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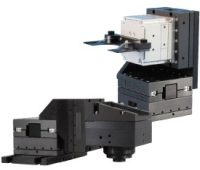
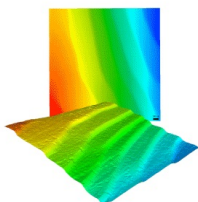
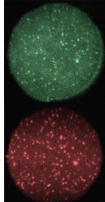
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ABSTRACT

The role of turbulence in setting boundary plasma conditions is presently a key uncertainty in projecting to fusion energy reactors. To robustly diagnose edge turbulence, we develop and demonstrate a technique to translate brightness measurements of HeI line radiation into local plasma fluctuations via a novel integrated deep learning framework that combines neutral transport physics and collisional radiative theory for the $3^3D - 2^3P$ transition in atomic helium with unbounded correlation constraints between the electron density and temperature. The tenets for experimental validity are reviewed, illustrating that this turbulence analysis for ionized gases is transferable to both magnetized and unmagnetized environments with arbitrary geometries. Based on fast camera data on the Alcator C-Mod tokamak, we present the first two-dimensional time-dependent experimental measurements of the turbulent electron density, electron temperature, and neutral density, revealing shadowing effects in a fusion plasma using a single spectral line.

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I. INTRODUCTION

Diagnosing edge plasmas is an essential task for testing turbulence models and better understanding plasma fueling and confinement in fusion devices. Gas puff imaging (GPI) of turbulence is a widely applied experimental diagnostic that captures line emission based on the interaction of neutrals with the hot plasma. As a technique with decades of application in a range of settings,¹ optical imaging of fluctuations provides a view of turbulent plasma transport. This transport is critical to the operation and energy gain of nuclear fusion reactors, but interpretation (e.g., velocimetry²) of these fluctuations to directly test reduced physics models is not always straightforward. By tracing the atomic theory underlying the nonlinear dynamics of observed line emission, we outline a novel spectroscopic method for understanding turbulent fluctuations based on high-resolution visible imaging of plasma-neutral interactions.

The plasma edge in magnetic fusion devices is characterized by neighboring regions: confined plasmas where temperatures can exceed 10^6 K and the colder scrape-off layer (SOL) where gaseous particles may not be completely ionized. These regions exist tightly coupled to one another and inseparable in many respects. Consequently, accounting for neutral transport in conjunction with ion and electron turbulence is essential in wholly analyzing boundary plasma fluctuations. Edge turbulence is characterized by a broadband spectrum with perturbation amplitudes of order unity and frequencies ranging up to 1 MHz. Edge localized modes and intermittent coherent structures convecting across open field lines can be responsible for significant particle losses and plasma-wall interactions that strongly influence operations. To model the vast dynamical scales present in fusion plasmas accordingly requires sufficiently complex modeling techniques. In this work, we introduce custom neural architectures within a multi-network deep learning framework that bounds output to abide by collisional radiative theory^{3,4}

and searches for solutions consistent with continuity constraints on neutral transport.⁵ Learning nonlinear physics via optimization in this way provides a new way to examine edge turbulence using experimental data from GPI. While our methodology is not fixed to any device, this paper focuses on two-dimensional experimental brightness measurements from open flux surfaces on the Alcator C-Mod tokamak,^{6,7} where we find a good signal-to-noise ratio of localized light emission. Recent advancements in validation techniques of reduced turbulence theories^{8,9} emphasize the importance of comprehensive diagnostic coverage of electron dynamics on turbulent spatial and temporal scales. To this end, we describe the first calculations of the two-dimensional turbulent electron density, electron temperature, and neutral density fields that self-consistently include fluctuation-induced ionization using only observations of the 587.6 nm line via fast camera imaging. With several possible extensions to the deep learning framework identified, our experimental analysis technique paves new ways to systematically diagnose edge plasma turbulence.

To demonstrate this framework, we evaluate the validity of the collisional radiative theory in conditions relevant to fusion plasmas for atomic helium line emission in Sec. II, an overview of the experimental setup for GPI on the Alcator C-Mod tokamak in Sec. III, outline a custom physics-informed machine learning optimization technique designed for turbulence imaging in Sec. IV, present results from the analysis applied to experimental fast camera data in Sec. V, and conclude with a summary and future outlook in Sec. VI.

II. TIME-DEPENDENT ANALYSIS OF QUANTUM STATES IN ATOMIC HELIUM

The electronic transition from 3^3D to 2^3P quantum states in atomic helium results in photon emission with a rest frame wavelength of 587.6 nm. Atomic physics modeling of this line radiation in a plasma correspondingly requires tracking all relevant electron transition pathways that can populate or depopulate 3^3D . Our starting point in this analysis is to consider the full rate equations where the evolution of a quantum state p follows

$$\begin{aligned} \frac{dn(p)}{dt} = & \sum_{q \neq p} \{C(q, p)n_e + A(q, p)\}n(q) \\ & - \left\{ \sum_{q \neq p} C(p, q)n_e + \sum_{q < p} A(p, q) + S(p)n_e \right\} n(p) \\ & + \{\alpha(p)n_e + \beta(p) + \beta_d(p)\}n_i n_e, \end{aligned} \quad (1)$$

where $n(p)$ is the population density of the $p = n^{2S+1}L$ state, in which n is the principal quantum number, S is the spin, and L is the orbital angular momentum quantum number. Similarly, q is another quantum state with the notation $q < p$ indicating that q lies energetically below p . Equation (1) includes the spontaneous transition probability from p to q given by the Einstein A coefficient $A(p, q)$, electron impact transitions $C(p, q)$, electron impact ionization $S(p)$, three-body recombination $\alpha(p)$, radiative recombination $\beta(p)$, and dielectronic recombination $\beta_d(p)$, with n_e and n_i denoting the electron density and hydrogen-like He^+ density, respectively. All aforementioned rate coefficients except $A(p, q)$ have a dependence on the electron temperature (T_e) that arises from averaging cross sections over a Maxwellian velocity distribution for electrons, which are

based on calculations with the convergent close-coupling (CCC)^{10–12} and R -matrix with pseudostates (RMPS)¹³ methods using high precision calculations of helium wavefunctions.^{14,15} For application in a numerical framework, we follow Goto,⁴ Zholobenko *et al.*¹⁶ to model atomic helium with a corresponding energy level diagram visualized in Fig. 1. All quantum states with $L \leq 2$ are resolved for $n < 8$, while states with $L \geq 3$ are bundled together into a single level denoted as “ $F+$.” For $n \geq 8$, L is not resolved, while those with $n \geq 11$ are approximated as hydrogenic levels with statistical weights twice those of hydrogen. Quantum states up to $n = 26$ are included, with $n \geq 21$ being given by the Saha–Boltzmann equilibrium.^{3,16,17} For application in magnetized plasmas (e.g., tokamaks), where rate coefficients vary with magnetic field strength due to wavefunction mixing, spin–orbit interactions are included to account for mixing between the singlet and triplet fine structure levels.^{4,16} Finite fields largely influence the modeling of metastable species and higher orbital quantum numbers. To quantify radiation trapping effects, the dimensionless optical depth for a Doppler-broadened line transition between states $j \rightarrow k$ can be expressed as¹⁸

$$\tau_{j \rightarrow k} = 5.4 \times 10^{-3} f_{j \rightarrow k} \lambda_{jk} n_j (\mu_j / T_j)^{\frac{1}{2}} L, \quad (2)$$

where $f_{j \rightarrow k}$ is the absorption oscillator strength, λ_{jk} (nm) is the line center wavelength, μ_j is the mass ratio of the emitting species relative to a proton, L (cm) is the physical depth of the gas along the viewing chord, and n_j (10^{13} cm^{-3}) and T_j (eV) are the density

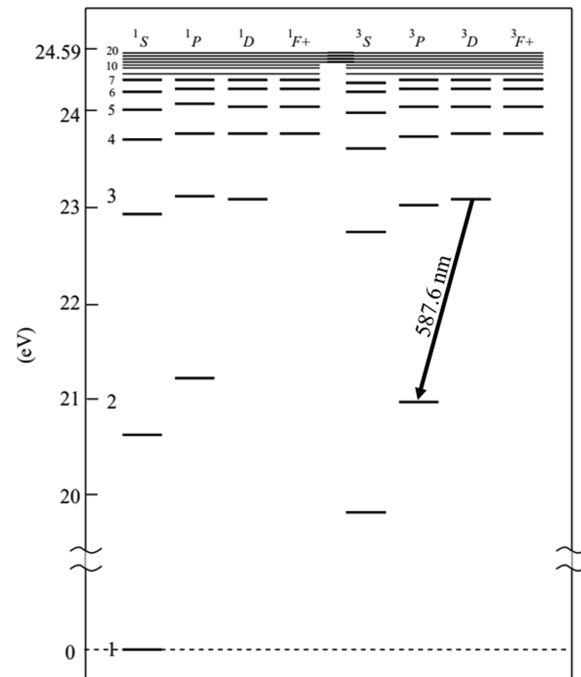


FIG. 1. Energy level diagram for atomic helium considered in the calculations. An arrow connects $3^3D \rightarrow 2^3P$, which is the origin of the 587.6 nm photon emission. The labels 1^3F+ denote the quantum states representing all levels with $L \geq 3$. Reprinted with permission from M. Goto, *J. Quant. Spectrosc. Radiat. Transfer* **76**, 331–344 (2003). Copyright 2003 Elsevier.

and temperature, respectively, of particles in state j . For 587.6 nm line emission with a thermal helium puff in conditions relevant to magnetic confinement fusion devices, where $f_{2^3P \rightarrow 3^3D} \sim 0.6$,¹⁹ $\tau_{2^3P \rightarrow 3^3D} \ll 1$ resulting in the plasma edge region being optically thin for our spectroscopic analysis of a localized gas puff.^{1,16}

The rate equation (1) for an optically thin plasma can be equivalently expressed in matrix form as^{16,20}

$$\frac{d\mathbf{n}}{dt} = \mathbf{M}(n_e, T_e)\mathbf{n} + \mathbf{\Gamma}(n_e, T_e, n_i), \quad (3)$$

where \mathbf{n} is a vector of the N atomic states, \mathbf{M} represents the $N \times N$ matrix of rates for collisional ionization, excitation, de-excitation, radiative decay, and recombination as above, and $\mathbf{\Gamma}$ symbolizes sources. Since time-evolving every state in atomic helium is computationally expensive, effective atomic physics models known as collisional radiative (CR) theories are often constructed. This involves separating the N states into P and Q spaces of sizes N_P and N_Q , respectively, such that (3) becomes

$$\frac{d}{dt} \begin{bmatrix} \mathbf{n}_P \\ \mathbf{n}_Q \end{bmatrix} = \begin{bmatrix} \mathbf{M}_P & \mathbf{M}_{PQ} \\ \mathbf{M}_{QP} & \mathbf{M}_Q \end{bmatrix} \begin{bmatrix} \mathbf{n}_P \\ \mathbf{n}_Q \end{bmatrix} + \begin{bmatrix} \mathbf{\Gamma}_P \\ \mathbf{\Gamma}_Q \end{bmatrix} = \begin{bmatrix} \frac{d\mathbf{n}_P}{dt} \\ 0 \end{bmatrix}. \quad (4)$$

