16.90 Spring 2013
Midterm Exam

I certify that:

- Prior to taking this exam, I did not discuss the content of the exam with anyone that had already taken it.
- In the 30 minute preparation period for this exam, I did not use any resources in preparing my response.
- After taking this exam, I will not discuss the content of the exam with anyone until after receiving an email from the instructors that it is acceptable to do so.

Signature:

Please bring this signed form to the oral, along with any notes you generated in the 30 minute preparation period. You can use these notes in your oral response.

Question 1

This problem is a detective game. I have implemented a well-known ODE time-integration method in Matlab to solve the following problem:

$$\frac{du}{dt} = \begin{pmatrix} a & 1 \\ -1 & b \end{pmatrix} u \,, \qquad \text{with initial condition} \qquad u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \,, \tag{1}$$

for real values a and b. You will be able to command me to run the script for any combinations of a, b, time step size Δt, and the number of time steps N. You can then ask me to plot and manipulate the numerical solution in any way you want. You just cannot look at the source code. Your job is to identify what the method is. You also need to explain the justification of your answer.

Question 2

Traffic flow on freeways can be modeled as the following partial differential equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial U\rho}{\partial x} = 0 \tag{2}$$

where the solution $\rho(x,t)$ is the density of cars. It represents the number of cars per unit length of the freeway. U is the speed of the cars and is assumed to be a given constant in this problem. We want to solve the partial differential equation for some given initial condition and boundary condition $\rho(x=0,t) = \rho_{in}(t)$ using numerical method. Suppose the domain $0 < x < L$ is subdivided into N cells (or equivalently, N elements).

1. Describe how you would formulate and solve this problem using a Finite Difference method. What do the unknowns represent? How are these unknowns determined?

2. Explain how the problem formulation and solution approach would be different using a Finite Volume method.

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