When Efficiency meets Equity in Congestion Pricing and Revenue Refunding Schemes

Devansh Jalota
Stanford University
Stanford, California, USA
djalota@stanford.edu

Kiril Solovey
Stanford University
Stanford, California, USA
kirilsol@stanford.edu

Karthik Gopalakrishnan
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA
karthikg@mit.edu

Stephen Zoepf
Lacuna AI
Palo Alto, California, USA
stephen.zoepf@lacuna.ai

Hamsa Balakrishnan
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA
hamsa@mit.edu

Marco Pavone
Stanford University
Stanford, California, USA
pavone@stanford.edu

ABSTRACT

Congestion pricing has long been hailed as a means to mitigate traffic congestion; however, its practical adoption has been limited due to social inequity issues, e.g., low-income users are priced out off certain roads. This issue has spurred interest in the design of equitable mechanisms that refund the collected toll revenues to users. Although revenue refunding has been extensively studied, there has been no characterization of how such schemes can be designed to simultaneously achieve system efficiency and equity objectives.

In this work, we bridge this gap through the study of *congestion pricing and revenue refunding* (CPRR) schemes in non-atomic congestion games. We first develop CPRR schemes, which in comparison to the untolled case, simultaneously (i) increase system efficiency and (ii) decrease wealth inequality, while being (iii) *user-favorable*: irrespective of their initial wealth or values-of-time (which may differ across users) users would experience a lower travel cost after the implementation of the proposed scheme. We then characterize the set of optimal user-favorable CPRR schemes that simultaneously maximize system efficiency and minimize wealth inequality. These results assume a well-studied behavior model of users minimizing a linear function of their travel times and tolls, without considering refunds. Overall, our work demonstrates that through appropriate refunding policies we can achieve system efficiency while reducing wealth inequality.

CCS CONCEPTS

• **Applied computing** → **Transportation**; • **Theory of computation** → **Solution concepts in game theory**; *Convex optimization*; Linear programming.

KEYWORDS

Congestion Games, Traffic Routing, Wealth Inequality

EAAMO '21, October 5–9, 2021, –, NY, USA

ACM ISBN 978-1-4503-8553-4/21/10…$15.00
https://doi.org/10.1145/3465416.3483296

ACM Reference Format:
Devansh Jalota, Kiril Solovey, Karthik Gopalakrishnan, Stephen Zoepf, Hamsa Balakrishnan, and Marco Pavone. 2021. When Efficiency meets Equity in Congestion Pricing and Revenue Refunding Schemes. In *Equity and Access in Algorithms, Mechanisms, and Optimization (EAAMO '21), October 5–9, 2021, –, NY, USA.* ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3465416.3483296

1 INTRODUCTION

The study of road congestion pricing is central to transportation economics and traces back to 1920 with the seminal work of Pigou [27]. Since then, the marginal cost pricing of roads, where users pay for the externalities they impose on others, has been widely accepted as a mechanism to alleviate traffic congestion. In particular, congestion pricing can be used to steer users away from the user equilibrium (UE) traffic pattern, which forms when users selfishly minimize their own travel times [29], towards the system optimum (SO) [30]. Despite the system-wide benefits of congestion pricing, its practical adoption has been limited [32]. A primary driving force behind the public opposition to congestion pricing has been the resultant inequity, e.g., high income users are likely to get the most benefit with shorter travel times while low income users suffer exceedingly large travel times since they avoid the high toll roads. Several empirical works have noted the regressive nature of congestion pricing [13, 24], which has often been viewed as "a tax on the working class [26]." Further, a recent theoretical work [16] has characterized the influence of road tolls on the Gini coefficient, a measure of wealth inequality. Most notably, the latter paper [16] developed an *Inequity Theorem* for users travelling between the same origin-destination (O-D) pair, and proved that any form of road tolls increases wealth inequality.

The lack of support for congestion pricing due to its social inequity issues [20, 34] has led to a growing interest in the design of equitable congestion-pricing schemes [36] that refund the collected toll revenues to users. Our work is centered on the design of congestion pricing and revenue refunding (CPRR) schemes that improve system performance, reduce wealth inequality, and benefit every user irrespective of their wealth or value-of-time. We view our work as paving the way for the design of practical, sustainable, and publicly acceptable congestion pricing schemes.

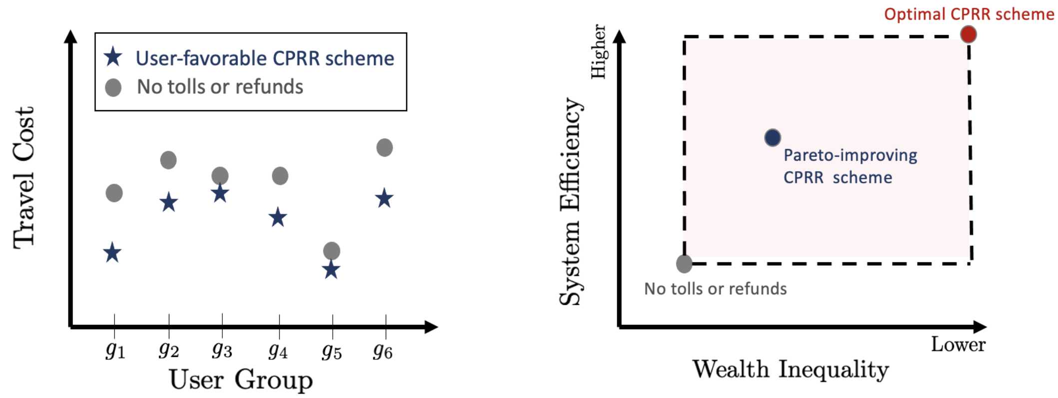

Figure 1: We develop congestion pricing with revenue refunding (CPRR) schemes that improve both system efficiency and wealth inequality, while being *favorable* to all users. On the left we illustrate the concept of a user-favorable CPRR scheme through a scatter plot. The horizontal X axis shows six distinct user groups, where all users in a given group have the same value-of-time, income, and O-D pair, and the vertical Y axis shows the travel cost of each user group. Two sets of points are depicted on this diagram to illustrate the travel cost of each user group (i) after the implementation of a user-favourable CPRR scheme, and (ii) under the untolled setting. In particular, for any user-favorable CPRR scheme, every user group, irrespective of their initial income, has a lower travel cost after the implementation of the scheme compared with the untolled setting. On the right we illustrate the concept of *Pareto-improving* CPRR schemes through a diagram. The horizontal X axis shows the wealth inequality of the income distribution of users after the implementation of a CPRR scheme, while the vertical Y axis shows the system efficiency. We depict a point corresponding to the level of system efficiency and the wealth inequality of the untolled setting as well as a point representing the optimal CPRR scheme. Then, any *Pareto-improving* CPRR scheme is depicted by a point in the rectangular region, whose opposite corners are the optimal solution and no tolls or refunds (untolled) solution. This rectangular region denotes the set of all points with a higher system efficiency and lower wealth inequality compared to the untolled case. We also illustrate an *optimal* user-favorable CPRR scheme, which we characterize in this work, and show that it simultaneously achieves the highest system efficiency and the lowest wealth inequality.

Contributions. In this work, we present the first study of the wealth-inequality effects of CPRR schemes in non-atomic congestion games and devise CPRR schemes that simultaneously reduce the total system cost, i.e., the value-of-time weighted travel times of all users, without increasing the level of wealth inequality. We consider the setting of heterogeneous users, with differing values-of-time and income, who seek to minimize their individual travel cost in the system. As in previous work [16], we incorporate the income elasticity of travel time, i.e., the lost income due to a loss of time, to reason about the income distribution of users before and after the imposition of a CPRR scheme.

To capture the behavior of selfish users, we study the effect of the Nash equilibria induced by CPRR schemes on wealth inequality for non-atomic congestion games. We consider an *exogenous equilibrium* setting, wherein users minimize a linear function of their travel time and tolls, without considering refunds, as in [18], for which we obtain the following results:

(1) *We develop CPRR schemes that improve both system efficiency and wealth inequality, while being favorable to all users.* We establish the existence of a CPRR scheme that, compared with the untolled outcome, (i) is user-favorable, i.e., every user group, irrespective of their initial wealth, has a lower travel cost after the implementation of the scheme, (ii) lowers total system cost, and (iii) decreases wealth inequality (see Figure 1). We call such CPPR schemes *Pareto improving*.

(2) *We characterize the set of optimal CPRR schemes that are favorable to all users.* In particular, we establish that the optimal CPRR schemes are those that simultaneously minimize total system cost and level of wealth inequality among all CPRR schemes that are favorable to any user (see Figure 1).

In the extended version of our paper [22] we also study CPRR schemes in the context of *endogenous equilibria*, wherein users also consider refunds in their travel cost minimization. Our work demonstrates that if we utilize the collected toll revenues to devise appropriate refunding policies then we can achieve system efficiency whilst also progressing towards reduced inequality. Further, in doing so, we ensure that our designed schemes are publicly acceptable since we guarantee that each user is at least as well off as before the introduction of the CPRR scheme. As a result, we view our work as a significant step in shifting the discussion around congestion pricing from one focused on the societal inequity impacts of road tolls to one that centers around *how* to best preserve equity through the distribution of toll revenues.

