ESSAYS IN UNEMPLOYMENT AND ECONOMIC ACTIVITY

by

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ABSTRACT

This thesis comprises three essays in unemployment and economic activity. The first essay deals with the microfoundations of employment and inflation theory. The standard implicit contract model implies an employment rule which is sub-optimal for the firm ex post. The consequences of allowing the firm to decide employment unilaterally after product market uncertainty is resolved is investigated. It is shown that there will always be instances where labour is paid less than its marginal product and there is an excess demand for labour, and that in all such states the wage is constant. For other states it receives its marginal product exactly, but involuntary unemployment may occur as in the standard implicit contract model. Further if the production elasticity is constant then the wage in lay-off states is invariant to the level of employment.The results are shown to extend to a model with labour mobility but a degree of skill specialisation.

The second essay describes an attempt to apply the non-clearing market paradigm to the United Kingdom over the period 1965 to 1979. Sectoral aggregation across micro-markets is employed to avoid "bang-bang" switching usually associated with disequilibrium models. Unemployment data is used to help identify the labour demand and supply schedules and a variant of McCallum's instrumental variable technique is used for modelling expectations. The results are somewhat mixed with perverse spillover effects from labour market disequilibrium, although there is evidence that changes in labour market tightness may be important.

The third essay compares the proposal by Meade and Tobin to set targets for nominal GNP with the current practice of setting monetary targets. It is shown in the context of both "New Classical" and "Keynesian" models that for a wide range of plausible parameter values nominal GNP control produces a lower output variance. It is shown that these results may be substantially modified if only inexact control of the target variable is possible due to information lags and in particular that a volatile demand for money function no longer necessarily favours nominal GNP control. The importance of this result depends on the time horizon for which targets are set. It is argued that fiscal policy, and in particular expenditure taxes, rather than monetary policy should be used as the prime instrument for acheiving nominal GNP control.

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CONTENTS

CHAPTER I

OPTIMAL WAGE-BARGAINS

I INTRODUCTION

Recent attempts to provide a choice-theoretic foundation for Keynesian macroeconomics have focussed on the role of the wage bargain between firms and workers. Deficient markets in contingent claims limit opportunities for workers to shed risk. On account of this market failure, firms, it is argued, find it profitable to compete for workers in terms of future employment and wages as well as the current wage. The resulting contract provides workers with partial insurance as well as employment compensation. A feature of these models is the possible occurence of involuntary unemployment.

The standard contracting model of Azariadis (1975) and Baily (1974) leads to not only a compensation rule, but also an employment rule which differs from the usual equalisation of the marginal product to the wage. In this paper we examine the implications of contracts (wage-bargains) which specify a wage schedule in advance, but leave the firm to determine the optimal employment level when future uncertainties are resolved. Since most employment arrangements appear to be of this form, the practical relevance of such contracts is clear. The main theoretical result is that, under certain assumptions about the firms production technology, the possible states of the world fall into three groups. In two of these the wage is invariant to the state of nature, with involuntary unemployment in one and excess demand for labour in the other. In the remaining category there is market clearing with full employment and a wage equal to the marginal product.

In the next section we present a review of the standard contract model and discuss alternative motives for contracts. In section III we develop the basic model and derive some fundamental results concerning the optimal wage-bargain. Section IV considers the importance of the assumption of labour immobility and Section V discusses the implications for macroeconomic modelling and policy. This is followed by a brief summary of the conclusions.

II MOTIVATION

The standard one-period contracting model derives from the seminal work of Azariadis (1975) and Baily (1974), and is surveyed in Azariadis (1979). Firms face uncertain product market conditions, but must make labour hiring decisions before this uncertainty is resolved. They are assumed to be risk-neutral profit-maximisers. Workers have a utility function depending only on consumption (i.e. wage income since there is no saving) which is strictly concave. Labour is homogenous and capital is fixed. Then the firms optimisation problem may be expressed:

(1) $$\max_{w_s,\rho_s,L} \sum \phi_s [p_s f(\rho_s L) - w_s \rho_s L]$$

subject to:

(2) $$\sum \phi_s [u(w_s)\rho_s + u(\tilde{w})(1-\rho_s)] \geq U \text{ and } 0 \leq \rho_s \leq 1$$

where ϕ_s = probability of state s occuring

p_s = real product price in state s

w_s = real wage in state s

ρ_s = employment rate in state s

$\tilde{w}$ = real reservation wage

L = number of workers hired

The production function is assumed to be well-behaved with $f'(L)>0$ and $f''(L)<0$. The reservation wage $\tilde{w}$ includes both unemployment benefit and the return from increased leisure.

The first-order conditions for this problem imply that $u'(w_s)$ is invariant to the state of nature, and hence that the real wage is constant. Either there is full employment in state s or else unemployment is determined by the condition:

$$(3) \quad p_s f'(\rho_s L) - w + [u(w) - u(\tilde{w})]/u'(w) = 0$$

where w = invariant real wage

The full employment contract $\{w^*, L^*\}$ is therefore sub-optimal if:

$$(4) \quad \mathrm{Min}\{p_s\} f'(L^*) - w^* < -[u(w^*) - u(\tilde{w})]/u'(w^*)$$

where w^* = full employment wage $(=u^{-1}(U))$

$f'(L^*) = w^*/\sum \phi_s p_s$

The essential feature of this result is that labour income is partially stabilised with workers receiving a premium over their marginal product in bad states of nature in return for paying an indemnity in good states. Employment is no longer determined by ex post profit maximisation considerations, but by (3) instead. Since fix-price macro models generally assume that employment is still determined by marginal productivity conditions it would appear that implicit contracting does not provide a logical foundation for disequilibrium models of the Barro and Grossman (1971) and Malinvaud (1977) type. Unemployment when it occurs is voluntary ex ante, but involuntary ex post (in the sense that labour supply exceeds labour demand at the contract wage). The assumption of labour immobility when the state of the world is revealed is essential to the one-period model. If labour is free to move then an auction market in surplus labour will develop, with those firms in beneficial states of nature taking on extra workers. If contracted workers can quit for a higher wage, then market forces will ensure that contracted workers receive a wage at least as great as the additional workers. Hence there is no way the firm can collect the indemnity from workers in good states. This assumption, however, may be relaxed in a multi-period model (see Holström (1980)) where workers receive less than their marginal product in the current period in return for insurance against adverse conditions in future periods.

One objection to the Azariadis-Baily model lies in the form of the employment rule in unemployment states (3), which implies a divergence between the wage and the marginal product of labour. Although such a rule is optimal ex ante, there is an incentive for the firm either to misrepresent the state of nature or else simply renege on the contract when the uncertainty about product market conditions is resolved. While it could be argued that employers might still prefer to abide by the contract, even though it is not the most profitable course ex post, in order to build a favourable reputation and facilitate future hirings, in practice contract negotiations usually concentrate on the wage schedule to the virtual exclusion of manning policy, and workers rarely have the power to monitor the employment rule. The optimal plan is therefore temporally inconsistent and it is no longer rational for workers to enter into Azariadis-Baily contracts if they believe the firm will not follow the agreed employment rule.

One possibility is to specify a wage schedule contingent on the observed ex post level of employment, and this is the course followed by Calvo and Phelps (1977), Azariadis (1979) and Chari (1980). However none of these authors have succeded in providing anything more than a partial characterisation of the optimal employment-contingent contract. In addition the practical relevance of such contracts is unclear since they imply that the wage paid to one worker is a function of how many of his fellow workers are employed. Observed contracts in general do not appear to be of this form; rather the wage is independent of the firms manning policy (but not necessarily of product market conditions). It is worth pointing out that the existence of premium rates for overtime working does not provide an example of such an employment contingent contract since the wage paid to a worker depends only on his own hours worked. This provides one motivation for studying contracts in which the wage rate, which may be state dependent, is specified in advance and employment is determined unilaterally by firms once the state of nature is revealed. While this ignores the scope for using the wage schedule itself as an instrument in obtaining the optimal degree of risk-sharing, it seems a more accurate reflection of the way wage and employment decisions are in practice made. As such the model may be seen as lying somewhere between that of Leontief (1946) who considers the consequence of fixing a wage in advance that is the same for all states of nature, and the employment-contingent contracts studied by the authors cited above.

An additional reason for studying this particular type of contract is provided by motivations for long-term labour contracts other than risk-shifting. Viewing labour contracts primarily as a means of providing workers with insurance against income fluctuations presents difficulties since the partial nature of the insurance offered is crucial to the nature of the results. If severance payments (z_s) to workers who are laid off are introduced the first-order conditions continue to imply that w_s is invariant to the state of nature and also that:

$$(5) \qquad u'(w_s) = u'(\hat{w}+z_s)$$

Workers receive a guaranteed income with the reservation wage $\hat{w}$ being "topped-up" by employers. Thus there will be no involuntary unemployment. While risk-aversion on the part of firms implies that the wage is no longer state invariant, it does not alter (5). Similarly the introduction of liquidity or profitability constraints imply that "topping-up" should occur to the extent of the constraint. In practice severance payments seem to be the exception rather than the rule, and where they have been introduced are largely the consequence of government legislation rather than voluntary agreement between employers and the work-force. This does seem to argue against risk-sharing as the prime motivation for long-term labour contracts.

However, there are a number of other arguments for the existence of implicit labour contracts, each of which suggest a pre-announced wage schedule, but leave employment to the discretion of firms. Wage negotiation is a time-consuming and costly business and uneconomic to undertake frequently. There may therefore be economies to be gained from setting employee compensation in advance (Okun (1975)). Another rationale has been provided by Phelps (1977) who suggests that with mobility costs firms will be tempted to pay employees less than the going wage. By fixing compensation in advance the employee ensures that he is not exploited in this manner. Conversely, more efficient production methods are also likely to be less flexible, and adoption of the most efficient technique may leave the firm vulnerable to excessive wage demands by workers in the future. Consequently firms may also find it desirable to fix remuneration in advance. Indeed where ex post possibilities for factor substitution are limited the preferred duration of wage contracts may be very long indeed. This may be one reason why contract renewals are frequently little more than a formality.

III THE MODEL

The setting for the model is the same as that of the standard contracting model set forth in (1) and (2); namely, risk-neutral firms facing uncertain product market conditions and employing risk-averse workers. Hiring decisions are made before the state of nature is revealed. The firm specifies a wage schedule, which may or may not be state dependent, in advance. However, instead of specifying employment rules like (3) in advance, the firm is free to adopt a profit-maximising strategy with respect to manning levels when the state of nature is revealed. In the contracts studied here the wage is not contingent on the level of employment actually chosen, although since the wage and employment are both dependent on the true state of nature there may well be an observed correlation between them. Herein lies the main difference from the employment-contingent contracts studied by Calvo and Phelps (1977), Azariadis (1979) and Chari (1980). In their models workers are unable to observe the state of nature. The standard implicit contract model discussed above implies that there are some states in which the real wage exceeds the marginal product (see equation (3)); there is therefore an incentive for firms to misrepresent the true state of nature and lay off more workers than agreed. These authors suggest that the wage schedule should be made contingent on the level of employment; the wage then acts as a signal to workers of the true state of nature. A complete charecterisation of the solution to this problem is so far unavailable, but the results of Calvo and Phelps, and Chari both suggest that higher levels of employment should be associated with higher levels of wages[1]. However, the relevance of these contracts was questioned above since observed contracts in general do not appear to be of this form. It should be emphasised that although the wage is not contingent on the level of employment it does not imply the absence of an observed relation between wages and employment, only that the ex post wage for a given state of nature is not a function of the firms manning decision.

When the state of nature s has been revealed the firms optimisation problem is simply:

(6) $$\max_{\rho_s} [p_s f(\rho_s L) - w_s \rho_s L]$$

subject to:

(7) $$0 \leqslant \rho_s \leqslant 1$$

Since $f'(0)=\infty$ it follows that $\rho_s>0$ and hence the lower bound in (7) is redundant. The first-order condition is, of course:

(8) $$p_s f_s' - w_s - \upsilon_s = 0$$

where $f_s = f(\rho_s L)$

and υ_s is a non-negative multiplier on the upper bound in (7). Thus $\rho_s<1$ implies $p_s f_s'=w_s$ and $p_s f_s'>w_s$ implies $\rho_s=1$. This defines ρ_s as a function of the agreed wage w_s and the contracted labour force L:

(9) $$\rho_s = \rho_s(w_s, L)$$

The firms ex ante optimisation problem (A) is then:

(10) $$\max_{w_s, L} \sum \phi_s [p_s f(\rho_s L) - w_s \rho_s L]$$

subject to:

(11) $$\sum \phi_s [u(w_s)\rho_s + u(\bar{w})(1-\rho_s)] \geq U$$

(12) $$w_s \geq \bar{w}$$

where ρ_s is given by equation (9).

Equation (11) is the usual constraint that the expected utility of the contract be at least as great as the market alternative. Equation (12) ensures that the contract wage exceeds the reservation wage in all states of nature. In fact this is not a particularly convenient problem to analyse because (9) is not differentiable at $\rho_s=1$ and $p_s f_s'=w_s$ and hence neither are the objective function (10) or the constraint function (11). Instead we consider the problem (B):

(10') $$\max_{w_s, \rho_s, L} \sum \phi_s [p_s f(\rho_s L) - w_s \rho_s L]$$

subject to:

(11) $\sum\phi_s[u(w_s)\rho_s+u(\bar{w})(1-\rho_s)] \geq U$

(12) $w_s \geq \bar{w}$

(13) $p_s f'(\rho_s L) \geq w_s$

(14) $0 \leq \rho_s \leq 1$

In addition we assume:

(15) $u(\bar{w}) < U \leq \lim_{w\to\infty} u(w)$

First, since $f'(L)\to 0$ as $L\to\infty$ and $\bar{w}>0$, the choice variables can be restricted to a compact set. The constraint set (11) to (14) is closed. Hence by continuity the problem (B) does have a solution. Also since $f'(0)=\infty$ it follows that $\rho_s>0$ for all s. Finally since ϕ_s is strictly positive (12) to (14) may be rewritten:

(12a) $\phi_s w_s \geq \phi_s\bar{w}$

(13a) $\phi_s p_s f'(\rho_s L) \geq \phi_s w_s$

(14a) $\phi_s\rho_s \leq \phi_s$

The first-order conditions for this problem are:

(16) $-\rho_s L+\lambda\rho_s u_s'+\xi_s-\mu_s = 0$

(17) $(p_s f_s'-w_s)L+\lambda(u_s-\bar{u})+\mu_s p_s f_s''L-\nu_s = 0$

(18) $\sum\phi_s\rho_s(p_s f_s'-w_s)+\sum\phi_s\rho_s\mu_s p_s f_s'' = 0$

where $u_s = u(w_s)$

$\tilde{u} = u(\tilde{w})$

and λ, ξ_s, μ_s, ν_s, are non-negative multipliers on the constraints (11), (12a), (13a), and (14a) respectively.

<u>PROPOSITION</u> $\{w_s, L, \rho_s(w_s, L)\}$ is a solution to problem (B) if and only if $\{w_s, L\}$ is a solution to problem (A).

<u>Proof</u> Since $\rho_s(w_s, L)$ satisfies (13) and (14) $\{w_s, L, \rho_s(w_s, L)\}$ is clearly feasible for problem (B). Thus it is only necessary to demonstrate that the first-order conditions for problem (B) imply the same decision rule for ρ as equation (9). Consider any state for which $p_s f_s' > w_s$. Then for these states $\mu_s = 0$, and equation (17) implies $\nu_s > 0$ since $(u_s - \tilde{u}) \geqslant 0$. Hence from (13a) $\rho_s = 1$, and any state with $p_s f_s' > w_s$ must be a full employment state. Otherwise ρ_s solves $p_s f_s' = w_s$. This is exactly the rule embodied in (9).

Conversely if $\{w_s, L, \rho_s\}$ solves problem (B), then since the employment rule (9) is satisfied $\{w_s, L\}$ must be feasible for problem (A). But if there is some other contract $\{w_s', L'\}$ which is better then by the first part $\{w_s', L', \rho_s(w_s', L')\}$ is a solution to (B), which contradicts $\{w_s, L, \rho_s\}$ optimal. Q.E.D.

We may deduce the following proposition concerning the agreed wage:

LEMMA 1 (i) There is at least one full employment state s for which the marginal revenue product exceeds the real wage;

(ii) For all such states the real wage $\overline{w}$ is invariant to the state of nature;

(iii) $w_s = \mathrm{Min}\{\mathrm{Max}\{p_s f_s', \tilde{w}\}, \overline{w}\}$.

Proof (i) We have already shown that any state in which $p_s f_s' > w_s$ must be a full employment state with $\rho_s=1$. Suppose $p_s f_s' = w_s$ for all s. Then $\mu_s=0$ for all s by (18). Also by (15) there is some state for which $w_s > \tilde{w}$; without loss of generality label this state 1. Hence $\xi_1=0$ and $\lambda=L/u_1'$. Then $\xi_s=0$ $(s\neq 1)$ implies $w_s=w_1$ by concavity and ρ_s solves $p_s f_s' = w_1$. Now either $p_s<p_1$ or $p_s>p_1$. If $p_s<p_1$ then since $p_s f_s' = w_s = w_1 = p_1 f_1'$ it follows that $f_s' > f_1'$ and $\rho_s<1$; hence $\nu_s=0$. Then (17) implies that $u_s=\tilde{u}$. Thus $\tilde{w}=w_s=w_1>\tilde{w}$ which is a contradiction. Analagously if $p_s>p_1$ then $\rho_1<1$ and $\nu_1=0$. Thus $u_1=\tilde{u}$ by (17) and $\tilde{w}=w_1>\tilde{w}$ which is again a contradiction. Hence if there exist at least two states r and t such that $\xi_r=\xi_t=0$. then there is some state s such that $p_s f_s' > w_s$.

It remains to demonstrate that $\xi_s>0$ $(s\neq 1)$ is not an optimal solution. Since $\mu_s=0$ and $\lambda=L/u_1'$, equation (16) implies $L\rho_s u_s'/u_1' < L\rho_s$; hence $u_s' < u_1'$. Thus $w_s>w_1>\tilde{w}$ and $\xi_s=0$. Thus $\xi_s>0$ $(s\neq 1)$ is not an optimum. This completes the proof of (i).

(ii) Consider states for which $p_s f_s' > w_s$. For these states $\mu_s=0$ and $\rho_s=1$. If $\xi_s>0$ then $w_s=\tilde{w}$. But equation (16) implies $\xi_s=(L-\lambda u_s')$ and hence that $u_s'<u_1'$ since $\lambda=L/u_1'$. Thus $w_s>w_1>\tilde{w}=w_s$ which is a contradiction. Hence $\xi_s=0$ and $u_s'=u_1'$ which implies $w_s=w_1=\overline{w}$ by concavity. This concludes the proof of (ii).

(iii) $w_s \geq \hat{w}$ follows from (12). We have already shown above that if $w_s<p_s f_s'$ then $\rho_s=1$ and $w_s=\overline{w}$. Hence for all other states $w_s=p_s f_s'$. It remains to show that for these states $w_s \leq \overline{w}$. Suppose the contrary and that $w_s>\overline{w}$. Then $\xi_s=0$ and from (16) $\mu_s=\rho_s L(u_s'/\overline{u}'-1)$ where $\overline{u}=u(\overline{w})$. Hence $u_s' \geq \overline{u}'$ and $w_s \leq \overline{w}$ which is a contradiction. This completes the proof of (iii). Q.E.D.

It is quite easy to demonstrate that the constraint in (12) never bites i.e. $\xi_s \equiv 0$. First, equation (15) and Lemma 1(iii) imply $\overline{w}>\hat{w}$. Hence if there is a state in which $\xi_s>0$ then $p_s f_s'=\hat{w}$. Equation (17) then implies $\mu_s=0$ and using $\lambda=L/\overline{u}'$ equation (16) implies $(u_s'-\overline{u}')<0$ and hence $w_s>\overline{w}$ which is a contradiction.

The next question of interest is under what conditions will unemployment occur. The following result follows directly from (16) and (17):

LEMMA 2 Unemployment will occur in state s if:

$$u(w_s^*)-u(\tilde{w}) < \sigma(u'(w_s^*)-u'(\overline{w}))w_s^*$$

where $w_s^* = p_s f'(L)$

$\sigma = -f''(L)L/f'(L)$

Proof For any state with $w_s^*<\overline{w}$ we know by Lemma 1 that $p_s f_s' = w_s$. Since $\xi_s \equiv 0$ and $\lambda = L/\overline{u}'$ we may substitute for μ_s using (16) in (17):

$$(19) \quad (u_s-\tilde{u})/\overline{u}'+\rho_s(u_s'-\overline{u}')p_s f_s''L/\overline{u}'-\nu_s = 0$$

Thus unemployment will be optimal if the left-hand side of the above expression evaluated at $w_s=w_s^*$ as $\rho_s \to 1^-$ (i.e. $\nu_s=0$) is negative. The inequality follows by substitution. For states with $w_s^* \geqslant \overline{w}$ this condition can never be satisfied, but we already know these are full employment states. Q.E.D.

Whether the full employment contract will be sub-optimal is somewhat harder to verify than in the standard implicit contract model (see equation (4) above). It is no longer straightforward to obtain the full employment contract $\{\overline{w}^*, L^*\}$ which solves:

$$(20) \quad U = u(\overline{w}^*) \sum_{s \in S} \phi s + \sum_{s \notin S} \phi s u(p_s f'(L^*))$$

and

$$(21)\quad u'(\overline{w}^*) \sum_{s\in S} \phi_s(p_s f'(L^*)-\overline{w}^*) - \sum_{s\notin S} \phi_s \sigma p_s f'(L^*)[u'(p_s f'(L^*))-u'(\overline{w}^*)] = 0$$

where $S = \{s \,|\, p_s f_s' > w_s\}$

These two equations define continuous loci in $\{\overline{w}^*, L^*\}$ space. Along (20):

$$(22)\quad (\overline{u}' \sum_{s\in S} \phi_s)\frac{d\overline{w}^*}{dL^*} = -f^{*''} \sum_{s\notin S} \phi_s p_s u_s^{*'}$$

Hence (20) is positively sloped. Differentiating (21) and evaluating using (21):

$$(23)\quad [\overline{u}^{*'} \sum_{s\in S} \phi_s - (\sigma \overline{u}'' f^{*'}/\overline{u}^{*'}) \sum_{s\notin S} \phi_s p_s u_s^{*'}]\frac{d\overline{w}^*}{dL^*}$$

$$= f^{*''}[(\overline{u}^{*'}\overline{w}^*/f^{*'}) \sum_{s\in S} \phi_s - \sigma f^{*'} \sum_{s\notin S} \phi_s p_s^2 u_s^{*''}]$$

$$-\frac{d\sigma}{dL^*}[f^{*'} \sum_{s\notin S} \phi_s p_s (u_s^{*'} - \overline{u}')]$$

where $f^* = f(L^*)$

$u_s^* = u(w_s^*)$

Thus a sufficient condition for (21) to be negatively sloped is that $d\sigma/dL^* \geqslant 0$, which is therefore also a sufficient condition for uniqueness of the full employment contract. Any particular full employment contract will be sub-optimal if:

(24) $u(\text{Min}\{w_s^*\})-u(\hat{w}) < \sigma(u'(\text{Min}\{w_s^*\}-u'(\overline{w}^*))\text{Min}\{w_s^*\}$

From equation (19) and Lemma 1 the wage in unemployment states satisfies:

(25) $(u_s-\hat{u})+(\overline{u}'-u_s')w_s\sigma_s = 0$

where $\sigma_s = -f_s''\rho_s L/f_s'$

Implicit differentiation of this expression yields:

(26) $$\frac{dw_s}{d\rho_s} = \frac{w_s(u_s'-\overline{u}')\partial\sigma_s/\partial\rho_s}{[u_s'(1-\sigma_s)+\sigma_s(\overline{u}'-w_s u_s'')]}$$

The sign of (26) is ambiguous; thus in general we cannot associate higher levels of employment with higher levels of real wages. However, a sufficient condition to ensure this result is that $\sigma_s \leqslant 1$ and $\partial\sigma_s/\partial\rho_s \geqslant 0$. Further interesting results can be obtained if we assume a constant elasticity production function $f(L)=L^{1-\sigma}$ $(0<\sigma<1)$. In that case $\sigma_s \equiv \sigma$ for all ρ_s and we therefore obtain the following result:

<u>LEMMA 3</u> With a constant elasticity production function $f(L)=L^{1-\sigma}$ $(0<\sigma<1)$, the real wage in underemployment states is invariant to the level of unemployment.

