Essays on International Trade and Sovereign Debt

by

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Abstract

This thesis studies the way intratemporal trade matters for intertemporal trade, focusing on three different interactions through sovereign debt. The thesis is divided in 3 chapters.

In the first chapter, I show that existing evidence suggests that sovereign defaults disrupt international trade. As a consequence, countries that are more open have more to lose from a sovereign default and are less inclined to renege on their debt. In turn, lenders should trust more open countries and charge them lower interest rate. In most cases, a country should also borrow more debt the more open it is. The first chapter formalizes this idea in a simple sovereign debt model *à la* Eaton and Gersovitz (1981). It also provides evidence using gravitational instrumental variables from Frankel and Romer (1999) and Feyrer (2019) as a source for exogenous variation for trade openness.

In the second chapter of the thesis, I develop a new model of crisis contagion through international trade. We focus on sovereign debt crises in a multi-country economy with endogenous default *à la* Eaton and Gersovitz (1981). The starting point of our analysis is the observation that sovereign defaults do not only reduce international borrowing, but also international trade flows: international trade is a commitment device to repay debt as a disruption in trade is one of the costs of default. As a consequence, when a country defaults, it reduces gains from trade in the rest of the world and it raises the incentives to default everywhere else. After providing some suggestive evidence for this kind of contagion through trade, we show how our model can rationalize default waves. Our model also predicts that more trade openness lowers the risks of a worldwide crisis, and it also has normative implications about tariffs and macroprudential policies because there is excess debt. A tax on debt and free-trade agreements with special tariff derogations for countries that want to default improve welfare from intertemporal transfers.

In the third chapter, I wonder why sovereign spreads of a sovereign country appear to depend more on economic conditions in the rest of the world rather than those of the country itself. To shed light on this puzzle, I propose an Eaton-Gersovitz sovereign debt model with international trade and terms of trade effects. I assume that there is an exogenous foreign demand for the domestic good that can vary over time and that trade costs increase whenever the sovereign government defaults. After calibrating my model on recent data, I show that a large share of the volatility of spreads can be explained by movements in the foreign demand for domestic goods.

I owe a very large debt of gratitude to my thesis supervisor Arnaud Costinot, for his outstanding advice and his support during the entire process. In spite of a disturbing pandemic, Arnaud managed to be very present. I am very grateful to Iván Werning, for his incredibly helpful insights, his kindness and his personal advice - I will try to be wise enough to apply them. Professors Dave Donaldson and David Atkin devoted a lot of time to help me, and I want to tell them thank you for that. More generally, the entire community of MIT was an incredible source for improvement and emulation in my work: I want to thank all the participants of trade tea and macro lunch seminars, faculty and students.

I dedicate this thesis to my wife Florence, whom I have met at the same time as economics, at 18. Florence and economics became two incredibly important partners for my life, but they were never equal: Florence has always supported me even when I felt down with economics, while I never experienced feeling down with Florence. The thesis is about the way commitment can be compromised by distance, but I am happy that we proved the thesis wrong: Atlantic Ocean was not enough to separate us.

Contents

Chapter 1

Sovereign Debt and International Trade

1.1 Introduction

The main peculiarity of sovereign debt contracts is that repayments are not easily enforceable by the lender: a sovereign country with a strong enough army and divided enough lenders can default without expecting dire consequences. In the absence of enforceable contracts, a good borrower is someone with whom the lender has frequent business relations, as suggested by the repeated game literature. A borrower afraid of paying the cost of losing those relationships would be incentivized to repay debt (for an early version of this argument, see Hume). From the point of view of sovereign borrowing, a form of relation with the outside world can serve as a commitment device. An obvious form of such a reputational cost is the interruption of sovereign borrowing. However, Bulow and Rogoff (1989) proved an theorem about impossibility of sovereign debt if the only cost of default is the impossibility of borrowing in the future. We must therefore assume that some kind of relation with the outside world gets interrupted after a sovereign default, making it worth for a sovereign debtor repaying its debt under normal circumstances.

What kind of relationship with the outside world gets interrupted after a sovereign default exactly? Does it get interrupted for external reasons (other countries deciding to sanction defaulters) or internal reasons (destruction of the financial local markets, relying on sovereign debt)? The answer is not entirely clear from historical precedents nor from the literature, but an obvious candidate is international trade, because it summarizes relations with the outside world from a static point of view. As we show in figure 1, periods of decreasing commercial integration have coincided with global default waves since 1800. It suggests that during defaults, international trade decreases or vice-versa.

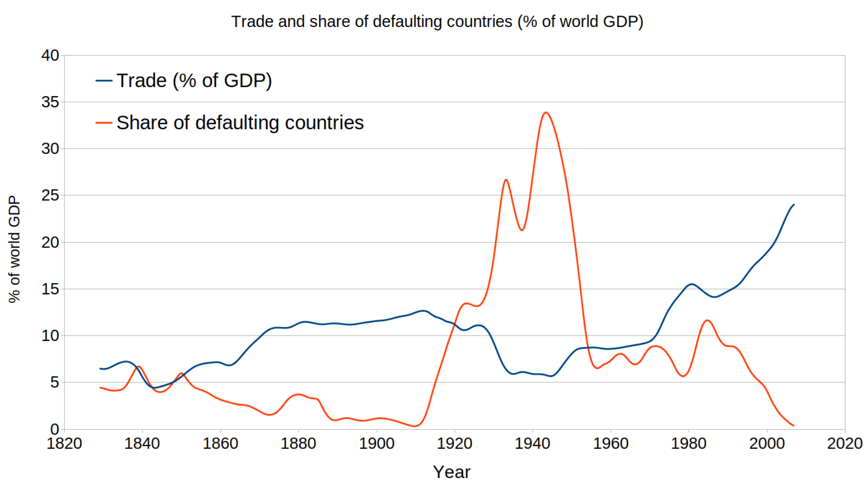

Figure 1.1: Trade and default from 1815 to 2007 (HP-filtered). Sources: Reinhart and Rogoff (2009), Fouquin et al. (2016)

This paper argues that international trade is an important component of non-reputational default costs. It argues that trade gets interrupted partially in the wake of a sovereign default. As a consequence, default is more costly for large traders. Indeed, larger gross trade flows imply that more is at stake when a government decides to go into financial autarky and to default. The defaulting country's inhabitants and firms can face tighter international constraints or trust issues which can affect their ability to trade internationally. Thus, governments in countries more open to trade should find it easier to borrow from international lenders: trade acts as a commitment device for these borrowers. We argue in favor of this mechanism with a simplified Eaton and Gersovitz (1981) model and provide empirical evidence in favor of it with cross-country regressions using instrumental variables for trade inspired by Frankel and Romer (1999) and Feyrer (2019) .

This paper completes our understanding of sovereign default costs but it also has important normative consequences. Indeed, a direct consequence of this paper's argument is that protectionist policies restricting both imports and exports should deteriorate government's ability to borrow.[1] Moreover, the large decrease in transport costs that has been observed since World War 2 can explain the development of sovereign debt markets: easier transport means more trade and more sensitivity to autarky as a result, therefore more commitment.

Section 2 starts with motivating evidence that sovereign defaults lead to a shrinkage in trade. It revisits the findings in Rose (2005) with updated data and more general controls. It finds that periods of default seem to coincide with declines in trade. It also finds that during a sovereign default, bilateral trade between a defaulter and its partners decreases by 10-50% depending on the specification.

Section 3 formalizes the idea with a simple model inspired by Eaton and Gersovitz (1981) where trade autarky is the cost of sovereign default. It finds that openness, defined as lower trade costs, improves a government's ability to borrow and also its actual borrowings in most cases.

Section 4 presents empirical results. Because of endogeneity concerns that trade variations depends on sovereign debt finance, we need to use instrumental variables, inspired by Frankel and Romer (1999) and Feyrer (2019). We define them using geographical predictors of trade and time variation in the relative importance of

[1]Assuming these policies are suboptimal from a static point of view and there are no dynamic externalities: in this case, a protectionist policy is seen as a self-inflicted damage. Therefore, the cost of this kind of policies increases when one takes into account its effects on sovereign debt crises. The existence of an optimal positive tariff could change the direction of our claim. We do not explore these issues in this paper.

trade and sea distances and exploit them for regression analysis. They confirm the results Section 3: more openness leads to cheaper credit costs and to larger levels of debt as well. An increase in total volume of trade by 10% is shown to lead up to a 40 b.p. decrease in CDS spreads, and to a 3% increase in debt.

Literature Review One of the questions in the sovereign debt literature is why a sovereign should repay when there is no clear mechanism to enforce either repayment or punishment from the point of view of investors. Private firms may be constrained to go bankrupt and their assets are then shared between their debtors when they default in developed financial markets. Direct invasions of defaulting countries by creditors have not been frequent since 1945, although they used to be frequent, as shown in Mitchener and Weidenmier (2010).[2] Government's assets cannot be seized easily. In that case, why should a sovereign borrower ever repay debt at any moment? Our suggestion to that old question is that international trade is a casualty of sovereign default, either because of sanctions or because of reliance of trade on sovereign debt finance (we stay agnostic about this mechanism).

Eaton and Gersovitz (1981), in a seminal paper, argue that reputation concerns may explain government's willingness to borrow. But Bulow and Rogoff (1989) that we cited earlier proved that the own model of Eaton and Gersovitz (1981) was not consistent with positive levels of borrowing, and one assume that there is direct cost for defaulting apart from financial autarky. Bulow and Rogoff (1989), like Kaletsky (1985) or Cole and Kehoe (1998), suggested the risk of trade wars or trade interruption, either because of retaliation, trade finance interruption or reputational spillovers was such a direct penalty of default. Mendoza and Yue (2012) directly used trade interruption as the cost of default, and they attributed it to trade credit, but focused on the dynamic implications of this assumption. In their model, trade finance deteriorates in bad times and the commercial interest rate is equal to the sovereign debt

[2]For example, the small state of Newfounland, as a consequence of its default in 1933, lost its sovereignty to Canada.

interest rate. As consequence, incentives to default in bad times get amplified. They did not study the effect of trade openness on sovereign debt finance as we do in this paper and focused more on the dynamic aspects of this assumption.

Most other sovereign debt papers took this cost of default as a given black box and rather focused on net trade flows rather than gross trade flows: current account and its relation to business cycles should indeed matter for sovereign debt, as underscored by Aguiar and Gopinath (2007) or Aguiar and Gopinath (2006). But relations between debtor country and the rest of the world is not summarized by debt or net trade flows. It also relies on *gross* trade flows. There are more general reputation concerns that are not about *intertemporal* trade, but also about *intratemporal* trade: Cole and Kehoe (1998) argued there might be reputation spillovers on other activities, such as trade, but they have not been studied widely. There has also been a trade literature focused on the links between intertemporal and intratemporal trade: Eaton et al. (2016), Reyes-Heroles et al. (2016). Kikkawa and Sasahara (2020) study more explicitly the relation between trade and sovereign default. In their model, default is associated with a negative productivity shock, as in Arellano (2008). This negative productivity shock limits countries' incentives' to default. In the presence of trade, Kikkawa and Sasahara (2020) note that the same productivity is also associated with terms-of-trade effects that affect both the value of a country's endowment as well as the value of its debt and, in turn, its probability to default. In contrast, default in our model is associated with a demand rather than a supply shock: countries that default lose foreign demand, while their endowments remain unchanged. This implies, in particular, that more openness to trade always creates less incentives to default in our model.

Fitzgerald (2012) also studied the link between risk sharing between countries and trade costs, following a suggestion made by Obstfeld and Rogoff (2000) that international macroeconomic puzzle might be attributed to trade costs. However, these papers do not feature defaultable sovereign debt.

The trade disruption occurring after sovereign default has been documented in several papers, prominently in Rose (2005), whose evidence we replicate later; similar contributions include Manasse and Roubini (2009). Martinez and Sandleris (2011), Kohlscheen and O'Connell (2008), Borensztein and Panizza (2009), Zymek (2012) found similar results, arguing that trade credit was the driver of this effect, rather than direct sanctions. On the microeconomic level, Gopinath and Neiman (2014), Borensztein and Panizza (2010), Arteta and Hale (2008) , Hébert and Schreger (2017) found in different contexts that exporting firms were disproportionately hurt by sovereign default, which is quite consistent with our hypothesis.

1.2 Motivational Evidence: Trade Collapse After Default

In this Section, we update findings in Rose (2005) including more recent years, with a different method: instead of defining default as an event, we are going to distinguish default phrases (from default to the end of restructuring) as in Reinhart and Rogoff (2009). We are also going to use more data points and to allow for more general controls: for example, a bilateral pair fixed effect and time-varying regional fixed effects, instead of geographical predictors of trade and simple year fixed effects.

1.2.1 Data and Specification

To define sovereign defaults, we use data by Reinhart and Rogoff (2009), available on their website and updated up to 2012. Their data starts in 1800 and allows use to include some early sovereign defaults. In their data, a country is considered defaulting as long as it did not find an agreement with creditors (on average, this period lasted 7 years). Therefore, restructuring to date the end of default has a broader end than Rose (2005) who used Paris debt renegotiations to define defaulting countries. Rose (2005) found lasting effects that were similar from one year to the other: however, the size of the effect of default on trade did not vary significantly in his findings, so that we do not study the dynamic effects of default. We also use CEPII data from Fouquin et al. (2016) that give historical series of bilateral trade data and allow us to go far as back as 1800 to estimate the effect of sovereign default on bilateral trade data.

We test the following equation with different sets of controls for all pairs of countries (i, j) and all years t:

$$\ln Exports_{i,j,t} = \gamma^e D_{i,t} + \gamma^i D_{j,t} + \beta Controls_{i,j,t} + \varepsilon_{i,j,s,t}, \tag{1.1}$$

where $Exports_{i,j,t}$ is exports from country i to country j at year t, of which we take the log, except when we want to include null observations, in which case we use inverse hyperbolic sine, equivalent to log for large values but equal to 0 in 0^3. $D_{i,t}$ is a dummy variable indicating whether a country is still defaulting, $Controls_{i,j,t}$ is a set of controls including at least a pair fixed effect $\alpha_{i,j}$ taking into account all possible fixed predictors of trade and a year fixed effect α_t taking into account variation. We allow for several other types of controls, as a time varying pair fixed effect $\alpha_{i,j,c(t)}$ defined for different bins of data (every 20 years), regional year fixed effects $\alpha_{R(i),t}$ and $\alpha_{R(j),t}$ for large regions.[4] We also allow for more flexibility to the structure by including the possibility of time-varying bilateral trade functions: if p is a function that associates a period to each year (for example, decades, every 20 years), we can control for time-varying pair fixed effects $\alpha_{i,j,p(t)}$ and still find significant effect of default on imports.

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[3] Fouquin et al. (2016), who provide the CEPII dataset, claim that null bilateral trade data correspond when bilateral trade data could indeed be estimated to be 0, although it might in some cases also be due to lack of evidence. We allow both interpretations as we either include or exclude observations where bilateral trade flow is "null" in the results below. Including null observations lessens the effect of default but does not change our effect qualitatively. We include regressions with null observations to stay conservative.

[4] The regions we define are Europe, Asia, Middle East, Atlantic Ocean, Africa, North America, Latin America.

1.2.2 Results

We run equation (1.1) with different covariates and specifications and show our results in table 1.1. We find results similar to those in Rose (2005). The decrease of imports after default is between 10% and more than 50%. We observe the effect of default on imports is larger than on exports but exports still decrease significantly after default, even in not favorable conditions (time-varying fixed effect).

One important question for significance is whether we should include observations of 0 bilateral trade as literally meaning 0 trade or as a mistake. Not including these observations reduces the size of the effect, which would make sense if null observations indeed corresponded to no trade: defaults seem to impact the extensive margin of trade).[5] When we include null observations, we use the inverse hyperbolic sine of exports rather than the log, to include more easily null observations.

[5] This macroeconomic evidence would be the macroeconomic equivalent of what Gopinath and Neiman (2014) find at the firm level in Argentina after 2001 default in Argentina: a large number of firms completely stopped importing certain kinds of inputs. It would mean that defaulting countries stop importing from some trade partners with whom they were trading less before.

Table 1.1: Effect of sovereign default on bilateral trade

	Dependent variable:					
	Exports (log or hyperbolic arcsine)					
	(1)	(2)	(3)	(4)	(5)	(6)
Default (origin)	-0.643^{***}	-0.438^{***}	-0.447^{***}	-0.126^{***}	-0.117^{***}	-0.027^{***}
	(0.016)	(0.017)	(0.018)	(0.009)	(0.018)	(0.009)
Default (destination)	-0.904^{***}	-0.534^{***}	-0.521^{***}	-0.149^{***}	-0.195^{***}	-0.108^{***}
	(0.014)	(0.016)	(0.016)	(0.008)	(0.017)	(0.009)
		Controls				
GDP (log, destination)	No	Yes	Yes	Yes	Yes	Yes
GDP (log, origin)	No	Yes	Yes	Yes	Yes	Yes
Pair F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Time-Varying Pair F.E.	No	No	No	No	Yes	Yes
Data Before 1950	Yes	Yes	No	No	No	No
Null=0	Yes	Yes	Yes	No	Yes	No
Observations	837,067	686,030	637,316	427,185	637,316	427,185
R^2	0.736	0.748	0.750	0.836	0.839	0.895

Note: *p<0.1; **p<0.05; ***p<0.01

1.3 Two-Period Model

In this Section, we present a model where a small open economy that trades with the rest of the world and borrows from it. In case of default, the country enters into financial autarky (which should not matter in the two-period case, but results can easily be extended to include the cost of financial autarky), and more importantly, partial trade autarky: as a consequence of default, trade costs increase. To make the exposition simpler, we assume the cost of default is going to be complete trade autarky.

The timing of the model is as follows:

- In the first period, government chooses how much to borrow. The price of bonds q it emits depends on its probability of default, which depends on debt B: government takes as given the price of bonds as a function of debt, $q : B \mapsto \mathbb{R}_+$. Government then spends its income and the amount borrowed in goods' consumption.

- In the second period, government learns the new value of GDP and uncertainty is solved out. At this point, it decides whether to default or not. If it defaults, it cannot benefit from international trade any more but debt burden is alleviated. If it repays, it can trade and benefit from commercial integration as before.

In this simple model, default is associated with an increase in the trade costs to infinity, but we see it as a simplification: in a more general setting, there would be a proportional increase in trade costs. This is the reduced form version of the assumption that the disruption in trade finance makes trade more costly after a sovereign default. The amount of disruption is likely to be increasing in the total volume trade.

We choose a 2-period model for the sake of simplicity: the 2-period model is the simplest possible way to capture debt accumulation, default risk and repayment conditions. Although consumption smoothing motives and reputational concerns are not very strong in a 2-period model - they are absent in the second period - the 2-period setting still captures the elements of sovereign debt we care about in this paper. Some of our results are still valid in a version of the model with infinite periods however.

Moreover, even in dynamic calibrations, non-reputational default costs matter a lot: for example, Arellano (2008), often cited as a benchmark for sovereign debt calibrations, managed to get satisfying quantitative results mainly through specific assumptions on non-reputational default costs. It is therefore interesting to study default cost for itself in the most direct framework, in view of the calibration literature.

1.3.1 Assumptions and Primitives

- Static Consumption and Gains from Trade There are two periods $t = 1, 2$. Aggregate consumption C_t at each period t is given by:

$$C_t := A(c_t, c_t^\star),$$

where A is an aggregator with constant returns to scale.[6] Trade is motivated by the inability of the country to produce foreign varieties of consumption goods as in Armington (1969). However, it could indifferently be motivated by comparative advantage motives, as in Ricardian trade models.

We make this assumption for the sake of simplicity, but it does not matter: the only relevant point for our results is that gains from trade can be inferred from the variation in imports, as in Arkolakis et al. (2012). Therefore, all trade models embedded in the results of Arkolakis et al. (2012) would work in our framework[7] for the first basic proposition: more open countries can borrow a larger amount of debt.

- Utility There are two periods $t = 1, 2$. The small open economy with a representative agent takes all prices as given and maximizes utility:

$$U_i = u(C_1) + \frac{1}{1+\rho}\mathbb{E}u(C_2),$$

given the budget constraints and default constraints we are defining below. We are going to assume for the sake of simplicity that the representative agent is also the

[6] The most common example of such an aggregator would be CES: $A(c_t, c_t^\star) = (\alpha^{\frac{1}{\sigma}} c_{t,D}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} c_{t,F}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$

[7] It includes Eaton and Kortum (2002) and Melitz (2003) among others.

government, which is the most common assumption in the sovereign debt literature. From a decentralized market perspective where the inhabitants of the country would take the quantity of sovereign debt as given, it is equivalent to assuming that government subsidizes or taxes borrowing to make agents internalize, as it was proved in Na et al. (2018).

- Budget constraint and trade costs Domestic good is assumed to be the *numéraire*. It is produced with an inelastic effective labor supply (alternatively, endowment or output) Y_t in t. The price of the foreign good, as perceived by the domestic country, $p(\tau, Y_t)$ is assumed to depend on endowment and trade costs. We assume our economy is small relative to the rest of the world and can take the macroeconomic international conditions as certain.[8] We shall assume that:

$$\frac{\partial p(\tau, Y)}{\partial \tau} > 0.$$

In other words, the price of the foreign goods increases in trade costs. In some of our results, we will be more radical and assume that there are no terms of trade: in this case, $p = p(\tau)$ depends on trade costs (for example, $p(\tau) = \tau^2 p_0$ for some exogenoux p_0) but not on output. This simplifying assumption is useful for our results about borrowing decisions of the government, because it spares us involved considerations about terms of trade.[9] Reciprocally, the perceived domestic selling price of the good is $\frac{1}{\tau}$, but we still assume that the domestic price is the *numéraire* - the perceived good price should translate into terms of trade effects.[10] Y_1 and τ_1 are given, but Y_2 is random and τ should depend on default's decision as we are going to specify.

[8] The only purpose of this assumption is for the clarity of exposition and avoid the accumulation of state variables. Most results are, at least quantitatively, robust to the addition of foreign shocks.

[9] In other words, we will in some cases (propositions 2 and 3) exclude from the analysis foreign exchange crises and terms of trade effects.

[10] This should simplify notations but the results are robust to several specifications about the *numéraire*. However, the choice of the *numéraire* matters for the analysis however because it determines the quantity on which debt is indexed. In this context, this assumption is equivalent to the assumption that a country borrows in its own currency.

The budget constraint for government that repays debt at period t writes:

$$p(\tau, Y_t)c_{t,F} + c_{t,D} + B_{t-1} = Y_t + q_t(B_t, \tau)B_t,$$

where B_{t-1} is debt inherited from the previous period, B_t is the face value of newly emitted bonds and q_t the actual price of those new bonds. The price of bonds q_t is a function of their face value B_t because the price of bonds depend on government's incentives to default in the next period and therefore on τ, which is a structural parameter of the analysis.

- Timing of the Model and Default Decision Bond schedule q_t and inherited debt B_{t-1} are considered given at the beginning of period t by the government. Price q_t of government bonds is determined by the financial markets. If there are only 2 periods, $q_2 \equiv 0$: no money should be borrowed before the end of the world. However, the model can easily be extended to infinity.

At the beginning of period t, government learns the value of Y_t. It chooses whether to default or to repay debt. If the government defaults, then it does not pay debt, but cannot borrow any more, and faces larger trade costs $\tau_D > \tau$. Budget constraint becomes:

$$p(\tau^D, Y_t)c_t^{\star} + c_t = Y_t.$$

The rest of this Section will consider the simplifying extreme case where $\tau^D = +\infty$, so that $p(\tau^D, Y_t) = +\infty$. In this case, the country enters into trade autarky when it defaults:

$$c_t = Y_t$$
$$c_t^{\star} = 0.$$

Based on the utility it gets from defaulting or repaying debt, government chooses whether to default or not. If initial debt is 0, then government should choose how much to borrow at the first period and decide whether or not to default based on the realization of GDP Y_2 in period 2.

- Financial Markets We assume that investors are risk-neutral.[11] This assumption can be considered acceptable however, if we assume that the economy is small and open: if so, then the economy's endogenous risks should be inconsequential to the lenders' portfolio risks. In other words, matters regarding risk diversification can be considered incorporated in an exogenous safe rate $r^\star$.

Because financial markets are competitive, the price $q(B_1)$ of a government bond depends on new debt B_1 only and is computed according to the corrected probability of default:

$$q(B_1) = \frac{1 - \mathbb{P}(C^R_{j,2} < C^{aut}_2)}{1 + r^\star}.$$

Here, the assumption that investors are risk-neutral can be lifted with no harm to our results to the case where there is a risk-aversion factor φ:

$$q(B) = \frac{1 - \varphi\mathbb{P}(\text{Default})}{1 + r^\star},$$

which does not modify the results of the paper. More generally, one could adopt:

$$q(B) = f(\mathbb{P}(\text{Default})),$$

where f is a positive decreasing function. We will stick to the first case for our exposition although many results are robust to this general formulation.

[11] Sovereign debt literature mostly considers risk-neutral investors. However, some models take into account time-varying risk-aversion. Since the effects of risk aversion can be considered GDP shocks or shocks on default cost, their inclusion is an issue for calibrations mostly. It should not matter too much for our study as we are interested in the local effect of a decrease in trade costs. In this case, risk aversion can be considered a simple multiplier φ to apply on the probability of default when we compute bonds' price.

- Equilibrium Definition With all the elements above, we implicitly presented the definition of an equilibrium in the 2-period model, which can be extended to N periods. For the sake of generality, we present the definition of an equilibrium in $N \in \mathbb{N} \cup \{+\infty\}$ periods.

Definition. *Let $N \in \mathbb{N}$, $(Y_t)_{t\in[0,N]}$ be a Markov process, $p(\tau, Y)$ be a value for price function and τ be a value for trade costs. A competitive equilibrium associated with trade costs τ is given by a sequence of value functions $(V_t(B,Y), V_t^R(B,Y), V_t^D(Y))_{t\in[0,N]}$, policy function for borrowing $(b_t(B,Y))_{t\in[0,N]}$, policy function for default $(D_t(B,Y))_{t\in[0,N]}$, and lending functions $(q_t(B,Y))_{t\in[0,N]}$ such that:*

- The value functions solve the recursive equations:

$$V_t^R(B,Y) = \max_{B', c_t, c_t^\star} u(A(c_t, c_t^\star)) + \beta \mathbb{E}(V_{t+1}(B', Y_{t+1})|Y_t = Y) \quad s.t. \; p(\tau, Y)c_t^\star + c_t + B = Y + q_t(B', Y)B'$$

$$\begin{aligned} V_t(B,Y) &= \max\{V_t^R(B,Y), V_t^D(Y)\} \\ V_t^D(Y) &= u(Y \times A(1,0)) + \beta \mathbb{E}(V_{t+1}^D(Y_{t+1})|Y_t = Y). \end{aligned}$$

- Policy functions solve the government's optimization problem::

$$\begin{aligned} D_t(B,Y) &= \mathbb{I}\{V_t^R(B,Y) < V_t^D(Y)\} \\ b_t(B,Y) &\in \arg\max_{B', c_t, c_t^\star} u(A(c_t, c_t^\star)) + \beta \mathbb{E}(V_{t+1}(B', Y_{t+1})|Y_t = Y). \quad s.t. \; p(\tau, Y)c_t^\star + c_t + B = Y + q_t(B', Y)B' \end{aligned}$$

- Financial markets are competitive:

$$q_t(B', Y) = \frac{\mathbb{P}(D_t(B', Y_{t+1}) = 0 | Y_t = Y)}{1 + r}.$$

We used the convention that $V_{N+1} \equiv V_{N+1}^R \equiv V_{N+1}^D \equiv 0$.

In the model where $N = +\infty$, the definition is the same except that none of the value, policy and lending functions depend on time.

1.3.2 Trade Costs, Probability of Default and Debt

In this Section, we are going to look at the effect of trade costs (hence trade openness by contraposition) on the probability of default and on the level of debt. We prove that the probability of default is going to decrease in trade openness.[12] We easily show it to be true for a fixed level of debt, but also when we do not control for debt: countries should adopt a safer behavior as they get more open. We also discuss whether a similar kind of results can apply to debt levels: does face value of debt increase in the total value of debt? While we cannot conclude unambiguously on this level, we show that under some plausible technical conditions about the distribution of GDP shock, debt should increase as a country gets more open (that is, debt should decrease as trade costs increase).

Trade Costs, Probability of Default

In the second period of the model, government has borrowed B_1 at the previous period (negative B_t would mean net savings). It now learns Y_2, which was distributed according to a given probability distribution with density f.

After learning Y_2 , government chooses whether to default or not.[13] If the country chooses to default, the cost for defaulting is the interruption of international trade. A defaulting government is stuck in autarky and it can only consume its own good,

[12] Equivalently, the probability of default should be increasing in trade costs.

[13] Here, the large size and the low volatility of productivity in country F that faces no productivity shock guarantee that it will not default. Indeed, the economy is deterministic from its point of view. This assumption may be interpreted as the simplification of a world in which wealthy entrepreneurs who buy a lot of insurances invest in sovereign bonds. A default of this country would be problematic since it would entail a global disaster for world trade: in the absence of specialization between small islands, all indebted countries would immediately default if trade with the central country were interrupted. Another way to rule that possibility out would simply be to assume that the central country is more patient than all the islands, as measured by the discount rate: $\rho_F < \inf_{i \in [0,1]} \rho_i$. We explore collective incentives to default in a companion paper.

so that:

$$\begin{aligned} C_2^{aut} &= A(Y_2, 0) \\ &= Y_2 A(1, 0). \end{aligned}$$

Not defaulting yields the following aggregate consumption:

$$C_{j,2}^{R} = (Y_2 - B_j)v(1, p(\tau, Y_2)),$$

where $\nu(.,.)$ is the indirect aggregate consumption function for given prices: $\nu(x, y) = \max_{c^D} A(c^D, \frac{1-xc^D}{y})$.