By taking the Q space to be time-independent, under the expectation that they evolve on timescales faster than those of plasma turbulence fluctuations, this allows one to fold the dynamics of the Q space into effective rates that depend on \mathbf{n}_P . This can be written as

$$\mathbf{n}_Q = -\mathbf{M}_Q^{-1}(\mathbf{M}_{QP}\mathbf{n}_P + \mathbf{\Gamma}_Q), \quad (5)$$

$$\begin{aligned} \frac{d}{dt} \mathbf{n}_P &= (\mathbf{M}_P - \mathbf{M}_{PQ}\mathbf{M}_Q^{-1}\mathbf{M}_{QP})\mathbf{n}_P \\ &\quad - \mathbf{M}_{PQ}\mathbf{M}_Q^{-1}\mathbf{\Gamma}_Q + \mathbf{\Gamma}_P \\ &= \mathbf{M}_{\text{eff}}\mathbf{n}_P + \mathbf{\Gamma}_{\text{eff}}. \end{aligned} \quad (6)$$

However, the applicability of such a separation in dynamical space needs to be quantitatively tested. In particular, for the constructed CR model to be applicable, it should satisfy Greenland's criteria,^{20–22} which require evaluating the normalized eigenvalues and eigenvectors of $\mathbf{M}(n_e, T_e)$. The N eigenvectors are arranged as the columns of an $N \times N$ matrix \mathbf{T} , in order of increasing eigenvalue, λ , and can be partitioned into four submatrices

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_P & \mathbf{T}_{PQ} \\ \mathbf{T}_{QP} & \mathbf{T}_Q \end{bmatrix}. \quad (7)$$

In terms of these quantities, Greenland's criteria require that (i) $\|\mathbf{T}_{QP}\| \ll 1$ and (ii) $\|\mathbf{T}_{QP}\mathbf{T}_P^{-1}\| \ll 1$. From this point onwards, we will adopt in our evaluation an $N_P = 1$ CR model where the P space consists of only the ground state for atomic helium being dynamically evolved. In this formulation, meta-stable species (e.g., 2^1S , 2^3S) are taken to be in a steady state. Greenland's criteria for the $N_P = 1$ CR theory were previously examined in a range of conditions relevant to fusion plasmas and found to widely satisfy (i) and (ii),²⁰ but there is an additional unresolved practical condition: (iii) the shortest timescales over which P space states are evolved should

be larger than the inverse of the smallest Q space eigenvalue, i.e., $\frac{\partial}{\partial t} < |\lambda_Q|$. In more concrete terms, phenomena on timescales faster than $\tau_Q \equiv 1/|\lambda_Q|$ are not resolved. As a result, τ_Q represents the slowest timescale in Q space, which is not tracked, and the ground state of atomic helium should be evolved on timescales slower than τ_Q for the separation of the two dynamical spaces to be consistent since all timescales faster than τ_Q are effectively instantaneous. For the CR formulation to be subsequently applicable in the spectroscopic analysis of plasma turbulence, the autocorrelation time of n_e (τ_{n_e}) and T_e (τ_{T_e}) must be larger than τ_Q . Additionally, the exposure time of the experimental imaging diagnostic, τ_{GPI} , should satisfy the timescale criterion of

$$\tau_Q < \tau_{GPI} < \tau_{n_e}, \tau_{T_e} \quad (8)$$

for consistency. This ensures the experimentally observed line emission in a single exposure time is based on neutrals nominally excited by a unique n_e and T_e instead of a range of contributing magnitudes. Using revised cross sections from Ralchenko *et al.*²³ and Zholobenko *et al.*,²⁴ we report τ_Q under the $N_P = 1$ CR formulation in Fig. 2 at a range of n_e and T_e relevant to fusion plasmas. This quantity demarcates the temporal domain of validity. An important trend from the plot is that as n_e increases, the limit on the temporal resolution of turbulent fluctuation measurements improves. For high plasma density fluctuations, such as coherent filamentary structures, the resolution is roughly $\tau_Q \lesssim 1 \mu\text{s}$ for even $n_e \sim 10^{13} \text{ cm}^{-3}$. As n_e increases in higher field devices, the theoretical limit for resolving temporal scales improves. This aids the application of our GPI analysis for studying plasma turbulence in new regimes on upcoming tokamaks. A lower limit on spatial resolution for turbulence diagnostic imaging is set by $v_{\text{HeI}}/A(3^3D, 2^3P)$, provided that it is shorter than $v_{\text{HeI}}\tau_{n_e}$ —or $v_{\text{HeI}}\tau_{T_e}$, if smaller—where v_{HeI} is the particulate velocity of the atomic helium. The validity criteria for the $N_P = 1$ CR formulation are generally satisfied in analyzing the $3^3D \rightarrow 2^3P$ transition for fusion plasmas of sufficient density, but one should

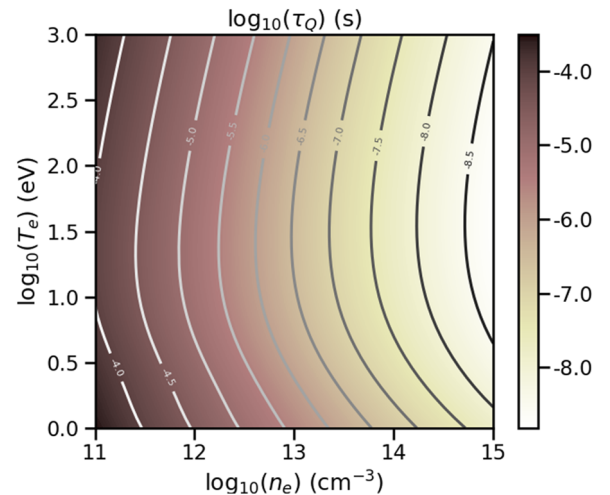


FIG. 2. A contour plot of τ_Q for the $N_P = 1$ CR model scanned over a range of relevant electron densities and temperatures for magnetically confined fusion plasmas. A logarithmic scale is applied on all axes, including the color bar.

take care when checking validity in different scenarios. For example, if applying CR theory to cameras imaging different electronic transitions (e.g., for analysis of line ratios^{25–27}) with long exposure times where $\tau_{n_e}, \tau_{T_e} < \tau_{GPI}$, the formulated CR theory is technically invalid as the condition given by Eq. (8) is no longer met. This could potentially cause misalignment of n_e and T_e profiles when comparing existing experimental diagnostics toward closed flux surfaces, where plasma fluctuations are temporally faster than the observed autocorrelation time of far SOL turbulence.²⁸ Farther in the SOL as the plasma pressure drops, one should also be careful to check that $\tau_Q < \tau_{n_e}, \tau_{T_e}$. Diagnosing edge fluctuations, thus, necessitates sufficiently high resolution for both the experimental diagnostic and applied CR theory.

The $N_p = 1$ CR formulation permits any excited state population density in Q space to be written as

$$\mathbf{n}_Q|_q = n(q) = R_0(q)n_e n_i + R_1(q)n_e n(1^1S), \quad (9)$$

where $R_0(q)$ and $R_1(q)$ are known as population coefficients associated with recombination and electron impact physics. The temporal evolution of the ground state, the only species in P space for this CR model, follows

$$\frac{d}{dt} \mathbf{n}_P = \frac{d}{dt} n(1^1S) = \alpha_{CR} n_e n_i - S_{CR} n_e n(1^1S), \quad (10)$$

where α_{CR} and S_{CR} are the recombination and ionization rate coefficients, respectively. To generate photon emissivity coefficients from this CR model, Eq. (9) is multiplied by the Einstein A coefficient for the given radiative transition. For the 587.6 nm line, $A(3^3D, 2^3P) = 2 \times 10^7 \text{ s}^{-1}$. If $q = 3^3D$, by multiplying Eq. (9) with the corresponding spontaneous decay rate, one can compute

$$\text{PEC}^{\text{exc}} = R_1(3^3D)A(3^3D, 2^3P), \quad (11)$$

$$\text{PEC}^{\text{rec}} = R_0(3^3D)A(3^3D, 2^3P). \quad (12)$$

Contours of all coefficients along with their dependence on n_e and T_e are visualized at a magnetic field of $B = 5 \text{ T}$ in Figs. 3 and 4. Given these rates, one can further simplify the expressions for Eqs. (9) and (10) when modeling 587.6 nm line emission in the presence of edge plasma turbulence by removing the effects of volumetric recombination, i.e., $\text{PEC}^{\text{exc}} \gg \text{PEC}^{\text{rec}}$ and $S_{CR} \gg \alpha_{CR}$, which are negligible for edge fusion plasmas unless $n_i \gg n_0 \equiv n(1^1S)$, i.e., only if the HeII density is far greater than the ground state neutral helium density. The effects of charge-exchange are also neglected as the reaction rate is small compared to electron impact ionization for atomic helium as long as $5 \text{ eV} < T_e < 5 \text{ keV}$.²⁹ Note that this is not necessarily true for other atomic or molecular species, e.g., deuterium,³⁰ but allows for an expression of 587.6 nm photon emissivity given by

$$I = C n_0 n_e \text{PEC}^{\text{exc}}(n_e, T_e) = C n_0 f(n_e, T_e), \quad (13)$$

where $f(n_e, T_e)$ can be interpreted as the photon emission rate per neutral consistent with the $N_p = 1$ CR model. Using an oft-applied exponential model of $f(n_e, T_e) \propto n_e^{\alpha_n} T_e^{\alpha_T}$ and treating α_n and α_T as constants could yield erroneous emissivity predictions where fluctuations of order unity are beyond the perturbative regime. Therefore, it is important to retain the full range of dependency on n_e and T_e . A constant factor C is introduced in (13) to account for the calibration of the instrument used to measure the line radiation, including effects introduced by the finite thickness of the observed emission cloud.

