SHIPPING STRATEGIES IN MULTIMODAL NETWORKS EXHIBITING ECONOMIES OF SCALES

by

·ANA BERGER

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McGill University, Montreal
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Signature of Author__
Operations Research
Interdepartmental Program
May 13, 1988

Certified by____
Professor Harilaos Koutsopoulos
Assistant Professor of Civil Engineering
Thesis Supervisor

Certified by____
Professor Yosef Sheffi
Professor of Civil Engineering
Thesis Supervisor

Accepted by____
Professor Amedeo R. Odoni
Professor of Aeronautics and Astronautics
and of Civil Engineering
Co-director of Operations Research Center

Accepted by____
Professor Ole S. Madsen
Chairman, Departmental Committee
on Graduate Students
Department of Civil Engineering

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Ilana Berger

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ABSTRACT

The problem of a shipper having to ship freight between a set of origins and destinations and wanting to do so in the most economical manner is studied. Economy for the shipper is a function of transportation cost, in-transit inventory cost, and inventory carrying costs incurred in shipping. The shipper can devise a freight transportation strategy to minimize these costs. A freight transportation strategy is a plan detailing exactly how freight is transported. The plan prescribes whether freight should be routed directly or consolidated with other freight via transportation terminals. Additionally, the plan determines the transportation modes and frequencies of service to be employed on each route segment.

The Shipper's Problem involves two interdependent decisions. There is the problem of finding the optimal shipping policies for the segments of a given routing strategy, such that the minimum total logistics cost of the strategy can be evaluated. Heuristics for identifying these policies are developed and compared. Given the ability to evaluate the minimum total logistics cost of a given routing strategy, the second problem involves identifying the overall minimum cost strategy. This problem is modeled as a multi-commodity network flow problem with concave costs. Algorithms for identifying the minimum total logistics cost freight transportation strategy, addressing both problems simultaneously, are developed and evaluated.

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Dr. Yosef Sheffi

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1. Introduction

The recent deregulation of airlines, motor carriers, and railroads in the United States has shifted the balance of power in the transportation market. Shippers are now exerting more control over the transportation of their supplies and products. Prior to deregulation, little distinction could be drawn between carriers because they all charged the same price and offered similar service. Deregulation has given shippers new freedom to negotiate for transportation services tailored to meet their needs.

A shipper having to ship freight between a set of freight origins and destinations would like to do so in the most economical manner. Economy for the shipper is a function of transportation cost, inventory and handling costs, and level of service costs such as safety stock costs, incurred in shipping. The shipper can devise a shipping strategy to minimize these costs, thereby attaining economy. A shipping strategy is a plan detailing exactly how freight will be transported. The plan prescribes the routes and transportation modes to be employed for various freight shipments, as well as the frequency of service on these routes.

More formally, a shipper's objective is the determination of the freight transportation strategy which minimizes its total logistics cost. A freight transportation strategy is composed of two different levels of shipping decisions. The higher level decisions involve the selection, for freight bound between each origin and destination pair, of a manner in which to ship the freight. The manner is defined by the mode and carrier combination chosen to transport freight via a given route. Possible routes for freight could include transporting it directly between its origin and destination, or consolidating it with other freight travelling between points in the same general vicinity. Higher level shipping decisions will be referred to as routing decisions for freight flows. Lower level shipping decisions involve determining the shipment size and frequency to be employed for a given freight routing. Lower level decisions define the shipping policy on a given route. Decisions are at a higher or lower level in the sense that mathematically this is a two level optimization problem. The optimal shipping policies for different freight routings must be determined such that the optimal freight transportation strategy can be identified.

This thesis addresses the problem of identifying a shipper's minimum total logistics cost freight transportation strategy for shipping freight between sets of origins and destinations. The routing alternatives that will be considered towards this end will include the shipping of freight flows directly between their origins and destination, or their consolidation for shipment via a specific transportation mode having one terminal central to the set of origins and another central to the set of destinations of the freight. Consolidated routing of freight flows is considered such that where economies of scale in shipping exist, they will be exploited in the effort to reduce the shipper's total logistics cost. This problem will be referred to as the Shipper's Problem.

A complete description of the Shipper's Problem is presented in chapter 2 of this thesis. This chapter also characterizes the problem as a multi-commodity network flow problem with arc costs that are concave functions of the volume of freight transported on them. Finally, the chapter reviews the literature summarizing efforts to solve concave network problems.

The subject of chapter 3 of this thesis is the modeling of total logistics cost for the Shipper's Problem. The objective of determining the minimum total logistics cost freight transportation strategy entails finding the minimum total logistics cost freight routing/shipping policy combination. The expression for the total logistics cost of a freight transportation strategy is derived, and heuristics for determining near-optimal shipping policies to employ for each of its route segments are developed. This will be referred to as objective function evaluation.

In chapter 4, algorithms which use one of the heuristic schemes developed in chapter 3 for the evaluation of the objective function, are developed for finding solutions to the Shipper's Problem. The algorithms include a branch-and-bound approach, devised to solve the problem such that the performance of two heuristics, one designed specifically for the Shipper's Problem can be evaluated.

Chapter 5 concludes this thesis and suggests directions for further research related to the Shipper's Problem.

2. The Shipper's Problem

As described in chapter 1, the problem being addressed is that of a shipper having to determine the most economical freight transportation strategy for shipping freight between sets of origins and destinations. The shipper has available the alternatives of routing individual freight flows directly between their respective origins and destinations, and routing them via transportation terminals such that they are consolidated. Consolidation of freight flows at terminals centrally located with respect to the origins and destinations is considered such that economies of scale in shipment can be exploited where they exist. The minimum cost routing for freight being shipped between a particular origin-destination pair is a function of the transportation services available, as well as of the volumes of freight being shipped on the route segments comprising the direct and consolidated routes.

In section 2.1 of this chapter, the Shipper's Problem is cast as a multi-commodity network flow problem with arc costs that are concave functions of the freight transported on them. Section 2.2 will review some approaches found in the literature for solving concave network problems.

2.1 A Multi-Commodity Network Flow Problem With Concave Arc Costs

The problem that has been described to motivate the discussion thus far has involved developing a freight transportation strategy for a single shipper. The Shipper's Problem can actually represent the shipping problems of multiple shippers collaborating in determining good freight transportation strategies for shipping between their respective origins and destinations. As such, it is important to preserve the identity of the freight flows throughout any solution procedures for this problem. The Shipper's Problem is thus a multi-commodity problem, with freight flows distinguished by their origin and destination modeled as different commodities.

From the description of the Shipper's Problem, it is clear that it can be modeled as a network problem consisting of a set of origin nodes and a set of destination nodes located on either side of a pair of transshipment nodes. There is a one to one correspondence between the origins and destinations for freight in the Shipper's Problem and the origin and destination nodes in the network problem. The transportation terminals of the Shipper's Problem correspond to the transshipment nodes of the network problem. There is a positive constant rate of supply of a commodity at its origin, which is assumed equal to the positive constant rate of demand for the commodity at its destination. There is no supply or demand for the commodity at the transshipment nodes.

The existence of arcs in the network analogue of the Shipper's Problem, represents the availability of transportation services between nodes. Since it is almost always possible to lease a truck to move freight between two locations, this analysis will assume that the arcs comprising both direct and consolidated routes for freight exist in the network. On the other hand, where more than one kind of transportation service is offered between a pair of nodes, only the dominating minimum cost alternative is included in the network, such that exactly one arc connects a pair of nodes. The dominating arc for a direct routing between an origin and destination is easily determined, as will be shown in chapter 3. Identifying the dominating arcs for route segments of consolidating routings is a much harder problem but will be shown to be possible as well in chapter 3.

Arcs connect the origins to the destinations of the network such that direct routing is a feasible alternative for every freight flow (figure 2.1). Arcs connect origins to the first transshipment node, the two transshipment nodes to each other, and the second transshipment node to the destinations in the network (figure 2.2). Where the transshipment nodes act as consolidating and deconsolidating transportation terminals respectively, these arcs ensure that consolidated routing is a feasible alternative for every commodity. Henceforth, the transportation terminal to which the origins are connected will be referred to as the freight consolidation center, while the transportation terminal to which the destinations are connected nodes will be referred to as the freight deconsolidation center. All of the arcs in the network are uncapacitated since, although a vehicle has a fixed capacity, there is no limit on the frequency of service between nodes.

The Shipper's problem has thus far been characterized as a multicommodity network flow problem. Substantial literature is available

Figure 2.1: The Network of Direct Routes for Freight

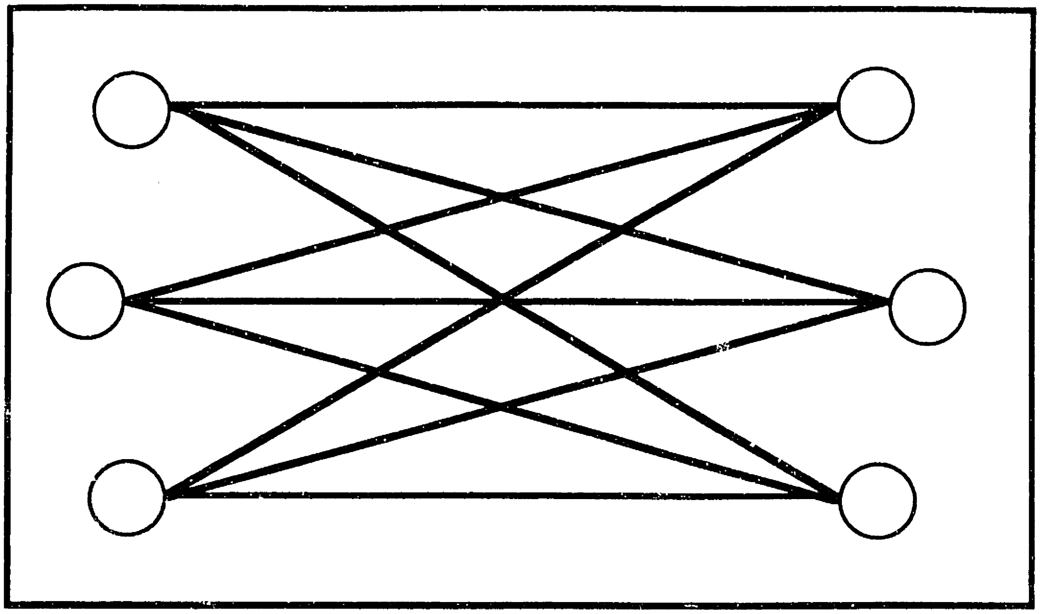
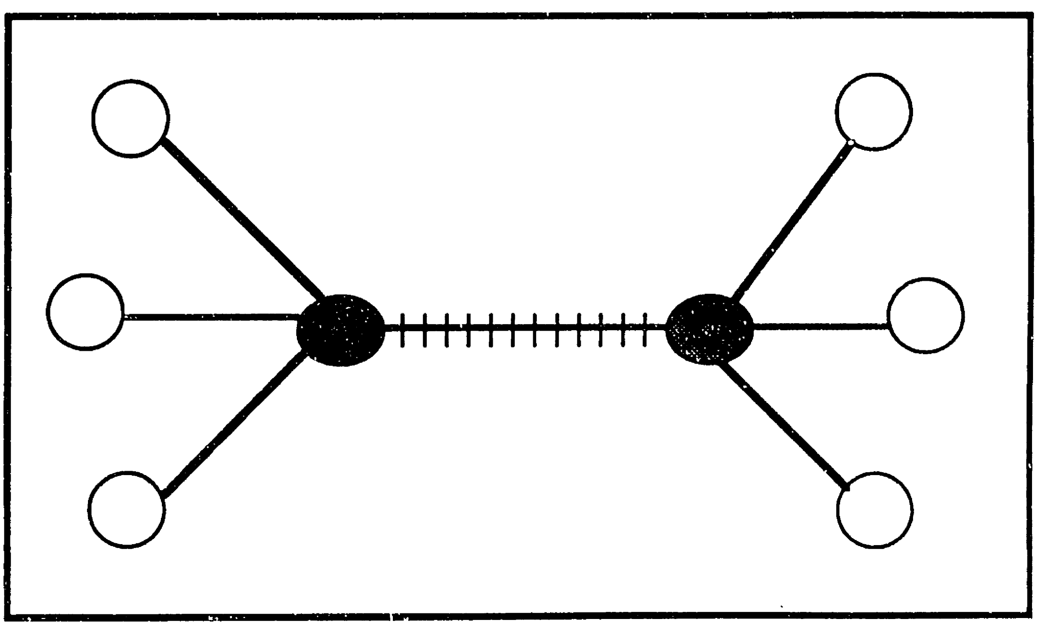


Figure 2.2: The Network of Consolidated Routes for Freight



on solving network problems with linear objective functions. Magnanti [MAGN84] reviews the literature which includes research on the "travelling salesman" problem and research on vehicle routing. The majority of this work focuses on designing shortest path algorithms. Network optimization manifests itself as minimization of transportation cost and in-transit inventory costs, both linear functions of travel distance. This literature typically neglects inventory costs related to shipment size because including them would necessitate solving network problems with arc costs that are concave functions of the volume of freight flowing on them. Concave network problems are difficult nonlinear programming problems. The key difficulty in solving them is that such problems have many locally optimal solutions, the number of which grow exponentially in the size of the problem.

The Shipper's Problem involves evaluating the relative attractiveness of freight transportation strategies based on total logistics cost - transportation cost, in-transit inventory cost and inventory carrying cost. The last component of this cost is an inventory charge related to shipment size. The arc costs of the network corresponding to the Shipper's Problem are thus concave functions of the volume of freight flowing on them, for reasons that will be made explicit in chapter 3 (figure 2.3). This multi-commodity network flow problem is then a concave network problem having multiple local optima. This is demonstrated in figure 2.4, where the costs of different transportation strategies or network solutions to a three origin - two destina-

Figure 2.3: Total Logistics Cost of Shipping on an Arc

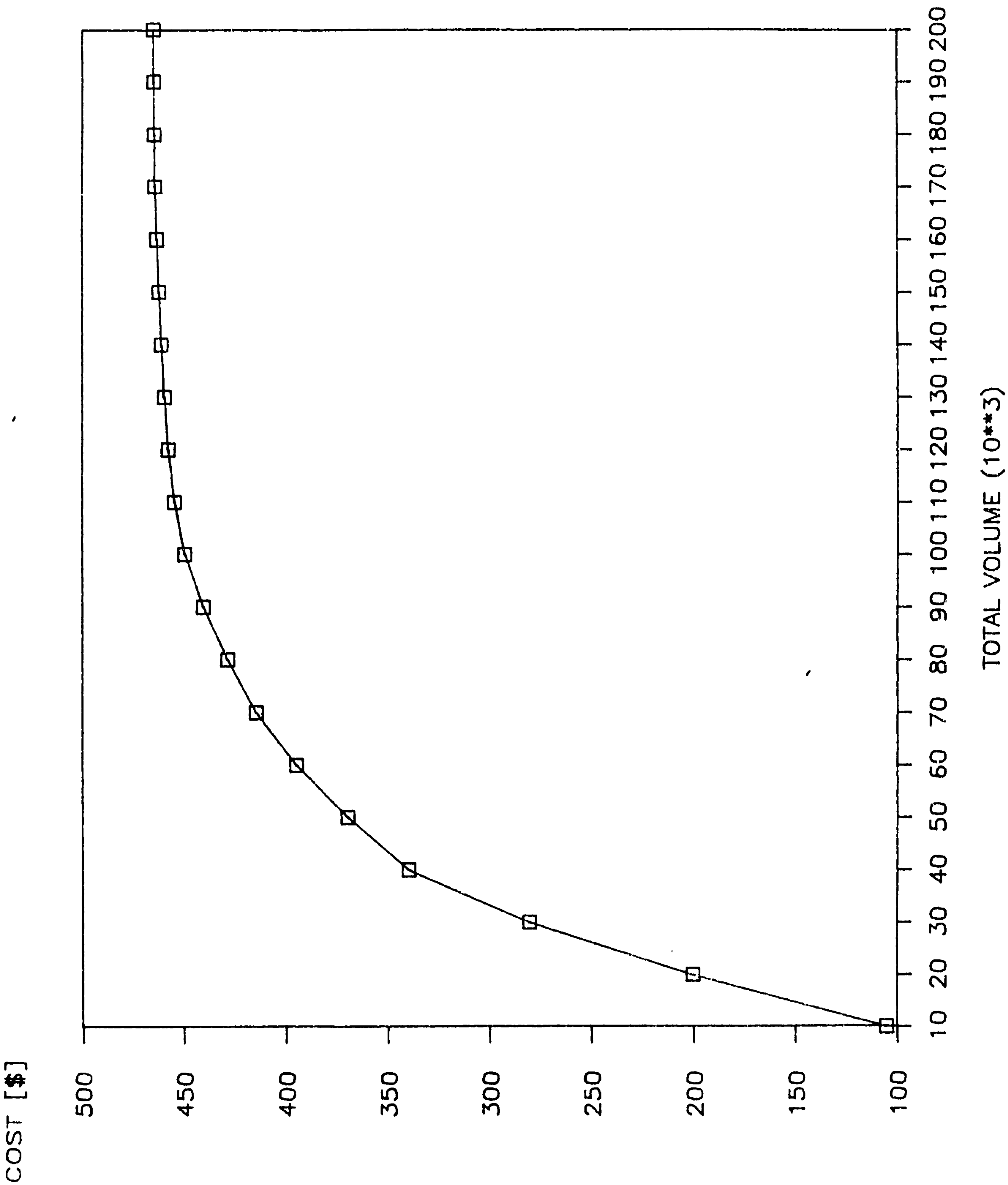
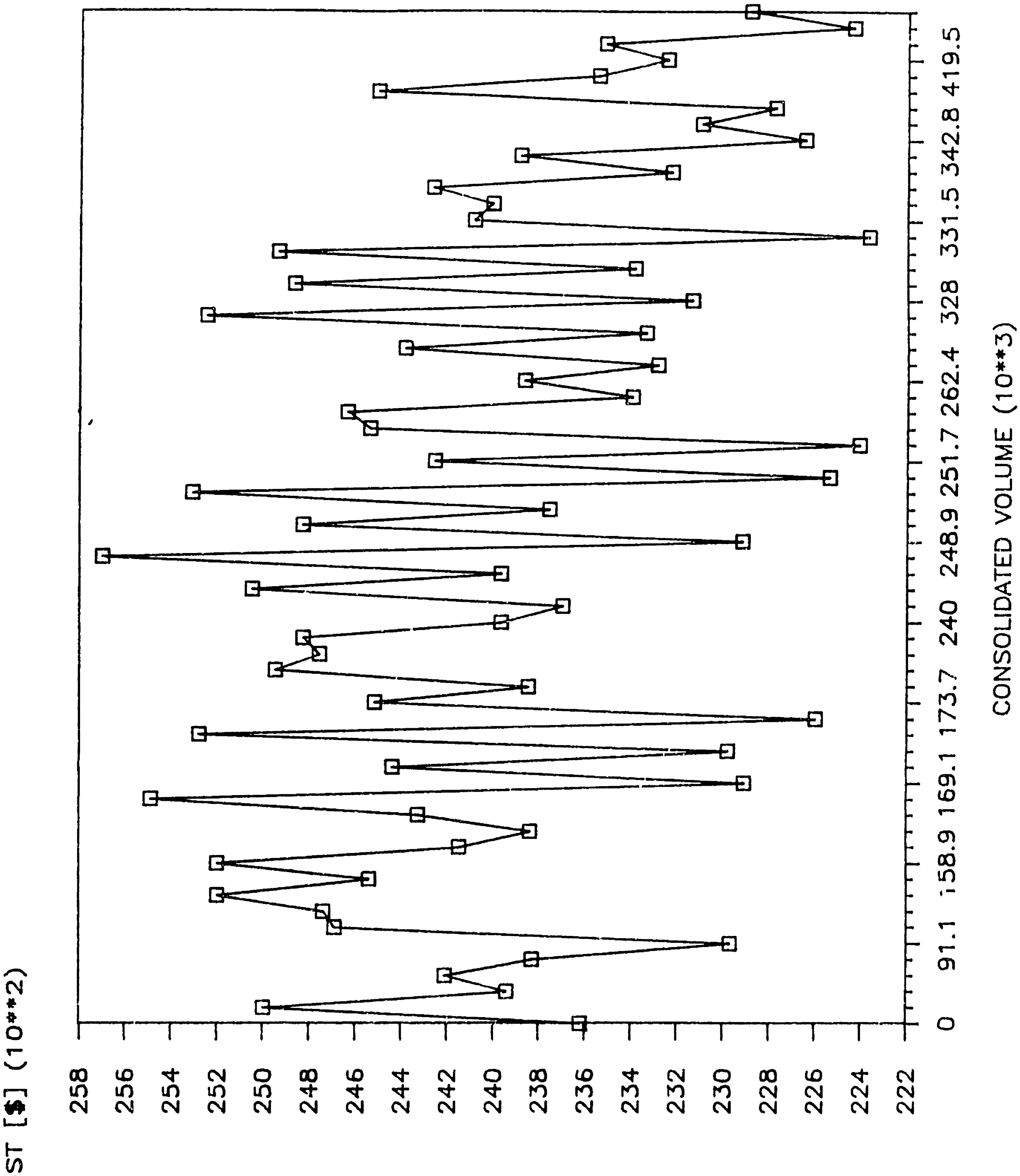


Figure 2.4: Potential Solution to a Shipper's Problem



tion - six commodity Shipper's Problem are graphed as a function of consolidated volume.

