From Data to Assessment Models, Demonstrated through a Digital Twin of Marine Risers

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Abstract

Assessing the fatigue damage in marine risers due to vortex-induced vibrations (VIV) serves as a comprehensive example of using machine learning methods to derive assessment models of complex systems. A complete characterization of response of such complex systems is usually unavailable despite massive experimental data and computation results. These algorithms can use multi-fidelity data sets from multiple sources, including real-time sensor data from the field, systematic experimental data, and simulation data. Here we develop a three-pronged approach to demonstrate how tools in machine learning are employed to develop data-driven models that can be used for accurate and efficient fatigue damage predictions for marine risers subject to VIV.

Introduction

The dynamic response of marine risers placed in ocean currents is caused by the formation of vortices in their wake. The flow instability that results in vortex shedding is a complex phenomenon, especially because the riser configuration is often geometrically complex, and may include buoyancy modules, strakes, secondary pipes, or fairing devices, while ocean currents are sheared and unsteady often resulting in transient multi-frequency vibrations, containing high harmonics (Zheng et al. 2014; Triantafyllou and Bourguet 2016). This makes the accurate prediction of vortex-induced vibrations (VIV) and the resulting fatigue damage complicated for semi-empirical prediction tools, while the use of fully resolved CFD is prohibitively expensive for realistic Reynolds numbers and outright impossible for designing a marine riser.

Here we employ a completely different procedure by developing machine learning tools to construct a digital twin of a marine riser (DigiMaR). The DigiMaR utilizes various sources of training data, including field data, experimental data, CFD simulations, extracted databases, semi-empirical codes such as VIVA, as well as existing knowledge of the underlying physical models. We also show that a well-trained digital twin can efficiently use the streaming data from a few field sensors to provide an accurate reconstruction of motion and provide fatigue damage prediction.

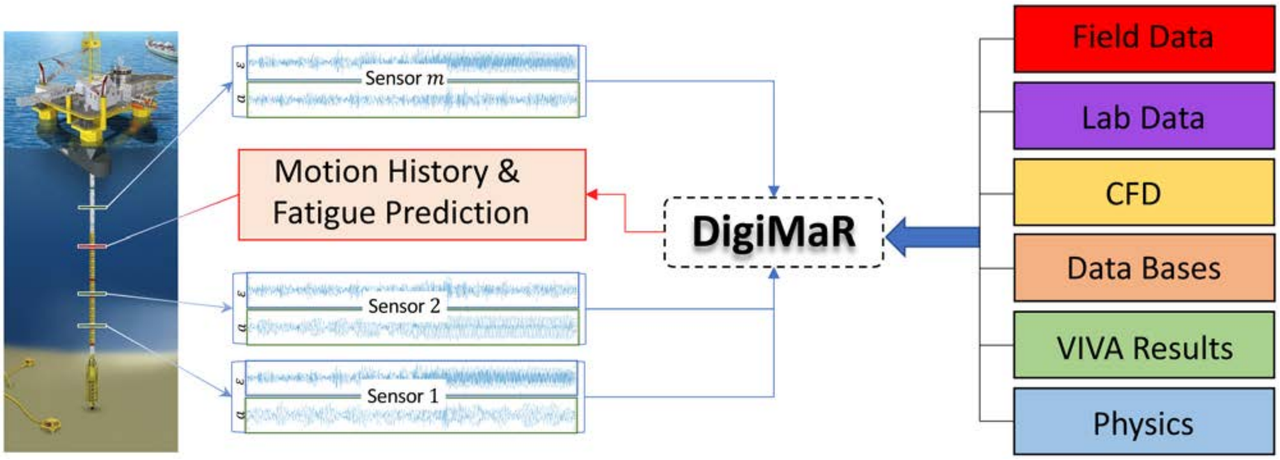

Figure 1—DigiMaR: a digital twin of marine risers. A data-driven assessment model based on several sources of multi-fidelity data, underlying physical model, and streaming filed sensor measurements.

