Transient Behavior and Predictability in Manipulating Complex Objects

Transient Behavior and Predictability in Manipulating Complex Objects

Rashida Nayeem, Salah Bazzi, Neville Hogan, and Dagmar Sternad

***Abstract*— Relatively little work in human and robot control has examined the control of complex objects with intrinsic dynamics, such as carrying a cup of coffee, a task that presents little problems for humans. This study examined how humans move a cup-of-coffee with a view to identify principles that may be useful for robot control. The specific focus was on how humans choose initial conditions to safely reach a steady state. We hypothesized that subjects choose initial conditions that minimized the transient duration to reach the steady state faster as it presented more predictable dynamics. In the experiment the cup of coffee was reduced to a 2-D cup with a sliding ball inside which was simulated in a virtual environment. Human subjects interacted with this virtual object via a robotic manipulandum that provided haptic feedback. Participants moved the cup between two targets without losing the ball; they were instructed to explore different initial conditions before initiating the continuous interaction. Results showed that subjects converged to a small set of initial conditions that decreased their transient durations and achieved a predictable steady state faster. Simulations with a simple feedforward controller and inverse dynamics calculations confirmed that these initial conditions indeed led to shorter transients and less complex interaction forces. These results may inform robot control of complex objects where the effects of initial conditions need further investigation.**

I. INTRODUCTION

Humans are exquisitely adept at using tools and interacting with dynamically complex objects. Tool use ranges from the simple swinging of a hammer to highly complex actions, such as cracking a whip. A seemingly mundane example of physically interacting with a complex object is carrying a cup of coffee: when the human moves the cup, it exerts a force on the coffee, which in turn, applies forces back on the cup and the hand. Despite these nonlinear interaction forces, humans are extremely skilled in such tasks, which is surprising considering their slow neural transmission rates and their high levels of intrinsic noise [1]–[4]. Better understanding of how humans manage complex objects with such dexterity may be instructive for robotic control and manipulation [5].

The most frequently studied task in robotic manipulation is the pick-and-place paradigm, and much of this research has been confined to rigid objects [6]. Manipulating flexible or underactuated objects present significant challenges: Successful control would require models of deformation and complex interaction forces, which require daunting computations for both humans and robots [7]. Moreover, robotic manipulation has also focused on the pre-contact stage, i.e., on planning the grasp, rather than the subsequent interaction, except when the object is placed at another position [8]. Goal-directed tool use with continuous physical interaction has received relatively little attention to date.

Current approaches in robotic control and motion generation have predominantly focused on steady-state behavior and stability. However, when initiating an action from a steady state, the system inevitably starts from initial conditions and passes through a transient state, which may be unstable. The transient state between steady states, remains to be thoroughly investigated, nevertheless further study of transient dynamics would be of great use to the field of robotics [9]–[11]. Since stability is critical for robot controllers, the approach has been to build locally stabilizing control policies that eliminate the effect of initial conditions *i.e.*, funneling initial conditions into the desired behavior [12] [13]. These control policies effectively create basins of attraction that span the entire state space. However, it is also possible to control a dynamical system by appropriately setting its initial conditions.

Recent studies in motor neuroscience have provided some evidence that the brain generates movements by setting appropriate initial conditions [14] [15]. Specifically, the neural networks in the brain responsible for movement execution behave as dynamical systems that are initialized with a desired state and then driven into patterns of collective activity [16]. In this spirit, Ernesti et al. generated a control policy for a humanoid robot wiping a windshield that selected initial conditions to tune transient behaviors using dynamic motion primitives [17]. This present study examined how humans start movements and whether they chose appropriate initial conditions to set up predictable interactions.

Transient dynamics can create unpredictable interaction forces; mastering this start-up transient to reach a desired steady state is not trivial. The human motor system faces significant limitations in its hardware: transmission speed peaks at ≈ 100 m/s and bandwidth in the human muscular response rarely exceeds 5 Hz [18]. Variability and noise are ubiquitous in the human system, with an approximate precision in timing of 9 ms [19] [18]. These features challenge the use of feedback control in continuous physical interaction because loop times are not fast enough for adaptation and error corrections [20] [21]. We hypothesize that humans deal with these shortcomings in the neuromotor system by making interactions with objects predictable.

Previous work in our lab showed that in continuous

Rashida Nayeem, Salah Bazzi and Dagmar Sternad are in the Department of Electrical and Computer Engineering and the Department of Biology, Northeastern University, Boston, Massachusetts 02115. Neville Hogan is in the the Department of Mechanical Engineering, MIT, Cambridge 02139. Corresponding author email: nayeem.r@husky.neu.edu. This work was supported by the CRCNS-1723998 grant awarded to D. Sternad. N. Hogan was supported in part by NSF NRI 1637824, NSF CRCNS 1724135, NSF M3X 1826097, NIH R01-HD087089.

