

**ESSAYS IN MONETARY ECONOMICS**

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## ESSAYS IN MONETARY ECONOMICS

Toshiki Yotsuzuka

### ABSTRACT

This dissertation consists of three essays, each of which deals with either or both of two of the most important topics in monetary economics: government deficits and inflation.

The first essay critically examines the validity of the common claim that Ricardian equivalence fails if capital markets are imperfect. Since credit rationing models based on asymmetric information are widely considered as a theoretical basis for such a claim, we concentrate on the question of whether models of informationally imperfect capital markets indeed yield debt nonneutrality results. We present three alternative models of adverse selection and analyze the effects of debt finance in these models. It is shown that a debt-financed tax cut can lead to Pareto improvement in some cases. In the theoretically most preferable model, however, Ricardian equivalence survives in spite of genuine imperfections in the capital market. The results point to the importance of specifying the exact nature of imperfection.

The second essay is an attempt to improve our understanding of the linkage among budget deficits, money growth and inflation by applying the Laplace transform technique to the so-called "tight money paradox" of Sargent and Wallace. The central question is the response of inflation to temporary tightening of money in a model where the path of money growth is dynamically constrained by predetermined fiscal policy through seigniorage revenue requirements. Using a Sidrauski-type monetary optimizing model, we obtain an exact dynamic constraint for monetary policy and closed-form solutions for the equilibrium path of inflation. The effects of various policies on inflation and welfare are examined both qualitatively and quantitatively.

The third essay is an empirical investigation of the Fisher relationship between nominal interest rates and inflation. It derives a specification of the Fisher relationship based on the consumption capital asset pricing model, and identifies the conditional variance-covariance matrix of inflation and consumption growth, along with the conditional expectation of consumption growth, as the missing variables in conventional formulations. In particular, the covariance of inflation with consumption growth is shown to be the correct measure of systematic inflation risk. Econometric tests, mainly based on a bivariate version of the ARCH model, show no evidence of a time-varying conditional variance-covariance structure, and therefore, do not support the conjecture that rejections of the Fisher hypothesis may be attributed to this type of misspecification.

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**ESSAY I**

**RICARDIAN EQUIVALENCE IN THE PRESENCE OF  
CAPITAL MARKET IMPERFECTIONS**

## **1. Introduction**

The effects of government deficits are among the most important, yet controversial, topics in macroeconomics. Major issues involved are the efficacy of a debt-financed tax cut as a stabilization policy in the short run, and its adverse effects on capital accumulation in the long run. The Ricardian equivalence theorem, revived by Barro(1974), states that whether government spending is financed by taxes or bonds is inconsequential if the following assumptions are satisfied: (1) Successive generations of rational consumers are linked through operative intergenerational transfers, so that consumption decisions can be modeled as being made by a representative consumer with infinite horizons; (2) taxes are nondistortionary (lump-sum); and (3) capital markets are perfect. Under these conditions, any intertemporal reallocation of taxes would be both useless and harmless.<sup>1</sup>

Virtually all well-articulated arguments against the Ricardian doctrine are based on theoretical and empirical criticisms of these three assumptions.<sup>2</sup> A particularly large amount of work has recently been devoted to examining the first assumption, and the effort has met with remarkable success in clarifying the nature and implications of gift and bequest motives.<sup>3</sup> The simulation results reported by Poterba and Summers(1987), however, suggest that the issue of finite vs. infinite horizons is unlikely to matter as far as short-run effects of budget deficits are concerned. Even if consumers have no motive for intergenerational transfers, the average lifespan seems long enough, relative to the typical time horizon of deficit policies, to make infinite horizon models a good approximation.

Another potential source of debt nonneutrality is the non-lump-sum nature of the actual tax system. One of the relatively few established results on this issue stems from the insurance aspect of postponed taxes levied on uncertain future income. If taxes are an increasing function of income rather than lump-sum, then substitution of future taxes for current taxes will reduce the variance of future after-tax income and increase current consumption. Barsky, Mankiw and Zeldes(1986) have shown that such an effect can be quantitatively significant for realistic parameter values. An

important qualification to this argument, however, is that postponement of taxes may decrease, rather than increase, current consumption if it entails intrinsic uncertainty about future tax liabilities uncorrelated with future income [Chan(1983)]. Chan also argues that the income insurance scheme is self-financing and can be established separately from the choice of the debt-tax mix.

What appears to be a more compelling "Keynesian" argument against Ricardian equivalence comes from the observation that a significant number of consumers seem to face liquidity constraints, where "liquidity constraints" refer to credit rationing or differential interest rates on lending and borrowing. Since the neutrality proposition derives from full intertemporal optimization by consumers, such imperfection in capital markets is widely believed to cause nonneutrality. In particular, this argument has been a major motivation behind the large body of recent empirical work designed to detect liquidity constraints from consumption data.<sup>4</sup>

One obvious source of liquidity constraints would be transaction costs. However, in order to ensure expansionary effects of tax cuts, one has to assume that the government is more efficient than the private sector in providing de facto loans. This assumption is not very likely to be warranted and, in any case, sounds rather meager as a theoretical basis for significant policy effectiveness.<sup>5</sup> Another source of imperfection, generally considered as a prime suspect, is informational asymmetry between lenders and borrowers. Unobservable risk characteristics of borrowers give rise to the problem of adverse selection, and in many cases, to the phenomenon of credit rationing. Recent literature on such informational imperfections in capital markets (most notably, Jaffee and Russell(1976) and Stiglitz and Weiss(1981)) has often been cited to refute Ricardian equivalence on theoretical grounds.<sup>6</sup>

It is, therefore, only natural to ask the following question: Do the models of informationally imperfect capital markets provide a theoretical basis for debt nonneutrality? Unfortunately, the macroeconomic literature on the effects of liquidity constraints provides little insight into this question since most of it has imposed arbitrary forms of borrowing constraints without careful treatment of the source of such constraints. Among the few exceptions are the papers by



King(1984) and Hayashi(1985), which offered first answers to the above question in the context of adverse selection. King's model, with an endogenously generated wedge between lending and borrowing rates but no credit rationing, gave a nonneutrality result. Hayashi, on the other hand, considered a model of credit rationing similar to the one by Jaffee and Russell, and obtained Ricardian equivalence. The relationship between these apparently conflicting results, however, remains unclear even when it is recognized that Hayashi's result does not generalize naturally.

This essay is an attempt toward a more integrated analysis of Ricardian equivalence in the presence of adverse selection in the consumer loan market. The analysis abstracts from all other issues associated with Ricardian equivalence in order to focus on the problem of asymmetric information. We examine three alternative models of adverse selection with a common basic structure. In all these models, it is assumed that borrowers have private information regarding their probability of default.

Implications of asymmetric information differ depending on whether some kind of signal can be used to infer hidden information. In our context, loan contracts purchased by a consumer can serve as a potential signal of his risk characteristics. We first consider a case in which the entire set of contracts purchased by a consumer, possibly from several lenders, is not observable to others. The implicit assumption is that lenders do not communicate with each other regarding their customers' indebtedness. This model with no communication, which roughly corresponds to King's model, shares its basic insights with Akerlof(1970). We discuss competitive equilibria in such a model and obtain nonneutrality. We then turn to a Jaffee-Russell type model in which the contract purchases can be perfectly monitored. All lenders are assumed to share all available information about their customers. This assumption of full communication leads to a signaling equilibrium à la Rothschild and Stiglitz(1976) and Wilson(1977). Our model contains Hayashi's as a special case, and his neutrality result has to be replaced by a rather complicated verdict on Ricardian equivalence. Both of these models, with and without communication, have some interesting welfare implications.

These two models, however, are unsatisfactory in the sense that lenders' incentives to

communicate with each other are not explicitly considered. We argue that, as in Jaynes'(1978) model of insurance markets, it is always profitable for some (but not all) lenders to guarantee that they will never divulge the names of their customers to other lenders. In the third model, this point is incorporated to derive an equilibrium with endogenous communication, in which one group of lenders chooses to disclose information on their customers while the other group does not. Ricardian equivalence survives in this model.

The plan of this essay is as follows. The next section lays out a basic framework for analysis and characterizes loan demand functions. Sections III through V then examine the implications of three alternative models, which are built on this common framework but have different communication structures. Section VI contains concluding remarks.

## **2. Basic Framework**

We construct a simple model of a two-period small open economy inhabited by two types of consumers. A fraction  $\theta$  of consumers belong to type A, and the rest to type B. Their total number is normalized to unity. All consumers live for the duration of the economy so that there are no issues associated with finite horizons or intergenerational transfers.<sup>7</sup> Output is exogenous: the first period is a "recession," while the second period is correctly anticipated to be a "boom." There is no aggregate uncertainty, but each individual's income in the second period is stochastic and depends on whether he is "employed" or "unemployed." The first-period (after-tax) income is  $Y_L$  for both types of consumers, and is sufficiently "low" so that all consumers are borrowers in the first period. The second-period income for a type  $i$  ( $i=A,B$ ) consumer is assumed to be  $Y_H$  with probability  $p_i$  and  $Y_L (< Y_H)$  with probability  $(1-p_i)$ . The subsistence level of consumption is set at zero for convenience. For most of the essay, we further assume that  $Y_L$  is equal to zero (the

subsistence level), which considerably simplifies the exposition without loss of generality.

Since negative consumption is not feasible, a consumer will default and repay nothing if he receives zero in the second period. We assume  $p_A$  is greater than  $p_B$ , so that type B consumers are more likely to default than type A consumers. The type of any particular consumer is known only to himself. The lender cannot distinguish between high-risk and low-risk types until their market behaviour (possibly) reveals their type.

It is assumed that income insurance markets do not exist. Two kinds of justification can be offered for this assumption. First, introducing income insurance has the effect of largely shifting the problem of adverse selection from capital markets to insurance markets, at least in the particular setting adopted here. If the equilibrium in a model with insurance markets is such that both types of consumers hold positive amounts of insurance, then no consumer defaults on loans, and therefore, we would have no capital market imperfection to analyze.<sup>8</sup> Second, the neutrality results in the essay go through even when income insurance has no role because there is no subjective uncertainty (i.e.,  $p_A=1$ ,  $p_B=0$ ). This suggests that the insurance aspect implicit in loan contracts is not the major force driving the results.

The riskless real interest rate (gross) is fixed at the world interest rate  $R^*$ . (Namely, the net interest rate is  $R^*-1$ .) The economy has two kinds of institutions: banks and a government. The banks, which make loans to consumers, eliminate all non-systematic risk by diversification. The banking industry is competitive and allows free entry, so that the return on the aggregate loan asset ("deposit"), which is riskless, is  $R^*$  by arbitrage. Since all consumers in this economy want to be borrowers, the role of the lender is played by the rest of the world. The government issues bonds abroad against its future domestic tax revenue. Their rate of return is also equal to  $R^*$  due to the absence of aggregate risk.

The government, for a given fixed path of expenditures, can alter lump-sum taxes and transfers by changing the amount of debt it issues. Let  $G$  be a cut in lump-sum taxes (or an increase in

lump-sum transfers), financed by debt, in the first period. Both types of consumers get the same amount because the government, like banks, cannot distinguish between the two types.  $G$  being zero means balanced budget. In the second period, the government taxes the consumers in order to repay the debt. The intertemporal budget constraint of the government requires that the amount of taxes collected from each non-bankrupt individual in the second period must be equal to  $R_0G$ , where

$$(1) \quad R_0 = R^* / [\theta p_A + (1-\theta)p_B].$$

In other words, the government would be making compulsory loans with the interest rate  $R_0$ .

Let  $X_i$  be borrowings by the type  $i$  consumer ( $i=A,B$ ) in the private loan market and  $R$  be the interest rate charged by the lender. For the moment, we treat  $R$  as a constant, although the loan repayment schedule will be nonlinear in one of the models considered later. Given time-separable utility over consumption in two periods, the type  $i$  consumer's problem is to choose  $X_i$  so as to maximize

$$(2) \quad V_i(X_i; R, G) = U(Y_1 + G + X_i) + \beta [p_i U(Y_H - R_0G - RX_i) + (1 - p_i)U(0)]$$

where  $U$  is increasing and strictly concave, and  $\beta$  is a discount factor. The first-order condition is

$$U'(Y_1 + G + X_i) = \beta p_i R U'(Y_H - R_0G - RX_i)$$

for an interior optimum. The optimum will be at the corner ( $X_i=0$ ) if

$$(3) \quad R^* \leq m_i(G) \leq R \quad \text{where} \quad m_i(G) = \frac{U(Y_L + G)}{\beta p_i U'(Y_H - R_0 G)} .$$

Let  $M_i$  denote the set of points  $(R, G)$  which satisfy (3).<sup>9</sup> The loan demand function of type  $i$  consumers  $X_i^*(R, G)$  has the following characteristics.

$$(4a) \quad X_i^*(R, G) = 0 \quad \text{for} \quad (R, G) \in M_i$$

$$(4b) \quad \partial X_i^*/\partial R < 0 \quad \text{and} \quad \partial X_i^*/\partial G < 0 \quad \text{for} \quad (R, G) \notin M_i$$

Loan demand is a decreasing function of the loan interest rate and the cut in taxes. A corner solution (no borrowing) obtains if the interest rate is "too high" or if the size of the tax cut is "too large". We also note that the extent to which tax cuts reduce loan demand depends on the relative sizes of  $R$  and  $R_0$ . If  $R=R_0$ , then we have  $\partial X_i^*/\partial G = -1$ , which means that a tax cut simply replace private loans and do not affect consumption. The set  $M_A$  contains  $M_B$  so that there is a range of interest rates for any given  $G$  where  $X_A$  is zero but  $X_B$  is strictly positive. This makes possible a situation in which only high-risk borrowers ("lemons") are in the market. We can also show the following:

$$(5) \quad X_B^*(R, G) > X_A^*(R, G) \quad \text{for all} \quad (R, G) \notin M_B.$$

**Except when  $X_A=X_B=0$ , high-risk individuals always wish to borrow more than low-risk individuals. This difference in preference over loan contracts will serve as a potential**

signal of the risk type of an individual.

### 3. A Model with No Communication

#### 3.1 Adverse Selection

We start from the analytically simplest case, namely, a Walrasian equilibrium in the consumer loan market, where loan contracts purchased by any particular individual cannot be monitored by others. In the context of a competitive loan market with many lenders, this assumption is equivalent to precluding communication among lenders. Since contract purchases are the only potential signal of an individual's risk characteristics, the model in this section allows no possibility that the lender may distinguish among different types of consumers. Consumers with different risk characteristics will be charged the same interest rate, which must compensate for the average economy-wide default risk.<sup>10</sup> Credit rationing is not feasible since the borrower can go to as many banks as he wishes without being found out.

We first examine how the loan interest rate  $R$  is determined in a Walrasian equilibrium of the loan market.<sup>11</sup> Recall that a fraction  $\theta$  of consumers belong to type A. Thus, the total quantity of loans to the whole population of consumers amounts to  $\theta X_A + (1-\theta)X_B$ , while the expected repayment is  $R[\theta p_A X_A + (1-\theta)p_B X_B]$ . The expected gross rate of return is

$$\Gamma(R, G) = \frac{R[\theta p_A X_A^*(R, G) + (1-\theta)p_B X_B^*(R, G)]}{\theta X_A^*(R, G) + (1-\theta)X_B^*(R, G)}.$$

Since there is no aggregate risk,  $R$  is determined by an arbitrage condition which ensures that a well-diversified lender has a zero profit:

$$(6) \quad R^* = \Gamma(R, G).$$

We can show the existence of equilibria by observing that the equation (6) has at least one solution.

Define

$$(7) \quad R_i = R^*/p_i \quad (i = A, B).$$

Then, it follows from (5) that

$$\Gamma(R_B, G) \geq R^* \quad \text{and} \quad \Gamma(R_0, G) \leq R^*.$$

Since  $\Gamma$  is continuous, there exists at least one borrowing rate  $R^e$  between  $R_0$  and  $R_B$  which makes  $\Gamma(R^e, G)$  equal to  $R^*$ . Furthermore, there is no equilibrium interest rate outside of this interval  $[R_0, R_B]$ . An intuitive explanation would be as follows.  $R_B$  is a rate at which banks break even when the only customers are high-risk (type B) consumers. In a competitive market, the equilibrium rate cannot exceed  $R_B$ . On the other hand, banks can break even at  $R_0$  only if  $X_A = X_B$ . Since loan demand by a low-risk individual is less than that by a high-risk individual, banks must make negative profits if  $R^e$  is set below  $R_0$ . Multiple equilibria cannot be ruled out.<sup>12</sup> However, equilibria can be Pareto-ranked, the one with the lowest interest rate dominating others, so that if banks can set interest rates (instead of an auctioneer), then there will be a unique equilibrium with the minimum solution  $R^e$  to (5).

It is convenient for later exercises to define adverse selection equilibrium, in which  $X_A$  is equal

to zero and  $X_B$  is positive, i.e., only high-risk borrowers appear in the loan market. It was shown in (4) that loan demand by the type A consumer,  $X_A^*(R,G)$ , will be zero for  $R \geq m_A(G)$ . If (6) has no solution for  $R$  smaller than  $m_A(G)$ , then type A consumers will not borrow at all in equilibrium. Since all loans will be defaulted on with probability  $(1-p_B)$  in this case, the equilibrium interest rate should be equal to  $R_B$ . Formally, a unique adverse selection equilibrium characterized by

$$(8) \quad R^e = R_B, \quad X_A = 0 \quad \text{and} \quad X_B = X_B^*(R_B, G)$$

obtains if the following conditions hold:

$$[A1] \quad \Gamma(R, G) < R^* \quad \text{for all } R < m_A(G)$$

$$[A2] \quad X_B^*(R_B, G) > 0, \quad \text{or equivalently, } m_B(G) > R_B$$

### 3.2 The Effects of Debt Finance

We now examine the effects of debt-financed tax cuts, which can be viewed as compulsory loans with an implicit interest rate  $R_0$ . We have shown that this implicit interest rate on "government loans" is less than the equilibrium rate  $R^e$ . Thus, the government will be providing cheaper, though limited, loans than banks. The reason this is possible is that the government can control how much it "lends" to each individual, and thus, does not suffer from unobservability of borrowings. Private banks, being price takers, have to lend more to high-risk individuals in the aggregate than to low-risk individuals at a market interest rate. If each bank fixes the size of its loan so that all of its applicants borrow the same amount, then each of them will end up with high-risk customers whose number is more than proportional to  $\theta$ .



The simplest case to consider, though somewhat special, would be when the pre-tax-cut ( $G=0$ ) equilibrium is an adverse selection equilibrium, i.e., the conditions [A1] and [A2] hold for  $G=0$ . To make the exercise interesting, we assume that type A consumers are willing to borrow a positive amount at  $R_0$  when  $G=0$ :

$$[A3] \quad X_A^*(R_0, 0) > 0, \text{ or equivalently, } m_A(0) > R_0.$$

Under these assumptions, a tax cut has straightforward effects on consumption and can be shown to be Pareto-improving.

