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THE HYDROLOGY OF FRACTURED ROCKS: A LITERATURE REVIEW

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Report Number 304

Prepared under the support of the
Sandia National Laboratories
and
the National Science Foundation

December, 1985

R85-14

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ABSTRACT

Recent literature on the quantitative description of flow in fractured rocks is reviewed with emphasis on modeling approaches and their conceptual framework. The relationships of modeling results to laboratory and field observations is also emphasized. The review is organized in terms of the following three categories: fracture characterization, hydraulics of fractured rocks, and solute transport in fractured media. It is found that there are several probabilistic models which seem to adequately characterize three-dimensional fracture geometry, but it is not clear how well these models represent fracture interconnection. The theory for hydraulic behavior of extensively fractured systems is well established but no workable theory has been developed for non-extensive three-dimensional fracture networks. The question of when the flow in a discrete fracture network can be treated as a hydraulic continuum remains unresolved. Matrix diffusion models of solute transport are conceptually attractive and have been developed extensively. However, it has not been shown by direct field observations that the matrix diffusion mechanism is important in the field. No theoretical approach has been advanced for treating solute transport in three-dimensional fracture networks.

ACKNOWLEDGMENTS

This work was supported in part by Sandia National Laboratories (Contract No. NRC-04-83-174) and the National Science Foundation (Grant No. ECE-8311786).

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CHAPTER 1

INTRODUCTION

Fractured rock hydrology is largely an unresolved field. In spite of the fact that it has many applications (water supply, hot dry rock energy extraction, high level radioactive waste isolation) and researchers have been working on these problems for over fifteen years, large gaps exist in the general understanding of the field and what is known is scattered through many different papers. In this report the available literature is examined in the context of conceptual frameworks, modeling approaches and the relationship of theories to published lab and field data.

We begin with the lowest-order problem, that of characterizing a fractured formation. Methods are discussed for determining such bulk parameters as conductivity and porosity and parameters relating to fractures, such as spacing and aperture. Typical field data for these parameters is presented, especially when it indicates the form of a probability distribution.

A higher-order problem is finding the flow through such a formation. Ultimately, what we would like is a theory which predicts values of the bulk parameters from the fracture parameters. This is done for many special cases of fracture network geometry, but not for the general case. It is not even clear if such a theory must always exist, as will become apparent from the discussion of when a fractured regime may be represented by an equivalent porous medium.

The highest-order problem is the analysis of solute transport. The knowledge in this area is very limited, being restricted to continuum analyses which ignore the fractured nature of the medium, and matrix diffusion models with simple geometry.

CHAPTER 2

CHARACTERIZATION OF FRACTURED FORMATIONS

2.1 Bulk Parameters

The bulk parameters which one most commonly measures in the field are hydraulic conductivity and porosity.

The hydraulic conductivity of small volumes of rock is usually measured using some type of borehole test. The tests used do not differ much from porous medium tests and, in fact, give values of conductivity as if the fractured medium were replaced by an equivalent anisotropic porous medium. Wilson et al. (1979) summarized these tests, of which there were four types:

i. pump test

ii. constant head injection test (packer test)

iii. slug test

iv. pulse test

Figure 1 shows the ranges of hydraulic conductivities which can be measured by each test.

In the pump test and its variants water is pumped from the aquifer at a constant rate, and drawdown in the pumped well and in an observation well are observed. Standard type curves for confined, phreatic, leaky, etc., aquifers can then be used to find the bulk hydraulic conductivity of the fractured reservoir. Snow (1966) and Hsieh et al. (1983) described ways to measure hydraulic anisotropy with packer tests.

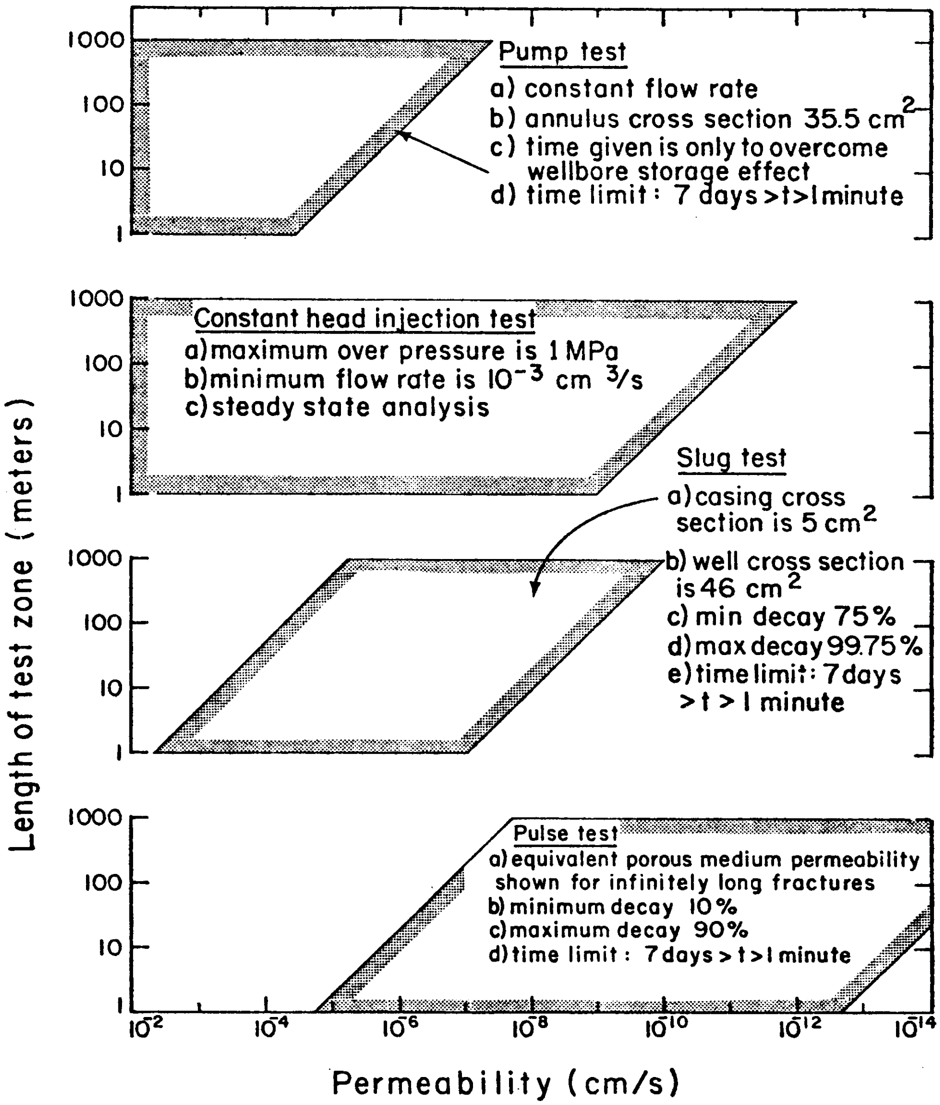


Figure 2.1 Ranges of hydraulic conductivity (permeability) for various borehole tests (from Wilson et al., 1979).

Constant head injection tests are used very widely in fractured environments. The excess pressure in the well is kept constant until steady flow is achieved. Flow rate should vary linearly with pressure, with hydraulic conductivity calculated from the slope. Maini et al. (1972) described several practical suggestions for use of this test.

In slug tests, a certain amount of head is added to the wellbore and the hydraulic conductivity is found from the transient response as this head is dissipated. The pulse test is conceptually the same, but is designed for much lower conductivities. The pulse test has been analyzed theoretically (Wang et al. 1978; Hsieh et al. 1981; Neuzil et al. 1981) but it is not clear how well it performs in the field. Davison et al. (1979) measured the hydraulic conductivity of several sections of borehole using the pulse test and packer tests. Figure 2 shows the results. The pulse test did not consistently give the same values as the packer test. Forster and Gale (1981) tried the test at Stripa, Sweden, but also could not get good results. They discussed several reasons why this may be.

Different techniques must be used when determining the hydraulic conductivity of large volumes of rock. Gale et al. (1982) describe a large-scale test which was run at Stripa. Seepage into a mine tunnel was measured by observing the changes in moisture of the ventilation air. Thus, by measuring or estimating the local gradient around the tunnel, an estimate of hydraulic conductivity may be obtained. The results are given by Wilson et al. (1983) who reported an equivalent porous medium conductivity of about 10^{-10} m/s in a volume of 2×10^5 m^3 of rock. They also observed a "skin"

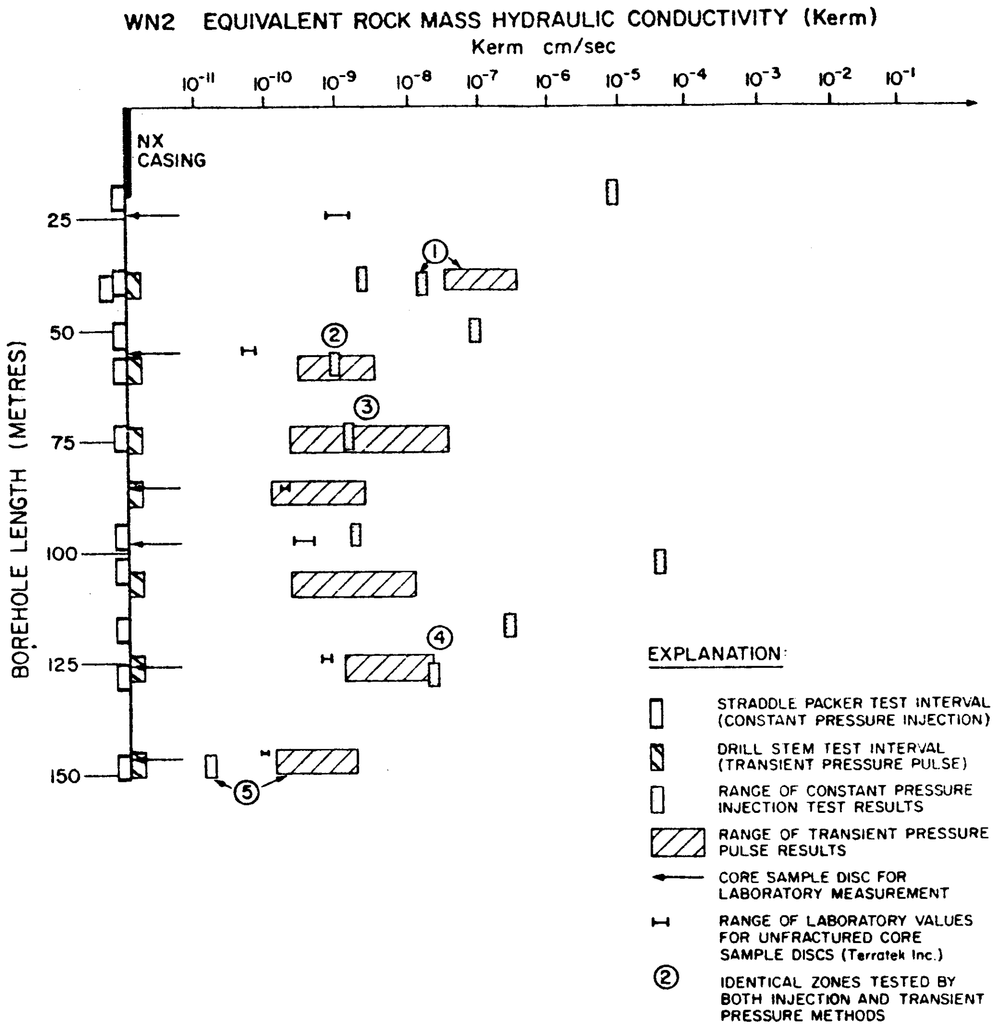


Figure 2.2 Comparison of the equivalent rock mass hydraulic conductivity values obtained from transient pressure pulse tests, constant pressure fluid injection tests and laboratory permeability tests of unfractured core samples (from Davison, et al., 1979).

of lower conductivity rock immediately surrounding their tunnel. The authors speculated that this skin could have been caused by two-phase flow near the tunnel, compressive stresses in the rock and/or chemical precipitates left by evaporating water. It would be interesting to compare the results to some aggregate value of the many packer tests run in the same volume.

Lindblom (1979) described a similar experiment at a liquid natural gas storage cavern elsewhere in Sweden. The conductivity of a large mass of overlying rock was measured in three ways. First, seepage into the cavern was measured under natural conditions. The gradient was estimated using flow nets, and thus an estimate of hydraulic conductivity was obtained. The second measure of hydraulic conductivity involved measuring the seepage into the cavern when the vertical gradient above it was increased. This was done by injection at constant head into many horizontal boreholes located a short distance ($\approx$15 m) above and spanning the horizontal extent of the cavern. The measures of conductivity for these two methods were 1.1×10^{-8} m/s and 4.2×10^{-8} m/s, exhibiting good agreement. Their third measure of hydraulic conductivity involved taking the arithmetic mean of the conductivities found with packer tests at small intervals along a vertical and an inclined borehole above the cavern. This average value agreed with the first two methods, although the agreement may be accidental, as the geometric mean should probably have been used.

Burgess et al. (1979) have provided a summary of field data relating hydraulic conductivity with depth (Figure 3) and fitted the empirical function

$$\log K = -5.57 + 0.362 (\log Z) - 0.978 (\log Z)^2 + 0.162 (\log Z)^3 \quad (1)$$

where K is the in conductivity m/s and Z is the depth in meters below the overburden of the fractured rocks. Maini and Hocking (1977), however, pointed out the inadvisability of extrapolating data of this type below 150 m.

Specific storage can also be measured using standard transient-type well tests. Hsieh et al. (1983) reported values of specific storage ranging from 2×10^{-7} to 2×10^{-5} m^{-1}. Carlsson et al. (1979) reported values measured at Stripa ranging from 2×10^{-8} to 5×10^{-8} m^{-1}.

There are two kinds of porosity in a fractured reservoir. The primary porosity is due to the spaces between the grains of the fractured material, and the secondary porosity, or fracture porosity, comes from the volume of the fractures in the material. It is usually the fracture porosity which accounts for most of the flow through such a region. Therefore, tracer tests based on continuum analyses, by far the most common measurement of effective porosity, measure only fracture porosity.