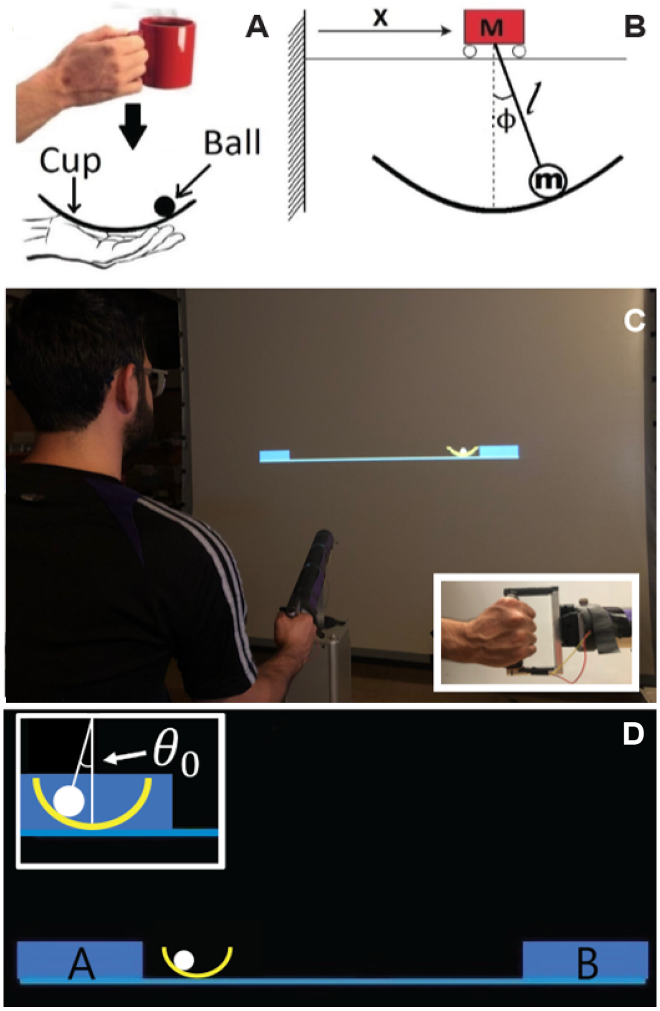

Fig. 1. (A) Real and simplified task. (B) Mechanical model. (C) Virtual environment showing a participant holding the HapticMaster robot to move the cup. The inset shows the subject's grip the robot handle. (D) Screen display. The inset shows the definition of the ball angle, where the clockwise direction was defined as negative.

rhythmic interactions, subjects indeed increased predictability of the object dynamics by selecting hand impedance, even though this required increased mechanical effort [22]–[24]. However, in the previous studies initial conditions of the object were fixed, and the analysis only focused on steady state behavior. The present study aimed to answer the following question: when given the option to choose them, do humans explore and exploit initial conditions? We investigated how humans physically interact with a non-rigid object in a continuous manner, transporting a 'cup of coffee'. Three specific hypotheses were tested: 1) Subjects will converge to a subset of all possible initial conditions. 2) Subjects will shorten their transient duration. 3) Subjects will maximize the predictability of the task dynamics. Results from the human experiments were compared with results from two simulations of the cart-and-pendulum system: inverse dynamics control and forward simulations using a coupled model with hand dynamics [25]. These findings may inform new control strategies for robotic manipulation of complex objects.

II. Methods

A. Mechanical Model

A cup of coffee is underactuated since the coffee motion is coupled to motion of the cup and cannot be controlled directly. For the virtual rendering, the real task was simplified to a two-dimensional semi-circular cup with a ball inside displayed on a large projection screen (Fig.1C). This simplification maintained the essential features of this task: physical interaction, underactuation, and nonlinearity [4]. The semi-circular cup was confined to moving on a horizontal line with a ball sliding inside. Under the premise that the ball was sliding and not rolling, it was identical to the well-known model of a cart with a suspended pendulum: the ball corresponded to the pendulum bob, the cup position corresponded to the cart position, and the arc of the cup corresponded to the semi-circular path of the pendulum, where the clockwise direction was defined as negative (Fig.1). Subjects moved the cup-and-ball system on a horizontal line using a robotic manipulandum. The robot transmitted the interaction force between the cup and ball to the subject. The equations of motion are:

$$(m_c + m_p)\ddot{X} = m_p d[\dot{\theta}^2 sin\theta - \ddot{\theta} cos\theta] + F_{inter} \quad (1)$$

$$m_p d[\dot{\theta}^2 sin\theta - \ddot{\theta} cos\theta] = F_{ball} \quad (2)$$

$$\ddot{\theta} = -\frac{\ddot{X}}{l} cos\theta - \frac{g}{l} sin\theta \quad (3)$$

where X is the cart position, θ is the pendulum angle, F_{inter} is the force applied by the human on the cart, and F_{ball} is the force applied by the pendulum on the cart. Parameters of the system were: cart mass m_c, pendulum mass m_p, pendulum length l, and gravitational acceleration g. The parameter values m_c = 2.40 kg, m_p = 0.60 kg, and l = 0.45 m were chosen because the cart and pendulum masses were light enough to avoid fatigue, but heavy enough such that subjects felt the forces generated by the ball.

B. Apparatus and Data Acquisition

Participants were seated on an adjustable chair 2.15 m in front of the screen (2.40 x 2.40 m) and grasped a handle on a 3-DOF robotic manipulandum (Fig.1C). The force applied by the participants on the handle, F_{inter}, controlled the position of the virtual cup X with a point input port, eliminating the grasping forces on the cup [23] [26]. The movements of the robotic arm were restricted to horizontal translations parallel to the participants' frontal plane to ensure a one-dimensional motion of the cup as in the model. Participants felt the interaction force (system inertia and F_{ball}) via the force feedback provided by the robot [26]. A custom-written C++ program based on the HapticAPI (Moog FCS Control Systems) computed the ball kinematics, controlled the virtual display and fed the force back to the subject. The visual display showed two blue rectangular targets on a horizontal line delimiting the desired displacement of the cup; a yellow

