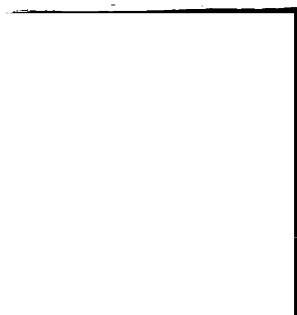


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PEASANT AND CAPITALIST AGRICULTURE
IN THE
DEVELOPING COUNTRY

by

KENT P. ANDERSON

B.A., The University of Michigan
(1964)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF
PHILOSOPHY

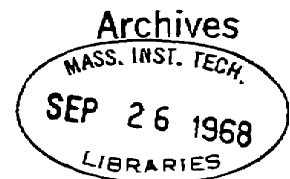
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Kent P. Anderson

Submitted to the Department of Economics on August , 1968,
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Economics.

ABSTRACT

This thesis is a theoretical and empirical study of the condition of dualism within the agricultural sector of a developing country. Chapter I develops the theoretical conditions under which dualistic equilibrium will exist and some of the welfare properties of equilibrium all largely within the framework of a static world of unchanging technologies and stocks of productive factors. In Chapter II analysis is extended to the case of changing technologies and factor endowments under a set of factor market imperfections which insure the existence of static dualistic equilibrium. Chapter III is devoted to answering the question of what is an optimum organization of the agricultural sector, as it is described by the theory in Chapter II. In Chapter IV some of the effects of the intersectoral relationships between agriculture and non-agriculture are given more explicit analysis. Chapter V surveys the characteristics of dualistic agriculture in Malaya as it is evidenced in the rubber industry there.

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I, alone, of course, am responsible for any errors or lacunae.

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CHAPTER I

THE EXISTENCE AND NATURE OF AGRICULTURAL DUALISM:

STATIC THEORY

The concept of economic dualism is founded on the notion that one sector of an economy may possess special properties or be related to the rest of the economy in a particular way not readily accounted for by conventional theory. Dualistic theories have thus focussed upon special intersectoral relations and asymmetries. That dualism may be an important feature of the developing economy is now widely accepted, and the properties of the dual economy have been explored at some length by a number of writers.^{1/} Up to this time the idea of focussing on special relations and asymmetries has been applied mainly to the case of a two-sector economy consisting of traditional or peasant agriculture and modern capitalist non-agriculture or "manufacturing," under the assumption that each sector was homogeneous. It is clear, however, that the breakdown of traditional versus modern enterprise may, in fact, find wide application within both the agricultural and non-agricultural sectors. The object of this chapter is to study on a theoretical level the implications of the existence of a dualistic structure within the agricultural sector of a developing country.

First, a model of dualistic agriculture developed by A.K. Sen will be presented and discussed.^{2/} Then, using his model as a starting point, new assumptions will be made, and the properties of the

model under various alterations, usually increasing in complexity, and, it is hoped, in proximity to reality, will be studied. The primary questions to which answers will be sought are the following: Under what conditions will the market mechanism, unimpeded by government policy, thought not necessarily perfectly competitive, produce a natural displacement of traditional by modern agriculture or vice-versa? Under what circumstances will displacement, either natural or policy induced, be desirable from an economic welfare or development standpoint?

Consider now a slightly modified version of A.K. Sen's static model of dualistic agriculture. Make the following assumptions: Traditional agriculture consists of family utility maximizing farmers, called peasants, who use only family labor. Modern agriculture is comprised of profit maximizing capitalist enterprises which hire labor in a perfectly competitive labor market. Output is homogeneous and identical in the peasant and capitalist sectors and is either sold in a perfectly competitive market at a fixed price, which may be set equal to one without loss of generality, or directly consumed. Production functions are identical and neoclassical.^{3/} (Note that due to the assumption of constant returns to scale implied by a neoclassical production function, it is impossible to say anything about the relative scales of operation of the two kinds of enterprises. We shall discuss later the influence of such factors as a minimal scale of capital required for modern agricultural practice.) All capitalist firms are identical, as are also all peasant units. Land and capital

goods in each sector are treated as a single composite good, called capital. The capital stock in each sector is fixed and may not be transferred.

Next, define the following symbols: Let

P_p = peasant population

N_p = peasant labor force = $a \cdot P_p$, a constant, $0 < a < 1$,

L_p = peasant labor expenditure,

$V = V(L_p/N_p) =$ (positive) disutility of labor expenditure per worker, $V'(\cdot) > 0$, $V''(\cdot) > 0$,

$Q_p = F(K_p, L_p) =$ peasant production as a function of factor inputs, ($F(\cdot)$ is neoclassical),

$K_p =$ peasant capital stock,

$U = U(Q_p/P_p) =$ utility of output per head, $U'(\cdot) > 0$, $U''(\cdot) < 0$,
and

$$f\left(\frac{L_p}{K_p}\right) = \frac{F(L_p, K_p)}{K_p} = \text{output per unit of capital.}$$

Peasant allocations of labor are determined by maximizing a family welfare function in which all individuals are treated identically. Now K_p , N_p , and P_p are constant so that family welfare may be altered only by varying L_p . Assume that family welfare is measured by the utility of output per head multiplied by the number of heads minus the disutility of labor expenditure per worker times the number of workers. The expression for family welfare, W , may then be written as follows:

$$(1.1) \quad W = P_p \cdot U \left[f \left(\frac{L_p}{K_p} \right) \cdot \frac{K_p}{P_p} \right] - N_p \cdot V \left(\frac{L_p}{N_p} \right).$$

The first order condition for a maximum is thus

$$\frac{\partial W}{\partial L_p} = 0 = P_p \cdot U' \cdot \frac{K_p}{P_p} f' \left(\frac{L_p}{K_p} \right) \frac{1}{K_p} - N_p V' \left(\frac{L_p}{N_p} \right) \frac{1}{N_p}.$$

Simplifying and rearranging terms, we have

$$(1.2) \quad f' \left(\frac{L_p}{K_p} \right) = \frac{V' \left(\frac{L_p}{N_p} \right)}{U' \left[f \left(\frac{L_p}{K_p} \right) \cdot \frac{K_p}{P_p} \right]}.$$

This condition states that the marginal productivity of labor must equal the marginal disutility of the individual work load divided by the marginal utility of per capita output. Since K_p , N_p , and P_p are fixed, peasant equilibrium is independent of the wage rate in capitalist agriculture or any other variable of capitalist agriculture. Equation (1.2) completely describes the equilibrium position of the peasant agricultural sector when no intersectoral flows of factors may occur.

Now, following Sen, suppose that workers may move between the capitalist and peasant sectors. Let N_c , L_c , K_c , and P_c be the capitalist variables corresponding to the peasant variables, N_p , L_p , K_p , and P_p , respectively. Let $N_c = a \cdot P_c$, i.e., let the worker-dependency ratio be the same in both sectors. Although it is to be expected that in many real situations this ratio will not be the same in both sectors, we shall abstract from this aspect of the problem in

order to have a more manageable model. Sen interprets the existence of a "wage gap" to mean that when the labor market is in equilibrium it is true that

$$f'(L_c/K_c) > f'(L_p/K_p),$$

i.e., the marginal productivity of labor is higher in the capitalist sector than in the peasant sector. If it is assumed that the wage gap may be expressed in terms of a constant proportional rather than absolute difference, then the equilibrium condition for the labor market may be written as follows:

$$(1.3) \quad f'(L_c/K_c) = \gamma \cdot f'(L_p/K_p), \quad \gamma > 1.$$

Since the results of subsequent analysis are not seriously affected, it is assumed that γ is constant.

Sen introduces no analytical relationship which would produce a wage gap endogenously, nor, as has just been noted, do we here, although it will be done later in the thesis ^{4/} There are a number of plausible reasons why a wage gap might exist. For example, there may be higher costs of living or a psychological disutility associated with working on capitalist farms instead of on the family plots. Minimum wage legislation or some other institutional arrangement such as a legal requirement to provide housing for laborers and their families may also produce a gap. This is easy to demonstrate. Suppose

that peasant and capitalist capital stocks are fixed, and that labor flows freely between sectors until the marginal product of labor (wage) in both sectors is the same. Suppose now that the government imposes a minimum wage in the capitalist sector which exceeds the previously attained equilibrium wage. Then capitalists will hire less labor than before to the extent that they behave as profit maximizers. The unemployed labor from the capitalist sector will be thrown back onto the peasant sector and will tend to depress the wage rate there by bidding down (explicitly, or implicitly by rejoining families on the old family land) the peasant wage (marginal product of peasant labor). In addition, there may exist real and psychological costs associated with the act of moving from the family unit to the capitalist farm or estate. Although the worker-dependency ratio has been assumed constant here, it is apparent that if that ratio is lower in the capitalist sector by virtue of institutional restrictions, for example, limitations on the amount of work performed by children or females, then the income per worker in the capitalist sector must be higher in order to provide the same income per head as on the family farm. Finally, if laborers on capitalist estates must work longer hours per day or with a more consistent effort per hour than on the family plots then income per worker may be expected to be higher for workers on capitalist farms than for those who stay with the family.

Sen concludes that for a given wage gap and given non-transferable capital stocks there exists a determinate dual equilibrium of peasant

and capitalist agriculture. But that this is not true is easily demonstrated. If labor is allowed to move freely so as to satisfy equation (1.3), if peasants allocate their labor according to condition (1.2), and if we have, in addition a third equation, stating that the total work force must be employed,

$$(1.4) \quad N_c + N_p = N,$$

where N is the total work force and is fixed and equal to $a \cdot P$, P being total population, then the system of equations describing the agricultural economy is underdetermined, since there are four unknowns, L_c , L_p , N_c , and N_p , and only three equations. In order to close the system it is necessary to make an assumption about how the work load in capitalist agriculture, L_c/N_c , is determined. Suppose that $L_c/N_c = \beta_c$, an institutionally determined parameter. When this is done, it is then easily demonstrated that static short-run equilibrium exists.

For convenience, the four equations of the model are brought together and rewritten below:

$$(1.2) \quad f'(L_p/K_p) = v'(L_p/N_p) = U'[f(L_p/K_p) \cdot a \cdot K_p/N_p]$$

$$(1.3) \quad \gamma f'(L_p/K_p) = f'(L_c/K_c)$$

$$(1.4) \quad N_c + N_p = N$$

$$(1.5) \quad L_c = \beta_c N_c$$

Using (1.4) and (1.5) to eliminate L_c and N_c , the labor market equation (1.3) may be written as follows:

$$(1.3') \quad f' \left(\frac{N-N_p}{K_c/\beta_c} \right) = \gamma f' \left(\frac{L_p}{K_p} \right).$$

Assume that $f'(0) = \infty$ and $f'(\infty) = 0$. Then by inspection of equation (1.3') it is evident that if $L_p = 0$, then $N_p = N$; and if $N_p = 0$, then $L_p = \bar{L}_p > 0$. It is also clear that dN_p/dL_p is negative when this equation holds.

Turning next to equation (1.2), observe that it is possible to compute dL_p/dN_p when that equation holds, i.e., when the family is obeying its welfare maximizing criterion. When this is done it is found that

$$\frac{dL_p}{dN_p} = \left[\left(\frac{\alpha f' U''}{N_p} - \frac{V''}{K_p} \right) \div \left(\frac{\alpha f' U'' K_p - V'' L_p}{N_p^2} \right) \right]^{-1}.$$

Since f'' and U'' are negative, V'' is positive, and f' is non-negative, it is clear that dL_p/dN_p is positive. If the family grows, so also does total family labor expenditure. If it is then assumed that $V'(0) = U'(\infty) = 0$ and $V'(\infty) = U'(0) = \infty$, it follows that equation (1.2) must emanate from the origin since it cannot hold when one variable is non-zero and the other is approaching zero.

It is now an easy task to graph these two equations. This has been done in Figure 1.1. Given the assumptions above, it is clear that there will exist one and only one intersection of the two curves and

that the equilibrium values of N_p and L_p are non-negative. The question of whether this equilibrium is stable will be ignored. It will be asked and answered for some later versions of the model.

Which agricultural sector will predominate depends on the size of the wage gap, γ , the work load in capitalist agriculture, β_c , and the fraction of population in the labor force, a . A large γ or β_c implies that a relatively large proportion of the population will comprise peasant agriculture, and small values of γ and β_c imply the opposite. The smaller is a , the heavier will be the work load in peasant agriculture and the greater will be the size of capitalist agriculture.

It has already been noted that equilibrium must involve a positive value for N_p . It should also be pointed out that the equilibrium value of N_c will be positive, since it is obvious from inspection of Figure 1.1 that $N_p^* < N$. This result is intuitively

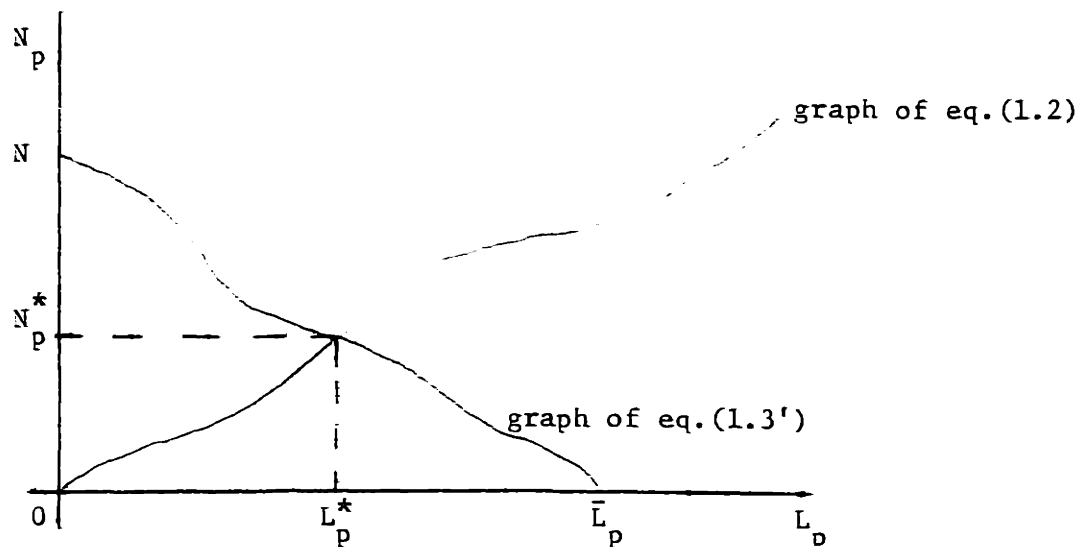


FIGURE 1.1

satisfying since an all capitalist or an all peasant equilibrium would imply the abandonment of the other sector's capital stock, which has been assumed to be non-transferable, in what is thus a short run kind of model.

Sen next considers what would happen when a perfect capital (land) market is introduced. He observes: "As long as the marginal productivity of land is higher for peasant farmers than for capitalist farmers [due to the wage gap], it will be in the interest of the capitalist farmer to rent his land out to peasants. The process of transfer will continue until either the labor-cost [wage] gap vanishes or, alternatively, all land owned by the capitalists is rented out."^{5/} Sen notes that the functioning of this mechanism is predicated on the assumption that there are no institutional barriers to such transfer.

The existence of a perfect capital market implies that in equilibrium, the marginal productivity of capital must be the same in both sectors. This requirement is expressed in equation (1.6) below. Equation (1.7) states simply that the capital stock must be fully employed, since K is the aggregate capital stock.

$$(1.6) \quad f\left(\frac{L_c}{K_c}\right) - \frac{L_c}{K_c} f'\left(\frac{L_c}{K_c}\right) = f\left(\frac{L_p}{K_p}\right) - \frac{L_p}{K_p} f'\left(\frac{L_p}{K_p}\right)$$

$$(1.7) \quad K_c + K_p = K.$$

It would appear that the model has been extended just appropriately to allow K_p and K_c to be determined endogenously. But equations (1.2)-(1.6) do not form a consistent system of equations. If equation (1.6) holds, then equation (1.3) holds if and only if γ is equal to one. In this case equations (1.3) and (1.6) imply and are implied by one another. When $\gamma \neq 1$, the system has no equilibrium, except in the important sense that no further shifting of capital and labor can occur after all have been transferred to the peasant sector.

The following discussion of disequilibrium behavior does not constitute a rigorous proof, but it is highly suggestive of the Sen result. Suppose that workers move from peasant family farms to capitalist farms when $f'(L_c/K_c) > f'(L_p/K_p) \cdot \gamma$ and in the reverse direction when the inequality is reversed and that capitalists rent capital to peasants when $f(L_p/K_p) - (L_p/K_p) \cdot f'(L_p/K_p) > f(L_c/K_c) - (L_c/K_c) \cdot f'(L_c/K_c)$, and peasants rent capital to capitalists when the inequality is reversed. Suppose that initially equation (1.6) is satisfied. This means that $f'(L_c/K_c) = f'(L_p/K_p)$ so that labor will flow to the peasant sector. Suppose this process continues until L_c/K_c has fallen and L_p/K_p has risen sufficiently to bring equation (1.3) into equality. (It is assumed that K_c and K_p have been held constant while this occurs.) Now equation (1.6) is no longer satisfied. In fact, since L_c/K_c has fallen and L_p/K_p risen, the left-hand side (LHS) of that equation is less than the right-hand side (RHS), and capitalists will want to rent capital to peasants.^{6/} Suppose that this takes place, L_p and L_c being held constant so that

no labor moves, until equation (1.6) is again satisfied. A situation similar to the initial one exists, but now both labor and capital have been transferred to peasants. This process will then be repeated until eventually all labor and capital have been transferred to peasants.

When $\gamma = 1$, i.e., there is no wage gap, the system has no determinate equilibrium, since either equation (1.3) or (1.6) is redundant. There will be an infinity of possible equilibrium allocations of capital and labor between the two sectors which satisfy equations (1.2)-(1.6). This would not necessarily be the case if production functions were not identical or of the constant returns to scale type. The effects of such modifying assumptions will not be investigated at this point, however.

The above Sen result showing peasant displacement of capitalists is not particularly satisfying. It requires a peculiar set of assumptions, in that the labor market is supposed to function imperfectly as a result of institutional flaws thought to be prevalent in developing countries while the capital market is imagined to be free of institutional defects, so that peasants and capitalists observe and are moved to eliminate any difference in capital productivity between sectors.

Suppose instead that there exists an imperfection in the capital market such that when the market is in equilibrium, the rate of return on capital is higher in the peasant sector than in the capitalist. In particular, suppose that the difference in the equilibrium rates

of return is a constant absolute amount, Δ . Then the capital market equilibrium equation will appear as follows:

$$(1.6') \quad f\left(\frac{L}{K_c}\right) - \frac{L}{K_c} f'\left(\frac{L}{K_c}\right) = f'\left(\frac{L}{K_p}\right) - \frac{L}{K_p} f'\left(\frac{L}{K_p}\right) - \Delta, \quad \Delta > 0.$$

The existence of a Δ can be rationalized on the grounds of such factors as the greater risk associated with financing peasant investment and the lack of institutional arrangements to transfer funds to relatively savings-poor peasant agriculture. Since peasants often operate at or near subsistence income levels, they are likely to be able to accumulate only modest savings, and the vagaries of weather and market conditions are likely to have significant effects on the surplus above and beyond subsistence consumption requirements that would be available to meet investment financing obligations.

If capitalists enjoy greater opportunities for investment then the assumption that the rate of return on capital is lower in peasant agriculture in equilibrium may be more appropriate than the one assumed here. For example, better access to current knowledge may allow capitalists to exploit the latest improvements in technology, where peasants in their ignorance or fear of innovative risk may fail to take advantage of them. The gap in rates of return would reflect the difficulty of transferring savings from opportunity-poor peasant agriculture to opportunity-rich capitalist agriculture. Note that this opportunity effect is qualitatively the reverse of the savings effect. That is, the relative "poverty" of savings in peasant agriculture may

be offset by a relative "poverty" of investment opportunity there. If such is the case, then even if there exist institutional difficulties in the transfer of capital between sectors, it may be possible for rates of return to be similar in both sectors or to vary in either direction, as noted above. We shall assume that the savings effect outweighs the opportunity effect, so that $\Delta > 0$, but it is important to realize that the case $\Delta < 0$ could be observed.

In order to facilitate further analysis the following simplifying assumption will be made: Suppose that $V'(L_p/N_p)$ behaves as shown in Figure 1.2. At $(L_p/N_p)^*$, $V''(\cdot)$ is assumed to be infinitely large. For $(L_p/N_p) < (L_p/N_p)^*$ assume that $V''(\cdot)$ is zero. Then, regardless of the size of $f'(\cdot) \cdot U'(\cdot)$, the family welfare maximizing level of L_p will be given by the equation

$$L_p = (L_p/N_p)^* N_p = \beta_p N_p.$$

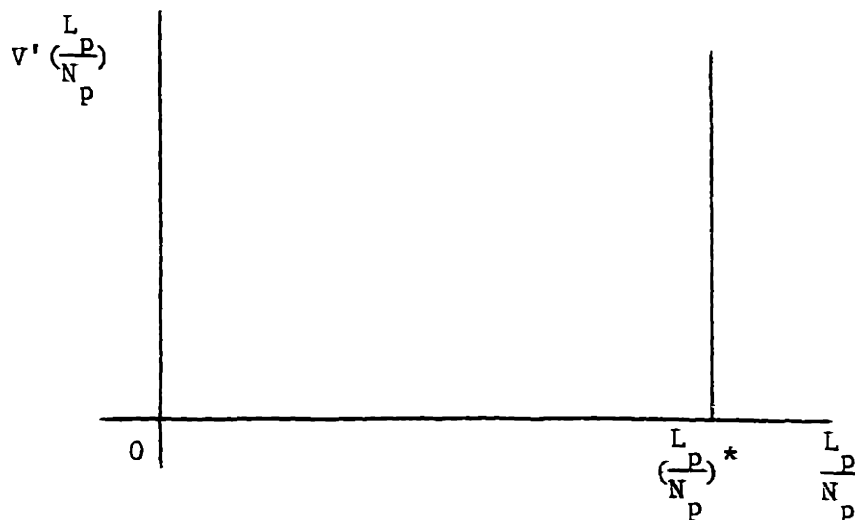


FIGURE 1.2

This simplifying assumption implies that the family maximizes its welfare by expending an amount of labor proportional to the number of individuals in the family. Assume further that $\beta_c = \beta_p$, i.e., that the individual work load is the same in each sector. Then in order to increase the labor-capital ratio in either sector it is necessary to increase the size of the labor force since increasing or decreasing the number of hours worked per man is ruled out. This assumption probably does not do great violence to analysis of long-run behavior, since over moderate stretches of time there is unlikely to be any significant alteration of the individual work load, although it might exhibit a trend over the very long run, perhaps over the course of decades. Since now labor expenditure is directly proportional to the size of the labor force it will be useful to redefine the production function so that N_i , $i = c, p$, appears as one of its arguments rather than L_i , $i = c, p$.

Let this new production function be defined as follows:

$$G(K_i, N_i) = F(K_i, \beta_i L_i) \quad i = c, p.$$

The reader may verify that this new production function also possesses neoclassical properties. Further, define an "intensive unit" production function in the following way: Let

$$g(k_i) = g\left(\frac{K_i}{N_i}\right) = G\left(\frac{K_i}{N_i}, 1\right) \quad i = c, p.$$

Now defining two new variables, n_c and n_p ,

$$n_c = \frac{N_c}{N}$$

$$n_p = \frac{N_p}{N},$$

once can reconstruct the static model of dualistic agriculture in the following way:

$$(1.8) \quad g(k_c) - k_c g'(k_c) = \gamma [g(k_p) - k_p g'(k_p)]$$

$$(1.9) \quad g'(k_c) = g'(k_p) - \Delta$$

$$(1.10) \quad n_c + n_p = 1$$

$$(1.11) \quad k_c n_c + k_p n_p = k, \text{ where } k = \frac{K}{N}.$$

Equations (1.8)-(1.11) correspond to equations (1.3), (1.6'), (1.4), and (1.7), respectively. There are four equations and four unknowns, $k_c, k_p, n_c,$ and $n_p,$ but depending upon the gap parameters, Δ and $\gamma,$ the exact properties of the production function, $g(\cdot),$ and the endowments of capital and labor, K and $N,$ a solution to the system of equations may or may not exist.

Since equations (1.8) and (1.9) contain only k_c and $k_p,$ possible equilibrium values for those variables may be obtained by solving the two equations. If, given these values, it is impossible to select values of n_c and n_p to satisfy the factor employment constraints, equations (1.10) and (1.11), then it may be concluded that no dualistic equilibrium can exist. Either peasant or capitalist agriculture must

disappear. This conclusion applies à fortiori if equations (1.8) and (1.9) admit of no solution.

In order to examine the issue of the existence of dualistic equilibrium it will be helpful to consider the two market equilibrium curves in k_c, k_p space, i.e., the loci of (k_c, k_p) pairs for which each of equations (1.8) and (1.9) is satisfied. Consider first the latter equation. It is readily verified that along the equilibrium locus $dk_p/dk_c > 0$ and $k_p < k_c$. When $k_c = \infty$, $g'(k_p) = \Delta$ if it is assumed that $g'(\infty) = 0$, and when $k_c = 0$, $g'(k_p) = 0$ and hence $k_p = 0$. Since $dk_p/dk_c > 0$, we may write k_p as a function of k_c . Let the capital market equilibrium curve be written $k_p = \bar{k}(k_c)$. Consider next the labor market equilibrium curve. It is again easily verified that $dk_p/dk_c > 0$ and $k_p < k_c$. It is also obvious that $k_p = 0$ when $k_c = 0$. As in the case of the capital market, the labor market equilibrium curve may be expressed in terms of k_p as a function of k_c . Let this function be written $k_p = \tilde{k}(k_c)$. In Figure 1.3, two factor market

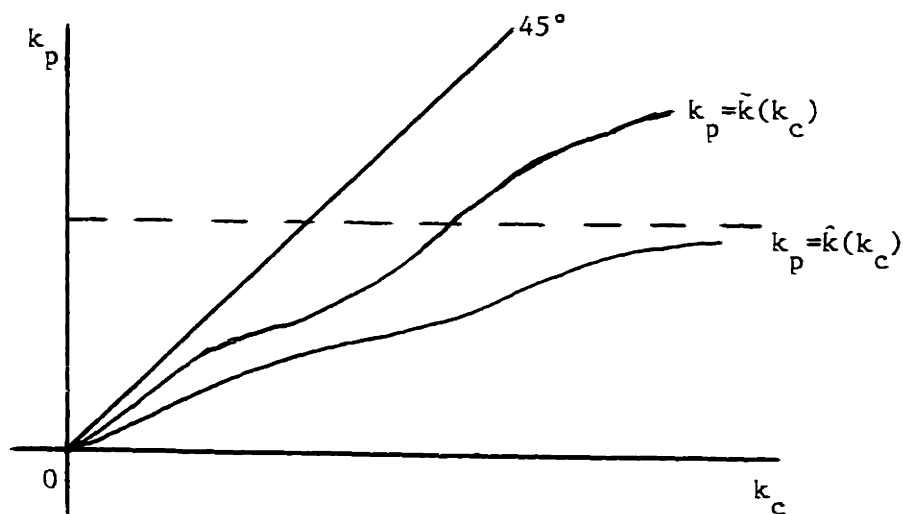


FIGURE 1.3

equilibrium curves have been drawn for a case where $g'(\cdot)$ has the limiting property mentioned above. Note that the two curves intersect only at the origin. It is clear that no guarantee of the existence of dualistic equilibrium is provided by the assumption of neoclassical production functions, even when the question of factor endowment compatibility is ignored. Nor is there any guarantee that multiple equilibria will not exist. In Figure 1.4 a pair of curves drawn in accordance with the known properties of the market equilibrium curves

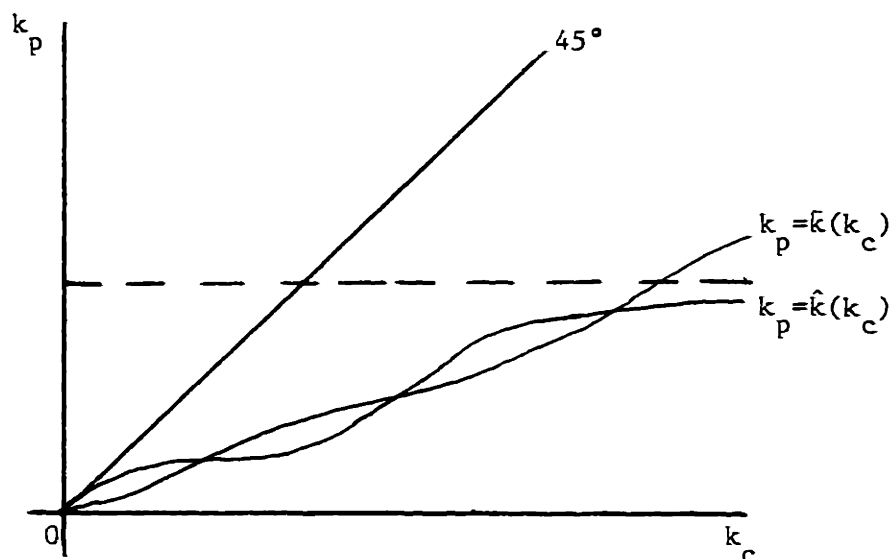


FIGURE 1.4

intersect more than once.

In order to get a better understanding of possible situations, we shall not consider a case where the production function assumes an explicit analytical form. Specifically, assume that $g(k_i) = k_i^{1-\alpha}$, $i=c,p$,

$0 < \alpha < 1$, α constant. In this case of a Cobb-Douglas production function it is easily shown that the equations of the two market equilibrium curves appear as follows:

$$k_p = \hat{k}(k_c) = \left(\frac{\Delta}{1-\alpha} + k_c^{-\alpha} \right)^{-\frac{1}{\alpha}}$$

$$k_p = \bar{k}(k_c) = \omega k_c, \quad \omega = \left(\frac{1}{\gamma} \right)^{1-\alpha} < 1.$$

Solving for the equilibrium values of k_c and k_p by means of these two equations it is found that

$$k_c^* = \left[\frac{1-\alpha}{\Delta} (\omega^{-\alpha} - 1) \right]^{\frac{1}{\alpha}}, \quad k_p^* = \omega k_c^*,$$

where the stars denote equilibrium values. Note that one and only intersection of the market equilibrium curves is possible, since $\bar{k}(k_c)$ is a straight line emanating from the origin and $\hat{k}(k_c)$ is bounded above by $\left(\frac{1-\alpha}{\Delta} \right)^{\frac{1}{\alpha}}$ with $\hat{k}''(k_c) < 0$.

Whether the point (k_c^*, k_p^*) is in fact in equilibrium depends on whether it lies within the set of (k_c, k_p) pairs defined by equations (1.10) and (1.11). Eliminating n_p from equation (1.11) by use of equation (1.10), it is seen that the aggregate capital-labor ratio must be able to be expressed as a weighted average of the capital-labor ratios in each sector, so that, given k , k_p and k_c must be such that

$$k = n_c k_c + (1-n_c) k_p, \quad 0 \leq n_c \leq 1.$$

Figure 1.5 shows the permissible set of (k_c, k_p) pairs for k set equal to an arbitrary \bar{k} lying in the region $k_p^* < \bar{k} < k_c^*$ so that a dualistic equilibrium does in fact exist. The permissible set consists of the two "open" rectangles containing the hash marks on the boundary lines. The question of what happens when \bar{k} does not lie on the closed interval (k_p^*, k_c^*) can be answered best after the question of the stability of dualistic equilibrium is explored, since it must be determined how the agricultural economy behaves in disequilibrium.

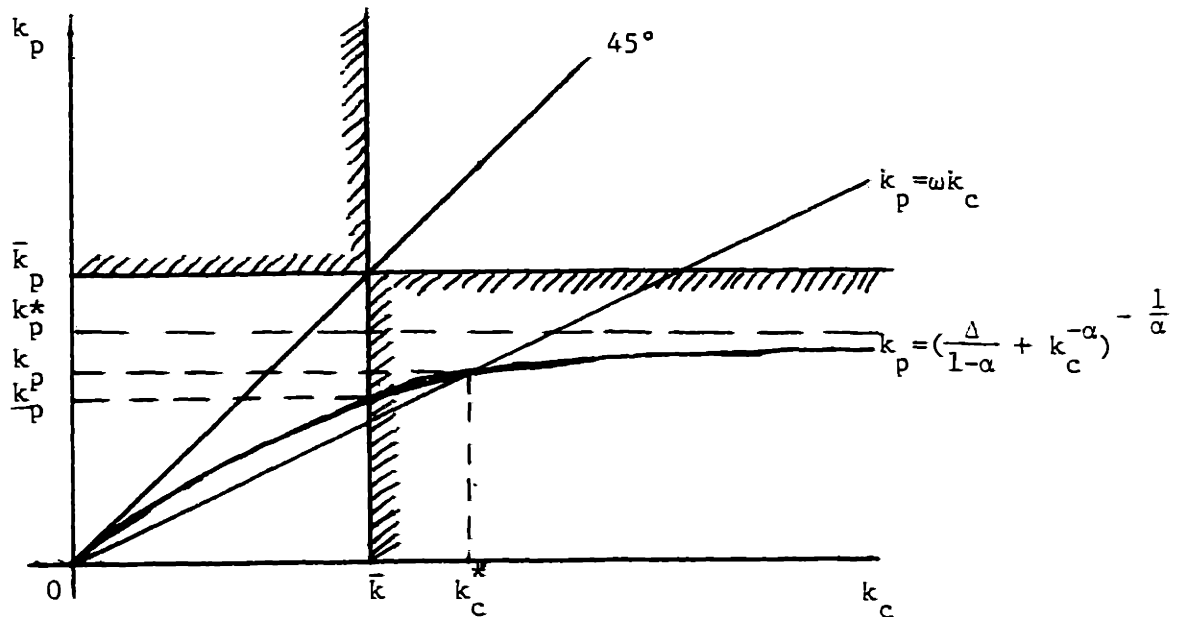


FIGURE 1.5

If it is assumed that labor and capital flow to the sector offering higher factor rewards, subject to the barriers represented by the gap parameters, γ and Δ , at rates which are directly proportional to the differences in sectoral rewards then the following pair of non-

linear differential equations describes the disequilibrium behavior of the agricultural economy:

$$(1.12) \quad \dot{N}_c = b_N [G_N(K_c, N_c) - \gamma G_N(K - K_c, N - N_c)], \quad b_N > 0$$

$$(1.13) \quad \dot{K}_c = b_K \cdot [G_K(K_c, N_c) - G_K(K - K_c, N - N_c) + \Delta], \quad b_K > 0,$$

where $G_x = \partial G / \partial x$. For reasons of convenience, which will become apparent, the "extensive" forms of the relevant variables and functions are used here.

The questions of stability which we want to answer are the following: For any initial K_c^0, N_c^0 will the agricultural economy proceed to the dualistic equilibrium values K_c^*, N_c^* ? If not, will K_c, N_c take on some other stationary values, such as 0,0 or K, N ? To answer these it will be useful to construct a phase diagram in (K_c, N_c) space. To do this, the loci of (K_c, N_c) points for which \dot{K}_c and \dot{N}_c are zero must be determined. The loci, it will be noted, are in fact market equilibrium curves corresponding to those derived in (k_c, k_p) space. Use will be made of this relationship.

In general, it is possible to show that $dN_c/dK_c > 0$ along each locus or stationary if $G_N, G_K, G_{NK} > 0$ and $G_{KK} < 0$. Since G is Cobb-Douglas by assumption, it has these properties. Now from the market equilibrium curves in (k_p, k_c) space it is clear that the corresponding stationaries in (K_c, N_c) space intersect once and only once. Further, since the market equilibrium curves in Figure 1.5 lie only in the

portion of the permissible set below the 45° line, both stationary curves in Figure 1.6 must lie below the diagonal $00'$.

The labor market stationary can be constructed easily. Setting $\dot{N}_c = 0$ and rearranging equation (1.12) yields

$$\omega \cdot \left(\frac{N}{N_c} - 1 \right) = \left(\frac{K}{K_c} - 1 \right),$$

so that if $K_c = K$, then $N_c = N$, and if $K_c = 0$, then $N_c = 0$.

Differentiating the above equation twice with respect to K_c yields:

$$\frac{dN_c}{dK_c} = \frac{1}{\omega} \cdot \frac{K}{N} \left(\frac{N}{K_c} \right)^2 = \omega \cdot \frac{K}{N} \cdot \left(\frac{N - N_c}{K - K_c} \right)^2,$$

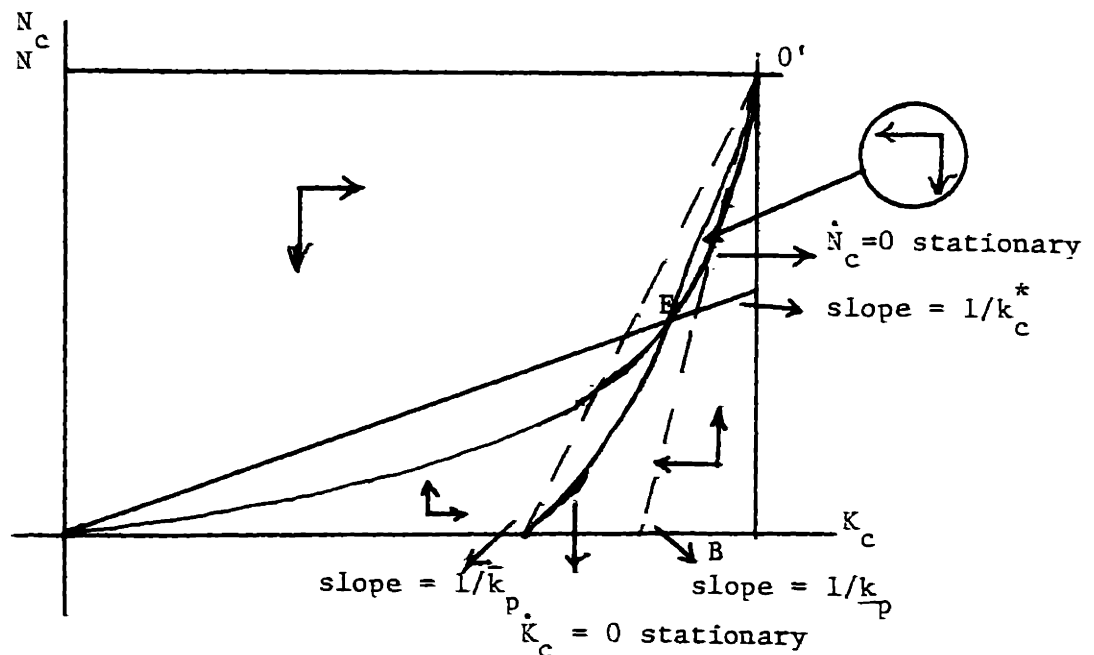


FIGURE 1.6

so that dN_c/dK_c at $K_c=N_c=0$ is equal to $\omega(N/K)$ and at $K_c=K$, $N_c=N$ is equal to $1/\omega(N/K)$, and $dN_c/dK_c > 0$ for all permissible K_c, N_c , and

$$\frac{d^2N_c}{dK_c^2} = 2\omega \frac{K}{N} \left(\frac{N-N_c}{K-K_c}\right)^2 \frac{1}{K-K_c} \left\{1 - \frac{K}{N} \left(\frac{N_c}{K_c}\right)\right\},$$

so that $\text{sign} \frac{d^2N_c}{dK_c^2} = \text{sign} (k_c - k) > 0$ below the diagonal in Figure 1.6.

It is not easy to derive the properties of the capital market stationary directly from equation (1.13), but they can be determined with the aid of Figure 1.5. In that diagram it can be seen that as $k_c \rightarrow \bar{k}$, $k_p \rightarrow \underline{k}_p$, and as $k_c \rightarrow \infty$, $k_p \rightarrow \bar{k}_p$ so that in Figure 1.6, the capital market stationary must lie within the triangle $AO'B$. As indicated above it will increase monotonically from A to O' , crossing the labor market stationary once and only once.

We can now examine the phase diagram shown in Figure 1.6. It is easily verified that the arrows correctly describe the directions of trajectories starting from any permissible K_c^0, N_c^0 : Note that $\text{sign} \dot{N}_c = \text{sign} [w(k_c) - \gamma w(k_p)]$ and $\text{sign} \dot{K}_c = \text{sign} [g'(k_c) + \Delta - g'(k_p)]$, where $w(k_i)$ $i=c,p$, is the marginal productivity of labor and is equal to $g(k_i) - k_i g'(k_i)$. (Observe that this information is essentially that contained in differential equations (1.12) and (1.13), but here again the intensive forms of the variables and functions are employed.) Consider first the labor market. If (K_c, N_c) lies above the $\dot{N}_c = 0$ stationary, then k_c is lower and hence k_p higher than is required for equilibrium. Since $w(k_i)$ is monotonically increasing, it follows

that $w(k_c) < \gamma w(k_p)$ and so that $\dot{N}_c < 0$. It is readily seen that by arguing in a similar fashion, \dot{N}_c can be shown to be positive if (K_c, \bar{N}_c) lies below the stationary. Consider now the capital market. If (K_c, N_c) lies to the left of that stationary curve, then k_c is lower and k_p is higher than is required for equilibrium. Since $g(k_i)$, $i=c,p$, is monotonically decreasing, it is true that $g'(k_c) + \Delta > g'(k_p)$ so that $\dot{K}_c > 0$. Mutatis mutandis, it can be shown that when (K_c, N_c) lies to the right of the stationary, $\dot{K}_c < 0$.

From Figure 1.6 it is immediately verified that the equilibrium point, (K_c^*, N_c^*) , indicated as point E in the diagram is globally stable. For any initial allocation of the fixed stocks of capital and labor between the peasant and capitalist sectors, it is assured that factor allocations will eventually attain their dualistic equilibrium levels and be maintained there.

Such an equilibrium may in fact characterize the situation in some less developed countries. Due to subsistence level incomes and a scarcity of savings and investment goods, the aggregate stocks of capital and labor may remain fairly constant for years. If institutional barriers to the flows of productive factors, other than those reflected in γ and Δ , are not significant, then the agricultural economy could achieve this kind of stagnant dualistic state. Later we shall see how population growth and capital accumulation may alter this result.

In order to do so, we must first consider what happens if K does not lie on the closed interval (k_p^*, k_c^*) . Suppose first that

$k > k_c^*$, as shown in Figure 1.7. On the labor market stationary, it is seen that when $k_c = k$, then $k_p = k_p^b$, and that if $k_c \geq k_c^a$, then $k_p = k$. On the capital market stationary, $k_c = k$ implies that $k_p = k_p^a$ and $k_c = \infty$ implies that $k_p = k_p^{\max}$. With this information, the corresponding stationary curves in K_c, N_c space can be drawn. This

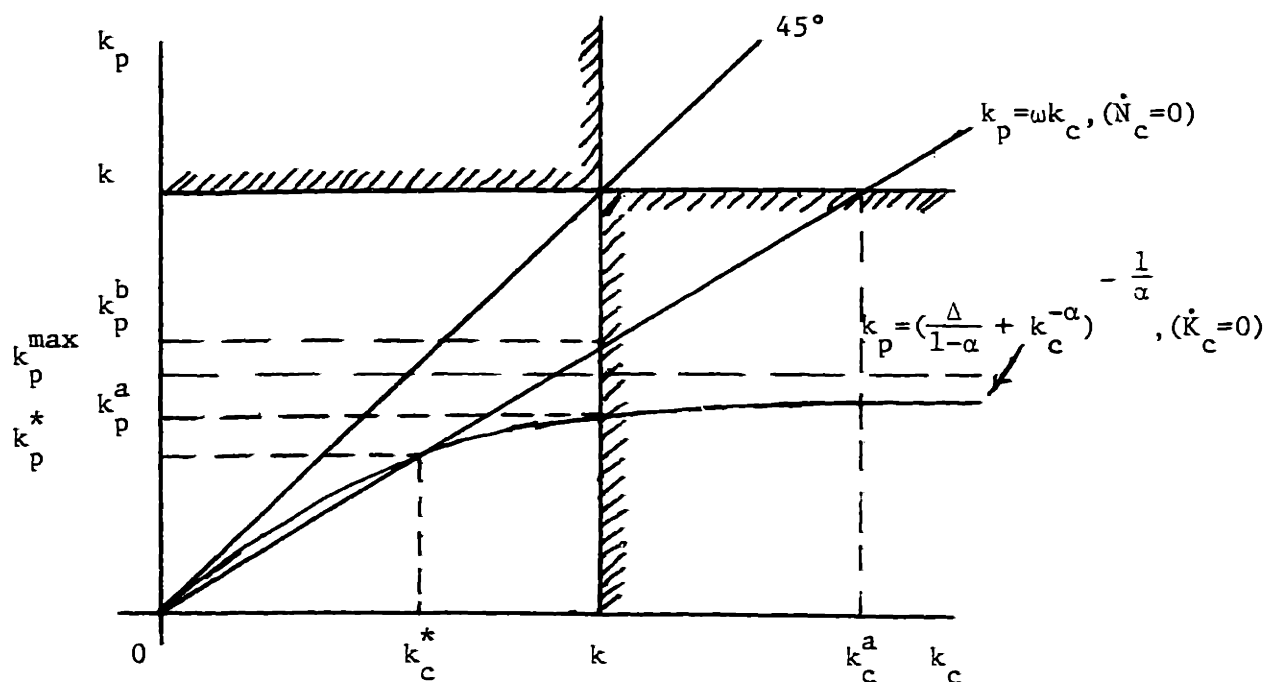


FIGURE 1.7

has been done in Figure 1.8. In Figure 1.9, the lines showing the limiting slopes have been omitted and the arrows showing the movement of the system have been added. Observe that when $k > k_c^*$, the agricultural economy is globally stable at an all capitalist equilibrium point.

