THE SOLUTION OF VACUUM TUBE CIRCUIT PROBLEMS BY MEANS OF THE DIFFERENTIAL ANALYZER

by

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A c k n o w l e d g m e n t

The author wishes to express his appreciation of the continued and kindly interest of Professor E. L. Bowles, under whose supervision the investigation which is the subject of this thesis was made. The author is indebted to Mr. F. M. Gager, who suggested the study, and to Mr. S. H. Caldwell and his assistants for their indispensable aid in setting up and operating the differential analyzer.

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I. INTRODUCTION

1. The Formal Solution Of Vacuum Tube Circuit Problems:- In general, neglecting the effects of contact voltages and variations in filament emission, the operation of a three-electrode thermionic vacuum tube is completely determined by the two fundamental relations:

$$(1) \qquad i_p = f(e_G, e_p)\ ^*$$

$$(2) \qquad i_G = f'(e_G, e_p)$$

which represent the static characteristic surfaces of the tube. The first of these relations expresses the plate current as a function of grid and plate potentials. The second relation gives the grid current as some other function of the grid and plate potentials.

Theoretically, any circuit involving a three-electrode vacuum tube can be completely and rigorously solved using the above fundamental relationships. Unfortunately, however, the characteristic surfaces of triodes are usually quite complicated affairs and they can be analytically represented only by means of infinite series of one form or another. While solutions based on series representations of the characteristic surfaces of a tube may be valuable for obtaining certain types of information, in general they are extremely cumbersome to handle and do not give a clear picture of the operation of the circuit.

*See Appendix A for list of symbols.

The series expansion representation of the characteristics of a vacuum tube is really of practical value only in the relatively simple case of an amplifier having a purely resistive load. In this case the dynamic plate-current characteristic of the tube and circuit can be either experimentally determined or it can be calculated from the static characteristic curves. The portion of this characteristic to which the operation of the tube is confined can then be represented by some convenient form of series expansion, and a rigorous analytical treatment of the behavior of the tube and its external circuit can be made. In the more general case where the external plate impedance is complex, the dynamic characteristic cannot be readily predetermined and rigorous treatment of the problem is no longer practicable.

In the case of an oscillating vacuum tube circuit the dynamic characteristics are usually quite complicated and extend over very considerable portions of the characteristic surfaces. Since they cannot be predetermined, a rigorous analytical treatment of the problem would require complete representation of the characteristic surfaces by series expansions. The problem thus becomes so involved that rigorous analytical treatment is practically impossible.

Because of the tremendous difficulty of rigorously treating vacuum tube circuit problems it has become customary, for the purpose of elementary analysis, to assume that the various parameters of the tube (as defined in terms of the static characteristic curves) are quite constant. The effects of grid currents are frequently neglected and the tube is often replaced by an "equivalent" circuit comprising a fictitious generator having an internal voltage (μe_G) and an internal resistance (R_p), where (μ) is the amplification constant of the tube; (R_p) is the differential plate resistance, and (e_G) is the grid potential. The use of these simplifying assumptions frequently leads to results which are very far indeed from representing the circuit conditions which actually exist. The only justification for the use of such assumptions is that they do enable an analytical approach to problems which it would otherwise be practically impossible to solve.

2. The Machine Solution Of Vacuum Tube Circuit Problems: - It has been indicated that the rigorous solution of a vacuum tube circuit problem is logically based upon the static characteristics of the tube. These characteristics are obtainable only from experimental data and are best represented in the form

of plotted curves. This being the case, the logical method of attacking a vacuum tube circuit problem is, perhaps, by some graphical or mechanical process whereby a solution can be obtained by working directly from the plotted curves. Such a method of solution is possible by means of the M.I.T. differential analyzer, which is a machine for solving ordinary differential equations.

By means of the differential analyzer it is possible to work directly from the static characteristic curves of a vacuum tube and to obtain, in the form of a plotted curve, a solution of a particular circuit problem for a specified set of boundary conditions. Although a solution obtained in this manner is devoid of generality it is nevertheless of great interest and value if for no other reason because of the clear picture it gives of the behavior of the vacuum tube and its external circuit.

3. The Object Of This Study: - Briefly stated, it is the object of this study to generally investigate the practicability of solving thermionic vacuum tube circuit problems by means of the differential analyzer and to attempt to obtain, by means of the machine, rigorous solutions of the following two typical types of three-electrode vacuum tube circuits:

(1) a "tuned-plate" oscillator circuit

(2) a one-stage amplifier circuit having a tuned parallel circuit as an external plate impedance.

The steady-state performance of the oscillator for various circuit conditions will be studied. Both the transient and steady-state performance of the amplifier circuit, for various operating conditions, will be investigated. It is hoped, by means of the results obtained from these investigations, to present a clearer picture of the performance of a three-electrode vacuum tube than has heretofore been possible.

4. The General Plan of The Paper:- The body of this paper will be divided into four chapters. The first of these, which will immediately follow this introductory section, will comprise a brief description of the differential analyzer and a short discussion of its use and limitations. The next section, which is to constitute the third chapter of this thesis, will be devoted to a general discussion of the types of vacuum tube circuit problems which can be solved on the differential analyzer. The reasons therefor and some of the more general aspects of the problem will be treated. The fourth chapter will consist of a complete discussion of the tuned-plate oscillator problem. The steps in the machine solution will be recorded; the

difficulties encountered and the methods of overcoming them will be described. The results obtained from the machine will be discussed, interpreted, and compared with experimental data. The fifth chapter will comprise a similar treatment of the amplifier problem. In the final chapter a general discussion of the results of the study and their significance will be given and suggestions for further investigations will be made.

* * *

I I. THE DIFFERENTIAL ANALYZER

1. General Remarks :- The differential analyzer is a machine for solving ordinary differential equations. In its present form, the machine is capable of solving equations of any order up to the sixth and with any reasonable degree of complexity. As many as three simultaneous second order equations can be handled. The coefficients of the various terms may be either constant or variable. The machine provides solutions in the form of plotted curves for specified boundary conditions.

The differential analyzer is the outgrowth of a series of developments which have been carried on during the past five or six years in the Department of Electrical Engineering of the Massachusetts Institute of Technology. The development, the mechanical details, and the use of the differential analyzer have been very fully discussed in an article by Dr. V. Bush entitled: "The Differential Analyzer. A New Machine For Solving Differential Equations." *. This article was published in the "Journal of the Franklin Institute" for October, 1931, and the reader is referred to it for a more complete account of the machine than it is feasible to present here.

* The material in this section has been drawn largely from this article. See bibliography for additional references.

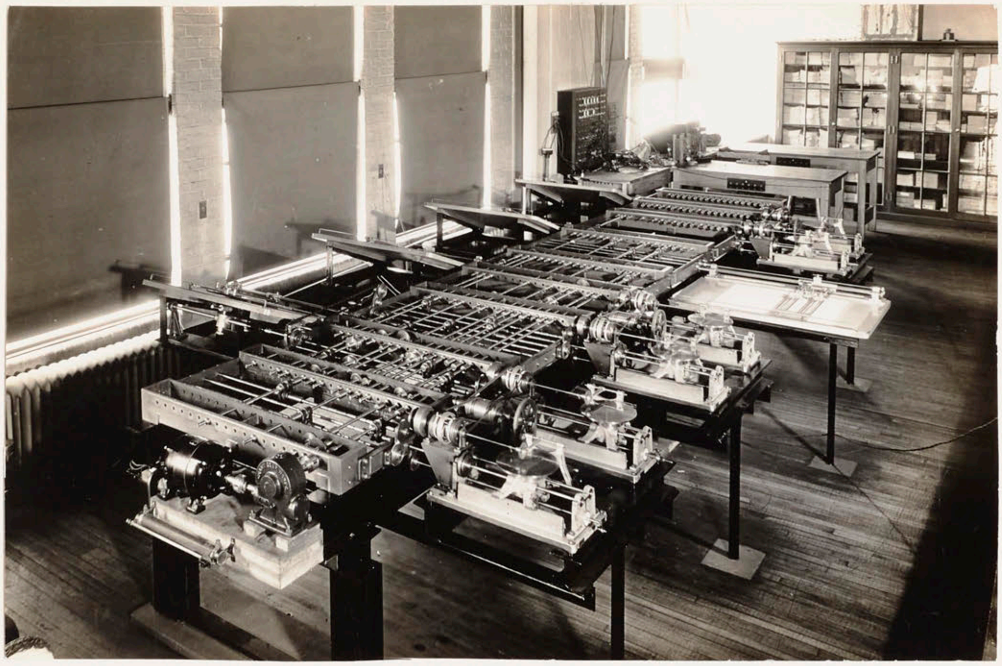

Fig. 1. The Differential Analyzer

2. A Brief Description Of The Machine:- A photograph of the differential analyzer is shown in Fig. 1. A diagram of the layout of the machine appears in Fig. 19. (see page 48.) As is apparent from these illustrations, the differential analyzer consists essentially of a very flexible system of longitudinal and transverse bus shafts; four "input tables"; an "output table"; and six Kelvin disc-and-wheel mechanical integrators. Each of the integrators is directly coupled to a two-stage mechanical torque amplifier, the purpose of which is to amplify the extremely small torque provided by the integrator wheel to such an extent that it is ample for driving the shafting and other parts of the machine in

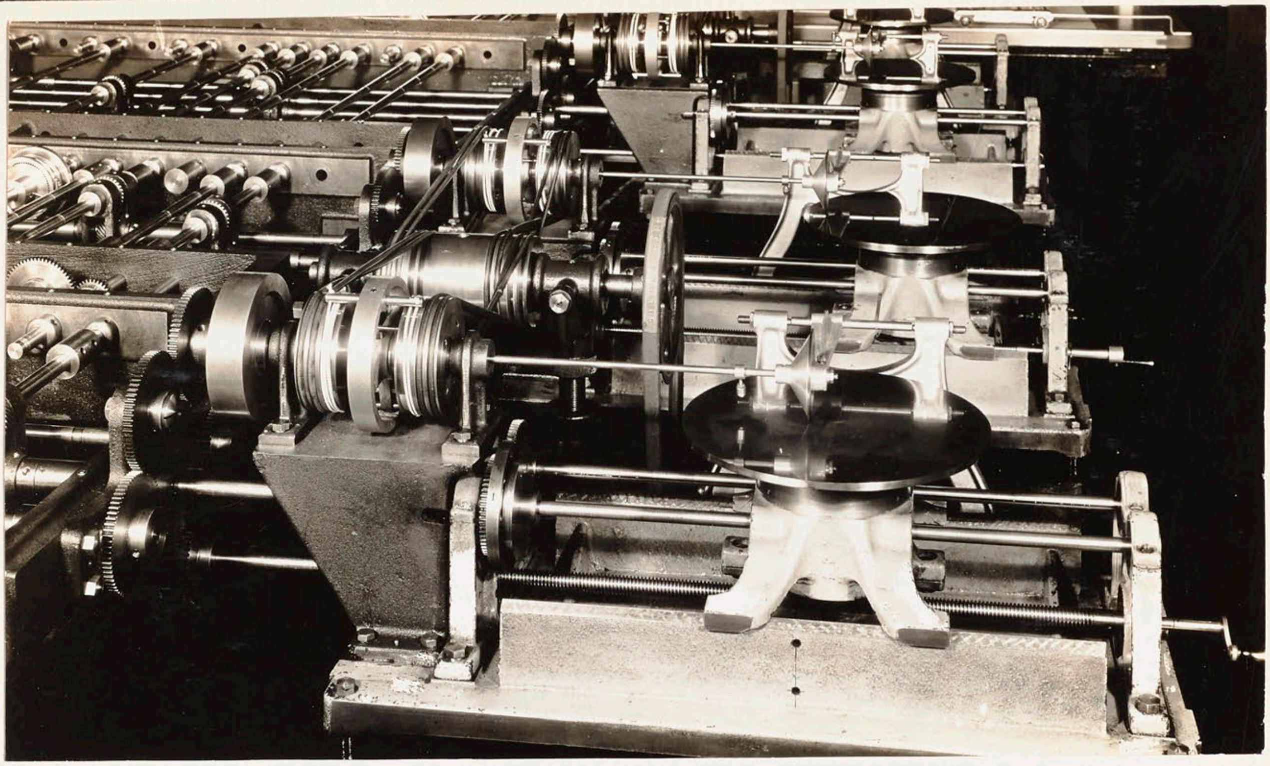

Fig. 2. An Integrator

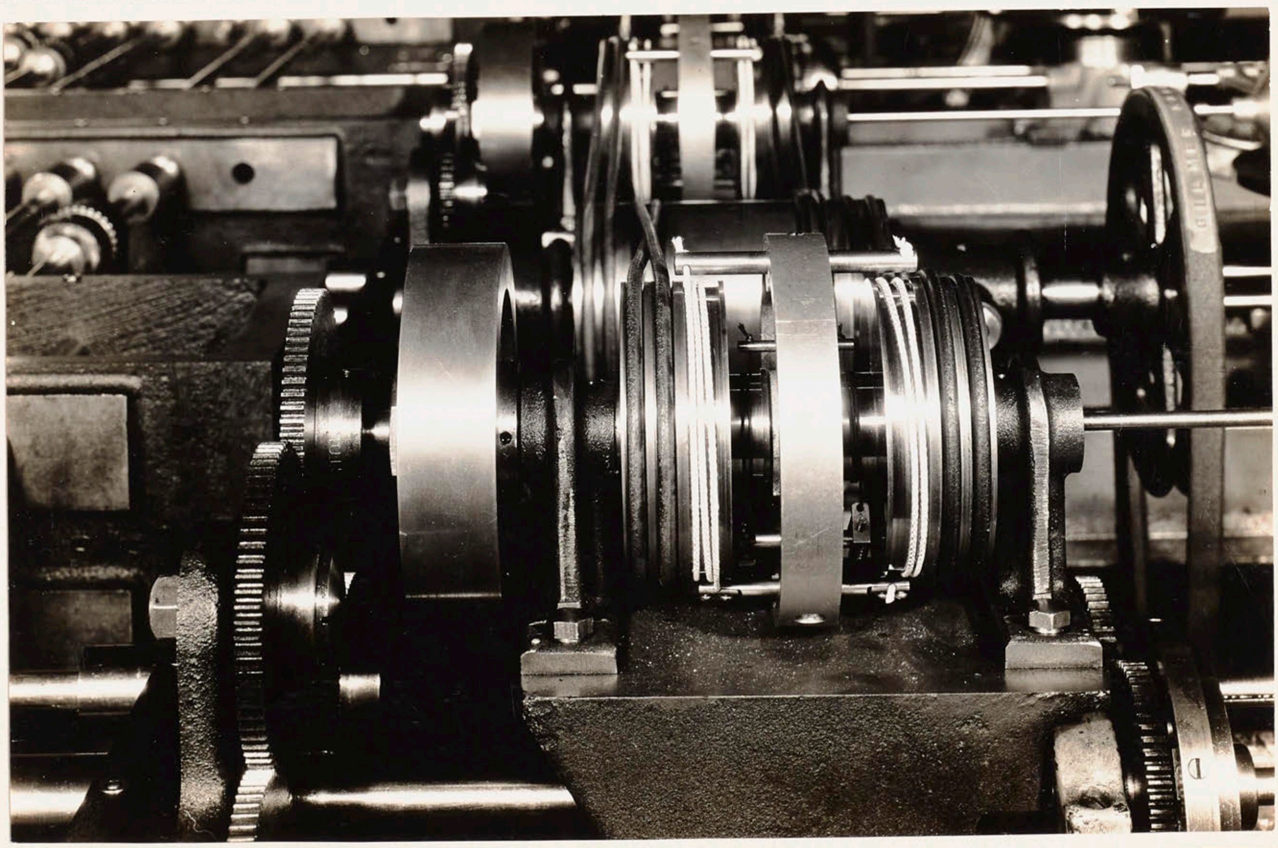

Fig. 3. A Torque Amplifier

whatever manner may be necessary for the solution of the particular problem.

The general arrangement of an integrator is apparent from Fig. 2. Briefly, the device may be considered as a unit having three shafts, the angular displacements of which are (u), (v), and (w); so connected that at every instant:

$$u = \frac{1}{32}\int w \, dv$$

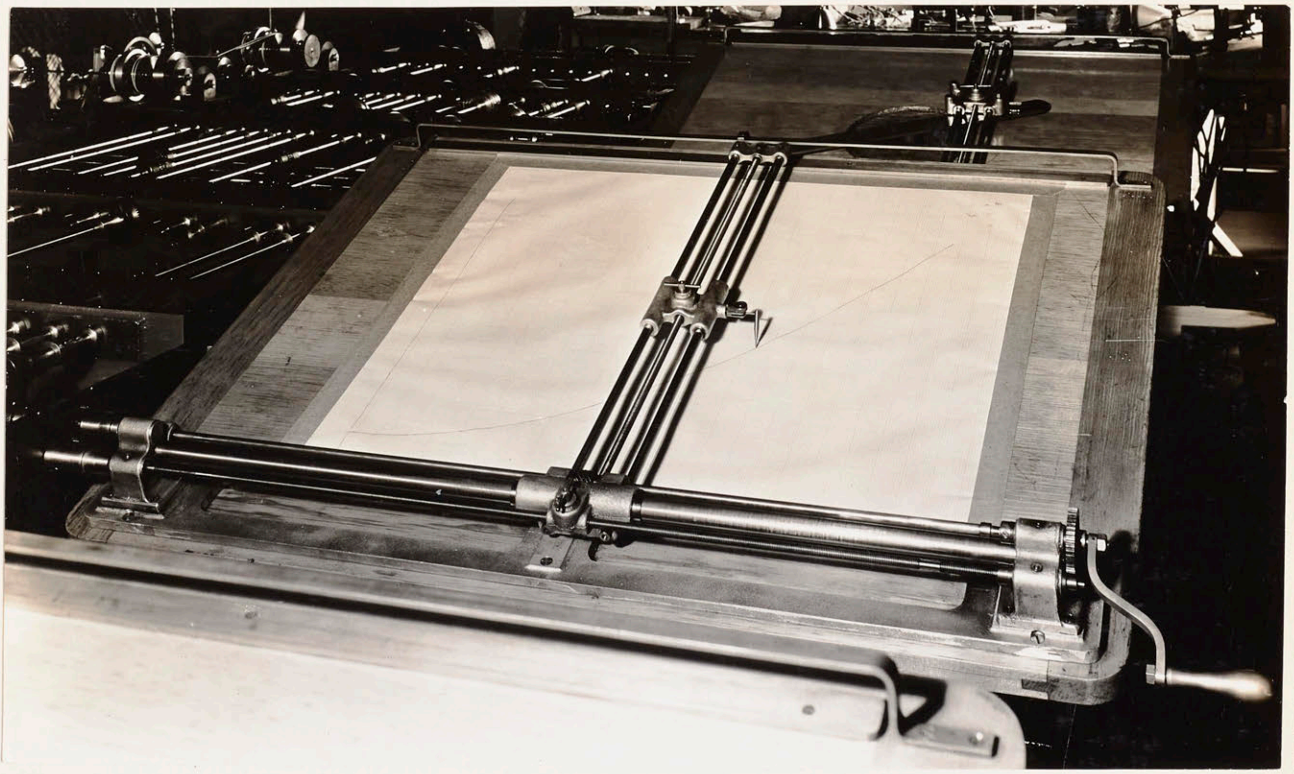

Fig. 4. An Input Table

An input table, Fig. 4., has two shafts, one of which displaces a pointer horizontally in the direction of abscissas, and the other vertically in the direction of ordinates. The shaft producing the horizontal displacement is driven by the machine while the other one is controlled manually by means of a crank. Thus if the rotations of the shafts causing vertical and horizontal displacements be denoted respectively by "p" and "q", it is seen that the input table can be used to feed into the machine any function :

$$p = f(q)$$

by simply plotting the function of the input table and manually controlling the vertical displacement of the pointer so as to always keep it on the curve.

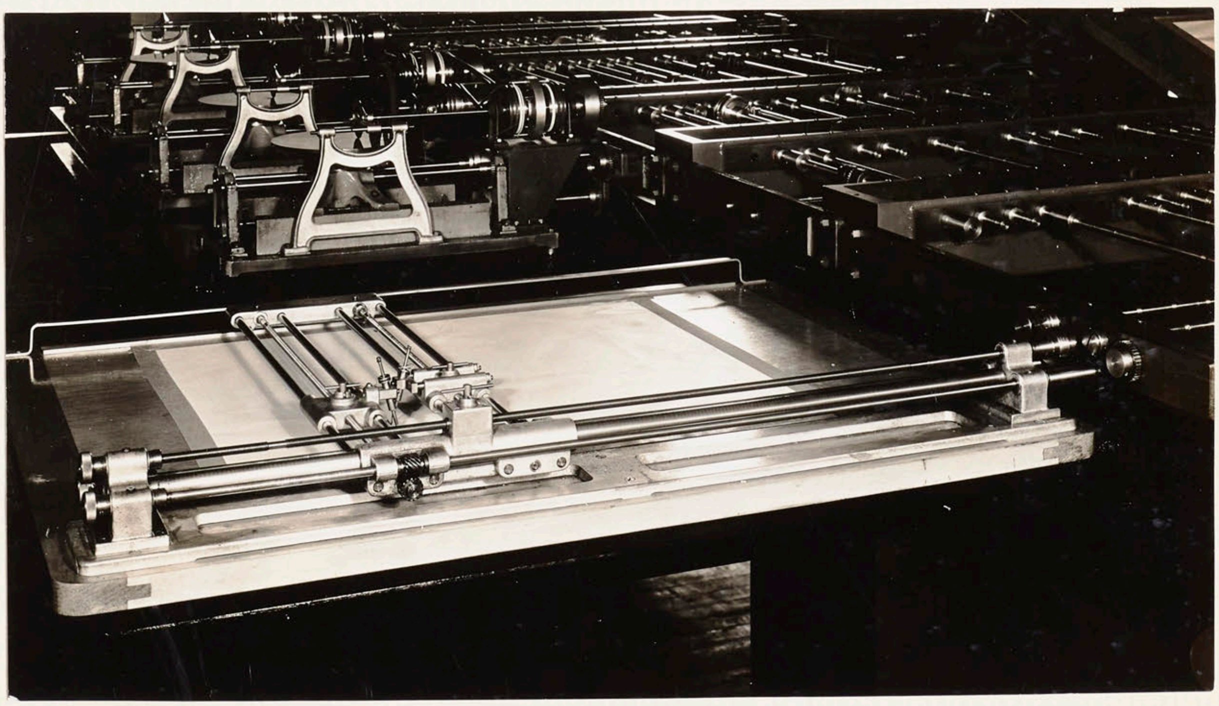

Fig. 5. The Output Table

The output table, Fig. 5., has three shafts, one of which displaces a carriage horizontally and the other two of which impart vertical displacements to recording pencils mounted on the carriage. Two quantities can thus be simultaneously recorded as a function of a chosen variable.

By means of right and left handed spiral gear boxes, longitudinal and transverse shafts can be interconnected as necessary. Sets of spur gears of simple ratios (1:1, 1:2, 1:4, and 2:3) are provided for the connection of adjacent longitudinal shafts. Differential gears or "adders" are provided for connecting three adjacent longitudinal shafts so that the total number of rotations of one will be the sum of the rotations of the other two. By means of "frontlash units", which can be inserted at convenient points, the errors introduced by backlash in geared drives can be exactly compensated for. The standard symbols for the various parts of the differential analyzer and their functions are indicated in Fig. 6.

Very generally, "the procedure of placing an equation on the machine is somewhat as follows: A bus shaft is assigned to each significant quantity appearing in the equation. The several relations existing between these are then set up by means of connections to the operating units: a functional relation by connecting the two corresponding shafts to an input table,

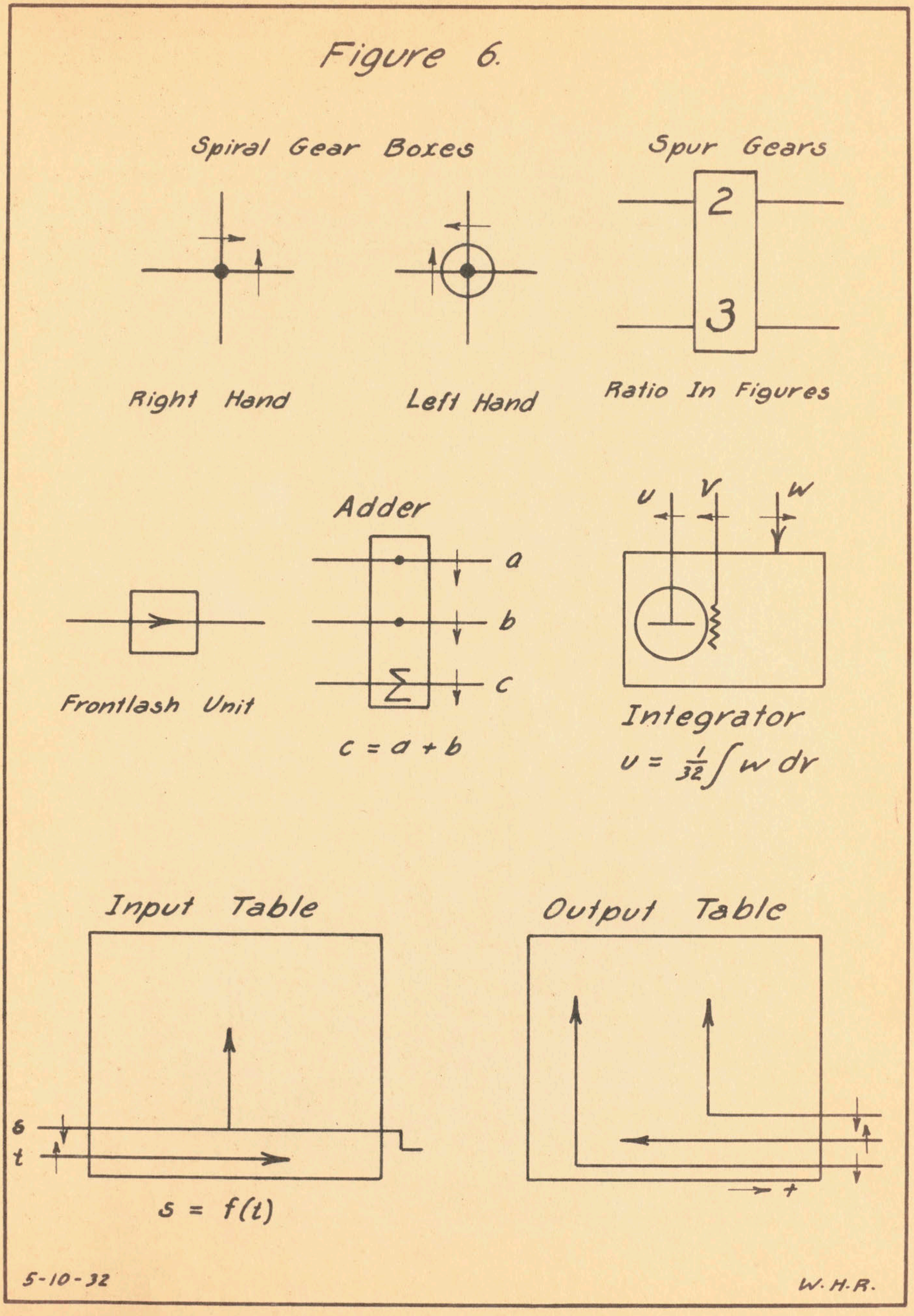
Spiral Gear Boxes
Right Hand
Left Hand
Spur Gears
2
3
Ratio In Figures
Frontlash Unit
Adder
a
b
c
Σ
c = a + b
u
v
w
Integrator
u = 1/32 ∫ w dv
Input Table
s
t
s = f(t)
Output Table
t
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Figure 6.

a sum by placing an adder in position, an integral relationship by an integrator, and so on. When all the relationships which are involved have been thus represented a final connection is made which represents the equality expressed in the equation. ....
When this has been done the machine is locked, and the rotation of the independent-variable shaft will drive everything else, thus forcing the machine to move in accordance with the expressed relationship of the equation." *

The very important question of selecting scales, limits, and gear ratios for the solution of a specific problem will be discussed and illustrated later in connection with the description of the solutions of the two problems which were studied in the course of this investigation.

* * *

* From page 459 of the article by Dr. V. Bush. Loc. cit.

III. THE TYPES OF VACUUM TUBE CIRCUIT PROBLEMS WHICH CAN BE SOLVED ON THE DIFFERENTIAL ANALYZER.

1. The General Method Of Attack:- As has already been pointed out, a rigorous treatment of a circuit involving a thermionic vacuum tube would logically be made in terms of the functions:

(1) $$i_p = f(e_G, e_p)$$

(2) $$i_G = f'(e_G, e_p)$$

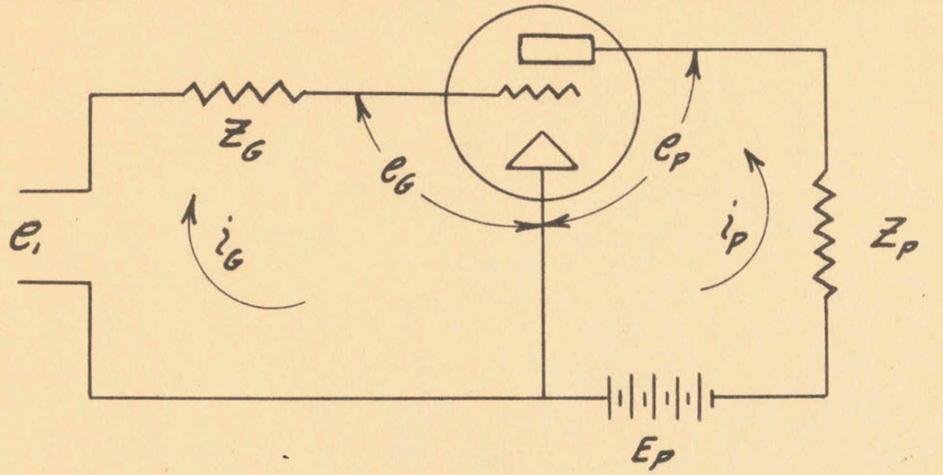


Figure 7.

which represent the static characteristic surfaces of the device. If the circuit containing the tube be represented in general form, as indicated in Fig. 7., the equations which completely specify its performance can be written as follows:

(1) $$i_p = f(e_G, e_p)$$

(2) $$i_G = f'(e_G, e_p)$$

(3) $$e_1 = Z_G(D)i_G + e_G$$

(4) $$E_p = Z_p(D)i_p + e_p$$

where e_1 represents a source of voltage in the external grid-filament circuit; where the Z's are general complex impedances and where D is the time differentiator. The solution of the circuit must be made from these equations. If the plate and grid current functions can be satisfactorily represented by input plots, and if all of the terms which enter into the voltage expressions due to the external impedances in the grid and plate circuits can be obtained, a perfectly rigorous solution of the problem can be effected by means of the M.I.T. differential analyzer.

