AN ASSESSMENT OF CREEP FORMULATIONS
FOR CONCRETE STRUCTURES

by

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ABSTRACT

The time-dependent behavior of concrete under load is studied, including shrinkage and creep deformations and their partial recovery. Prevailing theories of the mechanisms of concrete creep are presented and evaluated. Influential factors and their effects on creep are described.

Visco-elastic material and physical models for creep analysis are reviewed, and the proposed concrete creep equations presented. Approximate numerical solution methods are also examined and evaluated.

Finally, the solution technique for the concrete creep problem is described, using a finite element analysis of an axisymmetric thick-walled cylinder, and thin-walled sphere.

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INTRODUCTION

In some engineering materials, such as steel, strength and the stress-strain relationship are independent of rate and duration of loading (within the usual ranges of temperature, rate of stress, etc.). In contrast, however, there is a pronounced influence of time on the behavior of concrete under load. Concrete continues to deform with time when subjected to a sustained load. It is said to undergo creep. Concrete creep is a visco-elastic phenomenon, and thus it exhibits both instantaneous elastic and delayed viscous deformations, which are partially recoverable.

In conventional structures, with stress levels below one third of the ultimate concrete strength and generally not extreme temperatures, creep is only a minor problem. However, when a concrete structure is subjected to high temperatures, and elevated temperature gradients, for long periods of time, the temperature dependent creep properties of concrete cause stress redistribution which can lead to major problems. For example, in concrete structures subjected to cyclic heating, cracking can occur on cooling after a relatively short period of mild heating.

These creep effects on the stress distribution through a concrete pressure vessel have become a major analysis problem. As the phenomenon of concrete creep is not yet totally explained, considerable damage to the vessel could result if the effects of creep are not properly accounted for. Costly and sophisticated structural analysis procedures lose their accuracy and their effectiveness if the material behavior is not understood and modeled correctly. For this reason it becomes very important to develop an understanding of the phenomenon of concrete creep, so that the behavior of the concrete structure under stress may be modeled and analyzed accurately.

In Chapter 1 the behavior of concrete under stress is described. Time-dependent deformations due to shrinkage and creep are examined, including their recovery upon removal of the stress. A short historical note is also offered.

The mechanisms of concrete creep are examined in Chapter 2. The prevailing theories are presented and evaluated.

In Chapter 3 the factors which influence creep in concrete structures are described. Environmental and material influences are examined.

The major import of this study concerns concrete creep formulations. In Chapter 4 material and physical models used to represent the visco-elastic phenomenon are examined. Many of the creep equations which have been proposed are given. Also, approximate numerical solution methods for the analysis of concrete creep are described and evaluated on the basis of their ease of use and accuracy of solution.

Finally, the solution technique for the creep problem is described, using a finite element analysis of an axisymmetric thick-walled cylinder, and a thin-walled sphere.

CHAPTER 1

CREEP AND SHRINKAGE OF CONCRETE

1.1 Introduction

Since concrete is part crystalline and part amorphous, it exhibits properties common to both phases, that is, under working stresses it undergoes both instantaneous elastic and delayed plastic, or viscous, deformations. Thus, depending on the stress value, the stress state and the environment, there are elastic, delayed elastic, viscous and plastic components. As a result, in considering concrete behavior under stress, we encounter two distinct types of deformation; that which occurs on the application of the load and that which occurs with the passage of time while the load continues to act. The former is instantaneous strain, and the latter is creep strain.

Under a sustained load concrete undergoes an initial, instantaneous elastic strain, ε_{ii}. This initial strain is followed by a time-dependent strain consisting partly of the strain due to shrinkage or environmental effects, ε_s, and partly of a stress-dependent strain, called creep, ε_c. Thus the total strain is written

$$\varepsilon_T = \varepsilon_{ii} + \varepsilon_s + \varepsilon_c \tag{1.1}$$

The time-dependent strain increases in magnitude at a decreasing rate, until a limiting value is reached.

Upon removal of stress at some time t_1, there is an instantaneous recovery, ε_{iR}, which is usually smaller than ε_{ii}. There is also a relatively small, time-dependent recovery, ε_{tR}, called creep recovery (or delayed elasticity) which reaches a limiting value ε_R. As a result, there remains an irrecoverable, or residual, strain which is sometimes referred to as permanent set. Thus, at any time $t > t_1$ we have

$$\varepsilon_T = \varepsilon_{ii} + \varepsilon_s + \varepsilon_c \text{ (at } t_1) - \varepsilon_{iR} - \varepsilon_{tR} \tag{1.2}$$

Creep and shrinkage are not independent phenomena, but since they occur simultaneously in many structures it has been convenient to treat the two together. For this reason, the term "creep" is often used in engineering practice to denote the phenomenon of shrinkage and of creep together.

1.2 Historical Perspective

In 1905 Woolson described the ability of concrete in a steel tube to "flow" under a high axial stress, and the first paper on creep, then described as nonelastic deformation of concrete, was published (62). Hatt, of Purdue University, published the first data of creep on reinforced concrete in the 1907 proceedings of the ASTM (23). Although he made no reference to shrinkage of concrete, his results do show the presence of large scale nonelastic deformations under load. Hatt's comment on this behavior, "These results taken together show a sort of plasticity in concrete by which it yields under the action of a load applied for a long time, or applied a number of times".

Although the nonelastic behavior of shrinkage was observed earlier than Hatt's discoveries of creep behavior, the structural significance of shrinkage was not recognized until 1911 by White, in a paper to the ASTM (61). In his paper he spoke of stresses developing due to shrinkage. White's observations caused some concern since the ability of creep to relieve these shrinkage stresses was not yet recognized.

McMillan, in 1915, published one of the earliest studies reporting the time-dependent deformation of both loaded and unloaded concrete (36). From that time to the present, the relation of creep to shrinkage has been a problem, both from a theoretical point of view and for design purposes.

Many others were involved in the history of the early observations and in the development of the theory of concrete creep. By 1917 the ability of concrete to undergo both elastic and creep recovery was observed by Smith (56). On the basis of these early observations, the broad format of deformations of concrete under sustained loads and subsequent unloading was established.

At the present time, the number of publications dealing with creep and shrinkage of concrete is increasing. This does not mean that creep and shrinkage are now solved problems, but that they continue to loom large in the design of modern concrete structures such as prestressed concrete structures, highly statically indeterminate structures, shells, nuclear pressure vessels, mass concrete, structures of high flexibility, long columns, and even tall buildings.

1.3 Shrinkage

Concrete undergoes volume changes independent of externally imposed stresses and of temperature changes. These volume changes are commonly referred to as shrinkage, even though negative shrinkage, i.e., swelling, can also occur.

Shrinkage arises from basically two causes: loss of water on drying, and volume changes on carbonation. The former will be referred to as shrinkage, and the latter as carbonation shrinkage (5).

When loss of water to the ambient medium (unsaturated air) takes place, deformation occurs. A part of this deformation is reversible under alternating wet and dry storage conditions, and is referred to as moisture movement. The term irreversible shrinkage is used for that part of the deformation which is not recovered on subsequent rewetting. The process of moisture diffusion from the interior of the concrete toward its surface is very slow and complex. The surface dries more rapidly than the interior, and as a result "free" shrinkage of concrete tends to develop primarily in the outer periphery of the section. Tensile stresses are induced in the outer fibers and compressive stresses in the inner fibers, due to the nonuniform distribution of this free shrinkage and the requirement for plane strain. The uniform "apparent" shrinkage is the combined result of the free shrinkage and the instantaneous and creep deformations, which are caused by the induced stresses. Therefore, free unrestrained shrinkage can only take place in these sections of concrete where uniform drying is achieved very quickly. However, the term free shrinkage is frequently used to describe the shrinkage in plain concrete unrestrained by external containment (such as forms), or internal reinforcement.

Loss of water and shrinkage are in a cause-and-effect situation, but their relation is not a simple one. When concrete begins to dry, the free water held in the capillaries is the first to be lost. However, this loss of water causes practically no shrinkage. As drying continues, absorbed water is lost and the resulting volume change of unrestrained cement paste is approximately equal to the loss of a water layer one molecule thick from the surface of all gel particles. The "thickness" of a water molecule is about 1% of the gel particle size, therefore we would expect a linear change in dimensions of cement paste on complete drying to be on the order of 1%. Values up to 0.4% have been observed, but the overall change in the volume of drying concrete is less than the volume of water removed (5).

Although the loss of water occurs only from the cement paste, for engineering purposes, the overall shrinkage of the concrete is measured. This is much smaller than the free shrinkage of neat paste, due to the restraining effect of the aggregate and the nondrying inner portion. For design purposes, shrinkage is considered as an ordinary linear strain, and is added to the elastic and creep strains to determine deformations, curvature and deflection.

Shrinkage is greatly influenced by the magnitude of the surface area of cement paste being desorbed. As a result, high-pressure-steamcured cement paste, which is microcrystalline and has a low specific surface, shrinks only 1/10 to 1/5 as much as a similar paste cured normally.

Aggregate, due to its restraining effect on the free shrinkage of neat paste, is an important influencing factor of drying shrinkage. The volumetric content of aggregate is the greatest factor influencing the magnitude of shrinkage developed by concrete. For example, changing the maximum aggregate size from 1/4 inch to 6 inches means that the aggregate content can rise from 60% to 80% of the total volume of concrete. This results in a decrease in shrinkage to 40% of the value with the smaller aggregate (5).

The extent of restraint offered by the aggregate depends on its elastic properties, and there exists a qualitative relation between shrinkage and the modulus of elasticity of the aggregate used.

Although an increase in water content appears to be a primary factor in increasing shrinkage, in fact the influence is only in its role in reducing the volume content of the restraining aggregate. Therefore, the relation between water content and shrinkage is not a fundamental one.

The fineness of the cement does not have an effect on the magnitude of the concrete shrinkage, however higher fineness can accelerate the shrinkage. This results in an increase in cracking. Chemical composition of the cement is not of large importance to shrinkage. For example, shrinkage of concrete made with high alumina cement is of the same magnitude as when normal Portland cement is used, although it takes place much more rapidly (31). The addition of calcium chloride increases shrinkage 10 to 50%, but this is probably due to the fact that a finer gel is produced, and because of greater carbonation. Air entrainment also does not appear to influence shrinkage (28).

Shrinkage occurs no matter what the age at which drying begins, and continues for many years. At long ages, however, the rate of shrinkage is so low that it is not significant. Although the rate of shrinkage is affected by many factors, as described above, for the usual range of structural concretes exposed to relative humidity of 50 to 70%, the rate of shrinkage is (5):

14 to 34% of the 20-year shrinkage occurs in 2 weeks;

40 to 80% of the 20-year shrinkage occurs in 3 months;

66 to 80% of the 20-year shrinkage occurs in 1 year.

The magnitude of shrinkage also depends on the humidity of storage, increasing with low relative humidity, but is unaffected by the rate of drying.

Since the observed shrinkage is governed by the extent of drying that can take place, the size of the concrete member undergoing drying is a significant factor. The size effect can be accounted for indirectly by the ratio of the drying surface to the volume of concrete enclosed within. Ultimate shrinkage decreases as volume-surface ratio increases (22).

Swelling takes place when concrete is cured and stored for prolonged periods in water. This swelling is about six times smaller than shrinkage in air at a relative humidity of 70%, and eight times smaller than shrinkage at 50% humidity (58). Swelling takes place more rapidly than shrinkage and is usually completed in 6 to 12 months, whereas shrinkage increases for several years.

Swelling is caused by water absorption of the cement gel and is accompanied by an increase in weight. The gel particles are forced apart by the absorbed water molecules, and this creates a swelling pressure. The surface tension of the gel is decreased by the ingress of water, and this causes additional small expansion (45). Although drying shrinkage is not completely recoverable, concrete which has been dried in air with a given relative humidity will swell if subsequently placed in an environment of higher humidity (such as water). Usually, the irreversible part of shrinkage is about 0.3 to 0.6 of the drying shrinkage, with the lower value being more common. Reversible deformation, or moisture movement results from subsequent cycles of drying and wetting. Lightweight concrete has a higher moisture movement than concrete made with normal weight aggregate. Also, the magnitude of the moisture movement varies with humidity and the composition of concrete, being smaller the larger the aggregate content (5).

As was mentioned at the beginning of this section, concrete undergoes not only drying shrinkage, but also carbonation shrinkage; the two are quite distinct in nature.

The chemical process of carbonation is as follows. In the presence of moisture, CO_2 in the atmosphere reacts with hydrated cement minerals (the agent being carbonic acid). $Ca(OH)_2$ carbonates to $CaCO_3$, but other cement compounds are also affected, hydrated silica, alumina, and ferric oxide being produced.

Carbonation shrinkage is probably caused by the dissolving of crystals of $Ca(OH)_2$ under the compressive stress imposed by the drying shrinkage, and the depositing of $CaCO_3$ in spaces free from stress. As a result, the compressibility of the cement paste is temporarily increased.

The moisture content of the concrete and the relative humidity of the ambient medium affect the rate of carbonation. Also, the specimen size is a factor, since the moisture released by the reaction must diffuse out in order to preserve the hygral equilibrium between the inside of the specimen and the outside atmosphere. If this diffusion is too slow, the diffusion of CO_2 into the paste is nearly stopped due to the increase of the vapor pressure within the concrete. Carbonation increases the shrinkage at intermediate humidities, but not at 100% or 25%. At 25% humidity, there is insufficient water in the pores of the cement paste for CO_2 to form carbonic acid. At 100%, when the pores are full of water, the diffusion of CO_2 into the paste is very slow. It is also possible that the diffusion of calcium ions from the paste leads to percipitation of $CaCO_3$ which clogs the surface pores

(59).

Figure 1 shows the relation between shrinkage and time for specimens stored at different relative humidities (5).

There are several methods for the prediction of shrinkage, and many are of a similar nature. The European Concrete Committee (8) has proposed the following method for estimating shrinkage deformation. The effective shrinkage strain of an unreinforced concrete prism is defined as

$$\varepsilon_{sh} = k_b k_e k_t \varepsilon_h \tag{1.3}$$

where

k_b depends on the composition of the concrete,

k_e depends on the effective thickness of the member, and is defined as the area of the section divided by one-half of the perimeter in contract with the atmosphere,

k_t depends on the duration of drying and the effective thickness, and

ε_h depends on the relative humidity.

The values of these coefficients, for various conditions, can be found using available tables and graphs (8).

1.4 Creep Behavior

Creep occurs only when concrete is subjected to stress, either external or internal, and can be defined as the increase in strain, with time, under a sustained stress. This stress can be very low,

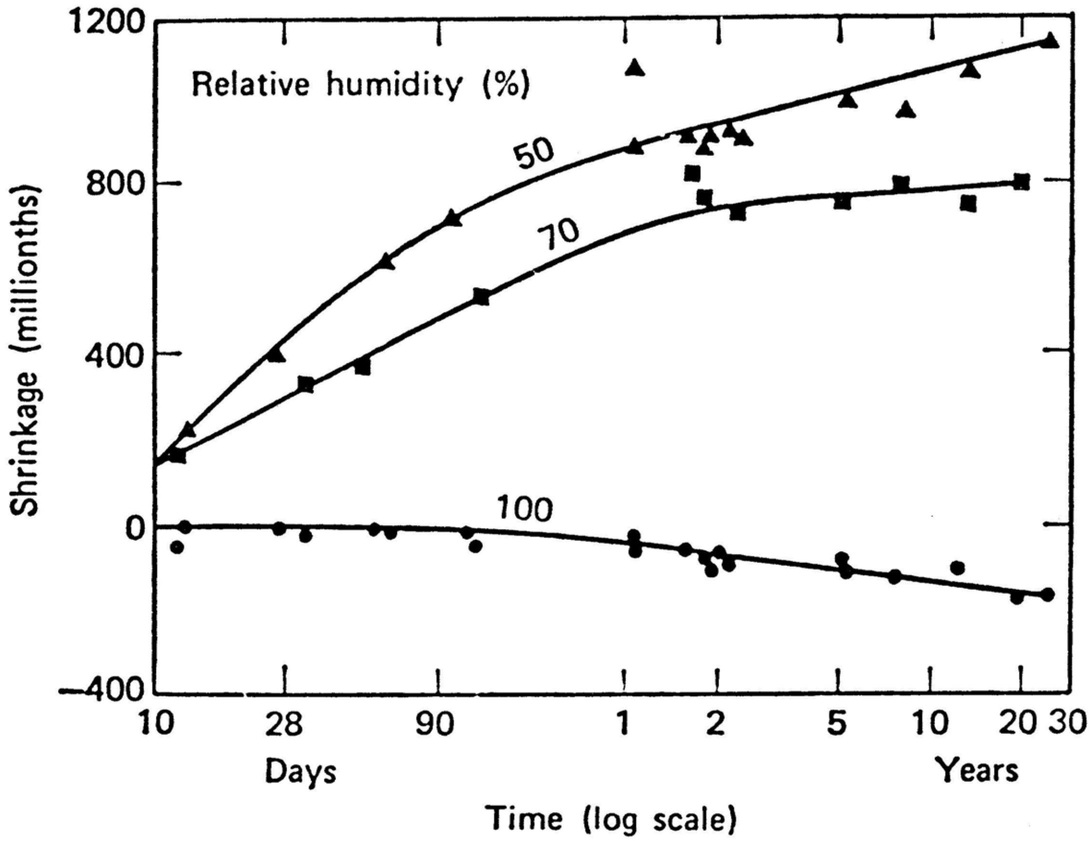


Figure 1 Relation Between Shrinkage and Time for Specimens Stored at Different Relative Humidities

almost approaching a zero value. In general, deformation due to creep is larger than the elastic deformation. For this reason, creep represents an important part of the deformations in concrete. Creep causes displacements and stresses in the structure, however, only in prestressed concrete and slender columns is the strength of the structure adversely affected, or in conditions of high temperature gradients.

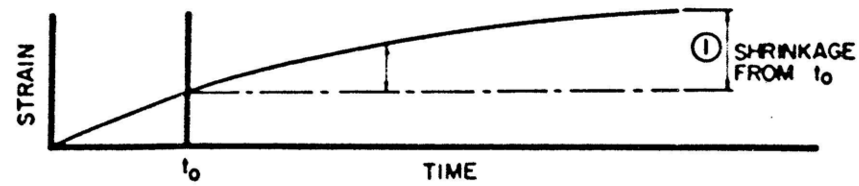
Although the rate of creep is affected by many factors, creep-time curves are all of similar shape. For the usual range of structural concretes, loaded at 28 and 90 days and stored at a relative humidity of 50-100%, the rate of creep is (58):

18 - 35% of the 20-year creep occurs in 2 weeks

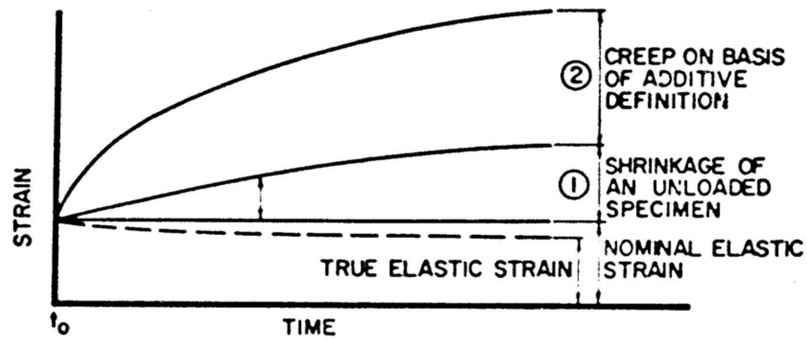
40 - 70% of the 20-year creep occurs in 3 months

64 - 80% of the 20-year creep occurs in 1 year

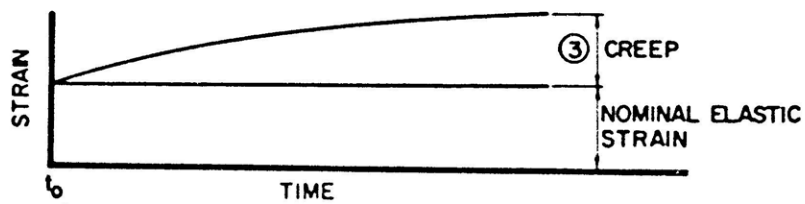
Figure 2 defines the various components of deformation of concrete (42). Figure 2(a) shows the nature of shrinkage alone, and Figure 2(c) defines the nature of creep in the absence of shrinkage or swelling. If a specimen is drying while under load it is usually assumed that creep and shrinkage are additive, as shown in Figure 2(b). Thus, the overall increase in strain of a loaded and drying member is assumed to consist of shrinkage (equal in magnitude to a similar unstressed member), and of creep. However, this assumption is not entirely accurate. Creep and shrinkage are not independent phenomena to which the principle of superposition can be applied. In fact, the effect of shrinkage on creep is to increase the magnitude of creep (41). But in many structures creep and shrinkage occur simultaneously and



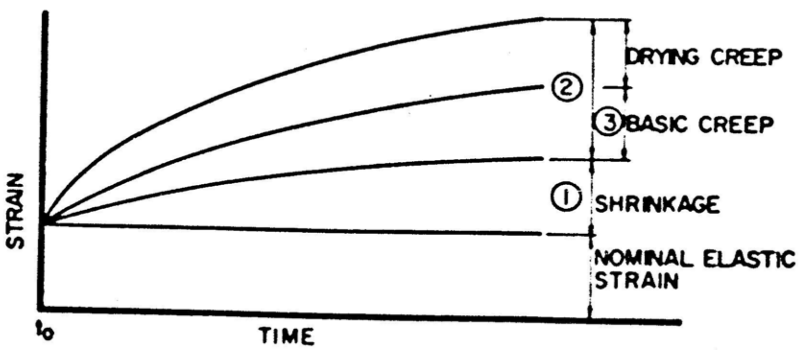
a) SHRINKAGE OF AN UNLOADED SPECIMEN



b) CHANGE IN STRAIN OF A LOADED AND DRYING SPECIMEN



c) CREEP OF A LOADED SPECIMEN IN HYGRAL EQUILIBRIUM WITH THE AMBIENT MEDIUM



d) CHANGE IN STRAIN OF A LOADED AND DRYING SPECIMEN

Figure 2 Definition of Creep Terms

the treatment of the two together is, from the practical and engineering standpoint, often convenient. Hence, while the additive apporach is generally followed, it should be noted that under drying conditions an additional creep, known as drying creep, occurs. When no moisture movement to or from the ambient medium occurs, creep is referred to as true or basic creep (1) [Figure 2(d)]. In general, creep strain is made up of two parts

$$\varepsilon_c = \varepsilon_{BASIC} + \varepsilon_{DRYING} \tag{1.4}$$

Basic creep appears to be made up of a viscous flow part which is totally irrecoverable, and a delayed elastic part which is partially recoverable (1).

$$\varepsilon_{BASIC} = \sigma\beta[\alpha_1(1 - e^{-t/\tau_1}) + \alpha_2(1 - e^{-t/\tau_2}) + \phi t] \tag{1.5}$$

where β is a gel compliance factor. Basic creep is independent of specimen size, and usually also of composition, size and grading of the aggregate, and type of cement used. Only the volumetric composition of the concrete (i.e., β) is involved. A temperature increase results in a higher basic creep, and concrete in relative humidities much below 50% may exhibit lower basic creep. Basic creep is similar for concretes under axial stress and shear stress.