An increase in  $G$ , as long as it does not exceed  $X_A^*(R_0, 0)$ , simply increases the type A's first-period consumption  $Y_1 + G + X_A$  by exactly the same amount. Borrowings by the type A remain at zero since  $X_A$  is non-increasing with respect to  $G$ : if  $X_A^*(R, 0) = 0$ , then,  $X_A^*(R, G) = 0$  for all  $G > 0$ . The marginal propensity to consume out of a tax cut is one. The type B consumer's response would be to decrease the amount of borrowings to partially offset the increase in government loans since

$$-1 < \partial X_B^*(R, G) / \partial G < 0 \quad \text{for } R > R_0.$$

Therefore, the type B's first-period consumption does not increase on a one-for-one basis in response to a tax cut. Let  $C_{ij}(G)$  be consumption by non-bankrupt type  $i$  consumers in period  $j$ . (Second-period consumption by bankrupt consumers is always zero.) Then,

$$C_{A1}(G) = Y_1 + G \quad (dC_{A1}/dG = 1)$$

$$C_{B1}(G) = Y_1 + G + X_B^*(R_B, G) \quad (0 < dC_{B1}/dG < 1)$$

$$C_{A2}(G) = Y_H - R_0G$$

$$C_{B2}(G) = Y_H - R_0G - R_B X_B^*(R_B, G)$$

Figure 1 summarizes the discussion above. The horizontal axis gives first-period consumption which is the sum of the reduction  $G$  in taxes and private borrowings  $X$ , while the vertical axis gives the sum of tax payments  $R_0G$  and loan repayments  $Z$  in the second period. Suppose first-period taxes are cut by  $G_1 > 0$ . When  $G=0$ , the budget constraint is represented by  $OE$  (whose slope is  $R_B$ ), while it becomes  $OA'F$  as  $G$  is increased to  $G_1$  (the slope of  $OA'$  is  $R_0$ ). The indifference curves, defined as

$$U(Y_1 + G + X) + \beta p_i U(Y_H - R_0G - Z) = \text{constant},$$

are upward sloping with the slope

$$\frac{U'(Y_1 + G + X)}{\beta p_i U'(Y_H - R_0G - Z)} > 0$$

and are concave (differentiate the above expression again). Note that the type B indifference curve is steeper than the type A indifference curve at every point. The tax cut increases first-period consumption of type A from zero to  $G_1$ , which corresponds to a move from  $O$  to  $A'$ . Type B consumers are at  $B$  if  $G=0$ . When  $G$  is increased to  $G_1$ , they gain access to  $A'$ , but they further borrow at the interest rate  $R_B$  to reach  $B'$ . It can be seen that consumption by both types is increased.

The Figure also shows that both types are made better off by this policy. Algebraically, it can be seen as follows. Let

$$V_i^*(R, G) = \max_{X_i} [V_i(X_i; R, G)]$$

where  $V_i$  is defined in (2). Then,

$$(9) \quad \partial V_i^*(R_B, G)/\partial G \geq \beta(1 - p_i)(R_B - R_0)U'(Y_H - R_0G - R_BX_i) > 0,$$

with an equality for type B. The policy of increasing  $G$  improves welfare of both type A and type B consumers, and therefore, we conclude that an adverse selection pre-tax-cut equilibrium offers room for a Pareto-improving debt issue. For the case in which both types borrow in the initial equilibrium (hence  $R^e \neq R_B$ ), this result does not necessarily hold since an increase in  $G$  will have an adverse effect of raising the equilibrium interest rate. In general, we have the following result.

**Proposition 1** *Suppose  $R^e$  is the unique equilibrium interest rate. Then, a small increase in  $G$  is Pareto improving if*

$$R^e - R_0 > \frac{\Gamma_2(R^e, G)}{\Gamma_1(R^e, G)} X_i^*(R^e, G)$$

*holds for  $i=A, B$ , where  $R^e$  is the equilibrium interest rate before the increase in  $G$  and  $\Gamma_j$*

*( $j=1,2$ ) is the partial derivative of  $\Gamma$  with respect to the  $j$ -th argument.<sup>13</sup>*

To see the intuition behind this proposition, note that  $\Gamma_2/\Gamma_1$  is equal to  $dR^e/dG$  (differentiate the both sides of (6)). Then, the left hand side is the benefit of a marginal increase in "cheap" government loans, while the right hand side is the increase in loan repayment through a change in the interest rate.

### 3.3 Problems with the Model

There are at least two serious problems with the preceding analysis. First, the "Walrasian" or price-competition equilibrium breaks down if we assume that banks can offer price-quantity contracts (contracts that specify both interest rates and loan sizes), which seems to be a more natural assumption. In this alternative setting, one bank (call it M) which offers a loan contract of a limited size  $X_A^*(R_0, G)$  per person at the interest rate  $R_0$  will attract all consumers and will break even. Since type A consumers demand no more loans at an interest rate equal to or higher than  $R_0$ , the remaining banks will supply "supplementary" loans to type B consumers at the rate  $R_B$ . No other bank will supply the same contract as bank M because it would make losses by attracting only the high-risk consumers who borrow from more than one bank without being detected.

In this monopolistic equilibrium, bank M plays the role of the government, or in other words, there is no role for the government. Debt neutrality obtains. However, this equilibrium is not strictly a Nash equilibrium since, given that other banks supply loans at  $R_B$ , bank M has an incentive to charge an interest rate slightly higher than  $R_0$  and make positive profits. This action, however, will induce another bank to take over the position of bank M by offering a lower interest rate. The market structure is unstable. These observations suggest that models with no communication suffer from the lack of an appropriate equilibrium concept.

Another caveat in the model is directly related to the unobservability of borrowings. Namely, how can we prevent all the borrowers from borrowing as much as possible (possibly infinite) in the

first period and going bankrupt with probability one in the second period? This problem can be avoided if a certain restriction is imposed on the form of the utility function, or if there is a prohibitively high penalty on defaulting when employed, but these solutions are clearly unsatisfactory.<sup>14</sup>

#### **4. A Model with Full Communication**

##### **4.1 Signaling Equilibrium**

As we have seen in Section II, consumers of different risk types have different preference orderings over the set of possible loan contracts. This fact raises the possibility that contracts purchased by a consumer serve as a potential signal of his risk characteristics. Banks may be able to design a set of contracts which would induce customers to reveal their characteristics. Or high-risk borrowers may, as in Jaffee and Russell(1976), choose to mimic the behavior of low-risk borrowers so as not to reveal themselves. This mechanism, known as signaling, is made viable by the assumption that all lenders communicate with each other regarding their customers' purchases, enabling themselves to observe the borrowing behavior of each consumer. The implications of signaling equilibria that arise in such a model are explored here in the context of Ricardian equivalence.

Banks compete with each other by offers of price-quantity contracts, of the form  $(R, X)$ , that specify both an interest rate  $R$  and an amount  $X$  any consumer can borrow at that rate.<sup>15</sup> Contracts of this form clearly dominate price-only contracts when borrowings are observable. A Nash equilibrium in this model is defined as a set of contracts such that, when consumers choose contracts to maximize expected utility, (1) no contract in the equilibrium set makes negative profits, and (2) there is no contract outside the equilibrium set that, if offered, makes a positive profit. We assume that each bank offers only one contract, so that there is no cross subsidization across contracts. This assumption can be relaxed by slightly modifying the Nash equilibrium concept [see

Wilson(1977)], but is retained for simplicity. We also assume free entry so that any collusive behavior is ruled out.

An equilibrium can only take one of the two forms: a pooling equilibrium and a separating equilibrium. If a set of contracts (separating contracts) can be designed to make high-risk borrowers reveal themselves, then, a separating equilibrium obtains; otherwise, we have a pooling equilibrium with a pooling contract. We first need to characterize the two alternative forms of contracts to define these equilibria. Separating contracts are a set of two contracts,  $[R_A, X_S(G)]$  (to be purchased by the low-risk) and  $[R_B, X_B^*(R_B, G)]$  (to be purchased by the high-risk), where  $R_i$  is defined as  $R^*/p_i$ , and  $X_S(G)$  is the solution to

$$\text{Max}_X V_A(X; R_A, G) \text{ subject to } V_B(X; R_A, G) \leq V_B^*(R_B, G).$$

It should be recalled that

$$V_B^*(R_B, G) = V_B[X_B^*(R_B, G); R_B, G],$$

which is the utility level of a type B consumer associated with his optimal borrowings at the interest rate  $R_B$ . Thus,  $X_S(G)$  gives optimal borrowings by type A under the constraint that type B consumers are better off by revealing themselves than by mimicking type A.

A pooling contract, on the other hand, makes both types borrow the same amount at the same interest rate. It takes the form  $[R_0, X_P(G)]$ , where  $X_P(G)$  maximizes  $V_A(X; R_0, G)$  without constraint, or equivalently,

$$(10) \quad X_P(G) = X_A^*(R_0, G)$$

$R_0$ , defined in (1), is the interest rate which makes the pooling contract break even. Since the government "loans," which bear the implicit interest rate  $R_0$ , are a perfect substitute for the pooling contract, the following simple relationships hold.

$$(11a) \quad X_P(G) = X_P(0) - G$$

$$(11b) \quad V_i[X_P(G); R_0, G] = V_i[X_P(0); R_0, 0]. \quad (i = A, B)$$

The latter expression will be abbreviated as  $V_i[X_P(0)]$  hereafter.

A separating equilibrium obtains if and only if type A consumers prefer a separating contract over a pooling one:

$$[SE] \quad V_A[X_S(G); R_A, G] \geq V_A[X_P(0)].$$

Loan demand by each type in this equilibrium is given by

$$(12) \quad X_A = X_S(G), \quad X_B = X_B^*(R_B, G).$$

Figure 2 depicts a typical separating equilibrium. The indifference curves are upward-sloping and concave as in Figure 1. The two separating contracts are represented by the points A and B, and are purchased by type A and type B individuals, respectively. The point P represents a pooling contract. The condition for a separating equilibrium, [SE], requires that the type A indifference curve through the point P should cut the  $Z=R_A X$  line to the left of the point A. This equilibrium can be shown to be a Nash equilibrium.

Figure 3 gives an example of a pooling equilibrium, which could obtain if [SE] is violated (type

A consumers prefer the pooling contract). The point P represents the pooling contract with interest rate  $R_0$ , where the type A consumer's utility is at an unconstrained maximum given  $R_0$ , while type B consumers are "credit rationed." Type A consumers are better off at P than at A, which represents the most preferable separating contract available to them. Thus, we have

$$(13) \quad X_A = X_B = X_P(G)$$

This pooling equilibrium basically corresponds to the well-known model of credit rationing by Jaffee and Russell (or its refined version by Smith(1983)). However, as Rothschild and Stiglitz(1976) pointed out (and as Smith notes), there is no Nash pooling equilibrium in a model of this type. Namely, there is incentive for banks to deviate from the equilibrium and offer contracts such as  $\alpha$  which would attract only low-risk types. Since existing contracts yield nonnegative profits only when both types purchase them, such deviation will make existing contracts unprofitable and destroy the equilibrium. In view of this nonexistence problem, some alternative quasi-dynamic equilibrium concepts have been proposed. Wilson's E2 equilibrium is one of them, and the above pooling equilibrium in fact exists as an E2 equilibrium. It assumes that a bank offers a new contract only if it makes nonnegative profits after all the contracts that become unprofitable because of the new offer are withdrawn. Wilson showed that an E2 equilibrium always exists and can be either a separating one or a pooling one. The E2 separating equilibrium is identical to the Nash separating equilibrium, and thus, need not be considered separately.

#### 4.2 The Effects of Debt Finance

We are now ready to examine the consequences of debt finance. We first consider the case in which a Jaffee-Russell (E2) pooling equilibrium obtains for  $G=0$ . The point P in Figure 4



represents this original equilibrium. Suppose, as in Section III, the government provides a debt-financed tax cut of size  $G$  to all consumers in the first period and repays the debt  $R_0G$  through a tax increase in the second period. In the Figure, this policy of "mandatory non-discriminatory government lending" is represented by  $C$  (if  $G=G_1$ ) or  $D$  (if  $G=G_2$ ). The private loan market operates given these new endowment patterns.

Due to continuity, the condition [SE] will continue to be violated for a small increase in  $G$ , say from zero to  $G_1$ , so that the equilibrium will be unchanged at  $P$ . First-period consumption by each type of consumer stays constant since the size of the pooled loan in the private market "shrinks" in response to an increase in the pooled loan supplied by the government. Thus, Ricardian equivalence holds locally. However, if  $G$  is further increased to  $G_2$ , [SE] will now be satisfied and we will have a separating equilibrium represented by  $A'$  and  $B'$ . The slope of  $DA'$  is  $R_A$ , while that of  $DB'$  is  $R_B$ . The type  $B$  individual's first-period consumption will be greater at  $B'$  than to at  $P$ . The type  $A$  individual's first-period consumption, on the other hand, could decrease initially although it will eventually exceed the original level as  $G$  is further increased.

Welfare implications are less ambiguous. The Figure suggests that the post-tax-cut separating equilibrium is Pareto superior to the pre-tax-cut pooling equilibrium. In fact, the following proposition holds.<sup>16</sup>

***Proposition 2*** *Let  $G^\circ$  be such that  $V_B^*(R_B, G^\circ) = V_B[X_P(0)]$ . Then, for any  $G \in [G^\circ, X_P(0)]$ , (i) the condition [SE] is satisfied so that a separating equilibrium obtains; and (ii) this equilibrium Pareto dominates the original pooling equilibrium.*

It suffices to show (ii) since Pareto domination implies that type  $A$  consumers prefer the separating

equilibrium (*i.e.*, [SE] holds). We showed in (9) that  $V_B^*(R_B, G)$  is increasing in  $G$ . This implies  $G^0 < X_P(0)$  since  $V_B^*(R_B, X_P(0)) > V_B[X_P(0)]$ . Hence,  $V_B^*(R_B, G)$  is no smaller than  $V_B[X_P(0)]$  for all  $G \in [G^0, X_P(0)]$ . Therefore,  $P$  must be above the type B indifference curve through  $A'$ . This implies that  $P$  must also be above the type A indifference curve through  $A'$  since it is flatter than the type B indifference curve. Thus, both types of consumers are made better off. Government lending plus the new separating loan contracts in the private market Pareto dominates the original pooling equilibrium.

This government-induced separation does not occur in the special case where  $p_B = 0$ . The original pooling equilibrium represents the only possible allocation regardless of the size of  $G$  since no bank would lend to type B consumers if they are so identified. Thus, the "local" Ricardian equivalence discussed above becomes a global result.<sup>17</sup>

Now we turn to the case of a separating equilibrium. Figure 5 shows a debt finance policy (represented by  $C$ ) and two separating equilibria, one before the policy (the pair  $A$  and  $B$ ) and the other after the policy ( $A'$  and  $B'$ ). By the tax cut, the government effectively provides a pooled loan  $OC$ , to be supplemented by private loans  $CA'$  and  $CB'$ . Both types of consumers increase their first-period consumption. Since the government's loan is a substitute for private loans, borrowings in the private market are reduced, and therefore, the magnitude of the increase in consumption is less than that of the tax cut. There is no general welfare result available in this case. However, it is possible to construct an example of Pareto improvement, and in fact, the relative location of indifference curves in the Figure indicates that this particular policy is Pareto improving.

The message of this section can be summarized as follows. The well-known model of signaling equilibrium, applied to loan markets with asymmetric information, does not support the conventional view that a tax cut should increase liquidity-constrained individuals' consumption on a dollar-for-dollar basis. In general, liquidity constraints change endogenously in response to policy

actions, and in particular, it is possible that the resulting change totally neutralizes what the government does. The neutrality result in this section is rather limited, but a stronger result obtains in the next section.

## **5. A Model with Endogenous Communication**

### **5.1 Incentives for Communication**

In the previous two sections, rather ad hoc assumptions were imposed regarding the sharing of information among banks: no communication and full communication. Natural questions arise: is there any incentive for banks to withhold customer identity information? Or alternatively, is there any incentive to share information at all? What kind of equilibrium, if any, would emerge if banks share information only if it is profitable to do so? The model in this section, based on recent work by Jaynes (1978) and Hellwig (1986), incorporates these considerations and yields some surprising policy implications.

We first discuss the questions above in an intuitive manner by considering the following game in the loan market: In the first stage of the game, banks make loan contract offers which consist of (1) the terms  $(R, X)$ , (2) an exclusivity clause regarding other loan contract purchases, and (3) a list of banks to whom the purchase of the contract by a consumer will be communicated; In the second stage, the consumer chooses a combination of contracts with an understanding that any detected violation of an exclusivity clause results in an automatic cancellation of the contract.

The first question of whether there is any incentive to withhold information can most easily be answered by referring back to Figure 2. This is the case where a Nash separating equilibrium exists with full communication. It is clear from the Figure that any type B consumer can be made better off if allowed to purchase a loan contract A plus a contract on the line OB. This is not a feasible choice if all banks know about his contract purchases. However, in our present setting, there is an incentive for a new bank to enter the market, offer a loan at the interest rate  $R_B$  (or slightly above it)

and agree not to divulge the names of its customers to other banks. This strategy makes the contract A unprofitable, and therefore destroys the separating equilibrium.

This observation does not imply that there will be a complete absence of communication. As an example, suppose two banks offer the same pooling contract  $[R_0, X_P(G)]$ . Further suppose that high-risk individuals prefer  $[R_0, 2X_P(G)]$  to  $[R_0, X_P(G)]$ , which is a reasonable assumption. If the two banks do not share customer information, high-risk borrowers will attempt to purchase contracts from both banks. The banks will suffer losses since each of them will have a higher share of high-risk customers than what would make it break even. Thus, there is an obvious incentive for these banks to send customer information to each other. In fact, all banks offering pooling contracts have an incentive to share information, and doing otherwise cannot be a rational behavior.

