME ARRAY
      I=1
      T(I)=0.0
      DO 10 I=1,1999
      T(I+1)=T(I)+1.0/RATE
   10 CONTINUE
      NREAD=5
C     READ IN THE CALIBRATION VOLTAGES (2/CHANNEL) AND THE
C     CALIBRATION HEIGHTS
      READ(NREAD,15)V11,V12,V21,V22,V31,V32,V41,V42,V51,V52,V61,V62,V71,
      V72,CH1,CH2
   15 FORMAT(14I4,2F10.0)
      WRITE(6,205)
      WRITE(6,16)V11,V12,V21,V22,V31,V32,V41,V42,V51,V52,V61,V62,V71,V72
      ,CH1,CH2
   16 FORMAT(1X,14(I4,1X),2F8.4)
      WRITE(6,205)
C     V=CALIBRATION VOLTAGES FOR EACH CHANNEL
C     CH=CALIBRATION HEIGHT
C     DETERMINE WHICH CHANNEL OF DATA IS ABOUT TO BE READ IN
   19 READ(NREAD,20)CHANL
   20 FORMAT(I2)
      WRITE(6,23) CHANL
   23 FORMAT(20X,I2)
      IF(7-CHANL)25,25,30
   25 GO TO 19
   30 J=1
      K=J+15
      NCHANL=CHANL+1
   35 GO TO (40,60,80,100,120,140,160), NCHANL
```

```
C       READ IN AND PRINT DATA SEQUENTIALLY BY CHANNEL
   40  READ(NREAD,45) (C1(I), I=J,K)
   45  FORMAT(4X,16I4)
       WRITE(6,45) (C1(NC), NC=J,K)
       IF(C1(K-15))50,55,50
   50  J=J+16
       K=J+15
       GO TO 40
   55  GO TO 19
   60  READ(NREAD,45) (C2(I), I=J,K)
       WRITE(6,45) (C2(NC), NC=J,K)
       IF(C2(K-15))70,75,70
   70  J=J+16
       K=J+15
       GO TO 60
   75  GO TO 19
   80  READ(NREAD,45) (C3(I), I=J,K)
       WRITE(6,45) (C3(NC), NC=J,K)
       IF(C3(K-15))90,95,90
   90  J=J+16
       K=J+15
       GO TO 80
   95  GO TO 19
  100  READ(NREAD,45) (C4(I), I=J,K)
       WRITE(6,45) (C4(NC), NC=J,K)
       IF(C4(K-15))110,115,110
  110  J=J+16
       K=J+15
       GO TO 100
  115  GO TO 19
  120  READ(NREAD,45) (C5(I), I=J,K)
       WRITE(6,45) (C5(NC), NC=J,K)
       IF(C5(K-15))130,135,130
  130  J=J+16
       K=J+15
       GO TO 120
  135  GO TO 19
  140  READ(NREAD,45) (C6(I), I=J,K)
       WRITE(6,45) (C6(NC), NC=J,K)
       IF(C6(K-15))150,155,150
  150  J=J+16
       K=J+15
```