Organization. This paper is organized as follows. Section 2 reviews related literature. We then present a model of traffic flow as well as metrics to evaluate the inequality of the wealth distribution and the efficiency of a traffic assignment in Section 3. We then prove the existence of Pareto improving and optimal CPRR schemes for the exogenous setting in Sections 4, and 5, respectively. Finally, we present a discussion of how our work fits into the broader conversation around equitable transportation in Section 6 and provide directions for future work in Section 7.

2 RELATED WORK

The design of mechanisms that satisfy both system efficiency and user fairness desiderata has been a centerpiece of algorithm design for a range of applications including resource allocation, classification tasks for machine learning algorithms and fair traffic routing. For instance, Bertsimas et al. [6] quantified the loss in efficiency in resource allocation settings when the allocation outcomes are required to satisfy certain fairness criteria. For machine learning classification tasks, Dwork et al. [12] studied group-based fairness notions to prevent discrimination against individuals belonging to disadvantaged groups. In the context of traffic routing, Jahn et al. [21] introduced a fairness-constrained traffic-assignment problem to achieve a balance between the total travel time of a traffic assignment and the level of fairness, i.e., the maximum discrepancy between the travel times of users travelling between the same O-D pair [28], that it provides. Subsequent work on fair traffic routing has focused on developing algorithms to solve the fairness constrained traffic assignment problem [2–4], whilst obtaining methods to price roads to enforce the fairness constrained flows [23].

Resolving the efficiency and equity trade-off is particularly important for allocation mechanisms involving monetary transfers given the welfare impacts of such mechanisms on low-income groups. Although achieving system efficiency involves allocating goods to users with the highest willingness to pay, in many settings, e.g., cancer treatment, the needs of users are not well expressed by their willingness to pay [35]. Since Weitzman's seminal work [35] on accounting for agent's needs in allocation decisions, there has been a rich line of work on taking into account redistributive considerations in the allocation of scarce resources to users. For instance, Besley and Coate [7] analyzed the free provision of a low-quality public good to low-income users by taxing individuals that consume the same good of a higher quality in the private market. More recently, Condorelli [9] studied the allocation of identical objects to agents with the objective of maximizing agent's values that may be different from their willingness to pay.

In the context of congestion pricing, revenue redistribution has long been considered as a means to alleviate the inequity issues of congestion pricing [31]. Several revenue redistribution strategies have been proposed in the literature, such as the lump-sum transfer of toll revenues to users [17]. In Vickrey's bottleneck congestion model [33]—a benchmark representation of peak-period traffic congestion on a single lane—Arnott et al. [5] investigated how a uniform lump-sum payment of toll revenues can be used to make heterogeneous users better off than prior to the implementation of the tolls and refunds. To extend the application of revenue redistribution schemes to a two parallel-routes setting, Adler and Cetin [1] designed a mechanism wherein the revenue collected from users on the more desirable route was directly transferred to users travelling on the less desirable route. In more general networks with a single O-D pair, Eliasson [13] established the existence of a tolling mechanism with uniform revenue refunds that reduced the travel cost for each user while also decreasing the total system travel time as compared to before the tolling reform. The extension of this result to general road networks with a multiple O-D pair travel demand and heterogeneous users was investigated by Guo and Yang [18]. Our work builds on [18] by characterizing the influence of CPRR schemes on wealth inequality.

3 PRELIMINARIES

In this section, we introduce basic definitions and concepts on traffic flow, congestion pricing and revenue refunding (CPRR) schemes, and efficiency and wealth-inequality metrics through which we evaluate the quality of CPRR schemes.

3.1 Elements of Traffic Flow

We model the road network as a directed graph $G = (V, E)$, with the vertex and edge sets V and E, respectively. Each edge $e \in E$ has a flow-dependent travel-time function $t_e : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, which maps x_e, the traffic flow rate on edge e, to the travel time $t_e(x_e)$. As is standard in the literature, we assume that the function t_e, for each $e \in E$, is differentiable, convex and monotonically increasing.

Users make trips in the transportation network and belong to a discrete set of user groups based on their (i) value-of-time, (ii) income, and (iii) O-D pair. Let $\mathcal{G}$ denote the set of all user groups, and let $v_g > 0$, $q_g > 0$ and $w_g = (s_g, t_g)$ denote the value-of-time, income and O-D pair represented by an origin s_g and destination t_g, respectively, for each user in group $g \in \mathcal{G}$. The total travel demand d_g of user group g represents the amount of flow to be routed on a set of directed paths $\mathcal{P}_g$, which is the set of all simple paths connecting O-D pair w_g.

A path flow pattern $\boldsymbol{f} = \{f_{P,g} : g \in \mathcal{G}, P \in \mathcal{P}_g\}$ specifies for each user group g, the amount of flow $f_{P,g}$ routed on a path $P \in \mathcal{P}_g$, where $f_{P,g} \geq 0$. In particular, a flow $\boldsymbol{f}$ must satisfy the user demand, i.e., $\sum_{P \in \mathcal{P}_g} f_{P,g} = d_g$, for all $g \in \mathcal{G}$. We denote the set of all non-negative flows that satisfy this constraint as Ω.

The corresponding edge flows associated with a path flow $\boldsymbol{f} = \{f_{P,g} : g \in \mathcal{G}, P \in \mathcal{P}_g\}$ is represented as (i) $\sum_{P \in \mathcal{P}_g : e \in P} f_{P,g} = x_e^g$, for all $e \in E$, and (ii) $\sum_{g \in \mathcal{G}} x_e^g = x_e$, for all $e \in E$, where $e \in P$ denotes whether edge e is in path P, while x_e^g represents the flow of users in group g on edge e. For conciseness, we denote $\boldsymbol{x} = \{x_e\}_{e \in E}$ as the vector of edge flows and $\boldsymbol{x}^g = \{x_e^g\}_{e \in E}$ denote the vector of edge flows for user group g.

3.2 Congestion Pricing and Revenue Refunding Schemes

A congestion pricing and revenue refunding (CPRR) scheme is defined by a tuple $(\boldsymbol{\tau}, \boldsymbol{r})$, where (i) $\boldsymbol{\tau} = \{\tau_e : e \in E\}$ is a vector of edge prices (or tolls), and (ii) $\boldsymbol{r} = \{r_g : g \in \mathcal{G}\}$ is a vector of group-specific revenue refunds, where each user in group g receives a lump-sum transfer of r_g. In other words, everybody pays the same toll for using an edge independent of their group, and all users with the same income, value-of-time and O-D pair get the same refund, irrespective of the actual route. Under the CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ and a vector of edge flows $\boldsymbol{x}$, the total value of tolls collected is given by $\Pi = \sum_{e \in E} \tau_e x_e$. In this work, we consider CPPR schemes such that the total sum of the revenue refunds equals the total sum of the revenue collected from the edge tolls, i.e., $\sum_{g \in \mathcal{G}} r_g d_g = \Pi$. In addition, we consider revenue refunding schemes that depend only on the groups $\mathcal{G}$ and the total revenue Π induced by a flow $\boldsymbol{f}$, but not on the specific paths that the users take under $\boldsymbol{f}$. We leave the study of more complex refund schemes for future work.

The total travel cost incurred by the user includes a linear function of their travel time and tolls, which is a commonly-used modelling approach [8, 14], and a component which reflects the refund received, which aligns with [18].

Definition 1 (User Travel Cost)**.** Consider a CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ and a flow pattern $\boldsymbol{f}$ with edge flow $\boldsymbol{x}$, and suppose that a user belongs to a group $g \in \mathcal{G}$. Then, the total cost incurred by a user when traversing a path $P \in \mathcal{P}_g$ is

$$\mu_P^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r}) = \sum_{e \in P} \left(v_g t_e(x_e) + \tau_e \right) - r_g. \tag{1}$$

With slight abuse of notation, we will denote $\mu_P^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{0})$ as a path cost that does not include refund, and $\mu_P^g(\boldsymbol{f}, \boldsymbol{0}, \boldsymbol{0})$ as a path cost that does not account for tolls or refunds, where $\boldsymbol{0}$ is a vector of zeros. Throughout this paper we will consider in many cases *equilibrium* flow patterns which emerge from the collective behavior of self-interested users. Relevant to the discussion here is that equilibrium flows equalize the user travel cost of all the users of a given group. That is, if $\boldsymbol{f}$ is an equilibrium for a CPRR scheme then $\mu_P^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r}) = \mu_Q^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$ for any group $g \in \mathcal{G}$ and any two paths $P, Q \in \mathcal{P}_g$ such that $f_{P,g}, f_{Q,g} > 0$. In such a case we drop the path dependence in the notation and use $\mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$ to denote the travel cost of any user within the group g.

