Systematic risk, debt maturity, and the term structure of credit spreads

Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads*

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Abstract

We document several facts about corporate debt maturity: (1) debt maturity is procyclical; (2) higher-beta firms tend to have longer debt maturity; (3) shorter maturity amplifies the sensitivity of credit spreads to aggregate shocks. We build a dynamic capital structure model that explains these facts. In the model, leverage and maturity choices are highly interdependent, which reflects the tradeoffs of liquidity discounts of long-term debt, repayment risks of short-term debt, and the benefit of short-term debt as a commitment device for timely leverage adjustments. Additionally, the model quantifies the effects of maturity dynamics on the term structure of credit spreads.

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1. Introduction

The aggregate corporate debt maturity has a clear cyclical pattern: the average debt maturity is longer in economic expansions than in recessions. Fig. 1 displays the trend and cyclical components of the share of long-term debt for nonfinancial firms from 1952 to 2018. The cyclical component falls in every recession in the sample, with an average drop of 4% from peak to trough.[1] For individual firms, the maturity variation over time can be even stronger. For example, during the financial crisis of 2008-09, 26% of U.S. non-financial public firms saw their long-term debt share fall by 20% or more.

What explains the cyclical variations in corporate debt maturity? How do the maturity dynamics differ across firms, and how do they affect corporate bond pricing? In this paper, we document new stylized facts linking firms' maturity dynamics and credit risk to firms' systematic risk exposure. We then build a dynamic capital structure model that endogenizes firms' leverage and maturity choices over the business cycle. The calibrated model accounts for several key empirical facts about debt maturity. It also provides new insights on the interactions between the dynamics of leverage ratio and debt maturity, as well as the impact of maturity dynamics on the term structure of credit spreads.

[Insert Fig. 1 near here]

Empirically, we document that firms with higher systematic risk choose longer debt maturity and maintain a more stable maturity structure over the business cycle. When macroeconomic conditions worsen (e.g., during recessions), average debt maturity shortens, while the cross-sectional relation between a firm's systematic risk exposure and debt maturity steepens. These findings are robust to different measures of systematic risk and proxies for debt maturity. Furthermore, having a larger fraction of long-term debt coming due in the 2008-09 financial crisis makes the increase in a firm's credit spreads significantly stronger in response to the crisis shock. This amplification effect is more pronounced at the shorter end of the credit curve, as well as for firms with higher leverage and higher systematic risk exposure.

We build a dynamic capital structure model to explain these findings. We depart from the standard approach to model finite maturity debt in continuous time (see e.g., Leland, 1994b; Leland and Toft, 1996; Goldstein et al., 2001) in two aspects. First, debt maturity is lumpy, with all the debt outstanding maturing simultaneously instead of gradually over time.

[1] We do not study the long-term trend in debt maturity in this paper. Greenwood et al. (2010) argue that this trend is consistent with firms acting as macro liquidity providers. Custodio, Ferreira, and Laureano (2013) argue that the secular decline in the maturity of public firms was generated by firms with higher information asymmetry and by new public firms in the 1980s and 1990s.

Second, firms are not required to roll over the matured debt immediately. Instead, we allow firms to optimally adjust their capital structure when existing debt matures. In addition, firms can also restructure by calling back the existing debt. In both cases, after repaying existing debt in full, firms can readjust their debt level and maturity based on the prevailing condition of the firm and the economy.

The model features the standard tradeoff between tax benefits of debt and costs of financial distress. The optimal debt maturity depends on the tradeoffs of three factors. First, long-term debt faces higher liquidity discount than short-term debt. Second, shorter debt maturity increases the frequency of debt repayment in the short run, which in turn raises the costs of external financing (the costs of raising funds to pay back the matured debt) and the ex-ante costs of financial distress (due to debt overhang). Third, conditional on remaining solvent in the short run, short-term debt can help the firm gain flexibility in its leverage over time. It enables equity holders to commit to retire existing debt at face value and then optimally readjust its leverage, hence overcoming the "leverage ratchet effect" as pointed out by Admati et al. (2018). Consequently, the firm can either increase or decrease its debt level in a more timely fashion when its fundamental or the economic condition changes.

The relative importance of the three factors depends on firms' systematic risk exposure, leverage ratio, as well as the aggregate economic condition, which generates variations in the maturity choice in the cross section and over the business cycle. Firms with high systematic risk exposure are more likely to encounter financial distress in aggregate bad times, all else equal, which makes their ex-ante costs of financial distress higher. As a result, these firms might dislike short-term debt due to the higher burden of debt repayment, yet they might prefer short-term debt because they value the long-run flexibility in leverage adjustments. We show that which of the two considerations above dominates depends on the level of external financing costs. When the costs of repaying old debt (by issuing equity) or issuing new debt are not too small, firms with higher systematic risk exposure will prefer longer debt maturity, and the opposite can occur when the external financing cost is sufficiently low. In addition, the higher liquidity costs for long-term debt lead firms to shorten debt maturity in bad times.

To obtain time-consistent capital structure policies across the business cycle, we characterize firm policies through the Nash equilibrium of a dynamic game between equity holders and their "future selves." The model is tractable, with analytical solutions for the value of debt and equity (up to a system of nonlinear equations). We calibrate the model to the data, with the key model parameters estimated by targeting moments regarding default risk and capital structure, including debt maturity. To facilitate the estimation procedure, we also devise efficient algorithms to compute model-implied stationary distributions for firm cash flows through the Kolmogorov forward equation.

The calibrated model generates realistic predictions for leverage, debt maturity, default probabilities, credit spreads, and equity pricing. It can account for the positive cross-sectional relation between firms' systematic risk exposure and debt maturity in the data, as well as the steepening of this cross-sectional relationship and the drop in firms' debt maturity during bad aggregate times.

We use the model to analyze the impact of debt maturity on the term structure of credit spreads. The model shows that changing the level of debt maturity can significantly impact both the level and shape of the term structure of credit spreads. In particular, a shortening of debt maturity can simultaneously increase credit spreads at short horizons and decrease credit spreads at longer horizons. This is again due to the dual effects of shortening the debt maturity on (1) raising debt repayment risks in the short run and (2) making leverage more adaptive in the long run. The model also shows that short debt maturity can amplify the sensitivity of firm credit spreads to aggregate shocks, especially for firms with higher leverage or higher exposure to systematic risk. This helps explain our empirical findings regarding the behavior of credit spreads during the 2008-09 financial crisis.

Related literature. Our paper adds to the empirical literature on corporate debt maturity by focusing on firm heterogeneity in systematic risk exposure. Barclay and Smith (1995) find that firms with higher asset volatility choose shorter debt maturity. They do not separately examine the effects of systematic and idiosyncratic risk. Baker, Greenwood, and Wurgler (2003) argue that firms choose debt maturity by timing inflation, the short rate, and the term spread to minimize the cost of capital. On the cyclical dynamics of debt maturity, Erel, Julio, Kim, and Weisbach (2012) show that new debt issuances shift towards shorter maturity and more security during times of poor macroeconomic conditions. Mian and Santos (2018) show that the maturity of syndicated loans is pro-cyclical, especially for credit-worthy firms. They argue that firms actively managed their loan maturity before the financial crisis through early refinancing of outstanding loans.

Our model contributes to dynamic capital structure modeling.[2] Several recent papers, including Dangl and Zechner (2016), Geelen (2016), Benzoni et al. (2019), and DeMarzo and He (2020), have proposed different models of leverage dynamics that allow for voluntary debt reduction outside of bankruptcy, which is well-documented empirically (see, for example, Graham and Harvey, 2001; Hovakimian et al., 2001; Leary and Roberts, 2005) but typically assumed away in existing models in the continuous time setting. Our modeling of lumpy

[2]Among the earlier contributions are Fischer et al. (1989), Leland (1994b), Leland and Toft (1996), Hennessy and Whited (2005, 2007), and Goldstein et al. (2001). Recent work on capital structure and macroeconomic conditions include Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a,b), Chen (2010), Chen and Manso (2017), and Gomes and Schmid (2018), among others.

debt maturity follows Geelen (2016), who emphasizes the ability to adjust leverage as a main reason for firms to issue finite maturity debt. Our model additionally examines firms' leverage and debt maturity dynamics both in the cross section and over the business cycle, and we demonstrate the quantitative effects of maturity dynamics on the term structure of credit spreads.

Our paper also contributes to the studies of the term structure of credit spreads.[3] Structural models can endogenously link default risk to firms' capital structure decisions, including leverage and maturity structure. For simplicity, earlier models mostly restrict the maturity structure to be time-invariant. Our model allows the maturity structure to change endogenously over the business cycle and quantifies the impact of maturity dynamics on corporate bond pricing.

The maturity tradeoff in our model involves liquidity discount of long-term debt, which can be microfounded via search frictions (see, for example, He and Milbradt, 2014; Chen et al., 2018); repayment risk of short-term debt, and the flexibility of leverage adjustments with short-term debt. Other theories of debt maturity include information asymmetry and adverse selection (Flannery, 1986; Diamond, 1991), debt overhang (Myers, 1977), and macro liquidity provision (Greenwood et al., 2010), among others.

2. Empirical evidence

In this section, we present evidence on the links between firms' systematic risk exposures and debt maturity choices. Then, using the 2008 financial crisis as a shock, we also examine the impact of debt maturity structure on the term structure of credit risk.

2.1. *Data*

We merge the data from COMPUSTAT annual industrial files and CRSP for the period 1974 to 2017. The choice for the start of the sample is due to the fact that the balance sheet information needed to construct our proxies for debt maturity is only available starting in 1974. As is standard in the literature, we exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4999), and quasi-public firms (SIC codes greater than 8999), and we require firms in our sample to have total debt that represents at least 5% of their assets.[4] All the variables are winsorized at the 1% and 99% level. Finally, we remove firm-year

[3] Earlier contributions include structural models by Leland and Toft (1996), Leland (1998), Collin-Dufresne and Goldstein (2001), and reduced-form models by Jarrow, Lando, and Turnbull (1997), Lando (1998), Duffie and Singleton (1999), among others.

[4] Lowering the threshold to 3% generates very similar results.

observations with extreme year-to-year changes in the capital structure, defined as having changes in book leverage or long-term debt share in the lowest or highest 1%, because these extreme adjustments are likely due to major corporate events such as mergers, acquisitions, and spin-offs.

Debt maturity measures. COMPUSTAT provides information on the amount of debt in 6 maturity categories: debt due in less than 1 year (*dlc*), which is the sum of long-term debt due in 1 year ($dd1$) and other short-term debt; debt due in years two to five ($dd2$, $dd3$, $dd4$, and $dd5$); and debt due in more than 5 years, computed as long-term debt due in more than 1 year (*dltt*) minus the sum of long-term debt due in years two to five ($dd2 + dd3 + dd4 + dd5$). Our benchmark measure of debt maturity is the long-term debt share, which is the percentage of total debt that are due in more than 3 years ($ldebt3y$). For robustness, we also construct several alternative measures of debt maturity, $ldebtny$, by classifying long-term debt as debt due in more than n years, where $n = 1, 2, 4, 5$.

Risk measures. Our primary measure of firms' exposure to systematic risk is asset market beta (*mktbeta*). We follow Bharath and Shumway (2008) to back out asset betas from equity betas using the Merton (1974) model. Equity market betas are computed using past 36 months of equity returns.[5] In the process of computing asset betas, we also obtain the systematic and idiosyncratic asset volatilities from equity volatilities based on the CAPM. In addition, following Acharya, Almeida, and Campello (2013), we compute the asset bank beta (*bankbeta*), which measures firms' exposures to the fluctuations of a banking portfolio, and the asset tail beta (*tailbeta*), which captures firms' exposures to large negative shocks to the market portfolio.

Since the asset betas above are unlevered from the corresponding equity betas, they could be mechanically (inversely) related to firms' leverage, which might affect our inference regarding firms' maturity choices. To address this concern, we also consider the cash flow beta (*cfbeta*). We compute the cash flow beta as the covariance between firm-level and aggregate cash flow growth rates divided by the variance of aggregate growth rates. To avoid the issue of negative cash flows, we use annual changes in cash flows normalized by total assets from the previous year to measure the annual cash flow growth rates. The betas are computed using 20-year windows.

Previous empirical studies find that debt maturity decisions are related to several firm characteristics, including firm size (log market assets, or *mkat*), abnormal earnings (*abnearn*),[6]

[5] Computing equity betas with past 12 or 24 months of equity returns generates similar results.

[6] Following Barclay and Smith (1995), we define "abnormal earnings" as the change in earnings from year

book leverage (*bklev*), market-to-book ratio (*mk2bk*), asset maturity (*assetmat*) and profit volatility (*profitvol*). We control for these firm characteristics in our main regressions.

[Insert Table 1 near here]

Table 1 provides the summary statistics for the variables used in our paper. The detailed descriptions of these variables are in the Internet Appendix. The median firm has 87% of the debt due in more than 1 year, 60% due in more than 3 years, and 33% due in more than 5 years. There is also considerable cross-sectional variation in debt maturity. The standard deviation of the long-term debt share *ldebt3y* (the percentage of debt due in more than 3 years) is 32%, with an interquartile range from 28% to 80%. The median systematic and idiosyncratic asset volatilities are 12% and 30%, respectively. The correlations among the different risk measures are reported in Panel B.

2.2. Debt maturity

In this section, we examine the cross-sectional relation between systematic risk exposures and debt maturity. We also present evidence that this relation changes with the macroeconomic conditions.

To examine the relation between debt maturity and systematic risk exposures across firms, we run panel regressions with the following general specification:

$$ldebt3y_{ijt} = \alpha_t + \alpha_j + \beta_1 risk_{ijt} + \beta_2' X_{ij,t-1} + \varepsilon_{i,t}, \quad (1)$$

where $ldebt3y_{ijt}$ is the long-term debt share for firm i from industry j in year t; $risk_{ijt}$ represents various measures of firms' systematic risk exposures; X_{ijt} represents firm-specific controls, including total asset volatility (*assetvol*), market assets (*mkat*), abnormal earnings (*abnearn*), book leverage (*bklev*), market-to-book ratio (*mk2bk*), asset maturity (*assetmat*) and profit volatility (*profitvol*). Following the literature, we lag the control variables by one year in the regression.

[Insert Table 2 near here]

Table 2 reports the results of the panel regressions. We include year and industry (2-digit SIC code) fixed effects along with additional controls. Following Petersen (2009) and Thompson (2011), we compute the standard errors by two-way clustering the observations by industry and year, as well as adjusting for heteroskedasticity. The coefficient for the

t to $t+1$ normalized by market equity at the end of year t.

asset market beta is positive and highly significant across various specifications. Its size increases after controlling for asset volatility (column (2)), and further increases to 0.097 after controlling for book leverage (column (3)), which implies that a one-standard deviation increase in asset market beta is associated with a 6.1% increase in the long-term debt share.

Previously studies have found that firms with high volatility (either asset volatility or cash flow volatility) tend to have shorter debt maturity (for example, see Barclay and Smith, 1995; Guedes and Opler, 1996; Stohs and Mauer, 1996). Our results are consistent with these findings, and we further show that this negative relation between volatility and debt maturity is driven by idiosyncratic volatility (column (4)).[7] This result is consistent with the theory of debt maturity based on information asymmetry (see Flannery, 1986; Diamond, 1991), which is more naturally associated with firm-specific uncertainty instead of aggregate uncertainty. This result also explains why controlling for asset volatility strengthens the coefficient on asset market beta. Firms with high market beta tend to also have high idiosyncratic volatility, which offsets the effect of systematic risk on debt maturity.

In column (5), we add additional controls to the regression. The coefficient of the asset market beta is smaller but remains significant. The decrease in the coefficient is mainly due to the addition of firm size (*mkat*), which is positively correlated with asset market beta and negatively correlated with asset volatility. Columns (6) through (8) report the results when we replace asset market beta with asset bank beta, asset tail beta, and cash flow beta, respectively. The coefficient estimates on these alternative systematic risk measures are all positive and statistically significant. Taken together, the results from Table 2 consistently imply that firms with high systematic risk exposures tend to have longer debt maturity.

[Insert Fig. 2 near here]

To investigate how the cross-sectional relation between debt maturity and systematic risk changes over time, we first plot in Fig. 2 the time series of the coefficients on the systematic and idiosyncratic volatilities in Fama-MacBeth regressions.[8] The 95% confidence intervals are computed using heteroskedasticity-consistent standard errors. Panel A shows that the coefficient for the systematic asset volatility is positive for all years and is significant for the majority of the sample. Panel B shows that the coefficient on the idiosyncratic asset volatility is significantly negative throughout the sample.

We have conducted a series of robustness checks for the empirical results above. The positive relation between beta and debt maturity holds for alternative measures of debt

[7] In the cross section, holding asset market beta fixed while changing total asset volatility is equivalent to holding systematic volatility (based on CAPM) fixed while changing idiosyncratic volatility.

[8] The full results of the Fama-MacBeth regressions are quite similar.

maturity (Table B.1). While our measure of long-term debt share is based on the stock of debt, the positive relation between beta and debt maturity also holds at the extensive margin. Specifically, using the public bond issuance data from FISD for 1990 to 2017, we find that high systematic-risk firms are more likely to issue long-term bonds (Table B.2). Finally, controlling for cash holdings mildly strengthens the effect of firms' systematic risk exposure on debt maturity (see the Internet Appendix).

Impact of macroeconomic conditions. Next, we specifically examine how macroeconomic conditions affect the relation between debt maturity and systematic risk. As proxy for macroeconomic conditions, we construct a firm-specific recession dummy, which equals one if the fiscal year-end month for the firm is in an NBER recession and zero otherwise. Then, we examine the impact of business cycles on debt maturity by adding a recession dummy and an interaction term between the recession dummy and the systematic risk measure to the regression. We no longer include year fixed effects but still control for the secular trend in maturity as shown in Fig. 1 Panel A. We do so by including a quadratic time trend based on the the average long-term debt share across firms, and we assume that all firms have the same loading on the time trend.

[**Insert Table 3 near here**]

The first three columns of Table 3 show the results for the full sample, where we use asset market beta, bank beta, and tail risk beta as the measure for systematic risk exposure, respectively. The coefficient estimate of the recession dummy is negative and significant. It suggests that long-term debt share is lower on average during recessions, consistent with the results in Fig. 1 Panel B. The coefficients for the interaction terms between the three asset beta measures and the recession dummy are all positive and significant, implying that the positive relation between firms' systematic risk exposures and debt maturity is stronger in recessions.[9] Together, these two effects of the recession dummy imply that low-beta firms reduce their debt maturity more than do high-beta firms from expansions to recessions.

Finally, in columns (4) through (6) of Table 3, we present the results for the sample that excludes the 2008 financial crisis. The main results are either unchanged or stronger in this sub-sample.

In the analysis above, we only allow the impact of business cycles on debt maturity to depend on firms' systematic risk exposure. However, changes in macroeconomic conditions could also affect the relation between debt maturity and other firm characteristics. We find

[9] In unreported results, we find that this result is particularly strong among more constrained firms, as proxied by smaller size and the lack of a rating.

that, in addition to low-beta firms, firms with low idiosyncratic volatility, large size, low market-to-book ratio and long asset maturity reduce their debt maturity more in recessions. These results are available in the Internet Appendix.

2.3. Term structure of credit spreads

Having examined the relation between firms' systematic risk exposures and debt maturity, we now turn to the effect of debt maturity on credit risk. Specifically, we would like to see how firms' maturity choices might amplify or dampen the sensitivity of credit spreads to aggregate shocks. Following Almeida et al. (2011), we treat the 2008 financial crisis as a shock to the macroeconomic conditions and exploit the cross-sectional variation in debt maturity, particularly the fraction long-term debt maturing in 2008.

We obtain firm-level credit default swap (CDS) spreads with maturity of 1 year, 5 years, and 10 years from Markit, and match the data with the COMPUSTAT information. Changes in the CDS spreads during the crisis are measured as the differences in the averages of daily CDS spreads between fiscal year 2007 and 2008. We use the fiscal year 2007 balance sheet information to compute the fraction of long-term debt that matures in 2008, $ldebt08 = dd1/(dd1 + dltt)$. The larger this measure, the shorter the effective maturity, and the more pressure the firm faces in debt repayment or refinancing during the crisis. Following Almeida et al. (2011), we treat August 2007 as the onset of the financial crisis and focus on firms that have the 2007 fiscal year-end month between September 2007 and January 2008. Among the 375 firms with CDS data in fiscal year 2007 and 2008, 87% meet this criterion.

We examine the cross-sectional relation between the changes in CDS spreads from 2007 to 2008 (based on 1 year, 5 year, and 10 year CDS spreads) and firms' maturity measures $ldebt08$. The financial crisis could exacerbate default risk through other firm characteristics besides maturity. Consequently, we control for firm characteristics including asset market beta, market leverage, asset volatility, firm size, market-to-book ratio, profitability, tangibility, equity return (past 12 months), credit rating (from the Standard & Poor's, converted to a numerical scale), and industry dummies (1-digit SIC code) in the regression. In addition, to explore how the maturity effects differ across firms, we split the sample into two halves based on the pre-crisis book leverage, market leverage, and cash flow beta, respectively, and run the cross-sectional regression within each sub-sample.