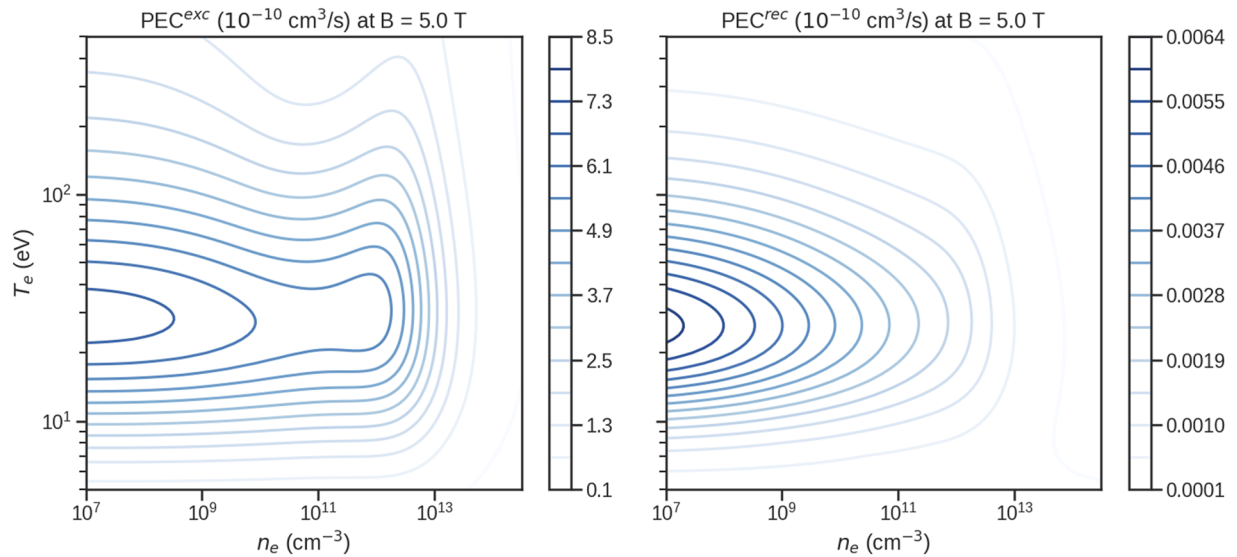


FIG. 3. Photon emissivity coefficients for the Hel 587.6 nm line based on electron impact excitation (left) and recombination (right). These quantities are derived from the $N_p = 1$ CR model's population coefficients and follow Eqs. (11) and (12).

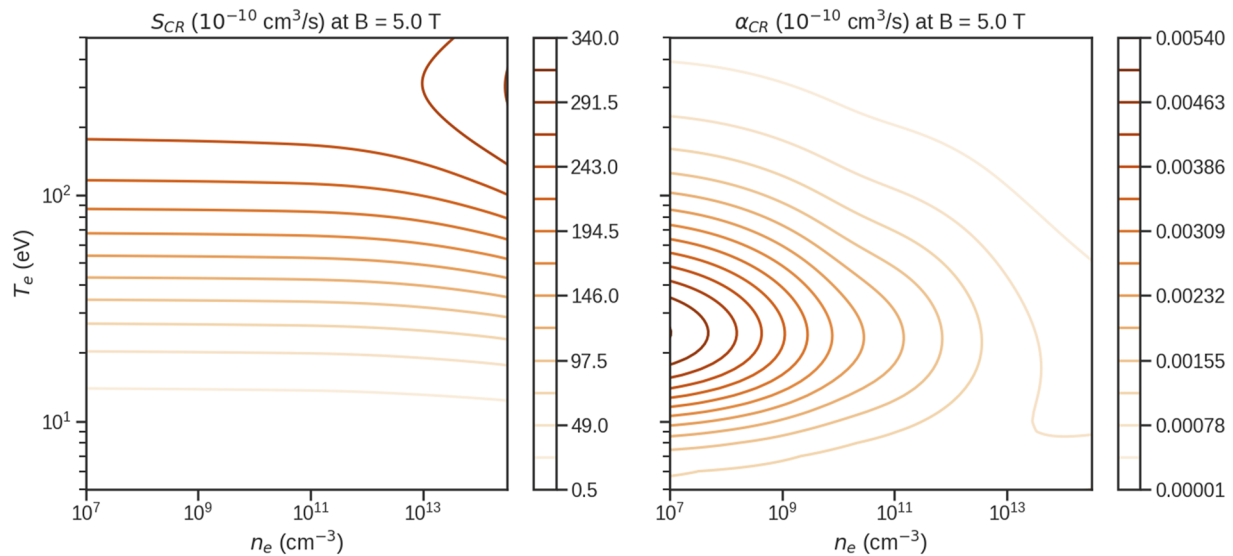


FIG. 4. Ionization (left) and recombination (right) rate coefficients derived from the $N_p = 1$ CR model. These quantities represent sinks and sources in Eq. (10) for atomic helium when considering their transport in fusion plasmas.

III. EXPERIMENTAL TURBULENCE IMAGING OF HELIUM LINE EMISSION

Our experimental analysis technique is generic to any plasma discharge on Alcator C-Mod where good fast camera data exist for the 587.6 nm line. The plasma discharge chosen for this work constitutes 1 120 711 021. This is a majority deuterium, lower single null diverted ohmic plasma with an on-axis toroidal magnetic field of 5.4 T and plasma current of 0.83 MA. The tokamak itself has a major radius of 0.68 m and a minor radius of 0.22 m. The discharge has independent diagnostic measurements from a main chamber scanning probe equipped with a mirror Langmuir probe (MLP) biasing system run in a swept mode in the edge plasma.³¹ Based on Thomson scattering and electron cyclotron emission diagnostic measurements, the core electron density and temperature are $2.0 \times 10^{20} \text{ m}^{-3}$ and 1.5 keV, respectively.

In the present work, the GPI diagnostic on the Alcator C-Mod tokamak^{32,33} was configured to capture visible light at a wavelength of 587.6 nm arising from the dynamic interaction of edge plasma turbulence with neutral helium puffed locally to the imaged region. This is a commonly used technique akin to other plasma diagnostics, such as beam emission spectroscopy (BES).³⁴ Helium is an ideal choice for two-dimensional turbulence imaging for several reasons: its low atomic number results in radiative losses minimally perturbing the plasma state; its larger ionization energy allows for greater neutral penetration than thermal deuterium; its lack of molecular interactions reduces complexity in modeling; and its neutrality keeps its transport independent of external magnetic fields. The localized distribution of atomic helium also provides a greater contrast to the background emissivity in fusion plasmas that primarily fuel with hydrogen isotopes. HeI emission was imaged onto a Phantom 710 fast camera, installed on Alcator C-Mod in 2009 to view the out-board midplane region.³³ The camera has a maximum framing rate

of 400 000 frames/s at 2.1 μs -exposure/frame when 64×64 pixels are being read out, and each pixel is $\sim 20 \times 20 \mu\text{m}^2$. The diagnostic's resultant temporal resolution is 2.5 μs as it takes 0.4 μs to read values from the pixel array. The fast camera has a built-in positive offset of ~ 80 counts, which is subtracted from all GPI signals before analysis

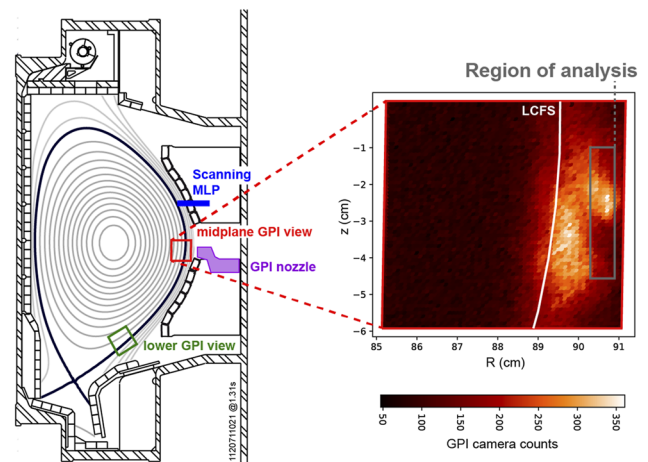


FIG. 5. Visualization of the experimental GPI setup on a poloidal cross section of a lower single null diverted plasma discharge (1 120 711 021) on Alcator C-Mod. In this study, we use measurements from the midplane fast camera with a 587.6 nm optical filter with full width at half maximum of 11.4 nm that has a largely field-aligned view of edge fluctuations. The expansion at the right shows raw counts measured by the fast camera at $t = 1.312 858 \text{ s}$ and includes overlays of both the last closed flux surface and the approximate domain of the analysis described in Sec. V.

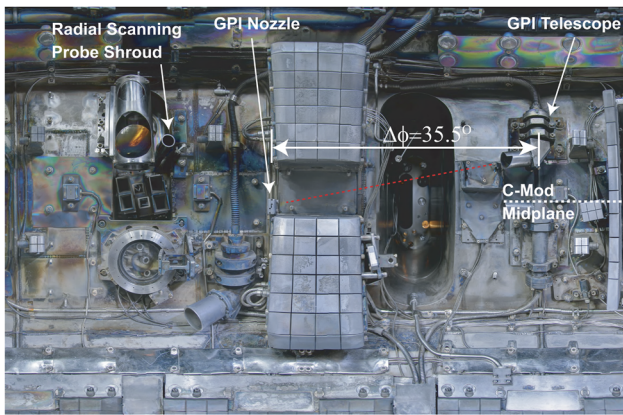


FIG. 6. A section of a panoramic photograph of the Alcator C-Mod outer wall, showing approximately one-quarter of the device and centered on the split poloidal limiter next to the gas puff imaging measurement location. Labels indicate the position of the GPI nozzle, its imaging telescope, and an approximate line of sight (red dashed). The position of the radial scanning probe, which provides the mirror Langmuir probe measurements, is also exhibited.

of the experimental data.³³ Based on the manufacturer's specifications and sample bench tests, the fast camera measurements are expected to vary linearly with light level over the pixels analyzed. The brightness is thus offset in absolute magnitude by a constant scale factor and accounted for in the framework.

A coherent fiber bundle/image guide was used to couple light from viewing optics mounted on the outer wall of the vacuum vessel to the Phantom camera detector array. The optics imaged a roughly $60 \times 60 \text{ mm}^2$ region in the (R, Z) -plane just in front of a gas puff nozzle through a vacuum window onto the image guide. The viewing chords pointed downward at a fixed angle of 11.0° below horizontal toward the vertically stacked 4-hole gas nozzle displaced from the telescope by $\sim 35.5^\circ$ in toroidal angle. The central ray of the imaged view, thus, pierced the gas puff plane approximately parallel with the local magnetic field line.³³ This aligns the GPI optics with field-aligned fluctuations for typical operational parameters of an on-axis toroidal field of 5.4 T and a plasma current of 1.0 MA. For discharge 1 120 711 021 conducted at a plasma current of 0.83 MA, the viewing chords are oriented at an angle of $\sim 2^\circ$ to the local field. Spatial blurring due to this angular misalignment, θ_B , consequently limits resolution to $\Delta x = L_{\parallel} \tan \theta_B$, where L_{\parallel} is the emission cloud's length parallel to the local magnetic field line. For L_{\parallel} between 5 and 40 mm, the smearing will be 0.2–1.4 mm in addition to the 1 mm pixel spot size in the image plane. Since the gas cloud expands after exiting the 4-hole nozzle and the local magnetic field's pitch angle varies, the smearing increases for those chords farther away from the nozzle depending on the collimation of the gas cloud.^{1,35} With this setup and under these plasma conditions, we, thus, estimate the spatial resolution over the portion of the field-of-view that we analyze to be ~ 1 –2 mm. Figure 5 displays a visualization of the experimental setup's poloidal cross section with the camera telescope in Fig. 6.