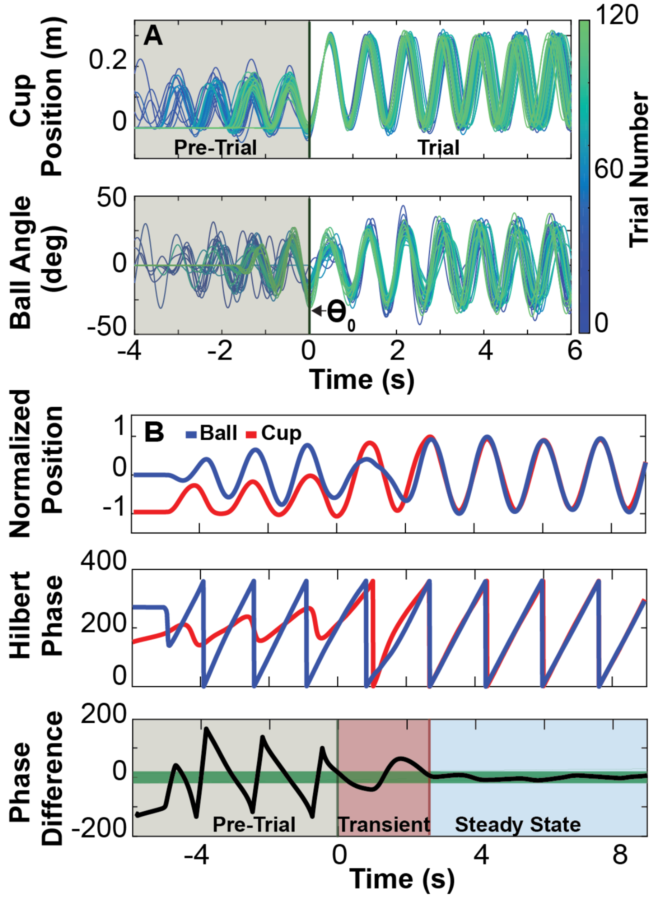

Fig. 2. (A) Time series of cup position and ball angle of a block of 120 trials of one subject aligned by the moment of initiation. (B) Method for transient calculation on one trial of one subject: based on cup and ball position continuous Hilbert phase was calculated. The difference between cup phase and ball phase was used to determine the beginning of the steady state of the trial.

semi-circle represented the cup and a small white circle represented the ball (Fig.1 D). The ball would escape from the cup if the ball angle exceeded ± 50 degrees. The visual displacement of the cup was 4 times larger than the physical displacement of the robot. The cup displayed on the screen was 7.5 times smaller than the simulated cup to convey a plausible size. These parameters were chosen such that the visuomotor gain was as close to 1:1 as possible so that it would not stretch the perception of realism. The applied force F_{inter}, the cup position X, velocity $\dot{X}$, and acceleration $\ddot{X}$, and the computed angular position θ, velocity $\dot{\theta}$, and acceleration $\ddot{\theta}$ of the ball were recorded at 120 Hz.

C. Experimental Task

At the beginning of each trial, the cup was positioned in Box A with the ball resting at the bottom of the cup (0 deg) (Fig.1). Prior to starting the rhythmic task subjects were encouraged to explore the best initial conditions by 'jiggling' the cup back and forth for as long as they wanted; this was called the 'Pre-Trial' period (Fig.2A). Once they felt ready to start the trial, they moved the cup towards Box B and continued moving in rhythmic fashion between the two boxes without losing the ball ($|\theta| < 50$ deg). A metronome started when the participant first reached Box B and continued to pace their movements at 0.6 Hz for 15 s. The distance between the centers of Box A and Box B was 0.3 m. Importantly, subjects were not explicitly told to shorten their transient duration. The experiment consisted of 4 blocks of 30 trials each. 13 college-age adults volunteered for the experiment (8 male). They gave written informed consent before the experiment, as approved by the Northeastern University Institutional Review Board.

III. Data Analysis

The variables that tested the hypotheses were initial ball angle, transient duration and predictability of the trial segment.

1) Initial Ball Angles (θ_0): Initial conditions of the cup and ball were defined at the time when the cup position was at its left-most excursion before subjects started the continuous rhythmic movements (Fig.2A). All movements prior to that moment were considered the 'Pre-Trial' period and movements after that point were considered to be in the trial. The most important variable was the ball angle at this instant, θ_0. Cup velocity at this maximum excursion was zero by definition; ball velocity and cup position were evaluated, but proved to be small and have minimal effect on the subsequent trial, both in simulation and experiment.

2) Transient Duration: To calculate the duration of the transient, a criterion for steady state first had to be determined [27]. For the movement frequency of 0.6 Hz, mathematical analysis showed that the ball and cup position were in-phase (i.e. had zero phase difference) at steady state. To identify when the transient trajectories approached steady state, the instantaneous phase differences between cup and ball position were computed using Hilbert transforms (Fig.2B) [28]. Since human trajectories are prone to variability, a threshold had to be set to define the onset of relative zero phase difference between cup and ball. This threshold was ± 27 deg (± 15 percent of 180 deg). The time point when the phase difference entered and remained within this threshold defined the end of the transient and start of the steady state (Fig.2B).