3.3 System Efficiency and Wealth Inequality Metrics

We evaluate the quality of a CPRR scheme using two metrics: (i) system efficiency, which is measured through the total system cost, and (ii) wealth inequality.

Total System Cost: We measure the efficiency of the system through the total system cost, which, for any feasible path flow $\boldsymbol{f}$ with corresponding edge flows $\boldsymbol{x}$ and group specific edge flows $\boldsymbol{x}^g$, is the sum of travel times weighted by the users' values-of-time [8, 14, 18], i.e., $C(\boldsymbol{f}) := \sum_{e \in E} \sum_{g \in \mathcal{G}} v_g x_e^g t_e(x_e)$. We denote by $C^* := \min_{\boldsymbol{f} \in \Omega} C(\boldsymbol{f})$ the widely studied cost-based system optimum.

Wealth Inequality: We measure the impact of a CPPR scheme on wealth inequality in the following manner. For a profile of incomes $\mathbf{q} = \{q_g : g \in \mathcal{G}\}$, we let a function $W : \mathbb{R}_{\geq 0}^{|\mathcal{G}|} \to \mathbb{R}_{\geq 0}$ measure the level of wealth inequality of society. We say that an income distribution $\tilde{\boldsymbol{q}}$ has a lower level of wealth inequality than $\mathbf{q}$ if and only if $W(\tilde{\boldsymbol{q}}) \leq W(\mathbf{q})$.

In this work, we assume that the wealth-inequality measure $W(\cdot)$ satisfies the following properties:

(1) Scale Independence: The wealth-inequality measure remains unchanged after rescaling incomes by the same positive constant, i.e., $W(\lambda \mathbf{q}) = W(\mathbf{q})$ for any $\lambda > 0$.
(2) Regressive (Progressive) Taxes Increase (Decrease) Inequality: The wealth-inequality measure increases (decreases) if the incomes of users are scaled by constants that increase (decrease) as the income increases (decreases).

We refer the readers to the extended version of this paper [22] for a more detailed description of the above properties. The above properties are well defined for any wealth inequality distribution when the incomes of all users are strictly positive, which we assume in this work. We note that the above properties are fairly natural [10, 16] and hold for commonly used wealth-inequality measures, such as the discrete Gini coefficient, which we elucidate in detail in the online version of our paper [22]. Furthermore, we note that the above properties jointly imply an important property of the wealth-inequality measure W, which we elucidate in detail in Appendix A.1.

For the wealth inequality measure W we investigate the influence of a flow $\boldsymbol{f}$ for a given CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ on the income distribution of users. To this end, we define the income profile of users before making their trip as the *ex-ante income distribution* $\boldsymbol{q}^0 > \boldsymbol{0}$ and that after making their trip as the *ex-post income distribution*, which is defined as follows.

Definition 2 (Ex-Post Income Distribution)**.** For a given CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ and an equilibrium flow $\boldsymbol{f}$, the induced ex-post income distribution of users is denoted by $\boldsymbol{q}(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$ and defined as follows. For a given group g, we have that $q_g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r}) := q_g^0 - \beta \mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$, where $\boldsymbol{q}^0$ is the ex-ante income distribution and β is a small constant representing the relative importance of the congestion game to an individual's well-being [16].

Since the trip made by users is one among a suite of factors influencing the income of users, we assume that the constant β is small enough so that the ex-post income of all users is strictly positive. The positive income assumption ensures that the above defined wealth inequality properties (including scale independence) hold.

To conclude this section, we note that in this paper we consider time-invariant travel demand that is fixed for all user groups and assume fractional flows, both of which are standard assumptions in the traffic routing literature [25], as well as in game theory in the context of non-atomic congestion games [28]. Furthermore, similar to much of the prior literature in traffic routing with heterogeneous groups of users [8, 14, 16], we assume that the different attributes (i.e., the income, value-of-time and O-D pair) of the user groups are known, and can be used in the design of CPRR schemes.

4 PARETO IMPROVING CPRR SCHEMES

The social inequity issue surrounding the regressive nature of congestion pricing has been documented in several empirical and theoretical works, while also having spurred political opposition to its implementation in practice. In this section, we show that if the tolls collected from congestion pricing are refunded to users in an appropriate way then the wealth inequality effects of congestion pricing can be reversed. Throughout this section and the next we assume that user behavior is characterized through the *exogenous equilibrium* model wherein users minimize a linear function of their travel time and tolls, without considering refunds.

After formally defining exogenous equilibrium below, we develop a CPRR scheme that simultaneously decreases the total system cost of all users while not increasing the level of wealth inequality relative to the untolled outcome, a property which we refer to as *Pareto improving*. Moreover, when designing the scheme, we ensure that it is politically acceptable for implementation by guaranteeing that each individual user is at least as well off in terms of the travel cost μ^g under the CPRR scheme than that without the implementation of congestion pricing or refunds.

In the extended version of this paper [22], we also consider the important special case of travel demand when users travel between the same O-D pair, and have values-of-time that are in proportion to their income. In this setting, we establish the existence of a Pareto improving CPRR scheme that results in an ex-post income distribution that has a lower wealth inequality as compared to that of the ex-ante income distribution, which is a stronger result than the more general case with multiple O-D pairs considered above.

4.1 Exogenous Equilibrium

To capture the strategic behavior of users, we present below the standard model of Nash equilibrium with heterogeneous users, which we call exogenous equilibrium. The exogenous setting is commonly studied in the context of non-atomic congestion games without [8, 14] or with refunds [18]. As the name suggests, in exogenous equilibrium revenue refunds are assumed to be *exogenous* and do not influence the behavior and route choice of users in the transportation network. That is, users minimize a linear function of their travel time and tolls, without considering refunds.

We note that such a model of user behavior can be quite realistic in certain settings, especially since accounting for refunds when making route choices may often be too complex and involve quite sophisticated decision making on the part of users. Furthermore, for users to reason about how their path choice will influence their refund, they must know the refunding policy, which may typically not be known in practice, thereby making the notion of an exogenous equilibrium more appropriate in such settings. The following definition formalizes the notion of an exogenous equilibrium, which only depends on the congestion pricing component $\boldsymbol{\tau}$ of a CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$.

Definition 3 (Exogenous Equilibrium)**.** For a given congestion-pricing scheme $\boldsymbol{\tau}$, a path flow pattern $\boldsymbol{f}$ is an exogenous equilibrium if for each group $g \in \mathcal{G}$ it holds that $f_{P,g} > 0$ for some path $P \in \mathcal{P}_g$ if and only if

$$\mu_P^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{0}) \leq \mu_Q^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{0}), \quad \forall Q \in \mathcal{P}_g.$$

In such a case, we say in short that $\boldsymbol{f}$ is an exogenous $\boldsymbol{\tau}$-equilibrium.

We reiterate that the above notion of an exogenous equilibrium is the standard Nash equilibrium concept used in non-atomic congestion games. In this work, we refer to this equilibrium concept as *exogenous* to explicitly distinguish it from the *endogenous* setting when coalitions of users also account for refunds when making travel decisions (see the extended version of this paper [22] for more details on the endogenous setting). A key property of any exogenous $\boldsymbol{\tau}$-equilibrium $\boldsymbol{f}$ is that all users within a given group $g \in \mathcal{G}$ incur the same travel cost without refunds, irrespective of the path on which they travel. Hence, we drop the path dependence in the notation and denote the user travel cost without refunds for any user in group g at flow $\boldsymbol{f}$ as $\mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{0})$. Additionally, since the refund r_g is the same for all users, the travel cost with refunds is denoted as $\mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$.

Another useful property of exogenous equilibrium is that for a given congestion-pricing scheme $\boldsymbol{\tau}$, the resulting total system cost, user travel cost, and ex-post income distribution are invariant under the different $\boldsymbol{\tau}$-equilibria (see Problem (2) and Appendix A.5 for a discussion). That is for any two $\boldsymbol{\tau}$-equilibria $\boldsymbol{f}, \boldsymbol{f}'$ it holds that $C(\boldsymbol{f}) = C(\boldsymbol{f}')$, $\mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{0}) = \mu^g(\boldsymbol{f}', \boldsymbol{\tau}, \boldsymbol{0})$, and $\boldsymbol{q}(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r}) = \boldsymbol{q}(\boldsymbol{f}', \boldsymbol{\tau}, \boldsymbol{r})$. Thus, we will use the simplified notation $C_{\boldsymbol{\tau}} := C(\boldsymbol{f})$, $\mu^g(\boldsymbol{\tau}, \boldsymbol{r}) := \mu^g(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$, and $\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r}) := \boldsymbol{q}(\boldsymbol{f}, \boldsymbol{\tau}, \boldsymbol{r})$ for some exogenous $\boldsymbol{\tau}$-equilibrium $\boldsymbol{f}$, when considering the exogenous equilibrium model. In this context, note that $C_{\boldsymbol{0}}$ corresponds to the untolled total system cost.