Lundstrom and Stille (1978) used tracer techniques at Stripa, reporting an effective porosity of 1.3×10^{-4}. Webster et al. (1970) used a two-well tracer test at the Savannah River Plant in South Carolina, obtaining an effective porosity of 8×10^{-4}. From the breakthrough curve

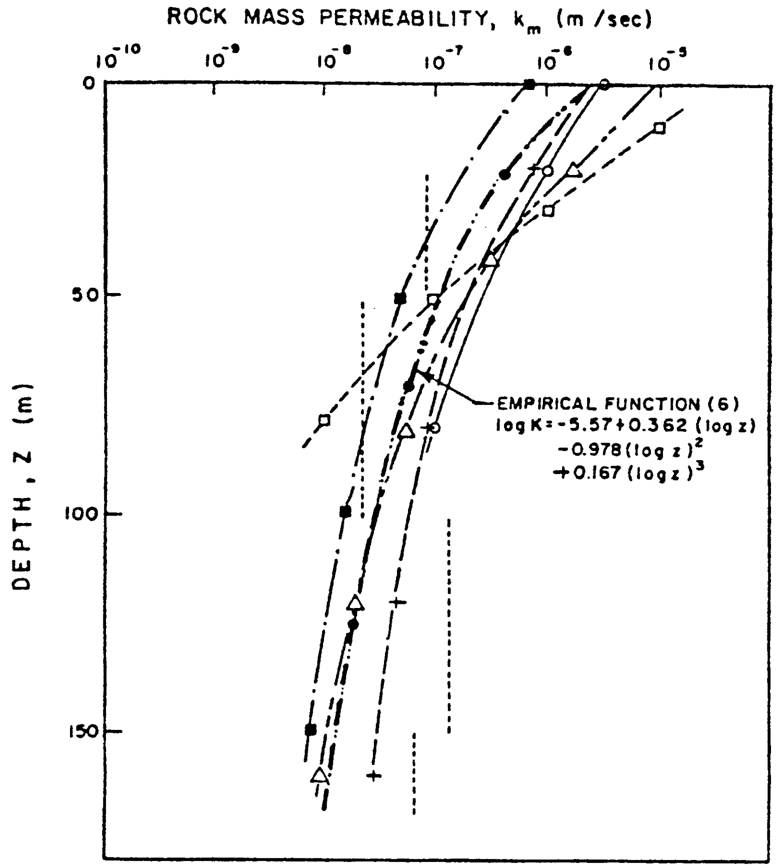


Figure 2.3 Hydraulic conductivity versus depth (from Burgess et al., 1979).

(Figure 15), we see that the 50% breakthrough is twice as soon as they estimated. Thus we suspect that the actual effective porosity is somewhat lower. Bartolami et al. (1979) monitored infiltration of environmental isotopes through Mont Blanc, France, to obtain an estimate of effective porosity in the range of 8.9×10^{-3} to 1.6×10^{-2}, much higher than the above data. Finally, Whincup and Domahidy (1982) reported values of specific yield for phreatic fractured reservoirs ranging from 2.4% to 8.6%. These high values are most likely due to extensive weathering commonly found in the upper regions of rock formations.

Unfortunately, no attempt has been made, to our knowledge, to correlate porosity measurements based on tracer tests, with direct calculations based on known or assumed fracture geometry.

2.2 Fracture Parameters

We consider here the methods reported in the literature for measuring five types of data commonly used in deterministic or probabilistic representations of fracture networks. These are the location, orientation, spacing, aperture and size of fractures or fracture sets.

The simplest method of locating fractures is by direct observation on an outcropping or excavation. Fractures in boreholes may be located using TV cameras. Alternatively, cores or integral samples (Rocha and Franciss, 1977) may be inspected.

Packer tests can be used to find fractures in a borehole, with a non-zero flow indicating the presence of one or more fractures. However, Marine (1979) pointed out that individual fractures are difficult to isolate due to limited resolution in packer placement. He described a method of locating fractures by the loss of gamma emitters into joints during injection. Nelson et al. (1980) used this method at Stripa, but encountered difficulties due to the natural occurrence of ^{222}Ra.

Orientation is usually observed visually in outcrops. Typically, equal-area stereographs are made of pole densities to illustrate the trends found in a particular area (Bianchi and Snow, 1968; Saari, 1979; Gale, et al., 1982; Forster and Gale, 1981; Kiraly, 1969). Figure 4 shows one such stereograph from Bianchi and Snow (1968), exhibiting the characteristic grouping of fractures into sets with similar orientations.

Baecher (1983) described a geometric bias in orientation sampling. Joints which are parallel to a sampling line are less likely to be observed than those which are perpendicular (Figure 5). To account for this, a weighting factor of $1/\sin\alpha$ should be used, where α is the angle between the sampling line and the normal to the fracture.

Kiraly (1969) presented a method of calculating the directional variance of pole density about a mean orientation. Baecher (1983), based on data from 25 joint surveys (15,000 joints), concluded that no analytical form of probability density function (PDF) fit field orientation data very well, but that the Bingham and bivariate (elliptical) Fisher probability density functions fit best.

Fracture spacing is an important parameter for estimates of porosity, hydraulic conductivity and solute transport. It can be measured directly on outcrops or excavations or by inspection of cores. In addition, Snow (1970) developed a method for finding fracture spacing based on the frequency of zero-flow packer tests on isolated sections of a borehole.

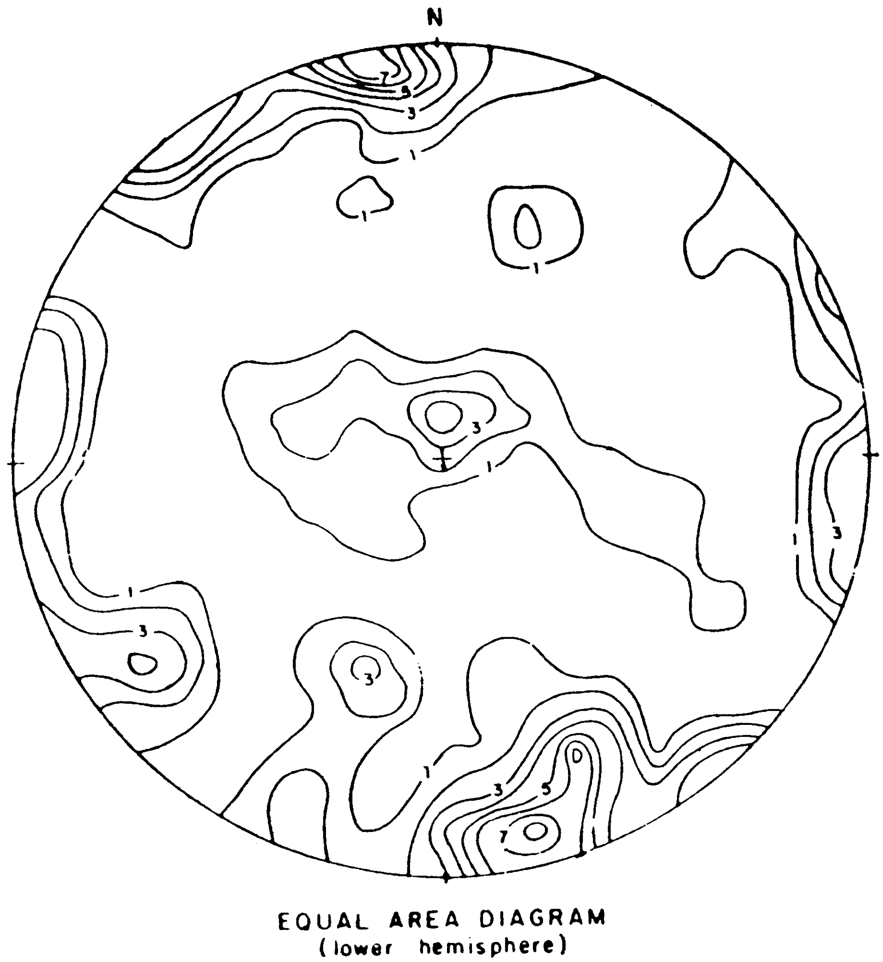


Figure 2.4 Equal area stereographs showing three approximately normal joint sets, A, B and C (from Bianchi and Snow, 1968).

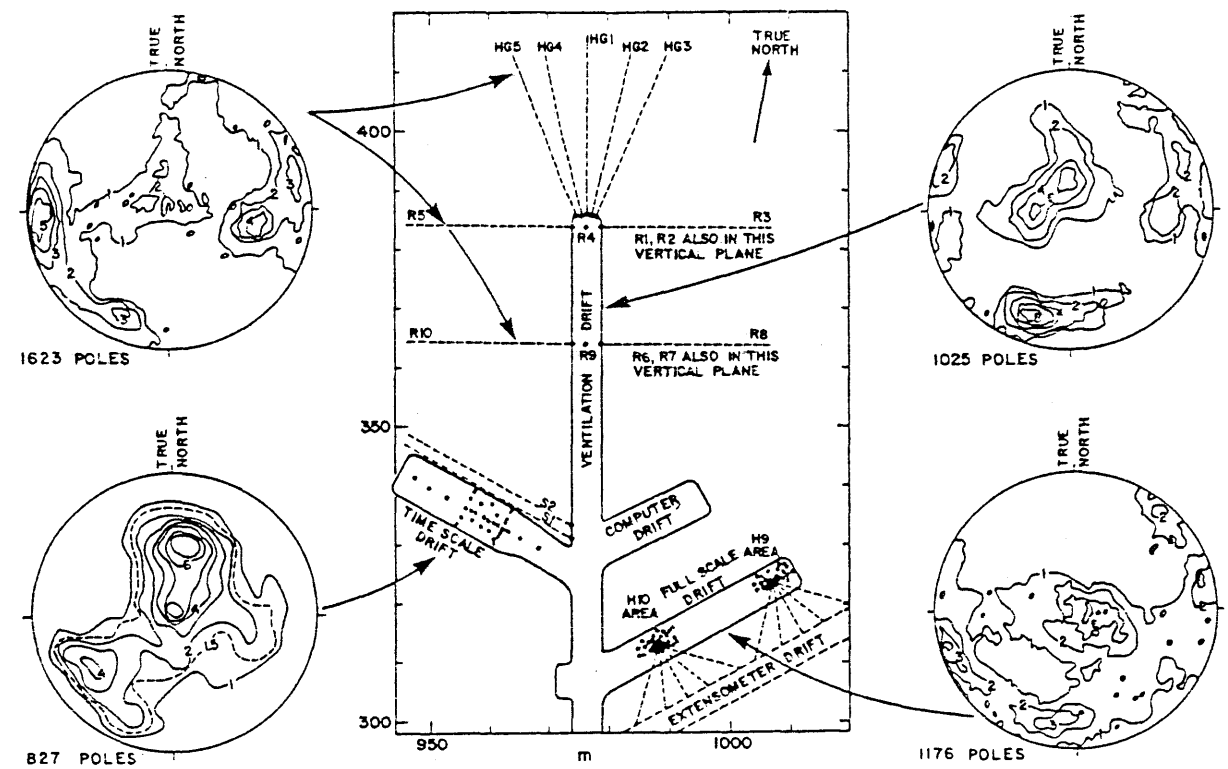


Figure 2.5 Contoured stereonets of poles to joint planes measured in different areas of the Stripa, Sweden site. The lower left stereonet based on exclusively vertical boreholes shows the geometric bias. The contoured values are in percent of points per one percent surface area (Gale et al., 1982).

Baecher's (1983) survey found that 92% of the data on spacing fit the exponential probability distribution. In his paper, though, he does not distinguish between three different definitions of spacing:

(i) the separation of the intersections of any joints with a sampling line,

(ii) the separation of the intersections of the joints in one joint set with a sampling line, and

(iii) the spacing defined by (ii), multiplied by the cosine of the angle between the sampling line and the pole of the average plane of the joint set.

Hudson and Priest (1979) also found exponential fits with their spacing data, but they did not apply any statistical inference tests. On the contrary Gale et al. (1982) reported that the exponential distribution did not satisfy goodness-of-fit tests but that the lognormal distribution did. They used the third definition of spacing. Hudson and Priest (1983) presented an analysis capable of reconciling the three definitions. Lapointe (1980) evaluated spatial correlation scales for fracture frequency.

Field measurement of fracture apertures may only be done indirectly, using borehole hydraulic tests. Assuming only one fracture intersects the test interval, a packer test will yield an aperture as a function of the hydraulic conductivity by using the "cubic law." If more than one fracture actually intersects the test interval, then this method will overestimate the aperture of either fracture (Gale 1982). The pulse test mentioned above should in theory be able to calculate fracture apertures from the transient pressure decay curve but, again, it has yet to work in the field. Bower (1983) presented a similar analysis capable of measuring apertures from the amplitude and phase of well tides.

In the lab, Witherspoon et al. (1980) used strain measurements on whole and jointed rock to infer apertures.

Snow (1968a, 1970) has been the major source of field data on apertures. Figure 6 shows the general trend of decreasing aperture with depth. In addition, apertures invariably follow a lognormal distribution, although the mean and variance change with depth (Figure 7).

All of the data reported in the literature on fracture size, or persistence, is given in the form of trace lengths, the intersections of fractures with sampling planes. No one has ever measured the area of a fracture, although this would seem to be the primary indication of the persistence of a fracture. Admittedly, it is not obvious how this could be done. Therefore, we are limited to measuring trace lengths on outcroppings and excavations.

Baecher and Lanney (1978) describe three biases in trace length surveys: geometric bias, censoring bias and truncation bias. Geometric bias represents the fact that longer trace lengths are more likely to be sampled. This bias is linear and has the curious effect of transforming most probability density functions into approximately lognormal form (Figure 8). Thus, lognormality may be an artifact of sampling procedures. Censoring bias accounts for the fact that the longest trace lengths cannot be completely measured, as one or two ends of the trace may be out of the

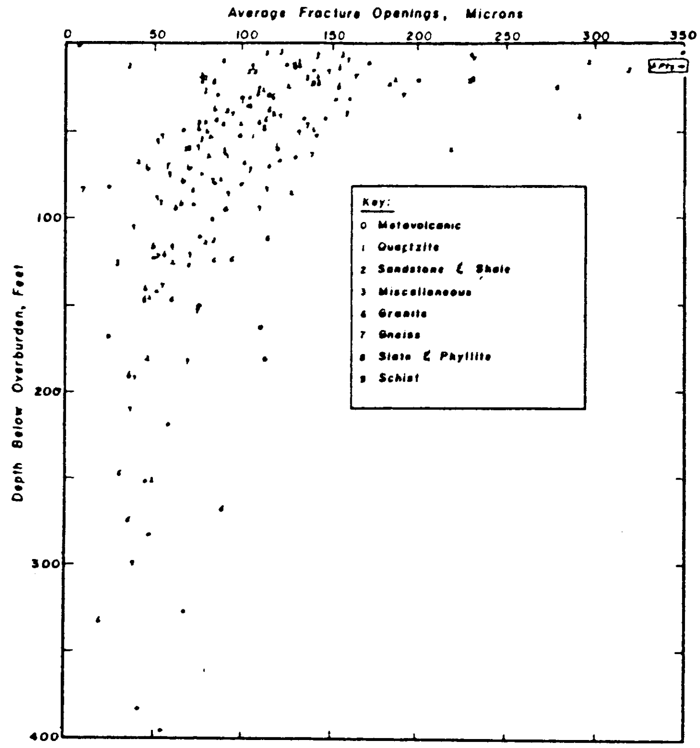
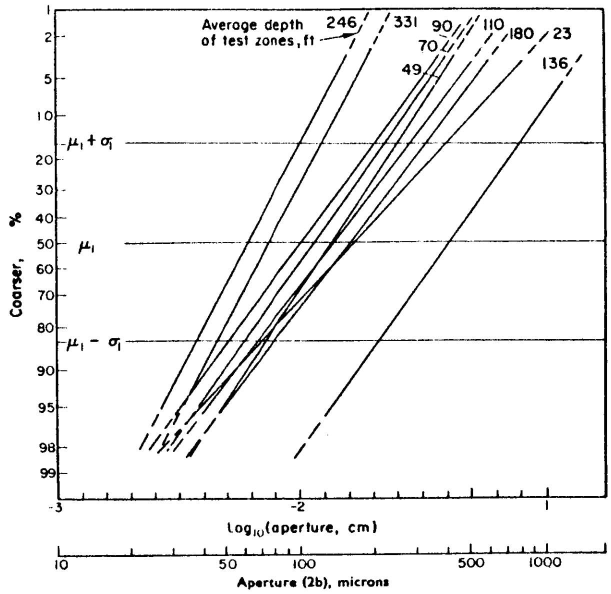


Figure 2.6 Decrease of fracture aperture with increasing depth (from Snow, 1968c).