Several machine learning algorithms have been developed in the literature to predict the life span of the structure through the changes in the parameters. To the authors' best knowledge, most of the existing methods are developed as black boxes that return parameters by only feeding experimental data, and therefore are ignorant of the underlying physics. Physics-informed neural networks (PINNs) (Raissi et al. 2019), however, provide the unique flexibility of encoding the mathematical model into the neural network. The deep learning of VIV problem using PINN formulation has been studied in (Kharazmi et al. 2020) to provide a predictive model by estimating the structural damping. A multi-fidelity PINN for predicting VIV problem in modal space has also been developed in (Meng et al. 2020) that can enhance the predicted accuracy compared to single-fidelity modeling. This framework is capable of assimilating dense inexpensive low-fidelity data, obtained from the semi-empirical code VIVA, with a small set of high-fidelity data, consisting of displacement measurements obtained from either fully-resolved 3D LES or experiments on large-aspect-ratio flexible risers. Other formulations of PINN have been also developed recently; see e.g. (Kharazmi et al. 2019a, 2020b; Jagtap et al. 2020).

Here, we focus on developing a three-pronged approach to reconstruct the dynamics in the entire spacetime domain by using several different sources of data. In the first approach, we enhance the capabilities of semi-empirical codes by developing efficient databases through active learning. In the second approach, we apply the *LSTM-ModNet* framework to reconstruct and analyze the entire motion of a riser in deep water from sensor measurements via modal decomposition in space and sequence learning capability of recurrent neural networks in time. Our formulation provides a tool that efficiently combines different types of sensor measurements, such as strain and acceleration. In the third approach, we introduce a higher level of abstraction and approximate the nonlinear operator that maps the inflow current velocity to the RMS function of the riser response. In particular, we employ the newly developed neural network *DeepONet* as a black box to learn the mapping between the input parameters (the inflow velocity, riser bending stiffness, and tension as a function of water depth) to the output parameters (strain, amplitude, and exciting frequencies as a function of water depth). In these approaches, we use the data from the NDP High Mode VIV Test to train the networks. Taken together, the three approaches demonstrate how machine learning methods and deep neural networks can be used to infer the riser dynamics from data, which if combined with the parametrized governing equations and streaming online data are used to develop robust and efficient digital twins of marine risers.

Enhance the Capabilities of Semi-empirical Codes through Active Learning

Active Learning of Hydrodynamic Database using the Intelligent Towing Tank

Semi-empirical codes, such as VIVA, Shear7, and VIVANA rely critically on hydrodynamic data encoded into databases to provide estimates of the vortex-induced forces. Databases are constructed to depend on the non-dimensional frequency and amplitude of response, as well as the geometry of the crosssection of the riser, e.g., bare versus straked, or a faired section. However, a multitude of other parameters influences the databases, including Reynolds number, multi-frequency response, the effect of in-line motions, surface roughness, and external turbulence conditions. It is virtually impossible to conduct systematic data to derive databases for semi-empirical codes as it would require an astronomical number of experiments. For this reason, we employ active learning to reduce by orders of magnitude the effort to derive databases and introduce the possibility to derive databases from field data.

Initially, databases were derived by conducting experiments on cylinders vibrating only in the crossflow (CF) direction at prescribed frequencies and amplitudes. In particular, studies focused on the mean drag coefficient C_d, the lift coefficient in-phase with the velocity C_{lv}, and the added mass coefficient in the CF direction C_{my} as a function of the true reduced frequency $f_r = fD/U$ and non-dimensional CF amplitude A_y/D, where U is the prescribed fluid velocity; f is the prescribed motion frequency; A_y is the prescribed motion amplitude; and D is the cylinder model diameter (Gopalkrishnan 1993). However, when adding in-line (IL) motion, it is found that the phase θ between the IL and the CF motions has a strong influence on fluid forces (Dahl 2008). Measured hydrodynamic coefficients of a rigid cylinder undergoing forced vibrations are helpful to understand the nature of rigid cylinder free vibrations (Wang et al. 2020a), but also serve as the hydrodynamic databases for the state-of-art semi-empirical software (such as (Triantafyllou and Triantafyllou 1999)) that assumes the validity of strip theory (Wang et al. 2020b). Prediction of the hydrodynamic force distribution along the riser using rigid cylinder forced vibration experiment was found to lead to accurate prediction of the riser structural response (Fan and Wang et al. 2019). In order to systematically study rigid cylinder forced vibration forces, we have developed at MIT Sea Grant the Intelligent Towing Tank (*ITT*), an automated experimental facility guided by active learning (empowered by the Gaussian Process Regression (GPR)), capable of learning to understand complex fluid-structure phenomena (Fan and Jodin et al. 2020) automatically, shown in Fig. 2. Using the ITT, we studied different problems of rigid cylinder forced vibration experiments, including single cylinder combined-CF-and-IL vibrations, the Reynolds number effect (Fan and Jodin et al. 2020), as well as dual tandem cylinder CF-only and combined-CF-and-IL vibrations (Lin and Fan et al. 2020).

