AMPLIFICATION OF GENERALIZED SURFACE WAVES

by

EVANGELOS MICHALOPOULOS

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Signature of Author Signature redacted
Department of Civil Engineering, May 20, 1976

Certified by Signature redacted
Thesis Supervisor

Accepted by Signature redacted
Chairman, Departmental Committee on Graduate
Students in the Department of Civil
Engineering

ABSTRACT

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The effect of a horizontally stratified deposit of soil layers in amplifying and filtering Generalized Surface Waves is studied. A condition of Plane Waves is considered and the soil is assumed to be a linear, viscoelastic material.

Displacements and amplification functions for an elastic half-space and a uniform soil layer resting on the half-space (representing the rock) are obtained. Results are given for SV waves travelling upwards through the rock at arbitrary angles of incidence and for stress waves generated at the surface by unit line loads (normal and shear).

The application of the one dimensional amplification theory in obtaining displacements and amplification function is examined. The theory can be used for determining significant frequencies in amplification studies of the motion.

Thesis Supervisor Josée M. Roesset

Title Professor of Civil Engineering

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Στούς γονεῖς μου

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LIST OF SYMBOLS

$\sigma_x, \sigma_y, \sigma_z$	-	Normal Stresses in x,y, and z Directions
$\tau_{xz}, \tau_{xy}, \tau_{yz}$	-	Shearing Stresses on x-z, x-y and y-z Planes
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	-	Normal Strains in x,y, and z Directions
$\gamma_{xz}, \gamma_{xy}, \gamma_{yz}$	-	Shearing Strains on x-z, x-y, and y-z Planes
ρ	-	Mass Density
E	-	Young's Modulus of Elasticity
G	-	Modulus of Elasticity in Shear
v	-	Poisson's Ratio
λ	-	Lame constant
π	-	3.14159 ...
β	-	percent of critical damping
ω	-	frequency in rad/sec
f	-	frequency in cps
f_n	-	natural frequency of material
$\{\Omega\}$	-	Rotation vector (Ω_x, Ω_y, Ω_z)
e	-	volumetric strain (= $\varepsilon_x + \varepsilon_y + \varepsilon_z$)
u,v,w	-	Particle Displacements in x,y, and z Directions
P	-	Dilational Wave
S	-	Shear Wave
SV	-	Shear wave in the Vertical Plane (x-z)
SH	-	Shear wave in the Horizontal Plane (x-y)
A_p	-	Amplitude of P-wave in negative z-direction

A_p'	- Amplitude of P-wave in positive z-direction
A_{SV}	- Amplitude of SV-wave in negative z-direction
A_{SV}'	- Amplitude of SV Wave in positive
A_{SH}	- Amplitude of SH Wave
A	- vector of amplitudes (A_p, A_{SV}, A_p', A_{SV}')
α_p	- angle of incidence of P wave
α_s	- angle of incidence of SV wave
ℓ,m,n	- Directional Cosines for P wave in x,y, and z directions
ℓ',m',n'	- Directional Cosines for SV waves in x,y, and z directions
ℓ'',m",n"	- Directional Cosines for SH waves in x,y, and z directions
f_p, f_s, f_g	- function of x,y,z and t for P, SV, SH waves respectively
f(x,t)	- function of x and t
V_p,V_s	- Dilatational and shear velocities respectively
u_p,v_p,w_s	- Displacements, u,v,w due to P wave
u_s,v_s,w_s	- Displacements u,v,w due to Shear wave
h	- Depth of layer
T	- Matrix of coefficients for top of layer
T_r	- T matrix for rock
H	- transformation matrix
F	- Matrix product of THT^{-1} matrices
G	- Matrix product of T_r^{-1} F

CHAPTER 1

INTRODUCTION

The effects of local soils conditions on the dynamic response of structures to earthquake motions has been recognized for some time. Traditionally this problem has been analyzed by decomposing it into two parts:

1) soil amplification; and

2) soil-structure interaction.

The first part examines the effects of the soil on the characteristics (amplitude and frequency) of earthquake motions. The second part is further subdivided into the determination of soil stiffnesses from the response of rigid massless foundations under harmonic excitations and the dynamic analysis of structures resting on "springs" with the obtained soil stiffnesses under the motion computed in 1.

Both soil amplification and soil structure interaction belong mathematically to the family of wave propagation problems in continuum, with mixed boundary conditions (force and displacement compatibilities). While it is possible for any particular situation to solve the total problem in one step, the importance of different parameters is better understood by conducting parametric studies on each part separately.

The solution of the wave problem is a difficult one due to the complexity of the boundary conditions and the representation of key parameters (i.e., the geometry of the constituents, uncertainties in soil properties, etc.). Various methods, such as finite elements, discrete or continuous models, have been used to attack this problem. Analytical solutions, though, have been possible only for a limited number of cases with simple geometries.

Here the interest lies in such solutions for the soil amplification case. Historically the soil amplification problem started from the analysis of one dimensional amplification of SH waves propagating vertically through the soil. The soil was considered first as an elastic half-space but later included horizontally layered profiles (9). Then the studies proceeded into consideration of SH waves at arbitrary angles of incidence (10) and were extended to plane P waves at arbitrary angles of incidence and plane SV waves at angles less than the critical (3).

This work is a logical continuation of studies presented in references 9, 10 and 3. First an analytical solution is given for P, SV and SH waves propagating in three dimensions. The solution is obtained by direct integration of the differential equations of motions in terms of amplitudes, for the three dimensional case (chapter 2) as opposed to potentials used in References 1 and 3 for plane waves. This is applied to Plane Waves propagating in both an elastic half space and a layered profile. The waves are the result of:

1) incoming waves propagating upwards through the bottom of the stratum at arbitrary angles of incidence (Chapters 3 and 4); and

2) stress waves generated at the surface and propagating downwards through the soil (Chapter 5).

The boundary conditions are imposed directly or by means of Fourier Transforms, depending on the case considered.

Displacements and amplification ratios are given in the first case for SV waves at arbitrary angles of incidence with the emphasis being on angles greater than the critical. Results are obtained for an elastic half space and for a uniform layer of soil resting on a half space. A one dimensional geometry is imposed, that is the motion is independent of the horizontal coordinate. Thus the motion is function of depth as well as frequency (Chapter 4).

In case 2 the displacements and amplification ratios are obtained for unit line loads (normal and shear). They are now functions of frequency and both vertical and horizontal coordinates. Amplification ratios are obtained only for the points directly under the load.

CHAPTER 2

BASIC FORMULATION IN THREE DIMENSIONS

2.1 Wave Equations

The dynamic equilibrium equations for the three-dimensional case in cartesian coordinates are:

$$
\begin{aligned}
&\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} = \rho \ddot{u} \\
&\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} = \rho \ddot{v} \\
&\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_x}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} = \rho \ddot{w}
\end{aligned}
\tag{2.1}
$$

where σ_x, σ_y and σ_z are the normal stresses, τ_{xy}, τ_{xz} and τ_{yz} are the shear stresses and u, v and w are the displacements in the x,y and z directions respectively. ρ is the mass density of the material. For definition of the coordinate system see Fig. 2.1.

The strain displacement relations for small deformations (linear geometry) are:

$$
\begin{aligned}
&\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \\
&\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
\end{aligned}
\tag{2.2}
$$

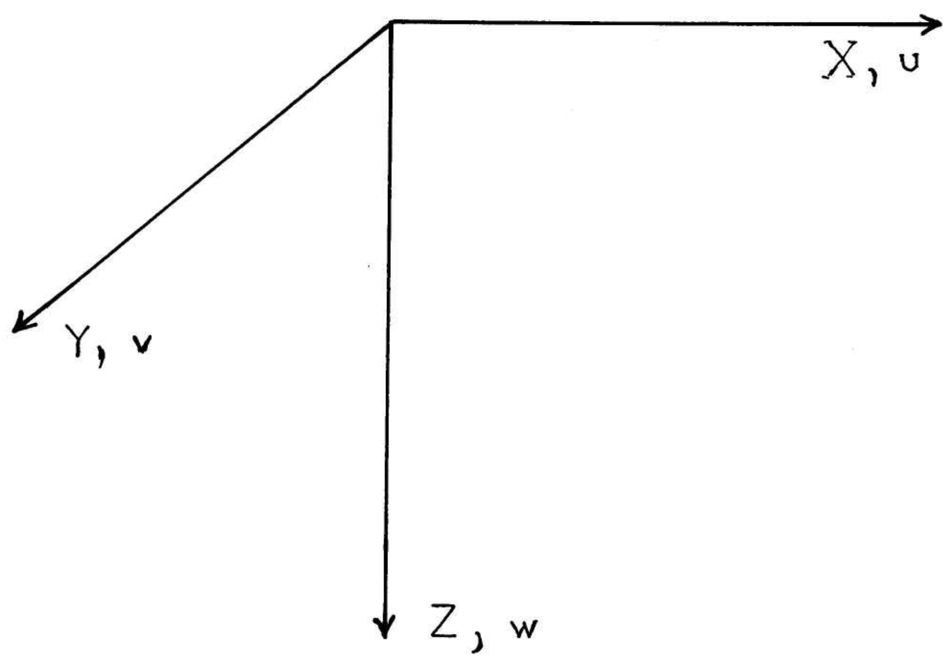


Fig. 2.1 Definition of coordinate system

Finally the stress-strain relations (constitutive equation) for a linear elastic, homogeneous isotropic material are given by:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z)$$

$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z)$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \tag{2.3a}$$

$$\gamma_{xz} = \frac{1}{G}\tau_{xz}$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$

where E is Young's modulus of elasticity, v is Poisson's ratio and G is the shear modulus.

Alternatively solving for the stresses in terms of the strains Eq. (3a) becomes:

$$\sigma_x = \lambda \ e + 2 \ G \ \varepsilon_x$$

$$\sigma_y = \lambda \ e + 2 \ G \ \varepsilon_y$$

$$\sigma_z = \lambda \ e + 2 \ G \ \varepsilon_z \tag{2.3b}$$

$$\tau_{xy} = G \ \gamma_{xy}$$

$$\tau_{xz} = G\gamma_{xz}$$

$$\tau_{yz} = G\gamma_{yz}$$

where λ and G are Lame's constants and e is the volumetric strain;

$$\lambda = \frac{vE}{(1+v)(1-2v)}$$

$$G = \frac{E}{2(1+v)}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\partial_u}{\partial_x} + \frac{\partial_v}{\partial_y} + \frac{\partial_w}{\partial_z}$$

By substituting Eq. (3b) into Eq. (1) the equations of motion in terms of displacements (stiffness formulation) are obtained.

$$(\lambda + G)\ \frac{\partial_e}{\partial_x} + G\ \nabla^2 u = \rho\ddot{u}$$

$$(\lambda + G)\ \frac{\partial_e}{\partial_y} + G\ \nabla^2 v = \rho\ddot{v}$$

$$(\lambda + G)\ \frac{\partial_e}{\partial_z} + G\ \nabla^2 w = \rho\ddot{w}$$

with $\nabla^2 = \frac{\partial}{\partial_x{}^2} + \frac{\partial}{\partial_y{}^2} + \frac{\partial}{\partial_z{}^2}$ being the Laplace operator.

Introducing a rotation vector $\{\Omega\}$ with components Ω_x, Ω_y, Ω_z

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

the equations of motion become:

$$(\lambda + 2G)\frac{\partial e}{\partial x} + 2G\left(\frac{\partial \Omega_y}{\partial z} - \frac{\partial \Omega_z}{\partial y}\right) = \rho\ddot{u}$$

$$(\lambda + 2G)\frac{\partial e}{\partial y} + 2G\left(\frac{\partial \Omega_z}{\partial x} - \frac{\partial \Omega_x}{\partial t}\right) = \rho\ddot{v}$$

$$(\lambda + 2G)\frac{\partial e}{\partial z} + 2G\left(\frac{\partial \Omega_x}{\partial y} - \frac{\partial \Omega_y}{\partial x}\right) = \rho\ddot{w}$$

or in vector form

$$(\lambda + 2G)\,\{\text{grad } e\} + 2G \text{ rot } \{\Omega\} = \rho\{\ddot{U}\}$$

It is possible to uncouple the volumetric strain e and the rotation vector $\{\Omega\}$. This can be accomplished by differentiating the first equation with respect to x, the second equation with respect to y and the third with respect to z and adding them to uncouple the volumetric strain e. In a similar manner differentiating the first equation with respect to y, the second with respect to x etc., the rotation vector $\{\Omega\}$ is uncoupled. The above procedure leads to

$$(\lambda + 2G)\ \nabla^2 e = \rho \frac{\partial^2 e}{\partial t^2} = \rho \ddot{e}$$

$$G\nabla^2 \Omega_x = \rho \frac{\partial^2 \Omega_x}{\partial_t^2} = \rho \ddot{\Omega}_x \qquad (2.4)$$

$$G\nabla^2 \Omega_y = \rho \frac{\partial^2 \Omega_y}{\partial t^2} = \rho \ddot{\Omega}_y$$

$$G\nabla^2 \Omega_z = \rho \frac{\partial^2 \Omega_z}{t^2} = \rho \Omega_z$$

or alternatively in vector form

$$G\nabla^2\ \{\Omega\} = \rho\ \{\ddot{\Omega}\}$$

with the additional condition

$$\mathrm{div}\ \{\Omega\} = \frac{\partial \Omega_x}{\partial_x} + \frac{\partial \Omega_y}{\partial_y} + \frac{\partial \Omega_z}{\partial_z} = 0$$

Calling

$$V_p^{\ 2} = \frac{\lambda + 2G}{\rho} \text{ and } V_s^{\ 2} = \frac{G}{\rho}$$

$$\nabla^2 e = \frac{1}{V_p^{\ 2}}\ \ddot{e} \qquad (2.5)$$

$$\nabla^2\ \{\Omega\} = \frac{1}{V_s^{\ 2}}\ \{\ddot{\Omega}\}$$

Equations (2.5) are the three dimensional Wave Equations for a linear elastic, homogeneous and isotropic material. V_p and V_s are the velocities of propagation of dilational (P) waves and shear (S) waves respectively.

2.2 Solution of the Wave Equations

The general solution of the wave equation

$$\nabla^2 u = \frac{1}{V^2} \ddot{u}$$

can be obtained employing D'Alembert's solution or by separation of variables (3,5,6,7,8) and is of the form

$$u = F(\ell x + my + nz \pm vt)$$

with

$$\ell^2 + n^2 + m^2 = 1$$

In this work only steady state harmonic motion is considered and thus the solution is taken to be of the complex exponential form.

Consequently the general solution of the wave equation (2.5) for a steady state harmonic motion with frequency ω becomes

$$e = A \exp \left[\frac{i\omega}{V_p} (V_p t - \ell x - my - nz)\right]$$

with $\quad \ell^2 + m^2 + n^2 = 1$

and

$$\{\Omega\} = \{B\} \exp \left[\frac{i\omega}{V_s} (V_s t - \ell' x - m' y - n' z)\right] \quad (2.6)$$

with $\quad \ell'^2 + m'^2 + n'^2 = 1$

$$\ell' Bx + m' By + n' Bz = 0$$

The last constraining equation is the result of the dependence between the rotation vector components imposed by the condition div $\{\Omega\} = o$ (Eq. 2.4).

The constants (ℓ,m,n) and (ℓ',m',n') are vectors with norms of 1. If the three components have modulus equal to or less than unity, they can be interpreted as director cosines and they represent then unit vectors indicating the direction of propagation of body waves (dilatational and shear waves respectively).

Considering first the dilatational (p) wave and calling

$$f_p = \exp\left[\frac{i\omega}{V_p}(V_p t - \ell x - my - nz)\right]$$

$$A_p = A\,\frac{i\,V_p}{\omega}$$

from the definition of e

$$\begin{aligned} u_p &= A_p\,\ell\,f_p \\ v_p &= A_p\,m\,f_p \\ w_p &= A_p\,n\,f_p \end{aligned} \tag{2.7}$$

which indicates that the motion u_p, v_p, w_p due to a P-wave propagating in the direction (ℓ,m,n) takes place entirely along that direction, with amplitude A_p and velocity of propagation v_p.

Similarly for the shear(S) wave defining

$$f_s = \exp\left[\frac{i\omega}{V_s}(V_s t - \ell' x - m' y - n' z)\right]$$

$$\{B\}_s = 2\,\frac{i\,V_s}{\omega}\,\{B\}$$

from the definition of the rotation vector$\{\Omega\}$

$$u_s = (n'\ Bys - m'\ Bzs)\ f_s$$

$$v_s = (\ell'\ Bzs - n'\ Bxs)\ f_s$$

$$w_s = (m'\ Bxs - \ell'\ Bys)\ f_s$$

From these equations it can be seen that the displacement vector (u_s, v_s, w_s) is orthogonal to the vector (ℓ', m', n') indicating the motion has no components along the direction of propagation. Consequently it is possible to find components of the motion in two orthogonal directions in a plane perpendicular to the direction of propagation.

Alternatively defining

$$A_{SH} = \frac{2\ i\ V_s}{\omega}\ \frac{Bz}{\sqrt{\ell'^2+m'^2}}$$

$$A_{SV} = \frac{2\ i\ V_s}{\omega}\ \frac{\ell'\ By - m'\ Bx}{\sqrt{\ell'^2+m'^2}}$$

$$u_S = \left(\frac{\ell' n'}{\sqrt{\ell'^2 + m'^2}} A_{SV} - \frac{m'}{\sqrt{\ell'^2 + m'^2}} A_{SH} \right) f_s$$

$$v_S = \left(\frac{m' n'}{\sqrt{\ell'^2 + m'^2}} A_{SV} + \frac{\ell'}{\sqrt{\ell'^2 + m'^2}} A_{SH} \right) f_s \qquad (2.8)$$

$$w_S = -\sqrt{\ell'^2 + m'^2}\, A_{SV}\, f_s$$

Combining equations 2.7 and 2.8 the total motion due to both P and S waves

$$u = u_p + u_s$$

$$v = v_p + v_s \qquad (2.9)$$

$$w = w_p + w_s$$

Summarizing the previous results the motion in an infinite space occurs:

a) in the direction of propagation - for a Dilatation (P) wave with amplitude A_p and velocity of propagation v_p.

b) in a vertical plane, perpendicular to the direction of propagation - for a shear (SV) wave with amplitude A_{SV} and velocity v_s.

c) in a horizontal plane, orthogonal to the direction of propagation - for a shear wave (SH) with amplitude A_{SH} and velocity v_s.

These are illustrated in figure 2.2 for a wave propagating in the x-z plane. The arrows describing the displacement are in two directions to demonstrate that the motion is periodic.

It is interesting to notice that if $\ell' = m' = 0$, $n' = \pm 1$, which corresponds to a direction of propagation coinciding with the z-axis ($\alpha=0°$), a distinction cannot be properly made between SH and SV waves.

The equations developed in this section are valid for both real and complex values of ℓ, m, n and ℓ', m', n' provided they satisfy equation 2.6. When all of these coefficients are real the case of Body Waves, which occur in an infinite medium, is obtained. If some of the coefficients are complex the equations represent Generalized Surface Waves (Generalized Love Waves if there is only shear distortion, Generalized Rayleigh Waves when there are both volumetric changes and shear distortions). The existence of Generalized Surface Waves arises from the boundary conditions or from discontinuities in material properties.

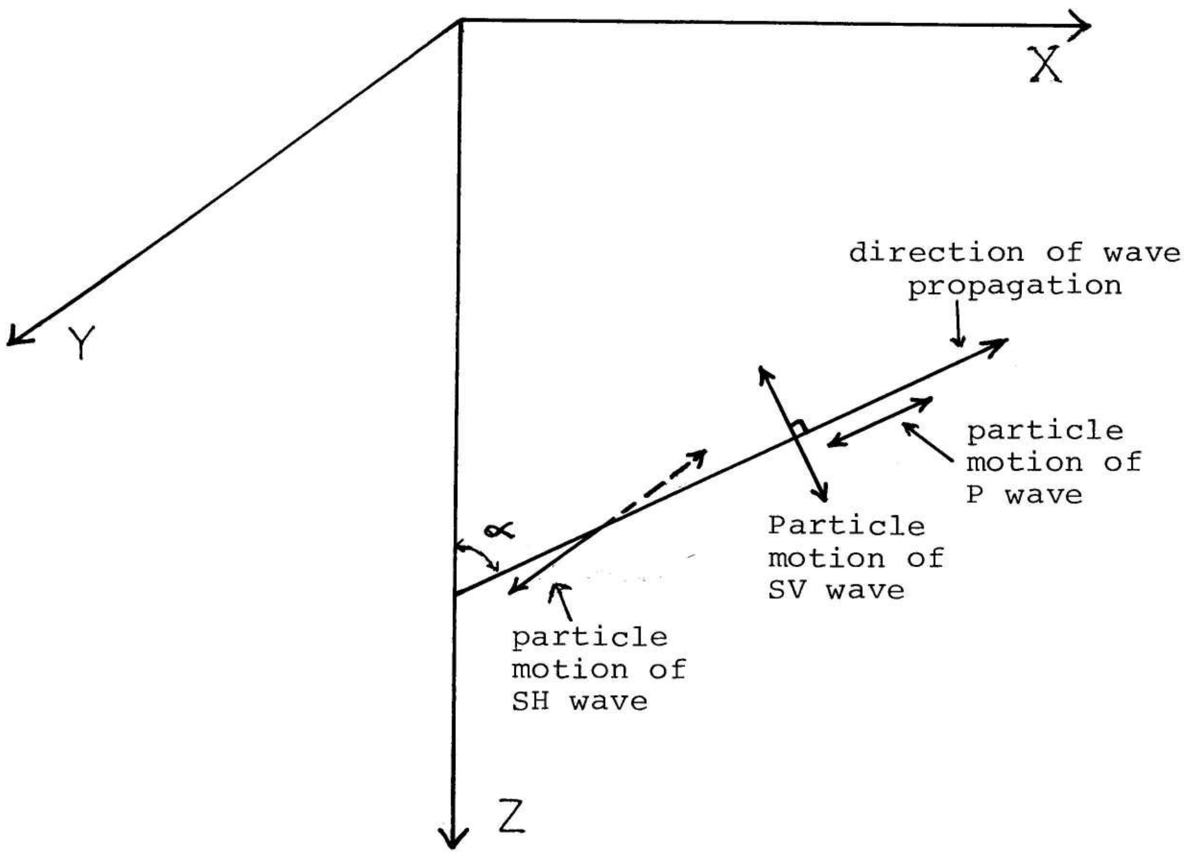


Fig. 2.2 Particle motion for P, SV and SH waves

CHAPTER 3

PLANE WAVES

3.1 Equations of Motion for Plane Waves

The condition of Plane Waves (x-z plane) can be directly obtained from the Three Dimensional Case (Chapter 2) by making $\varepsilon_y = 0$, $m = m' = 0$ and $\frac{\partial}{\partial_y} = 0$.

Then the dynamic equilibrium equations become

$$\frac{\partial\sigma_x}{\partial_x} + \frac{\partial\tau_{xz}}{\partial_z} = \rho\ddot{u}$$

$$\frac{\partial\tau_{xy}}{\partial_x} + \frac{\partial\tau_{yz}}{\partial_z} = \rho\ddot{v} \tag{3.1}$$

$$\frac{\partial\tau_{xz}}{\partial_x} + \frac{\partial\sigma_z}{\partial_z} = \rho\ddot{w}$$

The strain-displacement relations are

$$\varepsilon_x = \frac{\partial_u}{\partial_x} \qquad \gamma_{xz} = \frac{\partial_u}{\partial_z} + \frac{\partial_w}{\partial_x}$$

$$\varepsilon_z = \frac{\partial_w}{\partial_z} \qquad \gamma_{yz} = \frac{\partial_v}{\partial_z} \tag{3.2}$$

$$\varepsilon_y = 0 \qquad \gamma_{xy} = \frac{\partial_v}{\partial_x}$$

The stress-strain relations are

$$\begin{aligned}
\sigma_x &= \lambda e + 2\,G\,\varepsilon_x \\
\sigma_y &= \lambda e \\
\sigma_z &= \lambda e + 2\,G\,\varepsilon_z \\
\tau_{xz} &= G\,\gamma_{xz} \\
\tau_{xy} &= G\,\gamma_{xy} \\
\tau_{yz} &= G\,\gamma_{yz}
\end{aligned} \tag{3.3}$$

where now $e = \varepsilon_x + \varepsilon_z$ since $\varepsilon_y = 0$.

The equations of motion for P and SV are

$$\begin{aligned}
(\lambda + 2G)\,\nabla^2 e &= \rho\ddot{e} \\
G\,\nabla^2\,\Omega_y &= \rho\ddot{\Omega}_y
\end{aligned} \tag{3.4}$$

and for SH waves are

$$G\,\nabla^2 v = \rho\ddot{v}$$

where now

$$\nabla^2 = \frac{\partial^2}{\partial_x^2} + \frac{\partial^2}{\partial_z^2}$$

The general solution of a steady state harmonic motion is given by Equations 2.7, 2.8 and 2.9. Substituting m = m' = 0

$$\begin{aligned}
u &= \ell A_p\,f_s + n'\,A_{SV}\,f_s \\
v &= A_{SH}\,g_s \\
w &= n\,A_p\,f_p - \ell'\,A_{SV}\,f_s
\end{aligned} \tag{3.5}$$

where $\ell^2 + n^2 = 1 \quad \ell'^2 + n'^2 = 1 \quad \ell''^2 + n''^2 = 1$

$$f_p = \exp\left[\frac{i\omega}{V_p}(V_p t - \ell x - nz)\right]$$

$$f_s = \exp\left[\frac{i\omega}{V_s}(V_s t - \ell' x - n' z)\right]$$

$$g_s = \exp\left[\frac{i\omega}{V_s}(V_s t - \ell'' x - n'' z)\right]$$

These equations show that the study of P and SV waves must be considered simultaneously since it involves coupled in-plane displacements. On the other hand, the study of SH waves is independent of P and SV wave and can be performed separately.

The motion described by Equation 3.5 can be due to a system of plane waves propagating through an arbitrary layered medium. The system of waves, in turn, can be the result of

a) incoming waves travelling upwards through the bottom of the stratum profile.

b) stress waves generating at the surface and propagating downwards through the stratum profile.

Because the boundary conditions and the characteristics, in the two situations, are different they are considered separately. The first case is examined in the following sections and Chapter 4 and the second in Chapter 5.

3.2 One Dimensional Geometry

The study of plane waves propagating upwards through the bottom of the layered profile can be further simplified by assuming the same variation in time of all displacement components in the horizontal direction (x-axis). For this to be valid one must have a one dimensional geometry, that is the variation in soil properties must be a function only of the vertical coordinate (horizontally layered stratum profile).

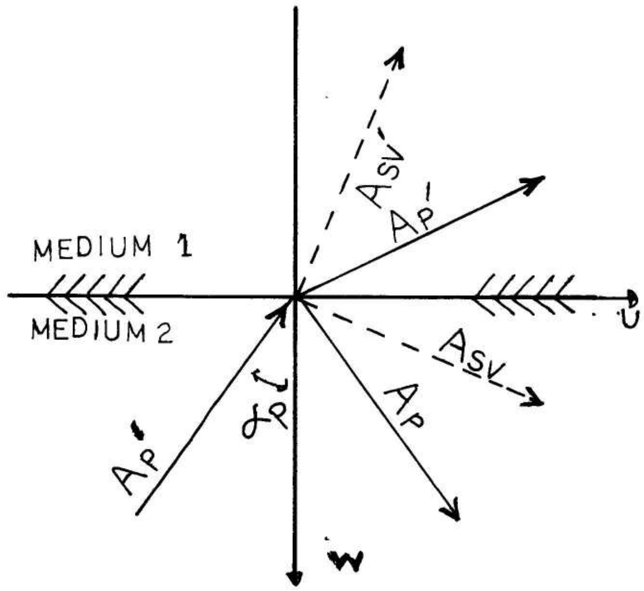
In mathematical terms the above assumption implies that f_p and f_s must have the same variation with respect to x, that is

$$\frac{i\omega\ell}{V_p} = \frac{i\omega\ell'}{V_s} \quad \text{or} \quad \frac{\ell}{V_p} = \frac{\ell'}{V_s} \tag{3.6}$$

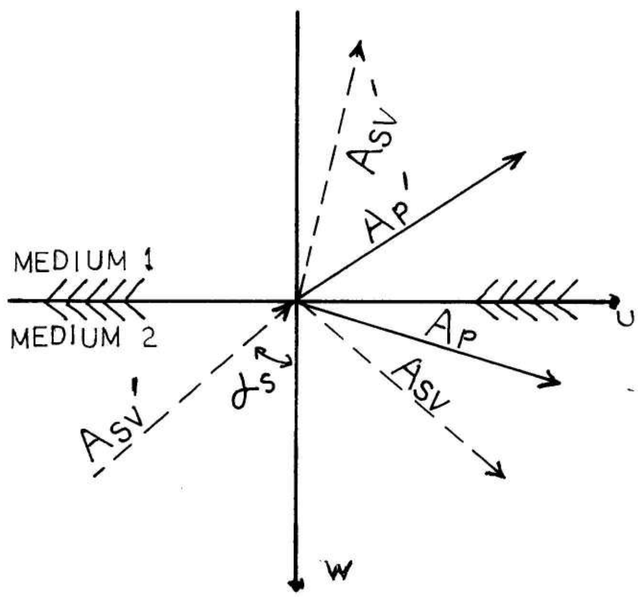
When P and S waves arrive at an interface, every incident dilatational or shear wave will produce two reflected and two refracted waves. There will be therefore a system of dilatational and shear waves propagating in the positive and negative x and z directions. (Figure 3.1)

The total motion of a point within any layer with constant properties must be obtained by adding the components of all waves in the proper directions in conjunction with Equations 3.5 and 3.6.

Considering first the case of P and SV waves and defining A_p as the amplitude of a P wave travelling in the negative z direction, A_p' as the amplitude of a similar wave travelling in the positive z direction and A_{SV}, A_{SV}' representing



incident P-wave



Incident S-wave

Fig. 3.1 Reflected and Refracted P and S waves the amplitudes of SV waves

$$u = [\ell\, A_p \exp\left(\frac{i\omega}{V_p} nz\right) + \ell\, A_p' \exp\left(-\frac{i\omega}{V_p} nz\right)$$

$$- n' A_{SV} \exp\left(\frac{i\omega}{V_s} n'z\right) + n' A_{SV}' \exp\left(-\frac{i\omega}{V_s} n'z\right)] \cdot$$

$$\cdot f(x,t) \qquad (3.7)$$

$$w = [- n\, A_p \exp\left(\frac{i\omega}{V_p} nz\right) + n\, A_p' \exp\left(-\frac{i\omega}{V_p} nz\right)$$

$$- \ell' A_{SV} \exp\left(\frac{i\omega}{V_s} n'z\right) - \ell' A_{SV}' \exp\left(-\frac{i\omega}{V_s} n'z\right)]$$

$$\cdot f(x,t)$$

with

$$\frac{\ell}{V_p} = \frac{\ell'}{V_s} \qquad n = \sqrt{1 - \ell^2} \qquad n' = \sqrt{1 - \ell'^2}$$

and $\frac{\ell}{V_p}$ constant with depth.

ℓ or ℓ' are arbitrary and if they possess a value between 0 and 1 they represent a train of P and SV waves at various angles. In this case the condition $\frac{\ell}{V_p} = \frac{\ell'}{V_s}$ can be written as $\frac{\sin\alpha_p}{V_p} = \frac{\sin\alpha_s}{V_s}$ where α_p and α_s are the angles of P and SV waves respectively, measured from the z-axis.

Since the dilatational wave velocity, V_p, is larger than the shear velocity, V_s, an incident P wave at any angle will always produce reflected and refracted P and SV waves.

On the other hand, for an incident SV wave, ℓ can be greater than 1 making $n = \sqrt{1 - \ell^2}$ imaginary. The angle, α_s, at which n first becomes imaginary is termed the critical angle (α_{crit}). For $\alpha_s > \alpha_{crit}$, calling $m = \sqrt{\ell^2 - 1}$

$$n = \pm mi$$

The function appearing in Equation 3.2 multiplying the A_p term is then

$$e^{\frac{i\omega}{V_p} nz} = e^{\pm \frac{\omega}{V_p} mz}$$

In order for the solution to be bound, as z tends to infinity, it is required that

$$n = -mi \ .$$

The term $\exp\left(\frac{i\omega}{V_p} nz\right)$ with n real represents a periodic shape whereas $\exp\left(- \frac{\omega}{V_p} mz\right)$ represents an exponential decay of amplitude with depth. Thus the condition $\ell > 1$ gives rise to Generalized Surface Waves. The same occurs if both ℓ and ℓ' are larger than 1.

Investigating further the solution it is observed that the motion is periodic and is described by the function f(x,t). The function reproduces itself at a point $x' = x + \Delta x$ at a time $t' = t + \frac{\ell \Delta x}{V_p}$. This furnishes an additional significance of the parameter ℓ, being associated with the apparent velocity of propagation in the horizontal direction $\frac{V_p}{\ell}$.