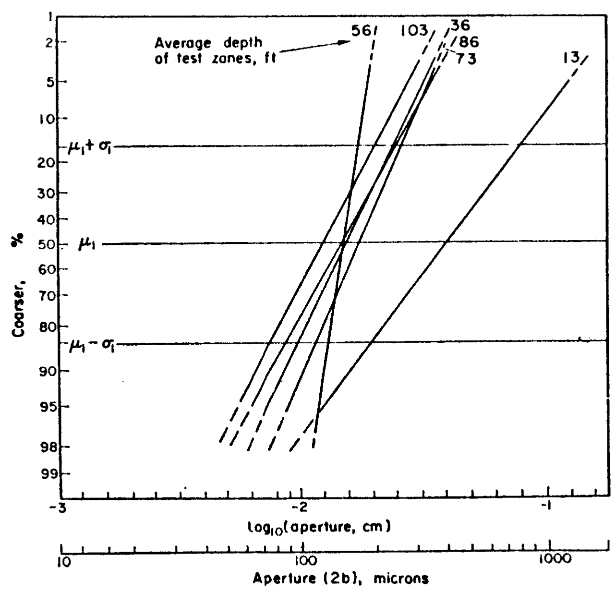


Figure 2.7 Cumulative probability distributions from two different sites; show effect of depth and lognormality of aperture distributions (from Snow, 1970).

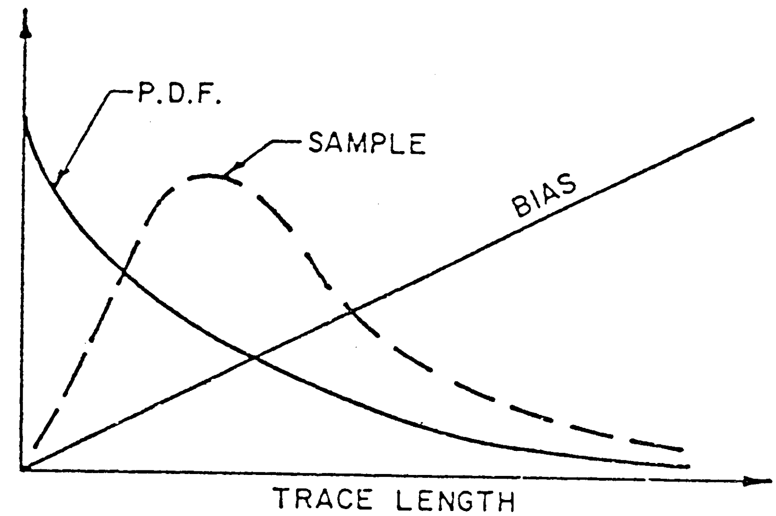


Figure 2.8 The influence of linear bias producing a lognormal-shaped sample distribution (from Baecher, 1983).

sampling region (or out of view). This would tend to distort sample means and variances. Truncation bias, relatively less important than the other biases, accounts for the possibility that very small joints may be neglected.

Baecher (1983) concluded that the log-normal distribution best fits unbiased trace length data. On the other hand, Gale et al. (1982) accounted for biases in sampling trace lengths at Stripa, and found that the exponential distribution fit very well.

The whole point of trace length surveys is to gain some information on the persistence of the fractures in a region and, hence, on the degree of inter-connectivity. Thus, some researchers have found it desirable to develop three-dimensional fracture models whose parameters (hopefully few in number) may be inferred from the trace length probability density function.

Veneziano (1978) proposed a model of jointing based on a Poisson plane process in space. Each plane is divided into polygons by random lines (also Poisson processes) and each polygon is assumed to represent either intact rock or a discontinuity (i.e., fracture). This model would produce exponential spacing and trace length probability density functions.

CHAPTER 3

FRACTURED ROCK HYDRAULICS

3.1 Single Fracture Hydraulics

The simplest model of the flow through a single fracture is the analogy with Poiseuille flow between two infinite smooth parallel plates. This is the model proposed by Snow in a series of papers in the late 1960's (Snow, 1966, 1968a, 1968b, 1968c, 1969, 1970; Bianchi and Snow, 1968). In this type of flow, the vertically-averaged velocity, $\bar{v}_i$, is given by

$$\bar{v}_i = \frac{b^2 g}{3\nu} J_i \qquad (3)$$

where b is the fracture's half-aperture, g is the gravitational acceleration, ν is the fluid's kinematic viscosity and J_i is the hydraulic gradient.

Thus, for a unit width of fracture, the total flow is given by

$$Q_i = \bar{v}_i b = \frac{2b^3 g}{3\nu} J_i, \qquad (4)$$

which is the well-known "cubic law" relating flow to aperture. We are then led to the definition of fracture hydraulic conductivity, K_f, as

$$K_f = \frac{b^2 g}{3\nu} \qquad (5)$$

The other type of model commonly used is that of circles positioned randomly in space. Baecher et al. (1978) assumed that the joints are randomly distributed in space, and also randomly oriented. Then, if the radii are assumed to be lognormally distributed, trace lengths will be approximately lognormal also. Dienes (1979), on the other hand, assumed an exponential distribution for the radii. He then solved the inverse problem: Given $P(\ell)$, the number of trace lengths greater than length ℓ, the PDF of joint radii, $n(c)$, is given by

$$n(c) = \frac{32}{\pi^2 c} \frac{d}{dc} \int_0^\infty c^3 \frac{P(\ell)\, d\ell}{\ell(\ell^2 - 4c^2)^{3/2}} , \qquad (2)$$

which may be integrated numerically. This approach seems promising.

Note that in this section, we have emphasized the probabilistic nature of the field data. This is important, because probability density functions of fracture parameters contain a lot of information, yet do not attempt to explicitly account for each individual fracture, a hopeless task. In order to condense this information of parameter distributions, we provide Table I, which lists the distributional forms found from actual data, as reported by the papers cited.

Table 3.1 Fracture Parameter Distributions

Authors	Trace Length	Spacing	Orientation	Aperture
Baecher (1983)	lognormal	exponential	bivariate Fisher; Bingham	--
Bianchi and Snow (1968)	--	--	--	lognormal
Gale et al. (1982)	exponential	lognormal	--	--
Hudson and Priest (1979)	--	exponential	--	--
Kiraly (1969)	--	--	non-analytic	--

This is the hydraulic conductivity of only that volume of space occupied by fractures. Bulk hydraulic conductivity for the entire medium is considered later.

For a fracture oriented arbitrarily in three-dimensional space, let n_i be the unit vector normal to the fracture plane. If I_i is the hydraulic gradient across a volume of rock containing the fracture, then J_j is given by the projection of I_i on the fracture plane (Figure 9):

$$J_j = (\delta_{ij} - n_i n_j)\, I_i, \qquad (6)$$

and thus

$$\bar{v}_j = \frac{b^2 g}{3\nu} (\delta_{ij} - n_i n_j)\, I_i \qquad (7)$$

gives the average fluid velocity vector for an arbitrary orientation of the fracture and the hydraulic gradient.

In the case that the fracture walls are considered to have a finite roughness, Castillo et al. (1972) suggests that the known relationships between friction factor and Reynolds number for flow in circular pipes be used, with the definition of the Reynolds number as

$$R = \frac{4b\bar{v}}{\nu} \qquad (8)$$

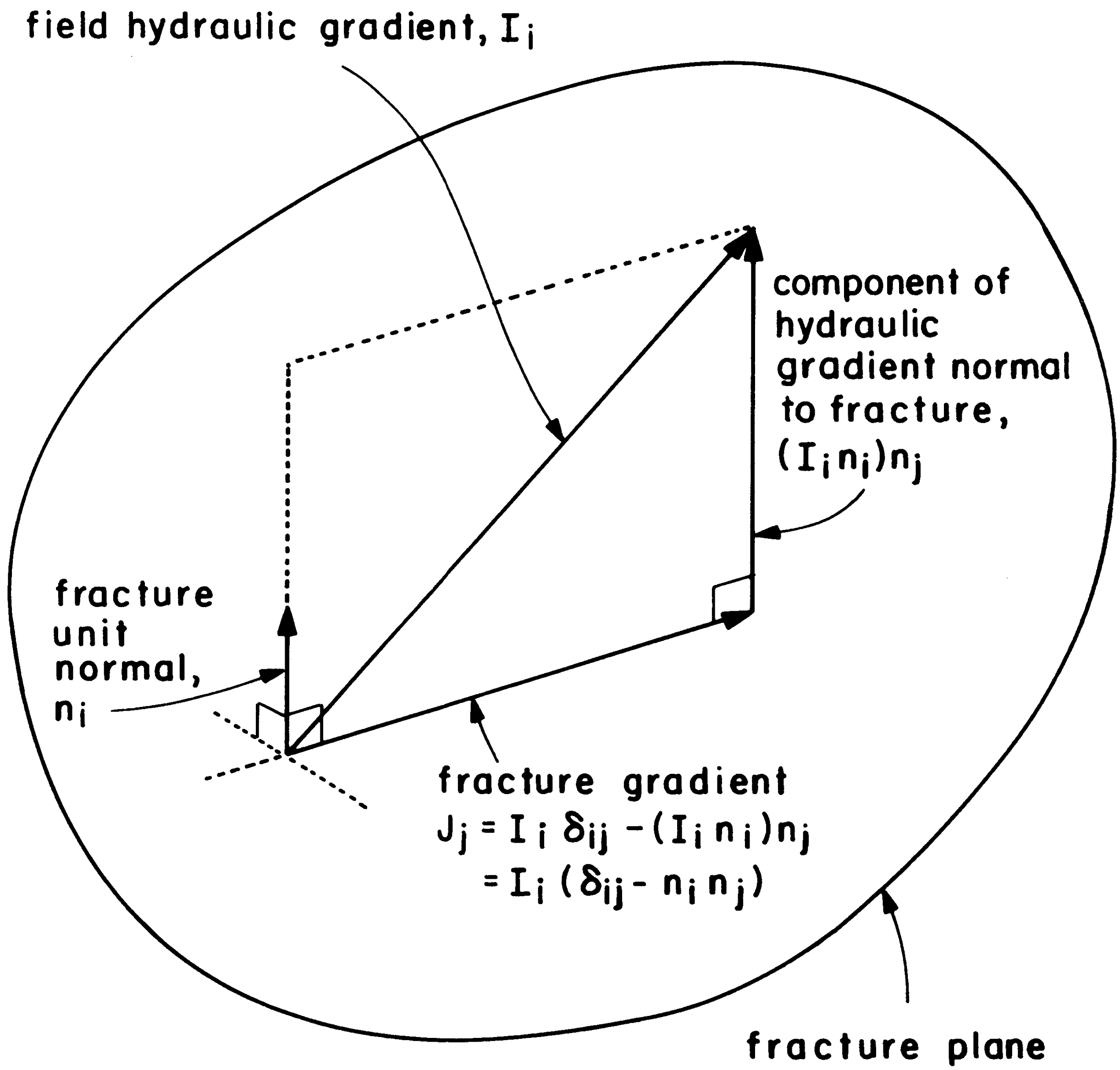


Figure 3.1 Projection of the hydraulic gradient I_j; onto the plane of the fracture with unit normal vector n_i^j (after Snow, 1969).

Alternatively, Rocha and Francis (1977) report Louis' (1969) empirical result

$$K_f = \frac{b^2 g}{3\nu} \left(\frac{1}{1 + 8.8\ r^{1.5}}\right) \tag{9}$$

where r is the relative roughness of the fracture walls.

Of course, we know that fracture walls are not perfectly parallel, since a fracture will conduct flow even while the walls are subjected to normal stress. There must, then, be places of contact where this stress is endured, while other areas are not in contact, permitting flow.

Pratt et al. (1977), studied the stress dependence of flow in three granite fractures in the field. They found that even after the fracture was closed, in that its modulus of elasticity was the same as for intact rock, a significant amount of flow was still occurring. At low stress, the fracture flow varied enormously, presumably due to the cubic relationship between flow and aperture. However, for stresses above 30 bars, there was little or no decrease in flow for large increases in stress (Figure 10).

Laboratory tests by Nelson and Handin (1977), Kranz et al. (1979), Gangi (1979) and Witherspoon et al. (1980) have all confirmed this behavior.

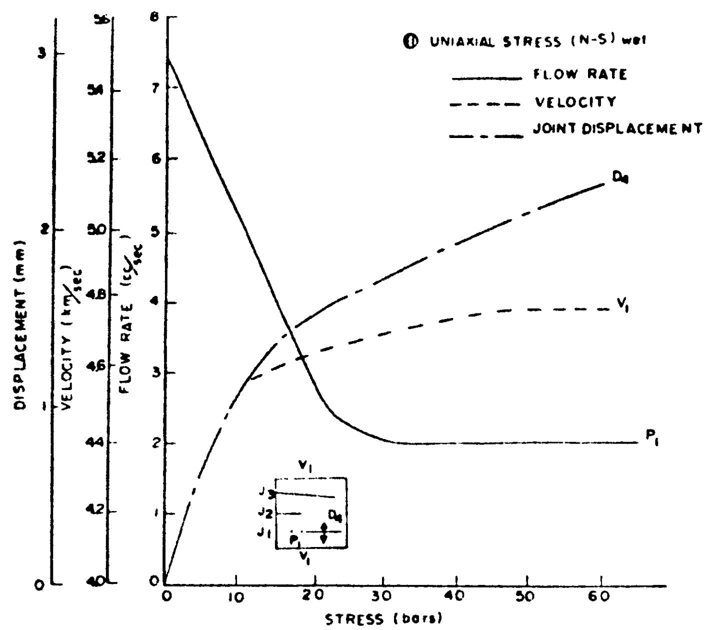


Figure 3.2 Stress dependence of flow in a fracture (from Pratt et al., 1977).