2.2 Solution Approaches For The Concave Network Problem

The remainder of this chapter reviews some of the approaches that have been used to find minimum cost flow solutions in networks where the flow cost is additive and concave in the flow on each arc. Section 2.2.1 describes the exact methods that have been used to find optimal solutions to these problems. It is not surprising that these methods translate into exponential time algorithms for the general problem because concave network problems have been determined to be NP-hard [ERIC86] even in planar graphs. Section 2.2.2 discusses solution methods devised for problems with special structure cost function or network configuration. Section 2.2.3 describes heuristic methods that have been devised to efficiently find good solutions in concave networks. Section 2.2.4 discusses conclusions drawn from the review of the literature as pertaining to the Shipper's Problem. These conclusions have motivated the work of this thesis.

2.2.1 Exact Methods For Concave Network Problems

As described above, finding the optimal solution in concave networks is difficult because of the existence of multiple local optima for the problem. It is the goal of exact solution methods to discern the global minimum from among the local minima. From the theory of mathematical programming, it is known that the existence of a finite global minimum over the set of feasible network flows implies the existence of an extreme flow which is an optimal flow. Further, there is a finite number of extreme flows over this set of feasible flows. A complete enumeration of all of the extreme flows would then identify the optimal flow assuming it existed. Enumerating every extreme flow in a concave network can be a very laborious process. Sections 2.2.1.1 and 2.2.1.2 describe dynamic programming and branch-and-bound approaches respectively which take advantage of different problem characteristics in finding optimal flows. In the worst case, both solution techniques could result in complete enumeration - identifying all extreme flows of a network in order to find an optimal flow.

2.2.1.1 Dynamic Programming Approaches

Dynamic programming models developed for the concave network problem take advantage of a characterization of extreme flow solutions. Zangwill [ZANG86] motivates this characterization using the example of a single commodity-single origin-multiple destination acyclic network.

Zangwill considers a network [N,A] consisting of a finite set N of nodes e,f,g,... and a subset of arcs A that are ordered pairs (e,f) of nodes from N. Further, r_e is the amount of material originally at node e such that if $r_e>0$ the node is an origin, if $r_e=0$ the node is a transshipment node, and if $r_e<0$ the node is a destination for flow. Finally, x_{ef} denotes the amount of flow sent along arc (e,f).

Zangwill defines a chain as a sequence of nodes and arcs in the network, $e_1,(e_1,e_2),e_2,\ldots,e_{n-1},(e_{n-1},e_n),e_n$, to be represented by

$E=(e_1,e_2,\ldots,e_n)$, where $e_i \epsilon N$ for $i=1,\ldots,n$ and $(e_i,e_{i+1}) \epsilon A$ for $i=1,\ldots,n-1$. E is a source to e_n chain if e_1 is the source and if all of the nodes of E are distinct. The distinctness requirement means $e_i \neq e_j$ for $i \neq j$ which implies that no subset of arcs and nodes from E can form a cycle.

Zangwill next characterizes extreme flows in an acyclic network with single source s and D destinations. The network contains a collection of D s-d chains $(E^1,E^2,\ldots E^D)$, of the form $E^k=(e_0{}^k,e_1{}^k,\ldots,e_{n\_k}{}^k)$, where $e_0{}^k=s$ and $e_{n\_k}{}^k=d_k$ for $k=1,\ldots,D$. Such a collection is called an arborescence if for any two chains E^{k_1} and E^{k_2} in the collection, there exists an integer i such that for $i \leq i_1$, $e_i{}^{k\_1}=e_i{}^{k\_2}$ while for any i and j, $n_{k\_1} \geq i > i_1$, $n_{k\_2} \geq j > i_1$, $e_i{}^{k\_1} \neq e_j{}^{k\_2}$. Hence in an arborescence, two or more s-d chains may share some initial sequence of nodes and arcs but after the node at which a difference in the chains is first detected, the sequence will never again share a node or arc. A flow x is called an arborescence flow if it is feasible and if $x_{ef}>0$ on the arcs (e,f) of each s-d chain in the arborescence and only on those arcs. Zangwill proves that for the network under consideration, a flow x is an extreme flow if and only if it is an arborescence flow.

Letting $(E^1,\ldots,E^D)$ be the s-d chains forming an arborescence in a single origin - multiple destination acyclic network, Zangwill defines the set H_e as the set of all indices k for which node e is in E^k : $H_e=(k|e \epsilon E^k, k=1,\ldots,D)$. Node e is then contained in s-d chains E^k $k \epsilon H_e$

and no others. Zangwill proceeds to prove that the flow value at any node e is $\Sigma_{k \epsilon H\_e} r_{d\_k}^1$ where $r_{d\_k}^1 = -r_{d\_k}$, k=1,...,D and the summation is zero if H_e is the null set. Further, given the extreme flow x, considering a node e and assuming that there is a node f such that $x_{fe}>0$, then $x_{fe} = \Sigma_{k \epsilon H\_e} r_{d\_k}^1$. This can be interpreted to mean that all freight leaves the origin node s, it continues until some node where it branches into two or more flows. Each branch continues until it further subdivides into smaller branches. This subdivision continues until each sub-branch reaches its end, a particular destination d_k.

Zangwill now uses this arborescence flow characterization of extreme flows to construct algorithms to solve special structure concave networks. The fact that a single source-single destination acyclic network will have as solution an extreme flow in a single s-d chain grows directly out of the arborescence flow characterization. By scaling the values of r in the network to r_s=1 and r_d=-1, respectively, the problem can be reformulated as a shortest path problem and hence solved using a shortest path algorithm, in polynomial time.

Zangwill shows how to construct a dynamic programming algorithm to solve single source-multiple destination acyclic networks. The arborescence flow characterization of an extreme flow leads directly to recursive relations used in the dynamic program. The recursion relations are derived form the fact that the flow on a particular arc of an extreme flow will be equal to the sum of the demands of some subset of the destinations. Unfortunately, this is an exponential time algorithm. Where the number of destinations in the network is D, the algorithm has a $O(|A||N|^D)$ running time.

Zangwill shows that all of the results developed for the single source-multiple destination acyclic problem hold also for the multiple source-single destination acyclic concave network. This stems from the fact that the new problem is simply the original problem but with the direction of shipment in each arc reversed. Exchanging the roles of sources and destinations in the problem preserves the arborescence flow characterization of an extreme flow. In addition, although the results above were developed for single commodity problems, for the special structure networks presented, Zangwill shows that multicommodity problems reduce to single commodity problems.

Erickson, et al [ERIK86] develop a dynamic programming algorithm called the send-and-split method based on the arborescence characterization of extreme flow solutions formalized by Zangwill. The method can be applied to networks with structure more general than that permitted by Zangwill - for example, networks are not restricted to be acyclic. The running time of the send-and-split method is $O(|N|2^{-1}3^D+s2^D)$ where $s=|N|\log_2|N|+3|A|$. This method is thus exponential in the number of destinations D.

2.2.1.2 Branch-and-Bound Approaches

Branch-and-bound models developed for the concave network problem differ from dynamic programming models in that the focus is shifted towards the cost functions that are involved in finding the minimum cost flow solution.

Soland [SOLA74] develops a branch-and-bound algorithm in the context of a facility location problem. Branching in his scheme corresponds to the setting of new limits for flow on a single network arc. Bounding corresponds to determining a lower bound on the optimal solution value in the feasible region constrained by the limits set in branching. At node N^k of the branch-and-bound tree, then, the flow vector x^k is a solution constrained by the limit vectors l^k and L^k such that $c^k = \{x^k \mid l^k \le x^k \le L^k\}$.

Soland defines $f^i(x_i)$ to be the cost of sending a flow of volume x_i on arc i of the network. Then finding the minimum cost flow solution of the network corresponds to the objective:

$$\text{minimize } f(x) = \Sigma_{i=1,\ldots,n} f_i(x_i).$$

Given a solution flow x^k at node N^k of the tree, the upper bound on the optimal solution value is the best solution found through node N^k, $UB^k = \min\{f(x^h): h=0,1,\ldots,k\}$. In order to find a lower bound at node N^k, Soland introduces the linear function $\varphi_i^k(x_i)$ which is defined by the points $[l_i^k, f_i(l_i^k)]$ and $[L_i^k, f_i(L_i^k)]$. By the concavity of f_i,

$$\varphi_i^k(x_i) \le f_i(x_i) \quad \text{if} \quad x_i \epsilon [l_i^k, L_i^k], \quad \text{and}$$

$$\varphi_i^k(x_i) \ge f_i(x_i) \quad \text{otherwise}$$

Let $$\varphi^k(x) = \Sigma_{i=1,\ldots,n} \varphi_i^k(x_i),$$

$$\varphi^k(x) \le f(x) \quad \text{if} \quad x \epsilon [l^k, L^k] = c^k$$

A lower bound at node N^k then corresponds to the minimum $\varphi^k(x)$ such that x is a feasible network flow.

At the end of stage k of the algorithm, if the lower bound associated with the node N^k is smaller than the least upper bound found, branching will occur at N^k, creating nodes N^{2k+1} and N^{2k+2}. If x^k is the solution at N^k, the next step is to find i_j that maximizes the difference $f_i(x_i^k) - \varphi_i^k(x_i^k)$ $i=1,\ldots,n$ and divide the corresponding interval $[l_{i\_j}{}^k, L_{i\_j}{}^k]$ into two intervals $[l_{i\_j}{}^k, x_{i\_j}{}^k]$ and $[x_{i\_j}{}^k, L_{i\_j}{}^k]$. Nodes N^{2k+1} and N^{2k+2} then have feasible regions constrained by c^{2k+1} and c^{2k+2}, where c^{2k+1} and c^{2k+2} differ from c^k only in that $[l_{i\_j}{}^k, x_{i\_j}{}^k]$ and $[x_{i\_j}{}^k, L_{i\_j}{}^k]$ replace $[l_{i\_j}{}^k, L_{i\_j}{}^k]$.

While the procedure continues, nodes are generated by branching as described above, their associated upper and lower bounds evaluated using the actual cost functions and linearizations of these cost functions respectively, and the set of feasible network flows becomes increasingly constrained. The procedure terminates if there are no nodes left having a lower bound smaller than the least upper bound, from which to branch - an optimal solution has been found.

Soland proves that his algorithm terminates at an optimal solution in a finite number of iterations. The final upper bound will be an optimal solution by the validity of the upper and lower bounds and the fact that no solution x in c is ever excluded from consideration. The finiteness of the algorithm results from the assumption that each x^k is one of a finite number of extreme flows in the concave network. The characterization of an extreme flow, as developed by Zangwill, is thus involved in solving for the flow x^k which minimizes $\varphi^k(x)$, thereby providing the lower bound and leading to the evaluation of the upper bound at node N^k. In the worst case, Soland's algorithm could require up to $2^{D|A|}$ minimum-linear-cost network flow problems to be solved.

2.2.2 Approaches For Certain Special Structure Problems

Solution methods have been devised to determine minimum cost flows for some concave network problems with specialized cost function or network configuration.

Barr, et al [BARR81] have developed branch-and-bound algorithms to optimally solve very large problems where costs are not actually concave but are linear except for a fixed charge. Balakrishnan and Graves [BALA85] have developed a solution approach which assumes a piecewise-linear concave cost function and uses Lagrangian relaxation as the basis for a heuristic procedure in networks with this cost structure.

Gallo, et al [GALL80] develop a branch-and-bound algorithm which uses as a subroutine, a vertex-following algorithm by Gallo and Sodini [GALL79] for finding a local optimum for the general concave-cost multi-commodity flow problem. The branch-and-bound algorithm is presented to solve the class of problems that can be configured as one origin-multiple destination networks. Methods for finding optimal consolidation strategies in networks that can be decomposed into smaller independent subnetworks, and in which minimum cost flows can be determined separately, are devised by Blumenfeld, et al [BLUM85], and by Hall [HALL85].

2.2.3 Heuristic Methods for the General Problem

Heuristic approaches to determining a minimum cost flow in a concave network have been pursued for solving large problems having no special structure. The goal of these methods is to find good local optima when the prospects for finding a global minimum are dismal due to the size of a problem.

Yaged [YAGE71], in the context of communication networks, presents a number of criteria necessary for global optimality which form the basis of heuristic algorithms he devises. The first criterion is that any locally optimal solution must have flow between each origin-destination pair assigned to a single path. This is equivalent to the extreme flow characterization of a minimum cost network flow presented by Zangwill. The second criterion is that the cost of a network flow solution can not be reduced by transferring a differential quantity of flow from one path to another.

Defining:

$\underline{F}_{ij}$: the optimal flow on arc (i,j),

by the first and second criterion, the optimal network solution can be found by solving the following shortest path problem for each origin-destination pair:

$$\min_{p \epsilon P(o,d)} \Sigma_{(i,j) \epsilon P} C_{ij}(\underline{F}_{ij}).$$

Equivalently, defining:

$\underline{C}_{ij}(\underline{F})$: the marginal cost on arc (i,j) given the flow $\underline{F}_{ij}$

$f_{ij}[\underline{C}(\underline{F})]$: the marginal flow on arc (i,j) given $\underline{C}(\underline{F})$,

the optimal solution to

$$\min_{\underline{F}} \Sigma_{(i,j)\epsilon A}\, C_{ij}(f_{ij}[\underline{C}(\underline{F})])$$

will yield the optimal network solution but will yield a faster algorithm when the number of arcs is much smaller than the number of origin-destination pairs. In this second formulation, a shortest path problem must be solved for each value of $\underline{F}$. The search procedure for the $\underline{F}$'s can be shortened by Yaged's third optimality criterion which states that $\underline{F}_{ij} = f_{ij}[\underline{C}_{ij}(\underline{F})]$ for all $(i,j)\epsilon A$. This third criterion can be interpreted as a statement of economic equilibrium.

Every flow vector $\underline{F}$ satisfying the third criterion is a local optimum. Yaged develops a fast procedure that converges to one of these local optima. The procedure begins with an initial guess for $\underline{F}$. Based on this $\underline{F}$, $\underline{C}_{ij}(\underline{F})$ is computed, from which the shortest path is found for each origin-destination pair. Once these paths are determined, a new value of $\underline{F}$ is calculated and the process is repeated. Yaged solves problems using both marginal and average cost values for $\underline{C}_{ij}(\underline{F})$ and finds that average costs yield better results.

Jordan [JORD86] devises a heuristic, based on the work of Yaged, for concave network problems arising in the shipment of freight. In his algorithm, the volume of flow carried by each network arc is set to a large value. The average cost per unit to send flow on each arc is evaluated. Based on these average costs, a flow is routed via the shortest path between its origin and destination. If the routing of the flows is unchanged from the previous iteration, the procedure terminates. If not, the flow on each network arc resulting from the shortest path solutions is evaluated. The procedure now repeats - calculation of the corresponding average cost of using an arc, solving the shortest path problems of each origin-destination pair and checking the termination criterion.

2.2.4 Conclusions Drawn From a Review of the Literature

As described in the first section of this chapter, the Shipper's Problem can be cast as a multi-commodity concave network flow problem. A review of the literature yields no specialized techniques for a network having this configuration. Moreover, more general solution methods for this problem do not appear promising in terms of worst case running times due to the multi-commodity nature of the network. There does exist, however, one characteristic of the network configuration being addressed, which can be exploited. Specifically, that flow between any origin-destination pair can be routed only one of two ways - either directly, or consolidated via the transportation terminals. Further, by the extreme flow characterization of solutions in concave networks in an optimal solution, all of the flow between a given origin-destination pair will be routed on the same path. This must be kept in mind in evaluating the usefulness of any of the solution techniques presented in sections 2.2.1 and 2.2.3 above.

The exact solution methods have extremely bad worst case running times for concave network problems. Branch-and-bound, however, appears more promising than dynamic programming for the Shipper's Problem. The fact that there are only two permissible routings between the origin and destination of each flow can be used to generate lower bounds in a Soland-type scheme. Dynamic programming models including Erickson, et al's send-and-split method will be highly inefficient given this network configuration because the permissible consolidated routing of every flow interacts. These models would most likely result in complete enumeration of solutions in order to find one that is optimal.

Jordan's heuristic might offer another means of finding a good solution in networks of the type being modeled. The two permissible routings for any given flow in the network facilitate a trivial solution for the shortest path problem to be solved for every flow at every iteration of the algorithm. The heuristic will thus probably run very quickly for this problem although it may result in solutions that are sub-optimal since average arc costs are used instead of actual arc costs in determining flow routings.

A review of the literature thus yields two potential solution approaches to the Shipper's Problem. The first, branch-and-bound, will yield optimal solutions but may have very bad running times. The second, Jordan's heuristic, should have good running times but may result in solutions that are far from optimal. This motivates the search for a third approach to the Shipper's Problem - a heuristic exploiting the structure inherent in the network problem to be solved, including both the nature of the route choice for each flow, as well as the behavior of the objective function operative in this network. A description of the heuristic devised, as well as measures of its performance relative to those of branch-and-bound and Jordan schemes tailored to solve the Shipper's Problem, are found in chapter 4.

3. The Shipper's Problem Objective

As defined in chapter 1, the objective of the Shipper's Problem is the determination of the minimum total logistics cost transportation strategy for shipping a set of freight flows between their respective origins and destinations. Section 3.1 of this chapter characterizes total logistics cost. Section 3.2 presents a methodology for determining the optimal shipping policy (frequency of service and shipment size) for freight flows routed directly in a freight transportation strategy. Section 3.3 presents the derivation of a model to evaluate the total logistics cost associated with freight flows routed consolidated given a shipping policy (frequency of service and shipment size for each consolidated route segment). In section 3.4 heuristics for identifying near-optimal shipping policies for consolidated flows using the model presented in Section 3.3 are developed and their performance evaluated.