Turning now to the case of SH waves the corresponding type solution would be of the form

$$V = [A'_{SH} \exp(-\frac{i\omega}{V_s} n) + A_{SH} \exp(\frac{i\omega}{V_s} n)] f(x,t)$$

with

$$n = \sqrt{1 - \ell^2}$$

and $\frac{\ell}{V_s}$ constant with depth.

The limitation of the foregoing solutions is the variation with respect to x. A more general case where the boundary conditions, at z = constant, had an arbitrary variation with respect to x could be solved, however, as a superposition of these simple solutions with different values of ℓ.

3.3 Motion and Stresses as a Function of Depth

In order to compute the motions and stresses in one or more layers of soil resting on an elastic half space (representing the rock), due to a train of waves with frequency ω travelling upwards through the rock, boundary conditions must be introduced.

The stresses σ_z, τ_{xz} are given by

$$\sigma_z = \lambda e + 2 \varepsilon_x = \lambda(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) + 2 G \frac{\partial u}{\partial x}$$

$$\tau_{xz} = G \gamma_{xz} = G(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z})$$

Differentiating Eq. (3.7)

$$\sigma_z = - \frac{i\omega}{V_p} [(\lambda + 2 Gn^2) A_p \exp (\frac{i\omega}{V_p} nz)$$

$$+ (\lambda + 2 Gn^2) A_p' \exp (- \frac{i\omega}{V_p} nz)$$

$$+ 2 G\ell n' A_{SV} \exp (\frac{i\omega}{V_p} n'z)$$

$$- 2 G\ell n' A'_{SV} \exp (- \frac{i\omega}{V_s} n'z)] f(x,t)$$

and

$$\tau_{xz} = \frac{i\omega}{V_s} [2 G\ell' n A_p \exp (\frac{i\omega}{V_p} nz)$$

$$- 2 G\ell' n A_p' \exp (- \frac{i\omega}{V_p} nz)$$

$$+ G(\ell'^2 - n'^2) A_{SV} \exp (\frac{i\omega}{V_s} n'z)$$

$$+ G(\ell'^2 - n'^2) A_{SV}' \exp (- \frac{i\omega}{V_s} n'z)] f(x,t).$$

These equations together with Eq. (3.7) provide the displacements and stresses in terms of the amplitudes A_p, A_p', A_{SV}, A_{SV}'. Defining h as the depth of a layer and dropping the term f(x,t) the above equations become:

a) For the top of the layer

$$X_o = T A \tag{3.8}$$

b) For the bottom of the layer

$$X_h = BA = THA \tag{3.9}$$

where the subscript o is used to denote the top of the layer and h the bottom.

The explicit expressions of Equations (3.8) and (3.9) are given in the next two pages.

Imposing continuity of stresses and displacements at the interface between two layers, j and j'+1,

$$(X_h)_j = (X_o)_{j+1}$$

In order to satisfy these equations for any x,

$$\left(\frac{\ell}{V_p}\right)_j = \left(\frac{\ell}{V_p}\right)_{j+1} = \left(\frac{\ell'}{V_s}\right)_j = \left(\frac{\ell'}{V_s}\right)_{j+1}$$

(Shell's law of refraction)

From Eqs. (3.8) and (3.9)

$$(X_o)_{j+1} = T_{j+1}\ A_{j+1} = T_j\ H_j\ A_j$$

Therefore

$$A_{j+1} = T_{j+1}^{-1}\ T_j\ H_j\ A_j$$

Proceeding down from layer to layer and noting that at the very first (top) layer $A_1 = T_1^{-1}\ (X_0)_1$, then for the n^{th} layer

$$A_n = T_n^{-1}\ T_{n-1}\ H_{n-1} \cdots T_2^{-1}\ T_1\ H_1\ A_1$$

$$(X_h)_n = T_n\ H_n\ A_n = T_n\ H_n\ T_n^{-1}\ T_{n-1}\ H_{n-1}\ T_{n-1}^{-1} \cdots$$
$$T_1\ H_1\ T_1^{-1}\ (X_o)_1$$

EQUATION 3.8

$$
\begin{bmatrix} u \\ w \\ \sigma \\ \tau \end{bmatrix}_0 =
\begin{bmatrix}
\ell & -n' & \ell & n' \\
-n & -\ell' & n & -\ell' \\
-\frac{i\omega}{V_p}(\lambda+2Gn^2) & -\frac{i\omega}{V_p} 2G\ell n' & -\frac{i\omega}{V_p}(\lambda+2Gn^2) & \frac{i\omega}{V_p} 2G\ell n' \\
\frac{i\omega}{V_s} 2G\ell' n & \frac{i\omega}{V_s} G(\ell'^2-n'^2) & -\frac{i\omega}{V_s} 2G\ell' n & \frac{i\omega}{V_s} G(\ell'^2-n'^2)
\end{bmatrix}
\begin{bmatrix} A_P \\ A_{SV} \\ A_P' \\ A_{SV}' \end{bmatrix}
$$

MATRIX H

$$
\left|\begin{array}{c:c:c:c}
\exp\left(\frac{i\omega}{V_p}\, n h\right) & 0 & 0 & 0 \\
\hdashline
0 & \exp\left(\frac{i\omega}{V_s}\, n' h\right) & 0 & 0 \\
\hdashline
0 & 0 & \exp\left(-\frac{i\omega}{V_p}\, n h\right) & 0 \\
\hdashline
0 & 0 & 0 & \exp\left(-\frac{i\omega}{V_s}\, n' h\right)
\end{array}\right|
$$

Calling $$F_n = \prod_{i=1}^{n} (T_i \; H_i \; T_i^{-1})$$

$$(X_h)_n = F_n \cdot (X_o)_1 \qquad (3.10)$$

or

$$\begin{Bmatrix} u_h \\ w_h \\ \sigma_h \\ \tau_h \end{Bmatrix}_n = F_n \cdot \begin{Bmatrix} u_o \\ w_o \\ \sigma_o \\ \tau_o \end{Bmatrix}_1$$

Equation 3.10 relates stresses and displacements at the surface to those occurring at the bottom of any layer of the stratum. It is only required to calculate the matrices T_i and H_i for each layer. These are specified by Eqs. (3.8) and (3.9) in terms of the layer properties (moduli $\lambda + 2\,G$, G, depth and angular parameters ℓ, n, ℓ', n').

An explicit expression for T^{-1} can also be obtained. Rather than inverting a 4x4 matrix, if one works with the parameters $A_p + A_p'$, $A_p - A_p'$, $A_{SV} + A_{SV}'$ and $A_{SV} - A_{SV}'$ only a 2x2 matrix needs to be inverted.

Writing Eq. (3.8) in terms of the above parameters 2 uncoupled systems of two equations are obtained.

$$\begin{Bmatrix} u \\ \sigma \end{Bmatrix} = \begin{vmatrix} \ell & -n \\ -\frac{i\omega}{V_p}(\lambda + 2\,Gn^2) & -\frac{i\omega}{V_p}\,2\,G\ell n' \end{vmatrix} \begin{Bmatrix} A_p + A_p' \\ A_{SV} - A_{SV}' \end{Bmatrix}$$

$$\begin{Bmatrix} w \\ \tau \end{Bmatrix} = \begin{vmatrix} -n & -\ell' \\ \frac{i\omega}{V_s}\,2\,G\ell' n & \frac{i\omega}{V_s}\,G(\ell'^2 - n'^2) \end{vmatrix} \begin{Bmatrix} A_p - A_p' \\ A_{SV} + A_{SV}' \end{Bmatrix}$$

Inverting the two 2x2 matrices separately the above equations become:

$$\begin{Bmatrix} A_p + A_p' \\ A_{SV} - A_{SV}' \end{Bmatrix} = \frac{1}{D_1} \begin{vmatrix} -\frac{i\omega}{V_p}\,2\,G\ell n' & n' \\ \frac{i\omega}{V_p}(\lambda + 2\,Gn^2) & \ell \end{vmatrix} \begin{Bmatrix} u \\ \sigma \end{Bmatrix}$$

$$\begin{Bmatrix} A_p - A_p' \\ A_{SV} + A_{SV}' \end{Bmatrix} = \frac{1}{D_2} \begin{vmatrix} \frac{i\omega}{V_s}\,G\,(\ell'^2 - n'^2) & \ell' \\ -\frac{i\omega}{V_s}\,2\,G\ell' n & n \end{vmatrix} \begin{Bmatrix} w \\ \tau \end{Bmatrix}$$

where D_1 and D_2 are the determinants given by

$$D_1 = -\frac{i\omega}{V_p}\,n'\,(\lambda + 2G)$$

$$D_2 = \frac{i\omega}{V_s}\,Gn$$

Combining these equations the matrix T^{-1} is obtained. The complete relation is given in the next page.

MATPIX T^{-1}

$$
\begin{bmatrix} A_p \\ A_{SV} \\ A_p' \\ A_{SV}' \end{bmatrix}
= \frac{1}{2}
\begin{bmatrix}
\dfrac{2G\ell}{\lambda+2G} & \dfrac{\ell'^2 - n'^2}{n} & -\dfrac{1}{\frac{i\omega}{V_p}(\lambda+2G)} & \dfrac{\ell'}{\frac{i\omega}{V_s}(Gn)} \\
-\dfrac{\lambda+2Gn^2}{n'(\lambda+2G)} & -2\ell' & -\dfrac{\ell}{\frac{i\omega}{V_p} n'(\lambda+2G)} & -\dfrac{1}{\frac{i\omega}{V_s} G} \\
\dfrac{2G\ell}{\lambda+2G} & -\dfrac{\ell'^2 - n'^2}{n} & -\dfrac{1}{\frac{i\omega}{V_p}(\lambda+2G)} & -\dfrac{\ell'}{\frac{i\omega}{V_s}(Gn)} \\
\dfrac{\lambda+2Gn^2}{n'(\lambda+2G)} & -2\ell' & \dfrac{\ell}{\frac{i\omega}{V_p} n'(\lambda+2G)} & -\dfrac{1}{\frac{i\omega}{V_s} G}
\end{bmatrix}
\begin{bmatrix} u \\ w \\ \sigma \\ \tau \end{bmatrix}
$$

3.4 Determination of Amplification Functions

3.4.a Definition of Amplification Functions

From Eq. (3.10) the displacements and stresses at any level within the soil deposit can be obtained in terms of the displacements and stresses at the surface. Alternatively one can solve the inverse problem and determine the amplitudes of motion and stresses at the surface or within any point of the soil stratum produced by a specified harmonic motion at any given depth.

If a harmonic motion is specified at bedrock, the motion produced at the free surface can be related to the input motion simply by amplification ratios which are functions of the frequency ω. Two different amplification ratios can be defined (9, 10).

The first amplification ratio, called elastic rock amplification, is defined as the ratio of the amplitude of displacement at free surface to the amplitude of displacement that would occur at the top of the rock if there was no soil (hypothetical outcropping of rock).

The second amplification ratio, called rigid rock amplification, is defined as the ratio of the amplitude of displacement at the free surface of the soil to the corresponding amplitude at bedrock, that is at the interface between the rock and the bottom layer of soil. This motion is different from the motion of the outcropping of rock since it is affected by the presence of soil. It would coincide with the elastic case if the rock were infinitely stiff.

Since two amplification ratios have been defined and since there are two motions involved (vertical and horizontal) one obtains a total of four different amplification functions.

3.4.b <u>Rigid Rock Amplification</u>

Partioning the 4x4 F-matrix into four 2x2 matrices Eq. (3.10) becomes

$$\left\{\begin{array}{c} u_h \\ w_h \\ \hline \sigma_h \\ \tau_h \end{array}\right\}_n = \left|\begin{array}{c:c} F_{11} & F_{12} \\ \hdashline F_{21} & F_{22} \end{array}\right| \left\{\begin{array}{c} u_o \\ w_o \\ \hline \sigma_o \\ \tau_o \end{array}\right\}_1$$

Calling

$$(U_h)_n = U_s = \left\{\begin{array}{c} u_h \\ w_h \end{array}\right\}_n$$

$$U_o = \left\{\begin{array}{c} u_o \\ w_o \end{array}\right\}_1 \qquad S_o = \left\{\begin{array}{c} \sigma_o \\ \sigma_o \end{array}\right\}_1$$

$$U_s = F_{11}\, U_o + F_{12}\, S_o$$

The subscript n indicating the number of the layer considered has been dropped and substituted by the subscript s to represent the interface between the bottom layer of soil and rock.

Since the stresses at the free surface of the soil deposit are zero, $S_o = 0$

$$U_s = F_{11} U_o$$

or

$$U_o = F_{11}^{-1} U_s \tag{3.11}$$

Thus by specifying the amplitude of the motion at bedrock, U_s, the amplitude of the motion at the surface of the soil, U_o, can be determined. The horizontal and vertical amplification ratios for the rigid rock case are then

$$\frac{u_o}{u_s} \text{ and } \frac{w_o}{w_s} .$$

3.4c Elastic Rock Amplification

In a similar way the motion that would occur at bedrock if there were no soil on top (outcropping of rock), U_s, can be related to the motion at the top of the soil deposit, U_o.

Again from Eq. (3.10)

$$(X_h)_n = F_n \cdot (X_o)_1$$

or

$$(Xh)_n = T_n H_n A_n$$

also

$$(X_o)_n = T_n A_n$$

Introducing the subscript r to represent the rock layer and n the bottom layer of the soil then the above equations lead to

$$(X_o)_r = (X_h)_n = T_r \, A_r = F_n \, (X_o)_1$$

therefore

$$A_r = T_r^{-1} \, F_n \cdot (X_o)_1 = G \, (X_o)_1$$

where the G matrix defined by the above expression relates the amplitudes of the waves in the rock to the motion and stresses at the surface of the soil.

Partioning the G-matrix, similarly to the F-matrix and noting that the stresses at the free surface are zero

$$\begin{Bmatrix} A_p \\ A_{SV} \end{Bmatrix}_r = G_{11} \, U_o \tag{3.12}$$

If there were no soil on top of the rock (elastic half space) the motion would be described by Eq. (3.8) which in the present notation is

$$\begin{Bmatrix} U_s \\ \hline S_s \end{Bmatrix} = \begin{Bmatrix} u \\ w \\ \hline \sigma \\ \tau \end{Bmatrix}_s = \left| \begin{array}{c|c} T_{(11)_r} & T_{(12)_r} \\ \hline T_{(21)_r} & T_{(22)_r} \end{array} \right| \begin{Bmatrix} A_p \\ A_{SV} \\ \hline A_p' \\ A_{SV}' \end{Bmatrix}_r$$

Since S_s would be zero

$$\underline{O} = T_{(21)_r} \begin{Bmatrix} A_p \\ A_{SV} \end{Bmatrix} + T_{(22)_r} \begin{Bmatrix} A_p' \\ A_{SV}' \end{Bmatrix}_r$$

or

$$\begin{Bmatrix} A_p' \\ A_{SV}' \end{Bmatrix} = - T^{-1}_{(22)_r} \cdot T_{(21)_r} \cdot \begin{Bmatrix} A_p \\ A_{SV} \end{Bmatrix}_r \tag{3.13}$$

and

$$U_s = [T_{(11)_r} - T_{(12)_r} T^{-1}_{(22)_r} T_{(21)_r}] \begin{Bmatrix} A_p \\ A_{SV} \end{Bmatrix}_r$$

$$= R \begin{Bmatrix} A_p \\ A_{SV} \end{Bmatrix}_r$$

substituting Eq. (3.13)

$$U_s = R\, G_{11}\, U_o$$

or

$$U_o = G^{-1}_{11}\, R^{-1}\, U_s \; . \tag{3.14}$$

This expression provides the amplification ratios for the elastic rock case.

3.4d Description of Computer Program

A computer program was written in Fortran IV utilizing the formulas developed in the preceding sections. By specifying the amplitudes of P and S-waves (A_p, A_{SV}) and the angle incidence of the latter (α_s), the program computes amplitudes of stresses and displacements (vertical and horizontal) at the interfaces of the layers. The program also plots (with a Stromberg Carlson plotter, SC 4020) the four amplification ratios as function of frequency.

Any type of soil and rock profile can be studied by specifying the parameters that describe the stratum (height, shear wave velocities, Poisson's ratio, fraction of critical damping and unit weight).

The program proceeds from top to bottom and calculates the T, H, T^{-1} and F matrices for each layer and finally the G-matrix. Knowing these the motion, stress and amplification ratios are calculated from the relation developed in the previous sections. This procedure is repeated for a selected number of frequencies, as required to obtain a good representation of the above quantities.

3.5 Consideration of Damping

In the foregoing formulation a linear elastic material was assumed. In reality, however, all materials and particularly soils are nonlinear and experience an internal dissipation of energy when undergoing cyclic loadings. This effect is taken into account by assuming the material to be linear viscoelastic with a viscosity function of frequency. A constant viscosity coefficient corresponds to the usual concept of viscous damping and produces a loss of energy per cycle which increases with frequency.

Experimental results indicate however that the energy dissipation in soils is almost independent of frequency (while a function of strain). For an assumed or expected amplitude of motion, this situation is better reproduced by considering a viscosity coefficient inversely proportional to frequency.

While the equations of motion can be developed by taking into account the viscosity coefficient, the same effect is obtained by working with complex soil parameters of the form

$$\lambda^* = \lambda \; (1 + 2 \, i \, \beta)$$

$$G^* = G \; (1 + 2 \, i \, \beta)$$

where β is the amount of critical damping considered.

Thus all the equations developed in the preceding sections are valid provided all related parameters (i.e., V_p, V_s, ℓ, n, ℓ', n' etc.) are also considered as complex quantities. The interpretation of ℓ, n, ℓ', n' as director cosines becomes difficult but they may be thought of as mathematical parameters related by the condition $\ell^2, n^2 = 1$, $\ell'^2 + n'^2 = 1$.

CHAPTER 4

AMPLIFICATION OF SV WAVES

4.1 Cases Considered

Although the program described in section (3.4d) is general and applies to both P and SV waves, in this work only SV waves will be considered. Jones (3) obtained amplification functions for P waves at an arbitrary angle of incidence and SV waves with angles of incidence less than the critical. Here the emphasis has been in examining amplifications for SV waves propagating with angles of incidence greater than the critical.

For the purpose of comparison the properties of the stratum profile were maintained the same as those used in References (4, 9, 10). These are:

	V_s (ft/sec)	ν	β	γ (lb/ft^3)
soil	750	0.25055	0.05	125
rock	4500	0.2857	0	140

It should be noticed that only a single value of damping was considered. The effect of damping for one-dimensional amplification was studied by V.C. Liang (5).

Two stratum profiles were examined in this work:

a) <u>Elastic Half Space</u> - A stratum composed of layers all of them with the same properties as the underlying rock.

b) Uniform soil layer upon rock - A 100 ft. uniform soil layer resting on elastic rock.

Of particular interest in this study was the consideration of SV waves incoming with angles larger than the critical. The critical angle occurs when the parameter $n = \sqrt{1 - \ell^2}$ switches from being real to imaginary, that is when $\ell = 1$. Since

$$\ell = \frac{V_p}{V_s} \ell' = \frac{V_p}{V_s} \sin \alpha_{crit} = 1$$

$$\sin \alpha_{crit} = \frac{V_s}{V_p}$$

the corresponding dilatational wave velocities for the soil and rock are 1300 ft/sec and 8000 ft/sec respectively. Substituting the shear and dilatational wave velocities for the rock

$$\sin \alpha_{crit} = \frac{4500}{8000}$$

or

$$\alpha_{crit} = 34.23° \quad .$$

4.2 Presentation and Discussion of Results

4.2a Motion - Elastic Half-Space

Figures 4.1 and 4.2 show the amplitudes of the horizontal and vertical motion, respectively, for the Elastic Half-Space. Results are presented for angles of incidence of 0, 30, 40, 45, 40, 60° and frequencies of 1.75, 3.25, 5.75 and 9.25 cps down to a depth of 100 ft.

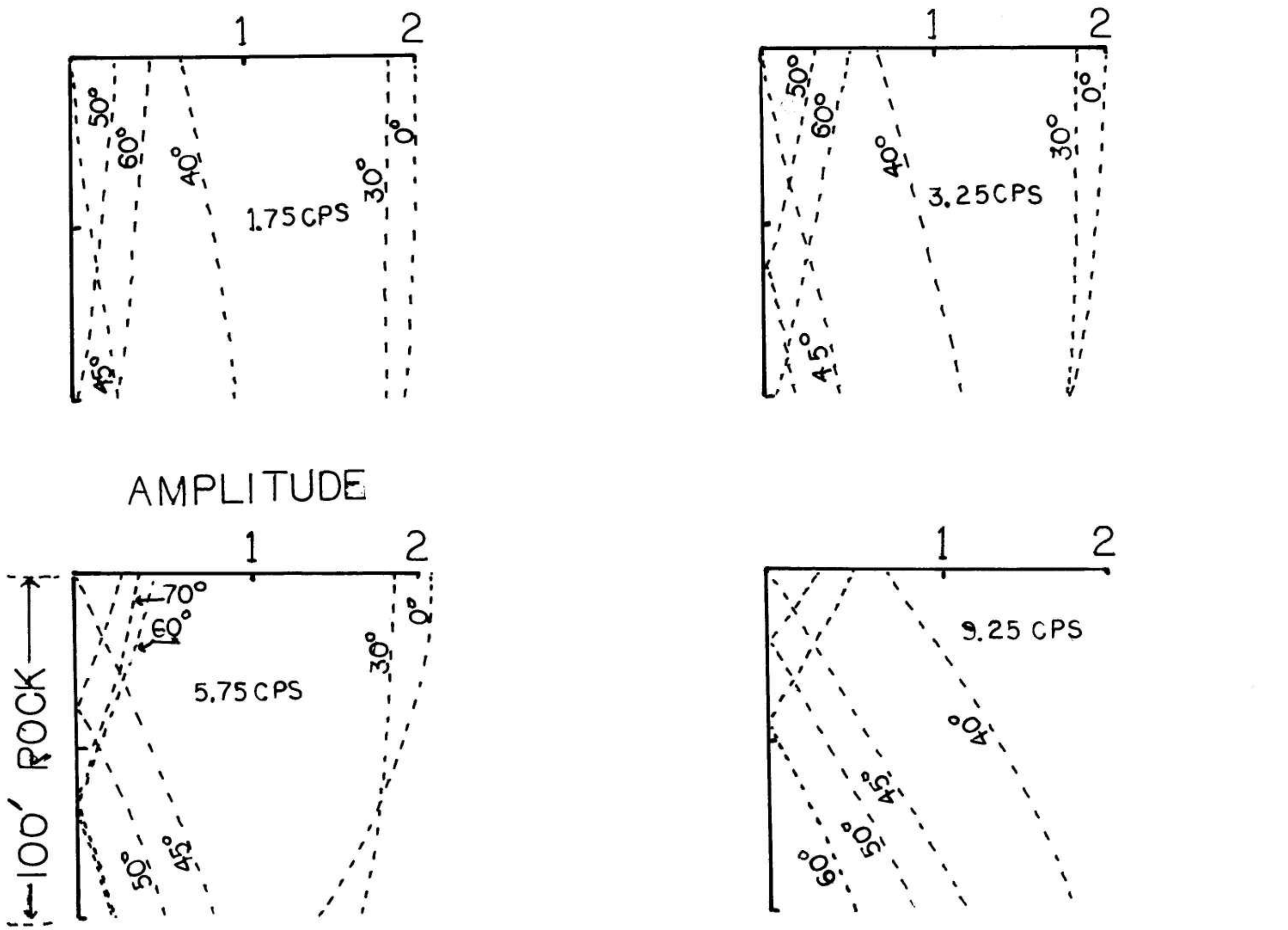


Fig. 4.1 Horizontal motion vs. Depth-Elastic Half Space

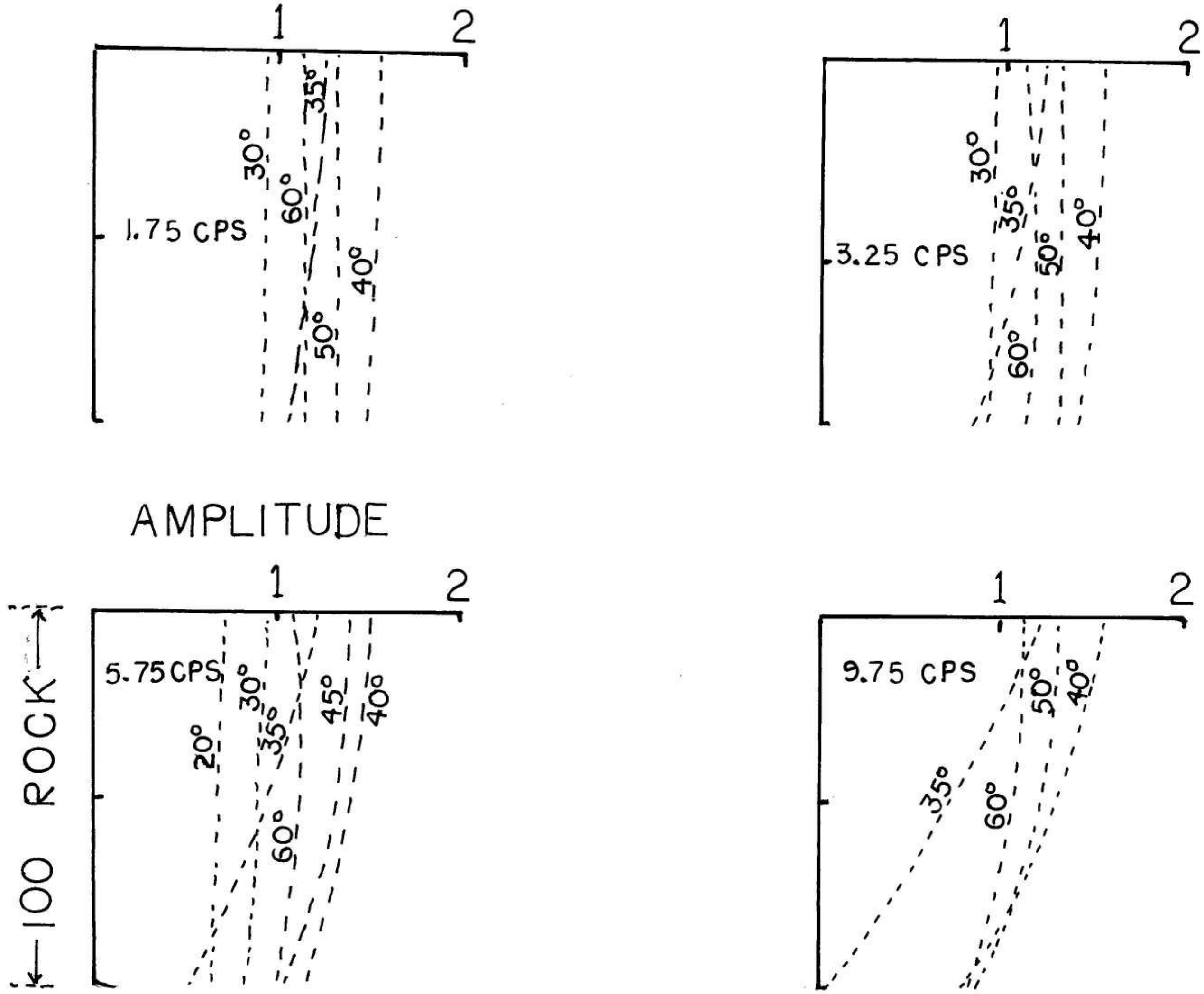


Fig. 4.2 Vertical motion vs. Depth-Elastic Half Space

The motion varies nearly periodically with depth with a wavelength (λ) that is given by

$$\lambda = \frac{V_s}{fn'}, \quad \text{for shear waves}$$

$$\lambda = \frac{V_p}{fn} \quad \text{for dilatational waves}$$

For normal incidence ($\alpha_s = 0°$) since $n'=n=1$ (see table 4.1) the wavelengths are

Frequency	Shear Wavelength	Dilat. Wavelength
1.75 cps	2571 ft	4571 ft
3.25 cps	1384 ft	2561 ft
5.75 cps	782 ft	1391 ft
9.25 cps	486 ft	865 ft

One of the effects of the angle of incidence is to change the wavelength through the parameters n and n'. The values of ℓ, ℓ', n and n' are shown in table 4.1.

The second effect of the angle of incidence is to modify the shape of the curve describing the motion by introducing a phase shift. For angles of incidence less than the critical the maximum amplitude occurs at the surface, whereas for $\alpha_s > \alpha_{crit}$ the depth at which the maximum amplitude occurs varies with the angle of incidence.

This phenomenon is better illustrated by looking at the explicit equations of the motion for the elastic half-space. The horizontal motion as a function of depth is given by:

$$u = \ell \; [(A_p + A_p') \cos \frac{\omega}{V_p} nz]$$

$$+ i \; (A_p - A_p') \sin \frac{\omega}{V_p} nz]$$

$$+ n' \; [(A_{SV}' - A_{SV}) \cos \frac{\omega}{V_s} n'z] \tag{4.1}$$

$$- i \; (A_{SV} + A_{SV}') \sin \frac{\omega}{V_s} n'z]$$

For the case of normal incidence substituting the appropriate parameters from tables 4.1 and 4.2 (A_p=0, A_p'=0, A_{SV}=1, A_{SV}'=-1,n'=1) the amplitude of the horizontal motion becomes

$$|u| = 2 \cos \frac{\omega}{V_s} n'z$$

On the other hand for α_s = 45° substituting ($A_p = A_p'$ = 0, $A_{SV} = A_{SV}'$ = 1, n' = 0.70711)

$$|u| = 2 n' \sin \frac{\omega}{V_s} n'z = 1.414 \sin \frac{\omega}{V_s} n'z$$

These results are shown in fig. 4.1, as obtained from the computer program using a number of rock layers.