3) Mutual Information: Predictability was mathematically operationalized by mutual information (MI) between the input and output of the system. High MI indicates a high degree of certainty [29]. Unlike cross correlation, MI assesses both linear and nonlinear dependencies [30] [31]. In the present context, MI quantified how much the cart trajectory predicted the interaction force. The cart trajectory was represented by its' phase in state space, $\phi(t) = arctan\left[\frac{\dot{X}}{(2\pi f X)}\right]$. $F_{inter}(t)$ was defined above. MI was;

$$MI(\phi, F_{inter}) = \iint P(\phi, F_{inter}) ln\left[\frac{P(\phi, F_{inter})}{P(\phi)P(F_{inter})}\right] d\phi dF \quad (4)$$

where P denotes the probability density functions for $\phi(t)$ and $F_{inter}(t)$ [29]. The probability density functions were estimated by linear interpolation of nonlinear Gaussian smoothing kernels, using Silverman's method for finding the parameters [32]. MI was calculated from the beginning to the end of the trial. It is a dimensionless quantity and is displayed on a natural log scale (nat).

IV. Results

A. Experimental Results

Maintaining the target frequency of 0.6 Hz was challenging; the average frequency that subjects attained was 0.58 Hz with a range of 0.55 Hz to 0.63 Hz. Note that 2 of 13 subjects had more than 40 percent failures, i.e. they lost the ball, and were excluded from analysis. The percentage of failed trials across the remaining subjects was 16.7 percent. Their average peak-to-peak amplitude was 0.308 m.

1) Initial Ball Angles (θ_0): The average initial ball angles across subjects changed from -9.69 deg in the first 5 trials (standard deviation 9.94 deg) to an average of -22.28 deg in the last 5 trials (standard deviation 10.59 deg). This change was statistically significant ($p = .0066$). Fig.3A shows the gradual change of ball angles across subjects that seemed to converge and reach an asymptote after about 60 trials. This result supported Hypothesis 1 that subjects converged to a preferred initial condition. The two other variables at initiation, cup position and ball velocity, were negligible: The mean X_0 was -0.157 $\pm$ .019 m (the centers of Box A and Box B were located at -0.15 m and 0.15 m respectively), the mean $\dot{\theta}_0$ was -15.12 deg/s $\pm$ 31.65 deg/s.

2) Transient Duration: The transient duration decreased over the 120 trials, supporting Hypothesis 2 (Fig.3B). In the first 5 trials, transient duration was on average 8.02 s (standard deviation 4.85 s) decreasing to an average of 3.67 s (standard deviation 3.10 s) in the last 5 trials. This significant decrease ($p = 0.0024$) shows that subjects reduced the transient duration, although this was not an explicit goal of the task.

3) Mutual Information: Mutual information (MI) between the cup kinematics and the interaction force increased from 1.137 nat in the first 5 trials to 1.385 nat in the last 5 trials (Fig.3C). The t-test confirmed the statistical significance of this change ($p = 6.8501e^{-05}$). The maximum achievable value of MI is $\approx$ 1.8 nat [22]. This increase supported Hypothesis 3 that participants sought to increase predictability of their interactions with the system.

B. Simulation Results and Hypothesis Testing

To further understand the subjects' choice of initial conditions given the known dynamics of the complex object, inverse dynamic and forward dynamic simulations were conducted. In the simulations initial ball velocity was $\dot{\theta}_0 = 0$ deg/s, it was found that varying $\dot{\theta}_0$ did not make a significant impact upon the simulation results.

1) Effect of initial conditions on interaction force: Inverse dynamics calculations were conducted for different initial ball angles (θ_0) to determine which interaction forces $F_{inter}(t)$ were needed to obtain the instructed sinusoidal cup displacement. This analysis did not infer a specific controller, instead it calculated the needed input force, $F_{inter}(t)$, for

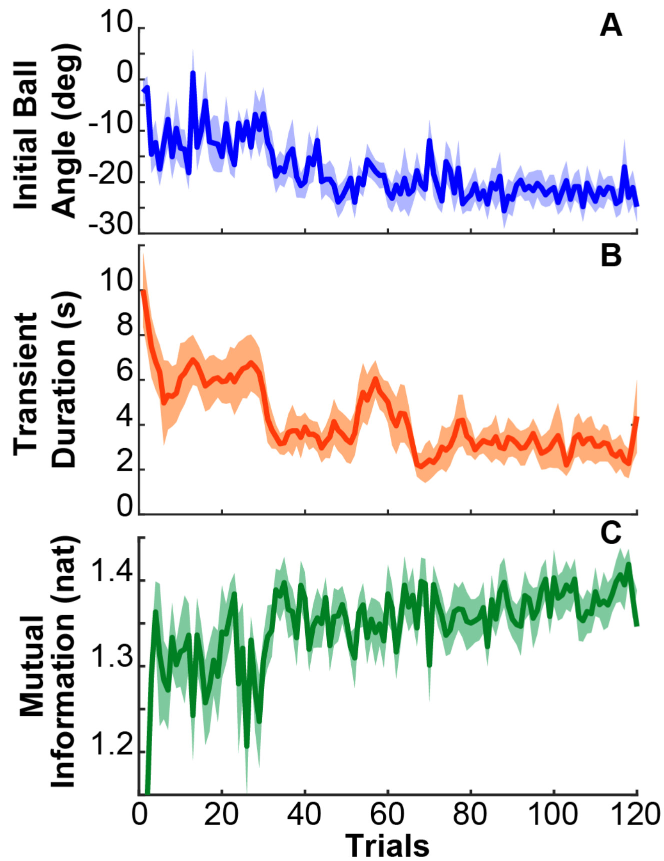

Fig. 3. (A) Initial ball angle averaged across subjects over trials. (B) Transient durations averaged across subjects over trials. (C) Mutual Information averaged over subjects plotted over trials on a natural log scale. In all three panels the shaded bands around the mean denote one standard error

