AN ADVANTAGE MODEL OF RISKY CHOICE

by

ELDAR B. SHAFLIR

B.A., Cognitive Science
Brown University
[1984]

SUBMITTED TO THE DEPARTMENT OF
BRAIN AND COGNITIVE SCIENCES
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1988

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Signature of Author ____________________________
Department of Brain and Cognitive Sciences
July 28, 1988

Certified by ____________________________
Daniel N. Osherson
Professor of Brain and Cognitive Sciences
Thesis Supervisor

Accepted by ____________________________
Emilio Bizzi
Head, Department of Brain and Cognitive Sciences
AUG 22 1988

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Submitted to the Department of
Brain and Cognitive Sciences on July 28, 1988
in partial fulfillment of the requirements
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Cognitive Science

ABSTRACT

A descriptive model of choice between monetary lotteries --
called the Advantage Model of Choice -- is proposed. According to
the model, people compare lotteries separately on the dimension of
gains and on the dimension of losses. In making these comparisons,
people employ both "absolute" and "comparative" strategies that
are subsequently combined to yield a choice.

The model is evaluated on both qualitative and quantitative
grounds, and compared to two popular alternative theories of risky
choice: Prospect Theory and Utility Theory. As part of the
qualitative evaluation, a number of well documented phenomena are
reviewed, that characterize people's choices between lotteries. It
is shown that only the Advantage Model is consistent with all the
phenomena.

As part of the quantitative evaluation, three experimental
tests of the model are reported, involving both "simple" and
"mixed" lotteries. In the context of these lotteries, the model
appears superior to both Prospect Theory and Utility Theory in
predicting group preference, and generally more successful in
predicting individual choice.

It is suggested that the Advantage Model captures one of the
underlying processes that guide human choice behavior in risky
situations. Examples of the model's relevance to nonmonetary
domains are provided.

Thesis Supervisor: Dr. Daniel N. Osherson
Title: Professor of Brain and Cognitive Sciences
IN MEMORY OF

RICHARD B. MILLWARD
Acknowledgements

I thank M.I.T.'s Department of Brain and Cognitive Sciences and the Center for Cognitive Science for four memorable years. They, together with related academic departments and a diverse affiliated community, have provided a wonderful environment in which to become acquainted with Cognitive Science at its best. The research reported here was further supported by NSF Grants #8609201 and #8705444.

I especially thank Dan Osherson for being a superb advisor, colleague, and friend. Stepping into Dan's office that September morning, 1984, has been undoubtedly the most fortunate decision of my graduate career. Next to Dan, no one deserves more thanks than my second advisor and dear friend, Ed Smith. The intellect, warmth, and dedication of Dan and Ed -- whether in Paris, Ann Arbor, Concord, or Cambridge -- have made my graduate life exceptional. My appreciation for them both is profound.

I also thank the other members of my committee, Sue Carey and Molly Potter, for an impressively careful reading of the dissertation, and for helpful comments then and at other times. Thanks also to Steve Pinker who was always willing to listen and help.

Special thanks go to Noam Chomsky for his interest and support, as well as for providing immense inspiration throughout the years.

My work was greatly influenced by that of Amos Tversky, who I thank for his time and encouragement during my months spent at Stanford.

Jan Ellertsen was of enormous help during my stay at M.I.T. Paul Kanla was essential to conducting the University of Michigan experiments, and Jigna Desai did a nice job on some of the tables and graphs. Sherri Seymour, Kate Whidden, Kathy Murphy, Pat Claffey, and Deb Keehn, were always willing to help in moments of difficulty. I thank them all.

Thanks to Mimi, Phil, and Eric for their care and for providing a home in America. And to Anne, Benny, and Yolande for their warm hospitality and friendship.

Special thanks to Noemi and Bernard whose love and generosity have made much of this possible, to my father who has maintained an active interest in my research and thought of how it could be made better, and to my mother and brother for always being there.

To dear friends: Ron, Udi, Yossi, Larry, Laurie, David L., David W., Beth, and John. They all contributed enormously to making these years more stimulating and fun.

With much love to my parents, Sharoni, Edja, Genia, Menachem, Noemi, Bernard, Ivonne, Roger, Daphne, Yossi, Aya, and Shir. Their love and support were vital to my ability to go on.

Finally, to Amy, who advised and encouraged and amused and simply made all the difference, and to whom this work is dedicated with love.
More than any other time in history, mankind faces a crossroads. One path leads to despair and utter hopelessness. The other, to total extinction. Let us pray we have the wisdom to choose correctly.

Woody Allen
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Chapter 1

Introduction

1.1 Risky choice in the cognitive sciences

The need to make choices and decisions in the face of uncertain outcomes permeates everyday life. The theories we adopt to explain how choices and decisions are made occupy a central role in our conception of human rationality. In light of this, the study of choice and decision making under uncertainty has come to occupy an important place in the social sciences in general, and in the cognitive sciences in particular.

Students of rationality in the cognitive sciences have conducted their studies roughly along the lines outlined in the tree of Figure 1.1. Each leaf in the tree represents a rich field of research. On the one hand is the study of how people decide what is true. This breaks down roughly into inductive and deductive reasoning. The study of inductive reasoning investigates 1) how people judge the probability that certain statements are true (see Kahneman, Slovic, and Tversky, 1982), and 2) how they judge that certain arguments are strong (see, e.g., Rips, 1975; Osherson, Smith, and Shafir, 1986; as well as Wilke et al., 1988). The study of deductive reasoning focuses on people's judgments that certain
Figure 1.1: The Tree of Rationality

Statements are necessarily true and that certain arguments are valid (see, e.g., Osherson, 1975; Johnson-Laird, 1975; Rips, 1983).

On the other branch of the tree is the study of how people choose what is good. This breaks down into choice under certainty, where you are certain to obtain the option you choose (see, e.g., Tversky, 1972; Luce, 1959), and choice under uncertainty, where there is some chance that you will not obtain the chosen option. Choice under uncertainty breaks down
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further into ambiguous choice, where the chances of obtaining the option you choose are unclear (Ellsberg, 1961; Einhorn and Hogarth, 1985), and risky choice, where you know exactly what the chances are of obtaining the chosen option. Finally, of course, any choice problem may involve monetary or nonmonetary outcomes.

Partly due to its affinity with questions of decision making in economics, the bulk of work on risky choice has been concerned with monetary outcomes. Although choice paradigms of more elaborate structure are needed to represent the variety of decisions typically encountered, researchers have studied extensively the behavior of subjects presented with choices between monetary lotteries. Such lotteries seem to represent "things desirable" while to some extent avoiding individual, idiosyncratic differences that might arise when other kinds of awards (e.g., "a trip to Europe") are used. The present paper will remain largely within that tradition, focusing mainly on the bottom right-most leaf of the rationality tree, namely, on risky choice involving monetary outcomes.

There are a number of reasons for the present focus. First, as pointed out by a number of investigators (see, e.g., Huber, 1980; Tversky and Kahneman, 1981), the most comprehensive version of a choice problem includes 1) the alternatives among which one must choose, 2) the values attached to their outcomes by the decision maker, and 3) the contingencies, or probabilities that relate the alternatives' outcomes to their having been chosen. Risky choice involving monetary outcomes captures all three of these basic types of information that characterize choice and decision problems. As a consequence -- and this is a second
reason for the present focus -- it has been one of the most active areas of research into human choice and decision making. Finally, understanding how individuals make choices under risk may have direct relevance for improving decisions in business (Libby and Fishburn, 1977), public policy (Slovic, 1978), and medicine (McNeil et al., 1982). Although the bulk of the present paper deals with monetary outcomes, we return to a discussion of choice in nonmonetary domains in the concluding chapter.

1.2 Absolute and comparative theories of choice

In an attempt to explain choices among monetary lotteries, economists have proposed theories largely influenced by normative considerations. Several kinds of choice problems have been described, however, in which people's preferences systematically violate the axioms of the theories proposed. (In later chapters, we will have an opportunity to review some of the proposed theories and their descriptive failures.) As a consequence, it has been argued that the normatively inspired theories are inadequate as descriptive models, and a search for alternative, descriptive accounts of human choice behavior has ensued. One such alternative account is proposed in the present paper.

Consider then a choice between one lottery that offers a 50 percent chance to win $2000 and another that offers a 75 percent chance to win $1000. We may distinguish two approaches to understanding the preferences exhibited by naive subjects faced with such choice problems (Tversky, 1969). These approaches will be called "absolute" and "comparative". Both proceed by assigning a hypothetical attractiveness coefficient to each
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lottery in a given choice problem, and predicting that the lottery assigned the numerically higher coefficient will be preferred. Within the absolute approach the attractiveness of a lottery is assumed to be independent of the alternative with which it is paired. In contrast, theories within the comparative approach evaluate the attractiveness of lotteries only in the context of a specified choice problem, the attractiveness of a given lottery depending on the alternative to which it is compared. The present paper offers a model of choice that combines the two approaches.

The most influential example of an absolute theory is the Theory of Expected Utility (von Neumann and Morgenstern, 1947). This theory is discussed in some detail in Chapter 2. Within Expected Utility Theory the attractiveness of a lottery \((d, p)\) that offers a less-than-certain chance \(p\) to win a specified sum of money \(d\) is given as \(u(d) \cdot p\), where \(u\) is an empirically estimated function from monetary assets to real numbers (called "utilities"). Observe that Utility Theory must be supplemented with external assumptions about the form of the function \(u\) before it can be used to make predictions about subjects' choices between a given pair of lotteries. Thus, \(u\) in Expected Utility Theory may be conceived as a free function-parameter. Notwithstanding this freedom in the specification of \(u\), it is widely appreciated (and reviewed extensively in Chapter 4) that the Theory of Expected Utility provides an inadequate description of subjects' choices in risky situations. The descriptive shortcomings of the theory have been exposed by Allais (1953), Kahneman and Tversky (1979; Tversky and Kahneman, 1981; 1986), and Lichtenstein and Slovic (1971; Slovic and Lichtenstein, 1983), among others. Slovic,
Lichtenstein and Fischhoff (1988) provide a guide to the relevant literature.

Another example of an absolute theory is Kahneman and Tversky's (1979) Prospect Theory, also discussed in detail in the next chapter. Within Prospect Theory, the attractiveness of \((d,p)\) is given as \(u(d)\pi(p)\), where now \(u\) is defined on monetary gains or losses and \(\pi\) is an empirically estimated function from probabilities to non-negative real numbers (called "decision weights"). Kahneman and Tversky (1979) show that Prospect Theory avoids many of the false predictions of Expected Utility Theory, and leads to new and surprising predictions confirmed by experiment (and illustrated later in this paper). Despite these successes, and largely due to its absolute character, Prospect Theory seems open to certain descriptive difficulties to be discussed in Chapter 4.

The absolute approach has also been brought into question by recent experimental evidence. Russo and Dosher (1983) present eye-movement data and verbal protocols to demonstrate that subjects first evaluate differences between lotteries on the separate dimensions of probability and payoff and then combine these estimates to yield a choice. These findings agree with other studies that use decision tracing methods and report evidence for comparative heuristics in preference judgments. Reviews of the relevant literature are provided by Russo and Dosher (1983) and by Schoemaker (1982). Russo and Dosher (1983, p.694) conclude that the data are "difficult or impossible to explain with a holistic [i.e., absolute] strategy." They go on to sketch plausible hypotheses
within the comparative framework but stop short of advancing an explicit rule for calculating the attractiveness of lotteries in a choice problem.

Other theoretical alternatives to Expected Utility Theory have incorporated a comparative framework. Foremost among these is Expected Regret theory, developed independently by Bell (1982; 1983), Fishburn (1982), and Loomes and Sugden (1982). Although there are important differences between the formulations offered by these authors, the following remarks apply to all three versions of the theory.

Regret theory replaces the absolute preference function \( F(L) \) that attributes attractiveness to single lotteries by a function \( F(L_1, L_2) \) defined over pairs of lotteries. An individual's level of satisfaction is thus viewed as determined not only by what is obtained as a consequence of having made a choice, but also by what might have been obtained had another choice been made. In effect, Regret Theory combines an absolute measure (viz., the utility of what was obtained) with a comparative measure (viz., the difference between what was and what could have been obtained). The result is a mathematically sophisticated theory that avoids many of the false predictions of Expected Utility Theory. On the other hand, the various versions of Expected Regret Theory share the disadvantage of being even more elusive than Utility Theory about predicting what decisions people will make when faced with specific choice problems. Predictions of this latter sort follow from the theory only in the presence of strong assumptions that go beyond its general picture of the subject's bivariate (Fishburn) or multiatribute (Bell and Loomes and Sugden) utility function. The space of candidates for these
functions is even less constrained than the space of unary functions within Expected Utility Theory.

Regret Theory captures an appealing psychological intuition concerning the regret or rejoicing that the decision maker experiences upon considering the outcome relevant to his choice. Apart from this intuition, however, the theory embodies no hypotheses about the mental processes and representations that underlie peoples' choices between lotteries. In this respect, it remains within the Expected Utility Theory tradition because the latter is equally reluctant to specify underlying mental processes.

The foregoing discussion cites only a fraction of the theories of choice under risk proposed in recent years. Each theory succeeds in deducing some important choice phenomena while leaving others unexplained. No theory appears to explain the entire spectrum of choice phenomena brought to light thus far. For more thorough review of the explanatory adequacy and inherent limitations of various choice theories, see Machina (1983, 1987), Battalio, Kagel, and Komin (1988), and Camerer (1988).

The present paper advances a descriptive model of choice between lotteries that invokes people's perception of the advantages accruing to each of the alternatives they face. The model -- called the Advantage Model of Choice -- construes the attractiveness of a lottery as a multiplicative combination of absolute and comparative components which result from judgmental processes of an elementary character. The Advantage Model of Choice (henceforth, "the Advantage Model") applies to
both simple and mixed monetary lotteries. It captures a simple and psychologically explicit hypothesis which, in later chapters, is tested both qualitatively and quantitatively. While it will be seen to face some unresolved difficulties, the model is highly parsimonious, posits a choice strategy of clear comparative character, and is well-supported by experimental test. It may thus provide insight into fundamental processes underlying human choice behavior.

Our presentation proceeds as follows. The last section of this chapter briefly reviews some necessary terminology. Chapter 2 reviews in detail the Advantage Model's most important predecessors among descriptive theories of risky choice. In particular, we describe both the general formulation of Expected Utility Theory and Prospect Theory, as well as the specific representatives of each that will be tested in later chapters. Chapter 3 introduces the Advantage Model. In Chapter 4 we evaluate the model from a qualitative point of view. Among other topics, we show how the Advantage Model helps to make sense of choice phenomena like intransitivity that are inexplicable within an absolute framework. Chapters 5 through 7 are dedicated to a quantitative evaluation of the Advantage Model. While in Chapter 5 we discuss various methodological considerations, in Chapter 6 the results of three experiments are shown to favor the model over comparable versions of Expected Utility Theory and Prospect Theory. Quantitative comparison of the model to Regret Theory was not attempted since clear candidates for this theory's curve-parameters do not suggest themselves. Additional analyses relevant to the Advantage Model are presented in Chapter 7. Chapter 8 is devoted to a discussion of possible refinements of the Advantage Model, and concludes
with some illustrations of how the model may be extended to nonmonetary domains.

1.3 Terminology

We adopt the following terminology. By a "positive lottery" is meant a less-than-certain chance \( p \) to win a specified sum of money \( d \). Such a lottery will be represented by the pair \((+d,p)\). By a "negative lottery" is meant a less-than-certain chance \( p \) to lose a specified sum of money \( d \). Such a lottery will be represented by the pair \((-d,p)\). Positive and negative lotteries in the sense just defined will be called "simple" lotteries. To designate a simple lottery without specifying whether it is positive or negative, we use: \((d,p)\). By a "simple choice problem" is meant an invitation to choose between a pair of simple lotteries. Such a pair will be denoted: \([(d_1,p_1),(d_2,p_2)]\). Table 1 summarizes every simple choice problem appearing in Kahneman and Tversky's (1979; Tversky and Kahneman, 1981) well-known discussion of risky choice.

The conjunction of two simple lotteries -- a positive lottery and a negative lottery -- whose probabilities add up to 1, that is, a less-than-certain chance \( p \) to win a specified sum of money \( d \) along with a less-than-certain chance \( 1-p \) to lose a specified sum of money \( e \), will be called a "mixed" lottery. In a mixed lottery the chance to win and the chance to lose always add up to 1. Such a lottery will be represented by the trio \((d,p,e)\), where \( p \) is the chance to win a specified sum of money \( d \) and \( 1-p \) is the chance to lose a specified sum of money \( e \). By a "mixed choice problem" is meant an invitation to choose between a pair of mixed lotteries. Such a pair will be denoted: \([(d_1,p_1,e_1),(d_2,p_2,e_2)]\). Notice,
finally, that simple choice problems are just special cases of mixed choice problems: a positive (simple) choice problem is a mixed choice problem of the form \([(d_1, p_1, 0), (d_2, p_2, 0)]\), and a negative (simple) choice problem is a mixed choice problem of the form \([(0, p_1, e_1), (0, p_2, e_2)]\).
Chapter 2

Important Predecessors

In later chapters, our proposed model of risky choice will be compared to alternative accounts of human choice behavior in risky situations. In particular, we will evaluate the Advantage Model against two of its most important predecessors, Utility Theory and Prospect Theory. In the present chapter, we provide a brief overview of these two popular theories, along with details concerning the particular version of each that will be used for later, quantitative evaluation.

2.1 Utility Theory

The study of risky choice involving monetary outcomes dates back to the 18th century when court mathematicians advised the French nobility on how to gamble. The then-popular theory of expected value soon led to obvious puzzles. It was pointed out, for example, that contrary to the spirit of the expected value model, most people did not feel indifferent with respect to all fair bets. Most people, for example, would reject the opportunity to toss a fair coin for an even chance to win or lose $1000. These, along with more ingenious puzzles like the St. Petersburg paradox, led to the formulation, by Daniel Bernoulli in 1738, of the expected utility principle. This principle proposed to replace the objective value of a payoff with the payoff's subjective utility for the decision maker.
Thus, for each person, there is assumed to be some function from money to 'utiles' that represents the utility to the person of the sum of money involved. It was further assumed that the utility of a gamble equaled the expected utility of its outcomes. According to this principle, a person may reject a coin toss that offers him an equal chance to win or lose $1000 because, for him, the utility of a $1000-gain is lower than the disutility of a $1000-loss. With the introduction of expected utility theory it became possible for different people to have different utilities for money, and hence to exhibit different preferences among risky choices. This led not only to richer normative theories but to the attempt at descriptive theories of risky choice as well.

More than 200 years after Bernoulli, the theory of expected utility received important support from a modern version due to von Neumann and Morgenstern (1947). These authors present a set of axioms concerning people's preferences among gambles, that can be regarded as maxims of rational behavior. Thus, for example, one of the axioms (weak ordering) requires of people to have an opinion (preference or indifference) about any two competing gambles, and further requires that those opinions be transitive. If you prefer gamble A to gamble B, and gamble B to gamble C, then modern utility theory requires that you prefer gamble A to gamble C. This axiom has appeared to virtually all commentators to offer a necessary condition on rational choice. The other axioms seem equally compelling. Von Neumann and Morgenstern prove a theorem stating that if an individual's preferences satisfy the proposed axioms then his choice behavior can be described as maximizing his expected utility.
Because of the intuitively compelling nature of its underlying axioms, modern utility theory (henceforth, "Utility Theory") has been interpreted as providing not only a convincing normative model, but a descriptive account of human choice behavior as well. Since the early fifties, there have been many attempts to test the descriptive validity of the theory. Apart from occasional difficulties whose nature at the time was hard to decipher, it can be said that until the early 1970's Utility Theory appeared to provide an adequate account of most of the data involving risky choice with monetary outcomes. Coombs, Dawes, and Tversky (1970) review some of Utility Theory's descriptive tests throughout the years. According to these authors, "the results indicate that...the utility of money can be described as a power function of money with different exponents for positive and negative outcomes." (p. 137). Kahneman and Tversky (1982) similarly recommend the power function as a good description of the utility of payoffs. Furthermore, because people have been repeatedly shown to overweight losses relative to gains, these authors agree on the need to differentially estimate the exact shape of the function in the domain of gains and in the domain of losses.

In our evaluation of Utility Theory in Chapters 6 and 7, we investigate precisely the family of utility functions suggested by the authors above. Specifically, we consider the following power utility function, denoted $U_{c_\alpha c_L}$, which employs two real parameters, $c_\alpha$ (serving as the exponent for gains) and $c_L$ (serving as the exponent for losses):

$$U_{c_\alpha c_L}(x) = [x^{c_\alpha}] \text{ if } x \geq 0$$

$$U_{c_\alpha c_L}(x) = -[|x|^{c_L}] \text{ if } x < 0$$
Just as hypothesized by Utility Theory, a mixed lottery \((d_1,p_1,e_1)\) is evaluated according to the formula:

\[ U_{c_1}(d_1)(p_1) + U_{c_2}(e_1)(1-p_1). \]

It should be noted that this formulation departs somewhat from the tradition of Utility Theory in that it computes the attractiveness of lotteries in terms of changes in wealth (i.e., gains and losses) rather than in terms of final assets. The notion of a reference point in choice behavior is due to more recent developments in the theory of choice, which brings us to the Advantage Model's second important predecessor, namely, Prospect Theory.

2.2 Prospect Theory

In the last decade, the work of Daniel Kahneman and Amos Tversky (1979, 1982; Tversky and Kahneman, 1986) has uncovered a set of systematic descriptive failures of Utility Theory. The sporadic descriptive inadequacies of Utility Theory in earlier years were now shown to be due to descriptive failures inherent to the theory. A more thorough review of these failures is provided in Chapter 4. This work led Kahneman and Tversky to propose an alternative account of human choice behavior concerning monetary outcomes, known as Prospect Theory.

An essential feature of Prospect Theory is that the carriers of value are considered to be changes in wealth rather than final monetary assets. The psychological validity of this claim is nicely illustrated by Tversky and Kahneman (1986), who posed the following two problems to two
separate groups of subjects (where the bracketed numbers following each option indicate the percentage of respondents who chose that option):

Problem 1: Assume yourself richer by $300 than you are today.
You have to choose between
a sure gain of $100 [72%]
50% chance to gain $200 and 50% chance to gain nothing [28%]

Problem 2: Assume yourself richer by $500 than you are today.
You have to choose between
a sure loss of $100 [36%]
50% chance to lose nothing and 50% chance to lose $200 [64%]

The two problems are essentially identical. In both cases the subject faces a choice between $400 for sure and an even chance at $500 or $300. Framing the problems in terms of losses or gains, however, had a substantial effect on preferences. Evidently, what subjects are evaluating are not final monetary assets, but rather changes relative to the current wealth, or reference point. As mentioned earlier, this notion of a reference point has actually been incorporated into the version of Utility Theory that we will be investigating. Furthermore, the Advantage Model will be seen naturally to capture the psychological reality of a reference point, since it address only gains and losses relative to current wealth, and never final monetary assets.

Prospect Theory hypothesizes a value function for payoffs, and a decision weight function for probabilities. The value function, much as the power utility function discussed in the context of the previous section, is assumed to be generally concave for gains and convex for losses, and commonly steeper for losses than for gains. The decision weight function applies to the probabilities associated with a gamble. It measures the impact of probabilistic events on the desirability of the
gambles, and not merely the perceived likelihood of those events. Thus, instead of computing the attractiveness of a gamble using its naked probabilities, as does Utility Theory, Prospect Theory transforms these probabilities, via the decision weight function, into their subjective values for the decision maker.

Briefly stated, the decision weight function overweights small probabilities relative to large ones. It also has the property, known as subproportionality, that for a fixed ratio of probabilities, the ratio of the corresponding decision weights is closer to unity when the probabilities are low than when they are high. This allows Prospect Theory to predict Kahneman and Tversky’s finding that while most people prefer the first lottery in a choice between (\$6000, .01) and (\$3000, .02), the majority prefers the second lottery in the choice between (\$6000, .40) and (\$3000, .80). The payoffs in the two choices are identical. And while the ratio of probabilities has remained the same, the impact of an 80% chance over that of a 40% chance is greater than that of a 2% chance over that of a 1% chance, which is a direct outcome of the assumption of subproportionality of the decision weight function.

Kahneman and Tversky never suggest specific candidates for their value function or for their decision weight function (which they call \(\pi\)). Observe, however, that in the context of our quantitative tests of the various theories (reported in chapters 5 through 7), all theories must have a comparable number of free parameters. For, a theory with a greater number of free parameters may do better than theories that are more constrained, due to purely mathematical reasons, not to its greater adequacy as a descriptive theory of human behavior. Therefore, in
conformity with the Advantage Model's and Utility Theory's two parameters, we test a 2-parameter version of Prospect Theory. For Prospect Theory's value function we use the 1-parameter rendition of the power utility function discussed in the previous section:

\[ U_c(x) = \text{sign}(x)|x|^{\gamma}. \]

(Note, however, that the limitation of this function to a single parameter results in its inability to yield a steeper curve for losses than for gains, as originally hypothesized by Prospect Theory.) For Prospect Theory's decision-weight function, \( \pi \), we have devised the following family of functions, parameterized by \( w \):

\[ \pi_w(p) = wp^2 + (0.8 - w)p + 0.1. \]

This formulation of \( \pi \) captures the essential features of the function suggested by Kahneman and Tversky (1979), at least in the range of probabilities used in the present study. This may be witnessed by a visual comparison of the present function with the schematic suggestion of Kahneman and Tversky, as in Figure 4.1. According to Prospect Theory, the mixed lottery \((d_1, p_1, e_1)\) is evaluated according to the formula:

\[ U_c(d_1)\pi_w(p_1) + U_c(e_1)\pi_w(1-p_1). \]

Of course, the above formulations are only two of many possible versions of Utility Theory and Prospect Theory. They are intended mainly to provide a standard against which to compare the predictive capabilities of the Advantage Model. We have analyzed our data (to be reported in later chapters) according to other Utility functions, including expected monetary value, exponential and various logarithmic
A: The decision-weight function for Prospect Theory suggested by Kahneman and Tversky (1979).

B: One of the family of decision-weight functions investigated by the current version of Prospect Theory (w=.4).

C: A sample of other decision-weight functions in the family of functions investigated by the current version of Prospect Theory.

Figure 2.1: Prospect Theory's $\pi$ function
functions (see, e.g., Shafir, Osherson, and Smith, 1988, for a 1-parameter analysis employing the exponential utility function \( u(x) = \text{sign}(x)[1-e^{-|x|}] \), suggested by Raiffa, 1968, and Keeney and Raiffa, 1976). However, none of these alternative versions fared as well empirically as the present version of the power function. Thus, it should be noted that the quantitative tests to be reported are conservative. We compared the Advantage Model only against the most successful comparable versions of the alternative theories.
Chapter 3

Theory

The Advantage Model is motivated by the intuition that when choosing between lotteries, people employ both absolute and comparative strategies, the results of which are subsequently combined. According to the model, a person's absolute strategy consists of a rough evaluation of each lottery's "size", to be captured by Expected Monetary Value, $d*p$. His comparative strategy consists of an evaluation of (a) the difference in payoff between the two lotteries, and (b) their difference in probabilities. These strategies, according to the model, are employed separately along the dimensions of losses and gains. They determine the attractivenesses of positive and negative lotteries separately, and are then combined to yield the attractiveness of mixed lotteries. We now express these intuitive ideas formally.