4.2 User-Favorable Pareto Improving CPRR Schemes

To ensure that the CPRR schemes we develop are politically acceptable, we consider schemes that result in equilibrium outcomes wherein each user is at least as well off as compared to that under the untolled user equilibrium outcome, a property we refer to as user-favorable (see Figure 1).

Definition 4 (User-Favorable CPRR Schemes)**.** A CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ is user-favorable if for any (exogenous) $\boldsymbol{\tau}$-equilibrium the travel cost of any user group g does not increase with respect to any untolled $\boldsymbol{0}$-equilibrium $\boldsymbol{f}^0$, i.e., $\mu^g(\boldsymbol{\tau}, \boldsymbol{r}) \leq \mu^g(\boldsymbol{0}, \boldsymbol{0})$.

We note that the the above definition and the following result (Proposition 1) can readily be extended to incorporate the notion of a user-favorable CPRR scheme relative to any status-quo traffic equilibrium pattern, which is not necessarily equal to the untolled case, e.g., the traffic pattern in a city that has already implemented some form of congestion pricing. Thus, considering the untolled user equilibrium $\boldsymbol{f}^0$ in the above definition is without loss of generality.

We now present the main result of this section. In particular, we establish that any pricing scheme $\boldsymbol{\tau}$ that improves the system efficiency compared to the untolled case, can be paired with a revenue refunding scheme $\boldsymbol{r}$ such that the wealth inequality relative to the ex-post income distribution under the untolled setting is not increased, i.e., the CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ is Pareto improving (see Figure 1) and user-favorable. Note that designing CPRR schemes with a lower wealth inequality and total system cost as compared to the untolled user equilibrium outcome is desirable since the CPRR scheme improves upon both the system efficiency and equity metrics of the status-quo traffic equilibrium pattern.

Proposition 1 (Existence of Pareto Improving CPRR Scheme). *Let $\boldsymbol{\tau}$ be a congestion-pricing scheme such that $C_{\boldsymbol{\tau}} \leq C_{\boldsymbol{0}}$, where $C_{\boldsymbol{0}}$ is the untolled total system cost. Then there exists a refund scheme $\boldsymbol{r}$ such that $(\boldsymbol{\tau}, \boldsymbol{r})$ is user-favorable and does not increase wealth inequality, i.e., $W(\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r})) \leq W(\boldsymbol{q}(\boldsymbol{0}, \boldsymbol{0}))$. That is, $(\boldsymbol{\tau}, \boldsymbol{r})$ is* Pareto improving.

For a proof of Proposition 1, see Appendix A.2. Note that Proposition 1 relies on the key observation that an exogenous equilibrium is completely defined through the road tolls $\boldsymbol{\tau}$, and is thus oblivious of the refund $\boldsymbol{r}$.

Proposition 1 establishes the existence of a user-favorable CPRR scheme that simultaneously decreases the total system cost and reduces the wealth inequality relative to that of the untolled outcome. We present an important consequence of this result for the setting when all users travel between the same O-D pair and have values-of-time that are proportional to their incomes in the extended version of this paper [22]. In this setting, we establish the existence of a revenue refunding scheme that decreases the wealth inequality relative to the ex-ante income distribution, which is a stronger result than Proposition 1. However, this result does not hold in general for users travelling between different O-D pairs. In particular, for the multiple O-D pair setting, we show in Proposition 2 that there are travel demand instances when no CPRR scheme can reduce income inequality relative to that of the ex-ante income distribution.

Proposition 2 (Increase in Income Inequality for Multiple O-D Pairs). *There exists a two O-D pair setting such that for any user-favorable CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ it holds that $W(\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r})) \geq W(\boldsymbol{q}^0)$.*

For a proof of Proposition 2, see Appendix A.3. Given that there may be multiple O-D pair instances when it may not be possible to achieve a lower wealth inequality relative to the ex-ante income distribution, we devise CPRR schemes that reduce the wealth inequality relative to the ex-post income distribution under the untolled user equilibrium outcome. Note that doing so is reasonable since we look to design CPRR schemes that improve on the status quo traffic pattern, which is typically described by the untolled user equilibrium setting.

5 OPTIMAL CPRR SCHEMES

In this section, we prove the existence of optimal CPRR schemes that achieve a total system cost and wealth inequality that cannot be improved by any other user-favorable CPRR scheme. In particular, we establish that the optimal CPRR schemes are those that induce exogenous equilibrium flows with the minimum total system cost while also resulting in ex-post income distributions with the lowest wealth inequality among all user-favorable CPRR schemes (see Figure 1).

We first present the main result of this section, which characterizes the set of optimal CPRR schemes.

Theorem 1 (Optimal CPRR Scheme). *There exists a user-favorable CPRR scheme $(\boldsymbol{\tau}^*, \boldsymbol{r}^*)$ such that for any user-favorable CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ it holds that $C_{\boldsymbol{\tau}^*} \leq C_{\boldsymbol{\tau}}$ and $W(\boldsymbol{q}(\boldsymbol{\tau}^*, \boldsymbol{r}^*)) \leq W(\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r}))$.*

The proof of this theorem relies on two intermediate results that are of independent interest. First, the under any user-favorable CPRR scheme, each user's ex-post income is at least that of the user under the untolled case.

Lemma 1 (Ex-post Income Distribution). *Let $\boldsymbol{\tau}$ be tolls such that $C_{\boldsymbol{\tau}} \leq C_{\mathbf{0}}$. Then, under any set of refunds $\boldsymbol{r}$ such that the CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ is user-favorable, the ex-post income of any user belonging to group g is $q_g(\boldsymbol{\tau}, \boldsymbol{r}) = q_g(\mathbf{0}, \mathbf{0}) + \beta c_g$, where the transfer value c_g is non-negative and satisfies the relation $\sum_{g \in \mathcal{G}} c_g d_g = C_{\mathbf{0}} - C_{\boldsymbol{\tau}}$.*

For a proof of Lemma 1, see Appendix A.4. The second result required to prove Theorem 1 relies on the observation that there is a monotonic relationship between the minimum achievable wealth-inequality and the total system cost.

Lemma 2 (Monotonicity of Refunds). *Suppose that there are two congestion-pricing schemes $\boldsymbol{\tau}_A$ and $\boldsymbol{\tau}_B$ with total system costs satisfying $C_{\boldsymbol{\tau}_A} \leq C_{\boldsymbol{\tau}_B} \leq C_{\mathbf{0}}$. Then there exists a revenue refunding scheme $\boldsymbol{r}_A$ such that $(\boldsymbol{\tau}_A, \boldsymbol{r}_A)$ is user-favorable and achieves a lower wealth inequality measure than any user-favorable CPRR scheme $(\boldsymbol{\tau}_B, \boldsymbol{r}_B)$ for any revenue refunds $\boldsymbol{r}_B$, i.e., $W(\boldsymbol{q}(\boldsymbol{\tau}_A, \boldsymbol{r}_A)) \leq W(\boldsymbol{q}(\boldsymbol{\tau}_B, \boldsymbol{r}_B))$.*

For a proof of Lemma 2, see Appendix A.6. The above result establishes that a smaller total system cost yields a larger amount of remaining refund $C_{\mathbf{0}} - C_{\boldsymbol{\tau}}$ after satisfying the user-favorable condition, which, in turn, results in a greater degree of freedom in distributing these refunds to achieve an overall lower level of wealth inequality.

Finally, Theorem 1 follows directly by the monotonicity relation established in Lemma 2, and prescribes a two-step procedure to find a optimal CPRR scheme that is also user-favorable. In particular, choose a congestion pricing scheme $\boldsymbol{\tau}^*$ such that the total travel cost is minimized, i.e., $C_{\boldsymbol{\tau}^*} = C^*$. Next, select the revenue refunding scheme $\boldsymbol{r}^*$ to be such that the expression $W(\boldsymbol{q}(\boldsymbol{\tau}^*, \boldsymbol{r}^*))$ is minimized and $(\boldsymbol{\tau}^*, \boldsymbol{r}^*)$ is user-favorable through an appropriate selection of transfers c_g. Now, let $(\boldsymbol{\tau}, \boldsymbol{r})$ be some user-favorable CPRR scheme. By definition of $\boldsymbol{\tau}^*$, it holds that $C_{\boldsymbol{\tau}^*} \leq C_{\boldsymbol{\tau}}$. Moreover, Lemma 2 ensures that $W(\boldsymbol{q}(\boldsymbol{\tau}^*, \boldsymbol{r}^*)) \leq W(\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r}))$ is satisfied.

Significance of Theorem 1. The result of Theorem 1 establishes that the optimal CPRR scheme simultaneously achieves the highest efficiency whilst also reducing wealth inequality to the maximum degree possible among the class of all user-favourable CPRR schemes. This finding is counter-intuitive since equity and efficiency are typically at odds but Theorem 1 establishes that no such tradeoff between system efficiency and wealth inequality exists. The reason for this is that the remaining refund after satisfying the user-favourable condition increases as the total system cost decreases (Lemma 2), thereby giving greater leverage in the design of the refunding scheme to achieve a lower wealth inequality.