The input tables of the differential analyzer have been designed to accomodate plotted curves expressing quantities as functions of one variable only. In order to represent the i_p, i_G functions of a vacuum tube it would be necessary to plot a very complete family of curves - each of which would express i_p or i_G, as the case may be, as a function of one of the independent variables for a constant value of the other independent variable. For example, families of curves expressing the currents as functions of the grid voltage e_G for various constant values of plate voltage e_p might be plotted. If a sufficient number of such curves were drawn, the functions could be quite accurately represented.

Assume now for the moment that the machine is set up for the solution of a specific triode circuit problem and that an operator is following an input plot which comprises a family of curves giving the plate current of the tube as a function of the grid voltage - each curve of the family being drawn for a different constant value of plate voltage. As the solution of the problem proceeds, both the grid and plate voltages of the tube change continuously. The change in grid voltage automatically causes the machine to displace the input table pointer in the horizontal direction. If now the operator of the input table knows the value of the plate voltage at every instant of time it is a relatively simple matter for him to manipulate the input table crank so as to always keep the index over that point on the input plot which corresponds to the particular instantaneous values of plate and grid voltages. If this is done, the input table feeds into the machine the variable quantity i_p as a function of e_G and e_p. If the pointer on the input table is replaced by a pencil, as the solution of the problem proceeds the dynamic characteristic of the tube and circuit will be traced out.

In order that the operator of the i_p input plot can know the value of the plate voltage (e_p) at any instant of time, the following expedient may be resorted to: Let one of the output pencils be arranged to record e_p as a function of time and let the record be traced out upon a piece of coordinate paper which has previously been provided with a scale from which values of e_p can be read directly with considerable accuracy - say to the nearest volt. Now let a person be stationed at the output table for the purpose of continuously reading off the values through which e_p undergoes and let that person be in constant communication with the operator of the input plot. The operator of the input plot can thereby be kept advised at every instant of the value of the plate voltage on the tube and he can control his crank accordingly. By having the values of e_p called out continuously, and by exercising a little care in following the plot, this method of obtaining i_p as a function of both e_G and e_p can be made to work surprisingly well. In order that communication between the man reading values of e_p from the output plot and the man operating the input plot be free of interference from room noises, it is advisable that a simple talking circuit comprising standard operators telephone sets with breast transmitters and head-band receivers be provided.

Thus the chief difficulty in the differential analyzer solution of triode circuit problems can be overcome. That is to say, it is practicable to arrange a standard input table to give either the plate or the grid current as a function of two variables; the plate voltage (e_p) and the grid voltage (e_G).

The second requirement to be met before a triode circuit can be solved on the machine is that all of the terms comprising the expression for the back-voltage due to the external circuit impedances be obtainable. That is, all derivatives must be obtainable by addition or subtraction of the various terms entering into the circuit equations. This is an extremely important requirement since the machine is unable to perform differentiations. Aside from the operations of addition, subtraction, multiplication, and division, the machine is capable only of integration. It follows from this fact that if, for example, the time rate of change of a function being supplied from an input table is absolutely required, the problem is incapable of solution. Thus if in a vacuum tube circuit problem it is necessary to have the time rate of change of either i_p or i_G, the problem cannot, in general, be solved by means of the machine. This is the chief criterion in determining whether or not a vacuum tube circuit problem can be solved by means of the differential analyzer.

It might seem, upon first consideration, that the necessity of obtaining a derivative of a quantity being fed to the machine from an input plot could be obviated by integrating or otherwise transforming the original voltage equations. In the cases considered in the course of this investigation, however, such was not found to be the case. Whenever the time rate of change of a plate or grid current entered into an equation, the problem was found to be incapable of machine solution.

Throughout this paper it has been tacitly assumed that the static characteristic curves of a vacuum tube completely and exactly determine its behavior under dynamic conditions. This assumption is practically necessary no matter how the vacuum tube circuit problem be attacked. If the action of the tube is purely thermionic - as it is substantially in modern high vacuum tubes - and if the operating frequency is low, this assumption is entirely justifiable. When the operating frequency is so high that the interelectrode capacities of the tube are comparable in magnitude to those in the external circuit, the static and dynamic characteristics of the tube proper will no longer coincide. In this case it is necessary to combine the internal tube capacities with the impedances of the external circuit before a solution of the problem can be made from the static characteristic curves.

The best available method of obtaining the static characteristic curves of a tube seems to be the "point-by-point" method. If in taking data for the characteristic curves of a tube by this method care is exercised to prevent excessive heating of the tube parts, quite representative curves can no doubt be obtained. When a very complete set of characteristic curves is needed, and the current values corresponding to extremely high grid and plate voltages are required, special precautions must be taken. Meters which respond very quickly to changes in applied currents or voltages should be used and, when making the measurements, the plate and grid voltages should be impressed upon the tube absolutely no longer than is necessary to obtain readings. There should also be an appreciable time interval between successive readings to prevent the average operating temperature of the tube from becoming too high.

It is possible that an oscillograph might be arranged to give a horizontal axis proportional to the grid voltage applied to a tube. The plate current could then be passed through a vibrator and a trace of i_p as a function of e_G , for a constant value of e_p , could then be obtained. The chief objection to such an arrangement is, perhaps, that the curves obtained would necessarily be drawn to a very small scale. The method might nevertheless be very effectively used to check the general shape of curves obtained by the point-by-point method. Such a check would be especially desirable in the regions of low plate voltages and extremely high grid voltages where saturation effects of one sort or another become manifest.

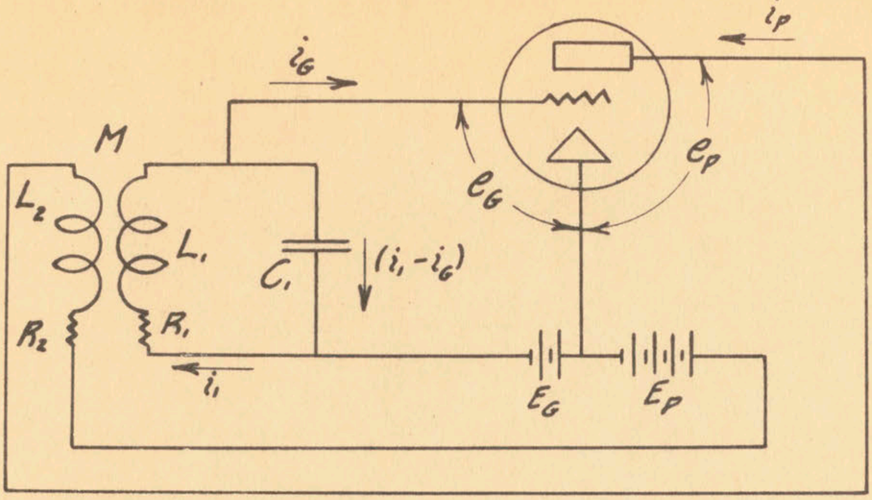
2. Circuits Involving Triodes: The types of three-electrode thermionic vacuum tube circuits which were considered in the course of this investigation, to discover possibilities of solving them by means of the differential analyzer, will now be briefly discussed.

The oscillator circuits considered were the "tuned-grid", "tuned-plate", "Hartley", and "Colpitts". These circuits, in elementary form, and the differential equations which describe their behavior are given in Figs. 8 to 11, inclusive.

As indicated in Fig.8 the equations for the tuned-grid circuit involve terms containing the time rate of change of plate current (i_p). For this reason, as previously explained, a rigorous solution of the circuit cannot be made upon the differential analyzer. In order to solve the problem by means of the machine it would be necessary to neglect the plate current and that, of course, would be an absurd thing to do.

Similarly, the equations describing the performance of the tuned-plate oscillator (Fig.9) include terms

Fig. 8. Tuned Grid Cct.

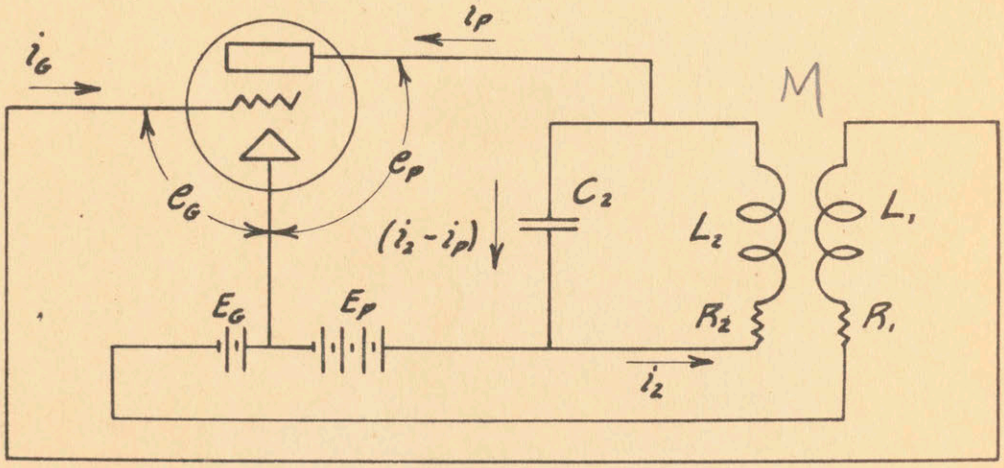


$$E_G = e_G - \frac{1}{C_1}\int (i_1 - i_G)\,dt$$

$$0 = R_1 i_1 + L_1 \frac{di_1}{dt} \pm M \frac{di_p}{dt} + \frac{1}{C_1}\int (i_1 - i_G)\,dt$$

$$E_p = R_2 i_p + L_2 \frac{di_p}{dt} \pm M \frac{di_1}{dt} + e_p$$

Fig. 9. Tuned Plate Cct.



$$E_G = R_1 i_G + L_1 \frac{di_G}{dt} \pm M \frac{di_2}{dt} + e_G$$

$$0 = R_2 i_2 + L_2 \frac{di_2}{dt} \pm M \frac{di_G}{dt} + \frac{1}{C_2}\int (i_2 - i_p)\,dt$$

$$E_p = e_p - \frac{1}{C_2}\int (i_2 - i_p)\,dt$$

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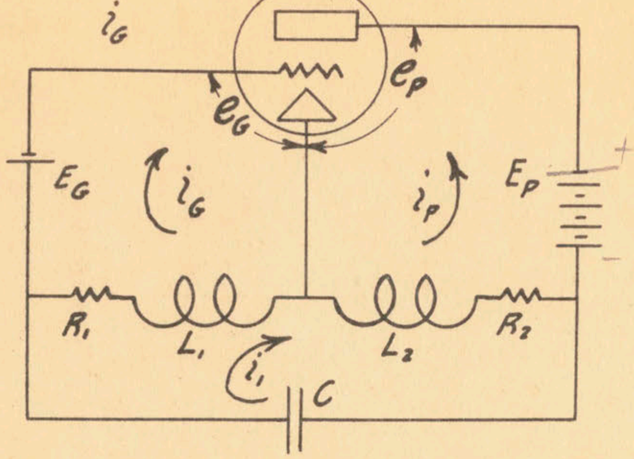
Fig. 10. Hartley Cct. (No Mutual Ind.)

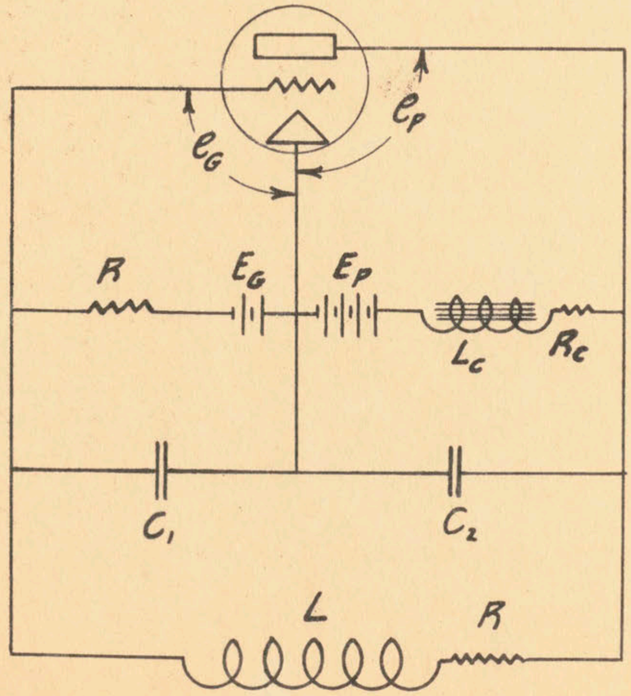


$$E_P = R_2(i_1 + i_P) + L_2 \frac{d}{dt}(i_1 + i_P) + e_P$$

$$E_G = R_1(i_G - i_1) + L_1 \frac{d}{dt}(i_G - i_1) + e_G$$

$$0 = R_1(i_1 - i_G) + L_1 \frac{d}{dt}(i_1 - i_G) + R_2(i_1 + i_P) + L_2 \frac{d}{dt}(i_1 + i_P) + \frac{1}{C}\int i_1\, dt$$

Fig. 11. Colpitts Circuit



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containing the rate of change of grid current (i_G) with respect to time and a rigorous machine solution of the problem cannot, therefore, be obtained. If the effects of the grid current be neglected, however, the problem lends itself very nicely to machine solution.

In the case of the Hartley oscillator circuit (Fig. 10) the voltage equations involve the derivatives of both i_G and i_p with respect to time, and rigorous treatment is out of the question. Moreover, when the coils L_1 and L_2 are considered to have mutual inductance the equations contain too many terms to be solvable on the present differential analyzer.

The equations of the Colpitts oscillator circuit (Fig. 11) involve the time rate of change of plate current. Moreover, the circuit has five meshes and the differential equations which determine its performance therefore involve a considerable number of terms. Rigorous solution of the Colpitts oscillator circuit on the present differential analyzer is quite impracticable.

Thus it appears that none of the commoner triode oscillator circuits can be solved rigorously by means of the differential analyzer. At the outset of this investigation it was hoped to obtain rigorous solutions of at least one of the types of circuits just considered.

The possibilities of obtaining machine solutions of several elementary forms of amplifier circuits were also investigated. The circuits considered are

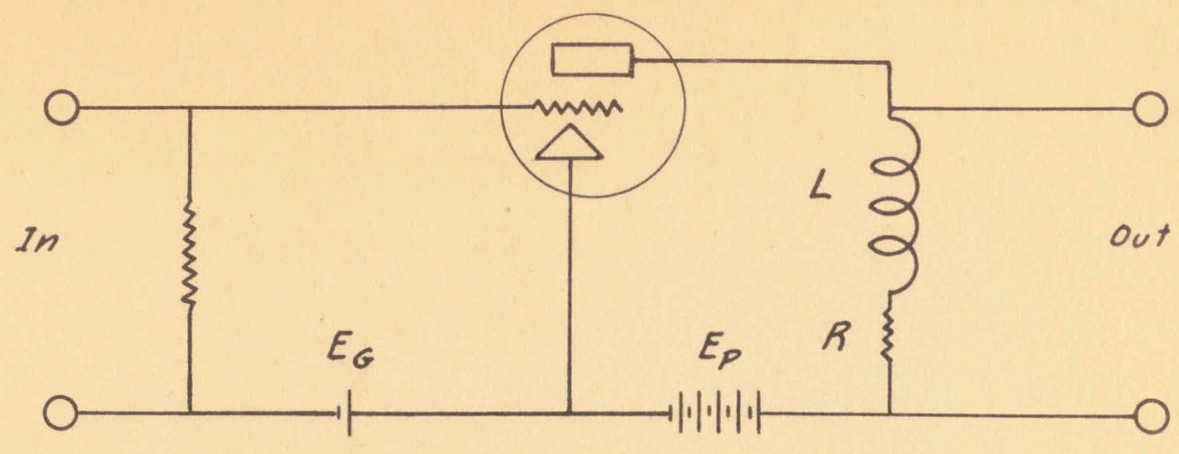


Figure 12.

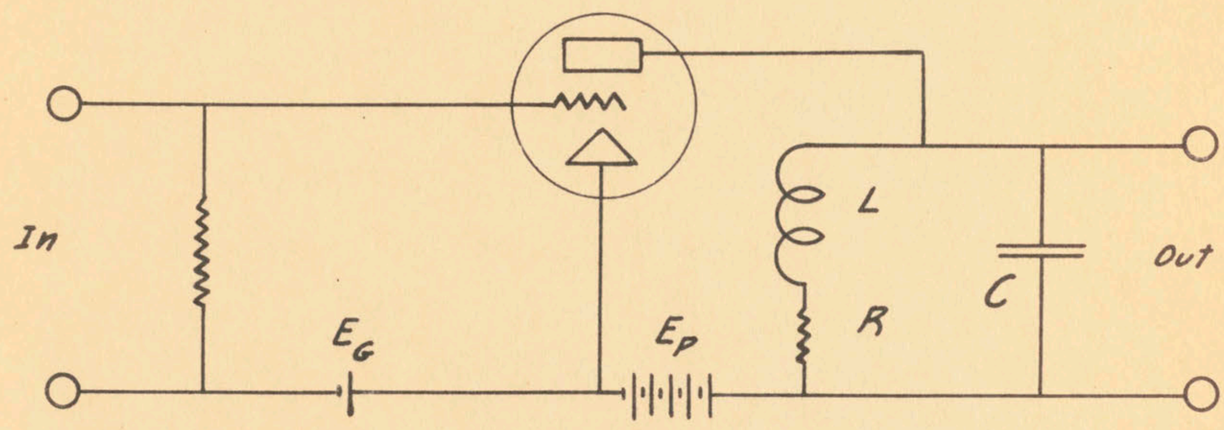


Figure 13.

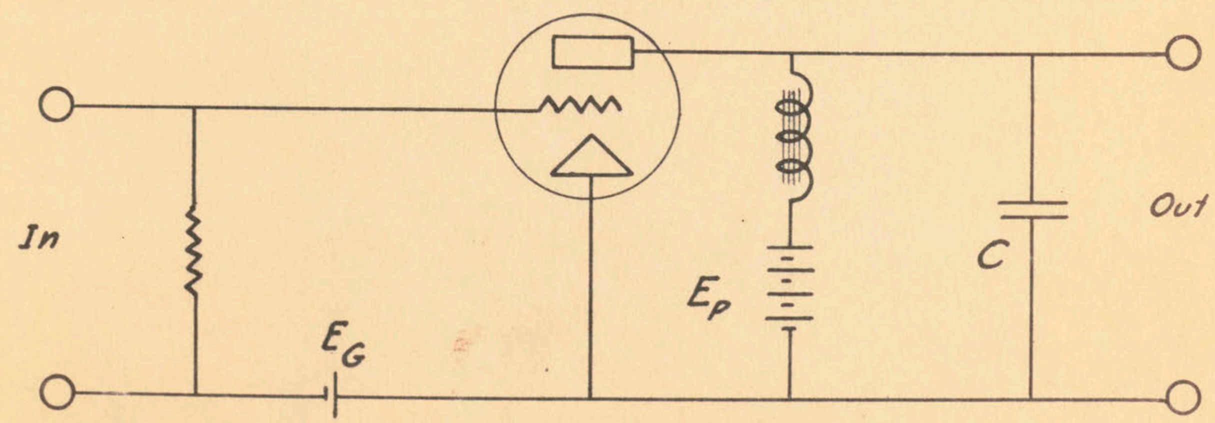


Figure 14.

schematically represented by Figs. 12 to 14. The circuit having simply a coil and a resistance in series for a load impedance cannot be solved for the same reason that plate current can't be taken into account in a tuned-grid oscillator - the rate of change of i_p with respect to time is required. It is unfortunate that this circuit cannot be handled since by making the resistance (R) large and the inductance (L) small the condition of a loaded amplifier having an output transformer with appreciable leakage reactance could be simulated.
A study of the steady-state and transient behavior of such a circuit would be both valuable and interesting.

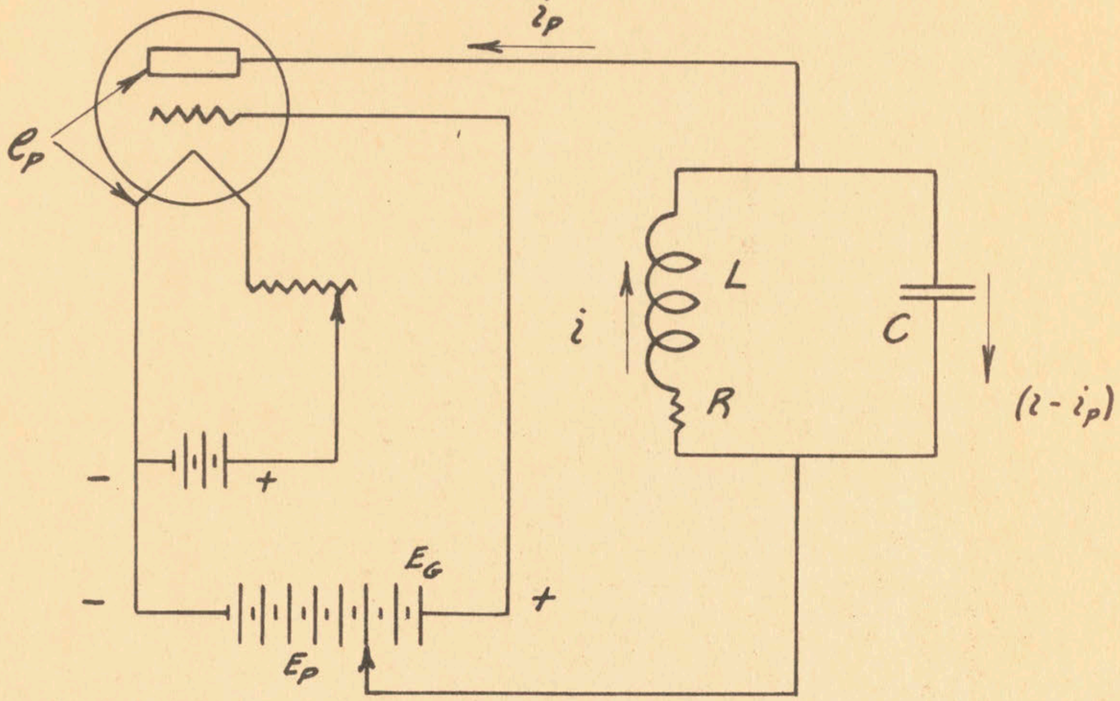
Fig. 13 represents a one stage amplifier having a tuned parallel circuit as a load impedance. This circuit is closely analagous to the tuned plate oscillator circuit (Fig. 9). The essential and important difference between the two being that the grid excitation voltage for the amplifier is taken from an external source and that by suitably choosing the grid biasing battery (E_G), grid current can be eliminated - practically speaking. Since the equations specifying the behavior of the circuit do not involve derivatives of i_p , a perfectly rigorous solution can be made by means of the differential analyzer. The circuit represented in Fig. 14 can also be rigorously treated since it is merely a special case of the one just discussed.

The possibility of obtaining differential analyzer solutions of various types of detector and modulator circuits involving triodes has also been briefly considered. In general the equations of such circuits invariably involve derivatives of either plate or grid currents, or both, and machine solutions therefore cannot be obtained.

3. The Dynatron Oscillator:- The dynatron oscillator is an important and interesting example of a vacuum tube circuit which is admirably adapted to differential analyzer solution. The circuit diagram of a typical dynatron oscillator and its differential equations are shown in Fig. 15. In the case of the dynatron oscillator there is no external impedance in the grid circuit which means that the grid potential is constant and quite independent of the grid current. (It is assumed that the grid battery has negligible internal impedance.) When the circuit is oscillating, the operating point of the tube therefore follows a single $i_p - e_p$ curve (such as the one represented at the bottom of Fig. 15) and the dynamic and static characteristics coincide. As far as the machine is concerned, the plate current of the tube is thus a function of one variable only (e_p) and one of the chief difficulties in the solution of conventional triode

Fig. 15. The Dynatron Oscillator

Cct. Diagram :-

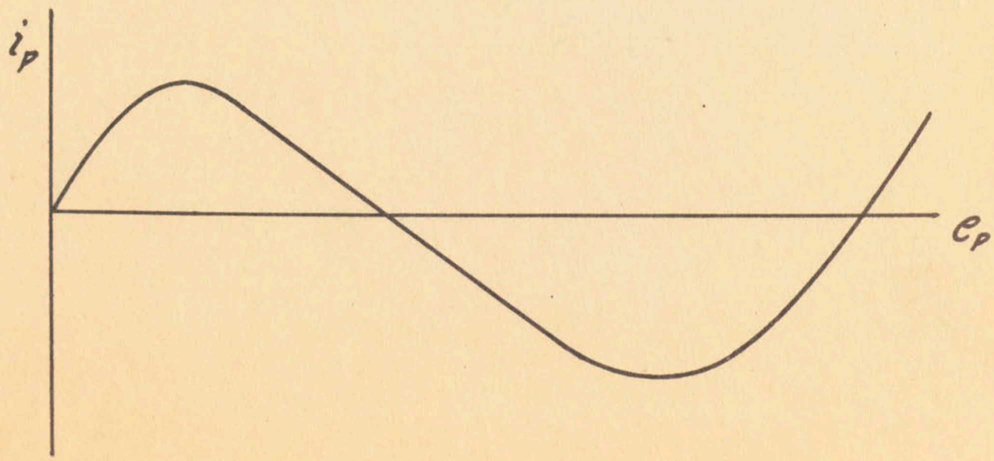


Cct. Equations :-

$$E_p = e_p - \frac{1}{C}\int (i - i_p)\, dt$$

$$0 = Ri + L\frac{di}{dt} + \frac{1}{C}\int (i - i_p)\, dt$$

Typical Dynatron Characteristic :-



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circuit problems is eliminated. For further information on the solution of a dynatron oscillator circuit by means of the differential analyzer, the reader is referred to a thesis by J.E. McGraw, entitled "Experimental and Integraph Study of the Dynatron Oscillator Operating at Low Frequencies". *

4. Summary:- From the foregoing considerations it is apparent that there are relatively few types of vacuum tube circuit problems which can be solved by means of the present differential analyzer. Of these, only the amplifier with a tuned parallel circuit for a load impedance and the dynatron oscillator can be rigorously treated. For the purpose of this thesis investigation it was decided to attempt machine solutions of a tuned-plate oscillator circuit (in which the grid current would be neglected) and of an amplifier having a tuned parallel circuit for a load impedance. The oscillator problem was selected largely because of its interest and relative simplicity. The amplifier problem was chosen as being a particularly interesting example of a vacuum tube circuit which can be rigorously solved by means of the differential analyzer.

* * * *

* M.S. thesis, course VI. Submitted June, 1932.

IV. THE TUNED-PLATE OSCILLATOR PROBLEM :-

1. Preliminary Experimental Work:- The decision to study the performance of a tuned plate oscillator circuit having been made, the next logical steps were the selection of a suitable vacuum tube and the choice of a working frequency. It was decided that the tube should be a Radiotron UX 112-A, this selection being made because the characteristics of that tube generally render it fit for use in an oscillator of small power output and also because of its moderate battery requirements. The manufacturers' data on the tube are given in Table I.

Table I

DATA ON RADIOTRON UX 112-A

Intended Use: Power Amplifier

Filament Volts: 5.00
Filament Amps. 0.25

E_p	E_G	mills I_p	ohms R_p	k	m.m. G_m	milli watts Output
135	- 9	7.0	5000	8.0	1600	120
157.5	-10.5	9.5	4700	8.0	1700	195

It was decided that the operating frequency of the oscillator should be low in order that the internal capacities of the tube would be quite negligible. An operating frequency of about sixty cycles per second was thought to be satisfactory. Two large air-cored coils, provided with taps, were procured and arranged so that their mutual inductance could be varied at will by simply altering the distance between them. Suitable condensers were provided and the circuit shown in Fig. 16 (page 34) was arranged. With the aid of a small portable oscillograph a hasty study was made to determine the probable maximum values of plate and coil currents to be encountered in covering the range of operating conditions which might be of interest. As a result of this preliminary study it was decided that the differential analyzer should be set up to accomodate approximately the ranges of values of circuit parameters and variables indicated in Table II. (See next page.)

Using the point-by-point method a very complete set of static characteristic curves for a UX 112-A tube was next obtained. Enough data were taken to plot curves covering the ranges of current and voltage indicated in Table II. A photostat of the characteristic curves obtained appears in Fig. 24, page 60. The experimental data from which the curves were plotted are tabulated in appendix B. Having obtained the static curves of the tube and the necessary information on the range of parameters and variables to be covered in the differential analyzer solution of the problem, the next step was the determination of a suitable machine set-up.

TABLE II

QUANTITY	RANGE OF VALUES TO BE COVERED
E_p	Constant at 140 volts
E_G	Constant at -10 volts
L	0.20 to 1.50 henrys
C	35.00 to 5.00 microfarads
M	0.00 to 0.70 henry
R	0.00 to 150 ohms
i_p	0.00 to 0.160 ampere
i_2	0.00 to 0.350 ampere
(i_2-i_p)	0.00 to 0.350 ampere
$\frac{di_2}{dt}$	- 175 to 175 amps./sec.
e_c	- 200 to 200 volts
e_p	0.00 to 340 volts
e_G	- 80 to 80 volts

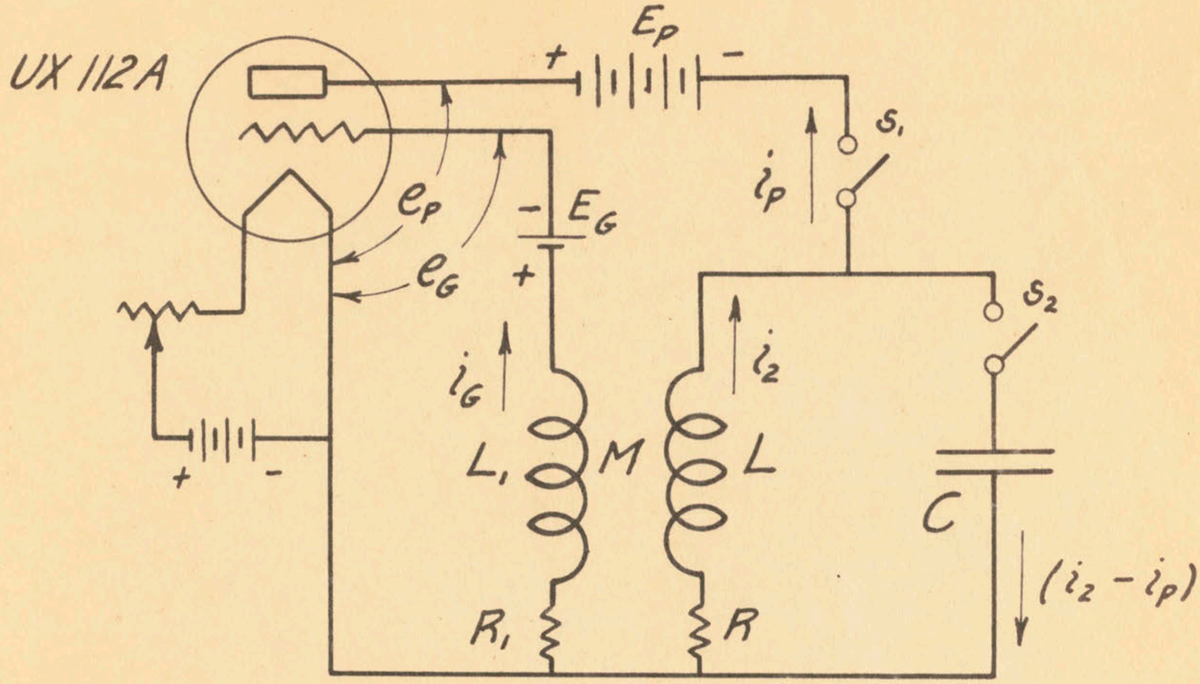
2. Preliminary Steps in the Machine Set-Up:-

The tuned-plate oscillator circuit used and the differential equations which determine its behavior are shown in Fig. 16. From the set of differential equations at the bottom of Fig. 16 the preliminary machine connection diagram on page 35 was prepared.