Drying creep has time variation characteristics similar to free shrinkage, and is influenced by the same organismic and environmental factors. It appears to be irrecoverable with respect to stress, but may undergo partial recovery upon restoration of the original moisture content (1).

$$\varepsilon_{DRYING} = \sigma\beta\gamma\varepsilon_s[a + (b/t)] \tag{1.6}$$

where a and b are constants, and ε_s is the free drying shrinkage strain at a given environmental humidity.

Creep occurs in three stages (38). "Primary creep" is the stage during which the strain rate decreases. During this stage slip occurs on closely spaced adjacent planes. When the strain rate becomes constant, the "secondary stage" of creep is attained. During this stage the slip planes bend and develop kinks, and eventually a sub-grain structure results. In terms of types of deformation, delayed elastic deformation can be considered as primary creep, and viscous deformation (i.e., residual deformation) as secondary creep. When the "final state" is reached, the strain rate accelerates. This is also known as tertiary creep. The manner in which the specimen is loaded is responsible for this increase in strain rate during the final stage. At high strain rates necking of the specimen is responsible for the final stage behavior, due to the accompanying stress intensification. At low strain rates the increasing rate of strain during the final stage is the result of microcracks forming at the grain boundaries, accompanied by internal stress intensification. The three stages of creep are shown in Figure 3 (38).

The effects of creep on concrete structures are larger deflections and redistributions of stresses (10, 52). Under conditions of non-uniform stress and temperature, stress redistribution and relaxation

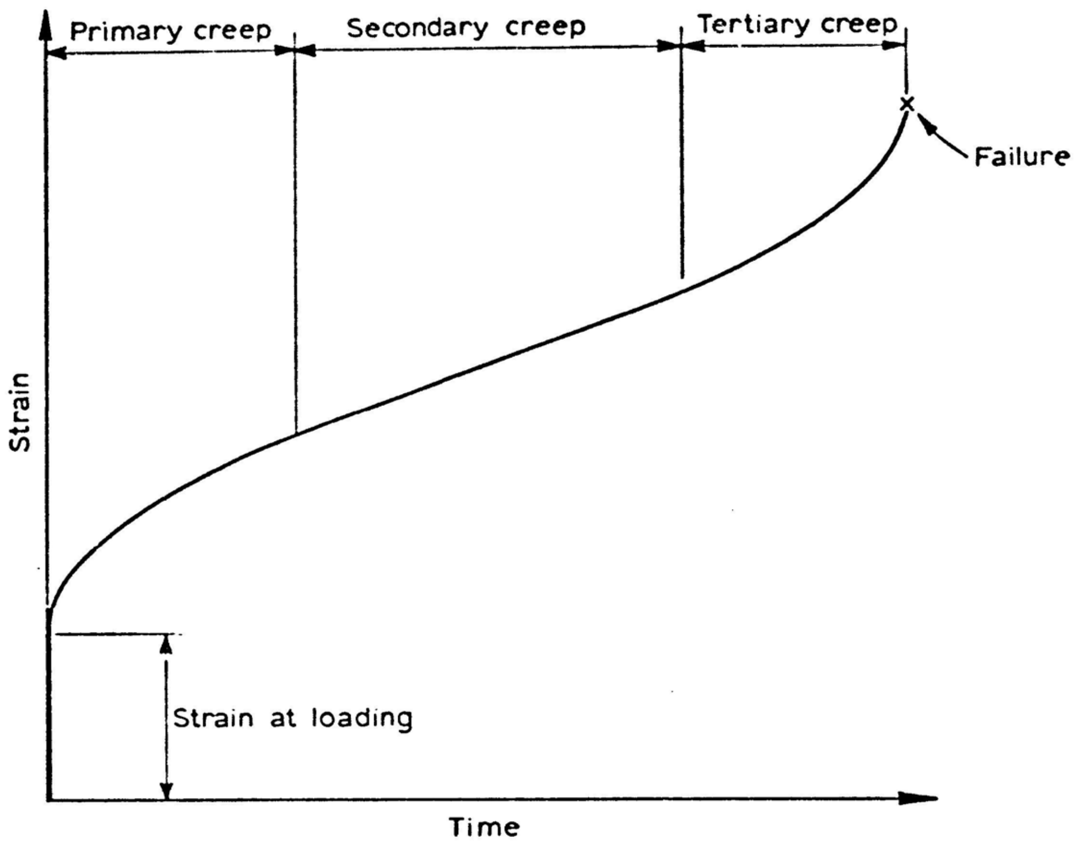


Figure 3 Idealized Creep Curve

takes place, and stresses become time dependent quantities. If the concrete is subjected to a constant strain, there will be a progressive decrease in stress with time. Figure 4 shows this decrease in stress with time (5).

1.5 Creep Recovery

When the sustained load is removed from concrete which has undergone creep, there is an instantaneous recovery, ε_{iR}, which represents the elastic strain corresponding to the stress removed and the elastic modulus at the given age. Generally, this recovered elastic strain is somewhat lower than the instantaneous elastic strain at loading, ε_{ii}. Following this instantaneous recovery is a gradual, relatively small time-dependent decrease in strain called creep recovery, ε_{tR}, which reaches a limiting value, $\varepsilon_{\infty R}$. As a result, there remains an irrecoverable strain, or permanent set. At any time $t > t_1$, where t_1 is the time when the load is removed, the total strain in the concrete is

$$\varepsilon_T = \varepsilon_{ii} + \varepsilon_s + \varepsilon_c \text{ (at } t_1) - \varepsilon_{iR} - \varepsilon_{tR} \tag{1.7}$$

For typical concrete mixes, creep recovery is approximately 10 - 20% of the creep strain, but the higher the applied stress, the lower the percent recovery (49). If the concrete is reloaded at a later time, instantaneous and creep deformations develop again, as shown in Figure 2. The shapes of the creep curve and the creep recovery curve

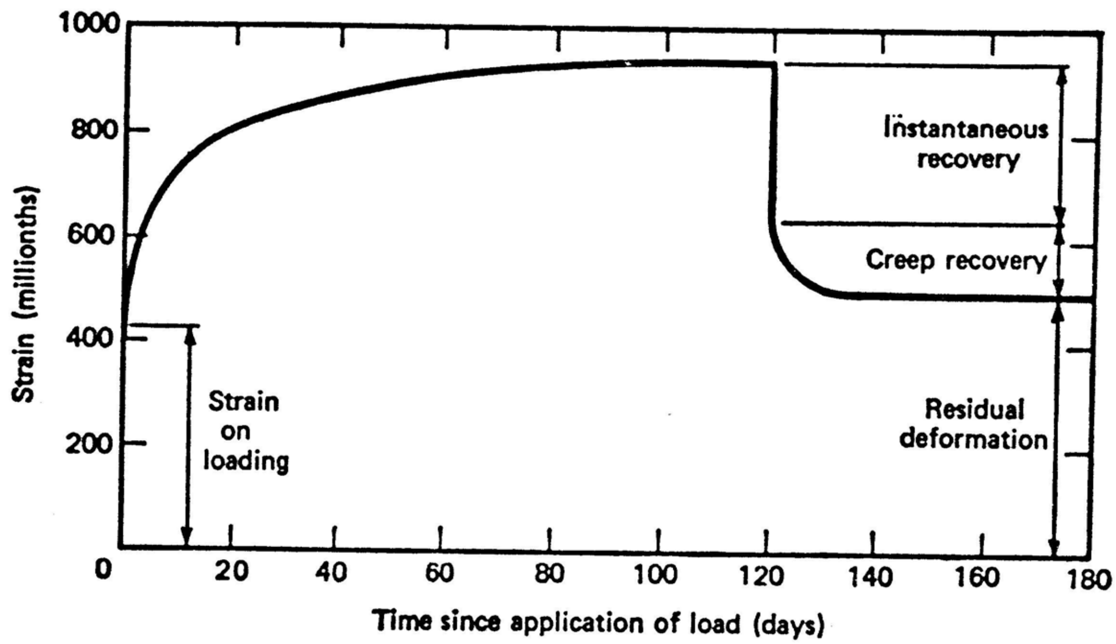


Figure 4 Typical Creep Curve

are similar, but the recovery approaches its maximum value much more rapidly than does the creep. The curves in Figure 5 show this relationship (38).

Both the instantaneous recovery and the total time-dependent recovery are linear functions of stress up to at least 65% of the ultimate strength, except for a very rich 1:1 mix which Ali et.al. found exhibited nonlinearity of the time-dependent recovery above 50% of the ultimate strength (1). Hornby found that creep recovery increases with temperature, and is 65% greater at 75°C than at 25°C (24).

Ali et.al. suggested a complex rheological model describing creep and creep recovery, including the partial recovery of the seepage effect. For a concrete unloaded at t_1, the time-dependent recovery is

$$\varepsilon_{tR} = \sigma\alpha_K(1-e^{-t/\tau_K})(1-e^{-(t-t_1)/\tau_K}) + \sigma C_R\alpha_1(1-e^{-t_1/\tau_1})(1-e^{-(t-t_1)/\tau_1}) \tag{1.8}$$

where C_R is a coefficient representing the amount of creep due to seepage which is recoverable. If it is completely recoverable $C_R = 1$, if completely irrecoverable $C_R = 0$ (1). The maximum creep recovery is given by

$$\varepsilon_{\infty R} = \sigma\alpha_K(1 - e^{-t_1/\tau_K}) + \sigma C_R\, \alpha_1(1 - e^{-t_1/\tau_1}) \tag{1.9}$$

In summary, creep recovery is due to interaction between the elastic and viscous phases of the concrete. It is time-dependent due to the internal redistribution of moisture which takes place

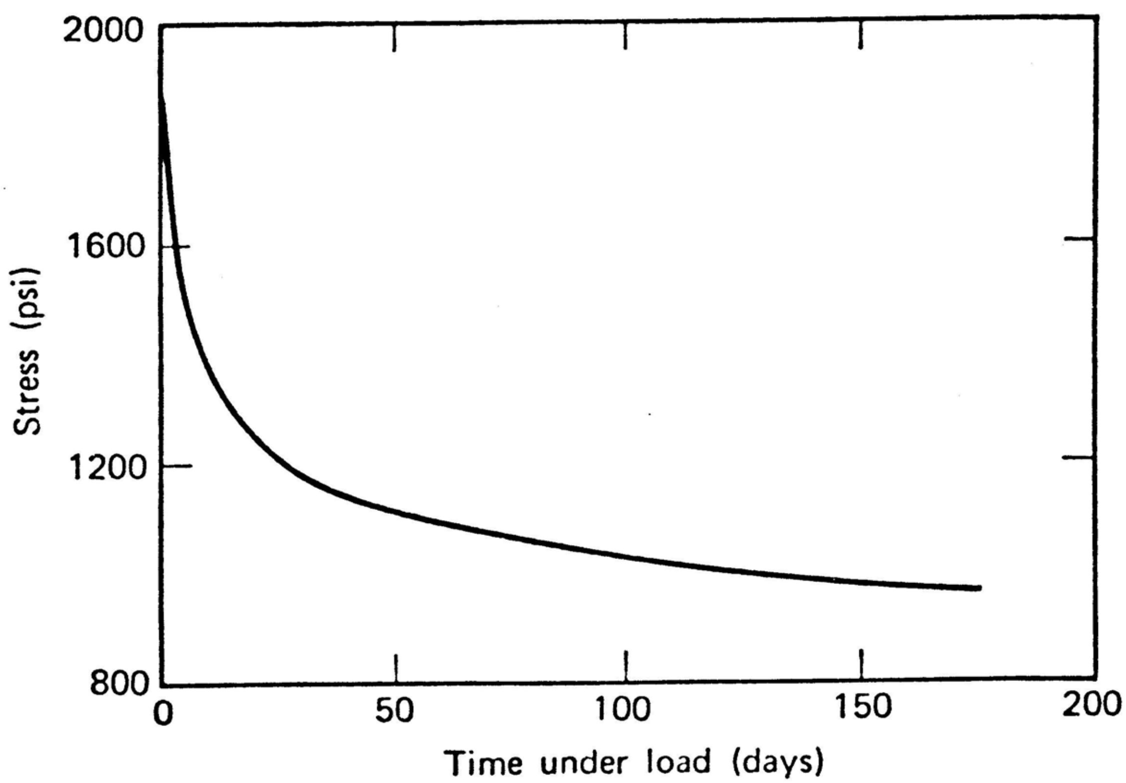


Figure 5 Relaxation of Stress Under a Constant Compressive Strain of 400×10^{-6}

slowly upon removal of the load. The recovery is both elastic (particles returning to their original positions and configurations), and inelastic (particles taking up new positions and configurations.

In an actual reactor vessel, pressurizing and heating can be expected to unload much of the concrete and cause creep recovery. However, it is unlikely to have any significant long-term effects under steady conditions unless it influences initial crack formation. Under cyclic loading conditions, however, creep recovery will occur at every cycle, and could be important.

In general, creep and shrinkage, and subsequent recovery, have the following adverse effects:

1. Steel reinforcement located in compression areas (of beams and columns) is subjected to severe stress increases which may reach the yield point of mild steel.

2. In pretensioned and post-tensioned concrete structures there is a gradual loss of prestress.

3. In statically indeterminate structures additional stresses or secondary moments may be created.

4. In columns, particularly slender columns, creep can increase the lateral displacement, thus decreasing the buckling load factor.

5. Most importantly, creep and shrinkage cause large stresses in reactor containment structures, where high temperatures and temperature gradients exist.

CHAPTER 2

CREEP THEORIES

2.1 Introduction

Although the field is narrowing, the mechanism of concrete creep is still subject to some controversy. The difficulty lies in the fact that a satisfactory theory of creep must explain, in a unified way, the behavior of creep under the various environmental conditions and states of stress which influence it. One can not have a theory which assumes different physical mechanisms for each set of conditions. For this reason it is difficult to propose definite conclusions on the mechanism of creep. Perhaps the only general statement that can be made is that the presence of some evaporable water is necessary for creep to occur. However, creep behavior at high temperatures suggests that at that stage the water no longer plays a role, and the gel itself is subject to creep-deformation.

2.2 Mechanisms

Although seepage of water to the outside of concrete may take place in drying creep, the occurrence of creep in mass concrete suggests that this process is not essential to basic creep. However, internal seepage of water from the absorbed layers to voids, such as capillary voids, is possible. Internal seepage is possible under any storage condition, and the fact that creep of non-shrinking specimens is independent of the ambient humidity indicates that the fundamental cause of creep in air and water is the same.

The creep-time curve exhibits a definite decrease in its slope. This may occur due to the same mechanism throughout, however it is reasonable to assume that after many years under load the thickness of the absorbed water layers is reduced to such an extent that no further reduction could take place under the same stress. Yet creep after more than 30 years has been observed (58). Therefore, it is probable that the slow, long-term part old creep is due to causes other than seepage, but that the deformation can occur only in the presence of some evaporable water (42). This suggests viscous flow, or sliding between the gel particles.

Because of coarsening of the gel particles associated with the formation of new bonds and stabilization in the deformed position, only a small part of creep due to seepage is reversible. At high stresses a part of the overall measured creep may be due to growth in microcracks, but at working loads a significant contribution of microcracking to creep is unlikely. The six prevailing hypotheses which attempt to explain the mechanism of creep are presented and discussed below. They are

1. Plastic deformations
2. Viscous flow
3. Seepage of gel water
4. Delayed elasticity
5. Nonuniform shrinkage
6. Intercrystalline deformations

2.3 Plastic Deformation Theory

The plastic theory suggests that creep is a result of crystalline flow, that is, slipping of material along planes. This theory was one of the first to explain the creep phenomenon in concrete, and at that time creep was known as "plastic flow" (41).

After a limiting stress is exceeded, plastic deformations occur. The plastic deformations are irrecoverable and nonlinear with applied stress, and result from intracrystalline slips and local rupture of the hardened cement paste. Although creep does have an irrecoverable part, it does occur at very low stresses, and creep is linear with stress up to about 50% of the ultimate. Neither this behavior, nor the sensitivity of creep to moisture and moisture movement can be explained in terms of plastic deformation mechanisms. Thus, this hypothesis might contribute significantly only at stress levels near the ultimate. Today investigators account less for the crystalline slipping as a main factor causing creep.

2.4 Viscous Flow Theory

Thomas (57), Glanville (14), Reiner (48), and Freudenthal (11) have considered hardened cement paste as a viscous fluid surrounding the loose and relatively rigid aggregate particles. The viscous theory suggests that creep is the result of viscous flow of the concrete against and around the aggregate particles, with transfer of more loads from the cement gel to the aggregate.

This concept offers plausible explanations for linearity of creep strain with stress, the absence of a limiting stress for creep to occur, stress relaxation at a constant strain, the sensitivity of creep to temperature, and the largely irreversible nature of long-term creep. However, it can not fully explain creep recovery upon stress removal, change of volume during creep, progressive reduction of the creep rate with time, and the sensitivity of creep to moisture change.

2.5 Seepage Theory

Hardened cement paste has been considered as a limited swelling gel, whose equilibrium with its solid skeleton and external load is determined by the vapor pressure of the gel water (46). The gel water seepage theory (34), which is similar to the theory of consolidation in soil mechanics, envisages a disturbance of this equilibrium under applied load and its gradual re-establishment by the exchange of moisture with the environment. Creep is the volume change accompanying the resulting moisture movement, that is, creep is the result of the seepage of water outside of the cement gel.

When external loads are applied on the concrete, the pressure on the water is increased. More loads are progressively applied on the solid material as the water flows outside the cement. This results in a volume decrease. This theory is parallel to the theory of concrete shrinkage, except that in shrinkage water is expelled from the gel by drying, not by loads.

The gel water seepage theory appears to explain the marked increase in creep under compression with simultaneous drying, but if this theory were true we would expect a specimen under sustained compression to undergo a greater gel moisture loss than an unloaded specimen, roughly in proportion to the corresponding creep and shrinkage deformations. However, several tests report little or no effect of applied load on the moisture lost by a drying specimen (39). Also, exchange of moisture results in increased creep irrespective of the direction of such exchange of the applied stress (20). Significant creep occurs even in the absence of moisture exchange. Hence, these observations weaken the conventional seepage hypothesis.

2.6 Delayed Elasticity Theory

The morphology of hydrated cement indicates the presence of both crystalline and noncrystalline components of colloidal size with the associated absorbed moisture. Under load, the gel could behave as a composite body of elastic and viscous phases which could interact, resulting in delayed elastic behavior. To a limited extent, concrete creep does exhibit such a behavior, however, this mechanism can not offer an explanation for the influence of moisture exchange on creep.

Freyssinet has proposed another delayed elastic mechanism to explain creep deformations in terms of the changes in the surface tension forces arising in the capillary pores of hardened cement paste (13). However, this hypothesis has been questioned (46). Also, creep deformation of concrete with practically empty capillaries is not significantly lower than that for saturated concrete.

2.7 Nonuniform Shrinkage Theory

The presence of differential shrinkage stresses due to nonuniform drying has been considered partly, or wholly, responsible for the phenomenon of creep in concrete (43). However, the concept that creep is entirely the result of restrained shrinkage has been seriously challenged (1). Only a small part of the increase in creep with simultaneous drying, especially at high stresses, may be explained on this basis.

2.8 Intercrystalline Deformation Theory

Imperfectly formed crystal lattices suffer viscous deformation under sustained stress, and these zones can exist not only where the crystalline components of the cement gel grow into each other, but also at the gel-aggregate interface (20). Although these deformations progress very slowly, the almost constant rate of flow could result in sizeable deformations after a long time. However, creep rate has been observed to decrease with time not remain constant.