<u>Proof</u> $\partial\sigma_s/\partial\rho_s=0$ and hence by (26) $dw_s/d\rho_s=0$. Q.E.D.

We are now ready to state the central result of the paper concerning the optimal wage schedule under these conditions:

THEOREM 1 With a constant elasticity production function the agreed wage in unemployment states ($\underline{w}$) will be invariant to the state of nature, but will display upward flexibility at full employment subject to a maximum of $\overline{w}$.

Proof First we need to demonstrate that there is no full employment state with a wage less than the unemployment wage $\underline{w}$. Suppose $w_s < \underline{w}$. Then by Lemma 1(iii) $w_s < \overline{w}$ and hence $w_s = p_s f'(L)$ by Lemma 1(ii). Then from (16) and (17) :

$$(27) \qquad (u_s - \tilde{u}) + (\overline{u}' - u_s')w_s\sigma = \nu_s\overline{u}' \geq 0$$

Now we know that $\underline{w}$ solves:

$$(28) \qquad (\underline{u} - \tilde{u}) + (\overline{u}' - \underline{u}')\underline{w}\sigma = 0$$

where $\underline{u} = u(\underline{w})$.

But the left-hand side of (27) is monotonic increasing in w_s. Hence $\underline{w} \leq w_s$. The theorem then follows from Lemmas 1 to 3. Q.E.D.

In this case the optimal contract $\{\overline{w}, L\}$ will therefore satisfy:

$$(29) \quad U = \bar{u} \sum_{s \in S} \phi_s + \sum_{s \notin S \cup T} \phi_s u_s + \sum_{s \in T} \phi_s [\rho_s \underline{u} + (1-\rho_s)\tilde{u}]$$

and:

$$(30) \quad \bar{u}' \sum_{s \in S} \phi_s (p_s f' - \bar{w}) - \sigma f' \sum_{s \notin S \cup T} \phi_s p_s (u_s' - \bar{u}') - \sigma \underline{w} \sum_{s \in T} \phi_s \rho_s (\underline{u}' - \bar{u}') = 0$$

where $T = \{s/\rho_s < 1\}$

$f = f(L)$

$w_s = p_s f_s'$ for $s \notin S$

and $\underline{w}$ solves (28).

As for the full employment contract discussed above, these two equations describe loci in $\{\bar{w}, L\}$ space. However, it is no longer possible to establish simple sufficient conditions for uniqueness which now also depend on the nature of the utility function.

We thus have states of nature divided into three groups. For states in S the marginal product exceeds the wage, which is nevertheless independent of demand; there is therefore an excess demand for labour. For states in T the wage, which is also independent of demand, equals the marginal product but there is involuntary unemployment; there is therefore an excess supply of labour. For intervening states there is market clearing with full employment and equality between the wage and the marginal product. It is natural to ask if all three sets can be non-empty and a moments consideration suggests that this is certainly possible. S is certainly non-empty by Lemma 1, and equation (24) suggests that under some circumstances T will be non-empty. With a continuum of prices it follows that the intermediate region with market clearing will also be non-empty. A specific numerical example in which all three regimes occur is presented in the appendix.

Analysis of the impact of changes in technology is straightforward. Differentiating (28):

$$(31) \qquad \partial\underline{w}/\partial\sigma = (\underline{u}'-\overline{u}')\underline{w}/[\underline{u}'(1-\sigma)+\sigma(\overline{u}'-\underline{wu}'')] \geqslant 0$$

Thus sharply diminishing returns to labour will, ceteris paribus, decrease the variability of the wage. Similarly we may obtain the response to changes in the level of unemployment benefit:

$$(32) \qquad \partial\underline{w}/\partial\tilde{w} = \tilde{u}'/[\underline{u}'(1-\sigma)+\sigma(\overline{u}'-\underline{wu}'')] > 0$$

Thus, as might be expected, increases in the reservation wage make unemployment less unattractive leading to lower employment and a higher wage in underemployment states.

Analysis of the impact of the degree of risk-aversion on the part of workers is more complex, and it is helpful to assume a particular form for the utility function. First, consider the quadratic utility function:

(33) $u(w) = a+w-bw^2$

In addition to apply the foregoing analysis we must ensure that marginal utility is positive over the range $(\hat{w},\bar{w}]$ so that $b\bar{w}\leqslant 1/2$. Then (28) becomes:

(34) $\underline{w}^2 b(2\sigma-1)+\underline{w}(1-2\sigma\bar{w}b)+\hat{w}(b\hat{w}-1) = 0$

and hence $\underline{w}$ solves:

(35) $\underline{w} = [(2\sigma\bar{w}b-1)+\sqrt{A}]/2b(2\sigma-1)$

where $A = (1-2\sigma\bar{w}b)^2+4b\hat{w}(2\sigma-1)(1-b\hat{w})$

and the positive root[2] is taken to ensure that $\underline{w}\to\hat{w}$ as $b\to 0$ (risk-neutrality).

Differentiating (34) with respect to b:

(36) $\partial\underline{w}/\partial b = [2\sigma\underline{w}\bar{w}-\hat{w}^2-\underline{w}^2(2\sigma-1)]/[2\underline{w}b(2\sigma-1)+1-2\sigma b\bar{w}]$

Hence using (34) and (35):

(37) $\partial\underline{w}/\partial b = (\underline{w}-\hat{w})\sqrt{A}/b > 0$

Thus increasing risk-aversion reduces wage variability. However, this result does not appear to be particularly robust as the following example demonstrates.

Consider the constant relative risk-aversion utility function:

$$(38a)\quad u(w) = w^{1-\beta}/(1-\beta) \qquad (\beta>0, \beta\neq 1)$$

$$(38b)\quad u(w) = \ln w \qquad (\beta=1)$$

Then (28) becomes:

$$(39a)\quad \sigma(1-\beta)\psi^{\beta}-\theta^{\beta-1} = \sigma(1-\beta)-1 \qquad (\beta\neq 1)$$

$$(39b)\quad \sigma\psi+\ln\theta = \sigma \qquad (\beta=1)$$

where $\psi = (\underline{w}/\overline{w})$

$\theta = (\underline{w}/\varpi)$

First it may be noted that as $\beta\to 0$, we must have $\theta\to 1$ for (39a) to hold and hence $\underline{w}\to\varpi$. Further, there can be no finite positive β for which either $\psi=1$ or $\theta=1$. Differentiating (39a) we obtain:

$$(40a)\quad \frac{\partial \underline{w}}{\partial \beta} = \frac{\underline{w}[\sigma\psi^{\beta}-\sigma(1-\beta)\psi^{\beta}\ln\psi-\sigma+\theta^{\beta-1}\ln\theta]}{(1-\beta)(\beta\sigma\psi^{\beta}+\theta^{\beta-1})} \qquad (\beta\neq 1)$$

$$(40b)\quad \frac{\partial \underline{w}}{\partial \beta} = \underline{w}\ln\underline{w}[\sigma(\psi-1)+\ln\theta]/(1+\sigma\psi) = 0 \qquad (\beta=1)$$

Evaluating (40a) as $\beta\to 0$:

$$(41)\quad \partial\underline{w}/\partial\beta = \varpi\sigma\ln(\overline{w}/\varpi) > 0 \quad (\beta=0)$$

Since $\underline{w}$ is a continuous function of β, it follows that $\underline{w}$ is bounded above and below by $\bar{w}$ and $\hat{w}$ respectively. Unfortunately it does not seem possible to sign the numerator of (40a) unambiguously, but it seems unlikely that it would change signs as β goes through unity. Hence $\beta=1$ is probably a turning point rather than an inflection (whether it is a maximum or a minimum is unclear). Further, consideration of (39a) suggests that $\underline{w}=\bar{w}$ and $\underline{w}=\hat{w}$ are the only possible limit points as $\beta\to\infty$. If $\underline{w}\to\bar{w}$ as $\beta\to\infty$ it must do so from below as $\theta^{\beta-1}\ell n\psi$ (>0) will dominate the remaining terms in the numerator of (40a). Since the denominator is negative for $\beta>1$ it follows that $\partial\underline{w}/\partial\beta<0$ which is inconsistent with $\underline{w}\to\bar{w}$ from below. Hence $\underline{w}\to\hat{w}$ as $\beta\to\infty$. Thus both low and high degrees of risk-aversion imply a high degree of variation between the unemployment wage and the maximal full employment wage. An intuitive explanation of this result is that with a high degree of risk-aversion on the part of workers the possibility of a fall in income due to being unemployed becomes relatively more important. It should be emphasised, however, that this analysis is conditional on the maximal wage $\bar{w}$ rather than the level of utility U, so that changing β also changes the utility level of workers. A more natural question might be to ask what the impact of changes in β would be keeping contract utility fixed (i.e. allowing $\bar{w}$ to change). This is analytically intractable, but it seems reasonable to suggest that the maximal wage $\bar{w}$ might also fall, so that wage variability might not increase with the degree of risk-aversion.

IV LABOUR MOBILITY

The assumption of a degree of labour immobility turns out to be rather essential to the results of the previous analysis. It is also essential to the Azariadis-Baily model considered in the Section II, but as we noted there it may be relaxed in a multi-period model of the sort considered by Holström (1980). He has a two-period model in which workers may buy insurance against adversity in the second period by accepting a lower wage in the first. However, if the worker can find a higher wage elsewhere in the second period he is free to quit. Firms may take on extra labour in the second period at the market wage (rather than the contractual wage offered to those already employed). Although the phenomenon of involuntary unemployment naturally disappears in this case of perfect labour mobility, it is still possible to characterise the contract wage as $w_s = \text{Max}\{\underline{w}, w_s^+\}$ where w_s^+ is the market wage obtainable in the second period and $\underline{w}$ is some constant floor wage. It does not seem simple to extend the wage bargains of the previous section in the same way.

Some insights, however may be gained by allowing a limited degree of labour mobility. Let us drop the assumption of an homogenous labour force and instead assume that contracted workers either have, or acquire through experience, firm specific skills which make them more productive than new workers hired from the ranks of the unemployed. When the state of nature is revealed the firm can either lay off contracted workers or hire additional workers at the going wage depending on the level of demand. The reservation wage $\tilde{w}$ now represents this market alternative rather than unemployment benefit. A contracted worker who is laid off is free to take work elsewhere at the wage paid to unskilled workers ($\tilde{w}$), but will be unable to find a job paying as high as his previous wage because his skills are specialised.

The firms ex post optimisation problem is therefore:

(42) $$\max_{\rho_s, L_s^\circ} \; [p_s f(\rho_s L + aL_s^\circ) - w_s \rho_s L - \tilde{w} L_s^\circ]$$

subject to:

(43) $0 \leqslant \rho_s \leqslant 1$

(44) $L_s^\circ \geqslant 0$

where L_s° = number of additional workers hired.

The coefficient $a<1$ represents the effectiveness of new workers in terms of skilled (contracted) workers. The first-order conditions are

(45) $(p_s f_s' - w_s)L - \lambda + \mu = 0$

(46) $ap_s f_s' - \tilde{w} + \nu = 0$

where λ, μ, and ν are non-negative multipliers on the constraints (43) and (44).

Hence $p_s f'_s \geq w_s$ if $\rho_s=1$, $p_s f'_s \leq w_s$ if $\rho_s=0$, and $p_s f'_s \leq \tilde{w}/a$ with equality if $L^{\circ}_s>0$. Thus the firm will replace contracted workers with new workers if $w_s>\tilde{w}/a$. To avoid indeterminacy we assume contracted workers have preference if $w_s=\tilde{w}/a$. As before this defines decision rules for ρ_s and L°_s:

(47) $\rho_s = \rho_s(w_s, \tilde{w}, L)$

(48) $L^{\circ}_s = L^{\circ}_s(w_s, \tilde{w}, L)$

The firms ex ante optimisation problem is (C) is:

(49) $\max_{w_s, L} \quad \sum \phi_s [p_s f(\rho_s L + a L^{\circ}_s) - w_s \rho_s L - \tilde{w} L^{\circ}_s]$

subject to:

(50) $\sum \phi_s [u(w_s)\rho_s + u(\tilde{w})(1-\rho_s)] \geq U$

(51) $w_s \geq \tilde{w}$

where ρ_s and L°_s are given by equation (47) and (48).

We immediately notice that it can never be optimal for the labour force and the firm to agree to so high a wage w_s that all contracted workers are replaced by unskilled workers ($\rho_s=0$). This could only happen if $w_s>\tilde{w}/a$, but then setting $\tilde{w}<w_s\leq\tilde{w}/a$ would increase both profits and the utility of the contract. Hence $w_s\leq\tilde{w}/a$. An important corollary is that $L^{\circ}_s>0$ only if $\rho_s=1$. As before this problem is difficult to analyse on account of the non-differentiability of (47) at $p_s f_s' = w_s$ and $\rho_s = 0$ or $\rho_s = 1$, so instead we consider the problem (D):

$$(49') \quad \underset{w_s, \rho_s, L, L_s^\circ}{\text{Max}} \quad \sum \phi_s [p_s f(\rho_s L + aL_s^\circ) - w_s \rho_s L - \bar{w} L_s^\circ]$$

subject to:

$$(50) \quad \sum \phi_s [u(w_s)\rho_s + u(\bar{w})(1-\rho_s)] \geqslant U$$

$$(51) \quad \phi_s w_s \geqslant \phi_s \bar{w}$$

$$(52) \quad \phi_s p_s f_s' \geqslant \phi_s w_s$$

$$(53) \quad \phi_s a p_s f_s' \leqslant \phi_s \bar{w}$$

$$(54) \quad \phi_s \rho_s \leqslant \phi_s$$

$$(55) \quad \phi_s L_s^\circ \geqslant 0$$

where we have multiplied through by ϕ_s for analytical convenience.

Conditions (52) and (53) ensure the firms ex post optimisation condition is met. In addition we modify (15) to:

$$(56) \quad u(\bar{w}) < U < u(\bar{w}/a)$$

This ensures that at least some workers will be offered contracts. The first-order conditions for this problem are:

$$(57) \quad -\rho_s L + \lambda \rho_s u_s' + \xi_s - \mu_s = 0$$

$$(58) \quad (p_s f_s' - w_s)L + \lambda(u_s - \tilde{u}) + (\mu_s - \eta_s a) p_s f_s'' L - \nu_s = 0$$

(59) $ap_s f_s' - \tilde{w} + \gamma_s = 0$

(60) $\sum\phi_s\rho_s(p_s f_s' - w_s) + \sum\phi_s\rho_s(\mu_s - \eta_s a)p_s f_s'' = 0$

where $\lambda, \xi_s, \mu_s, \eta_s, \nu_s$ and γ_s are non-negative multipliers on the constraints (50) to (55).

First, since $p_s f_s' > w_s$ implies $\mu_s = 0$, any states for which this is true must be full employment states with $\rho_s = 1$ by (58). Also if $L_s^\circ > 0$ then $p_s f_s' = \tilde{w}/a$, and $p_s f_s' < \tilde{w}/a$ implies $L_s^\circ = 0$. Hence we have the same decision rules for ρ_s and L_s° as (47) and (48) and problems (C) and (D) are equivalent. It is no longer possible to demonstrate that there must be states of nature in which there is an excess demand for skilled labour[3] i.e. $p_s f_s' > w_s$. However, <u>if</u> they exist we may use the argument in the proof of Lemma 1(ii) to demonstrate that the wage across such states will be invariant. Notice that it is not possible to have an excess demand for unskilled labour.

The condition for the existence of unemployment corresponding to Lemma 2 is now much more complex on account of the impossibility of deriving a simple expression for λ in terms of the wage paid for skilled labour in states of excess demand for labour and the presence of η_s in (58). However, if the production function is of the constant elasticity variety the essential features of Theorem 1 remain.

<u>THEOREM 2</u> With heterogeneous labour and a constant elasticity production function $f(L)=L^{1-\sigma}$, the wage in lay-off states ($\underline{w}$) exceeds the reservation wage $\tilde{w}$ and is invariant to the state of nature, but displays upward flexibility at full employment.

<u>Proof</u> If there are states for which $p_s f_s' > w_s = \overline{w}$ then the argument of Lemma 1(iii) can be applied to show that for all other states $w_s < \overline{w}$. For these states $p_s f_s' = w_s$. Now there is at least one state with $w_s > \tilde{w}$ by (56); label this state 1. Any state in which $\xi_s > 0$ must also have $p_s f_s' = \tilde{w}$. Then (57) implies:

$$(61) \qquad \rho_s L(u_s'/u_1' - 1) = \mu_s - \xi_s - \mu_1 \rho_s u_s'/\rho_1 u_1'$$

$\xi_s > 0$ implies $w_s = \tilde{w}$ and $u_s' > u_1'$. Hence $\mu_s > 0$, but then (58) implies $\eta_s > 0$ and $w_s = \tilde{w}/a$ which is a contradiction. Hence $\xi_s \equiv 0$.

We have already remarked that states for which $p_s f_s' > w_s$ must be full employment states. For states with $p_s f_s' = w_s$ equations (57) and (58) imply:

$$(62) \qquad (u_s - \tilde{u}) - w_s \sigma(u_s' - L/\lambda) = (L p_s f_s'' \eta_s a + \nu_s)/\lambda$$

The left-hand side of this equation (call it g_s) is invariant to ρ_s and monotonic increasing in w_s. For layoff states $\nu_s=0$ and there are two possibilities with respect to w_s: either $\eta_s>0$ and $w_s=\tilde{w}/a$; or $w_s<\tilde{w}/a$ and $\eta_s=0$. Any two states falling into the first category must necessarily have the same wage $\tilde{w}/a$. For any states in the second group $g_s=0$ and by the monotonicity of g_s the solution w_s is unique. It remains to demonstrate that at most one set of states is non-empty.

Consider two states s and t with $\eta_s>0$ so that $w_s=\tilde{w}/a$, and $w_t<\tilde{w}/a$ so that $\eta_t=0$. Then $g_t>g_s$ and hence $w_t>w_s$ since g_s is increasing; this is a contradiction. Hence there is a unique wage in unemployment states.

Finally since $\nu_s\geq 0$ and g_s is increasing, the wage in full employment states with $p_s f_s'=w_s$ must be at least as great as that in lay-off states. A corollary of this and (56) is that the wage in lay-off states $\underline{w}<\tilde{w}/a$. Q.E.D.

In this model there can technically be no involuntary unemployment since laid off workers can always find employment at the wage offered to unskilled workers $\tilde{w}$. However, ex post such workers would prefer to remain with a firm where their special skills are useful and take a cut in wages, which would nevertheless remain above the market alternative $\tilde{w}$. In this sense the unemployment could be said to be involuntary. This is an aspect of the unemployment problem ignored in macroeconomic models which treat labour as homogenous. There may always be a sufficient supply of poorly paid menial jobs to ensure full employment, but there may nevertheless be an imbalance between the demand and supply of skilled labour. Such a mismatch between demand and supply involves welfare losses and is obviously a cause for concern even though there may be a wage which clears the overall labour market.

A natural extension of the model is to place it in a general equilibrium context. We shall have more to say on the macroeconomic implications in the next section. However, it is appropriate to consider briefly the consequences of allowing the reservation wage $\tilde{w}$ available after the state of nature is revealed to be state dependent and determined by the interaction of demand and supply in the market for unemployed workers. If we assume perfect foresight of the market determined reservation wage $\tilde{w}_s$ in state s, it is no longer necessarily true that $w_s > \tilde{w}_s$ and for states in which $\tilde{w}_s < w_s < \tilde{w}_s/a$ equation (62) needs to be modified to:

$$(63) \quad (u_s - \tilde{u}_s) - w_s \sigma(u_s' - L/\lambda) = 0$$

Since the left-hand side is monotonic decreasing in $\tilde{w}_s$, the real wage in unemployment states will be positively related to the reservation wage, so that there will be a degree of responsiveness of the wage to general market conditions.

V MACROECONOMIC IMPLICATIONS

One important difference between the model of this paper and the standard contracting model is that in unemployment states labour receives its marginal product. Thus for regimes where $p_s f'(L) < \underline{w}$ or $p_s f'(L) > \overline{w}$ the results of this paper do seem to provide a genuine microfoundation for fix-price macroeconomic models of the Barro-Grossman-Malinvaud type. However, for those states of nature such that $\underline{w} \leq p_s f'(L) \leq \overline{w}$ the standard auction model with market clearing is appropriate. Although the model assumes that the employment decision is discrete and hours are fixed, one way that this upward flexibility at full employment may in practice be achieved is by the payment of premium rates for overtime working.

It is tempting to suggest that the fixed-wage type results of implicit contracting provide a rationale for cyclical changes in aggregate unemployment. In the case of the model discussed in this paper the limited upward flexibility of wages at full employment also suggests an explanation of procyclical real wage behaviour. Assuming all firms are identical except for the state of nature in which they find themselves, the larger is the proportion of firms experiencing buoyant goods market conditions, then the higher is the level of employment and wages. However, this ignores the fact that it is the firms <u>real</u> selling price that matters, and that the aggregate price level used to evaluate this is a function of the individual goods prices. To put it loosely, all firms cannot experience adverse states of nature simultaneously. In the absence of sales constraints,correlated changes in unemployment can only occur if there is an imperfect link between individual prices and the aggregate price level.

A small open economy where changes in the demand for domestically produced goods have little effect on the aggregate price level is one case where this might occur. Another example is provided by a two-sector economy producing manufactures, which are luxuries, and food, which is a necessity. The manufacturing sector offers its workers contracts of the sort examined above, while in the agricultural sector wages are determined in the spot market. There is no mobility between sectors and agricultural wages are generally lower than those in manufacturing. Then high levels of aggregate demand will be associated with a high relative price of manufactures and low aggregate unemployment.