The arguments above suggest that, if there is an equilibrium, it would be supported by two groups of banks, one sharing customer identity information and the other refusing to divulge it. The solution proposed by Jaynes goes basically as follows. Since a separating equilibrium is impossible, the equilibrium should involve some degree of pooling. Start from the pooling equilibrium of the previous section, in which both types purchase the same pooling contract. Type B consumers are "credit rationed" in this case, and are willing to pay a higher interest rate  $R_B$  to borrow some more. That is, there is an incentive for some banks to offer additional loans along the line OH in Figure 6, with the promise that they will not communicate the names of their customers to those banks offering the pooling contract P. This enables type B consumers to combine the two contracts P and H and attain the point L. Thus, all consumers purchase the pooling contract  $[R_0, X_P(G)]$ , and type B consumers also purchase a supplementary contract  $[R_B, X_L]$ , where  $X_L$  maximizes

$$(14) \quad U[Y_1 + G + X_P(G) + X_L] + \beta p_B U[Y_H - R_0 G - R_0 X_P(G) - R_B X_L].$$

**Banks offering the pooling contract share information on their customers, while those offering the supplementary contract do not, at least not with the suppliers of pooling contracts.<sup>18</sup>**

**This solution, though not a sequential equilibrium [of Kreps and Wilson(1982)] for the game described above, has been shown by Hellwig to be a sequential equilibrium for the following four-stage game.<sup>19</sup>**

- (a) At the first stage, banks announce loan contract offers (R,X) and whatever exclusivity requirements are imposed.**
- (b) Given the constellation of contract offers, consumers choose a contract or a combination of contracts.**
- (c) Banks then decide what customer information they want to divulge and which other banks they want to send this information to.**
- (d) Finally, banks that have received such information from other banks choose whether or not to enforce their exclusivity conditions.**

**To see how this formulation works, consider an offer  $\alpha$  in Figure 6, which by itself attracts only type A consumers and makes positive profits. We have already seen that a deviation with such an offer destroys the pooling equilibrium in the full communication case. In the current four-stage game, however, the following off-the-equilibrium-path behavior ensures that this deviation does not upset the equilibrium: After the first stage in which some bank offered the contract  $\alpha$ , all consumers purchase this contract along with appropriate amounts of the pooling contract (both types) and the supplementary contract (type B only). In the third stage, the suppliers of the pooling contract send customer information to each other, but none of them sends any information to the defector that offered  $\alpha$ . In the fourth stage, the banks offering the pooling contracts do not enforce exclusivity**

conditions on buyers of  $\alpha$ .

Along this off-the-equilibrium path, both types of consumers find it utility-increasing to purchase  $\alpha$  and an appropriate fraction of the pooling contract. The defecting bank, lacking contract purchase information, has no way of preventing this behavior, and therefore, makes negative profits. In anticipation of this, it will not make such an offer. It is not difficult to see that all other kinds of deviation fail to upset the equilibrium. Furthermore, the preceding discussions of the first game also apply to this game and preclude any other solution. Therefore, we have established a unique sequential equilibrium.

## 5.2 Ricardian Equivalence

Now we are ready to show that a change in  $G$  has no effect on consumption. We know that, in equilibrium,

$$X_A = X_P(G) = X_P(0) - G$$

$$X_B = X_P(G) + X_L = X_P(0) + X_L - G$$

for  $G \in [0, X_P(0)]$ . The crucial difference with the full communication case is that a change in  $G$  never threatens the existence of the pooling contract. Note that  $X_L$  is invariant with respect to  $G$  since (11a) and (14) imply

$$X_L = \operatorname{argmax}_X \{U[Y_1 + X_P(0) + X] + \beta p_B U[Y_H - R_0 X_P(0) - R_B X]\}.$$

It follows that first-period consumption of each type does not depend on  $G$ :

$$C_{A1}(G) = G + X_A = X_P(0)$$

$$C_{B1}(G) = G + X_B = X_P(0) + X_L$$

The same is true for consumption in the following period:

$$C_{A2}(G) = Y - R_0G - R_0X_P(G) = Y - R_0X_P(0)$$

$$C_{B2}(G) = Y - R_0G - R_0X_P(G) - R_BX_L = Y - R_0X_P(0) - R_BX_L$$

Hence, we have the following result.

**Proposition 3** Ricardian equivalence holds in the model with endogenous communication.

Namely, for any  $G_1, G_2 \in [0, X_P(0)]$ , we have  $C_{ij}(G_1) = C_{ij}(G_2)$ . ( $i = A, B; j = 1, 2$ )

An alternative way to look at this proposition is to use the fact that, given the structure of equilibrium contracts, total loan repayments  $Z$  by either type can be written as a function of  $X$  of the following form:

$$(15) \quad Z = \begin{matrix} R_0X & (X \leq X_P(G)) \\ R_0X_P(G) + R_B[X - X_P(G)] & (X > X_P(G)) \end{matrix}$$

This implies that the sum of tax payments and loan repayments in the second period is given by

$$R_0G + Z = \Phi(X + G)$$

where

$$\Phi(X + G) = R_0(X + G) \quad (X + G \leq X_P(0))$$

$$R_0X_P(0) + R_B[X + G - X_P(0)] \quad (X + G > X_P(0)).$$

Using this relationship, we can rewrite the consumer's optimization problem as

$$\text{Max } U(Y_1 + X_i + G) + \beta p_i U[Y_H - \Phi(X_i + G)]. \quad (i = A, B)$$

It should be evident from this expression that the consumer, faced with a change in  $G$ , can achieve the same optimum as before through an exactly offsetting change in  $X_i$ . In other words, a change in  $G$  does not alter the intertemporal budget constraint relevant for the consumer.

Figure 7 shows the argument graphically. If the government cuts taxes by  $G_1$  (represented by a move from  $O$  to  $C$ ), banks offering the pooling contract will reduce the size of their loan from  $OP$  to  $CP$ , and the remaining banks will provide the same supplementary loan contract  $PL$  as before. A tax cut replaces the pooling contract on a dollar-for-dollar basis and leaves the budget constraint and optimal consumption unchanged. The result holds even if  $Y_L$  (income when unemployed) is not zero as we have assumed. In terms of Figure 7, a positive  $Y_L$  means that the graph  $OPL$  should start from a point with coordinates  $(Y_L/R^*, Y_L)$  instead of the origin. Loans up to  $Y_L/R^*$  are risk free and will carry the riskless interest rate  $R^*$ . Since such loans are immune to the adverse selection problem, it does not matter who provides this portion of loan supply. Therefore, an increase in  $G$  is neutral not only in the current model but also in the model with endogenous communication if  $R^*G$



does not exceed  $Y_L$ . The neutrality result also remains valid for any finite number of risk types. If there are  $N$  types, then, there will be  $N$  kinds of contracts offered in the market, with the  $j$ -th contract pooling  $j$  highest risk types. A bank will share customer identity information only with those banks that offer the same contract as it does. The contract that pools all  $N$  types will play the role of the pooling contract in this section.

The significance of the result becomes clear if we consider the case in which  $p_A$  is unity so that type A consumers never go bankrupt. Since there is no risk of default, type A consumers would have access to loans at the riskless rate  $R^*$  in a competitive market with symmetric information. There will be neither credit rationing nor a wedge between lending and borrowing rates for them. Under asymmetric information, however, these individuals are indistinguishable from risky customers (type B) and are charged an interest rate higher than  $R^*$ . We have a wedge between lending and borrowing rates where there should be none. Furthermore, we also have credit rationing of sorts for type B consumers, since they cannot borrow as much as they wish at the pooling rate  $R_0$ . In spite of these genuine imperfections in the capital market, Ricardian equivalence still goes through. Tax cuts with a correctly anticipated future tax increase simply replace private lending and have no real effects.

The intuition behind debt neutrality may be described as follows. By assumption, the government does not have any informational advantage over the private sector, so that the only weapon it possesses to affect real allocation is its ability to force the pooling contract (or its fraction) on all consumers. In the model with no communication, this weapon proved effective because the private sector could not offer the pooling contract. In the model with full communication, the pooling contract either did not exist in equilibrium (a separating equilibrium) or, if it did, was eventually replaced by separating contracts as the size of a tax cut increased. By contrast, in the present model with endogenous communication, the private loan market supports the pooling contract regardless of the intertemporal reallocation of taxes. The size of the pooling contract offered

by banks will adjust so that the total size of the two pooling contracts, governmental and private, remains constant.

So far, we have only considered the case where  $G$  does not exceed  $X_p(0)$ . If  $G$  is greater than  $X_p(0)$ , an increase in  $G$  will increase first-period consumption of both types up to a point. This effect comes from the fact that the interest rate  $R_0$  implicit in the tax cut is higher than the lending rate  $R^*$ , which implies that type A consumers cannot exactly undo "forced lending" by the government. Figure 7 shows that, with a tax cut of  $G_2$ , consumers of types A and B will move to  $A'$  and  $B'$ , respectively. This "nonneutrality" actually has little to do with informationally imperfect capital markets since the same mechanism applies to the symmetric information case as well. The policy is simply a subsidy for high-risk individuals at the expense of low-risk individuals, and therefore, no Pareto improvement is possible.

## **6. Concluding Remarks**

The question asked at the beginning was: do the models of informationally imperfect capital markets provide a theoretical basis for debt nonneutrality? The answer depends crucially on the extent to which information is shared among lenders. Among the three models with different assumptions on communication, the first two yield nonneutrality results (although Ricardian equivalence may hold "locally" in the second model), along with a further implication that it is possible in many cases to achieve Pareto improvement by a debt-financed tax cut. However, these models have the theoretical weakness of imposing arbitrary restrictions on the extent of communication. When incentives for communication are explicitly considered, the conclusions are reversed. Ricardian equivalence survives.

Although our model is highly stylized, it illustrates the point that adverse selection in capital markets, supposedly the prime cause of market imperfections, does not imply a failure of Ricardian

**equivalence. The all-too-common remark that the validity of Ricardian equivalence hinges upon perfect capital markets seems to be unfounded. The essay also makes a more general point that, even if market "imperfections" exist, the government may not be able to take advantage of them since private market outcomes are likely to change in response to the government's actions.**

**It would be premature, however, to conclude that capital market imperfections do not matter. For example, certain institutional arrangements or constraints may render one of the first two models more relevant than the theoretically preferable model with endogenous communication.**

**Furthermore, even our preferred model might yield different results if moral hazard is introduced as an alternative form of informational asymmetry. What kind of imperfection, if any, leads to a profitable way out of Ricardian equivalence still largely remains an open question.**

