A SUPERCONDUCTING MEMORY SWITCH BASED ON
THE PRINCIPLES OF
FLUX-FLOW RESISTIVITY

by

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Abstract

This thesis presents a preliminary design for and theoretical treatment of a novel superconductive current-switch/memory-cell based on the phenomenon of flux-flow resistivity. The Superconducting Flux-Flow Resistance Switch (SUFFRS) prototype is a thin-film device utilizing vanadium and niobium with 5.0 micron line-width resolution. Simple current-switching is expected for low-frequency operational currents. Non-Destructive ReadOut (NDRO) memory operation should be attainable using high-frequency currents. Both modes operate by using a niobium control-line to nucleate vortices into an in-line vanadium vortex trap, through which sense-currents are passed. It is the flux-flow resistance that these vortices present to a sense-current that effects the current-switching or the sensing of a "1", respectively for the low or high frequency cases. In both modes, there exists a pinning threshold current \( I_p \): sense-currents of smaller magnitudes experience no resistivity. Estimates for operational currents, impedances, and switching times are examined; they are also compared to some currently favored superconductive memory cells and switching devices.

Thesis Supervisor: Doctor Jonathan B. Green
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Dedication

I wish to dedicate this thesis, and the culmination that it represents of my undergraduate career at MIT,


to John Kerekes. "Dad". My friend.

and to the memory of Theodore Carl Kraenzel.
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Chapter 1

Introduction

In my studies as an MIT undergraduate, I became interested in the topic of superconductivity. During my participation in the Undergraduate Research Opportunities Program, I studied a "dead" topic known as vortex phenomena and flux-flow dynamics. In studying this, I noted a particular effect that has commonly been considered an unwanted effect: flux-flow resistivity, an equilibrium state of superconductors that exhibits resistance under certain conditions.

Over the last decade, little research has been done on flux-flow resistivity; it is considered well-understood with respect to the current applications. Studies done on the topic primarily relate to the occurrence of this resistive effect in huge superconducting electromagnets, where the effect arises from the extremely large currents involved. In this case of mammoth currents and fields, the focus of research has been on how to prevent the resistive effect from occurring. However, little research has focused on how to apply flux-flow resistivity as a useful tool. Most often, researchers study the effect to gain insight to the properties and epitaxy of the materials they are studying. It is surprising that no-one has thought to utilize this effect in superconductive circuitry; hence, this thesis.

In this thesis, I will present a device design for a SUperconducting Flux-Flow Resistance Switch (SUFFRS). The SUFFRS utilizes flux-flow resistivity as a mechanism to switch an applied sense current; under certain conditions, it can also employ the manifestation of the vortex as the basis for information storage. Chapter 2 presents a terse discussion of related superconductivity theory that provides a foundation for discussion of the SUFFRS. Chapter 3 is a review of other superconductive memories with discussion on the advantages and disadvantages of each. Chapter 4 gives a qualitative presentation of a
prototype design for the SUFFRS. Chapter 5 is a theoretical treatment on the expected behavior and operational parameters of such a prototype. Finally, Chapter 6 compares the SUFFRS against the other devices discussed in Chapter 3 and draws some tentative conclusions.

Appendix A is a brief discussion on the microchip design and lithography techniques that are utilized in Group 86 of MIT Lincoln Laboratory, which determined a framework of limitations and possibilities for the SUFFRS design. Appendix B contains a Fortran program designed to predict theoretical device parameters, based on input of material characteristics and spacing of thin-film layers. Appendix C presents the experimentally determined properties of thin-film vanadium deposited at MIT Lincoln Laboratory. Finally, Appendix D presents certain mathematical derivations pertinent to this thesis.

Unfortunately, due to time constraints, the scope of this thesis had to be limited to that of the design of the device only. Work on the SUFFRS prototype may be continued at Lincoln Laboratory, hopefully leading to a valuable culmination.
Chapter 2

Related Background Theory

2.1 Introduction

The majority of the material presented here was extracted from R. P. Huebener's *Magnetic Flux Structures In Superconductors* [14], which is an excellent, complete source of information on the topic. Unless specifically cited otherwise, information found in this chapter comes directly from this source; what is important about this chapter is that it is a distillation from the information found in Huebener's book and other sources, designed to succinctly introduce the specific theory necessary to support the derivations in Chapter 5.

2.2 Bulk Phenomena

One of the basic properties of the superconductor, known as the Meissner effect, is the complete expulsion of magnetic flux from the interior of the material. This state will persist unless the Gibbs free energy of the superconducting state is exceeded by the free energy cost for deforming the magnetic field. For the ideal Type-I superconductor, superconductivity completely disappears or is *quenched* for magnetic field intensities greater than $H_c$, where the temperature dependence of $H_c$ is described by the following relation:

$$ H_c(T) = H_c(0)(1-(T/T_c)^2) $$  \hspace{1cm} (2.1)

Here $T$ is temperature in degrees Kelvin, $T_c$ is the critical temperature for zero applied field, and $H_c(0)$ is the critical field intensity at $T=0$.

The expulsion of flux just described is not perfect, however. At the surface where the superconducting state terminates, the magnetic field penetrates a distance $\lambda$ into the
superconductor. Likewise, the superconducting state dies off exponentially in a distance $\xi$. Both of these are indicated in Figure 2-1.

![Figure 2-1: Variation of the density of superconducting electrons, $n_s$, and of the magnetic field, $H$, near a S/N interface [14]](image)

The parameters $\xi$ and $\lambda$ are known as the coherence length and the magnetic penetration depth respectively. Both of these parameters are very important in the description of any magnetic phenomena involving superconductors and can be theoretically described by the following relations:¹

(pure)

$$\xi(T) = 0.74 \frac{\xi_0}{(1 - T/T_c)^{1/2}} \quad (2.2)$$

or

(dirty)

$$\xi(T, l) = 0.855 \frac{(\xi_0 l)^{1/2}}{(1 - T/T_c)^{1/2}} \quad (2.3)$$

Here $\xi_0$ is the Pippard coherence length at $T=0$ and $l$ is the electron mean-free-path.

The penetration depth takes the form:

(pure)

$$\lambda(T) = \frac{\lambda_L(0)}{\sqrt{2} (1 - T/T_c)^{1/2}} \equiv \lambda_{\text{pure}} \quad (2.4)$$

or

(dirty)

$$\lambda(T, l) = \lambda_{\text{pure}} \left( \frac{\xi_0}{1.33 l} \right)^{1/2} \quad (2.5)$$

¹The pure case is when the electron mean-free-path $l$ is much greater than the coherence length $\xi$; the dirty case is when $l$ is comparable to (or smaller than) $\xi$. 
Here $\lambda_L(0)$ is the London penetration depth at $T=0$.

The above equations were derived as part of the \textit{Ginzburg-Landau theory}, a phenomenological theory that describes superconductivity utilizing thermodynamic arguments. Another parameter that this very successful theory provides is known as the \textit{Ginzburg-Landau parameter}.

\begin{equation}
\kappa(l) \equiv \frac{\lambda}{\xi}
\end{equation}

In general, when $\kappa < 1/\sqrt{2}$, a superconductor will exhibit Type-I behavior, that is, all superconductivity immediately and completely disappears when the thermodynamic critical field $H_c$ is exceeded. However, when $\kappa > 1/\sqrt{2}$, Type-II superconductivity is observed.

Type-II superconductors exhibit a different quench transition: at a field value $H_{c1}$, magnetic field penetrates into the superconductor in concentrated bundles or flux tubes. These tubes are regions of normal-state and are surrounded by supercurrents that are a result of Maxwell's equations. As the field intensity is increased, more flux-tubes penetrate, until superconductivity completely disappears at $H_{c2}$. Such flux tubes are commonly referred to as vortices, and the thermodynamic state corresponding to $H_{c1} < H < H_{c2}$ is known as the Abrikosov vortex-state or the \textit{mixed-state}.

Once again, the Ginzburg-Landau theory gives effective analytical expressions for the lower and upper bulk critical fields. Here, they are in terms of the thermodynamic critical field given in equation 2.1;

\begin{equation}
H_{c1}(T,l) = \frac{H_c \ln(\kappa)}{\sqrt{2}\kappa}
\end{equation}

and

\begin{equation}
H_{c2}(T,l) = \sqrt{2}\kappa H_c
\end{equation}

Note that the $T$ dependence enters due to $H_c$, and the $l$ dependence comes from the possibility of variations in $\kappa$ with the electron mean free path.
The expression for the lower critical field is actually incomplete, as it does not take into account the effective energy barrier presented by the surface of a superconductor to the entry or exit of a vortex. Near the surface, the considerations of the vortex self-energy, the interaction with the external magnetic field, and the vortex interaction with its self-image, combine to create a local energy barrier that must be overcome. Thus, in order for vortices to start penetrating, the field must be greater than

\[ H_{\text{enB}} = \frac{H_c}{\sqrt{2}} \]  \hspace{1cm} (2.9)

where \( H_{\text{enB}} \) is known as the *bulk entry field*. Note that this relation applies only for Type-II superconductors.

The manifestation of the vortex also occurs in Type-I superconductors when demagnetization effects due to sample geometry are taken into account. Such a state exists for \( H_c(1-D) < H < H_c \), where \( D \) is the demagnetization coefficient corresponding to the shape of the superconductor. This state is commonly referred to as the *intermediate-state*.

An important distinction between the mixed and the intermediate states is the amount of flux passed by an individual vortex. Due to the quantum condition imposed by the phase of the superconducting order parameter, magnetic flux enclosed by a superconductive region must come in quantized units of

\[ \phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7} \text{ G cm}^2 \]  \hspace{1cm} (2.10)

where \( h \) is Plank’s constant, \( c \) is the speed of light, and \( e \) is the charge of the electron.

The distinction between the two states is that each intermediate-state vortex passes many such quanta, whereas each mixed-state vortex passes only one such flux quantum. Since it is a thermodynamic requirement that the center of a vortex must maintain a field intensity of \( H_c \), one immediately sees that vortices in the mixed state are much smaller than they are in the intermediate state.
2.3 Thin Films

All of the previously discussed properties shall now be considered for the special case of thin films. Thin films are defined here as continuous planes of material with thicknesses typically under a micron.

In such a case, many properties are altered due to the change in the electron mean free path \( l \):

\[
\frac{1}{l_{\text{effective}}} = \frac{1}{l_{\text{bulk}}} + \frac{\nu}{d}
\]

where \( l_{\text{bulk}} \) is the electron mean free path in the bulk material, \( d \) is the thickness of the film, and \( \nu \) is an empirically determined constant. Note that, for \( d < < l_{\text{bulk}} \), the relation \( l = d/\nu \) becomes useful.

Utilizing the Drude model for metals [2], it is possible to determine \( l \) from measuring the resistivity of a particular sample of material.

\[
n_e = 6.022 \times 10^{23} \frac{Z \rho_m}{W}
\]

(2.12)

Here, \( n_e \) is the number of conduction electrons per unit volume, \( Z \) is the valence of the material, \( \rho_m \) is the density of the material (g/cm\(^3\)), and \( W \) is the atomic weight of the material (amu).

\[
r_s = \left( \frac{3}{4\pi n_e} \right)^{1/3}
\]

(2.13)

In this equation, \( r_s \) is a characteristic length (cm) associated with the density of electrons in the material.

\[
l = \frac{(r_s/a_0)^2}{\rho_n} \times 92 \text{ angstroms}
\]

(2.14)

where \( a_0 \) is the Bohr radius (cm), \( \rho_n \) is the resistivity (\( \mu \Omega \)-cm) and \( l \) is the resultant electron mean free path in angstroms.

Since thin films are often subject to a high degree of epitaxial disorder, it is likely
that $l$ will be much less than $l_{bulk}$. Thus, in a thin film, $l$ will either be governed by the thickness of the film or the epitaxy. There is no analytical way to determine which in each particular case; hence, $l$ is best determined by measuring the resistivity and applying the previous three equations.

Small values for $l$ affect both $\xi$ and $\lambda$. The "dirty" case, in equations 2.3 and 2.5, is actually the case where some condition of the material reduces the electron mean free path to the point where it is of the same order-of-magnitude as the coherence length. Certainly, if $l < \xi$, even the behavior of superconducting electrons would be modified. In the case of thin films, the limitation on $l$ caused by geometry or epitaxy usually makes the use of the "dirty" case equations appropriate.

Examination of the penetration depth for this case shows that $\lambda(T,l) \propto l^{-1/2}$ and will increase with decreasing film thickness. In contrast, the coherence length decreases with film thickness as $\xi(T,l) \propto l^{1/2}$.

Tinkham [29] demonstrated that all thin films below a critical thickness exhibit Type-II vortices. For the moment ignoring demagnetization effects, such thin films enter the mixed state for field values

$$H_{en} < H < H_{c\perp}$$

where $H$ is oriented perpendicular to the plane of the film and fills the surrounding space, $H_{en}$ is either $H_c$ or $H_{c1}$ for Type-I or Type-II materials respectively, and $H_{c\perp}$ is the perpendicular critical field. $H_{c\perp}$ is given by

$$H_{c\perp}(T,l) = \frac{0.715\sqrt{2}\lambda(0)H_c}{l}$$

Demagnetization effects modify this range as

$$H_{en}(1-D) < H < H_{c\perp}$$

where $D$ is the demagnetization coefficient.

In the special case of a strip with a rectangular cross-section, $D$ has been calculated to be
\[ D = (1 + \frac{z_0}{x_0})^{-1} \]  \hspace{1cm} (2.18)

where \( z_0 \) is the thickness and \( x_0 \) is the width of the strip. It is useful to note that, for Type-II superconductors, \( H_{c\perp} = H_{c2} \).

Another important aspect of the behavior of thin-film superconductors is a different entry-field at the edges of the film from that given in equation 2.9. Again, as in the case of \( H_{\text{enB}} \), here the actual entry field value is affected by a Gibbs free energy barrier that manifests at the edge of the film. For an isolated thin film that is passing a current, the energy contained in the distortion of the magnetic field-lines outside the body of the superconductor becomes a significant factor. Because of the high demagnetization effects, practically all of the current flows along the edges of the strip, where the local field intensity is highest.

It has been demonstrated [7] that the entry field at the edge of the film can be expressed as

\[ H_{\text{enE}} = H_{\text{en}} \left( 1 + \frac{2(a_n x_0)^{1/2}}{\pi z_0} \right) \]  \hspace{1cm} (2.19)

All units here are in cgs, and \( a_n \) represents the cross-sectional width of the vortices that start to enter at \( H = H_{\text{enE}} \). Note that \( a_n \) can be estimated as \( 2z_0 \) (q.v. page 16).

The energy barrier associated with the thin-film geometry also affects the critical current that would create a self-field sufficient to start nucleation of mobile vortices along the film edges, which is the beginning of the onset of the resistive state. This also was derived to be [7]

\[ I_{\text{self}} = \frac{cz_0 H_{\text{en}}}{4} \left( 1 + \frac{2(a_n x_0)^{1/2}}{\pi z_0} \right) \]  \hspace{1cm} (2.20)

However, these last two expressions are based on the assumption that there are no
external electromagnetic factors\textsuperscript{2} affecting the film. In such a case, it is clear that the current-density distribution is extremely non-uniform across the strip.

If the current-carrying strip is placed in close proximity to a superconducting ground-plane (essentially another thin film, parallel to the plane of the strip, much wider, and with a higher $H_c$), then the ground plane will set up an image-current to counter the field effects of the strip. In such a configuration, the field between the strip and the ground-plane is forced to be parallel to the plane of the films, which in turn forces the current distribution within the strip to be uniform [6]. In many thin-film devices, this is deliberately done, as a uniform current density is much easier to deal with.

In the case of a superconducting strip that has a uniform current-density distribution, regardless of how it is achieved, the maximum critical current density has been derived to be [29]

$$J_c = \frac{cH_{en}}{3\sqrt{6}\lambda} \tag{2.21}$$

Also of interest, in the case of uniform current density, is the parallel critical field, which can be expressed as [29]

$$H_{c\parallel} = 2\sqrt{6}H_{en}\lambda/z_0 \tag{2.22}$$

2.4 The Vortex

The manifestation known as the vortex has a well defined structure. The very center of the vortex is in the normal state, otherwise magnetic flux could not pass through it. Surrounding the normal core are supercurrents that circulate as an equilibrium result of the passage of flux through the core. A rough estimate for the radius of the core is $\xi$; i.e. the coherence length is the smallest distance over which the superconducting state can change.

\textsuperscript{2}Such as other nearby superconductors or ambient magnetic fields
Each vortex has an energy associated with it. The contributions to this energy are stored in three parts: the magnetic field energy, the kinetic energy of the currents, and the condensation energy associated with the normal core. The energy contribution from the first two parts can be expressed per unit length as

\[ E_{m,k} = \frac{H_c^2 4\pi \xi^2 \ln(\lambda/\xi)}{8\pi} \]  \hspace{1cm} (2.23)

The destruction of superconductivity in the normal core provides a contribution of approximately \((H_c^2/8\pi)\pi \xi^2\) per unit length: this is the condensation energy of the superconducting state. Thus, the total amount of energy per unit length associated with an individual vortex is

\[ E_0 = \frac{H_c^2 \xi^2}{8} \left(1 + 4 \ln(\kappa)\right) \]  \hspace{1cm} (2.24)

In addition to the static properties just described, the vortex exhibits important time dependent properties. One of these is current induced flux-flow. When a current is passed through a region containing vortices, these vortices will move in response. Steady-state flux motion is described by the following flux-flow equation:

\[
\frac{\mathbf{j} \otimes \mathbf{\phi}}{c} - gn_s e \frac{\mathbf{v}_s \otimes \mathbf{\phi}}{c} - \eta \mathbf{v}_s - \mathbf{f} = 0
\]  \hspace{1cm} (2.25)

In the first term, \(\mathbf{j}\) is the directed current density, \(\mathbf{\phi}\) is the flux through the vortex. This term represents the Lorentz force acting on a vortex and was derived from Maxwell's equations, according to classical magneto-hydrodynamics. Note that this force acts to move the vortices in a direction perpendicular to the current.

The second term, known as the Magnus force, results in vortex motion in the direction of the current. Here \(\mathbf{v}_s\) is the velocity of the moving vortex, \(n_s\) is the density of the superconducting electrons and \(g\) is a constant much smaller than unity. Due to the scaling factor \(g\), this term is a relatively small contribution and, thus, can often be neglected.
Figure 2-2: Forces acting on a vortex in steady-state current-induced flux motion.

The third term is a damping force that represents any and all dissipative processes, such as eddy-current losses in the normal core. It is directly proportional to the vortex velocity. The viscosity coefficient $\eta$ is a scalar constant that is determined by material characteristics (q.v. page 19).

The last term on the left side represent the pinning force. Pinning will be discussed in a later section. Here, it suffices to say that $f_p$ is of constant magnitude for all vortex velocities and is always directly opposing vortex motion.

One of the interesting things about the flux-flow equation is the fact that, though it describes a superconducting phenomenon, there is a dissipative term. This leads to yet another important aspect of the vortex: a moving vortex produces a voltage that can be
detected across the superconductor. Said another way, a current passing through a superconductor containing vortices will experience a resistance. This can be described by the following empirical relation:

$$\rho_f = \frac{H_{\text{applied}}}{H_{c2}(0)} \rho_n$$  \hspace{1cm} (2.26)

Here $\rho_f$ is the flux-flow resistivity and $\rho_n$ is the normal state resistivity. The ratio of field intensities is approximately the fraction of conductor volume that is occupied by the vortex cores. This relation was derived to be valid at temperatures $T<\lessgtr T_c$ only, yet it serves to indicate that the dissipative effects are closely related to the total amount of normal-state volume. Indeed, materials that do not follow the above relation, most often yield higher resistivities than that described by equation 2.26.

The third term in the flux-flow equation has contains the viscosity coefficient $\eta$, which determines the magnitude of the vortex-velocity for a given set of applied forces. An analytic expression for $\eta$ in terms of material constants is

$$\eta = \frac{\phi_0 H_{c2}(0)}{\rho_n c^2}$$  \hspace{1cm} (2.27)

This equation represents a theoretical minimum for $\eta$. Experimentally, $\eta$ usually has considerably larger values.

The vortex velocity can be determined, if one neglects the relatively small Magnus force contribution, by the following special case of the flux-flow equation:

$$v_\phi = \frac{1}{\eta} \left( \frac{J\phi}{c} - f_p \right)$$  \hspace{1cm} (2.28)

One notes that the velocity is scaled by two factors: the viscosity coefficient and the pinning force. Large pinning forces will require large operating currents, and low viscosity leads to high vortex velocities.

However, it has been shown [14] that there is an upper limit to the velocity of the vortex: $v_\phi \leq v_s/2$, where $v_s$ is the super-electron velocity. Taking the standard definition for the supercurrent density, we find that the expression 2.28 has an upper bound given by
\[ v_s \leq \frac{J_s}{2en_s} \]  \hspace{1cm} (2.29)

where \( J_s \) is the supercurrent density, \( e \) is the electron charge, and \( n_s \) is the superelectron density [14] [16].