Gangi (1978) developed an analysis which uses an assumed distribution of asperity heights within a fracture to calculate fracture conductivity as a function of stress. He assumed the distribution

$$n(\ell) = \left(\frac{\ell_o - \ell}{\ell_o}\right)^{(1/m - 1)} \qquad 0 < m < 1 \tag{10}$$

where $n(\ell)$ is the fraction of asperities longer than ℓ, ℓ_o is the height of the largest asperity and m is a parameter of the distribution. Then,

$$K_f = K_{fo} \left[1 - (\sigma/E_f)^m\right]^3 \tag{11}$$

where σ is the effective normal stress, K_{fo} is the fracture hydraulic conductivity at zero stress and E_f is the fracture modulus of elasticity, related to the intact rock's modulus, E, by

$$E_f = E\,(A_c/A), \tag{12}$$

where A_c/A is the percentage of the fracture face which is in contact. This function seems to fit the data of Nelson (1975) quite nicely (Figure 11). We would be more convinced, though, if this theory were compared to other data as well.

Witherspoon et al. (1979) have noted a scale effect on the asymptotic value of K_f for high stress. It appears that the larger the sample, the higher the asymptotic value. They hypothesize that this is due to the

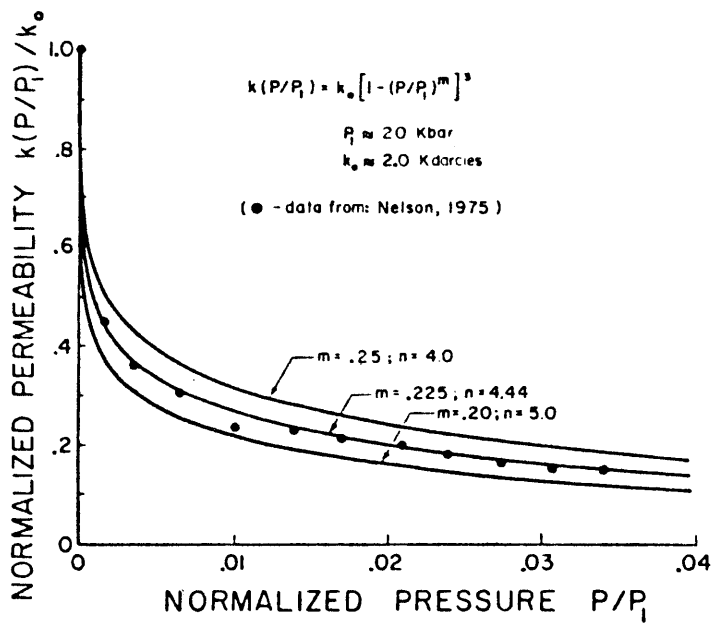


Figure 3.3 Comparison of Gangi theory with Nelson's data (from Gangi, 1978).

possibility that the smaller samples do not contain a statistically significant sample of the fracture's asperity height distribution. A larger specimen is more likely to sample asperities from the tail of the distribution, which would tend to keep the fracture propped wider for the same stress, thus producing a greater fracture conductivity. This type of information may be useful for extrapolating to field-scale effective probability density functions.

Since the fracture aperture, and hence flow, is dependent upon the effective stress applied, then the fluid pressure in the fracture is important, and not merely its gradient. For this reason, it appears that a theory of fracture hydraulics is needed which couples fluid flow and fracture mechanics. Indeed, this seems to be the case because, for the same head difference in a packer test, a fracture will accept more water upon injection then it will yield upon withdrawal. Snow (1968c) reviewed this and other evidence that fluid pressure affects conductivity in the field. In that paper he also attempted a coupled theory based on the conditions of constant vertical total stress and zero bulk horizontal strain. With this model, then, for a set of equal infinite fractures with spacing 2B, and whose normal makes an angle θ with the vertical, the change in aperture due to a change in fluid pressure is given by

$$\Delta(2b) = \frac{\Delta p}{C_f}\left(1 - \frac{E \sin^3\theta}{E \sin\theta + C_f 2B}\right) \tag{13}$$

where p is the fluid pressure and C_f is a fracture modulus. This model assumes, though, that the pressure is the same everywhere, which is strictly correct only for vertical flow.

To date, the only analyses which couple stress and flow at each point in a fracture are numerical studies (Noorishad et al., 1972; Gale et al., 1974; Noorishad and Doe, 1982; Noorishad et al., 1982). The first two papers dealt only with steady-state conditions. The latter two, however, used Biot's general theory of consolidation, and its analogy for fractures, to solve both steady-state and transient problems. Typical results follow: increasing effective stress causes fracture closure, further increasing head drop along a fracture, while decreasing the effective stress has the opposite effect.

With the aperture in a fracture varying from point to point, it becomes unclear as to which value to use to calculate flow. One alternative is to define the "effective aperture" as

$$b_{eff} = \left(\frac{3Q\nu}{2gJ}\right)^{1/3} , \tag{14}$$

but this is really a tautology. Witherspoon et al. (1980) sought to find another way to measure the aperture in the lab, and in doing so found an independent check of the cubic law. Their method was to subject a fractured specimen to uniaxial stress while measuring the strain both across the fracture and portions of the intact rock. By subtracting the deformation of the intact rock from the total deformation, it was possible to deduce the deformation in the fracture. Figure 12 shows the relationship between this deformation and the fracture's aperture. The total aperture, 2b, is equal to the sum of the apparent aperture, $2b_d$,

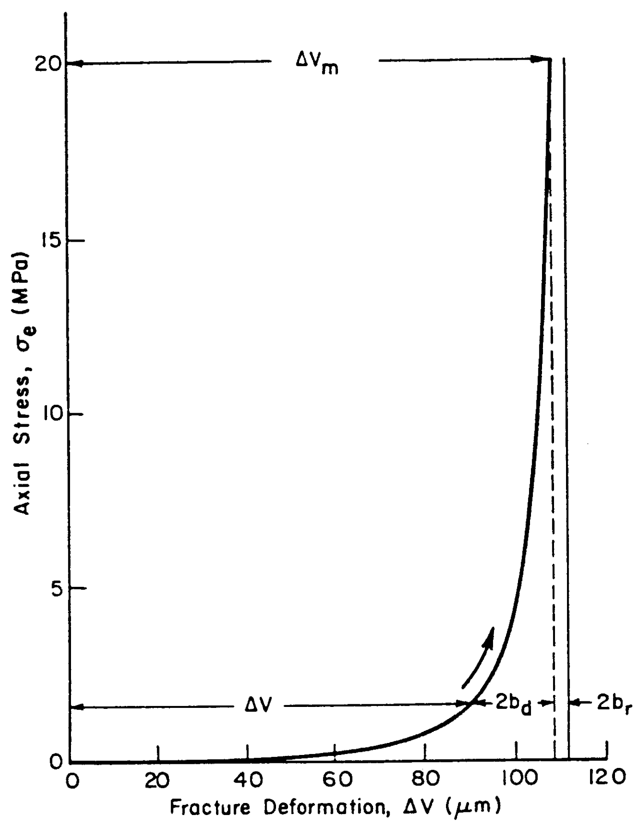


Figure 3.4 Fracture aperture related to axial stress (from Witherspoon et al., 1980).

and the residual aperture, $2b_r$. When the fracture is closed, $2b_d = 0$, but there is still flow, corresponding to the residual aperture. The cubic law was then rewritten as

$$\frac{Q}{J} = \frac{g}{12\nu} \left(2b_d + 2b_r\right)^n \tag{15}$$

where n and $2b_r$ are the unknowns, to be estimated from the stress-flow data. The result was that in every case, the exponent n came very close to 3, thus validating the cubic law for fractures whose walls are in contact at places.

Another approach is to attempt to specify the aperture at every point in the fracture and then solve for the flow analytically. Simplifying assumptions are in order. For example, Neuzil and Tracy (1981) have assumed that the aperture may be considered to vary orthogonally to the flow, but is constant in the direction parallel to the flow. Then, the effective aperture turns out to be the cube root of the arithmetic mean of the aperture distribution cubed:

$$b_{eff} = \langle b^3 \rangle^{1/3} \tag{16}$$

Tsang and Witherspoon (1981) also found this result for variation orthogonal to the flow. One may similarly analyze the effect of variation along the flow path as a series system, concluding that

$$b_{eff} = \langle b^{-3} \rangle^{-1/3} \tag{17}$$

However, these analyses are only partially correct since they do not allow for flow or gradient direction to vary from the mean, whereas it is well-known that in a two-dimensional hydraulic conductivity field, flow is diverted around areas of low conductivty. This two-dimensionality is illustrated by the dye pattern in Figure 13. It seems likely that the two-dimensional stochastic analysis of Mizell et al. (1982), can be adapted to this situation. This analysis would be more realistic by allowing for perturbations in velocity transverse to the mean flow.

3.2 Flow in Ensembles of Fractures

One of the most important questions involving the hydraulics of fracture networks is under what circumstances the fractured medium exhibits continuum behavior. This is important because if continuum behavior is evident, then for the purposes of analysis, all the tools available for porous medium flow may be used. This greatly simplifies the problem because otherwise it may be necessary to calculate the effect of each individual fracture. Not only is this a difficult problem, but the quantity and quality of field data will rarely if ever support such an effort.

For a fractured regime to be representable by an equivalent porous medium (continuum) model, two conditions (at least) must be satisfied:

(i) a representative elementary volume (REV) must exist, and

(ii) that REV must be much smaller than the domain of the flow problem.

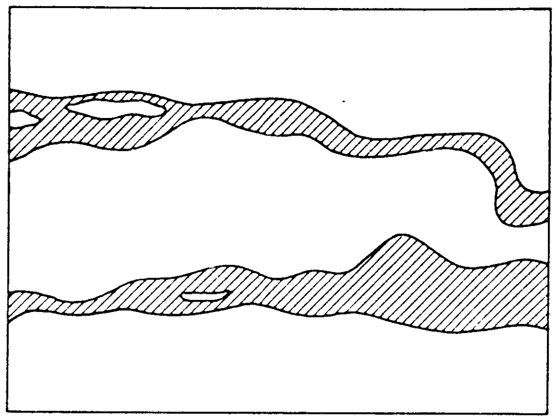


Figure 3.5 Dyestreaks in a fracture from photograph by Maini (1971) (from Neuzil and Tracy, 1981).

In an infinite domain, of course, if the first condition is satisfied, then the second is also.

One such infinite medium is the model proposed by Snow in a series of papers (Snow, 1968a, 1969; Bianchi and Snow, 1968). Let a set of fractures of aperture 2b all have the same normal vector n_i and be evenly spaced a distance d apart from each other. Then the flow through any volume with a length scale much larger than d may be calculated using the equivalent hydraulic conductivity tensor

$$K_{ij} = \frac{2b^3 g}{3\nu d} (\delta_{ij} - n_i n_j) , \tag{18}$$

which is obtained by averaging the flux due to one fracture over the spacing distance d. The natural extension to this is to consider p = 1, 2, ..., P sets of fractures, with spacings d_p, orientations n_{ip} and apertures $2b_p$. Then,

$$K_{ij} = \frac{2g}{3\nu} \sum_{p=1}^{P} \frac{b_p^3}{d_p} (\delta_{ij} - n_{ip} n_{jp}) \tag{19}$$

Use of this last equation assumes, in addition, that there is no interference between the flows in two fractures where they intersect (Snow, 1969). Wilson and Witherspoon (1976) ran an experiment to test this assumption. Their apparatus consisted of two intersecting pipes. The flow in one pipe was measured, with and without cross-flow, as a function of the Reynolds number. They concluded that for flows in the laminar range (most fracture situations) the head lost at intersections of fractures is indeed negligible.

It is evident from our previous discussion that, while joints come in sets, all the joints within a set are not parallel, but have orientations dispersed about a mean. Apertures are dispersed as well. Snow (1969) extended his model to account for this. Let a sampling vector, D_i, in a borehole intersect P fracture sets and n = 1, 2, ..., N_p representatives from the pth set. Snow's major assumption is that each individual fracture, with unit normal N_{ipn} and aperture $2b_{pn}$ is repeated with a spacing

$$d_{pn} = n_{ipn} D_i . \qquad (20)$$

Then, the conductivity tensor is given by

$$K_{ij} = \frac{2g}{3\nu} \sum_{p=1}^{P} \sum_{n=1}^{N_p} \frac{b^3_{pn}}{d_{pn}} (\delta_{ij} - n_{ipn} n_{jpn}) , \qquad (21)$$

which is equivalent to equation (23) in Snow (1969). Snow ran many Monte Carlo simulations of this equation, deriving the orientations of the principal permeabilities and their cumulative distributions for various fracture geometries.

However, this model is really just another version of the model with P sets of parallel fractures, except that now the number of sets is equal to $\sum_{p=1}^{P} N_p$, i.e., the total number of fractures intersected by the borehole.

Another twist to the problem is the consideration of deformation due to fluid pressure. Subject to the reservations noted earlier, Snow's (1968c) analysis for a single fracture set can be applied to any combination of extensive sets, as long as any increase or decrease in pressure is uniform throughout the domain. In general, since aperture changes will be different for different fracture orientations, the continuum principal conductivities will change and the principal axes will rotate. That this is the only model which attempts to couple the stress and flow for an ensemble of fractures is not surprising, due to the extreme complications involved.

All of the models discussed so far on this section have dealt with infinite domains. Boundaries have had no effect, and the hydraulic gradient has been taken as a given. In the real world, however, boundaries often do affect the flow problem.

For a finite domain, a fracture which is extensive, i.e., which completely spans the region, behaves as if it were infinite. However, even extensive fractures, if in a finite domain, may not behave as a continuum if the scale of the problem is not much larger than some characteristic scale of spacing (for example, the mean block size).

In the even more realistic case of nonextensive fractures in a finite domain, it is not even clear, for the general three-dimensional case, how to calculate the flow through the region, even if the mean gradient is constant. In two dimensions, though, network theory avails us. The principles of conservation of mass at intersections and zero net head drop around a circuit provide enough information to compute the flow through every fracture (assuming the locations, aperture, etc., are known) and thus through the region as a whole.

When does a network of finite (nonextensive) fractures, in two or three dimensions, exhibit continuum behavior? The same conditions must be met as for extensive fractures: the REV must exist, and it must be small compared to the flow domain. No one knows for sure under what conditions an REV will exist, given a network of finite fractures.

Long et al. (1982) discussed this problem in detail. For an REV to exist in an inherently heterogeneous medium, there must be some scale at which the average of these heterogeneities is invariant with a small change in the scale. This is the scale of the REV. Then, since the volume may be considered homogeneous, a symmetric conductivity tensor will predict the flux through the REV for an arbitrary direction of the gradient.

Sagar and Runchal (1982) attempted to show that, subject to certain limitations and assumptions, an equivalent hydraulic conductivity matrix for a region containing finite fractures will be both asymmetric and non-tensorial. We find their analysis lacking, however, since they have assumed that the flow in each fracture is independent of the flows in their other fractures, and may be found by projecting the hydraulic gradient upon the fracture plane. This works for extensive fracture networks, but for finite fractures it violates continuity. They then summed the contribution of each fracture to the flow through the region and found an equivalent K_{ij} for a uniform region which would predict the same flow. They have thus assumed that it is known a priori which fractures actually conduct fluid, whereas this should be one of the results of the analysis. These criticisms appear to place the validity of their results in doubt.