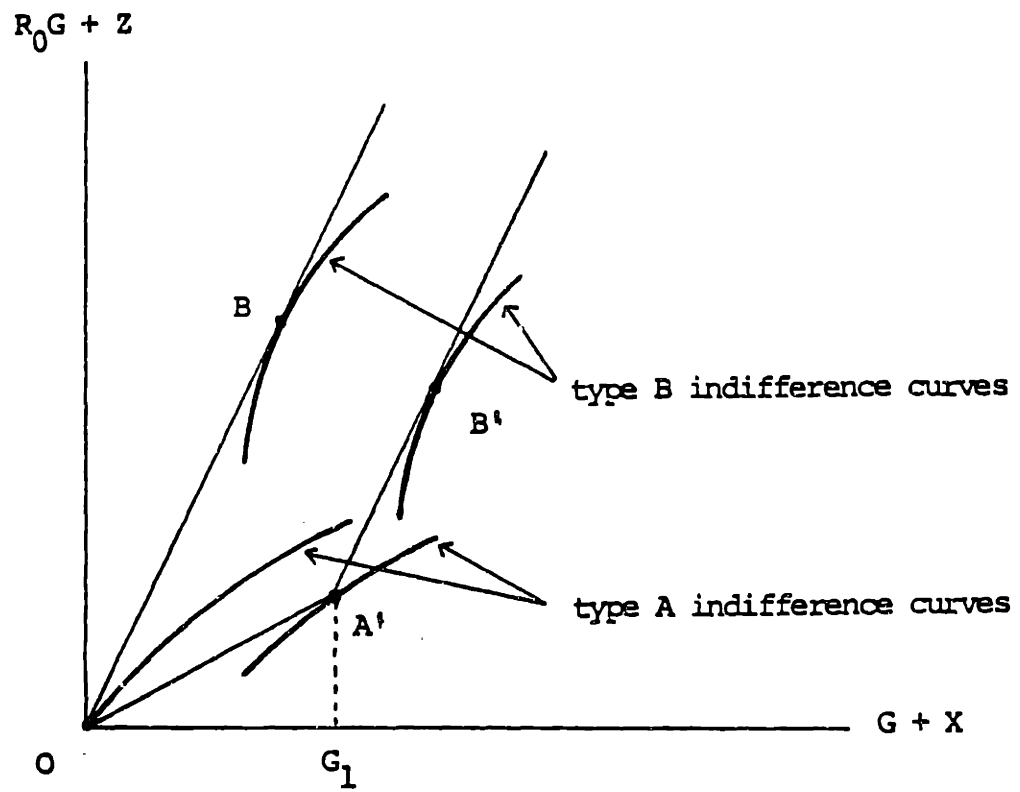


Figure 1

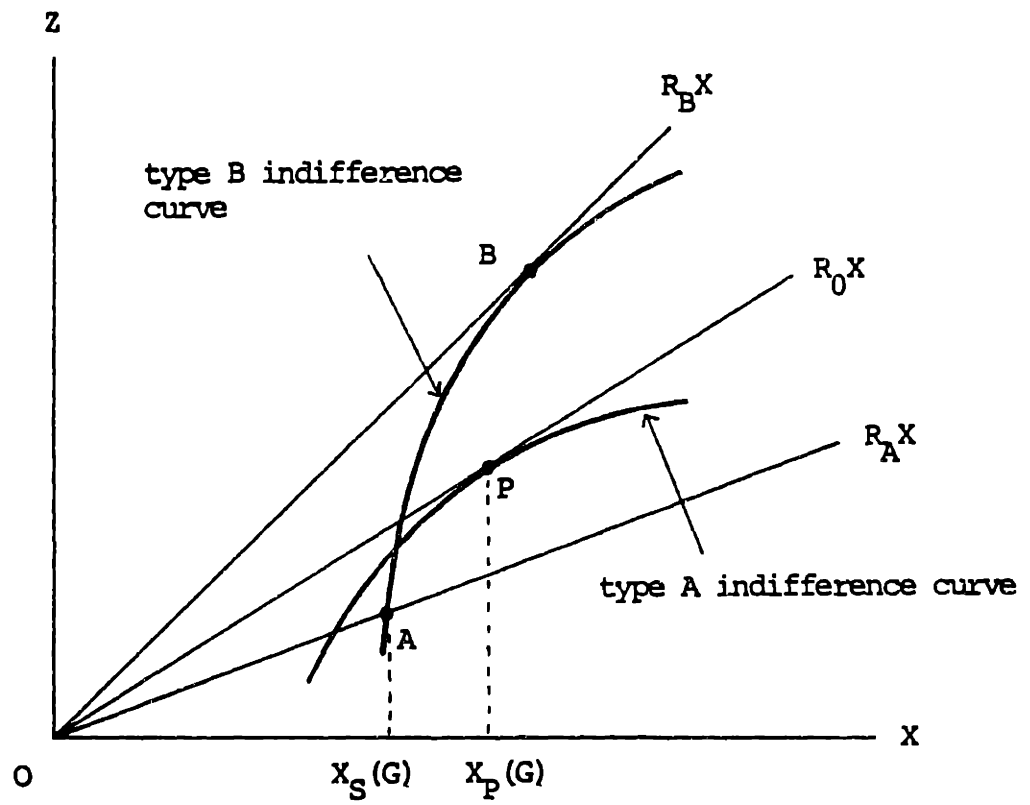


Figure 2

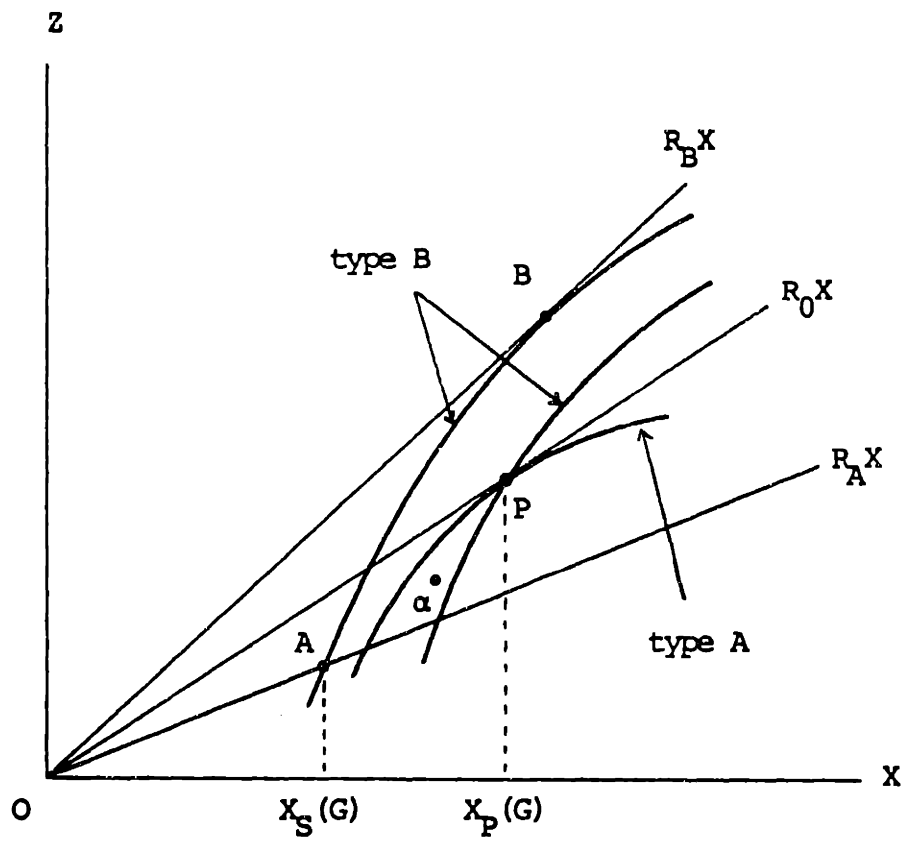


Figure 3

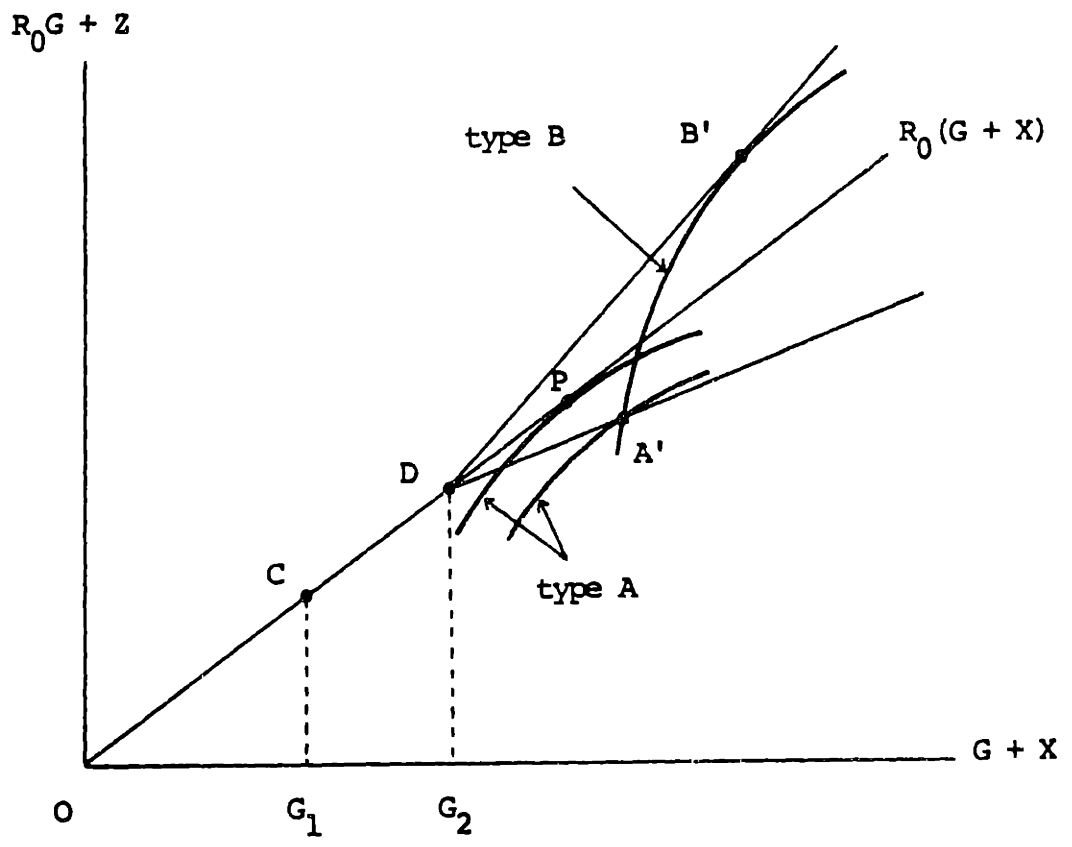


Figure 4

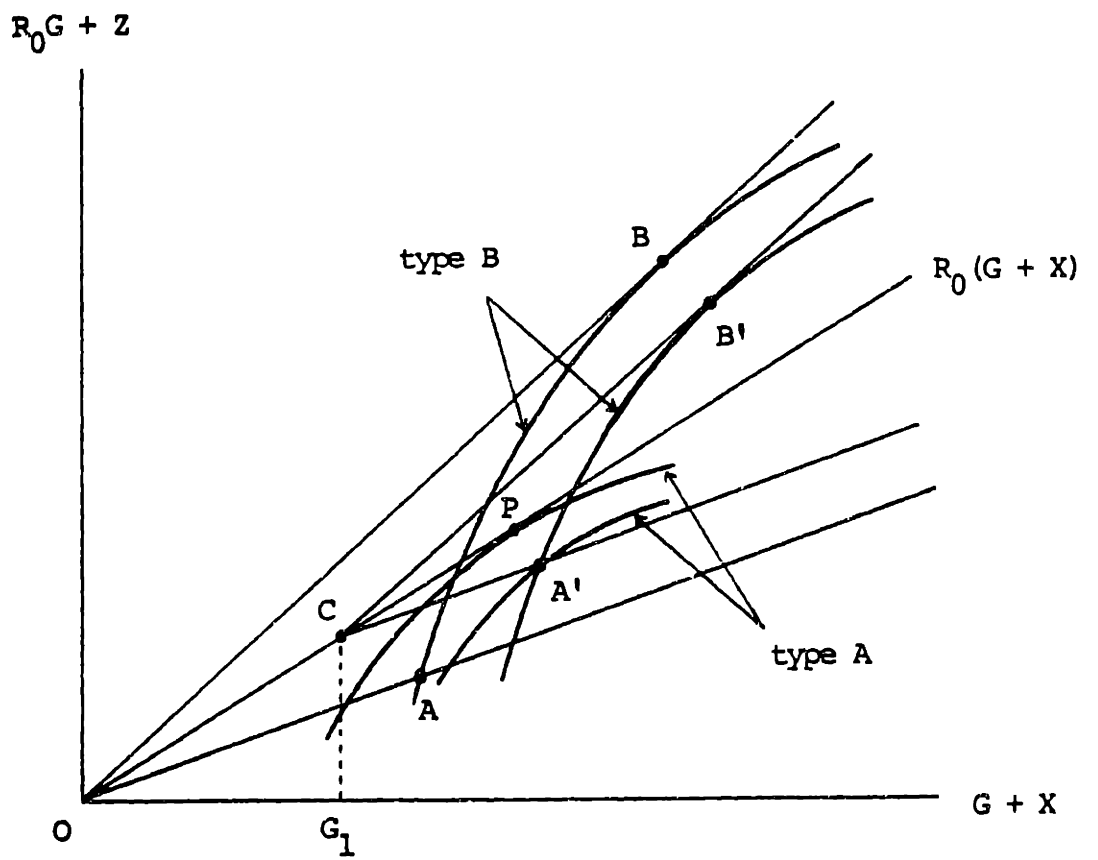


Figure 5



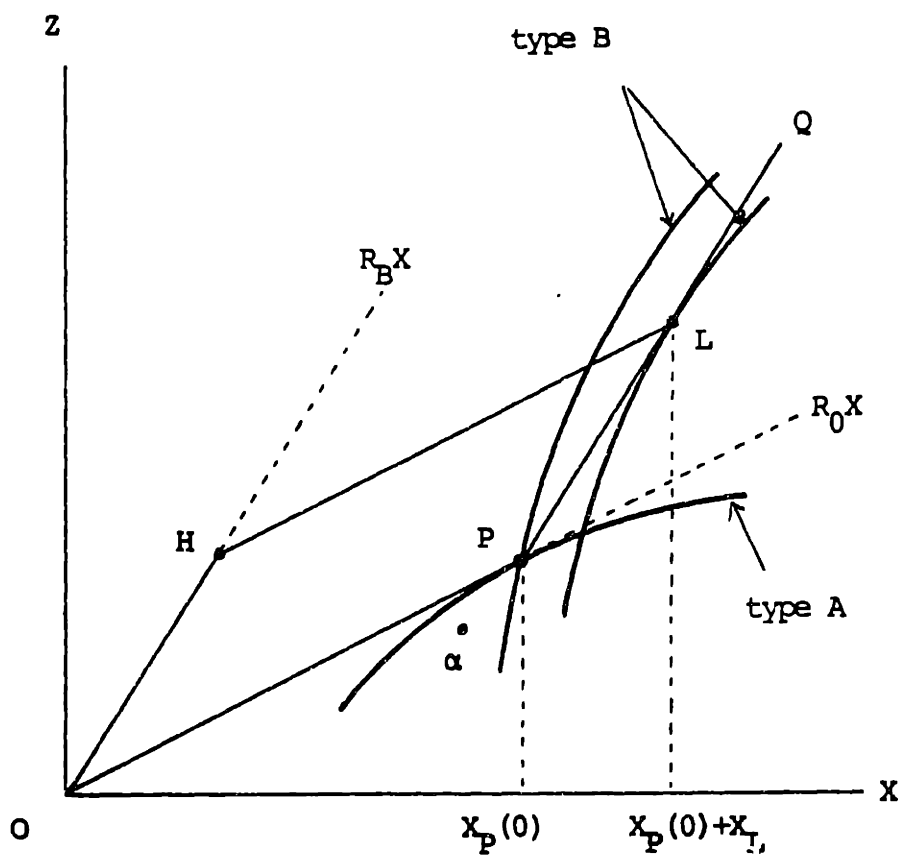


Figure 6

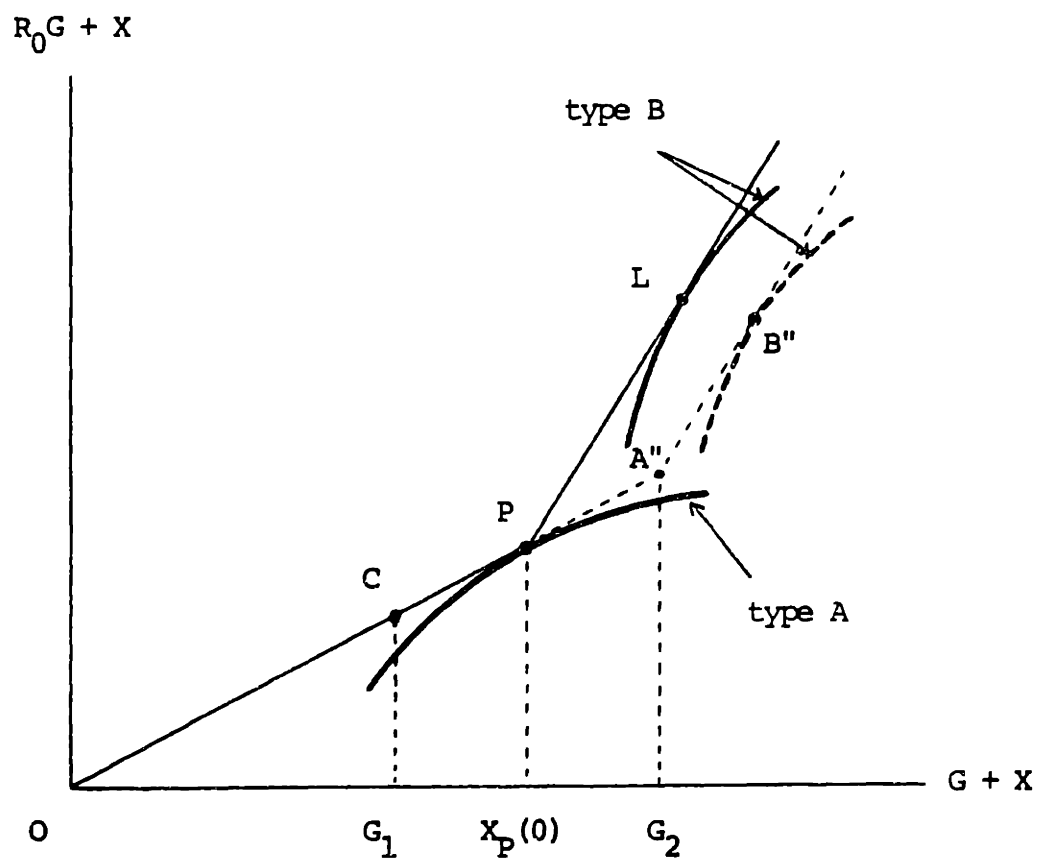


Figure 7

## NOTES

1. Barro(1986) and Bernheim(1987) provide extensive surveys of the large body of literature on this topic.
2. It is also often suggested that consumers may not be far-sighted rational optimizers, but the implications of such myopia have hardly been examined systematically.
3. See, for example, Weil(1984), Abel(1985), Bernheim, Shleifer and Summers(1985) and Bernheim and Bagwell(1986).
4. See Hayashi(1985) for a survey of this literature.
5. This argument assumes proportional transaction cost. If there is a significant fixed-cost element, a tax cut can be contractionary.
6. A recent example of such citation is found in Buiters(1985, p42).
7. The model can also be interpreted as describing a stationary equilibrium in an overlapping generations model, in which the young are liquidity constrained.
8. Since bankruptcy guarantees a certain minimum level of consumption, the only reason for the consumer to hold insurance is to provide himself with a level of consumption higher than this minimum when he is "unemployed." Therefore, holding of insurance precludes bankruptcy.
9. We do not consider the case  $m_i(G) < R^*$ , which occurs when  $G$  is very large. It would imply that the type  $i$  consumer is a lender rather than a borrower.
10. There would be more than one interest rate if lenders compete by offering price-quantity contracts. This point will be discussed at the end of the section.
11. This section's analysis is based on a modified version of King's(1984) model. The model, though closely related to the existing literature [see, e.g., Pauly(1974) and Abel(1986)], turns out to be rather problematic as we shall point out at the end of the section.
12. In the case of log utility, for example, there can be between one and three equilibria.
13. A sketch of the proof is as follows. The utility level in equilibrium is

$$V_i^* = U[Y_1 + G + X_i^*(R^e, G)] + \beta \{p_i U[Y_H - R_0 G - R^e X_i^*(R^e, G)] + (1 - p_i) U(0)\}.$$

Using the envelope theorem, we obtain

$$\begin{aligned} dV_i^*/dG &= U'(Y_1 + G + X_i^*) - \beta p_i [R_0 + (dR^e/dG)X_i^*] U'(Y_H - R_0 G - R^e X_i^*) \\ &= \beta p_i [R^e - R_0 - (dR^e/dG)X_i^*] U'(Y_H - R_0 G - R^e X_i^*) \\ &= \beta p_i [R^e - R_0 - (\Gamma_2/\Gamma_1)X_i^*] U'(Y_H - R_0 G - R^e X_i^*), \end{aligned}$$

whose sign is the same as that of the expression in the square bracket.

14. The restriction on preference must ensure that there is no incentive to default when "employed": formally,

$$\lim_{C \rightarrow \infty} U(C) + \beta U(0) < V_i[X_i^*(R, G); R, G] \quad i = A, B.$$

15. The linearity of loan contracts here is a result of using a particular probability distribution, and equilibrium contracts would specify nonlinear schedules for a more general distribution. Such a generalization is conceptually straightforward but cumbersome to carry out.

16. This proposition is parallel to those in Wilson(1977) and Eckstein, Eichenbaum and Peled(1985).

17. Hayashi's model, which produced global neutrality, essentially corresponds to this special case with a further assumption that  $p_A = 1$ .

18. Those banks that provide supplementary loans may share information within their group in order to prevent consumers from borrowing an "infinite" amount.

19. The following discussion of the game draws on Hellwig.

## REFERENCES

- Abel, Andrew B., 1985, Gifts and bequests: Comments and extension, mimeo., Harvard University,
- Abel, Andrew B., 1986, Capital accumulation and uncertain lifetimes with adverse selection, *Econometrica* 54, 1079-1097.
- Akerlof, George A., 1970, The market for 'lemons': Qualitative uncertainty and the market mechanism, *Quarterly Journal of Economics* 84, 488-500.
- Barro, Robert J., 1974, Are government bonds net wealth? *Journal of Political Economy* 82, 1095-1117.
- Barro, Robert J., 1986, The neoclassical approach to fiscal policy, mimeo., University of Rochester, forthcoming in: Robert J. Barro ed., *Handbook of modern business cycle theory*, Wiley, New York.
- Barsky, Robert B., N. Gregory Mankiw and Stephen P. Zeldes, 1986, Ricardian consumers with Keynesian propensities, *American Economic Review* 76, 676-691.
- Bernheim, B. Douglas, 1987, Ricardian equivalence: An evaluation of theory and evidence, in: Stanley Fischer, ed., *NBER Macroeconomics Annual 1987*, forthcoming.
- Bernheim, B. Douglas and Kyle Bagwell, 1986, Is everything neutral? NBER Working Paper No. 2086.
- Bernheim, B. Douglas, Andrei Shleifer and Lawrence Summers, 1985, "The strategic bequest motive, *Journal of Political Economy* 93, 1045-1076.
- Buiter, Willem H., 1985, A guide to public sector debt and deficits, *Economic Policy* 1, 14-79.
- Chan, Louis K.C., 1983, Uncertainty and the neutrality of government financing policy, *Journal of Monetary Economics* 11, 351-372.
- Eckstein, Zvi, Martin Eichenbaum and Dan Peled, 1985, Uncertain lifetimes and the welfare enhancing properties of annuity markets and social security, *Journal of Public Economics* 26, 303-326.
- Hayashi, Fumio, 1986, Tests for liquidity constraints: A critical survey, NBER Working Paper No. 1720.
- Hellwig, Martin F., 1986, A note on the specification of inter-firm communication in insurance markets with adverse selection, mimeo., University of Bonn.
- Jaffee, Dwight M. and Thomas Russell, 1976, Imperfect information, uncertainty, and credit rationing, *Quarterly Journal of Economics* 4, 651-666.
- Jaynes, Gerald D., 1978, Equilibria in monopolistically competitive insurance markets, *Journal of Economic Theory* 19, 394-422.
- King, Mervyn, 1984, Tax policy and the business cycle, mimeo., MIT.

- Kreps, David M. and Robert Wilson, 1982, Sequential equilibria, *Econometrica* 50, 863-894.**
- Pauly, Mark V., 1974, Overinsurance and public provision of insurance, *Quarterly Journal of Economics* 88, 44-62.**
- Poterba, James M. and Lawrence H. Summers, 1987, Finite lifetimes and the effects of budget deficits on national savings, *Journal of Monetary Economics*, this issue.**
- Rothschild, Michael, and Joseph Stiglitz, 1976, Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *Quarterly Journal of Economics* 90, 629-649.**
- Smith, Bruce, 1983, Limited information, credit rationing, and optimal government lending policy, *American Economic Review*, 73, 305-318.**
- Stiglitz, Joseph and Andrew Weiss, 1981, Credit rationing in markets with imperfect information, *American Economic Review* 71, 393-411.**
- Weil, Philippe, 1984, 'Love thy children': Reflections on the Barro debt neutrality theorem, mimeo., Harvard University.**
- Wilson, Charles, 1977, A model of insurance markets with incomplete information, *Journal of Economic Theory*, 16, 167-207.**

**ESSAY II**

**BUDGET DEFICITS, MONETARY POLICY AND INFLATION:  
THE "TIGHT MONEY PARADOX" RECONSIDERED**

## **1. Introduction**

In recent years, our understanding of the linkage among budget deficits, money growth and inflation has been significantly improved by the contributions of Sargent and Wallace (1981; SW hereafter) and several other authors. SW's celebrated article examined the response of inflation to temporary tightening of monetary policy in a model where the path of money growth is dynamically constrained by fiscal policy through seigniorage revenue requirements. Their analysis assumed that the path of primary budget deficits is "predetermined" vis-à-vis the path of money growth, and that the exogenous real interest rate exceeds the growth rate of the economy. Since these assumptions imply that the government's intertemporal budget constraint generally restricts the choice of monetary policy, SW argued, (1) a temporary reduction in money growth is likely to produce higher inflation in the long run due to a higher level of debt service eventually required, and (2) in certain "spectacular" cases, such a policy may immediately raise inflation without having even temporary success in disinflation. These results are sometimes collectively called the "tight money paradox."

Subsequent literature on this "paradox" has been primarily concerned with two major issues. One is the existence and the exact nature of the link between deficits and money growth. It has been suggested [e.g., Weil (1987)] that, in Sidrauski's (1967) representative-dynasty model of a monetary economy, the government's intertemporal budget constraint indeed defines the set of admissible monetary policies, although no explicit formula for the constraint on monetary policy has been obtained so far. Weil compared this with an economy inhabited by mutually disconnected overlapping dynasties and showed that, in the latter, there is less need for the monetization of debt and the set of admissible policies is larger.