3.1 The attractiveness of lotteries

All popular theories of risky choice hypothesize some function that attributes attractiveness coefficients to lotteries figuring in choice problems. The lottery attributed the numerically higher coefficient is predicted to be preferred. As was discussed in Chapter 1, the functions hypothesized by the various theories differ as to whether they compute a lottery's attractiveness in a comparative or an absolute manner.
Furthermore, they differ in the degree of psychological reality that they attribute to the computed attractivenesses. In Utility Theory, for example, a lottery's attractiveness coefficient is bestowed with little psychological reality inasmuch as any positive monotone transformation of all coefficients is deemed equally valid.

In the Advantage Model, the attractiveness of lotteries is computed via a function denoted AMk. This function takes as arguments a lottery and a simple choice problem in which the lottery appears, and returns a real number that represents the attractiveness of that lottery in that problem. AMk will be seen to compute a lottery's attractiveness via an elementary combination of absolute and comparative calculations. And the Advantage Model will be seen to attribute a high degree of psychological reality to the resulting coefficient. A lottery's attractiveness coefficient, according the Advantage Model, actually represents the relative attractiveness to the decision maker of that lottery compared to the other lottery in the choice problem. A lottery whose coefficient is twice as large as another's is assumed to be twice as attractive as the other.

In what follows, we describe the Advantage Model and its AMk-function, first as they apply to lotteries in simple choice problems, and then as they extend to lotteries in mixed choice problems. After a precise formulation has been given of the way in which the Advantage Model evaluates the attractiveness of lotteries, a brief section (3.2) summarizes the model's predictions regarding choice.
3.1.1 Simple choice problems

We begin a precise formulation of the model by focusing on simple choice problems. Observe that in order for these problems to be of any theoretical interest one lottery should not strictly dominate the other. For example, the two lotteries may not be such that one is positive and the other negative, since the positive will always be trivially preferred by all people. Also, the two lotteries should not offer the same payoff since the lottery with the larger probability will always be preferred if the payoffs represent gains and never be preferred if they represent losses. For similar reasons, the lotteries may not offer identical probabilities.

Since the Advantage Model addresses the conflict that arises from lotteries' advantages on payoffs and probabilities, the model will only be defined for simple choice problems where such conflict is present, that is, for pairs of lotteries that exhibit both payoff and probability advantages. Such pairs of lotteries will be called "conflictual". Pairs of lotteries that do not exhibit either a payoff or a probability advantage will be termed "nonconflictual" and will remain outside the domain of the Advantage Model. We return to a lengthy discussion of nonconflictual choice problems in Section 7.2.

One final category of lotteries must be considered. We will call "trivial" any lottery that offers 0 payoffs or 0 probabilities. The Advantage Model will assign an attractiveness of 0 to any trivial lottery.
Consider then a person who is engaged in determining his preference between the two (nontrivial and conflictual) lotteries figuring in the simple choice problem \(((d_1,p_1),(d_2,p_2))\), (where \(p_2 > p_1\)). According to the model, the person attempts both a rough absolute evaluation of the two lotteries, as well as a heuristic comparative evaluation. To represent the absolute component of the subject's judgment, we define: \(EMV_1 = d_1p_1\); \(EMV_2 = d_2p_2\). These measures provide the subject with a rough estimate of the "size" of the lotteries involved. On the other hand, for the comparative component, we let \(p_2 - p_1\) represent the "probability advantage" of the lottery \((d_2,p_2)\), and we let \((d_1-d_2)/d_1\) represent the "payoff advantage" of the lottery \((d_1,p_1)\). Here the subject is assumed to compare the two competing lotteries along the dimensions of payoffs and of probabilities. In line with the notion of dimensional commensurability (see, e.g., Slovic and MacPhillamy, 1974), it is suggested that comparing the two alternatives on payoffs and then comparing them on probabilities is easier than attempting to integrate probability information and payoff information. Notice that the payoff advantage has been normalized by \(d_1\) because in these situations people have been shown to be more sensitive to relative rather than absolute payoff magnitudes, as is discussed and demonstrated in Chapter 4. (Other means of normalizing are possible; the present scheme was selected on the kind of qualitative grounds also to be discussed in Chapter 4.) Observe that a lottery's probability advantage is determined by the difference between probabilities while the competing lottery's payoff advantage is ultimately determined by the payoffs'
ratio. It should be noted as well that, for all conflictual lotteries, the expressions $p_2 - p_1$ and $(d_1 - d_2)/d_1$ are always positive numbers for both positive and negative lotteries. As a consequence, these two "advantages" do not always represent desirable aspects of a lottery. In the case of negative lotteries, they are undesirable (and might best be conceived as "disadvantages").

Notice, finally, that the two advantages are qualitatively different: one is a probability advantage and the other a payoff advantage. As a means for comparing these two qualitatively different advantages, we introduce a unitless parameter into our model that represents the relative weight of payoffs and probabilities. This factor takes the form of a multiplicative coefficient, $k$, and is attached to the payoff advantage $(d_1 - d_2)/d_1$ of the lottery $(d_1, p_1)$.

Assembling the absolute and comparative components of the model, we arrive at the following function which captures our motivating intuitions regarding the attractiveness of lottery $(d_1, p_1)$ in simple choice problem $[(d_1, p_1), (d_2, p_2)]$. The function, denoted $\text{AM}k$, takes as arguments a conflictual lottery and a simple choice problem in which the lottery appears, and returns a real number that represents the attractiveness of that lottery in that choice problem:
The AMk-function:

\[ \text{AMk}((d_1,p_1);((d_1,p_1),(d_2,p_2))) \text{ is undefined if } \{(d_1,p_1),(d_2,p_2)\} \]

is nonconflictual; Otherwise:

\[ \text{AMk}((d_1,p_1);((d_1,p_1),(d_2,p_2))) = 0 \text{ if } (d_1,p_1) \text{ is trivial;} \]

\[ = \text{EMV}((d_1,d_2)/d_1)k \text{ if } p_1 < p_2; \]

\[ = \text{EMV}(p_1-p_2) \text{ if } p_1 > p_2. \]

We may refer to \(\text{AMk}((d_1,p_1);((d_1,p_1),(d_2,p_2)))\) as the attractiveness (due to \(k\)) of lottery \((d_1,p_1)\) in choice problem \(\{(d_1,p_1),(d_2,p_2)\}\). Thus, according to the Advantage Model, the attractiveness of the lottery offering the higher probability (to gain or lose) in a simple choice problem equals the lottery's EMV multiplied by its probability advantage. Similarly, the attractiveness of the lottery offering the larger payoff (either a loss or a gain) equals its EMV times its payoff advantage. Finally, the parameter \(k\) represents the relative weight of payoffs and probabilities. As is illustrated in Chapter 4 (see especially 4.2), the value of \(k\) is predicted to be smaller than 1 for most people. Since \(k\) represents the relative weight of payoffs to probabilities, this is consistent with findings that people generally give more weight to probability than to payoffs in choosing between lotteries (see, e.g., Slovic and Lichtenstein, 1968).

Now recall (as discussed in Chapter 2 and further illustrated in Chapter 4) that a person's relative weight of payoffs to probabilities is likely to differ when the payoffs represent gains compared to when the payoffs constitute losses. For example, a person may focus his attention on the amounts to be lost when losses are concerned, but care more about
the chances involved when gains are at stake. Because the relative weight of payoffs and probabilities may differ according to whether the payoffs constitute losses or gains, we need to introduce two k-parameters: $k_\alpha$ in the case where the payoffs constitute gains, and $k_\lambda$ in the case where they constitute losses. (We will continue to refer to the parameters as $"k"$ when we do not intend to distinguish between $k_\alpha$ and $k_\lambda$.) Recall that the version of Utility Theory advanced in Chapter 2 also hypothesizes a differential weighting of losses and gains. In the Advantage Model, the attractiveness of lotteries in all simple choice problems continues to be computed using the parameterized function $AM_k$. The relative weight of payoffs and probabilities, however, may differ between positive and negative choice problems. Therefore, in the case of positive choice problems, the function uses the parameter $k_\alpha$, and in the case of negative choice problems it uses $k_\lambda$. As will be illustrated in Chapter 4 (see especially 4.3), the value of $k_\lambda$ is predicted to be greater than that of $k_\alpha$ for most people. According to our model, these weighting factors may differ from person to person, but each person has exactly two such factors, $k_\alpha$ and $k_\lambda$, applicable to all simple choice problems. For other decision situations (e.g., assessing the monetary value of a lottery, or determining its probability equivalent) the value of these weighting factors is assumed to vary in a systematic manner to be discussed in Section 3.4.
Incorporating the above discussion into our formulation of the model leads to the following empirical claim concerning the attractiveness of lotteries in simple choice problems:

**The attractiveness of lotteries in simple choice problems:**

For every person $S$ there are $k_o, k_l > 0$ such that for any simple choice problem $\{(d_1,p_1),(d_2,p_2)\}$, the attractiveness for $S$ of $(d_1,p_1)$ in the context of $\{(d_1,p_1),(d_2,p_2)\}$ equals:

$AMk_o((d_1,p_1);\{(d_1,p_1),(d_2,p_2)\})$ if $d_1, d_2 \geq 0$,

$AMk_l((d_1,p_1);\{(d_1,p_1),(d_2,p_2)\})$ if $d_1, d_2 < 0$, and is undefined otherwise.

3.1.2 Mixed choice problems

We shall now extend the Advantage Model's account of the attractiveness of lotteries from simple to mixed choice problems. As with the simple problems, because the Advantage Model is concerned with the conflict that arises due to payoff and probability advantages, it only applies to conflictual pairs of mixed lotteries, i.e., to mixed lotteries that differ both in probabilities and payoffs. Consider then the attractiveness of lottery $(d_1,p_1,e_1)$ in mixed choice problem $\{(d_1,p_1,e_1),(d_2,p_2,e_2)\}$. The lottery $(d_1,p_1,e_1)$ may be thought of as a composite of two simple lotteries: a positive lottery $((d_1,p_1))$ and a negative lottery $(e_1,1-p_1)$. Its attractiveness, according to the Advantage Model, is simply the sum of the attractivenesses of these two parts. As numerous psychological phenomena (see Chapters 2 and 4) make
clear, people accord different treatment to risky outcomes involving gains and to those involving losses. According to the model, a person presented with a mixed choice problem evaluates its mixed lotteries separately on the dimension of gains and on the dimension of losses. The evaluation on the gain-dimension of the problem above reduces to what looks like a choice problem involving two simple lotteries: \((d_1,p_1)\) versus \((d_2,p_2)\). Thus, the superiority -- on gains -- of lottery \((d_1,p_1,e_1)\) over lottery \((d_2,p_2,e_2)\) is large to the extent that \((d_1,p_1)\) is preferred over \((d_2,p_2)\). Similarly, the evaluation on the loss-dimension reduces to a choice problem involving another two simple lotteries: \((e_1,1-p_1)\) versus \((e_2,1-p_2)\). Here, the superiority -- on losses -- of lottery \((d_1,p_1,e_1)\) over lottery \((d_2,p_2,e_2)\) is large to the extent that \((e_1,1-p_1)\) is preferred over \((e_2,1-p_2)\). Finally, the attractiveness of each mixed lottery consists of the sum of its evaluations on the dimension of gains and on the dimension of losses. Thus, the attractiveness of \((d_1,p_1,e_1)\) consists of the attractiveness of \((d_1,p_1)\) (relative to \((d_2,p_2)\)) plus the attractiveness of \((e_1,1-p_1)\) (relative to \((e_2,1-p_2)\)).

3.1.2.1 AMK_{\alpha,k} in mixed choice problems

To summarize the preceding discussion, according to the Advantage Model the attractiveness of mixed lotteries consists of the sum of attractivenesses of their component, simple lotteries. In other words, the function \(AMK\) is applied to two simple lotteries in order to compute the attractiveness of a mixed lottery. Notice, however, that while the attractiveness of each simple lottery was computed using only one of the
subject's two k's (specifically, \( k_a \) for positive lotteries and \( k_L \) for negative lotteries), each such computation for a mixed lottery requires both the subject's k's: \( k_a \) for the gain-dimension and \( k_L \) for the loss-dimension. Hence, we define the function \( AMk_a k_L \). This function, when applied to a mixed lottery in a choice problem, computes the attractiveness of this lottery's positive component (i.e., a positive lottery) using \( k_a \), and the attractiveness of its negative component (i.e., a negative lottery) using \( k_L \). The \( AMk_a k_L \) function is defined as follows.

The \( AMk_a k_L \)-function:

\[
AMk_a k_L((d_1, p_1, e_1); ((d_1, p_1, e_1), (d_2, p_2, e_2))) = \\
AMk_a((d_1, p_1); ((d_1, p_1), (d_2, p_2))) + \\
AMk_L((e_1, 1-p_1); ((e_1, 1-p_1), (e_2, 1-p_2))).
\]

The following formulation summarizes our account of the attractiveness of lotteries in mixed choice problems:

The attractiveness of lotteries in mixed choice problems:

For every person \( S \), there are \( k_a, k_L > 0 \) such that the attractiveness for \( S \) of lottery \((d_1, p_1, e_1)\) in the context of choice problem \([((d_1, p_1, e_1), (d_2, p_2, e_2))]\) equals:

\[
AMk_a k_L((d_1, p_1, e_1); ((d_1, p_1, e_1), (d_2, p_2, e_2))).
\]

Notice that according to the Advantage Model a person's \( k_a \) and \( k_L \) are the same for simple and mixed lotteries. According to the model, a
person's relative weight of payoffs to probabilities, \(k_\alpha\) and \(k_L\) for gains and for losses, respectively, is the same in simple and in mixed choice problems. In fact, as is pointed out in the next section, simple problems may be thought of as mixed problems in disguise.

3.1.3 The simple as mixed choice problems

Simple choice problems are just a special kind of mixed choice problems. A positive (simple) choice problem is a mixed choice problem of the form \([(d_1,p_1,0),(d_2,p_2,0)]\), and a negative (simple) choice problem is a mixed choice problem of the form \([(0,p_1,e_1),(0,p_2,e_2)]\). It is therefore appropriate that the Advantage Model's account of the attractiveness of simple lotteries is just a special case of its evaluation of mixed lotteries. Consider, for example, a positive choice problem represented as \([(d_1,p_1,0),(d_2,p_2,0)]\) and treated as a mixed choice problem. It is easy to verify that its evaluation along the loss-dimension -- using \(AMk_L\) -- reduces to zero (since both \(e_1\) and \(e_2\) are zero) and that the problem is finally evaluated only along the gain-dimension, as would have been the case had it been treated as a simple choice problem. A similar outcome -- due to the reduction to zero of the gain-dimension (using \(AMk_\alpha\)) -- may be observed for negative choice problems.

Simple choice problems have occupied an important role in the study of choice. According to our model, moreover, they play a pivotal role in the evaluation of mixed problems. For these reasons, they were accorded a separate presentation in these pages. It is important to note, however, that an account of the advantage model framed solely in terms of mixed
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choice problems would suffice to cover problems both simple and mixed. Such an account, summarizing our model of choice, occupies the next section.

3.2 Choice

Now that we have a precise formulation of the manner in which Person S evaluates the attractiveness of lotteries, we may briefly capture S's behavior when confronted with choice problems:

The Advantage Model of Choice:

For every person S, there are k_\alpha, k_\lambda > 0, such that for any mixed choice problem \[((d_1,p_1,e_1),(d_2,p_2,e_2))] (where for simple choice problems either d_1,d_2 or e_1,e_2 = 0),

S chooses \((d_1,p_1,e_1)\) if

\[\text{AM}_k\alpha k_\lambda((d_1,p_1,e_1);[(d_1,p_1,e_1),(d_2,p_2,e_2)]) > \text{AM}_k\alpha k_\lambda((d_2,p_2,e_2);[(d_1,p_1,e_1),(d_2,p_2,e_2)])\]

S chooses \((d_2,p_2,e_2)\) if

\[\text{AM}_k\alpha k_\lambda((d_1,p_1,e_1);[(d_1,p_1,e_1),(d_2,p_2,e_2)]) < \text{AM}_k\alpha k_\lambda((d_2,p_2,e_2);[(d_1,p_1,e_1),(d_2,p_2,e_2)])\]

S is indifferent if

\[\text{AM}_k\alpha k_\lambda((d_1,p_1,e_1);[(d_1,p_1,e_1),(d_2,p_2,e_2)]) = \text{AM}_k\alpha k_\lambda((d_2,p_2,e_2);[(d_1,p_1,e_1),(d_2,p_2,e_2)]).

The reader may verify that the Advantage Model correctly predicts all eight simple choice problems of Table 1 using any k_\alpha, k_\lambda such that \(.25 < k_\alpha, k_\lambda < .90\. We illustrate the calculations involved in problem (1), using k_\alpha = .50:
Problem 1: \([(2500, .33, 0), (2400, .34, 0)]\)

\[\text{AMk}_{\alpha k_L}((2500, .33, 0); [(2500, .33, 0), (2400, .34, 0)]) = \]

\[\text{AMk}_{\alpha}((2500, .33); [(2500, .33), (2400, .34)]) + \]
\[\text{AMk}_{L}((0, .67); [(0, .67), (0, .66)])\]

\[= (2500)(.33)(100/2500) + 0\]
\[= (2500)(.33)(.5)\]
\[= 16.5\]

\[\text{AMk}_{\alpha k_L}((2400, .34, 0); [(2500, .33, 0), (2400, .34, 0)]) = \]

\[\text{AMk}_{\alpha}((2400, .34); [(2500, .33), (2400, .34)]) + \]
\[\text{AMk}_{L}((0, .66); [(0, .67), (0, .66)])\]

\[= (2400)(.34)(.34 -.33) + 0\]
\[= (2400)(.34)(.01)\]
\[= 8.16\]

Hence, the Advantage Model (with \(k_{\alpha} = .50\)) predicts that (2500, .33) will be preferred to (2400, .34) -- which agrees with a significant majority of Kahneman and Tversky's subjects. Any choice of \(k_{\alpha}, k_L\) in the [.25,.90] interval applies in similar fashion to the remaining problems in Table 1. It will be seen that these values of \(k_{\alpha},k_L\) are consistent with the empirical estimates derived from the experiments to be reported in Chapter 6.

3.3 Alternative formulations of the model

We now briefly present a number of alternative formulations of the Advantage Model. For simplicity of exposition, we will focus on positive (simple) choice problems although much of the discussion extends to other choice problems as well. Consider the present version of the Advantage Model as it applies to the choice problem \([(d_1, p_1), (d_2, p_2)]\), where \(p_2 > p_1 > 0\). According to the Advantage Model, \(\text{AMk}_{\alpha}((d_1, p_1); [(d_1, p_1), (d_2, p_2)])\)
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\[ EMV_1((d_1-d_2)/d_1)k, \text{ and } AMk((d_2,p_2);((d_1,p_1),(d_2,p_2))) = EMV_2(p_2-p_1). \]

Thus, according to the Advantage Model, this choice problem leads the subject to the comparison: \( EMV_1((d_1-d_2)/d_1)k \) versus \( EMV_2(p_2-p_1) \). As described in Section 2.1, these two measures capture a combination of absolute and comparative evaluations, further enriched by \( k \), a parameter that represents the relative weight for the subject of payoffs and probabilities.

By rearrangement of terms, this comparison of the Advantage Model may be cast equivalently as follows: \( [EMV_1/EMV_2]k \) versus \( (p_2-p_1)/((d_1-d_2)/d_1) \). Observe that \( EMV_1/EMV_2 \) may be conceived as the advantage of \( (d_1,p_1) \) over \( (d_2,p_2) \) within the absolute perspective. On the other hand, \( (p_2-p_1)/((d_1-d_2)/d_1) \) is the advantage of \( (d_2,p_2) \) over \( (d_1,p_1) \) within the comparative perspective. The evaluation of the two simple lotteries now depends upon the relative sizes of these two kinds of advantages. In this framework, the parameter \( k \) may be interpreted as the relative importance for \( S \) of absolute versus comparative considerations. In what follows, we shall rely on the original formulation of the Advantage Model -- where \( k \) represents the relative weight of payoffs and probabilities -- because it is more faithful to choice protocols we have collected (e.g., "I compared the difference in percents with the difference in monetary benefits to see which one was more advantageous."), and because the relative weight of payoffs and probabilities has been shown previously to affect preference (e.g., Slovic and Lichtenstein, 1968).

Another rearrangement of terms is also worth mentioning. The Advantage Model may be formulated as a purely comparative model: \( ((d_1-d_2)/d_2)k \) and \( (p_2/p_1)(p_2-p_1) \) being the attractiveness coefficients of
(d₁,p₁) and (d₂,p₂), respectively. Notice, however, that according to this formulation the independent evaluation of each simple lottery plays no role in the choice process. This not only contradicts the assumptions that have guided absolute theories, but it also conflicts with empirical studies that suggest both "interdimensional" and "intradimensional" strategies. (See, e.g., Payne (1976), Payne and Braunstein (1978), and Russo and Dosher (1983).) For this reason, we prefer to formulate the algebra of the Advantage Model in such a way as to exhibit absolute as well as comparative terms. It is this formulation that represents the intuition that choice typically involves a compromise between comparative and absolute strategies.

Finally, consider the role of k in the original formulation of the model. According to this formulation, k -- a parameter that represents the subject's relative weight of payoffs to probabilities -- is attached to the payoff advantage of the lottery with the larger payoff. Alternatively, of course, the inverse of k could be attached to the probability advantage of the opposite lottery. Thus, instead of multiplying the attractiveness coefficient of a lottery by, say, (k=) 1/2, one could multiply the coefficient of the competing lottery by 2. Both methods would capture the same payoff-to-probability ratio (namely, 1-to-2), and, as long as used consistently, would remain indistinguishable as far as the model's predictions are concerned. They would, however, generate different values for the lotteries' attractiveness coefficients. Weighting one lottery by k=2 rather than the other lottery by k=1/2 yields coefficients that bear the same ratio but are twice as large.
Of course, our decision to attach $k$ to the payoff rather than the probability advantage is not motivated on empirical grounds. (In fact, the most reasonable assumption is that both probabilities and payoffs are attached weights that, together, yield the ratio captured by $k$.) And while this decision does not effect the model's predictions in any way, it does bear on the status of the obtained attractiveness coefficients. While we take the rest of the hypothesized terms (e.g., $p_2-p_1$, $EMV_1$, etc.) as approximating psychologically real computations, we cannot, due to the random placement of $k$, reify the resulting attractiveness coefficients. The attractiveness coefficients in the Advantage Model are "real" up to a similarity transformation (cf. Roberts, 1979, p.64-5). We can say that one lottery is twice as attractive as another, but not how attractive (in terms of a resulting coefficient) it actually is.

3.4 Monetary value

An important decision paradigm, different from choice problems, is that of determining a subject's monetary value for a lottery. In this brief section, we illustrate how the Advantage Model envisions the estimation of a lottery's monetary value. The next section provides some additional insight into the monetary value estimation process.

The monetary value of a lottery (either simple or mixed) is defined as that amount of money which, if received for sure, is as attractive as playing the lottery. To illustrate, if receiving $30 (for certain) is equally attractive to you as playing the lottery (100,.50), than the monetary value of this lottery for you is $30. Within the perspective of the Advantage Model, it is natural to consider the monetary value of a
simple lottery \((d,p)\) for a person \(S\) to be that sum \(x\) of money that renders \(S\) indifferent between \((d,p)\) and \((x,1)\). Observe that, according to the model, \(x\) may be calculated from the equation \(dp((d-x)/d)k = x(1-p)\). Similar remarks apply to the monetary value of mixed lotteries. The monetary value of a mixed lottery, according to the model, equals the sum of the monetary values of the two simple lotteries of which it is composed. Thus, the monetary value of mixed lottery \((d,p,e)\) equals the sum of the monetary values of \((d,p)\) and of \((e,1-p)\). In other words, it equals the sum of \(x\) and \(z\) such that \((d,p) = (x,1)\), and \((e,1-p) = (z,1)\). Similar to before, the monetary value of \((d,p,e)\) may be calculated by adding the amounts \(x\) and \(z\) obtained from \(dp((d-x)/d)k_\alpha = x(1-p)\) and \(e(1-p)((e-z)/e)k_L = z(p)\), respectively. The monetary value estimation paradigm is used extensively in a number of decision-research contexts and will be discussed further, especially in Chapter 4.

3.5 The k parameters

We return now to the question of the stability of the parameters \(k_\alpha\) and \(k_L\) for a given person. The Advantage Model asserts that a person's choices among lotteries are all governed by a single, fixed \(k_\alpha,k_L\)-pair. Consider, however, the following observations. Tversky, Sattath, and Slovic (1988) describe numerous instances in which strategically equivalent methods of preference-elicitation yield systematically different preferences. To accommodate these inconsistencies, Tversky and his colleagues develop a hierarchy of contingent weighting models in which the tradeoff between attributes depends on the method of elicitation. This is based on a general principle of "compatibility", according to which the weighting of an input is enhanced by its
compatibility with the output. According to this notion, the weight of any feature of the object under evaluation is enhanced to the extent that the feature is compatible with the required response. Thus, because both the monetary value of a lottery and the payoffs offered by the lottery are expressed in the same units, compatibility implies that a lottery's payoffs are weighted more heavily in monetary value estimation than in choice.

What are the implications of the principle of compatibility to the stability of a person's k in the Advantage Model? Consider, for example, the paradigm of monetary value estimation. Recall from the previous section that a person's monetary value of simple lottery (d,p) is defined as that sum x of money that renders the person indifferent between (d,p) and (x,1). Recall further, that, according to the Advantage Model, x may actually be calculated from the equation dp[(d-x)/d]k = x(1-p). Now according to the compatibility principle, the payoffs in this equation -- an equation in the context of monetary value estimation -- are weighted more heavily than in the context of choice. In terms of the Advantage Model, this enhanced importance of payoffs means that k -- the relative weight of payoffs to probabilities -- has increased in the monetary value paradigm compared to the choice paradigm. In summary, since the procedure of determining monetary value focuses the subject's attention on payoffs, the model assumes -- in line with Tversky, Sattath, and Slovic's compatibility principle -- that the importance of payoffs (relative to probabilities) is enhanced in this context. This means that both $k_o$ and $k_L$ have increased in the monetary value paradigm compared to the choice paradigm.
Similar reasoning applies to what is known as the probability equivalent paradigm. Here, a person is asked to determine a probability $y$ such that a given lottery $(d_1, p_1)$ is equivalent for him to lottery $(d_2, y)$. To illustrate, given $(100, .50)$ and $(40, y)$, you may decide that for you the probability equivalent $y$ is .80, meaning that for you an 80% chance to win $40 is as attractive as a 50% chance to win $100. Here attention is focused on the probabilities, thus enhancing their importance relative to payoffs. As a result, in terms of the Advantage Model (and consistent with the compatibility principle), the values of $k_a$ and $k_L$ are expected to decrease.

Subjects' differential weighting of features may depend not only on response compatibility, but on other characteristics of the decision context that may shift the focus of attention. (For a review, see, e.g., Payne, 1982.) Thus, for example, a subject's weighting of probabilities relative to payoffs may differ when the probabilities are very high or extremely low. For greater generality, we may summarize this discussion by the following principle:

(*) In a choice context in which payoffs (either positive or negative) are made particularly salient, the values of a person's $k_a$ and $k_L$ typically increase (thereby increasing the relative weight of payoffs); conversely, if probabilities are salient, the values of $k_a$ and $k_L$ typically decrease (thereby increasing the relative weight of probabilities).