It will be observed that on the preliminary connection diagram the coil inductance has been assigned a value of one henry and no special provisions have been

Oscillator Problem

Circuit Diagram And Equations



If i_G is neglected the cct. equations are:

(1) $$Ri_2 + L\frac{di_2}{dt} + \frac{1}{C}\int(i_2 - i_p)\,dt = 0$$

(2) $$e_p + L\frac{di_2}{dt} + Ri_2 = E_p$$

(3) $$e_p - \frac{1}{C}\int(i_2 - i_p)\,dt = E_p$$

(4) $$e_G \pm M\frac{di_2}{dt} = -E_G$$

For the machine the equations are written:

(1) $$L\frac{di_2}{dt} = -Ri_2 - \frac{1}{C}\int(i_2 - i_p)\,dt$$

(3) $$e_p = E_p + \frac{1}{C}\int(i_2 - i_p)\,dt$$

(4) $$e_G = -E_G + M\frac{di_2}{dt}$$

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Fig. 16

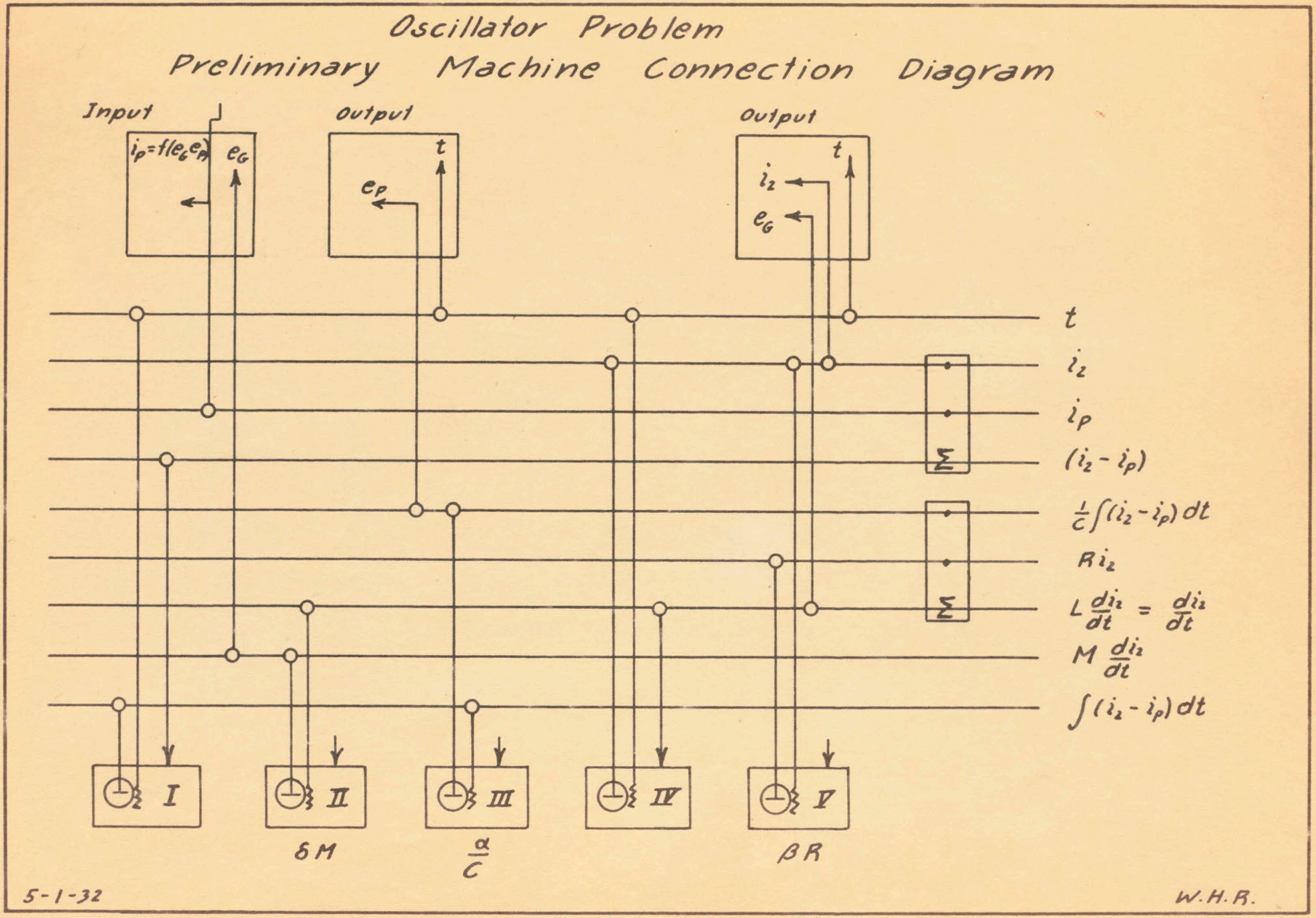
Oscillator Problem
Preliminary Machine Connection Diagram
Input
$i_p = f(e_G e_P)$
e_G
Output
t
e_P
Output
t
i_2
e_G
t
i_2
i_p
$(i_2 - i_p)$
$\frac{1}{C}\int(i_2 - i_p)\,dt$
$R i_2$
$L\frac{di_2}{dt} = \frac{di_2}{dt}$
$M\frac{di_2}{dt}$
$\int(i_2 - i_p)\,dt$
Σ
Σ
I
II
III
IV
V
δM
$\frac{\alpha}{C}$
βR
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Fig. 17

made to permit alteration of this value without making extensive changes in the entire set-up. Originally it was planned to obtain machine solutions of the oscillator problem for the range of values of L and C indicated in Table II. (Page 33.) When an attempt was made to find a suitable machine lay-out for the problem it was discovered to be impracticable to make the set-up as flexible as originally intended. To keep the time of solution of the problem within reason and also to effect certain other important simplifications, it was necessary to assign a constant value to the coil inductance. For convenience this value was made equal to unity.

From the preliminary connection diagram a second very similar diagram was prepared. This "intermediate" connection diagram appears in Fig. 18 (page 37). The chief difference between this and the first diagram is that there have been introduced a great number of undetermined constant factors and gear ratios. Thus the time shaft has been labled "At"; the i_2 shaft has been labled "Bi_2" and so on. Wherever two shafts are coupled through a set of gears, the gear ratio has been introduced as a factor. For example, the "n_1At" shaft is coupled to the "At" shaft by a set of gears having a ratio $(1:n_1)$. When two shafts are connected to an integrator the factor of the integrator (1/32) is introduced. When integrators are used to introduce circuit constants an additional factor

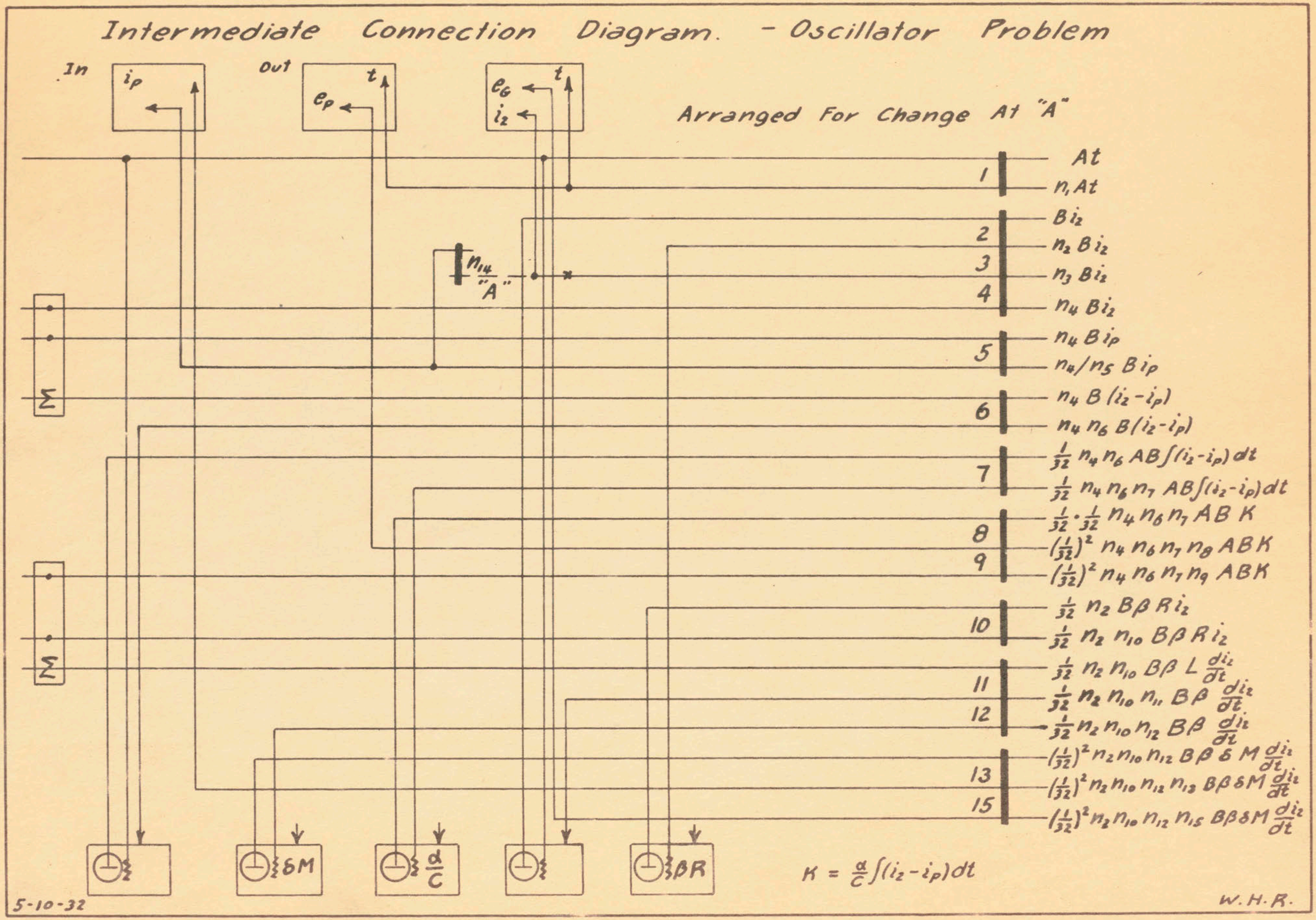
Intermediate Connection Diagram. - Oscillator Problem
In
i_p
Out
e_p
t
e_G
i_2
t
Arranged For Change At "A"
At
$n_1 At$
$B i_2$
$n_2 B i_2$
$n_3 B i_2$
$n_4 B i_2$
$n_4 B i_p$
$n_4/n_5 B i_p$
$n_4 B (i_2 - i_p)$
$n_4 n_6 B (i_2 - i_p)$
$\frac{1}{32} n_4 n_6 AB \int (i_2 - i_p) dt$
$\frac{1}{32} n_4 n_6 n_7 AB \int (i_2 - i_p) dt$
$\frac{1}{32} \cdot \frac{1}{32} n_4 n_6 n_7 AB K$
$(\frac{1}{32})^2 n_4 n_6 n_7 n_8 ABK$
$(\frac{1}{32})^2 n_4 n_6 n_7 n_9 ABK$
$\frac{1}{32} n_2 B\beta R i_2$
$\frac{1}{32} n_2 n_{10} B\beta R i_2$
$\frac{1}{32} n_2 n_{10} B\beta L \frac{di_2}{dt}$
$\frac{1}{32} n_2 n_{10} n_{11} B\beta \frac{di_2}{dt}$
$\frac{1}{32} n_2 n_{10} n_{12} B\beta \frac{di_2}{dt}$
$(\frac{1}{32})^2 n_2 n_{10} n_{12} B\beta \delta M \frac{di_2}{dt}$
$(\frac{1}{32})^2 n_2 n_{10} n_{12} n_{13} B\beta \delta M \frac{di_2}{dt}$
$(\frac{1}{32})^2 n_2 n_{10} n_{12} n_{15} B\beta \delta M \frac{di_2}{dt}$
1
2
3
4
5
6
7
8
9
10
11
12
13
15
n_{14}
"A"
Σ
Σ
δM
$\frac{\alpha}{C}$
βR
$K = \frac{\alpha}{C} \int (i_2 - i_p) dt$
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Fig. 18

representing the number of turns of the displacement shaft per unit value of the constant is also included. Thus in the case of integrator II in Fig. 17, the complete constant is (6/32). In preparing the intermediate connection diagram sufficient gear changes were introduced to insure complete flexibility yet at the same time every attempt was made to keep the number of undetermined factors at a minimum.

When the final connection which expresses the equality was made there was introduced a relationship between shafts already labled and from this was obtained an equation involving undetermined constants which had to be satisfied. In this problem the final connection was made through the integrator which forms

$$\int \frac{di_2}{dt} dt = i_2$$

and the "closing" equation is:

$$\frac{1}{32} \times \frac{1}{32} n_2 n_{10} n_{11} A \; B\beta = B$$

whence:

$$n_2 n_{10} n_{11} = \frac{32^2}{A\beta}$$

The constants which enter into this equation had to be selected so as to satisfy it.

In Fig. 18 the factors of the various quantities to which shafts have been assigned represent the number of turns of the shaft per unit value of the quantity .

Thus in the case of the "At" shaft, "A" rotations correspond to one second of time and in the case of the "n_2Bi_2" shaft, "n_2B" rotations correspond to unit value of i_2 , that is to one ampere.

Adders are so constructed that all three of the shafts connected to them must have the same number of turns per unit. This requirement introduces among the undetermined factors additional equations to be satisfied. Thus in the case of the lower adder represented on page 37 the following relation had to be satisfied:

$$\frac{1}{32^2} \; n_4 n_6 n_7 n_9 A \, B \alpha = \frac{1}{32} \; n_2 n_{10} B \beta$$

whence:

$$\frac{n_4 n_6 n_7 n_9}{n_2 n_{10}} = \frac{32\beta}{A\alpha}$$

In this particular problem the above equation and the "closing equation" previously set down are the only equalities which must be satisfied in the choice of constants.

In addition to the above relations there are a number of inequalities which must be fulfilled. These involve the maximum values of the various circuit parameters and variables and must be satisfied in order that nothing will go out of range in the course of solution of the problem. For example, the maximum number of turns, in either direction, which it is permissible to give shafts displacing integrator carriages is thirty-eight.

Input table pointers and output table pencils can be displaced vertically only 360 turns and horizontally only 480 turns. In selecting the scales and gear ratios for the solution of a problem, care must be exercised to insure that none of the displacements will be driven beyond its allowable maximum value.

From the foregoing and a consideration of the connection diagram on page 37, and the table of maximum values on page 33, it is evident that for the particular problem here being described the following are the equalities and inequalities which had to be satisfied:

Equalities;

(1) $$n_2 n_{10} n_{11} = \frac{32^2}{A\beta}$$

(2) $$\frac{n_4 n_6 n_7 n_9}{n_2 n_{10}} = \frac{32\beta}{A\alpha}$$

Inequalities;

(3) $n_3 B\,(.350) \leq 180$

(4) $(n_4/n_5)\ B\,(.160) \leq 360$

(5) $n_4 n_6\ B\,(.350) \leq 38$

(6) $\frac{1}{32^2}\, n_4 n_6 n_7 n_8\ A\,B\,\alpha\,(200) \leq 180$

(7) $\frac{1}{32}\, n_2 n_{10} n_{11}\ B\,\beta\,(175) \leq 38$

(8) $\frac{1}{32^2}\, n_2 n_{10} n_{12} n_{13}\ B\,\delta\,\beta\ (80) \leq 240$

(9) $\alpha\ (.2 \times 10^6) \leq 38$

(10) $\beta\ (150) \leq 38$

(11) $\delta\ (0.7) \leq 38$

In choosing gear ratios to satisfy a set of equations such as the above all ratios should be kept as small as possible and all must be obtainable from combinations of the four values of simple gear ratios which are available for use on the machine. *

3. Computation of Gear Ratios and Scales. The Final Connection Diagram:- A set of gear ratios and scales compatible with the above equalities and inequalities was calculated. The maximum value of "t" (for one "solution") was made 0.08 seconds, giving an output time scale of 0.08/24 or 0.0025 seconds per inch. A value of (1/24) was arbitrarily assigned to n_1 . The value of A was then determined from the relation:

$$n_1 A (0.08) = 480$$

Substituting the value of n_1:

$$\frac{1}{24} A (0.08) = 480$$

whence:

$$A = 144,000.$$

Since the maximum speed of the time shaft is approximately 480 R.P.M. this meant that the time required for a solution (i.e. the time required for the output pencil to travel across the paper once) was fixed at about 24 minutes .

Equation (1) and inequality (10) were next considered. According to equation (1):

$$n_2 n_{10} n_{11} = \frac{32 \times 32}{A \beta}$$

* Gears in the following ratios are available: 1:1, 1:2, 1:4, and 2:3. 2:5, 7:8

A value of (1/32) was assigned to the product $(n_2 n_{10} n_{11})$. Since A had already been made equal to 144,000 the value of β was thus automatically fixed at:

$$\beta = \frac{32 \times 32 \times 32}{144,000} = \frac{256}{1125} \quad \text{turns/ohm.}$$

Now the value of β used also had to satisfy relation (10):

(10) $\beta\,(150) \leqq 38$

Now:

$$\frac{256}{1125} \times 150 \cong 34.2$$

and the inequality is satisfied.

Relations (2) and (9) were next given consideration. According to (2):

(2) $$\frac{n_4 n_6 n_7 n_9}{n_2\, n_{10}} = \frac{32\beta}{A\,\alpha}$$

A value of (1/3) was assigned to this equation, whereupon the value of (α) became fixed at:

$$\alpha = \frac{32\beta \times 3}{A} = \frac{32 \times 256 \times 3}{144000 \times 1125} = \frac{64}{421875}$$

Now relation (9) also had to be satisfied:

(9) $\alpha\,(.2 \times 10^6) \leqq 38$

Since:

$$\frac{64}{421875} \times (.2 \times 10^6) \cong 30.4$$

(9) was satisfied and the computed value of (α) could be used.

The following relations were next considered:

(5) $n_4 n_6\, B\,(.350) \leqq 38$

(4) $(n_4/n_5)\, B\,(.160) \leqq 360$

Relation (4) was allowed to become an equality and a value of (9/4) was assigned to the quantity (n_4/n_5). The value of B then had to become:

$$B = 2250 \times \frac{4}{9} = 1000 \text{ turns / ampere.}$$

A value of (1/12) was assigned to the product $(n_4 n_6)$ whereupon:

$$n_4 n_6 B (.350) = \frac{1000 \times .350}{12} = 29.2$$

and relation (5) was satisfied by a wide margin.

The following relations were next considered:

(8) $\frac{1}{32^2} n_2 n_{10} n_{12} n_{13} B \delta \beta (80) \leq 240$

(11) $\delta (0.7) \leq 38$

Relation (8) was allowed to become an equality and a value of $(\frac{1}{4})$ was assigned to the product $(n_2 n_{10} n_{12} n_{13})$. The value of (δ) therefore became:

$$\delta = \frac{32 \times 32 \times 240 \times 1125 \times 4}{1000 \times 256 \times 80} = 54 \text{ turns/hy.}$$

Since:

$$54 \times 0.7 = 37.8$$

relation (11) was satisfied and the calculated value of (δ) could be used.

Relation (6) was next given consideration.

(6) $\frac{1}{32^2} n_4 n_6 n_7 n_8 A B \alpha (200) \leq 180$

In this the only two constants which had not yet been assigned values were n_7 and n_8. The value of unity was given to n_7 whereupon it became necessary that n_8 have a value less than

$$\frac{32 \times 32 \times 180 \times 12 \times 421875}{144000 \times 1000 \times 200 \times 64} = .507$$

To restrict the height of the e_p output plot it was decided to let n_8 equal (1/3).

There then remained only two inequalities to be considered; (3) and (7). In order to restrict the height of the i_2 output plot to a convenient size, n_3 was made equal to (1/4), a value which easily satisfied relation (3). All of the values entering into relation (7) had by this time been assigned. It was therefore necessary only to determine whether or not the relationship was satisfied. The inequality is:

$$(7) \quad \frac{1}{32} n_2 n_{10} n_{11} B \beta (175) \leq 38 \quad .$$

Making numerical substitutions we have:

$$\frac{1}{32} \times \frac{1}{32} \times 1000 \times \frac{256}{1125} \times 175 = 38.9$$

and it is seen that the inequality is not quite satisfied. The relation is so nearly satisfied, however, that it was considered unnecessary to revise the calculations.

Summarizing the results thus far obtained for gear ratios we have that:

$$n_1 = \frac{1}{24} \qquad n_3 = \frac{1}{4}$$

$$n_7 = 1 \qquad n_8 = \frac{1}{3}$$

$$n_2 n_{10} n_{11} = \frac{1}{32} \qquad n_2 n_{10} n_{12} n_{13} = \frac{1}{4}$$

$n_2 n_{10} = \frac{1}{8}$ $\qquad$ $n_4 n_6 = \frac{1}{12}$

$\frac{n_4 n_6 n_7 n_9}{n_2 n_{10}} = \frac{1}{3}$ $\qquad$ $\frac{n_4}{n_5} = \frac{9}{4}$

n_{14} and n_{15} as yet undetermined.

By inspection it was an easy manner to find simple gear ratios which satisfied all of the above equations. For convenience in scales, n_{14} and n_{15} were respectively made equal to (1/4) and (1/2).

The set of gear ratios and scales which were finally decided upon are listed in Table III. (Page 46.)

The scales of the various input and output plots are determined by the various gear ratios and other constants as is indicated on the intermediate connection diagram, page 37. Thus, for the i_2 output plot the scale is determined as follows: The i_2 shaft makes (n_3B) or ($\frac{1}{4}$x1000) or 250 turns per ampere. Since the output table lead screw has 20 threads to the inch, the scale of i_2 is:

$$\frac{250}{20} \text{ inches per ampere}$$

or:

$$0.080 \text{ amperes per inch}$$

The values of the other output and input scales were similarly determined. The results are given in the following summary:

Input scales:

i_p : (8/900) or 0.00889 amperes/inch

TABLE III

Gear Ratios And Constants - Oscillator

$n_1 = \frac{1}{24} = (\frac{1}{4} \times \frac{1}{4} \times \frac{2}{3})$

$n_2 = \frac{1}{4}$

$n_3 = \frac{1}{4}$

$n_4 = \frac{1}{8} = (\frac{1}{2} \times \frac{1}{4})$

$n_5 = \frac{1}{18} = (\frac{1}{4} \times \frac{1}{2} \times \frac{2}{3} \times \frac{2}{3})$

$n_6 = \frac{2}{3}$

$n_7 = 1$

$n_8 = \frac{1}{3} = (\frac{1}{2} \times \frac{2}{3})$

$n_9 = \frac{1}{2}$

$n_{10} = \frac{1}{2}$

$n_{11} = \frac{1}{4}$

$n_{12} = 1$

$n_{13} = 2$

$n_{14} = \frac{1}{4}$

$n_{15} = \frac{1}{2}$

$A = 144,000$ turns per second

$B = 1000$ turns per ampere

$\alpha = (64/421,875)$ turns per unit $(1/C)$

$\beta = (256/1125)$ turns per ohm

$\delta = 54$ turns per henry

e_G : (20/3) or 6.667 volts / inch

Output scales:

t : 0.0025 seconds / inch

i_2 : 0.080 amperes / inch

i_p : (32/900) or 0.03556 amps. / inch

e_p : 33.75 volts / inch

e_G : (40/3) or 13.33 volts / inch

When a satisfactory set of gear ratios and scales was obtained, the final machine connection diagram shown in Fig. 19 (page 48) was prepared. This diagram is a schematic representation of the plan view of the machine and shows the actual location of each unit. All gear ratios are clearly indicated upon it and the location of frontlash units and all other information necessary for connecting the machine is given. In preparing the diagram, frontlash units are inserted in every drive where the presence of backlash might introduce serious errors. The diagram is practically self-explanatory, except, perhaps, for this one point. The proper values of n_{13} and n_{15} were found to be respectively 2 and $\frac{1}{2}$. On the machine lay-out sheet apparently they have been made half these values. Such is not the case however, because a pair of two-to-one "step-up" gears is permanently installed in the output drive of integrator II. These two step-up gears are incorporated in a special large torque amplifier which has been provided to facilitate

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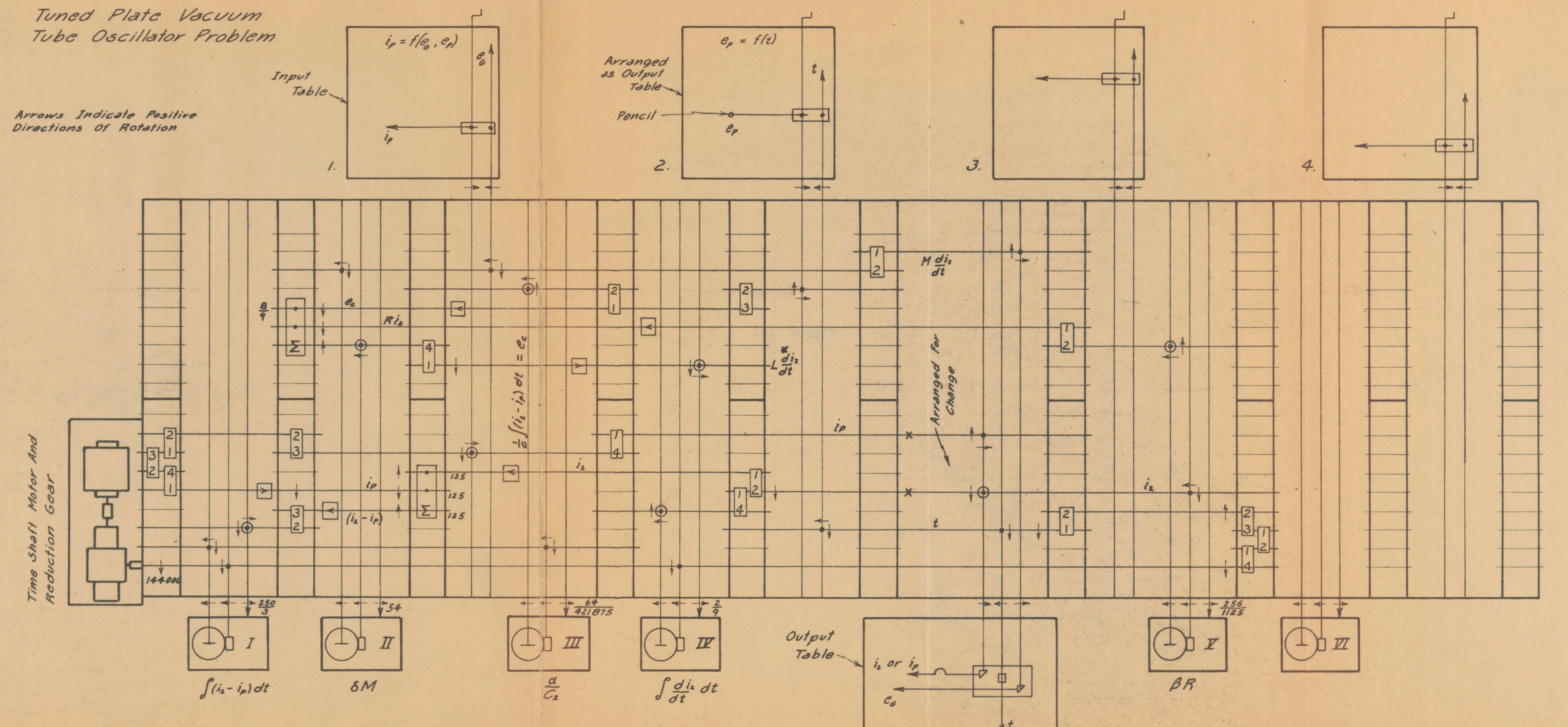
Connection Diagram For The Differential Analyzer



April 20, 1932

* $L = 1.00$

W. Radford

the handling of exceptionally heavy loads. Integrator II is the only one which is equipped with three stages of torque amplification.

After the connection diagram was finished, the $i_p = f(e_G, e_p)$ input plot was prepared to the proper scale. A number of sheets of output table paper were also provided with e_p voltage scales, horizontal lines being drawn across the sheets at intervals at five volts. When this work was finished the problem was ready for the machine. A photostat of a tracing of the i_p input plot appears in Fig. 24. (See page 60.)

4. Machine Solution of the Problem:- The differential analyzer was connected exactly as indicated on the diagram in Fig. 19. (Page 48.) The i_p input plot was arranged on the first input table. The pointer on the second input table was replaced by a pencil held in an improvised mounting; the crank was removed; and the table was connected as indicated on the diagram so that it would record e_p as a function of time. A simple talking circuit comprising a battery and two Bell System operators' telephone sets with head-band receivers and breast transmitters was provided so that the operator of the first input table could be in constant and reliable communication with the man stationed at the e_p output plot. In following the input plot, the system of operation previously discussed on pages 17 and 18 was employed.

After integrator displacement zeros had been checked; after frontlash units had been set to provide proper correction; and after other necessary minor adjustments had been made, everything was finally in readiness and an actual solution of the problem was undertaken. Referring to the circuit diagram in Fig. 16 (page 34), the first solution was made for the case where switch "S_1" is initially closed and "S_2" is suddenly closed at time $(t = 0)$. For these starting conditions the initial value of i_2 is equal to i_p, which is determined by the coil resistance R and the plate and grid battery voltages. The condenser is initially uncharged, but upon the instant of closing the switch becomes charged to a potential equal to the voltage drop across the coil - i.e. to (R) times the initial value of i_p. Since R is extremely small compared to the internal plate-filament resistance of the tube, substantially all of the battery voltage (E_p) initially appears across the tube. The plate current is therefore substantially that determined from the i_p input plot for the values:

$$e_G = E_G$$

$$e_p = E_p .$$

E_p was made equal to 140 volts and E_G to -10 volts. The corresponding value of i_p is 0.0073 ampere and the initial drop across the coil (for R = 30) is:

$$30 \times 0.0073 = 0.219 \text{ volts.}$$

The initial rate of change of the coil current is zero. A value of 7.409 microfarads was assigned to C and the mutual inductance (M) was made equal to 0.40 hy. Integrators II, III, and IV were given displacements corresponding to the selected values of M, C, and R. Integrators I and IV were set on zero. The e_p output pencil was set at the point corresponding to $(t= 0, e_p= 140)$ (the initial value of the condenser voltage being negligible). The e_G output pencil was set at the point (0, -10) and the i_2 output pencil was set at (0, .0073). The i_p input plot pointer was then set at (-10, 0) and cranked up to the point $(e_G= -10, e_p = 140)$. Integrator I was thereby given the proper initial displacement. The machine was then started.