One last property of vortices should be mentioned: the interaction between two neighboring vortices. In general, two vortices of the same flux-vector orientation will repel each other. Oppositely oriented vortices will tend to attract each other and annihilate.

2.5 Pinning

The fourth term in the flux-flow equation represented something called pinning. A pinning site occurs when some epitaxial or geometric flaw occurs in the material. At this site, superconductivity is weakened by the flaw. Since part of the contribution to the energy of the vortex is due to its normal core, coincidence of the core and the pinning site will minimize the total free energy: effectively, the vortex sees a potential well at such a site. The strength of the pinning force is maximized if the effective area of the site is roughly the same as the cross-sectional area of the vortex core.

A theoretical treatment of pinning interactions covers two elements: the interaction of the vortex core with the pinning site, and the interaction of the magnetic energy of the vortex with a pinning site. For the individual vortex, the core interaction is expressed as

\[ f_{p\, core} = \frac{\phi_0^{1/2} H_c^{3/2}}{16\sqrt{2}\pi \kappa^2} \]  \hspace{1cm} (2.30)

Here \( f_{p\, core} \) is given as the force per unit (vortex) length in cgs units.

The thermodynamic treatment of the magnetic interaction makes use of the surface energy barrier mentioned earlier on page 12. The results of this treatment give an analytic expression for the maximum contribution due to magnetic interactions, \( f_{p\, mag} \):
\[
\frac{f_{p, \text{core}}}{f_{p, \text{mag}}} = \frac{\kappa}{4 \ln(\kappa)}
\]  

Equation (2.31)

The two pinning interactions just covered are not the only such interactions, but they do dominate the other possible contributions from such effects as strain fields. The previous expressions can be useful in predicting the maximum pinning that could occur in a material:

\[
f_p = f_{p, \text{core}} + f_{p, \text{mag}}
\]  

Equation (2.32)

Note that this expression gives the maximum pinning force for a single vortex. Maximum pinning occurs when the radius of the pinning site is roughly the same as the radius of the vortex. Additionally, in the case of a lattice of vortices, the pinning will be maximized if the spacing between vortices is roughly the same as the spacing between pinning sites. The previous expressions are useful in suggesting what the upper limits for the pinning forces will be. However, empirical information [17] on materials often reveal pinning forces orders-of-magnitude lower than that predicted by these equations.

2.6 Forces Acting on a Vortex near the Edge of a Strip

In addition to the pinning forces just discussed, for the particular case of a superconducting strip (unaffected by external influences: i.e. ground plane), there are also forces acting on the vortex that encourage it to move so as to minimize the total Gibbs free energy of the system. It has been demonstrated that a vortex, embedded in a strip that is carrying no current, is metastable: if the pinning forces are small or non-existent, the vortex will leave the film.

Clem et. al. [7] derived the expressions for the Gibbs free energy associated with such a metastable state. In their derivation, by taking the gradient of the free-energy expression, they came up with the following expression for the forces that act to expel a vortex from a thin-film:
\[ f_{\text{out}} = f_{\text{int}} + f_{\text{ext}} \quad \text{(2.33)} \]

Here \( f_{\text{out}} \) is the total force (per unit length of vortex) acting to push the vortex out of the film, and the contributions to this arise from the energies associated with the fluxon in the interior and the exterior of the superconductor. The two component forces are

\[ f_{\text{int}} = \frac{\phi_0 H_{c1} x z_0}{\pi x_0^2} \left(1 - \frac{4 x^2}{x_0^2}\right)^{-1/2} \quad \text{(2.34)} \]

and

\[ f_{\text{ext}} = \frac{\phi_0^2}{8 \pi^2} (x_0/2 - x)^{-2} \quad \text{(2.35)} \]

where all values are defined as before, and \( x \) is the distance of the center of the vortex from the strip’s center-axis. These equations are most valid for the regions near the edge of the film that aren’t within \( 2\lambda \) of the edge.

Now the validity of these equations for a strip-line in the presence of a ground-plane is definitely questionable. However, it is unlikely that the addition of a ground-plane should change \( f_{\text{out}} \) by more than an order-of-magnitude. Hence, 2.33 may still be useful in predicting whether or not the pinning forces in a strip will be sufficient to keep an embedded vortex from exiting.

2.7 Time of Flux Penetration

The speed with which flux penetrates into a superconductor has been treated theoretically for several situations. In most of the treatments, the approach used assumes an isothermal process, and that shielding eddy-currents just barely maintain the critical field at the moving phase boundary. By equating the energy dissipated by normal eddy-currents during flux penetration with the excess free energy of magnetic field deformation, a general expression for the time of flux penetration has been derived:
\[ \tau = \frac{16\pi R_0^2(1-D)}{9\rho_n c^2(h-(1-D))} \]  \hspace{1cm} (2.36)

where \( \rho_n \) is the resistivity in the normal-state, \( R_0 \) is the radius of the superconducting "disk" to which the field is applied, \( h (=H/H_c) \) is the reduced applied magnetic field, and \( D \) is the demagnetization coefficient of the disk with respect to the orientation of the field. Even though this expression was derived for the geometry of a disk, it can also be used to give an indication as to the order-of-magnitude of \( \tau \) for other shapes under similar conditions.

In the case of vortices moving into a superconductor, there actually is an upper limit to the speed at which they can move, which in turn suggests a possible lower limit to the time it takes for flux penetration to finally reach an equilibrium distribution. This upper limit to the vortex velocity was mentioned earlier; thus a lower bound to expression 2.36 may be

\[ \tau \simeq \frac{x_0}{v_{\phi, max}} \]  \hspace{1cm} (2.37)

where \( x_0 \) is the distance that the vortices must travel in order to attain equilibrium, and \( v_{\phi, max} \) is found by substituting the critical (quenching) current density into equation 2.29. However, this last expression is derived (here in this thesis, only) as an extension to equation 2.29. It must be realized that the theoretical approaches that led to 2.36 and 2.37 are quite different, and thus the limit implied by 2.37 may be suspect.\(^3\)

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\(^3\)In various uses of the expressions provided by 2.36 and 2.37, the author has found a vast disparity in the results. Equation 2.36 often indicates vortex velocities far greater than the speed of light, whereas 2.37 predicts velocities on the order of the speed of sound. Some thought as to the cause for such a predictive disparity led the author to examine the differences in the fundamental premises of the two approaches leading to these equations. A fundamental difference is that the energy-balance approach (c.f. 2.36) does not require that vortices have a continuity of existence, whereas the BCS-theory based approach leading to 2.37 assumes that vortices will always have a continuity of existence (continuity of existence here refers to the following concept: if a vortex is to move from point A to point B, it must pass physically through all intervening space over a finite period of time). Since superconductivity is a quantum phenomenon, it is entirely possible that the constraint provided by the continuity of existence may not apply: e.g. if it becomes energetically favorable for the vortex to effectively "tunnel" past portions of its necessary path, dissolving at point A to reappear at point B. Whether or not 2.36 or 2.37 will be valid for a general situation is unclear, since both have supporting experimental evidence (Huebener [14] presents these in detail). Thus, each particular situation must be carefully examined to determine which equation is best suited for application.
2.8 Comments

The theory that has been presented here is far from an exhaustive treatment on the topic of vortex phenomena. However, the information presented in this chapter will serve as a basis for subsequent discussions on the use of the vortex in circuit construction.
Chapter 3

A Brief Review of Superconductive Switches & Memories

3.1 Introduction

In this chapter, some examples of superconductive current-switches and memory cells will be discussed, with an eye for the operational parameters of these devices. This chapter is by no means an exhaustive study of the different types of switching and memory schemes available, but instead were chosen to give a basis of comparison for the prototype design presented later in this thesis. The first part of this chapter deals with cryotrons and Josephson junctions as switches, followed by a section on memory schemes that utilize these switches. Discussion centers on device behavior, with little development on the actual theory behind the operation: such theory is tangential to the topic of this thesis. Excellent introductions, as well as in-depth theoretical information, can be found in most books on superconductivity [30] [14] [27] [29]. Additionally, there are several articles available that give comprehensive reviews of presently available devices [8] [32].

The third section of this chapter addresses three memory schemes that utilize the vortex as the basis of information storage. They are the result of survey of past literature, covering applicable of thin-film device development, for superconductive switches or memories utilizing vortices. No devices were turned up that utilize the vortex itself to effect the switching of currents.

This chapter finishes with a list of performance parameters that determine the practicality of actual switches and memories.

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4The Josephson junction is often referred to as "the J-J", a recognizable acronym; following this precedent, discussions in this thesis will often use this acronym to facilitate brevity.
3.2 Cryotron Switches

The cryotron acts as a current switch by taking advantage of the change in state that a superconductor experiences as ambient magnetic fields vary above and below $H_c$. Essentially, application of an external magnetic field drives a current-carrying superconductor into the normal-state, thus providing resistance. It is this normal-state resistivity that effects the switching of currents to alternate paths.

As thin-film devices, cryotrons have had two favored designs. One of these is the crossed-film cryotron, depicted in Figure 3-1. In the figure, the control-line crosses the sense-line. When a sufficiently high current passes through the control, the sense-line will be driven normal where the films cross.

![Diagram of a crossed-film cryotron](image)

**Figure 3-1: A crossed-film cryotron [21]**

Figure 3-2 depicts a sketch of the in-line cryotron. The choice to orient the control-line along the axis of the sense-line helps to reduce device inductances. It also makes the
quenching characteristics of the gate more uniform than in the crossed film version, since the fields of both the control and sense currents are oriented in the same directions. Both of these versions of the cryotron have benefited by the addition of a ground-plane, which greatly reduces the total device inductance. A ground-plane also tends to focus the control-field into the region between the control and the ground-plane: increasing the intensity there, while significantly reducing field intensities outside of this region (q.v. Figure 4-6).

![Diagram of an in-line cryotron](image)

**Figure 3-2:** A sketch of an *in-line cryotron* [27]

Cryotron switches have been reported to operate on the order of 10 microseconds, with operating currents on the order of milliAmps$^5$ [21]. Ultimately, the cryotron's performance is limited by the time it takes for the gate film to make its transition to the normal-state. This is fairly independent of device geometries, as it is an intrinsic aspect of superconductivity. This limit has been estimated to be 10 nanoseconds [27]; it is not expected that improved lithography techniques or other related thin-film technologies will be able to surmount this limit, once reached.

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3.3 Josephson Junctions

On the other hand, the Josephson junction makes its transition to a resistive state in better than 800 picoseconds (this transition time is primarily determined by the capacitance of the junction, rather than the time it takes for the quantum state to change) [27]. It is also capable of yielding higher resistances than the cryotron. The present drawback of the J-J is the need to reproducibly create very thin high-quality (= 10 to 100 angstroms) insulating barriers: the junction itself. Such quality is at the limit of current capabilities in thin-film device technology.

![Josephson junction](image)

**Figure 3-3: Sketch of a Josephson junction [27]**

Figure 3-3 displays a sketch of the J-J. The junction itself is the point at which the two tin conductors are joined by a thin insulating oxide barrier. Up to a threshold current (see Figure 3-4) of around 10 μA, there is no resistance; superconducting pairs of electrons are able to tunnel across this barrier for all currents up to this threshold value. The threshold is determined by the superconductive energy gap of the junction superconductor, and this threshold can be significantly depressed by the magnetic field of an active control-line.
3.4 Persistent Current Memories

The fact that superconductors can act as lossless inductors, suggests that information can be stored as a persistent current in a superconductive loop. Other information storage schemes have, of course, been demonstrated\(^6\), but testing of these other schemes showed the persistent current-loop schemes to be superior in the areas of fabrication ease, switching times, and device dimensions.

Figure 3-5 is a schematic of a flip-flop version of a such a current-loop memory. A "1" is stored by a current loop in the clockwise direction, and a "0" is represented by current looping in the opposite direction. If a "1" is stored, the self-field of a sense-current will constructively add with the field of the circulating current to drive the switch in the sense-

---

\(^6\)The persistatron, the Crowe cell, and the continuous-sheet memory are just a few of these schemes: they are discussed in detail in Newhouse's Applied Superconductivity [21]
line resistive, thus non-destructively sensing the information. If a "0" is stored, these fields will add destructively, and the switch in the sense-line will stay resistanceless.

![Diagram of current flow during writing in a "flip-flop" cell.]

**Figure 3-5:** Illustration of current flow during writing in a "flip-flop" cell.

A logic "1" is initially stored in the cell (a). Upon application of a Y current (b) and coincident X current (c), current transfer occurs (c) such that at the end of the process, a logic "0" remains in the cell (d). Writing a "1" consists of a symmetric reversal of this process. After Zappe [32].

One notes that the switches in this figure are denoted by x’s, which is the commonly accepted circuit symbol for the ideal J-J. Indeed, this particular illustration was taken from an article on Josephson memories [32]; in this same article, actual experimental work using this scheme is discussed, citing current transfer times of 600 picoseconds. This scheme has also been realized with cryotrons as the switching elements [21] with current transfer times in the 10’s of microseconds.

Figure 3-6 depicts another scheme, referred to as the 1.0 mode cell. In this case, a "0" is represented by the absence of a circulating current, and a "1" by the presence. Triple coincidence of the X and Y and Y’ currents will cause the write-gate to switch. The entire Y current is transferred to the right conduction path and upon removal of the Y current, a circulating current is left in the loop. A zero is written by applying currents X and Y’ but not Y; this drives the gate resistive and dissipates the the loop-current. Sensing of a "1" is the same as in the "flip-flop" cell. A "0" will mean that the sense-gate is only affected by its
own self-field, and thus the sense-gate will not switch.\textsuperscript{7}

\begin{center}
\includegraphics[width=0.5\textwidth]{figure3-6.png}
\end{center}

\textbf{Figure 3-6: The 1.0 mode cell}

This scheme has certain advantages over the first one (such as needing fewer switches), but the margins of the sense-current are diminished; switching times are, of course, similar. Here, again, it is possible to utilize cryotrons instead of J-J’s, but not desirable.

3.5 Vortex Memories

This section presents three different proposals of memory schemes that utilize the vortex. An important fact that they all have in common is that they all need well-placed J-J’s, which are used to detect the presence of the vortex’s flux. None of these schemes attempt to utilize the vortex’s resistive properties.

\textsuperscript{7}The resistor $R_e$ in this schematic acts to reduce capacitive resonances in the J-J that serves as the gate. [32]
3.5.1 The Vortex-File Memory

A common method of storing information in semiconductor-based computers employs magnetic-bubbles. In 1979, Werner Bachtold [3] proposed the vortex-file memory scheme that treats vortices as magnetic bubbles. Vortices are nucleated into the beginning of a "dandelion" trough, depicted in Figure 3-7. They are then moved along the trough by alternating an applied current in the y direction. Since vortices move mostly at right angles to the current, they will constantly be moving from one side to the other: it is the dandelion pattern that is responsible for guiding them down the trough.

This dandelion pattern is essentially a region of superconductor that is thicker than the trough. The vortices will be guided along this shape, since it is energetically unfavorable for them to enter the thicker region (c.f. page 17). For example, a current in the +y direction will cause the +z vortices to move in the +x direction. If they start at point A in the figure, they will move until they reach the guiding structure. They will then continue with a modified direction to point B, staying there until the current reverses. Reversal of the current will send these vortices again to the opposite guiding structure, which then shunts them down to point C. It is the presence (or absence) of these discrete traveling groups of vortices that represent a "1" (or "0"). Sensing of the information requires that J-J's be placed at each sawtooth indentations (for example, at point B); the nearby presence of a group of vortices will significantly depress the threshold current of the J-J, thus indicating a "1" to an applied sense-current.

No experimental work on the vortex-file has been published, possibly due to the many difficulties and unknowns apparent in this device.

1. Bachtold's method of generating vortices requires a fairly complex thin-film construct (not discussed here).

2. The scheme will only work for extremely low pinning forces, and little research has been done on the minimization of pinning forces.

3. The dandelion structure is slightly difficult to make using current lithography techniques, particularly when trying to make it as small as possible.
3.5.2 The Abrikosov Vortex Memory

The Abrikosov vortex memory [20] is similar in several ways to the vortex-file. Again, the presence or absence of vortices is used to indicate a "1" or a "0". Also, detection of the information depends on carefully placed J-J’s. Furthermore, vortices are localized by creating free-energy barriers through variations in the thickness of the superconducting film. Figure 3-8 shows a schematic structure of the Abrikosov vortex memory.
$I_w$ is used to magnetically nucleate vortices into the vortex storage region. This storage region is much thinner than the surrounding superconducting layer and is composed of a type-II material: the locally reduced thickness makes it energetically favorable for vortices to nucleate in the storage region before nucleating in the thicker surrounding superconductor.

![Diagram](image)

Figure 3-8: Schematic structure of an Abrikosov vortex memory cell [20]

A "1" is stored by writing a batch of vortices oriented in the $+z$ direction. A "0" is represented by a batch of oppositely oriented vortices, written by operating $I_w$ in the reverse direction. The two-junction interferometer\(^8\) is used to detect the presence and orientation of nearby magnetic fields: i.e., the trapped vortices. Write currents were reported to be of the order of 10-40 mA, with read currents of around .1 mA. $I_B$ and $I_c$ are respectively, a bias current and a control current for the interferometer, used simultaneously to read the state of the cell.\(^9\)

An important factor in determining the level of $I_w$ for this device is the magnitude of the pinning forces in the storage region. High pinning forces tend to make the vortices stay near the write-line. As was the case with Bachtold, the authors of this article also discussed

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\(^8\)Interferometers are current-switches constructed out of 2 or 3 J-J’s. They are much more sensitive detectors of magnetic fields. [30]

\(^9\)If either $I_B$ or $I_c$ is absent, the interferometer read-gate stays resistanceless: a "half-select" case. The need to have two separate addressing currents, in order to sense cell contents, is an important property for cells that fit into a memory-array.
the necessity for low pinning forces, since lower pinning forces improved write current margins.

The primary advantage this cell has is the fact that it is physically smaller than current-loop memory cells; at the time of publication, this 30x60 \( \mu \text{m}^2 \) (using 5 \( \mu \text{m} \) linewidths) was reported as being one fourteenth the size of conventional cells. As lithographic resolution improves, this cell can be expected to retain its size advantage over current-loop cells. No information was reported as to operational read or write time, nor the power levels involved.

3.5.3 "A Superconducting Vortex-Memory System"

Developing on an idea originally proposed by Hebard and Fiory [11], J. Parisi and R.P. Huebener [22] published work on a memory system based on bistable vortices or flux tubes. Actual experimental work was reported on the use of large multi-quanta flux (MFQ) tubes, with theoretical extrapolation to the same system using single-quantum flux (SFQ) tubes. Figure 3-9 helps illustrate the method of operation.

![Figure 3-9: Model Geometry for a Bistable Vortex Memory Cell](image)

In the figure, a vortex is trapped by a strong pinning site (4) in a superconducting ground-plane (3). The insulated (2) superconducting control-line (1) passes directly above the pinning site, and it is quite close to the ground-plane. A current passing through the control will create a magnetic field that interacts with the vortex-field, making it more
energetically favorable for the vortex’s flux to pass to one side or the other of the control. Removal of the current leaves the vortex flux diverted to only one side, since the control-line’s Meissner effect creates an energy barrier to spontaneous redistribution of the vortex’s flux. This results in the two energetically stable flux-paths that are illustrated in the figure.

In order for this to actually operate as a memory, a J-J (not shown) is required for each bistable vortex; this J-J must be placed with extremely precise accuracy, in order to detect the very localized and quite small amount of flux. Ideally, the J-J would be just to one side of the control; when the flux is passing through its nearer stability position, the J-J would detect this as a significant alteration in its threshold current. When the flux is passing to the far side of the control, the field in the region of the J-J would not significantly alter that threshold.

Parisi and Huebener did not actually attempt to create the ideal case. Instead, they modeled the situation using multi-fluxon tubes and a high-resolution magneto-optical detection method: no J-J’s were employed. Based on the modeling experiments, they produced the following table of values showing the results of their experiments and an extrapolation to the ideal, single-fluxon case.

All of the values in the table are self-explanatory, except the switching velocity, which refers to the actual speed with which the vortex moves from one stability state to the other.

Indeed, if the values listed on the right side of this table could be achieved, this memory scheme would far surpass any others. However, this extrapolation is based on thin-film technology that has yet to be achieved. The authors approach this problem by presenting their scheme as an excellent device simply awaiting the associated technologies. However, this wait may indeed be long.