```
      GO TO 140
  155 GO TO 19
  160 READ(NREAD,45) (C7(I), I=J,K)
      WRITE(6,45) (C7(NC), NC=J,K)
      IF(C7(K-15))170,175,170
  170 J=J+16
      K=J+15
      GO TO 160
C     INVERT AND OFFSET THE DATA
  175 DO 200 I=1,K
      C1(I)=-C1(I)+512
      C2(I)=-C2(I)+512
      C3(I)=-C3(I)+512
      C4(I)=-C4(I)+512
      C5(I)=-C5(I)+512
      C6(I)=-C6(I)+512
      C7(I)=-C7(I)+512
  200 CONTINUE
      WRITE(6,205)
  205 FORMAT(//)
C     USE CALIBRATION VOLTAGES AND HEIGHTS TO CONVERT THE INTEGER DATA
C     INTO PHYSICAL HEIGHTS
      DO 210 I=1,K
      H1(I)=C1(I)*(CH1-CH2)/(V11-V12)+(CH1-V11*(CH1-CH2)/(V11-V12))
      H2(I)=C2(I)*(CH1-CH2)/(V21-V22)+(CH1-V21*(CH1-CH2)/(V21-V22))
      H3(I)=C3(I)*(CH1-CH2)/(V31-V32)+(CH1-V31*(CH1-CH2)/(V31-V32))
      H4(I)=C4(I)*(CH1-CH2)/(V41-V42)+(CH1-V41*(CH1-CH2)/(V41-V42))
      H5(I)=C5(I)*(CH1-CH2)/(V51-V52)+(CH1-V51*(CH1-CH2)/(V51-V52))
      H6(I)=C6(I)*(CH1-CH2)/(V61-V62)+(CH1-V61*(CH1-CH2)/(V61-V62))
      H7(I)=C7(I)*(CH1-CH2)/(V71-V72)+(CH1-V71*(CH1-CH2)/(V71-V72))
  210 CONTINUE
      WRITE(6,220) (H1(MM), MM=1,K)
      WRITE(6,205)
      WRITE(6,220) (H2(MM), MM=1,K)
      WRITE(6,205)
      WRITE(6,220) (H3(MM), MM=1,K)
      WRITE(6,205)
      WRITE(6,220) (H4(MM), MM=1,K)
      WRITE(6,205)
      WRITE(6,220) (H5(MM), MM=1,K)
      WRITE(6,205)
      WRITE(6,220) (H6(MM), MM=1,K)
```

```
      WRITE(6,205)
      WRITE(6,220) (H7(MM), MM=1,K)
  220 FORMAT(5X,16F6.3)
C     OFFSET TIMES SO THAT THE EXPERIMENT STARTS AT T=0
      IT=1
  230 IF(H7(IT+1)-H7(IT))240,235,240
  235 IT=IT+1
      GO TO 230
  240 KS=K-IT
C     ELIMINATE ANY NEGATIVE HEIGHTS DUE TO PHOTOCELL NON-LINEARITY
      DO 250 M=1,KS
      HITE1(M)=H1(IT)
      IF(HITE1(M).LT.0.0)HITE1(M)=-HITE1(M)
      HITE2(M)=H2(IT)
      IF(HITE2(M).LT.0.0)HITE2(M)=-HITE2(M)
      HITE3(M)=H3(IT)
      IF(HITE3(M).LT.0.0)HITE3(M)=-HITE3(M)
      HITE4(M)=H4(IT)
      IF(HITE4(M).LT.0.0)HITE4(M)=-HITE4(M)
      HITE5(M)=H5(IT)
      IF(HITE5(M).LT.0.0)HITE5(M)=-HITE5(M)
      HITE6(M)=H6(IT)
      IF(HITE6(M).LT.0.0)HITE6(M)=-HITE6(M)
      HITE7(M)=H7(IT)
      IF(HITE7(M).LT.0.0)HITE7(M)=-HITE7(M)
      IT=IT+1
  250 CONTINUE
      READ(5,255) (DIST(L), L=1,7)
  255 FORMAT(7F10.0)
      WRITE(6,260)
  260 FORMAT(20X,'PHOTOCELL PLACEMENT')
      WRITE(6,265) (DIST(L),L=1,7)
  265 FORMAT(10X,7F6.1)
C     PLOT HEIGHTS AS A FUNCTION OF TIME
      CALL STOIDV('M8221-9002',9,0)
      CALL GRID1V(1,0.0,15.0,0.0,2.0,0.25,0.05,4,5,4,5,2,4)
      CALL APLOTV(1000,T,HITE1,1,1,1,42,IERR)
      CALL APLOTV(1000,T,HITE2,1,1,1,44,IERR)
      CALL APLOTV(1000,T,HITE3,1,1,1,16,IERR)
      CALL APLOTV(1000,T,HITE4,1,1,1,55,IERR)
      CALL APLOTV(1000,T,HITE5,1,1,1,63,IERR)
      CALL APLOTV(1000,T,HITE6,1,1,1,26,IERR)
```