As noted earlier, helium gas is injected into the vessel via four vertically displaced plasma-facing capillaries located at $Z = -4.2, -3.4, -2.6,$ and -1.9 cm , which are mounted in a port on a shelf just

below the outer midplane sitting in the shadow of two outboard limiters. The position $Z = 0$ corresponds to the vertical location of the magnetic axis. The gas tubes' orifices are positioned at $R = 91.94 \text{ cm}$ with the channel exit diameter being 3 mm. The helium atoms are supplied by the Neutral gas INJECTION Array (NINJA) storage and delivery system³⁶ that has a pneumatically controlled valve at the plenum that is connected to a 3.48 m-long, 1 mm-diameter capillary that feeds the four diverging gas tubes. Previous measurements indicate that the gas cloud exiting a single 1 mm-diameter capillary expands with an angular half-width of 25° in both the poloidal and toroidal directions. This is the basis for our estimate of the 1–2 mm spatial resolution given above. Due to the tubes' spatial displacement, the helium gas puff is intended to be relatively uniform in the vertical direction. By definition, there is a shock at (or near) the vacuum–nozzle interface for this sonic flow since only particles moving downstream can escape, and there is consequently no information being communicated to upstream particles.³⁷ The neutral dynamics, thus, transition from a fluid regime in the gas tube to a kinetic regime upon entering the tokamak from the nozzle. The HeI exiting the diverging nozzles is approximately modeled by a drifting, cut-off Maxwellian distribution with a mean radial velocity of $v_x \sim -900 \text{ m/s}$ and a mean vertical velocity of $v_y \sim -20 \text{ m/s}$ since the direction of the non-choked flow in the gas tubes is roughly 2.4° away from being purely radial in orientation.

To examine the experimental relevance of applying the $N_P = 1$ CR theory outlined in Sec. II for the analysis of edge plasma turbulence on Alcator C-Mod, we review a few key characteristic parameters of interest based on scanning MLP measurements of n_e and T_e in plasma discharge 1 120 711 021. Magnetically disconnected from the GPI field of view, the scanning MLP is located at $Z = 11.1 \text{ cm}$ roughly 20° in toroidal angle from the GPI view and radially traverses the tokamak plasma from the far edge to just inside the last closed flux surface (LCFS) with a temporal resolution of $0.3 \mu\text{s}$. Measurements mapped to the midplane radius are visualized in Fig. 7 based on a probe plunge nearly coincident temporally with our GPI analysis of this plasma discharge. While the probe bias is inherently perturbative due to the collection of charged particles, we assume that its effects on local plasma conditions are negligible.³⁸ From the MLP data, we can obtain autocorrelation times of fluctuations near the LCFS and ~ 8 –10 mm radially outward into the SOL when mapped to the midplane radius. Toward closed flux surfaces, τ_{n_e} and τ_{T_e} are ~ 4.2 and $6.1 \mu\text{s}$, respectively. In the far SOL, τ_{n_e} and τ_{T_e} increase to 15.6 and $22.9 \mu\text{s}$, respectively. Since the probe has a finite velocity and the autocorrelation length of fluctuations is finite, these estimates of τ_{n_e} and τ_{T_e} act as conservative lower bounds as long as there is no aliasing nor phase-alignment between the probe's motion and turbulence structures. We note that the fast camera exposure time of $2.1 \mu\text{s}$ is expected to be suitable for the analysis of edge plasma fluctuations in this ohmic discharge, although faster cameras could be helpful in analyzing plasma conditions. Further, for turbulence near the LCFS where $n_e \gtrsim 10^{19} \text{ m}^{-3}$ and $T_e \gtrsim 20 \text{ eV}$, then $\tau_Q < 1 \mu\text{s}$, and the condition of $\tau_Q < \tau_{\text{exp}} < \tau_{n_e}, \tau_{T_e}$ is well-satisfied. For fluctuations farther out into the SOL, this condition is still generally valid, especially in the treatment of high pressure filaments, but one should be careful when n_e drops below $2.5 \times 10^{18} \text{ m}^{-3}$ in fusion plasmas. For spectroscopic techniques analyzing line intensities, each optical camera's exposure time needs to be suitably adjusted to satisfy the timescale condition. This is especially important toward

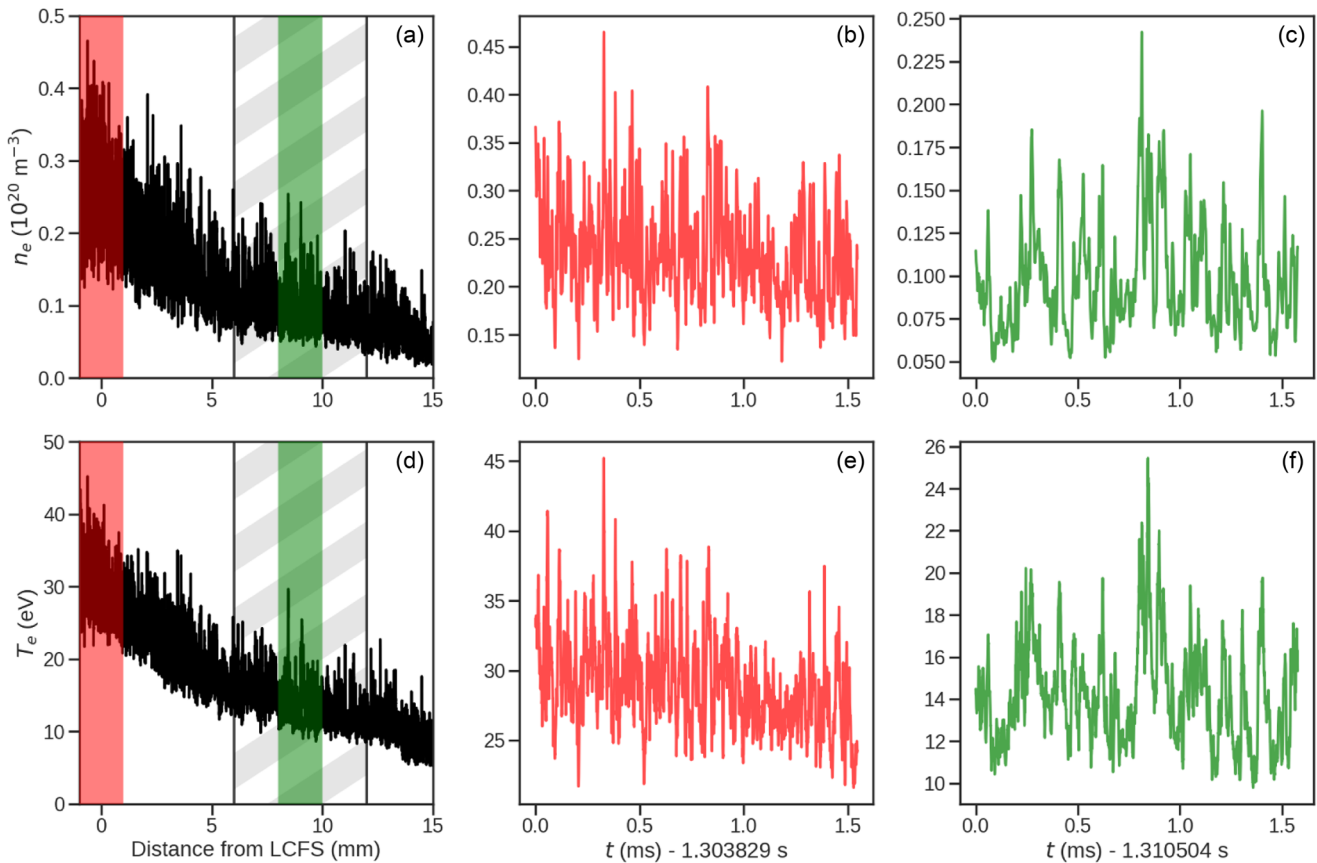


FIG. 7. Experimental n_e and T_e measurements in discharge 1120711021 from the independent scanning mirror Langmuir probe. The full probe plunge duration is $1.288 < t \text{ (s)} < 1.318$. As the probe is scanning back from closed flux surfaces, time series of n_e and T_e are plotted to compute autocorrelation times. At distances near the LCFS (red), τ_{n_e} and τ_{T_e} are ~ 4.2 and $6.1 \mu\text{s}$, respectively. Farther into the SOL (green), τ_{n_e} and τ_{T_e} increase to 15.6 and $22.9 \mu\text{s}$, respectively. For reference, the gray shaded regions in (a) and (d) roughly correspond to the radial extent of the GPI data analyzed.

closed flux surfaces where long camera exposure periods and shorter autocorrelation times would render time-dependent examination of brightness ratios arising from turbulent fluctuations as inconsistent.

Our framework outlined in the next sections can be applied to regions with arbitrary geometries (e.g., X-point, divertor) if using sufficiently planar helium beams where the width of the collimated gas is smaller than the parallel autocorrelation length of the plasma fluctuations in the direction of the viewing chords. Since the viewing chords are roughly field-aligned over the pixels being analyzed, this parallel scale condition is expected to be satisfied. Finally, we note that the signal-to-noise ratio degrades in the inboard portion of the field-of-view, which includes plasma close to or on closed flux surfaces where the electron pressure and ionization rate increase sharply.^{39,40} Accordingly, we analyze fluctuations a few millimeters away from the LCFS on a two-dimensional (R, Z)-grid co-located at the nominal gas puff plane. In future work, if greater neutral penetration can be achieved such that high signal-to-noise can be attained on closed flux surfaces, the capability to then probe pedestal dynamics also exists where priming our optimization framework on available one-dimensional data may help.⁴¹ This opportunity may

already be viable on devices with smaller line-integrated n_e , and the methodology can extend to unmagnetized plasmas, too.