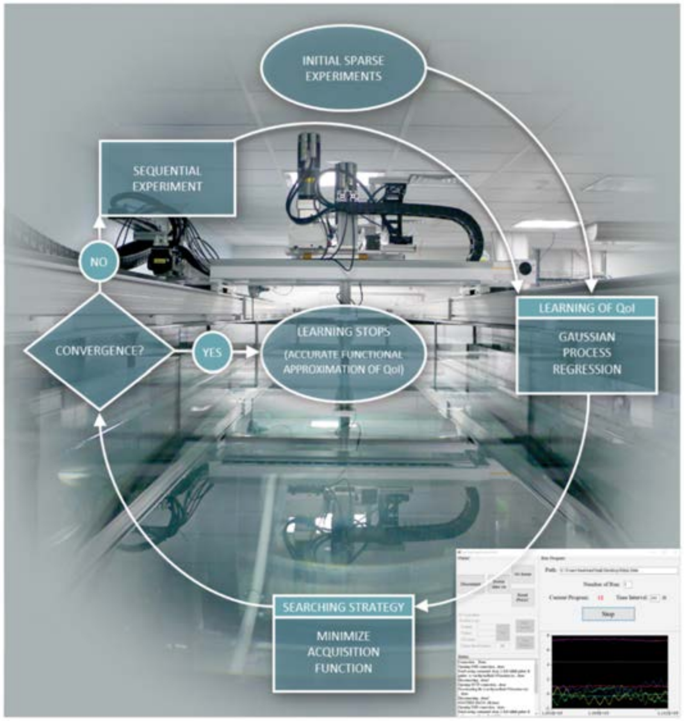

Figure 2—Front view of ITT with the key steps for sequential learning of complex fluid-structure dynamics. The process of the ITT commences once a hypothesis is proposed, then performs the adaptive sequential experiment to learn target hydrodynamic coefficients, interrupted only by pause periods between experiments to avoid cross-contamination of the results between successive experiments. Upon convergence, the results of learned hydrodynamic coefficients are further post-processed to examine the validity of the hypothesis.

Parametric Construction of Hydrodynamic Database

ITT has shown to be a powerful tool to construct the complex hydrodynamic database for forced vibrating rigid cylinder undergoing various prescribed motions. However, these hydrodynamic coefficients are sensitive to several parameters, including Reynolds number (Fan and Triantafyllou 2017), riser configuration (Xu et al. 2013), surface roughness (Chang et al. 2011), inflow condition (Han et al. 2018), etc. Hence, the systematic development of a hydrodynamic database is virtually impossible. Long-term effects such as equipment aging and bio-fouling inevitably alter the hydrodynamic coefficients throughout the lifetime of a riser in the field, making long-term riser prediction and monitoring even more challenging. By incorporating knowledge from large quantities of experimental data with fluid dynamics principles, we propose a parametric form for VIV hydrodynamic databases that are capable of capturing the dominant behavior of a large variety of experiments including forced and free vibrations of rigid cylinders. The parametric form for hydrodynamic databases is low-dimensional and interpretable, shown in Fig. 3.