As the angle of incidence increases from 0° a phase angle is introduced into the periodic shape of the motion which becomes 90° for the case of α_s=45° (sin function). Thus the point at which the maximum amplitude occurs varies with α_s. Also for $\alpha_s > \alpha_{crit}$ a term with exponentially

TABLE 4.1. Parameters for the Elastic Half-Space

α_s	ℓ		n		ℓ'		n'	
	real	imag.	real	imag.	real	imag.	real	imag.
0°	0	0	1	0	0	0	1	0
30°	0.8889	0	0.45813	0	0.5	0	0.8660	0
40°	1.1427	0	0	-0.5530	0.6428	0	0.7660	0
45°	1.2570	0	0	-0.7617	0.7071	0	0.70711	0
50°	1.370	0	0	-0.9244	0.7660	0	0.6428	0
60°	1.5396	0	0	-1.1706	0.8661	0	0.500	0
70°	1.6706	0	0	-1.3382	0.4397	0	0.34202	0
80°	1.75076	0	0	-1.437	0.9848	0	0.17365	0

TABLE 42. Amplitudes of the Waves

α_s	A_P		A_{SV}		A_P'		A_{SV}'	
	real	imag.(i)	real	imag.	real	imag.	real	imag.
0	0	0	1	0	0	0.0	-1.0	0
10	0	0	1	0	-.38	0	-.87	0
20	0	0	1	0	-.71	0	-.50	0
30	0	0	1	0	-1.0	0	-.057	0
35	0	0	1	0	-1.5	-1.5	.033	-1.0
40	0	0	1	0	-.037	-.49	+.99	-0.15
45	0	0	1	0	0	0	1.0	0
50	0	0	1	0	.0094	+.24	+1.0	-.077
60	0	0	1	0	.12	.46	.88	-.48
70	0	0	1	0	.28	.43	.41	-.91
80	0	0	1	0	.30	.18	-.45	-.89

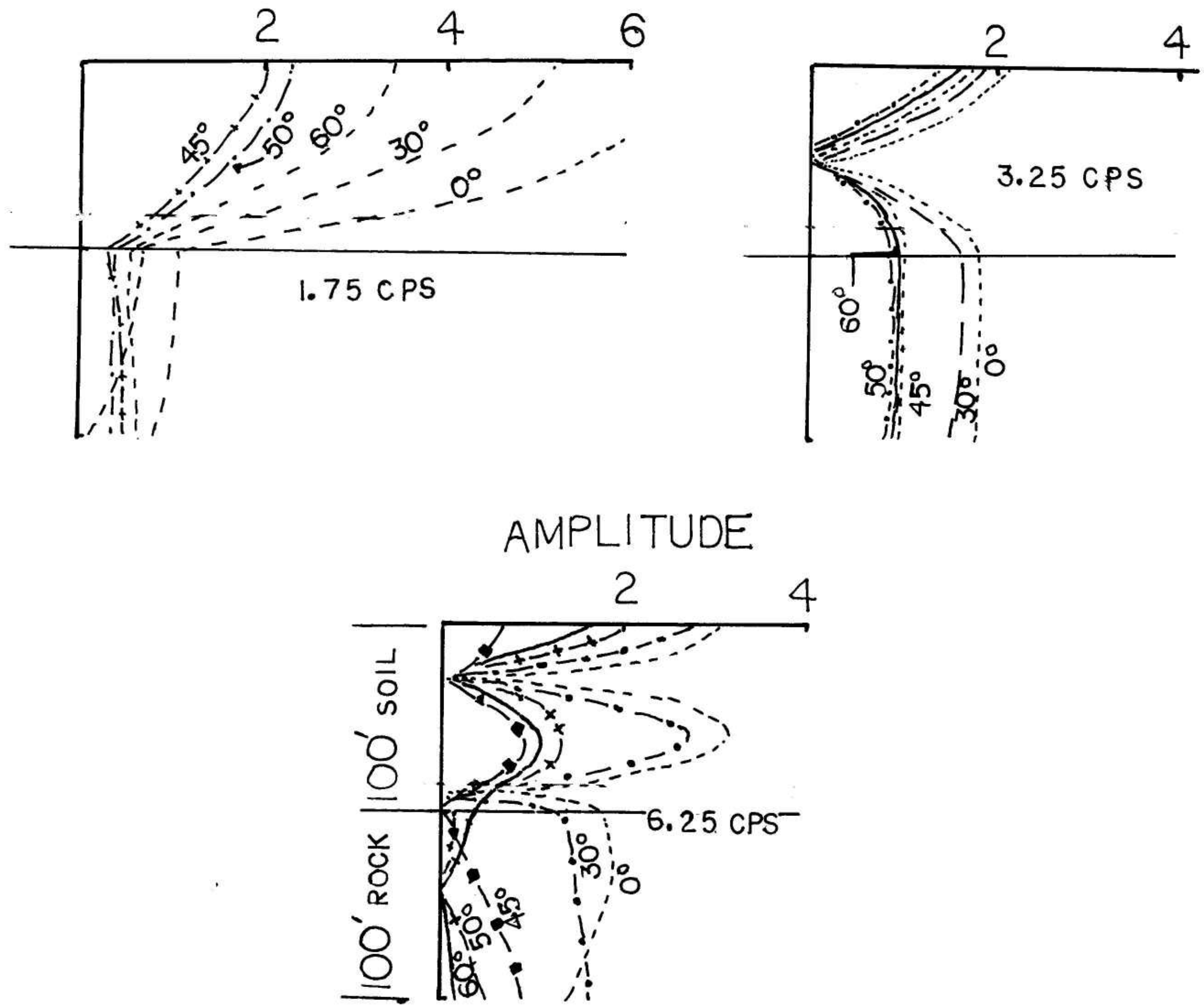


Fig. 4.3 Horizontal Motion vs. Depth-Uniform soil layer upon rock

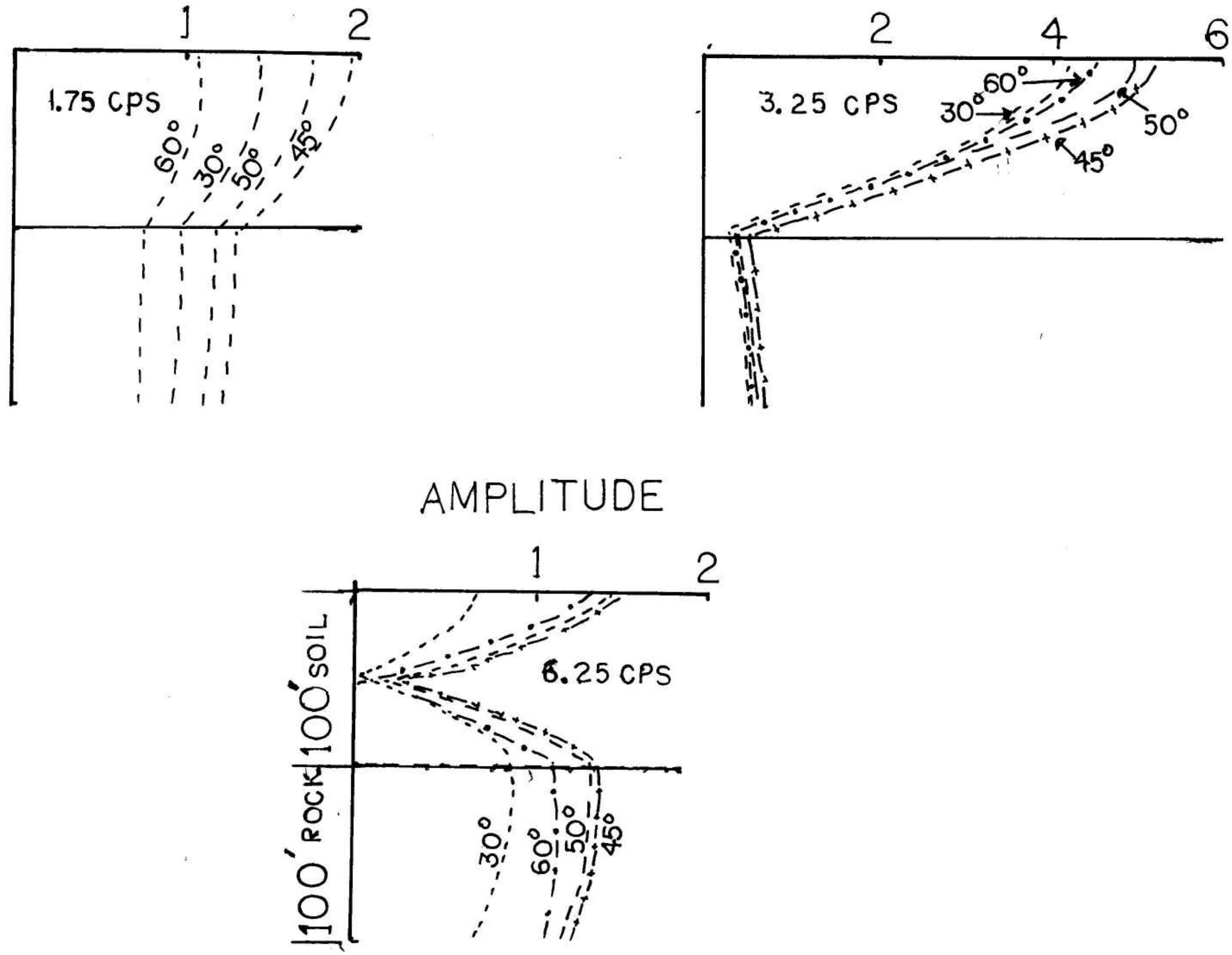


Fig. 4.4 Vertical Motion vs. Depth-Uniform soil layer upon rock

decaying amplitude is added to the motion due to the imaginary components of A_p', A_{SV}' and n (see tables 4.1, 4.2).

Similar results can be obtained for the vertical motion.

4.2b Motion - Uniform Soil Layer upon Rock

Figures 4.3 and 4.4 show the horizontal and vertical motion for a 100 ft. uniform soil layer upon a rock base. Results are shown for a depth of 200 ft. (100 ft. into rock base).

The motion in the rock exhibits the same characteristics observed in the elastic half-space. In the soil layer the motion is amplified and the wavelength of the propagating waves changes. The ratio of the wavelength in the soil to that of the rock is

$$\frac{\lambda(\text{soil})}{\lambda(\text{rock})} = \frac{V(\text{soil})}{V(\text{rock})}$$

which corresponds to $\frac{1}{6}$ for the shear wave and $\frac{1}{1.778}$ for the dilatational.

The depth in the soil layer at which the maximum amplitude occurs changes only slightly with the angle of incidence. The variation is small because the wavelengths in the soil does not vary greatly with α_s.

4.2c Particle Motion as a Function of Time

The formulation in section 3.4 provides only the amplitudes of the motion. The term f(x,t) was dropped since it was constant for all points on a horizontal plane. Including now the time component of f(x,t) the particle motion is described by

$$u = (u_R + i\, u_I)\, e^{i\omega t}$$

$$w = (w_R + i\, w_I)\, e^{i\omega t}$$

where the subscripts R and I refer to the real and imaginary components, respectively, of the amplitudes at a point.

Taking the real parts of the above equation we obtain

$$u = u_R \cos\omega t - u_I \sin\omega t$$

$$w = w_R \cos\omega t - w_I \sin\omega t .$$

The combined motion is shown in figures 4.5 and 4.6 at surface of the uniform soil layer for frequencies (1.75 cps, 6.25 cps) and $\alpha_s = (0°, 30°, 45°, 50°)$.

It is seen that the particle motion is an ellipse. The shape of this ellipse is determined by the relative magnitudes of the real and imaginary components of the motion. It should be noticed that for $\alpha_s = 0°$ there is only horizontal motion since, as mentioned in section 2.2, at normal incidence p and SV waves are uncoupled.

4.2d Amplification Functions

Figures 4.7 - 4.28 show the amplification ratios as a function of frequency for the 100 ft. uniform soil profile. By definition there is no amplification (amplification ratio = 1) in the elastic half-space.

Comparing first the amplification ratios for $\alpha = 0°$ and 30° with references (3) and (10), respectively (a different formulation was used in those references) the same results are obtained.

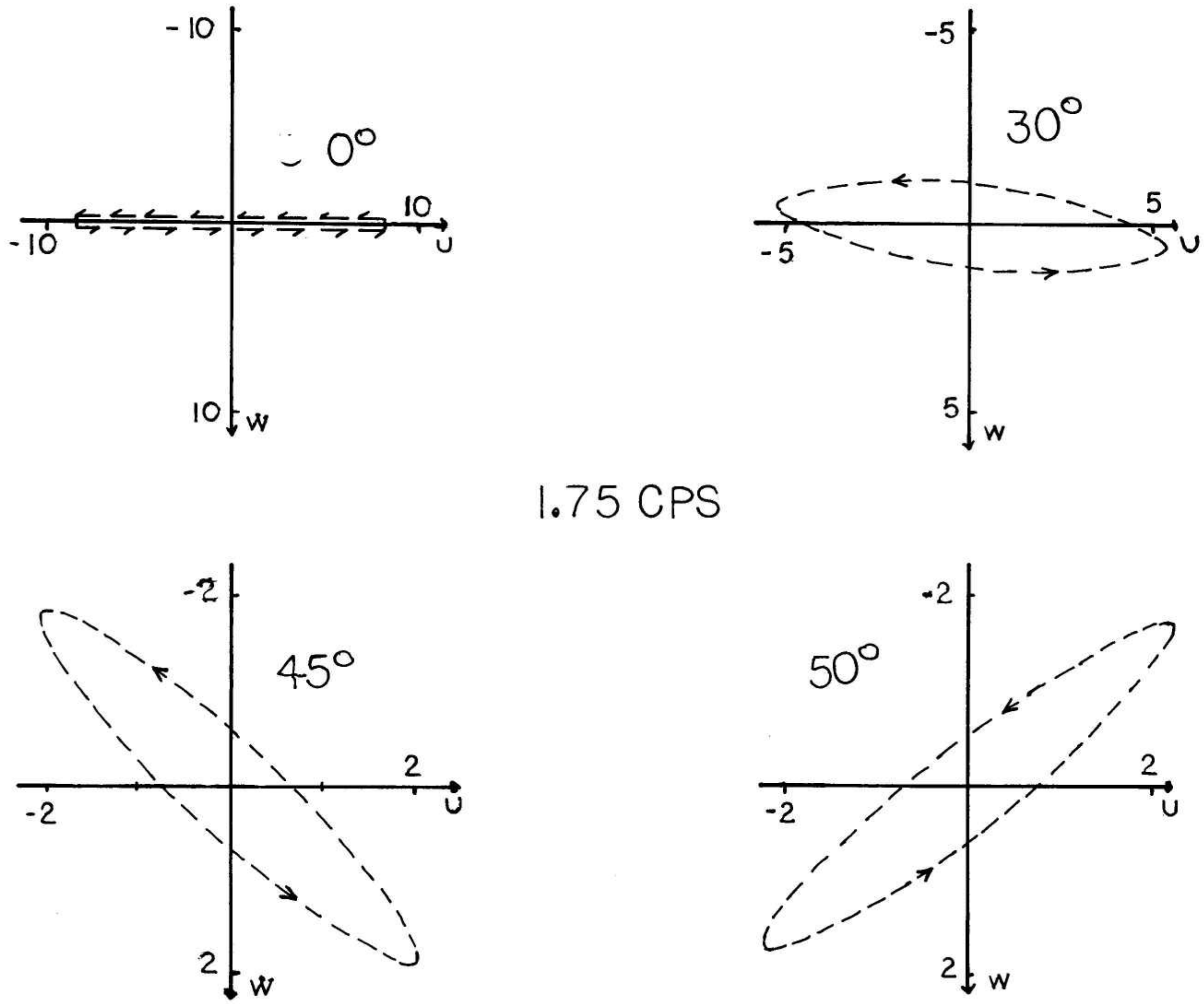


Fig. 4.5 Particle motion at surface of soil (f=1.75 cps)

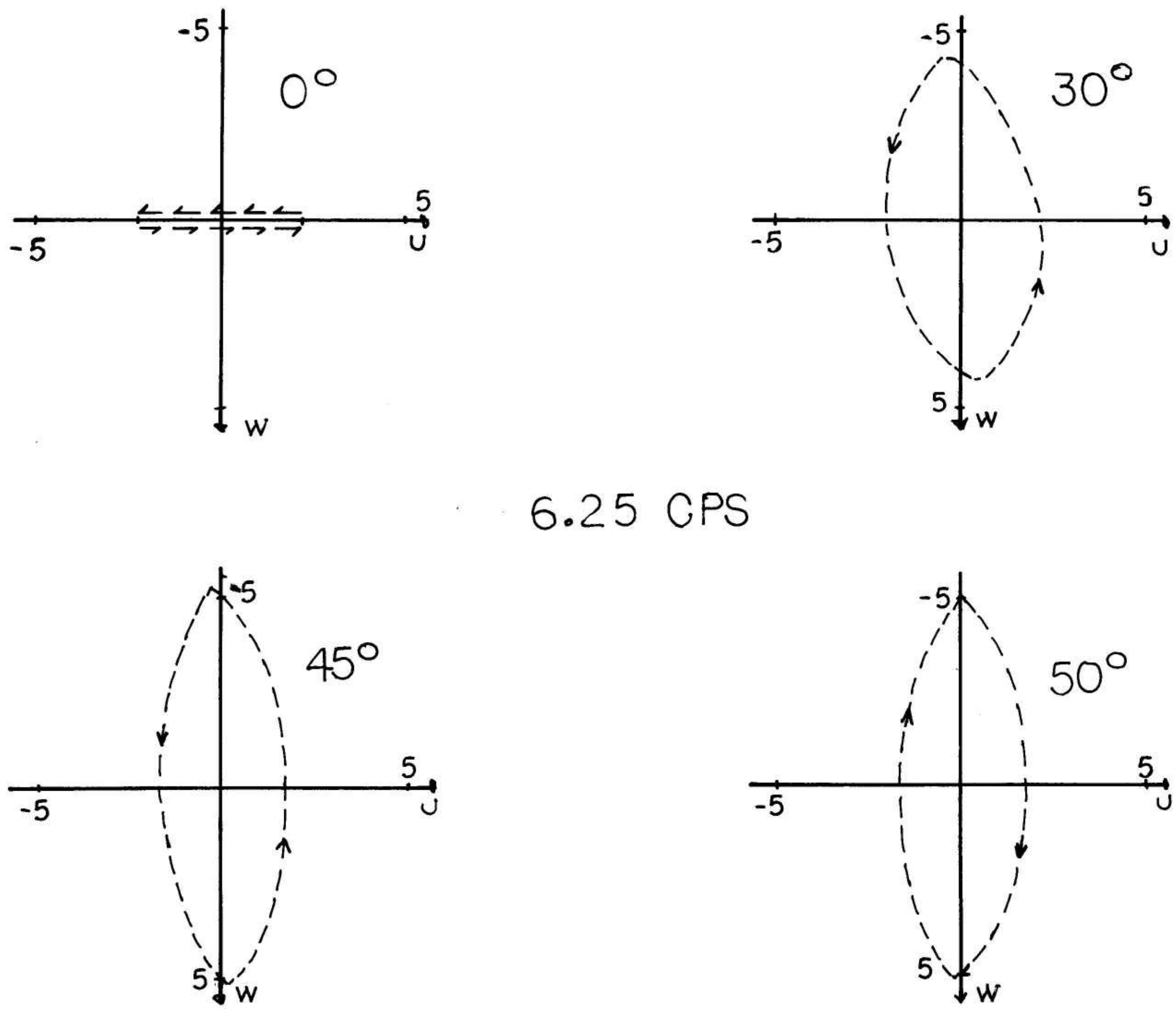


Fig. 4.6 Particle motion at surface of soil (f=6.25 cps)