6 DISCUSSION

A core tenet of sustainable transportation entails achieving a balance between *economic*, *equity* and *environmental* goals [19]. The results demonstrated in this paper challenge the traditional notion that these goals are in tension with each other by making progress towards achieving each of these goals simultaneously. In particular, our work directly addresses the economic and equity goals through the development of CPRR schemes that both minimize the total system cost and reverse the wealth inequality effects of congestion pricing. Furthermore, the schemes we develop achieve another economic goal—all users are left at least as well off under the CPRR schemes as compared to that prior to any implementation of congestion pricing or refunds. This property suggests that users would *favor* this pricing and refunding scheme. Finally, as the environmental impact of a scheme is often proportional to the total travel time of all users, the total system cost objective, which we seek to minimize within optimal CPRR schemes (Theorem 1), can be treated as an imperfect proxy for the total environmental pollution in the system. Environmental goals can be more directly incorporated within a CPRR scheme through appropriate congestion pricing schemes, e.g., aiming to minimize air pollution, while potentially improving total system cost and wealth inequality (Proposition 1).

Our work demonstrated that if we look at congestion pricing from the lens of refunding the collected tolls then we can not only achieve system efficiency but also reduce wealth inequality. As a result, we view our work as a significant step in shifting the discussion around congestion pricing from one that has focused on the inequity impacts of road tolls to one that centers around how to best distribute the revenues collected to different sections of society. While refunding toll revenues is not novel, our work provided a characterization of how such schemes can be designed to simultaneously achieve system efficiency and equity objectives. Furthermore, in doing so, we ensured that all users are at least as well off as compared to before the introduction of the CPRR scheme, thereby making it publicly acceptable to all users.

We believe that the results of our work pave the way for the design of sustainable, publicly-acceptable congestion-pricing schemes, but significant practical challenges remain. For instance, we assume centralized knowledge of the values-of-time of each user group. In practice these may not be known, and could confound successful implementation of an optimal CPRR scheme. Furthermore, we consider CPRR schemes involving direct refunds to users while not accounting for system designs with cross subsidies across multiple forms of transport, e.g., subsidies to improve the transit infrastructure. It is also important to note the degree to which the CPRR scheme is successful relies on the full implementation of the tolls *and* refunds. If policymakers implement the congestion pricing scheme but fail to deliver refunds, low-income users of the system will be made worse off, facing higher costs, worse travel times, or both. Underprivileged residents would have legitimate claims that the system was not working, undermining public trust in the system. Thus the onus is on policy makers to manage the entire life cycle of the CPRR scheme and ensure its successful and sustainable implementation. The difference between an equitable, optimal congestion pricing scheme and one that disproportionately burdens the poor depends significantly on how the toll revenue is spent.

7 CONCLUSION AND FUTURE WORK

In this paper, we studied and designed user-favorable congestion pricing and revenue refunding (CPRR) schemes that mitigate the regressive wealth inequality effects of congestion pricing. In particular, we developed CPRR schemes that improved both system efficiency and wealth inequality, while being favorable for all users, as compared to the untolled outcome. We further characterized the set of optimal CPRR schemes.

There are several interesting directions for further research. The first would be to relax some of the commonly-used assumptions in transportation research and game theory, to improve the applicability to practice. One example is to consider nonlinear user travel cost functions. In addition, we currently assume time-invariant travel demand and traffic flows, which motivates the possible generalization to dynamic settings, e.g., through the incorporation of the cell transmission model [11]. We have also assumed that the only decisions made by users are route choices, whereas in reality there are other options, such as changing departure time or travel mode. A possible way to overcome this limitation is by incorporating elastic-demand models into our traffic-assignment formulations [15, 25].

It would also be interesting to extend these results to the setting of anonymous revenue refunding schemes that do not rely on any knowledge of user's value-of-time. It would also be worthwhile to investigate a broader class of group specific differential congestion pricing mechanisms, e.g., path specific prices which may differ by user group, beyond those involving lump-sum transfers of the collected revenues to users. Finally, an even more general class of refunding mechanisms can be explored wherein some portion of the collected revenues is used to cover operational costs or improve transportation infrastructure, e.g., cross subsidies to improve public transit.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation, CAREER Award CMMI1454737, and by the Stanford Institute for Human-Centered AI. The authors thank Alon Eden and Kira Goldner for insightful discussions on non-atomic congestion games.

REFERENCES

[1] Jeffrey L. Adler and Mecit Cetin. 2001. A direct redistribution model of congestion pricing. *Transportation Research Part B: Methodological* 35, 5 (2001), 447–460.
[2] E. Angelelli, I. Arsik, V. Morandi, M. Savelsbergh, and M. Speranza. 2016. Proactive route guidance to avoid congestion. *Transportation Research Part B: Methodological* 94 (2016), 1–21.
[3] E. Angelelli, V. Morandi, M. Savelsbergh, and M. Speranza. 2020. System optimal routing of traffic flows with user constraints using linear programming. *European Journal of Operational Research* (2020).
[4] E. Angelelli, V. Morandi, and M. Speranza. 2018. Congestion avoiding heuristic path generation for the proactive route guidance. *Computers & Operations Research* 99 (2018), 234–248.
[5] Richard Arnott, André de Palma, and Robin Lindsey. 1994. The Welfare Effects of Congestion Tolls with Heterogeneous Commuters. *Journal of Transport Economics and Policy* 28, 2 (1994), 139–161.
[6] D. Bertsimas, Vivek Farias, and N. Trichakis. 2011. The Price of Fairness. *Operations Research* 59, 1 (2011), 17–31.
[7] Timothy Besley and Stephen Coate. 1991. Public Provision of Private Goods and the Redistribution of Income. *The American Economic Review* 81, 4 (1991), 979–984.
[8] Richard Cole, Yevgeniy Dodis, and Tim Roughgarden. 2003. Pricing Network Edges for Heterogeneous Selfish Users. In *Symposium on Theory of Computing* (San Diego, CA, USA). Association for Computing Machinery, 521–530.
[9] Daniele Condorelli. 2013. Market and non-market mechanisms for the optimal allocation of scarce resources. *Games and Economic Behavior* 82 (2013), 582–591.
[10] Era Dabla-Norris, Kalpana Kochhar, Nujin Suphaphiphat, Frantisek Ricka, and Evridiki Tsounta. 2015. *Causes and consequences of income inequality: A global perspective.* International Monetary Fund.
[11] Carlos F Daganzo. 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological* 28, 4 (1994), 269–287.
[12] Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Richard Zemel. 2012. Fairness through Awareness. In *Innovations in Theoretical Computer Science Conference* (Cambridge, Massachusetts). Association for Computing Machinery, New York, NY, USA, 214–226.
[13] Jonas Eliasson. 2001. Road pricing with limited information and heterogeneous users: A successful case. *The annals of regional science* 35, 4 (2001), 595–604.
[14] L. Fleischer, K. Jain, and M. Mahdian. 2004. Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games. In *Symposium on Foundations of Computer Science.* IEEE, 277–285.
[15] Nathan H Gartner. 1980. Optimal traffic assignment with elastic demands: A review part I. Analysis framework. *Transportation Science* 14, 2 (1980), 174–191.
[16] Kurtulus Gemici, Elias Koutsoupias, Barnabé Monnot, Christos H. Papadimitriou, and Georgios Piliouras. 2019. Wealth Inequality and the Price of Anarchy. In *International Symposium on Theoretical Aspects of Computer Science (Leibniz International Proceedings in Informatics, Vol. 126).* Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 31:1–31:16.
[17] Phil B Goodwin. 1989. The rule of three: a possible solution to the political problem of competing objectives for road pricing. *Traffic engineering & control* 30, 10 (1989), 495–497.