Principle (*) is also consistent with the context dependency of feature-weighting as demonstrated by Tversky and Gati (1978). Within the latter demonstrations, the importance that subjects attach to a given feature in a stimulus is shown to vary with the kind of judgment required.
and with the nature of the other stimuli in view. Furthermore, an increase in the relative weight of payoffs during monetary value estimation, as proposed by principle (*), is consonant with mechanisms proposed by other researchers. Thus, Lichtenstein and Slovic (1971) suggest that greater anchoring on payoffs occurs when monetary values are assigned than when choices between lotteries are made.

To illustrate the use of principle (*) in explaining specific compatibility effects in decision phenomena, consider the following experimental demonstration. When first asked to determine the monetary equivalent \( x \) that renders him indifferent between, e.g., (5000,.25) and (\( x, .75 \)), a person \( S \) decides that for him \( x \) is, say, 2000. But when later asked to determine the probability equivalent \( y \) that renders him indifferent between (5000,\( y \)) and (2000,.75), \( S \) decides that for him \( y \) is .50. A systematic pattern of preferences of this kind -- where the probabilities estimated in the latter stage are higher than those which figured in the former -- is reported by Delquié, de Neufville and Mangnann (1987), who gave people the two tasks separated by a two week interval. The indifferences indicated by \( S \) above yield the following inconsistent equivalences:

\[
(5000,.25) \approx (2000,.75) \text{ and } (5000,.50) \approx (2000,.75),
\]

which, according to the Advantage Model, indicate \( k_\alpha \) values of 1.0 in the first judgment and .25 in the second.

The foregoing shift in \( k_\alpha \)-values is predicted by principle (*), according to which the value of \( S \)'s \( k_\alpha \) in the first stage -- where \( S \) focuses on payoffs -- should be greater than that of the second stage --
where $S$ focuses on probabilities. The Advantage Model supplemented by principle (*) thus predicts the inconsistent indifference judgments exhibited by Delquie et al.'s subjects.

We shall observe other examples of the use of principle (*) in later chapters. In those chapters, each use of principle (*) to supplement the basic formulation of the Advantage Model will be explicitly noted.
Chapter 4

Qualitative Evaluation

The present chapter provides a qualitative evaluation of the Advantage Model. To help situate the model within the field of descriptive choice theories, we shall compare the Advantage Model to its two important predecessors discussed in Chapter 2, Utility Theory and Prospect Theory. The comparisons, however, will consider the theories in their general form -- not in any way limited to the particular versions presented in that chapter. In what follows, we describe ten, well documented phenomena that characterize people's choices between monetary lotteries. Some of the phenomena concern either simple or mixed lotteries, while others apply to lotteries of both kinds. It will be seen that only the Advantage Model is consistent with all ten phenomena. A brief summary of the qualitative evaluation concludes this chapter, while an illustration of some of these phenomena's relevance to nonmonetary domains occurs in Chapter 8, Section 2.

4.1 Limited Solvability

We begin with a general phenomenon that may be conceived as a basic desideratum of any successful theory of choice. Given a simple choice problem \(((d_1,p_1),(d_2,p_2))\) in which three of the four terms are determined, it is often possible to set the fourth term at some value so
as to make either one of the lotteries preferred over the other. For example, given \([(5000, .50), (d_2, .80)]\) there is a \(d_2\) (e.g., 10) that will make the left-hand lottery preferred over the right (for most people), and another \(d_2\) (e.g., 4500) that will make the right-hand lottery preferred over the left. In the same fashion, given \([(5000, .50), (2000, p_2)]\) there is a \(p_2\) (e.g., .95) that will make the right-hand lottery preferred over the left, and another \(p_2\) (e.g., .55) that will make the left-hand lottery preferred over the right. Similar examples can be generated for undetermined \(d_1\) or \(p_1\). Solvability is limited, however, because of choice problems for which the undetermined term cannot be "solved". Thus, given \([(5000, .70), (25, p_2)]\) there is no \(p_2\) that will make the right-hand lottery preferred over the left. Similar comments apply to mixed choice problems as well.

Utility Theory and Prospect Theory both predict solvability in appropriate cases. Given \([(5000, .50), (d_2, .80)]\), for example, both theories predict the existence of a \(d_2\) (close enough to 5000) that will raise the value of the right-hand lottery above that of the left-hand lottery (i.e., \(\pi(.80)*u(d_2) > \pi(.50)*u(5000)\), where for Utility Theory \(\pi\) is the identity function). The Advantage Model also predicts solvability. To see this, consider choice problems of the form \([(+d_1, p_1), (+d_2, p_2)]\) where \(p_2 > p_1\). As \(d_2\) approaches zero, so does EMV_2(p_2-p_1), whereas EMV_1((d_1-d_2)/d_1)k_0 increases. Hence, preference must shift to \((d_1, p_1)\). On the other hand, as \(d_2\) approaches \(d_1\), the term EMV_1((d_1-d_2)/d_1)k_0 approaches zero, whereas EMV_2(p_2-p_1) increases. Hence, preference must shift to \((d_2, p_2)\). A similar logic applies to the other cases. It may also
be seen that all three theories are consistent with the limited nature of solvability.

It is interesting to observe, on the other hand, that a purely comparative version of the Advantage Model -- stripped of the terms EMV₁ and EMV₂ -- fails to predict patent cases of solvability. To see this, suppose that κ₀ = .6 and consider the problem \([(3200,p₁),(1000,.4)]\). No value of p₁ -- even 0 -- renders \([(3200-1000)/3200]\)κ₀ less than .4-p₁. Hence, a purely comparative version of the Advantage Model yields the false prediction that (3200,p₁) is preferred to (1000,.4) for all probabilities p₁. Similar examples can be constructed for any chosen value of κ₀.

To summarize, limited solvability is a fundamental characteristic of human choice, which all three theories under investigation naturally predict.

4.2 Risk aversion and risk seeking

Consider a choice between the simple lottery \((1000,.80)\) and the alternative of receiving $800 for sure. A large majority of people prefer the sure gain over the gamble, despite the fact that the two have equal expected monetary value (namely, 1.0 * 800 and .80 * 1000, respectively). A preference for a sure outcome over a gamble that has higher or equal expected monetary value (EMV) is called risk averse.

In the same vein, consider a choice between the simple lottery \((-1000,.80)\) and the alternative of losing $800 for sure. Now, a large majority of subjects prefer the gamble over the sure loss, despite the
fact that the two, again, have equal EMV's. A preference for a gamble over a sure outcome with equal or higher EMV is called risk seeking. As the examples above illustrate, people's choices are generally characterized by risk aversion in the domain of gains and risk seeking in the domain of losses.¹

Utility Theory and Prospect Theory are able to predict these attitudes towards risk by assuming that the value function of payoffs is generally concave for gains and convex for losses. From the concavity of gains, for example, it follows that the subjective value attached to a gain of $800 is more than $80 of the value attached to a $1000 gain, which predicts a risk averse preference. A similar result, yielding risk seeking preferences, follows in the domain of losses.

The Advantage Model also predicts the appropriate attitudes towards risk. These follow from the model by way of its weighting parameters \( k_a \) and \( k_L \), which are assumed to be smaller than 1 for most people. Any comparison of a simple lottery against a sure outcome of equal EMV leads the Advantage Model to predict equal payoff and probability advantages for the lottery and the sure outcome, respectively. To illustrate, according to the Advantage Model, a choice between the lottery \((1000,.80)\) and the sure outcome of $800 leads to a comparison between \(EMV_1((1000-800)/1000)k_a\) and \(EMV_2(1.0-.80)\) (where \(EMV_1\) and \(EMV_2\) are the EMV's of the gamble and the sure outcome, respectively). Since the EMV's

¹ Risk aversion and risk seeking for gains and losses, respectively, have been confirmed by many investigators. See, e.g., Fishburn and Kochenberger (1979), Payne, Laughhunn and Crum (1980), Hershey and Schoemaker (1980a), and Slovic, Fischhoff, and Lichtenstein, (1982), among others.
are identical, the comparison reduces to \(0.20(k_\alpha)\) versus \(0.20\). For any value of \(k_\alpha < 1\), the Advantage Model predicts a choice of the sure outcome over the lottery. Hence, for any positive choice problem consisting of a simple lottery and a sure outcome of equal EMV the model predicts a risk averse preference.

A similar equality of payoff and probability advantages results from the comparison of a negative lottery and a sure loss of equal EMV. Thus, a choice between \((-1000, .80)\) and \(-\$800\) leads to a comparison between \(PMV_1(0.20)k_L\) and \(EMV_2(0.20)\). Since the EMV's are now equal but negative, any \(k_L < 1\) leads to a choice of the gamble over the sure loss. Hence, for any negative choice problem consisting of a simple lottery and a sure outcome of equal EMV the model predicts a risk seeking preference.

It should be pointed out, of course, that risk averse choice behavior in the case of gains and risk seeking behavior in the case of losses are not limited to choices between lotteries and sure outcomes with equal EMV. All three theories naturally extend their treatment above to pairs of lotteries whose EMV's are not equal.

To summarize, risk aversion and risk seeking characterize people's attitudes towards choice problems involving gains and losses, respectively, and are predicted by all three theories under investigation.

4.3 Loss aversion

Consider the offer to bet on a fair coin for equal stakes. Most people decline this offer because they find the loss of a particular sum
of money to be more painful than they find the gain of an equal sum enjoyable. Thus, given an option to play the lottery \((100,.50,-100)\), most people opt not to play because, for them, the attractiveness of a $100 gain is not sufficient to compensate for the aversiveness of a $100 loss. People's feeling that a loss causes more pain than an equal gain causes pleasure is known as loss aversion. Generally, this notion -- the notion that "losses loom larger than gains" -- need not be confined to payoffs of equal magnitude. (Thus, for example, a person may find the loss of $100 to be more aversive than he finds the gain of $150 attractive.) For clarity of exposition, however, we shall focus in this section on cases that involve equal gains and losses.

Utility Theory and Prospect Theory predict loss aversion by stipulating a value function for payoffs that is steeper for losses than for gains. Thus, the subjective value of -$100 is assumed to be more negative than the subjective value of $100 is positive. This, of course, predicts people's reluctance to accept a fair chance at the two.

Similarly, the Advantage Model predicts loss aversion via the parameters \(k_a\) and \(k_L\), where \(k_L\) -- the relative weight of losses to probabilities -- is assumed to be greater than \(k_a\), the corresponding weight for gains. To observe this, consider the mixed lottery \((100,.50,-100)\), which is composed of simple lotteries \((100,.50)\) and \((-100,.50)\). Recall that the monetary value of a lottery is defined as that sum of money, \(m\), that renders the person indifferent between the lottery and \(m\) at certainty. More specifically, recall (from Section 3.4) that according to the Advantage Model, the monetary value of lottery \((100,.50)\) equals \(m_1\) such that \((100,.50) = (m_1,1)\), and that the monetary
value of \((-100,.50)\) equals \(m_2\) such that \((-100,.50) = (m_2,1)\). According to the model, the values of \(m_1\) and \(m_2\) can be calculated from these equalities, using \(k_\alpha\) in the case of \(m_1\) (a gain) and \(k_L\) in the case of \(m_2\) (a loss). It now follows from the arithmetic of the model that for any values of \(k_\alpha\) and \(k_L\) such that \(k_\alpha < k_L\), \(|m_2| > |m_1|\). Since \(m_2\) is a negative amount and \(m_1\) a positive one, the lottery \((-100,.50)\) is predicted to be more aversive to the person than the lottery \((100,.50)\) is attractive. Hence, with its loss-dimension more aversive than its gain-dimension is attractive, the lottery \((100,.50,-100)\) is predicted by the Advantage Model to be rejected by most subjects, thus leading them to exhibit loss averse behavior.

To summarize, the phenomenon of loss aversion -- the fact that for most people losses loom larger than gains -- is predicted by all three theories under consideration.

4.4 Noninvariance

All analyses of rational choice incorporate the notion of invariance. Invariance requires that people's preferences between alternatives should not depend on the manner in which these alternatives are described (assuming, of course, that ultimately the same information is provided). Different representations of the same choice problem should yield the same preferences.

Kahneman and Tversky, however, have repeatedly demonstrated the failure of invariance in people's choices (see, e.g., Kahneman and Tversky, 1982, 1984; Tversky and Kahneman, 1986). An example of such noninvariance, mentioned briefly in Chapter 2, is illustrated in the
following problems (where the bracketed numbers following each option indicate the percentage of respondents who chose that option):

Problem 1: Assume yourself richer by $300 than you are today. You have to choose between
a sure gain of $100 [72%]
50% chance to gain $200 and 50% chance to gain nothing [28%]

Problem 2: Assume yourself richer by $500 than you are today. You have to choose between
a sure loss of $100 [36%]
50% chance to lose nothing and 50% chance to lose $200 [64%]

The two problems are essentially identical. In both cases the subject faces a choice between $400 for sure and an even chance at $500 or $300. Despite the fact that these problems offer identical choices, however, their different descriptions -- one in terms of gains and the other in terms of losses -- had a substantial effect on subjects' preferences. In particular, as discussed in Section 7.2, subjects made a risk averse choice when the problem was framed as a gain, and a risk seeking choice when it was framed as a loss.

This noninvariant pattern of preferences violates Utility Theory which, in its standard formulation, requires subjects' choices to respect invariance. In particular, according to the standard formulation of Utility Theory, choices are made over anticipated final wealth, not over changes in wealth. The two problems above offer identical chances at identical final wealths and thus cannot be predicted to differ by the standard formulation of Utility Theory. Notice, however, that the revised version of Utility Theory presented in Chapter 2 and tested in Chapters 5-7 addresses changes in wealth rather than final assets. This revision of Utility Theory -- borrowed, as mentioned earlier, from Prospect Theory -- was applied precisely in order to make it possible for the theory to
address the notion of gains and losses independently of total wealth. This revised version predicts the choices illustrated above in a fashion similar to that of Prospect Theory. We discuss Prospect Theory next.

Prospect Theory is explicitly defined on gains and losses rather than on total wealth. It is able to predict the pattern of choices exhibited above by assuming that subjects' decisions in these problems focus on the choices available and do not integrate available information about present wealth. Ignoring minor changes in total wealth, subjects are predicted to make their choices exhibiting the usual risk aversion for gains and risk seeking for losses. Thus, they are predicted to make their choices in the two problems in a way that leads -- in the final outcome -- to extensionally incongruent preferences.

The Advantage Model is similarly defined over gains and losses rather than total wealth levels. Similar to Prospect Theory, the model assumes that subjects' decisions in the problems above focus entirely on the choice available and do not integrate information regarding current wealth. As with Prospect Theory, this leads the Advantage Model to predict noninvariant choice behavior. In fact, for the choices presented in Problems 1 and 2 above, any values of $k_\alpha$, $k_L < 1$, lead the model to predict exactly the pattern of preferences exhibited by the majority of subjects. We leave the verification of this fact to the reader.

It is interesting to note, finally, that subjects' noninvariant choice behavior often leads them to violate other fundamental principles of rational choice. One such principle is the dominance principle. Perhaps the most obvious and compelling principle of rational choice, the
dominance principle states that if one option is better than the other on one dimension and is at least as good as the other on all the rest, then that option should be chosen. Thus, for example, given a choice between:

A: 25% chance to win $240 and 75% chance to lose $760.
B: 25% chance to win $250 and 75% chance to lose $750.

the dominance principle predicts that all subjects should prefer option (B) over option (A).

Consider, however, the following two choices, one involving gains and the other losses, presented by Tversky and Kahneman (1981) to a group of 150 undergraduate students:

Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.

Decision (i). Choose between:
C: a sure gain of $240 [84%]
D: 25% chance to gain $1000 and 75% chance to gain nothing [16%]

Decision (ii). Choose between:
E: a sure loss of $750 [13%]
F: 75% chance to lose $1000 and 25% chance to lose nothing [87%]

The percentage of students who chose each option is indicated in brackets. As expected from the previous discussion, the majority choice in decision (i) -- the decision involving gains -- is risk averse, while the majority choice in decision (ii) -- involving losses -- is risk seeking. Furthermore, it follows from the percentages above that at least 70% of the subjects chose both lotteries C and F. Because the subjects considered the two decisions simultaneously, they expressed, in effect, a preference for the combination of lotteries C and F over the combination D and E. Notice, however, that lotteries C and F combined yield the equivalent of lottery A, while lotteries D and E combined yield the
equivalent of lottery B. Thus, while subjects express one preference when the options are presented in a condensed form, they express the opposite preference when the options appear in a different format. This particular instance of noninvariance, moreover, leads subjects to choose a dominated alternative, thus failing to respect the most compelling principle of rational choice.

Notice, incidentally, that both Prospect Theory and the Advantage Model (as well as the revised version of Utility Theory) are able to predict the outcome above. In the first case (the choice between A and B) all the theories trivially predict a choice of the dominating alternative. In the following two choices (Decisions i and ii) the theories predict a majority risk averse choice in the first (gain) case and a majority risk seeking choice in the second. Thus, for example, given any \( k_a, k_L < .95 \), the Advantage Model predicts the choices of options B, C, and F, in the three problems, just as was exhibited by the majority of subjects.

To summarize, both Prospect Theory and the Advantage Model are consistent with the noninvariant nature of people's choice behavior. The classical version of Utility Theory cannot predict this phenomenon, while its revised version presented here can.

4.5 Cost-Loss

A special case of noninvariance is illustrated by what Kahneman and Tversky (1984) call the cost-loss phenomenon. This phenomenon arises when an aversive situation faced by a decision maker can be framed as involving either a cost or a loss. In such cases, the cost-loss
discrepancy can lead to noninvariant behavior. Consider, for example, the purchase of insurance. The price of insurance is usually regarded as a cost intended to insure against particular risks. Alternatively, however, the purchase of insurance may be framed as a choice between a sure loss and a risk at a greater loss. Slovic, Fischhoff, and Lichtenstein (1982) enquired about their subjects' willingness to pay $50 for insurance against a 25% risk of losing $200. Then, these authors presented subjects with a choice between a sure loss of $50 and a 25% chance to lose $200. While in the insurance condition only 35% of the subjects refused to pay the $50, in the choice condition 80% of the subjects expressed a risk seeking preference for the gamble over the sure loss. Similar results, showing noninvariant patterns of preference between paying costs and choosing losses, are reported by Hershey and Schoemaker (1980a) and by Schoemaker and Kunreuther (1979).

Neither Prospect Theory nor Utility Theory can predict this phenomenon since neither theory has a principled way to distinguish between what are perceived as costs and what are perceived as losses. The Advantage Model, on the other hand, separates the situations above into a situation of choice in one case and of monetary value estimation in the other. The subjects presented with a choice between a sure loss of $50 and a 25% chance to lose $200, are assumed to entertain a simple choice problem of the form \((-200,.25),(-50,1)\). On the other hand, subjects in the insurance condition are assumed to evaluate whether, for them, a 25% chance to lose $200 is worth more or less than -$50. They engage, in other words, in the monetary value estimation, \((-200,.25) = (x,1)\). The majority's willingness to pay the $50 indicates that for them \(x\) is lower.
than -50. They prefer to pay $50 than to accept a lottery whose value for them is lower. Recall, finally, that since the subjects in the insurance condition are asked to consider a price, their attention -- according to principle (*) -- is assumed to focus on payoffs, thereby raising the value of their k. Thus, according to the Advantage Model, one scenario is a simple choice situation, and the other is a situation of monetary value estimation involving a higher $k_L$. The two situations lead to noninvariant preferences since in the former subjects choose the gamble, but in the latter they assign it a lower monetary value than that assigned to the sure loss. In fact, the reader may verify that the Advantage Model with any $k_L < 1$ for choice, and any $k_L > 1$ for monetary value estimation predicts exactly the pattern of preferences exhibited by the majority of subjects in the cost-loss phenomenon above.

An interesting variant of the cost-loss phenomenon may be observed in mixed lotteries, involving gains as well as losses. Consider, for example, the following pair of problems which were posed, separated by a short filler question, to 132 subjects by Kahneman and Tversky (1984):

Problem 1: Would you accept a gamble that offers a 10% chance to win $95 and a 90% chance to lose $5?

Problem 2: Would you pay $5 to participate in a lottery that offers a 10% chance to win $100 and a 90% chance to win nothing?

Although it is easy to verify that the two problems offer objectively identical options, 55 of the respondents expressed different preferences in the two versions. Of these, 42 rejected the gamble in problem 1 but accepted the objectively equivalent lottery of problem 2. As in the previous case, spending x amount of money as a cost seems less
aversive to some subjects than incurring an equal amount as a loss. Thus, while these subjects rejected the lottery (95,.10,-5), they were willing to pay $5 for the lottery (100,.10). Observe that, unlike the previous case, here both problems are instances of monetary value estimation. The latter, simple lottery was worth more than $5 to the subjects, while the former, mixed lottery had a negative monetary value overall. (Recall from Section 3.4 that the monetary value of a mixed lottery equals the sum of the values of its component, simple lotteries.) Notice, however, that while payoffs are discussed in the second problem, thus leading the subject to focus his attention on payoffs and -- according to principle (*) -- raising the value of his k, no change of k occurs in the first problem. The monetary value of (95,.10,-5) in the first problem is simply the sum of m₁ and m₂ such that (95,.10) = (m₁,1) and (-5,.90) = (m₂,1), where the kₒ and kₗ figuring in the first and second calculations, respectively, are the subject's regular kₒ and kₗ. The monetary value of (100,.10) in the second problem is that sum of money, m, such that (100,.10) = (m,1). Because the subject's attention is now focused on payoffs, however, the value of kₒ is predicted to go up. In fact, with regular values of, say, kₒ = .4 and kₗ = .6, and an elevated value of kₒ = .5, the Advantage Model predicts the pattern of preferences exhibited by the majority of the noninvariant subjects above. The range of k-values that leads the model to predict this particular pattern of preferences is significantly more restricted than that required to predict the previous instance of the cost-loss phenomenon. It is encouraging to observe, therefore, that the proportion of subjects who exhibited the latter pattern is significantly smaller than that which exhibited the previous pattern.
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To summarize, while the Advantage Model is consistent with the cost-loss phenomenon as exhibited on a couple of well documented choice problems, neither Utility Theory nor Prospect Theory can predict this phenomenon.

4.6 Constant difference

Consider simple choice problem \([(d_1,p_1),(d_2,p_2)]\), where \(p_2 > p_1\). If person S prefers \((d_1,p_1)\) in this problem, it is possible to find a sum \(m\) of money such that S prefers \((m+d_2,p_2)\) in \([(m+d_1,p_1),(m+d_2,p_2)]\). For example, if S prefers the left-hand lottery in \([(300,.30),(50,.80)]\), he is likely to prefer the right-hand lottery in \([(1300,.30),(1050,.80)]\). A similar shift of preference -- but in the opposite direction -- can be shown for negative choice problems.

Prospect Theory and Utility Theory are able to predict the constant difference phenomenon by assuming that the value function of payoffs is generally concave for gains and convex for losses. Given such a function, as payoffs increase by the same amount their difference in value decreases. Since the probabilities are left unchanged, this yields a change in the relative attractiveness of the two lotteries, in the required direction for gains and for losses.

The Advantage Model also predicts the constant difference phenomenon because it is based on payoff ratios rather than payoff differences. For any simple choice problem \([(d_1,p_1),(d_2,p_2)]\) (where \(p_2 > p_1\)), as we keep the difference between the two payoffs constant and increase their size, the ratio \((d_1-d_2)/d_1\) decreases. At the same time, the absolute value of \(EMV_2\) increases faster than that of \(EMV_1\) since \(p_2 > p_1\). Hence, as payoffs...
are increased preference must eventually shift to the right-hand lottery 
in a positive problem, and to a left-hand lottery in a negative problem. 
We illustrate with the example above. According to the Advantage Model, 
[(300,.30),(50,.80)] gives rise to a comparison between 
(300)(.30)[(300-50)/300]ka and (50)(.80)(.80-.30), i.e., between 75ka and 
20. On the other hand, [(1300,.30),(1050,.80)] gives rise to a comparison 
between (1300)(.30)[(1300-1050)/1300]ka and (1050)(.80)(.80-.30), i.e., 
between 75ka and 420. For a large range of ka's (specifically, .27 < ka < 
5.6) the left-hand lottery will be preferred in the first problem but the 
right-hand lottery in the second, which predicts the reversal.

Thus, the constant difference phenomenon follows from all three 
theories' treatment of payoffs.

4.7 Reflection

The next phenomenon has been called the "reflection effect" by 
Kahneman and Tversky. It is illustrated by the following pair of 
problems (taken from Table 1):

\[ [(4000,.20),(3000,.25)] \quad [(4000,.20),(3000,.25)] \]

These simple choice problems are identical except that one involves 
positive and the other negative lotteries. The reflection phenomenon 
refers to the fact that people shift their preferences between these 
problems.

Utility Theory and Prospect Theory can account for this finding by 
means of their hypothesized value function, which is assumed to be 
(roughly) symmetrical around the zero, or "reference" point of the curve.
For example, a preference for the left-hand lottery in \([(4000, .20), (3000, .25)]\) entails that \(\pi(.20)u(4000) > \pi(.25)u(3000)\) (for Utility Theory \(\pi\) is the identity function). This, in turn, entails that \(u(4000)/u(3000) > \pi(.25)/\pi(.20)\). Now, symmetry around the zero point suggests that \(u(-4000)/u(-3000) > \pi(.25)/\pi(.20)\). But since the values of \(u\) are now negative, \(\pi(.20)u(-4000) < \pi(.25)u(-3000)\), thus predicting the shift of preference.

The Advantage Model also predicts the reflection effect. For, in the transition from \([(+d_1,p_1), (+d_2,p_2)]\) to \([(-d_1,p_1), (-d_2,p_2)]\), the attractiveness coefficients \(\text{EMV}_1((d_1-d_2)/d_1)k\) and \(\text{EMV}_2(p_2-p_1)\) both shift from positive to negative sign. The direction of the inequality between these coefficients is thus likely to reverse as well. Notice, however, that since the coefficients are compared using \(k_0\) in the case of positive lotteries and \(k_L\) in the case of negative lotteries, the Advantage Model does not predict necessary reflection for every pair of positive-negative variants. This is consistent with Hershey and Schoemaker's (1980b) results indicating that while the reflection effect is quite common, it is by no means pervasive. ²

To summarize, the reflection effect is consistent with all three theories' accounts of choice between simple lotteries.

---

2. The predicted generality of the reflection effect according to Prospect Theory depends on the specific shapes proposed for the value and probability weighting functions. For a detailed discussion, see Hershey and Schoemaker, 1980b.
4.8 Common ratio

The common ratio phenomenon is illustrated by problems 3 and 4 of Table 1. While most people prefer the right-hand lottery in the simple choice problem $[(6000,.45),(3000,.90)]$, they prefer the left-hand lottery in $[(6000,.001),(3000,.002)]$. Notice that the probability ratios in the two problems are the same (namely, 2:1).

The foregoing preference shift cannot be predicted by Utility Theory since in both problems according to that theory $u(3000)$ is multiplied by twice as much as is $u(6000)$. Prospect Theory, on the other hand, does account for the common ratio phenomenon. The introduction of "decision weights" via the function $\pi$ allows $\pi(.90)/\pi(.45)$ to be greater than $\pi(.002)/\pi(.001)$. Now, $u(3000)$ is multiplied by a greater weight relative to $u(6000)$ in the problem involving the higher probabilities than in the problem with the low probabilities. This fact allows derivation of the common ratio phenomenon. (The common ratio phenomenon is predicted by certain other absolute theories, which may also be conceived as weakenings of Utility Theory; see, e.g., Machina, 1982).