Long et al. (1982) used two-dimensional numerical analyses to address the problem of continuum behavior for finite fractures. They generated realizations of two-dimensional fracture networks and then measured the equivalent conductivity in the direction of the gradient, K_g, as the gradient was rotated 360°. A porous medium would have $K_g^{-1/2}$ plot as an ellipse. This is shown in Figure 14 for two sets of extensive fractures. They estimate the degree of continuum behavior for the finite-fracture networks according to the degree of ellipticity of the $K_g^{-1/2}$ plot. They then studied the effects of aperture and orientation distributions, scale and joint density. They concluded that a network of fractures with distributed apertures behaves less like an equivalent homogeneous medium than a network with uniform apertures. This makes sense because varying the apertures increases the heterogeneity in the system. A network with distributed orientations, though, was found to behave more like a continuum, due to the increased number of fracture intersections. The effects of density and scale are interdependent. At a small enough scale, finite fractures become extensive, enhancing continuum behavior. However, at this small scale, the fractures may not be dense enough that

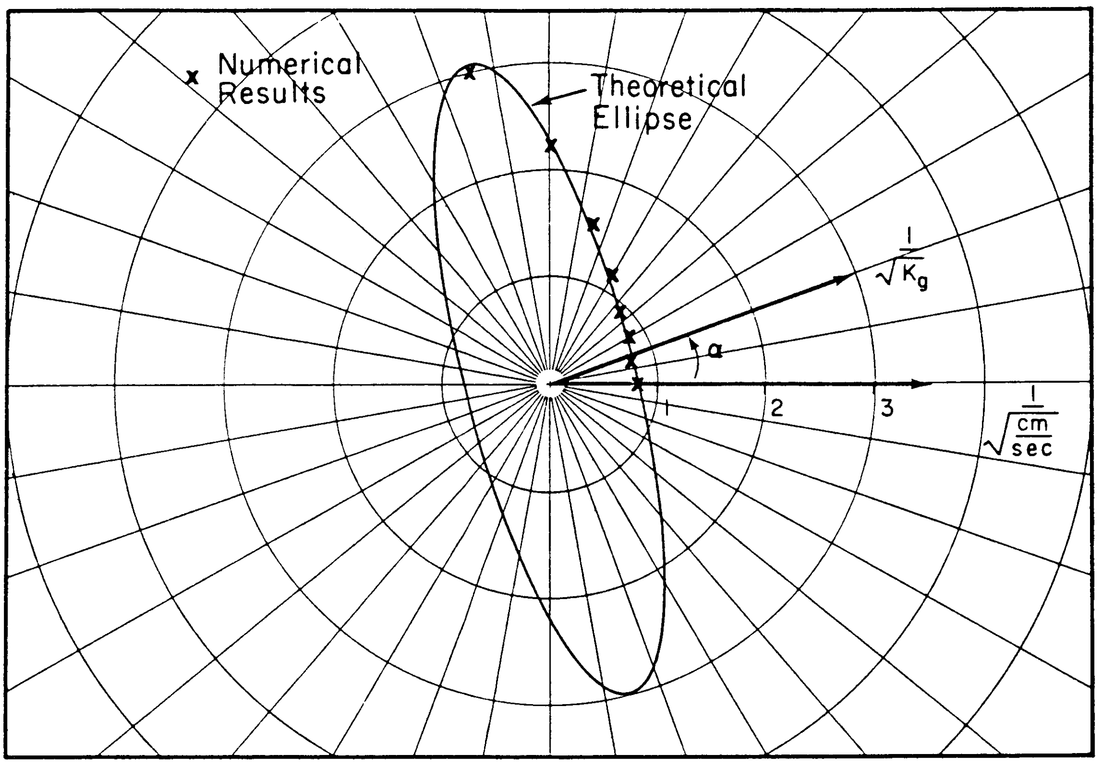


Figure 3.6 Hydraulic conductivity ellipse for infinite fractures (from Long et al., 1982).

the volume tested contains a significant statistical sample. Then the effective conductivity will be sensitive to scale changes, violating the homogeneity requirement for an REV.

It should be noted here that there seems to be two length scales inherent in a fracture system. One is the average length of a fracture in a set and one is the average spacing between fractures in that set. These should both be considered when assessing density and scale effects, a point which Long et al. (1982) seem to have neglected. We would advise a more rigorous definition of fracture density.

We know of no study to date which compares the bulk conductivity tensor as calculated from fracture data to that measured in the field.

Other aspects of fractured rock hydrology which have been studied are phreatic conditions (Castillo et al., 1972a; Leach, 1982), multi-phase flow (Braester, 1972), and well hydraulics (Boulton and Streltsova, 1977a, 1977b, 1978; Gureghian 1975; Leach, 1982; Nguyen, 1983; Strelsova, 1976; Warren and Root, 1963).

CHAPTER 4

SOLUTE TRANSPORT IN FRACTURED ROCKS

4.1 Continuum Models

The simplest model for chemical transport is based on the assumption that the fractured formation behaves as a continuum. Then, values for dispersivity and porosity may be found using standard tracer tests. Webster et al. (1970) performed a simple two-well test at the Savannah River Plant in South Carolina, finding a dispersivity of 440 feet for a test length of 1765 feet. The breakthrough curve is shown in Figure 15. We believe that a lower value for dispersivity should have been found from this data, which breaks through significantly more slowly, at first, than their fitted curve. Grove and Beetem (1971) employed a similar method in a fractured carbonate aquifer, obtaining a dispersivity of 125 feet with a 180 foot well spacing test. Claasen and Cordes (1975) reported a dispersivity of 15 m for a test length of 120 meters. Gelhar (1982) reported a dispersivity of 2 feet for a test with 56 feet well spacing in basalt at the Hanford site in Washington. Lallemand-Barres and Peaudecerf (1979) reviewed dispersivity data as a function of scale. Figure 16 shows their plot of the data. The salient feature of this plot is its considerable scatter about an otherwise observable trend. Lundstrom et al. (1978) also reported a scale effect, this time for two tests of different size, both at the same site.

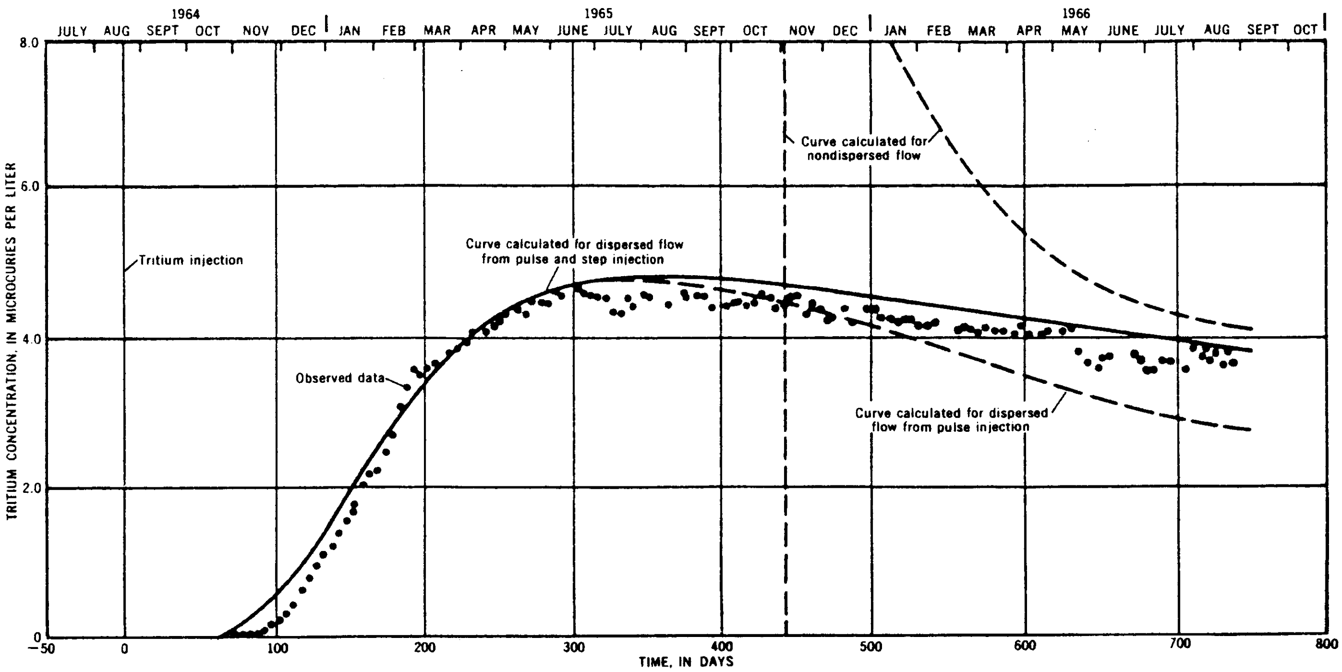


Figure 4.1 Breakthrough curve for the two-well recirculating tracer test at the Savannah River Plant, South Carolina (from Webster et al., 1970).

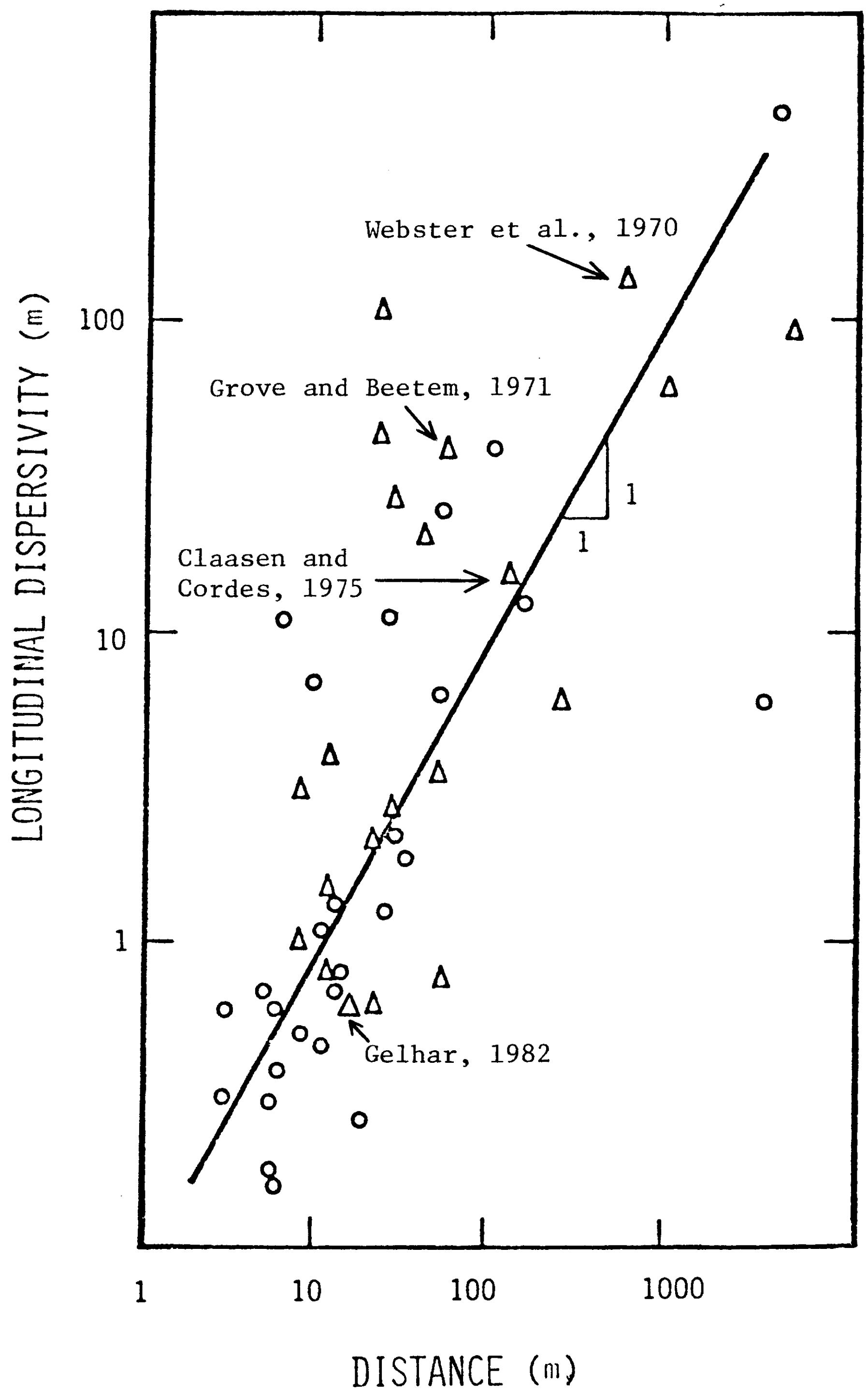


Figure 4.2 Field observations of longitudinal dispersivity; o sand, gravel and sandstone; Δ limestone, basalt, granite and schist; after Lallemand-Barres and Peaudecerf (1979).

4.2 Discrete Models

We first consider the case of solute transport in a single fracture. Not only is this the logical first step, but most of the literature deals with this simple case. This is most likely due to the complete knowledge of advection transport terms, whereas with more than one fracture, the flux problem hasn't really been solved, let alone that of predicting velocities.

In a single fracture with impermeable non-porous walls, we expect that the assumed parabolic velocity profile will tend to disperse any solute longitudinally. In addition, we expect that, after an appropriate start-up time, molecular diffusion and aperture heterogeneities will reduce the transverse gradient across the aperture and cause the longitudinal dispersion coefficient, D, to reach a constant value, along the lines of Taylor's (1953) analysis. This value is, from the analysis of Elder (1965),

$$D = \frac{(2b)^2 \bar{v}^2}{210 D_m} \tag{22}$$

where 2b is the aperture width, $\bar{v}$ is the average velocity and D_m is the coefficient of molecular diffusion. Karadi et al. (1972) presented a method of calculating the average concentration in a fracture at a point downstream of an arbitrary time-varying input boundary condition, assuming a constant longitudinal dispersivity.

Very little field or lab data exists on single-fracture dispersivities. Gustaffson and Klockars (1981) performed a two-well tracer test in fractured rock at Studsvik, Sweden, finding dispersivities on the order of 1 m for an interwell distance of 30 m. Carlsson et al. (1979), also performed a two-well tracer test but did not report a dispersivity. Laboratory tests by Grisak et al. (1980) and Neretnieks (1980) were inconclusive regarding dispersivities, but nevertheless found values of of 0.15 m and 0.025 m, respectively.

Much recent research has explored the idea of matrix diffusion (Barker, 1982; Barker and Foster, 1981; Erickson, 1981; Grisak et al. 1980; Grisak and Pickens, 1980, 1981; Neretnieks, 1980, 1983; Neretnieks et al. 1982; Rasmuson and Neretnieks, 1981; Sudicky and Frind, 1982; Tang et al. 1981; Uffink, 1983). Huyakorn et al. (1983b), Noorishad and Mehran (1982), and Rasmuson et al. (1982) presented numerical studies of this topic. Each model has different features, but the essence is the same. Mathematically, the model consists of two coupled one-dimensional second-order differential equations. One describes transport longitudinally on the fracture and the other describes diffusion transversely, but without advection, into the porous rock matrix bounding the fracture.