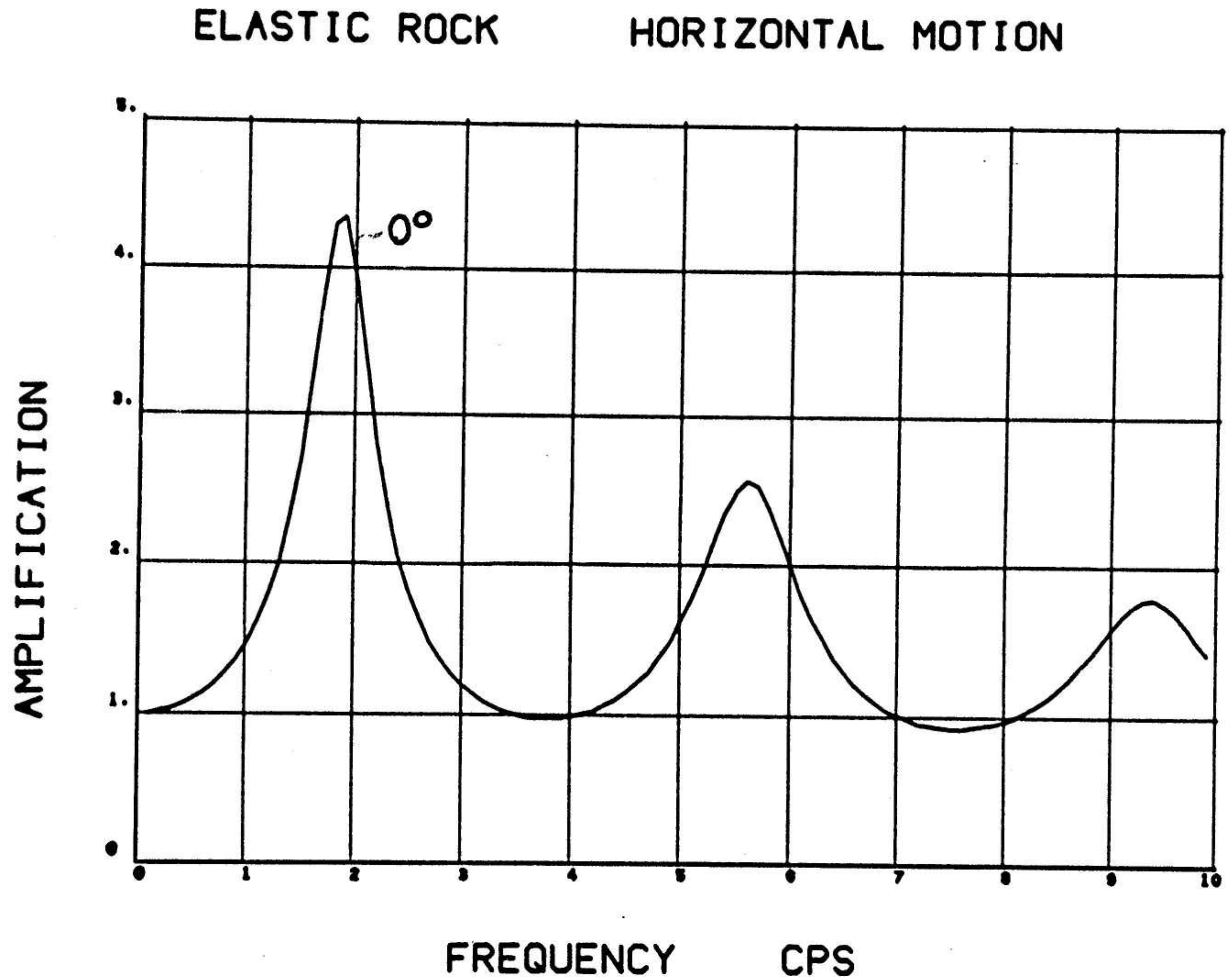


Fig. 4.7 Amplification functions - SV wave

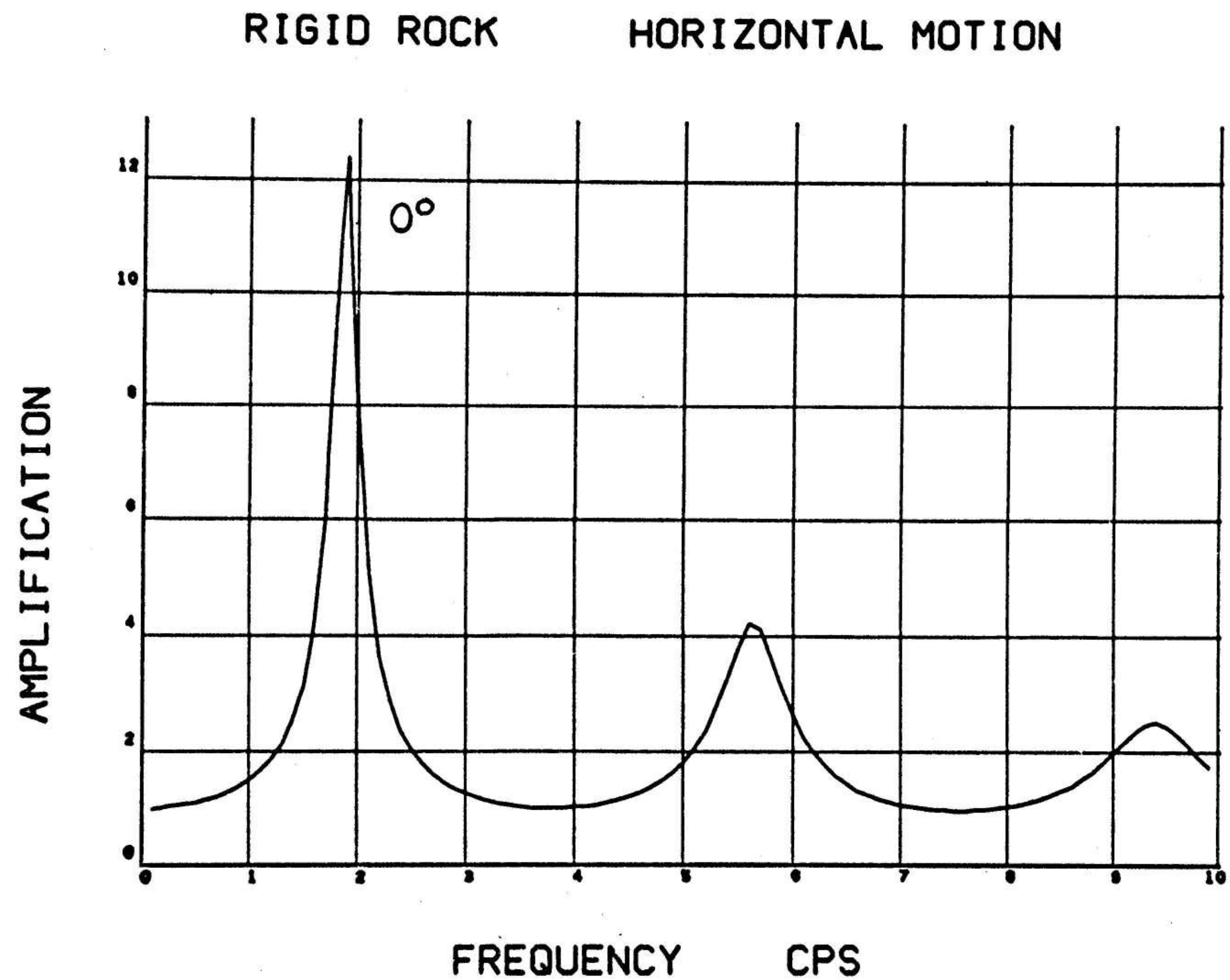


Fig. 4.8 Amplification functions - SV wave

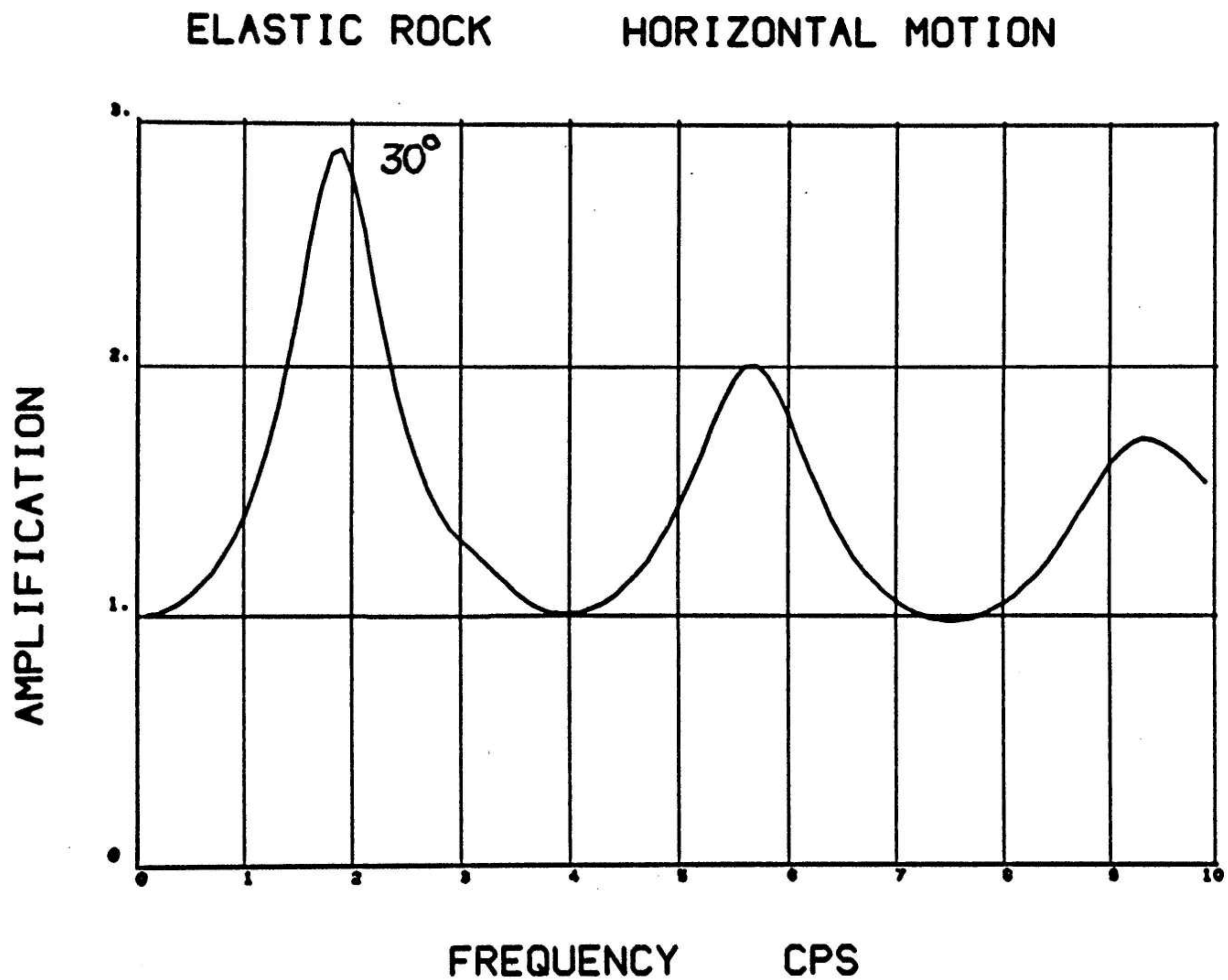


Fig. 4.9 Amplification functions - SV wave

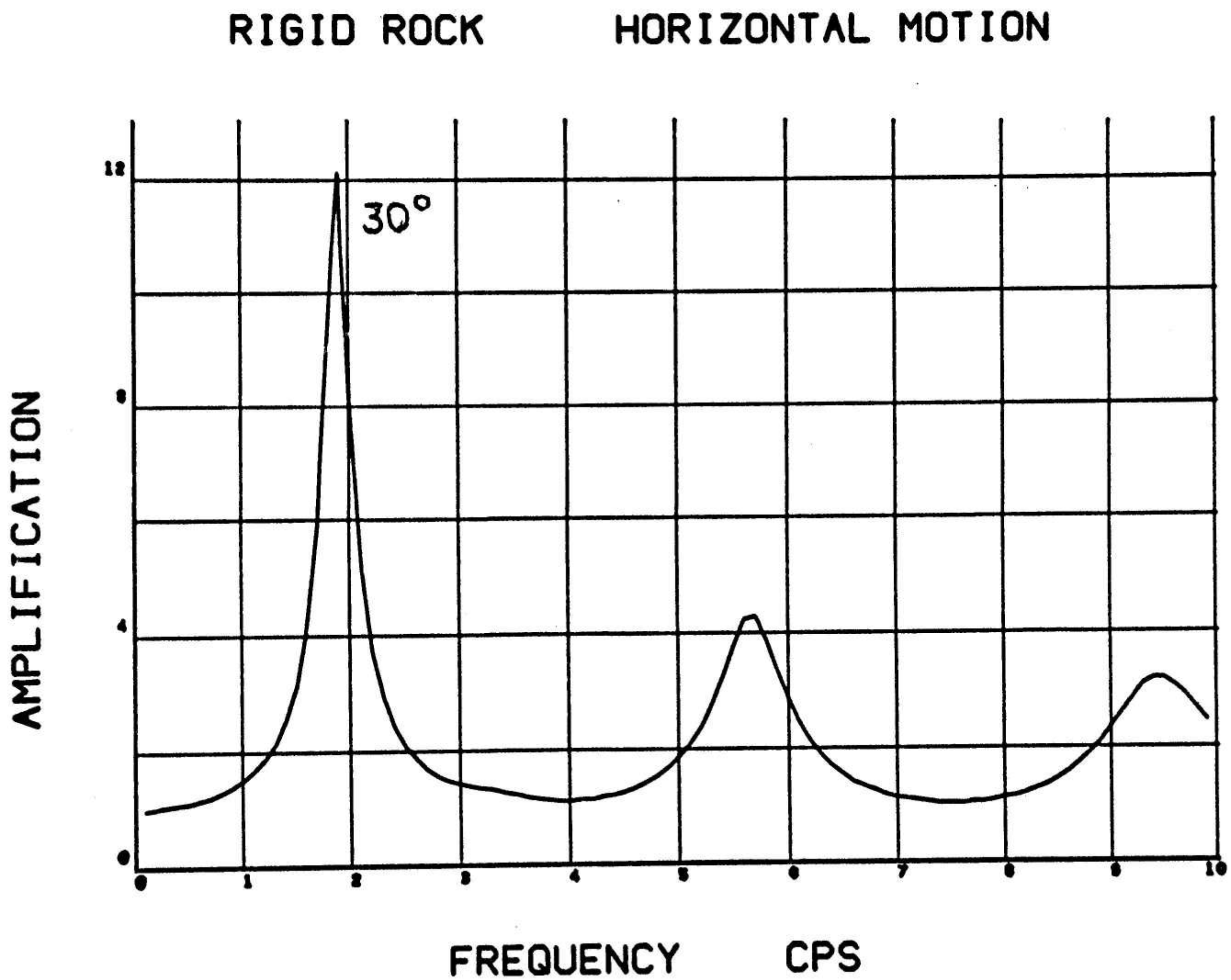


Fig. 4.10 Amplification functions - SV wave

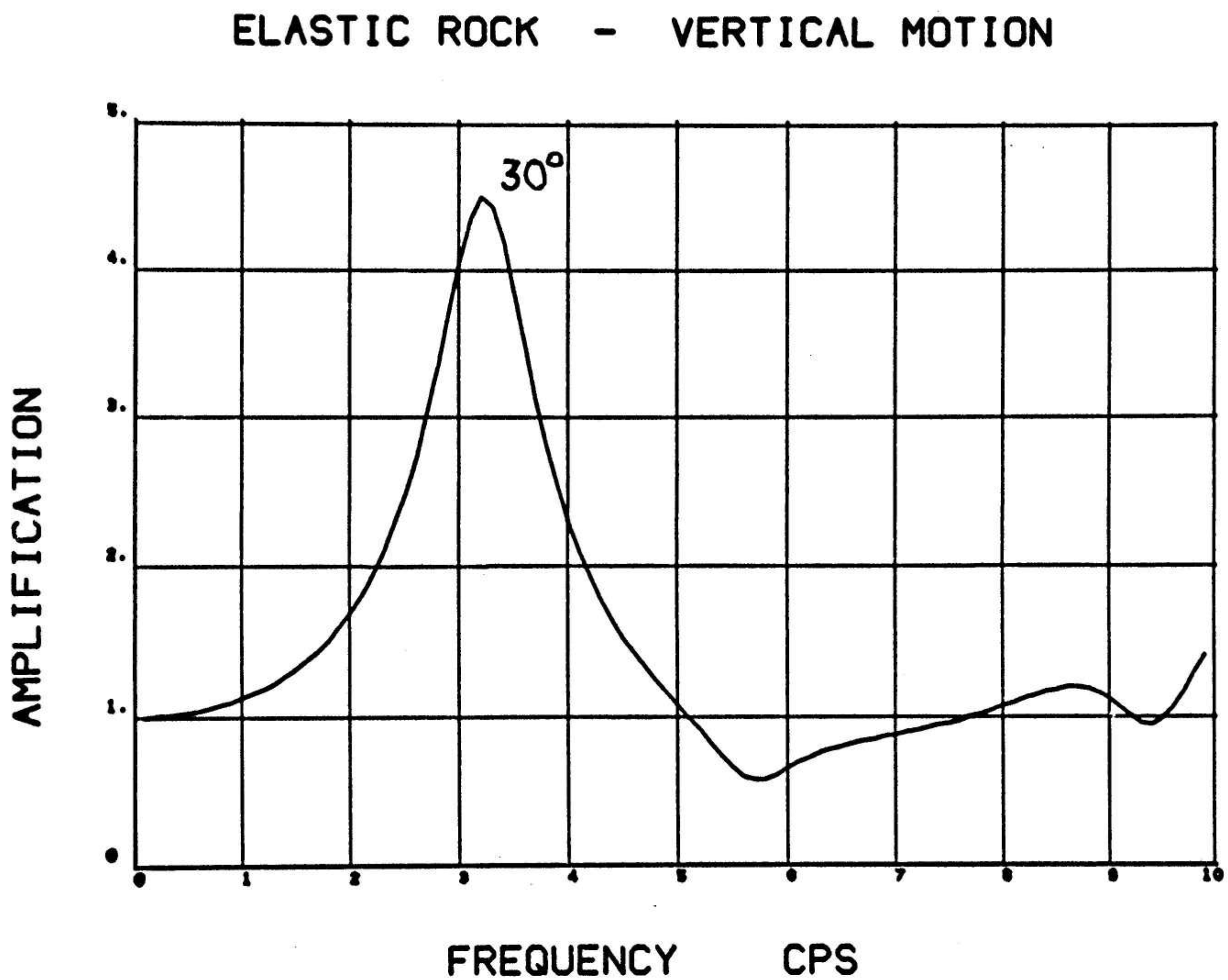


Fig. 4.11 Amplification functions - SV wave

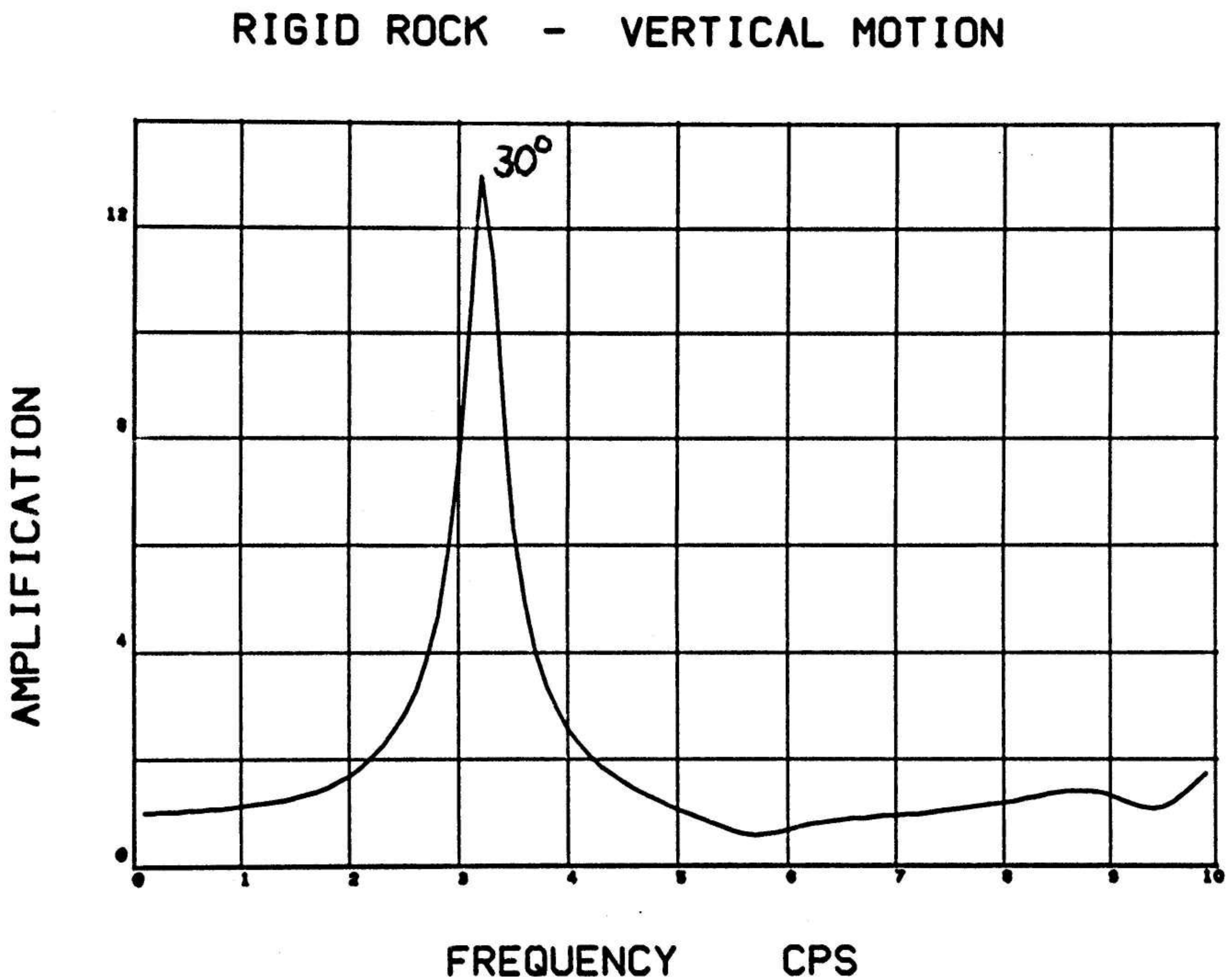


Fig. 4.12 Amplification functions - SV wave

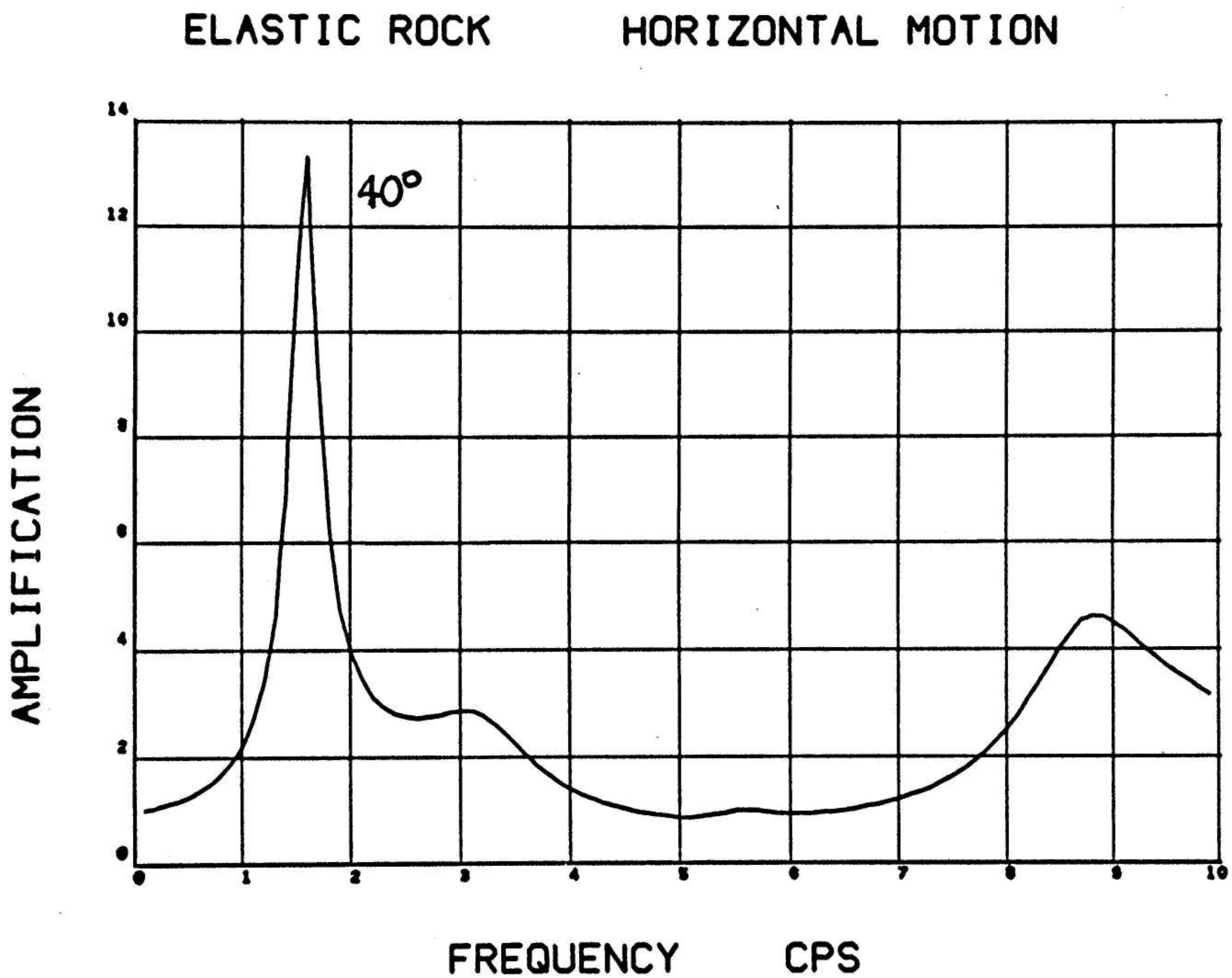


Fig. 4.13 Amplification functions - SV wave

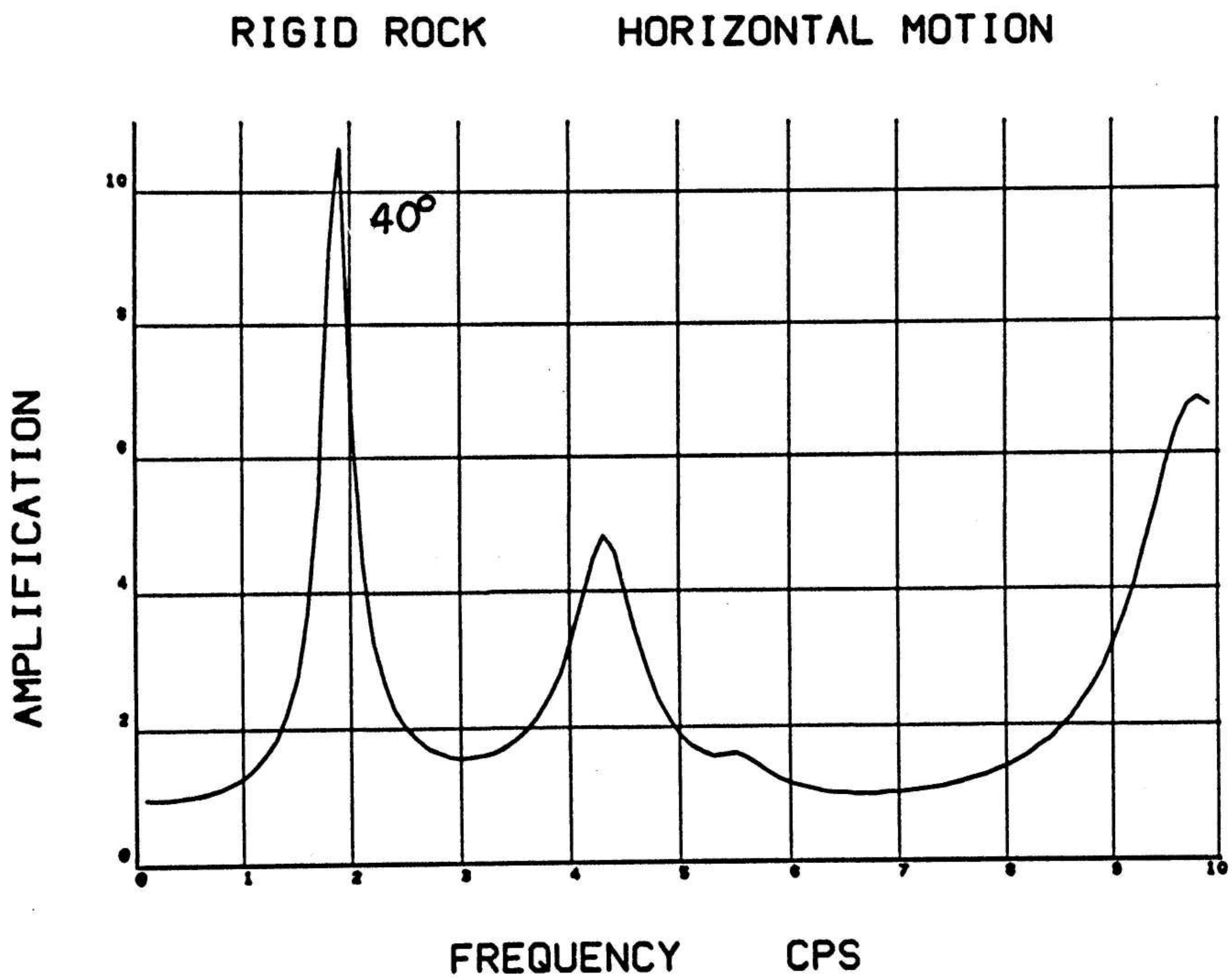


Fig. 4.14 Amplification functions - SV wave

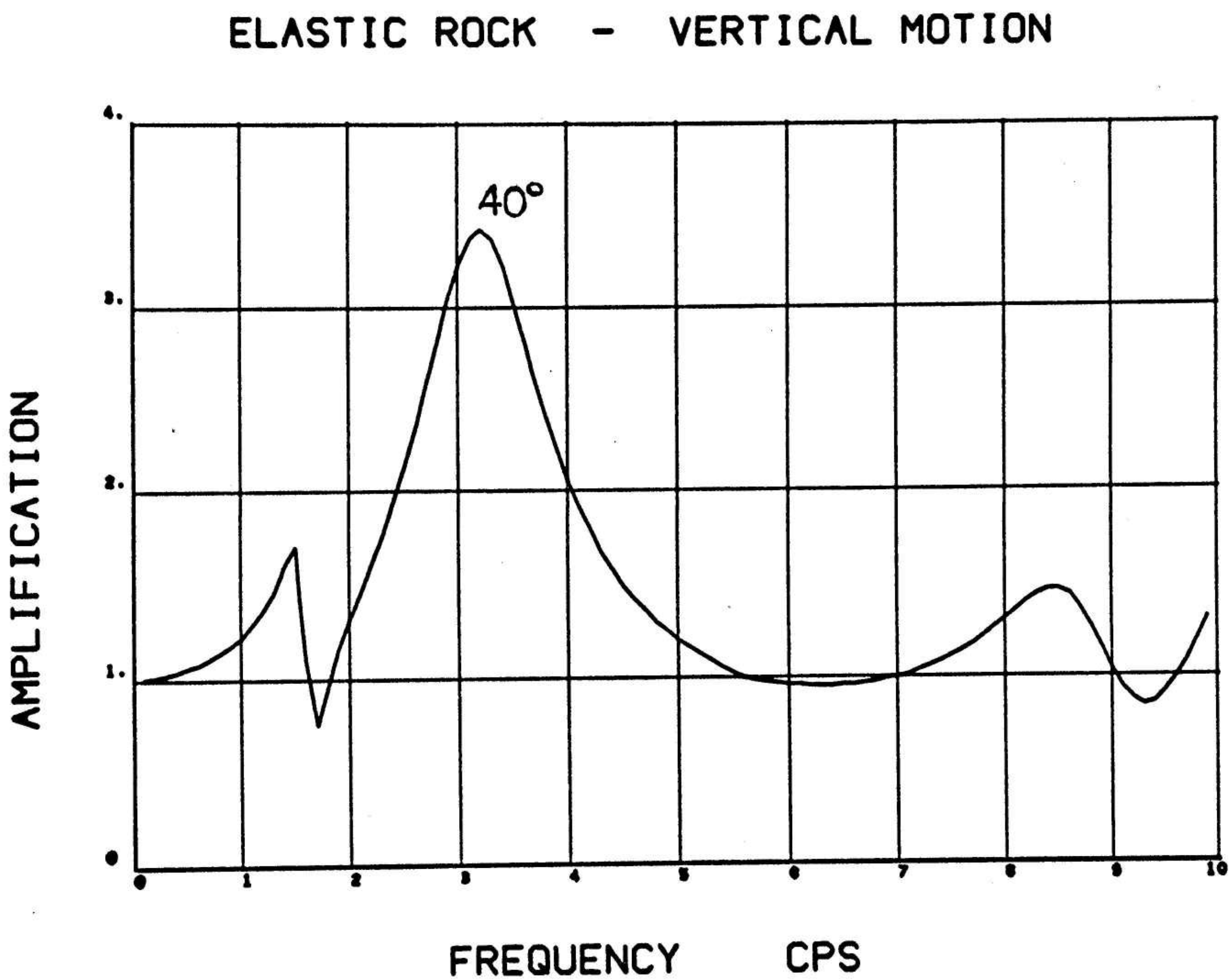


Fig. 4.15 Amplification functions - SV wave

RIGID ROCK - VERTICAL MOTION

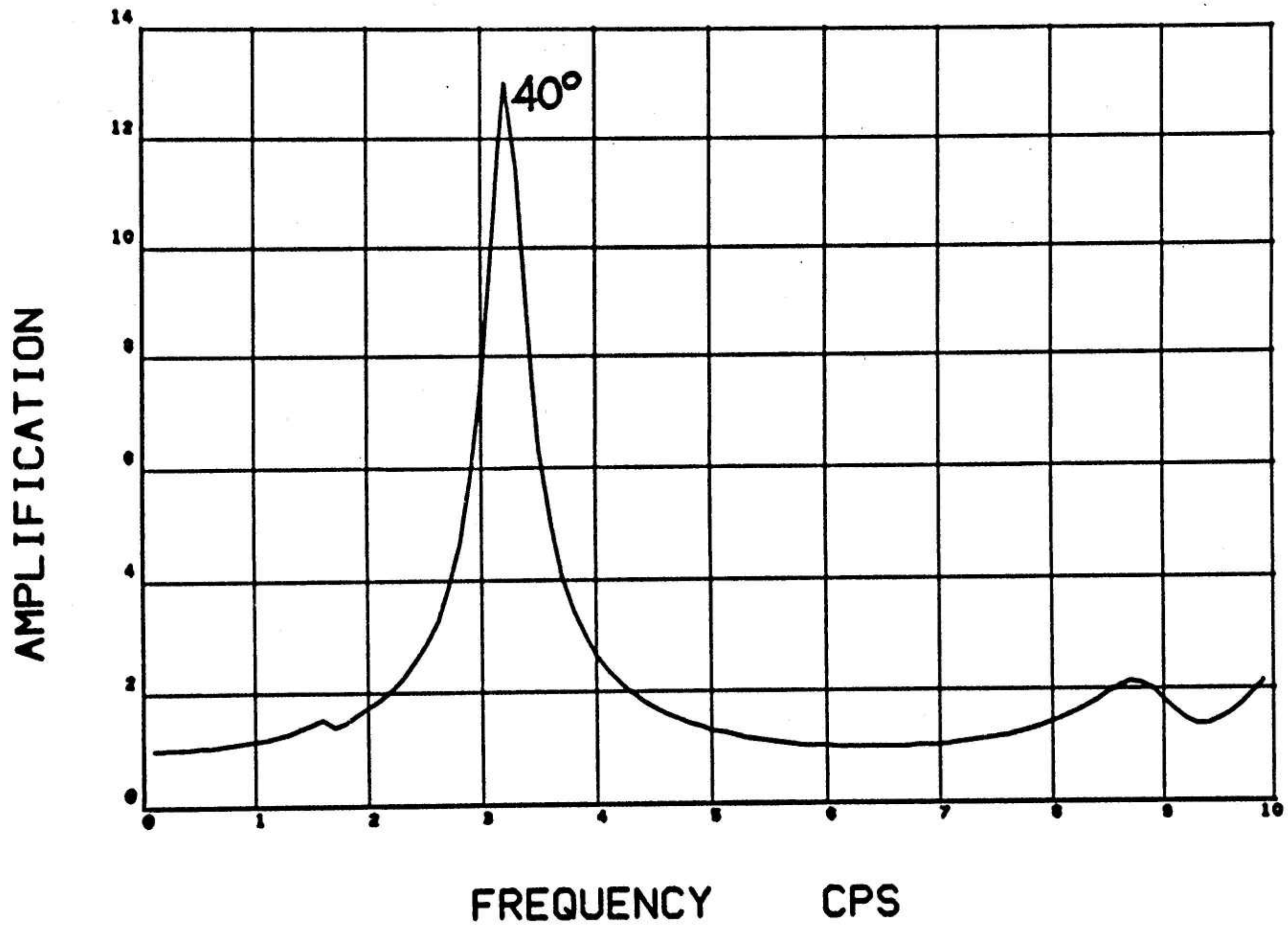


Fig. 4.16 Amplification functions - SV wave

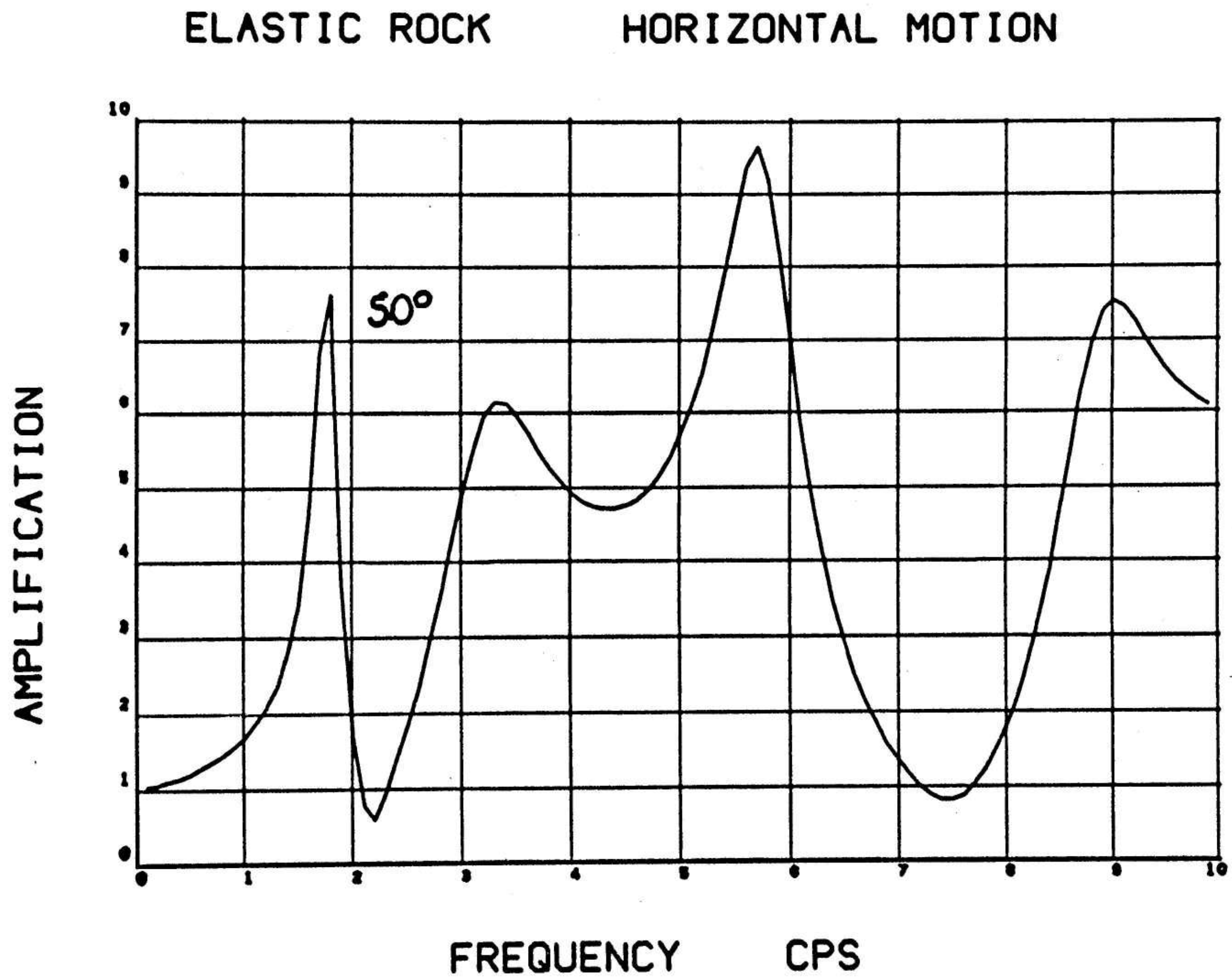


Fig. 4.17 Amplification functions - SV wave

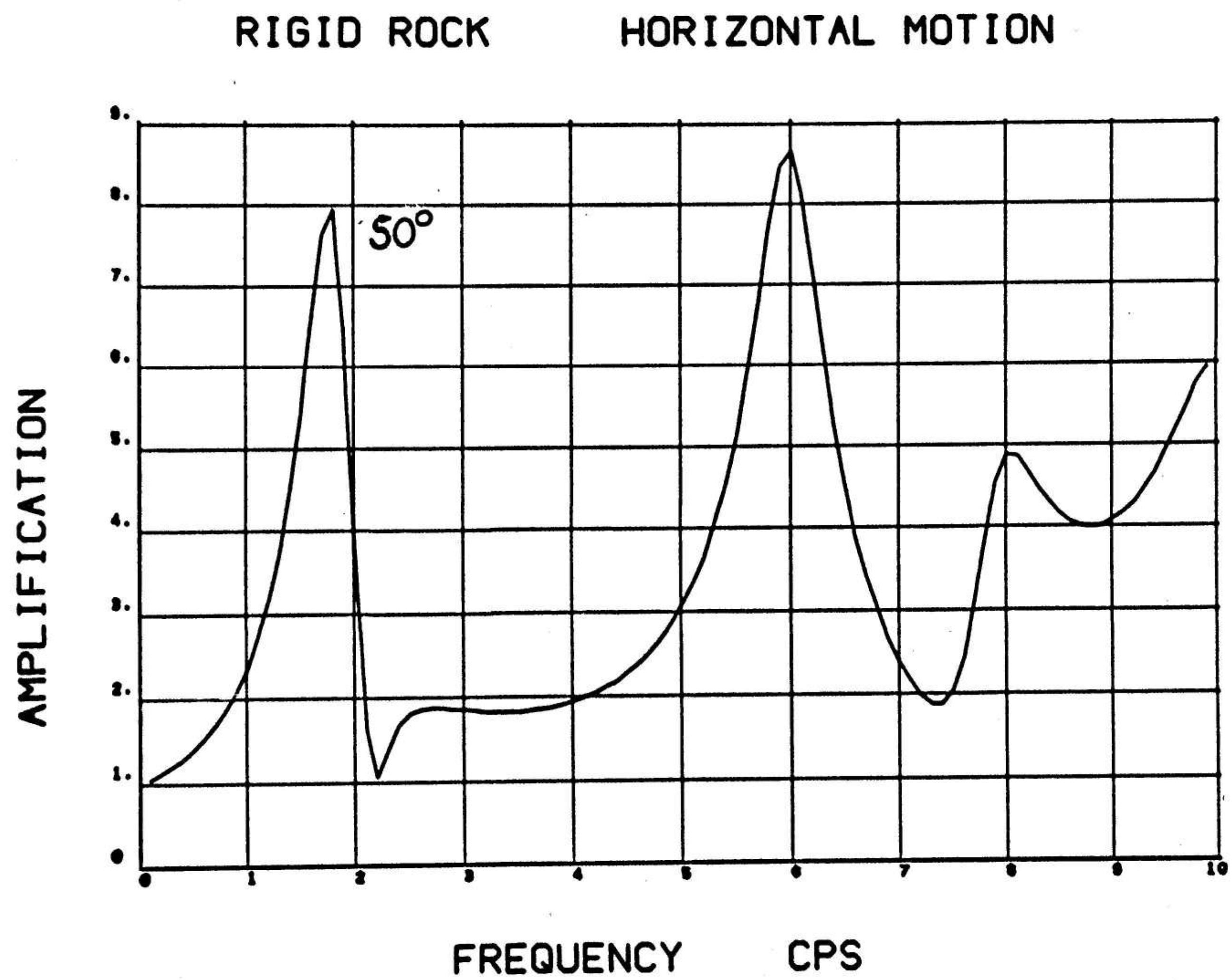


Fig. 4.18 Amplification functions - SV wave

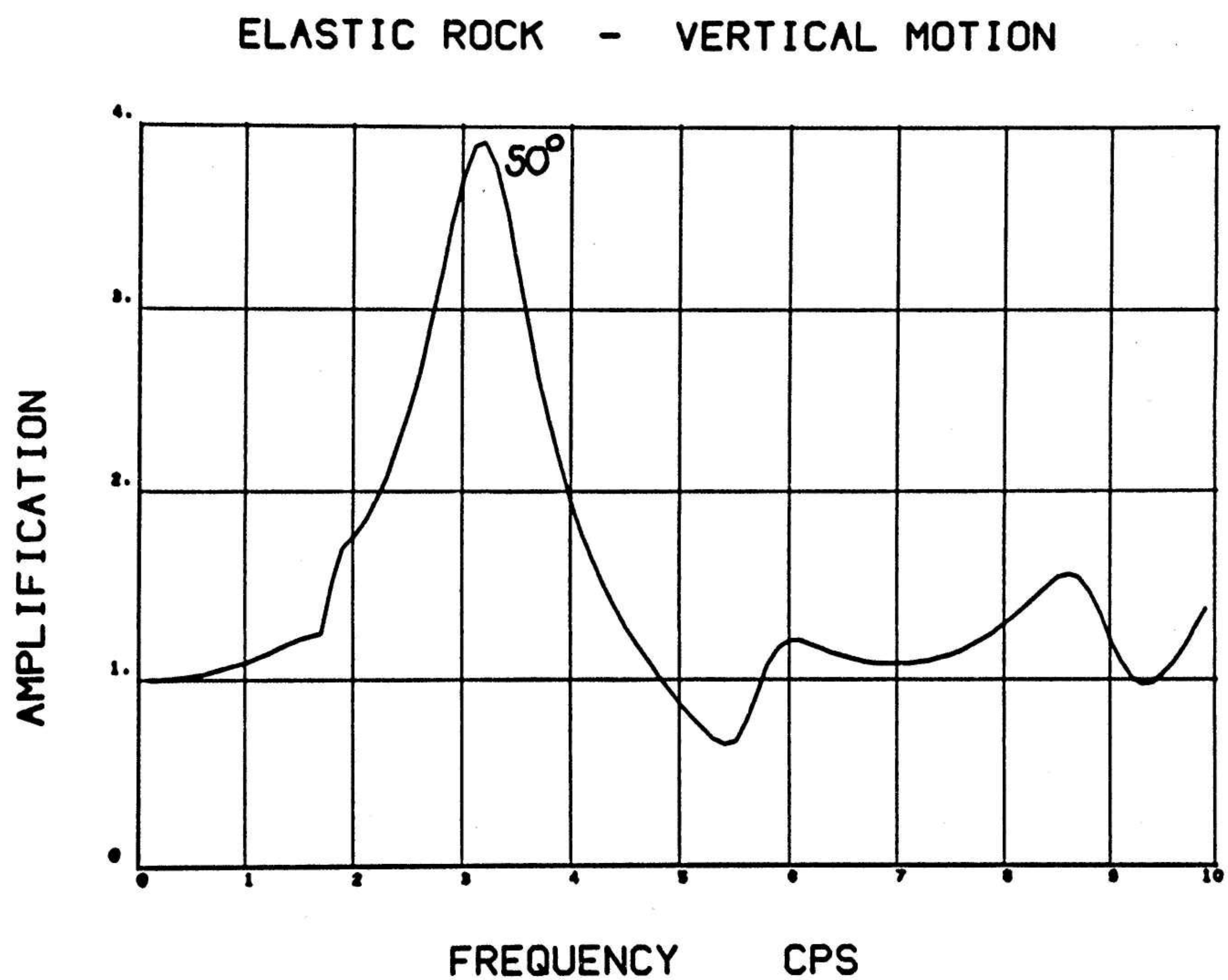


Fig. 4.19 Amplification functions - SV wave

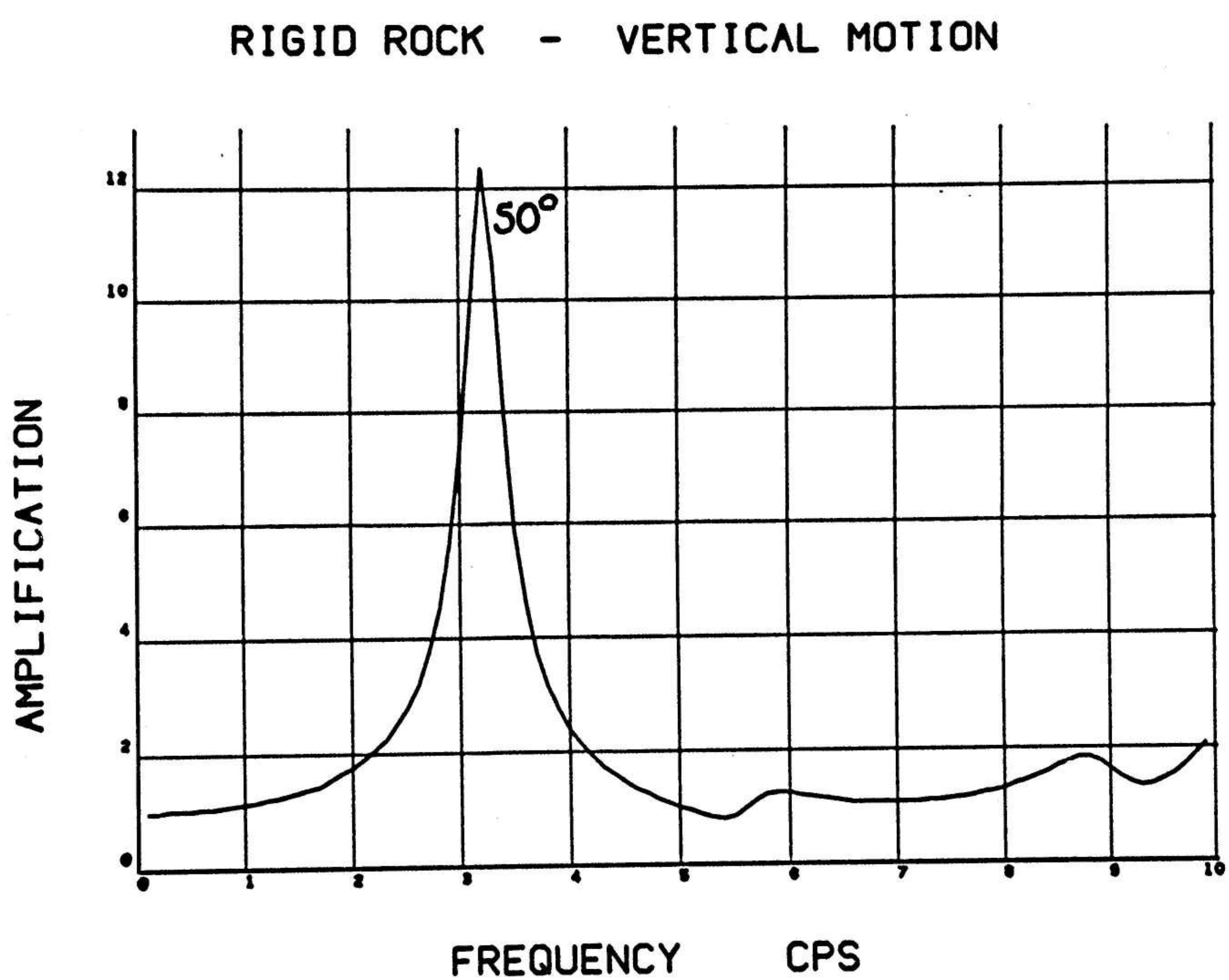