3.1 Total Logistics Cost

Total logistics cost has been defined as the sum of transportation cost, in-transit inventory cost and inventory carrying cost. These costs are included in this analysis because they figure in every transportation problem, whereas such costs as handling costs are context specific.

Determination of the optimal freight transportation strategy involves identifying every potential freight routing and the shipping policies yielding its minimum total logistics cost. Comparison of strategies on the basis of total logistics cost is valid because as it is defined here, it is the sum of costs incurred through the selection of an entire transportation strategy - including the costs related to both routing and shipping policy decisions. These comparisons will prove fundamental to the algorithms for finding optimal freight transportation strategies in chapter 4.

The determination of the shipping policy that minimizes total logistics cost for a given freight routing is equivalent to identifying the policy that minimizes the sum of transportation and inventory costs. Given that freight to be shipped becomes available at an origin at a fixed rate, in order to decrease transportation costs, less frequent, larger shipments of freight would have to be made. Conversely, in order to decrease inventory costs, more frequent, smaller shipments of freight would have to be made. Reductions in both transportation and inventory costs are thus competing objectives when given a fixed volume of freight to ship per period. This suggests that a shipping policy that optimally trades off transportation and inventory charges will minimize the total logistics cost of a given freight transportation strategy. These observations lead to the Economic Order Quantity (EOQ) type analysis that will be employed in the determination of shipping policies for direct and consolidated route segments of a freight transportation strategy.

The derivation of models for estimating the minimum total logistics cost over all possible shipping policies for a given freight routing will be presented next. Chapter 4 will deal with the structured manner in which different transportation strategies can be compared.

3.2 Shipping Policies for Direct Routes

Eskandari [ESKA87] uses an EOQ-type analysis to determine the optimal shipping policy for shipping a freight flow directly between its origin and destination via a single or multiple modes. The most basic model developed in his thesis will be presented here such that the notation and assumptions embodied therein and relevant to the work in this thesis can be introduced.

The following notation will be used for the cost computations:

s_{ij}: The rate of supply of a commodity at origin i to be shipped to destination j. This quantity is equal to the demand at destination j for the commodity being shipped from origin i, (freight unit/time unit).

V: Value of the commodity being shipped, ($/freight unit).

I: Inventory carrying charge, (% value/time unit)

a_{ij}^m: Freight transportation charge from origin i to destination j using mode m, ($/shipment).

t_{ij}^m: Total transit time for freight travelling from origin i to destination j via mode m, (time unit).

u^m: Vehicle capacity of mode m, (freight unit/shipment).

x_{ij}^m: Shipment size for freight travelling from origin i to destination j via mode m, (freight unit/shipment).

It is assumed that the volumes s_{ij} remain constant over the period. The value V and inventory carrying charge I are characteristic of the commodity being routed between the origins and destinations. The transportation charges a_{ij}^m are taken to be fixed per shipment regardless of the size of the shipment. The transit time t_{ij}^m includes both travel and transloading times for a particular transportation route and mode combination. Finally, for the sake of simplifying the discussion, it is assumed that transshipment does not occur on direct routes between origins and destinations although this assumption has been relaxed by Eskandari.

Given the above parameters and assumptions, the following relationships become apparent:

s_{ij}/x_{ij}^m: The frequency of service between origin i and destination j given mode m.

x_{ij}^m/s_{ij}: The average inter-departure time for shipments travelling between origin i and destination j given mode m.

$x_{ij}^m/2s_{ij}$: The average time a unit spends in inventory at origin i or destination j given the deterministic supply and demand assumption made above. A freight unit thus spends an average total of x_{ij}^k/s_{ij} time units at origin i <u>and</u> destination j.

The total logistics cost of shipping one unit directly from origin i to destination j given mode m is the sum of per unit transportation, in-transit inventory and inventory carrying costs respectively:

$$c_{ij}^m = a_{ij}^m/x_{ij}^m + VIt_{ij}^m + VIx_{ij}^m/s_{ij}$$

Taking the first derivative of this cost function with respect to shipment size and setting it equal to zero, yields the shipment size that minimizes total logistics cost:

$$\partial c_{ij}^m/\partial x_{ij}^m = -a_{ij}^m/(x_{ij}^m)^2 + VI/s_{ij} = 0$$
$$x_{ij}^{m*} = \sqrt{a_{ij}^m s_{ij}/VI}$$

Vehicle capacity constrains shipment size, however, and thus

$$x_{ij}^{m*} = \min(u^m, \sqrt{a_{ij}^m s_{ij}/VI})$$

is in reality the actual expression for optimal shipment size.

The shipping policy that minimizes the total logistics cost of a given direct freight routing between origin i and destination j can now be characterized as having shipping frequency s_{ij}/x_{ij}^{m*}, and headway between shipments x_{ij}^{m*}/s_{ij}. The total logistics cost associated with this policy is

$$c_{ij}^{m*} = \begin{cases} 2\sqrt{a_{ij}^m s_{ij} VI} + VIt_{ij}^m s_{ij}, & \text{for } s_{ij}^m \le (u^m)^2 VI/a_{ij}^m \\ a_{ij}^m s_{ij}/u^m + VIt_{ij}^m s_{ij} + VIu^m, & \text{for } s_{ij}^m \ge (u^m)^2 VI/a_{ij}^m. \end{cases}$$

The above function is clearly concave.

The optimal direct routing for freight travelling between origin i and destination j is the one for which total logistics cost is minimized over all transportation modes. Let us denote by m_D the optimal transportation mode for freight flow i-j on direct route i-j,

$$\{m_D \mid c_{ij}^{m\_D*} = \min_m [c_{ij}^{m*}] \}$$

Its associated optimal shipping policy is then characterized by shipment size $x_{ij}^{m\_D*}$ and yields minimum total logistics cost per unit given by $c_{ij}^{m\_D*}$.

The fact that an optimal direct routing for a particular freight flow can be determined in the absence of consideration of the routings for any of the other network freight flows is important. It is for this reason that the dominating direct arcs can be discerned and are the only ones that need be included in the Shipper's Problem network as asserted in chapter 2. The independence of these evaluations from all other freight transportation strategy calculations makes it attractive to perform them once for each freight flow, as a pre-processing step to the algorithms of chapter 4.

3.3 Total Logistics Cost of Consolidated Freight

In deriving the optimal shipping policies for the flows consolidated in a particular freight transportation strategy, the same trade-off with respect to transportation and inventory costs exists as in the direct routing case. Here, however, the derivation is complicated by the fact that the best shipping policy for each freight flow cannot be considered and solved for in isolation because by definition, consolidated routes for freight flows overlap. Instead shipping policies will have to be devised for consolidated route segments, with special attention paid to the results of the interaction of the policies for adjacent route segments.

The route segments comprising consolidation routes consist of the arcs between the origins and the consolidating transportation terminal, the inter-transportation terminal arc, and the arcs between the deconsolidating transportation terminal and the destinations. Freight consolidated from each origin can vary widely in volume between origins. In addition, the distances of the origins from the transportation terminal at which consolidation occurs, are different. It stands to reason then, that total logistics costs for these arcs will be different as . and hence, that good shipping policies for the origin-consolidation arcs of consolidated routes will differ. This means that shipments of different size will be arriving at a transportation terminal for consolidation with differing frequency from their respective origins. At the consolidation center, some of these shipments will be consolidated and shipped out immediately on the consolidation mode. Others will have to wait in inventory for more shipments to arrive in order to make up the shipment size that characterizes a good shipping policy for the inter-transportation terminal arc. Upon arrival at the deconsolidating transportation terminal, freight is shipped to its destinations. Again good shipping policies for the deconsolidation-destination arcs will differ because the volumes of freight being shipped on each as well as the transportation charges associated with each are different. The amount of time a consolidated freight shipment spends at the deconsolidation center in inventory depends on the shipment frequency comprising a good shipping policy for the particular deconsolidation-destination arc being examined.

Given the interdependencies of the shipping policies on the three sets of arcs comprising consolidation routes, an expression for evaluating the total logistics cost of these routes will be developed. As in the direct routing case, the total logistics cost of consolidated routings is the sum of transportation, in-transit inventory and inventory carrying costs. In the case of consolidated routings, however, there are inventory carrying costs incurred at four locations -at the two transportation terminals as well as at the origin and destination of a flow. The parameters and assumptions used in this analysis will be the same as those used in the direct case. In addition, the following notation will be used:

$s_{i.} = \Sigma_j s_{ij}$: The rate of supply at origin i of freight to be routed consolidated.

$s_{.j} = \Sigma_i s_{ij}$: The rate of demand at destination j for freight routed consolidated.

$s_{..} = \Sigma_i \Sigma_j s_{ij}$: The total volume of freight to be routed consolidated.

The transportation cost component of the total logistics cost for consolidated freight is easily derived. It is simply the sum of transportation charges for freight being transported on origin-consolidation arcs, on the consolidation-deconsolidation arc and on the deconsolidation-destination arcs respectively:

$$\Sigma_i a_{ic} s_{i.}/x_{ic} + a_{cd} s_{..}/x_{cd} + \Sigma_j a_{dj} s_{.j}/x_{dj}$$

where the indices c and d denote the consolidation and deconsolidation centers respectively.

The in-transit inventory cost component is the sum of the in-transit inventory charges for freight being transported on origin-consolidation arcs, on the consolidation-deconsolidation arc, and on arc, and on the deconsolidation-destination arcs respectively:

$$\Sigma_i \; VIt_{ic}s_{i.} + VIt_{cd}s_{..} + \Sigma_j \; VIt_{dj}s_{.j}$$

Inventory carrying cost is the remaining component of total logistics cost to evaluate. Inventory carrying charges at the origins and destinations of consolidated routes are evaluated in the same manner as those for direct routings. It is assumed that the average time a unit of freight spends in inventory at origin i is $x_{ic}/2s_{i.}$, and at destination j is $x_{dj}/2s_{.j}$. The inventory charges incurred at these locations are functions of these times respectively. The fact that these charges are calculated on a per flow instead of a per node basis implies that the production of the freight flows at an origin or demand for the freight flows at a destination are independent. This is an assumption that can be relaxed.

The difficulties inherent in finding minimum total logistics cost shipping policies for freight transportation strategies involving consolidated flows, arise in conjunction with the evaluation of inventory carrying costs at the transportation terminals. As described above, the time spent at the consolidating transportation terminal is a function of the shipping policies for freight transported on the origin-consolidation arcs as well as the consolidation-deconsolidation arc. Similarly the time spent by freight at the deconsolidation center is a function of the shipping policies for freight transported on the deconsolidation-destination arcs as well as the consolidation-deconsolidation arc. Any expression for evaluating inventory carrying cost must incorporate the interdependencies of shipping policies on adjacent arcs of consolidation routes.

An examination of the ratio of shipping frequencies on adjacent arcs of consolidation routes will reveal how the shipping policies for freight flows transported on these arcs interact. Analyzing first the situation at the consolidation center, we define:

$s_{i.}/x_{ic}$: The frequency that characterizes the shipping policy for freight travelling between origin i and the consolidation center.

$s_{..}/x_{cd}$: The frequency that characterizes the shipping policy for freight travelling on the consolidation-deconsolidation arc.

$s_{i.}x_{cd}/s_{..}x_{ic}$: The ratio of the frequencies of service on the origin i-consolidation and consolidation-deconsolidation arcs.

Let k_{ic}/k_{cd}^{i} be equal to this ratio of frequencies where k_{ic} and k_{cd}^{i} have a greatest common factor of one. Since the headway between shipments is the reciprocal of shipping frequency on an arc, k_{cd}^{i}/k_{ic} is the ratio of headways between shipments on the origin i-consolidation and consolidation-deconsolidation arcs.

The following definitions and theorems will be used to relate the ratio k_{cd}^i/k_{ic} to time spent by freight at the consolidation center:

<u>Definition 1</u>: Given k_{ic} a positive integer, $g \equiv h \bmod k_{ic}$ if and only if g and h have the same remainder when divided by k_{ic}. Alternatively, $g \equiv h \bmod k_{ic}$ if and only if $(h-g)/k_{ic}$ is a positive integer.

<u>Theorem 1</u>: Given k_{ic} and k_{cd}^i relatively prime, positive integers, $(k_{cd}^i)g \equiv (k_{cd}^i)h \bmod k_{ic}$ if and only if $h \equiv g \bmod k_{ic}$.

<u>Proof</u>: $\Rightarrow$ If $h \equiv g \bmod k_{ic}$ then $(k_{cd}^i)g \equiv (k_{cd}^i)h \bmod k_{ic}$

- $h \equiv g \bmod k_{ic}$

$\lfloor h/k_{ic} \rfloor k_{ic} + g = h$

- multiplying through by k_{cd}^i,

$k_{cd}^i \lfloor h/k_{ic} \rfloor k_{ic} + (k_{cd}^i)g = (k_{cd}^i)h$

- since k_{ic} and k_{cd}^i are relatively prime.

$ak_{cd}^i h/k_{ic} \rfloor k_{ic} + (k_{cd}^i)g = (k_{cd}^i)h$

and $(k_{cd}^i)g \equiv (k_{cd}^i)h \bmod k_{ic}$ ■

$\Leftarrow$ if $(k_{cd}^i)g \equiv (k_{cd}^i)h \bmod k_{ic}$ then $h \equiv g \bmod k_{ic}$.

- $(k_{cd}^i)g \equiv (k_{cd}^i)h \bmod k_{ic}$

$\lfloor k_{cd}^i h/k_{ic} \rfloor k_{ic} + (k_{cd}^i)g = (k_{cd}^i)h$

multiplying through by $1/k_{cd}^i$,

$1/k_{cd}^i \lfloor k_{cd}^i h/k_{ic} \rfloor k_{ic} + 1/k_{cd}^i(k_{cd}^i)g$

$= (1/k_{cd}^i)(k_{cd}^i)h$

$\lfloor k_{cd}^i h/k_{ic}k_{cd}^i \rfloor k_{ic} + g = h$

- since k_{ic} and k_{cd}^{i} are relatively prime.

and, $h \equiv g \bmod k_{ic}$ ■

<u>Definition 2</u>: The relation mod k_{ic} on I separates the integers into k_{ic} equivalence classes [0], [1], ..., $[k_{ic}-1]$ called residue classes, where $[h] = \{g: g \in I, h \equiv g \bmod k_{ic}\}$.

<u>Theorem 2</u>: Given k_{ic} and k_{cd}^{i} relatively prime, positive integers, the elements of $\{k_{cd}^{i}, 2k_{cd}^{i}, 3k_{cd}^{i}, \ldots, k_{ic}k_{cd}^{i}\}$ have distinct remainders when divided by k_{ic}.

<u>Proof</u>: Assume the contrary,

- for some $0<g<h\leq k_{ic}$,

gk_{cd}^{i} and hk_{cd}^{i} have the same remainder when divided by k_{ic} and thus are in the same residue class (definition 2),

- Then $gk_{cd}^{i} \equiv hk_{cd}^{i} \bmod k_{ic}$ by definition 1.

- Since k_{cd}^{i} and k_{ic} are relatively prime,

$g \equiv h \bmod k_{ic}$ by theorem 1 and $(h-g)/k_{ic}$ is a positive integer by definition 1,

- But $0<g<h\leq k_{ic} \Rightarrow 0<h-g<k_{ic} \Rightarrow 0<(h-g)/k_{ic} < 1$,

which is a contradiction since there are no integers $\in$ (0,1).

Thus where shipments arrive at the consolidation center from origin i uniformly every k_{cd}^{i}/k_{ic} time units, a sequence k_{cd}^{i}/k_{ic},

$2k_{cd}^i/k_{ic}$, $3k_{cd}^i/k_{ic}$, ..., $k_{ic}k_{cd}^i/k_{ic}$ of time units is generated. By theorem 2 this yields a sequence of distinct remainders. Here these remainders represent the distinct fractions of headway characterizing the consolidation-deconsolidation shipping policy, at which arrivals from origin i occur. Then the sequence 0, $k_{cd}^i(k_{ic}-1)/k_{ic}$, $k_{cd}^i(k_{ic}-2)/k_{ic}, \ldots, 2k_{cd}^i/k_{ic}, k_{cd}^i/k_{ic}$ represents the sequence of distinct fractions of headway between consolidation-deconsolidation arc shipments, that arrivals from origin i will wait in inventory at the consolidation center. The total waiting time for shipments arriving at the consolidation center from origin i, is then, the sum of all fractions of consolidation-deconsolidation headway multiplied by the headway between consolidation-deconsolidation arc shipments:

$$[\sum_{n=0}^{k_{ic}-1} n/k_{ic}]x_{cd}/s_{..} = [1/k_{ic}\sum_{n=0}^{k_{ic}-1} n]x_{cd}/s_{..} = [(1/k_{ic})[(k_{ic}-1)(k_{ic})/2]x_{cd}/s_{..}$$

$$= (k_{ic}-1)x_{cd}/2s_{..}$$

The average time spent at the consolidating transportation terminal by freight shipments arriving from origin i is then [LEE88]:

$$1/k_{ic}[(k_{ic}-1)x_{cd}/2s_{..}] = x_{cd}/2s_{..}[1-1/k_{ic}].$$

Using the same methodology presented thus far, it is possible to examine the manner in which the shipping policies for freight on the consolidation-deconsolidation and deconsolidation-destination arcs interact such that the time spent by freight at the deconsolidation center can be determined. Let:

$s_{..}x_{dj}/s_{.j}x_{cd}$: The ratio of the frequencies of service on the consolidation-deconsolidation and deconsolidation-destination j arcs.

Let k_{cd}^j/k_{dj} be equal to this ratio of frequencies where k_{dj} and k_{cd}^j have a greatest common factor of one. k_{dj}/k_{cd}^j is the ratio of headways between shipments on the consolidation-deconsolidation and deconsolidation-destination j arcs.

The total waiting time for shipments departing from the deconsolidation center to destination j, is then, the sum of all fractions of deconsolidation-destination j headway, multiplied by the headway between deconsolidation-destination j arc shipments:

$$[\sum_{n=0}^{k_{cd}-1} n/k_{cd}^j]x_{dj}/s_{.j} = (k_{cd}^j-1)x_{dj}/2s_{.j}$$

The average time spent at the deconsolidation center by freight shipments departing to destination j is then [LEE88]:

$$1/k_{cd}^j[(k_{cd}^j-1)x_{dj}/2s_{.j}] = x_{dj}/2s_{.j}[1-1/k_{cd}^j]$$

The inventory carrying cost component of the total logistics cost of consolidated freight can now be written as the sum of charges incurred at origins shipping consolidated freight, at the consolidation and deconsolidation centers, and at the destinations receiving consolidated freight respectively:

$$\sum_i VI(x_{ic}/2s_{i.})s_{i.} + \sum_i VI(x_{cd}/2s_{..}[1-1/k_{ic}])s_{i.}$$
$$+ \sum_j VI(x_{dj}/2s_{.j}[1-1/k_{cd}^j]s_{.j})s_{.j} + \sum_j VI(x_{dj}/2s_{.j})s_{.j}$$
$$= VI/2[[\Sigma_i\ x_{ic} + x_{cd}s_{i.}/s_{..}(1-1/k_{ic})] + \Sigma_j\ [x_{dj} + x_{dj}(1-1/k_{cd}^j)]].$$

The total logistics cost of consolidated routings (CC) is the sum of the transportation costs, in-transit inventory costs and inventory carrying costs associated with the freight flows routed consolidated:

$$CC = [\sum_i a_{ic}s_{i.}/x_{ic} + a_{cd}s_{..}/x_{cd} + \sum_j a_{dj}s_{.j}/x_{dj}]$$
$$+ VI[\sum_i t_{ic}s_{i.} + t_{cd}s_{..} + \sum_j t_{dj}s_{.j}$$
$$+ VI/2[\sum_i [x_{ic} + x_{cd}s_{i.}/s_{..}(1-1/k_{ic})] + \sum_j [x_{dj} + x_{dj}(1-1/k_{cd}^j)]].$$

Minimizing the total logistics cost of consolidated routings entails simultaneously solving for the shipment sizes x_{ic} for all i, x_{cd}, and x_{dj} for all j, that minimize CC.