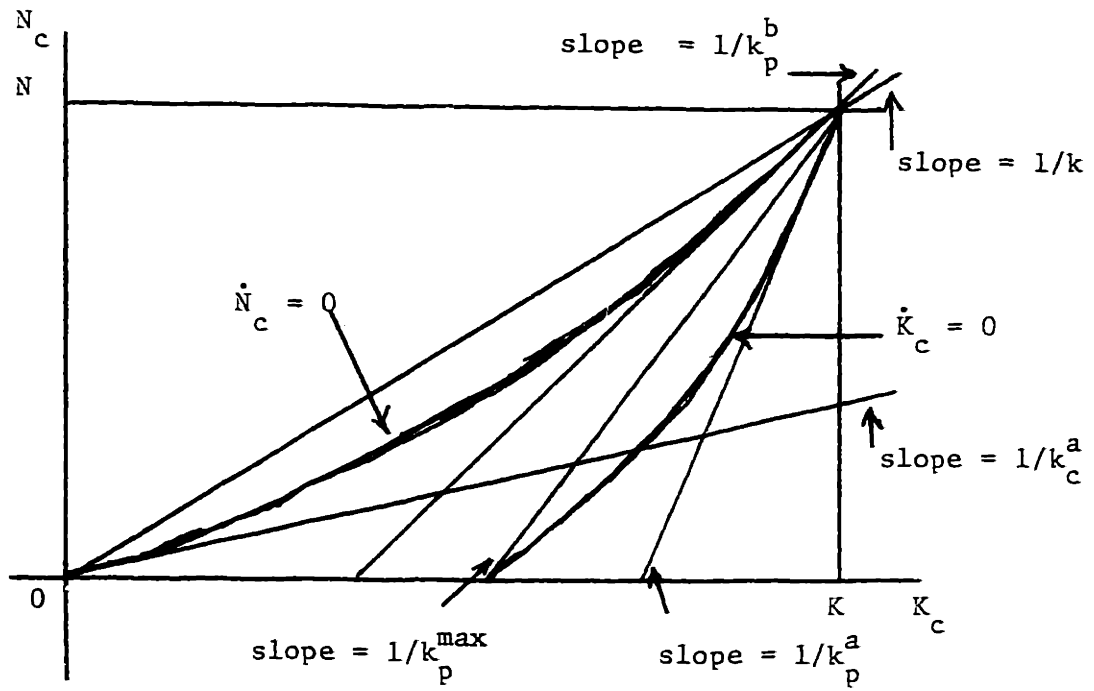


FIGURE 1.8

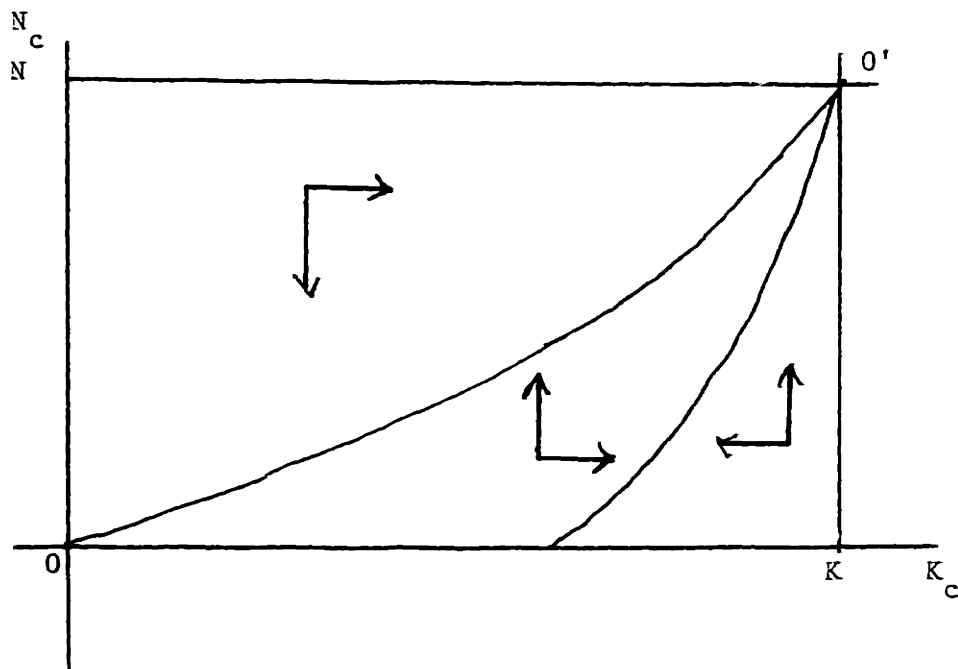


FIGURE 1.9

By an analogous process it can be demonstrated that when $k < k_p^*$, the system is globally stable at the all peasant equilibrium point. These results of the Cobb-Douglas case are intuitively satisfying. Peasants, who undervalue labor and tend to use relatively more labor intensive methods thrive in a capital-poor economy and capitalists thrive in a capital-rich environment. If net investment in the agricultural sector exceeds the rate of population growth, k will grow. Under such conditions, it would be possible for agriculture to move from a phase where only peasants exist to one in which (say after a burst of exogenous foreign investment), both form coexist, to one in which only capitalists exist, if the capitalist firms reinvest rapidly enough to keep k rising.

These results were, of course, obtained under a very special set of assumptions. Production functions were identical and Cobb-Douglas. The wage gap was a constant proportional factor, γ , and the capital rate of return gap was a constant absolute factor, Δ . These assumptions are all critical to the results. For example, if both gaps are assumed to be proportional, then it is easily proved that both the factor market stationaries in (k_c, k_p) space are straight lines emanating from the origin. If $\gamma > \Delta^{1-\alpha/\alpha}$, then the capital market stationary lies above the labor market stationary and the situation is similar to the case of $k < k_p^*$ above. Peasants thrive; capitalists disappear. If $\gamma < \Delta^{1-\alpha/\alpha}$, then the situation resembles the earlier case of $k > k_c^*$, since the all capitalist point is a globally stable equilibrium.

As a second example, consider the effects of assuming the production functions to be other than Cobb-Douglas. (Retain the assumption that they are identical, however.) If it is assumed that $g(k_i) = (ak_i^{-\beta} + b)^{-1/\beta}$, where $1/1+\beta = \sigma$, the Hicksian elasticity of substitution, then $g(\cdot)$ is CES (constant elasticity of substitution), and the labor market equilibrium curve is given by

$$k_p = \left[\frac{(ak_c^{-\beta} + b)^{\beta/1+\beta} - b^{-\frac{1}{\beta}}}{a} \right] = \bar{k}(k_c).$$

It is already known that $\bar{dk}/dk_c > 0$. The reader may verify that if $\beta > 0$ ($\sigma < 1$), then

$$k_p \rightarrow 0 \text{ as } k_c \rightarrow 0 \text{ and}$$

$$k_p \rightarrow \left[\frac{b}{a} (\gamma^{\beta/1+\beta} - 1) \right]^{-\frac{1}{\beta}} > 0 \text{ as } k_c \rightarrow \infty.$$

If $-1 < \beta < 0$ ($\sigma > 1$), then

$$k_p \rightarrow 0 \text{ as } k_c \rightarrow \left[\frac{b}{a} (\gamma^{\beta/1+\beta} - 1) \right]^{-\frac{1}{\beta}}$$

$$k_p \rightarrow \infty \text{ as } k_c \rightarrow \infty.$$

Calculating \bar{dk}/dk_c , it is found that

$$\frac{\bar{dk}}{dk_c} = a^{\left(\frac{1}{\beta} + 1\right)} \gamma^{\beta/1+\beta} k_c^{-(\beta+1)} \left[(ak_c^{-\beta} + b)^{\beta/1+\beta} - b \right]^{-\left(\frac{1}{\beta} + 1\right)},$$

so that it may be determined that

$$\begin{aligned} \text{sign } \frac{d^2 \bar{k}}{dk_c^2} &= \text{sign } -\left(\frac{1+\beta}{\beta}\right) \left\{ k_c^\beta \left[(ak_c^{-\beta} + b)^{\beta/1+\beta} - b \right] \right\}^{-\frac{1+\beta}{\beta}} (\gamma^{\beta/1+\beta} - 1) b \beta k_c^{\beta-1} \\ &= \text{sign } -\left(\frac{1+\beta}{\beta}\right) \end{aligned}$$

so that $\frac{d^2\hat{k}}{dk_c^2} < 0$ as $\beta > 0$. Note finally that $\tilde{k}'(0) = \gamma^{-\frac{1}{1+\beta}}$. The capital market equilibrium curve, when the rate of return gap is absolute, is

$$k_p = \left\{ \frac{\left[\frac{\Delta}{a} + (a + bk_c^\beta)^{-\frac{1+\beta}{\beta}} - \frac{\beta}{1+\beta} - a \right]^{\frac{1}{\beta}}}{b} \right\} = \hat{k}(k_c).$$

When $\sigma < 1$, it may be shown that

$$\hat{k}(0) = \left[(1 + \Delta a^{\frac{1}{\beta}})^{-\frac{\beta}{1+\beta}} - 1 \right]^{\frac{1}{\beta}} \left(\frac{a}{b} \right)^{\frac{1}{\beta}} < 0$$

$$k(\infty) = \left[\frac{(a/\Delta)^{\frac{\beta}{1+\beta}} - a}{b} \right]^{\frac{1}{\beta}} > 0 \text{ if and only if } a \cdot \Delta^\beta < 1.$$

$$\frac{d\hat{k}}{dk_c} \geq 0 \text{ as } \left[\frac{\Delta}{a} + (a + bk_c^\beta)^{-\frac{1+\beta}{\beta}} - \frac{\beta}{1+\beta} \right] \geq a$$

so that if $a\Delta^\beta > 1$, $\hat{k}(\infty) \rightarrow -\infty$. Thus the curve $\hat{k}(k_c)$ increases asymptotically toward a maximum at $k_c = \infty$ or reaches a finite maximum at some finite value of k_c .

Note that both $\tilde{k}(k_c)$ and $\hat{k}(k_c)$ are bounded above. In addition, observe that since $\hat{k}(0) < 0$, there will exist a minimum level of k_c for which it will be possible to obtain equilibrium in the capital market. Under these conditions it is possible that there exists no (k_c, k_p) pair for which equilibrium might exist in both factor markets. Such a case is pictured in Figure 1.10. However, contrary to the Cobb-Douglas example, it is easily shown that factor market equilibrium will always exist if the capital market gap is relative rather than absolute, that is to say, if the equilibrium rate of return in peasant

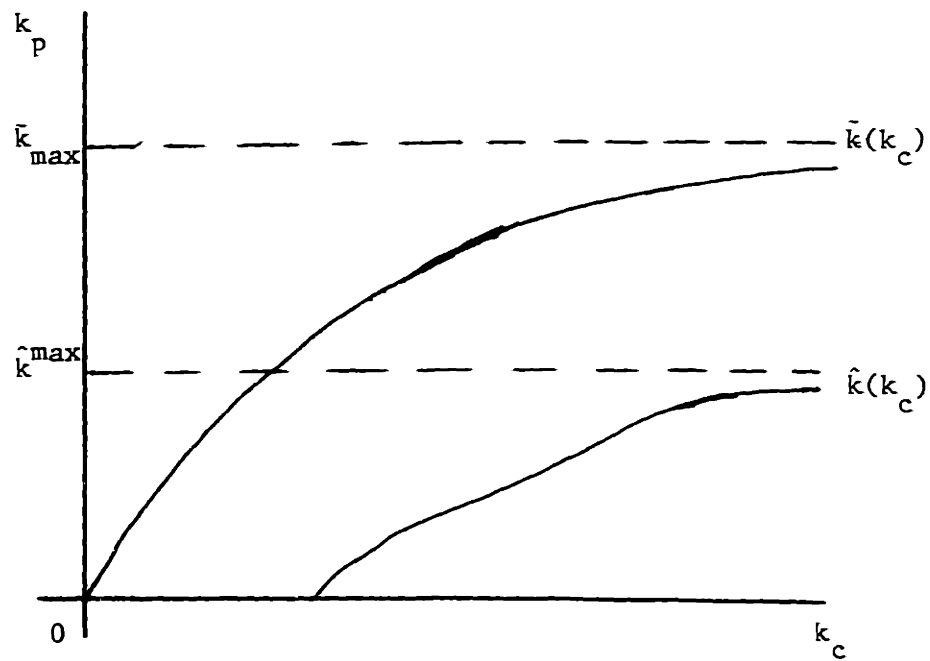


FIGURE 1.10

agriculture is some multiple of that in capitalist agriculture: Let

$$f'(k_p) = \Delta f'(k_c), \quad \Delta > 1,$$

when the capital market is in equilibrium. Recall that the labor market is in equilibrium when

$$\gamma w(k_p) = w(k_c).$$

It must then be true when both markets are in equilibrium that

$$R(k_p) = \gamma \Delta \cdot R(k_c),$$

where R is rental-wage ratio. For a CES production function, this last equation is written

$$\frac{a}{b} k_p^{-(1+\beta)} = \gamma \Delta \cdot \frac{a}{b} k_c^{-(1+\beta)}.$$

Rearranging, this becomes

$$k_p = (\Delta \gamma)^{-\frac{1}{1+\beta}} k_c.$$

The graph of this equation is a straight line through the origin having a positive slope less than unity. It is clear that factor market equilibrium obtains if the above equation is satisfied at the same time as the labor market equation. By the nature of these two equations it is easy to prove that there exists one and only one such equilibrium point. When $\beta > 0$, ($\sigma < 1$), the slope of $\bar{k}(k_c)$ at $k_c = 0$ exceeds $(\Delta \gamma)^{-1/1+\beta}$, since when $\sigma < 1$, $\bar{k}'(0) = \gamma^{-1/1+\beta}$, and $\Delta > 1$. Due to the fact that $\bar{k}''(k_c) < 0$ when $\beta > 0$, we know that the $\bar{k}(k_c) = k_p$ curve will cross any straight line from the origin once and only once. When $-1 < \beta < 0$, ($\sigma > 1$), it is clear that any straight line from the origin will lie above $\bar{k}(k_c)$ if k_c is below a critical size since $\bar{k}(k_c) \leq 0$ for $k_c \leq \bar{k}_c$. But since now $\bar{k}''(k_c) > 0$, it is again obvious that the two curves will intersect once and only once.

Consider next the capital market equilibrium curve, $k_p = \hat{k}(k_c)$, now that Δ has been interpreted as a proportional gap exceeding unity.

This curve is now given by

$$k_p = \left[\frac{(a+bk_c^\beta)\Delta^{-\beta/1+\beta} - a \frac{1}{\beta}}{b} \right] = \hat{k}(k_c).$$

It follows that

$$\frac{d\hat{k}}{dk_c} = \left[\frac{(a+bk_c^\beta)\Delta^{-\beta/1+\beta} - a \frac{1-\beta}{\beta}}{b} \right] k_c^{\beta-1} \Delta^{-\beta/1+\beta} > 0$$

and that

$$\begin{aligned} \text{sign } \frac{d^2\hat{k}}{dk_c^2} &= \text{sign } -(1-\beta) \left[\frac{(ak_c^{-\beta}+b)\Delta^{-\beta/1+\beta} - ak_c^{-\beta} \frac{1-2\beta}{\beta}}{b} \right] (\Delta^{-\beta/1+\beta}) \cdot k_c^{-(\beta+1)} \\ &= \text{sign } (\beta-1) (\Delta^{-\beta/1+\beta} - 1), \end{aligned}$$

so that

$$\frac{d^2\hat{k}}{dk_c^2} < 0 \text{ if } \beta > 1 \text{ or } \beta < 0, \text{ and}$$

$$\frac{d^2\hat{k}}{dk_c^2} > 0 \text{ if } 0 < \beta < 1.$$

Further more it is readily verifiable that if $\beta > 0$, ($\sigma < 1$), then

$$k_p \rightarrow 0 \text{ as } k_c \rightarrow \left[\left(\frac{a}{b} \right) (\Delta^{\beta/1+\beta} - 1) \right]^{\frac{1}{\beta}}, \text{ and}$$

$$k_p \rightarrow \infty \text{ as } k_c \rightarrow \infty$$

and that if $-1 < \beta < 0$, ($\sigma > 1$), then

$$k_p \rightarrow 0 \text{ as } k_c \rightarrow 0, \text{ and}$$

$$k_p \rightarrow \left[\left(\frac{a}{b} \right) (\Delta^{-\beta/1+\beta} - 1) \right]^{\frac{1}{\beta}} \text{ as } k_c \rightarrow \infty.$$

Consider now the case of dualistic equilibrium when $\sigma < 1$. It has already been proved that such an equilibrium must exist. We are interested in examining next the stability of this equilibrium. The reader may verify that when production functions are CES with $\sigma < 1$ and both gaps relative, the graphs of the two market equilibrium curves in (k_c, k_p) space will appear as in Figure 1.11. Notice that the capital market equilibrium curve, $k_p = \hat{k}(k_c)$, cuts the labor market equilibrium curve from below, instead of from above as in the Cobb-Douglas and $\sigma > 1$ cases. Knowing this, it is possible to demonstrate

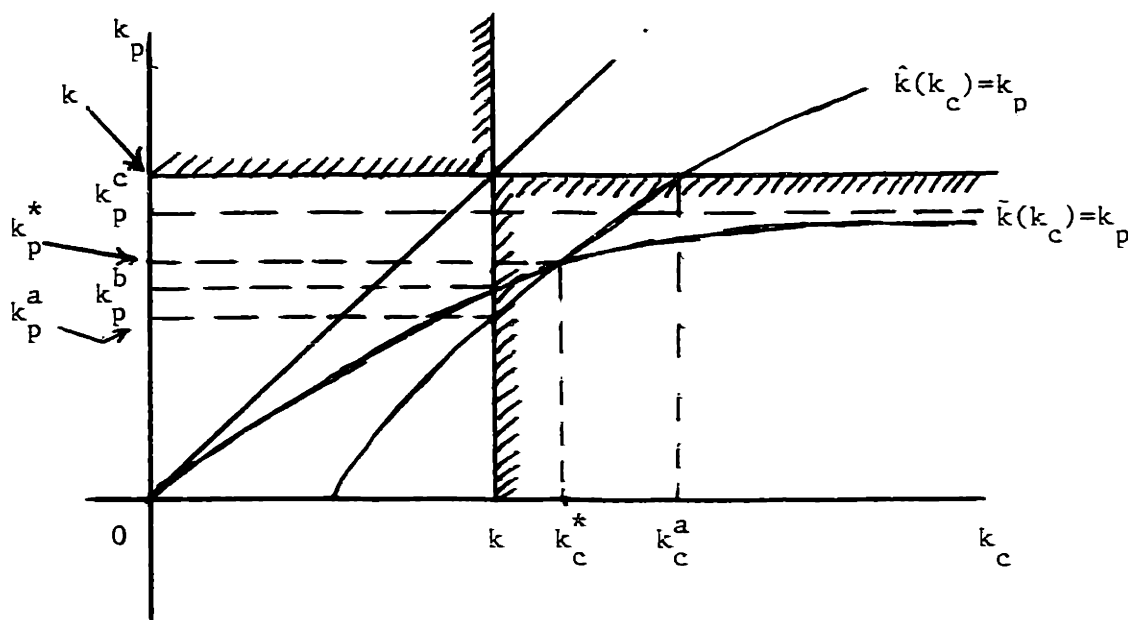


FIGURE 1.11

that if endowments are such that dualistic equilibrium exists, it will be a saddlepoint. Depending on the initial allocations of factors between peasants and capitalists, the agricultural economy will move to either the all capitalist or the all peasant point, except if these allocations should fall on one of the unique stable arms of the system. Then it will move toward the dualistic equilibrium unless or until random disturbances push it off the convergent trajectory. ^{1/}

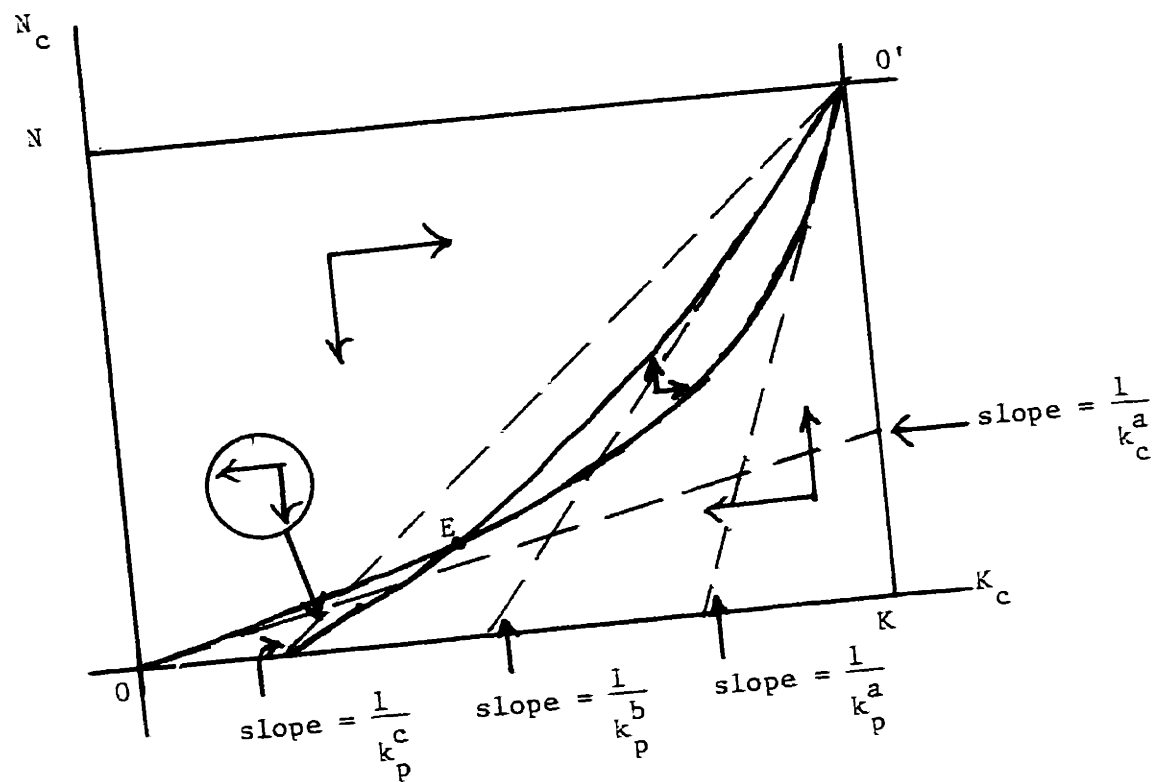


FIGURE 1.12

Figure 1.12 shows clearly the saddle point property of the system. As in the Cobb-Douglas case, the locations of the stationary

curves are derived from the corresponding diagram in (k_c, k_p) space, Figure 1.11 in this case. The dotted lines in Figure 1.12 show the limiting slopes of k_c and k_p as determined from Figure 1.11. Recall that dN_c/dK_c is strictly positive on both stationaries in (K_c, N_c) space and that these stationaries intersect exactly once since they intersect exactly once in Figure 1.11.

It is interesting to note that if the agricultural sector is capital poor, $(k < k_p^*)$, capitalists will replace peasants and if it is capital rich, $(k > k_c^*)$, peasants will replace capitalists. This is exactly the opposite conclusion of that obtained in the Cobb-Douglas example. It could easily be shown that the CES case of $\sigma > 1$ has the same global stability properties as the Cobb-Douglas case ($\sigma = 1$). Thus the perverse (in the sense of not agreeing with intuition) case is restricted to $\sigma < 1$. These CES results allow a slight generalization to the broader class of all neoclassical production functions: Any dualistic equilibrium will be locally stable if and only if $\sigma > 1$ in a small neighborhood about the equilibrium point. If we take a small enough neighborhood, σ may be treated as a constant. A final observation, perhaps worth making, is that similar results are obtained if, as in the Cobb-Douglas version, the capital rate of return gap is assumed to be absolute, except that more cases arise. As noted above, there may be no factor market equilibrium regardless of endowments or there may be multiple factor market equilibria, all, some, or none of which may exist for given factor endowments. Suffice it to say that it makes a great deal of difference what kind of production functions and factor market gaps are assumed.

Suppose now that the peasant and capitalist production functions are not the same in both sectors. Specifically, suppose again that they differ by a Hicks-neutral multiplicative efficiency factor and that they are otherwise of identical Cobb-Douglas form. Call this efficiency factor θ , and assume it to exceed unity so that the peasant production function is $q_p = k_p^{1-\alpha}$ and that of the capitalists is $q_c = \theta k_c^{1-\alpha}$, where q_i , $i=c,p$, is output per worker. The efficiency factor may be thought of as representing capitalists' access to superior technical knowledge or of as a crude way of accounting for a jump in productivity achieved through increasing returns to scale.

Since $q_p = g(k_p)$ and $q_c = \theta g(k_c)$ are both constant returns to scale production functions, they have no properties which would indicate the scale or size of an individual productive unit. These production functions may be viewed as industry or sectoral functions. If it is assumed that there exist decreasing returns to scale for individual productive units, then the industry or sectoral production function will still exhibit approximate constant returns to scale properties if decreasing returns to scale keep each individual unit from producing a large output relative to total sectoral output and if sectoral output is increased or decreased in the long run by the creation or dissolution of productive units. It is now clear how θ can account for increasing returns to scale. Organization on a peasant basis may bring about diminishing returns to scale at a low level of factor utilization due to the type of management and labor allocation system (family welfare maximizing) associated with peasant organization.

Operation on a capitalist basis with its modern managerial methods may permit a larger scale of operations. There is also a not strictly technical factor, but rather one which reflects an institutional rigidity and is, in effect, an economic factor. Namely, it may be necessary to organize on a capitalist basis in order to obtain sufficient capital finances for large scale operations. The price of capital finances to the individual peasant unit becomes virtually infinite at some level. Since capitalists' organizational form may permit them to operate at a larger scale for managerial and institutional reasons, it must also turn out that they reap the benefits of whatever technical economies of scale accrue to such capitalist operation. For example, they may be able to purchase more efficient "lumpy" capital goods which would be of too great a capacity and hence of too great a cost for small scale operation. As long as the individual capitalist firm ultimately encounters diminishing returns to scale at some point, there is no difficulty.

Under these new assumptions the market equilibrium conditions in (k_c, k_p) space become

$$k_p = \left(\frac{\theta}{\gamma}\right)^{\frac{1}{1-\alpha}} k_c \quad \text{and}$$

$$k_p = \left(\frac{\Delta}{1-\alpha} + \theta k_c^{-\alpha}\right)^{-\frac{1}{\alpha}},$$

for the labor and capital markets, respectively, where the gap in the latter market is again assumed to be absolute. It is easily proved that

if $(\gamma^\alpha/\theta) < 1$, then no dualistic equilibrium will exist for any aggregate factor endowment ratio, k : Solving the above two equations for k_c^* , we obtain

$$k_c^* = \left\{ \left(\frac{1-\alpha}{\Delta} \right) \theta \left[\left(\frac{\gamma^\alpha}{\theta} \right)^{\frac{1}{1-\alpha}} - 1 \right] \right\}^{\frac{1}{\alpha}},$$

so that $k_c^* > 0$ if and only if $(\gamma^\alpha/\theta) > 1$. Note that $dk_c^*/d\theta < 0$, so that for a given endowment, k , raising θ increases the likelihood that $k_c^* < k$, i.e., that capitalists will displace peasants. Lowering θ has the opposite result. This result is intuitively satisfying since it indicates that superior technology pays off in terms of survivability. It is interesting to note, however, that if $k < k_p^*$, then even though capitalists may enjoy an absolute technological advantage they will in fact be displaced by peasants, a disturbing possibility for any developing country which may be badly in need of increased agricultural output. This result is due to the fact that the aggregate capital-labor ratio is so low that it is impossible, given the imperfections in the factor markets, for both factor markets to be in equilibrium at any dualistic point. The direction of displacement is intuitively indicated by the fact that whenever one factor market is in equilibrium, the other is not, and the factors in that market are flowing to the peasant sector. Less intuitively satisfying is the effect of θ in the CES case with both gaps relative. Then the equilibrium value of k_c is given by

$$k_c^* = \left[\frac{a}{b} \cdot \frac{(\Delta\theta)^{\beta/1+\beta} - 1}{1 - (\theta/\beta)^{\beta/1+\beta}} \right]^{\frac{1}{\beta}}.$$

When $\beta > 0$, ($\sigma < 1$), $k_c^* > 0$ if and only if $\gamma > \theta$. It is easily verified that when $\gamma > \theta$, $dk_c^*/d\theta > 0$. Thus raising θ increases the likelihood that $k_c^* > k$, and hence the likelihood that capitalists displace peasants. When $-1 < \beta < 0$, ($\sigma > 1$), $k_c^* > 0$ if and only if $\gamma < \theta$. It may be seen that when $\gamma < \theta$, $dk_c^*/d\theta > 0$. Again, raising θ increases the likelihood that $k_c^* > k$, and thus in this case, the likelihood that peasants displace capitalists. It is thus possible that a superior technology may actually be a hindrance to survival when certain kinds of market imperfections exist.

Welfare questions will be discussed in greater detail later, after the dynamics of saving, investment, and population growth have been introduced. For the moment, however, there remains another assumption whose restrictiveness it is desirable to discuss beforehand.

The above analysis of factor market stability of equilibrium has been predicated on the assumption that flows of the factors of production between peasant and capitalist sectors have been governed by differential equations (1.12) and (1.13). These equations imply that there exist no barriers to factor mobility, except for the gap parameters. No frictions exist; there is no failure of communication of market signals. The peasant family must be imagined to be willing to diminish or augment its size according to marginal productivity differences, sometimes even when it is altering the size of its capital holdings in the opposite direction. A similar observation holds true for capitalists.

Consider, for example, a situation where $\dot{N}_p > 0$ and $\dot{K}_p < 0$. Peasants can be conceived of as either selling or renting little sections of each family plot while augmenting family size by taking on individuals from the capitalist labor force and keeping the total number of families and plots constant, or alternatively as reducing the number of families and plots by selling or renting the plots to capitalists and by incorporating dissolved families and ex-laborers from the capitalist sector into the remaining families. The first of these concepts is empirically unsatisfying since it implies that capitalists would be willing to buy or rent fragments of already small and probably fragmented family plots. Both conceptions imply that families will be willing to admit strangers into the family enterprise, and they both assume a remarkable astuteness and operational flexibility on the part of peasants, who would be willing to sell or rent their land to capitalists and join new families. Since it seems unlikely that such fluid factor market conditions could exist in developing countries, alternative assumptions will be considered.

For example, it could be assumed that there are gaps in each factor market for movements of factors in either direction. That is, labor will move from the peasant sector to the capitalist sector if, say, $w_c > \gamma w_p$, where w_i is the wage or consumption per worker, $i=c,p$, and γ is a gap parameter; but labor will return to the peasant sector only if, say $w_c < w_p$. This assumption sets up a kind of dual barrier so that for certain ranges of variable values, no movement of labor will occur at all. Such would be the case here if $\gamma w_p > w_c > w_p$. A

similar observation could be made about the capital market. In many cases, such assumptions can explain the persistence of dualism. Consider the static CES model discussed on pp. 28-35. Recall that it possessed an unstable type of dualistic equilibrium. In Figure 1.13, the phase diagram in K_c, N_c space has been drawn to include two sets of stationary lines for each market. The regions between each set denote areas of factor reward differences inadequate to promote movement. Notice that many initial conditions will now lead to stable dualistic solutions. In the next chapter we shall introduce an even stronger assumption about capital mobility, which will insure the existence of a stable static dualistic equilibrium.

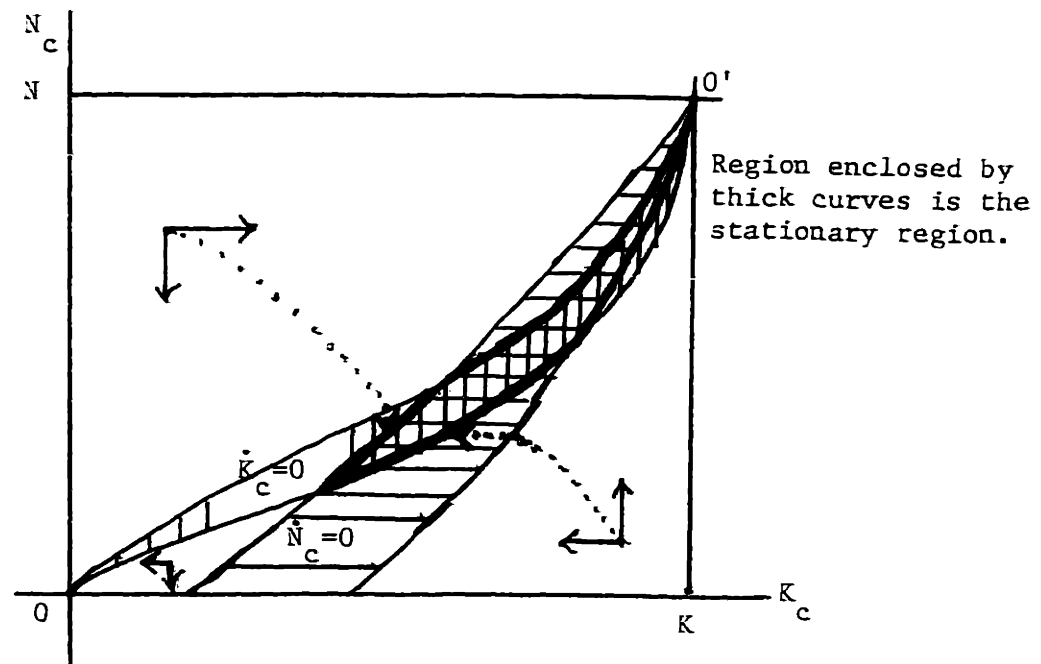


FIGURE 1.13

For the time being, however, continue to assume that differential equations (1.12) and (1.13) apply, and consider what happens when the capital stock is fixed, but agricultural population is allowed to grow. This is the case of gradual land crowding in a densely populated region. It is assumed that population in each sector grows at a rate, ϵ_i , $i=c,p$, which is a function of per worker consumption in each sector, c_i , $i=c,p$. Assume that $\epsilon_i = \epsilon_i(c_i)$ is a non-decreasing function of c_i and has a negative value at $c_i = 0$ and a maximum, ϵ_i^{\max} , for $c_i > \bar{c}_i$. For the moment, per worker consumption, c_i , will be left undefined. Suppose that capital decays in an evaporative fashion at a constant relative rate, δ , in both sectors. Define net agricultural product as total product less replacement-maintenance outlays on capital, which are assumed to be equal to depreciation. Production functions are assumed to be Cobb-Douglas and identical except for a multiplicative efficiency factor, θ . It is assumed that factor markets are always in equilibrium, since it is known from previous analysis that dualistic equilibrium is globally stable, factor endowments permitting. Of course, it must then be assumed that the initial aggregate capital-labor ratio, $k(0)$, lies in the appropriate region of values. The wage gap is relative, and the rate of return on capital gap is absolute as before. Note that the variables n_c, n_p , and k will now be functions of time. They will nevertheless continue to be written as above. Finally, observe that the following analysis is relevant only to cases where a stable dualistic equilibrium may exist, and hence not to the ($\sigma < 1$) CES case, since it possesses unstable

factor markets and may be expected to become a unisectoral system before population has changed significantly.

Since the factor markets are assumed always to be in equilibrium, k_p and k_c are fixed at their equilibrium values, k_p^* and k_c^* , as long as $k_p^* \leq k \leq k_c^*$. Thus while dualistic equilibrium persists, net per worker agricultural product is constant. If it is assumed that per worker consumption is a function of net per worker product, then it, too, is constant and thus so is the relative rate of population growth in each sector. Call these rates ϵ_p^* and ϵ_c^* , for peasants and capitalists, respectively. Now $k = K/N$, so that $\dot{k} = -k \cdot (\dot{N}/N)$. But $(\dot{N}/N) = \epsilon_p^* n_p + \epsilon_c^* n_c$ if $k_p^* \leq k \leq k_c^*$ and $\dot{N}/N = \epsilon_p(c_p)n_p + \epsilon_c(c_c)n_c$ if $k > k_c^*$ or $k < k_p^*$. Therefore, we may write the following differential equation:

$$(1.14) \quad \dot{k} = \begin{cases} -k \cdot (\epsilon_p^* n_p + \epsilon_c^* n_c), & k_p^* \leq k \leq k_c^* \\ -k \cdot (\epsilon_p(c_p)n_p + \epsilon_c(c_c)n_c), & k < k_p^* \text{ or } k > k_c^* \end{cases}$$

Now from previous results it is known that

$$k_c^* = \left\{ \left(\frac{1-\alpha}{\Delta} \right) \left[\left(\frac{\gamma^\alpha}{\theta} \right)^{\frac{1}{1-\alpha}} - 1 \right] \right\} \text{ and}$$

$$k_p^* = \left(\frac{\theta}{\gamma} \right)^{\frac{1}{1-\alpha}} k_c^*.$$

Recall that k_c^* is positive if and only if $\gamma^\alpha > \theta$ and that if this is true, $k_p^* < k_c^*$. If $\gamma^\alpha < \theta$, no dualistic equilibrium is possible and

all production will be undertaken by capitalists. In this case, capitalists are so much more efficient than peasants that no matter how high an implicit wage peasants receive, capitalists will be willing to hire labor at γ times that wage. An analogous observation applies to rentals or sales of capital goods. If peasants and capitalists are of equal efficiency, then the existence of a wage gap and rental gap insures that dualistic equilibrium is possible.

Using equations (1.10) and (1.11), n_p and n_c may be written as follows for the situation where $k_p^* \leq k \leq k_c^*$:

$$n_p = \frac{k_c^* - k}{k_c^* - k_p^*} \quad \text{and} \quad n_c = \frac{k - k_p^*}{k_c^* - k_p^*}.$$

Now differential equation (1.14) may be written as shown:

$$(1.15) \quad \dot{k} = \left\{ \begin{array}{l} -k \left[\frac{(\epsilon_p^* k_c^* - \epsilon_c^* k_p^*) + (\epsilon_c^* - \epsilon_p^*)k}{k_c^* - k_p^*} \right], \quad k_p^* \leq k \leq k_c^* \\ -k \cdot \epsilon_p(c_p), \quad k < k_p^* \\ -k \cdot \epsilon_c(c_c), \quad k > k_c^*. \end{array} \right.$$

Notice that for k in the dualistic region, equation (1.15) involves only constants and k . Thus for the dualistic phase of the trajectory this equation completely describes the motion of the system.

Before considering the effects of population growth, it would be desirable to examine dualistic equilibrium per worker output and welfare levels. Gross per worker output will be greater in the

capitalist sector if and only if $\gamma > 1$, since $\theta k_c^{*1-\alpha} = \gamma k_p^{*1-\alpha}$. Net per worker output or income is gross output minus replacement expenditures per worker, i.e., $q_i - \delta k_i = y_i$, $i=c,p$. Using the dualistic equilibrium values of k_c and k_p , it can be shown that

$$y_c^* > y_p^* \text{ if and only if } \gamma > 1 + \frac{\delta}{\Delta} \left[\left(\frac{\gamma}{\theta} \right)^{\frac{1}{1-\alpha}} - 1 \right] \left[1 - \theta \left(\frac{\gamma}{\theta} \right)^{-\frac{\alpha}{1-\alpha}} \right].$$

To get a rough feel for this inequality, note that it becomes $\gamma < 1.62$ for $\Delta = \delta$, $\theta = 1$, and $\alpha = \frac{1}{2}$. If all net income is consumed, then $y_i = c_i$, and welfare is higher in the capitalist sector if and only if $y_c^* > y_p^*$. However, it is important to consider the distribution of income since much of the capitalist per worker net output may be distributed to a small capital owning elite or to foreign investors. Suppose then we ask how per capita peasant income compares with the per capita income of capitalist workers, hereafter called laborers to distinguish them from peasants. Assume that all peasant net output is consumed, i.e., $y_p = c_p$, and that laborers receive their competitive wage share, α , of gross output, and that all wages are consumed, i.e., $c_c = \alpha q_c$. Then laborers' per worker income exceeds that of peasants if and only if

$$\alpha \theta k_c^{*1-\alpha} > k_p^{*1-\alpha} - \delta k_p^*.$$

Using the values of k_p^* and k_c^* in terms of their parameters, this

condition becomes

$$\frac{(\alpha\gamma - 1)\Delta}{(1-\alpha)} \left[1 - \left(\frac{\theta}{\gamma}\right)^{\frac{1}{1-\alpha}} \right]^{-1} + \delta > 0.$$

Note that a sufficient condition is $\alpha\gamma > 1$. For $\Delta = \delta$, $\theta = 1$, and $\alpha = \frac{1}{2}$, the condition is $\gamma > 1.62$. Notice that for this collection of parameters, it is not possible for laborers' per worker income to exceed that of peasants if net per worker output is higher in the capitalist sector.

An interesting result of these observations is that although the labor market is in equilibrium, per capita incomes of peasants and laborers may differ in either direction. This raises several issues about interpreting equilibrium in the labor market. If laborers are assumed to be ex-peasants, who retain title to their original lands, which they now rent to either peasants or capitalists, or alternatively have title to some equivalent asset, then it may be quite reasonable for laborers' income to be less than peasant per capita income. (A similar argument would apply to the case where ex-peasants who are now laborers had rented their land and now as laborers are spared this cost.) The crux of the matter is that in the peasant sector each family member may be thought to be both laborer and part-owner. When peasants become laborers they may leave the family farm as individuals, essentially relinquishing their share in the ownership of the family's capital stock to those of the family who remain. In this case it is to be expected that wage income in the capitalist sector would be at

least as great as peasant per worker income and greater than it to the extent that the laborer feels a loss of utility for having lost his position as a part-owner of family property or that he accounts for some of the wage gap creating factors discussed above. But it is possible that when peasants become laborers they do so as a family unit, selling or renting the entire family capital stock and moving en masse. Then the earlier observation about laborers who retain title to capital assets applies. In reality, it is likely that some laborers enjoy rental income or the equivalent present discounted value sum while others have relinquished their ownership shares.

A final observation on the comparison of peasant and laborers' per worker incomes is worth making. In those comparisons it was assumed that per worker peasant income was equal to the entire per capita net product of peasants. But if peasants are renting or have purchased capital goods recently, then a portion of their incomes will be drained off in the form of rental or debt amortization payments. All this is simply to say that the interpretation of the relative sizes of the two variables which have been labelled income turns on what is assumed about the distribution of ownership of land-capital resources.

Another issue in interpreting the model arises when it is seen that the existence of depreciation expenditures may imply negative net capitalist profits. If it is assumed that all laborers' income is consumed and as much capitalist income, which is equal to gross profits due to the assumption of perfect competition in the capital market,

is invested as is needed to meet replacement and maintenance expenditures, then capitalists' income will provide an adequate amount of savings resources if and only if

$$(1-\alpha) \theta k_c^{*1-\alpha} - \delta k_c^* > 0.$$

When the equivalent expression in terms of structural parameters is substituted for k_c^* , this condition becomes

$$(\Delta/\delta) \geq (\gamma^\alpha/\theta)^{\frac{1}{1-\alpha}}.$$

Since the RHS of this inequality is greater than one, it is clear that (Δ/δ) must exceed unity. Note also that since the requirement that y_p^* be greater than zero is

$$(\Delta/\delta) \geq (1-\alpha) [1 - (\theta/\gamma^\alpha)^{\frac{1}{1-\alpha}}]$$

and since the RHS of this inequality is less than one, it is assured that peasants enjoy positive net income if capitalists make a non-negative net profit.

These observations suggest that the concept of dualistic equilibrium as presented thus far has a weakness in ignoring savings-investment behavior. If dualistic equilibrium occurs at a point where capitalist income is inadequate to meet depreciation requirements and/or peasant per worker net income is below subsistence level so that some income must be diverted from replacement investment to

consumption, then it is hardly correct to refer to such situations as equilibria, since the size of the capital stock would be declining over time even if population were static.

In order to further improve the reader's grasp of some of the welfare conditions presented above, we shall consider once more a specific case. Suppose that $\theta = 1$, i.e., that peasants and capitalists have identical technologies, and that $\alpha = \frac{1}{2}$, i.e., that the share of wages in total product is $\frac{1}{2}$ under purely competitive conditions. Assume that in dual equilibrium capitalists have just enough income to meet replacement requirements so that $(\delta/\Delta) = (\theta/\gamma^\alpha) \frac{1}{1-\alpha}$. Then the condition that $y_c^* > y_p^*$ reduces to $\gamma > 1$. That is, the existence of any wage gap insures that capitalists have a higher net per worker output and gross per worker output than peasants. The condition that $c_c^* > c_p^* (=y_p^*)$ reduces to $\gamma < 2.4$ (approximately), so that if $1 < \gamma < 2.4$, there is no conflict in the goals of maximizing gross or net output and maximizing the per capita consumption of laborers and peasants. In this case, it is socially desirable that capitalists grow at the expense of peasants while dual equilibrium persists.

Suppose then that the agricultural economy is so structured that in dualistic equilibrium the levels of gross and net per worker output and per worker consumption are higher in the capitalist sector. Suppose also that capitalist income net of replacement outlays is non-negative. We have just seen that such a case is possible. Recall now the assumptions made earlier about population growth. If it is

assumed that c_c^* and y_p^* are high enough to result in positive net population growth in the dualistic phase, then given any $k(0)$, $k_p^* \leq k(0) \leq k_c^*$, \dot{k} will be negative and n_p will approach and eventually become equal to one, i.e., peasants will displace capitalists absolutely and both per worker output and consumption will fall. When k falls below k_p^* , then \dot{k} will be equal to $-k \cdot \epsilon_p(y_p)$. Since $y_p = k_p^{1-\alpha} - \delta k_p$, differential equation (1.15) may be written as follows for the all peasant phase:

$$(1.15') \quad \dot{k} = k \cdot \epsilon_p (k^{1-\alpha} - \delta k).$$

Now at $k = k_p^*$, we know that $\epsilon_p > 0$ by assumption so that $\dot{k} < 0$, and k must fall. As k falls, eventually $k^{1-\alpha} - \delta k$ must fall, and as peasant net income falls, the relative rate of population growth, ϵ_p , must fall. It is clear that the system will tend toward a point of zero population growth and with an aggregate capital-labor ratio given by a solution to the following equation

$$\epsilon_p (\bar{k}^{1-\alpha} - \delta \bar{k}) = 0.$$

This is a case of a kind of Malthusian equilibrium. Population growth reduces incomes to a subsistence level. It is interesting to observe that periodic withdrawals of labor from such an agricultural system would leave total agricultural output unaffected in the long run since a removal of labor at the Malthusian equilibrium point would

temporarily raise k and hence induce positive population growth until k again falls to \bar{k} , thus bringing total output back to the Malthusian equilibrium level.

It is clear that per worker consumption (identical to net per worker output in the peasant sector) must be lower in the terminal equilibrium state than when dualistic equilibrium existed because population growth becomes non-positive only when per worker consumption has fallen below the dualistic equilibrium level by virtue of the assumption that population growth is positive in the dualistic state. It should be noted, however, that it is possible for per worker consumption to rise temporarily as k declines from k_p^* to \bar{k} . This is easily seen with the aid of Figure 1.14. If k_p^* lies to the right of $k^\#$ as in that diagram, then it is obvious that y_p must rise temporarily as k falls.

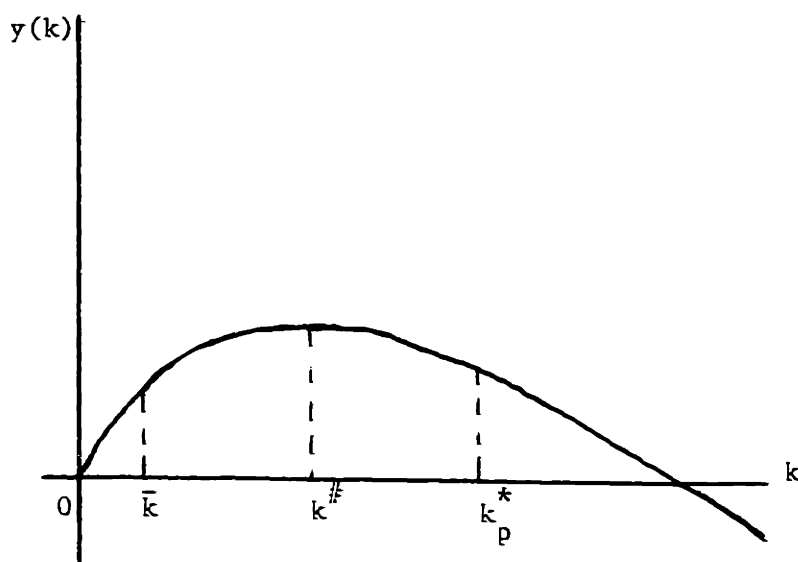


FIGURE 1.14

The conclusions of this model are necessarily gloomy. With no stimulus such as technological progress, positive net capital accumulation, a net population drain to growing non-agricultural sectors, or improving agricultural terms of trade there is little hope for raising agricultural income above the subsistence level. The agricultural economy is forced into an all peasant subsistence equilibrium.