2.9 Conclusions

None of the above theories provide a convincing explanation of the sensitivity of creep to moisture change, although they do appear to explain certain aspects of creep behavior. The seepage mechanism offers the most promising theory in this direction, but not in the conventional form.

Ali and Kesler suggest an hypothesis based on a reinterpretation of the seepage mechanism and its integration with the visco-elastic behavior of the cement gel (1). Creep deformation is explained partly in terms of the modification, by the applied stress, of the shrinkage or swelling resulting from changes in moisture content, and partly in terms of visco-elastic deformation of the structural elements of the gel.

The following behavior under various conditions of moisture exchange and applied load can be anticipated:

1. Free shrinkage is less than shrinkage under compression and more than shrinkage under tension.
2. Free swelling is more than swelling under compression and less than swelling under tension.

Hence, creep is expected to increase with moisture exchange, irrespective of the mutual directions of the load and the moisture movement. Such increase in creep entails no significant change in the moisture movement relation to an unloaded control specimen.

Now, creep does occur in the absence of any moisture exchange, although large creep deformations occur when there is simultaneous moisture loss. This creep is the result of mechanisms other than the stress-modification of shrinkage and swelling. Delayed elastic action and intercrystalline deformations, that is visco-elasticity, appear to best account for creep in the absence of moisture exchange. The interaction of the crystalline and amorphous phases of the cement gel and the absorbed moisture could account for the partly recoverable and partly nonrecoverable response under sustained load.

Almost all of the observed characteristics of concrete creep under moderate stress levels may be explained in terms of the stress-modification of shrinkage or swelling to variation of gel moisture, and the visco-elastic deformation of the gel and its associated moisture. Thus, concrete creep may be considered, broadly speaking, as composed of two practically independent components, caused by distinctly different groups of mechanisms:

1. Basic creep, e.g. that part which can occur independent of moisture exchange. This corresponds to the visco-elastic behavior of the gel. The viscous flow part is totally irrecoverable, while the delayed elastic part is partially recoverable.
2. Drying or wetting creep, e.g. the additional creep over the basic which occurs due to simultaneous moisture exchange with the environment. Such creep is the result of the modification of shrinkage or swelling by the applied stress. It appears to be irrecoverable with respect to stress, but may undergo partial recovery upon restoration of the original moisture content.

In summary, time-dependent deformations under load appear to arise in concrete mainly from the imperfectly crystalline colloidal components of the hydrated cement and the associated absorbed moisture. Also, the aggregate may contribute to this deformation.

Basic creep may be considered as a process of molecular diffusion and shear deformation of the gel and the absorbed water under load, not entailing any loss or gain of total moisture content. Interaction with the crystalline components results in the partly viscous and partly delayed elastic behavior exhibited. These mechanisms are temperature dependent.

Drying creep may be considered as due to a mechanism similar to that involved in free shrinkage due to desiccation. The removal of water by evaporation brings the extremely large surfaces of the colloidal structure closer, thus mobilizing strong surface forces, resulting in shrinkage of the gel structure. With no applied stress this shrinkage is much smaller than the amount of gel moisture withdrawn. Applied stress can be seen as modifying the extent of the shrinkage, without affecting the loss of moisture.

CHAPTER 3

SIGNIFICANT CREEP PARAMETERS

3.1 Introduction

In Chapter 2 the difficulty in finding a single theory which accurately explains the behavior of concrete creep under various conditions and states of stress was described. This difficulty is due to the fact that there are many variables which affect concrete creep. Several of the more important influencing factors are listed below:

1. Temperature
2. Stress-strength ratio
3. Ambient humidity
4. Age at initial loading (or dgreee of hydration)
5. Water-cement ratio
6. Concrete strength
7. Curing
8. Composition of cement
9. Concrete mix proportions
10. Aggregate
11. Admixtures
12. State of stress
13. Shape and size

The significant factors affecting concrete creep, and the extent of their influence on basic and drying creep, are listed in Table 1 (1).

TABLE 1—SIGNIFICANT FACTORS AFFECTING CREEP OF CONCRETE UNDER MODERATE STRESS

Significant factors affecting creep	Influence on basic creep		Influence on drying creep
	Delayed elastic action, partly recoverable on removal of stress	Viscous flow, irrecoverable on removal of stress	Stress-modified shrinkage, partly recoverable on restoration of moisture
Mix proportions	Primary	Primary	Primary
Degree of hydration	Primary	Primary	Primary
Moisture content	Secondary	. . . .	Secondary
Moisture exchange	. . . .	. . . .	Primary
Temperature	Primary	Primary	Secondary
Aggregate properties			
1. rheology	Secondary	Secondary	Secondary
2. permeability	. . . .	. . . .	Primary
3. surface texture	. . . .	. . . .	Secondary
4. shrinkage, swelling	. . . .	. . . .	Secondary
Stress magnitude	Primary	Primary	Primary
Stress distribution	. . . .	. . . .	Primary
Specimen size	. . . .	. . . .	Primary

3.2 Temperature

The influence of temperature on creep is of interest in connection with the use of concrete pressure vessels, particularly in prestressed concrete pressure vessels. The effect of temperature on concrete creep was studied by Hannant (19). In the tests performed, the temperature of the concrete was varied from 23°C to 93°C under several loading conditions. Results showed that the creep strain varied linearly up to 77°C, and non-linearly above that point. The creep at 77°C was about 4.5 times that at 23°C. Measurements from the Wylfa prestressed concrete reactor vessel showed that creep at 150°F was 2.8 to 4.6 times greater than creep at 70°F (52).

The work of Ross and England (10) shows that the rate of creep increases with temperature up to about 70°C, and thereafter decreases somewhat up to about 100°C. At higher temperatures the rate of creep increases again such that a high creep is attained. Figure 6 (5) illustrates these findings.

Sarne suggests that the decrease of creep, observed by some investigators, at high temperature is probably due to the specimen size, and is not an accurate reflection of the creep behavior (52). In massive structures, where drying is much slower even at high temperatures, the increase in creep should be used at all times.

The difference between the temperature during the period of loading, and during the period preceeding loading must be recognized. Whereas the former has a direct affect on creep behavior, the latter

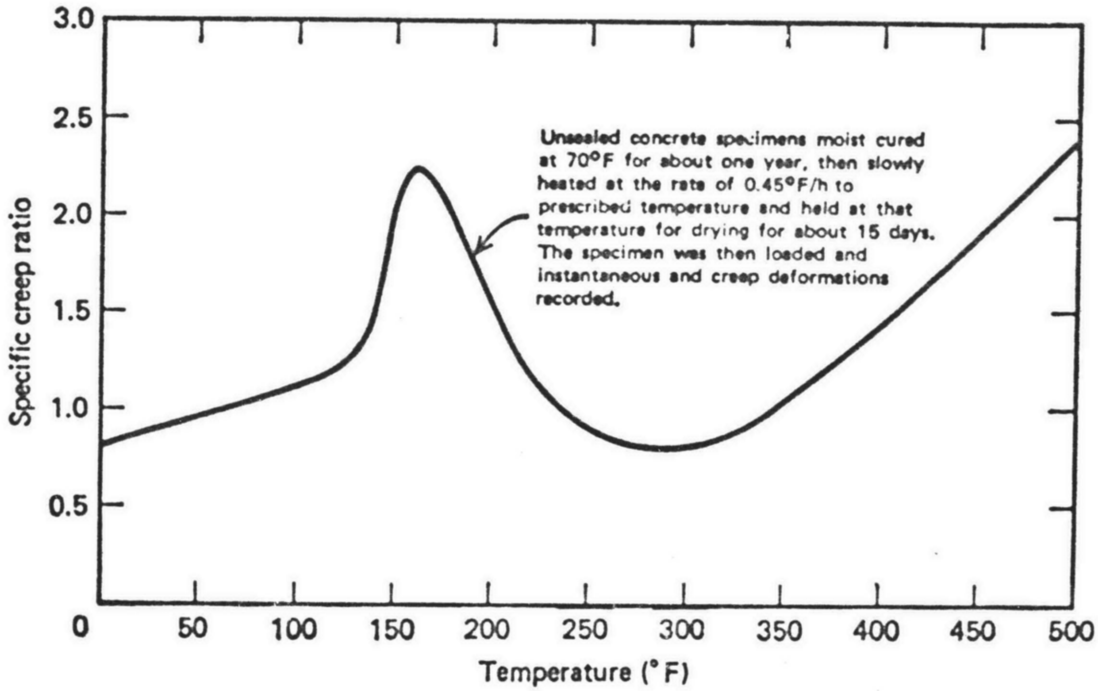


Figure 6 Specific Creep at Different Temperatures, as a Ratio of Specific Creep at 70°F After 1 Year Under Load

incluences the basic principles of concrete, such as its maturity, the structure of the gel, etc.

3.3 Stress-Strength Ratio

Within the range of working stresses, experiments have shown creep to be proportional to the applied stress, except with specimens loaded at a very early age (15, 27). The proportionality has a lower limit of virtually zero stress, and an upper limit of approximately 40-60% of the ultimate strength of the concrete. However, different observers suggest this upper limit may be anywhere from 23% to 75%. It seems safe to conclude, however, that within the range of working stresses, the proportionality holds good.

Tests made by Jensen and Richart show the proportionality of creep to stress-strength ratio to exist up to about 0.6, with increased creep at higher ratios (27). On the average they found:

Stress-strength ratio	0.2	0.4	0.6	0.7	0.8	0.9
Creep (arbitrary units)	0.2	0.4	0.6	0.83	1.23	2.06

Gopalakrishan et. al. found that for constant temperature, creep varies linearly with the stress-strength ratio. Their results suggest the following creep strain factor to account for the stress-strength ratio (52):

$\sigma < 0.35\ f'_c$ FACTOR = 1.0

$\sigma > 0.35\ f'_c$ FACTOR = $1.0 + (\sigma - 0.35\ f'_c)/0.45\ f'_c$

This factor will double the creep strain if the stress-strength ratio is equal to 0.8.

When the stress-strength ratio of a concrete compression specimen is 40-60%, severe internal microcracking occurs. As a result, once the cracking has accelerated, the creep behavior changes. This upper limit of proportionality rises with an increase in the strength of the concrete, i.e., it rises with the duration of the load.

Above the limit of proportionality, creep increases with an increase in stress, at an increasing rate. There exists a stress level above which creep produces time failure (about 70-80% of the short-term static strength). Figure 7 (5) shows the development of strain for different stress-strength ratios.

3.4 Ambient Humidity

Numerous tests have shown that creep increases with a decrease in the relative humidity of the surrounding medium. Creep may be 2 to 3 times greater at a relative humidity of 50% than at 100%, as illustrated in Figure 8 (58). However, two points should be noted.

First, ambient relative humidity affects creep if drying takes place while the specimen is under load. But if the concrete has reached hygral equilibrium prior to loading, the magnitude of creep is independent of the relative humidity of the surrounding madium (42). Therefore, it appears that it is not the ambient humidity that is a factor in creep, but the process of drying while the concrete is

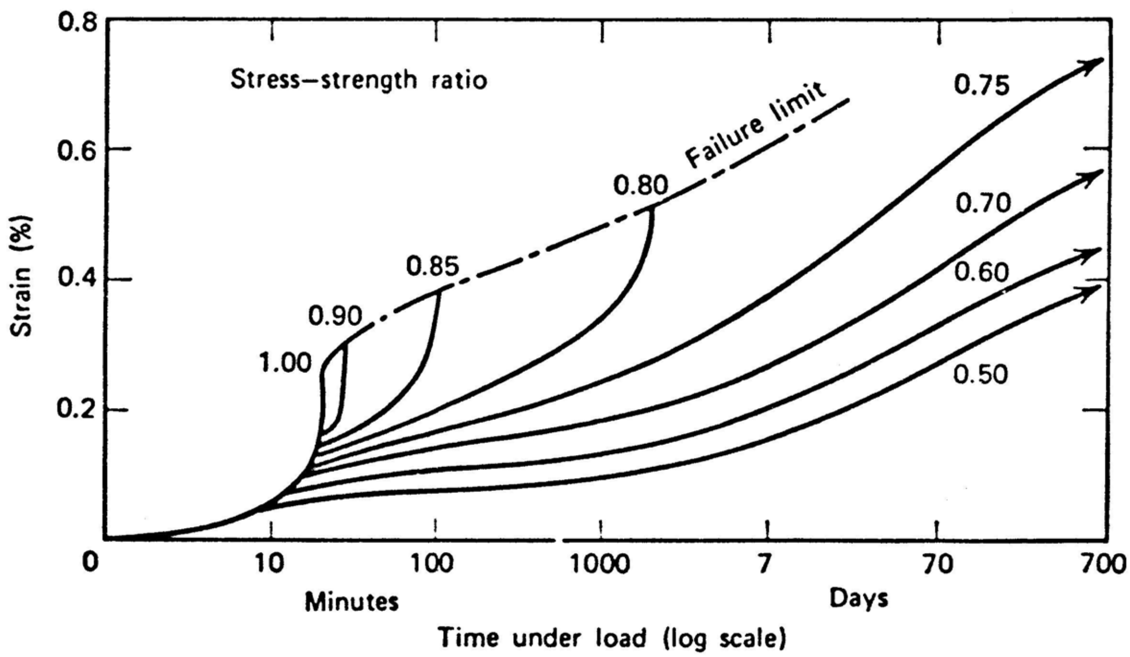


Figure 7 The Strain-Time Relation for Different Stress-Strength Ratios

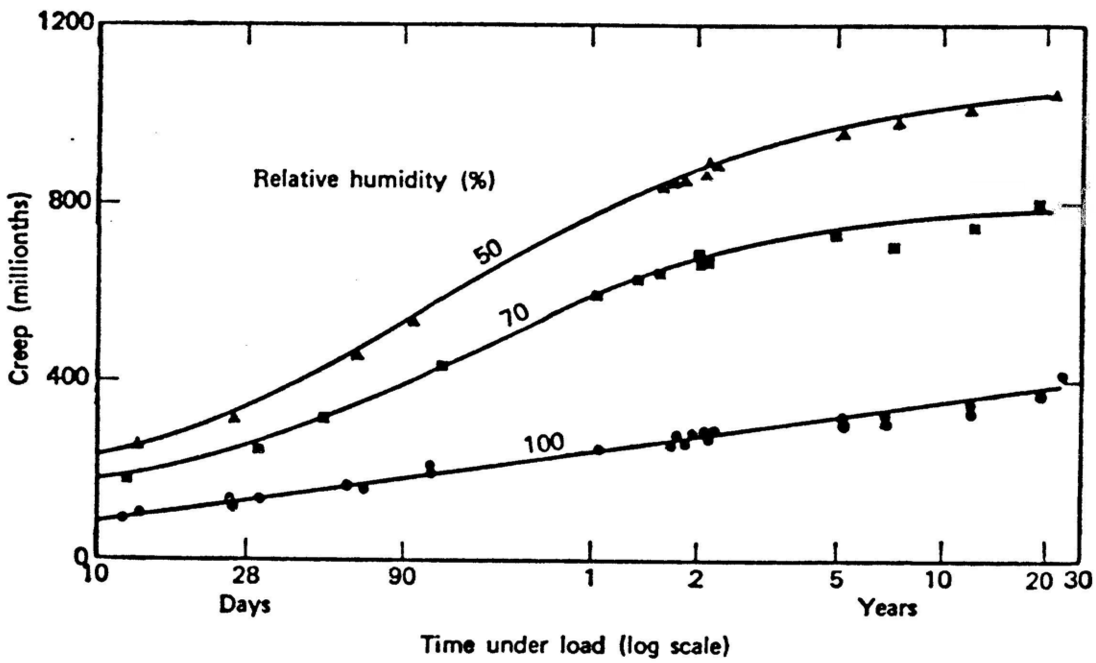


Figure 8 Creep as a Function of Relative Humidities

subject to creep.

Secondly, Hansen found, in 1958, that alternating the ambient relative humidity between two limits results in a greater creep than that obtained at a constant humidity within those limits. An effect of this behavior is that laboratory tests under constant humidity underestimate the actual creep under conditions of practical exposure.

3.5 Other Factors

Any comparison of creep behavior must take into account the degree of hydration of the cement at the time of application of the load, since different type cements have different rates of hydration, even though they have similar ultimate strengths. Comparison should be made under a load where the stress-strength ratio is the same. Under these conditions the type of cement, i.e., its composition or fineness, does not affect creep, in the first approximation (38).

The age of the concrete influences creep in so far as it influences the degree of hydration and the development of strength. Ross and Neville have shown creep to correlate well with maturity (40). Under conditions where no sensible variation in the degree of hydration occurs, the age at loading ceases to influence creep. For example, the influence of the age at loading is much smaller in the case of dry-cured concrete. Also, at later ages the rate of creep becomes independent of the age at loading.

Creep increases with an increase in the water-cement ratio, as Figure 9 shows (5). Creep is approximately proportional to the square of the water-cement ratio. Both the water-cement ratio and the aggregate-cement ratio influence concrete creep. Although both factors control the water content of the mix, the influence of the aggregate-cement ratio on creep is the lesser of the two.

With this influence of the water-cement ratio, and because the strength of structural concrete is a practical concern, relating creep to strength is both convenient and fairly reliable. Table 2 shows typical values which were observed by Klieger (29).

TABLE 2 - CREEP OF CONCRETES OF DIFFERENT STRENGTHS

Compressive Strength at Time of Application of Load, psi	Ultimate Specific Creep 10^{-6} per psi	Ultimate Creep at a Stress-Strength Ratio of 0.3, 10^{-6}
2000	1.40	933
4000	0.80	1067
6000	0.55	1100
8000	0.40	1067

The quality of the cement paste has a direct influence on creep. This can be expressed approximately by saying that for a constant cement paste content, and the same applied stress, creep is inversely proportional to the strength of the concrete. Thus, strength is a

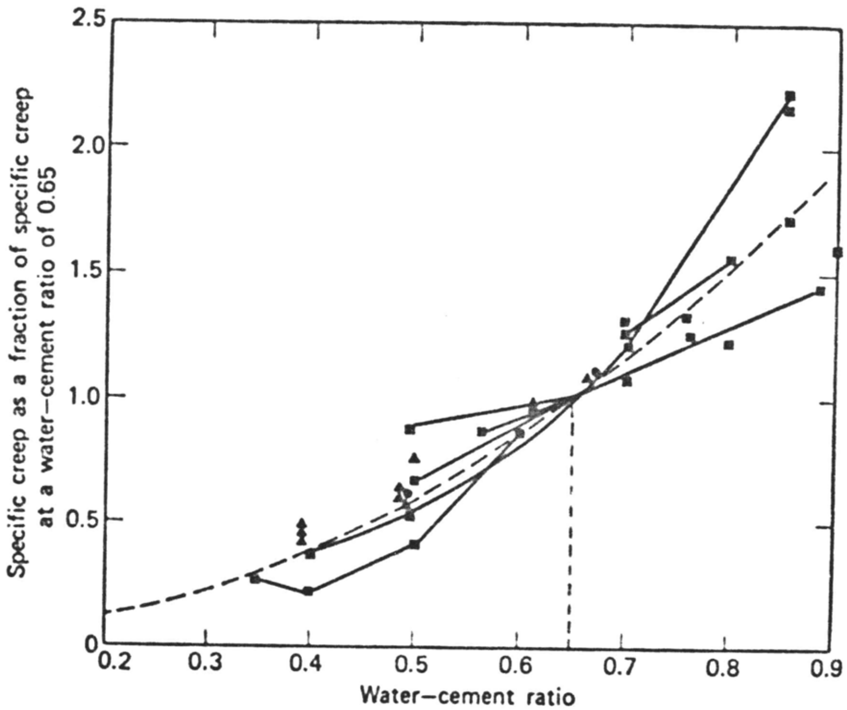


Figure 9 Specific Creep as a Function of Water-Cement Ratio

convenient, but approximate, measure of the state of the cement paste, i.e., its composition and degree of hydration.

Although normal weight aggregate is not likely to creep to an appreciable extent, it does influence concrete creep. Since cement paste is subject to creep, and aggregate is not, the effect of the aggregate is to reduce the effective creep of concrete. Also, the higher the modulus of elasticity of the aggregate, the greater the restraint offered by the aggregate to the potential creep of the cement paste. The porosity and absorption of the aggregate influence creep, as they effect the transfer of moisture within the concrete (42).

Although use of lightweight aggregate results in much higher creep than normal weight aggregate, there is no fundamental difference between the two as far as creep is concrened. The higher creep of concrete made with lightweight aggregate reflects only the lower modulus of elasticity of the aggregate. There is no inherent difference in the behavior of coated and uncoated aggregate, or between those obtained by different manufacturing processes. However, all aggregates do not lead to the same creep.

Not enough is known about the effects of entrained air, admixtures, and pozzlans, however it appears that entrained air probably increases creep. The reason is that entrained air can be considered as aggregate with zero elastic modulus (28).

Creep under uniaxial tension, its magnitude and creep-time curves, is similar to creep in compression. The behavior is also similar to creep under torsional loading.

Measured creep decreases with an increase in the size of the specimen, but at a thickness of greater than about three feet the size effect is no longer noticeable (42). The influence of size on creep is greatest during the initial period after the application of the load. Beyond several weeks the rate of creep is the same in specimens of all sizes. The size effect applies not to the basic creep, but to the increase in drying creep.