3.4 Heuristics for Finding Shipping Policies for Consolidated Freight

In section 3.3 of this chapter, it was discovered that given freight flows to be routed consolidated in a transportation strategy, solving for the minimum cost shipping policies on all of the corresponding route segments is a difficult problem. Developing heuristics to identify these shipping policies is the motivation for this section. Four heuristic approaches for finding near-optimal shipping policies yielding good minimum total logistics cost of consolidated freight (CC) follow. All of the approaches make assumptions with respect to how shipment sizes for freight on particular sets of arcs can be determined. These assumptions facilitate the evaluation of CC by providing the means with which the relatively prime numbers summarizing the effects of shipping policies on adjacent links can be determined. Heuristics are based on the following shipment size assumptions:

1) Fix the shipment size for freight on origin-consolidation and deconsolidation-destination arcs at vehicle capacity, and find the optimal shipment size for freight transported on the consolidation-deconsolidation arc, ie:

$$x_{ic} = u \text{ for all } i$$

$$x_{dj} = u \text{ for all } j$$

$$x_{cd} = \min \{u, \sqrt{a_{cd}s_{..}/VI}\}$$

These shipment sizes will yield a reasonable value for CC in instances where there are large enough volumes of freight being shipped on the origin-consolidation and deconsolidation-destination arcs to capacitate the vehicles operating on them, but where the number of origins and destinations is small such that the consolidation mode is not capacitated.

2) Fix the shipment size for freight transported on the consolidation-deconsolidation arc at vehicle capacity and determine the optimal shipment sizes for freight travelling on the origin-consolidation and deconsolidation-destination arcs, ie:

$$x_{cd} = u$$

$$x_{ic} = \min(u, \sqrt{a_{ic} s_{i.}/VI})$$

$$x_{dj} = \min(u, \sqrt{a_{dj} s_{.j}/VI})$$

These shipment sizes will yield a reasonable value for CC in instances where there might not be enough volume shipped by an origin or received by a destination to capacitate vehicles of the mode serving them, but over all of the origins shipping to all of the destinations, there is a big enough volume of freight to capacitate the consolidation mode.

3) Set the shipment sizes for freight transported on all of the arcs comprising consolidation routes at their independently optimal values, ie:

$$x_{ic} = \min(u, \sqrt{a_{ic} s_{i.}/VI})$$

$$x_{cd} = \min(u, \sqrt{a_{cd} s_{..}/VI})$$

$$x_{dj} = \min(u, \sqrt{a_{dj} s_{.j}/VI})$$

These shipment sizes will yield a good value for CC in instances where there is a limited number of origins and destinations in the network, such that all of the shipment size decisions being made independently for freight travelling on origin-consolidation and on deconsolidation-destination arcs do not result in large inventory buildups at the consolidation and deconsolidation centers respectively.

In the three approaches described above, once the shipment sizes have been set by the assumptions characterizing the heuristic, the ratios $s_{i.}x_{cd}/s_{..}x_{ic}$ yield the ratios k_{ic}/k_{cd}^{i} for the shipping policy of each origin-consolidation arc, and the k_{ic}'s can be determined for all i. Similarly, the ratios $s_{..}s_{dj}/s_{.j}x_{cd}$ yield the ratios k_{cd}^{j}/k_{dj} for the shipping policy of each deconsolidation-destination arc, and the k_{cd}^{j}'s can be determined for all j. Total logistics cost of consolidated flow (CC) under any of the three approaches can now easily be computed.

The fourth heuristic approach for minimizing CC is quite different from the others. It attempts to optimize the shipping policies on adjacent arcs with respect to each other such that the total logistics cost of consolidated flows is minimized. Therefore, in contrast to the first three approaches, the interaction between the arcs are taken into account in the evaluation of optimal shipping frequencies. This is done by first setting the shipment size on the consolidation-deconsolidation arc to its independently optimal value. Next, the inventory carrying charge term contributed by the shipping policy out of each origin at the consolidation center, is incorporated into the optimal shipment size calculation of its respective origin-consolidation arc. Similarly, the inventory carrying charge term contributed by the shipping policy into each destination at the deconsolidation center, is incorporated into the optimal shipment size calculation of its respective deconsolidation-destination arc. This attempt at coordinating the shipping policies of adjacent arcs should result in decreased inventory carrying charges. Algorithmically, the procedure is as follows:

- Determine the optimal shipment size on the consolidation arc,

  $$x_{cd} = \min(u, \sqrt{a_{cd}s_{..}/VI})$$

- Incorporate the inventory carrying cost term of CC at the consolidation center attributable to shipments from origin i, into the optimal shipment size calculation for the freight transported on the origin-consolidation arc from i, for all origins i:

  the total logistics cost per unit for freight on this arc is

  $c_{ic} = a_{ic}/x_{ic} + VIt_{ic} + VI/2[x_{ic}/s_{i.} + x_{cd}/s_{..}[1-1/k_{ic}]]$

  but $k_{ic}/k_{cd}^{i} = s_{i.}x_{cd}/s_{..}x_{ic}$,

  thus $k_{ic} = s_{i.}x_{cd}k_{cd}^{i}/s_{..}x_{ic}$

  and $c_{ic} = a_{ic}/x_{ic} + VIt_{ic} + VI/2[x_{ic}/s_{i.} + x_{cd}/s_{..} - x_{ic}/s_{i.}k_{cd}^{i}]$.

  To find the shipment size that minimizes c_{ic}:

  $$\partial c_{ic}/\partial x_{ic} = -a_{ic}/x_{ic}^2 + (VI/2s_{i.})[1-1/k_{cd}^{i}] = 0$$

  thus $x_{ic}^* = \min(u, \sqrt{2a_{ic}s_{i.}/(VI[1-1/k_{cd}^{i}])})$

  Perform the following algorithm to evaluate k_{ic}:

```
kcd^i := 0;

REPEAT
   kcd^i := kcd^i + 1
   Use kcd^i to evaluate xic*
   Use xic* to evaluate kic
UNTIL kic is integer
```

When k_{ic} is integer, k_{cd}^{i} is integer, these numbers are relatively prime and a good shipping policy for the freight transported on the origin-consolidation arc from i will hopefully have been identified. This step is performed to identify the k_{ic} for each origin shipping consolidated freight.

- Similarly, incorporate the inventory carrying cost term of CC at the deconsolidation center attributable to shipments received by destination j, into the optimal shipment size calculation for the freight transported on the deconsolidation-destination arc to j, for all destinations j:

  the total logistics cost per unit for freight on this arc is

  $$c_{dj} = a_{dj}/x_{dj} + VIt_{dj} + VI/2[x_{dj}/s_{.j} + x_{dj}/s.j[1-1/k_{cd}^{j}]]$$

  To find the shipment size that minimizes c_{dj}:

  $$\partial c_{dj}/\partial x_{dj} = -a_{dj}/x_{dj}^{2} + (VI/2s_{.j})[2-1/k_{cd}^{j}] = 0$$

  thus $x_{dj}^{*} = \min(u, \sqrt{2a_{dj}s_{.j}/VI[2-1/k_{cd}^{j}]})$

  Use an algorithm as above to determine when k_{dj} and k_{cd}^{j} are integer, these numbers are relatively prime, and hopefully yield a good shipping policy for the freight transported on the deconsolidation-destination j arc. This step is performed to identify the k_{cd}^{j} for each destination receiving consolidated freight.

- Finally, by using the shipment sizes, k_{ic}'s and k_{cd}^{j}'s determined above, evaluate CC.

The fourth heuristic is an iterative algorithm having lengthy execution time and questionable convergence properties. It attempts, however, to optimize the shipping policies of adjacent route segments with respect to each other and thus should provide the best shipping policies. This heuristic is not practical for objective evaluation but can be used to generate a lower bound on the total logistics cost of a freight transportation strategy against which to compare the solution of the other three heuristics. A simulation of the four heuristics for finding total logistics cost of consolidated freight in randomly generated networks with respect to numbers of origins and destinations, configuration of the network, and volumes of freight shipped between any origin and destination was used to evaluate heuristic performance. Different sets of runs were made corresponding to different value (V) commodities. The results are presented graphically in figures 3.1 and 3.2.

A number of conclusions can be drawn from figures 3.1 and 3.2 regarding the relative performance of the heuristics. Clearly, the fourth heuristic outperforms the other three in determining the consolidated route segment shipping policies minimizing the total logistics cost. The next best heuristics were the third, the second, and the first was the worst. Thus when shipping policies on adjacent arcs are coordinated, there is the least inventory buildup at the consolidation and deconsolidation centers, resulting in the attainment of the minimum total logistics cost. Optimizing the shipping policies of the consolidated route segments independently does result in at least some inventory buildup at the consolidation and deconsolidation centers

Figure 3.1: Heuristic Performance in Identifying Consolidated Route Shipping Policies (V=$1)

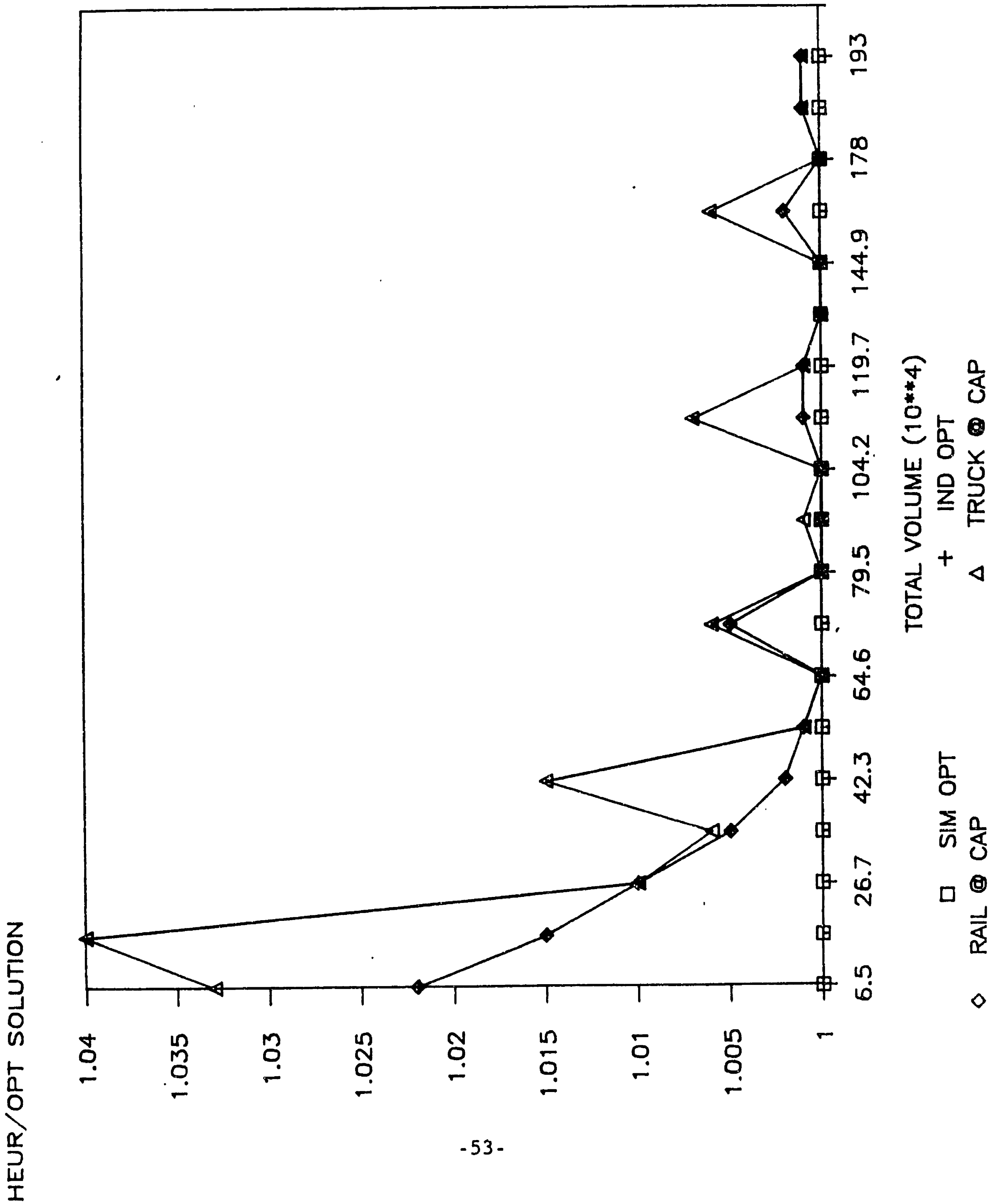
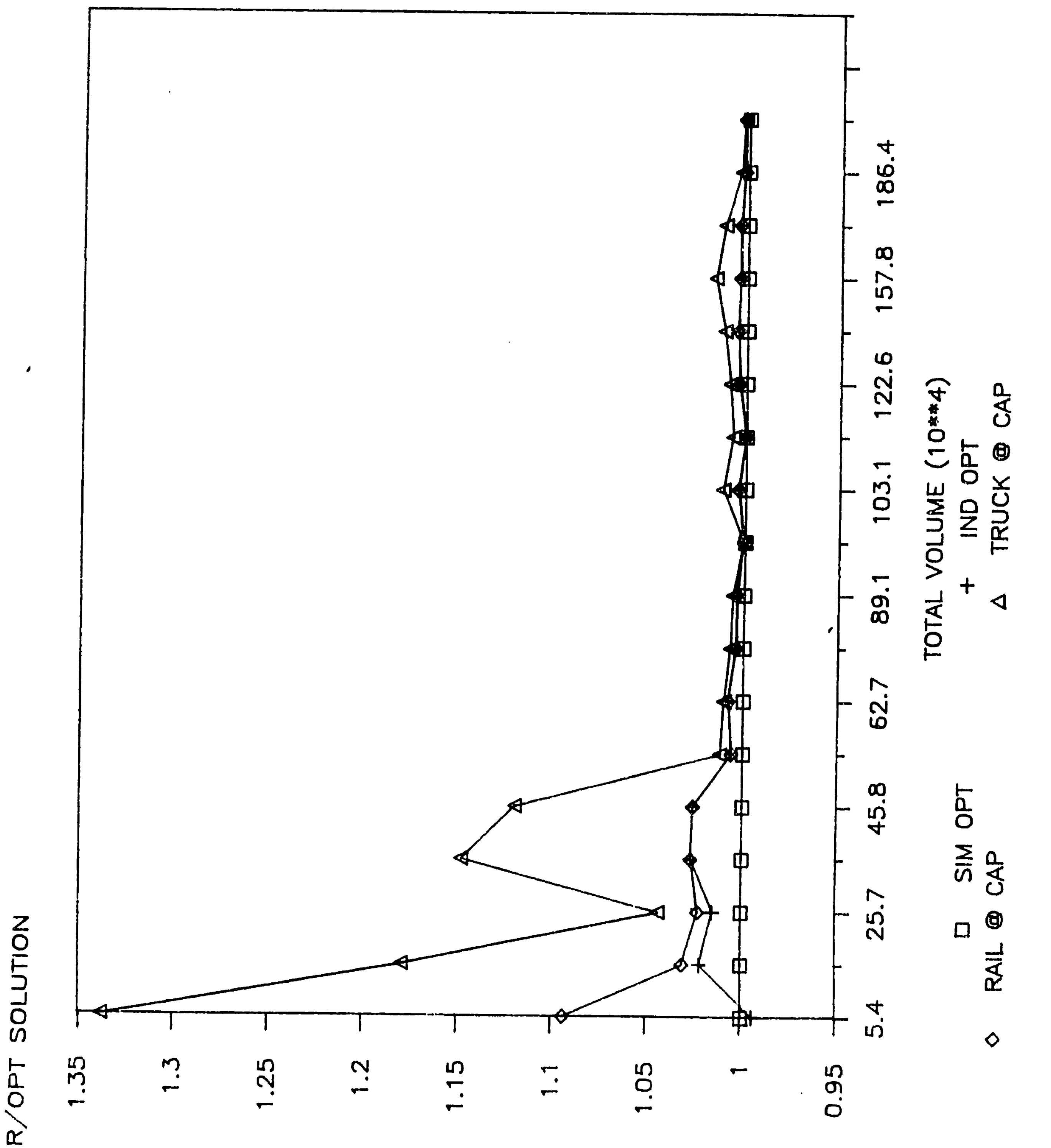


Figure 3.2: Heuristic Performance in Identifying Consolidated Route Shipping Policies (V=$5)



since it does not attain the lowest costs. The impact of the inventory carrying costs associated with this buildup on the minimum total logistics cost is small when commodity value is low, judging from relative performances in 3.1 as compared to 3.2. The assumption that the transportation mode of the consolidation-deconsolidation arc is capacitated seems more reflective of reality than the assumption that the modes of the origin-consolidation and deconsolidation-destination arcs are capacitated, since the second heuristic outperforms the first. As the volume of freight to be routed consolidated increases, the modes on all of the arcs become capacitated and as a result all of the heuristics presented converge to the same solutions.

As alluded to earlier, comparisons of the total logistics cost of different freight transportation strategies are fundamental to solution procedures for the Shipper's Problem. Cost evaluations in which calculating the cost of consolidated freight is vital take place very frequently and as such must be performed quickly and reliably. For these reasons the third heuristic, which performs comparably with the fourth heuristic but does not entail lengthy iterative procedures, will be the one employed in solving the Shipper's Problem. Further, the fact that it is the 'independently optimal' heuristic that is used in objective evaluation means that only dominating consolidation route arcs need be included in the network in a situation analogous to that of direct route arcs.

4. Solving the Shipper's Problem

The review of the literature in Chapter 2 yields two approaches to obtaining a solution to the Shipper's Problem: branch-and-bound and Jordan's heuristic. Section 4.1 of this chapter discusses efficient implementation strategies for these solution techniques, given the network representation of the problem developed in chapter 2, and the scheme for evaluating the objective function resulting from chapter 3. A third algorithm designed specifically to exploit the structure of the Shipper's Problem is also presented. Section 4.2 describes a simulation model which tests the performance of the different approaches on randomly generated networks. Finally, the performance of the three algorithms in solving the Shipper's Problem is evaluated.

4.1 Algorithms for Solving the Shipper's Problem

This section begins with a discussion of some properties of the Shipper's Problem that are instrumental in devising efficient algorithms.