• The system depends on extremely exact placement of very small J-J’s: it is hard enough just to fabricate a J-J, let alone achieve precise placement on the scale of a vortex-field, \(\approx \lambda\) (usually between 500 and 1000 angstroms).
<table>
<thead>
<tr>
<th>Memory Characteristic</th>
<th>MFQ Study</th>
<th>SFQ Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Size</td>
<td>$45 \mu m \times 45 \mu m$</td>
<td>$3 \mu m \times 3 \mu m$</td>
</tr>
<tr>
<td>Packing density</td>
<td>$\approx 5 \times 10^4 \text{ cm}^{-2}$</td>
<td>$\approx 10^7 \text{ cm}^{-2}$</td>
</tr>
<tr>
<td>Minimum line width</td>
<td>$6 \mu m$</td>
<td>$0.25 \mu m$</td>
</tr>
<tr>
<td>Control current</td>
<td>$25 \text{ mA}$</td>
<td>$\approx 2.5 \text{ mA}$</td>
</tr>
<tr>
<td>Control-current density</td>
<td>$\approx 2 \times 10^6 \text{ A/cm}^2$</td>
<td>$\approx 10^7 \text{ A/cm}^2$</td>
</tr>
<tr>
<td>Switching time</td>
<td>$350 \text{ ns}$</td>
<td>$\approx 80 \text{ ps}$</td>
</tr>
<tr>
<td>Switching velocity</td>
<td>$\approx 25 \text{ m/s}$</td>
<td>$3000 \text{ m/s}$</td>
</tr>
<tr>
<td>Switching voltage</td>
<td>$\approx 1 \mu V$</td>
<td>$\approx 25 \mu V$</td>
</tr>
<tr>
<td>Power dissipation</td>
<td>$\approx 10^{-8} \text{ J/s}$</td>
<td>$\approx 5 \times 10^{-8} \text{ J/s}$</td>
</tr>
<tr>
<td>Energy Dissipation</td>
<td>$\approx 10^{-14} \text{ J}$</td>
<td>$\approx 5 \times 10^{-18} \text{ J}$</td>
</tr>
</tbody>
</table>

Table 3-I: Characteristics of a superconducting SFQ vortex-memory cell, extrapolated from an experimentally tested MFQ model system; after *Parisi and Huebener* [22]

- Likewise, a controlled array of very strong pinning points in the ground-plane is required, again on a very small scale with extremely precise placement.

- A reliable method is required of initializing one (and only one) vortex to each pinning site, without any stray vortices getting trapped in the other parts of the ground-plane.

- The array of control-lines must have placement as precise as the placement of the pinning-points; otherwise, one of the two bistable points will become considerably more energetically favorable, diminishing operational memory characteristics.

Indeed, once thin-film technology is capable of the above feats, this scheme will be effective. However, when the technology reaches that stage, many other devices will have greatly improved, too, with the new techniques.
3.6 Device Characterization

The last article reviewed provided a list of device characteristics that parameterize the usefulness of a memory cell. It is unfortunate that few of the other papers reviewed were as complete with operational parameter descriptions. Along the lines of Table 3-1, the following is a list of performance considerations that generally characterize a device. This will be referred to later in this thesis.

1. Device dimensions, contrasted to line-width resolution.
2. Difficulty of fabrication
3. Operational current magnitudes
4. Switching time
5. Switching voltages (or impedances)
6. Power and Energy dissipation
7. Stability
8. Expected improvement of the above listed parameters, with respect to improvement of related technologies

Each of the current-switches and memory-schemes presented in this chapter have strengths and weaknesses in differing categories; e.g. the J-J is superior to the cryotron, but not on all counts (i.e. #2). As differing applications stress different characteristics in the list, each device ends up having a fair chance at being preferred for a particular application.
Chapter 4

Vortices as the Basis for Switching and Memory

4.1 Introduction

In the last chapter, the design and performance considerations of several types of superconductive memory cells were discussed. In each case, at least one J-J per 'bit' of information was necessary. Essentially, the J-J serves the function of a resistive gate that can be controlled by an external field or an applied current. However, the actual information is often stored as circulating currents in a conducting loop. The J-J itself does not actually store any information, but is placed so that it will shunt current at the appropriate times: i.e. read or write operations.

The thrust of this chapter is to qualitatively present another approach to memory and current-switching in supercomputer technology. This approach focuses primarily on the use of flux-flow resistivity as the basis for a resistive gate, and the manifestation of the vortex as the basis for information storage: a prototype switch/memory design is presented. Note that the following discussion subsumes the fact that any circuit element will be a thin-film device.

4.2 Switching Of Currents

One of the most basic circuit elements necessary for any form of computer is the current-switch: a device that, under certain controllable circumstances, is capable of shunting current from one path to another. Presently there are only two superconductive devices capable of current switching: the cryotron and the J-J. Both of these switch the current by producing a resistive effect which will encourage an applied sensing current to
shunt to a path of less (or usually, zero) resistance. The essence of the operation here is the controlled creation of some resistive effect within a current-path. The specific methods of obtaining this resistive effect in the two devices just mentioned utilize respectively, normal-state resistivity and quantum-tunneling. However, as of yet, there are no devices proposed that utilize flux-flow resistivity as the requisite resistive effect needed to switch a current.

A very simple idealized example illustrates the basic idea. In Figure 4-1, two thin-film conduction paths are set up in parallel. Both paths are assumed to have the same inductances and both are assumed to be superconducting. An applied sense current $I$, in such a case, would split up equally between the two paths.

If, however, vortices (which are free to move) are present in the material of the left path, the applied current will encounter a resistance there: in the steady state situation (after the inductive transients die away), the current would completely shunt to the other path. In this simple description, we find the basis for a current-switch. The circuit element responsible is that portion of the left film that is capable of storing vortices, which I will refer to from now on as a SUperconducting Flux-Flow Resistance Switch: hereafter, SUFFRS.

The description just presented is actually a gross simplification. It doesn’t take into account pinning effects within the material of the film, time-dependent effects, nor even how the vortices get into the material in the first place. These are problems that are dealt with when one starts to design the actual working model of the ideal circuit.

4.3 Flux-Flow Resistance as a Switching Mechanism

Already, a great deal is known about the phenomenon of flux-flow (c.f. Chapter 2). This information can be used to suggest what properties a SUFFRS would have.

1. There will be a threshold $I_p$ (see Fig. 4-2) for the sense current below which there is no resistance. This threshold is determined by the pinning forces of
Figure 4-1: Parallel superconductive current paths.

the material, which vary widely with the choice of material and deposition techniques.

2. Above the threshold \( I_p \), the sense current will experience a resistance that is roughly proportional to the cross-section of material in the normal-state: this normal-state is the collection of vortex cores, and thus is closely related to the total number of vortices and the size of their cores. A clear ceiling for this resistance is the case where the trap (the conduction path that contains the vortices) is completely quenched.
3. The time it takes to make the switch resistive is simply the time it takes for vortices to penetrate into the material: this can be, and most often is, a quicker process than completely quenching the material.

4. When the vortices are presenting a resistance to the sense current, they move at right angles to the current, and they move very fast indeed. This is important to keep in mind, since the direction of movement means the vortices exit the trap at a very high rate. This suggests that the vortices may need to be continually refreshed by the control-line during a switching cycle, for low frequency applications.
5. If the current applied to the vortices is alternating at high enough frequencies, the vortices will not exit the film; the frequency threshold will be determined by factors such as the vortex velocity as a function of applied current, pinning forces in the film, and the distance the vortex must travel to exit the film (this is discussed in detail in Chapter 5). This will provide a different use for the SUFFRS, as the vortices will not need to be refreshed. Additionally, in this case, it has been shown that the threshold current often is considerably depressed for high frequencies of applied currents (discussions by Huebener [14] on the various experimental effects on the variation of pinning forces with respect to a.c. currents and fields).

6. The time it takes for the current to switch to another path\textsuperscript{10} can be estimated as \( \tau = L/R \) (where \( L \) is the inductance of the device, and \( R \) is the resistance provided by the switching mechanism), and this time combines with the time period mentioned in \#3 to produce the total cycle-time of the device.

7. The amount of current switched will be the total sense current applied \textit{minus} the threshold current. One would expect the current to decay as \( e^{-Rt/L} \) until it reaches \( I_p \), at which point the vortices stop moving, setting the SUFFRS resistance to zero (see fig. 4-3). It is interesting to note that this behavior leads to roughly the same I-V characteristics (with respect to curvature and shape) as that of a J-J. This threshold effect in the SUFFRS would then serve the same utility as it has in the usages of the J-J (e.g. multiple addressing currents to a single memory location).

4.4 A Prototype SUFFRS

4.4.1 Design Considerations

Here is a list of the design considerations that led to the prototype SUFFRS presented in this section.

- A geometry for the control-line(s) must be chosen so that the control current creates a locally intense magnetic field and therefore a localized group of vortices.

- A vortex trap must be created, capable of holding vortices and passing a sense current.

- Vortices in the trap should be primarily oriented perpendicular to the trap surface (in general, this is energetically favorable after the removal of the applied field, however, any pinning forces will act to "freeze" the vortices in the original applied field’s orientation).

\textsuperscript{10}The time component will be referred to hereafter as \textit{the steering time}, since it is the time it takes for the resistive effect to steer the current to a new path.
Figure 4-3: Expected sense current behavior with time

- There must be an alternative current path for the applied sense current.
- The effects of a ground plane (or its absence) need to be considered.
- The order and thickness of layers is constrained by the necessity of continuous films in the face of step-coverage problems (see appendix A).
- The cross-sectional area that the trap presents to the sense current should be made as small as possible, thereby maximizing the resistance.
• The current density of a sense current in the trap should be fairly uniform, since the vortices will tend towards an even distribution across the width of the trap and each individual vortex moves in response to the current passing through its local region. An uneven distribution could lead to a variety of hysteresis effects.

• The control-line should never enter a resistive state. This means choosing a material with either a significantly higher $H_c$ than the trap-material or with extremely high pinning forces.

4.4.2 A Possible Design

The current design for the prototype SUFFRS is shown in Figure 4-4. The deposition of materials should be as follows:

1. 1000 angstroms of Nb, patterned: titled Hole, Nb1.
2. 1000 angstroms of SiO, unpatterned: not shown.
3. 500-750 angstroms of V, patterned: titled Trap, V.
4. 1000 angstroms of SiO, patterned: titled Protection, SiO.
5. 2000 angstroms of Nb, patterned: titled Control, Nb2.

It is assumed that the minimum lithography linewidth resolution will be 5 microns, since that is what is routinely available at Lincoln Laboratory. Additionally, the materials were chosen because they are available for deposition at Lincoln, as well as being likely candidates for satisfying material property constraints: e.g. Nb has a much higher $H_c$ than V.

The first layer of material is a ground-plane, serving to shield out stray magnetic fields and to reduce the inductances of other parts of the circuit: i.e. transmission-lines. It is important, however, that the ground-plane have a hole in it, situated below the active portion of the SUFFRS. This is to prevent the distortion of the control-field that would occur from image-current effects in the ground-plane. Such images significantly reduce the perpendicular components of the magnetic field, which would minimize the total number of
If a J-J is used to test the SUFFRS prototype, it would be located near the alternate current path to sense a switched current.

If testing is done using an external (to the chip) circuit, then the alternate current path does bend right to join up, but instead continues directly to the testing circuit.

Figure 4-4: SUFFRS Prototype
usable vortices in the trap.\textsuperscript{11}

The second layer is simply an unpatterned insulation layer. The third layer is the vortex trap itself. It is as thin and narrow as is reasonably possible, to maximize the effective resistance. Vanadium was chosen, as it has a fairly high intrinsic resistance (for a metal), does not easily oxidize, has a significantly lower $H_c$ than niobium, and is available for deposition at Lincoln.

The fourth layer is deposited to protect the vanadium layer from the processes involved in the final deposition. The SiO is patterned to cover only the vanadium, leaving two end-contacts for the final deposition (the Nb "alternate path"). Since the same chemicals are used to etch and pattern the niobium as the vanadium, it becomes necessary to put this protection cover on top of the V.

The fifth and final deposition provides both the niobium control-line as well as a

\textsuperscript{11}If a current is passed through the control-line, the magnetic field around it looks like the sketch in Figure 4-5 on page 50. Note that the field becomes most intense near the edges of the control. Additionally, the orientation of the field at the edges is primarily in the $z$-direction. This anisotropy is needed to write the vortices into the proper trap-region with the proper orientation. The hole in the ground-plane is there to preserve this field distribution. If the ground plane were continuous, image effects would make the field distribution look like Figure 4-6 on page 51, with the most intense field being parallel to the trap. The choice of this patterned ground plane thus avoids the difficulties caused by image-effects, while still reducing inductances and screening out 'stray' magnetic fields.
niobium "alternate current path". One will notice that, due to the limited deposition of the second insulating layer, the niobium control-line will be "on the same level" as the vanadium: both of these metals rest upon the first layer of insulation. It is desirable to have both the trap and the control in the same x-y plane, as this will orient nucleating vortices in the z-direction only, though exactitude should be unnecessary due to the ratio of widths to thicknesses. The previous SiO layer could be deposited as an unpatterned sheet, without significantly changing the expected vortex-nucleation behavior. The choice to pattern the second SiO layer, however, is an easy way to remove just one more approximation from the theoretical treatments of the device's operation.

The following table is a summary of the dimensions of the device, utilized later in Chapter 5.

In this table, the Symbol column gives the variable references that apply to these parameters later in this thesis. Clearly, these are not all of the dimensions that are apparent in Figure 4-2 (for instance, the size of the hole in the ground-plane); they are, however, the dimensions that are critical to the behavior of the device, as will be seen.

---

12In this prototype, the design that is examined has the alternate current path forming, along with the vanadium trap, a square conduction loop (50x50μm). This is the best starting prototype, but it may require slight modifications based on the testing techniques. Essentially, a choice must be made as to the "known" method that will be used to test the unknowns of the SUFFRS: e.g. How does the experimenter determine if the current has actually switched? In order to gather experimental data on the operational parameters of the SUFFRS, it necessary to use known detection techniques for such an endeavor. There are three approaches that the author can suggest.

(1) Put a J-J near the alternate current path, to determine when the current has switched (similar to the memory-cell designs covered in Chapter 3).

(2) Magneto-optics are a powerful method for characterizing the behavior of prototype vortex-devices: unfortunately, the facilities for this are unavailable at Lincoln Laboratory and, thus, are only practical if one were to locate a lab already set up with such techniques.

(3) Use transistor/amplifier set-ups external to the superconducting circuits: this would require that the alternate current path include the detectors, which will greatly increase the effective inductance. However, this is the easiest setup to adapt to in order to perform preliminary experimental verification on the theory presented in Chapter 5; if/when such verification is gained, then more discerning techniques would be justifiable.
<table>
<thead>
<tr>
<th>Symbol(s)</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
<td>1,000 A</td>
<td>Thickness layer 1</td>
</tr>
<tr>
<td>$z_2$</td>
<td>1,000 A</td>
<td>Thickness layer 2</td>
</tr>
<tr>
<td>$z_3, z_t$</td>
<td>500 A</td>
<td>Thickness layer 3</td>
</tr>
<tr>
<td>$z_4$</td>
<td>1,000 A</td>
<td>Thickness layer 4</td>
</tr>
<tr>
<td>$z_5, z_c$</td>
<td>2,000 A</td>
<td>Thickness layer 5</td>
</tr>
<tr>
<td>$x_c$</td>
<td>10 μm</td>
<td>Width of control</td>
</tr>
<tr>
<td>$x_t$</td>
<td>5 μm</td>
<td>Width of trap</td>
</tr>
<tr>
<td>$y_t$</td>
<td>50 μm</td>
<td>Length of trap</td>
</tr>
<tr>
<td>$x_{tc}$</td>
<td>5 μm</td>
<td>spacing, trap-to-control</td>
</tr>
</tbody>
</table>

Table 4-1: Summary of SUFFRS dimensions

There are two possible ways of utilizing this design, differing according to the frequency at which the device is operated. As was mentioned earlier, at low frequencies, the vortices will exit the film. In this situation the SUFFRS will be able to only operate as a simple current switch, analogous to the cryotron. At higher frequencies, the vortices will not move sufficiently far enough in one direction to exit the film. Hence, a previously "written" batch of vortices can be sensed several times, and the SUFFRS acts as a high-speed memory. The low frequency case is addressed here first.

4.4.3 The SUFFRS as a Switch

The method of switching is very simple. An applied control current, moving in the $-y$ direction, creates an anisotropic field (Fig. 4-5) that concentrates flux in the region of the trap. The field in this region is mostly oriented in the $+z$ direction and is decreasing monotonically in intensity as one moves away from the control line. A sense current is simultaneously applied in the $+y$ direction; it's own self-field contributes to the gradient of
the magnetic field.\textsuperscript{13} This gradient of field-intensity will cause vortices to be most likely to enter the trap from the +x side. The direction of the sense current will cause depinned vortices to move in the -x direction, thus exiting the other side of the trap. Hence, once the sense current exceeds the pinning forces, there will be a dynamic flow of vortices from one edge of the trap to the other. Thus, the control current must stay active for the entire switching time.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\caption{Sketch of control-current field distribution: cross-sectional view of the active control-line.}
\end{figure}

The fact that the control and the sense currents must be active simultaneously is what limits this device to a simple switch, instead of a memory device. This constraint comes from the high velocities at which the vortices move to exit the film. However, even a simple

\textsuperscript{13}On the +x side of the trap, the z-field components of the control-field and the sense current’s self-field will add constructively, since both are oriented in the +z direction. On the -x side of the trap, the self-field has an opposite orientation, and will add destructively.
current-switch is of interest, if it has enough properties to recommend it: one example of a "simple current-switch" is the J-J. The SUFFRS is easier to fabricate than the J-J, since the SUFFRS does not need thin tunneling barriers. Furthermore, any current-switch can be utilized in the persistent-current memories presented in Chapter 3.

At first glance, the SUFFRS may appear to be a modified cryotron. This illusion is dispelled when one realizes that the time it takes to write a vortex is much smaller than the time it takes to completely quench the trap, and that these time factors appear in the final switching time of the device. Additionally, the SUFFRS has a threshold current (determined by punning forces) up to which no resistance is encountered: the "threshold" for a cryotron is essentially its critical current for transition to the normal-state. A drastic difference in the essential mechanisms leading to similar, but far from identical, behaviors.
4.5 High-frequency Operation

4.5.1 Storage of Information

As was seen in Chapter 3, a favored method for storing information in superconductive technology is the creation of a circular current in a superconducting loop. The J-J and the cryotron are used to shunt current in the proper direction. Clearly, if the low-frequency prototype suggested in the previous section works, we now have another possible switch that could replace the current switches in those memory schemes. Whether or not this would be an improvement (with respect to response-time, device dimensions, etc.) is something that must be decided after experimental work with a SUFFRS prototype. In any case, the method of storing the information would be the same.

The other method of storing information described in Chapter 3 was based on vortices: the actual presence or absence of a vortex signified the stored information. Even though the vortex is technically just another circulating current, it has certain intrinsic differences. The vortex itself requires no special real-estate arrangement in order to represent information.\textsuperscript{14} Since the vortex is an extremely small, quantized, and directionalized manifestation, it has an excellent chance of becoming the favored information 'bit' in the future of superconductive computers. The designs in Chapter 3 that used vortices, however, all depended on a J-J to detect the presence of the vortices.

4.5.2 Memory Without a Josephson Junction?

A vortex is capable of representing information. A vortex is also capable of signalling its presence via flux-flow resistivity. These two facts suggest the possibility of a memory that doesn't depend on a J-J in order to function.

\textsuperscript{14} Though localization of the vortex sometimes can require specialized real-estate, as was the case in Chapter 3.
If the prototype SUFFRS is operated with high-frequency a.c. sense currents, then it may not be necessary to apply the control current and the sense current simultaneously. If this constraint can be lifted, then the device can act as a memory-device. Indeed, it is the vortex-motion, and the high vortex velocity that will determine the appropriate frequency to operate at. Assuming that this frequency can be determined and achieved, the SUFFRS would then act as a high-speed NDRO memory.

During the write-cycle, the control current would be pulsed for the duration necessary for a batch of vortices (a "1") to enter the trap. After the control-pulse, the vortices will tend to stay in the trap due to pinning forces. At a later time, an applied sense current of sufficient magnitude would depin the vortices. Since the sense current will be operating at very high frequencies the trapped vortices will oscillate between the edges of the trap, never quite exiting, and providing a resistance to the sense current. Clearly, if no vortices are present (a "0"), there will be no resistance, and thus no redirection of the sense current. In order to clear the memory, a simple d.c. sense current would suffice.

4.6 Comments

It is important to point out that the designs proposed in this chapter are kept deliberately simple. This is because there are several experiments that need to be done to properly characterize device behavior. Magnitudes and reproducibility of pinning forces is one major example. The viscosity coefficient $\eta$, which heavily influences the vortex velocities, needs to be examined experimentally. Simple prototypes are ideal for finding out these unknowns, while still providing the opportunity to demonstrate that the switching and storage techniques actually work. Eventually, the appropriate materials or deposition techniques can be found to properly tailor the operational characteristics of the prototypes.
At that point, more complex designs\textsuperscript{15} may be appropriate.