Oscillations began very gradually indeed, the operating point of the tube describing an ellipse-like spiral. So gradually did the oscillations build up that the amplitude was still extremely small after the output pencil had completely traversed the paper several times and a dozen or so cycles had been executed. The building-up process was so slow that it was decided to abandon the run and to make a fresh start, assuming an initial charge on the condenser.

The general procedure of starting with an assumed initial condenser charge is as follows: Let the voltage to which the condenser is initially charged be "V", and let the switches "S_1" and "S_2" (Fig. 16) be closed simultaneously at time $(t = 0)$. From the differential equations for the circuit it is evident that initially the various quantities have values as follow:

$$\frac{di_2}{dt} = -\frac{V}{L} = -V \text{ amps./sec. } (L = 1)$$

$$i_2 = 0$$

$$i_p = f(\text{initial } e_G \text{, initial } e_p)$$

$$e_p = (E_p + V) \text{ volts}$$

$$e_G = (E_G - MV) \text{ volts}$$

The steps in the starting procedure may be briefly summarized as follows:

1. Give integrators II, III, and V displacements to introduce the desired values of M, C, and R.

2. Set the displacement of integrator I at zero.

3. Displace integrator IV by an amount corresponding to $(-V)$ thus taking care of the initial rate of change of i_2.

4. Set i_2 and i_p output pencils on zero.

5. Set e_p output pencil on the point corresponding to $t = 0$ and $e_p = (E_p + V)$.

6. Set e_G output pencil on the point corresponding to $t = 0$ and $e_G = (-10 - MV)$.

7. Disconnect the horizontal drive on the i_p plot and set the pointer over the point corresponding to $i_p = 0$ and $e_G = (-10 - MV)$. Reconnect and crank the pointer up to the curve corresponding to the initial value of $e_p = (E_p + V)$. The machine is now ready for use.

The starting procedure outlined above was found to be so satisfactory that it was used in practically all of the solutions which were obtained. By exercising a little judgment in the selection of the value of V (the voltage to which the condenser is initially charged) it was often possible to attain steady-state conditions of oscillation within one or two cycles.

The circuit conditions for which solutions were made and the results obtained are summarized in Table IV. (see page 54.) Data on the integrator displacements corresponding to various values of constants and slopes are contained in the first part of appendix C.

Typical samples of the results obtained from the differential analyzer are given by Figs. 20 to 23 inclusive. All of these illustrations are photostats of tracings made from the original curves drawn by the output pencils of the machine. All scales, peak values, and circuit data are given on the drawings.

In Fig. 20 forty equally spaced ordinates were erected for one cycle and the values of e_G, e_p, and i_p at these ordinates were measured. From the data thus obtained it was possible to plot, on the $i_p = f(e_G, e_p)$ curve sheet, the excursion of the operating point of the tube for one complete cycle. The dynamic characteristic of the tube and circuit for the particular set of operating conditions used in Run No. 3. was thereby obtained. The dynamic characteristics for Runs Nos. 5 and 11 were similarly obtained. The data scaled from the curves at the various ordinates are tabulated in Appendix D.

TABLE IV

Results Of Differential Analyzer Solutions
(Oscillator Problem)

Run No.	hys. M	mfd. C	ohms R	secs. T	c.p.s. f	c.p.s. f' *
1	.50	15.117	50.00	-	-	-
2	.55	15.117	50.00	.0242	41.32	40.75
3	.50	7.409	50.00	.0170	58.82	58.2
4	.40	7.409	50.00	.0170	58.82	58.2
5	.35	7.409	50.00	.0170	58.82	58.2
6	.30	7.409	50.00	-	-	-
7	.40	7.409	70.00	-	-	-
8	.40	7.409	60.00	.0170	58.82	58.2
9	.40	7.409	40.00	.0170	58.82	58.2
10	.40	7.409	30.00	.01702	58.74	58.2
11	.40	7.409	20.00	.0171	58.48	58.2
12	.40	7.409	10.00	-	-	-

Run No.	max. e_G	min. e_G	max. e_p	min. e_p	max. i_2	min. i_2
1	-	(Oscillations died out rapidly)				
2	55.9	-53.1	236	37.5	.497	-.321
3	52.0	-71.3	262	16	.367	-.297
4	36.3	-55.3	252	23	.343	-.276
5	25.3	-45.3	239	36	.303	-.245
6	-	(Oscillations died out gradually)				
7	-	(Oscillations died out gradually)				
8	31.5	-50.7	240	33	.312	-.244
9	41.2	-60.25	265	13	.371	-.314
10	44.9	-63.8	274	5	.392	-.345
11	50.0	-69.2	288	-8	.418	-.387
12	-	(Amplitudes excessive after few cycles)				

Note:

* The values in this column were calculated from the relation:

$$f' = \frac{1}{2\pi\sqrt{LC}}$$

All runs were started with assumed initial condenser charges.
Machine results for run No. 3 appear in Figure 20
Machine results for run No.11 appear in Figure 21
In all runs the following values were constant:

E_p at 140 v. E_G at -10 v. L at 1.00 hy.

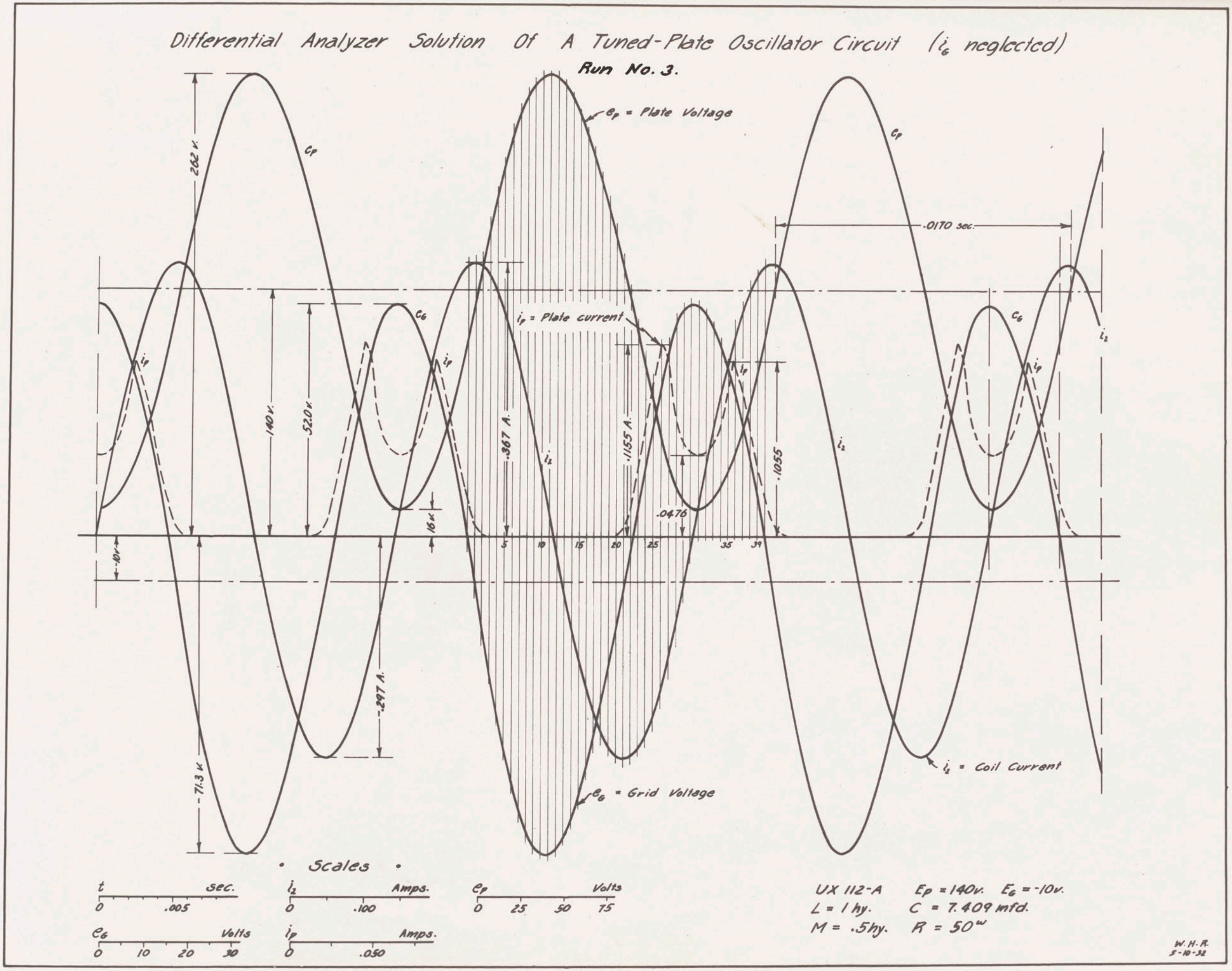

Differential Analyzer Solution Of A Tuned-Plate Oscillator Circuit (i_G neglected)
Run No. 3.
e_p = Plate Voltage
i_p = Plate current
i_2 = Coil Current
e_G = Grid Voltage
262 v.
140 v.
52.0 v.
16 v.
10 v.
71.3 v.
.367 A.
.297 A.
.1155 A.
.1055
.0476
.0170 sec.
5
10
15
20
25
35
39
Scales
t sec. 0 .005
i_2 Amps. 0 .100
e_p Volts 0 25 50 75
e_G Volts 0 10 20 30
i_p Amps. 0 .050
UX 112-A E_p = 140v. E_G = -10v.
L = 1 hy. C = 7.409 mfd.
M = .5 hy. R = 50ω
W.H.R.
5-10-32

Fig. 20.

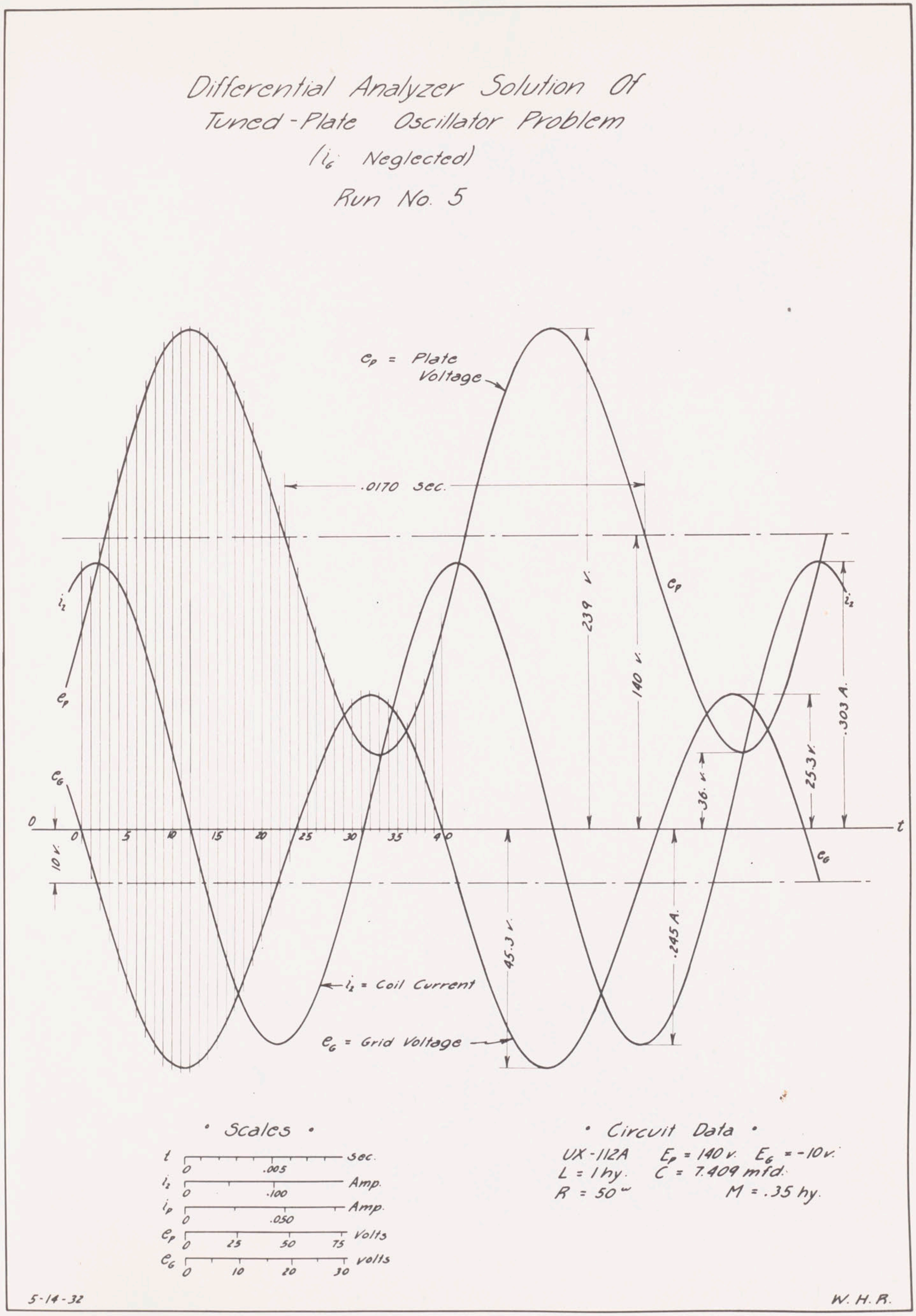

Differential Analyzer Solution Of
Tuned-Plate Oscillator Problem
(i_G Neglected)
Run No. 5
e_p = Plate Voltage
.0170 sec.
e_p
i_2
239 v.
140 v.
.303 A.
25.3 v.
36. v.
0
0 5 10 15 20 25 30 35 40
t
10 v.
e_G
45.3 v.
.245 A.
i_2 = Coil Current
e_G = Grid Voltage
• Scales •
t 0 .005 sec.
i_2 0 .100 Amp.
i_p 0 .050 Amp.
e_p 0 25 50 75 Volts
e_G 0 10 20 30 volts
• Circuit Data •
UX-112A E_p = 140 v. E_G = -10 v.
L = 1 hy. C = 7.409 mfd.
R = 50 ω M = .35 hy.
5-14-32
W. H. R.

Fig. 21

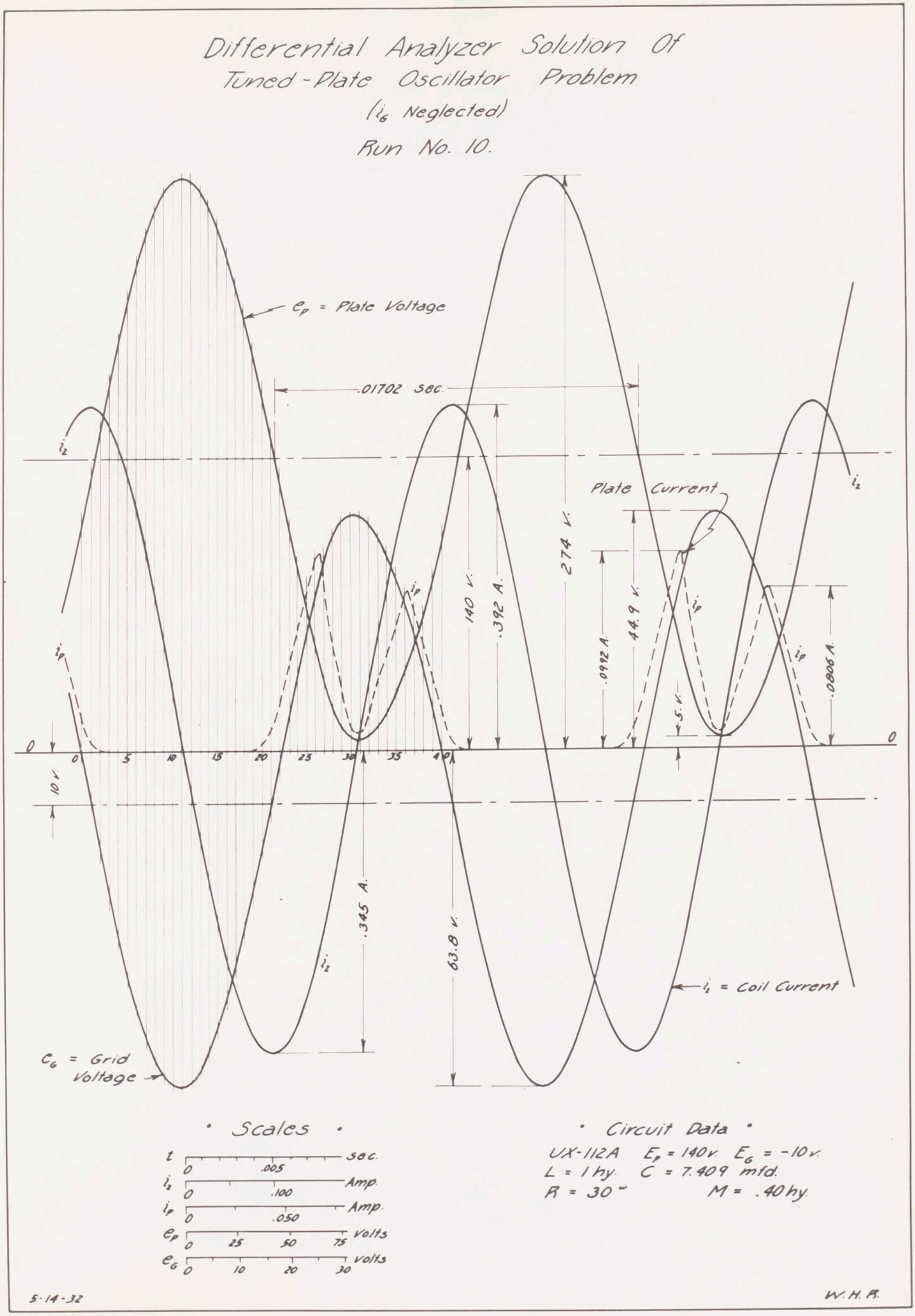

Differential Analyzer Solution Of
Tuned-Plate Oscillator Problem
(iG Neglected)
Run No. 10.
ep = Plate Voltage
.01702 sec.
Plate Current
274 v.
140 v.
.392 A.
.0992 A.
44.9 v.
.5 v.
.0806 A.
10 v.
.345 A.
63.8 v.
i2 = Coil Current
eG = Grid Voltage
• Scales •
t 0 .005 sec.
i2 0 .100 Amp.
ip 0 .050 Amp.
ep 0 25 50 75 Volts
eG 0 10 20 30 Volts
• Circuit Data •
UX-112A Ep = 140v. EG = -10v.
L = 1 hy. C = 7.409 mfd.
R = 30 ~ M = .40 hy.
5-14-32
W.H.R.

Fig. 22

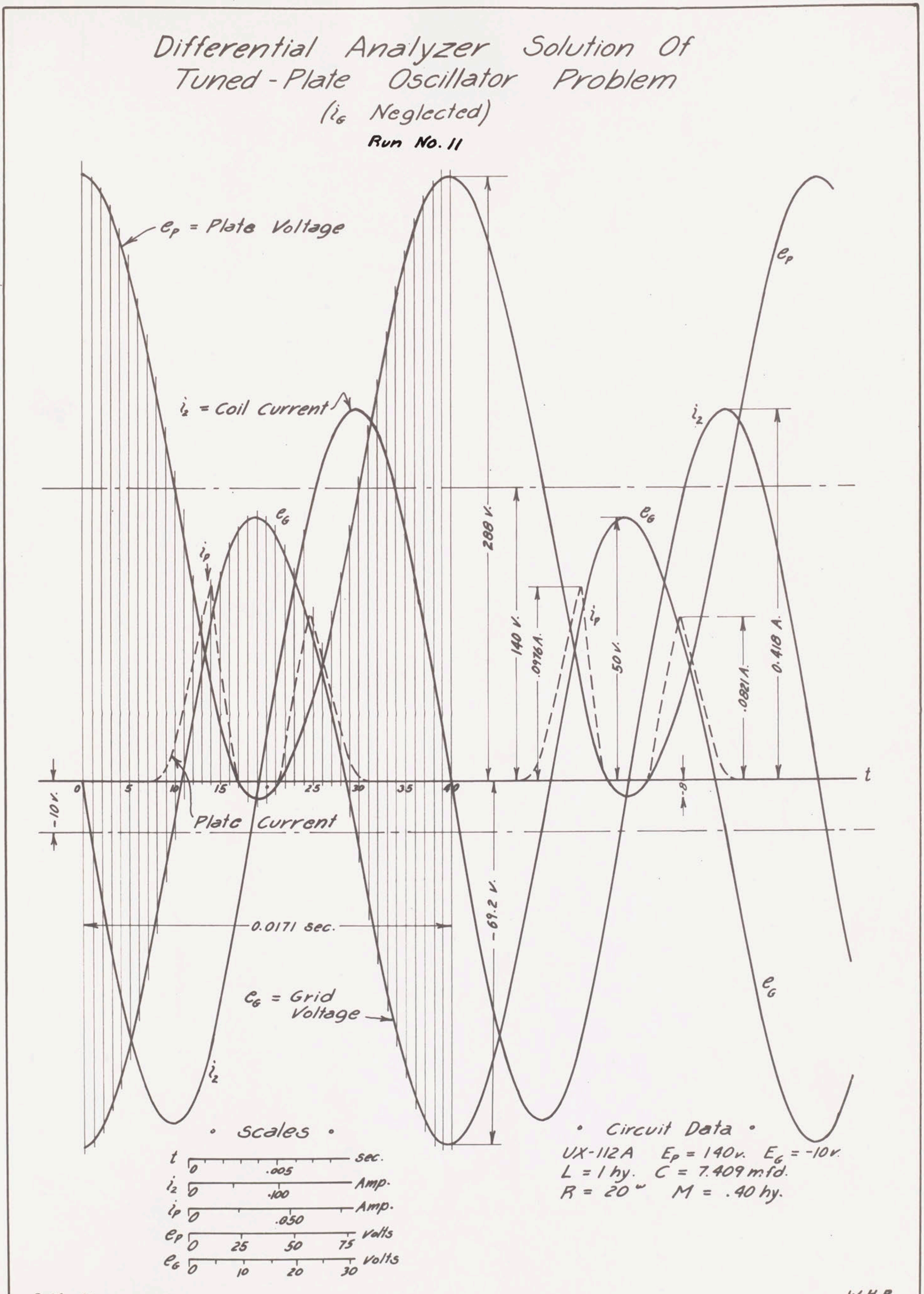

Differential Analyzer Solution Of
Tuned-Plate Oscillator Problem
(i_G Neglected)
Run No. 11
e_p = Plate Voltage
i_2 = Coil Current
e_G
i_p
Plate Current
e_G = Grid Voltage
i_2
e_p
e_G
288 v.
140 v.
.0976 A.
50 v.
.0821 A.
0.418 A.
-10 v.
-69.2 v.
0.0171 sec.
t
0 5 10 15 20 25 30 35 40
• Scales •
t sec. 0 .005
i_2 Amp. 0 .100
i_p Amp. 0 .050
e_P Volts 0 25 50 75
e_G Volts 0 10 20 30
• Circuit Data •
UX-112A E_P = 140 v. E_G = -10 v.
L = 1 hy. C = 7.409 mfd.
R = 20 ω M = .40 hy.
5-14-32
W.H.R.

Fig. 23

The dynamic characteristics, as reconstructed from these data, appear in Figs. 24 and 25. (See next two pages.)

The dynamic characteristic for Run No. 3 (shown in Fig. 24) was also plotted on a much larger copy of the static characteristic curves and measurements were made thereon to obtain data for determining the cyclical variation of the following tube parameters:

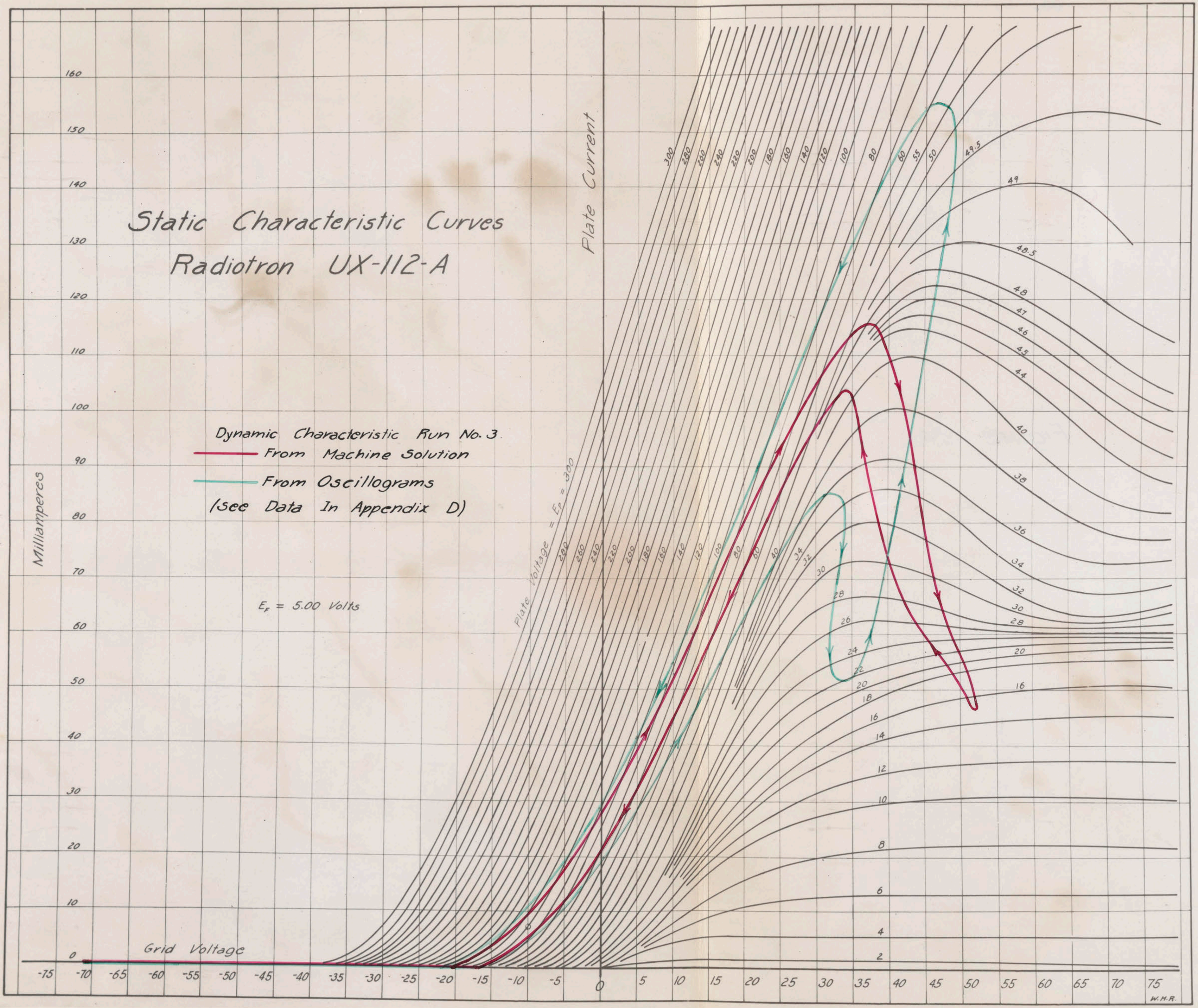
(a) The amplification constant (μ)
(b) The mutual conductance (G_m)
(c) The differential plate resistance (R_p) .

The results of this work are tabulated in part (1) of appendix D. Additional information on the method of obtaining the results is also given in the appendix. The curves shown in Fig. 26 (page 62) indicate the manner in which the tube parameters vary during the operating cycle. These curves strikingly reveal how far we come from representing the actual facts when in the analytical treatment of an oscillator circuit, we assume that the tube parameters are constants.

5. Experimental Work To Check The Machine Results:-

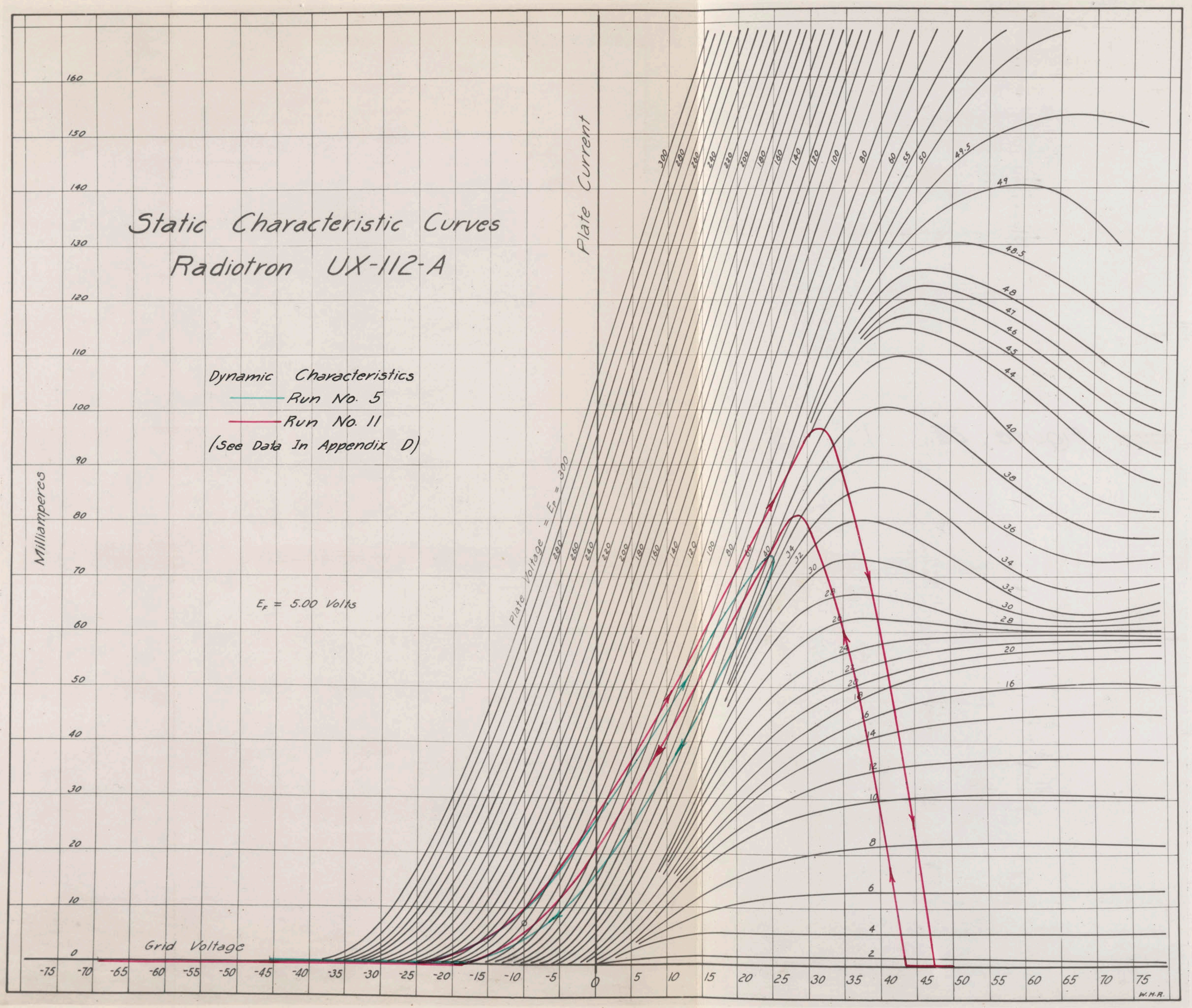
In order to substantiate the results obtained from the differential analyzer and also to qualitatively determine the effects of having neglected the grid current in the machine solution of the problem, an oscillator was set up and some oscillograms were obtained. The tube whose characteristics had been used on the differential analyzer was employed in the oscillator circuit. The constants of

Figure 24.