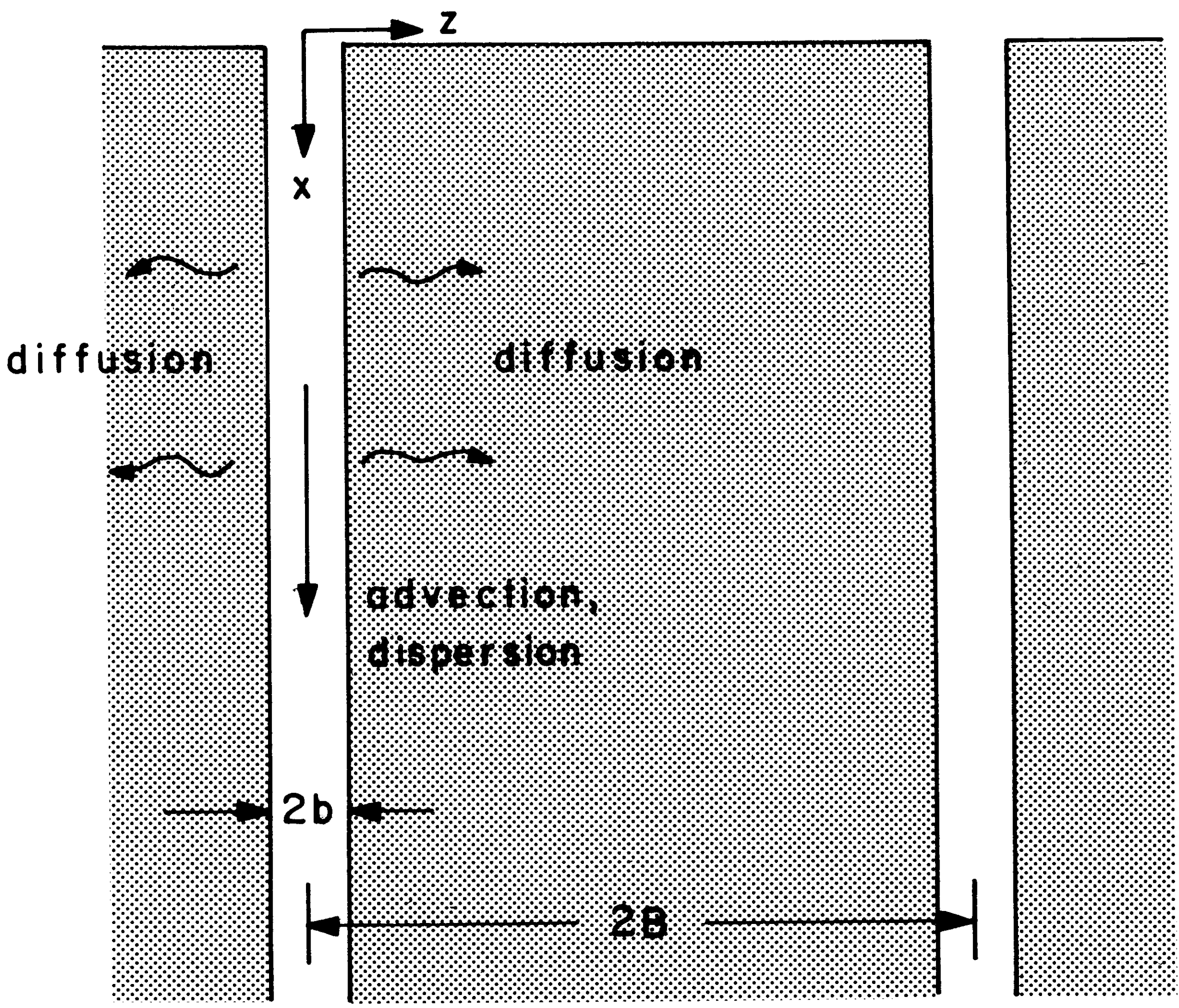


Figure 4.3 Idealized geometry for the matrix diffusion model.

Referring to the geometry of Figure 17, the mass balance equations, with all the features added, are:

$$\frac{\partial c_f}{\partial t} + v_x' \frac{\partial c_f}{\partial x} = D_L' \frac{\partial^2 c_f}{\partial x^2} - \lambda c_f - \frac{2D'_m}{b} \frac{\partial c_m}{\partial z}\bigg|_{z=b}$$

$$\frac{\partial c_m}{\partial t} = D_m' \frac{\partial^2 c_m}{\partial z^2} - \lambda c_m \qquad (23)$$

$$v_x' = v_x/R_{df}; \quad D_L' = D_L/R_{df}; \quad D_m' = D_m/R_{dm}$$

where $c_f(x,t)$ is the contaminant concentration in the fracture, $c_m(x,z,t)$ is the concentration in the rock matrix, v_x is the average velocity in the fraction, D_L is the longitudinal dispersion coefficient in the fracture, R_{df} is the retardation coefficient for solution on the fracture walls, D_m is the effective matrix diffusion coefficient, R_{dm} is the matrix retardation factor and λ is the species decay constant. Decay chains can also be accounted for (Kanki et al., 1980).

The initial and boundary conditions are:

$$c_f(x,0) = c_m(x,z,0) = 0$$

$$c_f(x,t) = c_m(x,b,t)$$

$$c_f(\infty,t) = c_m(\infty,z,t) = 0 \qquad (24)$$

$$c_f(0,t) = g(t)$$

with one of the following:

$$c_m(x,\infty,t) = 0 \qquad \text{(a)}$$

$$\frac{\partial}{\partial z} c_m(x,B,t) = 0 \, , \qquad \text{(b)} \qquad (25)$$

where g(t) is an arbitrary release scenario and 2B is the spacing between fractures in a set. Condition (b) is used in cases when the lateral boundary has an effect, i.e., when

$$B < \sqrt{D'_m \tau}$$

where τ is the travel time for a pulse of contaminant to pass through the fracture. Otherwise, condition (a) may be used. Table 2 compares the features and solution techniques for the several matrix diffusion models reported.

It is interesting to compare the matrix diffusion models to the traditional advection-dispersion model. In general, the matrix diffusion model predicts much later and much lower breakthroughs (Figure 18), due to a retarding effect whereby the front of a pulse in the fracture loses mass to the matrix while the tail of a pulse receives mass from the matrix.

Whether this can be represented by a single retardation factor is not clear. Barker and Foster (1981) showed that when $D'_m \to \infty$, the concentration is constant in the z-direction, the mass being instantaneously shared between the fracture and matrix pore space. Then, the peak of a pulse would travel with a velocity, v'_{xp}, given by

$$v'_{xp} = \frac{v_x}{R_{df}} \left(\frac{b}{\phi B + b}\right) \qquad (26)$$

where ϕ is the porosity of the matrix. Then the fracture may be considered to have an effective retardation factor, R_{df}^*, given by

$$R_{df}^* \simeq R_{df} \left(\frac{\phi B + b}{b}\right). \qquad (27)$$

Table 4.1 Summary of Matrix Diffusion Papers

Author(s)	solution technique	*a*	*b*	*c*	*d*	comments
Barker, 1982	analytical	T	yes	1.21	yes	-
Barker and Foster, 1981	analytical, numerical	T	yes	1.21	no	chromatography analogy, infiltration
Erickson, 1981	analytical	T	no	-	no	spherical diffusion
Grisak and Pickens, 1980	analytical, numerical	T	yes	1.21	no	-
Grisak and Pickens, 1981	analytical	T	no	1.22	no	-
Grisak et. al., 1980	laboratory	T	yes	1.21	no	-
Huyakorn et. al., (1983a)	numerical	T	yes	1.21	yes	spherical or rectilinear diffusion
Huyakorn et. al., (1983b)	numerical	T	yes	1.21	yes	decay chains, spherical or rectilinear diffusion

[a]steady (S) or transient (T)?
[b]longitudinal dispersivity in fracture?
[c]matrix boundary condition

Table 4.1 (continued)

Author(s)	solution technique	*a*	*b*	*c*	*d*	comments
Kanki et. al., 1980	analytical	T	no	1.22	yes	decay chains
Neretnieks, 1980	analytical	T	no	1.22	yes	decay chains
Neretnieks, 1983	analytical	T	-	1.22	no	advection-dispersion comparison
Neretnieks et. al., 1982	laboratory	T	-	1.21	yes	channeling
Noorishad and Mehran, 1982	numerical	T	yes	1.21	no	
Rasmuson and Neretnieks, 1981	analytical	T	yes	-	yes	spherical diffusion
Rasmuson et. al., 1982	numerical	T	yes	-	yes	spherical diffusion
Sudicky and Frind, 1982	analytical	S, T	yes	1.22	no	

[a]steady (S) or transient (T)?
[b]longitudinal dispersivity in fracture?
[c]matrix boundary condition
[d]decay?

Table 4.1 (continued)

Author(s)	solution technique	*a*	*b*	*c*	*d*	comments
Sudicky and Frind, 1984	analytical	T	no	1.22	yes	two-member decay chain
Tang et. al., 1981	analytical	S, T	yes	1.21	no	
Uffink, 1983	analytical	T	no	1.22, 1.21	no	heat flow

[a]steady (S) or transient (T)?
[b]longitudinal dispersivity in fracture?
[c]matrix boundary condition
[d]decay?

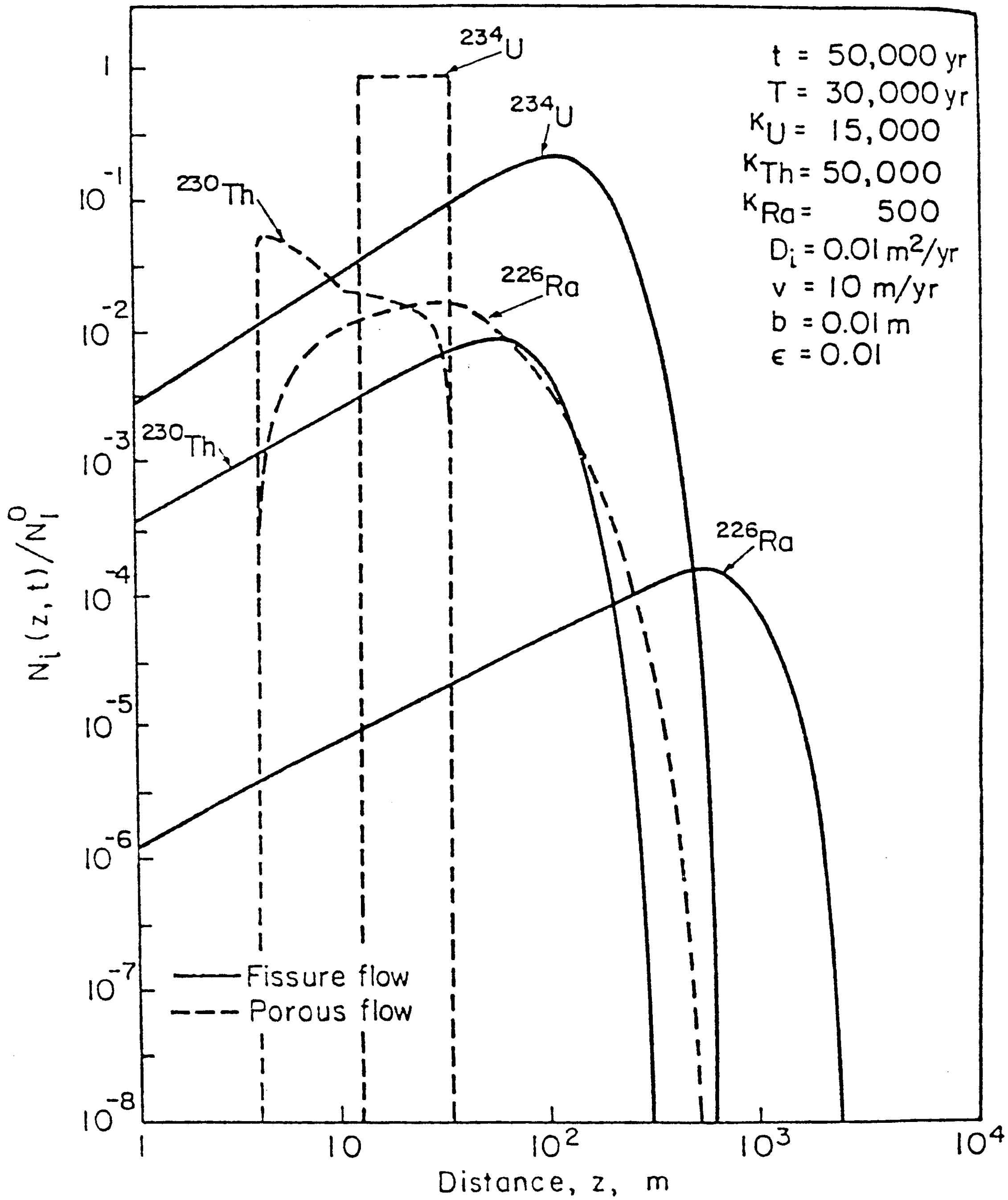


Figure 4.4 Radionuclide decay chain concentration distributions for advection-dispersion (dashed line) and matrix diffusion (solid line) models (from Kanki et al., 1980).

When D'_m is low enough to affect the problem, though, an originally Gaussian profile in the fracture becomes increasingly skewed towards the upstream side. Neretnieks (1983) discussed this trend for the case when $D_L = 0$. The breakthrough curve is shown in Figure 19. This curve is characterized by a peak at

$$t = \tau + \frac{4Dm' \tau^2}{b^2} , \tag{28}$$

which cannot be represented by a simple retardation factor.

Another characteristic of this curve is that is has an infinite variance. Therefore, no advection-dispersion solution can represent this breakthrough. As the lateral boundary becomes important (i.e., for small B, large D'_m or large τ), we would expect that Taylor's (1953) analysis would provide an effective dispersion coefficient capable of predicting the spreading about the peak of the mean concentration across the width 2B. As $B \to 0$, $D_m' \to \infty$ or $\tau \to \infty$, the effective dispersion coefficient approaches D_L.