[Insert Table 4 near here]

The regression results are in Table 4. All else equal, a firm that has a larger portion of its long-term debt maturing in 2008 would experience a larger increase in CDS spreads. In the full sample, a one-standard deviation increase in $ldebt08$ corresponds to a 48 bps, 39 bps and 32 bps increase in the CDS spreads with maturity of 1 year, 5 years, and 10 years respectively in 2008. These results are qualitatively consistent with the empirical findings in Hu (2010).

[Insert Fig. 3 near here]

The sub-sample results for splits based on book leverage and cash flow beta are summarized in Fig. 3, which plot the implied effects of a one-standard deviation increase in $ldebt08$ on the changes in CDS spreads from 2007 to 2008. Panel A of Fig. 3 shows that a one-standard deviation increase in $ldebt08$ raises the 1-year, 5-year, and 10-year CDS spreads by 79 bps, 75 bps, and 53 bps respectively for firms with above-median book leverage, while the corresponding increase in the CDS spreads is 32 bps, 23 bps, and 21 bps respectively for firms with below-median book leverage. The results are qualitatively similar when we sort firms based on market leverage, but the differences in the maturity effects between the high- and low-leverage group are even bigger. Panel B shows that a one-standard deviation increase in $ldebt08$ raises 1-year, 5-year, and 10-year CDS spreads by 71 bps, 66 bps, and 43 bps respectively for firms with above-median cash flow beta, while the corresponding increase in the CDS spreads is negligible for firms with below-median cash flow beta.

One might be concerned with the endogeneity of the $ldebt08$ measure. Mian and Santos (2018) show that firms with good credit quality actively managed the maturity of syndicated loans before the financial crisis through early refinancing of outstanding loans. This would imply that those firms with high rollover risk according to $ldebt08$ could be the firms with low quality, which would explain the larger increase in their credit spreads during the crisis. To address this concern, we instrument $ldebt08$ with $\widehat{ldebt08}$, which is based on balance sheet information from fiscal years 2004 ($dd4/(dd1 + dltt)$), 2005 ($dd3/(dd1 + dltt)$), or 2006 ($dd2/(dd1+dltt)$). We obtain almost identical results based on the fiscal year 2006 information (see the Internet Appendix). The results are slightly weaker for 2005 and no longer significant for 2004.

In summary, our empirical analysis produces the following findings. (1) Firms with higher systematic risk exposures choose longer debt maturity. (2) The sensitivity of debt maturity to systematic risk exposure becomes stronger after controlling for leverage. (3) Debt maturity shortens in recessions, while the sensitivity of debt maturity to systematic risk exposure rises. (4) A shorter debt maturity before entering into a crisis amplifies the sensitivity of credit spreads to the crisis shock. This effect of is stronger among firms with high leverage or high systematic risk exposure.

3. Model

Having presented the empirical evidence connecting firms' systematic risk exposures to debt maturity and credit risk, we now construct a dynamic capital structure model to explain these findings. The model not only generates endogenous firm leverage and maturity adjustments over the business cycle, but also shows how the debt dynamics affect credit risk. We first introduce the macroeconomic environment Section 3.1, and then describe the firm's problem in Section 3.2. Afterwards, Section 3.3 characterizes the model's solution and Section 3.4 explains the tradeoffs inherent in debt maturity choice.

3.1. The economy

Following Chen (2010), we model the aggregate state of the economy by a continuous-time Markov chain with the state at time t denoted by $s_t \in \{G, B\}$. State G represents a state of economic expansion, which is characterized by high expected growth rates, low economic uncertainty, and low risk premium, while the opposite is true in the recession state B. The physical transition intensities from state G to B and from B to G are $\overline{\pi}_G$ and $\overline{\pi}_B$, respectively. They imply that the probability that the economy switches from state G to B (or from B to G) in a small time interval Δ is approximately $\overline{\pi}_G\Delta$ (or $\overline{\pi}_B\Delta$).

Firms generate cash flows that are subject to large aggregate shocks that change the state of the economy, small systematic shocks, as well as firm-specific shocks. Specifically, a firm's cash flow y_t follows the process

$$\frac{dy_t}{y_t} = \overline{\mu}(s_t)dt + \sigma_\Lambda(s_t)dZ_t^\Lambda + \sigma_f(s_t)dZ_t^f . \tag{2}$$

The two independent standard Brownian motions Z_t^Λ and Z_t^f generate systematic and firm-specific cash-flow shocks, respectively. The conditional expected growth rate of cash flows is $\overline{\mu}(s_t)$, while $\sigma_\Lambda(s_t)$ and $\sigma_f(s_t)$ denote the systematic and idiosyncratic conditional volatility of cash flows, respectively. Although a change in the aggregate state s_t does not lead to any immediate change in the level of cash flows, it changes the dynamics of y_t by altering its conditional growth rate and volatility. As a note on notation, we denote the corresponding value of a process $\zeta(s_t)$ that is only dependent on the state s_t by ζ_s.

We assume that markets are complete, and there is a unique stochastic discount factor (SDF) Λ_t, which follows the process:

$$\frac{d\Lambda_t}{\Lambda_{t^-}} = -r\left(s_{t^-}\right)dt - \eta\left(s_{t^-}\right)dZ_t^\Lambda + \delta_G\left(s_{t^-}\right)\left(e^\kappa - 1\right)dM_t^G - \delta_B\left(s_{t^-}\right)\left(1 - e^{-\kappa}\right)dM_t^B, \tag{3}$$

with

$$\delta_G(G) = \delta_B(B) = 1, \quad \delta_G(B) = \delta_B(G) = 0, \tag{4}$$

where $r(s_t)$ is the state-dependent risk-free rate, and $\eta(s_t)$ is the market price of risk for the aggregate Brownian shocks dZ_t^Λ. The compensated Poisson processes $dM_t^s \equiv dN_t^s - \overline{\pi}_s dt$ reflect the changes of the aggregate state (away from state s), while κ determines the size of the jump in the discount factor when the aggregate state changes. Chen (2010) presents a continuous-time version of the long-run risk model of Bansal and Yaron (2004) that generates a stochastic discount factor of the form in Eq. (3).

3.2. The firm's problem

Our model of the capital structure dynamics builds on Leland (1994a), Leland and Toft (1996), and Goldstein et al. (2001) but differs in two key aspects.[10] First, we model debt maturity as lumpy. Instead of having debt retiring at a constant rate over time, we assume that all the outstanding debt will mature simultaneously. Lumpiness in maturity structure is a prevalent feature in the data (see Choi et al., 2018). Moreover, as we show later, this feature is also important for the model to generate meaningful default risk while matching the observed debt maturity. Second, we allow the firm to reoptimize on its capital structure whenever existing debt matures. This feature differs from the standard assumption that firms are committed to roll over any retired debt, and it ties the maturity choice with the frequency of capital structure adjustments.[11]

Firms in our model take on debt due to the tax benefits (interest expenses are tax-deductible), although default will generate costs of financial distress. The effective tax rate on corporate income is τ. In bankruptcy, the absolute priority rule applies, with debt holders recovering a fraction α_s of the defaulted firm's unlevered assets in state s and equity holders receiving nothing.

Debt maturity and restructuring. We model debt maturity as a Poisson process. Different from the standard assumption in the literature of having debt retired at a constant fractional rate (see for example, Leland, 1994a, 1998), we assume that the timings of maturity are perfectly correlated across all units of outstanding debt (i.e., they are driven by the same Poisson process). Suppose debt is issued at time 0, with initial face value P_0 and annualized

[10] We are very grateful to an anonymous referee for encouraging us to explore this modeling approach.

[11] Dangl and Zechner (2016), Geelen (2016), DeMarzo and He (2020), and Benzoni et al. (2019) have also studied models of leverage dynamics without full commitment. Goldstein et al. (2001) study a model in which firms have the option to increase but not decrease the level of debt. Strebulaev (2007) considers the possibility for distressed firms to reduce debt through costly asset sales.

maturity intensity m, and the coupon rate b_0 is chosen such that the debt is priced at par at issuance. Then, the maturity date of all outstanding debt is random and follows an exponential distribution with expected time to maturity of $1/m$. The face value and coupon rate of debt will remain constant prior to the arrival of debt maturity.

When existing debt matures, the firm can restructure by issuing new debt after paying off the matured debt. We assume that the firm first pays off the matured debt using funds from equity issuance before issuing new debt. This assumption helps preserve the scaling property, which we discuss in more detail in Section 3.3.

Prior to debt maturity, firms may want to lever up if their leverage ratios fall sufficiently following a sequence of positive cash flow shocks. Following Goldstein et al. (2001), we model this form of restructuring by assuming that debt is callable at par. To adjust the debt level or maturity, a firm first calls back all the outstanding debt, and then issues new debt with the desired quantity and maturity. Requiring that the firm buys back all the existing debt allows us to only keep track of one vintage of debt from the most recent restructuring. As in Goldstein et al. (2001), we assume that firms can precommit to the condition for calling back the debt in the bond indenture. Specifically, restructuring is triggered whenever the interest coverage y_t/b_t reaches a pre-specified level γ, which is chosen optimally by the firm.[12]

External financing costs. Either when existing debt matures or when the firm calls back existing debt, the firm repays the principal by issuing equity. We assume that such lumpy equity issuance is costly, with a proportional cost of θ per unit of equity issued. Not only will the equity issuance costs affect the optimal leverage choice (they raise the costs of servicing debt), they are also an important factor for firms' maturity choice. This is because shorter debt maturity raises the frequency of principal repayments, which makes a firm incur more external financing costs. The requirement that firms first repay all existing debt before issuing new debt renders the proportional equity issuance cost a quasi-fixed adjustment cost, which generates lumpiness in debt restructuring outside of debt maturity. Alternatively, having a proportional debt issuance cost in place of the equity issuance cost will generate similar results.

Capital structure dynamics over the business cycle. To understand the capital structure dynamics, let's consider a firm in between two adjacent debt restructuring points. Without loss of generality, we normalize the time of the most recent restructuring as $t = 0$. Then, the firm last restructured its debt at time 0 in state s_0 when the cash flow is y_0.

Let τ denote the random time at which the next restructuring occurs, which could be due

[12]For simplicity, we assume the buyback threshold γ does not depend on the current macro state s_t.

to either debt maturing or buyback. Before τ, while the macro state s_t could be different from s_0, the most recent restructuring state will remain s_0. We denote this most recent restructuring state by $\mathcal{R}_t$, so that $\mathcal{R}_t = \mathcal{R}_0 = s_0$ for any $0 \leq t < \tau$. It is necessary to keep track of both s_t and $\mathcal{R}_t$, which is summarized by the augmented state $(s_t, \mathcal{R}_t)$.

The firm optimally chooses the face value of debt, coupon rate, maturity intensity, and buyback threshold at time 0 as a function of the firm's cash flow y_0 and the restructuring state $\mathcal{R}_0 = s_0$. We denote the optimal choices by $P(y_0, \mathcal{R}_0)$, $b(y_0, \mathcal{R}_0)$, $m(y_0, \mathcal{R}_0)$, and $\gamma(y_0, \mathcal{R}_0)$, respectively. These values will remain unchanged before τ.

The default policy is characterized by a set of default boundaries optimally chosen by equity holders ex post. Due to the lumpiness in maturity structure, default can occur in two ways. First, at any time before debt restructuring ($t < \tau$), default is triggered whenever the cash flow y_t is at or below the threshold $y_D(y_0, s_t, \mathcal{R}_t)$ at time t in the augmented state $(s_t, \mathcal{R}_t)$. Second, at the instant of maturity, default is triggered whenever y_t is at or below the threshold $y_D^m(y_0, s_t, \mathcal{R}_t)$ at time t in the augmented state $(s_t, \mathcal{R}_t)$, where the superscript m denotes time of maturity. Technically, both types of default can be a jump-to-default event. Prior to debt restructuring, a jump-to-default can be triggered by a jump-up in the default boundary due to a switch of the aggregate state. Upon debt maturity, a jump-to-default can be triggered by a jump-up in the default boundary due to debt maturity itself if $y_D^m(y_0, s_t, \mathcal{R}_t) > y_D(y_0, s_t, \mathcal{R}_t)$.

Restructuring is triggered at time τ if either debt matures and is successfully paid off (i.e., when default is not triggered), or if debt is bought back. As explained earlier, the latter occurs whenever interest coverage y_t/b_t reaches the threshold $\gamma(y_0, \mathcal{R}_0)$ at time t. This is equivalent to triggering restructuring based on a restructuring boundary for the cash flow,

$$y_U(y_0, s_t, \mathcal{R}_t) = b(y_0, \mathcal{R}_0)\gamma(y_0, \mathcal{R}_0). \tag{5}$$

After restructuring, the restructuring state will be updated from s_0 to s_τ, and the face value, coupon, maturity intensity, and restructuring threshold for debt will all be adjusted. Together, the macro and restructuring states determine four unique augmented states,[13] and Fig. 4 illustrates the transition dynamics across these states.

[Insert Fig. 4 near here]

Corporate bond liquidity. The last part of the model is a liquidity spread for corporate bonds. We assume corporate bonds are discounted at an additional spread relative to equity and other liquid assets. There is ample evidence that corporate bonds are less liquid, especially

[13]More generally, a model with N macroeconomic states will result in N^2 augmented states in our model.

at longer maturity (see, for example, Bao et al., 2011; Chen et al., 2018). As He and Milbradt (2014) and Chen et al. (2018) show, such liquidity spreads can be micro-founded in a search model of Duffie et al. (2005, 2007). For simplicity, we model the liquidity spread in reduced form:

$$l(m,s) = \ell_s/m^2, \tag{6}$$

where $\ell_s \geq 0$. The key property of the liquidity spread is that it is increasing (and quadratic) in debt maturity ($1/m$). In addition, a larger liquidity spread in bad times can be captured by setting $\ell_B > \ell_G$.

3.3. *Model solution*

Having described the model setup, we now turn to the solution. We first discuss the valuation of debt and equity taking the firm policy for capital structure and default as given. Then we characterize the optimal firm policy.

Scaling property. In the model, the cash flow process is log-normal conditional on the aggregate state. In addition, the equity issuance and bankruptcy costs all scale with firm size. Consequently, a (state-dependent) scaling property applies as in Goldstein et al. (2001) and Chen (2010). The intuition is that, after paying back existing debt, equity holders essentially face the same capital structure problem at each point of restructuring, except that the firm size and the macro state might be different. Thus, conditional on the macro state, the value of debt and equity at the time of restructuring, the optimal choice for the amount of debt, and the default and restructuring boundaries will all be homogeneous in firm size, or equivalently the cash flow upon restructuring. This scaling property helps turn the dynamic optimization problem into a static one.

3.3.1. *Valuation of debt and equity*

Under the risk-neutral probability measure implied by the SDF in Eq. (3), the firm's cash flow process has expected growth rate and total volatility

$$\mu(s_t) = \overline{\mu}(s_t) - \sigma_\Lambda(s_t)\eta(s_t), \quad \sigma(s_t) = \sqrt{\sigma_\Lambda^2(s_t) + \sigma_f^2(s_t)}. \tag{7}$$

In addition, the risk-neutral transition intensities between the macro states are given by $\pi_G = e^\kappa \overline{\pi}_G$ and $\pi_B = e^{-\kappa}\overline{\pi}_B$. With $\kappa > 0$, the risk-neutral transition intensity from state G to B is higher than the physical intensity, while the risk-neutral intensity from state B to G is lower than the physical intensity. Jointly, they imply that the bad state is more likely to occur and also tends to last longer under the risk-neutral measure than under the physical measure.

Value of unlevered firm. As a building block for pricing debt and equity, consider an unlevered claim on the perpetual stream of cash flows y specified in Eq. (2) (ignoring taxes). Assuming the claim is liquid and hence its future cash flows are discounted using the risk-free rate under the risk-neutral measure, its value $V(y,s)$ satisfies a system of ordinary differential equations,

$$r_s V(y,s) = y + \mu_s y \frac{\partial V}{\partial y}(y,s) + \frac{1}{2}\sigma_s^2 y^2 \frac{\partial^2 V}{\partial y^2}(y,s) + \pi_s \left(V(y,s^c) - V(y,s)\right), \tag{8}$$

where $s^c \in \{G,B\} \setminus \{s\}$. The solution is $V(y,s) = v(s)y$, where the state-dependent price-dividend ratio $\mathbf{v}$ is given by

$$\mathbf{v} \;=\; \begin{pmatrix} r_G - \mu_G + \pi_G & -\pi_G \\ -\pi_B & r_B - \mu_B + \pi_B \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{9}$$

Eq. (9) is a generalized Gordon growth formula, which takes into account the state-dependent risk-free rate r_s and risk-neutral expected growth rate μ_s, as well as possible future transitions between the states. In the special case with no transition between the states ($\pi_G = \pi_B = 0$), Eq. (9) reduces to the standard Gordon growth formula.

Debt and equity pricing. Next, we turn to the valuation of debt and equity for the levered firm. Take the debt principle $P(y_0,\mathcal{R}_0)$, coupon $b(y_0,\mathcal{R}_0)$, maturity intensity $m(y_0,\mathcal{R}_0)$, default boundaries $y_D(y_0,s_t,\mathcal{R}_t)$ and $y_D^m(y_0,s_t,\mathcal{R}_t)$, and the restructuring boundary $y_U(y_0,s_t,\mathcal{R}_t)$ as given, the total value of outstanding debt $D(y_t,s_t,\mathcal{R}_t)$ satisfies the following system of ODEs (for brevity of notation, we drop the subscript t from here on),

$$\begin{aligned}(r_s + l(m(y_0,\mathcal{R}_0),s))D\left(y,s,\mathcal{R}\right) &= b(y_0,\mathcal{R}_0) + \mu_s y \frac{\partial D}{\partial y}\left(y,s,\mathcal{R}\right) + \frac{1}{2}\sigma_s y^2 \frac{\partial^2 D}{\partial y^2}\left(y,s,\mathcal{R}\right) \\ &+\pi_s\left(D\left(y,s^c,\mathcal{R}\right) - D\left(y,s,\mathcal{R}\right)\right) + m(y_0,\mathcal{R}_0)\left(D^m\left(y,s,\mathcal{R}\right) - D\left(y,s,\mathcal{R}\right)\right).\end{aligned} \tag{10}$$

The first three terms on the right hand side of Eq. (10) are standard. They capture the return to debt value due to coupon payment and changes in cash flows. The fourth term reflects the change in debt value due to a change in the macro state, which arrives at rate π_s. The final term reflects the change in debt value when debt matures, which arrives at rate $m(y_0,\mathcal{R}_0)$. D^m is the value of debt at maturity. In the absence of default, it is equal to the outstanding principal. If default occurs, bond holders recover a fraction of the unlevered firm after taxes.

$$D^m(y, s, \mathcal{R}) = \begin{cases} P(y_0, \mathcal{R}_0) & \text{for } y > y_D^m(y_0, s, \mathcal{R}) \\ \alpha_s(1-\tau)v_s y & \text{for } y \leq y_D^m(y_0, s, \mathcal{R}) \end{cases} \tag{11}$$

where v_s is the price-dividend ratio in state s from Eq. (9).

The boundary condition for debt value when default occurs before maturity is

$$D(y, s, \mathcal{R}) = \alpha_s(1-\tau)v_s y \qquad \text{for } y \leq y_D(y_0, s, \mathcal{R}), \tag{12}$$

as debt holders recover a fraction α_s of the unlevered assets of the defaulted firm. The boundary condition for debt value at the next point of debt buyback is

$$D(y, s, \mathcal{R}) = P(y_0, \mathcal{R}_0) \qquad \text{for } y \geq y_U(y_0, s, \mathcal{R}), \tag{13}$$

which reflects the fact that outstanding debt is called at face value.