IV. DEEP LEARNING OF TIME-DEPENDENT NEUTRAL TRANSPORT PHYSICS AND COLLISIONAL RADIATIVE THEORY

Combining the theory governing atomic emission and neutral transport with experimental turbulence measurements via fast camera imaging into an integrated analysis framework requires sufficiently sophisticated modeling techniques. Neural networks are differentiable computational programs that can provide natural representations for physical systems determined by differential-algebraic equations. They extend ordinary regression models into robust universal function approximators with generalized constraints that are highly effective in solving inverse optimization problems with sufficient training given their high plasticity. We outline a novel multi-network deep learning framework custom-built for the analysis of 587.6 nm helium line emission in fusion plasmas to uncover n_e , T_e , and n_0 . The networks only receive experimental

brightness measurements from GPI while being optimized against the $N_p = 1$ CR theory for photon emissivity along with the continuity equation for neutral transport that accounts for the ionization of helium atoms on turbulent scales. In this way, we combine training upon both mathematical laws and observational data. To begin, we represent the unobserved quantities n_e , T_e , and n_0 each with their neural network. The initial layer inputs correspond to the local spatiotemporal points (x, y, t) , with the (x, y) -coordinate being equivalent to (R, Z) , from the approximately two-dimensional domain viewed by the fast camera in the poloidal plane of the gas puff nozzle. The only output of each network is the respective dynamical variable being represented. Every network's inner architecture consists of five hidden layers with 150 neurons per hidden layer and hyperbolic tangent activation functions (σ) using Xavier initialization.⁴² To provide reasonable intervals for the optimization bounds, the networks for n_e , T_e , and n_0 are constrained via output activation functions to be between $2.5 \times 10^{18} < n_e \text{ (m}^{-3}\text{)} < 7.5 \times 10^{19}$, $2.5 < T_e \text{ (eV)} < 150.0$, and $0.1 < n_0 \text{ (arb.units)} < 10.0$. While required for numerical stability, care must be taken since the solutions existing near these limits may not be fully converged. We note that the learnt constant calibration factor, C , is similarly represented by a network but does not vary spatially nor temporally. Physically, this results in n_0 being determined up to a constant scaling. The scalar constant also accounts for the two-dimensional approximation of the localized gas puff, which has a finite toroidal width from the helium atoms exiting the capillaries. By assuming n_e , T_e , and n_0 to be roughly uniform along the camera's sightline, the effect of this finite volume is absorbed when learning the calibration factor. While the two-dimensional approximation is reasonable for sufficiently planar gas injection, the deep framework can technically be generalized toward natively handling three-dimensional space since it employs a continuous domain without any discretization. This is a future extension.

Our optimization is conducted in stages. To begin learning CR theory, we construct novel neural network structures where the outputs of the n_e and T_e networks serve as inputs to a new architecture representing the photon emissivity per neutral, $f \equiv f(n_e, T_e)$. The connectivity of the neurons conjoining n_e and T_e toward the network's output, f , is visualized in Fig. 8. These weights and biases are trained against $n_e \text{PEC}^{\text{exc}}(n_e, T_e)$, which is derived from the $N_p = 1$ CR theory. The corresponding emissivity coefficient is plotted in Fig. 3. The ionization rate per neutral, $n_e S_{\text{CR}}(n_e, T_e)$, which is based on the coefficient plotted in Fig. 4, is similarly represented by an architecture with n_e and T_e serving as inputs. All the training of the two architectures representing $f(n_e, T_e)$ and $n_e S_{\text{CR}}(n_e, T_e)$ is conducted in the first stage prior to any optimization against the fast camera data. This ensures the next stages involving training with collisional radiative constraints take place under an integrated optimization framework with all quantities being represented by neural networks. For numerical purposes, n_e and T_e are normalized by 10^{19} m^{-3} and 50 eV, respectively, and time is converted to units of microseconds during the optimization. For low temperature plasmas where $T_e < 2 \text{ eV}$, training with the networks and output CR coefficients from^{4,16} based on fitted electron impact cross sections should be carefully checked due to potential corrections to fits for collision strengths at such low energies. We also prime only the n_e and T_e networks against constants of 10^{19} m^{-3} and 50 eV, respectively, for initialization during this first stage. The priming of n_e and T_e and

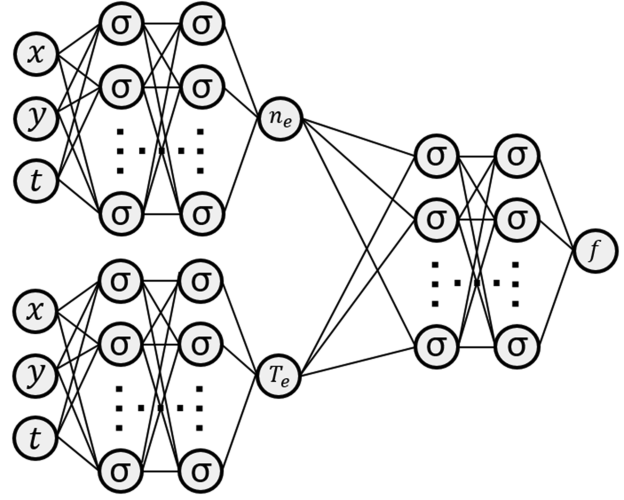


FIG. 8. Structure of networks to represent $f(n_e, T_e) = n_e \text{PEC}^{\text{exc}}(n_e, T_e)$, which is one of the terms composing the total emissivity function, $I = C n_0 f(n_e, T_e)$. The ionization rate per neutral, $n_e S_{\text{CR}}(n_e, T_e)$, is similarly represented when applied in the transport equation and is important to account for the “shadowing” of neutrals.^{5,32,43} The left side of the overall network consists of the networks for the predicted n_e and T_e , while the right side output represents the photon emissivity per neutral given by the $N_p = 1$ CR theory.

learning of CR coefficients by their respective networks takes place over the first 5 of 20 total hours of training on 32 cores with Intel Haswell-EP.

Next, we train the n_e , T_e , and C networks against Eq. (13) such that the predicted brightness intensity consistent with CR theory matches experimental measurements from the fast camera. Based on past high-resolution MLP data, turbulent fluctuations propagating in the edge were observed to exhibit strong correlations between the electron density and electron temperature.⁴⁴ Additional constraints are thus placed in our optimizer such that solutions where n_e and T_e are correlated are favored. This helps to avoid the learning of trivial solutions. Namely, the full loss function being collectively trained upon in this second stage is

$$\mathcal{L}^{C, n_e, T_e} = \frac{1}{N_0} \sum_{i=1}^{N_0} (\mathcal{L}_{\text{GPI}} + C_1 \mathcal{L}_{\text{corr}} + C_2 \mathcal{L}_{\text{relcorr}}), \quad (14)$$

where

$$\mathcal{L}_{\text{GPI}} = |I^*(x_0^i, y_0^i, t_0^i) - I_0|^2, \quad (15)$$

$$\mathcal{L}_{\text{corr}} = -[n_e^*(x_0^i, y_0^i, t_0^i) - \langle n_e^*(x_0^i, y_0^i, t_0^i) \rangle] \times [T_e^*(x_0^i, y_0^i, t_0^i) - \langle T_e^*(x_0^i, y_0^i, t_0^i) \rangle], \quad (16)$$

$$\mathcal{L}_{\text{relcorr}} = \frac{|\langle n_e^*(x_0^i, y_0^i, t_0^i) \rangle - \langle n_e^*(x_0^i, y_0^i, t_0^i) \rangle|^2}{|\langle T_e^*(x_0^i, y_0^i, t_0^i) \rangle - \langle T_e^*(x_0^i, y_0^i, t_0^i) \rangle|^2} + \frac{|\langle T_e^*(x_0^i, y_0^i, t_0^i) \rangle - \langle T_e^*(x_0^i, y_0^i, t_0^i) \rangle|^2}{|\langle n_e^*(x_0^i, y_0^i, t_0^i) \rangle - \langle n_e^*(x_0^i, y_0^i, t_0^i) \rangle|^2}, \quad (17)$$

with $I^*(x_0^i, y_0^i, t_0^i)$ following Eq. (13), and the points $\{x_0^i, y_0^i, t_0^i, I_0^i\}_{i=1}^{N_0}$ corresponding to the set of observed data from GPI. Here, we use

the notation that superscripts on \mathcal{L} identify the multiple networks being simultaneously trained during optimization of the respective loss function, e.g., $\mathcal{L}^{C, n_e, T_e}$ indicates that the networks for C , n_e , and T_e are being jointly optimized against this particular loss function. We note that the results from the converged solutions reported in Sec. V are largely unchanged by removing (17) in the optimization framework, although keeping it was found to enhance stability and, thus, the total number of realizations that converge. Better physics-informed optimization constraints may exist and should be investigated going forward to advance this turbulence analysis. While the coefficients C_1 and C_2 in Eq. (14) can be adaptively adjusted for optimal training in each iteration, they are set to constants of 1000 and 1, respectively, in this framework. The variables with asterisks symbolize predictions by their respective networks at the spatiotemporal points being evaluated during training. The notation $\langle X \rangle$ denotes the batch sample mean of X . This second training stage lasts for 100 min.

The next stage involves optimizing the n_0 network against both Eq. (15) and its transport equation that accounts for neutral drifts and fluctuation-induced ionization of helium. Namely, in implicit form,

$$f_{n_0} = \frac{\partial n_0}{\partial t} + \frac{\partial(n_0 v_x)}{\partial x} + \frac{\partial(n_0 v_y)}{\partial y} + n_0 n_e S_{CR}, \quad (18)$$

where, based on three-dimensional Monte Carlo neutral transport simulations of this region, closures of $v_x \sim -900$ m/s and $v_y \sim -20$ m/s are applied for modeling HeI as it exits the capillaries into the GPI frame of view.⁴⁵ This approximation of a drifting, cut-off Maxwellian for HeI may be reasonable for a narrow radial region, but the true velocity distribution characterizing helium gas particles becomes increasingly skewed the farther one goes away from the gas nozzles. Modeling other atomic and molecular species (e.g., deuterium) in this way may be inadequate as charge-exchange and recombination effects on trajectories are increasingly important. In addition, neutral-neutral collisions and their impacts on velocity closures are presently neglected in this treatment. This allows for a scaling constant to be factored out of Eq. (18), i.e., permitted by its linearity in n_0 . If using a sufficiently high spectral resolution spectrometer to view the emission cloud, the Doppler shift can be experimentally measured. We leave this task for further exploring momentum transport physics and potentially even learning the velocity closure directly from the GPI data within our optimization framework for future work.