Let us assume for example that preferences are CES. In this case, government should default whenever debt is more than:

$$B_2 := Y_2\big(1 - (1 - IM^{\star})^{\frac{1}{\sigma-1}}\big), \tag{1.2}$$

where B_2 is the debt stock at the end of the first period, when the economy is still open (so that international prices determine the value of GDP) and IM the share of imports in GDP that would correspond to balanced trade or, equivalently, the share of imports in the final consumption:

$$IM^{\star} = IM/(1 - x),$$

where IM is imports in value as a share of GDP and x is trade balance (equivalently in the model, current account) at time t.

Note that this computation was possible to reach using the result that $\varDelta \ln W = -\ln(1 - IM^{\star})/\varepsilon$, where $\varDelta \ln W$ is the difference between welfare in free trade and welfare in autarky, ε is the inverse elasticity of substitution.This result is established in Arkolakis et al. (2012). This result is equivalent for any aggregator A with a nonunitary elasticity of substitution: it would also apply to a non-Armington context. It should also hold as we allow terms of trade (as defined by price p) to vary.

Then, conditional on having a trade model embedded in Arkolakis et al. (2012), more open countries should be able to sustain a larger debt-to-GDP ratio, *ceteris paribus* (including the level of debt).

We can deduce from this result that more open countries should have lower costs of borrowing, everything else equal. Indeed, in line with sovereign debt models as Eaton and Gersovitz (1981), competitive financial markets with risk-neutral preferences lend money to a sovereign government.

Because bonds' price is determined by the probability of default, we can immediately deduce the following proposition:

Proposition 1. *Assume that the economy is a sovereign debt model in $T \in \mathbb{N}$ periods as described above where gains from trade can be computed as in Arkolakis et al. (2012). Conditional on the level of debt B, a more open country, that is a country with higher import share in final consumption, should be charged with a lower interest rate. Any change in trade costs that makes a country more open should improve also decrease the interest rate it faces. The result stays true for $T = +\infty$ if we assume that number of states for GDP is finite.*

We should test this proposition later in the empirical part. One can ask a more general question: what would happen to default probability as trade cost vary, without fixing the level of debt? As we are going to see it in section 3.2.2, debt should vary as the cost of default decreases. For example, if the government is very impatient, it could not care about next period consumption and borrow as much as possible in the current period. In this case, an increase in trade costs would allow this government to borrow more today, without decreasing the probability to default. While this extreme case is possible, can also prove that in general, the probability of default should not increase as trade costs decrease if utility is concave and if the discount factor is positive. However, our analytic proof applies only to the case with constant returns to scale.

Proposition 2. *Assume that we are a two-period sovereign debt model as described above with exogenous prices. Assume that government maximizes its utility and that instantaneous utility function is concave. As a country gets more open, it should face a lower interest rate, unconditional on the level of debt B.*

These two results have a direct corollary about trade tariffs. Let us consider an Armingtonian context, in which the government can impose a tariff on its trade: it should affect its ability to borrow. However, the role of trade tariffs is subtler than the role of trade costs and depend on whether tariffs are below or above the optimal rate. Assume that a country increases its tariffs. If the tariffs are above the optimal tariff, any increase in tariff should diminish the country's ability to borrow, but the results would reverse with tariffs below the optimal level: increasing trade tariffs to the optimal level could increase a government's capacity to borrow sovereign debt because it would improve its ability to trade. However, the optimal tariff is 0 whenever terms of trade effects are non-existent, which we assume in proposition 2 (but not in proposition 1). These results are also more sensitive to some of the specific assumptions we made: for example, we made the assumptions that debt did not impact terms of trade and that debt was indexed on the price of the domestic goods. If one those assumptions broke, then tariff and borrowing policy of the government could potentially interact in a non-intuitive manner.

Borrowing

At the beginning of the model, government chooses how much to borrow so as to maximize utility after inheriting debt B_0:

$$\begin{aligned}
&\max u\big(A(c_1, c_1^\star)\big) + \beta \mathbb{E} u\big(A(c_2, c_2^\star)\big).\\
&\text{s.t.} p(\tau, Y_1)c_1^\star + c_1 + B_0 = Y_1 + q(B_1)B_1\\
&\text{and} p(\tau, Y_2)c_2^\star + c_2 + B_1 = Y_2 \text{ or } \Big\{c_2 = L_2 \text{ and } c_2^\star = 0\Big\}
\end{aligned}$$

In a deterministic model, government should be interested in borrowing if and only if we assume $\frac{\beta-1}{\beta} =: \rho < r + \gamma \frac{L_2}{L_1}$ where γ is local relative risk aversion. In quantitative exercises, authors always assume that $\rho < r$. In other words, emerging countries' governments are assumed to be impatient:[14] otherwise, they would prefer to save at a better safe rate. We therefore assume that β is low enough to create positive borrowing.

To simplify notations, denote:

$$1 + g(\tau) := \frac{v(1, p(\tau, Y))}{A(1, 0)},$$

and also assume that terms of trade are not affected by the domestic country's GDP. Here, $g(\tau)$ summarizes gains from trade. The problem then writes:

$$\max_B V(B, g(\tau)) := u\Big((1+g(\tau))(1+q(B, g(\tau))B)\Big) + \beta \mathbb{E} u\Big(\max((1+g(\tau))(L-B); L)\Big).$$

We want to see what happens to the level of debt as trade costs decrease. Given the framing of the model, a decrease in trade cost is equivalent to an increase in default cost. In standard calibrations of the infinite-period version of this problem such as Arellano (2008), the average level of debt increases when the cost of default increases.

[14] Or, alternatively, governments are assumed to expect high enough future growth so that consumption smoothing would imply borrowing.

This problem does not allow a simple analytic characterization of solutions without further specification. Using the implicit function theorem, we give a local condition for debt to be decreasing in trade costs at the optimum in the appendix but it does not allow straightforward conclusions. However, if we assume that utility function is linear, we can establish the following result.

Proposition 3. *Let f be the density associated with the distribution of next period GDP shock. Let B_τ be the optimal level of borrowing corresponding to a given level of trade costs τ. If f is such that $x := \frac{Bg(\tau)}{1+g(\tau)}$, f is locally continuously differentiable around x and:*

$$(2-(1+r)\beta)f(x)+xf'(x)>0. \tag{1.3}$$

Then a local decrease in trade costs τ involves an increase in the optimal level of debt B.

The condition above, although it looks technical, makes economic sense. Indeed, one can prove that revenue function $B \mapsto q(B,\delta)\times B$ has the following double derivative:

$$\frac{\partial^2(q(B,\tau)B)}{\partial B\partial\tau} =^{\text{sign}} -(2f(x)+xf'(x)).$$

It means that the condition above is simply related to the Laffer curve of bond supply: does the revenue-maximizing level of bonds increase or decrease in τ? In the proposition above, this is simply corrected by discount factor, because more debt today implies less consumption tomorrow (in cases when debt are repaid at least). Then, this parametric assumption seems natural: it simply states that the revenue-maximizing level of debt decreases in the cost of default.

In the more general case with non-linear utility, formulas are more complicated (see appendix). However, whether debt increases as trade costs decrease depends on the interaction between three effects that we can summarize below:

- The direct price effect or substitution effect, the same as in the linear utility case:

it is $\frac{\partial^2 q(B,\tau)}{\partial B \partial \tau}$ times marginal utility. Under the same kind of technical assumption as in the proposition, this term should be positive and push debt to be decreasing in trade costs.

- Contemporaneous consumption smoothing or income effect: larger price of government bonds increases consumption. Thus it reduces contemporaneous marginal utility and encourages more savings for tomorrow.

- Future consumption smoothing : this term is the marginal utility in the second period discounted by the discount factor. If default cost increases, there are more states of the world where government repays debt tomorrow, therefore government should be more reluctant to borrow. This is the effect we would get if default cost increased but the borrowing function stood the same.

The negative effects of trade costs on debt should be stronger as the government is risk-averse or values future consumption (high β) or has low growth expectations. Overall, which effects dominate is an open empirical question, although simulations suggest that debt decrease in trade costs in most cases.

1.4 Data and Instruments

In this Section, we are going to test propositions 1, 2 and 3 the data. In Section 4.2, using data from 2007 and 2019, we find some evidence suggestive that probability of default comoves with trade openness in the short run and in the long run: we use direct regression and instrumental regression with geographical variation in the relevant distance (air sea and trade sea) as an instrument for series on debt. In Section 4.3, we find some evidence in line with proposition 3 that debt increases when a country gets more open, using variation in the effect of geography as in Feyrer (2019). In Section 4.1, we present the data we are going to use first.

1.4.1 Data

In the following empirical analysis, we use sovereign Credit Default Swap (CDS) data collected by Datastream on a daily basis, for 69 countries, from 2007 to the end of 2018. CDS are an insurance for bond holders against default. The main kind of events is default or restructuring of debt. CDS holders pay an insurance fee, called CDS premium every semester and, if the corresponding entity defaults, CDS sellers pay back the bonds, up to what the entity does not pay back. More precisely, CDS give insurance against "credit events", more general than debt, as defined by the International Swaps & Derivatives Association. For example, when Greece restructured its debt in 2011 and 2012, Greece did not officially default but holders of former bonds lost some of their value and the CDS holders got reimbursed after a period of institutional hesitation in 2012. In this case, CDS covered 3.2 billion dollars insured against a Greek default (to compare to more than 400 billion dollars of Greek debt). When they are activated, CDS take into account partial repayments from government, that they do not cover.

The interest of using CDS data rather than bond yields, besides its large availability, is that CDS markets are more liquid and more precise indicators or risks perceived in financial markets. We excluded from the data a few suspicious time series with very low availability of data: Iraq, Ukraine, Malaysia and Singapore (two of them being involved in a military conflict over the period). Including the spare available data from these countries did not change our results.

Thanks to CDS wide availability, we can successfully average spreads over each year and get good estimates of risks. The corresponding estimate of the associated sovereign risk should be more precise. While the CDS is priced on secondary market and may not reflect the cost of borrowing the country faces, due to maturation mismatch and strategic timing of borrowing, it reflects the probability of default. If the probability of default of the sovereign is constant and equal to P, and with a null recovery rate, then the relation between CDS premium λ and the instantaneous probability of default should be given by:

$$\lambda = -\ln(1 - P).$$

This is a simplification and more sophisticated models take into account maturity and more complicated risk functions. We abstract from them as we sense they should not affect our results: using 1-year or 10-year maturities did not affect our results.

We exclude from the analysis countries whose CDS spreads went above a 5,000 threshold because they are synonymous with near default and we want to avoid reciprocal causality issues in our analysis. In some regressions, we used a 500 b.p. threshold and got similar results.

Total flows of trade (including services) and debt are collected on yearly basis by the IMF and World Penn in publicly available data. We use World Penn Database for other general macroeconomic indicators. World Penn includes all countries and years included in IMF Global Debt Database. IMF Global Debt Database, that we completed with other debt indicators from IMF and World Bank for years when data was missing, includes data for 175 countries for years spanning from 1950 to 2018, including most countries from the CDS database. Bilateral flows of trade come from CEPII, given in Fouquin et al. (2016).

Because the mechanism at stake in the cost is assumed to be channeled by finance, we take into account total government debt rather than just debt owed to foreigners. Indeed, even a purely internal default might disrupt external finance and we do not attempt to discriminate both experiences.

1.4.2 Instruments

In Section 5, we want to show that more trade openness leads to a decrease in CDS premium. Besides direct OLS, we construct a gravitational instrument for trade, inspired by Frankel and Romer (1999) as described below, in Section 4.2.1. It should help us avoid some of the most obvious issues with the direct use of trade in the regression, although it is not immune to critics made by Rodrik et al. (2004)[15]. To address them, we will also use the instrument by Feyrer (2019) that we are going to present in Section 4.2.2

Frankel-Romer Instrument

To reconstruct the Frankel-Romer and Feyrer modified instruments, we used CEPII's data with bilateral trade in merchandises between countries from Fouquin et al. (2016). We also used geographical (distance between countries, area, borders, language) and demographic data from Head et al. (2010) to reconstitute the geographical variables.

As we want to directly address the question to whether a change in trade policy in the long run would affect a country's ability to borrow funds from the sovereign markets, we use the same instrument as in Frankel and Romer (1999) to evaluate the impact of trade on the terms of direct borrowing as measured by CDS spreads. This instrument relies on the intuition, given by gravitational models and almost universally observed in trade data, that bilateral trade between two countries depends on their distance and on their size. As a consequence, a small country surrounded by large and rich neighbors such as the Netherlands should trade more than a large country in an isolated island such as Australia, although both countries are rich. Frankel and Romer (1999) build their geographical instrument based on gravitational theories, prevalent in trade models.

More precisely, bilateral trade $T_{i,j}$ (as measured by the sum of imports and ex-

[15]The most common critic is that this indicator is correlated with distance to equator, supposedly reflecting other causes. As we use GDP in control rather than , it should not be as much of a matter as in the original paper by Frankel and Romer (1999).

ports) between country i and j is assumed to behave that way:

$$\ln(\frac{T_{i,j}}{Y_i}) = a_0 + a_1 \ln d_{i,j} + a_2 \ln N_i + a_3 \ln N_j + a_4 B_{i,j} + a_5 B_{i,j} \ln d_{i,j}$$
$$+ a_6 B_{i,j} \ln N_i + a_7 B_{i,j} \ln N_j + a_8 l_{i,j} + a_9 A_i + a_{10} A_j + \varepsilon_{i,j},$$

where $d_{i,j}$ is the bilateral distance between countries i and j[16], N_i the population of country i, A_i the area of country i, $B_{i,j}$ a dummy indicating whether they share a common border, $l_{i,j}$ a dummy indicating that countries i and j share a common language. The results of this regression are summarized in table **??** (see appendix). Without surprise, distance matters a lot to explain bilateral trade. The total R^2 is less than 50%, in part because we did not include GDP of trade partners in the regression to avoid potential biases in the next regressions, since level of development may be an explanatory variable.

Using the predictors given by this last regression, one can therefore predict the total trade level of one country using only the geographical variables:

$$\widehat{Trade}_i^{FR} := \sum_{j \neq i} \frac{\widehat{Trade}_{i,j,t_0}^{FR}}{GDP_{i,t_0}} = \sum_{j \neq i} \exp(\hat{a}_0 \log d_{i,j} + \hat{\beta} X_{i,j}).$$

To compute the instrument, we use only one year: we use 2007 as a reference point, before the beginning of our CDS series to ensure the instrument is exogenous.

Feyrer Instrument

The problem of the previous instrument is that it is fixed for each country. Then, it cannot be used for diff-in-diff regressions or with country fixed effects. To avoid that issue, we will also use the same time-varying gravitational instrument as in Feyrer (2019): this instrument is based on the idea that there are changes over time in the

[16] Distance between countries i and j is measured as the distance between the capitals of the two countries.

importance of geographical variables for trade. Indeed, sea distance matters relatively less today than in 1950, at least relative to air distance: the greater availability of planes for trade changes the impact of geography over time, as Feyrer (2019) explains in his paper: some goods, especially electronics and luxury leather goods, are often exchanged through air distance, which can represent 20% of the trade for some countries. This change in the importance of air trade heterogeneously impacted countries over time. A country such as the United States did not greatly benefit from air travel from the point of view of trade: to give the most salient example, sea distance between the US and most countries in the world coincides with air distance, while this would not be true between Europe and Eastern Asia: there are also significant variations within large regions.

We exclude neighbor countries to compute bilateral trade, and the distance from any country to a country with no maritime borders is computed through the closest port. Using total bilateral trade flows in goods, we estimate the following panel regression:

$$\log(Trade_{i,j,t}) = a_{i,j} + a_t + \beta_t^{sea} dist_{i,j}^{sea} + \beta_t^{air} dist_{i,j}^{air} + u_{i,j},$$

where $dist_{i,j}^{sea}$ is sea distance as computed by Feyrer (2019), $dist_{i,j}^{air}$ is the air population-weighted distance between countries (see Mayer and Zignago (2011)). The bilateral fixed effect $a_{i,j}$ takes into account all the constant determinants of trade a gravitational equation would normally control for, while a_t controls for time changes. The time-varying parameters on sea distance and air distance give some variation to the instrument. We can compute the instrument:

$$\widehat{Trade}_{i,t}^{Feyrer} = \sum_{j \neq i} \exp(\hat{a}_{i,j} + \hat{a}_t + \hat{\beta}_{c(t)}^{sea} dist_{i,j}^{sea} + \hat{\beta}_{c(t)}^{air} dist_{i,j}^{air}),$$

where $c(t)$ defines a time bin (decades). Therefore, the instrument exhibits some time variance for each country, which can be attributed to partial shift of trade from sea to air travel. As a consequence, this instrument is compatible with country fixed effects, unlike the previous one, which makes it more robust to critics.

1.5 Results

In this Section, we are going to test propositions 1, 2 and 3 using the data and the instruments presented in the previous Section. In Section 5.1, using data from 2007 to 2019, we find some evidence suggestive that probability of default comoves with trade openness in the short run and in the long run: we use direct regression and instrumental regression with geographical variation in the relevant distance (air sea and trade sea) as an instrument for series on debt. In Section 5.2, we find some evidence in line with proposition 3 that debt increases when a country gets more open, using variation in the effect of geography as in Feyrer (2019).

1.5.1 Trade and CDS Spreads

In this Section, we test for the result of proposition 1: *Conditional on the level of debt B, a more open country should face better a lower interest rate* and of proposition 2 as well: *a more open country should face a lower interest rate.* As a proxy for interest rates, we are going to use CDS premia. More precisely, we are going to test for:

$$CDS_{i,t} = -\gamma \log Trade_{i,t} + \gamma X_{i,t} + u_{i,t}, \tag{1.4}$$

where i is an index for country i, t for year t, $Trade_{i,t}$ trade openness (as a percentage of GDP), $X_{i,t}$ a set of controls including fixed effects and possibly debt-to-GDP ratio, $u_{i,t}$ an error term. We are also going to test for This precise functional form can be derived from a two-period model with specific assumptions regarding the distribution (see appendix).

We first directly run 1.4 with some additional controls: we add a time fixed effect which reflects the importance of global trade at the period of the regression, and a country fixed effect which captures the fact that countries have different growth processes (and might reflect other characteristics that are absent in the model). In the regression, it should more generally capture other institutional or macroeconomic aspects of the country that may be important to determine the threshold of default. We use trade openness ratio to compute $IM^{\star}$.

We run the following regression in the first column of table 2 and in the appendix for more complete controls:

$$CDS = \alpha \log IM_{i,t} + \beta X_{i,t} + \delta_j + \delta_t + u_{j,t}. \tag{1.5}$$

The first two columns of table 2 gives the result of the regression suggested by equation 1.5 and confirms our expectations. Because we have country and year fixed effects, the regression measures the effect of short-run variation in trade on CDS spreads. To deal with long-run variations, we are going use a gravitational instrument for trade.

We give the results of this regression in 1.B.4.

The results are in line with our expectations for the sign: debt has a positive impact on CDS premia (larger debt implies larger cost of borrowing), while larger trade flows as measured by trade openness are correlated with lower CDS premia, that is better credit ratings.

Long Run: Frankel-Romer Instrument In the previous paragraph, we showed that in the short run, variations in trade seem to create betters terms of credit. in this paragraph, we answer two concerns from the previous paragraph: does trade also favor terms of borrowing in the long run? Is the previous result due to endogeneity issues that may occur, such as reverse causality? Then, we use Frankel-Romer's trade predictor as an instrument for trade in the following way:

$$CDS_{i,t} = \gamma Trade_{i,t} + \delta \log Debt_{i,t} + \beta GDP_{i,t} + \alpha_t + +\varepsilon_{i,t}$$
$$Trade_{i,t} = c.\widehat{Trade}_i^{FR} + d \log Debt_{i,t} + b.GDP_{i,t} + a_t + u_{i,t},$$

where the first equation is the reduced form of the IV and the second equation is the first stage, $D_{i,t}$ is debt-to-GDP ratio of country i at time t, $GDP_{i,t}$ is output.

If one assumes that the geographical variables determining trade affect financial institutions and countries' credibility only through their effect of trade, this predicted trade share can therefore be an instrument for trade in this paper's analysis. The identification assumption is that variations in $\hat{Trade}_i^{FR}$ are not correlated with institutional quality otherwise than through GDP (and other covariates). Frankel and Romer (1999) used this instrument to evaluate the benefits of trade on growth[17]. We show the results of this IV regression in table 1.2. We cannot include country fixed effects because the instrument is time-invariant for each country, as for the original Frankel-Romer instrument. For the same reason, we cluster by year only, and not by countries[18]. The estimates given by this instrument are very close to the direct OLS ones, which might attenuate reverse causality concerns: indeed, the instrument is defined only thanks to 2007 data.

In columns 1, 2, 4 and 5, we add a control for oil countries, thanks to specific time fixed effects for oil-producing countries, as listed by the Direction of Trade of Statistics (DOTS) of the International Monetary Fund (IMF)[19]. We do it to deal with

[17]We will use the same proxy as Frankel and Romer (1999), substracting a few variables that we think may cause endogeneity issue such as regions, or the fact that a country is landlocked: indeed, they are likely to be directly correlated with financial institutions. Also, unlike Frankel and Romer (1999), we include area in the bilateral trade regression and not in our direct regressions. we run the regressions defining the proxy in 2007, which is the beginning of the period for the rest of the empirical analysis. Therefore, the proxy should not capture any variation posterior to 2007.

[18]However, clustering by year and country gave significant results at the 5% threshold.

[19]These countries are Algeria, Angola, the Republic of Azerbaijan, the Kingdom of Bahrain, Brunewe Darussalam, Chad, the Republic of Congo, Ecuador, Equatorial Guinea, Gabon, the Islamic Republic of Iran, Iraq, Kazakhstan, Kuwait, Libya, Nigeria, Oman, Qatar, Russian Federation, Saudwe Arabia, the Republic of South Sudan, Timor-Leste, Trinidad and Tobago, Turk-

variations in commodities' prices that affects gains from trade in a (oil is an easy to trade good that might be affected by default differently from non-commodity goods). We also control for trade balance in some specifications. These controls do not seem to matter as much as restriction to "safe" countries: we restrict the specification of column 5 to "safe countries" whose CDS never exceeded 500 b.p. This specification allows us to partly deal with the worry that some results might be driven by reverse causality.

In our estimation, the effect of trade is more important than the effect of debt (when we measure it), which is striking as debt is the first motive invoked in sovereign debt crises, for example by rating agencies. In these estimations, a 10% increase in trade leads to a 15 to 40 b.p. (basis points) decrease in spreads: the doubling of trade-to-GDP ratio through trade agreements could have large effects on sovereign borrowing according to this estimation;, up to 400 basis points. For example, in 2014, Italy trades twice as much of its GDP as Argentina. Then, according to our estimate, if Argentina traded as much as Italy in the beginning of 2014, its CDS premium could have been up to 400 b.p. lower (more than a fifth of the difference, although Argentina has a very peculiar institutional setting). The results are robust to a restriction of the regression to one given year.

1.5.2 Debt and Trade

In this Section, we want to show that, consistently with assertion 3, debt increases in trade openness. First, we run the instrumental variable regression induced by Frankel-Romer, as in the previous Section. The reduced form and first stage of the

menistan, United Arab Emirates, the RepúbliCurrent AccountBolivariana de Venezuela and the Republic of Yemen. See http://datahelp.imf.org/knowledgebase/articles/516096-which-countries-comprise-export-earnings-fuel-a.

Table 1.2: CDS and Frankel-Romer's instrument: reduced form IV. Standard errors cluster by year.

	Dependent variable:				
	cds				
	(1)	(2)	(3)	(4)	(5)
Trade (Percent of GDP, log)	-165.925^{**}	-368.460^{**}	-319.348^{***}	-339.927^{***}	-177.503^{***}
	(73.004)	(155.068)	(55.685)	(66.325)	(43.760)
Debt (percent of GDP, log)		50.347			26.450^{**}
		(54.684)			(11.151)
GDP (log)	-65.726^{***}	-234.815^{**}	-89.851^{***}	-95.230^{***}	-66.562^{***}
	(10.613)	(81.545)	(10.341)	(13.670)	(9.807)
Current Account (Percent of GDP)				1.601	0.610
				(1.678)	(1.516)
Instrument for Trade	No	No	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	No	No	No
Year and Oil Fixed Effects	Yes	Yes	No	Yes	Yes
Safe Countries Only	No	No	No	No	Yes
Observations	704	703	541	537	418
R^2	0.216	0.680	0.161	0.150	0.233
Note:					*p<0.1; **p<0.05; ***p<0.01

regression are:

$$CDS_{i,t} = \boldsymbol{\gamma}\, Trade_{i,t} + \delta \log Debt_{i,t} + \beta\, GDP_{i,t} + \alpha_t + \varepsilon_{i,t}$$
$$Trade_{i,t} = c\,\widehat{Trade}_i^{FR} + d \log Debt_{i,t} + b\, GDP_{i,t} + a_t + e_{i,t},$$

with the Frankel-Romer instrument defined thanks to bilateral trade data from 2007. The results of the regression confirm the cases we studied in the previous Section: more openness as defined by geographic factors is associated with larger debt stocks. However, the period is short and does not allow us to control for long-run changes in the trade capacity. Ideally, we would apply the Feyrer instrument to the regressions from the previous Section: but the time span for which CDS data are available is too short for the instrument to be a good predictor of trade.

Unlike CDS, debt data are available over the long run: a lot of countries have debt data from 1950, and even further for some western countries. Moreover, debt is typically slow-moving, so that long-run relations make more sense. We test for the long-run relationship between trade costs and debt in the long run.

Using the Feyrer instrument defined in Section 4.2.2, we can run a new instrumental regression. The reduced form and the first stage regressions are given by:

$$Debt_{i,t} = \boldsymbol{\gamma}\, Trade_{i,t} + \alpha_{GDP} GDP_{i,t} + \alpha_i + \alpha_t + \varepsilon_{i,t}$$
$$Trade_{i,t} = c\,\widehat{Trade}_{i,t}^{Feyrer} + a_{GDP} GDP_{i,t} + a_i + a_t + e_{i,t}.$$

By construction, the first stage is very robust (see appendix). The exclusion restriction hypothesis associated with this IV regression is that difference between air distance and sea distance did matter only through the impact on trade. Note that panel could also be replaced with a difference-in-difference method, instead of including a country fixed effect. Changing the method did not affect the results.

In table 1.3, we show the results of the reduced-form of this regression. In order to control for the rise of average imbalances, as in Reyes-Heroles et al. (2016), we propose a fourth variable in column (3), which the average of absolute value of trade balance in each country over different decades[20]. Our results suggest that countries that trade more thanks to air and sea variation also happen to borrow more sovereign debt. The result is not driven by the size of global imbalances and seems robust. A 10% increase in trade predicts a 3% increase in debt-to-GDP ratio, in line with our computational results from Section 4.

Table 1.3: Trade openness and Debt-to-GDP in the long run: Reduced Form IV using Feyrer's instrument

	Debt-to-GDP ratio		
Fixed Effects	Year+Country	Year+Oil+Country	Year+Oil+Country
Trade-to-GDP, log (Instrumented)	0.319***	0.326***	0.315***
	(0.059)	(0.054)	(0.057)
Real GDP (log)		−1.129***	
		(0.107)	
GDP (log)		0.401***	−0.190***
		(0.054)	(0.025)
Average Imbalances			−0.107
			(0.104)
Observations	2,918	2,847	2,847
R^2	0.258	0.355	0.283
Note:			*p<0.1; **p<0.05; ***p<0.01

1.6 Conclusion

In this paper, we argued that more open countries should be able to commit more easily to repay debt. After showing that defaulting countries seem to trade less as a consequence of default, we argued that the trade interruption was a realistic representation of what default cost could be. We investigated two consequences:

[20] We define Average trade balance as $aca_{i,t} = \sum_{\tau:c(\tau)=c(t)} |TB_{i,t}|$ where $TB_{i,t}$ is trade balance of country i at time t.

more open countries are considered safer by markets, and more open countries seem to borrow more in the long run, and we have shown empirically that they were plausible. Our results suggest that a 10% increase in the total volume of trade to GDP should lead up to a 40 b.p. (basis points) decrease in CDS premia, for a given level of debt. Moreover, it should lead to a 3% increase in debt-to-GDP ratio. With those estimates, we can argue that if Argentina had been trading as much as Italy relative to its GDP in early 2014, its CDS premium would have been up to 400 b.p. lower, which is quite large: to give an example, increase in trade volume between China and Argentina might have played an important role in the build-up or Argentinian debt before 2000: according to our theory, it might have created favorable terms of credit for Argentina and led the country to borrow more.

Roos (2019) noticed that, as of 2019, the share of world defaulters was surprisingly low (defaulting countries were 0.2% of world GDP only), and that even very fragile countries preferred to repay large debt burdens rather than to default. He argued that it was because the power of lenders and financial systems from rich countries had dramatically increased. Our theory can be considered a complementary explanation: the fear of an interruption of trade may have become much stronger today, after the deep international integration of goods' markets. This paper gives hints at trade as an important commitment device for sovereign international finance. Countries with anti-tariff policies do not only send a signal to markets about their economic management: they tie their hands with their gains from trade. Larger dependence on trade means that sovereign debt crises might be less likely but also more dramatic. This phenomenon could also explain the covariation of CDS sovereign premia observed by Longstaff et al. (2011) and Pan and Singleton (2008): the cycle of world trade could partly determine comovement of spreads.

We think two topics might be worth investigating for future research.The first is how trade channels might impact financial systems. A country's international liabilities can be paid back because of the country's dependence on international trade: how would a crisis in neighbor countries affect trade of other countries? The second is the extent to which trade depends on finance: did it change over time, is there a way to increase it? For example, if default interrupts trade only through an interruption of trade finance, dependence on trade finance is double-edged: it increases the ability to borrow ex ante, but hurts defaulting countries ex post. What has been the evolution of trade finance? Is it the reason why some countries chose not to default in recent years?

The study of sovereign debt could more generally give us a better understanding of the gains from trade: some countries accept to pay very large debt burdens inherited from previous years. If we assume these governments are rational, the size of these burdens should give us an insight of what the real gains from trade and financial integration are.