Similar to debt, the value of equity, $E(y, s, \mathcal{R})$ satisfies the following system of ODEs:

$$\begin{aligned} r_s E(y, s, \mathcal{R}) = {} & (1-\tau)(y - b(y_0, \mathcal{R}_0)) + \mu_s y \frac{\partial E}{\partial y}(y, s, \mathcal{R}) + \frac{1}{2}\sigma_s^2 y^2 \frac{\partial^2 E}{\partial y^2}(y, s, \mathcal{R}) \\ & + \pi_s \left(E(y, s^c, \mathcal{R}) - E(y, s, \mathcal{R})\right) + m(y_0, \mathcal{R}_0)\left(E^m(y, s, \mathcal{R}) - E(y, s, \mathcal{R})\right). \end{aligned} \tag{14}$$

The first term on the right hand side of Eq. (14) is the net cash flow after interest expenses and taxes. Similar to Eq. (10), the last term of Eq. (14) reflects the change in equity value upon debt maturity. E^m is the value of equity when debt matures. In the event of default, equity holders receive nothing due to the absolute priority rule,

$$E^m(y, s, \mathcal{R}) = 0 \qquad \text{for } y \leq y_D^m(y_0, s, \mathcal{R}). \tag{15}$$

In the absence of default, equity holders repay the matured debt in full by issuing new equity (subject to proportional issuance costs). Afterwards, they effectively face an identical problem as they do at time 0 in restructuring, except that the cash flow has changed. Due to the scaling property, this continuation value will be a scaled version of the firm value (debt plus equity) at time 0 provided that the macro state is the same, with the scaling factor equal to the ratio of cash flow at the two restructuring points. Thus, we have

$$E^m(y, s, \mathcal{R}) = \frac{y}{y_0}\left[D(y_0, s, s) + E(y_0, s, s)\right] - (1+\theta)P(y_0, \mathcal{R}_0) \quad \text{for } y > y_D^m(y_0, s, \mathcal{R}), \tag{16}$$

where the first term on the right hand side is the scaled firm value, while the second term is the total cost of paying back the retired debt for equity holders.

Because equity holders recover nothing in default, equity value will again be zero when default occurs before debt maturity,

$$E(y, s, \mathcal{R}) = 0 \qquad \text{for } y \leq y_D(y_0, s, \mathcal{R}). \tag{17}$$

Finally, analogous to Eq. (16), the equity value immediately following a debt buyback but before new debt issuance is

$$E(y, s, \mathcal{R}) = \frac{y}{y_0}\left[D(y_0, s, s) + E(y_0, s, s)\right] - (1+\theta)P(y_0, \mathcal{R}_0) \quad \text{for } y > y_U(y_0, s, \mathcal{R}). \tag{18}$$

Optimal default policy. For any given capital structure policy, equity holders choose the default boundaries optimally ex post. Before debt matures, the optimal default boundaries are determined by the standard smooth-pasting conditions (one for each augmented state):

$$\frac{\partial E}{\partial y}(y_D(y_0, s, \mathcal{R}), s, \mathcal{R}) = 0, \quad (s, \mathcal{R}) \in \{G, B\} \times \{G, B\}. \tag{19}$$

When debt matures, the optimal default boundary is the point at which equity holders are indifferent between defaulting (and obtaining a value of zero) and repaying the matured debt (and obtaining E^m),

$$E^m(y_D^m(y_0, s, \mathcal{R}), s, \mathcal{R}) = 0, \quad (s, \mathcal{R}) \in \{G, B\} \times \{G, B\}. \tag{20}$$

Change of variables. The scaling property allows us to remove the dependence of the firm's problem on its initial size (i.e., initial cash flow y_0). We define the scaled cash flow $\widehat{y}_t \equiv y_t/y_0$, which has initial value $\widehat{y}_0 = 1$ immediately after a restructuring. The scaled default boundaries are

$$\widehat{y}_D(s, \mathcal{R}) \equiv \frac{y_D(y_0, s, \mathcal{R})}{y_0} \;\text{ and }\; \widehat{y}_D^m(s, \mathcal{R}) \equiv \frac{y_D^m(y_0, s, \mathcal{R})}{y_0}, \tag{21}$$

which, duo to the scaling property, will be independent of initial cash flow y_0.

The scaling property also implies that we can express the optimal face value, coupon rate, maturity intensity, and buyback threshold at time 0 as

$$P(y_0, \mathcal{R}_0) = y_0 \widehat{P}_{\mathcal{R}_0}, \quad b(y_0, \mathcal{R}_0) = y_0 \widehat{b}_{\mathcal{R}_0}, \quad m(y_0, \mathcal{R}_0) = m_{\mathcal{R}_0}, \quad \gamma(y_0, \mathcal{R}_0) = \gamma_{\mathcal{R}_0}, \tag{22}$$

where $(\widehat{P}_{\mathcal{R}}, \widehat{b}_{\mathcal{R}}, m_{\mathcal{R}}, \gamma_{\mathcal{R}})$ will be functions of the restructuring state only. Consequently, it follows from Eq. (5) that the restructuring boundary $y_U(y_0, s, \mathcal{R})$ is also proportional to y_0,

$$y_U(y_0, s, \mathcal{R}) = \widehat{y}_U(\mathcal{R}) y_0, \quad \text{where} \quad \widehat{y}_U(\mathcal{R}) = \widehat{b}_{\mathcal{R}} \gamma_{\mathcal{R}}. \tag{23}$$

Together, Eq. (16) and Condition Eq. (20) imply

$$\widehat{y}_D^m(s, \mathcal{R}) = \frac{(1+\theta) P(y_0, \mathcal{R}_0)}{D(y_0, s, s) + E(y_0, s, s)} = \frac{(1+\theta) D(y_0, s, s)}{D(y_0, s, s) + E(y_0, s, s)} \frac{\widehat{P}_{\mathcal{R}}}{\widehat{P}_s}. \tag{24}$$

The second equality on the right-hand side of Eq. (24) follows from the fact that debt is issued at par (i.e., $D(y_0, \mathcal{R}_0, \mathcal{R}_0) = P(y_0, \mathcal{R}_0)$). Thus, the optimal scaled default boundary at maturity will be equal to initial market leverage multiplied by $1 + \theta$ (gross equity issuance cost) and the ratio of the scaled face value of debt between the restructuring state $\mathcal{R}$ and current macro state s.

Similarly, we can also scale the value of debt and equity, with $\widehat{E} = E/y_0$ and $\widehat{D} = D/y_0$, to remove their dependence on y_0. The resulting system of ODEs characterizing debt and equity value become independent of initial cash flow. The solutions for the system of ODEs are given in Appendix A.1.

From the postulated optimal face value of debt in Eq. (22), we can see the added flexibility of the leverage dynamics in this model. When debt matures or is called back at time τ, the ratio between the debt level immediately after and before restructuring, which we refer to as the debt adjustment ratio, is

$$\frac{P(y_\tau, s_\tau, \mathcal{R}_\tau)}{P(y_{\tau^-}, s_{\tau^-}, \mathcal{R}_{\tau^-})} = \frac{\widehat{P}_{s_\tau}}{\widehat{P}_{s_0}} \frac{y_\tau}{y_0}. \tag{25}$$

Depending on the level of cash flow and the macro state at the two adjacent restructuring points (y_0 vs. y_τ, s_0 vs. s_τ), the debt adjustment ratio can be either above or below 1.

3.3.2. Optimal capital structure decisions

We now characterize the optimal and time-consistent capital structure policies. The requirement that debt be issued at par implies that $\widehat{b}_{\mathcal{R}}$ is a function of $\widehat{P}_{\mathcal{R}}$.[14] Thus, the capital structure policy in the restructuring state $\mathcal{R}$ includes debt level, maturity intensity,

[14] We describe how we solve for the par coupon rate $\widehat{b}_{\mathcal{R}}$ in Appendix A.2.

and buyback threshold, as summarized by

$$\boldsymbol{\chi}_{\mathcal{R}} \equiv (\widehat{P}_{\mathcal{R}}, m_{\mathcal{R}}, \gamma_{\mathcal{R}}). \tag{26}$$

The capital structure policies in the two restructuring states are interdependent. For example, time consistency requires that, when choosing the optimal policies for a restructuring that takes place in state G, equity holders need to take the optimal policies for a future restructuring in state B as given. Formally, we model time-consistent optimal firm policies as the Nash equilibrium of a dynamic game played between equity holders restructuring in state G against their "future selves" restructuring in state B.

Specifically, at time $t = 0$ when the macro state is $s_0 = s$, the optimal capital structure policy is chosen to maximize the initial value of the firm, taking the policy in the other restructuring state s^c, $\boldsymbol{\chi}_{s^c}$, as given,

$$\max_{\boldsymbol{\chi}_s} E(y_0, s, s; \boldsymbol{\chi}_s, \boldsymbol{\chi}_{s^c}) + D(y_0, s, s; \boldsymbol{\chi}_s, \boldsymbol{\chi}_{s^c}). \tag{27}$$

The optimal time-consistent capital structure is the solution to the following Nash equilibrium:

Definition 1 (Nash equilibrium for time-consistent capital structure)**.** *A time-consistent and optimal capital structure is a pair* $(\boldsymbol{\chi}_G, \boldsymbol{\chi}_B)$*. Each* $\boldsymbol{\chi}_s$ *is the solution to problem Eq. (27), taking* $\boldsymbol{\chi}_{s^c}$ *as given and subject to the smooth-pasting conditions Eq. (19) and the indifference conditions Eq. (20) for the default boundaries.*

We solve for the Nash equilibrium for Eq. (27) numerically by searching for a fixed point in a high-dimensional space (including the default boundaries, there are a total of 14 choice variables across the two states). Appendix A.2 describes the numerical algorithm.

3.4. Maturity tradeoffs: liquidity, refinancing risk, and flexibility

There are three factors that affect the maturity choices in our model. First, long-term debt faces higher liquidity discounts than short-term debt. Second, shorter debt maturity raises repayment risk. The shorter the maturity, the more frequently the firm will need to repay the matured debt. It not only raises the expected costs of external financing (due to costly equity issuance), but also raises the probability of default (due to debt overhang) and in turn the ex-ante costs of financial distress.[15]

A third factor is that short-term debt can help mitigate the "leverage ratchet effect," as pointed out by Admati et al. (2018). Since the firm restructures each time after paying off the

[15]The rollover risk in He and Xiong (2012) is a particular form of refinancing risk when firms are required to roll over all the matured debt.

matured debt, short-term debt effectively enables equity holders to commit to retire the debt at face value and then adjust the leverage to the level that is optimal given the prevailing condition of the firm and the economy. For example, a firm with short-term debt can pursue a strategy whereby it levers up as its financial condition strengthens and delevers in a timely fashion either when its cash flows become lower or when the macro state deteriorates. Such flexibility could allow the firm to enjoy more tax shield with limited default risk.

When the costs of equity and debt issuance are sufficiently high, the strategy of using short-term debt to gain flexibility on leverage becomes prohibitively expensive. In such cases, firms choose debt maturity by mainly trading off the liquidity discounts of long-term debt against the refinancing risks of short-term debt. This tradeoff is influenced by firms' systematic risk exposures and macroeconomic conditions. All else equal, firms with higher exposures to systematic risk are more concerned about the refinancing risks, because they are more likely to incur the costs of financial distress induced by debt repayment in aggregate bad times. As a result, debt maturity will increase with firms' systematic risk exposures in the cross section. Furthermore, since risk prices rise in recessions, the cross-sectional relation between debt maturity and systematic risk exposure should become stronger during such times.

An entirely different cross-sectional pattern on debt maturity choice could emerge when the costs of equity and debt issuance are sufficiently low. In such cases, firms might want to issue short-term debt to obtain more flexibility on leverage. In the cross section, firms with higher systematic-risk exposures might value the flexibility more, which would lead debt maturity to decrease with firms' systematic risk exposures.

4. Quantitative analysis

In this section, we examine the quantitative implications of the model. We first describe the estimation procedure in Section 4.1. Then we examine the model-implied optimal maturity and leverage choices in the cross section and over the business cycle in Section 4.2. Finally, we examine the impact of maturity dynamics on the term structure of credit spreads in Section 4.3.

4.1. Calibration

We estimate the model parameters in two steps. We first exogenously specify a subset of parameters that are standard in the literature. The remaining parameters are then estimated using the method of moments, with a set of target moments that are central to our model. To compute the moments efficiently, we derive the stationary distribution for firm cash flows in each macro state through the Kolmogorov forward equation (see Appendix A.3.1 for details).

4.1.1. Exogenously specified parameters

We set the physical transition intensities for the aggregate states of the economy to $\widehat{\pi}_G = 0.1$ and $\widehat{\pi}_B = 0.5$, which imply that an expansion is expected to last for 10 years, while a recession is expected to last for 2 years. To calibrate the stochastic discount factor, we choose the riskfree rate r_s, the market prices of risk for Brownian shocks η_s, and the market price of risk for state transition κ to match the first two moments of their counterparts in the SDF in Chen (2010).

Similarly, we calibrate the expected growth rate $\overline{\mu}_s$ and systematic volatility $\sigma_{\Lambda,s}$ for the benchmark firm based on Chen (2010), which in turn are calibrated to the corporate profit data from the National Income and Product Accounts. In particular, the average systematic cash flow volatility of the benchmark firm is 13.9%. Later on, we introduce firm heterogeneity in systematic risk exposure by varying the level of systematic cash flow volatility.

The bankruptcy recovery rates in the two states are $\alpha_G = 0.7$ and $\alpha_B = 0.6$, respectively. The cyclical variation in the recovery rate has important effects on the ex ante bankruptcy costs. We set the effective corporate tax rate to $\tau = 0.2$. To compute market betas inside the model, we specify the dividend process for the market portfolio to be a levered version of the cash-flow process Eq. (2) absent idiosyncratic shocks. We choose the leverage factor such that the unlevered market beta for the benchmark firm is 0.8, the medium asset beta for U.S. public firms.

To examine the model's implications for the term structure of credit spreads, we take the firm's optimal default policy (default boundaries) as given and price fictitious bonds with different maturities (see Appendix A.3.2 for details). These bonds are assumed to default at the same time as the firm, and their recovery rates are set to 40% in state G and 10% in state B, consistent with the business-cycle variation in bond recovery rates in the data.

4.1.2. Estimated parameters

The four remaining parameters – the equity issuance cost parameter θ, the liquidity spread parameters in the two macro states, ℓ_G and ℓ_B, and the idiosyncratic cash flow volatility, σ_f – are estimated using the generalized method of moments (GMM). The estimation targets six moments. The first two moments are (1) an average market leverage of 30% for BBB rated firms (the median market leverage for Baa-rated firms; see, for example, Chen et al., 2018); (2) an average 10-year default rate of 5% for BBB rated firms (based on Moody's default data from 1920 to 2017). We define a BBB-rated firm in the model as one with interest coverage between the 57.3th and 80.6th-percentile of the unconditional distribution for the benchmark firm.[16] Averaging over these firms gives us the model-implied average market leverage and default rate for BBB firms.

The remaining four moments pertain to debt maturity. To map the debt maturity in the model into the data, we measure debt maturity in the model using "effective maturity," which is the expected time until either the debt matures or is called back, whichever occurs earlier. Formally, effective maturity is defined as

$$M_{eff,t} \equiv \mathbb{E}_t \left[\min\{\tau_M, \tau_U\} \right], \tag{28}$$

where τ_M is the time taken for debt to mature, and τ_U is the time until the next debt buyback. In Appendix A.3.3, we characterize the effective maturity as the solution to a system of ODEs. In contrast, the "promised maturity" is the expected time that the debt matures in the absence of debt buybacks and is simply equal to $1/m_{\mathcal{R}}$.

[Insert Fig. 5 near here]

Fig. 5 illustrates the distinction between promised and effective maturity through a simulated path of the benchmark firm. The promised maturity (Panel C) is constant conditional on the restructuring state. In contrast, the effective maturity (Panel D) is time varying: it shrinks to zero when the cash flow reaches the restructuring boundary at $t = 9.6$ years, and increases to the promised maturity as the cash flow decreases towards the default boundary $y_{D,t}$ around year 19 (see Panel A).

Our third targeted moment is (3) an effective maturity of 7 years for debt issued in state G. This is based on the median value of 0.866 for the fraction of long-term debt that matures in more than a year in the data (see Table 1), which we then convert to an annualized effective maturity intensity using the relation $0.866 = e^{-m_{eff}}$, resulting in $1/m_{eff} = 7$. We also target (4) an effective maturity of 6.5 years for debt issued in state B, (5) a 1.1-year difference in the effective maturity at issue between a high-beta firm (with market beta at the 75th percentile) and a low-beta firm (with market beta at the 25th percentile) in state G, and (6) a 1.5-year difference in the effective maturity at issue between the same two firms in state B. The target moments (4), (5), and (6) are based on our results from regressing long-term debt share on firms' asset beta in Table 3.[17] We set the average systematic cash flow volatility for

[16] Based on the empirical distribution of S&P credit ratings from 1985 to 2017, 19.4% of the S&P-rated firms have a rating above BBB, while 57.3% of the firms have a rating below BBB. By defining BBB firms this way, we are focusing on firm heterogeneity in interest coverage while ignoring other forms of heterogeneity that could affect credit ratings.

[17] More specifically, we obtain these target moments from the relation $ldebt3y = -0.044 \times rec + 0.051 \times$

high- and low-beta firms to be 20.9% and 7%, respectively; these values are at approximately the 75th-percentile and 25th-percentile of the distribution for systematic asset volatilities in the data (see Table 1).

[Insert Table 5 near here]

In the estimation, we place a weight of 5 on moments (1), (3), and (4) (the average default rate and the effective maturity in the two states); the remaining moments receive a unit weight. The results of the estimation are reported Table 5; the empirical values for the six targeted moments and their correspondig values in the model are shown in the first two rows of Table 6. The estimation produces a good fit for the six target moments, especially the three with higher weights.

[Insert Table 6 near here]

The estimated equity issuance cost parameter is $\theta = 0.65\%$, which is modest compared to the average issuance costs of 4-6% for seasoned equity offerings estimated by Altinkilic and Hansen (2015). The estimated liquidity cost parameters are $\ell_G = 0.02$ bps and $\ell_B = 0.18$ bps. They imply liquidity spreads of 0.02 (0.2), 0.5 (4.6), and 2 (18.3) bps in state G (B) for bonds with maturities of 1, 5, and 10 years, respectively. In comparison, when we follow the procedure of Longstaff et al. (2005) to estimate the liquidity component of corporate bond spreads for the period 2004—2010 (see the Internet Appendix for details), we obtain average liquidity spreads of 0, 4, and 12 bps for bonds with maturities of 1, 5, and 10 years during normal times, while the liquidity spreads are substantially higher (1, 65, and 145 bps) during the recession/crisis period of December 2007 to June 2009. We conduct sensitivity analysis for ℓ_G, and ℓ_B, as well as several other parameters, later in this section. Finally, the estimated value for idiosyncratic cash flow volatility σ_f is 26.8%.

Sensitivity analysis. We have conducted sensitivity analysis to examine how the parameters are identified by the different moments, the results of which are shown in Table 6.

First, lower idiosyncratic cash flow volatility σ_f (see rows 1a-b) corresponds to lower default rate. Consequently, firms increase leverage, choose shorter debt maturity in both states, and debt maturity becomes more sensitive to systematic risk exposure in the cross section. Next, rows 2a-b show that a decrease in equity issuance costs θ makes debt rollover less costly, which also results in higher leverage, shorter debt maturity, and a stronger positive

$\beta_{mkt} + 0.026 \times \beta_{mkt} \times rec + controls$ estimated in column (1) of Table 3, and by converting between $ldebt3y$, the fraction of long-term debt due in 3 or more years, and the effective debt maturity intensity through the relation $ldebt3y = e^{-3m_{eff}}$. For example, we obtain target moment (4) from $ldebt3y_B = ldebt3y_G - 0.044 + 0.26 \times \beta_{mkt}$ and setting $\beta_{mkt} = 0.894$ to be the mean value in our sample.

relation between firm beta and debt maturity. However, as we show later in Section 4.2.1, when θ_B is sufficiently small, the relation between beta and debt maturity can turn negative, as riskier firms use short-term debt to make their leverage more flexible over time. Finally, rows 3a-b and 4a-b show that lowering the liquidity cost parameter ℓ_s makes long-term debt less costly relative to short-term debt, which increases the level of debt maturity and the difference in maturity between the high- and low-beta firm.

4.1.3. Results for the benchmark firm

Table 7 shows the key results for the benchmark firm. The top panel shows the firm's capital structure decisions. They include the initial interest coverage, market leverage, and average debt maturity, which are conditional on the restructuring state. There are also four scaled default boundaries prior to debt maturity and four scaled default boundaries upon debt maturity (one for each of the augmented state), as well as two scaled restructuring boundaries.

[Insert Table 7 near here]

The initial interest coverage is 1.91 when the firm restructures in state G. It rises to 3.09 in state B as the firm becomes more conservative in choosing leverage in the bad state. Despite the fact that the valuation ratio for equity tends to drop more in state B than does that of debt, which partially offsets the gap in interest coverage between the two states, the initial market leverage is still pro-cyclical (33.1% in state G vs. 27.2% in state B). The effective maturities are 7.20 and 6.13 years when restructuring in state G and B, respectively. The corresponding promised maturities are 10.15 and 8.98 years in state G and B, respectively. The gap between promised and effective maturity implies an average annual probability of restructuring of 0.04 ($= 1/7.20 - 1/10.15$) and 0.05 ($= 1/6.13 - 1/8.98$) for newly issued debt in state G and B, respectively.