Work at the Portland Cement Association Laboratories indicates that both creep and shrinkage are functions of the surface-volume ratio. Thus it may be concluded that when a free surface is absent, creep is unaffected by the size of the member. In fact, in mass concrete, size effects are not present.

The rate of creep decreases progressively with time. The average increase in creep with time is shown in Table 3 (58).

TABLE 3 - AVERAGE INCREASE IN CREEP

Creep after 1 year	1.00
Creep after 2 years	1.14
Creep after 5 years	1.20
Creep after 10 years	1.26
Creep after 20 years	1.33
Creep after 30 years	1.36

The longest period for which creep data are available is around 30 years, and here a small but measurable rate of creep was observed (58). It is notpossible to say whether the rate ever vanishes to zero, in which case creep approaches asymptotically a limiting value, or if the rate becomes stabilized at some value, in which case the creep increases indefinitely.

CHAPTER 4

ANALYTICAL FORMULATIONS

4.1 Introduction

Due to the large number of variables which influence creep of concrete, it is impossible to make a single mathematical model which accurately accounts for all of them. Many creep expressions have been suggested which attempt to account for some of these variables, and under certain conditions, they can predict creep behavior with varying degrees of accuracy. Material and physical properties are included by varying fixed parameters in the creep equation to fit experimental results. Also, since creep of concrete is known to be a visco-elastic phenomenon (63), both material and physical models can be used to represent creep behavior.

4.2 Visco-Elastic Material Behavior

The visco-elastic behavior of creep of concrete means that creep is a function of not only the strains and stresses in the concrete at the time of the creep, but that it is also a function of the history of the strains and stresses in the concrete. Linear visco-elastic material models adequately predict behavior, when stresses are less than approximately 50% of the ultimate strength.

Since a unique relation between stress and strain, or between stress rate and strain rate, characterizes an elastic material, and a relation between stress and strain rate characterizes a viscous fluid, relations between stress, strain, stress rate and strain rate necessarily characterize creeping concrete which contains both elastic solid and viscous fluid components. The elastic component has perfect "memory" of its initial state, while the viscous component has no memory at all. Hence, the visco-elastic material has an imperfect memory of limited duration.

Two simple physical models of a visco-elastic material are (12):

(a) An incompressible viscous fluid with a high concentration of elastic particles suspended in it, where an applied stress produces viscous flow and elastic deformation of the solid particles.

(b) An elastic sponge with its pores filled with an incompressible viscous fluid. In this case, an applied stress produces elastic deformation of the sponge which increases gradually as the fluid is squeezed out of the pores.

The mechanical behavior of creeping concrete can be idealized in terms of combinations of these two models. Over a limited time scale the characteristic features of the visco-elastic behavior of creep of concrete can be illustrated by simple superposition of a linear elastic and a linear viscous one-dimensional relation.

Considering a substance described by model (a), the total rate of flow of this model at any time t is the flow rate of the viscous fluid augmented by the strain rate imposed on the solid particles. This is the case since the load is carried by the suspended particles and the surrounding fluids. Hence, the total strain rate for model (a) is obtained from the equations describing the elastic strain rate and the incompressible, linear, viscous fluid in uniaxial flow (12).

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\lambda}\sigma = \frac{1}{E}[\frac{d\sigma}{dt} + \frac{1}{\tau}\sigma] \tag{4.1}$$

where λ is the coefficient of viscous traction, and τ denotes a material time parameter, $\tau = \lambda/E$.

Integrating Equation 4.1 for constant stress (σ = constant, and $d\sigma/dt = 0$), the equation is obtained

$$\varepsilon = \sigma t/\lambda + \varepsilon(0) \tag{4.2}$$

or

$$\varepsilon = \frac{\sigma}{E}(1 + \frac{t}{\tau}) = \frac{\sigma}{E}\phi(t) \tag{4.3}$$

where the integration constant $\varepsilon(0) = \sigma/E$ represents the instantaneous elastic strain at time $t = 0$. The second term represents a strain which is increasing (linearly) with time, that is, creep. Equation 4.3 represents the simplest creep equation, and $\phi(t)$ characterizes the material in creep; it is known as the creep function. The time constant τ characterizes the initial speed of stress relaxation. It is a measure of the imperfect memory of the medium, and represents the time at which the stress has decreased to 1/e of its initial intensity. When $\tau \to 0$ the relaxation is instantaneous (viscous fluid with no memory), and when $\tau \to \infty$ no stress relaxation occurs (elastic solid with perfect memory).

The equation for the visco-elastic material described by model (b) is obtained by considering that the sponge and the fluid can not deform independently, but they must carry the load by joint deformation. Hence, the total stress is the sum of the elastic stress and the viscous stress (12)

$$\sigma = E\varepsilon + \lambda\frac{d\varepsilon}{dt} = E(\varepsilon + \tau \frac{d\varepsilon}{dt}) \tag{4.4}$$

This visco-elastic equation is similar to Equation 4.1 obtained for model (a) in that it also has a time parameter . However, it differs from Equation 4.1 in that the reduction to an elastic medium is obtained by setting $d\varepsilon/dt = 0$, instead of $d\varepsilon/dt = \infty$.

4.3 Physical Models

The linear visco-elastic material models described in Section 4.2 can be represented by a physical system of springs and dashpots arranged in various combinations (63). The two basic arrangements are a spring and a dashpot in series or parallel (44). The former is known as the Maxwell element (Figure 10) and represents a material described by model (a) above. The latter is known as the Kelvin (Voigt) element (Figure 10), and represents a material described by model (b) above.

The Maxwell element, described by Lewis et. al. (33), represents creep at constant stress, stress relaxation at constant strain, and has a strain-rate-dependent stress-strain diagram. Upon loading after

MAXWELL ELEMENT

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial \sigma}{\partial t} \cdot \frac{1}{E} + \frac{1}{\eta} \sigma$$

$$\sigma = Ae^{-(E/\eta)t}$$

KELVIN ELEMENT

$$\sigma = E\varepsilon + \eta \frac{\partial \varepsilon}{\partial t}$$

$$\varepsilon = (1-e^{-(E/\eta)t}) \frac{\sigma_o}{E}$$

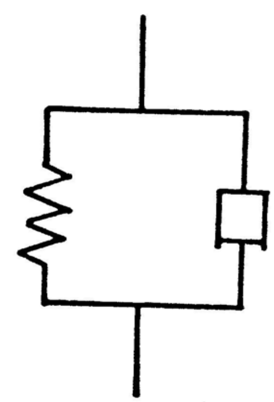


Figure 10 Visco-Elastic Models

a creep test, only the elastic strain is recovered (Figure 11). These are the principal features of a rate sensitive material under quasi-static conditions.

Since Maxwell models represent initial elastic response, plus permanent creep, they do not allow for creep recovery after load reversal. This is the principal shortcoming of this model, and can cause problems when analyzing structures which have a cyclic load history, such as a pressure vessel which undergoes annual shut down for refueling. One possibility is to use a percentage of the instantaneous elastic strain for creep recovery (2), or a percentage of the creep strain could be used (52). Another possibility, suggested by the creep equation presented by Argyris et. al. (2), is to use a linear function of time under load to calculate the non-recoverable part.

The use of a series of Kelvin elements was described by Zienkiewicz et. al. (66). For a Kelvin element, the stress across the element is σ, and the relative displacement is the creep strain ε_c (52). The creep expression is of the form (52)

$$\frac{d\varepsilon_c}{dt} = a\,\sigma - b\varepsilon_c \tag{4.5}$$

and, for several elements

$$\Delta\varepsilon_c = [(\textstyle\sum a_i)\,\sigma - \sum b_i\,\varepsilon_{ci}]\,\Delta t \tag{4.6}$$

The creep strains and stresses decay exponentially. In the exponential

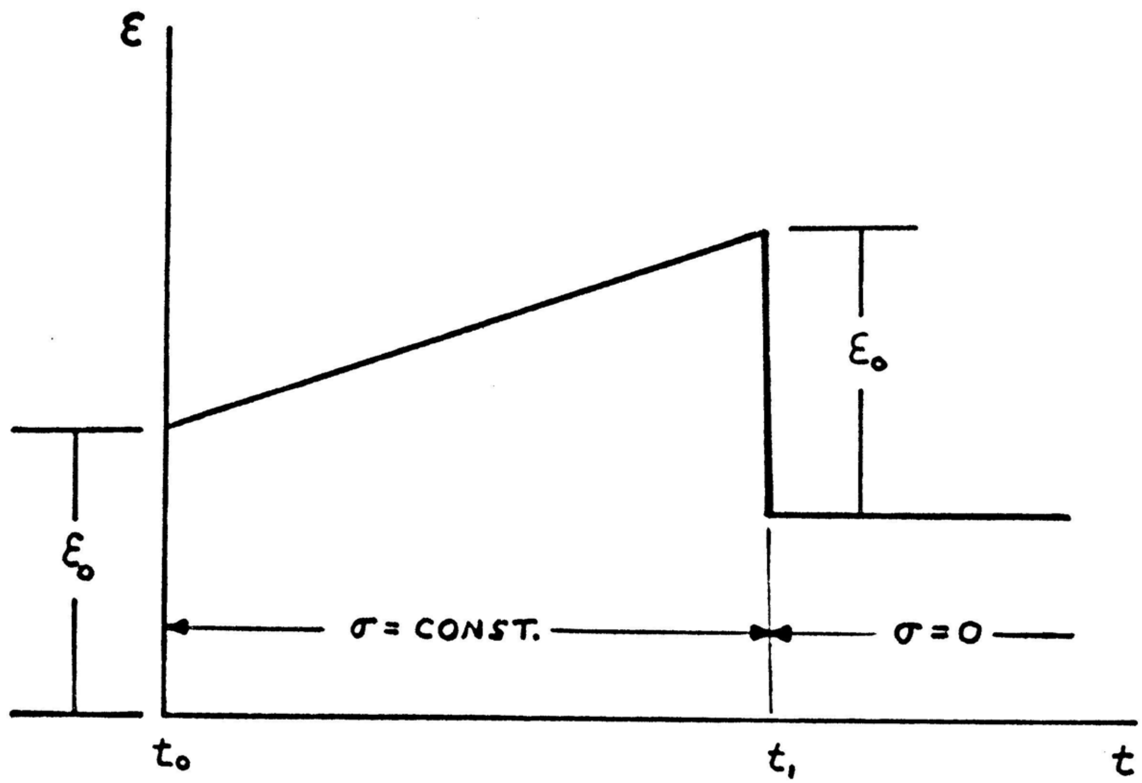


Figure 11 Response of Maxwell Element

form, the creep equation is

$$(\varepsilon_{ci})_{t+\Delta t} = (\varepsilon_{ci})_t \; e^{-bi\Delta t} + \frac{a_i}{b_i}\,\sigma(1 - e^{-bi\Delta t}) \qquad (4.7)$$

where a_i and b_i are material properties for the i^{th} Kelvin element.

If the creep equation can be written in this form, then the concrete behavior can be analyzed numerically using Kelvin elements (52). The degree of accuracy of the model depends on the number of Kelvin elements used in series. In general, two elements are adequate to represent the visco-elastic creep behavior of a concrete structure (64, 65, 66).

The Kelvin element shows no stress relaxation. It exhibits a delayed elastic response, which, upon unloading after a constant stress creep test, is totally recoverable at time $t \rightarrow \infty$ (44) (Figure 12). Althouth it can not be used to represent initial elastic response, results using Kelvin elements can be added to those obtained from an elastic analysis. Results using both Maxwell elements and Kelvin elements have been shown to give good agreement with results measured from existing structures and with those of other creep models (33, 52).

A model called a Burgess body has been proposed by Argyris et.al. (2). It consists of one Maxwell element in series with a number of Kelvin elements. Its equation is of the form

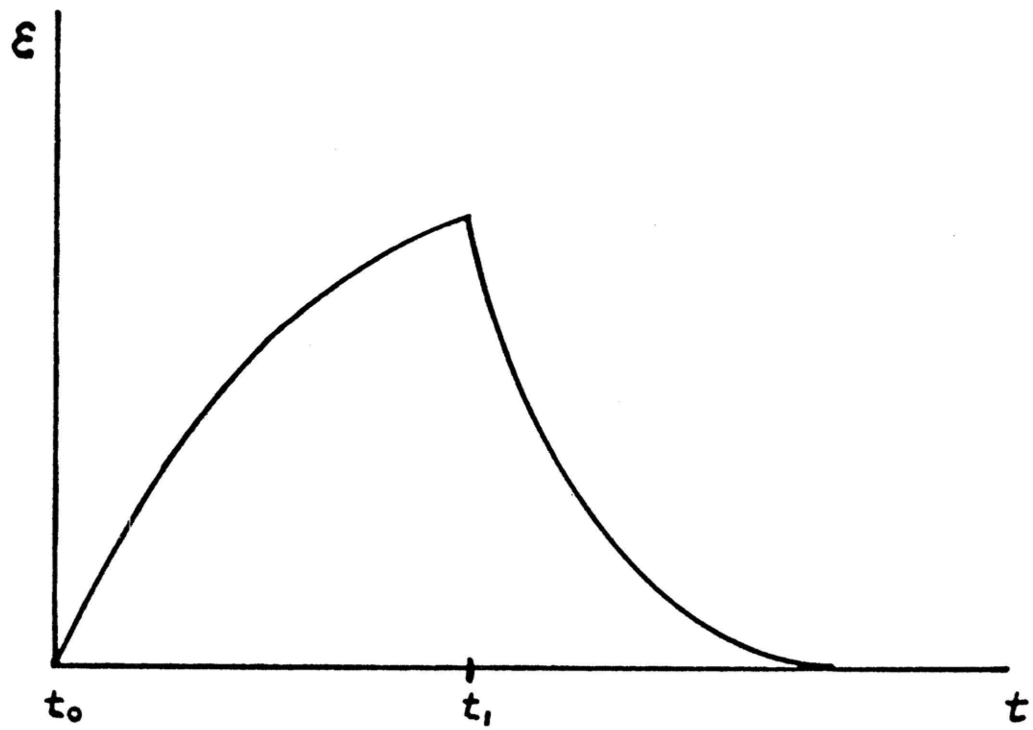


Figure 12 Response of Kelvin Element

$$\varepsilon(t) = \frac{\sigma(t)}{E(t,\tau)} + B_1^2 \int_0^t \Psi[T(\tau),t]\sigma(\tau)d\tau$$

$$+ \sum_{\alpha=2}^{n} B_\alpha^2 \int_0^t \Phi[T(t),\tau]e^{-c_\alpha[\mathcal{S}(t) - \mathcal{S}(\tau)]} \sigma(\tau)d\tau \qquad (4.8)$$

The first part corresponds to the instantaneous elastic response, the second to permanent flow, and the third to delayed elastic strain which is recoverable. Age and temperature are included as parameters, and if delayed elasticity and permanent flow are affected by age and temperature in the same manner, then

$$\Phi[T(t),\tau] = \Psi[T(\tau),t] \qquad (4.9)$$

Results obtained using this model are in good agreement with those from other experimental concrete tests. However, this agreement only confirms the effectiveness of curve fitting to experimental data, not the creep law itself.

4.4 Concrete Creep Equations

Many other creep-time equations have been suggested, most of which are of a hyperbolic or an exponential type. These have been developed mainly by experimentation, instead of by attempting to explain creep as a visco-elastic material. In some cases, creep is expressed by a "standard" curve, modified by factors for particular mix proportions and storage conditions.

In 1960 England represented the behavior of heated concrete beams by a visco-elastic model consisting of a spring of elastic modulus E, in series with a dashpot of time-dependent viscosity $\eta(t)$. He proposed the following relationship (26)

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \sigma f(\Theta)\,\frac{dc(t)}{dt} \tag{4.10}$$

where $f(\Theta)$ is a function of temperature, Θ

and $c(t)$ is the specific thermal dreep of concrete

$$\frac{dc(t)}{dt} = \frac{1}{\eta(t)}$$

Roll developed a mathematical model to describe the mechanism of creep (7), and on the basis of his model proposed the following expressions for creep rate and creep strain

$$\dot{\varepsilon}_c = Ce^{-b}e^{-t/T_M} + s\phi_1(e^{-t/\tau_1} + 100e^{-100t/\tau_1}) \tag{4.11}$$

$$\varepsilon_c = Ce^{-b}\,T_M(1-e^{-t/T_M}) + s\alpha_1(2-e^{-t/\tau_1} - e^{-100t/\tau_1}) \tag{4.12}$$

where

s = uniaxial sustained compressive stress

t = time under sustained stress

$C = (\text{time})^{-1}$

and all other parameters are model constants which are mix- and size-dependent.

The model constants were then estimated using the results of tests on creep, creep rate and creep recovery, and a sample of Roll's results follows:

TIME (days)	OBSERVED (10^{-4} in/in.)		CALCULATED
5	2.6	3.0	2.5
10	3.1	3.6	3.0
25	4.5	4.9	4.3
80	6.2	6.9	6.7
101	6.8	7.6	7.2
164	7.8	9.0	8.2
210	8.0	9.3	8.5

A study by Lewis et. al. (32) in 1969 used a creep equation proposed by Hanson of the form

$$\varepsilon_t = \frac{\sigma}{E} + \sigma \cdot F(K) \log_e (1 + t) \tag{4.13}$$

where F(K) is a function of age and temperature. Analytical results gave a lower bound to measurements taken from the Oldbury pressure vessel (9).

Another creep equation used in the analysis of an existing structure is of the form

$$\varepsilon_s = at^n \tag{4.14}$$

where ε_s is the specific creep strain with regard to stress, and a and n are functions of temperature and age. Results correlated with measured stresses.

Saugy (53) performed a nonlinear analysis using a creep equation of the form

$$\varepsilon = \varepsilon_i(1 + a(t-\tau)^n) \tag{4.15}$$

where a and n are functions of temperature and age, and ε_i is the strain for the present loading. Results showed that stresses were redistributed, increasing the integrity of the vessel up to about 1.5 times the design pressure.

Lorman proposed the following hyperbolic expression (34), which causes creep to approach a finite limiting value, $c_\infty = m\sigma$,

$$c = \frac{mt}{n+t}\,\sigma \tag{4.16}$$

where t is the time since application of the load, and m and n are constants.

Another hyperbolic expression was suggested by Ross (50), which leads to a limiting value of $c_\infty = 1/b$.

$$c = \frac{t}{a+bt} \tag{4.17}$$

where a and b are constants.

Thomas proposed an exponential expression (57) which leads to values of the ultimate creep, c_∞, closely agreeing with those of Lorman and Ross

$$c = c_\infty[1-e^{-A(t+d)^g-dg}] \tag{4.18}$$

where A, d, and g are experimentally determined constants. Thomas found that the ratio of the limiting creep to that occurring during the first year under load varies little, and suggested that it does not exceed 4/3 for specimens loaded at an age of 28 days. For specimens loaded at later ages the creep after one year will be smaller, and thus the ratio is an increasing function of the age at loading, approaching a limiting value of $c_{\infty} = 1.45c_1$. However, the disadvantage to using this expression is that in order to find the limiting value of creep, it is necessary to know the creep after one year under load, which is not very convenient in practice.

McHenry's exponential expression also assumes that creep is proportional to the amount of potential creep remaining (35). The specific creep is given by

$$c = (\alpha+\beta e^{pT})(1-e^{rt}) \tag{4.19}$$

where T is the age at the time of application of load (T > 5 days), t is the time since the application of load, and α, β, p, and r are constants.

A simple exponential equation was suggested by Shank (54)

$$c = at^{1/b} \tag{4.20}$$

where a is a constant, and b is a coefficient dependent on the concrete properties. Shank's equation is easy to use, however it can only be used to estimate creep up to about one year under load, since for longer periods the creep increases at too great a rate. Also, the expression postulates an indefinite increase in creep.

A similar approach was adopted by Saliger (51)

$$c = \alpha_t \cdot \sigma \tag{4.21}$$

where α_t is a creep coefficient of the form

$$\alpha_t = A \sqrt[3]{t} \tag{4.22}$$

Although this expression does not cause creep to reach a finite limit, Saliger assumes that the ultimate creep is reached at an age of 30 months. However, this would mean that concrete loaded at an age of 30 months would show no creep, which is not the case. Therefore, Saliger's equation can not be used for loading ages greater than a few months.

Saliger also suggested that concrete subjected to a sustained load will respond elastically to any additional live loads. That is, live loads produce only elastic deformations. He also postulated that strains produced by a given stress are independent of any stress applied either before or after that stress. McHenry first postulated this principal of superposition, and applied it to creep recovery. Although the principal of superposition introduces a fixed bias, it is convenient for most practical purposes.

The U.S. Bureau of Reclamation made an extensive study of creep of concrete (67), and they found that specific, or ultimate, creep can be expressed as

$$c = F(T)\log(t+1) \tag{4.23}$$

where T is the age at which the load is applied, F(T) is a function representing the rate of creep with time, and t is the time in days. In these studies creep was estimated from the change in the elastic properties of the concrete. Thus, for a given mix, the modulus of elasticity (as a function of time) is a useful parameter in estimating creep. However, as in many of the previous expressions, the necessary information can be obtained only after a long period of time.

Greenbaum and Rubenstein used an equation of the following form to analyze stresses in a thick-walled pressure vessel (17)

$$\varepsilon_c = a\sigma^b t^c \tag{4.24}$$

where a, b, and c are constants. They used two different equations, depending on specific structure and material conditions.

$$\varepsilon_c = 6.4 \times 10^{-18}\sigma_e^{4.4}t \tag{4.24a}$$

$$\varepsilon_c = 6.4 \times 10^{-18}\sigma_e^{4.4}t^{0.7} \tag{4.24b}$$

The results showed good agreement when compared to closed form solutions. Creep caused redistribution of stresses, and a large reduction of high stress concentrations.