Appendix

Appendix 1.A Theory Appendix

1.A.1 Proposition 1: Proof

Equation details for the CES case, we compute the level of debt $B^D(Y_2, \tau)$ at which a government is indifferent between defaulting and repaying debt in the final period:

$$\begin{aligned}
\frac{B^D(Y_2,\tau)}{Y_2} &= \frac{(\alpha + (1-\alpha)(p(\tau,Y_2))^{1-\sigma})^{\frac{1}{\sigma-1}} - \alpha^{\frac{1}{\sigma-1}}}{(\alpha + (1-\alpha)(p(\tau,Y_2))^{1-\sigma})^{\frac{1}{\sigma-1}}} \\
&= 1 - (1 + \underbrace{\frac{1-\alpha}{\alpha}\frac{1}{p^{1-\sigma}}}_{=\frac{IM^\star}{1-IM^\star}})^{\frac{1}{1-\sigma}} \\
&= 1 - (1 - IM^\star(Y_2,\tau))^{\frac{1}{\sigma-1}} \qquad (1.6)
\end{aligned}$$

The general case stems from the application of the central result used in Arkolakis et al. (2012). In their framework, if elasticity of substitution stays constant, gains from trade always write as a function of imports as above, and one gets:

$$\begin{aligned}
\frac{B^D(Y_2,\tau)}{Y_2} &= \frac{1 + g(Y_2,\tau)}{g(Y_2,\tau)} \\
&= 1 - (1 - IM^\star(Y_2,\tau))^{\frac{1}{\sigma-1}}
\end{aligned}$$

where $1+g=(1-IM^{\star})^{\frac{1}{\varepsilon}}$ and $IM^{\star}$ is the import share after taking trade surplus into account (that occurs because of debt). Both functions depend on endowment (because of terms of trade and substitution effects) and τ. Therefore, as long as trade decreases in bilateral trade costs, our proposition should hold:

$$\frac{\partial \frac{B^D(Y_2,\tau)}{Y_2}}{\partial IM^{\star}} > 0$$

and the proposition follows as long as we have: $\frac{\partial IM^{\star}}{\partial \tau} < 0$, which should be true for bilateral trade costs in a trade model. Larger trade costs imply less imports which implies less default costs. Once we apply this to the pricing of financial markets, that corresponds to the default probability, we get the result: for any level of future GDP, sustainable debt is larger next period when a country gets more open. Incidentally, we also proved that, for the same level of debt and endowment, a country that is more open has higher utility (this will turn out to be useful for the generalization of the proof in the next paragraph).

The proposition easily generalizes to a standard Eaton-Gersovitz model with trade (assuming again that terms of trade are not affected by debt) and infinite periods. To prove it, we are going to assume that, at the equilibrium, sustainable debt increases in trade openness, and proceed by induction.

Let Y be any value for the endowment. The value associated with repayment of debt in a model with $N+1$ periods is:

$$V^{R,\tau}_{N+1}(B,Y) = \max_{B'} u((Y-B+q^{\tau}_N(B',Y)B')v(1,p(\tau,Y)))+\beta\mathbb{E}\big(\max\{V^{R,\tau}_N(B',Y_1),V^D_N(Y_1)|Y_0=Y\big)$$

and the value associated with default does not depend on τ and is given by:

$$V^D_{N+1}(L) = \mathbb{E}\big(\sum_{t=0}^{N+1}\beta^t u(L_t)|L_0=L\big)$$

As we proved it for $N = 0$, we assume for the induction that:

$$\forall (B', L'),\ \frac{\partial V_N^{R,\tau}(B', L')}{\partial \tau} < 0$$

and the amount of sustainable debt decreases in trade costs:

$$\frac{\partial \frac{B_N^D(Y,\tau)}{Y}}{\partial \tau} < 0$$

where B_N^D is defined as the solution to:

$$V_N^D(Y) = V_N^{R,\tau}(B_N^D, Y)$$

Now we want to prove that:

$$\frac{\partial B_{N+1}^D(Y,\tau)}{\partial \tau} < 0$$

and

$$\frac{\partial V_N^{R,\tau}(B', Y)}{\partial \tau} < 0$$

where sustainable level of debt is defined as the unique solution to:

$$V_{N+1}^D(Y) = V_{N+1}^{R,\tau}(B^D, Y)$$

To prove our point, it is enough to prove that $V_{N+1}^{R,\tau}(B, Y)$ increases in τ for any B, because V_{N+1}^D does not depend on τ. Let $\tau' > \tau$. By induction assumption, we know that, for any B' and Y', we have:

$$V_N^{R,\tau'}(B', Y') < V_N^{R,\tau}(B', Y')$$

Because financial markets are competitive and the price of bonds decreases in the probability of default, it implies that

$$\forall B', q_N^{\tau'}(B', Y) \leq q_N^{\tau}(B', Y)$$

By the property of trade models, we also have:

$$v(1, p(\tau', Y)) \leq v(1, p(\tau', Y))$$

Now, because consumption has to be positive, we deduce that, for every B' that makes consumption of government with cost τ positive:

$$(Y - B + q_N(B', \tau')B')v(1, p(\tau', Y)) < (Y - B + q_N(B', \tau)B')v(1, p(\tau', Y))$$

Combining the inequalities, we can conclude that:

$$\forall B \text{ s.t. } Y - B + q_N(B', \tau)B' \geq 0,$$

$$\begin{aligned} u((Y - B + q_N(B', \tau)B')v(1, p(\tau, Y))) \geq\ & u((Y - B + q_N(B', \tau')B')v(1, p(\tau, Y))) \\ & + \beta\mathbb{E}\Big(\max\{V_N^{R,\tau}(B', Y_1), V_N^D(Y_1)|Y_0 = Y\Big) \\ & - \beta\mathbb{E}\Big(\max\{V_N^{R,\tau}(B', Y_1), V_N^D(Y_1)|Y_0 = Y\Big) \end{aligned}$$

Moreover, because $Y - B + q_N(B', \tau)B' \leq 0 \implies Y - B + q_N(B', \tau)B' \leq 0$, and negative consumption would not be part of the choice of a rational government, we can apply the previous inequality at the maximum and conclude:

$$\forall (Y, B), V_{N+1}^{R,\tau}(B, Y) \geq V_{N+1}^{R,\tau'}(B, Y)$$

which is enough to conclude the induction.

To finish our proof, we need to go to the limit as N goes to infinity. In the limit-case, we converge to an Eaton-Gersovitz equilibrium and our properties cannot revert to the limit by continuity, if the functions are well-behaved. However, there can be multiple equilibria, so that the equilibrium defined by the iteration is not the only one. This won't be true any more if one assumes that GDP is a Markov process with a finite number of states. Finiteness of Markov states ensures that the equilibrium is unique, as was proved in Auclert and Rognlie (2016).[21]

None of the arguments applied above would prevent us from including GDP of other countries, which should affect the domestic country to the extent they affect terms of trade. More generally, the result is robust to additional state variables. The only limitation would be to assume that the number of states is finite in the case of infinite periods (and it is only a sufficient condition to ensure the equilibrium is unique).

Note that the result might be ambiguous about a general increase of trade costs in the world economy. We only studied the effect of one country's bilateral trade costs. In the case of a general increase in trade costs, it is conceivable that one country could not benefit from more openness: for example, an increase in openness can benefit to a direct competitor of our sovereign country and reduce its gains from trade under certain circumstances.

1.A.2 Proposition 3: Proof

In this paragraph, we derive the formula from proposition 3 and also show the more general formula and detail the discussion about forces in motion to know whether the face value of debt increases as trade openness increases in the model.

More generally, if one assumes that preferences only consist in an aggregator with

[21] Their framework allows to have a utility that is state-dependent, which means that our assumption that terms of trade depend on GDP does not prevent uniquesness of equilibrium.

constant returns, then define

$$\nu(x,y) = \max_{c^D \in [0,1]} A(c^D, \frac{1-xc^D}{y})$$
$$1+g = \frac{\nu(\tau, p/\tau)}{A(1,0)}$$

With that notation, government defaults if and only if:

$$L \leq (1+g)(L-B)$$
$$L \leq \frac{B}{g}(1+g)$$
$$\iff (1-\delta)L \geq L-B$$
$$\iff L \leq \frac{B}{\delta}$$

where $\delta := \frac{g}{1+g}$ is the cost of default in standard models. δ depends on τ. We use this simpler notation. One obviously finds that δ increases in τ. Is the optimal borrowing quantity larger in this case? To answer that question, one only needs to compute what happens when δ increases.

At the first period, government maximizes (after normalizing the GDP of the first period to 1 and the interest rate to 0):

$$V(B,\delta) = u(1+q_\delta(B)B) + \beta\mathbb{E}u\big(\max((1-\delta)L; L-B)\big)$$
$$= u(1+\mathbb{P}(L > \frac{B}{\delta}) \times B) + \beta\int_0^{\frac{B}{\delta}} u((1-\delta)L)f(L)dL + \beta\int_{\frac{B}{\delta}}^{+\infty} u(L-B)f(L)dL$$

To prove this, we are going to use Topkis' theorem. Since we assume the distributional function is smooth enough, we can compute cross-derivatives, using the equilibrium condition to compute the function q. We compute

$$\frac{\partial}{\partial B}V(B,\delta) = \frac{\partial(q(B,\delta)B)}{\partial B}u'(1+q(B,\delta)B) - \beta\int_{\frac{B}{\delta}}^{+\infty} u'(L-B)f(L)dL$$

$$\theta(B,\delta) := \frac{\partial^2}{\partial B\partial\delta}V(B,\delta) = \frac{\partial^2(q(B,\delta)B)}{\partial B\partial\delta}u'(1+q(B,\delta)B) + \frac{\partial(q(B,\delta)B)}{\partial B}\frac{\partial(q(B,\delta)B)}{\partial\delta}u''(1+q(B,\delta)B)$$
$$- \beta\frac{\partial(q(B,\delta)B)}{\partial\delta}u'((1-\delta)\frac{B}{\delta})$$

One can notice that this quantity is equal to 0 whenever debt is negative or when B/δ is strictly less than the lower bound of the support of the distribution of GDP. In such a case, a change in the cost of default (equivalently, a change in trade costs) should not affect the will to borrow. Indeed, if the optimal level of borrowing is negative or strictly below the threshold for a positive probability of default, it means that the government is not constrained by default risk: it could happen for example if β is large enough, or, equivalenty, if the government expects low GDP GDP growth at the next period. The cost of default is irrelevant in this case: in an economy with pure commitment, government would borrow the same quantities.

Back to the general case, let A be absolute risk-aversion for a given level of consumption:

$$\theta(B,\delta) = \frac{B}{\delta^2}u'(1+q(B,\delta)B) \times \Big((\frac{B}{\delta}f'(\frac{B}{\delta}) + 2f(\frac{B}{\delta}) - \beta\frac{u'(\frac{B}{\delta})}{u'(1+q(B,\delta)B)}f(\frac{B}{\delta}))$$
$$- (q(B) - \frac{B}{\delta}f(\frac{B}{\delta}))f(\frac{B}{\delta})A(1+q(B,\delta)B)\Big)$$

If u is linear, $u'$is constant and positive and $A = u'' = 0$, so that θ is positive if and only if:

$$(2-\beta)f(x) + xf'(x) > 0$$

with $B > 0$ (which is assumed to be true, because probability of default is positive). This is equation of the proposition, modulo the normalization of interest rate.

One can notice that:

$$\begin{aligned}\frac{\partial^2}{\partial B \partial \delta} q(B)B &= 2\frac{B}{\delta^2} f(\frac{B}{\delta}) + \frac{B^2}{\delta^3} f'(B) \\ &= \frac{B}{\delta^2}(f(\frac{B}{\delta}) + \frac{B}{\delta} f'(\frac{B}{\delta}))\end{aligned}$$

A more general local condition guaranteeing that a larger default cost (which is equivalent to lower trade costs) implies more debt is the equation:

$$\theta(B, \delta) > 0$$

One can note that, in an infinite-time model, as β goes to 0, the solution converges to the one of the two-period model. Therefore, when β is low enough, this result should extend to infinite time period.

In the expression:

$$\frac{\partial(q(B,\delta)B)}{\partial B} u'(1 + q(B,\delta)B) - \beta \int_{\frac{B}{\delta}}^{+\infty} u'(L - B) f(L) dL = 0$$

default cost appears in 3 different ways:

- As a factor impacting price of debt today in $\frac{\partial(q(B,\delta)B)}{\partial B}$. As long as $\frac{\partial^2(q(B,\delta)B)}{\partial B \partial \delta}$ is positive, this effect should increase debt. All the standard distributions we have tested are such that this assumption is true for the range of relevant welfare-maximizing debt levels. This is the meaning of the term $\frac{B}{\delta} f'(\frac{B}{\delta}) + 2f(\frac{B}{\delta})$ in the formula for $\theta(B, \delta)$. This is the substitution effect.

- It appears in the final period's consumption: larger default costs mean that there are more cases where debt should be repaid. This is the meaning for the term $\beta \frac{u'(\frac{B}{\delta})}{u'(1+q(B,\delta)B)} f(\frac{B}{\delta})$ in the formula for $\theta(B, \delta)$.

- Inside the marginal utility $u'(1 + q(B, \delta)B)$ with an unambiguous negative effect on debt: better borrowing conditions today increase consumption today, and therefore lead to a decrease in marginal utility today and shift the consumption smoothing towards more consumption tomorrow. This is the meaning of the term $-(q(B) - \frac{B}{\delta}f(\frac{B}{\delta}))f(\frac{B}{\delta})A(1+q(B,\delta)B)$

-The second and third effects are clearly negative: if the cost of default of default increases, it means debt should have to be repaid in more states of the world in the next period. Then, borrower with a larger default cost should pay more attention to debt regarding its future consumption. The size of that effect should naturally get multiplied by β. This is the meaning of the term $\beta\frac{u'(\frac{B}{\delta})}{u'(1+q(B,\delta)B)}f(\frac{B}{\delta})$ in the formula for $\theta(B,\delta)$.

Example: Pareto Distribution Consider the case with a Pareto distribution with parameter γ and $L_{\min}$. With such a distribution for second period's GDP, there is a threshold $B_{\min}$ such that:

$$\forall B \leq B_{\min} q(B)B = \frac{B}{1+r}$$

Moreover, $q(B)B$ decreases in B whenever $B \geq B_{\min}$. The maximal amount a utility-maximizer government would like to borrow is B_{min}, because it is the revenue-maximizing level of debt. Moreover, when total borrowing is lower than $B_{\min}$, it means that government does not feel constrained by commitment problem. As $B_{\min}$increases in $g(\tau)$, the actual amount borrowed should be non-increasing in trading costs. Therefore, in this case, debt increases or stays constant as the country becomes more open.

This reasoning for this special case is more general than it looks: it can be applied to all governments that borrow little quantities of debt and maintain a fixed probability of default. One can imagine that the world is perceived by government as giving binary outcomes, bad or wrong, so that a discrete distribution perfectly summarizes the forecasts of agents and government. For those governments, an increase in openness is going to imply an expansion of the ability to borrow at the safe interest rate: this can in no case decrease the total level of debt.

1.A.3 Proposition 4: Proof

In this Section, we prove that the probability of default should increase in trade costs (equivalently, decrease in trade openness or default costs). We use the same notation as in the appendix Section above, and prove that when δ increases, the probability of default decrease. We assume that the cumulative distribution function of the GDP in the final period is increasing.

To prove it, we define an equivalent dual maximization problem where government maximizes its utility as a function of the probability of default and apply theorem 1 in Topkis (1978). Let P be the probability of default.

To keep exposition as simple as possible, we suppose $r = 0$ so that:

$$q(B,\delta) = 1 - P = \mathbb{P}(Y \geq \frac{B}{\delta}) = 1 - F(\frac{B}{\delta})$$

Then we can write B as:

$$B = \delta F^{-1}(P)$$

As long as F^{-1} is uniquely defined. If it is not uniquely defined, it means a local increase in debt B should lead to a no impact on the probability of default, so that the proposition would still hold. For now, we assume F^{-1} is uniquely defined and differentiable.

We write the new maximization problem of the government which maximizes its utility as a function of the probability of default at the next period, depending on the default cost:

$$V(P,\delta) := u\big(1+\delta F^{-1}(P)(1-P)\big)+\beta\int_0^{+\infty} f(Y)\max\{u(Y-\delta F^{-1}(P)),u((1-\delta)Y)\}dY$$

First order condition implies:

$$\frac{\partial V(P,\delta)}{\partial P} = 0$$

which can also br written:

$$\delta\Big(F^{(-1)\prime}(P)(1-P)-F^{-1}(P)\Big)u'\big(1+\delta F^{-1}(P)(1-P)\big)-\beta\delta\int_{F^{-1}(P)}^{+\infty}F^{(-1)\prime}(P)f(Y)u'(Y-\delta F^{-1}(P))dy=0$$

where $F^{(-1)\prime}$ is the derivative of F^{-1}. Finally, one can compute:

$$\begin{aligned}\frac{\partial^2 V(P,\delta)}{\partial P\partial\delta} &= ((F^{(-1)\prime}(P)(1-P)-F^{-1}(P))u'\big(1+\delta F^{-1}(P)(1-P)\big)\\ &-\beta\int_{F^{-1}(P)}^{+\infty}F^{(-1)\prime}(P)f(Y)u'(Y-\delta F^{-1}(P))dy\\ &+\delta F^{-1}(P)(1-P)\Big(F^{(-1)\prime}(P)(1-P)-F^{-1}(P)\Big)u''\big(1+\delta F^{-1}(P)(1-P)\big)\\ &+\beta F^{-1}(P)\int_{F^{-1}(P)}^{+\infty}F^{(-1)\prime}(P)f(Y)u''(Y-\delta F^{-1}(P))dy\end{aligned}$$

From the first order condition, one can observe that, at the optimum, the two first terms should cancel out:

$$\begin{aligned}\frac{\partial^2 V(P,\delta)}{\partial P\partial\delta} &= \delta F^{-1}(P)(1-P)\Big(F^{(-1)\prime}(P)(1-P)-F^{-1}(P)\Big)u''\big(1+\delta F^{-1}(P)(1-P)\big)\\ &+\beta F^{-1}(P)\int_{F^{-1}(P)}^{+\infty}F^{(-1)\prime}(P)f(Y)u''(Y-\delta F^{-1}(P))dy\end{aligned}$$

We know that the term $F^{(-1)\prime}(P)(1-P)-F^{-1}(P)$ cannot be negative: otherwise, a decrease in the default probability (equivalent to a decrease in borrowing) would imply more revenues today: this option would be improving consumption today and tomorrow. The integral on the right-hand side is negative because u is concave. As a consequence the term $\frac{\partial^2 V(P,\delta)}{\partial P\partial\delta}$ is negative at the optimum if the utility function is concave. Hence, as consequence of Topkis' theorem, if the utility function is concave, the probability of default should be decreasing in default cost.

Appendix 1.B Empirical Appendix

1.B.1 Default Risk and Trade: a Log Formula

In this paragraph, we present hypothesis under which the equation tested in Section 5.2 becomes a structural regression.

As seen earlier, the gains from trade are a good summary of each government's willingness to repay its debt in the model, and they can also be computed indirectly thanks to a sufficient statistics approach. We use a simplifying assumption (local Pareto) to derive an approximation that we can directly test in the data.

Let $L_{j,t}$ be the GDP of country j at time t. The probability of default of a government that borrower $B_{j,t}$ at the next period should then be:

$$P^D = \mathbb{P}(\frac{B_{j,t}}{Y_{j,t+1}} > 1 - (1 - IM^{\star})^{\varepsilon})$$

If you assume that $\varepsilon = 1$ [22], then the probability of default is simply given by:

$$P^D = \mathbb{P}(\tilde{Y}_{j,t+1} < \frac{b_{j,t}}{IM^{\star}})$$

where $b_{j,t} = B_{j,t}/L_{j,t}$. Combining this with previous assumption, the CDS premium for risky countries should be given by:

$$CDS = -\log(1 - \mathbb{P}(\tilde{Y}_{j,t+1} < \frac{b_{j,t}}{IM^{\star}}))$$

Assume now that $\tilde{L}_{j,t+1}$ is distributed according to a Pareto distribution (at least locally) with parameters $C_j C_t$ and γ, then:

$$CDS = \gamma \log b_{j,t} - \gamma \log IM^{\star} + \log C_j + \log C_t$$

[22] This assumption is equivalent to assuming that the elasticity of substitution between international goods is $\sigma = 2$, a lower bound of the estimates, generally between 4 and 10.

The fact the coefficients for $b_{j,t}$ and $IM^{\star}_{j,t}$ are the same stem from our assumptions. With different functional forms and different elasticity of substitution for, one can find different results.

1.B.2 Table 5: Frankel-Romer's definition Regression

	Dependent variable:
	Trade between Reporter and Partner (over reporter's GDP)
Distance (log)	−0.700***
	(0.016)
Common Border	3.920***
	(1.229)
Distance if Common Borser	0.519***
	(0.143)
Common official language	0.381***
	(0.056)
Common language	0.453***
	(0.056)
Population (log) of partner	0.474***
	(0.006)
Population (log) of reporter	−0.386***
	(0.006)
Area (reporter)	−39.936***
	(5.756)
Area (partner)	73.696***
	(5.843)
Population if common border (partner)	−0.198***

1.B.3 First Stage Regression

Table 1.B.2: First Stage Regression for Section 5.3 - Difference (Reference year is 1970).

	Dependent variable:
	Trade (Difference)
Feyrer's Instrument	3.938***
	(0.256)
GDP (Difference)	0.071***
	(0.015)
Observations	4,234
R^2	0.594
Adjusted R^2	0.589
Residual Std. Error	0.552 (df = 4182)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 1.B.3: First Stage Regression for Section 5.3 - Panel Regression (Country Fixed Effects)

	Dependent variable:
	Trade (log)
Feyrer Instrument (log)	1.010***
	(0.019)
GDP (log)	0.100***
	(0.014)
Observations	4,606
R^2	0.962
Adjusted R^2	0.961
Residual Std. Error	0.400 (df = 4435)
Note:	*p<0.1; **p<0.05; ***p<0.01

1.B.4 OLS: CDS and Trade Openness

Table 1.B.4: CDS and Trade Openness: OLS.

	Dependent variable:			
	CDS Spread			
	(1)	(2)	(3)	(4)
Trade to GDP (log)	−164.813**	−368.460**	−367.577**	−374.310**
	(74.829)	(155.068)	(151.559)	(159.186)
Debt-to-GDP (log)	39.539	50.347	64.624	57.932
	(22.684)	(54.684)	(61.923)	(60.944)
GDP (log)	−67.775***	−234.815**	−222.626**	−216.599**
	(10.707)	(81.545)	(90.237)	(88.021)
Trade Balance				−0.514
(Percentage of GDP)				(2.593)
Year Fixed Effects	Yes	Yes	Yes	Yes
Country Fixed Effects	No	Yes	Yes	Yes
Year and Oil Fixed Effects	No	No	Yes	Yes
Observations	703	703	703	699
R^2	0.225	0.680	0.681	0.682
Note:	*p<0.1; **p<0.05; ***p<0.01			

Chapter 2

Trade, Incentives to Default and Contagion

2.1 Introduction

International contagion became an important concern in macroeconomics after the 1990s currency and sovereign debt crises, especially the 1997 Asian crisis. It generated an important literature on the different linkages that could create a crisis. It is agreed that, while commercial and financial integration might have slowed down, they have not decreased by any measure for a few decades, as Antràs (2021) summarized. This might create worries about contagion risks and large-scale crises.

However, in the wake of the great Recession, there were surprisingly little sovereign defaults: only seven countries defaulted between 2008 and 2021, which is less than in any preceding decades.[1] Roos (2019) noticed this pattern and argued it was because of an ideological change in the financial institutions and elites in the world. In this paper, I argue one of the reasons why countries might have decided to commit to repayment is the increased commercial interdependence that increases the stakes of default. In

[1] Argentina defaulted twice, Greece once or twice according to the definition, Venezuela, Ecuador, Côte d'Ivoire, Venezuela, Barbados and Lebanon once.

this paper, I use the example of international trade to show that interdependence between countries is not only a factor of contagion between countries, but also a factor of stability. In the case of sovereign debt, I argue that incentives to repay debt should depend on how dependent a country is on the rest of the world, which means that more interdependence can create more stability. However, countries are exposed to other countries' decisions to default and do not internalize the harm they can cause to other countries. First, by borrowing more, a country increases its risks to default and to stop trading with its partners in the future, which would hurt them. Second, by borrowing more, a country increases the risk of a global default chain tomorrow, which should decrease the price at which all countries emit debt. I show that while more interdependence increases welfare and financial risk-sharing, it also creates motives for a worldwide regulation of debt. Although the argument focuses especially on gains from trade, it could be argued about financial linkages: what matters is that the cost of default increases as a country gets more integrated with the rest of the world.

In Section 2, we present a literature review to compare our contagion mechanism and its policy implications with papers from the contagion literature. In Section 3, we present some motivating evidence. Thanks to the special circumstances of the Argentinian default in June and July 2014, we argue that there is evidence of contagion happening through trade. We also provide a factor analysis that suggests international trade is an important factor to explain comovement in CDS spreads. In Section 4, we present a simple special case of our more general model, that shows the impact of interdependence in a symmetric global game. In Section 5, we present our general model of default. We present the mechanism through which default can happen with contagion, and we also present the kind of multiple equilibria crises that can occur. While trade creates the possibility of non-fundamental crises, it also makes sovereign debt safer. In Section 6, we summarize our results and conclude.

2.2 Literature Review

Financial and commercial contagion. There was an abundant literature about the risks of contagion after the 1997 Asian financial crisis. In this literature, the main trade channel often has to do with current account rather than static gains from trade.

If country A faces a crisis, its imports are going to dramatically decrease, more than its exports, due to its currency depreciation. Hence, its main trading partner B is going to face a current account shock. Current account is considered one of the drivers of financial crises, for several obvious reasons: it is the equivalent of net income, and the net present value of current account must be at least equal to the total amount of debt. From a short-term point of view, the current account is also the equivalent of international liquidities available to a country. We do not emphasize this current account hypothesis in this paper - although it has its own importance.

One thing we observed in Serfaty (2020), after updating findings in Rose (2005), is that, while countries with a deficit readjust their current account after a crisis, their total volume of trade, irrespective of the trade deficit, also decreases. Countries readjust their trade balance as a result of financial autarky after a crisis, but their exports also decrease substantially. The most likely explanation is that both imports and exports decrease as a result of the interruption in trade finance.[2] It would actually be sufficient that only import finance costs decrease to explain this kind of pattern. As a consequence, it means that not only sale-exposed exporting firms in B, but also input-exposed importing firms in B are going to suffer from a default in A. In other words, ongoing crises and foreign defaults imply a decrease in the actual cost of default.

[2] The two leading assumptions about the reason why trade decreases more than proportionally to GDP after a default are 1) punishment of lenders 2) interruption of trade credit due to financial autarky and lack of trust. Martinez and Sandleris (2011) pursue an intuition that Rose (2005) was already hinting at and show that the punishment assumption is flawed because the interruption in trade is actually weaker with its creditors, who would be willing to punish the country, than with other non-creditor countries.

The contagion literature was concerned with the precise definition of the broad term “contagion”, the channels of the said contagion as well as possible normative implications of contagion. Many papers distinguished interdependence from contagion, as in Claessens and Forbes (2013). Interdependence would also be called fundamental-based contagion in other settings: this is all that has to do with “normal” interdependence between countries. Exposure to common shocks, real trade and bilateral financial linkages are sometimes considered only “interdependence”.

Contagion would be something more, intervening in crises and often modeled as multiple equilibria. According to some authors, there is contagion when market inefficiencies linking countries to each other amplify a shock. Examples are given by financial constraints but our mechanism happens to be such a case of contagion narrowly defined. For example, Allen and Gale (2000b) have a model where the structure of financial holdings affect the crisis, and show that complete lending networks (everybody owes money to everybody) are more robust than incomplete ones (each country relies on a few very strong bilateral holdings). The crises happening through a default would be labeled “interdependence”, unless there is also a shortage of liquidities amplifying a crisis in the network: the shortage of liquidities would create “contagion” because it creates a self-amplifying recession.

Claessens and Forbes (2013) provided an early collection of papers about contagion after the 1997 crisis. In their review, Claessens et al. (2001) define two categories of contagion: first, the one resulting from “normal” economic interdependence or “fundamental-based contagion” (which corresponds to interdependence); second, the “volatility-based contagion” (which corresponds to the narrowly defined contagion), and which occurs because of financial market imperfections. This second class of contagion results from investor’s behavior, whether irrational or due to some informational (cost to acquire information and rational herding), incentives (hedge funds are agents that do not fully internalize the risks and benefits of their principal, as in Allen and Gale (2000a)) or coordination (multiple equilibria emerge from strategic complementarities between investors as in Obstfeld (1996)) constraints. Trade plays a role only as a vector of "fundamental-based" contagion, in the same way as common shocks or financial linkages: banks in country B that invested in country A are going to face important losses after a crisis in A, which will deteriorate their lending conditions or even make it necessary for the government to bail them out. Two categories of risks related to trade are underscored in this synthesis of the literature. First, the current account exposure that we explained above. Second, the risks of excessive competitive devaluations, whose best historical example can be found at a global level in the 1930s (see Irwin (2011)). This second kind of risks with competitive devaluation is theoretically described in Gerlach and Smets (1995). Their model of a currency crisis is inspired by first-generation crisis model Krugman (1979) but with a multi-country setting. In Gerlach and Smets (1995), two borrowing peripheral countries A and B peg their currency to a third country C. These two countries A and B are subject to speculative attacks from country C as their central banks are creating excess credit and reducing their reserves. If A has to depreciate its currency due to an attack, B is going to face a stronger pressure to depreciate if A and B are trading partners: indeed, the demand for B's goods decreases as A depreciates/stops pegging its currency - it can cause a decrease in B's GDP as well as a decrease in its shadow exchange rate. The more A and B trade, the stronger the effect. On the opposite side, the more distant the periphery countries are from the core country C, the less important contagion is between A and B. Also, this contagion channel relies on nominal wage rigidities that create demand mechanisms in this model, that are going to be absent from ours: the stronger these effects, the stronger contagion, which would not exist without them. At the end, though currency crisis, this model also emphasizes an approach based more on current account more than on gains from trade, absent from most papers.