Fig. 4.20 Amplification functions - SV wave

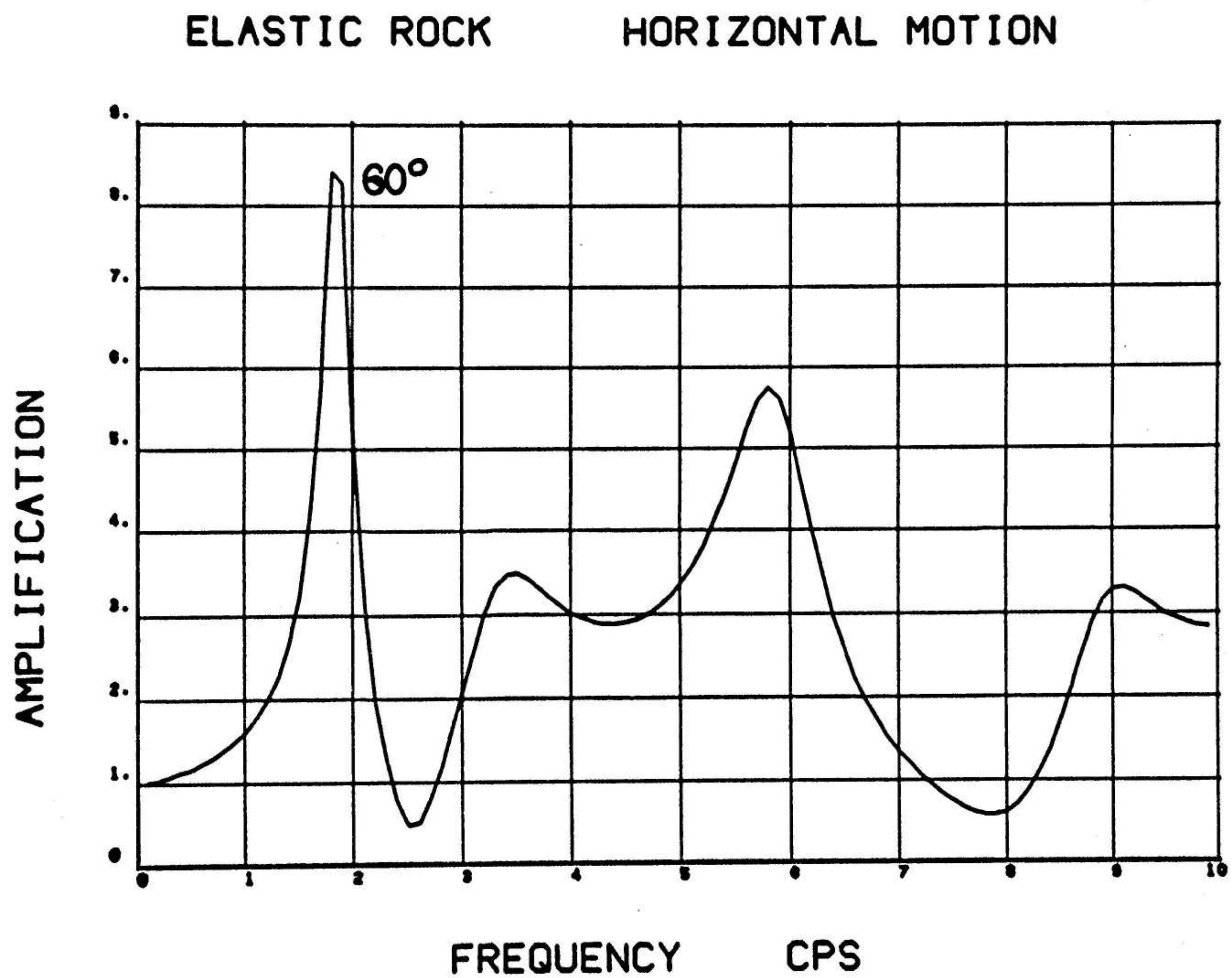


Fig. 4.21 Amplification functions - SV waves

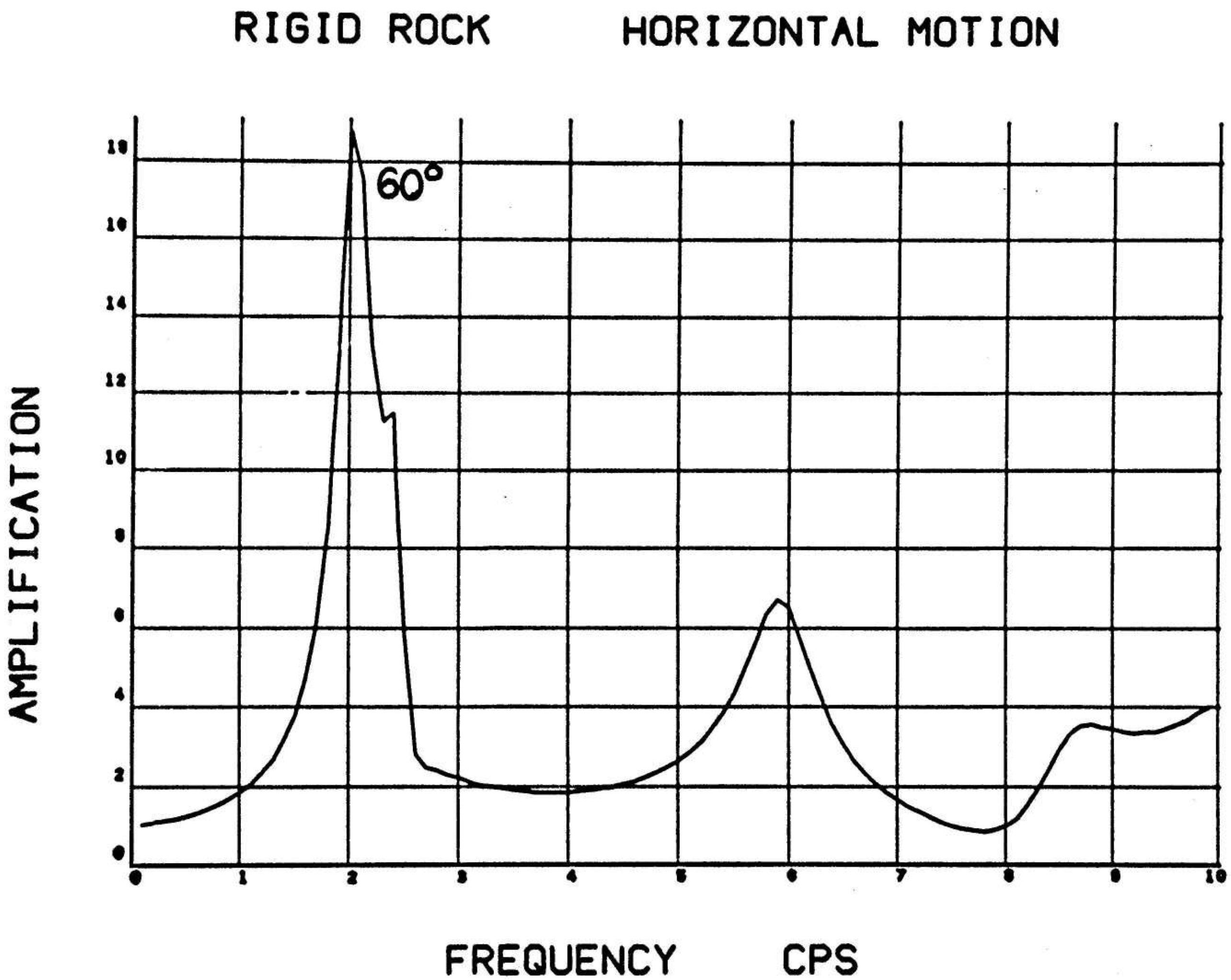


Fig. 4.22 Amplification functions - SV waves

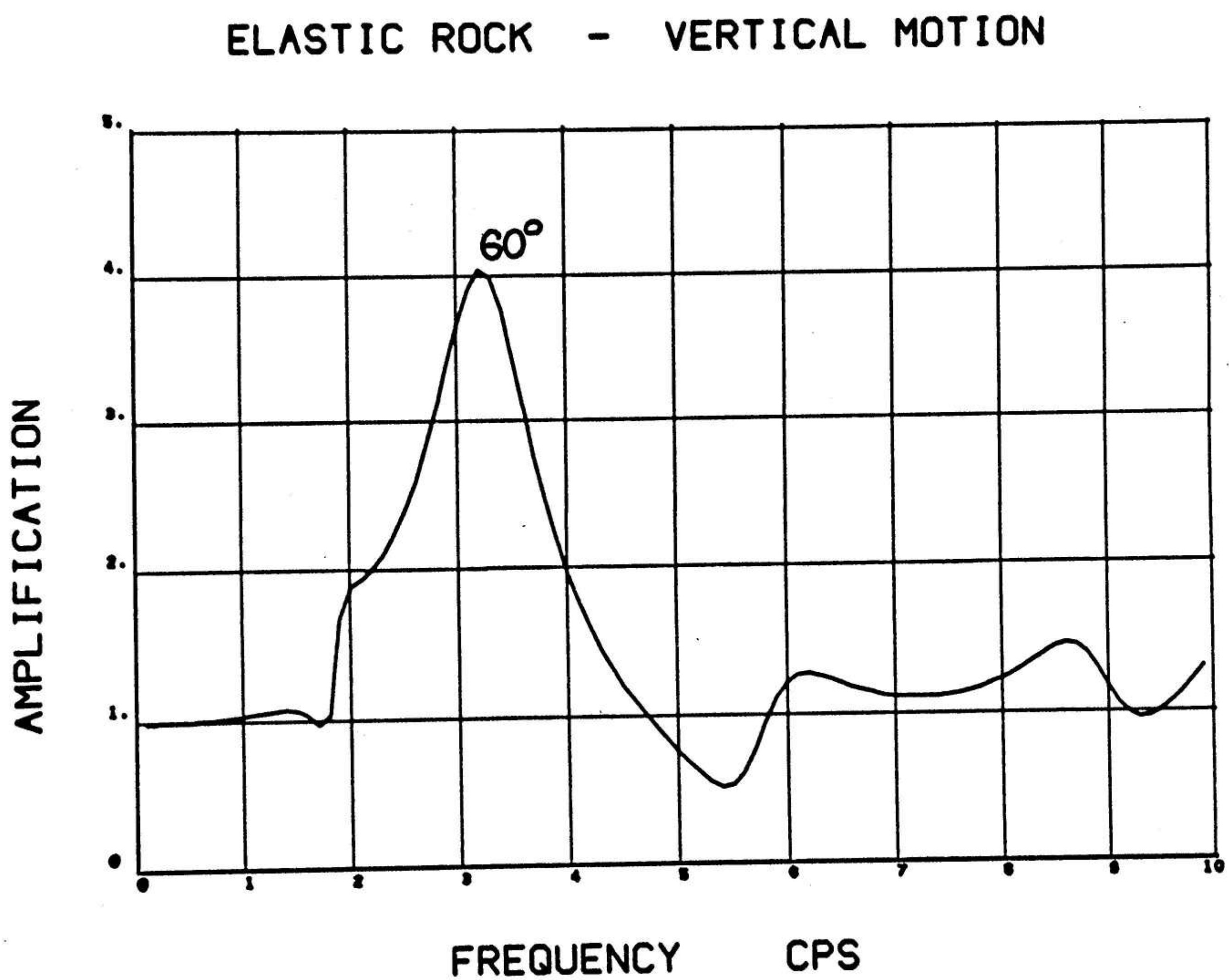


Fig. 4-23 Amplification functions - SV wave

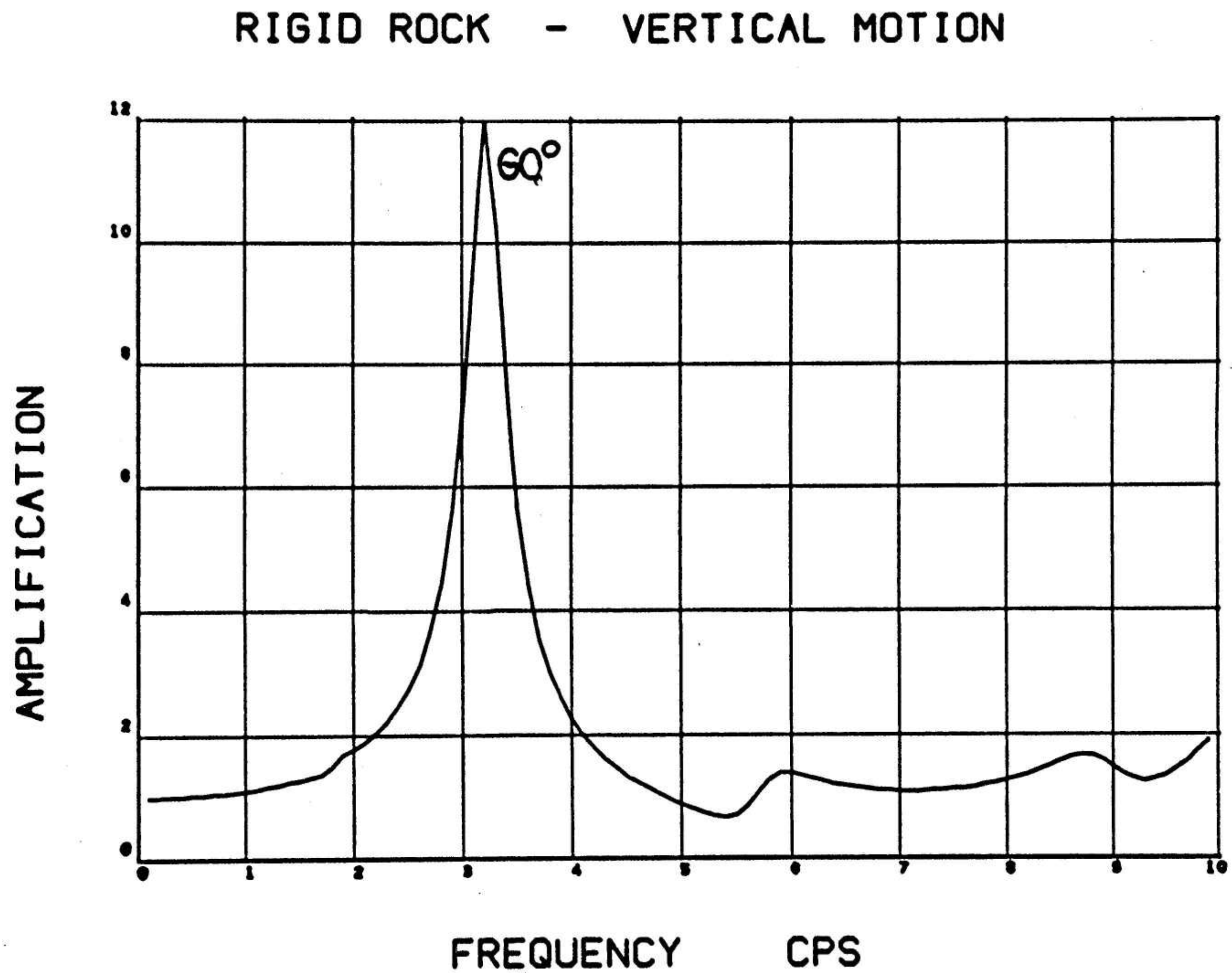


Fig. 4-24 Amplification functions - SV wave

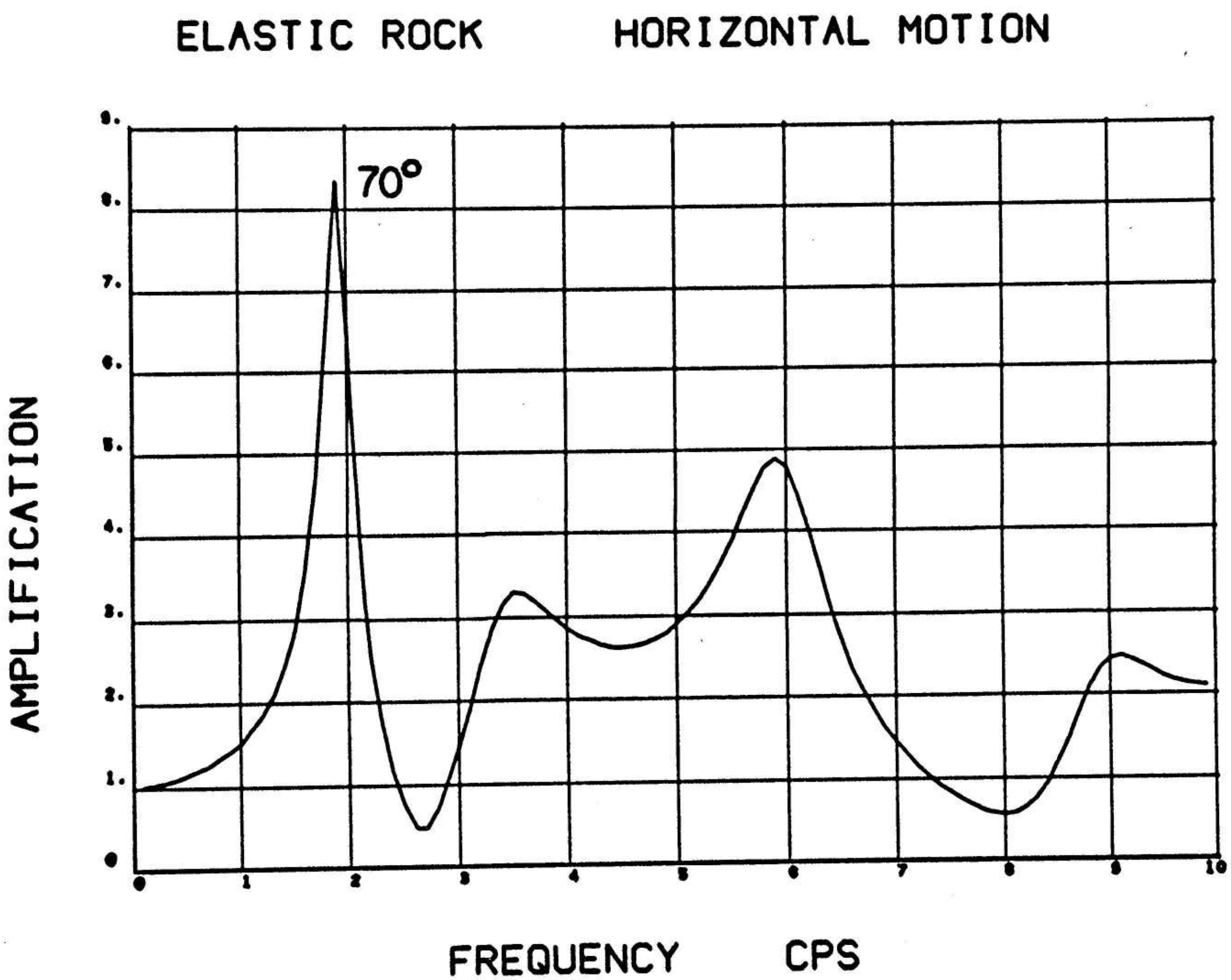


Fig. 4-25. Amplification functions - SV wave

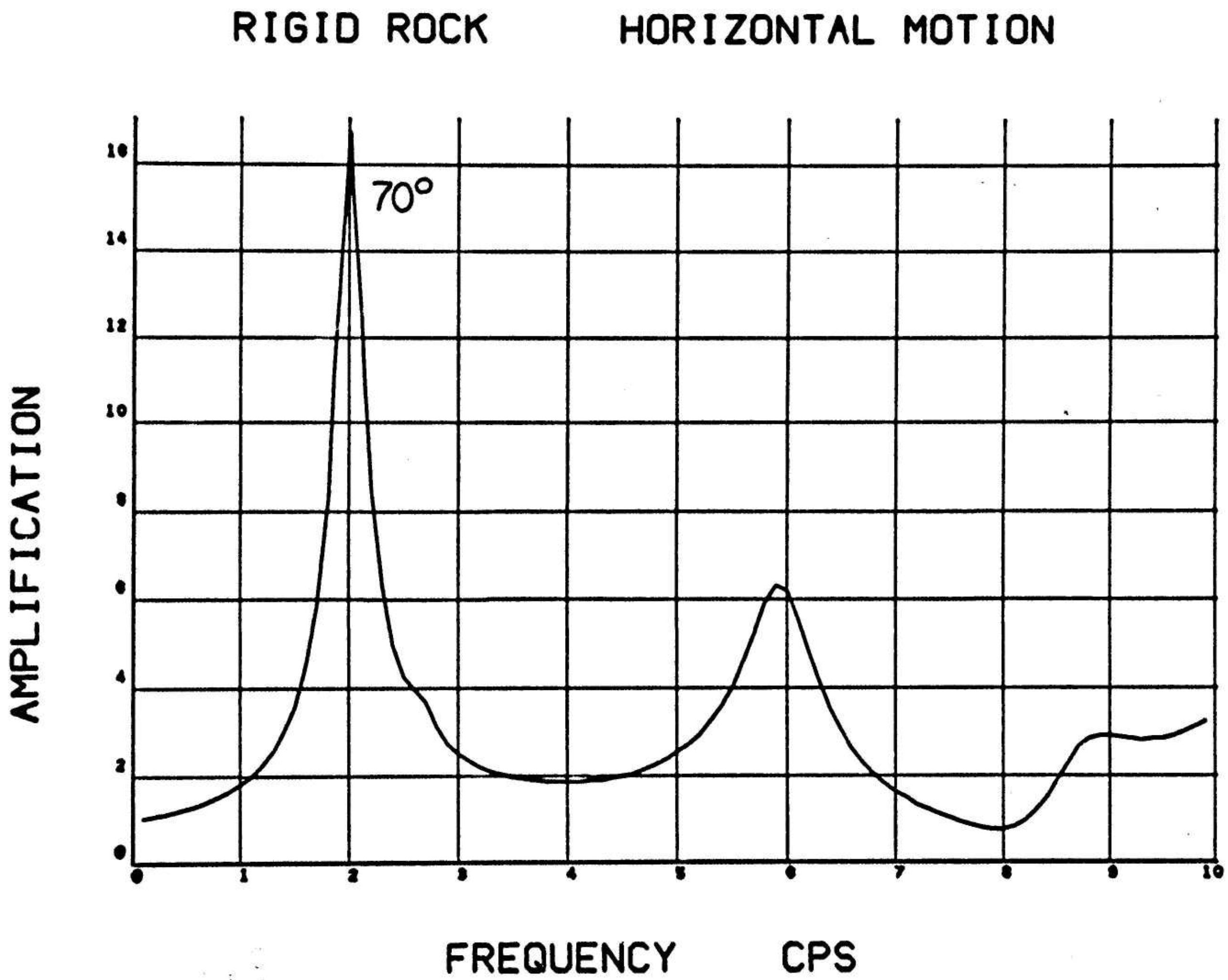


Fig. 4-26. Amplification functions - SV wave

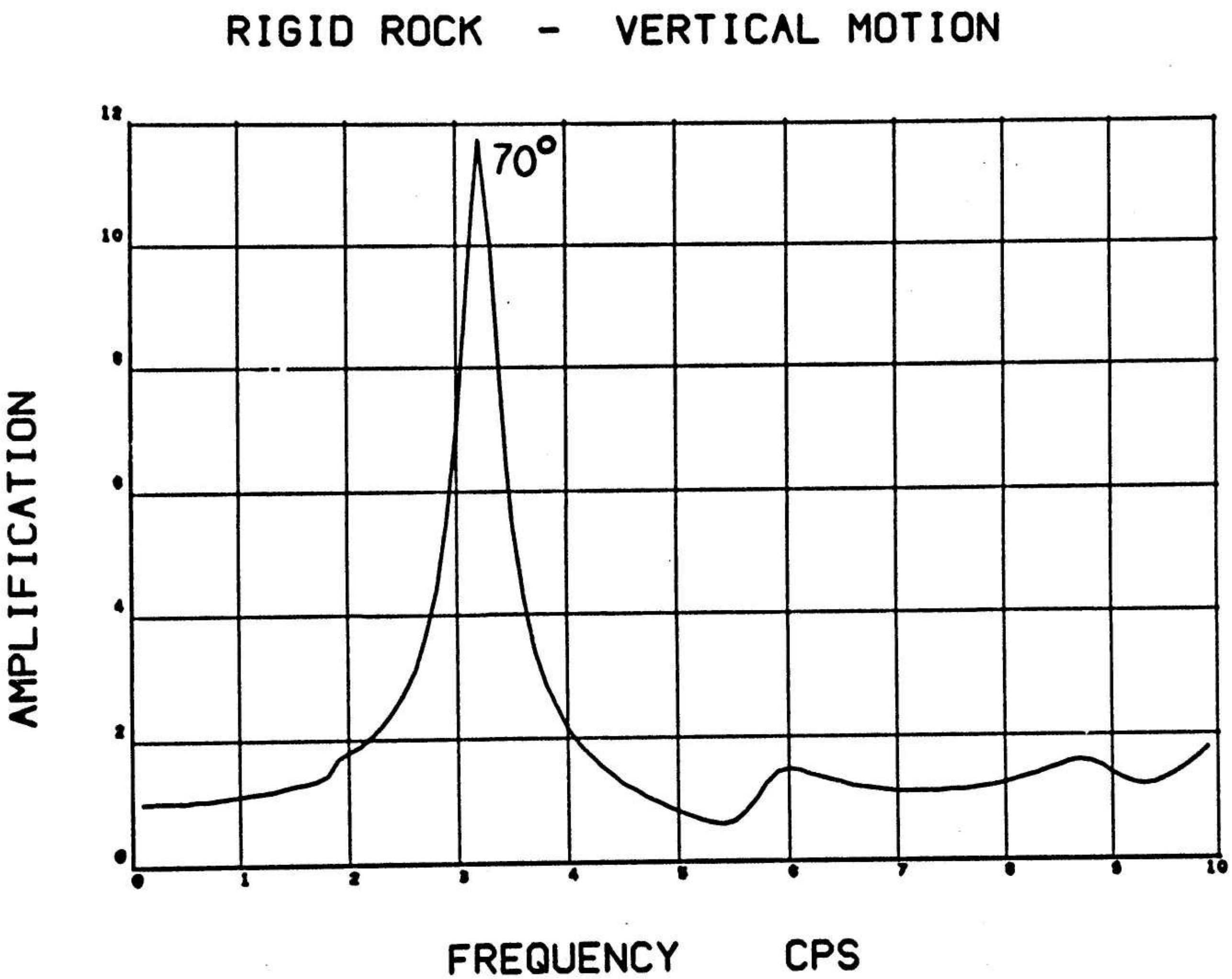


Fig. 4-27 Amplification functions - SV wave

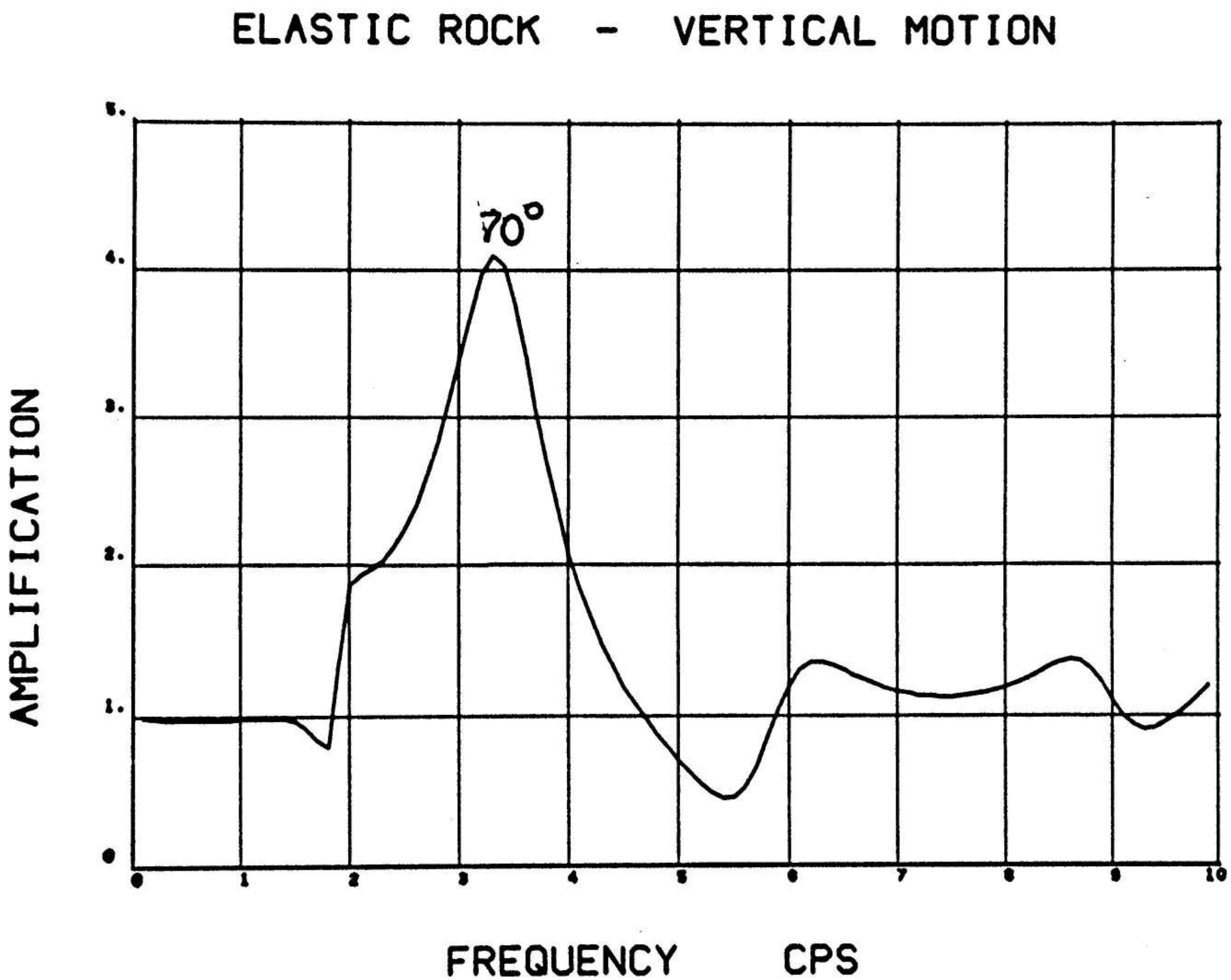


Fig. 4-28 Amplification functions - SV wave

Due to the uncoupling of p and SV waves at $\alpha_s = 0$, one dimensional wave propagation theory can be used to calculate the natural frequencies of the soil. They are determined from

$$f_n = \frac{2n-1}{4} \frac{V}{H}$$

Substituting the wave velocities and the height of the soil we obtain

Shear Wave	Dilatational Wave
1.875 cps	3.25 cps
5.625 cps	9.75 cps
9.375 cps	

As seen in the case of SH waves (10) the natural frequencies are slightly modified by the angle of incidence.