The other issue is how monetary policy affects inflation when it is dynamically constrained by a deficit-money link of some form. Since SW's results were obtained in a model with rather



arbitrary assumptions such as mutually exclusive sets of money holders and "capital" holders, several attempts at generalization have been made. In studies most closely related to ours, Liviatan (1984) and Drazen (1985) reformulated the problem in the context of a more standard Sidrauski-type optimizing model and essentially confirmed SW's results. Drazen and Helpman (1986) extended the model to include certain types of uncertainty regarding future policies. All of these studies, however, limit the analysis to utility functions that are separable in consumption and money, and closed-form solutions have been obtained only for the case of a logarithmic utility function [Liviatan]. Furthermore, only a very simple form of monetary policy (which we call the SW policy experiment) has been considered in the literature.

In this essay, we use the Laplace transform technique in order to discard the assumption of separable instantaneous utility and to consider a broader class of policy experiments. The model employed is a simple monetary optimizing model similar to those in Liviatan and Drazen. It is shown that, when the model has a unique convergent equilibrium path, predetermined fiscal policy indeed constrains the choice of nominal money growth. We present an exact formula for the constraint and obtain closed-form solutions for the equilibrium path of inflation in a model with a general utility function (non-separable in consumption and money). The effects of anticipated and unanticipated SW experiments on inflation are examined both qualitatively and quantitatively. We further consider welfare implications of various policies and show that maintaining a constant money growth rate is not an optimal policy except in a very special case.

The essay is organized as follows. The next section presents a simple intertemporal optimizing model of an exchange economy with government debt and money. Section 3 develops a technique for analyzing the local behavior of the system and presents general closed-form solutions and other basic results, while the following section applies the technique to the special case of SW-type policy experiments. Welfare implications are examined in Section 5. Section 6 contains concluding remarks.

## **2. The Model**

The economy is inhabited by a large number of infinite-lived agents, each of whom maximizes

$$(1) \quad \int_0^{\infty} u[c(t), m(t)] e^{-\delta t} dt$$

subject to

$$(2a) \quad c(t) + \dot{a}(t) = r(t)b(t) - \pi(t)m(t) + y(t) - x(t)$$

$$(2b) \quad a(t) = b(t) + m(t)$$

where  $u$  is concave with  $u_c, u_m > 0$  and  $u_{cc}, u_{mm} < 0$ ,  $\delta$  the subjective discount rate,  $c(t)$  consumption at time  $t$ ,  $a(t)$  the total wealth,  $m(t)$  real money holdings,  $b(t)$  the holdings of indexed bonds,  $r(t)$  the real interest rate,  $\pi(t)$  the rate of inflation,  $y(t)$  the exogenous flow endowment of perishable consumption goods, and  $x(t)$  lump-sum taxes. There is no capital. All variables denote per capita values, and the time index  $t$  will be omitted whenever there can be no confusion.

The first-order conditions for the maximization problem are given by (2) and

$$(3a) \quad u_c - \lambda = 0$$

$$(3b) \quad u_m - (r + \pi)\lambda = 0$$

$$(3c) \quad \dot{\lambda} = (\delta - r)\lambda$$

where  $\lambda$  is the Lagrange multiplier on the constraint (2a). It is assumed that the transversality condition at  $t=\infty$  is satisfied for the total wealth. Namely,

$$(3d) \quad \lim_{t \rightarrow \infty} \lambda(t) a(t) e^{-\delta t} = 0.$$

Goods market equilibrium requires that  $y = c + g$ , where  $g$  is the government purchases of goods and services, and both  $y$  and  $g$  are assumed to be constant at  $y^*$  and  $g^*$ , respectively. Therefore,  $c$  is also constant at  $c^* = y^* - g^*$  in equilibrium and (3c) reduces to

$$(3c') \quad u_{cm} \dot{m} = (\delta - r) u_c$$

The aggregate nominal money stock grows at rate  $\mu$  so that

$$(4) \quad \dot{m} = (\mu - \pi) m$$

Combining (3a) through (3c') and (4) yields

$$(5a) \quad \pi = \pi(m, \mu) = \frac{u_m - \delta u_c + \mu m u_{cm}}{u_c + m u_{cm}}$$

$$(5b) \quad r + \pi = i(m) = \frac{u_m}{u_c}$$

As is apparent from (5), the real interest rate is not constant along transition paths because the marginal utility of consumption is generally affected by the real money balances. If consumption and money are Edgeworth-complementary ( $u_{cm} > 0$ ), for example, increasing real balances imply increasing marginal utility of consumption ( $d\lambda/dt > 0$ ), which induces the consumer to shift consumption from the current period to a future date and therefore depresses the real interest rate.

The government's budget constraint is given by

$$(6) \quad \begin{aligned} \dot{b} &= rb - \mu m + g^* - x \\ &= rb - \mu m + d \end{aligned}$$

where  $d (= g^* - x)$  denotes the primary deficit. Government bonds are short-term real bonds that mature instantaneously and are continuously rolled over. Ruling out Ponzi games played by the government, we can integrate (6) to obtain the intertemporal government budget constraint:

$$(7) \quad b(0) = \int_0^{\infty} [\mu(t)m(t) - d(t)]e^{-\int_0^t r(s)ds} dt$$

Substituting (5) into (4) and (6), we have a system of two differential equations describing the evolution of the stock of government debt and money:

$$(8a) \quad \dot{b} = [i(m) - \pi(m, \mu)]b - \mu m + d$$

$$(8b) \quad \dot{m} = [\mu - \pi(m, \mu)]m$$

The steady state, given constant levels (asterisked) of  $d$  and  $\mu$ , is characterized by the following equations.<sup>1</sup>

$$(9a) \quad r^* = \delta$$

$$(9b) \quad \pi^* = \mu^*$$

$$(9c) \quad \frac{u_m(c^*, m^*)}{u_c(c^*, m^*)} = \delta + \mu^*$$

$$(9d) \quad \delta b^* + d^* = \mu^* m^*$$

Conditions (9a) through (9c) are self-explanatory. As in the original Sidrauski model, the real interest rate is invariant across steady states whether or not the utility function is separable in consumption and money. (9d) states that the (per capita) government deficit, inclusive of interest

payments on its outstanding debt, must be financed solely by money seigniorage in the steady state.

### 3. Dynamic Effects of Monetary Policy

Monetary policy in this essay is defined as a choice of the path of money growth,  $\{\mu(t)\}$ , given a fixed path of primary deficits predetermined by fiscal policy. In particular, we consider the following class of policy experiments: starting from a steady state associated with  $g^*$ ,  $d^*$  and  $\mu^*$ , the government announces, at time  $t=0$ , that it will change the growth rate of money to

$$\mu(t) = \mu^* + \varepsilon h(t)$$

for  $t \geq 0$ , where  $\varepsilon$  is a constant and  $h(t)$  is chosen so that the intertemporal government budget constraint (7) is satisfied. It is assumed that there are no open market operations that cause a discrete change in the level of outstanding government debt. Now, for any particular  $\varepsilon$ , the dynamics of the system for  $t \geq 0$  is described by

$$(10a) \quad \frac{\partial b(t, \varepsilon)}{\partial t} = \{i[m(t, \varepsilon)] - \pi[m(t, \varepsilon), \mu^* + \varepsilon h(t)]\} b(t, \varepsilon) - [\mu^* + \varepsilon h(t)] m(t, \varepsilon) + d^*$$

$$(10b) \quad \frac{\partial m(t, \varepsilon)}{\partial t} = \{\mu^* + \varepsilon h(t) - \pi[m(t, \varepsilon), \mu^* + \varepsilon h(t)]\} m(t, \varepsilon)$$

Due to the nonlinearity of the system, we cannot calculate the equilibrium response to a change in  $\varepsilon$  from an arbitrary value. However, as in Judd (1985, 1987), we may do so for an infinitesimal increase in  $\varepsilon$  from zero by defining<sup>2</sup>

$$\mathbf{x}_\epsilon(t) = \left. \frac{\partial \mathbf{x}(t, \epsilon)}{\partial \epsilon} \right|_{\epsilon=0} \quad (\mathbf{x} = \mathbf{b}, \dot{\mathbf{b}}, \mathbf{m}, \dot{\mathbf{m}}, \pi)$$

and differentiating (10) with respect to  $\epsilon$  to obtain

$$\begin{bmatrix} \dot{\mathbf{b}}_\epsilon \\ \dot{\mathbf{m}}_\epsilon \end{bmatrix} = \begin{bmatrix} \delta & [i'(m^*) - \pi_m(m^*, \mu^*)]b^* - \mu^* \\ 0 & -\pi_m(m^*, \mu^*)m^* \end{bmatrix} \begin{bmatrix} \mathbf{b}_\epsilon \\ \mathbf{m}_\epsilon \end{bmatrix} + \begin{bmatrix} -[\pi_\mu(m^*, \mu^*) + m^*]h \\ [1 - \pi_\mu(m^*, \mu^*)]m^*h \end{bmatrix}$$

Using (5), we can rewrite this as

$$(11) \quad \begin{bmatrix} \dot{\mathbf{b}}_\epsilon \\ \dot{\mathbf{m}}_\epsilon \end{bmatrix} = \begin{bmatrix} \delta & -(\mu^* + \theta\sigma q^*) \\ 0 & (1 - \theta)\sigma \end{bmatrix} \begin{bmatrix} \mathbf{b}_\epsilon \\ \mathbf{m}_\epsilon \end{bmatrix} + \begin{bmatrix} -(1 + \theta q^*)m^*h \\ (1 - \theta)m^*h \end{bmatrix}$$

where

$$\theta = \frac{m^*u_{cm}}{u_c + m^*u_{cm}}, \quad \sigma = -\frac{m^*(u_c u_{mm} - u_m u_{cm})}{(u_c)^2}, \quad q^* = \frac{b^*}{m^*}$$

The derivatives of  $u$  above are evaluated at  $(c^*, m^*)$ . It is assumed throughout this essay that  $\sigma > 0$ , namely, that consumption is normal.

Since the matrix in (11), which we shall call  $A$ , is a constant matrix, the explicit solution to (11) can be obtained by use of the Laplace transform technique. The Laplace transform of a function  $x(t)$  is defined as

$$X(s) = L[x](s) = \int_0^\infty x(t)e^{-st} dt$$

and has the property

$$L[\dot{x}](s) = sL[x](s) - x(0).$$

Let  $B_\epsilon(s)$ ,  $M_\epsilon(s)$  and  $H(s)$  be the Laplace transforms of  $b_\epsilon(t)$ ,  $m_\epsilon(t)$ , and  $h(t)$ , respectively.

Then, from (11), we have

$$(12) \quad \begin{bmatrix} sB_\epsilon(s) \\ sM_\epsilon(s) \end{bmatrix} = A \begin{bmatrix} B_\epsilon(s) \\ M_\epsilon(s) \end{bmatrix} + \begin{bmatrix} -(1 + \theta q^*)m^*H(s) \\ (1 - \theta)m^*H(s) + m_\epsilon(0) \end{bmatrix}$$

We have imposed the condition  $b_\epsilon(0) = 0$  since the real value of short-term indexed bonds cannot "jump" at  $t=0$ .  $m_\epsilon(0)$ , on the other hand, is not necessarily zero because the price level may jump in response to the announcement of the policy change. Solving (12) for  $B_\epsilon(s)$  and  $M_\epsilon(s)$  yields

$$(13a) \quad B_\epsilon(s) = \frac{[(1 - \theta)(\sigma - \mu^*) - (1 + \theta q^*)s]m^*H(s) - (\mu^* + \theta \sigma q^*)m_\epsilon(0)}{(s - \delta)[s - (1 - \theta)\sigma]}$$

and

$$(13b) \quad M_\epsilon(s) = \frac{(1 - \theta)m^*H(s) + m_\epsilon(0)}{s - (1 - \theta)\sigma}$$

The system has two eigenvalues,  $\delta$  and  $(1 - \theta)\sigma$ , and the steady state is either an unstable node

(if  $\theta < 1$ ) or a saddle point (if  $\theta > 1$ ). In a standard monetary optimizing model without capital accumulation, the case  $\theta < 1$  is equivalent to Obstfeld's (1984) condition for the local uniqueness of equilibrium, while  $\theta > 1$  is associated with multiple stable equilibria even when the steady state is unique.<sup>3</sup> We avoid dealing with "pathological" cases by restricting our analysis to the case  $\theta < 1$ . One sufficient condition for this to be the case is that consumption and money are Edgeworth-complementary.<sup>4</sup> Special cases in which  $\theta$  is less than one include, for example, utility functions that are separable in consumption and money ( $\theta = 0$ ). For a utility function of the form

$$(14) \quad u(c, m) = \frac{(c^\alpha m^\beta)^{1-\gamma}}{1-\gamma}, \quad 0 < \alpha, \beta < 1, \quad \alpha + \beta = 1, \quad \gamma > 0,$$

which we use in later exercises, the necessary and sufficient condition for  $\theta < 1$  is  $\gamma < 1 + (1/\beta)$ .

Although the system is apparently unstable when  $\theta$  is less than unity, stability can be ensured by an appropriate restriction on  $h(t)$ . More specifically, we require the solutions to be bounded, and use the fact that  $b_\epsilon(t)$  and  $m_\epsilon(t)$  are bounded if and only if both  $B_\epsilon(s)$  and  $M_\epsilon(s)$  are finite for all  $s > 0$ . First, we note that for  $B_\epsilon(\delta)$  to be finite, the numerator on the right hand side of (13a) must vanish for  $s = \delta$ . This implies

$$(15) \quad m_\epsilon(0) = J m^* H(\delta), \quad \text{where} \quad J = - \frac{(1 - \theta)(\delta + \mu^* - \sigma) + \delta\theta(1 + q^*)}{\mu^* + \theta\sigma q^*}$$



(15) states that the jump in real money balances at  $t=0$  is proportional to the Laplace transform of  $h(t)$  evaluated at  $s=\delta$ . It is worth noting that we can derive this equation alternatively by differentiating the intertemporal budget constraint for the government (7) with respect to  $\varepsilon$  and evaluating the resulting expression at  $\varepsilon = 0$ . In this sense, the condition that  $B_\varepsilon(\delta)$  is finite is a local equivalent of the no-Ponzi game condition for the government.

As  $\theta$  is assumed to be less than unity, we have an additional condition

$$(16) \quad m_\varepsilon(0) = - (1 - \theta) m^* H[(1 - \theta)\sigma]$$

which follows from the restriction that  $B_\varepsilon[(1-\theta)\sigma]$  and  $M_\varepsilon[(1-\theta)\sigma]$  must also be finite. Combining (15) and (16) yields the following restriction on the function  $h(t)$ :

$$(17) \quad \int_0^\infty h(t) [J e^{-\delta t} + (1 - \theta) e^{-(1-\theta)\sigma t}] dt = 0.$$

Equation (17) defines a set of permissible monetary policies in the sense that, if this condition is not satisfied, the government debt and/or the money stock will grow without bound and the economy will not converge to the steady state.<sup>5</sup> Note that the expression inside the bracket is positive for all  $t$  if  $J$  is positive (which tends to be the case for a broad range of parameters), and that, in such cases,  $h(t)$  must switch its sign at least once for the integral to vanish. In other words, temporary tightening of monetary policy has to be followed by a period of high money growth. On the other hand, if  $J$  is negative and the expression inside the bracket does not have the same sign for all  $t$ , then permanent disinflation may be possible. The formula is valid only for small variations in

policy, but it is nonetheless the first explicit formula with any generality for the intertemporal constraint on monetary policy.

Equations (15) (or (16)) and (17) allow us to examine the response of the system to any change in the path of money growth compatible with a given level of the primary deficit. We first integrate the differential equation for  $m_\epsilon$  (or, alternatively, take the inverse Laplace transform of (13b)) to obtain

$$(18) \quad m_\epsilon(t) = m_\epsilon(0)e^{(1-\theta)\sigma t} + (1-\theta)m^* \int_0^t h(\tau)e^{(1-\theta)\sigma(t-\tau)} d\tau,$$

which can be rewritten, using (16), as

$$(19) \quad m_\epsilon(t) = - (1-\theta)m^* \int_t^\infty h(\tau)e^{(1-\theta)\sigma(t-\tau)} d\tau.$$

The rate of inflation, as we know from (5a), is a function of the real money stock and the rate of money growth. In terms of deviations from the steady state, this relationship takes the following form:

$$(20) \quad \begin{aligned} \pi_\epsilon(t) &= \pi_m(m^*, \mu^*)m_\epsilon(t) + \pi_\mu(m^*, \mu^*)h(t) \\ &= - \frac{(1-\theta)\sigma}{m^*} m_\epsilon(t) + \theta h(t) \end{aligned}$$

Substituting (19) into (20), we have

$$(21) \quad \pi_\epsilon(t) = (1-\theta)^2 \sigma \int_t^\infty h(\tau)e^{(1-\theta)\sigma(t-\tau)} d\tau + \theta h(t).$$

Thus, we find that the equilibrium rate of inflation reflects two factors: the current money growth rate, and the "present discounted value" of future money growth rates, where the discount rate is equal to  $(1-\theta)\sigma$ , one of the eigenvalues of the system. Past history, including past monetary policy, does not affect current inflation.

The following proposition summarizes important results obtained in this section.