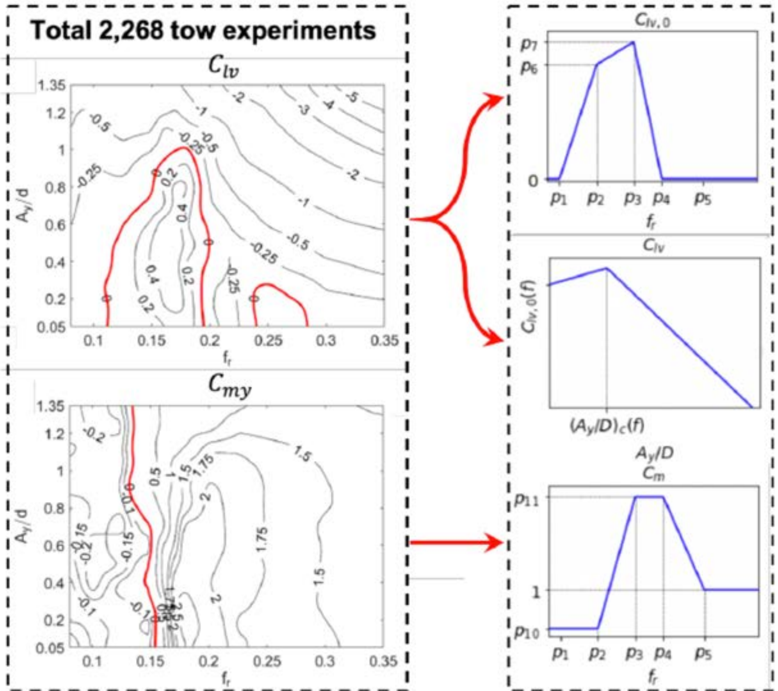

Figure 3—The simplification of hydrodynamic coefficient database constructed by large number of rigid forced vibration experiments [7], using simple piecewise linear models with a set of parameters to be learned from the data (right).

We learn optimal database parameters using machine learning to compare predictions with experimental data. Given a set of parameters for the hydrodynamic databases and arbitrary flow conditions, we are able to reconstruct the behavior of rigid cylinder CF-Only free vibrations using forward models. For a rigid cylinder of mass m, diameter D, and length L, mounted on springs with spring constant k and dashpots with damping constant b in a uniform current of velocity U, the governing equation of motion takes the form $m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=F_l(t)$, where $F_l(t)$ represents the oscillatory lift force acting from the fluid on the rigid cylinder. Assuming the system will achieve a harmonic oscillation, namely $y(t)=\mathrm{Re}\{A_y e^{i\omega t}\}$, where A_y and ω are the steady vibration amplitude and frequency, and $Re\{*\}$ is the real part of the solution, the lift force can be modeled as $F_l(t)=Re\left\{\left[C_{my}\rho_f\nabla+i\left(\frac{\rho_f U^2}{2}\right)DLC_{lv}\right]e^{i\omega t}\right\}$, where ρ_f is the fluid density and $\nabla=\frac{\pi}{4}D^2L$ is the cylinder displacement fluid volume. By substituting $F_l(t)$ and separating the real and imaginary parts, we reach the set of equations in a non-dimensional form predicting the rigid cylinder CF-only VIV response as follows,

$$V_r(U_r)=U_r\sqrt{\frac{m^*+C_{my}}{m^*+1}},\quad A^*(U_r)=\frac{A_y(U_r)}{D}=\frac{C_{lv}U_r^2}{4\pi^3(m^*+1)\zeta},\tag{1}$$

where V_r is the true reduced velocity, the inverse of the true reduced frequency, $U_r = U/(f_n D)$ is the nominal reduced velocity, f_n is the natural frequency of the cylinder in the still water, m^* is the mass ratio and ζ is the structural damping ratio. Given such a forward prediction model and the experimental dataset, we can form the objective function that we seek to minimize as follows,

$$J(\mathbf{p})=\sum_{j=1}^{m}\frac{\left(f_{r,j}-\widehat{f_{r,J}}\left(U_{r,j},m_j^*;\mathbf{p}\right)\right)^2}{\sigma_{f_r}^2}+\frac{\left(A_j^*-\widehat{A_j^*}\left(U_{r,j},m_j^*,\zeta_j;\mathbf{p}\right)\right)^2}{\sigma_{A^*}^2},\tag{2}$$

where $\hat{f}_r$ and $\widehat{A^*}$ is the prediction from the forward model, σ^2_{fr} and $\sigma^2_{A^*}$ are the variance used to normalize the error in the objective function. Therefore, the optimal parameters can be obtained for the parametric hydrodynamic databases as $\hat{\mathbf{p}} = \text{ary}\min_{\mathbf{p}}[J(\mathbf{p}) + \mathcal{R}(\mathbf{p})]$, where $\mathscr{R}(\mathbf{p})$ indicates the regularization term used in the optimization. Here, we demonstrate the application of the proposed methodology to the experimental datasets for rigid cylinder VIVs using force feedback apparatus (Smogeli 2003). Smogeli et al. (2003) studied the CF-Only free vibration of a rigid cylinder over a large range of U_r with a fixed Re = 20,000. The reconstructed results of C_{lv} and C_{my} are plotted in Fig. 4. The result shows that learned C_{lv} contour is similar to past CF-Only forced vibration result (Gopalkrishnan 1993) at a different Re = 10000. Furthermore, we plot in Fig. 5 for the comparison between the experimental result and prediction using optimal parametric hydrodynamic database shown in Fig. 4. It is found that the predicted structural response (red) is in good agreement with the experimental result (blue) for both displacement and frequency response. It is noteworthy that with an increase of ζ, the maximum A_y/D decreases.