The following equation suggested by England and Ross and given in Reference 66 was used by Sarne (52) in a nonlinear finite element analysis of concrete structures

$$\varepsilon_c = 4.0T[(1-e^{-1.5t})+(1-e^{-0.035t})]10^{-6}\sigma \tag{4.25}$$

where T is the temperature in °C, t is the time in days, and σ is the stress in Ksi. In this case, the value of concrete creep strains is a function of the duration of the load and the concrete temperature.

Hannant approximated creep strain with a log curve in two parts (26)

$$c(t) = 1.51 \times 10^{-9} \log_{10} (1+t) \quad \text{for } 0 \leq t \leq 50 \text{ days} \tag{4.26a}$$

$$c(t) = [3.62 \times \log_{10} (1+t) - 3.58] \times 10^{-9} \quad \text{for } t > 50 \text{ days} \tag{4.26b}$$

However, the constants used in these equations render them valid for only a specific concrete mix, that used in the Oldbury pressure vessels.

The above creep equations require that constants be determined empirically for the specific material and physical conditions being studied. That is, limited time creep tests must be made using the actual mix and storage conditions. The longer the time of the tests, the better the predictions will be. For a 60 day test the error is about 15%. However, the time necessary to obtain reasonably accurate results is very often not convenient, or possible, for practical purposes. Attempts have been made, using the creep data available in technical literature, to calculate coefficients and parameters for creep prediction under any conditions.

Jones et.al. propose a "standard" creep curve (28) which can be modified for the particular slump, air content, cement type and content, percent fines, relative humidity of storage, thickness of member, and age at loading, by means of correction factors. Although this method was developed for lightweight concrete, it appears to be valid for normal weight concrete.

Wagner used a similar approach (60). "Standard" values of ultimate specific creep are modified by various coefficients to suit the particular conditions. However, Wagner's predicted values of ultimate creep compare poorly with 4/3 of the measured one year creep, due in part to the properties of the aggregate, which are not considered in his curves.

In summary, if it is possible to make 60 day creep tests, any of the equations described above can be used, depending on the desired facility of use and accuracy, to give satisfactory results. If no tests can be made, the methods developed by Jones and Wagner must be used. These yield results which may not be sufficiently accurate in structures sensitive to creep, with errors of ± 30% commonplace.

4.5 Numerical Solution Methods

Several methods of numerical solution of the creep equation have been presented in technical literature. The oldest method is the effective modulus method. Because it is one of the simplest methods, its use is widespread (33, 65). The method consists of a single elastic solution using an effective (or sustained) Young's modulus,

$$E_{eff} = 1/J(t,t_o) = E(t_o)/[1 + \phi(t,t_o)] \tag{4.27}$$

where $J(t,t_o)$ is the creep function, and is equal to the strain at time t (including the elastic strain) caused by a unit constant stress acting since t_o. $\phi(t,t_o)$ is the creep coefficient and is equal to $E(t_o)J(t,t_o)-1$. Creep strains are calculated for a given stress as

$$\varepsilon_c = \frac{1}{E_{eff}} \cdot \sigma \tag{4.28}$$

In Reference 65 Zienkiewicz gives $1/E_{eff}$ as

$$\frac{1}{E_{eff}} = \frac{1}{E_o(t)} + \int_0^t c(T,t,\tau) \frac{\partial}{\partial\tau} (\sigma) d\tau \tag{4.29}$$

where $c(T,t,\tau) = (1 - e^{a(t-\tau)}) \frac{1}{E_1(T)}$

For a long-time load

$$\frac{1}{E} = \frac{1}{E_o(T)} + \frac{1}{E_1(T)} \tag{4.30}$$

The advantage that only one elastic solution is necessary to calculate strains is offset by several disadvantages. When aging is negligible, such as for old concrete, excellant accuracy is obtained. However, when aging is a factor and when stress variations occur, the assumption that stresses remain constant leads to overestimation of creep. Under pressure loads where stress changes are usually small, acceptable answers are obtained. Under temperature loads where stress changes more than 30% of the initial values are possible, the error will be large (47). Also, the effective modulus method incorrectly predicts all creep as fully recoverable.

In 1967 Frost presented the age-adjusted effective modulus method. In this method computation of changes from t_o to t due to creep is reduced to a single elastic analysis with inelastic strains, as in the effective modulus method. The effective modulus is adjusted by means of an aging coefficient to yield more accurate results using the age-adjusted effective modulus. This method gives exact solutions when the stress is constant (creep test), when the strain is constant (stress relaxation test), and when $\varepsilon = \varepsilon_1 \ \phi(t,t_o)$, which is typical of straining structures by differential creep or shrinkage and of buckling deflections. The age-adjusted effective modulus method is the closest approximate method to the exact solution.

The steady-state stress solution gives a stationary state of stress which would be achieved after a long period of time. A drawback to this method is that actual creep strains may not be large enough to cause stresses to approach a steady-state condition. The steady-state bound may lead to tensile values which are too high during stress reversals. (25)

The rate of creep method was proposed by Glanville (4), but was first applied to more complicated problems by Dischinger. (In German it is known as "Dischinger's Method" and in Russian as the "Theory of Aging"). This incremental procedure has the advantage that boundary conditions, body forces, thermal strains, and material properties can be changed at each time increment. The drawbacks are that no delayed recovery is modeled, creep due to stress changes is underestimated, and a negligible creep is predicted for loads applied to very old concrete. In contrast to the effective modulus method, however, the rate of creep method gives good results for loads applied to very young concrete.

The solution technique assumes that for each time step, the changes in strains and stresses are small compared to the total strains and stresses which exist in the structure before the creep occurs. The steps used for solution are:

1. An elastic solution is first obtained for the applied load.
2. Using the stress from the elastic solution, the creep strains are calculated from the creep law.
3. These creep strains are then applied to the structure using equivalent body forces.
4. New displacements, strains, and stresses are found. These new creep strains and stresses are considered as initial strains and stresses for the next iteration.
5. Step 2 is then repeated for each time increment using the new stress values.

This method will not diverge if the incremental creep strains are less than the total elastic strains before creep occurs.

Two methods of accumulating creep strains are commonly used (17). The time hardening creep law assumes the creep rate is dependent on the instantaneous stress and temperature and the time since the load was applied. The strain hardening law assumes the creep rate is dependent on the instantaneous stress, temperature and accumulated creep strain. The strain hardening law is more accurate, but the time hardening law presents fewer analytical difficulties. However, for constant loads, both laws provide virtually identical results, with the strain hardening law giving slower relaxation of stresses.

When large deformations are involved, the number of time steps necessary may make the rate of creep method impractical because of the number of time increments necessary. Therefore, when using this method the allowable size of time increments should be considered. In general, the time increments can increase at a rapid rate after the first few days. In any case, the large computer storage space needed may make this method unattractive for large problems.

England used a rate of flow method to represent the creep compliance function as a sum of a delayed elastic component, which is recoverable, and flow, which is not recoverable (4)

$$J(t,t') \simeq \frac{1}{E_d} + \frac{\phi_f(t) - \phi_f(t')}{E(t_o)} \tag{4.31}$$

where

$$\frac{1}{E_d} = \frac{1}{E(t')} + \frac{\phi_d}{E(28)}$$

and ϕ_d, ϕ_f are creep coefficients for delayed elastic strain and flow, respectively. Although this method appears to yield good results for creep recovery after sudden complete unloading, only simple problems have been solved to date.

Arutyunian (3) proposed the following approximation

$$J(t,t') \simeq \frac{1}{E(t')} + \frac{\phi_u(t')}{E(t')} (1 - e^{-(t-t')/\tau_1}) \tag{4.32}$$

where for long-time creep effects good values are

$$\tau_1 = 50 \text{ days} \quad \text{and} \quad \phi_u = \phi(\infty,7)1.25t'^{-0.118}$$

This corresponds to an age-dependent Kelvin model coupled in series with an age-dependent spring. Relaxation problems can be reduced to first-order differential equations for internal force rates or displacement rates, and a similar equation relates strain rates and stresses. The Arutyunian method has found widespread use in Eastern Europe, since in contrast with the effective modulus and the rate of creep methods, the proper ratio between the creep of young aging and old non-aging concrete can be taken into account. However, the analysis is much more complicated than that required for other methods, and has not always proven to be the most accurate.

In general, the age-adjusted modulus method is found to be the most accurate of those presented here, and, with the effective modulus method, is the simplest to use. These methods reduce the creep problem to a single elastic analysis, and unlike the rate of creep, rate of flow and other methods, no differential equations need to be solved. The rate of flow method, with effective modulus treatment of delayed elastic strain, appears as the next best method, and should be used when the table of aging coefficients required by the age-adjusted effective modulus method is unavailable. In the case of prestress loss, all methods give relatively equal results. In slender columns, with axial loads exceeding the long-time buckling load, the prediction of all approximate methods is poor.

The differences between methods are significant in cases of stress relaxation, shrinkage stresses, creep buckling deflections, and straining of structures by differential creep due to nonuniformity of concrete age. For stress redistributions in cracked reinforced concrete cross sections, the differences are unimportant. Also, contrary to widespread opinion, the effective modulus and rate of creep methods do not always give opposite bounds on the exact solution, as in the case of creep buckling (4).

All of the methods described above are linear, and satisfy the principle of superposition. As a result, two kinds of error are encountered in the prediction of creep effects. The first error originates in the stress-strain law, and the second is due to the approximate nature of the method of analysis. Short of a nonlinear creep law, nothing better than superposition is possible.

McHenry has developed a superposition law which provides reasonable prediction of strain variation with time, provided the concrete stress does not exceed about 50% of its ultimate strength. The superposition model tends to overestimate creep recovery, usually by about 12% (6). The major advantage to the model is that once the creep under a load has been determined to the point where additional calculations will yield little change, there is no need to store the particular creep history. The only problems arise when loads are short-term, with durations of less than about 90 days, and it must be determined whether creep of the load removal will give under or over conservative results. The problem could possibly be overcome by using a longer time under load for the initial load, or a shorter creep recovery of the load removal. This assumes that any additional changes past this time for each load will offset each other. When using superposition for old concrete, problems arise since the change in age during loading will have little effect on the creep curve. Upon removal of the load, a similar creep curve may be generated, thus negating the original creep.

4.6 Multiaxial Creep

All of the preceeding creep equations and solution methods have been for the uniaxial case, however multiaxial creep states are important in many structures, such as pressure vessels. Biaxial and triaxial creep states can be considered by using a creep Poisson's ratio. Not much data is available on the change of creep Poisson's ratio with temperature and time, but it has been considered to be a constant (19, 52). In general, a value equal to the elastic Poisson's ratio has been suggested (6, 52). Some variation in creep Poisson's ratio between uniaxial and multiaxial states of stresses are indicated by the results given in Reference 15, with the lower creep for the multiaxial state of stresses. For triaxial compression, creep Poisson's ratio is largest in the direction of the smallest principal compressive stress.

Previous tests have been made which report the Poisson's ratio of creep of concrete to be from $\nu_c = 0.0$ to $\nu_c = 0.5$, however these values are only valid under the specific conditions of the individual tests (37). Hanna (18) and Ross (50) found that lateral creep had no influence on creep in the other direction of loading. Stress $\sigma_2 \leq \sigma_1$ should have no influence on creep in the direction of σ_1. But σ_1 should influence creep in the direction of the lower load.

Meyer's tests showed different results, however (37). He found that for drying specimens, creep Poisson's ratio was considerably below the value for the elastic Poisson's ratio. For specimens with constant degree of water saturation, creep Poisson's ratio was equal to or higher than the elastic Poisson's ratio. Hence, Meyer concluded that the Poisson's ratio for creep of concrete is not a material constant, but a value dependent on drying conditions, i.e., environmental conditions. Others suggested that lateral creep, contrary to longitudinal creep, is a constant, influenced little by curing conditions. Meyer suggested that the higher the curing temperature, the lower the creep Poisson's ratio, and he proposed that for design purposes the final creep Poisson's ratio be taken to be 0.1. However, it is suggested by many others to be taken as the elastic Poisson's ratio.

Triaxial creep is calculated by interrelating the triaxial principal stresses, using a creep Poisson's ratio equal to the elastic ratio, in the creep equation

$$\sigma = \sigma_i - \nu(\sigma_j + \sigma_k) \tag{4.33}$$

The use of this method has been justified by comparison with experimental results (68).

Kraus (30) used an expression of the form

$$\dot{\varepsilon}_1^c = \frac{\dot{\varepsilon}^{c*}}{\sigma^*} [\sigma_1 - \frac{1}{2} (\sigma_2 + \sigma_3)] \tag{4.34}$$

with the experimentally determined creep law

$$\dot{\varepsilon}^{c*} = \dot{\varepsilon}^{c*}(\sigma^*, \varepsilon^{c*}, t) \tag{4.35}$$

where

$$\sigma^* = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \tag{4.35a}$$

$$\varepsilon^{c*} = \frac{\sqrt{2}}{3} [(\varepsilon_1^c - \varepsilon_2^c)^2 + (\varepsilon_2^c - \varepsilon_3^c)^2 + (\varepsilon_3^c - \varepsilon_1^c)^2]^{1/2} \tag{4.35b}$$

$$\dot{\varepsilon}^{c*} + \frac{\sqrt{2}}{3} [(\dot{\varepsilon}_1^c - \dot{\varepsilon}_2^c)^2 + (\dot{\varepsilon}_2^c - \dot{\varepsilon}_3^c)^2 + (\dot{\varepsilon}_3^c - \dot{\varepsilon}_1^c)^2]^{1/2} \tag{4.35c}$$

Equation 4.35 is written in different forms, corresponding to the steady-state creep law,

$$\dot{\varepsilon}^{c*} = \dot{\varepsilon}^{c*}(\sigma^*) \tag{4.36}$$

or

$$\dot{\varepsilon}^{c*} = B\sigma^{*n} \tag{4.37}$$

where for constant stress σ^*

$$\varepsilon^{c*} = A\sigma^{*n} t^m \tag{4.38}$$

the time hardening creep law,

$$\dot{\varepsilon}^{c*} = A\sigma^{*n}\, m t^{m-1} \tag{4.39}$$

and the strain hardening creep law,

$$\dot{\varepsilon}^{c*} = mA^{1/m} \sigma^{*n/m} \varepsilon^{c*(1-1/m)} \tag{4.40}$$

where A, B, m, and n are obtained from uniaxial tests.

In 1967 Hannant (19) proposed the following equations for the total dilatational and deviatoric strain components ε_{ii} and e_{ij}

$$\frac{\partial \varepsilon_{ii}}{\partial t} = (1 - 2\nu)\left[\frac{1}{E} \frac{\partial \sigma_{ii}}{\partial t} + \sigma_{ii} \; f(\Theta) \; \frac{\partial c}{\partial t}\right] \tag{4.41a}$$

$$\frac{\partial e_{ij}}{\partial t} = (1 + \nu)\left[\frac{1}{E} \frac{\partial S_{ij}}{\partial t} + S_{ij} \; f(\Theta) \; \frac{\partial c}{\partial t}\right] \tag{4.41b}$$

where σ_{ii} is the stress tension, S_{ij} is the stress deviator tensor, $f(\Theta)$ is a function of the temperature Θ, and $e_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij}$. The time variable can be eliminated and Equations 4.41a and 4.41b expressed in terms of the specific creep c

$$\frac{\partial \varepsilon_{ii}}{\partial c} = (1 - 2\nu)\left[\frac{1}{E} \frac{\partial}{\partial c} + f(\Theta)\right]\sigma_{ii} \tag{4.42a}$$

$$\frac{\partial e_{ij}}{\partial c} = (1 + \nu)\left[\frac{1}{E} \frac{\partial}{\partial c} + f(\Theta)\right]S_{ij} \tag{4.42b}$$

Although much data presently exists on uniaxial creep, relatively little is known about multiaxial creep behavior. Further work must be done in this area before any of these models, parameters, or expressions can be used with confidence in describing this phenomenon under various material and environmental conditions.

CHAPTER 5

NUMERICAL SOLUTION SCHEMES

5.1 Solution Technique

The creep behavior of a structure is determined by the finite element method using an incremental procedure. After the load or its increment is applied to the structure, and iteration completed, bringing the structure to equilibrium, a time increment is assumed to occur and the effects of creep during that time increment can be calculated.

During the time increment, creep strains are found by using a selected creep law. These creep strains are considered as initial strains for the next iteration. A consistent nodal load vector is built from the creep strains, and the displacements and stresses due to this load vector are found.

The solution technique for the concrete creep problem is summarized as follows:

1. For each time interval the total strain increment is made up of 3 parts

$$\Delta\varepsilon = \Delta\varepsilon_E + \Delta\varepsilon_{TH} + \Delta\varepsilon_C \tag{5.1}$$

where $\Delta\varepsilon_E$ is the incremental elastic strain,

$\Delta\varepsilon_{TH}$ is the incremental thermal strain

$$\Delta\varepsilon_{TH} = \alpha\Delta T \tag{5.1a}$$

and $\Delta\varepsilon_C$ is the incremental creep strain, and can be selected for the particular problem and conditions

$$\Delta\varepsilon_C = f(\sigma, t, T) \quad (5.1b)$$

T is the temperature, and can be a function of time and position

$$T = f'(t,r) \quad (5.1c)$$

2. The instantaneous elastic strains and stresses due to the external loading are calculated. These are considered as initial strains and stresses for the next time interval. Thus, at $t = 0$, there exists, due to the external loading, ε_o and σ_o.

3. Assuming that the stresses σ_o remain constant over the interval Δt, the incremental strains due to creep and temperature effects are calculated using Equations 5.1, 5.1a, 5.1b, and 5.1c.

4. Using the strains found in Step 3, the total change of the nodal point displacements are determined.

$$K\Delta u^n = \int B^T D(\Delta\varepsilon_C + \Delta\varepsilon_{TH}) \quad (5.2)$$

5. The total change in strain is then calculated using

$$\Delta\varepsilon = B\Delta u^n \quad (5.3)$$

6. The incremental stresses are elastically determined

$$\Delta\sigma = DB\Delta u^n - D[\Delta\varepsilon_C] - D[\Delta\varepsilon_{TH}] \quad (5.4)$$

7. If the stress increments found in Step 5 are larger than a preset fraction of the existing stresses, Steps 4-6 are repeated

using a smaller Δt. If the stresses are smaller than the preset fraction, the stresses at the end of the time interval are determined by adding the stress increments from Step 6 to the previously calculated stresses,

$$\sigma_{t+\Delta t} = \sigma_t + \Delta\sigma_t \tag{5.5}$$

8. Another time increment is added, and Steps 3-7 are repeated. The analysis continues for a desired time $t = \Sigma\Delta t$, or until a steady state is reached.

5.2 Present Status of Program Development

A program using a general curvilinear linear strain isoparametric finite element has been developed for the solution of displacements and stresses of axisymmetric structures. Displacements and stresses resulting from both external and creep loadings can be calculated, for both cylindrical and spherical structures.

The program considers the size and shape of the structure, the material properties, the magnitude and duration of the loads, and the creep formulation to be used. In this way, the reinforcing and/or prestressing effects of the actual structure can be included by modifying material properties and loading conditions. The user can also select the increment of time and the creep expression to be used, to meet the conditions of the particular problem.

Structures in which creep effects are typically of importance can be analyzed using the geometric parameters in the program. The response of a concrete pressure vessel to creep at higher temperatures and pressures can be approximated by using a thick-walled cylinder and an appropriate creep formulation. Concrete creep effects on thin shells can also be determined.

Several of the most widely used creep equations have been incorporated into the program, including those suggested by Greenbaum and Rubenstein, Lorman, Ross, and Shank. These expressions account for the influence of age of concrete, temperature, duration of load, and magnitude and type of load. The Greenbaum and Rubenstein expressions allow a multiaxial creep analysis. Any other creep expressions can be easily incorporated into the program.

For the general creep problem, the initial displacements and stresses due to the sustained load are calculated, and then the displacements and stresses resulting from the concrete creep are determined for each time step. The time increments are increased after the first few intervals to obtain a less costly analysis for long-timeloads. The time intervals increase according to the following equation,

$$\Delta t = \Delta t + 10i \qquad (5.6)$$

where i is the interval number.

Due to the time and monetary limitations, the incremental portion of the program has not been fully debugged. At present, the initial response of the structure due to the sustained load can be determined, and the consistent nodal load vector resulting from the creep strains can be assembled. The analysis procedure for the program is described in Figure 13. A listing of the program appears in Appendix A.