Previous welfare discussion relating to this model have considered only the dualistic phase. An interesting question to ponder is the following: Suppose that by some means it were possible to force all peasants off their land and into the capitalist labor force. If it is assumed that the population growth functions are the same in both sectors, i.e., $\epsilon_p(y_p) = \epsilon_c(c_c)$ if $y_p = c_c$, then it will remain true that population growth will vanish at the same per worker consumption level as before, say \bar{c} . But since now all capital is in the hands of capitalists, the aggregate capital-labor ratio will stabilize at a new value $\bar{k} = (\bar{c}/\alpha\theta)\frac{1}{1-\alpha}$ in contrast to the old value $\bar{k} = (c + \delta\bar{k})\frac{1}{1-\alpha}$. Thus gross and net per worker output will be different under the all capitalist regime. Capitalist per worker gross output will exceed that of peasants if and only if $\theta\bar{k}^{1-\alpha} > \bar{k}^{1-\alpha}$ or, making substitutions for \bar{k} and \bar{k} , if and only if

$$\bar{c} < \left(\frac{1-\alpha}{\alpha\delta}\right)^{\frac{1}{\alpha}} - 1 \quad \left(1 - \frac{1-\alpha}{\alpha}\right).$$

Capitalist per worker net output will exceed that of peasants if and

only if $\bar{y} = \bar{k}^{1-\alpha} - \delta\bar{k} > \tilde{c}$, or

$$\tilde{c} < (\alpha\theta)^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\alpha\delta}\right)^{\frac{1-\alpha}{\alpha}}.$$

These results imply that if \tilde{c} is low enough (and hence if \bar{k} is low enough), then putting all production into the hands of capitalists would make it possible for peasants to be equally well off while per worker gross and net output rise. If \tilde{c} is not low enough, then imposition of a capitalist regime, even if it possesses a superior technology, would not only be unwise, it might be impossible, for it could result in a situation where capitalist profits were inadequate to meet replacement requirements on the capital stock. In fact, when $\bar{y} < \tilde{c}$, this must be the case, since the condition that $\bar{y} > \tilde{c}$ is the same as the condition that $(1-\alpha)\bar{k}^{1-\alpha} - \delta\bar{k}$ be positive.

It is perhaps interesting to observe that the agricultural surplus or amount of agricultural output exchanged for non-agricultural goods may rise or fall when a capitalist regime is imposed. Since the size of the capital stock is fixed, the agricultural surplus arising from replacement investment will be the same under either regime if it is assumed that the same proportion of such investment goods must come from the non-agricultural sectors, regardless of who buys them. If in both sectors the same proportion of net output is allocated to consumption of domestic non-agricultural goods, then agricultural surplus per worker will rise when a capitalist regime is imposed and only if c is low enough to allow $\bar{y} > \tilde{c}$. But whether the absolute

magnitude of the surplus will rise depends on what happens to total population. If $\theta = 1$, then in order for \bar{y} to exceed \bar{c} it must be true that $\bar{k} > \bar{k}$, i.e., that total agricultural population be lower in the imposed capitalist regime. Hence the lesser number of workers may offset the rise in surplus per worker. If $\theta > 1$, then it may be true that both $\bar{y} > \bar{c}$ and $\bar{k} < \bar{k}$ and thus that the absolute surplus from agriculture may rise under capitalist management. In general, the absolute agricultural surplus will rise under imposed capitalism if and only if

$$(\bar{y}/\bar{k}) - (\bar{y}/\bar{k}) > 0.$$

Substituting for \bar{y} , \bar{k} , and \bar{y} , this condition becomes

$$\bar{k} < \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha}} (1 - \alpha\theta^{\frac{1}{\alpha}}) \text{ or}$$

$$\bar{c} < \left(\frac{1}{\delta}\right)^{\frac{1}{\alpha} - 1} (1 - \alpha\theta^{\frac{1}{\alpha}}) [(1 - \alpha\theta^{\frac{1}{\alpha}})^{-\alpha} - 1].$$

Notice that again there is a maximum \bar{k} or \bar{c} which will permit a result favorable to imposed capitalism.

To summarize the results of the model with population growth and no capital accumulation, peasants will displace capitalists if not interfered with. If a capitalist form of organization is imposed, laborers will be equally well off as they were as peasants, since equilibrium per worker income is determined by the neo-Malthusian population growth mechanism. Depending on conditions, absolute

agricultural output and/or the absolute agricultural surplus will rise or fall if capitalist organization is imposed. Also, the existence of a capitalist regime may create a new source of savings for development purposes.

Now suppose that there exists a positive population drain to the non-agricultural sectors. If this drain is large enough, it is obvious that k might stabilize at a higher income level. Net population growth may achieve a zero rate of increase at a level of k high enough to permit the existence of dualistic equilibrium. This would occur in the case analyzed here if there exists an n_p , $0 < n_p < 1$, such that $n_p \epsilon_p^* + (1 - n_p) \epsilon_c^* = \epsilon_d$, where ϵ_d is the relative rate of population outflow.

The above analysis of equilibrium in the agricultural sector under conditions of variable population was conducted on the basis of the assumption that production functions were Cobb-Douglas. When the general class of constant returns to scale, diminishing returns production functions is allowed, anything can happen. Recall from the investigation of the existence of factor market equilibrium and the examples of possible situations shown in Figures 1.3 and 1.4 that there could be no equilibrium or several equilibria, each of which might be stable, semi-stable, or unstable. Depending on the factor endowment ratio, k , none, one, or more of these equilibria might be consistent with factor availabilities.

Recall that it has been assumed that the time required to attain factor market equilibrium is very small relative to the time required

to alter population significantly, so that one may safely consider the factor market adjustment process to have worked itself out (assuming it to have some stable stationary point). If the ultimate state of this process is unisectoral, then the displacement issue is settled already, although it may be reopened if k later changes so as to permit such a possibility. If the initial equilibrium is dualistic and stable, then, as above, peasants will displace capitalists if and only if net agricultural population is growing. If the possibility of both a dualistic equilibrium and a unisectoral one exists, and they are locally stable, then the initial intersectoral allocations of the productive factors will uniquely determine whether the dualistic point is approached.^{8/}

Given the assumption of pre-adjusted factor market equilibrium, the agricultural sector's dynamic behavior may be studied in a series of discrete steps: Examine $k(t_0)$ and the allocations of capital and labor between the sectors at time t_0 . These allocations indicate unambiguously the nature of factor market equilibrium. The equilibrium allocations of factors determine, through the effect on population growth, whether k rises or falls. As k changes then these steps must be repeated. Anything goes on this general level. In any practical situation it is very important to establish the nature of market imperfections or gaps and to determine what kinds of production functions obtain.

Although it would be possible now to trace out the effects of such further complications as technological progress, capital accumulation

under specific savings-investment assumptions, and interaction with the non-agricultural sector using the model developed above, this will not be done. Instead we shall in the next chapter use a modified version of this model which rests on a different view of factor market interaction and which perhaps more closely describes the typical situation in developing countries.

FOOTNOTES

[1] See, for example, W.A. Lewis, "Economic Development with Unlimited Supplies of Labour," Manchester School, XXII (May, 1954), pp. 139-91; P.N. Rosenstein-Rodan, "Problems of Industrialization of Eastern and Southeastern Europe," Economic Journal, LIII (June - September, 1943), pp. 202-11; D.W. Jorgenson, "The Development of the Dual Economy," Economic Journal, LXXI (September, 1961); and J.C.H. Fei and Ranis, Development of the Labor Surplus Economy: Theory and Policy (Homewood, Ill., 1964).

[2] A.K. Sen, "Peasants and Dualism With or Without Surplus Labor," Journal of Political Economy, LXXIV (October, 1966), pp. 425-450. See especially pp. 425-8, 438-441.

[3] By neoclassical the usual constant returns to scale, diminishing returns production function is meant: $\lambda F(K,L) = F(\lambda K, \lambda L)$;
 $\partial F / \partial K, \partial F / \partial L > 0$; $\partial^2 F / \partial K^2, \partial^2 F / \partial L^2 < 0$.

[4] Cf. pp. 61-63.

[5] Sen, p. 441.

[6] From now on the abbreviations LHS and RHS will be used.

[7] The stable arms of the system will be unique if and only if the RHS's of differential equations (1.12) and (1.13) are Lipschitzian. It is sufficient that they be twice continuously differentiable.

[8] Again, this is true if and only if the RHS's of the differential equations are Lipschitzian.

CHAPTER II

EXTENSIONS OF THE THEORY TO COVER THE DYNAMICS OF SAVING AND INVESTMENT

A striking feature of capital markets in developing countries is that they are usually quite imperfect, if they exist at all in some sectors of the economy. The capital investment markets for peasants and capitalists may function separately so that the investing of current savings does not shift freely between capitalist and peasant enterprises according to differences in the rates of return on capital. This is because there may be significant institutional barriers to getting savings out of one sector for investment in the other. Such barriers may be due to tradition, ignorance, or political factors, but economic variables may also play a role. For example, there are probably very high costs attached to financing peasant investment for reasons of risk and the small scale of operations and fragmentation of holdings, a point which was discussed earlier. Peasant investment is usually financed by internal savings, if not internal to the family then most likely internal to the peasant economy, if included in this are the rural moneylenders. On the other hand, capitalist investment is most likely to be financed by foreign sources or from the profits or savings of workers in existing capitalist enterprises.

But failure of capital creation to reflect differences in rates of return may be ameliorated if there exist reasonably well-functioning

sale or rental markets for existing land-capital goods. However, in many developing countries these markets are also quite imperfect, if extant. For example, capitalists are usually unwilling to purchase or rent individual, scattered smallholdings since capitalists typically operate on relatively large compact tracts of land. The difficulties and costs of buying up or renting many smallholdings in one contiguous area must be great. If other options are open to the capitalist, e.g., in the form of developing previously unused land or increasing capital intensity on existing land, they are likely to be more profitable than acquisitions of peasant land. Similarly, if the option to move on to uncleared land exists for the peasant, he too will probably find this more profitable than trying to buy or rent a plot from a large capitalist enterprise. As in the case of the investment market, it is also likely to be true in the market for existing capital that tradition, taboo, inertia, ignorance, politics, and uncertainty, plus perhaps the lack of effective intermediating institutions, all may work to inhibit reallocation of capital according to differences in rates of return.

Therefore, in this chapter it will be assumed that neither capital goods nor savings move between sectors as a result of differences in rates of return. The savings of each sector stay in each sector, as do existing capital goods, so that the capital stock in each sector may disappear only through failure to save and invest enough in each sector to offset the depreciation of capital in each sector. Whether this assumption is closer to the truth than

the assumption that capital flows freely and rapidly, subject possibly to the effect of a gap, according to differences in rates of return is no doubt a moot point. It is the writer's belief that while both assumptions do violence to many real situations, both are probably worth using, and that perhaps more emphasis should be placed on the non-flowing assumption cases, since casual empirical observations seems to suggest that if rates of return do make a difference, they must take a long time to influence capital allocations significantly in the developing world, and hence it would be interesting to know what other forces might be at work in the meantime. That is to say, the assumption of completely imperfect or disjoint capital markets may more adequately caricature reality than one permitting fluid adjustments.

Since capital does not leave the peasant sector when workers leave the family unit to join the capitalist labor force, the decision of a peasant to move may be interpreted in such a way as to define labor market equilibrium differently. Assume that a peasant will move from one sector to the other if his expected disposable income after moving, modified in the appropriate way by an income gap parameter (γ), exceeds his current disposable income. (The definition of disposable income will be made clear below.) Expected disposable income is assumed to be simply the currently observed level of disposable income in the other sector. While a worker is a member of the family unit, he receives his individual share of the imputed net rental income on the jointly owned family capital stock in addition to his marginal labor

product. When he leaves the family he no longer receives this rental income. If net rental income is equal to the full competitively imputed share of capital, then the above is equivalent to saying that each family worker receives his average product. But it is assumed here that the peasant may not receive as disposable income his full average product, either because of family debt obligations on rented or purchased capital or because of family recognition of capital maintenance and, perhaps, capital creation requirements. If the head of the family (according to family wishes) skims off a proportion, σ_p , of family output for debt service or capital expenditures, then the net imputed rental income accruing to each family working member is the total competitively imputed rental income less the amount skimmed off per worker by the family head. (If σ_p equals the elasticity of output with respect to capital at the family's point of operation on the production function, then all imputed rental income is absorbed for capital expenditures.) Thus, the individual peasant may be thought to regard $(1 - \sigma_p)$ times his average product as his disposable income. When the worker considers his alternative as a laborer, he will take as the measure of disposable income the full wage, equal to the marginal product of labor in the capitalist sector, since savings for purposes of capital accumulation and maintenance are no longer demanded of him directly, as when he was effectively part owner of the family enterprise, but are assumed to be supplied by capitalist profits. It will be assumed that the entire disposable incomes of peasants and laborers are consumed so that per worker

consumption and disposable income are identical.

The equation of labor market equilibrium is now written as shown:

$$(2.1) \quad \gamma(1-\sigma_p)g(k_p) = \theta w(k_c),$$

where, it will be recalled, $w(k_c) = g(k_c) - k_c g'(k_c)$. Since $g(\cdot)$ is monotonically increasing and since $g(k_c) > w(k_c)$, it follows that $g(k_p) < w(k_c)$ if $\gamma(1-\sigma_p) > \theta$. If $\gamma(1-\sigma_p) < \theta$, it is possible that $g(k_p) < g(k_c)$. In any case, there will be a minimum value of $\gamma(1-\sigma_p)/\theta$ for which it is true that $g(k_p) < g(k_c)$ and hence for which the labor intensiveness of production in the peasant sector exceeds that in the capitalist sector, ($k_p < k_c$). If $\gamma(1-\sigma_p) > 1$, then it is assured that the marginal productivity of labor in the capitalist sector always exceeds that in the peasant sector. A necessary and sufficient condition for a positive labor productivity gap is that $\gamma(1-\sigma_p) > 1 - \pi_p$, where π_p is the imputed competitive share of capital in peasant output. If $\sigma_p < \pi_p$, as is to be expected, then it is sufficient that γ not be less than one.

It should be clear that if population growth and net capital accumulation are zero, the model is essentially the same as the fixed capital allocation model discussed in Chapter I on pages 3 - 7, except that the form of the labor market equilibrium condition has been altered. Since the allocation of capital stocks is fixed, there will always exist a dual equilibrium if $g'(0) = \infty$ and $g'(\infty) = 0$, and

dual equilibrium is still likely to exist when these extreme point conditions on the marginal product of capital are relaxed. A flow of workers back to the family unit will raise wages and lower peasant net average product, thus eventually eliminating the incentive to return to the family unit. A flow of peasants into the capitalist labor force will reduce wages and raise peasant average product, thus reducing the incentive to move in that direction. Since now it is not possible to explain the displacement of one sector by the other strictly in terms of factor market behavior, the influence of other elements of economic behavior must be considered for example, technological progress and savings-investment behavior.

As in the analysis conducted in the freely flowing capital case, the effect of agricultural population growth under static technological conditions and with no positive net capital accumulation must be to depress agricultural per capita consumption to the subsistence level, at which point the population valve is shut off. If at or above subsistence level incomes the replacement of depreciated capital can be maintained for some positive size of the capital stock, then the subsistence equilibrium will be dualistic, but relative displacement may have occurred in that the shares of population in each sector may have changed.

Let us now consider how the process of capital accumulation may affect the interaction of peasant and capitalist. It will be observed that the following analysis of dynamic behavior is based on the asymptotic movement of the agricultural economy toward steady states.

It might appear objectionable that this technique be employed in a study of agriculture since when population is growing, it is necessarily true that steady state behavior requires an equivalent rate of growth of the land-capital stock. This would seem to be an unrealistic requirement on agricultural behavior since the stock of available arable land is not in general limitless, nor is the substitutability of non-land capital for land likely to be perfect, or even close to it, for all possible aggregations of land and non-land capital.

Nonetheless we shall assume that the land-capital stock may grow without limit, since it is possible to put a number of meaningful interpretations on the analysis thus conducted. Consider first the idea of substitutability and limitless land-capital accumulation. In some backward countries there exists substantial uncleared but arable land, so that for long periods of time the agricultural economy may experience no barrier to land-capital accumulation, even with limited substitutability among capital goods and land. Furthermore, there do in fact exist many non-labor, non-land inputs which may easily increase the effective land-capital stock, especially in developing countries where their use is now quite limited. Some of these inputs are fertilizers, improved seed types, pesticides, irrigation facilities and other land improvements such as drainage, and, above all, machinery and equipment. Also, it is likely that as time passes, technological progress will improve the substitutability of non-land capital goods for land. If such progress keeps ahead of the rate of population growth, then fixed land represents no problem.

A second factor to consider is the exogenously given population growth function. In most of the ensuing work, a non-zero rate of population growth is assumed, but it must be apparent that the rate of population growth must ultimately reach zero since the world is finite. If governments act to prevent population growth from being shut off in a Malthusian way, then a static population could permit such long-run stationary states as described below with the only investment then being of a replacement worth. There is no difficulty in assuming fixed land, even with zero non-land capital substitutability in this case.

Finally, it remains true that the analysis can be applied over limited time. That is, even if a steady state cannot be approached asymptotically, it may be possible for the agricultural economy to move toward a steady state for a finite period of time. The models to be presented below will tell us a great deal about non-steady-state behavior, as well as about the steady state itself.

Suppose that the same neoclassical production functions obtain as before, that the labor market equilibrium is described by equation (2.1), and that there is no technological progress. Assume that depreciation occurs in the form of radio-active decay at a fixed relative rate, δ , in both sectors and that the relative rate of population growth is fixed exogenously at a value ϵ . Capitalists are assumed to save and invest all profits, laborers to consume all income, and peasants to save and invest a fixed proportion, σ_p , of gross output. If the labor market is assumed always to be in equilibrium, then the

behavior of the agricultural economic system will be described by equations (2.2) and (2.3), below, which state that saving per worker in each sector is equal to investment per worker, and by equations (1.10) and (2.1). Note that the assumption of disjoint capital markets is now employed

$$(2.2) \quad \theta k_c g'(k_c) = \dot{k}_c + \left(\frac{\dot{n}_c}{n_c} + n\right) k_c,$$

$$(2.3) \quad \sigma_p g(k_p) = \dot{k}_p + \left(\frac{\dot{n}_p}{n_p} + n\right) k_p.$$

$$\delta + \epsilon = n$$

Consider first the existence of static equilibrium. Let $k_p = h(k_c)$ be the functional relationship between k_p and k_c which applies when equation (2.1) is satisfied. In static equilibrium, the capital stocks in each sector are fixed as is total population. Let $v_p = K_p/N$ and $v_c = K_c/N$. It must be true by definition that $n_p k_p = v_p$ and $n_c k_c = v_c$. From these equations and equation (1.10) it can be deduced that $1 - v_c/k_c = v_p/h(k_c)$. Since $h'(k_c) > 0$ it is obvious that there exists a unique $k_c^* > 0$ which satisfies this equation in k_c . Using the labor market equilibrium requirement, the unique equilibrium value of k_p , k_p^* , is immediately obtainable. Now n_p^* and n_c^* may be obtained from the resource constraints. Since there is only one market, there is no problem of stability if it is assumed that laborers move to the sector of higher rewards, modified by the gap parameter.

Now let us examine the dynamic behavior of the system.

Eliminating n_p and k_p from equation (2.3) by means of equations (1.10) and (2.1), the following system of differential equations can be obtained:

$$(2.4) \quad \dot{n}_c = \frac{n_c(1-n_c)}{n_c + (1-n_c)\epsilon k_p, k_c} \{ \epsilon k_p, k_c [\theta g'(k_c) - \eta] - [\sigma_p \frac{g(h(k_c))}{h(k_c)} - \eta] \}$$

$$(2.5) \quad \dot{k}_c = \frac{k_c}{n_c + (1-n_c)\epsilon k_p, k_c} \{ n_c [\theta g'(k_c) - \eta] + (1-n_c) [\sigma_p \frac{g(h(k_c))}{h(k_c)} - \eta] \}.$$

The term $\epsilon k_p, k_c$ is the elasticity of k_p with respect to k_c as determined by the function $k_p = h(k_c) = h$, which is obtained by solving equation (2.1) for k_p . From equation (2.5) it can be seen that

$$\dot{k}_c \geq 0 \text{ as } \theta g'(k_c) - \eta \geq - \left(\frac{1-n_c}{n_c} \right) [\sigma_p \frac{g(h)}{h} - \eta].$$

Notice that $\dot{k}_c = 0$ only when $[\theta g'(k_c) - \eta]$ and $[\sigma_p g(h)/h - \eta]$ of the above inequation are of opposite sign. Examining next equation (2.4), it is easily seen that

$$\dot{n}_c \geq 0 \text{ as } \epsilon k_p, k_c [\theta g'(k_c) - \eta] \geq \sigma_p [g(h)/h] - \eta.$$

Notice that only when $[\theta g'(k_c) - \eta]$ and $[\sigma_p g(h)/h - \eta]$ of the above inequation are of the same sign with $\dot{n}_c = 0$. It is clear then that no long-run dualistic equilibrium may exist since there will exist no value of k_c for which the two sides of the above two inequations will be at once of the same sign and of opposite sign, and since in general there is no reason to expect $g'(k_c) = \sigma_p \cdot g(h(k_c))/h(k_c) =$ for any given value of k_c . Thus while the existence of disjoint land-capital markets insures that dualistic equilibrium will exist in the short run, i.e., for fixed capital stocks, the introduction of the process of capital accumulation necessitates the relative, and may bring about the absolute, displacement of one sector by the other over time.

It should be clear that the use of a Cobb-Douglas production function to further illustrate the properties of the capital accumulation model is not as severe a restriction as in the case of some of the earlier models. If, then, we let $g(k_i) = k_i^{1-\alpha}$, where α is the constant elasticity of output with respect to labor, then equation (2.1) becomes

$$(2.1') \quad \gamma(1-\sigma_p)k_p^{1-\alpha} = \theta\alpha k_c^{1-\alpha},$$

and solving for k_p in terms of k_c , one obtains

$$(2.1'') \quad k_p = \omega k_c = h(k_c),$$

where ω is a constant and is equal to $[\alpha\theta/(1-\sigma_p)\gamma]^{\frac{1}{1-\alpha}}$. Note that $k_p < k_c$ if and only if $\omega < 1$. Next, using equation (2.1"), it is easy to show that $\epsilon k_p, k_c$ is identically equal to one. Setting $\epsilon k_p, k_c = 1$ and substituting ωk_c for $h(k_c)$ in equations (2.4) and (2.5), the following two differential equations are obtained for the Cobb-Douglas case:

$$(2.6) \quad \dot{n}_c = n_c(1-n_c)[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]k_c^{-\alpha}$$

$$(2.7) \quad \dot{k}_c = [n_c(1-\alpha)\theta + (1-n_c)\sigma_p \omega^{-\alpha}]k_c^{1-\alpha} - nk_c.$$

From equation (2.6) it is evident that $\text{sign } \dot{n}_c = \text{sign } [(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]$. This latter expression is the difference between savings ratios adjusted for the difference in the marginal productivity of capital between the two sectors. (Since the income share of capital is constant with a Cobb-Douglas production function, the assumption that all profits are invested is equivalent to the assumption that a fixed savings ratio equal to the profit share obtains.) By substituting the expression $[\alpha\theta/(1-\sigma_p)\gamma]^{\frac{1}{1-\alpha}}$ for ω and making a few simple manipulations it can be shown that:

$$\text{sign } \dot{n}_c = \text{sign } \left[\left(\frac{1-\alpha}{\sigma_p} \right)^{1-\alpha} \left(\frac{\alpha}{1-\sigma_p} \right)^{\alpha} - \frac{\gamma}{\theta} \right].$$

From this it can be seen that the wage gap and the efficiency gap are qualitatively offsetting, increases in each favoring and hindering,

respectively, the likelihood of peasant dominance. According as $\alpha/(1-\alpha) \gtrless (1-\sigma_p)/\sigma_p$, raising σ_p will hinder or help peasants' relative position. If we make the seemingly plausible assumption that $\sigma_p < (1-\alpha)$, then $\alpha/(1-\alpha) < (1-\sigma_p)/\sigma_p$, and peasants will improve their chances of survival by raising σ_p to as close to $(1-\alpha)$ as is possible.

The term $\omega^{-\alpha}/\theta$ is the ratio of the marginal productivities of capital in the two sectors. It is clearly possible to have a negative \dot{n}_c if this ratio is high enough. Knowing that $\omega^{-\alpha}/\theta$ is this ratio, it is easy to interpret the expression $[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]$. The term $(1-\alpha)$ is the proportion of capitalist output saved, and $(1-\alpha)$ times the marginal productivity of capital in that sector is the additional output made available by invested capitalist savings. In the peasant sector, σ_p times the marginal productivity of capital in that sector is the increased output made possible there. Thus if $(1-\alpha)\theta > \sigma_p \omega^{-\alpha}$, one unit of current capitalist production will result in greater future output than one unit of current peasant output. If $(1-\alpha)\theta < \sigma_p \omega^{-\alpha}$, the opposite holds true.

Equation (2.7) indicates that the steady state capital-labor ratio for any arbitrary distribution of labor between the two sectors is a function of the modified savings ratios weighted by the proportion of the labor force in each sector. Figure 2.1 shows the $\dot{k}_c = 0$ stationary in (k_c, n_c) space under the alternative assumptions that $(1-\alpha)\theta >$ and $< \sigma_p \omega^{-\alpha}$, the solid curve representing the first case. It is readily verified that if k_c lies to the left of the stationary,

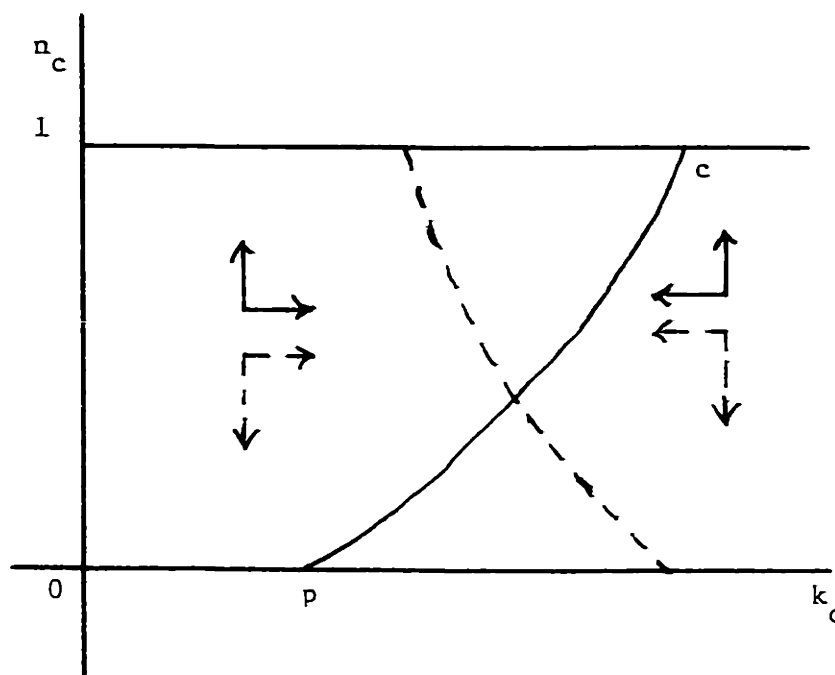


FIGURE 2.1

it is increasing ($\dot{k}_c > 0$) and that if it lies to the right, it is falling. The vertical arrows in the phase diagram, Figure 2.1, indicate the direction of motion of n_c corresponding to the two cases, the solid arrows referring to the case $(1-\alpha)\theta > \sigma_p \omega^{-\alpha}$. From the phase diagram it is clear that the agricultural economy moves toward either a virtually all-capitalist or a virtually all-peasant equilibrium and that this equilibrium is globally stable.

Up to this point it has only been asked in what direction relative displacement would occur. Relative displacement does not imply absolute displacement. In fact, both sectors may grow in absolute terms. Suppose that capitalists are displacing peasants relatively. It is easily shown that k_c will approach an equilibrium value $k_c^* = [(1-\alpha)\theta/\omega]^{1/\alpha}$.

Similarly, k_p, n_p , and n_c will approach their equilibrium values $k_c^*, 0$, and 1 , respectively. Knowing this, the peasant savings-investment equation may be written as follows when k_c^* is substituted for k_c .

$$\frac{\dot{k}_p}{k_p} + \frac{\dot{n}_p}{n_p} + \frac{\sigma \omega^{-\alpha}}{(1-\alpha)\theta} = \eta.$$

Now $\eta = \delta + \epsilon$, \dot{k}_p is approaching 0 and $\dot{n}_p/n_p = \dot{N}_p/N_p + \epsilon$, so the above equation may be written

$$\frac{\dot{N}_p}{N_p} + \delta = \frac{\sigma \omega^{-\alpha}}{(1-\alpha)\theta} (\delta + \epsilon).$$

It is clear that \dot{N}_p/N_p will be negative as k_c and n_c approach their equilibrium values if and only if

$$(1-\alpha)\theta/\sigma \omega^{-\alpha} > 1 + \epsilon/\delta.$$

Note that this condition for absolute displacement is a stronger version of the condition for relative displacement:

$$(1-\alpha)\theta/\sigma \omega^{-\alpha} > 1.$$

It is perhaps intuitively obvious that absolute displacement is more likely the lower the rate of population growth and the higher the rate of capital decay.

In an analogous fashion it may be shown that capitalists will be displaced absolutely if and only if

$$\sigma_p \omega^{-\alpha} (1-\alpha)\theta > 1 + \epsilon/\delta.$$

Observe that the same remarks about ϵ and δ apply here.

The foregoing analysis has been conducted on the assumption that the rates of depreciation are the same in both sectors. This assumption will now be temporarily relaxed to show how displacement behavior may be affected. Suppose that capital decays at relative rates δ_c and δ_p in the capitalist and peasant sectors respectively. Define $\eta_i = \epsilon + \delta_i$, $i=c,p$, then it is easily demonstrated that the differential equation system in k_c and n_c becomes

$$\dot{n}_c = n_c(1-n_c)\{[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]k_c^{-\alpha} + (\eta_p - \eta_c)\}$$

$$\dot{k}_c = [n_c(1-\alpha)\theta + (1-n_c)\sigma_p \omega^{-\alpha}]k_c^{-\alpha} - [n_c\eta_c + (1-n_c)\eta_p]k_c.$$

It is immediately apparent that displacement in the direction indicated by $[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]$ is reinforced or inhibited according as the sign of $(\eta_p - \eta_c)$ agrees with the sign of the former expression. If the signs are opposite, then there exists some \hat{k}_c ,

$$\hat{k}_c = \left[\frac{(1-\alpha)\theta - \sigma_p \omega^{-\alpha}}{\eta_c - \eta_p} \right]^{\frac{1}{\alpha}},$$

such that $\dot{n}_c = 0$. The question of interest is whether there also

exists some \bar{k}_c such that $0 < n_c < 1$ and $\dot{k}_c = 0$ for which it is also true that $\bar{k}_c = \hat{k}_c$. If so, then there will exist a point of dynamic dualistic equilibrium. The values of k_c for which $\dot{k}_c = 0$ are given by the following equation in n_c :

$$\bar{k}_c = \left[\frac{(1-\alpha)\theta n_c + \sigma_p \omega^{-\alpha}(1-n_c)}{\eta_c n_c + \eta_p(1-n_c)} \right]^{\frac{1}{\alpha}}.$$

Dynamic dualistic equilibrium will exist if and only if for some value of n_c , $0 < n_c < 1$, $\hat{k}_c = \bar{k}_c$, or substituting the RHS's of the above two equations, if and only if

$$\frac{[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}]n_c + \sigma_p \omega^{-\alpha}}{(\eta_c - \eta_p)n_c + \eta_p} = \frac{(1-\alpha)\theta - \sigma_p \omega^{-\alpha}}{\eta_c - \eta_p}$$

where n_c lies on the open interval $(0,1)$. By performing algebraic manipulations on the above equation it is found that the variable n_c drops out, and the condition becomes simply

$$(1-\alpha)\theta/\sigma_p \omega^{-\alpha} = \eta_c/\eta_p.$$

Now there is no reason to expect that the relevant parameters will satisfy this condition. What is of interest then is how displacement would occur if the above condition is not satisfied. It follows easily from the expressions for \hat{k}_c and \bar{k}_c that $\bar{k}_c = \bar{k}_c(n_c) > \hat{k}_c$ according as $(1-\alpha)\theta/\sigma_p \omega^{-\alpha} < \eta_c/\eta_p$ and that $d\bar{k}_c/dn_c > 0$ according as $\bar{k}_c(n_c) > \hat{k}_c$. for $0 < n_c < 1$.

As can be seen in the two phase diagrams, Figures 2.2a and 2.2b, the conclusions of the uniform depreciation case are only slightly modified. Capitalists displace peasants if and only if $(1-\alpha)\theta/\sigma_p \omega^{-\alpha} > \eta_c/\eta_p$, and the reverse occurs if and only if $(1-\alpha)\theta/\sigma_p \omega^{-\alpha} < \eta_c/\eta_p$. From now on it will be assumed that the η_i , $i=c,p$, are equal, though it should be borne in mind that a relatively larger depreciation rate in one sector will reduce its likelihood of survival and will also imply a lower steady state capital-labor ratio for that sector, should it exist alone, thus possibly modifying some of the welfare observations to be made below.

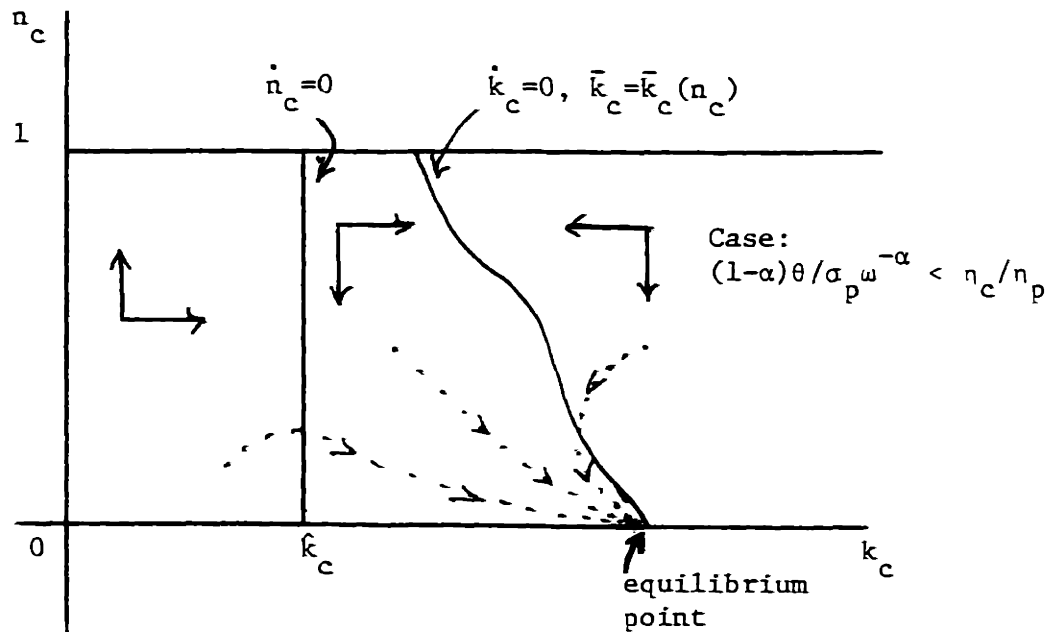


FIGURE 2.2a

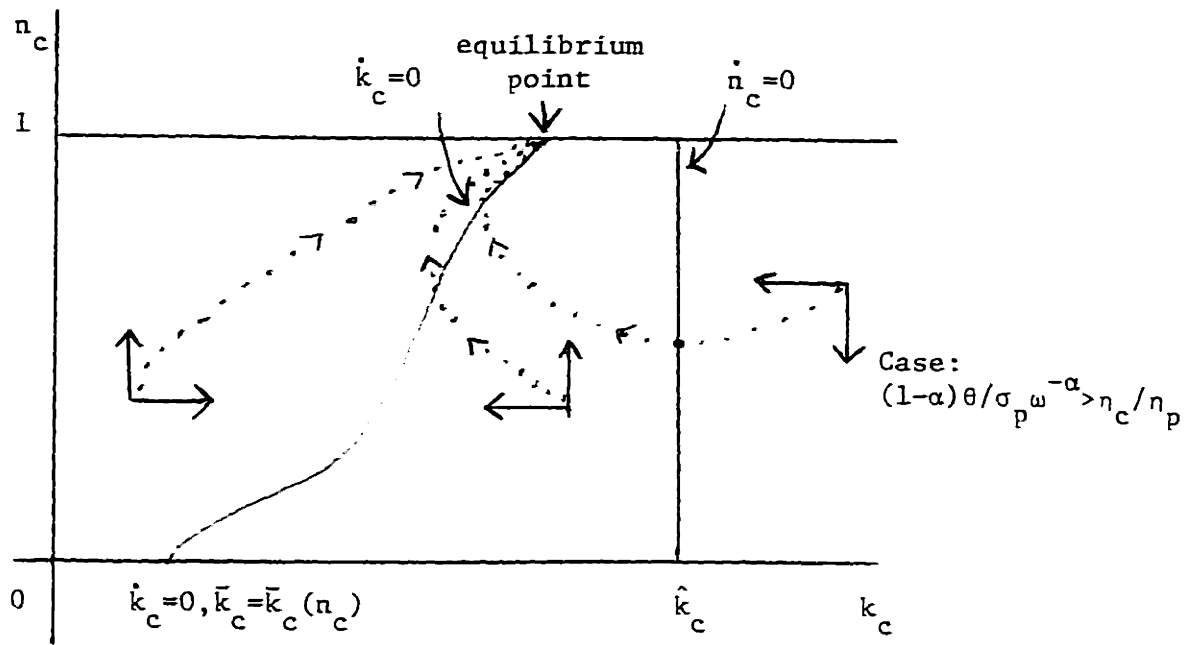


FIGURE 2.2b

By comparing properties of all-capitalist and all-peasant equilibria it is possible to make some simple observations about welfare. In an all-capitalist steady state, the equilibrium capital-labor ratio, k_c^* , is equal to $[(1-\alpha)\theta/\eta]^{1/\alpha}$, as can be verified by setting the RHS of equation (2.7) equal to zero and letting n_c take on the value unity. Steady state per worker output and consumption will be $\theta k_c^{*1-\alpha} = \theta[(1-\alpha)\theta/\eta]^{1-\alpha}$ and $\alpha\theta k_c^{*1-\alpha} = \alpha\theta[(1-\alpha)\theta/\eta]^{1-\alpha}$. Per worker output and consumption in the peasant sector are equal to $(\sigma_p/\eta)^{1-\alpha}$ and $(1-\sigma_p)(\sigma_p/\eta)^{1-\alpha}$, respectively. The ratios of steady state all-capitalist output and consumption to those of the all-peasant steady state are thus

$$r_q = \theta^{1/\alpha} \left(\frac{1-\alpha}{\sigma_p}\right)^{1/\alpha} - 1 \quad \text{and} \quad r_c = \theta^{1/\alpha} \frac{\alpha}{1-\sigma_p} \left(\frac{1-\alpha}{\sigma_p}\right)^{1/\alpha} - 1,$$

respectively.

Since $\theta > 1$ and $(1-\alpha) > \sigma_p$ by assumption, it follows immediately that $r_q > 1$ regardless of the direction of displacement. But why should the relative magnitudes of output per worker be of interest? The answer is that to the extent that the agricultural surplus, that is, the amount of agricultural output offered for exchange to the non-agricultural sector, is an increasing function of total output, given the terms of trade between agricultural and non-agricultural goods, then the magnitude of output per worker becomes of interest, since a larger output means that the agricultural sector may support the food needs of a larger non-agricultural sector. It is well known that the availability of food for the growing manufacturing sector is often a crucial constraint in the economic development process. Returning then to our comparison, the fact that $r_q > 1$ means that even if the economy moves toward an all-peasant steady state, an all-capitalist steady state will yield a higher output per worker than will an all-peasant one, even if both sectors have equally efficient technologies. All that is required is that peasants be less thrifty than capitalists. Suppose now that $(1-\alpha)\theta > \alpha \omega^{-\alpha}$ so that $\dot{n}_c > 0$, i.e., peasants are displaced relatively. Aggregate per worker agricultural output, q , may be written as follows when the economy is in a steady state:

$$(2.8) \quad q = \left[\frac{n_c (1-\alpha)\theta + (1-n_c)\sigma_p \omega^{-\alpha} \frac{1-\alpha}{\gamma}}{n} \right]^\alpha \cdot \left[n_c \theta + (1-n_c) \frac{\alpha\theta}{\gamma(1-\sigma_p)} \right].$$

(The term steady state is used here to mean any state in which $\dot{k}_c = 0$, but in which \dot{n}_c may be $\neq 0$. Only in the all-capitalist or all-peasant

steady states is $\dot{n}_c = 0$.) Since $\gamma \geq 1$ and $\dot{n}_c > 0$ by assumption, it can be seen immediately upon inspection of the above equation that q is an increasing function of n_c , so that all-capitalist steady state output per worker is superior to that of all steady states, since n_c , and hence q , achieves its maximum in that state.

The ratio of per worker consumptions in the two unisectoral or "pure" steady states, r_c , is also greater than one, regardless of the direction of displacement, under the assumptions made. All capitalist per worker consumption is $\theta^{\frac{1}{\alpha}} \alpha [(1-\alpha)/\eta]^{\frac{1-\alpha}{\alpha}}$ and that of peasants is $(1-\sigma_p)(\sigma_p/\eta)^{\frac{1-\alpha}{\alpha}}$. Since $(1-\alpha)$ is the savings ratio and is also the ratio of profits to output, it follows that setting $\sigma_p = (1-\alpha)$ will maximize peasant steady state consumption, since it is clear that $(1-\alpha)$ is the Golden Rule savings ratio introduced by Phelps.^{1/} Since $\sigma_p \neq (1-\alpha)$ it follows that $r_c > 1$ even if $\theta = 1$. Now suppose that $(1-\alpha)\theta > \sigma_p^{-\alpha}$. Aggregate per worker consumption, c , is the steady state may be expressed as

$$(2.9) \quad c = \alpha \theta \left[\frac{n_c (1-\alpha)\theta + (1-n_c)\sigma_p^{-\alpha}}{\eta} \right]^{\frac{1-\alpha}{\alpha}} \cdot [n_c + (1-n_c) \cdot \frac{1}{\gamma}].$$

As in the case of q , it is readily seen that c achieves its maximum at $n_c = 1$, so that per worker consumption in the all-capitalist steady state is superior to that in any other steady state. Thus when capitalists displace peasants, under the assumptions made above about parameter values, it follows that the agricultural economy moves to the best of all possible steady states.

It must be pointed out, however, that relative displacement of peasants, given any initial allocation of factors between the sectors, does not insure that per worker output and consumption will rise as equilibrium is approached. The following two equations define c and q , respectively, in terms of n_c and k_c :

$$c = [n_c + (1-n_c)/\gamma] \alpha \theta k_c^{1-\alpha}$$

$$q = [n_c + (1-n_c)(\alpha/\gamma(1-\sigma_p))] \theta k_c^{1-\alpha}.$$

If $k_c(0)$ falls to the right of the $\dot{k}_c = 0$ stationary in Figure 1.15, (solid lines), k_c will fall at the same time that n_c is rising, so that despite the fact that for every given k_c , a rise in n_c , under the assumptions made, will cause a rise in both c and q , nevertheless there is no assurance that they will not actually fall as steady state equilibrium is approached. If it be asked how initial capital stocks could be so large in the first place as to permit $k_c(0)$ to lie to the right of its stationary, the answer must be in terms of some exogenous factor such as a burst of foreign investment prior to the time when we as model builders look in upon the agricultural economy. Such an explanation would be quite plausible in many cases where foreign-owned capitalist firms have engaged in primary agricultural production for export.

One further point on the question of peasant versus capitalist savings and investment ratios should be mentioned here. It has, quite plausibly, been assumed that peasants save a lower proportion of output or gross income, $\sigma_p < (1-\alpha)$. It has further been assumed that all

savings were invested in agricultural capital investment. This latter assumption may be especially weak as regards capitalist investment since, for example, if capital is foreign owned, a substantial proportion of profits may be repatriated rather than invested in agriculture. But even if capital is domestically owned it may still be true that not all profits are reinvested in agriculture since land-owners may decide to invest in non-agricultural undertakings, which may appear more profitable, or to consume a portion of profits in the form of high living. A later alternative version of the model now under discussion will take account of these factors, but this limitation should be kept in mind in the displacement and welfare discussions related to the model now under investigation.

If it is now supposed that $(1-\alpha)\theta < \sigma_p \omega^{-\alpha}$, so that $\dot{n}_c < 0$, and peasants displace capitalists, then as noted above, the agricultural economy moves toward a steady state which is definitely inferior to the all-capitalist steady state. But it is no longer possible to say that the all-capitalist steady state is superior to all other steady states. Looking at equations (2.8) and (2.9), it is clear that raising n_c will lower the first bracketed term and raise the second, so that it becomes possible for q and c to be maximized at some values n_c^q and n_c^c respectively, where $0 < n_c^j < 1$, $j=q,c$. In any case, it remains true that the all-peasant steady state is undesirable since at least the all-capitalist and possibly some mixed steady states are superior in terms of both output and consumption per worker.

It must again be noted that given arbitrary initial values for k_c

and n_c , there is no assurance that c and q will not rise as the all-peasant equilibrium is approached. This would occur when $k_c(0)$ lies to the left of the $\dot{k}_c = 0$ stationary in Figure 2.1, (dotted lines). In this case, k_c will be continually rising and may rise sufficiently to offset the depressing effects of a fall in n_c .

It appears intuitively obvious that if capitalists save and invest at the Golden Rule rate and are perhaps technologically superior, while peasants save at less than the Golden Rule rate, then an all-capitalist steady state should be a kind of optimum for the agricultural sector. What is surprising is that under some conditions, e.g., given a large enough wage gap, peasants will displace capitalists and hence move the agricultural economy to a less than optimal position. It is interesting to note that when peasants tend to displace capitalists, a dualistic steady state, if it could only be attained and maintained, would possibly produce a higher level of consumption and/or output per worker than a virtually unisectoral one.

Now let us consider what happens when technological progress is introduced and population growth is made an endogenous variable. Suppose that the relative rate of population expansion is a function of consumption per worker in each sector, and in particular, suppose that capitalist laborers receiving a wage, $\alpha\theta k_c^{1-\alpha}$, will reproduce at the same relative rate as peasants receiving a net average product of $(1-\sigma_p)k_p^{1-\alpha} = \frac{1}{\gamma}\alpha\theta k_c^{1-\alpha}$, so that when the labor market is in equilibrium, population growth occurs at the same relative rate in both sectors. Now since the labor market is assumed always to be in equilibrium, it follows that the relative rate of population increase

can be expressed as a function simply of the capitalist wage, $\alpha\theta k_c^{1-\alpha}$.

Therefore we may write

$$(2.10) \quad \varepsilon = \varepsilon(\alpha\theta k_c^{1-\alpha}) = \varepsilon(k_c^{1-\alpha}),$$

where it is assumed that $\varepsilon'(\cdot) > 0$, $k_c < k_c^0$; $\varepsilon(\cdot) = \bar{\varepsilon}$, $k_c \geq k_c^0$; and $\varepsilon(0) < 0$.