```
      CALL APLOTV(1000,T,HITE7,1,1,1,58,IERR)
      CALL PRINTV(-12,12HTIME-SECONDS,450,10)
      CALL APRNTV(0,-14,-9,9HHEIGHT-CM,10,700)
C     NEXT, PLOT SLICK PROFILE (HEIGHT VS.POSITION) EVERY 1/2 SECOND
C     CONVERT THE 7 HEIGHT ARRAYS INTO ONE 2 DIMENSIONAL ARRAY
      DO 270 K1=1,KS
      HITE(1,K1)=HITE1(K1)
      HITE(2,K1)=HITE2(K1)
      HITE(3,K1)=HITE3(K1)
      HITE(4,K1)=HITE4(K1)
      HITE(5,K1)=HITE5(K1)
      HITE(6,K1)=HITE6(K1)
      HITE(7,K1)=HITE7(K1)
  270 CONTINUE
      DO 280 K3=1,30
      DO 280 K2=1,7
      PROF(K3,K2)=HITE(K2,25*K3)
  280 CONTINUE
C     PROF(K3,K2)=PROFILE(NUMBER,POSITION)
      CALL GRIDIV(1,0.0,DIST(7),0.0,2.0,1.0,0.05,5,5,5,5,2,4)
      DO 285 K4=1,10
      NS(K4)=K4-1
  285 CONTINUE
      DO 288 K4=11,19
      NS(K4)=K4+6
  288 CONTINUE
      DO 290 K4=20,28
      NS(K4)=K4+13
  290 CONTINUE
      DO 292 K4=29,30
      NS(K4)=K4+21
  292 CONTINUE
      DO 295 K5=1,30
      DO 295 K6=1,7
      CALL POINTV(DIST(K6),PROF(K5,K6),NS(K5))
  295 CONTINUE
      CALL PRINTV(-11,11HPOSITION-CM,600,10)
      CALL APRNTV(0,-14,-9,9HHEIGHT-CM,10,700)
C     NEXT PLOT HTILDA VS.R (SELF-SIMILAR HEIGHT SOLUTION)
C     LO=STARTING LENGTH          HO=STARTING HEIGHT
C     SIGMA=NET SPREADING COEFFICIENT              ROE=WATER DENSITY
C     NU=WATER VISCOSITY
```

```
      READ(5,300) LO,HO,SIGMA,RCE,NU
  300 FORMAT(5F10.0)
      WRITE(6,305) LO
      WRITE(6,306) HO
      WRITE(6,307) SIGMA
  305 FORMAT(10X,'STARTING LENGTH =',F6.1)
  306 FORMAT(10X,'STARTING HEIGHT =',F6.1)
  307 FORMAT(10X,'SPREADING COEFFICIENT =',F6.1)
      CONST=((SIGMA**2.0)/(NU*RCE**2.0))**.25
      DO 310 K7=1,7
      DO 310 K8=2,KS
      R(K7,K8)=DIST(K7)*(1.0/CONST)*(1.0/T(K8)**.75)
  310 CONTINUE
      DO 320 K7=1,7
      DO 320 K8=2,KS
      HTILDA(K7,K8)=HITE(K7,K8)*DIST(K7)/(R(K7,K8)*LO*HO)
  320 CONTINUE
      CALL SMXYV(0,1)
      CALL GRID1V(1,0.1,1.9,0.1,100.0,0.1,1.0,9,1,1,1,3,3)
      DO 330 K7=1,7
      DO 330 K8=2,KS
      CALL POINTV(R(K7,K8),HTILDA(K7,K8),NS(K7+1))
  330 CONTINUE
      CALL PRINTV(-1,1HR,500,10)
      CALL APRNTV(0,-14,-6,6HHTILDA,10,600)
      CALL PLTND(N)
      CALL EXIT
      STOP
      END
```