the specified kinematic output. Eq. 1 was solved to obtain the force profile $F_{inter}(t)$ that produced the sinusoidal cup trajectory with peak-to-peak amplitude A and frequency f: $X(t) = (A/2)sin(2\pi ft + \pi/2)$, where t was time. In this case $f = 0.58$, the subjects' mean frequency, and $A = 0.3$ m. $\dot{X}(t)$ and $X(t)$ were sinusoidal, however the kinematics of the ball $\theta(t)$ and $\dot{\theta}(t)$ were not. The form of the $F_{inter}(t)$ profile depended on the initial values of θ_0 and $\dot{\theta}_0$, although $\dot{\theta}_0$ was set to 0 deg/s for this analysis. Fig.4 (A, B) displays two example profiles produced with two different θ_0 that produced the same sinusoidal cup trajectories $X(t)$ but with different initial ball angles. $F_{inter}(t)$, generated with $\theta_0 \approx 17$ degrees, (Fig.4B) renders a profile with significantly less regularity and values between 20 N and -30 N. In contrast, the left $F_{inter}(t)$ profile using $\theta_0 \approx -28.4$ deg (Fig.4A) resulted in a completely predictable oscillatory force profile.

To convey this behavior of $F_{inter}(t)$, the force profile $F_{inter}(t)$ was strobed at the maxima of the periodic cup profile. The strobed force values were then mapped onto the force-axis to produce a marginal distribution. This analysis was repeated for every θ_0. These distributions were then plotted against the initial ball angles in Fig.4C. This produced a diagram resembling the bifurcation diagram for a period-doubling route to chaos in nonlinear systems [33] [34].