\textsuperscript{15}Some design changes would need to be made in order to render the SUFFRS usable as an array element in a memory or switching array. However, it is premature to pursue this more complex design, before a simple prototype has been experimentally studied.
Chapter 5

Theoretical Device Parameters of the SUFFRS

5.1 Introduction

In this chapter, the theory presented in Chapter 2 is adapted to the device design given for the SUFFRS in Chapter 4, so as to give some theoretical estimates on the operational parameters of a prototype. It must be kept in mind that nearly all of the values and equations presented in this chapter are adaptations, approximations, and worst-case analyses; the applicable theories are not sufficiently exact for better than rough order-of-magnitude estimates. However, at this point in developing a prototype, such estimates are quite valuable.

5.2 Environment and State-Variables

Before proceeding with any derivations, certain assumptions should be outlined: (1) It is assumed that the ambient temperature is 4.2 Kelvin. (2) The films are expected to have a high degree of epitaxial disorder, leading to short electron mean-free-paths. This situation requires that the equations taken from Chapter 2 be the "dirty" case equations. (3) Coordinate references, throughout this chapter, are as shown in Figure 4-4, with the y-axis (x=0, z=0) aligned with the axis of the trap.

Table 5-I provides a ready reference to the intrinsic parameters of the SUFFRS. These values are specific to the materials chosen, and they (along with the dimensions of the device) form the basic data set necessary to yield numerical values from the derivations
presented in this thesis.

<table>
<thead>
<tr>
<th></th>
<th>V</th>
<th>Nb</th>
<th></th>
<th>V</th>
<th>Nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$:</td>
<td>4.9 K</td>
<td>9.0 K</td>
<td></td>
<td>1310 oe</td>
<td>1944 oe</td>
</tr>
<tr>
<td>$\xi_0$:</td>
<td>460 A</td>
<td>390 A</td>
<td>$\lambda_L(0)$:</td>
<td>374 A</td>
<td>315 A</td>
</tr>
<tr>
<td>$W$:</td>
<td>50.94 amu</td>
<td>92.91 amu</td>
<td>$Z_{valence}$:</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\rho_m$:</td>
<td>5.98 g/cm$^3$</td>
<td>8.57 g/cm$^3$</td>
<td>$\rho_n$:</td>
<td>15.9 $\mu\Omega$-cm</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 5-I: Intrinsic parameters of the prototypes' materials

Both of the $T_c$'s listed are slightly smaller than the bulk value: this is an aspect of superconductivity in thin films. Vanadium actually has more than one valence. The value chosen is in keeping with the worst-case-analysis approach; of the possible values, this one leads to smaller estimates for the normal core of the vortex.

Table 5-II gives several state-variables that can be calculated immediately from the previous values, using the theory from Chapter 2. The electron mean-free-path has been calculated utilizing the Drude model [2], using the experimental value for the resistivity of vanadium. As no experimental value is directly available for niobium films, it is assumed here that it will be of the roughly the same order-of-magnitude ($\approx 10 \mu\Omega$-cm) as that of the vanadium.\(^\text{17}\) The values given in Table 5-II are utilized throughout this chapter.

---

\(^{16}\) All values are taken from published sources [17][31], except for the resistivity. The intrinsic resistivity of bulk vanadium is inapplicable here, due to the high disorder of the films. The resistivity presented in the table has been obtained experimentally, as is discussed in Appendix C. No specific experimental value for the resistivity of niobium films is available.

\(^{17}\) This assumption is based on valuable discussions with personnel in Group 86 of MIT's Lincoln Laboratory.
<table>
<thead>
<tr>
<th></th>
<th>V</th>
<th>Nb</th>
<th>V</th>
<th>Nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_c(T)$:</td>
<td>347.6 oe</td>
<td>1,521 oe</td>
<td>1,506 oe</td>
<td>2,221 oe</td>
</tr>
<tr>
<td>$H_{c1}(T,l)$:</td>
<td>59.5 oe</td>
<td>336 oe</td>
<td>4,496 oe</td>
<td>11,627 oe</td>
</tr>
<tr>
<td>$\xi(T,l)$:</td>
<td>262 A</td>
<td>149 A</td>
<td>2,403 A</td>
<td>808 A</td>
</tr>
<tr>
<td>$l$:</td>
<td>29.3 A</td>
<td>41.7 A</td>
<td>9.15</td>
<td>5.41</td>
</tr>
<tr>
<td>$D_{\text{demagnetization}}$</td>
<td>.995</td>
<td>---</td>
<td>$\eta$:</td>
<td>2.18x10^{-20}</td>
</tr>
</tbody>
</table>

Table 5-II: State variables for the prototype SUFFRS

5.3 Analytic Magnetic Field Expressions

When a current passes through the control-line (see Figure 5-1), the field result in the area occupied by the trap will be oriented in the z-direction. The control-field will pass through the trap, in the steady-state, with almost no attenuation, since the trap is much thinner than its magnetic penetration depth [30]. The control-field intensity will decrease monotonically as one proceeds in the negative x-direction.

In order to proceed properly, it is important to have an expression for the magnetic field distribution of a current-carrying strip.\(^\text{18}\) The following equations are general expressions for the x and z components of magnetic induction for a rectangular conductor, bounded by the planes $x=a$, $x=-a$, $z=b$, and $z=-b$, which is carrying a current $I$ of uniform density in the -y direction.

\(^\text{18}\)In the AIP Handbook [10], there are several equations for magnetic field distribution of a conductor. I was greatly surprised to discover that the two appropriate equations had been entered into the handbook incorrectly. On page 5-27, one finds two equations that resemble the equations presented here very closely. However, if tested, those equations fail: a very quick and simple test is to set the thickness of the strip to zero; for that case, the expressions must reduce to certain limiting equations given at the very top of 5-27. Since this test fails, and there is no reference given in the Handbook as to the equations' source, it became necessary for this author to rederive the appropriate equations from limiting cases. The actual derivation is presented in Appendix D.
\[ B_x = \frac{-\mu J}{2\pi A} \left[ z_3 \ln(r_3/r_4) - z_1 \ln(r_2/r_1) + |x_1| \alpha_1 - x_3 \alpha_3 \right] \]  
\[ B_z = \frac{\mu J}{2\pi A} \left[ x_3 \ln(r_3/r_4) - x_1 \ln(r_2/r_1) + |z_1| \alpha_4 - z_3 \alpha_2 \right] \]

These equations apply to the entire positive x-z quadrant. The variables \( r_1, r_2, r_3, r_4 \) are the distances from the corners of the rectangular cross-section to some field point in the positive quadrant; \( r_1 \) is the distance from the nearest corner, and the rest are from the other corners clockwise about the z axis. The angles \( \alpha \) are respectively the angles between successive r's: e.g. \( \alpha_1 \) is the angle between \( r_1 \) & \( r_2 \). The values \( x_{1,3} \) and \( z_{1,3} \) are respectively the vector components of \( r_{1,3} \).

The above equations are valid for uniform current densities. Can they be used to represent the field results of the control and sense currents in the SUFFRS? It is clear upon examining tables 4-I and 5-II that the magnetic penetration depth of the vanadium exceeds its thickness, and that the penetration depth of the niobium is also comparable to its thickness. Under such conditions, the current density is fairly uniform \([30]\).\(^{19}\) Thus, the rest of this chapter will proceed under the assumption that all currents are uniformly distributed.

5.4 Low Operating Frequencies: Switching Application

As was outlined in Chapter 4, it is expected that the SUFFRS can operate as a simple current switch for low-frequency control and sense currents. Switching occurs when the sense current exceeds pinning forces in the trap, while a coincident control current dynamically nucleates vortices into the trap. From this short description, one obvious boundary value appears: the pinning threshold value for the sense current is effectively the operational minimum.

\(^{19}\)See Appendix D
5.4.1 Minimum Amplitude for a Sense Current

In Chapter 2, equation 2.32 gives an expression for the maximum pinning force on a vortex per unit length. If we define \( n = B/\Phi_0 \), where \( B \) is the magnetization of the trap, the pinning force density can be expressed as [14]

\[
F_p = \frac{BH^2\zeta}{8\Phi_0} \left( 1 + 4 \ln(\kappa)/\kappa \right) \tag{5.3}
\]

The threshold current density necessary to free the vortices is found by setting the vortex velocity to zero in the *flux-flow equation* 2.25, giving the following relation\(^\text{20}\)

\[
J_p = \frac{cF_p}{B} \tag{5.4}
\]

The threshold current, then, can be expressed as

\[
I_p^V = \frac{A_r c H^2\zeta}{8\Phi_0} \left( 1 + 4 \ln(\kappa)/\kappa \right) \tag{5.5}
\]

where \( A_r \) is the cross-sectional area of the trap. This equation is given in *esu* units; to convert to *mks*, one must divide by a conversion factor of \( 3 \times 10^9 \). Note that this final value is independent of the magnetization \( \overline{B} \); that is why an expression for the magnetization was not pursued.

Equation 5.5 loses validity as the frequency of operating currents increases, since pinning forces will tend to decrease [14]. Additionally, this threshold represents the theoretical maximum: experimentally, this could turn out to be orders of magnitude smaller, since the theory that led to equation 5.3 was constructed on the assumption that all magnitudes of interactions will be maximized (further information on this can be found via the references at the end of this thesis [[1],[15],[18],[27],[28]]). Regardless, 5.5 does give an upper boundary to the threshold current, useful as a worst-case analysis.

\(^{20}\text{Assuming that } B \text{ and } J \text{ are directed perpendicular to each other.}\)
5.4.2 Operational Magnitudes for the Control Current

The lowest magnitude for the control current that is capable of causing the SUFFRS to switch is determined by the need to exceed \( H_{c1} \) somewhere inside the trap. Clearly, this will occur first at the edge of the trap nearest the control line. Additionally, the self-field of the sense current must be taken into account.

An upper boundary to the control current comes from the requirement that \( H_{c2} \) not be reached anywhere inside the trap. Again, this will occur first at the nearest edge of the trap, and, again, the field of the sense current comes into play. If \( H_{c2} \) is reached at this edge, then it is likely to start an avalanche effect\(^{21}\) leading to a total quenching of the trap.

These boundary conditions can be mathematically expressed as follows:\(^{22}\)

\[
I_{c \, op-lo} = \Gamma \pi A_c \left\{ \left( H_{c1}^V / \mu \right) - B_{zz}(x_\lambda) \right\} \tag{5.6}
\]

\[
I_{c \, op-hi} = \Gamma \pi A_c \left\{ \left( H_{c2}^V / \mu \right) - B_{zz}(x_\lambda) \right\} \tag{5.7}
\]

where \( x_\lambda \) is the point in the trap \( \lambda \) from the edge nearest the control, \( A_c \) is the cross-sectional area of the trap, and where \( \Gamma \) is a geometrical factor defined by equations 5.8 & 5.2 and Figure 5-1.

\[
\Gamma^{-1} = \left\{ x_{3c} \ln(r_{3c}/r_{4c}) - x_{1c} \ln(r_{2c}/r_{1c}) + |z_{1c}| \alpha_{4c} - z_{3c} \alpha_{2c} \right\} \tag{5.8}
\]

\(^{21}\)If any significant portion of the trap, that is not surrounded by a continuous superconductive path, becomes normal, the sense current will avoid this normal (and non-vortex) region. This leads to a compression of the current or an increase in the current density in the remaining superconductor, which intensifies the magnetic field at the phase boundary, forcing more of the material to enter the normal state: the standard vicious circle that leads to the complete quenching of a superconductor.

\(^{22}\)The gradient of the self-field at the edges is evened out by the existence of the magnetic penetration depth. Indeed, vortices are unstable if they are closer than \( \lambda \) to the edge. [7] Thus, the test-conditions are defined for a point in the trap \( \lambda \) away from the edge: this greatly improves the approximation for the self-field and is physically more accurate.
Figure 5-1: Geometry involved in determining the field result along the x-axis of the trap
5.4.3 Other Upper Bounds on Operational Current Magnitudes

It is important to examine other possible upper boundaries on the operational currents. The first and most obvious of these is the critical current of both the control and the trap: the current, passing through either, which is capable of causing a self-quench. These can be expressed (in the case of uniform current distributions [29]) as

\[ I^N_{\text{crit}} = \frac{cA H^2_{c(Nb)}}{3\sqrt{6}\pi \lambda_{Nb}} \]  \hspace{1cm} (5.9)

\[ I^V_{\text{crit}} = \frac{cA H^2_{c(Nb)}}{3\sqrt{6}\pi \lambda_{Nb}} \]  \hspace{1cm} (5.10)

Here, as before, conversion from esu to mks will be necessary. These are maximum values and are based on the assumption that the only field is the self-field. Since the operation being considered here is that of coincident currents, the bounds set by 5.9 and 5.10 will be considerably depressed; unfortunately, this depression effect has no simple analytic description\(^{23}\), for the case at hand, and must be determined experimentally. Regardless, these two equations are useful to give a sense of the quenching characteristics of the materials.

Another upper bound on the control current results from the possibility that, during operation, vortices might nucleate in the control itself. Near the edges of the control, the field is very intense, making nucleation likely. Since the control should operate without entering a resistive state, we must examine the threshold current of the control; an ideal control would have infinitely high pinning forces. In the case of niobium, we can predict an operational maximum, following the the same approach used for equation 5.5.

---

\(^{23}\) Equations 5.9 and 5.10 were derived using a thermodynamic energy-balance approach, starting from the assumption that the conductor under consideration is "isolated", experiencing no fields from external sources. Though superposition of fields holds in principle, modification of these particular equations to accommodate the presence of external fields would require a full rederivation starting with the original assumptions. The benefits and applicability of such a rederivation, compared to simple rough estimates, do not currently warrant the effort.
\[ J_p^{Nb} = \frac{A_c c H^2 \xi}{8 \phi_0} \left(1 + 4 \ln(\kappa)/\kappa \right) \]  

(5.11)

where all of the state-variables are those of the Nb control.

Note that this is a maximum, and that the upper bound on the operational limit should indeed be much smaller. Further discussion on the topic of pinning forces with respect to the SUFFRS is presented at the end of this chapter.

5.4.4 Resistance

Equation 2.26 on page 19 gave a means of estimating flux-flow resistivity. However, that particular expression is primarily for use at temperatures much less than \( T_c \) and with bulk Type-II superconductors; as such, it is likely to be inaccurate as an estimate for the case at hand.\(^{24}\) The principle behind equation 2.26, however, gives a plausible approach to deriving an estimate that can be used for the SUFFRS; i.e. the resistivity is directly proportional to the amount of material that is in the normal state, or

\[ \rho_f = \%_{n/s} \rho_n \]  

(5.12)

where \( \%_{n/s} \) is the (decimal) percentage amount of normal-state per unit volume in the trap. This percentage can be expressed as

\[ \%_{n/s} = \frac{B_{ave} y_i x_i}{\phi_0} \frac{\pi \xi^2}{y_i x_i} \]  

(5.13)

The first ratio represents the total number of vortices that are nucleated in the trap. The second ratio is the cross-sectional area (looking down at the trap, see Figure 5-2) of a single vortex over the total cross-sectional area of the active region.\(^{25}\) Since it is expected

---

\(^{24}\)It is encouraging to recall that 2.26 is an empirically determined lower bound for the resistivity. In most cases, the resistivity is expected to be higher. [14]

\(^{25}\)Henceforth, the term "active region" will be used to refer to the portion of the trap in which vortices are capable of nucleating, bounded by the planes \( x = \pm (x_i/2) \), \( y = 0 \), \( y = y_i \), \( z = \pm (z_i/2) \), where \( z_i \) is the trap thickness, \( y_i \) is the length of trap material in which vortices nucleate, and \( x_i \) is the width of the trap.
that the vortex structure will be uniform from the top of the active region to the bottom, this expression not only describes the percentage of cross-sectional area (looking down) that is in the normal state, but also represents the percentage of normal-state volume in the active region.

\[
B_{ave} = \frac{1}{m} \sum_{n=0, x=x_n}^{m, x=x_N} \left| \vec{B}(x,z=0) \right|, \quad m = \frac{x_i - 2x_N}{x_i}
\]  

(5.14)

The summation is only along the x-direction, since the field does not vary significantly in the y-direction. Note, too, that the contributions to \( \vec{B} \) come primarily from the z-field component. The increments are chosen to be \( x_i = \xi y \), since this is the shortest distance of change for the superconductive state in the trap.

A fairly subtle point comes up here. Earlier, it was noted that the trap is so thin that

\[\text{A summation is presented here, because the corresponding integral is difficult and even harder to include into the Fortran program that is presented in Appendix B.}\]
fields pass through with hardly any attenuation in intensity. This indicates that, right before the first vortex nucleates, there is an amount of flux $\Phi^-$ passing through the trap already. When vortices nucleate from greater field intensities, they nucleate mostly out of the added flux: the part of the trap that is still superconducting will continue to pass flux on its own. This reasoning suggests that the first ratio in 5.13 needs correction. Equation 5.15 is a corrected version:

$$
\%_{nls} = \frac{B_{ave}y_t^r x_t^r - \Phi^-}{\Phi_0} \pi z_t^2
$$

(5.15)

where $\Phi^- = \mu H_{c1} y_t x_t$. This equation is approximate, since the actual value of $\Phi^-$ should decrease with decreasing superconducting volume. However, it is best to use this approximation for now, in keeping with the worst-case aspect of this analysis.

Combining equations 5.12 and 5.15 gives an expression for the expected resistance that the SUFFRS can present to a sense current:

$$
R = \%_{nls} \rho_n \frac{y_t}{x_t z_t}
$$

(5.16)

It is important to note that this is, indeed, a pessimistic estimate of the resistance, and that it is reasonable to expect experimental values that are significantly higher, partly because of the estimate chosen for 5.15 and partly because several experimental studies have measured resistances much higher than the predictions obtained by theory (this is covered in detail by Huebener[15] in his presentation of our equation 2.26).

5.4.5 Inductance

The inductances involved with the SUFFRS have two components: the inductance of the lines leading to and from the cell, and the inductance of the cell itself. Since the ground-plane is absent in the second case, the inductance (per unit length) there should be greater. Working again from the assumption of a uniform current density, the total inductance of the rectangular cell for this prototype can be expressed as [10]

$$
L \approx 4\left[ a \ln \frac{4ab}{p(a+d)} + b \ln \frac{4ab}{p(b+d)} + 2d + \frac{1}{2}(a+b) + 0.223p \right](1 \times 10^{-7})
$$

(5.17)
where \( L \) is in Henrys, \( p \) is the perimeter of the rectangle that is formed by a \( x-z \) cross-section of the trap (or \( p = 2x_r + 2z_r \)), \( a \) \& \( b \) are the \( x-y \) dimensions of the entire cell (i.e. including the alternate current path), and \( d \) is the diagonal of the cell \( d = (a^2 + b^2)^{0.5} \).

For those circuit elements (in this case, transmission lines) that are over the ground-plane, the following expression should be used [30]:

\[
\frac{L}{y} = \frac{\mu z_i}{x_i} \left\{ 1 + \left( \frac{\lambda_i}{z_i} \right) + \left( \frac{\lambda_{nh}}{z_i} \right) \right\}
\]

\[5.18\]

which gives the inductance per unit length.

For the purposes of estimating the time dependencies of this prototype, we shall use equation 5.17 for the inductance of the cell.

5.4.6 Switching Time

The total switching time of the SUGFRS has two components; the steering time and the nucleation time. The steering time refers to the time it takes for an applied sense current to reach an equilibrium distribution. In the case at hand, this is defined as the time it takes for a sense current passing through the trap to drop off to \( 1/e \) of its original value, or [6]

\[
\tau_s = \frac{L}{R}
\]

\[5.19\]

The nucleation time refers to the time it takes to turn "on" the resistive effect; in this particular case, the time it takes for the vortices to nucleate in the trap. This can be approximated by using equation 2.36:

\[
\tau_n = \frac{16\pi(1-D)x_i^2}{9\rho_s c^2(h-(1-D))}
\]

\[5.20\]

Even though this equation was originally derived for a disk of radius \( R_0 \), it should still give a fair estimate as to the order-of-magnitude nucleation time, with the width of the film substituted for \( R_0 \).
We recall, from pages 19 and 23 of Chapter 2, that the vortex velocity has a maximum value of \( v_\phi = v_s / 2 \), where \( v_s \) is the superelectron velocity associated with the transport current. Even though there is no transport current here, a possible upper limit to the nucleation time comes from the fact that vortices can never move (continuously) faster than the half of the critical depairing velocity\(^{27} \), \( v_{s_{crit}} \). For the case at hand, there are two approaches. One is to assume that the vortices must enter from the edge of the trap and move in a continuous manner into the trap until an equilibrium distribution is reached. Using this assumption, a lower limit for the nucleation time can be expressed as

\[
\tau_n \geq \frac{2 e n_s}{J_{s_{crit}}^V} \tag{5.21}
\]

where \( J_{s_{crit}}^V \) is the current density associated with equation 5.10.