For non-zero angles of incidence an incoming SV wave will generate both shear and dilatational waves in the soil, and will produce both horizontal and vertical motion. Therefore as illustrated in these figures the amplification function will display both sets of natural frequencies. The significance, though, of each natural frequency depends greatly on the angle of incidence.

For angles of incidence less than the critical the overall shape of the curves is similar for all angles with the peaks (occurring at the natural frequency) decreasing with increasing angle and frequency (3).

For $\alpha_s > \alpha_{crit}$ the overall shapes are different with the peak values being highly dependent on the angle of incidence. For $\alpha_s = 45°$, the elastic rock horizontal amplification becomes infinite since in this case the motion at the surface of the elastic half space is zero (Fig. 4.1) Also for $\alpha_s > \alpha_{crit}$ the elastic rock amplification is not always smaller than the corresponding function for rigid rock as observed for $\alpha_s < \alpha_{crit}$ (3).

In summary it can be concluded that one dimensional wave propagation theory can be used to provide a good approximation of the natural frequencies for all angles of incidence. It will still provide a reasonable estimate of the shape and magnitude of the amplification functions for angles of incidence smaller than the critical, but for $\alpha_s > \alpha_{crit}$ the actual peak values will vary greatly with the angle of incidence.

The same conclusions can be derived by examining the variation of the motion with respect to depth (figs. 4.1-4.4). This should be expected since the amplification relates the motions at various points.

CHAPTER 5

SURFACE WAVES

5.1 Basic Formulation

In this chapter, plane stress waves generated by a line load applied at the surface and propagating within the layered medium are considered. The situation is similar to that studied in chapters 3 and 4 with different boundary conditions.

Equation (3.10) relates the stresses and displacements at the surface to the corresponding ones at the interfaces of the layers. Instead of specifying the motion at the bottom of the soil profile and calculating the stresses and displacements at any point in the stratum, these are determined from the stresses, $\sigma(x)_o$ and $\tau(x)_o$ given at the surface. The one dimensional geometry (section 3.2) condition is also relaxed so that the stresses and displacements are functions of the z as well as the x axis.

Two solutions as in the previous case can be obtained:

a) Rigid Rock Case

From the equation

$$(X_h)_n = F_n \; (X_o)_1 \tag{3.10}$$

or in partioned form

$$\begin{Bmatrix} U_n \\ \hdashline S_h \end{Bmatrix}_n = \left|\begin{array}{c:c} F_{11} & F_{12} \\ \hdashline F_{21} & F_{22} \end{array}\right| \begin{Bmatrix} U_o \\ \hdashline S_o \end{Bmatrix}_1$$

substituting the zero displacements at bedrock, $(U_h)_n = 0$,

$$F_{11}\, U_o + F_{12}\, S_o = 0$$

or

$$U_o = -F_{11}^{-1}\, F_{12}\, S_o \tag{5.1}$$

where

$$U_o = \begin{Bmatrix} u(x)_o \\ w(x)_o \end{Bmatrix}_1$$

$$S_o = \begin{Bmatrix} \sigma(x)_o \\ \tau(x)_o \end{Bmatrix}_1$$

represent the displacements and stresses at the surface.

b) <u>Elastic Rock</u>

Similarly for the elastic rock the relation

$$A_r = G\,(X_o)_1$$

holds. Since there are no incoming waves from the bottom of the stratum profile, that is $A_p = A_{SV} = 0$

$$G_{11}\, U_o + G_{12}\, S_o = 0$$

or

$$U_o = -G_{11}^{-1}\, G_{12}\, S_o \tag{5.2}$$

Equations (5.1) and (5.2) relate the stresses and displacements at the surface for the rigid and elastic rock, respectively. Having these the stresses and displacements at any depth can be obtained from Eq. (3.10).

The difficulty of imposing the boundary conditions, S_o, is overcome by the use of Fourier transforms, that is

$$\sigma(x)_o = \frac{1}{2\Pi} \int_{-\infty}^{+\infty} S(\xi)_o \; e^{i\xi x} \, d\xi$$

with

$$S(\xi)_o = \int_{-\infty}^{+\infty} \sigma(x)_o \; e^{-\xi x} \, dx$$

and similarly for $\tau(x)_o$, $T(\xi)_o$.

One can solve Equations (5.1), (5.2) and (3.10) for any particular ξ, by setting for each layer

$$\left(\frac{\omega \ell}{V_p}\right)_j = \left(\frac{\omega \ell'}{V_s}\right)_j = -\xi$$

leading to $U(\xi)$, $W(\xi)$, $S(\xi)$, $T(\xi)$.

Then the displacements are obtained from

$$u(x) = \frac{1}{2\Pi} \int_{-\infty}^{+\infty} U(\xi) \; e^{i\xi x} \, d\xi$$

$$w(x) = \frac{1}{2\Pi} \int_{-\infty}^{+\infty} W(\xi) \; e^{i\xi x} \, d\xi$$

and similarly for the stresses.

5.2 Definition of Amplification Functions

The definition of amplification functions in this case becomes more difficult since for each loading condition (normal and shear loads) there exist both horizontal and vertical motions which are functions of the frequency and spatial coordinates x and z. Thus for every point on the horizontal axis similar amplification functions as in section 3.4 could be defined.

Limiting the discussion to the points under the loads (x=0), an elastic rock amplification could be defined as the ratio of the amplitude of the displacement at the surface of the soil, under the load, to the corresponding amplitude of the outcropping rock, that is at the free surface of the rock without any soil on top (assuming the load applied at the rock).

Because two motions are involved for each loading condition, amplification functions could be defined for each combination of load and motion. By selecting the point under the load, though, the horizontal amplification corresponds to the shear load and the vertical to the normal load. Due to symmetry (or antisymmetry) the shear load produces no vertical motion at this point and similarly the normal load produces no horizontal motion.

Finally the amplification ratios were normalized by dividing with the corresponding displacement at small frequencies.

5.3 Cases Studied

The same two stratum profiles described in section 4.1 were subjected to unit harmonic stresses (normal and shear), applied at the origin of the coordinate system. That is, the boundary stresses are

$$\sigma(x=0)_o = 1 \qquad \tau(x=0)_o = 1$$

$$\sigma(x\neq 0)_o = 0 \qquad \sigma(x\neq 0)_o = 0$$

Displacements are obtained as a function of x and z for the elastic and rigid rock cases. Amplification functions are given only for the elastic rock case.

5.4 Description of Computer Program

A second computer program in FORTRAN IV was written utilizing the relations developed in the preceeding sections. It calculates the amplitudes of the displacements as a function of x and z due to the specified boundary conditions. It also plots (with a Stromberg Carlson plotter) the amplification ratios as a function of frequency. Input parameters are the necessary properties to specify the stratum profile and the case considered (rigid rock or elastic rock).

Considering unit stresses, applied at the origin of the coordinate axis, it obtains the Fourier Transforms by the use of the Cooley-Tukey algorithm. It then calculates the parameters ℓ, n, ℓ', n' for each layer and each horizontal point from the relations

$$\left(\frac{\omega \ell}{V_p}\right)_{j'} = \left(\frac{\omega \ell'}{V_s}\right)_{j'} = -\xi$$

and

$$\xi = 2\, n\Pi \Delta x$$

where n represents the number of points used for the Fourier Transform and Δx is the length increment along the x-axis. Based on Ref. 1 the above values were selected as

$$n = 256 \qquad \Delta x = \frac{V_s}{10f} = \frac{\lambda}{10}$$

The length increment is a function of the wavelength since at high frequencies, or small wavelengths, the motion fluctuates more rapidly and therefore more points are required to reproduce the motion. It was determined (R. 1) that about ten points per wavelength give a good representation of the motion.

Having determined the values of ℓ, n, ℓ', n' the program calculates the corresponding matrices T, H, T^{-1}, F and G for each layer.

Finally the displacements and amplifications are calculated using sections 5.1, 5.2 and the Cooley-Tucker algorithm.

A listing of the program with the input format is given in Appendix A.

5.5 Presentation and Discussion of Results

Figure 5.1 shows the amplitude of the vertical motion at the surface of a 100 ft. uniform soil layer resting on elastic or rigid rock as a function of the horizontal distance from the line of application of the load. The motion is the outcome of a unit normal periodic line load with frequency of 2 cps.

The maximum amplitude occurs under the load, and is given in the above figure as $w_{x=0}$ for both elastic and rigid rock. For the elastic rock the motion is nearly periodic with each consequetive peak along the x-direction decreasing due to damping. Most of the decay in the motion with respect to distance occurs near the line of application of the load. For the rigid rock case the motion reduces to essentially zero at a short distance.

Similarly Fig. 5.2 shows the amplitude of the horizontal motion due to a unit shear line load as a function of the horizontal distance. It applies to the same frequency and soil profile as in Fig. 5.1 and results are shown for both rigid and elastic rock. The curve for the elastic rock exhibits the same characteristics as the corresponding one for the vertical motion in the previous figure. The rigid rock, however in this case, produces larger horizontal displacements than the elastic rock. This is the effect of the frequency (2 cps) which corresponds approximately to the first natural frequency of the soil layer in shear (see Chapter 4). At the

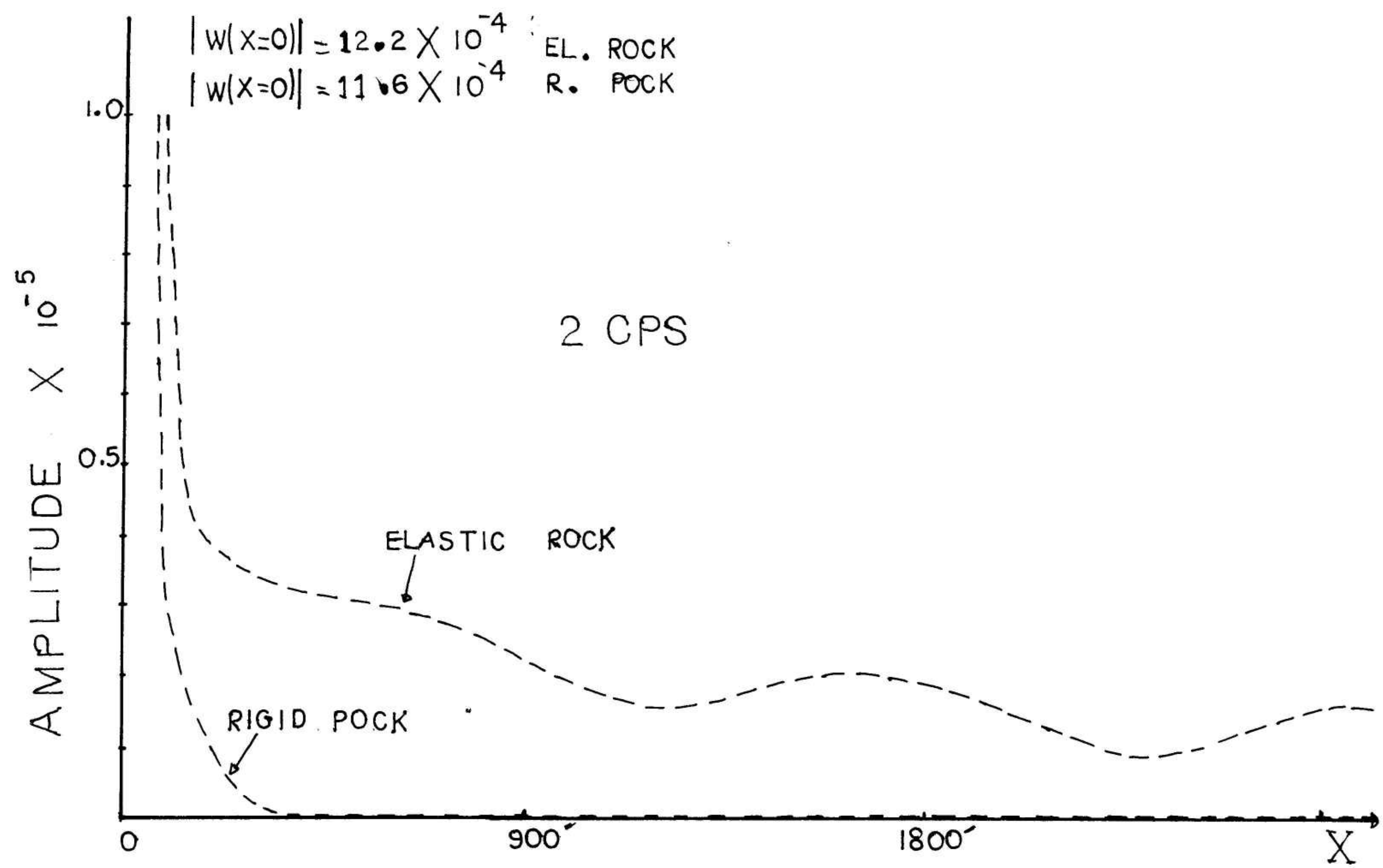


Fig. 5.1 Vertical motion at the surface due to unit normal line load

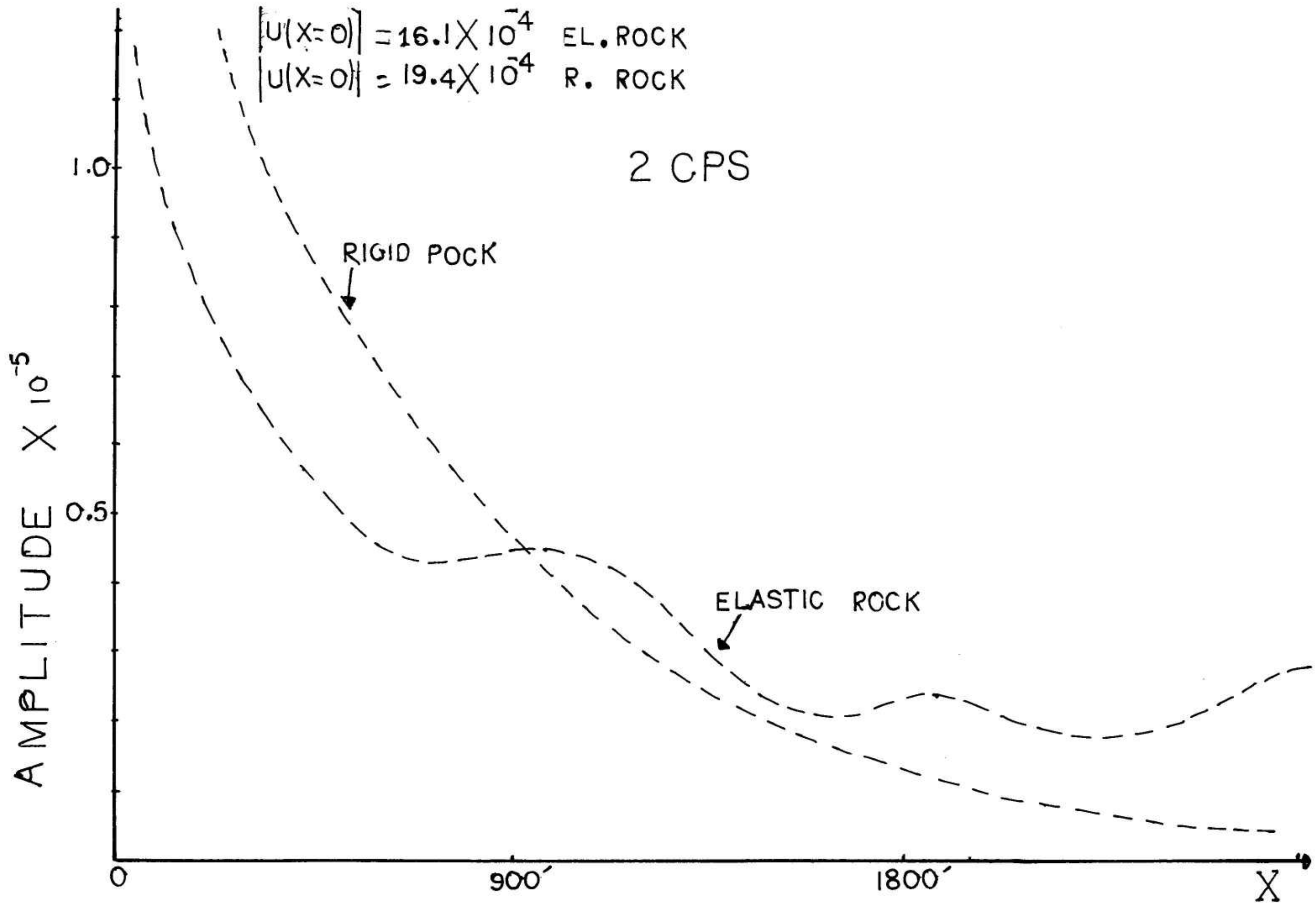


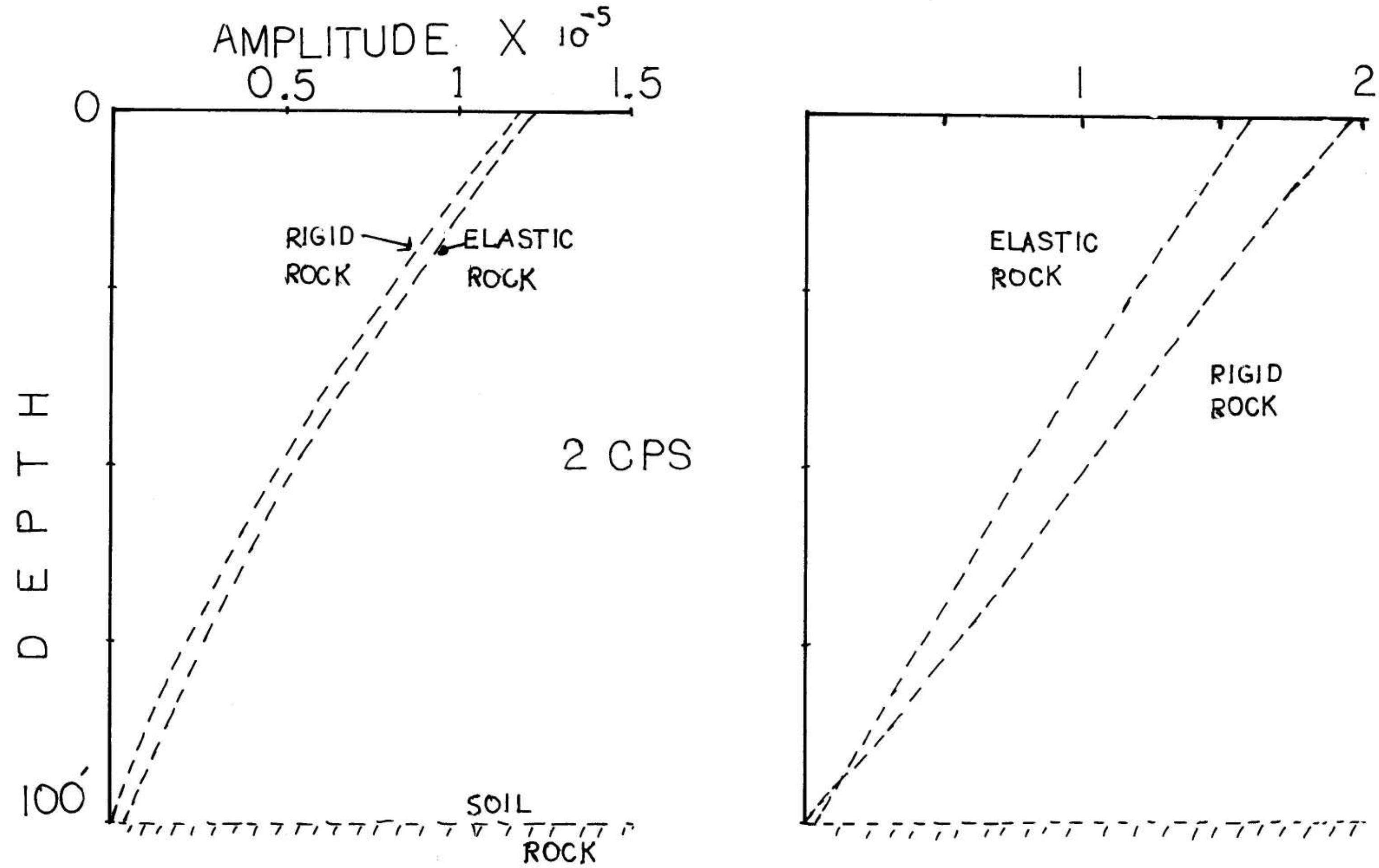
Fig. 5.2 Horizontal motion at the surface due to unit line shear load

resonant frequency damping plays an important role on the motion. Thus in the elastic rock case since energy is dissipated through the rock the motion is smaller. Rigid rock implies no energy is lost through the soil-rock interface. The same phenomenon is not observed in Fig. 5.1 for the vertical motion because in this case the first resonance does not occur until a frequency of 3.25 cps (see Chapter 4 for natural frequencies in dilatation).

Fig. 5.3 shows the amplitude of the motion as a function of depth, in the soil layer, directly under the load. It is observed that the motion decreases rapidly with depth even for the case of elastic rock (rigid rock motion is zero at the interface of soil and rock by definition).

It can be concluded, therefore, that when a stress wave is generated at the surface the greatest effect is near the location of the loads. The decay of the motion with respect to distance depends on the type of load considered, frequency and properties of the underlying rock (elastic or rigid).

Figures 5.4 and 5.5 show the elastic rock amplifications for horizontal and vertical displacements of the points directly under the application of the load. The shape of the curves is similar to those obtained in Chapter 4. The peaks occur near the natural frequencies as obtained from one dimensional amplification theory. For the horizontal motion the dominant peaks correspond to natural frequencies in shear since



Vertical motion due to unit normal load

Horizontal motion due to unit shear load

Fig. 5.3 Motion vs. depth at w(x=0)

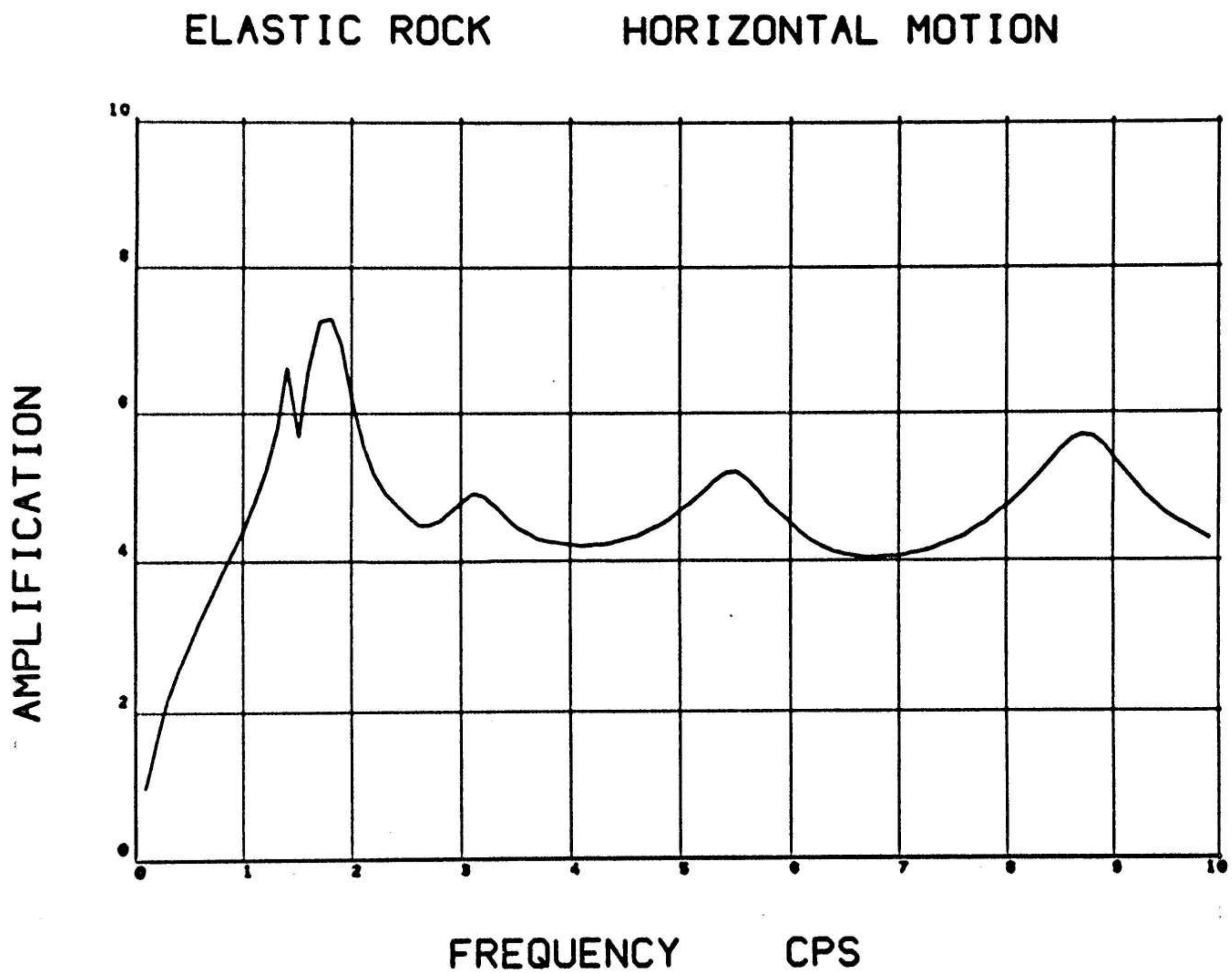


Fig. 5.4 Amplifation functions for surface wave

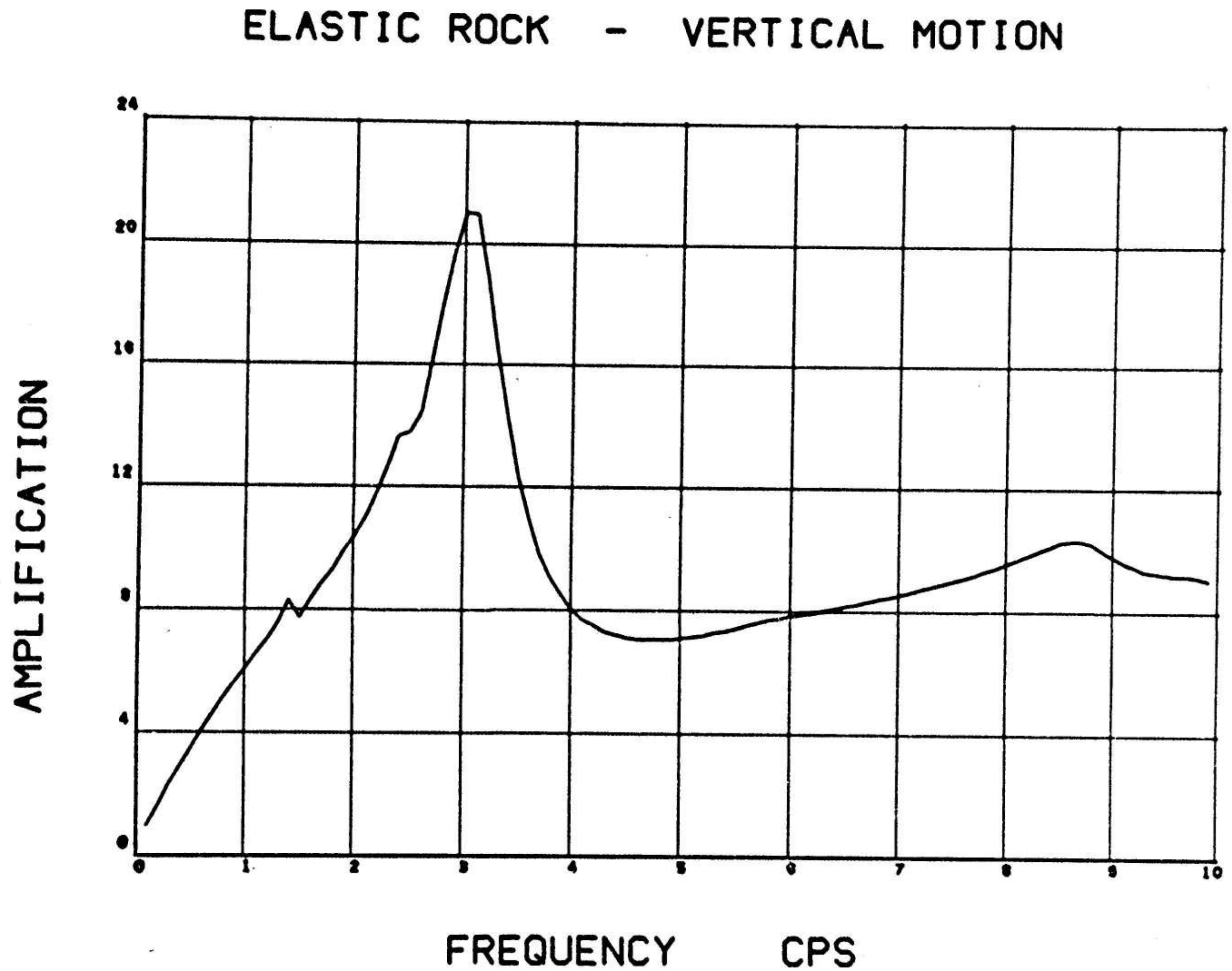


Fig. 5.5 Amplification functions for surface wave

the vertical motion of the point considered due a shear load is zero due to symmetry. Similarly for the vertical motion the significant peaks correspond to natural frequencies in dilatation.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

This work is a logical continuation of the studies performed in references 3,9 and 10. Displacements and amplification functions were presented for an elastic half-space and a uniform soil layer resting on bedrock. Two situations of plane waves were considered:

1) SV waves propagating upwards through the bedrock at an arbitrary angle of incidence.
2) Shear waves generated at the surface by unit periodic line loads (normal and shear).

A comparison of the results obtained in the first situation with those of the references shows that the one dimensional amplification theory can be used to predict the natural frequencies of the soil layer. The effect of the angle of incidence is to modify slightly these frequencies. The overall shape of the amplification functions is similar only for SV waves with angles of incidence less than the critical. For SV waves with larger angles the magnitude and the shape of the amplification functions depends significantly on the angle of incidence.

The same observation applies to the variation of displacements with depth. The overall shape of the curves is similar only for $\alpha_s < \alpha_{crit}$. The motion is nearly periodic with respect to depth. The effect of the angle of incidence is to influence the wavelength and to introduce a phase shift in the curve describing the motion vs. depth.

In the second case, surface waves, the natural frequencies, as obtained from one dimensional propagation theory, can be used to determine the peaks at the amplification functions. The significance of each frequency depends on the motion and load considered. As for the motion it decays rapidly with the distance, measured from the point of application of the line load, due to damping.

While this work has been concerned with displacements and amplification, the stresses can also be calculated. This should be the next logical step in the series of studies of soil amplification. Also, here the soil was assumed to be a linear viscoelastic material. It has been recognized, however, that an important factor in the application of soil amplification theories to practical situations is the consideration of nonlinear soil behavior. The variation of the amplification with the level of excitation for different types of waves (and cycles of incidence) should be studied.

APPENDIX A

COMPUTER PROGRAM

A.1 Computer Program - Case 1

a) Input Format

In order to use computer program described in Section 3.4d , the following input scheme must be followed.

The first data card contains an integer right justified to card column 10 which represents the number of the layers (n) wished to specify the soil profile.