An alternative explanation of general changes in unemployment could be formulated within the incomplete information paradigm. If firms know their own product price, but not the general level of prices, then overestimation of the aggregate price level will lead to an overly pessimistic inference about the true state of nature. Consequently output and employment will be lower than if the state of nature were correctly ascertained. We thus have an alternative explanation of the Lucas (1973) aggregate supply function based on firms misperceptions of product market conditions, rather than resulting from intertemporal sustitution of labour supply due to workers misperceptions of the true state of the labour market as in Lucas' original development[4]. This formulation will give rise to involuntary unemployment, whereas in Lucas' model unemployment is primarily a search phenomenon.

VI CONCLUSIONS

In this paper we have argued that employment contracts in the real economy are not of the sort usually modelled in the implicit contract literature, but instead specify a wage schedule in advance and leave employment to the discretion of employers. We then went on to examine the properties of such contracts and showed that there will always be instances where labour is paid less than its marginal product and there is therefore an excess demand for labour. For other states it receives exactly its marginal product, but involuntary unemployment can occur as in the standard contracting model. Further, if the production function has a constant elasticity, then the wage in lay-off states is constant, with the spread between the maximum full employment wage and the wage in unemployment states determined by the production elasticity, the reservation wage, and the degree of risk-aversion.

We then went on to examine the consequences of dropping the assumption of labour immobility, but assuming a degree of skill specialisation and the previous conclusions were substantially unaltered. The analysis also demonstrated that there is a sense in which involuntary unemployment can be said to exist even though the labour market clears. The results do appear to provide a genuine microfoundation for fix-price macro models, and perhaps give additional insights into the way wages respond to economic conditions.

APPENDIX

The optimal contract satisfies (28), (29) and (30). In addition to assuming a constant elasticity production function with $\sigma=.5$, assume that $u(w)=\ln w$ and $\tilde{w}=1$. There are three equi-probable states of nature with prices $p_1=3$, $p_2=2$, $p_3=1$. Then (28) implies:

(A1) $\quad \ln \underline{w} = .5(1-\underline{w}/\overline{w})$

Rather than assume a value of the contract U and solve for $\{\overline{w},L\}$ it is somewhat easier to assume a value for $(\underline{w}/\overline{w})$ or $\underline{w}$ and then derive the implied value of the contract. Therefore assume $\overline{w}=2\underline{w}$. Hence $\underline{w}=\exp(.25)=1.28$ and $\overline{w}=2\exp(.25)=2.57$. Equation (30) then yields a quadratic in $\sqrt{L}$:

(A2) $\quad 24\exp(.5)L-16\exp(.25)\sqrt{L}+1 = 0$

This yields roots of .6 and .07. The latter root implies $p_s f_s' > \overline{w}$ for all three states which clearly cannot be the optimum. The former root implies:

(A3) $\quad p_1 f'(L) = 3.23 > \overline{w} = 2.57$

$\quad 2.57 = \overline{w} > p_2 f'(L) = 2.15 > \underline{w} = 1.28$

$\quad 1.28 = \underline{w} > p_3 f'(L) = 1.08$

Hence if state 1 occurs there is excess demand for labour, if state 2 occurs there is market clearing, and if state 3 occurs there is excess supply of labour, with a level of employment given by:

(A4) $p_3 f'(\rho_3 L) = \underline{w} = 1.28$

Thus ρ_3=.7. The corresponding level of utility is U=.63 or a certainty-equivalent wage of w=1.87.

FOOTNOTES

1 Azariadis (1979) claims that the optimal wage is constant (p.29-30). His demonstration of the proposition seems to ignore the scope for employment changes when the wage schedule is changed. Chari (1980) points out that "incentive-compatible" contracts which ensure there is no incentive for the firm to misrepresent the state of nature do not necessarily imply ex post profit maximisation if the states of nature are discrete rather than continuous. If this is the case, however, there is still an incentive not to follow the agreed employment rule ex post.

2. Note that A is strictly positive since $\partial A/\partial\sigma>0$ for $0\leqslant\sigma\leqslant 1$ and hence reaches a minimum of $1-4b\tilde{w}(1-b\tilde{w})$ at $\sigma=0$; this is positive since $b\tilde{w}<b\overline{w}\leqslant 1/2$.

3. Since $p_s f_s' \equiv w_s$ no longer implies $\mu_s\equiv 0$.

4 This is very much in the spirit of Fischer (1977).

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CHAPTER II

A DISEQUILIBRIUM ECONOMETRIC MODEL OF THE UNITED KINGDOM

I INTRODUCTION

The contemporary macroeconomics literature has seen two main lines of development in recent years. On the one hand there is the "equilibrium with imperfect information" school exemplified in the work of Lucas et al. (see e.g. Phelps et al.(1970), Lucas (1973), Sargent(1979)). The adherents of this approach take the view that markets continuously clear, and that cyclical variations in activity and employment are the consequence of the actions of agents with incomplete information confusing aggregate and relative price shocks. Variations in employment are due to the voluntary intertemporal substitution of labour by workers and unemployment is a secondary phenomenon due primarily to search behaviour (Mortensen (1970)). However, such models do appear to be contradicted by the observation that most observed unemployment is involuntary in nature. In the words of Solow (1980):

> "Even if the workers in question have misread the future, they are merely mistaken, not confused or mystified about their own motives. It is thus legitimate to wonder why the unemployed do not feel themselves to be engaged in voluntary intertemporal substitution, and why they queue up in such numbers when legitimate jobs of their usual kind are offered during a recession."

The alternative paradigm with a heritage stretching back to Keynes and developed in recent years by, among others, Clower

(1965), Barro and Grossman (1971), Malinvaud (1975) and Muellbauer and Portes (1978) is the "temporary equilibrium with quantity rationing" school, also known less accurately, but more succinctly, as the "disequilibrium" approach. Here trades take place at prices other than the Walrasian market-clearing price vector with the consequence that some agents face quantity rationing in the amount that they can buy or sell. This inability to execute desired trades then spills over into the demand and supply functions in other markets so that "effective" demands may differ from the "notional", or Walrasian demands. The Keynesian demand multiplier is a particular case of this spillover process. In Western economies the market most obviously failing to clear is that for labour, but chronic excess demand in the goods market is a feature of Communist bloc countries and the approach has been fruitfully applied here by Portes, Winter and Burkett (1980).

The major critics of this approach (e.g. Barro (1980)) have focussed on the inability of the theory to explain why prices do not move to make possible mutually advantageous trades. However, recent work on the role of implicit labour contracts as a vehicle for spreading risk between workers and firms have suggested circumstances under which wages will be inflexible to changes in product market conditions and involuntary lay-offs will occur (Azariadis (1980) provides a comprehensive summary of the field). An alternative explanation for price rigidities can be drawn from the absence of a Walrasian auctioneer. Drazen (1980) has pointed out that there is nothing rational in price adjustment rules which respond to excess effective demands or supplies when an economy is away from equilibrium, yet these provide the signals which in a decentralised economy lead to price changes. Indeed whether such rules will necessarily lead to a movement towards the Walrasian equilibrium will depend on the nature of the rationing scheme in force. For instance in the model of Honkapohja and Ito (1979) who employ a stochastic rationing scheme, a Keynesian unemployment equilibrium can occur at the Walrasian price vector, so that price adjustment rules based on effective demands may actually lead the economy away from the equilibrium price vector. Even with effective demands of the sort employed in this paper which take disequilibrium in all other markets except the market in question as given (Benassy (1975)), price adjustment rules based on effective excess demand will not necessarily lead the economy monotonically towards Walrasian equilibrium1. If information on disequilibrium takes time to diffuse through the economy then prices may adjust very slowly.

Whereas the incomplete information paradigm has lead to a number of empirical applications e.g. Lucas (1973), Sargent (1976) and Barro (1978)) to name but a few, applications of the non-clearing market paradigm have been more limited, at least partly on account of the need to allow for switches in regimes. Two recent applications of disequilibrium models of the labour market are Rosen and Quandt (1978), and Muellbauer and Winter

(1980). The first two authors set the observed level of employment equal to the minimum of supply and demand with observations on the changes in wages used to help identify which regime is in effect. Aside from the estimation complexity of such switching regime models, they are incapable of explaining the co-existence at an aggregate level of unemployment and vacancies. In this paper we follow the approach of Muellbauer and Winter of aggregating across sectoral labour markets, some of which may be in excess demand and others in excess supply. A consequence of this is that in the aggregate the minimum condition does not hold and unemployment and unfulfilled vacancies may be observed simultaneously. Such an approach seems essential if the non-clearing market paradigm is to be applied successfully to empirical data. At the same time estimation procedures are considerably simplified.

This non-clearing labour market provides a central feature of a model of a small open economy in which producers in imperfectly competitive product markets make production and pricing decisions, and workers make labour supply and consumption decisions, taking into account the impact of labour market disequilibrium. This is then applied to the United Kingdom over the period 1964 to 1979. As well as incorporating an explicit treatment of a labour market in disequilibrium, an attempt is also made to incorporate expectations explicitly and so to separate out which aspects of the model dynamics are due to structural features such as adjustment costs and stock effects, and which are a consequence of expectations formation mechanisms. The isolation of these separate sources of dynamics is essential to the appraisal of the efficacy of economic policy as Lucas (1976) has so forcefully pointed out. It is hoped that the methods of this paper may provide a useful alternative to the traditional "reduced-form adaptive expectations" and the more recent "equilibrium rational expectations" (see Sargent(1981)) methods.

The next section develops the theoretical framework of the model, derives the firm and household behavioural functions, and sets out the model of sectoral labour markets. The following section discusses the econometric methodology adopted and develops a portmanteau diagnostic test for serial correlation in dynamic simultaneous equation models. After a brief discussion of the data we then turn to a presentation of the empirical estimates, and conclude with an appraisal of the results and suggestions for future work.

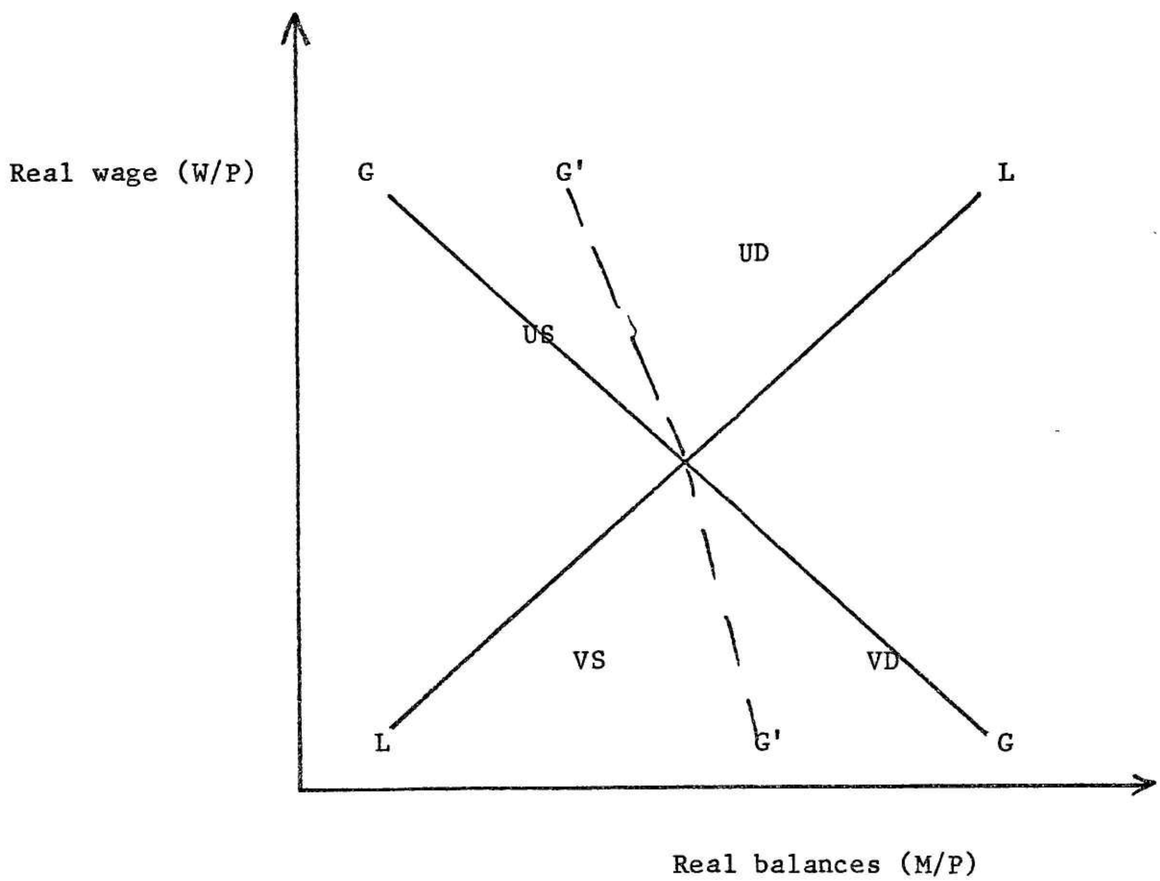
II THE MODEL

The structure employed here is broadly that of Dixit (1978) who develops a model of a small open economy producing a single tradeable good. In this model disequilibrium in the labour market leads to households or firms facing rationing in the amount of labour they can supply or obtain. This has repercussions for their demand for, or supply of, the consumption good. A divergence between the effective domestic demand for the consumption good and the effective domestic supply will not have repercussion effects on labour demand and supply, but instead spills over into the balance of payments. Thus excess effective domestic demand (supply) will lead to a balance of payments deficit (surplus).

This is illustrated in the Malinvaud diagram in Figure 1 which plots the notional labour market equilibrium (LL) and goods market equilibrium (GG) schedules in real wage-real money balance space. The locus G'G' corresponds to the effective goods market equilibrium schedule which allows for the effects of rationing in the labour market. LL and G'G' divide wage-price vectors into four regions: unemployment with a trade deficit (UD) or surplus (US); and excess demand for labour (vacancies) with a trade deficit (VD) or surplus (VS).

Our extension of this model consists of dropping the assumption of price-taking firms producing a single tradeable good

FIGURE 1



and replacing it by a world of monopolistic competition, and taking an explicitly intertemporal view of firms and households. A further extension would be to incorporate a non-traded goods sector along the lines of Neary (1978), but this introduces additional intrasectoral variables for wage and price variables on which data is unavailable, as well as necessitating consideration of the question of whether the non-traded goods market clears and what impact disequilibrium in this market has on agents behaviour in the labour and traded goods markets.

(a) FIRMS

Domestic producers face an uncertain and less than infinitely elastic demand schedule which is a function of the levels of world demand and home demand, and the price of competitors products. Unlike Muellbauer and Winter (1980) no explicit distinction is made between foreign and domestic markets, although the analysis could be extended without difficulty, but at the cost of complicating the model. They face a well-behaved production technology which utilises labour and raw materials and face given prices for these inputs. In addition there are costs of adjusting employment (hiring and training costs) and prices (the costs of notifying distributors of price changes and the loss of goodwill). The good is assumed to be storable, so that initial inventory holdings will also be a factor in determining the levels of output, prices, and employment. At time t the firm selects an optimal plan which solves:

$$(1) \quad \max_{Y_s, P_s, L_s} E\left[\sum_{s=t}^{\infty} \beta^{s-t}\left(P_s X_s - Q_s M_s - W_s L_s - g_1(\Delta L_s) - g_2(\Delta P_s) - g_3(S_s)\right)\right]$$

subject to:

$$(2) \quad Y_s = f(M_s, L_s, t) \quad f_1, f_2 > 0 \; f_{11}, f_{22} < 0$$

$$(3) \quad X_s = h(WD_s, HD_s, P_s, P_s^*)$$

$$(4) \quad S_s = S_{s-1} + Y_s - X_s$$

$$(5) \quad S_s \geqslant 0$$

$$(6) \quad L_s \leqslant L_s^{\circ} \quad s = t, \ldots, \infty$$

where P_s = sale price at time s

Y_s = output at time s

X_s = sales at time s

L_s = employment at time s

L_s° = labour ration at time s

M_s = raw material inputs at time s

W_s = wage at time s

Q_s = price of raw materials at time s

S_s = initial level of inventories at time s

P_s^* = competitors price at time s

WD_s = world demand at time s

HD_s = home demand at time s

Δ = difference operator

β = discount factor

g_1 and g_2 represent adjustment cost functions with the properties $g_1(0)=g_2(0)=0$ and $g_1'(x),g_2'(x)\gtrless 0$ as $x\gtrless 0$. g_3 is an inventory holding cost function with $g_3\geqslant 0$, $g_3'>0$. (2) is the production function, (3) the demand schedule for the firms product, (4) the inventory accounting identity and (5) the constraint that inventories can never fall below zero (this could be dropped if unfulfilled orders are admitted). Equation (6) allows for possible spillovers from expected future labour market constraints to current decisions. We further assume that firms hold point expectations about future demand and wages and prices so that the solution may be written in terms of expected values of these variables alone.

Even assuming simple functional forms for the production and cost functions and the demand schedule, an analytic solution to this problem is hopelessly complicated. It will, however, take the general form:

$$(7) \qquad (Y_t, P_t, L_t) = F({}_tP^*_s, {}_tW_s, {}_tQ_s, {}_tWD_s, {}_tHD_s, {}_tV_s, S, L_{t-1}, P_{t-1}, t)$$

Here F is a vector-valued function, V_t is the excess demand for labour, ${}_tX_s$ denotes the expected value of X_s at time t, and s runs from t to ∞. In the empirical work we shall allow for the possibility that production, pricing, and employment decisions may be taken in advance of uncertainty about the current levels of demand and prices being resolved so that ${}_tP^*_t$ may differ from P^*_t, etc.

To make (6) empirically implementable we approximate the vector function F by a local linearisation. Since theory suggests that the output and employment plans are homogenous of degree zero in costs and competitors prices and output prices are homogenous of degree one a log-linearisation rather than simple linearisation is chosen to facilitate testing of these restrictions. This also has the virtue of yielding elasticity estimates directly. In principle the choice of logarithmic, linear or other functional form is an empirical matter, but it seems unlikely to affect the results radically. Whether any such simple form is adequate in the face of constraints such as (5) is less clear, but the question is not pursued further.

Adding stochastic error terms to reflect errors in executing the optimal plan and omitted (hopefully orthogonal) variables, assuming that $\ell n(L_t+V_t) \simeq \ell nL_t + V_t/L_t \simeq \ell nL_t + V_t/\overline{L}$ (where $\overline{L}$ is the mean value of employment[2]), and applying the homogeneity restrictions discussed above:

(8) $y_t = \alpha_1 + X_t\beta_1 + \gamma_1 V_t + \varepsilon_{1t}$

(9) $(p_t - {}_tp^*_t) = \alpha_2 + X_t\beta_2 + \gamma_2 V_t + \varepsilon_{2t}$

(10) $\ell_t = \alpha_3 + X_t\beta_3 + \gamma_3 V_t + \varepsilon_{3t}$

where lower-case letters denote natural logarithms and

$X_t = (({}_tw_s - {}_tp^*_s), ({}_tq_s - {}_tp^*_s), {}_twd_s, {}_thd_s, {}_tV_{s+1}, s_t, \ell_{t-1}, (p_{t-1} - p^*_t), t)$

Since labour market rationing can only reduce output below its desired level we would expect the signs of γ_1 and γ_2 to be negative and positive respectively. The sign of γ_3 is by definition negative, with $\gamma_3 = -1/\overline{L}$. If this system were to be estimated by OLS the specification would place great weight on the completeness of the vector X_t, for the omission of any variables from this set would lead naturally to a correlation between vacancies and the error term. However, since V_t is an endogenous variable we shall be using an instrumental variable technique which renders this objection invalid.

These three equations in output, the relative price of output and employment constitute the production side of the model. The domestic price is set to clear the goods market ex ante, but may fail to do so ex post on account of expectational errors. Any excess of realised over anticipated demand is assumed to be met from one of two sources. First, producers may run down inventory levels. Mills (1962) has demonstrated that in the context of a model like the above firms are more likely to hold sufficient inventories to meet any possible demand if inventory storage costs are low and the more saleable goods are in future periods. Alternatively, if insufficient inventories are available the unanticipated excess demand is assumed to be met by a fall in net exports. While this is clearly not fully consistent with a model with differentiated goods, it is probably not an unreasonable characterisation of a Western capitalist economy like the United Kingdom. The role of this assumption is to ensure that there are no spillover effects from unsatisfied consumption demand onto labour supply. Instead the primary spillover effect of goods market disequilibrium is into future production and pricing decisions and into the trade balance.

(b) HOUSEHOLDS

The households optimisation problem may be expressed as:

$$(11)\quad \max_{C_s, L_s} \; E\left[\sum_{s=t}^{\infty} \gamma^{s-t} U(C_s, L_s)\right]$$

subject to:

$$(12)\quad \sum_{s=t}^{\infty} \delta^{s-t} W_s L_s + M_t = \sum_{s=t}^{\infty} \delta^{s-t} C_s$$

$$(13)\quad L_s \leqslant L_s^{\circ} \quad s=t,\ldots,\infty$$

where $U(C,L)$ = concave utility function in consumption and leisure with the usual properties.

C_s = consumption at time s

$\delta = 1/(1+r)$ where r = interest rate

γ = discount factor

M_t = initial wealth holdings (including the present discounted value of any exogenous income such as rent receipts), and all variables are measured in real terms.

In this case it is instructive to assume a particular form for the utility function, namely the Cobb-Douglas form $U=C^{\alpha}(L^*-L)^{\beta}$ where L^* is the households labour endowment. Manipulation of the first-order conditions for the problem (11) to (13) yield the following expressions for consumer demand and labour supply in terms of the financial wealth and the present value of the labour endowment:

$$(14) \quad C_t = K(W_s, L_s^\circ) NW_t$$

$$(15) \quad L_t = (\beta/\alpha)[W_t + \mu_t/\lambda] K(W_s, L_s^\circ) NW_t$$

where $NW_t = M_t + L^*(\sum \delta^s W_s)$

$$K(W_s, L_s^\circ) = \sum \delta^s \left[\frac{\phi^s (W_s + \mu_s/\lambda)}{(W_t + \mu_t/\lambda)} \right]^{\beta/\alpha+\beta-1} \left[1 + \frac{\beta}{\alpha} \left(\frac{W_s}{W_s + \mu_s/\lambda} \right) \right]$$

λ and μ_s are Lagrange multipliers on the constraints (12) and (13), (13) has been multiplied through by δ for analytical convenience, and $\phi = (\delta/\gamma)^{1-1/\beta}$

The multiplier λ represents, of course, the marginal utility of wealth endowments, and μ_s is the (discounted) shadow value of the unemployment constraint. However, life-cycle consumption functions are usually expressed in terms of financial wealth and the present value of labour incomes i.e. conditional on the actual quantity of labour supplied. It is illuminating to re-express (14) in terms of observed levels of employment. In that case :

$$(16) \quad C_t = K^*(W_s, L_s^\circ) NW_t^\circ$$

where $NW_t^\circ = M_t + \sum \delta^s W_s L_s$

$$K^*(W_s, L_s^\circ) = \sum \delta^s \left[\frac{\phi^s (W_s + \mu_s/\lambda)}{(W_t + \mu_t/\lambda)} \right]^{\beta/\alpha+\beta-1}$$

Equation (16) is a conventional life-cycle consumption function. However, it can clearly be seen that the propensity to consume depends on the multipliers μ_s and therefore on whether the consumer faces rationing in the labour market or not. Thus the fact that there is some set of real wage-rates at which the consumer would choose exactly the ration L_t° is not a justification for the empirical practice of estimating consumption functions conditional on the level of income if there are some sub-periods in which agents are rationed, since the propensity to consume will depend on whether he is rationed or not. Unemployment has two effects: first, there is an "income" effect on net worth NW_t°, which is captured by traditional consumption functions; and second there is a "substitution" effect on $K^*(W_s, L_s^\circ)$, which is ignored. Thus it is quite possible that unemployment may actually lead to increased consumption.