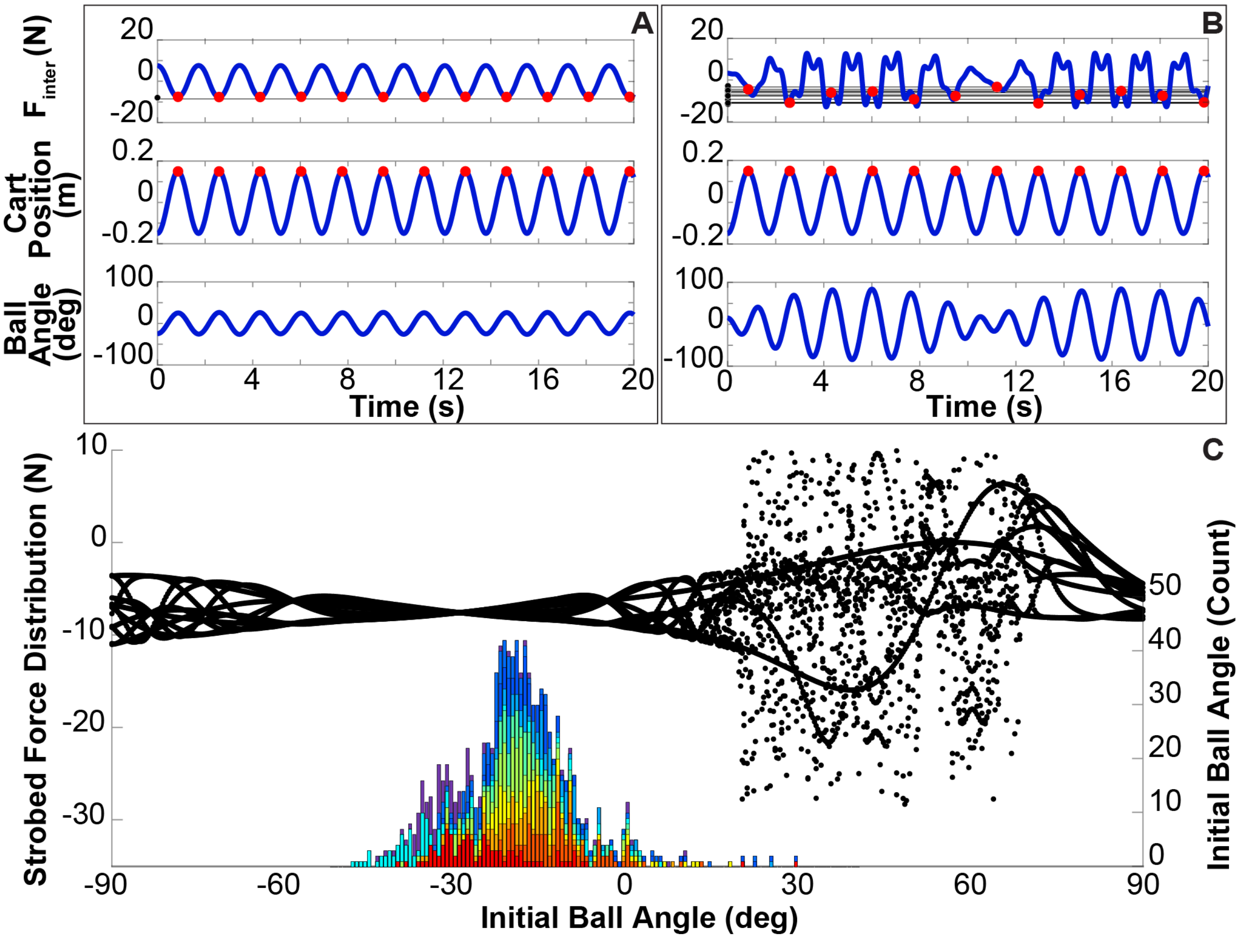

Fig. 4. (A) Example profile of a subject's interaction force $F_{inter}(t)$, cart position $X(t)$, and ball angle $\theta(t)$, for rhythmic cup displacement starting with $\theta_0 \approx -28.4$ deg; cup frequency and amplitude were 0.58 Hz and 0.3 m (peak-to-peak). (B) Example profile of interaction force $F_{inter}(t)$, cart position $X(t)$, and ball angle $\theta(t)$, for a trial initiated with $\theta_0 \approx 17$ deg; cup frequency and amplitude were 0.58 Hz and 0.3 m (peak-to-peak). (C) Diagram of strobed forces to summarize the complexity of interaction forces for different initial conditions: At every maximum of $X(t)$, the value of F_{inter} was determined, as displayed by the red points at each peak of $X(t)$ in Fig.4 (A,B). The strobed force values were mapped onto the vertical force axis to obtain the marginal distribution of the strobed force profile. These distributions were obtained for each simulation with different θ_0. The diagram in Fig.4C shows the F_{inter} distributions as a function of initial ball angle, θ_0. The figure also includes a histogram of θ_0 for all trials pooled over all subjects, different subjects appear in different colors. The peak of the histogram aligns with initial ball angles that render the low variability in the strobed forces.

The evolution of complex behavior from simple dynamics has been extensively examined in research on nonlinear dynamics [25], [35]–[37]. This analysis found the most predictable input force profile at $\theta_0 \approx -28.4$ deg. To illustrate how subjects' preferred initial angles, θ_0, compared to those predicted by inverse dynamics, the bottom of Fig.4C includes a histogram of θ_0 for all trials pooled over all subjects (different subjects appear in different colors). The initial angles subjects most frequently chose aligned with θ_0 that produced the least complex force patterns $F_{inter}(t)$ in the inverse dynamics simulation. This provided support for Hypothesis 3 that subjects chose initial conditions that favored simple input forces with more predictable dynamics.

2) Effect of initial conditions on transient duration: To derive predictions about transient duration, forward dynamics simulations were performed. In extension of a previous study, the cup-and-ball system was coupled to a simple model of hand impedance (Fig.5) [22]. The hand interactive dynamics were represented by a spring K, in parallel with a damper B. The desired trajectory was denoted as $X_{des}(t)$, $\dot{X}_{des}(t)$, which are the sinusoids that describe cup displacement and velocity for the appropriate frequency and amplitude of the cup displacement: $X_{des}(t) = (A/2)sin(2\pi ft + \pi/2)$. In the experiment the actual cart trajectory $X(t)$ differed from $X_{des}(t)$, most likely due to the pendulum forces acting as a perturbation. The hand impedance functioned as a simple proportional derivative controller and aided in minimizing the effects of this perturbation [22]. The equations of motion of this coupled model are:

$$(m_c + m_p)\ddot{X} = m_p d[\dot{\theta}^2 sin\theta - \ddot{\theta} cos\theta] + F_{inter} \quad (5)$$

$$\ddot{\theta} = -\frac{\ddot{X}}{l} cos\theta - \frac{g}{l} sin\theta \quad (6)$$

$$F_{inter} = -K(X - X_{des}) - B(\dot{X} - \dot{X}_{des}) \quad (7)$$

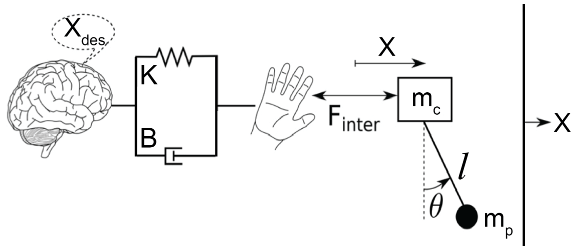

Fig. 5. The forward model with simple hand impedance coupled to the cart-pendulum-system. The interactive dynamics are represented by a spring K, in parallel with a damper B. The desired trajectory is denoted as $X_{des}(t)$.

The hand impedance parameters K and B were considered constant during a trial. To obtain an estimate for these two parameters from the experimental human data, for each trial of each subject an optimization procedure identified the values of K and B that best approximated the experimental cup and ball trajectories. The simulated f and A were matched to the exact frequency and amplitude for the particular trial. The objective function C was the root mean square difference between all kinematic traces over the entire trial, for more detail refer to the work by Maurice et al. [22].

To predict transient durations for different initial conditions, this model was forward simulated for different initial conditions. As the calculations proved to be sensitive to cup frequency, they were performed for a range of frequencies observed in the experimental data. The cup amplitude was set to the average experimentally observed value of 0.3 m. The results also varied with different stiffness and damping values; hence, calculations were performed for a range of K and B values estimated from the experimental data. Sweeping through all these parameters within the range of subjects' observed behavior, the transient duration was calculated for each simulation run. The same analysis procedures as in the experimental data were used to obtain the transient durations. Fig.6 summarizes the transient durations for the relevant frequencies and initial ball angles for K= 40 N/m and B= 50 Ns/m. These stiffness and damping values were the modes in the estimates from the subject data. The dark blue color represents the areas that predicted the shortest transient durations.

To compare simulation and experiment, individual subjects' trials were superimposed (red points). Only subject trials that had estimated stiffness and damping in the range of $K = 40 \pm 10$ N/m and $B = 50 \pm 20$ Ns/m were included. The figure shows trials in which the optimization-based determination for the impedance parameters were highly reliable ($C \leq .10$). Comparing the data with respect to the simulated predictions shows that subjects chose initial conditions that coincided with those in the model that produced the shortest transient durations. These results provided further support for Hypotheses 1, 2 and 3. Subjects preferred initial conditions that shortened the transients and reached a predictable steady state faster.