The review of the literature indicates that the concave network problem is NP-hard. The review, however, also yields the result that an optimal flow in a concave network in an extreme flow. As alluded to in chapter 2, this second fact mitigates some of the implications of the first for the network configuration being examined here. In the Shipper's Problem a freight flow can only be routed one of two ways - directly between its origin and destination, or consolidated via a pair of transportation terminals. By the extreme flow characterization of an optimal solution, all of the freight being routed between a particular origin and destination is routed via one of the alternative paths. The optimal solution in the network is then a set of routing decisions involving each network freight flow, and the Shipper's Problem is thus cast as a combinatorial problem. For a Shipper's Problem involving the routing of F flows, this perspective yields 2^F extreme flow solutions, nonetheless it is used here to drive the development of algorithms for solving the Shipper's Problem.

The concavity of the total logistics cost of shipping on an arc as a function of the volume of freight transported on it, implies diminishing marginal cost for routing freight on an arc. Increasing the volume routed decreases both the marginal cost per unit and the average cost per unit of shipping freight on an arc. Minimizing the marginal and average total logistics costs per unit of shipping freight on an arc, then, are equivalent to maximizing the volume of freight transported on the arc. This yields potential search directions for improved solutions in algorithms developed to solve the Shipper's Problem.

The independence of evaluating the total logistics cost of routing a freight flow directly between its origin and destination, from all other freight transportation strategy calculations, was identified in chapter 3. The evaluation of the direct routing cost, DC(i,j), for each freight flow i-j involved in a Shipper's Problem, is assumed to be performed as a preprocessing step to the execution of any of the algorithms described. These values are used by the algorithms to measure the relative attractiveness of routing particular freight flows consolidated under different consolidation scenarios.

Three solution techniques tailored to solve the Shipper's Problem are presented next. The first, a branch-and-bound scheme to be examined in section 4.1.1, guarantees that the optimal freight transportation strategy is identified. The other two solution techniques presented do not necessarily find this globally optimal network freight transportation strategy, but instead hopefully determine near-optimal strategies. The first of these, a heuristic scheme based on the work of Jordan is discussed in section 4.1.2. The second, a new heuristic designed to exploit the restricted configuration of the network under study for the Shipper's Problem, is described in section 4.1.3. All of the algorithms use the scheme for evaluating the objective where shipping policies on consolidation route segments are optimized independently.

4.1.1 A Branch-and-Bound Scheme for the Shipper's Problem

A branch-and-bound scheme tailored to solve the Shipper's Problem can be derived from the problem properties discussed above. Branching rules follow directly from the extreme flow property of potential network solutions. As indicated above, however, there are 2^F potential extreme flow solutions to the Shipper's Problem. It is thus important to devise good bounding rules to make this method at all practical for even small problems. Bounding rules follow from the concavity of the arc costs of consolidated route segments.

The casting of the Shipper's Problem as a combinatorial problem using the extreme flow property of potential network solutions, makes it possible to adopt the branching scheme of the first-in-first-out, zero-one knapsack problem by Horowitz and Sahni [HORO78]. The branching scheme results in the construction of a tree with levels 0,1,2,...,F. Each arc of the tree represents a routing decision for a particular freight flow. There are 2^N nodes at level N of the tree for $0 \leq N \leq F$, since two arcs emanate from each node of levels 0,1,2,...,F-1, signifying the possibility of routing the next flow considered in the scheme either direct or consolidated. A node at level N has N predecessors such that N freight flow routing decisions are made in attaining N, and F-N routing decisions remain in attaining a complete freight transportation strategy. In following a branch of the tree from its root at level 0 to its leaf at level F, then, the routing of each freight flow is fixed as arcs are traversed such that an entire freight transportation strategy is available at the leaf. The scheme described thus far results in 2^F leaves - an enumeration of every extreme flow solution to the Shipper's Problem. It is up to the bounding procedure described next to prevent complete enumeration from occurring.

Instead of branching down to all of the leaves of the tree generated by the scheme presented above, it is desirable to perform calculations at tree nodes indicating from which nodes continued branching might yield the optimal solution, and from which nodes it will not. These calculations are the evaluations of lower and upper bounds on the total logistics cost of any of the leaf nodes for which a node serves as a predecessor. The least upper bound total logistics cost solution is saved as the incumbent at each iteration , such that if at any node a lower bound on total logistics cost is greater than this value, no further branching will be necessary from this node. This is the determination that a freight transportation strategy involving the routing decisions made to this point cannot be an optimal strategy since a solution that is better than any potential solutions involving these decisions has already been identified.

The concavity of arc costs of consolidated route segments is instrumental in determining lower and upper bounds on total logistics cost. At a node of level N, $0 \le N \le F$ of the branch-and-bound tree, routings of N of F freight flows have been fixed, while the possible routings of F-N remaining freight flows result in 2^{F-N} leaf nodes emanating from this node. Given the N fixed routings, if the F-N remaining flows are routed consolidated, the minimum average total logistics cost per unit is attained on the consolidated route segments, by concavity. The average total logistics cost per unit of routing these flows direct was obtained in preprocessing. A comparison of the direct and consolidated costs of routing for each flow allows the minimum average cost of shipping this freight flow to be contributed to a lower bound. This linearization or averaging of costs over the 2^{F-N} feasible freight transportation strategies emanating from this node, is conceptually similar to the scheme used by Soland for finding lower bounds. The identification of the minimum cost routing for each freight flow is also used in the determination of an upper bound at the node.

The evaluation of lower and upper bounds at a node of level N where $0 \leq N \leq F$ is outlined below:

step 1: Initialize the state of the network with the N fixed flows i-j:

FOR each of the N fixed flows:

IF flow i-j is to be routed consolidated,

Add the flow volume s_{ij} to the volumes $s_{i.}$, $s_{..}$, and $s_{.j}$, corresponding to the origin i -consolidation, consolidation-deconsolidation and deconsolidation-destination j consolidated route segments along which this flow will be transported.

ELSE

Add the cost of directly routing the flow, DC(i,j), to the lower bound.

END

END

step 2: Route each of the not-yet-fixed freight flows i-j consolidated:

FOR each of the F-N not-yet-fixed flows:

Add the flow volume s_{ij} to the volumes $s_{i.}$, $s_{..}$, and $s_{.j}$, corresponding to the origin i -consolidation, consolidation-deconsolidation and deconsolidation-destination j consolidated route segments along which this flow will be transported.

END

step 3: Determine the minimum average total logistics cost per unit, AC(i,j), of routing each of the freight flows i-j currently being consolidated. This is the sum of the minimum average total logistics costs per unit on the origin i-consolidation, consolidation-deconsolidation and deconsolidation-destination j arcs:

FOR each of the flows being routed consolidated:

$$
\begin{aligned}
AC(i,j) := & (a_{ic}/x_{ic} + VIt_{ic} \\
& + VI/2[x_{ic}/s_{i.} + x_{cd}/s_{cd}(1-1/k_{ic})]) \\
& + (a_{cd}/x_{cd} + VIt_{cd}) \\
& + (a_{dj}/x_{dj} + VIt_{dj} \\
& + VI/2[x_{dj}/s_{.j} + x_{dj}/s_{.j}(1-1/k_{cd}^{j})])
\end{aligned}
$$

IF flow i-j is one of the N fixed flows,

Add $AC(i,j)*s_{ij}$ to the lower bound

END

END

step 4: Complete the evaluation of the lower bound by identifying the minimum average cost routing for the freight flows not-yet-fixed in routing:

FOR each of the F-N not-yet-fixed flows:

IF $AC(i,j) \le DC(i,j)/s_{ij}$,

Add $AC(i,j)*s_{ij}$ to the lower bound

ELSE

Add DC(i,j) to the lower bound

END

END

step 5: Evaluate the upper bound by calculating the actual total logistics cost of the freight transportation strategy comprised of the N constrained freight flow routings, and the F-N flows unconstrained in routing at their minimum average cost routings as identified in step 4.

This bounding scheme generates bounds that are initially quite loose. As tree nodes at levels further down the tree are attained, the number of not-yet-fixed flows decreases and much tighter bounds are obtained. By the structured manner in which branching occurs, the fact that upper bounds are valid since they correspond to feasible freight transportation strategies, and the fact that lower bounds are valid by concavity, this branch-and-bound scheme will yield the minimum total logistics cost freight transportation strategy.

4.1.2 A Jordan Heuristic Scheme for the Shipper's Problem

An implementation scheme for Jordan's heuristic tailored to solve the Shipper's Problem can use the extreme flow property of optimal solutions and the concavity of arc costs to converge to what is hoped to be a good freight transportation strategy. The extreme flow property allows routing decisions to be made on a flow by flow basis. The routing decisions are based on a scheme very similar to that for lower bound evaluation in the branch and bound algorithm - one based on the concavity of the total logistics cost of shipping on an arc. The algorithm proceeds as follows, with every freight flow initially not-yet-fixed as to its routing:

step 1: Route each of the not-yet-fixed freight flows i-j consolidated:

FOR each of the not-yet-fixed flows:

Add the flow volume s_{ij} to the volumes $s_{i.}$, $s_{..}$, and $s_{.j}$, corresponding to the origin i -consolidation, consolidation-deconsolidation and deconsolidation-destination j consolidated route segments along which this flow will be transported.

END

step 2: Determine the minimum average total logistics cost per unit, AC(i,j), of routing each of the not-yet-fixed freight flows i-j consolidated. This is the sum of the minimum average total logistics costs per unit on the origin i-consolidation, consolidation-deconsolidation and deconsolidation-destination j arcs:

FOR each of the flows being routed consolidated:

$$
\begin{aligned}
AC(i,j) := & (a_{ic}/x_{ic} + VIt_{ic} \\
& + VI/2[x_{ic}/s_{i.} + x_{cd}/s_{cd}(1-1/k_{ic})]) \\
& + (a_{cd}/x_{cd} + VIt_{cd}) \\
& + (a_{dj}/x_{dj} + VIt_{dj} \\
& + VI/2[x_{dj}/s_{.j} + x_{dj}/s_{.j}(1-1/k_{cd}^{j})])
\end{aligned}
$$

END

step 3: Identify the minimum average cost routing for the freight flows not-yet-fixed in routing:

FOR each of the not-yet-fixed flows:

IF AC(i,j) > DC(i,j)/s_{ij},
Constrain flow (i,j) to be routed direct
END
END

IF none of the flow routings have changed since the last iteration,

```
        STOP
    ELSE
        GOTO step 1
    END
```

Jordan's heuristic finds the locally optimal freight transportation strategy at which no freight flow's average total logistics cost per unit of consolidated routing exceeds that of its direct routing.

4.1.3 A Different Scheme for the Shipper's Problem

In sections 4.1.1 and 4.1.2 of this chapter, solution approaches to the Shipper's Problem are developed in which the concave functions representing arc costs are linearized to provide search directions for improved solutions. The approach developed here relies not on average arc costs of shipping freight but on marginal costs of doing so. This approach, then, is motivated by the assertion made earlier - that in maximizing the volume of freight transported on an arc, the minimum marginal costs of shipping on the arc are attained. The algorithm developed here draws on this perspective of arc costs, on the extreme flow property of potential network solutions and on aspects of the network configuration, to solve the Shipper's Problem.

By the extreme flow property, the determination of an optimal freight transportation strategy solution in a Shipper's Problem is the identification of a set of freight flow routings that minimizes total logistics cost. Here, the routing of a particular freight flow is based on a comparison of the marginal total logistics cost of routing it consolidated and the cost of routing it direct. The determination of a marginal cost of consolidation is dependent on the functional form of the total logistics cost of consolidated freight.

Given the concavity of arc costs it might seem reasonable to expect the total logistics cost of consolidated freight - a function that is the sum of consolidation route segment arc costs - to be concave as well. This concavity would yield a structured manner in which a minimum total logistics cost freight transportation strategy could be pursued. Where an arbitrary flow is designated i_1-j_1 let:

$C_{pq}(s)$: The total logistics cost of shipping freight of volume s on consolidated route segment p-q.

$CC^+(i_1,j_1)$:
$$= C_{i\_1c}(s_{i\_1.}+s_{i\_1j\_1}) + C_{cd}(s_{..}+s_{i\_1j\_1}) + C_{dj\_1}(s_{.j\_1}+s_{i\_1j\_1}) + \sum_{i,i\neq i_1} C_{ic}(s_{i.}) + \sum_{j,j\neq j_1} C_{dj}(s_{.j})$$

The total logistics cost of consolidated freight where freight flow i_1-j_1 is consolidated.

$CC^-(i_1,j_1)$:
$$= C_{i\_1c}(s_{i\_1.}) + C_{cd}(s_{..}) + C_{dj\_1}(s_{.j\_1}) + \sum_{i,i\neq i_1} C_{ic}(s_{i.}) + \sum_{j,j\neq j_1} C_{dj}(s_{.j})$$

The total logistics cost of consolidated freight with freight flow i_1-j_1 not consolidated.

$MC(i_1,j_1)$:
$$= C_{i\_1c}(s_{i\_1.}+s_{i\_1j\_1}) - C_{i\_1c}(s_{i\_1.}) + C_{cd}(s_{..}+s_{i\_1j\_1}) - C_{cd}(s_{..}) + C_{dj\_1}(s_{.j\_1}+s_{i\_1j\_1}) - C_{dj\_1}(s_{.j\_1})$$

The marginal cost $CC^+(i_1,j_1) - CC^-(i_1,j_1)$ of consolidating the freight flow i_1-j_1.

$SA(i_1,j_1)$: $= MC(i_1,j_1) - DC(i_1,j_1)$

The savings associated with rerouting freight flow i_1-j_1 from its consolidated route to its direct route.

When $SA(i_1,j_1)>0$, $MC(i_1,j_1)>DC(i_1,j_1)$ and it is attractive to reroute consolidated freight flow i_1-j_1 direct. When $SA(i_1,j_1)<0$, $MC(i_1,j_1)<DC(i_1,j_1)$ and it is not attractive to perform this rerouting. When $SA(i_1,j_1)=0$, $MC(i_1,j_1)=DC(i_1,j_1)$, and the shipper would be indifferent about performing the rerouting.

The concept of savings suggests a manner in which a good freight transportation strategy might be identified. Route every freight flow consolidated, perform the calculations necessary to determine the savings associated with rerouting each flow i-j, and reroute flows for which $SA(i,j)>0$. Observations with respect to the relationship between marginal costs and savings are made next such that an algorithm based in these concepts can be more fully characterized.

The savings realized by rerouting a freight flow is a function of the volume of freight routed consolidated. By the concavity of the total logistics cost of consolidated freight, the marginal costs associated with consolidating any freight flow are at their lowest when the remainder of the shipper's freight is routed consolidated. Savings realized, then, are necessarily lower at this level of consolidated freight than when less freight is routed consolidated. This stems from the fact that for the constant cost of routing a particular flow direct, the marginal costs associated with consolidating the flow are greater when there is a lesser level of consolidation and thus the value MC(i,j)-DC(i,j) increases. This observation has some important implications.

The fact that rerouting a flow i_1-j_1 direct does not yield positive savings at a particular level of freight consolidation does not mean that it is routed consolidated in the optimal freight transportation strategy. Letting the superscripts of the volumes s indicate different levels of consolidation with 1 representing a lesser level of consolidation than 0 such that $s_{i\_1.}^1 \leq s_{i\_1.}^0$, $s_{..}^0 < s_{..}^0$, and $s_{.j\_1}^1 \leq s_{.j\_1}^0$, the following is possible:

$$C_{i\_1c}(s_{i\_1.}^1 + s_{i\_1j\_1}) - C_{i\_1c}(s_{i\_1.}^1) \geq C_{i\_1c}(s_{i\_1.}^0 + s_{i\_1j\_1}) - C_{i\_1c}(s_{i\_1.}^0)$$

$$C_{cd}(s_{..}^1 + s_{i\_1j\_1}) - C_{cd}(s_{..}^1) > C_{cd}(s_{..}^0 + s_{i\_1j\_1}) - C_{cd}(s_{..}^0)$$

$$C_{dj\_1}(s_{.j\_1}^1 + s_{i\_1j\_1}) - C_{dj\_1}(s_{.j\_1}^1) \geq C_{dj\_1}(s_{.j\_1}^0 + s_{i\_1j\_1}) - C_{dj\_1}(s_{.j\_1}^0)$$

$$\Rightarrow MC(i_1,j_1)^1 > MC(i_1,j_1)^0$$

If $MC(i_1,j_1)^1 > DC(i_1,j_1) > MC(i_1,j_1)^0$, then, freight flow i_1-j_1 should be routed consolidated at level 0 but direct at level 1 of consolidation. The fact that positive savings are not attained from rerouting a particular flow direct at a particular level of freight consolidation, then, does not mean that if other flows are rerouted it will remain most attractive to route this flow consolidated.

The discussion above can be extended to the consideration of multiple reroutings. At a particular level of consolidation, the rerouting of any individual freight flow might not result in the realization of positive savings, however the rerouting of some combination of flows from consolidated to direct might. This is true because the removal of a larger volume of freight from consolidation than that of any of the individual freight flows, can result in a large enough increase in marginal costs to offset the direct costs of these flows (figure 4.1).

Perhaps the most important implication of the relationship between marginal costs and savings is that it can be shown that a freight flow rerouted direct at some level of freight consolidation will always be routed direct in the optimal freight transportation strategy. This is the case because as argued above, positive savings can only become more positive if initially all freight flows are routed consolidated, the removal of these flows from consolidation is considered, and the consolidation of new flows is not. Using the same notation conventions as above, more formally:

If at level of consolidation 0, flow i_1-j_1

has $MC(i_1,j_1)^0 > DC(i_1,j_1)$ and should be routed direct

And at level of consolidation 1, flow i_1-j_1

has $MC(i_1,j_1)^1 < DC(i_1,j_1)$ and should be routed consolidated

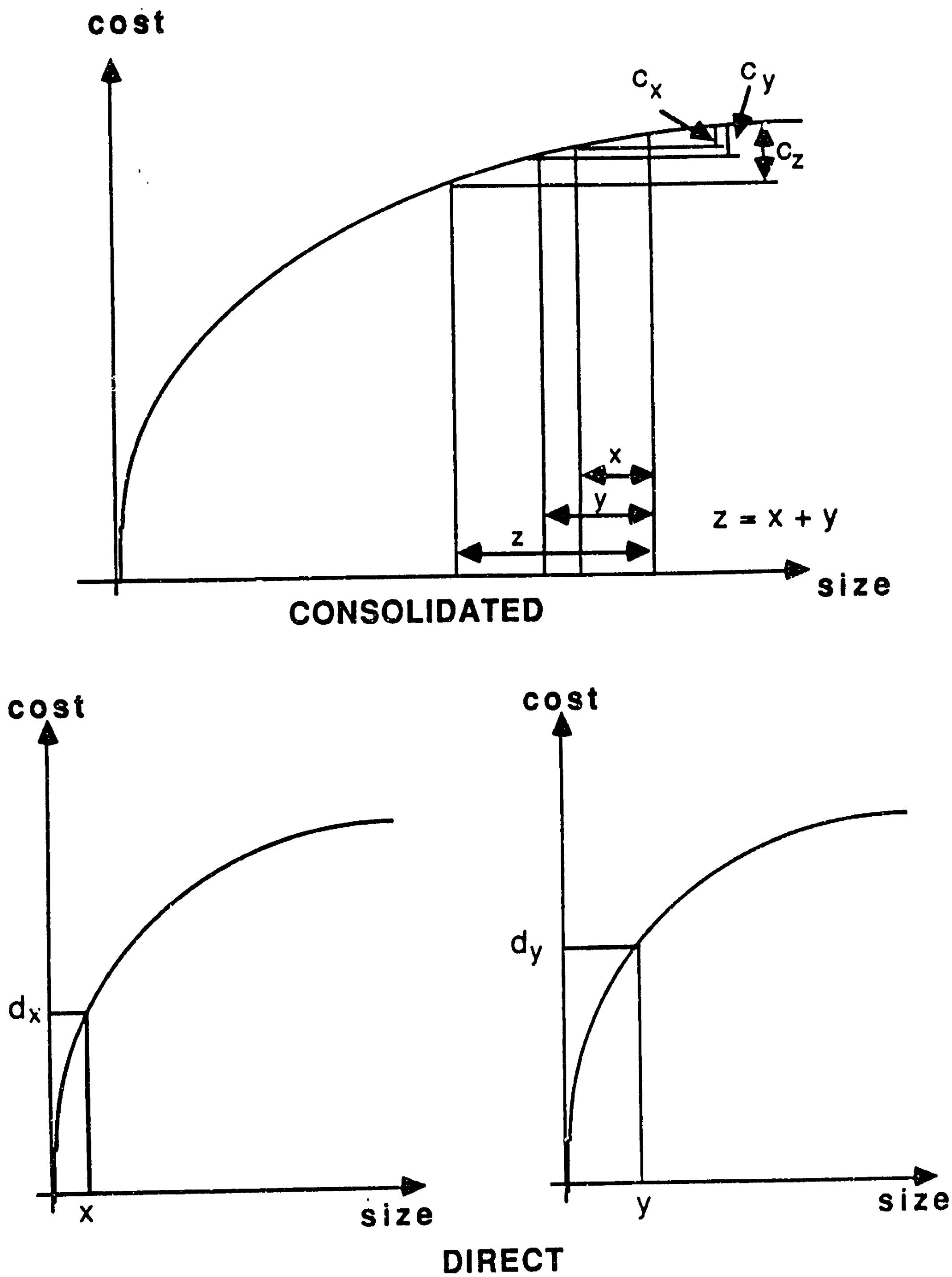
Then, there is a contradiction since by concavity,

$$MC(i_1,j_1)^1 > MC(i_1,j_1)$$

Thus if it becomes attractive to reroute a freight flow direct, it will remain attractive to do so.