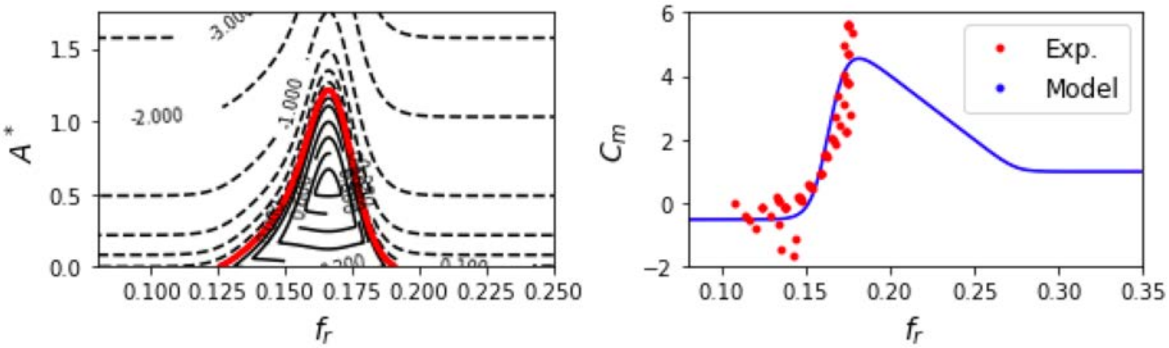

Figure 4—Optimal parametric hydrodynamic database of C_{lv} (left) and C_{my} (right) learned from the rigid cylinder CF-only free vibrations at Re = 20000 [23].

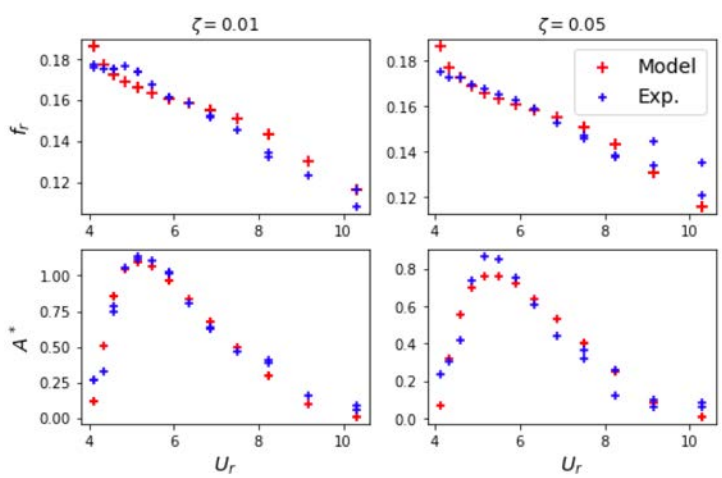

Figure 5—Comparison between the experimental result (blue cross) and forward model prediction (red cross) using optimal parametric hydrodynamic database in Fig 4 for f_r (first row) and A^* (second row) of two damping ratios: ζ = 0.01 (left column) and ζ = 0.05 (right column).

LSTM-ModNet

The LSTM-ModNet is a framework to reconstruct and analyze the motion of a riser in deep water from sensor measurements. It combines the sequence learning in time with modal decomposition in space to reconstruct the response in the entire space-time domain. We essentially postulate the system response in the form of $\varepsilon(z,t) = \sum_{n=1}^{N} C_n(t)\phi_n(z)$ where $\phi_n(z)$ and $c_n(t)$ are the prescribed modes and the modal coordinates, respectively. In particular, we choose the long short-term memory (LSTM) networks in the sequence learning in time to learn the modal coordinates $C_n(t)$ and use Fourier modes $\phi_n(Z) = \sin\left(\frac{n\pi}{L}Z\right)$ for decomposition in space.