This, of course, ignores the possibility that the unemployed may not have unlimited access to capital markets and are unlikely to be able to borrow substantially against future income. In this case the effect of unemployment is likely to be to reduce current consumption. The existence of substantial unemployment benefits will, however, mitigate this effect.

To make (14) and (15) empirically implementable we assume that households, like firms, hold point expectations about the future levels of real wages and unemployment and linearise. At the same time we allow for any effect anticipated inflation may have on the intertemporal allocation of durables expenditure by incorporating price expectations in a somewhat ad hoc manner:

(17) $C_t = \alpha_4 + X_t\beta_4 + U_t\gamma_4 + \varepsilon_{4t}$

(18) $L_t = \alpha_4 + X_t\beta_5 - U_t + \varepsilon_{5t}$

where $X_t = ({}_tW_s, {}_tPC_s, {}_tU_{s+1}, M_t)$

PC_s = consumer price at time s

U_s = level of unemployment at time s.

This formulation assumes that all agents face an identical degree of rationing. While such an assumption is convenient, it ignores the fact that in the real world typically there are some agents who are rationed and some who are not. Aggregation would then lead to interactive terms where the propensity to consume out of the various exogenous variables is itself a function of the unemployment level. Treating this objection properly, however, requires data on the financial wealth and wage expectations of rationed and unrationed agents separately, and introduces an additional level of complexity into the model. Instead we justify (17) and (18) as local approximations to the true non-linear form. (For an attempt to treat this problem in the context of a traditional consumption function see Bean (1978).) This concludes the exposition of the household side of the model.

(c) AGGREGATION ACROSS SECTORAL LABOUR MARKETS

Most existing implementations of non-clearing market models assume that observations lie on either the supply or the demand curve. This leads to "bang-bang" switches in regimes, and rules out the possibility of the co-existence of unemployment and vacancies. Muellbauer and Winter (1980), however, have developed a model of sectoral labour markets in which firms and workers have limited access to more than one market. As a consequence at any instant some markets may be in a state of excess demand and others in a state of excess supply. Consequently in the aggregate both vacancies and unemployment may be observed.

Let the demand and supply in the the jth labour market be given by:

(19) $L_j^d = L^d + \eta_j$

(20) $L_j^s = L^s + \varepsilon_j$

Here L_j^d and L_j^s are labour demand and supply in the jth market, L^d and L^s are the mean demands and supplies across markets, and η_j and ε_j are random cross-section deviations. Labour demand and supply are contingent on the degree of rationing in all markets (including expected future rationing) except the one in question. They therefore correspond to the Benassy-Clower formulation of effective demands and supplies.

Suppose (ε_j, η_j) have zero expectation and joint cumulative density $F(\varepsilon, \eta)$; then the expected level of unemployment in any market is given by:

$$(21) \quad U = \iint_{\varepsilon-\eta>L^d-L^s} (\varepsilon-\eta+L^s-L^d)dF = U(L^d-L^s)$$

the expected level of vacancies by:

$$(22) \quad V = \iint_{\varepsilon-\eta<L^d-L^s} (\eta-\varepsilon+L^d-L^s)dF = V(L^d-L^s)$$

and the expected level of employment by:

$$(23) \quad L = \iint_{\varepsilon-\eta>L^d-L^s} (L^d+\eta)dF + \iint_{\varepsilon-\eta<L^d-L^s} (L^s+\varepsilon)dF$$

$$= L^d - V = L^s - U$$

The aggregate values of each of these variables is then obtained by simply multiplying by the number of markets. One can either assume a particular functional form for F or else choose it implicitly by assuming a convenient functional form for the unemployment-vacancy trade-off. Here the latter course is adopted and we choose a rectangular hyperbola UV=K. This particular functional form has frequently been used in previous investigations of the unemployment-vacancy relationship and seems to fit the data tolerably well. A more flexible form might be desirable, but degrees of freedom considerations mitigate against this. In principle (21), (22) and (23) comprise a three-equation system with non-linear cross-equation restrictions. In practice the vacancy equation (22) is omitted from the model and the vacancy variable V replaced by K/U for two reasons. First the recorded vacancies series in the United Kingdom is a poor guide to the true level of vacancies because multiple vacancies at any single firm are recorded as a single vacancy, and firms seeking labour may register vacancies at more than one employment office. The other argument is theoretical and suggests that even if vacancies were measured correctly they do not provide a good measure of labour market tightness in so far as firms hiring policies may be related to future needs as well as current ones, especially where a significant training period is necessary.

This concludes the exposition of the theoretical underpinnings of the model. We now turn to a discussion of the modelling of expectations and some econometric issues before returning to the model with a detailed discussion of its empirical implementation.

III ECONOMETRICS

Time series analysts such as Granger and Newbold (1974) have emphasised the deficiencies of much applied econometric work which pays inadequate attention to the dynamic and stochastic structure of time series relationships. More recently, following the seminal work of Sargan (1964), Hendry and others (see Davidson et al. (1978), Hendry and Mizon (1978), and Hendry (1980)) have advocated a modelling methodology which combines the insights of time-series analysis with the information about long-run economic structures yielded by economic theory. The approach has been dubbed "general to specific" modelling and may be charecterised as the "intended over-parameterisation (of models) prior to data-based simplification" (Hendry (1980)). While both these approaches have proved relatively successful in producing equations suitable for use in forecasting, their ability to illuminate the structure of relationships, beyond summarising certain salient features of the data-generation process, seems limited. In particular models such as the consumption function of Davidson et al. and the investment equation of Bean (1981) cannot distinguish between dynamics which are an inherent part of the structure, such as those due to adjustment costs, and those that are the consequence of expectations formation mechanisms. Since many theorists would argue that a large part of the dynamics present in time series regressions are the result of the latter[3] it is important to ascertain what aspects of the equation dynamics are due to expectations formation mechanisms if the resulting estimates are to be used for policy analysis.

One approach to the explicit modelling of expectations in rational expectations equilibrium models has been to derive optimal linear predictors of the expectational variables in question and then employ these in the behavioural relationship. For instance in the case of the consumption function theory might suggest that consumption is a particular distributed lead on expected future income. If agents are rational and the generation of incomes is described by a linear stationary stochastic process, then cross-equation restrictions may be derived between the consumption and income equations, which may then be applied as a test of the theory (Sargent (1978)). However, apart from any reservations one might have about the availability to agents of the information necessary to make, and their capacity to calculate these optimal forecasts, such a procedure is clearly inappropriate when markets do not clear and switches in regimes may occur, since the variables on which expectations are to be formed are no longer generated by linear stationary stochastic processes.

In this paper we adopt an alternative approach of treating expectational variables as latent variables which realised values measure with error and using instrumental variables. Suppose some variable is a function of expected future values:

(24) $y_t = {}_t x_{t+j}\alpha + X_t\beta + \varepsilon_t$

where X_t is a vector of exogenous variables.

Suppose we assume that agents hold expectations of future variables that are unbiased (but not necessarily minimum variance):

(25) $x_{t+j} = {}_t x_{t+j} + u_t$

with $E(u_t) = 0$

Note that this is a weaker assumption than rational expectations and u_t may be correlated with information available at time t. Simply substituting the actual future value into the behavioural equation is likely to introduce two sources of bias: first, there is a classical measurement error bias; and second, the expectational error u_t may well be correlated with some of the other included explanatory variables X_t. However, if we have some additional variables z_t with the property:

(26) $z_t = {}_t x_{t+j} + v_t$

with $E(v_t) = 0$

$E(u_t v_t) = 0$

$E(v_t \varepsilon_t) = 0$

Then (24) can be consistently estimated by instrumental variables with the realised future value replacing the unobservable expectation and z_t used as an instrument.

This approach is similar to McCallum (1976), but whereas he uses an extrapolative predictor as an instrument for the observed value of the variable about which expectations are formed we employ forecasts published by the, broadly Keynesian, National Institute for Economic and Social Research (NIESR). Since these incorporate special factors such as the existence of incomes policies which are not captured by simple autoregressive predictors they should be more highly correlated with the unobservable expectational variables and therefore provide more efficient estimates. There seems to be no good prior reason for expecting the forecast divergence v_t to be correlated with the equation error ε_t. The assumption that the divergence between agents expectations and the NIESR forecasts v_t, and the agents forecast error u_t are uncorrelated is more questionable. Equations (25) and (26) imply:

$$(27) \quad z_t = x_{t+j} - u_t + v_t$$

Hence if $E(u_t v_t)=0$ the variance of NIESR forecasts must exceed the variance of individuals projections. The implication is that agents do not ignore the NIESR forecasts when casting their own projections. Since they receive wide press coverage this may not be unreasonable. However, if there are additional instruments available (in this case lagged values of the demand and price variables) then an asymptotic test of the validity of the chosen instruments is provided by the statistic:

(28) $e'P_z e/s^2 \overset{A}{\sim} \chi^2(m-k)$

where $P_z = Z(Z'Z)^{-1}Z'$ and Z is a T×m matrix of instruments

e = estimated residual vector

s^2 = consistent estimate of the equation error variance

k = number of explanatory variables.

As well as providing a test of the legitimacy of the chosen instruments, this procedure also provides a test of the overidentifying restrictions implied by excluding the additional explanatory variables from the estimated equation. This proves a useful test in the ensuing empirical work.

As already noted, time series econometricians have drawn attention to the importance of careful modelling of the dynamic and stochastic structure. Although the procedures of this paper lie within the spirit of the "general to specific" approach, the methods used in previous work in which initial model estimates are deliberately overparameterised with, say, five lags on all explanatory variables, prior to factorisation of the lag polynomials into equation and error dynamics (Hendry and Mizon) or applying restrictions derived from some underlying economic theory (Davidson et al. and Bean) is of limited applicability here because of the large number of explanatory variables in the behavioural equations, especially the production side, and the relatively small sample size (fifty-eight observations from 1965Q1 to 1979Q2) which is dictated by the availability of the NIESR forecasts. In addition when using instrumental variable techniques special care is needed to avoid losing the desirable properties of the estimator through having too many instruments. Attention is therefore restricted to at most second-order dynamics. However, to guard against biases introduced by neglecting higher order dynamics we extend Godfrey's (1978) portmanteau Lagrange Multiplier test for serial correlation in dynamic equations to a simultaneous equations context. In the appendix it is shown that a test of the null hypothesis of white noise errors against the alternative of up to pth order autocorrelation is provided by the following test statistic:

$$(29) \quad \pi(p) = (e'P_z e_p)(e_p' P_z (I-P_{\hat{X}}) P_z e_p)^{-1} (e_p' P_z e)/s^2 \overset{A}{\sim} \chi^2(p)$$

where e_p' is the $(T \times p)$ matrix formed by the first p lags of the residual vector e

$$P_{\hat{X}} = P_z X(X'P_z X)^{-1} X'P_z$$

This concludes the discussion of econometric issues.

IV DATA

We now return to the model set out in Section II. Detailed definitions and data sources are given in the appendix, but a number of comments regarding the empirical implementation are appropriate here. First, Sims (1974) and Wallis (1974) have noted the dangers of using data which are independently seasonally adjusted. The Central Statistical Office, in common with other statistical agencies seasonally adjusts series by a moving average process which is likely to distort the dynamic structure of empirical relationships and lead to inconsistent estimates. Consequently seasonally unadjusted data is employed throughout, and three seasonal dummies included in each equation. If an evolving seasonal pattern is present some fourth-order serial correlation might be expected.

The production sector was set out in equations (8),(9) and (10). Output Y_t is taken as real Gross Domestic Product, the output price P_t as the GDP deflator and employment L_t as the total number of employees. This definitionincludes the public sector, which may be subject to very different decision criteria. Unfortunately there is little alternative since there is insufficient data on the public sector to enable it to be separated. Home demand is defined as the sum of consumption, investment and government expenditures, with the latter two taken as exogenous to the model. Since a measure of demand that is relevant to producers is required, an adjustment for indirect taxes is necessary. we assume that all indirect taxes can be attributed to one of these components (in practice most of them fall directly on consumption) and net them out accordingly. World demand is similarly defined as the sum of home demand in a number of major OECD countries. (Lack of data precludes any adjustment for indirect taxes here.)

Wage-rates (GW_t) are adjusted for non-wage labour costs such as National Insurance which have risen substantially as a fraction of direct wage costs in recent years. For raw materials we need an indicator of inputs external to the economy and the price of imports of fuels and other basic materials is appropriate. Competitors prices are taken as the price of imports of finished manufactures. The inventory variable only refers to stocks of finished goods and therefore excludes raw materials and work in progress. Since only the change in inventories is available we use the approximation

$$(30) \quad \ell n S_t = \ell n(S_o + \sum S_s) \simeq \ell n S_o + \sum_{s=1}^{s=t} S_s$$

where S_o is the level of inventories at the start of the data period.

A strike variable (DS_t) measuring the man-days lost due to industrial action and the (logarithm of the) ratio of company sector liquid assets to liabilities (A_t) are included as additional explanatory variables.

The household sector was set out in equations (17) and (18). Consumption here refers to total consumers expenditure and no attempt is made to model durable expenditures separately. The labour supply equation is modelled with unemployment U_t as the explanatory variable and measures total registered unemployed excluding school-leavers since the latter are a seasonal and largely transitory phenomenon. As such this explains the labour force participation decision rather than labour supply. We simply assume the two are directly related.

A difficulty arises because the unemployment variable measures only those who register for unemployment benefit. It therefore ignores a substantial group of workers who may be actively seeking a job, but do not register. In addition there may be a "discouraged worker" effect by which potential workers may decide not to participate in the labour force if the probability of getting a job is small. In so far as measured unemployment and the discouraged worker effect are proportional to "true" unemployment (L^s-L) there is no difficulty since this merely has the effect of scaling the estimated parameters. However, it seems plausible that the discouraged worker effect may also be related to whether labour market tightness is increasing or decreasing and therefore to the rate of change of unemployment as well. Lagged unemployment is therefore included as an additional variable in the participation (unemployment) equation. A lagged endogenous variable is also included in the consumption equation to allow for any habit persistence effects on expenditure.

No overall wealth data is available yet for the United Kingdom, so as a proxy we use personal sector real net liquid assets. Arguably this is actually a more appropriate measure for the bulk of households who own little financial wealth and whose non-financial wealth such as housing may be insufficiently liquid. However, the personal sector does include life assurance and pension funds who hold substantial quantities of liquid assets and reallocate their portfolios frequently. There is therefore a potential measurement error problem here, but it is ignored in the empirical work for want of a suitable instrument.

Since the real marginal net wage rate (NW_t) is the appropriate earnings indicator, the wage index is multiplied by the standard rate of income tax, which is the appropriate marginal rate for most households, and deflated by the consumer price index. The full complexity of the income tax and benefits system is much too difficult to model at this aggregate level, but we approximate it by a negative income tax system. The value of the basic allowance may then be imputed from data on employment incomes, income taxes, and the standard tax rate. To this exogenous basic allowance is then added interest, dividends and rent receipts (net of tax at the standard rate) and deflated to form a measure of real exogenous income (N_t). This is then included as an additional explanatory variable in the household sector equations.

Finally dummies are included in the expenditure equations to allow for anticipated sales tax changes in the Budgets of 1968 and 1973, and the numbers of prime age males and females plus a time trend are included in the participation equation to allow for demographic effects. Finally to allow for the well-documented shift in the unemployment-vacancy relationship (see Nickell (1979)) that took place in the late sixties and variously attributed to changes in demographic trends, the value of unemployment benefits and redundancy legislation, a shift dummy is incorporated in all the equations but that for expenditure.

As noted in the preceeding section, NIESR forecasts are employed as instruments for the variables whose future values appear in the model. Forecasting bodies usually concentrate on projecting growth rates. Since the data on which these forecasts were based are frequently revised, using forecasts of simply the variable levels is not appropriate. Instead we computed the growth rates implied by the forecasts and then applied these to the final data in the period prior to which the forecast was made to obtain forecasts of the levels of the variables in question corrected for data revision. For world demand only year-on-year forecasts are available and these have been converted to quarterly paths by interpolation using a spline technique, computing the quarterly growth rates and then applying the foregoing technique. To turn the earnings projections into gross and net wage forecasts we assume that the ratio of labour costs to wage rates and the personal tax rate are expected to remain unchanged.

There is a difficulty with the forecasts of import prices in that no published forecasts are available prior to 1969. (Forecasts of other variables start on a regular basis in 1965Q1, which therefore marks the start of the estimation period.) From then until end-1971 and again from 1976Q2 until the present only a total import price forecast is available. For the remainder of the period (1972Q1 until 1976Q1) the two categories are distinguished, but for the periods 1972Q1 to 1973Q3 and 1975Q2 to 1976Q1 only semi-annual forecasts are available. For these two latter periods the same interpolation technique has been used as on the world demand variable. To obtain separate forecasts for the period when no distinction was made or when simply no forecasts were available we constructed univariate autoregressive forecasts for the requisite number of periods ahead for each series using second-order processes, updating the coefficients each period (i.e. a recursive regression). The univariate forecast is plausible here because import prices are likely to be exogenous to the domestic economy. The actual series were then regressed on the univariate and NIESR forecasts, allowing for different coefficients over the different sub-periods, and the predicted values employed as instruments. This procedure was used rather than directly including both univariate and NIESR forecasts as instruments (each defined over appropriate sub-periods) to limit the total number of instruments.

Unfortunately a continuous time-series of unemployment forecasts does not exist4. Instead anticipatory effects due to expected future rationing are proxied by the current and lagged unemployment in the household equations or "theoretical" vacancies (K/U) in the production equations.

Continuous forecasts for most of the variables are available up to five quarters ahead, including the current period. This limits consideration to distributed leads on expected variables to five quarters, including the current one. For the sample size under consideration this is more than adequate, and in fact for the production sector we limit attention to only four quarters ahead to conserve degrees of freedom. The empirical implementation of the model is therefore as summarised in Table 1. This is a simultaneous equation sytem which is non-linear in the variables and single and multiple equation non-linear instrumental variable methods (see Amemiya (1974,1977)) can be used to obtain consistent, although not efficient, estimates of the parameters. In principle one could add a trade balance equation to close the model, but since the focus of the investigation is on the quantitative importance of spillover effects and the role of expectations we do not pursue this further.

TABLE 1: SUMMARY OF MODEL

DEPENDENT VARIABLE	INDEPENDENT VARIABLES
Production Sector	
$(p-p^*)$, ℓ, y	Constant, seasonals, wd_{+s}, hd_{+s}, $(q-p^*)_{+s}$, $(gw-p^*)_{+s}$, ℓ_{-1}, $(p_{-1}-p^*)$, $(1/U)$, $\Delta(1/U)$, a, S, DS, t. $s = 0,\ldots,3$.
Household Sector	
C	Constant, seasonals, NW_{+s}, PC_{+s}, M, N, C_{-1}, U, ΔU, D68, D73.
U	Constant, seasonals, NW_{+s}, PC_{+s}, M, N, U_{-1}, L, t, DM, DF. $s = 0,\ldots,4$

V EMPIRICAL ESTIMATES

As well as reflecting omitted variables which may or may not appear in a number of equations, the error terms in the model of Table 1 also incorporate the effects of expectational errors ($wd_{t+s} - {}_t wd_{t+s}$), etc. Thus one would certainly expect that the block of equations describing the production sector and that describing the household sector would each have contemporaneously correlated error terms. Further, since expectations in the economy will be generally optimistic or pessimistic non-zero correlations between errors in the production equations and those in the household sector are likely. This suggests that multiple equation methods such as Non-Linear Three-Stage Least Squares (NL3SLS) will provide a gain in efficiency and raise the power of any test procedures. However, initial simplification of the highly parameterised equations in Table 1 is carried out using single equation methods - specifically Non-Linear Two-Stage Least Squares (NL2SLS). This procedure will still provide consistent estimates of correctly specified equations, whereas NL3SLS would not if any of the other equations were incorrectly specified. It also increases the degrees of freedom available in the first stage regressions, since all the exogenous variables which appear in the model must be included as instruments to ensure consistency, and consequently allows us to consider a greater range of dynamic structures[5].

NL3SLS and NL2SLS permit the use of any transformations of the exogenous variables as instruments, the ideal instrument for an endogenous variable y being $E(y|X)$ where X is a set of exogenous variables (which may be difficult to compute for a non-linear system such as this). In this case we confine ourselves to the use of only the exogenous variables themselves to conserve degrees of freedom. To further reduce the number of instruments, instead of employing a set of variables determining labour supply (demand) in the production (household) equations we construct single instruments for U and L by taking the predicted value of a regression of these variables on a subset of the exogenous variables determining labour demand and supply[6,7].

Initial estimates of the model included the forecast price of imports of manufactures ${}_tp^*_{t+s}$ as well as the forecast values of demand, etc., in the instrument set. The specification test in equation (27) provides a test of the overidentifying restrictions, which here amounts to the assumption of price homogeneity. The test statistics, distributed as $\chi^2(4)$ under the null of correct specification, are 4.07, 1.66 and 2.11 for the price, employment and output equations respectively. Thus the assumption of homogeneity is not rejected in each case. While these tests may be of rather low power because of the overparameterisation, it does seem sensible to apply restrictions based on theoretical presumptions before applying any further data-based restrictions. These estimates are generally fairly poorly defined

TABLE 2: PRODUCTION SECTOR, SINGLE EQUATION ESTIMATES

INDEPENDENT VARIABLE	DEPENDENT VARIABLE (p-p*)	ℓ	y
wdp†	.24 (0.43)	.384 (1.43)	-.866 (1.21)
wdt†	-.808 (2.07)	.022 (0.11)	.814 (1.62)
hdp†	.162 (0.64)	-.147 (1.18)	.806 (2.45)
hdt†	-.092 (0.28)	-.132 (0.83)	1.197 (2.83)
(q-p*)p†	-.027 (0.65)	.016 (0.76)	-.116 (2.13)
(q-p*)t†	.022 (0.63)	.024 (1.39)	-.01 (0.21)
(gw-p*)p†	.492 (2.74)	.132 (1.51)	-.268 (1.15)
(gw-p*)t†	.285 (2.29)	.095 (1.56)	-.177 (1.10)
ℓ_{-1}	-.174 (0.47)	.993 (5.48)	-1.05 (2.19)
$(p_{-1}-p^*)$†	.352 (2.10)	-.11 (1.34)	.138 (0.64)
(1/U)†	.567 (0.06)	.599 (0.13)	25.7 (2.12)
Δ(1/U)†	16.93 (1.30)	7.95 (1.25)	-.601 (0.04)
a	-.031 (2.03)	.009 (1.20)	.019 (0.97)
S	.12 (1.17)	-.056 (1.10)	-.278 (2.04)
DS	-.869 (0.97)	-.153 (0.35)	-.02 (1.72)
t	-.0071 (1.18)	-.0045 (1.52)	.017 (2.21)

TABLE 2(Cont.)