[18] Xiaolei Guo and Hai Yang. 2010. Pareto-improving congestion pricing and revenue refunding with multiple user classes. *Transportation Research Part B: Methodological* 44, 8 (2010), 972–982.
[19] Ralph P. Hall and Joseph M. Sussman. [n. d.]. Promoting the Concept of Sustainable Transportation within the Federal System - The Need to Reinvent the U.S. http://dspace.mit.edu/handle/1721.1/102801
[20] S. Jaensirisak, M. Wardman, and A. D. May. 2005. Explaining Variations in Public Acceptability of Road Pricing Schemes. *Journal of Transport Economics and Policy* 39, 2 (2005), 127–153.
[21] O. Jahn, R. Möhring, A. Schulz, and N. Stier-Moses. 2005. System-Optimal Routing of Traffic Flows with User Constraints in Networks with Congestion. *Operations Research* 53, 4 (2005), 600–616.
[22] Devansh Jalota, Kiril Solovey, Karthik Gopalakrishnan, Stephen Zoepf, Hamsa Balakrishnan, and Marco Pavone. 2021. When Efficiency meets Equity in Congestion Pricing and Revenue Refunding Schemes. *CoRR* abs/2106.10407 (2021). arXiv:2106.10407
[23] Devansh Jalota, Kiril Solovey, Stephen Zoepf, and Marco Pavone. 2021. Balancing Fairness and Efficiency in Traffic Routing via Interpolated Traffic Assignment. *CoRR* abs/2104.00098 (2021). arXiv:2104.00098
[24] Michael Manville and Emily Goldman. 2018. Would Congestion Pricing Harm the Poor? Do Free Roads Help the Poor? *Journal of Planning Education and Research* 38, 3 (2018), 329–344.
[25] Michael Patriksson. 2015. *The Traffic Assignment Problem: Models and Methods.* Dover Publications.
[26] Azi Paybarah. 2019. Congestion Pricing: Mass Transit Savior or Tax on the Working Class? New York Times.
[27] A. Pigou. 1912. *Wealth and Welfare.* London, Macmillan and Co.
[28] T. Roughgarden. 2002. How unfair is optimal routing?. In *Symposium on Discrete Algorithms.* ACM/SIAM, 203–204.
[29] T. Roughgarden and É. Tardos. 2002. How Bad is Selfish Routing? *J. ACM* 49, 2 (2002), 236–259.
[30] Y. Sheffi. 1985. *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods.* Prentice-Hall.
[31] Kenneth A Small. 1992. Using the revenues from congestion pricing. *Transportation* 19, 4 (1992), 359–381.
[32] Kenneth A. Small and Jia Yan. 2001. The Value of "Value Pricing" of Roads: Second-Best Pricing and Product Differentiation. *Journal of Urban Economics* 49, 2 (2001), 310 – 336.
[33] William S Vickrey. 1969. Congestion theory and transport investment. *The American Economic Review* 59, 2 (1969), 251–260.
[34] Martin Wachs. 2005. Then and now: The evolution of congestion pricing in transportation and where we stand today. In *Transportation Research Board Conference Proceedings.*
[35] Martin L. Weitzman. 1977. Is the Price System or Rationing More Efficient in Getting a Commodity to Those Who Need it Most? *Bell Journal of Economics* 8, 2 (1977), 517–524.
[36] Di Wu, Yafeng Yin, Siriphong Lawphongpanich, and Hai Yang. 2012. Design of more equitable congestion pricing and tradable credit schemes for multimodal transportation networks. *Transportation Research Part B: Methodological* 46, 9 (2012), 1273–1287.
[37] Hai Yang and Hai-Jun Huang. 2004. The multi-class, multi-criteria traffic network equilibrium and systems optimum problem. *Transportation Research Part B: Methodological* 38, 1 (2004), 1–15.

A PROOFS

A.1 Constant Income Transfer Property

In this section, we present the constant income transfer property and show that it follows directly from the regressive and progressive tax properties of the wealth inequality measure W.

Constant Income Transfer Property: If the initial income distribution is $\mathbf{q}$ and each user is transferred a non-negative (non-positive) amount of money λ $(-\lambda)$ where $0 \leq \lambda < \min_{g \in \mathcal{G}} q_g$, then the wealth inequality cannot increase (decrease). That is, $W(\mathbf{q} + \lambda \mathbf{1}) \leq W(\mathbf{q})$ and $W(\mathbf{q} - \lambda \mathbf{1}) \geq W(\mathbf{q})$, where $\mathbf{1}$ is a vector of ones.

We now prove how the above property follows from the regressive and progressive tax properties of the wealth inequality measure W. In particular, we show that if the initial income distribution is $\boldsymbol{q}$ and each person is transferred a non-positive amount of money $-\lambda$, where $0 \leq \lambda < \min_{g \in \mathcal{G}} q_g$, then the wealth inequality cannot decrease, i.e., $W(\mathbf{q} - \lambda \mathbf{1}) \geq W(\mathbf{q})$.

We note that at the new income distribution $\bar{\boldsymbol{q}} = \mathbf{q} - \lambda \mathbf{1}$, each user in group g has the following income:

$$\bar{q}_g = q_g - \lambda = q_g \left(1 - \frac{\lambda}{q_g}\right).$$

Note that if $q_g \leq q_{g'}$ for any two groups g, g', then $1 - \frac{\lambda}{q_g} \leq 1 - \frac{\lambda}{q_{g'}}$. Thus, by the regressive tax property, we observe that $W(\mathbf{q} - \lambda \mathbf{1}) \geq W(\mathbf{q})$. We finally note that the claim that $W(\mathbf{q} + \lambda \mathbf{1}) \leq W(\mathbf{q})$ for any $0 \leq \lambda < \min_{g \in \mathcal{G}} q_g$ follows by a similar analysis wherein we use the progressive tax property. This proves our claim that the wealth inequality measure W satisfies the constant income transfer property.

A.2 Proof of Proposition 1

We now prove Proposition 1 by leveraging a class of user-favorable CPRR schemes that were developed recently [18, Theorem 1].

Lemma 3 (Existence of user-favorable CPRR Scheme [18]). *Let $\boldsymbol{\tau}$ be a congestion pricing scheme such that $C_{\boldsymbol{\tau}} \leq C_{\mathbf{0}}$. Then, for any $\alpha_g \geq 0$ with $\sum_{g \in \mathcal{G}} \alpha_g = 1$, the CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ with refunds is given by*

$$r_g = \mu^g(\boldsymbol{\tau}, \mathbf{0}) - \mu^g(\mathbf{0}, \mathbf{0}) + \frac{\alpha_g}{d_g}(C_{\mathbf{0}} - C_{\boldsymbol{\tau}}),$$

for each group g, is user-favorable.

The above lemma states that as long as the edge tolls $\boldsymbol{\tau}$ reduce the total system cost there exists a method to refund revenues that makes every user at least as well off as compared to that under the untolled case. We now leverage Lemma 3 to prove Proposition 1.

Proof of Proposition 1. For the collected toll revenues, we construct a special case of the revenue refunding scheme from Lemma 3. In particular, consider the refunding scheme where $\alpha_g = \frac{d_g}{\sum_{g \in \mathcal{G}} d_g}$, which gives the refund

$$r_g = \mu^g(\boldsymbol{\tau}, \mathbf{0}) - \mu^g(\mathbf{0}, \mathbf{0}) + \frac{1}{\sum_{g \in \mathcal{G}} d_g}(C_{\mathbf{0}} - C_{\boldsymbol{\tau}})$$

to each user in group g. We now show that under this refunding scheme, the ex-post income distribution $\hat{\boldsymbol{q}} = \boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r})$ has a lower wealth inequality than that of the untolled user equilibrium ex-post income distribution $\tilde{\boldsymbol{q}} = \boldsymbol{q}(\mathbf{0}, \mathbf{0})$, i.e., we show that $W(\hat{\mathbf{q}}) \leq W(\tilde{\boldsymbol{q}})$.

To see this, we begin by considering the ex-ante income distribution $\boldsymbol{q}^0$. Under the untolled user equilibrium, users in group g incur a travel cost $\mu^g(\mathbf{0}, \mathbf{0})$, and thus the ex-post income distribution of users in group g is given by $\tilde{q}_g = q_g^0 - \beta \mu^g(\mathbf{0}, \mathbf{0})$, where β is the scaling factor as in Definition 2. On the other hand, under the CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$, the ex-post income distribution of users in group g is

$$\begin{aligned}
\hat{q}_g &= q_g^0 - \beta \left(\mu^g(\boldsymbol{\tau}, \mathbf{0}) - r_g\right) \\
&= q_g^0 - \beta \left(\mu^g(\mathbf{0}, \mathbf{0}) - \frac{1}{\sum_{g \in \mathcal{G}} d_g}(C_{\mathbf{0}} - C_{\boldsymbol{\tau}})\right) \\
&= \tilde{q}_g + \beta \frac{1}{\sum_{g \in \mathcal{G}} d_g}(C_{\mathbf{0}} - C_{\boldsymbol{\tau}}),
\end{aligned}$$

where we used that $\tilde{q}_g = q_g^0 - \beta\mu^g(\mathbf{0},\mathbf{0})$ to derive the last equality. Since the above relation is true for all groups g, we observe that $\hat{\mathbf{q}} = \tilde{\boldsymbol{q}} + \lambda\mathbf{1}$, where $\lambda = \frac{\beta}{\sum_{g\in\mathcal{G}} d_g}(C_\mathbf{0} - C_\boldsymbol{\tau}) \geq 0$. Finally, the result that $W(\hat{\mathbf{q}}) \leq W(\tilde{\boldsymbol{q}})$ follows by the constant income transfer property (Appendix A.1), establishing our claim. □

A.3 Proof of Proposition 2

We show that there exists a two O-D pair setting such that for any user-favorable CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ it holds that $W(\boldsymbol{q}(\boldsymbol{\tau}, \boldsymbol{r})) \geq W(\boldsymbol{q}^0)$.