However, in order to use this equation, we must have an expression for \( n_s \). Since \( n_s \) is the number density of superconducting electrons, a very rough estimate can be made as

\[
n_s(T) = n_e (1 - (T/T_c)^2) \tag{5.22}
\]

where \( n_e \) is the number density of conduction electrons (see equation 2.12).

The second approach to determining this nucleation time just mentioned is to postulate that the vortices may tunnel or "jump"\(^{28} \) to traverse the distance, or that the vortices are not constrained to nucleate only at the edge of the trap. From this second point of view, 5.20 has greater validity than 5.21. The author, for this particular case, has chosen to utilize 5.20 for estimating the nucleation times. This choice is based upon an assumption that has already been utilized: since the trap is much thinner than its magnetic penetration depth, the assumption has been made (and several equations have already been chosen with

\(^{27}\)The critical depairing velocity refers to the maximum drift velocity that can be associated with the superelectron fluid, before the Cooper pairs start to break up. Further information on this topic resides in London's *Superfluids* [16]

\(^{28}\)There is some justification to this: see Huebener's book [14] for discussions on thermally activated flux-jumps and flux-creep
this basis) that the imposed magnetic field at all times permeates the trap with insignificant attenuation of intensity in the z direction. This assumption, applied to the choice between equations 5.20 and 5.21, suggests as likely the possibility that the vortices will nucleate within the body of the trap instead of just at the edge, and that the local total field-intensity will determine the such nucleation. The lifting of the constraint requiring the vortices to enter from the edge of the trap and requiring that they move continuously to reach equilibrium argues strongly for the use of equation 5.20, since it was derived from thermodynamic arguments.

The total switching time characterizing the SUFFRS is simply the sum of the two component times just treated:

\[ T = \tau_s + \tau_n \]  \{5.23\}

5.4.7 Power Dissipation

The low-frequency limit, of course, is that of d.c. currents. In determining the power dissipation of the SUFFRS, for low frequencies, we only consider the resistance and ignore the inductance,\(^\text{29}\) yielding

\[ P = \frac{i^2}{R} \]  \{5.24\}

which is the amount of power dissipated in joules/second.

Thus the total amount of energy dissipated in one switching cycle is

\[ W = P \, T \]  \{5.25\}

\(^{29}\)Of course, this is a simplification to the extreme limit of a very slow rise in the application of the sense current. It is useful, though, to provide an estimate of the least amount of power that will be dissipated. Later in this chapter, a frequency dependent version is presented that should yield more accurate results.
5.4.8 Values and Results

Calculation of the following results was achieved using the Fortran program that is presented in Appendix B, which in turn, utilizes the equations presented in this chapter.

Table 5-III is a collection of values calculated from the equations presented in this section. It is assumed that the materials are vanadium and niobium, and the state-variables given in Table 5-II have been utilized. Examination of the table indicates that the SUFFRS can indeed work as a current switch, provided that the depression of the critical currents (c.f. equations 5-9 & 5-10) is not too pronounced. For those values that are dependent on the magnitude of the sense current, the operational value for the sense-current has been set to 0.95\(I_{\text{crit}}^{\text{V}}\). \(^{30}\) All of the values in the table are given positive: it must be remembered that the control current is actually flowing in the "+y" direction, and that the sense-current is flowing in the opposite direction.

<table>
<thead>
<tr>
<th>(I_{\text{crit}}^{\text{V}})</th>
<th>15.7 mA</th>
<th>(I_{\text{crit}}^{\text{Nb}})</th>
<th>1.63 A</th>
<th>(I_{p}^{\text{V}})</th>
<th>12.4 mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_{p}^{\text{Nb}})</td>
<td>1.45 A</td>
<td>(B_{ave})</td>
<td>1081 G</td>
<td>(\sum B_{z}(x))</td>
<td>21,619 G</td>
</tr>
<tr>
<td>m</td>
<td>20</td>
<td>(\Phi_{-}\Phi_{-})</td>
<td>2.55 mG-cm(^2)</td>
<td>(%_{n/s})</td>
<td>.107</td>
</tr>
<tr>
<td>R</td>
<td>3.4 Ω</td>
<td>L</td>
<td>1.62x10(^{-10}) H</td>
<td>(\tau_{s})</td>
<td>4.76x10(^{-11}) sec</td>
</tr>
<tr>
<td>(\tau_{n})</td>
<td>3.12x10(^{-25}) sec</td>
<td>P</td>
<td>.143 J/sec</td>
<td>W</td>
<td>6.82x10(^{-12}) J</td>
</tr>
<tr>
<td>(I_{c \text{op-lo}})</td>
<td>41.8 mA</td>
<td>(I_{c \text{op-hi}})</td>
<td>1.45 A</td>
<td>(F_{p}^{\text{V}})</td>
<td>4.076x10(^8) dynes/cm(^3)</td>
</tr>
</tbody>
</table>

Table 5-III: Numerical Results for the Low-Frequency SUFFRS

\((I_{s \text{op}} = 0.95I_{\text{crit}}^{\text{V}})\).

\(^{30}\)It will be necessary for the sense current to operate above the true threshold current, since the maximum amount of current switched is \(I_{s \text{op}} = I_{p \text{true}}^{\text{V}}\): the operational sense current minus the true threshold. The actual pinning forces should turn out to be much smaller than \(F_{p}\) in the table.
5.5 High Operating Frequencies: Memory Application

As was described qualitatively in Chapter 4, there should exist an operational frequency, above which the SUFFRS will act as an NDRO memory; if the sense current alternates swiftly enough, vortices that are already in the film don't exit. This removes the requirement of coincident control and sense currents: the vortices can be written into the trap at a previous time, since they do not need refreshing during sensing operation.\(^{31}\)

This section gives theoretical expressions for this cut-off frequency, as well as how the previously discussed operational characteristics should change in the high-frequency case.

5.5.1 Cut-Off Frequency For Memory Operation

The cut-off frequency will be determined by the time it takes for an operationally significant amount of the vortices to exit the film, or

\[ T_{flight} = \frac{x_f}{v_\phi} \quad \{5.26\} \]

where \( T_{flight} \) is the "time of flight" of the vortices, or the time it takes for them to reach the edge of the film; \( v_\phi \) is the vortex velocity, and \( x_f \) is the distance the vortices must travel to exit the film.

In order for equation 5.26 to be of any use, the statement "operationally significant amount of vortices" must be defined. If we consider the trap to be uniformly permeated with vortices, the time it takes for vortices nearest the edge to leave the trap is practically zero. In order to determine the cut-off frequency, a decision must be made as to the amount of the active region containing vortices that will be sacrificed in order to obtain NDRO operation. If we are willing to sacrifice half of the vortices, and thus half of the resistance,

\(^{31}\)It is assumed here that, in the absence of an applied sense current, the pinning forces are sufficient to keep the vortices in the trap.
then the distance \( x_f \) in 5.26 becomes \( x_f/2 \). An applied sense current that alternates at the appropriate frequency will drive out of the trap all of the vortices in half of the trap. The rest of the vortices will oscillate back and forth with the alternating current, never quite exiting the trap.

For the rest of this high-frequency treatment, the assumption that \( x_f = x_f/2 \) will be used. Thus, the cut-off frequency can be expressed as\(^{32}\)

\[
\omega = \frac{2v_\phi}{x_t}
\]  
\[\text{(5.27)}\]

### 5.5.2 Operating Range for Control Current

For NDRO memory operation, the vortices that are stored in the trap are written by a single control current pulse of sufficient length and intensity. The determination of the magnitude of the control current is done in a similar manner to that of the low-frequency case. The main difference is the absence of the self-field of the sense current. Taking equations 5.6 and 5.7, and setting the sense current’s self-field to zero, we find the following boundary values for the magnitude of the control current for the high-frequency SUFFRS:

\[
I_{c_{\text{op-lo}}} = \Gamma \pi A_c (H_{c1}^Y/\mu)
\]  
\[\text{(5.28)}\]

\[
I_{c_{\text{op-hi}}} = \Gamma \pi A_c (H_{c2}^Y/\mu)
\]  
\[\text{(5.29)}\]

### 5.5.3 Threshold Currents

The threshold currents that were derived for the low-frequency case provided a lower bound for the magnitude of the sense current and an upper bound for the control current. Both of these boundaries were functions of the pinning forces. In Chapter 2, it was

\(^{32}\text{Here, it is clear that the assumption that the vortices are moving in a continuous manner applies; in no way can it be argued that energy considerations may favor jumping or tunneling of vortices. This means that the vortex velocity will be limited, for the case of motion in response to an applied transport current, by the critical vortex velocity discussed in Chapter 2.}\)
mentioned that the effective pinning force in a material decreases with increasing frequency of applied current. This implies that the threshold currents in the SUFFRS high-frequency case may be significantly depressed. However, there is no theoretical formulation available that describes the variation of pinning forces with current frequency. Equation 5.4 is still technically correct\(^3\) for the high-frequency case, but the actual values will have to be determined experimentally. Indeed, 5.4 is useful for systematic study of the variation of the pinning forces with current frequency.

5.5.4 Critical Currents

Two other upper magnitude boundaries for the sense and control currents were derived for the low-frequency case: the critical currents that lead to self-quenching. Since self-quenching is effectively caused by the current's self-field, operation of the SUFFRS at high current frequencies should not significantly alter the values described by equations 5.9 and 5.10. However, since the sense and control currents are not operated simultaneously, 5.9 and 5.10 are far more valid in the high-frequency case than in the low-frequency regime.

5.5.5 Time of Write-Operation

An operational parameter of any memory device is the time it takes to write a "1" or a "0". For the SUFFRS, the time it takes to write a "1" is the time it takes to nucleate one batch of vortices into the trap: the nucleation time discussed previously in determining the low-frequency switching time. Thus, equations 5.20 & 5.21 can be used to estimate the time it takes to write a "1".

For the SUFFRS, there are two possible ways of writing a "0". One way is to pulse the control current in the reverse of its normal direction, so as to write oppositely oriented

\(^3\)Since it does not actually specify \(F_p\), simply the relation between \(I\) and \(F_p\)
vortices into the trap: writing a "-1" on top of a "+1". This method depends on the vortices in the trap annihilating with an equal number of oppositely oriented vortices. However, there are two possible flaws with in method. First, it has already been assumed that vortices in the trap are not free to flow, unless there is a sense current present. Hence, it is likely that there will be a significant number of vortices that will not annihilate, unless a current is passed through the trap during the zeroing cycle. Second, it is not reasonable to expect that the control current will create exactly the same number of vortices with every writing operation. Without such exactitude, the trap would tend to retain some vortices, leading to a sloppy "0". Exactly how sloppy is something that bears experimental determination: this method of writing a "0" has the advantage of being roughly as fast as the nucleation time described by equation 5.20, which would make it desirable for use if the sloppiness is not excessive.

A clean "0" can be written by pulsing the sense current at a frequency lower than the cut-off frequency. If the magnitude of this pulse is sufficient to depin the vortices, they will exit the film: all of them. The time it takes to write this clean "0" can be estimated as

\[ \tau_{w0} = \frac{x_f}{v_\phi} \]  

This is simply twice the inverse of the cut-off frequency, and the current responsible for writing the zero need only be active for this period of time. Note that this has a lower boundary value that is determined by the limit on \( v_\phi \) discussed in Chapter 2 and in context with equation 5.21. It is clear, in this case, that the vortices are moving in response to an applied transport current, which directly argues for the expectation of continuous vortex motion. Thus, the time for a clean zero will be much slower than the sloppy zero just discussed. Whether the clean method would be best for all zeroing, no zeroing, or "reset the zero" cycles in actual uses can only be speculated upon effectively after experimental information is gathered to confirm or deny the theoretical calculations for \( t_{0-0} \).
5.5.6 Time of Read-Operation

Application of a sense current operating just above the cut-off frequency will drive roughly half of the vortices out of the trap. The remaining vortices will cause a redirection of the current to an alternate path. The time it takes for the current passing through the trap to drop to 1/e of its original value is a good estimate for the effective read-time of the memory operation of the SUFFRS. We recognize this as the steering time discussed in the low-frequency case, and thus can estimate the time of one read-operation using 5.19.

5.5.7 Power Dissipation

The expression for power dissipation derived earlier did not take into account time dependencies. For the case of an alternating sense current, the average power dissipation\textsuperscript{34} can be expressed as \[ P = \frac{i^2 R}{2} \] (5.31)

5.5.8 Vortex Stability

In order for the SUFFRS to act as a memory, it is crucial that the vortices are stable enough to stay in the trap in the absence of any operating currents. This stability is determined by comparing the pinning forces with the Gibbs free energy exit-forces described in section . As a very rough criterion, the SUFFRS will have stable vortex storage properties for

\[ f_{out} < f_p \] (5.32)

where \( f_p \) is described by equation 2.32 and \( f_{out} \) by 2.33.

\textsuperscript{34}This equation does not take into account the part of the sense-current cycle during which the vortices are not flowing: the periods where the magnitude of the sense-current has dropped below the pinning threshold. Thus, the expression is too large by about a factor of two; acceptable for now, since we are interested mostly in order-of-magnitude. The same considerations applies to the use of 5.19 for the read-time which, again, is sufficient for order-of-magnitude.
5.5.9 Calculated Values

The following table presents values calculated from the equations presented for the high-frequency case. These values were calculated utilizing the Fortran program presented in Appendix B, and they have all been converted to mks units.

<table>
<thead>
<tr>
<th>$\omega_{\text{cut-off}}$</th>
<th>1.982x10⁶ sec⁻¹</th>
<th>$v_{\text{d}}$</th>
<th>495.5 cm/sec</th>
<th>$I_{\text{crit, Y}}$</th>
<th>15.7 mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\text{crit, Nb}}^N$</td>
<td>1.63 A</td>
<td>$I_p^Y$</td>
<td>≤12.4 mA</td>
<td>$I_p^{Nb}$</td>
<td>≤1.45 A</td>
</tr>
<tr>
<td>$R$</td>
<td>1.7 Ω</td>
<td>$I_{c,\text{op-lo}}$</td>
<td>55.6 mA</td>
<td>$I_{c,\text{op-hi}}$</td>
<td>1.45 A</td>
</tr>
<tr>
<td>$\tau_{w''1''}$</td>
<td>3.12x10⁻²⁵ sec</td>
<td>$\ast \tau_{w''0''}$</td>
<td>1.009x10⁻⁶</td>
<td>$\tau_r$</td>
<td>4.76x10⁻¹¹ sec</td>
</tr>
<tr>
<td>$P$</td>
<td>17.9 mJ/sec</td>
<td>$W$</td>
<td>8.53x10⁻¹³ J</td>
<td>$F_{\text{out}}$</td>
<td>1368.3 dynes/cm³</td>
</tr>
</tbody>
</table>

Table 5-IV: Numerical Results for the High-Frequency SUFFRS, 

$(I_{\text{op}} = 0.95I_{\text{crit, Y}})$.

5.6 Comments

The general framework of the derivations presented in this chapter indicates that, with proper choice of materials and deposition parameters, a working SUFFRS prototype is possible. A sense as to the order-of-magnitudes for the operational parameters has been gained, which will serve as a guide for any subsequent experimental work. Additionally, the treatment has been valuable in isolating those unknowns that need experimental analysis.

5.6.1 Pinning and the Viscosity Coefficient

There are two major unknowns that will ultimately determine the practical usefulness of the SUFFRS; pinning forces and the viscosity coefficient.

The pinning forces determine some very important boundaries for the magnitudes of the operational currents, as well as directly affecting the vortex velocity. The theory (see Section 2.5) on pinning is only capable of describing the maximum pinning values.
However, experimentally, pinning varies greatly with respect to material and deposition techniques [14]. For the most part, research on pinning forces has been focused on how to reproducibly maximize these forces. As Bachtold pointed out [3], equivalent research on the minimization of pinning forces should open up a new frontier for superconductive circuits.

The viscosity coefficient also plays an important role in determining the vortex velocity in the presence of a transport current. The theory involved is only capable of giving a gross lower-bound estimate to the viscosity. Little experimental information is available on this, though what there is tends to indicate that the viscosity is often much higher, leading to lower vortex velocities. As both the viscosity and the magnitude of the pinning forces directly affect the cut-off frequency for NDRO operation, gathering more experimental information on these two effects could be extremely beneficial.

5.6.2 Nucleation Time

The velocity of vortices is also a concern with respect to the time it takes for vortices to nucleate in the trap. This chapter has served to argue that determination of the nucleation time is equivalent to determining the write-time of the SUFFRS. Yet, as was first discussed in the end of Chapter 2 (page 23), there are two equally valid equations that may be used in for such estimates. The choice between these equations is determined by whether or not the vortices will nucleate via continuous motion, starting from the edge of the trap, moving in a continuous manner until equilibrium is reached. In this chapter, the energy balance approach has been argued as valid for the nucleation of vortices into the SUFFRS trap in response to the control-field, whereas the continuous motion approach has been argued as valid for those parameters (e.g. $\omega$) involving vortex motion in response to an applied transport current. The final justification of these arguments will come only with experimental data.
5.6.3 Experimental Followup

Further progress in ascertaining the viability of the SUFFRS can only be obtained through experimental work. The prototype design is well suited for these first experiments. The deliberately simple design lends itself to extremely straightforward testing. Using this prototype, several informative experiments can be done in quick succession, yielding information on

- Actual operational parameters for the SUFFRS, both as a switch, and (if possible) as a memory.\(^{35}\)
- The pinning forces in the trap and the control.
- The variation of the pinning forces with operating frequency.
- The viscosity coefficient.
- The nucleation time.

A choice will need to be made, at the early stages, as to the method to be used for detection of current-switching (c.f. page 48), which will directly affect the SUFFRS operation in that the effective inductance will change. In the early tests, it is sufficient to use whichever detection method is simplest, since the effect of the unfavorable (large) inductances will not inhibit extension of the experimental results to prediction of behavior for situations of favorable inductances. Furthermore, the applicability of the theoretical derivations in this chapter will still be put to the test, whether or not the inductance is "unfavorable".

Since it is necessary to try several different materials, differing cell lengths, different inductance set-ups, and differing depositional parameters, the very simplicity of the design will greatly facilitate these experiments. As experimental information is gathered, the derivations presented in this chapter can be adapted to give better guidance for further experimental testing and refining of an operating SUFFRS.

\(^{35}\)It may take several experimental runs to determine the appropriate materials and deposition conditions that will increase the viscosity sufficiently to make memory operation viable.
Chapter 6

Comparisons and Conclusions

6.1 Introduction

Now that we have order-of-magnitude estimates on the operational characteristics of the SUFFRS, we can proceed to compare the expected prototype to the devices reviewed in Chapter 3. Because the values from Chapter 5 are very rough estimates, the comparisons here will be mostly qualitative. Indeed, it is difficult to properly represent a device that has yet to be constructed and tested, against devices that have had many years of thorough research and development. Despite this difficulty, the following discussion does prove helpful, in that it suggests the expected competitive strengths and weaknesses of the SUFFRS. Such educated guessing is a necessary prerequisite in determining whether it is worthwhile to commit further resources to the creation and testing of prototypes.

6.2 Comparison as a Switch

As there are two distinct operational modes of the SUFFRS, there are two categories of comparison: (1) as a switch and (2) as a memory.

The devices that can be used for comparison with the SUFFRS as a switch are the cryotron and the J-J. The following list is a comparative summary of the information given in Chapters 3 and 5.

- **Switching Times:** With the current state-of-the-art lithography techniques, the J-J has a demonstrably excellent overall switching time \((\tau \leq 10^{-10}\text{sec})\), currently limited mostly by the capacitance of the junction-barrier. The cryotron is limited as a switch by the time it takes for it to reach a resistive state; the time it takes for an N/S phase boundary to propagate through the material (c.f. maximum vortex velocity on p. 23). The SUFFRS, according to the estimates in Chapter 5, has the potential to be as fast, if not faster than the J-J. Both the J-J and the SUFFRS have a steering time that is governed by the
ratio L/R: they are both equally affected by device inductances. The J-J gets its value for R from the quantum gap potential, and the SUFFRS from the number of mobile vortices. In general, the SUFFRS has to occupy more physical space than the J-J to get the same resistance values. However, the SUFFRS has a tunable resistance, since the length of the active region can be varied to alter the resistance. This is a feature that the J-J does not have. With respect to the nucleation time, the J-J switches from the resistive state with a quantum transition tempered by the capacitance of the junction, whereas the SUFFRS is expected to switch with the quantum nucleation of vortices. Whether the SUFFRS' nucleation time will be tempered by additional factors is unknown at this time; interestingly, however, capacitance (which limits the J-J’s response) does not seem to be a logical candidate, though the current-rise time in the control-line (determined by the control-line’s self-inductance) is a definite candidate).

- **Operational Current Magnitudes:** Here the J-J is superior, requiring tens of μA to reach a resistive state, whereas both the cryotron and the SUFFRS need in the mA to A range. The amount of current that can be usefully switched to an alternate path is the total amount of current that can be supported by the transmission lines (without exceeding the critical current) minus this minimum switching threshold. Thus, with respect to the amount of current needed to switch and the amount of current that can be switched, the J-J is superior.