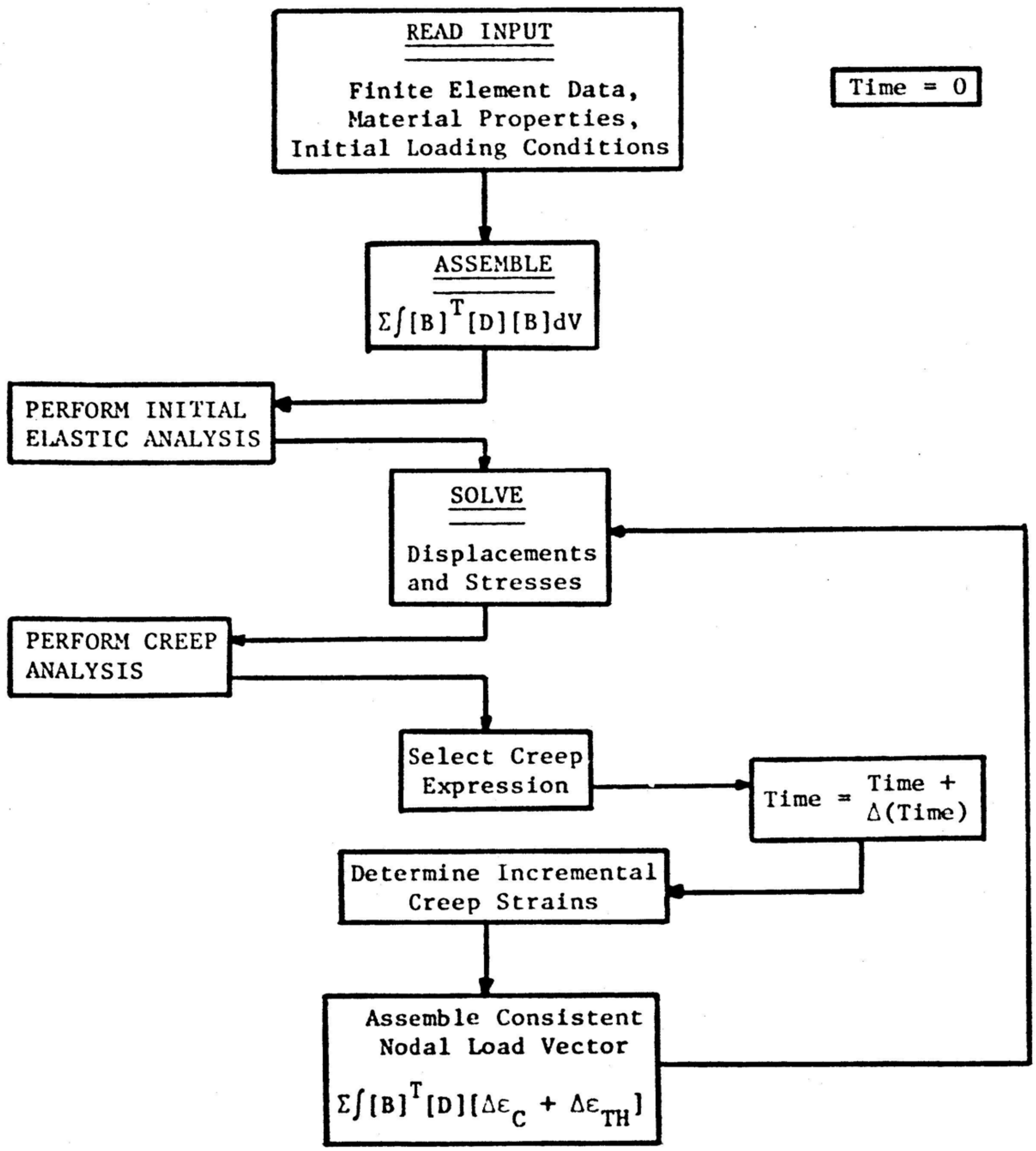


Figure 13 Program Analysis Procedure

DISCUSSION AND REMARKS

The behavior of reinforced and prestressed concrete members under the action of creep is considerably better understood than several years ago. Rational methods of allowing for the restraining action of reinforcement have been developed. Strains and stresses in concrete and steel can be calculated, and hence axial shortening and curvature of members can also be calculated.

The effects of creep in arches, shells, composite steel, concrete and precast concrete, in-situ concrete members, as well as continuous beams, have been evaluated. The problem of creep buckling has also become better understood. With the proper knowledge of concrete creep behavior under extreme conditions, the designer is able to make provision in design so as to minimize the adverse effects of creep. This is especially important in such structures as concrete pressure vessels.

There presently exists much data on the visco-elastic behavior of uniaxial creep. However, further experimental data is needed before multiaxial creep behavior is understood. More importantly, especially for the analysis of pressure vessels, further work needs to be done in the area of concrete creep formulations. This study, and the resulting program, serves as a first step towards the development of a means of assessing creep formulations. Once the present formulations are assessed, an accurate, yet easy to use, expression should be developed for use in the analysis of complex structures in which creep effects are important. This expression must account for the many environmental and material parameters which influence the mechanisms of concrete creep. Also, further work can be done in extending the present program to account for nonlinear creep behavior.