Suppose now that technological progress is disembodied and occurs in Hicks-neutral form according to a non-decreasing function of time, $A(t)$, in both sectors as in earlier examples. Since production functions are Cobb-Douglas, this Hicks-neutral technological progress can be reinterpreted as being either capital or labor augmenting. Accordingly, if we think of it as labor augmenting, it will occur as indicated by the function $B(t) = [A(t)]^{\frac{1}{\alpha}}$. If labor is now measured in efficiency units, then the relative growth rate of "effective population" is $\dot{B}/B + \varepsilon$, where now $k_i = K_i/B(t) \cdot L_i$, $i=c,p$, and $\varepsilon = \varepsilon(B \cdot k_c^{1-\alpha})$. This alteration in the population growth function is brought about by the fact that when technological progress occurs, incomes rise when the effective capital-labor ratio is constant.

As a result of these changes in assumptions, differential equations (2.6) and (2.7) now appear as shown below:

$$(2.6') \quad \dot{n}_c = n_c(1-n_c)[(1-\alpha)\theta - \sigma_p \omega^{-\alpha}] k_c^{-\alpha}$$

$$(2.7') \quad \dot{k}_c = [n_c(1-\alpha)\theta + (1-n_c)\sigma_p \omega^{-\alpha}] k_c^{1-\alpha} - [\delta + \dot{B}/B + \varepsilon(B k_c^{1-\alpha})] \cdot k_c.$$

Notice first that the sign of \dot{n}_c is unaltered. Relative displacement will take place in the same direction as before. If \dot{B}/B is bounded above by some maximum rate, ρ , then $k_c^*(t)$, the steady state value of k_c toward which the system is moving at any point is bounded from below by some minimum, $k_c^{\min}(t) = [\bar{\sigma}(t)/(\delta + \bar{\epsilon} + \rho)]$, where $\bar{\sigma}(t) = [n_c(1-\alpha)\theta + (1-n_c)\sigma_p \omega^{-\alpha}]$. As time passes and displacement occurs, $\bar{\sigma}(t)$ approaches $(1-\alpha)\theta$ or $\sigma_p \omega^{-\alpha}$, whichever is greater. In this Cobb-Douglas model, then, the rates of technological change and population growth do not play a significant role. It is fairly obvious why. Technological change occurs at the same relative rate in both sectors. Peasants simply lag behind to an extent determined by θ . If technological progress occurs at a more rapid relative rate in one sector, then this can be interpreted as a change in θ . It is clear that if progress takes place more rapidly in one sector, then the less rapidly progressing sector must eventually be displaced. Changes in the rate of population growth serve only to alter the steady state capital-labor ratio toward which the economy is moving, and since this ratio does not affect savings ratios and the ratio of capital marginal productivities, population growth can play no significant role in displacement when capital goods do not move between sectors. If the relative rates of population growth differed as between sectors, then the displacement mechanism would still be unaffected, but the possible beneficial effects of capitalist displacement of peasants could be offset if the steady state population growth rate in an all-capitalist regime were significantly greater than in an all-peasant

regime due to a more fecund population growth function. Even when the population growth functions are identical, the higher population growth rate occurring when k_c rises will have a depressing effect on the otherwise beneficial results produced by greater savings capability ($\sigma_p < 1-\alpha$) and superior technology ($\theta > 1$) of capitalists. This effect could be strong enough to reverse some of the welfare observations made earlier. Strictly speaking, however, such an effect would be relevant in the identical reproduction rate case only if technological change were zero, since otherwise technological progress would insure that in both the all-peasant and all-capitalist regimes the relative rate of population expansion is pushed to its maximum, $\bar{\epsilon}$.

Having now gotten a feeling for the behavior of the capital accumulation model in the Cobb-Douglas case, let us now refer to the original formulation using a general neoclassical production function. Examine again equation (2.4) on page 68, and note again how the sign of \dot{k}_c depends on the magnitudes of $[\theta g'(k_c) - n]$, $\{[\sigma_p g(h(k_c))]/h(k_c) - n\}$, and n_c . To determine the exact nature of this relationship, refer to Figure 2.3 below. It is easily verified that the functions graphed are monotonic. Observe that the curves have been drawn on the assumption that $\sigma_p g(h(k_c))/h(k_c)$ equals n at a lower value of k_c than that for which $\theta g'(k_c)$ is equal to n . This need not be the case, and what happens when the opposite is assumed will be discussed presently. In accordance with previous analysis, if $k_c < k_c^a$, $\dot{k}_c > 0$, and if $k_c > k_c^b$, $\dot{k}_c < 0$. The $\dot{k}_c = 0$ stationary is thus defined only on the interval (k_c^a, k_c^b) . In the graph, the solid upward sloping curve is

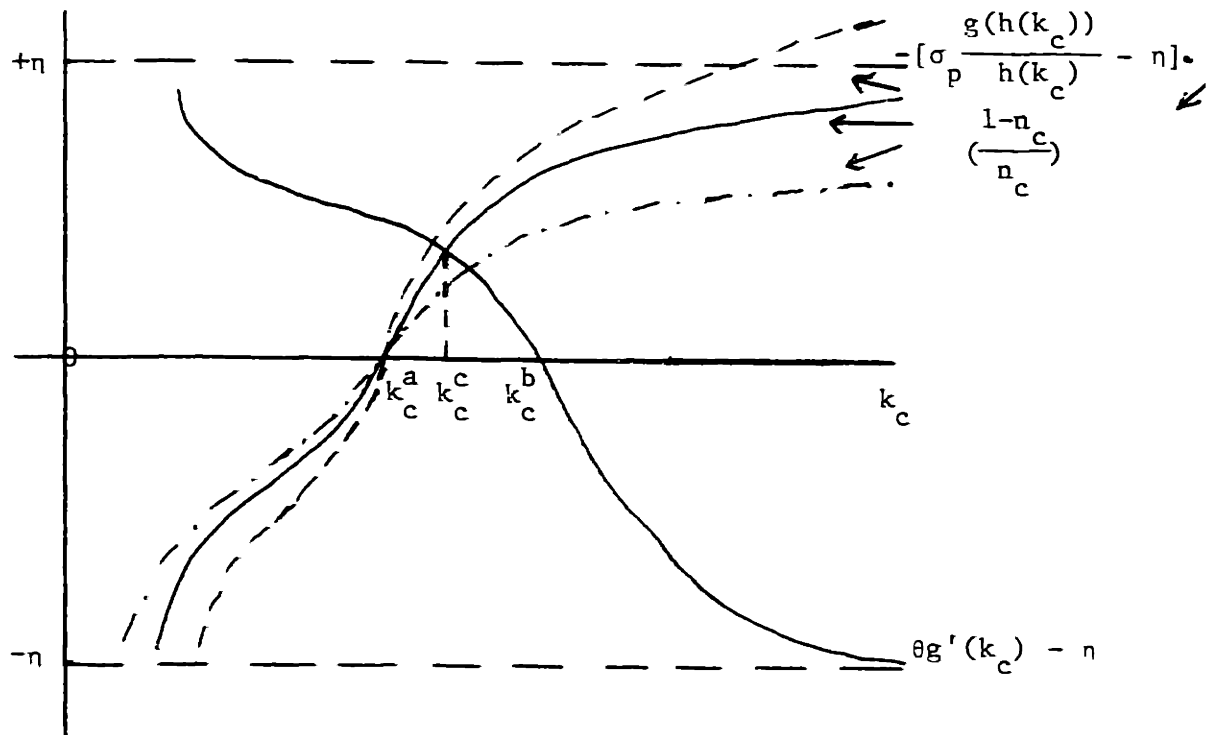


FIGURE 2.3

assumed to represent the case $n_c = \frac{1}{2}$, so that $(1-n_c)/n_c = 1$. In this case the value of k_c for which $\dot{k}_c = 0$ is k_c^c . If $n_c < \frac{1}{2}$, then $(1-n_c)/n_c > 1$ so for any such n_c the curve will shift proportionally about the point k_c^a as indicated by the dashed line. If $n_c > \frac{1}{2}$, then $(1-n_c)/n_c < 1$, and for any such n_c the curve will shift as indicated by the dot-dashed line. Since both the upward and downward sloping curves are monotonic and continuous, the value of k_c for which $\dot{k}_c = 0$ will rise monotonically from k_c^a to k_c^b as n_c goes from 0 to 1. If the graphs had been constructed so that the value of k_c for which $\theta g'(k_c) = n$ were lower than that for which $\sigma_p g(h(k_c))/h(k_c) = n$, then it would be true that $k_c^b < k_c^a$, and k_c would fall monotonically from k_c^a to k_c^b as n_c rose from 0 to 1.

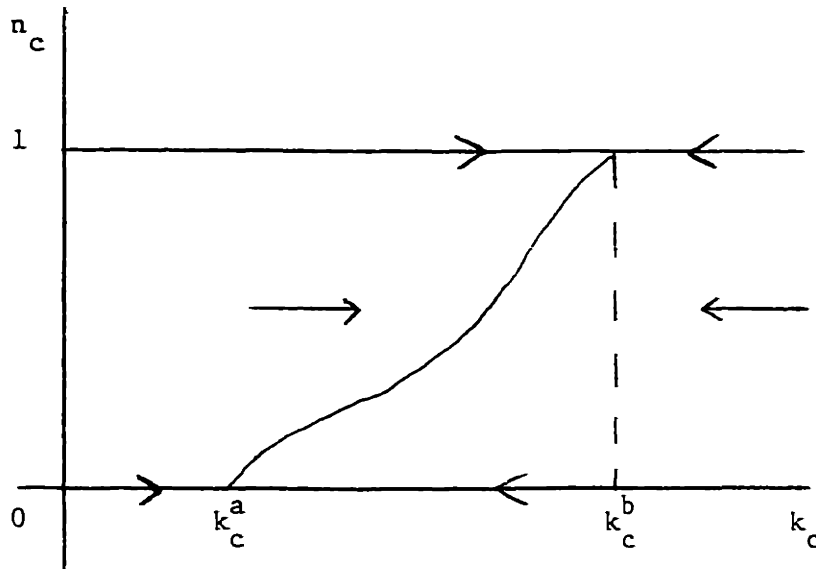


FIGURE 2.4

Figure 2.4 shows the $\dot{k}_c = 0$ stationary. The reader may verify that if an arbitrary point (k_c^*, n_c^*) does not lie on the stationary, then the sign \dot{k}_c is such as to move toward the stationary. (Note that it has been assumed that k_c^b exceeds k_c^a .) It is clear that whatever happens to n_c , which is confined to the closed interval $(0,1)$, k_c must approach some point on the $\dot{k}_c = 0$ stationary. Therefore, in order to determine the direction in which displacement will occur, it is sufficient to examine the behavior of \dot{n}_c for k_c restricted to the stationary curve. It follows from equation (2.5) that

$$(2.11) \quad \dot{n}_c = n_c(1-n_c) \left\{ \theta g'(k_c) - \sigma \frac{g(h(k_c))}{h(k_c)} - (1 - \epsilon k_p, k_c) \frac{\dot{k}_c}{k_c} \right\} .$$

When $\dot{k}_c = 0$, this reduces to

$$(2.12) \quad \dot{n}_c = n_c(1-n_c)\left\{\theta g'(k_c) - \sigma_p \frac{g(h(k_c))}{h(k_c)}\right\}.$$

Using equation (2.1) to eliminate $\sigma_p g(h(k_c))/h(k_c)$, the following equation for \dot{n}_c is obtained:

$$(2.13) \quad \dot{n}_c = n_c[\theta g'(k_c) - \eta].$$

Note that $\text{sign } \dot{n}_c = \text{sign } \theta g'(k_c) - \eta$, where k_c is constrained to lie on the $\dot{k}_c = 0$ stationary. Refer now to Figure 2.3. The set of k_c for which \dot{k}_c may equal 0 is the closed interval (k_c^a, k_c^b) . Since the two curves intersect in the first quadrant, it follows that if the two curves apply, $\theta g'(k_c) - \eta > 0$, and hence $\dot{n}_c > 0$. If the two curves in the diagram had been drawn to intersect in the fourth quadrant, the case of $k_c^b < k_c^a$, then the opposite result would have been obtained; i.e., $\theta g'(k_c) - \eta < 0$ and hence $\dot{n}_c < 0$.

Since both the curves in Figure 2.3 are monotonic, the location of their intersection in the first or fourth quadrant will be determined by which curve crosses the k_c axis at a higher value of k_c , i.e., by whether $\theta g'(k_c) - \eta$ or $\sigma_p g(h(k_c))/h(k_c) - \eta$ reaches 0 first as k_c increases. For example, if the latter expression reaches 0 first then $\dot{n}_c > 0$ on the $\dot{k}_c = 0$ stationary, and it is proved that capitalists will displace peasants relatively.

Using the information obtained above it is possible to estimate how changes in the parameters, θ, η, σ_p , and γ may affect the tendencies toward displacement in the general case of neoclassical production functions. Since the direction of displacement is indicated by the sign of $\theta g'(k_c) - \eta$ when the following constraints are in effect:

$$\theta g'(k_c) - \eta = - \left[\sigma_p \frac{g(k_p)}{k_p} - \eta \right] \text{ and}$$

$$\gamma(1-\sigma_p)g(k_p) = \theta w(k_c),$$

the derivative of $\theta g'(k_c) - \eta$ with respect to each of the above parameters, calculated under the imposition of the above constraints, will provide an indication of how changing a parameter may influence displacement. But, it must be warned, the signs of such derivatives do not prove that selecting a sufficiently large or small value of some parameter will cause $\theta g'(k_c) - \eta$ to acquire the desired sign, since that expression, constrained as noted, may be bounded with respect to increases or decreases in the parameter. Let $T(k_c) = \theta g'(k_c) - \eta$; then it can be shown that when the two constraining equations shown above are in effect,

$$dT/d\theta > 0, \quad dT/d\gamma < 0,$$

$$dT/d\sigma_p \gtrless 0 \text{ as } \sigma_p \gtrless k_p g'(k_p)/g(k_p) = \pi_p, \text{ and}$$

$$dT/d\eta \gtrless 0 \text{ as } k_p g'(k_p)/w(k_p) \gtrless [\sigma_p/\gamma(1-\sigma_p)] \cdot k_c/k_p.$$

Note that the first three of these derivatives agree qualitatively with the results obtained in the Cobb-Douglas case when those parameters were varied, since in that case $\pi_p = (1-\alpha)$. That is, lowering θ , or raising γ or σ_p , (if $\sigma_p < \pi_p$ in the case of σ_p), will improve the likelihood of a viable peasant agriculture. The possible weakness of this derivative test on $T(k_c)$ is indicated in the case of $dT/d\eta$.

With a Cobb-Douglas production function it can be shown that $dT/d\eta \gtrless 0$ as $(1-\alpha)\theta \gtrless \sigma_p \omega^{-\alpha}$, but from the results obtained in the analysis of population growth and technological progress changes in η do not qualitatively affect the direction of displacement. Even though the size of $T(k_c)$ may be altered by changes in η , its sign is always independent of that parameter.

What about welfare in this general case? In Figure 2.5 the two solid line curves of Figure 2.3 have been reproduced, and a third curve, $z = -[\sigma_p g(k_p)/k_p - \eta]$, has been added. Since the two original curves intersect in the first quadrant, it is known that capitalists displace peasants. Since $k_p = h(k_c) < k_c$ under the assumption of the model, it follows that the third curve lies to the left of the original upward sloping curve. Now it is clear that $k_c^* > k_p^*$ in the diagram, where k_c^* and k_p^* are the steady state capital-labor ratios in an all-capitalist and an all-peasant regime respectively. Since it has been assumed that capitalists invest all profits, then the all-capitalist steady state is a Golden Rule one in which steady state consumption per worker is maximized, so that even if θ is as low as one, it is assured that workers are better off in a capitalist steady state.

Furthermore, since $k_c^* > k_p^*$, it is clear that even when $\theta = 1$, output per worker is greater under total capitalism. Thus, as in the Cobb-Douglas case, it is true that when displacement favors capitalists, the resulting shifts lead to a steady state superior to an all-peasant one.

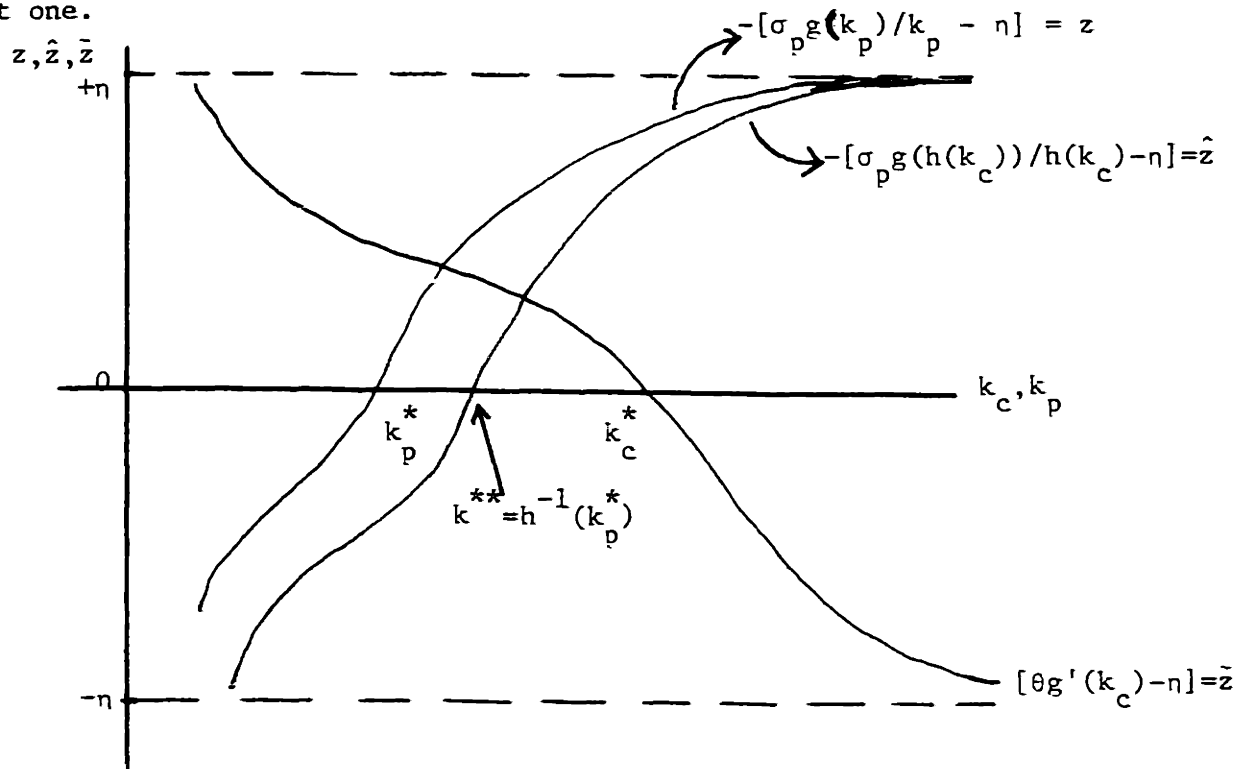


FIGURE 2.5

In Figure 2.6 the same three curves have been constructed on the inverse assumption, to wit, that peasants displace capitalists. The two curves which are functions of k_c intersect in the fourth quadrant. It is now true that $h^{-1}(k_p^*) > k_c^*$, but since $h(k_c) < k_c$, it follows that $k_p^* < h^{-1}(k_p^*)$ so that there is no assurance that $k_p^* > k_c^*$. Furthermore, even if $k_p^* > k_c^*$, it is not certain that per worker output in the all-peasant steady state is larger than in the all-capitalist one since θ may be greater than one and in fact may be sufficiently

large to insure that $\theta g(k_c^*) > g(k_p^*)$ when $k_p^* > k_c^*$. Moreover, since the all-capitalist steady state is a Golden Rule one, it insures that even under conditions of identical technology ($\theta=1$), per worker consumption is higher in the purely capitalist steady state. These results closely parallel those obtained in the Cobb-Douglas example.

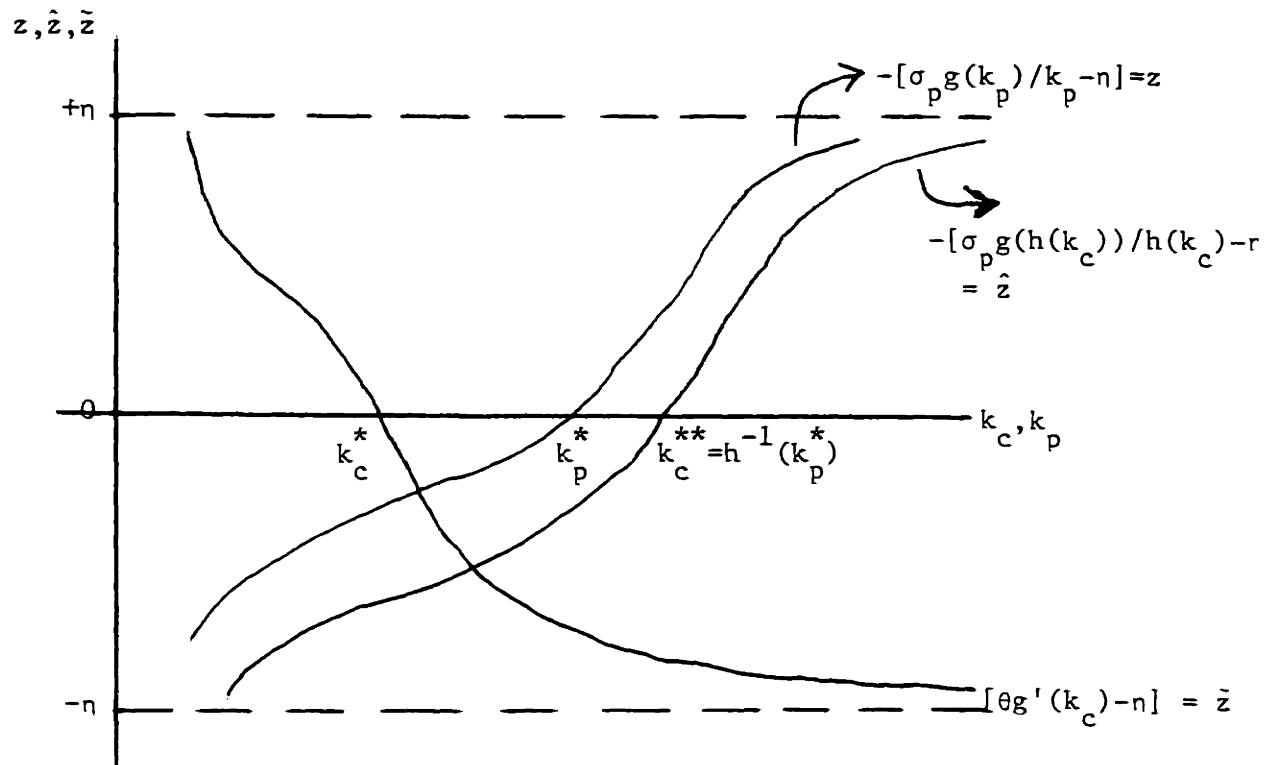


FIGURE 2.6

As in that special case, it is true that when capitalists displace peasants, both output and consumption per worker are maximized over all steady states. Look again at Figures 2.3 and 2.4. Since the $\dot{k}_c = 0$ stationary is upward sloping when peasants are displaced, it is obvious that k_c and n_c achieve their maxima at $n_c = 1$ and $k_c = k_c^b$. It can be shown that steady state output and consumption per worker

are given by the two following equations, respectively:

$$q = \theta [n_c g(k_c) + (1-n_c)w(k_c)]/\gamma$$

$$c = \theta w(k_c) [n_c + (1-n_c)/\gamma].$$

Since $g(\cdot)$ and $w(\cdot)$ are both monotonically increasing functions of k_c and $g(k_c) > w(k_c)$, it follows that the all-capitalist steady state is superior to any other.

When peasants displace capitalists, the $\dot{k}_c = 0$ stationary is then downward sloping, so that the effects of n_c and k_c work in opposite directions. It is possible that some dualistic steady states will result in a higher q or c than are attainable in the all-capitalist equilibrium.

It would be useful to investigate at this point how dependent the above welfare conclusions are upon the particular savings function assumed for the capitalist sector. We shall consider only the Cobb-Douglas case. Assume that in the capitalist sector a fixed proportion, σ_c , of output is saved and invested. Some of this saving may come from the laborers in the capitalist sector, but it will still be assumed that peasants compare their consumption level with the full capitalist wage, since individual laborers may be thought of as being free to dissolve their savings at any time for consumption purposes, whereas peasant saving must be seen as being in the nature of a not easily redeemable quasi-fixed claim on income since their

savings, once invested, may not be readily dissolved and if not maintained will result in loss of future income or default on debt obligations. Under this new assumption the two differential equations of the Cobb-Douglas case are modified as shown:

$$(2.6'') \quad \dot{n}_c = n_c(1-n_c)[\sigma_c\theta - \sigma_p\omega^{-\alpha}]k_c^{-\alpha}$$

$$(2.7'') \quad \dot{k}_c = [n_c\sigma_c\theta + (1-n_c)\sigma_p\omega^{-\alpha}]k_c^{1-\alpha} - nk_c.$$

The ratio of the "pure" steady state magnitudes of q and c are then given by the following equations:

$$r_q = \theta^{\frac{1}{\alpha}} (\sigma_c/\sigma_p)^{\frac{1-\alpha}{\alpha}}$$

$$r_c = \theta^{\frac{1}{\alpha}} [(1-\sigma_c)/(1-\sigma_p)] (\sigma_c/\sigma_p)^{\frac{1-\alpha}{\alpha}}.$$

If $\sigma_c > \sigma_p$, then $r_q > 1$. Of course, r_q may exceed unity even when $\sigma_c < \sigma_p$ if θ is large enough. In order that r_c be greater than unity it must be true that $r_q > (1-\sigma_p)/(1-\sigma_c)$. It is possible that too much thriftiness in the capitalist sector will result in relatively lower per worker consumption there, although I expect this is not very likely. Steady state q and c for any given n_c are given in the next two equations:

$$q = \theta \left[\frac{n_c\sigma_c\theta + (1-n_c)\sigma_p\omega^{-\alpha}}{n} \right]^{\frac{1-\alpha}{\alpha}} \cdot [n_c + (1-n_c)\frac{\alpha}{\gamma(1-\sigma_p)}]$$

$$c = \alpha \theta \left[\frac{n_c \sigma_c \theta + (1-n_c) \sigma_p \omega^{-\alpha}}{\alpha} \right]^{\frac{1-\alpha}{\alpha}} \cdot [n_c (1-\sigma_c) + (1-n_c) \frac{\alpha}{\gamma}].$$

If it is assumed, as before, that $\sigma_p < (1-\alpha)$, then if $\dot{n}_c > 0$, i.e., $\sigma_c \theta - \sigma_p \omega^{-\alpha} > 0$, it follows that q is maximized over all steady states, as is also c , if thriftiness in the capitalist sector does not cause σ_c to exceed $(1-\alpha) + \alpha[(\gamma-1)/\gamma]$. Note that even if peasants are more thrifty than capitalists, ($\sigma_p > \sigma_c$), capitalists may still displace peasants if they enjoy a sufficiently large technology gap. If $\sigma_c \theta - \sigma_p \omega^{-\alpha} < 0$, then q and c may or may not be maximized in either pure steady state. It is possible that dualistic steady states could produce larger q 's and c 's than either pure one.

FOOTNOTE

[1] E.S. Phelps, "The Golden Rule of Accumulation: A Fable for Growthmen," American Economic Review, LI (September, 1961), pp. 638-643.

CHAPTER III

OPTIMAL ORGANIZATIONAL POLICY IN A DUALISTIC AGRICULTURAL SECTOR

Up to this point we have studied the characteristics of a dualistic agricultural economy under various assumptions about factor market behavior, and we have inquired into the economic desirability of such behavior. It was seen that the factors of production may or may not be allocated in such a way as to result in economically desirable states. In this chapter, we shall take welfare maximization as a goal and ask the following question: If there existed a means of allocating land-capital freely between the peasant and capitalist sectors, how would it be allocated over time so as to maximize welfare in the agricultural sector?

Although capital may be freely shifted between sectors, other structural aspects of the agricultural economy are assumed to be unalterable: Each of the two sectors has its characteristic savings function, and the labor market functions as before: i.e., labor moves freely between the sectors, subject to the interference of such imperfections as discussed earlier. It is possible to give an institutional interpretation to the free allocation of capital in this economy. One may suppose that there exists a government or planning agency which may assign capital to each sector by confiscation and donation. Such confiscation may be assumed to have no effect on savings behavior, since as will be seen later, government inter-

ference will occur in one or two bursts in most cases and will always result in all production being undertaken by one class of agricultural productive units or the other. Another way of regarding the shifting of capital is simply as an intellectual exercise: Given various assumptions about capital market behavior we have observed various displacement patterns to which we have attached limited welfare judgements. Using the results of the optimizing model, specifically its displacement and welfare implications, we may gain greater insight into the significance of the earlier results. In any case, we shall describe the system as though it were controlled by some governmental agency.

It is assumed that an optimum growth path for the agricultural economy is one which maximizes W , the present discounted value of the infinite stream of aggregate per worker consumption, where the instantaneous rate of discount, ρ , is constant and positive. Then

$$(3.1) \quad W = \int_0^{\infty} c(t) e^{-\rho t} dt,$$

where $c(t)$ is aggregate per worker consumption at time t . It should be noted, however, that optimal growth (in some sense) of the agricultural economy does not necessarily imply anything about optimal growth for the entire economy of a developing country, since optimization of the growth paths of the agricultural and non-agricultural portions of such an economy is not in general a separable process. This limiting aspect of the optimizing problem to be laid out below is less serious

the greater the extent to which flows of capital between agriculture and non-agriculture are inhibited, the terms of trade between agriculture and non-agriculture are fixed, the net relative rate of flow of population between agriculture and non-agriculture is fixed and/or small, and the non-agricultural sector is quantitatively smaller than the agricultural sector.

For simplicity assume that production in both agricultural sectors is governed by the same Cobb-Douglas production function except that capitalists may enjoy a Hicks-neutral technological advantage. Thus

$$q_p = k_p^{1-\alpha}; \quad q_c = \theta k_c^{1-\alpha}; \quad \theta \geq 1,$$

where the subscripts c and p refer, as usual, to capitalists and peasants, respectively; k_i is the capital-labor ratio in sector i; and q_i is output per worker in sector i. Workers may move freely between the two sectors, and the incentive for such movement will be zero when consumption per worker in the peasant sector is equal to $1/\gamma$, ($\gamma > 1$), times the wage in the capitalist sector, that wage being totally consumed. Mathematically, the labor market is in equilibrium when

$$(1-\sigma_p) k_p^{1-\alpha} = \frac{\alpha\theta}{\gamma} k_c^{1-\alpha},$$

or when

$$(3.2) \quad k_p = \omega k_c, \quad \omega = [\alpha\theta/\gamma(1-\sigma_p)]^{\frac{1}{1-\alpha}}.$$

It is assumed that labor flows so swiftly between sectors that this market may be held to be always in equilibrium. Then equation (3.2) holds at all times.

Since marginal products are always positive, the economy may be assumed to be at full employment of both productive factors at all times. Thus,

$$(3.3) \quad n_p + n_c = 1$$

$$(3.4) \quad k_p n_p + k_c n_c = k,$$

where n_i is the proportion of workers employed in sector i , and k is the aggregate capital-labor ratio. Equation (3.3) states that the proportion of the work force in each sector must add up to one, and equation (3.4) states that the aggregate capital-labor ratio is a weighted average of the ratios in each sector. These two equations are equivalent to the equations of full employment.

Recall that the input of labor in both sectors is a fixed proportion of the number of workers in each sector, and the number of workers is proportional to total population. Capital is a land-capital aggregate. Since the rate of population growth is positive,

it is assumed that non-land capital goods are perfectly substitutable for land when all available land has been cultivated.

Since capital may be freely shifted it is possible to combine the sectoral savings functions to form an aggregate savings-investment function. In the peasant sector a fixed proportion of output, σ_p , is saved so that saving per worker is equal to $\sigma_p k_p^{1-\alpha}$. All capitalist profits are saved so that saving per worker is $(1-\alpha)\theta k_c^{1-\alpha}$. It is now easily shown that the savings-investment identity appears as follows:

$$(3.5) \quad n_p \sigma_p k_p^{1-\alpha} + n_c (1-\alpha)\theta k_c^{1-\alpha} = \dot{k} + \eta k,$$

where η is the sum of the exogenously fixed relative rates of population growth and capital decay.

Equations (3.2) - (3.5) represent the constraints on the government in formulating its policy. Notice that there are five variables: k_c, k_p, n_p, n_c , and k . Since the government may allocate capital, any one of the first four of these variables could be thought of as being directly controlled by the government. This is easily demonstrated: At any point in time k is fixed. If then capital is allocated between the two sectors, we may write

$$(3.6) \quad k_p n_p = v_p = K_p / N$$

$$(3.7) \quad k_c n_c = v_c = K_c / N.$$

The v_i , $i=c,p$, are constant since v_i is the ratio of capital in sector i to total population, both of which are fixed at any point in time. When v_p and v_c have been determined, equations (3.2), (3.3), (3.6), and (3.7) uniquely determine the values at time t for k_p, k_c, n_p , and n_c . Therefore, for analytical purposes, we may imagine that the government pegs the level of one of these four variables at each moment of time. Then the values of the remaining three variables are determined by equations (3.2) - (3.4). We shall assume that the government directly controls the variable n_c .

Using equations (3.2) - (3.4), we can eliminate n_p, k_p , and k_c from equation (3.5) to obtain the basic equation of motion of the system:

$$(3.8) \quad \dot{k} = \frac{(1-n_c)\sigma_p \omega^{-\alpha} + n_c(1-\alpha)\theta}{[(1-n_c) + n_c]^{1-\alpha}} k^{1-\alpha} - nk.$$

If we know the behavior of n_c and the initial value of k , k_p , then equation (3.8) determines the time path of k . Knowing the behavior of n_c and k , we can obviously infer the behavior of the other variables of the economy.

Turning now to the objective function, equation (3.1), it will be useful to write it also in terms of k and n_c alone. Consumption per worker in the peasant sector is $(1-\sigma_p)k_p^{1-\alpha}$ and in the capitalist sector is $\alpha\theta k_c^{1-\alpha}$, so that

$$c(t) = [(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c\alpha\theta]k_c^{1-\alpha},$$

since $n_c + n_p = 1$ and $k_p = \omega k_c$. Writing k_c as a function of k , this becomes

$$c(t) = \frac{(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c\alpha\theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha},$$

so that,

$$(3.1') \quad W = \int_0^{\infty} \frac{(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c\alpha\theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha} e^{-\rho t} dt.$$

Formally stated, the problem of the government is to maximize W with respect to $n_c(t)$, $0 \leq n_c(t) \leq 1$, subject to equation (3.8) and the initial condition $k(0) = k_0$.

This is a problem in the calculus of variations, which may be solved using the method derived by Pontryagin and associates,^{1/} provided that $n_c^*(t)$, the optimum time path of the control parameter is discontinuous at only a finite number of points. We shall now develop the necessary conditions for an optimum path of growth using this method.

First, construct the Hamiltonian, H , of the system:

$$H = \frac{(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c\alpha\theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha} e^{-\rho t} \\ + p e^{-\rho t} \left\{ \frac{(1-n_c)\sigma_p\omega^{1-\alpha} + n_c(1-\alpha)\theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha} - \eta k \right\},$$

where $pe^{-\rho t}$ is an undetermined multiplier analogous to the Lagrangian multiplier of the usual Kuhn-Tucker-Lagrange theory. The necessary conditions for an optimum are derived by performing certain operations on the Hamiltonian.

Next, choose n_c to maximize H at every instant of time. Upon examination of the Hamiltonian it is clear that H is maximized if and only if z is maximized, where

$$(3.9) \quad z = \frac{(1-n_c)[(1-\sigma_p)\omega^{1-\alpha} + p\sigma_p\omega^{1-\alpha}] + n_c[\alpha\theta + p(1-\alpha)\theta]}{[(1-n_c)\omega + n_c]^{1-\alpha}}.$$

Since p is fixed at each instant of time, z is maximized if and only if \hat{z} is maximized, where

$$\hat{z} = \frac{(1-n_c) + n_c\psi(p)}{[(1-n_c)\omega + n_c]^{1-\alpha}},$$

and

$$\psi(p) = \frac{\alpha\theta + p(1-\alpha)\theta}{(1-\sigma_p)\omega^{1-\alpha} + p\sigma_p\omega^{1-\alpha}}.$$

It is possible to give $\psi(p)$ and z some intuitive content, since p may be interpreted as the undiscounted social value or price of a unit of capital with a unit of consumption as the numeraire. This interpretation of p rests on the fact that

$$pe^{-\rho t} = \frac{\partial}{\partial k} \max_{n_c} \left[n_c(\tau) \int_t^{\infty} c(\tau) e^{-\rho\tau} d\tau \right]. \quad 2/$$

Now observe that we may write

$$\psi(p) = \frac{[\alpha + p(1-\alpha)]\theta k_c^{1-\alpha}}{[(1-\sigma_p) + p\sigma_p]k_p^{1-\alpha}},$$

since $k_p = \omega k_c$. Thus $\psi(p)$ is the ratio of the social value of output per worker in the capitalist sector to that in the peasant sector. Consider next equation (3.9). Since $k = [(1-n_c)\omega + n_c]k_c$, we may write equation (3.9) as follows:

$$z = \{(1-n_c)[(1-\sigma_p) + p\sigma_p]k_p^{1-\alpha} + n_c[\alpha + p(1-\alpha)]\theta k_c^{1-\alpha}\} / k^{1-\alpha}.$$

The numerator of the above expression is the social value of aggregate output per worker measured in terms of units of consumption. Since at any point in time k is constant, the selection of n_c to maximize \hat{H} at every instant of time is equivalent to maximizing the social value of aggregate output per worker at any point in time.

Returning now to the matter of maximizing \hat{z} with respect to n_c , we first calculate $\partial \hat{z} / \partial n_c$:

$$\frac{\partial \hat{z}}{\partial n_c} = \frac{[(1-n_c)\omega + n_c]^{1-\alpha}(\psi-1) - [(1-n_c) + \psi n_c](1-\alpha)[(1-n_c)\omega + n_c]^{-\alpha}(1-\omega)}{[(1-n_c)\omega + n_c]^{2(1-\alpha)}}.$$

Setting $\partial \hat{z} / \partial n_c = 0$, it is found that a local extremum occurs at

$n_c = \hat{n}_c$, where

$$(3.10) \quad \hat{n}_c = \frac{(1 - \omega\psi) - \alpha(1-\omega)}{\alpha(1-\omega)(\psi-1)} .$$

To determine whether \hat{n}_c is at a local minimum or maximum point, $\partial^2 \hat{z} / \partial n_c^2$ is calculated and is found to have its sign given by

$$\text{sign } \frac{\partial^2 \hat{z}}{\partial n_c^2} = \text{sign } \{2(1-\omega\psi) - \alpha(1-\omega)[(1-n_c) + \psi n_c]\} .$$

When \hat{n}_c is substituted for n_c it follows that

$$\text{sign } \frac{\partial^2 \hat{z}}{\partial n_c^2} \Bigg|_{n_c = \hat{n}_c} = \text{sign } (1-\omega\psi) .$$

Substituting for $\psi(p)$ and rearranging it is discovered that

$$\frac{\partial^2 \hat{z}}{\partial n_c^2} \Bigg|_{n_c = \hat{n}_c} > < 0 \text{ as } \frac{\omega^{-\alpha}}{\theta} > < \frac{\alpha + (1-\alpha)p}{1-\sigma_p + \sigma_p p} .$$

Recall that we have assumed that $\sigma_p < 1-\alpha$, i.e., that the peasant savings ratio is less than the competitively imputed profits-output ratio or capital share. We also assumed that ω was less than one, i.e., that the equilibrium capital-labor ratio in peasant agriculture was less than that in capitalist agriculture. In order that $\partial^2 \hat{z} / \partial n_c^2 |_{n_c = \hat{n}_c}$ be zero, it must be true that

$$\alpha / (1-\sigma_p) \leq \omega^{-\alpha} / \theta \leq (1-\alpha) / \sigma_p .$$

If $\omega^{-\alpha}/\theta > (1-\alpha)/\sigma_p$, then $\partial^2 \hat{z}/\partial n_c^2|_{n_c=\hat{n}_c}$ is positive for all non-negative values of p . If $\omega^{-\alpha}/\theta < \alpha/(1-\sigma_p)$, then it is negative for all non-negative values of p . If neither of these two situations pertains, then there will be some value of p , say \bar{p} for which

$$(3.11) \quad \frac{\omega^{-\alpha}}{\theta} = \frac{\alpha + (1-\alpha)\bar{p}}{1-\sigma_p + \sigma_p \bar{p}},$$

so that $\partial^2 \hat{z}/\partial n_c^2|_{n_c=\hat{n}_c} \gtrless 0$ as $p \lesseqgtr \bar{p}$. We may now judge whether the extremum implied by $n_c = \hat{n}_c$ is a local minimum or a local maximum.

But before we do so, let us narrow the problem by considering whether n_c lies in the feasible region $[0,1]$. The reader may verify that

$$\left. \frac{dp}{dn_c} \right|_{\partial \hat{z}/\partial n_c = 0} < 0,$$

so that the maximum value of p for which \hat{n}_c is in the feasible region is given by solving equation (3.10) for p when $\hat{n}_c = 0$. After doing this and rearranging, we obtain

$$(3.12) \quad \frac{\alpha + p(1-\alpha)}{(1-\sigma_p + p\sigma_p)} = \frac{\omega^{-\alpha}}{\theta} [1-\alpha(1-\omega)] < \frac{\omega^{-\alpha}}{\theta}.$$

Notice first that if $\omega^{-\alpha}/\theta < \alpha/(1-\sigma_p)$, then there is no non-negative value of p for which this equation may hold. Thus, when $\partial^2 \hat{z}/\partial n_c^2|_{n_c=\hat{n}_c}$ is negative for all non-negative values of p , we know that \hat{n}_c lies outside the feasible region. Define \bar{p} as that value of p for which

equation (3.12) holds, if it exists. From the definitions of \bar{p} and \bar{p} it is clear that $\bar{p} < \bar{p}$. Thus in this case when \hat{n}_c lies in the feasible region, it must be true that $p < \bar{p} < \bar{p}$, and from equation (3.11) it is clear that $\partial^2 \hat{z} / \partial n_c^2 |_{\partial \hat{z} / \partial n_c = 0}$ is positive so that \hat{n}_c determines a local minimum. Since equation (3.10) uniquely defines \hat{n}_c for each value of p , we know that when \hat{n}_c determines a local minimum, \hat{z} is maximized by setting n_c equal to one of its extreme values, zero or unity. We thus know that if $\omega^{-\alpha} \theta \geq \alpha / (1 - \sigma_p)$, and if \hat{n}_c lies on the closed interval $(0, 1)$, there exists either an interior minimum or no interior extremum, so that \hat{z} is maximized at $n_c = 0$ or $n_c = 1$; and we also know that if $\omega^{-\alpha} < \alpha / (1 - \sigma_p)$ there is no interior extremum for n_c in the feasible region, so that again \hat{z} is maximized at one of the limiting value of n_c . It is therefore proved that \hat{z} , and thus z and H , are always maximized at $n_c = 1$ or $n_c = 0$.

Referring to equation (3.9), it is readily seen that $z = (1 - \sigma_p) + p\sigma_p$ when $n_c = 0$ and that $z = \alpha\theta + p(1 - \sigma)\theta$ when $n_c = 1$. Since $(1 - \alpha) > \sigma_p$, it is clear that if p is large enough, $z(1) > z(0)$. It is also readily observable that if $\alpha\theta > (1 - \sigma_p)$, then $z(1) > z(0)$ for all p , ($p \geq 0$). When $\alpha\theta \leq 1 - \sigma_p$, then the solution to the equation,

$$(1 - \sigma_p) + p\sigma_p = \alpha\theta + p(1 - \sigma)\theta,$$

yields that p , say \hat{p} , at which $z(1) = z(0)$ and hence at which H will be maximized by setting n_c at 0 or 1, (but not in between). Figure 3.1

illustrates the optimal choice of n_c, n_c^* , as determined by the level of p , on the assumption that $(1-\sigma_p) > \alpha\theta$, so that $\hat{p} > 0$: If $p < \hat{p}$, set $n_c = 0$; if $p > \hat{p}$, set $n_c = 1$; and if $p = \hat{p}$ set $n_c = 0$ or 1 .

Having now determined the choice of n_c to maximize H at every instant of time, we shall now develop the remaining necessary conditions. The auxiliary variable p must satisfy the following condition:

$$(3.13) \quad \frac{d}{dt} (pe^{-\rho t}) = - \frac{\partial H}{\partial k}.$$

Calculating $\partial H/\partial k$, performing the differentiation with respect to time on the LHS of equation (3.13), substituting, and rearranging, the following differential equation in q, k , and n_c is obtained:

$$(3.14) \quad \dot{p} = (\rho+n)p - \frac{(1-\alpha)k^{-\alpha}}{[(1-n_c)\omega + n_c]^{1-\alpha}} \{ (1-n_c)[(1-\sigma_p) + p\sigma_p]\omega^{1-\alpha} + n_c[\alpha + p(1-\alpha)]\theta \}.$$

It remains only to place an endpoint condition on the behavior of p , namely that

$$\lim_{t \rightarrow \infty} pe^{-\rho t} = 0.$$

This condition is called the transversality condition, and given the interpretation of p as a price, can be interpreted as requiring the present discounted social value of capital to diminish toward zero as time recedes into infinity.

Differential equations (3.8) and (3.14), together with their respective endpoint conditions and the decision rule for n_c^* indicated in Figure 3.1 form a set of necessary conditions for an optimum and enable us to infer the nature of the welfare maximizing growth path.