	DEPENDENT VARIABLE		
	$(p-p^*)$	ℓ	y
SEE	.0094	.0046	.0121
$Z_1(11)$	14.4	1.66	11.3
$Z_2(1)$	2.83	1.40	0.33
$Z_3(4)$	3.56	2.34	8.09
$Z_4(4)$	9.18	1.96	25.07

Estimation period: 1965Q1-1979Q2

Numbers in parentheses are asymptotic standard errors. Dummies and seasonals have been omitted for brevity.

Z_1 is the specification test set out in equation (27), Z_2 and Z_3 are π-tests for first and first to fourth order serial correlation respectively, and Z_4 is a test of out of sample parameter stability. Each statistic is distributed as $\chi^2(n)$ under the null hypothesis.

† denotes a variable that has been instrumented. The additional instruments are:

fwdp, fwdt, fhdp, fhdt, $f(q-p^*)p$, $f(q-p^*)t$, $f(gw-p^*)p$, $f(gw-p^*)t$,
fp^*, $(1/fU)$, wd_{-1}, wd_{-2}, hd_{-1}, hd_{-1}, q_{-1},
q_{-2}, gw_{-1}, gw_{-2}, ℓ_{-2}, p_{-1}, p_{-2},
$(1/U_{-1})$

where an f prefix denotes the NIESR forecast of the variable in question, with the exception of fU which denotes the contructed instrument described in the text.

and for brevity not reported here. While a large number of valid reparameterisations would be possible one such set which attempts to draw a distinction between long-lived ("permanent") changes in demand, and those that are short-lived ("transitory"), is presented in Table 2. Here a p suffix indicates a "permanent" variable with $xp_t = \sum_{s=0}^{s=3} x_{t+s}/4$ i.e. the mean of x_t during the following year, and a t suffix indicates a "transitory" variable with $xt_t = (x_t - xp_t)$ i.e. the deviation of the current value of x from its permanent value. For each equation the restrictions are easily accepted, the asymptotic test statistics, distributed as $\chi^2(8)$ under the null, being 2.98, 1.39 and 3.26 respectively.

$Z_1(n)$ is the specification test described in (27) and distributed as $\chi^2(n)$ under the null hypothesis that the instruments are uncorrelated with the error term. $Z_2(1)$ is a π-test for first-order serial correlation (distributed as $\chi^2(1)$ under the null of serially uncorrelated errors) and $Z_3(4)$ is a π-test for first to fourth-order serial correlation (distributed as $\chi^2(4)$ under the null). Finally $Z_4(n)$ is an asymptotically valid test of out of sample parameter stability defined by:

$$(31) \quad Z_4(kT) = \sum (e'S^{-1}e) \overset{A}{\sim} \chi^2(kT)$$

where e = (k×1) vector of forecast errors associated with a set of k equations,

S = estimated (k×k) error variance-covariance matrix, and the summation is taken over the T forecast periods.

The out of sample period here is 1964Q1 to 1964Q4 so that T=4 and for single equation estimates k=1. This is a particularly stringent test of out of sample parameter stability since it ignores the part of the forecast error that is attributable to parameter uncertainty, and rejection of the null hypothesis in small samples with overparameterised models need not necessarily indicate model misspecification. By the same token completely incorrect models could be expected to pass this test in large samples if the stochastic process generating the data remain unchanged. Nevertheless it remains a useful diagnostic tool.

As well as the NIESR forecasts described above, the instrument set for these regressions include two lagged values of the demand, wage and raw material price variables, as well as additional lags on employment and the output price. The Z_1 statistic therefore provides a joint test of the hypothesis that none of these enter the structural equations. In no case is the hypothesis rejected at conventional significance levels, providing support for the dynamic structure chosen. The implication of this finding is that one cannot reject the hypothesis that the lags in conventional price and employment equations represent a mixture of adjustment costs (through ℓ_{-1} and p_{-1}) and expectational mechanisms, and are not a consequence of backward looking feedback mechanisms as advocated by Hendry and Spanos (1980). However, it should be emphasised that this may not be a particularly powerful test.

Although none of the π-tests indicate the presence of serial correlation at the 95% level, the marginal significance levels for the first-order test in the price equation and the portmanteau statistic in the output equation are sufficiently high to cause concern. Estimation by autoregressive instrumental variables is not particularly attractive since it introduces a large number of additional instruments. However, an alternative procedure, which yields estimates that are consistent irrespective of the order of the serial correlation, is to instrument the lagged endogenous variables using lagged exogenous variables. When this procedure is carried out the parameter estimates change very little, which suggests that serial correlation is unlikely to be a source of significant bias (this could merely indicate endogeneity of the instruments although the Z_1 statistic suggests this is unlikely). The parameter stability tests for the price and employment equations are fairly good, although the out of sample performance of the output equation is rather less satisfactory.

Turning now to the parameter estimates themselves, we note that there is little evidence of a positive effect of expected demand, either transitory or permanent, on prices, and that labour costs are the most important influence. Transitory changes in expected labour costs as well as permanent changes have a significant impact which is in contradiction to the "normal cost" pricing school. High levels of corporate liquidity tend to reduce prices but there is no evidence of any spillover from rationing in the labour market although there is a suggestion that increases in labour market tightness tend to raise them.

The employment equation is distinctly unsatisfactory with a number of perverse signs. Home demand has a significant negative impact, real labour costs have a positive impact which is almost significant and the coefficient on "theoretical vacancies" is small and positive instead of the expected negative sign.

Aside from the negative impact of permanent world demand the output equation accords well with prior notions witha significant impact from home demand and a negative effect from labour and raw material costs. High levels of inventories reduce output as do strikes. Unfortunately once again the theoretical vacancies variable is incorrectly signed indicating a positive spillover from labour market tightness to output.

Turning now to estimates of the household sector, unrestricted single equation estimates of the consumer demand and unemployment (participation) equations are presented in Table 3, and particular reparameterisations in Table 4. Since the instrument set, which includes lagged wages and prices, is the same for both the restricted and unrestricted estimates, a test of

TABLE 3: HOUSEHOLD SECTOR, SINGLE EQUATION ESTIMATES

INDEPENDENT VARIABLE	DEPENDENT VARIABLE C	U
NW†	38.23 (1.07)	7.76 (1.52)
NW_{+1}†	-12.76 (0.24)	-9.08 (1.11)
NW_{+2}†	-.017 (0.0)	1.33 (0.2)
NW_{+3}†	4.35 (0.12)	4.87 (0.77)
NW_{+4}†	31.49 (1.19)	-8.44 (1.93)
PC†	-3.77 (0.05)	22.26 (1.61)
PC_{+1}†	-89.12 (0.74)	-8.0 (0.46)
PC_{+2}†	136.18 (1.34)	-3.9 (0.25)
PC_{+3}†	-109.67 (1.04)	-13.3 (0.89)
PC_{+4}†	67.72 (1.04)	5.96 (0.42)
M	.031 (1.01)	.0016 (0.34)
N	-.36 (1.30)	-.014 (0.24)
C_{-1}	.23 (1.51)	
U†	.31 (0.37)	
ΔU†	-3.32 (3.62)	

TABLE 3(Cont.)

INDEPENDENT VARIABLE	DEPENDENT VARIABLE C	U
L†		-.20 (2.35)
U_{-1}		.55 (2.77)
t		-2.05 (0.46)
SEE	214.2	36.96
$Z_1(n)$	3.94 (n=6)	4.43 (n=5)
$Z_2(1)$	2.70	2.04
$Z_3(4)$	4.62	3.54
$Z_4(4)$	10.43	2.87

Estimation period: 1965Q1-1979Q2

Additional instruments for consumption equation are:

fNW, fNW_{+1}, fNW_{+2}, fNW_{+3}, fNW_{+4}, fPC, fPC_{+1}, fPC_{+2}, fPC_{+3}, fPC_{+4}, fU, U_{-1}, M_{-1}, C_{-2}, NW_{-1}, NW_{-2}, PC_{-1}, PC_{-2}.

Additional instruments for unemployment equation are:

fNW, fNW_{+1}, fNW_{+2}, fNW_{+3}, fNW_{+4}, fPC, fPC_{+1}, fPC_{+2}, fPC_{+3}, fPC_{+4}, fL, M_{-1}, NW_{-1}, NW_{-2}, PC_{-1}, PC_{-2}.

fL is the constructed instrument referred to in the text.

TABLE 4: HOUSEHOLD SECTOR, RESTRICTED SINGLE EQUATION ESTIMATES

INDEPENDENT VARIABLE	DEPENDENT VARIABLE C	DEPENDENT VARIABLE U
NW†	58.18 (4.22)	-2.69 (0.84)
∇_4NW†	33.63 (4.08)	-3.18 (1.92)
∇_4PC†	34.15 (2.82)	-5.82 (1.29)
M	.032 (1.92)	-.0035 (0.92)
N	-.272 (1.37)	-.013 (0.31)
C_{-1}	.245 (2.09)	
U†	.39 (2.34)	
ΔU†	-3.18 (4.40)	
U_{-1}		.664 (8.30)
L†		-.122 (2.07)
t		1.78 (0.59)
SEE	200.2	33.59
Z_1(n)	9.81 (n=13)	19.4 (n=12)
Z_2(1)	2.64	1.5
Z_3(4)	6.45	7.08
Z_4(4)	12.69	1.31

Estimation period: 1965Q1-1979Q2

Instruments are the same as in Table 3.

∇ is the forward difference operator; thus $\nabla x = x_{+1} - x$.

the validity of the restrictions is provided by the incremental change in the Z_1 statistic. This takes the value of 5.87 for the consumption equation and 14.97 for the unemployment equation, each distributed with seven degrees of freedom under the null. The significant value of the second of these ststistics is largely due to the use of $\nabla_4 PC$ rather than $\nabla_3 PC$ (where ∇ is the forward difference operator; thus $\nabla_4 PC = PC_{+4} - PC$). The former seems more reasonable on theoretical grounds and conforms with the structure of the consumption equation, although purely statistical grounds favour the latter. In addition there is some scope for alternative reparameterisations which take into account more complex functions of the leaded price level in the consumption equation, but these are not readily interpretable in terms of sensible economic decision variables. One notable feature of the unrestricted estimates is the absence of any weighted average of expected future real wages, such as would be expected from life-cycle models of the sort set out in section II.

The restricted estimates of Table 4 are generally well-defined and accord well with prior expectations. Expenditure is significantly positively related to the expected level and rate of change of real net wages and to the anticipated rate of inflation. The real wealth effect is significant and positive, although the exogenous income variable is incorrectly signed. The lagged dependent variable is significant implying rejection of the purest form of the life-cycle consumption model, but its coefficient is small relative to that found in conventional distributed lag consumption functions confirming the view that a substantial portion of the dynamics in these equations is attributable to expectational factors.

Unemployment has a significant positive impact on consumer spending, with an increase in the level of unemployment of 250,000 (approximately a one percentage point increase in the unemployment rate) leading to a rise in consumption of something over 2/3%. This is in contradiction to the "liquidity constrained" school which predicts an unambiguous negative spillover effect. On the other hand rising unemployment does seem to have a negative spillover effect on consumption as agents build up precautionary savings: the same rise in unemployment leads to a temporary fall in expenditure of around 5%. Conventional Keynesian consumption functions in which the income effect of unemployment is captured in disposable income variables also tend to yield these results (see Bean (1978)), but the positive levels effect of unemployment here is rather surprising - especially to a Keynesian like the author.

Turning now to the unemployment equation we find a negative, but insignificant, impact of real net wages suggesting if anything a backward bending labour supply curve. On the other hand rising real wages promote an intertemporal reallocation of labour supply. This is the mechanism put forward by Lucas and Rapping (1970) to explain variations in employment over the business cycle. However, although the coefficient is significant it is hardly of sufficient magnitude to explain observed variations in unemployment since an expected 5% rise in real net wages over the coming year will reduce unemployment in the long-run by only 47,000 or something like 0.2 percentage points.

Anticipated inflation apparently has a negative impact on labour supply. This seems slightly puzzling since if anything one might expect inflation to lead to increased labour force participation in order to finance the bringing forward of durable purchases. One possibility is that its influence may be associated with the tendency of inflation to erode the real value of unemployment benefits and pensions thus reducing the present discounted value of entering the labour force. Liquid assets and exogenous income both have the anticipated negative effect on labour force participation, but are not conventionally significant.

Current employment has the correct sign, and the highly significant coefficient on lagged unemployment suggests a strong discouraged worker effect. However, the size of this coefficient suggests that it may well be proxying other omitted variables. The implied long-run propensity to register for unemployment benefits (i.e. the ratio of measured to theoretical unemployment) is a little under 0.4 which is not unreasonable though perhaps a little on the low side.

The out of sample performance of the unemployment equation is excellent, but that for consumption poor with a distinct tendency to overforecast. None of the tests for serial correlation are conventionally significant, although the tests for first-order correlation for consumption and the portmanteau test for the unemployment equation are sufficiently near the 90% critical points to cause concern. However, re-estimation treating lagged consumption and lagged unemployment as endogenous rather than predetermined and using lagged exogenous variables as instruments produces little change in the parameter estimates.

As noted above there is scope for the use of systems estimators to increase efficiency if there are non-zero covariances between the equation errors. A Lagrange Multiplier test for non-diagonality of the variance-covariance matrix of the errors (see Engle (1979)) yields a significant value of $\chi^2(10)=19.82$, indicating the potential gain from systems methods. Table 5 reports NL3SLS estimates of the system (the exogenous income variable has also been omitted), where the instrument set in addition to the appropriate forecast variables includes just the exogenous variables appearing in the system. This yields consistent estimates of the parameters under the null hypothesis that all equations are correctly specified, but unlike 3SLS in the linear system they are no longer asymptotically efficient on account of the non-linearities. In principle one could use the resulting estimates to generate predicted values of the endogenous

TABLE 5A: PRODUCTION SECTOR, SYSTEM ESTIMATES

INDEPENDENT VARIABLE	DEPENDENT VARIABLE (p-p*)	ℓ	y
wdp†	.341 (0.70)	-.075 (0.5)	.069 (0.14)
wdt†	-.40 (1.14)	.216 (2.0)	.46 (1.28)
hdp†	.097 (0.48)	.073 (1.19)	.393 (1.90)
hdt†	-.496 (1.91)	.217 (2.71)	.18 (0.68)
(q-p*)p†	.004 (0.11)	-.015 (1.38)	-.015 (0.43)
(q-p*)t†	.075 (2.26)	.025 (2.41)	.007 (0.20)
(gw-p*)p†	.505 (3.14)	.001 (0.03)	-.142 (0.85)
(gw-p*)t†	.293 (2.65)	.01 (0.28)	-.093 (0.82)
ℓ_{-1}	.058 (0.19)	.755 (8.19)	-.476 (1.54)
$(p_{-1}-p^*)$†	.418 (2.65)	.015 (0.31)	.036 (0.22)
(1/U)†	-1.42 (0.19)	5.78 (2.46)	15.29 (1.91)
Δ(1/U)†	45.4 (3.44)	.192 (0.05)	25.34 (1.89)
a	-.034 (3.06)	.017 (5.01)	-.007 (0.64)
S	.183 (2.15)	-.018 (0.69)	-.849 (0.98)
DS	-1.469 (1.84)	.393 (1.6)	-.035 (4.35)
t	-.009 (1.78)	.001 (0.64)	.0058 (1.10)
SEE	.0113	.0034	.0128
$Z_4(4)$	6.36	25.54	3.22

TABLE 5(b): HOUSEHOLD SECTOR, SYSTEM ESTIMATES

INDEPENDENT VARIABLE	DEPENDENT VARIABLE C	DEPENDENT VARIABLE U
NW†	57.5 (5.08)	-1.29 (0.72)
∇_4NW†	28.16 (4.18)	-2.29 (1.84)
∇_4PC†	19.88 (2.43)	-6.72 (2.18)
M	.041 (3.48)	-.0041 (1.24)
C_{-1}	.292 (2.89)	
U†	.43 (3.14)	
ΔU†	-3.06 (5.20)	
U_{-1}		.674 (10.18)
L†		-.101 (3.67)
t		-1.16 (0.62)
SEE	196.6	34.65
$Z_4(4)$	30.29	1.66

Estimation period: 1965Q1-1979Q2

Additional instruments are:

fwdp, fwdt, fhdp, fhdt, f(q-p*)p, f(q-p*)t, f(gw-p*)p, f(gw-p*)t, (fp*-p_{-1}), fNW, fNW_{+4}, (fPC_{+4}-fPC).

labour market variables and then use these as instruments in the final regressions. This procedure is analagous to Brundy and Jorgenson's (1971) FIVE estimator for the linear system. However, since these predicted values are rather tedious to compute and the gain in efficiency relative to the estimates in Table 5 is unclear we do not follow this course.

Since the results of Tables 2 and 4 are also consistent under the null of correct specification, significant changes in parameter estimates may be taken as an indication of misspecification. Hausman (1978) has presented a formal test for this in the linear simultaneous equations case based on the quantity:

$$(32) \quad (\beta_{3SLS}-\beta_{2SLS})'[\mathrm{Var}(\beta_{2SLS})-\mathrm{Var}(\beta_{3SLS})]^{-1}(\beta_{3SLS}-\beta_{2SLS}) \overset{A}{\sim} \chi^2(k)$$

where k is the number of estimated parameters.

The development of this statistic relies on the fact that, under the null, 3SLS is asymptotically efficient and therefore has zero covariance with the 2SLS estimator. Although NL3SLS is not asymptotically efficient for the model considered here, it is efficient relative to NL2SLS and the Hausman test is still applicable. Unfortunately, however, the matrix term in square brackets is not positive definite so that in this instance the test statistic cannot be computed. However, an eyeball inspection of the results suggest that in the main they have not changed significantly, with the exception of the employment equation where home demand now appears with positive rather than negative coefficients, the positive labour cost coefficients are much reduced, the coefficient on lagged employment is somewhat smaller and the spillover effect from theoretical vacancies is now significantly positive. The source of this change in parameter estimates turns out to be not the use of a systems estimator, but rather the change in the instrument set which has taken place between Tables 2 and 5[8]. This finding, together with the positive wage and labour market spillover effects, seems to be a clear indication of an inadequate specification.

In an effort to track down the source of the misspecification we first conjectured that it might be associated with the widely documented fall in the underlying rate of growth of productivity in the middle and late seventies, which was not being adequately captured by those variables already included. We therefore incorporated an additional trend variable starting in 1973Q1, but this had little impact on the results.

The perverse sign on labour costs could be explained if causation ran from employment to wages rather than vice versa. We also noted at the outset that if the determinants of desired output were incomplete then we would naturally expect the theoretical vacancies variable (1/U) to be positively correlated with the error term ε_2 in the employment equation. This suggests a second hypothesis: namely that our instrumental variable procedure may have been unsuccessful in "purging" the right-hand side variables of their correlation with the error term. (Note that this does not provide an explanation of the positive effect from unemployment to consumption.) In a sample as small as this with a relatively large number of instruments the instrumental variable estimates may still be badly biased. Anderson and Sawa (1977) have examined the small sample properties of 2SLS for a linear system and find that even for samples between 50 and 100 the bias can be substantial if the number of excluded exogenous variables is large. In order to shed further light on this we ran the following much restricted regression:

$$(33) \quad \ell = \text{constant} + \text{seasonals} + \underset{(0.07)}{.005wdp\dagger} + \underset{(0.45)}{.024hdp\dagger} - \underset{(0.46)}{.008gwp\dagger} + \underset{(0.85)}{.018gwt\dagger} + \underset{(7.58)}{.726\ell_{-1}} + \underset{(2.14)}{6.12(1/U)\dagger} + \underset{(3.67)}{.013a} - \underset{(0.8)}{.002DU}$$

SEE=.0035 $Z_1(4)=1.65$ $Z_2(1)=0.35$ $Z_3(4)=1.83$ $Z_4(4)=12.41$

Additional instruments: fwdp, fhdp, fgwp, fgwt, fNW, fNW_{+4}, $f\nabla_4PC$, M, DM, DF.

where DU is the unemployment shift dummy, and DM and DF are the demographic dummies.

The coefficient on theoretical vacancies is significantly positive and of the same magnitude as the estimates of Table 5, which at least suggests that small sample bias may not be at the root of the problem. It would appear that only a larger sample can answer this question conclusively,however.

An out of sample parameter stability test over the period 1964Q1 to 1964Q4 for the model as a whole yields the highly significant value of $Z_4(20)=108.3$, which is mainly due to the poor performance of the employment and consumption equations. While the performance of the former is explicable in terms of the misspecification discussed above, it is not easy to find an immediate explanation for the overprediction of the consumption equation in terms of events unique to 1964. On the other hand we have already remarked above that this is a particularly stringent test, so that it may not indicate a major misspecification.

VI CONCLUSIONS

In the foregoing sections we have developed and estimated a small macroeconomic model of an open economy with a non-clearing labour market, focussing especially on the empirical importance of the spillover effects due to labour market disequilibrium and the role of expectations. There seemed to be little evidence of a contemporaneous spillover from unsatisfied labour demand to the behaviour of firms, with theoretical vacancies (1/U) appearing with perverse signs in all of the price, output, and employment equations. With respect to the first and last of these the absence of a quantitatively important spillover effect may not be too surprising since labour shortages can usually be met by working the existing labour force harder and/or more efficiently. However, vacancies, almost by definition, should appear with a negative sign in the employment equation. Together with the perversely signed real wage variable, this strongly suggests that the employment equation is badly misspecified. One difficulty may be the very high level of aggregation, and this warrrants further investigation, although there are both theoretical difficulties and data deficiencies to be overcome if a greater degree of disaggregation is to be attempted. On the other hand there is some suggestion that increases in labour market tightness tends to raise output and prices which presumably reflects the impact of expectations.

The household sector estimates are generally more satisfactory than the production sector, and the results accord well with prior expectations. The results suggest the presence of a modest positive spillover from unemployment to consumption which contradicts the view that workers who face labour market rationing also face liquidity constraints. Since if anything one would probably expect a small sample bias in a negative direction this does seem to be a fairly strong result. It should be emphasised, however, that the fact the unemployed may not experience liquidity constraints most probably reflects the generous levels of unemployment compensation in the United Kingdom, and not their ability to borrow against future earnings. One would certainly expect the magnitude of the coefficient on unemployment in the consumption equation to be sensitive variations in unemployment compensation. On the other hand there is strong evidence of a negative effect on expenditure of rising unemployment. Taken with the results on the production side it suggests that emphasis on the contemporaneous spillover consequences of rationing may have been somewhat misplaced, and the expectational effects of market disequilibrium somewhat underplayed in the theoretical literature on models with quantity rationing.