[Insert Fig. 6 near here]

Examining the ordering of the eight (scaled) default boundaries is helpful for understanding the model. In Fig. 6, we illustrate how equity value is affected when (1) the macro state changes (Panel A), (2) under two different restructuring states (Panel B), and (3) debt maturity arrives (Panel C), which in turn helps explain how the default boundaries will be affected. First, conditional on the restructuring state (and thus fixing the debt level), the default boundaries are higher in the macro state B (i.e., $\widehat{y}_D(B,\mathcal{R}) > \widehat{y}_D(G,\mathcal{R}), \widehat{y}_D^m(B,\mathcal{R}) > \widehat{y}_D^m(G,\mathcal{R})$). This is because higher discount rates and lower expected growth in state B reduce the value of equity, leading equity holders to exercise the default option earlier.

Second, conditional on the macro state, the default boundaries are higher if the restructuring state (state of the most recent restructuring) is G (i.e., $\widehat{y}_D(s, G) > \widehat{y}_D(s, B), \widehat{y}_D^m(s, G) > \widehat{y}_D^m(s, B)$). This is because the firm chooses higher debt level when restructuring in state G, which reduces the value of equity ex post and hence makes equity holders default earlier.

Third, conditional on the augmented state $(s, \mathcal{R})$, the default boundary is higher upon maturity than before (i.e., $\widehat{y}_D^m(s, \mathcal{R}) > \widehat{y}_D(s, \mathcal{R})$). As shown in Panel C of Fig. 6, the scaled value of equity before maturity, $\widehat{E}$, and upon maturity, $\widehat{E}^m$, are both increasing in cash flows and cross at a point near the initial cash flow level $\widehat{y}_0$.[18] When cash flow is sufficiently below $\widehat{y}_0$, there will be a transfer from equity to debt when retiring the risky debt at par (a form of debt overhang), and the arrival of debt maturity removes equity holders' option to wait. Consequently, the arrival of debt maturity reduces equity value when cash flow is low ($\widehat{E}^m < \widehat{E}$), which is why the default boundaries are higher at maturity.

The restructuring boundary is lower for debt issued in the macro state B (i.e., $\widehat{y}_U(B) < \widehat{y}_U(G)$). This is because the firm is more conservative with its financial leverage when restructuring in state B. Starting with a smaller amount of debt initially, the firm then relevers at an earlier point (after experiencing positive cash flow shocks) in order to replenish its tax shield sooner.

The bottom panel of Table 7 provides additional information about capital structure and default risk for BBB-rated firms in the model (computed by averaging over the conditional cash flow distribution relevant for BBB firms; see footnote 16). The average market leverage for BBB firms is counter-cyclical (28.5% in state G vs. 32.1% in state B). This is mainly due to the valuation effect: the value of equity declines more than that of debt when the macro state switches from G to B.

The average 10-year default probability is 4.8% in state G and 5.5% in state B. The average 10-year total credit spread is 93 bps in state G and 114 bps in state B, which are largely accounted for by the default components (90 bps in state G and 109 bps in state B; the default components are computed by pricing the bonds using the same default boundaries while removing the liquidity spreads). Finally, the conditional equity Sharpe ratio is 0.11 in state G and 0.198 in state B. These moments are consistent with the data.

A well-known limitation of diffusion-based credit risk models is the inability to generate significant default risks at shorter horizons (see Duffie and Lando, 2001), which applies to our model as well. For example, both the average 5-year default probability and the 5-year total credit spreads are significantly lower than in the data (see e.g., Chen et al., 2018). One way to improve the model's performance in this dimension is to introduce idiosyncratic jumps in

[18] This can be seen from Eq. (16). Without equity issuance costs ($\theta = 0$), $E^m(y_0, s, s) = E(y_0, s, s)$ because $D(y_0, s, s) = P(y_0, s)$.

cash flows (see e.g., Kou and Wang, 2003; Chen and Kou, 2009; Bai et al., 2019).[19] We leave this extension to future research.[20]

Fig. 5 illustrates the capital structure dynamics for the benchmark firm along a simulated path of cash flows. Panel A shows the cash flows y_t (solid line), the corresponding default boundaries before maturity $y_{D,t}$ (dotted line) and upon maturity $y^m_{D,t}$ (dash-dotted line), and the corresponding restructuring boundary $y_{U,t}$ (dashed line). In this simulation, default takes place around year 19, when the arrival of debt maturity causes the relevant default boundary to switch from $y_{D,t}$ to $y^m_{D,t}$, which is above the current cash flow level.

Panels B and C plot the face value and promised maturity of debt over time, respectively, both of which are constant in between restructuring points. The first incidence of debt restructuring occurs at $t = 5$ when debt matures. After repaying old debt, the firm issues more debt because its cash flow has grown since $t = 0$ ($y_t = 1.8$ vs. 1), but the new debt has the same promised maturity because the macro state has not changed. The second restructuring takes place at $t = 8$, again triggered by the arrival of debt maturity. Even though the cash flow level at that time has increased since the previous restructuring point (2.1 vs. 1.8), as a result of the macro state switching to $s_t = B$ (as denoted by the shade), the firm ultimately reduces its debt level and shortens the promised maturity. The third restructuring takes place following a debt buyback at $t = 9.6$ (when y_t reaches the restructuring boundary $y_{U,t}$; see Panel A). The firm keeps its maturity short as the macro state remains in state $s_t = B$, but significantly increases its leverage due to its cash flow doubling from the last restructuring point (4.2 vs. 2.1). The fourth restructuring takes place at $t = 13$ (due to debt maturity). At that time, the macro state has switched back to $s_t = G$, and the cash flow has significantly increased since the previous restructuring point (6 vs. 4.19). This leads to a lengthening of debt maturity and a significant increase in the debt level.

4.2. Maturity choice

Next, we examine the impact of systematic risk exposures on firms' maturity structure in the model. We parameterize firms with different cash flow betas by rescaling the systematic volatility of cash flows $(\sigma_\Lambda(G), \sigma_\Lambda(B))$ for the benchmark firm while keeping the idiosyncratic volatility of cash flows σ_f unchanged. We first examine the effective maturities across firms under optimal leverage. Then we isolate the effect of systematic risk on debt maturity by holding leverage constant across firms.

[19] The presence of aggregate state transitions and the lumpy maturity structure in our model does generate jumps in asset value in our model. However, this effect is weak quantitatively.

[20] The main challenge with this extension is that analytical solutions to the value of debt and equity are no longer available in the presence of cash flow jumps.

[Insert Fig. 7 near here]

Fig. 7 shows the results. First, Panel A shows firms' choices for the face value of debt. We see that high-beta firms choose lower leverage to reduce their expected costs of financial distress. Panel B plots firms' choice of effective maturity under optimal leverage. Controlling for the idiosyncratic cash-flow volatility, firms with higher systematic volatility choose longer average debt maturity. As the average systematic volatility rises from 7% to 21%, the optimal effective maturity at issue rises from 6.3 to 8.1 years when restructuring in state G, and from 5.5 to 6.8 years in state B.

This positive relation between systematic risk and debt maturity is consistent with the empirical finding in Table 2. The result is robust so long as the cost of equity injection θ is not too low (we analyze the low θ case in Section 4.2.1). As explained in Section 3.4, a sufficiently high equity issuance cost makes it prohibitively costly to use short-term debt to gain flexibility in leverage adjustments. In such cases, the main tradeoff for debt maturity is between the higher liquidity discounts for long-term debt and the higher financing costs and ex ante costs of financial distress for short-term debt. Holding leverage fixed, the costs of external financing and financial distress are larger for high-beta firms on a risk-adjusted basis, which leads these firms to choose longer debt maturity. When firms are allowed to choose their leverage, high-beta firms will choose lower leverage (see Panel A), which partially offsets the previous effect. Thus, the positive relation between systematic risk and debt maturity becomes even stronger when we fix the leverage (based on interest coverage ratio) across firms (see Panel C). This result is consistent with the finding in Table 2, where the relation between long-term debt share and asset market beta becomes more positive after controlling for firm leverage.

In both Panels B and C, we see that firms choose longer debt maturity in state G than in state B, consistent with the empirical finding in Table 3. In the model, this result is driven by lower leverage when restructuring in state B (see Panel A) and an increased liquidity discount for long-term debt in state B.

4.2.1. Maturity choice when equity issuance costs are small

In our calibration, we estimate equity issuance costs $\theta = 0.65\%$ to match the positive relation between debt maturity and systematic volatility in the data. We now set equity issuance costs to a low value ($\theta = 0.1\%$) and examine its impact on the capital structure decisions.

With lower equity issuance costs, the average market leverage of the benchmark firm rises from 29% (with $\theta = 0.65\%$) to 41%. Despite the higher leverage, the 10-year default rate falls from 5.0% to 3.3%. The combination of high leverage and low default risk is achieved by choosing a very short debt maturity: the effective maturity at issue is less than 9 months in both macro states.

[**Insert Fig. 8 near here**]

Fig. 8 illustrates how shorter debt maturity enhances the flexibility in leverage adjustments. Along a common simulated path of cash flows, we compare the dynamics of default boundaries before maturity and at maturity (Panels A and B), debt level (Panel C), and interest coverage (Panel D) for two firms that face different equity issuance costs but are otherwise identical.

The firm facing lower equity issuance costs adjusts its capital structure much more frequently. This is particularly visible when examining the face value of debt in Panel C. Each time when the face value of debt changes, it corresponds to an incidence of debt restructuring. When the cash flow grows from year 0 to year 3, the firm facing low equity costs ramps up its debt level accordingly while keeping its interest coverage relatively stable. In contrast, the firm facing high equity costs does not adjust its debt level during the same period, and its interest coverage rises significantly.

The contrast in firm behavior is even more interesting during episodes of cash flow declines. As discussed in Section 3.4, issuing short term debt enables the firm to commit to adjusting its debt level frequently, which enables the firm to delever in a timely fashion. Consequently, as cash flow becomes lower, default boundaries also become lower, preventing default risk from rising too much. In contrast, the firm with long-term debt will not adjust its capital structure before debt maturity. In fact, due to the "leverage ratchet effect" (Admati et al., 2018), this firm will have no incentive to delever even if it were given the option to restructure before debt maturity.

In the simulation, the two firms were hit by a series of negative shocks to cash flows between $t = 4$ and $t = 5.5$ years and even more so between $t = 9$ and $t = 14$ years. In both episodes, the firm with short-term debt is able to delever sufficiently to avoid default. In contrast, the debt level for the firm with long-term debt remains elevated, causing its interest coverage to decline (and leverage ratio to rise). At the beginning of year 10, the two firms have similar market leverage, with that for the firm with short-term being slightly higher (37% vs 32%); by the end of year 13, the market leverage remained essentially unchanged at 34% for the firm with short-term debt, compared to 82% for the other. Finally, default is triggered at $t = 14.1$ years for the firm with long-term debt when its debt matures.

[**Insert Fig. 9 near here**]

The implications of low equity issuance costs for capital structure choices in the cross section are shown in Fig. 9. As Panel A shows, the average maturities of all the firms are now below 0.7 years, and the relation between firms' systematic risk and maturity choice is negative. This is because high-beta firms place a greater value on having a flexible capital structure and therefore choose shorter debt maturity. Panel B shows that the negative relation between leverage and systematic risk still holds under low equity issuance costs, the same as under baseline calibration (see Panel A of Fig. 7), although all the firms choose higher debt level than before.

4.3. Maturity dynamics and credit risk

So far we have focused on how systematic risk and liquidity frictions affect the capital structure choices and debt maturity in particular. Existing structural credit risk models mostly consider the setting of time-invariant maturity structures (many of them only consider perpetual debt), yet it is quite intuitive that maturity dynamics can have significant impact on credit risk. Our structural model not only provides a tractable framework to analyze such effects, but also takes firms' endogenous responses, including both the capital structure and default policies, into account. We organize the results of our analysis around the following questions.

(i) How does debt maturity affect the term structure of credit spreads? As a first step, we examine the impact of the level of debt maturity on the term structure of credit spreads. Specifically, we exogenously set the initial debt level $(\widehat{P}_G, \widehat{P}_B)$ to be equal to that of the benchmark firm and fix the expected debt maturity to be the same in the two aggregate states ($1/m_G = 1/m_B$; this is to remove the effect of maturity dynamics). The default boundaries are then recomputed based on conditions Eq. (19) and Eq. (20). We compute the term structure of credit spreads in state G and B for different levels of debt maturity and under different leverage ratios (by varying the cash flow level).

[Insert Fig. 10 near here]

The results are shown in Fig. 10 for three levels of promised debt maturities: 2, 5, and 8 years. Panels A and B show the term structure of credit spreads for the low leverage firm (with interest coverage y/b equal to the median of the benchmark firm's interest rate coverage distribution). The credit spread curve is upward sloping. The incremental effect of shortening the debt maturity on credit spreads is ambiguous and depends on the horizon. Reducing the promise maturity from 8 to 2 years increases short-term (up to 5 years) credit spreads by up to 6 bps in state G and 8 bps in state B. The effect is rather small at the 1 to 2 year horizon, which is mainly due to the diffusion assumption for cash flows (see the earlier discussion in

Section 4.1.3). On the long end (beyond 10 years), shortening the debt maturity decreases credit spreads. The same 8-year to 2-year decrease in expected debt maturity decreases long-term credit spreads by up to 87 bps in state G and by up to 96 bps in state B.

What accounts for the opposite responses of credit spreads to maturity changes at short and long horizons? On the one hand, a shorter debt maturity increases the likelihood of debt repayment in the short run, which increases default risk. On the other hand, a shorter debt maturity results in more frequent debt adjustments. Therefore, conditional on survival in the short run, a firm with shorter maturity can have lower default risk in the longer run.

Panels C and D of Fig. 10 show the term structure of credit spreads for a high leverage firm (with interest coverage at the 10th percentile of the benchmark firm's interest rate coverage distribution). The credit curve becomes hump shaped, with the downward-sloping portion becoming more prominent as debt maturity shortens. The increase in credit spreads at the shorter horizons is more significant for the high-leverage firm when debt maturity shortens. The effect is the strongest at the horizon of around 1.5 years, where the credit spreads rise by up to 234 bps in state G and up to 525 bps in state B when reducing the expected debt maturity from 8 to 2 years. Intuitively, when leverage is high, the increase in short-term default risks associated with a shorter debt maturity becomes more prominent in determining the shape of the credit curve.

With high leverage, the model is also able to generate higher credit spreads as well as stronger responses of the credit spreads to changes in debt maturity at very short horizons. For example, at the 6-month horizon, the decrease in expected debt maturity from 8 to 2 years raises credit spreads by 88 bps in state G and 297 bps in state B.

It is well known that structural models can generate an upward-sloping term structure of credit spreads for low-leverage firms and a downward-sloping term structure for high-leverage firms. The new finding here is that changing the level of debt maturity not only affects the level of credit spreads, but also changes the shape of the credit curve. It does so by affecting short-term and long-term credit spreads differently, through its effects on the short-run burden of debt repayment and the flexibility of leverage adjustments in the longer run.

(ii) How much can maturity structures affect the sensitivity of credit spreads to aggregate shocks? As shown in Section 2.3 and Fig. 3, firms with more long-term debt coming due in the 2008 financial crisis experienced significantly larger increases in credit spreads during the crisis, and the effect is stronger for firms with high leverage and high cash-flow beta. We design the following thought experiment in the model to examine this question.

Consider two firms with the same cash flow process as the benchmark firm. The two firms have the same capital structure policy, with initial debt level $(\widehat{P}_G, \widehat{P}_B)$ and relevering thresholds (γ_G, γ_B) equal to that of the benchmark firm, and promised maturities of $1/m_G = 8$ and $1/m_B = 2$.[21] Again, the default boundaries are chosen optimally based on conditions Eq. (19) and Eq. (20). The only difference between the two firms is the restructuring state $\mathcal{R}_t$: due to the different realizations of the idiosyncratic shocks in the past, Firm 1 last restructured in state $\mathcal{R}_t = G$ and thus currently has longer-term debt (with maturity intensity m_G) and debt level $y_0\widehat{P}_G$, while Firm 2 last restructured in state $\mathcal{R}_t = B$ and currently has shorter-term debt (with maturity intensity m_B) and debt level $y_0\widehat{P}_B$. Finally, by choosing the appropriate y_t for the two firms, we keep their current interest coverage to be the same.

Now consider an instantaneous transition of the macro state from G to B. By comparing the changes in credit spreads for the two firms following this aggregate shock, we can quantify how much the different maturity structures at the time of the aggregate shock helps dampen/amplify the shock's effect on credit spreads.

[Insert Fig. 11 near here]

Fig. 11 plots the changes in credit spreads for the two firms at different horizons following the macro shock. In Panel A, the two firms have low ex-ante leverage (with interest coverage at the median level for the unconditional distribution of the benchmark firm). The impact of the macro shock on credit spreads is similar between the two firms and small across different horizons. In Panel B, where leverage is high (with interest coverage at the 10th percentile for the unconditional distribution of the benchmark firm), the impact of the macro shock is much larger. This is especially the case for Firm 2 (dash-dotted line), which has shorter debt maturity when the macro shock arrives. Specifically, Firm 1's credit spread rises by up to 179 bps, while Firm 2's credit spread rises by up to 639 bps, and the differences between the two firms are the most pronounced at the short horizons (1-4 years). These results are consistent with the empirical findings in Panel A of Fig. 3. It shows that short maturity can amplify the sensitivity of credit spreads to aggregate shocks, especially at the shorter horizon and for firms with higher leverage.

Besides higher leverage, our empirical results in Panel B of Fig. 3 also show that short maturity amplifies the response of credit spreads to aggregate shocks more for firms with higher systematic risk exposure. In Panel C of Fig. 11, we raise the average systematic volatility for the two firms above from the level of the benchmark firm (13.9%) to 21% (as we did in Fig. 7). The rest of the firm parameters and the financial policies (including interest coverage) are the same as those in Panel A. Indeed, with higher systematic volatility, the

[21] We choose a wider spread in debt maturity between the two states than the values chosen by the benchmark firm to better highlight the maturity effects.

responses of credit spreads to the macro shock become much stronger, especially for the firm with short-term debt at the time of the shock.

5. Concluding remarks

In this paper, we provide new empirical evidence linking firms' maturity choices to their systematic risk exposures and macroeconomic conditions. We then build a dynamic capital structure model to explain the joint dynamics of leverage and debt maturity over the business cycle, as well as their implications for the term structure of credit risk.

While we have focused on debt maturity in this paper, there are several other dimensions in which firms can manage their exposures to macroeconomic risks, such as cash holding, lines of credit, payout, and real investments (see, for example, Bolton et al., 2011, 2013; Acharya et al., 2013; Hugonnier et al., 2015). To understand the data, ultimately we need to jointly study the firms' decisions in all these dimensions. Moreover, our modeling of the maturity structure dynamics is still fairly stylized. It would be useful to extend the model to capture richer maturity structure dynamics.

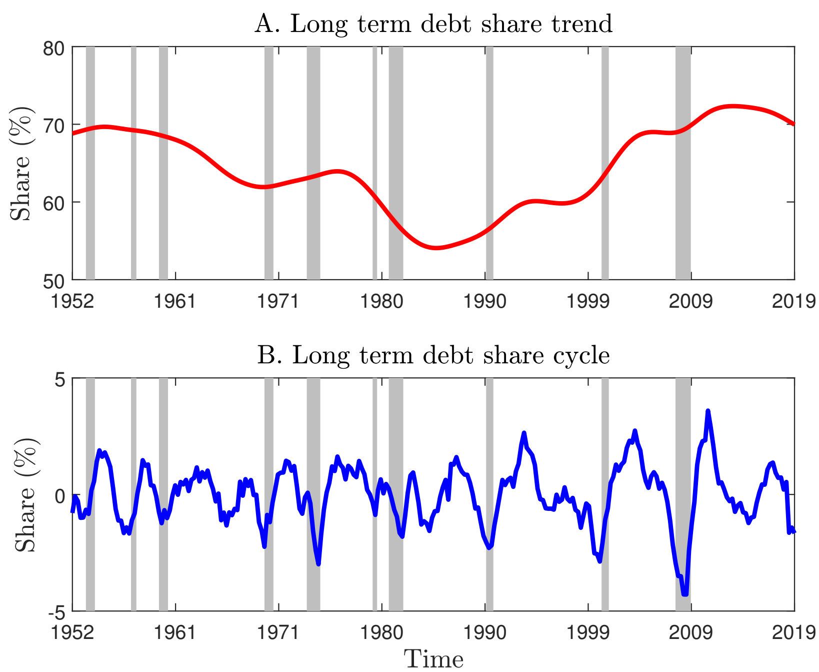

Fig. 1. **Long-term debt share for nonfinancial corporate business: 1952Q1–2018Q4.** The top panel plots the trend component (via the Hodrick-Prescott filter) of aggregate long-term debt share. The bottom panel plots the cyclical component. The shaded areas denote NBER-dated recessions. Source: Flow of Funds Accounts (Table L.103).