Our approach is different, as trade is going to play a role in both kinds of contagion. We argue that trade matters not only through direct contagion of a crisis (less trade hurts a commercial partner) but also through financial contagion: less trade incites partners to default. Moreover, we our mechanism opens the door multiple equilibria, owing to bad coordination between sovereign debtors at the moment to pay debt back, in the same fashion as Diamond and Dybvig (1983). The difference with the other setting is that the miscoordination in the bad equilibrium is the responsibility of the sovereign borrowers and not of the lenders or investors in our model.

Kaminsky et al. (2003) and Goldstein et al. (2000) argue there are three "unholy" components of contagious crises: large capital inflows that accumulate before the crisis (necessary ingredient amplified by terms of trade effects and procyclicality of current account), the unanticipated character of the crisis and a common lender between "ground zero" country and "contamined" countries. In their explanation, a crisis in country A creates disrupts its country A's creditor, say a bank C situated in a third country (Japan in the example of the Asian crisis). If this bank from country C also happens to be an important lender of country B and if there are financial constraints, the bank from C is going to restrict its lending to B or constrain B to accelerate repayments. This restriction in the lending ability of B has real repercussions and can therefore create a crisis. The fact the crisis is unanticipated seems important because we can think that, otherwise, bank C would have enough time to redirect its investments less painfully.

In a more recent literature review, Rigobon (2019) stresses heteroskedasticity issues to which he applied solutions in past contributions. He notices that, in the 1990s, for the Tequila crisis and the Russian Debt, some second wave or contaminated countries had little to do with the "ground zero" country (from Mexico to Argentina and from Russia to Argentina and Brazil).

In more general settings about the link between trade and finance, papers were focused on the production side of finance. Antras and Caballero (2009) argue that finance and trade might complementary: a more open country should face less stringent liquidity constraints, justifying higher capital openness.

From a more theoretical perspective, this paper is a multi-country application of the insights from our previous study, Serfaty (2020). It is related to models like Dixit (2003a), where other agents' decisions to join or quit a club (in our paper the club of defaulters or the club of payers) make it more or less attractive to be part of the club. It is also inspired by Dixit (2003b) for the structure of participants and the comparative statics. Geographical economic intuitions and models, as exposed in Fujita et al. (1999), also share some similarities with our model.

Empirics of Contagion: From a historical point of view, Bordo et al. (2007) (see Neal and Weidenmier (2003) for a summary and critics) survey contagion episodes and their narratives. Using a list of financial crises by Kindleberger (2000), the authors argue that in the 1890s, contagion was more mastered than in the 1990s, through restrictive policies in peripheral countries that we would assimilate to stringent macroprudential norms. Reinhart and Rogoff (2009) suggest that international crises happen through a wake-up call: investors and lenders overestimate the borrowing ability of other countries, and they suddenly realize they were wrong.[3] In our model, by contrast, frictions emerge from strategic complementarities between borrowers. Eichengreen et al. (1996) show that currency crises are "contagious", meaning that, once a country was hit by a currency crisis, its neighbors were more likely to face a currency crisis as well.

Many papers test the hypothesis of contagion in financial markets by looking at correlation of returns in financial markets. Among them, Forbes and Rigobon (2002) applied the distinction we discussed above between interdependence and contagion to foreign exchange markets. They notice that, at times of crises in which contagion is supposed to intervene (besides interdependence that always plays), markets become more volatile. As a consequence, estimates that do not take into account that heteroskedasticity will have a tendency to overestimate contagion. Heteroskedastic models show that "contagion" more narrowly defined does not intervene. Karolyi

[3]Basu (2002) finds empirical evidence in favor of such of informative explanation for contagion in post WW2 period.

(2003) arrives to the same conclusion as Forbes and Rigobon (2002) and criticizes at the very use of the term of contagion. In our model, the contagion mechanism affects the difference between the welfare under repayment and the welfare under default - it is a "fundamental" shock and can reflect what these authors define as interdependence, but I will use the word "contagion" with the same meaning as "interdependence". Using their definition of contagion, contagion appears in our model only to the extent that there can be multiple equilibria.

Forbes (2012) studies interdependence, and she noticed that more open economies are more dependent on common and foreign shocks than relatively autarkic economies, which is in line with our model. Corsetti et al. (2005) argued that there is some contagion, narrowly defined: Hong Kong crisis in 1997 triggered downfalls in 5 other "contaminated" economies. Rigobon (2019) surveyed methodological contributions: Rigobon and Sack (2004), Gravelle et al. (2006), Raddatz and Schmukler (2012), Pavlova and Rigobon (2007) work on relevant methodological methods to disentangle contagion .

Some papers in the same literature also found a significant effect of trade: for example, Glick and Rose (1999) find that trade relations are good predictors of currency crises. A paper like Forbes (2004), using firm-level data, found that, after the "Asian flu" or the "Russian virus" crises, the most specifically hurt firms from 46 other countries were the ones with trade exposure to the hit countries or working in similar sectors (respectively 25 and 8 points less in abnormal returns). Importantly in this pre-Great Recession debate about the relative importance of trade and finance as channels for crisis, Forbes and Chinn (2004) find that bilateral trade flows are much better at explaining stock comovement between countries than financial bilateral flows are.

Our study is focused on sovereign risk *per se* and its contagion channels and should therefore not look at stock markets independently of their effect on sovereign risk. Some papers are closer to our interests. For example, Debarsy et al. (2018) apply special regression techniques and argue that socio-economic information matters more than trade for the transmission of sovereign risk across countries.

Our paper is also related to a branch of papers studying the link between business cycles correlation and trade, such as Frankel and Rose (1998). This contribution was concerned with the optimal currency area definition and noticed that countries that traded more with each other also happened to have more correlated business cycles - a necessary condition to share efficiently a common currency. More recent contributions study the way trade creates interdependence between countries, such as Abeysinghe and Forbes (2005) who show in an Asian context that GDP shocks can propagate, using bilateral trade matrices with GARCH.

In other fields of economics, especially in educational economics, there is a broad study of peer effects that are in fact similar to contagion problems. Sacerdote (2001) shows that two students randomly assigned to the same room are likely to have more similar GPA in college. This is an example of "contagion" between students, and the econometric challenges faced by the contagion literature are quite similar to those faced in the peer effects literature: comments from Angrist (2014), Feld and Zölitz (2017) are useful for the empirical claims we make in Section III.

Sovereign Debt and International Macro: The reasons why countries repay debt have to do with their linkages with the outside world: a country that could reenter into autarky without harm would not hesitate to default after borrowing, even in a reputational model, as was proved by Bulow and Rogoff (1988). Therefore, a theoretically sound analysis of sovereign debt must rely on the idea of a cost of sovereign default and autarky. A natural candidate to summarize the relations with the outside world is international trade. In Serfaty (2020), I studied some evidence suggestive of a causal relation between trade openness and the ability to commit to repayments. I argued that more open countries could borrow for cheaper, had a tendency to default less and to borrow more. The reason for that tendency is that trade decreases after a default, probably due to an interruption in trade credit, as in

Amiti and Weinstein (2011). The evidence and the model in Serfaty (2020) was about the total level of trade openness in a given country and its relation with sovereign debt and spreads. In contract, my results in this paper are about the relation between the spreads from one country to the other.

More generally, our model deals with the macroprudential implications of integration. The consequences of open capital markets have been studied in . An example is Caballero and Simsek (2020). The authors show that the tendency of countries to temporarily retrench capital from foreign countries create incentives in all countries to restrict capital flows. However, at a global and uncoordinated level, restrictions increase the damage generated by the fire sales in the countries with retrenchment, and welfare decreases as a consequence of the policy. We have similar dynamics with tariff: generally, the best strategies that each country can individually use sometimes imply tariff, which have to be used cautiously.

Our model is also related to literature that explains comovement in finance through common factors. Miranda-Agrippino et al. (2015) argue that a global financial cycle explains much of the comovement of financial prices in the world. Specific evidence about sovereign debt spreads was discussed in Pan and Singleton (2008) and Longstaff et al. (2011).

2.3 Motivating Evidence

In this Section, we propose evidence that trade channel can explain contagion of sovereign debt crises: because sovereign debt interrupts international trade, the commercial partners of a defaulting country have more risk to default. We use an event study to identify some contagion happening through trade. In Section 3.1, we present the context of our event study, inspired by Hébert and Schreger (2017) whose objective was to estimate the cost of sovereign default in terms of economic loss. In this special default episode, we can argue that default was not caused by a shock common between the sovereign defaulter and its main commercial partners, which makes the identification of the shock credible, and we present our results in Section 3.2.

2.3.1 Argentinian Event Study

Context

The empirical issue with most empirical studies of contagion is that it is hard to disentangle common shocks from pure contagion. For example, assume that countries that trade a lot with each other do so because they trade differentiated inputs from a given sector, say to produce cars. If there is a huge negative demand shock for cars, and trade of differentiated inputs for cars defines trade between countries, then one might expect to observe contagion-like phenomena through trade although everything happens because of a common shock on the demand for cars. This same argument was used against Sacerdote (2001) in the literature about peer effects: the similarity of the score between two students randomly assigned to the same room might be due to contagion, but it might also be due to a common shock. For example, if the two students had a particularly noisy room close to a train station that made it more difficult to study in a quiet environment, then a regression about peer effects or contagion would be likely to interpret it as contagion if it could not observe common shocks.

In international economics, these issues are even more challenging, as it is very difficult to find a case where "pure contagion" can credibly be identified: international economic shocks are often multi-faceted and hard to precisely identify. We propose an event study inspired by Hébert and Schreger (2017) about the Argentinian default in 2014. As the authors explain:

> The case of *NML Capital, Ltd. v. Republic of Argentina* provides a natural experiment to identify the causal effect of sovereign default. Following Argentina's sovereign default in 2001, NML Capital, a hedge fund, pur-

> chased some of the defaulted bonds and refused to join other creditors in restructurings of the debt that occurred in 2005 and 2010. Instead, because the defaulted debt was issued under New York law, NML sued the Argentine government in US courts to receive full payment. To compel the Argentine government to repay the defaulted debt, the US courts blocked Argentina's ability to pay its restructured creditors, unless NML and the other holdout creditors also received payments. The Argentine government resisted paying the holdouts, even though the required payments would be small relative to the Argentine economy. As a result, legal rulings in favor of NML raised the probability that Argentina would default on its restructured bonds, while rulings in favor of Argentina lowered this probability. We argue that these legal rulings are exogenous shocks to the risk-neutral probability of default that allow us to identify the causal effect of sovereign default on the market value of Argentine firms. Our key identifying assumption is that the information revealed to market participants by these legal rulings affects firms' stock returns only through the effect on the sovereign's risk-neutral probability of default.

As these authors do it in the context of the internal Argentinian stock market, we propose a study on the effect of US Supreme court ruling on other sovereign CDS spreads at a daily scale. The US Supreme Court decision can be reasonably considered independent from South American business cycles. It was decided and publicly announced on June 16, 2014.[4] Its broader impact as a precedent that seems to break the hitherto traditional interpretation of the *pari passu* clause[5] present in most sovereign debt contracts in New York Law has been discussed by practitioners and

[4]see https://www.supremecourt.gov/opinions/13pdf/12-842_5hdk.pdf for the decision.

[5]This clause states that all creditors of a defaulting country have the same seniority for their debt contracts. In a same court system, a sovereign borrower has to reimburse each creditor the same way. In case of default, creditors get the same fraction of their claims paid back if they accept restructuring. The traditional interpretation was that, after a sovereign restructuring deal with a significant share of creditors, creditors refusing to be part of the deal were not allowed to use the *pari passu* clause to prevent new post-restructuring lenders from getting paid back.

jurists: Muse-Fisher (2014), Martindale (2019). If we abstract of the possible contagion through Argentina, this judgment should have increased the default cost and the recovery value of debt for sovereigns borrowing from markets under the law of the US federal court system, especially New York: it would have had led to a decrease in the value of CDS prices. However, when we controlled whether countries trading more with the US saw a difference in their CDS spreads before and after the default, we found a non-significant effect. We interpret this as the proof that the identification strategy is credible.

We use only the last ruling made by the Supreme Court of the United States rather than other rulings because it finally led to a selective default on New York Law debt and we are specifically interested in the consequences of default rather than in the consequences of default risk.[6] We are specifically interested in the effect of a sovereign default so we are going to restrict our attention to a short window time before and after the ruling. For example, in November 2015, Argentina elected a former businessman and pro-market President, Mauricio Macri: since his election might have affected commercial relations between Argentina and the rest of the world as well as financial markets' pricing, we do not want to include this episode in our analysis, hence the short period of time that is analyzed: we limit our time window to 250 days before and after the ruling. We use daily CDS data from Datastream, with 64 countries whose data was updated each day. Argentinian default not only became likely in the immediate aftermath of the ruling - it happened officially on July, 30 2014. The default occurred because of other legal consequences the repayment of the debt would have had[7] rather than simply because of the extra debt, but it still had consequences on the Argentinian economy, which entered into financial autarky

[6]However, our later model might be interpreted as the discrete version of a model where probability of default is the equivalent of a continuous decision or mixed strategy to partially default as evaluated by the country and financial markets simultaneously.

[7]Namely, the creditors who had accepted restructuring would have had a claim to be fully paid back if Argentina had paid NML claims at face value as it was required to do by Supreme Court. The consequences would have been much larger than the value of NML claims suggest - hence Argentina chose to default in this context.

vis-à-vis American financial markets, creating a downfall in trade credit. Hébert and Schreger (2017) proved that the value of many Argentinian firms significantly declined after the default, especially the value of exporting firms.

Results

We posit that a country's decision to default affects the incentives of its neighbors to default through trade. As a consequence of Argentinian default, we expect countries whose trade with Argentina was a larger share of their total trade (e.g. Uruguay) to face a larger increase their CDS than other countries. CDS are our measure for default risk: they correspond to insurance premia against the sovereign default of a country. The larger CDS spreads (or premia) are, the more likely a country is deemed to default. The interest of CDS spreads is that they are exchanged on day-to-day base in financial markets, which makes them more liquid: their price is then likely to be more meaningful. We use CDS spreads for bonds with a 5-year maturity because they are traditionally the most liquid ones, hence the ones with the more significant prices.

Assume that several economies default at time t. We are interested in the evolution of the spreads of other economies. We start from the following empirical specification:

$$CDS_{i,t+h} - CDS_{i,t} = \alpha_h + \beta_h \sum_{j \neq i} D_{j,t} T_{i,j} + \gamma_h X_{i,t} + \varepsilon_{i,h}, \tag{2.1}$$

where t is the time of default, h is the number of days around default, $CDS_{i,t'}$ is the CDS of country i at day t'

More explicitly, we are going to run the following difference-in-difference regression around the date of the ruling t (which is June 16, 2014): where h is the number of days, $CDS_{i,t+h}$ is the CDS spread of country i at day $t+h$, α_h is a day-fixed effect, $D_{j,t}$ is a dummy indicating whether country j defaulted at time t, $T_{i,j}$ is country j's share in the total trade of goods of country i at the year before the event t (2013 in our case), $X_{i,t}$ is a set of controls for i at time t: total trade of country i and GDP of country i in 2014 are included to take into account potential shocks to GDP. Our key coefficient of interest is β_h: if there is some contagion of sovereign default through trade channel, one should expect $\beta_h = 0$ for any $h \leq 0$ and $\beta_h > 0$ otherwise. The identification issue is that $\varepsilon_{i,t}$ is likely to be correlated with $D_{j,t}$ due to common shocks. That is the reason why the event study around the Supreme court ruling is interesting to us: the juridical shock that forced Argentina to default was likely to be exogenous to other macroeconomic conditions. Hence we test the following regression:

$$CDS_{i,t+h} - CDS_{i,t} = \alpha_h + \beta_h D_{A,t} T_{i,A} + \gamma_h X_{i,t} + \varepsilon_{i,h}.$$

Our assumption identification can be written the following way:

$$\forall i \neq A, h,\; D_{A,t} T_{i,A} \perp \varepsilon_{i,h}.$$

We allow the coefficients to vary and the shock to be fully heteroskedastic in order to take into account the fact that variance of CDS increases after a financial shock as it is explained in Rigobon (2019).

We summarize the results in figure 1. We can observe two things: first, we find no evidence of a pre-trend before the ruling, which is consistent with our identification assumption: countries trading with Argentina did not face a particular trend before the ruling. Second, as it would be consistent with our contagion hypothesis, countries trading more with Argentina faced a larger increase in their CDS spreads after Argentinian default. The increase seemed low in the days immediately after the ruling, which might be attributed to partial delay or registered transactions. But after a few days, we observe a clear positive effect. This result means that the default risk of commercial partners of Argentina was perceived as larger after the Supreme Court ruling. This regression confirms our contagion channel through trade.

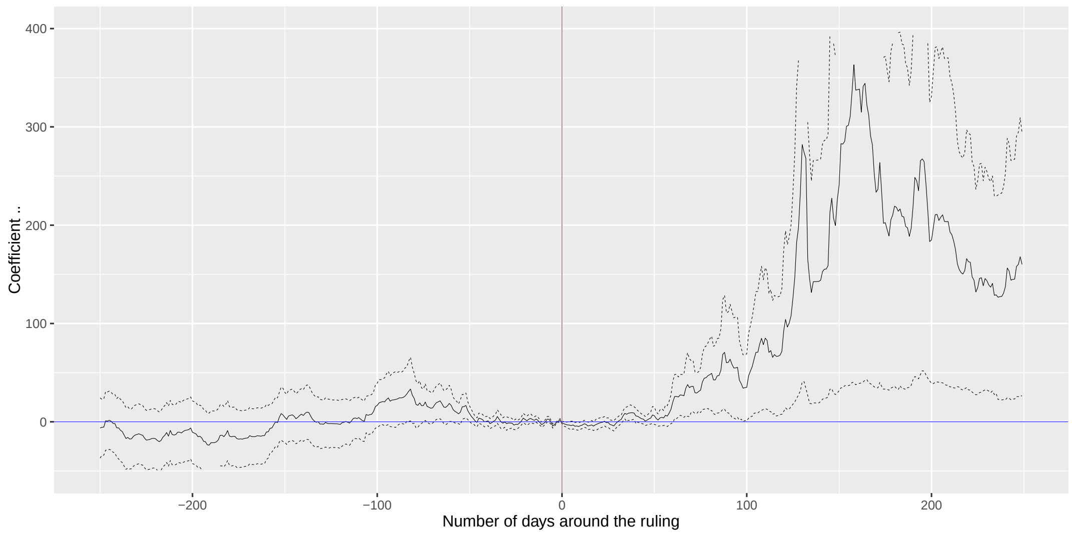

Figure 2.1: Effect of trade with Argentina around Supreme Court Decision: value of coefficient β_h from equation (2.1) over time.

How large is the effect empirically? The country for which Argentina is more important as a commercial partner is Uruguay. Uruguayan trade with Argentina was more than 12% of Uruguayan total trade in goods in 2014. Using a baseline estimate of β_h of 150, it means that Argentinian default caused an increase in CDS spreads in Uruguay equal to 20 b.p., roughly equal to the spread between France and Germany in 2019. While it does not necessarily imply a very large increase in the probability of default, we have to notice that Argentina was already deemed likely to default before the event and that it is also a relatively closed economy, with no great impact on the outside world. Moreover, Argentinian default in 2001 already caused Uruguayan default in 2001: Uruguayan and other South American governments are likely to have been cautious on their dependence on Argentina. It was therefore unlikely that we would still be able to find some significant and sizable effects of the ruling on other countries' spreads.

2.3.2 Factor's Analysis

Longstaff et al. (2011) presented evidence that there was strong comovement in sovereign spreads around the world. They associate it to the importance of risk premia and global financial cycles. We present motivating evidence that this could also be attributed to movements in international trade. Unfortunately, international trade data are only available at an annual scale for most of the countries of interest.

Let $CDS_{i,y}$ be the sovereign spreads of country i at year y: we average daily values of five-year credit default swap spreads. Let $(F_y)_{y\geq 0}$ be a proposed factor. We run the following regression with random fixed effects:

$$CDS_{i,y} = \alpha + \beta_i F_y + \varepsilon_{i,y},$$

and compute the associated R^2. In order to test whether the associated R^2 is large enough, we simulate a large number of independently and identically distributed white noise processes $(F_y^R)_{y\geq 0}$, which provides a test in order to test the quality of a factor. We compare factors listed in Aguiar et al. (2016) with international trade as a share of world GDP, that we obtain from CEPII. The results from these simulations are presented in table 1. CDS come from the same source as in the previous

Table 2.1: Comparison of different factors for CDS

	World trade	VIX	PE-ratio	LIBOR
R^2	0.598	0.636	0.534	0.331
p-value	0.002	< 0.001	0.093	0.996

We compare the explanatory power of international trade on yearly comovements to other common factors from the corporate finance literature. The most successful factor to explain comovement is the Volatility Index, VIX, which uses the price of calls and puts on Standard & Poor's 500. Price-Earnings ratio is simply a measure of the income from stock markets divided by total capitalization. LIBOR is the inter-bank interest rate.

We see that total world trade can be considered a very good explaining factor, with 60% of the variation of CDS spreads being explainable by trade alone. We interpret this as evidence that trade is as relevant as factors usually associated with investment fears to explain comovement of spreads, which suggests its role might have been overlooked to explain the global financial cycle: in the end, financial flows have to translate into trade flows and it should come as no surprise that trade influences finance and the other way around.

2.4 A Simple Model of Contagion

In this Section, we present a simplified version of the model where gains from trade occur. A continuum of countries $i \in [0, 1]$ with the same preferences and the same risks on their GDP borrow debt from foreign creditors. Debt is defaultable, and the cost of default decreases in the number of defaulters. At the second and final period, countries choose whether to default or not default, and default equilibria are defined as the Nash equilibria resulting from their unilaterally optimal policy choices. In the first period, they borrow debt, anticipating what will happen in the second period, and they choose how much debt to borrow, unilaterally once again.

Indeed, we assume default interrupts commercial relations. In Section 4.1, we present the fundamentals of the model, the first order conditions and some comparative statics. In Section 4.2, we present the social planner's problem and optimal policies.

2.4.1 Unilateral Incentives to Borrow and Default

Preferences and Default

At the second period, GDP Y_2 is distributed according to cumulative distribution function F. We assume this distribution is absolutely continuous with respect to the

Lebesgue measure and F has derivative f. We will omit the subscript.

At the second period, utility in case of default is simply equal to GDP, as the country does not have the ability to trade with the rest of the world.

$$V_2^D(Y) = Y. \tag{2.2}$$

Utility in case of repayment depends on GDP Y, debt B and rate of default of debtors Λ:

$$V_2^R(Y, B, \Lambda) = (Y - B)(1 + g(1 - p\Lambda)), \tag{2.3}$$

where g is an index for gains from trade, that is increasing in trade openness, and p is the proportion of debtors in the world. We assume actual gains from trade are simply proportional to the number of trade partners that exist. If a proportion Λ of debtors default and do not participate to international trade any more, as we assume that creditors never renounce, it means that a share $(1 - p\Lambda)$ of all countries are available as trade partners.

We assume that Λ is deterministic, as all countries face the same distribution of shocks independently of each other. As a consequence, the law of large numbers applies and it is possible to determine it in the first period with probability 1.

There is a fixed interest rate r taken as given by creditors. Government can emit debt B at price $q(B)$, that is going to depend on the probability of default at the equilibrium:

$$q(B) = \frac{\mathbb{P}(V_2^R(Y, B, \Lambda) > V_2^D(Y))}{1 + r}.$$

At the first period, a debtor receives income Y and borrows income $q(B)B$, so that its value function can be written:

$$V_1(B, \Lambda) = (1 + g)(Y + q(B)B) + \beta\mathbb{E}(V_2(Y, B, \Lambda)),$$

where

$$V_2(Y, B, \Lambda) = \max\{V_2^R(Y, B, \Lambda), V_2^D(Y)\},$$

so that the borrower's optimization problem is going to be:

$$\max_B (1+g)(Y + q(B)B) + \beta \mathbb{E}(V_2(Y, B, \Lambda)).$$

We are going to assume that:

$$(1+r)\beta < 1,$$

which ensures that sovereign countries are willing to borrow.

When we solve the model, we are interested in the probability of default. As a consequence, we are going to write the value function as a function of the probability of default of an individual country λ. This alternative will turn out to be useful for computations. First, we need to write the value of face value debt $B(\lambda, \Lambda)$ associated with default probability λ, given the expected aggregate rate of default Λ. We have the following formula:

$$B(\lambda, \Lambda) = F^{-1}(\lambda) \frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}. \tag{2.4}$$

Equation (2.4) is simply derived from finding which value of B equates V^R and V^D when $Y(\lambda) := F^{-1}(\lambda)$, which corresponds to the quantile associated with λ of the distribution of GDP. We use equations (2.2) and (2.3) to solve this equation. We know that debt is equal to $B(\lambda, \Lambda)$, a country should default if and only $Y \leq F^{-1}(\lambda)$, which implies that the probability of default is λ. We simply use the fact that incentives to default decrease in GDP Y.

As the default risk associated with $B(\lambda, \Lambda)$ is by definition λ, we know that:

$$q(B(\lambda, \Lambda)) = \frac{1-\lambda}{1+r}.$$

As a consequence, we can write the borrower's optimization problem the following way:

$$V_1 = \max_{\lambda}(1+g)(Y + \frac{1-\lambda}{1+r}B(\lambda,\Lambda)) + \beta\mathbb{E}(V_2(Y,B,\Lambda)).$$

The first order condition associated with this maximization problem is:

$$\begin{aligned}
& \frac{1+g}{1+r}\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}\big(\frac{1-\lambda}{f(Y(\lambda))} - Y(\lambda)\big) = \beta\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}\frac{1}{f(Y(\lambda))}\int_{\lambda}^{1} 1+g(1-p\Lambda)dl \\
\iff & \frac{1}{1+r}\frac{1+g}{1+g(1-p\Lambda)}\big(\frac{1-\lambda}{f(Y(\lambda))} - Y(\lambda)\big) = \beta(1-\lambda)\frac{1}{f(Y(\lambda))} \\
& \iff \frac{1+g}{1+g(1-p\Lambda)} - \beta(1+r) = \frac{1+g}{1+g(1-p\Lambda)}\frac{f(Y(\lambda))}{1-\lambda}Y(\lambda) \\
& \iff 1 - \frac{\beta(1+r)(1+g(1-p\Lambda))}{1+g} = \frac{f(Y(\lambda))}{1-\lambda}Y(\lambda).
\end{aligned}$$

To solve this problem, we need to solve for interest rate r as a function of Λ. We need to take into account the fact that returns to consumption depend on the aggregate level of inflation: if some countries default, the purchasing power of a given amount of money in the reserve currency decreases, because productivity decreases. As a consequence, we need to assume that the nominal interest rate adjusts to keep the real interest rate constant:

$$\frac{(1+g(1-p\Lambda))}{(1+g)(1+\rho)} = \frac{1}{1+r}, \tag{2.5}$$

where ρ is the discount rate of creditors, that is an exogenous parameter. As a consequence, nominal interest rate r should depend on Λ and the first order condition writes:

$$1 - \beta(1+\rho) = \frac{f(Y(\lambda))}{1-\lambda}Y(\lambda). \tag{2.6}$$

This first order definition allows us to define a best response function $\lambda^{\star}(\Lambda)$ for every value $\Lambda \in [0,1]$ if we assume that $\lambda \mapsto \frac{f(Y(\lambda))}{1-\lambda}Y(\lambda)$ is increasing. Because the quantile function is increasing, this function is increasing if and only if $y \mapsto \frac{f(y)}{1-\mathbb{P}(Y\leq y)}y$ is an increasing function over the support of y. We are going to assume it is monotonic.

Moreover, we are going to assume that:

$$\sup_{y\in \mathrm{Supp}(Y)} \frac{f(y)}{1-\mathbb{P}(Y\leq y)}y \geq 1$$
$$\inf_{y\in \mathrm{Supp}(Y)} \frac{f(y)}{1-\mathbb{P}(Y\leq y)}y \leq 1-\beta(1+\rho)$$

to ensure an equilibrium exists.

Nash Equilibrium in *Laissez-Faire*

From our previous results, we naturally define a Nash equilibrium:

Definition 1. *A global symmetric equilibrium for borrowing is a probability of default* Λ *such that* Λ *is a fixed point of the correspondence:*

$$\lambda^\star(\Lambda) = \arg\max_\lambda (1+g)(Y + \frac{1-\lambda}{1+r(\Lambda)}B(\lambda,\Lambda)) + \beta\mathbb{E}V_2(Y, B(\lambda,\Lambda),\Lambda),$$

which means that $\Lambda \in \lambda^\star(\Lambda)$*. Moreover, any* $\Lambda \in (0,1)$ *is an equilibrium if and only if:*

$$1-\beta(1+\rho) = \frac{f(Y(\Lambda))}{1-\Lambda}Y(\Lambda)$$
$$\textit{and} \frac{d}{d\Lambda}\frac{f(Y(\Lambda))}{1-\Lambda}Y(\Lambda) > 0$$

In other words, Λ *is a Nash equilibrium of the default game.*

This definition simply means that the situation of all countries is symmetric: they should borrow the same amount of debt and have the same probability of default. However, they impact each other because of gains from trade.

After defining equilibrium, we are interested in its comparative statics. We computed the first order condition (2.6) in the previous Section, which allows us to write immediately the following proposition:

Lemma 1. *Assume that the function:* $\lambda \mapsto \frac{f(Y(\lambda))}{1-\lambda} Y(\lambda)$ *is increasing and that real interest rate is constant, as in (2.5). All default probabilities equilibria* (Λ^{eq}) *form a lattice that does not depend on gains from trade. As a consequence, the total level of transfers and the size of debt increase in g for each extremal equilibrium, while the level of risk stays constant.*

A sufficient condition for the assumption to be true would be an increasing hazard rate, or an increasing density (which is locally true for all unimodal distributions at low quantiles). This condition is true for log-normal distributions which are the most standard distributions for GDP shocks.[8]

This result simply stems from rewriting the first order condition with our new equation where r is endogenous. Although default risk does not change as a consequence of more openness, there is still an increase in total financial transfers at the equilibrium in our case. Indeed, for the same level of risk, a government can borrow more debt. We are going to use that assumption (2.5) henceforth.