The null formulation following Eq. (18) is vital for training since all physical terms collectively sum to zero when the unknown dynamical variables in the equation are correctly solved to self-consistently account for neutral propagation and ionization. The physical theory is computationally expressed by differentiating the n_0 neural network with respect to its input spatiotemporal coordinates via the application of the chain rule through automatic differentiation.⁴⁶ By then multiplying and adding the graph outputs to construct representations of the physical constraints, the network for n_0 can be trained against (14) and (18) to satisfy the physical theory constraining the nonlinear connection between networks. This accounting of Eq. (18) is particularly essential since the Kubo number, which quantifies the strength of turbulent perturbations on neutral transport, is large ($\gtrsim 1$) for helium.^{45,47,48} There are no

explicit boundary conditions applied for n_0 , but instead, we train the network against the fast camera's experimentally measured intensities to learn how n_0 should be treated around the boundaries of the analyzed camera image. Namely, the loss function in this third following stage is given by

$$\mathcal{L}^{n_0} = \frac{1}{N_0} \sum_{i=1}^{N_0} \mathcal{L}_{GPI} + \frac{C_{f_{n_0}}}{N_f} \sum_{j=1}^{N_f} \mathcal{L}_{f_{n_0}} \quad (19)$$

with

$$\mathcal{L}_{f_{n_0}} = |f_{n_0}^*(x_f^j, y_f^j, t_f^j)|^2, \quad (20)$$

where $\{x_f^j, y_f^j, t_f^j\}_{j=1}^{N_f}$ denote the set of collocation points that can span any arbitrary domain but taken to be equivalent to the ones encompassed by $\{x_0^i, y_0^i, t_0^i, I_0^i\}_{i=1}^{N_0}$, and $f_{n_0}^*$ is the null partial differential equation prescribed by Eq. (18) in normalized form directly evaluated by the neural networks. For the remainder of the training time, Eqs. (14) and (19) are sequentially trained in repeating intervals of 100 min to iteratively find convergence in their respective networks. The only difference in later stages is that C is no longer a free parameter while training against Eq. (14), and $C_{f_{n_0}}$ in Eq. (19) is increased from 10^2 to 10^6 to improve the focused learning of neutral transport physics. If $C_{f_{n_0}}$ is increased any higher, we risk finding trivial solutions at a higher occurrence. Generalizing our training framework to adaptively update training coefficients⁴⁹ is an important pathway for future investigation. All loss functions are optimized with mini-batch sampling where $N_0 = N_f = 1000$ using the L-BFGS (limited-memory Broyden-Fletcher-Goldfarb-Shanno) algorithm—a quasi-Newton optimization algorithm.⁵⁰ In addition, points found to have difficulty converging (e.g., optimizer becomes stuck in local minima) were removed from training in subsequent stages to improve learning of turbulent fluctuations in the remaining regions of the spatiotemporal domain analyzed. In the end, this results in the multi-network framework training only on 8 (radial) \times 38 (vertical) pixels over 39 frames imaged by the fast camera. We note that by embedding $f(n_e, T_e)$ in Fig. 8, the emissivity predictions by the networks are forced to satisfy CR theory. Similarly, the ionization rate per neutral, $n_e S_{CR}$, is encoded in Eq. (18). We, thus, ensure that the unobserved n_e , T_e , and n_0 being learned are in agreement with the experimentally measured brightness while trying to satisfy the neutral transport physics for HeI that self-consistently includes time-dependent ionization in the presence of plasma turbulence. The repeated differentiation and summation of networks to represent every term in the ascribed loss functions resultantly constructs a far deeper computation graph representing the collective constraints beyond the eight hidden layers in each dynamical variable's network. The cumulative graph is therefore a truly deep approximation of the physics governing the observed 587.6 nm line emission.

Due to the stochastic nature of the initialization and multi-task training, learned solutions for n_e , T_e , n_0 , and C vary each time an individual optimization is run. This may arise due to a unique solution not necessarily existing given the above optimization constraints. Therefore, we run an ensemble of realizations and consider this collection of runs that roughly follow Gaussian statistics. Based on past testing within our optimization framework, the necessary criteria for convergence in normalized units are set to $\mathcal{L}_{GPI} < 10^{2.5}$,

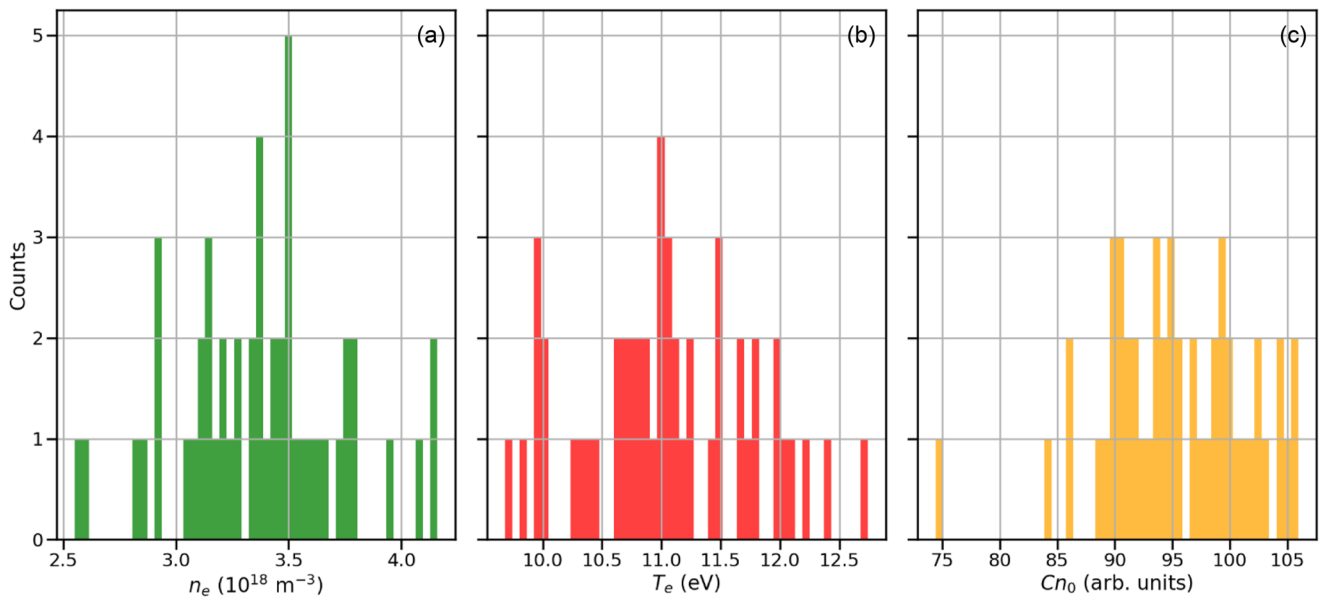


FIG. 9. Histogram of the n_e (a), T_e (b), and Cn_0 (c) fluctuations at $[R = 90.7 \text{ cm}, Z = -4.2 \text{ cm}, t = 1.312 850 \text{ s}]$, i.e., with corresponding normalized poloidal magnetic flux coordinate $\psi_n = 1.07$, based on 50 converged realizations when training against experimental GPI data from plasma discharge 1 120 711 021 in the optimization framework. The ensemble mean and standard deviation of these realizations are used to construct the results presented in Sec. V.

$\mathcal{L}_{corr} < -10^3$, and $\mathcal{L}_{f_{n_0}} < 10^{-3}$. Checks for spurious gradients, trivial solutions, and a low number of training iterations were additionally investigated for downselecting converged realizations. For analysis of C-Mod discharge 1 120 711 021, there were 800 runs with 50 sufficiently converging within our present analysis. The scatter in learned

turbulent fluctuations among these realizations is used to quantify uncertainty intervals associated with the optimization framework, and as an example, the distribution of inferred measurements at a particular spatial and temporal point is plotted in Fig. 9. It is also important to note that the loss functions never truly go to zero either

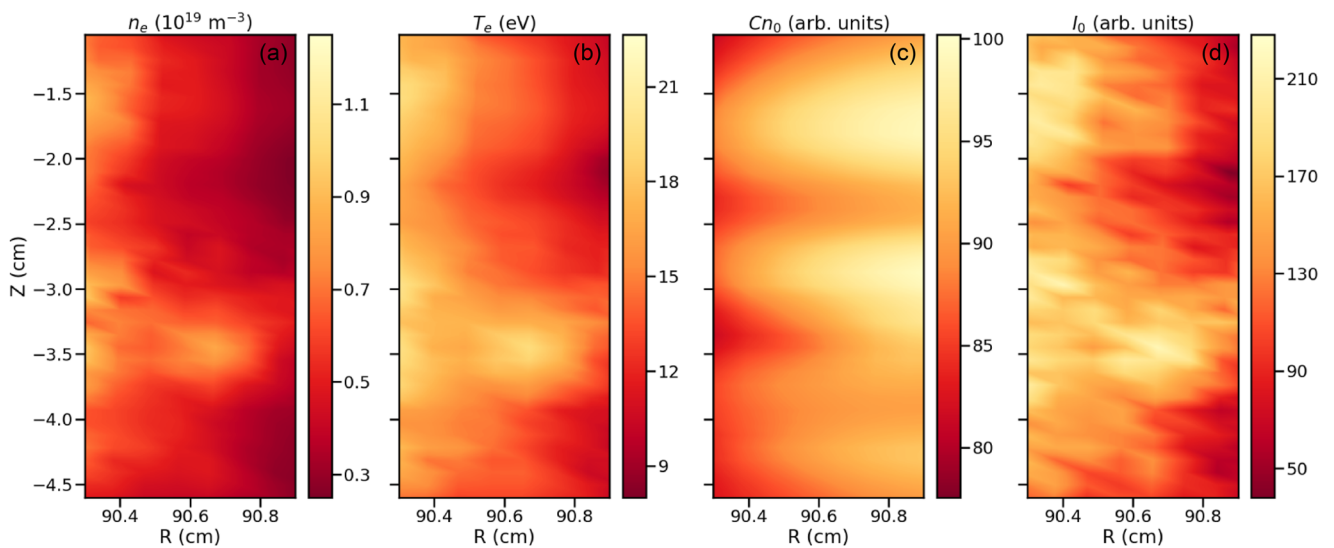


FIG. 10. The learned two-dimensional n_e (a), T_e (b), and Cn_0 (c) for plasma discharge 1 120 711 021 along with the experimentally observed 587.6 nm photon emission (d) at $t = 1.312 815 \text{ s}$. The learned measurements are based on the collective predictions within the deep learning framework training against the neutral transport physics and $N_p = 1$ CR theory constraints. Multimedia view: <https://doi.org/10.1063/5.0088216.1>.

and act to quantify potential discrepancies involved in modeling the physical system with deep networks, e.g., $\mathcal{L}_{f_{n_0}}$ can be understood as the outstanding error in approximating the neutral transport theory. Of these converged runs, the normalized mean loss functions at the end of training for the collection of realizations were found to be $\mathcal{L}_{GPI} = (1.44 \pm 0.42) \times 10^2$, $\mathcal{L}_{corr} = (-4.51 \pm 0.20) \times 10^3$, $\mathcal{L}_{relcorr} = 6.75 \pm 0.03$, and $\mathcal{L}_{f_{n_0}} = (3.57 \pm 3.79) \times 10^{-5}$. Values of the loss metrics remaining finite signify departures from exactly satisfying the training conditions being learnt. Identifying these errors allows for their iterative improvement while identifying even better loss functions for generalized training is left for future work.