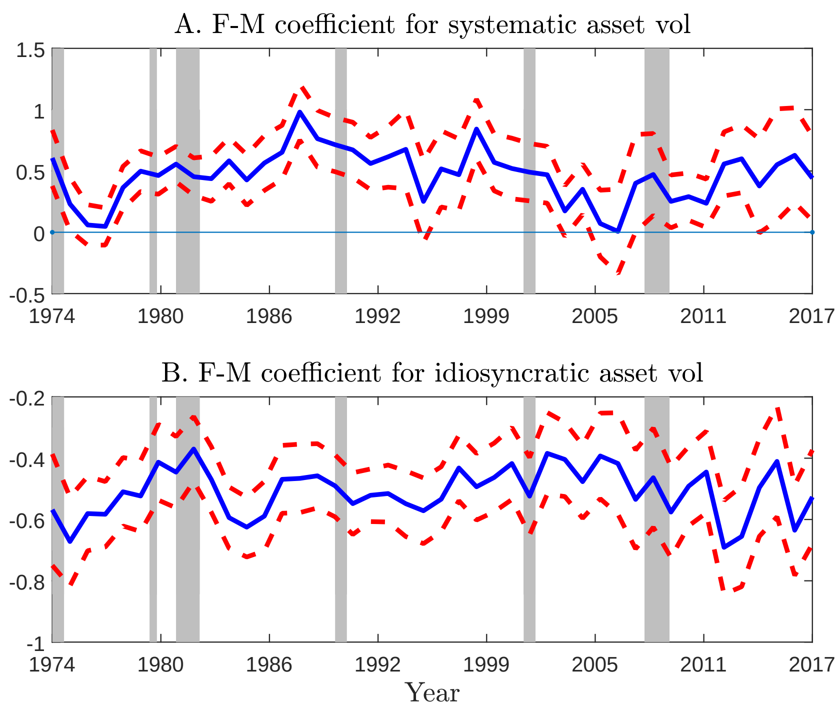

Fig. 2. **Time series of Fama-MacBeth coefficients for systematic and idiosyncratic volatility.** This graph plots time series of coefficient estimates in a cross-sectional regression of long-term debt shares on systematic and idiosyncratic asset volatility. The confidence intervals are at 95% level. The shaded areas denote NBER-dated recessions.

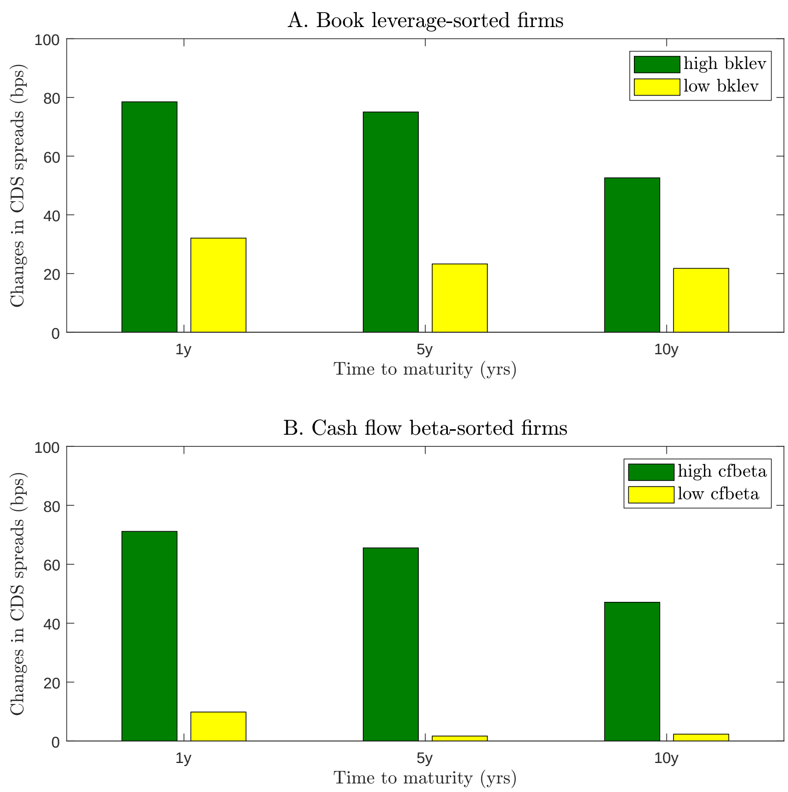

Fig. 3. **Impact of long-term debt structure on credit spreads.** This graph shows the impact of a one-standard deviation increase in the proportion of long-term debt that matures in 2008 on the changes in the CDS spreads between 2007 and 2008. Panels A and B display the result for firms sorted on book leverage and cash-flow beta, respectively.

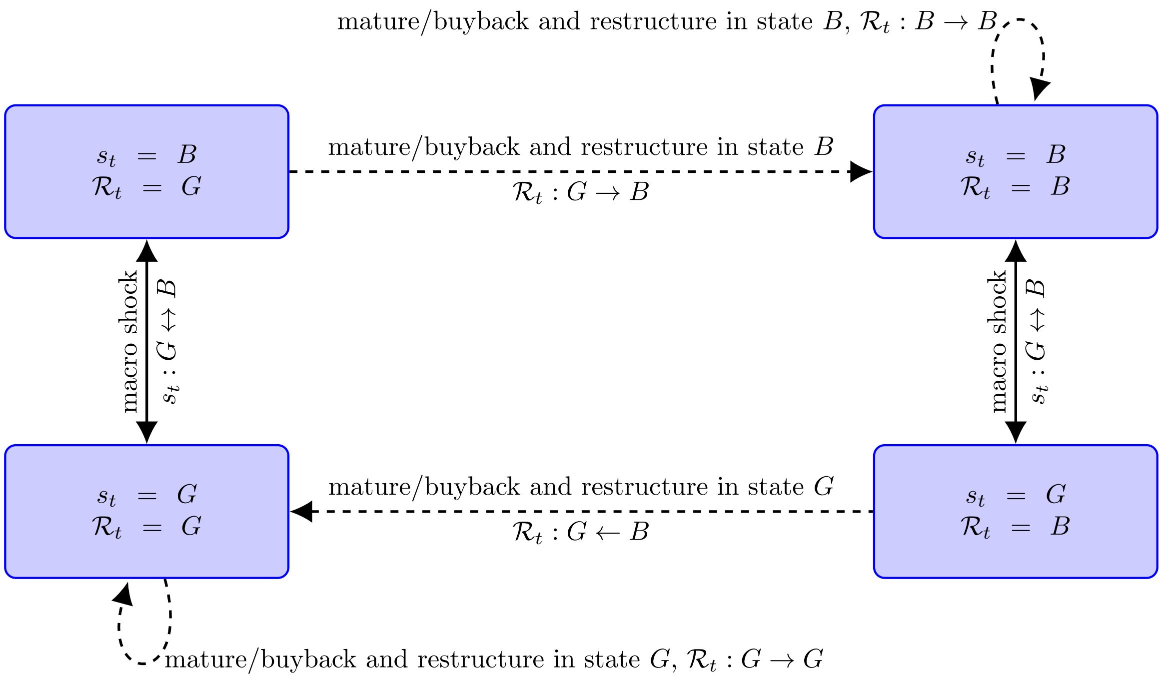

Fig. 4. **Transitions of the macro and restructuring states.** Solid arrows denote transitions due to a change in the macroeconomic state s_t; dashed arrows denote transitions due to successful debt restructuring after outstanding debt either matures or is called back, which can potentially change the restructuring state $\mathcal{R}_t$.

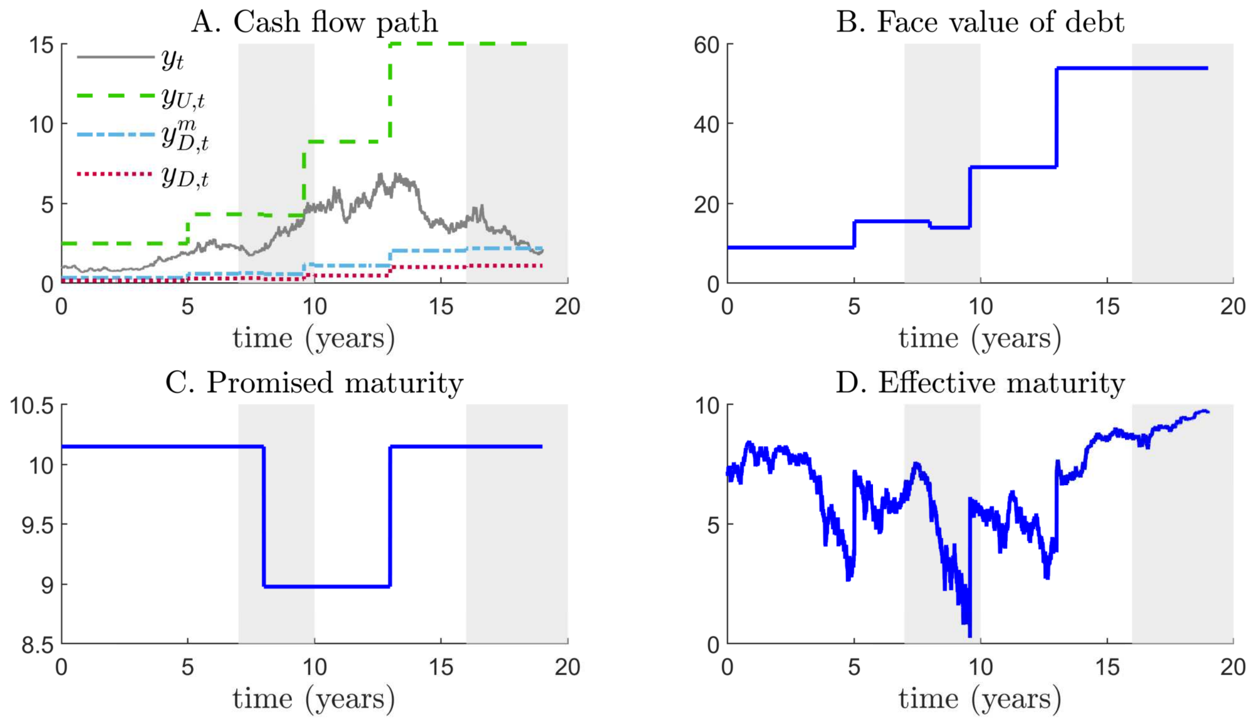

Fig. 5. **Simulated path of cash flows and debt maturity.** Panel A plots a simulated cash flow path y_t along with the default boundaries in and out of debt maturity dates, $y_{D,t}$ and $y^m_{D,t}$, respectively, and the restructuring boundary $y_{U,t}$. Panel B plots the face value of debt (P_t). Panels C and D plot promised and effective maturity, $1/m_t$ and $M_{eff,t}$, respectively. Shaded bands indicate times during which the macroeconomic state is B.

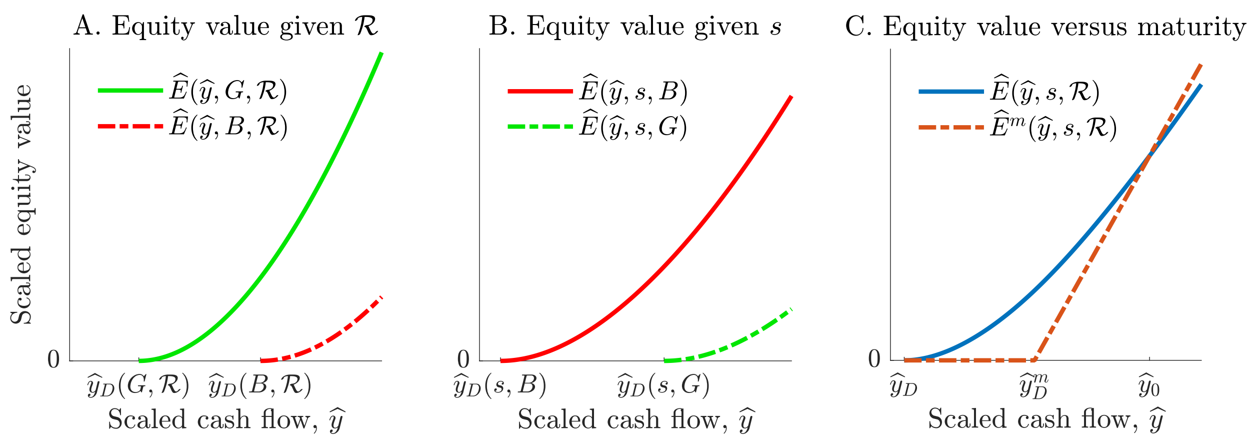

Fig. 6. **Ordering of default boundaries.** Panel A plots the equity value in both macro states after fixing the restructuring state. Panel B plots the equity value in the same macro state, but for capital structures chosen from two different restructuring states. Panel C compares the equity value before and upon debt maturing for a given augmented state.

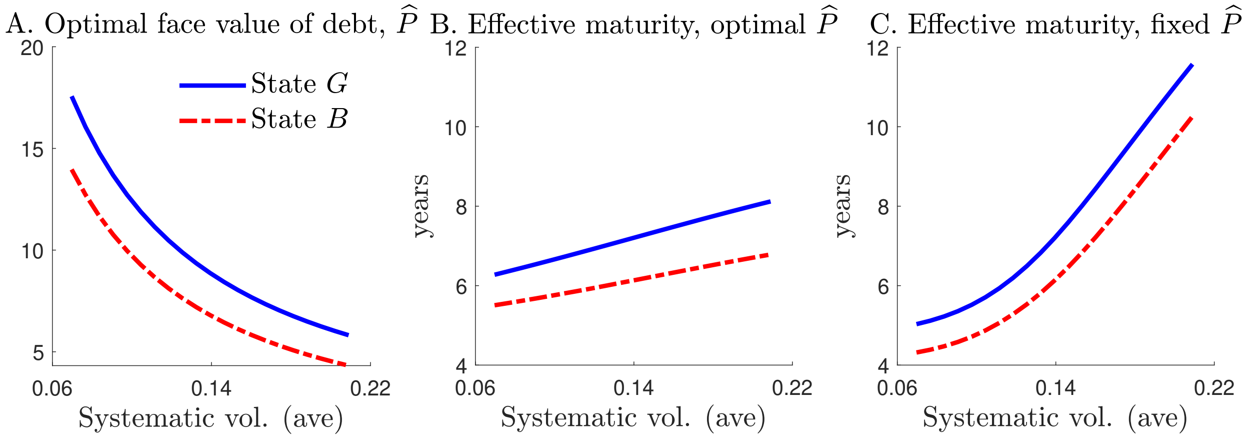

Fig. 7. **Optimal debt maturity.** We fix the idiosyncratic cash flow volatility while letting the systematic cash flow volatility vary and examine the resulting capital structure choices. Panels A plots the face value of debt for restructuring in the two states. Panels B plots the corresponding effective maturity at the time of restructuring. Panels C plots the effective maturity after fixing the debt levels at restructuring for all firms to be equal to that of the benchmark firm in each macro state. The benchmark firm has an average systematic volatility of 0.139 and asset beta of 0.8.

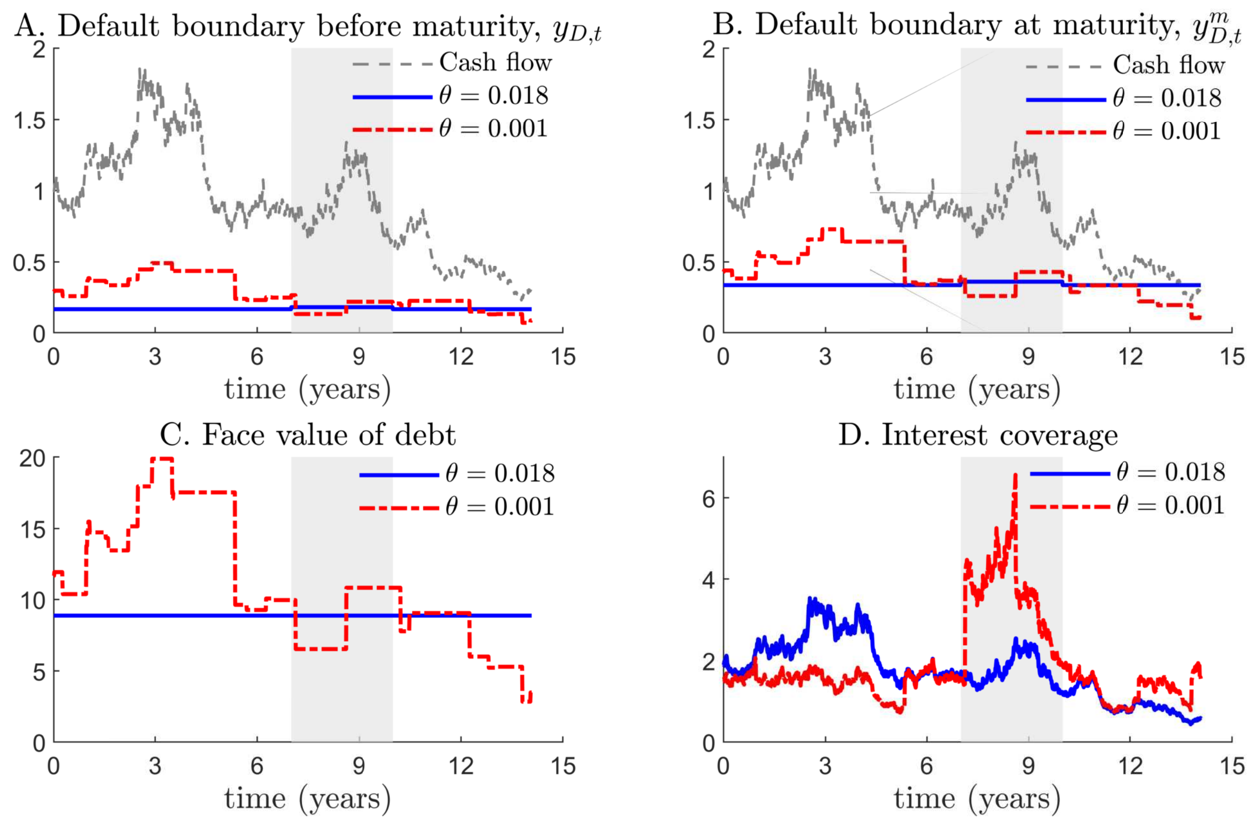

Fig. 8. **Leverage and default outcomes: baseline vs. low equity issuance cost.** Panel A compares the default boundary before maturity ($y_{D,t}$) for the benchmark firm facing high equity issuance costs ($\theta = 0.65\%$) to that of an otherwise identical firm facing low equity issuance costs ($\theta = 0.1\%$); the cash flow realizations are identical across the two firms. Panels B, C, and D plot the corresponding comparisons for the default boundary at maturity ($y^m_{D,t}$), the face value of debt (P_t), and the interest coverage ratio (y_t/b_t), respectively. Shaded bands indicate times during which the macroeconomic state is B.

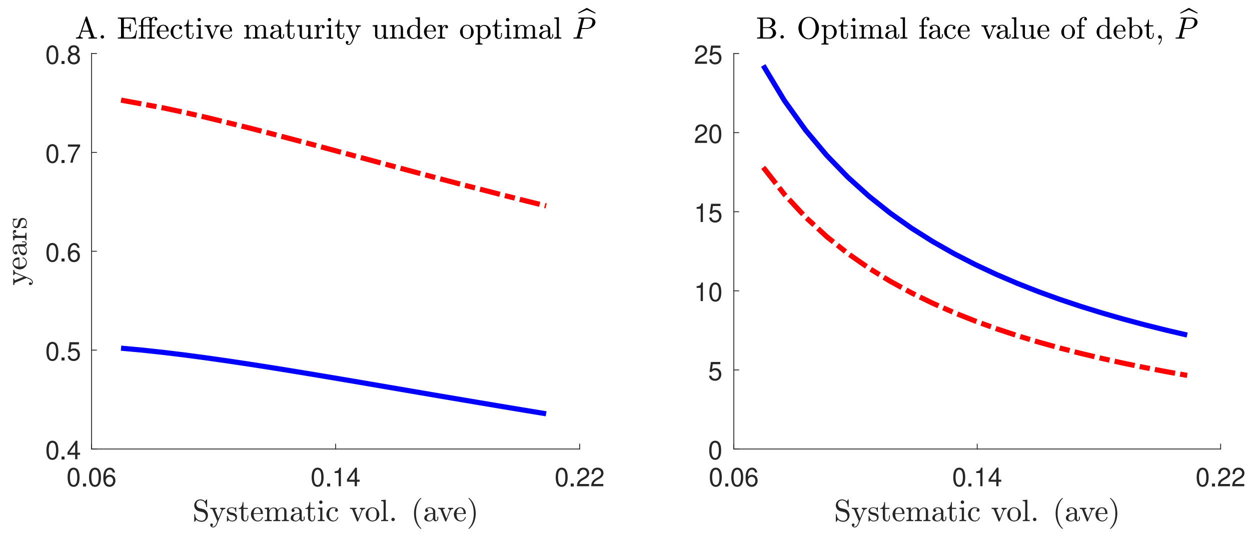

Fig. 9. **Optimal debt maturity for low equity issuance costs.** We fix the idiosyncratic cash flow volatility while letting the systematic cash flow volatility vary and examine the resulting capital structure choices. Panel A plots the optimal choice of effective maturity for debt issued in the two states. Panel B plots the corresponding optimal face value of debt. The benchmark firm has an average systematic volatility of 0.139 and asset beta of 0.8. The equity issuance costs parameter is set to $\theta = 0.1\%$.