If the success of the model in terms of isolating spillover effects is somewhat mixed, the approach to modelling expectations does seem reasonably successful. The greatest limitation here seeme to be the relatively short time period and the relatively small number of series for which instrumental forecasts, such as those produced by NIESR, are available. Nevertheless the approach does seem to present a useful complement to existing methods of modelling expectations.

Whether this approach to modelling markets in disequilibrium is likely to prove useful remains to be seen. However, it does seem to offer a method of modelling disequilibrium markets which is both more realistic and avoids the econometric complexities associated with existing attempts to implement the non-clearing market paradigm.

FOOTNOTES

1. For instance in the simple Malinvaud model there will be some wage-price vectors which yield an excess Walrasian demand for goods (labour), but produce an excess effective supply of goods (labour).

2. This is a reasonable approximation since $V_t \ll L_t$ and thus most of the variance of V_t/L_t is attributable to variations in V_t.

3. While this is the stance of most theorists Hendry and Spanos (1980) have argued that in the face of ignorance about the future backward-looking control rules are sensible and have interpreted the results of Davidson et al. and others in this light. The equation dynamics here reflect certain aspects of psychological behaviour and would not be expected to change if the process generating the exogenous variable changes.

4. For the latter part of the period there are year-end forecasts only, and for the earlier part published forecasts are infrequent.

5. This problem could be mitigated somewhat by carrying out NL3SLS on the production and household sector equations separately. Note, however, that the estimates for the production

sector would in that case be identical to the single-equation estimates since each equation contains the same set of explanatory variables.

6. Note that these constructed variables are used as <u>instruments</u> for U, 1/U, and L and not as replacements for them as in the second stage of a 2SLS regression. The latter procedure will in general not produce consistent estimates.

7. The explanatory variables in these auxiliary regressions are: constant, seasonals, fwdp, fhdp, f(gw-p*)p, ℓ_{-1}, fNW, fNW_{+1}, M, DU. For an explanation of these symbols see the text.

8. Note that the NL2SLS and NL3SLS estimates used for constructing the test statistic in (32) use the same instrument set, so that parameter changes due to the change in instruments between Table 2 and Table 4 can be diagnosed from the two sets of NL2SLS estimators.

APPENDIX: PORTMANTEAU TEST FOR SERIAL CORRELATION IN A DYNAMIC SIMULTANEOUS EQUATION MODEL

This section employs the methods of Godfrey (1976). The same statistic may derived using the results of Engle (1979). Let the model of interest be:

(A1) $y = X\beta + \varepsilon$

with

(A2) $\varepsilon = \rho(L)\varepsilon + u$

where X is a $(T\times k)$ matrix of explanatory variables including endogenous and lagged endogenous variables, β is a $(k\times 1)$ coefficient vector, ε and u are $(T\times 1)$ vectors of error terms with u independently distributed with zero mean and variance σ^2, and $\rho(L)$ is a polynomial of order p in the lag operator L with $\rho(0)=0$.

Sargan's (1959) AIV is an appropriate estimation technique if $\rho(L)$ is not identically zero. The AIV estimators are the minimisers of:

(A3) $S(\beta,\rho) = T^{-1}[(1-\rho(L))(y-X\beta)]'P_z[(1-\rho(L))(y-X\beta)]$

where ρ is the $(p\times 1)$ vector of polynomial coefficients in $\rho(L)$, Z is a $(T\times m)$ instrument matrix $(m\geq k+p)$ and P_z is the projection matrix defined above.

Following Godfrey (1976) define the following notation:

(A4) $\theta' = (\beta':\rho')$

(A5) $f_\beta(\theta) = \partial S(\theta)/\partial\beta$

(A6) $f_\rho(\theta) = \partial S(\theta)/\partial\rho$

(A7) $f_\theta(\theta)' = (f_\beta(\theta)':f_\rho(\theta)')$

(A8) $F_{\beta\beta}(\theta) = \partial^2 S(\theta)/\partial\beta\partial\beta'$

(A9) $F_{\beta\rho}(\theta) = \partial^2 S(\theta)/\partial\beta\partial\rho'$

(A10) $F_{\rho\rho}(\theta) = \partial^2 S(\theta)/\partial\rho\partial\rho'$

(A11) $F_{\theta\theta}(\theta) = \begin{bmatrix} F_{\beta\beta}(\theta) & F_{\beta\rho}(\theta) \\ F_{\rho\beta}(\theta) & F_{\rho\rho}(\theta) \end{bmatrix}$

Sargan has shown that:

(A12) $\sqrt{T}(\theta_{AIV}-\theta) \overset{A}{\sim} N(0, 2\sigma^2 \text{plim} F_{\theta\theta}(\theta)^{-1})$

where θ_{AIV} is the minimiser of $S(\beta,\rho)$.

Using this result the asymptotic distribution of $\sqrt{T}f_\theta(\theta)$ may be obtained from a Taylor expansion around θ_{AIV}:

(A13) $0 = f_\theta(\theta_{AIV}) = f_\theta(\theta) + F_{\theta\theta}(\theta)(\theta_{AIV}-\theta) + O(T^{-1})$

and:

(A14) $\sqrt{T}f_\theta(\theta) = -(F_{\theta\theta}(\theta))\sqrt{T}(\theta_{AIV}-\theta) + O(T^{-1/2})$

Hence omitting the θ argument where no confusion arises:

(A15) $\sqrt{T}f_\theta \overset{A}{\sim} N(0, 2\sigma^2 \text{plim} F_{\theta\theta})$

Under the null that $\rho(L)$ is identically zero the AIV estimator reduces to the simple IV estimator:

(A16) $\beta_{IV} = (X'P_zX)^{-1}(X'P_zy)$

Let $e = y - X\beta_{IV}$ be the associated residual vector, and consider the parameter vector ρ that minimises $S(\beta_{IV},\rho)$. This is simply:

(A17) $r = (e_p'P_ze_p)^{-1}(e_p'P_ze)$

where e_p' is the $(T\times p)$ matrix formed by the first p lags of the residual vector e.

It should be clear from (A17) that the test is only feasible if the degree of overidentification (m-k) is at least as great as the order of the autoregressive process p.

Under the null hypothesis of no serial correlation the asymptotic distribution of r may be obtained as follows. Consider the expansions:

(A18) $0 = f_\beta(\beta_{IV},0) = f_\beta(\beta,0) + F_{\beta\beta}(\beta_{IV}-\beta) + O(T^{-1})$

and

(A19) $0 = f_\rho(\beta_{IV},r) = f_\rho(\beta,0) + F_{\rho\beta}(\beta_{IV}-\beta) + F_{\rho\rho}r + O(T^{-1})$

Hence under the null:

(A20) $\sqrt{T}F_{\rho\rho}r = \sqrt{T}(F_{\rho\beta}F_{\beta\beta}^{-1}f_{\beta}-f_{\rho}) + O(T^{-1/2})$

Hence it follows that from the asymptotic distribution of $\sqrt{T}f_{\theta}$ derived above that:

(A21) $\sqrt{T}F_{\rho\rho}r \overset{A}{\sim} N(0,2\sigma^2 \text{plim}[F_{\rho\rho}-F_{\rho\beta}F_{\beta\beta}^{-1}F_{\beta\rho}])$

Now under the null it can easily be shown that:

(A22) $\text{plim}F_{\theta\theta} = 2\text{plim}T^{-1}\begin{bmatrix} X'P_zX & X'P_ze_p \\ e_p'P_zX & e_p'P_ze_p \end{bmatrix}$

Hence:

(A23) $\sqrt{T}r \overset{A}{\sim} N(0,\sigma^2A^{-1}BA^{-1})$

where $A = \text{plim}[(e_p'P_ze_p)/T]$

$B = \text{plim}[(e_p'P_z(I-P_{\hat{X}})P_ze_p)/T]$

$\hat{X} = P_zX$

We thus obtain the following test statistic:

(A24) $\pi(p) = (e'P_ze_p)(e_p'P_z(I-P_{\hat{X}})P_ze_p)^{-1}(e_p'P_ze)/s^2 \overset{A}{\sim} \chi^2(p)$

where s^2 is again a consistent esimator of the equation error variance.

A further simplification may be obtained by noting that since $(I-P_{\hat{X}})$ is idempotent and that $(I-P_{\hat{X}})e=e$, $\pi(p)$ is simply T times the coefficient of determination in a regression of e on $(I-P_{\hat{X}})P_Z e_p$, which is also T times the coefficient of determination of a regression of e on $P_Z X$ and $P_Z e_p$. Thus the form of the test statistic is identical to Godfrey's (1978) Lagrange Multiplier statistic except that both the explanatory variables and the residuals must first be projected on the instrument space. In practice we use (T-k) rather than T in compiling the statistic since in some cases the number of parameters to be estimated is large relative to the number of observations (i.e $e'e/(T-k)$ is used to estimate σ^2 rather than $e'e/T$).

DATA APPENDIX

This appendix describes the data definitions and sources. All recorded data is seasonally unadjusted. Forecast variables are only available on an adjusted basis. The following abbreviations apply: ET = Economic Trends Annual Supplement 1980; FS = Financial Statistics; AA = Annual Abstract of Statistics; MD = Monthly Digest of Statistics; DEG = Department of Employment Gazette.

A Ratio of corporate sector liquid assets to liquid liabilities (£M). Source ET.

C Consumers expenditure (£M, 1975 prices). Source ET.

D68 Budget dummy for 1968. Takes the value 1.0 in 1968Q1, -.67 in 1968Q2 and -.33 in 1968Q3.

D73 Budget dummy for 1973. Takes the value 1.0 in 1973Q1, -.67 in 1973Q2 and -.33 in 1973Q3.

DF Number of females aged 18-65 (annual, quarterly interpolated). Source AA.

DM Number of males aged 18-65 (annual, quarterly interpolated). Source AA.

DS Total working days lost due to strikes. Normalised by 10^{-6}. Source DEG.

DU Unemployment-vacancy shift dummy. Takes the value of zero until 1967Q4 and unity thereafter.

GW Labour cost index. Index of basic weekly wage rates (manual workers) in all industries and services times index of ratio of total labour costs to wage costs. Sources ET and DEG.

Forecast growth rates are for average earnings rather than wage rates and assume ratio of labour cost to wage rates remains unchanged.

HD Domestic demand = consumption + gross domestic fixed capital formation + government expenditure on goods and services - factor cost adjustment (all £M, 1975 prices). Source ET. Forecast values calculated in like manner.

L Employees in employment (thousands). Source ET.

M Personal sector net liquid assets at start of period (£M) deflated by consumer price deflator. Source FS and ET.

N Real exogenous income = [standard rate of income tax x (wages and salaries and forces pay + rent, dividends and interest) - taxes on income]/consumer price index. Source FS and ET.

NW Real marginal net wage = [Index of wage rates (manual workers) x (1-standard rate of income tax)]/consumer price index. Source ET.

Forecast growth rates are for average earnings rather than wage rates and assume that tax rates remain unchanged.

P GDP deflator (1975=100). Source ET.

P* Competitors prices. Unit value index for imports of food and manufactures. Source MD.

For explanation of forecasts see text.

Q Raw material prices. Unit value index for basics and fuel. Source MD.

For explanation of forecasts see text.

S Inventories of finished goods at start of quarter. Computed as total cumulated change in inventories excluding inventories of raw materials and work in progress held by manufacturers (£M, 1975 prices). Normalised by 10^{-4}. Source ET.

U Total registered unemployed excluding school leavers (thousands). Source ET.

WD World demand = consumption + investment + government expenditure (all at constant prices) in U. S., Canada, West Germany, Italy, Japan, and France. Source OECD Economic Indicators.

Forecast growth rates are for OECD GNP excluding U. K.; see text for further explanation.

Y Gross domestic product (£M, 1975 prices). Source ET.

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CHAPTER III

NOMINAL GNP VERSUS MONEY SUPPLY CONTROL: AN ANALYSIS OF THE MEADE-TOBIN PROPOSALS

I. INTRODUCTION

The practice of setting target growth rates for a year or more ahead for some measure of the money supply is now followed by most Western industrialized countries. However James Meade (1978) in his Nobel prize lecture, and James Tobin (1980) in his retrospective on stabilization policy have both suggested that since the velocity of circulation is volatile and unpredictable it would be more appropriate to set targets, or target ranges, for nominal incomes rather than the money supply. Full employment would then be obtained by the adoption of labour market controls designed to mimic competitive behaviour.

This paper presents a formal analysis of the consequences of adopting a strategy of controlling money expenditures (MV-control) and an assessment of its merits relative to the control of a monetary aggregate (M-control). We concentrate solely on this aspect of their proposals because neither Meade nor Tobin have indicated in detail the nature of their labour market policies. This certainly does not deny the importance of labour market policies to their overall strategy. However, because it is possible to envisage the adoption of nominal GNP targetry as a replacement for money supply targetry alone, the question of their relative merits is interesting.

The basic analysis is carried out in the next section. There it is demonstrated that in both a model of the sort advocated by the "New Classical" school and a "Keynesian" model with sluggish price adjustment that for a wide range of parameter values the MV-control rule is preferable. The consequences of information lags and an inability to hit targets exactly is examined in Section III. It is shown that if both M-control and MV-control is inexact there may be little to choose between them, but if MV-control is inexact and M-control is exact the ranking of the two policies may be reversed from the exact control case. The importance of this depends critically on the time horizon over which targets are to be set. Section IV considers the question of how control of nominal GNP might in practice be carried out and suggests the use of fiscal instruments, particularly variations in expenditure taxes, is likely to be preferable to monetary policy. This is followed by a brief summary of the conclusions.

II. AN INITIAL ANALYSIS WITH PERFECT CONTROLLABILITY

We initially assume that the authorities can control both the monetary aggregate and money expenditures perfectly over the relevant time horizon. First, we consider a neo-classical model with market-clearing and rational expectations of the sort employed by Sargent and Wallace (1975) and others:

(1) $y_t = a(p_t - {}_{t-1}p_t) + u_{1t}$

(2) $y_t = b(m_t - p_t) - c(i_t - {}_{t-1}p_{t+1} + {}_{t-1}p_t) + u_{2t}$

(3) $m_t - p_t = dy_t - ei_t + u_{3t}$

y_t, m_t, p_t and i_t are the logarithms of the level of output, money stock, price level and the nominal interest rate at time t respectively. y_t, m_t and p_t have been suitably normalized to make intercepts redundant. ${}_{t-1}p_t$ is the mathematical expectation of p_t conditional on information available at time t-1. u_{1t}, u_{2t} and u_{3t} are serially uncorrelated error terms with covariance matrix Σ. We shall often examine the special case where Σ is diagonal. The assumption of serial independence is perfectly general since an equation with serially correlated errors may always be transformed into an equivalent one with white-noise errors. The additional lags so introduced complicate the analysis but add no further insights. a, b, c, d and e are non-negative constants.

Equation (1) is an aggregate supply curve of the sort developed by Lucas (1973). Equation (2) is a conventional IS schedule and equation (3) is a demand for money function. Equations (2) and (3) may be combined to yield an aggregate demand schedule:

$$(4) \qquad \alpha y_t = \beta(m_t - p_t) + c({}_{t-1}p_{t+1} - {}_{t-1}p_t) + u_{2t} - (c/e)u_{3t}$$

where $\alpha = (1 + cd/e)$

$\beta = (b + c/e)$

Combining this with (1) we obtain:

$$(5) \qquad \alpha[a(p_t - {}_{t-1}p_t) + u_{1t}] = \beta(m_t - p_t) + c({}_{t-1}p_{t+1} - {}_{t-1}p_t) + u_{2t} - (c/e)u_{3t}$$

First consider the pursuit of a deterministic money supply rule. Taking expectations conditional on information available at time t-1:

$$(6) \qquad \beta({}_{t-1}m_t - {}_{t-1}p_t) + c({}_{t-1}p_{t+1} - {}_{t-1}p_t) = 0$$

Using the fact that for a deterministic rule ${}_{t-1}m_t = m_t$ this may be substituted into (5) to give:

$$(7) \qquad p_t - {}_{t-1}p_t = [u_{2t} - \alpha u_{1t} - (c/e)u_{3t}] / (\alpha a + \beta)$$

and from the aggregate supply curve:

$$(8) \qquad y_t = [\beta u_{1t} + a u_{2t} - (ac/e)u_{3t}] / (\alpha a + \beta)$$

Hence the variance of output under a deterministic money supply rule is:

$$(9) \quad Var(y_t) = \frac{[\beta^2\sigma_{11} + a^2\sigma_{22} + (ac/e)^2\sigma_{33} + 2a\beta\sigma_{12} - 2(\beta ac/e)\sigma_{13} - 2(a^2c/e)\sigma_{23}]}{(\alpha a + \beta)^2}$$

where σ_{ij} is the ijth element of Σ.

The variance of prices about their expected value (this is determined by the particular money supply rule chosen by the authorities) is given by:

$$(10) \quad Var(p_t) = \frac{[\alpha^2\sigma_{11} + \sigma_{22} + (c/e)^2\sigma_{33} - 2\alpha\sigma_{12} + 2(\alpha c/e)\sigma_{13} - 2(c/e)\sigma_{23}]}{(\alpha a + \beta)^2}$$

Now consider the pursuit of a deterministic control rule which fixes money expenditures $x_t = (y_t + p_t)$ rather than m_t. Taking expectations of the aggregate supply curve (1) immediately implies that ${}_{t-1}x_t = {}_{t-1}p_t$. Hence:

$$(11) \quad y_t = u_{1t} / (1 + a)$$

$$(12) \quad p_t = x_t - u_{1t} / (1 + a)$$

Thus an MV-rule insulates the economy from aggregate demand shocks, whether they originate in the goods market or money market. The variance of output and price is given by:

$$(13) \quad Var(y_t) = Var(p_t) = \sigma_{11} / (1+a)^2$$

Thus an MV-rule produces a lower output variance than an M-rule if:

$$(14) \quad \sigma_{11}(\alpha a + \beta)^2 < (1+a)^2 \, [\beta^2\sigma_{11} + a^2\sigma_{22} + (ac/e)^2\sigma_{33} + 2a\beta\sigma_{12} - 2(a\beta c/e)\sigma_{13} - 2(a^2c/e)\sigma_{23}]$$

Whether this condition is satisfied will depend on the magnitude of the elasticities and the various elements of Σ. If we assume that the off-diagonal elements are zero (14) reduces to:

$$(15) \quad (\alpha + \beta + 2\beta/a)(\beta - \alpha)\sigma_{11} + (1+a)^2 \, [\sigma_{22} + (c/e)^2\sigma_{33}] > 0$$

Hence a necessary condition for a monetary control rule to dominate is that $\alpha > \beta$. Now empirical studies suggest that the income elasticity of the demand for money is somewhat less than unity and the elasticity of demand with respect to real balances is also small, at least in the short run, so that this condition is likely to be fulfilled. However, since the maximum feasible value of $(\alpha - \beta)$ is unity an M-rule is only likely to dominate if the variance of supply shocks is much larger than the variances of shocks to goods and money demand, if the interest elasticity of money demand is high relative to the interest elasticity of demand for goods, or if the elasticity of the supply curve with respect to unanticipated price shocks is small. Thus not only does volatility of the velocity of circulation favour MV-control as Tobin argues, but also unpredictability of the IS schedule. Values of the variance of output under an MV-rule relative to that under an M-rule for a range of values of the parameters are presented in Table 1.

While the average rate of inflation is determined by the particular control rule chosen, it is also of interest to compare the variances of the unexpected component of the price level since in the Lucas story this will affect the informativeness of local prices and hence the elasticity of the supply curve. Assuming the off-diagonal elements are zero the MV control rule dominates if:

$$(16) \quad (\alpha + \beta + 2\alpha a)(\beta - \alpha)\sigma_{11} < (1 + a)^2 \; [\sigma_{22} + (c/e)^2\sigma_{33}]$$

A sufficient condition for this to be true is that $\alpha > \beta$ which is almost certainly true. Hence an MV-rule will lead to fewer "surprises" concerning the price level. In the Lucas model this will tend to make a market price more informative about the relative price and therefore raise the value of a. This can only strengthen the conclusions reached above concerning the superiority of an MV-rule.

We now drop the assumption of market-clearing which is central to models of the New Classical school. Instead of the Lucas supply

TABLE 1: VARIANCE OF OUTPUT UNDER MV-RULE RELATIVE TO M-RULE.

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; $d = 0.5$

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		1.17	1.05	0.98
0.1	0.5		1.03	0.99	0.96
0.5	0.1		0.99	0.89	0.75
0.5	0.5		0.89	0.82	0.73

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		1.19	1.06	0.99
0.1	0.5		1.04	1.0	0.96
0.5	0.1		1.29	1.04	0.85
0.5	0.5		1.02	0.9	0.79

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		1.13	1.03	0.97
0.1	0.5		1.02	0.98	0.95
0.5	0.1		0.68	0.69	0.62
0.5	0.5		0.72	0.69	0.62

function (1) assume that prices move only partially towards their market-clearing level:

$$(1') \quad p_t = \lambda p_t^* + (1-\lambda)p_t^+ + u_{1t} \qquad (0 \leq \lambda < 1)$$

Here p_t^* is the price consistent with "full employment" ($y_t = 0$) and p_t^+ is some initial and fixed value of the price level with the property that ${}_{t-1}p_t^+ = p_t^+$. Supply is infinitely elastic at p_t. One possibility might be that p_t^+ is set to equate expected demand and supply i.e. $p_t^+ = {}_{t-1}p_t^*$. In this case only unanticipated events matter and there is no role for activist monetary policy. An alternative case, discussed in greater detail below, is to assume a degree of price inertia because of adjustment costs and set $p_t^+ = p_{t-1}$. In that case activist monetary policy can offset the effects of lagged shocks. Initially, however, we shall not concern ourselves with how p_t^+ is determined and consider only the impact of shocks under the two control regimes on the variance of output about its expected value at t-1, ${}_{t-1}y_t$.[1]

From the aggregate demand schedule (4) it follows that:

$$(17) \quad p_t^* = m_t + (c/\beta)({}_{t-1}p_{t+1} - {}_{t-1}p_t) + (1/\beta)(u_{2t} - (c/e)u_{3t})$$

Substituting this into (1'):

$$(18)\quad p_t = \lambda m_t + (c\lambda/\beta)({}_{t-1}p_{t+1} - {}_{t-1}p_t) + (1-\lambda)p_t^+ + [u_{1t} + (\lambda/\beta)(u_{2t} - (c/e)u_{3t})]$$

Combining this with the aggregate demand schedule (4):

$$(19)\quad \alpha y_t = [\beta(1-\lambda)/\lambda](p_t - p_t^+) - (\beta/\lambda)u_{1t}$$

Hence:

$$(20)\quad \alpha(y_t - {}_{t-1}y_t) = [\beta(1-\lambda)/\lambda](p_t - {}_{t-1}p_t) - (\beta/\lambda)u_{1t}$$

Consider now the pursuit of a deterministic M-rule. Taking expectations of (18) at t-1 and subtracting yields:

$$(21)\quad (p_t - {}_{t-1}p_t) = u_{1t} + (\lambda/\beta)(u_{2t} - (c/e)u_{3t})$$

Substituting this into (20) then gives:

$$(22)\quad (y_t - {}_{t-1}y_t) = [(1-\lambda)(u_{2t} - (c/e)u_{3t}) - \beta u_{1t}] / \alpha$$

And:

$$(23)\quad Var(y_t - {}_{t-1}y_t) = [\beta^2\sigma_{11} + (1-\lambda)^2(\sigma_{22} - 2(c/e)\sigma_{23} + (c/e)^2\sigma_{33}) - 2\beta(1-\lambda)(\sigma_{12} - (c/e)\sigma_{13})] / \alpha^2$$

Now consider the case of a deterministic MV-rule. Using the identity $x_t = y_t + p_t$ equation (19) may be rearranged to yield:

$$(24) \quad [\alpha\lambda + \beta(1-\lambda)]y_t = \beta(1-\lambda)(x_t - p_t^+) - \beta u_{1t}$$

Taking expectations and subtracting:

$$(25) \quad y_t - {}_{t-1}y_t = -\beta u_{1t} / (\alpha\lambda + \beta(1-\lambda))$$

And:

$$(26) \quad \mathrm{Var}(y_t - {}_{t-1}y_t) = \beta^2\sigma_{11} / (\alpha\lambda + \beta(1-\lambda))^2$$

Thus assuming the off-diagonal elements of Σ are zero MV-control dominates if:

$$(27) \quad \beta^2[\alpha^2 - (\alpha\lambda + \beta(1-\lambda))^2]\sigma_{11} < (\alpha\lambda + \beta(1-\lambda))^2(1-\lambda)^2(\sigma_{22} + (c/e)^2\sigma_{33})$$

Thus, as before, a necessary condition for monetary control to be preferable is that $\alpha > \beta$. Since the maximum value of $(\alpha - \beta)$ is unity an M-rule is only likely to dominate if the variance of the price adjustment innovation σ_{11} is much larger than the variances of the shocks to the IS and LM schedules. Values of the variance ratio for a range of values of the parameters are presented in Table 2.

