16.90 Spring 2013
Final Exam

I certify that:

- Prior to taking this exam, I did not discuss the content of the exam with anyone that had already taken it.
- In the 30 minute preparation period for this exam, I did not use any resources in preparing my response.
- After taking this exam, I will not discuss the content of the exam with anyone until after receiving an email from the instructors that it is acceptable to do so.

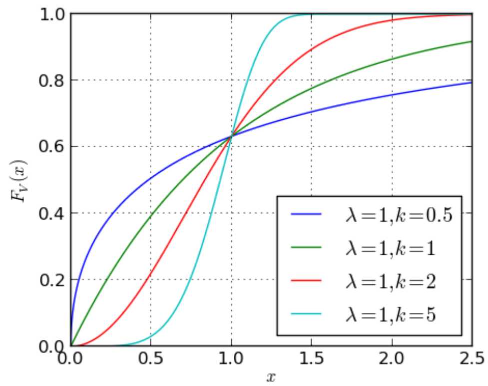
Signature:

Please bring this signed form to the oral, along with any notes you generated in the 30 minute preparation period. You can use these notes in your oral response.

Question 1

In designing the layout of an off-shore wind farm, we need to analyze the performance of each candidate design by computing the expected power generation rate under uncertain wind speed and uncertain wind direction. You are given a computational fluid dynamics (CFD) simulation code that can compute the power generation rate of a candidate design at any given wind speed and direction. Your job is to use a Monte Carlo method to compute the expected power generation rate.

(a) Horns Rev 1 wind farm, Denmark



(b) Public domain image, from Wikipedia.

(a) The uncertain wind speed V is described by a Weibull distribution, whose cumulative distribution function (CDF) is

$$F_V(x) = 1 - e^{-(x/\lambda)^k}$$

where λ and k are calibrated from historic meteorological data at the wind farm site. The uncertain wind direction Θ is described by a uniform $[0, 2\pi]$ distribution, and is independent of V. Discuss how one can generate random samples of V and Θ for the purpose of Monte Carlo simulation.

(b) The CFD simulation code is run at N randomly sampled pairs of V and Θ. The computed power generation rates are $P_1, P_2, \ldots, P_N$. What is your estimate of the expected power generation rate?

(c) Continuing on Question (b), how would you ascertain that the expected power generation rate lies within $\pm X$ Watts of your estimated value with 95% confidence?

(d) The CFD simulations are time consuming. Discuss how you can construct a response surface model (RSM). How can the RSM be used in estimating the expected power generation rate? (Note: Open ended discussion)

Question 2

Consider the advection equation:

$$\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} = 0 \;, \tag{1}$$

in a periodic domain of $x \in [0, 2\pi]$. Suppose the domain is discretized into N grid points

$$x_i = i\Delta x \;, \quad i = 1, \ldots, N \;, \quad \text{where} \quad \Delta x = \frac{2\pi}{N}$$

and the equation is discretized with the following finite difference scheme

$$\frac{\hat{w}_i^{(k+1)} - \hat{w}_i^{(k)}}{\Delta t} + U\frac{\hat{w}_{i+1}^{(k)} - \hat{w}_{i-1}^{(k)}}{2\Delta x} = \kappa\frac{\hat{w}_{i+1}^{(k)} + \hat{w}_{i-1}^{(k)} - 2\hat{w}_i^{(k)}}{\Delta x} \tag{2}$$

for some constant $\kappa > 0$, where $\hat{w}_i^{(k)}$ is the numerical solution at $x = x_i$ and $t = k\Delta t$. The initial condition $\hat{w}_i^{(0)}, i = 1, \ldots, N$ is given. The periodic boundary condition is enforce by $\hat{w}_{N+1}^{(k)} \equiv \hat{w}_1^{(k)}$ and $\hat{w}_0^{(k)} \equiv \hat{w}_N^{(k)}$.

(a) Is this a consistent scheme? Why or why not?

(b) Use von Neumann stability analysis to determine the amplification factors of this scheme for any given U, κ Δt and Δx.

(c) [Bonus] What is the maximum Δt at which the scheme is stable for any given combination of U, κ and Δx?