2.4.2 Social Planner

In the previous Section, we presented *laissez-faire* equilibrium, which describes what happens when countries borrow debt without internalizing the effect of their default risks on their trade partners. Because default directly affects the gains from trade of other countries, there are obvious externalities of borrowing in this model. We define the first best equilibrium below and look for a characterization of policies that could reach this equilibrium.

The social planner is going to be constrained. It cannot prevent countries from defaulting *ex post*, and it has to reduce their incentives *ex ante*. We assume that it cannot reduce debt *ex post* either, when countries face a bad shock. It would create some risks of moral hazard

[8] A counterexample is the Pareto distribution for which the function $\lambda \mapsto \frac{f(Y(\lambda))}{1-\lambda} Y(\lambda)$ is constant over the support. However, this is not a significant counterexample as it is easy to check that Pareto distributions give pathological results in the case of sovereign debt.

Proposition 4. *The centralized social planner's equilibrium is the value of* Λ *that solves:*

$$\max_{\Lambda}(1+g)(Y+\frac{1-\Lambda}{1+r}B(\Lambda,\Lambda))+\beta\mathbb{E}V_2(Y,B(\Lambda,\Lambda),\Lambda),$$

where

$$\mathbb{E}V_2(Y,B(\Lambda,\Lambda),\Lambda)=\int_0^{\Lambda}Y(l)dl+\int_{\Lambda}^{1}(Y(l)-B(\Lambda,\Lambda))(1+g(1-p\Lambda))dl.$$

The first order condition the social planner has to solve is:

$$\frac{1+g}{1+r}\big[(1-\Lambda)\frac{dB(\Lambda,\Lambda)}{d\Lambda}-F^{-1}(\Lambda)\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}\big]=\beta\frac{dB(\Lambda,\Lambda)}{d\Lambda}\int_{\Lambda}^{1}1+g(1-p\Lambda)dm+gp\beta\int_{\Lambda}^{1}(Y(l)-B(\Lambda,\Lambda$$

where:

$$\frac{dB(\Lambda,\Lambda)}{d\Lambda}=\frac{g(1-p\Lambda)}{f(Y(\Lambda))}\frac{1}{1+g(1-p\Lambda)}-\frac{Y(\Lambda)gp}{(1+g(1-p\Lambda))^2}.$$

This definition simply means that the social planner is going to internalize the impact of individual default probability λ on the aggregate default probability. This is a standard example of externality. The first order condition associated with the social planner's includes the decrease in the welfare of all trading countries.

Let us assume now that social planner can tax debt emission at rate t^b. Any country borrowing $q(B)B$ from financial markets would have to pay a tax $t^bq(B)B$. The *laissez-faire* equilibrium associated is characterized by the following first order condition at the equilibrium:

$$(1-t^b)\frac{g(1-p\Lambda)}{1+\rho}\big(\frac{1-\lambda}{f(Y(\lambda))}-Y(\lambda)\big)=\beta\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}\frac{1}{f(Y(\lambda))}\int_{\lambda}^{1}1+g(1-p\Lambda)dl$$
$$\iff(1-t^b-(1+\rho)\beta)=\frac{f(Y(\lambda))Y(\lambda)}{1-\lambda}(1-t^b).$$

Proposition 5. *Let* $\Lambda^{\star}$ *be a first best equilibrium obtained by the central planner. There is a tax on debt* $t^{b\star}(\Lambda,g)$ *that decentralizes the optimal equilibrium. It is defined* *by the following equation:*

$$t^{b\star}(\Lambda^\star, g)\frac{g(1-p\Lambda^\star)}{1+\rho}\big(\frac{1-\Lambda^\star}{f(Y(\Lambda^\star))} - Y(\Lambda^\star)\big) = \frac{(1-\Lambda^\star)F^{-1}(\Lambda^\star)gp}{1+g(1-p\Lambda^\star)}\big(\frac{1}{1+\rho} - \beta\big)$$
$$+ \frac{gp}{1+\rho}[(1-\Lambda^\star)F^{-1}(\Lambda^\star)\frac{g(1-p\Lambda^\star)}{1+g(1-p\Lambda^\star)}]$$
$$+ \beta gp\int_\Lambda^1 Y(l) - Y(\Lambda^\star)\frac{g(1-p\Lambda^\star)}{1+g(1-p\Lambda^\star)}dl.$$

Alternatively, one can write:

$$\frac{t^{b\star}}{1-t^{b\star}}\beta\frac{1-p\Lambda^\star}{f(Y(\Lambda^\star))} = \frac{F^{-1}(\Lambda^\star)p}{1+g(1-p\Lambda^\star)}\big(\frac{1}{1+\rho} - \beta\big)$$
$$+ \frac{p}{1+\rho}[(1-\Lambda^\star)F^{-1}(\Lambda^\star)\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}] \tag{2.7}$$
$$+ \beta p\int_\Lambda^1 (Y(l) - B(\Lambda^\star, \Lambda^\star))dl.$$

This proposition gives us a decomposition of the externality. On the first line of the right-hand–side of equation (2.7), we see the pecuniary externality of over-borrowing: countries that borrow too much decrease the borrowing capacity of other countries, so that all countries hurt each other when they borrow in excess: it is a pecuniary externality of debt over the risk level. The second line of (2.7) shows the effect on interest rates: safe nominal interest rate increases in the total rate of default to compensate for the larger price of goods when trade is interrupted at the next period. Due to the imperfection of the debt market and to the absence of commitment, these pecuniary externalities do not cancel out in this case. The third line shows the direct negative effect of excess borrowing on other countries: excess default hurts countries that decided to repay their debt. These externalities go into the same direction and imply the existence of excess borrowing among debtors.

We prove in the appendix that, as in the case of *laissez-faire*, the optimal policy does not depend on the level of trade. As a conclusion, we get the following proposition:

Proposition 6. *Welfare gains from the optimal policy increase in the level of openness g.*

This result means that the distortion in borrowing due to the externality increases as the level of openness increases at the same pace as the equilibrium quantity of debt as total level of openness increases. As a result, the gap between optimal default risk and the one we actually observe does not depend on the level of trade openness.

This result could be interpreted as a proof that the size of openness does not matter for policy. In our simplified linear model, we show they do not. However, the total welfare gains from optimal policy increase proportionally in the gains from trade which makes optimal tax more important when there is a large trade agreement. Let us assume that the political integration necessary to reach the first-best equilibrium as a policy with a fixed coordination and administrative cost c (that each country has to pay). Then this policy becomes worth the political effort when the total level of integration increases.

After discussing our simple model with gains from trade that are linear in the proportion of trade partners, one might wonder which assumptions are sensitive to a more realistic description of trade. Moreover, we are also interested in the effect of tariff policy and of free-trade agreements, as they are part of realistic policies that can be reached by countries. Hence, we need to include terms of trade effects in the model and a depiction of determinants of trade in order to discuss the orders of magnitude and see what optimal policies can be reached, and more generally we need to make gains from trade endogenous: should we expect a linear function as the one we have used, or a more concave one? These new assumptions require a model that is less tractable analytically but they will make it possible to answer new questions about optimal policy.

2.5 A Circle of Countries with Defaultable Debt

In this Section, we present a more general model, with an explicit model of trade and terms of trade: with these new assumptions, we will be able to see whether the results from the previous section are robust to more realistic features of trade, and also what the consequence of trade policy is.

To make exposition and analysis of the problem simpler, we will make the problem symmetric *ex ante.* The decision to default after an asymmetric shock is going to imply some asymmetries but only *ex post.* Our model is a multi-country model inspired by Eaton and Gersovitz (1981) where trade linkages serve to countries as a commitment device to repay their debt.

In the model, a continuum of symmetric economies trade with each other and borrow defaultable debt from each other.

There are two periods: at the first period, more impatient economies borrow from more patient economies. Debt is prices according to the probability of default. In the second period, borrowers can observe their GDP and they decide whether to default or to repay. The consequence of sovereign default is a reduction in trade opportunities.

In Section 5.1, we introduce the structure of the model. In Section 5.2, we define equilibria and present results about default decisions in the second period and default waves. We prove that when trade openness increases, the risk of sovereign default decreases In Section 5.3, we present some results about optimal policy, especially trade policy.

2.5.1 Assumption and Primitives

Preferences and Technology

Preferences

There are two periods $t = 1, 2$. There is a continuum of economies $i \in [0, 1]$. Each economy i is inhabited by a representative household who wants to maximize its intertemporal utility U_i:

$$U_i = C_{i,1} + \beta_i \mathbb{E} C_{i,2}$$

where $C_{t,i}$ is the aggregate consumption of country i at time t. All agents are risk neutral in the model and intertemporal elasticity of substitution is infinite.

There are two types of agents: patient agents $i \in [0,1]$ with discount factor $\beta_i = 1$ and impatient agents with discount factor $\beta_i = \beta < 1$. The share of patient agents is assumed to be $p = \frac{1}{2}$, so that there are as many borrowers as lenders.

Technology

Aggregate consumption is made of aggregate goods and domestic goods. Each economy is a small open economy because it is one among a continuum of infinitesimal economies, as in Gali and Monacelli (2005). Each economy i has an exogenous endowment Y_i, in the form of an idiosyncratic Armington tradable variety.

At each period t, the aggregate consumption of each economy i aggregates consumption $c_{i,D,t}$ of its domestic good and consumption $c_{i,F,T}$ of an international good:

$$C_{i,t} = \left(\alpha^{\frac{1}{\sigma}} c_{i,D,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} c_{i,F,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where σ is the elasticity of substitution between imported goods and exported goods. The international good itself is an aggregate of all varieties in the world:

$$c_{i,F,t} = \left(\int_0^1 c_{j,i,t}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}},$$

where $c_{j,i,t}$ is the total quantity of variety from country j consumed by country i at time t, and η is the elasticity of substitution between different imports. The international good is not tradable and its components must be bought locally in order to produce it.

The variety that each country produces is going to be important for its own consumption. On the contrary, each individual variety from foreign country should only have an infinitesimal importance for other countries.

Trade Costs and Default

In order to consume the international domestic good, countries have to trade with each other. There are iceberg trade costs. Trade costs between two countries i, j, $i \neq j$ are fixed and equal to a constant $\tau \geq 1$ but they will be infinite whenever a country defaults. In the appendix, we present a more general version where each country only loses a share $\theta \in [0, 1]$ of its commercial partners.

Default affects trade on the extensive margin in our model instead of the intensive margin. At the national level, default disrupts trade at the intensive margin mostly. However, at the individual firm level, extensive margin gets affected, as it has been proven in Gopinath and Neiman (2014). However, given the nature of our model, which margin of trade is affected by default is irrelevant: the stylized assumption would work the same way as an increase in trade.[9] As long as the decrease in trade that occurs because of a default is proportional to the existing volume of trade, our simplifying assumption should not change the results. However, this last assumption requires imposing specific assumption on the price effects of default.[10] That is why

Shocks

The labor endowment of each country in the first period is assumed to be equal to Y: $\forall i \in [0, 1]$, $Y_{i,t} \equiv 1$. In the second period, each country shares the same cumulative distribution function for the probability distribution of GDP Y_2. By the law of large

[9]Most results in the paper stay true when one makes the alternative assumption that trade costs between countries i and j are equal to:

$$\tau_{i,j} = \tau \times \tau_D^{D_i + D_j}$$

where D_ι is a dummy equal to 1 if country ι defaulted, 0 otherwise.

[10]For example, if trade costs increase by 5%, one should expect a decrease in foreign demand roughly equal to $5 \times \eta\%$. However, as the country would consume more of its own good, there would be a compensating

numbers, there is no aggregate uncertainty about what happens: symmetric countries with similar levels of debt or assets should face shocks all along the distribution of shocks: countries just ignore in advance whether they will be losers or winners.

This assumption allows us to focus on risk-sharing. There is no aggregate shock in the second period: we are interested about the appropriate levels of risk-sharing and the impact of risk-sharing on incentives to default. We exclude contagion that could arise due to fundamental common shocks in the model.

Financial Markets (First Period)

Each country can choose how much debt to borrow, or how much to lend at the first period. Debt must be repaid at the next period, but it is defaultable: each indebted country can choose not to repay at the next period. Financial markets are competitive. There can be multiple equilibria at the second period regarding payments, so we assume there should be a unique equilibrium selected. This assumption combined with the absence of global uncertainty regarding the distribution of GDP implies that total share of default in the next period can be determined.

We assume that there is no inherited debt at period 1 and all debt is repaid at period 2. We will therefore omit the subscript for debt: B_i is the face value of debt owed by country i to its creditors. We assume that all debt is intermediated by a competitive insurance, so that there is perfect risk-sharing between creditors.

Therefore, at the equilibrium, there is a safe interest r and the price at which a country emits its debt is only determined by r and the probability to repay debt:

$$q(B_i) = \frac{\mathbb{P}(D_i = 0)}{1 + r},$$

where D_i is a dummy for default, equal to 1 if country i defaults, 0 otherwise.

This formula relies on the absence of aggregate uncertainty. Indeed, if there were aggregate uncertainty, countries more susceptible to default in one state or another would benefit from different valuation and price would be non-linear in default probability.[11]

Because of debt, the definition of the *numéraire* is going to be especially important in the model: indeed, it is going to determine the growth of debt.

Default Decision

At period 2, countries with large discount factor have accumulated debt and must therefore either repay or default. If they choose to default, they should be alleviated from the burden of debt but they will face a partial decrease in their trade. A defaulting country loses a share θ of its trade partners: trade costs should switch to infinity for a share of these partners. This is similar to the assumption in Mendoza and Yue (2012). The micro-foundation for this mechanism would simply be that a share of imported inputs has to be financed with cash in advance. As a consequence, this kind of imports necessitates cash in advance, which can be disrupted by a sovereign default. An alternative assumption would be that countries face punishment for defaulting, but the idea has not been substantiated by evidence of trade tariffs or non-tariff barriers imposed after a default.[12]

Tariff

In the model, as we are going to make it clear in the next period, good's markets is competitive and the representative agent within each country is going to take prices

[11] This would be true even in our model with risk-neutrality. Indeed, the share of defaulters in the world impacts price levels available to creditors, which modifies the perceived real interest rate. Without aggregate certainty and linear utility, we would need a more general formula:

$$q(B_i) = \int_{x \in X} m(x) \frac{\mathbb{P}(D_i = 0|x)}{1+r} f(x) dx$$

where m is the stochastic discount factor and f is the density of the distribution of aggregate states $x \in X$. The issue for our analysis with this more general formula would be that the stochastic discount factor also depends on borrowing and default decisions.

[12] See also Martinez and Sandleris (2011) who discuss this possibility and show that the decrease in trade after a default does not seem to depend on punishment: trade with creditors, who could have an incentive to punish the defaulter, does not decrease more than trade with non-creditors.

as given. However, government can impose an import tariff, in order to extract a rent from the monopoly power it could exert given the uniqueness of each variety. Let $\delta > 0$[13] be the tariff chosen by government. The representative agent will perceive prices of each foreign variety $j \in [0,1]$ as $\delta_j \times p_j$, where p_j is the price of variety j. Unless stated otherwise, it should be assumed that all countries choose the same value for tariff δ.

The specific choice of an import tariff instead of, say, an export subsidy, should usually not matter at the individual level of each country: this is the Lerner theorem. However, countries own assets from each country, so that there might be a temptation to manipulate its price in order to reduce debt, making import tariffs and export subsidies asymmetric. This is going to rely on the *numéraire*: if debt of a country does not depend on domestic prices, import tariff and export subsidy will be equivalent, as it has been proven in Costinot and Werning (2019). We can then consider import subsidies without loss of generality.

Budget Constraint

Each country i first sets its intertemporal trade: let T_i denote financial transfers received by country i at the current period. T_i is positive when the country emits debt in the first period or gets reimbursed in the second period. T_i is negative when the country either pays back its debt or lends money. We can separate the problem and consider the intratemporal optimal consumption decisions for a given transfer.

Each representative household takes prices as given, so that the budget constraint of the household from country j becomes:

$$p_j Y_j + T_j + G_j = \int_0^1 \delta_j \tau_{i,j} p_i \times c_{i,j} di + p_j c_{j,D},$$

[13]One should typically expect $\delta \geq 1$ from a rational government. If the government fixed its tariff unilaterally, without any possible interaction with other governments, the optimal value for the tariff is $\delta = \frac{\eta}{\eta - 1}$.

where $G_j = (\delta_j - 1)\int_0^1 \tau_{i,j} p_i \times c_{i,j} di$ is the tariff income from the import tariff. The representative agent should consider G_j exogenous in her optimization problem.

Numéraire

We are going to assume that the numéraire is given by the price of the international good from the point of view of any trading country j:

$$p_j^\star = (\int_0^1 \tau_{i,j}^{1-\eta} p_i^{1-\eta} di)^{\frac{1}{1-\eta}} = 1.$$

By applying the law of large numbers and our assumption on trade costs, we might prove that, this quantity does not depend on j, as long as country j is not a defaulter. If $\Delta \subset [0,1]$[14] is the set of countries that default, then we can write:

$$\int_{\Delta^c} \tau^{1-\eta} p_i^{1-\eta} di = 1.$$

Equilibrium

In this section, we present definitions of equilibrium. First, we need to infer the equations for prices, that cannot be solved analytically.

Once the price vector is defined for different sets of GDP, default and debt, we can define equilibrium at the second period for a given level of debt. We assume the decision to repay or to default is a simultaneous game. There are strategic interactions between countries for repayment.

Although there might be multiple equilibria in the repayment/default game, we need to select one specific equilibrium in order to be able to define an equilibrium quantity of lending. With that assumption, we present a definition for the simultaneous game of borrowing, lending and trading.

[14]We will assume all along the paper this set is measurable.

Intratemporal Trade

We assume that each country chose how much to borrow or repay, so that there are transfers $(T_i)_{i\in[0,1]}$ for each country. These transfers should compensate each other for each period and state of the world:

$$\forall t \in \{1,2\}, \forall \omega \in \Omega,\ \int_0^1 T_{i,t}(\omega)di = 0.$$

The representative agent from each country j chooses domestic consumption and consumption from each country. The agent takes all prices as given. Her intratemporal maximization problem is:

$$\max\left(\alpha^{\frac{1}{\sigma}} c_{j,D,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(\int_0^1 c_{i,j,t}^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$
$$\text{s.t.} p_j Y_j + T_j + G_j = \int_0^1 \delta_j \tau_{i,j} p_i \times c_{i,j} di + p_j c_{j,D}$$

with $G_j = \int_0^1 (\delta_j - 1)\tau_{i,j} p_i \times c_{i,j} di$, considered exogenous in the model. Using this expression for every country $j \in [0,1]$ we can compute the total demand for any variety $i \in [0,1]$.

We also assume that factor productions can temporarily move to country j at cost τ^{imm}. Countries with lower than average Y should import factors of production and vice-versa. We will consider two extreme cases: full mobility of factors $(\tau^{\text{imm}} + 1)$ and null mobility of factors $(\tau^{\text{imm}} = +\infty)$. Let $l_{i,j}$ be the total number of factors going from country i to country j - it has the following properties:

$$l_{i,j} = -l_{i,j}$$
$$\int\int l_{i,j} dj di = 0.$$

After simplifying the terms for tariff income, we have the following implicit price equation:

$$Y_j + \int \frac{l_{i,j}}{\tau^{\text{imm}}} di = \frac{\alpha p_j^{-\sigma}}{\alpha p_j^{1-\sigma} + (1-\alpha)(\int_0^1 (\delta\tau p_\iota)^{1-\eta} d\iota)^{\frac{1-\sigma}{1-\eta}} \delta^{-1}} (p_j Y_j + T_j)$$
$$+ (1-\alpha) \int \frac{\tau_{i,j}^{1-\eta} (\delta p_j)^{-\eta} \big(\int_0^1 (\tau_{i,j} \delta p_\iota)^{1-\eta} d\iota \big)^{\frac{\eta-\sigma}{1-\eta}}}{\alpha p_i^{1-\sigma} + (1-\alpha)\delta^{-\sigma} (\int_0^1 (\tau_{i,j} p_\iota)^{1-\eta} d\iota)^{\frac{1-\sigma}{1-\eta}}} (p_i Y_i + T_i) di.$$

Combining this equation with our assumption about the *numéraire* yields, for a non-defaulting country:[15]

$$Y_j + \mu_j(\frac{p_j}{p_{\min}\tau^{\text{imm}}}) = \frac{\alpha p_j^{-\sigma}}{\alpha p_j^{1-\sigma} + (1-\alpha)\delta^{-\sigma}} (p_j Y_j + T_j)$$
$$+ (1-\alpha) \int \frac{\tau^{1-\sigma} \delta^{-\sigma} p_j{}^{-\eta}}{\alpha p_i^{1-\sigma} + (1-\alpha)\delta^{-\sigma}} (p_i Y_i + T_i) di, \qquad (2.8)$$

where μ is a function that describes optimal movement of factors of production. $\mu(x)$ is 0 for any $x \in [0, 1)$, infinite for any $x > 1$ and indeterminate at 1. We can notice that elasticity of substitution η plays no direct role in this equation and should appear only through the *numéraire.*

The left-hand side is increasing in p, while the right-hand-side expression is decreasing in p, so that the solution should be unique - however, each individual value p_j implicitly solves an equation that depends on the integral. We give an algorithm to compute the equilibrium in the appendix.

2.5.2 Equilibrium and Comparative Statics

In this Section, we present the definition of *laissez-faire* or competitive equilibria, when the social planner does not intervene. We see that there is potential for contagion in the model through multiple equilibria. However, we show that for a given level of debt, in spite of contagion risks, the world gets safer when integration increases.

[15]This is the equation in the case every country repays. We give the formula that takes into account default in the appendix.

Equilibria in the Second Period

Let us assume that there is a distribution $(B_i)_{i\in[0,1]}$ of debts, and a distribution (Y_i) of GDP. B_i is positive if i accumulated debt, it is negative otherwise. Because of our assumptions, we can prove that Y_i and B_i should be independent but this should not matter for definition in this case. Let $\Delta \in \mathcal{B}[0,1]$ be the set of countries that default. Let $(p_i)_{i\in[0,1]}$ be the equilibrium set price that solves 2.8 in the case for default. For any $i \in [0,1]$, let p_i^D(respectively $p_i^R(B_i)$) be the price corresponding to the value of the price of good i if country i were defaulting (respectively repaying debt B_i) and all other things would stay the same.

Let country i be a country with debt B_i. Country i's consumption in case of repayment is:

$$V^R(Y_i,\Delta,B_i) = \frac{p_i^R Y_i - B_i}{\left(\alpha p_{2,i}^{R\,1-\sigma}(B_i) + (1-\alpha)\delta^{-\sigma}\right)^{\frac{1}{1-\sigma}}}.$$

In case of default, it is:

$$V^D(Y_i,\Delta) = \alpha^{\frac{1}{1-\sigma}} Y_i.$$

Thanks to these functions, we can define equilibrium for repayment.

Definition 2. *Let $(B_i)_{i\in[0,1]}$ be a set of inherited debt from the previous period, such that $\int_0^1 B_i di = 0$. An equilibrium of default is given by a default set $\Delta \in \mathcal{B}[0,1]$ and a price vector $(p_i)_{i\in[0,1]}$ such that:*

- *$(p_i)_{i\in[0,1]}$ solves the price equations with default set Δ.*
- *$i \in \Delta \iff V^D(Y_i,\Delta) > V^D(Y_i,\Delta,B_i)$.*

Given the inherent symmetry of our model, we can conclude that default equilibria are determined by their measure $\lambda(\Delta)$, where λ is the Lebesgue measure. We should have:

$$0 \le \lambda(\Delta) \le \frac{1}{2},$$

because the share of debtors is exactly $\frac{1}{2}$.

Equilibrium in the First Period

To define an equilibrium in first period, we need to assume that there is a way to assert the probability to default in the second for given levels of debt.

Before going further in our discussion, we need to make it clear how equilibrium is going to be selected in competitive equilibria. There can be multiple default equilibria in the second period for the same level of debt B. As a consequence, when we model the choice of the borrowed amount in the first period, we need to be specific about which equilibrium is going to be selected. We choose the minimal default, equilibrium, which is also the best equilibrium in terms of welfare. This choice seems justified because the minimal default equilibrium is the more robust equilibrium: it is the only subgame perfect equilibrium when we assume decisions are sequential in the game.[16]

ASSUMPTION: in the second period, the minimal default equilibrium, which is the best equilibrium in terms of welfare, is always selected.

This assumption also makes it easier to compare competitive equilibria and what a social planner would do: indeed, a social planner would always choose the better equilibrium, while it is harder to make sure it happens in a competitive equilibrium. This is the role classically assigned to certain institutions, such as the lender of last resort in diamond1983bank.

This equilibrium is the minimal default equilibrium, that is the equilibrium for which the quantity of defaulters $\lambda(\Delta)$ is minimized. We prove in the appendix that this equilibrium is well defined.

A borrowing country i wants to maximize its utility, given by the sum of expected aggregate consumption at each period. Since everything is symmetric in the model, we can assume that all other countries borrow B_{-i}, which implies a certain default probability: $\lambda(\Delta)$ is fixed in the model.

[16] This result is true about models such as Diamond and Dybvig (1983): each "ambiguous" decision-maker anticipates the next ones will choose the better equilibrium if she does not default herself. In order to make the game sequential, we need to assume that the set of decision makers is at most countable (which is not the case of $[0, 1]$), which can be done by segmenting the distribution of GDP and going to the limit in our model.

Let $P_1(i) = \left(\alpha p_{1,i}^{D\,1-\sigma} + (1-\alpha)\delta^{-\sigma}\right)^{\frac{1}{1-\sigma}}$ be the aggregate price level. It is going to depend on the level of aggregate debt B_{-i}.[17] Moreover, for a given level of aggregate debt B_{-i}, borrowing countries are going to face the price schedule $q(., B_{-i})$ and solve:

$$\max_{C_1,C_2,B} C_1 + \beta C_2.$$
$$\text{s.t. } P_1(q(B, B_{-i})B_i)C_1 = p_{1,i}(B_{-i})Y_1 + q(B_i, B_{-i})B_i$$
$$C_2 = \mathbb{E}\max(V^D(Y_i,\Delta), V^R(Y_i,\Delta,B_i)|B_{-i})$$

where $P_1(B_{-i})$ is the total price index in period 1, that depends on B_{-i}, $p_{1,i}(B_{-i})$ is the price of the domestic variety produced by country i, $q(B_i, B_{-i})$ the price schedule for bonds.

One can notice that the price of domestic variety i $p_{1,i}(B_{-i})$ should depend on debt B_i and not only on B_{-i}. Indeed, own debt plays a role in the price equation. Indeed, more debt today increases sizes of transfers today. It implies more demand for the domestic good, and leads to an increase in its price as a consequence. However, we do not include it here because we assume it is not internalized by the country in its decision to borrow. Government could issue a tariff to improve terms of trade, and it could also issue a tariff to internalize the terms of trade effect of debt.

The problem of lenders is going to be similar. Lenders are going to intermediate their lending, so that each lending country would have the same exposure to international default risks. They will be willing to lend L at safe interest rate r, which they take as given:

$$\max_{C_1,C_2,B} C_1 + C_2.$$
$$\text{s.t.} P_1(-L_{-i})C_1 = p_{1,i}(L_{-i})Y_1 - \frac{1}{1+r}L_i$$
$$C_2 = \mathbb{E}(V^R(Y_i,\Delta,-L_i)|L_{-i})$$

[17] We omit to write P as a function of aggregate debt B_{-i} and of private debt B_i because the lender should not take it into account.

Return of sovereign lending is certain because there is no aggregate uncertainty in this model. At the equilibrium, in a symmetric equilibrium, we should have:

$$\forall i \in [0,1],\ (1-2\lambda(\Delta))B_i = L_i = (1-2\lambda(\Delta))B = L.$$

Debt and assets are the same for all countries and only depend on whether they are debtors or creditors. The quantity of face value debt that is going to be paid back is the portion that is not defaulted. This portion is equal to $1-2\lambda(\Delta)$. Indeed, the share of countries that repudiate their debt is equal to the size of default set $\lambda(\Delta)$ divided by the number of debtors, 1/2.

To close the model, we need to see what happens in financial markets. We assume they are perfectly competitive, so that price should correspond to default risk. We can therefore solve these problems to define equilibrium.

In the following definition, we use the dummy $\mathbb{L}_i$, equal to 1 if country i is a lender, to 0 otherwise.

Definition 3. *A first period equilibrium of the model is a safe rate r, a debt level B, a default risk λ^D and price functions for first and second period $p_1(\mathbb{L}_i)$ and $p_2(\mathbb{L}_i, Y, B)$ such that:*

- For any lending country $i \in [0,1]$, $L = (1-\lambda^D)B$ solves the consumer's problem, which implies:

$$P_{1,i}(-L) = \frac{1}{1+r}\mathbb{E}(P_{2,i}(L)),$$

where $P_{1,i}(-L) = P_{1,i}(p_1(\mathbb{L}_i))$ is the aggregate price function for a given level of price.

- For any borrowing country $i \in [0,1]$, B solves its consumer's problem.

- At period 2, (random) a default set Δ with measure size $\lambda(\Delta) = p\lambda^D$, and Δ is a default equilibrium.

Because we are going to derive results about the probability of default, it is useful to introduce some further notations. For any aggregate level of default risk, $\Lambda \in [0,1]$, we know the amount of borrowing decided by a lonely given country should not impact the rest of the world. As a consequence, aggregate global risk is associated with a global level of debt. Any country takes it as given, and we can write $B(\lambda, \Lambda)$ the amount of debt that is such that, given the risks taken by other countries, a small economy with debt $B(\lambda, \Lambda)$ has a probability λ to default. With such a notation, we can in fact define an equilibrium uniquely as a function of λ and Λ.

Social Planner's Objective and Macroprudential Policy

In this section, we present the difference between *laissez-faire* equilibrium and social planner's first best. We assume that the import tariff rate is the same for all countries and focus on macroprudential aspects.