The structure of LSTM-ModNet, shown in Fig. 7 is comprised of two main compartments: *i*) LSTM that outputs the values of modal coordinates in time and *ii*) modal reconstruction. LSTM is one of the most successful network architectures for sequence learning (Hochreiter and Schmidhuber 1997). It introduces the memory cell as a unit of computation that replaces the traditional nodes of neural networks. The typical LSTM regulates the flow of training information through three gates: the input gate that selectively adds information, the forget gate that removes information, and the output gate that passes the information. The embedded memory in LSTM networks is a key factor in successful motion reconstruction as the feed-forward neural networks fail to accurately learn long term high frequency signals. Space discretization techniques that use modal decomposition are often employed to obtain a reduced-order model of the system, which encapsulates most, if not all, of the fundamental dynamics of the original more complex system. The advantage of LSTM-ModNet is manyfold. It has the capability to efficiently combine different types of sensor measurements, i.e. strain and acceleration, into the training algorithm. It introduces a dynamic weight coefficient that can effectively balance the training between the strain and acceleration data, leading to a robust learning framework. It implements a spectral filtering of the reconstructed motion by adding extra derivative-based regulariser terms to avoid over-fitting. The trained LSTM-ModNet will yield a continuous space-time response of the riser, which can be used to reconstruct the force fields and obtain the hydrodynamic coefficients of the riser. We applied the developed framework in analyzing the experimental NDP data sets.

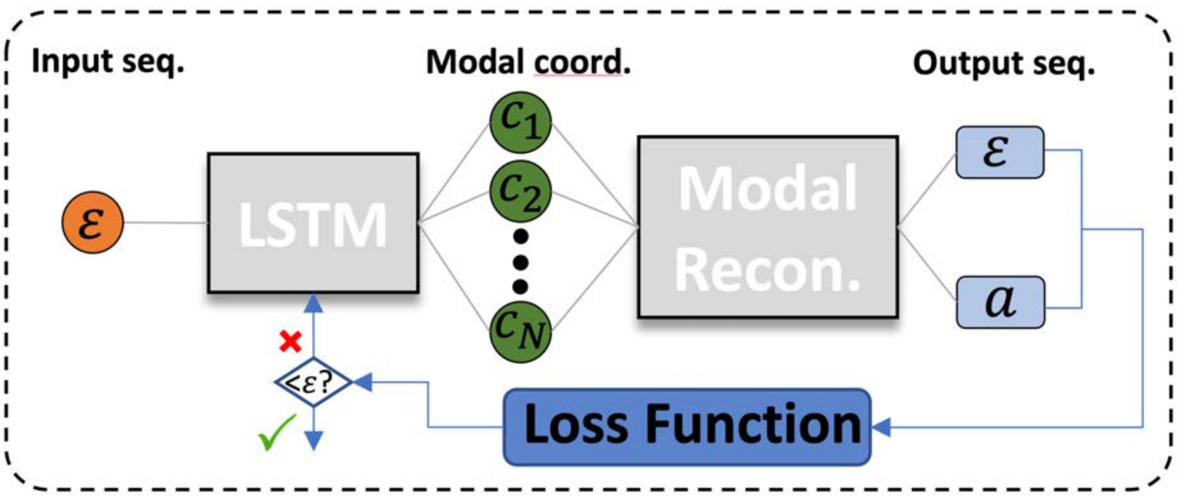

Figure 7—LSTM-ModNet structure. The LSTMS part to learn the modal coordinates $c_n(t)$ and the second part to reconstruct the motion.

We use the LSTM-ModNet to reconstruct the motion of a riser using the Norwegian Deep Water Program (NDP) data set. Here, we consider the uniform flow case number 2030. The results are shown in Fig. 8 for the cross flow (CF) motion. The input to LSTM-ModNet is the time-series measurement of strain only at 24 sensor locations along the riser. The output is the reconstructed strain and acceleration along the entire riser. The loss function is formed based on the mismatch between the LSTM-ModNet output and the available data. We note that we use a sliding window technique in learning the sequence of strain and acceleration as the available data is usually dense in time. This technique speeds up the training of the network without any significant decrease in the training accuracy. We see from the top panel in Fig. 8 that the LSTM-ModNet can accurately learn the non-smooth signals of strain and acceleration. The plot shows the results at one of the sensor locations at z/L=0.66. In the bottom panel in Fig. 8, we see that the reconstruction of the strain and acceleration along the entire riser matches very well with the training data. The results are shown by computing the standard deviation in time.