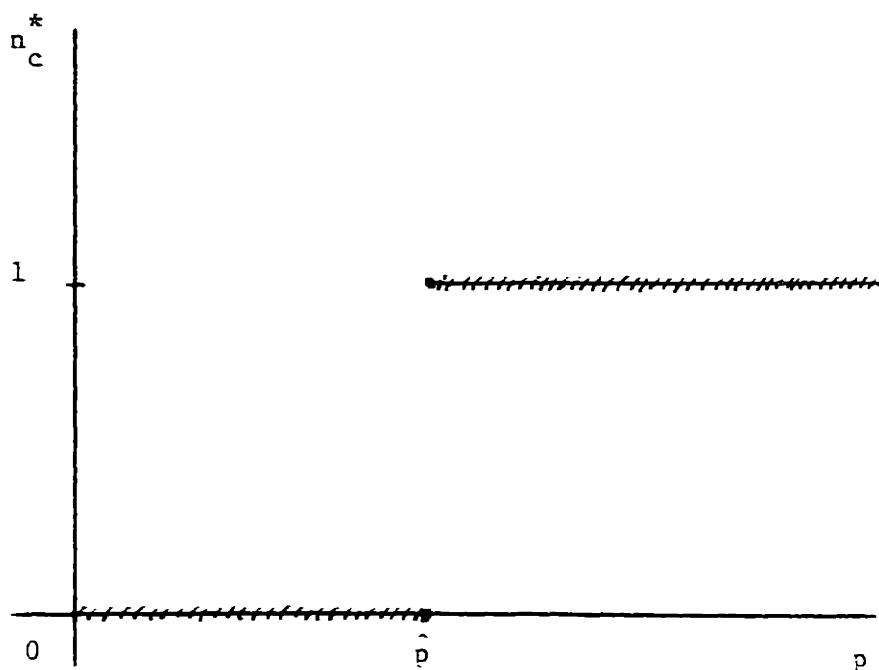


FIGURE 3.1

A phase diagram in (k, q) space will help to clarify the behavior of the optimally controlled system. Assume that $\alpha\theta < 1 - \sigma_p$ so that $\hat{p} > 0$. Divide the non-negative quadrant into two regions lying on either side

of the line $p = \hat{p}$. When $p > \hat{p}$, then $n_c^* = 1$, and differential equations (3.8) and (3.14) become simply

$$(3.15) \quad \dot{k} = (1-\alpha)\theta k^{1-\alpha} - \eta k \quad \text{and}$$

$$(3.16) \quad \dot{p} = [\rho + \eta - (1-\alpha)^2 k^{-\alpha}]p - \alpha\theta(1-\alpha)k^{-\alpha},$$

respectively. It is easy to see that the $\dot{k} = 0$ stationary is a straight line defined by the equation

$$k = [(1-\alpha)\theta/\eta]^{\frac{1}{\alpha}} = k^*,$$

and that the $\dot{p} = 0$ stationary is a curve defined by the equation

$$(3.17) \quad p = \alpha \div \left[\frac{\rho + \eta}{(1-\alpha)\theta k^{-\alpha}} - (1-\alpha) \right].$$

The reader may verify that $dp/dk < 0$, and that $d^2p/dk^2 > 0$ along this latter curve when $p > 0$. Notice that $p \rightarrow \infty$ as k approaches $k^\#$ from above, where $k^\#$ is obtained as the solution to

$$(\rho + \eta)/(1-\alpha)\theta k^{-\alpha} = 1-\alpha.$$

By referring to equation (3.16) it is easily determined that when $k > k^\#$, $\dot{p} > 0$ according as the point (p, k) lies above, on, or below the $\dot{p} = 0$ stationary. When $k < k^\#$, it is obvious that $\dot{p} < 0$ at all

points. From equation (3.15) we can see that $\dot{k} > 0$ as $k < k^*$. The reader may verify that $k^\# < k^*$ so that the $\dot{p} = 0$ and $\dot{k} = 0$ stationary points will always intersect in the first quadrant.

Turning next to the region of the phase plane where $p < \hat{p}$, we know that $n_c^* = 0$, and consequently differential equations (3.8) and (3.14) become

$$(3.18) \quad \dot{k} = \sigma_p k^{1-\alpha} - \eta k \quad \text{and}$$

$$(3.19) \quad \dot{p} = [\rho + \eta - \sigma_p (1-\alpha) k^{-\alpha}] p - (1-\alpha)(1-\sigma_p) k^{-\alpha},$$

respectively. The $\dot{k} = 0$ stationary is again a straight line defined as follows:

$$k = (\sigma_p / \eta)^{\frac{1}{\alpha}} = k^{**}.$$

Since $\sigma_p < (1-\alpha)\theta$, $k^{**} < k^*$. The $\dot{p} = 0$ stationary is a curve defined by the next equation

$$(3.20) \quad p = (1-\sigma_p) \div \left[\frac{\rho + \eta}{(1-\alpha)k^{-\alpha}} - \sigma_p \right].$$

Again we see that $dp/dk < 0$ and $d^2p/dk^2 > 0$ along this curve when $p > 0$ and that $p \rightarrow \infty$ as k approaches $k^{\#\#}$ from above, where $k^{\#\#}$ is obtained by solving

$$(\rho + \eta) - \sigma_p (1-\alpha) k^{-\alpha} = 0.$$

If $k < k^{##}$, then $\dot{p} < 0$; and if $k \geq k^{##}$, then $\dot{p} \geq 0$ as the point (k, p) lies above, on, or below the stationary. Finally, it is again evident that $k^{##} < k^{**}$ so that the $\dot{p} = 0$ and $\dot{k} = 0$ stationaries always intersect in the first quadrant.

Now let us consider how the two sets of stationary curves will lie relative to one another. We already know that the $\dot{k} = 0$ stationaries are vertical lines in (k, p) space. Since $k^{**} < k^*$, it is obvious that the $\dot{k} = 0$ stationary in the region $p \leq \hat{p}$, ($n_c^* = 0$), lies to the left of the $\dot{k} = 0$ stationary in the region $p \geq \hat{p}$, ($n_c^* = 1$), as shown in Figure 3.2. Next, rearranging and rewriting the equations of the

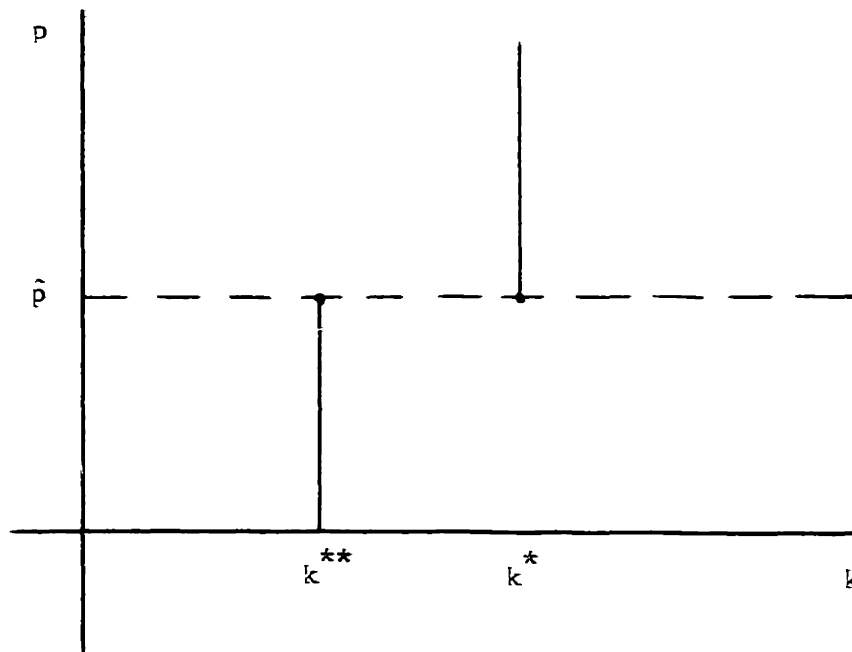


FIGURE 3.2

$\dot{p} = 0$ stationaries, it is made apparent that for any given value of p , the $\dot{p} = 0$ stationary for $n_c^* = 1$ lies to the left or right of that

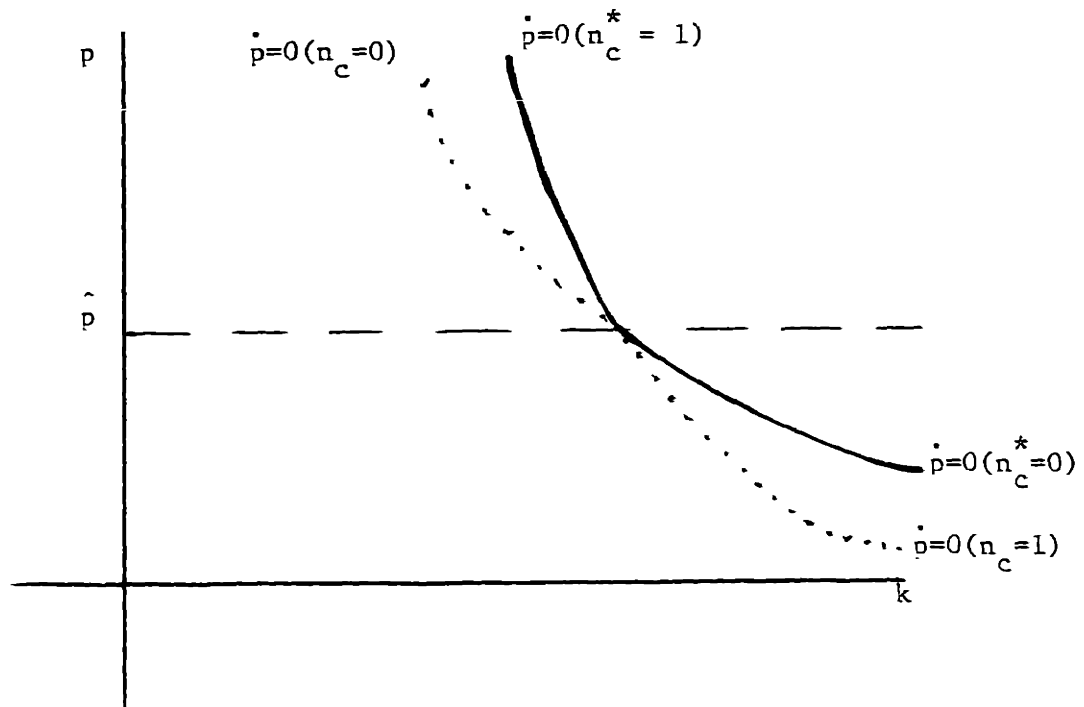


FIGURE 3.3

for $n_c = 0$ according as $p > \hat{p}$. When $p = \hat{p}$, the two $\dot{p} = 0$ stationary curves intersect. This relationship is shown in Figure 3.3. The solid

$$(3.17') \quad k^\alpha = \frac{1-\alpha}{\sigma+\eta} \cdot \frac{1}{p} \cdot [(1-\sigma_p) + p\sigma_p]$$

$$(3.20') \quad k^\alpha = \frac{1-\alpha}{\sigma+\eta} \cdot \frac{1}{p} \cdot [\alpha\theta + p(1-\sigma)\theta]$$

curve is the effective $\dot{p} = 0$ stationary for the whole range of non-negative p when n_c is optimally assigned.

In order to put Figures 3.2 and 3.3 together to obtain a phase diagram for the entire optimized system we must know more about where

the effective $\dot{p} = 0$ and $\dot{k} = 0$ stationaries may intersect. Since we know the stationary values k^* and k^{**} , we can calculate the corresponding stationary values p^* and p^{**} using equations (3.17) and (3.20). When this is done, we find that

$$p^* = \frac{\alpha\eta}{\sigma+\eta} \quad \text{and} \quad p^{**} = \frac{(1-\sigma)_\eta}{\sigma+\eta} \cdot \frac{1-\alpha}{\sigma_p}$$

Since $\sigma_p < 1-\alpha$, it follows that $p^{**} > p^*$. In order that the point (k^*, p^*) or (k^{**}, p^{**}) be consistent with the necessary conditions for an optimum, it is required that

$$p^* > \hat{p} \quad \text{and} \quad p^{**} < \hat{p}.$$

Since $p^{**} > p^*$, it is clear that if $p^* > \hat{p}$, then it is impossible that $p^{**} < \hat{p}$; and if $p^{**} < \hat{p}$, then it is untrue that $p^* > \hat{p}$. It is consequently quite contrary to possibilities that both (k^*, p^*) and (k^{**}, p^{**}) be stationaries of the same optimally controlled system. In fact, it is within the realm of feasibility that $p^{**} > \hat{p} > p^*$, in which case there is no stationary point in the system. Since it will become apparent that stationary points of the system are of interest, we shall now study in greater detail the conditions that

$$p^* > \hat{p} \quad \text{and} \quad p^{**} < \hat{p}.$$

It can be shown by substituting for p^* , p^{**} , and \hat{p} their equivalent expressions in the parameters of the system that

$$p^* > \hat{p} \text{ if and only if } 1 + \frac{\rho}{\eta} < \alpha \left[\frac{\theta(1-\alpha) - \sigma_p}{(1-\sigma_p) - \alpha\theta} \right]$$

$$p^{**} < \hat{p} \text{ if and only if } 1 + \frac{\rho}{\eta} > \frac{(1-\sigma_p)(1-\alpha)}{\sigma_p} \left[\frac{\theta(1-\alpha) - \sigma_p}{(1-\sigma_p) - \alpha\theta} \right]$$

Note first that if $1-\sigma_p < \alpha\theta$, the case of $n_c^* = 1$ for all non-negative values of p , then it is trivially true that $\hat{p} = 0$. Since p^* is always positive, it follows that $p^* > \hat{p}$, so that the point (k^*, p^*) is a stationary of the optimal system. If $1-\sigma_p > \alpha\theta$, then it is clear that if the rate of time discounting, ρ , is set high enough then it is always possible to have a case where $p^{**} < p$. Of interest, naturally, is whether it is possible for $p^* > \hat{p}$ if ρ is set low enough. It is clear that the answer is yes if and only if

$$1 < \alpha \left[\frac{\theta(1-\alpha) - \sigma_p}{(1-\sigma_p) - \alpha\theta} \right],$$

or rearranging, if and only if

$$1 - \alpha\theta < (1-\alpha)[\alpha\theta + \sigma_p].$$

Since $1 > 1 - \sigma_p > \alpha\theta$, it is clear that both sides of this inequation are positive and less than $1-\alpha$. But it cannot be stated in general whether the above inequation is satisfied. When $\theta = 1$, it is

demonstrable that the inequation is not satisfied, i.e., no matter how low ρ is set, p^* cannot be made to fall below \hat{p} : When $\theta = 1$ we have,

$$1-\alpha < (1-\alpha)(\alpha + \sigma_p) < (1-\alpha).$$

Observe finally that ρ may be in some middle range where $p^{**} > \hat{p} > p^*$, so that as discussed above, no stationary point exists in the optimally controlled system. Later we shall put some economic meat on these mathematical bones, when we discuss the various cases in detail, but first we must construct the phase diagrams which are relevant to them.

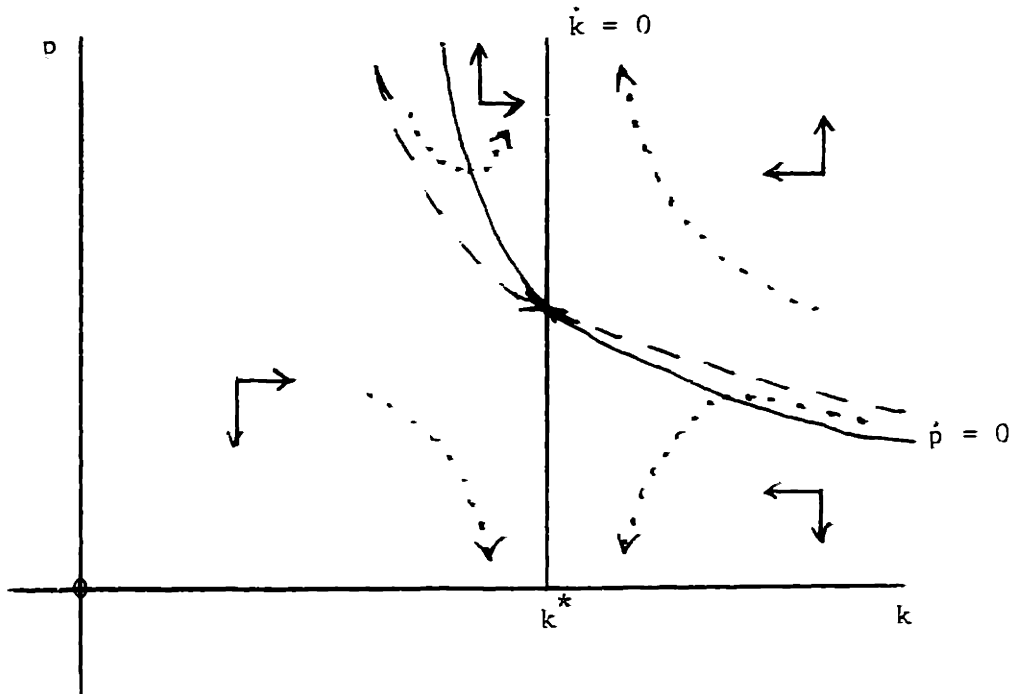
To summarize, there are four possible cases:

- I. $1-\sigma_p < \alpha\theta$; $n_c^* = 1$ for all $p \geq 0$;
the system has a stationary, (k^*, p^*) .
- II. $1-\sigma_p > \alpha\theta$; $\hat{p} > 0$ with $n_c^* = 1, 0$ as $p \gtrless \hat{p}$;
the system has a stationary, (k^*, p^*) , since
 $p^* > \hat{p}$; $n_c^* = 1$ at (k^*, p^*) by definition.
- III. $1-\sigma_p > \alpha\theta$; $\hat{p} > 0$ with $n_c^* = 1, 0$ as $p \gtrless \hat{p}$;
the system has no stationary, since $p^{**} > \hat{p} > p^*$.
- IV. $1-\sigma_p > \alpha\theta$; $\hat{p} > 0$ with $n_c^* = 1, 0$ as $p \gtrless \hat{p}$;
the system has a stationary, (k^{**}, p^{**}) , since
 $p^{**} < \hat{p}$; $n_c^* = 0$ at (k^{**}, p^{**}) by definition.

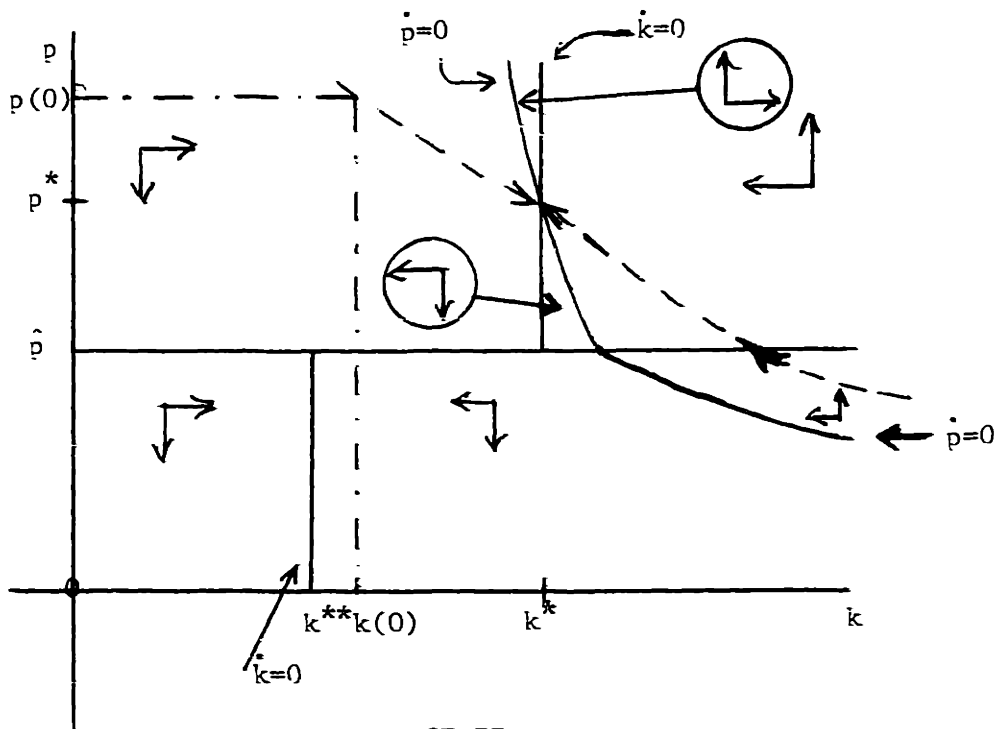
The phase diagrams for each of these four cases are shown in Figures 3.4 through 3.7, respectively. Since cases I. and III. have some peculiarities, let us consider first cases II. and IV. It can be shown that if the system does not move to (k^*, p^*) in case II., shown in Figure 3.5, or the (k^{**}, p^{**}) in case IV., shown in Figure 3.7, it will move in such a fashion that p approaches either plus or minus infinity. When p climbs toward plus infinity, it can be demonstrated that $pe^{-\rho t}$ also grows without limit so that the transversality condition is violated. When p falls toward minus infinity, not only may it be shown that transversality is violated, but also it is obvious that the non-negativity constraint on p is violated. This means that the necessary conditions in cases II. and IV., will be satisfied if and only if $p(0)$ is selected in such a way that (k^*, p^*) or (k^{**}, p^{**}) is approached since only then is it assured that

$$p > 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} pe^{-\rho t} = 0.$$

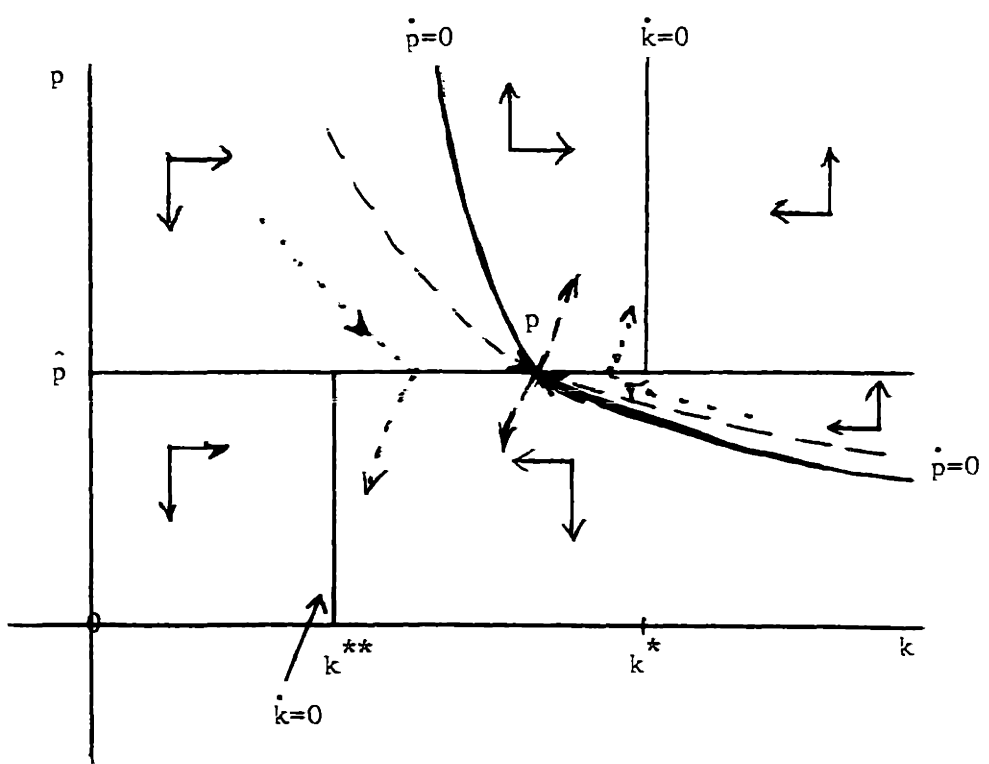
Now it can be demonstrated that the points (k^*, p^*) and (k^{**}, p^{**}) are saddlepoints and that there exists a unique converging trajectory in the (k, p) plane for each of these saddlepoints. In each case the convergent trajectory consists of the two stable arms shown by solid hash marked lines in the phase diagrams. We shall prove that (k^*, p^*) is a saddlepoint and shall simply assert that (k^{**}, p^{**}) is one. Since the RHS's of differential equations (3.15) and (3.16) are everywhere twice continuously differentiable in k and p and



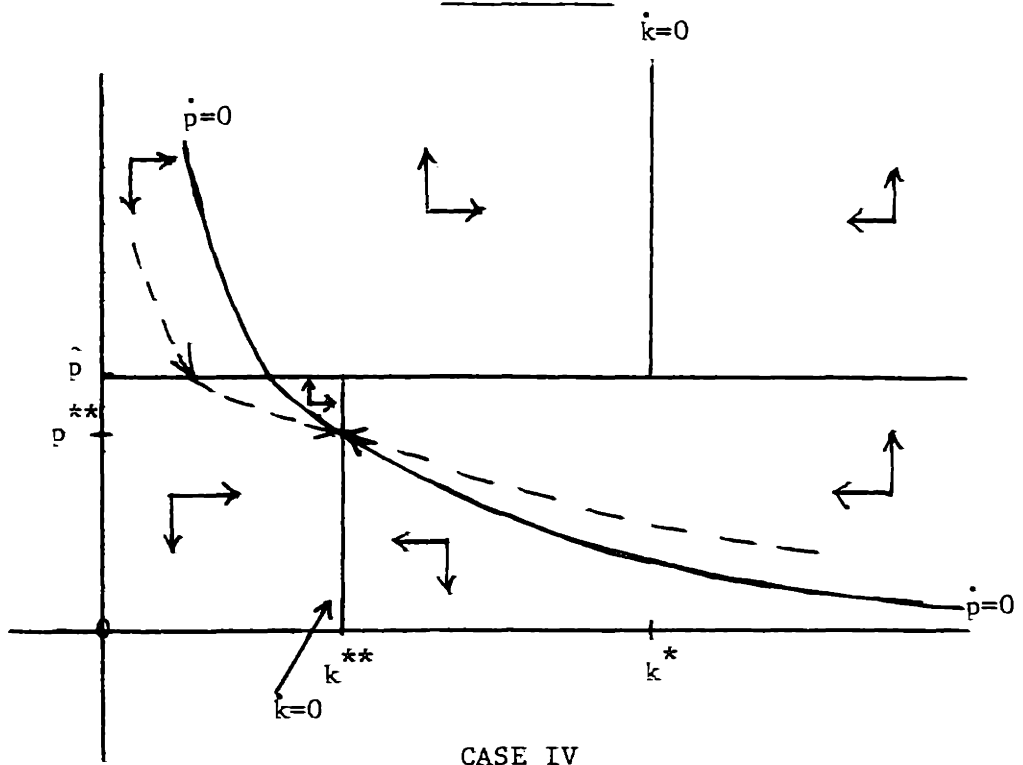
CASE I
FIGURE 3.4



CASE II
FIGURE 3.5



CASE III
 FIGURE 3.6



CASE IV
 FIGURE 3.7

since $n_c = n_c^* = 1$ is constant, it is sufficient to establish the saddlepoint properties of (k^*, p^*) that linear approximation of the Taylor's series expansion of the system about the point (k^*, p^*) has the saddlepoint property. Now this linear approximation to the two differential equations comprising our system is simply

$$\begin{pmatrix} \dot{k} \\ \dot{p} \end{pmatrix} = \begin{bmatrix} \partial \dot{k} / \partial k |^* & \partial \dot{k} / \partial p |^* \\ \partial \dot{p} / \partial k |^* & \partial \dot{p} / \partial p |^* \end{bmatrix} \cdot \begin{pmatrix} k - k^* \\ p - p^* \end{pmatrix},$$

where the vertical bars and stars indicate that the relevant partial derivatives are evaluated at the point (k^*, p^*) . When these partials are calculated and evaluated, it is found that

$$\partial \dot{k} / \partial k |^* = -\alpha n, \quad \partial \dot{k} / \partial p |^* = 0$$

$$\partial \dot{p} / \partial k |^* = b > 0, \quad \partial \dot{p} / \partial p |^* = \rho + \alpha n.$$

Thus our approximate linear system becomes

$$\begin{pmatrix} \dot{k} \\ \dot{p} \end{pmatrix} = \begin{bmatrix} -\alpha n & 0 \\ b & \rho + \alpha n \end{bmatrix} \cdot \begin{pmatrix} k - k^* \\ p - p^* \end{pmatrix} = C \cdot \begin{pmatrix} k - k^* \\ p - p^* \end{pmatrix}.$$

It is well known that this system will have a saddlepoint if the solution to the equation

$$\det[C - \lambda I] = 0$$

has real roots of opposite sign. Writing out this equation we have,

$$-(\alpha\eta + x)(\rho + \alpha\eta - x) = 0, \text{ or}$$

$$x^2 - \rho x - \alpha\eta(\rho + \alpha\eta), \text{ so that}$$

$$x = \frac{1}{2}[\rho \pm \sqrt{\rho^2 + 4\alpha\eta(\rho + \alpha\eta)}].$$

Since $\sqrt{\rho^2 + 4\alpha\eta(\rho + \alpha\eta)}$ is real and is great than ρ , we know that the two roots to the quadratic are real and of opposite sign. Thus we have a saddle point in the associated linear system and hence, in this case, in the non-linear system. Because of the differentiability properties of the RHS's of the non-linear system and the saddlepoint property, we know that specification of a particular $(k(0), p(0))$ initially will uniquely determine the trajectory of the system from $t = 0$ to $t = \infty$. Thus in order to approach (k^*, p^*) given any arbitrary $k(0)$, we must select $p(0)$ so that $(k(0), p(0))$ lies on one of the unique arms converging to (k^*, p^*) .^{3/}

As a step toward winding up the formal mathematics of the problem, we shall demonstrate the sufficiency of the Pontryagin necessary conditions in cases II., and IV.^{4/} Define an optimal program as a triple, $(\bar{k}(t), \bar{p}(t), n_c^*(t))$ which satisfies the necessary conditions of Pontryagin, et al., and a feasible program as any triple $(k(t), p(t), n_c(t))$, which satisfies equation (3.8) and the initial condition $k(0) = k_0$. We wish to show that

$$(3.21) \quad \int_0^{\infty} (\bar{c} - c) e^{-\rho t} dt \geq 0$$

where $\bar{c} = \bar{\tau} \bar{k}^{1-\alpha}$, $c = \tau k^{1-\alpha}$,

$$\tau = \frac{(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c \alpha \theta}{[(1-n_c)\omega + n_c]^{1-\alpha}}, \text{ and}$$

$$\hat{\tau} = \frac{(1-n_c^*)(1-\sigma_p)\omega^{1-\alpha} + n_c^* \alpha \theta}{[(1-n_c^*)\omega + n_c^*]}.$$

The variables c and \hat{c} are per capita consumption in the optimal and feasible programs, respectively. Showing that (3.21) holds is equivalent to showing that the following relationship holds:

$$\begin{aligned} & \int_0^{\infty} \left\{ \frac{(1-n_c^*)(1-\sigma_p)\omega^{1-\alpha} + n_c^* \alpha \theta}{[(1-n_c^*)\omega + n_c^*]^{1-\alpha}} \bar{k}^{1-\alpha} - \frac{(1-n_c)(1-\sigma_p)\omega^{1-\alpha} + n_c \alpha \theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha} \right\} \\ & + \bar{p} \left\{ \frac{(1-n_c^*)\sigma_p \omega^{1-\alpha} + n_c^* (1-\alpha)\theta}{[(1-n_c^*)\omega + n_c^*]^{1-\alpha}} \bar{k}^{1-\alpha} - \eta \bar{k} - \dot{\bar{k}} \right\} \\ & - \bar{p} \left\{ \frac{(1-n_c)\sigma_p \omega^{1-\alpha} + n_c (1-\alpha)\theta}{[(1-n_c)\omega + n_c]^{1-\alpha}} k^{1-\alpha} - \eta k - \dot{k} \right\} e^{-\rho t} dt \geq 0. \end{aligned}$$

This relationship is equivalent to that of inequation (3.21) since the expressions in braces which are premultiplied by \bar{p} are by definition identically zero. (See equation (3.8).) By rearranging terms in the above integral we obtain the following equivalent condition:

$$\int_0^{\infty} \left\{ \frac{(1-n_c^*)[(1-\sigma_p)\omega^{1-\alpha} + \tilde{p}\sigma_p\omega^{1-\alpha}] + n_c^*[\alpha\theta + \tilde{p}(1-\alpha)\theta]}{[(1-n_c^*)\omega + n_c^*]^{1-\alpha}} \right\} \bar{k}^{1-\alpha} \\ - \left\{ \frac{(1-n_c)[(1-\sigma_p)\omega^{1-\alpha} + \tilde{p}\sigma_p\omega^{1-\alpha}] + n_c[\alpha\theta + \tilde{p}(1-\alpha)\theta]}{[(1-n_c)\omega + n_c]^{1-\alpha}} \right\} k^{1-\alpha} \\ + \tilde{p}\eta(\bar{k} - \dot{\bar{k}}) + \tilde{p}(\dot{\bar{k}} - \dot{\bar{k}})]e^{-\rho t} dt \geq 0.$$

But observe that the first large expression in braces is $z(n_c^*)$ and that the second is $z(n_c)$, for $p = \tilde{p}$. (See page 174.) We know that $n_c = n_c^*$ maximizes z for all p so that $z(n_c^*) > z(n_c)$ and thus

$$z(n_c^*)\bar{k}^{1-\alpha} - z(n_c)k^{1-\alpha} > z(n_c^*)(\bar{k}^{1-\alpha} - k^{1-\alpha}).$$

Further, since $g(k) = k^{1-\alpha}$ is a concave function, it follows that

$$z(n_c^*)(\bar{k}^{1-\alpha} - k^{1-\alpha}) > z(n_c^*)(1-\alpha)\bar{k}^{-\alpha}(\bar{k} - k).$$

It will therefore suffice if we can show

$$\int_0^{\infty} \{ (\bar{k} - k) [z(n_c^*)(1-\alpha)\bar{k}^{-\alpha} - \eta\tilde{p}] + \tilde{p}(\dot{\bar{k}} - \dot{\bar{k}}) \} e^{-\rho t} dt \geq 0.$$

Consider first a portion of this integral:

$$\int_0^{\infty} \tilde{p}(\dot{\bar{k}} - \dot{\bar{k}}) e^{-\rho t} dt.$$

If we integrate this by parts, we obtain

$$(k-\bar{k}) \cdot \bar{p} e^{-\rho t} \Big|_0^{\infty} - \int_0^{\infty} (k-\bar{k}) (\dot{\bar{p}} - \rho \bar{p}) e^{-\rho t} dt.$$

Now the first expression of the above result is identically zero by the transversality and initial conditions, so that it will suffice to prove sufficiency if we can show that

$$\int_0^{\infty} (\bar{k}-k) [\dot{\bar{p}} - (\rho+\eta)\bar{p} + z(n_c^*) (1-\alpha)\bar{k}^{-\alpha}] e^{-\rho t} dt \geq 0.$$

But recall now equation (3.14) and the definition of $z(n_c^*)$. It is clear from these that

$$\dot{\bar{p}} = (\rho+\eta)\bar{p} - z(n_c^*) (1-\alpha)\bar{k}^{-\alpha}.$$

Thus we have

$$\int_0^{\infty} (\bar{k}-k) \cdot 0 \cdot e^{-\rho t} dt = 0,$$

and it is proved that the necessary conditions are also sufficient in cases II. and IV.

A sufficiency proof for case I. would be exactly the same as the above, but since $n_c^* = 1$ for all non-negative values of p , the Pontryagin necessary conditions are merely sufficient conditions since the transversality conditions on p are inapplicable. Any trajectory on which $p > 0$ at all times is sufficient to achieve optimality.

Case III. is one having special difficulties, since, as may be seen in Figure 3.6, there exists no stationary point to the system. For any initial choice of $p(0)$ given $k(0)$, it will always turn out that

$$\lim_{t \rightarrow \infty} p(t)e^{-\rho t} \neq 0,$$

so that transversality is violated. It appears that there may exist no optimum solution in this case where $p^{**} > \hat{p} > p^*$, although we have seen in case I. that transversality may be violated along an optimal trajectory. The difficulty is related to a conflict between the long-run "optimum" aggregate savings ratio and the instantaneous maximization rule on the Hamiltonian. In order to maximize H at every t it is necessary to set n_c either at zero or at unity and never in between. Now when $p^* > \hat{p}$, and hence there exists a long-run optimum steady state at (k^*, p^*) , it turns out that it is desirable to set the savings ratio of the economy as high as possible, i.e., equal to $(1-\alpha)\theta$. This is intuitively supported by the fact that (k^*, p^*) is a stationary in the optimal system only when ρ is sufficiently low. An analogous observation about the case of $p^{**} < \hat{p}$ with a low optimum steady-state savings ratio, σ_p , and a relatively high rate of discount, ρ , can be made. When $p^{**} > \hat{p} > p^*$, and hence ρ lies in some intermediate range, then it may be reasoned that the optimum long-run aggregate savings ratio ought to lie on the open interval $[\sigma_p, (1-\alpha)\theta]$. But the only aggregate savings ratios which are

consistent with optimization at each instant of time are in fact the endpoints of this open interval. This is due to the fact that at any given point in time, i.e., when k and p are fixed, the Hamiltonian, or the current social value of aggregate output, is a convex function of the control parameter n_c .

Examine Figure 3.6 again and note that point p would be a saddlepoint if only \dot{k} were equal to zero there. But the only way to cause \dot{k} to assume a null value at that point is to set $n_c = \bar{n}_c$, $0 < \bar{n}_c < 1$, which will produce that aggregate savings ratio which is required. Actually, if we violate one of the basic assumptions of the Pontryagin method, it is possible to imagine \dot{k} being kept equal to zero at p by allowing n_c to flip back and forth between 0 and 1 so fast (infinitely fast) that k never has a chance to grow or shrink since no amount of time is spent in either the regime $n_c = 0$ or the regime $n_c = 1$. Obviously such behavior has no analog in the real world. Further, it violates the restriction on the application of Pontryagin's method that the time path of the control variable be discontinuous at only a finite number of points. But in some sense, such a point as p has optimum properties, as the reader may readily grasp, since if n_c flips infinitely fast, p is a (generalized) saddlepoint which satisfies all the necessity and sufficiency properties for an optimum.

Note that the convexity of H in n_c for t fixed is due to the fact that $\omega < 1$, i.e., that the equilibrium capital labor ratio in the peasant sector is less than that in the capitalist sector. By

referring to equation (3.9), it is easily seen that when ω is equal to one, then z is a simple linear function of n_c and that when $p = \hat{p}$ in addition, z is invariant with respect to n_c , $0 \leq n_c \leq 1$, so that it is maximized for n_c^* lying anywhere on the closed interval $(0,1)$. This result implies that $\dot{k}_c = 0$ stationary is now continuous along the interval (k^{**}, k^*) and in that region lies on the horizontal line $p = \hat{p}$. Under such circumstances, the point p is in fact an optimum long-run steady state. This result is not surprising, since when $\omega = 1$, capital and labor are used in the same proportions in each sector, the only difference between them being their effective savings ratios. Thus no sacrifice is implied when the two sectors are combined in such a way as to obtain an aggregate savings ratio lying between those of each sector.

Exactly under what conditions will ω be equal to one? Recall that

$$\omega = [\alpha\theta/\gamma(1-\sigma_p)]^{\frac{1}{1-\alpha}}$$

so that $\omega = 1$ if and only if,

$$\alpha\theta = \gamma(1-\sigma_p).$$

If $\theta = \gamma$, so that the wage income gap exactly offsets the technology gap, we have

$$\alpha = (1-\sigma_p) \text{ or } (1-\alpha) = \sigma_p.$$

But we have assumed $\sigma_p < (1-\alpha)$, so that when $\theta = \gamma$, ω will never be equal to unity. Recall next that case III. applies only when,

$$\alpha\theta < (1-\sigma_p),$$

but since $\gamma \geq 1$, it can never be true when case III. applies that

$$\alpha\theta = \gamma(1-\sigma_p)$$

except in the borderline case where

$$\alpha = 1-\sigma_p, \quad \theta = \gamma = 1,$$

in which case all cases are identical since then both sectors are the same.

Under different labor market assumptions, case III. would be neither trivial nor of special difficulty. For example, if the marginal product of labor were equated as between the two sectors, then an optimal dualistic equilibrium along the lines of case III., $\omega = 1$, would be approached, (if the two sectors had differing savings ratios), for some intermediate range of discount rates.

Cases I., II., and IV. have no such difficulties of interpretation as case III., and we shall now move on to them. Case I. is of particular interest, since when it applies, then it is always optimal to organize the agricultural sector on a capitalist basis. Recall that $\alpha\theta > 1-\sigma_p$ in this case. Knowing this, it is easy to see

why it is never optimal to have peasants. Given any aggregate capital-labor ratio, k , it is clear that

$$\alpha \theta k^{1-\alpha} > (1-\sigma_p) k^{1-\alpha} \text{ and}$$

$$(1-\alpha) \theta k^{1-\alpha} > \sigma_p k^{1-\alpha},$$

so that, comparing the levels of per worker investment and consumption for the two cases of all capitalist and all peasant organization, we see both are higher when production is organized on a capitalist basis, and any dualistic mixing of organizational types would only "dilute" the optimum obtainable under pure capitalism. Note that such a case is possible only when

$$\theta > (1-\sigma_p)/\alpha > 1,$$

i.e., when there exist sufficient technological gains associated with capitalist production, as through economies of scale or superior knowledge.

It is easy to generalize this case to a situation of any neo-classical production function. If we wish to consider the possibility of ever having peasant agriculture exist on an optimal growth trajectory, we may ask the following questions: Is

$$\theta w(k) > (1-\sigma_p)g(k) \text{ and}$$

$$\theta\pi(k) > \sigma_p g(k), \text{ where } w(k) + \pi(k) = g(k)$$

for all relevant k ? (The relevant domain of k will be determined by k^* , the maximum sustainable capital-labor ratio of the system, if $k(0) \leq k^*$, and by k^{**} , the minimum sustainable capital-labor ratio, if $k(0) \geq k^{**}$, otherwise the minimum sustainable ratio will be defined by $k(0)$.) It is clear that θ must again exceed unity in order that these two conditions be met since if $w(k) \geq (1-\sigma_p)g(k)$, then $\pi(k) \leq \sigma_p g(k)$. If both conditions are satisfied, then as in the Cobb-Douglas case, both per worker consumption and per worker investment are maximized in an all capitalist regime. For example, in the case where production is governed by a CES production function of the form

$$g(k) = (ak^{-\beta} + b)^{-\frac{1}{\beta}}, \quad a+b = 1,$$

it is easily demonstrated that in the case of $\sigma < 1$ ($\beta > 0$), where σ is the Hicksian elasticity of substitution, that the above conditions are met when

$$\sigma_p < (a\theta\eta^\beta)^{\frac{1}{1+\beta}} \quad \text{and}$$

$$\theta > (1-\sigma_p)/(1-a\eta^\beta/\sigma_p^\beta),$$

where σ_p , θ , and η have their usual interpretations and $k \in [k^{**}, k^*]$.

It may be verified by the reader that these conditions may easily be satisfied for reasonable parameter values.

The above sufficiency conditions for the desirability of capitalist agriculture over the entire growth path may find some application in empirical situations where an evaluation of peasant versus capitalist enterprises is desired, if it is possible to estimate $\pi(k)$, $w(k)$, θ , and σ_p over the relevant ranges of k and time.

Turning now to cases II. and IV., we can again gain insight into the economics of the results by recalling that $1 - \sigma_p > \alpha\theta$ in these cases. Thus at any given aggregate capital-labor ratio, k , it happens that

$$\alpha\theta k^{1-\alpha} < (1 - \sigma_p) k^{1-\alpha} \text{ but}$$

$$(1 - \alpha)\theta k^{1-\alpha} > \sigma_p k^{1-\alpha},$$

so that per worker investment is higher in an all capitalist regime but per worker consumption is lower. It is thus intuitively clear why the optimum long-run steady state of the agricultural economy may be all capitalist when the time preference of consumption is relatively slight, i.e., ρ is low, and will be all peasant when there exists a strong preference for present over future consumption, i.e., ρ is high, since the choice between capitalist and peasant organization represents a choice between future and present consumption as implied

by their respective savings ratios. It is interesting to observe in the phase diagrams for these two cases, Figures 3.5 and 3.7, that when $k(0)$ is very low, it is always optimal to accumulate capital initially under an all capitalist regime. In fact, in case II. we see that if $k(0) < k^*$, a not implausible initial state, it is always desirable to remain organized on a purely capitalist basis. It appears that the high consumption ratio implied by peasant agriculture, together with its technological backwardness, are "luxuries" which may be tolerated only in an economy possessing a minimum stock of capital per man and having a relatively low degree of preference for present as opposed to future consumption. But it is important to recall that when $\delta = 1$, ρ may never be set low enough to permit an optimum at a purely capitalist steady state. Then either the peasant steady state is approached or the difficult case III. situation obtains.

Now these problems are, in a sense, illusory. We have assumed that the government has only one control variable, an optimum capital transfer mechanism. This assumption has been desirable since it will make possible some interesting comparisons with the descriptive models discussed above in which such things as the savings process and labor markets function without government interference. (Below we shall make such comparisons.) The reader will have observed by now that the crucial factors in the optimizing problem are two: the difference in savings ratios associated with the two forms of agricultural organization and the imperfections in the allocation

of labor between the sectors. This latter factor renders any dualistic equilibrium inefficient, thus forcing our optimum policy into a totally discriminating one as between sectors. The former factor, in conjunction with the rate of time preference, forms the basis for deciding how to discriminate. The illusory aspects of the problem stem from the perhaps unnecessary prevention of government interference in either the labor market or savings behavior. For example, if the government insures that the marginal product of labor in each sector is such that $k_p = k_c$, then it is possible to have optimal dualistic equilibria of the type which case III. shows are not otherwise permissible. But more important is the possibility of manipulating effective savings behavior. Suppose that $\theta(1-\alpha) > \sigma_p$ but $(1-\sigma_p) > \alpha\theta$. Then if it is possible for the government to levy an income tax on both sectors at some uniform rate, τ , and if it may then use the proceeds to augment either aggregate savings or consumption, then there will never be any reason to ever have peasant agriculture exist on an optimum trajectory if the following conditions hold: $\theta > 1$, and $(1-\sigma_p)(1-\tau_{\max}) + \tau_{\max} < \alpha\theta(1-\tau_{\max}) + \theta\tau_{\max}$, where τ_{\max} is the maximum feasible proportional rate of taxation, since then it will be possible by taxation, and subsidy to obtain a higher level of consumption or investment per worker under pure capitalist organization given that the other quantity is held the same under both pure forms of organization. This is due to the technological superiority of capitalism ($\theta > 1$). If $\theta = 1$, then there will be situations where peasant organization will be preferred, if $\tau_{\max} < 1$. These situations

will arise when the rate of discount is very high or the initial capital stock exceeds the long-run steady state level, and it is desirable to consume as much as possible. An optimal policy planner would be indifferent between peasant and capitalist forms of organization in the situations analogous to case III. for $\theta = 1$, since the desired optimum savings ratio could be obtained by tax and subsidy policy under either form of organization. Let it be emphasized that he would not be indifferent to a dualistic form of organization as long as there exists an imperfect labor market. He would still prefer either pure capitalist or pure peasant organization.

This should be of interest to planners in countries having dualistic agricultures in which it is felt that capitalists enjoy no particular technology advantage, since then it could concentrate on eliminating imperfections in the labor market, while promoting dualistic organization for political reasons or for possible economic reasons associated with the costs of land-capital reallocation. Of course, this kind of policy would work only if the government were able to effect the appropriate changes in the aggregate savings ratio through taxation and subsidy. Note that when $\theta = 1$, the existence of either a perfect labor market or a perfect capital market will insure the equivalency of productive efficiency in both sectors, if it is possible to obtain equilibrium in that perfect market, so that then the only difference between peasants and capitalists is in the range of attainable savings and consumption ratios, the peasants being able to provide a higher maximum consumption and the capitalists a

higher maximum savings ratio when τ_{\max} is less than one.