We remind that the objective of each borrowing country is to maximize the following objective function, while taking Λ as given:

$$\frac{pY + q(\lambda, \Lambda)B(\lambda, \Lambda)}{(\alpha p^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} + \beta \int_0^\lambda \alpha^{\frac{1}{\sigma-1}} y(l) dl + \beta \int_\lambda^1 \frac{p_{y(l)} y(l) - B(\lambda, \Lambda)}{(\alpha p_{y(l)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl,$$

and at the equilibrium $\lambda = \Lambda$.

Each lending country takes Λ as given and chooses $T = (1-\Lambda)B$ so as to maximize:

$$\frac{pY - \frac{1}{1+r}T}{(\alpha p^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} + \int_0^1 \frac{p_{y(l),\Lambda} y(l) + T}{(\alpha p_{y(l),\Lambda}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl.$$

This implies the following interest rate:

$$\frac{1}{1 + r(\Lambda)} = \int_0^1 \frac{(\alpha p^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}{(\alpha p_{y(l),\Lambda}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}. \tag{2.9}$$

Equation (2.9) simply means that the competitive safe interest rate is going to be determined by the relative intertemporal price of goods in periods 1 and 2. If prices in the first period are very low, then more income in the first period implies more enhanced consumption, hence more reluctance to lend money and a higher interest rate.

Before being able to characterize optimal equilibria that can be reached by the social planner, we need to define which policies are possible for the social planner.

We are going to restrict our attention to macroprudential policies and restrict the set of feasible policies to one tool: tax on debt, and compare it to a second best equilibrium where the planner can set the amount of debt. We assume the planner cannot redistribute income, and that it cannot prevent countries from defaulting:[18] default is a sovereign decision.

We are going to assume that the planner is a price taker in goods' markets. It means that the planner will not internalize the effect of its decisions on the price of goods. It means that the planner should not try to manipulate terms of trade or interest rate in favor of one of the agents, except when there is pecuniary externality caused by default. The reason why we are making this assumption is to avoid artificial results about terms of trade. Costinot et al. (2014) show that capital controls and interest rate's manipulation only matter to the extent that imbalances change from one period to the other. In our model, they change from the first to the second period, but this is partly an artificial outcome of the two-period structure. Would the model be stationary, the motives for regulation would change. Moreover, avoiding this assumption keeps the model tractable.[19]

We give a characterization of *laissez-faire* equilibria with macroprudential tax on debt t^b in the appendix. The *laissez-faire* equilibrium maximizes the welfare of borrowers under the resource constraints and the assumption that global default rate is exogenous - and in the absence of direct redistribution. Indeed, lenders enjoy no benefit from intratemporal trade at the individual level. Indeed, they earn no gains from intertemporal trade: the linear nature of the model and the price-setting means

[18] We discuss alternative tariff policies below

[19] Another possibility to keep the results of the model tractable was to assume that $\tau^{\text{imm}} = 1$ which would be another counterintuitive assumption - with this assumption, only tariffs could affect the price of goods. In a companion paper, we propose an analysis of a single country's problem when $\tau^{\text{imm}} = +\infty$.

borrowers earns all the gains from finance.

Thus, lenders would always be better off in the absence of intertemporal trade. This result stems from the assumption we made about borrowing, which is that borrowers internalize the effect of borrowing on their default risks while creditors do not - due to intermediaries and perfect competition on the market. This assumption is quite standard in sovereign debt models *à la* Eaton and Gersovitz (1981): sovereign borrowers always exert a monopoly power over their lenders, who only lend at a competitive interest rate. More generally, lenders are similar to black boxes in sovereign debt models, more interested in borrowers.[20]

We assume the global social planner could implement a tax on borrowing: more precisely, every time a country borrows an amount $T = q \times B$, it has to pay a tax $t^b T$ and receives $(1 - t^b)T$. The European Union has an Excessive Imbalance Procedure (EIP), part of the Macroeconomic Imbalance Procedure that was put in place in 2011. This program could apply a fine of 0.2% of GDP to a country running deficits above its commitments, after an evaluation of the situation of the country. Although it has never been applied, it is the real-world policy closest to our assumption: countries have to pay a fine when they borrow excessive amounts.[21] To a certain extent, the Stability and Growth Pact Rules can be thought of as a political version of the tax we propose in this model: countries that borrow larger quantities of debt have more political and economic constraints than the other ones.

The following result establishes that such a macroprudential tax on debt can improve welfare, whatever the weights on welfare between creditors and debtors are:

Theorem 1. *Let γ be a welfare weight on creditors. Then there is a function $\xi(\Lambda, \gamma, \tau)$*

[20] Aguiar et al. (2016) uses models with time-varying interest rates based on wealth shocks occurring to lenders. However, in these models, this is only a micro-foundation to explain comovement of spreads and risk premia. There is no welfare analysis including the welfare of the lenders.

[21] Formally, it would be written $t^b(qB) = \mathbb{I}\{qB \geq d\}t$ for a threshold d. The threshold d could be 3% imposed by Maastricht or 0.5% for countries with high level of debt, but the European commission has always politically careful. The real value of d is unknown and depends on circumstances.

such that the social planner solves:

$$\max \frac{1-\Lambda}{1+r}\frac{B(\Lambda,\Lambda)}{P_1} + \beta \int_0^1 \max\{\frac{p_2 Y(x) - B(\Lambda,\Lambda)}{P_{2,x}}, \alpha^{\frac{1}{\sigma-1}} Y(x)\} dx - \xi(\Lambda,\gamma,\tau),$$

and ξ represents the deadweight loss on gains from trade created in the world by trade disruptions caused by default, and ξ is increasing in Λ and decreasing in τ. Moreover, there is a tax on debt $t^b > 0$ that reaches this second best equilibrium and t^b solves at the optimum $\Lambda^\star$:

$$\begin{aligned}\frac{t_b}{1+r(\Lambda)}\Big((1-\Lambda^\star)B_1(\Lambda^\star,\Lambda^\star) - B(\lambda,\Lambda)\Big) = & \frac{-\frac{1-\Lambda}{1+r}B_2(\Lambda^\star,\Lambda^\star)}{P_b} \\ & - \beta \int_\Lambda^1 \frac{B_2(\Lambda^\star,\Lambda^\star)}{P_l} dl \\ & + \xi_\Lambda(\Lambda,\gamma,\tau),\end{aligned}$$

and $B_2(\Lambda^\star,\Lambda^\star) = \lim_{\epsilon\to+\infty} \frac{B(\Lambda^\star,\Lambda^\star+\epsilon) - B(\Lambda^\star,\Lambda^\star)}{\epsilon} < 0.$

This proposition gives the same decomposition of the externality as in the simple model from Section 4. There is a direct pecuniary externality, occurring because excess default destroys some value through its negative effect on gains from trade, and this effect is captured by function ξ that we define in the appendix. There is also a pecuniary externality on the price of debt, occurring because contagion of default risk is anticipated in the first period but not internalized in the second period. The deadweight loss associated with trade should be larger as the world is more open, which also means that gains from an efficient macroprudential tax should be higher.

2.5.3 Trade Costs and Default Waves

In this Section, we look at what happens at equilibrium when trade costs decrease. We want to show that total default decreases at the second period, whichever equilibrium we consider. One can wonder whether multiple equilibria mean that high interdependence is more likely to lead to a crisis, which is what is sometimes hinted at with comments about contagion. We show that, in our model, more trade (under the form of lower trade costs) means not only safer debt under normal times, but also safer debt even in the presence of coordination problems. Indeed, we show that even the worse equilibrium of the default game becomes safer when countries become more interdependent.

At the second period, any creditor i hesitates between defaulting and repaying debt. In order to be able to present clearer results about trade costs, let us write $V^R(B, \tau, \Delta)$ the utility in case of repayment and $V^D(\tau, \Delta)$ the utility in case of default, where τ represents trade costs, B the level of debt, and Δ the default set. Note that, because of the definition of the *numéraire*, the real value of debt B should depend on trade costs, but results are robust to different specifications. We can see that the game is a supermodular global, game from which we get the following result:[22]

Proposition 7. *The Default game is a supermodular game, i.e. for any fixed τ and B $V^D(\tau, \Delta) - V^R(B, \tau, \Delta)$ increases when Δ increases (in the sense of the inclusion partial order). Moreover, $V^D(\tau, \Delta) - V^R(B, \tau, \Delta)$ is increasing in τ. As a consequence, for any value of B and τ, there are two extremal equilibria default sets $\Delta_{\min} \subseteq \Delta_{\max}$ such that:*

- $\Delta_{\min}$ and $\Delta_{\max}$ are Nash equilibria of the default game.

- Any Nash equilibrium Δ is such that:

$$\Delta_{\min} \subseteq \Delta \subseteq \Delta_{\max}.$$

- The game has a dominant equilibrium if and only if $\Delta_{\min} = \Delta_{\max}$ (up to a zero-measure set).

Extremal equilibria $\Delta_{\min}$ and $\Delta_{\max}$ are increasing (in the sense of inclusion partial order) in trade costs τ and debt B. In other words, the total number of defaulters

[22]Supermodular games were introduced by Topkis (1979) and the results we are going to use were established by Milgrom and Roberts (1990).

should decrease as countries get more open due to lower transport costs or as debt decreases.

We can define extremal equilibria by iteration the following sequence $(\Delta_n)_{n\in\mathbb{N}}$:

- $\Delta_0 = \emptyset$

- $\forall n \in \mathbb{N}$, $\Delta_{n+1} = \{i \in [0,1),\, b_i > b_i^D(\Delta_n)\}$

Then the minimal equilibrium set is $\Delta_{\min} := \cup_{n\in\mathbb{N}}\Delta_n = \lim_{n\to+\infty} \uparrow \Delta_n$. Intuitively, Δ_1 is the set of "pure" defaulters or the first wave: they would have defaulted even if all countries in the world had not. The set $\Delta_{n+1}\backslash\Delta_n$ is the $n+1$-th default wave: it is the set of countries that defaulted because of previous defaults. The maximal default equilibrium is defined except the way by substituting $\Delta_0 = [0,1] \cap L$ where L is the set of all lenders with positive debt. The set $\Delta_{\max}\backslash\Delta_{\min}$ is the set of *ambiguous* countries.

In our game, lower trade costs entail larger default costs and lower debt implies lower benefits from default. As a logical consequence, default should be less likely for any individual country as trade costs decrease. The last part of the proposition proves that this is also true at the global level when we consider the Nash equilibrium. In other words, more trade makes the world a safer place in terms of sovereign finance.

2.5.4 Optimal Trade Policy

In the previous Section, we have shown that an increase in commercial interdependence between countries led to a safer environment for a given level of borrowing, and that it allowed more gains from intertemporal trade. However, intertemporal trade implies negative externalities as it can compromise intratemporal trade when a country experiences default. In this Section, we study what would happen if the world were regulated by a global planner wanting to maximize a weighted welfare of all countries.

There are externalities in this model that make the *laissez-faire* equilibrium inefficient. One of the externalities is that default disrupts trade and therefore reduces the set of inputs available for all other countries. It is therefore natural to consider trade policies as alternative.

Second Period We assume that each country is free to set its own import tariff. There is an optimal tariff that is determined by the elasticity of substitution between the imported the domestic good σ: $\delta^\star = \frac{\sigma}{\sigma-1}$. The following result is straightforward:

Proposition 8. *For every country $i \in [0,1]$, any unilateral deviation further away from the optimal tariff increases the probability of default in the second period for a given level of debt. More precisely, if $\delta \leq \delta^\star$ and δ decreases, or if $\delta \geq \delta^\star$ and δ increases, then probability of default increases. Assume that each country is free to set its tariff unilaterally. Then a free-trade agreement forcing all countries to free-trade with $\forall j \in [0,1]$, $\delta_j = 1$ would be a Pareto-improvement over trade wars with positive tariff $\delta_j = 1 + \frac{1}{\sigma}$.*

Let $\delta_0 > 1$ If all countries commit from the first period to free trade $\delta_1 = 1$, then the probability of default should decrease.

The first part of this result simply stems from the fact that gains from trade decrease as a country does not apply optimal tariff. The second part simply translates the fact that, although unilateral deviation from free trade is optimal, retaliations and tariff wars decrease gains from trade overall, which has consequences in the model in the ability to do financial transfers.

The results about trade tariffs are more ambiguous than in the previous section, because tariffs can benefit a country, unlike trade costs. A lower level of trade openness is associated with larger gains from trade resulting from terms-of-trade effects as long as the tariff stays unilateral and does not trigger any trade retaliations.

A more paradoxical result can be obtained if one is ready to extend options available to the planner at the moment when debtors have to pay back. Let us assume that there is a free trade agreement such that $\delta = 1$ for every country in the world, and that there is a credible mechanism through which any unilateral increase in tariff in country j would create a reciprocal tariff from all the trade partners of the country. If a country does not have incentives to repay debt, it might make sense for the social planner to allow this country to unilaterally increase its tariff. Default is a policy where a country chooses to be excluded from the rest of the world, and it is better for the rest of the world if the defaulting country increases its tariff instead of defaulting. We describe that policy in the appendix.

First Period In the first period, a free-trade agreement would still be an optimal policy relative to trade war. However, we can see that a generous default clause as the one we explored in the previous proposition can give incentives to countries to borrow debt in excess, because they internalize they will benefit from help. This feature can be corrected with an adaptive tariff that would make any country indifferent between default and repayment. Let Δ be the minimal equilibrium default set without adaptive tariff, and define $\delta^\Delta : \Delta \to [1, \frac{\sigma}{\sigma-1}]$ the following way for any $j \in \Delta$:

- If there is a δ such that $V_j^R = V_j^D$, then $\delta^\Delta(j) = \delta$
- Otherwise, $\delta^\Delta = \frac{\sigma+1}{\sigma}$

Then, define the new default set $\Delta' = \{j \in \Delta,\, V_j^R(\delta^\Delta(j)) < V^D\}$ and iterate the definition. This algorithm defines a function: $\mathcal{B}[0,1] \to \mathcal{B}[0,1]$ that admits a fixed point.

Such a policy improves the individual possibilities of each country but reinforces incentives to borrow. Indeed, the force that limits borrower's tendency to borrow is the repayment of debt in good states, not the welfare in default states. Indeed, assume a country sets its individual default risk λ, and the social planner makes the country indifferent between debt and default with a probability $\zeta(\lambda) \geq \lambda$:

$$\begin{aligned}(1-\lambda)B_\lambda(\lambda,\Lambda) - B &= \beta \int_{\zeta(\lambda)}^1 B_\lambda(\lambda,\Lambda)dx + (\zeta'(\lambda)-1)\alpha^{\frac{1}{1-\sigma}}Y(\zeta(\lambda)) \\ &\leq \beta \int_\lambda^1 B_\lambda(\lambda,\Lambda)dx,\end{aligned}$$

so that this policy can incidentally subsidize risk, which is already excessive in the model.

Depending on the way the social planner redistributes income that is reimbursed by the defaulters, this trade policy could play a role similar to a debt subsidy, and we prove in the next section that tax on debt is a welfare-improvement.

Let us assume that the planner redistributes the total income it receives from this policy as a lump-sum transfers, in order to avoid subsidizing excess borrowing: that way, lenders would internalize the impact of their lending on the global economy and the action of the sovereign lender. Such a policy would not distort prices relative to the *laissez-faire* equilibrium. Such a designed contract would ensure minimal losses from default. Allowing countries about to default to use a tariff in order to boost their terms of trade and gains from trade encourages them not to default. However, this kind of policy creates two kinds of moral hazard from the point of view of borrowers. The first moral hazard is *ex post:* welfare can increase in case of default. This moral hazard is suppressed by making sure that the country stays indifferent between default and repayment at the second period.[23] *Ex ante*, there is moral hazard created by the fact that the sovereign benefits from a lower price for the same level of price, although the social planner would prefer to limit its interventions and to tax debt. A solution is to force creditors of countries that ended up benefiting from the tariff policy not to get paid back so that the use of the tariff policy does not increase the price of debt in the first period.

2.6 Conclusion

We have seen empirically that international trade can create contagion of the risk of sovereign defaults and that trade is associated with the comovements of sovereign spreads.

[23]This kind of strategy is often used in restructuring models.

Although trade seems associated with comovement and contagion, we have seen that trade makes the world safer. Because trade makes it easier for open countries to commit to repay their debt, the intertemporal benefits of trade outweigh the contagion risks it creates. In a multi-country model, more trade makes global sovereign debt safer, not riskier, not only at the individual country level (as it was proved in Serfaty (2020)), but also at the collective level. However, contagion creates an excess of sovereign debt from the point of view of borrowers.

While trade improves welfare, trade also creates externalities that are likely to increase in the total level of openness. With trade, *laissez-faire* stops being an optimal macroprudential policy from the worldwide point of view, and a Pigouvian taxation of sovereign debt becomes more and more necessary. Moreover, tariff policies play an important role. An individual increase in tariff increases a country's gains from trade as long as there is no retaliation. Generally, a free-trade agreement should improve welfare, even if we ignore gains from intratemporal trade. However, allowing a country to increase its tariff can be a more efficient policy than letting this country default if there is a consistent control of the moral hazard this kind of rule allows.

These two results can seem paradoxical. Trade makes global sovereign debt safer, because sovereign debt uses commercial interdependence as a commitment device. It might be that, in a world with non-fundamental volatility or rational exuberance, this structure creates a risk of an irrational default (which is visible through multiple equilibria in the model). However, when we look at our comparative static results, greater interdependence mostly means greater safety. However, greater interdependence also means that countries rely more on what other countries do on sovereign markets: hence, countries weigh more on each other and this greater safety of sovereign debt market goes in pair with a greater need for financial regulation of sovereign debt. International trade openness improves welfare in the *laissez-faire* equilibrium, but it improves welfare in the global planner's equilibrium even more.

From another point of view, we have shown the importance of trade in sovereign debt crises and incentives to repay makes regional crises and amplification likely. A new kind of multiplicity of equilibria emerged, implying more interdependence in the decisions of different countries. Also, the size of global trade and its exposure to global financial shocks might be a very important issue from a global macroeconomic perspective, explaining the existence of the global financial cycle, because some parts of finance might in fact rely on trade.

We saw that sovereign defaults tended to become scarcer. It might be in fact because the same general openness that had made large financial flows possible also made commitment to repay sovereign debt much stronger. In a time when concerns about national independence and trade wars become more and more present, and after a substantial increase in the sovereign debt of most countries in 2020, our paper raises concerns that if a deglobalization happened, it could create an important sovereign debt crisis: real contagion might be when trade disappears.

Appendix

Appendix 2.A Simple Model

2.A.1 Simplified Model is a Special Case of the More General Model

Assume that σ is infinite, $\eta = 2$, and there is for every worker the possibility to immigrate and work in a foreign country at no cost: τ^{imm}. As a consequence, all prices in non-defaulting countries should be equal. Because σ is infinite, each country will either consume only the domestic good or the aggregate international good.

We also need to make the parametric assumption that:

$$\frac{(1-\alpha)^{\frac{1}{\sigma-1}}}{\alpha^{\frac{1}{\sigma-1}}} := 1 + g > 1,$$

to ensure that it is more profitable to consume the aggregate international goods when no other country defaults.

The price of every variety is constant as long as the country is integrated, so that we can assume that $p \equiv 1$.

This assumption helps us ensure that And welfare at second period for borrower can be written:

$$V^R(B, Y, P) = \alpha^{\frac{1}{\sigma-1}}(1+g)\frac{pY - B}{P(\Lambda)}.$$

Because $\Lambda = 2$, one can easily compute that:

$$P(\Lambda) = (1-\Lambda)^{-1}p = (1-\Lambda)^{-1},$$

and the value of default is given by:

$$V^D(Y) = \alpha^{\frac{1}{\sigma-1}}Y.$$

After a normalization, we have the same game as in Section 3.

2.A.2 Proof of Proposition on Trade Costs and Default

We remind the lemma and the proposition we want to prove:

Proposition. *Optimal default equilibrium and optimal tax on debt do not depend on total gains from trade. As a consequence, welfare gains from the optimal policy increase in the level of openness g.*

Proof. To prove our claim, we consider the previous results: whatever the level of taxes is, the *laissez-faire* equilibrium $\Lambda(t_b)$ does not depend on g but only on the discount factor β, on the real interest rate ρ and on the tax t^b, and it is decreasing in t^b. As a consequence, whatever the value of g is, the value of t_b necessary to make a given Λ an equilibrium value should be decreasing in Λ. As a consequence, to prove our claim, it is enough to prove that optimal equilibrium default probability $\Lambda^\star$ does not depend g under the conditions of the theorem. The first order condition is:

$$\begin{aligned}
&\frac{1+g(1-p\Lambda)}{1+\rho}[(1-\Lambda)\frac{dB(\Lambda,\Lambda)}{d\Lambda} - F^{-1}(\Lambda)\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}] \\
&\qquad - \frac{gp}{1+\rho}[(1-\Lambda)F^{-1}(\Lambda)\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}] \\
&-\beta\frac{dB(\Lambda,\Lambda)}{d\Lambda}\int_\Lambda^1(1+g(1-p\Lambda))dm - gp\beta\int_\Lambda^1(Y(l)-B(\Lambda,\Lambda))d\Lambda = 0.
\end{aligned}$$

where:

$$\frac{dB(\Lambda,\Lambda)}{d\Lambda} = \frac{g(1-p\Lambda)}{f(Y(\Lambda))}\frac{1}{1+g(1-p\Lambda)} - \frac{F^{-1}(\Lambda)gp}{(1+g(1-p\Lambda))^2}$$

Let us develop this equation in several parts to make it more reader-friendly. The first two lines can also be written:

$$\begin{aligned} FL(\Lambda,g) :&= \frac{g}{(1+\rho)}\big[(1-\Lambda)\frac{(1-p\Lambda)}{f(Y(\Lambda))} - (1-\Lambda)\frac{Y(\Lambda)p}{1+g(1-p\Lambda)} - F^{-1}(\Lambda)(1-p\Lambda)\big] \\ &- \frac{gp}{1+\rho}[(1-\Lambda)Y(\Lambda)\frac{g(1-p\Lambda)}{1+g(1-p\Lambda)}]. \end{aligned}$$

The third line writes:

$$TL(\Lambda,g) := \beta\Big(\frac{g(1-p\Lambda)}{f(Y(\Lambda))}(1-\Lambda) - \frac{Y(\Lambda)gp}{1+g(1-p\Lambda)}\Big)(1-\Lambda) + \beta gp\int_\Lambda^1 (Y(l) - \frac{Y(\Lambda)g(1-p\Lambda)}{1+g(1-p\Lambda)})dl.$$

Because the first order condition would hold at the point of interest and $g \neq 0$, we can divide by g before computing the derivative of $LHS(\Lambda,g) - RHS(\Lambda,g)$ with respect to g for comparative statics. Then we have:

$$\begin{aligned} \frac{\partial}{\partial g}(LHS - RHS) \propto &\Big((\frac{1}{1+\rho} - \beta)\frac{1-p\Lambda}{(1+g(1-p\Lambda))^2} \\ &- \frac{1}{1+\rho}\frac{1-p\Lambda}{(1+g(1-p\Lambda))^2} \\ &+ \beta\frac{1-p\Lambda}{(1+g(1-p\Lambda))^2}\Big) \\ &= 0, \end{aligned}$$

so that the optimal default rate should not depend on Λ.

The second part of the proposition can be derived in a straightforward manner. The welfare gains from the optimal policy are given by the difference between the *laissez-faire* equilibrium with Λ^c without taxes and the social planner's optimum $\Lambda^\star$. The values of Λ^c and $\Lambda^\star$ in each case do not depend on g. A simple computation shows that the gap between the two values increases in g: the size of the externality in terms of welfare becomes larger as g increases. □

Appendix 2.B Optimal Trade Policy

Proposition. *Assume that the cumulative distribution function of GDP F is continuous and that default risk is not equal to zero. Let us assume that there is a free-trade agreement such that $\forall j \in [0,1]$, $\delta_j = 1$. Let Δ_1 be the default set at the second period. Then a policy allowing all countries $j \in \Delta_1$ to unilaterally set any level of tariff $\delta_j \in (1, \frac{\sigma-1}{\sigma}]$ would entail a Pareto-improvement of default equilibrium: such a policy would reduce the size of Δ_2 and improve the welfare of all countries in the world.*

This result simply stems from the fact that gains from trade should increase in the number of countries that are not defaulting, as well as in unilateral tariff. A country close to indifference between default and repayment could in fact choose to repay if it were given the possibility to adopt an optimal tariff. As a consequence, it would repay its debt which would make its creditors better off. Moreover, the increase in tariff would hurt the other countries relative to a case where countries would choose to repay without tariff, but the alternative option in this case is default, which in our assumption has worse consequences on trade than tariff.

This proposition relies heavily on the assumption that default is worse for trade than optimal unilateral tariffs. Our precise assumption is that trade costs go to infinity after a default. If we instead assumed that trade costs get multiplied by a constant $\tau^D > 1$, the result would hold as long as $\tau^D \geq 1 + \frac{1}{\sigma}$.

We also have the following result:

Theorem. *For any level of debt B, let $\Delta^c(B)$ be the minimal default equilibrium associated with debt level B in the second period. Then, consider the policy that allows all countries $j \in \Delta^c(B)$ to raise their tariff up to the level where they are indifferent between default and repayment in the second period while redistributing the* *debt of these countries as a lump-sum transfer by taxing creditors. This amiable tariff policy leads to a Pareto-improvement relative to a standard free-trade agreement.*

This result extends the previous one to the problem of borrowing.

Appendix 2.C Characterization of *Laissez-Faire* Equilibrium

We give the characterization of the *laissez-faire* equilibrium that we use in Section 5.3.2:

Lemma. *Every* laissez-faire *equilibrium with tax t^b solves the following maximization problem:*

$$\max_{\lambda\geq 0}\frac{pY+(1-t^b)q(\lambda,\Lambda)B(\lambda,\Lambda)+G}{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\beta\int_0^{\lambda}\alpha^{\frac{1}{\sigma-1}}y(l)dl+\beta\int_{\lambda}^{1}\frac{p_{y(l)}y(l)-B(\lambda,\Lambda)}{(\alpha p_{y(l)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl.$$

$$s.t.\frac{1}{1+r(\Lambda)}=\int_0^1\frac{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}+G}{(\alpha p_{y(l),\Lambda}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}$$

$$s.t.Y_j=\frac{\alpha p_j^{-\sigma}}{\alpha p_j^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_jY_j+T_j)+(1-\alpha)\int_{\Delta^c}\frac{\tau^{1-\eta}\delta^{-\sigma}p_j{}^{-\eta}}{\alpha p_i^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_iY_i+T_i)di$$

and at the equilibrium $\lambda=\Lambda$ and $G=t^bq(\Lambda,\Lambda)B(\Lambda,\Lambda)$. For any Pareto-optimal equilibrium, there is $\gamma\in[0,+\infty]$ such that default risk Λ solves:

$$\max_{\lambda\geq 0}\frac{p_bY+q(\Lambda,\Lambda)B(\Lambda,\Lambda)}{(\alpha p_b^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\beta\int_0^{\Lambda}\alpha^{\frac{1}{\sigma-1}}y(l)dl+\beta\int_{\Lambda}^{1}\frac{p_{y(l)}y(l)-B(\Lambda,\Lambda)}{(\alpha p_{y(l)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl$$
$$+\gamma\frac{p_lY-q(\Lambda,\Lambda)B(\Lambda,\Lambda)}{(\alpha p_l^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\gamma\int\frac{p_{y(l)}y(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dx.$$

$$s.t.\frac{1}{1+r(\Lambda)}=\int_0^1\frac{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}{(\alpha p_{y(l),\Lambda}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}$$

$$s.t.Y_j=\frac{\alpha p_j^{-\sigma}}{\alpha p_j^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_jY_j+T_j+M(\mathbb{L}_j))+(1-\alpha)\int_{\Delta^c}\frac{\tau^{1-\eta}\delta^{-\sigma}p_j{}^{-\eta}}{\alpha p_i^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_iY_i+T_i+M(\mathbb{L}_j$$

Appendix 2.D The Planner's Problem

We will present the problem with $p = 1/2$ but all the reasoning stays the same independently on p.