The Advantage Model predicts the common ratio phenomenon because it is based on probability differences rather than probability ratios. While everything else remains essentially the same, the probability advantage of the right-hand lottery in the problems above changes from (.90-.45) in the first problem to (.002-.001) in the second. Thus, the advantage of this lottery is significantly smaller in the second problem than in the first, which for a wide range of $k_0$-values predicts a corresponding shift in preference. In their discussion of the common ratio effect, Kahneman
and Tversky (1979) advance the following generalization (here presented for positive lotteries; a similar generalization -- in the opposite direction -- applies to negative lotteries). If \((d_1, pq)\) is preferentially equivalent to \((d_2, p)\), then \((d_1, pqr)\) is preferred to \((d_2, pr)\), \(0 < p, q, r < 1\). This property is incorporated by Kahneman and Tversky into Prospect Theory. It can be shown formally that the above generalization is a necessary outcome of the Advantage Model.

The following phenomenon is related to the common ratio effect. People seem to overweigh outcomes that are considered certain compared to outcomes that are merely probable. Kahneman and Tversky (1979) dub this tendency "the certainty effect" and illustrate it with the following pair of problems, in both of which \(p_1\) is 80\% of \(p_2\).

\[
((4000, .20),(3000, .25)) \quad ((4000, .80),(3000, 1))
\]

Notice that these problems also instance the common ratio phenomenon since the probabilities in the first problem are each one-quarter of their counterparts in the second problem. It is not clear, therefore, whether the certainty effect is a separate feature of choice behavior or just a special case of the common ratio phenomenon. Following Kahneman

3. The proof is as follows:

If \((d_1, pq)\) is preferentially equivalent to \((d_2, p)\), then

\[
(1) \quad d_1(pq)/(d_1-d_2)/d_1k_2 = d_2(p)(p-pq).
\]

A comparison of \((d_1, pqr)\) with \((d_2, pr)\) yields:

\[
(2) \quad d_1r(pq)/(d_1-d_2)/d_1k_2 \quad \text{vs.} \quad d_2r^2(p)(p-pq), \quad \text{or:}
\]

The right-hand term in statement (2) is equal to the right-hand term in equation (1) multiplied by \(r^2\). The left-hand term in statement (2) is equal to the left-hand term in equation (1) multiplied by \(r\). Since \(r < 1\), \(r^2 < r\), and the left-hand term of statement (2) is larger than the right. Thus, \((d_1, pqr)\) is preferred over \((d_2, pr)\).
and Tversky, let us assume that the certainty effect is a separate phenomenon and consider how the Advantage Model can account for it.

By definition, the certainty effect occurs in cases where one of the alternatives is not a simple lottery but rather an outcome at certainty. The context, therefore, is no longer one of choice between simple lotteries but, rather, a situation where -- due to the certainty offered by one of the alternatives -- probabilities have been made salient. In this context, principle (*) states that the value of a person's k (either \( k_a \) or \( k_L \)) typically diminishes (thereby increasing the relative weight of probabilities). Thus, a choice problem \( (d_1,p_1),(d_2,1) \) is evaluated on the basis of a comparison between \( \text{EMV}_1[(d_1-d_2)/d_1](k') \) and \( \text{EMV}_2(1-p_1) \), where \( k' < k \), whereas problems with \( p_2 < 1 \) are evaluated -- as usual -- using k. The result is that for positive problems preference is biased towards \( (d_2,p_2) \) when \( p_2 = 1 \).

The foregoing use of principle (*) can be seen to have another satisfying consequence. Kahneman and Tversky (1979) show that the certainty effect is not just a special case of risk aversion. For, in negative lotteries subjects tend to exhibit risk seeking preferences in an attempt to avoid sure losses. This is illustrated in the following pattern of choices, which is the "reflection" of the previous one.

\[ [(-4000,.20),(-3000,.25)] \quad [(-4000,.80),(-3000,1)] \]

This reflection/certainty phenomenon is predicted by the Advantage Model supplemented by principle (*) since both the values of \( k_a \) and \( k_L \) are predicted to decrease in the positive and negative cases, respectively.
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In summary, while both Prospect Theory and the Advantage Model predict the common ratio phenomenon via their treatment of probabilities, Utility Theory cannot predict this phenomenon.

4.9 Intransitivity

Consistent intransitivity of preferences can be demonstrated in people's choices among lotteries. One such intransitivity, discovered by Tversky (1969), involves the following five lotteries:

a) (5.00, 7/24)
b) (4.75, 8/24)
c) (4.50, 9/24)
d) (4.25, 10/24)
e) (4.00, 11/24)

Many subjects prefer (a) to (b), (b) to (c), (c) to (d), (d) to (e), but (e) to (a).

Because both Utility Theory and Prospect Theory assign attractiveness coefficients to each lottery independently of the alternatives against which it is being compared, and because coefficients are then compared numerically, neither theory is able to predict intransitivity. In contrast, the Advantage Model -- in virtue of its comparative component -- predicts intransitivity in certain cases. According to the model, for example, the relative attractiveness of lottery (a) above differs when it is compared with lottery (b) from when

---

4. Tversky and Kahneman (1986) invoke "editing" strategies to explain intransitivity. Thus, for the example cited above, it might be assumed that the probabilities of adjacent lotteries are considered -- via editing -- to be identical, whereas the probabilities of lotteries (a) and (e) differ enough to affect evaluation and choice. Of course, as Tversky and Kahneman (1986, p.5273) point out, intransitivity of preference may result from more than one psychological mechanism.
it is compared with lottery (e). The reader may verify that the Tversky-intransitivity above follows from the Advantage Model with $k_A = .91$.

Although, to the best of our knowledge, intransitivity of preferences has never been demonstrated with mixed lotteries, we observe that the Advantage Model predicts cases of intransitivity for these lotteries as well. Consider, for example, the following three mixed lotteries:

$$f) \ (20,.20,-5)$$
$$g) \ (10,.40,-8)$$
$$h) \ (6,.60,-13)$$

The Advantage Model predicts that any subject whose $k_A = k_L = .5$ will prefer (f) to (g), (g) to (h), but (h) to (f). The arithmetic required to verify this claim is provided in Appendix D.

To summarize, due to their absolute character, neither Utility Theory nor Prospect Theory can predict intransitivity of preferences in people's choices. The Advantage Model, on the other hand, predicts intransitivity in certain cases, both for simple and for mixed lotteries.

4.10 Preference reversal

Preference reversal occurs when subjects indicate a preference for one lottery in a choice problem, but then assign a larger monetary value to the other. An example of preference reversal that we have repeatedly observed is as follows. Given $[(10,.60),(5,.80)]$ subjects often prefer the right-hand lottery but assign higher monetary value to the left-hand lottery. Numerous experimental studies have revealed consistent preference reversals in a majority of subjects. (See Slovic and Lichtenstein, 1983, for a review).
Because of their absolute character neither Utility Theory nor Prospect Theory can predict preference reversal. According to both theories, each lottery has an inherent attractiveness coefficient with respect to a given person. The lottery with the larger coefficient should both be preferred and have a larger monetary value. In contrast, we shall now see that preference reversal follows from the Advantage Model by considering -- as usual -- the monetary value of a lottery \((d,p)\) for a person \(S\) to be that sum \(x\) of money that renders \(S\) indifferent between \((d,p)\) and \((x,1)\). Consider a person \(S\) for whom \(k_\alpha = .25\). The Advantage Model predicts that \(S\) prefers the right-hand lottery in the problem \([(10,.60),(5,.80)]\). The following calculations demonstrate that the Advantage Model also predicts that \(S\)'s monetary value will be larger for the left-hand lottery than for the right.

\[
\begin{align*}
(10,.60) &= (x,1) \\
6(1-x/10)(.25) &= x(1-.60) \\
1.5 - .15x &= .4x \\
x &= 2.73
\end{align*}
\]

\[
\begin{align*}
(5,.80) &= (x,1) \\
4(1-x/5)(.25) &= x(1-.80) \\
1 - .2x &= .2x \\
x &= 2.5
\end{align*}
\]

Notice, moreover, that the above calculations are instances not of simple choice problems but of monetary value estimations. As discussed in Section 3.4, such estimation is assumed to focus the subject's attention on payoffs and thus -- according to principle (*) -- to increase the value of his \(k\)'s. It is easy to verify that a larger \(k_\alpha\) in the monetary value calculations above is likely to produce a still more pronounced preference reversal than that produced by the Advantage Model without principle (*). For example, suppose that \(k_\alpha\) rises from its original value of .25 in the context of the choice problem \([(10,.60),(5,.80)]\) to .80 in the associated monetary value estimation task. Then, calculations like
those above yield monetary values of 4.74 and 3.53 for the left- and right-hand lotteries, respectively. These differ from each other more than the monetary values predicted without principle (*) and thus yield a more pronounced reversal. More generally, it follows from the Advantage Model that as a person's k increases, the amount of his monetary value, x, for lottery (d,p) approaches d from below (i.e., gets bigger towards d for positive lotteries, and gets smaller towards d for negative ones).a

It is worth pointing out that principle (*) is crucial for the Advantage Model's explanation of preference reversal. For, without this principle, the model is forced into false predictions. Thus, it follows from the arithmetic of the Advantage Model (when stripped of principle (*)) that for any positive choice problem \(((d_1,p_1),(d_2,p_2))\) where \(p_2>p_1\) and \(EMV_1 = EMV_2\), if \(k < 1\), the model cannot assign a higher monetary value to \((d_1,p_1)\), and, therefore, will not allow any preference reversals

5. Let us show that, according to the model, no matter how large a person's k may be, his monetary value, x, for lottery (d,p) will always remain smaller (in absolute value) than d. The proof is as follows:

Since x is the monetary value of (d,p),
(d,p) = (x,1). Therefore, according to the Advantage Model,
dp[(d-x)/d]k = x(1-p),
pdk - pxk = x - xp,
pdk = x(1 - p + pk), and thus
x = d * \{ pk / (1 - p + pk) \}.
Now, for any k > 0, since 0 < p < 1, (1-p) must be positive and
\{ pk / ((1-p) + pk) \} < 1. Therefore, |d| > |x|. 

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once \((d_2, p_2)\) is chosen. * Invoking principle (*) allows reversals to occur in this kind of case.

Our account of preference reversals in simple choice problems may be partially tested with the help of data reported by Goldstein and Einhorn (1987). Among other tasks, these investigators had subjects choose between lotteries in simple choice problems, and also had the subjects determine the monetary value of each lottery appearing in a problem. Following, is the list of simple choice problems appearing in Goldstein and Einhorn (1987). All problems are listed so that the lottery offering the higher chance at a smaller payoff is on the right, and the lottery offering the greater payoff with a smaller probability is on the left.

----------

6. The proof is as follows.

The monetary equivalents of \((d_1, p_1)\) and \((d_2, p_2)\) are \(x_1\) and \(x_2\), respectively:

\((d_1, p_1) = (x_1, 1)\) and \((d_2, \frac{1}{2}) = (x_2, 1)\).

The arithmetic of the Advantage Model yields:

\[ p_1d_1k_α = x_1(1-p_1) + x_1p_1k_α \quad \text{and} \quad p_2d_2k_α = x_2(1-p_2) + x_2p_2k_α. \]

Since \(EMV_1 = EMV_2\), \(p_1d_1k_α = p_2d_2k_α\). Therefore,

\[ x_1(1-p_1) + x_1p_1k_α = x_2(1-p_2) + x_2p_2k_α, \]
and

\[ x_1 / x_2 = [(1-p_2) + p_2k_α] / [(1-p_1) + p_1k_α]. \]

Now, since \(p_2 > p_1 > 0\), for any \(k_α < 1\) the ratio on the right side of the equality must be \(< 1\). Therefore, \(x_1\) must be less than \(x_2\).

To show that when supplemented with principle (*) the model allows reversals in the formerly-denied direction, we provide the following example, where \(EMV_1 = EMV_2\), \(k_α = .7\), and where \(k_α\) is doubled when assigning monetary value.

\[ (40, .4), (20, .8)^* \]

\[ 16(40/40)k_α \text{ vs. } 16(.8-.4) \]

\[ 16\{1-x/40\}(1.4) = .6x \quad \text{and} \quad 16\{1-x/20\}(1.4) = .2x \]

\[ 8(.7) \text{ vs. } 6.4 \]

\[ x = 19.31 \quad \text{and} \quad x = 16.97 \]

Thus, while \((d_2, p_2)\) is preferred, \((d_1, p_1)\) is assigned a higher monetary value.

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Over all six problems, 40% of the subjects' responses yielded preference reversals of kind (♯):

(♯) choice of the right-hand lottery coupled with a higher monetary value for the left-hand lottery,

and only 2% of the subjects manifested preference reversals of the opposite sort (###):

(###) choice of the left-hand lottery coupled with a higher monetary value for the right-hand lottery.

These results are consistent with the Advantage Model as supplemented by principle (*). Recall that principle (*) requires a higher $k$ for monetary value estimation than for choice. And indeed, according to the Advantage Model, a person with $k_\alpha \geq .9$ for monetary value estimation and $k_\alpha \leq .5$ for choice must manifest preference reversal of kind (♯) in all six of the simple choice problems above. In addition, no subject with a $k_\alpha$ below .5 can manifest preference reversals of kind (###) on any of Goldstein and Einhorn's simple choice problems. To the extent that subjects' $k_\alpha$ tends to fall below .5, this result concords with the low frequency of this kind of preference reversals observed by Goldstein and Einhorn. We leave to the reader the arithmetic needed to verify the claims above.

As mentioned in the discussion of principle (*) (Section 3.4), an increase in the relative weight of payoffs during monetary value
estimation, as proposed by the principle, is consonant with proposals made by other researchers. Of particular relevance to the present analysis, is Tversky, Slovic, and Kahneman's (1988) rather subtle demonstration that the preference reversal phenomenon is not a simple case of intransitivity, as was initially assumed. Instead, these investigators argue that the preference reversal phenomenon results mainly from the overpricing of lotteries during monetary value estimation. As demonstrated above, an overpricing of lotteries is the precise effect of an increase in the subject's k.

Finally, we extend our illustration of the Advantage Model's account of preference reversal from simple to mixed choice problems. Following, is the list of mixed choice problems used in Experiment 1 of Lichtenstein and Slovic's (1971) well known exposition of preference reversal. As before, all problems are listed so that the lottery offering the higher chance at a smaller gain is on the right.

\[
\begin{align*}
((16.00, .33, -2.00), (4.00, .99, -1.00)) \\
((8.50, .40, -1.50), (2.50, .95, -.75)) \\
((6.50, .50, -1.00), (3.00, .95, -2.00)) \\
((5.25, .50, -1.50), (2.00, .90, -2.00)) \\
((9.00, .20, -.50), (2.00, .80, -1.00)) \\
((40.00, .10, -1.00), (4.00, .80, -.50))
\end{align*}
\]

Over seventy percent of Lichtenstein and Slovic's subjects consistently committed preference reversal on these problems. An overwhelming majority of these reversals were again of kind (\#) above. In fact, 73% of all subjects always assigned a higher monetary value to the left-hand lottery after having chosen the right-hand lottery.
Once again, these results are consistent with the Advantage Model as supplemented by principle (*). A wide range of $k_α, k_L$-pairs predict choice of the right-hand lotteries and -- when increased due to principle (*) -- a higher monetary value for the left-hand lotteries. For example, any subject with $k_α < .6$ and $k_L < 1$ for choice, and $k_α = k_L > 1.1$ for monetary value estimation, is predicted to exhibit preference reversal of the popular kind on all of Lichtenstein and Slovic's problems above. Appendix E illustrates the Advantage Model's prediction of preference reversal on the fourth problem.

In summary, because of their absolute character, neither Utility Theory nor Prospect Theory can predict the preference reversal phenomenon. The Advantage Model, on the other hand, predicts preference reversal for both simple and mixed lotteries.

4.11 Summary

In this chapter we compared the three competing theories of risky choice under investigation -- Utility theory, Prospect Theory, and the Advantage Model -- on ten phenomena that characterize people's choices between monetary lotteries. Utility Theory -- aided by a modification due to Prospect Theory -- is able to predict six of the phenomena, namely, limited solvability, risk seeking and risk aversion, loss aversion, noninvariance, constant difference, and the reflection effect. Prospect

7. Of course, supplementing Utility Theory or Prospect Theory with a notion similar to principle (*) may render these theories capable of predicting preference reversal as well. Given the nature of these theories, however, it is not clear what a psychologically motivated revision of this kind may look like, that will "supplement" rather than simply weaken these theories.
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Theory predicts seven phenomena, namely, the previous six plus common ratio. The Advantage Model predicts all ten, including the cost-loss phenomenon, intransitivity, and preference reversal. Of course, the Advantage Model cannot claim to provide an exhaustive account of these phenomena, since any one of them might rest upon multiple mechanisms (cf. footnote 4). This notwithstanding, the underlying conception of the Advantage Model (viz., a combination of absolute and comparative strategies) seems well supported, and qualitatively superior to those of its important competitors.

In order to become a seriously considered alternative, however, a theory of choice should perform well not only on qualitative grounds, but on quantitative grounds as well. To the extent that the Advantage Model proves inferior to its competitors on quantitative analyses, the search for a preferred descriptive theory will remain elusive. On the other hand, if the Advantage Model proves not to be quantitatively inferior to either of its important predecessors, then its qualitative superiority should provide sufficient grounds for declaring it a serious alternative description of human choice behavior. It is to a quantitative evaluation of the model that we turn next.
Chapter 5

Quantitative Evaluation,
Methodological Considerations

We turn now to a quantitative evaluation of the Advantage Model. Three experiments were conducted, designed to test our model against comparable versions of competing theories. Specifically, we compared the Advantage Model to the versions of Utility Theory and Prospect Theory described in Chapter 2. The theories were evaluated in terms of their ability to predict individual choice as well as group preference. These evaluations were conducted both on simple and on mixed choice problems.

The present chapter considers various methodological issues. Among other things, it describes in detail the manner in which we conducted both the within-subject as well as the group analyses. Chapter 6 presents the three 'experiments that were conducted and analyzes each of the theories' ability to predict the obtained results. In Chapter 7 we provide additional analyses -- using a 1-parameter version of the Advantage Mode -- of the results reported in Chapter 6. Also in Chapter 7, we present an additional set of choice problems that are "nonconflictual" and, therefore, lie outside the domain of the Advantage Model. We then discuss the ability of a relaxed version of the Advantage Model to predict these special problems. A general discussion, at the end
of Chapter 7, concludes our quantitative evaluation of the Advantage Model.

5.1 Preference rather than indifference

The Advantage Model yields strong predictions concerning indifference judgments. For example, in the case of simple lotteries, if person S is indifferent between lotteries \((d_1, p_1)\) and \((d_2, p_2)\), then he should be indifferent between any pair of lotteries \((d_3, p_1)\) and \((d_4, p_2)\) such that \((d_3 - d_4)/d_3 = (d_1 - d_2)/d_1\). For another example, according to the Advantage Model a person is predicted not to exhibit indifference among any three simple lotteries with equal expected value. Given lotteries \((d_1, p_1)\), \((d_2, p_2)\), and \((d_3, p_3)\), with \(d_1 p_1 = d_2 p_2 = d_3 p_3\) and \(p_3 > p_2 > p_1\), if person S is indifferent between \((d_1, p_1)\) and \((d_2, p_2)\) then, according to the Advantage Model, S must prefer \((d_3, p_3)\) over either of the former. (These two predictions are easily deduced from the arithmetic of the Advantage Model.) The intuition of indifference, however, is usually unstable for even the most cooperative subject. For this reason we have designed our experiments around the sturdier judgment of strict preference. Every problem presented to a subject yields a choice of exactly one lottery. For every problem, each theory -- the Advantage Model, Utility Theory, and Prospect Theory -- predicts which lottery should be preferred. A theory predicts a subject's choice correctly just in case it predicts preference for the lottery that the subject chooses. The next two sections describe how the theories' predictions were analyzed. The first concerns prediction of individual choice and the second describes group preference.
5.2 Within-subject analyses

Consider first the Advantage Model. Each pair of values assigned to the parameters $k_a$ and $k_L$ leads the Advantage Model to make specific predictions about which lottery should be preferred in each choice problem. At the same time, each subject chooses a specific lottery in each problem. Thus, each pair of values assigned to the parameters $k_a$ and $k_L$ leads the Advantage Model to make a definite number of true predictions about the choices of an individual subject. Because subjects did not have the option to indicate indifference between lotteries, any prediction of indifference by the Advantage Model, using a particular $k_a,k_L$-pair, was counted as a misprediction. Notice, therefore, that the test of the model was overly conservative in the sense that only predictions of strict preference by the model were ever counted as correct. (An alternative -- less conservative -- approach would have been to count as correct one half of the model's predictions of indifference.) We call a $k_a,k_L$-pair "optimizing" with respect to a given subject if no other pair of values $k_a,k_L$ leads the Advantage Model to a greater number of true predictions about that subject's choices. Preliminary searches of the parameter space indicated that a subject's optimizing $k_a$ and $k_L$ are almost certain to fall in the interval $[0, 3]$. Consequently, for each subject in each experiment we computed the lowest optimizing $k_a,k_L$-pair in the interval $[0, 3]$, proceeding by increments of .075. A subject's lowest optimizing $k_a,k_L$-pair was defined as the lowest, in the sense of lexicographic ordering, among the subject's optimizing pairs. (Lexicographic ordering, a standard computer theoretic notion, corresponds to the ordering of words in a dictionary. We first search the
pairs by their first coordinate, $k_\alpha$; for each $k_\alpha$, we search by the second coordinate, $k_L$. The first optimizing pair discovered this way is retained.) Every combination of values between 0 and 3 at intervals of .075 for both $k_\alpha$ and $k_L$ was thus attempted. This constituted a search through 1600 ($40 \times 40$) different value-pairs for each subject in each experiment. The number of true predictions made by the Advantage Model for each subject, relative to his optimizing $k_\alpha,k_L$-pair was recorded, along with the subject's lowest optimizing $k_\alpha,k_L$-pair.

Consider now Utility Theory. Parallel to the Advantage Model, each pair of values assigned to the parameters $c_\alpha$ and $c_L$ leads the present version of Utility Theory to make a definite number of true predictions about the choices of an individual subject. Again, predictions of indifference were counted as incorrect. Call a $c_\alpha,c_L$-pair "optimizing" with respect to a given subject if no other values of $c_\alpha$ and $c_L$ lead Utility Theory to a greater number of true predictions about that subject's choices. A rough sketch of the resulting utility curves indicates that a subject's optimizing $c_\alpha$ and $c_L$ are almost certain to fall in the interval $[0, 1.5]$. Consequently, for each subject in each experiment we computed the lowest optimizing $c_\alpha,c_L$-pair in the interval $[0, 1.5]$, proceeding by increments of .0375. Just as for the Advantage Model, this constitutes a search through 1600 different $c_\alpha,c_L$-pairs per subject per experiment. The number of true predictions made by the present version of Utility Theory for each subject, relative to his optimizing $c_\alpha,c_L$-pair was recorded, along with the subject's lowest, optimizing $c_\alpha,c_L$-pair.
The within-subject analyses for Prospect Theory followed exactly the same logic. Each pair of values assigned to the parameters $c$ and $w$ leads Prospect Theory to make a definite number of true predictions about the choices of an individual subject. As before, predictions of indifference were counted as incorrect. Again, we call a $c,w$-pair "optimizing" with respect to a given subject if no other values of $c$ and $w$ lead Prospect Theory to a greater number of true predictions about that subject's choices. For each subject we computed his lowest optimizing $c,w$-pair by searching for the lowest optimizing $c$ in the interval $[0, 1.5]$ proceeding by increments of 0.0375. and for the lowest optimizing $w$ in the interval $[-1.14, 1.11]$, proceeding by increments of 0.05625. The latter interval was motivated by requirements of monotonicity: values of $w$ outside this interval lead to nonmonotone functions, which clearly are rejected by Prospect Theory. As before, these values entail a search through 1600 different $c,w$-pairs per subject in each experiment. The number of true predictions made by the present version of Prospect Theory for each subject, relative to his optimizing $c,w$-pair was recorded, along with the subject's lowest, optimizing $c,w$-pair.

The within-subject analyses of each theory yield, for each subject, that subject's pair of optimizing parameters for that theory. By averaging over the optimizing parameters of all subjects in the experiment we obtain an estimate of the theory's optimizing parameters for the entire group. These, in turn, may help us predict group preference. The analyses of group preference are described in the following section.
5.3 Group analyses

We proceed now to a consideration of the theories as descriptions of group preference. We observe at the outset that the ability of a theory to predict group preferences ought not be confounded with its ability to predict the preferences of particular individuals in the group. (For discussion, see Luce, 1959.) Consequently, these group analyses do not contribute to the earlier, within-subject analyses, but rather bear on an independent characteristic of the theories, namely, their ability to predict group data. For the group analyses, we shall attempt to use the competing theories to predict the proportion of subjects opting for one or another lottery in a choice problem. In this use of the theories, the pair of optimizing parameters attributed to a group of subjects is the average optimizing parameter-pair obtained by each theory from the within-subject analysis of the experiment in question. The details of the group analyses are as follows.

The observed advantage of a given lottery in a given choice problem is defined to be the proportion of subjects who chose that lottery in that problem. For example, if the simple choice problem \(((2000,.50),(1000,.60))\) is presented to 100 subjects and 75 choose the left-hand lottery, than the observed advantage of that lottery in that problem is 75/100. Because of the binary-choice nature of our procedure, it is sufficient to focus attention on the observed advantage of one lottery in each choice problem (the observed advantage of the second lottery being 1 minus that of the first). Thus, to carry out our group tests of the theories we computed the observed advantage of one lottery in each choice problem (in particular, the left-hand lottery of each
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choice problem used in the experiments, as listed in Appendix B). We then correlated the observed advantage of these lotteries against their predicted advantage.

Recall from Chapters 2 and 3 that all theories under investigation assign a theoretical attractiveness (in the form of a coefficient) to each lottery in a choice problem. The predicted advantage of a lottery captures the extent to which that lottery's theoretical attractiveness is advantageous over the other lottery's attractiveness. The larger a lottery's theoretical attractiveness relative to its competitor in a choice problem, the larger its predicted advantage. The first obvious candidate that comes to mind for a way to determine a lottery's predicted advantage in a simple choice problem is to divide that lottery's theoretical attractiveness by the sum of the theoretical attractivenesses of both lotteries figuring in the problem. Thus, if the attractiveness coefficient of one lottery in a problem is twice that of another's, say, 20 versus 10, than its predicted advantage (namely, 20/30) will be twice the other's (whose predicted advantage is 10/30). On the other hand, if the two lotteries are assigned equal attractiveness coefficients (e.g., 10) than the predicted advantage of each is 1/2 (i.e., 10/20). In the case of mixed lotteries, however, this formula is insufficient. For, while in simple choice problems the lotteries' theoretical attractivenesses are either both positive or both negative, in the case of mixed problems it is possible for one lottery to be assigned a positive coefficient while the other is assigned a negative one. Cases of this kind require slight modification of the formula above.
Consider a mixed choice problem, and a theory that assigns to the two lotteries in this problem attractiveness coefficients of 10 and -10. Notice, first of all, that a simple addition of the attractiveness coefficients places a 0 in our original formula's denominator and no longer provides an adequate measure of the total amount of attractiveness in the problem. Thus, instead of simply adding the attractivenesses we now add their absolute values. This, once again, gives is a measure of the total amount of attractiveness (both positive and negative) present in the problem. Next, consider the predicted advantage attributed to the lottery whose attractiveness coefficient is 10. According to the present formulation, this lottery's predicted advantage is 10/20. In fact, it is 10/20 regardless of whether the competing lottery's coefficient is -10 or 10. But this, of course, is wrong. The predicted advantage of a lottery must be higher when its competitor has a negative attractiveness than when it has a positive one. In fact, a mixed lottery's predicted advantage must depend on the difference between its predicted advantage and that of its competitor.