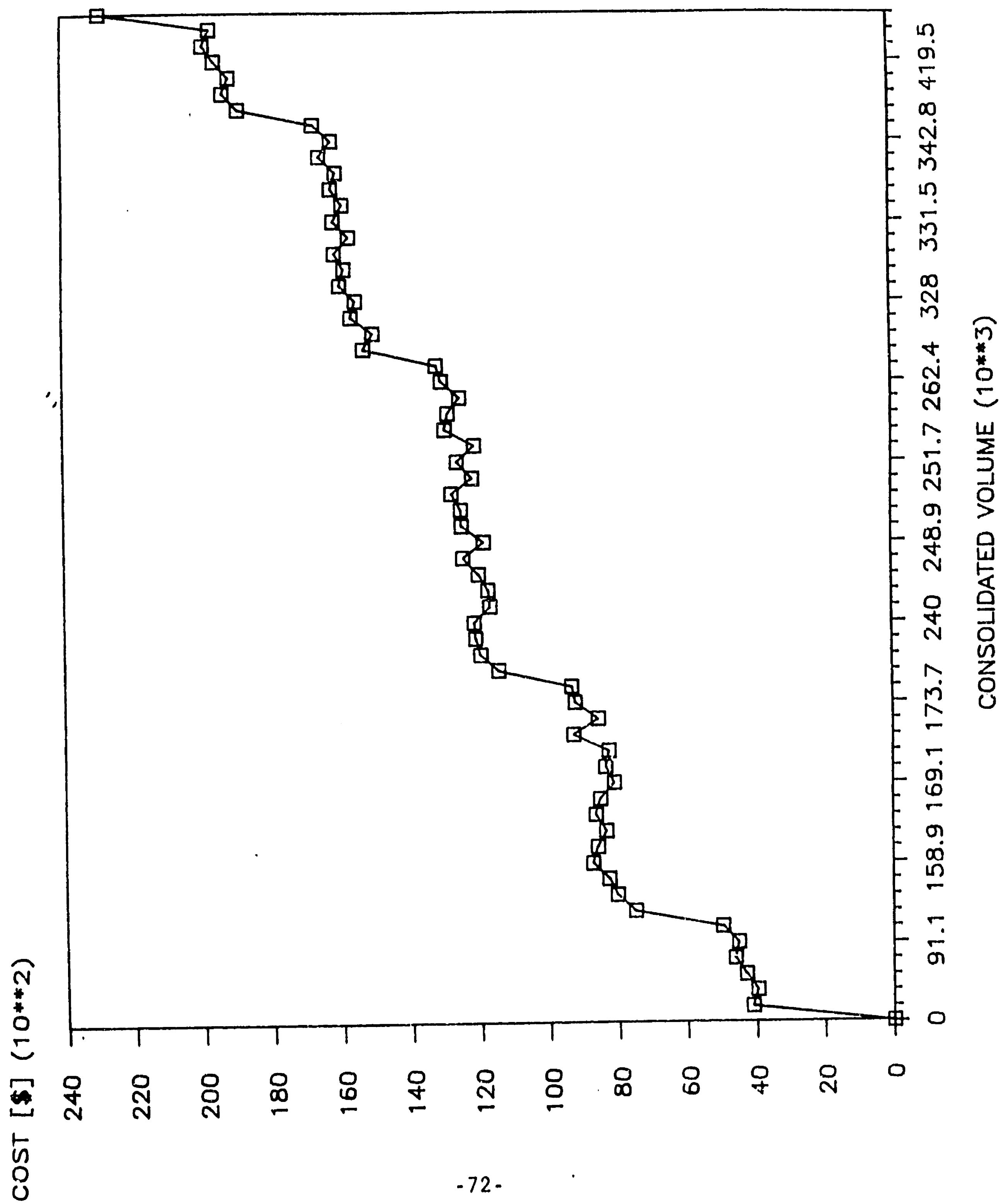
The entire analysis presented thus far rests on the concavity assumption of the total logistics cost of consolidated freight as a function of freight flow. This assumption is not strictly valid, however, since although the arc costs of consolidation route segments are concave when considered in isolation, the interaction of the shipping policies of these segments leads to non-concavity of the overall function when these segments comprise consolidation routes. This is evidenced in the graph of figure 4.2 of the total logistics cost of consolidated freight for a three origin-two destination-six commodity Shipper's Problem. Two types of non-concavity are manifested in the total logistics cost function. The first translates into the composition of the graph of disjoint segments. The consolidation of a variable number of freight flows involving a variable number of origins and destinations results in a variable number of origin and destination-associated terms contributed to the total logistics cost of consolidated freight function. The second type of non-concavity manifests itself within the segments of the graph and is due to the same factors that lead the optimal origin-consolidation shipping policies to differ among origins and optimal deconsolidation-destination ship-

Figure 4.1: Total Logistics Cost Reduction Via Rerouting a Pair of Freight Flows



$d_x > c_x$; $d_y > c_y$; $c_x + c_y < d_x + d_y < c_z$

Figure 4.2: Non-concavity of the Total Logistics Cost of Consolidated Freight



ping policies to differ among destinations- differences in volume of freight associated with the node S, and in node-transportation terminal distances.

The development of an iterative algorithm in which a comparison of the direct costs and the marginal costs of consolidation of freight flows determine the manner in which they should be routed, has been motivated. The algorithm identifies the optimal freight transportation strategy if it attempts to reroute combinations of one,two,three,...,F freight flows until total logistics cost cannot be reduced further by rerouting some combination of freight flows, by concavity. The running time of such an algorithm would be high because in the worst case, where no rerouting is attractive, the attempts at rerouting $O(2^F)$ combinations of freight flows would have to be made before this was discovered. In addition, the validity of the algorithm rests on the assumption that the total logistics cost of consolidation is concave - found not to hold in a strict sense. The assumption of concavity of the total logistics cost of consolidated freight is actually not as bad as would seem from figure 4.2. The problem on which this graph is based consists of only six freight flows. For problems having a larger number of freight flows, non-concavities appear much less significant. This is because proportionately the fluctuations in total logistics cost of consolidated freight become much smaller. What follows next is the development of a heuristic to solve the Shipper's Problem (referred to as the Shipper heuristic). It is a heuristic in two senses. First, it assumes concavity of the total logistics cost of consolidated freight. Second, it does not attempt to reroute all of the combinations of one, two, ..., F freight flows in order to attain the minimum total logistics cost freight transportation strategy.

Given the Shipper's Problem network, positive savings arise from rerouting a consolidated freight flow direct if the economies of scale realized in its consolidation do not compensate for the circuity of its consolidation route relative to its direct route. This is the situation if the origin-consolidation and/or deconsolidation-destination distances of the freight flow consolidation route are great relative to the origin-destination distance. Positive savings can also result when relatively large volumes of freight are associated with a node. In this case, economies of scale realized via consolidation of the flow are small compared to circuity costs of consolidated over direct routing. Positive savings, then, are a function of relative distance and relative volume, both of which can be used to characterize nodes. Nodes, as a function of their location in the network and the volume of their associated freight flows, can be looked upon as being more or less likely to be involved in direct routing. A ranking of nodes can thus be generated as a function of the savings associated with their flows. For the freight flows of nodes with higher ranking (higher savings), direct routing is probably attractive. While for the freight flows of nodes with lower ranking (lower savings), consolidated routing is probably attractive. These rankings will capture as a function of the nodes, the hard to quantify network characteristics of relative distance and relative volume. An algorithm driven by this ranking of nodes should promote greater efficiency because it attempts to reroute the freight flows with the greatest potential savings first. By concavity, this strategy should converge quickly.

The Shipper heuristic initially consolidates all of the network freight flows. It then calculates the savings that would be realized by rerouting each individual origin-destination flow directly. It orders the origin-destination flows according to decreasing associated savings. This ordering is used to rank origins and destinations in decreasing likelihood of having associated freight flows routed direct. This ranking is now used to drive the attempts to find a good freight transportation strategy. Beginning with the highest ranked node, either an origin or a destination, the rerouting of single freight flows in which this node is involved is attempted and performed when rerouting yields positive savings. If any freight flows involving this node remain consolidated after the attempted reroutings of single flows, reroute any pairs of freight flows involving this node where positive savings are yielded. When all of the favorable single and pair reroutings have been performed for the highest ranked node, perform the same procedure for the next ranked node. This process continues until every origin and destination has been examined.

The algorithm thus begins with the freight transportation strategy of routing everything consolidated, potentially passes through a sequence of other freight transportation strategies, and terminates with the best one of the sequence. Formally, the algorithm proceeds as follows:

step 1: Route each of the not-yet-fixed freight flows i-j consolidated and find the total logistics cost of consolidating every freight flow, CC:

FOR each flow:

Add the flow volume s_{ij} to the volumes $s_{i.}$, $s_{..}$, and $s_{.j}$, corresponding to the origin i -consolidation, consolidation-deconsolidation and deconsolidation-destination j consolidated route segments along which this flow will be transported.

END

Evaluate CC given the volumes s found

step 2: Evaluate the savings associated with rerouting each consolidated freight flow i-j direct individually. When all of the savings have been determined, order them from greatest to least:

FOR each of the flows being routed consolidated:

```
Evaluate CC_(i,j)
MC(i,j) := CC - CC_(i,j)
SA(i,j) := MC(i,j) - DC(i,j)
```

END

Order the SA(i,j) 's found from greatest to least

step 3: Generate rankings for the network origins and destinations where the flows are ordered such that

$SA(i_1,j_1) \geq SA(i_2,j_2) \geq \ldots \geq SA(i_F,j_F)$

Letting ORank[i] and DRank[j] be the rankings of origin i and destination j respectively, and OF_i, DF_j be the number of flows emanating from origin i and the number of flows terminating at destination j respectively:

```
ORank[i] := 0, for all origins i
DRank[j] := 0, for all destinations j

FOR P := 1 TO F:

  Where SA(iP,jP),
  Find origin i=iP and destination j=jP
  ORank[i] := ORank[i] + (F-P+1)
  OFi := OFi + 1
  DRank[j] := DRank[j] + (F-P+1)
  DFj := DFj +1

END

ORank[i]:= ORank[i]/OFi for all origins i
DRank[j]:= DRank[j]/DFj for all destinations j
```

step 4: Using the node ranking to drive the search, reroute single flows and then pairs of flows where favorable, to find successively lower total logistics cost freight transportation strategies. Where there are NO origins and ND destinations and the rankings have been ordered such that

$ORank[i_1] \geq ORank[i_2] \geq \ldots \geq ORank[i_{NO}]$

$DRank[j_1] \geq DRank[j_2] \geq \ldots \geq DRank[j_{ND}]$.

Further assume $DRank[j_1] \geq ORank[i_1] \geq \ldots$ Then the sequence of flows in which rerouting attempts will be made and in which rerouting will be performed (singly or in pairs of flows) if positive savings would result is:

1) Driving Node: j_1

i_1-j_1, i_2-j_1,...,i_{NO}-j_1

2) Driving Node: i_1

i_1-j_1, i_1-j_2,...,i_1-j_{ND}

etc.

until each origin and destination has been the examined.

This heuristic should perform quite well for the Shipper's Problem for a number of reasons. By attempting to reroute flows that emanate from the highest ranked origins and terminate in the highest ranked destinations first, the greatest savings reroutings are aimed for first - a strategy that seems promising. Sequential rerouting attempts involving the same node would seem to be efficient because the reroutings of these flows impact each other significantly since these flows share two consolidation route segments. In section 4.2, the performance of this heuristic is evaluated.

4.2 Simulation of the Algorithms on Randomly Generated Networks

In order to evaluate the performance of algorithms devised for the Shipper's Problem, the algorithms are applied to randomly generated networks. In this way comparative statistics with respect to the quality of the solutions attained by the algorithms, and their running times are obtained.

A simulation is developed incorporating three variants of the Shipper heuristic, the Jordan heuristic and a branch and bound scheme. Three variants of the Shipper heuristic are included such that it is possible to judge whether a scheme as complicated as the one described in section 4.1.3 is necessary, or whether performing only a subset of the steps therein yields as good a result. The first variant of the heuristic consolidates all origin-destination flows, proceeds by attempting to reroute individual freight flows until no more of these can be rerouted, and then attempts to reroute pairs of flows until no more of these can be rerouted. This heuristic will be referred to as SP for Singles-Pairs. The second variant of the heuristic performs a larger subset of the steps described in section 4.1.3. All freight flows are initially routed consolidated, and the potential savings that would be realized by rerouting each of the individual flows are evaluated. Freight flows are ordered in decreasing order of savings. Only now does the attempted rerouting of first individual and then pairs of ordered freight flows proceed. This heuristic will be referred to as SSP for Savings-Singles-Pairs. The third variant of the Shipper heuristic is the one described in section 4.1.3. It will be referred to as SRSP for Savings-Rank nodes-Singles-Pairs.

The Jordan heuristic scheme applied to the random networks is the one described in section 4.1.2, to be referred to as JOR. Finally, the branch-and-bound scheme is the one described in section 4.1.1 with one enhancement. Instead of sequencing the flow routing decisions for the different freight flows randomly when constructing the branch-and-bound tree, the concept of flows ordered by savings is borrowed from the Shipper heuristic and the sequence of flows considered in tree construction is that of greatest to least characteristic savings. In this way it is hoped that tighter upper bounds will be attained higher in the tree such that the bounding of branches with no potential to produce optimal freight transportation strategies can take place early on, reducing the tree size significantly. This solution approach will be referred to as BB.

During the course of simulation, networks are generated randomly with respect to the number of freight origins and freight destinations involved in shipping, the location of these nodes with respect to each other and the consolidating and deconsolidating transportation terminals (although the basic configuration of the network described in chapter 2 - sets of origins and destinations separated by two transshipment nodes, stays constant), and with respect to the freight flow volumes shipped between different origins and destinations.

Looking at the cost components being employed in the objective function, it seems reasonable to expect that flows of larger volume might generally be routed directly. This is expected since the proportionately smaller economies they realize via consolidation might not outweigh the circuity of their consolidated routes. In addition, freight flows of a higher value commodity might be more likely to be shipped directly since inventory costs become a larger portion of total logistics costs. Consolidated routes usually have higher inventory cost because of the time spent at the consolidation centers. Both of these phenomena are very much a function of all of the parameters characterizing Shipper's networks. Here, the value of the commodity and the order of magnitude of volumes shipped in the network are altered between simulations. In this way the performance of SP, SSP, SRSP, JOR, and BB can be evaluated in contexts where flows are more likely to be routed consolidated, and where flows are more likely to be routed direct.

The simulation and the algorithm were implemented on an AT class microcomputer. There are difficulties with the branch-and-bound algorithm in Shipper's Problems that involve many freight flows, especially where proportionately many of the freight flows should be routed directly in an optimal strategy. In many of these cases, branch-and-bound trees grow extremely large, causing memory limitations to be exceeded in the execution of the algorithm. For problems involving less that thirty-six freight flows and a commodity of value \$0.50 or of value \$1.00, no difficulties are encountered. Shipper's Problems consisting of less than thirty-six freight flows will thus be referred to as small problems. Shipper's Problems involving more than thirty-six but less than two hundred and twenty-five freight flows (a network where freight is shipped from each of fifteen origins to each of fifteen destinations) will be referred to as large problems. In order to overcome the BB difficulties in large problems, the best solution attained from the execution of the heuristics is used as the initial incumbent of the BB in the hope that this measure will reduce potential tree growth, and allow the BB algorithm to execute to completion. For Shipper's Problems involving a commodity of value $0.50, this was successful. For problems involving a commodity of value $1.00, however, more flows are likely to be routed direct as discussed above. The lower bounding scheme in BB provides loose bounds initially in these instances, permitting the BB tree to grow large as few branches can be bound. In these problems even using a good solution as a starting point wasn't enough to make BB practical.

The total logistics costs of the two extreme freight transportation strategies of routing every flow consolidated and routing no flow consolidated are calculated prior to attempting to solve the problem using any of the solution approaches. After the execution of any of the heuristics, the heuristic solution attained can be compared to these two initial solutions and the best solution retained. For this reason, the results to be presented next involve only Shipper's Problems where some intermediate freight transportation strategy is optimal. The fact that Jordan's heuristic sometimes reroutes some freight flows when all freight flows should be routed consolidated, and that the three variants of the Shipper heuristic often do not reroute enough freight flows direct when all freight flows should be routed direct, will thus be ignored.

The graphs of figures 4.3, 4.4, 4.5 and 4.6 indicate that given the optimal freight transportation strategy as determined by the branch-and-bound scheme, the Shipper heuristic variants perform extremely well, always identifying a strategy having cost that is within one percent of optimal and often identifying the optimal freight transportation strategy itself. The Jordan heuristic scheme performs less well on the same randomly generated networks, attaining strategies that have cost up to seven percent greater than the minimum total logistics cost. The graph of figure 4.6 seems to permit the same conclusions to be drawn with respect to algorithmic performance although here the optimal freight transportation strategies are unknown because of the difficulties with BB discussed above.