- **Power Dissipated:** This is really a by-product of the considerations involved with switching times, since the power dissipated is a direct result of the resistance and the time that the current is applied. Thus, the same "superiority order" that applies to the switching times can reasonably be expected to apply to the amount of power dissipated by the current-switch. However, it should be mentioned that the effective surface area (with respect to power/heat dissipation to the helium bath) of a SUFFRS in greater than that provided by a J-J of equal resistance.

- **Threshold Currents:** The J-J’s threshold current is well-defined by the energy-gap of the material used, and it is often quite small (on the order of tens of μA). The SUFFRS threshold current has a much wider range of possibilities, as it is determined by pinning forces which can vary widely: this is either a distinct disadvantage or advantage, depending on the control and reproducibility of pinning forces. The cryotron has no intrinsic threshold.

- **Size and Difficulty of Creation:** With current linewidths, the J-J is superior in size to both the cryotron and the SUFFRS. However, the cryotron and the SUFFRS are far easier to create, since they don’t require the extremely thin insulation barrier that the J-J needs.

- **Expected Improvements with Improved Thin-Film Technology:** As

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36 this would ultimately be the same limit on the J-J, if it was not limited by capacitance effects

37 This category is listed, since the non-linear behavior embodied by a threshold current is a useful tool for circuit designers
lithographic techniques improve, yielding smaller linewidths, the cryotron is not expected to improve significantly; even though its effective resistance will increase and its operating currents will decrease, it is limited by the time it takes for a complete transition to the normal state to occur. The J-J will not benefit from improved lithography, but other equally possible advances in associated technologies should benefit the J-J in that it will be easier to make; otherwise, its operational characteristics will tend to stay the same. Finally, the SUFFRS will experience drastic competitive improvement. (A) Thinner linewidths will allow shorter vortex-trap regions for equivalent resistances which will affect the switching time by reducing the minimum possible inductance. Even though a thinner line-width will cause an increase in $\omega_{\text{cut-off}}$, this can be offset by the fact that fewer vortices are needed to stay in the trap, since higher resistances are available. (B) The operating currents of the SUFFRS will improve directly with better lithography. (C) With smaller linewidths, yielding higher net resistance, the length of the SUFFRS could well be reduced significantly. Since the J-J’s primary advantage in size is its near-negligible length, we see that the SUFFRS would begin to compete in this category too. Other than these three categories, the rest of the SUFFRS characteristics are expected to stay roughly the same.

From this qualitative list, we see that the current-switch of preference, currently, is the J-J. However, the derivations of Chapter 5 distinctly suggest that the SUFFRS could indeed provide competition. One final aspect of the SUFFRS that makes it superior on all counts to the other two switches, is the fact that it can actually store information, where the J-J and the cryotron (without special added real-estate arrangements) can’t.

6.3 Comparison as a Memory

The following list compares the SUFFRS, operating as a memory, to the persistent current-loop memory schemes covered in Chapter 3.

- **Write-Time**: In write operations, the current-loop schemes are influenced by the time it takes for the internal switch to create a resistance, and the inductance of the loop. When the SUFFRS is used as a memory, the write-time is essentially the vortex nucleation time. If the vortex nucleation time does indeed end up on the order of $10^{-25}$ seconds (as resulted from equation 5.20), it is clear that the SUFFRS write-time would be far superior, since inductance does not come into play for the SUFFRS "write".

- **Read-Time**: Reading a current-loop memory consists of addressing the sense-gate with a current: if a "1" is present, the sense-gate will act to switch current to another path. With the SUFFRS, the SUFFRS is in itself the storage and the
sense-gate combined. Thus, the sensing time in both cases is simply
determined by the magnitude of the resistance that the sense-gate provides. The
J-J has a set resistance, whereas the SUFRRS can be "tuned" to have a higher
resistance, which would yield a better read-time.

- Operational Currents: In order for a current-loop memory to be able to
influence a sense-gate, a significant amount of current must be in the loop; the
exact amount varies drastically according to the relative positions of the loop
and the sense-gate, but is practically constrained to the range of mA to
A. Overall, the operational currents for a SUFRRS are then expected to be
roughly the same (mA to A). This category does not compare well in
qualitative generalizations, but is best left to experimental comparisons.

- Power Dissipation: In the current-loop scheme, there are two different
dissipative activities. The first is the redirection of current in a write-cycle,
which dissipates energy. The second is the redirection of current by the sense-
gate, which also dissipates energy. Since the SUFRRS serves as both the
memory and the sense-gate, it really has only one dissipative effect: the
redirection of the sense-current. There is, of course, the energy inductively lost
by the control-current, when writing the vortices into the trap. However, such a
loss is negligible in comparison to these other quantities.

- Stability: It has been demonstrated that current-loop schemes are close to
perfectly stable [30]. The SUFRRS is not quite so stable, and its stability
depends on the competition of internal pinning forces with a variety of other
forces. Thermal stability in the SUFRRS is likely, since it has such a high
surface to volume ratio, but this has to be demonstrated. However, it is indeed
well within the realm of possibility to create a SUFRRS with the appropriate
pinning forces to make it stable, while still having a decent range of operating
currents.

- Device Dimensions: A persistent current-loop memory will always have to be
of an area that is at least $9w^2 (3w \times 3w)$ (usually more), where $w$ is the available
linewidth resolution. The corresponding size for a SUFRRS memory is $wl$,
where $l$ is the length of the SUFRRS. The length of the SUFRRS is
determined by the total resistance that is needed to switch the sense current: as
linewidths improve, the length necessary will decrease, since resistances will
increase. Hence, we see that the SUFRRS has a distinct size advantage, now
and in the future.

- Difficulty of Fabrication: Clearly, there is no device geometry that is simpler
than the SUFRRS. Whether or not special techniques are needed to make to
properly control pinning-forces is a question that must be answered after some
experimentation.

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38 This does not count the real-estate for the alternate path of the switched sense-current

39 Another way of looking at it is that the current-loop scheme needs two alternate current-paths (one for the
sense-current and one to establish the current-loop) whereas the SUFRRS gets rid of the loop and makes the
sense-gate act as the memory: one alternate current-path.
One notes that the vortex memories from Chapter 3 have yet to be mentioned. This is due to the fact that none of these schemes are yet competitive with the persistent current-loop scheme: the above comparison is primarily to determine the overall competitiveness of the SUFFRS. However, there are some comments that should be made.

- **Device Dimensions**: The Abrikosov memory and the vortex-file, both, are much larger than the SUFFRS. The MFQ/SFQ memory scheme would actually be somewhat smaller per information bit than the SUFFRS, if the SFQ design could be realized with current technology. Even though the SUFFRS has yet to be realized, based upon the limits of current technology, the SUFFRS is at least competitive, if not superior.

- **Fabrication Difficulty**: Clearly, of all circuits that utilize vortices, the SUFFRS has the simplest construction requirements. The vortex-file memory and the Abrikosov memory require several lines and thickness gradients, while the MFQ/SFQ memory requires several difficult lithographic tricks (outlined previously on page 37).

- **Energy Dissipation, Operational Currents**: both of these categories are not useful for comparison, since none of the devices have been studied sufficiently. The expectation that they are similar within a few orders-of-magnitudes does not assist comparison.

- **Stability**: The SFQ/MFQ memory is the vortex memory that has the greatest possible problems with stability. However, all of these vortex devices (including the SUFFRS) depend upon pinning forces in some way for device operation and stability. Until more is known about the magnitudes and reproducibility of pinning forces in thin films, the only conclusion here is that all of them have an equally fair chance of being stable devices.

- **Switching Times**: With current technologies, the vortex-file and the MFQ memory systems are considerably slower than the SUFFRS (no time information was reported on the Abrikosov system). Huebener and Parish predicted, though, that a SFQ system would switch at speed of 80 picoseconds using 0.25 μm linewidths. A SUFFRS switch is currently estimated to operate at speeds between 10^{-11} and 10^{-25} seconds, without such an improvement in lithography: definitely competition.

From the above discussion, it is clear that there are several possible ways in which the SUFFRS could provide competition to other existing devices. More in-depth comparisons can only be useful after some experimental evidence has been gathered on a SUFFRS prototype. However, there are enough possible advantages to a working SUFFRS to encourage the investment of experimental research time and money.
6.4 Unknowns and Suggested Experimentation

In the first leg of experiments on a SUFFRS prototype, there are certain topics that need to be addressed immediately:

1. The magnitudes, reproducibility, and frequency dependencies of the pinning forces for both the trap and the control.

2. The values and behavior of the viscosity coefficient.

3. The actual upper and lower limits to the operational currents.

4. The cut-off frequency, and its dependence on pinning, the viscosity, and the resultant vortex-velocity.

5. The vortex nucleation time.

6. What the operational characteristics of the SUFFRS are (a) as a switch and (b) as a memory. If such operation does not occur, determination as to why the operation fails.

Systematic research with multiple prototypes of varying dimensions and materials can quickly yield a large body of information; the simplicity of the prototype design will greatly facilitate such research. The results of such an effort will not only determine the SUFFRS actual experimental properties, but also can benefit the development of other vortex-related devices (such as the ones in Chapter 3).

6.5 Conclusions

When all is said and done, this thesis is simply a great deal of creative educated guessing. This author has often been frustrated by the fact that his training has only been up to the Bachelor’s level: certain mathematical and physical formulations are beyond his training. Thus, it is likely that the theoretical developments in this thesis lack completeness or exactitude. However, it must be remembered that the overall goal of this thesis is not just to present a specific prototype design, but more importantly to present valid arguments and examples suggesting the use of vortices and flux-flow resistance as the basis for thin-film circuit operation. It is quite possible that experiments will show this prototype, or even the
entire design concept, to be unworkable. Yet the essential idea holds: the vortex is currently the smallest 'bit' of information available to superconductive circuit technology, and the vortex is capable of signalling its presence without intermediate circuit elements. This author fully expects it to become, in one way or another, an integral part of superconducting computer technology; perhaps his grandchildren will view the entire topic as "obvious".
Appendix A

The discussion on design for the devices presented in this thesis are based on the resources that were made available to me at MIT’s Lincoln Laboratory. In this appendix I wish to give a description of the processes involved with the lithography methods that are utilized there.

A.1 Deposition Constraints

The materials that can be deposited by the vacuum-systems in Group 86 are Ti, V, Nb, Pb, Al, and SiO. Of these materials, Nb was the obvious choice for control-line materials: due to its high $T_c$ and strength as a material. The choice of material for a trap was partly dictated by the necessity of utilizing an operating temperature of 4.2 K; this ruled out Al and Ti (Ti is primarily used as a resistor in superconductive circuit designs anyway). Pb was ruled out due to the fact that the trap needed to be as thin as possible, and such a thin film of Pb would oxidize away between deposition steps; hence V was chosen as the trap material.

The thinnest linewidth that can be reliably patterned in Group 86 is 5.0 microns.

A.2 Design Procedure

In order to ultimately create a working thin-film device, there are five steps that must occur.

The first is the utilization of the photomask design program called Chipgraph, created by Mentor Graphics Inc. This program allows several editing options that the user applies to create a color display that graphically resembles the view of each photomask. The configurations of the shapes and lines on each layer are edited separately, and each layer design will later dictate the construction of the corresponding lithography mask.
Once this design is finished, it is processed through a program called Dracula, created by ECAD Inc. This program fractures the shapes represented in the Chipgraph design into coordinate sets that represent many small rectangles. These rectangles, when pieced together are a puzzle-piece version of the original Chipgraph design. This breakdown of the graphics information is then recorded on computer tape.

This information is then formatted for the technician who operates the mask-machine, utilizing a "generic" pattern generator program; essentially this reconstructs the design from the taped information in a format that is useful for the technician. It also allows the designer to double-check that the Dracula translation occurred properly.

The next step is simply to feed the computer tape to the actual machine that creates the masks: the tape goes in, and after some technological magic (i.e. the author did not pursue the details of the mask-construction) , the masks (chrome-plated glass) come out.

After the lithography masks are made, depositions of each material in turn occurs on silicon wafers. Before another material is deposited, the wafer is coated with negative photoresist, then exposed to ultraviolet light that has been shaped in the appropriate patterns by the lithography mask that matches this particular deposition layer. Negative photoresist hardens in the areas that receive the U-V light, but the other portions of photoresist are subsequently washed off. The wafer is then placed in a plasma-etcher that will etch away the unprotected portions of the deposition layer, leaving the circuit design that applies to that layer. A quick chemical bath removes the hardened photoresist, after which the wafer is ready for the next film deposition.
A.3 Some Additional Considerations

When making a thin-film device, it is crucial to remember that each subsequent layer of material must be thicker than the previous layer. This becomes obvious when one thinks of a cross-sectional view of the device; the problem is that previous layers create steps and bumps in the effective surface that the next layer encounters during deposition. In order to have a continuous layer, step-coverage must be accounted for, and this is usually achieved by making each subsequent layer at least 500 angstroms thicker than the previous film. Such a requirement often acts as a design constraint, determining which materials get deposited first.

A.4 Samples

The following pages contain some hardcopy samples of the work that I have done at Lincoln Laboratory. The first is an example of the screen-view of a Chipgraph editing session. The second and third are samples of the pattern-generator's output.
Figure A-1: Chipgraph Edit Session

This is the graphical display that the user sees. Most editing operations are done by utilizing the mouse and numerous pop-up menus.
Figure A-2: Pattern Generator Output (for one layer), one page

This is a sample of the graphical output that is delivered to the technician in charge of creating the photomasks, to be delivered with the magnetic tape containing the Dracula breakdown.
Figure A-3: Pattern Generator Output (for one layer), another page
Appendix B
A Fortran Program for Determining SUFFRS Operational Parameters

This appendix contains a copy of a Fortran program that was written for the purpose of calculating the operational parameters of a SUFFRS based on the derivations done in Chapter V. It is a very general program, in that it can be applied to any choice of materials for the trap and the control-line, as well as any choice of thickness for the layers involved. It is, however, specifically adapted to the geometry of the SUFFRS design presented at the end of Chapter IV.

B.1 Inputs

The inputs are given in the following table. Most of the parameters are straightforward material constants.

In this table, the superscripted numbers in parentheses indicate the order in which these inputs are fed to the program. Several variables are self-explanatory as material parameters, but the rest need some commentary.\textsuperscript{40}

The variable $F_p$ is an optional input. If, as in the table, $F_p$ is set to 0, then the program calculates the theoretical (maximum) value for $F_p$. However, if the input value is non-zero, the program will work with the input value instead: to be used primarily if experimental information on $F_p$ is available.

The variables $z_{1.5}$ are, respectively, the thicknesses of the deposition layers as described in Chapter IV: i.e. $z_3$ is the thickness of the vanadium trap.

\textsuperscript{40}The parameter $x_{nc}$, which is the spacing between the control and the trap, is not part of the input list. Instead, it is embedded in the program code.
<table>
<thead>
<tr>
<th></th>
<th>vanadium</th>
<th>niobium</th>
<th>others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$</td>
<td>4.9$^{1}$</td>
<td>$T_c$</td>
<td>$z_1$</td>
</tr>
<tr>
<td>$H_c(0)$</td>
<td>1310$^{2}$</td>
<td>$H_c(0)$</td>
<td>2.0e-05$^{19}$</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>4.6e-06$^{3}$</td>
<td>$\xi_0$</td>
<td>1.0e-05$^{21}$</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>3.74e-06$^{4}$</td>
<td>$\lambda_L$</td>
<td>1.0e-04$^{22}$</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>1.59e-05$^{5}$</td>
<td>$\rho_n$</td>
<td>5.0e-03$^{23}$</td>
</tr>
<tr>
<td>$W$</td>
<td>50.9415$^{6}$</td>
<td>$W$</td>
<td>$y_t$</td>
</tr>
<tr>
<td>$Z$</td>
<td>2$^{7}$</td>
<td>$Z$</td>
<td>$x_c$</td>
</tr>
<tr>
<td>$F_p$</td>
<td>0$^{8}$</td>
<td>----</td>
<td>$x_t$</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>5.98$^{9}$</td>
<td>$\rho_m$</td>
<td>5.0e-04$^{25}$</td>
</tr>
<tr>
<td>xstep</td>
<td>2.5e-05$^{27}$</td>
<td>lchoice</td>
<td>1$^{28}$</td>
</tr>
<tr>
<td></td>
<td>lchoice</td>
<td>1$^{28}$</td>
<td>verbose</td>
</tr>
</tbody>
</table>

Table B-I: Inputs to Fortran Program

The input variable \textit{xstep} determines the increments used for the summation of $B$ over the trap width.

The input variable \textit{lchoice} determines which of the two possible inductance equations is used: one is for the case with a ground-plane, the other without; \textit{lchoice} should be set to "1", to toggle for the case without the ground plane.

The last input variable, \textit{verbose} is a toggle switch that governs the verbosity of the program output. If \textit{verbose} is set to "1", all possible variables of interest (state-variables, current distribution multipliers, device parameter estimates) will be output. If \textit{verbose} is set to "0", the only outputs are the device parameter estimates.
B.2 Source Code

program SUFFRS

declare variable types.

double precision, Tc, Hc0t, xi0t, lam0t, rho0t, atwt, Val, Fp, denst
double precision, Tcc, Hc0c, xi0c, lam0c, rho0c, atwc, Valc, densc
double precision, z1, z2, z3, z4, z5, xc, xt, yt
double precision, T
double precision, c, mu, a0, pi, rtwo, phi0, muM, charge
double precision, pM, sM, L1tM, L2tM, L, dist, temp1, temp2
double precision, nt, rst, lmt, nc, rsc, lmc, Velv, Velmax, Velw1
double precision, x1j, x2j, multiplierj, multiplierjc, Velnst, nst
double precision, IcminM, percent, fint, fext, Fout, xtest
double precision, Icrit, Icritc, IcrithM, Icrithc, IoptM
double precision, factor, calc, Velw0, timew0, phiExc, Energy
double precision, number, R, IcrthtM, tau, omega, Power

double precision, z1t, z3t, z1c, z1t, x1t, x3t, x3c, x1c
double precision, a12t, a23t, a34t, a41t, r1t, r2t, r3t, r4t
double precision, r1c, r2c, r3c, r4c, a12c, a23c, a34c, a41c
double precision, AtM, AcM, Bzc, Bxc, Bzt, Bxt, Btot, Blo, Bhi
double precision, xiTt, lamTt, kapt, HcTt, Hc1t, Hc2t, Hent, Hpt
double precision, xiTc, lamTc, kapc, HcTc, Hc1c, Hc2c, Henc, Hpc
double precision, C1t, C1c, Hc20t, Hcpart, Hc20c
double precision, c2t, c2c, At, eta, sum, IopcM
double precision, Demag, x, IptM, IpcM, xstep
double precision, timew1, Ac, hold
integer, lchoice, count, freqcase, verbose

read*, Tc, Hc0t, xi0t, lam0t, rho0t, atwt, Val, Fp, denst
read*, Tcc, Hc0c, xi0c, lam0c, rho0c, atwc, Valc, densc
read*, z1, z2, z3, z4, z5, yt, xc, xt, T, xstep, lchoice
read*, verbose

C******************************************************************************
C Set internal program constants
C******************************************************************************

freqcase = 0
c = 3.0e+10
mu = .9998
a0 = 5.29e-09
pi = 3.1416
muM = mu*(pi*4.0e-07)
rtwo = 2**(0.5)
phi0 = 2.07e-07
charge = 4.80294e-10

C******************************************************************************
C Print out all input values, regardless of verbose toggle
C******************************************************************************

print*, '*****All inputs are CGS units*****
print*, 'Tct = ', Tct, '; Hc0t = ', Hc0t, '; xiot = ', xiot
print*, 'lam0t = ', lam0t, '; rhot = ', rhot, '; atwt = ', atwt
print*, 'Valt = ', Valt, '; Fpt = ', Fpt, '; dens = ', dens
print*,
print*, 'Tcc = ', Tcc, '; Hc0c = ', Hc0c, '; xio0c = ', xio0c
print*, 'lam0c = ', lam0c, '; rhoc = ', rhoc, '; atwc = ', atwc
print*, 'Valc = ', Valc, '; densc = ', densc
print*,
print*, 'z1 = ', z1, '; z2 = ', z2
print*, 'z3 = ', z3
print*, 'z4 = ', z4, '; z5 = ', z5
print*, 'xc = ', xc, '; xt = ', xt
print*, 'yt = ', yt, '; T = ', T
print*,
print*, c, mu, muM, a0, pi, rtwo, phi0
print*,
print*, '$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>

C******************************************************************************
C Calculate the mean free electron paths utilizing the
C Drude model, as discussed in chapter 2
C******************************************************************************

nt = (6.022e+23)*Valt*dens/atwt
rst = (3.0/(4.0*pi*nt))**(1.0/3.0)
lmt = ((rst/a0)**2)*9.2e-13/rhot

nc = (6.022e+23)*Valc*densc/atwc
rsc = (3.0/(4.0*pi*nc))**(1.0/3.0)
lmc = (rsc/a0)**2*9.2e-13/rhoc