*Proposition 1. If the utility function is such that  $\theta$  is less than unity, then the steady state is an unstable node, and a bounded solution is obtained if and only if the path of  $h(t)$  is chosen so that the intertemporal constraint (16) is satisfied. The equilibrium response of inflation to a perturbation  $\{h(t)\}$  that satisfies (16) is given by equation (21).*

#### **4. The Sargent-Wallace Experiment**

We now apply the technique described above to a slightly generalized version of the simple policy experiment considered by SW, Liviatan and Drazen. In an environment where the dynamic constraint (16) is binding (i.e.,  $\theta < 1$ ), the monetary authority temporarily reduces money growth to fight inflation, but subsequently readjusts its policy to avoid debt explosion and achieve a new steady state. Our scenario generalizes the SW experiment by incorporating anticipated as well as unanticipated policies: at  $t=0$ , the government announces that the money growth rate will be lowered at  $t=T$ ; this policy of tight money is to be continued until  $t=T+L$ , at which point the government switches to a level of money growth that enables it to maintain its debt balance constant. Formally, we have  $h(t) = h^o(t)$  where

$$h^{\circ}(t) = \begin{cases} 0 & 0 \leq t < T \\ h_T < 0 & T \leq t < T + L \\ \psi(h_T; T, L) & t \geq T + L \end{cases}$$

where  $h_T$ ,  $T$  and  $L$  are exogenously given ( $T=0$  for the original SW experiment), but  $\psi(h_T)$  depends on all the parameters in the model. For this type of policy, the constraint (15) becomes

$$\begin{aligned} & \sigma J [h_T (e^{-\delta T} - e^{-\delta(T+L)}) + \psi(h_T) e^{\delta(T+L)}] \\ & = -\delta [h_T (e^{-(1-\theta)\sigma T} - e^{-(1-\theta)\sigma(T+L)}) + \psi(h_T) e^{-(1-\theta)\sigma(T+L)}], \end{aligned}$$

and can be solved to yield

$$(22 \text{ a}) \quad \psi(h_T; T, L) = h_T [1 - \phi(T, L)]$$

where

$$(22 \text{ b}) \quad \phi(T, L) = \frac{\sigma J e^{-\delta T} + \delta e^{-(1-\theta)\sigma T}}{\sigma J e^{-\delta(T+L)} + \delta e^{-(1-\theta)\sigma(T+L)}}$$

It is straightforward to show, using (22), that the path  $\{h^{\circ}(t)\}$  has a Laplace transform

$$(23) \quad H^{\circ}(s) = \frac{h_T}{s} [1 - \phi(T, L) e^{-sL}] e^{-sT}.$$

Combining this with (15), we find the proportionate change in the price level at  $t = 0$  to be

$$(24) \quad -\frac{m_{\epsilon}(0)}{m^*} = -\frac{h_T}{\sigma} [\phi(T, L) e^{-(1-\theta)\sigma L} - 1] e^{-(1-\theta)\sigma T}.$$

The equilibrium path of inflation can be directly calculated by substituting into (21):

$$(25) \quad \pi_{\varepsilon}(t) = \begin{cases} (1 - \theta)h_T[1 - \phi(T, L)e^{-(1-\theta)\sigma L}]e^{-(1-\theta)\sigma(T-t)} & 0 \leq t < T \\ h_T[1 - (1 - \theta)\phi(T, L)e^{-(1-\theta)\sigma(T+L-t)}] & T \leq t < T + L \\ h_T[1 - \phi(T, L)] & t \geq T + L \end{cases}$$

The next proposition follows from (25).<sup>6</sup>

*Proposition 2. An unanticipated SW policy experiment causes an immediate increase in inflation if and only if  $(1 - \theta)\phi(0, L) > e^{(1-\theta)\sigma L}$ ; if the policy change is announced  $T$  periods in advance, the announcement has an immediate inflationary effect if and only if  $\phi(T, L) > e^{(1-\theta)\sigma L}$ . In either case, the necessary and sufficient condition for the steady-state rate of inflation to rise is  $\phi(T, L) > 1$ .*

One corollary to Proposition 2 is that, since  $e^{(1-\theta)\sigma L} > 1$ , anticipated temporary monetary contraction that has an inflationary effect in the short run is also inflationary in the long run. This conclusion, however, does not necessarily hold for unanticipated policy experiments unless utility is separable in consumption and money. The dynamic response of inflation to an anticipated policy has three stages and typically looks like Figure 1. At  $t=0$ , when the announcement of a new monetary policy is made, the rate of inflation jumps from  $\mu^*$  to  $\mu^* + \pi_{\varepsilon}(0)$ , and then monotonically declines until time  $T$  because the relatively high money growth during this period (compared to the following period [ $T \leq t < T+L$ ]) is continuously removed from the discounted value formula (21) as  $t$  increases. When the policy change is implemented, it jumps again to another monotonic path, and at

exactly  $t=T+L$ , the steady state is achieved with the rate of inflation now constant and equal to the new money growth rate  $\mu^*+h_T[1-\phi(T,L)]$ . The rate of inflation jumps three times unless  $\theta$  is equal to zero, but the price level jumps only once at  $t = 0$ , as can be verified from (19). A jump in  $\pi$  after  $t=0$  is accompanied by an offsetting jump in the real interest rate so that the nominal interest rate (which is a function of real balances) remains a continuous function of time.

Figure 2 shows the transition path in the  $(b_\epsilon, m_\epsilon)$  space. Note that any steady state must be on the terminal surface SS, which represents

$$\delta b_\epsilon + (\sigma - \mu)m_\epsilon = 0.$$

The original equilibrium is at the origin and the dynamics before policy implementation ( $0 \leq t < T$ ) is dictated by the arrows around it, while the path for  $T \leq t < T+L$  follows the arrows around point D. The level of government debt is decreasing between points A and B because of higher seigniorage revenues generated by increased real balances. The continuous fall in real balances between B and C reflects the monotonically rising inflation. A new steady state is reached at point C, where the level of outstanding debt is higher and more money seigniorage is required to finance interest payments, which in turn means that the money growth rate is higher and the real money stock lower.

We have described the general features of a typical response, but the exact path of inflation turns out to be quite sensitive to the shape of the utility function. Below, we consider two special cases, one in which the instantaneous utility function is separable in consumption and money, and another in which it is a Cobb-Douglas function as in (14).

### *Case 1: Separable Utility*

For the case of an instantaneous utility function

$$(26) \quad u(c, m) = f(c) + v(m),$$

we have  $\theta = 0$  and  $\sigma = \rho(\delta + \mu^*)$ , where  $\rho$  is defined as  $-m^*v''(m^*)/v'(m^*)$ .  $\rho^{-1}$  is the intertemporal elasticity of substitution for money holdings, which can also be interpreted as the elasticity of money demand with respect to the nominal interest rate in the steady state. The simplest case to analyze is that of a logarithmic utility function ( $\rho = 1$  for all  $m^*$ ), which Liviatan used to reproduce SW's "spectacular" result for unanticipated monetary contraction. Since  $\phi(T, L) = e^{\rho(\delta + \mu^*)L}$ , we have  $\pi_\epsilon(t) = 0$  for all  $t \in [0, T]$  and  $\pi_\epsilon(t) > 0$  for  $t > T$  in the log utility case, meaning that temporary tightening of monetary policy, whether anticipated ( $T > 0$ ) or unanticipated ( $T = 0$ ), fails to produce lower inflation in the short run and leads to higher inflation in the long run. When  $T = 0$ , our result is identical to Liviatan's.

If  $\rho$  is greater than unity (i.e., money demand is relatively interest-inelastic), the SW policy experiment (both anticipated and unanticipated) will indeed reduce inflation in the short run, although not in the long run. If  $\rho$  is less than unity (money demand highly elastic), on the other hand, the results are less straightforward. It can be shown that the denominator of (22b) has a root  $\omega \in (\mu^*/(\delta + \mu^*), 1)$ , and that  $\pi_\epsilon(t) > 0$  for all  $t > 0$  if and only if  $\omega < \rho \leq 1$ . Drazen gave a (necessary) condition equivalent to  $\mu^*/(\delta + \mu^*) < \rho \leq 1$ , but the range in which SW's "spectacular" result obtains is in fact narrower.

Table 1 shows, for different values of  $\rho$ , the response of inflation to unanticipated ( $T = 0$ ) and anticipated ( $T = 3$ ) reduction in money growth by 1 percent point, sustained for 3 years. It can be

seen that, for values of  $\rho$  less than unity, the path of inflation is extremely sensitive to  $\rho$ . A more plausible parameter range, however, would be  $\rho > 1$ , given typical estimates of the long-run elasticity in the neighborhood of 0.2 ( $\rho = 5$ ). Within this range, disinflation can be successfully carried out in the short run, but not in the long run. The magnitude of the negative jump in inflation in response to policy implementation tends to increase with  $\rho$ , while the rise in the steady state rate of inflation becomes smaller as  $\rho$  increases. The intuition behind this result is that, when  $\rho$  is large, the rate of inflation never differs much from the rate of money growth because the consumer's real money holdings are approximately constant. Therefore, a reduction in money growth by 1 percent at  $t=0$  immediately leads to a drop in inflation by almost the same amount. On the other hand, when the government switches to a sustainable policy at  $t=L$ , the new money growth rate implied by the seigniorage revenue requirements is relatively low (and therefore inflation is low) since the money demand is not interest elastic. For this range of  $\rho$ , a delay in policy implementation tends to reduce the inflationary impact at announcement ( $t = 0$ ), while inflation at the time of policy implementation ( $t = T$ ) is largely independent of how long the policy change has been anticipated.

### *Case 2: Cobb-Douglas Utility*

If the instantaneous utility function has the form (14), we have

$$\theta = \frac{\beta(1 - \gamma)}{1 + \beta(1 - \gamma)}, \quad \sigma = \delta + \mu.$$

No simple analytical conditions are available in this case, but numerical results can be easily obtained. A sample of such results is shown in Table 2 for different values of  $\beta$  and  $\gamma$ . The initial



impact of unanticipated ( $T = 0$ ) temporary monetary contraction, though small for most admissible parameter values, tends to grow as the value of  $\beta$  or  $\gamma$  increases. When the policy is anticipated ( $T = 3$ ), the announcement tends to have a relatively large negative effect, while actual implementation does not. As in the separable utility case, the response at the time of policy implementation ( $t = T$ ) is largely independent of how long the policy change has been anticipated. The long-run rate of inflation increases as a result of such policies in most cases, and the size of the increase is negatively related to the values of  $\beta$  and  $\gamma$ .

### 5. A Note on the Theory of Optimal Seigniorage

Since inflation is a tax on holding money balances, it is natural to ask how a government can minimize the deadweight loss and other social costs associated with collection of this tax. Some economists have suggested that an (approximate) optimal rule in dynamic taxation is to keep tax rates constant over time [Barro(1979)]. Mankiw(1987) applied this idea to seigniorage collection and argued that inflation should follow a random walk process. In a deterministic model such as ours, his rule simply prescribes constant inflation.

It may be of some interest to see whether the standard monetary optimizing model, in which inflation affects welfare through reduced money holdings, supports this argument. First we note that, for a given path of money growth  $\{\mu^* + \epsilon h(t)\}$ , the consumer's utility can be considered as a function of  $\epsilon$ :

$$(26) \quad W(\epsilon) = \int_0^{\infty} u[c^*, m(t, \epsilon)] e^{-\delta t} dt$$

Differentiating (26) with respect to  $\epsilon$  and evaluating the resulting expression at  $\epsilon=0$  yields

$$(27) \quad W'(0) = u_m(c^*, m^*) \int_0^{\infty} m_{\epsilon}(t) e^{-\delta t} dt = u_m(c^*, m^*) M_{\epsilon}(\delta).$$

We combine this with (13b) and (15) to obtain the following relationship.

$$(28) \quad W'(0) = - \frac{(1 + \theta q^*) m^* u_m}{\mu + \theta \sigma q^*} H(\delta) = - \frac{(1 + \theta q^*) m^* u_m}{\mu + \theta \sigma q^*} \int_0^{\infty} h(t) e^{-\delta t} dt$$

Constant money growth (and therefore constant inflation) is optimal only if  $W'(0)=0$  for all permissible  $H(\delta)$ . Since  $(1 + \theta q^*)$  is not generally equal to zero and since  $u_m > 0$ , it follows from (28) that the constant money growth policy is suboptimal and consumer welfare can be increased by some appropriate deviation from it. If we remove the assumption  $u_m > 0$  and instead assume that there is a unique value of  $m$  such that  $u_m(c^*, m) = 0$ , then there is a possibility that maintaining a constant money growth rate may be optimal. For the policy of constant money growth to produce a steady state with  $u_m = 0$  (zero nominal interest rate), however, setting  $\mu(t) = -\delta$  for all  $t$  must be consistent with the government's intertemporal budget constraint. Except in this unlikely ("measure zero") event, a constant money growth rule fails to satisfy the first-order condition for optimality. Thus, we have the following proposition.

**Proposition 3.** *A policy of constant nominal money growth is optimal only if the associated steady state is characterized by zero nominal interest rate (Friedman's optimal quantity of money). If primary deficits are predetermined, however, it is generally not feasible to attain such a steady state*

*by maintaining a constant rate of money growth. Therefore, the policy of constant money growth is generally suboptimal in our model.*

The result that the monetary authority can bring about welfare improvement by deviating from the original steady state can be interpreted as a mirror image of the tight money paradox. Namely, if the long-run rate of inflation can be reduced at the expense of temporary inflation, it is possible that the gains from the former outweigh the losses from the latter. In general, there are unanticipated capital gains or losses at policy announcement, but the existence of such an effect is not crucial.<sup>7</sup> To demonstrate the point, let us consider the example of separable utility with  $\rho = 1$ .<sup>8</sup> In this case, substituting into (17) and (19), we may write down the equilibrium path of the real money stock as

$$m_{\epsilon}(t) = m^* \int_t^{\infty} h(\tau) e^{(\delta + \mu)(t - \tau)} d\tau$$

where  $h$  is chosen subject to

$$\int_0^{\infty} h(\tau) e^{-(\delta + \mu)\tau} d\tau = 0$$

It is easy to see that  $m(t)$  responds to a policy change only after its implementation: if  $h(t) = 0$  for  $0 \leq t < T$ , then  $m(t) = 0$  for that period regardless of  $T$ . There is no price level jump, or unexpected capital gains or losses, and furthermore, the path of  $m(t)$  after  $T$  does not depend on how long the policy change has been anticipated. In this sense, the monetary authority is not engaged in "cheating." Temporary monetary expansion (a "reverse" SW experiment) in this case makes the real money stock gradually grow to a higher level that corresponds to lower inflation.  $m(t)$  never goes below the initial level, and thus, we have a long-run welfare gain without any short-run loss.

The foregoing analysis does not tell us what the optimal policy is when a constant money growth rate of  $-\delta$  is incompatible with the predetermined level of the primary deficit. It does provide us, however, with the magnitude of the welfare gain or loss associated with a particular monetary policy. If  $u_m(c^*, m^*) \neq 0$ , (28) can be rewritten as

$$(28') \quad \text{MEG} = \frac{\delta W'(0)}{m^* u_m} = - \frac{\delta(1 + \theta q^*)}{\mu + \theta \sigma q^*} H(\delta).$$

MEG (money-equivalent gain) is a utility gain (or loss) expressed in terms of a proportionate gain (or loss) in real money balances. In other words, the impact of a policy change on the consumer's utility is equivalent to an increase in the real per capita money stock by  $\delta W'(0)/u_m$ .

Table 3 gives the values of MEG in the case of separable utility for  $L=3$  and  $T=0, 1$  and  $3$  under the assumption that the rate of money growth is temporarily increased at  $t = T$ , followed by an appropriate adjustment at  $t = T + L$ . If  $\rho > 1$ , temporary monetary expansion raises welfare while temporary contraction has the opposite effect. The welfare gain due to temporary expansion is greatest when it is unexpected, and is significantly reduced by an early announcement, especially for large values of  $\rho$ . This result is essentially analogous to the well-known result in public finance that anticipated taxes on the capital stock typically have greater efficiency costs than unanticipated ones. Depending on parameter values, there may be a strong incentive for the government to bring about surprise inflation instead of adhering to the constant money growth rule. It seems that an optimal policy is likely to be time inconsistent because of such an effect. It should be noted, however, that if the policy of keeping  $\mu(t) = -\delta$  (for all  $t$ ) happens to be feasible, then there is no incentive for the government to deviate from it. In other words, an optimal *constant* money growth

rule is time consistent.

## **6. Conclusion**

The primary accomplishment of this essay is the successful use of the Laplace transform technique in determining the short-run and long-run effects of temporary monetary contraction in a perfect foresight model where the path of nominal money growth is constrained by fiscal policy through seigniorage revenue requirements. We have presented the first explicit formula for the dynamic constraint on monetary policy and obtained closed-form solutions for the equilibrium path of inflation. The quantitative effects on inflation of anticipated and unanticipated tightening of money have been examined in the cases of separable and Cobb-Douglas utility functions for various parameter values. The results are sensitive to functional forms and parameter values, but on balance, Sargent and Wallace's "spectacular" case of uniformly higher inflation seems unlikely to occur. On the other hand, a higher steady state rate of inflation does result in the majority of cases. The essay also includes an analysis of welfare implications, which shows that a constant money growth rule is generally not optimal when the choice of the growth rate is dictated by the government's intertemporal budget constraint. Numerical results suggest that surprise inflation is likely to have welfare-enhancing effects.