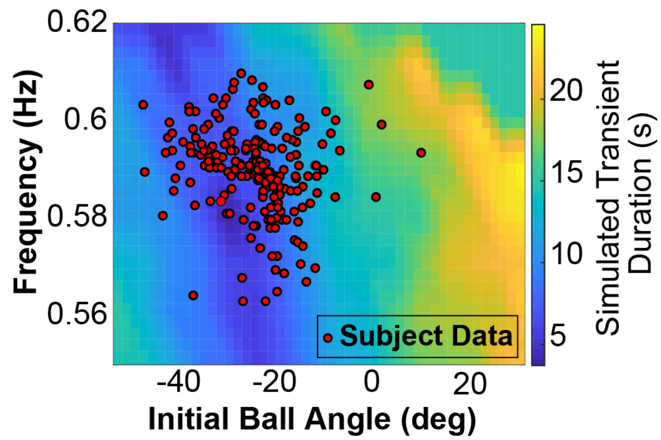

Fig. 6. Transient durations for different frequency and initial ball angles simulated with the forward simulation using K= 40 N/m and B= 50 Ns/m. Subject trials with estimated stiffness and damping in the range of $K = 40 \pm 10$ N/m and $B = 50 \pm 20$ Ns/m are superimposed.

V. Discussion and Conclusions

This study examined strategies that humans adopted when manipulating objects with underactuated dynamics, such as a cup of coffee. It showed that, when given a choice, subjects chose initial conditions that decreased their transients and reached a predictable steady state faster. Subjects chose initial ball angles that were associated with less complex interaction forces, demonstrating that subjects preferred predictable interaction forces. Mutual information (MI) increased over trials demonstrating that subjects interacted with the cup and ball in an increasingly predictable manner. The initial ball angles and frequencies that subjects chose matched initial ball angles and frequencies that decreased their transient duration in a forward simulation using impedance control. These results present first support for our hypothesis that humans can identify initial conditions that minimize transients and favor reaching a steady state, that enhances predictability, faster.

The results presented here may inspire new control strategies for robotic manipulation: strategies that exploit initial conditions instead of trying to cancel their effects. The notion of controlling a system by making small changes to its parameters has been explored for systems with chaotic dynamics. For example, Ott, Grebogi, and York used small perturbations to system parameters to attain stable behavior, rather than actively changing the systems dynamics [38] [39]. One could abstract these accessible system parameters to be the initial conditions of the system; control of a system is achieved by setting its initial conditions.

Based on these results and those from our earlier work, we propose a new guiding principle for robot control: exploit initial conditions to maximize predictability. It has been shown in previous work that human-robot interaction is facilitated when human movement features are considered in robotic control [40]. The findings from this present work provides insight on how to augment object interaction, and especially physical interaction and collaboration with humans. As physical human-robot interaction is becoming increasingly more common, further investigation into human motor control schemes will be crucial.

REFERENCES

[1] E. R. Kandel, J. H. Schwartz, and T. M. Jessell, *Principles of Neural Science*. McGraw-Hill New York, 2000, vol. 4.

[2] A. J. Nagengast, D. A. Braun, and D. M. Wolpert, "Optimal control predicts human performance on objects with internal degrees of freedom," *PLoS Computational Biology*, vol. 5, no. 6, p. e1000419, 2009.

[3] F. Danion, J. S. Diamond, and J. R. Flanagan, "The role of haptic feedback when manipulating nonrigid objects," *Journal of Neurophysiology*, vol. 107, no. 1, pp. 433–441, 2011.

[4] C. J. Hasson, T. Shen, and D. Sternad, "Energy margins in dynamic object manipulation," *Journal of Neurophysiology*, vol. 108, no. 5, pp. 1349–1365, 2012.

[5] M. T. Ciocarlie and P. K. Allen, "Hand posture subspaces for dexterous robotic grasping," *The International Journal of Robotics Research (IJRR)*, vol. 28, no. 7, pp. 851–867, 2009.

[6] N. Correll, K. E. Bekris, D. Berenson, O. Brock, A. Causo, K. Hauser, K. Okada, A. Rodriguez, J. M. Romano, and P. R. Wurman, "Analysis and observations from the first amazon picking challenge," *IEEE Transactions on Automation Science and Engineering*, vol. 15, no. 1, pp. 172–188, 2016.

[7] F. Lamiraux and L. E. Kavraki, "Planning paths for elastic objects under manipulation constraints," *The International Journal of Robotics Research*, vol. 20, no. 3, pp. 188–208, 2001.

[8] M. Dogar, K. Hsiao, M. Ciocarlie, and S. Srinivasa, "Physics-based grasp planning through clutter," in *Robotics: Science and Systems VIII*, N. Roy, P. Newman, and S. Srinivasa, Eds. MIT Press, 2013, ch. 9, pp. 57–64.

[9] C. Fisher, C. Hubicki, and A. Patel, "Do intermediate gaits matter when rapidly accelerating?" *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 3418–3424, 2019.

[10] M. Karlsson, A. Robertsson, and R. Johansson, "Detection and control of contact force transients in robotic manipulation without a force sensor," in *IEEE International Conference on Robotics and Automation (ICRA)*, 2018, pp. 1–9.