Grisak et al. (1980) ran a tracer experiment in the laboratory using both reactive and non-reactive solutes and a step input. Even for the non-reactive solute, simple advection-dispersion solutions could not fit the breakthrough curves, whereas matrix-diffusion solutions could, albeit roughly. Grisak et al. (1980) used an analytical model which used boundary

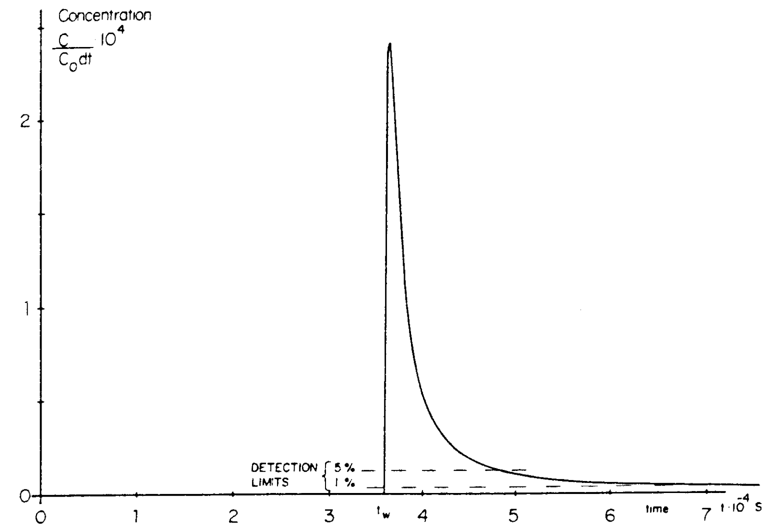
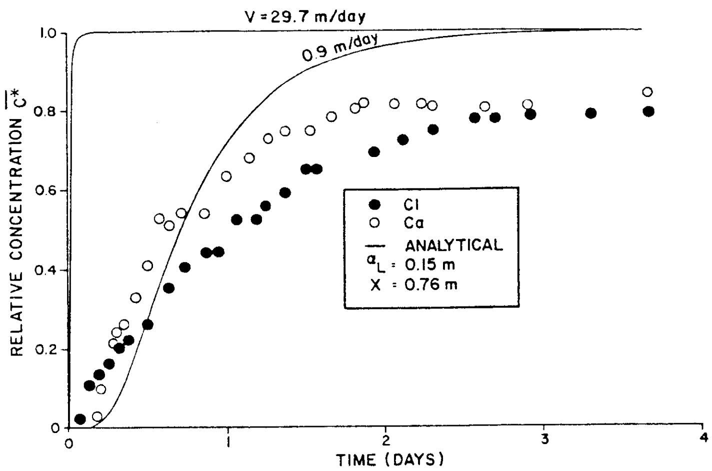


Figure 4.5 Concentration breakthrough curve for a pulse input in a fracture with matrix diffusion (from Neretnieks, 1983).

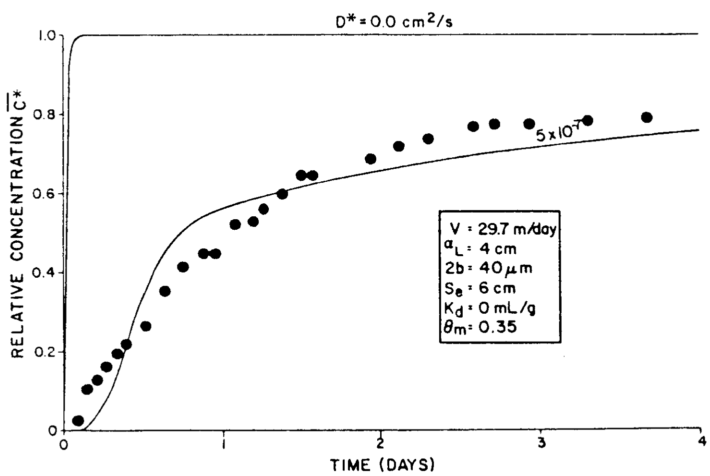
condition (b), with B = 3 cm. However, we don't think the lateral boundaries had an effect on the breakthrough, since the mean distance of diffusion into the matrix was only $(D_m'\tau)^{1/2} = 0.36$ cm. This is also shown by the fact that the breakthroughs reached only about 80% of C_0 (Figure 20), indicating that some mass had not yet left the matrix. We think longer tests need to be done, particularly if these results are to be extrapolated to field scale. Yet, some researchers are using this model to find breakthrough curves for points as far downstream as 4 km (Grisak and Pickens, 1981). This model may, however, be adapted to the case where a thin zone of highly fractured rock is surrounded by a large zone of relatively intact rock as in Uffink (1983). Then, however, there would undoubtedly be some longitudinal dispersion.

It would be appealing to be able to calculate this longitudinal dispersion coefficient as a function of the fracture parameters or their distributions. Unfortunately, no such analysis is available.

Neretnieks (1983) discussed a model with one set of parallel fractures with no matrix diffusion. Each fracture has a constant aperture, but the apertures in the set vary lognormally. As in all perfectly stratified models with no transverse dispersion, the dispersivity increases without bound as time and distance increase. Such models should be used cautiously because they may overestimate the magnitude of the dispersion coefficient at large distances and, hence, overpredict the degree of dilution. It seems possible that including more than one set in this model would alleviate the non-Fickian nature of the spreading, and might even be an



(a) Advection-dispersion model.



(b) Matrix diffusion model with chloride.

Figure 4.6 Comparison of model simulations (solid lines) with laboratory experiments (from Grisak et al., 1980).

acceptable representation of field-scale dispersion. Allowing for more than one fracture set should also diminish the effects of matrix diffusion as all the fracture sets will be competing for the same storage space.

Testor and Potter (1979) described a series of tracer tests in a hydraulically fractured granite. They concluded that the shapes of their breakthrough curves were due to the sum of a small number of discrete pathways. Although their curve fitting procedure does not provide a unique set of parameters, the variation in Peclet numbers from 0.0094 to 3.32 for different components of the same breakthrough curve is probably realistic.

Numerical experiments on solute transport through two-dimensional networks of discrete fractures have been reported by Castillo et al. (1972b) using constant aperture fractures which extend through the entire flow domain. Similar numerical experiments on two-dimensional orthogonal networks of non-extensive fractures with randomly varying apertures are reported by Schwartz et al. (1983) and Smith and Schwartz (1984). These numerical experiments produce interesting results which suggest significant departures from classical Fickian dispersion but, because of the small size of these networks and the oversimplified assumptions about fracture orientation, it can not be concluded that these two-dimensional results are representative of natural three-dimensional networks.

CHAPTER 5

COMMENTS AND RECOMMENDATIONS

In the area of fracture characterization there are several probabilistic models which seem to represent three-dimensional fracture geometry in ways which are consistent with statistics from joint surveys. However, it is not clear how realistically these existing models represent the degree of fracture interconnection especially in relatively sparse networks. There is a need for systematic investigations emphasizing the connectivity characteristics of these models since this feature will be critical in determining the flow properties of such networks. Percolation theory (e.g., Charlaix et al., 1984) may provide a framework for evaluating connectivity characteristics. Also the question of spatial correlation of fracture characteristics needs to be explored because if such persistence exists, it implies a greater degree of continuity of flow paths than would be produced by a completely random model.

In the area of fracture hydraulics, it is well established that extensively fractured systems can be represented in a continuum sense as an equivalent porous medium. However, in the case of non-extensive fracture systems it remains unclear, even for the relatively simple case of two-dimensional networks, what specific criteria must be satisified in order to represent the discrete network as a hydraulic continuum. In the case of realistic three-dimensional networks it seems that a workable flow simulator has not even been developed. Furthermore, it seems likely that three-dimensionality will play a critical role in the connectivity and continuum behavior fractured rock systems.

The understanding of solute transport in fractured systems is much more limited than that of the hydraulics. Continuum models, which are based on the classical advection-dispersion equation, seem to be adequate for some field situations, but little is known about the conditions under which these models are expected to be applicable. The so-called matrix diffusion model is conceptually attractive and has been the subject of numerous theoretical studies. Hypothetical calculations based on matrix diffusion models show that this mechanism could produce very significant retardation of solutes but there are no field experiments which demonstrate the importance of this mechanism in fractured rock. There is a need for carefully designed field experiments which evaluate the importance of matrix diffusion.

There is no established approach for treating solute transport in a realistic three-dimensional network of discrete fractures. Even the relatively simple problem of solute transport in three-dimensional infinitely extensive fractures has not been analyzed. Also, the effect of aperture variability on solute-transport within a single fracture have not been evaluated. This effect could be analyzed by treating the aperture as a spatial random field and solving the flow and solute transport equations for the fracture as stochastic differential equations.

REFERENCES

1. Baecher, G.B., "Statistical Analysis of Rock Mass Fracturing," Math. Geol., 15(2), 329-348, 1983.

2. Baecher, G.B., and N.A. Lanney, "Trace Length Biases in Joints Surveys," in 19th U.S. Symposium on Rock Mechanics, 1978.

3. Baecher, G.B., N.A. Lanney, and H.H. Einstein, "Statistical Description of Rock Fractures and Sampling," in 18th U.S. Symposium on Rock Mechanics, 1977.

4. Barker, J.A., "Laplace Transform Solutions for Solute transport in Fissured Aquifers," Adv. Wat. Res., 5, 98-104, 1982.

5. Barker, J.A., and S.D. Foster, "A Diffusion Exchange Model for Solute Movement in Fissured Porous Rock," Q.J. Eng. Geol. London, 14, 17-24, 1981.

6. Bear, J., Dynamics of Fluids in Porous Media, Elsevier, New York, 1972.

7. Bianchi, L., and D.T. Snow, "Permeability of Crystalline Rock Interpreted from Measured Orientations and Apertures of Fractures," Annals of the Arid Zone, 8(2), 231-245, 1968.

8. Bortolami, G.C., J-Ch. Fontes, D. Lale-Demoz, Ph. Olive, L. Quijano, and G.M. Zuppi, "Infiltration Rate Through the Crystalline Massif of Mont Blanc Evidenced by Environmental Isotope Measurements," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable-Rocks, Paris, pp. 237-248, 1979.

9. Boulton, N.S., and T.D. Streltsova, "Unsteady Flow to a Pumped Well in a Two-Layered Water-Bearing Formation," J. Hydr., 35, 245-250, 1977.

References (cont'd)

10. Boulton, N.S., and T.D. Streltsova, "Unsteady Flow to a Pumped Well in a Fissured Water-Bearing Formation," J. Hydr., 257-269, 1977.

11. Boulton, N.S., and T.D. Streltsova, "Unsteady Flow to a Pumped Well in a Fissured Aquifer with a Free Surface Maintained Constant," Water Resources Research, 14(3), 527-532, 1978.

12. Bower, D.R., "Bedrock Fracture Parameters from the Interpretation of Well Tides," J. Geophys. Res., 88(B6), 5025-5035, 1983.

13. Braester, C., "Simultaneous Flow of Immiscible Liquids through Porous Fissured Media," Soc. Petr. Eng. J., 297-305, 1972.

14. Burgess, A.S., R.G. Charlwood, E.L. Skiba, J.L. Ratigan, P.F. Guirk, H. Stille, and U.E. Lindblom, "Analysis of Groundwater Flow around a High-Level Waste Repository in Crystalline Rock," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 199-218, 1979.

15. Castillo, E., G.M. Karadi, and R.J. Krizek, "Unconfined Flow through Jointed Rock," Water Resour. Bull., 8(2), 266-281, 1972a.

16. Castillo, E., R.J. Krizeck, and G.M. Karadi, "Comparison of Dispersion Characteristics in Fissured Rocks," Proc. Symp. Fundam. Transp. Phenom. Porous Media, 2nd, 778-797, 1972b.

17. Carlsson, L., G. Gidlund, H. Hansson, and C.-E. Klockars, "Estimation of Hydraulic Conductivity in Swedish Precambrian Crystalline Rock," in Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 97-114, 1979.

18. Charlaix, E., E. Guyon, and N. Rivier, "A Criterion for Percolation Threshold in a Random Array of Plates," Solid State Communications 60 (11), 999-1002, 1984.

<u>References</u> (cont'd)

19. Claasen, H.C., and E.C. Cordes," Two-well Recirculating Tracer Test in Fractured Carbonate Rock, Nevada," <u>Hydrol. Sci. Bull.</u>, 20(3), 367-382, 1975.

20. Davison, C.C., G.E. Grisak, and D.W. Williams, "Field Permeability and Hydraulic Potential Measurements in Crystalline Rock and Solute Transport through Finely Fractured Media," in <u>Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks</u>, Paris, 139-154, 1979.

21. Dienes, J.K., "On the Inference of Crack Statistics from Observations on an Outcropping," in <u>20th U.S. Symposium on Rock Mechanics</u>, 1979.

22. Elder, J.W., "The Dispersion of Marker Fluid in Turbulent Shear Flow," <u>Journal of Fluid Mechanics</u>, 5, 544-560, 1965.

23. Erickson, K.L., "A Fundamental Approach to the Analysis of Radionuclide Transport Resulting from Fluid Flow through Jointed Media," Sandia National Laboratories report SAND80-0457, 1981.

24. Forster, C.B., and J.E. Gale, "Field Assessment of the Use of Borehole Pressure Transients to Measure the Permeability of Fractured Rock Masses," report LBL-11829, SAC-34, UC-70, Lawrence Berkeley Lab, Berkeley, California, 1981.

25. Gale, J.E., "Fundamental Hydraulic Characteristics of Fractures from Field and Laboratory Investigations," in <u>Papers of the Groundwater in Fractured Rock Conference</u>, AWRC, Canberra, Australia, 79-94, 1982.

26. Gale, J.E., A. Rouleau, and P.A. Witherspoon, "Hydrogeologic Characteristics of a Fractured Granite," in <u>Papers of the Groundwater in Fractured Rock Conference</u>, AWRC, Canberra, Australia, 95-108, 1982.

References (cont'd)

27. Gale, J.E., R.L. Taylor, P.A. Witherspoon, and M.S. Ayatollahi, "Flow in Rocks with Deformable Fractures," in Finite Elements in Flow Problems, J.T. Oden, O.C. Zienkiewicz, R.H. Gallagher and C. Taylor, eds., UAH Press, Huntsville, Alabama, 583-598, 1974.

28. Gangi, A.F., "Variation of Whole and Fractured Porous Rock Permeability with Confining Pressure," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 15, 249-257, 1978.

29. Gelhar, L.W., "Analysis of Two-Well Tracer Tests with a Pulse Input," report RHO-BW-CR-131P, Rockwell International Corporation, Richland, Washington, 1982.

30. Grisak, G.E., and J.F. Pickens, "Solute Transport through Fractured Media, 1. The Effect of Matrix Diffusion," Water Resources Research, 16(4), 719-730, 1980.

31. Grisak, G.E., and J.F. Pickens, "An Analytical Solution for Solute Transport through Fractured Media with Matrix Diffusion," J. Hydr., 52, 47-57, 1981.

32. Grisak, G.E., J.F. Pickens, and J.A. Cherry, "Solute Transport through Fractured Media, 2. Column Study of Fractured Till," Water Resources Research, 16(4), 731-739, 1980.

33. Grove, D.B., and W.A. Beetem, "Porosity and Dispersion Constant Calculations for a Fractured Carbonate Aquifer Using the Two Well Tracer Method," Water Resources Research, 7(1), 128-134, 1971.

34. Gureghian, A.B., "A Study by the Finite-Element Method of the Influence of Fractures in Confined Aquifers," Soc. Petr. Eng. J., 181-191, 1975.

References (cont'd)

35. Gustafsson, E., and C.-E. Klockars, "Studies on Groundwater Transport in fractured Crystalline Rock under Controlled Conditions using Nonradiocactive Tracers," KBS technical report 81-07, Uppsala, Sweden, 1981.