V. UNCOVERING DYNAMICS IN EXPERIMENTAL TURBULENCE IMAGING IN ALCATOR C-MOD

The learned turbulent n_e , T_e , and Cn_0 from our time-dependent analysis of fast camera imaging for plasma discharge 1120711021 using an ensemble of 50 optimizers are visualized in two-dimensional space along with experimentally observed GPI measurements in Fig. 10 (Multimedia view). A correlation matrix for the normalized relative fluctuations in two-dimensional space over the roughly 100 μs time window analyzed is displayed in Table I.

The positive fluctuations in brightness are largely correlated with n_e and T_e , and these regions tend to have depressed values of n_0 as the ionization rate is elevated. This results in a “shadowing effect” in atomic helium trajectories arising from fluctuation-induced ionization. The autocorrelation time of n_0 also decreases with radius, while it increases for n_e and T_e . Temporal variation with radius is visualized in Fig. 11 where, considering a one-dimensional slice of Fig. 10 (Multimedia view), the same physical quantities are plotted at $Z = -4.0$ cm. While correlations vary poloidally and radially, and precise dependencies across the turbulent variables change as n_e and

TABLE I. A correlation matrix of the turbulent measurements inferred and observed experimentally in plasma discharge 1120711021. For reference, I^* is the predicted emissivity given by Eq. (13), and I_0 is the experimentally observed brightness of the 587.6 nm line. Each quantity’s normalized fluctuation amplitude, i.e., $\{X\} = (X - \langle X \rangle) / \langle X \rangle$, is based on measurements over $90.3 < R$ (cm) < 90.9 , $-4.6 < Z$ (cm) < -1.0 , and $1.312799 < t_{GPI}$ (s) < 1.312896 .

	$\{n_e\}$	$\{T_e\}$	$\{n_0\}$	$\{I^*\}$	$\{I_0\}$
$\{n_e\}$	1.000	0.887	-0.325	0.843	0.822
$\{T_e\}$	0.887	1.000	-0.307	0.925	0.902
$\{n_0\}$	-0.325	-0.307	1.000	-0.051	-0.059
$\{I^*\}$	0.843	0.925	-0.051	1.000	0.971
$\{I_0\}$	0.822	0.902	-0.059	0.971	1.000

T_e increase, the observed line emission is found to be strongly correlated with electron pressure. The atomic helium density fluctuations do not vary directly proportional to I_0 in this far edge region on open field lines near the gas tubes. There is instead a weak negative correlation over this narrow radial extent arising from the largest brightness fluctuations corresponding to trajectories with elevated ionization rates causing a depletion, or shadowing, of HeI. The maximal n_0 fluctuation amplitudes tend to be roughly 30%–40% from peak-to-trough in this far edge region that sits away from the LCFS, where sharper equilibrium gradients and smaller relative fluctuation levels may result in different correlations. While relative fluctuations may be correlated from $90.3 < R$ (cm) < 90.9 as in Table I, connections between the turbulent quantities are nonlinear. To better visualize their interdependence, Fig. 12 displays histograms for n_e , T_e , and Cn_0 vertically along $R = 90.3$ cm (with ψ_n in the range 1.035–1.053). The fluctuations follow different statistical distributions and cannot necessarily be linearly mapped from the noisy HeI line intensity measured by the fast camera.

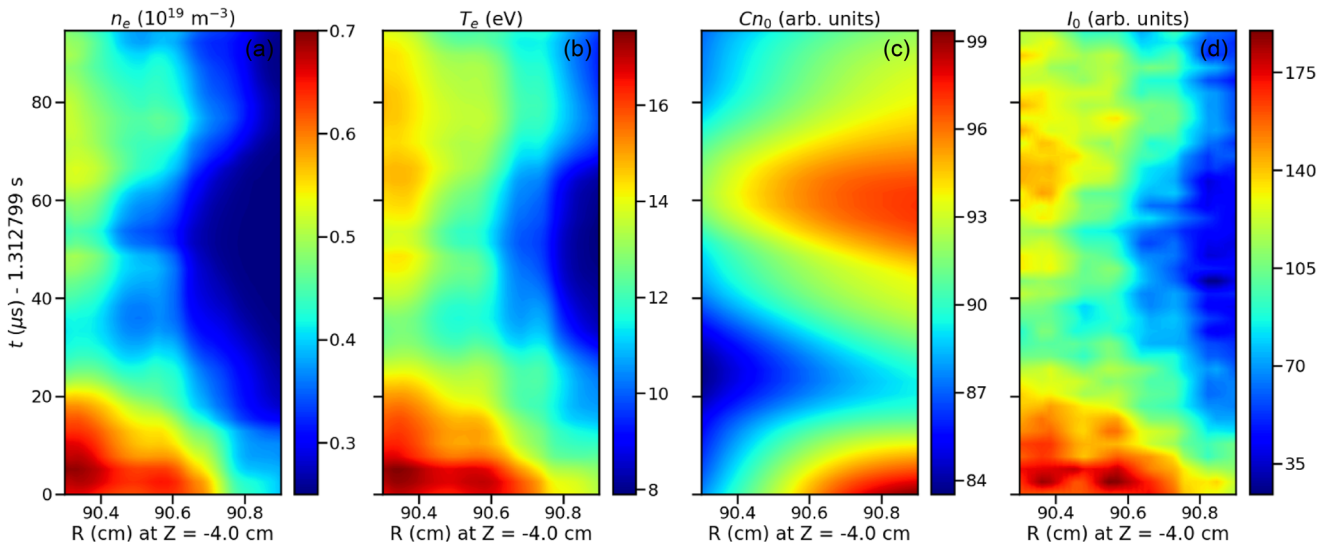


FIG. 11. The learned n_e (a), T_e (b), and Cn_0 (c) along with the experimentally observed 587.6 nm photon brightness (d) for plasma discharge 1120711021 at $Z = -4.0$ cm. These quantities are plotted as a function of radius and time.

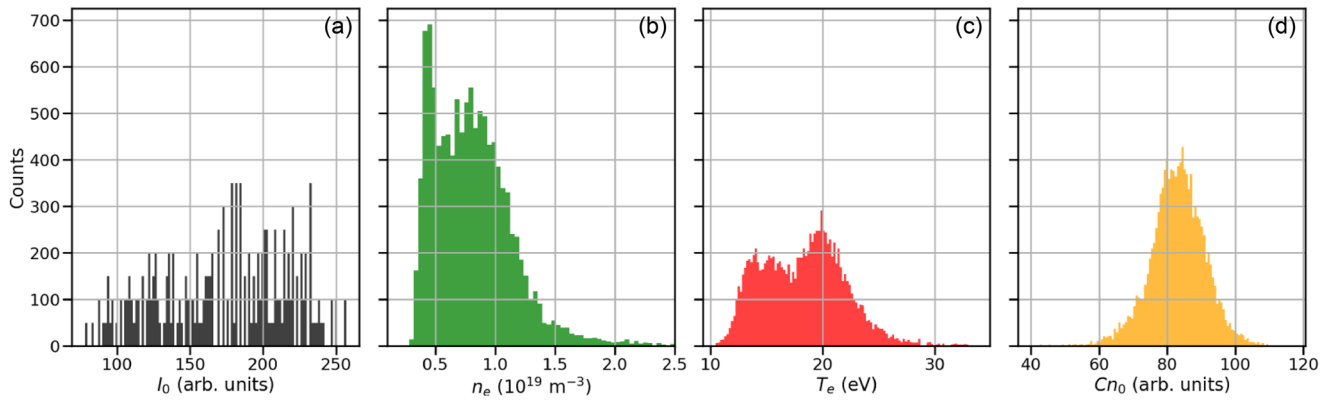


FIG. 12. Histograms displaying the distribution of the experimentally observed 587.6 nm line intensity (a) along with turbulent n_e (b), T_e (c), and Cn_0 (d) at $[R = 90.3 \text{ cm}, -4.6 < Z \text{ (cm)} < -1.0, 1.312799 < t_{\text{GPI}} \text{ (s)} < 1.312896]$.

We note that these learned n_e , T_e , and Cn_0 are consistent solutions with the collisional radiative and optimization constraints being trained upon, but not necessarily unique solutions. Accordingly, in Fig. 13, we display the predicted light emission from the ensemble of realizations against the fast camera's measurements. We also plot the mean outputs and uncertainty intervals for the turbulent n_e , T_e , and Cn_0 associated with the scatter of running an ensemble of our stochastic realizations. There is no temporal averaging of the profiles in Fig. 13. For GPI on Alcator C-Mod, sharp features exist in the experimental data

potentially associated with noise, while the learned line intensity from the collection of networks is smoother and consistent in both magnitude and shape with the observed brightness. These measurements enable novel research pathways into the two-dimensional tracking of experimental parameters (e.g., turbulent particle and heat fluxes both radially and poloidally) and calculation of fluctuating fields for turbulence model validation.^{8,9} They further provide the first quantitative estimates of a two-dimensional turbulent structure for neutrals in an experimental fusion plasma.