Next a group of n+1 containing the data for each soil layer (n) plus the last card the data of the underlying rock. The forms of these cards in decimal number is as follows:

For Soil Layers (n cards)

c.c.	$1 \longleftrightarrow 10$	$11 \longleftrightarrow 20$	$21 \longleftrightarrow 30$
	h	V_s	v
	$31 \longleftrightarrow 40$	$41 \longleftrightarrow 50$	
	β	γ	

For Rock Layer (1 card)

c.c.	$1 \longleftrightarrow 10$	$11 \longleftrightarrow 20$	$21 \longleftrightarrow 30$
		V_s	v
	$31 \longleftarrow 40$	$41 \longleftrightarrow 50$	
	β	γ	

Any number of angles and type of waves can be tested by putting

i) one card representing the number of angles considered. FORMAT (I10).

ii) two cards for each angle. The first card gives the angle considered, the amplitude of P-wave (A_p') and the amplitude of SV-wave (A_{SV}'). FORMAT (3F10.0). The second card containing the number of frequencies for which the analysis wished to be performed, the first frequency at which the analysis starts, and the increment of the frequency. FORMAT (I10, 2F10.0).

b) Listing of Program

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION H(20), VS(20), ANU(20), BETA(20), GAM(20)
      DIMENSION DAUX(2), CG(20), CL(20), CVP(20), CVS(20)
      DIMENSION AL(20), ALP(20), AN(20), ANP(20)
      DIMENSION EE(4), FMAT(4,4,20), GMAT(4,4)
      DIMENSION TOP(4,4), BOT(4,4)
      DIMENSION FREQ(100),RRHA(100),RRVA(100),ERHA(100),ERVA(100)
      DIMENSION TEX1(4),TEX2(4),TEX3(9),TEX4(8),TEX5(8),TEX6(8),NAME(2)
       COMPLEX*16 AUX,CG,CL,CVP,CVS,AL,AN,ALP,ANP
       COMPLEX*16 EE,AIM,GMAT,DET,UT,VT,UA,UB,US,VSI
       COMPLEX*16 TOP,BOT,FMAT
       COMPLEX*16 APP,ASVP
      EQUIVALENCE (AUX, DAUX(1))
      DATA NAME/'EVAN','MICH'/
      DATA TEX1/'FREQ','UENC','Y   ',' CPS'/
      DATA TEX2/'AMPL','IFIC','ATIO','N   '/
      DATA TEX3/'ELAS','TIC ','ROCK','    ',' HOR','IZON','TAL ','MOTI'
     1,'ON  '/
      DATA TEX4/'RIGI','D RO','CK  ','   H','ORIZ','ONTA','L MO','TION'/
      DATA TEX5/'ELAS','TIC ','ROCK','  - ',' VER','TICA','L MO','TION'/
      DATA TEX6/'RIGI','D RO','CK  ','-  V','ERTI','CAL ','MOTI','ON  '/
C
C ***********************************************************************
C       THIS PROGRAM IS FOR THE CASE OF INCOMING WAVES THROUGH THE BOTTOM
C       OF THE LAYERED PROFILE. IT CALCULATES THE DISPLACEMENTS AS A
C       FUNCTION OF FREQUENCY AND DEPTH.IT ALSO CALCULATES AND PLOTS
C       (WITH A CARLSON-STROMBERG PLOTTER) THE AMPLIFICATION RATIOS AS
C       A FUNCTION OF FREQUENCY
C ***********************************************************************
C
C..... SET VARIABLE AIM= SQRT(-1)=I
      DAUX(1)= 0.
      DAUX(2)= 1.
      AIM= AUX
C
C......READ NUMBER OF LAYERS OF SOIL TO BE STUDIED
```

```
C
      READ(5,100) NLAY
100   FORMAT(I10)
      NLAY1=NLAY+1
C...........................................................
      DO 10 I=1,NLAY1
C
C.....READ HEIGHT, SHEAR WAVE VELOCITY, POISSON%S RATIO, DAMPING
C......AND UNIT WEIGHT FOR EACH LAYER OF SOIL PLUS LAST DATA CARD FOR
C.....ELASTIC ROCK
C
      READ(5,101) H(I), VS(I), ANU(I), BETA(I), GAM(I)
101   FORMAT(5F10.0)
10    CONTINUE
C.....CALCULATE LAME%S CONSTANTS AND WAVE-VELOCITIES IN COMPLEX FORM
C...........................................................
      DO 11 I=1,NLAY1
      RO= GAM(I)/32.2
      G= RO*VS(I)*VS(I)
      E= 2.*G*ANU(I)/(1.-2.*ANU(I))
      DAUX(1)= G
      DAUX(2)= 2.*BETA(I)*G
      CG(I)  =AUX
      DAUX(1)= E
      DAUX(2)= 2.*BETA(I)*E
      CL(I)= AUX
      CVP(I)= (CL(I)+2.*CG(I))/RO
      CVS(I)= CG(I)/RO
       CVP(I)=CDSQRT(CVP(I))
11     CVS(I)=CDSQRT(CVS(I))
C
C...... READ ANGLE OF INCIDENCE OF SHEAR WAVE-WAVE IN DEGREES.
C...... READ ANGLE OF INCIDENCE OF SHEAR WAVE- IN DEGREES.
C.....ANGLE IS MEASURED FROM Z-AXES
C
      K=0
```

```
      NANGC=0
      READ(5,100) NANG
40    NANGC= NANGC+1
      READ (5,99) ALFA,AMPP,AMPS
99    FORMAT(3F10.0)
      ANG= ALFA* 3.14159/180.
       ALP(NLAY1)=DSIN(ANG)
       ANP(NLAY1)=1.-ALP(NLAY1)*ALP(NLAY1)
       ANP(NLAY1)=CDSQRT(ANP(NLAY 1))
      AL(NLAY1)= CVP(NLAY1)*ALP(NLAY1)/CVS(NLAY1)
      AN(NLAY1)= 1.-AL(NLAY1)*AL(NLAY1)
       AN(NLAY1)=CDSQRT(AN(NLAY1))
      AUX= AN(NLAY1)
      IF(DAUX(2)) 1,1,2
2     AN(NLAY1)= -AN(NLAY1)
1     CONTINUE
C...............................................................
      DO 12 I=1,NLAY
      ALP(I)= ALP(NLAY1)*CVS(I)/CVS(NLAY1)
      AL(I)= AL(NLAY1)*CVP(I)/CVP(NLAY1)
      ANP(I)= 1.-ALP(I)*ALP(I)
       ANP(I)=CDSQRT(ANP(I))
      AN(I)= 1.-AL(I)*AL(I)
       AN(I)=CDSQRT(AN(I))
      AUX= AN(I)
      IF(DAUX(2)) 12,12,13
13    AN(I)= -AN(I)
12    CONTINUE
C
C.......READ  NO. OF FREQUENCIES, INITIAL FREQ. AND INCREMENTAL FREQ.
C
      READ(5,102) NF, F1, DF
102   FORMAT(I10,2F10.0)
      DO 300 I=1,NF
      AI=I-1
300   FREQ(I)=F1+AI*DF
```

```
      WRITE(6,200) NLAY
200   FORMAT(1H1,50X,I5,2X,'LAYERS',//)
      WRITE(6,201)
201   FORMAT(2X,'LAYER',3X,'THICKNESS',1X,'SHEAR VEL',1X,'POISS RAT',3X,
     1'DAMPING',2X,'UNIT WEIGHT',/)
      WRITE(6,202)  (I,H(I),VS(I),ANU(I),BETA(I),GAM(I),I=1,NLAY)
202   FORMAT(I5,5X,5F10.2)
      WRITE(6,203)   VS(NLAY1),ANU(NLAY1),BETA(NLAY1),GAM(NLAY1)
203   FORMAT(2X,'ROCK',14X,4F10.2,//)
      WRITE(6,204) ALFA
204   FORMAT(10X,'ANGLE OF INCIDENCE OF    WAVE IN ROCK',F10.0,' DEG',//)
      WRITE(6,205)
205   FORMAT(2X,'LAYER',13X,'L',14X,'N',18X,'LP',18X,'NP',//)
      WRITE(6,206)  (I,AL(I),AN(I),ALP(I),ANP(I),I=1,NLAY)
206   FORMAT( I5,5X,8F10.5)
      WRITE(6,207)   AL(NLAY1),AN(NLAY1),ALP(NLAY1),ANP(NLAY1)
207    FORMAT(2X 'ROCK',4X,8F10.5,//)
      WRITE(6,208) AMPP
208   FORMAT(10X,'AMPLITUDE OF P WAVE IN ROCK',F10.2,/)
      WRITE(6,209) AMPS
209   FORMAT(10X,'AMPLITUDE OF S WAVE IN ROCK',F10.2,/)
C............................................................
       DO 1000 J=1, NF
      AI= J-1
      FR= F1 + AI*DF
      OM= 6.28318*FR
      WRITE(6,210) FR,OM
210   FORMAT(1H1,20X,'FREQUENCY',F6.2,' CPS ',F6.2,' RAD/SEC',//)
C............................................................
      DO 30 L=1,4
      DO 30 M=1,4
30    FMAT(L,M,1)=0.
      DO 31 L=1,4
31    FMAT(L,L,1)=1.
C............................................................
      DO 1001 I=1,NLAY1
```

```
C..... DEFINE T-MATRIX
       TOP(1,1)= AL(I)
       TOP(1,2)= -ANP(I)
       TOP(1,3)= AL(I)
       TOP(1,4)= ANP(I)
       TOP(2,1)= -AN(I)
       TOP(2,2)= -ALP(I)
       TOP(2,3)= AN(I)
       TOP(2,4)= -ALP(I)
       AUX= AIM*OM/CVP(I)
       TOP(3,1)= -AUX*(CL(I)+2.*CG(I)*AN(I)*AN(I))
       TOP(3,2)= -AUX*2.*CG(I)*AL(I)*ANP(I)
       TOP(3,3)= TOP(3,1)
       TOP(3,4)= -TOP(3,2)
       AUX= AIM*OM/CVS(I)
       TOP(4,1)= AUX*2.*CG(I)*ALP(I)*AN(I)
       TOP(4,2)= AUX*CG(I)*(ALP(I)*ALP(I)-ANP(I)*ANP(I))
       TOP(4,3)= -TOP(4,1)
       TOP(4,4)= TOP(4,2)
C..... DEFINE THE INVERSE OF T-MATRIX
       AUX= (CL(I)+2.*CG(I))*2.
       BOT(1,1)= 2.*CG(I)*AL(I)/AUX
       BOT(2,1)= -(CL(I)+2.*CG(I)*AN(I)*AN(I))/(AUX*ANP(I))
       BOT(3,1)= BOT(1,1)
       BOT(4,1)= -BOT(2,1)
       BOT(1,2)= (ALP(I)*ALP(I)-ANP(I)*ANP(I))/(2.*AN(I))
       BOT(2,2)= -ALP(I)
       BOT(3,2)= -BOT(1,2)
       BOT(4,2)= -ALP(I)
       BOT(1,3)= AIM*CVP(I)/(OM*AUX)
       BOT(2,3)= AIM*AL(I)*CVP(I)/(OM*ANP(I)*AUX)
       BOT(3,3)= BOT(1,3)
       BOT(4,3)= -BOT(2,3)
       AUX= 2.*AIM*OM*CG(I)/CVS(I)
       BOT(1,4)= ALP(I)/(AUX*AN(I))
       BOT(2,4)= -1./AUX
```

```
      BOT(3,4)= -BOT(1,4)
      BOT(4,4)= BOT(2,4)
C..... MULTIPLY T-MATRIX BY H-MATRIX
       IF(I-NLAY) 225,225,1001
225   CONTINUE
C..... DEFINE THE ' H-MATRIXH OR EE= DIAGONAL OF H-MATRIX
       AUX= -OM*AN(I)*H(I)/CVP(I)
      AUX= AUX*AIM
       EE(3)=CDEXP(AUX)
      EE(1)=   1./EE(3)
      AUX= -OM*ANP(I)*H(I)/CVS(I)
      AUX= AUX*AIM
       EE(4)=CDEXP(AUX)
      EE(2)= 1./EE(4)
C..............................................................
25    DO  20 L=1,4
      DO 20 M=1,4
20    TOP(L,M)= TOP(L,M)*EE(M)
      I1= I+1
C......CALCULATE THE F-MATRIX FOR EACH LAYER OF SOIL
      CALL MATMUL (TOP,BOT,BOT)
      CALL MATMUL(BOT, FMAT(1,1,I), FMAT(1,1,I1))
1001  CONTINUE
      CALL MATMUL(BOT,FMAT(1,1,NLAY1),GMAT)
      DET= GMAT(1,1)*GMAT(2,2)-GMAT(1,2)*GMAT(2,1)
C .......HERE UT AND VT CONTAIN THE HOR. & VERT. MOTION AT THE SURFACE
      UT= AMPP*GMAT(2,2)-AMPS*GMAT(1,2)
      VT= AMPS*GMAT(1,1)-AMPP*GMAT(2,1)
      UT= UT/DET
      VT= VT/DET
C...... CALCULATE APP AND ASVP IN TERMS OF AP AND ASV USING THE BOUNDARY
C...... CONDITION THAT THE STRESSES AT THE TOP ARE ZERO
      UA= TOP(3,1)*AMPP+TOP(3,2)*AMPS
      UB= TOP(4,1)*AMPP+TOP(4,2)*AMPS
      DET= TOP(3,3)*TOP(4,4)-TOP(3,4)*TOP(4,3)
      US=   (TOP(4,4)*UA-TOP(3,4)*UB)/DET
```

```
      VSI= (TOP(3,3)*UB-TOP(4,3)*UA)/DET
      APP= -US
      ASVP= -VSI
      WRITE(6,217)
217   FORMAT(17X,'AP',18X,'ASV',27X,'APP',26X,'ASVP',/)
      WRITE(6,218) AMPP,AMPS,APP,ASVP
218   FORMAT(10X,E10.2,10X,E10.2,10X,2E10.2,10X,2E10.2,//)
C...... CALCULATE THE DISPLACEMENTS U AND W BY KNOWING AP,APP,ASV,ASVP
      UA= TOP(1,3)*US+TOP(1,4)*VSI
      UB= TOP(2,3)*US+TOP(2,4)*VSI
C......HERE US AND VSI CONTAIN U AND W AT THE TOP OF THE ROCK
      US= TOP(1,1)*AMPP+TOP(1,2)*AMPS-UA
      VSI=TOP(2,1)*AMPP+TOP(2,2)*AMPS-UB
C...... IF THE ANGLE OF INCIDENCE IS ZERO WE HAVE MOTION ONLY IN ONE DIRECTION.
C......(HOR.OR VERT. FOR P AND SV-WAVES RESPECTIVELY). THEREFORE IN CALCULATING
C.......AMPLIFICATIONS WE  DIVIDE BY ZERO.  THIS PART AVOIDS THAT.
      IF(ALFA-0.0) 401,399,401
399   IF(AMPP-0.0) 398,400,398
398   IF(AMPS-0.0) 401,397,401
400   DAUX(1)=0.
      DAUX(2)=0.0
      UB=AUX
      IF(AMPS-0.0) 395,396,395
396    WRITE(6,409)
409   FORMAT(' **ERROR**. NO WAVE WAS INPUT(AP=ASV=0). TERMINATION OF
     1 THIS CASE CALLED.')
      GO TO 1003
395   UA=UT/US
      GO TO 402
397   DAUX(1)=0.0
      DAUX(2)=0.0
      UA=AUX
      UB=VT/VSI
      GO TO 402
401   UA=UT/US
      UB=VT/VSI
```

```
402    CONTINUE
       AUX= UA
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
C .......ERHA= ELASTIC ROCK HORIZONTAL AMPLIFICATION
       ERHA(J)=ABA
       WRITE(6,211) DAUX(1),DAUX(2),ABA
211     FORMAT(10X,'HOR AMPL',3F20.5,/)
       AUX= UB
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
C .......ERVA= ELASTIC ROCK VERTICAL AMPLIFICATION
       ERVA(J)=ABA
       WRITE(6,212) DAUX(1),DAUX(2),ABA
212    FORMAT(10X,'VER AMPL',3F20.5,///)
       WRITE(6,213)
213    FORMAT(10X,'HOR MOTION',13X,'REAL',13X,'IMAGINARY',11X,'AMPLITUDE
      *',/)
       AUX= UT
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
C........HERE AABA CONTAINS THE AMPLITUDE OF HOR. MOTION AT THE SURFACE
       AABA=ABA
       WRITE(6,214) DAUX(1),DAUX(2),ABA
214    FORMAT(20X,3(10X,E10.3))
C........THIS DO LOOP CALCULATES THE HORIZONTAL MOTION VS. DEPTH
       DO 1100 I=1, NLAY
       I1= I+1
       UA= FMAT(1,1,I1)*UT+FMAT(1,2,I1)*VT
       AUX= UA
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
       WRITE(6,214) DAUX(1),DAUX(2),ABA
1100   CONTINUE
C .......RRHA= RIGID ROCK HOR. AMPLIFICATION
       IF (ALFA-0.0) 420,421,420
```

```
421    IF (AMPP-0.0) 419,420,419
419    RRHA(J)=0.0
       GO TO 422
420    RRHA(J)=AABA/ABA
422    CONTINUE
       AUX=US
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
       WRITE(6,215) DAUX(1),DAUX(2),ABA
215    FORMAT(2X,'OUTCROP',11X,3(10X,E10.3),//)
       WRITE(6,216)
216    FORMAT(10X,'VER MOTION',13X,'REAL',13X,'IMAGINARY',11X,'AMPLITUDE
      *',/)
       AUX= VT
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
C .......HERE AABA CONTAINS THE AMPLITUDE OF THE VERT. MOTION AT THE SUFRACE
       AABA=ABA
C .......THIS DO LOOP CALCULATES THE VERT. MOTION VS. DEPTH
       WRITE(6,214) DAUX(1),DAUX(2),ABA
       DO 1200 I=1,NLAY
       I1=I+1
       UB= FMAT(2,1,I1)*UT+FMAT(2,2,I1)*VT
       AUX= UB
       ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
        ABA=DSQRT(ABA)
       WRITE(6,214) DAUX(1),DAUX(2),ABA
1200   CONTINUE
C .......RRVA= RIGID ROCK VERT. AMPLIFICATION
       IF(ALFA-0.0) 410,411,410
411    IF (AMPS-0.0) 414,410,414
414    RRVA(J)=0.0
       GO TO 412
410    RRVA(J)=AABA/ABA
412    CONTINUE
       AUX= VSI
```

```
      ABA= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
       ABA=DSQRT(ABA)
      WRITE(6,215) DAUX(1),DAUX(2),ABA
      WRITE(6,424)    RRHA(J)
424   FORMAT(' RIGID-HOR AMPL',F22.5,///)
       WRITE(6,425) RRVA(J)
425   FORMAT(' RIGID-VER AMPL',F22.5,///)
1000  CONTINUE
      CALL STOIDV (NAME,7,3)
      CALL PLOT(ERHA,FREQ,TEX1,TEX2,TEX3,225,36,NF,K)
      CALL PLOT(RRHA,FREQ,TEX1,TEX2,TEX4,260,32,NF,K)
      CALL PLOT(ERVA,FREQ,TEX1,TEX2,TEX5,260,32,NF,K)
      CALL PLOT(RRVA,FREQ,TEX1,TEX2,TEX6,260,32,NF,K)
      IF (NANGC-NANG) 40,41,41
41     CONTINUE
      CALL PLTND(N)
      CALL EXIT
      END
```

```
      SUBROUTINE MATMUL (A,B,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4), B(4,4), C(4,4), D(4)
       COMPLEX*16 A,B,C,D,SUM
C
C..... THIS SUBROUTINE MULTIPLIES TWO MATRICES.  C = A . B
      DO 10 I=1,4
      DO 11 J=1,4
      SUM= 0.
      DO  12 K=1,4
12    SUM = SUM+A(J,K)*B(K,I)
11    D(J)= SUM
      DO 10 J=1,4
10    C(J,I)= D(J)
      RETURN
      END
```

```
      SUBROUTINE PLOT(A,B,TIT1,TIT2,TIT3,N1,N2,N3,K)
      DIMENSION A(100),B(100),TIT1(10),TIT2(10),TIT3(10)
      CALL SETMIV(150,73,250,223)
       AMAX=0.
      DO 10 I=1,N3
      IF(AMAX-A(I)) 11,10,10
11    AMAX=A(I)
10    CONTINUE
      IF(AMAX-10.) 15,15,14
14    DY=2.0
      J=1
      GO TO 16
15    DY=1.0
      J=1
16    CONTINUE
      K=K+1
      IF(K-1) 20,20,21
20    L=2
      GO TO 23
21    L=4
23    CONTINUE
      AMAX=AMAX+1.
      IMAX=AMAX
      AMAX=IMAX
      BMAX=B(N3)+1.
      IMAX=BMAX
      BMAX=IMAX
      CALL GRID1V(L,0.0,BMAX,0.0,AMAX,1.0,DY,0,0,1,J,2,2)
      CALL RITE2V(410,200,1023,0,3,16,1,TIT1,IDUM)
      CALL RITE2V(100,375,1023,90,3,16,1,TIT2,IDUM)
      CALL RITE2V(N1,850,1023,0,3,N2,1,TIT3,IDUM)
       CALL GRAF1V(B,A,IERR,N3,1)
      RETURN
      END
```

A.2 Computer Program - Case 2

a) Input Format

In order to use the computer program described in section 5.5 the following input scheme must be followed.

The first cards contains the number of soil layers considered. FORMAT (I10).

Then a group of cards containing h, V_s, v, β and γ for each soil layer plus one card for the underlying rock. FORMAT (5F10.0).

The next card specifies the type of rock considered. If the number 1 appears in c.c 10, elastic rock is considered. If 0 appears in c.c. 10, rigid rock is considered. FORMAT (I10).

The last card contains the number of frequencies desired in the analysis, the first frequency at which the analysis starts, and the increment of the frequency. FORMAT (I10, 2F10.0).

b) Listing of Program

```
      IMPLICIT REAL*8(A-H,O-Z)
       REAL*4 ERHA,ERVA,RRHA,RRVA,FREQ
       REAL*4 NAME,TEX1,TEX2,TEX3,TEX4,TEX5,TEX6
       DIMENSION FREQ(100)
      DIMENSION H(20), VS(20), ANU(20), BETA(20), GAM(20)
      DIMENSION DAUX(2), CG(20), CL(20), CVP(20), CVS(20)
      DIMENSION AL(20), ALP(20), AN(20), ANP(20)
      DIMENSION EE(4), FMAT(4,4,20), GMAT(4,4)
      DIMENSION TCP(4,4), BOT(4,4)
      DIMENSION TEX1(4),TEX2(4),TEX3(9),TEX4(8),TEX5(8),TEX6(8),NAME(2)
      DIMENSION P(520),T(520),U1(520,5),V1(520,5),U2(520,5),V2(520,5)
      DIMENSION CP(260),CT(260),CU1(260,5),CV1(260,5),CU2(260,5)
       DIMENSION CV2(260,5)
       DIMENSION UR(520), CUR(260), VR(520),CVR(260)
      DIMENSION ERHA(100),ERVA(100),RRHA(100),RRVA(100)
      COMPLEX*16 AUX,CG,CL,CVP,CVS,AL,AN,ALP,ANP
      COMPLEX*16 EE,AIM,GMAT,DET,UT,VT,UA,UB,US,VSI
      COMPLEX*16 TCP,BOT,FMAT
      COMPLEX*16 APP,ASVP
      COMPLEX*16 CP,CT,CU1,CV1,CU2,CV2
      COMPLEX*16 CUR,CVR
      EQUIVALENCE (AUX, DAUX(1))
      EQUIVALENCE (CP(1),P(1)), (CT(1),T(1)),(CU1(1,1),U1(1,1))
      EQUIVALENCE (CV1(1,1),V1(1,1)), (CU2(1,1),U2(1,1)), (CV2(1,1),V2(1
     *,1))
      EQUIVALENCE (CUR(1),UR(1)), (CVR(1),VR(1))
      EQUIVALENCE (P(1),T(1))
      DATA NAME/'EVAN','MICH'/
      DATA TEX1/'FREQ','UENC','Y   ',' CPS'/
      DATA TEX2/'AMPL','IFIC','ATIO','N   '/
      DATA TEX3/'ELAS','TIC ','ROCK','    ', ' HOR','IZON','TAL ','MOTI'
     1,'ON  '/
      DATA TEX4/'RIGI','D RO','CK  ','   H','ORIZ','ONTA','L MO','TION'/
      DATA TEX5/'ELAS','TIC ','ROCK','  - ',' VER','TICA','L MO','TION'/
      DATA TEX6/'RIGI','D RO','CK  ','-  V','ERTI','CAL ','MOTI','ON  '/
C
```

```
C ********************************************************************
C       THIS PROGRAM IS FOR THE CASE OF STRESS WAVES GENERATED AT THE
C       SURFACE AND PROPAGATING THROUGH THE MEDIA.  IT CALCULATES THE
C       DISPLACEMENTS AS A FUNCTION OF FREQUENCY AND SPATIAL COORDINATES
C       X AND Z. IT ALSO CALCULATES AND PLOTS(WITH A  STROMBERG CARLSON
C       PLOTTER) THE AMPLIFICATION RATIOS AS A FUNCTION OF FREQUENCY.
C ********************************************************************
C
C..... SET VARIABLE AIM= SQRT(-1)=I
      DAUX(1)= 0.
      DAUX(2)= 1.
      AIM= AUX
C
C ********************************************************************
C......READ NUMBER OF LAYERS OF SOIL TO BE STUDIED
C ********************************************************************
C
      READ(5,100) NLAY
100   FORMAT(I10)
      NLAY1=NLAY+1
C...............................................................
      DO 10 I=1,NLAY1
C
C ********************************************************************
C.....READ HEIGHT, SHEAR WAVE VELOCITY, POISSON%S RATIO, DAMPING
C......AND UNIT WEIGHT FOR EACH LAYER OF SOIL PLUS LAST DATA CARD FOR
C.....ELASTIC ROCK
C ********************************************************************
C
      READ(5,101) H(I), VS(I), ANU(I), BETA(I), GAM(I)
101   FORMAT(5F10.0)
10    CONTINUE
C...............................................................
C ********************************************************************
C.....CALCULATE LAME%S CONSTANTS AND WAVE-VELOCITIES IN COMPLEX FORM
C ********************************************************************
```

```
C.............................................................
      DO 11 I=1,NLAY1
      RO= GAM(I)/32.2
      G= RO*VS(I)*VS(I)
      E= 2.*G*ANU(I)/(1.-2.*ANU(I))
      DAUX(1)= G
      DAUX(2)= 2.*BETA(I)*G
      CG(I)  =AUX
      DAUX(1)= E
      DAUX(2)= 2.*BETA(I)*E
      CL(I)= AUX
      CVP(I)=  (CL(I)+2.*CG(I))/RO
      CVS(I)= CG(I)/RO
      CVP(I)=CDSQRT(CVP(I))
11    CVS(I)=CDSQRT(CVS(I))
C.............................................................
      K=0
C ***************************************************************************
C.....  P=  VERTICAL STRESS, T= SHEAR STRESS
C......  INITIALIZE STRESSES TO ZERO AT EVERY POINT ON THE X-AXIS
C ***************************************************************************
C.............................................................
      DO 600 I=1,520
      P(I)=0.
C.............................................................
      DO 600 J=1,NLAY1
      U1(I,J)=0.
      U2(I,J)=0.
      V1(I,J)=0.
600   V2(I,J)=0.
C.............................................................
       NPP=256
      NPS=256
      NPT=256
      NCPS=2*NPS
      NCPT=2*NPT
```

```
      NPR=NPP/2
C
C ****************************************************************************
C.....   SET VERTICAL AND SHEAR STRESS TO UNIT AT MIDPOINT OF TH(
C......  X-AXIS(2*NPR-1).
C.....   (2*NPR-1) IS THE MIDPOINT BECAUSE  THE VARIABLE P & T ARE DEFINED AS
C.......  REAL  BUT THE DISPLACEMENTS ARE DEFINED AS COMPLEX. A COMPLEX VARIABLE
C.....   REQUIRES TWO  STORAGE LOCATION INSTEAD OF ONE AS IS THE CASE FOR REAL.
C ****************************************************************************
      P(1)=-1.
      CALL FOUR2 (P,NPP,1,-1,1)
      ANPP=NPP
C......  DSI=1/( TOTSL NO. OF POINTS)
      DSI=1./ANPP
C
C ****************************************************************************
C....... READ IROCK
C......  IROCK=0 REPRESENTS RIGID ROCK CASE
C......  IROCK=1  REPRESENTS ELASTIC ROCK CASE
C.....   READ NO. OF FREQUENCIES, INITIAL FREQ. AND INCREMETAL  FREQ.
C ****************************************************************************
C
      READ(5,100) IROCK
      IF(IROCK) 802,802,803
802   WRITE(6,810)
810   FORMAT(1H1,50X,'RIGID ROCK CASE')
      GO TO 815
803   WRITE(6,811)
811   FORMAT(1H1,50X,'ELASTIC ROCK CASE')
815   WRITE(6,816)
816   FORMAT(50X,'*****************',//)
      READ(5,102) NF,F1,DF
102   FORMAT(I10,2F10.0)
      WRITE(6,200) NLAY
200   FORMAT(50X,I5,2X,'LAYERS',//)
      WRITE(6,201)
```

```
201   FORMAT(2X,'LAYER',3X,'THICKNESS',1X,'SHEAR VEL',1X,'POISS RAT',3X,
     1'DAMPING',2X,'UNIT WEIGHT',/)
      WRITE(6,202) (I,H(I),VS(I),ANU(I),BETA(I),GAM(I),I=1,NLAY)
202   FORMAT(I5,5X,5F10.2)
      WRITE(6,203)  VS(NLAY1),ANU(NLAY1),BETA(NLAY1),GAM(NLAY1)
203   FORMAT(2X,'ROCK',14X,4F10.2,//)
C..............................................................
      DO 1000 JF=1, NF
      AI=JF-1
      FR= F1 + AI*DF
      FREQ(JF)=FR
      OM= 6.28318*FR
      ALIM=VS(1)/FR
C ***************************************************************************
C.....   SELECT INCREMENTAL LENGTH(DX)= VS/( 10*FR)
C......  TOTX= TOTAL LENGTH OF X-AXIS
C.....   DXI=1./( TOTAL LENGTH OF X-AXIS  )
C ***************************************************************************
      DX=ALIM/10.
      TOTX=ANPP*DX
      DXI=1./TOTX
C ***************************************************************************
C......  THIS DO LOOP CALCULATES THE MOTION AT EACH INTERFACE FOR A SPECIFIC
C......  POINT ON THE XI-AXIS (POINT  CALLED JOSE). IT STARTS FROM POINT 1
C......   AND ENDS AT MIDPOINT( NPR).
C ***************************************************************************
C..............................................................
      DO 700 JOSE=1,NPR
      AJ=JOSE-1
      MARY=1
      LUIS=JOSE
C ***************************************************************************
C......   XI= TRANSFORMED COORDINATE FROM FOURIER TRANSFORM
C.....   NOTE THE INCREMENT IN THIS NEW TRANSFORMED XI-AXIS IS DXI=N2*PI/(NPP*DX
C ***************************************************************************
      XI=AJ*DXI
```

```
710   CONTINUE
C..............................................................
      DO 30 L=1,4
      DO 30 M=1,4
30    FMAT(L,M,1)=0.
      DO 31 L=1,4
31    FMAT(L,L,1)=1.
      XI=6.2831853*XI
C ***************************************************************************************
C.....   THIS DO LOOP CALCULATES THE PARAMETERS L,LP,N,NP  FOR EACH LAYER.
C ***************************************************************************************
      DO 701 I=1,NLAY1
      AL(I)=-XI*CVP(I)/OM
      ALP(I)=-XI*CVS(I)/OM
      AN(I)=1.-AL(I)*AL(I)
      AN(I)=CDSQRT(AN(I))
      ANP(I)=1.-ALP(I)*ALP(I)
       ANP(I)=CDSQRT(ANP(I))
      AUX=OM*AN(I)/CVP(I)
      AA=DABS(DAUX(2))
      AA=DAUX(2)/AA
      AN(I)=-AN(I)*AA
      AUX=OM*ANP(I)/CVS(I)
      AA=DABS(DAUX(2))
      AA=DAUX(2)/AA
701   ANP(I)=-ANP(I)*AA
C..............................................................
      DO 1001 I=1,NLAY1
C ***************************************************************************************
C..... DEFINE T-MATRIX
C ***************************************************************************************
      TOP(1,1)= AL(I)
      TOP(1,2)= -ANP(I)
      TOP(1,3)= AL(I)
      TOP(1,4)= ANP(I)
      TOP(2,1)= -AN(I)
```