36. Hsieh, P.A., S.P. Neuman, and E.S. Simpson, "Pressure Testing of Fractured Rocks -- A Methodology Employing Three-Dimensional Cross-Hole Tests," report NUREG/CR-3213, U.S. Nuclear Regulatory Commission, 1983.

37. Hsieh, P.A., J.V. Tracy, C.E. Neuzil, J.D. Bredehoeft, and S.E. Silliman, "A Transient Laboratory Method for Determining the Hydraulic Properties of Tight Rocks - I. Theory," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 18(3), 245-252, 1981.

38. Hudson, J.A., and S.D. Priest, "Discontinuities and Rock Mass Geometry," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.," 161, 339-362, 1979.

39. Hudson, J.A., and S.D. Priest, "Discontinuity Frequency in Rock Masses," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 20(2), 73-89, 1983.

40. Huyakorn, P.S., B.H. Lester, and C.R. Faust, "Finite Element Techniques for Modeling Groundwater Flow in Fractured Aquifers," Water Resources Research, 19(4), 1019-1035, 1983.

41. Huyakorn, P.S., B.H. Lester, and J.W. Mercer, "An Efficient Finite Element Technique for Modeling Transport in Fractured Porous Media, 1. Single Species Transport," Wat. Res. Res., 19(3), 841-854, 1983b.

42. Kanki, T., A. Fujita, P.L. Chambre, and T.H. Pigford, "Radionuclide Transport through Fractured Rock," report LBL-13094, Lawrence Berkeley Lab, Berkeley, California, 1980.

43. Karadi, G.M., R.J. Krizek, and E. Castillo, "Hydrodynamic Dispersion in a Single Rock Joint," J. Appl. Phys., 43(12), 5013-5021, 1972.

References (cont'd)

44. Kiraly, L., "Statistical Analysis of Fractures (Orientation and Density)," Geologische Rundschau, 59, 125-151, 1969.

45. Kranz, R.L., A.D. Frankel, T. Engelder, and C.H. Scholz, "The Permeability of Whole and Jointed Barre Granite," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 16, 225-234, 1979.

46. Lallemand-Barres, A., and P. Peaudecerf, "Recherche des Relations entre la Valeur de la Dispersivité Macroscopique d'un Mileau Aquifere, ses autres Caracteristiques et les Conditions de Mesure," Bulletin 4, 2e Sér., Sec. III, Bar. de Rech. Geol. et Mineres, Orleans, France, 1978.

47. LaPointe, P.R., "Analysis of the Spatial Variation in Rock Mass Properties through Geostatistics," in 21st U.S. Symposium on Rock Mechanics, 1980.

48. Leach, L.M., "Application of a Modified Jacob's Correction to Pump Tests in Fractured Rock Bores," in Papers of the Groundwater in Fractured Rock Conference, AWRC, Canberra, Australia, 127-138, 1982.

49. Lindblom, V.E., "Comparison of Flow and Permeability Interpreted from In-Situ Measurements in a Granitic Rock," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 125-138, 1979.

50. Long, J.C.S., J.S. Remer, C.R. Wilson, and P.A. Witherspoon, "Porous Media Equivalents for Networks of Discontinuous Fractures," Water Resources Research, 18(3), 645-658, 1982.

51. Louis, C., "A Study of Ground Water Flow in Jointed Rock and its Influence on the Stability of Rock Masses," Rock Mechanics Report No. 10, Imperial College, London, 1969.

52. Lundstrom, O., C.-E. Klockars, K.E. Holenberg, and S. Westerberg, "In Situ Experiments on Nuclide Migration in Fractured Crystalline Rocks," KBS Tech. Rep. 110, Swedish Nuclear Supply Co., Stockholm, 1978.

53. Lundstrom, L., and H. Stille, "Large Scale Permeability Test of the Granite in the Stripa Mine and Thermal Conductivity Test," report LBL-7052, SAC-02, US-70, TID-4500-R66, Lawrence Berkeley Lab, Berkeley, California, 1978.

54. Maini, T., "In situ Hydraulic Parameters in Jointed Rock - Their Measurement and Interpretation," Ph.D. Thesis, Imp. Coll. of Sci. and Tech., London, 1971.

55. Maini, T., and G. Hocking, "An Examination of the Feasibility of Hydrologic Isolation of a High Level Waste Repository in Crystalline Rock," invited paper, Geologic Disposal of High-Level Radioactive Waste Session, Annual Meeting of the Geological Society of America, Seattle, 1977.

56. Maini, T., J. Noorishad, and J. Sharp, "Theoretical and Field Considerations on the Determination of In Situ Hydraulic Parameters in Fractured Rock," in Int'l. Soc. Rock Mech., Symp. Perc. Fiss. Rock, Stuttgart, Germany, pp. T1-E, 1-8, 1972.

57. Marine, I.W., "Determination of the Location of Fractures in Metamorphic Rock with In-Hole Tracers," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 85-96, 1979.

58. Mizell, S.A., A.L. Gutjahr, and L.W. Gelhar, "Stochastic Analysis of Spatial Variability in Two-Dimensional Steady Groundwater Flow Assuming Stationary and Nonstationary Heads," Water Resources Research, 18(4), 1053-1067, 1982.

References (cont'd)

59. Nelson, P.H., R. Rachiele, and A. Smith, "The Effect of Radon Transport in Groundwater upon Gamma-Ray Borehole Logs," report LBL-11180, SAC-30, UC-70, Lawrence Berkeley Lab, Berkeley, California, 1980.

60. Nelson, R., "Fracture Permeability in Porous Reservoirs: Experimental and Field Approach," Ph.D. thesis, Texas A & M Univ., College Station, Texas, 1975.

61. Nelson, R., and J. Handin, "Experimental Study of Fracture Permeability in Porous Rock," Amer. Assoc. Petr. Geol. Bull., 61(2), 227-236, 1977.

62. Neretnieks, I., "Diffusion in the Rock Matrix: An Important Factor in Radionuclide Retardation?," J. Geophys. Res., 85(B8), 4379-4397, 1980.

63. Neretnieks, I., "A Note on Fracture Flow Dispersion Mechanics in the Ground," Water Resources Research, 19(2), 364-370, 1983.

64. Neretnieks, I., T. Eriksen, and P. Tähtinen, "Tracer Movement in a Single Fissure in Granite Rock: Some Experimental Results and their Interpretation," Water Resources Research, 18(4), 849-858, 1982.

65. Neuzil, C.E., C. Cooley, S.E. Silliman, J.D. Bredehoeft, and P.A. Hsieh, "A Transient Laboratory Method for Determining the Hydraulic Properties of 'Tight' Rocks - II. Application," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 18, 253-258, 1981.

66. Neuzil, C.E., and J.V. Tracy, "Flow through Fractures," Water Resources Research, 17(1), 191-199, 1981.

67. Nguyen, V.V., "Direct Parameter Identification of Fractured Porous Medium," Adv. Wat. Res., 6, 11-13, 1983.

References (cont'd)

68. Noorishad, J., M.S. Ayatollahi, and P.A. Witherspoon, "A Final Element Method for Coupled Stress and Fluid Flow Analysis in Fractured Rock Masses," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 19(4), 185-193, 1982.

69. Noorishad, J., and T.W. Doe, "Numerical Simulation of Fluid Injection into Deformable Fractures," report LBL-12833, Lawrence Berkeley Laboratory, Berkeley, California, 1982.

70. Noorishad, J., and M. Mehran, "An Upstream Finite Element Method for Solution of the Transient Transport Equation in Fractured Porous Media," Water Resources Research, 18(3), 588-596, 1982.

71. Noorishad, J., P.A. Witherspoon, and T. Maini, "The Influence of Fluid Injection on the State of Stress in the Earth's Crust," in Proc. Int'l Soc. Rock Mech., Symp. Perc. Fiss. Rock, Stuttgart, Germany, pp. T2-H, 1-11, 1972.

72. O'Neill, K., and G.F. Pinder, "A Derivation of the Equations for Transport of Liquid and Heat in Three Dimensions in a Fractured Porous Medium," Adv. Wat. Res., 4, 150-164, 1981.

73. Pratt, H.R., H.S. Swolfs, W.F. Brace, A.D. Black, and J.W. Handin, "Elastic and Transport Properties of an In Situ Jointed Granite," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., 14, 35-45, 1977.

74. Rasmuson, A., T.N. Narasimhan, and I. Neretnieks, "Chemical Transport in a Fissured Rock: Verification of a Numerical Model," Water Resources Research, 18(5), 1479-1492, 1982.

75. Rasmuson, A., and I. Neretnieks, "Migration of Radionuclides in Fissured Rock: The Influence of Micropore Diffusion and Longitudinal Dispersion," J. Geophys. Res., 86(B5), 3749-3758, 1981.

References (cont'd)

76. Rocha, M., and F Franciss, "Determination of Permeability in Anisotropic Rock Masses from Integral Samples," in Structural and Geotechnical Mechanics, J.W. Hall, ed., Prentice-Hall, New York, 178-202, 1977.

77. Saari, K., "Analysis of Jointing of Granitic Rock Mass," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 261-274, 1979.

78. Sagar, B., and A. Runchal, "Permeability of Fractured Rock: Effect of Fracture Size and Data Uncertainties," Water Resources Research, 18(2), 266-274, 1982.

79. Shapiro, A.M., and J. Andersson, "Steady State Fluid Response in Fractured Rock: A Boundary Element Solution for a Coupled, Discrete Fracture Continuum Model," Water Resources Research, 19(4), 959-969, 1983.

80. Schwartz, F.W., Smith, L., and A.S. Crowe. "A Stochastic Analysis of Macroscopic Dispersion in Fractured Media," Water Resources Research, 19 (5), 1253-1265, 1983.

81. Smith, L., and F.W. Schwartz. "An Analysis of the Influence of Fracture Geometry on Mass Transport in Fractured Media," Water Resources Research, 20(9), 1241-1252, 1984.

82. Snow, D.T., "Three-Hole Pressure Test for Anisotropic Foundation Permeability," Felsmechanik und Ingenieurgeologie, 4(4), 298-316, 1966.

83. Snow, D.T., "Rock Fracture Spacings, Openings and Porosities," J. Soil Mech. Found. Div., ASCE, (SM1) 73-91, 1968a.

84. Snow, D.T., "Hydraulic Character of Fractured Metamorphic Rocks of the Front Range and Implications to the Rocky Mountain Arsenal Well," Colo. Sch. Min. Quart., 63(1), 167-200, 1968b.

References (cont'd)

85. Snow, D.T., "Fracture Deformation and Changes of Permeability and Storage upon Changes of Fluid Pressure," Colo. Sch. Min. Quart., 63(1), 201-244, 1968c.

86. Snow. D.T., "Anisotropic Permeability of Fractured Media," Water Resources Research, 5(6), 1273-1289, 1969.

87. Snow, D.T., "The Frequency and Apertures of Fractures in Rock," Int. J. Rock Mech. Min. Sci., 7, 23-40, 1970.

88. Streltsova, T.D., "Hydrodynamics of Groundwater Flow in a Fractured Formation," Water Resources Research, 12(3), 405-414, 1976.

89. Sudicky, E.A., and E.O. Frind, "Contaminant Transport in Fractured Porous Media: Analytical Solution for a Two-Member Decay Chain in a Single Fracture," Wat. Res. Res., 20(7), 1021-1029, 1984.

90. Sudicky, E.A., and E.O. Frind, "Contaminant Transport in Fractured Porous Media: Analytical Solutions for a System of Parallel Fractures," Water Resources Research, 18(6), 1634-1642, 1982.

91. Tang, D.H., E.O. Frind, and E.A. Sudicky, "Contaminant Transport in Fractured Porous Media: Analytical Solution for a Single Fracture," Water Resources Research, 17(3), 555-564, 1981.

92. Taylor, G.I., "The Disperson of Soluble Matter in Solvent Flowing Slowly through a Tube," Proc. R. Soc. London Ser. A. 219, 189-203, 1953.

93. Tester, J.W., and R.M. Potter, "Interwell Tracer Tests of a Hydraulically Fractured Granitic Geothermal Reservoir," SPE-AIME Conference, Las Vegas, Nevada, September 23-26, 1979.

94. Tsang, Y.W., and P.A. Witherspoon, "Hydromechanical Behavior of a Deformable Rock Fracture Subject to Normal Stress," J. Geophys. Res., 86(B10), 9287-9298, 1981.

95. Tsang, Y.W., and P.A. Witherspoon, "The Dependence of Fracture Mechanical and Fluid Flow Properties on Fracture Roughness and Sample Size," J. Geohys. Res., 88(B3), 2359-2366, 1983.

96. Uffink, G.J.M., "Dampening of Fluctuations in Groundwater Temperature by Heat Exchange Between the Aquifer and the Adjacent Layers," J. Hydr., 60, 311-328, 1983.

97. Veneziano, D., "Probabilistic Model of Joints in Rock," technical report, Department of Civil Engineering, M.I.T., Cambridge, Massachusetts, 1979.

98. Wang, J.S.Y., T.N. Narasimhan, C.F. Tsang, and P.A. Witherspoon, "Transient Flow in Tight Fractures," report LBL-7027, Lawrence Berkeley Lab, Berkeley, California, 1978.

99. Warren, J.E., and P.J. Root, "The Behavior of Naturally Fractured Reservoirs," So. Petr. Eng. J., 245-255, 1963.

100. Webster, D.S., J.F. Proctor, and I.W. Marine, "Two-Well Tracer Test in Fractured Crystalline Rock," USGS Water Supply Paper 1544-I, 1970.

101. Wilson, C.R., P.A. Witherspoon, J.C.S. Long, R.M. Galbraith, A.O. DuBois, and M.J. McPherson," Large-Scale Hydraulic Conductivity Measurements in Fractured Granite," Int. J. Rock Mech. Min. Sci. Geomech. Abstr., 20(6) 269-279, 1983.

102. Wilson, C.R., T.W. Doe, J.C.S. Long, and P.A. Witherspoon, "Permeability of Rock Masses for Nuclear Waste Repository Siting," in Proceedings of the Workshop on Low-Flow Low-Permeability Measurements in Largely Impermeable Rocks, Paris, 13-27, 1979.

103. Wilson, C.R., and P.A. Witherspoon, "Flow Interference Effects at Fracture Intersections," Water Resources Research, 12(1), 102-104, 1976.

References (cont'd)

104. Witherspoon, P.A., C.H. Amick, J.E. Gale, and K. Iwai, "Observations of a Potential Size Effect in Experimental Determination of the Hydraulic Properties of Fractures," Water Resources Research, 15(5), 1142-1146, 1979.

105. Witherspoon, P.A., J.S.Y. Wang, K. Iwai, and J.E. Gale, "Validity of Cubic Law for Fluid Flow in a Deformable Rock Fracture," Water Resources Research, 16(6), 1016-1024, 1980.

106. Whincup, P., and G. Domahidy, "The Agnew Nickel Project - No. 1 Mine Dewatering," in Papers of the Groundwater in Fractured Rock Conference, AWRC, Canberra, Australia, 261-272, 1982.