The foregoing discussion leads to the following formulation of a lottery's predicted advantage. Consider a (simple or mixed) choice problem. We shall call the two lotteries of this problem the left-hand and right-hand lotteries, and refer to their attractiveness coefficients as L and R, respectively. According to the present formulation, the predicted advantage of the left-hand lottery equals \( L - R / |L| + |R| \). Similarly, the predicted advantage of the right-hand lottery equals \( R - L / |L| + |R| \). The predicted advantage of a lottery is now the difference between that lottery's attractiveness and that of its competitor, divided
by the total amount of attractiveness in the problem. Observe that according to the present formulation, given a choice problem with lotteries whose attractiveness coefficients are 10 and -10, these lotteries' predicted advantages are 1 and -1, respectively. The lottery with the positive attractiveness is predicted to have total advantage over the lottery with the negative attractiveness, while the lottery with the negative attractiveness has total disadvantage. On the other hand, given lotteries with attractiveness coefficients of 10 and 5, these lotteries' predicted advantages are 1/3 and -1/3, respectively. The leading lottery's advantage consists of 1/3 of the total attractiveness present in the problem, while the second lottery has a disadvantage that equals 1/3 of the total attractiveness in the problem.

Notice that by this formula, according to all three theories, the predicted advantages of competing lotteries in a choice problem are always the same value and of opposite sign. This is because in calculating the predicted advantages of competing lotteries, the sum of attractivities in the problem (i.e., the denominator) remains unchanged, while the difference in attractiveness between lotteries (i.e., the numerator) reverses.

To summarize this discussion, the predicted advantage of a lottery (either simple or mixed) in a choice problem equals its theoretical attractiveness in that problem minus the theoretical attractiveness of the competing lottery divided by the sum of the absolute values of both theoretical attractivities.
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As noted earlier, the values of the parameters used in the calculations of theoretical attractiveness are the values of the average lowest optimizing parameter-pair of the experiment in question. These will be denoted \( k_\alpha(\text{av}) \) and \( k_L(\text{av}) \) for the Advantage Model, \( c_\alpha(\text{av}) \) and \( c_L(\text{av}) \) for Utility Theory, and \( c(\text{av}) \) and \( w(\text{av}) \) in the case of Prospect Theory.

Thus, with respect to the Advantage Model, predicted advantage is computed in the following way.

The predicted advantage (according to the Advantage Model) of \((d_1,p_1,e_1)\) in \(\{(d_1,p_1,e_1),(d_2,p_2,e_2)\}\) is

\[
\begin{align*}
\text{AM}_{k_\alpha(\text{av})}k_L(\text{av})\{(d_1,p_1,e_1);(d_1,p_1,e_1),(d_2,p_2,e_2)\} - \\
\text{AM}_{k_\alpha(\text{av})}k_L(\text{av})\{(d_2,p_2,e_2);(d_1,p_1,e_1),(d_2,p_2,e_2)\} \\
\end{align*}
\]

\[
\left|\text{AM}_{k_\alpha(\text{av})}k_L(\text{av})\{(d_1,p_1,e_1);(d_1,p_1,e_1),(d_2,p_2,e_2)\}\right| + \\
\left|\text{AM}_{k_\alpha(\text{av})}k_L(\text{av})\{(d_2,p_2,e_2);(d_1,p_1,e_1),(d_2,p_2,e_2)\}\right|.
\]

Predicted advantage for Utility Theory and Prospect Theory was calculated just as for the Advantage Model. To simplify notation, we define the term \(U(d,p,e)\) as the attractiveness of lottery \((d,p,e)\) according to Utility Theory when that theory is being used, and according to Prospect Theory when that theory is under investigation. Then:
The predicted advantage (according to Utility Theory and Prospect Theory) of \((d_1,p_1,e_1)\) in \([(d_1,p_1,e_1),(d_2,p_2,e_2)] =

\[
\frac{U(d_1,p_1,e_1) - U(d_2,p_2,e_2)}{|U(d_1,p_1,e_1)| + |U(d_2,p_2,e_2)|}
\]

For each theory, we have two measures pertaining to a lottery in a choice problem: the lottery's predicted advantage, which captures the proportion of that lottery's theoretical attractiveness in the problem, and the lottery's observed advantage, which represents the proportion of subjects who chose that lottery in that problem. The latter is a measure of the lottery's relative attractiveness to the group as a whole; the former is a measure of the lottery's relative attractiveness according to the theory. On this basis we calculated the correlation between the observed and predicted advantage of lotteries in choice problems.

5.4 Status of the obtained \(k_0,k_L\)-pair

Consider our search, in the context of the Advantage Model's within-subject analysis, for a subject's optimizing \(k_0,k_L\)-pair. Since a subject's optimizing \(k_0,k_L\)-pair is defined as that pair of values of \(k_0\) and \(k_L\) that lead the Advantage Model to the greatest number of correct predictions about that subject's choices, it is possible for a subject to have more than one such optimizing pair. In other words, there could be a number of \(k_0,k_L\)-pairs such that each leads to the same number of correct predictions for that subject and no other pair leads to a greater number. In fact, if one searches at small enough increments of \(k_0\) and \(k_L\) this
situation is bound to occur, since certain small increments simply will not affect the model's predictions and thus lead to an equal number of correct ones. By convention, we have focused on a subject's lowest optimizing $k_\alpha,k_L$-pair. This is the optimizing pair first arrived at in an increasing, lexicographically ordered search of $k_\alpha$ and $k_L$. Notice, however, that even this pair is somewhat arbitrary, since the search can proceed by first anchoring on either $k_\alpha$ or $k_L$. In our case, the search was structured so that for each value of one parameter ($k_\alpha$), all values of the second parameter ($k_L$) are attempted. Then, the first parameter ($k_\alpha$) is incremented, and all values of the second parameter ($k_L$) are tested again. Once an optimizing number of correct predictions has been reached, that $k_\alpha,k_L$-pair is retained (that is, the current $k_\alpha,k_L$-pair is replaced only by a pair that yields a greater number of correct predictions). The number of correct predictions recorded by this method is robust and does not depend on the particular form of the search. The optimizing $k_\alpha k_L$-pair, however, could have been a different one had the search for the two parameters been embedded in reversed order - first $k_L$ and then $k_\alpha$. (In particular, a reversed order could lead to a lower $k_L$.) Of course, similar comments apply to the within-subject analyses in the contexts of Utility Theory and Prospect Theory.

As the foregoing discussion indicates, our obtained values of a subject's $k_\alpha$ and $k_L$ in the context of the Advantage Model are to some degree arbitrary, as is their relative size, which is also biased by the nature of the search. Thus, although a person's $k_\alpha,k_L$-pair has a clear psychological interpretation in the Advantage Model, we cannot, in our present investigation, reify the $k_\alpha,k_L$-pairs actually obtained. Notice,
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furthermore, that we cannot for that reason generalize the \( k_o, k_L \)-pair obtained for one set of a subject's choices to another set of his choices, since the subject's "real" \( k_o k_L \)-pair may not be the lowest optimizing one obtained for the first set. As was suggested by the qualitative phenomena of Chapter 4 (and by the problems in Table 1), to the extent that the Advantage Model is right a person's values of \( k_o \) and \( k_L \) are bound to fall in the interval \([.2, .9]\). This interval is at least consistent with the optimizing \( k_o k_L \)-pairs obtained in our experiments to be reported in the following chapter.
Chapter 6

Quantitative Evaluation,
Experimental Analyses

This chapter reports three experiments designed to evaluate our model against comparable versions of competing theories. In particular, we employ the results obtained in these choice experiments to test the present version of the Advantage Model against the versions of Utility Theory and Prospect Theory presented in Chapter 2. The theories are evaluated in terms of their ability to predict both individual choice as well as group preference data, as discussed in Chapter 5.

The first experiment involves only simple choice problems, whereas the other two investigate a combination of both simple and mixed choice problems. In Experiment 2, choice problems were presented in a pictorial fashion, while Experiment 3 replicated the same problems verbally. We conclude this chapter with a brief summary of the three experiments. Additional analyses of data reported in the present chapter (as well as novel data) are provided in Chapter 7.
6.1 Experiment 1: Simple choice problems

In this experiment we presented subjects with simple choice problems only. Experiments 2 and 3 involved both simple and mixed choice problems.

6.1.1 Experimental Method

6.1.1.1 Design and materials

Three sets of 24 simple choice problems were used. No problem figured in more than one set. All 72 problems are listed, by set, in Appendix B.1.

In Set 1 payoffs range from $750-$1750. Twelve problems (numbers 1-12 in Appendix B.1) were constructed so as to include all combinations of 30%, 40%, and 50% payoff differences and .10, .20, .30, and .40 probability differences between lotteries. To illustrate, problem 1 -- viz., [(1600,.25),(800,.35)] -- represents a 50% payoff difference and a .10 probability difference. The remaining 12 problems (numbered 13-24) are variants of problems 1-12, respectively. Thus, problem 13 is identical to problem 1 except that it involves negative rather than positive lotteries; problem 14 is identical to problem 2 except that the probabilities are halved; problem 16 is the negative and halved version of problem 4, etc.
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Set 2 consists of the same problems as Set 1 except that payoffs are uniformly decreased by a factor of 10. Note that both the Advantage Model and the present versions of Utility Theory and Prospect Theory should be equally applicable to all simple choice problems, regardless of payoff-size. Furthermore, conflicting claims have been made in recent years regarding the more "realistic" nature of problems involving small payoffs, as opposed to the "increased incentives for making a motivated decision" provided by problems offering larger payoffs (see, e.g., Pommerehne et al., 1982; Slovic, 1975). For these reasons, choice problems of varying payoff magnitudes were used.

Set 3 ranges over payoffs from $5-$17. Problems 1-12 constituted new problems and problems 13-24 were, as before, variants of the first 12.

The range of probabilities in the three experiments is .10-.85. We did not include extreme probabilities (such as .05 or .95) in order to avoid "editing" effects. Presented with a probability of .05, for example, the subject might decide that the chance of winning is "essentially zero" and base her choice entirely on that. We return to the topic of editing in the Chapter 8.

For each set, the 24 choice problems appeared on separate pages, assembled into a 24-page booklet. The problems (e.g., problem 1 of set 1) appeared in the following format:
Choose between:
25% chance to win $1600
35% chance to win $800

The original 12 problems formed an uninterrupted half of the booklet and their 12 variants formed the other half. This was done so as to avoid juxtaposition of two variants of the same problem. The order of the two halves was counterbalanced across booklets. Within each half, the order of problems was randomized for each subject. Finally, within each problem (i.e., on a single page), the order of the two competing lotteries was counterbalanced.

6.1.1.2 Subjects

The subjects were 160 M.I.T. undergraduate volunteers, recruited from a variety of classes and paid for their participation.

6.1.1.3 Procedure

Each subject received one booklet corresponding to one of the three sets of choice problems. The administration of the three sets followed the same procedure. Subjects were first presented with written instructions, in which they were asked to choose, from each pair of monetary lotteries that they encounter, the lottery that they prefer to have. These instructions are reproduced in their entirety in Appendix C.1. Next, each subject was handed a single booklet and asked to work through it at his own speed without referring back to earlier problems. Typically, subjects worked for 10-15 minutes. No bets were actually
played. After completing their booklets, subjects were asked to indicate if they had used any predetermined, mechanical procedure to arrive at their choices, rather than responding intuitively. Data from subjects who had decided at the outset to use some mechanical procedure (e.g., "always choose higher probabilities", "always choose higher payoff", etc.) were discarded. (These subjects, when later presented with particular examples, all agreed that the strategy they had adopted does not reflect their true preferences.) Three to five subjects were thereby eliminated from each set. Apart from those eliminated, 53 subjects completed set 1, 57 completed set 2, and 50 completed set 3.

6.1.2 Results and discussion

The analyses of the present experiment follow the logic outlined in Chapter 5. For methodological details regarding the within-subject and group analyses, refer back to Sections 5.2 and 5.3, respectively.

6.1.2.1 Preliminary analysis

The bracketed number next to each problem in Appendix B.1 indicates the number of subjects who chose the left-hand lottery. Since a forced-choice procedure was employed, the remaining subjects chose the right-hand lottery. Substantial numbers of subjects exhibited the reflection and common ratio effects. This can be seen in Appendix B.1 by comparing, for example, problems 1 and 13, 5 and 17, 8 and 20, and 12 and 24 of set 1. Note further that decreasing all payoffs by a factor of 10 (Sets 1 vs. 2) had no significant effect on subjects' preferences between lotteries.
This implies what is known in the economic literature as "proportional risk aversion" (e.g., Keeney and Raiffa, 1976), and is predicted by the Advantage Model.

6.1.2.2 Within-subject tests of theories

Consider first the Advantage Model. Recall that a \( k_\alpha, k_L \)-pair is considered "optimizing" with respect to a given subject if no other pair of values \( k_\alpha, k_L \) leads the Advantage Model to a greater number of true predictions about that subject's (in this case, 24) choices. For each subject we computed his lowest optimizing \( k_\alpha, k_L \)-pair in the interval \([0, 3]\), proceeding by increments of .075. The number of true predictions made by the Advantage Model for each subject, relative to his optimizing \( k_\alpha, k_L \)-pair was recorded, along with the subject's lowest optimizing \( k_\alpha, k_L \)-pair.

There were no significant differences in the model's performance on the three sets of choice problems (average number of correct predictions were 21.28, 21.19, and 21.24, for Sets 1, 2, and 3, respectively). This supports the model's claim that payoff ranges (apart from highly unusual ones which may lead to qualitatively different ways of arriving at decisions) are immaterial to its ability to account for choice behavior.

Because there were no significant differences in the model's performance on the three sets of choice problems, the three sets will henceforth be presented as a single group yielding a total of 160 subjects each of which made 24 choices. In this large group, the average number of correct predictions per subject was 21.24 (s.d.=1.46). The
average, lowest, optimizing values for the $k_a,k_L$-pair were .406 (s.d.=.247) for $k_a$, and .337 (s.d.=.282) for $k_L$.

Finally, we generated 160 fictitious subjects (53, 57, and 50, for sets 1, 2, and 3, respectively) and randomly assigned 24 choices to each subject. We then repeated the analysis on these random data (searching the interval [0,3] by steps of .075 for an optimizing $k_a,k_L$-pair for each subject). The average number of correct predictions per randomly generated subject was 16.44 (standard deviation: 1.50). The average lowest optimizing $k_a$ was .631 (sd:.647), and the average lowest optimizing $k_L$ was .725 (sd:.701). The average number of correct predictions obtained by the Advantage Model for the randomly generated subjects is significantly lower than the average obtained for the 160 real subjects ($p < .001$, t-test).

Consider now Utility Theory. Parallel to the Advantage Model, we call a $c_a,c_L$-pair "optimizing" with respect to a given subject if no other values of $c_a$ and $c_L$ lead Utility Theory to a greater number of true predictions about that subject's 24 choices. Similar to the Advantage Model, for each subject we computed his lowest optimizing $c_a,c_L$-pair in the interval [0, 1.5], proceeding by increments of .0375. For each subject, the number of true predictions made by Utility Theory, relative to that subject's optimizing $c_a,c_L$-pair was recorded, along with the subject's lowest optimizing $c_a,c_L$-pair. Across all three problem sets, the average number of correct predictions per subject was 20.68. The average, lowest, optimizing values for the $c_a,c_L$-pair across all three sets were .713 (s.d.=.245) for $c_a$, and .592 (s.d.=.289) for $c_L$. 

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The Advantage Model's average number of true predictions per subject was significantly higher than that of Utility Theory \( t = 4.26; p < .001 \).

Finally, consider Prospect Theory. As for the previous two theories, we call a \( c,w \)-pair "optimizing" with respect to a given subject if no other values of \( c \) and \( w \) lead Prospect Theory to a greater number of true predictions about that subject's 24 choices. For each subject, we computed his lowest optimizing \( c,w \)-pair by searching for the lowest optimizing \( c \) in the interval \([0, 1.5]\) proceeding by increments of \(.0375\), and for the lowest optimizing \( w \) in the interval \([-1.14, 1.11]\) proceeding by increments of \(.05625\). The number of true predictions made by the present version of Prospect Theory for each subject, relative to his optimizing \( c,w \)-pair was recorded, along with the subject's lowest optimizing \( c,w \)-pair. Across the three sets, the average number of correct predictions per subject was 21.39. The average, lowest, optimizing values for the \( c,w \)-pair were \(.582 \) (s.d. = \(.189\)) for \( c \), and \(-.115 \) (s.d. = \(.605\)) for \( w \).

Prospect Theory's average number of true predictions per subject was somewhat higher than that of the Advantage Model but the difference did not reach significance \( t = 1.6 \). On the other hand, Prospect Theory's average number of true predictions per subject was significantly higher than that of Utility Theory \( t = 6.23; p < .001 \).

To summarize, both the Advantage Model and Prospect Theory did significantly better than Utility Theory in predicting individuals' choices. There was, however, no significant difference in the ability to
predict individual choices between Prospect Theory and the Advantage Model. In the following section we attempt to distinguish between the theories, based on their ability to predict group data.

6.1.2.3 Group tests of theories

We proceed now to a test of the Advantage Model as a description of group preference. We remind the reader that in the group analyses we attempt to use the competing theories to predict the proportion of subjects opting for one or another lottery in a choice problem (see Section 5.3). In this use of the theories, the pair of optimizing parameters attributed to a group of subjects is the average optimizing parameter-pair obtained by each theory in the within-subject analysis. (Thus, $k_a$ and $k_L$ for the present group analysis are 0.406, and 0.337, respectively; $c_a$ and $c_L$ are 0.713 and 0.592, respectively; and $c$ and $w$ are 0.582 and -0.115, respectively.) For each theory, we have two measures pertaining to a lottery in a choice problem: the lottery's predicted advantage, which captures the proportion of that lottery's theoretical attractiveness in the problem, and the lottery's observed advantage, which represents the proportion of subjects who chose that lottery in that problem.

We correlated, for the left-hand lotteries of all choice problems in Appendix B.1, between the lotteries' observed and predicted advantages. This correlation involves 72 paired numbers. For the Advantage Model, the obtained correlation was 0.96. The obtained correlation for Prospect Theory was 0.89, while the obtained correlation for Utility Theory was
Prospect Theory does significantly better than Utility Theory (t = 3.63; p < .001, significance test between dependent correlations, Bruning and Kintz, 1977, p.215). The Advantage Model predicts group data significantly better than both Prospect Theory and Utility Theory (t = 4.59 and 6.49, respectively; p < .001 in both cases).

6.1.3 Experiment 1: Summary

This experiment has shown that, in the domain of simple choice problems, both Prospect Theory and the Advantage Model predict individual choices significantly better than Utility Theory, but do not differ significantly from each other. In predicting group data, however, the Advantage Model was shown to be superior both to Prospect Theory and to Utility Theory. We proceed now to tests of the model that involve both simple and mixed choice problems.
6.2 Experiment 2: Simple and mixed choice problems, Pictorial display

A second experiment was conducted, this time designed to evaluate the predictive capabilities of our model on a combination of both simple and mixed choice problems. Because of the increased complexity inherent to mixed choice problems, we provided the subjects with a visual aid. Each lottery was represented as a circle (or a "pie"), with its probabilities occupying proportional slices of the pie. The payoffs and probabilities appeared inside their respective slices. Thus, the mixed lottery ($25,.54,-$23) would be presented in the following fashion:

![Mixed Lottery Diagram]

while the simple lottery ($21,.36) would appear as follows:

![Simple Lottery Diagram]
6.2.1 Experimental Method

6.2.1.1 Design and materials

The experiment consisted of 72 choice problems. Twenty four of these problems lie outside the purview of the current version of the Advantage Model and will be presented and discussed in Section 7.2. Of the remaining 48 problems, 24 were simple choice problems and 24 were mixed choice problems. These are listed in Appendices B.2 and B.3, respectively.

Payoffs in this experiment ranged from $2-$20. The simple problems were constructed in the following manner. The value of the higher payoff (either a loss or a gain) was 150%, 160%, 170%, 180%, 190%, or 200% the value of the lower payoff. The probability differences were .10, .20, .30, or .40. The simple choice problems were constructed so as to yield all combinations of these payoff and probability differences.

The mixed choice problems were constructed in the following manner. Only probabilities that were multiples of .10, and between .20 and .80 were used. Thus, in order to construct mixed lotteries, there were 7 possible probability combinations (since the probabilities must add up to 1). Each possible combination was matched with all six other combinations to yield 21 different choice problems. For the remaining 3 problems, some probability combinations were arbitrarily repeated. Finally, consider a mixed lottery that offers a higher chance to gain than the competing
lottery. This lottery can have both its payoffs (gain and loss) greater than the competing lottery's, smaller than the competing lottery's, or it can offer a smaller gain and a greater loss (notice that the opposite -- a greater gain and a smaller loss -- is not permissible since then that lottery would dominate the other). All possible such payoff-relations were interspersed among the different probability combinations.

The range of probabilities over all problems in this experiment was .15-.85. As in the previous experiment, we did not include extreme probabilities in order to avoid "editing" effects.

The 48 choice problems appeared on separate pages, assembled into a booklet. The booklet -- containing 72 pages -- included some additional choice problems that do not form part of this experiment, and which will be discussed in Section 7.2. The problems appeared in the following format:

Choose between:

For each subject, the order of the choice problems was randomized. Within each problem (i.e., on a single page) the order of the two competing lotteries was counterbalanced.
6.2.1.2 Subjects

The subjects were 78 University of Michigan undergraduate volunteers, recruited by phone and paid for their participation.

6.2.1.3 Procedure

Subjects were first presented with written instructions, in which they were asked to choose, from each pair of lotteries that they encounter, the lottery that they prefer to have. These instructions are reproduced in their entirety in Appendix C.2. Next, each subject was handed a single booklet and asked to work through it at his own speed without referring back to earlier problems. Typically, subjects worked for approximately 30 minutes. After completing the booklet, subjects were asked to indicate if they had used any predetermined, mechanical procedure to arrive at their choices, rather than responding intuitively. Because no subject gave an unequivocal indication of such procedure, no subject's data were discarded. Finally, as had been explained to them in the instructions, subjects had the option to actually play for money the lottery that they had chosen in a randomly picked choice problem. This was done so as to increase subjects' motivation to choose the options that they genuinely preferred.¹

¹There is, however, ample evidence that subjects' choices tend not to differ significantly between situations involving hypothetical payoffs and situations where real payoffs are offered. See, e.g., Lichtenstein and Slovic (1971), Grether and Plott (1979), as well as Schoemaker (1982) for a review.
6.2.2 Results and discussion

Next to each problem in Appendices B.2 and B.3 is indicated the number of subjects who chose the left-hand lottery in that problem. Since a forced-choice procedure was employed, the remaining subjects chose the right-hand lottery. The within-subject and group analyses of the present experiment follow essentially the same logic as those of the previous experiment. For methodological details regarding these analyses, refer back to Sections 5.3 and 5.4.

6.2.2.1 Within-subject tests of theories

Recall that, in the Advantage Model, a $k_\alpha, k_L$-pair is considered "optimizing" with respect to a given subject if no other pair of values $k_\alpha, k_L$ leads the Advantage Model to a greater number of true predictions about that subject's choices. For each subject we computed his lowest optimizing $k_\alpha, k_L$-pair in the interval [0, 3], proceeding by increments of .075, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by the Advantage Model, relative to the subject's optimizing $k_\alpha, k_L$-pair was recorded, along with the subject's lowest optimizing $k_\alpha, k_L$-pair in that case. Thus, for each subject, we obtained (1) the greatest number of true predictions made by the Advantage model for that subject's 24 choices among simple lotteries, along the $k_\alpha, k_L$-pair that yields that number of predictions; (2) the greatest number of true predictions made by the Advantage model for the subject's 24 choices among mixed
lotteries, and the $k_α,k_L$-pair that yields those predictions; and (3) the greatest number of true predictions made by the Advantage model for the subject's 48 choices among both simple and mixed lotteries combined, along with the $k_α,k_L$-pair that yields those predictions. (Observe that the greatest number of true predictions made by the Advantage Model for a subject's total 48 choices will not necessarily be the sum of the number of true predictions made on the two separate sets of 24 choices. An analysis of all 48 choices requires a single (optimizing) $k_α,k_L$-pair, whereas the 24-problem analyses can each utilize a different, optimizing $k_α,k_L$-pair.)

Table 2-A presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing $k_α,k_L$-pair. For the simple choice problems, the average number of correct predictions per subject was 20.85. For the mixed problems, it was 20.21, and for all 48 problems combined the average number of correct predictions was 38.60.

Finally, we randomly generated another 100 fictitious subjects, and repeated the analysis (using the same searches as for the real subjects) on these random data. Both for simple problems and for mixed problems, the average number of correct predictions per randomly generated subject (16.42 and 16.35, respectively) was significantly lower than that obtained for the real subjects ($p < .001$ in both cases, t-test).

The Advantage Model's percentage of correct predictions per subject was somewhat lower for the set of simple and mixed problems combined, than for the simple and mixed problems when analyzed separately. Of
course, one reason for this may be that in the former case a single $k_0,k_L$-pair is used to predict 48 choices, while in the latter cases each $k_0,k_L$-pair is used to predict only 24. It is also possible, however, that the two kinds of problems -- the simple and the mixed -- lead subjects to weigh payoffs and probabilities differently. Such context-dependent shifts in weights would be consistent with other contingent weighting notions (as discussed in Section 3.5), and with Slovic and Lichtenstein's (1968) discussion of the relative importance of probabilities and payoffs in risky choice. Although the Advantage Model may be easily weakened to incorporate this notion, the present study investigates a stronger version. Notice that the present version of the Advantage Model does not allow for such change in relative weights - it explicitly hypothesizes a single parameter-pair for all of a subject's choices.

Consider now Utility Theory. As before, we call a $c_0,c_L$-pair "optimizing" with respect to a given subject if no other values of $c_0$ and $c_L$ lead Utility Theory to a greater number of true predictions about that subject's choices. Similar to the Advantage Model, for each subject we computed his lowest optimizing $c_0,c_L$-pair in the interval [0, 1.5], proceeding by increments of .0375, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by Utility Theory, relative to the subject's optimizing $c_0,c_L$-pair was recorded, along with the subject's lowest optimizing $c_0,c_L$-pair in that case. Table 2-B presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing $c_0,c_L$-pair. Utility Theory's average number of correct predictions per subject was
20.26, 19.99, and 38.67, for the simple problems, the mixed problems, and all 48 problems combined, respectively.

For the simple choice problems, the Advantage Model's average number of true predictions per subject was significantly higher than that of Utility Theory (t = 3.48; p < .001). For the mixed problems, the Advantage Model's average number of true predictions per subject was higher than that of Utility Theory, but failed to reach significance on a 2-tailed test (t=1.79; p < .10). Finally, for the simple and mixed choice problems combined, Utility Theory's average number of true predictions per subject was a little higher than that of the Advantage Model, but this was not significant (t=.289).