C*****************************************************************************
C Calculate the state-variables
C**

Clt = sqrt(1.0-(T/Tct))
Clc = sqrt(1.0-(T/Tcc))

xit = 0.855*(sqrt(xi0t*lmt))/Clt
lamTt = lam0t*(sqrt(xi0t/(1.33*lmt)))/(rtwo*Clt)
kapt = lamTt/xiT
Hct = Hc0t*(1-(T/Tct)**2)
Hc1t = (Hct/(rtwo*kapt))*log(kapt)
Hc2t = rtwo*kapt*HcTt
Hct = HcTt/rtwo
Hpt = 0.715*rtwo*lam0t*HcTt/lmt
Hc20t = Hc0t*rtwo*lam0t/xi0t
Hcpart = 2.0*(sqrt(6.0))*HcTt*lamT/t3

xitc = 0.855*(sqrt(xi0c*lmc))/Clc
lamTc = lam0c*(sqrt(xi0c/(1.33*lmc)))/(rtwo*Clc)
kapc = lamTc/xiTc
Hctc = Hc0c*(1-(T/Tcc)**2)
Hc1c = (Hctc/(rtwo*kapc))*log(kapc)
Hc2c = rtwo*kapc*HcTc
Henc = HcTc/rtwo
Hpc = 0.715*rtwo*lam0c*HcTc/lmc
Hc20c = Hc0c*rtwo*lam0c/xi0c
Hcparc = 2.0*(sqrt(6.0))*HcTc*lamTc/z5

C***
C Estimate the density of superconducting electrons at the
C operational temperature
C***
nst = nt*(1-(T/Tct)**2)

C*****************************************************************************
C Calculate the inductance of a SUFRS cell having the dimensions
C as specified by the inputs
C**
pM = (2.0*xt+2.0*z3)*0.01
sM = yt*0.01
dM = sqrt(2.0*sM**2)
temp1=2.0*sM*log((4.0*sM**2)/(pM*(sM+dM)))
temp2=2.0*dM+sM+(0.223*pM)
L1tM = 4.0e-07*(temp1 + temp2)
dist = z2
temp1 = (muM*dist/xt)
temp2 = (1+(lamTt/dist)+(lamTc/dist))
L2tM = temp1*temp2

C**
The option L2tM is not completely supported, use L1tM
C**

if (lchoice .ne. 2) then
L = L1tM
else L = L2tM
endif

C*****************************************************************************
C Calculate areas
C*****************************************************************************

At = xt*z3
Ac = xc*z5
AtM = At*1.0e-04
AcM = Ac*1.0e-04

C*****************************************************************************
C Calculate some constants and the viscosity coefficient
C*****************************************************************************

c2t = (1.0+(4.0*log(kapt))/kapt)
c2c = (1.0+(4.0*log(kapc))/kapc)
etac = (phi0*Hc20t)/(rhot*c*c)

C*****************************************************************************
C Estimate the time it takes to write a vortex
C*****************************************************************************

Demag = 1/(1+z3/xt)
timewl = pi*(1-Demag)*(xt**2)/(rhot*c**2)
Velw1 = xt/timewl

C*****************************************************************************
C Calculate the current distribution multipliers
C*****************************************************************************

x1j = -1*((xt/2)-lamTt**2/(2*z3))
x2j = -1*x1j
temp1 = (xt/2)*(asin(2*x2j/xt)-asin(2*x1j/xt))
temp2 = 2.33*sqrt(xt/z3)*lamTt*(0.39346934)
multiplierj = (temp1+temp2)/xt
x1j= -1*((xc/2)-lamTc**2/(2*z5))
x2j= -1*x1j
temp1=(xc/2)*(asin(2*x2j/xc)-asin(2*x1j/xc))
temp2= 2.33*sqrt(xc/z5)*lamTc*(0.39346934)
multiplierjc = (temp1+temp2)/xc

C*****************************************************************************
C Calculate the critical current densities (i.e. max)
C*****************************************************************************

Jcritz=(c/3.0e+09)*HcTt/(3*sqrt(6.0)*pi*lamTt)
Jcricz=(c/3.0e+09)*HcTc/(3*sqrt(6.0)*pi*lamTc)
IcrizM= At*Jcriz
IcriztM= Ac*Jcrizt

C*****************************************************************************
C Calculate the pinning threshold currents
C*****************************************************************************

JplotM = (c*(HcTt**2)*xiTt*c2t/(8*phi0*3.0e+09))
IptM = multiplierjt*At*JplotM
IpcM = multiplierjc*Ac*c*(HcTc**2)*xiTc*c2c/(8*phi0*3.0e+09)
JoptM = (0.95*Jcrizt)

C*****************************************************************************
C Print out any one-time values of interest
C*****************************************************************************

if (verbose .eq. 1) then
  print*, '**********************************************outputs**********
  print*, 'nt, rst, lmt'
  print*, 'nt,rst,lmt'
  print*, 'nc, rsc, lmc'
  print*, 'nc, rsc, lmc'
  print*, 'pM, sM, L1tM, L2tM'
  print*, 'pM, sM, L1tM, L2tM'
  print*, 'xiTt, lamTt, kapt'
  print*, 'xiTt,lamTt,kapt
  print*, 'HcTt, Hc1t, Hc2t'
  print*, 'HcTt,Hc1t,Hc2t'
  print*, 'Hpt, Hc20t, Hcpart'
  print*, 'Hpt,Hc20t,Hcpart'
  print*, 'xiTc, lamTc, kaptc'
  print*, 'xiTc, lamTc, kapc'
  print*, 'HcTc, Hc1c, Hc2c'
  Print*, 'HcTc, Hc1c, Hc2c'
  print*, 'Hpc, Hc20c, Hcparc'
  print*, 'Hpc, Hc20c, Hcparc
C Calculate Bave iteratively
C********

10  x = xt/2 +xstep -lamTt
    sumn = 0
    count = 0

20  x = x - xstep

C**
C Note that all dimensions are subject to the constraint of
C only examining the field along the x-axis.
C**

toggl et = 1
z3t = z3/2
z1t = -z3/2
x3t = x + (xt/2)
x1t = x - (xt/2)
r1t = sqrt(z1t**2 + x1t**2)
r3t = sqrt(z3t**2 + x3t**2)

C**
C If r1 is greater than r3, the B expressions are not valid. Hence,
C it is necessary to use the symmetry of the situation, and "reflect"
C the parameters around the z-axis so as to determine the field
C results properly.
C**

    if (r1t .gt. r3t) then
        hold = r1t
        r1t = r3t
        r3t = hold
togglet = -1
hold =ogglet*x1t
x1t = togglet*x3t
x3t = hold

dendif
r2t= r1t
r4t= r3t

a12t= acos((((z3**2)-(r1t**2)-(r2t**2))/(-2*r1t*r2t)))
a34t= acos((((z3**2)-(r3t**2)-(r4t**2))/(-2*r3t*r4t)))
a23t= acos((((x2t**2)-(r2t**2)-(r3t**2))/(-2*r2t*r3t)))
a41t= acos((((x2t**2)-(r4t**2)-(r1t**2))/(-2*r4t*r1t)))

z3c= z5/2
z1c= -z5/2
x3c = x-(xt/2 + 1.0e-05 + xc)
x1c = x-(xt/2 + 1.0e-05)

r1c= sqrt(z1c**2+x1c**2)
r3c= sqrt(z3c**2+x3c**2)
r2c= sqrt(z3c**2+x1c**2)
r4c= sqrt(z1c**2+x3c**2)

a12c= acos((((z5**2)-(r1c**2)-(r2c**2))/(-2*r1c*r2c)))
a34c= acos((((z5**2)-(r3c**2)-(r4c**2))/(-2*r3c*r4c)))
a23c= acos((((x2c**2)-(r2c**2)-(r3c**2))/(-2*r2c*r3c)))
a41c= acos((((x2c**2)-(r4c**2)-(r1c**2))/(-2*r4c*r1c)))

calc = (z3t*log(r4t/r1t)-z1t*log(r3t/r2t))
factor=(.01)*((calc -(x3t*a34t -abs(x1t)*a12t ))
Bzt = togglet*muM*IptM*factor*(1.0d+04)/(2*pi*AtM)

calc = (z3c*1og(r4c/r1c)-z1c*1og(r3c/r2c))
factor=(.01)*((calc -(x3c*a34c -abs(x1c)*a12c ))

C***
C   Calculate the operational control-current (Hc2 at edge)
C**

if (count .eq. 0) then
  if (freqcase .ne.1) then
    IopcM=(-2.0d-04*(Hc2t-abs(Bzt))*pi*AcM/(muM*factor))
  else
    IopcM=(-2.0d-04*(Hc2t)*pi*AcM/(muM*factor))
  endif
if (abs(IopcM) .gt. IpcM) then
  print*, 'IopcM = ', IopcM
IopcM = -1.0*IpcM
endif
endif
Bzc = -1.0*muM*IopcM*factor*(1.0d+04)/(2*pi*AcM)
hold = factor

calc=(x1t*log(r2t/r1t)-x3t*log(r3t/r4t))
factor=(.01)*(calc - ( z3t*a23t - abs(z1t)*a41t))
Bxt = muM*(1.0)*IptM*factor*(1.0d+04)/(2*pi*AtM)

calc=(x1c*log(r2c/r1c)-x3c*log(r3c/r4c))
factor=(.01)*(calc - ( z3c*a23c - abs(z1c)*a41c))
Bxc = -(1)*muM*IopcM*factor*(1.0d+04)/(2*pi*AcM)

C**** These lines of code were used to double-check the
C more complex B expressions. They are the limiting case.
C It is left here for possible future reference & comparison
C
C
C Bxc = muM*IopcM*log(r3c/r1c)*(1.0d+06)/(2*pi*z)
C Bxc = muM*IopcM*a23c*(1.0d+06)/(2*pi*xc)
C Bzc = muM*IopcM*a34c*(1.0d+06)/(2*pi*z5)
C Bzc = -1.0*muM*IopcM*log(r3c/r2c)*(1.0d+06)/(2*pi*xc)
C print*, 'lim', Bzc, Bxc
C print*, 'xxx', x

C******************************************************************************
if (freqcase .ne. 1) then
   Btot=sqrt(((Bxc+Bxt)**2+(Bzc+Bzt)**2))
else
   Btot=sqrt(((Bxc)**2+(Bzc)**2))
endif
sum = sum + Btot
count = count + 1

C******************************************************************************
C Calculate the minimum control-current
C**

if (count .eq. 1) then
   if (freqcase .ne. 1) then
      IcminM=(-2.0d-04*pi*AcM*(Hc1t-abs(Bzt)))/(hold*muM)
   else
      IcminM=(-2.0*pi*AcM*(Hc1t))/(hold*muM*1.0d+04)
   endif
   Bhi=Btot
endif
C*** test for end of vortex nucleation region
C*** or the end of the trap material
C**
    if (x .gt. ((-1.0*xt/2) + lamTt)) then
      if (Btot .gt. (mu*Hc1t)) then
        go to 20
      endif
    endif

C*******************************************************************************
C*******************************************************************************

Blo=Btot
xend = x
Bave = (sum/count)
C*******************************************************************************
C Calculate the max pinning force in the trap
C*******************************************************************************

    c3t = ((Bave*(HcTt**2)*xiTt)/(8.0*phi0))
    Fpt = c3t*c2t
    if (Fp .ne. 0) then
      Fpt = Fp
    endif

C*******************************************************************************
C Calculate the operational velocity of the vortices (assuming no
C limit), the velocity of vortices entering the film (no limit),
C and the time it takes to write a zero.
C*******************************************************************************

    Velv = (((3.0e+09)*JoptM*phi0/c)-(phi0*Fpt/Bave))/eta
    Velw0 = (((3.0e+09)*(0.95*Jcritt)*phi0/c)-(phi0*Fpt/Bave))/eta
    timew0 = xt/Velw0

C*******************************************************************************
C Recalculate a more exact expression for timew1... based
C on equation 5.20
C*******************************************************************************

    Demag = 1 - (1/(1+z3/xt))
    temp1 = ((Bave/HcTt)-Demag)*9.0*rhot*c**2
    timew1 = 16.0*pi*Demag*(xt**2)/temp1

C*******************************************************************************
C Calculate the Resistance
C*******************************************************************************
\[ \text{phiExc} = (B\text{ave-mu} \times Hc1t) \times y(\times t/2) - \text{xend} \]
number = \text{phiExc}/\phi0
percent = \text{(number} \times \pi \times (\pi t^{**2})/(\pi t \times xt))

if (percent .gt. 1) then
  percent = 1
endif

R = \text{percent} \times \rho(\times t/At)

C** The characteristic current decay time:

tau = L/R

if (freqcase .ne. 1) then
  Power = IptM*(R**2)
else
  R = R/2.0
  tau = L/R
  Power = IptM*(R**2)/2.0
endif

C******
C Calculate the estimated operational supercurrent velocity.
C This determines a maximum to the operational velocity of the
C vortices, Velmax.
C**

Velnst = (3.0e+09*IptM)/(charge*nst)
Velmax = Velnst/2.0
if (Velw0 .gt. (Velmax)) then
  if (verbose .eq. 1) then
    print*, 'Velocity constraint.. Velw0 was:', Velw0
  endif
  Velw0 = Velmax
  timew0 = xt/Velw0
endif

C*** In the following case, the maximum is determined by the critical
C depairing velocity of the superelectrons
C**

Velnst = (3.0e+09*(Jcritt))/(charge*nst)
Velmax = Velnst/2.0
if (Velw1 .gt. Velmax) then
  if (verbose .eq. 1) then
    print*, 'Velocity constraint.. Velw1 was:', Velw1
  endif
  Velw1 = Velmax
endif

Energy = (timewl+tau)*Power

if (verbose.eq. 1) then
  print*, '+++++xend=', xend
  print*, '%%%%%%%SUM%%%%%%%SUM%%%%%%%SUM%%%%%%%SUM%%%%%%%PERCENT', percent
  print*, 'Bave, sum, count'
  print*, 'Bave, sum, count
  print*, 'phiExc, number, >>> R <<<'
  print*, 'phiExc, number, R'
  print*, 'Fpt, L,' } } tau{ { }
  print*, FptL, tau
  print*, 'Velw0, )timew0(, Power'
  print*, Velw0, timew0, Power
  print*, 'IcminM, IcrthtM'
  print*, IcminM, IcrthtM
  print*, 'Blo, bhi, PowerPerSwitch'
  print*, BloBhiEnergy
  print*, 'lopcM'
  print*, IopcM
  print*, '%%%%%%%%%%%%%%%%%%%%%%%%'  
endif

if (freqcase .ne. 1) then
  print*, '
  print*, '-----------------------------------'
  print*, '-----------------------------------'
  print*, '***** Low-Frequency Case*****'
  print*, 'The critical sense-current is (Amps):',IcritM
  print*, 'The critical control-current is (Amps):',IcritcM
  print*, 'The trap threshold-current is (Amps):', IptM
  print*, 'The control threshold-current is (Amps):', IpcM
  print*, 'The average magnetization Bave is (G):', Bave
  print*, 'The iterated sum of Bz(x) is:', sum
  print*, 'The number of iterations m is:', count
  print*, 'The excess flux is:', phiExc
  print*, 'The total number of vortices nucleated is:', number
  print*, 'The percent of normal volume in the trap is: ', percent
  print*, 'The resistance R is (ohms): ', R
  print*, 'The cell inductance is (Henries):', L
  print*, 'The current steering time is:', tau
  print*, 'The vortex nucleation time is:', timew1
  print*, 'The power dissipated is (j/sec):', Power
  print*, 'The energy expended in 1 switching is (j): ', Energy
  print*, 'Min practical operating control current(A):', IcminM
  print*, 'Max practical operating control current(A):', IopcM
  print*, '-----------------------------------'
  print*, '-----------------------------------'
print*, 
def

C*******************************************************************************
C Modify values for High-frequency...
C The toggle freqcase determines whether low-frequency or high-
C frequency case equations are used.
C*********

if (freqcase .ne. 1) then
    freqcase = 1
    go to 10
endif

C*********
C Determine cuttof frequency
C****
Velnst = (3.0e+09*JoptM)/(charge*nst)
Velmx = Velnst/2.0
if (Velm .gt. (Velmx)) then
    if (verbose .eq. 1) then
        print*, 'Velocity constraint. Velm was:', Velm
    endif
    Velm = Velmx
endif
omega = 1/(0.5*(xt/Velm))
xt = (xt/2)-2*lamTt
fint = phi0*Hc1t*xtest*z3/(sqrt(1-(4*(xt)*xt)**2)*pi*xt**2)
fret = (phi0**2)/(8*(pi*((xt/2)-xtest))**2)
Fout = (Bave/phi0)*(fret+fint)

print*, 
print*, 
print*, >>>>>>>>>>> High-Frequency Case <<<<<<<<<<<<<<<<
print*, 'The cut-off frequency is (1/sec):', omega
print*, 'The vortex velocity is (cm/sec):', Velw0
print*, 'The critical sense-current is (Amps):', IcrttiM
print*, 'The critical control-current is (Amps):', IcritcM
print*, 'The trap threshold-current is (Amps):', IptM
print*, 'The control threshold-current is (Amps):', IpcM
print*, 'The resistance R is (ohms):', R
print*, 'The cell inductance is (Henries):', L
print*, 'The read-time is: ', tau
print*, 'The write-time for a +1 is: ', timew1
print*, 'The power dissipated is (j/sec):', Power
print*, 'The energy expended in 1 read is (j): ', Energy
print*, 'The write-time for a 0 is:', timew0
print*, 'Min practical operating control current(A):', IcminM
print*, 'Max practical operating control current*(A):', IopcM
print*, 'The force in absence of a current, Fout(dynes/cm3):', Fout

end

B.3 Sample Outputs

The following is an example of *verbose* output, given the inputs outlined earlier.