Static Characteristic Curves
Radiotron UX-112-A
Dynamic Characteristic Run No. 3
From Machine Solution
From Oscillograms
(see Data In Appendix D)
E_F = 5.00 Volts
Plate Current
Milliamperes
Grid Voltage
Plate Voltage = E_P = 300
W.H.R.

Figure 25.



Static Characteristic Curves
Radiotron UX-112-A
Dynamic Characteristics
Run No. 5
Run No. 11
(See Data In Appendix D)
E_F = 5.00 Volts
Plate Current
Plate Voltage = E_P = 300
Milliamperes
Grid Voltage
W.M.R.

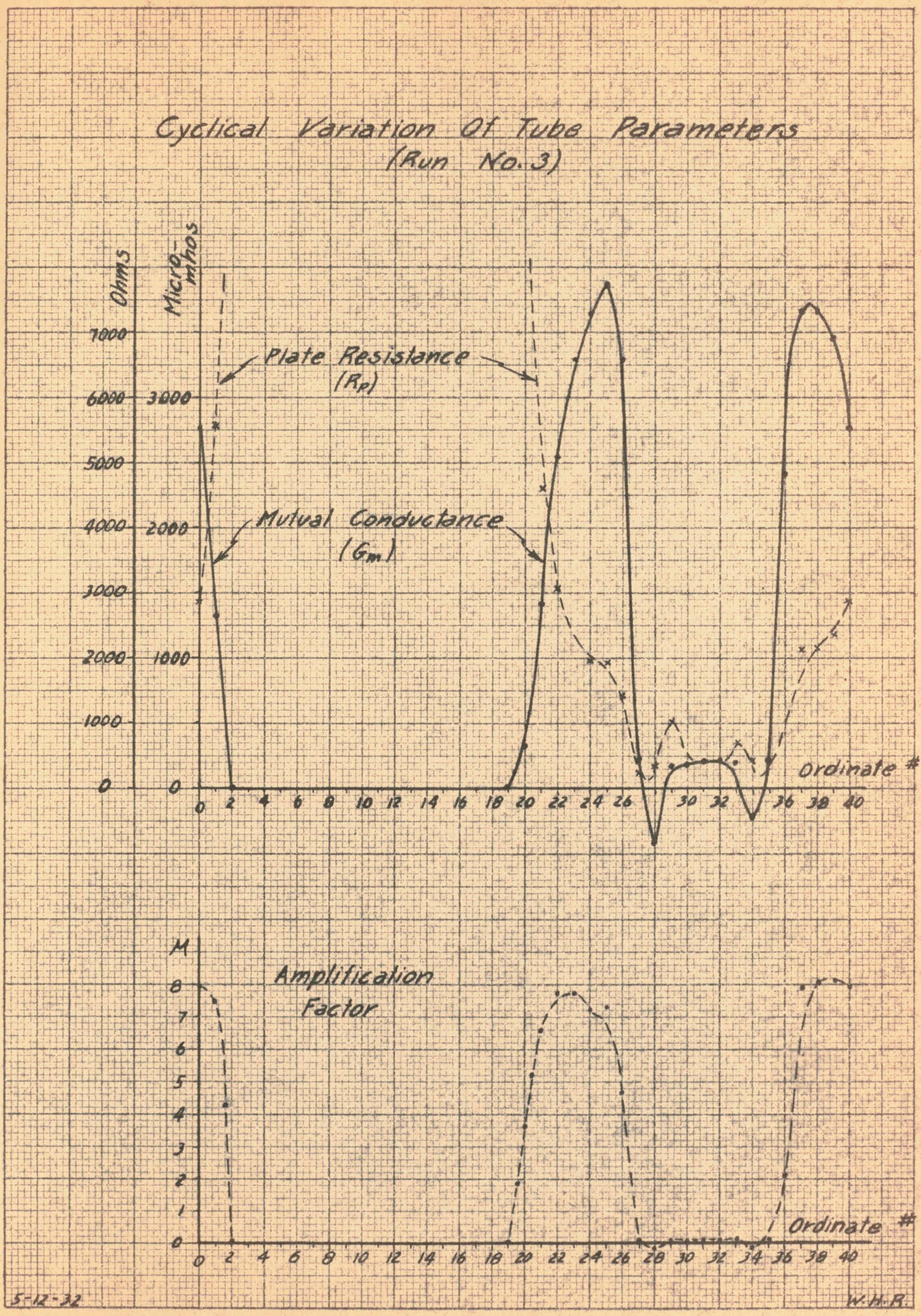

Cyclical Variation Of Tube Parameters
(Run No. 3)
Ohms
Micro-mhos
Plate Resistance
(Rp)
Mutual Conductance
(Gm)
Ordinate #
Amplification
Factor
M
Ordinate #
5-12-32
W.H.R.

Fig. 26

the circuit were equal to those used in machine solution No. 3. * Oscillograms of the plate, grid, coil, and condenser currents, and the alternating components of grid and plate voltages were obtained. These oscillograms are shown in Figs. 27, 28, and 29. (See pages 64 - 66.)

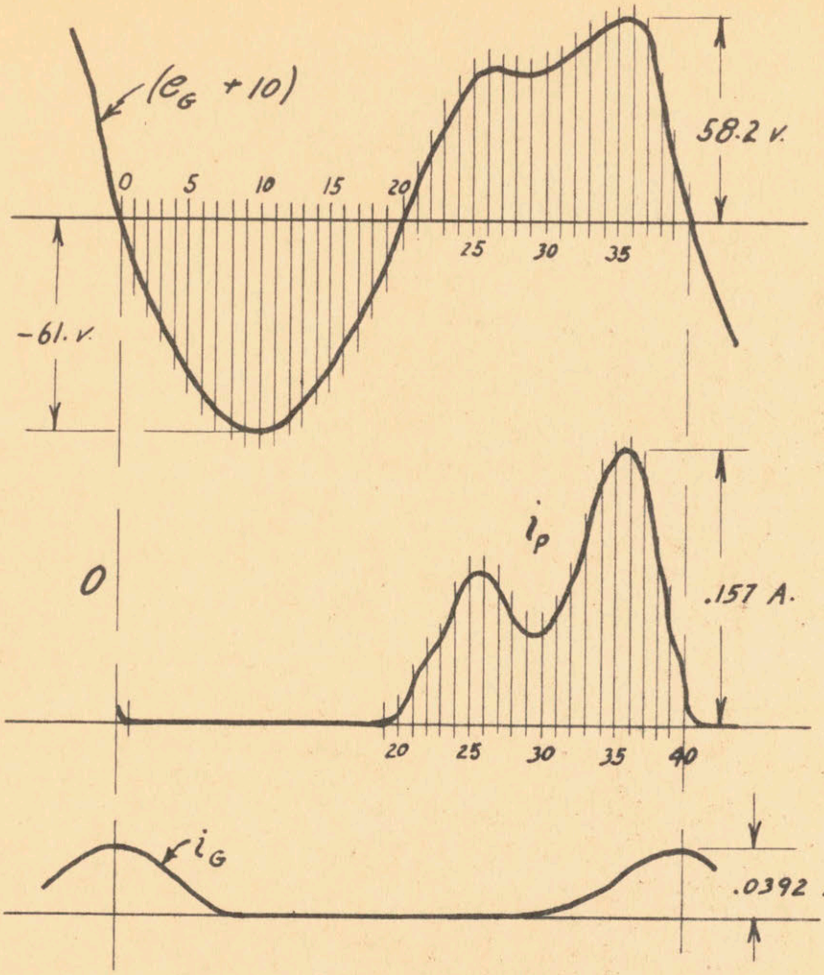
A similar set of oscillograms was obtained for circuit conditions corresponding to those used in machine solution No. 11. These appear in Figs. 30 and 31.

All of the oscillograms were taken with a small portable General Electric, two-element oscillograph. The time scales are unknown. A one-stage, transformer coupled, push-pull amplifier was used in obtaining the oscillograms of the various voltages. The input impedance of this amplifier was at all times greater than a million and a half ohms. The distortion introduced by the amplifier seemed to be small when the input and output traces of a fairly ragged 60 cycle wave were compared. The chief difficulty in obtaining the oscillograms of voltages was due to 60 cycle inductive interference, elimination of which seemed to be impossible without resorting to elaborate shielding of the oscillator coils. On some of the voltage oscillograms the distorting effects of this interference are quite noticeable.

On the oscillograms corresponding to machine solution No. 3, forty equally spaced ordinates were erected for one cycle and the values of i_p, e_G, and e_p

* Except in the case of the inductance.

Oscillograms
Tuned-Plate Oscillator Circuit



• Scales •

e_G : 1" = 57 volts

i_p : 1" = .112 Amps.

i_G : 1" = .112 Amps.

Corresponds To
D.A. Solution No. 3

• Circuit Data •

UX-112-A Tube

L = 1.072 hys.

C = 7.409 mfd.

M = 0.5 hys.

R = 50 $^\omega$

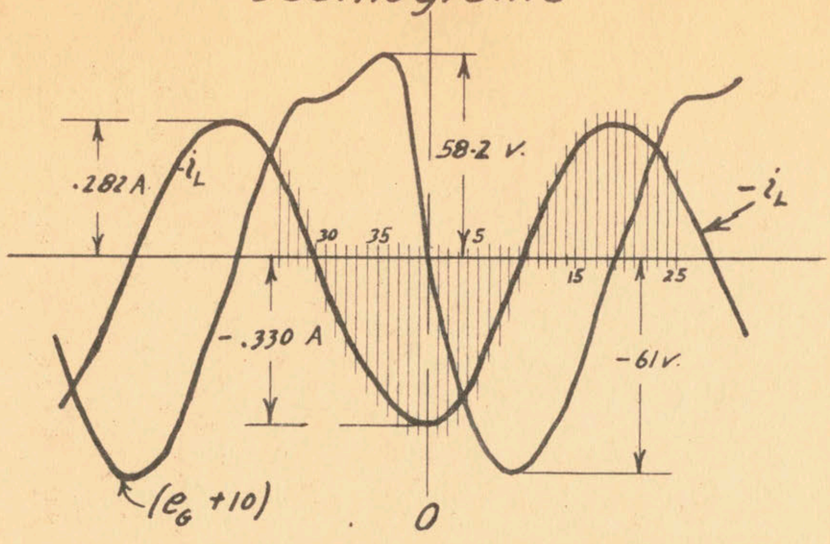
E_P = 140 v. E_G = -10. v.

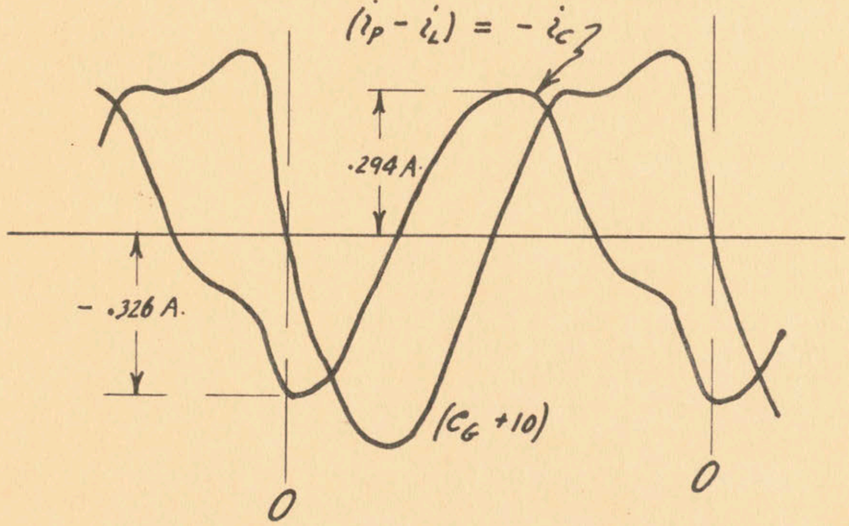
4-20-32

W.H.R.

Fig. 27

Oscillograms





• Scales •

e_G : 1" = 57 volts

i_L : 1" = .402 Amps.

i_c : 1" = .402 Amps.

Corresponds To
D.A. Solution No. 3.

• Circuit Data •

UX-112-A Tube

L = 1.072 hys.

C = 7.409 mfd.

M = 0.5 hys.

R = 50 $^{\omega}$

E_p = 140 v. E_G = -10 v.

4-20-32 W. H. R.

Fig. 28

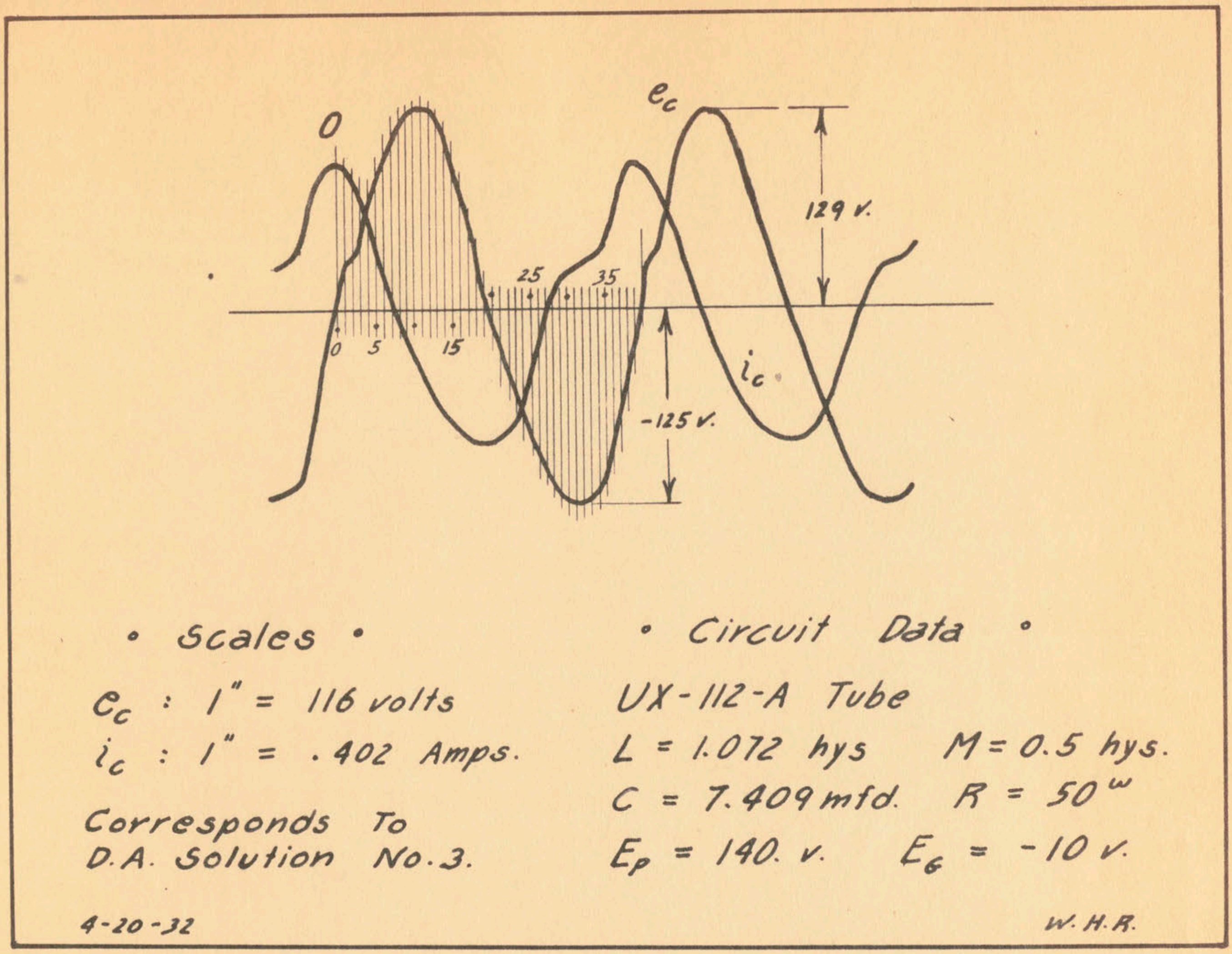
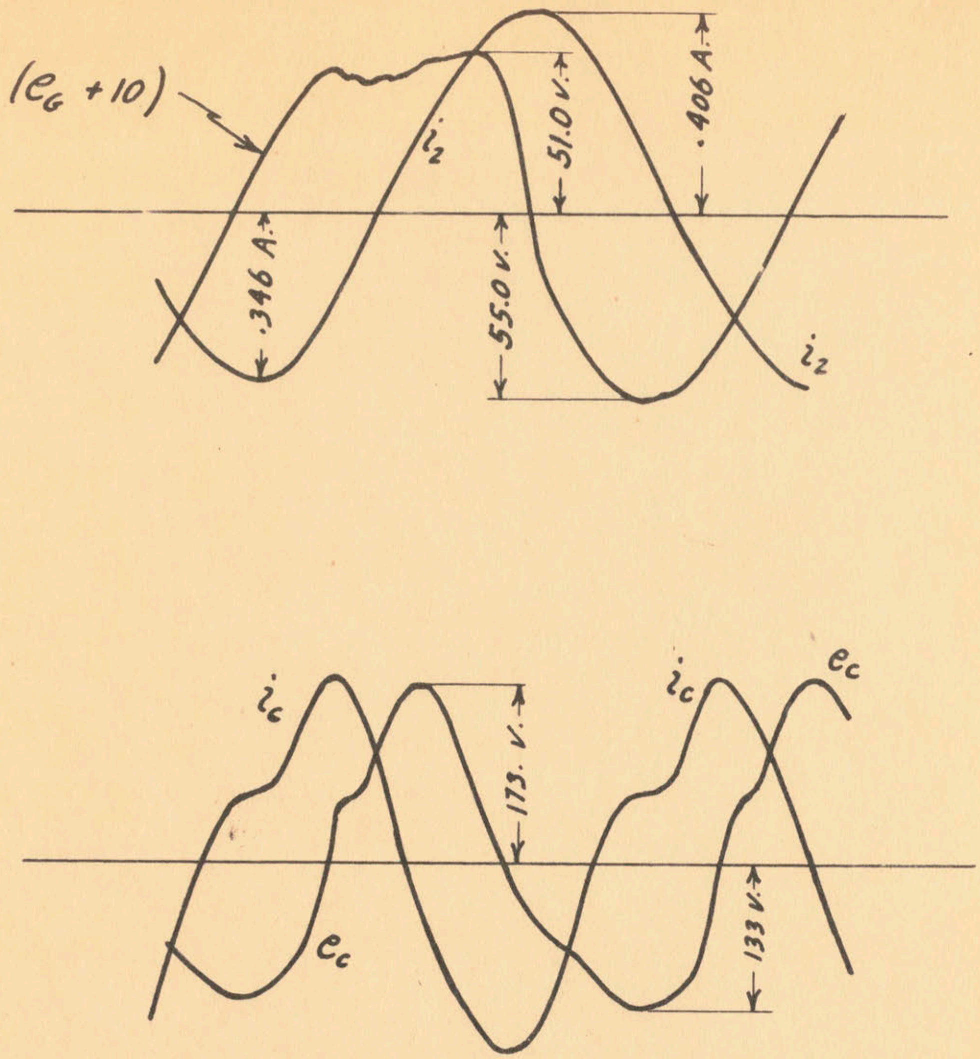


Fig. 29.

at these ordinates were scaled off. The results of this work are tabulated in part (2) of appendix C. The data thus obtained were used to plot the excursion of the operating point on the family of static characteristic curves. This excursion is drawn in green ink in Fig. 24, page 60.

6. Comparison Of Machine Results and Oscillograms:- To facilitate comparisons, the results of machine solution No. 3 and the corresponding oscillograms have been replotted to convenient scales on the same set of axes. The results of this work appear in Figs. 32 and 33.

Oscillograms
Tuned - Plate Oscillator Circuit



• Scales •

e_G : 1" = 58.5 volts
e_c : 1" = 184. volts
i_2 : 1" = .402 Amps.
i_c : 1" = .402 Amps.

Corresponds To
D.A. Solution No. 11.

• Circuit Data •

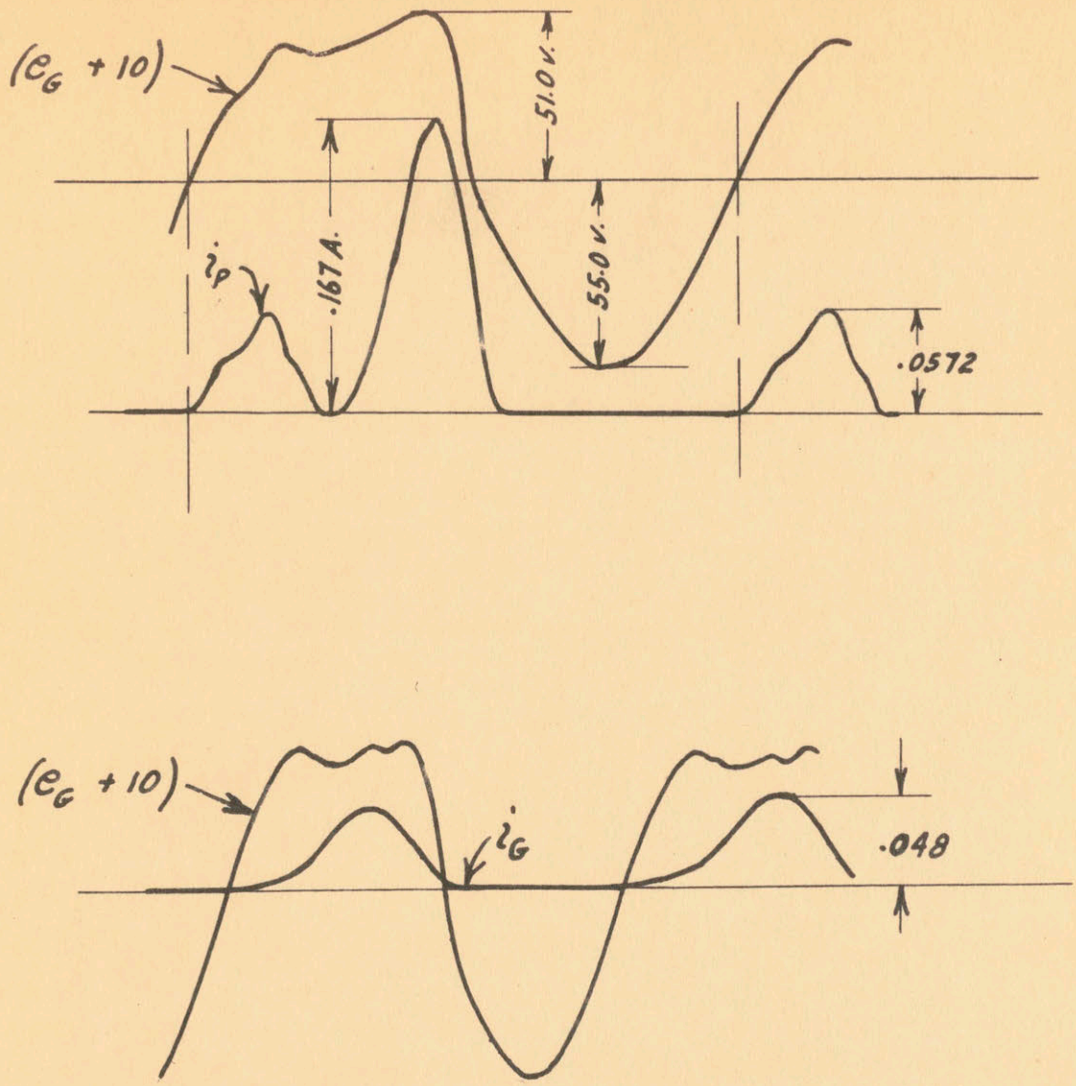
UX 112A Tube
L = 1.072 hys.
C = 7.409 mfd.
M = 0.4 hys.
$R = 20^{\omega}$
$E_P = 140$ v. $E_G = -10$ v.

4-20-32

W.H.R.

Fig. 30

Oscillograms
Tuned - Plate Oscillator Circuit



• Scales •

e_G : 1" = 58.5 volts
i_p : 1" = .112 Amps.
i_G : 1" = .112 Amps.

Corresponds To
D.A. solution No. 11

• Circuit Data •

UX-112A Tube
L = 1.072 hys.
C = 7.409 mfd.
M = 0.4 hys.
R = 20$^{\omega}$
E_p = 140 v. E_G = −10 v.

4-20-32

W. H. R.

Fig. 31.

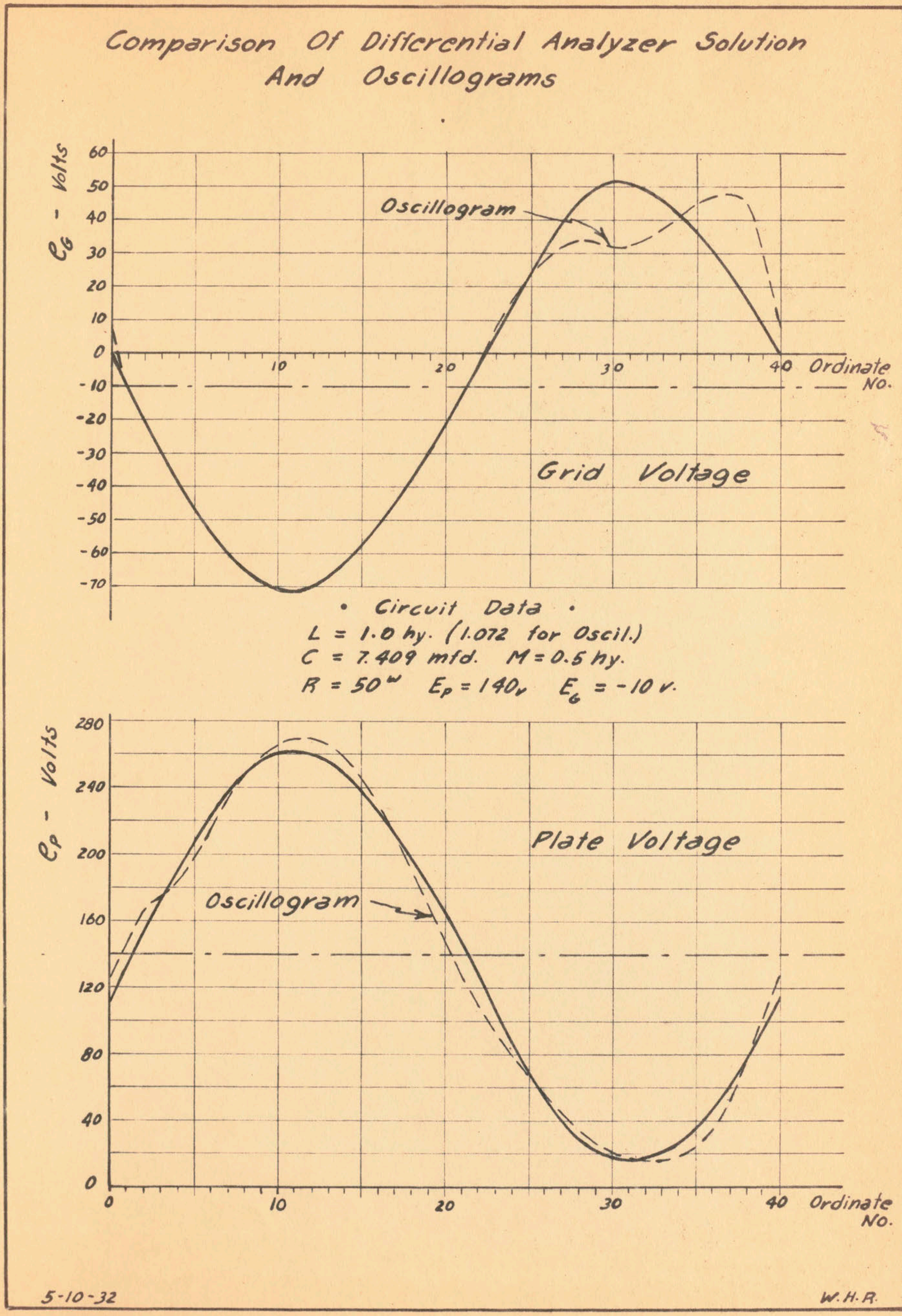

Comparison Of Differential Analyzer Solution
And Oscillograms
e_G - Volts
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
10
20
30
40
Ordinate No.
Oscillogram
Grid Voltage
• Circuit Data •
L = 1.0 hy. (1.072 for Oscil.)
C = 7.409 mfd. M = 0.5 hy.
R = 50^{w} E_p = 140$_v$ E_G = -10 v.
e_p - Volts
280
240
200
160
120
80
40
0
0
10
20
30
40
Ordinate No.
Plate Voltage
Oscillogram
5-10-32
W.H.R.