Figure 1

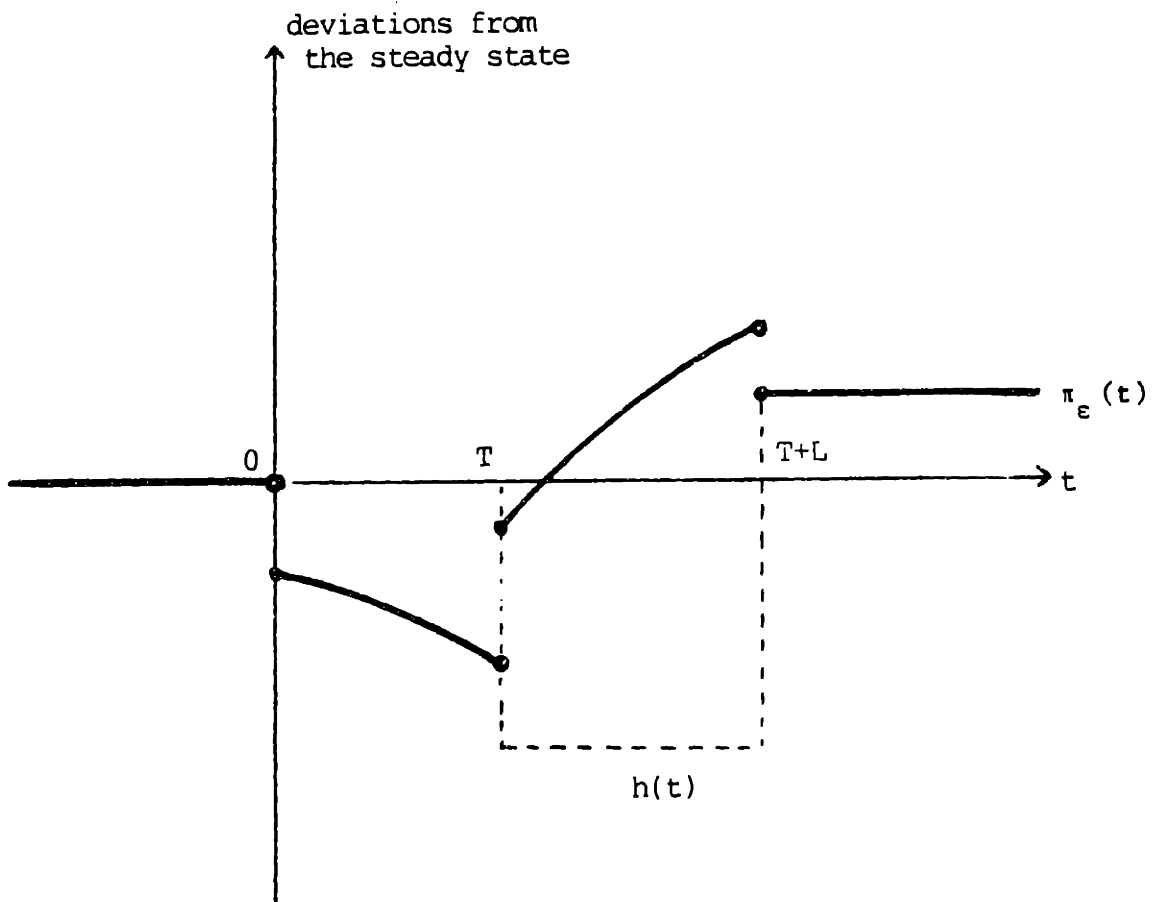
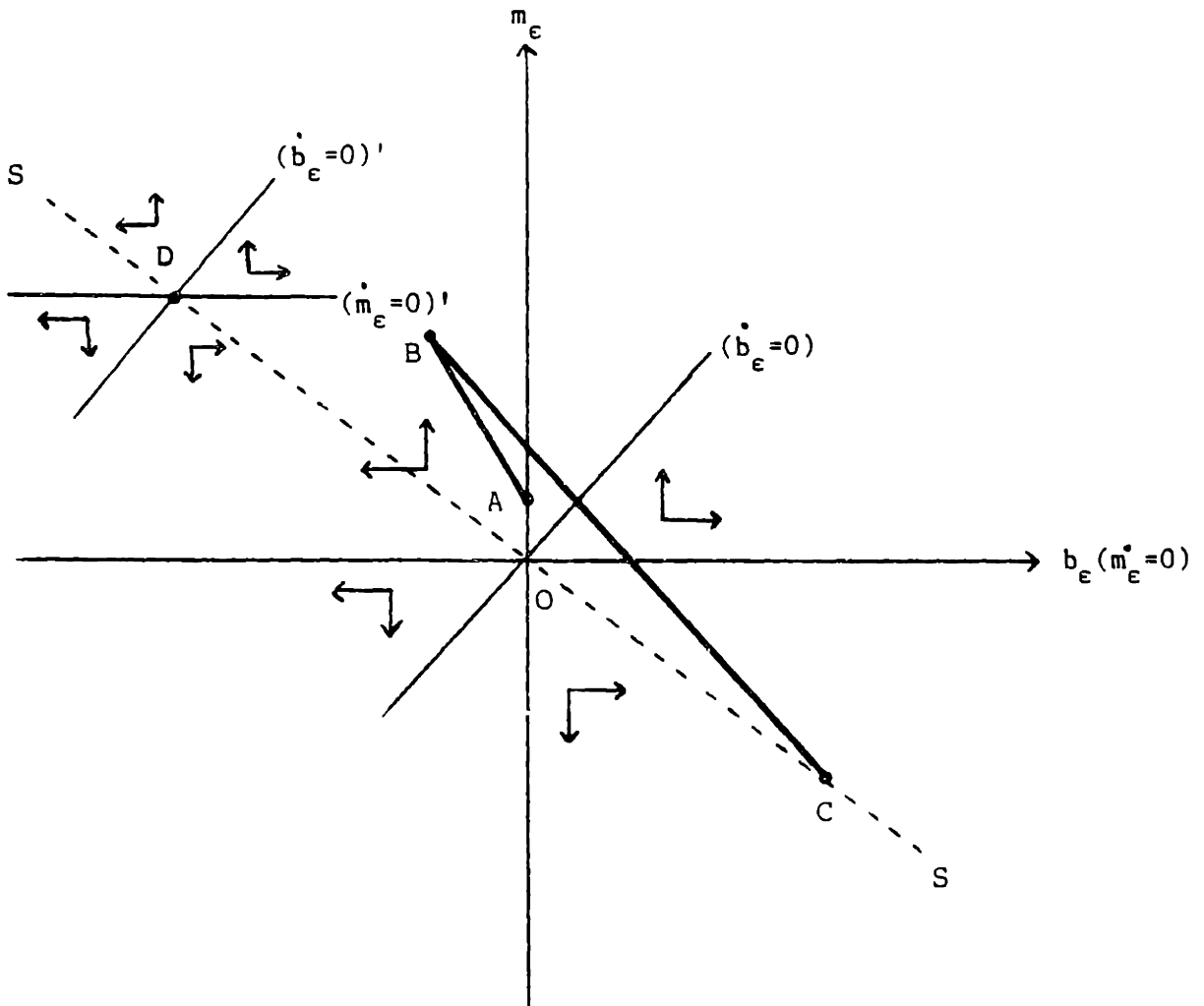


Figure 2



**TABLE 1**

**The Response of Inflation to Anticipated and Unanticipated  
Temporary Reduction in Money Growth by 1 Percent\***

- Separable Utility -

Key Parameters:  $h_T = -1\%$ ,  $L = 3$

$\rho$	T = 0		T = 3		
	$\pi_\epsilon(0)$	$\pi_\epsilon(L)$	$\pi_\epsilon(0)$	$\pi_\epsilon(T)$	$\pi_\epsilon(T+L)$
0.5	- 0.28	- 0.12	- 0.19	- 0.24	- 0.07
0.8	- 1.79	- 2.08	- 0.51	- 0.69	- 0.58
0.9	0.44	1.05	0.95	1.35	2.34
1.0	0.0	0.48	0.0	0.0	0.48
2.0	- 0.48	0.14	- 0.22	- 0.48	0.12
5.0	- 0.84	0.10	- 0.12	- 0.84	0.10
10.0	- 0.98	0.10	- 0.02	- 0.98	0.09

\* Changes in inflation are in percentage points.  $\mu^* = 10\%$ ,  $\delta = 3\%$ ,  $q^* = 1$  assumed.



**TABLE 2****The Response of Inflation to Anticipated and Unanticipated  
Temporary Reduction in Money Growth by 1 Percent\*****- Cobb-Douglas Utility -****Key Parameters:  $h_T = -1\%$ ,  $L = 3$** 

$\beta$	$\gamma$	$T = 0$		$T = 3$		
		$\pi_\epsilon(0)$	$\pi_\epsilon(L)$	$\pi_\epsilon(0)$	$\pi_\epsilon(T)$	$\pi_\epsilon(T+L)$
0.1	0.5	0.00	0.51	0.04	0.01	0.54
	2.0	0.01	0.40	- 0.08	- 0.02	0.36
	4.0	0.00	0.22	- 0.26	- 0.03	0.18
	8.0	- 0.27	- 0.19	- 0.66	- 0.08	0.02
0.2	0.5	- 0.01	0.55	0.08	0.03	0.61
	2.0	0.01	0.31	- 0.17	- 0.03	0.26
	4.0	- 0.13	- 0.08	- 0.58	- 0.03	0.03
0.4	0.5	- 0.02	0.62	0.17	0.06	0.77
	2.0	- 0.02	0.13	- 0.36	- 0.03	0.11

**\* Changes in inflation are in percentage points.  $\mu^* = 10\%$ ,  $\delta = 3\%$ ,  $q^* = 1$  assumed.**

**TABLE 3**

**The Money-Equivalent Gain (MEG) due to Anticipated and Unanticipated  
Temporary Increase in Money Growth by 1 Percent\***

**- Separable Utility -**

**Key Parameters:  $h_T = 1\%$ ,  $L = 3$**

$\rho$	T = 0	T = 1	T = 3
0.5	- 1.96	- 1.73	- 1.37
0.3	- 19.83	- 11.39	- 5.61
0.9	8.73	10.26	18.73
1.0	3.45	3.40	3.20
2.0	0.42	0.33	0.20
5.0	0.08	0.04	0.01
10.0	0.02	0.01	0.00

\* Proportionate equivalent changes in real balances in percentage points.  $\mu^* = 10\%$ ,  $\delta = 3\%$ ,  $q^* = 1$  assumed.

## NOTES

1. Uniqueness of the steady state is ensured by normality of consumption, which we explicitly assume later. See, for example, Brock (1974).
2. These derivatives exist if  $u(\cdot)$  is twice continuously differentiable. See Oniki (1973).
3. The problem of multiple convergent equilibria in optimizing monetary models has also been discussed by Calvo (1979) and Gray (1984). In Fischer's (1979) model with capital accumulation, on the other hand, normality of consumption implies a unique convergent equilibrium path. It is also known that, when money seigniorage is used to finance deficits, the uniqueness and stability of equilibria strongly depend on the fiscal and monetary policy regimes [Bruno and Fischer (1987)].
4. Feenstra (1986) shows that an indirect utility function with such a property can be approximately derived from a model with cash-in-advance constraints.
5. Note that the restriction (17) is independent of  $\epsilon$ . Namely, if  $\{\epsilon h(t)\}$  is an admissible policy, then so is  $\{\epsilon' h(t)\}$  ( $\epsilon' \neq \epsilon$ ).
6. The initial response of inflation to unanticipated policy is obtained by first calculating  $\pi_{\epsilon}(T)$  using (25) and then substituting  $T = 0$ .
7. See Auernheimer (1974) for a treatment of unanticipated capital gains or losses in the analysis of seigniorage.
8. This class of utility functions includes, but is not restricted to, logarithmic utility.

## REFERENCES

- Auernheimer, Leonardo, 1974, "The Honest Government's Guide to the Revenue from the Creation of Money," *Journal of Political Economy*, 82, 598-606.
- Barro, Robert, 1979, "On the Determination of the Public Debt," *Journal of Political Economy*, 87, 940-971.
- Brock, William, 1974, "Money and Growth: The Case of Long Run Perfect Foresight," *International Economic Review*, 15, 750-777.
- Bruno, Michel and Stanley Fischer, 1987, "Seigniorage, Operating rules and the High Inflation Trap," National Bureau of Economic Research, Working Paper No. 2413.
- Calvo, Guillermo, 1979, "On Models of Money and Perfect Foresight," *International Economic Review*, 20, 83-103.
- Drazen, Allan, 1985, "Tight Money and Inflation: Further Results," *Journal of Monetary Economics*, 15, 113-120.
- and Elhanan Helpman, 1986, "Inflationary Consequences of Anticipated Macroeconomic Policies," National Bureau of Economic Research, Working Paper No. 2006.
- Feenstra, Robert, 1986, "Functional Equivalence between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics*, 17, 271-291.
- Fischer, Stanley, 1979, "Capital Accumulation on the Transition Path in a Monetary Optimizing Model," *Econometrica*, 47, 1433-1439.
- Gray, Jo Anna, "Dynamic Instability in Rational Expectations Models: An Attempt to Clarify," *International Economic Review*, 25, 93-122.
- Judd, Kenneth, 1985, "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model," *Journal of Political Economy*, 93, 298-319.
- , 1987, "Debt and Distortionary Taxation in a Simple Perfect Foresight Model," *Journal of Monetary Economics*, 20, 51-72.
- Liviatan, Nissan, 1984, "Tight Money and Inflation," *Journal of Monetary Economics*, 13, 5-15.
- Mankiw, N. Gregory, 1987, "The Optimal Collection of Seigniorage: Theory and Evidence," *Journal of Monetary Economics*, 20, 327-341.
- Obstfeld, Maurice, 1984, "Multiple Stable Equilibria in an Optimizing Perfect-Foresight Model," *Econometrica*, 52, 223-228.
- Oniki, Hajime, 1973, "Comparative Dynamics (Sensitivity Analysis) in Optimal Control Theory," *Journal of Economic Theory*, 6, 265-283.

Sargent, Thomas and Neil Wallace, 1981, "Some Unpleasant Monetarist Arithmetic," Federal Reserve Bank of Minneapolis, *Quarterly Review*, Fall, 1-17.

Sidrauski, Miguel, 1967, "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review, Papers and Proceedings*, 57, 534-544.

Weil, Philippe, 1987, "Permanent Budget Deficits and Inflation," *Journal of Monetary Economics*, 20, 393-410.

**ESSAY III**

**INTEREST RATES AND INFLATION:  
A NEW APPROACH**

## **1. Introduction**

The impact of inflation on interest rates has long attracted the attention of economists. The Fisher hypothesis states that the nominal interest rate should fully reflect any change in the expected rate of inflation, leaving the expected real interest rate to be determined solely by "real" variables.

Mishkin(1981), Summers(1983) and many others have dealt with the question of whether such a relation can be found in the data, and have overwhelmingly shown that the Fisher effect is far from being perfect.<sup>1</sup>

Most of the studies focusing on the Fisher effect have assumed constant equilibrium real interest rates. Such constancy cannot be derived from any reasonable model of market equilibrium, and therefore, tests based on such an assumption is likely to suffer from a potentially serious misspecification. As Summers points out, the *ex ante* real interest rate can be quite variable in a relatively standard model of macroeconomic equilibrium. The relationship between movements in nominal interest rates and inflation depends on the source of exogenous shocks. Although the long-run steady-state relationship would be simple and stable, the short-run relationship tends to be complex and seemingly unstable. And the long-run specification is clearly inadequate if we are to test this relationship using actual data, which do not represent steady states.

Another source of dissatisfaction with conventional tests of the Fisher relationship is that most of them ignore the risk associated with uncertainty in inflation. Given the volatility of inflation we experienced in the recent past, it is natural to ask whether inflation risk systematically affected the real interest rate. Fama(1976) tested the statistical significance of a measure of inflation variability and reported negative results. However, his measure does not distinguish between systematic and nonsystematic risk, and more importantly, it is based on the assumption that the Fisher effect is perfect. Bodie, Kane and McDonald(1985), using the mean-variance CAPM framework, suggest that inflation risk in short-term Treasury bill rates is likely to be negligible. They only calculate unconditional means and variances, however, and do not address the question of whether

conditional variances, which any meaningful measure of risk should involve, have varied over time.

In this essay, we consider a simple specification of the Fisher relationship derived from the consumption capital asset pricing model (CAPM), and test for its empirical relevance. The specification is valid both in the short run and in the long run, and incorporates the appropriate measure of systematic inflation risk, which is the conditional covariance of the rate of inflation with the rate of consumption growth. Tests for time-varying risk are carried out using, mainly, a bivariate version of the autoregressive conditional heteroscedasticity (ARCH) model developed by Engle(1982) and Bollerslev(1986). The next section develops a theoretical framework, and Section 3 discusses econometric methods and results. Concluding remarks are in Section 4.

## **2. The Fisher Hypothesis and the Consumption CAPM**

Consider a representative consumer who maximizes the expected utility of an intertemporal consumption stream by trading in frictionless asset markets. Assuming additive time-separability of utility over a single good, the problem for the consumer is to maximize  $E_t [\sum_i \beta^i U(C_{t+i})]$  subject to an intertemporal budget constraint, where  $\beta$  is the discount factor,  $U$  is a concave utility function,  $C_t$  is consumption in period  $t$ , and  $E_t$  is the conditional expectation operator given the information set  $(\Omega_t)$  at time  $t$ . Further assuming that  $U$  is isoelastic with a coefficient of relative risk aversion  $\gamma$ , the first-order condition is given by

$$\beta E_t [(C_{t+1}/C_t)^{-\gamma}(1 + R_t')(P_t/P_{t+1})] = 1$$

where  $R_t'$  is a one-period nominal rate of return from period  $t$  to  $t+1$  on any security and  $P_t$  is the



price of the consumption good in period  $t$ .<sup>2</sup> In the case of 1-period pure discount bonds,  $R_t'$  is known at time  $t$  so that

$$(1) \quad 1 + R_t' = \{\beta E_t [(C_{t+1}/C_t)^{-\gamma} (P_t/P_{t+1})]\}^{-1}$$

holds.

Although this equation cannot be interpreted as representing a causal relationship, it imposes a restriction on the joint stochastic process  $\{R_t', C_t, P_t\}$ . One way to exploit this restriction is to postulate that the rates of inflation and consumption growth follow a stationary VAR process, and then to see whether predictions from this VAR process explain the movements of  $R_t'$ . Let  $\pi_{t+1} = \log(P_{t+1}/P_t)$  and  $g_{t+1} = \log(C_{t+1}/C_t)$ . We assume that  $\pi_{t+1}$  and  $g_{t+1}$  are conditionally normally distributed, namely,

$$(2) \quad X_{t+1} | \Omega_t \sim N[E_t(X_{t+1}), \text{Var}_t(X_{t+1})]$$

where  $X_{t+1} = (\pi_{t+1}, g_{t+1})'$  and  $\text{Var}_t(\cdot)$  is the conditional variance(-covariance) operator. Then, from (1), we obtain a linear expression for the nominal (continuously compounded) interest rate:

$$(3) \quad R_t = -\log\beta + E_t(\pi_{t+1}) + \gamma E_t(g_{t+1}) \\ - (1/2)\text{Var}_t(\pi_{t+1}) - \gamma\text{Cov}_t(\pi_{t+1}, g_{t+1}) - (\gamma^2/2)\text{Var}_t(g_{t+1})$$

where  $R_t = \log(1 + R_t')$ . The nominal interest rate is the sum of the following components: (i) the discount rate, (ii) the expected rate of inflation, (iii) the expected rate of consumption growth times the coefficient of relative risk aversion, and (iv) the variance-covariance terms.

Note that  $E_t(\pi_{t+1})$  has a unit coefficient in (3), implying that the nominal interest rate, *ceteris paribus*, should fully reflect changes in the expected rate of inflation. We interpret this as representing the Fisher relationship. The third term is a well-known consequence of intertemporal substitution. Namely, consumption growth is expected to be large when the real interest rate is high. The variance of inflation,  $\text{Var}_t(\pi_{t+1})$ , appears not because of risk aversion, but simply because of the nonlinearity in the definition of the expected real rate of return. As pointed out by Fischer(1975) in an analogous continuous time framework, the correct measure of the expected real rate of return is  $R_t - E_t(\pi_{t+1}) + (1/2)\text{Var}_t(\pi_{t+1})$  when inflation is stochastic.

In equation (3), it is the covariance term that can be naturally interpreted as the risk premium due to inflation uncertainty. To see this, consider a real rate of return  $r_t'$  on riskless real bonds. The first-order condition associated with this security will be

$$(4) \quad r_t = -\log\beta + \gamma E_t(g_{t+1}) - (1/2)\gamma^2 V_t(g_{t+1})$$

where  $r_t = \log(1 + r_t')$ . Substituting (4) into (3) yields an expression for the expected real rate of return on nominal riskless bonds:

$$(5) \quad \log\{E_t[(1 + R_t)P_t/P_{t+1}]\} = R_t - E_t(\pi_{t+1}) + (1/2)\text{Var}_t(\pi_{t+1})$$

$$= r_t - \gamma \text{Cov}_t(\pi_{t+1}, g_{t+1})$$

Thus, the expected real rate of return on nominal riskless bonds exceeds the real riskless interest rate by  $-\gamma \text{Cov}_t(\pi_{t+1}, g_{t+1})$ . The intuition behind this is as follows. Suppose  $\text{Cov}_t(\pi_{t+1}, g_{t+1})$  is negative. Then, inflation is unexpectedly high, on average, when consumption growth is unexpectedly low. Such an asset is risky because its real return will be low in bad times. Hence, the expected return on this asset must be higher than on riskless real bonds. Equation (5) shows that the premium required to compensate for this risk is proportional to this covariance, where the proportionality factor is the coefficient of relative risk aversion. Strong risk aversion implies a large risk premium.