The three variants of the Shipper heuristic are all based on the premise that total logistics cost of consolidated freight is a concave function of volume of freight consolidated. The success of these heuristics suggests that this assumption was not a bad one. Strict concavity does not actually hold, however, as was asserted previously. This is evidenced by the results of the graph of figure 4.3. This graph indicates that there are cases when one of the three variants of the Shipper heuristic will outperform the others. These are cases when the suboptimal heuristics have rerouted 'too much' consolidated freight direct. They have skipped over the optimal level of consolidation by considering the rerouting of freight flows in the wrong order. Looking back at the irregularity of the segments in the graph

Figure 4.3: Algorithmic Performance in Solving the Shipper's Problem (small problems, V=$.50)

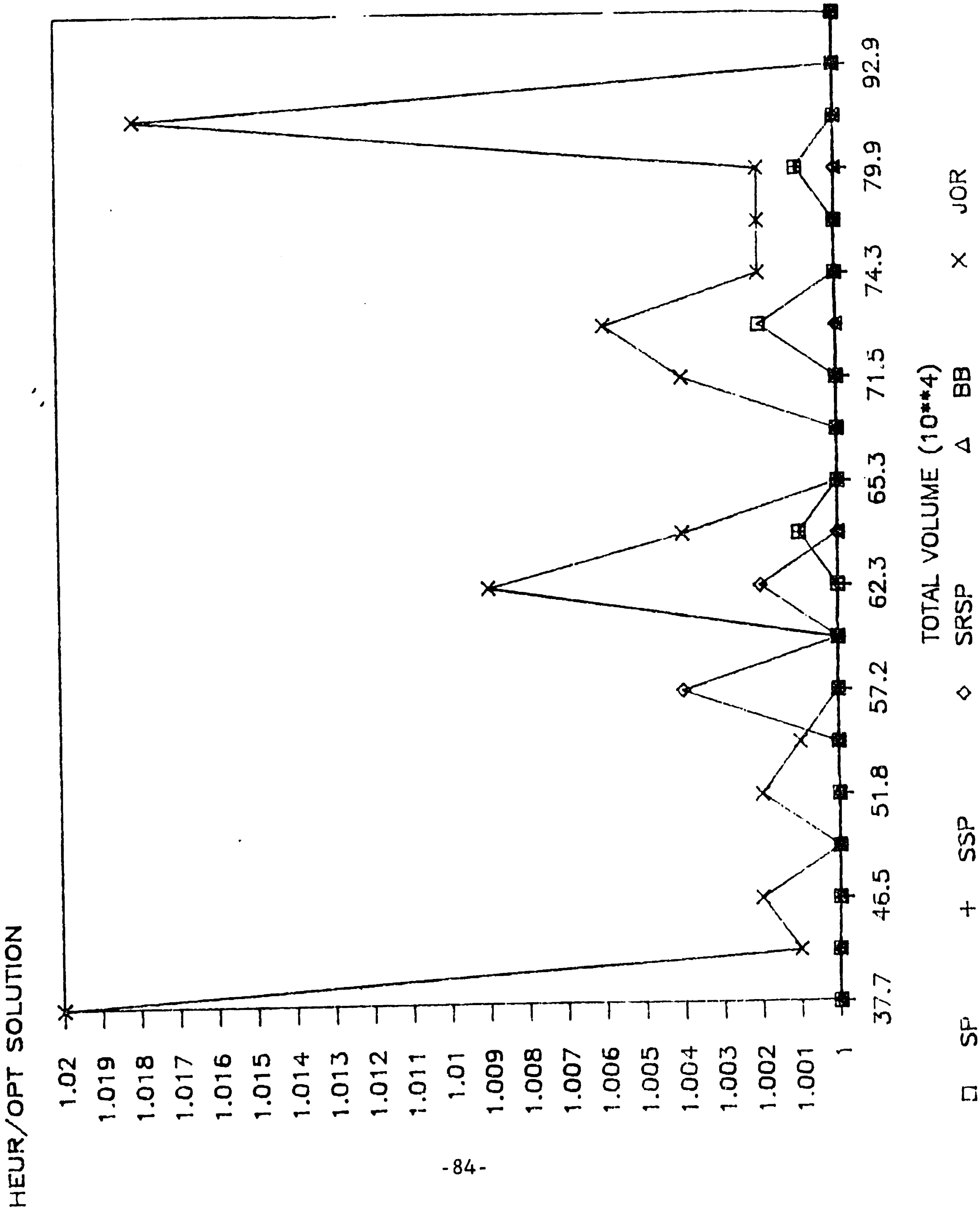


Figure 4.4: Algorithmic Performance in Solving the Shipper's Problem (small problems, V=$1.00)

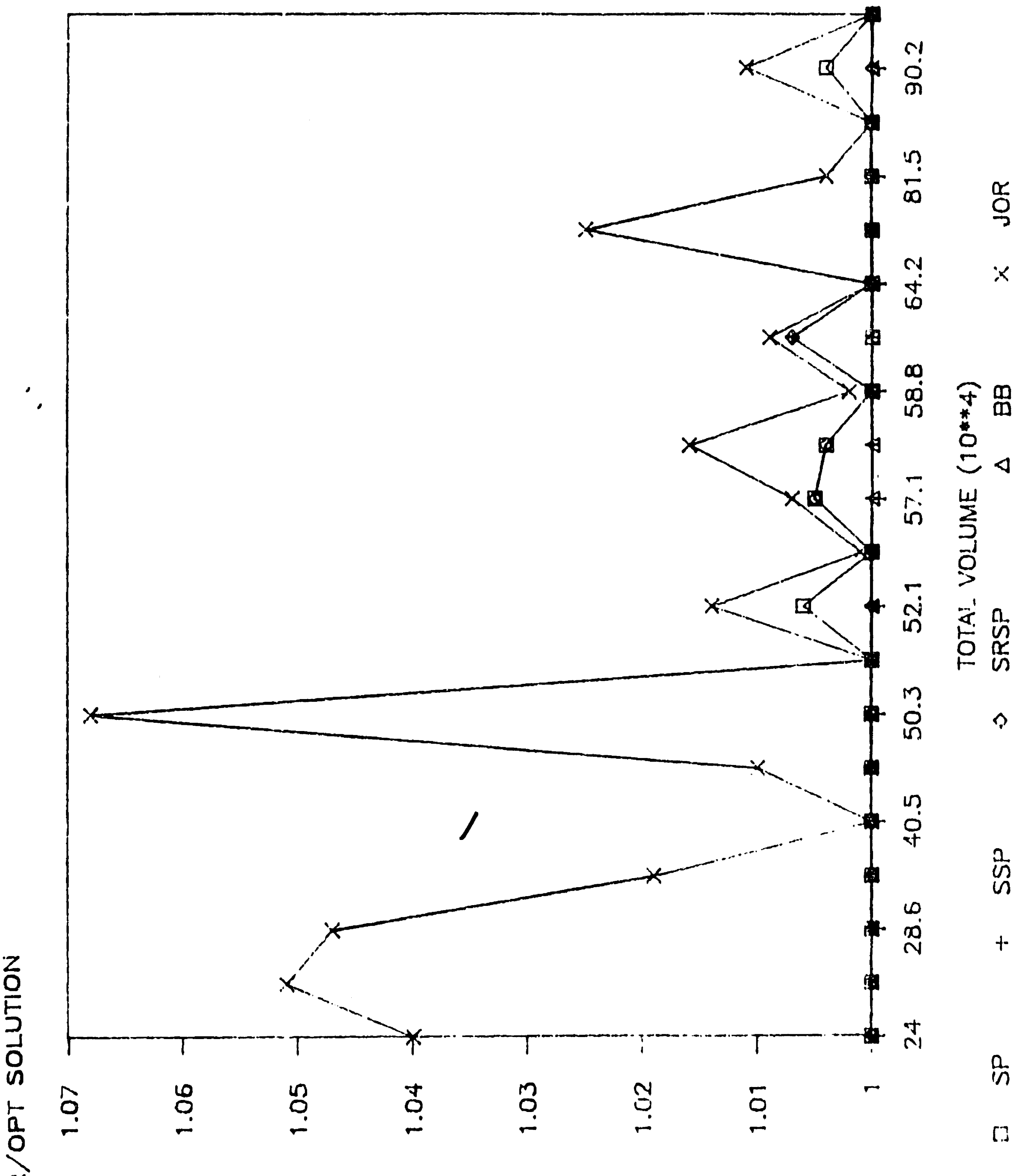


Figure 4.5: Algorithmic Performance in Solving the Shipper's Problem (large problems, V-$.50)

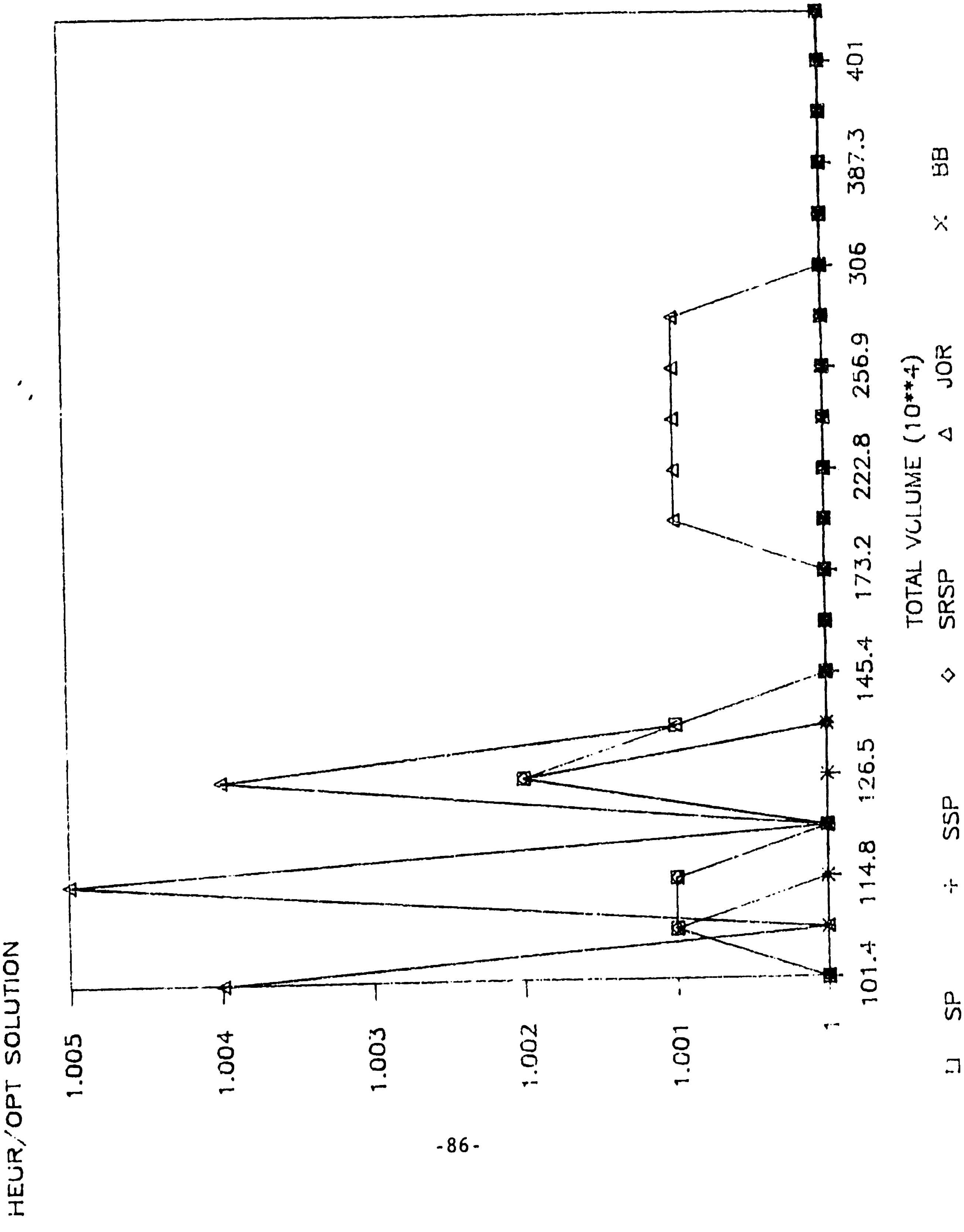
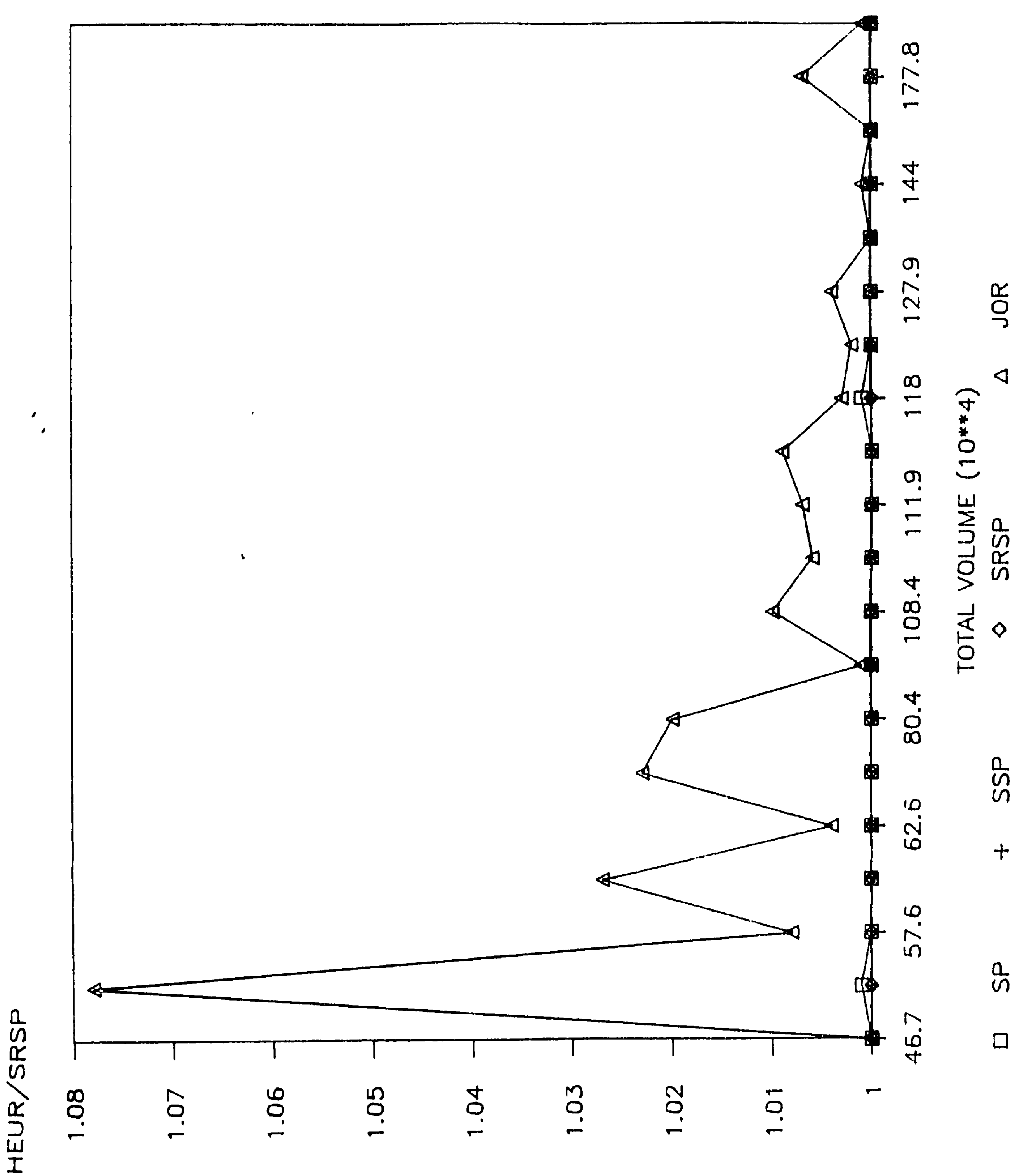


Figure 4.6: Algorithmic Performance in Solving the Shipper's Problem (large problems, V=$1.00)



of figure 4.2, it becomes clear that the proposed heuristics could get trapped in a suboptimal local minimum via an unfortunate sequence of successful freight reroutings. Fortunately, lack of strict concavity does not seem to significantly affect the performance of the heuristics. Further, as Shipper problem size increases, concavity is more reflective of reality as is evidenced by the heuristic results for large problems graphed in figure 4.5. In the largest fourteen problems solved, the identical solutions which were also the optimal solutions were attained by each of the three heuristics.

Running times of algorithms are a function of the number of freight flows involved in a particular Shipper's Problem as well as the proportion of flows that should be routed directly in an optimal freight transportation strategy. This is the case because as consolidated flows are rerouted directly, they drop from consideration, thereby reducing the size of the problem to be solved. Jordan's heuristic runs consistently faster than any other algorithm for the Shipper's Problem. Its longest execution time was three seconds, even for problems it took some of the other algorithms hours to solve. The next fastest algorithm is SRSP which ran consistently faster than any of the other algorithms as is evidenced in figures 4.7, 4.8, 4.9, and 4.10. It is interesting to note in figures 4.9 and 4.10, that all of the Shipper heuristic variant running times displayed a higher rate of growth than that of SRSP. The running time of SSP is usually lower than that of SP, although the two running times are generally of the

Figure 4.7: Execution Time of Algorithms for Solving the Shipper's Problem (small problems, V=$.50)

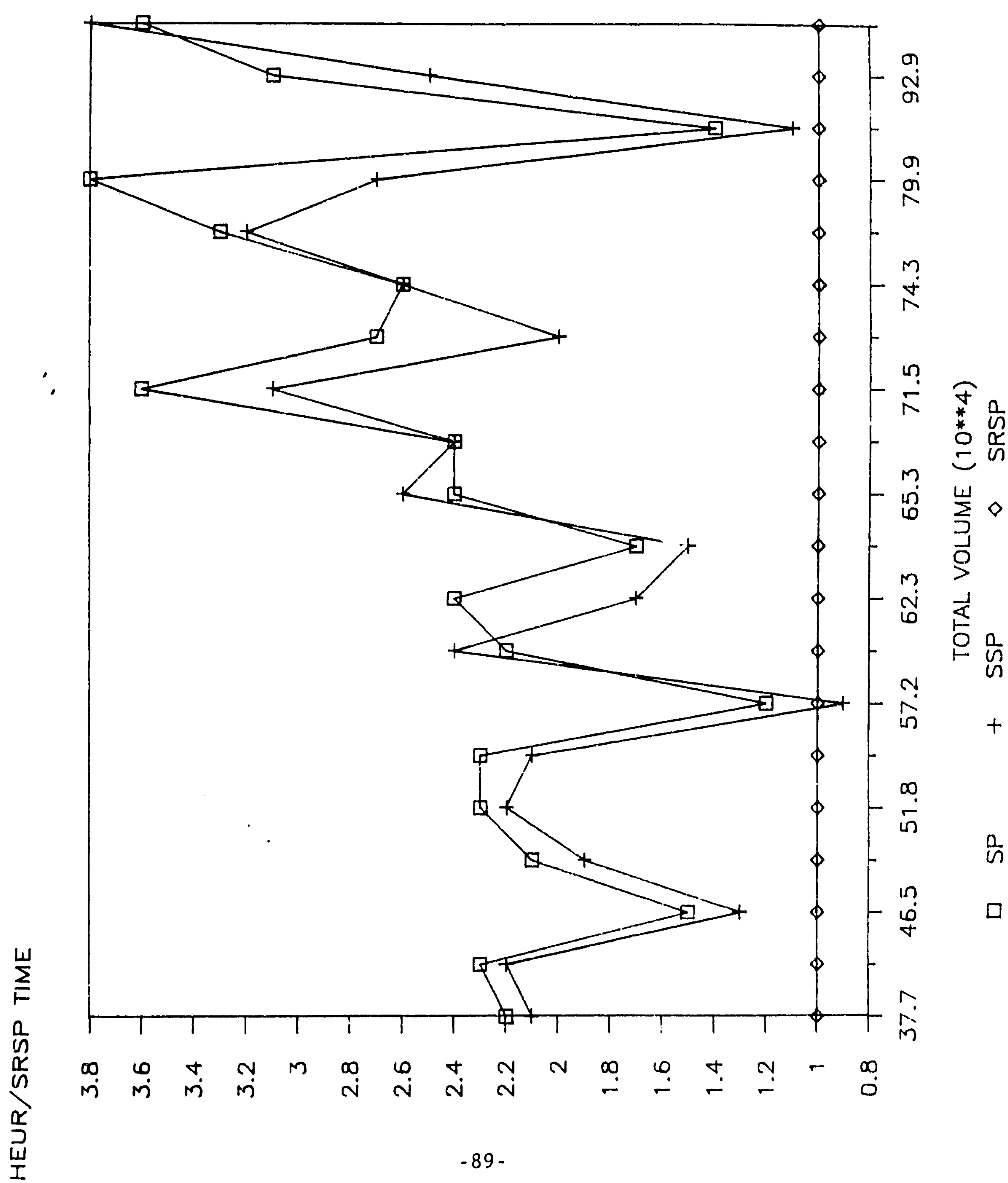


Figure 4.8: Execution Time of Algorithms for Solving the Shipper's Problem (small problems, V=$1.00)