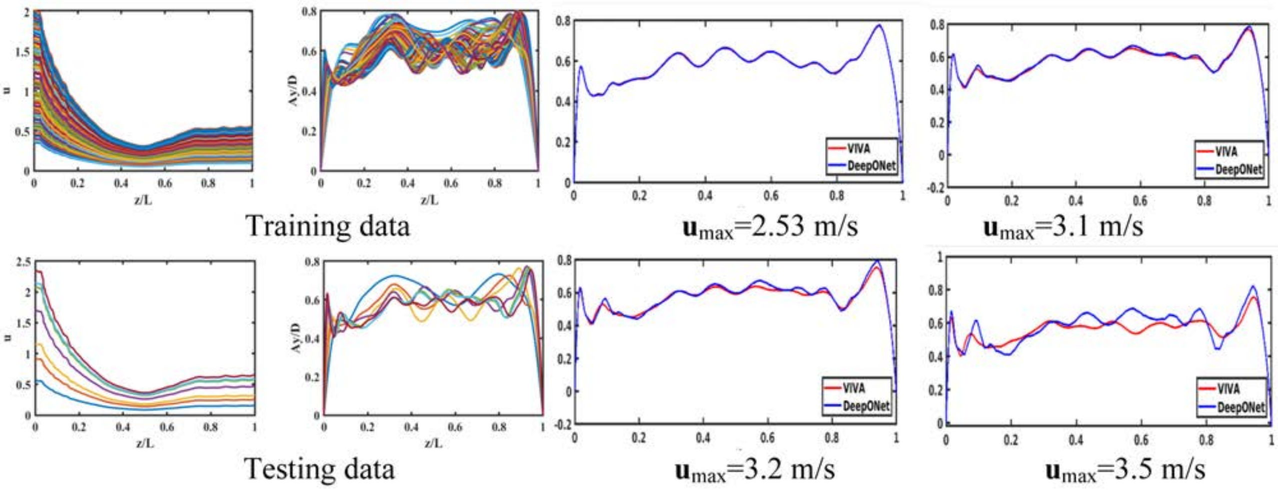

Figure 8—DeepONet training and prediction of VIV amplitude. Left four figures, the training and testing data; right four figures, the predictions on the *unseen* data.

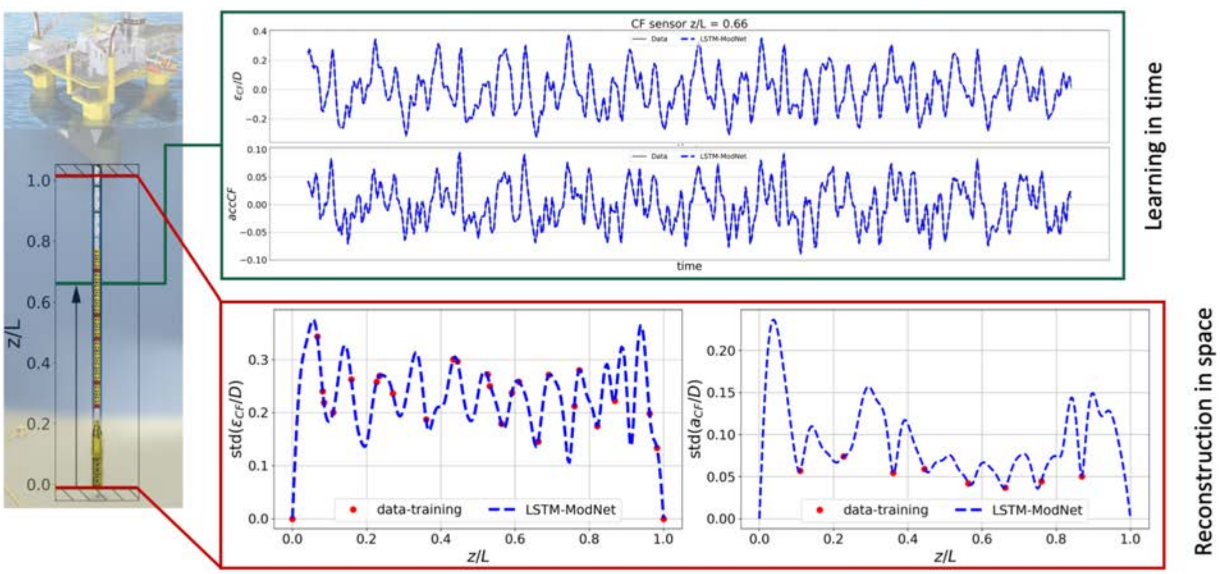

Figure 9—LSTM-ModNet: motion reconstruction using NDP data set case number 2030. Top: accurate learning of the signal in time shown at one of the training sensors. Bottom: reconstruction of strain and acceleration over the entire riser by computing the standard deviation in time.