Consider now Prospect Theory. Recall that a c,w-pair is "optimizing" with respect to a given subject if no other values of c and w lead Prospect Theory to a greater number of true predictions about that subject's 24 choices. As for the other theories, for each subject we computed his optimizing c,w-pair, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by Prospect Theory for each subject, relative to his optimizing c,w-pair was recorded, along with the subject's lowest, optimizing c,w-pair. Table 2-C presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing c,w-pair. Prospect Theory's average number of correct predictions per subject was 20.62, 19.41, and 37.85, for the simple problems, the mixed problems, and all 48 problems combined, respectively.
The Advantage Model's average number of true predictions per subject was higher than that of Prospect Theory in all three comparisons. For the simple choice problems this difference in averages failed to reach significance (t=1.30). However, for the mixed choice problems and for all 48 problems combined, the Advantage Model's average number of true predictions per subject was significantly higher than that of Prospect Theory (t = 4.24 and 3.03, respectively; p < .01 in both cases).

Finally, let us briefly turn to a comparison of Utility Theory and Prospect Theory. For the simple choice problems, Prospect theory's average number of correct predictions per subject was significantly higher than that of Utility Theory (t=2.27, p < .05). For the mixed problems and for all 48 problems combined, Utility Theory's average number of true predictions per subject was significantly higher than that of Prospect Theory (t = 3.25, and 3.98, respectively; p < .01 in both cases). The comparisons of all three theories' individual choice predictions are summarized in Table 3.

6.2.2.2 Group tests of theories

We proceed once again to a test of the Advantage Model as a description of group preference. As before (cf. 6.1.2.3), we test the Advantage Model's ability to predict the proportion of subjects opting for one or another lottery in a choice problem. We do this by correlating, for the left-hand lottery in each problem of Appendices B.2 and B.3, between the lotteries' observed and predicted advantages. The values figuring in the calculations of predicted advantage were, as
before, the average lowest optimizing $k_a,k_L$-pair (for the Advantage Model), $c_a,c_L$-pair (for Utility Theory), or $c,w$-pair (for Prospect Theory). For the simple choice problems as well as for the mixed choice problems these correlations involve 24 paired numbers, while for all choice problems combined they involve 48 paired numbers.

The correlations obtained are summarized in Table 4. For the Advantage Model, the obtained correlations were .96 for the simple choice problems, .85 for the mixed choice problems, and .75 for all problems combined. For Utility Theory, the corresponding numbers were .81, .76, and .63. For Prospect Theory, these numbers were .88, .82, and .59, respectively. The Advantage Model's obtained correlations were higher than those of both competing theories in all three cases. For the simple choice problems and for all problems combined, the Advantage Model's obtained correlations were significantly higher (p < .05 in all cases), while for the mixed problems alone the difference failed to reach significance. The comparisons of all three theories' group preference predictions are summarized in Table 5.

6.2.3 Experiment 2: Summary

In this experiment, the Advantage Model never does less well than either Utility Theory or Prospect Theory, both in predicting individual choice and in predicting group preference. The competing theories were compared along three sets of choice problems: simple choice problems, mixed choice problems, and simple and mixed choice problems combined. In predicting individual choice, the Advantage Model did significantly
better than Prospect Theory on two of these sets, and significantly better than Utility Theory on one. It also did better on two of the remaining comparisons, and less well on the third, but these were not significant. The theories were then compared -- along the same three sets of problems -- on their ability to predict group preference. Here, the Advantage Model did better than both Utility Theory and Prospect Theory on all comparisons. The differences on four of the six comparisons were statistically significant.
6.3 Experiment 3: Simple and mixed choice problems, Verbal display

A third experiment was conducted, intended to replicate the previous experiment using a different mode of presentation. In this experiment, rather than presenting the lotteries of a choice problem in a pictorial fashion as was done in Experiment 2, the problems were described in writing, as in Experiment 1.

6.3.1 Experimental Method

6.3.1.1 Design and materials

The choice problems used in this experiment were the same 48 problems used in Experiment 2 (and listed in Appendices B.2 and B.3). As before, the 48 choice problems appeared on separate pages, assembled into a 72-page booklet that included other choice problems to be discussed in Section 7.2. The problems (e.g., mixed problem 1) appeared in the following format (which follows that of Experiment 1):

Choose between:
20% chance to win $20 and 80% chance to lose $8 ___
30% chance to win $10 and 70% chance to lose $5 ___

For each subject, the order of the choice problems was randomized. Within each problem (i.e., on a single page) the order of the two competing lotteries was counterbalanced.
6.3.1.2 Subjects

The subjects were 62 M.I.T. undergraduate volunteers, recruited by phone and paid for their participation.

6.3.1.3 Procedure

Subjects were first presented with written instructions, in which they were asked to choose, from each pair of lotteries that they encounter, the lottery that they prefer to have. These instructions are reproduced in their entirety in Appendix C.3. Next, each subject was handed a single booklet and asked to work through it at his own speed without referring back to earlier problems. Typically, subjects worked for approximately 30 minutes. After completing the booklet, subjects were asked to indicate if they had used any predetermined, mechanical procedure to arrive at their choices, rather than responding intuitively. Because no subject gave an unequivocal indication of such procedure, no subject's data were discarded. Finally, as had been explained in the instructions, subjects had the option to actually play for money the lottery that they had chosen in a randomly picked choice problem.

6.3.2 Results and discussion

Next to each problem in Appendices B.2 and B.3 is indicated the number of subjects who chose the left-hand lottery in that problem. Since a forced-choice procedure was employed, the remaining subjects chose the
right-hand lottery. The within-subject and group analyses of the present experiment follow essentially the same logic as those of the previous two experiments. For methodological details regarding these analyses, refer back to Sections 5.2 and 5.3.

6.3.2.1 Within-subject tests of theories

Consider first the Advantage Model. As in Experiment 2, for each subject we computed his lowest optimizing \( k_0, k_L \)-pair in the interval \([0, 3]\), proceeding by increments of 0.075, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by the Advantage Model, relative to the subject's optimizing \( k_0, k_L \)-pair was recorded, along with the subject's lowest optimizing \( k_0, k_L \)-pair in that case.

Table 6-A presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing \( k_0, k_L \)-pair. For the simple choice problems, the average number of correct predictions per subject was 22.0. For the mixed problems, the average number of correct predictions was 20.29, and for all 48 problems combined, the average number of predictions was 38.76.

For the simple choice problems, the average number of correct predictions obtained by the Advantage Model in the present experiment -- where lotteries were presented verbally -- was significantly greater than that obtained in the previous experiment, where they were presented pictorially. This, however, was not the case for either the mixed

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problems or the simple and mixed problems combined. Moreover, for the simple problems, the number of predictions obtained in the first experiment -- where the lotteries were also presented verbally -- was not significantly greater than that obtained in the second. This, together with the various results concerning the group analyses of the Advantage Model (which were identical for the present and the previous experiment) leads us to conclude that the pictorial and verbal presentations essentially constituted replications of one another with no discernible differences for the Advantage Model due to the mode of presentation. The conclusion that the mode of presentation had little influence on subjects' choices is further supported by the fact that a similar proportion of subjects chose corresponding lotteries in the two experiments (correlation of .94).

Consider now Utility Theory. As before, for each subject we computed his lowest optimizing $c_a, c_L$-pair in the interval [0, 1.5], proceeding by increments of .0375, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by Utility Theory, relative to the subject's optimizing $c_a, c_L$-pair was recorded, along with the subject's lowest optimizing $c_a, c_L$-pair in that case. Table 6-B presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing $c_a, c_L$-pair. Utility Theory's average number of correct predictions per subject was 21.76, 20.34, and 39.58, for the simple problems, the mixed problems, and all 48 problems combined, respectively.
The Advantage Model's average number of correct predictions per subject was higher than that of Utility Theory for the simple problems, and lower than that of Utility Theory for the mixed problems. Neither difference, however, was statistically significant (t = 1.65 and .33, respectively). On the simple and mixed choice problems combined, Utility Theory did significantly better than the Advantage Model (t = 4.2; p < .001).

Next, consider Prospect Theory. For each subject we computed his optimizing c,w-pair, separately for the simple problems, for the mixed problems, and for all 48 problems combined. In each case, the number of true predictions made by Prospect Theory for each subject, relative to his optimizing c,w-pair was recorded, along with the subject's lowest, optimizing c,w-pair. Table 6-C presents the average number of correct predictions per subject thereby obtained for each set of problems, along with the average, lowest, optimizing c,w-pair. Prospect Theory's average number of correct predictions per subject was 21.79, 19.76, and 38.61, for the simple problems, the mixed problems, and all problems combined, respectively.

The Advantage Model's average number of true predictions per subject was higher than that of Prospect Theory in all three comparisons. For the mixed choice problems, this difference was statistically significant (t = 3.35; p < .01), while for the simple problems and for all problems combined the difference in averages failed to reach significance (t = .98 and .49, respectively).
Finally, let us briefly compare Utility Theory and Prospect Theory. In the simple problems, Prospect Theory's average number of correct predictions per subject was higher than that of Utility Theory, but failed to reach significance ($t=1.47$). For the mixed problems and for all problems combined, Utility Theory's average number of correct predictions per subject was significantly higher than that of Prospect Theory ($t = 3.39$ and $3.76$, respectively; $p < .01$ in both cases). The comparisons of all three theories' individual choice predictions are summarized in Table 7.

To conclude, notice that both in Experiments 2 and 3 all theories predicted the simple choice problems better than they predicted the mixed problems or the simple and mixed choice problems combined. One reason for this may simply be that subjects' choices on the simple problems are easier for them to make and thus lead to data that exhibits less noise. It is also possible, however, that subjects made their choices on the simple problems more carefully than they did on the other, more complicated problems. Interesting evidence from studies on decision time and task complexity (see, e.g., Kiesler, 1966; Hogarth, 1975) suggest a greater motivation in people to be optimal in their decisions when simple choice problems are involved than when more complex choices are required.

6.3.2.2 Group tests of theories

We now test the Advantage Model's ability to predict group preference. As before (cf. Sections 6.1.2.3 and 6.2.2.2), we test the Advantage Model's ability to predict the proportion of subjects opting
for one or another lottery in a choice problem. To do this we correlate, for the left-hand lottery in each problem of Appendices B.2 and B.3, between the lotteries' observed and predicted advantages, using the average lowest optimizing \( k_a, k_L \)-pair (for the Advantage Model), \( c_a, c_L \)-pair (for Utility Theory), or \( c, w \)-pair (for Prospect Theory). For the simple choice problems as well as for the mixed choice problems these correlations involve 24 paired numbers, while for all choice problems combined they involve 48 paired numbers.

The correlations obtained are summarized in Table 8. For the Advantage Model, the obtained correlations were .97 for the simple choice problems, .87 for the mixed choice problems, and .74 for all problems combined. For Utility Theory, the corresponding numbers were .89, .87, and .58. For Prospect Theory, these numbers were .92, .81, and .57, respectively. Except for obtaining a correlation equal to that of Utility Theory on the mixed problems, the Advantage Model's obtained correlations were higher than those of both competing theories in all three cases. For the simple choice problems and for all 48 problems combined, the Advantage Model's obtained correlations were significantly higher \( p < .05 \) in all cases), while for the mixed problems the difference failed to reach significance. The comparisons of all three theories' group preference predictions are summarized in Table 9.

6.3.3 Experiment 3: Summary

In this experiment, unlike the previous one, the Advantage Model does significantly less well than Utility Theory in predicting individual
choice on the set of simple and mixed problems combined. On all other comparisons, however, the model does as well as its competitors or better. In predicting individual choice, the Advantage Model did significantly better than Prospect Theory on the mixed problems. In predicting group preference, the Advantage Model did significantly better than Utility Theory and Prospect Theory, both on the simple problems and on all 48 choice problems combined.

6.4 Summarizing the experiments

Three experiments compared the Advantage Model to Utility Theory and to Prospect Theory. The theories were evaluated on their ability to predict individual choice as well as group preference, on simple as well as on mixed choice problems. We first summarize the results concerning individual choice and then those related to group preference.

6.4.1 Individual choice

In Experiment 1, which involved simple choice problems only, the Advantage Model did significantly better than Utility Theory and did not differ significantly from Prospect Theory. Experiments 2 and 3 included both simple and mixed choice problems. In Experiment 2 the Advantage Model predicted individual choices better than both Utility Theory and Prospect Theory, on both the simple choice problems and the mixed choice problems (two of these comparisons were significant at the .001 level, while the other two failed to reach significance). For the simple and mixed choice problems combined, the Advantage Model did significantly better than Prospect Theory, and did not differ significantly from
Utility Theory. In Experiment 3, the Advantage Model did better than Prospect Theory on the simple problems, the mixed problems, and all problems combined (although only for the mixed problems was this difference significant). Against Utility Theory, the Advantage Model did not differ significantly on either the simple or the mixed problems, but did significantly less well on the simple and mixed problems combined. This last comparison was the only one in all three experiments where either theory did significantly better than the Advantage Model in predicting individual choice.

6.4.2 Group preference

In Experiment 1, involving only simple choice problems, the Advantage Model predicted group preference significantly better than both Utility Theory and Prospect Theory. In Experiment 2, the Advantage Model predicted group preference better than both competing theories for the simple problems, for the mixed problems, and for the simple and mixed problems combined. Finally, in Experiment 3, the Advantage Model predicted group preference as well as Utility Theory for the mixed lotteries, and did better than both theories on the remaining comparisons. Altogether, out of a total of 14 comparisons involving group preference (seven against Utility Theory and seven against Prospect Theory), the Advantage Model tied on one comparison and did better on the remaining 13 (10 of which were statistically significant).
6.4.3 Conclusion

It was seen in Chapter 4 that both Utility Theory and Prospect Theory have fatal descriptive flaws arising from their absolute character. In that context, the Advantage Model was shown to provide a better account of certain qualitative phenomena than either of its competing theories. The data presented in this chapter indicates that the Advantage Model is at least as successful as Utility Theory and Prospect Theory in predicting individual choice, and quantitatively superior in predicting group preference. Following some additional analyses in Chapter 7, we return, in Chapter 8, to a discussion of some needed refinements of the model, and to examples of how the model may be extended to nonmonetary domains.
Chapter 7

Quantitative Evaluation, Additional Analyses

The present chapter presents additional analyses relevant to the Advantage Model. In the first part of the chapter (Section 7.1) we evaluate a single-parameter version of the Advantage Model (where $k_o = k_L$) on the data obtained in Chapter 6. This single-parameter version is evaluated on its ability to predict both individual choice as well as group preference. We then compare the performance of the single-parameter version of the model to its regular, 2-parameter version. The single-parameter version is shown to be an inferior predictor of individual choice, but as successful as the 2-parameter version in predicting group preference.

In the second part of the chapter (Section 7.2), we present a set of nonconflictual choice problems. The mixed lotteries in these problems have equal probabilities and differ only in their payoffs. Because they are nonconflictual, the Advantage Model in its present formulation is undefined on these problems. We show how the model can be extended to include this kind of choice problems, and then evaluate the ability of the extended version of the model to predict choices in them. It will be seen that the model does significantly less well on the nonconflictual than on regular choice problems. We conclude that the Advantage Model's
inability to predict choice in nonconflictual problems is consonant with
the psychological assumptions that underlie the model.

7.1 A single-parameter version of
the Advantage Model

A guiding assumption of the Advantage Model is that people
differentially weigh payoff and probability advantages when making
choices between monetary lotteries. Since it has been repeatedly
observed, however, that people treat losses and gains differently, the
model makes the additional assumption that people's relative weighting of
payoffs to probabilities differs when the payoffs represent gains from
when they represent losses. To capture this notion, the Advantage Model
introduces the parameters $k_\alpha$ and $k_\lambda$ which represent, for each subject,
his relative weight of payoffs and probabilities in the case of gains and
in the case of losses, respectively. According to this formulation of the
model, a person's $k_\alpha$ and $k_\lambda$ differ to the extent that his weighting of
payoffs to probabilities differs for gains and for losses. If a person's
relative weight of payoffs to probabilities does not differ significantly
for losses and for gains, then his values for $k_\alpha$ and $k_\lambda$ will be similar.
And to the extent that his $k_\alpha$ and $k_\lambda$ are similar, a single-parameter
version of the Advantage Model (where $k_\alpha = k_\lambda$) should predict the
person's choices as well as the regular, 2-parameter version.

Thus, in order to investigate the Advantage Model's claim that
people's choices are guided by two different parameters -- one used for
gains and the other for losses -- we analyzed the data of Chapter 6 using
a single-parameter version of the model -- where the same parameter is
used for both losses and gains. To the extent that people's choices can be characterized by an equal weighting of probabilities relative to both losses and gains, the single-parameter version of the Advantage Model should predict their choices as well as the 2-parameter version. On the other hand, if the Advantage Model's assumption of different weighting parameters for losses and for gains is correct, then the restriction to a single parameter per person should significantly reduce the model's ability to predict choice behavior. In what follows we describe the application of the single-parameter version of the Advantage Model to the data obtained in the three experiments of Chapter 6, first as it applies to individual choice, and then as it relates to the prediction of group preference.

7.1.1 Individual choice according to the single-parameter version

The single-parameter version of the Advantage Model is identical to the regular, 2-parameter version as presented in Chapter 3, except for the additional restriction that \( k_0 = k_L \). Thus, instead of employing two parameters -- \( k_0 \) for gains and \( k_L \) for losses -- the present version employs a single parameter -- call it \( k \) -- that figures both in the calculations involving gains and in those involving losses. Parallel to before, we call a \( k \)-value optimizing with respect to a given subject if no other value of \( k \) leads the present version of the Advantage Model to a greater number of true predictions about that subject's choices. Preliminary searches of the parameter space indicated that a subject's optimizing \( k \) is almost certain to fall in the interval \([0, 2]\). Consequently, for each subject in each experiment of Chapter 6 we
computed the lowest optimizing $k$ in the interval $[0, 2]$ proceeding by increments of .02. Although (due to the lack of combinatorial interaction between two distinct parameters) this yields fewer searches per subject than had been conducted in the previous, 2-parameter analyses, notice that the present search is finer — involving 100 incremental values of $k$ rather than the 40 values per parameter investigated before. For each subject, the number of correct predictions made by the Advantage Model relative to the subject's optimizing $k$ was recorded, along with the subject's lowest optimizing $k$.

Table 10-A presents the average number of correct predictions per subject thereby obtained in each experiment, along with the average lowest optimizing $k$. In Experiment 1, across all three problem sets, the single-parameter version of the Advantage Model made an average of 20.71 (s.d.=1.66) correct predictions per subject. This average number of correct predictions per subject was significantly smaller than that made by the regular, 2-parameter version (namely, 21.24; $t = 7.02$, $p < .001$).

In Experiment 2, the average number of correct predictions per subject made by the single-parameter version of the Advantage Model was 19.76 (s.d.=1.91), 18.71 (s.d.=2.73), and 36.68 (s.d.=4.41), for the simple choice problems, mixed choice problems, and simple and mixed problems combined, respectively. In all three cases, the average number of predictions made by the single-parameter version was significantly smaller than the average number of predictions made by the regular, 2-parameter version of the Advantage Model (namely, 20.85, 20.21, and 38.60, respectively; $t = 6.51$, 7.0, and 7.48, for the simple problems,
mixed problems, and simple and mixed problems combined, respectively; p < .001 in all cases).

In Experiment 3, the average number of correct predictions per subject made by the single-parameter version of the Advantage Model was 21.13 (s.d.=1.88), 19.0 (s.d.=1.98), and 37.18 (s.d.=3.49), for the simple choice problems, mixed choice problems, and simple and mixed problems combined, respectively. Once again, in all three cases, the average number of predictions made by the single-parameter version was significantly smaller than the average number of predictions made by the regular, 2-parameter version of the Advantage Model (namely, 22.0, 20.29, and 38.76, respectively; t = 4.25, 7.98, and 5.39, for the simple problems, mixed problems, and simple and mixed problems combined, respectively; p < .001 in all cases).

On all comparisons in all three experiments, the single-parameter version of the Advantage Model predicts individual choices significantly less well than does the regular, 2-parameter version of the model. Thus, the Advantage Model's assumption of two different parameters per person -- one to represent his relative weight of payoffs to probabilities for gains and the other to represent this weight for losses -- leads to better prediction of individual choice than when a single parameter is assumed. This of course agrees with the long standing observation that people accord different treatments to losses and to gains. We next consider the implications of the above discussion to the Advantage Model's prediction of group preference.
7.1.2 Group preference according to the single-parameter version

As discussed in the preceding section, to the extent that the Advantage Model is correct, people's choices are characterized by two different weighting parameters; one, $k_\alpha$, represents a person's relative importance of payoffs and probabilities when gains are involved, and the other, $k_L$, represents his relative importance of payoffs and probabilities when losses are at stake. In the present section we investigate the relevance of these two different parameters to the Advantage Model's prediction of group preference.

Recall (Section 5.3) that in its group analysis the Advantage Model attempts to predict, for each choice problem, the proportion of subjects who will opt for one or another lottery in that problem. In this use of the model, the optimizing parameters attributed to a group of subjects are the average optimizing parameters obtained by the model in its within-subject analysis, and are denoted $k_\alpha(\text{av})$ and $k_L(\text{av})$.

Consider now people's relative size of $k_\alpha$ and $k_L$. If for most people one of these, say $k_L$, is substantially greater than the other ($k_\alpha$), then it is likely to be true about the group in general that its $k_L(\text{av})$ is greater than its $k_\alpha(\text{av})$. In that case, in order to predict for each choice problem the proportion of subjects who chose each lottery, the model should attribute a greater value to $k_L(\text{av})$ than to $k_\alpha(\text{av})$. That is, just as in the case of individual choice, group preference would be better predicted using different values for $k_\alpha$ and $k_L$. A single-parameter version of the Advantage Model, where $k_\alpha = k_L$ (and, therefore, $k_\alpha(\text{av}) = k_L(\text{av})$)
would predict group preference significantly less well than the regular, 2-parameter version.

Consider, on the other hand, the possibility that people's average values of \( k_0 \) and \( k_L \) do not differ significantly over the entire group. As discussed earlier (Section 4.3) the observation that a majority of people are loss averse leads the Advantage Model to predict that for most people \( k_L \) is greater than \( k_0 \). The difference between these people's \( k_L \) and \( k_0 \) may not be large, however. And, more importantly, it may be offset by those people for whom \( k_0 \) is greater than \( k_L \). (Especially, for example, if the latter, smaller group tends to be characterized by a greater difference between the two parameters than the former, larger group.) Recall from Section 4.3 that any person who rejects the fair bet \((100,.50,-100)\) is predicted by the Advantage Model to have a larger \( k_L \) than \( k_0 \). Observe, on the other hand, that any person who accepts this bet is predicted to have a larger \( k_0 \) than \( k_L \). And there may certainly be a substantial number of subjects in the group who would, in fact, accept this gamble. To the extent that some subjects' greater \( k_0 \) counteracts the rest of the group's greater \( k_L \), the average values of \( k_0 \) and \( k_L \) over the entire group (i.e., \( k_0(\text{av}) \) and \( k_L(\text{av}) \)) may be similar.

Notice that according to this scenario people's individual values of \( k_0 \) and \( k_L \) continue to be significantly different; it is only over the entire group that the values of the two become similar. And if this is the case, then a single-parameter version of the Advantage Model (where \( k_0 = k_L \), and, therefore, \( k_0(\text{av}) = k_L(\text{av}) \)) may predict group preference about as well as the regular, 2-parameter version.
We used the single-parameter version of the Advantage Model to predict group preference in exactly the same way that the regular, 2-parameter version was used in Chapter 6 (and as is described in detail in Section 5.3). For the calculation of lotteries' predicted advantage, however, instead of a pair of optimizing parameters \( k_o(\text{av}) \) and \( k_L(\text{av}) \), each group was attributed a single parameter \( k(\text{av}) = k_o(\text{av}) = k_L(\text{av}) \), which was the average lowest optimizing \( k \) obtained by the single-parameter version of the model in its within-subject analysis.

As before, we correlated for the left-hand lotteries of all choice problems in Appendices B.2 and B.3, between the lotteries' observed and predicted advantages. The correlations obtained are summarized in Table 10-B. The correlation obtained by the single-parameter version of the Advantage Model in Experiment 1 was .96, identical to that obtained by the regular, 2-parameter version of the model.

In Experiment 2, the correlations obtained by the single-parameter version of the Advantage Model were .95 for the simple choice problems, .84 for the mixed problems, and .77 for the simple and mixed choice problems combined. While for the simple problems the correlation obtained by the single-parameter version of the model was slightly lower than that obtained by the regular, 2-parameter version (which was .96, \( t = 3.21, p < .01 \)), the other two correlations were not significantly different between the single- and the 2-parameter versions.

In Experiment 3, the correlations obtained by the single-parameter version of the Advantage Model were .97, .86, and .72, for the simple problems, for the mixed problems, and for the simple and mixed problems
combined, respectively. These correlations were essentially the same as those obtained by the regular, 2-parameter version of the Advantage Model (the small differences were statistically insignificant).

Out of seven comparisons concerning the prediction of group preference, the single-parameter version of the Advantage Model did slightly less well than the regular, 2-parameter version on one comparison, and equally well on the remaining six. It appears, therefore, that the single-parameter version of the Advantage Model is approximately as good a predictor of group preference as the regular, 2-parameter version. As discussed above, to the extent that the Advantage Model is right, this indicates that the majority's greater value of $k_L$ relative to $k_\alpha$ is to a large degree counteracted by the rest of the subjects' greater value of $k_\alpha$ relative to $k_L$. The relative size of the parameters $k_\alpha$ and $k_L$ appears to differ from person to person in a manner that yields roughly equal average values of the two parameters in the group as a whole.

Notice, finally, the implications of these results for the use of the Advantage Model to predict group preference. While in Chapter 6 it was seen that the Advantage Model predicts group preference significantly better than both Utility Theory and Prospect Theory, it now appears that a particularly limited, single-parameter version of the model predicts group preference just as well. Thus, armed with a particularly simple version of the Advantage Model -- where a single parameter is used to characterize a group's relative importance of payoffs and probabilities (regardless of whether gains or losses are at stake) -- one can predict the group's preferences better than with the 2-parameter versions of either Utility Theory or Prospect Theory.
7.2 Nonconflictual choice problems

The Advantage Model is assumed to address the conflict that arises from lotteries' advantages on probabilities and payoffs. For that reason, the model is not defined for problems where these advantages -- either a payoff advantage or a probability advantage -- are absent. In Chapter 3 we called such problems "nonconflictual". Because nonconflictual choice problems do not exhibit either a payoff or a probability advantage, a relative weighting of payoff and probability advantages to determine choice is not applicable in these problems. To the extent that the underlying psychological assumptions of the Advantage Model are correct -- that is, to the extent that choices between lotteries are determined by a differential weighting of payoff and probability advantages -- a person will make his choices among nonconflictual choice problems in a different fashion from that which guides his choices among regularly conflicting lotteries. Thus, to the extent that the Advantage Model is right, nonconflictual choice problems may form an interesting boundary condition on its applicability.

In this section, we relax a formal assumption of the Advantage Model so as to make it applicable to nonconflictual choice problems. We then present a set of nonconflictual choice problems and test the model's ability to predict both individual choice as well as group preference in the context of these problems. The model's ability to predict nonconflictual choice problems is seen to be substantially inferior to its ability to predict choices among regularly conflicting lotteries. In a brief conclusion, we raise the possibility that the Advantage Model
represents one of a number of heuristic procedures that people employ to arrive at their choices.