A special case of interest is where the conditional variance-covariance matrix is time-invariant. Then, (5) can be rewritten as

$$(6) \quad R_t = \alpha + E_t(\pi_{t+1}) + \gamma E_t(g_{t+1})$$

where  $\alpha$  includes the constant variance terms. Furthermore, if consumption follows an approximate random walk as Hall(1978) suggests, then the third term above can also be treated as a constant, and we have

$$(7) \quad R_t = \alpha' + E_t(\pi_{t+1})$$

where  $\alpha' = \alpha + \gamma E(g_{t+1})$ . This corresponds to the conventional specification of the Fisher

relationship, and the above argument makes clear the implicit assumptions behind it.

Various empirical studies have indicated that consumption growth, though not exactly a white noise, is difficult to predict. Namely, its predictable movements are limited in magnitude. This observation suggests that testing the constancy of the conditional variance-covariance structure would be a natural strategy for detecting a possible misspecification in (7). The degree to which the variance terms matter depends on the coefficient of relative risk aversion ( $\gamma$ ). Table 1 shows the values of  $(1/2)\text{Var}_t(\pi_{t+1} + \gamma g_{t+1})$  for plausible conditional distributions and for  $\gamma = 2$  and  $\gamma = 5$ .

The two sets of distributional parameters make a difference of almost 5 percentage points if  $\gamma = 5$ , implying a potentially significant role for a time-varying variance in causing variations in the expected real rate of interest. On the other hand, the same experiment leads to a difference of only slightly more than 1 percentage point if  $\gamma = 2$ .

Examining changes in the conditional variance-covariance matrix provides a simple way of relating short-run and long-run relationships between interest rates and inflation. It is often maintained that the Fisher relationship should hold between long-run averages of interest rate and inflation. Summers(1983), for example, tried to circumvent the problem of short-run instability by running band-spectrum regressions, filtering out high-frequency variations. From the equation (3), one can see that Summers' procedure can be justified if the conditional variance-covariance terms are stationary and if their movements are mostly in high frequencies. In that case, most of the variations in these "extra" terms are filtered out by his method, and the specification (7) will be approximately correct. It would also be testable provided the expected rate of inflation is not constant. On the other hand, if movements in these terms are largely in low frequencies, then band spectrum regressions are not free from bias. The rest of the essay attempts to estimate systematic movements in the conditional variance-covariance terms in the hope of deciding which of the above two cases is a better description of the data.

### **3. Tests for Time-Varying Risk**

Any statistical inference on the stochastic process generating  $\text{Var}_t(\pi_{t+1})$ ,  $\text{Cov}_t(\pi_{t+1}, g_{t+1})$  and  $\text{Var}_t(g_{t+1})$  requires estimation of forecast equations for  $\pi_{t+1}$  and  $g_{t+1}$  as a first step. We use vector autoregressions (VAR) for this purpose, and then apply the autoregressive conditional heteroscedasticity (ARCH) model for tracking the evolution of the conditional variance-covariance matrix of the error terms.<sup>3</sup> It should be noted that, for this procedure to be economically meaningful, the VAR predictions have to be a good approximation to the representative consumer's expectations.

The linear VAR-ARCH model used here has the following general form:

$$(8a) \quad \Phi(L)X_t = \varepsilon_t$$

$$(8b) \quad \text{Var}_t(\varepsilon_{t+1}) = A_0 + \varepsilon_t A_1 \varepsilon_t' + \dots + \varepsilon_{t-n} A_n \varepsilon_{t-n}'$$

where  $\Phi(L)$  is a matrix of lag polynomials,  $\varepsilon_t$  is a vector martingale difference with a conditional normal distribution, and  $A_i$  ( $i=1, \dots, n$ ) are  $2 \times 2$  symmetric matrices.

The ARCH model has an intuitive appeal since it incorporates the notion that, if large forecast errors have been observed in the recent past, one would expect the next-period forecast variance to be also large. With the ARCH formulation, it is easy to measure the persistence of shocks to the conditional variances and to verify stationarity of the variance-covariance terms. Also, it enables one to avoid theorizing the evolution of conditional variances, which would be quite difficult unless very restrictive assumptions are made. The ARCH model does not require any such theory.

Furthermore, its effectiveness has been demonstrated by several studies, including Engle(1983) and Bollerslev, Engle and Wooldridge(1986).

Estimation was carried out using seasonally adjusted quarterly per-capita consumption of (i) nondurable goods, (ii) services and (iii) the sum of these two. Mid-quarter-month data were used to minimize the effect of the potential durability of consumption. A corresponding consumption deflator was used for each of the categories. The interest rate used is the mid-quarter monthly average of the rate of return on 3-month Treasury bills traded in the secondary market. Note that estimation using the nondurables data only, for example, is justified if and only if utility is additively separable over nondurables and other goods. For general non-separable utility functions with many goods, a simple relationship such as (3) does not hold, although it may not be a bad approximation.<sup>4</sup>

First, the forecast equations for the  $\pi_t$  and  $g_t$  were chosen by searching for a specification that minimizes the Akaike Information Criterion (AIC), which is defined as  $-2(\log \text{likelihood}) + 2(\text{number of parameters})$ .<sup>5</sup> Joint maximum likelihood estimation was carried out ignoring potential heteroscedasticity. Rewrite (8a) as

$$(9a) \quad \pi_t = \Phi_{11}(L)\pi_t + \Phi_{12}(L)g_t + \varepsilon_{1t}$$

$$(9b) \quad g_t = \Phi_{21}(L)\pi_t + \Phi_{22}(L)g_t + \varepsilon_{2t}$$

where the orders of the polynomials  $\Phi_{11}(L)$ ,  $\Phi_{12}(L)$ ,  $\Phi_{21}(L)$ ,  $\Phi_{22}(L)$  are a, b, c and d, respectively. Minimizing the AIC yielded  $(a, b, c, d) = (4, 0, 1, 1)$  for nondurables and nondurables plus services, and  $(3, 0, 1, 2)$  for services. Table 2 summarizes the results. Lagged consumption growth can be seen to have no significant explanatory power for inflation, and thus,

inflation is essentially a univariate AR process. Consumption growth can be forecast with short lags of its own past values and inflation.

Provided the regularity and symmetry conditions of Engle(1982) are satisfied, the information matrix is block diagonal, implying that the estimation of the VAR forecast equations and the ARCH equations can be considered separately without loss of asymptotic efficiency. In particular, the ARCH coefficients can be estimated with full efficiency using any consistent estimates of the VAR parameters. We restrict the matrices  $A_i$  in (8b) to be diagonal, so that it can be rewritten as

$$(10a) \quad \text{Var}_{t-1}(\varepsilon_{1t}) = \theta_0 + \theta_1 \varepsilon_{1,t-1}^2 + \dots + \theta_4 \varepsilon_{1,t-n}^2$$

$$(10b) \quad \text{Var}_{t-1}(\varepsilon_{2t}) = \omega_0 + \omega_1 \varepsilon_{2,t-1}^2 + \dots + \omega_4 \varepsilon_{2,t-n}^2$$

$$(10c) \quad \text{Cov}_{t-1}(\varepsilon_{1t}, \varepsilon_{2t}) = \lambda_0 + \lambda_1 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + \dots + \lambda_4 \varepsilon_{1,t-n} \varepsilon_{2,t-n}$$

for the n-th order ARCH model. We would have to constrain the coefficients  $\theta_i$  and  $\omega_i$  to be nonnegative if we want to ensure that they are never negative. Doing so is also necessary in order to satisfy Engle's regularity conditions. One easy way to avoid negative variance forecasts is to use an exponential function formulation, namely,

$$(11a) \quad \text{Var}_{t-1}(\varepsilon_{1t}) = \exp [\theta_0 + \theta_1 \varepsilon_{1,t-1}^2 + \dots + \theta_4 \varepsilon_{1,t-4}^2]$$

$$(11b) \quad \text{Var}_{t-1}(\varepsilon_{2t}) = \exp [\omega_0 + \omega_1 \varepsilon_{2,t-1}^2 + \dots + \omega_4 \varepsilon_{2,t-4}^2].$$

This modification is unnecessary for the covariance equation, which can take negative values. Both

linear and exponential specifications were estimated, using the residuals obtained from the estimation of (9), where nonnegativity constraints were ignored for the estimation of the linear form (10).

We are interested in testing the hypothesis of no ARCH for each of the variances and the covariance. Namely, the null hypotheses are

$$H_1: \theta_i = 0, \quad i = 1, \dots, n$$

$$H_2: \omega_i = 0, \quad i = 1, \dots, n$$

$$H_3: \lambda_i = 0, \quad i = 1, \dots, n.$$

Since the model (9) has homoscedastic errors under the null, the Lagrange multiplier test, as in Breusch and Pagan(1978), is asymptotically equivalent to the likelihood ratio test. It can be implemented simply by calculating the sample size  $T$  times  $R^2$  from regressions (10) or (11), which is asymptotically distributed as  $\chi^2$  with  $n$  degrees of freedom under the null hypothesis. Table 3 and 4 give the test statistics for the first- and forth-order ARCH model ( $n = 1, 4$ ), along with the critical values of  $\chi^2$  for 5, 10 and 25 percent significance levels. None of the above hypotheses can be rejected at any reasonable levels of significance.

The results show that the conditional variance-covariance structure does not vary over time in the way postulated by the ARCH model. Other functional forms were also experimented with, but the test statistics came nowhere close to rejecting the null hypothesis of no ARCH. It is still possible, however, that heteroscedasticity which does not take the form of ARCH exists. To test for this possibility, White's(1980) direct test for general heteroscedasticity was performed for each of the variances. This test is designed to detect any heteroscedasticity which invalidates use of the usual



standard errors, and has the advantage that a particular form of heteroscedasticity need not be specified. The asymptotic distribution of the test statistics is again the  $\chi^2$  distribution. As can be seen in Table 5, the null hypothesis of homoscedasticity cannot be rejected even at the 25 percent significance level. Thus, both the ARCH model and White's test find no evidence at all against a constant variance-covariance structure.

If these results are accepted, two implications immediately follow. First, the failure of the Fisher relationship to hold has little to do with time-varying risk. In other words, omitting the conditional variance terms is not a source of misspecification. We have to look for something else. Second, the results suggest that frequent rejections of the consumption CAPM (see Hansen and Singleton(1983), for example) may be closely related to the failure of the Fisher relationship. The specification (6) of the consumption CAPM has now turned out to be correct, but it is not very different from the conventional formulation of the Fisher relationship (7), given the small variability of the  $E_t(g_{t+1})$  term. Thus, if the coefficient on  $E_t(\pi_{t+1})$  in (7) is substantially different from unity, we can expect that the consumption CAPM would have a very small chance of success.

The tests carried out above, however, do not escape some fundamental difficulties. We have made the assumption that the information set of the representative consumer can be well approximated by the VAR formulation. This is a very restrictive assumption, and alternative ways of forecasting should be considered before strong conclusions are drawn. Using commercial forecasts seems to be one reasonable alternative. Another obvious problem is that true consumption is not observable. We can only observe consumption expenditures, and there is no exact correspondence between the two since most goods and services have some durable characteristics. Data for short-term changes in consumption may also suffer from nonnegligible measurement errors.



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## **NOTES**

- 1. Fama's(1975) result to the contrary is now widely believed to be an artifact of the choice of sample period.**
- 2. This equation can easily be turned into a relationship between the expected return on an asset and the covariance of the asset's return with the marginal rate of substitution (between current and future consumption). In this sense, our framework is a discrete-time version of Breeden's(1979) consumption CAPM.**
- 3. This two-step procedure can be iterated to yield a maximum likelihood estimator. See Engle(1982).**
- 4. Breeden(1986) derives a similar relationship for a Cobb-Douglas utility function with many goods in a continuous time framework.**
- 5. See, for example, Neftci(1982) for a discussion of the AIC.**

## REFERENCES

- Bodie, Zvi, Alex Kane and Robert McDonald, "Inflation and the Role of Bonds in Investor Portfolios," in Benjamin M. Friedman, ed., *Corporate Capital Structures in the United States*, Chicago, The University of Chicago Press, 1985.
- Bollerslev, Tim, "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, 31(1986), 307-327.
- , Robert F. Engle and Jeffrey M. Wooldridge, "A Capital Asset Pricing Model with Time Varying Covariances," mimeo., 1986.
- Breedon, Douglas T., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7(1979), 265-296.
- , "Consumption, Production, Inflation and Interest Rates: A Synthesis," *Journal of Financial Economics*, 16(1986), 3-39.
- Breusch, Trevor S. and Adrian R. Pagan, "A Simple Test for Heteroscedasticity and Random Coefficient Variation," *Econometrica*, 46(1978), 1287-1294.
- Engle, Robert F., "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50(1982), 987-1007.
- , "Estimation of the Variance of U.S. Inflation Based on the ARCH Model," *Journal of Money, Credit and Banking*, 15(1983), 286-301.
- Fama, Eugene F., "Short-Term Interest Rates as Predictors of Inflation," *American Economic Review*, 65(1975), 269-282.
- , "Inflation Uncertainty and Expected Returns on Treasury Bills," *Journal of Political Economy*, 83(1976), 427-448.
- Fischer, Stanley, "The Demand for Index Bonds," *Journal of Political Economy*, 83(1975), 509-534.
- Hall, Robert E., "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86(1978), 971-988.
- Hansen, Lars P. and Kenneth J. Singleton, "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, 91(1983), 249-265.
- Mishkin, Frederic S., "The Real Interest Rate: An Empirical Investigation," in K. Brunner and A. H. Meltzer ed., *Carnegie-Rochester Conference Series on Public Policy*, Vol. 15, 1981.
- Neftci, Salih N., "Specification of Economic Time Series Models Using Akaike's Criterion," *Journal of the American Statistical Association*, 77(1982), 537-540
- Summers, Lawrence H., "The Non-Adjustment of Nominal Interest Rates: A Study of the Fisher Effect," in J. Tobin ed., *Macroeconomics, Prices and Quantities*, Washington, The Brookings

**Institution, 1983.**

**White, Halbert, "A Heteroscedasticity Consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity," *Econometrica*, 48(1980), 817-837.**

**TABLE 1**Values of  $(1/2)\text{Var}_t(\pi_{t+1} + \gamma g_{t+1})$ 

$\gamma$	$\sigma_\pi$	$\sigma_g$	$\rho$	$(1/2)\text{Var}_t(\pi_{t+1} + \gamma g_{t+1})$
5	0.05	0.06	0.5	5.38 %
5	0.01	0.02	- 0.5	0.46 %
2	0.05	0.06	0.5	1.15 %
2	0.01	0.02	- 0.5	0.07 %

Note:  $\sigma_\pi = [\text{Var}_t(\pi_{t+1})]^{1/2}$  (standard deviation of the rate of inflation)

$\sigma_g = [\text{Var}_t(g_{t+1})]^{1/2}$  (standard deviation of the rate of consumption growth)

$\rho = \text{Cov}_t(\pi_{t+1}, g_{t+1})/\sigma_\pi\sigma_g$  (correlation coefficient)

**TABLE 2**

Estimated coefficients in VAR forecast equations (ML)

	(i) Nondurables	(ii) Services	(iii) Nondurables + Services
<b>[A] Dependent variable: <math>\pi_t</math></b>			
constant	0.011 (2.17)	0.005 (1.64)	0.007 (1.85)
$\pi_{t-1}$	0.404 (4.17)	0.463 (4.99)	0.469 (4.78)
$\pi_{t-2}$	0.259 (2.68)	0.284 (2.89)	0.240 (2.36)
$\pi_{t-3}$	0.354 (3.65)	0.167 (1.85)	0.322 (3.17)
$\pi_{t-4}$	- 0.244 (2.54)	-----	- 0.162 (1.68)
<b>[B] Dependent variable: <math>g_t</math></b>			
constant	0.031 (4.62)	0.039 (5.38)	0.035 (6.07)
$\pi_{t-1}$	- 0.246 (2.46)	- 0.169 (2.22)	- 0.198 (2.40)
$g_{t-1}$	- 0.245 (2.48)	- 0.244 (2.52)	- 0.160 (1.60)
$g_{t-2}$	-----	0.180 (2.00)	-----

Note: t-statistics are in parentheses. Sample period: 1961.I - 1985.III.

**TABLE 3**  
 $\chi^2$  test statistics for the first-order ARCH model

	(i) Nondurables	(ii) Services	(iii) Nondurables + Services
<b>[A] <math>\text{Var}_t(\pi_{t+1})</math></b>			
Linear	0.03	0.05	0.45
Exponential	0.04	0.04	0.49
<b>[B] <math>\text{Var}_t(g_{t+1})</math></b>			
Linear	0.39	0.0	0.22
Exponential	0.45	0.0	0.22
<b>[C] <math>\text{Cov}_t(\pi_{t+1}, g_{t+1})</math></b>			
Linear	0.02	0.02	0.44

Note: The critical values are 3.84, 2.71 and 1.32 for the significance levels 5, 10 and 25 percent, respectively. Sample period: 1962.I - 1985 III.

**TABLE 4**  
 $\chi^2$  test statistics for the fourth-order ARCH model

	(i) Nondurables	(ii) Services	(iii) Nondurables + Services
<b>[A] <math>\text{Var}_t(\pi_{t+1})</math></b>			
Linear	2.98	0.23	2.05
Exponential	4.58	0.22	1.72
<b>[B] <math>\text{Var}_t(g_{t+1})</math></b>			
Linear	1.07	4.00	0.57
Exponential	0.94	4.89	0.48
<b>[C] <math>\text{Cov}_t(\pi_{t+1}, g_{t+1})</math></b>			
Linear	2.16	1.03	1.84

Note: The critical values are 9.49, 7.78 and 5.39 for the significance levels 5, 10 and 25 percent, respectively. Sample period: 1962.I - 1985 III.

**TABLE 5**

**White's test for general heteroscedasticity**

	$\text{Var}_t(\pi_{t+1})$	$\text{Var}_t(g_{t+1})$
(i) Nondurables	7.53	0.84
(ii) Services	8.81	4.86
(iii) Nondurables + Services	12.37	5.67
$\chi^2_{0.10}$	21.1	14.7
$\chi^2_{0.25}$	17.1	11.4
Degree of freedom	14	9

**Sample period: 1962.I - 1985.III.**