We begin by formally defining the instance depicted in Figure 2. Consider a graph with four nodes, v_1, v_2, v_3, v_4 and three edges $e_1 = (v_1, v_2)$, $e_2 = (v_3, v_4)$ and $e_3 = (v_3, v_4)$, where there are two possible ways to get from v_3 to v_4. We define the travel time on edge e_1 as $t_1(x_1) = \frac{x_1}{2}$, that on edge e_2 as $t_2(x_2) = x_2$ and that on edge e_3 as $t_3(x_3) = 1$. Further consider two user types, one with a high income q_H and value-of-time ωq_H that make trips between O-D pair $w_H = (v_1, v_2)$, and the other with a low income q_L and value-of-time ωq_L that make trips between O-D pair $w_L = (v_3, v_4)$. Let the demand of the high income users be $d_H = 1$ and that of the low income users be $d_L = 1$. Then at the untolled user equilibrium outcome it follows that all high income users traverse their only edge e_1, while all the low income users traverse the edge e_2. At this equilibrium flow, the cost to the high income users is $\omega q_H \frac{1}{2}$, since the travel time of the edge e_1 is $\frac{1}{2}$, and that to the low income users is ωq_L, since the travel time on edge e_2 is one.

Next, we note that under any CPRR scheme $(\boldsymbol{\tau}, \mathbf{r})$ users in the high income group will continue to use edge e_1 since this is the only available edge on which they can travel. Thus, for this scheme to be user-favorable it must be that any tolls collected from the high income users is directly refunded back within the groups. To see this, if there were tolls collected from high-income users that were given to low income users then some high income users would incur strictly higher costs than at the untolled $\mathbf{0}$-equilibrium outcome. We similarly observe that all collected refunds from the low income groups must be completely refunded to users within the low income group to ensure that the CPRR scheme is user-favorable. Note that the above argument stems from the fact that the travel paths of the two user groups are completely disjoint, and so any CPRR scheme $(\boldsymbol{\tau}, \mathbf{r})$ must refund all the collected revenues from each user group directly back to that user group to ensure that the scheme is user-favorable.

Thus, we have for any user-favorable CPRR scheme $(\boldsymbol{\tau}, \mathbf{r})$ that all the users incur the same costs as that under the $\mathbf{0}$-equilibrium outcome. Now, under the untolled user equilibrium, we observe that the ex-post income of the high income group is $q_H = q_H - \beta\frac{\omega q_H}{2} = q_H(1 - \beta\frac{\omega}{2})$ and the ex-post income of the low income group is $q_L = q_L - \beta\omega q_L = q_L(1 - \beta\omega)$. The above analysis implies that the untolled user equilibrium outcome results in a regressive tax, i.e., lower income users are charged a greater fraction of their incomes than higher income users. Since the function W satisfies the property that regressive taxes increase inequality, we have that the wealth inequality of the ex-post income distribution is greater than that of the ex-ante income distribution. □

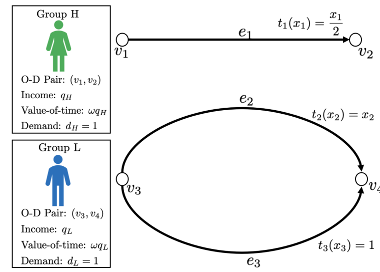

Figure 2: A two O-D pair and two user group instance for which the wealth inequality of the ex-post income distribution under any congestion pricing and revenue refunding (CPRR) scheme is at least the wealth inequality of the ex-ante income distribution. In particular, the first user group, i.e., user group H, has an income level of q_H, value-of-time of ωq_H for some $\omega > 0$, demand $d_H = 1$ and O-D pair (v_1, v_2). Here the origin and destination vertices v_1 and v_2, respectively, are connected by a single edge e_1 with a travel time of $t_1(x_1) = \frac{x_1}{2}$. The second user group, i.e., user group L, has an income level of q_L, value-of-time of ωq_L for the same $\omega > 0$ as for user group H, demand $d_L = 1$ and O-D pair (v_3, v_4). Here the origin and destination vertices v_3 and v_4, respectively, are connected by a two edges e_2 and e_3 with a travel time of $t_2(x_2) = x_2$ and $t_3(x_3) = 1$. Under these defined attributes for the different user groups, the user group H with a higher income and value-of-time incurs a strictly lower cost as a proportion of their income as compared to user group L, indicating the regressive nature of any valid CPRR scheme.

A.4 Proof of Lemma 1

Denote the ex-post income of group g as $\hat{q}_g = q_g(\boldsymbol{\tau}, \boldsymbol{r})$. We now prove the ex-post income relation using the definition of a user-favorable CPRR scheme. In particular, for any user-favorable CPRR scheme $(\boldsymbol{\tau}, \boldsymbol{r})$ the user travel cost does not increase from the untolled case, i.e., $\mu^g(\boldsymbol{\tau}, \boldsymbol{r}) \leq \mu^g(\mathbf{0}, \mathbf{0})$. As it holds that $\mu^g(\boldsymbol{\tau}, \boldsymbol{r}) = \mu^g(\boldsymbol{\tau}, \mathbf{0}) - r_g$, we observe that for some $c_g \geq 0$ the following relation must hold for each user in group g: $\mu^g(\boldsymbol{\tau}, \mathbf{0}) - r_g + c_g = \mu^g(\mathbf{0}, \mathbf{0})$. Then, for an ex-ante income distribution $\boldsymbol{q}^0$, the ex-post income of each user belonging to group g is given by

$$\hat{q}_g = q_g^0 - \beta\left(\mu^g(\boldsymbol{\tau}, \mathbf{0}) - r_g\right) = q_g^0 - \beta\mu^g(\mathbf{0}, \mathbf{0}) + \beta c_g = q_g(\mathbf{0}, \mathbf{0}) + \beta c_g,$$

where the second equality follows since $\mu^g(\boldsymbol{\tau}, \mathbf{0}) - r_g = \mu^g(\mathbf{0}, \mathbf{0}) - c_g$ and the last equality follows from the observation that the ex-post income of users in group g for the untolled setting is given by $q_g(\mathbf{0}, \mathbf{0}) = q_g^0 - \beta\mu^g(\mathbf{0}, \mathbf{0})$.

Next, to show that $\sum_{g\in\mathcal{G}} c_g d_g = C_\mathbf{0} - C_\boldsymbol{\tau}$ we characterize the quantities $C_\mathbf{0}$ and $C_\boldsymbol{\tau}$. In particular, observe that by definition $C_\mathbf{0} = C(\boldsymbol{f}^0)$ and $C_\boldsymbol{\tau} = C(\boldsymbol{f})$, where $\boldsymbol{f}^0$ is the untolled $\mathbf{0}$-equilibrium and $\boldsymbol{f}$ is an exogenous $\boldsymbol{\tau}$-equilibrium. Now, note that both flows $\boldsymbol{f}^0$ and $\boldsymbol{f}$ can be expressed in closed form. In particular, for a given pricing scheme $\boldsymbol{\tau}'$ the exogenous $\boldsymbol{\tau}'$-equilibrium $\boldsymbol{h}(\boldsymbol{\tau}')$ can be written as

$$\boldsymbol{h}(\boldsymbol{\tau}') = \operatorname*{arg\,min}_{\boldsymbol{h}'\in\Omega} \sum_{e\in E} \int_0^{x(\boldsymbol{h}')_e} t_e(\omega)d\omega + \sum_{e\in E}\sum_{g\in\mathcal{G}} \frac{1}{v_g} x(\boldsymbol{h}')_e^g \tau_e, \quad (2)$$

where $\boldsymbol{x}(\boldsymbol{f}')$ denotes the edge representation of a path flow $\boldsymbol{f}'$. We note that this program corresponds to the *multi-class user-equilibrium optimization problem* [37].

Given this representation of the flow $\boldsymbol{h}(\boldsymbol{\tau}')$, we derive the following relation that relates the total system cost $C_{\boldsymbol{\tau}'}$ to the amount of collected revenues, by analyzing the KKT conditions of this minimization problem. In particular, it holds that

$$C_{\boldsymbol{\tau}'} = \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}', \mathbf{0}) d_g - \sum_{e \in E} \tau'_e x(\boldsymbol{h}(\boldsymbol{\tau}'))_e. \quad (3)$$

Note that the edge flow $\boldsymbol{x}(\boldsymbol{h}(\boldsymbol{\tau}'))$ is unique by the strict convexity of the travel-time function. We defer the proof of Equation (3) to Appendix A.5.