[11] A. Stolt, M. Linderoth, A. Robertsson, and R. Johansson, "Detection of contact force transients in robotic assembly," in *IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 962–968.

[12] R. Tedrake, I. R. Manchester, M. Tobenkin, and J. W. Roberts, "Lqr-trees: Feedback motion planning via sums-of-squares verification," *The International Journal of Robotics Research*, vol. 29, no. 8, pp. 1038–1052, 2010.

[13] R. R. Burridge, A. A. Rizzi, and D. E. Koditschek, "Sequential composition of dynamically dexterous robot behaviors," *The International Journal of Robotics Research*, vol. 18, no. 6, pp. 534–555, 1999.

[14] A. Afshar, G. Santhanam, M. Y. Byron, S. I. Ryu, M. Sahani, and K. V. Shenoy, "Single-trial neural correlates of arm movement preparation," *Neuron*, vol. 71, no. 3, pp. 555–564, 2011.

[15] K. V. Shenoy, M. T. Kaufman, M. Sahani, and M. M. Churchland, "A dynamical systems view of motor preparation: implications for neural prosthetic system design," in *Progress in Brain Research*. Elsevier, 2011, vol. 192, pp. 33–58.

[16] G. Hennequin, T. P. Vogels, and W. Gerstner, "Optimal control of transient dynamics in balanced networks supports generation of complex movements," *Neuron*, vol. 82, no. 6, pp. 1394–1406, 2014.

[17] J. Ernesti, L. Righetti, M. Do, T. Asfour, and S. Schaal, "Encoding of periodic and their transient motions by a single dynamic movement primitive," in *IEEE-RAS International Conference on Humanoid Robots*, 2012, pp. 57–64.

[18] R. G. Cohen and D. Sternad, "State space analysis of timing: exploiting task redundancy to reduce sensitivity to timing," *Journal of Neurophysiology*, vol. 107, no. 2, pp. 618–627, 2011.

[19] A. A. Faisal, L. P. Selen, and D. M. Wolpert, "Noise in the nervous system," *Nature Reviews Neuroscience*, vol. 9, no. 4, p. 292, 2008.

[20] M. Lone and A. Cooke, "Review of pilot modelling techniques," in *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*, 2010, p. 297.

[21] M. Kawato, "Internal models for motor control and trajectory planning," *Current Opinion in Neurobiology*, vol. 9, no. 6, pp. 718–727, 1999.

[22] P. Maurice, N. Hogan, and D. Sternad, "Predictability, force and (anti-) resonance in complex object control," *Journal of Neurophysiology*, vol. 120, no. 2, pp. 765–780, 2018.

[23] B. Nasseroleslami, C. J. Hasson, and D. Sternad, "Rhythmic manipulation of objects with complex dynamics: predictability over chaos," *PLoS Computational Biology*, vol. 10, no. 10, p. e1003900, 2014.

[24] D. Sternad and C. J. Hasson, "Predictability and robustness in the manipulation of dynamically complex objects," in *Progress in Motor Control*. Springer, 2016, pp. 55–77.

[25] T. Insperger, J. Milton, and G. Stépán, "Acceleration feedback improves balancing against reflex delay," *Journal of the Royal Society Interface*, vol. 10, no. 79, p. 20120763, 2013.

[26] R. Van der Linde and P. Lammertse, "Hapticmaster - a generic force controlled robot for human interaction," *Industrial Robot: An International Journal*, vol. 30, no. 6, pp. 515–524, 2003.

[27] C. L. Phillips and R. D. Habor, *Feedback Control Systems*. Simon & Schuster, Inc., 1995.

[28] S. L. Hahn, *Hilbert Transforms in Signal Processing*. Artech House Boston, 1996, vol. 2.

[29] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, 2012.

[30] R. Kleeman, "Information theory and dynamical system predictability," *Entropy*, vol. 13, no. 3, pp. 612–649, 2011.

[31] T. DelSole, "Predictability and information theory: Measures of predictability," *Journal of the Atmospheric Sciences*, vol. 61, no. 20, pp. 2425–2440, 2004.

[32] B. W. Silverman, *Density Estimation for Statistics and Data Analysis*. Routledge, 2018.

[33] R. C. Hilborn *et al.*, *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers*. Oxford University Press, 2000.

[34] S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. CRC Press, 2018.

[35] E. N. Lorenz and K. Haman, "The essence of chaos," *Pure and Applied Geophysics*, vol. 147, no. 3, p. 244, 1996.

[36] R. M. May, "Simple mathematical models with very complicated dynamics," in *The Theory of Chaotic Attractors*. Springer, 2004, pp. 85–93.

[37] S. A. Campbell, J. Belair, T. Ohira, and J. Milton, "Complex dynamics and multistability in a damped harmonic oscillator with delayed negative feedback," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 5, no. 4, pp. 640–645, 1995.

[38] C. Ott, B. Henze, G. Hettich, T. N. Seyde, M. A. Roa, V. Lippi, and T. Mergner, "Good posture, good balance: comparison of bioinspired and model-based approaches for posture control of humanoid robots," *IEEE Robotics & Automation Magazine*, vol. 23, no. 1, pp. 22–33, 2016.

[39] E. Ott and M. Spano, "Controlling chaos," in *AIP Conference Proceedings*, vol. 375, no. 1, 1996, pp. 92–103.

[40] P. Maurice, M. E. Huber, N. Hogan, and D. Sternad, "Velocity-curvature patterns limit human–robot physical interaction," *IEEE Robotics and Automation Letters*, vol. 3, no. 1, pp. 249–256, 2018.