One further important point needs to be made, although it may already be obvious to the reader. By virtue of the assumptions made about static resource use by peasant families, namely that they provide the same labor effort per unit of population as they would in a competitive labor market and that each identical family owns the same amount of capital which it uses at maximum capacity, we have in effect imposed a purely competitive result on the peasant sector: Given the population and capital stock in an isolated peasant sector, output will be maximized in the same way as if the sector consisted of profit maximizing capitalist firms. Since, in fact, all families are not likely to be identical or supply the same amount of labor as they would in a competitive labor market, it may be wrong to attribute equal allocative efficiency to both capitalists and peasants when these sectors exist in isolation (so that the inefficiencies of dualistic organization are ruled out). Thus even when technologies are identical, it may be useful to consider a non-unitary value of θ , and to interpret its magnitude as a reflection of relative allocative inefficiencies within each of the two sectors.

These observations apply equally well to the case where production functions are of the more general neoclassical type. If taxation and subsidy permit sufficient variation of the effective savings ratio in the capitalist sector, then it will always be optimal to use capitalist organization if $\theta > 1$. When $\theta = 1$, and $\tau_{\max} < 1$, there may be occasions where peasants are preferred due to their

higher maximum consumption ratio (under taxation and subsidy). Case III. type situations will be prevented by using either all peasants or all capitalists with appropriate taxation policy. There is one apparent difference: Since now k_p/k_c is no longer a constant, it is mathematically possible that H becomes concave in n_c so that static maximization of social product occurs at a dualistic point. However, economic reasoning proves that this is false, since it is clear that a planner would be indifferent between a unisectoral agricultural organization and a dualistic one only if he could, under dualism, assure equality of the marginal product of labor in each sector (given savings rates). Inequality of the marginal products can only reduce total productivity at any given aggregate capital-labor ratio (when $\theta = 1$), since it is obvious that output is maximized only when the marginal product of labor is identical in each sector. Thus, as in the Cobb-Douglas case, it is clear that a dualistic long-run optimal steady state would be possible only under conditions of equal technological efficiency and perfect factor markets.

Let us now think back to the descriptive analysis in the preceding chapters. Our optimizing formulation can tell us a good deal about the welfare properties of the various descriptive cases considered earlier. But let it be remembered that the welfare results which we have obtained from the optimizing model are not necessarily valid if we place the agricultural sector into the context of a multisectoral developing economy, so that some of the earlier welfare judgements may retain partial validity vis-a-vis our

latest welfare results. Keeping this in mind we shall now assess the welfare results of the descriptive models in terms of the results obtainable under welfare maximization.

If the descriptive models of dualistic agriculture approach the reality of agricultural policy making in developing countries in their assumed inflexible savings behavior, which is independent of governmental control, then it is clear that pure peasant agriculture may be economically desirable because of its savings properties at some points and pure capitalist agriculture at other times because of its savings behavior. Now we have observed that when both capital and labor markets are fluid but imperfect, there will be conditions under which pure capitalism or pure peasant organization will prevail under conditions of static equilibrium. It is equally clear, however, that there is no clear cut correspondance between who will prevail and who ought to prevail according to the criterion of welfare maximization. Similarly, we have observed how many kinds of factor market imperfections can result in dualistic organization under conditions of static equilibrium and which may persist over long periods in dynamic growth states. But we have just seen how these dualistic states, brought about through market imperfections always represent a sub-optimum situation, not only in terms of allocative efficiency of the factors of production, but also in terms of the desired aggregate savings ratio. Reflect for a moment that in the Cobb-Douglas case where $(1-\sigma_p) < \alpha\theta$, so that capitalists are always preferred (even when savings rates are fixed, remember), it is possible

in the non-flowing capital descriptive case to have the wage gap so high that during the whole sub-optimum trajectory not only is the agricultural economy always dualistic, and hence inefficiently organized, but also capitalists are being displaced relatively, or even absolutely, in spite of the fact that they are preferred.

We should briefly consider how the existence of a non-agricultural sector may affect our results. It remains true, obviously, that if $\theta > 1$, it is desirable to concentrate all production in the hands of capitalists from the standpoint of maximizing agricultural output for a given capital and labor stock in agriculture. Whether, in fact, it will be optimal in a given case to concentrate production in this way will depend on how easily the capitalist and peasant savings-investment behavior may be altered, first in terms of the magnitude of the savings ratio, and second in terms of the allocation of investment activity between sectors. Given the priority usually assigned to investment in such non-agricultural activities as manufacturing it is, for example, conceivable that despite capitalist agriculture's possible technological and savings superiority, peasant agriculture could be preferred if it were possible to channel more peasant savings into non-agricultural investment than capitalist savings. But now, even given equal technologies ($\theta = 1$), peasant agriculture is likely to be less preferable due to the fact that allocative inefficiency will exist in the labor markets, since the marginal products of labor in agriculture and non-agriculture will not be equated if there exists

a wage gap associated with peasant agriculture. The presumption against the desirability of peasants runs higher, the greater the extent to which the capitalist savings ratio and technology exceeds that of peasants and to which the direction of capitalist investment can be or will be directed to "high priority" sectors, since in general it is to be expected that the aggregate capital-labor ratio in developing countries is quite low and that there will be a premium on rapid capital accumulation, especially in manufacturing. Pure peasant or dualistic agriculture will be more likely to be desirable if peasants are of equal technological efficiency, if it is possible to obtain sufficient savings through taxation, and if factor market imperfections can be eliminated.

As a final note, it may be of use to point out that capitalist technological superiority need not be Hicks-neutral to apply the theorems derived above. It is sufficient that $g(k_p) \lesseqgtr h(k_c)$, for $k_p = k_c$, where $g(\cdot)$ and $h(\cdot)$ are the peasant and capitalist production functions, respectively. This would be the case under either a Hicks-neutral or non-neutral factor augmenting technological advantage.

FOOTNOTES

- [1] Pontryagin, et al., The Mathematical Theory of Optimal Processes, translated by K.N. Tirovogoff, (New York: Wiley and Co., 1962).
- [2] See Arrow, "Applications of Control Theory to Economic Growth," unpublished manuscript, (M.I.T., 1967), pp. 1-15.
- [3] See Karl Shell, "Optimal Programs of Capital Accumulation for an Economy in which there is Exogenous Technical Change," in K. Shell, ed., Essays on the Theory of Optimal Economic Growth, (Cambridge: M.I.T. Press, 1967), pp. 5-7, including note.
- [4] For this method of proof I follow Shell, op. cit., pp. 11-13.

CHAPTER IV

EXTENSIONS OF THE THEORY TO COVER EXPLICIT INTERACTIONS BETWEEN AGRICULTURE AND THE REST OF THE ECONOMY

In previous chapters the agricultural economy has been studied largely in isolation. It has been assumed that the terms of trade between agricultural and non-agricultural goods were fixed and immutable, that capital did not flow freely into or out of the agricultural economy, and that the relative rate of population drain to non-agricultural sectors was small or constant. In what follows, these assumptions will be relaxed to see what effect they have on the viability of each form of agricultural organization and on the conditions of economic welfare. In this section it is again assumed that there is no government and that the general two sector disjoint capital market model applies.

First, we shall consider what would happen if all of the above assumptions continued to hold except the one about capital flows. Suppose instead that while capitalists invest all profits, they invest in agricultural enterprises only when the rate of return on capital in such enterprises exceeds some externally given interest rate, i . Further, suppose that should i lie below the rate of return on capital in capitalist agriculture, then capital resources will flow into agriculture from the outside until i and the rate of return on capital are brought into equality. Assume finally that this

mechanism works so fast relative to the rates of change of the endogenous variables of the model that this capital market may be assumed always to be in equilibrium, i.e., so that the rate of return on capital in capitalist agriculture is always equal to i .

Since then $\theta g'(k_c) = i = \text{constant}$, it follows that $\dot{k}_c = 0$ in the capitalist saving-investment equation (2.2). (See p. 67 Chapter II.) Substituting i for $\theta g'(k_c)$ in this equation, the following result is obtained after rearranging:

$$i - \eta = \dot{n}_c / n_c.$$

Under these circumstances, the direction of relative displacement is independent of what goes on in the peasant sector. If the fixed rate of return exceeds the relative rate of depreciation plus the net relative rate of population growth, the capitalist sector will grow relative to the peasant sector, regardless of the peasant savings rate or the rate of technological progress in peasant agriculture. Equilibrium in the labor market is still governed by equation (2.1) on page 63. From this equation it is seen that the ratio of peasant to laborers' per worker consumption is $\frac{1}{\gamma}$; since k_c and k_p are fixed, it follows that if displacement favors capitalists, then welfare increases; if it favors peasants, welfare decreases. This remains true even if both organizational types are of equal technological status.

If population growth is not kept at a fixed relative rate but rather is assumed to increase when welfare improves, then the dis-

placement mechanism will be affected. Suppose that initially $i-\eta > 0$, so that capitalists are becoming predominant, but since $\gamma > 1$ welfare improves and ϵ , and hence η , rises. If ϵ rises far enough, displacement will terminate as $(i-\eta) \rightarrow 0$. If peasant technology should improve, no change in the rate of displacement will occur. Since the ratio of peasant to laborers' welfare is constant and the latter is also constant, regardless of the size of the technology gap, the rate of population growth can change only when labor is transferred to capitalist agriculture. Under such circumstances, there would be a zero social rate of return to investment in technological change in peasant agriculture, as long as peasants have no chance of obtaining an absolutely superior technology.

Restoring the assumption that capital does not flow freely between the non-agricultural sector and capitalist agricultural sector according to the rate of return on capital there, suppose instead that, as before, all capitalist profits are invested and that peasants continue to save and invest a proportion, σ_p , of their total output. In addition, assume that the terms of trade between agricultural and non-agricultural goods, p^{-1} , may vary. In regard to capitalist investment, assume that capitalists may invest a proportion of their profits outside agriculture and that this proportion will rise as p rises. The basis of this assumption can be explained as follows: When p rises, the rate of return on capital invested in agriculture falls relative to that invested in non-agriculture, measured in terms of the price of non-agricultural goods. Capitalists then find investment

outside agriculture to be more attractive. The proportion of capitalist output invested in agriculture may be written as σ_c where

$$\sigma_c = \sigma_c(p), \sigma_c'(\cdot) < 0, \lim_{p \rightarrow 0} \sigma_c(p) = 1-\alpha.$$

It is assumed that peasants will also react to changes in the terms of trade so that the peasant savings-investment ratio may also be written as a function of p :

$$\sigma_p = \sigma_p(p), \sigma_p'(\cdot) \gtrless 0, \lim_{p \rightarrow 0} \sigma_p(p) = \sigma_p^0.$$

Whether σ_p rises or falls when p shifts depends on the interaction of two effects: Raising p lowers real income and hence reduces the ability of peasants to save anything after subsistence requirements have been met. Also, raising p makes non-agricultural goods, including investment goods, less attractive relative to agricultural goods. On the other hand, peasants may try to maintain real investment at roughly the same level when p rises and thus will increase the savings-investment ratio. In such circumstances peasants prefer to reduce current consumption relatively more in order to maintain the capital stock from which to produce future income.

In this section the ratios σ_c and σ_p will be called investment ratios, since capitalists actually may save more than they invest in agriculture.

Next assume that a proportion of aggregate agricultural output is exchanged for non-agricultural goods and that this proportion is given by

$$b = b(p), \quad b(p) \geq \sigma_c(p), \sigma_p(p).$$

The inequality restriction states that this proportion must equal or exceed the larger of the two investment ratios, since it is assumed that all investment goods are produced externally. The supply of "exports" to non-agricultural sectors is then given by

$$S = b(p) \cdot q \cdot N_0 e^{\epsilon t},$$

where N_0 is the initial level of the labor force. Suppose there exists a market demand function for agricultural output defined as shown:

$$D = d(p) E(t) \cdot e^{\epsilon t}, \quad d'(\cdot) > 0,$$

where $E(t) \cdot e^{\epsilon t}$ is an exogenous time trend function. If the relevant exogenous variables are growing at a relative rate which exceeds ϵ , then it is assumed that $E(t)$ is rising. If they are growing at a rate less than ϵ , $E(t)$ will fall. Setting D equal to S , under the assumption that the market is always in equilibrium, the following

equation results:

$$d(p)/b(p) = [N_o/E(t)] \cdot q.$$

It will be assumed that the LHS of this equation has a positive derivative with respect to p so that one may write

$$(4.1) \quad p = p(q/E), \quad p'(\cdot) > 0.$$

This assumption implies that even if $b(p)$ rises when p increases $d(p)$ will always increase by a greater relative amount. Of course, if the more likely reaction is assumed, i.e., that $b'(p) < 0$, then there is no difficulty.

The condition of labor market equilibrium will be simplified in order to make the problem more manageable. It will be assumed that the entire average product of peasants is equated to the marginal product of laborers adjusted by a gap factor. Any effect the savings ratio of peasants might have is assumed to be incorporated in a constant way in the gap parameter. Finally, it is again assumed that both peasant and capitalist production functions are identical except for the efficiency parameter, θ , and that they are Cobb-Douglas. The condition of labor market equilibrium may then be written as:

$$(4.2) \quad \gamma k_p^{1-\alpha} = \alpha \theta k_c^{1-\alpha}, \quad w = (\alpha \theta / \gamma) \frac{1}{1-\alpha}.$$

The savings-investment functions for peasants and capitalists, respectively, now appear as follows:

$$(4.3) \quad \frac{\sigma_p(p)}{p} \cdot k_p^{1-\alpha} = \dot{k}_p + \left(\frac{\dot{n}_p}{n_p} + \eta\right) k_p$$

$$(4.4) \quad \frac{\theta\sigma_c(p)}{p} \cdot k_c^{1-\alpha} = \dot{k}_c + \left(\frac{\dot{n}_c}{n_c} + \eta\right) k_c.$$

Equations (4.1) - (4.4), plus the defining equation for q and the summation constraint for n_p and n_c fully describe the agricultural economy under the new assumptions. When all variables except k_c, n_c , and p have been eliminated, the result is the following set of equations:

$$(4.5) \quad \dot{n}_c = n_c(1-n_c)k_c^{-\alpha} \{ \theta\sigma_c(p) - \sigma_p(p)\omega^{-\alpha} \} p^{-1}$$

$$(4.6) \quad \dot{k}_c = [n_c\theta\sigma_c(p) + (1-n_c)\sigma_p(p)\omega^{-\alpha}] p^{-1} k_c^{1-\alpha} - \eta k_c$$

$$(4.7) \quad p = p\left(\frac{1}{E} \cdot \theta k_c^{1-\alpha} \cdot [n_c + (1-n_c)\frac{\alpha}{\gamma}]\right).$$

As usual, to examine the behavior of the system, it will be useful to construct a phase diagram in (k_c, n_c) space, such as has been done in Figure 4.1. If $E(t)$ is shifting continually as time passes, a different phase diagram will be relevant for each point in time, since each stationary curve is relevant only for a particular value of E . We shall treat E as a constant parameter and then ask

what happens when it shifts, since nothing is lost in such an approach. Considering first the $\dot{n}_c = 0$ curve, it is seen from equation (4.5) that $\text{sign } \dot{n}_c = \text{sign} [\theta \sigma_c(p) - \sigma_p \omega^{-\alpha}]$. It may safely be assumed that for p relatively small, $\theta \sigma_c > \sigma_p \omega^{-\alpha}$, since it is most likely that $\sigma_c^{\max} > \sigma_p(p)$ and as $p \rightarrow 0$, $\sigma_c \rightarrow \sigma_c^{\max}$. Under such circumstances, the only way in which \dot{n}_c could be made to become non-positive would be for $\sigma_c(p)/\sigma_p(p)$ to fall as p rises. Suppose this is true and that for $p = \bar{p}$, \dot{n}_c will be precisely 0. If $p > \bar{p}$, \dot{n}_c will be negative, and if $p < \bar{p}$, \dot{n}_c will be positive. The nature of the $\dot{n}_c = 0$ stationary is easily determined by setting the LHS of equation (4.7) equal to \bar{p} , since the resulting equation is the $\dot{n}_c = 0$ stationary. Since $\alpha < \gamma$, increases in n_c , (k_c), must be compensated by decreases in k_c , (n_c), i.e., along the curve, $dk_c/dn_c < 0$. Note that a decrease in E would shift the curve to the left, while an increase would shift it to the right.

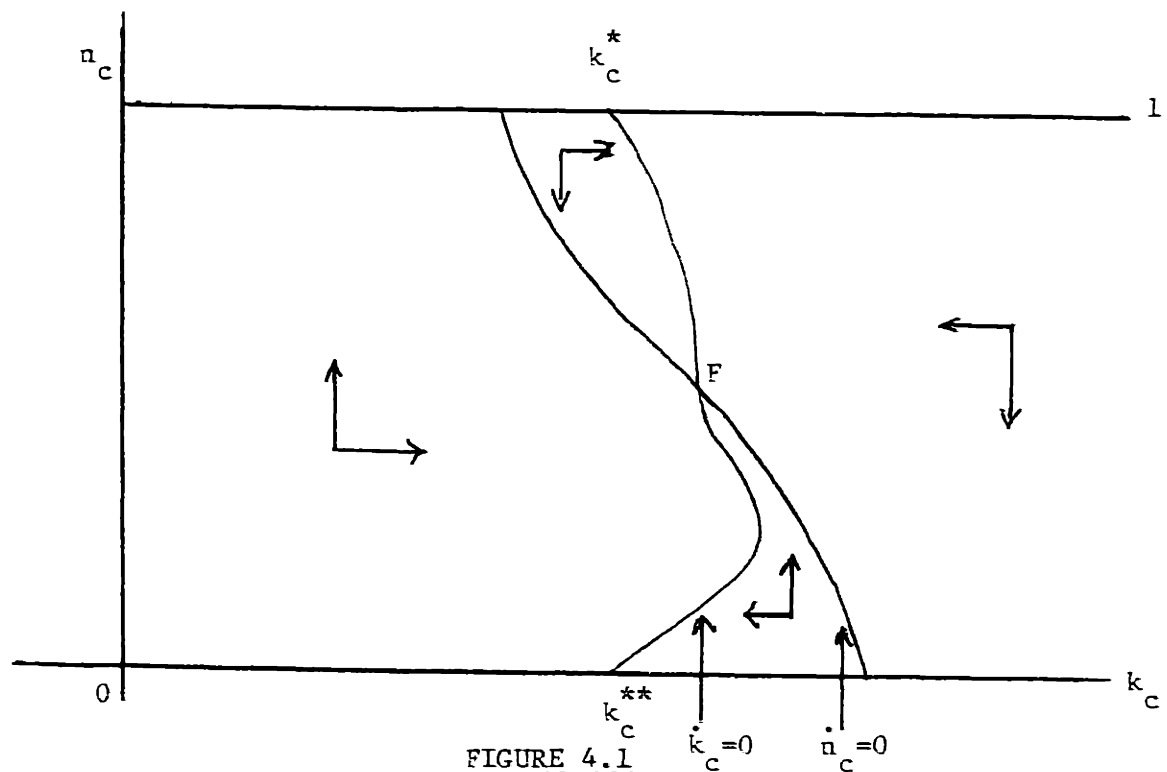
Consider next the $\dot{k}_c = 0$ stationary curve. Setting the LHS of (4.6) equal to zero and substituting the RHS of equation (4.7) for p , the following expression for dn_c/dk_c is obtained upon differentiation:

$$\frac{dn_c}{dk_c} = \frac{\eta p \alpha k_c^{\alpha-1} + [\eta k_c^\alpha p' - n_c \theta \sigma_c' - (1-n_c) \sigma_p' \omega^{-\alpha}] [n_c + (1-n_c) \cdot \frac{\gamma}{\gamma}] \frac{\theta}{A} (1-\alpha) k_c^{-\alpha}}{\theta \sigma_c - \sigma_p + \frac{\theta}{A} k_c^{1-\alpha} \{ n_c \theta \sigma_c' + (1-n_c) \sigma_p' \omega^{-\alpha} - \frac{p'}{p} [n_c \theta \sigma_c + (1-n_c) \sigma_p \omega^{-\alpha}] \}}$$

If σ_c' , σ_p' , and $-p'$ are all assumed to be negative, then the numerator of this expression is positive. Assume this to be the case, then the sign of dn_c/dk_c is determined by the sign of the denominator of the

above expression. If $p > \bar{p}$, i.e., if an arbitrary point in (k_c, n_c) space lies to the right of the $\dot{n}_c = 0$ stationary, then $\theta\sigma_c - \sigma_p < 0$, and the denominator is negative since the large term enclosed in braces is negative under the assumptions about σ'_p , σ'_c , and p' . If $p < \bar{p}$, then $\theta\sigma_c - \sigma_p > 0$ and the denominator may be positive or negative. Thus, even with fairly strong restrictions, i.e., on the signs of σ'_p , σ'_c , and p' , it is not possible to state that the $\dot{k}_c = 0$ stationary curve will necessarily slope one way or the other.

The phase diagram, Figure 4.1, may be used to illustrate the possibilities. Notice that in the illustrated case the $\dot{k}_c = 0$ stationary, as we go from $n_c = 1$ to $n_c = 0$, has first positive and then negative slopes. The maximum steady state k_c is not at either pure equilibrium point. It has been assumed that the $\dot{k}_c = 0$ stationary



intersects the k_c axis to the left of the $\dot{n}_c = 0$ stationary and the $n_c = 1$ horizontal to the right of the $\dot{n}_c = 0$ stationary. This assumption is arbitrary, and it determines whether the equilibrium point F will be globally stable or unstable, since examination of equation (4.6) shows that regardless of the shape of the $\dot{k}_c = 0$ curve, if a point lies to the right, $\dot{k}_c < 0$, and if it lies to the left, $\dot{k}_c > 0$. (The reader may verify instability for the case of intersections with the $n_c = 1$ horizontal and k_c axis in the reverse order.) Also, it is clear from equation (4.7) that decreases in E increase p , ceteris paribus. Now the aggregate investment ratio, in terms of non-agricultural goods is $[n_c \theta \sigma_c(p) + (1-n_c) \sigma_p(p) \omega^{-\alpha}] p^{-1}$, and under the assumptions made, must fall when p rises, so that the $\dot{k}_c = 0$ stationary shifts to the left when E decreases, ceteris paribus. Mutatis mutandis, it can be seen that when E rises, the curve will shift to the right. It is clear, when the discussion of the $\dot{n}_c = 0$ stationary is recalled, that both stationaries tend to shift in the same direction when E changes, so that there is no way of determining easily whether secular shifts in the terms of trade will lead to situations where the two stationaries do not intersect. Finally, in the phase diagram, the stationary curves have been drawn so as to intersect only once, although it is clear that they might possibly intersect more than once in a combination of stable and unstable equilibria.

It is all too clear that the introduction of varying investment ratios and terms of trade has thrown into limbo the earlier quite

clear cut results about relative displacement. But this is true only if the ratios may vary widely and freely. If it is known, for example, that the capitalist investment ratio always exceeds the peasant ratio by a substantial margin, then there is a strong presumption in favor of capitalists, regardless of what happens to the terms of trade. It is equally difficult to make welfare assessments of the two pure steady states. The ratio of output per worker is $(\gamma/\alpha)(k_c^*/k_c^{**})^{1-\alpha}$ in terms of agricultural goods and $(\gamma/\alpha)(k_c^*/k_c^{**})^{1-\alpha}(p^{**}/p^*)$ in terms of non-agricultural goods. The ratio of consumption per worker is $[\gamma/(1-\sigma_p^{**})] \cdot (k_c^*/k_c^{**})^{1-\alpha}(p^{**}/p^*)$. (In each case the capitalist quantity is the numerator. One star attached to a variable indicates a value pertaining to the all-capitalist steady state and two a value pertaining to the all-peasant state.) From the discussion of the phase diagram it is clear that k_c^* may or may not exceed k_c^{**} . If $k_c^* < k_c^{**}$, there is no guarantee that the ratio of output per worker in terms of agricultural goods is greater than unity. Even if $k_c^* > k_c^{**}$, there is then no guarantee that the ratio remains greater than one when measured in terms of non-agricultural goods, since if $q^*/q^{**} > 1$, then $p^*/p^{**} > 1$. A similar observation is relevant to the ratio of per worker consumption in each sector. Finally, note that there exists a good possibility that some sort of dualistic steady state will be optimal in the sense of maximizing per worker consumption or output, since as the all-capitalist equilibrium is approached the possible decline in the terms of trade will reduce the value of agricultural output and depress investment ratios. On the other hand, the all-peasant point may have the disadvantages of

a low investment ratio regardless of the size of p and of a lower level of technology.

These results can be made more tractable at the cost of further restrictions, which effectively render the model more similar to the case of fixed investment ratios and terms of trade. If it is assumed that investment ratios are fixed, but the terms of trade may vary, then the question of displacement is settled as before by simple examination of the expression $(\theta\sigma_c - \sigma_p\omega^{-\alpha})$. (Alternatively, it could be assumed that σ_c/σ_p varies only slightly over the range of attainable terms of trade, so that regardless of the value of p , displacement would always occur in one direction or the other.) The question of welfare comparisons would continue to be affected by changing terms of trade. For example, if we assume constant investment ratios and that $p(q/E) = (q/E)^\mu$, then $c^*/c^{**} > 1$ if and only if

$$\theta \left(\frac{\sigma_c}{\sigma_p}\right)^{1-\alpha} \cdot \left(\frac{\alpha}{1-\sigma_p}\right)^{\alpha+\mu(1-\alpha)} > 1.$$

If μ is large enough, this condition may not be satisfied even though θ and (σ_c/σ_p) are large enough to insure that $\theta\sigma_c - \sigma_p\omega^{-\alpha} > 0$.

Up to this point, all the analysis has been based on explicit study of only two sectors. The rest of the economy and of the world has been treated largely as an exogenous factor. Now a purely competitive domestic non-agricultural goods sector organized on a capitalist basis will be introduced into the framework of the Cobb-Douglas capital accumulation model with fixed savings ratios and

terms of trade in order to see what role intersectoral factor flows may play in the interaction of peasant and capitalist agriculture.

Suppose that production in the non-agricultural sector is governed by a Cobb-Douglas production function having an elasticity of output with respect to labor equal to β . Then we may write

$$(4.8) \quad q_m = \theta_m k_m^{1-\beta},$$

where q and k have the usual interpretation, m is a subscript denoting the non-agricultural sector, and θ_m is the level of technology parameter in that sector. Assume that the economy is open to international trade and that it is small in relation to the volume of total world trade so that the terms of trade between agriculture and non-agriculture may be treated as fixed exogenously and equal to p^{-1} , where as before p has units: units of agricultural output per non-agricultural good.

The condition of intersectoral capitalist labor market equilibrium is assumed to be the equality of the value of the purely competitive wage, and it is further assumed that equilibrium in this market always obtains. Thus the relevant equation is

$$(4.9) \quad r\theta_c k_c^{1-\alpha} = p \cdot \beta \theta_m k_m^{1-\beta}.$$

The subscript c has been added to the technology parameter, θ , on the LHS to distinguish it further from θ_m and to provide symmetry.

Similarly, it is assumed that capital, both new and existing, may flow freely from one capitalist sector to the other according to differences in the rate of return on capital. This market is also assumed always to be in equilibrium, and the relevant equation is

$$(4.10) \quad (1-\alpha)\theta_c k_c^{-\alpha} = p \cdot (1-\beta)\theta_m k_m^{-\beta}.$$

The peasant-capitalist labor market equilibrium condition remains unchanged as does also the peasant savings-investment equation. The capitalist saving-investment constraint will include both sectors since capital flows freely between them. As before, it is assumed that all profits are invested and all wages consumed. There is no inflow or outflow of foreign capital, nor does labor emigrate or immigrate. The rate of depreciation in the non-agricultural sector is the same as that in agriculture, so that the sum of this rate and the relative rate of population growth is also \bar{n} in non-agriculture. Under these assumptions, the savings-investment constraint on the capitalist sectors is

$$(4.11) \quad p^{-1} \bar{n}_c (1-\alpha)\theta_c k_c^{1-\alpha} + \bar{n}_m (1-\beta)\theta_m k_m^{1-\beta}$$

$$= \left(\frac{\dot{k}_c}{k_c} + \frac{\dot{n}_c}{n_c} + \eta \right) k_c n_c + \left(\frac{\dot{k}_m}{k_m} + \frac{\dot{n}_m}{n_m} + \eta \right) k_m n_m.$$

It remains only to modify the labor force constraint to account for labor in non-agriculture. The modified equation states simply

that the proportion of labor in each of the three sectors must add up to one:

$$(4.12) \quad n_p + n_c + n_m = 1.$$

Equations (4.9) - (4.12), (2.1'), and (2.3), when $k_p^{1-\alpha}$ is substituted for $g(k_p)$ in this last equation, comprise a closed system which fully determines the behavior of the economy. Notice that the three factor market equilibrium conditions, equations (2.1'), (4.9) and (4.10) immediately determine fixed equilibrium values for k_c, k_p , and k_m , since the only variables appearing in these three equations are just the ones mentioned. In fact, since just k_c and k_m appear in equations (4.9) and (4.10), it is clear that the equilibrium values of k_c and k_m , and hence the equilibrium values of wage rates, rents, and output and consumption per worker in the two capitalist sectors, are independent of what goes on in the peasant sector. Solving the equations for the equilibrium values k_c^*, k_p^* , and k_m^* , in terms of the parameters, we have

$$k_c^* = \left[\left(p \frac{\theta}{\theta_c} \right) \left(\frac{1-\beta}{1-\alpha} \right)^{1-\beta} \cdot \left(\frac{\beta}{\alpha} \right)^3 \right]^{\frac{1}{\beta-\alpha}}$$

$$k_m^* = \frac{1-\beta}{1-\alpha} \cdot \frac{\alpha}{\beta} \cdot k_c^*$$

$$k_p^* = \left[\frac{\alpha \theta_c}{\gamma(1-\sigma_p)} \right]^{\frac{1}{1-\alpha}} \cdot k_c^*$$

Notice that k_p^* bears the same relation to k_c^* as in the two sector model and that $k_m^* > k_c^*$ as $\alpha > \beta$.

Since it has been assumed that the equations which determine k_c^* , k_m^* , and k_p^* always obtain, it follows that $\dot{k}_c = \dot{k}_m = \dot{k}_p = 0$. By use of equation (4.12) one of the three labor force allocation variables may be eliminated so that only the two savings-investment equations describe the behavior of the entire system. This has been done below, n_p having been eliminated. Also, since the k_i^* , $i=c,m,p$, imply immediately the values q_i^* for each sector, these values have been introduced for greater convenience of notation. The savings-investment equations then appear as follows after re-arranging:

$$(4.13) \quad [p^{-1}(1-\alpha)q_c^* - \eta k_c^*]n_c + [(1-\beta)q_m^* - \eta k_m^*]n_m = k_m^* \dot{n}_m + k_c^* \dot{n}_c$$

$$(4.14) \quad (1 - p^{-1}\sigma_p q_p^* - \eta k_p^*)(1 - n_c - n_m) = -k_p^*(\dot{n}_m + \dot{n}_c).$$

Observe that, given equilibrium in the factor markets, the direction of the flow of peasant labor is immediately established, since in equation (4.14), $\text{sign LHS} = \text{sign } \dot{n}_p = \text{sign } (p^{-1}\sigma_p q_p^* - \eta k_p^*)$. The question simply put is this: At the equilibrium capital-labor ratio, will peasants save enough out of current output to meet both depreciation (replacement) requirements and new capital needs for the increment in population? If they save too little, some of the incremental population will have to be drained off in order to keep

k_p constant. Conversely, if they save more than the amount required for replacement and population growth, the peasant labor force must be augmented by draining off members of the capitalist labor force. These labor force flows of course occur naturally as a result of income incentives. For example, if peasants saved too little, but did not undergo any adjustment in population, the capital labor ratio would fall and thus reduce peasant per worker consumption and output. The wage income in the capitalist sectors would become more attractive, and peasants would flow to the capitalists until the capital labor ratio again rose to its equilibrium level.

Again examine equations (4.13) and (4.14). The three expressions, $[p^{-1}(1-\alpha)q_c^* - nk_c^*]$, $[p^{-1}\sigma_p q_p^* - nk_p^*]$, and $[(1-\beta)q_m^* - nk_m^*]$, may be interpreted as "capital deepening" investment per worker, since each expression represents capital accumulation per worker net of replacement and of new capital at the equilibrium capital labor ratio required by labor force growth within the sector and would thus represent capital deepening if additional labor were not drawn in from the other sectors. When the factor markets are in equilibrium, capital deepening investment in each sector is constant and may be labelled i_c^* , i_p^* , and i_m^* , for each of the three sectors. Substituting the i_i^* for their equivalent expressions in equation (4.13) and (4.14), the following two differential equations in n_m and n_c are obtained after manipulating.

$$(4.15) \quad \dot{n}_m = \frac{1}{k_m^* - k_c^*} [i_c^* n_c + i_m^* n_m + i_p^* (1 - n_c - n_m) \frac{k_c^*}{k_p^*}].$$

$$(4.16) \quad \dot{n}_c = \frac{1}{k_c^* - k_m^*} [i_c^* n_c + i_m^* n_m + i_p^* (1 - n_c - n_m) \frac{k_p^*}{k_c^*}].$$

From these two equations, the differential equation for \dot{n}_p may be written simply as

$$(4.17) \quad \dot{n}_p = (i_p^*/k_p^*) n_p.$$

Returning to differential equations (4.15) and (4.16), it is readily seen that they may be written in the following form:

$$(4.15') \quad \dot{n}_m = a_m n_c + b_m n_m + c_m$$

$$(4.16') \quad \dot{n}_c = a_c n_c + b_c n_m + c_c,$$

where the a_i , b_i , and c_i are derived constants. For example,

$$a_m = \frac{i_c^*/k_c^* - i_p^*/k_p^*}{k_m^*/k_c^* - 1}.$$

Since equations (4.15') and (4.16') form a system of linear differential equations, the properties of the system can be determined by examining the roots of the characteristic equation of this system, which are two in number and are given immediately below:

$$x_1 = (1/2) [a_m + b_c + \sqrt{(a_m - b_c)^2 + 4a_c b_m}]$$

$$x_2 = (1/2)[a_m + b_c - \sqrt{(a_m - b_c)^2 + 4a_c b_m}]$$

If $-4a_c b_m > (a_m - b_c)^2$, the system will oscillate, since x_1 and x_2 will be complex. The oscillations will be damped if and only if $a_m + b_c < 0$. If x_1 and x_2 are real and negative, then n_c and n_m will approach steady values, as in the damped oscillations case. If x_1 and x_2 are real and of differing or positive sign, the system is unstable.

Unfortunately, there exists a difficulty in directly applying these results. Equations (4.15') and (4.16') accurately describe the economy when n_m , n_c , and $(1-n_m - n_c)$ are non-negative, but there is no presumption that these differential equations will imply trajectories satisfying the non-negativity constraints for every set of initial conditions also satisfying them. In fact, it is evident that the unstable trajectories will inevitably violate them. When this occurs, the economy reverts to a two sector economy whose behavior is described by models of the type discussed already.

Given this difficulty, it is nevertheless possible to discuss certain interesting cases. For example, if all of the i_i^* are non-negative, then one capitalist sector will be displaced by the other two sectors, since by inspection of equations (4.15) and (4.16) it is easily seen that $\text{sign } \dot{n}_c = -\text{sign } \dot{n}_m$. Note that $\dot{n}_c > 0$ if and only if $k_c^* > k_m^*$. From the parametric expression for k_m^* , it is evident that this occurs if and only if $\beta > \alpha$. Thus, which capitalist sector will disappear depends on the technology in the two sectors.

The sector having a relatively higher elasticity of output with respect to capital will survive. Note that this is true regardless of the level of p or the θ_i , but of course only to the extent that alterations in these parameters do not cause any of the i_i^* to become negative. Why is this the case? The answer is not clear. By assumption, the marginal productivity of labor and the rate of return on capital are the same in both sectors. It cannot simply rest on the fact that the surviving sector has a higher savings ratio, since it may also have a higher depreciation rate. In addition, the level of p affects the effective savings capability in agriculture. For some reason, when all i_i^* are non-negative, it pays to be the capitalist with the higher equilibrium capital-labor ratio.

As noted earlier, equations (4.15) and (4.16) will no longer accurately reflect the behavior of the economy after one of the capitalist sectors has disappeared. When this occurs, the capital labor ratios in the two remaining sectors are no longer fixed, and if the capitalist agricultural sector remains, then the two sector model already discussed is applicable. If the non-agricultural sector survives, then a similar dualistic model applies, this time of a more often discussed sort. It is interesting to note that peasant agriculture may be viable and grow relative to the rest of the economy when two capitalist sectors exist, but may not be when only one is functioning.

If i_p^* is negative, it is evident from equation (4.17) that as long as both capitalist sectors exist, \dot{n}_p is negative, and

$n_p \rightarrow 0$ as a limit. Assuming that both capitalist sectors remain active, i.e., continue each to employ a positive fraction of the total labor force, it is trivially true that peasants will be displaced. When n_p is close to 0 and the economy is effectively a two sector system, the equation, $n_c + n_m = 1$, may be used to eliminate n_m from equation (4.16) yielding

$$(4.18) \quad \dot{n}_c = [i_c^* n_c + i_m^* (1-n_c)] - (k_c^* - k_m^*).$$

By inspection it is readily verified that n_c , and thus n_m , approaches a stable solution if and only if $\text{sign } i_c^* = \text{sign } (-i_m^*)$ and $\text{sign } (k_m^* - k_c^*) = \text{sign } i_c^*$. Thus, there is no a priori certainty that both capitalist sectors will remain viable when peasants are displaced.

In any case it is evident that the behavior of the economic system depends crucially on the signs of the i_i^* . For this reason, the equilibrium values of the signs of these three variables are shown below in terms of the structural parameters of the model:

$$\text{sign } i_p^* = \text{sign} \left\{ \left(\frac{\sigma}{n} \right)^{\frac{1}{\alpha}} (1-\sigma)^{-p} \frac{\beta}{\alpha(\beta-\alpha)} \cdot \frac{1}{\gamma} \cdot \theta_c^{\frac{\beta-1}{\alpha}} \cdot \theta_m^{\frac{\beta-1}{\alpha}} \cdot \alpha \cdot \left[\left(\frac{1-\beta}{1-\alpha} \right)^{1-\beta} \left(\frac{\beta}{\alpha} \right)^{\beta} \right]^{\frac{1}{\beta-\alpha}} \right\}$$

$$\text{sign } i_c^* = \text{sign} \left\{ \left(\frac{1-\alpha}{n} \right)^{\frac{1}{\alpha}} - p \frac{\beta}{\alpha(\beta-\alpha)} \cdot \theta_c^{\frac{-\beta}{\alpha(\beta-\alpha)}} \cdot \theta_m^{\frac{1}{\beta-\alpha}} \cdot \left[\left(\frac{1-\beta}{1-\alpha} \right)^{1-\beta} \left(\frac{\beta}{\alpha} \right)^{\beta} \right]^{\frac{1}{\beta-\alpha}} \right\}$$

$$\text{sign } i_m^* = \text{sign} \left\{ \left(\frac{1-\beta}{n} \right)^{\frac{1}{\alpha}} - p \frac{1}{\beta-\alpha} \cdot \theta_c^{\frac{1}{\beta-\alpha}} \cdot \theta_m^{\frac{2\alpha-\beta}{\beta-\alpha}} \cdot \left[\left(\frac{1-\beta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\beta}{\alpha} \right)^{\alpha} \right]^{\frac{1}{\beta-\alpha}} \right\}$$

Notice how important it is to know the sign of $\beta - \alpha$ in order to evaluate the effects of changes in other parameters. For example, if $\beta > \alpha$, raising p will increase the likelihood that each sign is negative, the effect of a given change in p being more strongly felt in agriculture than in non-agriculture. If $\beta < \alpha$, raising p actually improves the chances that peasants will persist, and changes in p will have a stronger depressing effect on the sign of i_m^* than on that of i_p^* or i_c^* . Technological progress will also have widely differing effects depending on the sign of $\beta - \alpha$. When $\beta - \alpha > 0$, i_c^* and i_m^* are more likely to be positive if technological progress occurs in capitalist agriculture and i_p^* is more likely to be negative. This conclusion is reversed if $\beta - \alpha < 0$. Similar observations can be made about technological progress in non-agriculture. Increases in either the relative rate of population growth or the rate of depreciation will under any circumstances unambiguously reduce the likelihood that the i_i^* are positive. Note also that if $\sigma_p < \frac{1}{(1+\alpha)}$, increases in σ_p will unambiguously improve the chances that $i_p^* > 0$ and that any accretion in the size of the wage gap, γ , will also improve such chances. Technological progress in the peasant sector will have the same effect on displacement tendencies as proportionally uniform technological regression in both capitalist sectors. It may be verified by the reader that multiplying both θ_c and θ_m by a proportional constant f , $f < 1$, leads to an unambiguous reduction in the absolute value of the negative elements of i_p^* and an unambiguous increase in the

absolute value of the negative element of i_c^* . The effect on the absolute value of the negative component of i_m^* will be upward if and only if $[2\alpha - (1+\beta)]/(\beta-\alpha) < 0$; otherwise it will be non-upward.

As long as all three sectors are functioning, per worker consumption and output in each sector are fixed due to the fixity of the capital labor ratios. Aggregate output or consumption per worker will increase or decline according to the direction of change in the n_i . It is possible to calculate the values of the output and welfare magnitudes, although this will not be done. Suffice it to say that there is no guarantee that output or welfare per worker will increase as a result of natural displacement tendencies. Note, for example, that since $\gamma > 1$, per worker consumption is necessarily higher in the capitalist sectors. But if $i_p^* > 0$, then during the three sector phase, at least, the peasant sector will grow relatively at the expense of the capitalist sectors and consequently welfare will be reduced. The same observation applies to total output per worker as well if $\alpha/\gamma(1-\sigma_p)$ and $\beta/\gamma(1-\sigma_p)$ are both less than unity. If one of the capitalist sectors is displaced first, then the same kinds of welfare considerations apply as in the discussion of welfare in the two sector model. Thus, to repeat, there is no reason to expect that the natural displacement mechanism will function to the benefit of economic welfare.

Before making further extensions of the three sector model, several points which were glossed over earlier should be mentioned.

First, given the equilibrium values of the capital labor ratios, k_i^* , there is only a certain range of values of the aggregate capital-labor ratio which will permit the existence of factor market equilibrium with full employment of all factors of production. In the above analysis it was tacitly assumed that the initial endowments of capital and labor in each sector were consistent with the achievement of static full employment equilibrium. If the initial endowments satisfy the full employment requirement, then the subsequent dynamic behavior of the economy will be consistent with that indicated by equations (4.15) and (4.16). In the case of static equilibrium, the savings-investment equations (2.3) and (4.11) are replaced by the following factor employment constraints:

$$k_p n_p = v = \text{constant}$$

$$k_c n_c + k_m n_m + k_p n_p = k = \text{constant}$$

where $v = K_p/N$, the ratio of the fixed stock of peasant capital to the total labor force and $k = (K_p + K_c + K_m)/N$, the aggregate capital-labor ratio. Assuming then that production functions are such as to permit three market equilibrium at and only at some point (k_p^*, k_c^*, k_m^*) , and we have seen this to be possible with Cobb-Douglas production functions, then a static three sector equilibrium exists if and only if

$$n_p^* = v/k_p^* < 1, \text{ and}$$

$$k_i^* < \frac{k-v}{1-n_p^*} < k_j^*, \quad i,j = c,m \text{ or } m,c.$$

Note that since the peasant capital market is not connected with the capitalist capital market, static equilibrium always includes peasants, i.e., $n_p^* > 0$. Furthermore static equilibrium will always be dualistic, since at least one capitalist sector will exist due to the fact that the capitalist capital stock is non-transferable to peasants and may be shifted freely only from one capitalist sector to another. Thus, the results of static factor market interaction must always lead to a situation where the three sector model applies or a two sector model of the type already discussed applies. Note that n_p^* is inversely proportional to k_p^* , so that the lower is the capital intensity of peasant agriculture in three sector equilibrium, the more likely is it that three sector equilibrium cannot exist. It is interesting to note, finally, that if some of the capitalist capital goods were specific to the sector in which they were initially employed, then in a static world, all three sectors would of necessity continue to function if production functions satisfied the Inada conditions or at least came near to satisfying them. Static three sector equilibrium, however, would still not exist unless the conditions outlined above held. In this case some factor markets would be in disequilibrium and/or there would be less than full employment of all factors.

A second point concerns the assumption of Cobb-Douglas production functions. This assumption is used above to show the

existence and uniqueness of the k_i associated with three market static equilibrium and in determining some of the properties of the k_i^* and i_i^* . The properties of the model as implied by equations (4.15) and (4.16) remain the same if generalized neoclassical production functions are used, provided that there exist unique k_i^* which satisfy the market equilibrium conditions. It is not readily apparent that three-market equilibrium values of the k_i will exist or be unique for any arbitrary choice of three neoclassical production functions. If no such k_i^* exist, one capitalist sector will disappear under static market conditions and the relevant dynamic model becomes two sectoral.

Finally, recall that the two labor markets and the capitalist capital market were assumed always to be in equilibrium in the dynamic model. In the two sector model there was only one market, so that its stability could be readily assured. When three markets exist, this is no longer the case. The assumption that they are always in equilibrium is thus a somewhat stronger assumption than in the two sector model. If three sector equilibrium exists but is unstable, then the system reverts to a two sector case immediately in the static phase.