We now leverage Equation (3) to obtain $C_{\boldsymbol{\tau}} = \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g - \sum_{e \in E} \tau_e x(\boldsymbol{f})_e$, where $\boldsymbol{x}(\boldsymbol{f}) = \boldsymbol{x}(\boldsymbol{h}(\boldsymbol{\tau}))$. Furthermore, from Equation (3) for the untolled setting, we obtain that $C_{\mathbf{0}} = \sum_{g \in \mathcal{G}} \mu^g(\mathbf{0}, \mathbf{0}) d_g$. Finally, using these two relations and leveraging the fact that $c_g = \mu^g(\mathbf{0}, \mathbf{0}) - \mu^g(\boldsymbol{\tau}, \mathbf{0}) + r_g$ we get

$$\begin{aligned}
\sum_{g \in \mathcal{G}} c_g d_g &= \sum_{g \in \mathcal{G}} (\mu^g(\mathbf{0}, \mathbf{0}) - \mu^g(\boldsymbol{\tau}, \mathbf{0}) + r_g) d_g, \\
&= \sum_{g \in \mathcal{G}} \mu^g(\mathbf{0}, \mathbf{0}) d_g - \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g + \sum_{g \in \mathcal{G}} r_g d_g, \\
&= C_{\mathbf{0}} - \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g + \sum_{e \in E} \tau_e x(\boldsymbol{f})_e, \\
&= C_{\mathbf{0}} - C_{\boldsymbol{\tau}}.
\end{aligned}$$

Here we used the properties $C_{\mathbf{0}} = \sum_{g \in \mathcal{G}} \mu^g(\mathbf{0}, \mathbf{0})$, $\sum_{g \in \mathcal{G}} r_g d_g = \sum_{e \in E} \tau_e x(\boldsymbol{f})_e$, and $C_{\boldsymbol{\tau}} = \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g - \sum_{e \in E} \tau_e x(\boldsymbol{f})_e$. This proves our claim. □

A.5 Proof of Equation 3

In this section, we use the first order necessary and sufficient KKT conditions of the well studied multi-class user equilibrium optimization problem [37]

$$\boldsymbol{f} = \underset{\boldsymbol{f}' \in \Omega}{\arg\min} \sum_{e \in E} \int_0^{x'_e} t_e(\omega) d\omega + \sum_{e \in E} \sum_{g \in \mathcal{G}} \frac{1}{v_g} x'^g_e \tau_e,$$

to prove that the following holds:

$$C_{\boldsymbol{\tau}} = \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g - \sum_{e \in E} \tau_e x_e. \quad (4)$$

Here $\boldsymbol{\tau}$ is congestion-pricing scheme and $\boldsymbol{f}$ is an exogenous $\boldsymbol{\tau}$-equilibrium with edge flow representation $\boldsymbol{x}$. Note that the edge flows $\boldsymbol{x}$ are unique by the strict convexity of the travel time function.

The following exogenous-equilibrium conditions follow directly from the KKT conditions of the above optimization problem:

$$\begin{aligned}
\sum_{e \in P} \left(v_g t_e(x_e) + \tau_e \right) &= \mu^g(\boldsymbol{\tau}, \mathbf{0}), \quad \text{if } f_{P,g} > 0, P \in \mathcal{P}_g, g \in \mathcal{G}, \\
\sum_{e \in P} \left(v_g t_e(x_e) + \tau_e \right) &\geq \mu^g(\boldsymbol{\tau}, \mathbf{0}), \quad \text{if } f_{P,g} = 0, P \in \mathcal{P}_g, g \in \mathcal{G}.
\end{aligned}$$

From the above equilibrium conditions and the fact that the sum of the path flows for any group adds up to d_g, i.e., $\sum_{P \in \mathcal{P}_g} f_{P,g} = d_g$, we obtain that:

$$\begin{aligned}
\sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g &= \sum_{g \in \mathcal{G}} \sum_{P \in \mathcal{P}_g} f_{P,g} \mu^g(\boldsymbol{\tau}, \mathbf{0}) \\
&= \sum_{g \in \mathcal{G}} \sum_{P \in \mathcal{P}_g} f_{P,g} \sum_{e \in E} \left(v_g t_e(x_e) + \tau_e \right) \delta_{e,P} \\
&= \sum_{e \in E} \sum_{g \in \mathcal{G}} \sum_{P \in \mathcal{P}_g} f_{P,g} \left(v_g t_e(x_e) + \tau_e \right) \delta_{e,P}, \\
&= \sum_{e \in E} \sum_{g \in \mathcal{G}} \sum_{P \in \mathcal{P}_g : e \in P} f_{P,g} \left(v_g t_e(x_e) + \tau_e \right) \\
&= \sum_{e \in E} \sum_{g \in \mathcal{G}} \left(v_g t_e(x_e) + \tau_e \right) \sum_{P \in \mathcal{P}_g : e \in P} f_{P,g}, \\
&= \sum_{e \in E} \sum_{g \in \mathcal{G}} x^g_e \left(v_g t_e(x_e) + \tau_e \right) \\
&= \sum_{e \in E} \sum_{g \in \mathcal{G}} x^g_e v_g t_e(x_e) + \sum_{e \in E} x_e \tau_e
\end{aligned}$$

where $\delta_{e,P} = 1$ if edge $e \in P$ and otherwise it is 0. Note that the above analysis implies Equation (3) since

$$C_{\boldsymbol{\tau}} = \sum_{e \in E} \sum_{g \in \mathcal{G}} x^g_e v_g t_e(x_e) = \sum_{g \in \mathcal{G}} \mu^g(\boldsymbol{\tau}, \mathbf{0}) d_g - \sum_{e \in E} x_e \tau_e.$$

This proves our claim.

Remark 1. We note that since the total tolls collected and user travel costs $\mu^g(\boldsymbol{\tau}, \mathbf{0})$ are unique at any equilibrium flow [18], the total travel cost $C_{\boldsymbol{\tau}}$ is also unique for any equilibrium induced by the edge tolls $\boldsymbol{\tau}$. Furthermore, the ex-post income of each user group g is also the same under any equilibrium induced by the edge tolls $\boldsymbol{\tau}$ since the user travel cost $\mu^g(\boldsymbol{\tau}, \mathbf{0})$ is unique at any equilibrium flow [18].

A.6 Proof of Lemma 2

We prove this claim by constructing for each revenue refunding scheme $\boldsymbol{r}_B$ under the tolling scheme $\boldsymbol{\tau}_B$, a revenue refunding scheme $\boldsymbol{r}_A$ under the tolling scheme $\boldsymbol{\tau}_A$ that achieves a lower wealth inequality measure. To this end, we first introduce some notation. Let c^A_g and c^B_g be non-negative transfers for each group g as in Lemma 1, where $\sum_{g \in \mathcal{G}} c^A_g d_g = C_{\mathbf{0}} - C_{\boldsymbol{\tau}_A}$ and $\sum_{g \in \mathcal{G}} c^B_g d_g = C_{\mathbf{0}} - C_{\boldsymbol{\tau}_B}$ must hold for the feasibility of the scheme.

Then, by Lemma 1 we have that the ex-post income of users in group g can be expressed as: $q_g(\boldsymbol{\tau}_A, \boldsymbol{r}_A) = q_g(\mathbf{0}, \mathbf{0}) + \beta c^A_g$ and $q_g(\boldsymbol{\tau}_B, \boldsymbol{r}_B) = q_g(\mathbf{0}, \mathbf{0}) + \beta c^B_g$. Let $c^A_g = c^B_g + \frac{1}{\sum_{g \in \mathcal{G}} d_g} (C_{\boldsymbol{\tau}_B} - C_{\boldsymbol{\tau}_A})$. We now show that the refunding $\boldsymbol{r}_A$ is feasible.

$$\begin{aligned}
\sum_{g \in \mathcal{G}} c^A_g d_g &= \sum_{g \in \mathcal{G}} \left(c^B_g d_g + \frac{d_g}{\sum_{g \in \mathcal{G}} d_g} \left(C_{\boldsymbol{\tau}_B} - C_{\boldsymbol{\tau}_A} \right) \right) \\
&= \sum_{g \in \mathcal{G}} c^B_g d_g + C_{\boldsymbol{\tau}_B} - C_{\boldsymbol{\tau}_A} \\
&= C_{\mathbf{0}} - C_{\boldsymbol{\tau}_B} + C_{\boldsymbol{\tau}_B} - C_{\boldsymbol{\tau}_A} = C_{\mathbf{0}} - C_{\boldsymbol{\tau}_A},
\end{aligned}$$

Here we leveraged the fact that $\sum_{g \in \mathcal{G}} c^B_g d_g = C_{\mathbf{0}} - C_{\boldsymbol{\tau}_B}$.

Under the above defined non-negative transfer c^A_g, the ex-post income distribution under the CPRR scheme $(\boldsymbol{\tau}_A, \boldsymbol{r}_A)$ is the same as the ex-post income distribution under the CPRR scheme $(\boldsymbol{\tau}_B, \boldsymbol{r}_B)$ plus a constant non-negative transfer, which is equal for all users. That is, we have $\boldsymbol{q}(\boldsymbol{\tau}_A, \boldsymbol{r}_A) = \boldsymbol{q}(\boldsymbol{\tau}_B, \boldsymbol{r}_B) + \lambda \mathbf{1}$ for $\lambda = \frac{\beta}{\sum_{g \in \mathcal{G}} d_g} (C_{\boldsymbol{\tau}_B} - C_{\boldsymbol{\tau}_A}) \geq 0$. Finally, by the constant income transfer property (Appendix A.1) it follows that $W(\boldsymbol{q}(\boldsymbol{\tau}_A, \boldsymbol{r}_A)) \leq W(\boldsymbol{q}(\boldsymbol{\tau}_B, \boldsymbol{r}_B))$. □