7.2.1 EP choice problems

Nonconflictual choice problems are often theoretically uninteresting, inasmuch as the lack of advantage on one dimension often leads to a relation of dominance between the lotteries. For example, two distinct positive lotteries that offer identical payoffs are necessarily such that the one that offers the greater probability dominates the other. In some cases, however, a dimension may present no conflict and yet the choice may not be obvious. One such case are mixed lotteries that offer the same probabilities to gain and to lose, but that differ in their payoffs. An example is the choice problem \([(20,.20,-5),(12,.20,-4)]\). Henceforth, such problems will be called equal-probability (or EP) choice problems. In an EP choice problem, although the probabilities offered by the two competing lotteries are identical (in the case above, 20% chance to win and 80% chance to lose) the choice is not obvious. One of the lotteries offers a higher potential gain but also a higher potential loss. Notice that EP problems involve no probability advantages and are, therefore, nonconflictual.

According to our present formulation, this kind of choice problem -- an EP choice problem -- lies outside the domain of the Advantage Model. Since one of the two kinds of advantages is absent, the model's assumption of a differential weighting of advantages is inapplicable. To the extent that the intuitions of the Advantage Model are correct, people
will evaluate choices of this kind in a different fashion from that which
guides their choices among regularly conflicting lotteries and,
therefore, in a different fashion from that suggested by the Advantage
Model.

To test this idea, we must extend the Advantage Model to lotteries
of the EP kind in the most natural way possible. Such an extension is
provided in the following section.

7.2.2 An extended version of the AMk-function

Recall (Section 3.1.1.1) that the Advantage Model's AMk-function is
formulated so as to render the model undefined for nonconflictual choice
problems. The exact formulation of the original AMk-function is
reproduced below.

The AMk-function:

\[
\text{AMk}
\begin{pmatrix}
(d_1, p_1); \\
\end{pmatrix}
\begin{pmatrix}
(d_2, p_2);
\end{pmatrix}
\]

is undefined if \[(d_1, p_1),(d_2, p_2)\]
is nonconflictual; Otherwise:

\[
\text{AMk}
\begin{pmatrix}
(d_1, p_1); \\
\end{pmatrix}
\begin{pmatrix}
(d_2, p_2);
\end{pmatrix}
\] = 0 if \(d_1, p_1\) is trivial;

\[
= \text{EMV}_1 \left( \frac{(d_2 - d_1)}{d_1} \right) k \text{ if } p_1 < p_2;
\]

\[
= \text{EMV}_1 (p_1 - p_2) \text{ if } p_1 > p_2.
\]

The clause that declares the AMk-function undefined for
nonconflictual choice problems was motivated by the kind of psychological
considerations -- regarding the differential weighting of advantages --
discussed above. Technically, however, this clause can be removed so as
to make the model applicable to nonconflictual choice problems of the EP
type. The removal of the clause yields the following, extended version of the AMk-function.

The extended version of the AMk-function:

\[
\text{AMk}\left((d_1, p_1);\{(d_1, p_1), (d_2, p_2)\}\right) = 0 \quad \text{if} \quad (d_1, p_1) \text{ is trivial};
\]

\[
= \text{EMV}_1\left((d_1 - d_2)/d_1\right)k \quad \text{if} \quad p_1 < p_2;
\]

\[
= \text{EMV}_1(p_1 - p_2) \quad \text{if} \quad p_1 > p_2.
\]

Otherwise, if \( p_1 = p_2 \):

\[
\text{AMk}\left((d_1, p_1);\{(d_1, p_1), (d_2, p_2)\}\right) = \text{EMV}_1\left((d_1 - d_2)/d_1\right)k \quad \text{if} \quad d_1 > d_2;
\]

\[
= \text{EMV}_1(p_1 - p_2) \quad \text{if} \quad d_1 \leq d_2.
\]

Notice that the present, extended version of the AMk-function applies to nonconflictual choice problems as well as to conflictual ones. If the two lotteries differ in probabilities, than the lottery offering the higher probability is assigned the probability advantage, while the other lottery is assigned the payoff advantage (which could be 0, if the payoffs are identical). If the two lotteries offer equal probabilities, than the extended AMk-function attributes a payoff advantage to the lottery with the higher payoff and a 0 advantage to the other lottery. Finally, if the lotteries offer equal payoffs and equal probabilities, both are assigned 0 advantages.

Consider now a version of the Advantage Model -- call it the extended version -- that employs the extended rather than the standard formulation of the AMk-function. In particular, consider the extended version of the Advantage Model as it applies to the EP problems above.

Observe that although the evaluation of lotteries in an EP problem yields no probability advantages, it does produce payoff advantages. And
the differential weighting of the payoff advantages due to \( k_a \) and \( k_L \) yields specific predictions about which lottery should be preferred. Thus, for example, the reader may verify that the EP problem \(((20, .20, -5), (12, .20, -4))\) yields a comparison between \((1.6)k_a\) and \((.8)k_L\), for a choice of the left- and right-hand options, respectively.

Additional experimental studies were conducted in order to test the ability of the extended version of the Advantage Model to predict people's choices among EP problems. To the extent that the psychological intuitions underlying the Advantage Model are correct, people's strategies of resolving the conflict that arises in EP problems may be quite different from those used when regularly conflicting lotteries are entertained. Thus, although the algebra of the model yields coherent predictions when applied to EP problems, it seems plausible that the Advantage Model -- which formally is not considered to apply to such problems -- may do significantly less well on these than on regular choice problems.

In what follows, we describe two experiments designed to evaluate the model's ability (in its extended version) to predict both individual choice as well as group preference on nonconflictual choice problems of the EP type.
7.2.3 Experimental evaluation

7.2.3.1 Method

Recall from Chapter 6 that both Experiments 2 and 3 included an additional 24 choice problems not discussed in the context of those experiments. The 24 problems -- randomly mixed among the rest of the problems of Experiments 2 and 3 -- were EP problems. All 24 EP problems are listed in Appendix B.4. Of course, the subjects and the procedure followed in the administration of these problems are exactly as described in Sections 6.2.1 and 6.3.1 of Experiments 2 and 3, respectively. In Experiment 2 the 24 EP problems were administered in a pictorial fashion, while in Experiment 3 they were presented verbally.

7.2.3.2 Analyses

The within-subject and group analyses of the present two experiments follow exactly the same logic as the analyses of Experiments 2 and 3 of Chapter 6. For details concerning these analyses, refer back to Sections 5.2 and 5.3. For the within-subject analysis, we call a \( k_\alpha, k_L \)-pair optimizing with respect to a given subject if no other \( k_\alpha, k_L \)-pair leads the Advantage Model to a greater number of correct predictions about that subject's 24 EP choices. As before, we computed each subject's lowest optimizing \( k_\alpha, k_L \)-pair in the interval \([0, 3]\) proceeding by increments of \( .075 \). The number of correct predictions made by the Advantage Model for
each subject, relative to his optimizing $k_\alpha,k_L$-pair was recorded, along with the subject's optimizing $k_\alpha,k_L$-pair.

For the group analysis, as before (see Sections 6.2.2.2 and 6.3.2.2), we test the Advantage Model's ability to predict the proportion of subjects opting for one or another lottery in an EP choice problem. To do this, we correlated the lotteries' observed and predicted advantages, using the average lowest optimizing $k_\alpha,k_L$-pair obtained from the within-subject analysis.

These within-subject and group analyses were applied twice: once to the EP problems presented visually in Experiment 2, and once to these problems presented verbally in Experiment 3. Finally, we also conducted all the relevant analyses on the EP problems for Utility Theory and for Prospect Theory. The details of these analyses, as well as the search-intervals and the increments used, were identical to those of the experiments in Chapter 6.

7.2.3.3 Results and discussion

Table 11-A presents the average number of correct predictions per subject obtained by each theory for the EP problems, along with the average, lowest optimizing parameter-pair. For the EP choice problems of Experiment 2, the average number of correct predictions per subject made by the Advantage Model was 18.09. For the EP problems of Experiment 3, this number was 17.37. In both experiments, the average number of correct predictions per subject made by the Advantage Model for the EP problems was significantly smaller than the number of correct predictions made by
the model for either the simple choice problems or the mixed choice problems (which was 20.85 and 20.21, respectively, in Experiment 2, and 22.0 and 20.29, respectively, in Experiment 3; p < .001 in all cases, t-test).

It is interesting to note in this context that in both experiments, with one exception that did not reach statistical significance, both Utility Theory and Prospect Theory also predicted the EP problems significantly less well than they predicted the simple and mixed choice problems. Utility Theory's average number of correct predictions per subject on the EP problems was 18.54 and 18.21 for Experiments 2 and 3, respectively. Prospect Theory's average number of correct predictions per subject was 18.86 and 19.68 for Experiments 2 and 3, respectively. The average number of correct predictions per subject made by both Utility Theory and Prospect Theory for the EP problems was significantly greater than that made by the Advantage Model (p < .05 in all cases).

The correlations between lotteries' observed and predicted advantage obtained by each theory in the two experiments are summarized in Table 11-B. The correlations obtained by the Advantage Model in its group analyses of the EP problems were .41 and .39 for Experiments 2 and 3, respectively. These are significantly lower than those obtained by the model for both the simple and the mixed problems in both experiments (p < .001 in all cases).

Utility Theory's obtained correlations were .33 and .35 for Experiments 2 and 3, respectively. Prospect Theory's correlations were .65 and .68 for Experiments 2 and 3, respectively. Neither theory
differed significantly from the Advantage Model, but Prospect Theory did significantly better than Utility Theory.

Finally, it is worth pointing out that both the Advantage Model's within-subject and group analyses did not differ significantly between Experiments 2 and 3. This corroborates similar findings in Chapter 6 (see Section 6.3.2.1), which indicate that the pictorial and verbal displays had no discernible effects on subjects' choices as predicted by the model. As with the problems of Chapter 6, the conclusion that subjects' choices were not significantly influenced by the mode of presentation are further supported by the fact that a similar proportion of subjects chose corresponding lotteries in the problems of Experiments 2 and 3 (correlation of .98).

7.2.4 Conclusion

In this section, we considered a set of nonconflictual choice problems that fall outside the Advantage Model's domain. We then extended the model so as to render it applicable to these problems, and evaluated its ability to predict both individual choice as well as group preference, with regard to these nonconflictual problems. The results of two experiments showed the extended model to be significantly less successful on the nonconflictual problems than it was on either the simple or the mixed problems of Chapter 6.

To the extent that the Advantage Model provides a correct description of the processes that underlie people's choices between conflictual lotteries, it is not surprising that the model is less
adequate when nonconflictual choices are involved. An increasing body of literature (partly discussed in Chapters 3 and 4 and 6, and reviewed by Einhorn and Hogarth, 1981, and Payne, 1982) indicates that information processing in choice and decision making, as in other areas of cognition, is highly contingent on the task, the context, and the stimuli involved. Following a comprehensive review of "contingent decision behavior", for example, John Payne (1982) concludes that "the finding that decision behavior is sensitive to seemingly minor changes in task and context is one of the major results of years of decision research" (p.395). What decision strategies are used is related to the way problems are structured. Choice behavior is likely to consist of multiple systems, of numerous "heuristics", that interact in various ways. Payne and Braunstein (1971) and Payne (1975) present a number of studies which lead them to conclude that individuals often make an initial judgment about what kind of gambles (e.g., attractive or unattractive) are involved before deciding what choice rule to use. (Of course, this selection of a choice rule need not be a conscious process; see Payne, 1982, for discussion.) Along the same lines, it seems quite possible that people employ one choice rule when certain relations between competing lotteries obtain, and another when the options bear a different relation. It seems highly plausible, in this context, that the processes by which decisions among nonconflictual choice problems are made are quite different from those which guide decisions among conflictual lotteries.

Conflictual choice problems give rise to a conflict between payoff and probability advantages. According to the view espoused above, the Advantage Model may provide a good first approximation of the processes
that underlie the resolution of such conflict. Choice problems of a different nature are resolved in a different manner. (According to the results reported above EP problems are probably not resolved according to Utility Theory or Prospect Theory either, since both these theories -- like the Advantage Model -- do significantly less well on these problems.)

It is important to note, finally, that the nature of the conflict addressed by the Advantage Model -- namely, one option's advantage on one dimension and a competing option's advantage on another -- is extremely common, and by no means limited just to monetary domains. In fact, in the following chapter, along with a discussion of certain possible refinements of the model, we provide examples of how the model may be used to predict choices involving medical treatment and human lives.
Chapter 8

Conclusion

It was seen in Chapter 4 that, both for simple choice problems and for mixed choice problems, the Advantage Model provides a better account of certain qualitative choice phenomena than do either Utility Theory or Prospect Theory. The data presented in the last three chapters indicates that the model is at least as successful as comparable versions of Utility Theory and Prospect Theory in predicting individual choice, and significantly more successful than both theories in predicting group preference. The underlying conception of the Advantage Model (viz., a compromise between absolute and comparative choice strategies, as discussed in Chapter 3) seems well supported and worthy of further development. With that in mind, the next section (Section 8.1) discusses some needed refinements of the model. The final section of this chapter (Section 8.2) provides illustrations of the model's possible extensions to nonmonetary domains.

8.1 Editing and other refinements

In its present form, the Advantage Model appears too crude to handle choice problems in which probability differences are small. This is highlighted by the following example communicated to us by Amos Tversky. Consider \(((101,.49),(99,.51))\). Most people, it seems, are indifferent
between the two lotteries. For the Advantage Model to predict this, $k_\alpha$ must be approximately equal to 1. Now consider $[(6000,.001),(5000,.002)]$. Most people, it seems, prefer the right-hand lottery to the left, which implies a $k_\alpha$ smaller than .01. Not only are the two required $k_\alpha$-values outside most people's predicted range, but they also conflict with the claim of a single $k_\alpha$ per person.

We believe that the psychological mechanism underlying this kind of example is that people misrepresent small probabilities and small probability differences. Such misrepresentation might take two forms. First, editing processes of the kind discussed by Kahneman and Tversky (1979) may intervene to change the character of the lottery internally represented by the subject. Thus, subjects may edit the problem $[(101,.49),(99,.51)]$ to yield a pair of lotteries with essentially identical payoffs and probabilities. (Cf. Russo and Dosher's (1983) findings that small differences between alternatives are often ignored.) Given this, the Advantage Model predicts a "zero" advantage for each lottery and, hence, indifference between them, regardless of a subject's $k_\alpha$. Second, we concur with Kahneman and Tversky (1979, p.281) that "very low probabilities are generally overweighted" by most subjects. Thus, the probabilities in a choice problem like $[(6000,.001),(5000,.002)]$ might be perceived as considerably larger than .001 and .002. The reader may verify that such an increase in probabilities is compatible with a larger value of $k_\alpha$ for choice of the right-hand lottery. It may thus be seen that the existence of these two forms of probability-misrepresentation mitigates or eliminates the difficulty raised by the example discussed above.
Related to this example is a more general difficulty for the Advantage Model. Given a choice between two positive lotteries, \((d_1, p_1)\) and \((d_2, p_2)\), whose expected monetary values are equal, and where \(p_2 > p_1\), the Advantage Model can predict choice of \((d_1, p_1)\) only when \(k_\alpha > p_2\). For example, in order to predict choice of the left-hand option in the problem \([(5000, .001),(5, 1)]\), \(k_\alpha\) must be greater than 1. Similar results -- but in the opposite direction -- obtain for negative lotteries, using \(k_L\). Thus, in its present form, the Advantage Model is incapable of predicting extreme cases of gambling and insurance, which typically consist of a choice between a small gain/loss at certainty versus a very large gain/loss at a very low probability.

These considerations motivate refinement of the Advantage Model in order to allow it to handle lotteries with extreme probabilities. The needed refinements might involve both editing mechanisms and a "decision-weight" function like the \(\pi\) invoked in Kahneman and Tversky's Prospect Theory. Even a utility transformation of dollars might be incorporated into our model. Such revisions do not compromise the underlying psychological tenets of the model, namely, that choice in risky

\[1.\] The proof is as follows.
Assume a choice between \((d_1, p_1)\) and \((d_2, p_2)\), where \(d_1 p_1 = d_2 p_2\) and \(p_2 > p_1\).
We know that:
\[d_1 = \frac{(d_2 p_2)}{p_1}.\]
Now, according to the Advantage Model, \((d_1, p_1)\) will be preferred only if
\[d_1 p_1 k_\alpha \left(\frac{d_1 - d_2}{d_1}\right) > d_2 p_2 (p_2 - p_1).\]
Canceling \(d_1\) on the left-hand side of the inequality and substituting for the remaining \(d_1\) using (1), yields:
\[k_\alpha p_1 \left(\frac{d_2 p_2}{p_1} - d_2\right) > d_2 p_2 (p_2 - p_1).\]
\(k_\alpha > p_2\) then follows from (2) by elementary algebra. We are indebted to A. Tversky for this observation.
situations involves a combination of absolute and comparative strategies. The latter revisions affect only the manner in which choice problems are represented. The revised Advantage Model would process the resulting representations according to the same principles as before.

8.2 Extension to nonmonetary domains

We conclude with some illustrations of how the Advantage Model may be extended to nonmonetary domains. Tversky and Kahneman (1981, p.453) offered subjects a choice between two possible programs to combat a disease:

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.

A second group of subjects was given the same cover story with the following descriptions of the alternative programs:

If Program C is adopted, 400 people will die.

If Program D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

The outcomes presented to the two groups are essentially identical. They differ only in that the former are framed in terms of the number of lives saved, whereas the latter are framed in terms of the number of
lives lost. This difference in framing, however, was shown by Kahneman and Tversky to have a decisive effect on people's choices, since a significant majority in the first group chose Program A, whereas a significant majority in the second group opted for Program D.

In an attempt to apply the Advantage Model to the present context, it is natural to replace negative and positive monetary payoffs with human lives lost and saved. For example, under the present interpretation, the alternative \((-100, .30)\) signifies a 30% chance that 100 people will die. Now let us use the Advantage Model, interpreted in this fashion, to compare the alternative programs offered in the Tversky-Kahneman problem above. Programs A and B yield the following comparison:

Program A: \((200, 1)\)  Program B: \((600, 1/3)\)

\[
[(200, 1), (600, 1/3)]
\]

Attractiveness of Program A:
\[
AMk_a((200, 1); [(200, 1), (600, 1/3)]) = 200(1)(1 - 1/3) = 133.33
\]

Attractiveness of Program B:
\[
AMk_a((600, 1/3); [(200, 1), (600, 1/3)]) = 600(1/3)(400/600)k_a = 133.33k_a
\]

On the other hand, Programs C and D yield the following comparison:
Program C: \((-400, 1)\)  
Program D: \((-600, 2/3)\)

\[\{(-400, 1), (-600, 2/3)\}\]

Attractiveness of Program C:
\[\text{AMk}_L((-400, 1);((-400, 1),(-600, 2/3))) = -400(1)(1-2/3) = -133.33\]

Attractiveness of Program D:
\[\text{AMk}_L((-600, 2/3);((-400, 1),(-600, 2/3))) = -600(2/3)(200/600)k_L = -133.33k_L\]

For any \(k_\alpha, k_L < 1\) it follows from the Advantage Model that Program A is preferred to Program B, and that Program D is preferred to Program C. This is exactly the pattern of preferences exhibited by the majority of Tversky and Kahneman's subjects.

Consider next the following scenario, presented by Tversky and Kahneman (1986) to 72 physicians attending a meeting of the California Medical Association. (Similar responses were obtained by Tversky and Kahneman from a group of 180 college students).

In the treatment of tumors there is sometimes a choice between two types of therapies: (i) a radical treatment such as extensive surgery, which involves some risk of imminent death, (ii) a moderate treatment, such as limited surgery or radiation therapy. Each of the following problems describes the possible outcome of two alternative treatments, for three different cases. In considering each case, suppose the patient is a 40-year-old male. Assume that without treatment death is imminent (within a month) and that only one of the treatments can be applied. Please indicate the treatment you would prefer in each case.

Case 1

Treatment A: 20% chance of imminent death and 80% chance of normal life, with an expected longevity of 30 years. [35%]

Treatment B: certainty of a normal life, with an expected longevity of 18 years. [65%]
Case 2

Treatment C: 80% chance of imminent death and 20% chance of normal life, with an expected longevity of 30 years. [68%]

Treatment D: 75% chance of imminent death and 25% chance of normal life, with an expected longevity of 18 years. [32%]

Case 3

Consider a new case where there is a 25% chance that the tumor is treatable and a 75% chance that it is not. If the tumor is not treatable, death is imminent. If the tumor is treatable, the outcomes of the treatment are as follows:

Treatment E: 20% chance of imminent death and 80% chance of normal life, with an expected longevity of 30 years. [32%]

Treatment F: certainty of a normal life, with an expected longevity of 18 years. [68%]

The bracketed numbers indicate the proportion of subjects who preferred each alternative. Observe that the ratio of the chances to lead a normal life is identical in Cases 1 and 2 (i.e., in both cases the probability of leading a normal life following the first treatment is 4/5 that of the second treatment). The majority's preference, however, reverses between the two cases. Also observe that while, ultimately, Cases 2 and 3 offer objectively identical chances at identical outcomes, again the majority's preference in the two cases reverses. In fact, the physicians in this experiment expressed identical preference in Cases 1 and 3, indicating that their decision was based entirely on the choice between treatments, with no consideration given (in Case 3) to the prior odds.

Similar to before, in an attempt to apply the Advantage Model to the present context, it is natural to replace positive payoffs with number of years of expected longevity. For example, under the present
interpretation, the alternative (30,.20) signifies an 20% chance of a life with an expected longevity of 30 years. Now let us use the Advantage Model, interpreted in this manner, to compare the alternative treatments above. Treatments A and B yield the following comparison:

Treatment A: (30,.80)   Treatment B: (18,1)
[(30,.80),(18,1)]

Attractiveness of Treatment A:
\[ AMk_\alpha((30,.80);[(30,.80),(18,1)]) = 30(.80)(12/30)k_\alpha \]
= 9.6k_\alpha

Attractiveness of Treatment B:
\[ AMk_\alpha((18,1);[(30,.80),(18,1)]) = 18(1)(1-.80) \]
= 3.6

On the other hand, Treatments C and D yield the following comparison:

Treatment C: (30,.20)   Treatment D: (18,.25)
[(30,.20),(18,.25)]

Attractiveness of Treatment C:
\[ AMk_\alpha((30,.20);[(30,.20),(18,.25)]) = 30(.20)(12/30)k_\alpha \]
= 2.4k_\alpha

Attractiveness of Treatment D:
\[ AMk_\alpha((18,.25);[(30,.20),(18,.25)]) = 18(.25)(.25-.20) \]
= .225

Finally, Treatments E and F yield the following comparison (which is a replica of the A-B comparison):
Treatment E: (30, .80)  Treatment F: (18, 1)

\[((30, .80), (18, 1))\]

Attractiveness of Treatment E:
\[\text{AM}_k((30, .80); ((30, .80), (18, 1))) = 30(.80)(12-30)k_o\]
\[= 9.6k_o\]

Attractiveness of Treatment F:
\[\text{AM}_k((18, 1); ((30, .80), (18, 1))) = 18(1)(1-.80)\]
\[= 3.6\]

The reader may now verify that the Advantage Model is consistent with the physicians' choices in the problems above, using any \(.095 < k_o < .375\). Any value of \(k_o\) in that range, (which may substantially overlap people's relative weight of expected longevity to probability of normal life), leads the Advantage Model to predict that Treatment B will be preferred to Treatment A, that Treatment C will be preferred to Treatment D, and that Treatment F will be preffered to Treatment E. This is exactly the pattern of preferences exhibited by the group of physicians above.

Notice that Cases 2 and 3 exhibit a classic occurrence of the noninvariance phenomenon (Section 4.4). While they offer extensionally identical outcomes, the background information is not integrated when making the choice in Case 3, which leads to a choice opposite from that of Case 2. Similarly, Cases 1 and 2 manifest a typical common ratio phenomenon (Section 4.8). While the ratio of the probabilities remains unchanged, the probabilities' systematic reduction leads the physicians to shift their preference. Finally, a quick review of the disease-problem presented earlier, will show it to instantiate a variant of the reflection effect (Section 4.7).
The qualitative phenomena discussed earlier in the context of monetary lotteries are thus seen to extend to nonmonetary domains as well. The same reflection effect that was shown to characterize people's preferences between monetary losses and monetary gains, is here seen to affect their preferences in problems involving human lives lost and saved. Similarly, the same common ratio and noninvariance phenomena that were shown to characterize people's choices between monetary gambles, here appear to influence their choices between medical treatments. To the extent that the Advantage Model provides an adequate description of human choice behavior in the context of monetary lotteries, these parallels suggest that it may lead to a better understanding of human choice and decision behavior in other domains as well. In particular, it may illuminate the mental processes that underlie such behavior in a variety of circumstances.
References


Appendix A

Tables
A.1 Table 1: Kahneman and Tversky's Simple Choice Problems

All Simple Choice Problems from Kahneman and Tversky (1979; 1981)*

(Asterisk indicates lottery preferred by a significant majority of subjects.)

<table>
<thead>
<tr>
<th></th>
<th>d₁</th>
<th>p₁</th>
<th>d₂</th>
<th>p₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2500, .33) *</td>
<td>(2400, .34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(4000, .20) *</td>
<td>(3000, .25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(6000, .45)</td>
<td>(3000, .90) *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(6000, .001) *</td>
<td>(3000, .002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(-4000, .20)</td>
<td>(-3000, .25) *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(-6000, .45) *</td>
<td>(-3000, .90)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(-6000, .001)</td>
<td>(-3000, .002) *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(45, .20) *</td>
<td>(30, .25)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The payoffs in problems 1-7 refer to Israeli currency. To appreciate the amounts involved: the median net monthly income for a family at the time was about 3,000 Israeli pounds. Problem 8 is in dollars.
### Table 2: Individual Choice Predictions, Experiment 2

This table gives the average number of correct predictions made by each theory for each set of choice problems in Experiment 2, along with the average, lowest optimizing parameter-pair for the theory in question.

#### A) Advantage Model

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing $k_a,k_L$-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>20.85 (sd:1.69)</td>
<td>$k_a = .533 (sd:.460)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_L = .420 (sd:.353)$</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>20.21 (sd:2.11)</td>
<td>$k_a = .738 (sd:.690)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_L = .811 (sd:.706)$</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>38.60 (sd:4.18)</td>
<td>$k_a = .644 (sd:.501)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_L = .580 (sd:.503)$</td>
</tr>
</tbody>
</table>

#### B) Utility Theory

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing $c_a,c_L$-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>20.26 (sd:2.11)</td>
<td>$c_a = .665 (sd:.335)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_L = .644 (sd:.355)$</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>19.99 (sd:2.29)</td>
<td>$c_a = .733 (sd:.431)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_L = .810 (sd:.464)$</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>38.67 (sd:4.38)</td>
<td>$c_a = .717 (sd:.355)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_L = .814 (sd:.349)$</td>
</tr>
</tbody>
</table>

#### C) Prospect Theory

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing $c,w$-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>20.62 (sd:2.03)</td>
<td>$c = .570 (sd:.236)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w = -.212 (sd:.626)$</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>19.41 (sd:2.58)</td>
<td>$c = .713 (sd:.355)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w = -.517 (sd:.712)$</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>37.85 (sd:4.44)</td>
<td>$c = .706 (sd:.314)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w = -.137 (sd:.612)$</td>
</tr>
</tbody>
</table>
A.3 Table 3: Comparison of the Theories' Individual Choice Predictions, Experiment 2

This table summarizes the tests of significance between the theories' average number of correct predictions for each set of choice problems in Experiment 2. Arrows point from higher to lower scores.

```
Advantage Model
-------------------
Simple Choice Problems (20.85)
Mixed Choice Problems (20.21)
Simple and Mixed Choice (38.60) Problems
----------
Utility Theory
Simple Choice Problems (20.26)
Mixed Choice Problems (19.99)
Simple and Mixed Choice (38.67) Problems
----------
Prospect Theory
Simple Choice Problems (20.62)
Mixed Choice Problems (19.41)
Simple and Mixed Choice (37.85) Problems
```

A.4 Table 4: Group Preference Predictions, Experiment 2

This table gives the correlations between lotteries' observed and predicted advantage, obtained by each theory for each set of choice problems in Experiment 2.