<u>APPENDIX A</u>

Listing of Program

```
                    AN ASSESSMENT OF CREEP FORMULATIONS
                         FOR CONCRETE STRUCTURES

                                   BY

                            JOSEPH A. MARTORE

                   MASSACHUSETTS INSTITUTE OF TECHNOLOGY

      MASTER'S THESIS                                       FEBRUARY, 1976



C
C     PROGRAM INITIAL
C
C     DETERMINES PROBLEM SIZE AND RUNNING PARAMETERS. ALSO FIXES ALL
C        VARIABLY-DIMENSIONED ARRAYS.
C
      DIMENSION PFORCE(16),S(8),Z(8),BC1(8),BC2(8),KEYBC(8)
      DIMENSION UDISP(8),WDISP(8),UFORCE(8),WFORCE(8),TDISP(16)
      DIMENSION NODE(8,9),XKSYS(16,17),DISP(16),FORCE(16)
      DIMENSION PFORCE(16,17),STRESS(8,5)
C
C THE ABOVE ARRAYS MUST BE DIMENSIONED BY THE USER IN THIS ROUTINE ONLY
C
C     DEFINITION OF VARIABLES
C
C     NE                  NUMBER OF ELEMENTS
C     NEN                 ELEMENT NUMBER IN CONNECTIVITY LISTING
C     NN                  NUMBER OF NODES
C     NNE                 TWICE THE NUMBER OF NODES
C     NNNE                TWICE THE NUMBER OF NODES PLUS ONE
C     NPRNT               PRINTOUT CONSISTS OF AN INPUT EDIT AND ANY
C                         OUTPUT WHICH IS SELECTED BY NPRNT. THE NPRNT
C                         VALUES AND CORRESPONDING OUTPUT ARE:
```

```
C                                  0   SOLVED DISPLACEMENTS                            MAIN0037
C                                  1   SOLVED STRESSES                                 MAIN0038
C                                  2   REDUCED STIFFNESS MATRIX                        MAIN0039
C                                  3   UNREDUCED STIFFNESS MATRIX AND FORCE            MAIN0040
C                                      DISPLACEMENT BOUNDARY CONDITIONS                MAIN0041
C                                  4   ELEMENTAL STIFFNESS MATRICES                    MAIN0042
C                                  5   REFERENCE SURFACE PARAMETERS FROM               MAIN0043
C                                      SUBROUTINE GEOM                                 MAIN0044
C                                  6   B MATRIX, FACTORS USED TO GENERATE IT,          MAIN0045
C                                      AND COMPONENTS OF ELEMENT STIFFNESS AT          MAIN0046
C                                      EACH INTEGRATION POINT                          MAIN0047
C                                  7   FULL DEBUG INCLUDING REDUCED STIFFNESS          MAIN0048
C                                      MATRIX DETERMINANT                              MAIN0049
C     S(NN)                        ARC LENGTH COORDINATE ALONG THE REFERENCE           MAIN0050
C                                  CURVE CORRESPONDING TO EACH NODE                    MAIN0051
C     Z(NN)                        NORMAL COORDINATE FOR EACH NODE                     MAIN0052
C     BC1(NN), BC2(NN)             TWO FORCE OR DISPLACEMENT BOUNDARY                  MAIN0053
C                                  CONDITIONS IN THE S AND N DIRECTIONS, RESPECTIVELY  MAIN0054
C     KEYBC(NN)                    BOUNDARY CONDITION KEY DETERMINING THE              MAIN0055
C                                  NATURE OF THE BOUNDARY CONDITIONS AT A NODE         MAIN0056
C                                                                                      MAIN0057
C               KEYBC(I)           BC1(I)          BC2(I)                              MAIN0058
C                  0                FORCE           FORCE                              MAIN0059
C                  1                FORCE        DISPLACEMENT                          MAIN0060
C                  2             DISPLACEMENT       FORCE                              MAIN0061
C                  3             DISPLACEMENT    DISPLACEMENT                          MAIN0062
C                                                                                      MAIN0063
C     NODE(NE,9)                   ARRAY KEEPING TRACK OF THE ACTUAL NODE NUMBERS      MAIN0064
C                                  FOR EACH ELEMENT WITH A NINTH POSITION FOR AN       MAIN0065
C                                  AS YET NOT PROGRAMMED MATERIAL SELECTION KEY        MAIN0066
C     DISP(NNE)                    FINAL SOLVED DISPLACEMENT VECTOR                    MAIN0067
C     UDISP(NN),                   SUBMATRICES OF THE PARTITIONED DISPLACEMENT         MAIN0068
C        WDISP(NN)                 VECTOR CONTAINING NODAL DISPLACEMENTS IN THE        MAIN0069
C                                  S AND N DIRECTIONS, RESPECTIVELY                    MAIN0070
C     FORCE(NNE)                   LOAD VECTOR WHICH CONTAINS EXTERNAL LOADS AND       MAIN0071
C                                  UNKNOWN REACTIONS WHERE THE DISPLACEMENTS           MAIN0072
```

```
C                         ARE PRESCRIBED                                  MAIN0073
C     UFORCE(NN),         SUBMATRICES OF THE PARTITIONED LOAD VECTOR      MAIN0074
C         WFORCE(NN)      CONTAINING NODAL FORCES IN THE S AND N          MAIN0075
C                         DIRECTIONS, RESPECTIVELY                        MAIN0076
C     XKSYS(NNE,NNNE)     ASSEMBLED,REDUCED, AND AUGMENTED MASTER         MAIN0077
C                         STIFFNESS MATRIX                                MAIN0078
C     PFORCE(NNE),        DUMMY ARRAYS USED BY THE SUBROUTINES WHICH      MAIN0079
C         FFORCE(NNE,NNNE)  SOLVE THE FINAL REDUCED SYSTEM OF EQUATIONS   MAIN0080
C     STRESS(NN,5)        STRESS VECTOR - THE FIRST FOUR POSITIONS FOR    MAIN0081
C                         EACH NODE ARE OCCUPIED BY THE AVERAGE NODAL     MAIN0082
C                         STRESS VALUE FOR ALL ELEMENTS INCIDENT UPON     MAIN0083
C                         THAT NODE (SN, SS, TSN, STHETA). THE FIFTH      MAIN0084
C                         POSITION IS USED IN KEEPING TRACK OF THE        MAIN0085
C                         AVERAGING PROCESS.                              MAIN0086
C     B(4,16)             ELEMENTAL B MATRIX                              MAIN0087
C     D(4,4)              RIGIDITY MATRIX D                               MAIN0088
C                         IF D(1,3) = 1.0, THE MATERIAL IS ASSUMED        MAIN0089
C                         TO BE ISOTROPIC WITH                            MAIN0090
C                              E = D(1,1)                                 MAIN0091
C                              POISSON'S RATIO = D(1,2)                   MAIN0092
C     DB(4,16)            PRODUCT OF THE MATRICES, D*B                    MAIN0093
C     BT(16,4)            MATRIX B TRANSPOSED                             MAIN0094
C     C(16,4)             PRODUCT OF MATRICES, BT*D                       MAIN0095
C     ELSTIF(16,16)       PRODUCT OF MATRICES, BT*D*B                     MAIN0096
C     ELST(16,16)         FINAL INTEGRATED ELEMENT STIFFNESS              MAIN0097
C     XSTRES(4)           STRESS COMPONENTS AT A NODE FOR A               MAIN0098
C                         PARTICULAR ELEMENT                              MAIN0099
C     XDISP(16)           APPROXIMATE DISPLACEMENT VECTOR USED IN         MAIN0100
C                         GENERATING XSTRES FOR AN ELEMENT                MAIN0101
C     KEYSTR              SIGNALS SUBROUTINE BFORM TO EVALUATE THE        MAIN0102
C                         B MATRIX AT NODAL POINTS                        MAIN0103
C     XNT(8)              TRANSPOSE OF INTERPOLATION POLYNOMIAL VECTOR N  MAIN0104
C     DUM1,DUM2           DUMMY VARIABLES WHICH TRANSFER 0, -1, OR 1      MAIN0105
C                         VALUES FOR CURVILINEAR COORDINATES, S AND N,    MAIN0106
C                         TO BFORM WHEN SIGNALED BY KEYSTR                MAIN0107
C                                                                         MAIN0108
```

```
      INTEGER QR,QB                                                     MAIN0109
      QR=8                                                              MAIN0110
      QB=5                                                              MAIN0111
      WRITE(QB,3001)                                                    MAIN0112
 3001 FORMAT('1',1X,'NOTE:  1. THE UNIT OF LENGTH IS INCHES.',//,8X,'2. MAIN0113
     1THE UNIT OF FORCE IS POUNDS.',//,8X,'3. TEMPERATURES ARE IN DEGREE MAIN0114
     2S CENTIGRADE.',//)                                                MAIN0115
      READ(QR,1000) NE,NN,NGEOM,NPRNT                                   MAIN0116
 1000 FORMAT(4I5)                                                       MAIN0117
      WRITE(QB,2000)                                                    MAIN0118
 2000 FORMAT(///,20('*'),4X,'FINITE ELEMENT DATA',4X,65('*'))           MAIN0119
      WRITE(QB,2001) NE                                                 MAIN0120
 2001 FORMAT  (//,13X,'NUMBER OF ELEMENTS',17('*'),I5)                  MAIN0121
      WRITE(QB,6001) NN                                                 MAIN0122
 6001 FORMAT(/,13X,'NUMBER OF NODAL POINTS',13('*'),I5,///)             MAIN0123
      NNE=2*NN                                                          MAIN0124
      NNNE=2*NN+1                                                       MAIN0125
      CALL CLISP(NE,NN,NGEOM,NNE,NNNE,PFORCE,S,Z,BC1,BC2,KEYBC,NODE,XKSY MAIN0126
     1S,DISP,FORCE,FFORCE,STRESS,NPRNT,UDISP,WDISP,UFORCE,WFORCE,TDISP) MAIN0127
 5    CONTINUE                                                          MAIN0128
      STOP                                                              MAIN0129
      END                                                               MAIN0130
```

```
      SUBROUTINE CLISP(NE,NN,NGEOM,NNE,NNNE,PFORCE,S,Z,BC1,BC2,KEYBC,NOD
     1E,XKSYS,DISP,FORCE,FFORCE,STRESS,NPRNT,UDISP,WDISP,UFORCE,WFORCE,T
     2DISP)
C
C     PROCESSES INFORMATION - INPUT OF GEOMETRY, BOUNDARY CONDITIONS,
C        ELEMENT NODE NUMBERING, AND MATERIAL RIGIDITY
C
      DIMENSION PFORCE(NNE),S(NN),Z(NN),BC1(NN),BC2(NN),KEYBC(NN)
      DIMENSION NODE(NE,9),XKSYS(NNE,NNNE),DISP(NNE),FORCE(NNE)
      DIMENSION FFORCE(NNE,NNNE),STRESS(NN,5)
      DIMENSION UDISP(NN),WDISP(NN),UFORCE(NN),WFORCE(NN),TDISP(16)
      DIMENSION SN(8),ZN(8),RON(8),ROIN(8),R1N(8),R2N(8),X3IN(8),RN(8)
      DIMENSION B(4,16),PSIS(8),PHIS(8),XNI(8),ELSTIF(16,16),BT(16,4)
      DIMENSION D(4,4),C(16,4),DB(4,16),XSTRES(4),ELST(16,16),PSI(8)
      DIMENSION PHI(8)
      DIMENSION TSTRES(NN,5),CRP(NN),CRPNV(NNE,NN)
      INTEGER QR,QB
      DUM1=0.0
      DUM2=0.0
      DUM3=0.0
      QR=8
      QB=5
      KEYSTR=0
      NIJ=0
      IJK=0
      T=0.0
      DO 8888 I=1,NN
      DO 8888 J=1,5
 8888 TSTRES (I,J)=0.0
C
C       INPUT
C
      WRITE(QB,5130)
 5130 FORMAT(///,1X,10('*'),4X,'NODAL POINT COORDINATES       AND      BO
     1UNDARY CONDITIONS',4X,40('*')/)
      WRITE(QB,5131)
```

```
5131 FORMAT(/1X, 8X,'NODE POINT',7X,'S', 9X,'Z',7X,'KEYBC',8X,'BC1',11X
    2,'BC2',/)
     DO 1 I=1,NN
     READ (QR,1001) S(I),Z(I),BC1(I),BC2(I),KEYBC(I)
1001 FORMAT(2F10.4,2E10.6,I5)
     WRITE(QB,2002) I,S(I),Z(I),KEYBC(I),BC1(I),BC2(I)
2002 FORMAT(1X, 9X,I5,7X,F7.2,3X,F7.2,3X,I5,4X,E13.4,2X ,E13.4)
1    CONTINUE
     WRITE(QB,5133)
5133 FORMAT     (//////,1X,20('*'),4X,'ELEMENT CONNECTIVITIES',4X,60('*'
    1)/)
     WRITE(QB,2003)
2003 FORMAT  (/,5X,'ELEMENT',3X,'NODE1',2X,'NODE2',2X,'NODE3',2X,'NODE4
    1',2X,'NODE5',2X,'NODE6',2X,'NODE7',2X,'NODE8',/)
     DO 2 I=1,NE
     READ(QR,1002)  NEN,(NODE(NEN,J),J=1,8)
1002 FORMAT(10I5)
     WRITE(QB,2004)  NEN,(NODE(NEN,J),J=1,8)
2004 FORMAT(6X,I3,6X,I3,4X,I3,4X,I3,4X,I3,4X,I3,4X,I3,4X,I3,4X,I3)
2    CONTINUE
     READ(QR,1003)  ((D(I,J),J=1,4),I=1,4)
1003 FORMAT(4F15.6,/,4F15.6,/,4F15.6,/,4F15.6)
     CONTINUE
     IF(D(1,3).NE.1.0)  GO TO 3
     WRITE(QB,3001)
3001 FORMAT('1',///,11X,'MATERIAL IS ASSUMED TO BE ISOTROPIC:',////,19X
    1,'MATERIAL PROPERTIES',//,16X,'MODULUS OF',6X,'POISSON''S',/,16X,'
    2ELASTICITY',8X,'RATIO',/)
     WRITE(QB,3002)  D(1,1),D(1,2)
3002 FORMAT(11X,2E15.4)
3    CONTINUE
C
C       ELEMENT STIFFNESS GENERATION
C
     DO 50 I=1,NNE
     DO 50 J=1,NNNE
```

```
      XKSYS(I,J)=0.0                                                    CLSP0073
   50 CONTINUE                                                          CLSP0074
      DO 55 KK=1,NE                                                     CLSP0075
      DO 4 I=1,16                                                       CLSP0076
      DO 4 J=1,16                                                       CLSP0077
      ELST(I,J)=0.0                                                     CLSP0078
    4 CONTINUE                                                          CLSP0079
      DO 5 J=1,8                                                        CLSP0080
      NNODE=NODE(KK,J)                                                  CLSP0081
      SN(J)=S(NNODE)                                                    CLSP0082
      ZN(J)=Z(NNODE)                                                    CLSP0083
    5 CONTINUE                                                          CLSP0084
      CALL GEOM(KK,NGEOM,SN,ZN,RON,R0IN,R1N,R2N,X3IN,RN,NPRNT,NIJ)      CLSP0085
      CALL BFORM(SN,ZN,RN,R1N,R2N,RON,R0IN,X3IN,KEYSTR,DUM1,DUM2,B,D,ELS CLSP0086
     1T,NGEOM,NPRNT,NIJ,C)                                              CLSP0087
      NIJ=1                                                             CLSP0088
C                                                                       CLSP0089
C          ASSEMBLY OF ELEMENT STIFFNESSES IN MASTER STIFFNESS MATRIX   CLSP0090
C                                                                       CLSP0091
      DO 54 I=1,8                                                       CLSP0092
      DO 54 J=1,8                                                       CLSP0093
      NAN=NODE(KK,I)                                                    CLSP0094
      NBN=NODE(KK,J)                                                    CLSP0095
      XKSYS(NAN,NBN)=XKSYS(NAN,NBN)+ELST(I,J)                           CLSP0096
   54 CONTINUE                                                          CLSP0097
      DO 51 I=1,8                                                       CLSP0098
      DO 51 J=9,16                                                      CLSP0099
      JJ=J-8                                                            CLSP0100
      NAN=NODE(KK,I)                                                    CLSP0101
      NBN=NODE(KK,JJ)+NN                                                CLSP0102
      XKSYS(NAN,NBN)=XKSYS(NAN,NBN)+ELST(I,J)                           CLSP0103
   51 CONTINUE                                                          CLSP0104
      DO 52 I=9,16                                                      CLSP0105
      DO 52 J=1,8                                                       CLSP0106
      II=I-8                                                            CLSP0107
      NAN=NODE(KK,II)+NN                                                CLSP0108
```

```
      NBN=NODE(KK,J)                                                    CLSP0109
      XKSYS(NAN,NBN)=XKSYS(NAN,NBN)+ELST(I,J)                           CLSP0110
 52   CONTINUE                                                          CLSP0111
      DO 53 I=9,16                                                      CLSP0112
      DO 53 J=9,16                                                      CLSP0113
      II=I-8                                                            CLSP0114
      JJ=J-8                                                            CLSP0115
      NAN=NODE(KK,II)+NN                                                CLSP0116
      NBN =NODE(KK,JJ)+NN                                               CLSP0117
      XKSYS(NAN,NBN)=XKSYS(NAN,NBN)+ELST(I,J)                           CLSP0118
 53   CONTINUE                                                          CLSP0119
 55   CONTINUE                                                          CLSP0120
      IF(NPRNT.GE.3) WRITE(QB,2043) NNE,NNE                             CLSP0121
 2043 FORMAT('1', /,38X,'THE ASSEMBLED MASTER STIFFNESS MATRIX (',I2,'X'CLSP0122
     1,I2,')',//)                                                       CLSP0123
      IF(NPRNT.GE.3) WRITE(QB,2014) ((XKSYS(I,J),J=1,NNE),I=1,NNE)      CLSP0124
 2014 FORMAT(16(8(1X,E11.4),/,25X,8(E11.4,1X),//))                      CLSP0125
C                                                                       CLSP0126
C       BOUNDARY CONDITIONS                                             CLSP0127
C                                                                       CLSP0128
      DO 60 I=1,NNE                                                     CLSP0129
      DISP(I)=0.0                                                       CLSP0130
      FORCE(I)=0.0                                                      CLSP0131
 60   CONTINUE                                                          CLSP0132
      DO 61 I=1,NN                                                      CLSP0133
      UDISP(I)=0.0                                                      CLSP0134
      WDISP(I)=0.0                                                      CLSP0135
 61   CONTINUE                                                          CLSP0136
      J=-1                                                              CLSP0137
      DO 70 I=1,NN                                                      CLSP0138
      J=J+2                                                             CLSP0139
      KB=KEYBC(I)                                                       CLSP0140
      KB=KB+1                                                           CLSP0141
      GO TO (65,66,67,68),KB                                            CLSP0142
 65   FORCE(J)=BC1(I)                                                   CLSP0143
      KF=J+1                                                            CLSP0144
```

```
      FORCE(KF)=BC2(I)                                                  CLSP0145
      GO TO 69                                                          CLSP0146
   66 FORCE(J)=BC1(I)                                                   CLSP0147
      KF=J+1                                                            CLSP0148
      DISP(KF)=BC2(I)                                                   CLSP0149
      GO TO 69                                                          CLSP0150
   67 DISP(J)=BC1(I)                                                    CLSP0151
      KF=J+1                                                            CLSP0152
      FORCE(KF)=BC2(I)                                                  CLSP0153
      GO TO 69                                                          CLSP0154
   68 DISP(J)=BC1(I)                                                    CLSP0155
      KF=J+1                                                            CLSP0156
      DISP(KF)=BC2(I)                                                   CLSP0157
   69 CONTINUE                                                          CLSP0158
   70 CONTINUE                                                          CLSP0159
      DO 64 I=1,NN                                                      CLSP0160
      J=2*I-1                                                           CLSP0161
      UDISP(I)=DISP(J)                                                  CLSP0162
      UFORCE(I)=FORCE(J)                                                CLSP0163
      J=2*I                                                             CLSP0164
      WDISP(I)=DISP(J)                                                  CLSP0165
      WFORCE(I)=FORCE(J)                                                CLSP0166
   64 CONTINUE                                                          CLSP0167
      DO 63 I=1,NN                                                      CLSP0168
      J=I+NN                                                            CLSP0169
      DISP(I)=UDISP(I)                                                  CLSP0170
      FORCE(I)=UFORCE(I)                                                CLSP0171
      DISP(J)=WDISP(I)                                                  CLSP0172
      FORCE(J)=WFORCE(I)                                                CLSP0173
   63 CONTINUE                                                          CLSP0174
      IF(NPRNT.GE.3) WRITE(OB,2008)                                     CLSP0175
 2008 FORMAT('1',///,5X,'ASSEMBLED DISPLACEMENT AND LOAD VECTORS FROM BO  CLSP0176
     1UNDARY CONDITIONS',//)                                            CLSP0177
      IF(NPRNT.GE.3) WRITE(OB,2009)                                     CLSP0178
 2009 FORMAT(24X,'DISPLACEMENT',7X,'FORCE',/)                           CLSP0179
      DO 71 I=1,NN2                                                     CLSP0180
```

```
      IF(NPRNT.GE.3) WRITE(OB,2010) DISP(I),FORCE(I)
 2010 FORMAT(24X,E11.4,4X,E11.4)
 71   CONTINUE
C
C       STIFFNESS REDUCTION
C
      DO 76 I=1,NNE
      IF(I.GT.NN) GO TO 73
      IF(KEYBC(I).EQ.0) GO TO 76
      IF(KEYBC(I).EQ.1) GO TO 76
      IF(KEYBC(I).EQ.2) GO TO 75
      IF(KEYBC(I).EQ.3) GO TO 75
 73   CONTINUE
      JJK=I-NN
      IF(KEYBC(JJK).EQ.0) GO TO 76
      IF(KEYBC(JJK).EQ.1) GO TO 75
      IF(KEYBC(JJK).EQ.2) GO TO 76
      IF(KEYBC(JJK).EQ.3) GO TO 75
 75   CONTINUE
      DO 72 J=1,NNE
      IF(J.EQ.I) GO TO 72
      FORCE(J)=FORCE(J)-XKSYS(J,I)*DISP(I)
 72   CONTINUE
      DO 74 K=1,NNE
      XKSYS(K,I)=0.0
      XKSYS(I,K)=0.0
 74   CONTINUE
      XKSYS(I,I)=1.0
      FORCE(I)=DISP(I)
 76   CONTINUE
      J=NNNE
      DO 77 I=1,NNE
      XKSYS(I,J)=FORCE(I)
 77   CONTINUE
      IA=NNE+2
      JA=NNE+2
```

```
      IB=NNE+2
      JB=NNE+2
      IF(NPRNT.GE.2) WRITE(QB,2011) NNE,NNE
 2011 FORMAT('1',/,42X,'THE REDUCED STIFFNESS MATRIX (',I2,'X',I2,')',//
     1)
      IF(NPRNT.GE.2) WRITE(QB,2014) ((XKSYS(IJ,JI),JI=1,NNE),IJ=1,NNE)
      IF(NPRNT.GE.2) WRITE(QB,2041)
 2041 FORMAT('1',/,10X,'THE REDUCED LOAD VECTOR',//)
      IF(NPRNT.GE.2) WRITE(QB,2042) (FORCE(IJ),IJ=1,NNE)
 2042 FORMAT((12X,E19.12,/))
 78   CONTINUE
C
C       SOLUTION FOR DISPLACEMENTS
C
      CALL MATSOL(XKSYS,FORCE,DISP,FFORCE,FFORCE,NNE,0,0,0,0,NNNE,DET)
      WRITE(QB,2019)
 2019 FORMAT('1',27X,'THE SOLVED DISPLACEMENTS',////,12X,'NODE',12X,'UDI
     1SP',21X,'WDISP',/)
      DO 79 IJ=1,NN
      IJJ=IJ+NN
      WRITE(QB,2020) IJ,DISP(IJ),DISP(IJJ)
 2020 FORMAT(/,10X,I5,5X,F20.15,5X,F20.15)
 79   CONTINUE
      IF(NPRNT.GE.7) WRITE(QB,2040) DET
 2040 FORMAT(50X,////,'THE DETERMINANT IS ',E20.5,///)
C
C       STRESS CALCULATIONS
C
      DO 81 I=1,NN
      DO 81 J=1,5
      STRESS(I,J)=0.0
 81   CONTINUE
      IF(NPRNT.EQ.0) GO TO 112
      KEYSTR=1
      DO 100 K=1,NE
      XI=-1.0
```

```
      ETA=-1.0                                                          CLSP0253
      DO 82 LL=1,8                                                      CLSP0254
      NNODE=NODE(K,LL)                                                  CLSP0255
      SN(LL)=S(NNODE)                                                   CLSP0256
      ZN(LL)=Z(NNODE)                                                   CLSP0257
   82 CONTINUE                                                          CLSP0258