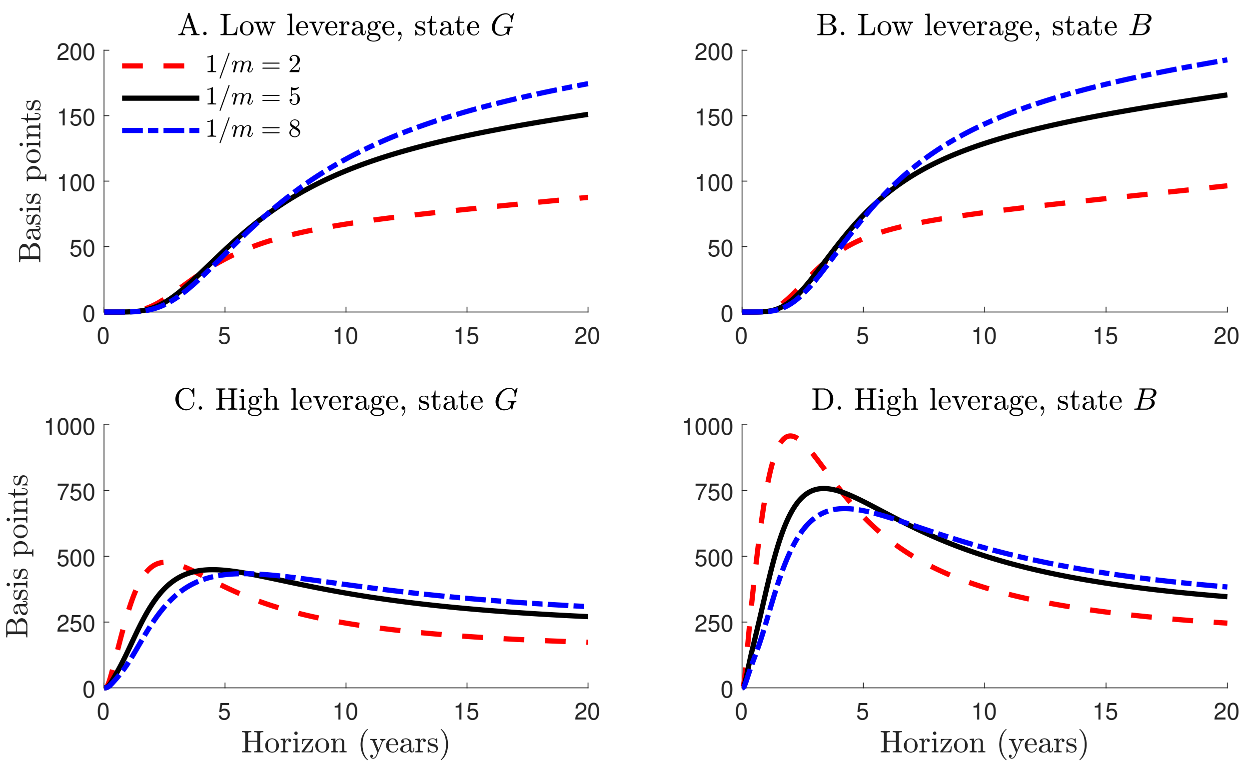

Fig. 10. **Maturity choice and the term structure of credit spreads.** This figure plots the term structure of credit spreads as debt maturity choice varies. Debt maturity choice is fixed across states so that $1/m_G = 1/m_B$. In Panels A and B, the firm's initial interest coverage is fixed at 1.83 (median). In Panels C and D, the firm's initial interest coverage is fixed at 0.84 (10th percentile). The restructuring state is $\mathcal{R} = G$ in all plots.

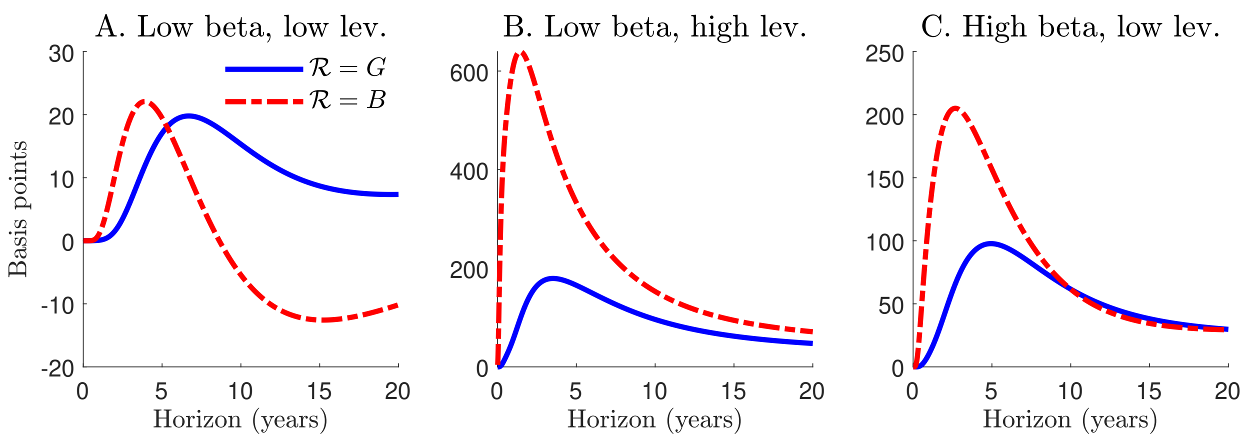

Fig. 11. **The amplification effect of pro-cyclical maturity on credit spreads.** This figure plots the changes in credit spreads following a recessionary shock for both Firm 1 (solid line) and Firm 2 (dash-dotted line). Panels A and B are for initial values of interest coverage at the 50th and 10th percentiles of the benchmark firm's unconditional interest coverage distribution, respectively. Panel C is for a high beta firm (average systematic volatility of 21%) with interest coverage at the 50th percentile.

Table 1: **Summary statistics and correlations.** This table presents summary statistics of firm-level variables (Panel A) and Pearson correlations among risk measures (Panel B). *ldebtny* is the percentage of total debt that matures more than n year(s), with $n = 1, 2, 3, 4, 5$. *mkat* is firm size computed as the logarithm of the GDP deflator-adjusted market value of total assets, defined as the sum of the book value of debt and the market value of equity. *abnearn* is abnormal earnings computed as earnings per share in year $t + 1$ minus earnings per share in year t, divided by year t share price. *bklev* is the book leverage defined as the ratio of total debt to total assets. *mk2bk* is the market-to-book ratio computed as the market value of total assets divided by the book value of total assets. *assetmat* is the value-weighted average of the maturities of current assets and property, plant and equipment. *profitvol* is the volatility of past 5 years of profit growth computed as the earnings growth divided by total assets. *equitybeta* is computed using past 36 months of equity returns and value-weighted market returns. *mktbeta*, *bankbeta*, and *tailbeta*, respectively, are the Merton model implied asset beta computed using equity market beta, equity bank beta, and equity tail beta following Acharya et al. (2013). *cfbeta* is the cash flow beta computed as the covariance between firm-level and aggregate cash flow changes divided by the variance of aggregate cash flow changes using past 20 years of data. *assetvol* is a measure of asset volatility computed using equity volatility and a Merton formula. *sysvol* and *idiovol* are the measure of firm-level systematic and idiosyncratic volatility of asset returns respectively.

A. Summary statistics

	mean	std	median	25%	75%	obs
ldebt1y	0.748	0.285	0.866	0.633	0.962	104,869
ldebt2y	0.644	0.309	0.741	0.455	0.893	79,422
ldebt3y	0.538	0.321	0.599	0.275	0.804	79,422
ldebt4y	0.443	0.317	0.461	0.136	0.703	79,422
ldebt5y	0.351	0.300	0.325	0.037	0.588	79,422
mkat	5.895	2.118	5.727	4.273	7.391	103,266
abnearn	0.013	0.256	0.008	-0.036	0.047	93,510
bklev	0.300	0.173	0.272	0.166	0.401	104,869
mk2bk	1.615	1.170	1.245	0.967	1.782	103,266
assetmat	8.221	6.309	6.661	3.865	10.610	100,830
profitvol	0.061	0.063	0.041	0.023	0.073	75,948
equitybeta	1.143	0.747	1.081	0.669	1.538	73,425
mktbeta	0.894	0.632	0.818	0.469	1.220	73,425
bankbeta	0.525	0.447	0.489	0.245	0.767	73,425
tailbeta	0.703	0.599	0.662	0.305	1.051	73,102
cfbeta	1.709	2.529	1.241	0.017	3.126	23,504
assetvol	0.378	0.184	0.332	0.25	0.455	73,425
sysvol	0.137	0.094	0.120	0.069	0.186	73,425
idiovol	0.340	0.180	0.295	0.215	0.416	73,425

B. Correlations among risk measures

	equitybeta	*mktbeta*	*bankbeta*	*tailbeta*	*cfbeta*
mktbeta	0.914				
bankbeta	0.695	0.749			
tailbeta	0.400	0.496	0.360		
cfbeta	0.112	0.085	0.062	0.016	
sysvol	0.773	0.833	0.694	0.432	0.090

Table 2: **Debt maturity and systematic risk.** This table reports the results of panel regressions of the long-term debt share on various measures of firms' systematic risk exposures. Industry and year fixed effects along with additional firm level controls are also included in the regressions. Standard errors of the coefficients are clustered at the industry and year level and adjusted for heteroskedasticity. The t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
mktbeta	0.025***	0.083***	0.097***		0.045***			
	(3.72)	(13.28)	(14.49)		(8.83)			
sysvol				0.492***				
				(12.52)				
idiovol				-0.486***				
				(-29.49)				
bankbeta						0.044***		
						(4.88)		
tailbeta							0.041***	
							(6.96)	
cfbeta								0.005***
								(3.13)
assetvol		-0.509***	-0.493***		-0.057***	-0.007	-0.013	
		(-27.65)	(-27.17)		(-2.60)	(-0.30)	(-0.62)	
mkat					0.048***	0.050***	0.048***	0.045***
					(14.76)	(15.09)	(14.97)	(10.62)
abnearn					-0.018***	-0.018***	-0.017**	-0.035***
					(-2.71)	(-2.81)	(-2.55)	(-3.89)
bklev			0.236***	0.242***	0.262***	0.254***	0.258***	0.275***
			(6.58)	(6.76)	(7.25)	(7.06)	(6.85)	(7.26)
mk2bk					-0.020***	-0.019***	-0.022***	-0.042***
					(-4.47)	(-4.09)	(-4.64)	(-5.22)
assetmat					0.005***	0.005***	0.005***	0.002***
					(7.56)	(7.36)	(7.62)	(2.68)
profitvol					-0.333***	-0.338***	-0.349***	-0.330***
					(-3.66)	(-3.74)	(-3.79)	(-2.67)
N	58,785	58,785	54,082	54,082	48,200	48,200	48,057	16,810
R^2	0.086	0.140	0.151	0.151	0.228	0.226	0.228	0.193

Table 3: **Debt maturity: impact of macroeconomic conditions.** This table presents panel regressions of long-term debt shares on various measures of firms' systematic risk exposures, a recession dummy, and their interactions. Additional firm level controls along with industry fixed effects and a quadratic time trend are also included in the regressions. The recession dummy is firm-specific and equals one if the fiscal year-end month is in an NBER recession and zero otherwise. The regression is estimated for both the full sample and a sub-sample excluding the 2008 financial crisis (2007-2009). Standard errors of the coefficients are clustered at the industry and year level and adjusted for heteroskedasticity. The t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

	Full sample: 1974-2017			Sub-sample: excluding 2007-2009		
	(1)	(2)	(3)	(4)	(5)	(6)
rec	-0.044***	-0.038***	-0.035**	-0.053***	-0.053***	-0.036**
	(-3.21)	(-2.77)	(-2.40)	(-4.42)	(-3.87)	(-2.09)
mktbeta	0.051***			0.053***		
	(9.34)			(9.02)		
$mktbeta \times rec$	0.026**			0.035***		
	(2.16)			(8.89)		
bankbeta		0.049***			0.050***	
		(5.31)			(5.27)	
$bankbeta \times rec$		0.037*			0.056***	
		(1.87)			(5.17)	
tailbeta			0.045***			0.046***
			(6.55)			(6.34)
$tailbeta \times rec$			0.015**			0.013**
			(2.24)			(2.25)
assetvol	-0.106***	-0.055**	-0.057**	-0.101***	-0.052**	-0.049**
	(-4.24)	(-2.33)	(-2.42)	(-4.15)	(-2.26)	(-2.17)
mkat	0.047***	0.049***	0.046***	0.046***	0.048***	0.045***
	(14.94)	(15.14)	(14.86)	(14.50)	(14.84)	(14.57)
abnearn	-0.019**	-0.020***	-0.018**	-0.017**	-0.018**	-0.016**
	(-2.52)	(-2.64)	(-2.40)	(-2.21)	(-2.28)	(-2.07)
bklev	0.256***	0.243***	0.248***	0.254***	0.242***	0.244***
	(7.07)	(6.71)	(6.53)	(7.00)	(6.68)	(6.41)
mk2bk	-0.019***	-0.016***	-0.020***	-0.019***	-0.016***	-0.020***
	(-4.34)	(-3.70)	(-4.42)	(-4.55)	(-3.88)	(-4.62)
assetmat	0.005***	0.005***	0.005***	0.005***	0.005***	0.005***
	(7.58)	(7.31)	(7.59)	(7.73)	(7.45)	(7.75)
profitvol	-0.300***	-0.305***	-0.317***	-0.318***	-0.319***	-0.336***
	(-3.57)	(-3.67)	(-3.77)	(-3.96)	(-4.01)	(-4.17)
N	48,200	48,200	48,057	45,575	45,575	45,435
R^2	0.222	0.219	0.220	0.225	0.222	0.223

Table 4: **Long-term debt structure and credit spreads.** This table presents cross-sectional regressions of yearly changes of CDS spreads from fiscal year 2007 to 2008 on the proportion of firms' long-term debt maturing in 2008. Additional firm level controls (asset market beta, asset volatility, firm size, market leverage, market-to-book ratio, profit, tangibility, past 12-month equity return and S&P credit rating) along with industry fixed effects are also included in the regressions. The regressions are estimated for the entire sample and separately for sub-samples of firms formed on the basis of firm characteristics at the end of fiscal year 2007. For the three firm characteristics, the sub-samples comprise firms with market leverage, book leverage, and cash flow beta above and below the sample median, respectively. Standard errors of the coefficients are adjusted for heteroskedasticity. The t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

	All	Book leverage		Market leverage		Cash flow beta	
		High	Low	High	Low	High	Low
				A. Changes in 1-year CDS spreads			
*ldebt*08	0.044***	0.068*	0.031**	0.111**	0.013	0.070***	0.012
	(2.66)	(1.74)	(2.04)	(2.03)	(1.51)	(3.07)	(0.51)
N	272	130	142	130	142	83	89
R^2	0.533	0.597	0.391	0.574	0.371	0.682	0.612
				B. Changes in 5-year CDS spreads			
*ldebt*08	0.035***	0.065**	0.022	0.103**	0.010	0.065***	0.002
	(2.64)	(2.25)	(1.57)	(2.39)	(1.06)	(2.96)	(0.12)
N	284	139	145	137	147	88	91
R^2	0.518	0.579	0.361	0.556	0.367	0.679	0.619
				C. Changes in 10-year CDS spreads			
*ldebt*08	0.029**	0.046*	0.021*	0.075**	0.009	0.046***	0.003
	(2.56)	(1.91)	(1.69)	(2.01)	(1.29)	(2.77)	(0.21)
N	273	132	141	130	143	85	86
R^2	0.537	0.611	0.326	0.577	0.386	0.697	0.624

Table 5: **Baseline model parameters.** Panel A summarizes the exogenously specified parameters. Exogenously fixed parameters that do not vary across the states include: $\kappa = \ln 2.5$, $\tau = 0.2$. All parameters are annualized if applicable. The asset beta for the unlevered firm is 0.8 and is calculated based on a market dividend process with a with a leverage ratio of $\phi = 1.25$ relative to the cashflows of the baseline firm. Panel B summarizes the estimated parameters. Panel C compares various model-implied moments with their targeted counterparts.

A. Exogenously specified parameters		
	State G	State B
Aggregate state transition intensities: $\overline{\pi}_s$	0.1	0.5
Riskfree rate: r_s	5.6%	2.6%
Market price of Brownian risk: η_s	0.16	0.24
Cash flow expected growth rate: $\overline{\mu}_s$	6.2%	1.6%
Cash flow systematic volatility: $\sigma_{\Lambda,s}$	13.6%	15.6%
Recovery rate: α_s	0.7	0.6
B. Estimated parameters		
Liquidity cost parameter, state G: ℓ_G	0.0203×10^{-4}	
Liquidity cost parameter, state B: ℓ_B	0.1828×10^{-4}	
Cash flow idiosyncratic volatility: σ_f	0.2683	
Equity issuance cost: θ	0.0065	

Table 6: **Parameters vs. targeted moments.** This table illustrates the dependence of targeted moments on various parameters. Targeted moments include (1) average market leverage for a BBB rated firm, (2) average 10 year default rate for a BBB rated firm, (3,4) the effective maturity at issue in states G and B, and (5,6) the difference in the effective maturities at issue between the high- and low-beta firm in states G and B.

	(1) Ave. mkt. leverage	(2) Ave. 10y default	(3) Maturity, state G	(4) Maturity, state B	(5) Diff. mat., state G	(6) Diff. mat., state B
Target moments	0.300	5.00	7.00	6.50	1.10	1.50
Baseline model	0.291	4.95	7.20	6.13	1.85	1.28
			Sensitivity analysis			
(1a) $\sigma_f = 0.75\times$Base	0.310	1.95	6.35	5.46	2.19	1.41
(1b) $\sigma_f = 1.25\times$Base	0.280	10.39	7.99	6.78	1.39	0.98
(2a) $\theta = 0.75\times$Base	0.296	5.18	6.57	5.57	1.97	1.35
(2b) $\theta = 1.25\times$Base	0.286	4.70	7.72	6.61	1.76	1.22
(3a) $\ell_G = 0.75\times$Base	0.292	4.93	7.28	6.14	1.86	1.28
(3b) $\ell_G = 1.25\times$Base	0.290	4.88	7.12	6.11	1.83	1.28
(4a) $\ell_B = 0.75\times$Base	0.296	5.04	7.73	6.74	1.92	1.33
(4b) $\ell_B = 1.25\times$Base	0.287	4.77	6.78	5.66	1.78	1.22

Table 7: **Baseline model results.** This table summarizes results from the baseline calibration.

Capital structure decision for the benchmark firm	Restructuring state	
	$\mathcal{R} = G$	$\mathcal{R} = B$
Interest coverage at issue: $y_0/b(y_0, \mathcal{R})$	1.91	3.09
Market leverage at issue: $D_0/(D_0 + E_0)$	33.1%	27.2%
Promised maturity: $1/m_{\mathcal{R}}$	10.15	8.98
Effective maturity at issue: $\mathbb{E}_0[\min\{\tau_M, \tau_U\}]$	7.20	6.13
Scaled default boundary: $\widehat{y}_D(G, \mathcal{R})$	0.165	0.110
Scaled default boundary: $\widehat{y}_D(B, \mathcal{R})$	0.179	0.120
Scaled default boundary upon debt maturing: $\widehat{y}^m_D(G, \mathcal{R})$	0.333	0.255
Scaled default boundary upon debt maturing: $\widehat{y}^m_D(B, \mathcal{R})$	0.358	0.274
Scaled restructuring boundary: $\widehat{y}_U(\mathcal{R})$	2.470	2.074

Moments averaged over BBB firms	Aggregate state	
	$s = G$	$s = B$
Market leverage: $D/(D + E)$	28.5%	32.1%
Expected 5 year default rate	0.6%	0.9%
Expected 10 year default rate	4.8%	5.5%
5 year credit spread (default component; in bps)	23.0	39.0
5 year credit spread (total; in bps)	23.5	40.3
10 year credit spread (default component; in bps)	90.3	108.7
10 year credit spread (total; in bps)	92.8	113.5
Conditional equity Sharpe ratio	10.9%	19.8%

Appendix A. Model solution

In this section, we provide details for solving the model. Throughout this section, we work with the log scaled cash flow $\widehat{x}_t \equiv \log \widehat{y}_t$, which has initial value $\widehat{x}_0 = 0$. Appendix A.1 characterizes the solution for a given capital structure. Appendix A.2 provides a numerical algorithm for finding the optimal time-consistent capital structure corresponding to the Nash equilibrium Definition 1. To simplify notation in this section, we define the differential operator

$$\begin{aligned} \mathscr{L}^{\mathcal{P}} f(\widehat{x}, s, \mathcal{R}) \;\; &\equiv \;\; \left(\overline{\mu}_s - \frac{1}{2}\sigma_s^2\right) \frac{\partial}{\partial \widehat{x}} f(\widehat{x}, s, \mathcal{R}) + \frac{1}{2}\sigma_s^2 \frac{\partial^2}{\partial \widehat{x}^2} f(\widehat{x}, s, \mathcal{R}) \\ &+ \overline{\pi}_s \left[f(\widehat{x}, s^c, \mathcal{R}) - f(\widehat{x}, s, \mathcal{R})\right], \end{aligned} \tag{A.1}$$

along with its risk-neutral counterpart

$$\begin{aligned} \mathscr{L}^{\mathcal{Q}} f(\widehat{x}, s, \mathcal{R}) \;\; &\equiv \;\; \left(\mu_s - \frac{1}{2}\sigma_s^2\right) \frac{\partial}{\partial \widehat{x}} f(\widehat{x}, s, \mathcal{R}) + \frac{1}{2}\sigma_s^2 \frac{\partial^2}{\partial \widehat{x}^2} f(\widehat{x}, s, \mathcal{R}) \\ &+ \pi_s \left[f(\widehat{x}, s^c, \mathcal{R}) - f(\widehat{x}, s, \mathcal{R})\right]. \end{aligned} \tag{A.2}$$

A.1. Solution for a given capital structure

We first state a general result that will be the basis for analytically pricing both equity and debt.