```
      TOP(2,2)= -ALP(I)
      TOP(2,3)= AN(I)
      TOP(2,4)= -ALP(I)
      AUX= AIM*OM/CVP(I)
      TOP(3,1)= -AUX*(CL(I)+2.*CG(I)*AN(I)*AN(I))
      TOP(3,2)= -AUX*2.*CG(I)*AL(I)*ANP(I)
      TOP(3,3)= TOP(3,1)
      TOP(3,4)= -TOP(3,2)
      AUX= AIM*OM/CVS(I)
      TOP(4,1)= AUX*2.*CG(I)*ALP(I)*AN(I)
      TOP(4,2)= AUX*CG(I)*(ALP(I)*ALP(I)-ANP(I)*ANP(I))
      TOP(4,3)= -TOP(4,1)
      TOP(4,4)= TOP(4,2)
C ***********************************************************************
C..... DEFINE THE INVERSE OF I-MATRIX
C ***********************************************************************
      AUX= (CL(I)+2.*CG(I))*2.
      BOT(1,1)= 2.*CG(I)*AL(I)/AUX
      BOT(2,1)= -(CL(I)+2.*CG(I)*AN(I)*AN(I))/(AUX*ANP(I))
      BOT(3,1)= BOT(1,1)
      BOT(4,1)= -BOT(2,1)
      BOT(1,2)= (ALP(I)*ALP(I)-ANP(I)*ANP(I))/(2.*AN(I))
      BOT(2,2)= -ALP(I)
      BOT(3,2)= -BOT(1,2)
      BOT(4,2)= -ALP(I)
      BOT(1,3)= AIM*CVP(I)/(OM*AUX)
      BOT(2,3)= AIM*AL(I)*CVP(I)/(OM*ANP(I)*AUX)
      BOT(3,3)= BOT(1,3)
      BOT(4,3)= -BOT(2,3)
      AUX= 2.*AIM*OM*CG(I)/CVS(I)
      BOT(1,4)= ALP(I)/(AUX*AN(I))
      BOT(2,4)= -1./AUX
      BOT(3,4)= -BOT(1,4)
      BOT(4,4)= BOT(2,4)
       IF(I-NLAY) 225,225,1001
225   CONTINUE
```

```
C *********************************************************************************
C..... DEFINE THE ' H-MATRIXH CR EE= DIAGONAL OF H-MATRIX
C..... MULTIPLY T-MATRIX BY H-MATRIX
C *********************************************************************************
        AUX= -OM*AN(I)*H(I)/CVP(I)
       AUX= AUX*AIM
       EE(3)=CDEXP(AUX)
       EE(1)=  1./EE(3)
       AUX= -OM*ANP(I)*H(I)/CVS(I)
       AUX= AUX*AIM
       EE(4)=CDEXP(AUX)
       EE(2)= 1./EE(4)
25     DO  20 L=1,4
       DO 20 M=1,4
20     TOP(L,M)= TCP(L,M)*EE(M)
       I1= I+1
C *********************************************************************************
C......CALCULATE THE F-MATRIX FCR EACH LAYER CF SOIL
C *********************************************************************************
       CALL MATMUL (TOP,BOT,BOT)
       CALL MATMUL(BOT, FMAT(1,1,I), FMAT(1,1,I1))
1001   CONTINUE
C...................................................................
       CALL MATMUL(BOT,FMAT(1,1,NLAY1),GMAT)
       IF(IROCK) 702,702,703
C *********************************************************************************
C........ THIS PART CALCULATES THE MOTION FOR THE RIGID-ROCK CASE
C *********************************************************************************
702    DET=FMAT(1,1,NLAY1)*FMAT(2,2,NLAY1)-FMAT(1,2,NLAY1)*FMAT(2,1,NLAY1
      *)
       UT=FMAT(1,3,NLAY1)*CP(LUIS)
       VT=FMAT(2,3,NLAY1)*CP(LUIS)
       UA=-UT*FMAT(2,2,NLAY1)+VT*FMAT(1,2,NLAY1)
       UB=UT*FMAT(2,1,NLAY1)-VT*FMAT(1,1,NLAY1)
       CU1(LUIS,1)=UA/DET
       CV1(LUIS,1)=UB/DET
```

```
      UT=FMAT(1,4,NLAY1)*CT(LUIS)
      VT=FMAT(2,4,NLAY1)*CT(LUIS)
      UA=-UT*FMAT(2,2,NLAY1)+VT*FMAT(1,2,NLAY1)
      UB=UT*FMAT(2,1,NLAY1)-VT*FMAT(1,1,NLAY1)
      CU2(LUIS,1)=UA/DET
      CV2(LUIS,1)=UB/DET
      GO TO 704
C ******************************************************************************
C .....  THIS PART CALCULATES THE MOTION FOR THE ELASTIC CASE.
C ******************************************************************************
703   DET=GMAT(1,1)*GMAT(2,2)-GMAT(1,2)*GMAT(2,1)
      UT=GMAT(1,3)*CP(LUIS)
      VT=GMAT(2,3)*CP(LUIS)
      UA=-UT*GMAT(2,2)+VT*GMAT(1,2)
      UB=UT*GMAT(2,1)-VT*GMAT(1,1)
C ******************************************************************************
C.....    HERE CU1 & CV1 CONTAIN THE HOR. & VERT. MOTION AT THE SURFACE
C.....     FOR THE POINT (JOSE=LUIS) ON THE XI-AXIS.
C ******************************************************************************
      CU1(LUIS,1)=UA/DET
      CV1(LUIS,1)=UB/DET
      UT=GMAT(1,4)*CT(LUIS)
      VT=GMAT(2,4)*CT(LUIS)
      UA=-UT*GMAT(2,2)+VT*GMAT(1,2)
      UB=UT*GMAT(2,1)-VT*GMAT(1,1)
      CU2(LUIS,1)=UA/DET
      CV2(LUIS,1)=UB/DET
704   IF(NLAY-1) 707,706,706
C ***************************************************************************
C.....    KNOWING THE MOTION AT THE SURFACE THE MOTION AT THE INTERFACES IS
C......     OBTAINED
C ******************************************************************************
706   DO 705 I=2,NLAY1
      CU1(LUIS,I)=FMAT(1,1,I)*CU1(LUIS,1)+FMAT(1,2,I)*CV1(LUIS,1)+FMAT(1
     *,3,I)*CP(LUIS)
      CV1(LUIS,I)=FMAT(2,1,I)*CU1(LUIS,1)+FMAT(2,2,I)*CV1(LUIS,1)+FMAT(2
```

```
     *,3,I)*CP(LUIS)
      CU2(LUIS ,I)=FMAT(1,1,I)*CU2(LUIS,1)+FMAT(1,2,I)*CV2(LUIS,1)+FMAT(
     *1,4,I)*CT(LUIS)
705   CV2(LUIS,I)=FMAT(2,1,I)*CU2(LUIS,1)+FMAT(2,2,I)*CV2(LUIS,1)+FMAT(2
     *,4,I)*CT(LUIS)
C ***********************************************************************
C......  THIS PART CALCULATES THE MOTION FOR THE ELASTIC HALF-SPACE.
C.....   CVR= VERTICAL MOTION AT POINT LUIS ON THE XI-AXIS DUE TO A UNIT VERT.
C.....   STRESS
C.....   CUR= HORIZONTAL MOTION AT EACH POINT ON THE XI-AXIS DUE TO A SHEAR STR.
C ***********************************************************************
707   DET=BOT(1,1)*BOT(2,2)-BOT(1,2)*BOT(2,1)
      UT=BOT(1,3)*CP(LUIS)
      VT=BOT(2,3)*CP(LUIS)
      UB=UT*BOT(2,1)-VT*BOT(1,1)
      CVR(LUIS)=UB/DET
      UT=BOT(1,4)*CT(LUIS)
      VT=BOT(2,4)*CT(LUIS)
      UA=-UT*BOT(2,2)+VT*BOT(1,2)
       CUR(LUIS)=UA/DET
C ***********************************************************************
C.....   THIS PART USES THE SYMMETRY ABOUT THE MIDPOINT TO CALCULATE THE MOTION
C.....   AT THE OTHER END OF THE XI-AXIS STARTING FROM THE LAST POINT(NPP)
C ***********************************************************************
      GO TO (708,700),MARY
708    MARY=2
      AJ=JOSE
      XI=-AJ*DXI
      LUIS=NPP+1-JOSE
      GO TO 710
700   CONTINUE
C ***********************************************************************
C.....    AT THIS POINT THE MOTION IS TRANSFORMED FROM THE XI-AXIS(FOURIER-
C......  TRANSF.) BACK INTO THE ORIGINAL X-AXIS BY MEANS OF FOURIER TRASFOMS.
C ***********************************************************************
      DO 711 I=1,NLAY1
```

```
       CALL FOUR2 (U1(1,I),NPP,1,1,1)
       CALL FOUR2 (V1(1,I),NPP,1,1,1)
       CALL FOUR2 (U2(1,I),NPP,1,1,1)
711    CALL FOUR2 (V2(1,I),NPP,1,1,1)
      DO 712 I=1,NLAY1
      DO 712 J=1,NPP
       U1(J,I)=U1(J,I)*DSI
      U2(J,I)=U2(J,I)*DSI
      V1(J,I)=V1(J,I)*DSI
712   V2(J,I)=V2(J,I)*DSI
C ***************************************************************************
C......  CU1(J,2) CONTAINS  U-DISPLACEMENT OF POINT J AT LAYER INTERFACE I
C......  DUE TO A UNIT VERTICAL STRESS
C....... SIMILARLY CV1 CONTAINS W(VERTICAL) DISPLACEMENT
C.....   CU2 & CV2 ARE THE DISPLACEMENTS DUE TO A UNIT SHEAR STRESS.
C ***************************************************************************
      CALL FOUR2 (UR,NPP,1,1,1)
      CALL FOUR2 (VR,NPP,1,1,1)
      CUR(1)=CUR(1)*DSI
      CVR(1)=CVR(1)*DSI
      WRITE(6,210) FR,OM
210   FORMAT(1H1,20X,'FREQUENCY',F6.2,' CPS ',F6.2,' RAD/SEC',//)
      WRITE(6,880) DX
880   FORMAT(/,1X,'  DX FOR THIS FREQUENCY IS  ',F10.3)
      DO 920 I=1,NLAY1
      WRITE(6,910) I
910   FORMAT(1H1,20X,'INTERFACE NO   ',I2,/20X'**************',///)
      WRITE(6,911)
911   FORMAT(20X,'UNIT VERTICAL STRESS',/20X'********************',//)
      WRITE(6,912)
912   FORMAT(' POINT NO  ',' X-COORD. ',20X,'HORIZONTAL MOTION',42X,'VER
     1TICAL MOTION')
      DO 901 J=1,NPR
      AJ=J-1
      X=AJ*DX
      AUX=CU1(J,I)
```

```
      ACU1= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
      ACU1=DSQRT(ACU1)
      AUX=CV1(J,I)
      ACV1= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
      ACV1=DSQRT(ACV1)
901   WRITE(6,913) J,X,CU1(J,I),ACU1,CV1(J,I),ACV1
913   FORMAT(2X,I3,5X,F10.3,5X,3(5X,E10.3),10X,3(5X,E10.3))
      WRITE(6,914)
914   FORMAT(///,20X,'UNIT SHEAR STRESS',/20X'*****************',//)
      WRITE(6,912)
      DO 902 J=1,NPR
      AJ=J-1
      X=AJ*DX
      AUX=CU2(J,I)
      ACU2= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
      ACU2=DSQRT(ACU2)
      AUX=CV2(J,I)
      ACV2= DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
      ACV2=DSQRT(ACV2)
902   WRITE(6,913) J,X,CU2(J,I),ACU2,CV2(J,I),ACV2
920   CONTINUE
      N=0
717    N=N+1
      GO TO (713,714,715,716),N
713    AUX=CU2(1,1)/CUR(1)
718   AMPL=DAUX(1)*DAUX(1)+DAUX(2)*DAUX(2)
      AMPL=DSQRT(AMPL)
      GO TO (723,724,725,726),N
723     ERHA(JF)=AMPL
       IF(JF.EQ.1) SERHA=ERHA(1)
       ERHA(JF)=ERHA(JF)/SERHA
      GO TO 717
714    AUX=CV1(1,1)/CVR(1)
      GO TO 718
724     ERVA(JF)=AMPL
       IF(JF.EQ.1) SERVA=ERVA(1)
```

```
      ERVA(JF)=ERVA(JF)/SERVA
     GO TO 717
715  AUX=CU2(1,1)/CU2(1,NLAY1)
     GO TO 718
725    RRHA(JF)=AMPL
      IF(JF.EQ.1) SRRHA=RRHA(1)
       RRHA(JF)=RRHA(JF)/SRRHA
     GO TO 717
716  AUX=CV1(1,1)/CV1(1,NLAY1)
     GO TO 718
726    RRVA(JF)=AMPL
      IF(JF.EQ.1) SRRVA=RRVA(1)
       RRVA(JF)=RRVA(JF)/SRRVA
1000 CONTINUE
     CALL STOIDV (NAME,7,3)
     CALL PLOT(ERHA,FREQ,TEX1,TEX2,TEX3,225,36,NF,K)
     CALL PLOT(RRHA,FREQ,TEX1,TEX2,TEX4,260,32,NF,K)
     CALL PLOT(ERVA,FREQ,TEX1,TEX2,TEX5,260,32,NF,K)
     CALL PLOT(RRVA,FREQ,TEX1,TEX2,TEX6,260,32,NF,K)
     CALL PLTND(N)
     CALL EXIT
     END
```

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4), B(4,4), C(4,4), D(4)
      COMPLEX*16  A,B,C,D,SUM
C
C..... THIS SUBROUTINE MULTIPLIES TWO MATRICES.  C = A . B
      DO 10 I=1,4
      DO 11 J=1,4
      SUM= 0.
      DO  12 K=1,4
12    SUM = SUM+A(J,K)*B(K,I)
11    D(J)= SUM
      DO 10 J=1,4
10    C(J,I)= D(J)
      RETURN
      END
```

```
      SUBROUTINE PLOT(A,B,TIT1,TIT2,TIT3,N1,N2,N3,K)
      DIMENSION A(100),B(100),TIT1(10),TIT2(10),TIT3(10)
       CALL SETMIV(150,73,250,223)
       AMAX=0.
      J=1
      DO 10 I=1,N3
      IF(AMAX-A(I)) 11,10,10
11    AMAX=A(I)
10    CONTINUE
C.......  ADVANCE FRAME (L=4), EXCEPT FIRST FRAME(L=2)
      K=K+1
      IF(K-1) 20,20,21
20    L=2
      GO TO 23
21    L=4
23    CONTINUE
C......  SELECT INCREMENT DY AND MAXIMUM VALUES ON HOR. AND VERT. LINES (AMAX,
C....... BMAX)
C......  DY IS SELECTED SO THAT WE HAVE ABOUT  6 (NSQ) VERTICAL LINES. IF A
C......  DIFFERENT VALUE OF NSQ IS DESIRED  JUST CHANGE NSQ CARD
       NSQ=6
       ANSQ=NSQ
       IMAX=AMAX/ANSQ
       IDY=IMAX+1
       BMAX=NSQ*IDY
       IDIFF=BMAX-AMAX
       IF(IDIFF-IDY) 4,4,5
5      NSQ=NSQ-1
4      CONTINUE
       AMAX=NSQ*IDY
        DY=IDY
      BMAX=B(N3)+1.
      IMAX=BMAX
      BMAX=IMAX
      CALL GRID1V(L,0.0,BMAX,0.0,AMAX,1.0,DY,0,0,1,J,2,2)
      CALL RITE2V(410,200,1023,0,3,16,1,TIT1,IDUM)
```

```
CALL RITE2V(100,375,1023,90,3,16,1,TIT2,IDUM)
CALL RITE2V(N1,850,1023,0,3,N2,1,TIT3,IDUM)
 CALL GRAF1V(B,A,IERR,N3,1)
RETURN
END
```

```
      SUBROUTINE FOUR2 (DATA,N,NDIM,ISIGN,IFORM)                  FF2   1
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DATA(1), N(1)                                     FF2  29
      NTOT=1                                                      FF2  30
      DO 10 IDIM=1,NDIM                                           FF2  31
10    NTOT=NTOT*N(IDIM)                                           FF2  32
      IF (IFORM) 70,20,20                                         FF2  33
20    NREM=NTOT                                                   FF2  34
      DO 60 IDIM=1,NDIM                                           FF2  35
      NREM=NREM/N(IDIM)                                           FF2  36
      NPREV=NTOT/(N(IDIM)*NREM)                                   FF2  37
      NCURR=N(IDIM)                                               FF2  38
      IF (IDIM-1+IFORM) 30,30,40                                  FF2  39
30    NCURR=NCURR/2                                               FF2  40
40    CALL BITRV (DATA,NPREV,NCURR,NREM)                          FF2  41
      CALL COOL2 (DATA,NPREV,NCURR,NREM,ISIGN)                    FF2  42
      IF (IDIM-1+IFORM) 50,50,60                                  FF2  43
50    CALL FIXRL (DATA,N(1),NREM,ISIGN,IFORM)                     FF2  44
      NTOT=(NTOT/N(1))*(N(1)/2+1)                                 FF2  45
60    CONTINUE                                                    FF2  46
      RETURN                                                      FF2  47
70    NTOT=(NTOT/N(1))*(N(1)/2+1)                                 FF2  48
      NREM=1                                                      FF2  49
      DO 100 JDIM=1,NDIM                                          FF2  50
      IDIM=NDIM+1-JDIM                                            FF2  51
      NCURR=N(IDIM)                                               FF2  52
      IF (IDIM-1) 80,80,90                                        FF2  53
80    NCURR=NCURR/2                                               FF2  54
      CALL FIXRL (DATA,N(1),NREM,ISIGN,IFORM)                     FF2  55
      NTOT=NTOT/(N(1)/2+1)*N(1)                                   FF2  56
90    NPREV=NTOT/(N(IDIM)*NREM)                                   FF2  57
      CALL BITRV (DATA,NPREV,NCURR,NREM)                          FF2  58
      CALL COOL2 (DATA,NPREV,NCURR,NREM,ISIGN)                    FF2  59
100   NREM=NREM*N(IDIM)                                           FF2  60
      RETURN                                                      FF2  61
      END                                                         FF2  62-
```

```
      SUBROUTINE BITRV (DATA,NPREV,N,NREM)                              BIT   1
      IMPLICIT REAL*8(A-H,O-Z)
C     SHUFFLE THE DATA BY 'BIT REVERSAL'.                               BIT   2
C     DIMENSION DATA(NPREV,N,NREM)                                      BIT   3
C     DATA(I1,I2REV,I3) = DATA(I1,I2,I3), FOR ALL I1 FROM 1 TO NPREV,   BIT   4
C     ALL I2 FROM 1 TO N (WHICH MUST BE A POWER OF TWO), AND ALL I3     BIT   5
C     FROM 1 TO NREM, WHERE I2REV-1 IS THE BITWISE REVERSAL OF I2-1.    BIT   6
C     FOR EXAMPLE, N = 32, I2-1 = 10011 AND I2REV-1 = 11001.            BIT   7
      DIMENSION DATA(1)                                                 BIT   8
      IP0=2                                                             BIT   9
      IP1=IP0*NPREV                                                     BIT  10
      IP4=IP1*N                                                         BIT  11
      IP5=IP4*NREM                                                      BIT  12
      I4REV=1                                                           BIT  13
      DO 60 I4=1,IP4,IP1                                                BIT  14
      IF (I4-I4REV) 10,30,30                                            BIT  15
   10 I1MAX=I4+IP1-IP0                                                  BIT  16
      DO 20 I1=I4,I1MAX,IP0                                             BIT  17
      DO 20 I5=I1,IP5,IP4                                               BIT  18
      I5REV=I4REV+I5-I4                                                 BIT  19
      TEMPR=DATA(I5)                                                    BIT  20
      TEMPI=DATA(I5+1)                                                  BIT  21
      DATA(I5)=DATA(I5REV)                                              BIT  22
      DATA(I5+1)=DATA(I5REV+1)                                          BIT  23
      DATA(I5REV)=TEMPR                                                 BIT  24
   20 DATA(I5REV+1)=TEMPI                                               BIT  25
   30 IP2=IP4/2                                                         BIT  26
   40 IF (I4REV-IP2) 60,60,50                                           BIT  27
   50 I4REV=I4REV-IP2                                                   BIT  28
      IP2=IP2/2                                                         BIT  29
      IF (IP2-IP1) 60,40,40                                             BIT  30
   60 I4REV=I4REV+IP2                                                   BIT  31
      RETURN                                                            BIT  32
      END
```

```
      SUBROUTINE COOL2 (DATA,NPREV,N,NREM,ISIGN)
      IMPLICIT REAL*8(A-H,O-Z)
C     FOURIER TRANSFORM OF LENGTH N BY THE COOLEY-TUKEY ALGORITHM.
C     BIT-REVERSED TO NORMAL ORDER.
C     DIMENSION DATA(NPREV,N,NREM)
C     COMPLEX DATA
C     DATA(I1,J2,I3) = SUM(DATA(I1,I2,I3)*EXP(ISIGN*2*PI*I*((I2-1)*
C     (J2-1)/N))), SUMMED OVER I2 = 1 TO N FOR ALL I1 FROM 1 TO NPREV,
C     J2 FROM 1 TO N AND I3 FROM 1 TO NREM.  N MUST BE A POWER OF TWO.
C     FACTORING N BY FOURS SAVES ABOUT TWENTY FIVE PERCENT OVER FACTOR-
C     ING BY TWOS.
C     NOTE--IT IS NOT NECESSARY TO REWRITE THIS SUBROUTINE INTO COMPLEX
C     NOTATION SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
C     IMAGINARY PARTS IN ADJACENT STORAGE LOCATIONS.  IT MUST ALSO
C     STORE ARRAYS WITH THE FIRST SUBSCRIPT INCREASING FASTEST.
      DIMENSION DATA(1)
      TWOPI=6.2831853072*DFLOAT(ISIGN)
      IP0=2
      IP1=IP0*NPREV
      IP4=IP1*N
      IP5=IP4*NREM
      IP2=IP1
      NPART=N
 10   IF (NPART-2) 50,30,20
 20   NPART=NPART/4
      GO TO 10
C     DO A FOURIER TRANSFORM OF LENGTH TWO
 30   IP3=IP2*2
      DO 40 I1=1,IP1,IP0
      DO 40 I5=I1,IP5,IP3
      J0=I5
      J1=J0+IP2
      TEMPR=DATA(J1)
      TEMPI=DATA(J1+1)
      DATA(J1)=DATA(J0)-TEMPR
      DATA(J1+1)=DATA(J0+1)-TEMPI
```

```
      DATA(J0)=DATA(J0)+TEMPR
 40   DATA(J0+1)=DATA(J0+1)+TEMPI
      GO TO 140
C     DO A FOURIER TRANSFORM OF LENGTH FOUR (FROM BIT REVERSED ORDER)
 50   IP3=IP2*4
      THETA=TWOPI/DFLOAT(IP3/IP1)
      SINTH=DSIN(THETA/2)
      WSTPR=-2.*SINTH*SINTH
C     COS(THETA)-1, FOR ACCURACY.
      WSTPI=DSIN(THETA)
      WR=1.
      WI=0.
      DO 130 I2=1,IP2,IP1
      IF (I2-1) 70,70,60
 60   W2R=WR*WR-WI*WI
      W2I=2.*WR*WI
      W3R=W2R*WR-W2I*WI
      W3I=W2R*WI+W2I*WR
 70   I1MAX=I2+IP1-IP0
      DO 120 I1=I2,I1MAX,IP0
      DO 120 I5=I1,IP5,IP3
      J0=I5
      J1=J0+IP2
      J2=J1+IP2
      J3=J2+IP2
      IF (I2-1) 90,90,80
C     APPLY THE PHASE SHIFT FACTORS
 80   TEMPR=DATA(J1)
      DATA(J1)=W2R*TEMPR-W2I*DATA(J1+1)
      DATA(J1+1)=W2R*DATA(J1+1)+W2I*TEMPR
      TEMPR=DATA(J2)
      DATA(J2)=WR*TEMPR-WI*DATA(J2+1)
      DATA(J2+1)=WR*DATA(J2+1)+WI*TEMPR
      TEMPR=DATA(J3)
      DATA(J3)=W3R*TEMPR-W3I*DATA(J3+1)
      DATA(J3+1)=W3R*DATA(J3+1)+W3I*TEMPR
```

```
90    T0R=DATA(J0)+DATA(J1)
      T0I=DATA(J0+1)+DATA(J1+1)
      T1R=DATA(J0)-DATA(J1)
      T1I=DATA(J0+1)-DATA(J1+1)
      T2R=DATA(J2)+DATA(J3)
      T2I=DATA(J2+1)+DATA(J3+1)
      T3R=DATA(J2)-DATA(J3)
      T3I=DATA(J2+1)-DATA(J3+1)
      DATA(J0)=T0R+T2R
      DATA(J0+1)=T0I+T2I
      DATA(J2)=T0R-T2R
      DATA(J2+1)=T0I-T2I
      IF (ISIGN) 100,100,110
100   T3R=-T3R
      T3I=-T3I
110   DATA(J1)=T1R-T3I
      DATA(J1+1)=T1I+T3R
      DATA(J3)=T1R+T3I
120   DATA(J3+1)=T1I-T3R
      TEMPR=WR
      WR=WSTPR*TEMPR-WSTPI*WI+TEMPR
130   WI=WSTPR*WI+WSTPI*TEMPR+WI
140   IP2=IP3
      IF (IP3-IP4) 50,150,150
150   RETURN
      END
```

```
      SUBROUTINE FIXRL (DATA,N,NREM,ISIGN,IFORM)
      IMPLICIT REAL*8(A-H,O-Z)
C     FOR IFORM = 0, CONVERT THE TRANSFORM OF A DOUBLED-UP REAL ARRAY,
C     CONSIDERED COMPLEX, INTO ITS TRUE TRANSFORM.  SUPPLY ONLY THE
C     FIRST HALF OF THE COMPLEX TRANSFORM, AS THE SECOND HALF HAS
C     CONJUGATE SYMMETRY.  FOR IFORM = -1, CONVERT THE FIRST HALF
C     OF THE TRUE TRANSFORM INTO THE TRANSFORM OF A DOUBLED-UP REAL
C     ARRAY.  N MUST BE EVEN.
C     USING COMPLEX NOTATION AND SUBSCRIPTS STARTING AT ZERO, THE
C     TRANSFORMATION IS--
C     DIMENSION DATA(N,NREM)
C     ZSTP = EXP(ISIGN*2*PI*I/N)
C     DO 10 I2=0,NREM-1
C     DATA(0,I2) = CONJ(DATA(0,I2))*(1+I)
C     DO 10 I1=1,N/4
C     Z = (1+(2*IFORM+1)*I*ZSTP**I1)/2
C     I1CNJ = N/2-I1
C     DIF = DATA(I1,I2)-CONJ(DATA(I1CNJ,I2))
C     TEMP = Z*DIF
C     DATA(I1,I2) = (DATA(I1,I2)-TEMP)*(1-IFORM)
C 10  DATA(I1CNJ,I2) = (DATA(I1CNJ,I2)+CONJ(TEMP))*(1-IFORM)
C     IF I1=I1CNJ, THE CALCULATION FOR THAT VALUE COLLAPSES INTO
C     A SIMPLE CONJUGATION OF DATA(I1,I2).
      DIMENSION DATA(1)
      TWOPI=6.2831853072*DFLOAT(ISIGN)
      IP0=2
      IP1=IP0*(N/2)
      IP2=IP1*NREM
      IF (IFORM) 10,70,70
C     PACK THE REAL INPUT VALUES (TWO PER COLUMN)
 10   J1=IP1+1
      DATA(2)=DATA(J1)
      IF (NREM-1) 70,70,20
 20   J1=J1+IP0
      I2MIN=IP1+1
      DO 60 I2=I2MIN,IP2,IP1
```

```
      DATA(I2)=DATA(J1)                                        FIX  36
      J1=J1+IP0                                                FIX  37
      IF (N-2) 50,50,30                                        FIX  38
30    I1MIN=I2+IP0                                             FIX  39
      I1MAX=I2+IP1-IP0                                         FIX  40
      DO 40 I1=I1MIN,I1MAX,IP0                                 FIX  41
      DATA(I1)=DATA(J1)                                        FIX  42
      DATA(I1+1)=DATA(J1+1)                                    FIX  43
40    J1=J1+IP0                                                FIX  44
50    DATA(I2+1)=DATA(J1)                                      FIX  45
60    J1=J1+IP0                                                FIX  46
70    DO 80 I2=1,IP2,IP1                                       FIX  47
      TEMPR=DATA(I2)                                           FIX  48
      DATA(I2)=DATA(I2)+DATA(I2+1)                             FIX  49
80    DATA(I2+1)=TEMPR-DATA(I2+1)                              FIX  50
      IF (N-2) 200,200,90                                      FIX  51
90    THETA=TWOPI/DFLOAT(N)
      SINTH=DSIN(THETA/2)
      ZSTPR=-2.*SINTH*SINTH                                    FIX  54
      ZSTPI=DSIN(THETA)
      ZR=(1.-ZSTPI)/2.                                         FIX  56
      ZI=(1.+ZSTPR)/2.                                         FIX  57
      IF (IFORM) 100,110,110                                   FIX  58
100   ZR=1.-ZR                                                 FIX  59
      ZI=-ZI                                                   FIX  60
110   I1MIN=IP0+1                                              FIX  61
      I1MAX=IP0*(N/4)+1                                        FIX  62
      DO 190 I1=I1MIN,I1MAX,IP0                                FIX  63
      DO 180 I2=I1,IP2,IP1                                     FIX  64
      I2CNJ=IP0*(N/2+1)-2*I1+I2                                FIX  65
      IF (I2-I2CNJ) 150,120,120                                FIX  66
120   IF (ISIGN*(2*IFORM+1)) 130,140,140                       FIX  67
130   DATA(I2+1)=-DATA(I2+1)                                   FIX  68
140   IF (IFORM) 170,180,180                                   FIX  69
150   DIFR=DATA(I2)-DATA(I2CNJ)                                FIX  70
      DIFI=DATA(I2+1)+DATA(I2CNJ+1)                            FIX  71
```

```
      TEMPR=DIFR*ZR-DIFI*ZI                                             FIX  72
      TEMPI=DIFR*ZI+DIFI*ZR                                             FIX  73
      DATA(I2)=DATA(I2)-TEMPR                                           FIX  74
      DATA(I2+1)=DATA(I2+1)-TEMPI                                       FIX  75
      DATA(I2CNJ)=DATA(I2CNJ)+TEMPR                                     FIX  76
      DATA(I2CNJ+1)=DATA(I2CNJ+1)-TEMPI                                 FIX  77
      IF (IFORM) 160,180,180                                            FIX  78
  160 DATA(I2CNJ)=DATA(I2CNJ)+DATA(I2CNJ)                               FIX  79
      DATA(I2CNJ+1)=DATA(I2CNJ+1)+DATA(I2CNJ+1)                         FIX  80
  170 DATA(I2)=DATA(I2)+DATA(I2)                                        FIX  81
      DATA(I2+1)=DATA(I2+1)+DATA(I2+1)                                  FIX  82
  180 CONTINUE                                                          FIX  83
      TEMPR=ZR-.5                                                       FIX  84
      ZR=ZSTPR*TEMPR-ZSTPI*ZI+ZR                                        FIX  85
  190 ZI=ZSTPR*ZI+ZSTPI*TEMPR+ZI                                        FIX  86
C     RECURSION SAVES TIME, AT A SLIGHT LOSS IN ACCURACY.  IF AVAILABLE,FIX  87
C     USE DOUBLE PRECISION TO COMPUTE ZR AND ZI.                        FIX  88
  200 IF (IFORM) 270,210,210                                            FIX  89
C     UNPACK THE REAL TRANSFORM VALUES (TWO PER COLUMN)                 FIX  90
  210 I2=IP2+1                                                          FIX  91
      I1=I2                                                             FIX  92
      J1=IP0*(N/2+1)*NREM+1                                             FIX  93
      GO TO 250                                                         FIX  94
  220 DATA(J1)=DATA(I1)                                                 FIX  95
      DATA(J1+1)=DATA(I1+1)                                             FIX  96
      I1=I1-IP0                                                         FIX  97
      J1=J1-IP0                                                         FIX  98
  230 IF (I2-I1) 220,240,240                                            FIX  99
  240 DATA(J1)=DATA(I1)                                                 FIX 100
      DATA(J1+1)=0.                                                     FIX 101
  250 I2=I2-IP1                                                         FIX 102
      J1=J1-IP0                                                         FIX 103
      DATA(J1)=DATA(I2+1)                                               FIX 104
      DATA(J1+1)=0.                                                     FIX 105
      I1=I1-IP0                                                         FIX 106
      J1=J1-IP0                                                         FIX 107
```

```
      IF (I2-1) 260,260,230                                           FIX 108
  260 DATA(2)=0.                                                      FIX 109
  270 RETURN                                                          FIX 110
      END                                                             FIX 111-
```

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