We know the first order condition of the Planner's Problem can be written this way:

$$\max_{\lambda\geq 0}\frac{p_bY+q(\Lambda,\Lambda)B(\Lambda,\Lambda)}{(\alpha p_b^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\beta\int_0^{\Lambda}\alpha^{\frac{1}{\sigma-1}}y(l)dl+\beta\int_{\Lambda}^{1}\frac{p_{y(l)}y(l)-B(\Lambda,\Lambda)}{(\alpha p_{y(l)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl$$
$$+\gamma\frac{p_lY-q(\Lambda,\Lambda)B(\Lambda,\Lambda)}{(\alpha p_l^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\gamma\int\frac{p_{y(l)}y(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dx.$$
$$\text{s.t.}\frac{1}{1+r(\Lambda)}=\int_0^1\frac{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}{(\alpha p_{y(l),\Lambda}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}$$
$$\text{s.t.}Y_j=\frac{\alpha p_j^{-\sigma}}{\alpha p_j^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_jY_j+T_j+M(\mathbb{L}_j))+(1-\alpha)\int_{\Delta^c}\frac{\tau^{1-\eta}\delta^{-\sigma}p_j{}^{-\eta}}{\alpha p_i^{1-\sigma}+(1-\alpha)\delta^{-\sigma}}(p_iY_i+T_i+M(\mathbb{L}_j$$

We can rewrite it in a way that synthesizes how functions depend on parameters:

$$\max_{\Lambda\geq 0,M}\frac{p_b(M(0)+(1-\Lambda)B(\Lambda))Y+\frac{1-\Lambda}{1+r(\Lambda,M)}B(\Lambda)+M(0)+G}{(\alpha p_b^{1-\sigma}(M(0)+(1-\Lambda)B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\beta\int_0^{\Lambda}\alpha^{\frac{1}{\sigma-1}}y(l)dl$$
$$+\beta\int_{\Lambda}^{1}\frac{p(y(l),-B(\Lambda))y(l)-B(\Lambda,\Lambda)}{(\alpha p^{1-\sigma}((y(l),-B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl$$
$$+\gamma\frac{p_lY(M(1)-\frac{1-\Lambda}{1+r(\Lambda,M)}B(\Lambda))-\frac{1-\Lambda}{1+r(\Lambda,M)}B(\Lambda,\Lambda)+M(1)+G}{(\alpha p_l^{1-\sigma}(y(l),-B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}$$
$$+\gamma\int\frac{p(y(l),(1-\Lambda)B(\Lambda)))y(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dx.$$
$$\text{s.t.}M(1)=-M(0)$$

As long as:

$$1+\frac{\partial r}{\partial M(1)}\frac{1-\Lambda}{(1+r(\Lambda,M))^2}B(\Lambda)\neq 0,$$

which is true as $\frac{\partial r}{\partial M(1)} < 0$, we can show that the first order condition on M implies we can focus on the second period for the optimal policy:

$$\max_{\Lambda} \beta \int_0^{\Lambda} \alpha^{\frac{1}{\sigma-1}} y(l) dl + \beta \int_{\Lambda}^{1} \frac{p(y(l), -B(\Lambda))y(l) - B(\Lambda, \Lambda)}{(\alpha p^{1-\sigma}((y(l), -B(\Lambda)) + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl$$
$$+ \gamma \int \frac{p(y(l), (1-\Lambda)B(\Lambda)))y(x) + (1-\Lambda)B(\Lambda, \Lambda)}{(\alpha p_{y(x)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dx,$$

so that the first order condition is:

$$\beta \int_{\Lambda}^{1} -(1-\sigma) \frac{dp_l^b}{d\Lambda} p^{-\sigma} \frac{p(y(l), -B(\Lambda))y(l) - B(\Lambda, \Lambda)}{(\alpha p^{1-\sigma}((y(l), -B(\Lambda)) + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}+1}} dl$$
$$+\beta \int_{\Lambda}^{1} \frac{dp_l^b}{d\Lambda} \frac{y(l)}{(\alpha p^{1-\sigma}((y(l), -B(\Lambda)) + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}+1}} dl$$
$$-\beta \int_{\Lambda}^{1} \frac{dB}{d\Lambda} \frac{1}{(\alpha p^{1-\sigma}((y(l), -B(\Lambda)) + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}+1}} dl$$
$$+\gamma \int -(1-\sigma) \frac{dp_l^l}{d\Lambda} p^{-\sigma} \frac{p(y(l), (1-\Lambda)B(\Lambda)))y(x) + (1-\Lambda)B(\Lambda, \Lambda)}{(\alpha p_{y(x)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl$$
$$+\gamma \int \frac{dp_l^l}{d\Lambda} \frac{y(x)}{(\alpha p_{y(x)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl$$
$$+\gamma \int \frac{(1-\Lambda)\frac{dB}{d\Lambda} - B}{(\alpha p_{y(x)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl = 0.$$

Overall, effects of debt on welfare seem negative: prices should decrease as an effect of the transfer $dp_l^b/d\Lambda > 0$

Lump-sum transfers with zero debt would be more efficient policies than debt.

When γ is low enough, it can be proved heuristically that this expression is going to be decreasing in Λ so that the maximum equilibrium. At the opposite, when γ tends to infinity, everything depends on the expression $\gamma \int \frac{p(y(l),(1-\Lambda)B(\Lambda)))y(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dx$ which should have an inverted-U shape.

Assume planner takes prices as given. Then we have:

$$\begin{aligned}\max_{\Lambda\geq 0}&\frac{p_bY+\frac{1-\Lambda}{1+r}B(\Lambda)}{(\alpha p_b^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}+\beta\int_0^\Lambda\alpha^{\frac{1}{\sigma-1}}y(l)dl\\ &+\beta\int_\Lambda^1\frac{py(l)-B(\Lambda,\Lambda)}{(\alpha p^{1-\sigma}((y(l),-B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl\\ &+\gamma\frac{p_lY(M(1)-\frac{1-\Lambda}{1+r}B(\Lambda))-\frac{1-\Lambda}{1+r}B(\Lambda,\Lambda)}{(\alpha p_l^{1-\sigma}(y(l),-B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\\ &+\gamma\int\frac{p_xy(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dx.\end{aligned}$$

We have:

$$\begin{aligned}0=&\frac{\frac{1-\Lambda}{1+r}B'(\Lambda)-\frac{1}{1+r}B(\Lambda)}{(\alpha p_b^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\\ &+\beta\int_\Lambda^1\frac{-B'(\Lambda,\Lambda)}{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dl\\ &+\gamma\frac{-\frac{1-\Lambda}{1+r}B'(\Lambda,\Lambda)+\frac{1}{1+r}B(\Lambda)}{(\alpha p_l^{1-\sigma}(y(l),-B(\Lambda))+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\\ &+\gamma\int\frac{(1-\Lambda)B'(\Lambda,\Lambda)-B(\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}dx\\ &+\beta\int_\Lambda^1\frac{dp}{d\Lambda}\frac{\partial}{\partial p}\Big(\frac{py(l)-B(\Lambda)}{(\alpha p^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\Big)dl\\ &+\gamma\int\frac{dp}{d\Lambda}\frac{\partial}{\partial p}\Big(\frac{p_xy(x)+(1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma}+(1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\Big)dx.\end{aligned}$$

By definition of the interest rate r, the third and the fourth line should cancel out.

We have:

$$
\begin{aligned}
0 =& \frac{\frac{1-\Lambda}{1+r}B'(\Lambda) - \frac{1}{1+r}B(\Lambda)}{(\alpha p_b^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} \\
&+ \beta \int_\Lambda^1 \frac{-B'(\Lambda)}{(\alpha p^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}} dl \\
&+ \beta \int_\Lambda^1 \frac{dp}{d\Lambda}\frac{\partial}{\partial p}\Big(\frac{py(l) - B(\Lambda)}{(\alpha p^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\Big) dl \\
&+ \gamma \int \frac{dp}{d\Lambda}\frac{\partial}{\partial p}\Big(\frac{p_x y(x) + (1-\Lambda)B(\Lambda,\Lambda)}{(\alpha p_{y(x)}^{1-\sigma} + (1-\alpha)\delta^{-\sigma})^{\frac{1}{1-\sigma}}}\Big) dx.
\end{aligned}
$$

Losses from trade in each country can be computed from the total reduction in the import penetration ratio, and they are represented in the second and the third line. There is another externality, which is the pecuniary externality on debt, inside the derivative $B'(\Lambda) = \frac{d}{d\Lambda}B(\Lambda,\Lambda)$. In the competitive equilibrium, countries only consider the partial derivative $B_1(\Lambda,\Lambda) = \frac{\partial}{\lambda_{\lambda=\Lambda}}B(\lambda,\Lambda)$, so that there is an externality represented by $B_2(\Lambda,\Lambda) = B'(\Lambda) - B_1(\Lambda,\Lambda) = \frac{\partial}{\partial\Lambda}B(\lambda,\Lambda)$. Then, defining $\xi(\Lambda,\gamma,g)$ as the opposite of the primitive of the third and the fourth lines give the result.

ξ can be interpreted as a function that describes losses from trade occurring due to default in the second period. It represents the direct externalities in the model, or the deadweight loss associated with default.

Chapter 3

A Non-Sovereign Determinant of Sovereign Risk: International Trade

3.1 Introduction

In macroeconomics as well as in the financial press, it is quite common to use the adjective “sovereign” to talk about public debt. The French philosopher Jean Bodin (1576) is credited for inventing the notion of sovereignty, which he defines as “perpetuate and absolute power”. Sovereign debt is sovereign to the extent that government can choose to default over it at any moment, whatever happens in the world. That’s also where sovereignty stops: sovereign governments cannot control investors’ willingness to lend money. Worse, even the national parameters that could potentially be under the grasp of the government - such as output growth, surpluses - do not seem to affect so much the price of sovereign debt.

A government’s willingness to reimburse debt is not sovereign will depend on factors totally outside its control. It has been established that debt did not depend on some serious macroeconomic factors that play a preeminent role in sovereign debt models: GDP growth or debt level do very little to explain the variations of Credit Default Swap spreads. Aguiar et al. (2016) computed the average R^2 of different explanatory variables on sovereign debt, and they found that factors independent on their GDP or debt had more explanatory power for every investigated country.[1] These findings were line with Pan and Singleton (2008) and Longstaff et al. (2011) who find that CDS spreads comove in a lot of countries. The proposed explanation often has to do with preferences of lenders - whose risk aversion can temporarily increase after a shock to their wealth.

In this paper, I propose a quantiative model of sovereign debt with foreign demand for the domestic goods, and I use that assumption to explain why sovereign risk does not depend on sovereign factors only. I assume that a country is willing to repay sovereign debt to the extent that default entails larger trade costs hence less trade. This reduction in trade after a default has been suggested as a factor increasing incentives to repay by Bulow and Rogoff (1989) and proved empirically by Rose (2005), Martinez and Sandleris (2011). I show that as a consequence foreign demand for domestic goods and shocks to trade must play an important role to explain variations in spreads. I propose a sovereign debt model *à la* Eaton and Gersovitz (1981) with intratemporal trade. There is a foreign demand for domestic goods which, alongside the domestic demand, determines the price of the domestic good and the terms for trade. As a consequence, incentives to default should depend not only on the total level of debt and on GDP process, as in a standard sovereign debt model, but also on foreign demand.

This addition, under the form of a new state variable, seems quite natural to us. After all, sovereign debt models are designed to discuss the behavior of net (or intertemporal) trade and current account imbalances, besides sovereign debt crises. One knows the interdependence between gross (or intratemporal) trade flows and net

[1] The authors use a variance decomposition algorithm by Grömping (2007) and compare the explained variance of different parameters, completing two national variables with 2 international factors. The first factor has more explanatory variable than national fundamentals in all countries they tested: Argentina, Brazil, Bulgaria, Chile, Colombia, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Turkey and Ukraine.

trade flows: in the absence of any intratemporal trade flows, there cannot be any net trade. In all sovereign debt models since Eaton and Gersovitz (1981), international trade has only been present under the form of intertemporal trade: there is only one kind of homogeneous commodity to consume over different periods of time.[2] As a consequence, such models do not give natural tools to address terms of trade, or even currency shocks - confounded with real GDP shocks. This paper is a first step in this direction which is a natural development for the literature.

As a result of this addition of intratemporal and foreign demand, we can explain new kinds of variation - we show they behave the same way as shocks on default costs and interest rates, and that a large share of spreads comovement is explained by changes in international trade parameters. In our model, the long-run level of trade also matters for the total level of sustainable of debt and the willingness of a country to borrow and lend.

This addition of trade matters for positive and descriptive aspects of the model, but it also changes normative aspects of the model. We can also propose counter-factual exercises about optimal trade policy and show that they matter to determine sustainable levels of debt as well as sovereign default: because a higher dependence on international trade reduces incentives to default, more trade openness should play a positive role, not only because of static gains from trade, but also because of greater level of risk-sharing in sovereign debt.

The rest of the paper is presented as follows. To end the Introduction, we present a literature review to discuss sovereign debt literature, the models as well as the empirical findings. In Section II, we present our sovereign debt model with trade. In Section III, we present our calibration assumptions and results. In Section IV, we conclude.

[2] An exception is Mendoza and Yue (2012), that we discuss in the literature review.

Literature Review

This paper is part of the theoretical and quantitative sovereign debt literature that was born with the seminal paper by Eaton and Gersovitz (1981). This paper proposed a reputational debt model, where a country repays sovereign debt in order to keep a good reputation, which allows it keep borrowing debt. Bulow and Rogoff (1989) famously proved it was not enough though and one needed to add non-reputational costs of default, for example a decrease in productivity. Among the examples of economic consequences that could play this role, Bulow and Rogoff (1989) quoted trade wars. Cole and Kehoe (1998) proposed a reputational debt model where reputation has spillovers on other economic activities, such as trade: this is close to our approach.

Aguiar and Gopinath (2006) and Arellano (2008) gave important contributions to the literature, focusing on quantitative techniques: shocks to growth rather than temporary shocks to GDP lead to default events, and default cost needs to be procyclical, essentially being larger in good times, for the model to make large levels of debt similar to those observe in the reality sustainable. Other features were added for sovereign debt models: Chatterjee and Eyigungor (2012) added maturity to sovereign debt models, which we use in this paper. Most recent updates to the model, including shocks to the risk aversion of lenders, are discussed by Aguiar et al. (2016).

Two papers in this literature adopt an approach that might seem comparable to ours.Kikkawa and Sasahara (2020) assume there is international trade in a sovereign debt but do not assume that trade disruption is one of the costs of default -- they assume that trade openness makes debt repayment less costly because is labeled in terms of the final output produced thanks to foreign inputs. Moreover, they are not interested in explaining “non-sovereign” variations in spreads with a sovereign debt model as we aim at doing in this paper. Mendoza and Yue (2012) propose a model where the cost of default is endogenous and depends on trade: national firms, in order to produce a composite final good, need to import some inputs and pay in advance. However, paying in advance relies on sovereign debt finance: firms have to pay a larger cost as spreads are larger. The cost of decrease consists in the impossibility to use trade credit for certain intermediate inputs: as a consequence, default cost decreases when spreads increase, after a bad shock. Mendoza and Yue (2012) show that this mechanism generates amplification of crises, and they are a good microfoundation of the otherwise *ad hoc* assumptions suggested by Arellano (2008). However, this paper studies the impact of trade finance rather than the direct impact of international trade, which is the focus of this paper. Mendoza and Yue (2012) do not try to answer our main concerns, which are the non-sovereign variations of sovereign spreads, and they do not propose an exogenous variable to explain these variations as we do. Moreover, they do not wonder what the consequences of more openness or a free-trade agreement might be. Although the models seem similar, they use sensibly different assumptions and answer different questions.

This paper also builds on the empirical literature on sovereign debt. Kaletsky (1985) was one of the first to stress that large volumes of trade credit could make default more costly because trade credit could be interrupted after a sovereign default, because of its reliance on national banks and sovereign lending. Most relevant findings for our purpose are given by Rose (2005) and Martinez and Sandleris (2011): trade decreases after sovereign defaults, which makes it logical to assume that trade might be responsible for productivity losses in the wake of a sovereign default. In their general evaluation of the cost of Argentinian default in 2014, Hébert and Schreger (2017) noticed that default shock disproportionately hurt Argentinian exporting firms. In the earlier Argentinian default, Gopinath and Neiman (2014) used firms' data to prove that firms relied more heavily on domestic inputs than on foreign inputs after the 2000-2001 default. More generally, studies about trade finance show how one of the consequences of financial crises is to disrupt international trade because of the dependence of trading firms on trade credit: Amiti and Weinstein (2011), **?** or Manova (2013).

The most relevant empirical papers for the calibration exercise in this paper come from the finance literature about sovereign spreads: we already quoted Pan and Singleton (2008) and Longstaff et al. (2011) that proved that national determinants were not good predictors of the evolution of sovereign spreads and that there was a lot of commonality in the movement of sovereign spreads. Hilscher and Nosbusch (2010) in a paper looking for macroeconomic determinants of sovereign risks find that terms of trade volatility is usually correlated with spreads. In this paper, this volatility should in the parameters governing the distribution of an underlying variable, foreign demand. As terms of trade are endogenous and depend, for example, on transfers at a given period, our model will help understand this correlation better.

Our paper also concerns papers in the international trade literature that address the topics associated with intertemporal trade. Costinot et al. (2014) discussed capital controls from a trade point of view, where terms-of-trade can be manipulated through capital controls. In our paper, sovereign debt and intertemporal trade also entail terms-of-trade manipulation because a country receiving financial transfers through a deficit will also consume more of its own good. Reyes-Heroles et al. (2016) and Eaton et al. (2016) propose a model of intertemporal and intratemporal trade, but without defaultable debt.

In Serfaty (2020), I have shown that the evidence in Rose (2005) held in more general data set and I provided evidence that suggested a causal relationship between trade openness and lower spreads as well as larger quantities of sovereign debt, by looking at countries with varying trade openness over time. In Serfaty (2021), I gave suggestive evidence that countries could default together in waves through a contagion effect: a default in a neighbor country B can reduce gains from trade in country A, and therefore increases incentives to default, which are larger as the country is more dependent on the outside world. While this papers uses some of the insights of Serfaty (2020), it answers a very different question: why does the variation in spreads seem not to depend on sovereign characteristics? In the model of Serfaty (2020), a country's variation in GDP explain all the movement in spreads. In Serfaty (2021), I argued that an international macroprudential tax on debt would be optimal because of interdependence. This paper uses the trade hypothesis in a quantitative model to bring a solution to other puzzles inspired by the sovereign debt literature.

3.2 A Sovereign Debt Model with Terms of Trade

In this Section, we present our calibration model. In Section 2.1, we present the primitives of the model: preferences and technology. In Section 2.2, we define intratemporal trade and see how prices of goods are determined at each period. In Section 2.3, we present intratemporal and intertemporal trade. In Section 2.4, we define equilibrium in the model.

3.2.1 Preferences, Technology and Stochastic Shocks

Domestic economy has random endowment $(Y_t)_{t\geq 0}$ that is a stochastic process. The endowment is given in terms of the country's specific good that it will trade with the rest of the world. We are interested in a small open economy trading with the rest of the world. We assume that the rest of the world is a large economy behaving like a single country. This assumption serves as a simplification for a world economy with a large number of countries: the assumption restricts it to mean that what happens in the domestic economy should not affect the relative prices of goods between 2 foreign countries.

This is an Armington economy: each country produces its own good. Besides the domestic good, there is foreign good. The price of foreign goods is considered given by the domestic economy: it is the *numéraire*, which should matter as there will be debt in the model.

There are trade costs τ (for both imports and exports) and export taxes δ that we consider fixed and given. However, the behavior of the economy can modify the trade costs through default, as we will see below.

- Static Consumption and gains from trade

A country wants to maximize its utility:

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t u(C_t),$$

where total aggregate consumption $(C_t)_{t\geq 0}$ is defined, for every period t, as:

$$C_t = m_0(c_t, c_t^\star),$$

which means it is an aggregate mix of domestic good's consumption $(c_t)_{t\geq 0}$ and foreign consumption $(c_t^\star)_{t\geq 0}$.[3]

There is an intratemporal resource constraint and an intertemporal budget constraint. To simplify the presentation, we first present the intratemporal resource constraint taking as given transfers and then present the borrowing conditions in this sovereign debt model.

Foreign demand

There is a foreign demand for the domestic good that is going to determine its price - hence the welfare of the domestic economy. The domestic economy is a small open economy that does not affect relative prices from one country to another. However, it can affect through tariff the price of its own good and manipulate its terms of trade. Indeed, it exerts a monopoly power over its domestic good.

[3] It is easy to see how this equation can include the more general case with any number of countries N. Assume each country $i \in [1, N]$ produces a good $(c_{1,t}, ..., c_{N,t})_{t\geq 0}$. We assume that preferences are weakly separable so that aggregate consumption can also be written:

$$C_t = m(c_t, c_t^\star)$$

where $c_t^\star = m^\star(c_{1,t}, ..., c_{N,t})$ is an aggregate of consumption of foreign goods. If the relative price of all foreign countries is assumed to be fixed, then, up to a normalization, the trade model will be equivalent to a 2-country trade model.

Moreover, the specific choice of Armington trade should not matter as we are mostly interested in gains from trade. Arkolakis et al. (2012) prove that, conditional on similar imports and a constant elasticity of substitution, using a Ricardian or an Armington trade model should not matter to estimate the gains from trade.

We also assume that all foreign economies have CES preferences with the same elasticity of substitution σ. As a consequence, total foreign demand for the domestic good should be equal to:

$$d_t^\star p^{-\sigma}.$$

This functional form stems from our assumption that our economy is small relative to the rest of the world.[4] In a model with several countries, each one having its output $Y_{i,t}$, its own-good price level $p_{i,t}$ and aggregate price level $P_{i,t}$, we would have:

$$d_t^\star = \sum_{i=1}^{N} d_{i,t}(p) = \sum_{i=1}^{N} \frac{p_{i,t} Y_{i,t}}{P_{i,t}} \tau_i^{1-\sigma}.$$

Because of the intertemporal nature of the model, we would need to assume that government keeps track of the foreign demand of each country to predict the evolution of terms of trade.

For the sake of tractability, we are going to assume that aggregate demand $(d_t^\star)_{t\geq 0}$ is a Markov process. This is not always true for the sum of Markov processes,[5] but it is a reasonable assumption that is going to allow significant progresses in terms of tractability. We are also going to assume that, in the long run, $(d_t)_{t\geq 0}$ follows the tracks of the output process. We assume that there is an underlying process $(\xi_t)_{t\geq 0}$

[4] Under standard CES assumptions, the functional form $d_{i,t}$ of the demand of foreign country i for the domestic good should be given by:

$$d_{i,t}(p, \tau_{i,t}, \tau_{i,t}^F) := \frac{p_{i,t} Y_{i,t}}{P_{i,t}} (1 + \tau_{i,t}^d)^{1-\sigma^\star} (1 + \tau_{i,t}^F)^{-\sigma^\star} p^{-\sigma^\star}$$

where $p_{i,t}$ is the price of the Armington good of economy i - potentially including a tariff from the domestic economy, $Y_{i,t}$ is GDP, $\tau_{i,t}^d$ is an iceberg trade cost, $\tau_{t,i}^F$ is an import tariff imposed by country i, $\sigma^\star$ is the elasticity of substitution between different goods' consumption for country i, $P_{i,t}$ is the aggregate price index faced by country i at time t. We included as variables in the demand function parameters that depend on the domestic economy: its price, the tariff the domestic economy faces, which can decrease thanks to free-trade agreements, and the trade costs, which might increase if the economy defaults. Variable $d_t^\star$ is then an aggregate weighted by standard gravitational determinants of trade. It includes potentially relevant political variable, such as the import tariff imposed by foreign countries.

[5] In this case, learning past observations of total demand can be useful to predict which countries have low or high demand at the moment, if they have different sizes. As a consequence, it might impact predictions about which shocks are going to evolve.

such that:

$$d_t^\star = \xi_t Y_t,$$

and $(\xi_t)_{t\geq 0}$ is stationary (while Y_t is not necessarily stationary). This assumption is necessary in order to be able to solve the model numerically.[6] Furthermore, we make it because we are interested in the relation between debt and total openness of a country: any other assumption would modify the long-run value of trade openness.[7]

Stochastic Structure

As we have seen, there are two exogenous state variables $(Y_t)_{t\in\mathbb{N}}$ and $(d_t^\star)_{t\in\mathbb{N}}$. The stochastic structure is going to rely on a Markov process $(s_t)_{t\in\mathbb{N}} \in \mathcal{S}^{\mathbb{N}}$ where S would typically be a subset of $\mathbb{R}^d$. For calibrations, we will assume that S is finite.

We simply assume there are functions Y and $d^\star$ such that: $(Y_t)_{t\in\mathbb{N}} = (Y(s_t))_{t\in\mathbb{N}}$ and $(d_t^\star)_{t\in\mathbb{N}} = (d^\star(s_t))_{t\in\mathbb{N}}$. As a consequence of our assumption, we will be able to write value functions as a function of s_t directly.

A specific case of this structure is when $(Y_t, d_t^\star)$ is a Markov process.[8]

3.2.2 Intratemporal Trade

Trade Costs and Tariff There are trade costs τ that are assumed to be constant over time, except when there is a default, as we will explain below.

We also assume that the government chooses a tariff level on exports equal to δ. As a consequence, foreign demand for exports should be defined by the perceived

[6] The assumption is not necessary strictly speaking to solve the model numerically but it sensibly reduces the number of states we have to consider for state variables.

[7] If we introduce long-run growth, this assumption means that $(d_t^\star)$ has the same long-run trends as (Y_t). When (Y_t) is stationary, this assumption only implies that $(d_t^\star)$ should be stationary. Additional state variables have a very heavy computational costs and we are interested in the aggregate foreign demand in this paper.

[8] The purpose of this general definition is to include all cases that would usually appear in a calibration exercise. For example, when we consider persistent growth shocks similar to those modeled by Aguiar and Gopinath (2006), (Y_t) is not a Markov chain so that we need a more general definition for a Markov equilibrium.

price δp:

$$D_t(p) = d_t^\star \tau^{1-\sigma}\delta^{-\sigma}p^{-\sigma}.$$

Using a tariff on exports is, for our purpose, equivalent to a tariff for imports, as it has been demonstrated in Costinot and Werning (2019). The only thing that could break the equivalence would be if foreign assets were labeled in domestic currency. It turns out that we assume that debt is monetized in the foreign currency. However, if foreign debt were labeled in domestic currency rather than in the reserve currency, import tariff and export tax would not be equivalent any more. Indeed, in such a case, government would have a tendency to favor export taxes or import tariff depending on whether it would be borrowing or lending money: the policy choice affects the value of the current account in this case. However interesting such an asymmetry might be, we exclude it from the analysis: the model is a better fit for countries borrowing their money in foreign currencies.

Budget Constraint and Resource Constraint Let T be the size of financial transfers from others countries in the world to country 1 at time t - equivalently, T is the current account deficit, that should depend on inherited debt, new debt emission and the price of new debt emissions as we will discuss in the next section. The budget constraint is:

$$p_t c_t + \tau c_t^\star = p_t Y_t + R_t + T,$$

where $R_t = (\delta - 1)p_t(Y_t - c_t)$ is the revenue from tariff. The resource constraint for the economy's own good can be written:

$$\begin{aligned} & D(\tau p_t) + c_t = Y_t \\ \iff\ & p_t D(\tau p_t) = \tau c_t^\star - R_t - T. \end{aligned}$$

In the model, the price of other goods is considered exogenous. We consider our economy is small enough not to have any effect on relative prices between foreign countries. As a consequence of this assumption, we do not need to take into account resource constraints in foreign countries. If we fix the level of intertemporal transfers T and the revenue from tariff $R(p_t)$, the domestic demand for the domestic good should be equal to the following function of p:

$$D^d(p) = \frac{p^{-\sigma}}{p^{1-\sigma} + \tau^{D\,1-\sigma}} \Big[pY + T + R \Big].$$

Summing these two demands and computing the revenue from tariff should give us the complete price equation that defines the implicit price equation $p(s, T)$.

3.2.3 Default and Financial Markets

As a reminder, government maximizes the expected of utility of its future aggregate consumption:

$$\max \sum_{t \geq 0} \beta^t u(C_t),$$

where C_t is a CES aggregate of domestic consumption and foreign good's consumption:

$$C_t = \left(\alpha^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} c_t^{\star\,\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

Default Utility

In case of default, government gets temporarily out of financial markets and has to finance its own trade which results in an increase in trade costs from τ to $\tau^D \geq \tau$ until the end of default. Moreover, the country faces a temporary loss of productivity $x \in [0, 1)$.

Hence, the default budget constraint becomes:

$$p^D(s)c + \tau^D c^\star = p^D(s)(1-x)Y,$$

where function $p^D(Y, d^\star)$ is the value of p that solves:

$$\frac{p^{-\sigma}}{p^{1-\sigma} + \tau^{D\,1-\sigma}}(p(1-x)Y + d^\star(\delta - 1)\delta^{-\sigma}\tau^{1-\sigma}p^{1-\sigma^\star}) + d^\star p^{-\sigma}\tau^{D\,-\sigma} = (1-x)Y. \quad (3.1)$$

At each period during default, government can get out of default with probability λ - which happens independently with the evolution of other variables. From that, we conclude that total aggregate consumption should be equal to:

$$\frac{(p^D(Y, d^\star)(1-x)Y + T + d^\star(\delta - 1)\delta^{-\sigma}(p\tau^D)^{1-\sigma})}{P^D(Y, d^\star)},$$

and $P^D(Y, d^\star) := \left(p^D(Y, d^\star)^{1-\sigma} + \tau^{D\,1-\sigma}\right)^{\frac{1}{1-\sigma}}$.

Indeed, the trade cost increase applies to both imports and exports. Utility from default is then given by solving this recursive equation:

$$V^D(Y_t, d^\star_t) = \max_{s.t.} u(C_t) + \beta\mathbb{E}\big[(1-\lambda)V^D(Y_{t+1}, d^\star_t) + \lambda V(Y_{t+1}, d^\star_{t+1}, \bar{B})|Y_t, d^\star_t\big].$$

Because of the possibility to reenter into financial markets, this value function depends on the value function associated with debt repayment, V,that shall be defined recursively in the following paragraph.

Repayment Utility

The domestic economy inherits debt from the previous period, B_{t-1}, indexed on the price of foreign goods (which is the numéraire). Given this level of debt, government either defaults or emits new debt B_t taking into account debt schedule price. Government has to choose B_t so has to maximize its expected utility. In case of repayment, it solves the following recursive equation:

$$\begin{aligned} V^R(B_{t-1}, s_t) = & \max_{B_t, C_t, T \text{s.t.}} u(C_t) + \beta \mathbb{E}_t V(B_t, s_{t+1}), \\ \text{s.t.} C_t = & \frac{p(s_t, T)Y(s_t) + T + d^\star(s_t)(\delta - 1)\delta^{-\sigma}(\tau p)^{1-\sigma}}{P(s_t, T)} \\ T = & q(B_t, Y_t, d_t^\star)(B_t - (1-\psi)B_{t+1}) - (\psi + r)B_{t-1} \end{aligned}$$

and we define the price function implicitly as the solution to:

$$Y_t = \frac{p^{-\sigma}\Big[pY_t + q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (\psi + r)B_{t-1} + d_t^\star(\delta - 1)\delta^{-\sigma}p^{1-\sigma}\tau^{1-\sigma})\Big]}{p^{1-\sigma} + \tau^{D\,1-\sigma}} \tag{3.2}$$

$$+ d_t^\star p^{-\sigma^\star}(1+\tau)^{-\sigma^\star}.$$

Default Decision

The government should choose to default whenever the value of default is more than the value of repayment. Let $D(B_{t-1}, Y_t, d_t^\star)$ be the default decision. Then we should have:

$$D(B_{t-1}, Y_t, d_t^\star) = \mathbb{I}\{V^R(B_{t-1}, Y_t, d_t^\star) < V^D(Y_t, d_t^\star)\}.$$

Price of Bonds

Government takes the price schedule of bonds as given. We assume all bonds have the same maturity, indexed by parameter ψ. The period after borrowing, government has to pay back the interests of its past debt as well as a fraction $\psi \in [0, 1]$ of it. When $\psi = 1$, we have a standard sovereign debt model with one-period bonds. When $\psi = 0$, all bonds consist in perpetuities.