Fig. 32

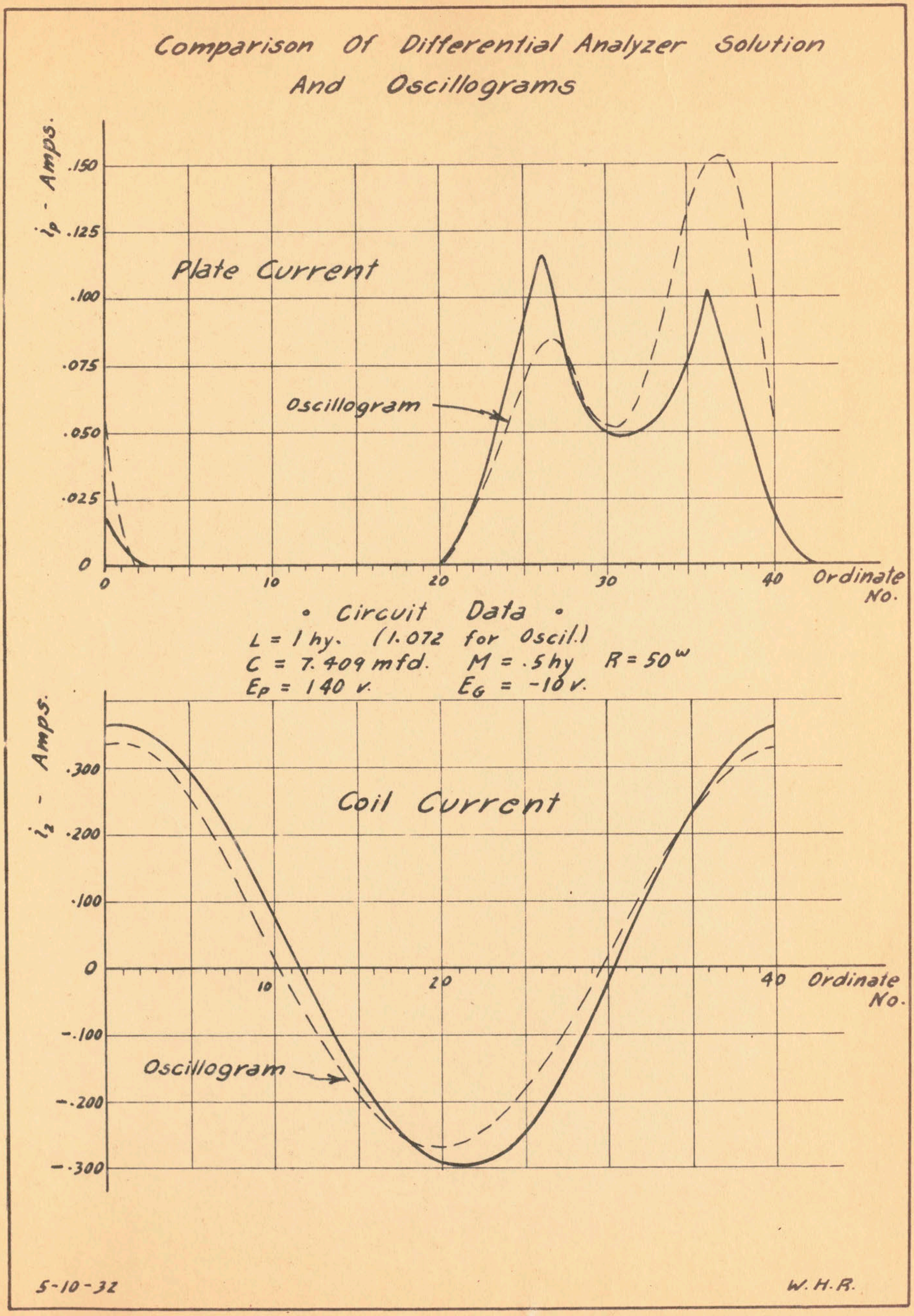

Comparison Of Differential Analyzer Solution
And Oscillograms
i_p - Amps.
.150
.125
.100
.075
.050
.025
0
Plate Current
Oscillogram
0
10
20
30
40
Ordinate No.
• Circuit Data •
L = 1 hy. (1.072 for Oscil.)
C = 7.409 mfd. M = .5 hy R = 50 w
Ep = 140 v. EG = -10 v.
i_2 - Amps.
.300
.200
.100
0
-.100
-.200
-.300
Coil Current
10
20
40
Ordinate No.
Oscillogram
5-10-32
W.H.R.

Fig. 33

The machine results are indicated by the solid black lines; the oscillograms by the broken lines.

It is evident from the curves in Figs. 32 and 33 that the neglection of grid current in the machine solution considerably affected the character of the results. The oscillogram reveals that during the positive half of the e_G cycle, when the grid current flows, the grid voltage wave is considerably distorted. The nature of the distortion is such as to bring the peak value of grid voltage later in the cycle - at a time when the plate voltage is considerably higher. This circumstance largely accounts for the differences in the appearances of the machine and oscillograph plate current records. It also accounts for the fact that in the machine solution the operating point of the tube appears to describe the dynamic characteristic in a clockwise direction whereas actually, as revealed by the oscillograms, the point should appear to move in a counter-clockwise direction. (Compare the dynamic characteristics plotted in Fig. 24, page 60.)

* * *

V. THE AMPLIFIER PROBLEM

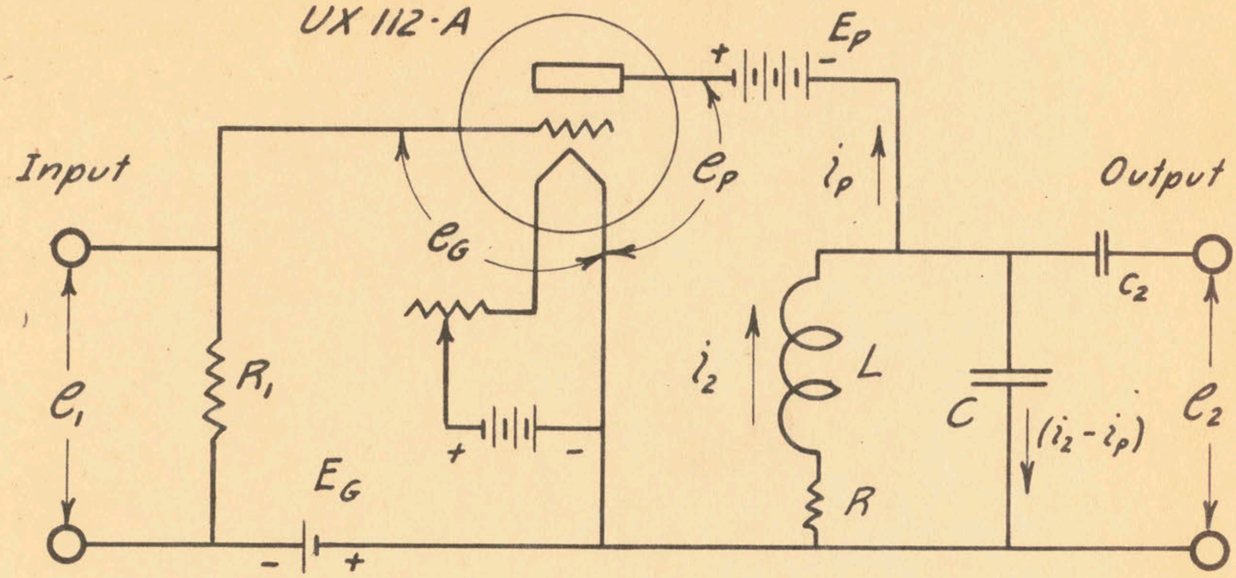
1. Preliminary:- The second type of thermionic vacuum tube circuit which was studied by means of the differential analyzer in the course of this investigation was a one-stage amplifier having a tuned parallel circuit for an external plate impedance. The amplifier circuit and the differential equations which determine its performance are shown in Fig. 34. (See page 73.)
The transient and steady-state behavior of the circuit was studied for various operating conditions. The same characteristic curves which had been used in the oscillator problem were also used in this investigation, after having been replotted to suitable scales. The relatively low operating frequency of one hundred cycles per second was chosen in order that the internal capacities of the tube would be negligible. It was decided, early in the course of the investigation, that the amplifier input voltage should be:

$$e_1 = 10 \sin 200\pi t$$

and that in studying the behavior of the circuit it would be interesting to vary the inductance (L) of the parallel circuit from approximately 0.3 to 2.0 henrys and the capacity (C) from 8 to 2 microfarads. Battery voltages of $E_p = 160$ and $E_G = -10$ were arbitrarily selected.
On the basis of these assumptions it was possible, from

Amplifier Problem

Circuit Diagram And Equations



The Circuit equations are:

(1) $e_G = -E_G + e_1$

(2) $0 = Ri_2 + L\frac{di_2}{dt} + \frac{1}{C}\int(i_2 - i_p)\,dt$

(3) $E_p = e_p - \frac{1}{C}\int(i_2 - i_p)\,dt$

For the machine the equations are written:

(1) $e_G = -E_G + e_1$

(2) $L\frac{di_2}{dt} = -Ri_2 - \frac{1}{C}\int(i_2 - i_p)\,dt$

(3) $e_p = E_p + \frac{1}{C}\int(i_2 - i_p)\,dt$

5-15-32 W. H. R.

Fig. 34

rough calculations, to discover the probably ranges of values of the different variables. These are summarized in Table V.

TABLE V

(Data for Amplifier Problem)

QUANTITY		RANGE OF VALUES TO BE COVERED
E_p	....	Constant at 160 volts
E_G	....	Constant at either -10 or -20 v.
L	....	2.00 to 0.30 henrys
C	....	2.00 to 8.00 microfarads
R	....	0.00 to 200.0 ohms
i_p	....	0.00 to 0.040 ampere
i_2	....	-0.600 to 0.600 ampere
(i_2-i_p)	....	-0.600 to 0.600 ampere
e_G	....	-34.0 to 14.0 volts
e_p	....	40.0 to 280 volts
$\frac{di_2}{dt}$	....	-370 to 370 amps. per sec.

2. The Machine Set-Up:- Using the equations given on page 73, the preliminary machine connection diagram shown in Fig. 35 (next page) was prepared. From this the intermediate connection diagram (Fig. 36) was drawn and the equations to be satisfied in the selection of the gear ratios and constants were determined.

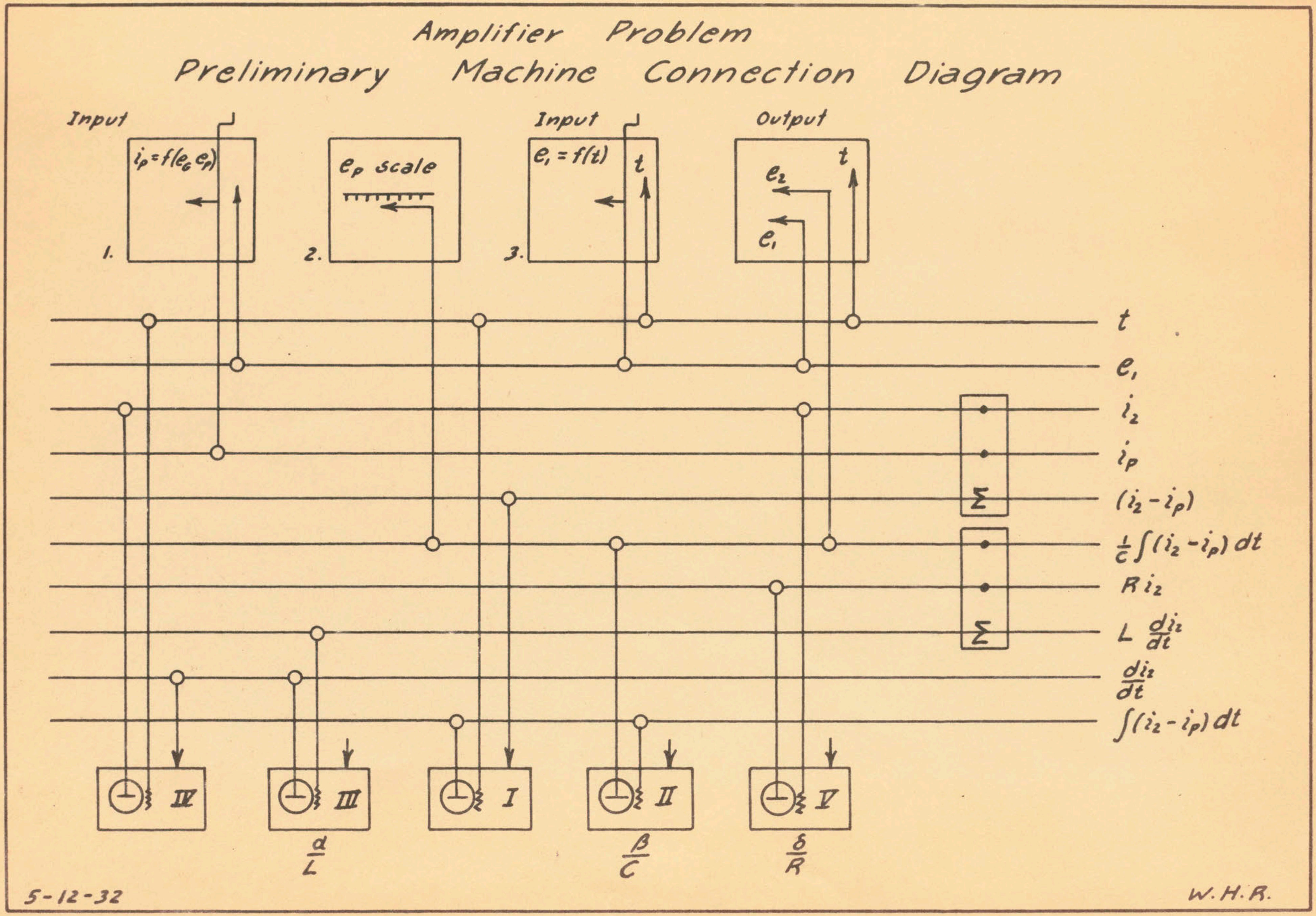
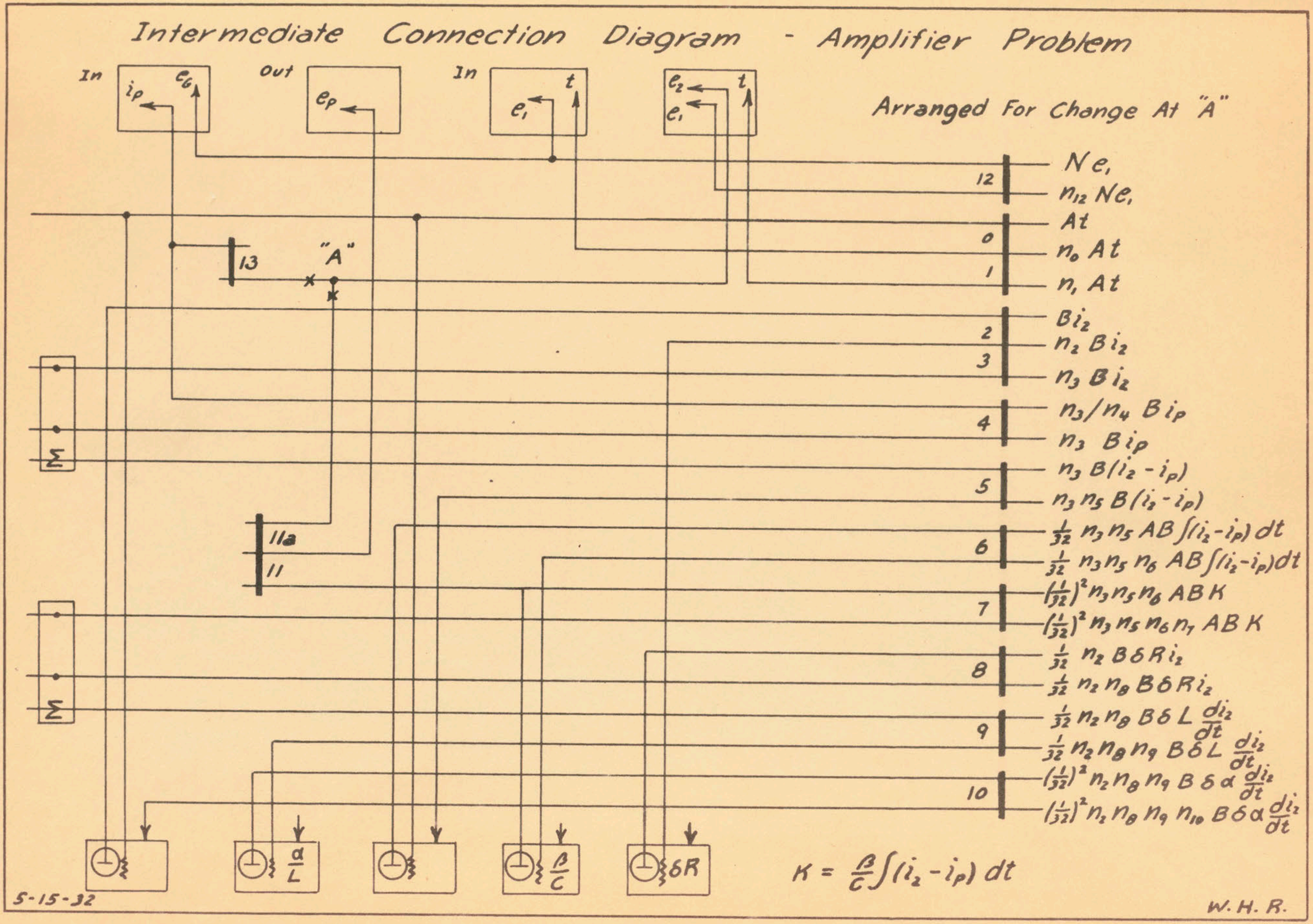

Amplifier Problem
Preliminary Machine Connection Diagram
Input
Input
Output
$i_p = f(e_G\, e_p)$
e_p scale
$e_1 = f(t)$
t
e_2
e_1
1.
2.
3.
t
e_1
i_2
i_p
$(i_2 - i_p)$
$\frac{1}{C}\int(i_2 - i_p)\,dt$
$R\,i_2$
$L\,\frac{di_2}{dt}$
$\frac{di_2}{dt}$
$\int(i_2 - i_p)\,dt$
Σ
Σ
IV
III
I
II
V
$\frac{\alpha}{L}$
$\frac{\beta}{C}$
$\frac{\delta}{R}$
5-12-32
W.H.R.

Fig. 35.



Intermediate Connection Diagram - Amplifier Problem
Arranged For Change At "A"
In
out
In
"A"
13
11a
11
12
0
1
2
3
4
5
6
7
8
9
10
Ne,
n12 Ne,
At
n0 At
n1 At
Bi2
n2 Bi2
n3 Bi2
n3/n4 Bip
n3 Bip
n3 B(i2 - ip)
n3 n5 B(i2 - ip)
K = β/C ∫(i2 - ip) dt
5-15-32
W. H. R.

Fig. 36

Using the data given in table V, the various inequalities which had to be satisfied were also written down. The relationships which had to be fulfilled in the selection of gear ratios and scales are as follows:

Equalities:

(1) $$n_2 n_8 n_9 n_{10} = \frac{32^3}{A \delta \alpha}$$

(2) $$\frac{n_3 n_5 n_6 n_7}{n_2 n_8} = \frac{32\,\delta}{A\,\beta}$$

Inequalities:

(3) $N\,(10) \leq 180$

(4) $n_{12}\,N\,(10) \leq 180$

(5) $(n_3/n_4)\,B\,(.040) \leq 360$

(6) $n_3 n_5 B(.600) \leq 38$

(7) $\frac{1}{32^2}\;n_2 n_8 n_9 n_{10} B \delta \alpha\,(370) \leq 38$

(8) $\frac{\alpha}{.3} \leq 38$

(9) $\frac{\beta}{2 \times 10^{-6}} \leq 38$

(10) $\delta\,(200) \leq 38$

Working with these relationships, and following the same general procedure that was described in the discussion of the tuned-plate oscillator problem, * a suitable set of gear ratios and constants was found. The results of this work are summarized by Table VI. (See next page.)

*Page 40 etc.

TABLE VI

Gear Ratios and Constants - Amplifier Problem

$n_0 = \frac{1}{4}$

$n_1 = \frac{1}{16} = (\frac{1}{4} \times \frac{1}{4})$

$n_2 = 1$

$n_3 = \frac{1}{4}$

$n_4 = \frac{1}{72} = (\frac{1}{4} x \frac{1}{4} x \frac{1}{2} x \frac{2}{3} x \frac{2}{3})$

$n_5 = \frac{1}{2}$

$n_6 = 1$

$n_7 = 2$

$n_8 = \frac{1}{2}$

$n_9 = \frac{1}{2}$

$n_{10} = \frac{1}{2}$

$n_{11} = \frac{3}{2}$

$n_{11a} = 1$

$n_{12} = \frac{1}{2}$

$n_{13} = \frac{1}{4}$

A = 153,600 turns per second

B = 500 turns per ampere

N = 10 turns per volt

α = (512/57) turns per unit (1/L)

β = (38/480,000) turns per unit value (1/C)

δ = (38/200) turns per ohm

The input and output table scales, as determined from the constants in Table VI and the relationships indicated on the intermediate connection diagram (see page 76) were found to be as follows:

Input plots;

$i_p = f(e_G, e_p)$ plot;

i_p : (1/450) or 0.002222 amps./inch

e_G : 2 volts / inch

$e_1 = 10 \sin 200\pi t$ plot;

e_1 : 2 volts / inch

t : (1/1920) or 0.00052083 sec./in.
(19.2 inches = .01 second)

Output Plots;

t : 24 inches = .05 seconds or
1 inch = 0.002082 s.

e_1 : 4 volts / inch

e_2 : (512/19) or 26.95 volts / inch

i_p : (4/450) or 0.008888 amps./ inch

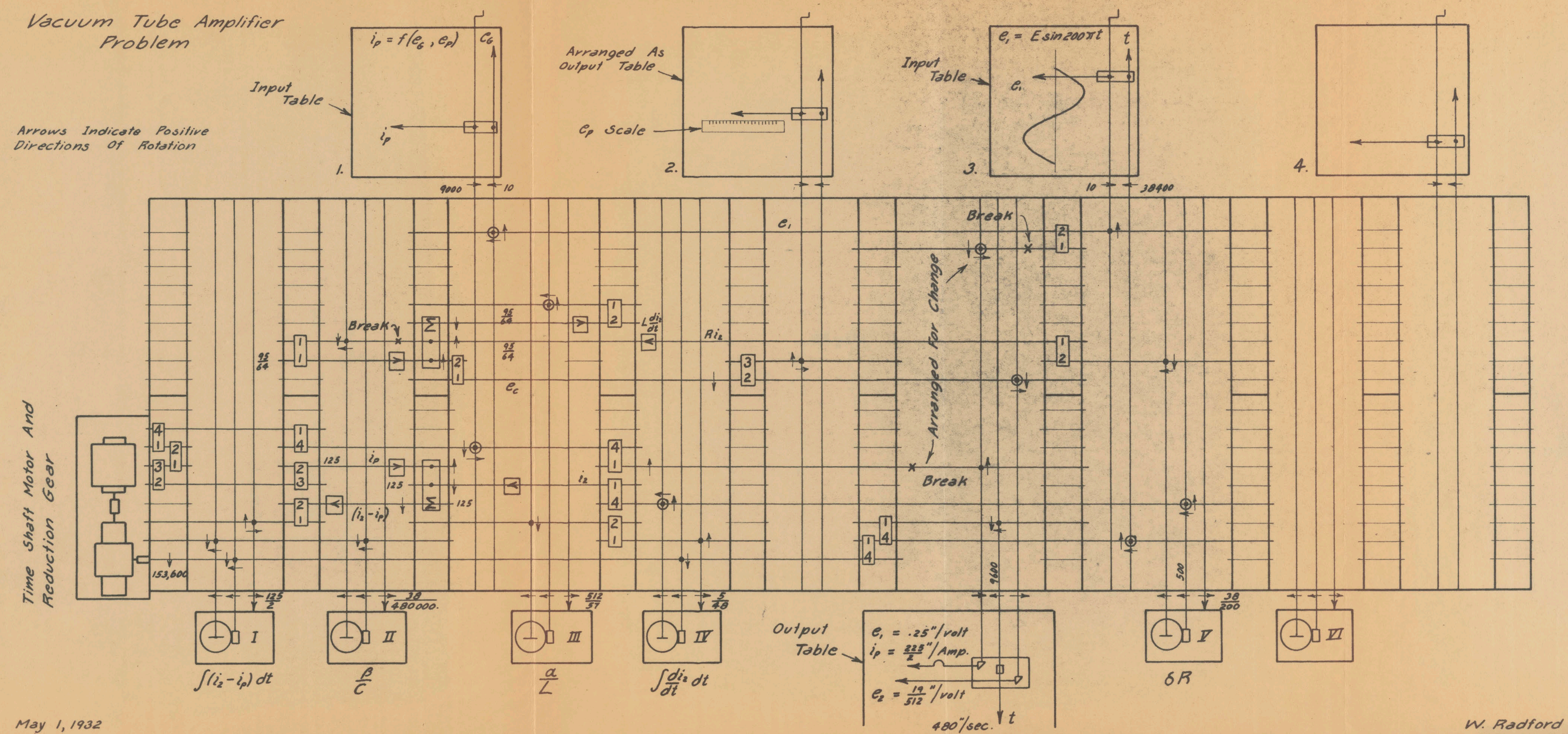
The "e_p Scale";

e_p : (1024/57) or 17.965 volts / inch

Using the gear ratios listed in Table VI, the final machine connection diagram shown in Fig. 37 was prepared. (See next page.) As indicated on the diagram, the second input table was provided with a paper scale from which values of plate voltage (e_p) could be read directly. This scale could be read quite accurately to the nearest volt. The horizontal drive of the table was left disconnected so that the pointer simply travelled vertically up and down the scale, indicating the

Fig. 37

Connection Diagram For The Differential Analyzer



May 1, 1932

W. Radford

instantaneous values of plate voltage. This arrangement, instead of the one employed in the machine solution of the oscillator problem, was used because of its greater simplicity and also because an output record of e_p as a function of time was not desired. The input voltage wave ($e_1 = 10 \sin 200\pi t$) was drawn to a very large scale (so that it would be easy to follow) and only one cycle was plotted. In operating the machine, when the end of this cycle was reached, the machine was shut down and the input table carriage was released and reset at the beginning of the cycle.

3. Machine Solution Of The Problem:- A photostat of the actual i_p input plot appears in Fig. 44. (See page 93). The data from which this plot was made are tabulated in Appendix B. The method used in following the plot was identical with that employed in the solution of the oscillator problem. *

All of the solutions obtained were made for the condition where the circuit is initially at rest (the tube filament being turned on and normal battery voltages being impressed) and a voltage $e_1 = 10 \sin 200\pi t$ is suddenly applied to the input terminals at time $t = 0$. (Refer to the circuit diagram on page 73.) For these

* Explained on page 49.

starting conditions the initial values of the various quantities are as follows:

$$i_p = f(e_G, e_p)$$

$$e_G = E_G \quad (-10 \text{ or } -20)$$

$$e_p = E_p \quad \text{(neglecting small resistance drop in the coil)}$$

$$i_2 = i_p$$

$$e_c = \text{condenser voltage} = (-R \times \text{initial } i_p)$$

$$e_2 = 0$$

$$\frac{di_2}{dt} = 0$$

The procedure in starting the machine for these initial conditions is briefly as follows: Both of the output table pencils are set on zero. The pointer on input table number 2 is set to indicate the proper value (E_p) of e_p on the voltage scale provided there. The displacements of integrators I and IV are set to zero and those of integrators II, III, and V are adjusted to give the desired values of C, L, and R. (Refer to the machine lay-out sheet on page 80.) The e_1 input plot pointer is set on zero. The i_p input plot pointer is first set at (E_G, 0) and then is cranked upward until the curve corresponding to the initial value of e_p is reached. Doing this gives integrator I and the i_p output pencil the proper initial displacements. When these adjustments have been made the machine is ready for use.

TABLE VII

Results Of Differential Analyzer Solutions
(Amplifier Problem)

Run No.	volts E_G	hys. L	mfd. C	ohms R	N*	c.p.s. f
1	-10	.6340	4	50	6	100
2	-10	.6340	4	25	9	100
3	-10	.6340	4	15	11	100
4	-10	.6340	4	10	13	100
5	-10	.3170	8	5	18	100
6	-10	.8450	3	13.32	10	100
7	-10	1.269	2	20.00	8	100
8	-10	1.013	2.5	15.97	8	100
9	-20	.6340	4	10		100
10	-10	.3170	2	5	6	200
11	-20	.3170	2	5		200
12	-10	.6340	3	10		-
13	-10	.6340	5	10		-

Run No.	max. e_2	min. e_2	max. i_p	min. i_p	$\omega L/R$	Z
1	37.8	-38.6	.0285	.00533	7.95	3300 - j415
2	51.5	-52.3	.0217	.0054	15.90	6330 - j398
3	62.3	-61.5	.0181	.0073	26.54	10540 - j443
4	72.3	-63.9	.01325	.00978	39.8	15830 -j396
5	61.3	-51.8	.0207	.00693	39.8	7930 - j 0.**
6	68.7	-71.5	.0145	.00862	39.8	21150 - j 0.
7	75.0	-76.3	.0171	.0079	39.8	25340 - j 0.
8	71.0	-71.0	.0171	.0079	39.8	25340 - j 0.
9	35.0	- 31.4	.00453	.0000	39.8	15830 - j396
10	19.40	-16.45	.03552	.0000	39.8	6.3 - j .032
11	24.6	-22.4	.00747	.0000	39.8	6.3 - j .032
12	29.7	-28.3	.0382	.00088	39.8	159 j 1575
13	28.1	-29.1	.0386	.00088	39.8	158 - j 1576

Note:

* N equals duration of transient in cycles.
**Quadrature component too small to be obtained on slide rule
Values of f satisfy the relation:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

In all runs the value of E_p was 160 volts.

Z is the impedance of the tuned circuit at 100 c.p.s.

Solutions of the problem were obtained for various combinations of circuit conditions. The runs made; the circuit data; and the results obtained are summarized in table VII. As noted therein, most of the values of L and C were selected to satisfy the relation :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

with f equal to 100 cycles per second. Although in a tuned parallel circuit having dissipation the values of L and C which satisfy the above equation are not equal to those which cause the impedance of the parallel combination to be a maximum, if the circuit resistance is small they differ from them but slightly. The values of "Z" given in Table VII are the calculated impedances of the parallel circuit at a frequency of 100 cycles per second. These values were computed from the known relationship: *

$$Z = \frac{R + j\omega\left[L - C(R^2 + L^2\omega^2)\right]}{R^2 C^2 \omega^2 + (LC\omega^2 - 1)^2}$$

Figure 38 is a photostat of a tracing made from the machine output plot for run No. 2A. It is a typical example of the results obtained from the machine. Prints made from tracings of the steady-state solutions obtained in other runs appear as Figs. 39 to 43 inclusive.

4. Discussion of Results:- In all of the solutions which were made with the parallel circuit "tuned" to the

* From K.S.Johnson, Page 174. See bibliography.