Now consider the particular case of lagged price adjustment where $p_t^+ = p_{t-1}$. This introduces serially correlated deviations from

TABLE 2: VARIANCE OF OUTPUT (ABOUT $_{t-1}y_t$) UNDER MV-RULE RELATIVE TO THAT UNDER M-RULE FOR PRICE EQUATION (1')

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; $d = 0.5$

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		0.92	0.84	0.60
0.3	0.5		0.84	0.70	0.52
0.7	0.1		1.07	1.03	0.88
0.7	0.5		1.02	0.93	0.81

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.34	1.08	0.73
0.3	0.5		1.04	0.82	0.61
0.7	0.1		1.21	1.10	0.92
0.7	0.5		1.07	0.96	0.84

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		0.56	0.58	0.44
0.3	0.5		0.61	0.53	0.41
0.7	0.1		0.86	0.91	0.81
0.7	0.5		0.92	0.86	0.76

the "full employment" level of output and enables us to examine the implications of setting immutable long-term targets for money supply or nominal GNP. It is immediately apparent from equation (19) that lagged price adjustment produces an exploitable trade-off between the level of activity and the rate of inflation. Since higher rates of inflation place a greater premium on price flexibility it seems reasonable to suggest that λ might be an increasing function of the rate of inflation, so that the long-run trade-off might be considerably less pronounced or even non-existent. However whether this is so is immaterial since we will be concerned with the relative performance of money supply and nominal GNP rules designed to achieve the same mean levels of inflation and output. This enables us to concentrate on characteristics which are a consequence of the type of control rule rather than the particular rule followed by the authorities. To simplify the algebra and without loss of generality we shall examine the case where the monetary authorities set $m_t = 0$ for all t with M-control, and $x_t = 0$ for all t with MV-control. In both cases this produces a mean value of output $\bar{y}_t = 0$ and a mean value of the price level of $\bar{p}_t = 0$.

It is easiest to obtain the reduced form expressions for p_t and y_t in terms of the exogenous shocks and the policy variables by using the method of undetermined coefficients. For the case of a deterministic M-rule assume a solution of the form:

$$(28) \quad p_t = \sum_{-\infty}^{\infty} \pi_i m_{t-i} + \sum_{0}^{\infty} \rho_i v_{t-i}$$

where $v_t = [u_{1t} + (\lambda/\beta)(u_{2t} - (c/e)u_{3t})]$

Substituting this into equation (18) defining p_t:

(29) $$\sum_{-\infty}^{\infty} \pi_i m_{t-i} + \sum_{0}^{\infty} \rho_i v_{t-i} = \lambda m_t + (1-\lambda)[\sum_{-\infty}^{\infty} \pi_i m_{t-1-i} + \sum_{0}^{\infty} \rho_i v_{t-1-i}]$$
$$+ (c\lambda/\beta)[\sum_{-\infty}^{\infty} \pi_i m_{t+1-i} + \sum_{2}^{\infty} \rho_i v_{t+1-i}]$$
$$- (c\lambda/\beta)[\sum_{-\infty}^{\infty} \pi_i m_{t-i} + \sum_{1}^{\infty} \rho_i v_{t-i}] + v_t$$

Equating coefficients:

(30a) $(1-\lambda)\pi_{i-1} - (1+\gamma)\pi_i + \gamma\pi_{i+1} = 0$ $(i \neq 0)$

(30b) $(1-\lambda)\pi_{-1} - (1+\gamma)\pi_0 + \gamma\pi_1 = -\lambda$ $(i = 0)$

(30c) $(1-\lambda)\rho_{i-1} - (1+\gamma)\rho_i + \gamma\rho_{i+1} = 0$ $(i \neq 0)$

(30d) $\rho_0 = 1$ $(i = 0)$

where $\gamma = (c\lambda/\beta)$

Since we have deliberately chosen the case $m_t = 0$ for all t we may ignore (30a) and (30b) and solve merely for the ρ_i. Other monetary rules will affect the mean values of prices and output but, since they do not affect ρ_i, will not affect the stochastic parts of p_t and y_t and therefore also the variance of y_t about its expected value. The second-order difference equation (30c) yields the solution:

(31) $\rho_i = c_1 k_1^i + c_2 k_2^i$

where $k_1, k_2 = [(1+\gamma) \pm \sqrt{(1+\gamma)^2 - 4\gamma(1-\lambda)}] / 2\gamma$

and c_1 and c_2 are constants with $c_1 + c_2 = 1$.

To determine ρ_i we need another initial condition and we assume that the process generating p_t is of finite variance so that the ρ_i are bounded. Since the larger root k_1 always exceeds unity it follows that $c_1 = 0$ and hence that $c_2 = 1$. Hence, for $m_t = 0$ for all t:

$$(32) \quad p_t = \sum_{o}^{\infty} k^i v_{t-i} = v_t/(1-kL)$$

where k $(=k_2)$ is the smaller root of the quadratic and L is the lag operator.

From equation (19) it then follows that:

$$(33) \quad y_t = [\beta(1-\lambda)/\alpha\lambda](1-L)v_t / (1-kL) - (\beta/\alpha\lambda)u_{1t}$$

$$= \beta[(1-\lambda)v_t - u_{1t}] / \alpha\lambda - [\beta(1-\lambda)(1-k)/\alpha\lambda]v_{t-1} / (1-kL)$$

The mean value of y_t is therefore zero and for the case of Σ diagonal the variance of output about this value is given by:

$$(34) \quad Var(y_t) = \{\beta^2[\lambda^2(1+k) + (1-\lambda)^2(1-k)] / \lambda^2\alpha^2(1+k)\}\sigma_{11}$$

$$+ [2(1-\lambda)^2 / \alpha^2(1+k)](\sigma_{22} + (c/e)^2\sigma_{33})$$

Now consider the implications of setting a long-run target for MV. Using the identity $x_t = y_t + p_t$ equation (19) may be rewritten:

(35) $p_t = (1-\phi)x_t + \phi p_{t-1} + \phi u_{1t}/(1-\lambda)$

where $\phi = \beta(1-\lambda)/[\alpha\lambda + \beta(1-\lambda)]$

Assume a solution of the form:

(36) $p_t = \sum_{-\infty}^{\infty} \pi_i x_{t-i} + \sum_{o}^{\infty} \rho_i u_{1t-i}$

Substituting this into (35) and equating coefficients yields:

(37a) $\pi_i - \phi\pi_{i-1} = 0$ $(i \neq 0)$

(37b) $\pi_0 - \phi\pi_{-1} = (1-\phi)$ $(i = 0)$

(37c) $\rho_i - \phi\rho_{i-1} = 0$ $(i \neq 0)$

(37d) $\rho_o = \phi / (1-\lambda)$ $(i = 0)$

Once again since we have deliberately chosen the particular rule $x_t = 0$ for all t we need only solve for the ρ_i. Other rules will alter the deterministic component of p_t and y_t but will not affect the variance of the stochastic component. The solution to the first-order difference equation (37c) is given by:

(38) $\rho_i = \phi^{i+1} / (1-\lambda)$

Therefore:

(39) $p_t = [\phi / (1-\lambda)]u_{1t} / (1-\phi L)$

And since $y_t + p_t = 0$:

(40) $$y_t = -[\phi / (1-\lambda)]u_{1t} / (1-\phi L)$$

Hence this particular MV-rule produces a mean value of output of zero and the variance of output about this value is:

(41) $$\text{Var}(y_t) = \beta^2\sigma_{11} / [\alpha^2\lambda^2 + 2\alpha\beta\lambda(1-\lambda)]$$

Once again we have the result that M-control is only likely to be preferable if the variance of the price innovation is much larger than the variance of the shocks to the IS and LM schedules. Values of the variance ratio for a range of parameter values are presented in Table 3.

It should be noted that immutable long-term targets for monetary or nominal GNP growth in this sticky price case are both inferior to feedback rules which attempt to nullify the influence of past shocks. In this case the optimal policy, assuming that $y_t = 0$ is the desired output level, is to set either the money supply or nominal GNP so that ${}_{t-1}p_t^* = p_{t-1}$ thus eliminating the impact of past shocks on current events. The variance of output about its mean value is then given by equations (23) and (26) respectively. However, if for some (political?) reason the authorities find it desirable to set long-term targets then for most values of the parameters MV-targetry is preferable.

TABLE 3: VARIANCE OF OUTPUT (ABOUT MEAN) UNDER MV-RULE RELATIVE TO THAT UNDER M-RULE FOR PRICE EQUATION (1') WITH $p_t^+ = p_{t-1}$

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; $d = 0.5$; $e = 0.2$;

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		0.85	0.85	0.71
0.3	0.5		0.87	0.78	0.66
0.7	0.1		0.91	0.94	0.85
0.7	0.5		0.95	0.89	0.81

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.14	1.01	0.81
0.3	0.5		1.00	0.87	0.73
0.7	0.1		1.07	1.03	0.91
0.7	0.5		1.02	0.94	0.85

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		0.57	0.64	0.57
0.3	0.5		0.68	0.65	0.56
0.7	0.1		0.69	0.80	0.75
0.7	0.5		0.83	0.81	0.74

A recent alternative explanation of unemployment emphasizes the role of implicit labour contracts rather than the failure of prices to clear markets. Macroeconomic models incorporating such a feature have been analyzed by Fischer (1977) and Taylor (1980). In Taylor's model overlapping labour contracts are combined with mark-up price behaviour by firms. The resulting model yields serially correlated fluctuations in prices and output and a role for feedback monetary policy. He postulates an equation describing the wage-setting behaviour of firms and unions:

$$(42) \quad w_t = \sum_{s=1}^{N-1} b_s w_{t-s} + \sum_{s=1}^{N-1} b_s ({}_{t-1}w_{t+s}) + \frac{h}{N} \sum_{s=0}^{N-1} {}_{t-1}e_{t+s} + \varepsilon_t$$

where w_t = wage at time t

b_s = (1-s/N) / (N-1) i.e. a triangular distribution

e_t = excess demand in labour market at time t

N is the contract length and ε_t is a random error. This is embedded in a simple macroeconomic model:

$$(43) \quad y_t = m_t - p_t + v_t$$

$$(44) \quad p_t = \frac{1}{N} \sum_{s=0}^{N-1} w_{t-s}$$

$$(45) \quad e_t = g_2 y_t$$

Equation (43) is a simple money demand function where v_t is a random error, equation (44) is a mark-up equation and (45) an inverted production function relating the excess demand for labour to output. Taylor then considers monetary rules of the form:

(46) $m_t = g_3 p_t$

We are particularly interested in rules where there is no contemporaneous feedback i.e. $g_3 = 0$. In this case:

(47) $y_t = -p_t + v_t$

and

(48) ${}_{t-1}y_{t+s} = -{}_{t-1}p_{t+s} = \frac{-1}{N} \sum_{i=0}^{N-1} {}_{t-1}w_{t+s-i}$

Substituting (45) in (42) and then using (48) yields an expression for w_t in terms of the innovations to the contracting equation:

(49) $A(L)w_t = \varepsilon_t$

where A(L) is a polynomial in the lag operator

The coefficients of A(L) depend on the values of the parameters in the system; the reader should consult Taylor's article for further details. From (47) it follows that:

(50) $y_t = v_t - [D(L)/A(L)]\varepsilon_t$

where D(L) is the moving average operator $\frac{1}{N} \sum_{s=0}^{N-1} L^s$

Now consider the case of an MV-rule. In this case, given an appropriate normalization:

(51) $y_t + p_t = 0$

Thus (48) is unaltered and Taylor's analysis for the case of a monetary rule is unaffected. Hence the stochastic process generating y_t is given by:

(52) $y_t = -[D(L)/A(L)]\varepsilon_t$

If aggregate demand shocks are uncorrelated with current and past wage innovations it must necessarily be true that the output variance under a monetary control rule must exceed that under a money expenditure rule by $Var(v_t)$, while yielding an identical price performance. In this case, however, there is also a feedback rule which dominates the MV-rule.

III. CONTROLLABILITY

The previous analysis assumed that the authorities could achieve any desired target level of money expenditures. Meade, in his Nobel lecture, and Tobin suggest that it would be both impractical and undesirable to remove from governments the weapon of discretionary fiscal policy. For most of the time the monetary authorities would be responsible for fine tuning money expenditures given the fiscal policy of the government. If an additional instrument were required to affect the consumption-investment mix (or the exchange rate in an open economy) Tobin has suggested using the structure of the tax system to affect the return on savings. Poole (1980), however, has questioned whether it is feasible for the authorities to control MV. Since "the income velocity of money is rather volatile and difficult to predict...it is not reasonable to hold (the monetary) authority responsible for annual (nominal) GNP fluctuations" Poole argues. He goes on to suggest that "Precisely because there is so little consensus on macroeconomic theory, any of a number of different policies may be advocated as being not only consistent with the targets but also as absolutely necessary to their achievement. Economists will argue the matter ex ante and ex post, and only on those rare occasions when all agree will it be possible to show that GNP missed its announced target range because the policymakers made a mistake."

How fatal are these arguments to the conclusions drawn above? There seem to be two points at issue here. The income velocity of money may be unpredictable because of the stochastic shocks in the system, yet the effects of a policy change may be known with certainty. This unpredictability only causes difficulties in so far as the current value of the target variable is unknown. As a consequence it may be impossible to achieve precise day-to-day control of the target variable and leads to what we shall term inexact control. The second quote relates to the uncertainty of the effects of monetary policy on expenditures and raises the question of whether there are other instruments, particularly fiscal policy, available whose effects are more certain. The problem of information lags and inexact control are discussed in this section and the choice of instrument in the next.

The problem of information delays in principle also arises with the pursuit of monetary targets since in reality the authorities usually set the interest rate(s) in order to achieve a desired monetary target. In most cases the link between this interest rate and the monetary aggregate in question is stochastic so that monetary control is necessarily inexact. However, the setting of monetary targets in both the United States and the United Kingdom has led to an improvement in the quality and frequency of data necessary to monitor the money supply e.g. the movement from quarterly to monthly data and the development of more effective instruments for control. It would seem reasonable to suggest that if the authorities were to embrace the idea of control of nominal GNP rather than the money supply then appropriate improvements could be made in the accuracy and timeliness of nominal GNP data. However, because GNP is a more diverse and heterogeneous quantity than some monetary aggregate it seems reasonable to suggest that nominal GNP data will always be prone to delays in collection and future revision.

In the following analysis we shall assume that the monetary authorities set interest rates and know the impact of interest changes on the economy exactly, but do not have current information on the target variable. At the start of the period the authorities therefore set the interest rate(s) so that the expected and desired value of the target variable coincide. Targets are set for the same time horizon as the information lag. We shall also examine the case of exact M-control and inexact MV-control. This is an interesting question since it may be impossible to reduce the information lag for nominal GNP to that of the monetary aggregates. Note that the fact that the authorities set the interest rate to hit a monetary or expenditure target removes the price level indeterminacy which arises in the neo-classical market-clearing model when the authorities follow an interest rate pegging rule (see Sargent and Wallace (1975) for the implications of simple interest rate rules and McCallum (1980) for an analysis of the case where the interest rate is used to hit a monetary target.)

Consider first the case of an M-rule. Suppose the authorities wish to attain some target value of the money supply m_t^*. Taking expectations of (3) implies that, for the New Classical model, the appropriate level of the interest rate is:

$$(53)\quad i_t = ({}_{t-1}p_t - m_t^*)/e$$

Substituting this in (2) and (3) and eliminating m_t from the resulting equation yields:

$$(54)\quad (1-bd)y_t = \beta(m_t^* - {}_{t-1}p_t) + c({}_{t-1}p_{t+1} - {}_{t-1}p_t) + u_{2t} + bu_{3t}$$

Using (6) and noting that ${}_{t-1}m_t = m_t^*$ yields:

$$(55)\quad y_t = (u_{2t} + bu_{3t})/(1-bd)$$

and the variance of output is given by:

$$(56)\quad \mathrm{Var}(y_t) = (\sigma_{22} + b^2\sigma_{33} + 2b\sigma_{23})/(1-bd)^2$$

We thus have the result that with inexact execution of the monetary rule, supply shocks are reflected entirely in prices and not in output. A corollary is that if the variance of supply shocks is large relative to the variances of shocks to goods and money market demand an inexact M-rule may be preferable to an exact one! The reason for this apparently paradoxical result is that the error due to inexact control which results in unanticipated monetary changes may offset the "system" error given by equation (8). Specifically, substituting (6) into (5) and combining with (1) yields:

$$(57)\quad y_t = [a\beta/(\alpha a+\beta)](m_t - {}_{t-1}m_t) + [\beta u_{1t} + au_{2t} - (ac/e)u_{3t}]/(\alpha a+\beta)$$

Using (1), (3) and (55) gives:

$$(58)\quad m_t - {}_{t-1}m_t = -u_{1t}/a + [(1+ad)u_{2t} + (a+b)u_{3t}] / a(1-bd)$$

Thus with inexact control the unanticipated monetary shock will exactly offset supply shocks, amplify IS shocks and offset the effect of shifts in the demand for money. (Note that it may more than offset them so that the output variance due to LM shifts may actually increase.) The irrelevance of supply shocks in the inexact control case follows immediately from the fact that with nominal and expected real rates fixed equations (2) and (3) form a self-contained sub-system determining output and real balances, whereas in the case of exact control all three equations defining the system must be solved simultaneously.

Now suppose the authorities set interest rates in order to hit a target value of nominal GNP, x_t^*. Taking expectations of (1) implies that ${}_{t-1}p_t = {}_{t-1}x_t = x_t^*$. Eliminating m_t from (2) and (3), taking expectations and using this result yields the appropriate value for i_t as:

$$(59)\quad i_t = c(x_{t+1}^* - x_t^*) / \beta e$$

Substituting this in (2) and (3) and eliminating m_t yields:

$$(60)\quad y_t = (u_{2t} + bu_{3t}) / (1-bd)$$

This is identical to (55) and hence both the M-rule and the MV-rule produce identical output variances. The reason for this is clear since in both cases the interest rate is fixed and can therefore be analyzed as a fixed interest rate rule, with the proviso that the form of the rule leads to a determinate price level. Since only unanticipated events matter, output and prices must behave identically under the two regimes. It follows that an inexact MV-rule dominates an exact M-rule in those cases where an inexact M-rule dominates an exact one.

To examine this further it is once again convenient to assume that Σ is diagonal. In that case an inexact MV-rule dominates an exact M-rule if:

$$(61) \quad (\sigma_{22} + b^2\sigma_{33})(\alpha a + \beta)^2 < (1 - bd)^2[\beta^2\sigma_{11} + a^2\sigma_{22} + (ac/e)^2\sigma_{33}]$$

Values of the ratio of the output variance under the MV-rule compared to that under an exact M-rule for a range of parameter values are presented in Table 4. While the M-rule dominates for most values, an inexact MV-rule may be preferable if the relative variance of supply shocks is large and the output elasticity is small. These are precisely the conditions which tend to favour the M-rule in the case of exact control. As before, however, a high interest elasticity in the IS schedule relative to that in the demand for money function tends to make MV-control better. Whether a volatile LM schedule favours MV-control depends on the particular values of the parameters- see Table 4(d).

TABLE 4: VARIANCE OF OUTPUT UNDER INEXACT MV-RULE RELATIVE TO EXACT M-RULE.

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; $d = 0.5$;

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		1.58	1.42	1.33
0.1	0.5		2.78	2.67	2.57
0.5	0.1		2.50	2.24	1.9
0.5	0.5		4.47	4.09	3.63

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		0.80	0.72	0.67
0.1	0.5		1.13	1.34	1.29
0.5	0.1		1.63	1.62	1.07
0.5	0.5		2.54	2.25	1.98

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		3.06	2.80	2.63
0.1	0.5		4.43	5.28	5.10
0.5	0.1		3.41	3.49	3.11
0.5	0.5		7.22	6.92	6.22

(d) $2\sigma_{11} = 2\sigma_{22} = \sigma_{33}$;

a	b	c/e=	0.5	1.0	2.0
0.1	0.1		2.41	1.42	1.33
0.1	0.5		3.33	3.23	3.07
0.5	0.1		2.31	1.97	1.63
0.5	0.5		5.12	4.50	3.84

We next consider the implications of inexact control in the context of the model without market-clearing. Poole (1970) has implicitly examined the question of the relative variance (about ${}_{t-1}y_t$) of inexact MV (or M) control and exact M-control for the fix-price case $p_t = p_t^+$ (i.e. $\lambda = u_{1t} = 0$) in his comparison of fixed money supply and fixed interest rate rules and the analysis here may be viewed as a generalization. Substituting (2) in (3) and taking expectations yields the appropriate level of the interest rate for an M-rule as:

$$(62) \quad i_t = [(bd-1)(m_t^* - {}_{t-1}p_t) + cd({}_{t-1}p_{t+1} - {}_{t-1}p_t)] / \alpha e$$

Eliminating m_t from (2) and (3) and taking expectations yields:

$$(63) \quad (1 - bd)(y_t - {}_{t-1}y_t) = u_{2t} + bu_{3t}$$

Hence the variance of output about its expected value is exactly the same as in the market-clearing model (equation (56)). This is not an unexpected result since we noted above that in the inexact control case the values of output and real balances were independent of the supply curve. Once again since only unanticipated events determine $(y_t - {}_{t-1}y_t)$ it follows that inexact MV-control must yield an identical expression for the output variance. This is to be compared with (23) which gives the variance of output about its expected value at t-1 for exact M-control. Assuming Σ is diagonal inexact MV (or M) control dominates exact M-control if:

$$(64) \quad (1 - bd)^2\beta^2\sigma_{11} > [\alpha^2 - (1 - \lambda)^2(1 - bd)^2]\sigma_{22} + [\alpha^2 b^2 - (1 - \lambda)^2(1 - bd)^2(c/e)^2]\sigma_{33}$$

Thus conditions favourable to inexact MV-control are a high interest elasticity of goods demand relative to money demand and a high variance of price innovations relative to the variance of IS and LM shocks, the opposite of the case of exact MV-control. Values of the variance ratio for a range of parameter values are presented in Table 5.