*****All inputs are CGS units*****
Tct = 4.900000000000000 ; Hc0t = 1310.00000000000 ; xi0t = 4.600000000000000d-06
lam0t = 3.740000000000000d-06 ; rhoht = 1.590000000000000d-05 ; atwt = 50.9415000000000
Valt = 2.000000000000000 ; Fpt = 0. ; denst = 5.980000000000000

Tcc = 9.000000000000000 ; Hc0c = 1944.000000000000 ; xi0c = 3.900000000000000d-06
lam0c = 3.150000000000000d-06 ; rhoc = 1.000000000000000d-05 ; atwc = 92.9064000000000
Valc = 3.000000000000000 ; densc = 8.570000000000000

z1 = 5.000000000000000d-04 ; z2 = 1.000000000000000d-05
z3 = 5.000000000000000d-06
z4 = 1.000000000000000d-05 ; z5 = 2.000000000000000d-05
xc = 1.000000000000000d-03 ; xt = 5.000000000000000d-04
yt = 5.000000000000000d-03 ; T = 4.200000000000000

30000000000.00 0.99980000000000 1.2563886720000d-06 5.2900000000000d-09
3.14160000000000 1.4142135381699 2.0700000000000d-07

$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>$>

nt, rst, lmt
1.4138397966295d+23 1.1907942376238d-08 2.9319246696430d-07
nc, rsc, lmc
1.6664688331482d+23 1.1272953782471d-08 4.1778412749807d-07
pM, sM, L1tM, L2tM
1.0100000000000d-05 5.0000000000000u0d-05 3.5083655427008d-11
1.0581774166873d-07
xiTt, lamTt, kapt
\begin{verbatim}
2.6270627982283d-06 2.4031579893748d-05 9.1476990614595
HcTt, Hc1t, Hc2t
3.4755102040816 59.466407385221 4496.1979907063
Hpt, Hc20t, Hcpart
4.882.8932228300 1506.2603932412 8183.4517014669
xiTc, lamTc, kapc
1.494428096734d-06 8.0802856566968d-06 5.4069417639599
HcTc, Hc1c, Hc2c
1.520.64000000000 335.62226837034 11627.680573841
Hpc, Hc20c, Hpcarc
1.1573.272927360 2220.5328262402 3009.74

Jcritc
6264529.5282364 81517698.928181
lcr
crc
6264529.5282364 81517698.928181
IpcM
0.16661323820591d-02 1.6303539785636
lptM
0.4242487677583d-02 1.4473720648089 3770935
eta,

\textbf{multiplierj}
=
3.1788999561205 1.5423530447850

\textbf{IopcM} = -4.1928597929604
\textbf{Velocity constraint.. Velw0 was:} 2.0714255128178d+18
\textbf{Velocity constraint.. Velw1 was:} 9.2011077158136d+20
+++++end= -2.49032e-04
\textbf{PERCENT} 0.10678548511637
\textbf{Bave, sum, count}
1.080.94 2.618.826216899 20
\textbf{phiExc, number, } >>> R <<<
2.5487707938859d-03 12312.902385922 3.3957784267006
\textbf{Fpt, L}, temp
4.08030e+08 3.5083655427008d-11 2.0640917962990d-11
\textbf{Velw0, }>>>timew0((, Power
495.50573960469 1.009070470975d-06 0.1432683269491
\textbf{IcminM, IcrthtM}
-4.179786398678d-02 0.
\textbf{Blo, Bhi, }PowerPerSwitch
869.94257698739 1561.7700483169 6.8231459623408d-12
\textbf{IopcM}
-1.4473720648089

------------------------------------------------------------------------

\end{verbatim}
***** Low-Frequency Case *****
The critical sense-current is (Amps): 1.5661323820591d-02
The critical control-current is (Amps): 1.6303539785636
The trap threshold-current is (Amps): 1.2424287677583d-02
The control threshold-current is (Amps): 1.4473720648089
The average magnetization Bave is (G): 1080.94
The iterated sum of Bz(x) is: 21618.826216889
The number of iterations m is: 20
The excess flux is: 2.5487707938859d-03
The total number of vortices nucleated is: 12312.902385922
The percent of normal volume in the trap is: 0.10678548511637
The resistance R is (ohms): 3.3957784267006
The cell inductance is (Henries): 1.6164579808633d-10
The current steering time is: 4.7579040144266d-11
The vortex nucleation time is: 3.1159100497981d-25
The power dissipated is (j/sec): 0.14326832669491
The energy expended in 1 switching is (j): 6.8231459623408d-12
Min practical operating control current(A): -4.1797863938678d-02
Max practical operating control current(A): -1.4473720648089

IopcM = -4.2066994484697
Velocity constraint.. Velw0 was: 2.0714253469228d+18
+++++xend= -2.49032e-04
%%%%%% PERCENT 0.10690024606474
Bave, sum, count
1082.04 21640.780259058 20
phiExc, number, >>> R <<<
2.5515099241448d-03 12326.134899250 1.6997139124294
Fpt, L, ))tau( ))
4.08030e+08 3.5083655427008d-11 2.0640917962990d-11
Velw0, )timew0(, Power
495.50573960469 1.009700470975d-06 1.7947053664275d-02
IcminM, IcrhtM
-5.5637519447978d-02 0.
Blo, Bhi, PowerPerSwitch
893.25488228567 1546.9779691453 8.5289324529260d-13
IopcM
-1.4473720648089

Velocity constraint.. Velv was: 2.0714253469228d+18

>>>>>>>>>> High-Frequency Case <<<<<<<<<<<<<
The cut-off frequency is (1/sec): 1982022.9584188
The vortex velocity is (cm/sec): 495.50573960469
The critical sense-current is (Amps): 1.5661323820591d-02
The critical control-current is (Amps): 1.6303539785636
The trap threshold-current is (Amps): 1.2424287677583d-02
The control threshold-current is (Amps): 1.4473720648089
The resistance R is (ohms): 1.6997139124294
The cell inductance is (Henries): 1.6164579808633d-10
The read-time is: 4.7579040144266d-11
The write-time for a +1 is: 3.1159100497981d-25
The power dissipated is (j/sec): 1.7947053664275d-02
The energy expended in 1 read is (j): 8.5239324529260d-13
The write-time for a 0 is: 1.0090700470975d-06
Min practical operating control current(A): -5.5637519447978d-02
Max practical operating control current(A): -1.4473720648089
The force in absence of a current, Fout(dynes/cm3): 1368.3108642226
Appendix C

Resistivity of Thin-Film Vanadium at Lincoln Laboratory

In order to determine whether vanadium would be a suitable material for the trap portion of the SUFFRS, sample depositions of roughly .3 micron thick films were deposited. Deposition took place in a vacuum of better that $1 \times 10^{-7}$ Torr. The films were then patterned with a photoresist and plasma-etching process, after which the samples were put through a $T_c$ test, which utilizes a standard four-point probe resistance determination, and the testing is monitored by a computer that outputs the resistance as a function of temperature. The results of this is illustrated in Figure C.1. Figure C.2 is the same data in the form of the material's resistivity: this is determined utilizing the thickness and width of the patterned current paths. The last figure shows the results of a thickness measurement on the vanadium samples.
Figure C-1: Resistance vs. Temperature, Experimental

Vanadium thin-film. Dimensions: 5.145 cm long, 100 µm wide, 2800 angstroms thick. Deposition date 7/14/87.
RESISTIVITY VS TEMPERATURE

RESISTIVITY (\mu\text{OHM-cm})

TEMPERATURE (K)

Figure C.2: Vanadium Resistivity vs. Temperature. Experimental deposition date 7/4/87.

Vanadium thin film. Dimensions: 5.145 cm long, 100 \mu m wide, 2600 angstroms thick.
Figure C-3: Thickness Measurement of Vanadium Film

This measurement was done using one of the vanadium samples, after lithographic patterning for the resistance measurements had been finished. The device utilized to get the above results, a *Sloan Dektak II*, measures the thickness of a film by passing an extremely sensitive needle-probe across the surface of the substrate: this probe is pushed upwards by the thickness of the film and the machine registers the relative information in the graphical form shown above.
Appendix D
Mathematical Derivations

D.1 Introduction

This appendix serves to present those mathematical derivations that are necessary to the thesis, but are sufficiently large or distracting that they would detract from the flow of ideas in Chapter V.

D.2 Field Result of Rectangular Current-Carrying Strip

This section presents the reasoning that led to equations 5.1 and 5.2. Figure D-1 is a photocopy of a page in the AIP handbook. The first two equations are limiting case expressions for the field result of a conducting strip, carrying current in the +z direction, with a width of 2a. The strip lies in the x-z plane and is centered on the z axis; the equations apply to the limit where the thickness of the strip is much less than its width.

The next two equations on this page are similar in form and purpose to equations 5.1 and 5.2. However, regardless of the method of interpreting the somewhat obtuse instructions, these two AIP equations fail. A simple verification comes from setting the thickness b to zero: they do not reduce to the limiting cases. Additionally, two very simple boundary conditions, \( B_y(y=0)=0 \) & \( B_y(x=0)=0 \), are not satisfied by these equations.

The first possibility that comes to mind, of course, is that there has been a typographical error. However, creative attempts at switching around the subscripts for these equations yield no positive results; for whatever reason, they are incorrect. Since Chapter 5 depends on having an expression for at least the z-field, and since the limiting case equations simply aren't appropriate for the dimensions of the SUFFRS, the following
FORMULAS

The magnetic induction components are

\[ B_y = \frac{\mu I}{4\pi a} \ln \frac{r_2}{r_1}, \quad B_z = -\frac{\mu I}{4\pi a} \alpha \]

A conductor of rectangular section of area \( A \) is bounded by the planes \( x = a, \ z = -a, \ y = b, \ \) and \( y = -b \) and carries a uniformly distributed current \( I \) in the \( z \) direction. The distances from a field point in the positive quadrant to the corners, starting with the nearest and proceeding clockwise about the \( z \) axis, are \( r_1, r_2, r_3, \) and \( r_4. \) The angles between successive \( r \)'s are \( \alpha_1, \ \alpha_2, \ \alpha_3, \ \) and \( \alpha_4, \) and the \( z \) and \( y \) components of \( r_1 \) and \( r_4 \) are \( x_1, y_1, \) and \( x_4, y_4. \) If all the above quantities are taken positive, the magnetic-induction components are

\[ B_x = -\frac{1}{2} \mu I (\pi A)^{-1} \left( y_2 x_1 + x_2 y_1 + x_1 \ln \frac{r_2}{r_1} - y_1 \ln \frac{r_2}{r_1} \right) \]
\[ B_y = \frac{1}{2} \mu I (\pi A)^{-1} \left( x_2 x_1 - x_1 x_2 + y_2 \ln \frac{r_2}{r_4} - y_1 \ln \frac{r_2}{r_4} \right) \]

The space inside and outside the conductor has the same permeability \( \mu. \)

The magnetic induction outside the conductors of a long bifilar line that consists of a cylinder whose axis is \( y = a \) which carries a uniformly distributed \( x \)-directed current \( I \) and another cylinder whose axis is \( y = -a \) that carries the same current in the opposite direction is

Figure D-1: Photocopy of AIP Handbook, page 5-27

reasoning was used to construct equations 5.1 & 5.2. No claims are made as to the mathematical rigor of the derivation; however, the physics of the limiting conditions, a little 'hand-waving', and some number crunching strongly indicate that the 5.1 & 5.2 are the correct expressions.

First, let us clearly list the boundary conditions and limiting cases, using the coordinate axes defined in Chapter 4 (c.f. Figure 5-1). The two basic boundary conditions are a result of the rectangular symmetry and the physical requirement of flux conservation:

\[ B_x(x=0)=0 \ \& \ B_x(z=0)=0 \]

Equations D.1 and D.2 are the limiting case equations, with the coordinates transposed, and their conditions for validity (in the positive \( x-z \) quadrant only) are specified. D.1 applies to the case where the thickness \( z_0 \) approaches zero; see Figures D-2 and D-3. D.2 represents the symmetric counterpart, where \( x_0 \) approaches zero. In either case, the strip effectively has a negligible thickness. Note that both conditions must be
satisfied for these equations to hold validity.\textsuperscript{41}

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Figure D-2: Cross-Section of a Rectangular Current-Carrying Strip, distant view

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\textsuperscript{41}It is assumed that these limiting case equations are correct, despite the fact that the general case equations given in the handbook are not. This assumption is borne out at least by the fact that the limiting case equations do predict decreasing magnetic fields as one moves away from the strip-line. They otherwise end up giving field results in close approximation to what is "classically" expected (q.v. figures D-4 & D-5).
Figure D-3: Cross-Section of a Rectangular Current-Carrying Strip, z-axis expanded

\[ B_x = -\frac{\mu I_y}{4\pi a} \alpha_{4,2}, \quad B_z = \frac{\mu I_y}{4\pi a} \ln\left(\frac{r_{4,3}}{r_{1,2}}\right) \]

\[ \frac{b}{a} \to 0, \quad r \gg b \]

[D.1]

\[ B_x = -\frac{\mu I_y}{4\pi b} \ln\left(\frac{r_{3,2}}{r_{1,4}}\right), \quad B_z = \frac{\mu I_y}{4\pi b} \alpha_{1,3} \]

\[ \frac{a}{b} \to 0, \quad r \gg a \]

[D.2]
Equations 5.1 and 5.2 were largely derived through trial and error, using the boundary conditions, and the expectation that the final versions would resemble the AIP versions. Equations 5.1 and 5.2 are reproduced here, so that the reader can readily see that they do, indeed, reduce properly with respect to the limits.

\[
B_x = \frac{-\mu_f}{2\pi A} \left\{ x_1 \ln(r_2/r_1) - x_2 \ln(r_3/r_4) + |z_1| \alpha_4 - z_3 \alpha_2 \right\} \tag{D.3}
\]

\[
B_z = \frac{\mu_f}{2\pi A} \left\{ z_3 \ln(r_4/r_1) - z_1 \ln(r_3/r_2) + |x_1| \alpha_1 - x_2 \alpha_3 \right\} \tag{D.4}
\]

where the input variables are as defined on page 61. Note that the variables \( x \) & \( z \) are the only geometric variables that can have a negative value.

Equations D.3 & D.4 were constructed so that they would reduce to the limiting cases, following generally the structure suggested by the AIP versions. In the limit where \( b \to 0 \) & \( b < r_1 \), certain simplifications occur: \( \alpha_1 = \alpha_3 = 0, \alpha_4 = \alpha_2, r_1 = r_2, \text{ and } r_3 = r_4 \). In order for this to reduce properly, the following condition must be true: \( 2b = z_3 - z_1 \), since \( A = 4ba \). Thus, \( b \) is canceled out mathematically in the limit. The same process applies to testing the other limiting case, canceling \( a \) out in the limit. Thus, these limiting cases yield the sign constraints on terms in the general equations.

The fact that D.3 & D.4 do reduce to the limiting cases is necessary, but not sufficient, to defining their structures. Further constraints come from applying the boundary conditions: The condition \( B_x(y = 0) = 0 \) applies to all points on the \( x \)-axis, which sets \( r_1 = r_2, r_4 = r_3, \alpha_2 = \alpha_4 \). Substitution of these equalities yields the requirement of the absolute value sign around \( z_1 \). The same reasoning with the other boundary condition leads to the other absolute-value statement.\(^{[42]} \) Note that there are no other absolute value-signs in the expressions, since there are no boundary conditions that require them: i.e. for \( b \to 0, B_x \) doesn’t have to be zero on the \( z \)-axis.

\(^{[42]} \) This may seem to contradict the sign constraints given by the limiting cases. However, it must be remembered that, in the case of finite thicknesses, the limiting cases are not valid where the boundary conditions are operative. The absolute value sign reflects the competing requirements imposed by these disparate conditions.
The constraints imposed by the limiting cases and the boundary conditions have almost completely specified the form for the general expressions. However, neither of these sets of constraints give any information on the weighting of the terms: e.g. for any of the extreme cases just treated, there is no difference if the last two terms in D.4 are \(|x_1|\alpha_3 - x_2\alpha_1\) or \(|x_1|\alpha_1 - x_3\alpha_3\).

However, only a few possible permutations are left as options; iterative testing of these differing possibilities yielded the versions presented here. The general case candidates were examined with \(x_0=30z_0\) in the portion of the positive x-z quadrant bounded by \(x=3x_0\) and \(z=3x_0\); the resultant values were compared with limiting case results for the same width. Of the possible candidate expressions, D.3 & D.4 yielded agreement with the limiting case equations (in their region of validity) to four decimal places. At best, the other candidates were off by at least a factor of 3. Figures D-4 & D-5 shows two views of a three-dimensional graph\(^{43}\) representing the variation of D.4 in the positive quadrant. Qualitatively, the shape of the field intensity distribution exhibits all of the properties expected.

1. The field intensity decreases more slowly in the +x direction than in the +z direction.

2. The intensity smoothly approaches zero at the z-axis

3. The most intense region is located near the conductor edge, \(x=x_0/2\), as expected.

It is on the basis of this process of elimination and iterative confirmation, that equations D.3 and D.4 are held forth as the correct expressions for the field result of a rectangular conductor.

\(^{43}\)Calculated for \(x_0=30z_0\).
Figure D-4: A 3-D Representation of the $B_z$ Intensity, one view

D.2.1 Support Source Code

The iterative testing mentioned was achieved by using the following short fortran program, "stripfields". Being much shorter than the program presented in Appendix B, this program lent itself well to the quick changes and code-adjustments necessary to try the various permutations. The code presented here is the program's last incarnation, structured to deliver the values that led to equations D.3 & D.4 and Figures D-4 & D-5.

```
c*******************************************************************************
program stripfields

double precision, z5, xc, AcM, r1c,r2c,r3c,r4c
double precision, factor,calc,z,x,r1l,r2l
double precision, a12c, a23c,a34c,a41c,big
real,Bzc,Bzl,Bxc,Bxl
xc = 1.0d-03/50
```
Figure D-5: A 3-D Representation of the $B_z$ Intensity, another view

\[
z5 = 1.0d-03
\]
\[
xstep = z5/9.17537777767
\]

C** toggle for outside inputs.
\[
\text{outsource} = 0
\]
if (outsource .eq. 1) then
\[
\text{read*}, xc, z5, xstep
\]
endif
if (xc .gt. z5) then
\[
\text{big} = xc
\]
else
\[
\text{big} = z5
\]
endif
\[
\text{AcM} = (z5*xc)
\]
\[
z = 0.0
\]
10 \ x = 0.0

20 \ z3c = z+(z5/2.0)
\[
z1c = z-(z5/2.0)
\]
x3c = x+(xc/2.0)
x1c = x-(xc/2.0)

r1c= sqrt(z1c**2+x1c**2)
r3c= sqrt(z3c**2+x3c**2)
r2c= sqrt(z3c**2+x1c**2)
r4c= sqrt(z1c**2+x3c**2)

a12c= acos((((z5**2)-(r1c**2)-(r2c**2))/(-2*r1c*r2c))
a34c= acos((((z5**2)-(r3c**2)-(r4c**2))/(-2*r3c*r4c))
a23c= acos((((xc**2)-(r2c**2)-(r3c**2))/(-2*r2c*r3c))
a41c= acos((((xc**2)-(r4c**2)-(r1c**2))/(-2*r4c*r1c))

calc = (z3c*log(r4c/r1c)-z1c*log(r3c/r2c))
factor=(calc +(-x3c*a34c +abs(x1c)*a12c ))
Bzc = factor/AcM

calc= (x1c*log(r2c/r1c)-x3c*log(r3c/r4c))
factor=(calc - ( z3c*a23c - abs(z1c)*a41c))
Bxc = -factor/AcM

if (z5 .gt. xc) then
  r1l=sqrt((z1c**2+(x**2))
  r2l=sqrt((z3c**2+(x**2))
  calc= acos((((z5**2)-(r1l**2)-(r2l**2))/(-2*r1l*r2l)))
  Bxl = log(r2l/r1l)/z5
  Bzl = calc/z5
else
  r1l=sqrt((z**2+(x1c**2))
  r2l=sqrt((z**2+(x3c**2))
  calc= acos((((xc**2)-(r1l**2)-(r2l**2))/(-2*r1l*r2l)))
  Bxl = calc/xc
  Bzl = log(r2l/r1l)/xc
endif

print*,Bzl,Bzc,Bxl,Bxc,x,z

x = x + xstep
if (x .lt. (3.0*big)) then
  go to 20
endif

z = z + xstep
if(z .lt. (3.0*big)) then
  go to 10
endif

end
D.3 The Current-Distribution For SUFFRS Operating Currents

This section presents a simple derivation of the current distribution as it applies to the prototype design for the SUFFRS. The original current distribution equations were taken from *Principles of Superconductive Devices* by VanDuzer [30]. This derivation simply applies it to the case at hand.

The cross-sectional current density through a superconducting strip-line, with thickness \( z_t \leq \lambda \) and width \( x_t \), is described by D.5 and D.6, where D.5 is valid for the center of the film, and D.6 is valid for the edges; D.5 and D.6 are joined at the points \( x = \pm x_\alpha \)

\[
J_s(x) = J_s(0) \left( 1 - \frac{2x}{x_t^2} \right)^{-1/2} \tag{D.5}
\]

\[
J_s(x) = J_s(x/2) e^{-x_i/2 + x/(\xi_2 \lambda^2)} \tag{D.6}
\]

Here \( z_t \) is the thickness of the strip, \( x_t \) is the width of the trap, and \( a \) is an empirically determined constant of order unity.

Now the expression for the current dependence on \( J \) is normally \( I = JA \), where \( A \) is the cross-sectional area of the strip. However, this assumes that the current density is completely uniform. More exactly, the current is the integral of the current density over the cross-sectional area. For the case under examination, the very thinness of the film allows us to assume that there is no variation of the current-density in the \( z \) direction. [30] Thus we can express the current through the strip as follows:

\[
\frac{I_s}{z_t} = \int_{-x_\alpha}^{x_\alpha} J_s(x) \, dx + \int_{-x_t/2}^{x_t/2} J_s(x/2) e^{-x/2 + x/(\xi_2 \lambda^2)} \, dx + \int_{x_\alpha}^{x_t/2} J_s(x/2) e^{-x/2 + x/(\xi_2 \lambda^2)} \, dx \tag{D.7}
\]

where

\[
\pm x_\alpha = \pm \left( \frac{1}{2} x_t - \frac{a \lambda^2}{2 z_t} \right) \tag{D.8}
\]

Solving this integral, we can define a dimensionless constant \( \Lambda \) as follows
\[ I_s = J_s(0) A_t \Lambda \]  \hspace{1cm} (D.9)

with

\[ \Lambda \equiv \frac{1}{x_t} (x_t/2) \sin^{-1} \left( \frac{2x_t}{x_t} \right) x_t a \sqrt{x_t z_t} - \frac{2.33 z_t \sqrt{x_t z_t}}{a^{3/2} \lambda^3} (e^{-\nu^2} - 1) \]  \hspace{1cm} (D.10)

This value \( \Lambda \) allows us to determine the current-density at the center of the trap from the value of the applied current, which is quite important when determining the minimum current to depin embedded vortices. Additionally, the current density anywhere in the film can be determined from the knowledge of \( J_s(0) \) using equations D.5 and D.6. Simple substitution of the subscript "c" for "t" in the previous equations yields the appropriate expressions for the control-line.

All of the derivations in Chapter 5 operate under the assumption of uniform current densities. For the SUFRS dimensions treated in Chapters 4 and 5, the current distribution multipliers for the trap and the control, respectively are

\[ \Lambda_t = 1.318 \quad \Lambda_c = 1.542 \]

From these values, we see that the approximation to uniform current densities is fairly good: better for the trap than the control. However, uniform current densities are far more important in the trap than in the control, because of the required local current-densities needed to depin vortices. The indication that the current density is less uniform in the control can have positive effects. If more current is concentrated in the edges of the control, the field result near the edges of the control will be higher for lower currents, which improves the operational range for control & write currents.
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