Proposition 1. *Let $\mathbf{f} : \mathbb{R} \to \mathbb{R}^n$ be a vector-valued function satisfying the following system of ordinary differential equations:*

$$\mathbf{A}\mathbf{f}(x) \;\; = \;\; \mathbf{a}_0 + \sum_{i=1}^{I} g_i \left(\mathbf{a}_i e^{b_i x}\right) + \mathbf{B}\mathbf{f}'(x) + \mathbf{C}\mathbf{f}''(x), \tag{A.3}$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n\times n}$ are real-valued matrices, $\mathbf{C} \in \mathbb{R}^{n\times n}$ is a real and non-singular *matrix, $I \in \mathbb{N}$, $\mathbf{a}_0 \in \mathbb{R}^n$, $\mathbf{a}_i \in \mathbb{C}^n$, $b_i \in \mathbb{C}$, and g_i is either the operator $Re(\cdot)$ (the real part of a complex number) or $Im(\cdot)$ (the imaginary part). Then $\mathbf{f}(x)$ takes the following form:*

$$\begin{aligned} \mathbf{f}(x) \;\; &= \;\; \mathbf{f}_0 + \sum_{i=1}^{I} g_i \left(\mathbf{f}_i e^{b_i x}\right) + \sum_{j=1}^{n_{\mathbb{R}}} \omega_j^{\mathbb{R}} \mathbf{v}_j^{\mathbb{R}} e^{\lambda_j^{\mathbb{R}} x} \\ &+ \sum_{k=1}^{n_{\mathbb{C}}/2} \left(\omega_k^{\mathbb{C},Re} Re\left(\mathbf{v}_k^{\mathbb{C}} e^{\lambda_k^{\mathbb{C}} x}\right) + \omega_k^{\mathbb{C},Im} Im\left(\mathbf{v}_k^{\mathbb{C}} e^{\lambda_k^{\mathbb{C}} x}\right)\right), \end{aligned} \tag{A.4}$$

with $\mathbf{f}_i$ $(i = 0, ..., I)$ defined by

$$\mathbf{A}\mathbf{f}_0 \;\; = \;\; \mathbf{a}_0, \tag{A.5}$$

$$\left(\mathbf{A} - b_i \mathbf{B} - b_i^2 \mathbf{C}\right) \mathbf{f}_i \;\; = \;\; \mathbf{a}_i, \quad i = 1, ..., I, \tag{A.6}$$

and $\omega_j^{\mathbb{R}}, \omega_k^{\mathbb{C},Re}, \omega_k^{\mathbb{C},Im} \in \mathbb{R}$ *are real-valued coefficients.*

The pairs $\left(\lambda_j^{\mathbb{R}}, \mathbf{v}_j^{\mathbb{R}}\right)$ *are the real solutions to the following Quadratic Eigenvalue Problem (QEP):*

$$\mathbf{A}\mathbf{v} = \lambda \mathbf{B}\mathbf{v} + \lambda^2 \mathbf{C}\mathbf{v}. \tag{A.7}$$

while the pairs $\left(\lambda_k^{\mathbb{C}}, \mathbf{v}_k^{\mathbb{C}}\right)$ *are the complex solutions to the QEP with* $Im\left(\lambda_k^{\mathbb{C}}\right) > 0$*. In total, there are* $2n = n_{\mathbb{R}} + n_{\mathbb{C}}$ *pairs of such solutions (unique up to scaling) with* $n_{\mathbb{C}}$ *being even. Furthermore, in the special case where* $\mathbf{A} = (a_{ij})$ *satisfies*

$$a_{ij} \geq 0, \quad (i \neq j) \tag{A.8}$$

$$\sum_k a_{ik} < 0, \quad \forall\, i. \tag{A.9}$$

Exactly half of the eigenvalues for the QEP will be located in the left (complex) half plane with the other half in the right half plane.

Proof. The conditions Eq. (A.5), Eq. (A.6) and Eq. (A.7) are obtained after substituting Eq. (A.4) into Eq. (A.3). Next, the assumptions for $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ imply that there are $2n$ finite eigenvalues to the QEP Eq. (A.7), with eigenvalue-eigenvector pairs either being real or in conjugate pairs (see Tisseur and Meerbergen (2001)). Since the weights $\omega_k^{\mathbb{C},Re}$ and $\omega_k^{\mathbb{C},Im}$ can be freely chosen, for each conjugate pair of solutions to Eq. (A.7), we may keep the solution with $\mathrm{Im}\left(\lambda_k^{\mathbb{C}}\right) > 0$ w.l.o.g. Finally, the result for the location of the eigenvalues when conditions Eq. (A.8) and Eq. (A.9) are satisfied is shown in Barlow, Rogers, and Williams (1980). □

Next, we derive the analytical solutions to debt and equity. The solution forms are similar to those obtained in Jobert and Rogers (2006) and Chen (2010), but our characterization is more general in that it allows for (possibly) complex eigenvalue-eigenvector solutions to the QEP.

Debt and equity valuation. The system characterizing scaled debt value, $\widehat{D} \equiv D/y_0$, is given by

$$\begin{aligned} \left[r_s + l(m_{\mathcal{R}_0}, s)\right] \widehat{D}(\widehat{x}, s, \mathcal{R}) &= \widehat{b}_{\mathcal{R}_0} + m_{\mathcal{R}_0} \left[\widehat{D}^m(\widehat{x}, s, \mathcal{R}) - \widehat{D}(\widehat{x}, s, \mathcal{R})\right] \\ &\quad + \mathscr{L}^{\mathcal{Q}} \widehat{D}(\widehat{x}, s, \mathcal{R}), \text{ for } \widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R}), \end{aligned} \tag{A.10}$$

$$\widehat{D}(\widehat{x}, s, \mathcal{R}) = \alpha_s (1-\tau) v_s e^{\widehat{x}}, \text{ for } \widehat{x} \leq \widehat{x}_D(s, \mathcal{R}), \tag{A.11}$$

$$\widehat{D}(\widehat{x}, s, \mathcal{R}) = \widehat{P}_{\mathcal{R}_0}, \text{ for } \widehat{x} \geq \widehat{x}_U(\mathcal{R}), \tag{A.12}$$

where $\widehat{x}_D \equiv \log \widehat{y}_D$ denotes the log scaled default boundary, and the scaled debt value conditional on debt maturing, $\widehat{D}^m \equiv D^m/y_0$, is given by

$$\widehat{D}^m(\widehat{x}, s, \mathcal{R}) = \begin{cases} \widehat{P}_{\mathcal{R}_0} & \text{if } \widehat{x} > \widehat{x}_D^m(s, \mathcal{R}), \\ \alpha_s(1-\tau) v_s e^{\widehat{x}} & \text{if } \widehat{x} \leq \widehat{x}_D^m(s, \mathcal{R}), \end{cases} \tag{A.13}$$

where $\widehat{x}_D^m \equiv \log \widehat{y}_D^m$ is the log scaled default boundary conditional on debt maturing.

Similarly, the system characterizing scaled equity value, $\widehat{E} \equiv E/y_0$, is given by

$$\begin{aligned} r_s \widehat{E}(\widehat{x}, s, \mathcal{R}) &= (1-\tau)(e^{\widehat{x}} - \widehat{b}_{\mathcal{R}_0}) + m_{\mathcal{R}_0} \left[\widehat{E}^m(\widehat{x}, s, \mathcal{R}) - \widehat{E}(\widehat{x}, s, \mathcal{R}) \right] \\ &\quad + \mathscr{L}^{Q} \widehat{E}(\widehat{x}, s, \mathcal{R}), \text{ for } \widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R}), \end{aligned} \tag{A.14}$$

$$\widehat{E}(\widehat{x}, s, \mathcal{R}) = 0, \text{ for } \widehat{x} \leq \widehat{x}_D(s, \mathcal{R}), \tag{A.15}$$

$$\widehat{E}(\widehat{x}, s, \mathcal{R}) = e^{\widehat{x}} \left[\widehat{D}(\widehat{x}_0, s, s) + \widehat{E}(\widehat{x}_0, s, s) \right] - (1+\theta) \widehat{P}_{\mathcal{R}_0}, \text{ for } \widehat{x} \geq \widehat{x}_U(\mathcal{R}), \tag{A.16}$$

where the scaled equity value conditional on debt maturing, $\widehat{E}^m \equiv E^m/y_0$, is given by

$$\widehat{E}^m(\widehat{x}, s, \mathcal{R}) = \begin{cases} e^{\widehat{x}} \left[\widehat{D}(\widehat{x}_0, s, s) + \widehat{E}(\widehat{x}_0, s, s) \right] - (1+\theta) \widehat{P}_{\mathcal{R}_0} & \text{if } \widehat{x} > \widehat{x}_D^m(s, \mathcal{R}), \\ 0 & \text{if } \widehat{x} \leq \widehat{x}_D^m(s, \mathcal{R}). \end{cases} \tag{A.17}$$

To solve the system of equations (Eq. A.10 - Eq. A.12) and (Eq. A.14 - Eq. A.16) for a given set of default boundaries, we order the collection of ten default and restructuring boundaries $\{\widehat{x}_D(s, \mathcal{R})\}_{s, \mathcal{R} \in \{G, B\}} \cup \{\widehat{x}_{D,mature}(s, \mathcal{R})\}_{s, \mathcal{R} \in \{G, B\}} \cup \{\widehat{x}_U(\mathcal{R})\}_{\mathcal{R} \in \{G, B\}}$ in increasing order $\widehat{x}_1 \leq \widehat{x}_2 \leq \cdots \leq \widehat{x}_{10}$. This defines eleven regions $\mathcal{R}_1 = (-\infty, \widehat{x}_1)$, $\mathcal{R}_2 = (\widehat{x}_1, \widehat{x}_2)$, ..., $\mathcal{R}_{10} = (\widehat{x}_9, \widehat{x}_{10})$, and $\mathcal{R}_{11} = (\widehat{x}_{10}, +\infty)$. We denote by $\widehat{D}_{[i]}(\widehat{x}, s, \mathcal{R})$ and $\widehat{E}_{[i]}(\widehat{x}, s, \mathcal{R})$ the solution to (Eq. A.10 - Eq. A.12) and (Eq. A.14 - Eq.A.16), respectively, in region $\mathcal{R}_i$ for state $(s, \mathcal{R})$. For each region $\mathcal{R}_i$, the states $(s, \mathcal{R})$ can be partitioned into three groups. The first group consists of states $(s, \mathcal{R})$ for which default occurs throughout region $\mathcal{R}_i$ (i.e. $\widehat{x} \leq \widehat{x}_D(s, \mathcal{R})$ for all $x \in \mathcal{R}_i$). For these states, the solutions for scaled debt, $\widehat{D}_{[i]}(\widehat{x}, s, \mathcal{R})$, and equity, $\widehat{R}_{[i]}(\widehat{x}, s, \mathcal{R})$, are given by their respective boundary conditions Eq. (A.11) and Eq. (A.15). The second group consists of states $(s, \mathcal{R})$ for which restructuring occurs throughout region $\mathcal{R}_i$ (i.e. $\widehat{x} \geq \widehat{x}_U(\mathcal{R})$ for all $x \in \mathcal{R}_i$). For these states, the solutions for scaled debt and equity are given by boundary conditions Eq. (A.12) and Eq. (A.16), respectively. The third group consists of the set active states $(s, \mathcal{R})$ for which neither default nor restructuring occurs (i.e. $\widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R})$ for all $\widehat{x} \in \mathcal{R}_i$). System (Eq. A.10 - Eq.A.12) for these active states, stacked as a vector, takes the form $\mathbf{W}_{D[i]} \widehat{\mathbf{D}}_{[i]}(\widehat{x}) = \widehat{\mathbf{d}}_{0[i]} + \widehat{\mathbf{d}}_{1[i]} e^{\widehat{x}} + \mathbf{U}_{D[i]} \widehat{\mathbf{D}}'_{[i]}(\widehat{x}) + \mathbf{V}_{D[i]} \widehat{\mathbf{D}}''_{[i]}(\widehat{x})$. Similarly, the system (Eq. A.14 - Eq. A.16) for active states, stacked as a vector, also has form $\mathbf{W}_{E[i]} \widehat{\mathbf{E}}_{[i]}(\widehat{x}) = \widehat{\mathbf{e}}_{0[i]} + \widehat{\mathbf{e}}_{1[i]} e^{\widehat{x}} + \mathbf{U}_{E[i]} \widehat{\mathbf{E}}'_{[i]}(\widehat{x}) + \mathbf{V}_{E[i]} \widehat{\mathbf{E}}''_{[i]}(\widehat{x})$. The solution follows directly from Proposition 1, and takes the form

$$\begin{aligned} \widehat{\mathbf{D}}_{[i]}(\widehat{x}) &= \widehat{\mathbf{D}}_{0[i]} + \widehat{\mathbf{D}}_{1[i]} e^{\widehat{x}} + \sum_j \omega^{\mathbb{R}}_{D[i],j} \mathbf{v}^{\mathbb{R}}_{D[i],j} \exp\left(\lambda^{\mathbb{R}}_{D[i],j} \widehat{x}\right) \\ &\quad + \sum_k \left[\omega^{\mathbb{C},Re}_{D[i],k} Re\left(\mathbf{v}^{\mathbb{C}}_{D[i],k} \exp\left(\lambda^{\mathbb{C}}_{D[i],k} \widehat{x}\right)\right) + \omega^{\mathbb{C},Im}_{D[i],k} Im\left(\mathbf{v}^{\mathbb{C}}_{D[i],k} \exp\left(\lambda^{\mathbb{C}}_{D[i],k} \widehat{x}\right)\right) \right] \end{aligned} \tag{A.18}$$

for scaled debt, and

$$\widehat{\mathbf{E}}_{[i]}(\widehat{x}) = \widehat{\mathbf{E}}_{0[i]} + \widehat{\mathbf{E}}_{1[i]}e^{\widehat{x}} + \sum_j \omega^{\mathbb{R}}_{E[i],j}\mathbf{v}^{\mathbb{R}}_{E[i],j}\exp\left(\lambda^{\mathbb{R}}_{E[i],j}\widehat{x}\right) + \sum_k \left[\omega^{\mathbb{C},Re}_{E[i],k} Re\left(\mathbf{v}^{\mathbb{C}}_{E[i],k}\exp\left(\lambda^{\mathbb{C}}_{E[i],k}\widehat{x}\right)\right) + \omega^{\mathbb{C},Im}_{E[i],k} Im\left(\mathbf{v}^{\mathbb{C}}_{E[i],k}\exp\left(\lambda^{\mathbb{C}}_{E[i],k}\widehat{x}\right)\right)\right] \quad \text{(A.19)}$$

for scaled equity. Finally, the weights for debt and equity, $\left\{\omega^{\mathbb{R}}_{D[i],j}\right\}_j \cup \left\{\omega^{\mathbb{C},Re}_{D[i],k}, \omega^{\mathbb{C},Im}_{D[i],k}\right\}_k$ and $\left\{\omega^{\mathbb{R}}_{D[i],j}\right\}_j \cup \left\{\omega^{\mathbb{C},Re}_{D[i],k}, \omega^{\mathbb{C},Im}_{D[i],k}\right\}_k$, are uniquely determined by requiring that the respective solutions for scaled debt and equity be continuously differentiable across all eleven regions. That is, we require $\widehat{D}_{[i]}(\widehat{x}_i, s, \mathcal{R}) = \widehat{D}_{[i+1]}(\widehat{x}_i, s, \mathcal{R})$ to hold at all ten boundary points $\widehat{x}_i$, and for $\widehat{D}'_{[i]}(\widehat{x}_i, s, \mathcal{R}) = \widehat{D}'_{[i+1]}(\widehat{x}_i, s, \mathcal{R})$ to hold at all boundary points $\widehat{x}_i$ for which $\widehat{x}_D(s, \mathcal{R}) < \widehat{x}_i < \widehat{x}_U(\mathcal{R})$. An analogous set of requirements apply for equity.

We solve for the default boundaries via a set of nonlinear equations. In particular, conditional on debt maturing, the scaled log default boundary is given by the first instance for which Eq. (A.17) equals zero and equals $\widehat{x}^m_D(s, \mathcal{R}) = \log\left(\frac{(1+\theta)\widehat{P}_{\mathcal{R}}}{\widehat{D}(\widehat{x}_0,s,s)+\widehat{E}(\widehat{x}_0,s,s)}\right)$; when debt does not mature, the scaled log default boundary satisfies the smooth pasting condition $\widehat{E}_{\widehat{x}}(\widehat{x}_D(s, \mathcal{R}), s, \mathcal{R}) = 0$ in each of the four augmented states.

A.2. Numerical algorithm for the optimal capital structure

We use the following procedure to locate a Nash equilibrium of Definition 1.

1. Step 1: Obtaining initial guesses. We obtain initial guesses for capital structure in the two states $s \in \{G, B\}$ by solving for the optimal capital structure in a version of our model in which the macroeconomic state is permanent and never switches. The average $\boldsymbol{\chi}^0_{\mathcal{R}} = \omega_0 \boldsymbol{\chi}^{perm}_{\mathcal{R}} + (1-\omega_0)\frac{1}{2}\left(\boldsymbol{\chi}^{perm}_G + \boldsymbol{\chi}^{perm}_B\right)$ serves as the initial guess for Step 2, where $\omega_0 \in (0, 1)$ is a weighting parameter and $\boldsymbol{\chi}^{perm}_{\mathcal{R}} = (P^{perm}_{\mathcal{R}}, m^{perm}_{\mathcal{R}})$ denotes the solution to the "permanent state problem" in which the macroeconomic state never switches away from state $\mathcal{R}$.
2. Step 2: Iterate for the Nash equilibrium. Given capital structure $(\boldsymbol{\chi}^n_G, \boldsymbol{\chi}^n_B)$, obtain the optimal responses $(\boldsymbol{\chi}^{n,resp}_G, \boldsymbol{\chi}^{n,resp}_B)$ according to Definition 1 for the Nash equilibrium. Terminate this step if $(\boldsymbol{\chi}^{n,resp}_G, \boldsymbol{\chi}^{n,resp}_B)$ and $(\boldsymbol{\chi}^n_G, \boldsymbol{\chi}^n_B)$ is sufficiently close. Otherwise, update $\boldsymbol{\chi}^{n+1}_{\mathcal{R}} = \omega_{update}\boldsymbol{\chi}^{n,resp}_{\mathcal{R}} + (1-\omega_{update})\boldsymbol{\chi}^n_{\mathcal{R}}$ and continue the iteration. Here, $\omega_{update} \in (0, 1)$ is an update weight.

   In the numerical implementation of the maximization problem Eq. (27) for state s, we first use the scaling property to scale out the initial cash flow level y_0. We then enforce debt issuance at par by converting the maximization problem Eq. (27) into a constrained maximization problem by including $\widehat{D}(\widehat{x}_0, s, s; \boldsymbol{\chi}_s, \widehat{b}_s, \boldsymbol{\chi}_{s^c}, \widehat{b}_{s^c}) = \widehat{P}_s$ as an additional constraint. In the resulting constrained maximization problem, the coupon rate for state s, $\widehat{b}_s$, is treated as an additional choice variable, while the coupon rate in the complementary state, $\widehat{b}_{s^c}$, is fixed and taken from the previous iteration.