<table>
<thead>
<tr>
<th></th>
<th>Simple choice problems</th>
<th>Mixed choice problems</th>
<th>Simple and mixed choice problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Advantage Model</td>
<td>.96</td>
<td>.85</td>
<td>.75</td>
</tr>
<tr>
<td>Utility Theory</td>
<td>.81</td>
<td>.76</td>
<td>.63</td>
</tr>
<tr>
<td>Prospect Theory</td>
<td>.88</td>
<td>.82</td>
<td>.59</td>
</tr>
</tbody>
</table>
A.5 Table 5: Comparison of the Theories' Group Preference Predictions, Experiment 2

This table summarizes the tests of significance between the theories' obtained correlations of lotteries' observed and predicted advantages for each set of choice problems in Experiment 2. Arrows point from higher to lower correlations.
A.6 Table 6: Individual Choice Predictions, Experiment 3

This table gives the average number of correct predictions made by each theory for each set of choice problems in Experiment 3, along with the average, lowest optimizing parameter-pair for the theory in question.

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing ( k_a, k_L )-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>22.00 (sd:1.20)</td>
<td>( k_a = .368 ) (sd:.225) ( k_L = .379 ) (sd:.279)</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>20.29 (sd:1.82)</td>
<td>( k_a = .740 ) (sd:.565) ( k_L = .997 ) (sd:.726)</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>38.76 (sd:3.01)</td>
<td>( k_a = .452 ) (sd:.235) ( k_L = .604 ) (sd:.459)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing ( c_a, c_L )-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>21.76 (sd:1.35)</td>
<td>( c_a = .618 ) (sd:.289) ( c_L = .668 ) (sd:.340)</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>20.34 (sd:1.67)</td>
<td>( c_a = .884 ) (sd:.273) ( c_L = .982 ) (sd:.388)</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>39.58 (sd:3.09)</td>
<td>( c_a = .740 ) (sd:.290) ( c_L = .846 ) (sd:.338)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set of choice problems</th>
<th>Average number of correct predictions</th>
<th>Average, lowest optimizing ( c, w )-pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>21.79 (sd:1.77)</td>
<td>( c = .540 ) (sd:.197) ( w = -.144 ) (sd:.600)</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>19.76 (sd:1.94)</td>
<td>( c = .696 ) (sd:.282) ( w = -.790 ) (sd:.614)</td>
</tr>
<tr>
<td>Simple and mixed problems combined</td>
<td>38.61 (sd:3.28)</td>
<td>( c = .641 ) (sd:.243) ( w = -.223 ) (sd:.545)</td>
</tr>
</tbody>
</table>
A.7 Table 7: Comparison of the Theories' Individual Choice Predictions, Experiment 3

This table summarizes the tests of significance between the theories' average number of correct predictions for each set of choice problems in Experiment 3. Arrows point from higher to lower scores.

<table>
<thead>
<tr>
<th>Advantage Model</th>
<th>Simple Choice Problems (22.00)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mixed Choice Problems (20.29)</td>
</tr>
<tr>
<td></td>
<td>Simple and Mixed Choice (38.76) Problems</td>
</tr>
<tr>
<td>Utility Theory</td>
<td>Simple Choice Problems (21.76)</td>
</tr>
<tr>
<td></td>
<td>Mixed Choice Problems (20.34)</td>
</tr>
<tr>
<td></td>
<td>Simple and Mixed Choice (39.58) Problems</td>
</tr>
<tr>
<td>Prospect Theory</td>
<td>Simple Choice Problems (21.79)</td>
</tr>
<tr>
<td></td>
<td>Mixed Choice Problems (19.76)</td>
</tr>
<tr>
<td></td>
<td>Simple and Mixed Choice (38.61) Problems</td>
</tr>
</tbody>
</table>
A.8 Table 8: Group Preference Predictions, Experiment 3

This table gives the correlations between lotteries' observed and predicted advantage, obtained by each theory for each set of choice problems in Experiment 3.

<table>
<thead>
<tr>
<th></th>
<th>Simple choice problems</th>
<th>Mixed choice problems</th>
<th>Simple and mixed choice problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Advantage Model</td>
<td>.97</td>
<td>.87</td>
<td>.74</td>
</tr>
<tr>
<td>Utility Theory</td>
<td>.89</td>
<td>.87</td>
<td>.58</td>
</tr>
<tr>
<td>Prospect Theory</td>
<td>.92</td>
<td>.81</td>
<td>.57</td>
</tr>
</tbody>
</table>
A.9 Table 9: Comparison of the Theories' Group Preference Predictions, Experiment 3

This table summarizes the tests of significance between the theories' obtained correlations of lotteries' observed and predicted advantages for each set of choice problems in Experiment 3. Arrows point from higher to lower correlations.
### A.10 Table 10: Individual Choice and Group Preference Predictions Made by the Single-Parameter Version of the Advantage Model

A)

This table gives the average number of correct predictions made by the single-parameter version of the Advantage Model for each set of choice problems in each experiment, along with the average lowest optimizing k for that set.

<table>
<thead>
<tr>
<th></th>
<th>Exp. 1</th>
<th>Exp. 2</th>
<th>Exp. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>20.71 (sd:1.66)</td>
<td>19.76 (sd:1.91)</td>
<td>21.13 (sd:1.88)</td>
</tr>
<tr>
<td></td>
<td>k=.380 (sd:.109)</td>
<td>k=.489 (sd:.360)</td>
<td>k=.377 (sd:.210)</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>18.71 (sd:2.73)</td>
<td>19.00 (sd:1.98)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>k=.695 (sd:.496)</td>
<td>k=.838 (sd:.442)</td>
<td></td>
</tr>
<tr>
<td>Simple and mixed</td>
<td>36.68 (sd:4.41)</td>
<td>37.18 (sd:3.49)</td>
<td></td>
</tr>
<tr>
<td>problems combined</td>
<td>k=.562 (sd:.348)</td>
<td>k=.493 (sd:.266)</td>
<td></td>
</tr>
</tbody>
</table>

B)

This table gives the correlations between lotteries' observed and predicted advantage obtained by the single-parameter version of the Advantage Model for each set of choice problems in each experiment.

<table>
<thead>
<tr>
<th></th>
<th>Exp. 1</th>
<th>Exp. 2</th>
<th>Exp. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple problems</td>
<td>.96</td>
<td>.95</td>
<td>.97</td>
</tr>
<tr>
<td>Mixed problems</td>
<td></td>
<td>.84</td>
<td>.86</td>
</tr>
<tr>
<td>Simple and mixed</td>
<td></td>
<td>.77</td>
<td>.72</td>
</tr>
<tr>
<td>problems combined</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A.11 Table 11: Individual Choice and Group Preference Predictions Made by the Theories on the Nonconfictual (EP) Choice Problems

A)

This table gives the average number of correct predictions made by each theory for the nonconfictual (EP) choice problems in experiments 2 and 3, along with the average lowest optimizing parameter-pair.

<table>
<thead>
<tr>
<th>Theory</th>
<th>Exp. 2</th>
<th>Exp. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Advantage Model</td>
<td>18.09 (sd:2.56)</td>
<td>17.37 (sd:2.30)</td>
</tr>
<tr>
<td></td>
<td>k_a=.207 (sd:.192)</td>
<td>k_a=.243 (sd:.169)</td>
</tr>
<tr>
<td></td>
<td>k_L=.286 (sd:.309)</td>
<td>k_L=.301 (sd:.207)</td>
</tr>
<tr>
<td>Utility Theory</td>
<td>18.54 (sd:2.38)</td>
<td>18.21 (sd:2.03)</td>
</tr>
<tr>
<td></td>
<td>c_a=.569 (sd:.415)</td>
<td>c_a=.568 (sd:.434)</td>
</tr>
<tr>
<td></td>
<td>c_L=.649 (sd:.503)</td>
<td>c_L=.638 (sd:.526)</td>
</tr>
<tr>
<td>Prospect Theory</td>
<td>18.86 (sd:2.58)</td>
<td>19.68 (sd:1.91)</td>
</tr>
<tr>
<td></td>
<td>c=.512 (sd:.369)</td>
<td>c=.495 (sd:.349)</td>
</tr>
<tr>
<td></td>
<td>w=-.549 (sd:.765)</td>
<td>w=-.769 (sd:.537)</td>
</tr>
</tbody>
</table>

B)

This table gives the correlations between lotteries' observed and predicted advantage obtained by each theory for the nonconfictual (EP) choice problems in experiments 2 and 3.

<table>
<thead>
<tr>
<th>Theory</th>
<th>Exp. 2</th>
<th>Exp. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Advantage Model</td>
<td>.41</td>
<td>.39</td>
</tr>
<tr>
<td>Utility Theory</td>
<td>.33</td>
<td>.35</td>
</tr>
<tr>
<td>Prospect Theory</td>
<td>.65</td>
<td>.68</td>
</tr>
</tbody>
</table>
Appendix B

CHOICE PROBLEMS USED IN EXPERIMENTS
### B.1 Simple Choice Problems Used in Experiment 1

(In brackets: number of subjects who chose left-hand lottery.)

#### Set 1 (N=53)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1600, .25)(800, .35) [52]</td>
<td>13</td>
<td>(-1600, .25)(-800, .35) [5]</td>
</tr>
<tr>
<td>2</td>
<td>(1500, .30)(750, .50) [28]</td>
<td>14</td>
<td>(1500, .15)(750, .25) [38]</td>
</tr>
<tr>
<td>3</td>
<td>(1700, .35)(850, .65) [12]</td>
<td>15</td>
<td>(-1700, .45)(-850, .85) [41]</td>
</tr>
<tr>
<td>4</td>
<td>(1500, .40)(750, .80) [9]</td>
<td>16</td>
<td>(-1500, .20)(-750, .40) [31]</td>
</tr>
<tr>
<td>5</td>
<td>(1500, .30)(900, .40) [49]</td>
<td>17</td>
<td>(1500, .60)(900, .80) [19]</td>
</tr>
<tr>
<td>6</td>
<td>(1250, .55)(750, .75) [19]</td>
<td>18</td>
<td>(-1250, .55)(-750, .75) [22]</td>
</tr>
<tr>
<td>7</td>
<td>(1300, .50)(800, .80) [7]</td>
<td>19</td>
<td>(1300, .15)(800, .25) [20]</td>
</tr>
<tr>
<td>8</td>
<td>(1750, .30)(1050, .70) [1]</td>
<td>20</td>
<td>(-1750, .30)(-1050, .70) [53]</td>
</tr>
<tr>
<td>9</td>
<td>(1650, .50)(1150, .60) [43]</td>
<td>21</td>
<td>(1650, .25)(1150, .30) [39]</td>
</tr>
<tr>
<td>10</td>
<td>(1350, .40)(950, .60) [17]</td>
<td>22</td>
<td>(1350, .10)(950, .15) [38]</td>
</tr>
<tr>
<td>11</td>
<td>(1700, .20)(1200, .50) [2]</td>
<td>23</td>
<td>(-1700, .20)(-1200, .50) [51]</td>
</tr>
<tr>
<td>12</td>
<td>(1400, .40)(1000, .80) [1]</td>
<td>24</td>
<td>(-1400, .40)(-1000, .80) [53]</td>
</tr>
</tbody>
</table>

#### Set 2 (N=57)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(160, .25)(80, .35) [57]</td>
<td>13</td>
<td>(-160, .25)(-80, .35) [4]</td>
</tr>
<tr>
<td>2</td>
<td>(150, .30)(75, .50) [33]</td>
<td>14</td>
<td>(150, .15)(75, .25) [40]</td>
</tr>
<tr>
<td>3</td>
<td>(170, .35)(85, .65) [18]</td>
<td>15</td>
<td>(-170, .45)(-85, .85) [43]</td>
</tr>
<tr>
<td>4</td>
<td>(150, .40)(75, .80) [8]</td>
<td>16</td>
<td>(-150, .20)(-75, .40) [33]</td>
</tr>
<tr>
<td>5</td>
<td>(150, .30)(90, .40) [54]</td>
<td>17</td>
<td>(150, .60)(90, .80) [23]</td>
</tr>
<tr>
<td>6</td>
<td>(125, .55)(75, .75) [26]</td>
<td>18</td>
<td>(-125, .55)(-75, .75) [37]</td>
</tr>
<tr>
<td>7</td>
<td>(130, .50)(80, .80) [10]</td>
<td>19</td>
<td>(130, .15)(80, .25) [28]</td>
</tr>
<tr>
<td>8</td>
<td>(175, .30)(105, .70) [3]</td>
<td>20</td>
<td>(-175, .30)(-105, .70) [56]</td>
</tr>
<tr>
<td>9</td>
<td>(165, .50)(115, .60) [51]</td>
<td>21</td>
<td>(165, .25)(115, .30) [48]</td>
</tr>
<tr>
<td>10</td>
<td>(135, .40)(95, .60) [21]</td>
<td>22</td>
<td>(135, .10)(95, .15) [43]</td>
</tr>
<tr>
<td>11</td>
<td>(170, .20)(120, .50) [1]</td>
<td>23</td>
<td>(-170, .20)(-120, .50) [55]</td>
</tr>
<tr>
<td>12</td>
<td>(140, .40)(100, .80) [0]</td>
<td>24</td>
<td>(-140, .40)(-100, .80) [54]</td>
</tr>
</tbody>
</table>

#### Set 3 (N=50)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(13, .50)(8, .70) [25]</td>
<td>13</td>
<td>(13, .30)(8, .40) [37]</td>
</tr>
<tr>
<td>2</td>
<td>(15, .60)(9, .80) [28]</td>
<td>14</td>
<td>(15, .30)(9, .40) [40]</td>
</tr>
<tr>
<td>3</td>
<td>(10, .60)(5, .80) [37]</td>
<td>15</td>
<td>(10, .30)(5, .40) [47]</td>
</tr>
<tr>
<td>4</td>
<td>(11, .40)(7, .60) [16]</td>
<td>16</td>
<td>(17, .40)(13, .60) [8]</td>
</tr>
<tr>
<td>5</td>
<td>(14, .50)(9, .85) [7]</td>
<td>17</td>
<td>(14, .20)(9, .35) [28]</td>
</tr>
<tr>
<td>6</td>
<td>(17, .50)(9, .70) [39]</td>
<td>18</td>
<td>(-17, .50)(-9, .70) [18]</td>
</tr>
<tr>
<td>7</td>
<td>(16, .30)(5, .50) [43]</td>
<td>19</td>
<td>(-16, .30)(-5, .50) [10]</td>
</tr>
<tr>
<td>8</td>
<td>(16, .40)(10, .70) [5]</td>
<td>20</td>
<td>(-16, .40)(-10, .70) [46]</td>
</tr>
<tr>
<td>9</td>
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<td>(11, .70)(5, .80) [49]</td>
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### B.2 Simple Choice Problems Used in Experiments 2 and 3

(To the right: number of subjects who chose left-hand lottery in each experiment)

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### B.3 Mixed Choice Problems Used in Experiments 2 and 3

(To the right: number of subjects who chose left-hand lottery in each experiment)

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### B.4 Nonconflictingual (EP) Choice
Problems Used in Experiments 2 and 3

(To the right: number of subjects who chose left-hand lottery in each experiment)

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Appendix C

INSTRUCTIONS GIVEN TO SUBJECTS IN EXPERIMENTS
C.1 Instructions Given to Subjects in Experiment 1

This is an experiment on how people make decisions in uncertain situations. We will represent such situations as chances to win or lose sums of money. For example, one such situation might be:

36% chance to win $60

You will be presented with pairs of situations of this kind and asked to choose the one you would prefer to be in. For example, you might be asked to choose between a situation with a 58% chance to win $30 and another situation with a 32% chance to win $60. This problem would be represented as follows:

58% chance to win $30 ___
32% chance to win $60 ___

You are to place an "X" in the blank corresponding to the situation you would prefer to be in. Sometimes the choice will be between two possible gains (as above); other times it will be between two possible losses, illustrated as follows:

28% chance to lose $50 ___
37% chance to lose $40 ___

For each problem, please choose exactly one of the two situations indicated (no ties allowed).
C.2 Instructions Given to Subjects in Experiment 2

This is an experiment on how people make decisions in uncertain situations. We will represent such situations as chances to win or lose sums of money. For example, one such situation might be a 36% chance to win $21. It will be represented by the following circle, where one slice (which consists of 36% of the circle's area) represents a 36% chance to win $21 and the remainder of the circle (64% of the area) represents the chance to win or lose nothing.

You will be presented with pairs of situations of this kind and asked to choose the one you would prefer to be in. For example, you might be asked to choose between a situation with a 36% chance to win $21 and another situation with a 16% chance to win $29. This problem would be represented as follows:

You are to place an 'X' in the blank corresponding to the situation you would prefer to be in. Sometimes the choice will be between two possible gains (as above), other times it will be between two possible losses. Illustrated as follows:

Again, you are to choose the circle that represents the situation you would rather be in. Some situations, moreover, will include both possible gains and losses. For example:

As before, you are to place an 'X' in the blank corresponding to the situation you would prefer to be in. For each problem, please choose exactly one of the two situations indicated (no ties allowed).

Please try to indicate your true choices. At the end of this experiment we will select one of the problems, and you will have the option to play it for money. If you do choose to play, you will have to stick with the situation that you originally selected.
C.3 Instructions Given to Subjects in Experiment 3

This is an experiment on how people make decisions in uncertain situations. We will represent such situations as chances to win or lose sums of money. For example, one such situation might be:

**Situation A:** a 36% chance to win $60

You will be presented with pairs of situations of this kind and asked to choose the one you would prefer to be in. For example, you might be asked to choose between a situation with a 58% chance to win $30 and another situation with a 32% chance to win $60. This problem would be represented as follows:

**Choose between:**
- Situation A: a 58% chance to win $30 ___
- Situation B: a 32% chance to win $60 ___

You are to place an "X" in the blank corresponding to the situation you would prefer to be in. Sometimes the choice will be between two possible gains (as above); other times it will be between two possible losses, illustrated as follows:

**Choose between:**
- Situation A: a 28% chance to lose $50 ___
- Situation B: a 37% chance to lose $40 ___

Some situations, moreover, will include both possible gains and losses. For example:

**Choose between:**
- Situation A: a 72% chance to win $10 and a 28% chance to lose $5 ___
- Situation B: a 54% chance to win $15 and a 46% chance to lose $8 ___

Notice, by the way, that in these latter cases the probabilities of winning and losing always add up to 100%. As before, you are to place an "X" in the blank corresponding to the situation you would prefer to be in. For each problem, please choose exactly one of the two situations—indicated (no ties allowed).

Please try to indicate your true choices. At the end of this experiment we will select one of the problems, and you will have the option to play it for money. If you do choose to play, you will have to stick with the situation that you originally selected.
Appendix D

Intransitivity among mixed lotteries

We now demonstrate that, for any person with $k_a = k_L = .5$, the Advantage Model predicts an intransitive pattern of preferences among the following three mixed lotteries:

$$(20, .20, -5)$$
$$(10, .40, -8)$$
$$(6, .60, -13).$$

The two mixed lotteries in the choice problem $[(20, .20, -5), (10, .40, -8)]$ are assigned the following attractiveness coefficients by the Advantage Model:

$$AMk_a, k_L((20, .20, -5); [(20, .20, -5), (10, .40, -8)]) =$$
$$AMk_a((20, .20); [(20, .20), (10, .40)]) + AMk_L((-5, .80); [(-5, .80), (-8, .60)]) =$$
$$.40(.20)(10/20)k_a + (-5)(.80)(.60 - .60) = 2k_a - .80 = .20$$

$$AMk_a, k_L((10, .40, -8); [(20, .20, -5), (10, .40, -8)]) =$$
$$AMk_a((10, .40); [(20, .20), (10, .40)]) + AMk_L((-8, .60); [(-5, .80), (-8, .60)]) =$$
$$.40(.40 - .20) + (-8)(.60)(-3/-8)k_L = .80 - 1.8k_L = -.10$$

The two mixed lotteries in the choice problem $[(10, .40, -8), (6, .60, -13)]$ are assigned the following attractiveness coefficients:

$$AMk_a, k_L((10, .40, -8); [(10, .40, -8), (6, .60, -13)]) =$$
$$AMk_a((10, .40); [(10, .40), (6, .60)]) + AMk_L((-8, .60); [(-8, .60), (-13, .40)]) =$$
$$.40(4/10)k_a + (-8)(.60)(.60 - .40) = 1.6k_a - .96 = -.16$$

$$AMk_a, k_L((6, .60, -13); [(10, .40, -8), (6, .60, -13)]) =$$
$$AMk_a((6, .60); [(10, .40), (6, .60)]) + AMk_L((-13, .40); [(-8, .60), (-13, .40)]) =$$
$$.60(.60 - .40) + (-13)(.40)(-5/-13)k_L = .72 - 2k_L = -.28$$
The mixed lotteries in the choice problem \{(20, 20, -5), (6, 60, -13)\} are assigned the following attractivenesses:

\[
\text{AMk}_k \text{a}_{kL}(20, 20, -5); \{(20, 20, -5), (6, 60, -13)\} = \\
\text{AMk}_0((20, 20); \{(20, 20), (6, 60)\}) + \\
\text{AMk}_L((-5, .80); \{(-5, .80), (-13, .40)\}) = \\
20(.20)(14/20)k_a + (-5)(.80)(.80-.40) = 28k_a - 1.6 = -.20
\]

\[
\text{AMk}_k \text{a}_{kL}((6, 60, -13); \{(20, 20, -5), (6, 60, -13)\} = \\
\text{AMk}_0((6, 60); \{(20, 20), (6, 60)\}) + \\
\text{AMk}_L((-13, .40); \{(-5, .80), (-13, .40)\}) = \\
6(.60)(.60-.20) + (-13)(.40)(-8/-13)k_L = 1.44 - 3.2k_L = -.16
\]

Notice that the lottery \((20, 20, -5)\) is assigned a positive attractiveness -- larger than the competing lottery's -- in the first comparison, but a negative attractiveness -- lower than the competing lottery's -- in the third comparison. In fact, for any person with \(k_a = k_L = .5\), the calculations above predict that \((20, 20, -5)\) is preferred over \((10, 40, -8)\), that \((10, 40, -8)\) is preferred over \((6, 60, -13)\), but that \((6, 60, -13)\) is preferred over \((20, 20, -5)\). Hence, the Advantage Model predicts an intransitivity of preferences.
Appendix E

Preference Reversal in Mixed Choice Problems

We now demonstrate that a person with, say, \( k_\alpha = k_L = .5 \) for choice, and \( k_\alpha = k_L = 1.2 \) for monetary value estimation, is predicted by the Advantage Model to exhibit preference reversal on Lichtenstein and Slovic's (1971) mixed choice problem number 4. Similar calculations (using the same \( k_\alpha, k_L \) values) reveal preference reversal in the rest of Lichtenstein and Slovic's choice problems.

According to the Advantage Model, when faced with the choice problem \([(5.25, .50, -1.50), (2.00, .90, -2.00)]\), a person with \( k_\alpha = k_L = .5 \) assigns the following attractiveness coefficients to the two mixed lotteries figuring in the problem:

\[
AMk_\alpha k_L((5.25, .50, -1.50);((5.25, .50, -1.50), (2.00, .90, -2.00))) = \\
AMk_\alpha((5.25, .50);(5.25, .50), (2.00, .90)) + \\
AMk_L((-1.50, .50);((-1.50, .50), (-2.00, .10))) = \\
5.25(.50)(3.25/5.25)k_\alpha + (-1.50)(.50)(.50-.10) = \\
1.625k_\alpha - .30 = .5125
\]

\[
AMk_\alpha k_L((2.00, .90, -2.00);((5.25, .50, -1.50), (2.00, .90, -2.00))) = \\
AMk_\alpha((2.00, .90);(5.25, .50), (2.00, .90)) + \\
AMk_L((-2.00, .10);((-1.50, .50), (-2.00, .10))) = \\
2.00(.90)(.90-.50) + (-2.00)(.10)(.50/-2.00)k_L = \\
.72 - .05k_L = .695
\]

On the other hand, according to the model, a person whose \( k_\alpha = k_L = 1.2 \) during monetary value estimation, assigns the following monetary values to the two lotteries:

The monetary value of \((5.25, .50, -1.50)\) equals the sum of the monetary values of \((5.25, .50)\) and of \((-1.50, .50)\). In other words, it equals \((x + y)\) such that \((5.25, .50) = (x,1)\) and \((-1.50, .50) = (y,1)\).
An Advantage Model of Risky Choice

\[(5.25, .50) = (x, 1)\]
\[A_{MK}(5.25, .50; (5.25, .50), (x, 1)) = A_{MK}(x, 1; (5.25, .50), (x, 1))\]
\[5.25(.50)\frac{(5.25-x) / 5.25}{k_a} = x(1)(1-.50)\]
\[3.15(1 - x / 5.25) = x(.50)\]
\[3.15 - .60x = .50x\]
\[x = 2.86\]

\[(-1.50, .50) = (y, 1)\]
\[A_{MK}(y, 1; (-1.50, .50), (y, 1)) = A_{MK}(y, 1; (-1.50, .50), (y, 1))\]
\[-1.50(.50)\frac{(-1.50-y) / -1.50}{k_{L}} = y(1)(1-.50)\]
\[-.90(1 - y / -1.50) = y(.50)\]
\[-.90 - .60y = .50y\]
\[y = -.82\]

Thus, the monetary value of \((5.25, .50, -1.50)\) equals \(2.86 - .82 = 2.04\).

The monetary value of \((2.00, .90, -2.00)\) equals the sum of the monetary values of \((2.00, .90)\) and of \((-2.00, .10)\). In other words, it equals \((x + y)\) such that \((2.00, .90) = (x, 1)\) and \((-2.00, .10) = (y, 1)\).

\[(2.00, .90) = (x, 1)\]
\[A_{MK}(2.00, .90; (2.00, .90), (x, 1)) = A_{MK}(x, 1; (2.00, .90), (x, 1))\]
\[2.00(.90)\frac{(2.00-x) / 2.00}{k_a} = x(1)(1-.90)\]
\[2.16(1 - x / 2.00) = x(.10)\]
\[2.16 - 1.08x = .10x\]
\[x = 1.83\]

\[(-2.00, .10) = (y, 1)\]
\[A_{MK}(y, 1; (-2.00, .10), (y, 1)) = A_{MK}(y, 1; (-2.00, .10), (y, 1))\]
\[-2.00(.10)\frac{(-2.00-y) / -2.00}{k_{L}} = y(1)(1-.10)\]
\[-.24(1 - y / -2.00) = y(.90)\]
\[-.24 - .12y = .90y\]
\[y = -.24\]

Thus, the monetary value of \((2.00, .90, -2.00)\) equals \(1.83 - .24 = 1.59\).

Thus, while the Advantage Model predicts that the lottery \((2.00, .90, -2.00)\) will be chosen, its assigns a higher monetary value to the lottery \((5.25, .50, -1.50)\). This predicts a preference reversal of the form exhibited by the majority of Lichtenstein and Slovic's subjects.