Now consider the special case of lagged price adjustment $p_t^+ = p_{t-1}$ which with fixed target values of money supply or nominal GNP introduces serially correlated deviations from mean output levels. First we note that the equivalence of inexact M-control and inexact MV-control found above does not carry over here since the different regimes have different implications for p_t and hence for output in ensuing periods. Rather than examine the case where the authorities set target levels of the money supply/nominal GNP for all time periods in advance it seems more realistic to consider the case where the authorities actually fix a desired growth rate, so that control errors are not required to be offset in later periods. Consider first the case of inexact M-control and suppose the authorities set a monetary growth target path $\delta_t^* = m_t^* - m_{t-1}$ for all t. Eliminating y_t by substituting (2) into (3), taking expectations and subtracting gives:

TABLE 5: VARIANCE OF OUTPUT (ABOUT $_{t-1}y_t$) UNDER INEXACT MV-RULE RELATIVE TO THAT UNDER EXACT M-RULE FOR PRICE EQUATION (1')

$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; $d = 0.5$;

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.80	1.15	0.65
0.3	0.5		2.15	1.55	1.02
0.7	0.1		3.70	1.81	0.92
0.7	0.5		3.12	2.06	1.33

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.31	0.74	0.40
0.3	0.5		1.33	0.91	0.59
0.7	0.1		2.10	0.97	0.48
0.7	0.5		1.64	1.07	0.69

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		2.21	1.59	0.96
0.3	0.5		3.12	2.38	1.59
0.7	0.1		5.98	3.21	1.69
0.7	0.5		5.67	3.83	2.49

(d) $2\sigma_{11} = 2\sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.61	0.95	0.51
0.3	0.5		2.40	1.12	1.00
0.7	0.1		3.57	1.72	0.95
0.7	0.5		3.67	2.38	1.51

$$(65) \quad m_t - (m_{t-1} + \delta_t^*) = p_t - {}_{t-1}p_t + v_{1t}$$

where $v_{1t} = (du_{2t} + u_{3t}) / (1 - bd)$

Hence:

$$(66) \quad m_t = \sum_{o}^{\infty} (\delta_{t-i}^* + p_{t-i} - {}_{t-i-1}p_{t-i} + v_{1t-i})$$

Substituting into (18):

$$(67) \quad p_t = \lambda \sum_{o}^{\infty} (\delta_{t-i}^* + p_{t-i} - {}_{t-i-1}p_{t-i} + v_{1t-i}) + \gamma({}_{t-1}p_{t+1} - {}_{t-1}p_t) + (1 - \lambda)p_{t-1} + v_{2t}$$

where $v_{2t} = [u_{1t} + (\lambda/\beta)(u_{2t} - (c/e)u_{3t})]$

Once again we shall use the method of undetermined coefficients to obtain a solution for p_t. Assume a solution of the form:

$$(68) \quad p_t = \sum_{-\infty}^{\infty} \pi_i \delta_{t-i}^* + \sum_{o}^{\infty} \rho_i v_{1t-i} + \sum_{o}^{\infty} \tau_i v_{2t-i}$$

Again for simplicity, but without loss of generality, take the special case $\delta_t^* = 0$ for all t. Substituting into (67) and equating coefficients yields, for ρ_i and τ_i:

$$(69a) \quad (1 - \lambda)\rho_{i-1} - (1 + \gamma)\rho_i + \gamma\rho_{i+1} = -\lambda(1 + \rho_0) \qquad (i \neq 0)$$

$$(69b) \quad \rho_0 = \lambda / (1 - \lambda) \qquad (i = 0)$$

$$(69c)\quad (1-\lambda)\tau_{i-1} - (1+\gamma)\tau_i + \gamma\tau_{i+1} = -\lambda\tau_0 \qquad (i \neq 0)$$

$$(69d)\quad \tau_0 = 1/(1-\lambda) \qquad (i = 0)$$

The characteristic equations of (69a) and (69c) are the same as (30c) and the particular solution is $1/(1-\lambda)$ in each case. Taking the smaller root as before we obtain:

$$(70a)\quad \rho_i = 1/(1-\lambda) + c_1 k^i$$

$$(70b)\quad \tau_i = 1/(1-\lambda) + c_2 k^i$$

where c_1 and c_2 are constants and k was defined in (31).

Finally using (69b) and (69d) we may solve for the constants to obtain $c_1 = -1$ and $c_2 = 0$. Substituting into (19):

$$(71)\quad y_t = (\beta/\alpha\lambda)[\lambda v_{1t} + v_{2t} - u_{1t} + (1-k)(1-\lambda)v_{1t-1}/(1-kL)]$$

$$= (u_{2t} + bu_{3t})/(1-bd) + [\beta(1-k)(1-\lambda)/\alpha\lambda]v_{1t-1}/(1-kL)$$

Hence for Σ diagonal the variance of output about its mean value is given by:

$$(72)\quad \mathrm{Var}(y_t) = \{[\alpha^2\lambda^2(1+k) + \beta^2(1-\lambda)^2 d^2(1-k)]\sigma_{22}$$

$$+ [\alpha^2\lambda^2(1+k)b^2 + \beta^2(1-\lambda)^2(1-k)]\sigma_{33}\}/(1-bd)^2\alpha^2\lambda^2(1+k)$$

Now suppose the authorities fix target growth rates for nominal

GNP, $\delta^*_{t-1} = x^*_t - x_{t-1}$. Eliminating m_t by substituting (2) into (3), rewriting in terms of x_t rather than y_t, taking expectations and subtracting:

(73) $$x_t - (x_{t-1} + \delta^*_t) = p_t - {}_{t-1}p_t + v_{3t}$$

where $v_{3t} = (u_{2t} + bu_{3t}) / (1 - bd)$

Hence:

(74) $$x_t = \sum_{o}^{\infty} (\delta^*_{t-i} + p_{t-i} - {}_{t-1-i}p_{t-i} + v_{3t-i})$$

Substituting into equation (35):

(75) $$p_t = (1-\phi) \sum_{o}^{\infty} (\delta^*_{t-i} + p_{t-i} - {}_{t-i-1}p_{t-i} + v_{3t-i}) + \phi p_{t-1} + \phi u_{1t} / (1-\lambda)$$

Assume a solution of the form:

(76) $$p_t = \sum_{-\infty}^{\infty} \pi_i \delta^*_{t-i} + \sum_{o}^{\infty} \rho_i v_{3t-i} + \sum_{o}^{\infty} \tau_i u_{1t-i}$$

Again taking the special case $\delta^*_t = 0$ for all t, substituting into (75) and equating coefficients yields, for ρ_i and τ_i:

(77a) $\rho_i - \phi\rho_{i-1} = (1 - \phi)(1 + \rho_0)$ $(i \neq 0)$

(77b) $\rho_0 = (1 - \phi) / \phi$ $(i = 0)$

(77c) $\tau_i - \phi\tau_{i-1} = (1 - \phi)\tau_0$ $(i \neq 0)$

(77d) $\tau_0 = 1/(1-\lambda)$ $(i = 0)$

The characteristic equations of (77a) and (77c) are the same as (37c) and the particular solutions are $1/\phi$ and $1/(1-\lambda)$ respectively. Hence:

(78a) $\rho_i = 1/\phi + c_1\phi^i$

(78b) $\tau_i = 1/(1-\lambda) + c_2\phi^i$

Finally using (77b) and (77d) we find that $c_1 = -1$ and $c_2 = 0$. Substituting into (19):

(79) $y_t = [\beta(1-\lambda)(1-\phi)/\alpha\lambda\phi][v_{3t}/(1-\phi L)]$

Thus for Σ diagonal the variance of output about its mean is:

(80) $\text{Var}(y_t) = \beta^2(1-\lambda)^2(1-\phi)(\sigma_{22}+b^2\sigma_{33})/\alpha^2\lambda^2\phi^2(1-bd)^2(1+\phi)$

The conditions under which inexact MV-control dominates exact and inexact M-control follow immediately from equations (80), (34) and (72). However, the inequalities are complicated and not particularly illuminating. Values of the variance ratio for plausible parameter values are presented in Tables 6 and 7. In the case of the comparison with inexact M-control (Table 6) the variance of the price innovation is irrelevant and as expected a relatively volatile demand for money function together with a high relative interest elasticity

TABLE 6: VARIANCE OF OUTPUT (ABOUT MEAN) UNDER INEXACT MV-RULE RELATIVE TO THAT UNDER INEXACT M-RULE FOR PRICE EQUATION (1') WITH $p_t^+ = p_{t-1}$ $\sigma_{23} = 0$; $d = 0.5$; $e = 0.2$;

(a) $\sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.08	1.00	0.85
0.3	0.5		1.06	0.98	0.88
0.7	0.1		1.00	0.99	0.97
0.7	0.5		1.00	1.00	0.98

(b) $\sigma_{22} = 2\sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.18	1.18	1.10
0.3	0.5		1.22	1.18	1.10
0.7	0.1		1.01	1.02	1.02
0.7	0.5		1.03	1.03	1.03

(c) $2\sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		0.92	0.76	0.58
0.3	0.5		0.89	0.79	0.67
0.7	0.1		0.98	0.94	0.89
0.7	0.5		0.97	0.95	0.92

of the IS schedule and small real balance effect tend to favour MV-control. In the case of the comparison with exact M-control (Table 7) MV-control tends to dominate when the price innovations have relatively large variance--again the opposite of the exact control case. Whether a volatile demand for money favours MV-control depends on the precise parameter values--compare Table 7(d) with 7(a)--but a high interest elasticity of the IS schedule relative to that of the LM schedule and a small real balance effect always tend to favour MV-control for the range of parameter values considered.

What are the lessons of this analysis? Although a precise answer depends on the model employed[2] and the parameter values used, certain regularities emerge. We found in the case where exact control of both nominal GNP and the money supply is possible then MV-control is likely to be preferable the smaller the variance of supply shocks (price innovations in the model in which prices fail to clear markets) relative to IS and LM shocks and the higher the interest elasticity of the IS schedule relative to that of the LM schedule. Where only inexact MV-control is possible however, a small supply (price) shock variance relative to the variance of IS and LM shocks tends to favour M-control, although a high relative interest elasticity of the IS schedule still tends to argue for MV-control.

The most interesting feature of the results, however, is that a volatile demand for money alone does not seem to be sufficient to favour inexact MV-control over exact M-control. This is a consequence of the additional impact of unanticipated nominal GNP growth which

TABLE 7: VARIANCE OF OUTPUT (ABOUT MEAN) UNDER INEXACT MV-RULE RELATIVE TO THAT UNDER EXACT M-RULE FOR PRICE EQUATION (1') WITH $p_t^+ = p_{t-1}$ $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$; d = 0.5; e = 0.2;

(a) $\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.68	1.17	0.77
0.3	0.5		2.23	1.74	1.30
0.7	0.1		3.14	1.65	0.89
0.7	0.5		2.91	1.99	1.33

(b) $\sigma_{11} = 2\sigma_{22} = 2\sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.12	0.70	0.44
0.3	0.5		1.28	0.97	0.72
0.7	0.1		1.85	0.90	0.47
0.7	0.5		1.56	1.05	0.70

(c) $2\sigma_{11} = \sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		2.23	1.76	1.23
0.3	0.5		3.51	2.89	2.18
0.7	0.1		4.80	2.81	1.58
0.7	0.5		5.12	3.61	2.44

(d) $2\sigma_{11} = 2\sigma_{22} = \sigma_{33}$;

λ	b	c/e=	0.5	1.0	2.0
0.3	0.1		1.30	1.01	0.65
0.3	0.5		2.54	1.89	1.35
0.7	0.1		2.99	1.53	0.81
0.7	0.5		3.41	2.27	1.49

offsets the "system" error at the cost of introducing additional variance due to instrument error. For instance for the "new classical" model it follows immediately from the Lucas supply curve (1) and ${}_{t-1}x_t = {}_{t-1}p_t$ that:

$$(81) \quad y_t = a(x_t - {}_{t-1}x_t) / (1 + a) + u_{1t} / (1 + a)$$

and from (60) that:

$$(82) \quad x_t - {}_{t-1}x_t = -u_{1t}/a + (1 + a)(u_{2t} + bu_{3t}) / a(1 - bd)$$

Thus the unanticipated nominal GNP changes exactly offset the "system" error due to supply shocks, but introduce additional errors due to shifts in the IS and LM schedules.

Similarly for the sticky price model (1') by taking expectations of (24) and subtracting we obtain:

$$(83) \quad y_t - {}_{t-1}y_t = \phi(x_t - {}_{t-1}x_t) - \phi u_{1t} / (1 - \lambda)$$

and from (63) that:

$$(84) \quad x_t - {}_{t-1}x_t = u_{1t} / (1 - \lambda) + (u_{2t} + bu_{3t}) / \phi(1 - bd)$$

Thus the unanticipated nominal GNP changes again exactly offset the "system" error, but introduce additional errors due to the IS and LM shocks.

The relevance, however, of each set of results depends critically on the time horizon under consideration. The inexact controllability studied above arises entirely from information lags about the current state of the economy. If the reporting lag for monetary variables is much shorter than that for nominal GNP and the horizon for nominal GNP targets is of the same order as the reporting lag (say quarterly) then the comparisons of this section between exact M-control and inexact MV-control are appropriate. On the other hand if MV-targets are set for a time horizon longer than the reporting lag (say annually) then more precise control of nominal GNP could be expected and the comparisons of Section II are relevant. Thus it would appear that short-run nominal GNP targetry is unlikely to be desirable, but medium and long-term nominal GNP targetry is likely to be preferable to the current practice of monetary control.

IV. CHOICE OF INSTRUMENT

The previous discussion assumed the only source of uncertainty was the impact of exogenous variables as represented by the stochastic shocks. In reality, as Poole points out, there is no consensus on the appropriate model of the economy and empirical evidence on the influence of monetary variables on expenditure is conflicting. In the light of this Poole has argued that it is unreasonable to charge the monetary authorities with the task of controlling nominal expenditures. However, Meade (1981) in a later contribution has suggested that fiscal policy should be used as the primary tool for hitting a nominal expenditure target. Fiscal policy may be incorporated into the above analysis in a straightforward manner by rewriting (2) as:

$$(2') \quad y_t = b(m_t - p_t) - c(i_t - {}_{t-1}p_{t+1} + {}_{t-1}p_t) + fg_t + u_{2t}$$

where g_t is some measure of fiscal stance.

In so far as the parameters of the model are known with certainty this modification leaves unaltered the previous discussion and it is immaterial whether monetary or fiscal policies are used. The lack of a consensus model of the economy may be summarized as uncertainty about the parameters of the reduced form relationship for x_t in terms of current and future values of the policy instruments and a term representing the exogenous variables and stochastic shocks:

$$(85) \quad x_t = \sum_{s=0}^{\infty} \phi_{1s} i_{t+s} + \sum_{s=0}^{\infty} \phi_{2s} g_{t+s} + v_t$$

It is unfortunately not easy to derive explicit expressions for ϕ_i in terms of the parameters of the structural model in the case where the parameters themselves are uncertain because the expectation of future prices will depend on the particular distribution of the parameters. The reduced form will generally be non-linear but (85) can be regarded as a local approximation. Adjustment costs and information delays, etc., will introduce lagged values of the instruments as well.

Brainard (1967) has discussed the appropriate mix of policy instruments when the parameters are uncertain. He considers the simple case of one period and two policy instruments:

(86) $$x_t = \phi_1 i_t + \phi_2 g_t + v_t$$

For simplicity we will assume that $Cov(\phi_1 v_t) = Cov(\phi_2 v_t) = 0$. The authorities objective is to minimize the mean square error $E(x_t - x_t^*)^2$ between the target and its desired value. The first order conditions yield the following expressions for the optimal choice of instrument:

(87) $$i_t^* = (x_t^* - \bar{v}_t)(\bar{\phi}_1 Var(\phi_2) - \bar{\phi}_2 Cov(\phi_1 \phi_2)) / A$$

$$g_t^* = (x_t^* - \bar{v}_t)(\bar{\phi}_2 Var(\phi_1) - \bar{\phi}_1 Cov(\phi_1 \phi_2)) / A$$

where $A = ((\bar{\phi}_1)^2 + Var(\phi_1))((\bar{\phi}_2)^2 + Var(\phi_2)) - (\bar{\phi}_1 \bar{\phi}_2 + Cov(\phi_1 \phi_2))^2$ and bars denote means.

The easiest way to interpret these is to consider them in terms of their expected impact on the target variable, $\bar{\phi}_1 i_t^*$ and $\bar{\phi}_2 g_t^*$ respectively. The result is clearest in the special case $Cov(\phi_1 \phi_2) = 0$:

$$(88) \quad \bar{\phi}_1 i_t^* / \bar{\phi}_2 g_t^* = Var(\phi_2 / \bar{\phi}_2) / Var(\phi_1 / \bar{\phi}_1)$$

Thus the authorities should place greatest reliance on the instrument whose effect is proportionately more certain. The extension to the multi-period case where the reduced form is given by (85) and the objective function is $\sum_{s=0}^{\infty} \delta^s E(x_{t+s} - x_{t+s}^*)^2$ where δ is a discount factor is in principle a simple extension but algebraically complex and the resulting formulae add no new insights. Most commentators would argue that there is a greater certainty about the short-run impact of fiscal changes that interest rate changes on nominal GNP so the Brainard analysis supports the view that fiscal policy should be the prime instrument of MV-control. The relative uncertainty about the <u>short-run</u> impact[3] of monetary policy is relevant if fine-tuning of the target variable is contemplated.

There are a number of fiscal instruments which could be used to control MV. Since the variations in fiscal stance are often likely to be temporary and need to be introduced rapidly, purchases of goods and services by the government are unsuitable even though their effect is fairly predictable because the isolation of suitable projects is time-consuming and the expenditure is often difficult to reverse when required. Variations in the basic income tax allowance can be simply and speedily enacted under the PAYE arrangements in the United Kingdom. However, such tax variations are likely to be temporary and the life cycle/permanent income hypothesis of consumption suggests they would impinge primarily on savings. Empirical analysis of the 1975 tax rebate by Modigliani and Steindel (1977) supports this view. Consequently temporary income tax changes might have very little effect on nominal demand (in a Keynesian world) or result in a fall in investment (in a classical world). Thus variations in income taxes are likely to be ineffective or have possibly undesirable effects on the growth of productive potential.

Ideally one would like a tax impinging primarily on current consumption and a varying expenditure tax is a plausible candidate. In the United Kingdom such a tax already exists and the Chancellor of the Exchequer is empowered to alter the rate of the tax (purchase tax before 1973, Value-added tax thereafter) by 10 percent at will. The fact that variations in the rate are likely to be temporary would actually tend to increase the efficacy of tax changes since it would encourage the intertemporal redistribution of expenditure, particularly on durables, from high tax periods to periods of low tax rates. Against this there is the possibility of making consumption expenditures erratic by encouraging speculative purchases of goods if a rise in the tax rate is expected in the wake of above target GNP data or the postponement of expenditures if the rate is expected to fall. Such destabilizing (and potentially self-fulfilling) speculation could be mitigated somewhat by confining the tax variation to non-durables, although this would also tend to reduce its current potency, and by only publishing the latest national accounts data in tandem with the current setting of the tax rate. Destabilizing speculation is, however, an occupational hazard of attempting to control any economic magnitude by means other than a poll tax and the ill effects here seem no worse than, say, the hot capital flows associated with monetary control in an open economy like the United Kingdom.

Further difficulties could arise if prices fail to clear markets and are instead set as a mark-up on costs and if there is a degree of real wage rigidity in the labour market. With mark-up pricing increases in tax rates designed to curtail nominal expenditures will feed directly into prices tending to increase expenditure and necessitating further tax increases. Thus control of nominal GNP at market prices would have the effect of amplifying the impact of price shocks on activity. This could be avoided by setting the targets in terms of nominal GNP at factor cost rather than market prices; as a consequence price increases due to increases in tax rates would not need to be offset by falls in activity.

The problem of resistance to real wage cuts on the part of workers and unions is more complex. Both Meade and Tobin have advocated adopting labour market policies designed to achieve full employment as an adjunct to MV-control. Changes in real wages due to discretionary expenditure tax changes could presumably be disallowed as a valid reason for revising nominal wages depending on the particular institutional framework involved. On the other hand it is possible that the authorities could decide to pursue MV-targets without modification of existing wage-fixing arrangements. In that case one solution might be to couple expenditure tax changes with offsetting variations in income taxes so as to maintain post-tax real wages constant, or, equivalently, to institute a varying tax/subsidy on savings rather than expenditure. A savings tax would be simpler but could only be levied on savings passing through the existing financial institutions. Since agents would always have the option of simply hoarding it would seem that it would always be necessary to actually have a subsidy rather than a tax. Expenditure and income taxes on the other hand are collected at source so the scope for avoidance would be less and no additional legislation would be required in the United Kingdom at least. Against this it must be admitted that the introduction of a Taxes and Prices Index by the Thatcher administration in the British Budget of 1979 in an effort to persuade the unions that the switch from direct to indirect taxation did not imply a fall in real wages was hardly a spectacular success.

V. CONCLUSIONS

The analysis of section II suggests that for plausible values of the parameters MV-control dominates M-control in both "New Classical" and Keynesian models. While an inability to control MV exactly on account of information lags may negate this result, if targets are set for periods longer than the information lag this is unimportant. Policy effectiveness considerations suggest that control would be better achieved by fiscal rather than monetary instruments. There does not seem to be any reason to suppose that the performance of the authorities would be any worse with annual nominal GNP targets than with monetary targets. Consequently the pursuit of nominal GNP targets with the aid of fiscal instruments seems preferable to the existing pursuit of monetary targets. The analysis only establishes a case for the superiority of MV-targetry over M-targetry and not a case for MV-control per se. The resolution of this question requires not only a detailed specification of the economy but also elaboration of accompanying policies. Meade and Tobin both advocate the use of labour market policies to ensure full employment. In a world where there is a role for discretionary monetary and fiscal policy then MV-targetry may well be inferior to other activist policies.

FOOTNOTES

1. In the pure fix-price case where $p_t = p_t^+$ nominal GNP control fixes y_t as well as x_t so that $Var(y_t - {}_{t-1}y_t) = 0$.

2. It is not easy to analyze the implications of inexact controllability in the context of the Taylor model.

3. Both structural models and reduced form regressions suggest the short-run impact of monetary variables on output and prices is limited and relatively uncertain, while the impact of fiscal variables is immediate (although possibly short-lived).

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