      DO 100 L=1,8                                                      CLSP0259
      KNODE=NODE(K,L)                                                   CLSP0260
      CALL GEOM(KK,NGEOM,SN,ZN,RCN,B0IN,R1N,B2N,X3IN,RN,NPRNT,NIJ)      CLSP0261
      CALL BFORM(SN,ZN,RN,R1N,R2N,RCN,B0IN,X3IN,KEYSTR,XI,ETA,B,D,ELST,N CLSP0262
     1GEOM,NPRNT,NIJ,C)                                                 CLSP0263
   83 CONTINUE                                                          CLSP0264
      DO 85 II=1,4                                                      CLSP0265
      DO 85 JJ=1,16                                                     CLSP0266
      DB(II,JJ)=0.0                                                     CLSP0267
      DO 85 KJ=1,4                                                      CLSP0268
      DB(II,JJ)=DB(II,JJ)+D(II,KJ)*B(KJ,JJ)                             CLSP0269
   85 CONTINUE                                                          CLSP0270
      DO 86 KJ=1,8                                                      CLSP0271
      NNODE=NODE(K,KJ)                                                  CLSP0272
      TDISP(KJ)=DISP(NNODE)                                             CLSP0273
      JJ=KJ+8                                                           CLSP0274
      NNODE=NNODE+NN                                                    CLSP0275
      TDISP(JJ)=DISP(NNODE)                                             CLSP0276
   86 CONTINUE                                                          CLSP0277
      DO 88 II=1,4                                                      CLSP0278
      XSTRES(II)=0.0                                                    CLSP0279
      DO 88 JJ=1,16                                                     CLSP0280
      XSTRES(II)=XSTRES(II)+DB(II,JJ)*TDISP(JJ)                         CLSP0281
   88 CONTINUE                                                          CLSP0282
      DO 87 II=1,4                                                      CLSP0283
      STRESS(KNODE,II)=STRESS(KNODE,II)+XSTRES(II)                      CLSP0284
   87 CONTINUE                                                          CLSP0285
      STRESS(KNODE,5)=STRESS(KNODE,5)+1.0                               CLSP0286
      GO TO (91,92,93,94,95,96,97,98),L                                 CLSP0287
   91 XI=1.0                                                            CLSP0288
```

```
      ETA=-1.0
      GO TO 100
 92   XI=1.0
      ETA=1.0
      GO TO 100
 93   XI=-1.0
      ETA=1.0
      GO TO 100
 94   XI=0.0
      ETA=-1.0
      GO TO 100
 95   XI=1.0
      ETA=0.0
      GO TO 100
 96   XI=0.0
      ETA=1.0
      GO TO 100
 97   XI=-1.0
      ETA=0.0
      GO TO 100
 98   CONTINUE
 100  CONTINUE
C
C       AVERAGING THE STRESSES
C
      WRITE(QB,2029)
 2029 FORMAT('1',//,32X,'THE CALCULATED STRESSES',/)
      DO 110 I=1,NN
      DO 110 J=1,4
      STRESS(I,J)=STRESS(I,J)/STRESS(I,5)
 110  CONTINUE
      IJK=IJK+1
      DO 9999 I=1,NN
      DO 9999 J=1,4
 9999 TSTRES (I,J)=TSTRES (I,J)+STRESS(I,J)
      WRITE(QB,2031)
```

```
2031 FORMAT(///,5X,'NODE',8X,'NORMAL STRESS',4X,'TANGENT STRESS',4X,
    1'SHEAR STRESS',5X,'HOOP STRESS',//)
     DO 111 I=1,NN
     WRITE(03,2030) I,(TSTRES (I,J),J=1,4)
2030 FORMAT(5X,I3,5X,4(2X,E15.6),/)
111  CONTINUE
112  CONTINUE
     CALL SELECT(TSTRES ,IJK,T,DT,C,CRPNV)
     IF (IJK.LE.6) GO TO 60
C
     RETURN
     END
```

```
      SUBROUTINE GEOM(KK,NGEOM,SN,ZN,RON,ROIN,R1N,R2N,X3IN,RN,NPRNT,NIJ)
C
C     CALCULATES GEOMETRIC PARAMETERS OF THE REFERENCE SURFACE
C
      DIMENSION RON(8),ROIN(8),R1N(8),R2N(8),X3IN(8),RN(8),SN(8),ZN(8)
      INTEGER QR,QB
      QB=5
      QR=8
      PI=3.141592654
      IF(NPRNT.GE.5) WRITE(QB,2006)
 2006 FORMAT('1',//,50X,'FROM SUBROUTINE GEOM',///)
      GO TO (1,3),NGEOM
C
C       CYLINDRICAL SECTION
C
 1    CONTINUE
      IF(NIJ.EQ.0) READ(QR,1005) RREF
 1005 FORMAT(F15.6)
      DO 2 J=1,8
      RON(J)=RREF
      ROIN(J)=0.0
      R1N(J)=1.0E20
      R2N(J)=+RREF
      X3IN(J)=1.0
      RN(J)=RREF+ZN(J)
      IF(NPRNT.GE.5) WRITE(QB,2007) KK,J
 2007 FORMAT(5X,'EN= ',I5,2X,'NN= ',I5)
      IF(NPRNT.GE.5) WRITE(QB,2008) RON(J),ROIN(J),R1N(J),R2N(J),X3IN(J)
     1,RN(J)
 2008 FORMAT(5X,'RON= ',E11.4,2X,'ROIN= ',E11.4,2X,'R1N= ',E11.4,2X,'R2N
     1= ',E11.4,2X,'X3IN= ',E11.4,2X,'RN= ',E11.4,//)
 2    CONTINUE
      GO TO 100
C
C       SHERICAL SECTION
C
```

```
3     CONTINUE
      IF(NIJ.EQ.0) READ (QR,1005) RREF
      NIJ=1
      DO 4 J=1,8
      RON(J)=RREF*COS(SN(J)/RREF)
      ROIN(J)=-SIN(SN(J)/RREF)
      R1N(J)=RREF
      R2N(J)=RREF
      X3IN(J)=COS(SN(J)/RREF)
      RN(J)=(RREF+ZN(J))*COS(SN(J)/RREF)
      IF(NPRNT.GE.5) WRITE(QB,2007)KK,J
      IF(NPRNT.GE.5) WRITE(QB,2008) RON(J),ROIN(J),R1N(J),R2N(J),X3IN(J)
     1,RN(J)
4     CONTINUE
      GO TO 100
100   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BFORM(SN,ZN,RN,R1N,R2N,RON,ROIN,X3IN,KEYSTR,DUM1,DUM2,B
     1,D,ELST,NGEOM,NPRNT,NIJ)
C
C     DEFINES THE INTERPOLATION POLYNOMIALS AND THEIR DERIVATIVES,
C        SETS UP THE EQUATIONS FOR THE ELEMENTS, OPTIONALLY DEFINES
C        THE ISOTROPIC RIGIDITY MATRIX, AND NUMERICALLY INTEGRATES TO
C        FIND THE STIFFNESS MATRIX FOR EACH ELEMENT
C
      DIMENSION RON(8),ROIN(8),R1N(8),R2N(8),X3IN(8),RN(8),SN(8),ZN(8)
      DIMENSION XNT(8),PSIS(8),PHIS(8),B(4,16),D(4,4),BT(16,4),C(16,4)
      DIMENSION ELSTIF(16,16)
      DIMENSION ELST(16,16)
      DIMENSION PSI(8),PHI(8)
      INTEGER QR,QB
      PI=3.141592654
      H1=5./9.
      H2=8./9.
      H3=H1
      QR=8
      QB=5
C
C       B MATRIX
C
      DO 10 I=1,4
      DO 10 J=1,16
      B(I,J)=0.0
 10   CONTINUE
      KKK=1
      XI=-.774597
      ETA=XI
      IF(KEYSTR.EQ.0) GO TO 1
      XI=DUM1
      ETA=DUM2
 1    CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2000)
 2000 FORMAT('1',/,42X,'FROM SUBROUTINE BFORM',//)
```

```
11    CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2018) KKK
 2018 FORMAT(//,3X,27('*'),/,3X,'*',25X,'*',/,3X,'* INTEGRATION POINT NO
     1.',I2,' *',/,3X,'*',25X,'*',/,3X,27('*'),////)
      IF(NPRNT.GE.6) WRITE(QB,2001) XI,ETA
 2001 FORMAT(5X,'XI= ',F10.4,5X,'ETA= ',F10.4,/)
C
C       INTERPOLATION POLYNOMIALS AND DERIVATIVES
C
      XNT(1)=(-.25)*(1.-XI)*(1.-ETA)*(XI+ETA+1.)
      XNT(2)=(.25)*(1.+XI)*(1.-ETA)*(XI-ETA-1.)
      XNT(3)=(.25)*(1.+XI)*(1.+ETA)*(XI+ETA-1.)
      XNT(4)=(-.25)*(1.-XI)*(1.+ETA)*(XI-ETA+1.)
      XNT(5)=(.5)*(1.-XI**2)*(1.-ETA)
      XNT(6)=(.5)*(1.+XI)*(1.-ETA**2)
      XNT(7)=(.5)*(1.-XI**2)*(1.+ETA)
      XNT(8)=(.5)*(1.-XI)*(1.-ETA**2)
      PSIS(1)=(.25)*(1.-ETA)*(2.*XI+ETA)
      PSIS(2)=(.25)*(1.-ETA)*(2.*XI-ETA)
      PSIS(3)=(.25)*(1.+ETA)*(2.*XI+ETA)
      PSIS(4)=(.25)*(1.+ETA)*(2.*XI-ETA)
      PSIS(5)=-XI*(1.-ETA)
      PSIS(6)=(.5)*(1.-ETA**2)
      PSIS(7)=-XI*(1.+ETA)
      PSIS(8)=(-.5)*(1.-ETA**2)
      PHIS(1)=((XI+2.0*ETA)*(1.0-XI))/4.
      PHIS(2)=((2.*ETA-XI)*(1.+XI))/4.
      PHIS(3)=((XI+2.*ETA)*(1.+XI))/4.
      PHIS(4)=((2.*ETA-XI)*(1.-XI))/4.
      PHIS(5)=((XI**2)-1.0)/2.
      PHIS(6)=-ETA*(1.+XI)
      PHIS(7)=(.5)*(1.-XI**2)
      PHIS(8)=-ETA*(1.-XI)
      IF(NPRNT.GE.6) WRITE(QB,2015)
 2015 FORMAT(//,10X,'*1*',9X,'*2*',9X,'*3*',9X,'*4*',9X,'*5*',9X,'*6*',9
     1X,'*7*',9X,'*8*',/)
```

```
      IF(NPRNT.GE.6) WRITE(QB,2002) (XNT(I),I=1,8),(PSIS(I),I=1,8),(PHIS
     1(I),I=1,8)
 2002 FORMAT(2X,'XNT ',8(E11.4,1X),/,1X,'PSIS ',8(E11.4,1X),/,1X,'PHIS '
     1,8(E11.4,1X),/)
      A1=0.0
      DO 12 K=1,8
      A1=A1 + PSIS(K)*ZN(K)
 12   CONTINUE
      A2=0.0
      DO 13 K=1,8
      A2=A2 + PHIS(K)*SN(K)
 13   CONTINUE
      A=A1*A2
      B1=0.0
      DO 14 K=1,8
      B1=B1 + PHIS(K)*ZN(K)
 14   CONTINUE
      B2=0.0
      DO 15 K=1,8
      B2=B2 + PSIS(K)*SN(K)
 15   CONTINUE
      B3=B1*B2
      XJACOB=ABS(A-B3)
      DO 16 K=1,8
      PSI(K)=(1./XJACOB)*(A2*PSIS(K)-B2*PHIS(K))
 16   CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2003) (PSI(I),I=1,8)
 2003 FORMAT(2X,'PSI ',8(E11.4,1X))
      DO 17 K=1,8
      J=K+8
      B(1,J)=PSI(K)
 17   CONTINUE
      DO 18 K=1,8
      PHI(K)=(1./XJACOB)*(A1*PHIS(K)-B1*PSIS(K))
 18   CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2004) (PHI(I),I=1,8)
```

```
 2004 FORMAT(2X,'PHI ',8(E11.4,1X),///)
      D1=0.0
      D2=0.0
      D3=0.0
      D4=0.0
      D5=0.0
      D6=0.0
      D7=0.0
      D8=0.0
      DO 19 K=1,8
      D1=D1+(XNT(K)*ZN(K))
      D2=D2+(XNT(K)*R1N(K))
      D3=D3+(XNT(K)*R2N(K))
      D4=D4+(XNT(K)*R0IN(K))
      D5=D5+(XNT(K)*RON(K))
      D6=D6+(XNT(K)*X3IN(K))
      D7=D7+(XNT(K)*RN(K))
      D8=D8+(XNT(K)*SN(K))
 19   CONTINUE
      IF(NPRNT.GE.6) WRITE(OB,2016) XJACOB,D1,D2,D3,D4,D5,D6,D7,D8
 2016 FORMAT(20X,'XJACOB = ',E11.4,/,20X,'D1 = ',E11.4,/,20X,'D2 = ',E11
     1.4,/,20X,'D3 = ',E11.4,/,20X,'D4 = ',E11.4,/,20X,'D5 = ',E11.4,/,2
     20X,'D6 = ',E11.4,/,20X,'D7 = ',E11.4,/,20X,'D8 = ',E11.4,///)
      IF(NGEOM.EQ.4) GOTO 22
      IF(NGEOM.NE.2) GO TO 100
      DEN1=1.0
      DEN2=1.0+D1+D2
      DO 99 K=1,8
      B(2,K)=PHI(K)
      J=K+8
      B(2,J)=0.0
      B(3,K)=PSI(K)
 99   CONTINUE
      GO TO 102
 100  CONTINUE
      DEN1=1.0+D1/D2
```

```
      DEN2=D1+D2
      DO 101 K=1,8
      B(2,K)=PHI(K)*(1./DEN1)
      J=K+8
      B(2,J)=XNT(K)/DEN2
      B(3,K)=PSI(K)-XNT(K)/DEN2
  101 CONTINUE
  102 CONTINUE
      DO 20 K=1,8
      J=K+8
      B(3,J)=PHI(K)*(1./DEN1)
      B(4,K)=(-TAN(D8/D2)*XNT(K))/DEN2
      B(4,J)=XNT(K)/DEN2
   20 CONTINUE
      GO TO 24
   22 CONTINUE
      DO 23 K=1,8
      J=K+8
      B(2,K)=PHI(K)
      B(2,J)=0.0
      B(3,K)=PSI(K)
      B(3,J)=PHI(K)
      B(4,K)=XNT(K)*D4/(D5*(1.+D1/D3))
      B(4,J)=XNT(K)*D6/(D5*(1.+D1/D3))
      DEN1=1.+D1/D2
   23 CONTINUE
   24 CONTINUE
      IF(NPRNT.GE.6) WRITE(Q3,2005)
 2005 FORMAT(/,50X,'THE B MATRIX (4X16)',//)
      IF(NPRNT.GE.6) WRITE(Q3,2006) ((B(I,J),J=1,16),I=1,4)
 2006 FORMAT(4(8(1X,E11.4),/,25X,8(E11.4,1X),//))
      IF(KEYSTR.NE.0) GO TO 49
C
C       D MATRIX
C
      DDD=D(1,3)
```

```
      IF (NPRNT.LT.6) GO TO 8001
      IF(DDD.NE.1.0) WRITE(QB,2022)
 2022 FORMAT('1',23X,'RIGIDITY MATRIX, D, (4X4)',//)
      IF(DDD.NE.1.0) WRITE(QB,2017) ((D(I,J),J=1,4),I=1,4)
 2017 FORMAT(10X,4(E11.4,2X,E11.4,2X,E11.4,2X,E11.4,/,10X),//)
 8001 CONTINUE
      IF(DDD.NE.1.0) GO TO 25
      E=D(1,1)
      ENU=D(1,2)
      CON1=E/((1.+ENU)*(1.-2.*ENU))
      CON2=E/(2.*(1.+ENU))
      D(1,1)=CON1*(1.-ENU)
      D(1,2)=CON1*ENU
      D(1,3)=0.0
      D(1,4)=CON1*ENU
      D(2,1)=CON1*ENU
      D(2,2)=CON1*(1.-ENU)
      D(2,3)=0.0
      D(2,4)=CON1*ENU
      D(3,1)=0.0
      D(3,2)=0.0
      D(3,3)=CON2
      D(3,4)=0.0
      D(4,1)=CON1*ENU
      D(4,2)=CON1*ENU
      D(4,3)=0.0
      D(4,4)=CON1*(1.-ENU)
      IF(NIJ.EQ.0) WRITE(QB,2019)
 2019 FORMAT('1', 25X,'ISOTROPIC D MATRIX (4X4)',//)
      IF(NIJ.EQ.0) WRITE(QB,2017) ((D(I,J),J=1,4),I=1,4)
 25   CONTINUE
C
C       B TRANSPOSE MATRIX
C
      DO 26 I=1,4
      DO 26 J=1,16
```

```
      BT(J,I)=B(I,J)
   26 CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2030)
 2030 FORMAT(//,28X,'BT MATRIX (16X4)')
      IF(NPRNT.GE.6) WRITE(QB,2032) ((BT(I,J),J=1,4),I=1,16)
 2032 FORMAT(//,10X,16(E11.4,2X,E11.4,2X,E11.4,2X,E11.4,/,10X))
C
C        BT*D MATRIX
C
      DO 27 I=1,16
      DO 27 J=1,4
      C(I,J)=0.0
      DO 27 K=1,4
      C(I,J)=C(I,J) + BT(I,K)*D(K,J)
   27 CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2031)
 2031 FORMAT(////,27X,'BT*D MATRIX (16X4)')
      IF(NPRNT.GE.6) WRITE(QB,2032) ((C(I,J),J=1,4),I=1,16)
C
C        BT*D*B MATRIX
C
      DO 28 I=1,16
      DO 28 J=1,16
      ELSTIF(I,J)=0.0
      DO 28 K=1,4
      ELSTIF(I,J)=ELSTIF(I,J) + C(I,K)*B(K,J)
   28 CONTINUE
      IF(NPRNT.GE.6) WRITE(QB,2035)
 2035 FORMAT('1',//,50X,'BT*D*B MATRIX (16X16)',//)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELSTIF(I,J),J=1,16),I=1,16)
 2012 FORMAT(16(8(1X,E11.4),/,25X,8(E11.4,1X),//),'1')
C
C        NUMERICAL INTEGRATION FOR ELEMENT STIFFNESS
C
      GO TO (31,33,35,37,39,41,43,45,47),KKK
   31 DO 32 I=1,16
```

```
      DO 32 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H1*H1
32    CONTINUE
      KKK=KKK+1
      XI=-.774597
      ETA=0.0
      IF(NPRNT.GE.6) WRITE(QB,2036)
2036  FORMAT(//,37X,'THE ACCUMULATED ELEMENT STIFFNESS MATRIX (16X16)',/
     1/)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
33    DO 34 I=1,16
      DO 34 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H1*H2
34    CONTINUE
      KKK=KKK+1
      XI=-.774597
      ETA=-XI
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
35    DO 36 I=1,16
      DO 36 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H1*H3
36    CONTINUE
      KKK=KKK+1
      XI=0.0
      ETA=-.774597
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
37    DO 38 I=1,16
      DO 38 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H2*H1
38    CONTINUE
      KKK=KKK+1
```

```
      XI=0.0
      ETA=0.0
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GOTO 11
   39 DO 40 I=1,16
      DO 40 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H2*H2
   40 CONTINUE
      KKK=KKK+1
      XI=0.0
      ETA=.774597
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
   41 DO 42 I=1,16
      DO 42 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H2*H3
   42 CONTINUE
      KKK=KKK+1
      XI=.774597
      ETA=-.774597
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
   43 DO 44 I=1,16
      DO 44 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H3*H1
   44 CONTINUE
      KKK=KKK+1
      XI=.774597
      ETA=0.0
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
   45 DO 46 I=1,16
```

```
      DO 46 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H3*H2
 46   CONTINUE
      KKK=KKK+1
      XI=.774597
      ETA=.774597
      IF(NPRNT.GE.6) WRITE(QB,2036)
      IF(NPRNT.GE.6) WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
      GO TO 11
 47   DO 48 I=1,16
      DO 48 J=1,16
      ELST  (I,J)=ELST  (I,J) + ELSTIF(I,J)*D7*DEN1*XJACOB*H3*H3
 48   CONTINUE
      IF(NPRNT.GE.4)   WRITE(QB,2021)
 2021 FORMAT(1H1)
      IF(NPRNT.GE.4)   WRITE(QB,2013)
 2013 FORMAT(//,34X,'THE FINAL INTEGRATED ELEMENT STIFFNESS MATRIX (16X1
     16)',//)
      IF(NPRNT.GE.4)   WRITE(QB,2012) ((ELST  (I,J),J=1,16),I=1,16)
 49   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATSOL(AP,P,W,AA,B,NEQ,IP,KY,MOD,NPRIT,NNNE,DET)
C
C     SETS UP THE AUGMENTED STIFFNESS MATRIX AP(NNE,NNNE), AND TWO
C        AUXILIARY MATRICES: AA(NNE,NNNE), THE SIZE OF THE STIFFNESS MATRIX,
C        AND B(NNNE), A VECTOR THE SAME SIZE AS THE DISPLACEMENT VECTOR.
C        THE ARRAYS XKSYS, FORCE, AND DISP ARE RELABELED AP, P, AND W,
C        RESPECTIVELY.
C
      DIMENSION AP(NEQ,NNNE),P(NEQ),W(NNNE)
      DIMENSION AA(NEQ,NNNE),B(NEQ)
      N=NEQ
      NP=N+1
      DO 20 I=1,N
      AP(I,NP)=P(I)
 20   CONTINUE
      CALL TRIDIG(AP,W,AA,B,N,KY,MOD,NPRIT,NNNE,DET)
      CALL CHEKSM(AP,W,AA,B,N,IP,NNNE)
      RETURN
      END
```

```
      SUBROUTINE TRIDIG(AP,W,BTC,WI,N,KY,MOD,NPRIT,NNNE,DET)
C
C     PERFORMS AN IN-CORE REDUCTION OF THE NNE EQUATIONS AFTER HAVING
C        REDEFINED THE AA AND B ARRAYS AS BTC AND WI, RESPECTIVELY.
C        NOTE: WHEN NPRIT = 0, OUTPUT FROM TRIDIG AND CHEKSM IS SUPPRESSED;
C        WHEN NPRIT IS NOT EQUAL TO ZERO, THE OUTPUT IS CALLED FOR.
C
      DIMENSION AP(N,NNNE),W(NNNE)
      DIMENSION BTC(N,NNNE),WI(N)
      DIMENSION WWW(27)
      INTEGER QR,QB
      QR=8
      QB=5
C
C       FORM INITIAL A-P MATRIX
C
      NP=N+1
      DO 95 I=1,NP
 95   W(I)=0.0
      KEY=0
      NSP=NP
      NPP=NP-1
      IF(NPRIT)201,210,201
 201  WRITE(QB,1001) NPP,NP
 1001 FORMAT('1',50X,'INITIAL AP MATRIX (',I2,'X',I2,')',//)
      WRITE(QB,1000) ((AP(I,J),J=1,NP),I=1,NPP)
 1000 FORMAT(16(8(1X,E11.4),/,25X,9(E11.4,1X),//))
 210  CONTINUE
      IF(KY.EQ.0) GO TO 25
      GO TO 160
 25   CONTINUE
C
C       SET UP BTC MATRIX
C
C     FIRST COLUMN
C
```

```
      DO 30 I=1,NPP
      BTC(I,1)=AP(I,1)
 30   CONTINUE
C
C    TOP ROW
C
      DO 40 J=2,NP
      BTC(1,J)=AP(1,J)/AP(1,1)
 40   CONTINUE
C
C    SECOND ROW AND ON
C
      DO 100 I=2,NPP
      DO 80 J=2,NP
      IF(J.GT.I) GO TO 60
      JJ=J-1
      BT=0.0
      DO 50 K=1,JJ
      BT=BT+BTC(I,K)*BTC(K,J)
 50   CONTINUE
      BTC(I,J)=AP(I,J)-BT
      GO TO 80
 60   CONTINUE
      II=I-1
      BT=0.0
      DO 70 K=1,II
      BT=BT+BTC(I,K)*BTC(K,J)
 70   CONTINUE
      BTC(I,J)=(AP(I,J)-BT)/BTC(I,I)
 80   CONTINUE
 100  CONTINUE
      IF(NPRIT) 220,230,220
 220  WRITE(QB,1003) NPP,NP
 1003 FORMAT('1',50X,'FINAL BTC MATRIX (',I2,'X',I2,')',//)
      WRITE(QB,1000) ((BTC(I,J),J=1,NP),I=1,NPP)
C
```

```
C        FIND DEFLECTIONS
C
 230  CONTINUE
      DO 120 I=1,NPP
      K=NP-I
      TW=0.0
      NPPP=NPP-1
      DO 110 J=K,NPPP
      TW=TW+BTC(K,J+1)*W(J+1)
 110  CONTINUE
      W(K)=BTC(K,NP)-TW
 120  CONTINUE
      IF(KEY.EQ.0) GO TO 129
C
C        INVERT THE DEFLECTION MATRIX
C
      DO 240 I=1,NP
      WWW(I)=W(I)
 240  CONTINUE
      DO 125 I=1,NPP
      J=NP-I
      WI(I)=W(J)
 125  CONTINUE
      DO 126 I=1,NP
      W(I)=WI(I)
 126  CONTINUE
      IF(NPRIT) 260,270,260
 260  WRITE(Q3,5000) NP,NP
 5000 FORMAT('1',10X,'THE DEFLECTION MATRIX (',I2,'X1)',10X,'THE INVERTE
     1D DEFLECTION MATRIX (',I2,'X1)',//)
      DO 250 I=1,NP
      WRITE(Q3,5002) WWW(I),W(I)
 5002 FORMAT(19X,E11.4,29X,E11.4)
 250  CONTINUE
 270  CONTINUE
      KEY=2
```

```
      GO TO 161
  129 CONTINUE
      GO TO 200
  160 CONTINUE
C
C        FORM THE FLIP-FLOP MATRIX
C
C     PLACE AP MATRIX ONTO BTC MATRIX
C
      KEY=1
  161 CONTINUE
      DO 170 I=1,NPP
      DO 170 J=1,NPP
      K=NP-I
      L=NP-J
      BTC(I,J)=AP(K,L)
  170 CONTINUE
C
C     SET UP PRESSURE COEFFICIENTS
C
      DO 175 I=1,NPP
       J=NP-I
      BTC(I,NP)=AP(J,NP)
  175 CONTINUE
C
C     RE-INITIALIZE AP MATRIX
C
      DO 180 I=1,NPP
      DO 180 J=1,NP
      AP(I,J)=BTC(I,J)
  180 CONTINUE
      IF(NPRIT) 182,183,182
  182 CONTINUE
      WRITE(Q3,1002) NPP,NP
 1002 FORMAT('1',50X,'MODIFIED AP MATRIX (',I2,'X',I2,')',//)
      WRITE(Q3,1000) ((AP(I,J),J=1,NP),I=1,NPP)
```

```
  183 CONTINUE                                                          TRDG0145
      IF(KEY.EQ.2) GO TO 200                                            TRDG0146
      GO TO 25                                                          TRDG0147
  200 CONTINUE                                                          TRDG0148
      NP=NSP                                                            TRDG0149
      RETURN                                                            TRDG0150
      END                                                               TRDG0151
```

```
      SUBROUTINE CHEKSM(AP,W,D,Y,N,IP,NNNE)
C
C     USES THE SOLVED DISPLACEMENTS TO CALCULATE A LOAD VECTOR. THIS
C        IS COMPARED TO THE KNOWN LOAD VECTOR AND THE DIFFERENCES, OR
C        RESIDUALS, CALCULATED (GIVING A GENERAL INDICATION OF THE
C        ACCURACY OF THE SOLUTION).
C        NOTE: WHEN IP = 0, OUTPUT FROM CHECKSM IS REPRESSED;
C        WHEN IP IS NOT EQUAL TO ZERO, THE OUTPUT IS CALLED FOR.
C
      DIMENSION AP(N,NNNE),W(N),D(N),Y(N)
      INTEGER QR,QB
      QR=8
      QB=5
      IF(IP.EQ.0) RETURN
      DO 20 I=1,N
      SUM=0.0
      DO 10 K=1,N
      SUM=SUM+AP(I,K)*W(K)
 10   CONTINUE
      Y(I)=SUM
 20   CONTINUE
      WRITE(QB,1020)
 1020 FORMAT(////)
      WRITE(QB,1000)
 1000 FORMAT('1',17X,'C H E C K S U M  F O R  T H E  A X = P  M A T R X'
     1,/,16X,53('*'),////,21X,'Y',13X,'A P',15X,'Y - A P',/20X,3('*'),16
     2X,5('*'),13X,9('*'),//)
      DO 30 I=1,N
      D(I)=Y(I)-AP(I,N+1)
      WRITE(QB,1001) Y(I),AP(I,N+1),D(I)
 1001 FORMAT(10X,3E20.10)
 30   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SELECT (TSTRES ,IJK,T,DT,CRP,C,CRPNV)
C
C     SELECTS THE PARTICULAR CREEP EXPRESSION AND TIME INCREMENT TO BE
C        USED. CALCULATES THE CREEP STRAINS AND ASSEMBLES THE CONSISTENT
C        NODAL LOAD VECTOR. PROGRAM THEN RETURNS TO CLISP FOR ITERATION
C        AND CALCULATION OF CREEP DISPLACEMENTS AND STRESSES.
C
      DIMENSION TSTRES (NN,5),CRP(NN),XS(NN,5),SEFF(NN),C(16,4),CRPNV(NN
     1E,NN)
      INTEGER QR,QB
      QR=8
      QB=5
C
      IF (IJK.NE.1) GO TO 44
      READ (QR,99) JCODE,DT
   99 FORMAT (I2,F5.1)
   44 CONTINUE
      T=T+DT+10*IJK
      DO 30 I=1,NN
      DO 31 K=1,4
      XS(I,K)=TSTRESS(I,K)
   31 CONTINUE
      SEFF(I)=(1/SQRT(2))*(SQRT(XS(I,1)-XS(I,4))**2+(XS(I,4)-XS(I,2))**2
     1+(XS(I,2)-XS(I,1))**2+6(XS(I,3)**2)))
      S=SEFF(I)
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20),JCODE
    1 CRP(I)=6.4E-18*(S**4.4)*T
      IF(IJK.NE.1) GO TO 899
      IF(I.NE.1) GO TO 999
      WRITE(QB,101)
  101 FORMAT('1'/10X,'CREEP EXPRESSION:',7X,'GREENBAUM AND RUBENSTEIN,'/
     1/34X,'CEFF = (6.4E-18)(SEFF**4.4)T')
      GO TO 999
    2 CRP(I)=6.4E-18*(S**4.4)*(T**0.7)
      IF(IJK.NE.1) GO TO 899
      IF(I.NE.1) GO TO 999
```

```
      WRITE(Q3,201)
201   FORMAT('1'/10X,'CREEP EXPRESSION:',7X,'GREENBAUM AND RUBENSTEIN,'/
     2/34X,'CEFF = (6.4E-18) (SEFF**4.4) (T**0.7)')
      GO TO 999
3     CRP(I)=((0.47E-6*T)/(-28.+T))*S
      IF(IJK.NE.1) GO TO 899
      IF(I.NE.1) GO TO 990
      WRITE(QB,301)
301   FORMAT('1'/10X,'CREEP EXPRESSION:',7X,'LORMAN '//34X,
     3'C = T*S(0.47E-6)/(-28.0+T)')
      GO TO 991
4     CRP(I)=T/(0.06+0.165E-3*T)
      IF(IJK.NE.1) GO TO 899
      IF(I.NE.1) GO TO 990
      WRITE(Q3,401)
401   FORMAT('1'/10X,'CREEP EXPRESSION:',7X,'ROSS,'//34X,'C = T/(0.06+0.
     4165E-3*T)')
      GO TO 993
5     CRP(I)=60.E-6*(T**(1./0.82))
      IF(IJK.NE.1) GO TO 899
      IF(I.NE.1) GO TO 990
      WRITE(Q3,501)
501   FORMAT('1'/10X,'CREEP EXPRESSION:',7X,'SHANK,'//34X,
     5'C = (60.0E-6)(T**(1/0.82))')
      GO TO 993
6     CONTINUE
7     CONTINUE
8     CONTINUE
9     CONTINUE
10    CONTINUE
11    CONTINUE
12    CONTINUE
13    CONTINUE
14    CONTINUE
15    CONTINUE
16    CONTINUE
```

```
17    CONTINUE
18    CONTINUE
19    CONTINUE
20    CONTINUE
      GO TO 990
991   WRITE(QB,992)
992   FORMAT(1X///21X,'WHERE,',7X,'C = CREEP STRAIN'/34X,'T = TIME SINCE
     1 APPLICATION OF LOAD'/34X,'S = APPLIED STRESS')
      GO TO 990
993   WRITE(QB,994)
994   FORMAT(1X///21X,'WHERE,',7X,'C = CREEP STRAIN'/34X,'T = TIME SINCE
     1 APPLICATION OF LOAD')
      GO TO 990
999   WRITE(QB,998)
998   FORMAT (1X///21X,'WHERE,',7X,'CEFF = EFFECTIVE CREEP STRAIN'/34X,
     1'T    = TIME SINCE APPLICATION OF LOAD'/34X,'SEFF = EFFECTIVE STRE
     2SS'//41X,'SEFF = (1/SQRT(2))*(SQRT((SR-STHETA)**2 + (STHETA-SZ)**2
     3 + (SR-SZ)**2 + 6(SRZ**2)))'//48X,'SR     = STRESS IN RADIAL DIREC
     4TION'/48X,'STHETA = STRESS IN THETA DIRECTION'/48X,'SZ     = STRES
     5S IN Z DIRECTION'/48X,'SRZ    = SHEAR STRESS IN R-Z DIRECTION')
      GO TO 990
 899  CONTINUE
990   CONTINUE
 30   CONTINUE
      WRITE(QB,32) T
 32   FORMAT(//////////10X,'AT TIME T = ',I3,' DAYS,'/10X,'THE TOTAL DISP
     1LACEMENTS AND STRESSES ARE:')
      DO 9998 I=1,NNE
      DO 9998 J=1,NN
 9998 CRPNV(I,J)= C(I,J)*CRP(I)
C
C
C
C
C     PERFORM CREEP ANALYSIS USING SPECIFIED CREEP FORMULATION
C
```

```
C
C
C       END  OF  PROGRAM
C
      RETURN
      END
```

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