Because defaulting government cannot reenter into financial markets, we have $q = 0$ when after a government's default until it reenters into financial markets. We also assume that financial markets are competitive and risk-neutral. There is a fixed global interest rate r, and the price of bonds should be determined by the following dynamic equation:

$$q(B_t, s_t) = (\psi+r)\mathbb{P}(D(B_t, s_{t+1}) = 0)+\frac{1-\psi}{1+r}\mathbb{E}_t((1-D(B_t, s_{t+1}))q^D(B_{t+1}, s_{t+1})|B_t, s_t),$$

where the term inside the expectation is assumed to be equal to 0 when $D(B_t, s_{t+1}) = 0$.[9] When $\psi = 1$, this equation simply means that lenders correctly forecast default risk in the next period. When $\psi \in [0, 1)$, it also implies that lenders correctly forecast long-run default risks and the borrowing behavior of the government.

Intertemporal Budget Constraint

Combining all the previous elements, we can write the intertemporal budget constraint in case of repayment:

$$\begin{aligned} P(s_t, T)C_t &= p(s_t, T)Y(s_t) \\ &+ T \\ &+ (\delta - 1)d^\star \delta^{-\sigma}(\tau p)^{1-\sigma}(s_t, T), \\ \text{where } T &= q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (\psi + r)B_{t-1}. \end{aligned}$$

The variable T is the sum of financial transfers received by the domestic economy, or the capital account. Because there is only one kind of assets in our economy - sovereign bonds, these financial transfers are the opposite of the trade balance (which is also equal to the current account) in the model.

[9]Another equivalent way to write this recursive equation without this additional notation is to use conditional expectation:

$$q(B_t, s_t) = \mathbb{P}(D(B_t, s_{t+1}) = 0)\Big((\psi + r) + \frac{1-\psi}{1+r}\mathbb{E}_t(q(B_{t+1}, s_{t+1})|B_t, s_t, D(B_t, s_t) = 1)\Big)$$

We have to write it that way because the function q is not assumed to depend on default decisions but only on state variables that would be relevant for a default.

Comments As we discussed in the literature review, since the contribution of Arellano (2008), the quantitative sovereign debt literature has adopted some specific assumptions about defaults. Cost of default has been assumed to be non-linear. In our model, default creates a cost proportional to GDP, x, but is also implies larger trade costs, hence lower gains from trade. In Arellano (2008), GDP in case of default is calibrated to be equal to a constant.[10] As a consequence, costs of default are null, if not negative, when GDP gets low enough. While this assumption allows larger levels of debt and larger risks of default at the same time, it does not seem natural, especially as one still observes losses of efficiency and a decrease in *exports* after a sovereign default: it does suggest that default has a negative effect on either the productivity or the ability to trade. In Mendoza and Yue (2012), the cost of default is indexed directly on spreads: sovereign bonds are used for trade credit, which means that domestic importers of inputs might pay a larger price when spreads get larger. Because spreads are finally determined by the level of production, this assumption has the same effects on calibration as the non-linear default costs in Arellano (2008).

In contrast with these assumptions, we assume the direct costs of default are not decreasing in the country's production. Instead, we propose an observable variable that can change the cost of default over time: foreign demand. As a consequence, some large negative shocks to the net present value of production in the economy can sometimes result in default, in some other cases just create a hardship and some consolidation in debt - which makes the model less mechanic than some models in which a shock to growth almost always entails a sovereign default.[11]

Moreover, one could think that this model implies that default costs can be inferred

[10] More precisely, Arellano (2008) assumes that $y^D = 0.969 \times \mathbb{E}y$ where $\mathbb{E}y$ is the expectation of the stationary distribution of the AR(1) process that GDP is assumed to follow.

[11] In our calibrations, more than 99% of defaults occurred after a large shock to the net present value of production, but a large shock could occur without default. When we considered persistent growth shocks, a shock to GDP growth was likely to cause the default, in line with the calibration results from Aguiar and Gopinath (2006). Carré et al. (2019) show this is a general theoretical property of sovereign debt models: as we get closer to continuous time, shocks need to create a discontinuity in order to generate default risk. If it were not the case, government would have enough time to adjust its behavior before entering into the gray area where default is possible.

directly by looking at changes in imports, using the formula in Arkolakis et al. (2012) - and then we could compare gains from trade to the burden of debt. We show in the appendix that this simple approach is not exact when we assume terms of trade depend on financial transfers, since transfers increase the price of the domestic good. As a consequence, costs from default depend heavily on the level of financial transfers at a given period, which means that our model can also display amplification of crises, but without relying on the same kind of trade credit assumptions as Mendoza and Yue (2012).

3.2.4 Equilibrium definition

Definition 4. *Let ψ, λ, σ, $\tau < \tau^D$ be fixed parameters. Let $(s_t)_{t\geq 0} \in (\mathbb{R}^d)^{\mathbb{N}}$ be a Markov process on space $\mathcal{S}$ and $Y, d^\star$ two functions $\mathcal{S} \to \mathbb{R}_+^\star$. Define $Y_t = Y(s_t)$ and $d_t^\star := d(s_t)$. Let p and p^D price functions $p : Y \times d^\star \times T \in (\mathbb{R}_+^\star)^2 \times \mathbb{R} \mapsto p \in \mathbb{R}_+$ and $p^D : Y \times d^\star \in (\mathbb{R}_+^\star)^2$ that solve for 3.2 and 3.1, and let P and P^D be the associated aggregate price functions.*

A Markov equilibrium associated with these parameters and functions is given by (i) value functions $V^R(B_{t-1}, s_t)$, $V^D(s_t)$ (ii) policy function $b(B_{t-1}, s_t)$, (iii) Default decision $D(B_{t-1}, s_t)$ and (iv) bond price schedule $q(B_t, s_t)$ such that:

- When the government takes function q as given, V^D and V^R solve the following Bellman equation, for every $t \in \mathbb{N}, s_t \in \mathcal{S}$:

$$V^D(s_t) = \max_{c_t, c_t^\star} u(C_t) + \beta \mathbb{E}\big[(1-\lambda)V^D(s_{t+1}) + \lambda V(s_{t+1}, 0)|s_t\big], \tag{3.3}$$

$$s.t. P^D(s_t)C_t = p^D(s_t)(1-x)Y$$

and:

$$V^R(B_{t-1}, s_t) = \max_{B_t, C_t s.t.} u(C_t) + \beta \mathbb{E} V(B_t, s_{t+1}|s_t), \tag{3.4}$$
$$s.t. P(s_t, T)C_t = p(s_t, T)Y(s_t) + T + (\delta - 1)d^\star \delta^{-\sigma}(\tau p)^{1-\sigma}(s_t, T)$$
$$T = q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (r+\psi)B_{t-1}$$

where $V(B_{t-1}, s_t) := \max\{V^R(B_{t-1}, s_t), V^D(s_t\})$.

- Optimal saving policy $b(B_{t-1}, s_t)$ *solves the maximization in the Bellman equation 3.4.*

- Default decision $D(B_{t-1}, s_t)$ *is equal to 1 if and only if:*

$$V^R(B_{t-1}, s_t) < V^D(s_t),$$

and equal to 0 otherwise.

- Price schedule function correctly predicts the future likelihood of default:

$$\forall B_t, q(B_{t-1}, s_t) = \mathbb{P}\big(D(B_t, s_{t+1}) = 0\big)\Big(\frac{\psi + r}{1+r} + \frac{1-\psi}{1+r}\mathbb{E}\big[q(b(B_t, s_{t+1}), s_{t+1})|D(B_t, s_{t+1}) = 0\big]\Big).$$

3.3 Results

3.3.1 Shocks, State Variables and Numerical Solutions

Exogenous State Variables For the sake of generality, we have not fully described shocks to GDP and foreign demand in the previous section. We are going to make the same assumption as Aguiar and Gopinath (2006) and Aguiar et al. (2016) and assume that output Y_t at time t can be written:

$$Y_t = e^{z_t} \prod_{i=0}^{t} e^{\tilde{g}_t},$$

where $(z_t)_{t\in\mathbb{N}}$ and $(\tilde{g}_t)_{t\in\mathbb{N}}$ are two independent AR(1) processes, such that, for every $t \in \mathbb{N}$:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$
$$\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$$
$$\tilde{g}_t = m_g + \rho_g \tilde{g}_{t-1} + \varepsilon_t^g$$
$$\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2),$$

where $(\rho_z, \rho_g) \in [0,1)^2$ and $\sigma_z, \sigma_g \geq 0$ are four parameters that can be determined by observing GDP processes, and m_g is the average long-run growth rate. z_t is the stationary component, or the cyclical component of GDP, while g_t is the trend. As long as $\rho_g > 0$, a shock to the trend ε_t^g has much more persistent effects on GDP than a shock on the cyclical component. Indeed, past growth affects the level of GDP, but, more importantly, a shock on growth persists through higher distribution of future growth.

We will assume that foreign demand follows the same trend as GDP growth. Foreign demand also has a stationary component that follows an AR(1) process as well:

$$D_t = e^{d_t} \prod_{i=0}^{t} e^{\tilde{g}_t}$$
$$d_t = m_d + \rho_d d_{t-1} + \varepsilon_t^d$$
$$\varepsilon_t^d \sim \mathcal{N}(0, \sigma_d^2),$$

where ρ_d and σ_d are parameters determining the stationary component of foreign demand. As a consequence of our assumption, growth shocks should have little effect on the price of commodities.[12] We assume that growth shocks are the same for

[12]In the absence of intertemporal trade, this model clearly implies that any shock on $\tilde{g}_t$ has no effect on the price. However, growth affects intertemporal trade and changes the weight of inherited debt relative to current GDP and demand, so it still plays an indirect although second-order role.

tractability and consistency. With independently but identically distributed growth rates between D_t and Y_t, the ratio D_t/Y_t can land anywhere between 0 and infinity - which compels us to add more grid points than we could. Moreover, if the growth rate distributions of Y and D are different, the ratio might converge to 0 and $+\infty$ which is not realistic and makes the model non-stationary. We study in the model what happens when foreign demand for domestic goods increases relative to GDP.

Parametric Restrictions For numerical resolution, we use a CRRA utility function u with relative risk aversion parameter γ as in the rest of the literature:

$$\forall C \geq 0,\, u(C) = \frac{C^{1-\gamma}}{1-\gamma}.$$

Because this utility function is homogeneous of degree $1-\gamma$, we can simplify the terms inside and divide the terms in the Bellman equation by $\prod_{i=0}^{t} e^{\tilde{g}_t}$.

As a consequence, we have three exogenous state variables to follow: z_t, $\tilde{g}_t$ and d_t. 3 is usually the maximal possible number of exogenous state variables in endogenous sovereign debt models. There is an additional endogenous state variable: the stock of sovereign debt B_t.

To solve for the model, we first discretize the auto-regressive continuous processes for each variable, using the algorithm developed by Tauchen (1986).[13] We are restricting to 7 states per variable, a number for which the Tauchen algorithm is generally deemed precise enough. We allow 101 different values for debt on a grid scaled according to other variables in the model.

Because of the discretization, there is no ambiguity about the number of equilibria in the model. Indeed, our model is embedded in the case studied by Auclert and Rognlie (2016), who proved that Eaton-Gersovitz sovereign debt models with a finite number of states had a unique equilibrium.

[13] We used for several functions, including the discretization, the Python library QuantEcon from the https://quantecon.org/, which, among things.

3.3.2 Calibration

We have to assign a value to each parameter through estimation. We discuss the stakes associated with problematic variables by the same token. In order to compare our model with the quantitative sovereign debt literature, we align our estimates with Aguiar et al. (2016) when it is possible.

- $\gamma, r,$: We use the standard value that is equal to 2 in most models, in the sovereign debt literature as well as in many DSGE simulations. For the safe interest rate r, we use the value 1.5%, in line with average Treasury yield bonds in the last few decades.

- The elasticity of substitution σ is usually estimated before 5 and 10 as it is reminded by Anderson and Van Wincoop (2004) and we keep the lower estimate. In our model, Armington trade means that gains from trade, absent any intertemporal trade, have the general form described by Arkolakis et al. (2012): they depend on the level of imports' penetration ratio and on the elasticity of substitution. However, in the presence of net trade, the results can change, and we show numerically in the appendix that the ACR formula gives a lower bound for gains from trade in the presence of net trade (although the ACR formula gives the right order of magnitude). The ACR formula is usually considered a "disappointing" result, because it infers low gains from trade relative to empirical estimates (Frankel and Romer (1999) and Feyrer (2009)) or to what the importance of the subject suggests,[14] hence we use the more favorable value of the parameter.

- τ and m_d: we assign to this variable a value that is more than 1, typically 1.1. With this variable, The decision is partly arbitrary because the mean value m_d also affects the total volume of trade from the point of view of the domestic economy. Because the foreign good is the *numéraire*, a change in τ should affect the real value of debt from the point of view of the domestic economy, which can access the foreign good only after paying the iceberg costs τ. We also have to take into account the

[14]Some papers try to correct the trade models, so that larger gains from trade might emerge while keeping the gravitational approach.

role of the elasticity of substitution σ to see the total impact of trade costs on trade, while the effect of the parameter m_d on foreign demand function does not depend on σ. More generally, τ gives an assessment of the gains from trade. To avoid relying on some nominal effects of trade, we will explore the role of trade openness using m_d. We want to target the average gross trade flows thanks to this parameter. We use Mexico as an example, with an openness ratio around 40% in 2019.

- x, τ^D: when default costs are linear, the value usually chosen is 2%. Because we rely on default mechanism, we choose a proportional default loss that is 4 times less, equal to 0.5%. For τ^D, we choose $\tau^D = (1+f)\tau$, where $f > 0$ is chosen as a function of other parameters to obtain a decrease in trade after default similar to what we observe in the data, in Rose (2005) or in my updated findings Serfaty (2020): 20%. We choose $f = 1.1$ in the following regressions. The relative importance of x and f matter a lot and we will study how making them vary affects the dynamics of the paper.

- We fix other parameters such as ψ or θ the same way as it is suggested in the literature.

Parameter	Description	Value	Source
g	Average growth rate	2.42%	Mexico
σ	Elasticity of substitution	5.0	Anderson and Van Wincoop (2004)
γ	Risk aversion	2.0	SD literature
r	Safe interest rate	1.5%	Treasury Rates
τ	Trade Costs	-	-
θ	Probability of Redemption	0.1	Wright (2012)
x	Effect of default on prod.	0.005	Assumption
ψ	(Inverse) Maturity	0.125	Broner et al. (2013)
τ_D	Increase in trade costs	5%	Regressions

- Parameters for the distribution of GDP (cycles and growth) m_g, ρ_y, ρ_g, σ_y, σ_g

can be estimated directly from GDP series. We use the same values as in Aguiar et al. (2016) to be able to compare our results.

Parameter	Description	Value	Source
ρ_g	Autocorrelation of g	0.45	Based on GDP series
σ_g	Variance of shocks to g	0.011	-
ρ_y	Autocorrelation of y	0.85	-
σ_y	Variance of y	0.05	-

- Parameters about the distribution of demand shocks ρ_d and σ_g can be estimated from the data with a structural approach. Indeed, in our model, the value of exports-to-GDP (which is observable) should be equal to:

$$X_t = \frac{d_t^\star \delta^{-\sigma} \tau^{1-\sigma} p_t^{-\sigma}}{Y}$$
$$\implies d_t^\star \propto X_t p_t^\sigma Y_t$$

where Y_t is real GDP, X_t is exports to GDP ratio, and p_t is equal to terms of trade. The relation is proportional with a fixed factor that does not move because we assume that δ and τ are fixed over time. Using data on terms of trade from the World Bank and real GDP series, we run this regression for Mexico between 1980 and 2018 and find $\rho_d = 0.89$ and $\sigma_d = 0.20$, which makes this term more volatile than GDP or growth.

Parameter	Description	Value	Source
ρ_D	Autocorrelation of Foreign Demand	0.89	Computations on Mexico
σ_D	Variance of D	0.20	-

3.4 Quantitative Results

3.4.1 Foreign Demand Volatility and Spreads

First, we solve the model through a value-function iteration on $7 \times 7 \times 7 \times 101$ grid. We show in Figure 4.1 the cost of debt emitting depending on debt for high $(y_H, d^\star_H, g_H)$ or low values $(y_L, d^\star_L, g_L)$ of the parameter. We see that, with the parameters we used, much larger levels of debt are sustainable when the level of foreign demand is at a large value relative to the case when growth or stationary output is high: foreign demand drives the level of borrowing that is possible. We also observe some other features: for low levels of debt, price of debt is lower for countries with high growth than for countries for low growth. Indeed, although growth absorbs a part of debt in the long run, a growing country will also end up accumulating deficits, and because debt has a long maturity (over 8 periods), it should be anticipated by the financial markets.

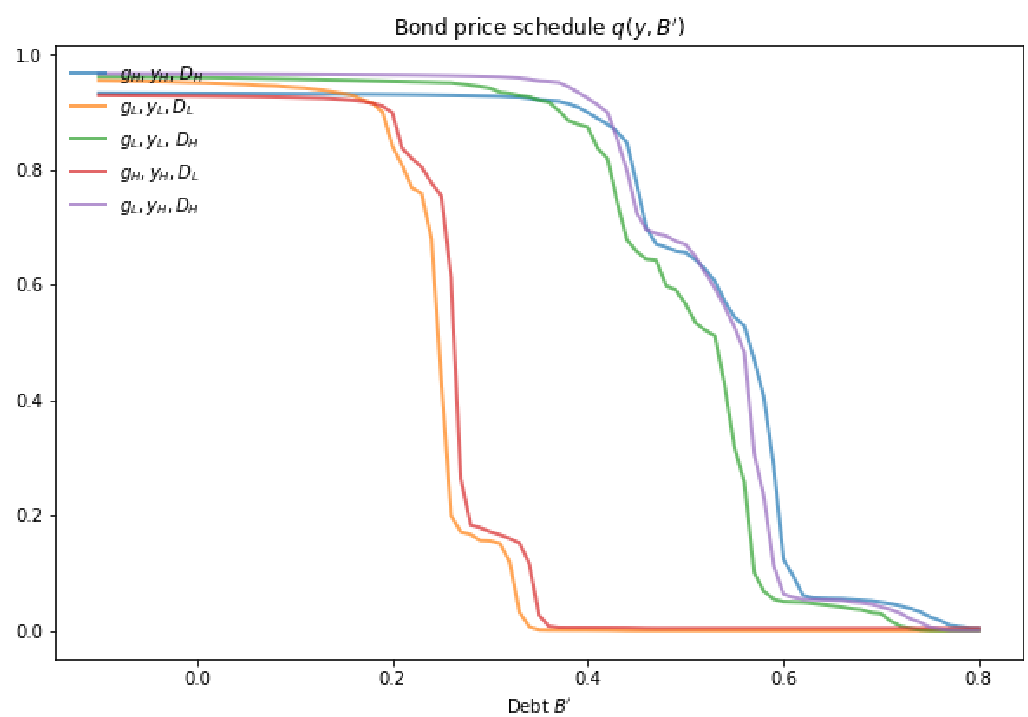

Figure 3.1: Borrowing schedule $q(s, B)$ for various levels of $y, d^\star, g$.

After solving the model, we simulate its behavior 100 times over $T = 10\,000$ periods (that are quarters), substracting the 2 000 periods that might depend on the initial conditions. We show some of the significant targets of our sovereign debt model in table 3.2. The model, which uses the same GDP structure as Aguiar and Gopinath (2006) is unsurprisingly good at matching some of the same moments, for example the counter-cyclicality of current account: countries accumulate debt in good times and accumulate surpluses in bad times because of the persistence of growth shocks. More importantly, the model creates a very powerful "exogenous" and non-sovereign kind of shock, that explains, with realistic parameters, more than 35% of the movement in spreads. In line with previous estimates, GDP explains almost no variation in the spreads - the event that causes defaults in the model is either a turning point in growth (a negative shock to g_L) that affects the net present value of output flows - or a large demand shock. Therefore, this model replicates the non-Sovereign variation that has been observed in other papers. The frequency of sovereign defaults that we did not try to match is at 3.1%, while the usual target in the literature is 3%. The level of debt-to-GDP ($B/(4Y)$ in the model) is low however relative to reality, although a bit higher than in other papers. As we did not introduce *ad hoc* mechanisms to reduce debt-to-GDP ratio in times of hardship, this result is not really surprising. It shows that the effect of trade on the cyclicality of default through terms of trade is not as strong as it would be required to explain the large levels of debt.

3.4.2 Openness to Trade and Sovereign Default's volatility

In table 2, we show what happens when the general of openness in the economy decreases: we use the same parameters as in Section 4.1, except for the general level of foreign demand that is lower, and we simulate the model 100 times for $T = 10\,000$ again. In this second economy, the level of exports-to-GDP is roughly 25% lower: imports represent 34% of GDP, rather than 43%. This reduction in trade openness

Table 3.1: Results of the Calibration

Variable	Value
Debt to GDP ratio	0.088
Mean Spread	0.102
S.d. of spread	0.064
Spread diff., 95th percentile	0.052
Frequency of Crises	0.031
Corr(Y,CA)	-0.239
Corr(Demand,CA)	-0.187
Corr(Δ Y, CA)	-0.244
Corr(ΔY, Δ Spread)	-0.110
Corr(CA, Δ Spread)	0.027
Corr(Imports, Δ Spread)	-0.483
Mean Trade Balance (in % of GDP)	0.017
Mean Imports (in % of GDP)	43
Corr($d^\star$, Spread)	-0.356
Corr(GDP, Spread)	-0.004
Terms of Trade	0.952
Corr(Spreads,Terms of trade)	-0.377

has a negative effect on terms of trade (that decrease by 10%), on debt levels (that go from 8.8% of GDP to 6%) and on the likelihood of crises, that increases from 3.1% to 3.8%.

We also see that the comovement between spreads and demand does not depend on the size of trade: dividing by 2 the foreign demand does not reduce it. Rather, it depends on the volatility of the foreign demand relative to output shocks, which is rather large in the actual data.

3.5 Conclusion

In this paper, we have calibrated a model that proved that foreign demand is a likely explanation for much of the movements in sovereign risks. We did a natural addition to a standard and updated sovereign debt model: international trade and

Table 3.2: Calibration with

Variable	Value
Debt to GDP ratio	0.060
Mean Spread	0.112
S.d. of spread	0.076
Spread diff., 95th percentile	0.062
Frequency of Crises	0.038
Corr(Y,CA)	-0.186
Corr(Demand,CA)	-0.144
Corr(ΔY, CA)	-0.255
Corr($\Delta Y, \Delta$Spread)	-0.121
Corr(CA, Δ Spread)	0.012
Corr(Imports, Δ Spread)	-0.427
Mean Trade Balance (in % of GDP)	0.013
Mean Imports (in % of GDP)	34.3
Corr($d^\star$, Spread)	-0.339
Corr(GDP, Spread)	-0.003
Terms of Trade	0.863
Corr(Spreads,Terms of trade)	-0.339

terms of trade considerations. Our model is a good tool to think about long-run consequences of trade policy in a context with defaultable debt. The commitment of countries and their will to go on paying back depends on their gains from trade. As a consequence, many sovereign governments should be very sensitive to global systemic shocks affecting the demand for their good, hence their gains from trade. This effect is quite sizable and is a good candidate to explain the mysteries of the comovements of sovereign spreads.

Appendix

Appendix 3.A Optimal Tariff

In this case, the domestic demand for the country's own good can be determined the following way:

$$D^d(p) = d^\star \tau^{1-\sigma} \delta^{-\sigma} p^{-\sigma}.$$

The objective of the country is to exploit the national monopoly that it exerts over the variety it produces, and that national competitive firms are unable to exploit. To estimate the optimal tariff, we can study the first best policy through a primal approach. A government takes into account the price at which it can sell its good and its national monopoly power. It wants to set its level of domestic and foreign goods' consumption so as to solve:

$$\max_{C,C^\star \geq 0} u(c, c^\star).$$
$$s.t. (Y - c) + T = \tau c^\star$$

The corresponding first order condition is:

$$\frac{\frac{\partial u}{\partial c}}{\frac{\partial u}{\partial c^\star}} = \frac{(1 - \frac{1}{\sigma})(Y - c)^{-\frac{1}{\sigma}}}{\tau} = \frac{(\frac{\sigma - 1}{\sigma}) p_s}{\tau},$$

where $p^{d\star}$ is the price of the domestic good as perceived by foreigners, and p the price of the domestic good as perceived by domestic demand.

Let δ be an export tax. Under a competitive equilibrium with tariff δ, consumption would solve the following first order condition:

$$\frac{\frac{\partial u}{\partial C}}{\frac{\partial u}{\partial C^\star}} = \frac{p}{\tau} = \frac{p^{d\star}}{\delta\tau}.$$

Indeed, because of the export tax, the domestic price p is equal to the price paid by foreigners divided by the level of tariff: $p^{d\star}/\delta$. When tariff δ is equal to $\frac{\sigma}{\sigma-1}$, the country maximizes its welfare.

Appendix 3.B Tariff Retaliation

To study counterfactuals and evaluate the impact of free-trade policies, we are also going to assume that the rest of the world can apply a tariff. Because the elasticity of substitution is assumed to be similar between the domestic economy and foreign economies, and because we do not want to model explicitly the foreign economies (the number of state variables is already high from a computational point of view), we are simply going to assume that foreign countries can retaliate to an export tax with an import tariff. If foreign economies retaliate with tariff δ, then foreign demand function $D_t^{\star\,\text{retaliation}}$ becomes:

$$D_t^{\star\,\text{retaliation}}(p) = \delta^{-\sigma} D_t^\star(p)$$

where $D_t^\star$ is the demand function in the absence of retaliations. This way, we can model a free-trade agreement in a simple and straightforward manner.

Appendix 3.C Static Gains from Intratemporal Trade and Intertemporal Trade

In the paper, we wondered whether we could apply Arkolakis et al. (2012) in spite of trade deficits. We show a graph that shows that we must be careful. Let T be a given level of transfers (labeled in terms of *numéraire*). We compare two economies: one in which trade costs are low ($\tau_1 = 1$) and receives T, one in which there are the same transfers T but with larger trade costs ($\tau_2 = 1$). More precisely, we want to compare instantaneous consumption in two cases. We know consumption should be higher in the state with no tariff, but does it depend on the level of transfers T? Figure 6.1 compares the results with a corrected ACR formula that would take into account gains from trade:

$$ACR = \frac{1}{1-\sigma}\log\left(\frac{1-\frac{IM_1}{p_1Y+T}}{1-\frac{IM_2}{p_2Y+T}}\right)$$

where IM_i represents imports in scenario i and p_iY_i is nominal GDP in scenario i, while T indicates financial transfers received by the country: hence the ratio $\frac{IM_1}{p_1Y+T}$ represents the import penetration ratio.

We see that the two quantities coincide when the net level of transfers is 0, consistently with the original paper. However, this stops being true whenever net trade is not equal to 0.[15]

[15] There might be a problem due to the fact that transfers do not represent the same value when trade costs change. Several different specifications have given different results regarding the general behavior of the curve, but not the fact that the ACR formula does not apply in the presence of net trade. In the representation of the graph, we assume T stays constant although the perceived price of the *numéraire* and foreign good increases from the point of view of the domestic economy from τ_1 to τ_2.

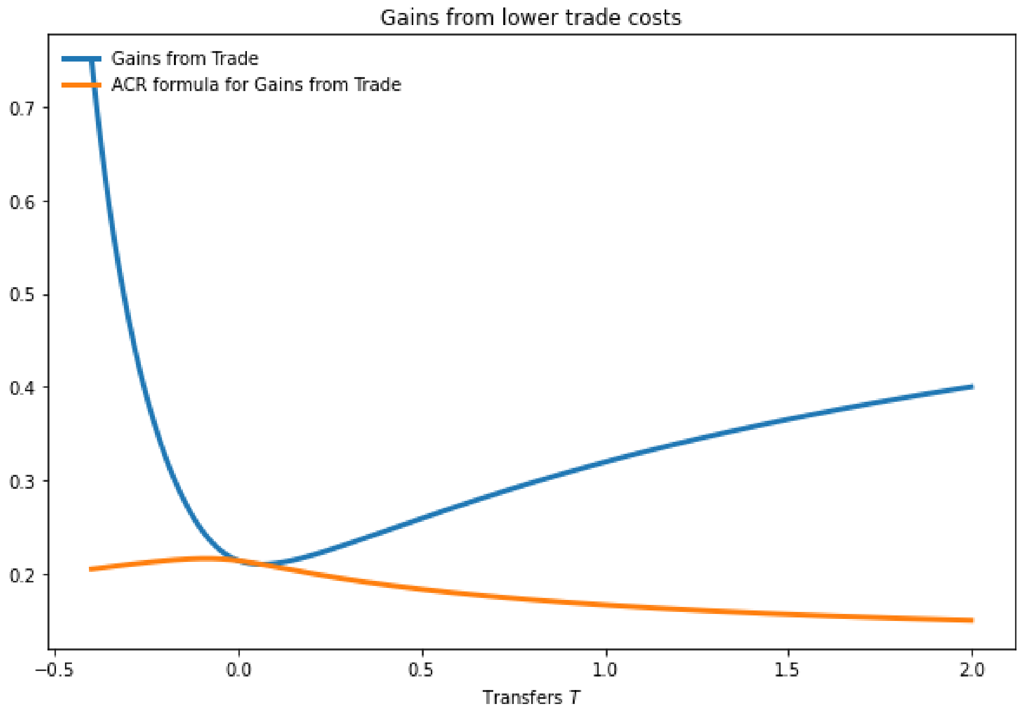

Figure 3.C.1: Gains from trade relative to a case with larger trade costs as a function of transfers T

We see that the effects of trade look magnified relative to the ACR formula whenever a country has a net surplus or a net deficit.

This should matter for our calibrations as a shock to foreign demand will affect differently a country accumulating deficits or surpluses - and their incentives to default as well. This feature, might add cyclicality to the costs of default, which implies terms of trade have a different effect from the trade mechanism in Mendoza and Yue (2012) where it was procyclical.

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