> Welcome to 16.90 iSession ...

- Instructor: Turn on **Webex** ,

  and distribute **MuddyCards** ...

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...etc...

...etc...

Review and outlook

- Local accuracy (including consistency)
- Global accuracy $V(t) \xrightarrow{\Delta t \to 0} u(t)$ what we want
- Zero stability $\frac{du}{dt} = 0$
- Eigenvalue stability

$$\frac{du}{dt} = \lambda u$$

- Newton Raphson iteration

easy

easy

Eigenvalue stability analysis

- http://math.mit.edu/mathlets/mathlets/eigenvalue-stability/

$$\frac{V^{n+1} - V^{n}}{\Delta t} = f(V^{n}) = \lambda V^{n}$$

$$\frac{V^{n+1}}{\Delta t} = \frac{V^{n}}{\Delta t} + \lambda V^{n}$$

$$V^{n+1} = (1 + \Delta t \lambda) V^{n}$$

Eigenvalue stability analysis

- http://math.mit.edu/mathlets/mathlets/eigenvalue-stability/
- Discuss stability for Forward Euler, Backwards Euler and Midpoint Rule, for the ODE:

$$\frac{dx}{dt} = \lambda x \qquad x = e^{\lambda t}$$

where λ can take any complex value

- For what λ and Δt is each scheme stable?

Eigenvalue stability analysis

- http://math.mit.edu/mathlets/mathlets/eigenvalue-stability/
- Discuss stability for Forward Euler, Backwards Euler and Midpoint Rule, for the coupled ODEs:

$$\frac{dx}{dt} = -y - \epsilon x$$

$$\frac{dy}{dt} = x$$

  where ϵ range from 0 to ∞
- For what ϵ and Δt is each scheme stable?

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\varepsilon & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$v^{n+1} = v^{n-1} + 2\Delta t \underbrace{f(v^{n})}_{\lambda v^{n}} \qquad \frac{dy}{dt} = \lambda y$$

$$v^{(n+1)} = v^{(n-1)} + 2\Delta t\, \lambda v^{(n)}$$

$$v^{(n+1)} = v^{(0)} z^{n+1} \qquad v^{(n)} = v^{(0)} z^{n} \qquad v^{(n-1)} = v^{(0)} z^{n-1}$$

$$z^{n+1} = z^{n-1} + 2\Delta t\, \lambda z^{n}$$

$$\boxed{z^{2} = 1 + 2\Delta t\, \lambda z} \qquad z = \frac{2\lambda\Delta t \pm \sqrt{4\lambda^{2}\Delta t^{2} + 4}}{2}$$

$$z = e^{i\theta}$$

$$e^{2i\theta} = 1 + 2\underbrace{(\alpha+\lambda)}\cdot e^{i\theta}$$

Newton Raphson for implicit scheme

$$\frac{du}{dt} = -u^2 \qquad u(0) = 1$$

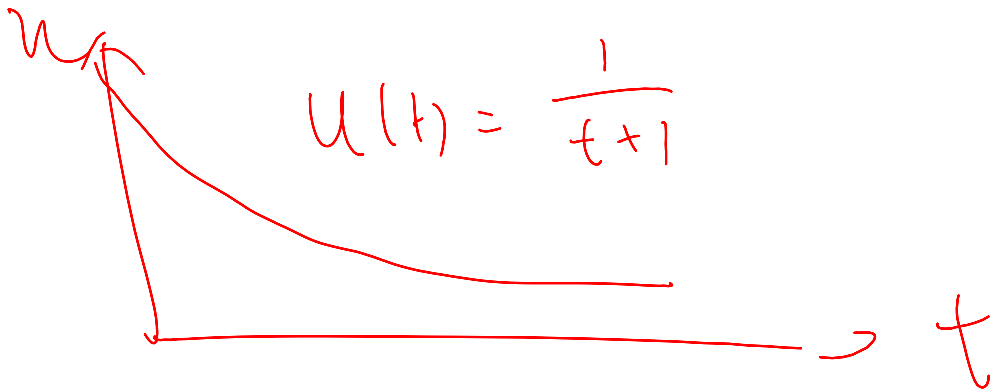
$\underset{\sim\sim}{u} \longrightarrow \underset{\sim\sim\sim}{u + v}$ where v is very small

$$\frac{d(u+v)}{dt} = -(u+v)^2$$

$$\frac{du}{dt} + \frac{dv}{dt} = -u^2 - 2uv - v^2 \quad \text{very very small}$$

$$\frac{dv}{dt} = -2u \cdot v$$

When is FE stable?



$$u(t) = \frac{1}{t+1}$$

$$-2 \le -2u\Delta t \le 0 \quad \text{for FE.}$$

$$\frac{dy}{dt} = -u^2 \qquad \text{BE?}$$

$f(u^n)$ Forward Euler

$$\frac{u^{n+1} - u^n}{\Delta t} = f(u^{n+1}) \quad \text{Backward Euler}$$

$$= -\left(u^{n+1}\right)^2$$

$$\frac{u^{n+1} - u^{n}}{\Delta t} = -\left(u^{n+1}\right)^2$$