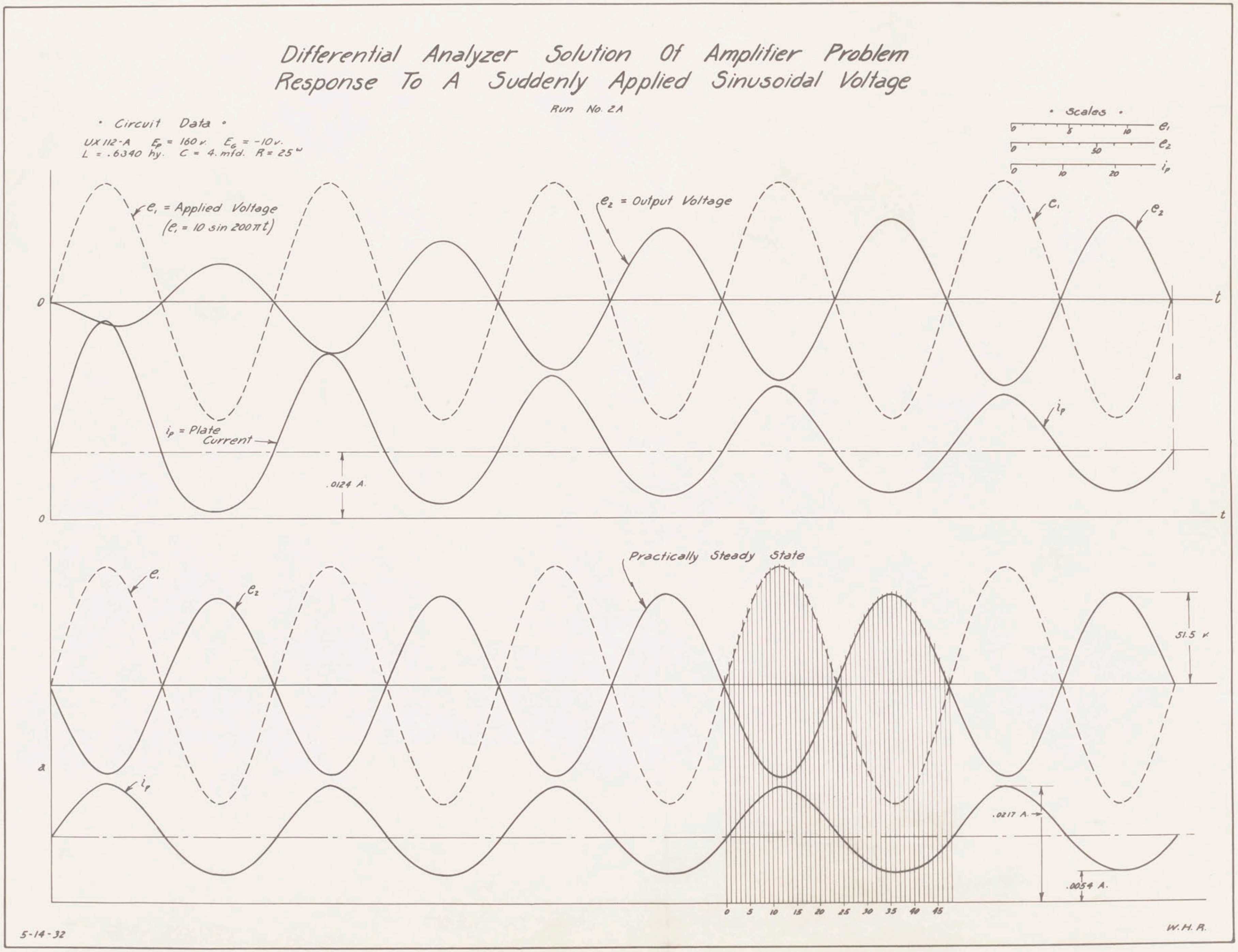

Differential Analyzer Solution Of Amplifier Problem
Response To A Suddenly Applied Sinusoidal Voltage
Run No. 2A
• Circuit Data •
UX 112-A E_p = 160 v. E_G = -10 v.
L = .6340 hy. C = 4. mfd. R = 25
• Scales •
e_1
e_2
i_p
e_1 = Applied Voltage
(e_1 = 10 sin 200πt)
e_2 = Output Voltage
i_p = Plate Current
.0124 A.
Practically Steady State
51.5 v.
.0217 A.
.0054 A.
5-14-32
W. H. R.

Fig. 38

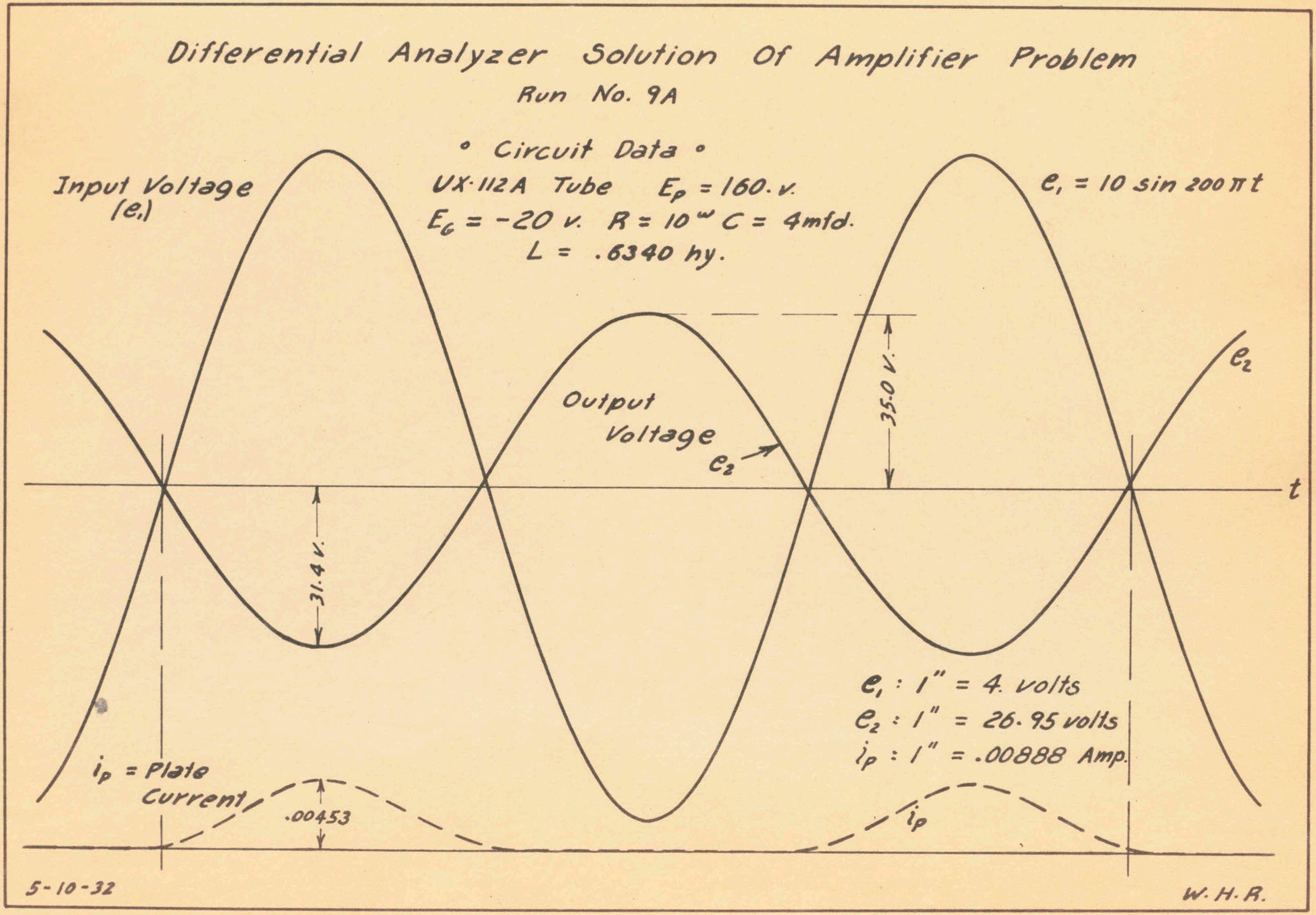

Differential Analyzer Solution Of Amplifier Problem
Run No. 9A
• Circuit Data •
UX-112A Tube Ep = 160. v.
EG = -20 v. R = 10 ω C = 4 mfd.
L = .6340 hy.
Input Voltage (e1)
e1 = 10 sin 200πt
Output Voltage
e2
35.0 V.
31.4 V.
t
e1 : 1" = 4. volts
e2 : 1" = 26.95 volts
ip : 1" = .00888 Amp.
ip = Plate Current
.00453
ip
5-10-32
W. H. R.

Fig. 39

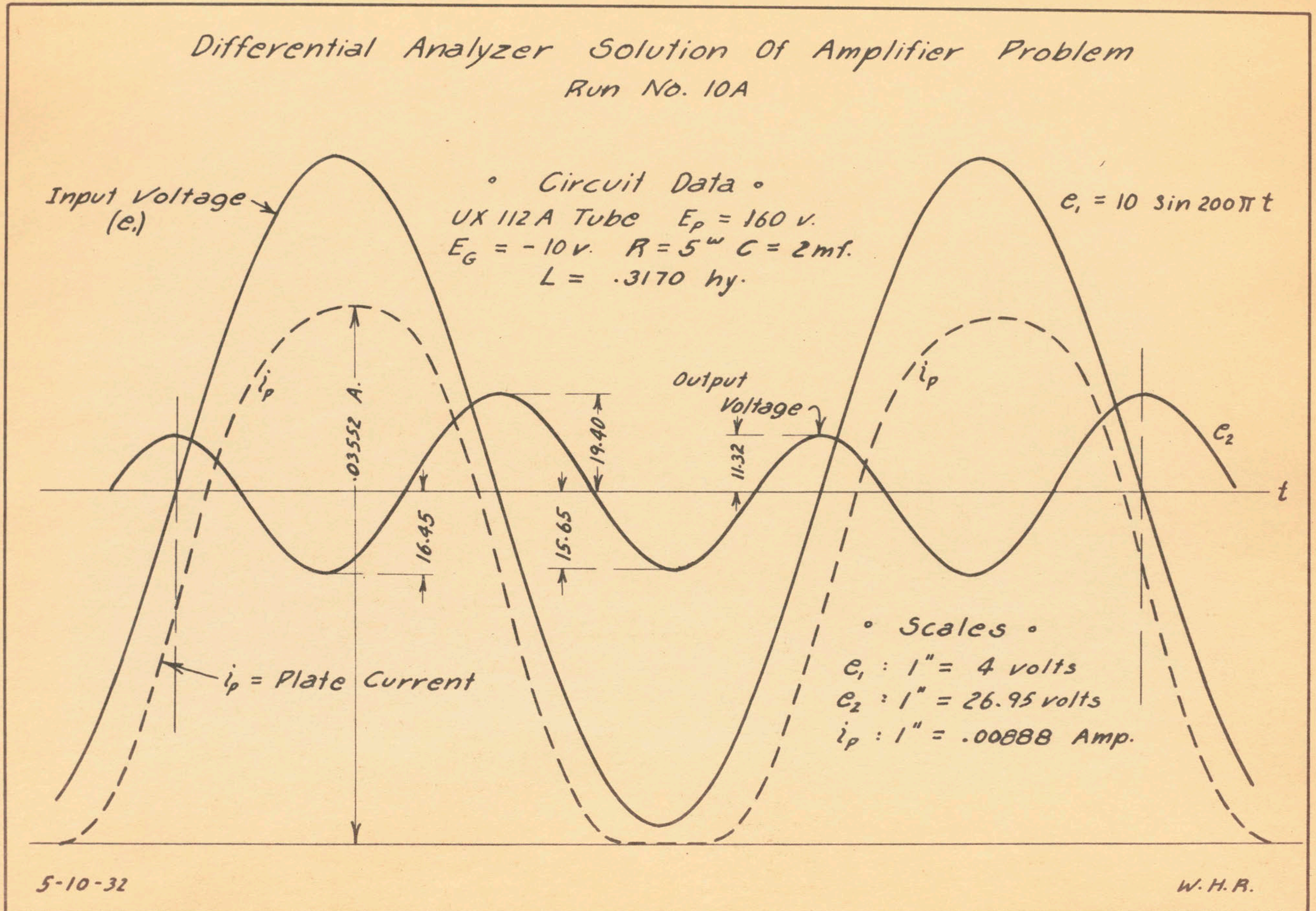

Differential Analyzer Solution Of Amplifier Problem
Run No. 10A
• Circuit Data •
UX 112 A Tube E_p = 160 v.
E_G = -10 v. R = 5^w C = 2 mf.
L = .3170 hy.
Input Voltage (e_1)
e_1 = 10 sin 200π t
i_p
.03552 A.
Output Voltage
19.40
11.32
16.45
15.65
e_2
t
i_p = Plate Current
• Scales •
e_1 : 1" = 4 volts
e_2 : 1" = 26.95 volts
i_p : 1" = .00888 Amp.
5-10-32
W. H. R.

Fig. 40

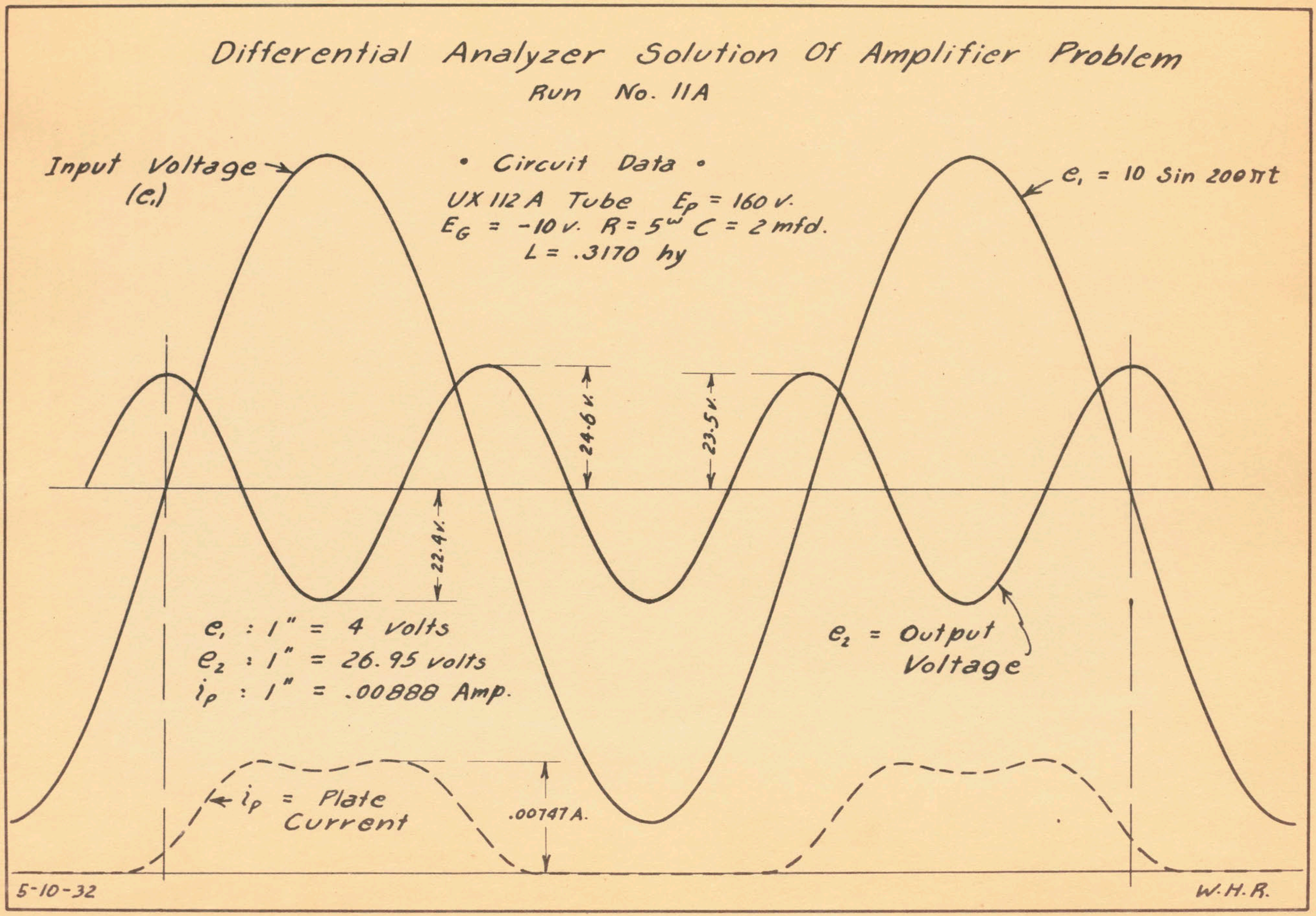

Differential Analyzer Solution Of Amplifier Problem
Run No. 11A
• Circuit Data •
UX 112 A Tube E_P = 160 v.
E_G = -10 v. R = 5^w C = 2 mfd.
L = .3170 hy
Input Voltage (e_1)
e_1 = 10 Sin 200πt
24.6 v.
23.5 v.
22.4 v.
e_1 : 1" = 4 volts
e_2 : 1" = 26.95 volts
i_p : 1" = .00888 Amp.
e_2 = Output Voltage
i_p = Plate Current
.00747 A.
5-10-32
W.H.R.

Fig. 41

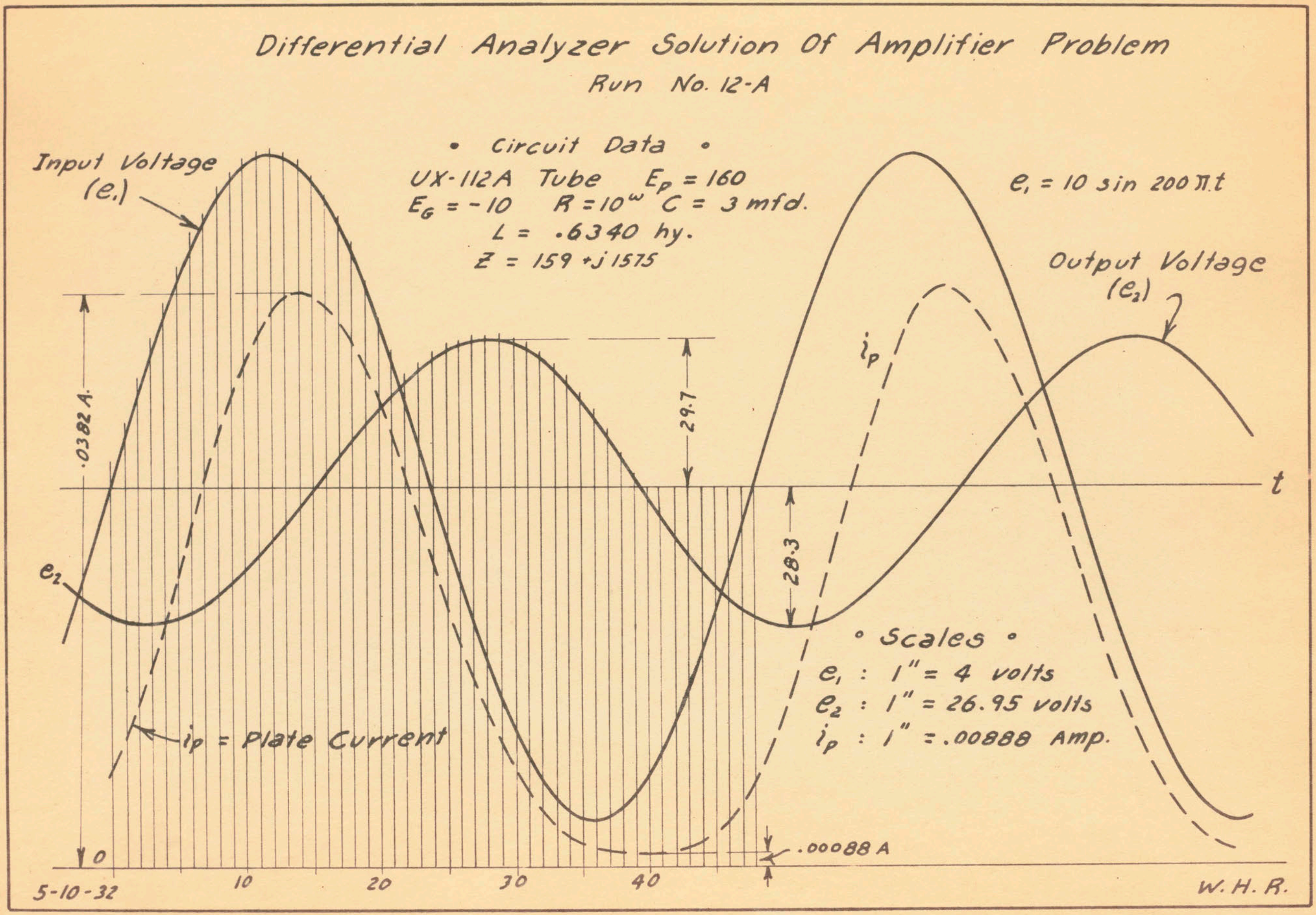

Differential Analyzer Solution Of Amplifier Problem
Run No. 12-A
• Circuit Data •
UX-112A Tube $E_p = 160$
$E_G = -10$ $R = 10^{w}$ $C = 3$ mfd.
$L = .6340$ hy.
$Z = 159 + j\,1575$
$e_1 = 10 \sin 200\pi t$
Input Voltage (e_1)
Output Voltage (e_2)
e_2
i_p
i_p = Plate Current
.0382 A.
29.7
28.3
.00088 A
t
0
10
20
30
40
• Scales •
e_1 : 1" = 4 volts
e_2 : 1" = 26.95 volts
i_p : 1" = .00888 Amp.
5-10-32
W. H. R.

Fig. 42

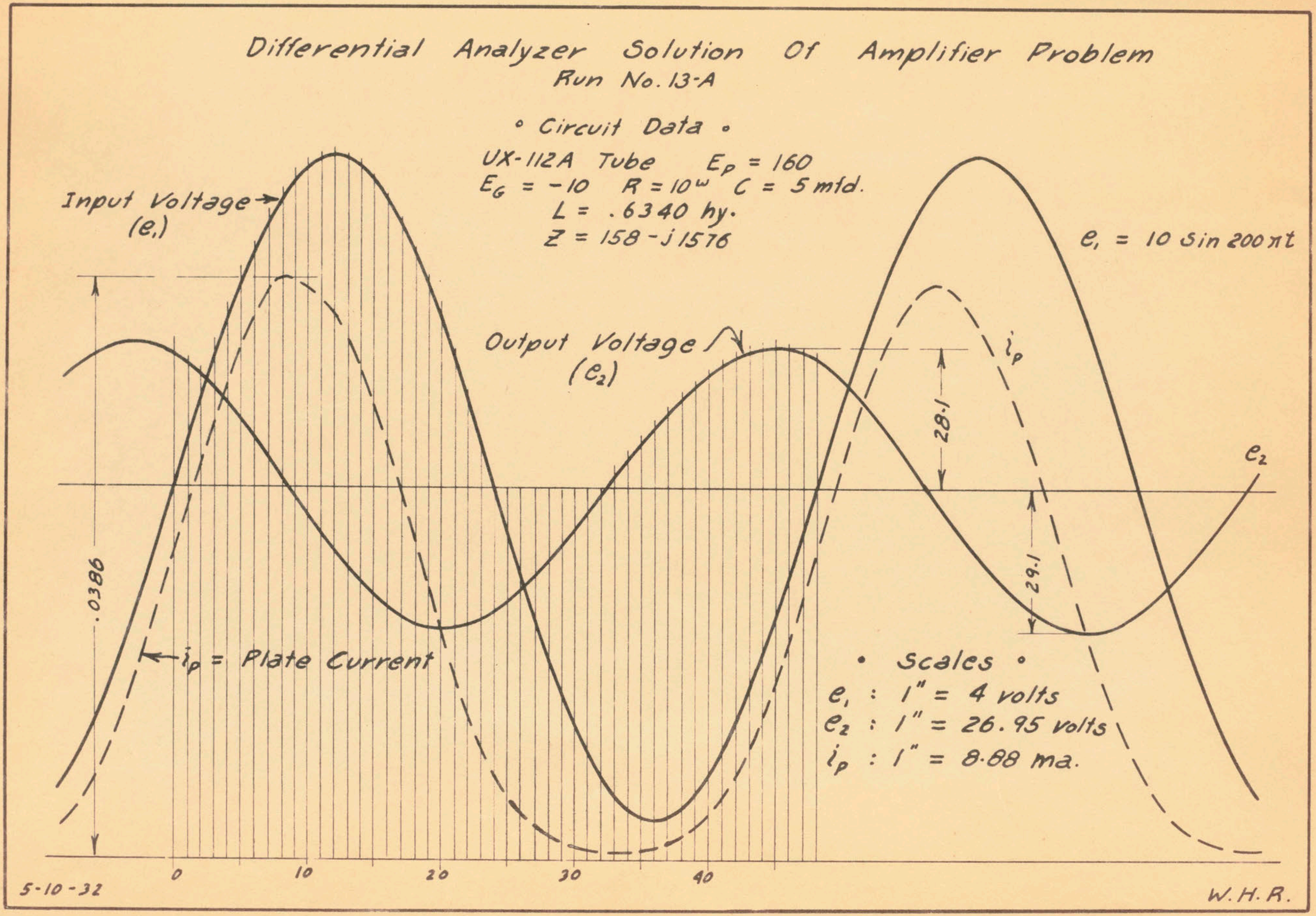

Differential Analyzer Solution Of Amplifier Problem
Run No. 13-A
• Circuit Data •
UX-112A Tube Ep = 160
EG = -10 R = 10w C = 5 mfd.
L = .6340 hy.
Z = 158 - j 1576
Input Voltage (e1)
Output Voltage (e2)
e1 = 10 sin 200πt
ip
e2
28.1
29.1
.0386
ip = Plate Current
• Scales •
e1 : 1" = 4 volts
e2 : 1" = 26.95 volts
ip : 1" = 8.88 ma.
0
10
20
30
40
5-10-32
W.H.R.

Fig. 43

fundamental frequency, the input and output voltages are substantially 180 degrees out of phase. In each case the plate current attained quite high values during the first cycle or two and then gradually settled down to a periodic variation about a constant value. In those cases where the impedance of the tuned circuit was high and the reactive component was small, (as in runs 5 - 8) the plate current was nearly sinusoidal and of very small amplitude. The dynamic characteristics in these cases were substantially straight lines and very nearly horizontal. In no case did the transient become inappreciable until at least half a dozen cycles had been completed.

The curves on page 86 (Fig. 39) illustrate the behavior of the amplifier circuit when the grid bias is such that no plate current flows when no voltage is impressed upon the input terminals. The curves on page 87 reveal the steady-state performance of the amplifier when the parallel circuit is tuned to double the frequency of the input voltage. The curves on page 88 show the response under similar conditions when the biasing battery is such that, with no input voltage, the plate current is zero. In both of these runs, at the time the solutions were made, the dynamic characteristics were observed to bear a resemblance to a very flat ellipse which has been seized by the ends and twisted through one complete turn. The characteristics crossed themselves twice and not at the same point. This fact might be surmised from the general shape of the output voltage waves.

Fig. 42 (page 89) illustrates the steady-state performance of the amplifier when the parallel circuit is adjusted to present an impedance

$$Z = 159 + j\,1575 \text{ ohms}$$

to the fundamental frequency. Fig. 43 (page 90) indicates the behavior when the parallel circuit impedance is

$$Z = 158 - j\,1576 \text{ ohms}$$

at one hundred cycles. These two solutions were made to determine the general effects of inductive and capacitative loads upon the performance of the amplifier. Values were scaled from the results of these two runs for the purpose of reconstructing the dynamic characteristics. The data obtained from the curves are tabulated in appendix E and the dynamic characteristics are plotted on the photostat appearing as Fig. 44 . (See next page.) The excursion of the operating point for the case of the inductive load is drawn in green ink. It is very interesting to observe that the directions in which the operating point describes the characteristics are contrary; the point moving clockwise when the load is capacitative and counter-clockwise when it is inductive. Except for this very important difference the two characteristics are similar.

* * *

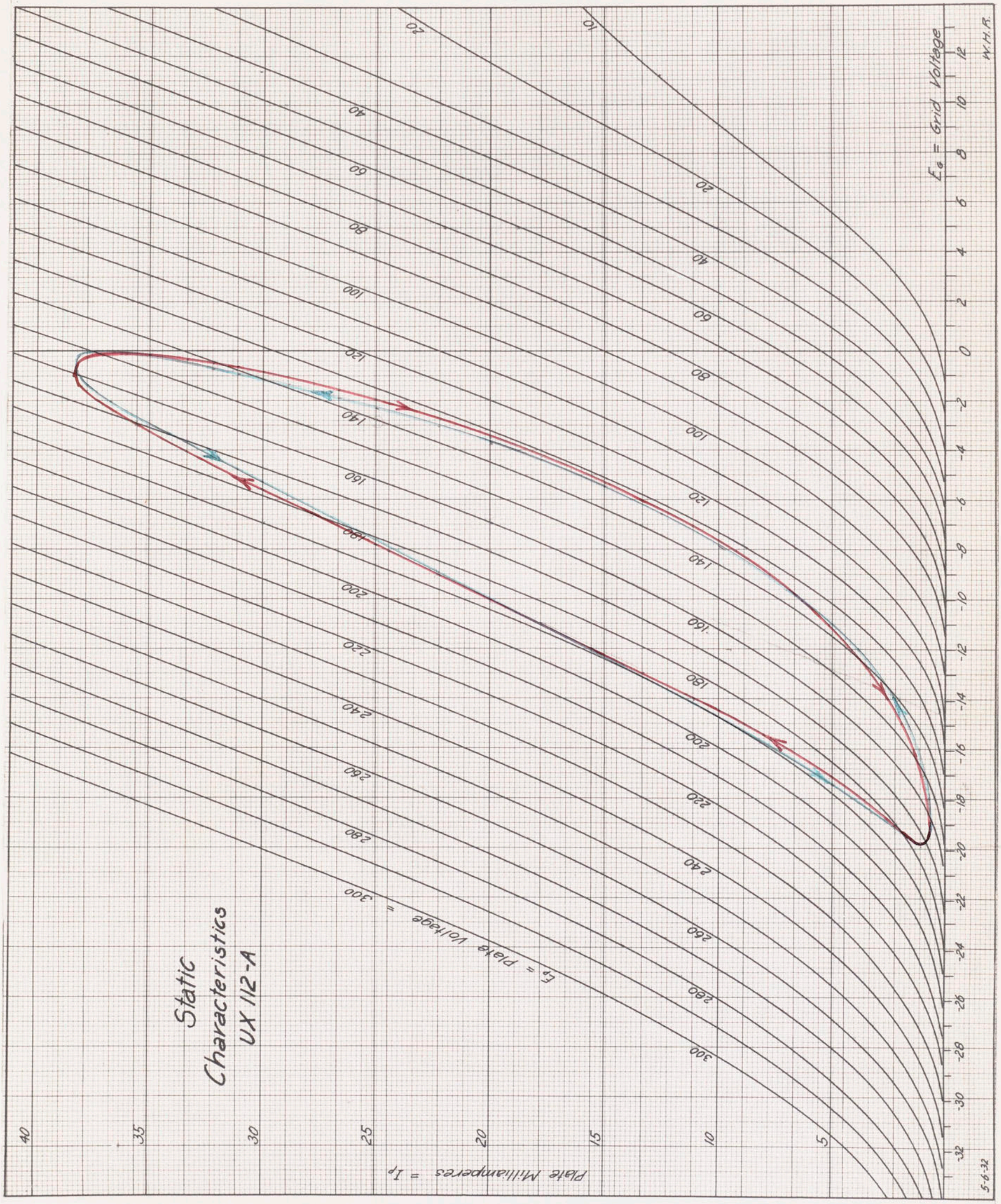


Figure 44

Amplifier Dynamic Characteristics

Run 13A   Z = 158 - j 1576

Run 12A   Z = 159 + j 1575

(Tabulated data in Appendix E)

VI. CONCLUSION

In general, the results which have been obtained conclusively indicate that it is entirely practicable to employ the differential analyzer in the solution of certain types of thermionic vacuum tube circuit problems. Unfortunately the number of different types of such problems which can be handled on the present differential analyzer is small. Evidently no problem which involves equations containing derivatives of plate or grid currents is susceptible to machine solution. Of the circuits which can be solved, apparently the dynatron oscillator and the amplifier with a tuned parallel circuit for an external plate impedance are the only ones which can be rigorously treated.