3. Step 3: Verify the Nash equilibrium. We obtain a candidate solution after Step 2 has converged. We check that the candidate solution is indeed a valid Nash equilibrium. To do so, for each capital structure choice state, we verify Definition 1 is satisfied by checking the objective function is indeed maximized at the candidate solution along all single-dimensional deviations over the choice space. The Internet Appendix illustrates this procedure for the benchmark firm.

To check for the possibility of multiple equilibria, we repeat the above algorithm for different initial starting points by varying ω_0, and the compare the final solutions. We did not find evidence for multiple equilibria using this check.

A.3. Computing additional statistics

After obtaining the optimal capital structure using the algorithm outlined in Appendix A.2, we compute a number of associated statistics for the firm. These computations involve using hte Feynman-Kac formula to convert conditional expectations to partial differential equations. We then numerically solve the resulting partial differential equations. To do so, we use finite difference methods to obtain discretized approximations to (variants of) the differential operators Eq. (A.1) and Eq. (A.2). See, for example, https://benjaminmoll.com/codes/ for an introduction to finite difference methods.

A.3.1. Stationary distribution

We require the stationary distribution in order to compute averages. The stationary distribution $f(\widehat{x}, s, \mathcal{R})$, can be obtained by solving the (stationary) Kolmogorov forward equation:

$$0 = (\mathscr{L}^{\mathcal{P},m})^{\dagger} f(\widehat{x}, s, \mathcal{R}), \tag{A.20}$$

where $(\mathscr{L}^{\mathcal{P},m})^{\dagger}$ denotes the adjoint of the differential operator $\mathscr{L}^{\mathcal{P},m}$, defined as

$$\mathscr{L}^{\mathcal{P},m} f(\widehat{x}, s, \mathcal{R}) \equiv m_{\mathcal{R}_0} \left[f(\widehat{x}_0, s, s) - f(\widehat{x}, s, \mathcal{R}) \right] + \mathscr{L}^{\mathcal{P}} f(\widehat{x}, s, \mathcal{R}) \tag{A.21}$$

for $\widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R})$. The law of motion associated with the differential operator $\mathscr{L}^{\mathcal{P},m}$ assumes that a newly defaulted firm is immediately reorganized. This implies that points at and below the default boundary $\widehat{x}_D(s, \mathcal{R})$ are reflected to $\widehat{x}_0$ and the state is reset to (s, s) when this occurs. In addition, maturing debt results either in a default or debt repayment and a subsequent restructuring; in either case, the log cash flow will be reset to $\widehat{x}_0$ and the state being set to (s, s). Similarly, restructuring results in log cash flows above $\widehat{x}_U(\mathcal{R})$ to be reflected to $\widehat{x}_0$ and the state to be reset to (s, s). Finally, the numerical solution for Eq. (A.20) requires a discretized approximation

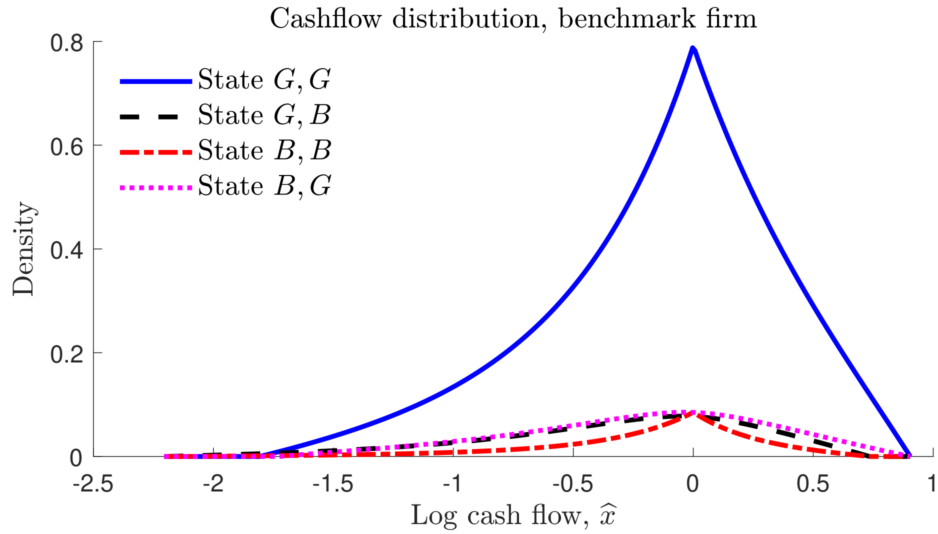

Fig. A.1. **Cash flow distribution, benchmark firm.**

to the adjoint operator $(\mathscr{L}^{\mathcal{P},m})^{\dagger}$. This can be done using the matrix transpose $(\mathbf{L}^{\mathcal{P},m})^T$, where the matrix $\mathbf{L}^{\mathcal{P},m}$ is a finite difference approximation for $\mathscr{L}^{\mathcal{P},m}$. Fig. A.1 shows the solution to Eq. (A.20) for the benchmark firm. The solution can then be used to compute model-implied moments (e.g., for BBB rated firms).

A.3.2. Term structure of default and credit risk

The τ year conditional cumulative default probability is given by

$$p_D(\tau, \widehat{x}, s, \mathcal{R}) = \mathbb{E}_{\widehat{x},s,\mathcal{R}}\left[\tau_D \leq \tau\right], \tag{A.22}$$

where $\tau_D \equiv \inf\{t \geq 0 : \ \widehat{x}_t \leq \widehat{x}_{D,t}\}$ is the time of default, with $\widehat{x}_{D,t} = \widehat{x}^m_D(s_t, \mathcal{R}_t)$ when debt matures and $\widehat{x}_{D,t} = \widehat{x}_D(s_t, \mathcal{R}_t)$ when debt does not mature. An application of Feynman-Kac to Eq. (A.22) results in the following PDE:

$$\begin{aligned}
\frac{\partial}{\partial \tau} p_D(\tau, \widehat{x}, s, \mathcal{R}) &= m_{\mathcal{R}_0}\left[p^m_D(\tau, \widehat{x}, s, \mathcal{R}) - p_D(\tau, \widehat{x}, s, \mathcal{R})\right] && \text{(A.23)} \\
&\quad + \mathscr{L}^{\mathcal{P}} p_D(\tau, \widehat{x}, s, \mathcal{R}), \text{ if } \widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R}), \\
p_D(\tau, \widehat{x}, s, \mathcal{R}) &= 1, \text{ if } \widehat{x} \leq \widehat{x}_D(s, \mathcal{R}), && \text{(A.24)} \\
p_D(\tau, \widehat{x}, s, \mathcal{R}) &= p_D(\tau, \widehat{x}_0, s, s), \text{ if } \widehat{x} \geq \widehat{x}_U(\mathcal{R}), && \text{(A.25)} \\
p_D(0, \widehat{x}, s, \mathcal{R}) &= 1\left\{\widehat{x} \leq \widehat{x}_D(s, \mathcal{R})\right\}, && \text{(A.26)}
\end{aligned}$$

where the default probability conditional on debt maturing is given by

$$p^m_D(\tau, \widehat{x}, s, \mathcal{R}) = \begin{cases} 1 & \text{if } \widehat{x} \leq \widehat{x}^m_D(s, \mathcal{R}), \\ p_D(\tau, \widehat{x}_0, s, s) & \text{if } \widehat{x} > \widehat{x}^m_D(s, \mathcal{R}). \end{cases} \tag{A.27}$$

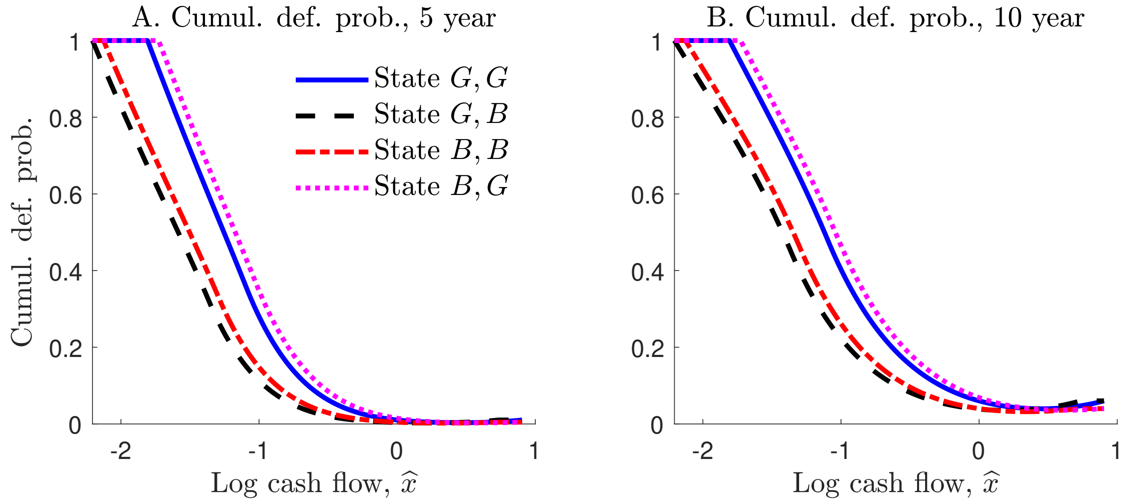

Fig. A.2. **Cumulative default probability, benchmark firm.**

We use a finite difference scheme to solve the PDE Eq. (A.23). Fig. A.2 displays the solution for the benchmark firm.

The price of a τ year defaultable zero coupon bond is given by

$$Z_D(\tau,\hat{x},s,\mathcal{R}) = \mathbb{E}^{\mathcal{Q}}\left[e^{-\int_0^{\tau\wedge\tau_D} r(s_u)+l(\tau-u,s_u)\,du}\left[1-(1-R(s_{\tau_D}))1\{\tau_D<\tau\}\right]\right], \tag{A.28}$$

where $R(s)$ is the recovery rate conditional on defaulting in state s. An application of Feynman-Kac to Eq. (A.28) gives

$$\begin{aligned}
[r_s+l(\tau,s)]Z_D(\tau,\hat{x},s,\mathcal{R}) &= -\frac{\partial}{\partial\tau}Z_D(\tau,\hat{x},s,\mathcal{R}) + m_{\mathcal{R}_0}\left[Z_D^m(\tau,\hat{x},s,\mathcal{R}) - Z_D(\tau,\hat{x},s,\mathcal{R})\right] \\
&\quad +\mathscr{L}^{\mathcal{Q}}Z_D(\tau,\hat{x},s,\mathcal{R}),\ \text{if } \hat{x}_D(s,\mathcal{R})<\hat{x}<\hat{x}_U(\mathcal{R}), &\text{(A.29)}\\
Z_D(\tau,\hat{x},s,\mathcal{R}) &= R(s)Z(\tau,s),\ \text{if } \hat{x}\le\hat{x}_D(s,\mathcal{R}), &\text{(A.30)}\\
Z_D(\tau,\hat{x},s,\mathcal{R}) &= Z_D(\tau,\hat{x}_0,s,s),\ \text{if } \hat{x}\ge\hat{x}_U(\mathcal{R}), &\text{(A.31)}\\
Z_D(0,\hat{x},s,\mathcal{R}) &= 1-(1-R(s))1\{\hat{x}\le\hat{x}_D(s,\mathcal{R})\}, &\text{(A.32)}
\end{aligned}$$

where

$$Z_D^m(\tau,\hat{x},s,\mathcal{R}) = \begin{cases} R(s)Z(\tau,s) & \text{if } \hat{x}\le\hat{x}_D^m(s,\mathcal{R}),\\ Z_D(\tau,\hat{x}_0,s,s) & \text{if } \hat{x}>\hat{x}_D^m(s,\mathcal{R}),\end{cases} \tag{A.33}$$

is the defaultable bond price conditional on debt maturing, and $Z(\tau,s)$ denotes the price of a non-defaultable τ year zero coupon bond, which satisfies

$$\begin{aligned}
[r_s+l(\tau,s)]Z(\tau,s) &= -\frac{\partial}{\partial\tau}Z(\tau,s)+\pi_s[Z(\tau,s^c)-Z(\tau,s)], &\text{(A.34)}\\
Z(0,s) &= 1. &\text{(A.35)}
\end{aligned}$$

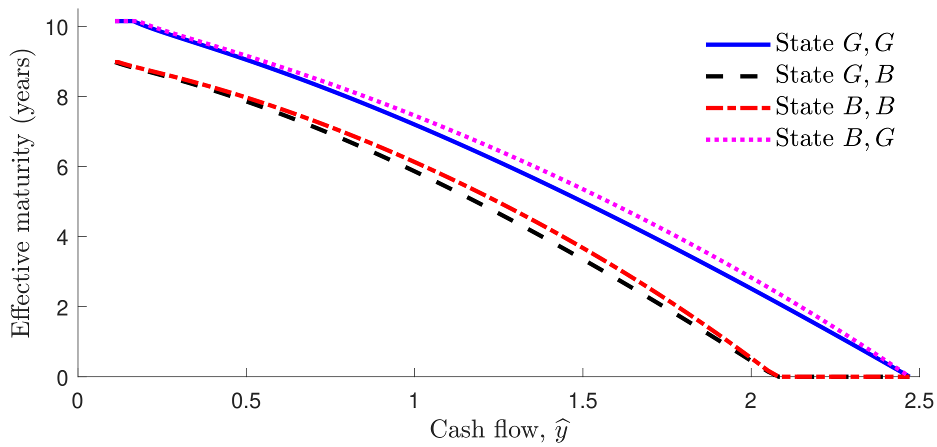

Fig. A.3. **Effective maturity, benchmark firm.**

We solve the PDE Eq. (A.29) using a finite difference scheme similar to that used for computing the default probabilities. Finally, we obtain the default component of credit spreads by computing zero coupon bond prices for defaultable bonds which are not subject to liquidity discounts (i.e., by setting $l = 0$).

A.3.3. Effective maturity

Under the scaled variables, effective maturity can be written as:

$$M_{eff}(\widehat{x}, s, \mathcal{R}) \equiv \mathbb{E}_{\widehat{x},s,\mathcal{R}}\left[\min\{\tau_M, \tau_U\}\right], \tag{A.36}$$

where τ_M arrives at rate $m_{\mathcal{R}}$ and $\tau_U = \inf\{t \geq 0 : \ \widehat{x}_t \geq \widehat{x}_U(\mathcal{R})\}$ is the time at which scaled cash flows first cross the restructuring boundary ($\tau_U = +\infty$ if the firm defaults). We can make use of the fact that τ_M follows an exponential distribution to express Eq. (A.36) as

$$M_{eff}(\widehat{x}, s, \mathcal{R}) = \mathbb{E}_{\widehat{x},s,\mathcal{R}}\left[\int_0^{\tau_U} m_{\mathcal{R}} e^{-m_{\mathcal{R}} u} u \, du + \tau_U e^{-m_{\mathcal{R}} \tau_U}\right] \tag{A.37}$$

$$= \mathbb{E}_{\widehat{x},s,\mathcal{R}}\left[\int_0^{\tau_U} e^{-m_{\mathcal{R}} u} \, du\right]. \tag{A.38}$$

Eq. (A.37) shows that we can equivalently interpret effective maturity as the expected face-value weighted average time until repayment.

We compute effective maturity by applying Feynman-Kac to convert Eq. (A.38) to the following system of ODEs:

$$m_{\mathcal{R}} M_{eff}(\widehat{x}, s, \mathcal{R}) = 1 + \mathscr{L}^{\mathcal{P}} M_{eff}(\widehat{x}, s, \mathcal{R}), \text{ if } \widehat{x}_D(s, \mathcal{R}) < \widehat{x} < \widehat{x}_U(\mathcal{R}), \tag{A.39}$$

$$M_{eff}(\widehat{x}, s, \mathcal{R}) = 1/m_{\mathcal{R}}, \text{ if } \widehat{x} \leq \widehat{x}_D(s, \mathcal{R}), \tag{A.40}$$

$$M_{eff}(\widehat{x}, s, \mathcal{R}) = 0, \text{ if } \widehat{x} \geq \widehat{x}_U(\mathcal{R}), \tag{A.41}$$

where $\mathscr{L}^{\mathcal{P}}$ is given by Eq. (A.1). Eq. (A.39) can be solved using the guess and verify methods outlined in Appendix A.1. Fig. A.3 displays the solution for effective maturity for the benchmark firm.

Appendix B. Robustness checks

This section includes two additional robustness checks. Table B.1 shows that the positive relation between beta and debt maturity holds for alternative measures of debt maturity. Table B.2 shows that the positive relation between beta and debt maturity also holds at the extensive margin, with high systematic-risk firms being more likely to issue long-term bonds.

Table B.1: **Alternative measures of long-term debt share.** This table presents panel regressions of alternative measures of long-term debt share (*ldebt1y*, *ldebt2y*, *ldebt4y*, and *ldebt5y*) on various measures of firms' systematic risk exposures, a recession dummy, and their interactions. Additional firm level controls along with industry fixed effects and a quadratic time trend are also included in the regressions.The recession dummy is firm-specific and equals one if the fiscal year-end month is in an NBER recession and zero otherwise. Standard errors of the coefficients are adjusted for heteroscedasticity and for clustering of observations at the firm and year level. The t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

	ldebt1y			*ldebt2y*		
rec	-0.033***	-0.025**	-0.026**	-0.043***	-0.032***	-0.033**
	(-3.26)	(-2.51)	(-2.51)	(-3.58)	(-2.72)	(-2.38)
mktbeta	0.042***			0.052***		
	(8.89)			(10.10)		
mktbeta × *rec*	0.023***			0.029***		
	(3.37)			(3.48)		
bankbeta		0.042***			0.053***	
		(7.36)			(6.84)	
bankbeta × *rec*		0.028**			0.034***	
		(2.53)			(2.68)	
tailbeta			0.038***			0.047***
			(8.91)			(8.18)
tailbeta × *rec*			0.016*			0.019**
			(1.84)			(1.99)
N	58,336	58,336	58,154	48,200	48,200	48,057
R^2	0.187	0.183	0.185	0.207	0.204	0.206
	ldebt4y			*ldebt5y*		
rec	-0.046***	-0.046***	-0.034***	-0.042***	-0.039***	-0.034***
	(-3.97)	(-4.00)	(-2.74)	(-3.59)	(-3.76)	(-2.90)
mktbeta	0.043***			0.034***		
	(7.83)			(6.09)		
mktbeta × *rec*	0.021			0.021		
	(1.64)			(1.61)		
bankbeta		0.041***			0.035***	
		(4.40)			(4.42)	
bankbeta × *rec*		0.039**			0.033*	
		(2.22)			(1.92)	
tailbeta			0.041***			0.028***
			(6.52)			(5.17)
tailbeta × *rec*			0.005			0.012
			(0.62)			(1.25)
N	48,200	48,200	48,057	48,200	48,200	48,057
R^2	0.220	0.217	0.219	0.216	0.215	0.215

Table B.2: **Maturity of corporate bond issuance.** This table presents panel regression results of value-weighted bond maturity of issued corporate bonds on asset beta, firm controls (total asset volatility, firm size, abnormal earnings, book leverage, market-to-book ratio, asset maturity, profit volatility, and credit rating), year and industry dummies . Corporate bond issuance data from 1990 to 2017 are obtained from Mergent Fixed Income Securities Database. Standard errors of the coefficients are adjusted for clustering of observations at both the industry and the year levels. Robust t-statistics are presented in parentheses below parameter estimates. Significance at the 10%, 5%, and 1% levels is indicated by *, **, ***, respectively.

	(1)	(2)	(3)	(4)
$mktbeta$	0.569*			
	(1.74)			
$bankbeta$		0.827*		
		(1.81)		
$tailbeta$			0.719**	
			(2.48)	
$cfbeta$				0.097
				(1.62)
$assetvol$	-2.187*	-1.780**	-1.705*	
	(-1.91)	(-2.01)	(-1.87)	
$mkat$	0.389***	0.396***	0.384**	0.523***
	(2.67)	(2.68)	(2.54)	(3.67)
$abnearn$	0.159	0.132	0.175	-1.104
	(0.41)	(0.34)	(0.44)	(-1.16)
$bklev$	-2.289***	-2.373***	-2.033**	-2.157
	(-2.73)	(-3.00)	(-2.36)	(-1.54)
$mk2bk$	-0.390**	-0.361**	-0.434**	-0.854***
	(-2.36)	(-2.16)	(-2.56)	(-3.17)
$assetmat$	0.041**	0.040*	0.041**	0.033
	(2.00)	(1.96)	(2.05)	(0.73)
$profitvol$	-2.887	-2.631	-3.015	-6.109
	(-1.08)	(-0.97)	(-1.12)	(-1.00)
$spltratg$	-0.376***	-0.371***	-0.382***	-0.327***
	(-5.18)	(-5.11)	(-5.20)	(-4.38)
N	4,453	4,453	4,447	2,491
R^2	0.169	0.169	0.169	0.178

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