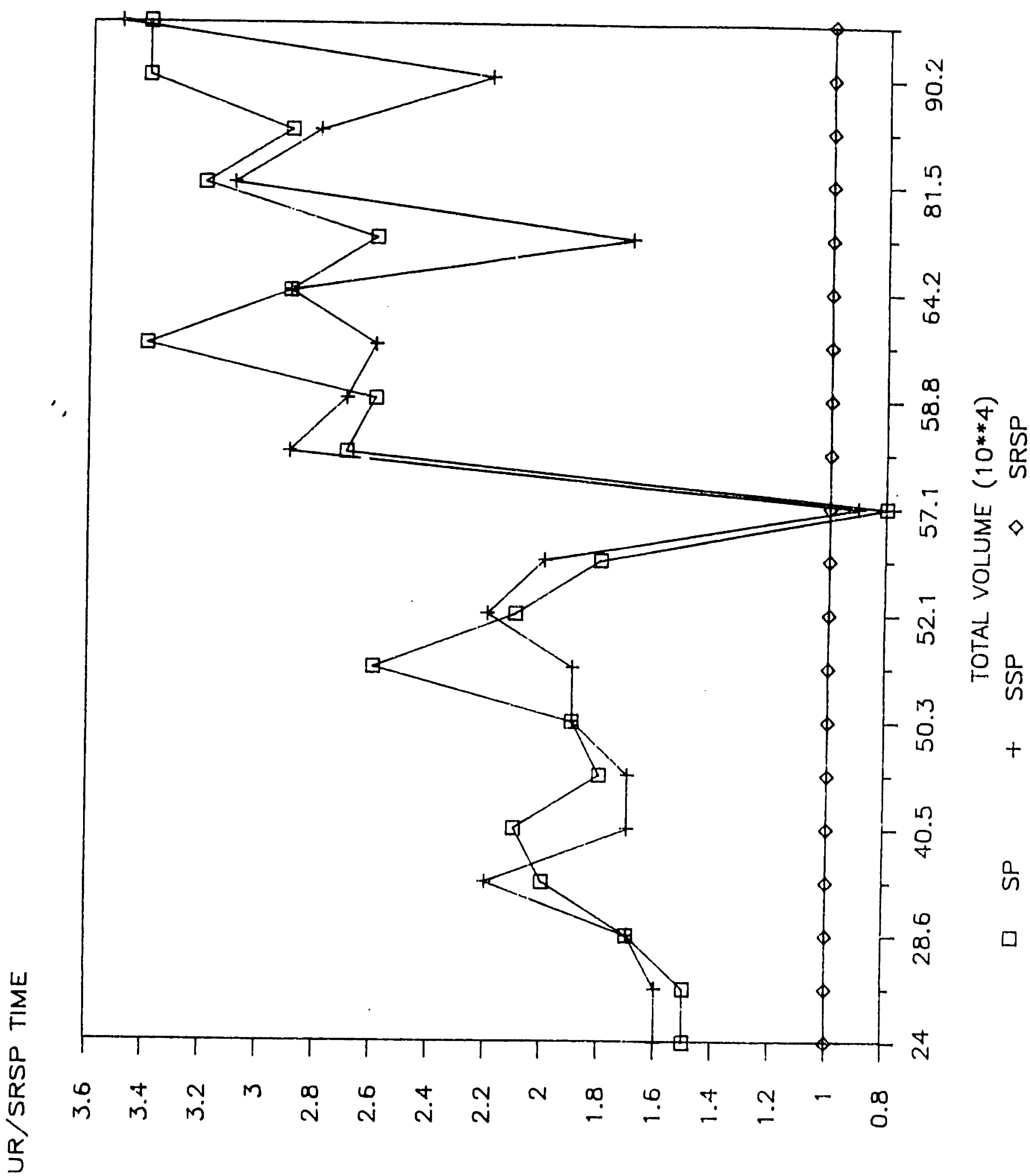


Figure 4.9: Execution Time of Algorithms for Solving the Shipper's Problem (large problems, V=$.50)

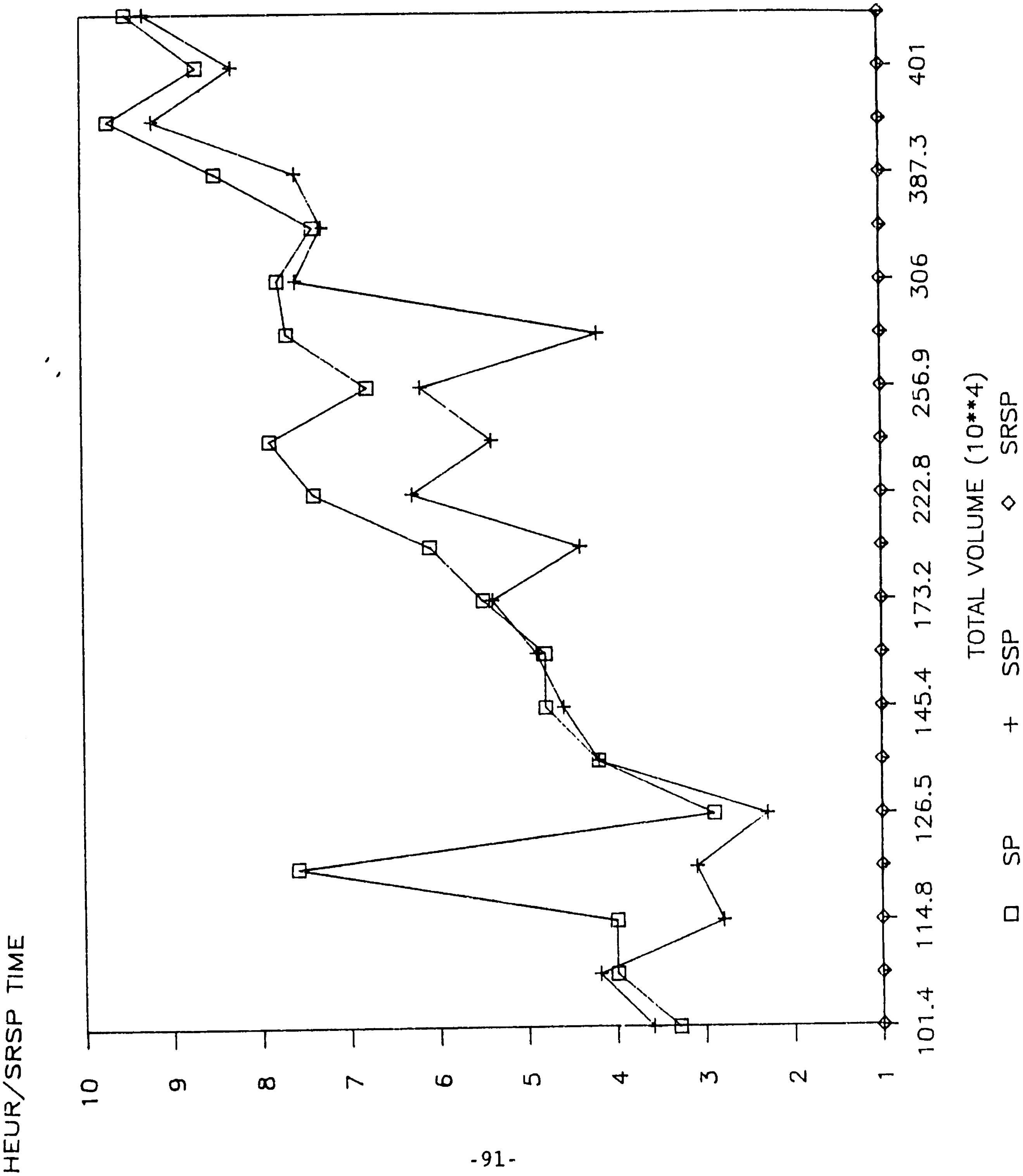
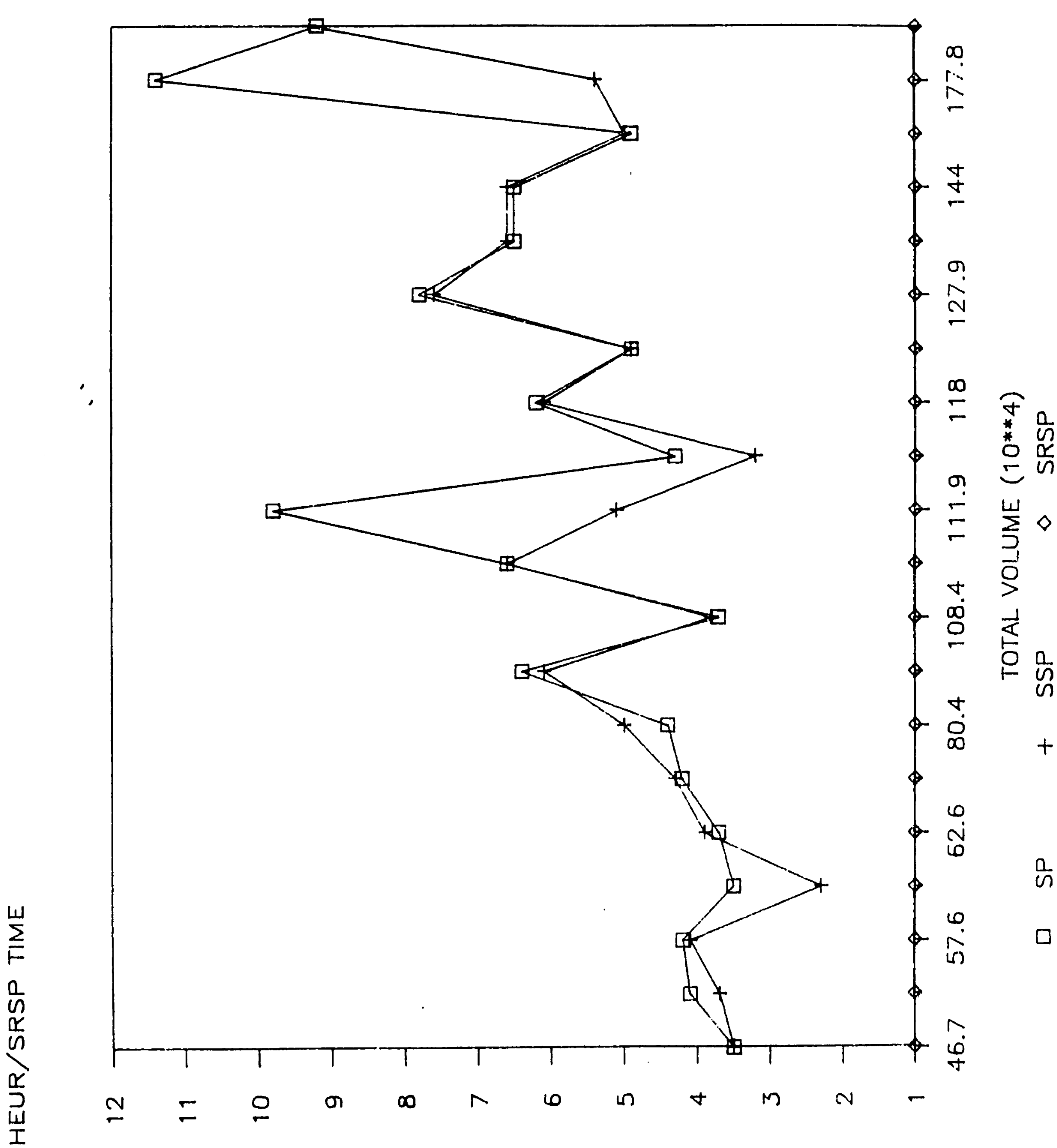


Figure 4.10: Execution Time of Algorithms for Solving the Shipper's Problem (large problems, V=$1.00)



same order. Finally, the running time of BB varies a lot from problem to problem but in general is greater than that of the other solution approaches when BB can be used at all.

Conclusions that can be drawn here are that for the Shipper's Problem, the marginal approach to seeking an optimal freight transportation strategy provides the best results. Within the algorithms that approach the Shipper's Problem via a marginal analysis, efficiencies are gained by driving rerouting attempts of consolidated freight flows by magnitude of savings. The most time efficient of these algorithms uses the concept of savings in conjunction with consolidated freight interaction, to drive rerouting attempts of consolidated freight using a hierarchy of nodes. The Jordan heuristic is extremely fast but attains inferior solutions. Branch-and-bound cannot be used in applications even for problems of small size.

5. Conclusion

The objective of the thesis was to examine the logistics problem from the Shipper's perspective. The particular problem which was examined (Shipper's Problem) is characterized by a set of production locations and a set of consumption locations for a given product(s). The location of two connected consolidation centers is also known. The Shipper is interested in determining the most cost effective transportation strategies using the existing service network. These transportation strategies are defined in terms of the route by which the freight should be shipped (direct or consolidated), the mode to be used, the frequency of service and shipment size. These decisions are not independent. Economies of scale associated with shipping large quantities of freight and trade-offs between transportation and inventory cost are important characteristics of the problem and are taken into account explicitly in both the formulation of the Shipper's problem and the solution approach developed.

5.1 Overview of Methodology and Conclusions

The Shipper's Problem has been formulated as a multi-commodity network flow problem with arc costs that are concave functions of the volume of flow they carry. Concave network problems are NP-hard. For the limited structure network of the Shipper's Problem, the extreme flow property of optimal solutions results in freight travelling along one of only two possible routes between any origin-destination pair. This is a characteristic taken advantage of by a branch-and-bound scheme, a variant of the heuristic of Jordan, and a heuristic devised for the Shipper's Problem.

In developing an efficient algorithmic process for the solution of the Shipper's Problem, another problem had to be addressed. The cost function associated with each routing strategy cannot be expressed in closed form. It was therefore necessary to develop a procedure to evaluate the cost associated with a given routing decision. While the estimation of the cost associated with direct arcs is relatively straightforward, to obtain an accurate estimate of cost corresponding to consolidated routes, a rather difficult minimization problem had to be solved. In this minimization problem the cost is estimated based on optimal decisions with respect to frequency of service and shipment size on each segment of every consolidated route and for a fixed level of flow. Obtaining an exact solution to this problem alone is a very difficult task and therefore heuristic algorithms were developed which efficiently identify good solutions. These algorithms were evaluated and the heuristic of independent optimization of the shipping policy for each route segment was found to provide a very good approximation to the optimal strategy. Since the total logistics cost of freight shipment is the basis upon which all freight transportation strategies can be compared, its evaluation is fundamental to the algorithms for solving the Shipper's Problem. This heuristic is thus the scheme used in objective evaluation during execution of the algorithms for solving the Shipper's problem.

In the branch-and-bound and Jordan heuristic schemes for the Shipper's problem, improvement of an incumbent freight transportation strategy is pursued by averaging total logistics cost. The Shipper heuristic, on the other hand, employs a marginal analysis of total logistics cost to direct its search for reduced cost strategies. A variant of the Shipper heuristic where the search is driven by nodes characterized as having relatively great associated savings, and hence that are likely candidates for having associated flows routed directly, performs very well.

The algorithms developed to solve the Shipper's Problem were evaluated using randomly generated Shipper networks. The results of these simulations point to the marginal approach to the Shipper's Problem as the best. Three variants of the Shipper heuristic outperform the Jordan heuristic - obtaining a solution within at most one percent of the optimal solution in all of the generated networks. The Jordan heuristic is faster than the other algorithms, however. The node-driven variant of the Shipper heuristic described above is the next fastest algorithm. Not surprisingly, the branch-and-bound scheme employed could not be relied upon to solve Shipper's Problems of practical significance due to time and memory requirements.

5.2 Extension and Direction for Further Research

The problem addressed in this thesis was limited in many respects. Some of the limitations relate to issues that were not addressed in an attempt to simplify discussion. Others relate to simplifying assumptions on problem parameters such that the analysis presented could proceed. The restriction of total logistics cost to consist only of transportation, in-transit inventory, and inventory carrying costs is a limitation of the first type. The analysis presented could easily be extended to include such costs as handling costs and level of service costs because these costs are linear in the volume of freight shipped. Simplifying assumptions, however, have been made with respect to commodity value, the rate of demand for a commodity, and the availability of transportation services on an arc connecting two nodes. Although the Shipper's Problem has been presented as a multi-commodity problem, the analysis has been restricted to the case where the commodities being shipped all have identical value (V). This is relaxed somewhat by the fact that V can represent the weighted average value of commodities being shipped in the network. A second assumption, that demand for freight at a destination is deterministic and exactly equal to the supply of the freight at its origin, is a simplification. Finally, the assumption that the availability of transportation services is such that shippers are completely in control of the frequency of their service is not valid. Restrictions such as mode schedules and limitations on mode capacity for a particular route have not been modeled. Further work can be done to generalize the analysis in all of these directions.

On a more theoretical level, a better understanding of the total logistics cost function would probably be beneficial. The heuristic solution approaches to the problem perform surprisingly well. The Jordan heuristic, despite displaying inferior performance, achieves a worst result in the simulation of less than eight percent from optimality. This may indicate that there is a relatively flat region surrounding the global minimum of the total logistics cost function. Perhaps some characterization of the function could yield insight into how to develop tighter lower bounds to make a branch-and-bound scheme practical, or evaluate the heuristics without the need of identifying the optimal solution through a branch-and-bound algorithm.

The merits of more complicated transportation alternatives can be considered even given the current analysis. Given the network configuration considered for the Shipper's Problem, perhaps a larger realization of economies of scale could occur if shipments being consolidated at a transportation terminal were collected in a vehicle and transported in unison to this consolidation center. Similarly, shipments being deconsolidated could leave a transportation terminal in unison and be dropped off at their respective destinations, again more fully realizing the potential of economies of scale. These origin or destination tours are called peddling routes. A result by Burns et al [BURN85] suggests that a good algorithm for generating peddling tours might attempt to group together nodes distant from the consolidation center but close to each other.

Finally, extension of this analysis to networks more general than that of the Shipper's Problem is another potential direction for future research. Networks involving more than one alternative for consolidation, and generally, a less restrictive choice of freight routings would better reflect reality.

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