The results which were obtained from the differential analyzer solutions of the tuned-plate oscillator problem strikingly reveal the absurdity of regarding the parameters of an oscillating tube as constants. They also indicate that the grid current has an appreciable effect upon the performance of the circuit and that it cannot be neglected in any analysis which pretends to be rigorous.

The results obtained in the study of the amplifier problem indicate, in accord with the elementary theory of triodes, that when the external impedance in the plate circuit of the tube is a resistance, the dynamic characteristic is practically a straight line. When the external plate impedance is complex, the dynamic characteristic is elliptical in shape. If the external reactance is negative, the operating point of the tube appears to describe the dynamic characteristic in the clockwise direction. When the reactance is positive the characteristic is described in the counter-clockwise direction.

By way of further investigation upon the subject of this thesis, it might be well to attempt to find methods of effecting rigorous machine solutions of those circuits which, at present, apparently cannot be handled. * It would be particularly desirable to find a method of obtaining rigorous solutions of the amplifier circuit having a simple inductive and resistive load. (See the schematic in Fig. 12, page 26.) It would also be desirable to further investigate the agreement between results obtained from the differential analyzer and actual circuit performance, as revealed by oscillograms. It is possible that additional study of the subject of this thesis might lead to a more rational, approximate, analytical method of treating thermionic vacuum tube circuit problems than is now available.

* * *

* Refer to paragraph 2 of chapter III, page 22 et seq.

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* * * *

A P P E N D I C E S

A P P E N D I X   A

A List Of The More Important Symbols

i_p	....	plate current of the tube
i_G	....	grid current of the tube
i	....	a coil current
e_p	....	plate potential - referred to negative side of the filament
e_G	....	grid potential - referred to negative side of the filament
e_c	....	a condenser voltage
L	....	coil inductance in henrys
C	....	capacity of a condenser in mfds.
M	....	mutual inductance in henrys
R	....	resistance in ohms
T	....	time of one period in seconds
f	....	frequency in cycles per second
R_p	....	differential plate resistance - ohms
G_m	....	mutual conductance in micromhos
μ	....	amplification constant of a tube
Z	....	impedance of a tuned parallel circuit

A P P E N D I X B

DATA FOR STATIC CHARACTERISTICS UX - 112 A *

E_p 2

E_G	i_p
0	-
5	1.3
10	1.4
15	1.4
20	1.4
30	1.4
40	1.4
50	1.4
60	1.4

E_p 4

E_G	i_p
5	2.81
10	5.0
15	5.7
20	5.8
25	5.8
30	5.85
35	6.0
40	6.0
50	6.0
60	6.1
65	5.9
70	6.0
80	6.0

E_p 6

E_G	i_p
5	3.8
10	8.0
15	10.4
20	11.6
25	12.4
30	12.8
35	13.1
40	13.2
45	13.2
50	13.3

E_p 10

E_G	i_p
-5	0
0	.3
5	4.4
7.5	7.8
10	10.9
12.5	14.3
15	17.7
20	23.5
25	27.5
30	30.5
35	32.8
40	34.1
45	35.2
50	36.2
55	37.0

E_p 14

E_G	i_p
10	14.0
15	21.8
20	26.9
25	32
30	36.2
35	39.2
40	41.5
45	43
50	44
55	44.5
60	45
75	45

E_p 18

E_G	i_p
10	15.8
15	25.5
20	32.1
22.5	35
25	37
27.5	37

E_p 18

E_G	i_p
30	41.5
32.5	43.8
35	46
37.6	48
40	49.7
42.5	51
50	53.5
55	54.5
60	55
65	55.5
70	55
75	55
80	54.4

E_p 22

E_G	i_p
10	18.5
15	29.5
20	40
25	45.5
30	49.1
35	51.5
40	53.8
45	56
50	57.5
55	58
60	58.5

E_p 24

E_G	i_p
10	18.6
15	31.0
20	41.8
22.5	46.8
25	49.3
27.5	52
30	53.2
32.5	54.3
35	55.2
37.5	55.9

* Filament voltage constant at 5 volts.
The Values of plate current (i_p) are in milliamperes

(Appendix B)

$E_p = 24$

E_G	i_p
40	56.4
42.5	57
45	57.6
47.5	58.5
50	58.5
52.5	58.7
55	58.7
55	62.3
57.5	62
60	61.5
62.5	61
70	63
75	63
80	63

$E_p = 28$

E_G	i_p
10	20
15	34
20	48
22.5	54
25	58.5
27.5	62
30	64.1
32.5	65.5
35	66.5
37.5	67
40	67
42.5	66
45	65
47.5	63.8
50	63
52.5	62
55	61
57.5	59.5
60	58

$E_p = 32$

E_G	i_p
20	51.9
26	65
30	75

$E_p = 32$

E_G	i_p
32.5	77.5
35	79
37.5	79
40	79
42.5	78.5
45	77.2
47.5	75.5
50	73
52.5	71
55	69
57.5	67
60	64

$E_p = 34$

E_G	i_p
20	53.1
25	67.2
30	79
32.5	82.5
35	84.5
37.5	85.5
40	85.5
42.5	85
45	84
47.5	82.5
50	80.2
53.5	78.5
55	74
57.5	71.5
60	70

$E_p = 36$

E_G	i_p
20	55
26	70
30	82
32.5	87
35	90
37.5	91
40	91.5
42.5	91
45	89.5
47.5	87.2
50	85.8
52.5	83
55.0	81

$E_p = 40$

E_G	i_p
10	25.5
20	57
30	91
32.5	95
35	103.5
37.5	107
40	109
42.5	110
45	109
47.5	107
50	105
53.5	102.5
55	101
57.5	97
60	94
62.5	85

$E_p = 44$

E_G	i_p
20	60
30	93.5
35	107
37.5	112
40	114
42.5	115
45	115
47.5	114
50	113
52.5	111
55	110
57.5	107
60	104
62.5	97

$E_p = 48$

E_G	i_p
30	97
35	112
37.5	117
40	121
42.5	124
45	126
47.5	126

(Appendix B)

$E_p = 48$

E_G	i_p
50	125
52.5	124
55	123
57.5	122
60	119

$E_p = 50$

E_g	i_p
-10	0
-7.5	.006
-5.0	.42
-2.5	2.2
-5	.42
0	5.8
5	15.8
10	29
15	44.5
20	61.5
25	80
30	98.2
35	115
40	130
45	142

$E_p = 60$

E_G	i_p
-10	0
-7.5	.068
-5	.90
-2.5	3.3
0	7.3
5	18.8
10	32.5
15	48.0
20	66
25	85
30	103.5
35	122
40	138
42.5	147

$E_p = 70$

E_G	i_p
-12.5	0
-10	.005
- 7.5	.34
- 5	1.93
- 2.5	5.0
0	9.7
5	22.0
10	36.1
15	53
20	70.9
22.5	80.2
25	90.0
27.5	99
30	109.1
32.5	118.5
35	127.
37.5	136
40	144

$E_p = 80$

E_G	i_p
-12.5	0
-10	0.082
- 7.5	0.94
- 5.	3.2
- 2.5	7.0
0	12.4
2.5	18.5
5	25.5
7.5	32.5
10	40
15	57.5
20	75.8
25	95.5
30	115.
35	132.5
37.5	144.

$E_p = 90$

E_G	i_p
-12.5	0.0072
-15	0.0000

$E_p = 100$

E_G	i_p
-10	0.96
- 7.5	3.2
- 5.0	7.0
- 2.5	12.2
0	18.3
2.5	25.5
5	33
7.5	40.8
10	48.5
15	67.0
12.5	57.6
17.5	76
20	86
25	106.5
27.5	117
30	127
32.5	138

$E_p = 90$

E_G	i_p
-10	0.36
- 7.5	1.9
- 5	4.9
- 2.5	9.3
0	15.2
2.5	21.9
5	29.1
7.5	36.1
10	44.7
15	62.0
20	80.9
25	100.9
30	121
35	140

$E_p = 110$

E_G	i_p
-17.5	0
-15	0.24
-12.5	0.5
-10	2.2
- 7.5	5.4

(Appendix B)

$E_p = 110$

E_G	i_p
- 5.0	9.8
- 2.5	15.7
0	22.5
2.5	30
5	37.5
7.5	45.5
10	54.2
12.5	63.0
15	72.5
20	92.5
25	113
30	133
32.5	143

$E_p = 120$

E_G	i_p
-20	0
-17.5	0.0013
-15	0.146
-12.5	1.1
-10	3.5
- 7.5	6.4
- 5	12.5
- 2.5	18.5
0	26
2.5	33.6
5	41.2
7.5	50
10	58
12.5	68
15	77.5
17.5	87.6
20	98.0
22.5	108
25.	117
27.5	128.5
30	140

$E_p = 130$

E_G	i_p
-20.0	0
-17.5	0.034
-15	0.54
-12.5	2.2

$E_p = 130$

E_G	i_p
-10	5.3
- 7.5	9.8
- 5.0	15.2
- 2.5	22.0
0	29.5
2.5	37.5
5	46
7.5	54.1
10	63
12.5	73
15	83
17.5	93
20	103
22.5	113.5
25	123.0
27.5	134
30	146

$E_p = 140$

E_G	i_p
-22.5	0
-20	0.0033
-17.5	0.197
-15	1.26
-12.5	3.6
-10	7.3
- 7.5	12.4
- 5	18.4
- 2.5	25.5
0	33.2
2.5	41.8
5	50
7.5	59
10	67.5
12.5	77.5
15	88.
17.5	98
20	109
22.5	119
25	130.5
27.5	142

$E_p = 150$

E_G	i_p
-22.5	0
-20	0.05
-17.5	0.62
-15	2.3
-12.5	5.4
-10	9.8
- 7.5	15.3
- 5	21.9
- 2.5	29.1
0	37.5
2.5	46
5	54.7
7.5	63
10	72.5
12.5	82.9
15.	93.1
17.5	103.5
20	114.5
22.5	126
25	137
27.5	148

$E_p = 160$

E_G	i_p
-25	0
-22.5	0.0072
-20	0.258
-17.5	1.35
-15	3.8
-12.5	7.5
-10	12.4
-12.5	7.5
- 7.5	18.4
- 5	25.5
- 2.5	33.5
0	42
2.5	50
5	59
7.5	68.5
10	77.6
12.5	88.8
15	99.1
17.5	109
20	119
22.5	131
25	142

(Appendix B)

$E_p = 170$

E_G	i_p
- 25	0
-22.5	0.077
-20	0.74
-17.5	2.4
-15	5.4
-12.5	9.8
-10	15.1
- 7.5	21.9
- 5	28
- 2.5	37
0	46.1
2.5	55
5	64
7.5	73.5
10	83
12.5	94.1
15	104
17.5	115
20	126
22.5	136
25	146

$E_p = 180$

E_G	i_p
-27.5	0
-25	0.0133
-22.5	0.30
-20	1.43
-17.5	3.7
-15	7.4
-12.5	12.2
-10	18
- 7.5	25
- 5	32.8
- 2.5	41
0	50.5
2.5	59
5	69
7.5	78
10	88.5
12.5	97.6
15	107.5
17.5	117.8
20	128.7
22.5	138.5
25	148.5

$E_p = 200$

E_G	i_p
-30	0
-27.5	0.025
-25	0.29
-22.5	1.3
-20	3.5
-17.5	7
-15	11.6
-12.5	17.3
-10	24.
- 7.5	31.2
- 5	39.2
- 2.5	48.5
0	58
2.5	68.5
5.	77.5
7.5	88
10	98
12.5	109
15	120.5
17.5	130
20	139
22.5	149
25	-

$E_p = 220$

E_G	i_p
-32.5	0
-30	0.028
-27.5	0.34
-25	1.4
-22.5	3.6
-20	7.0
-17.5	11.4
-15	17.0
-12.5	23.9
-10	31.1
- 7.5	39.1
- 5	48.4
0	67
2.5	77.5
5	87.5
7.5	97
10	108
12.5	119
15	129
17.5	139.5

$E_p = 240$

E_G	i_p
-35	0.0002
-32.5	0.038
-30	0.38
-27.5	1.61
-25	3.8
-22.5	7.4
-20	11.8
-17.5	17.1
-15	23.4
-10	38.9
- 7.5	47.5
- 5.	56.1
- 2.5	66.5
0	77
2.5	87.1
5	97.4

$E_p = 260$

E_G	i_p
-37.5	0.0005
-35	0.052
-32.5	0.44
-30	1.6
-27.5	3.9
-25	7.3
-22.5	11.8
-20	17.3
-17.5	23
-15	30.5
-12.5	38.2
-10	47
- 7.5	55.9
- 5	65.4
- 2.5	76
0	86.9

$E_p = 280$

E_G	i_p
-42.5	0
-40	0.001
-37.5	0.063
-35	0.51

(Appendix B)

E_p = 280

E_G	i_p
-32.5	1.7
-30	4.0
-27.5	7.5
-25	11.6
-22.5	17.1
-20	23.4
-17.5	30.2
-15	37.9
-12.5	46.3
-10	55.2
- 7.5	65
- 5	75
- 2.5	85.1

E_p = 300

E_G	i_p
-45	0
-42.5	0.0024
-40	0.079
-37.5	0.54
-35	1.80
-32.5	4.00
-30	7.5
-27.5	11.6
-25	17.2
-22.5	23
-20	30.2
-17.5	37.9
-15	46.2
-12.5	55
-10	64
- 7.5	75.1

E_p = 340

E_G	i_p
-50	0
-45	0
-40	2.0
-37.5	4.3
-35	7.8
-32.5	12.3
-30	17.6
-27.5	23.4
-25	29.6
-22.5	36.5
-20	45
-17.5	54
-15	63
-12.5	73

APPENDIX C

Containing Data On Integrator Displacements

Part 1. Data For The Oscillator Problem:

Integrator II *

($\delta = 54$)

M	Turns Displ.
.60	32.4
.55	29.7
.50	27.0
.45	24.3
.40	21.6
.35	18.9
.30	16.2

Integrator III *

($\alpha = 64/421875$)

C mfd.	1/C	Turns Displ.
7.409	134900	20.48
15.117	66170	10.02

Integrator V *

($\beta = 256/1125$)

R	Turns Displ.
10	2.275
20	4.50
30	6.83
40	9.10
50	11.38
60	13.65
70	15.93

Integrator IV

(Constant equals 2/9)

volts V	initial $\frac{di_2}{dt}$	Turns Displ.
10	-10	-2.22
15	-15	-3.33
20	-20	-4.44
25	-25	-5.55
30	-30	-6.66
40	-40	-8.88
60	-60	-13.33
120	-120	-26.65

Note:

* Used to introduce a constant.
The displacements of integrator I were automatically taken care of when the initial value of i_p was "cranked in". They were not recorded.

(Appendix C)

Part 2. Data For Amplifier Problem:

Integrator II *

(β = 38/480000)

C mfd.	Turns Displ.
8	9.90
6	13.20
5	15.82
4	19.80
3	26.40
2.5	31.68
2.0	39.60

Integrator III *

(α = 512/57)

L hys.	Turns Displ.
.3170	28.32
.4224	21.25
.5065	17.74
.6340	14.17
.8450	10.61
1.013	9.10
1.269	7.08

Integrator V *

(δ = 38/200)

R Ohms	Turns Displ.
5	0.95
10	1.90
13.32	2.530
15	2.85
15.97	3.036
20	3.80
25	4.75
50	9.50

Note:

* Used to introduce constant.
For all runs integrator IV was initially set on zero.
The displacements of integrator I were automatically taken care of when the initial value of i_p was "cranked in". They were not recorded.

A P P E N D I X   D

Containing Special Data Pertaining To Runs 3, 5, 11
(See Table IV)

Part 1. Data Obtained from results of Run No.3 - which are reproduced in Figure 20. (In body of paper) The values of e_G, e_p, i_p, and i_2 tabulated below were scaled from curves given by the machine. The ordinate numbers agree with those marked on Figures 20 and 32. The dynamic characteristic shown in Figure 24 was reconstructed from data given below. The values of tube parameters given below were calculated from data graphically obtained from the static curves. The values of the various parameters are plotted versus ordinate number in Figure 26.

Ord. No.	volts e_G	volts e_p	m.a. i_p	amps. i_2	micro-mhos G_m	ohms R_p	μ
0	0	111.5	21.4	.363	2790	2860	7.99
1	-10	132.1	7.5	.367	1333	5600	7.48
2	-19.7	152.0	1.6	.363	0	inf.	0
3	-28.8	173.4	0	.351	0	inf.	0
4	-39.4	192.4	0	.329	0	inf.	0
5	-47.4	209	0	.301	0	inf.	0
6	-54.6	227	0	.263	0	inf.	0
7	-61.2	240	0	.224	0	inf.	0
8	-65.6	250	0	.177	0	inf.	0
9	-69.1	256	0	.134	0	inf.	o
10	-70.8	260	0	.080	0	inf.	0
11	-71.3	263	0	.0304	0	inf.	0
12	-70.2	260.5	0	- .024	0	inf.	0
13	-67.8	257	0	- .0735	0	inf.	0
14	-63.8	249	0	- .121	0	inf.	0
15	-58.5	239	0	- .163	0	inf.	0
16	-51.9	227	0	- .200	0	inf.	0
17	-45.6	215.6	0	- .232	0	inf.	0
18	-38.2	201	0	- .262	0	inf.	0
19	-29.7	185	0	-.282	0	inf.	0
20	-21.05	169	1.4	- .294	332	24300	8.06
21	-12.0	152	6.75	-.298	1430	4590	6.56
22	- 3.47	135	22.4	-.295	2560	3030	7.76
23	+ 5.33	115	44.5	-.287	3300	2350	7.76
24	14.68	95.2	66.0	-.270	3650	1975	7.21
25	24.0	77.0	91.	-.246	3870	1905	7.36
26	33.4	56.7	113.	-.207	3300	1405	4.63
27	41.5	39.5	104.	-.164	235	244	.057
28	47.1	27.7	67.5	-.119	-410	342	- .140

(Appendix D)

Ord. No.	volts e_G	volts e_p	m.a. i_p	amps. i_g	micro-mhos G_m	ohms R_p	μ
29	50.4	20.3	55.	-.0712	188	1020	.192
30	52.0	16.5	49.7	-.0232	188	416	.078
31	51.9	15.2	48.4	+.0368	211	416	.088
32	49.8	16.9	50.2	+.0944	235	426	.100
33	46.7	20.6	55.	.144	211	689	.145
34	42.7	27.7	66.9	.190	-235	450	-.106
35	37.6	36.5	85.4	.230	211	375	.079
36	32.7	46.6	103	.265	2410	882	2.12
37	25.8	59.1	85.3	.302	3670	2165	7.94
38	18.67	74.9	60.7	.329	3670	2195	8.06
39	10.13	91.5	37.4	.349	3470	2350	8.15
40	0	111.4	21.4	.363	2790	2860	7.99

Information On Calculation Of Tube Parameters:

The values of G_m were computed from the slopes of the characteristic curves at the various points in the operating cycle. The work was done on a set of static curves made from the tracing of which Fig. 24 is a photostat. The original tracing measured 18"x24" so the graphical work could be done with accuracy. In determining G_m the slopes of the curves were measured at the desired points by means of a straight edge and protractor. The angle between the tangent to the curve and the horizontal axis was determined in each case. The scales to which the curves were plotted were such that the expression for G_m became:

$$G_m \text{ equals } 1333 \tan A \text{ micromhos}$$

where A is the angle the tangent line makes with the horizontal.

The values of R_p were also obtained directly from the curves. At each point the value of R_p was found by taking the ratio of small increments of e_p and i_p.

$$R_p \text{ equals } \frac{\Delta e_p}{\Delta i_p}$$

The values of the amplification factor (μ) were computed from the known values of G_m and R_p by means of the relation:

$$\mu \text{ equals } R_p \, G_m$$

In this paper the usual definitions of the tube parameters have been used.

(Appendix D)

Definitions of tube parameters:

Amplification factor $\mu = \left| \frac{de_p}{de_G} \right|$ for constant i_p

Differential plate resistance

$$R_p = \left(\frac{de_p}{di_p} \right) \quad \text{for constant } e_G$$

Mutual Conductance

$$G_m = \left(\frac{di_p}{de_G} \right) \quad \text{for constant } e_p$$

Part 2. The following data was scaled from the oscillograms shown in figures 27, 28, and 29. These oscillograms were taken for cct. conditions corresponding to those in Run No.3

Ord. No.	volts e_G	volts e_c	volts e_p	m.a. i_p	amps. i_2
0	-10	4.	144	11.2	.340
1	-24.25	27	127	0	.336
2	-31.6	35	175	0	.322
3	-38.5	46	186	0	.298
4	-47	58	198	0	.262
5	-54.5	81.3	221	0	.217
6	-60.7	94	234	0	.161
7	-65.8	108	248	0	.119
8	-68.1	119	259	0	.0704
9	-70.4	126	266	0	.0181
10	-71	129	269	0	-.0402
11	-69.8	129	269	0	-.0804
12	-67.	128	268	0	-.121
13	-62.4	121	261	0	-.159
14	-57.3	107	247	0	-.191
15	-53.3	90	230	0	-.221
16	-45.3	71.9	212	0	-.245
17	-39.1	55.7	196	0	-.262
18	-31.7	34	174	0	-.268
19	-22.5	8	148	0	-.270
20	-10	-10	130	6.72	-.264

(Appendix D)

Ord. No.	volts e_G	volts e_c	volts e_p	m.a. i_p	amps. i_2
21	+ 0.82	- 24	116	23.5	-.251
22	8.24	- 44	96	35.8	-.239
23	15.6	- 56	84	49.3	-.221
24	24.2	-67	73	70.5	-.181
25	30.4	- 81	59	81.8	-.161
26	33.3	- 93	47	85.1	-.131
27	34.4	-104	36	76.1	-.0804
28	32.2	-111	29	58.2	-.0302
29	31.6	-119	21	52.6	.0201
30	34.4	-123	17	53.8	.0765
31	35.6	-125	15	65.0	.121
32	39.0	-124	16	80.6	.167
33	41.8	-122	18	103.0	.199
34	45.8	-116	24	137.8	.229
35	47.6	-108	32	150.0	.259
36	48.2	- 87	53	156.0	.288
37	45.8	- 65	75	143.0	.318
38	29.9	- 37	103	95.2	.328
39	8.23	- 13	127	54.8	.338
40	-10.	4	144	11.2	.340

Part 3. The following Data were obtained from the results of Run No. 5 (see page 56) and were used to plot the dynamic characteristic shown in Fig. 25. (See page 61.) The ordinate numbers correspond to those on page 56.

Ord. No.	volts e_G	volts e_p
0	0	95
1	- 6.0	112
2	-12	129
3	-18	147
4	-23.7	165
5	-28.1	179
6	-33.2	196
7	-37.3	210
8	-40.6	221
9	-42.7	230
10	-42.7	230
11	-45.3	238

(Appendix D)

Ord. No.	volts e_G	volts e_p
12	-45.2	239
13	-44.7	238
14	-42.1	236
15	-39.6	229
16	-36.5	220
17	-32.8	212
18	-28.6	202
19	-24.0	188
20	-19.3	176
21	-14.0	163
22	- 9.2	147
23	- 3.33	138
24	1.33	199
25	6.0	105
26	10.0	97
27	14.4	79
28	18.1	67
29	21.2	56
30	23.4	47
31	24.7	41
32	25.3	37
33	24.8	36
34	23.4	37
35	21.3	41
36	18.0	48
37	14.6	56
38	9.95	67
39	4.67	80
40	0	95

Part 4. The following data were obtained from the results of Run No. 11 (see page 58 .) and were used to plot the dynamic characteristic shown in Fig. 25. (See page 61 .) The ordinate numbers correspond to those on page 58 .

Ord. No.	volts e_G	volts e_p
0	-69.2	288
1	-68.1	287
2	-65.3	282
3	-60.6	271
4	-56.0	259
5	-50.0	248
6	-42.0	226
7	-33.3	205
8	-24.7	185
9	-15.3	163
10	- 6.66	140

(Appendix D)

Ord. No.	volts e_G	volts e_p
11	2.67	116
12	12.00	91
13	20.7	68
14	29.7	47
15	38.3	27
16	43.9	11
17	47.3	0
18	49.3	- 6
19	50.0	- 8
20	48.9	- 7
21	46.7	- 3
22	42.6	5
23	38.0	15
24	32.7	27
25	26.6	39
26	20.4	55
27	12.66	73
28	5.00	95
29	- 3.33	115
30	-12.66	137
31	-22.7	163
32	-32.0	187
33	-40.7	209
34	-47.3	230
35	-54.3	247
36	-60.2	263
37	-64.5	275
38	-67.9	283
39	-69.0	287
40	-69.2	288

A P P E N D I X   E

Part 1. Data For Dynamic Characteristic Run No. 12A
Following values were scaled from curves in Fig. 42

Ord. No.	volts e_G	volts e_2	volts e_p	mills i_p
0	-10	-25.8	134.2	6.00
1	- 8.6	-26.95	133.0	8.66
2	- 7.4	-27.2	133.0	10.7
3	- 6.1	-27.1	133.0	13.2
4	- 5.0	-26.95	133.0	15.7
5	- 4.0	-26.2	134.0	19.2
6	- 3.0	-25.5	135.0	22.7
7	- 2.1	-23.6	136.0	25.8
8	- 1.4	-21.6	138.0	28.5
9	- 0.8	-19.0	141	31.1
10	- 0.5	-16.5	143	33.3
11	- 0.2	-13.5	146	35.1
12	0	-10.7	149	36.8
13	- 0.2	- 7.15	153	37.8
14	- 0.5	- 3.91	156	38.2
15	- 0.8	- 0.95	159	37.9
16	- 1.4	2.7	163	37.3
17	- 2.1	5.66	166	36.4
18	- 3.0	9.31	169	34.9
19	- 4.0	12.3	172	32.9
20	- 5.0	15.4	175	31.1
21	- 6.1	18.1	178	28.9
22	- 7.4	20.5	181	25.9
23	- 8.6	23.1	183	23.1
24	-10.0	25.5	186	20.1
25	-11.4	27.2	187	17.4
26	-12.6	28.3	188	14.9
27	-13.8	29.4	189	12.5
28	-15.0	29.5	190	10.3
29	-16.0	29.4	189	8.3
30	-17.0	28.6	189	6.45
31	-17.9	27.5	188	4.98
32	-18.6	25.8	186	3.64
33	-19.2	23.1	183	2.67
34	-19.5	20.5	181	2.00
35	-19.8	17.4	177	1.55
36	-20.0	13.6	174	1.11
37	-19.8	9.7	170	.93
38	-19.5	5.40	165	.89
39	-19.2	1.48	161	.89
40	-18.6	- 1.89	158	.89
41	-17.9	- 5.66	154	.89
42	-17.0	- 9.98	150	.93
43	-16.0	-13.5	146	1.29

(Appendix E)

Ord. No.	volts e_G	volts e_2	volts e_p	mills i_p
44	-15.0	-17.3	143	1.56
45	-13.9	-20.1	140	2.14
46	-12.6	-22.6	137	3.02
47	-11.4	-24.6	135	4.22
48	-10	-25.8	134	6.0

Part 2. Data for Dynamic Characteristic Run No. 13A
Following Values scaled from Curves in Fig. 43

Ord. No.	volts e_G	volts e_2	volts e_p	mills i_p
0	-10	26.3	186	20.4
1	- 8.6	24.4	184	23.5
2	- 7.4	22.8	183	26.2
3	- 6.1	19.55	179	28.8
4	-5.0	16.85	177	31.5
5	- 4.0	13.48	173.5	33.7
6	- 3.0	9.84	169.8	36.0
7	- 2.1	5.66	165.7	37.8
8	- 1.4	1.48	161.5	38.6
9	- 0.8	- 1.89	158.1	38.6
10	- 0.5	- 5.39	155	38.0
11	- 0.2	- 9.44	151	37.3
12	0	-12.40	148	36.2
13	- 0.2	-16.19	144	34.6
14	- 0.5	-18.88	141	32.8
15	- 0.8	-21.6	138	30.2
16	- 1.4	-24.1	136	27.5
17	- 2.1	-25.8	134	24.4
18	- 3.0	-27.4	133	21.3
19	- 4.0	-28.3	132	18.2
20	- 5.0	-28.3	132	15.2
21	- 6.1	-28.2	132	12.1
22	- 7.4	-28.2	132	9.8
23	- 8.6	-26.4	134	7.45
24	-10	-24.9	135	7.68
25	-11.4	-23.0	137	3.99
26	-12.6	-20.9	139	3.11
27	-13.8	-17.5	142	2.22
28	-15.0	-14.7	145	1.55
29	-16.0	-11.85	148	1.24
30	-17.0	- 8.09	152	0.89
31	-17.9	- 5.12	155	0.71
32	-18.6	- 0.10	159	0.53
33	-19.2	+ 2.02	162	0.53

(Appendix E)

Ord. No.	volts e_G	volts e_2	volts e_p	mills. i_p
34	-19.5	5.66	166	0.53
35	-19.8	8.35	168	0.62
36	-20	11.9	172	0.80
37	-19.8	14.8	175	0.93
38	-19.5	17.7	178	1.42
39	-19.2	20.2	180	2.18
40	-18.6	22.8	183	3.02
41	-17.9	24.3	184	3.99
42	-17.0	25.8	186	5.41
43	-16.0	27.1	187	7.46
44	-15.0	27.8	188	9.37
45	-13.9	28.3	188	11.80
46	-12.6	27.9	188	14.25
47	-11.4	27.1	187	17.31
48	-10.0	26.3	186	20.4

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