DeepONet

Here, we develop a purely data-driven approach for predicting the vortex-induced vibration (VIV). In theory, it is proved that the neural networks (NNs) not only can be used to approximate any continuous function (Hornik et al. 1989), but also approximate accurately any continuous nonlinear functional (Chen and Chen 1993) or operator (a mapping from a function to another function) (Chen and Chen 1995). Based on the approximation theorem for operators, a specific network architecture, namely the deep operator network (DeepONet) is proposed in (Lu et al. 2019), which also presents the theoretical analysis and demonstrates various examples to show the high accuracy and high efficiency of this formulation. It is shown that DeepONet can learn both explicit as well as implicit operators, e.g., in the form of PDEs. DeepONet has a unique structure. The network is made of two sub-networks, the *branch net* for the input function and the *trunk net* for the coordinate to evaluate the output function.

Here, we use DeepONets to build a black box that can map the inflow velocity profile and the amplitude along the flexible riser. We develop a fast prediction tool that can infer the amplitudes providing that the inflow velocity is given. To construct the training dataset for DeepONet, we consider105 cases of different sheared inflow profiles (while all other flow and structural parameters are the same) obtained by using the VIVA program (Triantafyllou and Triantafyllou 1999). As shown in Fig. 8, the input of the *branch net* is the inflow velocity functions at 900 locations that are uniformly distributed in the range [0,1], the output of the *trunk net* are the amplitudes at 300 locations that are also uniformly placed in the same range. In particular, the results of the 81 cases are used as the training data, while the rest 20 cases are used as test data. In this case, both the *branch net* and *trunk net* have 2 hidden layers with 300 neurons in each layer. Moreover, the learning rate is 5×10^{-4}, *ReLU* activation function, *Glorot normal* initializer are used and the DeepONet training stopped after 6000 epochs. It is worthy to mention that, as shown in Fig. 8 (left), the DeepOnet is trained using a dataset that has the highest value of the inflow velocity ($\mathbf{u}_{max}$) is in the range [0.5 m/s, 3.0 m/s]. After training, the DeepONet is used to predict the amplitudes of the unseen inflow velocity. As shown in Fig. 8 (right), the predictions agree with ground truth (VIVA calculations) very well when the inflow velocity is inside the training range or not in the range of far away.

CONCLUSIONS

We demonstrate the development of a framework for digital twin for marine risers, DigiMaR that utilizes a variety of training data, including field sensor data, systematic experimental data, CFD simulations, existing databases, semi-empirical codes such as VIVA, as well as knowledge of the underlying physical models. A well-trained digital twin is shown to efficiently use streaming data from only a few field sensors to provide an accurate reconstruction of motion and hence provide improved fatigue damage prediction.

A three-pronged approach was used to develop the algorithms appropriate for encapsulating the complex dynamics of vortex-induced vibrations. In the first approach, we target the enhancement of the databases of semi-empirical codes using active learning techniques. We show that both the development of a database can be achieved at a fraction of the time and cost previously required, while the methodology allows for the enhancement of databases using field data.

The second approach consists of the *LSTM-ModNet* framework. This is used to reconstruct and analyze the entire motion of a riser in deep water from sensor measurements using modal decomposition in space and sequence learning capability of recurrent neural networks in time. We show that it efficiently combines different types of sensor measurements, such as strain and acceleration, to reconstruct the entire motion and fatigue response.

In the third approach, we employ operator regression via the neural network *DeepONet*, which is used as a black box to learn the mapping between the input parameters, consisting of the inflow velocity, riser bending stiffness, and tension as a function of water depth, to the output parameters, viz. strain, amplitude, and exciting frequencies as a function of water depth.

We demonstrate the methodologies using data from the NDP High Mode VIV Test to train the networks, although the methodologies are applicable to any set of multi-fidelity data. The three approaches demonstrate how machine learning methods and deep neural networks can be used to infer the riser dynamics from data, which are then combined with the parametrized governing equations and streaming online sensor data to develop efficient and robust digital twins of marine risers.

Acknowledgments

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