The three sector dynamic model of a developing economy will be extended now in the following way: The domestic terms of trade between agriculture and non-agriculture will be assumed to be affected by domestic demand and supply. We shall assume that only agricultural output may be exported and that the terms of trade

between agricultural exports and imports are fixed exogenously and constant so that without loss of generality, they may be set equal to unity. Domestic non-agricultural production is assumed to be of an import replacing nature, not competitive on the world market, and to be protected by an import tariff, although the effects of government spending of the tariff receipts will be ignored. The price of domestic agricultural goods and imports (tariff included) in terms of domestic non-agricultural goods, p , is given by the following equation:

$$(4.19) \quad p = \frac{\hat{\sigma}_m q_m n_m}{\hat{\sigma}_c q_c n_c + \hat{\sigma}_p q_p n_p},$$

where $\hat{\sigma}_i$, $i=c,p,m$, is the proportion of non-agricultural (agricultural) output allocated to the purchase of imports and agricultural produce (domestically produced non-agricultural goods). Ideally, the $\hat{\sigma}_i$ should be treated as functions of p and the level of income, but they will be assumed constant here. Next, assume that $\sigma_m \leq 1$ of gross investment expenditure in the non-agricultural sector is allocated to the purchase of imports and that $\sigma_a \leq 1$ of gross investment expenditure in the agricultural sector is allocated to the purchase of domestically produced capital goods. It is assumed that the $\hat{\sigma}_i$ are large enough to include both intersectoral consumption and investment demands. Finally, retain the assumption that production functions are Cobb-Douglas, in fact, the same as in the earlier three sector model, and assume that depreciation occurs at the same rate, δ , in all three

sectors. The equations of the model may now be written down. Consider first the three equations describing the conditions of factor market equilibrium. These will be the same as in the earlier model; that is, equations (2.1'), (4.9), and (4.10) still apply except that p is now a variable instead of a parameter, and it appears on the opposite side of each equation due to its re-definition. Corresponding to this new variable is its definitional equation, equation (4.19). The full employment condition for labor remains the same; i.e., equation (4.12) still applies. It remains only to write down the two savings-investment equations for capitalists and peasants, respectively:

$$(4.20) \quad \vartheta_m (1-\beta) k_m^{1-\alpha} n_m [(\sigma_m/p) + (1-\sigma_m)] + \theta_c (1-\alpha) k_c^{1-\alpha} n_c [p\sigma_a + (1-\sigma_a)] \\ = \left(\frac{\dot{k}_m}{k_m} + \frac{\dot{n}_m}{n_m} + \eta\right) k_m n_m + \left(\frac{\dot{k}_c}{k_c} + \frac{\dot{n}_c}{n_c} + \eta\right) k_c n_c$$

$$(4.21) \quad \sigma_p k_p^{-\alpha} [p\sigma_a + (1-\sigma_a)] = \dot{k}_p/k_p + \dot{n}_p/n_p + \eta.$$

By algebraic manipulations of the equations listed above, the following system of equations may be obtained:

$$(a) \quad p = (\beta/\alpha) \cdot (\theta_m/\theta_c) \omega_2^{1-\beta} k_c^{\alpha-\beta}, \quad \omega_2 = \frac{\alpha}{1-\alpha} \cdot \frac{1-\beta}{3}$$

$$(b) \quad k_p = \omega_1 k_c, \quad \omega_1 = [\alpha\theta_c/\gamma(1-\sigma_p)]^{\frac{1}{1-\alpha}}$$

$$(c) \quad k_n = \omega_2 k_c$$

$$(d) \quad n_m = \frac{\phi_2 - (\phi_2 - \phi_1)n_p}{1 + \phi_2} \quad \left\{ \begin{array}{l} \phi_1 = (\hat{\sigma}_p / \hat{\sigma}_m) [\beta / \gamma (1 - \sigma_p)] \\ \phi_2 = (\hat{\sigma}_c / \hat{\sigma}_m) (\beta / \alpha) \end{array} \right.$$

$$(4.22) \quad (e) \quad n_c = [1 - (1 + \phi_1)n_p] / (1 + \phi_2)$$

$$(f) \quad \dot{n}_p = - \frac{1 + \phi_2}{1 + \omega_2 \sigma_2} \left\{ \left[\frac{\sigma_m}{1 - \alpha} (1 - \alpha) \theta_c - \sigma_p \omega_1^{-\alpha} \right] \omega_2 n_m + \left[(1 - \alpha) \theta_c - \sigma_p \omega_1^{-\alpha} \right] n_c \right. \\ \left. + (1 - \sigma_a) k_c^{-\alpha} + \left[\frac{1 - \sigma_m}{\sigma_a} (1 - \alpha) \theta_c - \sigma_p \omega_1^{-\alpha} \right] \frac{1 - \beta}{1 - \alpha} n_m + \left[(1 - \alpha) \theta_c - \sigma_p \omega_1^{-\alpha} \right] \right. \\ \left. + \frac{\beta}{\alpha} n_c \right\} \sigma_a \frac{\partial_m}{\partial_c} \omega_2^{1 - \beta} k_c^{-\beta} n_p$$

$$(g) \quad \dot{k}_c = \left(\frac{\dot{n}_p}{n_p} + \gamma \right) k_c + \left[\sigma_p \omega_1^{-\alpha} \frac{\beta}{\alpha} \frac{\partial_m}{\partial_c} \omega_2^{1 - \beta} k_c^{1 - \beta} \sigma_a + \sigma_p \omega_1^{-\alpha} (1 - \sigma_a) k_c^{1 - \alpha} \right]$$

Notice that equations (4.22) (f) and (g) could be written in the form,

$$\dot{n}_p = n_p(n_p, k_c)$$

$$\dot{k}_c = k_c(n_p, k_c)$$

by making simple substitutions for n_m and n_c in (f) and then for \dot{n}_p in (g), so that the behavior of the entire economy may be inferred from the behavior of n_p and k_c . Examining equation (4.22.f) more

closely it is readily seen that the sign of \dot{n}_p is determined by the sign of the large expression enclosed by brackets. It will be non-negative for all non-negative $n_i, k_i, i=c,p,m$, if $[(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}]$, $[\frac{\sigma_m}{1-\sigma_a} \cdot (1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}]$, and $[\frac{1-\sigma_m}{\sigma_a}(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}]$ are all non-negative, and it will be non-positive if all three expressions are non-positive. Note that these sufficiency conditions involve an expression which is already familiar, $[(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}]$, since ω_1 is equivalent to ω in the two sector model. There is obviously a strong presumption in favor of the idea capitalist versus peasant displacement will occur in the same direction as the two sector case. It is reassuring to note that parameters exert their influence in the same qualitative way as in the two sector case. Observe, however, that the term which pre-multiplies $(1-\alpha)\theta_c$ in the second expression is small when the term which pre-multiplies $(1-\alpha)\theta_c$ in the third expression is large, and vice versa. It can be shown that the net positive or negative effect of these two expressions on the sign of the entire large bracketed expression is

$$z = \omega_2 k_c^{-\alpha} \{ (1-\alpha)\theta_c [\sigma_m + p(1-\sigma_m)] - \sigma_p \omega_1^{-\alpha} [(1-\sigma_a) + p\sigma_a] \} n_m.$$

The relative importance of z is small if n_m is small relative to n_c . This is reasonable, since when n_m is relatively small, the system closely approximates the two sector case. If n_m is relatively large, the sizes of σ_m, σ_a , and p may play a significant role. Consider first three special cases where the size of p is irrelevant and where $\text{sign } z = \text{sign} [(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}]$. This will occur, for example,

if one of these three situations obtains: $\sigma_m=1, \sigma_a=0; \sigma_m=0, \sigma_a=1$:
 or $\sigma_m + \sigma_a = 1, \sigma_m$ and $\sigma_a \neq 1, 0$. The first case is one where all investment goods must be imported, or in the case of the agriculture sector only, may also be supplied internally to the sector. The second case is the situation where all investment goods are supplied by the domestic non-agricultural sector. In the third case, exactly σ_m of all investment goods are supplied by domestic non-agriculture and $(1-\sigma_m)$ of all non-agricultural investment goods are imported. Actually, it is quite possible to expect a developing country to come fairly close to one of these stylized patterns. For example it does not seem to be unreasonable to expect that, say, 80 percent of all capital goods in domestic non-agriculture must be imported and 20 percent of all capital goods used by agriculture are supplied by domestic industry. Furthermore, it is perhaps quite possible for $\sigma_m + \sigma_a$ to be near unity without actually being equal to it, provided that $p = p(k_c) = \bar{c}k_c^{\alpha-\beta}$, $\bar{c} = \text{constant}$, does not vary too widely, a result which depends in part on how close α is to β in magnitude.

Returning now to equation (4.22.d), observe that $\dot{n}_m > 0$, when $\dot{n}_p < 0$, if and only if $\phi_2 > \phi_1$, i.e., if $\hat{\sigma}_c / \hat{\sigma}_p > \alpha / \gamma(1-\sigma_p)$. The RHS of this inequality is less than one by hypothesis. If it is assumed that $\hat{\sigma}_c \geq \hat{\sigma}_p$, i.e., that the proportion of capitalist agricultural output exchanged for domestic non-agricultural output is at least as large as that exchanged of peasant output, (This seems reasonable since the investment ratio and consumption per worker are higher in capitalist agriculture.), then the inequality

is satisfied, $\phi_2 > \phi_1$ and, $\text{sign } \dot{n}_m = \text{sign } -\dot{n}_p$. From equation (4.22.e) it is readily apparent that $\text{sign } \dot{n}_c = \text{sign } -\dot{n}_p$. Thus $\text{sign } \dot{n}_c = \text{sign } \dot{n}_m = \text{sign } -\dot{n}_p$, and both capitalist sectors will grow or shrink together until the economy becomes dualistic, consisting either of capitalist non-agriculture and peasant agriculture (the case of $\dot{n}_p > 0$), or of an all-capitalist two sector economy (the case of $\dot{n}_p < 0$). When the two sector phase is entered, equation system (4.22) no longer applies, and again we must revert to a model of the earlier type to explain subsequent behavior. This extended version of the three sector model is in some sense intuitively more satisfying, since both capitalist sectors grow or shrink in relative size together, and when capitalism is viable, it is consistent with the existence of more than one capitalist sector due to the existence of intersectoral demands. Note that it is impossible for both capitalist sectors to disappear at once. This can be seen on inspection of equations (4.22.c and d). As n_p increases, n_m and n_p fall. Now for $n_p \leq 1$, $n_m \geq 0$; but if $1/(1+\phi_1) \leq n_p \leq 1$, $n_c \leq 0$, so that when n_p increases to $1/(1+\phi_1)$, $n_c = 0$, and the economy is two sectoral. Observe, finally, that if capitalist agriculture is displaced, it necessarily disappears altogether since n_p actually may take on the value $1/(1+\phi_1)$. If peasant agriculture is displaced, we are assured only that such displacement is relative; i.e., the peasant sector may continue to expand absolutely, although absolute displacement is possible. Even under conditions of absolute displacement, peasants disappear only virtually, since the peasant capital stock cannot be completely eliminated. It will decay toward zero over infinite time.

At any rate, if the stylized case ($\sigma_a + \sigma_m = 1$) applies exactly or perhaps approximately, then the prediction of the two sector model holds true in a qualitative sense for the three sector model. In situations where that case does not apply, then the role of p , the intersectoral terms of trade, is crucial. Consider the extreme cases $\sigma_a = \sigma_m = 0$ and $\sigma_a = \sigma_m = 1$: then z is given by

$$z = \omega_2 k_c^{-\alpha} [(1-\alpha)\theta_c p - \sigma_p \omega_1^{-\alpha}] n_m, \text{ and}$$

$$z = \omega_2 k_c^{-\alpha} [(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha} p] n_m, \text{ respectively.}$$

In the former case, where investment goods are supplied completely internally to each sector, or in the case of agriculture, also by imports, a high p favors displacement of peasants. But in the latter case, where all investment goods used by agriculture are supplied by non-agriculture, and all investment goods used by non-agriculture are imported, a high p favors displacement of capitalists. The prediction of the two sector model may be rendered invalid by the influence of domestic terms of trade.

It is not possible to say a great deal more without considering the total interaction of the economy implied by equation system (4.22). It is, however, possible to state that no trisectoral dynamic equilibrium can exist, i.e., that displacement must occur in a dynamic world. This can be demonstrated as follows: Let the stationary value of n_i be n_i^* . In order that the stationary point be trisectoral, it must

be true that $0 \leq n_p^* \leq 1/(1+\phi_1)$, in order that the constraints on the n_i be satisfied, ($\sum_i n_i = 1, n_i \geq 0$). Using equations (4.22.d and e) to eliminate n_p and n_c from equation (4.22.f), setting the LHS of this latter equation equal to zero, and solving for n_p^* , one obtains

$$(4.23) \quad n_p^* = \frac{\omega_2 \phi_2 - \psi(p^*)}{\omega_2(\phi_2 - \phi_1) - \psi(p^*)(1 + \phi_1)},$$

where

$$\psi(p^*) = - \frac{(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}}{(1-\alpha)\theta_c \left[\frac{\sigma_m + p^*(1-\sigma_m)}{(1-\sigma_a) + p^*\sigma_a} \right] - \sigma_p \omega_1^{-\alpha}} = \frac{\omega_2 n_c^*}{n_c^*}.$$

Observe that $\psi(p^*)$ must be non-negative if n_m^* and n_c^* are to be non-negative. Now substitute the RHS of equation (4.23) for n_p^* in the inequality restriction, so the question becomes

$$0 \leq \frac{\omega_2 \phi_2 - \psi(p^*)}{\omega_2(\phi_2 - \phi_1) - \psi(p^*)(1 + \phi_1)} \leq \frac{1}{1 + \phi_1}.$$

If $\omega_2(\phi_2 - \phi_1) > \psi(p^*)(1 + \phi_1)$, we may write

$$0 \leq \omega_2 \phi_2 - \psi(p^*) \leq \frac{\omega_2(\phi_2 - \phi_1)}{(1 + \phi_1)} - \psi(p^*)$$

or

$$0 \leq \phi_2 \phi_1 \leq -\phi_1.$$

Since ϕ_1 and ϕ_2 are positive, the right hand inequality cannot be

satisfied. If $\omega_2(\phi_2 - \phi_1) < \psi(p^*)$, we may write

$$0 \stackrel{?}{\geq} \omega_2 \phi_2 - \psi(p^*) \stackrel{?}{\geq} \frac{\omega_2(\sigma_2 - \sigma_1)}{1 + \phi_1} - \psi(p^*)$$

$$\text{or } 0 \stackrel{?}{\geq} \phi_2 \phi_1 \stackrel{?}{\geq} -\phi_1.$$

Since ϕ_1 and ϕ_2 are positive, the left hand inequality cannot be satisfied. Finally, if $\omega_2(\phi_2 - \phi_1) = \psi(p^*)(1 + \phi_1)$, it is also obvious that the original inequality restriction cannot be satisfied. Thus there can exist no trisectoral dynamic equilibrium in which $\sum \bar{n}_i^* = 1$ and $\bar{n}_i^* \geq 0$. Now this conclusion is quite independent of the magnitude of $\psi(p^*)$. This means that for any $p = p(k_c)$ it is impossible to observe a stationary point, $(\bar{n}_c, \bar{n}_p, \bar{n}_m)$, such that $\sum \bar{n}_i = 1$, and each $\bar{n}_i > 0$. That is, as long as three sectors exist, it is impossible to observe a change in the direction of displacement, since if at one phase $\dot{n}_p < 0$ and $\dot{n}_c, \dot{n}_m > 0$, then the only way to observe a later phase of $\dot{n}_p > 0$ and $\dot{n}_c, \dot{n}_m < 0$ is if there exists a point $\dot{n}_p = \dot{n}_c = \dot{n}_m = 0$, due to the fact that $\text{sign } \dot{n}_m = \text{sign } \dot{n}_c = \text{sign } -\dot{n}_p$. Of course, the same reasoning applies to $\overset{a}{\wedge}$ shift between phases in the reverse order. Since it is impossible to observe a point $\dot{n}_p = \dot{n}_c = \dot{n}_m = 0$ in the three sector phase, it is clear that displacement must always occur in one direction or the other.

Consider now one of the extreme cases in which p makes a difference. Suppose that $\sigma_m = \sigma_a = 0$. This is the case where the agricultural sector is self-sufficient in capital goods as is also

the non-agricultural sector. This could be interpreted as a case where each sector produces its own capital goods, or in the case of agriculture, imports. [If both sectors could import, the external terms of trade would determine the internal terms of trade (tariff modified), as in the version of the model discussed earlier.] The differential equation for n_p in this case is:

$$(4.24) \quad \frac{\dot{n}_p}{n_p} = - \frac{(1+\phi_2)}{1+\omega_2\phi_2} n_c \{ \omega_2 [(1-\alpha)\theta_c p - \sigma_p \omega_1^{-\alpha}] \frac{n_m}{n_c} + [(1-\alpha)\theta_c - \sigma_p \omega_1^{-\alpha}] k_c^{-\alpha} \}$$

If $(1-\alpha)\theta_c > \sigma_p \omega_1^{-\alpha}$, then at $t=0$, p must be such as to make $[(1-\alpha)\theta_c p - \sigma_p \omega_1^{-\alpha}]$ algebraically small enough, given n_m and n_c , in order that \dot{n}_p be positive at $t=0$.

Now by the conclusions on the necessity and irreversibility of displacement, the direction of displacement can be determined by examining the initial situation, i.e., by examining the properties of the static equilibrium at time $t=0$. The static equilibrium of the system at $t=0$ would be given by the solution to system (4.22) where equations (f) and (g) have been replaced by the following static constraints on the employment of factors:

$$k_p n_p = v_p = K_p(0)/N(0)$$

$$k_c n_c + k_m n_m = v_c = [K_c(0) + K_m(0)]/N(0).$$

Given the static equilibrium solution, the values of the relevant variables may be substituted into equation (4.22.f) and the direction of \dot{n}_p determined. Let us then examine some of the properties of static equilibrium. By eliminating all variables except k_c , it is found that

$$k_c(0) = v_c + v_p/\omega_1.$$

From this it follows immediately that

$$0 < n_p(0) = [1 + \omega_1(v_c/v_p)]^{-1} < 1$$

$$\text{and } p(0) = \bar{c}[v_c + (v_p/\omega_1)]^{\alpha-\beta}.$$

In order that static equilibrium be trisectoral it must be true that $n_p(0) < (1+\phi_1)^{-1}$, as indicated earlier. If $n_p(0) \geq (1+\phi_1)^{-1}$, the capitalist agricultural sector must disappear as static equilibrium is approached. (The question of stability of equilibrium is hedged. We simply assume that there exist institutions which insure stability.) Substituting for $n_p(0)$ its equivalent collection of parameters, we have the following condition for the static viability of capitalist agriculture:

$$\omega_1 v_c / v_p > \phi_1,$$

which becomes

$$\left(\frac{v_c}{v_p}\right)^{1-\alpha} > \left(\beta \frac{\hat{\sigma}_p}{\hat{\sigma}_m}\right)^{1-\alpha} \left[\frac{\gamma(1-\sigma_p)}{\alpha\theta_c}\right]^\alpha$$

in terms of the initial parameters. It is not likely that the RHS of this expression is far different from unity since $\beta\hat{\sigma}_p/\hat{\sigma}_m$ is probably a bit less than one, while $\gamma(1-\sigma_p)/\alpha\theta_c$ is probably a bit greater than one, so that if the size of the combined capitalist capital stocks exceeds that of peasants by, say, a factor of two or more, there is a good bet that trisectoral static equilibrium is possible. In very backward countries where v_c is possibly quite small relative to v_p , it would be unusual to expect to observe capitalist agriculture.

Referring now to the case $\sigma_m = \sigma_a = 0$, and making the full substitution of parameters and initial values in equation (4.24), it turns out that for this case

$$\text{sign } \dot{n}_p \Big|_{(t=0)} = \text{sign} \left\{ \frac{(1-\alpha)\theta_c}{\sigma_p \omega_1^{-\alpha}} - \frac{1 + \frac{\omega_1 v_c - \phi_1 v_p}{\omega_2 (\phi_2 \omega_1 v_c + \phi_1 v_p)}}{\bar{c} \left(v_c + \frac{v_p}{\omega_1} \right)^{\alpha-\beta} + \frac{\omega_1 v_c - \phi_1 v_p}{\omega_2 (\phi_2 \omega_1 v_c + \phi_1 v_p)}} \right\}$$

The sign of \dot{n}_p is uniquely determined by the values of the parameters and the initial endowments of capital and labor. (Similar results would hold for other cases, e.g., $\sigma_m = \sigma_a = 1$.) The reader may verify that at plausible levels for the initial values and parameters, the second term in the expression on the RHS of the above equation may

be significantly different from unity. It is necessary to conclude then that the two sector displacement condition may be carried over to the three sector economy only under special circumstances.

Although many possible extensions, alternations, and elaborations of the models already discussed are possible, we shall stop formal analysis at this point and attempt to draw a few generalizations from the results obtained thus far. Above all, it should be acknowledged that it is very difficult to generalize about the relative viabilities of peasant and capitalist agriculture. In a world of imperfections and non-profit-maximizing factor allocations (in the case of peasants), there is no solid case in favor of the profit maximizer (capitalists) or the technologically superior. Furthermore, there is no reason to expect natural displacement to function so as to maximize, in some sense, community welfare. It is frequently likely that some sort of government intervention into the interplay of markets will be necessary in order to insure favorable patterns of growth in the agricultural sector. It appears that the case of peasant versus capitalist in any real situation must be assessed without a precommitment to standard answers. There may be reasons for promoting the existence of peasant agriculture, given the appropriate milieu of parameter values and imperfections, although the preceding analysis suggests that the cards may be stacked against the long-run desirability of their existence if not against their long-run viability.

CHAPTER V

THE MALAYAN RUBBER INDUSTRY, AN EXAMPLE OF DUALISTIC AGRICULTURE

In this final chapter we shall turn from theoretical issues to examine a concrete case of agricultural dualism, the rubber producing industry of Malaya.¹ The analysis will proceed according to the following plan: the question of the existence of dualism in an economically meaningful sense will be explored. Next, the role of factor market imperfections in the creation of dualism in Malaya will be scrutinized. Then we shall turn to the matters of savings, investment, and technological change in the two rubber producing subsectors. Finally, based on the factual evidence and on the earlier theoretical investigations, some policy implications for Malayan rubber growing will be suggested.

Around two-thirds of all cultivated acreage in Malaya is devoted to rubber, and about 60 percent of agricultural employment is concentrated in that industry. Rubber production accounts for almost one-third of total Malayan employment and has been responsible for one-sixth to one-third of the total GNP of Malaya in the postwar period (1947--present), depending on the market price of rubber. Rubber exports have accounted for from slightly less than half to almost two-thirds of total Malayan exports, again depending on prices.² It is clear that what happens in the rubber industry will have a great influence on the Malayan economy. In fact, Lim Chong-Yah has shown that every spurt of growth experienced

by the Malayan economy since World War Two has been due mainly to changes in export earnings, rubber being the dominant factor in exports.³

Not only is the Malayan rubber industry of importance domestically, but also it holds a significant position with respect to total world supply. Table V.1 shows that Malaya's output has continued to grow during the postwar period and continues to account for a significant share of both total natural and synthetic production as well as of just natural rubber production, although it is clear that the rapid growth of synthetic rubber production and consumption after World War Two has reduced Malaya's individual dominance of world rubber supply. Most forecasts of Malaya's share in world rubber output predict a decline, although it is also accepted that the absolute quantity of Malayan production and exports will rise.⁴ (Production and exports are virtually identical.)

Table V.1

Malayan and World Rubber Production Compared

| Year | Malayan production long tons x10 ⁶ | Malayan % of world natural rubber production | Malayan % of total world rubber production |
|------|--|---|---|
| 1940 | .55 | 48 | 48 |
| 1950 | .75 | 75 | 30 |
| 1965 | .90 | 39 | 17 |

Source: Natural Rubber News, Feb. 1967, Table I, p. 2.

The crucial factors in the future viability of Malayan rubber are the relative costs of production of natural and synthetic rubber and the degree of technical substitutability between them. Currently, substitutability is not perfect, but indications are that continued technological progress will increase substitutability and hence the degree of economic competition between synthetic and natural rubber. As matters now stand, there is a high degree of price competition between synthetic and natural rubber in many uses, and as a result, the price of natural rubber is kept down by the fact that synthetic rubber is, given existing facilities, producible at a fairly constant marginal cost and in greater quantities than are now brought forth. The future appears to hold forth the prospect of lower natural rubber prices due to improved substitutability and secular downward shifts in the marginal cost curves of synthetic rubber producing firms. Notice in Table V.2 the steady decline in rubber prices since 1960. The price of 49.63¢/lb. in 1967 is the lowest since 1949 and is 15.73¢/lb. lower than the lowest annual average price over the period 1950-67. Malayan rubber production will thus remain economic only if it can continue to offset declining prices with improved technology and efficiency. We shall investigate these factors as they relate to the existence of dualism below. But let us examine first the structure of Malaya's rubber industry in greater detail.

The first matter of importance is to determine whether in fact we may properly speak of rubber production in Malaya as being dualistic. It would be thus useful to clarify at the outset what is meant by dualism.

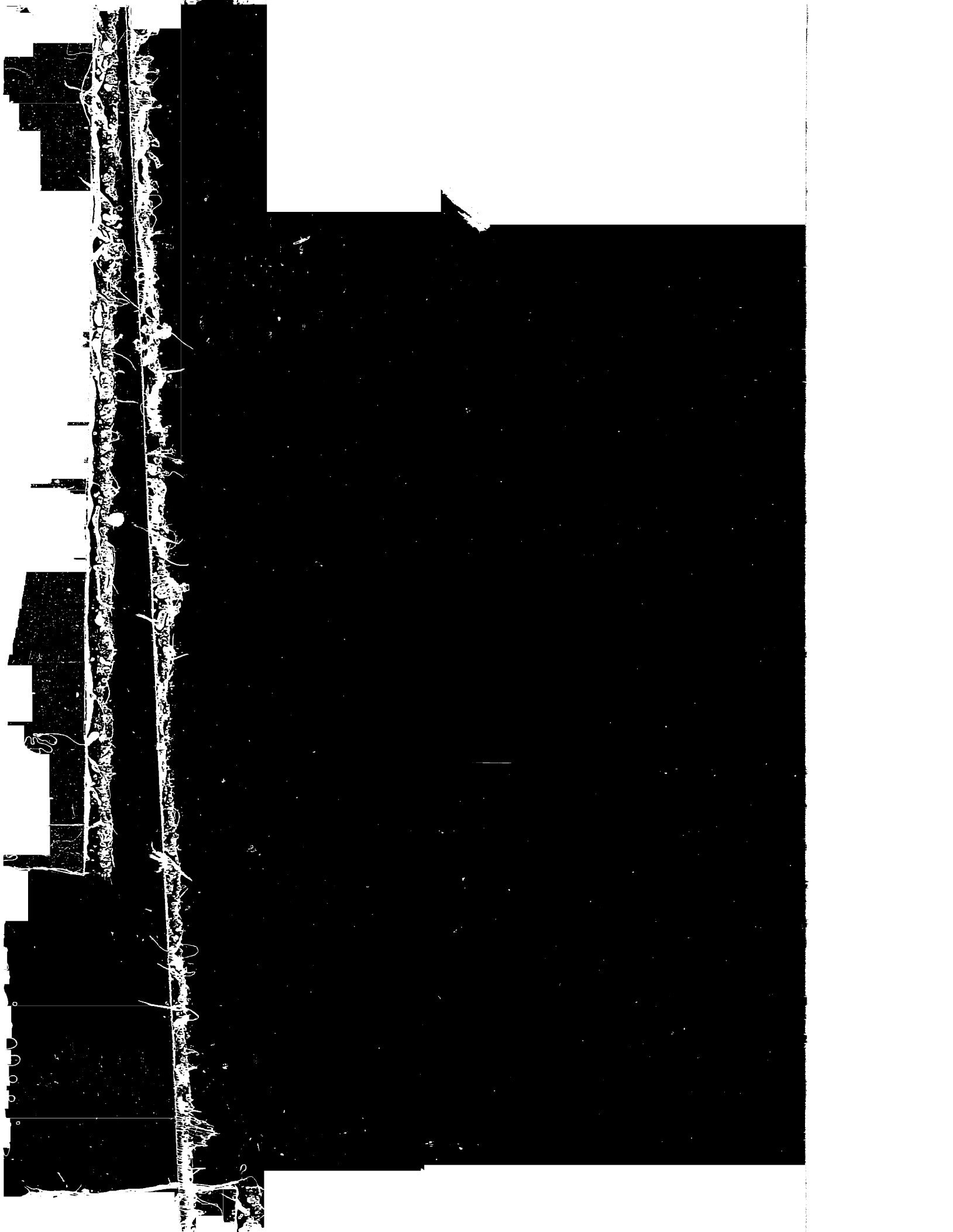
Table V.2

Natural Rubber Prices (Annual Averages 1958-67)
 (Malayan ¢/lb.; RSS #1 grade; f.o.b. in bales,
 Singapore; wholesale)

| Year | Price | Year | Price |
|------|--------|-------|-------|
| 1958 | 80.25 | 1963 | 72.42 |
| 1959 | 101.56 | 1964 | 68.14 |
| 1960 | 108.08 | 1965 | 70.02 |
| 1961 | 83.54 | 1966 | 65.36 |
| 1962 | 78.20 | 1967* | 49.63 |

* Month of December only. In 1967 there was a fairly steady decline of rubber prices from 59.19 in January to 49.63 in December. Source: Rubber Statistics Handbook, various years, Mo. Stat. B. of W. M., Jan., 1968.

Generally the term is used to refer to the production of some commodity or group of commodities by two distinct groups of productive units, each using a different combination of productive inputs. We have seen in earlier chapters how this may be possible when factor market imperfections alter the effective relative factor prices facing the two groups, and we have seen that this may be the case when the technologies of the two groups are identical or different. Of interest is the following question: If technologies were identical and neoclassical, and factor markets were perfect, so that factors of production were allocated in the same proportions by each group, is there any economic meaning to the continued use of the term dualism beyond that of referring to two different institutional forms of essentially the same kind of economic unit? The answer to this question is in the affirmative if the



ior of the two groups differs, as the welfare has shown. This, of course, is not dualism of the word, but it would seem to be of im- f the traditional dualism based on static factor cordingly, in this chapter, we shall speak of 1 which there exist two groups of productive ocates the factors of production in the static which manifests distinct savings-investment

at least, rubber production in Malaya has been classes of productive units, the smallholders s, and the estates. P. T. Bauer in his classic try wrote in 1947:

tion between estates and smallholdings est importance since the earliest days The estates are large or at least of several hundred or several thousand ted with substantial capital and employ- orces in receipt of a daily wage. The mallholding acreage is in the hands of rs, each with a holding of, say, two to ork with family labor, occasionally be- outside workers paid on a share basis. oducing territories [Malaya] much of ially classed as smallholdings is in f 15 - 100 acres each, usually tapped outside labor, either on a share basis, piece rates. . . A greater part of the by absentees. . . Even when the small- m holdings rely on outside labor, their reciably less than that of the estates nfining to tapping only."

, especially the European owned es-
 r from smallholdings and medium
 doption of an elaborate [manage-
 or the production of rubber . . ."5

e shall examine some of the statistical evi-
 er's observations. In 1947, smallholders ac-
 Malaya's total output of 695,000 long tons.
 approximately 46% of a total of 928,000 long
 During this period, the share of smallholders
 From 1947 to 1965 the acreage under cultivation
 m about 1,580,000 to 2,400,000 acres, while
 y estates declined slightly from 1,950,000
 00 acres in 1965, after reaching a peak of
 (As will be seen later, much of the increase
 ar period is traceable to the introduction of
 trees.) The labor force in the estate sector
 d 290,000 in 1947 to 270,000 in 1965. Similar
 are not possible in the smallholders' sector.
 ' labor force was found to be about 361,000
 s. Since the total agricultural labor force
 tant from 1947 to 1957 and the all estate
 labor force remained fairly constant over this
 e to assume that the smallholder labor force
 well, since there is no indication of a switch
 een rice and rubber cultivation, rice growing

Table V.3

Acreage* and Production of Rubber, Malaya, 1955

| Unit | No. of holdings or estates# | % of total planted acreage | % of total production |
|---|--------------------------------|-------------------------------|--------------------------|
| Smallholdings: less than 25 acres | 385,208 | 37 | 33 |
| 25 - 99.9 acres | <u>7,274</u> | 9 | 7 |
| Estates: | | | |
| 100 - 999 acres | 1,186 | 14 | 12 |
| 1000 acres and up | <u>603</u> | <u>40</u> | <u>48</u> |
| Total | ----- | 100 | 100 |

Source: Federation of Malaya, Report of the Mission of Inquiry into the Rubber Industry of Malaya, p. 4.

* Peasant acreages are based on area recorded as alienated for rubber, and there was no check to see if such land was actually planted.

The number of smallholdings refers to the number of plots. One smallholder may have several plots.

The above statistics on smallholdings relate to separate plots, more than one of which may belong to a single owner or operator. However, according to the 1960 agricultural census, the average size of a smallholder rubber farm is 6.3 acres, indicating that, although many smallholders may have more than one plot, nevertheless the average size of the combined holding remains small compared to estates.⁷

Table V.4

Size Distribution of Smallholdings

| Size Group | Total Acreage | Total Number of Holdings | Acreage per Holding |
|------------|---------------|--------------------------|---------------------|
| 0 - 24.9 | 1,138,100 | 299,900 | 3.8 |
| 25 - 99.9 | 231,700 | 5,400 | 42.9 |
| 0 - 99.9 | 1,369,800 | 305,300 | 4.5 |

Source: Lim Chong-Yah, The Economic Development of Modern Malaya, Appendix 4.6, p. 332.

Based on these results, it is clear that there is a fairly sharp institutional distinction based on size of acreage between the smallholdings and estates, but we are interested in looking deeper, for example, at differences in factor allocations and savings-investment behavior. Sector-wide data on acreage, employment, and output will give some indication as to differences in allocative behavior. Unfortunately, the only postwar years for which sufficient data are available are 1957, a census year, and 1962. Consequently, we shall confine most of our analysis of aggregates to those years. Table V.5 summarizes the relevant

data. From these data three ratios have been computed for estates and smallholdings, respectively, and for each of these two years, respectively. These ratios are displayed in Table V.6.

Table V.5

Malaya: Output, Acreage, and Labor Force
in the Rubber Industry, 1957 and 1962

| Group and Year | Output (long tons x 10 ³) | Acreage (acres x 10 ³) | Labor Force (persons x 10 ³) |
|----------------|--|---------------------------------------|---|
| 1957: | | | |
| Smallholders | 269 | 1,710 | 361 |
| Estates | 369 | 2,020 | 283 |
| 1962: | | | |
| Smallholders | 312 | 2,060 | 457 |
| Estates | 439 | 1,930 | 286 |

Sources: Lim Chong-Yah, The Economic Development of Modern Malaya, Table 7.15, Appendix 4.3; Federation of Malaya, Annual Report, 1960; Pierre Crosson, Economic Growth in Malaysia, Table 2.18.

Table V.6

Malaya: Land and Labor Productivity and Land Intensity
in the Rubber Industry, 1957 and 1962

| Item and Group | 1957 | 1962 |
|---------------------------------------|------|------|
| Output/Labor Force (long tons/worker) | | |
| Smallholders | .74 | .68 |
| Estates | 1.30 | 1.54 |
| Output/Acreage (long tons/acre) | | |
| Smallholders | .16 | .15 |
| Estates | .18 | .23 |
| Acreage/Labor Force (acres/worker) | | |
| Smallholders | 4.7 | 4.5 |
| Estates | 7.4 | 6.8 |

Source: Table V.5.

From examination of Table V.6 it is readily apparent that the techniques or conditions of production differ substantially as between sectors. The land-labor ratio in the estate sector is approximately 1 1/2 times that in the smallholders sector in both years. Notice also that output per worker in the estate sector is roughly twice that in the smallholders sector. If land and labor were the only factors of production and diminishing returns and constant returns to scale obtained, we would expect on the basis of these observations to find that output per acre of land would be substantially lower in the estate sector. As Table V.6 shows, such is not the case. Output per acre is actually greater in the estate sector. The main reason for such a result is that certain factors of production have been ignored. The productivity of a stand of rubber is greatly dependent upon the age of the trees and on the type of trees growing on that stand.

The relationship between age, type of tree and yield per acre is shown in Table V.7. The estimates contained in that table are based on the type of planting materials available in 1954 and assume "average" infestation, tapping skills, and fertilizer use. According to the Mission of Inquiry which published the data on which Table V.7 is based, the figures may apply to smallholdings as well as estates, since the densities of planting are not very different.⁸ It would seem that the Mission assumes equivalent labor inputs for the two sectors, although it is clear that smallholders production is relatively more labor intensive. Unfortunately, lack of time and data make it difficult to test statistically

whether the extra labor on smallholders' farms is redundant. However, as we shall see later, this is a possibility.

Turning now to the table, notice that high-yielding trees are over twice as productive at their peak as ordinary trees and that trees from 11 to 25 years old are substantially more productive than younger or older trees. Notice also that no yield figures are given for trees less than seven years old. This is due to the fact that a rubber tree requires from six to seven years to mature to the point where it may be tapped. Henceforth all acreage seven years old or older will be called mature, and all acreage consisting of trees which are less than seven years old will be called immature.

Table V.7

Estimates of Yields of Estate Rubber as
Dependent upon the Type of Planting Material and
the Age of Trees (lb. of rubber/tapped acre):

| Years since Planting | Ordinary Trees | Improved or High Yielding Trees |
|-------------------------|-------------------|---------------------------------------|
| 7 | 220 | 430 |
| 9 | 415 | 760 |
| 11 | 485 | 1000 |
| 13 | 535 | 1100 |
| 15 | 545 | 1190 |
| 17 | 540 | 1190 |
| 21 | 470 | 1115 |
| 25 | 420 | 1000 |
| 29 | 365 | 875 |
| 33* | 315 | 750 |
| 49* | 205 | 355 |

Note: *The figures for 33 and 49 years since planting are subject to large errors.

Source: Report of the Mission of Inquiry, Federation of Malaya, Appendix B, pp. 66-7.

In order to compare output per acre meaningfully, it is necessary to consider the age and quality of the rubber trees, and it follows that the land productivity figures given in Table V.6 are somewhat misleading. In order to sort out the effects of age and quality, let us examine the case of 1964. At the end of 1964 smallholders' acreage totalled $2,210 \times 10^3$ acres, and the estate acreage was $1,889 \times 10^3$ acres. The breakdown of estate acreage was as follows (acres $\times 10^3$):

| Mature Acreage | |
|----------------|----------|
| High Yielding | Ordinary |
| 909 | 451 |

| Immature Acreage | |
|------------------|----------|
| High Yielding | Ordinary |
| 511 | 28 |

The acreage of smallholders is not available in exactly the same breakdown, but the following information is obtainable (acres $\times 10^3$):

High Yielding Acreage

1,030

Immature Acreage

852

Since virtually all immature acreage is high yielding, it follows that only about 178×10^3 acres consisted of mature high yielding trees.⁹

Since immature trees are not tapped in any significant quantity, it is more useful to consider output per mature acre rather than output per acre. Output in 1964 consisted of 478×10^3 long tons produced by estates and 347×10^3 long tons produced by small holders. Table V.8 shows the calculated output/acre and output/mature acre for smallholders and estates, respectively. The figure of .35 long tons/mature acre on estates corresponds well with the figure of .36 long tons/acre tapped

Table V.8

Land Productivity* Based on
Total and Mature Acreage:
1964

| | Estates | Smallholdings |
|--------------------|---------|---------------|
| Output/Acre | .25 | .16 |
| Output/Mature Acre | .35 | .26 |

* Long tons/acre

Source: Computations from text.

on estates given in the Rubber Statistics Handbook.¹⁰ Also according to that publication, the estate yields per tapped acre on pure high-yielding stands in 1964 were .45 long tons/acre and on pure low-yielding stands were .19 long tons/acre. Now an interesting test to make is the following: Suppose that smallholders obtained the same yields as estates on holdings of equal yield quality in 1964. If so, then the difference in actual smallholder yield per mature acre and the hypothetical smallholder yield per mature acre will be zero if yields were in fact the same as between smallholders and estates, the only effect on aggregate yield being the difference in the proportion of mature acreage consisting of high-yielding trees. Since we have assumed that all immature peasant acreage was high yielding, it follows that only 13 percent of mature acreage in 1964 was high yielding. Using this proportion, we find that if smallholder yields were identical to those of estates, the aggregate output/mature acre should be about .23 long tons/mature acre. Since the actual figure was .26, we must conclude that either smallholders achieved higher yields per tapped acre, given the same quality trees as the estates, or there are erroneous elements in the manipulation or interpretation of the statistics.

If we assume that the data and use of them are correct, then one plausible explanation of higher smallholder yields per acre on equivalent quality holdings is the apparent greater labor intensity of production in the smallholders sector. However, there are, in fact, two possible sources of error in the use of the data. It was assumed that 100 percent

of the immature rubber smallholdings were in high yielding trees. Although it is certain that the vast preponderance of immature stands are high yielding, due to the fact that they must be of such quality in order to qualify for government planting subsidies, it is possible that a not insignificant proportion of immature stands may consist of ordinary yielding rubber trees. It is known that about 20 percent of the re- or newly planted acreage on estates during the period 1958-64 consisted of ordinary trees.¹¹ If it is assumed that peasants likewise planted 20 percent with ordinary trees, then it follows that about 26 percent rather than 13 percent of the mature smallholder acreage in 1964 consisted of high yielding material. If the hypothetical yield per mature acre using average estate yields on high and ordinary yielding stands is again calculated, the result is .26 long tons per acre, exactly the figure obtained from the direct calculations. Thus, it is quite possible that the difference between the aggregate yields of estates and smallholdings may be explained entirely in terms of total acreage and the proportion of acreage under high yielding trees.

This last result suggests the possibility that some smallholder workers may be redundant, since the greater labor intensity of smallholder production produces no greater output per acre when tree quality is the same as that of estates. Intuitive support for this argument is found in the fact that there is a maximum intensity of tapping which is consistent with continued tree productivity. Should the rate of tapping exceed this maximum, the trees will soon dry out. When an old stand is

about to be removed, then such "slaughter tapping" is economic, but only then, since there is a very poor trade-off between the alternatives of slaughter tapping, replanting, waiting, and finally tapping again, and of continuous tapping at lower intensity when such a trade-off concerns any but old trees. Since the only other important labor activities prior to the processing stage are weeding and field maintenance and collecting and transporting, it is clear that there will exist fairly rapid diminishing returns to extra labor applied to a given stand of rubber, and it is possible that, given the extra workers per unit of land, the smallholder sector may be operating at a much lower marginal productivity level than the estate sector.

The above argument requires solid empirical testing, which we have not time to get into. We shall remain content with presenting certain additional facts which affect the labor redundancy hypothesis. In a study of estate acreages which had been converted into smallholdings during the period 1951-60, it was found that there was less specialization of labor after subdivision. For example, tappers were expected to do some amount of weeding, and there were virtually no full-time field workers.¹² It is possible that this reduction in specialization also resulted in a loss of worker efficiency so that the effective amount of labor applied to a given piece of land by a given number of workers is reduced under smallholder organization, as compared to estate organization.

In a study conducted by Robert Ho, it was found that smallholders would be hard pressed to apply sufficient labor to obtain optimum economic

Continuing our investigation of static resource allocation, one further area of interest is the allocation of resources to management and processing activities. P. T. Bauer, in arguing the case for the smallholder efficiency, pointed out that peasants were able to obtain yields per mature acre equal or superior to those of estates on stands of equivalent quality trees with a lower stock of capital goods (other than trees and land) and without an elaborate management hierarchy such as is found on estates. He wrote:

"There is ... a ... contrast between the amount of equipment and materials absorbed by estate and smallholders' production, respectively. Estate requirements which are not needed on smallholdings include buildings for the housing of the labor force and the staff as well as various sheds which are sometimes elaborate; cars, lorries, and petrol; sheeting batteries for the production of sheet rubber, and small engines to operate them; lighting equipment for staff quarters; and many other items." ¹⁴

The absence of a management hierarchy on the smallholdings is obvious, and their lower capital intensity is confirmed by the experience of estate subdivision over the period 1950-61, during which about 190,000 acres of estate lands were subdivided. It was found that substantial proportions of the fixed capital on estates were allowed to deteriorate. For example, 28 percent of the factories and 50 percent of the smoke-houses (both relating to processing) were not in use and 70 percent of the roads were not being maintained.¹⁵ Most of the fixed capital involved represented investment either in management facilities, processin

and transportation facilities, or social overhead facilities such as schools, roads and hospitals. It is clear that the loss of social overhead facilities is a cost of smallholder organization unless it would be possible to obtain equivalent facilities through government taxation of the smallholders. In that case, the only difference would be that the financing of such investment was external rather than internal to the firm, provided that output and disposable incomes (after deductions for social overhead investment) were unchanged. The management hierarchy and associated facilities on estates may represent a cost of estate organization, a kind of diseconomy of scale, which might be offset by the (hypothesized) superior division of labor on estates. The economics of such a trade-off are not clear.

In regard to processing, it appears that estates may enjoy certain advantages. Their size permits the operation of efficiently sized mechanized facilities for processing, whereas most smallholders use simpler and less effective processing techniques or use the processing facilities of nearby estates or government-sponsored cooperatives. Cooperatives utilized by groups of smallholders would seem to be capable of producing high-grade rubber with as much efficiency as estate processing facilities, but currently most smallholders' rubber is of a lower grade than that of estates and is of less uniformity. Whereas 90 percent of estate output consists of grade I rubber, the preponderance of smallholders' output is of grades II through IV. How much emphasis is to be placed on superior processing and grading is not clear. During

by non-Asian-owned estates which have easy access to international capital markets as well as to the domestic capital market. Over 50 percent of the estate acreage is under the management of public corporations. The remainder are either private limited liability companies or are under simple private ownership.¹⁷ The estate sector is substantially unionized, and essentially all labor is hired under fixed wage agreements (as opposed to output sharing arrangements). The pattern which emerges is similar to modern capitalist industries all over the world.

Smallholders rubber is predominantly of the owner-operator type, as indicated by the 1960 Census of Agriculture. Table V.10 classifies smallholdings according to type of tenure of the principal operator and indicates the average size of the holdings of each tenure class. R. D. Hill has shown, however, that the high proportion of ownership indicated by the census may be a distortion: As many as 84,900 smallholding owners

Table V.10

Size and Operator Classification of Smallholdings

| Operator class owner | % of total | Average size of farm (acres) |
|------------------------|------------|---------------------------------|
| Owner | 80.1 | 6.17 |
| Temporary occupation) | | |
| Licencee) | 1.8 | 7.23 |
| Tenant) | 0.0 | --- |
| Other single tenure | 5.1 | 6.05 |
| Owner-tenant | 1.2 | 4.91 |
| Other mixed tenure | 11.8 | 7.07 |
| All classes | 100.0 | 6.30 |

Source: R. D. Hill, "Agricultural Land Tenure in West Malaysia," Malayan Economic Review (XII:1), April 1967, Tables 2 and 3, pp. 101, 102.

resident in towns were excluded from the census, and at least a fraction of these would have virtual tenants in the form of share-tappers on their land. Hill points out that detailed studies of certain areas reveal sometimes much higher percentages of nonowner-operators, and he concludes that tenancy both per se and in the form of share tapping is "far more widespread than the census data would suggest."¹⁸

Most smallholders use predominantly family labor, but there is some hiring, and the following description of smallholders' hiring practices is rather enlightening:

"One-third of all rubber farms surveyed in the 1960 Census of Agriculture reported using ... [hired] ... labor, whether on a full or on a part time basis. However, 82 percent of these farms paid for their additional labor not in cash but in kind, usually by means of a share cropping system known in Malaya as bagi dua (two shares). In rubber holdings, the system may take the form of sharing all work tasks and resulting incomes between two or more persons. A more common alternative may involve the actual partitioning of the holding, which is then worked as two or more separate units by the owner and his tenants, in return for a fixed proportion of their rubber output. It effectively circumvents regulations against land fragmentation, preserving the legal semblance of a property as an entity while in effect dividing it."¹⁹

The capital market relevant to smallholders is much different than that relevant to the estates which have access to all major domestic credit facilities such as the commercial banks and even foreign capital (especially sterling) sources. The principal source of capital to the smallholder is the rural moneylender. The cost of such capital financing tends to be high due to risk, the small scale of lending

operations, and possible monopolistic elements. It is also thought that the elasticity of supply of capital is rather low with respect to the interest rate.²⁰ Contrast this situation with the following statement concerning corporate funds: "Corporate funds are highly mobile between Malaya and the Sterling countries. Domestic and Sterling securities or company shares circulate with equal standing between Malaya and certain other Sterling countries."²¹

The difference in the methods and institutions of capital financing suggests that there are substantially higher costs of capital to smallholders. This contention is further supported by the fact that estates were able to finance replanting and new planting investment entirely from internal sources in the postwar period, whereas smallholders were unable to undertake such investment at a significant level until the imposition of government subsidy programs. (We shall discuss this aspect of the situation in greater detail later when the whole matter of saving and investment is examined more closely.) A lack of data precludes any conclusive comparisons of rates of return; however sufficient information exists to provide a basis for speculation. A study of 24 rubber producing corporations for the period 1954-58 indicated an average annual net profit after taxes of 11% on the average current value of all assets.²² The relevant data are summarized in Table V.11.

smallholdings produced 503 lb/acre. Although no figures were presented, the report stated that the cost pattern, in terms of the total cost per acre and of the proportions of total cost assigned to various categories, of the high yielding smallholdings was similar to the cost pattern of high yielding estates.