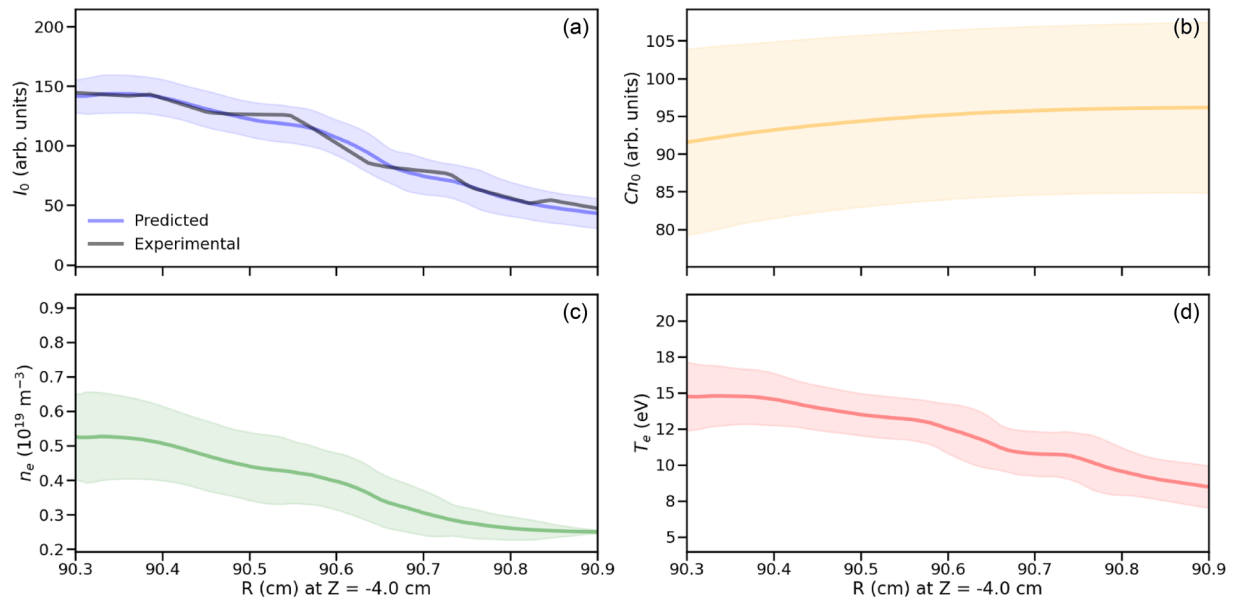


FIG. 13. Radial profiles of the inferred turbulent Cn_0 (b), n_e (c), and T_e (d) at $[Z = -4.0 \text{ cm}, t = 1.312866 \text{ s}]$ along with a trace of the experimentally observed and predicted GPI intensity profiles. The computed line emission is based on the deep learning framework following Eq. (13). The dark line in each plot corresponds to the average output of the ensemble of realizations, while the shaded uncertainty intervals correspond to scatter ($\pm 2\sigma$) arising from the independently trained networks.

To further examine the validity of these results, we juxtapose turbulent n_e and T_e from our GPI measurements of the single spectral line against an independent MLP with four electrodes. This scanning probe is plunged at times overlapping with our gas puff analysis, i.e., $1.287631 < t_{MLP}$ (s) < 1.317671 vs $1.312799 < t_{GPI}$ (s) < 1.312896 , although located at different positions toroidally and vertically. The MLP measures fluctuations in time as it scans through the edge plasma, and we consider its resultant constructed radial profile. For comparison, we time-average our turbulent measurements at different Z -locations to compare the n_e and T_e fluctuations. While the measurement regions spatially spanned by the two independent diagnostics are magnetically disconnected, we map the

fluctuations to common poloidal magnetic flux coordinates based on magnetohydrodynamic equilibrium reconstruction in the tokamak plasma using the EFIT code.⁵¹ Deconstructing the GPI fluctuations into the turbulent n_e , T_e , and n_0 instead of the raw brightness from atomic emission largely resolves diagnostic misalignment challenges,⁵² and, in contrast with the past analysis of plasma discharge 1120711021,⁵³ there is no radial shift applied to align the turbulent fluctuation profiles from these two independent experimental diagnostics. Profiles on these open field lines fluctuate strongly with perturbations of up to 10%–100%. For turbulence measurements by GPI in this two-dimensional spatial domain spanning $\sim 100 \mu\text{s}$, peak n_e and T_e fluctuations do not far exceed $2.5 \times 10^{19} \text{ m}^{-3}$ and 25 eV, respectively, which are roughly consistent with the MLP measurements in the far SOL. When evaluating the two sets of measurements side-by-side in Fig. 14, we find excellent agreement in magnitude and structure between the T_e measurements. For the electron density channel, the MLP n_e data are slightly elevated on average although still quantitatively consistent within the measurement bounds of the four electrodes. A potential contributing factor to this observed difference in n_e peaks could be natural variations in the poloidal structure of turbulent fluctuations over the narrow time window. Temporal averaging over longer durations may help reduce the imprint of intermittent plasma transport when comparing radial profiles at different vertical positions. One should also remember that, beyond the diagnostics viewing different spatiotemporal locations, systematic uncertainties extant in both the GPI and probe measurements can cause discrepancies left to be reconciled.^{54,55} For example, the MLP is intrinsically perturbative to local edge plasma conditions and experimental analysis of the probe edge sheath assumes electrons can be described by a single Maxwellian velocity distribution.³⁸ Additionally, while our training paradigm attempts to find consistent solutions within the applied optimization framework, questions of uniqueness and generalized constraints are still being explored.

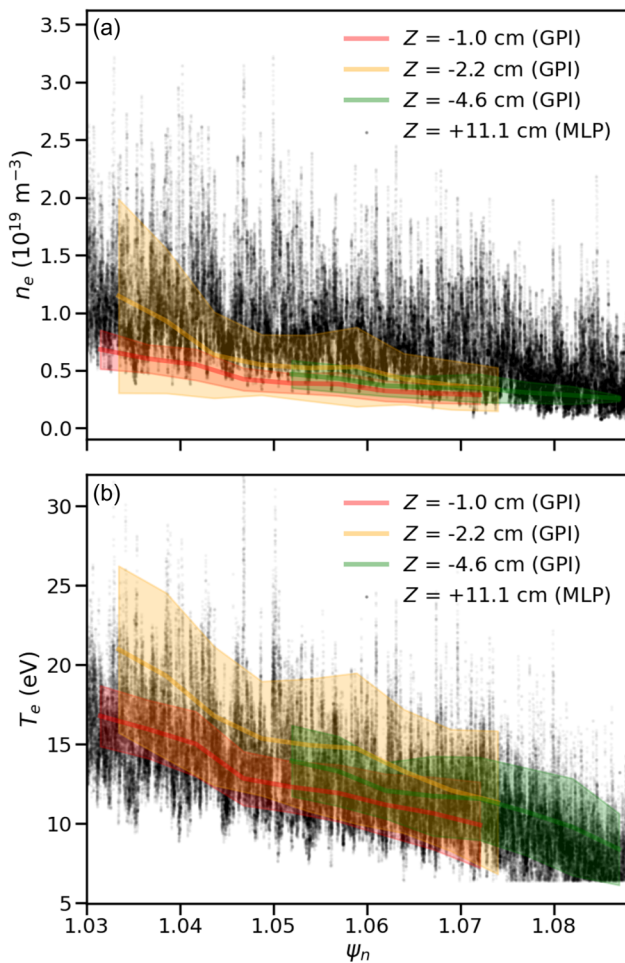


FIG. 14. For evaluation of radial profiles, we compute the time-averaged n_e (a) and T_e (b) over roughly $100 \mu\text{s}$ from the experimental GPI at different vertical locations. These quantities inferred from GPI are compared against independent scanning mirror Langmuir probe measurements from four electrodes that are not time-averaged. This MLP is located at $Z = +11.1$ cm with a radially moving probe head in the edge plasma. Shaded intervals on the time-averaged n_e and T_e inferred from GPI correspond to the temporal scatter ($\pm 2\sigma$) associated with the learned turbulent fluctuations in our optimization framework. All measurements are mapped to normalized poloidal magnetic flux coordinates, ψ_n .

VI. CONCLUSION

In summary, we have developed a novel time-dependent deep learning framework for uncovering the turbulent fluctuations of both the plasma quantities, n_e and T_e , as well as the neutrals underlying experimental imaging of HeI line radiation. Significantly, this allows the determination of two-dimensional fluctuation maps in the plasma boundary, revealing the detailed spatiotemporal structure and nonlinear dynamics. It thereby extends the usefulness of the gas puff imaging method. The computational technique intrinsically constrains solutions via collisional radiative theory and trains networks against neutral transport physics with constraints on correlations between n_e and T_e . This advancement has allowed for the first estimates of the two-dimensional turbulent n_e , T_e , and n_0 that reveal fluctuation-induced ionization effects in a fusion plasma based on optical imaging of just the 587.6 nm line. While our analysis is demonstrated on the edge of the Alcator C-Mod tokamak with quantitative agreement found with independent probe measurements, this technique is generalizable to ionized gases in a wide variety of conditions and geometries (e.g., stellarators, spheromaks, and magneto-inertial fusion).

A number of opportunities for future development exist. One key outstanding question is the identification of underlying numerical and physical factors contributing to non-uniqueness in outputs during optimization. From experimental noise to the chaotic properties of the turbulent system, finding sufficient conditions for precise convergence is the focus of ongoing research. Future extensions of the framework also include expanding the radial domain of coverage toward closed flux surfaces, which will require widening the queried bounds on n_e , T_e , n_0 , and improving the overall training paradigm via adaptive training and architecture structures.⁴⁹ For example, neutral density amplitudes can vary over orders of magnitude with steep shapes in background equilibrium profiles. Tactfully embedding this information during the training of the networks can aid with the overall physical modeling via optimization. In this way, better experimental constraints from one-dimensional data may help uncover further dynamics not otherwise directly probed.

Adaptation to other experiments is a logical next step, and translating our present technique to contemporary experimental devices using helium beams is a pathway that can be explored immediately for regions that are traditionally difficult to probe (e.g., X-point regions). Alternatively, our deep learning framework does not apply any discretization of the spatiotemporal domain—networks provide natural continuum representations for dynamical variables—and can thus be extended, in principle, to three-dimensional geometries as well to account for integrated light emission along the camera's lines-of-sight if using wide gas distributions as expected, for example, when running He plasmas on ITER.⁵⁶ Furthermore, this global turbulence imaging technique provides new ways to diagnose high pressure plasma events, e.g., disruptive instabilities, such as edge localized modes, that can be destructive to plasma facing components. Translating our framework for direct analysis of deuterium instead of helium is also possible with a few modifications, but requires investigation of relevant CR physics²² where charge exchange and molecular effects are no longer necessarily negligible.²⁹ One prospect is to couple the turbulent n_e and T_e learned within our framework with Monte Carlo neutral transport codes,⁵⁷ potentially allowing recovery of two-dimensional time-dependent estimates of atomic and molecular deuterium density and its emissivity, e.g., through the ultraviolet Ly α line. These could be compared directly to experimental measurements of line emission from deuterium.^{58,59} Such extended comparisons will be important in the testing of reduced edge plasma turbulence models.⁸

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

A. Mathews: Conceptualization (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **J. L. Terry:** Data curation (lead); Project administration (supporting); Supervision (supporting); Writing – review & editing (supporting). **S. G. Baek:** Formal analysis (supporting); Methodology (supporting); Writing – review & editing (supporting). **J. W. Hughes:** Project administration (supporting); Supervision (lead); Writing – review & editing (supporting). **A. Q. Kuang:** Data curation (supporting); Writing – review & editing (supporting). **B. LaBombard:** Data curation (supporting). **M. A. Miller:** Data curation (supporting). **D. Stotler:** Formal analysis (supporting); Methodology (supporting). **D. Reiter:** Formal analysis (supporting); Methodology (supporting). **W. Zhlobenko:** Formal analysis (supporting); Methodology (supporting). **M. Goto:** Formal analysis (equal); Methodology (equal)

All authors read and approved the final manuscript.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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