Table V.12

Yields, Gross Incomes, Costs, and Profits per Acre
on High Yielding Smallholdings in 1963 and on
Low Yielding Smallholdings in 1963-64

| Item | High Yielding Smallholdings | Low Yielding Smallholdings |
|---------------------------------|--------------------------------|-------------------------------|
| Production (lb./acre) | 1,064 | 503 |
| Gross Income (\$/acre) | 588.5 | 260.6 |
| Costs (\$/acre) | | |
| Tapping and collecting labor | 187.2 | 134.5 |
| Processing labor | 41.5 | 13.2 |
| Maintenance labor | 27.2 | 11.8 |
| Processing changes | 31.1 | 13.1 |
| Transportation | 0.7 | 1.7 |
| Materials | 32.4 | 15.2 |
| Minor equipment | 9.6 | 6.6 |
| Other (including depreciation) | 12.2 | 11.3 |
| Family return* (\$/acre) | 502.5 | 212.7 |
| Family profit# (\$/acre) | 246.6 | 53.2 |
| Average size of holding (acres) | 7.4 | 5.0 |
| Total family return (\$) | 3,718 | 1,063 |

* Gross income minus all nonlabor costs.

Gross income minus all costs.

Source: Rubber Research Institute of Malaya, Annual Report, 1965.

Labor costs were estimated at prevailing rates in the estate sector, as far as this author can determine. But if this is the case, there seems to be some possible conflict between this and some of the data on required labor inputs mentioned earlier, based on the case of low yielding smallholdings only, however. The above data indicate a total labor bill of \$1894 and \$798 per farm on high yielding and low yielding smallholdings, respectively. Now in 1963 the annual wage of estate tappers was \$1020 and of estate field workers was \$720.²⁴ These numbers suggest that the high-yielding smallholding could easily employ up to two full-time tapper-fieldworkers. Available studies on family labor input on the smallholding indicate that from 1.5 to 2.0 persons may work consistently and regularly over a year.²⁵ Thus these data on estate wages and family labor input correspond well to the figures given in Table V.12. On the other hand, the labor bill on the low yielding holdings, if calculated at estate wage rates indicates that a maximum of one full-time worker would be required to do all tapping and other work. In fact, based on the labor cost figures, one "full-time" worker would spend about 70 percent of his time tapping and collecting and about 10 percent of his time at other tasks, leaving the remaining 20 percent of his time idle. Such a breakdown would indicate about .8 full-time workers per 5 acres or 6.3 acres/worker. But recall the results of the study mentioned earlier, which indicated a workable limit of 4.8 acres/worker. Further, since smallholdings averaged 3.1 acres/worker in 1962, it would seem surprising to find so

many low-yielding smallholdings using as little as one worker on a plot of five acres. The most likely explanations to the conflict would seem to be that the actual labor input on the surveyed low yielding smallholdings was underestimated or it was undervalued, i.e., not measured at estate wage rates. Of course, there remains the possibility that such holdings were, in fact, worked with a relatively low input of labor.

Estimates of the rate of return on capital on the surveyed smallholdings are not directly available in the R.R.I. report. An idea of orders of magnitude may be obtained by examining land values on subdivided estates, since virtually all of the smallholders' capital is in the form of the land and accompanying trees. By comparing family profit per acre in Table V.12 divided by the value of rubber land per acre to the rate of return on estates, we obtain a rough idea of the relative rate of return on investment in the estate and smallholders' sector. Table V.13 shows much of the relevant data on the values of subdivided land. In general land planted with high yielding rubber fetched a much higher price than that planted with low yielding rubber. Also very old and immature stands tended to have lower prices. A not uncommon price range for mature, high yielding stands was \$2,000 - \$3,000 per acre, whereas ordinary yielding stands of medium age would tend to be clustered around \$1,000 per acre, although the variation within each yield and age group was high.²⁶ If we take \$2,000 and \$1,000 per acre as reasonable prices for high and low yielding stands,

Table V.13

Average Price per Acre of Subdivided Estate Lands
(659 transactions)

| Price (\$/Acre) | Number of Transactions |
|-----------------|------------------------|
| 0 - 199 | 5 |
| 200 - 399 | 107 |
| 400 - 599 | 83 |
| 600 - 799 | 133 |
| 800 - 999 | 88 |
| 1,000 - 1,499 | 133 |
| 1,500 - 1,999 | 53 |
| 2,000 - 2,499 | 36 |
| 2,500 and up | <u>21</u> |
| Total | 659 |

Average price: \$977 per acre

Source: Federation of Malaya, Report of the Subdivision of Estates Committee, Table 5.6, p. 81.

respectively, on mature but not aged holdings, probably the most relevant condition for the surveyed holdings in Table V.12, then the estimated rates of return on those holdings are 12 percent and 5.3 percent on high and low yielding smallholdings, respectively. If production functions are roughly constant returns to scale, and it is assumed that in both the estate and smallholding sectors labor receives its marginal product, then these rates of return indicate that the marginal productivity of capital is significantly lower in the smallholders' sector, since the before tax rate of profit in the estate sector based on Table V.11 was 16.2 percent, due to the fact that profits were taxed at a rate of 30 percent. Since this rate of profit was based on the value of the total assets, which include sterile cash and government securities, it is

likely that the rate of return on rubber growing was even greater. For example, if the after tax rate of return on financial assets was 5 percent, then the before tax rate of return on other capital investments was 20 percent.

Among the possible explanations for the apparent low marginal productivity of capital among smallholders are the following: Land values may have been set too high on the peasant holdings or too low in the survey of estates. The estate wage rate, assumed equal to the marginal productivity of labor on smallholdings, may actually exceed that productivity by a significant amount. This is supported by the aggregate data on factor intensities, but not by the micro-data on labor inputs. The average rate of return on capital in the estate sector may not equal the marginal productivity of capital, and the same may apply to smallholders. For example, based on the pattern of clearing and planting costs, it is possible to estimate the rate of return on the creation of an acre of high yielding rubber. Using the gross income per acre in Table V.12 as a measure of the marginal product of planting and adjusting this for the time pattern of yields, it is found that the rate of return on such investment is at least 17 percent, based on an estimated planting cost of \$805 per acre spread over a seven-year period.

The earlier mentioned study on the phenomenon of estate subdivision observed the following: "The fact that subdivided land fetches a relatively high price, as compared to the sale of large pieces of land,

makes profitability a very decisive factor in creating a supply of such land."²⁷ Since it is to be expected that market prices will reflect differences in the marginal productivity of land, at least to some extent if not exactly, the above statement lends credence to the view that capital may be relatively scarce in the smallholders' sector.

It must be concluded that the evidence on capital productivity is equivocal. The institutional forms of capital financing in the two sectors suggest that the cost of capital to smallholders is relatively high. We should expect this feature to reflect itself in a high marginal productivity of capital in smallholder agriculture. The available data have not allowed us thoroughly to confirm this.

What about wages and the marginal product of labor? The aggregate data suggest that they should be lower in peasant agriculture due to their higher labor intensity. There are also some institutional factors which would tend to support this argument. Labor mobility in rural areas seems to be rather low, and this may have a depressing effect on wages as population rises and fragmentation occurs. According to the Report of the Subdivision of Estates Committee:

" . . . the proof of [low] labor mobility is the low level of wages in subdivided areas. If rural labor were more mobile, then wages would have to be higher. New owners are able to exploit the labor partly because of its low mobility and partly because the trade unions and Ministry of Labour have not been able to extend their respective organizations to protect workers employed in small groups. . . There is probably a high degree of disguised unemployment as is evidenced by unfavorable wages and conditions of work and the increased amount of work extracted by employers."²⁷

The Committee based its analysis of wages only on those who remained in the subdivided regions, which totalled 187,000 acres over the period 1951-60, so that its results do not necessarily indicate anything definite about the overall wage level in these regions, either before or after subdivision. Further, the Committee did not indicate at what points in time the "before" and "after" measurements of the wages of particular individuals were taken. The results of the survey are summarized in Table V.14. On the basis of the data, the Committee concluded:

"Incomes are generally lower and the distribution is more unequal after subdivision. This is due to three main influences: a general lowering in wage rates, an increase in the number of persons in all income levels below \$80 per month after subdivision, and substantial decreases for many persons earning more than \$100 per month before subdivision."²⁸

Table V.14

The Cumulative Distribution of Monthly Earnings
Before and After Subdivision

| Monthly Earnings (\$ per Month) | Cumulative % of Earners | |
|------------------------------------|-------------------------|-------|
| | Before | After |
| 0 - 20 | 0.3 | 0.0 |
| 0 - 40 | 2.8 | 3.6 |
| 0 - 60 | 9.9 | 17.4 |
| 0 - 80 | 30.8 | 53.4 |
| 0 -100 | 71.5 | 79.3 |
| 0 -120 | 90.3 | 93.0 |
| 0 -160 | 98.4 | 97.9 |
| 0 -200 | 99.8 | 99.1 |

Source: Federation of Malaya, Report of the Subdivision of Estates Committee, Table 6.10, p. 115.

There are other important institutional factors, some of which have been mentioned earlier. The estates use labor which is hired at fixed wages. Due to the existence of fairly strong unionization in the estate sector, these wages could easily be set at above equilibrium levels, in the sense that if all workers in the rubber industry were allocated to equalize their value marginal product, that quantity would be exceeded by the estate wage level. Further, rubber holdings of 25 acres or more are required by law to provide certain amenities for workers. These amenities substantially enhance the nominal wage rates in the estate sector as indicated in Table V.15.

Table V.15
Amenities on Estates: 1965

| Item | Percent of Relevant Group | Group |
|---------------------------------------|---------------------------|----------------------|
| Free accomodation | 73 | Estate labor force |
| Free medical attention | 97 | " |
| Free medical treatment | 50 | " |
| Paid sick leave | 88 | " |
| Paid holidays | 89 | " |
| Free piped water | 90 | Resident labor force |
| Free electricity | 51 | " |
| Schools | 27 | All estates |
| Help children attend school elsewhere | 7 | " |
| Maternity allowance | 100 | " |

Source: Malaysia, Monthly Statistical Bulletin of West Malaysia, February, 1967.

The fact that smallholder production is undertaken largely by family labor, combined with the low mobility of rural labor, also represents a strong institutional influence possibly working against the efficient allocation of labor, since there may be a tendency for persons to remain with the family, even when they might obtain higher incomes by moving away from it.

A more quantitative test of relative labor marginal productivities may be had by comparing estimates of average value added per worker in the smallholders sector with the wage rate in the estate sector. If it is assumed that estates use labor fairly efficiently so that wages are close to labor's value marginal product, then a sufficient proof for the lower value marginal productivity of smallholder labor is that the average value added per worker in the smallholders sector be less than the estate wage level, provided that the smallholders are operating on a region of the production function where the average product of labor falls, given ceteris paribus increases in labor inputs, and provided that individual producers face reasonably price elastic demands for their output. We shall accept these conditions as satisfied, since they are quite reasonable.

The estimates on smallholder average value labor productivity are obtained as follows: The ratio between the f.o.b. Singapore wholesale price for RSS #3 rubber in 1963 and 1964 and the price received by smallholders implied by the data on income and production in Table V.12 is approximately .75. Assuming this ratio to be valid for 1957 and 1962,

and using the output per worker figures for those years given in Table V.6, the estimated gross annual income per worker is \$965 in 1957 and \$790 in 1962. Again using Table V.12 to obtain a rough estimate of intermediate goods and services, purchases, which we assume to be \$40 per acre, probably on the low side, and using the estimates of mature acreage per worker in Table V.9 of 4.1 and 3.1 for 1957 and 1962, respectively, we obtain the following estimates of value added per worker: \$801 and \$666 for 1957 and 1962, respectively.

In constructing these estimates, no attempt was made to account for changes in the price level, since this could be of significance for our purposes only in the computation of intermediate purchases. In any case, prices were not particularly volatile as indicated by Table V.16.

Table V.16

Retail Price Index -- Selected Years
(1959 = 100)

| Year | Index | Year | Index |
|------|-------|------|-------|
| 1957 | 104 | 1963 | 103 |
| 1962 | 100 | 1964 | 102 |

Source: Malaysia, Monthly Statistical Bulletin of West Malaysia, February, 1967.

The estimates are compared with the annual wages of tappers and weeders on estates in the same years in Table V.17. Tappers and weeders together account for almost 90 percent of all estate employment, and

Table V.17

Comparison of Estimated Value Added per Worker on Smallholdings with Annual Wages of Tappers and Weeders on Estates, 1957 and 1962
(All figures in Malayan dollars)

| Year | Annual Wage* | | Value Added per Worker on Smallholdings (est.) |
|------|--------------|---------|---|
| | Tappers | Weeders | |
| 1957 | 970 | 685 | 801 |
| 1962 | 1,040 | 710 | 666 |

* Excluding amenities, and assuming full 12-month employment per worker.

Sources: Federation of Malaya, Rubber Statistics Handbook, 1958, 1963; text.

tappers alone account for two-thirds. Since all of the remaining 10 per cent are foremen or other skilled workers, earning larger incomes, an index of the annual estate wage based on only tappers and weeders would be biased downward. Constructing such an index using the weights .75 and .25 for tappers and weeders, respectively, we obtain the following average annual wage estimates for 1957 and 1962: \$896 and \$955. These numbers suggest that the marginal product of labor is substantially higher in the estate sector and that the existence of substantial market imperfections is likely.

Having now examined in some detail the differences in factor usage between sectors and possible causes, let us turn to the question of saving and investment behavior. The author has been able to obtain little data on peasant saving. However, peasant real direct investment levels are closely ascertainable since they are manifested almost entirely in the form of rubber planting, and from the data on investment we may be able to infer something about savings levels as well. Data on estate saving and investment are available, but no information on saving by estate employees has come to light. As in the case of peasants, the major form of direct estate investment is rubber planting. Consequently, let us first survey the postwar pattern of planting in Malaya. The relevant data are summarized in Table V.18. New planting refers to the planting of land previously not under cultivation of rubber, and replanting refers to the planting of new trees on existing rubber land.

After World War Two it generally came to be recognized that replanting of existing rubber acreage was of utmost importance to the survival of the rubber industry in Malaya. The depression and planting restriction schemes of the 1930's and the interruption of the Japanese occupation during the first half of the 1940's had prevented sufficient replanting of rubber land, and by 1946 many trees were ageing and beginning to yield uneconomically low output.³⁰ About half the acreage on estates consisted of trees 21 years old or more, and one-third of trees over 25 years old. Over 70 percent of the acreage on smallholdings consisted of trees of age 20 or

Table V.18

Re- and New Planting by Estates and Smallholders
in Malaya, 1947-65
(All figures: acres x 10³)

| Year | New Planting | | Replanting | |
|-------------------|--------------|-------------------|------------|---------------|
| | Estates | Smallholdings | Estates | Smallholdings |
| 1947 | 1.3 | .1 | 24.8 | 4.4 |
| 1948 | 6.6 | 2.0 | 45.5 | 2.0 |
| 1949 | 7.0 | 2.0 | 52.8 | 2.4 |
| 1950 | 5.8 | 3.5 | 44.0 | 3.5 |
| 1951 | 14.8 | 5.5 | 58.2 | 3.8 |
| 1952 | 7.2 | 6.8 | 51.6 | 4.2 |
| 1953* | 4.7 | 6.3 | 29.8 | 29.5 |
| 1954# | 7.1 | 3.3 | 39.1 | 22.6 |
| 1955 | 10.0 | 8.1 ^x | 57.6 | 25.3 |
| 1956° | 14.7 | 13.1 | 78.4 | 46.5 |
| 1957 | 16.1 | 11.3 ⁺ | 76.3 | 49.8 |
| 1958 | 13.9 | 10.8 | 64.8 | 59.7 |
| 1959 | 14.4 | 20.9 | 68.2 | 69.1 |
| 1960 | 21.7 | 25.1 | 75.2 | 69.5 |
| 1961 | 17.7 | 67.4 | 70.3 | 57.3 |
| 1962 | 18.0 | 82.5 | 60.3 | 69.2 |
| 1963 | 8.7 | 100.3 | 58.7 | 83.4 |
| 1964 | 6.1 | 57.5 | 58.5 | 79.7 |
| 1965 [@] | 4.9 | 39.8 | 53.2 | 91.4 |

*Government replanting subsidy introduced: \$400 per acre replanted on smallholdings.

#Smallholder replanting subsidy raised to \$500 per acre.

°Smallholder replanting subsidy raised to \$600 per acre. Estate replanting subsidy introduced up to an acreage of not more than 21% of individual estate acreages on December 31, 1954.

@Smallholder replanting subsidy raised to \$750 per acre in October.

+Federal Land Development Authority created to resettle peasants on newly developed rubber holdings. (See Text.)

^xDuring the period 1955-62, funds were made available for new planting as well under the provisions of the replanting program. (See text.)

Sources: Rubber Statistical News Sheet and Rubber Statistics Handbook, various issues and years.

more, and almost half the acreage consisted of trees aged 26 or older. Furthermore, more than two-thirds of the mature estate acreage and virtually all of the mature smallholding acreage were of low yielding material.³¹ The depressing effect of postwar synthetic rubber competition on natural rubber demand was making it necessary for natural rubber producers to lower costs by planting high yield materials in order to survive the increasingly severe price competition.

During the period 1947-52, both new and replanting were not hindered by explicit government policy. There was substantial available uncultivated land for new rubber planting. (In 1963 it was estimated that only about 50 percent of Malaya's arable land was under cultivation.³²) Nevertheless, there are some institutional barriers to the cultivation of new land. All land not alienated to individuals for cultivation is vested in the hands of the various state governments, and such alienation often has encountered administrative bottlenecks and delays. Permission to use land for a purpose other than that for which it was originally alienated is sometimes hard to obtain. Portions of land are reserved for the use and ownership of Malays to the exclusion of the substantial indigenous Chinese and Indian minorities.³³

As will be seen later, the estates appeared to have no difficulty in replanting or new planting, since these activities could easily be financed through internal funds. The smallholders sector, however, faced serious difficulties in replanting. That this is so is reflected

clearly in the data on replanting during the period 1947-52. Over this period estates replanted about 277,000 acres, whereas smallholders replanted about 20,000 acres. A number of observers have pointed out clearly why smallholders were unable or unwilling to replant.

P. T. Bauer, writing in 1947, observed:

"There are two principal reasons for smallholders' unwillingness, or rather inability, to replant. There is first the lack of capital required to pay not only the heavy expenses involved . . . , but also to bridge the loss of income from the felling of the old trees to the maturity of the new stand. . . . Replanting can thus be undertaken only by producers with ample working capital, which the estates do, and the smallholders do not, possess.

"The second reason for the smallholders' inability to replant is the technical impossibility of replanting successfully part of a holding of a few acres, as the area replanted would be closely surrounded by mature trees which would intercept the sunlight and whose roots would compete for food with the undeveloped rootlets of the newly planted trees."³⁴

On this same subject of replanting, the Committee on the Sub-division of Estates observes:

"Below a certain level (about four acres) it is impossible for the farmer to replant without finding a whole new farm or enterprise from which he can obtain an income for a period of seven or eight years while his original land is replanted and while the new high yielding trees mature.

"In fact, the small producer has not replanted on a scale proportionate to his importance in relation to the total rubber acreage in the country.

"The units that can best replant are those above ten acres, and replanting . . . becomes increasingly 'economic' as the holding increases in size.

"On the large units, it is possible for one part to be tapped on an ordinary basis while a second part of the holding is slaughter tapped, and a third part is cleared and replanted. Carefully phased replanting plans allow intercropping of replanted areas during the first few years after initial planting of the rubber tree."³⁵

It would thus appear that the greatest obstacle to replanting is the inability to obtain capital financing to cover the direct costs of replanting and to bridge the loss of income during the maturation period. E. K. Fisk in a 1961 case study of a Malay reservation found that 81 percent of the smallholders who had commenced replanting owned more than one lot and could rely on income from other land until the new trees matured.³⁶ Above, we suggested that the rate of return on new planting investment could be higher than 17 percent for smallholders. This would seem to be high enough to be economic. (Of course, we don't actually know what the social opportunity cost of capital is, so we do not know for sure.) However, if the smallholder is replanting, this rate of return must be reduced substantially, since it does not account for any drop in current consumption. If the peasant must hypothetically lose income during the investment period, then it is obvious that the rate of return on replanting is uneconomic until the yields on existing trees are so low that very little income is foregone by chopping them down and replanting. It is clear that replanting becomes much easier for the

large scale enterprise which will have many stands of varying age. When the trees on a particular stand become uneconomic due to very low yields, the loss of income factor is small, and there is no general loss of income, since only a small percent of total acreage will be affected.

Again, due to the shortage of capital financing in the peasant sector, the undertaking of new planting may also be very difficult, institutional factors again playing the key role. Due to the high cost or complete unavailability of external financing, the smallholder may find it impossible to accumulate the necessary funds for land purchases and planting expenditures, since he may be unable to generate sufficient internal savings. According to the report on estate subdivision: "It is very unlikely that the typical small rubber farmer earning between \$60 and \$100 per month would be able to buy . . . [subdivided] . . . land." To support this contention the report points out that less than 40 percent of the new owners of subdivided land were smallholders or agricultural workers. The majority were petty capitalists in commerce or trading, professionals, industrial workers, housewives, students, or land owners.³⁷ On the other hand, the Mission of Inquiry reported: "Many smallholders' families are apparently prepared to invest their labor and cash reserves in creating new holdings. . . Indeed it is evident that the basic demand for land from smallholders is very high, reflecting in some localities what is in fact severe land-hunger."³⁸ It is interesting to note that during 1947-52, when estate replanting exceeded that of smallholders by a factor of over 23, new planting was only twice as large, sug-

gesting that smallholders found such planting to be economic in spite of possible institutional difficulties or relatively high capital costs.

By 1952 the Malayan government had become convinced of the need to accelerate replanting efforts in both the estate and smallholding sectors. A replanting tax of 4 1/2¢ per pound on exported rubber, applicable if the wholesale price of rubber was above 72¢ per pound, was introduced in 1952. This tax was called Schedule IV, and the estate share of this tax was refunded unconditionally. The smallholding share was placed in Fund B, for purposes which will be explained shortly. In addition to this tax, there was already in effect a Schedule II tax on exports, which was applied at an ad valorem rate of zero if the wholesale price of rubber was less than 60¢ per pound and at a rate given by

$$T = .045P - 27,$$

where T is the tax and P is the price, if P was greater than 60¢ per pound. Receipts from the Schedule II tax were placed into Fund A for estates and Fund B for smallholders, each separately managed. The receipts in Fund A would be refunded only against approved replanting expenditures. The tax receipts in Fund B were used to pay smallholders a subsidy of \$400 per acre replanted. In 1954 the subsidy was raised to \$500. The results of the smallholders subsidy are clear on inspection of Table V.16. In 1953, the first year of the replanting program, smallholder replanting rose to 29,500 acres compared to 20,300 for the whole period 1947-52.

As a result of the 1954 Report of the Mission of Inquiry, already cited several times in this chapter, changes in the tax and subsidy program were introduced. The Schedule II export tax was eliminated and the basic export tax rates (Schedule I) were raised when the price of rubber exceeded 80¢ per pound and lowered when the price was below 80¢. In addition, the government allocated grants of \$168 million and \$112 million for estate and smallholding replanting subsidies, respectively. The estate replanting subsidy was \$400 per acre for up to 20 percent of official acreage as of December 31, 1954, and was in effect until 1963. The smallholder subsidy was raised to \$600 per acre and was financed from Fund B as well as the grants. In October 1965, this subsidy was further increased to \$750 per acre.³⁹

The total grant of \$750 to smallholders is paid in seven installments during the immature period of tree growth. The administering board maintains standards regarding the proper preparation of land and the correct procedure of planting and maintenance. Extension service advisors supervise the replanting, and high yielding planting material is supplied from official government nurseries. Portions of the \$112 million smallholder grant have been used to finance Block New Planting Schemes under government sponsorship as well as replanting.⁴⁰

As may be expected, the rapid rise in new planting by smallholders after 1958 shown in Table V.16 is also due largely to government efforts. The Block New Planting Schemes were not the main source of new planting efforts. These came rather from the Federal Land Development Authority

(FLDA) which was established in 1957. Between 1957 and 1965 about 121,000 acres were developed under this program. The FLDA obtains land for re-settlement through the state governments. Such land is cleared and planted by contracted labor, and the settlers are moved in in phased groups after houses have been constructed and basic services are available. Each settler receives 10 1/4 acres, consisting of 1/4 acre for a house, 8 acres for rubber, and 2 acres for subsidiary crops. Resettled families are expected to work together under the supervision of an FLDA manager and his field staff. Settlers receive a subsistence allowance during the immature phase of tree growth based on the number of days' work done by each settler and his working dependents. The resettled smallholders are expected to repay the funds invested in the development of their rubber holding at a rate of 2/3 of the excess of their monthly family income over \$150 per month. The costs of administration and infrastructure investments are borne by the general government coffer. Preference is given to Malaysian nationals between the ages of 21 and 45 with children who are landless or who own less than two acres of land. Fragmentation of individual FLDA holdings is not permitted. Expected family incomes on mature, debt-free holdings is thought likely to be \$4,000 or more, thus comparing favorably with Malaya's finest smallholdings, based on expected yields of 1140 lb/acre/year.⁴¹

Two things are clear. First, the government has clearly operated vigorously to support the economic viability of smallholders' rubber and to a lesser extent that of the estates. We shall want to assess the

economic validity of such a policy shortly. Second, it seems likely that the smallholders sector is, under the present milieu of factor market imperfections, incapable of providing savings internally or of obtaining them from other sectors through intermediaries to a degree adequate to maintain their stock of rubber growing capital.

Although we have obtained no information on saving by estate workers, information on saving and investment by the estates themselves is available, and we shall now examine it. It should be noted, however, that even the estate data cover only the publicly incorporated non-Asian owned ("European") estates, and that not all observations about them will be applicable to the Asian owned estates. K. R. Chou, in his study of saving and investment in Malaya, observed that the bulk of new and replanting in the estate sector after World War Two was in fact done by the European estates. As to the source of the financing of their planting investments, he stated: ". . . except for the utilization of prewar reserves in the first two years, the funds came from depreciation allowances plus the ploughing back of current profits, which were several times the value of current new investments over the years."⁴² Chou estimated the annual average taxable profits of the entire rubber industry to be about \$160 million over the period 1949-1958 and assumed that the European estates must have accounted for over \$100 million of this since most of the smallholders did not pay income taxes and since the European estates based on their greater acreage and yields per acre than Asian estates should be expected to account for such a proportion of total profits. Company profit

tax rates were 20 percent until 1951, 30 percent from 1951 to 1959, and 40 percent from 1959 onward. Annual average gross rubber planting investment was \$31.4 million from 1951 through 1955 and \$40.6 million from 1956 through 1960, indicating a ratio of roughly .5 between gross investment and net profits over the postwar period. Unfortunately, Chou was not able directly to ascertain what proportion of gross investment was financed from depreciation allowances and what proportion from net profits.

An indication of these proportions is, however, given in the results of a survey of 24 European estates also discussed by Chou. The period covered by the survey is 1949-1958. It was found that about 44 percent of net profits were retained, but that only 30 percent of such retained profits were spent on increasing fixed assets. Most of the remainder went into liquid assets, so that, in effect, only about 16 percent of net profits were reinvested in the industry. Depreciation allowances averaged about 2 percent on the value of issued capital and accounted for roughly 40 percent of gross investment in fixed assets, the remaining 60 percent coming from retained profits. Chou concluded: "[The high rate of return on capital in rubber] . . . discouraged the rubber companies from investing their enormous amounts of liquid assets into other fields of production in Malaya, for which the rate of profit could not possibly reach the standard set by rubber estates."⁴³ Chou also noted that there were few obstacles to prevent the European estates from transferring their liquid funds to relatively more profitable investments abroad.

It would appear that the estate sector in general was quite capable of financing its own investment programs internally and that in the case of foreign owned estates, which predominate in the estate sector, there were substantial transfers abroad of savings or potential savings in the form of profits paid to foreign shareholders.

Before making any tentative assessments of the conditions of rubber production in Malaya and the policies pursued with respect to its dualistic organization, it will be helpful to place matters in context by considering briefly the general availabilities of capital and labor in the Malayan economy. As regards capital, Chou writes: "The lack of capital has been far less acute in Malaya than in other underdeveloped countries. It is the know-how to industrial projects . . . that is of crucial importance." Later he speaks of an "abundance of local capital," and cites the free flow of capital goods into the country due to the close ties of the Malayan dollar with sterling (until 1967) and Malaya's good balance of payments position.⁴⁴ Table V.19 shows domestic saving as a percent of GNP for the period 1955-64. Note that the aggregate savings ratio was above 16 percent for all years except 1957 and 1958 when it was 15.4 percent and 12.9 percent, respectively. These ratios compare favorably with relatively capital rich countries. For example, gross private saving in the United States was 16.1 percent and 16.5 percent of GNP in 1966 and 1967, respectively.

Although Malaya appears to be relatively well endowed with capital resources, there are indications that labor is abundant at a very

Table V.19

Domestic Saving as a Percent of GNP, 1955-64

| Year | Percent | Year | Percent |
|------|---------|------|---------|
| 1955 | 20.0 | 1960 | 20.9 |
| 1956 | 16.1 | 1961 | 17.2 |
| 1957 | 15.4 | 1962 | 16.9 |
| 1958 | 12.9 | 1963 | 16.3 |
| 1959 | 17.9 | 1964 | 16.3 |

Source: Pierre Crosson, Economic Growth in Malaysia (Washington: National Planning Association, 1966), Table 2.13, p.56.

low opportunity cost, so that on balance Malaya is undoubtedly a relatively more labor abundant and capital poor country than most developed countries. This is evidenced by two factors. First, there is a substantial amount of overt unemployment:

"Present levels of unemployment in Malaya stand at around 6 percent of the total work force, and are expected to remain fairly constant until 1970 at least. By that year a further 380,000 persons will swell the labor pool, of whom 165,000 will be in the primary sector. Recruitment of labor may be easier in the long-established communities of Western Malaya, where Islamic laws of inheritance have progressively enlarged the numbers of fragmented holdings and landless farmers."⁴⁵

Second, there are sectors in which labor appears to be underemployed. This is evident in urban areas which contain large numbers of hawkers, messenger boys, domestic servants, etc. We have examined evidence which suggests that labor may be either underemployed in smallholders rubber or overemployed in estate rubber. But the condition of underemployment

is most evident in the rice producing subsistence sector. The government has long pursued a policy of promoting domestic rice production, since Malaya has always had to import large quantities of that item, and the government has gone so far as to restrict the use of certain lands exclusively to rice production, even though it is clear that rice cultivation is the least profitable of all occupations in Malaya. This point was made clear by Bauer, writing in 1947, and has been confirmed as recently as 1967 by Lim Chong-Yah.⁴⁶ The rice sector is the second largest agricultural sector; it contains 19 percent of Malaya's labor force, 17 percent of its total acreage and produces 4 percent of Malaya's domestic product. It is entirely self sufficient and makes no contribution to government revenue. In 1960 gross output per worker in Malaya as a whole was \$232 per month. This compares with \$230 per month in the rubber industry and only \$48 per month in rice.

Having now examined static factor use and saving and investment in the rubber industry and the general picture of factor availabilities, what can we say about the economics of rubber production in Malaya? Harkening back to the results of Chapter III, recall that there are two aspects to the question of optimum organization of dualistic agriculture. The first concerns efficient resource use in a static sense, i.e., given the social prices of scarce factors, under what organizational configuration will output measured in terms of the social prices be maximized? It was found that if production functions were identical and there existed factor market imperfections, or if one sector were

technologically superior to the other at all factor combinations, then dualistic organization was never optimal provided that the government could manipulate savings-investment behavior without adverse effects on static factor allocative behavior. This brings us to the second aspect of resource use, saving and investment. The analysis of Chapter III showed that technological inferiority could be tolerated if savings-investment behavior could not be influenced significantly, since the technological loss could be offset by a more favorable time profile of consumption. We shall attempt to apply some of these results in a speculative way to Malayan rubber.

Consider first the matter of static resource use. We have seen fairly solid evidence that estates employ relatively more capital intensive methods than smallholders. This evidence may be interpreted in more than one way. For example, if we suppose that the relevant production function for rubber is neoclassical and identical in both sectors, then the differences in factor use must necessarily reflect imperfections in factor markets. But if technologies differ or if the production function is not neoclassical, it may be possible for two different methods of production to be about equally efficient economically at some special set of factor prices. If so, when we examine the differences in factor allocation ratios, we cannot be certain whether they reflect two equally viable methods possible at the particular wage-rentals configuration in the economy or if they reflect market imperfections. Further, if we start from the other end and cite evidence of market im-

perfections, we cannot tell to what extent differences in factor usage will reflect the imperfections as opposed to possible differences in technology.

On the basis of the observed data, we shall hypothesize a rubber technology of roughly the following characteristics. Suppose that production is governed by a technology showing diminishing marginal rates of substitution within certain bounds at all scales of operation. Suppose further that production is almost constant returns to scale, in the following ways: Smallholders produce at constant returns to scale up to a maximum scale of output, at which point the institutional characteristics of smallholder organization cause it to be necessary to organize on a capitalist, fully-labor-hiring basis. There are maximum and minimum capital-labor ratios beyond which one or the other factor is redundant. The maximum scale isoquant has been drawn in Figure V.1. Next, assume that estates or capitalists produce according to a constant returns to scale technology, but which is operable only above the maximum peasant scale of output and which also possesses minimum and maximum nonredundant capital-labor ratios. Now based on our empirical observations, assume that at the organization change-over scale the isoquants relate in the following way: for any level of capital stock above the minimum required by estates, less labor per unit of output is required than by smallholders due to the greater efficiency of the division of labor on estates. For any capital stock below the minimum required by estates, only smallholders organization is feasible. These relationships are illustrated in Figure V.2, which shows the isoquants of maximum small-

Figure V.1

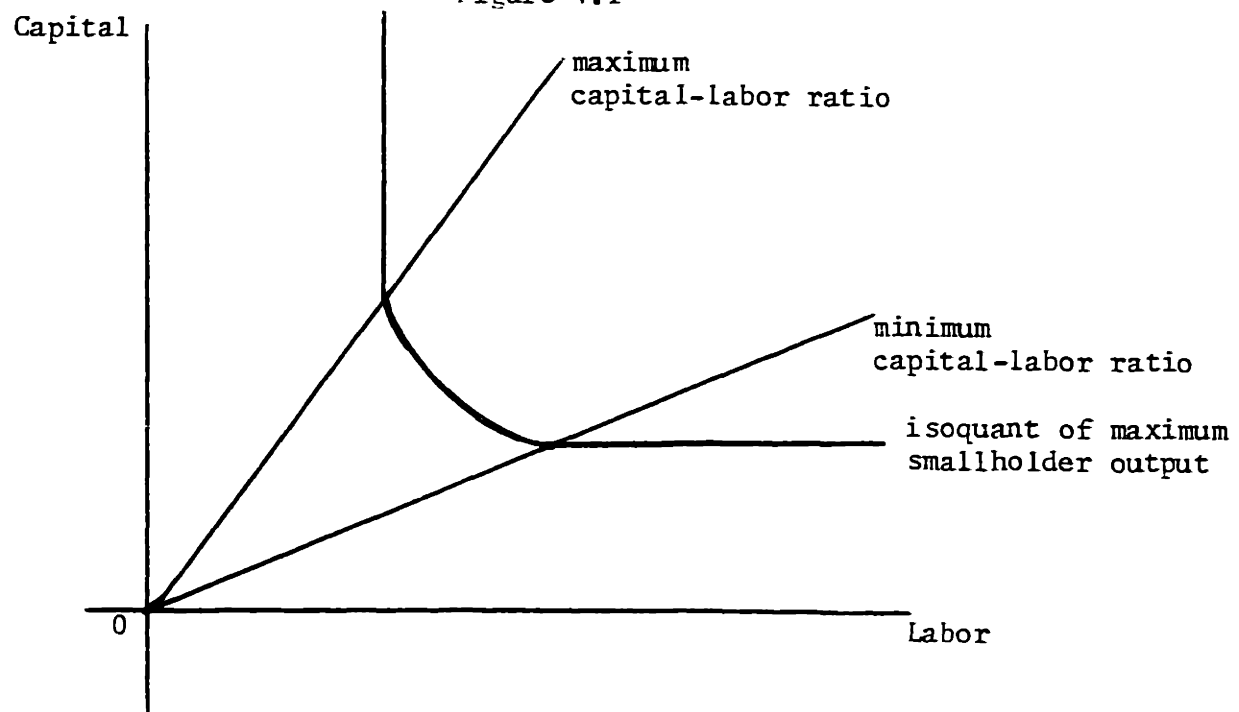
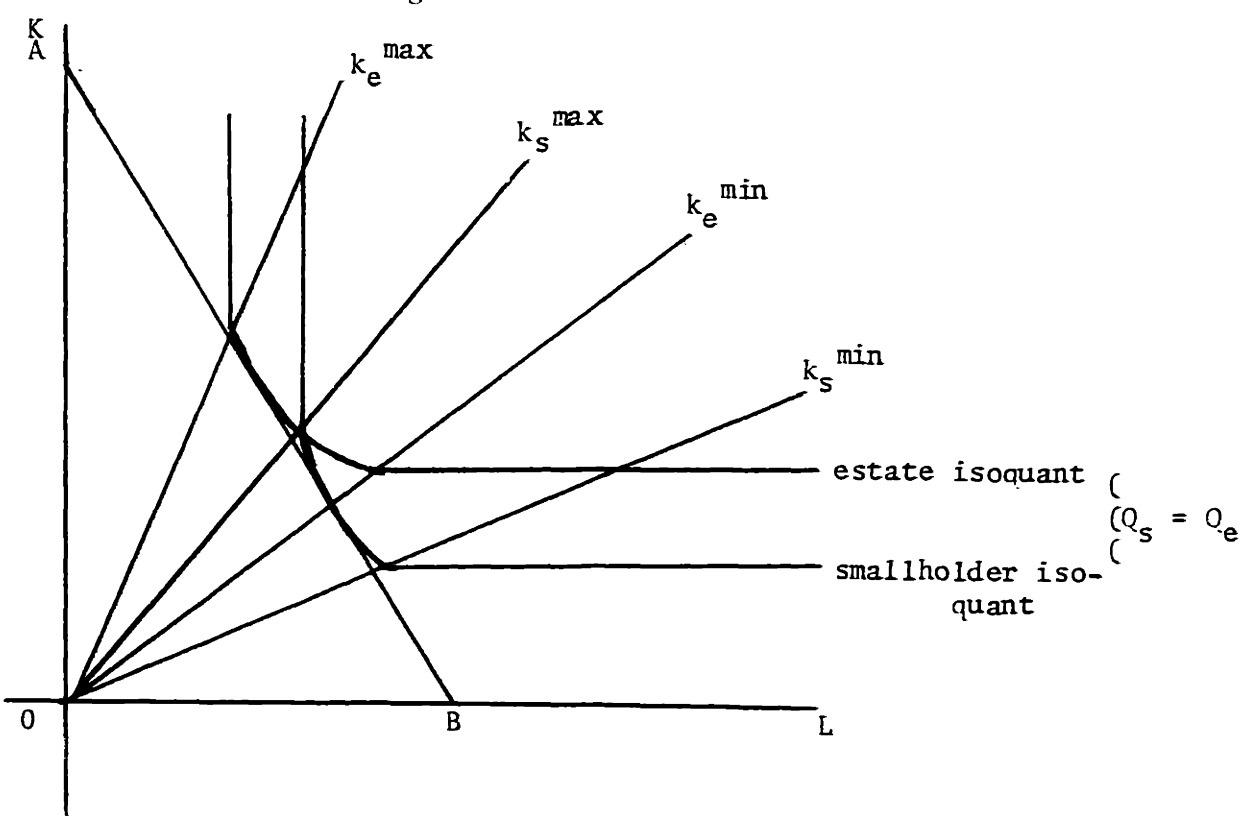


Figure V.2



holder output and minimum estate output. The symbols are defined as follows: K = capital, L = labor, e = estates, s = smallholdings, k = capital-labor ratio, \min = minimum, \max = maximum, Q = output.

If observed rubber technology is approximated roughly by the assumptions behind Figure V.2, then at various social prices for capital and labor one or the other form of organization will be economically superior (ignoring saving and investment for the moment). At the social prices implied by line AB in Figure V.2, both technologies would be equally efficient. What all this hypothesizing boils down to is that, depending on the relative social prices of capital and labor in Malaya, estate or smallholding organization may be preferred. It remains true that dualistic organization would be optimal only if social prices happened to be those implied by line AB. It must be emphasized, however, that these conclusions depend on an unproved hypothesis. To repeat, if technologies are identical and possess constant returns and diminishing marginal rates of substitution, then dualistic organization, if we may still call it that, is quite efficient, provided that both organizational types allocate factors as though they faced the same factor prices.

We must now bring in the matter of savings and investment. In an economy which is relatively labor abundant, static efficiency would suggest the use of relatively labor intensive techniques, but this priority could be altered if the relatively more capital intensive techniques also provided sufficient extra savings. The trade-off is between current

capital use and provision of future capital. Now the situation in Malayan rubber suggests that, government involvement aside, the advantages of labor-intensive smallholder techniques may not offset the obvious loss in savings. Without a great deal more data, it is difficult to estimate what the trade-offs between consumption and investment and between capital and labor use in Malaya might be. The problem is further complicated by the issue of government intervention. If the government could tax and subsidize in such a way as to alter the sectoral savings ratio and investment level without adversely affecting output, then the savings issue becomes unimportant, and the only relevant issue is static efficiency. It is obvious that the Malayan government and later the Malaysian have pursued a vigorous policy of savings investment augmentation, especially in the smallholders sector. It is not clear exactly how much adverse effect on output the additional export tax used to finance the investment subsidies has had. It is obvious, however, that government policy was and is able substantially to influence savings-investment behavior.

Due to inadequate data, it is really impossible to state whether there are solid economic grounds for encouraging one form of rubber producing as against the other. Theoretical and empirical evidence has suggested that there are potentially valid reasons for pursuing a strong policy biased in one direction or the other, but certainly this study cannot have advanced far enough along the way to provide a definite answer. Further, any consideration of optimum rubber policy must be

tied in more closely with other economic sectors in Malaya, since investment priorities and factor allocation decisions are dependent on what goes on outside the rubber industry. But time and space prevent, for the moment, a more exhaustive study.

Footnotes

1. Malaya refers to that portion of Malaysia which was known as the Federation of Malaya prior to the creation of Malaysia and includes all of the Malayan peninsula except Singapore. Malaysia's rubber-growing acreage is almost entirely concentrated in Malaya, which contains the vast majority of the population of Malaysia, as well.
2. Pierre Crosson, Economic Growth in Malaysia (Washington: National Planning Association, 1966), p. _____ and K. R. Chou, Saving and Investment in Malaya (Hong Kong: Academic Publications, 1966), p. 152.
3. Lim Chong-Yah, The Economic Development of Modern Malaya (Kuala Lumpur: Oxford University Press, 1967), pp. 13-25.
4. See, for example, C. R. Wharton, Jr., "Malayan Rubber Supply Conditions," in Silcock and Fisk, eds., The Political Economy of Independent Malaya (Canberra: The Australian National University Press, 1963), p. 138.
5. P. T. Bauer, The Rubber Industry (Cambridge: Harvard University Press, 1948), p. 4.
6. Data on output were obtained from the Federation of Malaya, Report of the Mission of Inquiry into the Rubber Industry (Kuala Lumpur: Government Press, 1954), p. 60 and from various issues of the Rubber Statistical Bulletin, published by the Secretariat of the International Rubber Study Group. Data on peasant acreage came from Lim Chong-Yah, op. cit., p. 329, and Malaysia, Monthly Statistical Bulletin of West Malaysia, various issues. Statistics on estate labor came from Federation of Malaya, Rubber Statistics Handbook, various issues (published annually). Estate acreage data also were obtained from this source and from Malaya, Report of the Mission . . . Information on smallholders labor was derived from Lim Chong-Yah, op. cit., pp. 200, 214, and Crosson, op. cit., p. 41.
7. R. D. Hill, "Agricultural Land Tenure in West Malaysia," Malayan Economic Review, Vol. XII, no. 1, April, 1967, p. 102.
8. Malaya, Mission Report, pp. 66-67.
9. Data on smallholders' acreage were obtained from Robert Ho, "Labor Inputs of Rubber Producing Smallholders in Malaya," Malayan Economic Review, Vol. XII, no. 1, April, 1967, p. 79. Data on estate acreage came from the Rubber Statistics Handbook, 1965.

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10. Malaya, Rubber Statistics Handbook, 1965.
11. Ibid.
12. Federation of Malaya, Report of the Subdivision of Estates Committee (Kuala Lumpur: Government Press, 1963), p. 28.
13. Ho, op. cit., pp. 79-89.
14. Bauer, op. cit., pp. 29, 201-202.
15. Malaya, Subdivision, pp. 84-92.
16. Ronald Ma, "Company Profits and Prices in the Rubber Industry in Malaya, 1947-58," Malayan Economic Review, Vol. IV, no. 2, October, 1959, p. 30.
17. Lim Chong-Yah, op. cit., p. 332, and Ma, op. cit., p. 30.
18. Hill, op. cit., pp. 101-104.
19. Ho, op. cit., p. 89.
20. Crosson, op. cit., p. 95.
21. Chou, op. cit., p. 107.
22. Chou, op. cit., p. 126.
23. Rubber Research Institute of Malaya, Annual Report, 1965.
24. Malaya, Rubber Statistics Handbook, 1964.
25. Ho, op. cit., pp. 81-82.
26. Malaya, Subdivision, pp. 81-83.
27. Ibid., p. 125.
28. Ibid., pp. 104-105.
29. Ibid., pp. 41, 119, and "Minority Report," p. 5.
30. See Lim Chong-Yah, op. cit., for a good brief history of the rubber industry in Malaya.
31. Malaya, Mission, p. 68.

32. Crosson, op. cit., p. 42.
33. Crosson, op. cit., p. 33.
34. Bauer, op. cit., pp. 173-174.
35. Malaya, Subdivision, pp. 96-97.
36. E. K. Fisk, "Productivity and Income from Rubber in an Established Malay Reservation," Malayan Economic Review, Vol. VI, no. 1, April, 1961, pp. 13-22.
37. Malaya, Subdivision, pp. 75, 148.
38. Malaya, Mission, pp. 32-33.
39. On taxation, see Yah, "Export Taxes on Rubber in Malaya, A Survey of Postwar Development," Malayan Economic Review, Vol. V, no. 2, October, 1960, pp. 46-58. On taxation and subsidies, see Lim Chong-Yah, op. cit., pp. 124-126.
40. L. C. Bateman, "Natural Rubber and Southeast Asia," pamphlet published by Natural Rubber Bureau, no date, pp. 8-9.
41. See Bateman, op. cit., pp. 8-9 and Natural Rubber Bureau, Natural Rubber News, September, 1967, pp. 1-10, and July, 1967, pp. 3-5.
42. Chou, op. cit., p. 155.
43. Ibid., p. 157.
44. Ibid., pp. 175-176.
45. Ho, op. cit., pp. 87-88.
46. Bauer, op. cit., pp. 60-62, and Lim Chung-Yah, op. cit., pp. 29-31 and 168-176.

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